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An individual's intrinsic mode preference and responsiveness to level-of-service variables affects her or his travel mode choice for a trip. The mode preference and responsiveness will, in general, vary across individuals based on observed (to an analyst) and unobserved (to an analyst) individual characteristics. The current paper formulates a multinomial logit-based model of travel mode choice that accommodates variations in mode preferences and responsiveness to level-of-service due to both observed and unobserved individual characteristics. The model parameters are estimated using a maximum simulated log-likelihood approach. The model is applied to examine urban work travel mode choice in a multiday sample of workers from the San Francisco Bay area.

Most work travel mode choice models are based on the random utility maximization framework of microeconomic theory. The random utility maximization framework assumes that an individual's choice of mode on any choice occasion is a reflection of underlying indirect utilities associated with each of the available modes and that the individual selects the alternative that provides her or him the highest utility (or least disutility). The indirect utility that an individual associates with each mode is not observed to the demand analyst, who then assumes that this utility is composed of three components: a) an intrinsic individual-specific mode bias term that varies across individuals and that represents the bias of the individual toward the mode due to observed and unobserved (to the analyst) individual factors (such as sex, lifestyle, and culture); b) the utility that the individual derives from observable (to an analyst) level-of-service characteristics offered by the mode for the individual's trip; and c) a mean-zero random term that captures the effect of unobserved modal characteristics or unknown measurement error in modal level-of-service attributes (more generally, this final third term represents the effects of all omitted variables that are not individual specific). Ideally, we should obtain individual-specific parameters for the first two utility components; that is, for the intrinsic mode biases and for the subjective evaluations of modal level-of-service attributes. However, the data used for mode choice estimation are usually cross-sectional or comprise very few observations on each individual. This precludes estimation at the individual level and constrains the modeler to pool the data across individuals. In such pooled estimations, the analyst should, in some way, accommodate taste differences (i.e., heterogeneity in intrinsic mode biases and heterogeneity in responsiveness to level-of-service attributes) across individuals. In particular, if the assumption of taste homogeneity is imposed when there is taste heterogeneity, the result is inconsistent model parameter estimates and even more severe inconsistent choice probability estimates (see CHAMBERLAIN, 1980; the reader is also referred to HSIAO, 1986 and DIGGLE, LIANG, and ZEGER, 1994 for a detailed discussion of heterogeneity bias in discrete-choice models).

Taste heterogeneity may be incorporated in travel mode choice models by introducing observed individual socio-economic characteristics as alternative-specific variables and by interacting level-of-service variables with observed individual characteristics (such as using a "travel cost over income" specification or using a market segmentation scheme). However, it is very likely that taste heterogeneity will remain even after accounting for differences in ob-

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served individual characteristics (see FISCHER and NAGIN, 1981). This taste heterogeneity due to unobserved individual attributes is generally ignored in travel mode choice modeling.

In this paper, we formulate a multinomial logit-based model of work travel mode choice that accommodates taste heterogeneity due to both observed and unobserved individual attributes. The formulation ensures the correct sign on the level-of-service parameters (for example, a negative coefficient on the time and cost variables) for all individuals. The model takes the form of a random-coefficients logit (or RCL) structure. The RCL structure has been known for a long time, but there have been few applications of this structure. The primary reason is that the choice probabilities in the RCL structure do not have a closed-form expression and generally involve high dimensional integration. However, in the past few years, the advent of simulation techniques to approximate integrals has facilitated the application of the RCL structure (see BHAT, 1998; BROWNSTONE and TRAIN, 1997; and TRAIN, 1998).

The mode choice model in this paper is estimated from repeated work travel mode choices of workers obtained from a multi-day travel survey conducted in the San Francisco Bay area. It is important to note that repeated mode choice data from workers is needed to accommodate unobserved variations in intrinsic mode biases across individuals. In conventional cross-sectional work mode choice models that use a single observation for each individual, it is impossible to separate the effect of unobserved heterogeneity in intrinsic bias from the effect of omitted variables that are generic to all choice occasions (see BHAT, 1998 for an application that allows variation in level-of-service responsiveness, but is unable to accommodate unobserved heterogeneity in intrinsic mode preferences because it uses cross-sectional data).

The rest of this paper is organized as follows. The next section discusses the formulation and estimation of the RCL model used in the paper. Section 2 presents the empirical results obtained from applying the model to an urban mode choice context. The final section provides a summary of the research findings.

1. MODEL FORMULATION

WE DEVELOP THE model formulation assuming that all alternatives are available on all choice occasions. Extension of the formulation to the case where only a subset of alternatives are available on some choice occasions is straightforward.

The utility U_{qit} that an individual q associates with an alternative i on choice occasion t may be written in the following form:

$$U_{qit} = \beta_q' x_{qit} + \epsilon_{qit} \tag{1}$$

where x_{qit} is a vector of observed variables (including alternative specific constants), β_q is a corresponding coefficient vector that may vary over individuals but does not vary across alternatives or time, and ϵ_{qit} is an unobserved extreme value random term that captures the idiosyncratic effect of all omitted variables that are not individual specific. ϵ_{qit} is assumed to be identically and independently distributed across all choice occasions and independent of β_q and x_{qit}.

A number of different specifications may be used for the coefficient vector β_q in Eq. 1. To facilitate the following discussion, we partition the coefficient vector β_q:

$$\beta_q = ([\beta_q^{\text{asc}}]', [\beta_q^{\text{ls}}]')', \tag{2}$$

where β_q^{asc} is the coefficient sub-vector on the alternative specific constants and β_q^{ls} is the coefficient sub-vector on the level-of-service variables. One possible specification is then to write each element of the β_q^{asc} as a deterministic function of an observed vector z_q of individual characteristics ($\beta_{qi}^{\text{asc}} = \beta_i^{\text{asc}} + \delta_i' z_q$), and to maintain a fixed value (across individuals) on the level-of-service coefficients ($\beta_q^{\text{ls}} = \beta^{\text{ls}}$). This specification corresponds to the standard multinomial logit (or MNL) model and is the one generally adopted in mode choice modeling. A second specification is similar to the first, except that it relaxes the assumption of homogeneity (across individuals) in response to level-of-service changes by specifying the level-of-service coefficient β_{qk}^{ls} associated with the kth level-of-service variable ($k = 1, 2, \ldots, K$) as a function of an observed vector w_{qk} of individual attributes, $\beta_{qk}^{\text{ls}} = \pm\exp(\beta_k^{\text{ls}} + \gamma_k' w_{qk})$. The + sign is applied for a non-negative response coefficient (such as the coefficient on frequency of service) and the − sign is applied for a non-positive response coefficient (such as the coefficient on travel time or travel cost). This second specification corresponds to a MNL model with parameters entering the utility non-linearly. We will refer to this specification as the deterministic coefficients logit (DCL) model. A third specification superimposes random (unobserved) heterogeneity over the deterministic (observed) heterogeneity of the second specification, $\beta_{qi}^{\text{asc}} = \beta_i^{\text{asc}} + \delta_i' z_q + \alpha_{qi}$ and $\beta_{qk}^{\text{ls}} = \pm\exp(\beta_k^{\text{ls}} + \gamma_k' w_{qk} + v_{qk})$, where α_{qi} and v_{qk} are assumed to be normally distributed across individuals. We will refer to this specification as the random coefficients logit (RCL) model. Our RCL model specification differs from (and is more general than) the RCL model specification used by REVELT and TRAIN (1997), JAIN, VILCASSIM, and CHINTAGUNTA, (1994), MEHNDIRATTA (1996), and Train (1998). Specifically, these other studies do not allow the distribution of the random taste coefficients to vary based on observed individual characteristics.

In the RCL specification of this paper, we assume that the elements in each of the random vectors α_q $(=[\alpha_{q1}, \alpha_{q2}, \ldots, \alpha_{q1}]')$ and $v_q(=[v_{q1}, v_{q2}, \ldots, v_{qK}]')$ are independent of the elements in the other vector, and that each element in a vector is independent from other elements in that vector.

The normal distribution assumption for the elements in the v_q vector in the RCL model implies a log-normal distribution for the level-of-service coefficients. Specifically, the kth level-of-service coefficient is log-normally distributed with the following properties (see JOHNSON and KOTZ, 1970): a) Median = $\exp(\omega_{qk})$, b) mode = $\exp(\omega_{qk})/\mu_k$, c) mean = $\exp(\omega_{qk})\mu_k^{1/2}$, and d) variance = $\exp(2\omega_{qk})\mu_k(\mu_k - 1)$, where $\omega_{qk} = \beta_k^{\mathrm{ls}} + \gamma_k w_{qk}$, $\mu_k = \exp\sigma_k^2)$, and σ_k^2 is the variance of the kth element of the vector v_q. A useful property of the log-normal distribution is that the ratio of two independent log-normally distributed variables is also log-normally distributed. Therefore, a log-normal distribution assumption for the level-of-service coefficients implies a log-normal distribution for the money value of time, which is obtained as the ratio of the travel time and travel cost coefficients (BEN-AKIVA, BOLDUC, and BRADLEY, 1993, in contrast, specify a log-normal distribution for the money value of time by imposing the a priori assumption that the cost coefficient is fixed, while allowing the travel time coefficient to be lognormally distributed; the specification in this paper is more general than the one used by those authors.

The coefficient vector β_q in the RCL model depends on both observed and unobserved individual attributes, as indicated earlier. The assumptions about the functional form of this dependence, and the distributional assumptions regarding the unobserved attributes, imply that β_q varies in the population with density $f(\tilde{\beta}|\theta^*)$, where θ^* is a vector of the true parameters (mean and variance) characterizing the distribution (to be precise in notation, we should subscript the distribution function f by an index for the elements of $\tilde{\beta}$, because different elements may follow different distributions; however, for convenience, we forego this notational formality). Conditional on $\tilde{\beta}$, we get the familiar MNL form for the probabilities,

$$\mathrm{P}_{qit}(\tilde{\beta}) = \frac{\exp(\tilde{\beta}\ x_{qit})}{\sum_j \exp(\tilde{\beta}\ x_{qit})}. \tag{3}$$

The unconditional probability of choosing alternative i on choice occasion t for a randomly selected individual q can now be obtained by integrating the conditional multinomial choice probabilities in Eq. 3 over all possible values of $\tilde{\beta}$,

$$\mathrm{P}_{qit}(\theta^*) = \int \mathrm{P}_{qit}(\tilde{\beta})\, f(\tilde{\beta}|\theta^*)\ \mathrm{d}\tilde{\beta}. \tag{4}$$

The disaggregate-level self- and cross-elasticities are cumbersome though straightforward to compute from the choice probability expression in Eq. 4. The aggregate-level elasticities may be computed from the disaggregate-level elasticities in the usual way (see BEN-AKIVA and LERMAN, 1985, page 113).

To develop the likelihood function for parameter estimation, we need the probability of each sample individual's sequence of observed travel mode choices. Let T_q denote the number of choice occasions observed for individual q. Conditional on $\tilde{\beta}$, the likelihood function for individual q's observed sequence of choices is

$$\mathrm{L}_q(\tilde{\beta}) = \prod_{t-1}^{T_q} \prod_{i-1}^{I} [\mathrm{P}_{qit}(\tilde{\beta})]^{y_{qit}},$$

where

$$y_{qit} = \begin{cases} 1 & \text{if the } q\text{th individual chooses} \\ & \quad \text{alternative } i \text{ on choice occasion } t \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The unconditional likelihood function of the choice sequence is

$$\mathrm{L}_q(\theta^*) = \int \mathrm{L}_q(\tilde{\beta})\, f(\tilde{\beta}|\theta^*)\ \mathrm{d}\tilde{\beta}. \tag{6}$$

The goal of the maximum likelihood procedure is to estimate θ^*. The log-likelihood function is $\mathscr{L}(\theta) = \Sigma_q \ln \mathrm{L}_q\ (\theta)$.

The log-likelihood function involves the evaluation of a multi-dimensional integral. Conventional quadrature techniques cannot compute the integrals with sufficient precision and speed for estimation via maximum likelihood when the dimensionality of the integration is greater than 2 (see Revelt and Train, 1997 and HAJIVASSILIOU and RUUD, 1994).

We apply Monte Carlo simulation techniques to approximate the integrals in the log-likelihood function and maximize the resulting simulated log-likelihood function. The simulation procedure is similar to the one used by Revelt and Train (1997). For a given value of the parameter vector θ, we draw a particular realization of $\tilde{\beta}$ from its distribution, and, subsequently compute the individual likelihood function $L_q(\tilde{\beta})$ (Eq. 5). We then repeat this process M times for each individual for the given value of the parameter vector θ. The individual likelihood function is then approximated by averaging over the different $L_q(\tilde{\beta})$ values.

$$SL_q(\theta) = \frac{1}{M}\sum_{m-1}^{M} L_q(\tilde{\beta}^m|\theta), \qquad (7)$$

where $SL_q(\theta)$ is the simulated likelihood function for the qth individual's sequence of choices given the parameter vector θ, $\tilde{\beta}^m|\theta$ is the mth draw from $f(\tilde{\beta}|\theta)$, and M is the number of repetitions (or draws of $\tilde{\beta}$). $SL_q(\theta)$ is an unbiased estimator of the actual likelihood function $L_q(\theta)$. It's variance decreases as M increases. It also has the appealing properties of being smooth (i.e., twice differentiable) and being strictly positive for any realization of the finite M draws. The former property is important because it implies that conventional gradient-based optimization methods can be used in the maximization of the simulated log-likelihood function. The latter property ensures that the simulated log-likelihood function is always defined.

The simulated log-likelihood function is constructed as

$$S\mathscr{L}(\theta) = \sum_q \ln[SL_q(\theta)]. \qquad (8)$$

The parameter vector θ is estimated as the vector value that maximizes the above simulated function. Under rather weak regularity conditions, the maximum simulated log-likelihood (MSL) estimator is consistent, asymptotically efficient, and asymptotically normal (see Hajivassiliou and Ruud, 1994 and Lee, 1992). However, the MSL estimator will generally be a biased simulation of the maximum log-likelihood estimator because of the logarithmic transformation in the log-likelihood function. This bias decreases as the number of repetitions increase. Brownstone and Train (1997) have shown the bias to be negligible with as few as 125 repetitions in the context of the RCL model. Earlier applications of the RCL model have used about 250–500 draws (see Revelt and Train, 1997; Brownstone and Train, 1997; and Bhat, 1998), and this range is now generally used as the norm for RCL model estimation. In the current paper, we use 1000 repetitions for accurate simulations of the individual log-likelihood functions and to reduce simulation variance of the MSL estimator.

All estimations and computations were carried out using the GAUSS programming language on a personal computer. Gradients of the simulated log-likelihood function with respect to the parameters were coded.

2. EMPIRICAL ANALYSIS OF URBAN MODE CHOICE

2.1 Data and Empirical Specification

In the empirical analysis of this paper, we apply the RCL model to examine the urban work travel mode choice behavior of commuters in the San Francisco Bay area. The data source for the analysis is the San Francisco Bay Area Household Travel Survey conducted by the Metropolitan Transportation Commission MTC in the Spring and Fall of 1990 (see White and Company, Inc., 1991 for details of survey sampling and administration procedures). This survey collected a multiple-weekday (either three-day or five-day) travel diary for some households, and it is this multi-day sample that is used here. In addition to the travel diary, the survey also collected individual and household socio-demographic information.

In this paper, we examine mode choice among five travel modes: drive alone, shared ride with 2 people, shared ride with 3 or more individuals, transit, and walk (these five modes account for about 99% of all modes chosen for the work trip in the Bay area). The sample is composed of 520 individuals, who are observed to make a total of 2806 home-based work trips. About 78% of individuals in the sample used the same commute mode on all their choice occasions, whereas the remainder used a mixture of commute modes. Among individuals who used some combination of modes, the most frequent combinations were drive alone and shared ride with 2 people (57% of combination users), drive alone and shared ride with 3 or more people (10% of combination users), and shared ride with 2 people and shared ride with 3 or more people (8% of combination users).

The motorized travel modes available for any home-based work trip were determined from household vehicle holding information (for the drive alone mode) and the origin and destination of the trip (for the transit mode). The walk mode was considered to be available to an individual if the walk time was less than 1 hour (this estimate was based on an examination of the distribution of walk times for those work trips for which the walk mode was cho-

TABLE I

Sample Statistics

Mode	Availability Shares[1]	Mode Choice Shares	Total Cost (in cents)[2]	In-Vehicle Time (in mins.)[2]	Out-of-Vehicle time (in mins.)[2]
Drive alone	0.94	0.73	77 (138)	10.33 (8.2)	3.30 (1.8)
Shared-ride-2	1.00	0.11	60 (110)	15.75 (9.0)	3.90 (2.3)
Shared-ride-3+	1.00	0.03	34 (62)[3]	17.40 (8.1)	3.90 (2.3)
Transit	0.87	0.10	97 (48)	16.30 (14.0)	25.82 (10.6)
Walk	0.42	0.03	0 (0)	0.00 (0.0)	26.34 (11.0)

[1]The availability shares represent the share of home-based work trips for which the mode is available.

[2]The numbers in the table for the modal level-of-service measures denote one-way mean (no parenthesis) and standard deviation (parenthesis) values across trips for which the mode is available. The relative magnitudes of the level-of-service variables across modes varies substantially based on the origin and destination of trips. These variations are not borne out by the mean statistics presented in the table. For example, the mean drive alone cost for trips destined to the San Francisco downtown area is \$6.60, whereas the corresponding mean transit cost is only \$1.49.

[3]The shared-ride mode with three or more individuals (shared ride-3+ mode) is exempt from most bridge and other tolls in the San Francisco Bay area; hence the substantially lower average cost of this mode compared to the drive alone and shared ride-2 modes.

sen; the maximum walk time if walk was chosen was 56 minutes).

Level-of-service data were generated for each zonal pair in the study area and by peak and off-peak periods. These data were appropriately appended to the home-based trips based on the origin–destination and time of day of trips. Three level-of-service variables were used in the current analysis: travel cost, in-vehicle travel time, and out-of-vehicle travel time. A detailed description of the procedures and assumptions used in arriving at the level-of-service data is beyond the scope of the current paper, but is available in PURVIS (1996). Table I presents the mode availability shares, mode choice shares, and the descriptive statistics for the level-of-service measures in the sample.

A number of variables associated with individual socio-demographics and trip characteristics were considered for accommodating observed taste heterogeneity. We arrived at the final variable specification based on a systematic process of eliminating variables found to be statistically insignificant in previous specifications. The variables in the final specification for capturing observed heterogeneity in intrinsic preferences included vehicles per worker in the household, a San Francisco downtown destination indicator that identified whether a trip terminated in the San Francisco downtown area, a Central Business District (CBD) destination flag that indicated whether a trip terminated in a CBD, and a San Francisco/Berkeley origin indicator that identified whether a trip originated in the superdistricts of San Francisco or Berkeley (the CBD districts include the San Francisco superdistricts, except the downtown superdistrict, which has an extremely high employment density and is identified separately, and the superdistricts of San Jose and Oakland; the superdistrict classification is based on a 34-system categorization developed by the Metropolitan Transportation Commission).

2.2 Empirical Results

We present the results for three models here: a) MNL model, b) the DCL model in which observed heterogeneity in the level-of-service coefficients is added to the MNL model while simultaneously ensuring negative coefficients on the cost and time variables for all individuals, and c) the RCL model in which unobserved heterogeneity is superimposed on the DCL model (see Section 1). The results are shown in Table II. We maintain the car mode as the base in accommodating differences in intrinsic mode preferences across individuals. To identify the model parameters in the RCL model, we also have to normalize the standard deviation of the distribution of one of the mode preference terms. Although many normalizations are possible, we adopt a normalization value of zero because the MNL model and the DCL model are nested within the resulting RCL model. An additional issue is that the estimation results for the RCL model are not invariant to the alternative whose standard deviation is normalized to zero (see Ben-Akiva and Bolduc, 1996). So, the analyst must attempt to normalize the standard deviation for the alternative with the minimum variance. In mode choice modeling, we expect the drive alone mode to have minimum variance in intrinsic preference (across individuals) because unobserved modal factors such as comfort and privacy are likely to vary least across individuals for the drive alone mode. Hence, we normalize the variance of the intrinsic preference term for the drive alone mode to zero.

The MNL model results in Table II show that, as the ratio of the number of vehicles to workers in a household increases, it is less likely that the transit

TABLE II
Urban Mode Choice Estimation Results

Parameter on . . .	MNL Parm.	MNL t-stat.	DCL Parm.	DCL t-stat.	RCL Parm.	RCL t-stat.
Intrinsic Mode Preferences						
Mode Constants						
SR—Mean	−2.135	−32.41	−2.031	−20.53	−5.340	−8.30
—S.D.	—	—	—	—	4.538	8.64
SR-3+—Mean	−3.338	−30.23	−3.191	−22.21	−8.945	−5.54
—S.D.	—	—	—	—	5.040	7.10
TR—Mean	−1.609	−5.27	−1.120	−3.11	−0.911	−0.27
—S.D.	—	—	—	—	4.716	4.40
Walk—Mean	−0.028	−0.08	0.877	1.91	3.291	1.02
—S.D.	—	—	—	—	3.762	4.11
Vehicles per worker						
Transit	−0.749	−4.80	−0.796	−3.96	−2.752	−1.45
Walk	−0.802	−4.21	−0.834	−4.42	−2.036	−3.50
SF downtown dest. indicator						
Transit	2.915	9.90	2.527	8.03	4.552	2.32
CBD destination indicator						
Transit	1.824	6.90	1.607	5.26	2.718	1.83
SF/Berkeley Origin indicator						
Walk	1.673	6.62	1.670	5.90	3.217	2.08
Response to Level-of-Service Measures						
Travel Cost						
Constant[1]	−0.002	−5.52	−7.020	—	−6.248	—
Female	—	—	1.174	2.91	1.800	2.01
Std. deviation	—	—	—	—	0.909	4.41
In-vehicle time						
Constant[1]	—	—	−3.954	—	−2.500	—
Std. deviation	—	—	—	—	0.709	2.15
Out-of-vehicle time						
Constant[1]	−0.071	−8.56	−2.088	—	−0.877	—
Travel distance	—	—	−0.083	−3.39	−0.071	−2.45
Std. deviation	—	—	—	—	0.609	4.94

[1]The coefficients on the level-of-service constants for the multinomial logit (MNL) model represent the direct (and invariant across individuals) effect of the level-of-service variable on modal utility. The coefficients reported on the constants and other variables for the deterministic coefficients logit (DCL) model are such that the exponent of the sum of the linear combination of the coefficients with the corresponding variables provides the sensitivity to level-of-service. The coefficients reported on the constants and other variables for the random coefficients logit (RCL) model are such that the exponent of the sum of the linear combination of the coefficients with the corresponding variables provides the median sensitivity to level-of-service (across individuals). We do not report any t-statistics for the constants in the DCL and RCL models because the only reasonable test of the constant parameters would be against a value of negative infinity. The estimated standard errors of the constant parameters are very small relative to their estimated magnitudes.

or walk modes will be chosen relative to the drive alone mode (there was no significant difference due to this variable on the preference for drive alone and the shared-ride modes). There is a preference for the transit mode over all other modes for trips that end in the San Francisco downtown area. This preference for transit is also present, albeit to a lesser extent, for trips that terminate in other CBD areas. The walk mode is preferred over other modes for trips originating in the San Francisco and Berkeley superdistricts. The level-of-service parameters have the expected signs for the cost and out-of-vehicle variables. The coefficient on the in-vehicle travel time variable, however, had a small positive value and was very insignificant; hence, it is not included in the MNL model.

The DCL model provides results similar to the MNL model for the non-level of service parameters. As indicated in Section 1, the negative sign on the cost and time variables is ensured in the DCL model by specifying the response parameters as $\beta_{qk}^{\mathrm{ls}} = -\exp(\beta_k^{\mathrm{ls}} + \gamma_k' w_{qk})$, where β_k^{ls} is a constant and w_{qk} is a relevant vector of individual attributes. Thus, a negative sign on the coefficient of an individual attribute corresponding to the time and cost response parameters implies a lower response sensitivity, and a positive effect indicates a higher response sensitivity. The results show that women are more sensitive to cost than men. We did not find any significant variation in the sensitivity to in-vehicle travel time due to individual characteristics. However, unlike in the MNL model, the implied parameter on the in-vehicle time variable in the DCL model has the appropriate sign (the implied parameter value is

TABLE III
Comparison of Response Coefficients and Implied Money Values of Time

Level-of-Service/Money Value of Time	Average Response Coefficient Values and Average Implied Values of Time Across Individuals				
			RCL		
	MNL	DCL	Mode	Median	Mean
Level-of-service variable					
Travel cost (cents)	−0.0020	−0.0019	−0.0030	−0.0068	−0.0103
In-vehicle time (minutes)	195	−0.0192	−0.0497	−0.0821	−0.1055
Out-of-vehicle time (minutes)	−0.0708	−0.0843	−0.2963	−0.2963	−0.3568
Money value of time					
In-vehicle time ($/hr)	—	8.45	3.94	14.89	28.93
Out-of-vehicle time ($/hr)	21.24	36.83	16.06	53.23	96.91

$-\exp(-3.9536) = -0.0192$). Also, the parameter on the in-vehicle time variable in the DCL model has a rather small standard error of 0.43. Thus, the standard error of the implied in-vehicle time parameter of -0.0192 is 0.00825, which indicates that the effect of in-vehicle time is statistically significant (the t-statistic with respect to zero is 2.32). Finally, the sensitivity to out-of-vehicle travel time decreases with travel distance.

The mean coefficients of the non-level of service variables in the RCL model are, in general, higher in magnitude than in the DCL model. This is because the RCL model decomposes the unobserved portion of utility into individual-specific heterogeneity and an idiosyncratic effect of all remaining omitted variables. The non-level of service parameters in the RCL model are normalized with respect to the variance of only the second unobserved component, whereas the parameters in the DCL model are normalized with respect to the variance of the sum of all unobserved components. The reader will note also that the estimated standard deviations characterizing the unobserved heterogeneity distributions are highly significant in the RCL model (except for the walk constant heterogeneity).

To obtain an intuitive characterization of the responsiveness to level-of-service variables among the different models, we compute the average of the response coefficients and implied money values of travel time across individuals in the sample for the DCL model. For the RCL model, we compute three summary measures to characterize the log-normal distribution of the response coefficients and implied money values of time. Specifically, we compute the mode (point at which the density function peaks), median (50th percentile value), and mean of the log-normally distributed response coefficients and money values of time conditional on the observed characteristics of each individual in the sample and then average these values across individuals. The results are shown in Table III.

Between the DCL and MNL models, the DCL model implies about the same (average) cost sensitivity and a higher (average) out-of-vehicle travel time sensitivity relative to the MNL. The RCL model shows variation in the average values of the mode, median, and mean for the response coefficients due to the significant unobserved variation in sensitivity across individuals. The higher value for the mean of the response coefficients relative to the median and mode (and for the median relative to the mode) in the RCL model is a result of the left skew and long right tail of the log-normal distribution. It is interesting to note that even the mode of each response coefficient in the RCL model is greater than the corresponding coefficient value implied by the DCL model. In Section 2.3, we examine the implications for policy analysis of the differences in responsiveness estimates among the MNL, DCL, and RCL models.

The (average) implied values of in-vehicle time from the DCL model (see bottom of Table III) is $8.45. The (average) value of out-of-vehicle travel time from the DCL model is larger than that from the MNL model. In the RCL model, the mode, median, and mean money values of time show considerable variation. This is again a reflection of the substantial unobserved variation in the money values of time across individuals. Comparing the results from the DCL and RCL models, we find that the (average) median value of the money values of time in the RCL model are larger than the corresponding (average) deterministic values from the DCL model. Thus, among individuals with the same observed characteristics, the RCL model shows that more than 50% of individuals have values of in-vehicle and out-of-vehicle times greater than the fixed values suggested by the DCL model. Because of the long right-hand tail of the log-normal distribution, the (average) mean values of time from the RCL model are much higher than the (average) values of time from the DCL model.

TABLE IV

Measures of Data Fit

Summary Statistic	MNL	DCL	RCL
Log-likelihood value at convergence[1]	−1832.14	−1814.94	−1054.33
Number of parameters[2]	7	10	17
Adjusted likelihood ratio index	0.1859	0.1922	0.5258

[1]The log-likelihood value at zero is −4012.13 and the log-likelihood value with only the intrinsic mode bias constants and no preference heterogeneity is −2259.03.

[2]The number of parameters excludes the intrinsic mode constants.

The differences in empirical results among the MNL, DCL, and RCL models suggest the need to apply formal statistical tests to determine the structure that is most consistent with the data. Table IV provides the log-likelihood value at convergence and the adjusted likelihood ratio index ($\bar{\rho}^2$) for the different models.

The models may be statistically compared using nested likelihood ratio tests. The result of such a test between the MNL and DCL models leads to the rejection of the MNL model; that is, the test provides strong evidence that there is variation in the responsiveness across individuals due to observed individual factors (the likelihood ratio test value is 34.4, which is larger than the χ^2 statistic with 2 degrees of freedom at any reasonable level of significance). A further likelihood ratio test between the DCL and RCL models leads to the clear rejection of the hypothesis that there is no preference and response heterogeneity due to unobserved individual characteristics (the test value is 1521, which is larger than the χ^2 statistic with 7 degrees of freedom at any reasonable level of significance). Indeed, the improvement in data fit is dramatic when we accommodate unobserved heterogeneity (earlier studies such as the ones by Ben-Akiva, Bolduc, and Bradley, 1993 and Revelt and Train, 1997 have also found substantial improvements in model fit after accommodating unobserved heterogeneity in model parameters). The empirical results, taken together, indicate that both the MNL and DCL models are mis-specified.

2.3 Policy Implications

Most transportation congestion management actions attempt to effect a change in mode choice during the peak period by influencing the level-of-service variables. For example, congestion-pricing and parking-pricing schemes rely on the use of monetary disincentives for use of the drive alone mode. Improvements to transit service may involve more frequent service and more extensive route coverage

TABLE V

Aggregate-Level Elasticity Effects on Drive Alone Mode Share in Response to Changes in Level-of-Service of Non-Walk Modes

Level-of-Service Variable	MNL	DCL	RCL
Drive alone mode			
Increase in cost	−0.0465	−0.0378	−0.0718
Increase in IVTT	0.0000	−0.0398	−0.0945
Increase in OVTT	−0.0535	−0.0622	−0.1121
Shared-ride mode with 2 people			
Decrease in cost	−0.0080	−0.0068	−0.0184
Decrease in IVTT	0.0000	−0.0309	−0.0763
Decrease in OVTT	−0.0240	−0.0274	−0.0597
Shared-ride mode with 3+ people			
Decrease in cost	−0.0016	−0.0013	−0.0047
Decrease in IVTT	−0.0000	−0.0109	−0.0305
Decrease in OVTT	−0.0076	−0.0085	−0.0217
Transit mode			
Decrease in cost	−0.0084	−0.0076	−0.0091
Decrease in IVTT	−0.0000	−0.0147	−0.0241
Decrease in OVTT	−0.0477	−0.0483	−0.0592

(thereby decreasing transit out-of-vehicle travel time by reducing wait time and walking time, respectively), or introduction of additional express services (thereby reducing in-vehicle travel time). Employer-based monetary incentives to use non-drive alone modes may involve subsidizing transit fares or lowering high-occupancy vehicle parking costs. Conversion of existing general lanes to high-occupancy vehicle use lanes on expressways increases in-vehicle travel time by the drive alone mode, while decreasing in-vehicle travel times by high-occupancy vehicles. The implementation of all of these policies can be reflected through changes in the appropriate level-of-service variables in travel mode choice models.

In this section, we present the substantive policy implications obtained from the MNL, DCL, and RCL models regarding the effect of changes in level-of-service of the different non-walk modes on drive alone mode share. The substantive implications are examined in terms of the aggregate-level self and cross-elasticity effects, which provide the proportional change in the expected market share of the drive alone mode in response to a uniform percentage change (across all individuals) in the level-of-service measures of non-walk modes.

Table V shows the elasticity effects. All the models show that the drive alone self-elasticities are much higher in magnitude than the cross-elasticities of other modes on the drive alone mode; this result emphasizes the potential effectiveness of solo-auto use disincentives in reducing drive alone mode share. The drive alone self-elasticities also indicate that an increase in out-of-vehicle travel time (for example, due to an employer-based trip-reduction plan that aims at making drive alone parking far removed from the work place) is the most effective means of reducing drive alone mode share.

Between the DCL and MNL models, the DCL model shows a lower cost self-elasticity and a higher out-of-vehicle time self-elasticity (the MNL in-vehicle time self-elasticity is zero because the in-vehicle time variable does not appear in the MNL model). The RCL model suggests larger self-elasticities for all level-of-service variables relative to the MNL and DCL models. The drive alone cost (out-of-vehicle time) self-elasticity suggested by the RCL model is 53% (108%) and 90% (80%) higher than the cost (out-of-vehicle time) self-elasticity from the MNL model and the DCL models, respectively. The in-vehicle time self-elasticity from the RCL model is 137% higher than that from the DCL model. In summary, the MNL and DCL models substantially underestimate the decrease in drive alone mode share in response to auto-use disincentives.

The cross-elasticities in Table V show that, in general, reducing the cost of non-drive alone modes will have a smaller impact on drive alone mode share than reducing travel times of non-drive alone modes. Further, improvements in the shared-ride with 2 people mode level-of-service appears to be very effective in reducing drive alone share. This finding is useful because it suggests exploring the possibility of extending the cost and time benefits that are presently limited to the shared-ride with 3+ people mode in the Bay area to the shared-ride with 2 people mode. Currently, tolls on most bridges and roads are waived, and a travel time bonus is provided (by allowing toll booth queues to be bypassed), for the shared-ride with 3+ people mode, but not for the shared-ride with 2 people mode.

The cross-elasticity estimates do not show much variation between the MNL and DCL models. However, as in the case of the self-elasticities, the RCL cross-elasticities are larger than those from the other two models. This is particularly so for the cross-elasticities of the drive alone mode with respect to improvements in the shared-ride modes, where the RCL cross-elasticities are about 120–180% higher than those from the DCL model. In summary, the MNL and DCL models considerably underestimate the decrease in drive alone mode share due to incentives for the use of non-drive alone modes, particularly those associated with improvements in the shared-ride modes.

3. SUMMARY AND CONCLUSIONS

THIS PAPER HAS formulated a multinomial-logit-based model of travel mode choice that accommodates observed and unobserved taste variations, and ensures the correct sign on the level-of-service measures for all individuals.

The maximum-likelihood estimation of the random-coefficients mode choice model requires the evaluation of a multi-dimensional integral for the choice probabilities. The multi-dimensional integral cannot be evaluated analytically because it does not have a closed-form solution. In the empirical application of this paper, there are three level-of-service variables and up to five available alternatives, necessitating the evaluation of a seven-dimensional integral for the choice probabilities. We use a simulated maximum-likelihood procedure in which the multi-dimensional integral is evaluated using Monte Carlo techniques.

We have applied the multinomial logit without variation in level-of-service coefficients (i.e., the MNL model), the multinomial logit model with variation in level-of-service coefficients due to observed individual attributes (i.e., the deterministic coefficients logit or DCL model), and the proposed MNL, which superimposes unobserved variation in intrinsic mode preferences and level-of-service responsiveness across individuals (i.e., the RCL model) to the estimation of multi-day urban travel mode choice in the San Francisco Bay area. The DCL and RCL models show significant differences in sensitivity to level-of-service variables based on observed traveler attributes, thus rejecting the level-of-service homogeneity assumption of the MNL model. In addition, the RCL model indicates significant unobserved variation (across individuals) in intrinsic mode preferences and level-of-service responsiveness. A comparison of the average response coefficients (across individuals in the sample) among the MNL, DCL, and RCL models shows that the RCL model implies substantially higher sensitivity to level-of-service variables and higher monetary values of time than do the other two models.

In an empirical comparison of data fit, we found that the RCL model provides a dramatic improvement in fit relative to the DCL model. The DCL model, in turn, rejects the MNL model based on a likelihood ratio test. Thus, the MNL and the DCL models are mis-specified.

The substantive policy implications of changes in modal level-of-service on travel mode shares are quite different among the MNL, DCL, and RCL models. In this paper, we have focused on the effect of level-of-service changes for the alternative modes on drive alone mode share. Our results indicate that the MNL and DCL models underestimate the decrease in drive alone market share in response to auto-use disincentives. The MNL and DCL models also underestimate the decrease in drive alone mode share in response to improvements in shared-ride and transit level-of-service. These results imply that the simpler MNL and DCL models provide inappropriate evaluations of policy actions aimed at alleviating urban traffic congestion problems.

Overall, the empirical results emphasize the need to accommodate observed and unobserved heterogeneity across individuals in urban mode choice modeling. Specifically, not accounting for such heterogeneity is likely to have serious consequences on data fit as well as on policy conclusions regarding the effectiveness of auto-use disincentive and rideshare-use incentive programs. Of course, the extent to which ignoring unobserved heterogeneity affects model fit and policy conclusions is dependent on the model specification. As the ability to explain variations in mode choice behavior as a function of observed individual characteristics improves, the consequences of not accommodating unobserved heterogeneity decrease. However, as in the current paper, there will almost always be quite substantial variations not explainable even by the best systematic specification of the effect of observed individual characteristics. Thus, it behooves the analyst to accommodate unobserved heterogeneity.

The random-coefficients logit model in the current paper is more general in specification than most previous applications of random-coefficients in economics and transportation. However, it is still restrictive in at least two respects. First, we have assumed specific distributional forms for the unobserved heterogeneity associated with the alternative specific constants and the level-of-service coefficients. A more general approach would adopt a non-parametric form for the unobserved heterogeneity. Problems with the non-parametric form, however, are that a) the non-negative restrictions on the cost/time parameters have to be imposed explicitly, which can lead to difficulty in estimation, and b) it does not provide an analytic distribution for the money value of travel time, an important consideration in economic and social-benefit cost analysis. Second, we have assumed the random-coefficients to be mutually independent. More generally, the random-coefficients can be specified to be correlated and originating from a multi-variate joint distribution. However, assuming correlated random components destroys the appealing result that the implied money values of travel time (obtained as the ratio of travel time coefficients and the travel cost coefficient) have a tractable log-normal distribution.

Notwithstanding the problems associated with adopting a non-parametric distributional form for model coefficients and correlated random components, these two generalizations are useful avenues for future empirical research in travel choice modeling.

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