

**Copyright**

**by**

**Feng Hong**

**2007**

**The Dissertation Committee for Feng Hong certifies that this is the approved  
version of the following dissertation:**

**MODELING HETEROGENEITY IN TRANSPORTATION  
INFRASTRUCTURE DETERIORATION: APPLICATION TO  
PAVEMENT**

**Committee:**

---

**Jorge A. Prozzi, Supervisor**

---

**Robert B. Gilbert**

---

**Yingyao Hu**

---

**Samer M. Madanat**

---

**C. Michael Walton**

**MODELING HETEROGENEITY IN TRANSPORTATION  
INFRASTRUCTURE DETERIORATION: APPLICATION TO  
PAVEMENT**

by

**Feng Hong, B.S.; M.S.**

**Dissertation**

Presented to the Faculty of the Graduate School of  
the University of Texas at Austin  
in Partial Fulfillment  
of the Requirements  
for the Degree of

**Doctor of Philosophy**

The University of Texas at Austin

May 2007

To  
My Parents, Nanqi Hong and Junying Lv  
and  
My Wife, Zhi Liu

## ACKNOWLEDGEMENTS

First of all, I would like to sincerely thank my advisor, Professor Jorge A. Prozzi for his advice, suggestion, and encouragement all the way through my doctoral studies at UT Austin. This dissertation would be impossible without his insightful guidance and continuous support.

Many thanks to other committee members of my dissertation. My horizon in the area of transportation system is enlarged by many valuable suggestions from Professor C. Michael Walton. This dissertation is enhanced through thought-provoking discussions with Professor Robert B. Gilbert. I also benefit from Professor Yingyao Hu's expertise on econometric modeling. I am greatly inspired by Professor Samer M. Madanat from UC Berkeley. His "dollar signs" idea serves as a cornerstone of the second part of this dissertation.

During my dissertation work, I appreciate many constructive discussions with some very good friends: Runhua Guo, Zheng Li, Jianmin Ma, Feng Wang, Xiaokun Wang, Ivan Damnjanovic, and Abdul Pinjari. In addition, this dissertation is polished by Jan Slack for her always willing to help me with editing and proofreading.

I appreciate financial support by the Eisenhower Graduate Fellowship from U.S. DOT. I also want to recognize the MnRoad personnel for sharing me with the dataset for carrying out my dissertation research.

Finally, I am grateful to my family for their love and support. To my parents, their best wish, although from thousands of miles away, is always my best gift. To my wife, thanks for holding my hands through this bumpy and smooth road.

**MODELING HETEROGENEITY IN TRANSPORTATION  
INFRASTRUCTURE DETERIORATION: APPLICATION TO  
PAVEMENT**

Publication No. \_\_\_\_\_

Feng Hong, Ph.D.

The University of Texas at Austin, 2007

Supervisor: Jorge A. Prozzi

One of the key elements for managing transportation infrastructure is to accurately capture and predict the performance of the facility through well established deterioration models. A sound deterioration model should incorporate 1) physical principle that reflects the deterioration mechanism; 2) relevant variables affecting the deterioration process; and 3) rigorous statistical approach to estimating the model. This dissertation aims at addressing these critical issues with focus on highway pavements.

Data collected from in-service pavement sections are adopted to capture the real-world pavement deterioration process. A widely used pavement performance indicator, riding quality in terms of International Roughness Index (IRI) is used. A nonlinear model with a hierarchical parameter structure is formulated to effectively account for both observed and unobserved heterogeneity.

The model is estimated through an econometric technique, Maximum Simulated Likelihood estimation. Simulation is employed to solve the computationally challenging

problem of multi-dimensional integration. Engineering implications based on estimation results are discussed. The findings are not only consistent with engineering judgment but also helpful to reveal and enhance understanding of the pavement deterioration mechanism. Furthermore, the proposed methodology provides flexibility to obtain both parameters reflecting deterioration for all units and each individual unit of the population.

The second part of the dissertation establishes and evaluates optimal maintenance policy on the basis of realistic deterioration models. The optimal policy is obtained so that the total cost, agency plus user cost, is minimized. A steady state resurfacing problem is investigated in the case study. In particular, the effect of model accuracy related to unobserved heterogeneity on total cost is discussed.

This study makes a contribution to transportation infrastructure management and design in the following sense. From a management viewpoint, the proposed methodology with hierarchical parameters can accommodate both network and project levels of management. It also facilitates decision making for budget planning and resource allocation. From a design viewpoint, model estimation results can be used to update the current AASHTO pavement design equation by incorporating other critical factors.

# TABLE OF CONTENTS

<b>Chapter 1: Introduction.....</b>	<b>1</b>
1.1 Background and motivation.....	1
1.2 Research objectives.....	5
1.3 Contributions.....	5
1.4 Dissertation layout.....	7
<b>Chapter 2: Literature Review.....</b>	<b>9</b>
2.1 Pavement performance.....	9
2.1.1 International roughness index.....	11
2.2 Factors/Variables affecting pavement performance.....	12
2.3 Existing models.....	17
2.4 Model estimation framework.....	20
2.5 Model application in pavement design and management.....	23
<b>Chapter 3: Data Sources and Characteristics.....</b>	<b>25</b>
3.1 Data sources for pavement deterioration models.....	25
3.2 A discussion on data source selection.....	28
3.3 Data source adopted in this study - Minnesota Road Test.....	28
3.4 Data characteristics.....	31
<b>Chapter 4: Model Specification.....</b>	<b>34</b>
4.1 AASHO model.....	34
4.2 General model.....	35
4.3 Model specification based on Mn/Road project.....	36
4.3.1 Dependent variable – riding quality.....	36
4.3.2 Explanatory variables.....	36
4.3.3 Final model specification.....	40



4.3.4	Incorporating unobserved heterogeneity.....	44
4.3.5	Incorporating spatial correlation.....	46
<b>Chapter 5: Parameter Estimation and Results.....</b>		<b>49</b>
5.1	Estimation approach for nonlinear models.....	49
5.2	Maximum Likelihood Estimation.....	50
5.2.1	Maximum Simulated Likelihood estimation.....	51
5.3	Individual-Level parameters.....	55
5.4	Estimation results and implications.....	59
5.5	Policy implications.....	65
<b>Chapter 6: Incorporating Deterioration Model in Pavement Management.....</b>		<b>67</b>
6.1	Introduction on optimal pavement resurfacing problem.....	67
6.1.1	Pavement deterioration model.....	69
6.1.2	User cost .....	71
6.1.3	Resurfacing effectiveness.....	72
6.1.4	Agency cost.....	72
6.2	A Steady-state optimal pavement resurfacing problem.....	72
6.3	Results and implications.....	76
6.3.1	A case study of optimal resurfacing policy for one section.....	76
6.3.2	Optimal resurfacing policy among all sections.....	77
6.3.3	Effect of unobserved heterogeneity on optimal policy for one section....	85
6.3.4	Effect of unobserved Heterogeneity on optimal policy for two sections in adjacent lanes.....	88
6.4	Summary.....	90
<b>Chapter 7: Conclusions.....</b>		<b>91</b>
7.1	Conclusion remarks.....	91

7.2	Further work.....	95
<b>Appendix A: Model Specification Test Using Maximum Likelihood Estimation Approach.....</b>		
		<b>98</b>
<b>Appendix B: Estimation of Correlation Matrix for Model Parameters.....</b>		
		<b>110</b>
<b>Appendix C: Deterioration Curves by Two Levels of Parameters (Population and Individual) for All Section.....</b>		
		<b>113</b>
<b>References.....</b>		
		<b>121</b>
<b>Vita.....</b>		
		<b>128</b>

## LIST OF TABLES

Table 2.1	Summary of Road Profile Statistics.....	12
Table 5.1	Population Level Parameter Estimation Results.....	55
Table 5.2	Individual-Level Parameters.....	58
Table 6.2	Example of Optimal Resurfacing Policy (Cell 1 Passing Lane).....	77
Table 6.3	Parameters Used in Deterioration Models in Optimization Problem.....	78
Table 6.4	Comparison of Optimal Resurfacing Policy under Two Levels of Parameters.....	87
Table 6.5	Resurfacing Policy for Two Sections of Adjacent Lanes.....	89
Table B1	Estimation of Correlation Matrix for the Parameters in Pooled Regression Model.....	110
Table B2	Estimation of Correlation Matrix for the Parameters in RE Model.....	111
Table B3	Estimation of Correlation Matrix for the Parameters in RP Model.....	112

## LIST OF FIGURES

Figure 3.1	Mn/Road Mainline Test Cells Layout.....	30
Figure 4.1	Cumulative ESALs Running On the Mainline Test Sections.....	39
Figure 4.2	Spatial Correlation vs. Distance for different $\lambda$ 's.....	48
Figure 5.1	Example of Performance Fit by Two Levels of Parameters (DL, Cell 17).....	59
Figure 5.2	Example of Performance Fit by Two Levels of Parameters (PL, Cell 17).....	60
Figure 5.3	Layer Coefficients Ratios across All Sections.....	62
Figure 6.1	Schematic Relationships between Costs and Cycle Length.....	68
Figure 6.2	System Enters Steady-State at the Time of the First Resurfacing (Li and Madanat, 2002).....	74
Figure 6.3	Relationship between Optimal Cost and Initial Roughness for Driving Lane.....	79
Figure 6.4	Relationship between Optimal Cost and Initial Roughness for Passing Lane.....	80
Figure 6.5	Relationship between Optimal Resurfacing Cycle and Initial Roughness for Driving Lane.....	81
Figure 6.6	Relationship between Optimal Resurfacing Cycle and Initial Roughness for Passing Lane.....	81
Figure 6.7	Relationship between Optimal Cost and Deterioration Rate ( $b$ ) for Driving Lane.....	83
Figure 6.8	Relationship between Optimal Cost and Deterioration Rate ( $b$ ) for Passing Lane.....	83
Figure 6.9	Relationship between Optimal Resurfacing Cycle and Deterioration Rate ( $b$ ) for Driving Lane.....	84
Figure 6.10	Relationship between Optimal Resurfacing Cycle and Deterioration Rate ( $b$ ) for Passing Lane.....	84

Figure 6.11	Two Resurfacing Policy Scenarios for One Given Pavement Section....	86
Figure C1	Performance Fit by Two Levels of Parameters (Driving Lane, Cells 1, 2, 3, and 4).....	113
Figure C2	Performance Fit by Two Levels of Parameters (Driving Lane, Cells 4, 15, 16, and 17).....	114
Figure C3	Performance Fit by Two Levels of Parameters (Driving Lane, Cells 18, 19, 20, and 21).....	115
Figure C4	Performance Fit by Two Levels of Parameters (Driving Lane, Cells 22 and 23).....	116
Figure C5	Performance Fit by Two Levels of Parameters (Passing Lane, Cells 1, 2, 3, and 4).....	117
Figure C6	Performance Fit by Two Levels of Parameters (Passing Lane, Cells 14, 15, 16, and 17).....	118
Figure C7	Performance Fit by Two Levels of Parameters (Passing Lane, Cells 18, 19, 20, and 21).....	119
Figure C8	Performance Fit by Two Levels of Parameters (Passing Lane, Cells 22 and 23).....	120

# Chapter 1: Introduction

## 1.1 Background and motivation

Transportation infrastructure facility deterioration is the process by which the condition of the facility changes with time. Modeling and predicting deterioration plays a critical role in planning and managing a transportation infrastructure system. Four interdependent components rely heavily on deterioration modeling: 1) management of the entire system, 2) determination of pricing and taxation, 3) establishment of planning, policy, and standards, and 4) design of facilities (Paterson, 1987).

Accurately predicting deterioration or changes in condition has been a major challenge in managing transportation infrastructure systems (Haas et al, 1994). Hierarchically, the management system can be divided into network and project levels. At the network level, budget justification and resource allocation are dependent on the predicted performance of the overall facilities. At the project level, estimating when a section of interest will become deficient directly affects the planning process of scheduling maintenance and rehabilitation work and the specific corrective actions.

On the one hand, transportation infrastructure is built to provide an indispensable service to its user: traffic. On the other hand, the infrastructure is inevitably consumed by the traffic as it runs on the system. Thus, the users (e.g. different types of vehicles) should pay for part of the cost for construction and management of the system. How to allocate the cost to different users equitably has been a concern among user groups and has been debated at every level of government. One of the major factors affecting equitable cost allocation is the need to understand how an infrastructure facility deteriorates. For example, highway construction cost can be divided into two components: load-related cost and non-load-related cost (FHWA, HCA 1997). Understanding how a pavement

deteriorates under different load configurations and magnitudes can help determine the relevant load-related cost attributed to different vehicle classes.

Combining both aspects from transportation infrastructure system and its users, the objective is to minimize the total cost, agency cost plus user cost, over the planning horizon. To protect the infrastructure system, the agency would like to minimize the damage to the infrastructure caused by users as much as possible. For example, load limits imposed on the individual truck axle types are established based on the estimation of their damage to highway facilities because overload contributes exponentially to facility deterioration (Huang, 2003). On the other hand, the user (trucks in particular), prefer higher legal load limits in order to benefit from increased transport capacity. However, heavily deteriorated road pavements resulting from load-induced damage will lead to more severe damage to transported goods and vehicles. From the perspective of interplay between transportation infrastructure and its user (i.e. traffic), capturing the change of the condition of the facility (deterioration) is necessary for the establishment of relevant transportation policy and the evaluation of the economic consequences (Small et al, 1989).

Sound deterioration models are also keys to the enhancement of facility design. For example, the current highway pavement design method, the AASHTO Pavement Design Guide (1993), was established based on the performance model developed following the AASHO Road Test. However, important tasks remain due to its evident deficiencies in the aspects of: 1) reflecting long-term deterioration conditions, 2) mixed traffic, 3) new design approach, and 4) sound statistical approach in model estimation.

A sound deterioration model should incorporate three essential elements: 1) a physical principle that governs the actual deterioration mechanism, 2) critical variables affecting the deterioration process, and 3) a rigorous approach for estimating model parameters. The detailed implication of these three aspects is presented next.

First of all, the process of facility deterioration is complex because it involves a series of factors, both observed and unobserved. It is impossible to exhaustively capture and address all of those factors in a deterioration model. However, effort should be made to incorporate the basic physical deterioration principle in the model so that the deterioration process is well represented and the causes of the deterioration are effectively revealed from an engineering point of view. This physical principle should be reflected through model structure. In this regard, each single component in a deterioration model should be correctly structured and physically meaningful instead of being merely aggregated without considering its physical role. For example, in a pavement deterioration model, structural layer information is usually incorporated in the term associated with deterioration rate. As a consequence, a pavement with stronger layers demonstrating higher capacity to resist deterioration is reflected through a lower deterioration rate. In addition, each layer's contribution to resisting deterioration can be reflected and quantified via a specified model structure.

Second, although it is not realistic to incorporate all factors in the deterioration model, the critical variables involving facility deterioration should be identified and accounted for. For highway pavements, the relevant variables may include structural and material information, design approach, traffic, time, environmental effect, and maintenance activity. More often than not, these variables are used in engineering practice to determine pavement performance because they have been identified as contributing significantly to the evolution of the performance. Missing one or more of these variables may introduce additional uncertainty and lead to inaccuracy of performance estimation since they do significantly impact the deterioration process.

Third, rigorous estimation approach is equally important in deterioration modeling. Model estimation is carried out by applying probability and statistics principles. Both linear and nonlinear models can be used for the estimation, with the latter requiring more mathematical effort. In terms of estimation results, not only optimal parameters should be



targeted, but also their statistical inferences are critical objectives in understanding and interpreting the models. A sound estimation approach aims at providing model estimators with three desirable properties: unbiased, consistent, and efficient.

In addition to the above three critical aspects in deterioration modeling, heterogeneity should also be considered since it is closely related to the second and third aspects but generally not well addressed in the models existing in the literature. In the context of deterioration modeling, heterogeneity can be defined as the performance difference across different individuals (such as facilities or facility segments). Heterogeneity can be categorized into two types: observed heterogeneity and unobserved heterogeneity. Both observed and unobserved heterogeneity exist by the very nature of transportation infrastructure.

Observed heterogeneity can be captured by the explanatory variables. A review of current literature shows that an extensive body of literature has focused on dealing with the observed heterogeneity since it is relatively simple to capture and incorporate into the models. The unobserved heterogeneity, however, has not been well accounted for and often it is even ignored. The source of unobserved heterogeneity comes from factors beyond those already well identified variables due to the unavailability of good data sources and the lack of an appropriate model estimation approach. The undesirable results of missing or unsuccessfully addressing unobserved heterogeneity may lead to biased and inconsistent parameter estimates (Hsiao, 2003). From a deterioration modeling point of view, this contributes negatively to model accuracy.

Therefore, the motivation to account for both observed and unobserved heterogeneity in this study is to:

- Identify facility performance uncertainty sources;
- Better accommodate reliability analysis;
- Facilitate capturing construction cost more accurately;

- Facilitate decision making on optimal maintenance schedule; and
- Facilitate pursuing facility (pavement) optimal structural design

## **1.2 Research objectives**

The objectives of this dissertation are described as follows:

1. Develop a methodology to account for both observed and unobserved heterogeneity in transportation infrastructure deterioration models.
2. Incorporate the established deterioration models in pavement system and identify the effect of unobserved heterogeneity on optimal management policy.

## **1.3 Contributions**

The major contributions of this dissertation are as follows:

### Deterioration Models

A statistical model for riding quality based on deterioration principles is developed. The model comprehensively incorporates a rich variety of variables relevant to the deterioration process of a pavement facility. The variables formulated in the model include: 1) construction, 2) design, 3) structure, 4) material, 5) time and traffic, 6) environment, and 7) maintenance. In addition, thanks to the panel data structure, unobserved heterogeneity associated with construction and material variability is highlighted through a hierarchical parameter structure.

Based on the estimation results of the *best* model, the Random Parameter (RP) specification, the major findings are:

1. The aforementioned unobserved heterogeneity is statistically significant and cannot be ignored;
2. The relative contribution to resist deterioration by unit thickness from individual structural layers, surface, base, and subbase is 4.80/1.40/1.00;
3. There is no significant difference between traditional Marshall Design of asphalt mixes with lower level of compaction energy such as 35 and 50 blows and Superpave Gyrotory Design, in terms of their effect on the roughness deterioration resistance of the mix, while the contribution to resist deterioration with 75 blows Marshall Design is roughly 1.23 times of that with Gyrotory Design;
4. The contribution to resist deterioration by surface layer with PG 58-28 asphalt binder is roughly 1.29 times of that with PG 64-22;
5. Sections on both driving and passing lanes demonstrate curvatures of larger than 1, with the former showing a larger curvature due to heavier traffic; and
6. Pavement performance spatial correlation is proven existent - the correlation decays with increasing distance between sections.

### Model Estimation Approach

To estimate the RP model, an econometric approach, Maximum Simulated Likelihood (MSL), is applied to address the unobserved heterogeneity problem. In this approach, randomness in the model's parameters can be removed by integration. Mathematically, the computationally challenging multi-dimension integral is equivalent to averaging, which is tractable through simulation technique. As a result, the distributions of random parameters are obtained. Furthermore, based on the established distributions and information from each individual section, the specific parameters for each section are estimated through a Bayesian approach. The appeal of the underlying methodology is that it provides the flexibility to accommodate both network and project levels of management with two levels of parameters.

## Deterioration Model in Optimal Management Policy

The realistic deterioration model is successfully incorporated in optimal policy decision making, with focus on a steady state resurfacing problem. In particular, a close-form solution is derived for integrating the deterioration model into the objective function. The effect of unobserved heterogeneity on optimal policy is thoroughly investigated. It is revealed that optimal policies from two levels of model parameters may differ significantly. In addition, the optimal policy is not sensitive to initial riding quality while it clearly depends on the deterioration rate. Optimal policy involving the joint consideration of two sections on adjacent lanes is also investigated since it more accurately reflects engineering practice than focusing on each section separately. Additional costs from section-based optimal policy are quantified with respect to both unobserved heterogeneity and joint-lane constraint.

### **1.4 Dissertation layout**

The remainder of this dissertation is organized as follows.

Chapter 2 presents a literature review on deterioration modeling in the context of pavement engineering. First, a series of key elements in deterioration models are discussed. Second, existing deterioration models are reviewed, identifying their strengths and weaknesses. Third, a variety of model estimation approaches and their properties are briefly described. Finally, previous work involving the application of performance models in pavement design and management is reviewed.

Chapter 3 reviews data issues in deterioration modeling, including 1) possible data sources for pavement deterioration modeling, with particular emphasis on the data source used in this dissertation: the Mn/Road project; and 2) data characteristics in terms of their structures and possible problems from the standpoint of statistical and model estimation.

Chapter 4 establishes the deterioration model based on the Mn/Road project's in-service flexible pavement sections. The model incorporates: 1) physical deterioration principle originated from the well-known AASHO Road Test, 2) comprehensive variables affecting the deterioration process, 3) a hierarchical parameter structure to represent unobserved heterogeneity associated with construction and material variability; and 4) spatial correlation.

Chapter 5 proposes a methodology to estimate the deterioration model formulated in Chapter 4. Due to the nonlinear characteristic of the underlying deterioration model, several commonly used nonlinear estimation approaches are first discussed, which leads to the adoption of the Maximum Simulated Likelihood (MSL) estimation approach. The detailed process of MSL is also provided, which produces population-level parameters. Furthermore, the individual-level parameters are obtained by applying the Bayesian theorem. Both levels of parameter estimates are presented, and findings associated with engineering implications are discussed.

Chapter 6 focuses on incorporating the developed model in a pavement system management problem. The problem of pavement resurfacing is introduced, including a steady state resurfacing problem in the case study. The optimal policies in a series of scenarios involving different deterioration model parameters are investigated. In particular, the effect of unobserved heterogeneity on optimal policy is highlighted.

Finally, Chapter 7 summarizes this dissertation study with conclusions and identifies future research needs in this area.

## Chapter 2: Literature Review

A good deterioration model involves a variety of aspects related to the facility or the system of facilities. In this chapter, the literature review primarily includes 1) the key components of a typical deterioration model, 2) the existing models, 3) approaches applied in model estimation, and 4) model application in pavement management. The investigation is carried out with focus on highway pavements.

### 2.1 Pavement performance

A deterioration model expresses the performance of a facility as a function of a series of relevant explanatory variables. It aims to identify and quantify the relationship between those variables and the performance of the facility, and serves to predict performance development. In the context of highway pavement, the performance is defined as its ability to provide a safe, smooth, and comfortable ride (Haas et al, 1994). To facilitate description from an engineering perspective, specific indicators are generally used to denote pavement performance. There are basically three types of performance indicators for pavement:

- Distress based. For asphalt pavements the major distress types include alligator or fatigue cracking, block cracking, joint reflection cracking from the underlying concrete slab, lane/shoulder drop off or heave, longitudinal and transverse cracking, pumping and water bleeding, rutting, and swell. For concrete pavements, the major distress types include blowup, corner break, faulting of transverse joints and cracks, longitudinal cracks, pumping and water bleeding, and spalling, (Huang, 2003). The causes of these distresses can be attributed to factors such as design, construction, material, traffic, environment and maintenance.
- Panel rating based. The most well known panel rating based pavement performance index dates back to the serviceability concept developed during the

- American Association of State Highway Officials (AASHO) Road Test (HRB, 1962). The present serviceability is defined as “*the ability of a specific section of pavement to serve high-speed, high-volume, mixed traffic in its existing conditions*”. A five-point scale ranging from 0 to 5 was used by the panel individuals to assess a specific pavement’s performance, with 0 denoting poor and 5 denoting excellent. The mean of those individual ratings is denoted as present serviceability rating (PSR). The present serviceability index (PSI) was thereafter established as a mathematical combination of physical measurements, roughness and distresses of a pavement to predict PSR for certain pavements within prescribed limits. Since the AASHO Road Test, PSI has been widely used in pavement design and management, and is one of the key input variables in the current American Association of State Highway and Transportation Officials (AASHTO) guide for the design of pavement structures (1993). However, one of the drawbacks of PSR is the expense involved in terms of rating personnel and time spent. Moreover, the subjective featured index lacks good repeatability and reproducibility properties since it may vary from panel to panel and time to time.
- Roughness or longitudinal profile based. Roughness is a pavement characteristic reflecting the longitudinal profile along the wheel paths. According to American Society for Testing and Materials (ASTM) Specification E867-82A (1982), roughness is defined as “*the deviation of a surface from a true planar surface with characteristic dimensions that affect vehicle dynamics, riding quality, dynamic loads and drainage.*” The application of roughness index has increased to the point that it has become a dominant criterion in describing pavement performance based on its various merits. First, roughness reflects both functional and structural performance of pavements. From the serviceability-performance perspective, roughness was found to be highly correlated with PSR and PSI. It was shown in the AASHO Road Test that over 90% of PSI was contributed by longitudinal profile in terms of slope variance (Haas et al, 1994). Second, roughness not only reflects pavement performance in terms of irregularity but

also relates to riding quality. Third, from an economic viewpoint, roughness is directly correlated with vehicle operating costs. A series of studies established the relationship between roughness and vehicle operating costs (e.g., Chesher and Harrison 1987; Archondo-Callao and Faiz, 1994). The results of these studies can be easily adopted in the estimation of vehicle operating costs. Fourth, high speed profilers are available to facilitate data collection for roughness. Generally, the instruments that measure road roughness fall into one of four categories: 1) absolute profile instruments; 2) moving-datum profile instruments; 3) vehicle-motion instruments; and 4) dynamic profile instruments (Paterson, 1987). The measurements are converted into profile statistics to describe pavement performance and riding quality for practical purposes. Table 2.1 summarizes the major types of profile statistics used in pavement engineering (Paterson, 1987). These different statistics can be converted into each other through established empirical equations.

### **2.1.1 International roughness index (IRI)**

International Roughness Index (IRI) merits discussion because, among the profile statistics aforementioned, it is the most widely used. As will be shown in the Chapter 3, this research aims at modeling pavement riding quality in terms of IRI.

The IRI was established in the Brazil Road Test (1982) sponsored by the World Bank, and was later adopted as a standard by the Federal Highway Administration (FHWA) in its Highway Performance Monitoring System (HPMS). The value of IRI can be determined through a quarter car model simulation. In addition to the general merits of using roughness to represent pavement performance, the desirable properties exclusive to the IRI are as follows:

- It reflects the characteristics of a moving vehicle's vertical motions and reveals both the vehicle's response and the occupant's perception of comfort.



- It has a range from zero to an open-ended amount – the lower the IRI, the higher the riding quality of a pavement.
- It has time-stable and reproducible summary statistics.

**Table 2.1 Summary of Road Profile Statistics**

Category	Acronym	Description	Source
Mathematical Simulation of Vehicle Response	RQCS	Reference Quarter Car Simulation with parameters representing passenger car	NCHRP Report 228
	QCS	Quarter Car Simulation with vehicle constants derived by K.J. Law (Inc.)	GMR Profilometers
	IRI	International Roughness Index	World Bank
Estimation of Vehicle Responses by Correlation to Wavelength Statistics	MO	Estimate of “Maysmeter Output”	Texas
	QI	Quarter Car Index	Brazil-UNDP study
	BI <sub>r</sub>	Estimate of Bump Integrator trailer by root mean square deviations (RMSD)	TRRL (Overseas Unit)
Statistics of Discrete Wavebands	PI	Root mean square elevation statistic from 0.5 to 2.4m wavelength band	NCHRP Report 275
	PU <sub>3.0</sub>	Variance of elevation from 3 m moving average	TRRL
	CP <sub>2.5</sub>	Average rectified elevation on 2.5m moving average base length	CRR Belgium
	W <sub>sw</sub> W <sub>MW</sub>	Mean square energy of profile signal in wavebands sw, mw, and lw.	LCPC France

## 2.2 Factors/Variables affecting pavement performance

Pavement deterioration is caused by the combined effects of traffic loading and environmental factors on the structure and materials. Therefore, among the factors

causing pavement deterioration, the following components, identified as playing a pivotal role in the deterioration process, will be discussed: construction, design, structure and material, environment, traffic, time, maintenance and rehabilitation, and other factors.

### Construction

Construction is directly related to the initial quality of a pavement, which in turn affects pavement performance over the pavement's life. In the context of asphalt pavement, one of the most important factors in construction contributing to the initial quality is compaction density. Under-compaction of the subgrade material leads to a series of problems such as rutting and cracking. Air voids are a critical control criterion for Hot Mix Asphalt (HMA). It is accepted that an air voids content of below approximately 8% is preferable to mitigate the effect of water permeability and accelerated oxidation of dense-graded mix (Roberts et al., 1996). Under-compacted HMA can be attributed to factors during construction such as low compaction effort (light rollers or too few passes) and inappropriate temperature control. Segregation is also a concern during pavement construction since it may lead to surface irregularity immediately after construction and distresses during the pavement service life. Another factor in construction associated with initial quality is the number of lifts adopted in the asphalt layer. It was observed (Prozzi, 2001) that pavement initial serviceability is positively correlated to surface layer thickness, the reason being that during construction, thicker pavements require more lifts, and each lift provides improved working conditions for the subsequent upper lift.

### Design

In general, design involves every aspect affecting pavement performance in a pavement system. In the context of modern pavement design, it is composed of two interdependent dimensions: structural design and material design. The structural design process includes structural information, subgrade modulus, and forecast traffic, the goal being to

determine each layer's thickness under predicted traffic during a given period of service life. Material design includes the determination of material type based on required properties and economic considerations. Asphalt mix design, in particular, requires the most effort in the material design process. There are basically two mix design approaches. The traditional approach is called Marshall design method. Recently a new approach, referred to as Superpave design method, was proposed in the Strategic Highway Research Program (SHRP). One of the pronounced differences between these two approaches is the method of compacting a test specimen for the selection of the optimal binder content. In the Marshall method, the specimen is formed by applying a given number of blows of a standard hammer, typically 35, 50 or 75. In the Superpave method, the specimen is formed by a gyratory compactor, which simulates in-field asphalt mix compaction conditions. In addition, the two approaches differ in terms of their means of determining optimal binder content. Currently, both approaches are employed throughout the United States with different states having individual preferences. It is expected that different design approaches may lead to different in-place asphalt mix performance and distress development.

### Structure and materials

A typical flexible pavement is composed of at least three structural layers from top to bottom: surface course (made of asphalt mix), base course, and subbase course (both base and subbase courses are made of either crushed stone, or other untreated or stabilized materials) (Huang, 2003). These layers are built on subgrade, which is made of compacted soil. Since these structures play a central role in supporting traffic, their properties, mainly reflected through thickness and modulus, bear direct impact on pavement performance. For example, heavily trafficked pavements with relatively weak structures usually experience premature deterioration. The ingredients associated with material properties include aggregate (gradation and other physical indexes), asphalt binder types, and their proportions in the mix.

## Environment

Environment is another critical factor in the pavement deterioration process. The main environmental factors include temperature and precipitation. The effect of environmental factors on pavement performance is usually reflected through a change in material properties due to their environment-sensitive characteristics. For example, for asphalt materials, aging due to the combined effect of water, oxygen, and sunshine, leads to the asphalt mix hardening, which contributes positively to rutting resistance but negatively to cracking resistance. With regard to granular materials, frost susceptibility is a critical concern among pavement engineers since water is accumulated during the freezing period and later released during thawing, weakening the untreated granular base and subbase layers.

## Traffic and time

A pavement structure is built to sustain traffic during its service life. It is consumed by traffic running on it along time. Traffic can be accounted for through traffic volume and axle loading. Considering the fact that the vehicle axle load directly causes pavement damage, a certain measure of axle loading is used in pavement design and analysis. Axle loads with different magnitudes and configurations, primarily single, tandem, tridem, and quadruple axles, are usually converted into standard single axle load (18-kip single axle with dual wheels). That is, in performance modeling, the accumulated number of equivalent single axle loads (ESALs) is customarily adopted as a variable to capture cumulative traffic, such as in the AASHTO 1993 Design Equation. From the probability standpoint, ESAL is simply the fourth moment of axle load spectra (Prozzi and Hong, 2006).

Alternatively, considering the fact that cumulative traffic is proportional to time, time can be used as a proxy of traffic. In addition, time is also an appropriate indicator of asphalt

aging. The high correlation between time and cumulative traffic makes the identification of both variables into the model impossible, unless the experiment was purposely designed to this effect.

### Maintenance

The purpose of maintenance and rehabilitation activities is to keep pavement condition at or above the minimum acceptable serviceability level. In practice, routine maintenance, rehabilitation, and resurfacing are used according to distress severity. Performing routine maintenance activities before the pavement has reached unacceptable condition contributes to extending the service life of the pavement and has been proven to be cost-effective. Rehabilitation and resurfacing are usually applied to pavements with severe distresses. During condition assessment, maintenance effects can be observed through a condition improvement jump and a slowing down in the deterioration rate.

### Other factors

The factors previously discussed are taken into account, to different extent, in existing deterioration models. However, this by no means excludes other factors affecting pavement deterioration. The reasons why the remaining factors are not addressed can be attributed to their 1) small impact in affecting performance, and 2) being unobserved. Significant variables such as aforementioned are identified based on existing engineering experience and research results. Other variables that minimally contribute to deterioration are generally ignored. However, unobserved factors should not be ignored since they may be significantly associated with pavement performance. In other words, variations in pavement performance across different units may not be well captured solely by the observed significant variables. Initial quality is a good example of this. Two pavement sections with all other elements (such as structure, material, environment, etc.) being the same may exhibit a difference in initial roughness immediately after construction due to a

difference in the quality of the construction. As a result, this difference will be propagated and reflected in the performance of the two pavement sections. Another example of an equally critical unobserved factor is material variability, which is a common phenomenon in pavement engineering. Identifying and, more importantly, addressing these factors in deterioration modeling can significantly increase our understanding of pavement deterioration mechanism.

### **2.3 Existing models**

There are three approaches for developing pavement performance models:

- Empirical approach, which is based on empirical analysis of the relationship between a pavement performance indicator (such as PSI, rut depth, percentage of cracking, etc.) and relevant variables (including structure, material, etc.) affecting the performance. In the empirical approach, regression analysis plays a key role while no mechanistic element is involved. Empirical approach has been widely used in both pavement design and management.
- Mechanistic approach, which focuses on mechanistic analysis of pavement structures through elastic layered theory or finite element method. Pavement responses, mainly strain and stress, are obtained through these approaches. However, due to the fact that these responses cannot be directly used to represent observed pavement performance, the mechanistic approach remains at the conceptual stage (Haas et al, 1994).
- Mechanistic-Empirical (M-E) approach, which is composed of two parts, namely, mechanistic and empirical. In the former, pavement response is obtained through mechanistic analysis. These responses then are correlated with pavement performance based on calibrated transfer function established via empirical analysis. In recent years, a monumental effort has been made toward the M-E pavement design, such as the M-E Design Guide under NCHRP Project 1-37 A ([www.trb.org/mepdg](http://www.trb.org/mepdg)).

To-date, much effort has been given to developing state-of-the-art pavement performance models to address the deterioration process. Either the empirical or mechanistic approach, or a combination of the two approaches, is utilized for performance modeling. For example, an empirical sigmoid curve was applied to fit the pavement deterioration process by Garcia-Diaz and Riggins (1984). A mechanistic approach was incorporated to develop the damage functions for rutting, fatigue cracking, and loss of pavement serviceability index (PSI) by Rauhut et al. (1983). After the World Bank road test in Brazil, Paterson (1987) established a series of empirical performance models on the basis of a comprehensive study of previous modeling efforts and the characteristic of the road test data. Currently, the most widely accepted model is the American Association of State Highway Transportation Officials (AASHTO) design equation (AASHTO, 1993):

$$\log W_{t18} = 9.36 \log(SN + 1) - 0.20 + \frac{\log[(4.2 - p_t)/(4.2 - 1.5)]}{0.4 + 1094/(4.2 - 1.5)^{5.19}} + 2.32 \log M_R - 8.07 \quad (2.1)$$

$$SN = a_1 D_1 + a_2 D_2 m_2 + a_3 D_3 m_3 \quad (2.2)$$

Where,  $W_{t18}$  is the allowable number of 18-kip single axle load applications to cause PSI to reduce from 4.2 to  $p_t$ ;  $p_t$  is the terminal serviceability;  $M_R$  is effective roadbed soil resilient modulus;  $SN$  is the structural number;  $a_1$ ,  $a_2$ , and  $a_3$  are layer coefficients for surface, base, and subbase layers, respectively;  $D_1$ ,  $D_2$ , and  $D_3$  are thicknesses of the three layers, respectively; and  $m_2$ , and  $m_3$  are drainage coefficients for the three layers, respectively.

In terms of model format, both linear and nonlinear models were developed in the previous studies. Most existing models adopted the linear format due largely to its simplicity. However, the linearity is criticized for being incapable of describing physical

principles and inaccurate in capturing the deterioration process (e.g., Prozzi, 2001). The nonlinear models were found to be more appropriate for representing performance deterioration due to the characteristics of traffic and environmental impacts on a pavement structure. The tradeoff of adopting the nonlinear model comes from the complexity in model estimation. In summary, both physical explanation and sound model estimation are critical problems that deserve further study.

These problems were improved through recent developments in nonlinear models with panel data (Archilla, 2000; Archilla and Madanat, 2001; Prozzi, 2001; Prozzi and Madanat, 2003; Hong and Prozzi, 2006). The nonlinear modeling is capable of better addressing performance deterioration process along time, while the panel data structure facilitates the differentiation of performance characteristics across pavement sections (i.e. heterogeneity). In such cases, the unobserved heterogeneity can be captured by the intercept term by means of random-effect (RE) models, while it is assumed that the other parameters are fixed.

Although the RE approach produces efficient parameter estimates, heterogeneity is not sufficiently captured. In addition to introducing the individual-specific characteristic in the intercept to account for the unobserved heterogeneity, heterogeneity is likely to remain in the model. Mathematically, unobserved heterogeneity can be captured by adopting a hierarchical model structure (Davidian and Giltinan, 1995; Greene 2002). According to the literature, there are two ways to establish hierarchical models to more comprehensively account for the unobserved heterogeneity. One way is direct, whereby a random variable is attached to a traditional model's parameter (representing mean). Unlike in the traditional model where the estimation result only provides information for the means, the hierarchical model provides information on both means and variations (in most cases, standard deviations). For example, a hierarchical Bayesian model was studied to address the unobserved heterogeneity from construction quality and axle load, respectively (Hong and Prozzi, 2006). The other way is indirect, whereby a random



variable is attached to an aggregate term in the model instead of one particular parameter. For example, in a recent study on asphalt pavement rutting by Archilla (2006), a multiplicative term is adopted to capture unobserved heterogeneity from the combination of load and pavement structure. Archilla also disclosed that the results were “*more in accordance with the priori expectation.*”

In summary, the unobserved heterogeneity should potentially be reflected not only through the intercept term but in the other regression parameters or terms. It is more realistic and reasonable to let the relevant parameters be random (hereafter referred to as random parameters model), because each section may possess unique characteristics reflecting its particular deterioration process.

## 2.4 Model estimation framework

From a statistical perspective, there are two basic approaches to model estimation, the classical and Bayesian approaches. The former is based on repeated sampling of distributions; its parameter estimation is solely based on observed data. The latter is based on the Bayesian theorem; its parameter estimation is based on both subjective belief and observed data. According to the Bayesian theorem, the updated parameter, also referred to as posterior, is yielded as (Gelman et al., 2004),

$$p(\underline{\theta}|y_1, y_2, \dots, \underline{x}_1, \underline{x}_2, \dots) = \frac{p(\underline{\theta})p(y_1, y_2, \dots, \underline{x}_1, \underline{x}_2, \dots)}{p(y_1, y_2, \dots, \underline{x}_1, \underline{x}_2, \dots|\underline{a})} \quad (2.3)$$

Where,

$y_1, y_2, \dots, \underline{x}_1, \underline{x}_2, \dots$  are observed data; and

$\underline{\theta}$  is parameter vector to be estimated.

In addition, another significant distinction lies in the way each approach treats parameters. The classical approach argues that the parameter is a fixed but unknown value (Greene, 2002). The Bayesian approach argues that instead of a fixed value, the individual parameter has its own distribution (Gelman et al., 2004). However, parameter estimates can be regarded as asymptotically equivalent between the two approaches, i.e., both estimates are asymptotically normal (Train, 2003; Gelman et al., 2004). Apart from the previously discussed distinctions and similarities between the underlying two estimation approaches, the literature reveals that there has been a great amount of debate concerning this interesting issue. No fixed guidelines have been provided as to which approach to take in practice. In this research, along the line of existing deterioration modeling in the context of pavement, the focus will be on the classical approach to model estimation.

In terms of parameter estimators, the estimation framework consists of three alternatives: parametric, semi-parametric, and non-parametric (Greene, 2002). Three examples cited in this study are Maximum Likelihood Estimation (MLE), Generalized Method of Moments (GMM), and Kernel Density Estimation for these alternatives, respectively. The estimators from parametric to non-parametric improve in terms of robustness but at the cost of weakening the conclusions drawn from the data, such as less efficiency (Greene, 2002). The framework of these three estimation alternatives is now presented.

#### Maximum Likelihood Estimation (MLE)

MLE has enjoyed popularity as the most common parametric estimator in econometric literature (Greene, 2002). Based on a specified probability density function, the likelihood function is established as,

$$f(y_1, y_2, \dots, \underline{x}_1, \underline{x}_2, \dots) = \prod_{i=1}^n f(y_i, \underline{x}_i | \underline{\theta}) \quad (2.4)$$

Where,

$f(\bullet)$  is pre-specified density function, such as normal distribution;

$y_1, y_2, \dots, \underline{x}_1, \underline{x}_2, \dots$  are observed data; and

$\underline{\theta}$  is parameter vector to be estimated.

The likelihood function is then maximized to obtain model estimates. MLE estimator is consistent, asymptotically efficient and normal should the model be correctly specified.

Whether the model is correctly specified can be tested through Information Matrix Equality (IME). The details of model specification and test will be discussed in Chapter 5 and Appendix A.

### Generalized Method of Moments (GMM)

In recent years, GMM has gained momentum in econometric model estimation.

Compared with parametric approach such as MLE, GMM removes the distributional assumption. The only assumption in GMM is concerned with the moment function,

$$E(m(y_i, \underline{x}_i, \underline{\theta})) = 0 \quad (2.5)$$

Where,

$m(\bullet)$  is the moment function, e.g. in linear condition,  $m(\bullet) = \underline{x}_i (y_i - \underline{x}_i' \underline{\theta})$ ;

$y_i, \underline{x}_i$  are observed data; and

$\underline{\theta}$  is parameter vector to be estimated.

### Kernel Density Estimation

The fundamental idea of Kernel density estimation is originated from the histogram. A simple example is the regression function of a variable  $y$  on a scalar  $x$ . The Kernel weighted regression estimator for conditional mean is,

$$\hat{\mu}(x^* | x_1, x_2, \dots, x_n, h) = \frac{\sum_{i=1}^n \frac{1}{h} K\left(\frac{x_i - x^*}{h}\right) y_i}{\sum_{i=1}^n \frac{1}{h} K\left(\frac{x_i - x^*}{h}\right)} \quad (2.6)$$

Where,

$x^*$  is a given value of the explanatory variable;

$x_i, y_i$  are observations;

$h$  is bandwidth; and

$K(\bullet)$  is the kernel function, such as normal, uniform, triangle distributions.

The modeler should decide which estimation method to use according to the objective and in the context of problem of interest. In this regard, it is important to point out that deterioration modeling aims at drawing conclusions from observed data. Models and estimation approaches should support this objective.

## 2.5 Model application in pavement design and management

The ultimate goal of developing deterioration models is to assist pavement designers and decision makers in managing pavement systems. There are a series of examples involving the application of deterioration models. Obviously, among them, the most influential is the one adopted in pavement design – the AASHTO Design Guide (1993) equation. It has been pointed out that pavement design is closely related and sensitive to model accuracy (Small and Winston, 1988). The two authors showed that close-to-optimal pavement design can be realized by employing improved estimated model parameters. One of their

key findings indicated that both rigid and flexible pavements designed under the suggested AASHTO equations yielded “*substantially less durable roads than would be optimal.*” Their findings were supported and fine-tuned in subsequent work based on duration modeling technique (Prozzi and Madanat, 2000; Madanat et al., 2002).

In pavement management, deterioration models serve as one of the critical inputs for determining optimal policy. Due to the complexity of deterioration models, they are usually mathematically simplified to ease the problem solving process. For instance, a simple deterioration model with Markovian properties was adopted in a resurfacing problem in Tsunokawa and Schofer (1994) and Li and Madanat (2002). This approach does not accurately reflect pavement deterioration reality (Ouyang and Madanat, 2004). An improved approach was carried out by incorporating a correction term to address historical impact in recent work by Ouyang and Madanat (2004). Although significant progress was made, the issue related to incorporating deterioration modeling in the decision making process remains to be improved to more closely reflect reality through more realistic models.

## **Chapter 3: Data Sources and Characteristics**

Data serve as the building blocks in the deterioration modeling. Therefore, it is a prerequisite to obtain good-quality data for model development and estimation. In this chapter, possible data sources for pavement deterioration models are explored. In particular, the data set used in this study is described. Data characteristics are then investigated from the perspective of their data structure. In addition, the effect of data problems on model estimation is discussed.

### **3.1 Data sources for pavement deterioration models**

Data for pavement deterioration models come from two sources, accelerated pavement tests and in-service pavements. Each data source has its own advantages and disadvantages. They can be complementary to each other in giving support to the understanding of pavement deterioration. Advantages of accelerated pavement testing include: 1) the factors affecting the deterioration process can be well controlled and accounted for through experimental design; 2) a relatively short period of time is required; and 3) a small financial investment is needed. A disadvantage is its inability to capture long-term pavement performance and deterioration mechanisms under actual traffic and environmental conditions. These concerns can be addressed by obtaining data from in-field pavement sections. However, to fully monitor pavement deterioration, the in-field condition assessments usually involves heavy financial commitment and a longer time span than its experimental counterpart.

Among the many data sources mentioned in literature, a list of representative cases are briefly discussed in the following paragraphs.

AASHO Road Test. The AASHO Road Test is one of the most influential road tests ever conducted. Interestingly, the motivation behind the AASHO Road Test was cost

allocation study, and the results proved to be a milestone of modern pavement and bridge design. The current AASHTO Design Guide (1993) is based on the AASHO Road Test, which was conducted during the period of November 1958 to December 1960 near Ottawa, Illinois (HRB, 1962). Both rigid and flexible pavement structures were tested. Six 2-lane loop tracks were constructed for the test, with loops 2 through 6 subject to traffic loading. Loop 1 was used to test environmental effects. Approximately 1,114,000 axle repetitions were applied on the test sections using different trucks covering three types of axle configurations, single axle with single wheel, single axle with dual wheels, and tandem axle.

Brazil Road Test. The Brazil Road Test is generally referred to as the Brazil-UNDP field study. The motivation for this study was to understand road deterioration, both for paved and unpaved road types (Paterson, 1987). The test was carried out in Brazil from 1976 to 1982, sponsored by the World Bank. During this study the characteristics of a series of pavement performance indices including roughness, cracking, and rut depth were investigated. Both flexible and semi rigid pavements were evaluated in the test. In addition, the non-freezing climates, mixed traffic loadings, and various maintenance standards were involved (Paterson, 1987). Partially based on the Brazil-UNDP study, the World Bank issued the Highway Design and Maintenance Standards Model, HDM-III (Watanatada et al. 1987).

LTPP Program. The Long-term Pavement Performance (LTPP) program was initiated as a part of the Strategic Highway Research Program (SHRP), which was established by the Transportation Research Board (TRB) of the National Research Council in the 1980s. The program is now sponsored by the Federal Highway Administration (FHWA) with the cooperation of AASHTO (FHWA 2003). The LTPP program focuses on in-service pavement sections. A total of approximately 2,400 LTPP sections cover most states or provinces in the US and Canada. The collected data for these sections are published and updated periodically through FHWA's DataPave Online ([www.datapave.com](http://www.datapave.com)). The

overall objective of the LTPP program is to monitor and evaluate long-term pavement performance under a variety of factors over a pavement's service life, usually over 20 years.

Minnesota Road Test. The Minnesota Road Test project (hereafter referred to as Mn/Road) was carried out by the Minnesota Department of Transportation to improve design, construction, and maintenance of transportation infrastructure in Minnesota's wet-freeze climate region. Mn/Road test sections also serve as LTPP sections. The test was started in 1993 and is still in operation. The systematically and comprehensively collected information makes the Mn/Road a sound data source for deterioration modeling. Detailed information on Mn/Road is presented in subsequent chapters because this study adopts Mn/Road as the case study for deterioration modeling.

NCAT Test Track. The NCAT Test Track was built at the National Center for Asphalt Technology (NCAT) in Alabama (NCAT Report 02-12). The major objective of this test is to evaluate the effect of different mix types – Superpave, Stone Matrix Asphalt (SMA), and Open Graded Friction Courses (OGFC) – and their properties on pavement performance. The test track, or loop, which is 1.7 miles (2.8 km) in length, is an accelerated loading facility consisting of 46 test sections that are subjected to controlled truck traffic. A total of 10 million ESALs were applied during the first test cycle that spanned a 2-year period after the test sections were built in 2000.

PMIS database. Many states DOTs have launched a Pavement Monitoring Information System (PMIS) to better manage their road facilities. Those individual systems, which focus on network level, can provide insight into how pavements behave under actual traffic and environmental conditions. However, further work is needed on existing systems to accommodate the development of a deterioration model in part due to the lack of sufficient variables and inspection duration and frequencies.



WesTrack Road Test. The WesTrack Road Test was conducted in Reno, Nevada in 1995. The objectives of this road test were: 1) to develop performance-related specifications for hot-mix asphalt construction; and 2) to provide early field verification of the SHRP Superpave Level III mix design procedures ([www.westrack.com](http://www.westrack.com)). The 1.8 mile (2.9 km) oval loop consisted of 13 test sections, which were located on the loop's tangent segments. All test sections shared the same structural design. In addition, those test sections were subject to controlled traffic, i.e. four triple trailer vehicle combinations. During a 2-year period of traffic applications, a total of around 4.9 million ESALs were applied on the test track.

### **3.2 A discussion on data source selection**

As stated previously, behind different data sources underlay the specific objectives of each individual program or project. A sound deterioration model is far more than mere performance fit; sufficient information is required in a pavement system to reflect the deterioration mechanism. In this regard, whether a given data source is feasible for the development of a sound deterioration model depends on the available information. For example, to capture the role of different layers in pavement deterioration resistance, structural variation in the sample is required. Similarly, to identify seasonal variation and relevant causes of pavement performance requires seasonal inspections and knowledge of the related environmental factors affecting the performance. In addition to data availability, a good-quality data source also plays a key role in deterioration modeling. The term “good-quality” means that the collected data correctly reflects physical phenomena and are relatively free of error. In this study, based on previous work (Prozzi, 2001; Von Quintus et al., 2003) and consultation with Mn/Road personnel, the Mn/Road project is identified as a good-quality data source and is thus adopted as the case study for developing and estimating the deterioration model.

### **3.3 Data source adopted in this study - Minnesota Road Test**

The Minnesota Road Test (Mn/Road) project area is located 40 miles (65 km) northwest of Minneapolis/St. Paul and consists of two segments, the Mainline and the Low Volume Roadway (LVR) (Mn/Road website). The Mainline is a 3.5-mile (5.6 km) 2-lane stretch of Interstate Highway 94 (I-94), while the LVR is parallel to I-94. The LVR was constructed as a loop with a length of 2.5 miles (4 km) and has been subjected to traffic with weight-controlled axle loads. In this study, in order to capture the real-world highway deterioration, the in-service segment, i.e. Mainline, is adopted in model development and deterioration analysis. The layout of the Mainline test sections is illustrated in Figure 3.1.

Both flexible and rigid pavements are tested in Mn/Road project, but the flexible pavement sections are of particular interest in this study. There are a total of 14 flexible pavement test cells with four belonging to the 5-Year plan (including pavement sections designed for 5-year design life) and 14 belonging to the 10-Year plan (including pavement sections designed for 10-year design life). Each cell is 500 feet long. Between two adjacent cells, there is a transitional section of around 60 feet in length.

In the factorial design, pavement structures vary from cell to cell, as shown in Figure 3.1. Details related to structure, material, design, environment, and traffic are discussed in Chapter 4. Each test cell covers two lanes (driving and passing lanes) with different traffic, setting the sample size of sections for modeling at 28. For each section, a series of observations along time have been obtained according to Mn/Road performance inspection since the initiation of traffic on the sections in 1994. This leads to a panel data structure ideally suited for the objectives of this dissertation. The characteristics of the panel data set are presented in the following section.

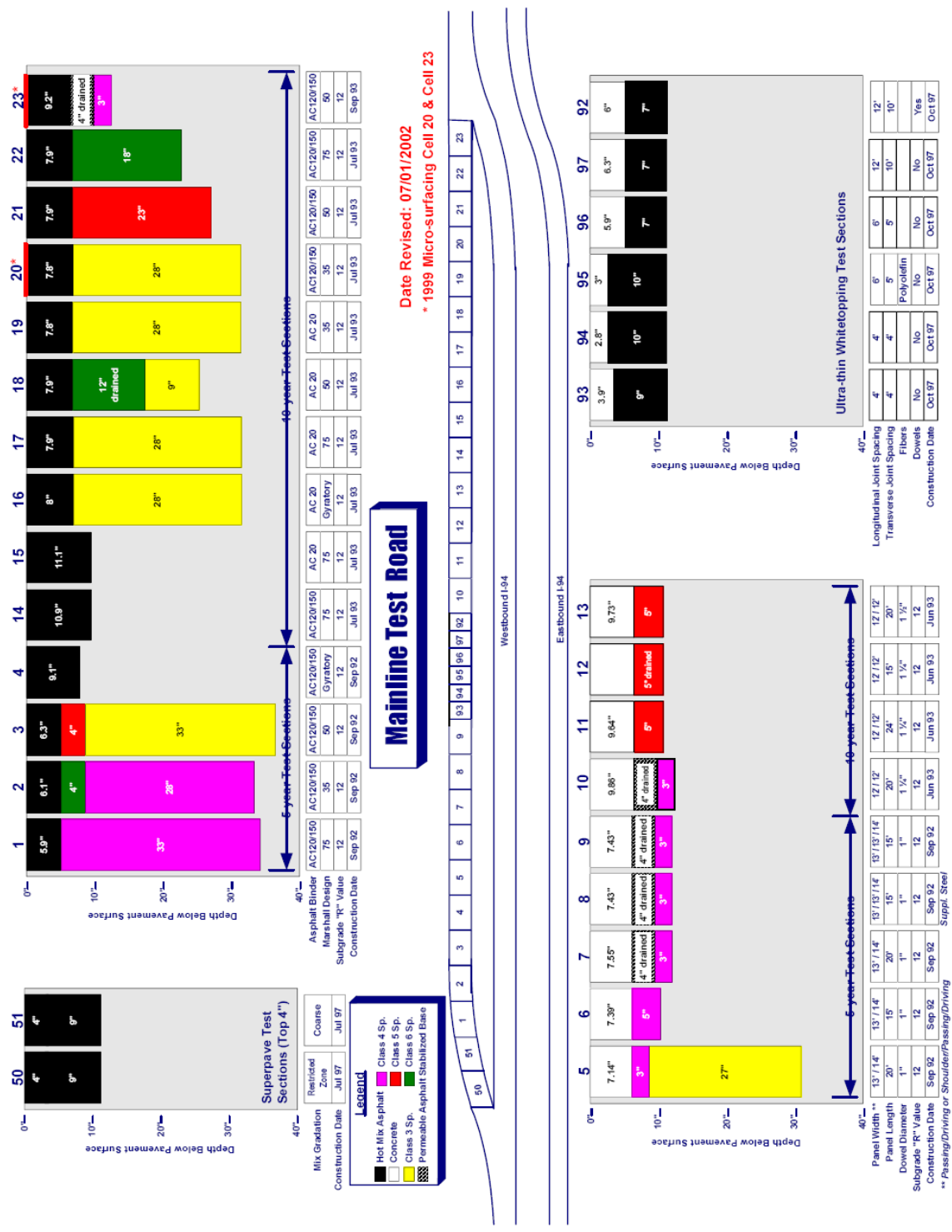


Figure 3.1 Mn/Road Mainline Test Cells Layout

### 3.4 Data characteristics

From a data structure viewpoint, there are basically two types of data: cross section and time series data. Cross section data consist of parallel observations on many units, i.e., one observation for each unit. Time series data focus on one unit, with finite or a countable infinite number of observations for that unit. The combination of these two data structures is referred to as panel data. It is clear that the former two are one-dimensional and the panel data are two-dimensional, which leads to their differing roles in capturing and explaining the causal relationship between dependent and explanatory variables.

From a model estimation perspective, the cross section data are incapable of capturing performance evolution with time while the time series data cannot incorporate parameters associated with variations across different units. For example, a deterioration model based on cross section data cannot be used to forecast pavement performance development along time, while a time series model cannot disclose the ability of different pavement structures to resist deterioration. The two deficiencies, however, can be addressed in a complementary fashion in a panel data-based model. In particular, the unique characteristic of panel data is that they provide the feasibility to account for unobserved heterogeneity.

The other issue involving data characteristics that is worth noting concerns commonly encountered problems in data sets from a statistical viewpoint. This issue generally includes three aspects: 1) multi-collinearity, 2) endogeneity, and 3) unobserved events typical of the data (Prozzi, 2001).

Multi-collinearity results from highly correlated explanatory variables, which may lead to paradoxical model estimation results. For example, the  $p$ -values for certain variables are large although those variables should be statistically significant according to pre-

judgment. In pavement engineering, a typical example of multi-collinearity is pavement age and accumulated traffic (Prozzi, 2001). That is, as time passes, traffic accumulates accordingly. Another example is concerned with accumulated traffic and pavement thickness. Usually, a highway with higher levels of traffic is designed with thicker pavement. There are several ways to treat the multi-collinearity problem. First, remove either of the variables if two variables are found to be highly correlated. These two collinear variables convey similar or the same information. Second, use a larger sample size to reduce the impact of multi-collinearity. For example, a data combination technique was used as an effective solution in deterioration modeling by Archilla (2000), Archilla and Madanat (2001), Prozzi (2001), and Prozzi and Madanat (2004). In summary, it should be noted that it is essential to check collinear conditions among explanatory variables before establishing model specification.

Endogeneity is another common problem occurring in empirical models. An explanatory variable is said to be endogenous if it is correlated with the error term (Wooldridge, 2001). Otherwise, the variable is referred to as exogenous. Usually, three sources contribute to endogeneity: omitted variables, measurement error, and simultaneity. Several good examples can be cited to facilitate understanding of endogeneity in pavement deterioration modeling. In modeling the crack index as a function of structural number and traffic, endogeneity was found to exist due to omitted variables such as environmental and subgrade characteristics (Madanat et al., 1995). The maintenance activity variable may also be endogenous since the maintenance schedule is usually carried out in response to pavement condition instead of being randomly conducted (Ben-Akiva and Ramaswamy, 1993). One way to avoid endogeneity is to adopt the data source from test sections under a predetermined and well controlled experimental design. The other way is to correct endogeneity bias through statistical technique, introducing instrumental variables (Greene, 2002; Wooldridge, 2001).

Regarding unobserved events, two common phenomena can be encountered in engineering, i.e. censoring and sample selectivity. In censoring condition, observations beyond a certain range are all converted into one single value. Examples of censored data in civil engineering can be found in pavement (Prozzi and Madanat, 2000) and groundwater (Finley, 2004). Sample selectivity bias occurs from nonrandom sampling. For example, in modeling maintenance effectiveness, using observations only from sections receiving treatment is subject to sample selectivity bias since those selected sections are not representative of the population of all sections (Madanat and Mishalani, 1998). To address the sample selectivity bias, a two-step estimation procedure is usually used, with the first step involving a choice model and the second step aiming to develop the parameter estimation of interest with appropriate corrections (Greene, 2002).

In summary, in addition to model development and estimation per se, data characteristics deserve an in-depth investigation in deterioration modeling. Some problems can be avoided through purposely selecting appropriate data sources (such as from a well-controlled road test). Alternatively, targeting the identified problems, relevant statistical measures should be taken to correct the potential bias aforementioned.

## Chapter 4: Model Specification

This chapter focuses on the process of developing the model specifications based on the data from the Mn/Road project. First, a general deterioration model form is presented. The general model form has been well established to reveal basic deterioration principles such as deterioration trends. Specific models rooted on the general model can vary among different backgrounds. Second, concrete information is incorporated as dependent and explanatory variables, and as parameters in the general form to represent the deterioration mechanism. In addition, the model parameter structure for the purpose of addressing unobserved heterogeneity is discussed.

### 4.1 AASHO model

The prototype of the pavement deterioration model can be traced to the AASHO model developed from the AASHO Road Test (HRB, 1962). Based on experimental data from the AASHO Road Test, a state-of-the-art pavement performance model was established (HRB, 1962), as is shown in Equation (4.1):

$$p_t = p_0 - (p_0 - p_f) \left( \frac{W}{\rho} \right)^\alpha \quad (4.1)$$

Where,

- $p_t$ : serviceability at time point  $t$ ;
- $p_0$ : serviceability at time  $t = 0$ , i.e. initial serviceability;
- $p_f$ : terminal serviceability;
- $W$ : cumulative axle load repetitions until time point  $t$ ;
- $\rho$ : cumulative axle load repetitions until failure; and
- $\alpha$ : parameter determining the curvature of performance model.

When estimating the AASHO model,  $\rho$  and  $\alpha$  were further expressed as functions of the variables associated with traffic and pavement structures. Deficiencies of the model were identified with regard to determining  $\rho$  and  $\alpha$  in both specification and parameter estimation aspects (Rauhut et al., 1983; Small and Winston, 1988; Prozzi and Madanat, 2000). These aspects mainly include mismatched units, serious statistical flaw and poor fit to the data.

## 4.2 General model

Following a similar deterioration principle of the AASHO model, a general deterioration model is proposed as,

$$p_t = a + bN_t^c + \mu \quad (4.2)$$

Where,

- $p_t$ : pavement serviceability or performance at time point  $t$ ;
- $N_t$ : measure of cumulative traffic or service time duration until time point  $t$ ;
- $a$ : parameter representing initial serviceability;
- $b$ : parameter representing deterioration rate;
- $c$ : parameter representing the curvature of deterioration curve; and
- $\mu$ : random error term.

As the initial condition,  $a$  represents the highest condition level for a given pavement section. As the deterioration rate,  $b$  is expected to have a positive sign in roughness context, meaning that performance in terms of IRI will increase with traffic applications and time. What follows is to determine the corresponding specification terms in Equation (4.2) by incorporating relevant factors accounting for pavement deterioration, such as



those factors involving pavement structures, environment, traffic, and others. Previous work by Prozzi (2001) and Prozzi and Madanat (2004) has successfully established and estimated a nonlinear riding quality model with focus on random effects. Their model is applied in this study with some further refinements based on the particular characteristics of the Mn/Road project. The details are explained in the following section.

### **4.3 Model specification based on Mn/Road project**

#### **4.3.1 Dependent variable – riding quality**

In the Mn/Road project, riding quality is readily available and expressed in terms of IRI. The as-constructed pavement has an average IRI of 0.62 m/km. The Minnesota Department of Transportation (MnDOT) considers the threshold of poor riding quality to be an IRI value of 2.5 m/km or above. Mn/Road personnel have periodically collected riding quality data since the beginning of the project. Two techniques are involved in riding quality data collection. From 1994 to July 1997, data were collected by a PaveTech van equipped with ultrasonic sensors. After July 1997, the PaveTech van was replaced with a Pathway van, equipped with laser sensors. A total of 1,027 observations are used as the sample in the deterioration model. The sample consists of panel data, which include both cross section and time series observations. Additionally, it was suggested in the Mn/Road research report (2002) entitled, “Hot-Mix Asphalt Mainline Test Cells Condition Report” that the IRI is sensitive to the severity of cracking in terms of crack width. For instance, the expansion of cracking width due to cold weather increases roughness by an average of 0.22 m/km as compared to hot weather conditions during mid-summer due to cracking contraction.

#### **4.3.2 Explanatory variables**

Pavement deterioration results from the combined effects of traffic loading and environmental factors on the structure and materials. Therefore, based on the availability of information, the proposed deterioration model includes the following key components as explanatory variables.

#### Pavement structure, materials, and design

1) Surface layer: The surface layer of each flexible pavement section is made of asphalt concrete. The Mn/Road experimental design in particular takes into account the possible variations in pavement design such as layer thickness, and mix design such as asphalt binder type and mix design method.

The surface layer thickness has a mean of 7.9 in. (20.0 cm) and standard deviation of 1.6 in. (4.0 cm). Two asphalt binders are used, referred to as AC-20 and PEN 120/150. For ease of comparison, the two binder grades can be converted to the same binder specification standard in terms of Superpave grading system, PG grade. AC-20 is equivalent to PG 64-22 and PEN 120/150 is equivalent to PG 58-28. It is implied that compared to an asphalt mix containing binder PG 58-28, the mix containing PG 64-22 is more resistant to high-temperature deformation such as rutting and less resistant to low-temperature deformations such as temperature-induced cracking. Considering the latitude of the Mn/Road project location – a wet-cold climate region – the PG 58-28 binder mix is believed to more effectively improved pavement performance. Whether this speculation is valid can be later tested through the relevant parameter estimation results of the deterioration model.

In addition to asphalt materials, the mix design method is investigated as a potential factor affecting pavement performance. Traditionally, mix design uses the Marshall method, in which the specimen used to determine optimum binder content is compacted by means of a standard hammer with an empirically determined number of blows. In the

Mn/Road project, three scenarios are adopted by using 35, 50 and 75 blows, respectively. In most cases, 75 blows are applied while in other cases 50 blows are applied, such as in Stone-Matrix-Asphalt (SMA) design. However, 35 blows are seldom adopted, except for low volume roads with limited traffic. In recent years, a new mix design approach developed through the Strategic Highway Research Program (SHRP), known as Superpave method, has become more widely used in many states. In this approach, the specimen is prepared through the gyratory compactor, which tends to simulate real asphalt mix compaction conditions during pavement construction.

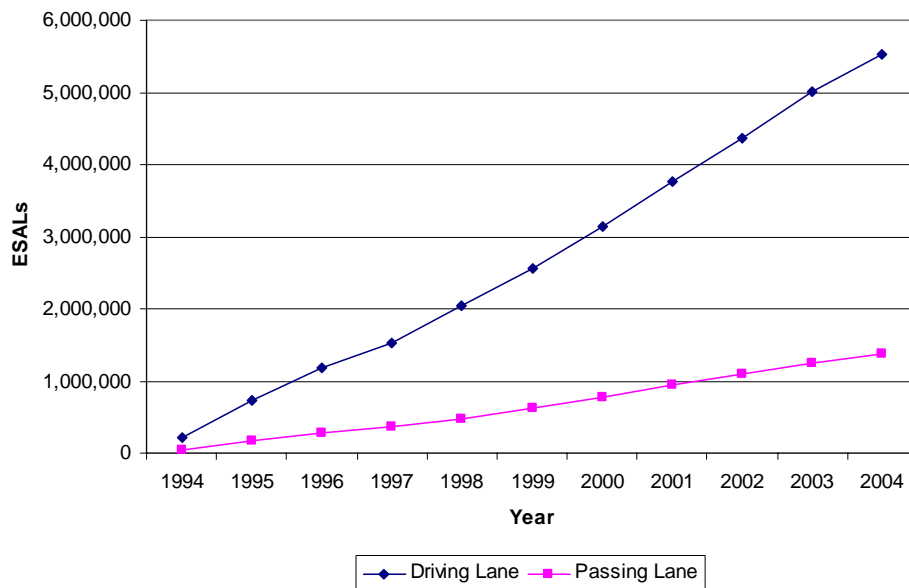
2) Base and subbase layers: Granular aggregates are used as the materials in the untreated base and subbase layers for the Mn/Road project. According to MnDOT, material specification classes 3, 4, 5, and 6, are involved. In order to match the existing studies concerning pavement structure, these four types of aggregates are classified into two categories based on their gradations. Classes 3 and 4 are combined into one category to serve as the subbase layer material due to their relatively fine gradation. Classes 5 and 6 serve as base layer material because they are coarser with higher resistance to shear deformation. In addition, it is indicated that not all cells have both base and subbase layers. The thickness of the base layer ranges from 0 to 23 in. (58.4 cm). The thickness of the subbase layer ranges from 0 to 33 in. (83.8 cm).

3) Subgrade: All of the flexible pavement cells are built on a silty-clay subgrade, with an “R-value” of 12 or A-6 according to the AASHTO Soil Classification.

### Traffic/ Time

The Mainline sections were subject to real traffic running on the test segment of I-94. Two weigh-in-motion (WIM) systems were installed to collect axle load information. First, a load cell WIM was installed, followed by a quartz piezo WIM for the purpose of comparing the two sensor technologies. Since each test section was not assigned with one

WIM system exclusively, it is assumed that all the sections in the driving lane and all the test sections in the passing lane experienced the same levels of traffic, respectively. The cumulative number of Equivalent Single Axle Load (ESALs) for both driving and passing lanes were provided by the Mn/Road project personnel. The calculation, based on the method proposed by the 1993 AASHTO Design Guide, shows that after approximately 11 years of service, by the end of 2004, the driving lane experienced over 5 million ESALs while the passing lane experienced over 1 million. The accumulated ESALs for either lane are depicted in Figure 4.1. It is shown that traffic levels increased approximately in a linear fashion on both lanes.



**Figure 4.1 Cumulative ESALs Running On the Mainline Test Sections**

Environment

Environment is another critical factor in the pavement deterioration process. The main environmental factors include temperature and precipitation. The effect of environmental factors on pavement performance is usually reflected through the change of material properties due to their environmentally sensitive characteristics. For asphalt materials,

aging leads to hardening of the asphalt mix, which is believed to contribute positively to rutting resistance but negatively to cracking resistance. With regard to granular materials, frost susceptibility is a critical concern among pavement engineers. The granular aggregate with high percentage of fines (passing # 200 (0.075 mm) sieve) is known to be susceptible to frost action during cold weather seasons due to its capacity to retain moisture. Frost heave causes new cracking or expansion of existing cracking. Among the four classes of granular materials, Class 3 has the highest percentage of fines (around 13%). The other three classes all have less than 10% of fines. Research conducted by the U.S. Army Cold Regions Research and Engineering Lab indicated that those classes with relatively low percentages of fines have very low frost susceptibility (Mn/Road Website). In addition, Mn/Road is located in a region that experiences freeze-thaw cycles, which contribute significantly to pavement deterioration should no effective protection measures be taken. Fortunately, in the Mn/Road project, load-restrictions are imposed during the thawing period (usually February through March) (Mn/Road Website).

### Maintenance

The purpose of maintenance is to extend pavement performance or reduce deterioration rates. During the condition assessment of the test sections, maintenance effects can be observed through a condition improvement jump or deterioration curvature change. In the Mn/Road project, the major maintenance activities include crack sealing and micro-surfacing (slurry seal).

### **4.3.3 Final model specification**

#### Specification for Structure, Material, and Design

In the parameter corresponding to the surface layer, in addition to layer thickness, two more factors are added: one for asphalt binder type and the other for mix design approach.

As previously discussed, two types of asphalt binders are used in the Mn/Road project with different effects on pavement performance improvement. In the current model, PEN 120/150 (PG 58-28) is included as a dummy variable with AC 20 (PG 64-22) as the reference. For the mix design approach, three dummy variables – 35Blow, 50Blow, and 75Blow – are included to capture the different effects on pavement performance between the three alternatives based on the Marshall method and the Superpave design method, respectively. The Superpave design method is treated as the reference. Thirty-five blows represent a relatively low level of compaction energy in preparing specimens, while 50 blows represent a median level and 75 blows represent the highest level of compaction energy. Therefore, by incorporating the information associated with pavement structure, material, and design, the term  $b$  can be expressed as,

$$b = \exp \left\{ \begin{array}{l} \alpha_{0i} + \exp(\gamma_{1i} AC120/150 + \psi_{1i} 35Blow + \psi_{2i} 50Blow + \psi_{3i} 75Blow) \alpha_{1i} H_{1i} \\ + \alpha_{2i} H_{2i} + \alpha_{3i} H_{3i} \end{array} \right\} \quad (4.3)$$

Where,

- $AC120/150$ : asphalt binder type PEN 120/150;
- $35Blow$ : Marshall mix design with 35 blows;
- $50Blow$ : Marshall mix design with 50 blows;
- $75Blow$ : Marshall mix design with 75 blows;
- $H_{1i}$ : surface layer thickness on test section  $i$ , in.;
- $H_{2i}$ : base layer thickness on test section  $i$ , in.;
- $H_{3i}$ : subbase layer thickness on test section  $i$ , in.;
- $\alpha_{0i}, \alpha_{1i}, \alpha_{2i}, \alpha_{3i}$ : parameters for pavement structures, to be estimated;
- $\gamma_{1i}$ : parameters for asphalt material, to be estimated; and
- $\psi_{1i}, \psi_{2i}, \psi_{3i}$ : parameters for mix design, to be estimated.

The exponential form for  $b$  is adopted to ensure that the deterioration rate is positive. That is, with the increase of cumulative traffic running or time, pavement roughness in terms of IRI should increase. The exponential term attached to surface layer coefficient  $\alpha_{1i}$  serves as a multiplicative factor to denote the difference between the alternatives of asphalt binder and mix designs and the reference, Superpave design with AC 20 (PG 64-22) as the binder. The signs of related parameters in the exponential term,  $\psi_{1i}, \psi_{2i}, \psi_{3i}$ , and  $\gamma_{1i}$  being positive, leading to an exponential of larger than one, indicates an improvement of that particular design or binder to the reference surface layer's ability to resist deterioration, while the negative signs, leading to a less-than-one factor, means the opposite. The ratio of  $\alpha_{1i}, \alpha_{2i}$ , and  $\alpha_{3i}$  reveals the ability of the unit thickness from three layers - surface, base, and subbase - to resist deterioration. Exponential of  $\alpha_{0i}$  indicates riding quality after one year's service when only subgrade is available.

#### Specification for traffic and time, and curvature

As was discussed in the literature review in Chapter 2, for in-service pavement, traffic and time can be perfectly correlated due to the fact that traffic increases linearly with time (Figure 4.1). Therefore, cumulative traffic or time, rather than both, is used in modeling. In this study, a variable representing the time since the pavement was open to traffic was adopted in the model. In addition, it is reasonable to believe that the curvature for different lanes is heterogeneous and affected by traffic levels. A dummy variable  $DL$  (referred to as driving lane) is introduced to differentiate the curvature associated with heavier traffic lane, driving lane from that of lighter traffic lane, passing lane. Moreover, to quantify curvature difference due to different traffic levels, the average annual difference in ESALs is attached to the dummy variable  $DL$ . It is calculated that the driving lane experienced 0.399 million more ESALs each year on average, as indicated in Figure 4.1. Thus, the term for traffic, time, and curvature  $c$  can be expressed as,

$$N_{it}^c = T_{it}^{(\phi_{0i} + 0.399\phi_{1i}DL)} \quad (4.4)$$

Where,

$T_{it}$  : service time duration until time point  $t$  for section  $i$ ;

$\phi_{0i}, \phi_{1i}$  : parameters to be estimated; and

$DL$  : dummy variable representing driving lane.

In Equation (4.4), considering the traffic level on the driving lane is heavier than on the passing lane, the parameter  $\phi_{1i}$  is expected to have a positive sign. The reason is that, due to heavier traffic, the driving lane experiences decreased riding quality or higher IRI, which is manifested through a larger curvature.

#### Specification for other factors: environment and maintenance

As previously mentioned, Class 3 granular aggregate is potentially susceptible to frost action during the winter season due to its high percentage of fines. Through observation of pavement performance, it was shown that those sections with Class 3 material as subbase directly under the surface layer experience a jump in IRI in cold weather. Thus, a compound variable consisting of frost-heave (in mm) multiplying a dummy variable representing Class 3 material directly under the surface layer is incorporated in the model. However, whether or not this environmental effect on performance measures is significant is later tested from the model estimation results.

Finally, the term representing maintenance activity is added to the model to capture its effect on pavement performance. A dummy variable is used to capture the abrupt performance jump due to maintenance intervention.



By integrating all the above components, the final specification of the deterioration model is obtained, as follows:

$$\begin{aligned}
 r_{it} = & \beta_{0i} + \\
 & \exp\{\beta_{1i} + \exp(\beta_{3i}AC120/150 + \beta_{4i}35Blow + \beta_{5i}50Blow + \beta_{6i}75Blow)\beta_{2i}H_{1i} + \beta_{7i}H_{2i} + \beta_{8i}H_{3i}\} \\
 & \times T_{it}^{(\beta_{9i}+0.399\beta_{10i}DL)} + \\
 & \beta_{11i}Frost \times SubSpec3 + \beta_{12i}Mnt + \mu_{it}
 \end{aligned}
 \tag{4.5}$$

Where,

- $r_{it}$ : pavement roughness on test section  $i$  at time  $t$ , in terms of IRI, m/km;
- $AC120/150$ : asphalt binder type PEN 120/150;
- $35Blow$ : Marshall mix design with 35 blows;
- $50Blow$ : Marshall mix design with 50 blows;
- $75Blow$ : Marshall mix design with 75 blows;
- $H_{1i}$ : surface layer thickness, in.;
- $H_{2i}$ : base layer thickness, in.;
- $H_{3i}$ : subbase layer thickness, in.;
- $T_{it}$ : service time duration until time point  $t$ ;
- $DL$ : dummy for right or driving lane;
- $Frost$ : frost heave, mm;
- $SubSpec3$ : dummy for subbase with material specification of Class 3;
- $Mnt$ : maintenance activity;
- $\beta_{0i} \sim \beta_{12i}$ : parameters to be estimated; and
- $\mu_{it}$ : error term.

#### 4.3.4 Incorporating unobserved heterogeneity

The structure for the parameters deserves a detailed discussion. As a major goal of this study, pavement performance heterogeneity is particularly addressed in this section. That is, an individual pavement section may demonstrate its specific deterioration characteristic, which can be reflected through the heterogeneity in model parameters. A hierarchical parameter structure is formulated particularly to address this issue.

Technically, the heterogeneity can be accounted for by imposing randomness over some of the parameters. What follows refers to the determination of parameter structures in Equation (4.5). The parameters can be divided into two categories, random and fixed, with random denoting the unobserved heterogeneity and fixed denoting the commonness shared by all individuals (e.g. in the traditional OLS regression approach).

First, those parameters considered varying are treated as random. As in the random effects model, the intercept  $\beta_{0i}$  is treated as random.  $\beta_{0i}$  represents the initial roughness of a given pavement section  $i$ . This value is usually considered to vary across pavement sections due to factors such as different pavement structures, construction quality, and other conditions during construction of the specific section. The fact that the initial IRI observations differ among the test sections in the Mn/Road project supports this hypothesis. Thus, the structure for  $\beta_{0i}$  is assumed to be:  $\beta_{0i} = \beta_0 + \delta_0 v_{0i}$ , with a deterministic term plus a term capturing the randomness. For conciseness, the implication of this structure is explained subsequently together with those for  $\beta_{2i}$ ,  $\beta_{7i}$ , and  $\beta_{8i}$ , which are similar to  $\beta_{0i}$  in structure. With regard to pavement structures and materials, the parameters for surface, base, and subbase layers are regarded as varying among sections. The reason is, except for the observed heterogeneity (e.g. different thicknesses across sections), the unobserved heterogeneity (such as material properties – among which the asphalt binder types are already accounted for – and drainage conditions) will produce different contributions to deterioration resistance. Therefore, the specifications for  $\beta_{2i}$ ,  $\beta_{7i}$ , and  $\beta_{8i}$  are  $\beta_{2i} = \beta_2 + \delta_2 v_{2i}$ ,  $\beta_{7i} = \beta_7 + \delta_7 v_{7i}$ , and  $\beta_{8i} = \beta_8 + \delta_8 v_{8i}$  respectively, with the deterministic and random terms in each parameter. For those

random parameters,  $\beta_{0i}$ ,  $\beta_{2i}$ ,  $\beta_{7i}$ , and  $\beta_{8i}$  the implication of the deterministic terms  $\beta_0$ ,  $\beta_2$ ,  $\beta_7$ , and  $\beta_8$  is the same as in the traditional approach: they are unknown but fixed values.  $v_{0i}$ ,  $v_{2i}$ ,  $v_{7i}$ , and  $v_{8i}$  are assumed to be standard normal random variables with their coefficients  $\delta_0$ ,  $\delta_2$ ,  $\delta_7$ , and  $\delta_8$  (standard deviation of  $\beta_{0i}$ ,  $\beta_{2i}$ ,  $\beta_{7i}$ , and  $\beta_{8i}$ , respectively) to be estimated. Whether or not the randomness of these parameters is significant is later tested through hypothesis test based on model estimation results. In addition, the random parameters are assumed to be independent.

Second, for the rest of the “slope” parameters, little evidence has been found to support their significant variability across pavement sections. Thus, for simplicity, those parameters are assumed to be fixed and only include the deterministic term as in traditional modeling approach:  $\beta_{1i} = \beta_1$ ,  $\beta_{3i} = \beta_3$ ,  $\beta_{4i} = \beta_4$ ,  $\beta_{5i} = \beta_5$ ,  $\beta_{6i} = \beta_6$ ,  $\beta_{9i} = \beta_9$ ,  $\beta_{10i} = \beta_{10}$ , and  $\beta_{11i} = \beta_{11}$ ,  $\beta_{12i} = \beta_{12}$ . In other words, the standard deviations for these parameters are all assumed to be equal to zero. If there is sufficient evidence to support the variation of these parameters, they can also be treated as random parameters with the same approach proposed before.

Considering that two types of parameters are adopted, the partition form of parameter vector is adopted to facilitate the following discussion,

$$\underline{\beta}_i = \left\{ \left[ \underline{\beta}_i^R \right]', \left[ \underline{\beta}_i^F \right]' \right\} \quad (4.6)$$

Where,  $\underline{\beta}_i^R = [\beta_{0i}, \beta_{2i}, \beta_{7i}, \beta_{8i}]'$ , including the random parameters, and

$$\underline{\beta}_i^F = [\beta_{1i}, \beta_{3i}, \beta_{4i}, \beta_{5i}, \beta_{6i}, \beta_{9i}, \beta_{10i}, \beta_{11i}, \beta_{12i}]' = [\beta_1, \beta_3, \beta_4, \beta_5, \beta_6, \beta_9, \beta_{10}, \beta_{11}, \beta_{12}]',$$

including the fixed parameters.

### 4.3.5 Incorporating spatial correlation

Spatial correlation or dependence may occur due to their proximity in distance. Anselin suggests three different ways to address spatial effects: 1) spatial stochastic process models, 2) direct representation, and 3) nonparametric approach (1999). Conveniently and commonly, the direct representation is used to address spatial correlation, which involves a distance function. The distance function is constructed so that spatial correlation of two observations is inversely related to the distance between their locations. For example, in a groundwater study, an exponential function was used to represent the correlation coefficients of contaminated concentration (Finley, 2004). In a study involving land use choice, inverse of distance was adopted by assuming the correlation is inversely proportional to distance (Wang and Kockelman, 2006). In this study, through the random intercept parameters in Equation (4.5), an exponential term is adopted to capture the spatial correlation among different pavement sections,

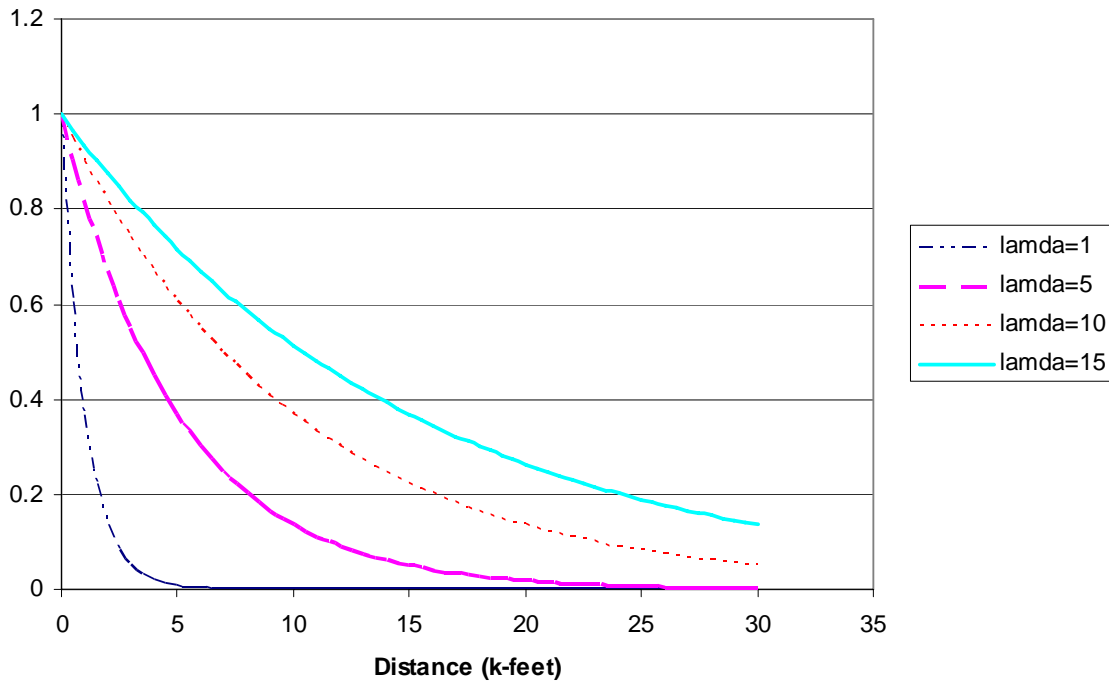
$$\text{corr}(\beta_{0i}, \beta_{0j}) = \exp\left(-\frac{d_{ij}}{\lambda}\right) \quad (4.7)$$

Where,

$d_{ij}$  : distance between two test sections (from centroids), in k-feet; and

$\lambda$  : parameter to be estimated.

$\lambda$  is to be determined based on estimation results. Figure 4.2 illustrates how the spatial correlation is sensitive to the distance given a variety of representative values of  $\lambda$ .



**Figure 4.2 Spatial Correlation vs. Distance for different  $\lambda$ 's**

## **Chapter 5: Parameter Estimation and Results**

This chapter describes the parameter estimation process. First, existing econometric approaches to estimating a typical nonlinear model are discussed. Then, the section elaborates further on the particular approach applied in this study for a nonlinear model with random parameters. The model estimation results - referred to as population level parameters - are provided next through applying the proposed estimation approach. Based on the estimated population level parameters, a step is extended to obtain the individual-level parameters through the Bayesian theorem. The implications based on model estimation results are discussed in detail. Finally, a summary is presented concerning the policy implication of two levels of model parameters.

### **5.1 Estimation approach for nonlinear models**

It is shown in Chapter 4 that the final model specification is highly nonlinear. To facilitate understanding, a simple case is discussed first by assuming that all the sections share the same regression parameters. That is, the randomness is removed from those regression parameters in the final model specification in Equation (4.5). Under such circumstances, the model becomes a typical nonlinear model encountered in some existing work. In the panel data context, this model can be estimated through pooled regression (Greene, 2002).

In econometrics, basically there are three approaches to estimating the parameters for a typical nonlinear model: nonlinear least square (NLLS), Maximum Likelihood Estimation (MLE), and Generalized Method of Moments (GMM). The first two approaches have seen many applications in engineering cases. The third approach is mainly applied in Economics due to its desirable property for application in the area. As is popular in current studies involving estimating random parameter models (e.g. Greene, 2002, 2004; Train, 2003), the MLE approach is adopted in this study. The MLE estimator

is more efficient than the other two approaches, or generally the most efficient estimator (Wooldridge, 2001). However, efficiency may come at the cost of nonrobustness (Wooldridge, 2001); the MLE estimator from a misspecified model is likely to be inconsistent. The specification test can be carried out through testing Information Matrix Equality (IME). Fortunately, the IME is proven valid for the model specification in this study. The details of model specification test and MLE estimator properties are presented in Appendix A.

## 5.2 Maximum Likelihood Estimation

Customarily, in MLE approach, the logarithm of the likelihood function is used to obtain the estimates and inferences of a given model. By taking logarithm, the log-likelihood function is denoted as,

$$LL = \frac{1}{n} \sum_{i=1}^n \log f(y_i; \underline{\theta}) \quad (5.1)$$

Where,

- $n$  : sample size;
- $y_i$  : dependent variable of observation  $i$ ; and
- $\underline{\theta}$  : parameter vector to be estimated.

The optimum parameter values are obtained using an iterative method. This method starts with a set of initial values, which are updated through the following algorithm,

$$\underline{\theta}_{r+1} = \underline{\theta}_r + \rho(H(\underline{\theta}_r))^{-1} g(\underline{\theta}_r) \quad (5.2)$$

Where,

- $\underline{\theta}_r$  : the current set of parameter values at step  $r$ ;

$\underline{\theta}_{r+1}$ : the updated set of parameter values at step r+1;

$\rho$ : search step length;

$H(\underline{\theta}_r)$ : Hessian matrix, which is

$$\frac{1}{n} \sum_{i=1}^n \frac{\partial^2}{\partial \underline{\theta} \partial \underline{\theta}'} \log f(y_i; \underline{\theta}_r)$$

$g(\underline{\theta}_r)$ : Gradient matrix, which is,

$$\frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \underline{\theta}} \log f(y_i; \underline{\theta}_r)$$

Convergence is achieved by a preset tolerance of the gradient of each parameter. After convergence, the asymptotic variance-covariance matrix of parameter estimates can be obtained as,

$$\left( -\frac{1}{n} \sum_{i=1}^n \frac{\partial^2}{\partial \underline{\theta} \partial \underline{\theta}'} \log f(y_i; \hat{\underline{\theta}}) \right)^{-1} \quad (5.3)$$

### 5.2.1 Maximum Simulated Likelihood estimation approach

Although the traditional MLE is capable of estimating a typical nonlinear model, it needs modification for solving a random parameter (RP) problem. Due to randomness incorporated in the regression parameters, the multi-dimensional integral is computationally challenging. In this regard, the simulation technique is integrated into traditional MLE. That is, to estimate such models with random parameters, the Maximum Simulated Likelihood (MSL) approach is applied (see Greene, 2002, 2004; Train, 2003). The detailed process of MSL is now described.

In the first step, the likelihood function based on panel data structure is established. The density function for an observation on pavement section  $i$  given time  $t$ ,  $y_{it}$  is,



$$f(y_{it}|\underline{X}_{it}, \underline{\beta}_i, \theta) = g(y_{it}, \underline{X}_{it}, \underline{\beta}_i, \theta) \quad (5.4)$$

Where,

- $\underline{X}_{it}$ : contains all the explanatory variables (see Equation (4.5));
- $\underline{\beta}_i$ : denotes the parameters as in Equation (4.5);
- $\theta$ : is a scalar denoting the standard deviation of the disturbance; and
- $g(y_{it}, \underline{X}_{it}, \underline{\beta}_i, \theta)$ : is the density function, a normal distribution herein.

For a given pavement section  $i$ , conditional on the random term in  $\underline{\beta}_i^R$ , the joint density (likelihood function) for the  $T_i$  observations at that section, i.e., the likelihood contribution of the  $i$ th section to the total is,

$$L_i(\underline{\bar{\beta}}, \underline{\delta}, \theta | \underline{y}_i, \underline{X}_i) = \prod_{t=1}^{T_i} g(y_{it}, \underline{X}_{it}, \underline{\beta}_i, \theta) \quad (5.5)$$

Where,

- $\underline{\bar{\beta}}$ : is the vector including the mean of  $\underline{\beta}_i$ ;
- $\underline{\delta}$ : is the vector including the standard deviation of elements in  $\underline{\beta}_i^R$ ; and
- $\underline{y}_i$ : is a vector including riding quality observations at section  $i$ .

Due to the randomness in  $\underline{\beta}_i^R$ , it is not feasible to maximize the likelihood function. The expectation of  $L_i$  over those random terms can be utilized (Greene, 2002, 2004) to obtain parameter estimates. After the random terms in  $\underline{\beta}_i^R$  being integrated out through its density  $f(\underline{\beta}_i^R)$  (whose parameters are its mean  $\underline{\bar{\beta}}^R$  and standard deviation  $\underline{\delta}$ ), the unconditional likelihood contributed by pavement section  $i$ , is:

$$L_i(\underline{\beta}, \underline{\delta}, \theta | y_i, \underline{X}_i) = \int_{\underline{\beta}_i^R} \prod_{t=1}^{T_i} g(y_{it}, \underline{X}_{it}, \underline{\beta}_i^R, \underline{\beta}_i^F, \theta) f(\underline{\beta}_i^R) d\underline{\beta}_i^R \quad (5.6)$$

Collecting all the contributions from the individual pavement sections, the log-likelihood is obtained as,

$$\ln L = \sum_{i=1}^n \ln \left\{ \int_{\underline{\beta}_i^R} \prod_{t=1}^{T_i} g(y_{it}, \underline{X}_{it}, \underline{\beta}_i^R, \underline{\beta}_i^F, \theta) f(\underline{\beta}_i^R) d\underline{\beta}_i^R \right\} \quad (5.7)$$

The second step is concerned with maximizing the log-likelihood function. It is shown that Equation (5.7) involves a high-dimensional integration in that  $\underline{\beta}_i^R$  being a vector includes randomness in all of its elements. There is no close-form solution to the integral.

Mathematically, the integral is equivalent to the expectation of  $\prod_{i=1}^{T_i} g(\bullet)$ , i.e.

$$E \left( \prod_{i=1}^{T_i} g(\bullet) \right),$$

which can be approximated by the simulated mean. Therefore, replacing

the integral by the simulated mean, the simulated log-likelihood is obtained as follows,

$$\ln L_S = \sum_{i=1}^n \ln \left\{ \frac{1}{M} \sum_{m=1}^M \prod_{t=1}^{T_i} g(y_{it}, \underline{X}_{it}, \underline{\beta}_i^{R,m}, \underline{\beta}_i^F) \right\} \quad (5.8)$$

Where,  $\underline{\beta}_i^{R,m}$  is the  $m$ th draw from  $f(\underline{\beta}_i^R)$ , totally  $M$  draws are adopted to simulate the integral in Equation (5.8).

The simulated log-likelihood  $\ln L_S$  is then maximized through iteration (see Equation (5.2)) to arrive at the parameter estimates and their asymptotic standard errors. It has been shown that under mild regularity conditions, the MSL estimator is consistent and asymptotically efficient and normal (Hajivassiliou and Rudd, 1994). In practice, the

estimator is inconsistent and biased because the log and the integral do not commute for the simulator due to Jensen's Inequality (Gourieroux and Monfort, 1996). The bias, however, decreases with the increase of number of simulation replications. Usually a moderate size of the number is sufficient to reduce the bias to an acceptable small level (Gourieroux and Monfort, 1996).

Since the MSL approach estimates the distributions of random coefficients based on the population, these estimates are hereafter referred to as population-level parameter results. By applying MSL with a relatively large number of simulation replications, 300 in this case, the parameter estimation results, means and *t*-statistics are obtained, as shown in Table 5.1. In addition, the correlation matrices of parameters are presented in Appendix B. The detailed explanation and implication of these estimation results are discussed in the subsequent text following the next section on the methodology for deriving individual-level parameters.

**Table 5.1 Population Level Parameter Estimation Results**

Model Alternatives		Pooled		RE		RP	
Variables	Parameters	Estimate	<i>t</i> -stat	Estimate	<i>t</i> -stat	Estimate	<i>t</i> -stat
Initial IRI	$\beta_0$	0.650	43.55	0.652	24.15	0.618	40.12
	$\delta_0$	-	-	0.153	8.15	0.114	11.58
Subgrade	$\beta_1$	-0.778	-3.52	-1.478	-6.09	-1.265	-4.21
Surface Layer	$\beta_2$	-0.179	-9.18	-0.109	-5.25	-0.144	-4.62
	$\delta_2$	-	-	-	-	0.022	14.29
AC120/150	$\beta_3$	0.339	16.86	0.490	9.43	0.246	5.38
35Blow	$\beta_4$	0.043	2.17	0.073	1.93	0.022	0.79
50Blow	$\beta_5$	-0.043	-1.98	0.058	1.40	0.021	0.66
75Blow	$\beta_6$	0.124	6.31	0.169	4.66	0.208	3.46
Base Layer	$\beta_7$	-0.049	-14.80	-0.031	-8.09	-0.042	-7.96
	$\delta_7$	-	-	-	-	0.008	8.19
Subbase Layer	$\beta_8$	-0.031	-13.49	-0.022	-8.45	-0.030	-9.68
	$\delta_8$	-	-	-	-	0.003	5.13
PL Curvature	$\beta_9$	1.741	34.60	1.712	40.82	1.760	54.80
DL Curvature Dummy	$\beta_{10}$	0.358	16.28	0.338	14.08	0.271	13.65
Frost Heave	$\beta_{11}$	0.074	3.33	0.110	5.85	0.115	7.91
Maintenance	$\beta_{12}$	-0.161	-5.77	-0.151	-5.81	-0.221	-9.61
Log-likelihood at Convergence		18.6		205.5		407.0	
Number of Observations		1027					

### 5.3 Individual-Level parameters

To this point, a deterioration model allowing for parameter heterogeneity and the corresponding estimation approach have been discussed. The estimated parameters (Table 5.1) provide a general view of pavement behavior at a population level.

Furthermore, it leads to another issue of interest: where does the parameter for a specific pavement section or structure lie in the distribution? For example, pavements with

different structures can possess different structural coefficients, which can differ from the mean-level values. In this sense, compared with the population-level parameters, the subpopulation level parameters are more informative and relevant to describe the behavior of a specific pavement section. The individual-specific parameters can be obtained through simulation in conjunction with the application of the Bayesian theorem (Greene, 2004).

Denote  $\underline{\beta}^F$  to be a vector including the fixed parameter estimates across the population (all pavement sections). Denote  $\underline{\beta}^R$  to be a vector including the random parameter estimates across the population. The means and standard deviations are available through MSL in the previous section (Table 5.1). These estimates are used as the prior for the subpopulation of interest (e.g. a pavement section  $i$ ). The mean of parameters for a subpopulation data set can be derived as:

$$E(\underline{\beta}_i^R) = E(\underline{\beta}^R | \underline{y}_i, \underline{X}_i) = \int_{\underline{\beta}^R} \underline{\beta}^R f(\underline{\beta}^R | \underline{y}_i, \underline{X}_i) d\underline{\beta}^R \quad (5.9)$$

The conditional density function  $f(\underline{\beta}^R | \underline{y}_i, \underline{X}_i)$  is unknown (where  $\underline{y}_i$  and  $\underline{X}_i$  are the time series observations of riding quality and explanatory variables at section  $i$ ). Apply the Bayesian theorem,

$$f(\underline{\beta}^R | \underline{y}_i, \underline{X}_i) = \frac{f(\underline{y}_i | \underline{\beta}^R, \underline{X}_i) f(\underline{\beta}^R)}{\int_{\underline{\beta}^R} f(\underline{y}_i | \underline{\beta}^R, \underline{X}_i) f(\underline{\beta}^R) d\underline{\beta}^R} \quad (5.10)$$

It is implied that  $f(\underline{y}_i | \underline{\beta}^R, \underline{X}_i)$  is the conditional density function by revisiting Equation (5.4).  $f(\underline{\beta}^R)$  is joint normal distribution with the parameters obtained through MSL. The

estimated mean  $E(\underline{\beta}_i^R)$  is then obtained by replacing Equation (5.10) into (5.9). Again, it is shown that there is no close-form solution for the high-dimension integration. Thus, the Monte Carlo simulation is employed to approximate the integral (Greene, 2004). As a result, the simulation estimator  $E(\underline{\beta}_i^R)$  is,

$$\hat{E}(\underline{\beta}_i^R) = \frac{\frac{1}{M} \sum_{m=1}^M \prod_{t=1}^{T_i} \underline{\beta}^{R,m} g(y_{it}, \underline{X}_{it}, \underline{\beta}^{R,m}, \underline{\beta}^F)}{\frac{1}{M} \sum_{m=1}^M \prod_{t=1}^{T_i} g(y_{it}, \underline{X}_{it}, \underline{\beta}^{R,m}, \underline{\beta}^F)} \quad (5.11)$$

Where,  $M$  is the total number of draws in the simulation;  $m$  represents the  $m$ th draw.

With the similar approach, the simulated second moment estimator for  $\underline{\beta}_i^R$  is,

$$\hat{E}((\underline{\beta}_i^R)^2) = \frac{\frac{1}{M} \sum_{m=1}^M \prod_{t=1}^{T_i} (\underline{\beta}^{R,m})^2 g(y_{it}, \underline{X}_{it}, \underline{\beta}^{R,m}, \underline{\beta}^F)}{\frac{1}{M} \sum_{m=1}^M \prod_{t=1}^{T_i} g(y_{it}, \underline{X}_{it}, \underline{\beta}^{R,m}, \underline{\beta}^F)} \quad (5.12)$$

Hence, the estimated standard deviation for  $\underline{\beta}_i^R$  can be obtained as follows,

$$\hat{S}(\underline{\beta}_i^R) = \sqrt{\hat{E}((\underline{\beta}_i^R)^2) - (\hat{E}(\underline{\beta}_i^R))^2} \quad (5.13)$$

By applying the above procedure, the individual-level parameter estimates are presented in Table 5.2.

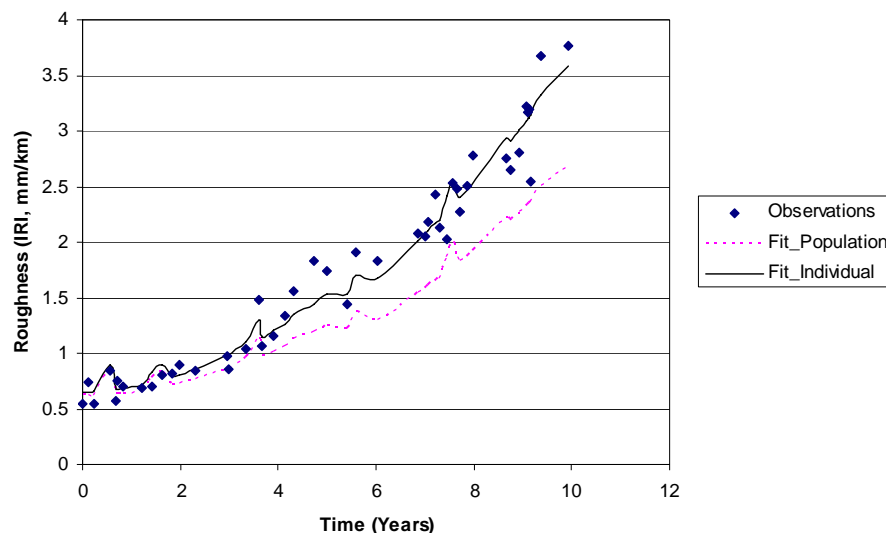
**Table 5.2 Individual-Level Parameters**

Section		Initial IRI		Surface Layer Coef.		Base Layer Coef.		Subbase Layer Coef.	
		$\beta_{0i}$		$\beta_{2i}$		$\beta_{7i}$		$\beta_{8i}$	
		Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	Cell1 DL <sup>†</sup>	0.603	0.038	-0.150	0.008	-0.044	0.007	-0.030	0.002
2	Cell2 DL	0.632	0.039	-0.123	0.009	-0.040	0.006	-0.028	0.002
3	Cell3 DL	0.827	0.047	-0.126	0.012	-0.039	0.006	-0.031	0.002
4	Cell4 DL	0.898	0.014	-0.149	0.001	-0.044	0.008	-0.029	0.002
5	Cell14 DL	0.529	0.027	-0.109	0.002	-0.036	0.011	-0.029	0.001
6	Cell15 DL	0.537	0.030	-0.142	0.003	-0.042	0.006	-0.030	0.002
7	Cell16 DL	0.525	0.029	-0.114	0.005	-0.042	0.007	-0.028	0.001
8	Cell17 DL	0.655	0.032	-0.119	0.005	-0.038	0.004	-0.026	0.001
9	Cell18 DL	0.594	0.035	-0.108	0.010	-0.032	0.005	-0.028	0.003
10	Cell19 DL	0.615	0.037	-0.127	0.007	-0.038	0.007	-0.027	0.002
11	Cell20 DL	0.449	0.039	-0.174	0.009	-0.047	0.011	-0.030	0.001
12	Cell21 DL	0.514	0.039	-0.154	0.011	-0.048	0.004	-0.030	0.003
13	Cell22 DL	0.575	0.042	-0.119	0.010	-0.038	0.007	-0.029	0.003
14	Cell23 DL	0.795	0.029	-0.159	0.005	-0.047	0.005	-0.031	0.003
15	Cell1 PL <sup>†</sup>	0.657	0.029	-0.144	0.009	-0.041	0.005	-0.030	0.003
16	Cell2 PL	0.687	0.025	-0.164	0.007	-0.052	0.006	-0.031	0.001
17	Cell3 PL	0.798	0.018	-0.142	0.010	-0.044	0.002	-0.030	0.003
18	Cell4 PL	0.879	0.047	-0.176	0.006	-0.045	0.005	-0.033	0.001
19	Cell14 PL	0.587	0.044	-0.113	0.003	-0.029	0.011	-0.029	0.003
20	Cell15 PL	0.640	0.028	-0.136	0.002	-0.038	0.008	-0.029	0.002
21	Cell16 PL	0.524	0.027	-0.134	0.007	-0.038	0.007	-0.029	0.002
22	Cell17 PL	0.574	0.031	-0.122	0.006	-0.039	0.007	-0.029	0.002
23	Cell18 PL	0.690	0.028	-0.120	0.009	-0.035	0.006	-0.030	0.002
24	Cell19 PL	0.528	0.023	-0.099	0.006	-0.037	0.005	-0.026	0.002
25	Cell20 PL	0.592	0.035	-0.178	0.012	-0.047	0.007	-0.032	0.002
26	Cell21 PL	0.581	0.036	-0.160	0.006	-0.049	0.004	-0.030	0.002
27	Cell22 PL	0.656	0.043	-0.146	0.008	-0.044	0.004	-0.029	0.002
28	Cell23 PL	0.803	0.033	-0.169	0.006	-0.051	0.008	-0.030	0.003

<sup>†</sup>: DL denotes driving lane; and PL denotes Passing Lane.

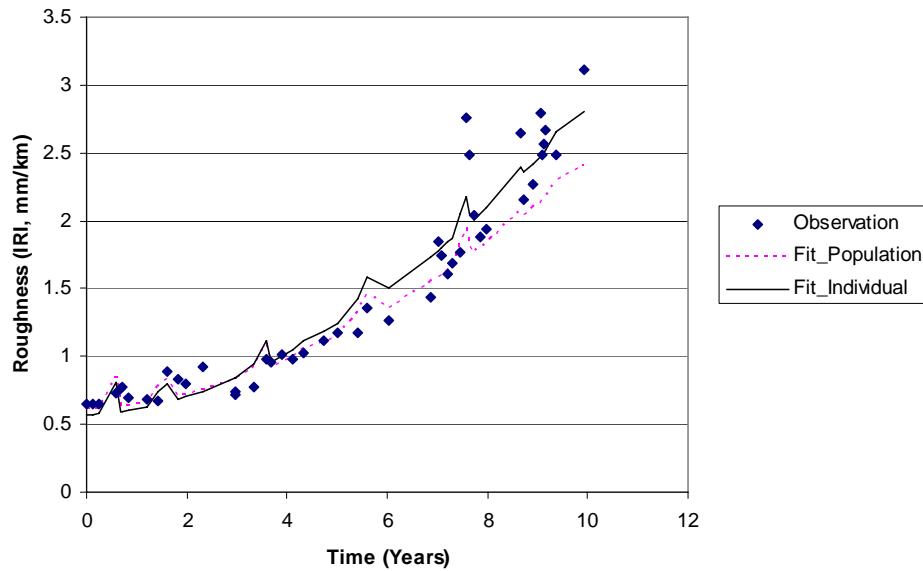
## 5.4 Estimation results and implications

Through implementing MSL, the estimation results are obtained (Table 5.1) and is referred to as random parameter (RP) modeling estimation result. In addition to RP result, the estimation results for the other two alternatives, pooled regression and RE models, are also presented in Table 5.1 for comparison. The pooled regression results are obtained by assuming the individuals across the population share the same parameters. The RE results are obtained by relaxing partial assumption from the pooled regression model – the intercept (i.e. initial roughness) varies across individuals – as has been done in previous studies in Archilla, 2000; Prozzi 2001). It is implied that from pooled regression to RE and to RP models, more heterogeneity in pavement performance is captured through the change of parameter structures toward more flexibility to represent the real-world condition. Appendix 3 present the fitted deterioration curves obtained from population-level parameters and individual-level parameters for both driving and passing lane sections of all cells. As examples, Figures 5.1 and 5.2 illustrate the results for cell No. 17. It is shown that the solid lines fit the observations more precisely than the dotted lines in that it is obtained based on its site-specific parameters.



**Figure 5.1 Example of Performance Fit by Two Levels of Parameters (DL, Cell 17)**





**Figure 5.2 Example of Performance Fit by Two Levels of Parameters (PL, Cell 17)**

The major findings and implications are discussed next.

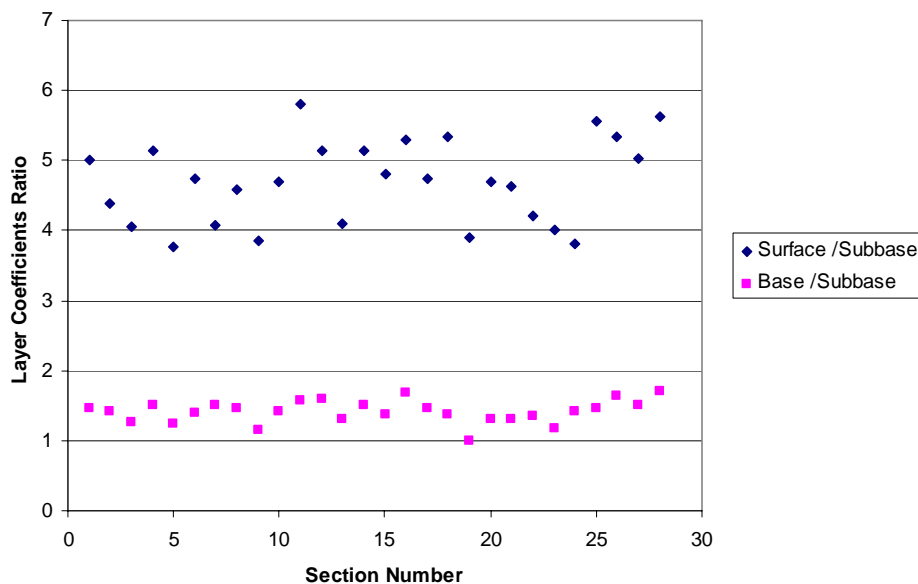
- Concerning the performance heterogeneity, it is shown in the RP estimation results that all the  $t$ -statistics for the four assumed-random parameters are significant at a 5% significance level (throughout this dissertation, 95% confidence interval are applied). In addition, the existence of heterogeneity can be tested through the likelihood ratio (LR) test based on the restricted and unrestricted specifications. The statistic applied for the test is  $-2(\ln L_R - \ln L_U)$ , where  $\ln L_R$  and  $\ln L_U$  are log-likelihood values at convergence for restricted and unrestricted models. It follows  $\chi^2$  distribution with  $n$  degrees of freedom, which equals the number of constraints. First, between the pooled regression and the RE model, the hypothesis of homogeneity of the intercept (in the former one) is evidently rejected since the calculated likelihood ratio statistic, 373.8, is significantly larger than the critical value  $\chi^2_1$  at 95% level. In the same way, the hypothesis of only addressing the unobserved heterogeneity through the intercept in the RE model instead of more relevant parameters in the RP model leads to

rejection of the RE model since the likelihood ratio 403.0 is also significantly larger than  $\chi_3^2$  at 95% level. Therefore, it is shown on the basis of statistical analysis that:

- a. the RP model fits the observations better than the other two alternative specifications;
  - b. the unobserved heterogeneity does exist across the pavement sections; and
  - c. the unobserved heterogeneity should be accounted for not only through the intercept as in RE model but through other relevant slope parameters.
- The relative contribution of unit thickness of each structural layer to deterioration resistance agrees with previous experience and engineering judgment. From the parameters in Table 5.1, the ratios of mean surface layer coefficients to that of base layer are: 3.43, 3.52, and 3.65 in RP, RE, and pooled regression models, respectively. These results make sense from a material property perspective: the largest contribution comes from the surface layer (with asphalt material), followed by base (with courser granular material) and subbase (with finer granular material) layers. The ratios of base layer coefficients to that of subbase layer are close but greater than one, i.e., 1.40, 1.41, and 1.58 in RP, RE, and pooled regression models, respectively. Furthermore, it is implied that compared with RP specification, given the same base layer, the other two overestimate the surface layer's capacity to resist deterioration, while given the same subbase layer, the contribution by the base layer is also overestimated. These results can be helpful in the pursuit of optimal pavement structural design.

Another interesting comparison on the implication of layer coefficients can be carried out between the underlying estimation results and that which is suggested in the 1993 AASHTO Design Guide. Setting the subbase layer coefficient as reference in each model, their individual ratios among three layer coefficients are: 4.00/1.27/1.00, 4.80/1.40/1.00, 4.95/1.41/1.00, and 5.77/1.58/1.00 for AASHTO, RP, RE, and pooled regression cases, respectively.

In addition to the previous discussion based on population-level parameters, the layer coefficients for each individual section are referred to in Table 5.2. The layer coefficients ratios with subbase layers as reference are illustrated in Figure 5.3. It is indicated that all sections are consistent with established knowledge regarding the relative contribution of unit thickness to deterioration resistance. Finally, it is important to point out that the underlying layer coefficients discussion is based on reference case in the model, i.e., pavements with AC 20 (PG 64-22) asphalt binder and Superpave Gyratory mix design. The coefficient for surface layer may vary should other alternatives of asphalt binder and design approach be considered. This issue will be discussed in detail in the next paragraphs.



**Figure 5.3 Layer Coefficients Ratios across All Sections**

- Regarding asphalt binder, in all three models, the uniformly positive signs together with significant *t*-statistics of the parameters for explanatory variable PEN 120/150 (PG 58-28) suggest that the asphalt mix with this type of binder demonstrates a higher capability of deterioration resistance than that with AC 20 (PG 64-22). Considering

the fact that the Mn/Road test is located in the cold region of the country, this result matches the material properties. Specifically, asphalt binder of PEN 120/150 (PG 58-28) exhibits better properties at low temperature conditions than AC 20 (PG 64-22). Furthermore, it is noted that an exponential form is imposed on the underlying explanatory variable and its parameter. The difference of the effect of PEN 120/150 (PG 58-28) binder from AC 20 (PG 64-22) on the ability of the unit surface layer to resist deterioration can be quantified by a “shift factor” in an exponential term of the parameter. As a result, the shift factors are 1.29, 1.63, and 1.40 for RP, RE, and pooled regression models, respectively.

- Similarly, the effect of different design approaches can also be quantified. It is shown that the *t*-statistics are all insignificant except in one case in the pooled regression model, for the two lower compaction levels of Marshall design, 35 and 50 blows in all three models. On the other hand, all statistics are significant under 75 blows of design. The positive signs indicate that, compared to Superpave design with gyratory compaction, the mix under Marshall design with 75 blows performs better. Recall that 75 blows Marshall design has been widely used in standard asphalt mix design based on engineering experience, targeted to design dense-grade asphalt mix with optimum binder content. Such widespread use, as implied in the traditional pavement design, places significant importance on finding a solution for premature pavement cracking failure. In other words, it is reasonable to consider that asphalt mix by 75 blows Marshall design addresses the cracking issue adequately, and may lead to improved pavement performance in terms of longitudinal profile or roughness due to its positive correlation with cracking. Superpave mixes, on the other hand, are expected to perform better in terms of transverse profile, rutting, but not necessarily in terms of cracking. Quantitatively, by taking exponential on  $\beta_6$ , the shift factors by 75 blows Marshall design from Superpave design are 1.23, 1.18, and 1.13 in RP, RE, and pooled regression models, respectively. Note that mix design and asphalt properties

are not directly addressed in the current AASHTO Design Guide (1993) equations. Hence, both shift factors could be used to improve current pavement design.

- Concerning deterioration curvatures, it is indicated that both values, for passing and driving lanes, are larger than one, which is consistent with the existing findings (e.g. Paterson, 1987), suggesting that pavement deterioration in terms of IRI increases in a higher order than the development of time. In addition, the parameter for curvature dummy variable,  $\beta_{10}$  being significant suggests that given other variables being equal, pavement lanes subject to different traffic levels experience different rates of deterioration. Furthermore, the positive sign of  $\beta_{10}$  implies that pavement sections with heavier traffic deteriorate faster than those with lighter traffic. The deterioration curvatures for passing lanes are 1.760, 1.712, and 1.741 while they are 2.031, 2.050, and 2.099 for driving lanes in RP, RE, and pooled regression models. The model focuses on two-lane highways, which accounts for the largest percentage of the nation's highway system. However, the same approach to differentiating varying lanes can be applied to other cases of multi-lane highways by simply adding more dummy variables.
- Regarding environment, it is indicated that frost-heave has a statistically significant impact on pavement performance. For those sections with frost-susceptible granular subbase, 1mm frost heave generates 0.115, 0.110, and 0.074 higher IRI according to the RP, RE, and pooled regression models, respectively. The effect of other environmental factors such as freeze-thawing is not clearly observed in the Mn/Road project due to load restrictions during this weather period (February through March).
- The  $t$ -statistic of the maintenance parameter manifests that the effect of maintenance activity is significant. The negative sign indicates that after maintenance, IRI dropped, suggesting that pavement conditions improved.

- It is important to point out that under the condition of separately employing time-series data of each section, most of the parameters in the established model (Equation (4.4)), are unidentifiable. For example, the layer coefficients could not be identified because there is no thickness variation for one particular section. However, this problem can be solved. Following the Bayesian approach discussed in the previous sections, the section-specific parameters are obtained. Table 5.2 shows the estimated parameters, mean and standard deviation for each individual section. It is shown that due to inevitable heterogeneity, the initial performance differs across the sections. Similarly, as is depicted in Figure 5.3, the ratios of layer coefficients between surface and subbase layers and between base and subbase layers for all sections are also found to vary across sections. To take this research a step further, the parameters for any group of sections can be obtained by applying the same approach as previously discussed to accommodate different levels of management required by the agencies. For instance, sections with similar pavement structure can be grouped to produce the parameters that reveal the characteristics of that particular structure.
- Regarding spatial correlation, the  $t$ -statistic for  $\lambda$  (see Equation 4.7), 4.68 suggests that pavement performance spatial correlation exists on the Mn/Road project Mainline sections. Moreover, the positive sign of the estimate indicates that there is a similarity in the pavement performance of sections that are closer in distance to each other. This finding is consistent with engineering judgment since sections in close proximity to each other are more likely to share similar characteristics such as construction quality and material properties.

## **5.5. Policy implications**

From an operational perspective, pavement management can be divided into network and project levels (Haas and Hudson, 1994). Both levels rely heavily on deterioration models in planning maintenance and rehabilitation activities. In this sense, models with

population-level and subpopulation or individual-level parameters can accommodate both levels of management. The former provides parameters representing a “general map” of pavement deterioration across the sections in the entire network of interest, while the latter focuses on the specific deterioration characteristic of one individual section for a given project.

## **Chapter 6: Incorporating Deterioration Model in Pavement Management**

One of the major tasks of managing transportation infrastructure is to optimize the maintenance schedule and plan appropriate activities. To realize this objective, the deterioration model plays a significant role since it predicts facility performance, which serves as a critical input of the optimization system. In this chapter, the deterioration model based on in-service pavement sections developed in Chapters 4 and 5 is incorporated into a resurfacing optimization system. The sensitivity of optimal policy solution to different accuracy levels of deterioration models is highlighted.

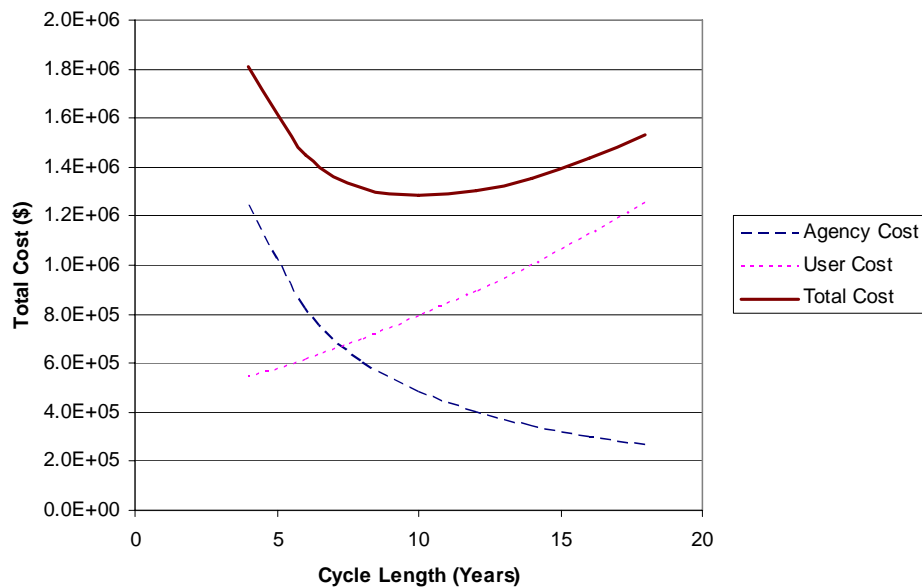
### **6.1 Introduction on optimal pavement resurfacing problem**

One aspect of critical economic consequence in managing pavement systems is the determination of a resurfacing policy, which includes frequency and intensity. Frequency is associated with the temporal interval between two consecutive resurfacing activities. Intensity generally refers to the thickness of a resurfacing layer.

Generally, pavement performance can be depicted with a saw-tooth type trajectory curve as the structure deteriorates and receives resurfacings (Tsunokawa and Schofer, 1994). An optimal problem can thus be formulated to minimize total social cost. The total social cost is composed of two parts: agency cost and user cost. From an engineering perspective, agency costs decrease as the resurfacing life cycle (the amount of time between resurfacing occurrences) increases; while user costs (including damage to vehicles and goods, delays, etc.) increases with a decrease in the frequency of resurfacing because the user is exposed to a pavement with more deterioration in riding quality. Thus, on the total cost curve there is possibly an optimal point, where the total cost is minimized. The relationships between agency and user costs and resurfacing life cycle length are illustrated in a schematic depiction in Figure 6.1. The optimal resurfacing



problem is targeted to find the optimal point. Tsunokawa and Schofer formulated an optimal control model to solve this problem (1994). In order to find a solution to this problem, a smoothed trend curve was applied to approximate the saw-tooth trajectory performance curve (Tsunokawa and Schofer, 1994). Based on their work, without requiring trend curve approximation, a steady-state solution was proposed under the condition of continuous pavement state and continuous time in Li and Madanat (2002). Furthermore, the solution for one facility was extended to multiple facilities in Ouyang and Madanat (2004).



**Figure 6.1 Schematic Relationships between Costs and Cycle Length**

Typically, user cost can be expressed as a function of the riding quality of the pavement in terms of roughness, denoted as  $C(s(t))$ , where  $s(t)$  is the roughness at time point  $t$ ; and the agency cost can be expressed as a function of maintenance intensity,  $M(w_n)$ , where  $w_n$  is the intensity, in this case overlay thickness. Mathematically, the total cost in terms of net present value for a pavement facility in an infinite time horizon can be described as (Tsunokawa and Schofer, 1994),

$$J = \sum_{n=1}^{\infty} \left\{ \int_{t_{n-1}}^{t_n} C(s(t))e^{-rt} dt + M(w_n)e^{-rt_n} \right\} \quad (6.1)$$

Subject to:

- 1) Deterioration function;
- 2) User cost function;
- 3) Resurfacing effectiveness; and
- 4) Agency cost functions;

Where,  $r$  is the discount rate;  $e^{-rt}$  represents the continuous discount factor;  $n$  designates the sequence of life cycles. In the context of optimal resurfacing policy, Equation (6.1) serves as the objective function, which is to be minimized under the above-mentioned constraints. These constraints are discussed next.

### 6.1.1 Pavement deterioration model

Initially, in the study by Tsunokawa and Schofer (1994) based on the field studies in Brazil and other countries, the following exponential function was used to approximate the roughness development along time,

$$s(t) = s(t_0) \exp(\beta(t - t_0)) \quad \text{for } t \geq t_0 \quad (6.2a)$$

or

$$s(t + 1) = s(t) \exp(\beta) \quad t = 0,1,2,\dots \quad (6.2b)$$

Where,  $\beta$  is a constant.

The deterioration model implies that pavement deterioration features Markovian properties because the next step condition is only dependent on the current condition. Although this underlying function eases mathematical programming, as pointed out by Ouyang and Madanat (2004), it is not realistic from a pavement viewpoint. A

counterexample is, all  $s(t)$  will be zero should  $s(t_0) = 0$ . To correct this deficiency, Ouyang and Madanat adopted an improved model proposed in Paterson (1987),

$$s(t) = (s(t_0) + f(t - t_0))\exp(\beta(t - t_0)) \quad t \geq t_0 \quad (6.3)$$

Where,  $f(t - t_0)$  is a function of pavement structure, environment, and traffic.

It is shown that the next step pavement condition is not only dependent on the current condition, but also on its history since  $t_0$ . This model improvement, however, makes the problem more complex. To accommodate the solution process, a constant  $\bar{f}$  is selected to approximate the function  $f(t - t_0)$  in Ouyang and Madanat (2004).

Although the improved model in Equation (6.3) is more realistic than the previous one in Equation (6.2), it still lacks of key elements from the viewpoint of deterioration modeling. First, the model specification is vague in terms of physical deterioration principle. For example, the term  $s(t_0) + f(t - t_0)$  per se is functionally capable of capturing the deterioration process, i.e. pavement performance at a given time  $t$  is equal to initial performance plus deteriorated performance. The exponential term is used to improve data fit but bears no explicit physical meaning. Second, it was manifested that the model parameters were not soundly estimated (Paterson, 1987). As a matter of fact, it was pointed out by Paterson that the initial model estimation results were unsatisfactory. In order to correct this problem and also improve goodness-of-fit, the model was re-estimated with “expanded data”, which were processed by adding weights to different parts of the data. This, however, leads to two further arguments: 1) higher goodness-of-fit per se does not reflect better model; and 2) expanded data do not necessarily reflect the reality. Furthermore, the model was estimated ignoring some statistical aspects such as unobserved heterogeneity.

Apart from the deterioration model per se, the existing work on optimal resurfacing generally simplified the models, e.g., by approximation, to accommodate problem solution. This may introduce errors to a certain extent and make the results less accurate.

Therefore, it is of critical importance that a more accurate and realistic deterioration model be incorporated in the optimal resurfacing policy solution process. The trend curve established in the deterioration model in this study is used as the performance function,

$$s(t) = s(0) + bt^c \quad (6.4a)$$

$$s(0) = s_0 = \beta_0 \quad (6.4b)$$

$$b = \exp \left\{ \begin{array}{l} \beta_1 + \exp(\beta_3 AC120/150 + \beta_4 35Blow + \beta_5 50Blow + \beta_6 75Blow) \beta_2 H_1 \\ + \beta_7 H_2 + \beta_8 H_3 \end{array} \right\} \quad (6.4c)$$

$$c = \beta_9 + 0.339\beta_{10}DL \quad (6.4d)$$

There are two sets of parameters  $\beta_i, i = 0, \dots, 10$  for each pavement section. One is the population-level, which is shared by all sections. The other is individual-level, which is particular for the section under consideration. The two sets of parameters were presented in Chapter 5.

### 6.1.2 User cost

The user cost is composed of two components: travel delay and vehicle operation cost. As is customary, travel delay cost is assumed constant and vehicle operation cost is discussed further in a resurfacing problem (Ouyang and Madanat, 2004). According to Tsunokawa and Schofer (1994), vehicle operation cost can be expressed as a linear function of roughness  $s(t)$ , in QI, (1 QI = 13 IRI, Paterson (1987)),

$$C(s(t)) = c_1 s(t) + c_2 \quad (6.5)$$

Where,  $c_1 = 1000$ .  $c_2$ , also a constant, is omitted since it does not affect optimization result.

### 6.1.3 Resurfacing effectiveness

Assume that at the  $n$ th resurfacing pavement the roughness is  $s_{2n}$  and  $s_{1n}$  immediately before and after the resurfacing activity, respectively. The roughness reduction due to resurfacing can be expressed as a function of resurfacing thickness and the roughness immediately before resurfacing (Tsunokawa and Schofer, 1994) as follows,

$$s_{2n} - s_{1n} = g_1 \sqrt{w_n} + g_2 s_{2n} + g_3 \quad (6.6)$$

Where,  $g_1 = 5.0$ ,  $g_2 = 0.78$ ,  $g_3 = -66.0$

### 6.1.4 Agency cost

Agency costs can be expressed as a linear function of resurfacing thickness. According to Tsunokawa and Schofer (1994), the agency cost is expressed as,

$$M(w_n) = m_1 w_n + m_2 \quad (6.7)$$

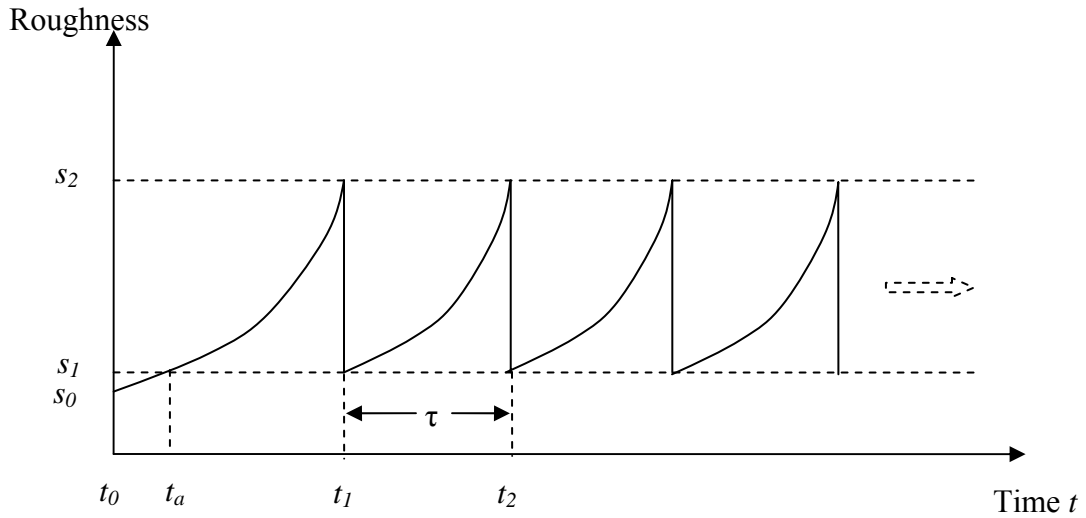
Where,  $m_1 = 3,000$ , and  $m_2 = 150,000$ .

## 6.2 A Steady-state optimal pavement resurfacing problem

The irregular saw-tooth pavement performance curve can be converted into a steady-state problem, as shown in Figure 6.2 (Li and Madanat, 2002). After the pavement enters the

steady-state, i.e. after first resurfacing in the figure, the same action will occur whenever the pavement reaches the same state. Two actions could be involved: doing nothing (pavement continuing deterioration) and resurfacing. After each resurfacing action, pavement roughness drops to the same level,  $s_1$ . It is implied that every resurfacing has the same thickness under steady-state condition. In a steady-state optimization problem,  $s_1$  is preset to denote the agency's requirement of pavement serviceability after resurfacing.

In addition, as illustrated in Figure 6.2, an infinite planning horizon is adopted so that the difficulty in calculating the salvage value is circumvented. Under steady-state condition, if an optimal standard is obtained for the first cycle of resurfacing, it is also optimal for all resurfacings that follow. Therefore, the task is to develop a standard, upper threshold of roughness, or optimal cycle time,  $\tau$ , so that the total cost is minimized. The basic approach herein followed was proposed in Li and Madanat (2002). The major difference in the underlying study comes from the deterioration model. Instead of using a simplified approximation model, a realistic deterioration model developed in this study is employed.



**Figure 6.2 System Enters Steady-State at the Time of the First Resurfacing (Li and Madanat, 2002)**

The net present value of total cost during the first cycle after the first resurfacing, including user cost between  $t_1$  and  $t_2$  and the second resurfacing, evaluated at  $t_1$ , can be expressed as,

$$j_1 = j_{1A} + j_{1U} \quad (6.8)$$

Where,

$j_{1A}$  is the agency cost; and

$j_{1U}$  is the user cost, to be discussed subsequently.

For the agency cost, first solve for  $w$ ,

$$w = \left( \frac{s_1((1-g_2)e^{f_1\tau} - 1) - g_3}{g_1} \right)^2 \quad (6.9)$$

By replacing into Equation (6.7) and evaluated at  $t_1$ ,

$$j_{1A} = \left( m_1 \left( \frac{s_1((1-g_2)e^{f_1\tau} - 1) - g_3}{g_1} \right)^2 + m_2 \right) e^{-r\tau} \quad (6.10)$$

For the user cost, evaluated at  $t_1$ , replace in the deterioration curve Equation (6.4a) into the integral part of Equation (6.1),

$$j_{1U} = \int_0^\tau c_1 C(s(t+t_a)) e^{-rt} dt \quad (6.11)$$

Where,

$t_a$  is the time roughness reaches a preset level by the highway agency,  $s_1$ . If  $s_1$  is given,  $t_a$  is determined accordingly through the established deterioration model.

After several steps of integral derivation, the following close-form solution is obtained,

$$j_{1U} = c_1 \left( \frac{s_0(1-e^{-r\tau})}{r} + \frac{be^{rt_a}}{r^{c+1}} \Gamma(c+1) (F(r(t_a+\tau), c+1) - F(rt_a, c+1)) \right) \quad (6.12)$$

Where,

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

$$F(x; \alpha) = \int_0^x \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)} dy \quad x > 0$$



$b, c$ , see Equations 6.4c and d.

Thus, the total cost of all cycles, evaluated at  $t_1$  can be obtained, as

$$J_1 = \frac{\left( c_1 \left( \frac{s_0(1-e^{-r\tau})}{r} + \frac{be^{rt_a}}{r^{c+1}} \Gamma(c+1)(F(r(t_a+\tau), c+1) - F(rt_a, c+1)) \right) + \left( m_1 \left( \frac{s_1((1-g_2)e^{f_1\tau} - 1) - g_3}{g_1} \right)^2 + m_2 \right) e^{-r\tau} \right)}{1 - e^{-r\tau}} \quad (6.13)$$

The next step is to determine the optimal resurfacing policy, cycle time,  $\tau$ , in the above equation by minimizing the total discounted cost  $J_1$ . The results and implications are presented in the next section.

## 6.3 Results and implications

### 6.3.1 A case study of optimal resurfacing policy for one section

First, as a representative, the optimal resurfacing policy for one section, Cell 1, passing lane, is presented. Except for the constants provided in the proceeding discussion on the optimization process, the two remaining constants are discussed next. One is related to the discount rate,  $r$ . A value of 7% is used based on previous studies by Tsunokawa and Schofer (1994) and Li and Madanat (2002). If the other discount rate is used, the same optimization approach applies. The other constant corresponds to the preset roughness,  $s_1$ , denoting riding quality immediately after resurfacing. A series of  $s_1$  values are used to investigate the sensitivity level of the optimal policy. By replacing all relevant constants (in the case study, the individual-level parameters for Cell 1, passing lane, are used) and minimizing Equation (6.13), the optimal policy is obtained for that section, as shown in Table 6.2.

**Table 6.2 Example of Optimal Resurfacing Policy (Cell 1 Passing Lane)**

$s_1$ (QI)	15	20	25	30	35	40
$s_2$ (QI)	92	96	98	100	102	104
$\tau$ (years)	16.5	15.1	13.9	12.8	11.8	10.9
$w$ (mm)	203.1	179.9	156.9	135.0	114.6	95.8
$J_1$ (\$)	811,168	917,378	1,004,469	1,079,773	1,146,731	1,207,448

It is indicated that the optimal cost is closely related to  $s_1$ . With increasing  $s_1$ , the optimal cost increases, suggesting that an economic choice is to reduce pavement roughness to a best possible level immediately after resurfacing. These findings are consistent with those reported in Li and Madanat (2002). In addition to the results for one particular pavement section, a more comprehensive study, involving all sections, is presented in next section.

### **6.3.2 Optimal resurfacing policy among all sections**

Employing the same approach as in the case study just discussed, the optimal policies for all pavement sections can be obtained. The differences in optimal policy among sections due to different inputs associated with their unique conditions, can be explained by considering Equations (6.4). Specifically, the difference originates from  $s(0)$ ,  $b$  and  $c$  in Equations (6.4).  $s(0)$  is the initial roughness (Table 5.2).  $b$  and  $c$  are related to parameters and explanatory variables in the deterioration models (Chapter 5). By replacing the related information,  $b$  and  $c$  for different sections are shown in Table 6.3. In particular, the values from both individual and population-level parameters are provided. The value for  $c$  remains the same for each individual travel lane under both levels of parameter conditions, as is denoted in the deterioration model in Chapter 5.

**Table 6.3 Parameters Used in Deterioration Models in Optimization Problem**

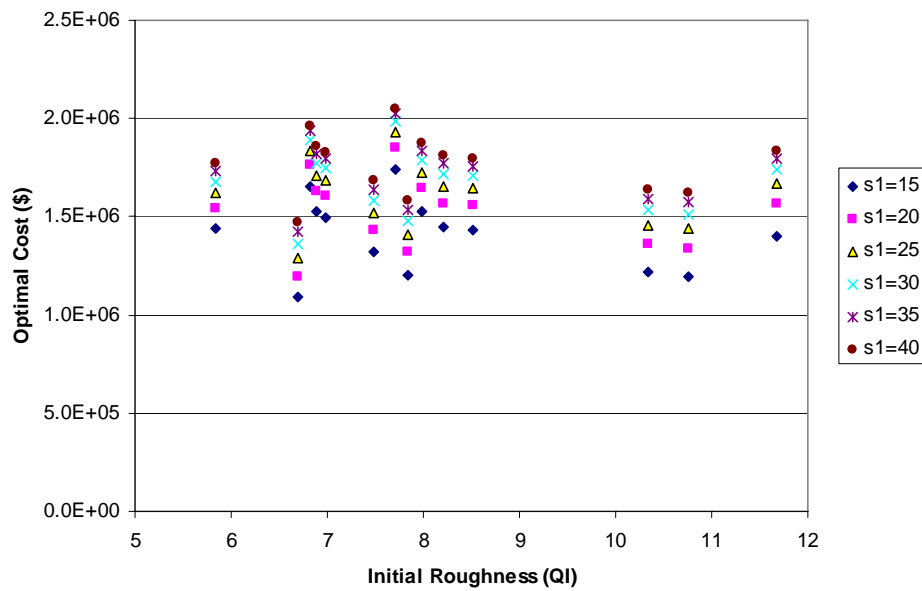
	<b>Parameters</b>				
	<b><math>s(0)</math> (QI)</b>		<b><math>b</math></b>		<b><math>c</math></b>
<b>Section #</b>	<b>Individual</b>	<b>Population</b>	<b>Individual</b>	<b>Population</b>	<b>Both</b>
Cell1 DL <sup>†</sup>	7.84	8.03	0.027	0.029	2.031
Cell2 DL	8.22	8.03	0.043	0.035	2.031
Cell3 DL	10.75	8.03	0.034	0.031	2.031
Cell4 DL	11.67	8.03	0.053	0.057	2.031
Cell14 DL	6.88	8.03	0.045	0.025	2.031
Cell15 DL	6.98	8.03	0.043	0.042	2.031
Cell16 DL	6.83	8.03	0.054	0.041	2.031
Cell17 DL	8.52	8.03	0.043	0.031	2.031
Cell18 DL	7.72	8.03	0.064	0.042	2.031
Cell19 DL	8.00	8.03	0.048	0.040	2.031
Cell20 DL	5.84	8.03	0.036	0.029	2.031
Cell21 DL	6.68	8.03	0.020	0.025	2.031
Cell22 DL	7.48	8.03	0.033	0.023	2.031
Cell23 DL	10.34	8.03	0.034	0.042	2.031
Cell1 PL <sup>†</sup>	8.54	8.03	0.029	0.029	1.760
Cell2 PL	8.93	8.03	0.028	0.035	1.760
Cell3 PL	10.37	8.03	0.031	0.031	1.760
Cell4 PL	11.43	8.03	0.040	0.057	1.760
Cell14 PL	7.63	8.03	0.042	0.025	1.760
Cell15 PL	8.32	8.03	0.047	0.042	1.760
Cell16 PL	6.81	8.03	0.044	0.041	1.760
Cell17 PL	7.46	8.03	0.039	0.031	1.760
Cell18 PL	8.97	8.03	0.055	0.042	1.760
Cell19 PL	6.86	8.03	0.062	0.040	1.760
Cell20 PL	7.70	8.03	0.019	0.029	1.760
Cell21 PL	7.55	8.03	0.018	0.025	1.760
Cell22 PL	8.53	8.03	0.021	0.023	1.760
Cell23 PL	10.44	8.03	0.031	0.042	1.760

<sup>†</sup>: DL denotes Driving Lane; and PL denotes Passing Lane.

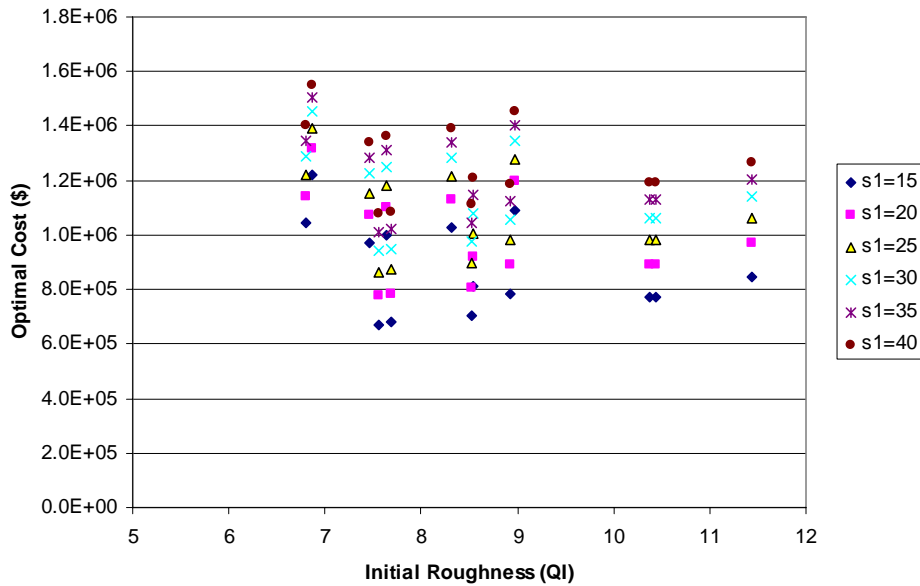
Three issues of interest are highlighted next: the sensitivity of optimal policy to initial roughness (representing construction quality), the deterioration rate  $b$  (mainly incorporating structural and material information), and the deterioration curvature  $c$  (related to traffic and time).

Sensitivity of optimal policy to initial riding quality

First, regarding the sensitivity of optimal cost to initial roughness, Figures 6.3 and 6.4 show their relationship under different preset roughness  $s_1$  scenarios for the driving and passing lanes, respectively. Both figures indicate no pronounced relationship between optimal cost and initial roughness.



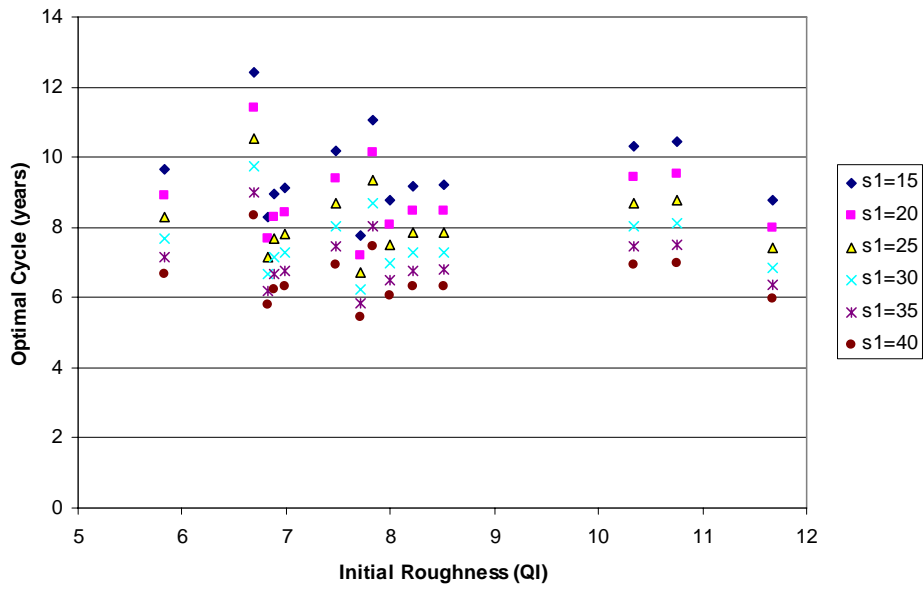
**Figure 6.3 Relationship between Optimal Cost and Initial Roughness for Driving Lane**



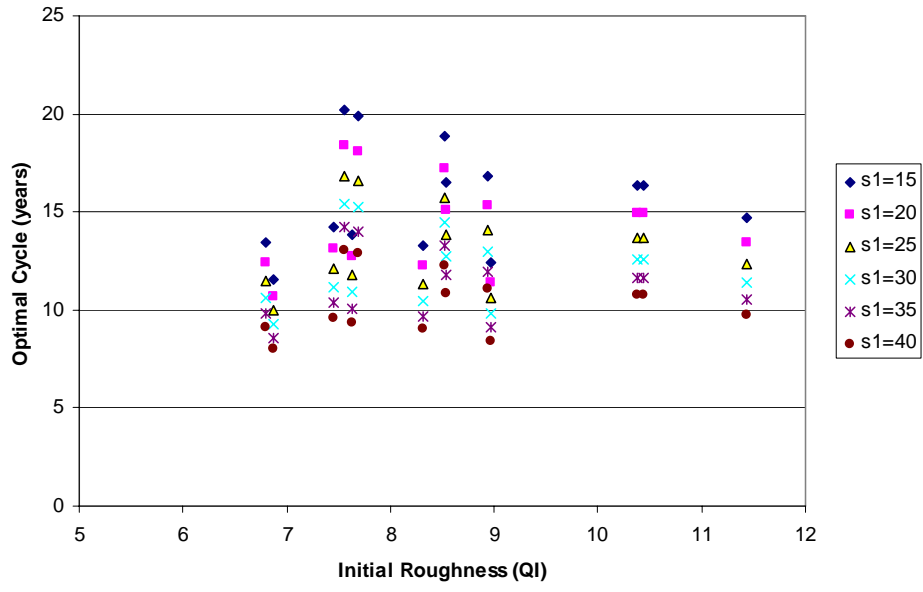
**Figure 6.4 Relationship between Optimal Cost and Initial Roughness for Passing Lane**

Similarly, regarding the sensitivity of optimal resurfacing cycle length to initial roughness, Figures 6.5 and 6.6 show this relationship under different preset roughness scenarios,  $s_1$ , for the driving and passing lanes, respectively. Again, it can be observed in both figures that the optimal resurfacing cycle is not sensitive to the initial roughness.

However, it should be pointed out that the underlying finding of optimal policy being insensitive to initial roughness applies to this particular condition. That is, the initial roughness covers a relatively narrow range, 0.45 to 0.90 IRI (from 5.8 to 11.7 QI). The result may differ if the initial roughness changes over a wider range.



**Figure 6.5 Relationship between Optimal Resurfacing Cycle and Initial Roughness for Driving Lane**

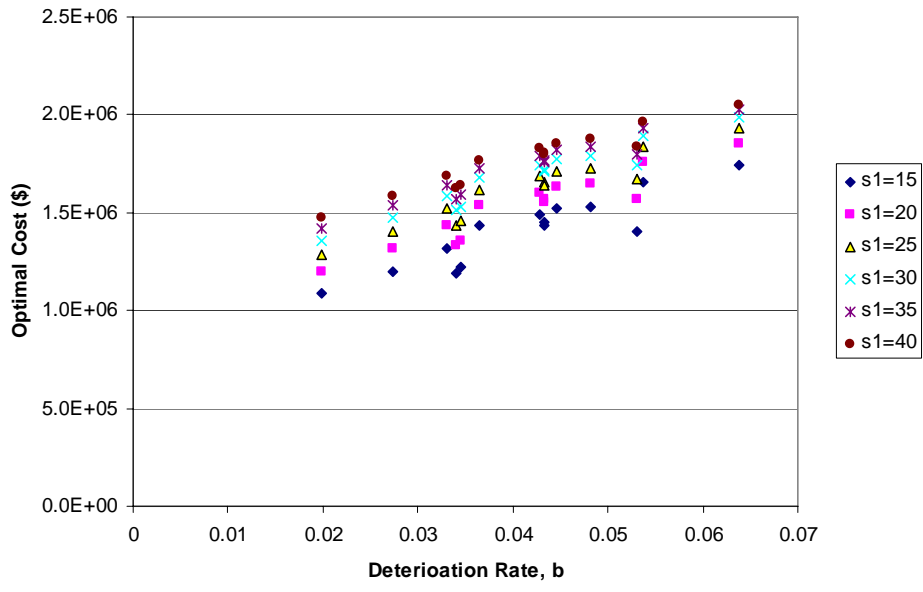


**Figure 6.6 Relationship between Optimal Resurfacing Cycle and Initial Roughness for Passing Lane**

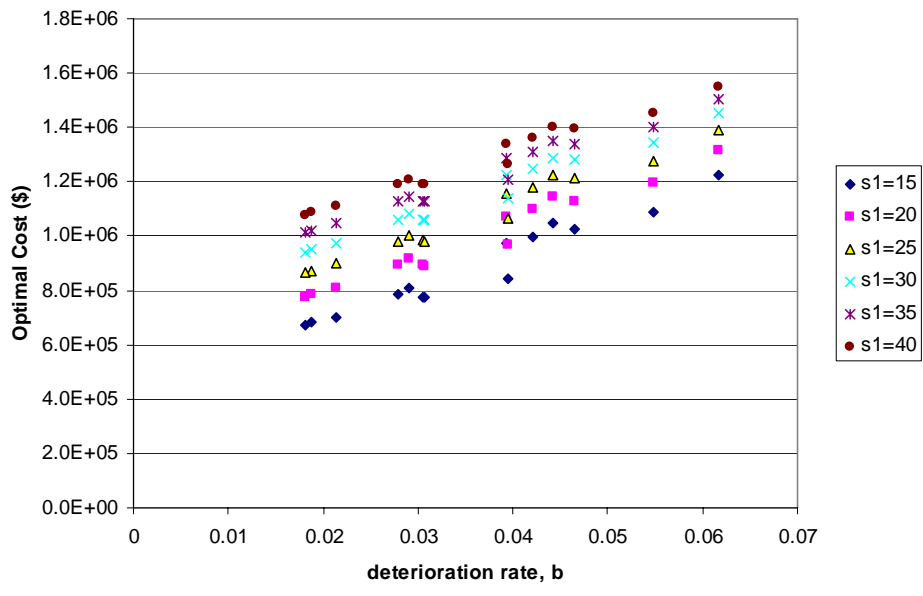
### Sensitivity of Optimal Policy to Deterioration Rate

Figures 6.7 and 6.8 show the sensitivity of optimal cost to deterioration rate,  $b$ , for the driving and passing lanes, respectively. It can be noted that for both lanes, with higher deterioration rates, the optimal cost increases for each present roughness scenario ( $s_1$ ), which makes sense from an engineering standpoint because higher deterioration rates lead to poor pavement condition, increasing total cost. It is also noted that the optimal cost is sensitive to deterioration rate. Take the scenario of given  $s_1 = 15$  QI as an example: for driving lane sections, optimal cost increases by a factor of around 1.6 when the deterioration rate increases by a factor of around 3; for passing lane sections with the same deterioration rate change, the optimal cost increases by a factor of around 1.8. This result implies that the optimal cost is more sensitive to the deterioration rate than the previous study in Ouyang and Madanat (2004), which found that a deterioration rate increase of around five times resulted in an optimal cost increase factor lower than two.

The sensitivity of optimal cycle to deterioration rate is shown in Figures 6.9 and 6.10. With increasing deterioration rate, optimal cycle length decreases for both lanes. Moreover, it is revealed that the optimal cycle is more sensitive for passing lane sections than driving lane sections.

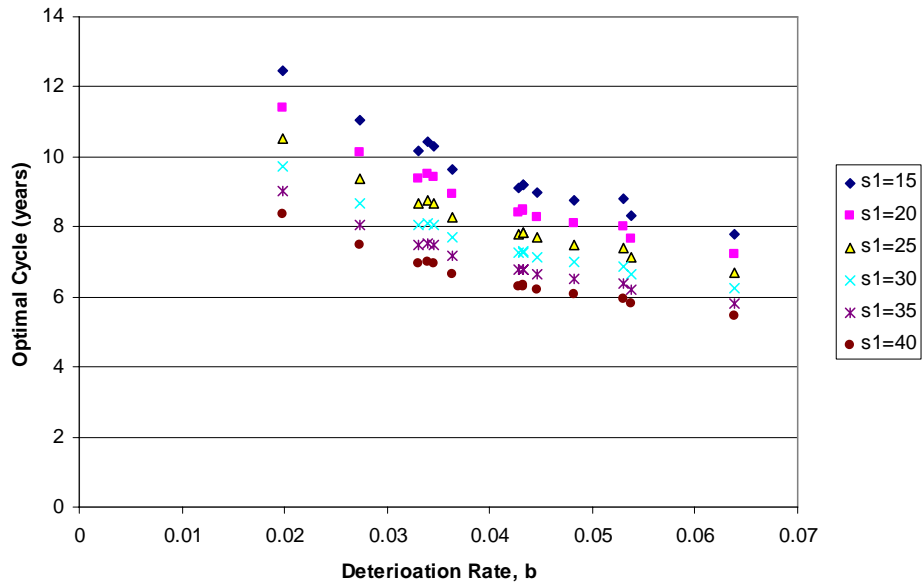


**Figure 6.7 Relationship between Optimal Cost and Deterioration Rate ( $b$ ) for Driving Lane**

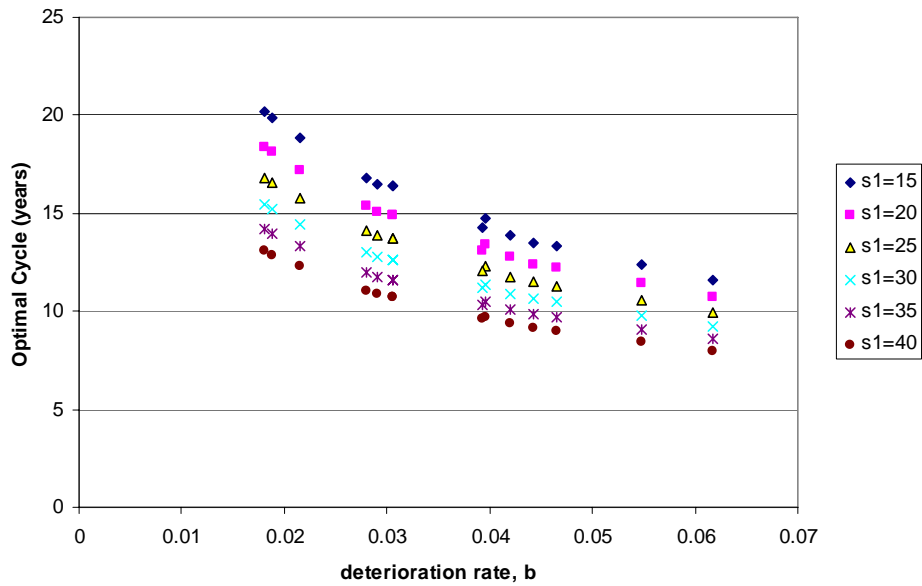


**Figure 6.8 Relationship between Optimal Cost and Deterioration Rate ( $b$ ) for Passing Lane**





**Figure 6.9 Relationship between Optimal Resurfacing Cycle and Deterioration Rate (b) for Driving Lane**



**Figure 6.10 Relationship between Optimal Resurfacing Cycle and Deterioration Rate (b) for Passing Lane**

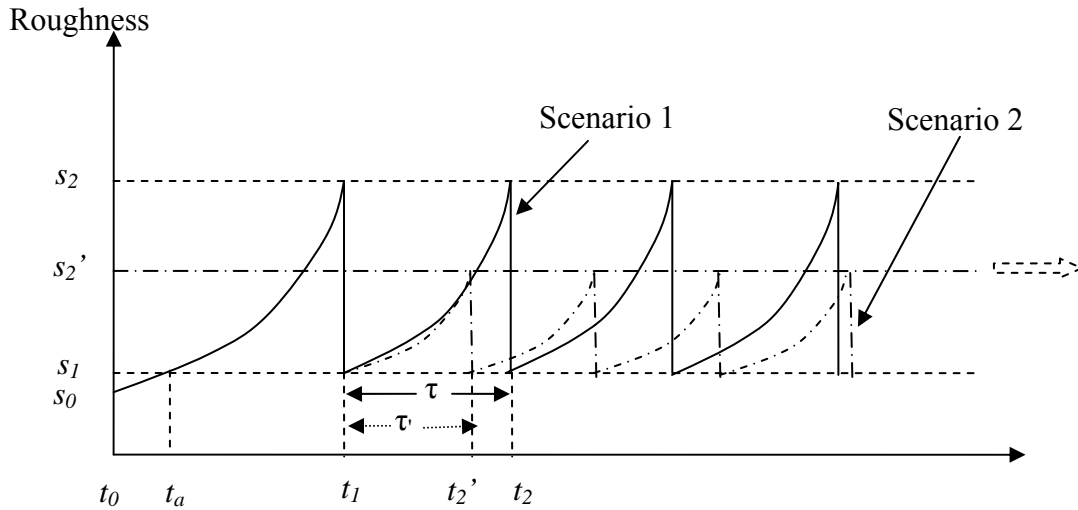
### Sensitivity of Optimal Policy to Deterioration Curvature

It can be seen in the above figures that given the same  $s_1$  and similar  $b$ , the sections on the driving lane have higher cost and shorter resurfacing cycle. The reason is because the driving lane experienced more traffic loading resulting in worse pavement conditions. Notice that the curvature,  $c$ , is 2.031 for driving lane and 1.760 for passing lane. On average, the optimal cost on the driving lane sections is 1.5 times as much as on the passing lane sections, while the optimal cycle is around 60 percent.

### **6.3.3 Effect of unobserved heterogeneity on optimal policy for one section**

It has been established in Chapter 5 that two sets of parameters are available to describe the deterioration for one given pavement section. Due to unobserved heterogeneity, a model with population-level parameters usually does not accurately represent deterioration (see the dotted curves in Figures 5.1 and 5.2 as examples). The performance curve by individual-level parameters fits the observations well due to the fact that its parameters are updated with the information from the section of interest. In other words, the individual-level parameters more closely represent the deterioration for that section. In a pavement resurfacing problem, the two levels of parameters may lead to two different policies. That is, each level of parameter leads to its specific optimal policy, as shown in Figure 6.11. Suppose Scenario 1 is the optimal policy based on individual level parameters, under which the section receives resurfacing with a cycle length of  $\tau$ . The optimal cycle based on population level parameters is  $\tau'$ , which usually differs from  $\tau$ . Since the observations are closely represented by the performance curve based on individual parameters, it is reasonable to use that curve to calculate the real total life cycle cost under a given resurfacing policy. Hence, if the policy under  $\tau$ , Scenario 1 in Figure 6.11, is optimal, the policy under  $\tau'$  Scenario 2 in Figure 6.11, leads to a different total cost, denoted as  $J_1'$  no less than the cost under  $\tau$ , denoted as  $J_1$ . In addition, considering that the total cost is sensitive to deterioration rate  $b$ , it is implied that the

larger difference between the  $b$ 's from two levels of performance curves, the larger deviation of total cost from the optimal value are produced by the Scenario 2, sub-optimal solution.



**Figure 6.11 Two Resurfacing Policy Scenarios for One Given Pavement Section**

Cell 14, driving lane, is adopted for case study to demonstrate the difference due to two levels of parameters. The results and comparison are shown in Table 6.4. It can be seen that difference does exist when applying optimal and sub-optimal cycle lengths to a same deterioration curve. Regarding optimal resurfacing cycle, the result based on population-level parameters is around 2 years longer than that from individual-level parameters. The total cost difference between the two scenarios indicates that should the mean of population level parameters be employed, the cost would be around 2 to 4% higher. It is important to note that the cost of maintaining the nation's transportation infrastructure reaches \$91 billion annually (FHWA, 2002). The underlying percentage variation makes a significant difference in cost allocation and should not be ignored. In addition, it is shown in Table 6.4 that with the increase of  $s_1$ , present roughness immediately after resurfacing the difference between the two scenarios decreases. However, the total cost

increases as  $s_1$  increases. To better understand the difference between the two underlying levels of performance curves, the optimal cost based on the population-level curve is also presented in Table 6.4, denoted as  $J_1''$ . It is indicated that if population level curve is adopted, the total cost is significantly underestimated. This result has a profound impact on budget planning. This conclusion applies to the condition that performance curve by population-level parameters lies below that of individual-level parameters. The opposite is true if the population-level curve lies above the individual-level curve.

**Table 6.4 Comparison of Optimal Resurfacing Policy under Two Levels of Parameters**

$s_1$ (QI)	15	20	25	30	35	40
$\tau$ (years)	9.0	8.3	7.7	7.2	6.7	6.2
$\tau'$ (years)	11.5	10.5	9.7	9.0	8.3	7.7
$J_1$ (\$)	1,523,905	1,633,427	1,712,692	1,772,693	1,819,080	1,855,462
$J_1'$ (\$)	1,582,205	1,686,126	1,760,757	1,816,658	1,859,343	1,892,369
$J_1''$ (\$)	1,146,271	1,264,921	1,355,187	1,427,970	1,488,504	1,540,005
( $J_1'$ to $J_1$ ) Cost Difference (%)	+3.8	+3.2	+2.8	+2.5	+2.2	+2.0
( $J_1''$ to $J_1$ ) Cost Difference (%)	-24.8	-22.6	-20.9	-19.4	-18.2	-17.0

The difference between these two scenarios can be even larger. Note that the underlying result is obtained from a sound deterioration model from the Mn/Road project data source. The data source provides comprehensive information associated with pavement deterioration than most other sources. For example, apart from basic pavement structure information, other critical variables such as design and material properties are available. These variables are found to be statistically significant in affecting the deterioration process. If this information were omitted, extra unobserved heterogeneity would be introduced into the deterioration model resulting in a larger difference between the two levels of performance curves. As a result, additional cost due to sub-optimal policy would be incurred.

### **6.3.4 Effect of unobserved Heterogeneity on optimal policy for two sections in adjacent lanes**

Another issue of interest in addressing the resurfacing problem is regarding the policy for adjacent lanes. The deterioration rates vary for two sections in adjacent lanes due to the impacts of varying traffic. Hence, the optimal resurfacing policy differs between them. However, in practice, resurfacing is more often than not, carried out for both driving and passing lanes at one particular highway segment. Therefore, instead of addressing the individual optimal policy separately, the alternative is to determine joint optimal policy under the constraint that both sections of the two lanes share the same cycle length. It can be predicted that under this joint optimal policy, the total cost is higher than the separate condition due to the introduced constraint. As an example, the two adjacent sections from Cell 14 are investigated. The results are shown in Table 6.5. The summation of two costs from both sections based on their individual optimal policies separately obtained is used as reference. The costs for the two sections for other alternatives are compared with the reference. It is indicated that both unobserved heterogeneity (resulting in two levels of parameters) and two-section joint consideration contribute to higher total cost. In this specific example, the increased cost due to unobserved heterogeneity accounts for higher percentage than from jointly considering two sections. Again, the increased cost may be largely due to the factors associated with the deterioration model.

**Table 6.5 Resurfacing Policy for Two Sections of Adjacent Lanes**

	$s_1$ (QI)	15	20	25	30	35	40
Two Sections Separately	Individual-Level Cost (\$)	2,5217,44	2,732,241	2,892,988	3,022,082	3,128,715	3,218,741
	Population-Level Cost (\$)/ Difference (%)	2,608,613 /+3.5	2,811,435 /+2.9	2,965,677 /+2.5	3,088,934 /+2.2	3,190,247 /+2.0	3,275,414 /+1.8
Two Sections Jointly	Individual-Level Cost (\$)/ Difference (%)	2,584,680 /+2.5	2,792,590 /+2.2	2,950,258 /+2.0	3,076,081 /+1.8	3,179,414 /+1.6	3,266,219 /+1.5
	Population-Level Cost (\$)/ Difference (%)	2,678,635 /+6.2	2,878,054 /+5.3	3,028,523 /+4.7	3,147,897 /+4.2	3,245,364 /+3.7	3,326,824 /+3.4

## 6.4 Summary

This chapter discussed the incorporation of the deterioration model developed in Chapter 5 in a pavement system management. A typical optimization problem in conjunction with pavement resurfacing policy was formulated. On the basis of the previous research, a steady state deterioration and resurfacing process were adopted. The objective was to minimize the total cost, consisting of user cost and agency cost. A close-form solution for total life cycle cost was derived. The sensitivity of optimal policy to three key factors in the deterioration model –initial riding quality, deterioration rate and curvature – was investigated in detail.

It was found that the optimal policy, total cost and resurfacing cycle length are insensitive to the initial roughness while they are highly dependent on the deterioration rate and curvature. In addition, the effect of model accuracy due to unobserved heterogeneity on optimal policy was quantified. It was revealed that the inaccuracy of the deterioration model from unobserved heterogeneity contributes to the increase of total cost. The cost increase varies with the difference between the performance curves by individual and population levels of parameters. If the magnitude of the unobserved heterogeneity is large due to insufficient information included in the model, sub-optimal policies may make a significant difference. It was also found that the total cost based on model parameters using the traditional regression approach can be significantly over- or under-estimated as compared to the minimum total cost. Last, to reflect the reality more closely, joint optimal policy for two sections in adjacent lanes was investigated. The result suggests the total cost of this joint optimal policy is higher than the separate condition due to one more constraint being added. In summary, a sound deterioration model can better accommodate cost estimation, budget planning and resource allocation in a transportation infrastructure system and result in significant savings to the transportation agency.

## Chapter 7: Conclusions

### 7.1 Concluding remarks

In this dissertation, a sound deterioration model is established based on recent data from in-service pavement sections. Three key aspects are included in the deterioration model: physical deterioration principle, critical variables, and rigorous estimation approach. In particular, the unobserved heterogeneity in deterioration modeling is highlighted. The motivation to account for unobserved heterogeneity comes from the fact that any deterioration model is inevitably incapable of capturing all relevant information in the system. This contributes to the inaccuracy of the facility's performance prediction. As a consequence, the maintenance policies are affected. This effect is clearly reflected through a pavement resurfacing decision-making problem by incorporating the established deterioration model in a formulated objective function on total life cycle cost. The major findings of this research are presented as follows.

First, the deterioration model specification is developed based on a basic but widely accepted deterioration model form in pavement engineering. The deterioration model improves the existing state-of-the-art models by incorporating some unique features. The specification comprehensively establishes a relationship in a nonlinear fashion between riding quality in terms of roughness and a variety of variables. These variables include: 1) pavement structure information in terms of each individual layer's ability to resist deterioration; 2) material information with focus on different asphalt binder's role in resisting deterioration; 3) design information regarding different asphalt mix design approaches, Marshall and Superpave; 4) traffic and time, which consumes the pavement facility; 5) environmental factors, which relates to pavement performance change through its effect on material properties; and 6) maintenance activity, which aims at improving the performance. In addition, the variations in performance between travel lanes that experience different traffic levels are also addressed in the model.



More importantly, to target the major goal in this research, a hierarchical model parameter structure is formulated to represent unobserved heterogeneity. The unobserved heterogeneity comes from two major sources: construction quality and material variability from section to section. The former is reflected through varying initial roughness among constructed pavement sections. The latter is reflected through the coefficients of pavement structure layers. The variability is regarded as random and mathematically formulated as a random variable. Thus, those related parameters include not only means, as in the traditional approach, but also standard deviations. As is customary, a normal distribution is assumed for each of the underlying parameters. The model of this feature is referred to as random parameter (RP) model.

Second, an econometric methodology is employed to estimate the specified deterioration model. This research supports the claim in recent studies that the integration of structured econometric techniques and basic engineering knowledge can be effectively used in deterioration model development and estimation. The application of econometric methodology in deterioration modeling moves one step further in this research. In particular, Maximum Simulated Likelihood (MSL) is adopted as the estimation approach. Simulation is used to tackle multi-dimension integral intricacy. Although, MSL has been studied frequently in recent econometric literature, it is applied for the first time in deterioration modeling with meaningful engineering implications. The main advantages of using MSL instead of a traditional approach such as OLS include: 1) it is capable of accounting for unobserved heterogeneity; and 2) parameters revealing deterioration for each individual homogeneous facility of interest can be obtained through Bayesian updating from MSL results. In engineering practice, the two levels of parameters obtained based on MSL can accommodate both network and project level pavement system management.

Third, by applying the MSL approach, parameter estimation results are obtained for the RP model. For comparison, the counterpart results in the other two alternatives, pooled

regression and random effects, are also presented. Through statistical test (likelihood ratio test based on likelihood values at convergence), it is suggested that the RP model is preferred. Regarding the engineering implications of the parameter estimates, the major findings are listed as follows.

- The causal relationship between carefully selected variables customarily encountered in pavement engineering and riding quality is well captured in the proposed deterioration model.
- Unobserved heterogeneity is found to be significant in the deterioration model. Variability in both construction quality and material property across pavement sections is proven to be existent and is quantified, which significantly aids in the decision making process for planning in highway construction, pavement service life estimation, and system reliability analysis.
- Layer coefficients ratios are found to be consistent with engineering judgment. Moreover, their quantitative relationship is obtained with data from in-service pavement sections, a finding that can serve to update and expand on traditional knowledge such as AASHTO's recommendation developed around half a century ago.
- From a roughness viewpoint, pavement with Marshall asphalt mix design with 75 blows demonstrate better performance than the Superpave designs.
- Asphalt binder type plays a significant role in pavement roughness development in the Northern states with an environment similar to that in Minnesota. Pavement roughness develops more slowly when softer asphalt binder are used.
- Pavement performance in terms of roughness features a curvature larger than one. Different lanes, such as driving and passing lanes, should be treated separately in deterioration modeling because their deterioration rates differ due to different traffic levels. The driving lane usually deteriorates faster than the passing lane.

- Frost-heave is found to have a statistically significant impact on roughness due to frost-susceptible subbase granular material.
- Maintenance is found to be effective in improving pavement performance, as roughness drops immediately and significantly after maintenance.
- Finally, all the effects due to these pavement performance aspects are quantified through well established mathematical terms in the deterioration model.

Last, to demonstrate the role of deterioration modeling in pavement system management, the realistic models with two levels of parameters are directly applied into a steady state resurfacing problem. The objective is to optimize resurfacing policy by minimizing total life cycle cost consisting of user cost and agency cost. The findings of the case study are presented as follows.

- Optimal policy is not sensitive to construction quality represented by initial ride quality in the Mn/Road project.
- Optimal policy is clearly dependent on deterioration rate. With an increase of deterioration rate, optimal cost increases while optimal cycle length decreases.
- Optimal policy differs between driving and passing lanes. On average, the cost of driving lane sections is 50 percent higher and the optimal cycle length is 60 percent of those in the passing lane.
- Optimal policy varies with different preset roughness immediately after resurfacing: with an increase of the preset value, total life cost increases while the cycle length decreases.
- Unobserved heterogeneity has a pronounced impact on optimal policy. Optimal policies under two levels of deterioration model parameters differ significantly from each other. Important savings can be obtained by adopting optimal policy based on individual-level parameters from the policy based on population-level parameters.
- Jointly considering two sections in adjacent lanes generates higher life cycle cost due to the added constraint.

In conclusion, the approach and findings can be used to facilitate decision making, budget planning and resource allocation in managing a pavement system and accrue important savings.

It is important to note that the framework of the deterioration model and estimation approach investigated in this dissertation can also be used in the analysis of other infrastructure facilities.

## **7.2 Further work**

First, the data source for this study, the Mn/Road project, represents a typical cold climate region in the United States. From a climate perspective, the nation can be roughly divided into four regions, North Atlantic, North Central, South, and West. Minnesota is located in the North Central region. To generalize the models to other climate regions such as Texas, funds should be dedicated for data collection and processing for modeling. Among the data sources discussed in Chapter 3, the most appealing corresponds to the LTPP Program. After investigating the latest Standard Data Release, (version 20), it is believed that for the majority of sections, there is a gap between the available information and that which is required in a sound deterioration model such as the one established in this dissertation. Due to possible lack of key variables, directly adopting the data may result in poorly-specified models or models with larger unobserved heterogeneity and poor prediction capabilities. For example, those sections without good-quality traffic data may lead to unexpected estimation results. However, it is anticipated that the goal of developing sound deterioration models with LTPP data source is closer to being reached because more and better quality data are becoming available with the improvement of inspection technology and data management.

Another source of good-quality data for deterioration modeling can be from a customized database, where pertinent information can be purposely targeted and collected. Currently, several states are leading a similar efforts to develop their own databases. For example, in an ongoing project sponsored by the Texas Department of Transportation, The University of Texas is developing a comprehensive database for flexible pavements to support Texas highway design and management.

Second, further work can be extended to reliability analysis based on the established deterioration models. It is implied in this research that the models with two levels of parameters result in different levels of uncertainty. Due to larger uncertainty, a model with population-level parameters is less accurate in performance prediction than its counterpart with individual-level parameters. As a consequence, two models with two levels of parameters lead to different implications in the reliability analysis. Particular interest is associated with the effect of unobserved heterogeneity on reliability of the system in question.

Third, although this research focuses on infrastructure management, the same model estimation approach can be applied to improve mechanistic–empirical (M-E) pavement design. In the M-E design, the empirical component is used to calibrate the models based on in-service pavement data, mainly LTPP. Although significant effort and funds have been dedicated to enhance mechanistic analysis in pavement engineering, the equally important part involving empirical modeling has not been well addressed. The rigorous statistical approach proposed in this research can meet this requirement.

Fourth, additional effort can be made to fine-tune the current deterioration models. In conditions other than those associated with the Mn/Road project, other subgrade properties may appear, which remains to be addressed in the model. In addition, other binders, such as different PG grades or modified asphalt can also be applied. To account for the different binders, corresponding terms should be added in the models in a proper

manner. Furthermore, the effect of resurfacing on pavement performance deserves to be investigated. It is more realistic to believe that deterioration rates and curvatures change after each resurfacing due to the change in the pavement structure. Whether this change and its effect on the optimal policy are significant is an interesting issue to be explored.

Last, but not least, one further step can be added by including the construction cost into the life-cycle cost analysis. In the current research, the optimal policy is investigated with a focus on the pavement management perspective. It is shown from the established deterioration model that pavement performance is heavily dependent on construction quality and material quantity (layer thickness) and quality. These factors are closely associated with the construction cost. Therefore, the optimal policy is not only related to maintenance cost but also to the cost of construction. This task can be fulfilled if the construction cost information is available. In particular, this effort can be applied to Public-Private-Partnerships (PPPs), such as Design Build Operate and Transfer (DBOT), which are being used more often in highway planning and construction in recent years. From the public side, such a model can help develop performance measures to establish acceptance criteria for the transfer of facilities under DBOT agreements. From the private side, it can facilitate decision making to determine construction schemes and planning maintenance.

## Appendix A: Model Specification Test Using Maximum Likelihood Estimation Approach

### A.1 Proof of information matrix equality when using MLE (Greene, 2002)

Let log-likelihood function from observed sample be denoted as,

$$\hat{Q}_n(\theta) = \frac{1}{n} \sum_{i=1}^n \log f(y_i; \theta),$$

For the population, the log-likelihood takes the following form,

$$Q(\theta) = E(\log f(y_i; \theta)) = \int \log f(y; \theta) f(y; \theta_0) dy$$

Where,  $\theta_0$  is the parameter maximizing the log-likelihood.

$$\begin{aligned} E\left(\frac{\partial}{\partial \theta} \log f(y_i; \theta_0)\right) &= \int \left(\frac{\partial}{\partial \theta} \log f(y_i; \theta_0)\right) f(y; \theta_0) dy \\ &= \int \left(\frac{\frac{\partial}{\partial \theta} f(y_i; \theta_0)}{f(y_i; \theta_0)}\right) f(y; \theta_0) dy = \int \frac{\partial}{\partial \theta} f(y; \theta_0) dy \\ &= \frac{\partial}{\partial \theta} \int f(y; \theta_0) dy = \frac{\partial}{\partial \theta} (1) = 0 \end{aligned}$$

$$\begin{aligned}
0 &= \int \left( \frac{\partial}{\partial \theta \theta \theta'} \log f(y_i; \theta_0) \right) f(y; \theta_0) dy + \int \left( \frac{\partial}{\partial \theta} \log f(y_i; \theta_0) \right) \frac{\partial}{\partial \theta'} f(y; \theta_0) dy \\
&= \int \left( \frac{\partial}{\partial \theta \theta \theta'} \log f(y_i; \theta_0) \right) f(y; \theta_0) dy + \int \left( \frac{\partial}{\partial \theta} \log f(y_i; \theta_0) \right) \left( \frac{\partial}{\partial \theta'} \log f(y; \theta_0) \right) f(y; \theta_0) dy \\
&= E \left( \frac{\partial}{\partial \theta \theta \theta'} \log f(y_i; \theta) \right) + E \left( \frac{\partial}{\partial \theta} \log f(y_i; \theta) \frac{\partial}{\partial \theta'} \log f(y_i; \theta) \right)
\end{aligned}$$

Let,

$$A = E \left( \frac{\partial}{\partial \theta} \log f(y_i; \theta) \frac{\partial}{\partial \theta'} \log f(y_i; \theta) \right)$$

$$B = E \left( \frac{\partial}{\partial \theta \theta \theta'} \log f(y_i; \theta) \right)$$

We obtain

IME:  $A + B = 0$ , which is information matrix equality.

## A.2 Model specification test

In the specification test, to ease derivation and presentation, a slightly simplified model specification is adopted as follows. In other words, the same model framework with Equation (4.5) is used. The change is made so that those same functional terms only appear once in the simplified specification. However, it can be shown that the full specification, Equation (4) is correctly specified through the same procedure provided in the following. The simplified specification is:

$$h(X) = \beta_0 + \exp(\beta_1 + \exp(\beta_2 x_2)) \beta_3 x_3 + \beta_4 x_4 x_5^{\beta_5} + \beta_6 x_6$$



As is customary, the normal distribution is applied. As a result, the likelihood for one given observation:

$$f(y; \theta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(h(X) - y)^2}{2\sigma^2}\right)$$

Where, parameters:  $\theta = [\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6]$  and  $\sigma^2$ .

The first order condition is:

$$\frac{\partial}{\partial \sigma^2} f(y; \theta) = -\frac{1}{2\sigma^2} + \frac{(h - y)^2}{2\sigma^4}$$

$$\frac{\partial}{\partial \beta} f(y; \theta) = -\frac{1}{2\sigma^2} \begin{pmatrix} 2(h - y) \\ 2(h - y) \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \\ 2(h - y) \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) \beta_3 x_3 x_2 \\ 2(h - y) \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_4 \\ 2(h - y) \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \log x_5 \\ 2(h - y) x_6 \end{pmatrix}$$

Thus, Let

$$g(y_i; \theta) = \frac{\partial}{\partial \theta} \log f(y_i; \theta) = \begin{pmatrix} \frac{\partial}{\partial \sigma^2} f(y; \theta) \\ \frac{\partial}{\partial \beta} f(y; \theta) \end{pmatrix}$$



$$\begin{aligned}
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) \beta_3 x_3 x_2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) x_3 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 x_4 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \log x_5 \\
& 4\varepsilon^2 \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_6
\end{aligned}$$

$$\begin{aligned}
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) \beta_3 x_3 x_2 \right)^2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 \right)^2 \beta_3 x_2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) \beta_3 x_3 x_2 x_4 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) \beta_3 x_3 x_2 \log x_5 \\
& 4\varepsilon^2 \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_6
\end{aligned}$$

$$\begin{aligned}
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 \right)^2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 \right)^2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) x_3 x_4 \\
& 4\varepsilon^2 \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 x_6
\end{aligned}$$

$$\begin{aligned}
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_4 \right)^2 \\
& 4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 x_4 \log x_5 \\
& 4\varepsilon^2 \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_4 x_6
\end{aligned}$$

$$\left. \begin{aligned} &4\varepsilon^2 \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \log x_5 \right)^2 \\ &4\varepsilon^2 \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \log x_5 x_6 \quad 4\varepsilon^2 x_6^2 \end{aligned} \right\}$$

By applying:  $E(\varepsilon) = 0$ ,  $E(\varepsilon^2) = \sigma^2$ ,  $E(\varepsilon^3) = 0$ , and  $E(\varepsilon^4) = 3\sigma^4$

$$E(g(y_i; \theta)g(y_i; \theta)') = \begin{pmatrix} \frac{1}{2\sigma^2} & & & & & & & & & & \\ 0 & 1 & & & & & & & & & \\ 0 & E\left(\exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5}\right) & & & & & & & & & \\ 0 & E\left(\exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) \beta_3 x_3 x_2\right) & & & & & & & & & \\ 0 & E\left(\exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3\right) & & & & & & & & & \\ 0 & E\left(\exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_4\right) & & & & & & & & & \\ 0 & E\left(\exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \log x_5\right) & & & & & & & & & \\ 0 & E(x_6) & & & & & & & & & \end{pmatrix}$$

$$\begin{aligned}
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \exp(\beta_2 x_2)\beta_3 x_3 x_2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \exp(\beta_2 x_2)x_3 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 x_4 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \log x_5 \\
& E\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} x_6
\end{aligned}$$

$$\begin{aligned}
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \exp(\beta_2 x_2)\beta_3 x_3 x_2\right)^2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \exp(\beta_2 x_2)x_3\right)^2 \beta_3 x_2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \exp(\beta_2 x_2)\beta_3 x_3 x_2 x_4 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \exp(\beta_2 x_2)\beta_3 x_3 x_2 \log x_5 \\
& E\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} x_6
\end{aligned}$$

$$\begin{aligned}
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \exp(\beta_2 x_2)x_3\right)^2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \exp(\beta_2 x_2)x_3\right)^2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 \exp(\beta_2 x_2)x_3 x_4 \\
& E\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \exp(\beta_2 x_2)x_3 x_6
\end{aligned}$$

$$\begin{aligned}
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} x_4\right)^2 \\
& E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5}\right)^2 x_4 \log x_5 \\
& E\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} x_4 x_6
\end{aligned}$$

$$\left. \begin{aligned} & E\left(\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \log x_5\right)^2\right) \\ & E\left(\exp(\beta_1 + \exp(\beta_2 x_2))\beta_3 x_3 + \beta_4 x_4\right)x_5^{\beta_5} \log x_5 x_6\right) \quad E(x_6^2) \end{aligned} \right\}$$

2) The Hessian of the log-likelihood is,



$$\begin{aligned}
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 + \varepsilon a \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) \beta_3 x_3 x_2 + \varepsilon a \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) x_3 + \varepsilon a \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 x_4 + \varepsilon a \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \log x_5 + \varepsilon a \\
& \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_6 + \varepsilon a
\end{aligned}
\quad
\begin{aligned}
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) \beta_3 x_3 x_2 \right)^2 + \varepsilon b \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 \right)^2 \beta_3 x_2 + \varepsilon b \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) \beta_3 x_3 x_2 x_4 + \varepsilon b \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) \beta_3 x_3 x_2 \log x_5 + \varepsilon b \\
& 4\varepsilon^2 \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_6 + \varepsilon b
\end{aligned}$$

$a = \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5}$ ,  $b$  is a similar term, which is not related to  $\varepsilon$ . Hence, the expectation is itself, and it implies that the expectation of multiplication of  $\varepsilon$  and  $a$  or  $b$  will be equal to zero.

$$\begin{aligned}
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 \right)^2 + \varepsilon d \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 \right)^2 + \varepsilon d \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 \exp(\beta_2 x_2) x_3 x_4 + \varepsilon d \\
& \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \exp(\beta_2 x_2) x_3 x_6 + \varepsilon d
\end{aligned}
\quad
\begin{aligned}
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_4 \right)^2 + \varepsilon e \\
& \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \right)^2 x_4 \log x_5 + \varepsilon e \\
& \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} x_4 x_6 + \varepsilon e
\end{aligned}$$



Where,  $d$  and  $e$  are similar term with  $a$  and  $b$ .

$$\left. \begin{aligned} & \left( \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \log x_5 \right)^2 + \mathcal{E}^f \\ & \exp(\beta_1 + \exp(\beta_2 x_2) \beta_3 x_3 + \beta_4 x_4) x_5^{\beta_5} \log x_5 x_6 + \mathcal{E}^f \quad x_6^2 \end{aligned} \right\}$$

Again, by applying:

$$E(\varepsilon) = 0, \quad E(\varepsilon^2) = \sigma^2$$

It is easy to find that

$$E(H(y_i; \theta)) = -E(g(y_i; \theta)g(y_i; \theta)')$$

That is,

$A + B = 0$  in above proof of IME.

Therefore, the model is correctly specified and MLE will produce consistent, and asymptotically normal and efficient estimates.

**Appendix B: Estimation of Correlation Matrix for the Parameters in  
RP Model**

**Table B1 Estimation of Correlation Matrix for the Parameters in Pooled Regression  
Model**

	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$
$\beta_0$	1						
$\beta_1$	-0.169	1					
$\beta_2$	-0.155	-0.882	1				
$\beta_3$	0.009	-0.751	0.852	1			
$\beta_4$	0.025	-0.363	0.442	0.413	1		
$\beta_5$	0.043	0.035	0.033	0.067	0.468	1	
$\beta_6$	0.053	-0.640	0.726	0.707	0.674	0.400	1
$\beta_7$	-0.233	-0.619	0.750	0.640	0.387	0.354	0.533
$\beta_8$	-0.228	-0.802	0.938	0.755	0.434	-0.008	0.609
$\beta_9$	0.647	-0.394	-0.068	-0.010	-0.023	-0.053	0.056
$\beta_{10}$	0.156	0.049	-0.184	-0.252	-0.132	-0.070	-0.196
$\beta_{11}$	-0.116	0.056	-0.098	-0.161	-0.068	-0.088	-0.140
$\beta_{12}$	0.129	0.112	-0.109	0.045	0.325	0.398	0.211

	$\beta_7$	$\beta_8$	$\beta_9$	$\beta_{10}$	$\beta_{11}$	$\beta_{12}$
$\beta_7$	1					
$\beta_8$	0.773	1				
$\beta_9$	-0.176	-0.175	1			
$\beta_{10}$	-0.086	-0.19	0.157	1		
$\beta_{11}$	-0.057	-0.112	0.059	0.053	1	
$\beta_{12}$	-0.111	-0.169	-0.015	-0.102	-0.067	1

**Table B2 Estimation of Correlation Matrix for the Parameters in RE Model**

	$\beta_0$	$\delta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$
$\beta_0$	1						
$\delta_0$	0.102	1					
$\beta_1$	-0.200	-0.205	1				
$\beta_2$	0.090	0.198	-0.926	1			
$\beta_3$	0.126	0.203	-0.859	0.949	1		
$\beta_4$	0.012	0.109	-0.555	0.628	0.575	1	
$\beta_5$	0.010	0.127	-0.451	0.530	0.497	0.662	1
$\beta_6$	0.006	0.103	-0.700	0.786	0.746	0.757	0.707
$\beta_7$	0.045	0.219	-0.696	0.762	0.749	0.554	0.699
$\beta_8$	0.058	0.216	-0.867	0.937	0.874	0.626	0.479
$\beta_9$	0.301	0.025	-0.377	0.024	-0.005	0.007	-0.011
$\beta_{10}$	-0.036	-0.037	0.096	-0.182	-0.238	-0.186	-0.175
$\beta_{11}$	-0.019	0.037	-0.018	-0.005	-0.012	-0.051	-0.059
$\beta_{12}$	0.016	0.058	-0.015	0.020	0.082	0.311	0.363

	$\beta_6$	$\beta_7$	$\beta_8$	$\beta_9$	$\beta_{10}$	$\beta_{11}$	$\beta_{12}$
$\beta_6$	1						
$\beta_7$	0.631	1					
$\beta_8$	0.688	0.783	1				
$\beta_9$	0.063	-0.057	-0.058	1			
$\beta_{10}$	-0.204	-0.092	-0.191	0.068	1		
$\beta_{11}$	-0.052	-0.009	0.000	0.052	0.031	1	
$\beta_{12}$	0.244	0.045	-0.021	-0.003	-0.154	-0.116	1

**Table B3 Estimation of Correlation Matrix for the Parameters in RP Model**

	$\beta_0$	$\delta_0$	$\beta_1$	$\beta_2$	$\delta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$
$\beta_0$	1								
$\delta_0$	-0.406	1							
$\beta_1$	0.28	-0.563	1						
$\beta_2$	-0.402	0.582	-0.972	1					
$\delta_2$	0.473	-0.399	0.713	-0.770	1				
$\beta_3$	-0.395	0.637	-0.878	0.928	-0.773	1			
$\beta_4$	0.026	-0.074	-0.189	0.182	-0.240	0.074	1		
$\beta_5$	-0.075	0.325	-0.404	0.417	-0.366	0.362	0.341	1	
$\beta_6$	-0.361	0.561	-0.944	0.969	-0.706	0.906	0.255	0.470	1
$\beta_7$	-0.388	0.562	-0.862	0.877	-0.752	0.782	0.258	0.616	0.834
$\delta_7$	0.302	-0.259	0.324	-0.365	0.462	-0.477	-0.113	0.091	-0.350
$\beta_8$	-0.422	0.530	-0.925	0.944	-0.781	0.811	0.276	0.431	0.886
$\delta_8$	0.254	-0.299	0.405	-0.448	0.279	-0.546	0.388	-0.119	-0.433
$\beta_9$	0.450	-0.038	-0.168	-0.044	0.149	-0.034	0.022	-0.027	0.012
$\beta_{10}$	-0.026	0.076	-0.102	0.082	-0.017	0.098	-0.372	-0.086	0.088
$\beta_{11}$	-0.133	0.116	-0.026	0.016	0.000	0.028	-0.035	-0.067	0.002
$\beta_{12}$	0.056	-0.003	0.145	-0.143	0.059	-0.117	0.335	0.248	-0.056
	$\beta_7$	$\delta_7$	$\beta_8$	$\delta_8$	$\beta_9$	$\beta_{10}$	$\beta_{11}$	$\beta_{12}$	
$\beta_7$	1								
$\delta_7$	-0.403	1							
$\beta_8$	0.898	-0.301	1						
$\delta_8$	-0.279	0.130	-0.358	1					
$\beta_9$	-0.067	-0.001	-0.115	0.018	1				
$\beta_{10}$	-0.015	0.180	-0.009	-0.257	0.038	1			
$\beta_{11}$	0.024	-0.065	0.000	-0.050	0.067	-0.014	1		
$\beta_{12}$	-0.101	0.089	-0.163	0.284	-0.045	-0.111	-0.120	1	

## Appendix C: Deterioration Curves by Two Levels of Parameters (Population and Individual) for All Sections

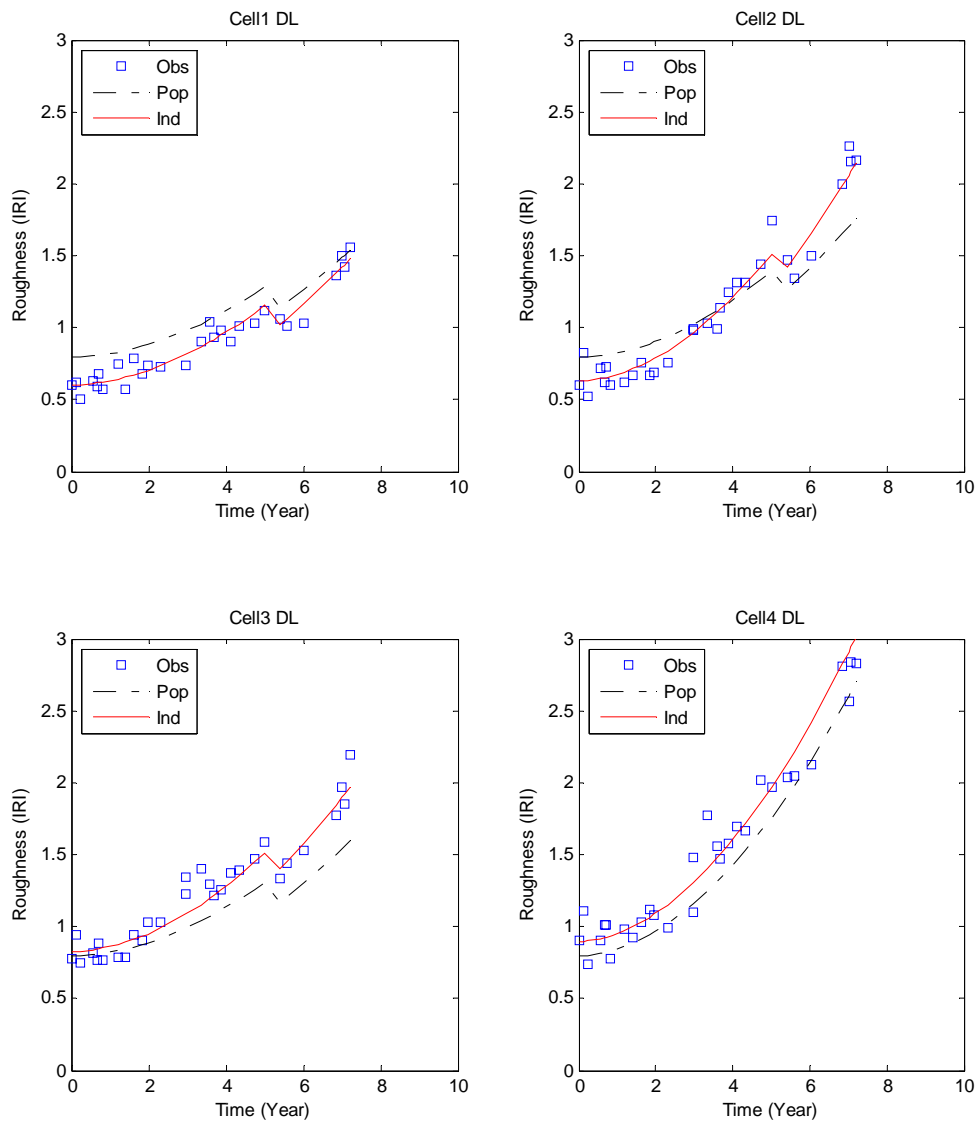
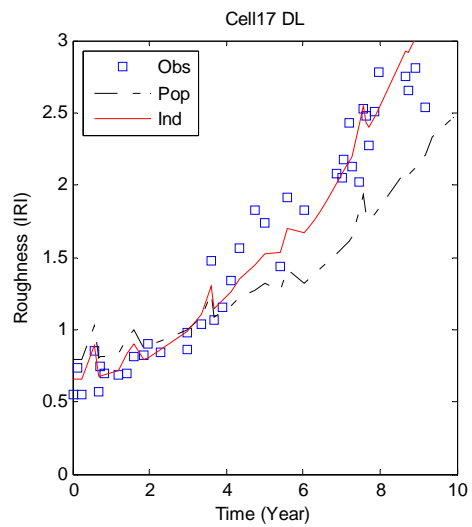
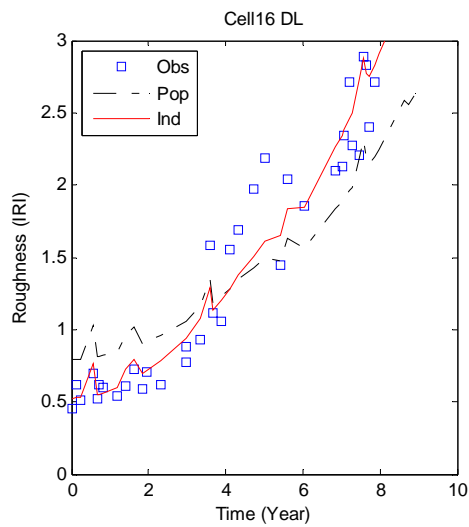
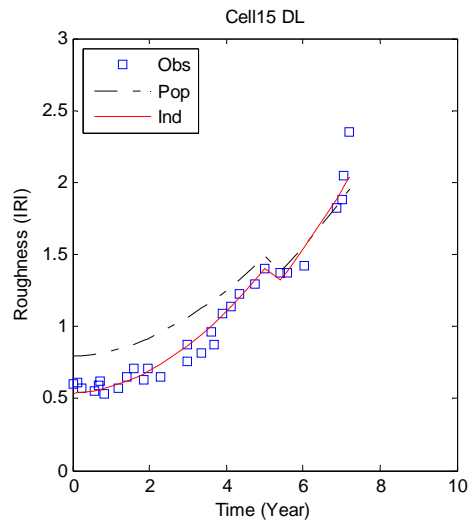
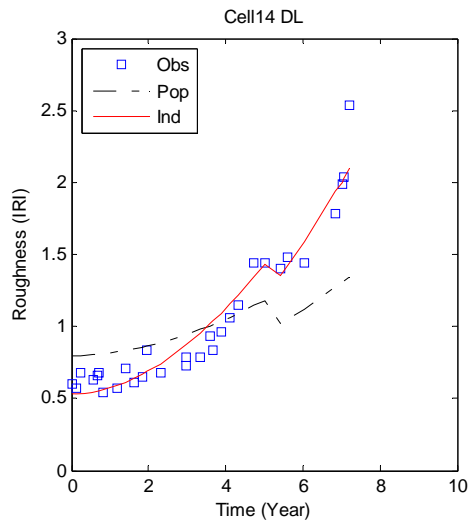
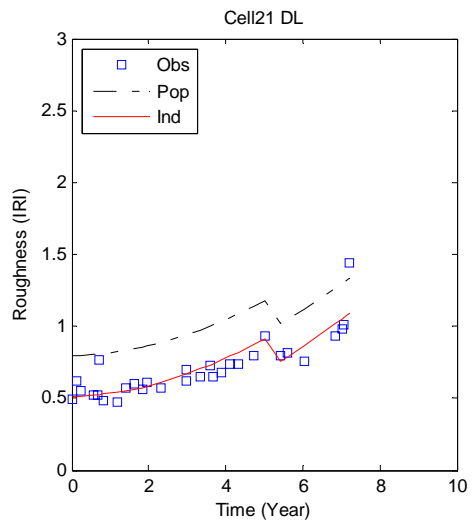
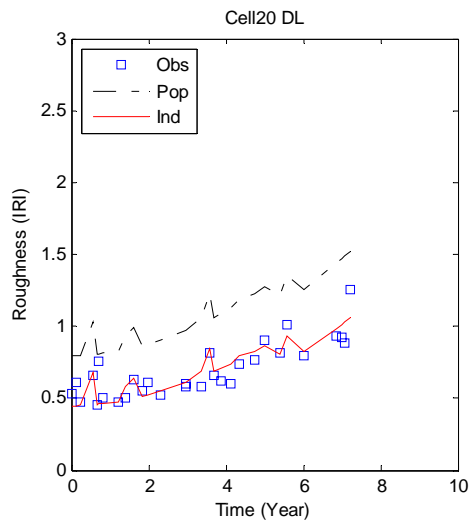
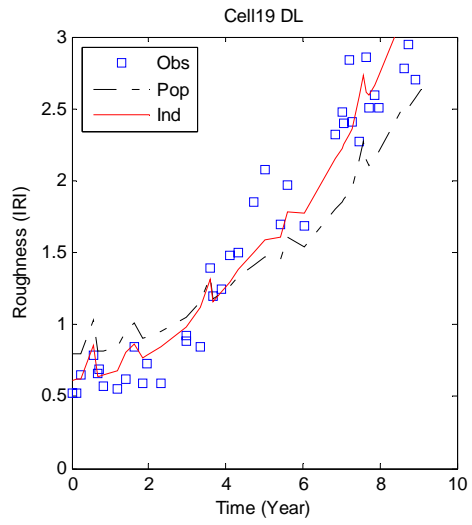
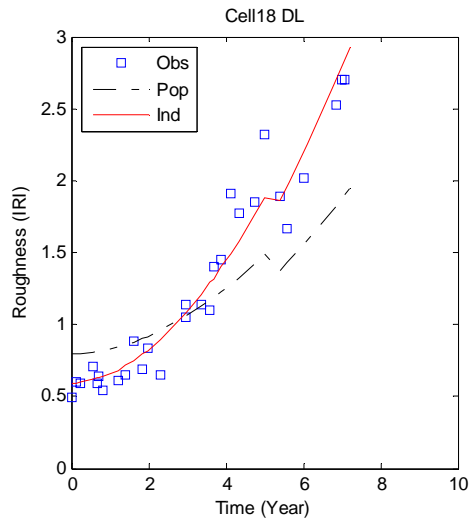


Figure C1 Performance Fit by Two Levels of Parameters (Driving Lane, Cells 1, 2, 3, and 4)

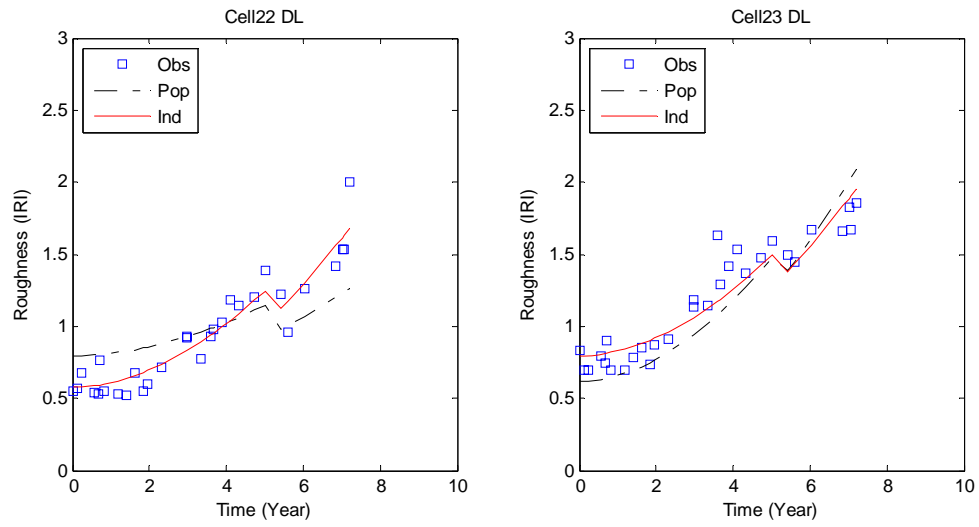


**Figure C2 Performance Fit by Two Levels of Parameters (Driving Lane, Cells 4, 15, 16, and 17)**

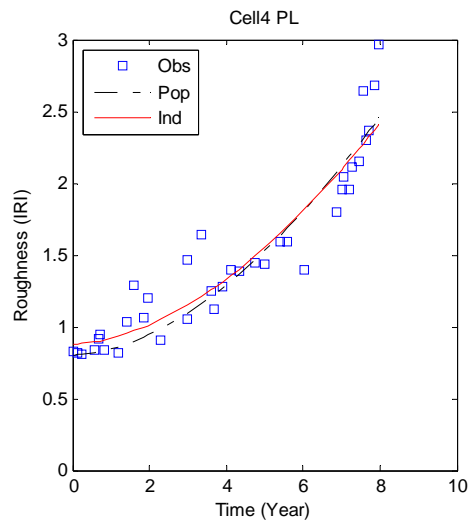
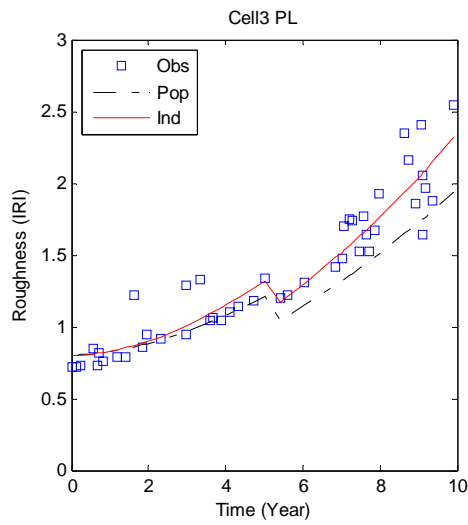
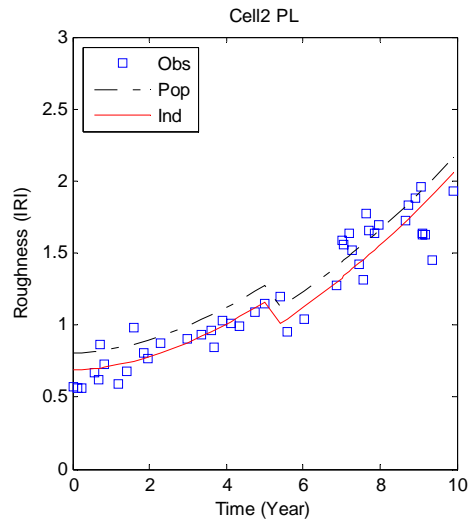
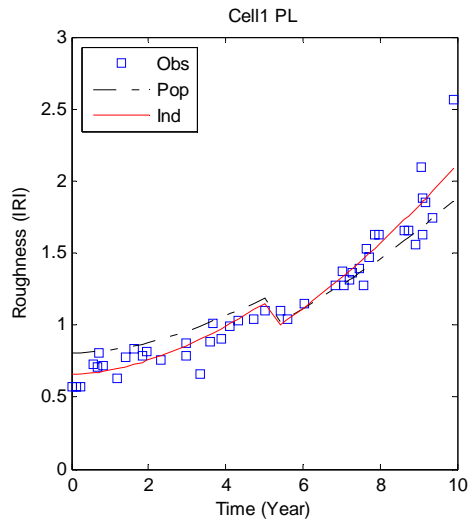


**Figure C3 Performance Fit by Two Levels of Parameters (Driving Lane, Cells 18, 19, 20, and 21)**

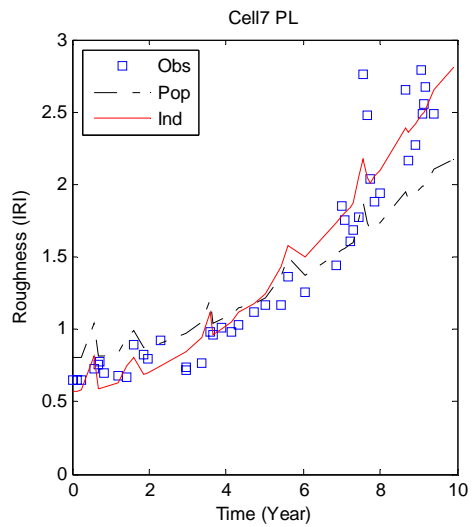
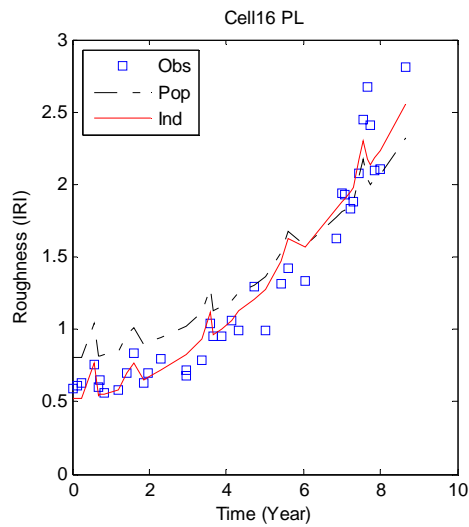
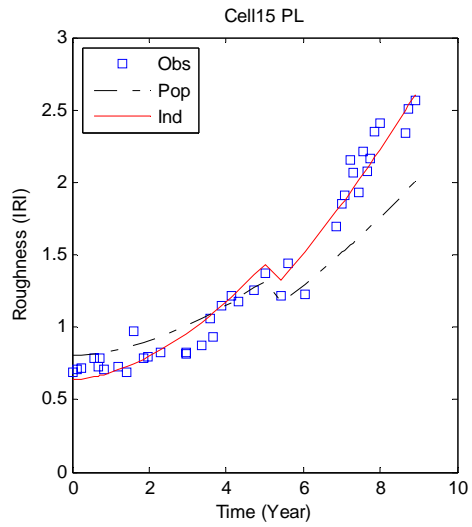
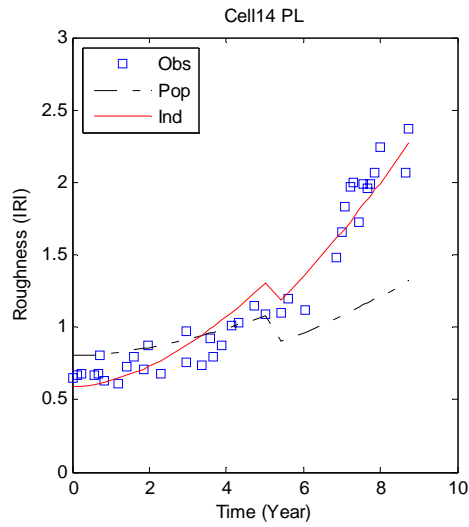




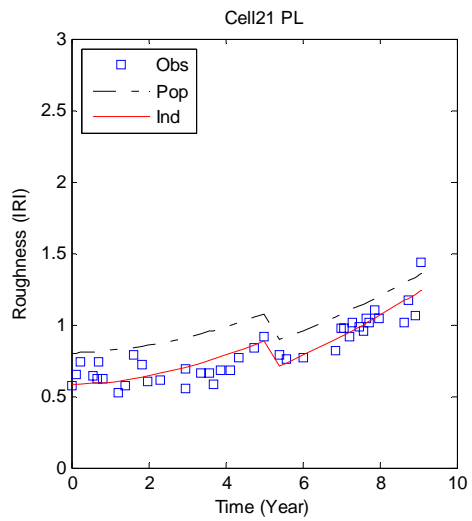
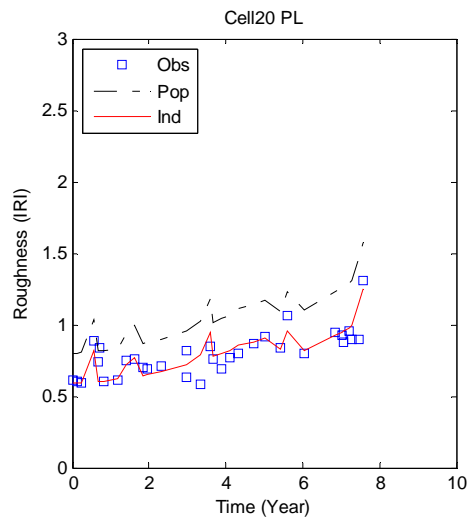
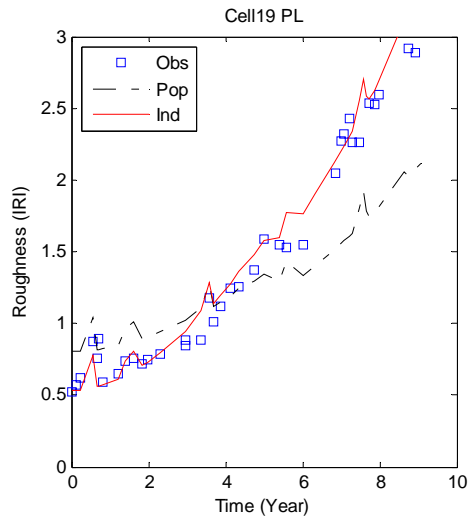
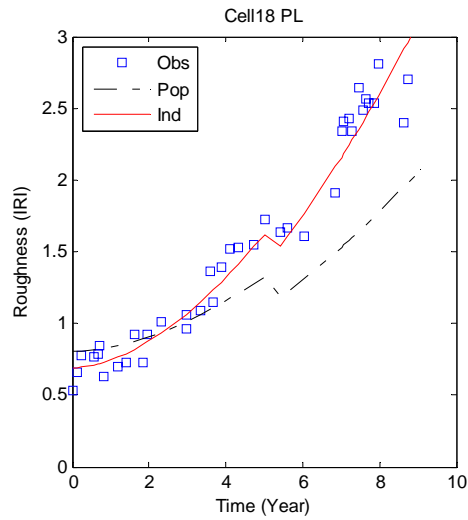
**Figure C4 Performance Fit by Two Levels of Parameters (Driving Lane, Cells 22 and 23)**



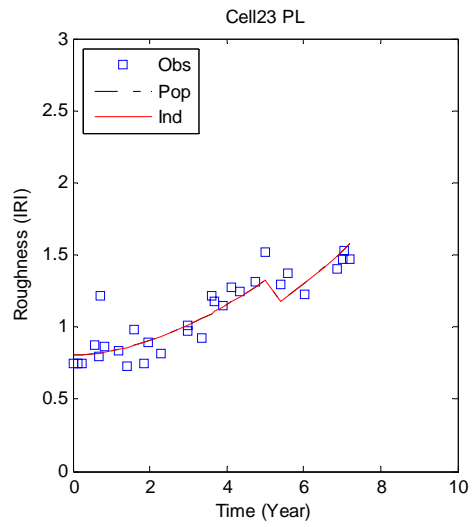
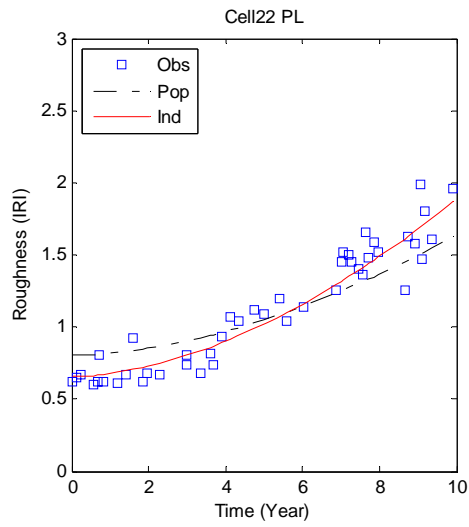
**Figure C5 Performance Fit by Two Levels of Parameters (Passing Lane, Cells 1, 2, 3, and 4)**



**Figure C6 Performance Fit by Two Levels of Parameters (Passing Lane, Cells 14, 15, 16, and 17)**



**Figure C7 Performance Fit by Two Levels of Parameters (Passing Lane, Cells 18, 19, 20, and 21)**



**Figure C8 Performance Fit by Two Levels of Parameters (Passing Lane, Cells 22 and 23)**

## References

American Association of State Highways and Transportation Officials (1993), AASHTO Guide for Design of Pavement Structures, AASHTO. Washington, D.C.

American Society for Testing and Materials (1982), Standard Definitions of Terms Relating to Traveled Surface Characteristics. ASTM Standard E867-82A. Philadelphia.

Anselin, L. (1999), Spatial Econometrics, Working Paper, Accessed May 2006:  
[http://www.csiss.org/learning\\_resources/content/papers/baltchap.pdf](http://www.csiss.org/learning_resources/content/papers/baltchap.pdf).

Archilla, A. R. (2000), Developing of Rutting Progression Models by Combining Data from Multiple Sources, Ph.D. Dissertation, University of California, Berkeley.

Archilla, A. R. and S. M. Madanat (2001), Development of Pavement Rutting Model by Combining Data from Different Experimental Sources, Journal of Transportation Engineering, Vol. 126, No. 4, American Society of Civil Engineers, pp. 291–299.

Archilla, A.R. (2006), Repeated Measurement Data Analysis in Pavement Deterioration Modeling, Journal of Infrastructure Systems, Vol. 12, No. 3, American Society of Civil Engineers, pp. 163-173.

Archondo-Callao, R. S. and A. Faiz (1994), Estimating Vehicle Operating Costs, Technical Paper No. 234, the International Bank for Reconstruction and Development/World Bank, Washington, D. C.

Ben-Akiva, M. and R. Ramaswamy (1993), An Approach for Predicting Latent Infrastructure Facility Deterioration, Transportation Science. Vol. 27, No. 2.

Chesher, A.D., and R. Harrison (1987), *Vehicle Operating Costs: Evidence from Developing Countries*, Highway Design and Maintenance Standards Series, The Johns Hopkins University Press, Baltimore, Maryland.

Davidian, M., and D. M. Giltinan (1995), *Nonlinear Models for Repeated Measurement Data*, Monographs on statistics and applied probability, Chapman & Hall, San Francisco.

Federal Highway Administration (1997), *Federal Highway Cost Allocation Study Final Report*, Washington, D.C., 1997, FHWA.

Federal Highway Administration (2003), *Long-Term Pavement Performance Information Management System: Pavement Performance Database User Reference Guide*, Report No. FHWA-RD-03-088, FHWA.

Federal Highway Administration, DataPave Online, [www.datapave.com](http://www.datapave.com), Accessed, October, 2006, FHWA.

Federal Highway Administration, (2002), *2002 Status of the Nation's Highways, Bridges and Transit: Report to Congress*, FHWA.

Finley, C. A. (2004), *Designing and Analyzing Test Programs with Censored Data for Civil Engineering Applications*, Ph.D. Dissertation, The University of Texas at Austin.

Garcia Diaz, A. and M. Riggins (1984), *Serviceability and Distress Methodology for Predicting Pavement Performance*, Transportation Research Record 997, Transportation Research Board, National Research Council, Washington, D.C., pp. 56–61.

Gelman, A., J.B. Carlin, H. S. Stern, and D. B. Rubin (2004), *Bayesian Data Analysis*, 2<sup>nd</sup> Edition, Chapman & Hall/CRC.

Gourieroux, C. and A. Monfort (1996), *Simulation-Based Econometric Methods*. Oxford University press.

Greene, W. (2002), *Econometric Analysis*, 5<sup>th</sup> Edition, Prentice Hall.

Greene, W. (2004), *Interpreting Estimated Parameters and Measuring Individual Heterogeneity in Random Coefficient Models*, Department of Economics, New York University, working paper.

Hajivassiliou, V. A. and P.A. Rudd (1994), *Classical Estimation Methods for LDV models Using Simulation*, in Engle, R. F. and D. L. McFadden *Handbook of Econometrics*, Chapter 40, (4).

Haas, R., W. R. Hudson, and J. Zaniewski (1994), *Modern Pavement Management*, Krieger Publishing Company, Malabar, Florida.

Highway Research Board (HRB) (1962), *The AASHO Road Test*. Special Report Nos. 61A through 61G and 73, Washington, D.C.

Hong F. and J. A. Prozzi (2006), *Estimation of Pavement Deterioration Using Bayesian Approach*, *ASCE Journal of Infrastructure Systems*, American Society of Civil Engineers, Vol. 12, No. 2, 2006.

Hsiao, C. (2003), *Analysis of Panel Data*, 2<sup>nd</sup> Edition, Cambridge University Press.

Huang, Y. H. (2003), *Pavement Analysis and Design*, 2<sup>nd</sup> Edition, Prentice Hall, Inc., New Jersey.



Li, Y. and S. M. Madanat (2002), A Steady-State Solution for the Optimal Pavement Resurfacing Problem, Transportation Research, Vol. 36A, pp. 525- 535.

Madanat, S. M., S. Bulusu, and A. Mahmoud (1995), Estimation of Infrastructure Distress Initiation and Progression Models, ASCE Journal of Infrastructure Systems, American Society of Civil Engineers, Vol. 1, No. 3. 1995.

Madanat, S. M. and R. Mishalani (1998), Selectivity Bias in Modeling Highway Pavement Maintenance Effectiveness, Journal of Infrastructure Systems, American Society of Civil Engineers, Vol. 4, No. 3. 1998

Madanat, S. M., J. A. Prozzi, and M. Han (2002), Effect of Performance Model Accuracy on Optimal Pavement Design, Journal of Computer-Aided Civil and Infrastructure Engineering, No. 17, pp. 22-30.

Minnesota Department of Transportation, Office of Materials and Road Research (2002), 2002 Mn/Road Hot-Mix Asphalt Mainline Test Cells Condition Report.

Minnesota Department of Transportation, Minnesota Road Test Project, Website: [http://www.mrr.dot.state.mn.us/research/MnROAD\\_Project/MnROADProject.asp](http://www.mrr.dot.state.mn.us/research/MnROAD_Project/MnROADProject.asp), Accessed, April, 2005.

National Center for Asphalt Technology, NCAT Test Track Design, Construction, and Performance, [www.eng.auburn.edu/center/ncat/reports/rep02-12.pdf](http://www.eng.auburn.edu/center/ncat/reports/rep02-12.pdf), Accessed, August, 2006.

National Cooperative Highway Research Program, Mechanistic-Empirical Design of New & Rehabilitated Pavement Structures, NCHRP 1-37a, Website: [www.trb.org/mepdg](http://www.trb.org/mepdg), Accessed, January, 2006.

Ouyang, Y. and S. M. Madanat (2004), Optimal Scheduling of Rehabilitation Activities for Multiple Pavement Facilities: Exact and Appropriate Solutions, Transportation Research, Vol. 38 A, pp. 347-365.

Paterson, W. D. O. (1987), Road Deterioration and Maintenance Effects: models for planning and management, The John Hopkins University Press, Baltimore, Md.

Prozzi, J. A. and S. M. Madanat (2000), Using Duration Models to Analyze Experimental Pavement Failure Data, Transportation Research Record 1699, Transportation Research Board, National Research Council, Washington, D.C., pp87-94.

Prozzi, J. A. (2001), Modeling Pavement Performance by Combining Field and Experimental Data, PhD dissertation, University of California, Berkeley, CA.

Prozzi, J. A., and S.M. Madanat (2003), Incremental Nonlinear Model for Predicting Pavement Serviceability, Journal of Transportation Engineering, American Society of Civil Engineers, 129(6), pp. 635–641.

Prozzi, J. A. and S. M. Madanat (2004), Development of Pavement Performance Models by Combining Experimental and Field Data, Journal of Infrastructure Systems, American Society of Civil Engineers, Vol. 10, No. 9. pp 9-22.

Prozzi J. and F. Hong (2006), Optimum Statistical Characterization of Axle Load Spectra Based on Load-Associated Damage, Forthcoming at International Journal of Pavement Engineering.

Roberts, F. L., Kandhal, E. R. Brown, D. Lee, and T. Kennedy (1996), *Hot Mix Asphalt Materials, Mixture Design, and Construction*, 2<sup>nd</sup> Edition, National Asphalt Pavement Association Research and Education Foundation, Lanham, Maryland.

Rauhut, J. B., R. L. Lytton, P. R. Jordhal, and W. J. Kenis, (1983), *Damage Functions for Rutting, Fatigue Cracking and Loss of Serviceability in Flexible Pavements*, Transportation Research Record 943, Transportation Research Board, National Research Council, Washington, D.C., pp. 1–9.

Small, K. and C. Winston (1988), *Optimal Highway Durability*, the Academic Economic Review, Vol. 78, No. 3, pp. 560-569.

Small, K., C. Winston, and C. Evans (1989), *Road Work: A New Highway Pricing and Investment Policy*, The Brookings Institution, Washington, D. C.

Train, K. (2003), *Discrete Choice Methods with Simulation*, Cambridge University Press.

Tsunokawa, K. and J. L. Schofer (1994), *Trend Curve Optimal Control Model for Highway pavement Maintenance: Case Study an Evaluation*, Transportation Research, Vol. 28A, pp. 151 – 166.

Watanatada, T., C. G. Harral, W. D. O. Paterson, A. M. Dhareshwar, A. Bhandari, and K. Tsunokawa (1987), *The Highway Design and Maintenance Standards Model*, Vol. 1, Description of the HDM-III Model, The World Bank, Washington, DC.

Von Quintus, H. L., C. Schwartz, R. H. McCuen, and D. Andrei (2003), *Jackknife Testing- An Experimental Approach to Refine Model Calibration and Validation*, National Cooperative Highway Research Program, Research Results Digest 283.

Wang, X. and K. Kockelman K (2006), Tracking Land Cover Change in a Mixed Logit Model: Recognizing Temporal and Spatial Effects, Forthcoming in Transportation Research Record, Transportation Research Board, National Research Council, Washington, D.C.

WesTrack Road Test, Website: [http://www.westrack.com/wt\\_02.htm](http://www.westrack.com/wt_02.htm), Accessed, May, 2006.

Wooldridge, J. M. (2001), Econometric Analysis of Cross Section and Panel Data, The MIT Press, Cambridge, Massachusetts.

## **Vita**

Feng Hong was born in Yixing, Jiangsu Province, China, on August 27, 1977. He is the only son of Junying Lv and Nanqi Hong. Feng Hong entered Southeast University, Nanjing, China in 1995 and received the Bachelor's degree in 1999. After that, he worked as a research staff at the Highway Research Institute at Southeast University for two years. He studied as a master student from 2001 to 2002 at the graduate school of Southeast University. In 2002 he entered the University of Texas at Austin to pursue the Ph.D. degree.

Permanent Address: 287 Nanzhuang, Fengyi, Yixing, Jiangsu, China 214252

This dissertation was typed by the author.