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Three Essays on Openness, International Pricing, and Optimal Monetary Policy

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Three Essays on Openness, International Pricing, and Optimal Monetary Policy

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This work is dedicated to my sweet wife, Marty,
and to my three children, William, Jane, and Katherine
who have supported me and endured through
seven years of graduate school.

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Three Essays on Openness, International Pricing, and Optimal Monetary Policy

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The unifying theme of this dissertation is to ask questions about how pricing inefficiencies and institutional characteristics interact to influence the aggregate real outcomes of countries in an open economy setting in which each country's monetary policy is set optimally. Chapter 1 tries to answer the question of whether openness is inflationary using a two-country general equilibrium model with optimal monetary policy that is explicitly derived from microeconomic foundations. Imperfect competition plays a key role and is modeled as a degree of inelasticity of substitution among differentiated goods. I find that a country's inflation rate increases with its degree of openness and that this inflationary effect is dampened by the degree of imperfect competition.

In Chapter 2, I ask the same question of whether openness is inflationary, but I change the imperfect competition structure from Chapter 1 so that workers supply differentiated labor to a competitive final goods producer. This more closely follows the theoretical story cited by much of the empirical literature on openness and inflation. However, the interesting result in Chapter 2 is that the implications for optimal monetary policy and real outcomes are the same as in Chapter 1. That is, the source of the imperfect competition does not matter. Chapter 2 then goes on to evaluate much of the empirical literature on the basis of whether it controls for imperfect competition among goods producers and among suppliers of labor. Finally Chapter 2 includes an empirical test of the theory. Using a sample from 1987 to 2002, the data confirm the implication of my model that increased openness can be inflationary—a result that contradicts much of the previous empirical literature

Lastly, Chapter 3 deals with the question of how a monetary authority should respond to foreign monetary policy. The model relaxes the assumption of rational expectations in order to generate steady state equilibria that are neither overly inflationary nor independent of foreign monetary policy. The resulting monetary policy rules are well below the upper bound of money growth and are an increasing function of the history of foreign monetary policy.

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Chapter 1

Is Openness Inflationary? Imperfect Competition and Monetary Market Power

1.1 Introduction

The goal of this paper is to try to answer the question of whether or not increased openness to international markets is inflationary using a structural international general equilibrium model derived from microeconomic foundations. This question has been the subject of a large body of research beginning as early as 1962 and continuing to the present. Most of these papers have been empirical in focus and provide strong evidence of a negative relationship between openness and inflation. However, much less work exists that structurally models this relationship beginning with the behavior of individual agents. This paper is intended as an attempt at furthering the theoretical understanding of some specific channels through which economic openness may influence a country's inflation rate.

A major focus of this work and one of its main innovations is how the level of imperfect competition, both within a country and between countries, affects the relationship between openness and inflation. Some of the literature has begun to assess the relationship between imperfect competition and inflation in open economies, but this is the first paper that specifically models how imperfect competition affects the relationship between openness and inflation.

To address this question, I use a two-country overlapping generations (OLG) model in which agents are born in each country in each period and live for only two periods. The agents use their labor to produce a differentiated good in the first period of their lives for which they enjoy a degree of market power, and they sell the good to consumers from both countries in exchange for the producers' country's currency. A monetary authority in each country chooses and commits to a money growth rate at the beginning of time and implements that policy through non-proportional transfers to the consumers of its own country in each period so as to maximize the welfare of its citizens.

The results derived from this model run counter to most of the findings from the literature addressing the question of the effect of openness on inflation. I find that an increased level of openness actually increases the steady-state equilibrium inflation rate in a country. In a closed economy and in environments in which money is not neutral, increased money growth generates inflation which provides a leisure subsidy and levies a consumption tax. However, in the environment laid out in this paper, increased openness to international trade opens up two new channels through which a country's inflation rate benefits its citizens.

First, increased openness reduces the burden of the inflation tax borne by the citizens of the inflating country in that they spend a larger portion of their currency holdings on Foreign goods. Second, inflation causes the terms of trade to appreciate in favor of the Home country. That is, the price of exports increase in relation to the price of imports. These two benefits working together result in a country's real wage increasing in response to higher Home inflation levels. These benefits are generated by a degree of market power enjoyed by each monetary authority in the international markets due to the assumption that consumers in each country prefer some consumption combination of its own country's production, the assumption that a consumer's expenditure share on the other country's goods is inelastic to some degree, and the institutional assumption that consumers must hold both countries' currencies in order to consume both of their goods. The problem of the monetary authority then becomes choosing the money growth rate and the associated rate of inflation so as to balance the resulting consumption tax with the real-wage benefit (consumption tax burden shift plus terms of trade appreciation).

In addition, I find that the level of imperfect competition among the producers within a country acts as a perfect substitute for the market power enjoyed by a country's monetary authority. That is, an increased level of imperfect competition among producers within a country reduces the benefits that result from inflation generated by that country's monetary authority. Put differently, a fixed amount of international monopoly rents are available to the citizens of each country given the structure of the model, and whatever percentage of those rents are not obtained through the pricing behavior of each country's producers is obtained by that country's monetary authority changing the inflation rate through the money growth rate. So this model predicts a negative relation between a country's inflation rate and the level of imperfect competition, given the degree of openness to international markets. Thus, the channel through which openness affects inflation is the international market power that a country enjoys.

The structure of the paper is as follows. In Section 1.2, I survey the literature that has addressed the question of openness and inflation. Section 1.3 presents the model and its equilibrium properties. Section 1.4 presents the key results from the model, and Section 1.5 concludes.

1.2 Literature

This paper's place in the international monetary literature is to provide a simple attempt at a micro-founded structural model of openness, inflation, and imperfect competition in order to try to match the relationship between openness and inflation documented in the empirical literature. The oldest branch of the theoretical literature uses a structural model that is an international version of Barro and Gordon (1983) which predicts that, other things equal, openness leads to a lower inflation rate. But a newer branch of the literature can be loosely grouped under the rubric of "new open economy macroeconomics" (NOEM) models, and predicts that, other things equal, more openness leads to a higher inflation rate. The modeling approach I use in this paper will follow the NOEM style for reasons that I will detail below.

One of the earliest empirical papers addressing the question of the relationship between openness and inflation, although somewhat indirectly, is Triffin and Grubel (1962). Using data from six European countries during the 1950s, they provide evidence that inflationary pressures are more correlated, and thus less independent, across countries that are more integrated. They propose that, among countries that are more open and integrated, inflation generated by a monetary authority can have more of an effect on the balance of payments, than on inflation. However, they only mention in passing that this balance of payments effect can only be short-term, and they assume no optimizing behavior by the government, consumers, or firms.

In his famous AEA Presidential address, Friedman (1968) proposed that monetary policy should target inflation or money growth rates. But he also added indirectly that exchange rate targeting could be more desirable if imports were a bigger share of GDP, thus implying a potential connection between openness and inflation.[1]

The first structural model directly addressing the question of openness and inflation is Rogoff (1985). His approach is to extend the Barro and Gordon (1983) framework to a two-country Mundell-Fleming model. As in Barro and Gordon, a labor market friction causes the optimal time-consistent policy of the monetary authority to be increased inflation in order to raise the level of employment. However, in Rogoff's international version, the increased inflation has an extra cost in that optimal employment is a function of the real exchange rate and that the real exchange rate depreciates with higher inflation. Thus the optimal time-consistent inflation rate chosen by a monetary authority is lower as the deteriorating effect on the exchange rate increases. More openness leads to a lower equilibrium inflation rate in this time consistent environment.

The empirical literature testing the effect of openness on inflation primarily cites the model and conclusions of Rogoff (1985). The most important empirical paper that addresses this question is Romer (1993). He cites the Rogoff prediction that, in his time-consistent environment, more openness should lead to lower inflation. In his regressions, Romer controls for endogeneity, includes political controls, development level controls, regional controls, and uses many different samples of countries over the post-Bretton Woods period from 1973 to the early 1990s. Romer's empirical findings lend support to the theoretical results of Rogoff (1985) in that he finds robust evidence of a negative relationship between openness and inflation and that the negative relationship becomes weaker in countries with less independent central banks and more political instability.[2]

[1] On page 15, Friedman makes the contrapositive statement that, with only 5 percent of U.S. resources devoted to international trade in 1967, "it would be better to let the market, through floating exchange rates, adjust to world conditions."

[2] A number of empirical papers follow up on Romer (1993), and most of them either confirm his

Figure 1.1: Import share vs. CPI for 30 OECD countries: annual avg. for 1982 to 2005

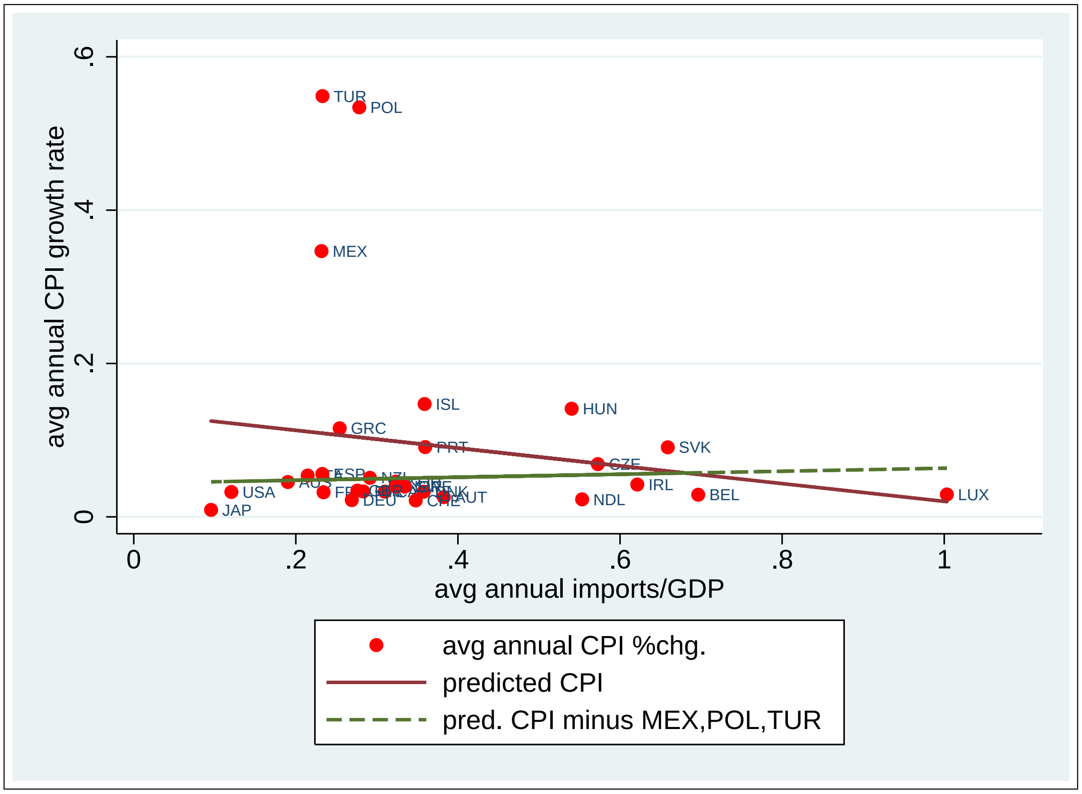


Figure 1.1 shows a scatterplot of the average annual import share and average annual CPI growth rate for the 30 OECD countries over the period from 1982 to 2005. This picture is similar to figures in Romer (1993) and Wynne and Kersting (2007) and is common to this empirical literature. However, the conclusions to take from Figure 1.1 are not obvious. A slight negative correlation exists between import share and inflation over the sample period (solid line), but that negative relationship becomes positive when I drop the three high-inflation outliers of Mexico, Poland, and Turkey (dotted line). Restricting the sample to the G7 countries produces a positive correlation nearly identical to that of the whole sample minus Mexico, Poland, and Turkey. When the sample period is shortened to more recent periods,

finding of a negative relationship or find that the relationship is not statistically significant. Wynne and Kersting (2007) provide a good survey of the empirical literature as well as some of their own analyses.

the negative relationship with all the countries and the positive relationship without the high-inflation countries both diminish to the point where the two predicted value lines for the year 2005 are nearly indistinguishable and are both slightly positive. However, none of the slopes in any specification is significantly different from zero.[3]

The "natural rate" approach of the model used in Rogoff (1985) has been criticized on a number of dimensions. Azariadis (1981) questions the Phillips curve assumption of dropping all but the first two terms of a Taylor series expansion of the aggregate supply equation around the expected logarithm of price. Also, the natural rate models on which so much of empirical monetary policy today is based, assume that the welfare of a representative agent is a quadratic loss function in the deviation of output from its natural rate and in the deviation of inflation from expected inflation. This type of disutility function is a step removed from maximization of individual's utility functions that is standard in most micro-founded macroeconomics.

Another key characteristic implicit in the Rogoff model is that the labor market friction that causes the optimal employment level to be higher than the level desired by the suppliers of labor could be caused by some form of monopoly power on the part of these suppliers such as a labor union. Thus, the monetary authority uses the inflationary money injection to induce higher demand which causes the owners of labor to supply more. Intuitively, the more open an economy is, the less market power the monopolistic labor suppliers enjoy and the less incentive a monetary authority has to inflate.

An alternative to the natural rate international models mentioned above for addressing the relationship between openness and inflation are some more recent works related to the NOEM models. A number of optimal monetary policy papers have come out in recently in this vein of the literature that address optimal inflation levels generated by a monetary authority in general equilibrium multi-country environments in which firms and consumers are acting optimally and the monetary authority is maximizing the utility of its citizens.

Cooley and Quadrini (2003) and Cooper and Kempf (2003) both use models in which the production market is perfectly competitive to answer the questions of whether and when countries gain from cooperating in currency unions. An attempt

[3]Other authors obtain statistically significant correlations by expanding the sample of countries and by controlling for other variables to isolate the effect of openness on inflation.

to categorize them might place them close to the "new open economy macroeconomic" (NOEM) models literature, except that they both feature perfectly competitive markets. Cooley and Quadrini (2003) employ a model in which Home final goods producers use inputs from both Home and Foreign intermediate goods producers, and then consumers in each country only consume the final goods produced in their own country. Monetary policy in Cooley and Quadrini is a country's monetary authority choosing a nominal interest rate on a bond that final goods producers in both countries purchase to finance the intermediate inputs from both countries.

Cooper and Kempf (2003) use a technique that is conceptually different but structurally similar in which consumers only care about final goods consumption and that the final goods consumption is an aggregation of a Home produced good and a Foreign produced good in an OLG setting. Monetary policy in Cooper and Kempf is a country's monetary authority choosing a currency growth rate. They impose two cash-in-advance constraints such that a Home consumer must pay for Home produced goods in his own currency and he must pay for Foreign produced goods in the Foreign currency.

In both papers, the standard consumption tax of inflation results. But, in the two-country setting with international trade, both papers find that the a degree of monetary market power—derived in Cooley and Quadrini (2003) from some degree of inelasticity in the demand for both Home and Foreign intermediate goods and derived in Cooper and Kempf (2003) from a degree of inelasticity in the demand for Home and Foreign final goods—generates an added benefit to inflation of being able to appreciate the terms of trade in favor of the inflating country. Cooley and Quadrini find that this inflationary bias in open economies is actually larger if the monetary authority cannot commit to a policy.

In a more traditional NOEM paper, Arseneau (2007) uses a model very similar to Corsetti and Pesenti (2001) that adds imperfectly competitive firms in each country. In an environment in which the monetary authority can commit to policy, Arseneau confirms the inflationary bias of monetary policy result from Cooley and Quadrini (2003) and Cooper and Kempf (2003). In addition, Arseneau shows that the degree of imperfect competition can dampen the inflationary bias and can even fully offset it such that the equilibrium inflation rate is zero or negative. However, none of the four NOEM papers discussed in the previous paragraphs attempts to answer the question of how the degree of openness in a country affects its equilibrium inflation level when monetary policy is set optimally.

Analogous to the interpretation of the mechanism of the "natural rate" models but with an opposite result, the following interpretation applies to these NOEM models with imperfectly competitive firms. In a closed economy, the monetary authority has an incentive to deflate in order to offset the inefficiently high price and low output level caused by the market power held by firms. However, this degree of market power is eroded as the country becomes more open and the elasticity of substitution between Home and Foreign consumption is less than the elasticity of substitution among the goods of a given country.

The goal of this paper is to use the micro-founded two-country model with optimal monetary policy in this paper that borrows heavily from the NOEM literature, instead of following the Mundell-Fleming "natural rate" approach, to try and match the relationship borne out in the data that openness is negatively correlated with inflation levels.

1.3 Model

Following Cooper and Kempf (2003), I use a two-country OLG general equilibrium framework with an independent monetary authority in each country that maximizes the welfare of its own citizens. In addition, similar to Arseneau (2007), the model includes imperfectly competitive producers in each country. The model includes no stochastic shocks and agents enjoy perfect foresight.

I will call the two countries Home and Foreign, which are not relative terms but are the names of the actual countries. Most of the exposition in this section will focus on the problem of Home agents and the Home monetary authority, but the Foreign problem is symmetric in almost every dimension. However, I will allow Home and Foreign countries to differ in their respective levels of openness to international trade in a way that I will specify. Within a country, I assume the equilibrium is symmetric, so I will drop any subscripting of individuals.

This stylized economy is made up of two countries, each of which has a monetary authority, producers, and consumers. The overlapping generations of agents live for two periods. In the first period of their lives, they produce differentiated goods in a monopolistically competitive environment and sell the goods to both Home and Foreign consumers in exchange for the producer's Home-currency. The producers then choose how much of their Home currency to hold and how much of the Foreign currency to hold given that they will use a portfolio of each respective currency to consume Home and Foreign goods in the second period of their lives.

The role of each country's monetary authority is to maximize the lifetime welfare of the representative agent in the Home country by giving a non-proportional transfer of Home currency to the consumers of its own country in each period. Money is held in this economy because it is the only store of value, and changes in the money supply are not neutral due to the transfers being non-proportional.[4] The two cash-in-advance constraints and consumer preferences generate demand for both currencies by a given consumer.

1.3.1 Money

The objective of the monetary authority in each country, which will be made more explicit in Section 1.3.4, is to choose a fixed (gross) money growth rate $x_t = x$ or $x_t^* = x^*$ at the beginning of time in such a way as to maximize the welfare of its own citizens. I assume here that the monetary authority is committed to its money growth rate and cannot deviate once it has chosen its money growth path.[5]

Let M_t and M_t^* be the aggregate supply of Home currency and Foreign currency, respectively, in period t. I normalize the initial supply of Home and Foreign currency to 1 and divide it equally among the period-1 consumers at the beginning of the period.

$$M_0 = M_0^* = 1 \quad \text{and} \quad m_0^h = m_0^f = m_0^{h*} = m_0^{f*} = \frac{1}{2} \tag{1.1}$$

where m_0^h and m_0^{f*} are the individual holdings of Home currency by Home consumers and Foreign currency by Foreign consumers, respectively, at the beginning of period 1. Each country's monetary authority makes non-proportional transfers of $(x-1)M_{t-1}^h$ to each Home consumer in period t and $(x^*-1)M_{t-1}^f$ to each Foreign consumer where x and x^* represent the respective constant gross money growth rates of each country. So aggregate supply of currency in each country obeys the

[4]See Azariadis (1981) for a proof that non-proportional monetary transfers are not neutral, even in a perfect foresight economy.

[5]The reason to avoid discretionary monetary policy in this paper is due to the resulting characteristic of multiple equilibria as described in Barro and Gordon (1983) and Ireland (1997). See also King and Wolman (2004) and Chatterjee, Cooper, and Ravikumar (1993).

following laws of motion.

$$M_{t+1} = xM_t \tag{1.2}$$

$$M^*_{t+1} = x^* M^*_t \tag{1.3}$$

This implies that the following relationships for τ_{t+1} and τ^*_{t+1} represent the non-proportional transfer to each Home consumer and to each Foreign consumer by their respective monetary authorities.

$$\tau_{t+1} = (x - 1)\, M_t \tag{1.4}$$

$$\tau^*_{t+1} = (x^* - 1)\, M_{t^*} \tag{1.5}$$

At the end of the first period of their lives, producers make a portfolio decision of how much of each type of currency to hold. They have just received either $p_t(z)n_t(z)$ in Home currency or $p_t(z^*)n_t(z^*)$ in Foreign currency from the sale of their differentiated goods. Now, before the end of the first period of life, producers in each country exchange some of their Home currency balances from sales revenues for Foreign currency balances at the exchange rate e_t as shown in the budget constraint equation (1.27). Let m^h_t and m^f_t represent each Home producer's portfolio choice between Home and Foreign currency, respectively, in period t. Because the monetary authority of each country only transfers currency to its own consumers, the laws of motion for individual currency balances are the following:

$$m^h_{t+1} = m^h_t + \tau_{t+1} \tag{1.6}$$

$$m^f_{t+1} = m^f_t \tag{1.7}$$

$$m^{f*}_{t+1} = m^{f*}_t + \tau^*_{t+1} \tag{1.8}$$

$$m^{h*}_{t+1} = m^{h*}_t \tag{1.9}$$

Because the equilibrium currency holdings within each country are symmetric, then m^h_t, m^f_t, m^{f*}_t, m^{h*}_t represent the aggregate amounts of each currency $(M^h_t, M^f_t, M^{h*}_t, M^{f*}_t)$ held in each country in each period.

1.3.2 Individuals

A unit measure of agents are born in each period in both the Home country (indexed by z) and the Foreign country (indexed by z^*). In the first period of their lives, individuals can either enjoy leisure l_t or provide labor $n_t(z)$ subject to their endowment of one unit of time.

$$l_t + n_t(z) = 1 \quad \forall t, z \tag{1.10}$$

Each individual also has access to a technology through which she can convert labor hours into a differentiated good indexed by the individual z for each Home producer and z^* for each Foreign producer.[6]

$$y_t(z) = f\left(n_t(z)\right) \quad \forall t, z \quad \text{where} \quad f\left(n_t(z)\right) = n_t(z) \tag{1.11}$$

I follow an international variation of the model of monopolistic competition of Dixit and Stiglitz (1977).[7] I assume that individuals only care about aggregate consumption, where each Home consumer's aggregate consumption of Home produced goods C^h_{t+1} and aggregate Home consumption of Foreign produced goods C^f_{t+1} are defined as:

$$C^h_{t+1} \equiv \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\, dz\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{1.12}$$

$$C^f_{t+1} \equiv \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\, dz^*\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{1.13}$$

where $\varepsilon \geq 0$ represents the elasticity of substitution among all the differentiated goods in country either the Home country or the Foreign country. Symmetric to the Home consumer, each Foreign consumer's aggregate consumption of Foreign produced goods C^{f*}_{t+1} and aggregate Foreign consumption of Home produced goods

[6] The equilibrium outcomes in Section 1.3.4 and results in Section 1.4 from this structure in which the imperfect competition is among producers of a differentiated good are equivalent to the equilibrium outcomes and results in a similar model in which the imperfect competition is among labor suppliers who provide differentiated labor to identical firms that are perfectly competitive.

[7] Good example of this type of international monetary model with monopolistic competition are Corsetti and Pesenti (2001) and Arseneau (2007). Appendix A-4 has a full derivation of the demand and price functions shown below that result from this monopolistic competition structure.

C^{h*}_{t+1} is defined as:

$$C^{f*}_{t+1} \equiv \left(\int_0^1 c^*_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{1.14}$$

$$C^{h*}_{t+1} \equiv \left(\int_0^1 c^*_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{1.15}$$

Total consumption by each Home and Foreign consumer is further aggregated over her aggregate consumption of Home produced goods and aggregate consumption of Foreign produced goods using an analogous CES aggregator of total consumption:

$$C_{t+1} \equiv \left[(1-\theta_h)^{\frac{1}{\rho}} \left(C^h_{t+1} \right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}} \left(C^f_{t+1} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_h \in \left[0, \frac{1}{2} \right] \tag{1.16}$$

$$C^*_{t+1} \equiv \left[(1-\theta_f)^{\frac{1}{\rho}} \left(C^{f*}_{t+1} \right)^{\frac{\rho-1}{\rho}} + \theta_f^{\frac{1}{\rho}} \left(C^{h*}_{t+1} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_f \in \left[0, \frac{1}{2} \right] \tag{1.17}$$

where θ_h and θ_f are the Home bias parameters for the Home country and the Foreign country, respectively, and $\rho \geq 0$ is the elasticity of substitution between a unit of Home consumption and a unit of Foreign consumption.[8] For the analysis in this paper, I will assume that the elasticity of substitution between a unit of Home aggregate consumption and a unit of Foreign aggregate consumption is equal to 1 ($\rho = 1$) which results in the following Cobb-Douglas aggregation at this level.

$$C_{t+1} \equiv \left(C^h_{t+1} \right)^{1-\theta_h} \left(C^f_{t+1} \right)^{\theta_h} \quad \text{for} \quad \theta_h \in \left[0, \frac{1}{2} \right] \tag{1.18}$$

$$C^*_{t+1} \equiv \left(C^{f*}_{t+1} \right)^{1-\theta_f} \left(C^{h*}_{t+1} \right)^{\theta_f} \quad \text{for} \quad \theta_f \in \left[0, \frac{1}{2} \right] \tag{1.19}$$

The Home and Foreign countries are symmetric in every dimension except for the Home bias parameters. This assumption seems to fit the empirical evidence that import shares differ across countries, as shown in Figure 1.1.

This functional form assumption is strong because it forces individuals to spend a fixed portion of their earnings on consumption of Home-produced goods.

[8] To be more specific, the parameter space of the two Home bias parameters is $\{\theta_h, \theta_f\} = \{0, 0\}$ or $\{\theta_h, \theta_f\} = \{(0, 0.5], (0, 0.5]\}$.

However, the general case results in a degenerate equilibrium. The Cobb-Douglas aggregator has the desirable pedagogical property of allowing for analytical solutions.[9] Whatever the specification, a key intuitive relationship is that the elasticity of substitution among differentiated goods within a country is different from and greater than the elasticity of substitution between aggregate and Foreign consumption $\varepsilon > \rho$. This is the main source of the international market power a monetary authority enjoys when a country becomes more open.

The following individual demand and price relationships result from the problem of an agent minimizing her expenditure given a certain level of aggregate consumption.[10]

$$\begin{aligned} c_{t+1}(z) &= \left(\frac{p_{t+1}(z)}{P^h_{t+1}}\right)^{-\varepsilon} C^h_{t+1} \quad \forall\, t, z \\ c_{t+1}(z^*) &= \left(\frac{p_{t+1}(z^*)}{P^f_{t+1}}\right)^{-\varepsilon} C^f_{t+1} \quad \forall\, t, z^* \end{aligned} \tag{1.20}$$

$$\begin{aligned} P^h_{t+1} &= \left(\int_0^1 p_{t+1}(z)^{1-\varepsilon}\, dz\right)^{\frac{1}{1-\varepsilon}} \quad \forall\, t \\ P^f_{t+1} &= \left(\int_0^1 p_{t+1}(z^*)^{1-\varepsilon}\, dz^*\right)^{\frac{1}{1-\varepsilon}} \quad \forall\, t \end{aligned} \tag{1.21}$$

$$P_{t+1} = \frac{1}{(1-\theta_h)^{1-\theta_h}\theta_h^{\theta_h}} \left(P^h_{t+1}\right)^{1-\theta_h} \left(e_t P^f_{t+1}\right)^{\theta_h} \tag{1.22}$$

where $p_{t+1}(z)$, P^h_{t+1}, and P_{t+1} are prices of individual consumption, aggregate country-specific consumption, and aggregate total consumption, respectively. Analogous to the derivation for the demand for individual differentiated goods z and z^* in (1.20), each Home consumer's demand for aggregate Home consumption and aggregate Foreign consumption, respectively, are the following:

$$C^h_{t+1} = (1-\theta_h)\left(\frac{P^h_{t+1}}{P_{t+1}}\right)^{-1} C_{t+1} \tag{1.23}$$

[9] Appendix A-6 provides some of the results of what happens when using the general form of the CES aggregator.

[10] The expenditure minimization problem is preferred to the utility maximization problem because the multiplier on the aggregate consumption constraint in the minimization problem has the interpretation of the aggregate price.

$$C^f_{t+1} = \theta_h \left(\frac{e_t P^f_{t+1}}{P_{t+1}}\right)^{-1} C_{t+1} \tag{1.24}$$

These two equations simply imply that the expenditure on Home aggregate consumption and the expenditure on Foreign aggregate consumption are constant shares of total expenditure. Another way of putting this is to divide (1.23) by (1.24), which gives the following relationship that describes the relationship between total expenditures on Home consumption to total expenditure on Foreign consumption.

$$\frac{P^h_{t+1} C^h_{t+1}}{e_t P^f_{t+1} C^f_{t+1}} = \frac{1-\theta_h}{\theta_h} \tag{1.25}$$

The ratio of total Home consumption to total Foreign consumption is a constant. That is, θ_h represents the Home expenditure share on Foreign consumption or the import share. Equations (1.20) through (1.25) result directly from the Dixit-Stiglitz monopolistic competition structure and from the CES aggregation.[11]

Individuals seek to maximize lifetime utility derived from disutility of work in the first period of life in order to sell a differentiated production good for own-country currency balances that are carried over to the second period of life in which the individual can spend those balances on consumption of both Home and Foreign goods. Because the monopolistically competitive producers can set the quantity demanded by choosing price in order to clear their goods, the consumer's problem is characterized by choosing how much to charge for her differentiated good $p_t(z)$ and then the portfolio decision of how much of her sales to keep in the form of Home currency m^h_t and how much to exchange for Foreign currency m^f_t.[12]

$$\begin{aligned} &\max_{m^h_t, m^f_t, p_t(z)} \quad u\left(C_{t+1}\right) - g\left(n_t(z)\right) \\ &\text{where} \quad u\left(C_{t+1}\right) = \frac{\left(C_{t+1}\right)^{1-\sigma} - 1}{1-\sigma} \quad \text{for} \quad \sigma > 0 \\ &\text{and} \quad g\left(n_t(z)\right) = \chi\left(n_t(z)\right)^{\xi} \quad \text{for} \quad \chi > 0 \quad \text{and} \quad \xi \geq 1 \end{aligned} \tag{1.26}$$

[11] The CES consumption aggregation in (1.12), (1.13), and (1.18) can also be interpreted as CES utility over Home and Foreign differentiated goods. Appendix A-5 details the properties of the CES aggregator.

[12] An implicit assumption in this setup is that the producer will meet demand, whatever it is. Thus the producer sets price $p_t(z)$ and then produces $n_t(z)$ to meet the resulting demand. Some other interesting cases arise in a model with shocks when producers are not required to meet demand.

$$\text{s.t.} \quad p_t(z)n_t(z) = m_t^h + e_t m_t^f \tag{1.27}$$

$$P_{t+1}^h C_{t+1}^h = m_t^h + \tau_{t+1} \tag{1.28}$$

$$P_{t+1}^f C_{t+1}^f = m_t^f \tag{1.29}$$

where (1.27) is the budget constraint reflecting the portfolio decision and (1.28) and (1.29) are cash-in-advance constraints.

The two cash-in-advance constraints can be thought of as a simplification of one equilibrium outcome of a richer environment in which governments or monetary authorities strategically choose what currencies to accept for exchange that takes place within their borders. Matsuyama, Kiyotaki, and Matsui (1993) present a random matching search model of money after the flavor of Kiyotaki and Wright (1989) in which blocks of agents (countries) choose which currencies to accept for local and international transactions based on the likelihood of that currency being accepted in future transactions. In one equilibrium, corresponding to the two cash-in-advance constraint environment of this paper, each block of agents (country) only accepts local currency for all local and international transactions.

Another equilibrium in the Matsuyama, Kiyotaki, and Matsui (1993) is the case in which vendors in both countries accept currency of both countries. This is analogous to the more standard approach in the NOEM literature as exemplified by Corsetti and Pesenti (2001). Their environment is one characterized by a single cash-in-advance constraint in which producers sell their goods in both countries and charge a price in terms of Home currency and a price in terms of Foreign currency. The exchange rate is then pinned down by an assumption of the law of one price.

The reason for choosing the two cash-in-advance constraints approach as shown in equations (1.28) and (1.29) instead of the more standard Corsetti and Pesenti (2001) method of one cash-in-advance constraint and the law of one price is that the method employed here gives rise to a portfolio decision. The law of one price is implicit in the two cash-in-advance constraint assumption because, by definition, vendors only accept one currency and therefore only charge one price. As will be in Section 1.3.3, the exchange rate here serves as a price that clears the currency exchange market rather than a mechanism for enforcing the law of one price. Furthermore, the currency portfolio decision is an interesting one that has not received much attention.[13] However, both the single CIA constraint with the law

[13] Add some references here to the currency portfolio choice literature, such as Engel and Mat-

of one price method and the dual CIA constraints with currency exchange market clearing method deliver the same results for optimal monetary policy.

Using the individual demand equations represented by (1.20), I define the total demand $d_t(z)$ for differentiated Home good z as the sum of the individual Home and Foreign demands:[14]

$$d_t(z) \equiv c_t(z) + c_t^*(z) = \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h} \tag{1.30}$$

I assume that producers always choose price to maximize utility given their knowledge of total demand $d_t(z)$ and then meet the demand.

$$n_t(z) = d_t(z) = \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h} \tag{1.31}$$

Using the cash-in-advance constraints (1.28) and (1.29), the money laws of motion (1.6) and (1.7), and the expressions for the non-proportional transfer in terms of the Home money growth rate (1.4), country-specific aggregate consumptions can be expressed in the following way:

$$C_{t+1}^h = \frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h} \tag{1.32}$$

$$C_{t+1}^f = \frac{m_t^f}{P_{t+1}^f} \tag{1.33}$$

The expression for Home aggregate total consumption is then:

$$C_{t+1} = \left(\frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h} \tag{1.34}$$

Using the portfolio constraint in (1.27) to substitute out either $m_{i,t}^h$ or $m_{i,t}^f$ and substituting in the expression for labor supply from (1.31), the maximization problem

sumoto (2006) and Evans and Lyons (2005).

[14] The derivation is given in Derivation 1 in Appendix A-3.

then becomes

$$\max_{m_t^f, p_t(z)} \frac{\left[\left(\left[\frac{p_t(z)}{P_t^h}\right]^{1-\varepsilon} \frac{xM_{t-1}}{P_{t+1}^h} - \frac{e_t m_t^f - (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h}\right]^{1-\sigma} - 1}{1-\sigma} \dots \\ -\chi\left[\left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h}\right]^{\xi} \tag{1.35}$$

The first order conditions with respect to m_t^f and $p_t(z)$, respectively, are:

$$\frac{P_{t+1}^h C_{t+1}^h}{e_t P_{t+1}^f C_{t+1}^f} = \frac{1-\theta_h}{\theta_h} \tag{1.36}$$

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z)}{P_{t+1}^h}\left(C_{t+1}^h\right)^{(1-\theta_h)(1-\sigma)-1}\left(C_{t+1}^f\right)^{\theta_h(1-\sigma)} = \chi\xi\left(n_t(z)\right)^{\xi-1} \tag{1.37}$$

where equation (1.36) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (1.37) equates the marginal benefit of raising price to its marginal cost in terms of reduced demand, increased utility of leisure, and the change in income in the next period of life. Because each agent within a country is identical, other than for a differentiated production good, the resulting individual equilibrium price $p_t(z)$ and the amount of total revenues held in Foreign currency m_t^f will be symmetric across individuals in a given country.

Notice that the first order condition for m_t^f in (1.36) is equivalent to the condition (1.25) that results from the two first order conditions in the imperfect competition expenditure minimization problem. This is because the optimal choice of m_t^f in period t is equivalent to the optimal choice of C_{t+1}^h and C_{t+1}^f in period $t+1$. These two decisions are equivalent and take the labor or pricing decision as given.

1.3.3 Market clearing conditions

This economy has three markets that must clear—the goods market, the money market, and the exchange market. The following paragraphs describe each market and the respective market clearing condition.

Goods Market. Both Home and Foreign consumers demand goods from both countries. Producers meet that demand by construction in this model. Let $n_t(z)$ represent the amount of production by each Home producer of differentiated good z. Goods market clearing requires that production equal the sum of all the Home demands $c_t(z)$ and Foreign demands $c_t^*(z)$ for differentiated good z.

$$n_t(z) = d_t(z) = c_t(z) + c_t^*(z) \quad \forall\, t, z \tag{1.38}$$

$$n_t(z^*) = d_t(z^*) = c_t(z^*) + c_t^*(z^*) \quad \forall\, t, z^* \tag{1.39}$$

where the the right-hand side of each equation is characterized by equation (1.30) and its Foreign country analogue. This market clearing condition is actually assumed in the individual maximization stage as shown in (1.31).

Money Market. Money market clearing simply requires that money supply equal money demand at the time that goods are purchased.

$$M_t = m_t^h + m_t^{h*} \quad \forall\, t \tag{1.40}$$

$$M_t^* = m_t^f + m_t^{f*} \quad \forall\, t \tag{1.41}$$

where M_t and M_t^* are the Home and Foreign aggregate money supplies, respectively, at time t.

Currency Exchange Market. After trade has taken place in the goods market, period-t producers go to the currency market and make a portfolio decision of how much of each currency to hold. The exchange rate e_t is the price that equates the amount of Foreign currency purchased with Home currency by Home producers with the amount of Home currency purchased by Foreign producers with Foreign currency.

$$e_t m_t^f = m_t^{h*} \quad \forall t \tag{1.42}$$

It is important to note that the exchange rate here is not pinned down by the assumption of the law of one price as in models with a single cash-in-advance constraint, such as Corsetti and Pesenti (2001) and Arseneau (2007). Here, the exchange rate is a price that clears the currency exchange market in period-t. Because of the two cash-in-advance constraints, the law of one price holds by definition. Using the cash-in-advance constraint (1.29) and its Foreign country analogue, it can be shown that exchange rate market clearing implies that the nominal value of imports equals the nominal value of exports.

$$e_t P^f_{t+1} C^f_{t+1} = P^h_{t+1} C^{h*}_{t+1} \quad \forall t \tag{1.43}$$

1.3.4 Equilibrium

This perfect foresight overlapping generations model has one unique nonautarkic steady state equilibrium. As noted in Section 1.3.1, I avoid discretionary monetary policy in this paper due to the resulting characteristic of multiple equilibria, most of which are unstable sunspot equilibria characterized by expectations traps.[15] Table 1.1 shows the conditions that must hold in a perfect foresight equilibrium. I define the steady state international equilibrium given both Home and Foreign monetary policy (x, x^*) as follows:

Definition 1.1 (Steady State International Equilibrium given x and x^*). A steady state international equilibrium, given Home and Foreign monetary policy (x, x^*) is the set of Home consumption of both Home and Foreign aggregate goods C^h and C^f, Home production n, Home portfolio holdings of both Home and Foreign currency m^h and m^f (or rather, as a percentage of initial Home holdings, ϕ and $1-\phi$), the Foreign counterparts $(C^{h*}, C^{f*}, n^*, m^{h*}, m^{f*})$, individual Home and Foreign prices $p_t(z)$ and $p_t(z^*)$, and exchange rate e_t such that:

- **Individual optimization**: Home and Foreign agents choose the price level of their differentiated good as well as their currency portfolio holdings in order to maximize lifetime utility in (1.26) and its Foreign counterpart subject to a budget constraint (1.27) and two cash-in-advance constraints (1.28) and (1.29).
- **Market Clearing** The goods markets (1.38) and (1.39), money markets (1.40) and (1.41), and currency exchange market (1.42) all clear.

Following Cooper and Kempf (2003), let ϕ_t represent the share of revenues $p_t(z)n_t(z)$ kept in the form of Home currency in period t, and let $1-\phi_t$ be the share of revenues exchanged for Foreign currency as characterized in the portfolio budget

[15] Ireland (1997) and King and Wolman (2004) are good references on multiple equilibria in models of discretionary monetary policy, which builds on the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983). See also Chatterjee, Cooper, and Ravikumar (1993).

Table 1.1: Equilibrium conditions given x and x^*

	Home country	Foreign country
(1.36)	$\frac{P^h_{t+1}C^h_{t+1}}{e_t P^f_{t+1}C^f_{t+1}} = \frac{1-\theta_h}{\theta_h}$	$\frac{e_t P^f_{t+1}C^{f*}_{t+1}}{P^h_{t+1}C^{h*}_{t+1}} = \frac{1-\theta_f}{\theta_f}$
(1.37)	$\frac{(1-\theta_h)}{\left(\frac{\varepsilon}{\varepsilon-1}\right)}\frac{p_t(z)}{P^h_{t+1}}\frac{\left(C^h_{t+1}\right)^{(1-\theta_h)(1-\sigma)-1}}{\left(C^f_{t+1}\right)^{-\theta_h(1-\sigma)}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$\frac{(1-\theta_f)}{\left(\frac{\varepsilon}{\varepsilon-1}\right)}\frac{p_t(z^*)}{P^f_{t+1}}\frac{\left(C^{f*}_{t+1}\right)^{(1-\theta_f)(1-\sigma)-1}}{\left(C^{h*}_{t+1}\right)^{-\theta_f(1-\sigma)}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(1.31)	$n_t(z) = \left(\frac{p_t(z)}{P^h_t}\right)^{-\varepsilon}\frac{xM_{t-1}}{P^h_t}$	$n_t(z^*) = \left(\frac{p_t(z^*)}{P^f_t}\right)^{-\varepsilon}\frac{x^*M^*_{t-1}}{P^f_t}$
(1.27)	$p_t(z)n_t(z) = m^h_t + e_t m^f_t$	$p_t(z^*)n_t(z^*) = m^{f*}_t + \frac{m^{h*}_t}{e_t}$
(1.32)	$C^h_{t+1} = \frac{m^h_t+(x-1)xM_{t-1}}{P^h_{t+1}}$	$C^{f*}_{t+1} = \frac{m^{f*}_t+(x^*-1)x^*M^*_{t-1}}{P^f_{t+1}}$
(1.33)	$C^f_{t+1} = \frac{m^f_t}{P^f_{t+1}}$	$C^{h*}_{t+1} = \frac{m^{h*}_t}{P^h_{t+1}}$
(1.18)	$C_{t+1} = \left(C^h_{t+1}\right)^{1-\theta_h}\left(C^f_{t+1}\right)^{\theta_h}$	$C^*_{t+1} = \left(C^{f*}_{t+1}\right)^{1-\theta_f}\left(C^{h*}_{t+1}\right)^{\theta_f}$
	Market clearing conditions	
(1.38)	$n_t(z) = c_t(z) + c^*_t(z)$	
(1.39)	$n_t(z^*) = c_t(z^*) + c^*_t(z^*)$	
(1.40)	$M_t = m^h_t + m^{h*}_t$	
(1.41)	$M^*_t = m^f_t + m^{f*}_t$	
(1.42)	$e_t m^f_t = m^{h*}_t$	

constraint (1.27). Then the following expressions hold.

$$p_t(z)n_t(z) - m^h_t = e_t m^f_t = (1-\phi_t)\,M_t \tag{1.44}$$

$$p_t(z^*)n_t(z^*) - m^{f*}_t = \frac{m^{h*}_t}{e_t} = (1-\phi^*_t)\,M^*_t \tag{1.45}$$

$$m^h_t + \tau_{t+1} = (\phi_t + x - 1)\,M_t \tag{1.46}$$

$$m^{f*}_t + \tau^*_{t+1} = (\phi^*_t + x^* - 1)\,M^*_t \tag{1.47}$$

Plugging (1.44), (1.45), (1.46), and (1.47) into the first order condition (1.36) and its Foreign country analogue, the unique nonautarkic steady state equilibrium share of currency from sales held for own-country consumption is given by:

$$\phi = 1 - \theta_h x \quad \forall\, x \in \left(0, \frac{1}{\theta_h}\right) \tag{1.48}$$

$$1 - \phi = \theta_h x \quad \forall\, x \in \left(0, \frac{1}{\theta_h}\right) \tag{1.49}$$

$$\phi^* = 1 - \theta_f x^* \quad \forall\, x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{1.50}$$

$$1 - \phi^* = \theta_f x^* \quad \forall\, x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{1.51}$$

From the aggregate money laws of motion in (1.2) and (1.3) and from the money market clearing conditions in (1.40) and (1.41), it is clear that the non-autarkic steady state equilibrium country-specific consumption inflation rates are:

$$\frac{P^h_{t+1}}{P^h_t} = x \tag{1.52}$$

$$\frac{P^f_{t+1}}{P^f_t} = x^* \tag{1.53}$$

Furthermore, using the definition of the Home country CPI level P_{t+1} from (1.22) and its Foreign country analogue, the expressions for the share Home country revenues traded for Foreign currency balances (1.49) and the share of Foreign country revenues traded for Home currency balances (1.51), and the currency exchange market clearing condition (1.42), the Home country CPI growth rate and the Foreign country CPI growth rates can be shown to be equal to their respective countries' money growth rates.[16]

$$\frac{P_{t+1}}{P_t} = x \tag{1.54}$$

$$\frac{P^*_{t+1}}{P^*_t} = x^* \tag{1.55}$$

Using (1.48), (1.49), (1.50), and (1.51), as well as the equilibrium inflation rates from (1.52) and (1.53), equilibrium consumption can be derived in terms of

[16] The derivation is given in Derivation 4 in Appendix A-3.

steady state employment from the cash-in-advance constraints as:

$$C^h = (1-\theta_h)n \tag{1.56}$$

$$C^f = \theta_f n^* \tag{1.57}$$

$$C^{f*} = (1-\theta_f)n^* \tag{1.58}$$

$$C^{h*} = \theta_h n \tag{1.59}$$

where the steady state employment levels n and n^* are characterized below in equations (1.62) and (1.63).

The expressions for the steady state international equilibrium employment is then found by solving the two equilibrium forms of the Home first order condition (1.37) and its Foreign analogue.

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x}\left[(1-\theta_h)n\right]^{(1-\theta_h)(1-\sigma)-1}\left[\theta_f n^*\right]^{\theta_h(1-\sigma)} = \chi\xi(n)^{\xi-1} \tag{1.60}$$

$$(1-\theta_f)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x^*}\left[(1-\theta_f)n^*\right]^{(1-\theta_f)(1-\sigma)-1}\left[\theta_h n\right]^{\theta_f(1-\sigma)} = \chi\xi(n^*)^{\xi-1} \tag{1.61}$$

Solving (1.61) for n^* and plugging it into (1.60), and doing the symmetric operation for the Foreign country gives the expressions for Home and Foreign equilibrium labor supply:

$$n\left(x,x^*\right) = \Omega_H\left(x\right)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x^*\right)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{1.62}$$

$$n^*\left(x^*,x\right) = \Omega_F\left(x^*\right)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x\right)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{1.63}$$

where the symbols in (1.62) and (1.63) summarize the parameters of the model in

Table 1.2: Properties of representative parameters

Symbol	Sign	$\frac{\partial(\cdot)}{\partial\theta_h}$	$\frac{\partial(\cdot)}{\partial\theta_f}$
Δ_h	$(-)$ always	$(+)$ when $\sigma > 1$	
Δ_f	$(-)$ always		$(+)$ when $\sigma > 1$
Σ_h	$(-)$ when $\sigma > 1$ and $\theta_h > 0$	$(-)$ when $\sigma > 1$	
Σ_f	$(-)$ when $\sigma > 1$ and $\theta_f > 0$		$(-)$ when $\sigma > 1$
Ω_h	$(+)$ when $\theta_f > 0$	$(-)$ when $\sigma > 1$ and $\theta_f > 0$	$(+)$ when $\sigma > 1$ and $\theta_h > 0$
Ω_f	$(+)$ when $\theta_h > 0$	$(+)$ when $\sigma > 1$ and $\theta_f > 0$	$(-)$ when $\sigma > 1$ and $\theta_h > 0$
$\Delta_h\Delta_f - \Sigma_h\Sigma_f$	$(+)$ always	$(-)$ when $\sigma > 1$	$(-)$ when $\sigma > 1$

Note: The results from this table are derived in Derivation 5 in Appendix A-3.

the following way:

$$\Delta_h = (1-\theta_h)(1-\sigma) - \xi \tag{1.64}$$

$$\Delta_f = (1-\theta_f)(1-\sigma) - \xi \tag{1.65}$$

$$\Sigma_h = \theta_h(1-\sigma) \tag{1.66}$$

$$\Sigma_f = \theta_f(1-\sigma) \tag{1.67}$$

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}} \tag{1.68}$$

$$\Omega_f = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}} \tag{1.69}$$

$$\Omega_H = (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{1.70}$$

$$\Omega_F = (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{1.71}$$

The signs of these expressions and their derivatives with respect to the openness parameters θ_h and θ_f are given in Table 1.2. From the signs of the representative parameters, it is clear that steady state equilibrium Home employment n decreases in x always and increases in x^* when $\sigma > 1$.

Looking at the equation for Home labor supply in (1.62), the sign of Σ_h determines how Foreign monetary policy affects the real economy in the Home country.

$$\Sigma_h = \begin{cases} > 0 & \text{if} \quad \theta_h \in (0, 0.5] \quad \text{and} \quad \sigma \in (0,1) \\ = 0 & \text{if} \quad \theta_h = 0 \quad \text{or} \quad \sigma = 1 \\ < 0 & \text{if} \quad \theta \in (0, 0.5] \quad \text{and} \quad \sigma > 1 \end{cases} \tag{1.72}$$

The third case is the most common in which $\Sigma_h < 0$, implying that Foreign inflation causes an increase in the equilibrium level of Home production and, therefore, an increase in equilibrium consumption of the Home good by both Home and Foreign consumers.

If one were to make the strong assumption that the coefficient of relative risk aversion σ were less than one, the first case in (1.72) occurs in which Foreign inflation causes a decrease in the equilibrium level of Home production. Lastly, it is interesting to notice the cases in which Foreign monetary policy has no real effect on the Home country ($\Sigma = 0$). Obviously, when the economies do not trade with each other, $\theta_h = 0$, Foreign monetary policy will be neutral. But it is interesting to note that the case of log utility ($\sigma = 1$) also induces the real neutrality of Foreign monetary policy.

The monetary authority in each country seeks to maximize the lifetime utility of a representative agent in this economy by choosing Home monetary policy x given Foreign monetary policy x^*. Define $V(x, x^*)$ as the lifetime utility of a representative agent. The objective of the Home monetary authority is then:

$$\max_x V(x, x^*) = \max_x \frac{\left([(1-\theta_h) n(x, x^*)]^{1-\theta_h} [\theta_f n^*(x^*, x)]^{\theta_h} \right)^{1-\sigma} - 1}{1-\sigma} - \chi n(x, x^*)^{\xi} \tag{1.73}$$

Definition 1.2 (**Home Country Steady State Monetary Equilibrium**)**.** A Home country steady state monetary equilibrium is a function for the optimal Home money growth rate $\hat{x}(x^*)$ given the Foreign money growth rate such that:

- the individual steady state equilibrium conditions from Definition 1.1 hold for each country,
- the Home monetary authority chooses x to maximize the lifetime utility of the representative agent of its country as in equation (1.73).

Definition 1.2 can be thought of as a monetary partial equilibrium in a world monetary environment because it implies a best response function for Home monetary policy that is a function of any level of Foreign monetary policy. Taking the derivative of (1.73), the resulting solution for optimal Home monetary policy is:[17]

$$\begin{aligned}\hat{x} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f-\theta_h\Sigma_f}\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi}\end{aligned} \tag{1.74}$$

The analogous solution for the Foreign monetary authority is:

$$\begin{aligned}\hat{x}^* &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}\end{aligned} \tag{1.75}$$

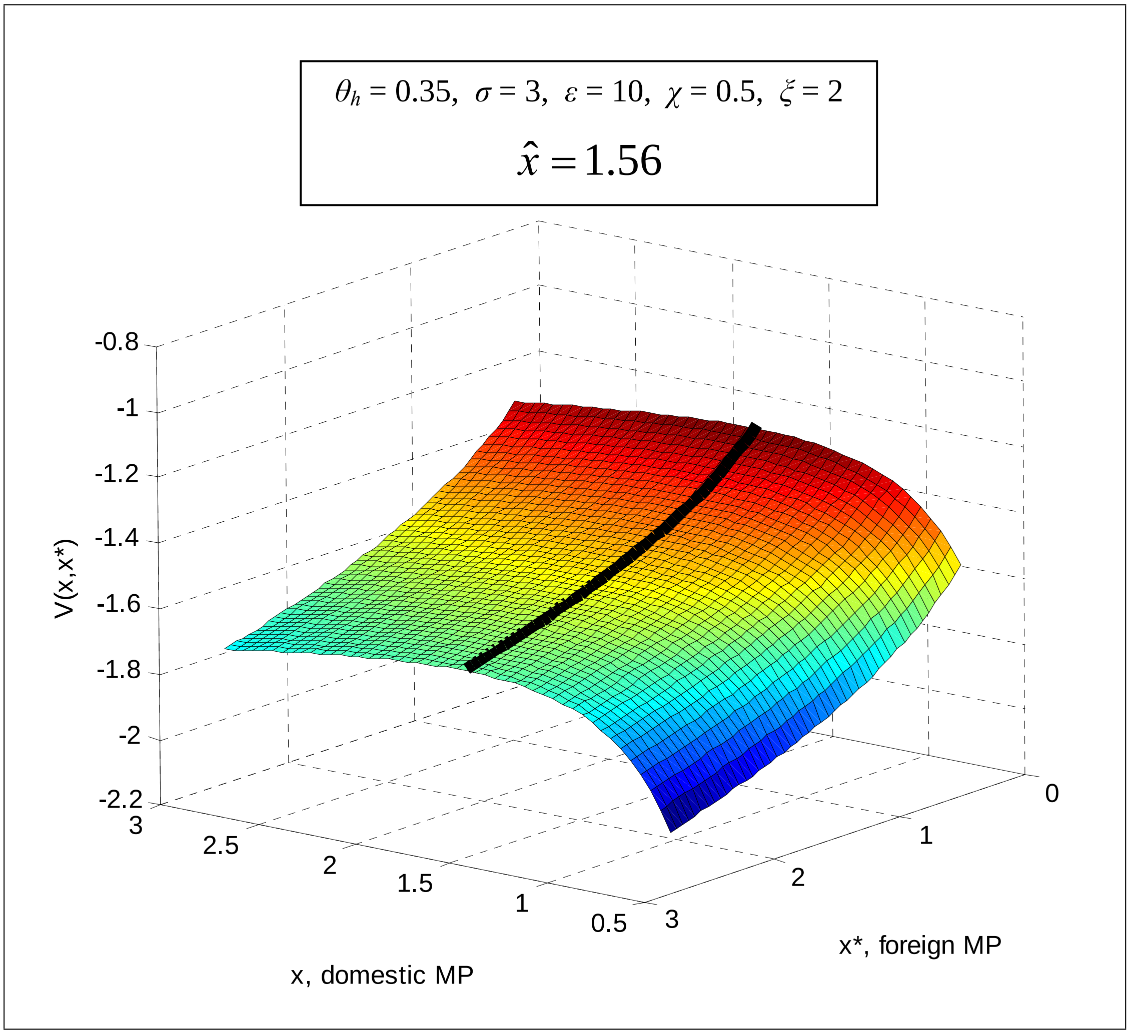
The first characteristic to note about the optimal Home monetary policy function in (1.74) is that it is independent of Foreign monetary policy x^*. That is, the optimal level of the Home money growth rate does not change with changes in the Foreign money growth rate and is a dominant strategy equilibrium.[18]

This dominant strategy equilibrium is shown in Figure 1.2 which plots the lifetime utility of a representative Home agent from (1.73) as a function of Home inflation x and Foreign inflation x^*. The parameters $(\theta, \sigma, \varepsilon, \chi, \xi)$ are simply chosen to reflect values estimated in the empirical literature in order to make a simple example. The dark line running across the top of Figure 1.2 represents the Home monetary policy best response function from (1.74). The optimal Home inflation level at the selected parameter values is a constant $\hat{x} = 1.56$, which is not overly high given that the duration of a period is a generation. Because each country's best response function for monetary policy is a dominant strategy equilibrium, the world Nash monetary equilibrium is the same as the country partial monetary equilibrium.

[17]See Derivation 6 in Appendix A-3.

[18]Derivation 6 in Appendix A-3 details why $\hat{x}$ is independent of x^*.

Figure 1.2: Home lifetime utility V as a function of x and x^*



1.3.5 Frictions

Before moving on to the results from Section 1.3.4, it is instructive to highlight the two frictions present in this model—money and imperfect competition—and their interplay with the level of openness. The two frictions are most easily isolated in a closed economy when the other friction is shut down. The inefficiencies caused by these two frictions are manifested in this setting as the "labor wedge" outlined in Chari, Kehoe, and McGrattan (2007).[19] The efficient allocation is found by solving the planner's problem of maximizing the utility of the period-t old from consumption minus the disutility of labor of the period-t young in the closed economy case $\theta_h = 0$, subject to the resource constraint.

$$\begin{aligned} &\max_{C_t^h, n_t} u\left(C_t^h\right) - g\left(n_t\right) \\ &\text{s.t.} \quad C_t^h = n_t \end{aligned} \tag{1.76}$$

The planner's solution steady state equilibrium is the following:

$$(C_{ps}, n_{ps}): \quad u'(C^h) = g'(n) \tag{1.77}$$

The deviation from the planner's solution created by the presence of imperfect competition is isolated by looking at the closed economy decentralized steady state solution where $\theta_h = 0$ in which the money growth rate is fixed at $x = 1$. The first order condition in (1.37) can be written as:

$$(C_{ic}, n_{ic}): \quad u'(C) = \Phi g'(n) \tag{1.78}$$

where $\Phi = \frac{\varepsilon}{\varepsilon - 1} \geq 1$ and (1.78) represents that marginal utility of consumption equals a markup over marginal cost. The monopoly power enjoyed by firms resulting from the imperfect substitutability ε of their goods allows producers to raise prices above the efficient level and lower output in order to maximize profits. Thus, $(C_{ic}, n_{ic}) \ll (C_{ps}, n_{ps})$, and (C_{ic}, n_{ic}) decreases as the degree imperfect competition Φ increases (as ε decreases).

[19] However, a key point on which this paper differs from Chari, Kehoe, and McGrattan (2007) is that money is set optimally in this paper and not stochastic. But Chari, Kehoe, and McGrattan (2007) do conclude that the labor-wedge channel does explain much of the observed variation in business cycles.

In like manner, the deviation from the planner's solution created by the money growth rate is isolated by looking at the closed economy decentralized steady state solution where $\theta_h = 0$ in which producers are perfectly competitive $\varepsilon = \infty$ ($\Phi = 1$). The first order condition in (1.37) can now be written as:

$$(C_{mp}, n_{mp}) : \quad \frac{1}{x}u'(C) = g'(n) \tag{1.79}$$

Equation (1.79) highlights the reason why expansionary monetary policy is thought of as an inflation tax. For higher money growth rates, the marginal benefit of an extra unit of labor decreases. Another way of looking at this problem is that the marginal productivity of labor is equal to 1, given the linear production technology. But the real wage in the closed economy is $\frac{1}{x}$. So for any money growth rate greater than 1, the real wage is less than the marginal productivity of labor. The result is that labor supplied is inefficiently low and $(C_{mp}, n_{mp}) \ll (C_{ps}, n_{ps})$ for all $x > 1$. Conversely, $(C_{mp}, n_{mp}) \gg (C_{ps}, n_{ps})$ for all $x < 1$. If the money growth rate is set optimally, the first best policy is $x = 1$ in this closed economy setting.

The interplay between openness, monetary policy and imperfect competition is seen when the closed economy frictions described preceding paragraphs are compared to their open economy counterparts. In the closed economy above, any money growth rate greater than the inverse of the markup gives a leisure subsidy that is dominated by an inflation tax, both of which are borne entirely by the agents of the closed country. However, when the two countries are open ($\theta_h, \theta_f > 0$), the inflation tax imposed by increasing the money growth rate is no longer borne solely by Home agents. Furthermore, increased money growth by the Home monetary authority actually increases the real wage through the terms of trade appreciation and increased preference weight on Foreign consumption. This added benefit of Home money growth is due to the international monopoly power of the Home monetary authority derived from the degree of inelastic demand fo imports by Foreign consumers.[20]

From the expressions for Home and Foreign employment in (1.62) and (1.63), the Home leisure subsidy results from the negative effect of an increase in x and the Foreign leisure tax results from the positive effect of an increase in x. The

[20]Recall that the constant expenditure share principle derives from the first order condition of the utility with the Cobb-Douglas aggregate consumption.

consumption tax of inflation can be seen by taking the derivative of equilibrium Home aggregate consumption C and Foreign aggregate consumption with respect to x.

$$\begin{aligned} C &= \left[(1-\theta_h)n\right]^{1-\theta_h}\left[\theta_f n^*\right]^{\theta_h} \\ &= (1-\theta_h)^{1-\theta_h}\theta_f^{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}}\right]^{\frac{\theta_h(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \cdots \\ &\qquad (x)^{\frac{(1-\theta_h)\Delta_f-\theta_h\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x^*)^{\frac{\theta_h\Delta_h-(1-\theta_h)\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \end{aligned} \tag{1.80}$$

$$\begin{aligned} C^* &= \left[(1-\theta_f)n^*\right]^{1-\theta_f}\left[\theta_h n\right]^{\theta_f} \\ &= (1-\theta_f)^{1-\theta_f}\theta_h^{\theta_f}\left[\frac{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}}\right]^{\frac{\theta_f(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \cdots \\ &\qquad (x^*)^{\frac{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(x)^{\frac{\theta_f\Delta_f-(1-\theta_f)\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \end{aligned} \tag{1.81}$$

The exponents on x in both (1.80) and (1.81) are both negative, but the exponent on x in (1.80) is larger in absolute value. That is, an increase in the Home money growth rate will cause a decrease in both the Home aggregate consumption C and Foreign aggregate consumption C^*, but the decrease in C is greater than the decrease in C^*. This latter fact is seen more clearly when steady state equilibrium relative aggregate consumption is expressed as follows:

$$\frac{C}{C^*} = \frac{(1-\theta_h)^{(1-\theta_h)}\theta_f^{\theta_h}}{(1-\theta_f)^{(1-\theta_f)}\theta_h^{\theta_f}}\left[\frac{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}\right]^{\frac{(1-\theta_h-\theta_f)(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{1.82}$$

The exponent on the bracketed term is negative, so an increase in x makes C decrease more than C^*. Thus, the inflation tax in the open economy is not just a decrease in equilibrium Home aggregate consumption C as in the closed economy case, but also a decrease in Foreign aggregate consumption C^* and an increase in Foreign employment n^*.

As was mentioned earlier, the Home leisure subsidy is the only benefit of inflation in the open economy that also exists in the closed economy. However, in contrast to the decrease in the real wage in a closed economy, an increase in the

Home money growth rate x increases the real wage in the open economy setting. The real wage in the open economy is the extra aggregate consumption from an extra unit of labor. Thus, the Home real wage is the derivative of Home aggregate consumption C with respect to n.

$$\begin{aligned} \frac{\partial C}{\partial n} &= (1-\theta_h)^{2-\theta_h}\theta_f^{\theta_h}\left(\frac{n^*}{n}\right)^{\theta_h} \\ &= (1-\theta_h)^{2-\theta_h}\theta_f^{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}\right]^{\frac{\theta_h(1-\sigma-\xi)}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \end{aligned} \tag{1.83}$$

Because the exponent on the bracketed term is negative, the effect of an increase in the Home money growth rate x is to increase the real wage. On the other hand, an increase in the Foreign money growth rate x^* is to decrease the real wage due to the positive effect of x^* on n.

This real-wage benefit of Home inflation is driven by two components. First, as has been documented by Corsetti and Pesenti (2001), Cooley and Quadrini (2003), Cooper and Kempf (2003), and Arseneau (2007), an increase in the Home money growth rate x causes the terms of trade to appreciate in favor of the Home country. The terms of trade for a given country is defined as the price of its exports in terms of its imports. In the steady state equilibrium, the terms of trade for the Home country can be expressed as follows:

$$ToT \equiv \frac{P_{t+1}^h}{e_t P_{t+1}^f} = \frac{\theta_f}{\theta_h}\left[\frac{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}\theta_f^{\theta_h(1-\sigma)}x^*}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}\theta_h^{\theta_f(1-\sigma)}x}\right]^{\frac{1-\sigma-\xi}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{1.84}$$

Again, because the exponent on the bracketed term is negative, the effect of an increase in the Home money growth rate x is to increase the cost of Home exports in terms of Home imports. On the other hand, an increase in the Foreign money growth rate x^* is to decrease the terms of trade. The second component of the real-wage benefit of Home inflation is simply that increased openness means that more weight is placed on Foreign consumption which is amplified by the terms-of-trade appreciation.

So the objective of the Home monetary authority is to set its money growth rate such that the benefits of the inflation caused by x (leisure subsidy and real-wage benefit) equal the costs (consumption tax). The real-wage benefit is a direct result of the monopoly power that the monetary authority enjoys in international markets. And this monopoly power derives from the degree of inelastic demand for Foreign goods, as shown in the first order condition for Foreign currency balances (1.36).

Lastly, looking at the expression for the optimal Home money growth rate x in (1.74), it is no surprise that as the degree of imperfect competition increases in the Home country, the country-specific welfare benefits that the monetary authority can obtain from increasing the money growth rate decrease. Intuitively, the monopoly rents from the imperfect competition structure replace the monopoly rents obtained by the monetary authority through increasing the money growth rate.

1.4 Results

The main question of this paper is whether openness is inflationary. The following proposition answers this question with regard to both absolute inflation rate (Home country CPI growth rate) and what I will define as relative inflation rate (Home country CPI growth rate over Foreign country CPI growth rate) or a real exchange rate.

Proposition 1.1 (Monetary response to changes in openness). The equilibrium optimal Home money growth rate $\hat{x}$ in (1.74) increases with more Home openness in the form of a higher level of θ_h and in response to more Foreign openness in the form of a higher level of θ_f. The argument for the Foreign country is symmetric. However, when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$. Conversely, when θ_f increases, the increase in $\hat{x}^*$ is greater than the increase in $\hat{x}$.

Proof. See Appendix A-1. □

Because the Home country CPI growth rate (P_{t+1}/P_t) is equal to the Home money growth rate x, an increase in θ_h increases Home country inflation as well as Foreign country inflation. From the perspective of the Home monetary authority, if the Home marginal utility of Home consumption decreases relative to the Home marginal utility of Foreign consumption as is the case when θ_h increases while θ_f remains constant (see first order condition (1.37)), Home country agents bear a smaller proportion of the inflation tax. In effect, higher θ_h increases the welfare benefits from higher money growth rates to the Home country and lowers the costs.

Consequently, the optimal response by the Home monetary authority is to raise the Home money growth rate or the CPI inflation rate in response to a higher degree of openness.

The next two propositions further explain how the level of imperfect competition among producers in a country, as parameterized by the elasticity of substitution among a country's differentiated goods ε, influences the optimal money growth rate x and the real outcomes of the economy in equilibrium.

Proposition 1.2 (**Deflationary bias of imperfect competition**)**.** Both the optimal Home money growth rate $\hat{x}$ and the optimal Foreign money growth rate $\hat{x}^*$ decrease as the level of imperfect competition increases (as ε decreases). Furthermore, there exist two critical within-country elasticities of substitution for the Home country and Foreign country $(\bar{\varepsilon}, \bar{\varepsilon}^*)$ such that $\hat{x} = 1$ when $\varepsilon = \bar{\varepsilon}$ and $\hat{x}^* = 1$ when $\varepsilon = \bar{\varepsilon}^*$. That is, these two critical levels of the imperfect competition parameter implement the Friedman Rule in the Home and Foreign country, respectively.

$$\bar{\varepsilon} = \frac{(1-\theta_f)(1-\sigma) - \xi}{\theta_h(1-\sigma-\xi)} \tag{1.85}$$

$$\bar{\varepsilon}^* = \frac{(1-\theta_h)(1-\sigma) - \xi}{\theta_f(1-\sigma-\xi)} \tag{1.86}$$

Proof. See Appendix A-1. □

This result that the level imperfect competition induces a deflationary bias in monetary policy has been shown recently by Arseneau (2007).

Lastly, Proposition 1.3 highlights the relationship between the level of market power held by producers within a country and the monopoly power held by the each monetary authority in international markets.

Proposition 1.3 (**Market power neutrality**)**.** In the case of symmetric countries $\theta_h = \theta_f$, the steady state equilibrium levels of employment n and n^* are not affected by the level of imperfect competition ε within both countries.

Proof. See Appendix A-1. □

Proposition 1.3 says that the real outcomes in each country $(n, n^*, C^h, C^f, C^{h*}, C^{f*})$ are the same regardless of whether the countries are characterized by perfect competition $\varepsilon = \infty$ or whether any degree of monopoly power is enjoyed by producers $\varepsilon < \infty$. The implication of this result is that if any monopoly rents available to Home or Foreign agents are not collected through producer price setting, the remainder will be collected by the monetary authority raising prices. As stated in

Proposition 1.2, a level of imperfect competition exists at which all the monopoly rents are collected through producer price setting along. That is, inflation generated by the monetary authority increasing the money growth rate is not needed.

These results provide an interesting interpretation of the empirical findings summarized in Figure 1.1. If one is looking at the negative relationship between openness and inflation from the entire sample predicted values, Propositions 1.1 through 1.3 suggest that the inflationary bias of openness is dominated by the deflationary bias of imperfect competition. That is, the level of imperfect competition is greater than the critical value at which optimal monetary policy causes zero inflation ($\varepsilon < \bar{\varepsilon}, \bar{\varepsilon}^*$). On the other hand, if one is looking at the positive relationship between openness and inflation that results when looking at low-inflation countries, the conclusion is that the inflationary bias of openness slightly dominates the deflationary bias of imperfect competition.

1.5 Conclusion

The main result of this work is that increased openness, as defined by the import share of GDP, is associated with a higher level of steady state equilibrium inflation. In a closed economy, the leisure subsidy of inflation is strictly dominated by the consumption tax, so the only role for the optimal money growth rate is to offset the inefficiencies of imperfect competition. However, as a country becomes more open, more of the burden of the consumption tax of inflation is borne by Foreign consumers, and the terms of trade and the real wage appreciate with increased inflation. These extra benefits from higher money growth rates cause an inflationary effect of openness in equilibrium.

However, another important finding of this paper is that, not only does the level of imperfect competition among producers in a given country dampen the incentive for a monetary authority to increase the money growth rate, but is a perfect substitute. That is, any monopoly rents that are available to the agents of a country that are not collected through price setting behavior of producers derived from the level of imperfect competition within the country are extracted by the monetary authority.

The result that openness is inflationary runs contrary to much previous work that has documented a negative correlation between various measures of the level of globalization or openness and inflation. However, much less work exists that explores this relationship through structural international models based on microeconomic foundations. This work is a first pass at studying, specifically, the imperfect competition and monetary market power channel.

Further work includes relaxing the strong assumption that the elasticity of substitution between aggregate Home-produced consumption and aggregate Foreign-produced consumption is unity $\rho = 1$, which results in the Cobb-Douglas form of the final consumption aggregator. Relaxing this assumption would break the constant expenditure share result and allow consumers to substitute away from expenditures on a country's production when the monetary authority raises the money growth rate. This may also break the dominant strategy equilibrium result in which the optimal monetary policy of each country is independent of the policy of the other country. Other extensions that may break the dominant strategy equilibrium result are to add pricing or exchange rate frictions such as time- or state-dependent pricing or pricing-to-market.

Also, this paper assumes that the two countries are asymmetric with respect to the level of openness θ. However, another dimension of asymmetry that might be interesting is the elasticity of substitution ε that parameterizes the level of imperfect competition. Furthermore, a vein of the literature exists that studies environments with endogenous markups in which the elasticity of substitution changes as firms enter and exit.[21]

And lastly, if the degree of openness has such important effects on the ability of the monetary authority to extract monopoly rents for its citizens, then how would an entity like a congressional body set openness policy optimally if it could? That is, what would be the equilibrium outcomes with endogenous openness θ?

[21] See Ferreira and Lloyd-Braga (2005), Ferreira and Dufourt (2006), and D'Aspremont, Ferreira, and Gérard-Varet (1996).

Chapter 2

Openness, Inflation, and Imperfect Competition: Theory and Evidence

2.1 Introduction

The nebulous term "globalization" is most often used to describe the increased economic integration of countries in the last 30 years. In making that term more specific, many branches of the international economics literature have studied questions about the effects of a countries openness on various economic outcomes. The goal of this paper is to provide evidence on the effect of openness on inflation.

First, I provide a theoretical model that illustrates a channel through which increase openness of a country to international trade can increase equilibrium inflation in that country. This positive effect of openness on inflation depends critically on the level of imperfect competition, both within countries and on the international market. Because domestic and foreign goods are imperfect substitutes, a monetary authority can export some of the inflation tax of money growth to foreign holders of their currency. In effect, a monetary authority enjoys a degree of monopoly power in international markets due to foreign holders of the domestic currency and some degree of inelasticity of substitution between domestic and foreign consumption.

However, this inflationary incentive is dampened by the degree of imperfect competition within the country. The intuition is that some rents exist in the international marketplace, and that the private holders of monopoly power within a country will obtain as much of those rents as possible given the level of imperfect competition in that country. And whatever portion of those rents are left over is taken by the monetary authority through its policy instrument.

In Section 2.2, I present the model. It is a two-country overlapping generations (OLG) model with imperfectly competitive labor markets, a monetary authority in each country that chooses a money growth rate to maximize the welfare of its citizens, and in which domestic and foreign consumption are imperfect substitutes. This model draws from Cooper and Kempf (2003) who use a two-country OLG model with perfectly competitive markets within each country, optimal monetary authorities, and imperfectly substitutable domestic and foreign goods to answer the question of when currency unions are optimal. However, one implicit result of their work is that increased openness is inflationary.

I follow the method of modeling imperfect competition within each country from Arseneau (2007) who uses a two-country infinite horizon model with imperfectly competitive goods markets, optimal monetary policy, and imperfectly substitutable domestic and foreign consumption goods to study the effects of imperfect competition on inflation in a multi-country environment. He finds that the increased imperfect competition (less competition) has a negative effect on equilibrium inflation rates. However, rather than model the degree of openness of a country, Arseneau (2007) models the size of a country. Corsetti and Pesenti (2001) use a similar model with stochastic money growth.

However, none of the above papers directly study the question of openness and inflation.[1] The fundamental theoretical underpinnings of this question come from Rogoff (1985), who uses a two-country adaptation of Barro and Gordon (1983) to illustrate a channel through which increased openness decreases equilibrium inflation. In a closed economy Barro-Gordon setting, the well known time-consistency problem generates a situation of suboptimally high inflation and employment and output at their natural rates. However, when the country becomes open to international trade, the time-consistent inflationary incentive of the monetary authority decreases because of its adverse effect on foreign demand for domestically produced goods.

[1]Evans (2007) and Wynne and Kersting (2007) provide summaries of the literature on openness and inflation.

The channel that I model is different from Rogoff (1985) in two major ways. First, I model optimal monetary policy in the simplest possible way—commitment to a constant money growth rate in order to maximize country welfare. This is merely a simplifying assumption instead of time-consistent monetary policy because the goal is not the effect of monetary policy but rather the effect of openness on inflation in the presence of optimal monetary policy. The second major difference is the structure of demand on international markets. Rogoff (1985) implicitly assumes that domestic and foreign goods are perfect substitutes, whereas I allow some degree of inelasticity of substitution between home and foreign goods. This is the key characteristic that generates the *beggar-thy-neighbor* incentive for a monetary authority to inflate in an international setting.[2]

In Section 2.3, I present some empirical tests of the effect of openness on inflation. The empirical methods of this paper follow Romer (1993). He tested the theoretical prediction of Rogoff (1985), by running regressions of a proxy for openness (average import share of GDP) on the average inflation rate for a sample of 114 countries over the period from 1973 to 1987. Across a number of different specifications, Romer found a robust negative relationship between openness and inflation. His result was further confirmed in follow-up studies by Lane (1997) and Terra (1998).

The striking result that comes from the empirical estimates in this paper is that the sign of the effect of openness on inflation becomes positive when looking at a the later sample period from 1988 to 2002. Another innovation of this paper in the empirical openness and inflation literature is that I control for the level of imperfect competition within each country. Using limited manufacturing and nonmanufacturing markup data from Høj, Jimenez, Maher, Nicoletti, and Wise (2007) and using more broadly available union membership rates and union coverage rates data, I include these proxies for countrywide imperfect competition levels as controls in the regression.

I find that the empirical tests in this paper provide supporting evidence of the prediction of my theoretical model that increased openness can result in increased inflation. This is not to make a normative statement that openness to international

[2]This international market power story fits well with the current situation in which the dollar is the most widely held currency in the world, and that this monopoly power allows the United States to maintain a large budget deficit.

trade is negative. Rather, the findings of this paper illustrate in more detail the costs and benefits to openness to international trade.

2.2 Model

The model in this section uses a two-country overlapping generations framework similar to Cooper and Kempf (2003). However, I add imperfectly competitive labor markets within each country following Arseneau (2007) and Corsetti and Pesenti (2001). However, the imperfect competition in this model is in the labor market rather than the goods producing market. Each country has a continuum of perfectly competitive firms that produce by hiring the differentiated labor at a contracted wage. Each country also has a monetary authority that commits to a constant money growth rate at the beginning of time in order to maximize the welfare of the citizens of its own country.

2.2.1 Money

The optimal monetary policy structure of this model is the simplest possible form. Money is held because it is the only store of value, and I assume that the monetary authority must commit to a constant money growth rate at the beginning of time. Money in this model is not neutral because of a price rigidity arising from wage contracts. The consumption tax levied by money growth in a steady state equilibrium reflects the expected increased deterioration of purchasing power over time on the part of consumers.

The objective of the monetary authority in each country, which will be made more explicit in Section 2.2.5, is to choose a fixed (gross) money growth rate $x_t = x$ or $x_t^* = x^*$ at the beginning of time in such a way as to maximize the welfare of its own citizens. I assume here that the monetary authority is committed to its money growth rate and cannot deviate once it has chosen its money growth path.[3] Money is held because it is the only store of value, and monetary policy is not neutral in the steady state because it drives a wedge between the marginal rate of substitution

[3]This is the simplest possible optimal monetary policy structure. The reason to avoid discretionary monetary policy in this paper is due to the resulting characteristic of multiple equilibria as shown in Ireland (1997). See also King and Wolman (2004) and Chatterjee, Cooper, and Ravikumar (1993).

and the real wage.[4]

Let M_t and M_t^* be the aggregate supply of Home currency and Foreign currency, respectively, in period t. I normalize the initial supply of Home and Foreign currency to 1 and divide it equally among the period-1 consumers at the beginning of the period.

$$M_0 = M_0^* = 1 \quad \text{and} \quad m_0^h = m_0^f = m_0^{h*} = m_0^{f*} = \frac{1}{2} \tag{2.1}$$

The variables m_0^h and m_0^{f*} are the individual holdings of Home currency by Home consumers and Foreign currency by Foreign consumers, respectively, at the beginning of period 1. Each country's monetary authority makes non-proportional transfers of $(x-1)M_{t-1}^h$ to each Home consumer in period t and $(x^*-1)M_{t-1}^f$ to each Foreign consumer where x and x^* represent the respective constant gross money growth rates of each country. So aggregate supply of currency in each country obeys the following laws of motion.

$$M_{t+1} = xM_t \tag{2.2}$$

$$M_{t+1}^* = x^* M_t^* \tag{2.3}$$

This implies that the following relationships for τ_{t+1} and τ_{t+1}^* represent the non-proportional transfer to each Home consumer and to each Foreign consumer by their respective monetary authorities.

$$\tau_{t+1} = (x-1)\, M_t \tag{2.4}$$

$$\tau_{t+1}^* = (x^*-1)\, M_t^* \tag{2.5}$$

At the end of the first period of their lives, workers make a portfolio decision of how much of each type of currency to hold. They have just received wages represented by either $p_t(z)n_t(z)$ in Home currency or $p_t(z^*)n_t(z^*)$ in Foreign currency for their differentiated labor hired by the identical firms in their own country. Before the end of the first period of life, workers in each country exchange some of their wages in own-country currency balances for other-country currency balances at the exchange rate e_t as shown in the budget constraint equation (2.17). Let m_t^h

[4]Intuitively, increased money growth taxes the purchasing power of the only store of value, which causes agents to take more leisure in the first period of their lives.

and m_t^f represent each Home worker's portfolio choice between Home and Foreign currency, respectively, in period t. Because the monetary authority of each country only transfers currency to its own consumers, the laws of motion for individual currency balances are the following:

$$m_{t+1}^h = m_t^h + \tau_{t+1} \tag{2.6}$$

$$m_{t+1}^f = m_t^f \tag{2.7}$$

$$m_{t+1}^{f*} = m_t^{f*} + \tau_{t+1}^* \tag{2.8}$$

$$m_{t+1}^{h*} = m_t^{h*} \tag{2.9}$$

Because the equilibrium currency holdings within each country are symmetric, then $m_t^h, m_t^f, m_t^{f*}, m_t^{h*}$ represent the aggregate amounts of each currency $(M_t^h, M_t^f, M_t^{h*}, M_t^{f*})$ held in each country in each period.

2.2.2 Firms

The Home and Foreign country each have a unit measure of identical infinitely lived firms that produce a consumption good y_t and y_t^*, respectively, each period. Within each country, the firms are perfectly competitive as characterized by a zero profit condition and can be treated as a single representative firm. The elasticity of substitution between Home and Foreign goods is given by the parameter $\rho \geq 0$, which enters into individual preferences described in Section 2.2.3.[5]

The representative firm in each country produces a final good y_t using differentiated labor $n_t(z)$ from its own country, where $z \in [0, 1]$ indexes the type of labor.

[5]For most of this paper, I assume that $\rho = 1$, which results in a Cobb-Douglas aggregator or utility function as shown in Equation 2.21. Appendix A-5 shows the general case of CES preferences in which $\rho \geq 0$ and shows some of its limiting values such as Leontief preferences ($\rho = 0$), Cobb-Douglas preferences ($\rho = 1$), and perfect substitutes ($\rho = \infty$).

The production technology takes the constant elasticity of substitution form:[6]

$$y_t \equiv \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{2.10}$$

$$y_t^* \equiv \left(\int_0^1 n_t(z^*)^{\frac{\varepsilon-1}{\varepsilon}} \, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{2.11}$$

where $\varepsilon \geq 1$ represents the elasticity of substitution among all the differentiated types of labor available to the firm in a given country.[7]

Note here that the imperfect competition is not on the part of firms. Instead, the suppliers of labor will possess the market power. However, a similar model used by Evans (2007) places the imperfect competition on the part of differentiated intermediate goods producers, and the results do not change.

The representative firm maximizes profits π_t by choosing how much of each type of labor within its own country $n_t(z)$ to hire in order to produce y_t units of the final consumption good given a market selling price for the good P_t^h and the negotiated wages for each of the different types of labor $w_t(z)$. Substituting the production function from (2.10) into the profit equation for y_t, the firm's problem becomes:

$$\max_{n_t(z)} \pi_t = P_t^h \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 w_t(z) n_t(z) \, dz \quad \forall\, t \tag{2.12}$$

The resulting function for labor demand is the following.[8]

$$n_t(z) = \left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \quad \forall\, t, z \tag{2.13}$$

[6]The CES production function was first introduced by Arrow, Chenery, Minhas, and Solow (1961) and was extended to the n-input case by Uzawa (1962) and McFadden (1963). The idea of the differentiated inputs resulting in imperfect competition was then formalized by Dixit and Stiglitz (1977). The difference here is that the good produced in each country is a composite of the differentiated inputs rather than aggregate consumption being a composite of differentiated goods consumption.

[7]This is analogous to a situation in which firms can hire CEOs, accountants, and janitors. Each is an imperfect substitute for the other. I have assumed that the elasticities of substitution among differentiated labor types in the two countries are equal $\varepsilon = \varepsilon^* \geq 0$. However, this assumption does not change the sign of the effect of openness on inflation. Furthermore, the elasticity must satisfy $\varepsilon \geq 1$ because any $\varepsilon \in [0, 1)$ implies a negative markup at which a firm would not produce.

[8]See Derivation 2 in Appendix A-3.

The expression for the price level of the consumption good P_t^h is pinned down by the zero profit condition.[9]

$$P_t^h = \left(\int_0^1 w_t(z)^{1-\varepsilon} \, dz \right)^{\frac{1}{1-\varepsilon}} \quad \forall\, t \tag{2.14}$$

The expression for an aggregate Home-country consumer price level P_t as a function of the price of Home-produced consumption P_t^h and Foreign-produced consumption P_t^f takes the following form:

$$P_t = \frac{1}{(1-\theta^h)^{1-\theta^h} (\theta^h)^{\theta^h}} \left(P_t^h \right)^{1-\theta^h} \left(e_{t-1} P_t^f \right)^{\theta^h} \tag{2.15}$$

where e_t is the exchange rate.[10]

2.2.3 Individuals

A unit measure of agents is born in each period in both the Home country (indexed by z) and the Foreign country (indexed by z^*). Each generation of agents lives for two periods. They work in the first period, and consume in the second period. From this point forward, I will show the problem of a Home-country agent. But the problem of a Foreign country agent is symmetric.

In the first period of their lives, individuals can either enjoy leisure l_t or provide labor $n_t(z)$ subject to their endowment of one unit of time.

$$l_t + n_t(z) = 1 \quad \forall t, z \tag{2.16}$$

Labor is not mobile across countries. Labor markets are imperfectly competitive in this model because each worker's labor is differentiated from that of all the other workers, analogous to Dixit and Stiglitz's (1977) differentiated goods model. As was shown in the firm production functions in equations (2.10) and (2.11) each differentiated type of labor is imperfectly substitutable among the other types of labor available within a given country for producing the homogeneous country-specific consumption good.

At the beginning of the first period of life, an individual knows the monetary

[9] See Derivation 3 in Appendix A-3.
[10] See equation (**??**) in Appendix A-4 for derivation.

policy of both the Home and Foreign country (x, x^*), the aggregate money supply in each country (M_t, M_t^*), the labor demand functions for own-country firms (2.13), and the resulting pricing equations in each country (2.14). Because individuals in the first period of life know the firm's labor demand function (2.13) and because these workers can set their wage contract at the beginning of the period, labor supply equals labor demand. That is, because changes in an individual's amount of labor supplied and contracted wage do not affect the amount of country-specific output and price level, respectively, the choice of wage level determines the labor supply amount.[11]

Once the wage $w_t(z)$ has been contracted between each worker and the firm, each worker supplies labor $n_t(z)$, and the representative firm produces the amount y_t of the country-specific consumption good according to the Dixit-Stiglitz CES production technology in (2.10). The firm then sells its output to consumers from both the Home and Foreign countries who have had their currency balances augmented by the non-proportional transfers from their respective monetary authorities. Firms then take these revenues in their own country's currency and pay their workers according to the contracted wage rate.[12] At the end of the period, workers take their earnings in their own-country currency and decide how much of it to trade for foreign currency balances:

$$w_t(z)n_t(z) = m_t^h + e_t m_t^f \tag{2.17}$$

where e_t is the exchange rate.

In the second period of life, the workers become consumers. They receive the non-proportional transfer from the monetary authority τ_{t+1} and they spend currency balances according to the following cash-in-advance constraints:

$$P_{t+1}^h c_{t+1}^h = m_t^h + \tau_{t+1} \tag{2.18}$$

$$P_{t+1}^f c_{t+1}^f = m_t^f \tag{2.19}$$

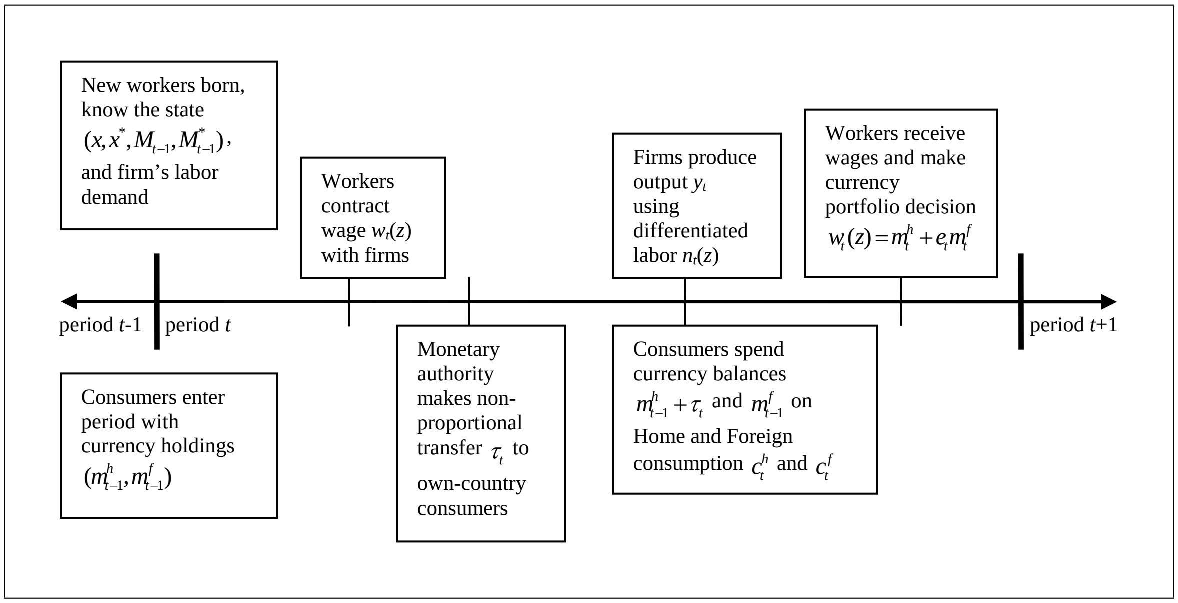
where c_{t+1}^h and c_{t+1}^f are the consumption of Home and Foreign goods, respectively,

[11]See Appendix A-1 for proof.

[12]These wage contracts are only considered a price friction in a stochastic or time-consistent monetary policy environment. Wage contracts are not a price friction in this perfect foresight, policy-commitment environment.

by a Home country agent. The prices of the Home and Foreign goods in terms of their respective currencies are P_{t+1}^h and P_{t+1}^f. The timing of this model is illustrated in Figure 2.1.

Figure 2.1: Timing of two-country OLG model



Lifetime utility of a Home-country agent is defined as an additively separable function over an aggregate consumption basket c_{t+1} and labor $n_t(z)$:

$$\begin{aligned} U\left(c_{t+1}, n_t(z)\right) &= u\left(c_{t+1}\right) - g\left(n_t(z)\right) \\ \text{where} \quad u\left(c_{t+1}\right) &= \frac{\left(c_{t+1}\right)^{1-\sigma} - 1}{1-\sigma} \quad \text{for} \quad \sigma > 0 \\ \text{and} \quad g\left(n_t(z)\right) &= \chi\left(n_t(z)\right)^{\xi} \quad \text{for} \quad \chi > 0 \quad \text{and} \quad \xi \geq 1 \end{aligned} \tag{2.20}$$

where σ represents the coefficient of relative risk aversion, and χ and ξ are the level and shape parameters, respectively, of the function for the disutility of labor. Individual aggregate consumption c_{t+1} is defined as a Cobb-Douglas constant elasticity of substitution aggregator over Home and Foreign consumption:

$$c_{t+1} \equiv \left(c_{t+1}^h\right)^{1-\theta_h} \left(c_{t+1}^f\right)^{\theta_h} \quad \forall t \quad \text{and} \quad \theta_h \in \left[0, \frac{1}{2}\right] \tag{2.21}$$

where c^h_{t+1} is individual consumption of the Home-produced good by the Home agent, c^f_{t+1} is individual consumption of the Foreign-produced good by the Home agent and θ^h is the home bias parameter. Because θ^h reflects the preference weighting that Home consumers place on Foreign consumption, it characterizes the degree of openness of the Home country.[13]

The Home consumer's demand for Home-produced and Foreign-produced consumption can be derived from the consumer's expenditure minimization problem in the second period of life, given the prices of Home-produced consumption P^h_{t+1} and Foreign-produced consumption P^f_{t+1} and given the level of aggregate consumption c_{t+1}.[14]

$$c^h_{t+1} = (1-\theta^h)\left(\frac{P^h_{t+1}}{P_{t+1}}\right)^{-1} c_{t+1} \tag{2.22}$$

$$c^f_{t+1} = \theta^h \left(\frac{e_t P^f_{t+1}}{P_{t+1}}\right)^{-1} c_{t+1} \tag{2.23}$$

Note that these consumption demand equations are analogous to the differentiated labor input demand equations in (2.13), except that they include the home-bias parameter. Also note that dividing (2.22) by (2.23) gives the constant expenditure share equation from the first order condition in (2.29). This simply because the demand equations from expenditure minimization and utility maximization are equivalent.

The objective of each individual in the Home country can be represented as a choice of a contracted wage rate $w_t(z)$ and a currency portfolio decision m^h_t and

[13]Foreign consumers' degree of openness θ^f is not forced to be symmetric to the Home consumers' parameter. As will be shown later, θ^h is a good way to parameterize the degree of openness because it equals the import share in equilibrium, which has been a proxy for openness in many previous empirical studies (e.g., Romer (1993), Lane (1997), and Terra (1998)).

[14]See equations (A.4.22) and (A.4.23) in Appendix A-4 for derivation.

m_t^f in order to maximize lifetime utility.

$$\max_{m_t^h, m_t^f, w_t(z)} \frac{\left[\left(c_{t+1}^h\right)^{1-\theta_h}\left(c_{t+1}^f\right)^{\theta_h}\right]^{1-\sigma} - 1}{1-\sigma} - \chi\left(n_t(z)\right)^{\xi} \tag{2.24}$$

$$\text{s.t.} \quad w_t(z)n_t(z) = m_t^h + e_t m_t^f \tag{2.17}$$

$$P_{t+1}^h c_{t+1}^h = m_t^h + \tau_{t+1} \tag{2.18}$$

$$P_{t+1}^f c_{t+1}^f = m_t^f \tag{2.19}$$

where (2.17) is the budget constraint reflecting the portfolio decision and (2.18) and (2.19) are the cash-in-advance constraints.

The two cash-in-advance constraints can be thought of as a simplification of one equilibrium outcome of a richer environment in which governments or monetary authorities strategically choose what currencies to accept for exchange that takes place within their borders. Matsuyama, Kiyotaki, and Matsui (1993) present a random matching search model of money after the flavor of Kiyotaki and Wright (1989) in which blocks of agents (countries) choose which currencies to accept for local and international transactions based on the likelihood of that currency being accepted in future transactions. In one equilibrium, corresponding to the two cash-in-advance constraint environment of this paper, each block of agents (country) only accepts local currency for all local and international transactions.

Another equilibrium in the Matsuyama, Kiyotaki, and Matsui (1993) is the case in which vendors in both countries accept currency of both countries. This is analogous to the more standard approach in the NOEM literature as exemplified by Corsetti and Pesenti (2001). Their environment is one characterized by a single cash-in-advance constraint in which producers sell their goods in both countries and charge a price in terms of Home currency and a price in terms of Foreign currency. The exchange rate is then pinned down by an assumption of the law of one price.

The reason for choosing the two cash-in-advance constraints approach as shown in equations (2.18) and (2.19) instead of the more standard Corsetti and Pesenti (2001) method of one cash-in-advance constraint and the law of one price is that the method employed here gives rise to a portfolio decision. The law of one price is implicit in the two cash-in-advance constraint assumption because, by definition, vendors only accept one currency and therefore only charge one price.

As will be in Section 2.2.4, the exchange rate here serves as a price that clears the currency exchange market rather than a mechanism for enforcing the law of one price. Furthermore, the currency portfolio decision is an interesting one that has not received much attention.[15] However, both the single CIA constraint with the law of one price method and the dual CIA constraints with currency exchange market clearing method deliver the same results for optimal monetary policy.

Using the cash-in-advance constraints (2.18) and (2.19), the money laws of motion (2.6) and (2.7), and the expressions for the non-proportional transfer in terms of the Home money growth rate (2.4), country-specific aggregate consumptions can be expressed in the following way:

$$c_{t+1}^h = \frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h} \tag{2.25}$$

$$c_{t+1}^f = \frac{m_t^f}{P_{t+1}^f} \tag{2.26}$$

The expression for Home aggregate total consumption is then:

$$c_{t+1} = \left(\frac{m_t^h + (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h} \tag{2.27}$$

Using the portfolio constraint in (2.17) to substitute out either m_t^h or m_t^f and substituting in the expression for labor demand from (2.13), the maximization problem then becomes

$$\max_{m_t^f, w_t(z)} \frac{\left[\left(\frac{w_t(z)^{1-\varepsilon}}{P_{t+1}^h \left(P_t^h\right)^{-\varepsilon}} y_t - \frac{e_t m_t^f - (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{1-\theta_h} \left(\frac{m_t^f}{P_{t+1}^f}\right)^{\theta_h}\right]^{1-\sigma} - 1}{1-\sigma} \cdots \\ -\chi \left[\left(\frac{w_t(z)}{P_t^h}\right)^{-\varepsilon} y_t\right]^{\xi} \tag{2.28}$$

[15] Good examples of the currency portfolio choice literature are Engel and Matsumoto (2006) and Evans and Lyons (2005).

The first order conditions with respect to m_t^f and $w_t(z)$, respectively, are:

$$\frac{P_{t+1}^h c_{t+1}^h}{e_t P_{t+1}^f c_{t+1}^f} = \frac{1-\theta_h}{\theta_h} \quad \forall t, z \tag{2.29}$$

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{w_t(z)}{P_{t+1}^h}\left(c_{t+1}^h\right)^{(1-\theta_h)(1-\sigma)-1}\left(c_{t+1}^f\right)^{\theta_h(1-\sigma)} = \chi\xi\left(n_t(z)\right)^{\xi-1} \quad \forall t, z \tag{2.30}$$

where equation (2.29) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (2.30) equates the real wage with the marginal rate of substitution between consumption and leisure. Because each agent within a country is identical, other than for a differentiated production good, the resulting individual equilibrium wage $w_t(z)$ and the amount of total revenues held in Foreign currency m_t^f will be symmetric across individuals in a given country.

Notice that the first order condition for m_t^f in (2.29) is implied by the consumption demand functions in (2.22) and (2.23) from the expenditure minimization problem This is because the optimal choice of m_t^f in period t is equivalent to the optimal choice of C_{t+1}^h and C_{t+1}^f in period $t+1$. These two decisions are equivalent and take the labor or pricing decision as given.

2.2.4 Market clearing conditions

This economy has three markets that must clear—the goods market, the money market, and the currency exchange market. The labor market clears by assumption because each differentiated labor supplier is essentially a monopolist who chooses a wage to charge $w_t(z)$ in order to provide labor at the point on the labor demand curve that gives the highest utility. The following paragraphs describe each market and the respective market clearing conditions.

Goods Market. Both Home and Foreign consumers demand goods from both countries. Aggregate supply in the Home and Foreign countries is simply the production of the representative firm y_t and y_t^*, respectively. Define d_t and d_t^* as the world demand for Home-produced goods and Foreign-produced goods, respectively. Goods market clearing requires that aggregate production in each country equal the world demand for that country's goods.

$$y_t = d_t = c_t^h + c_t^{h*} \quad \forall\, t \tag{2.31}$$

$$y_t^* = d_t^* = c_t^f + c_t^{f*} \quad \forall\, t \tag{2.32}$$

Money Market. Money market clearing simply requires that money supply equal money demand at the time that goods are purchased.

$$M_t = m_t^h + m_t^{h*} \quad \forall\, t \tag{2.33}$$

$$M_t^* = m_t^f + m_t^{f*} \quad \forall\, t \tag{2.34}$$

where M_t and M_t^* are the Home and Foreign aggregate money supplies, respectively, at time t.

Currency Exchange Market. After trade has taken place in the goods market, period-t laborers go to the currency market and make a portfolio decision of how much of each currency to hold. The exchange rate e_t is the price that equates the amount of Foreign currency purchased with Home currency by Home laborers with the amount of Home currency purchased by Foreign laborers with Foreign currency.

$$e_t m_t^f = m_t^{h*} \quad \forall t \tag{2.35}$$

It is important to note that the exchange rate here is not pinned down by the assumption of the law of one price as in models with a single cash-in-advance constraint, such as Corsetti and Pesenti (2001) and Arseneau (2007). Here, the exchange rate is a price that clears the currency exchange market in period-t. Because of the two cash-in-advance constraints, the law of one price holds by definition. Using the cash-in-advance constraint (2.19) and its Foreign country analogue, it can be shown that exchange rate market clearing implies that the nominal value of imports equals the nominal value of exports.

$$e_t P_{t+1}^f C_{t+1}^f = P_{t+1}^h C_{t+1}^{h*} \quad \forall t \tag{2.36}$$

2.2.5 Equilibrium

This perfect foresight overlapping generations model has one unique nonautarkic steady state equilibrium. As noted in Section 2.2.1, I avoid discretionary monetary policy in this paper due to the resulting characteristic of multiple equilibria, most of which are unstable sunspot equilibria characterized by expectations traps.[16] Table 2.1 shows the conditions that must hold in a perfect foresight equilibrium. I define the steady state international equilibrium given both Home and Foreign monetary policy (x, x^*) as follows:

Definition 2.1 (**Steady State International Equilibrium given** x **and** x^***).** A steady state international equilibrium, given Home and Foreign monetary policy (x, x^*) is the set of Home consumption of both Home and Foreign aggregate goods c^h and c^f, Home production y, Home labor n Home portfolio holdings of both Home and Foreign currency m^h and m^f (or rather, as a percentage of initial Home holdings, ϕ and $1 - \phi$), the Foreign counterparts $(c^{h*}, c^{f*}, y^*, n^*, m^{h*}, m^{f*})$, Home and Foreign prices and wages $(P^h, P^f, w(z), w(z^*))$, and exchange rate e_t such that:

- **Individual optimization**: Home and Foreign agents choose the wage rate for their differentiated labor input as well as their currency portfolio holdings in order to maximize lifetime utility in (2.24) and its Foreign counterpart subject to a budget constraint (2.17) and two cash-in-advance constraints (2.18) and (2.19).

- **Firm profit maximization**: Home and Foreign firms choose how much to produce and how much of each type of labor to use given contracted wages and a market price of their good in order to maximize profits according to (2.12). Wages are contracted in imperfectly competitive labor markets, but the price of the firms good is determined in a perfectly competitive environment according to a zero-profit condition.

- **Market Clearing** The goods markets (2.31) and (2.32), money markets (2.33) and (2.34), and currency exchange market (2.35) all clear.

Following Cooper and Kempf (2003), let ϕ_t represent the share of wage revenues $w_t(z)n_t(z)$ kept in the form of Home currency in period t, and let $1-\phi_t$ be the share of revenues exchanged for Foreign currency as characterized in the portfolio

[16] Ireland (1997) and King and Wolman (2004) are good current references on multiple equilibria in models of discretionary monetary policy, which builds on the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983). See also Chatterjee, Cooper, and Ravikumar (1993).

Table 2.1: Equilibrium conditions given x and x^*

	Home country	Foreign country
(2.29)	$\frac{P^h_{t+1}c^h_{t+1}}{e_t P^f_{t+1}c^f_{t+1}} = \frac{1-\theta_h}{\theta_h}$	$\frac{e_t P^f_{t+1}c^{f*}_{t+1}}{P^h_{t+1}c^{h*}_{t+1}} = \frac{1-\theta_f}{\theta_f}$
(2.30)	$\frac{(1-\theta_h)}{\left(\frac{\varepsilon}{\varepsilon-1}\right)}\frac{w_t(z)}{P^h_{t+1}}\frac{\left(c^h_{t+1}\right)^{(1-\theta_h)(1-\sigma)-1}}{\left(c^f_{t+1}\right)^{-\theta_h(1-\sigma)}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$\frac{(1-\theta_f)}{\left(\frac{\varepsilon}{\varepsilon-1}\right)}\frac{w_t(z^*)}{P^f_{t+1}}\frac{\left(c^{f*}_{t+1}\right)^{(1-\theta_f)(1-\sigma)-1}}{\left(c^{h*}_{t+1}\right)^{-\theta_f(1-\sigma)}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(2.13)	$n_t(z) = \left(\frac{w_t(z)}{P^h_t}\right)^{-\varepsilon} y_t$	$n_t(z^*) = \left(\frac{w_t(z^*)}{P^f_t}\right)^{-\varepsilon} y^*_t$
(2.17)	$w_t(z)n_t(z) = m^h_t + e_t m^f_t$	$w_t(z^*)n_t(z^*) = m^{f*}_t + \frac{m^{h*}_t}{e_t}$
(2.25)	$c^h_{t+1} = \frac{m^h_t + (x-1)xM_{t-1}}{P^h_{t+1}}$	$c^{f*}_{t+1} = \frac{m^{f*}_t + (x^*-1)x^*M^*_{t-1}}{P^f_{t+1}}$
(2.26)	$c^f_{t+1} = \frac{m^f_t}{P^f_{t+1}}$	$c^{h*}_{t+1} = \frac{m^{h*}_t}{P^h_{t+1}}$
(2.21)	$c_{t+1} = \left(c^h_{t+1}\right)^{1-\theta_h}\left(c^f_{t+1}\right)^{\theta_h}$	$c^*_{t+1} = \left(c^{f*}_{t+1}\right)^{1-\theta_f}\left(c^{h*}_{t+1}\right)^{\theta_f}$
	Market clearing conditions	
(2.31)	$y_t = c^h_t + c^{h*}_t$	
(2.32)	$y^*_t = c^f_t + c^{f*}_t$	
(2.33)	$M_t = m^h_t + m^{h*}_t$	
(2.34)	$M^*_t = m^f_t + m^{f*}_t$	
(2.35)	$e_t m^f_t = m^{h*}_t$	

budget constraint (2.17). Then the following expressions hold.

$$w_t(z)n_t(z) - m^h_t = e_t m^f_t = (1-\phi_t)\,M_t \tag{2.37}$$

$$w_t(z^*)n_t(z^*) - m^{f*}_t = \frac{m^{h*}_t}{e_t} = (1-\phi^*_t)\,M^*_t \tag{2.38}$$

$$m^h_t + \tau_{t+1} = (\phi_t + x - 1)\,M_t \tag{2.39}$$

$$m^{f*}_t + \tau^*_{t+1} = (\phi^*_t + x^* - 1)\,M^*_t \tag{2.40}$$

Plugging (2.37), (2.38), (2.39), and (2.40) into the first order condition (2.29) and its Foreign country analogue, the unique nonautarkic steady state equilibrium share of currency from sales held for own-country consumption is given by:

$$\phi = 1 - \theta_h x \quad \forall\, x \in \left(0, \frac{1}{\theta_h}\right) \tag{2.41}$$

$$1 - \phi = \theta_h x \quad \forall\, x \in \left(0, \frac{1}{\theta_h}\right) \tag{2.42}$$

$$\phi^* = 1 - \theta_f x^* \quad \forall\, x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{2.43}$$

$$1 - \phi^* = \theta_f x^* \quad \forall\, x^* \in \left(0, \frac{1}{\theta_f}\right) \tag{2.44}$$

From the aggregate money laws of motion in (2.2) and (2.3) and from the money market clearing conditions in (2.33) and (2.34), it is clear that the non-autarkic steady state equilibrium country-specific consumption inflation rates are:

$$\frac{P^h_{t+1}}{P^h_t} = x \tag{2.45}$$

$$\frac{P^f_{t+1}}{P^f_t} = x^* \tag{2.46}$$

Furthermore, using the definition of the Home country CPI level P_{t+1} from (2.15) and its Foreign country analogue, the expressions for the share Home country revenues traded for Foreign currency balances (2.42) and the share of Foreign country revenues traded for Home currency balances (2.44), and the currency exchange market clearing condition (2.35), the Home country CPI growth rate and the Foreign country CPI growth rates can be shown to be equal to their respective countries' money growth rates.[17]

$$\frac{P_{t+1}}{P_t} = x \tag{2.47}$$

$$\frac{P^*_{t+1}}{P^*_t} = x^* \tag{2.48}$$

Using (2.41), (2.42), (2.43), and (2.44), as well as the equilibrium inflation rates from (2.45) and (2.46), equilibrium consumption can be derived in terms of

[17]See Derivation 4 in Appendix A-3.

steady state employment from the cash-in-advance constraints as:

$$c^h = (1-\theta_h)n \tag{2.49}$$

$$c^f = \theta_f n^* \tag{2.50}$$

$$c^{f*} = (1-\theta_f)n^* \tag{2.51}$$

$$c^{h*} = \theta_h n \tag{2.52}$$

where the steady state employment levels n and n^* are characterized below in equations (2.55) and (2.56).

The expressions for the steady state international equilibrium employment are then found by solving the two equilibrium forms of the Home first order condition (2.30) and its Foreign analogue.

$$(1-\theta_h)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x}\left[(1-\theta_h)n\right]^{(1-\theta_h)(1-\sigma)-1}\left[\theta_f n^*\right]^{\theta_h(1-\sigma)} = \chi\xi(n)^{\xi-1} \tag{2.53}$$

$$(1-\theta_f)\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{x^*}\left[(1-\theta_f)n^*\right]^{(1-\theta_f)(1-\sigma)-1}\left[\theta_h n\right]^{\theta_f(1-\sigma)} = \chi\xi(n^*)^{\xi-1} \tag{2.54}$$

Solving (2.54) for n^* and plugging it into (2.53), and doing the symmetric operation for the Foreign country gives the expressions for Home and Foreign equilibrium labor supply:

$$n\left(x,x^*\right) = \Omega_H\left(x\right)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x^*\right)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{2.55}$$

$$n^*\left(x^*,x\right) = \Omega_F\left(x^*\right)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}\left(x\right)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{2.56}$$

where the symbols in (2.55) and (2.56) summarize the parameters of the model in

Table 2.2: Properties of representative parameters

Symbol	Sign	$\frac{\partial(\cdot)}{\partial\theta_h}$	$\frac{\partial(\cdot)}{\partial\theta_f}$
Δ_h	(−) always	(+) when $\sigma > 1$	
Δ_f	(−) always		(+) when $\sigma > 1$
Σ_h	(−) when $\sigma > 1$ and $\theta_h > 0$	(−) when $\sigma > 1$	
Σ_f	(−) when $\sigma > 1$ and $\theta_f > 0$		(−) when $\sigma > 1$
Ω_h	(+) when $\theta_f > 0$	(−) when $\sigma > 1$ and $\theta_f > 0$	(+) when $\sigma > 1$ and $\theta_h > 0$
Ω_f	(+) when $\theta_h > 0$	(+) when $\sigma > 1$ and $\theta_f > 0$	(−) when $\sigma > 1$ and $\theta_h > 0$
$\Delta_h\Delta_f - \Sigma_h\Sigma_f$	(+) always	(−) when $\sigma > 1$	(−) when $\sigma > 1$

Note: The results from this table are derived in Derivation 5 in Appendix A-3.

the following way:

$$\Delta_h = (1-\theta_h)(1-\sigma) - \xi \tag{2.57}$$

$$\Delta_f = (1-\theta_f)(1-\sigma) - \xi \tag{2.58}$$

$$\Sigma_h = \theta_h(1-\sigma) \tag{2.59}$$

$$\Sigma_f = \theta_f(1-\sigma) \tag{2.60}$$

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}} \tag{2.61}$$

$$\Omega_f = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}} \tag{2.62}$$

$$\Omega_H = (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{2.63}$$

$$\Omega_F = (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}} \tag{2.64}$$

The signs of these expressions and their derivatives with respect to the openness parameters θ_h and θ_f are given in Table 2.2. From the signs of the representative parameters, it is clear that steady state equilibrium Home employment n decreases in x always and increases in x^* when $\sigma > 1$.

Looking at the equation for Home labor supply in (2.55), the sign of Σ_h determines how Foreign monetary policy affects the real economy in the Home country.

$$\Sigma_h = \begin{cases} > 0 & \text{if} \quad \theta_h \in (0, 0.5] \quad \text{and} \quad \sigma \in (0,1) \\ = 0 & \text{if} \quad \theta_h = 0 \quad \text{or} \quad \sigma = 1 \\ < 0 & \text{if} \quad \theta \in (0, 0.5] \quad \text{and} \quad \sigma > 1 \end{cases} \tag{2.65}$$

The third case is the most common in which $\Sigma_h < 0$, implying that Foreign inflation causes an increase in the equilibrium level of Home production and, therefore, an increase in equilibrium consumption of the Home good by both Home and Foreign consumers.

If one were to make the strong assumption that the coefficient of relative risk aversion σ were less than one, the first case in (2.65) occurs in which Foreign inflation causes a decrease in the equilibrium level of Home production. Lastly, it is interesting to notice the cases in which Foreign monetary policy has no real effect on the Home country ($\Sigma = 0$). Obviously, when the economies do not trade with each other, $\theta_h = 0$, Foreign monetary policy will be neutral. But it is interesting to note that the case of log utility ($\sigma = 1$) also induces the real neutrality of Foreign monetary policy.

The monetary authority in each country seeks to maximize the lifetime utility of a representative agent in this economy by choosing Home monetary policy x given Foreign monetary policy x^*. Define $V(x, x^*)$ as the lifetime utility of a representative agent. The objective of the Home monetary authority is then:

$$\max_x V(x, x^*) = \max_x \frac{\left([(1-\theta_h) n(x, x^*)]^{1-\theta_h} [\theta_f n^*(x^*, x)]^{\theta_h}\right)^{1-\sigma} - 1}{1-\sigma} - \chi n(x, x^*)^{\xi} \tag{2.66}$$

Definition 2.2 (**Home Country Steady State Monetary Equilibrium**). A Home country steady state monetary equilibrium is a function for the optimal Home money growth rate $\hat{x}(x^*)$ given the Foreign money growth rate such that:

- the individual steady state equilibrium conditions from Definition 2.1 hold for each country,
- the Home monetary authority chooses x to maximize the lifetime utility of the representative agent of its country as in equation (2.66).

Definition 2.2 can be thought of as a monetary partial equilibrium in a world monetary environment because it implies a best response function for Home monetary policy that is a function of any level of Foreign monetary policy. Taking the derivative of (2.66), the resulting solution for optimal Home monetary policy is:[18]

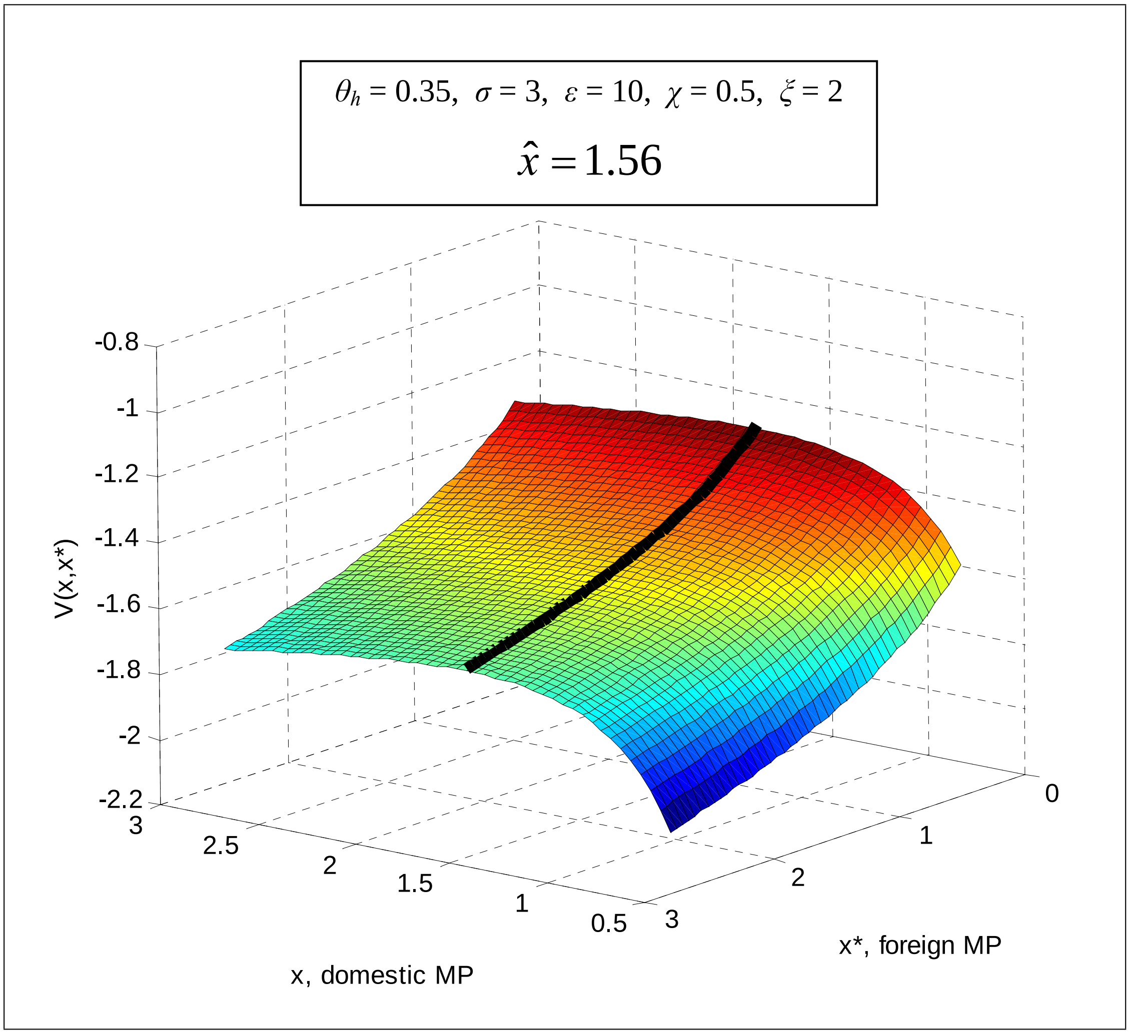
$$\begin{aligned}\hat{x} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f-\theta_h\Sigma_f} \\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi}\end{aligned} \tag{2.67}$$

The analogous solution for the Foreign monetary authority is:

$$\begin{aligned}\hat{x}^* &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h} \\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}\end{aligned} \tag{2.68}$$

[18]See Derivation 6 in Appendix A-3.

Figure 2.2: Home lifetime utility V as a function of x and x^*



The first characteristic to note about the optimal Home monetary policy function in (2.67) is that it is independent of Foreign monetary policy x^*. That is, the optimal level of the Home money growth rate does not change with changes in the Foreign money growth rate and is a dominant strategy equilibrium.[19]

This dominant strategy equilibrium is shown in Figure 2.2 which plots the lifetime utility of a representative Home agent from (2.66) as a function of Home inflation x and Foreign inflation x^*. The parameters $(\theta, \sigma, \varepsilon, \chi, \xi)$ are simply chosen to reflect values estimated in the empirical literature in order to make a simple example. The dark line running across the top of Figure 2.2 represents the Home monetary policy best response function from (2.67). The optimal Home inflation level at the selected parameter values is a constant $\hat{x} = 1.56$, which is not overly high given that the duration of a period is a generation. Because each country's best response function for monetary policy is a dominant strategy equilibrium, the world Nash monetary equilibrium is the same as the country partial monetary equilibrium.

The main question of this paper is whether openness is inflationary. The following proposition answers this question with regard to both absolute inflation rate (Home country consumer price growth rate) and the real exchange rate inflation (Home country consumer price growth rate over Foreign country consumer price growth rate).

Proposition 2.1 (**Monetary response to changes in openness**)**.** The equilibrium optimal Home money growth rate $\hat{x}$ in (2.67) increases with more Home openness in the form of a higher level of θ_h and in response to more Foreign openness in the form of a higher level of θ_f. The argument for the Foreign country is symmetric. However, when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$. Conversely, when θ_f increases, the increase in $\hat{x}^*$ is greater than the increase in $\hat{x}$.

Proof. See Appendix A-1. □

Because the Home country CPI growth rate (P_{t+1}/P_t) is equal to the Home money growth rate x, an increase in θ_h increases Home country inflation as well as Foreign country inflation. From the perspective of the Home monetary authority, if the Home marginal utility of Home consumption decreases relative to the Home marginal utility of Foreign consumption as is the case when θ_h increases while θ_f remains constant (see first order condition (2.30)), Home country agents bear a

[19]Derivation 6 in Appendix A-3 details why $\hat{x}$ is independent of x^*.

smaller proportion of the inflation tax. In effect, higher θ_h increases the welfare benefits from higher money growth rates to the Home country and lowers the costs. Consequently, the optimal response by the Home monetary authority is to raise the Home money growth rate or the CPI inflation rate in response to a higher degree of openness.

The next two propositions further explain how the level of imperfect competition in a country's labor market, as parameterized by the elasticity of substitution among the differentiated labor inputs ε, influences the optimal money growth rate x and the real outcomes of the economy in equilibrium.

Proposition 2.2 (Deflationary bias of imperfect competition). Both the optimal Home money growth rate $\hat{x}$ and the optimal Foreign money growth rate $\hat{x}^*$ decrease as the level of imperfect competition increases (as ε decreases). Furthermore, there exist two critical within-country elasticities of substitution for the Home country and Foreign country $(\bar{\varepsilon}, \bar{\varepsilon}^*)$ such that $\hat{x} = 1$ when $\varepsilon = \bar{\varepsilon}$ and $\hat{x}^* = 1$ when $\varepsilon = \bar{\varepsilon}^*$. That is, these two critical levels of the imperfect competition parameter implement the Friedman Rule in the Home and Foreign country, respectively.

$$\bar{\varepsilon} = \frac{(1-\theta_f)(1-\sigma)-\xi}{\theta_h(1-\sigma-\xi)} \tag{2.69}$$

$$\bar{\varepsilon}^* = \frac{(1-\theta_h)(1-\sigma)-\xi}{\theta_f(1-\sigma-\xi)} \tag{2.70}$$

Proof. See Appendix A-1. □

This result that the level imperfect competition induces a deflationary bias in monetary policy has been shown recently in a different model by Arseneau (2007).

Lastly, Proposition 2.3 highlights the relationship between the level of market power held by producers within a country and the monopoly power held by the each monetary authority in international markets.

Proposition 2.3 (Market power neutrality). In the case of symmetric countries $\theta_h = \theta_f$, the steady state equilibrium levels of employment n and n^* are not affected by the level of imperfect competition ε within both countries.

Proof. See Appendix A-1. □

Proposition 2.3 says that the real outcomes in each country $(n, n^*, c^h, c^f, c^{h*}, c^{f*})$ are the same regardless of whether the countries are characterized by perfect competition $\varepsilon = \infty$ or whether any degree of monopoly power is enjoyed by labor suppliers $\varepsilon < \infty$.

The implication of this result is that if any monopoly rents available to Home or Foreign agents are not collected through producer price setting, the remainder will be collected by the monetary authority raising prices. As stated in Proposition 2.2, a level of imperfect competition exists at which all the monopoly rents are collected through producer price setting alone. That is, inflation generated by the monetary authority increasing the money growth rate is not needed.

2.3 Empirical Test

The inflation data used here come from the IMF's International Financial Statistics. I use the GDP deflator whenever available, and I use the CPI otherwise. A proxy for openness is an issue is subject to more debate. This paper follows the convention that has been used in much of the previous literature, which is to treat the import share (imports as a share of GDP) as the proxy for a country's level of openness.[20] The results in this section are not very sensitive to whether exports are included in some way in the numerator of this measure.

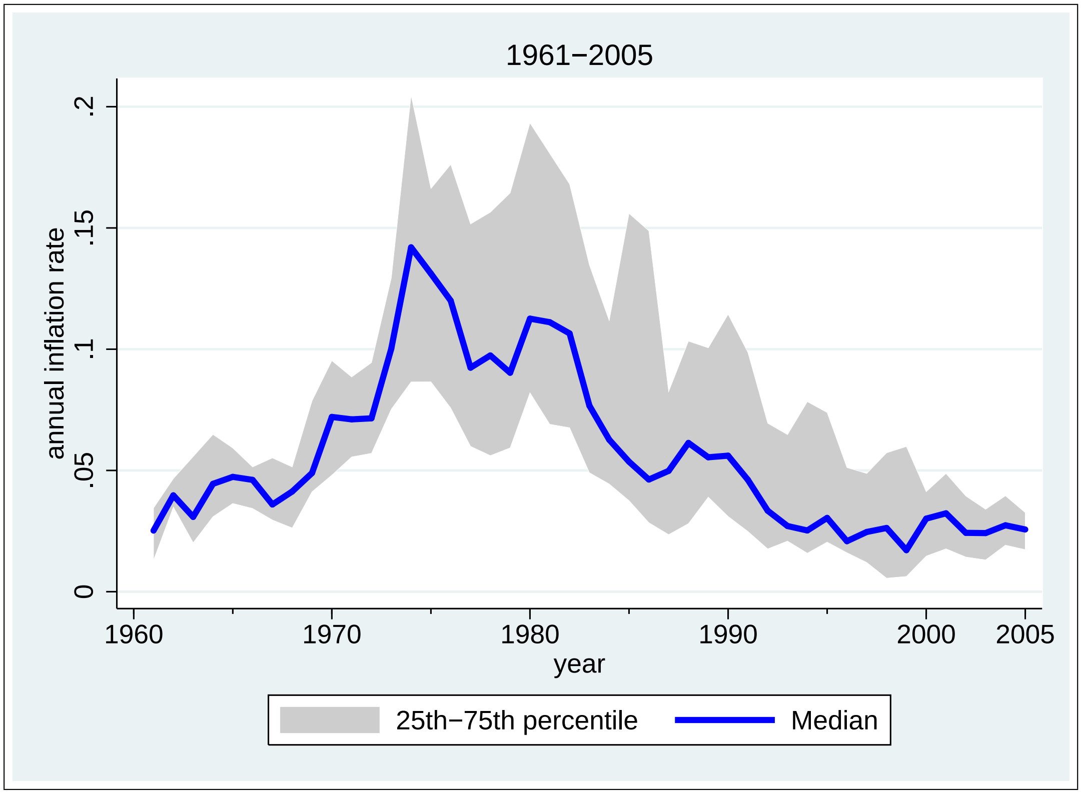
Figures 2.3 and 2.4 show the median inflation rate and median import share for the 30 OECD countries, as well as the 25-percent to 75-percent quartile range, over the period from 1960 to 2005. Notice two facts from the inflation picture in Figure 2.3. First, the median inflation rate has been declining among OECD countries since the early 1970s. In particular, the decade from the early 1970s to the early 1980s can be classified as a high-inflation period. The second fact is that the variance in inflation rates across countries has been declining as well since the early 1970s.

Figure 2.4 shows that the median import share for the 30 OECD countries has been rising since the early 1970s, but that the variance in the share has stayed relatively constant. Taken together, these two figures reflect a negative correlation across all the countries between openness and inflation. However, Figure 2.5 shows that this relationship is not obvious even with respect to simple correlation if one looks at a cross section of countries. Neither Figure 2.3 nor Figure 2.4 changes much if more countries are included.

Figure 2.5 shows the average annual inflation rate and the average annual import share for each of the 30 OECD countries averaged over the 1982 to 2005

[20]This is the convention used in Romer (1993) and Lane (1997).

Figure 2.3: Median inflation rate for 30 OECD countries



period. The solid downward sloping line is the linear fitted line for all 30 countries, reflecting a negative correlation between openness and inflation. However, when the high-inflation outliers of Mexico, Poland, and Turkey are excluded, the relationship becomes positive as evidenced by the upward sloping dashed line. Both relationships get closer to zero as the sample years become smaller and smaller. The point to take from Figure 2.5 is that the relationship between openness and inflation is not obvious, even when looking only at the simple correlation level.

Figure 2.4: Median import share for 30 OECD countries

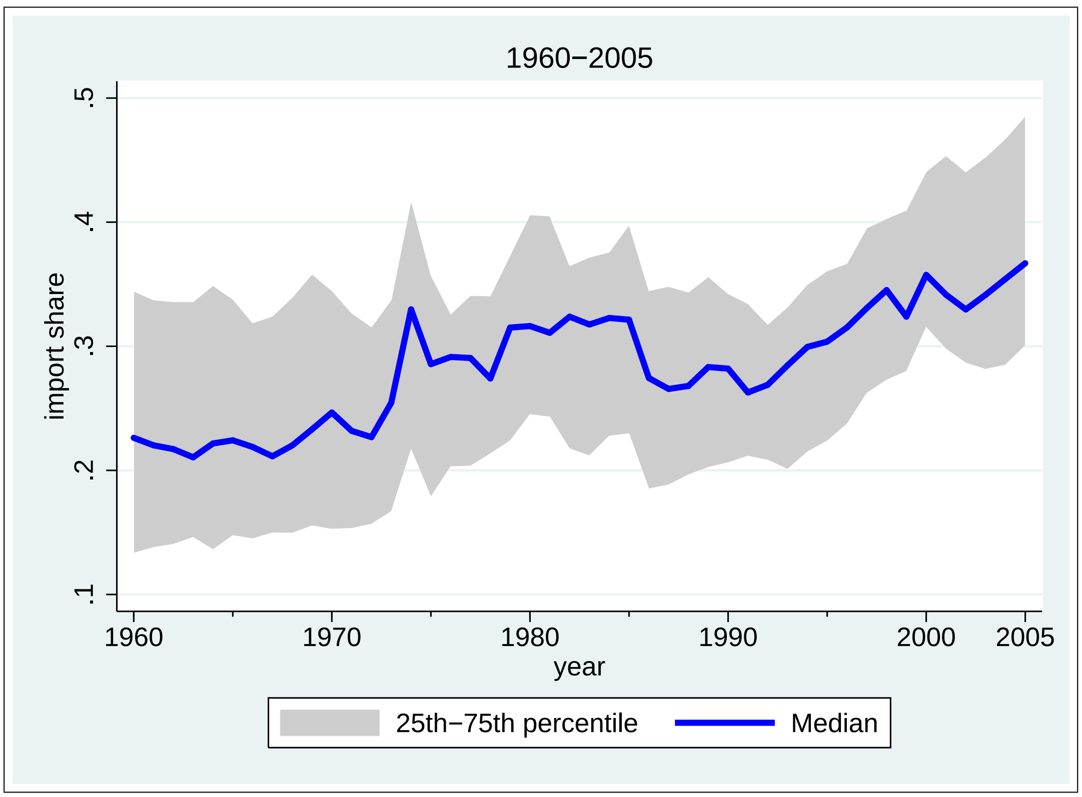
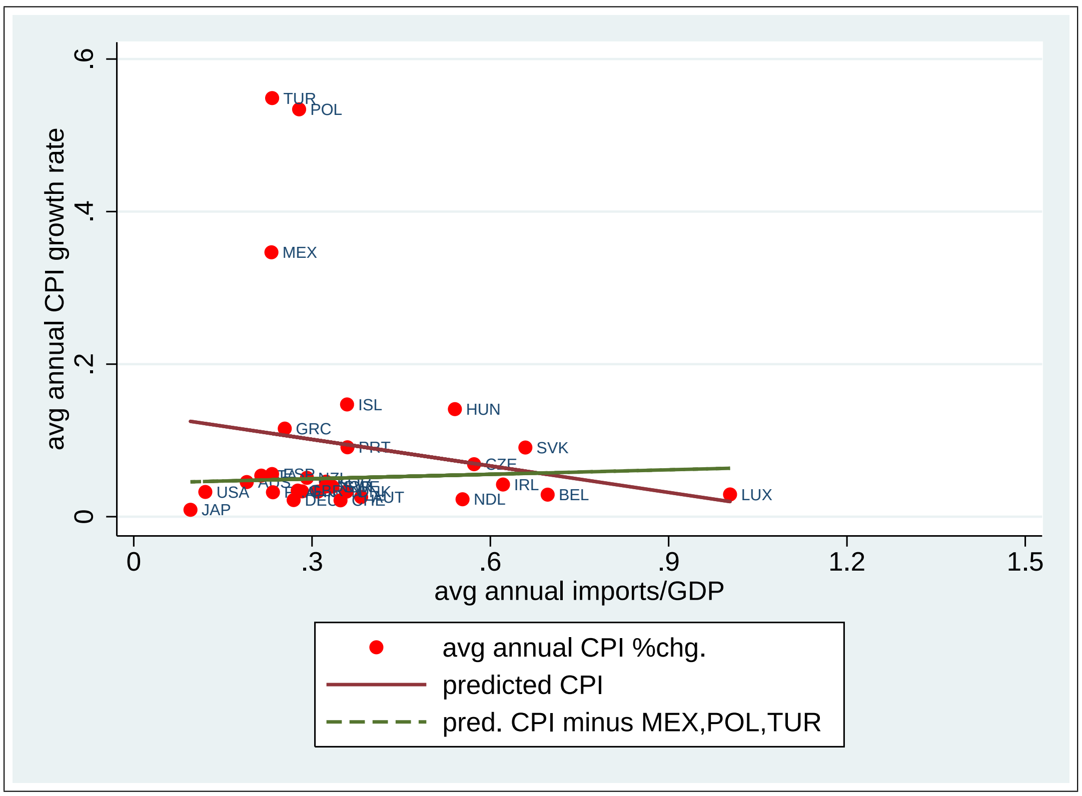


Figure 2.5: Import share vs. CPI for 30 OECD countries: annual avg. for 1982 to 2005



My first empirical test is to run Romer's (1993) regressions with the addition of controlling for the level of imperfect competition. The imperfect competition data come from two sources. First, I use estimates of both manufacturing and nonmanufacturing markups from Høj, Jimenez, Maher, Nicoletti, and Wise (2007) for 17 OECD countries. Obviously, this presents a problem because of the small sample size. The other imperfect competition data come from Blanchflower (2006), Visser (2006), for Economic Cooperation and Development (2004), and Organisation (1998).

The first column of Table 2.3 shows Romer's (1993) baseline specification with the same data and over the same sample period. The subsequent columns of Table 2.3 show the Romer regression augmented by the various controls for the level of imperfect competition. Using this sample period, the negative relationship between openness and inflation is robust to any specification I try.

Table 2.3: Regression of openness on inflation: 1973 to 1987

	log average inflation 1973-1987			
	Romer (1993)	Union Mem. Rt.	(1) Union Cov. Rt.	(2) Union Cov. Rt.
Import share (1973-1987)	-1.130*** (0.272)	-1.719*** (0.393)	-1.263* (0.715)	-1.828** (0.703)
Real per cpta. inc. (1980)	0.038 (0.070)	0.051 (0.162)	-0.912*** (0.218)	-0.037 (0.187)
OECD dummy	-0.446** (0.188)	-0.857*** (0.270)		-1.205** (0.453)
Union mem. rate (1978-1988)		0.013*** (0.005)		
Union cov. rate (1980-1990)			0.006 (0.004)	
Union cov. rate (1980-1995)				0.011** (0.006)
Countries	114	64	20	44
R-squared	0.167	0.335	0.532	0.352

However, Table 2.5 presents a different story. This table runs the Romer regression in a different sample period, 1988 to 2002. As was discussed in Figure 2.3, the period from Romer (1993) was characterized by abnormally high inflation and included the rejection of the pegged exchange rate system. Using the more current sample period, the negative relationship between openness and inflation disappears. Descriptive statistics for the variables in these regressions are reported in Table 2.4.

Table 2.4: Descriptive statistics of regression variables

Variable	Mean	Std. Dev.	Min.	Max.	N
avg. inflation (88-02)	0.158	0.229	0.004	1.360	124
avg. import share (88-02)	0.429	0.225	0.087	1.314	124
Real per cpta. inc. (95)	7,947	7,623	531	33,757	124
Union mem. rate (88-02)	24.0	17.0	2	80.9	84
Union cov. rate (90-00)	45.1	29.4	1.5	95.0	45
Mfct. markup (04)	24.4	5.6	16.0	38.0	16
Nonmfct. markup (04)	12.4	2.6	7.0	18.0	17

Table 2.5: Regression of openness on inflation: 1988 to 2002

	log average inflation 1988-2002				
	Romer updated	Union Mem. Rt.	Union Cov. Rt.	Mfct. Markup	Nonmfct. Markup
Import share (1988-2002)	-0.343 (0.423)	0.088 (0.512)	0.062 (0.549)	1.084 (0.680)	1.317* (0.666)
Real per cpta. inc. (1995)	-0.340*** (0.085)	-0.526*** (0.127)	-0.805*** (0.129)	-1.384 (0.873)	-1.293 (0.769)
Union mem. rate (1988-2002)		0.008 (0.008)			
Union cov. rate (1990-2000)			0.004 (0.005)		
Mfct. markup (2004)				0.004 (0.028)	
Nonmfct. markup (2004)					0.066 (0.060)
Countries	124	84	45	16	17
R-squared	0.134	0.191	0.546	0.248	0.340

In each of the specifications in which I control for the level of imperfect competition, the sign of the coefficient on openness is positive. This result must be taken with some caution because the Romer regression with the sample restricted to the corresponding imperfect competition sample also has a positive coefficient. So it is not necessarily controlling for imperfect competition that makes the relationship positive. The sample of countries declines appreciably with each different imperfect competition measure.

Also, none of the coefficients on openness in the more current sample period is statistically significant. These regressions must be taken as evidence that a positive relationship between openness and inflation may exist. Furthermore, additional data on imperfect competition would be valuable for this question as well as many others. And the additional data does not have to be for as broad a group of countries if some degree of panel dimension can be obtained.

2.4 Conclusion

This paper proposes a model that predicts that increased openness to trade in a country will increase its equilibrium inflation rate. Supporting evidence for this theoretical result is then provided by some empirical tests using more current data than the previous studies. These results run contrary to the negative relationship between openness and inflation proposed by theoretical and empirical work in the broadest vein of this literature.

This work begs the question of what is the optimal degree of openness for a country. This is a conceptually simple exercise to perform in this framework, although it is analytically quite involved. The parameter for openness in this paper θ incorporates both individual preferences and policy choices such as barriers to trade. But θ could be endogenized as fiscal decision that takes place before the monetary authority chooses the money growth rate.

Chapter 3

Expectations, Open Economies, and Time-consistent Monetary Policy

3.1 Introduction

This paper asks the question of how domestic monetary authority should optimally respond to foreign monetary policy. I approach this task using a dynamic general equilibrium (DGE) model derived from microeconomic foundations in which each country's monetary authority chooses the money growth rate with discretion in order to maximize the welfare of its own citizens. This question has proven difficult for two main reasons. Micro-founded DGE models in which the monetary authority has discretion have been shown to suffer from multiple equilibria. On the other hand, international DGE models in which the monetary authority can commit to a policy generate a unique equilibrium. However, these commitment equilibria are dominant strategy equilibria and are independent of foreign monetary policy. This paper seeks to study optimal monetary policy in an environment in which the monetary authority has discretion a unique equilibrium exists that is a function of foreign monetary policy.

Ireland (1997) defines the entire set of time-consistent or "sustainable" equilibria in a closed economy setting. His paper is a more fully specified version of Barro and Gordon (1983) in terms of individual preferences, production technologies, imperfect competition, and monetary objective . In particular, Ireland shows that a continuum of trigger-strategy reputational equilibria exist in this setting. But, similar to Barro and Gordon (1983), Ireland finds a unique non-reputational time-consistent equilibrium in which the monetary authority pushes inflation to its upper bound. This multiplicity of equilibria can be a problem because it implies either that all observed monetary policy is supported some underlying societal strategic threats or that the optimal solution of any monetary authority is to inflate to the maximum.

Ireland (2000) shows that the multiplicity of reputational equilibria, as well as the autarkic non-reputational equilibrium, is a result of the strong assumption of rational expectations. Ireland (2000) shows that relaxing the rational expectations assumption results in a unique non-autarkic steady-state equilibrium that corresponds to the Friedman Rule. The goal of this paper could be summarized as extending Ireland (2000) to a two-country environment.

A number of authors have studied the best response functions of monetary authorities in fully specified two-country DGE models in which monetary authorities can commit to their policy. Cooper and Kempf (2003), Arseneau (2007), and Evans (2007) all study two-country micro-founded DGE models of optimal monetary policy with commitment, and all find unique dominant strategy equilibria. The problem with this result is that anecdotal evidence suggests that monetary authorities are influenced by the actions of foreign monetary authorities and that strategic actions occur among them.

In order to answer the question of how a monetary authority should optimally respond to foreign monetary policy, I will use a model that is most similar to Arseneau (2007). But I will follow Ireland (2000) in that I will not allow each monetary authority to commit to a policy, and I will relax the assumption of rational expectations.

The results of this two-country model of discretionary optimal monetary policy are policy rules that are a function of foreign monetary policy, are not the upper bound of inflation, and do not rely on trigger strategies. The paper is organized as follows. Section 3.2 presents the model, expectations, and equilibrium definitions. Section 3.3 presents some numerical simulations of the time path of monetary policy under various specifications of the model. And Section 3.4 concludes.

3.2 Model

The model in this paper is a time-consistent version of Arseneau's (2007) two-country model of monetary policy under commitment. I will follow Ireland (2000) and relax the rational expectations assumption in order to obtain a unique nonautarkic steady-state monetary policy that is time-consistent. The two countries are Home and Foreign, which are names rather than relative terms.

Each country is populated by a unit measure of identical infinitely lived households and a unit measure of differentiated goods firms. Each country also has a benevolent monetary authority that chooses the growth rate of its country's money supply in order to maximize the welfare of its own citizens.

3.2.1 Money

The monetary authority in each country choose the money growth rate in order to maximize the welfare of the representative household in its respective country. The law of motion for money supply in each country can be represented by the following relationship:

$$M_{t+1}^{S} = x_t M_t^{S} \qquad \text{and} \qquad M_{t+1}^{*S} = x_t^{*} M_t^{*S} \tag{3.1}$$

where M_{t+1}^{S} is the money supply at the end of period t, $x_t > 0$ is the gross money growth rate chosen by the monetary authority during period t, and M_t^{S} is the money supply at the beginning of period t. Variables with stars denote Foreign country variables.

Each monetary authority distributes the change in money supply in each period t through a lump-sum transfer to the representative household in its own country.

$$(x_t - 1)M_t^{S} \qquad \text{and} \qquad (x_t^{*} - 1)M_t^{*S} \tag{3.2}$$

A useful normalization of the initial money stock in each country is $M_0^S = M_0^{*S} = 1$. I assume that Home country households only hold Home currency and Foreign country households only hold Foreign currency.[1]

In addition to currency, households in each country can purchase country-

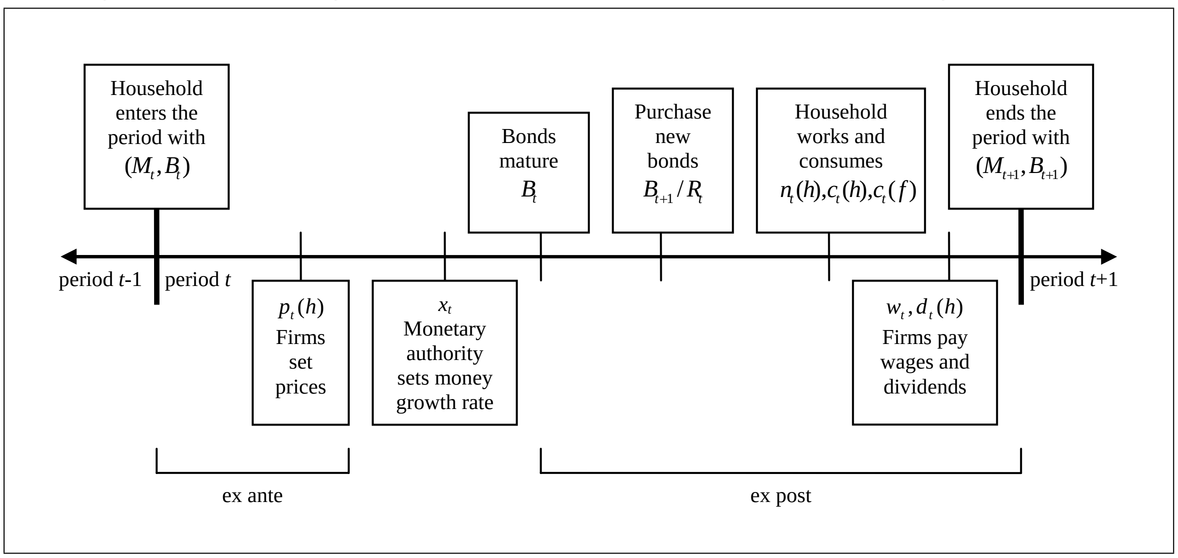
[1]This is not a strong assumption, given that currency can be freely exchanged in the currency market. An alternative specification is to allow households to hold both currencies and let the exchange rate be determined by currency exchange market clearing as in Evans (2007).

specific bonds for B_{t+1}/R_t in period t that return B_{t+1} in period $t+1$. The gross interest rate on bonds purchased in period t is then R_t. This model rules out international trade in bonds, but Arseneau (2007) shows that this constraint does not bind due to the functional form of the model. Other than the bonds and labor, this model assumes no barriers to trade. So the law of one price holds.

3.2.2 Timing

Figure 3.1 illustrates the timing of the model from the standpoint of the representative household in the Home country. The timing for the Foreign household is symmetric. The key characteristic of the model is that the events in each period can be grouped into pre-monetary decision and post-monetary decision. The firm pricing decision is ex ante, and the rest of the decisions are ex post.

Figure 3.1: Timing of the model for Home representative agent



The representative household in the Home country enters period t with Home currency balances M_t and unmatured bond holdings B_t from the previous period. The Home household enters the period knowing the prior N periods of money growth rates in both the Home and Foreign countries. So the past data in each household's information set is $\{x_{t-1}, x_{t-2}, ...x_{t-N}\}$ and $\{x^*_{t-1}, x^*_{t-2}, ...x^*_{t-N}\}$.[2]

[2]I could just as well assume that agents know the entire history of money growth rates, but their

At the beginning of each period, before the monetary authority chooses a money growth rate, each differentiated goods firm h sets the selling price of its good $p_t(h)$. As will be explained more fully in Section 3.2.4, a pricing friction is built into the model following Barro and Gordon (1983) in that the ex ante firm price $p_t(h)$ cannot be changed during period t. Once the firm has set its price, the monetary authority sets the money growth rate for the period x_t and makes a lump-sum transfer $(x_t - 1)M_t^S$ to the representative household of its country.

Once the monetary authority has set the money growth rate x_t, the household's bonds from the previous period mature and the household receives B_t in Home currency. At this point, the household splits into a worker and a shopper. The worker supplies labor to the various firms $n_t(h)$, and the firms produce goods with a linear technology using that labor $y_t(h) = n_t(h)$. The shopper then enters the marketplace and can either buy new bonds B_{t+1} with gross interest rate R_t costing B_{t+1}/R_t or it can purchase consumption of home and foreign differentiated goods, $c_t(h)$ and $c_t(f)$, respectively.

In this model, I assume a cash-in-advance constraint in that all purchases must be paid for in currency. So the cash-in-advance constraint on consumption is the following:

$$M_t + (x_t - 1)M_t^S + B_t - \frac{B_{t+1}}{R_t} \geq P_t c_t \tag{3.3}$$

where P_t and c_t are aggregations of price and consumption over Home and Foreign prices $[p_t(h), p_t(f)]$ and Home and Foreign consumption $[c_t(h), c_t(f)]$, respectively.

Lastly, after the consumption decisions are made, the firm pays each worker a nominal market wage W_t and pays the representative household a dividend $D_t(h)$. The household then leaves period t with Home currency balances M_{t+1} and bonds to mature next period B_{t+1}. The budget constraint for the representative household is the following:

$$M_t + (x_t - 1)M_t^S + B_t + W_t n_t + \int_0^1 D_t(h)\, dh \geq P_t c_t + \frac{B_{t+1}}{R_t} + M_{t+1} \tag{3.4}$$

expectations are formed using only the prior N periods of data.

3.2.3 Households

Each country is populated by a unit measure of identical infinitely lived households. These households maximize lifetime utility by choosing in each period t how much of each differentiated good from each country to consume $c_t(h)$ and $c_t(f)$, how much to work $n_t(h)$, how much wealth to hold in bonds B_{t+1}, and how much wealth to hold in currency M_{t+1}. Household lifetime utility from period t on is given by:

$$U_t = \sum_{j=0}^{\infty} \beta^j \left[ln(c_{t+j}) - \chi n_{t+j} \right] \quad \forall\, t \tag{3.5}$$

where $\beta \in (0,1)$ is the discount factor, $\chi > 0$ is a scale parameter in the disutility of labor function, c_t is aggregated consumption over consumption of both Home and Foreign differentiated goods, and n_t is aggregated labor over labor at Home differentiated goods firms. Using the notation of Arseneau (2007) and following the convention of Dixit and Stiglitz (1977), let $c_{H,t}$ be a CES aggregator of Home consumption by Home households, and let $c_{F,t}$ be a CES aggregator of Foreign consumption by Home households given by:

$$c_{H,t} \equiv \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and} \quad c_{F,t} \equiv \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{3.6}$$

where $\varepsilon > 1$ represents the constant elasticity of substitution between any pair of goods within a country.[3] Then let c_t be defined as the following Cobb-Douglas aggregator over aggregate Home consumption and aggregate Foreign consumption by the Home household:

$$c_t \equiv \left(c_{H,t}\right)^{1-\theta} \left(c_{F,t}\right)^{\theta} \quad \forall\, t \tag{3.7}$$

where the constant elasticity of substitution between Home and Foreign aggregate consumption is 1, and $\theta \in [0.5, 1]$ is a preference parameter that can be interpreted as the degree of openness of the Home country to Foreign goods.[4] The labor aggregator

[3] I impose symmetry in the elasticity of substitution ε both within and across countries. It would be an interesting exercise to relax this symmetry.

[4] Evans (2007) shows that θ represents the share of national income spent on imports (import share), which has often been used as a proxy for openness. See Romer (1993) and Wynne and Kersting (2007). Here also I impose symmetry across countries on the degree of openness, so $\theta_h = \theta_f$ and $c_t^* = \left(c_{F,t}^*\right)^{1-\theta} \left(c_{H,t}^*\right)^{\theta}$.

is defined as the following:

$$n_t \equiv \int_0^1 n_t(h)\, dh \quad \forall\, t \tag{3.8}$$

The following expressions for household demand for Home and Foreign differentiated goods and within-country aggregated goods can be derived from the definition of the consumption aggregators from (3.6) and (3.7) and by solving the household's expenditure minimization problem. The following expressions for aggregate prices also result from this expenditure minimization problem.[5]

$$c_t(h) = \left(\frac{p_t(h)}{p_{H,t}}\right)^{-\varepsilon} c_{H,t} \quad \text{and} \quad c_t(f) = \left(\frac{p_t(f)}{p_{F,t}}\right)^{-\varepsilon} c_{F,t} \tag{3.9}$$

$$c_{H,t} = (1-\theta)\left(\frac{p_{H,t}}{p_t}\right)^{-1} c_t \quad \text{and} \qquad c_{F,t} = \theta\left(\frac{p_{F,t}}{p_t}\right)^{-1} c_t \tag{3.10}$$

$$p_{H,t} = \left(\int_0^1 p_t(h)^{1-\varepsilon} dh\right)^{\frac{1}{1-\varepsilon}} \qquad \text{and} \qquad p_{F,t} = \left(\int_0^1 p_t(f)^{1-\varepsilon} df\right)^{\frac{1}{1-\varepsilon}} \tag{3.11}$$

$$p_t = \frac{1}{\gamma}\,(p_{H,t})^{1-\theta}\,(p_{F,t})^{\theta} \quad \text{where} \quad \gamma = (1-\theta)^{1-\theta}\theta^{\theta} \tag{3.12}$$

where the lower case letters for the prices above are a normalization by the money supply in period t that will be made clear in the household's utility maximization problem.

Let z_{t+j}^{t} and $z_{t+j}^{*,t}$ represent the representative Home household's expectation with probability 1 of the Home money growth rate x_{t+j} and the Foreign money growth rate x_{t+j}^{*} in period $t+j$ given period t information. Thus, $z_t^t = x_t$ and $z_t^{*,t} = x_t^*$ because the consumer's decision in period t comes after the money growth rates in each country have been established. The exact way in which expectations are formed given past information will be detailed in Section 3.2.5.

Given the demand functions above, the problem of the representative household is to maximize lifetime utility by choosing aggregate consumption c_t, aggregate labor supply n_t, normalized wealth to hold in bonds b_{t+1}, and normalized wealth to hold in currency m_{t+1}. This problem is given by dividing the cash-in-advance

[5]See Appendix A-7 for the derivation. Note that the demand functions can also be derived from the utility maximization problem, but the derivation of the prices is particularly intuitive in the expenditure minimization problem as they are simply the multipliers on the consumption level constraints.

constraint (3.3) and budget constraint (3.4) by the Home money supply M_t^S and let lower-case variables represent either real or normalized variables. Also, from this point on, I rewrite the variables with a superscript that represents the information held by the household.

$$\begin{aligned}
\max \quad & \sum_{j=0}^{\infty} \beta^j \left[ln(c_{t+j}^t) - \chi n_{t+j}^t\right] \\
& \text{s.t} \quad m_{t+j}^t + z_{t+j}^t - 1 + b_{t+j}^t - \frac{b_{t+j+1}^t z_{t+j}^t}{R_{t+j}^t} \geq p_{t+j}^t c_{t+j}^t \\
& \text{and} \quad m_{t+j}^t + z_{t+j}^t - 1 + b_{t+j}^t + w_{t+j}^t n_{t+j}^t + \int_0^1 d_{t+j}^t(h)dh \geq ... \\
& \qquad p_{t+j}^t c_{t+j}^t + \frac{b_{t+j+1}^t z_{t+j}^t}{R_{t+j}^t} + m_{t+j+1}^t z_{t+j}^t \quad \forall t
\end{aligned} \tag{3.13}$$

The solution to this problem, as derived in Appendix A-2, results in the following expressions. The analogous expressions for the Foreign representative household are symmetric.

$$c_t^t = \frac{x_t}{p_t^{t-1}} \tag{3.14}$$

$$w_t^t = \left(\frac{\chi}{\beta}\right) z_t^t z_{t+1}^t \tag{3.15}$$

$$R_t^t = \frac{z_{t+1}^t}{\beta} \tag{3.16}$$

These three equations fully describe household behavior and come out of the household maximization problem.

3.2.4 Firms

Each country is populated by a unit measure of firms, indexed by $h \in [0, 1]$ in the Home country and $f \in [0, 1]$ in the Foreign country. They each produce a differentiated good, the substitutability of which relative to the other differentiated goods in that country is represented by the households' preference parameter ε. This is the Dixit and Stiglitz (1977) mechanism in which the love of variety on the part of consumers coupled with imperfect substitutability generates an imperfectly competitive market in which firms are able to charge a markup over marginal cost.

Each firm chooses the price of its good $p_t(h)$ at the beginning of the period given its period $t-1$ information, its expectations about period-t what the monetary policy, and its effects on household demand. However, following the New Keynesian price friction laid out by Barro and Gordon (1983), firms cannot change their price after the period-t money growth rate is chosen. Firms must satisfy demand at the given prices, regardless of whether the money growth rate was close to the firms' expectation.

The output of each firm is governed by a linear production technology in labor $y_t(h) = n_t(h)$. Each firm pays the same market wage w_t to its domestic workforce. Again, labor is assumed to not be mobile. Because the firm meets demand in each period given its prices, this implies $y_t(h) = n_t(h) = c_t(h) + c_t^*(h)$. However, the firm must set its ex ante price with period $t-1$ information, before the monetary authorities in each country set their respective money growth rates. So the problem of the domestic firm can be written as:

$$\max_{p_{t+j}(h)} d_{t+j}^{t-1}(h) = \left[p_{t+j}(h) - w_{t+j}^{t-1}\right]\left[c_{t+j}^{t-1}(h) + c_{t+j}^{*,t-1}(h)\right] \tag{3.17}$$

where $d_{t+j}^{t-1}(h)$ represent the profits of firm h which are paid out as dividends to the representative household in the Home country.[6] Substituting in the expressions for w_{t+j}^{t-1}, $c_{t+j}^{t-1}(h)$, and $c_{t+j}^{*,t-1}(h)$ from Section 3.2.3 with the information iterated backward one period, the expression for the optimal price level for firm h in period t given period $t-1$ information is the following:

$$p_t^{t-1}(h) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} z_{t+1}^{t-1} \tag{3.18}$$

Notice that this means firm price is a markup over the expected wage $p_t^{t-1}(h) = \left(\frac{\varepsilon}{\varepsilon-1}\right) w_t^{t-1}$. Because the right-hand-side of (3.18) does not depend on h, $p_{H,t}^{t-1} = p_t^{t-1}(h)$.

$$p_{H,t}^{t-1} = p_t^{t-1}(h) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} z_{t+1}^{t-1} \tag{3.19}$$

[6]Note that I am assuming that capital markets are closed. Foreign households cannot own stock in Home firms and vice versa.

The symmetric Foreign firm problem produces the following result:

$$p_{F,t}^{*,t-1} = p_t^{*,t-1}(f) = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left(\frac{\chi}{\beta}\right) z_t^{*,t-1} z_{t+1}^{*,t-1} \tag{3.20}$$

The remaining prices $p_{F,t}^{t-1}$, $p_{H,t}^{*,t-1}$, p_t^{t-1}, $p_t^{*,t-1}$, are determined by the firm's expectation of the period t exchange rate e_t^{t-1} through the law of one price. The exchange rate is solved for by substituting all four market clearing conditions (3.30)-(3.33) described in Section 3.2.6 into the budget constraint (3.4).[7]

$$e_t^t = \frac{z_t^t}{z_t^{*,t}} \tag{3.21}$$

I follow Arseneau (2007) in assuming that there are no barriers to exchange in currency or goods, so the price of good h in the Home country must equal the price of good h in the Foreign country converted in to Home country units and likewise for the price of the Foreign produced goods.

$$p_t(h) = e_t p_t^*(h) \qquad \text{and} \qquad p_t(f) = e_t p_t^*(f) \tag{3.22}$$

Given the definitions of the aggregate price indices in (3.11) and (3.12), the law of one price at the individual price level (3.22) implies that the law of one price also holds for the aggregate price indices $p_{H,t} = e_t p_{H,t}^*$, $p_{F,t} = e_t p_{F,t}^*$, and $p_t = e_t p_t^*$. This is a no-arbitrage condition which insures that Home and Foreign households are indifferent between holding either country's currency. It is therefore without loss of generality to make the simplifying assumption that Home households only hold home currency and Foreign households only hold Foreign currency.

Now, using the law of one price (3.22) and the expression for the expected exchange rate that comes from iterating the information in (3.21) back one period

[7]Appendix A-2 shows that the budget constraint holds with equality.

e_t^{t-1}, the following expressions result for the remaining price indices:

$$p_{F,t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} z_{t+1}^{*,t-1} \tag{3.23}$$

$$p_{H,t}^{*} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\chi}{\beta}\right) z_t^{*,t-1} z_{t+1}^{*,t-1} \tag{3.24}$$

$$p_t = \frac{1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\chi}{\beta}\right) z_t^{t-1} \left(z_{t+1}^{t-1}\right)^{1-\theta} \left(z_{t+1}^{*,t-1}\right)^{\theta} \tag{3.25}$$

$$p_t^{*} = \frac{1}{\gamma}\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\chi}{\beta}\right) z_t^{*,t-1} \left(z_{t+1}^{*,t-1}\right)^{1-\theta} \left(z_{t+1}^{t-1}\right)^{\theta} \tag{3.26}$$

3.2.5 Expectations

A large literature has show that the method by which households and firms form their expectations of a policymaker's actions has a profound influence on the policy outcomes. Lucas (1972) showed that allowing agents to consider the policy maker's incentives in a forward-looking manner is a desirable mechanism in economic models. However, Barro and Gordon (1983) and, more recently, Ireland (1997) show that the strong assumption of rational expectations in models of monetary policy leads to a multiplicity of reputational trigger-strategy equilibria as well as a unique highly inflationary steady-state equilibrium.

One problem with the multiple equilibria generated under models of optimal monetary policy in which agents have rational expectations is that even a surprise decrease in the money growth rate will generate a large jump in expected inflation. Ireland (2000) shows in a closed economy model that relaxing the assumption of rational expectations in an intuitive way generates a unique stationary equilibrium that is less inflationary than the rational expectations equilibrium and sometimes equals the deflationary Friedman Rule.

I will impose adaptive expectations functions on this model that follow the same form as in Ireland (2000). Let $z_{t+j}^{t} = \psi_{t+j}^{t}$ and $z_{t+j}^{*,t} = \psi_{t+j}^{*,t}$, where ψ_{t+j}^{t} and $\psi_{t+j}^{*,t}$ are the expectations functions for the money growth rates in the Home country and the Foreign country, respectively, in period $t+j$ given period t information.[8] When $j = 0$, the expectation function is trivial because the information is known

[8] I am assuming here that Home and Foreign households and firms have the same information. That is, the Home households' information about past Foreign money growth rates is the same as Foreign households' information about past Foreign money growth rates. However, this might be an interesting assumption to relax.

($\psi_t^t = x_t$ and $\psi_{t+j}^{*,t} = x_t^*$). However, for $j > 0$, the expectations will be functions of $N < \infty$ periods of past money growth rates.

$$z_{t+j}^{t-1} = \psi_{t+j}^{t-1}\left(x_{t-1}, x_{t-2}, ...x_{t-N}\right) \quad \forall t \quad \text{and} \quad j = 0, 1, 2, ... \tag{3.27}$$

$$z_{t+j}^{*,t-1} = \psi_{t+j}^{*,t-1}\left(x_{t-1}^*, x_{t-2}^*, ...x_{t-N}^*\right) \quad \forall t \quad \text{and} \quad j = 0, 1, 2, ... \tag{3.28}$$

where the expectations functions are defined on $\psi_{t+j}^{t-1} : R_{++}^N \rightarrow R_{++}$ and $\psi_{t+j}^{*,t-1} : R_{++}^N \rightarrow R_{++}$.

Let ψ represent both ψ_{t+j}^{t-1} and $\psi_{t+j}^{*,t-1}$ for any t or any j. Ireland (2000) proposes the following four intuitive restrictions on the expectations functions ψ.

(R1) ψ is nondecreasing in each of its arguments.
(R2) $\psi\left(x, x, ...x\right) = x$ for all $x \in R_{++}$.
(R3) ψ is continuously differentiable on R_{++}^N.
(R4) $\psi\left(x_1, x_2, ...x_N\right) \geq \beta$ for all $\left(x_1, x_2, ...x_N\right) \in R_{++}^N$ and $x_i \geq \beta$.

Restriction (R1) simply insures that the expected money growth rate moves in the same direction as the actual money growth rate. Restriction (R2) implies that money growth expectations will converge to the actual money growth rate if the rate is held constant long enough. Ireland (2000) notes that (R1) and (R2) allow a monetary authority to build credibility over time. Restriction (R3) limits the extent to which expectations can jump, given any size policy surprise. Lastly, restriction (R4) ensures that a no-arbitrage condition that the interest rate be positive $R_t = z_{t+1}^t / \beta \geq 1$.

Taken together, restriction (R1)-(R4) impose properties on the backward looking expectations ψ that look like some behavior we see in practice. The exact specification of the expectations function used by Ireland (2000) is the following:

$$\psi_{t+j}^{t-1} = \prod_{k=1}^{N} x_{t-k}^{\alpha_k} \quad \forall t \quad \text{and} \quad j = 0, 1, 2, ... \tag{3.29}$$

where the α_k represent the elasticity of the $t-k$th money growth rate on the expected money growth rate ψ_{t+j}^{t-1} in period $t+j$ given period $t-1$ information. Restriction (R1) requires that $\alpha_k \geq 0$, and restriction (R2) implies that $\sum_{k=1}^{N} \alpha_k = 1$.

3.2.6 Equilibrium

This section will define both a private equilibrium in this two-country environment, given expectations and the monetary policy in both countries. I will then define the best response function of the Home monetary authority given the policy of the Foreign monetary authority. The focus on the Home monetary best response function is without loss of generality because the two countries are symmetric.

Four markets must clear in each country in equilibrium. First, the money market must clear in each country, which means that $M_t = M_t^S$. The bond market must clear in both countries, which means a zero net supply. Each goods market must clear, which was already assumed in the firm problem. And lastly, the labor market must clear in both the Home country and the Foreign country.

$$m_t = 1 \quad \text{and} \quad m_t^* = 1 \quad \forall t \tag{3.30}$$

$$b_t = 0 \quad \text{and} \quad b_t^* = 0 \quad \forall t \tag{3.31}$$

$$y_t(h) = c_t(h) + c_t^*(h) \quad \text{and} \quad y_t(f) = c_t(f) + c_t^*(f) \quad \forall t, h, f \tag{3.32}$$

$$n_t(h) = y_t(h) \quad \text{and} \quad n_t(f) = y_t(f) \quad \forall t, h, f \tag{3.33}$$

I now define a private equilibrium in this two-country economy, given the money growth rates in both the Home and Foreign countries.

Definition 3.1 (**Private Equilibrium given Home and Foreign Monetary Policy**). A private equilibrium given the money growth rates in the Home and Foreign countries consists of the following:

- expectations $z_{t+j}^{t-1} = \psi_{t+j}^{t-1}$ and $z_{t+j}^{*,t-1} = \psi_{t+j}^{*,t-1}$ for all t and $j = 0, 1, 2, ...$ as shown in (3.27) and (3.28) that satisfy restriction (R1)-(R4)
- a sequence of Home money growth rates $\{x_t\}_{t=0}^{\infty}$ and Foreign money growth rates $\{x_t^*\}_{t=0}^{\infty}$ such that $x_i \geq \beta$ and $x_i^* \geq \beta$
- The Home aggregate price index p_t^{t-1} and the Foreign aggregate price index $p_t^{*,t-1}$ are given by (3.25) and (3.26), respectively.
- Aggregate consumption by Home households c_t^t and aggregate consumption by Foreign households $c_t^{*,t}$ in period t are given by (3.14)
- Labor supply in the Home country n_t^t and labor supply in the Foreign country n_t^* are given by (3.32) and (3.33) and the expression for consumption demand and their Foreign analogues in (3.10) and (3.14)

Before showing the time-consistent problem of the Home monetary authority and its corresponding equilibrium, I will present the commitment equilibrium, which corresponds to the methods and findings of Arseneau (2007). If the Home and Foreign monetary authorities can commit to a money growth rate at the beginning of time, then $x_t = x$ and $x_t^* = x^*$ for all t. The problem of the Home monetary authority is then to choose a constant money growth rate x at $t = 0$ given x^* in order to maximize the welfare of the Home representative household.

$$\begin{aligned} \max_{x} \quad & \sum_{j=t}^{\infty} \beta^t \left[ln(c_t) - \chi n_t \right] \\ & \text{s.t} \quad c_t = \gamma \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\beta}{\chi} \right) (x)^{\theta - 1} (x^*)^{-\theta} \\ & \text{and} \quad n_t = \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\beta}{\chi} \right) \frac{1}{x} \\ & \text{and} \quad R_t \geq 1 \quad \forall t \end{aligned} \tag{3.34}$$

where the expressions for c_t and n_t in (3.34) come from their specifications in Definition 3.1 of the private equilibrium, but with x_t and z_{t+j}^t replaced by x and with x_t^* and $z_{t+j}^{*,t}$ replaced by x^*. A monetary equilibrium with commitment can then be defined as follows.

Definition 3.2 (**Monetary Nash Equilibrium with Commitment**)**.** A monetary Nash equilibrium with commitment consists of the following:

- All characteristics of Definition 3.1 of a private equilibrium hold.
- The sequence of Home money growth rates $\{x_t = x\}_{t=0}^{\infty}$ satisfy the maximization problem in (3.34).
- The sequence of Foreign money growth rates $\{x_t^* = x^*\}_{t=0}^{\infty}$ satisfy the Foreign maximization problem analogous to (3.34).
- In each period t, the Home money growth rate x is a best response the the Foreign money growth rate, and the Foreign money growth rate x^* is a best response to the Home money growth rate x.

The first step in finding the monetary Nash equilibrium is solving for the best response function that comes out of the maximization problem in (3.34). The best response function of both the Home and Foreign monetary authorities ends up being a dominant strategy equilibrium. So the best response function also represents the

Nash equilibrium. The monetary Nash equilibrium with commitment, shown below, is identical for the Home and Foreign countries because they are symmetric.[9]

$$\hat{x}^c = \hat{x}^{*,c} = \begin{cases} \frac{\beta}{1-\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right) & \text{if} \quad \frac{1}{1-\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right) > 1 \\ \beta & \text{if} \quad \frac{1}{1-\theta}\left(\frac{\varepsilon-1}{\varepsilon}\right) \leq 1 \end{cases} \tag{3.35}$$

Lastly I define a time-consistent Home Monetary equilibrium given Foreign money growth as a best response function to x^*. The Home monetary authority's maximization problem in the time consistent case is to choose x_t in every period to maximize the lifetime utility of the Home representative household.

$$\begin{aligned} \max_{x_t} \quad & \sum_{j=0}^{\infty} \beta^{t+j} \left[ln(c^t_{t+j}) - \chi n^t_{t+j}\right] \\ \text{s.t} \quad & c^t_t = x_t \left(\frac{\beta\gamma}{\chi}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\psi^{t-1}_t\right)^{\theta-2}\left(\psi^{*,t-1}_t\right)^{-\theta} \\ \text{and} \quad & n^*_t = \left(\frac{\varepsilon-1}{\varepsilon}\right)\left(\frac{\beta}{\chi}\right)\psi^{t-1}_t\left[(1-\theta)x_t\psi^{t-1}_t + \theta x^*_t\psi^{*,t-1}_t\right] \\ \text{and} \quad & R^*_t \geq 1 \\ \text{and} \quad & x_t \in [\beta, \bar{x}] \quad \forall t \end{aligned} \tag{3.36}$$

The definition of a time-consistent Home monetary equilibrium given x^* is given below.

Definition 3.3 (**Time-consistent Home Monetary Equilibrium given** x^***).** A time-consistent monetary equilibrium given the rate of Foreign money growth x^* consists of the following:

- All characteristics of Definition 3.1 of a private equilibrium hold.
- The sequence of Home money growth rates $\{x_t\}_{t=0}^{\infty}$ satisfy the maximization problem in (3.36) given the sequence of Foreign money growth rates $\{x^*_t = x^*\}_{t=0}^{\infty}$.

The next section provides a numerical example of a time-consistent Home monetary equilibrium given x^*.

[9]The derivation of this result is in Appendix **??**. The Nash equilibrium here is slightly different from that of Arseneau (2007) because he reverses that Cobb Douglas weights on the Foreign household consumption aggregator in order to interpret θ as country size instead of country openness as I do here.

3.3 Numerical Simulation

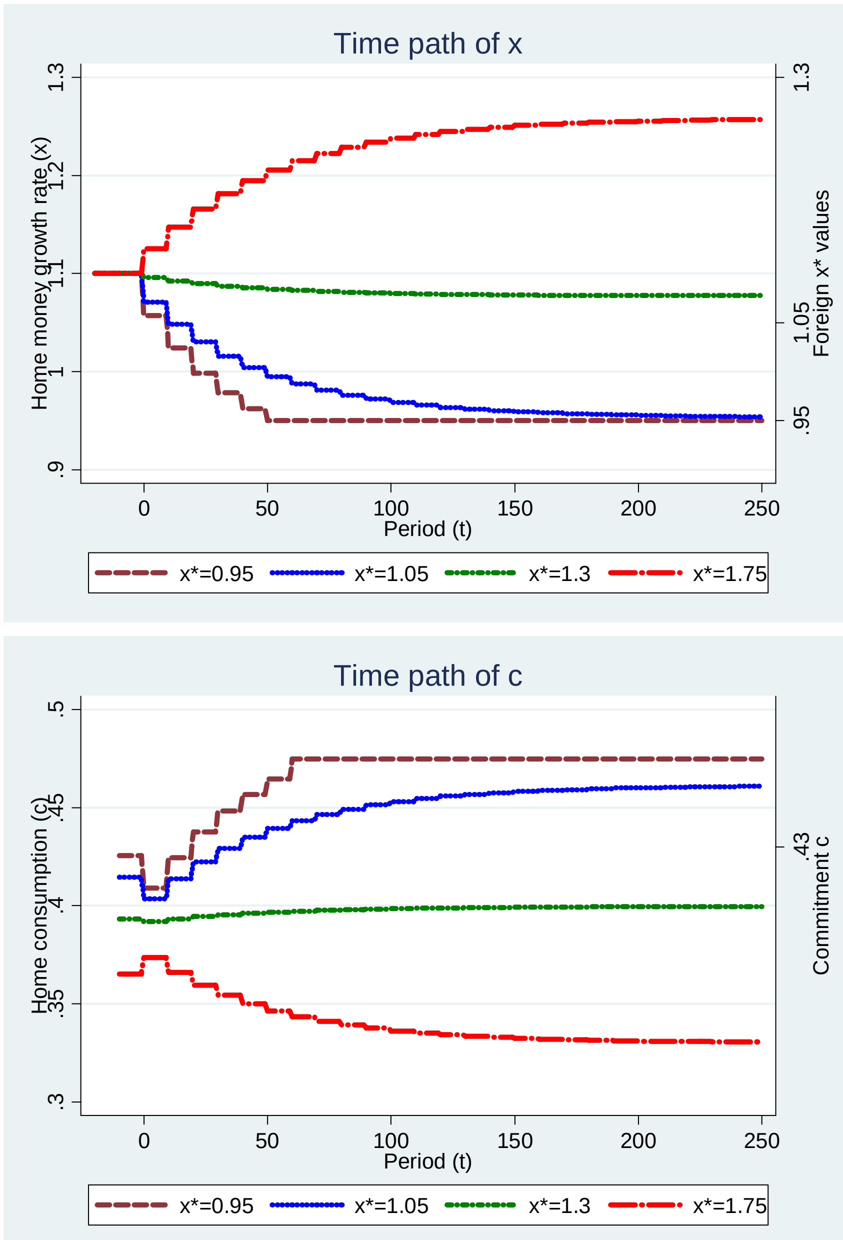
In this section I provide a numerical example of what the time path of a time-consistent Home monetary equilibrium looks like given different constant levels of Foreign money growth rates. This numerical exercise follows Ireland (2000). I set the openness parameter of the model in both countries to $\theta = 0.25$ implying that both countries have a 25-percent import share of GDP. The discount factor is set to $\beta = 0.95$. The elasticity of substitution is set to $\varepsilon = 6$, consistent with a 20-percent markup. The scale parameter of the disutility of labor function χ is set equal to 1. And the number of periods that households and firms use to formulate expectations is $N = 10$. The elasticities α_k in the expectations function are set to $\alpha_{10} = 1$ and $\alpha_k = 0$ for k = 1,2,...9.

With these parameters, the optimal money growth rate with commitment that corresponds to Definition 3.2 and constant money growth rate rule (3.35) is about 1.05. The experiment here is to start the Home money growth rate at a level not equal to the monetary Nash equilibrium with commitment (1.10 in this case) and leave it there long enough that all households expect it to stay there. Then let the Home monetary authority adjust the money growth rate x_t optimally each period according to Definition 3.3. Figure 3.2 shows the time path of the Home money growth rate starting at a money growth rate of 1.1 in periods $t < 0$ but then being allowed to change its rate optimally in periods $t \geq 0$. The lower panel shows the corresponding aggregate consumption levels. Each line corresponds to a different level of constant Foreign money growth rate.

The top panel of Figure 3.2 shows that it is optimal for the Home monetary authority to gradually reduce the Home money growth rate for Foreign money growth rates less than 1.3. The lower panel shows how this reduction of interest rates initially inflicts a cost to societal welfare. This illustrates the key point noted by Ireland (2000) that the discount factor β is a key parameter governing the ability of a monetary authority to be able to earn credibility. That is, households must be patient enough for the long-term benefits of inflation reduction to outweigh the short-term costs. Another key characteristic of these time-consistent Home monetary equilibrium paths is that the incentive to lower inflation diminishes as the Foreign country has higher inflation.

This strategic complementarity has predicts that when a country like the

Figure 3.2: Time path of x for various values of x^*



Note: this graph shows the time path of the Home gross money growth rate and the corresponding aggregate Home consumption. The parameters are set to the following: $(N = 10, \chi = 1, \varepsilon = 6, \beta = 0.95, \theta = 0.25)$

United States begins a period of increased money growth rates that other central banks should follow. However, in this exercise, the Foreign central bank has to have a money growth rate of 30 percent in every period as opposed to the Home money growth rate of 10 percent. This is a wider disparity than is usually seen in major trading partners.

This experiment is merely a first pass at studying situations in which Foreign monetary policy influences Home monetary policy. An obvious next step is to construct a time-consistent Nash equilibrium. But some good findings come from this exercise. First, the steady state values in this time-consistent monetary equilibrium have more to do with the level of Foreign monetary policy than they do with the Ramsey equilibrium that comes from the commitment policy. Also, a strategic complementarity seems to exist between the Home and Foreign monetary authority. As the Foreign money growth rate increases, the incentive for the Home monetary authority to inflate goes up. Lastly, these time-consistent equilibrium fall below the upper bound of the money growth rate and do not rely on trigger strategies.

3.4 Conclusion

The model outlined in this study provides an environment in which to study the effect of foreign monetary policy on domestic monetary policy in which the problems of multiple equilibria and dominant strategy equilibria are overcome. This result is achieved by following Ireland (2000) and relaxing the assumption of rational expectations in an intuitive way. The implications of the model are that a monetary authority can build credibility over time if households are patient enough for the long-term benefits to outweigh the short-term costs of adjusting monetary policy.

As mentioned in the previous section, an obvious extension of this work is to construct the time-consistent monetary Nash equilibrium of the two countries given different starting points. Also, the fact that the experiment of this paper starts each country at a suboptimal money growth rate begs the question of what might cause this. Adding shocks to this model that move the steady-state around would be instructive.

In addition, a large portion of the results is due to the form of the expectations functions ψ and ψ^*. One extension would be to try some more sophisticated forecasting methods that might be closer to what is seen in practice although still not as strong as the rational expectations assumption. Also, adding some imperfect information such that the Home household knows more about Home monetary policy than the Foreign household does.

APPENDIX

A-1 Proofs

Proof of Proposition 1.1: Monetary response to changes in openness. Taking the derivative of the expression for $\hat{x}$ in (1.74) with respect to θ_h and θ_f gives the following results:

$$\hat{x} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h \Sigma_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_f)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_h)\xi}$$

$$\frac{\partial \hat{x}}{\partial \theta_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_f(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f - \theta_h \Sigma_f]^2} > 0$$
$$\frac{\partial \hat{x}}{\partial \theta_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\theta_h(1-\sigma)(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f - \theta_h \Sigma_f]^2} > 0$$

Taking the derivative of the expression for $\hat{x}^*$ in (1.75) with respect to θ_f and θ_h gives the following results:

$$\hat{x}^* = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h}{(1-\theta_f)\Delta_h - \theta_f \Sigma_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{(1-\theta_h)(1-\sigma) - \xi}{(1-\theta_h-\theta_f)(1-\sigma) - (1-\theta_f)\xi}$$

$$\frac{\partial \hat{x}^*}{\partial \theta_f} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\Delta_h(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h - \theta_f \Sigma_h]^2} > 0$$
$$\frac{\partial \hat{x}^*}{\partial \theta_h} = \left(\frac{\varepsilon - 1}{\varepsilon}\right) \frac{\theta_f(1-\sigma)(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h - \theta_f \Sigma_h]^2} > 0$$

Now the proposition that when θ_h increases, the increase in $\hat{x}$ is greater than the increase in $\hat{x}^*$, simply means that $\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} > 0$.

$$
\begin{aligned}
\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} &= \frac{\partial\hat{x}}{\partial\theta_h}\left[\frac{1}{\hat{x}^*}\right] - \frac{\partial\hat{x}^*}{\partial\theta_h}\left[\frac{\hat{x}}{(\hat{x}^*)^2}\right] \\
&= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f(1-\sigma-\xi)}{[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}{\Delta_h}\ldots \\
&\quad -\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Sigma_f(1-\sigma-\xi)}{[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]^2}\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\Delta_f\left[(1-\theta_f)\Delta_h-\theta_f\Sigma_h\right]^2}{\Delta_h^2\left[(1-\theta_h)\Delta_f-\theta_h\Sigma_f\right]}\right) \\
&= \frac{\Delta_f(1-\sigma-\xi)[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]}{\Delta_h[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2} - \frac{\Delta_f\Sigma_f(1-\sigma-\xi)}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]} \\
&= \frac{\Delta_h\Delta_f(1-\sigma-\xi)[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]-\Delta_f\Sigma_f(1-\sigma-\xi)[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2} \\
&= \Delta_f(1-\sigma-\xi)\left(\frac{\Delta_h[(1-\theta_f)\Delta_h-\theta_f\Sigma_h]-\Sigma_f[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) \\
&= \Delta_f(1-\sigma-\xi)\left(\frac{\Delta_h[(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi]-\Sigma_f[(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) \\
\frac{\partial\left(\frac{\hat{x}}{\hat{x}^*}\right)}{\partial\theta_h} &= \Delta_f(1-\sigma-\xi)\left(\frac{(\Delta_h-\Sigma_f)(1-\theta_h-\theta_f)(1-\sigma)+\xi\left[\Sigma_f(1-\theta_h)-\Delta_h(1-\theta_f)\right]}{\Delta_h^2[(1-\theta_h)\Delta_f-\theta_h\Sigma_f]^2}\right) > 0
\end{aligned}
$$

The last line is true because $\Delta_h - \Sigma_f < 0$ and $\Sigma_f(1-\theta_h) - \Delta_h(1-\theta_f) > 0$. □

Proof of Proposition 1.2: Deflationary bias of imperfect competition. From (1.74) and (1.75):

$$
\begin{aligned}
\frac{\partial\hat{x}}{\partial\varepsilon} &= \left(\frac{1}{\varepsilon^2}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f-\theta_h\Sigma_f} > 0 \\
\frac{\partial\hat{x}^*}{\partial\varepsilon} &= \left(\frac{1}{\varepsilon^2}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h} > 0
\end{aligned}
$$

Then, to find the respective levels of ε that induce the Home and Foreign monetary authorities, respectively, to set their money growth rates equal to 1 is found by solving (1.74) and (1.75) for ε when $\hat{x} = 1$ and when $\hat{x}^* = 1$.

$$\bar{\varepsilon}: \quad 1 = \left(\frac{\bar{\varepsilon}-1}{\bar{\varepsilon}}\right) \frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi}$$
$$\bar{\varepsilon}^*: \quad 1 = \left(\frac{\bar{\varepsilon}^*-1}{\bar{\varepsilon}^*}\right) \frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}$$

Solving these two equations for $\bar{\varepsilon}$ and $\bar{\varepsilon}^*$, respectively, gives the results in (1.85) and (1.86).

$$\bar{\varepsilon} = \frac{\Delta_f}{\Sigma_h - \theta_h \xi} = \frac{(1-\theta_f)(1-\sigma)-\xi}{\theta_h(1-\sigma-\xi)}$$
$$\bar{\varepsilon}^* = \frac{\Delta_h}{\Sigma_f - \theta_f \xi} = \frac{(1-\theta_h)(1-\sigma)-\xi}{\theta_f(1-\sigma-\xi)}$$

□

Proof of Proposition 1.3: Market power neutrality. When the Home and Foreign Country are symmetric $\theta_h = \theta_f = \theta$, the equilibrium employment level is given by:

$$n = \left[\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\left(\frac{\varepsilon}{\varepsilon-1}\right)\right]^{\frac{\Delta-\Sigma}{\Delta^2-\Sigma^2}} (\hat{x})^{\frac{\Delta}{\Delta^2-\Sigma^2}} (\hat{x}^*)^{\frac{-\Sigma}{\Delta^2-\Sigma^2}}$$

where $\Delta = (1-\theta)(1-\sigma)-\xi$ and $\Sigma = \theta(1-\sigma)$. The expressions for the optimal money growth rates in this symmetric case are given by:

$$\hat{x} = \hat{x}^* = \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}$$

Now the equilibrium employment level can be written as:

$$n = n^* = \left[\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\right]^{\frac{1}{1-\sigma-\xi}} \left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\sigma-\xi}} \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{1-\sigma-\xi}} \left[\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}\right]^{\frac{1}{1-\sigma-\xi}}$$
$$= \left[\left(\frac{\chi\xi}{(1-\theta)^{(1-\theta)(1-\sigma)}\theta^{\theta(1-\sigma)}}\right)\left(\frac{\Delta}{(1-\theta)\Delta-\theta\Sigma}\right)\right]^{\frac{1}{1-\sigma-\xi}}$$

It is clear that neither n nor n^* is a function of the level of imperfect competition ε. And because the equilibrium consumption levels are simply constant fractions of the output level, consumption is also not affected by changes in the level of imperfect competition. □

A-2 Derivation of Solution to Chapter 3 Utility Maximization Problem

The maximization problem of the Home country representative household is to choose c^t_{t+j}, n^t_{t+j}, b^t_{t+j+1}, and m^t_{t+j+1} given firm prices, Home and Foreign money growth rates, and period-t information. The problem for the Foreign representative household is symmetric. The Home household maximization problem, with the constraints divided by M^S_t is the following:

$$\begin{aligned}
\max \quad & \sum_{j=0}^{\infty} \beta^j \left[ln(c^t_{t+j}) - \chi n^t_{t+j} \right] \\
& \text{s.t} \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} - \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} \geq p^t_{t+j} c^t_{t+j} \\
& \text{and} \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} + w^t_{t+j} n^t_{t+j} + \int_0^1 d^t_{t+j}(h) dh \geq ... \\
& \qquad p^t_{t+j} c^t_{t+j} + \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} + m^t_{t+j+1} z^t_{t+j} \quad \forall t
\end{aligned} \tag{A.2.1}$$

Taking the derivative of the Lagrangian with respect to the choice variables and with respect to the multipliers on the constraints, the first order conditions of this problem are the following:

$$\frac{\partial \mathcal{L}}{\partial c^t_{t+j}}: \quad \frac{1}{c^t_{t+j}} = \left(\mu^t_{t+j} + \lambda^t_{t+j}\right) p^t_{t+j} \tag{A.2.2}$$

$$\frac{\partial \mathcal{L}}{\partial n^t_{t+j}}: \quad \lambda^t_{t+j} w^t_{t+j} = \chi \tag{A.2.3}$$

$$\frac{\partial \mathcal{L}}{\partial b^t_{t+j+1}}: \quad \beta \left(\mu^t_{t+j+1} + \lambda^t_{t+j+1}\right) = \left(\mu^t_{t+j} + \lambda^t_{t+j}\right) \frac{z^t_{t+j}}{R^t_{t+j}} \tag{A.2.4}$$

$$\frac{\partial \mathcal{L}}{\partial m^t_{t+j+1}}: \quad \beta \left(\mu^t_{t+j+1} + \lambda^t_{t+j+1}\right) = \lambda^t_{t+j} z^t_{t+j} \tag{A.2.5}$$

$$\frac{\partial \mathcal{L}}{\partial \mu^t_{t+j}}: \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} - \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} = p^t_{t+j} c^t_{t+j} \tag{A.2.6}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda^t_{t+j}}: \quad m^t_{t+j} + z^t_{t+j} - 1 + b^t_{t+j} + w^t_{t+j} n^t_{t+j} + \int_0^1 d^t_{t+j}(h) dh = ... \tag{A.2.7}$$

$$p^t_{t+j} c^t_{t+j} + \frac{b^t_{t+j+1} z^t_{t+j}}{R^t_{t+j}} + m^t_{t+j+1} z^t_{t+j}$$

where μ^t_{t+j} and λ^t_{t+j} are the multipliers on the cash-in-advance constraint and the budget constraint, respectively. Equations (A.2.6) and (A.2.7) hold with equality only when $\mu^t_{t+j} \geq 0$ and $\lambda^t_{t+j} \geq 0$. From (A.2.3), it is clear that $\lambda^t_{t+j} > 0$ so the budget constraint holds with equality. At this point, I assume that $\mu^t_{t+j} > 0$ so that the cash-in-advance constraint holds with equality, but this will be shown to be correct after the other variables values are derived.

If the CIA constraint holds with equality, then substituting in the market clearing conditions $m^t_{t+j} = 1$ and $b^t_{t+j} = 0$ and solving for c^t_{t+j} gives the equilibrium condition for c^t_{t+j}:

$$c^t_{t+j} = \frac{z^t_{t+j}}{p^t_{t+j}} \tag{A.2.8}$$

The expressions for $c^t_{t+j}(h)$, $c^t_{t+j}(f)$, $c^t_{H,t+j}$, and $c^t_{F,t+j}$ can be found by substituting (A.2.8) into (3.9) and (3.10). Substituting (A.2.8) into (A.2.2), solving for $\mu^t_{t+j} + \lambda^t_{t+j}$ and substituting that expression into (A.2.5) gives the following equilibrium expression for λ^t_{t+j}:

$$\lambda^t_{t+j} = \frac{\beta}{z^t_{t+j} z^t_{t+j+1}} \tag{A.2.9}$$

Substituting (A.2.9) into (A.2.3) gives:

$$w_{t+j}^t = \frac{\chi}{\beta} z_{t+j}^t z_{t+j+1}^t \tag{A.2.10}$$

Again, substituting (A.2.8) into (A.2.2), solving for $\mu_{t+j}^t + \lambda_{t+j}^t$, and substituting that expression into (A.2.4) gives the following equilibrium expression for R_{t+j}^t:

$$R_{t+j}^t = \frac{z_{t+j+1}^t}{\beta} \tag{A.2.11}$$

A-3 Derivations

Derivation 1 (**Demand for differentiated good** z)**.** The demand function for individual differentiated good z from (1.30) is derived in the following way:

$$\begin{aligned} d_t(z) \equiv c_t(z) + c_t^*(z) &= \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} C_t^h + \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} C_t^{h*} \\ &= \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{m_t^h}{P_t^h} + \left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon} \frac{m_t^{h*}}{P_t^h} \\ &= \left(\frac{p_{i,t}(z)}{P_t^h}\right)^{-\varepsilon} \frac{m_t^h + m_t^{h*}}{P_t^h} = \left(\frac{p_{i,t}(z)}{P_t^h}\right)^{-\varepsilon} \frac{M_t}{P_t^h} \\ d_t(z) &= \left(\frac{p_{i,t}(z)}{P_t^h}\right)^{-\varepsilon} \frac{xM_{t-1}}{P_t^h} \end{aligned}$$

Derivation 2 (**Firm Demand for differentiated labor input** z)**.** The demand function for individual differentiated labor input $n_t(z)$ by firms from (2.13) is derived in the following way. Firms choose the amount of each type of differentiated labor $n_t(z)$ given the contracted wages $w_t(z)$ and the perfect competition selling price of the output P_t^h.

$$\begin{aligned} \max_{n_t(z)} \pi_t = P_t^h \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz\right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 w_t(z) n_t(z)\,dz \quad \forall\, t \\ \frac{\partial \pi_t}{\partial n_t(z)} \quad \Rightarrow \quad P_t^h \left(\int_0^1 n_t(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz\right)^{\frac{1}{\varepsilon-1}} n_t(z)^{-\frac{1}{\varepsilon}} - w_t(z) = 0 \quad \forall\, t, z \\ \Rightarrow \quad n_t(z) = \left(\frac{w_t(z)}{P_t^h}\right)^{-\varepsilon} y_t \quad \forall\, t, z \end{aligned}$$

Derivation 3 (**Firm output price** P_t^h)**.** The expression for the price level of the consumption good P_t^h is pinned down by the zero profit condition. Set the profit function equal to zero, substitute in the expression of differentiated input demand

(2.13), and solve for P_t^h.

$$\begin{aligned}
&P_t^h: \quad \pi_t = 0 \quad \forall t \\
&\Rightarrow \quad P_t^h \left(\int_0^1 \left[\left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \right]^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} = \int_0^1 w_t(z) \left(\frac{w_t(z)}{P_t^h} \right)^{-\varepsilon} y_t \, dz \quad \forall t \\
&\Rightarrow \quad \left(P_t^h \right)^{1+\varepsilon} y_t \left(\int_0^1 w_t(z)^{1-\varepsilon} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(P_t^h \right)^{\varepsilon} y_t \left(\int_0^1 w_t(z)^{1-\varepsilon} dz \right) \quad \forall t \\
&\Rightarrow \quad P_t^h = \left(\int_0^1 w_t(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}} \quad \forall t
\end{aligned}$$

Derivation 4 (**Steady State Equilibrium Home and Foreign CPI growth rates**)**.** The steady state equilibrium Home and Foreign country CPI growth rates as shown in (1.54) and (1.55) are derived in the following way. The Home CPI level is derived in Appendix A-4, and takes the following form as in (1.22):

$$P_{t+1} = \frac{1}{(1-\theta_h)^{1-\theta_h} \theta_h^{\theta_h}} \left(P_{t+1}^h \right)^{1-\theta_h} \left(e_t P_{t+1}^f \right)^{\theta_h}$$

Dividing P_{t+1} by P_t gives the following expression for the Home country CPI growth rate:

$$\begin{aligned}
\frac{P_{t+1}}{P_t} &= \left(\frac{P_{t+1}^h}{P_t^h} \right)^{1-\theta_h} \left(\frac{e_t P_{t+1}^f}{e_{t-1} P_t^f} \right)^{\theta_h} \\
&= (x)^{1-\theta_h} \left(\frac{e_t}{e_{t-1}} x^* \right)^{\theta_h}
\end{aligned}$$

Using the currency exchange market clearing condition (1.42) and plugging in the equilibrium expressions for m_t^f and m_t^{h*}, the steady state equilibrium expression for the growth rate of the exchange rate is:

$$\frac{e_t}{e_{t-1}} = \frac{x}{x^*}$$

Thus, the expression for the steady state equilibrium CPI growth rate in the Home country is:

$$\frac{P_{t+1}}{P_t} = x$$

And by symmetry, the steady state equilibrium CPI growth rate in the Foreign country is:

$$\frac{P^*_{t+1}}{P^*_t} = x^*$$

It is the steady-state exchange rate growth expression that cancels out the effects of the other country's prices in each CPI growth rate expression.

Derivation 5 (**Sign of parameter objects and derivatives with respect to θ_h and θ_f).** Here I derive the derivatives of the parameter summary objects with respect to θ_h and θ_f. A review of the objects and their representation is the following:

$$\Delta_h = (1-\theta_h)(1-\sigma) - \xi$$
$$\Delta_f = (1-\theta_f)(1-\sigma) - \xi$$
$$\Sigma_h = \theta_h(1-\sigma)$$
$$\Sigma_f = \theta_f(1-\sigma)$$
$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}}$$
$$\Omega_f = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_f)^{(1-\theta_f)(1-\sigma)}(\theta_h)^{\theta_f(1-\sigma)}}$$
$$\Omega_H = (\Omega_h)^{\frac{\Delta_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_f)^{\frac{-\Sigma_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}$$
$$\Omega_F = (\Omega_f)^{\frac{\Delta_h}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}(\Omega_h)^{\frac{-\Sigma_f}{\Delta_h\Delta_f-\Sigma_h\Sigma_f}}$$

The signs of the representative parameter objects and their derivatives with respect to θ_h and θ_f are the following:

$$\Delta_h < 0 \text{ always}, \quad \frac{\partial \Delta_h}{\partial \theta_h} = -(1-\sigma) > 0 \text{ when } \sigma > 1$$

$$\Delta_f < 0 \text{ always}, \quad \frac{\partial \Delta_f}{\partial \theta_f} = -(1-\sigma) > 0 \text{ when } \sigma > 1$$

$$\Sigma_h < 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0, \quad \frac{\partial \Sigma_h}{\partial \theta_h} = 1-\sigma < 0 \text{ when } \sigma > 1$$

$$\Sigma_f < 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0, \quad \frac{\partial \Sigma_f}{\partial \theta_f} = 1-\sigma < 0 \text{ when } \sigma > 1$$

$$\Omega_h > 0 \text{ when } \theta_f > 0,$$

$$\frac{\partial \Omega_h}{\partial \theta_h} = \Omega_h \frac{\partial log(\Omega_h)}{\partial \theta_h} = \Omega_h (1-\sigma) \left[1 + log\left(\frac{1-\theta_h}{\theta_f}\right)\right] < 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0$$

$$\frac{\partial \Omega_h}{\partial \theta_f} = \Omega_h \frac{\partial log(\Omega_h)}{\partial \theta_f} = -\Omega_h (1-\sigma) \frac{\theta_h}{\theta_f} > 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0$$

$$\Omega_f > 0 \text{ when } \theta_h > 0,$$

$$\frac{\partial \Omega_f}{\partial \theta_f} = \Omega_f \frac{\partial log(\Omega_f)}{\partial \theta_f} = \Omega_f (1-\sigma) \left[1 + log\left(\frac{1-\theta_f}{\theta_h}\right)\right] < 0 \text{ when } \sigma > 1 \text{ and } \theta_h > 0$$

$$\frac{\partial \Omega_f}{\partial \theta_h} = \Omega_f \frac{\partial log(\Omega_f)}{\partial \theta_h} = -\Omega_f (1-\sigma) \frac{\theta_f}{\theta_h} > 0 \text{ when } \sigma > 1 \text{ and } \theta_f > 0$$

$$\Delta_h \Delta_f - \Sigma_h \Sigma_f = (1-\theta_h-\theta_f)(1-\sigma)^2 - (1-\sigma)\xi(2-\theta_h-\theta_f) + \xi^2 > 0 \text{ always}$$

$$\frac{\partial(\Delta_h \Delta_f - \Sigma_h \Sigma_f)}{\partial \theta_h} = \frac{\partial(\Delta_h \Delta_f - \Sigma_h \Sigma_f)}{\partial \theta_f} = -(1-\sigma)(1-\sigma-\xi) < 0 \text{ when } \sigma > 1$$

Derivation 6 (**Optimal monetary rules**). The optimal monetary policy rules (1.74) and (1.75) are derived by having the monetary authority maximize the equilibrium utility of a representative agent in its own country with respect to its money growth rate. Below is the solution for the problem of the Home monetary authority, but the Foreign monetary authority's problem is symmetric.

$$\max_x V(x, x^*) = \max_x \frac{\left([(1-\theta_h)n]^{1-\theta_h} [\theta_f n^*]^{\theta_h}\right)^{1-\sigma} - 1}{1-\sigma} - \chi n^{\xi}$$

Taking the derivative of V with respect to x gives:

$$\frac{\partial V}{\partial x} = C^{-\sigma}\left[(1-\theta_h)^2\left(\frac{C^f}{C^h}\right)^{\theta_h}\frac{\partial n}{\partial x} + \theta_h\theta_f\left(\frac{C^h}{C^f}\right)^{1-\theta_h}\frac{\partial n^*}{\partial x}\right] - \chi\xi n^{\xi-1}\frac{\partial n}{\partial x}$$

where n, n^*, C^h, C^f, and C are given by (1.62), (1.63), (1.56), (1.57), and (1.18), respectively. Setting the derivative equal to zero, it can be rewritten:

$$(1-\theta_h)^{\xi}\left(C^h\right)^{\Delta_h}\left(C^f\right)^{\Sigma_h}\left[(1-\theta_h) + \theta_h\frac{n\frac{\partial n^*}{\partial x}}{n^*\frac{\partial n}{\partial x}}\right] = \chi\xi$$

where $\Delta_h = (1-\theta_h)(1-\sigma) - \xi$ and $\Sigma_h = \theta_h\,(1-\sigma)$. Writing C^h and C^f in terms of n and n^*, the expression can be rewritten in the following way:

$$(n)^{\Delta_h}\,(n^*)^{\Sigma_h}\left[(1-\theta_h) + \theta_h\frac{n\frac{\partial n^*}{\partial x}}{n^*\frac{\partial n}{\partial x}}\right] = \frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}}$$

The following two expressions are important for finding the solution and for understanding why the optimal monetary policy rules are independent of the policy choice of the other country.

$$\frac{n\frac{\partial n^*}{\partial x}}{n^*\frac{\partial n}{\partial x}} = \frac{-\Sigma_f}{\Delta_f} \quad \text{and} \quad (n)^{\Delta_h}\,(n^*)^{\Sigma_h} = x\Omega_h$$

where $\Delta_f = (1-\theta_f)(1-\sigma) - \xi$, $\Sigma_f = \theta_f\,(1-\sigma)$, and:

$$\Omega_h = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\chi\xi}{(1-\theta_h)^{(1-\theta_h)(1-\sigma)}(\theta_f)^{\theta_h(1-\sigma)}}$$

So now the optimal money growth rate for the Home country can be written in the following way:

$$\begin{aligned}\hat{x} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_f}{(1-\theta_h)\Delta_f - \theta_h\Sigma_f}\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_f)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_h)\xi}\end{aligned}$$

Using the analogous symmetric value function of the Foreign representative agent, one can solve the Foreign monetary authority's maximization problem and get the symmetric result that the optimal rate of Foreign money growth is given by:

$$\begin{aligned}\hat{x}^{*} &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\Delta_h}{(1-\theta_f)\Delta_h-\theta_f\Sigma_h}\\ &= \left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{(1-\theta_h)(1-\sigma)-\xi}{(1-\theta_h-\theta_f)(1-\sigma)-(1-\theta_f)\xi}\end{aligned}$$

The reason why $\hat{x}$ is not a function of x^* and why $\hat{x}^*$ is not a function of x is because the equilibrium derivative $\frac{\partial U(C)}{\partial x}$ divided by the equilibrium derivative $\frac{\partial g(n)}{\partial x}$ is independent of x^*. This reduces down to the two expressions above for $(n)^{\Delta_h}(n^*)^{\Sigma_h}$ and $\frac{n}{n^*}\frac{\partial n^*}{\partial x}\left(\frac{\partial n}{\partial x}\right)^{-1}$. Both of them are independent of x^*.

A-4 Derivation of Monopolistic Competition Demand and Price Equations in Two-Country OLG Model

In this section, I derive the individual differentiated-good demand equations for the two-country overlapping generations model described in this paper as well expressions for aggregate demand and aggregate prices. This follows the intuition of the closed economy derivation as first proposed by Dixit and Stiglitz (1977). The difference here is that aggregate consumption for each individual in time $t+1$ is a Home-biased composite of both Home and Foreign goods. I assume that consumers only care about Home-biased aggregate consumption C_{t+1} and that the elasticity of substitution among Home differentiated goods and the elasticity of substitution among Foreign differentiated goods is a constant ε that is symmetric across countries. However, I assume that the elasticity of substitution between a unit Home aggregate consumption and a unit of Foreign aggregate consumption is, in general, not equal to the elasticity of substitution among individually differentiated goods.

Let ρ be the constant elasticity of substitution between a unit of aggregate Home-country consumption and a unit of aggregate Foreign-country consumption. And let ε be the constant elasticity of substitution among the differentiated goods of each country. A realistic assumption is that the elasticity among the goods of a specific country is greater than the elasticity between a units of aggregate consumption from each country $\varepsilon > \rho$.

The form of the CES consumption aggregator with a Home-bias term θ is represented by the following two equations for the Home country and the Foreign country, respectively.

$$C_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^{h}_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^{f}_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{A.4.1}$$

$$C^{*}_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^{f*}_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^{h*}_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{A.4.2}$$

where $\rho \geq 0$ is the elasticity of substitution between Home and Foreign aggregate consumption, $\theta \in \left[0, \frac{1}{2}\right]$ parameterizes the degree of Home-bias in both countries, and C^{h}_{t+1}, C^{f}_{t+1}, C^{h*}_{t+1}, and C^{f*}_{t+1} represent aggregate consumption of Home produced and Foreign produced goods by Home and Foreign consumers, respectively. The exponent on the Home-bias parameter $1/\rho$ is merely an *ad hoc* functional form that makes the solutions more clean. An alternative would be an exponent of 1. From this point on, I only provide the derivation for the Home country, but the derivation for the Foreign country is completely symmetric.

If Home consumer purchases individual differentiated goods consumption $c_{t+1}(z)$ from Home producer z and $c_{t+1}(z^*)$ from Foreign producer z^*, aggregate consumption of goods from each country C^h_{t+1} and C^f_{t+1} can be defined by the Dixit-Stiglitz CES aggregator:

$$C^h_{t+1} \equiv \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\, dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \forall t \tag{A.4.3}$$

$$C^f_{t+1} \equiv \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\, dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \forall t \tag{A.4.4}$$

where $\varepsilon \geq 0$ represents the elasticity of substitution among all differentiated goods of a given country.[10] The individual demand equations for each differentiated good $c_{t+1}(z)$ and $c_{t+1}(z^*)$ for all z and z^* result from minimizing the cost of consuming given aggregate levels of consumption C^h_{t+1} and C^f_{t+1} by choosing the optimal consumption bundle $c_{t+1}(z)$ and $c_{t+1}(z^*)$ given individual prices $p_{t+1}(z)$ and $p_{t+1}(z^*)$.[11]

$$\begin{aligned} &\min_{c_{t+1}(z),c_{t+1}(z^*)} \int_0^1 p_{t+1}(z)c_{t+1}(z)\, dz + e_t \int_0^1 p_{t+1}(z^*)c_{t+1}(z^*)\, dz^* ... \\ &\text{subject to} \quad C^h_{t+1} \leq \left[\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\, dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &\qquad \text{and} \quad C^f_{t+1} \leq \left[\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\, dz^* \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned} \tag{A.4.5}$$

where the exchange rate e_t is lagged one period due to the portfolio decision being

[10] Appendix A-5 details some of the various forms that this CES aggregator can take resulting from different specifications of ε.

[11] The dual problem of maximizing the level of aggregate consumption subject to a budget constraint of expenditures being less than the currency held at the time of exchange does not yield the same result because the multiplier on the budget constraint does not have the interpretation as the price of an extra unit of aggregate consumption.

made in the period previous to consumption. The Lagrangian is the following:

$$\mathcal{L} = \int_0^1 p_{t+1}(z)c_{t+1}(z)\,dz + e_t \int_0^1 p_{t+1}(z^*)c_{t+1}(z^*)\,dz^* ... \\ + \lambda_h \left(C_{t+1}^h - \left[\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) ... \\ + \lambda_f \left(C_{t+1}^f - \left[\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) \quad \text{(A.4.6)}$$

Because the Lagrange multipliers λ_h and λ_f have the interpretation of being the marginal cost of an extra unit of aggregated country-specific consumption in terms of Home-country currency, λ_k is the price of aggregated country-specific consumption P_{t+1}^k for $k = h, f$. That is, P_t^h is the Home country price index of Home produced goods consumed at Home, and P_t^f is the import price index. The Lagrangian is now given by:

$$\mathcal{L} = \int_0^1 p_{t+1}(z)c_{t+1}(z)\,dz + e_t \int_0^1 p_{t+1}(z^*)c_{t+1}(z^*)\,dz^* ... \\ + P_{t+1}^h \left(C_{t+1}^h - \left[\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) ... \\ + e_t P_{t+1}^f \left(C_{t+1}^f - \left[\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right]^{\frac{\varepsilon}{\varepsilon-1}} \right) \quad \text{(A.4.7)}$$

Because the constraints always bind, the first order conditions are:

$$p_{t+1}(z) = P_{t+1}^h \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right)^{\frac{1}{\varepsilon-1}} c_{t+1}(z)^{-\frac{1}{\varepsilon}} \quad \forall\, t, z \quad \text{(A.4.8)}$$

$$p_{t+1}(z^*) = P_{t+1}^f \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right)^{\frac{1}{\varepsilon-1}} c_{t+1}(z^*)^{-\frac{1}{\varepsilon}} \quad \forall\, t, z^* \quad \text{(A.4.9)}$$

$$C_{t+1}^h = \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\,dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \quad \text{(A.4.10)}$$

$$C_{t+1}^f = \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\,dz^* \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall t \quad \text{(A.4.11)}$$

Solving for $c_{t+1}(z)$ and $c_{t+1}(z^*)$ in (A.4.18) and (A.4.19) and plugging in the constraints from (A.4.10) and (A.4.11) which are simply the definitions of aggregated country-specific consumption from (A.4.3) and (A.4.4), the demand for each country's individual differentiated goods take the following form:

$$c_{t+1}(z) = \left(\frac{p_{t+1}(z)}{P^h_{t+1}}\right)^{-\varepsilon} C^h_{t+1} \quad \forall t, z \tag{A.4.12}$$

$$c_{t+1}(z^*) = \left(\frac{p_{t+1}(z^*)}{P^f_{t+1}}\right)^{-\varepsilon} C^f_{t+1} \quad \forall t, z^* \tag{A.4.13}$$

Plugging (A.4.12) and (A.4.13) back into (A.4.10) and (A.4.11) and solving for P^h_{t+1} and P^f_{t+1}, respectively, gives the analogous expression for the price of aggregated country-specific consumption.

$$P^h_{t+1} = \left(\int_0^1 p_{t+1}(z)^{1-\varepsilon}\, dz\right)^{\frac{1}{1-\varepsilon}} \qquad \forall t \tag{A.4.14}$$

$$P^f_{t+1} = \left(\int_0^1 p_{t+1}(z^*)^{1-\varepsilon}\, dz^*\right)^{\frac{1}{1-\varepsilon}} \qquad \forall t \tag{A.4.15}$$

As in the cost minimization problem in (A.4.5), the Home consumer seeks to minimize total expenditure subject to a given level of aggregate consumption.

$$\min_{C^h_{t+1}, C^f_{t+1}} P^h_{t+1}C^h_{t+1} + e_t P^f_{t+1}C^f_{t+1} \quad \text{s.t.} \quad C_{t+1} \leq \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{A.4.16}$$

The Lagrangian for this problem is:

$$\mathcal{L} = P^h_{t+1}C^h_{t+1} + e_t P^f_{t+1}C^f_{t+1} + P_{t+1}\left(C_{t+1} - \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}\right) \tag{A.4.17}$$

where P_{t+1} is the multiplier on the constraint and represents the marginal cost of an extra unit of aggregate consumption. So P_{t+1} is interpreted as the price of aggregate consumption. The first order conditions are the following:

$$P^h_{t+1} = P_{t+1}\left[\frac{(1-\theta)C_{t+1}}{C^h_{t+1}}\right]^{\frac{1}{\rho}} \tag{A.4.18}$$

$$e_t P^f_{t+1} = P_{t+1}\left[\frac{\theta C_{t+1}}{C^f_{t+1}}\right]^{\frac{1}{\rho}} \tag{A.4.19}$$

$$C_{i,t+1} = \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{A.4.20}$$

Dividing (A.4.18) by (A.4.19) gives the following relationship:

$$\frac{P^h_{t+1}}{e_t P^f_{t+1}}\left(\frac{C^h_{t+1}}{C^f_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\rho}} \tag{A.4.21}$$

Notice that in the Cobb-Douglas or log utility case when $\rho = 1$, the ratio of Home consumption expenditure to Foreign consumption expenditure is a constant.[12] Also, note that solving (A.4.18) and (A.4.19) for C^h_{t+1} and C^f_{t+1}, respectively, gives Home demand equations for aggregate consumption of Home goods and aggregate consumption of Foreign goods.

$$C^h_{t+1} = (1-\theta)\left(\frac{P^h_{t+1}}{P_{t+1}}\right)^{-\rho} C_{t+1} \tag{A.4.22}$$

$$C^f_{t+1} = \theta\left(\frac{e_t P^f_{t+1}}{P_{t+1}}\right)^{-\rho} C_{t+1} \tag{A.4.23}$$

These demand equations are analogous to the individual demand equations in (A.4.12) and (A.4.13), except that they include the Home-bias parameter.

The expression for the aggregate price index P_{t+1} of the Home consumption

[12]The outcome when $\rho = 1$ is different from the case when $\rho \in (0, \infty)$ and $\rho \neq 1$ in a critical way. As is shown in Appendix A-6, in the case when $\rho \neq 1$, an international Bertrand duopoly situation develops between monetary authorities, and the world equilibrium is (x, x^*). [*This last sentence may not be correct.*]

over aggregate Home and Foreign consumption is found by rewriting (A.4.20) as:

$$C_{t+1} = \left[\left(\frac{1-\theta}{C^h_{t+1}} \right)^{\frac{1}{\rho}} C^h_{t+1} + \left(\frac{\theta}{C^f_{t+1}} \right)^{\frac{1}{\rho}} C^f_{t+1} \right]^{\frac{\rho}{\rho-1}} \tag{A.4.24}$$

Then, substituting the expressions for $\left([1-\theta]/C^h_{t+1}\right)^{1/\rho}$ and $\left(\theta/C^f_{t+1}\right)^{1/\rho}$ from (A.4.18) and (A.4.19) into (A.4.24) gives the expression for aggregate expenditures which is implied by the cost minimization problem in (A.4.16):

$$P_{t+1}C_{t+1} = P^h_{t+1}C^h_{t+1} + e_t P^f_{t+1} C^f_{t+1} \tag{A.4.25}$$

Now divide (A.4.25) by aggregate consumption C_{t+1} and plug in the expressions for C^h_{t+1}/C_{t+1} and C^f_{t+1}/C_{t+1} from (A.4.18) and (A.4.19). The resulting expression for aggregate price is:

$$P_{t+1} = \left[(1-\theta) \left(P^h_{t+1} \right)^{1-\rho} + \theta \left(e_t P^f_{t+1} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{A.4.26}$$

Note that this expression is the Home country CPI and is analogous to the within-country price aggregator in equation (A.4.14) but with the inclusion of the Home-bias parameter θ.

In the case of Cobb-Douglas aggregation over aggregate Home consumption and aggregate Foreign consumption ($\rho = 1$), the expression for aggregate price is:

$$P_{t+1} = \frac{1}{(1-\theta)^{1-\theta}\theta^{\theta}} \left(P^h_{t+1} \right)^{1-\theta} \left(e_t P^f_{t+1} \right)^{\theta} \tag{A.4.27}$$

and total aggregate expenditure is given by:

$$P_{t+1}C_{t+1} = \frac{1}{(1-\theta)^{1-\theta}\theta^{\theta}} \left(P^h_{t+1} C^h_{t+1} \right)^{1-\theta} \left(e_t P^f_{t+1} C^f_{t+1} \right)^{\theta} \tag{A.4.28}$$

A-5 Properties of International Model CES aggregator

In this paper, I assume a specific case of the general CES functional form for aggregate consumption of a given Home or Foreign consumer. As defined in Section 1.3.2, the aggregate consumption levels of the differentiated goods of the Home and Foreign countries are, respectively:

$$C^h_{t+1} \equiv \left(\int_0^1 c_{t+1}(z)^{\frac{\varepsilon-1}{\varepsilon}}\, dz\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{A.5.1}$$

$$C^f_{t+1} \equiv \left(\int_0^1 c_{t+1}(z^*)^{\frac{\varepsilon-1}{\varepsilon}}\, dz^*\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \forall\, t \tag{A.5.2}$$

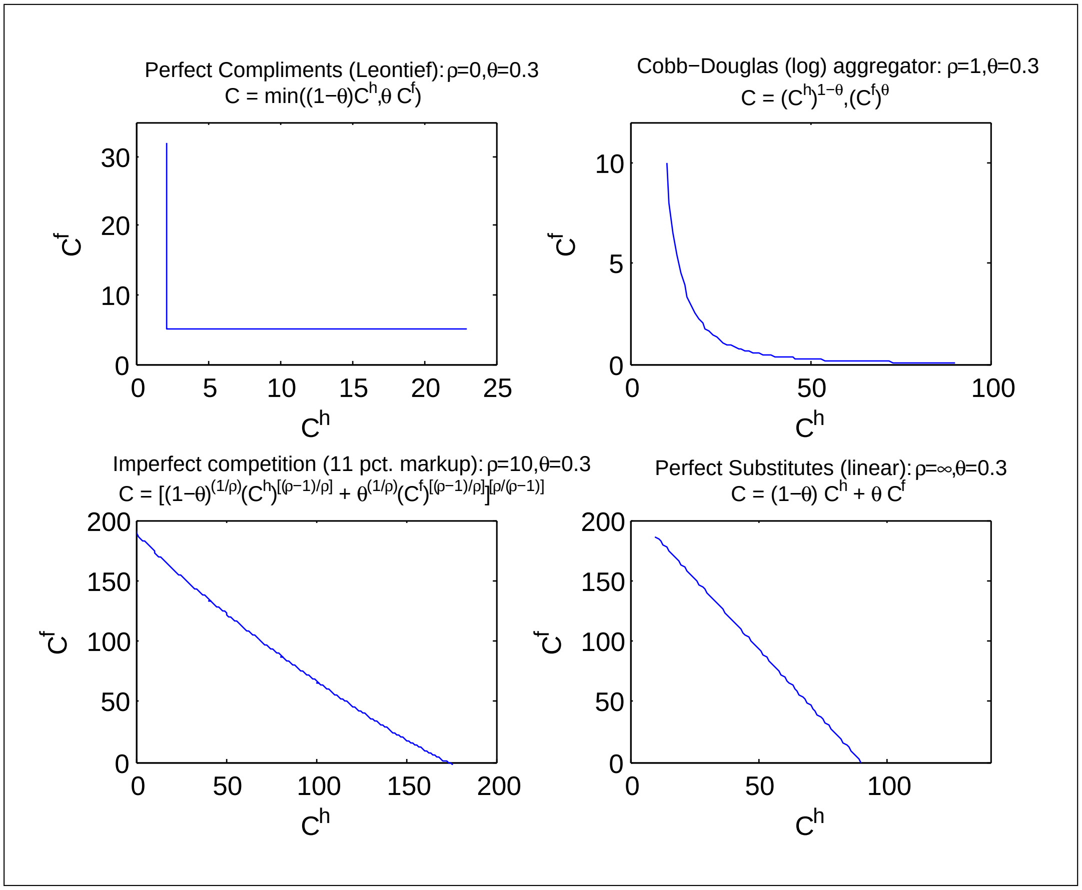
where ε is the constant elasticity of substitution among differentiated goods in either the Home or Foreign country. The aggregator over both Home aggregate consumption C^h_{t+1} and aggregate Foreign consumption C^f_{t+1} takes the same general CES form as in (A.5.1) and (A.5.2).

$$C_{t+1} \equiv \left[(1-\theta)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta \in \left[0, \frac{1}{2}\right] \tag{A.5.3}$$

where θ is a Home-bias parameter and $\rho \geq 0$ is the elasticity of substitution between a unit of Home consumption and a unit of Foreign consumption. The only restriction is that the elasticity of substitution between aggregate Home consumption and aggregate Foreign consumption is assumed to be less than or equal to the elasticity of substitution among the differentiated goods in either country ($\rho \leq \varepsilon$). In the analyses in Section 1.3, I assume a specific case of (A.5.3) in which the aggregator assumes a Cobb-Douglas form ($\rho = 1$). The general CES aggregator is an attractive form because it nests so many economically relevant cases.

Figure T-1 shows various specifications of the general CES aggregator function in (A.5.3). Taking the limit of (A.5.3) as $\rho \to 0$, a fixed level of aggregate consumption takes the Leontief form of perfect complements as shown in the first panel of Figure T-1. Using L'Hospital's rule when taking the limit of (A.5.3) as $\rho \to 1$, the aggregator function corresponding to unit elasticity ($\rho = 1$) is Cobb-Douglas or log utility as shown in the second panel in Figure T-1. Lastly, the fourth panel shows that the linear aggregator or perfect substitutes is the resulting aggregator function as $\rho \to \infty$. This reflects the case of perfect competition. Included

Figure T-1: Various specifications of general CES aggregator function



in the third panel of Figure T-1 shows the shape of the general CES aggregator function when the elasticity of substitution is at its often calibrated value of 10.

The key result here is that each constant consumption aggregator curve becomes flatter as the elasticity of substitution increases from the perfectly inelastic case of $\rho = 0$ to the perfectly elastic case of $\rho = \infty$. The exponent of $1/\rho$ on the Home bias terms is merely a convenience to make the resulting constant expenditure ratio a more simple expression.

It is important, however, to recognize that the common assumption of a logarithmic or Cobb-Douglas aggregator is implicitly assuming a unit elasticity of substitution between Home and Foreign aggregate consumption. As is shown in equation (1.36) of Section 1.3.2, the case of $\rho = 1$ implicitly makes the strong assumption that individuals exchange a constant share of their revenues for Foreign currency. Appendix A-6 addresses the solution to the model when the total consumption aggregator takes its general form ($\rho \geq 0$).

A-6 Solutions for General CES Aggregator

The purpose of this appendix is to document some of the solutions to the equilibrium problem when the CES aggregator over Home aggregate consumption is not restricted to the Cobb-Douglas case of unit elasticity of substitution ($\rho \neq 1$).

$$C_{t+1} \equiv \left[(1-\theta_h)^{\frac{1}{\rho}}\left(C_{t+1}^h\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}}\left(C_{t+1}^f\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \quad \text{for} \quad \theta_h \in \left(0, \frac{1}{2}\right] \tag{A.6.1}$$

The maximization problem analogous to (1.35) is the following:

$$\max_{m_t^f, p_t(z)} \frac{\left(\left[(1-\theta_h)^{\frac{1}{\rho}}\left(\left[\frac{p_t(z)}{P_t^h}\right]^{1-\varepsilon}\frac{xM_{t-1}}{P_{t+1}^h} - \frac{e_t m_t^f - (x-1)xM_{t-1}}{P_{t+1}^h}\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}}\left(\frac{m_t^f}{P_{t+1}^f}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}\right)^{1-\sigma} - 1}{1-\sigma} \dots$$

$$-\chi\left[\left(\frac{p_t(z)}{P_t^h}\right)^{-\varepsilon}\frac{xM_{t-1}}{P_t^h}\right]^{\xi} \tag{A.6.2}$$

where the two first order conditions, analogous to (1.36) and (1.37), are:

$$\frac{P_{t+1}^h}{e_t P_{t+1}^f}\left(\frac{C_{t+1}^h}{C_{t+1}^f}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_h}{\theta_h}\right)^{\frac{1}{\rho}} \tag{A.6.3}$$

$$(1-\theta_h)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z)}{P_{t+1}^h}\left(C_{t+1}\right)^{\frac{1}{\rho}-\sigma}\left(C_{t+1}^h\right)^{-\frac{1}{\rho}} = \chi\xi\left(n_t(z)\right)^{\xi-1} \tag{A.6.4}$$

where equation (A.6.3) equates the marginal cost of giving up a Home-currency unit of Home consumption for the marginal benefit of a Home-currency unit of Foreign consumption. Equation (A.6.4) equates the marginal benefit of raising price in terms of less demand and less disutility of labor to its marginal cost in terms of lost income in the next period of life. The market clearing conditions are the same as in Section 1.3.3. The equations that characterize an equilibrium in this case, given monetary policy (x, x^*) are shown in Table T-1.

The steady state equilibrium inflation rates are again equal to the money growth rates as in (1.52) and (1.53). However, because the first order condition for m_t^f in (A.6.3) no longer implies a constant expenditure share on Home and Foreign

Table T-1: Equilibrium conditions given x and x^* with general CES aggregator

	Home country	Foreign country
(1.36′)	$\frac{P^h_{t+1}}{e_t P^f_{t+1}}\left(\frac{C^h_{t+1}}{C^f_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_h}{\theta_h}\right)^{\frac{1}{\rho}}$	$\frac{e_t P^f_{t+1}}{P^h_{t+1}}\left(\frac{C^{f*}_{t+1}}{C^{h*}_{t+1}}\right)^{\frac{1}{\rho}} = \left(\frac{1-\theta_f}{\theta_f}\right)^{\frac{1}{\rho}}$
(1.37′)	$(1-\theta_h)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z)}{P^h_{t+1}}\frac{\left(C_{t+1}\right)^{\frac{1}{\rho}-\sigma}}{\left(C^h_{t+1}\right)^{\frac{1}{\rho}}} = \chi\xi\left(n_t(z)\right)^{\xi-1}$	$(1-\theta_f)^{\frac{1}{\rho}}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{p_t(z^*)}{P^f_{t+1}}\frac{\left(C^*_{t+1}\right)^{\frac{1}{\rho}-\sigma}}{\left(C^{f*}_{t+1}\right)^{\frac{1}{\rho}}} = \chi\xi\left(n_t(z^*)\right)^{\xi-1}$
(1.31)	$n_t(z) = \left(\frac{p_t(z)}{P^h_t}\right)^{-\varepsilon}\frac{xM_{t-1}}{P^h_t}$	$n_t(z^*) = \left(\frac{p_t(z^*)}{P^f_t}\right)^{-\varepsilon}\frac{x^* M^*_{t-1}}{P^f_t}$
(1.27)	$p_t(z)n_t(z) = m^h_t + e_t m^f_t$	$p_t(z^*)n_t(z^*) = m^{f*}_t + \frac{m^{h*}_t}{e_t}$
(1.32)	$C^h_{t+1} = \frac{m^h_t + (x-1)xM_{t-1}}{P^h_{t+1}}$	$C^{f*}_{t+1} = \frac{m^{f*}_t + (x^*-1)x^* M^*_{t-1}}{P^f_{t+1}}$
(1.33)	$C^f_{t+1} = \frac{m^f_t}{P^f_{t+1}}$	$C^{h*}_{t+1} = \frac{m^{h*}_t}{P^h_{t+1}}$
(1.16)	$C_{t+1} = \left[(1-\theta_h)^{\frac{1}{\rho}}\left(C^h_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta_h^{\frac{1}{\rho}}\left(C^f_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$	$C^*_{t+1} = \left[(1-\theta_f)^{\frac{1}{\rho}}\left(C^{f*}_{t+1}\right)^{\frac{\rho-1}{\rho}} + \theta_f^{\frac{1}{\rho}}\left(C^{h*}_{t+1}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$
	Market clearing conditions	
(1.38)	$n_t(z) = c_t(z) + c^*_t(z)$	
(1.39)	$n_t(z^*) = c_t(z^*) + c^*_t(z^*)$	
(1.40)	$M_t = m^h_t + m^{h*}_t$	
(1.41)	$M^*_t = m^f_t + m^{f*}_t$	
(1.42)	$e_t m^f_t = m^{h*}_t$	

consumption, individuals can substitute away from Foreign expenditure when the inflation tax of the Foreign country's monetary policy adversely affects them. A key point here is that, when $\rho = 1$ and the aggregator is Cobb-Douglas, agents are bound to hold a specific fraction of their revenues in Foreign currency. Thus, $\rho = 1$ renders the demand for Foreign currency inelastic. When $\rho \neq 1$ the elasticity of demand for Foreign currency becomes elastic.

As in Section 1.3.4, the steady state equilibrium inflation level is found by substituting the money market clearing conditions (1.40) and (1.41) and the currency exchange market clearing condition (1.42) into the portfolio constraint (1.27) and its Foreign analogue, and then iterating the constraint one period forward.

$$\frac{P^h_{t+1}}{P^h_t} = x \tag{A.6.5}$$

$$\frac{P^f_{t+1}}{P^f_t} = x^* \tag{A.6.6}$$

The steady state equilibrium exchange rate is found by plugging the expressions for the currency shares from (1.44) and (1.45) into the currency exchange market clearing condition (1.42).

$$e_t = \frac{(1-\phi)M_t}{(1-\phi^*)M^*_t} \tag{A.6.7}$$

Plugging in the expressions for the currency shares from (1.44) and (1.45), the equilibrium inflation rates (A.6.5) and (A.6.6), and using the currency exchange market clearing condition (1.42), the expressions for steady-state equilibrium aggregate consumption levels given x and x^* are the following:

$$C^h = \frac{(\phi + x - 1)n}{x} \tag{A.6.8}$$

$$C^f = \frac{(1-\phi^*)n^*}{x^*} \tag{A.6.9}$$

$$C^{f*} = \frac{(\phi^* + x^* - 1)n^*}{x^*} \tag{A.6.10}$$

$$C^{h^*} = \frac{(1-\phi)n}{x} \tag{A.6.11}$$

and the steady state equilibrium expressions for Home and Foreign employment are:

$$n = \frac{xM_{t-1}}{p_t(z)} \tag{A.6.12}$$

$$n^* = \frac{x^*M^*_{t-1}}{p_t(z^*)} \tag{A.6.13}$$

Now taking the steady state equilibrium values of (A.6.8) through (A.6.13) as well as the equilibrium characterizations for prices and the exchange rate from (**??**), (A.6.6), and (A.6.7), and substituting them into the two first order conditions for the Home country (A.6.3) and (A.6.4) and their two Foreign analogues, the steady state equilibrium is characterized by the following set of four equations in four unknowns $(\phi, p_t(z), \phi^*, p_t(z^*))$:

$$C^h\left(C^f\right)^{\rho-1}\left(C^{h*}\right)^{-\rho} = \frac{1-\theta_h}{\theta_h} \tag{A.6.14}$$

$$C^{f*}\left(C^{h*}\right)^{\rho-1}\left(C^f\right)^{-\rho} = \frac{1-\theta_f}{\theta_f} \tag{A.6.15}$$

$$\frac{(1-\theta_h)^{\frac{1}{\rho}}}{x}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\left(C\right)^{\frac{1}{\rho}-\sigma}}{\left(C^h\right)^{\frac{1}{\rho}}} = \chi\xi\left(\frac{xM_{t-1}}{p_t(z)}\right)^{\xi-1} \tag{A.6.16}$$

$$\frac{(1-\theta_f)^{\frac{1}{\rho}}}{x^*}\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{\left(C^*\right)^{\frac{1}{\rho}-\sigma}}{\left(C^{f*}\right)^{\frac{1}{\rho}}} = \chi\xi\left(\frac{x^*M^*_{t-1}}{p_t(z^*)}\right)^{\xi-1} \tag{A.6.17}$$

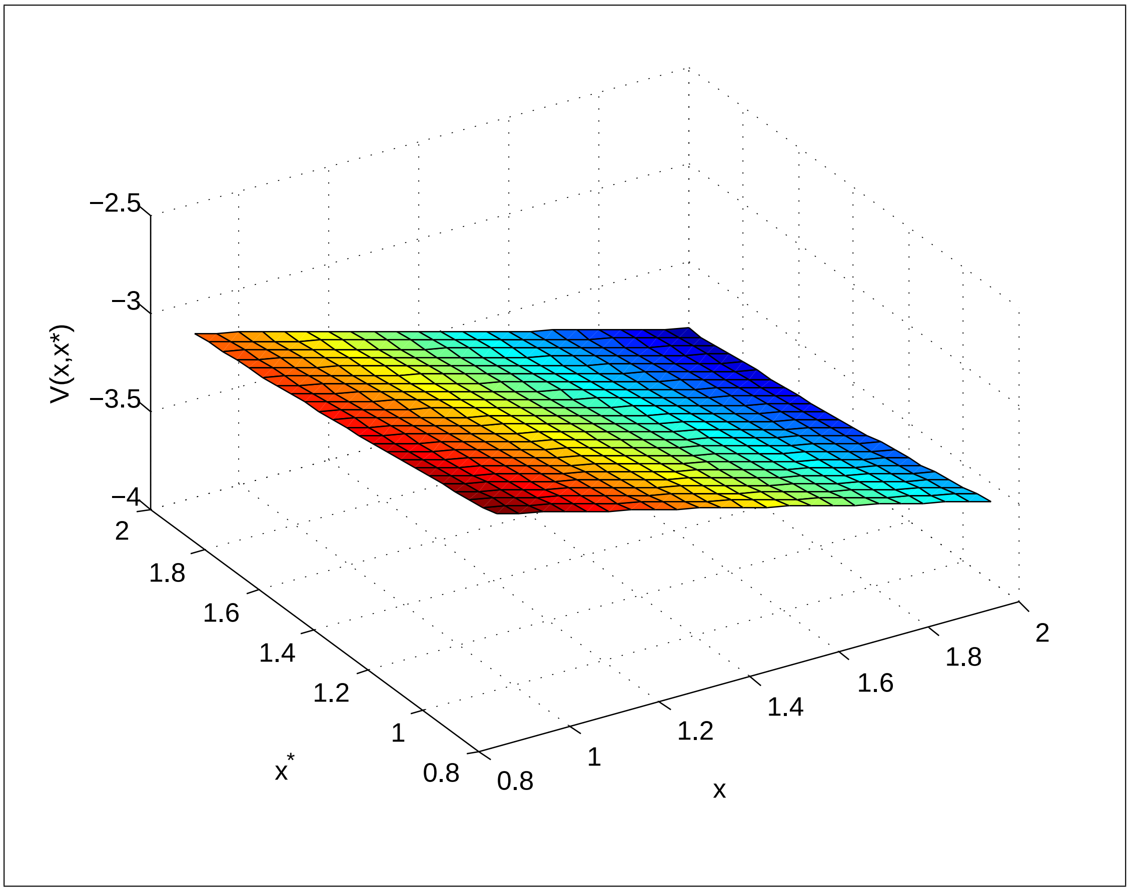
Thus, the policy functions are functions of the parameters, x and x^*, and state variables M_{t-1} and M^*_{t-1}. The state variables are normalized to 1 in this case without loss of generality.

Because the model with the general CES aggregator ($0 < \rho < \infty$ and $\rho \neq 1$) has no analytical solution, I solve it numerically.[13] Figure T-2 shows the value function $V(x, x^*$ of a representative agent in the Home country. It was calibrated such that $\theta = 0.75$, $\sigma = 3$, $\varepsilon = 10$, $\chi = 2$, $\xi = 2$, $\rho = 2$, and $x, x^* \in (0.9, 2.0)$.

The main difference here from the Cobb-Douglas aggregator case in the paper in which $\rho = 1$ is that the optimal expenditure share on Home currency ϕ is now a function of both x and x^*. So individuals can substitute away from Foreign currency expenditure if it becomes too expensive in terms of Home consumption. This induces an international Bertrand duopoly situation between the two monetary authorities. That is, the lower money growth rate a monetary authority chooses, the more attractive are the terms of trade for a Foreign country. It becomes a race to the bottom and the world equilibrium monetary policy is $(x = x* = 0)$.

[13] I discretize the (x, x^*) state space and use a Nelder-Mead simplex search method to find the solution at each point. The code for this computation is available upon request.

Figure T-2: $V(x, x^*)$ **in general CES case**



Calibrated parameters: $\theta_h = \theta_f = 0.75$, $\sigma = 3$, $\varepsilon = 10$, $\chi = 2$, $\xi = 2$, $\rho = 0.95$.

A-7 Derivation of Chapter 3 Demand and Price Equations from Cost Minimization Problem

Expressions for household demand functions for individual differentiated goods $c_t(h)$ and $c_t(f)$ and within-country aggregated goods $c_{H,t}$ and $c_{F,t}$ can be derived in terms of prices and aggregate consumption c_t using the household expenditure minimization problem. This method also provides expressions for aggregate price p_t in terms of the within-country aggregate prices $p_{H,t}$ and $p_{F,t}$, as well as expressions for the within-country aggregate prices $p_{H,t}$ and $p_{F,t}$ in terms of the within-country differentiated goods prices $p_t(h)$ and $p_t(f)$. I use an individual cost-minimization problem to derive the demand functions—even though the same functions result from utility maximization—because the cost-minimization problem provides an intuitive solution for the consumer price level. The Foreign demand and price equations are derived in the same way and are symmetric.

From the lifetime utility function in equation (3.5), households only care about aggregate consumption as defined in (3.7). Furthermore, the Dixit-Stiglitz CES country-specific consumption aggregators $c_{H,t}$ and $c_{F,t}$ are defined in (3.6). So the household demand functions for consumption of Home-produced differentiated goods $c_t(h)$ and Foreign-produced differentiated goods $c_t(f)$ can be derived by solving the problem of the household choosing how much of each type of differentiated good to consume, given the prices of each type of good $p_t(h)$ and $p_t(f)$ and given particular levels of aggregate country-specific consumption $c_{H,t}$ and $c_{F,t}$ in order to minimize total expenditures.[14]

$$\begin{aligned} \min_{c_t(h),c_t(f)} \quad & \int_0^1 p_t(h)c_t(h)\,dh + \int_0^1 p_t(f)c_t(f)\,df \\ \text{s.t.} \quad & c_{H,t} \leq \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}}\,dh\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ \text{and} \quad & c_{F,t} \leq \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}}\,df\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad \forall t \end{aligned} \tag{A.7.1}$$

[14]The dual problem of maximizing the level of aggregate consumption subject to a budget constraint of expenditures being less than the currency held at the time of exchange does not yield the same result because the multiplier on the budget constraint does not have the interpretation as the price of an extra unit of aggregate consumption.

The Lagrangian for this problem is:

$$\mathcal{L} = \int_0^1 p_t(h)c_t(h)\,dh + \int_0^1 p_t(f)c_t(f)\,df + ...$$
$$p_{H,t}\left[c_{H,t} - \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}}\,dh\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] + p_{F,t}\left[c_{F,t} - \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}}\,df\right)^{\frac{\varepsilon}{\varepsilon-1}}\right] \tag{A.7.2}$$

where $p_{H,t}$ and $p_{F,t}$ are the multipliers on the two constraints and represent the marginal cost of an extra unit of aggregate country-specific consumption. So $p_{H,t}$ and $p_{F,t}$ are interpreted as the index of Home produced goods prices and the index of Foreign-produced goods prices, respectively. The first order conditions are the following:

$$p_t(h) = p_{H,t}\left(\frac{c_{H,t}}{c_t(h)}\right)^{\frac{1}{\varepsilon}} \tag{A.7.3}$$
$$p_t(f) = p_{F,t}\left(\frac{c_{F,t}}{c_t(f)}\right)^{\frac{1}{\varepsilon}} \tag{A.7.4}$$
$$c_{H,t} = \left(\int_0^1 c_t(h)^{\frac{\varepsilon-1}{\varepsilon}}\,dh\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.7.5}$$
$$c_{F,t} = \left(\int_0^1 c_t(f)^{\frac{\varepsilon-1}{\varepsilon}}\,df\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.7.6}$$

Soling (A.7.3) and (A.7.4) for $c_t(h)$ and $c_t(f)$, respectively, give the following differentiated-good demand functions:

$$c_t(h) = \left(\frac{p_t(h)}{p_{H,t}}\right)^{-\varepsilon} c_{H,t} \tag{A.7.7}$$
$$c_t(f) = \left(\frac{p_t(f)}{p_{F,t}}\right)^{-\varepsilon} c_{F,t} \tag{A.7.8}$$

Substituting (A.7.7) and (A.7.8) back into (A.7.5) and (A.7.6) gives the following two expressions for $p_{H,t}$ and $p_{F,t}$ in terms of $p_t(h)$ and $p_t(f)$:

$$p_{H,t} = \left(\int_0^1 p_t(h)^{1-\varepsilon} dh\right)^{\frac{1}{1-\varepsilon}} \tag{A.7.9}$$

$$p_{F,t} = \left(\int_0^1 p_t(f)^{1-\varepsilon} df\right)^{\frac{1}{1-\varepsilon}} \tag{A.7.10}$$

Likewise, the household demand functions for consumption of Home-produced good $c_{H,t}$ and Foreign-produced good $c_{F,t}$ can be derived by solving the problem of the household choosing how much of each type of country-specific aggregate good to consume, given the prices of each type of good $p_{H,t}$ and $p_{F,t}$ and given a particular level of aggregate consumption c_t, in order to minimize total expenditures.

$$\min_{c_{H,t},c_{F,t}} p_{H,t}c_{H,t} + p_{F,t}c_{F,t} \quad \text{s.t.} \quad c_t \leq (c_{H,t})^{1-\theta} (c_{F,t})^{\theta} \quad \forall t \tag{A.7.11}$$

The Lagrangian for this problem is:

$$\mathcal{L} = p_{H,t}c_{H,t} + p_{H,t}c_{H,t} + p_t \left[c_t - (c_{H,t})^{1-\theta} (c_{F,t})^{\theta}\right] \tag{A.7.12}$$

where p_t is the multiplier on the constraint and represents the marginal cost of an extra unit of aggregate consumption. So P_t is interpreted as the price of aggregate consumption. The first order conditions are the following:

$$p_{H,t} = (1-\theta)p_t \left(\frac{c_{F,t}}{c_{H,t}}\right)^{\theta} \tag{A.7.13}$$

$$p_{F,t} = \theta p_t \left(\frac{c_{H,t}}{c_{F,t}}\right)^{1-\theta} \tag{A.7.14}$$

$$c_t = (c_{H,t})^{1-\theta} (c_{F,t})^{\theta} \tag{A.7.15}$$

Dividing (A.7.13) by (A.7.14) gives the following relationship:

$$\frac{p_{H,t}c_{H,t}}{p_{F,t}c_{F,t}} = \frac{1-\theta}{\theta} \tag{A.7.16}$$

Solving (A.7.13) and (A.7.14) for $c_{H,t}$ and $c_{F,t}$, respectively, and substituting in (A.7.15) gives Home demand equations for aggregate consumption of Home goods and aggregate consumption of Foreign goods.[15]

$$c_{H,t} = (1-\theta)\left(\frac{p_{H,t}}{p_t}\right)^{-1} c_t \tag{A.7.17}$$

$$c_{F,t} = \theta\left(\frac{p_{F,t}}{p_t}\right)^{-1} c_t \tag{A.7.18}$$

These demand equations are analogous to the differentiated-good demand equations in (A.7.7) and (A.7.8), except that they include the home-bias parameter. Substituting (A.7.17) and (A.7.18) back into (A.7.15) gives the expression of the consumer price index in the Home country p_t in terms of $p_{H,t}$ and $p_{F,t}$.

$$p_t = \frac{1}{\gamma}\left(p_{H,t}\right)^{1-\theta}\left(p_{F,t}\right)^{\theta} \quad \text{where} \quad \gamma = (1-\theta)^{1-\theta}\theta^{\theta} \tag{A.7.19}$$

Lastly, $c_t(h)$ and $c_t(f)$ can be expressed in terms of prices and c_t by substituting (A.7.17) and (A.7.18) into (A.7.7) and (A.7.8), which results in:

$$c_t(h) = (1-\theta)\left(\frac{p_t(h)}{p_{H,t}}\right)^{-\varepsilon}\left(\frac{p_{H,t}}{p_t}\right)^{-1} c_t \tag{A.7.20}$$

$$c_t(f) = \theta\left(\frac{p_t(f)}{p_{F,t}}\right)^{-\varepsilon}\left(\frac{p_{F,t}}{p_t}\right)^{-1} c_t \tag{A.7.21}$$

[15] Equations (A.7.16), (A.7.17), and (A.7.17) show that expenditures on imports are a constant fraction of GDP, represented by θ. This is the foundation for using θ as representing a country's degree of openness to international trade.

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