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CLASSICAL "ZENO" AND "ANTI-ZENO" EFFECT?

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Abstract

Classical "Zeno" And "Anti-Zeno" Effect ?

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If one continuously measures a decaying system, the system will appear to never decay that was called quantum Zeno effect. The continuous measurement is defined by a sequence of measurements whose time interval t between measurements approaches zero. Later many works chose the time interval t as finite (and greater than the Zeno time) which corresponds to making equal spaced measurements over a discrete time interval. With the discrete variable formulism one can derive the so-called Anti-Zeno effect. Our study is trying to contrast the results between continuous time interval measurement versus discrete time interval measurement. We demonstrate that we can obtain so-called "Zeno" and "Anti-Zeno" in a classical system if we apply the definition of *non-ideal* measurement.

Contents

1 Introduction

Unstable system have been intensively studied in quantum theory. A general formulism gave unstable system as pure exponential decay[1][2].However, Kalfin had shown that if the Hamiltonian was bounded from below, the decay at small and large time domain there should be deviations from strictly exponential decay law. In the very large t domain, the survival probability has a power- law decay[3].

1977, Misra and Sudarshan gave a seminal work [4]. A unstable quantum system at time $T = 0$ was denoted $\rho(0)$ as the initial matrix density states (undecayed states); and the time evolution of the states $\rho(0)$ follows the Schrodinger time develops as usual. If one takes a measurement at time $T = t$, the initial state will change to a new state $\rho'(t)$. At the limit $t \to 0$, the probability of decayed state is negligible. Hence, one may say, $\rho(0) \simeq \rho'(0)$; the measurement set the system back to the initial state. Under this consideration, if one repeatedly measures the system with each time interval $t \to 0$ then the system appears never decay that was called Quantum Zeno Paradox [4].

Later two specific cases of Zeno's paradox were investigated by Chiu, Sudarshan and Misra, Ghirardi et.al [5]. The quantum Zeno was a consequence of the first principle of quantum theory: $\frac{dP(T)}{dT} = 0$ at $T = 0$. That is, survival probability at small time t,the survival probability can be written as

$$P(t) = 1 - (\frac{\epsilon T}{n\hbar})^2 \tag{1}$$

For all *ideal* successive measurements, the survival probability at end of nth observation

$$\begin{aligned} P(nt) &= [P(t)]^n = \ [1 - (\frac{\epsilon T}{n\hbar})^2]^n \\ &= 1 - \frac{T^2}{n}(\frac{\epsilon}{\hbar})^2 + ... \end{aligned} \tag{2}$$

The time interval $t \to 0$ was regarded as $n \to \infty$ here; and for $n \to 0$ the $P(nt) \to 1$. This is, say, at $T = nt$ the survival probability is still one. The system appears never decay that was called Quantum Zeno Effect.

Prashant Valanju has showed the evidence of Quantum Zeno Effect in hadron-nucleus collisions in which several interactions occur in rapid succession.[6].The Quantum Zeno Effect was verified by Itano et al in a three level oscillation system. [7]

Quantum Zeno paradox has sparked a great of deal interest in the field today. Some of them claimed that new decay laws were found in unstable quantum system ; and suggested that the new property of the unstable quantum system might give the Zeno or/Anti-Zeno effect (or Inverse Zeno effect) [8][9].

In 2001 Physical Review Letters, M.C.Fischer et.la have claimed Zeno Effect and Anti Zeno Effect were observed in their experiment[10]; and various papers also claimed the existence of Zeno effect and Anti-Zeno effect by all kind of means[11][12] [13][14].

Those works suggested one can obtain Zeno effect and Anti-Zeno effect with more realistic measurement protocols. Namely, the time interval t is small (still greater then Zeno time) but not approaching to zero. In this thesis, we select and summarize some of these papers. we analysis the fundamental difference between the original theory and the extension of the Zeno and Anti-Zeno effect. More importantly,we show how the deviations lead to some contradictory results—we can obtain "Zeno" and "Anti-Zeno" effect in a classical radioactive decay system ! We also point out if the measurement is *non-ideal* for a quantum system, the probability of repeated measurement yields an inconsistence. Furthermore, we want to say the survival probability decreasing as number measurement increasing was results of selective measurement (due to *non-ideal* measurement) which essentially is the same as the Stern-Gerlach experiment.

2 The Extension of Zeno and Anti-Zeno Effect

2.1 Experimental model

In 2001,M.Raizen Group published a article on the Physical Review Letter [10]. In the Letter they claimed quantum Zeno and Anti-Zeno were observed in an unstable system. We are here giving a simple description of their experiment.

A group of ultracold sodium atoms were placed in periodic-optical trap formed by two optical beams. The time dependence of the optical potential traps were given as

$$V_0 cos[2k_L x - k_L a t^2] \qquad (3)$$

where the $V_0$is the amplitude of the potential, $k_L$is the wave number of the light forming the potential, x is the position in the laboratory frame, a is the acceleration, and t is time. The atoms in the trap were divided into three energy bands. The top band of atoms were completely free. The middle band of atoms were partially trapped, and the lowest band of atoms were considered as the ground state which were completely trapped. When the frequencies of the two beams are the same, the system was considered as a stable system, no tunneling occurs. However, one of the frequency was increasing while the other frequency was fixed. The atoms were accelerated along the trap. When the acceleration reach a_{tran}, the second band atoms were emptied out but the lowest energy band atoms remained inside the trap. When the frequency was continuously increasing up to the tunneling acceleration, a_{tun}, the atoms inside the trap start tunneling out. They argued that the atoms have different velocities, so trapped and tunneled atoms can be separated spatially when the trap was turned off. The ratio of the number of atoms tunneling out to the atoms remaining inside the trap was treated as the survival probability.

2.2 Measurement in the experiment

To obtain the Zeno and the "anti-Zeno" effects, the tunneling process was interrupted by decreasing the acceleration a_{tun} back to the initial acceleration a_{trans} and the interruption was then interpreted as a measurement. namely, the process of "state wave function collapse into the state vector".The number of remaining atoms inside trap along the initial condition acceleration a_{trans} were considered as an initial state. They assumed that the interruption periods were long enough ($40\mu s$,$50\mu s$) to separate out the atoms that tunnel out before and after each interruption into resolvable groups. When the trap was accelerated up to a_{tun} then atoms begin tunneling again.

They claimed that depends on the length of time interval, they observed the Zeno and the Anti-Zeno. When the tunneling time was $1\mu s$ the Zeno effect was observed. When the tunneling time was $5\mu s$ the so-called anti-Zeno effect was observed. The survival probability was obtained by the ratio of atoms that had tunneled out and those still remaining in the trap. The survival probability of the nth tunneling segment approximately equals n times the survival probability of the first tunneling segment. They compared the slope of survival probability curve with interrupted, Let's call it $P(nt)$. The slope of the "free" tunneling curve $P_f(T)$. If the curve of $P(nt)$ is about $P_f(T)$, it is Zeno effect. If the $P_f(T)$ is below the $P(nt)$, it is Anti-Zeno Effect.

2.3 Theoretical model

This experimental result was constructed as a theoretical model to verify the experimental results by Modi and Shaji. They claimed the experimental results were reproduced. We do see the discrepancies between the two models. However, we main focus is on a comment theme among those works which is the time interval t between a serise of measurement.

They consider an interacting field theory of four fields labeled A, B, C,and Θ *which is continuous field*. The processes in this model are

$$A \rightleftharpoons B \quad ,, \; B \rightleftharpoons C\Theta \tag{4}$$

The Hamiltonian for the model with these allowed process can be written as

$$H = H_0 + V \tag{5}$$

where,

$$H_0 = E_A a^+ a + E_b b^+ b + \int_0^\infty d\omega \, \omega \, \theta^+(\omega)\theta(\omega) \tag{6}$$

and

$$V = \Omega a^+ a + \Omega^* b^+ b + \int_0^\infty d\omega \, [f(\omega) \, b^+ c\theta(\omega) + f(\omega)^* c^+ \theta(\omega) \, b] \tag{7}$$

The $E_A$and E_B are denoted as the two discrete energy levels for the two bound bare states $|A\rangle$, $|B\rangle$. To see the dynamics of this system. They were trying to find the engeisolutions by introducing the effective Hamilton.

$$H = \begin{bmatrix} E_A & \Omega^* & 0 \\ \Omega & E_B & f^*(\omega) \\ 0 & f(\omega) & \omega\delta(\omega-\omega') \end{bmatrix} \tag{8}$$

the Schrodinger equation as usual

$$H\psi_\lambda = \lambda\psi_\lambda \tag{9}$$

They set the generic state as

$$\psi_\lambda = \begin{pmatrix} \mu_\lambda^A \\ \mu_\lambda^B \\ \phi_\lambda(\omega) \end{pmatrix} \tag{10}$$

By solved the above equation, they came to this expression

$$\mu_\lambda^A(Z - E_B - \frac{\Omega^2}{Z-E_A} - \int_0^\infty \frac{|f(\omega')|^2}{Z-\omega'}d\omega') = 0 \tag{11}$$

And they denoted the expression as below

$$\beta(\lambda) \equiv Z - E_B - \frac{\Omega^2}{Z-E_A} - \int_0^\infty \frac{|f(\omega')|^2}{Z-\omega'}d\omega' \tag{12}$$

They choose $\mu_\lambda^A \neq 0$, for a non-trivial solution. Then $\beta(\lambda) = 0$. and by solving this expression, they would obtain a engeisolutions for $0 \leq \omega \leq \infty$.

They focus on the solutions of the continuum states for $\beta(\lambda)$ has no real zero for proper value of the parameters.The solution of the bound states is now on the second sheet and has complex energy.

By solving the Schrodinger equation, they obtain the solutions and choose the delta function as "in" state with positive $i\epsilon$.

$$\psi_\lambda = \begin{pmatrix} \mu_\lambda^A \\ \mu_\lambda^B \\ \phi_\lambda(\omega) \end{pmatrix} = \begin{pmatrix} \frac{f(\lambda)}{\beta^+(\lambda)} \frac{\Omega^*}{\lambda - E_A} \\ \frac{f(\lambda)}{\beta^+(\lambda)} \\ \frac{f(\lambda)}{\beta^+(\lambda)} \frac{f(\omega)}{\lambda - \omega + i\epsilon} \end{pmatrix} \tag{13}$$

Once they obtain the solutions, survival probabilities of the two states were calculated. the survival probability of the state $|A\rangle$ was defined as below

$$P_A(t) = |\int_0^\infty d\lambda \langle A|e^{-iHt}|\psi_\lambda\rangle|^2 = |\int_0^\infty d\lambda e^{-i\lambda t}|\langle A|\psi_\lambda\rangle|^2|^2 \tag{14}$$

Similar for $|B\rangle$

$$P_B(t) = |\int_0^\infty d\lambda e^{-i\lambda t}|\langle B|\psi_\lambda\rangle|^2|^2 \tag{15}$$

The numerical solutions of $P_A(t)$ and $P_B(t)$ were obtained by choosing a practical form factor with fixed parameters

$$f(\omega) = \frac{\sigma\mu^2\sqrt{\omega}}{(\omega - \omega_0)^2 + \mu^2} \tag{16}$$

The $\sqrt{\omega}$ a phase -space factor. The width of $f(\omega)$ is controlled by μ and σ is its strength. Unfortunately, the most crucial steps in the calculations are not stated. Nor were the numerical solutions for $P_A(t)$ an $P_B(t)$. However, the two "free decay" curve $P_A(t)$ (Fig.4 in the reference [13]) and $P_B(t)$ were given, which fits the experimental results perfectly (Fig.4 in the reference [10]).

2.4 Measurement and the definition of Zeno and Anti-Zeno Effect.

They gave the explanation how the the graph (Fig.4 in the reference, the solid line) was generated as stated statement.

"We start with a bare state with unit amplitude and compute its survival probability till time t. At this point the measurement is assumed to reset the system. The initial bare state wave function had only one non-zero component when expressed in the basis of bare states. Time evolution of this state under the full Hamiltonian makes all three components non-zero in general. Resetting the system corresponds to setting the two new components that appeared as a result of the evolution back to zero. This new (un-normalized) state is the starting point for further evolution until the next interruption. This process is repeated several times to obtain the graph of the survival probability of the initial unstable state when it is subject to frequent interruptions".

For more instructive, we summary the above statement. They denoted the states vector as $|A\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ at time $T = 0$ and it follows the Schrodinger time evolution. After the system free evolved for period t . All the three components of the vector state were non-zero. However, they argued that the measurement has set the system back to the initial state.Thus,they set the second and third components as zero. It said, after a measurement at $T = t$, the state becomes

$$|A(t)\rangle = \begin{pmatrix} P^{(1)}(t) \\ 0 \\ 0 \end{pmatrix} = P^{(1)}(t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

where the

$$P^{(1)}(t) = |\int_0^\infty d\lambda \langle A|e^{-iHt}|\psi_\lambda\rangle|^2|^2 = |\int_0^\infty d\lambda e^{i\lambda t}|\langle A|\psi_\lambda\rangle|^2|^2 \tag{18}$$

Clearly, then they treated the state $|A(t)\rangle$ as the new state vector and then took another " measurement" again. To set the second and third components back to zeros was taken as " measurement cause the wave function collapse back to the initial state"

By repeating this process, the survival probability with n repeated measurement at time $T = nt$ was given as the below

$$P(nt) = [P(t)]^n \tag{19}$$

Apparently, the graphical results indicated that if

$$\frac{d[Log(P_f(T))]}{dT} > \frac{d[Log(P(nt))]}{d(nt)} \tag{20}$$

they called it QZE. if

$$\frac{d[Log(P_f(T))]}{dT} < \frac{d[log(P(nt))}{d(nt)} \tag{21}$$

is Anti-Zeno.

We also notice that Facchi and Pascazio also gave very similar definitions of Zeno and Anti-Zeno Effect[12]. Based on the definitions and the finite time interval, we show that we can obtain “Zeno effect” and “Anti-Zeno effect” in classical radioactive decay system in later section.

3 The Deviation of The Zeno and Anti-Zeno Effect

3.1 Misra-Sudarshan’s theorem

Consider an unstable quantum system, whose undecayed and decayed states from the Hilbert space $\mathcal{H}$ and whose evolution is described by the unitary operator $U(T) = exp(-iHT)$., where H is a time-independent semi-bounded Hamiltonian. Let $\mathcal{H}_{\mathcal{E}}$ be the subspace spanned by the undecayed states of the system and E be a projection operator onto $\mathcal{H}_{\mathcal{E}}$, so $E\mathcal{H}E = \mathcal{H}_{\mathcal{E}}$. In general ,we assume that E does not commute with the Hamiltonian, $[E, H] \neq 0$. Our Hilbert space decomposes $\mathcal{H} = \mathcal{H}_{\mathcal{E}} \oplus \mathcal{H}_{\mathcal{E}^\perp}$ and $E^\perp = \mathbb{I} - \mathcal{E}$ is the projection operator onto $\mathcal{H}_{\mathcal{E}^\perp}$. let ρ be a density state matrix of unstable quantum system on$\mathcal{H}$ we can write it with respect to this decomposition as

$$\rho = \begin{pmatrix} E\rho_{nn'}E & \star \\ \star & E^\perp\phi_{nn'}E^\perp \end{pmatrix} \tag{22}$$

where the off-diagonal terms (denoted by $\star$) are "interference" terms that are assumed to be negligible here and the index n and n' stands for multi-decayed and undecayed states. Let $\rho(0)$ be the initial states (undecayed states) at $T = 0$

$$\rho(0) = \begin{pmatrix} E\rho_{nn'}E & 0 \\ 0 & 0 \end{pmatrix} \tag{23}$$

the $\rho_{nn'}$are eigenstates of E. i.e. $\rho_{nn'} = E\rho_{nn'}E$. Also, $Tr[\rho(0)] = Tr[\rho_{nn'}] = 1$. For more instructive, we assume there is only one undecayed state,ρ_0 and one decayed state ϕ. The time development of $\rho(0)$ follows the Schrodinger time evolution. Under the unitary transformation, all of elements of the density matrix$\rho(T)$ are non-Zeno now.

If one take a measurement on the system at time $T = t$, the density matrix $\rho(t)$ now changes into a new density matrix $\rho'(t)$. The new density matrix $\rho'(t)$.(un-re-normalized) is diagonalized the density matrix $\rho(t)$ and was formed by the undecayed state

$$\rho_0(t) = EU(t)\rho_0 U^+(t)E \tag{24}$$

and the decayed state

$$\phi(t) = E^\perp \phi E^\perp \tag{25}$$

A measurement yields that the system is still in a particular state at time t (undecayed state in our case) with probability $P(t)$

$$P(t) = Tr[U(t)\rho(0)U^{+}(t)E] \tag{26}$$

where the probability $P(t)$ was called survival probability and $P_d(t)$ is the probability for the system is in decayed state at time t

$$P_d(t) = 1 - P(t) \tag{27}$$

In general, $P(t)$ is less then one.However, at very small time t , the unstable system decay probability has quadratic time depend as $t \to 0$. Thus, the decayed probability is negligible. The new density matrix $\rho'(t \to 0)$ approximately

$$\rho'(t \to 0) = \begin{pmatrix} E\rho_0 E & 0 \\ 0 & 0 \end{pmatrix} \tag{28}$$

And $Tr[\rho'(t \to 0)] \simeq Tr[\rho_0] = 1$.

Hence, the new density matrix is nearly same as the initial density matrix. One one may say " the measurement set the system back to the initial state"

$$\rho'(t \to 0) \simeq \rho(0) \tag{29}$$

Under the consideration, the process of measurement was regarded as

$$\rho_0 \to \rho' = E\rho_0 E \tag{30}$$

this is *ideal* measurement that allow one to take successive measurements, and allow the system to collapse at each measurement. However, the system undergoes the Schrodinger time evolution at intervening time interval . The state of $\rho_0$after nth measurement was given

$$\rho^{(n)}(t) = V(t)_n \rho_0 V(t)_n^{+} \tag{31}$$

$$V_n(t) \equiv [EU(t/n)E]^n \tag{32}$$

and the probability for nth measurement of the state $\rho^{(n)}(t)$ was given as

$$P(nt) = Tr[V_n(t)\rho(0)V_n^{*}(t)] \tag{33}$$

Also, $lim_{n\to\infty}V(t) = E$, the probability, $Tr(\rho E) = 1$ as $t \to 0$. The system appears never decay that was called Quantum Zeno Effect.

Subsequently the Quantum Zeno's paradox was studied in two specific models by Chui, Sudarshan, Misra[3]. The limit $t \to 0$ was regarded as $n \to \infty$. The first survival probability at the first observation is equal to

$$P(t) = 1 - (\frac{\epsilon T}{n\hbar})^2 \tag{34}$$

For all *ideal* successive measurements, the survival probability at end of nth observation

$$P(nt) = [P(t)]^n = [1 - (\frac{\epsilon T}{n\hbar})^2]^n \tag{35}$$

$$= \ 1 - \frac{T^2}{n}(\frac{\epsilon}{\hbar})^2 + ...$$

Where $nt = T$ .For $n \to \infty$,$P(T) \to 1$. The system appears never decay! This is original definition of quantum Zeno Effect.

3.2 Non-ideal measurement

According to the Misra-Sudarshan theorem, the decay probability has quadratic time depends at small t region. At $t \to 0$, the decay probability is negligible, therefore the survival probability is near one. the state after a measurement is same as the initial state under the approximation.

$$\rho_0 \to \rho' = E\rho_0 E \tag{36}$$

An operator of the repeated measurement was given in Misra-Sudarshan as

$$V_n(t) \equiv [EU(t/n)E]^n \tag{37}$$

as $n \to 0$. Here we need to keep that in mind that the operator is valid only if the measurement is *ideal.*

If the time interval t between sequences measurements are small (greater than Zeno time) but finite we called the measuremet are *non-ideal,* then the undecayed states $\rho_0$changes into $\rho_0'(t)$ as we stated before

$$\rho_0'(t) = EU(t)\rho_0 U^+(t)E \tag{38}$$

with the probability

$$P(t) = Tr[U(t)\rho_0 U^+(t)E] \tag{39}$$

and this probability is less then one. The new density state matrix $\rho'(t)$ that are made up by the undecayed state and decayed state. It is not same as the initial density state matrix$\rho(0)$ after the measurement. The statement "the measurement set the system back to the initial state" is no longer true for *non-ideal* measurement.

Furthermore, as Misra and Sudershan pointed out that these formulas do not yield the correct probability connection. The operator $V_n(T)$ is no longer valid if the measurements are *non-ideal*. The survival probability with repeated measurement was given

$$P(nt) \ = [P(t)]^n = \ [1-(\frac{\epsilon T}{n\hbar})^2]^n \tag{40}$$

If the n is large but finite, then $P(nt) = [P(t)]^n = [1-(\frac{\epsilon T}{n\hbar})^2]^n \sim exp[-\lambda(nt)]$, where $(\frac{\epsilon}{\hbar})^2 t$. This is a seemingly inconsistent result. However, one needs to realize the two variables are involved. The t is a continuous variable while the T is a discrete variable which is defined at the each points evenly space by time interval t.

However, in a classical system, e.g. a radioactive decay, the operation of repeated measurement is valid regardless the length of the time interval. And without involveing the projection postulate, we can obtain Zeno effect and Anti-Zeno effect from a classical system based on the new definitions.

4 Classical Zeno and Anti-Zeno

4.1 Classical radioactive decay;

We consider a classical multi-generation radioactive decay system. The mother particles N_1decay into daughter particlesN_2, and the N_2 continuously decay into N_3. As is well known the decay process of $N_1 \rightarrow N_2$ is purely exponential. However, with multiple generations, the process $N_1 \rightarrow N_3$ is not properly described by a pure exponential decay process.

We let the N_1decay with rate γ_1, N_2 with decay rate γ_2. and N_3 with decay rate γ_3. We let N_4 be the final decay product, which we directly measure. The survival rate of the system is the combination of the all three survival rates. The survival probability is denoted as $P(N_1 + N_2 + N_3, t)$. The system of equations describing classical radioactive decay for our three-level system is given by

$$\frac{dN_1}{dt} = -\gamma_1 N_1 \tag{41}$$

For the remaining two generations,

$$\frac{dN_2}{dt} = -\gamma_2 N_2 + \gamma_1 N_1 \tag{42}$$

$$\frac{dN_3}{dt} = -\gamma_3 N_3 + \gamma_2 N_2 \tag{43}$$

With the initial conditions $N_1(0) = m$, $N_2(0) = g$. $N_3(0) = 0$. Our solutions are

$$N_1(t) = g\exp(-\gamma_1 t) \tag{44}$$

$$N_2(t) = \left(\frac{e^{-t\gamma_1 - t\gamma_2}\left(-e^{t\gamma_1}g\gamma_1 + e^{t\gamma_2}g\gamma_1 - e^{t\gamma_1}m\gamma_1 + e^{t\gamma_1}m\gamma_2\right)}{\gamma_2 - \gamma_1}\right) \tag{45}$$

$$\begin{aligned} N_3(t) \;=\; & \left(e^{-t\gamma_1 - t\gamma_2 - t\gamma_3}\left(e^{t\gamma_1 + t\gamma_2}g\gamma_1^2\gamma_2 - e^{t\gamma_1+t\gamma_3}g\gamma_1^2\gamma_2 + e^{t\gamma_1+t\gamma_2}m\gamma_1^2\gamma_2 - \right.\right.\\ & e^{t\gamma_1+t\gamma_3}m\gamma_1^2\gamma_2 - e^{t\gamma_1+t\gamma_2}g\gamma_1\gamma_2^2 + e^{t\gamma_2+t\gamma_3}g\gamma_1\gamma_2^2 - e^{t\gamma_1+t\gamma_2}m\gamma_1\gamma_2^2 + \\ & e^{t\gamma_1+t\gamma_3}m\gamma_1\gamma_2^2 + e^{t\gamma_1+t\gamma_3}g\gamma_1\gamma_2\gamma_3 - e^{t\gamma_2+t\gamma_3}g\gamma_1\gamma_2\gamma_3 - \\ & e^{t\gamma_1+t\gamma_2}m\gamma_1\gamma_2\gamma_3 + e^{t\gamma_1+t\gamma_3}m\gamma_1\gamma_2\gamma_3 + e^{t\gamma_1+t\gamma_2}m\gamma_2^2\gamma_3 - \\ & \left.\left. e^{t\gamma_1+t\gamma_3}m\gamma_2^2\gamma_3\right)\right) / \left(\left(-\gamma_1+\gamma_2\right)\left(\gamma_2-\gamma_3\right)\left(-\gamma_1+\gamma_3\right)\right) * \left(\frac{1}{m+g}\right) \end{aligned} \tag{46}$$

Clearly, we can see the first generation decays exponentially, but N_2 and N_3 decay non-exponentially. To give a probabilistic interpretation, we rescaled$(N_1 + N_2 + N_3)$ by dividing the factor$\frac{1}{(m+g)}$ in order to normalized survival probability $P(N_1 + N_2 + N_3, T)$ to unity at $T = 0$. Although $P(N_1 + N_2 + N_3, T)$ is not purely exponential, we can define a time-dependent decay rate$\gamma(T)$ as

$$P(N_1 + N_2 + N_3, T) = e^{-\gamma(T)T} \tag{47}$$

From now on let us denote $P(N_1 + N_2 + N_3, t)$ as $P(T)$. Its derivative is

$$\frac{dP}{dT} = \left(\frac{dN_1}{dT} + \frac{dN_2}{dT} + \frac{dN_3}{dT}\right)\frac{1}{(m+g)} = -\frac{1}{(m+g)}\gamma_3 N_3(T). \tag{48}$$

From the above relation and the initial conditions we know $\frac{dP}{dT} = 0$ at $T = 0$. That is the main feature of the Quantum Zeno Effect (QZE) in Misra-Sudarshan's theorem but it is not crucial condition for the new definitions of "Zeno Effect". We are giving the conditions for "Zeno" and "Anti-Zeno" by the following paragraph.

4.2 Conditions of "Zeno" and "Anti-Zeno" effect

For more instructive, we summarize the geometric definition Eq.(20) and Eq.(21) into more systematic form. We noticed the similar method of defining Zeno and Anti-Zeno as given by Facchi and Pascazio[12]. We introduce the averaged survival probability $P_a(T)$ with decay rate γ_a over the life time of the whole system T.

$$P_a(T) = exp(-\gamma_a T) \tag{49}$$

where $\gamma_a = \frac{-Log[P(T)]}{T}$. Obviously, this is parallel to the $\frac{d[log(P(T))}{dT}$ in the Eq.(20).

We now compare the two survival probabilities, $P(T)$ and $P_a(T)$ to determine whether the "Zeno" and/or "Anti-Zeno" effect can occur. There are two possibilities. The first case is that if the curve of $P(t)$ is above the $P_a(t)$, and there is no intersection until $t = T$. In other words, $\gamma(t) < \gamma_a$ in the entire time domain $[0, T]$. Then we say the system has "Zeno Effect". We can further to show that if we take n repeating measurements of the survival probability over the small time regiont_1 for $0 < t_1 < T$ then we

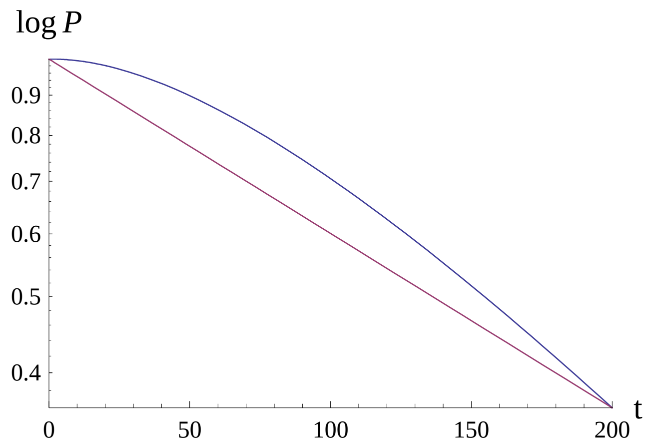

Figure 1: "Zeno effect" .$N_1(0) = 1000;\ N_2(0) = 0.\ \gamma_1 = 0.05;\ \gamma_2 = 1; \gamma_3 = 0.5.$

obtain the "Zeno effect".

The second case is that if there is an intersection point at time t. One may want to call the time t is the transition time where the curve $P(t)$ cross from "Zeno region" to "Anti-Zeno region". Now we ask whether such a time t exists. We can explicitly write down the analytic expression for the solution of t

$$\gamma_a t = Log[P(t)] \qquad \text{for } t < T \tag{50}$$

We want the t to be smaller than the T so we can have sufficient number of measurements n to show "Zeno effect" and "Anti-Zeno effect". In general, solutions of the above relation may or may not exist. It depends on the value of $P(T)$ which is determined by the initial conditions and the ratio of the γ_1, γ_2 and γ_3. However, finding a general conditions of "Zeno effect" and/or "Anti-Zeno effect" is not our interest. Our goal here is to show how the details of the system together with non-ideal measurement can lead some particular feature; such as "Zeno effect" and"Anti-Zeno effect" .

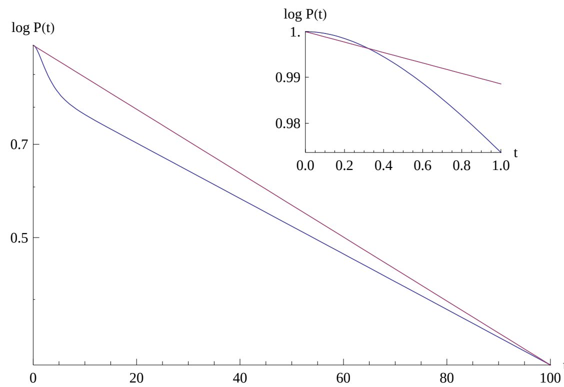

Figure 2: "Anti-Zeno" for $N_1(0) = 1000;\ N_2(0) = 200.\ \gamma_1 = 0.01;\ \gamma_2 = 1; \gamma_3 = 0.5$. The small graph is enlarged region at very time to show there is a intersection at small time t.

For example, we choose $N_1(0) = 5N_2(0)$ for the initial condition. We also let N_1have the longest life time and $\gamma_2 = 100\gamma_1$while $\gamma_3$is greater than γ_2. The time T is chosen to be $(\gamma_1)^{-1}$. This particular condition gives us the feature we are looking for: there is an intersection at time t which means we can obtain "Zeno effect" and "Anti-Zeno effect". The time interval $0 < t_1 < t$ is "Zeno effect" region; the time interval $t < t_2 < T$ is "Anti-Zeno effect" region. In this case both Zeno and Anti-Zeno can be obtained.

However, if we change the the ratios between the $\gamma_1, \gamma_2$and $\gamma_3$while the initial condition hold–for example; we set $\gamma_3 \leq \gamma_1$and $\gamma_2 \backsimeq 0.1\gamma_1$, then we don't see an intersection time. There is no "Anti-Zeno effect" for this condition.

There is another general case for the system has no "Anti-Zeno" feature. That is when the $N_2(0) = 0$,$N_1(0) = g$ and $N_3(0)$=0 while taking any arbitrary values for $\gamma_1, \gamma_2$and γ_3. We do have a intersection untill $t = T$.

However, we are still able to obtain the "Zeno" effect.

There is a special case, if we allow only one generation decay, e.g $N_1 \rightarrow N_2$. We know this is pure exponential decay. There is neither "Zeno" nor "Anti-Zeno".

These examples have shown the "Zeno effect" and "Anti-Zeno effect" depends on the details of the system. In fact, we can know how the system exhibit "Zeno effect" or "Anti-Zeno effect". We know the system is a combination of three generations decay, the first generation is pure exponential decay, the second and third generation has to grow at very short time. At the short time, the system is likely dominated by the second generation decay. Then at the short time, if the $\gamma_2 > \gamma_1$ the system decays faster than the average decay rate of whole systemγ_a that is likely we can obtain "Anti-Zeno effect". If γ_2 is smaller then γ_1, then the system decays slower than the average decay rate of the whole system that will give "Zeno effect".

5 Selective measurement and "Anti-Zeno"

As Misra and Sudarshan pointed out that the measurement in the quantum Zeno effect is *non-selective* measurement. However, in the new definition of Zeno and Anti-Zeno, all measurements were selective measurements due to the interval time t being greater than quantum Zeno time; that means when one preforms a measurement on a particular component, e.g. the decayed state. The survival probability is less than one. Therefore, for each measurement the probability is reduced due to the multiplication of probability that is essential and comment frame work to obtain “Anti-Zeno effect” in those papers we referred. We show another example how selective measurement can leading to “Anti-Zeno effect”.

Stern-Gerlach experiment is well known., An atomic beam goes through an SG apparatus when a measurement is taken on a component, let say the spin up component. The probability of an atom being spin up is $\frac{1}{2}$. Lets denote this as

$$P(1) = \frac{1}{2}. \tag{51}$$

Now consider the sequence of selective measurement. We put a SG apparatus, let called it 1, and measured the spin up component of the atomic beam. We assume here, after the measurement, the atomic beam become unpolarized again. And we put a SG apparatus 2th and measurement the component spin up, and so on. We want to know the probability of obtaining spin up on the nth SG apparatus when the beam coming out of the first SG apparatus is normalized to unity [12]. The probability of the nth measurement for spin up is given by probabilities multiplication;

$$P(1....n) = P(1) * .....P(n) = (\frac{1}{2})^n \tag{52}$$

After nth such measurements the probability was given

$$P(n) = [P(1)]^n \tag{53}$$

As $n \to \infty$, the probability of atoms spin up through all the SG apparatus is approaching to zero. If we compare the probability $P(n)$ to the probability $P(1)$, or even probability of the previous step, we know

$$P(n) < P(n-1) \tag{54}$$

This is to say, the probability with less measurement is greater then the probability with more measurement. Therefore, according the definition of Anti-Zeno Effect, we can say we obtained the Anti-Zeno Effect in Stern-Gerlach experiment.

6 Concluding Remarks

We compare one formalism with continuous variable t with another with discrete time interval. We show *non-ideal* measurements where the time interval t is finite (greater than the Zeno time) yield some contradictory results. We are able to produce the "Zeno" and "Anti-Zeno" effects in a classical radioactive decay system based on the definitions. However, the "Zeno" and "Anti-Zeno" effects are artifacts of the details of the system. We also point out that the survival probability decreases as the number of measurements increases was a result of selective measurement in the cases we studied here. This phenomenon can be seen in familiar experiments, such as the Stern-Gerlach experiment.

The quantum Zeno Effect was well definited and experimentally verified theorem. Any deviation from the theorem would lead to inconsisitance results. WE also find there is no a clear evident show the machinsime of

Qanutum Anti-Zeno. We will further investigate on the topic.

Appendix A

We used two difference notations in this thesis. One is the density matrix for Misra-Sudarshan theorem. To be consistent with the original work, we followed the notation was used in Modi and Shaji's work. The two notations essentially are the same. For example, The equation (15) is same as the equation (26):

Proof: $\rho_0 \equiv |A\rangle\langle A|$ and $E \equiv |A\rangle\langle A|$, $\quad \rho(t) = U(t)\rho(0)U^{+}(t)$, $\int d\lambda |\psi_\lambda\rangle\langle\psi_\lambda| = 1$

$$\begin{aligned}
P(t) &= Tr[U(t)\rho(0)U^{+}(t)E] \\
&= Tr[|exp(-iHt)|A\rangle\langle A|\, exp(iHt)|A\rangle] \\
&= Tr[\int d\lambda |\psi_\lambda\rangle\langle\psi_\lambda|exp(-iHt)|A\rangle\langle A|\, exp(iHt)|\psi_\lambda\rangle\langle\psi_\lambda|] \\
&= Tr[\int d\lambda |\psi_\lambda\rangle\langle\psi_\lambda||\langle A|exp(-iHt)|\psi_\lambda\rangle|^2] \\
&= |\langle A|exp(-iHt)|\psi_\lambda\rangle|^2
\end{aligned}$$

This is same as the equation (15)

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