The Dissertation Committee for Dan Lin Certifies that this is the approved version of the following dissertation:

EXPLORATION OF ROLE OF MARKET IN PERISHABLE GOODS

Committee:

Andrew B. Whinston, Supervisor

Maxwell B. Stinchcombe

David Sibley

Thomas Wiseman

Mina Dioun

EXPLORATION OF ROLE OF MARKET IN PERISHABLE GOODS

by

Dan Lin, B. Econ.; M.S.

Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

May 2007

Dedication

To my parents, Shimin Lin and Xiaomi Guo

Acknowledgements

I would like to acknowledge the following for their various contributions:
For excellent supervising and valuable comments: Dr. Andrew Whinston.
For excellent insightful reviews and support: Dr. Maxwell Stinchcombe, Dr. David Sibley, Dr. Thomas Wiseman and Dr. Mina Dioun.
For careful proof reading: Karen Evans, my dearest friend.
The last, but not the least, for his encouragement and faith in me: Zhuo Tan, my husband.

EXPLORATION OF ROLE OF MARKET IN PERISHABLE GOODS

Publication No._____________

Dan Lin, Ph.D

The University of Texas at Austin, 2007

Supervisor: Andrew B. Whinston

Firms face a big challenge in matching the supply of perishable goods with uncertain demand in real time. In practice, the traditional supply chain models are proved not efficiently enough to lower firms' risk exposure. The purpose of the dissertation is to provide the theoretical framework of roles of several stylized markets in firms' risk management. In particular, we explore the influence of the spot business-to-business exchange market, forward contract market and credit-default swap market respectively. The dissertation is divided into the following three chapters.

In chapter 1, we show that when the exchange market lacks perfect liquidity, a firm's capital structure has a greater influence on its output-level decisions, then the market is perfectly liquid. The impact may be even greater than that without an exchange market. This is primarily because the introduction of the exchange market causes firms to act strategically in absence of perfect liquidity.

In chapter 2, we study the essential relationship between producers' forward contracts and their supply strategies in business-to-business exchange market. Specifically, we focus on the application of the electricity power exchange market in the US. Our model reveals that the strategic incentive makes producers to join in forward contract market voluntarily and increases social welfare.

We show in chapter 1 that even when firms' risks are independent of each other, there is a chance that the realization of market uncertainty turns out to be the same. As a result, there is no exchange market as a platform to help firms hedge their risks. Therefore, we need other instruments in firms' risk management portfolio. In chapter 3, we propose a financial market, credit-default swap market, in which firms can temporarily transfer default risks to outside investors. However, the "lemon" problem may cause social cost.

TABLE OF CONTENTS

Chapter 1. Capital Structure, Demand Uncertainty and the B2B Exchange Market1

1. Introduction 1

2. Model and Analysis 5

2.1 No Exchange Market 5

2.2 Exchange Market with Multiple Firms 7

2.3 Exchange Market with Two Firms 9

2.3.1 The Exchange Market and Market-Clearing Price, w. 9

2.3.2 The Equilibrium Production Strategies.............................11

2.3.3 The Numerical Analysis 12

2.3.3.2 The Asymmetric Case..16

3 Discussion and Extension 19

3.1 Choice of Risk-Hedging Vehicles 19

3.2 Number of firms and market liquidity 20

3.3 Correlated demand 20

3.4 Bankruptcy Cost...21

3.5 Multi-Period Strategies. 22

4 Conclusion 22

Chapter 2. Forward Contracting, Supply Function Equilibrium, and Market Efficiency 24

1. Introduction 24

2. Model with no capacity constraint 29

2.1 Model Set-up...29

2.2 Spot Market...31

2.3 Contract stage...36

3. Forward contract with capacity constraints 37

3.1 Spot market 38

3.2 Contracting stage 39

3.3 The influence of capacity constraint on forward contract 40

3.3.1 Symmetric case 40

3.4.2 Asymmetric case 41
3.4.3 Market with N>2 producers 42
4. The effect of call option contract on spot market 43
5. Discussion 45
3.1 Observation of the forward contract 45
3.1 The forward contract exchange..45
6. Conclusion 46

Chapter 3. Credit-default swap and B2B Exchange Market liquidity 48
1. Introduction..48
2. Credit-credit swap 51
3. Benchmark Model: Without Credit-default Swap Market..52
3.1 Model Set-Up..52
3.2 The spot exchange market 54
4. With Credit-default swap market..57
4.1. Credit-default swap market. 57
4.2. Symmetric information 58
4.3. Asymmetric information 60
4.4. Welfare analysis..62
5. Discussion 63
6. Conclusion remarks 66

Appendix A: The Proofs for Chapter 1 68

Appendix B: The Proofs for Chapter 2 74

Appendix C: The Proofs for Chapter 3 80

References..88

Vita..92

Chapter 1. Capital Structure, Demand Uncertainty and the B2B Exchange Market

1. Introduction

In this paper, we analyze a new type of risk-hedging vehicle, a business-to-business (B2B) exchange market, and its effect on a firm's capital structure dependency. Operations management researchers generally ignore the influence of capital structure following Modigliani-Miller (1958) irrelevance propositions, which state that a firm's market valuation is independent of its financing and dividend pay-out decisions. Therefore, the propositions conclude, a firm can separate its financial and operational decisions. However, the propositions require strong assumptions that generally are not satisfied. Economics and corporate finance researchers have shown that a firm’s capital structure affects its production decisions and hence its market valuation (Titman 1984; Brander and Lewis 1986). Recognizing this impact is particularly important to a firm’s shareholders, managers, and competitors who seek to predict the firm’s operation decisions and determine its value.

Despite these theoretical results, many firms still separate their financial and production department decision processes (Wanzenried 2003). Researchers argue that the effect of capital structure is less powerful when the firm can better hedge its operational risks. (Froot et al 1993; Spano 2002). Financial engineering has made available a variety of risk-hedging vehicles for public trading, creating unprecedented opportunities for firms to hedge various operational risks.[1] Although capital structure concerns seem less relevant in this better "insured" world, the precise nature of the relationship deserves more careful study.

In a risk-hedging market, firms essentially buy or sell contingency contracts according to their own risk-hedging requirements. Firms that take opposite positions can effectively form a risk-sharing pool. However, researchers have noted that those hedging markets often fail to attract enough traders (Borio 2003; Moorthy 2003; Scobie 2003f). This lack of liquidity may result in increased risk-hedging and strategic interaction costs. In this paper, we show that lack of participation may make a firm's operational decisions even more dependent on its capital structure. We justify this possibility by analyzing a group of limited-liability manufacturing firms with certain debt levels and uncertain future customer demand.

We choose to study the business-to-business exchange market in this paper (our choice of hedging market is an effective example, but not the only one[2]). B2B exchange markets have grown in popularity with the development of the Internet. An exchange market provides a firm the platform to sell its excess production or supply for its unmet demand, alleviating its demand risk. As in most trading markets, the B2B market price is set to balance supply and demand. An outstanding example of a B2B exchange market is the cooperative electricity exchange in the power pool, such as in California (Wilson 2003) and Great Britain (Newbery 1992).

A firm's capital structure, especially its debt-to-equity ratio, may be set for a variety of reasons. Jensen and Meckling (1976) are among the first to demonstrate that an appropriate debt-to-equity ratio helps firms reduce agency costs. Hart (1993) summarizes the role of debt as a managerial discipline device. For example, it can force liquidation and prevent empire building. In addition, a firm's capital structure can signal its profitability to outside investors (Stiglitz and Weiss 1981). Brander and Lewis (1986) found that debt could also serve as a commitment to competition in a duopoly market. We choose to study the effect of a firm's debt on its operational decisions, rather than explain the reasons for its established capital structure, while our simulation results provide evidence that the optimal capital structure exists. Hence, we assume firms' debt levels are exogenous variables.

---

[1] These hedging platforms include B2B markets, forward markets, VoXs, event derivatives, and others.
[2] Shin (2004) showed that the relationship between two traders in a financial market lacking liquidity is mostly comprised of *strategic substitutes*.

The influence of capital structure varies according to different liquidation policies. Firms that adopt limited liability are most often studied. Under a limited-liability structure, the firm is controlled by equity holders unless it fails to pay back its contracted debt and claims bankruptcy. If that happens, control shifts to debt-holders. We assume the same liquidation policy to analyze the firm's strategic decisions in the exchange market. We show how hedging-market liquidity affects a firm's dependency on its capital structure. Other capital structures and liquidation policies may lead to different results. We suggest that investors should carefully monitor firms' outstanding debt levels.

This paper is the first to study the effect of hedging-market liquidity on firms' capital structures and operational decisions. We show that the use of a hedging market may not necessarily unbind firms' operational and financial decisions. Hedging-market liquidity plays an important role in this issue. On the surface, the increasing availability of new hedging tools creates more hedging opportunities for firms and thus more financial market prosperity. However, researchers have expressed concern over how liquidity vanishes as the market grows (Borio 2003). On one hand, the wider choice of hedging vehicles increases the complexity of identifying the best one. On the other, it increases market frictions, such as transaction costs, thereby diminishing pricing efficiency. Our research justifies the critical role of hedging-market liquidity in correctly determining firm value.

Our model includes multiple local monopolies facing idiosyncratic market demand risks. Each monopoly produces a homogenous, short-lived product and bears a certain level of debt. All the firms adopt a limited-liability structure. The exchange market opens after the firms determine production levels, but before they resolve local demand uncertainties.

We begin with a model in which no exchange market exists. A firm's production level may increase as it bears more debt, which is consistent with the results predicted in most limited-liability literature (Brander and Lewis 1986; Showalter 1995; Wanzenried

2003). We then investigate two models in which firms can trade in the exchange market. The only difference between them is the total number of participating firms (O'Hara 1995; Kyle 1989).

In the first model, there are an infinite number of identical firms. Each firm's trading volume is small compared to the total market size. In this extreme case, the market-clearing price almost certainly converges to the marginal cost of production. Each firm produces to maximize overall firm value, but the debt level does not affect production. Therefore, a fully liquid market is introduced and the equilibrium is consistent with the Modigliani-Miller (1958) irrelevance propositions.

Departing from the fully liquid market, our second model contains only two firms. Although the two-firm model is another extreme case, it captures a firm's strategic decisions when observing the other firm's outstanding debt. It also maintains theoretical simplicity and flexibility. The lack of liquidity in this model induces firms' to adjust their exchange market prices. This strategic incentive is affected by the firm's debt level, leading capital structure to become more influential. Most importantly, we quantitatively show that the affect of capital structure on firms' incentives to adjust market price may dominate the decreasing impact of the capital structure from hedging. This leads to an even stronger aggregate impact than in the case without the exchange market. We also find that production levels are lower in the exchange-market model because the exchange market guarantees more efficient use of the aggregate production. In addition, the firm's production rises with its own debt, but falls as the other firm holds more debt.

We conducted simulations to analyze firms' equilibrium strategies under a variety of debt levels. Our results suggest that the firm with significantly less debt can take advantage of its counterpart's high leverage, and save costs by lowering (or even ceasing) production. This is because the high-debt firm will produce more aggressively. The exchange market can allow the low-debt firm to save on operational costs and reduce its financial risk. The high-debt firm, in contrast, cannot get help from its counterpart if it has a shortage. It must insure itself by increasing production. In the extreme case -- in which one firm bears enormous debt and the other has none -- the exchange market's impact is significant: The full-equity firm can nearly stop producing and rely on the other firm's excess production.

The rest of the paper is organized as follows: In Section 2, we introduce the model and present our argument and major findings. In Section 3, we discuss possible extensions to and limitations of our model. We summarize and conclude in Section 4.

2. MODEL AND ANALYSIS

2.1 No Exchange Market

Consider N identical firms that are monopolies in their local markets. The firms produce homogenous, short-lived goods at a constant marginal cost, $c > 0$, and sell them at a uniform price, $P > c$. For firm $i \in I = \{1, 2, \cdots, N\}$, its local market demand, Q_i, is uncertain when it determines its production level, q_i. The firm's operational profit function is

$$\pi_i = \pi\left(q_i; Q_i, P, c\right) = P\min\{q_i, Q_i\} - cq_i \qquad \forall i = 1, \ldots, N \tag{1}$$

Prior to realization, Q_i is regarded as an independent random variable drawn from a common distribution function, $F(\cdot)$ with support $[0, \infty)$. This means that the local markets are isolated from each other. We assume that F is twice differentiable and strictly concave.

In terms of capital structure, we assume the firms are controlled by equity-holders who adopt the limited-liability corporate structure. Each firm has outstanding debt, which we denote the face value as B_i for firm *i*. The debt holders will seize control of the company if the equity holder fails to pay back B_i after selling the products.

Without an exchange market, each firm makes production decisions based on its local market demand. We can hence analyze each firm separately. The firm i equity-holder's profit is $(\pi_i - B_i)^+$. The equity holder chooses production level, q_i, to maximize the firm's expected profit:

$$\max_{q_i \geq 0} \ \mathrm{E}\left[(\pi_i - B_i)^+\right] = \int_0^\infty (\pi(q_i; P, c, Q_i) - B_i)^+ f(Q_i) dQ_i \quad \forall \ i = 1, \ldots, N \ (2)$$

Lemma 1[3]: Firm i is bankrupt when:

1. $q_i < \frac{B_i}{P-c}$; or

2. $q_i \geq \frac{B_i}{P-c}$; however, $Q_i < \hat{Q}_i = \frac{cq_i + B_i}{P}$.

Note that $(\pi_i(q_i; P, c, Q_i) - B_i)^+$ is either positive or zero. Lemma 1 implies that the firm can expect a strictly positive profit when producing more than $\frac{B_i}{P-c}$. We can thus regard $\frac{B_i}{P-c}$ as the lower bound of the firm's production. It increases with the firm's debt level. According to Lemma 1, we can rewrite the equity-holder's optimization problem as

$$\max_{q_i \geq \frac{B_i}{P-c}} \ \mathrm{E}\left[\pi_i - B_i\right] = \int_{\hat{Q}_i}^{q_i} (PQ_i - cq_i - B_i) f(Q_i) dQ_i + \int_{q_i}^{\infty} (Pq_i - cq_i - B_i) f(Q_i) dQ_i$$

$\forall \ i = 1, \ldots, N$ (3)

In order to rule out extreme equilibrium cases such as zero or infinite production levels, we must impose two more assumptions. First, we assume that $F\left(\frac{B_i}{P-c}\right) < 1$, which guarantees that the debt is not impossible to repay. Hence, the firms will be willing to produce. Second, we assume that $T(x) = \frac{-f'(x)}{f(x)}$ is non-decreasing in x[4]. This ensures that the probability of extremely high demand decreases fast enough to keep production from reaching infinity.[5] Denote q_i^{NO} as the optimal production level for firm i when there is no exchange market. Proposition 1 characterizes q_i^{NO}.

[3] All proofs are listed in the Appendix.

[4] $T(x)$ represents the rate of decreasing likelihood.

[5] If $f(x)$ does not decrease fast enough, the firm is so secure in limited liability that it always produces an infinite amount seeking huge profit. For example $f(x) = \frac{1}{x^2}$ for $x \in [1, +\infty)$. The first order condition will not equal zero unless $x = +\infty$.

Proposition 1: For any debt level B_i , q_i^{NO} uniquely exists, and $q_i^{NO} > \frac{B_i}{P-c}$.

Proposition 1 enables us to discuss the affect of debt levels on the operational decisions of limited-liability firms and compare that affect to other cases. This impact is summarized in Proposition 2.

Proposition 2: The optimal solution, q_i^{NO} , increases in the debt cost, i.e. $\frac{\partial q_i^{NO}}{\partial B_i} > 0$.

Proposition 2 shows that, in equilibrium, firm i increases production as it bears more debt. Denoting the optimal solutions for cases with and without limited liability as q^{NO} and q^{NL} , respectively, yields Lemma 2.

Lemma 2. $q^{NO} > q^{NL}$

The first order derivative of (3) can be written as

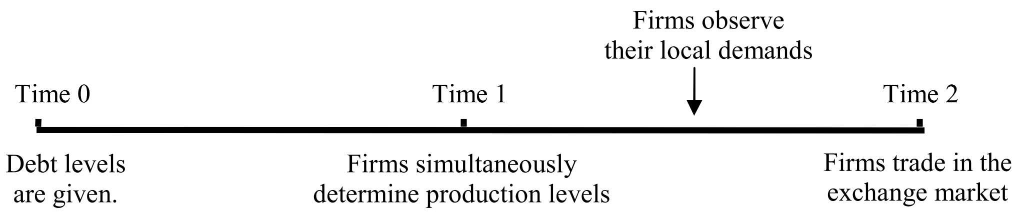
$$\left\{P\left[1-F\left(q_i\right)\right]-c\right\}+cF\left(\hat{Q}_i\right) \qquad (4)$$

The part in parentheses is identical to the one in the case without limited liability. The extra term in (3.4), $cF\left(\hat{Q}_i\right)$, represents the impact of limited liability when the firm includes a positive debt level in its capital structure. Since $cF\left(\hat{Q}_i\right)$ is strictly positive, we conclude that the limited-liability firm will choose a higher production level. We call this over-production the "limited liability effect."

2.2 Exchange Market with Multiple Firms

The exchange market opens at time 1. After the demand is realized, each firm simultaneously submits its trade plan. Without knowing the realization of demand for other firms each firm's quantity to exchange is $|q_i - Q_i|$. The market-clearing price, *w*, is decided by the total supply and demand in the market. The model is divided into three periods (see Figure 1): At time 0, firms' debt levels are set; at time 1, firms choose their production levels before they observe their local demand; at time 2, when the local demand is realized, the firms decide how to trade to maximize profit and then repay debt.

Figure 1: Timeline



We first consider an ideal case that includes a very large number of identical firms. As a result, each firm's production has a negligible impact on the market price. The Law of Large Numbers yields

$$\lim_{N\to\infty} \overline{Q} = EQ. \qquad (5)$$

Since the firms bear identical debt ex ante, we focus on the symmetric equilibrium. We suppress the subscript i for each firm and use superscript FL to denote the case with a large number of market participants. The equilibrium is summarized in Proposition 3.

Proposition 3: When $N \to \infty$,

1. the market clearing price $w = c$;
2. the equilibrium production is $q^{FL} = EQ$;
3. and $\partial q^{FL} / \partial B = 0$.

Proposition 3 shows that the market-clearing price is equal to the marginal cost of production. In addition, each firm's production level is the same as the expected demand in the equilibrium. Because of the stability of the average market demand, the firms can fully hedge their market risks. The market-clearing price will not deviate from the marginal cost so to avoid over-supply or shortage. The critical assumption is that each firm's trading volume is tiny compared to the total market size. No one firm expects to affect the market price, w, through its own order and, therefore, the exchange market achieves full liquidity. Number 3 of Proposition 3 shows that a firm's debt has no impact on its production choice; production and financial decisions are unbundled in a fully liquid market.

This model depends on the assumption of an infinite number of small firms. However, no actual B2B exchange market satisfies this condition. In the next section, we determine if the production and financial decisions will still be separated in a market that is not fully liquid.

2.3 Exchange Market with Two Firms

In this section, we study a case where there is insufficient liquidity in the exchange market. We keep the same model setup, except that N =2, to maintain comparability. We denote the two firms as i and j. The firms' strategies are captured by their production decisions, (q_i, q_j). We solve the equilibrium strategy, (q_i^*, q_j^*), by backward induction.

2.3.1 The Exchange Market and Market-Clearing Price, w.

One of six cases is possible after the firms make production decisions and realize the demand in their local market. These cases are summarized in Table 1:

Table 1. The Cases after Demand Realization

Case 1	$Q_i - q_i > q_j - Q_j > 0$
Case 2	$q_j - Q_j > Q_i - q_i > 0$
Case 3	$Q_j - q_j > q_i - Q > 0$
Case 4	$q_i - Q_i > Q_j - q_j > 0$
Case 5	$Q_i - q_i > 0$ & $Q_j - q_j > 0$
Case 6	$Q_i - q_i < 0$ & $Q_j - q_j < 0$

The two firms only exchange if one has excess products and the other has a shortage (Cases 1 through 4). The trade is not possible if both firms have excess or shortage simultaneously (Cases 5 and 6). Therefore, the firms' operational risks are not fully hedged.

In Cases 1 through 4, the exchange demand, $D(w)$, for the buying firm, i (without loss of generality) is

$$D(w)=\begin{cases} Q_i - q_i & if \quad 0 \le w < P \\ [0, Q_i - q_i] & if \quad w = P \\ 0 & if \quad w > P \end{cases} \qquad \forall i = 1,2 \tag{6}$$

Since the firm has local shortage, it wants to buy as much as possible when the price is lower than *P*. However, when the market-clearing price is *P*, the firm is indifferent toward buying. The firm will stop buying when the price rises above *P*. The selling firm's supply function is

$$S(w)=\begin{cases} q_j & if \quad w > P \\ q_j - Q_j & if \quad 0 \le w \le P \quad \forall j = 1,2 \\ [0, q_j - Q_j] & if \quad w = 0 \end{cases} \tag{7}$$

The firm is willing to sell at any positive price because it already paid the cost of production. There is no extra local demand and selling on the exchange market is the only way to earn extra money. The firm is indifferent toward selling when the market-clearing price is zero. Given the firms' trading strategies , the market-clearing price, *w,* is defined as

$$w=\begin{cases} P & If \ Q_i - q_i \ge q_j - Q_j \\ 0 & If \ Q_i - q_i < q_j - Q_j \end{cases} \qquad \forall i, j = 1,2 \tag{8}$$

A simple calculation shows that

Lemma 3. $\dfrac{dE_{Q_i,Q_j}W}{dq_i} < 0$

Lemma 3 illustrates that in the two-firm model, the market-clearing price is influenced by the firms' production strategies (q_i, q_j). The expected market-clearing price, $E_{Q_i,Q_j}W$, is

$$E_{Q_i,Q_j}W = P\Pr(Q_i + Q_j \ge q_i + q_j) \tag{9}$$

Given firms' local demands (Q_i, Q_j), firm *i* expects that market-clearing price decreases with the production level, since the increase in the production lowers the chance of the market price being high. As a result, the exchange market is not fully liquid.

2.3.2 The Equilibrium Production Strategies.

We now determine the firms' production decisions in period 1. A firm selects a production level to maximize its equity-holders' expected profits. Decision-makers know that the profit is contingent on the other firm's production decision and the possible realization of demand in both local markets. Depending on range that the optimal solution falls into, firm i's expected profit takes two forms:

$$\underset{q_i}{Max} \quad G_i = \begin{cases} G_i^1\left(q_i; q_j, B_i\right) & q_i < \dfrac{B_i}{P-c} \\ G_i^2\left(q_i; q_j, B_i\right) & q_i \geq \dfrac{B_i}{P-c} \end{cases} \tag{10}$$

Denoting the probability of bankruptcy in the cases with and without the exchange market as $\Gamma(Q_i, Q_j)$ and $F(Q_i)$, respectively, yields

Lemma 4: $\Gamma(Q_i, Q_j) < F(Q_i)$

(10) implies that a firm working within an exchange market may still expect a strictly positive profit, even if it produces less than $q_i < \frac{B_i}{P-c}$. This is because the firm can expect to buy products from the other firm if the latter has less demand. However, Lemma 1 shows that, without the exchange market, a firm always goes bankrupt when it produces less than $\frac{B_i}{P-c}$. The Lemma 4 also shows that a firm working inside the exchange market is less likely to go bankrupt if it chooses a production level more than $\frac{B_i}{P-c}$.

Lemma 4 indicates that the exchange market lets firms pool their demand risks and effectively reduce their operational risks. In fact, when the two firms are totally isolated, their probability of going bankrupt depends only on the actual local demand after they set up their production plans. Exchange markets give firms a channel to improve their profits after they know their local demands. The two firms help each deal with demand uncertainty through this channel.

2.3.3 The Numerical Analysis

We next determine the optimal production level for each firm. Unfortunately, the objective function is complex and non-linear; its properties can be analyzed only for an explicit distribution. Consider the exponential distribution. The demand distribution is now represented as $f(Q_i) = \lambda e^{-\lambda Q_i}$; $\forall\ i = 1,2$. The exponential distribution follows the characteristics of the distribution defined earlier in this paper. We assume that $\frac{1}{\lambda} > \frac{B_i}{P-c}$. $\forall\ i = 1,2$, which guarantees both that the expected demand is not too low and that the probability of demand will not fade out too quickly.

2.3.3.1 Symmetric Case

In this section, we only consider the optimal solution: When the two firms have debt with identical face values. $B_i = B_j = B$. We are only interested in symmetric, sub-game, perfect equilibrium ($q_i^{EX} = q_j^{EX} = q^{EX}$) here.

Lemma 5: If there exists a symmetric equilibrium such that $q_i^{EX} = q_j^{EX} = q^{EX}$, then $q^{EX} > \frac{B}{P-c}$ must hold.

The two firms cannot simultaneously choose production levels lower than $\frac{B}{P-c}$ in the equilibrium. This is because a firm will only scale back production if it knows that the other firm will produce enough for both of them. Since Lemma 3 rules out the existence of a symmetric equilibrium in Case 1, we focus only on Case 2. The first order derivative from Case 2 is

$$\begin{aligned}&(Pe^{-\lambda(q_i+q_j)})\left[1+\lambda(q_i+q_j)-\lambda^2\left(q_i+q_j\right)q_i+\lambda^2\frac{1}{2}\left(q_i+q_j\right)^2\right]-c\\&+e^{-\lambda(q_i+q_j)}\frac{P\lambda^2}{2}\left(\hat{Q}_i+\frac{2c}{P\lambda}\right)\hat{Q}_i+c\left[1-e^{-\lambda\hat{Q}_i}\right]\end{aligned} \tag{11}$$

Proposition 4: The optimal production q^{EX} uniquely exists.

(11) shows the existence of equilibrium. In addition, if we denote that q^{NL} is the optimal production level in the case without limited liability, then the result can be summarized as

Lemma 6: q^{EX} is greater than q^{NL} .

As with Lemma 2, we can prove that the first two terms of (11) are identical to the first-order derivative of the profit maximization problem in the case without limited liability. Therefore, the last two terms of (11) represent the limited-liability effect. Since the sum of the last two terms is always positive -- even when the firms can exchange in the market -- the limited-liability effect still leads to over-production.

Although Proposition 4 presents the existence and uniqueness of the optimal solution under a general condition, it is still unlikely that we can derive a closed-form solution. This is because the functional forms of the objective function and the first-order condition are complex. But, a general analytical solution is unnecessary because we seek only to justify the possibility that the influence of a firm's capital structure on its operational decisions is strengthened when its operational risks are reduced. Rather than performing a complicated mathematical proof in the next section, we use the simulation to show the comparative static analysis results.

We next compare the equilibrium of the current model to the one without an exchange market. Our goal is to show the change in both equilibrium production and the effect of the capital structure in an illiquid exchange market (P = 1 through 100). The combinations of P, c and B were randomly chosen under the constraint $\frac{1}{\lambda} > \frac{B}{P-c}$ in order to follow the assumption in the model[6]. We present simulation results and numerical examples[7] in the next section.

Result 1. $\frac{\partial q_i^{EX}}{\partial B_i} > 0$ and $\frac{\partial q_i^{EX}}{\partial B_i} > \frac{\partial q_i^{NO}}{\partial B_i}$ $for\ i = 1, 2$

Table 2.The Effect of the Firm's Own Debt

[6] We randomly chose 500 groups of parameters for this simulation.

[7] The numerical example uses the exponential distribution with $\lambda = 0.5$ and $P = 2c$. Authors will provide other simulation results by request.

	Without Exchange Market					With Exchange Market				
	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
$P=10$	0.200	0.200	0.200	0.200	0.200	0.302	0.315	0.324	0.339	0.355
$P=20$	0.100	0.100	0.100	0.100	0.100	0.148	0.151	0.154	0.157	0.161
$P=30$	0.067	0.067	0.067	0.067	0.067	0.098	0.099	0.101	0.102	0.103
$P=40$	0.050	0.050	0.050	0.050	0.050	0.073	0.074	0.075	0.075	0.076
$P=50$	0.039	0.039	0.039	0.039	0.039	0.058	0.059	0.059	0.060	0.060

Result 1 shows that the effect of debt on production is still positive with an exchange market. Production increases when the firm has more debt. More importantly, the results show that the debt effect is of greater magnitude in the exchange case. This is the core argument in this paper: Even if firms can reduce their risks through an exchange market, the effect of debt is likely higher than when there is no exchange market.

In equilibrium, firms' production levels are the result of their strategic interactions. We summarize the effect of the opponent's debt on a firm's production decisions in Result 2 and present part of our findings in Table 3.

Result 2. $\frac{\partial q_i^{EX}}{\partial B_j} < 0 \quad for\ i \neq j$

Table 3.The Effect of the Other Firm's Debt

	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
$P=10$	-0.185	-0.197	-0.207	-0.221	-0.237
$P=20$	-0.090	-0.093	-0.096	-0.099	-0.102
$P=30$	-0.059	-0.060	-0.061	-0.063	-0.064
$P=40$	-0.044	-0.045	-0.045	-0.046	-0.047
$P=50$	-0.035	-0.035	-0.036	-0.037	-0.037

Result 2 and Table 3 show that one firm's optimal production level is negatively correlated with the other firm's debt. In other words, the higher one firm's debt, the less the other produces.

According to Results 1 and 2, the firm's debt affects both its and its counterpart's production. Hence, when the market lacks full liquidity, the firms can strategically affect each other's production decisions. The optimal output level from Result 3 and Table 4 is the equilibrium of the strategic interaction between both firms.

Result 3: $q^{NO} > q^{EX}$ $for\ i = 1,2$.

Table 4.The Optimal Production

	Without Exchange Market					With Exchange Market				
	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
$P=10$	2.97	3.17	3.37	3.57	3.77	2.70	2.82	2.93	3.05	3.17
$P=20$	2.87	2.97	3.07	3.17	3.27	2.64	2.70	2.76	2.82	2.88
$P=30$	2.84	2.90	2.97	3.03	3.10	2.62	2.66	2.70	2.74	2.78
$P=40$	2.82	2.87	2.92	2.97	3.02	2.61	2.64	2.67	2.70	2.73
$P=50$	2.81	2.85	2.89	2.93	2.97	2.61	2.63	2.65	2.68	2.70

The equilibrium output is lower when the firms operate in an exchange market. On one hand, firm i knows that its production should be higher than the lower bound in order to cover its debt obligation. Firm i's debt has the same positive impact on its production as it has in the model with no exchange market (limited-liability effect). However, when the exchange market opens, Firm i's debt has a negative effect on firm j. The more debt firm i holds, the less firm j produces. (12) shows that firm i's marginal profit decreases as firm j increases production.

$$\frac{\partial^2 \pi}{\partial q_i \partial q_j} = \frac{\partial FOC_i}{\partial q_j} \overset{q_i = q_j}{=} -\lambda e^{-\lambda(q_i + q_j)} \left\{ \lambda \left(Pq_i - c\frac{cq_i + B_i}{P} \right) + \frac{1}{2}\lambda^2 P \left(\frac{cq_i + B_i}{P} \right)^2 \right\} < 0 \;\; for \;\; i \neq j$$

(12)

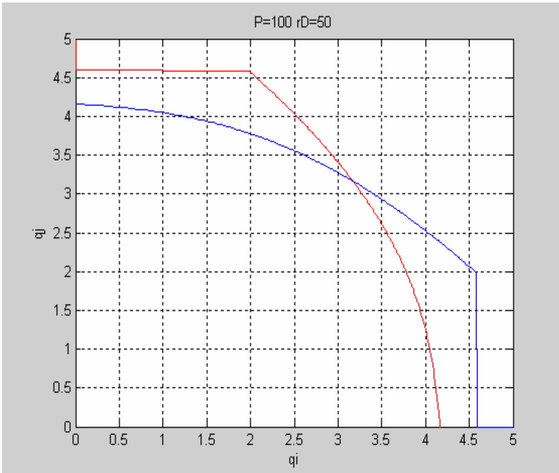
This result is because the expected market-clearing price increases as firm j's production drops. Firm i benefits from increasing production because it may have a better chance to sell its excess at price P. Overall, identical debt levels induce even higher production when the exchange market exists. At the same time, firm j's debt obligation has a negative effect on firm i's production decision. firm j has to produce enough to cover its identical debt level. Since the market lacks perfect liquidity, the rise in firm j's production increases the chance for firm i to buy excess products at low prices, which, in turn, leads to a decrease in firm i's production. As a result, firm i is less motivated to plan a very high level of production. The equilibrium output level is eventually the balance of these two forces.

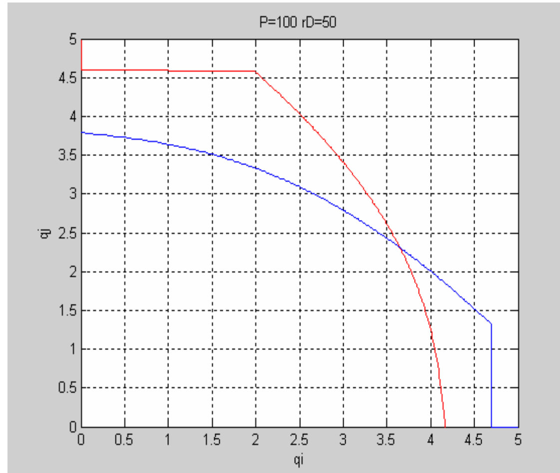
2.3.3.2 The Asymmetric Case

We assume that the two firms have different capital structures and $B_i > B_j \;\; for \;\; i \neq j$. In Figure 2, we compare the equilibrium between the symmetric case and the asymmetric case (in which we assume $B_i = 2B_j$).

Figure 2. The Best Response of Two Firms

(Left graph: $B_i = 50, B_j = 50$. Right graph: $B_i = 50, B_j = 25$)





According to Figure 2:

Result 4: If $B_i > B_j$, then $q_i^* > q_j^*$.

Table 5.The Optimal Output (P=100)

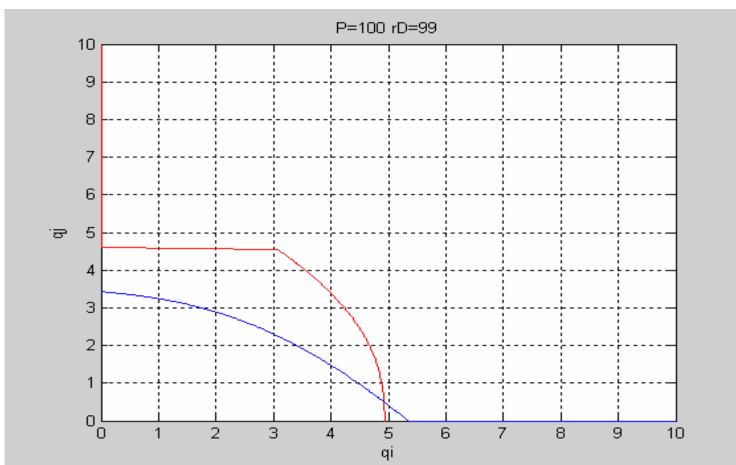
		$B_i = 10$	$B_i = 20$	$B_i = 30$	$B_i = 40$	$B_i = 50$
$B_j = B_i / 2$	q_i	2.79	3.08	3.36	3.63	3.88
	q_j	2.55	2.35	2.14	1.92	1.70

The observation from Table 5 follows the same logic as in the symmetric case. When one firm has more financial obligation than the other, it should maintain a higher level of production. As the low-leveraged firm takes advantage of its counterpart's high production commitment its own production approaches its lower boundary.

We argued that the symmetric equilibrium cannot exist when $q_i < \dfrac{B_i}{P-c}$ because both firms have the same ex ante obligation and so should behave the same way. This conclusion may not be true in the asymmetric case. Consider a case in which one firm has zero debt and the other firm bears a very high level of debt ($B_i > 0, B_j = 0$). The full-equity firm has the least incentive to produce. Hence, the debt-laden firm predicts that it will get little help fulfilling any shortage from the exchange. Meanwhile, there is a fairly good chance for the debt-laden firm to profit from the exchange market when the full-equity firm's local demand turns out to be high. Therefore, two forces drive the debt-laden firm to produce at a much higher level. Figure 3 shows that when a debt-laden firm produces aggressively while the optimal solution for the full-equity firm is close to zero.

Figure 3. The Best Response

($B_i = 99, B_j = 0$)



In this situation, the debt-laden firm commits to very high production in order to pay back its debt. Since the full-equity firm has a zero lower bound for production, it can simply shut down its machines and purchase from the other firm. Table 6 presents the simulation results for the effect of debt on the debt-laden firm. When the two firms have different capital structures, the table shows that the effect of the firm's debt on its production decision is still larger than in the case with the exchange market.

Table 6. The Effect of the Debt

With Exchange Market	Without Exchange Market
0.0190	0.0177

3 DISCUSSION AND EXTENSION

3.1 Choice of Risk-Hedging Vehicles

We choose to study a firm with a B2B exchange market hedging vehicle. A B2B exchange market effectively hedges demand uncertainty because it is flexible enough to absorb hedging volumes. As a result, it is used in industries with high demand volatility. However, when there are insufficient participants in the market, each one can affect the market price and other participants' payoffs. Borio (2003) named these "counterparty risks" and documented the exaberance of them in derivative markets. He also argues that counterparty risks are one of the major factors leading to market crises.

In our model, the firms strategically determine their production levels in an effort to affect the price in an insufficiently liquid market. According to Borio, this behavior is one source of counterparty risks. We extend Borio's argument by showing that this strategic behavior makes firms more vulnerable to their capital structures.

One may argue that firms can simply choose other hedging vehicles to avoid counterparty risks. For example, firms could agree to a fixed exchange price ex ante. However, this is generally not possible in a demand uncertainty model in which firms' production and local demand levels are private information (Oum, Oren, Deng 2005). Hence, an ex ante contract cannot explicitly describe when the firms should exchange. The contract literature, though unfinished, predicts that ex post renegotiation due to an incomplete contract may lead to an inefficient outcome.

Alternate hedging vehicles, such as options or futures, require that firms commit to certain hedging volumes (e.g. purchase a fixed number of call options). But, that type of hedging may not be fully efficient when demand is extremely volatile. In addition, counterparty risks may also exist in the derivative market (Borio 2003).

3.2 Number of firms and market liquidity

The exchange market effectively reduces firms' profit volatility by pooling their risks. Lack of liquidity in the market may cause strategic interaction among firms and strengthen the affect of their capital structures on their production decisions. Hedging is effective only with a sufficient number of market participants, so that firms can reduce their operational risks to the largest extent and be the least restricted by their capital structures. However, although the law of large number is effectively used in all kind of research works, we know that a world with infinite number of firms does not exist in general cases.

With limited number of firms in the market, there is a chance that the realization of firms demand turns out to be the same, i.e. all firms have excess supply or shortage, even if firms have idiosyncratic demand risks. As a result, the market fails, because of severe illiquidity. In traditional economic or financial researches, this "macro"-like shock is usually caused by a perfectly correlated risk faced by all firms in ex-post market (Persaud, 2003). However in this paper , we have showed that even with idiosyncratic shocks, this could happen[8], and the chance could be large especially when we deviate from the conventional assumption of fat tail distribution of demand risk and consider extreme events as ancillaries (Taleb 2001). Therefore, in their risk management decision, firms need some other hedging vehicles which and insure firms when this "macro" shock appears.

3.3 Correlated demand

Our analysis focuses on firms' idiosyncratic demand risks. We show that liquidity depends on the number of market traders. However, liquidity can also be affected by the diversity of hedging demand (Persaud 2001). When demand risks are negatively

[8] See case 5 & 6 in two firms model in page 10.

correlated,[9] the exchange market can clearly help the hedging counterparties and hence improve market liquidity. In this case, we predict a firm's capital structure would be less influential and the problems discussed in this paper would become less severe. The ideal Modigliani-Miller world is obtained when the risks are perfectly negatively correlated (the correlation is -1). However, finding a hedging counterparty with a negatively correlated risk is generally difficult – if not impossible.

Firms' operational risks are usually positively correlated due to some macro event. The war in Iraq affects gas prices, for example, or an economic downturn slows consumer spending. The exchange market appears to be less capable of hedging risks such as these. However, if the firms receive differential predictions about forthcoming risks, the exchange market price can reflect the aggregate prediction and help the firms better prepare. Guo et al (2005) explored the possibility of constructing a macro prediction market and theoretically justified the potential efficiency gain.

In addition, even if the exchange market maintains adequate liquidity it could cause firms to care less about their capital structures. For example, firms may maintain a high debt-to-equity ratio to increase the return per share. As a result, they become vulnerable to macro-level shocks, which may eventually lead to financial market crises.

3.4 Bankruptcy Cost

The limited-liability effect is an important issue in the literature (Brander & Lewis 1986; Wanzenried 2003). The limited-liability structure assumes no bankruptcy costs and may lead firms to take risks. For the simplicity of mathematical illustration, we consider it an example of the possible influence of capital structure. In practice, a bankruptcy imposes extra costs such as financial distress, inefficient liquidation, and managerial reputation loss. It is reasonable to predict that these costs may induce firms to follow a more conservative output strategy. However, the major result of our analysis stands: Debt

[9] For example, the demand peaks for electricity in the Northern and Southern United States arrive in Winter and Summer, respectively.

levels influence firms' production decisions and the influence may get stronger when the exchange market lacks liquidity.

3.5 Multi-Period Strategies.

We opened our paper with a one-period analysis to show the possible influence of exchange market liquidity. We assumed the product was short-lived and the firm's debt matured at the end of the period. In this case, firms use the exchange market to maximize end-of-period cash flow by either selling excess products or buying products to meet demand. One possible extension of this work is to look at storable goods associated with long-term debt. If the goods are storable, the firms can hold inventory for future periods, leading to less desire to trade in the exchange market. However, the firm will still want to sell if the inventory costs are high enough. In that case, our model and results hold. The firms' exchange plans are a trade-off between the benefit from holding inventory and the profit to be gained in the exchange market.

Future research should address firms with low inventory costs but differing debt-maturity dates. Firms with long-term debt may take advantage of those that have debts closer to maturity. We expect that the capital structure in an illiquid exchange market would be more influential under these circumstances.

4 CONCLUSION

We studied how firms' capital structures affect their production decisions in a business-to-business exchange market. Our example shows how the market can increase a capital structure's influence on production, even as it reduces the firm's operational risk. Firms behave strategically in an exchange market that lacks perfect liquidity. In a two-firm model, a firm's debt has positive effect on its own production decision and a negative effect on its counterpart's. Under certain circumstances, the positive effect is so powerful that the influence of capital structure becomes stronger than it would be without an exchange market.

We expect that our model can be applied to a variety of industries. One of the best prospects is the wholesale electricity market (Stoft 2002). In all de-regulated US electricity markets an individual utility serves its own demand (bilateral contracts) while simultaneously bidding in the balancing energy market. Although market designs may vary, the energy market lets firms buy or sell energy which helps them increase operational flexibility and serve the real-time load. Future researchers should look at the influence of financial decisions on a firm's production behaviors, especially for investor-owned utilities. Our paper addresses the importance of exchange market liquidity. As more risk-hedging vehicles become available, we call managerial attention to the essential relationship between capital structure and production.

Chapter 2. Forward Contracting, Supply Function Equilibrium, and Market Efficiency

1. INTRODUCTION

The non-storable characteristic of perishable goods makes unilateral withholding of production profitable. Hence, mitigation of market power is a big issue in the exchange market of perishable goods. It is widely acknowledgeable that that long-term contract arrangement serves an efficient vehicle to improve market efficiency[10]. However, the vision cannot be taken for granted. Mitigating market power in perishable goods exchange market calls for carefully studying the motivation and consequence of long-term contracting. An important question is whether the firms with perishable goods, have incentive to sign long-term contract voluntarily, rather than compulsorily.

This paper is inspired by the electricity market exchange. Over the past decade or more, most electricity markets in US have been in the process of restructuring. The goal of this restructuring emphasizes the introduction of competition into markets where competition will benefit ratepayers, the use of performance-based incentive regulation in areas that are not competitive, and the development of efficient prices based on marginal costs for regulated rates. However, the current electricity market is not always competitive. The price significantly departs from the competitive level, i.e. marginal cost, especially during peak time. The market power is common, even in the area of low market concentration. It is mostly because of the unique characteristics of electricity itself, and its demand and supply. Since electricity is a non-storable good, the producers' ability to exercise market power can not be smoothed by inventory management. The problem is even exacerbated because the demand of electricity is very inelastic and the supply is constrained by producers' available capacities.

---

[10] See Borenstein, S (2001)

A considerable amount of empirical researches provide evidence of the existence of market power in US power market. Borenstein, S., Bushnell, J. B., and Wolak, F. A. (2002) study California market during June 1998 to October 2000, and conclude that the 59% of increase in electricity expenditure was attributable to increased market power. Wolak, F. (2003) measures the five largest electricity suppliers in the California who had to exercise market power in the state's wholesale market almost in the same periods. The estimation results present the enormous increase in the amount market power exercised in the California market. Hortacsu and Puller (2005) examine producers' bidding behaviors in the balancing energy market in Texas. They show that single producer's bidding strategy can influence the distribution of market clearing price. Taking this influence into account, they show that the producers' bidding schedules significantly depart from the competitive one.

In order to alleviate the market power problem, many market designers try to modify the regulation policy to allow the forward trading. For instance, Wolak, F. (2001), Chairman of Market Surveillance Committee (MSC) of the California Independent System Operator (ISO) proposed a comprehensive market power mitigation plan, which impose that "any market participant that does not offer these two-year forward contracts would lose its market-based rate authority and be subject to cost-of-service rates for all of its sales of energy and ancillary services into the California market and surrounding markets in the Western US for at least this two-year period." (Wolak, F. 2001). Bushnell, Mansur and Saravia (2004) compare market equilibrium outcomes in California, New England, and PJM electricity market. Their estimation shows that the equilibrium price falls dramatically in New England and PJM market, when they include the forward contract between producers and retails in the objective function. Meanwhile, the price in California market keeps high because producers can not be allowed to sign forward contract with retails. However, the equilibriums become similar in all the markets after introducing this kind of vertical arrangement, especially during peak time.

In earlier literature, the motivation of long-term contracting often rests on the desire of hedging price or cost uncertainties by risk-averse firms. However, more recently, a number of studies show that strategic benefit in competition is sufficient even for risk-neutral firms to engaging long-term contracting. Allaz (1992) and Allaz and Vila(1993) are among the first to study the impact of forward contracting on oligopoly competition. They illustrates that firms can take up better position in competition through strategic forward. Willems (2005) use the similar framework as Allaz and Vila (1993) to show that the future market increases the market efficiency in Cournot competition.

In this paper, our research rules out the hedge-orientated incentive of long-term contracting. We assume that a group of risk-neutral producers interact in two consequent stages: forward contract stage and spot exchange market. Before the real time demand is realized, producers can sell forward contracts to retailers in the contract stage. In real time, the producers involve a uniform multi-unit auction in the spot market, with the structure similar to that in Wilson (1979). The producers submit supply functions, specifying the correspondent between all prices and quantities which they would like to offered ex-post.

Although Cournot game setting is useful to analyze the effect of forward contract in the literature, it is not the best structure to describe firms' supply strategies of perishable goods with high demand uncertainty in the exchange market (Baldick, Grant and Kahn, 2004). In Cournot setting, producers' strategy is to choose a fixed quantity ex-ante. However, it is not optimal for producers to commit to a simple or fixed quantity in a market with high demand risk. Instead, the supply functions (or supply schedules), which contain the quantity they want to provide at each level of price, allow producers' operation to be optimal corresponding to any realized level of market demand and guarantee the market safety by balancing demand and supply at any moment. Therefore, in general, firms are better off by employing supply function strategies instead of a fixed production level at any price.

Klemperer and Meyer (1989) are among the first to study supply function equilibria under uncertainty. They prove the existence of this equilibrium in symmetric setting. Moreover, they also show that the equilibrium is closed to the one in Cournot competition when the supply functions are very steep, and resembles the one in Bertrand competition when the supply functions are very flat. Turnbull (1983), Green and Newbery (1992) analyze the asymmetric duopoly firms' optimal supply functions with linear and quadratic marginal costs respectively. They show the linear supply function equilibrium is always in the set of solutions.

Our study reveals the strategic incentive for producers to sign forward contract with retailers. We find out that the position of the forward contract affects producers' bidding in spot market. Specifically, the more contracts a producer sells in the contract stage, the more aggressively the producer bid in the spot market in terms of quantity, and the more conservative supply strategies its opponents will apply in the spot market. Therefore, producers will voluntarily trade in the contract stage even without the hedging incentive, in order to gain strategic advantage in the competition. Most importantly, we show that the sale of forward contracts can decrease the market clearing price and mitigate the market power in electricity bidding.

Furthermore, out model also includes the influence of capacity constraints. Therefore, the study becomes more realistic, compared to the one in the literature of forward contracting before. We studies how the forward contracting is affected by producers' capacity constraints, so does the market efficient. First of all, producers with capacity constraints sell less forward contract than in the case when they have unlimited generation resources. Second, the producers with larger capacities do not sign as many forward contracts as producers with smaller ones. As a result, they hold up part of their capacities before the small size firm reaches its limitation. Hence, they take advantage of their higher generation capability to push up the market clearing price. The intuition is easily understood in the duopoly case. The producer with smaller capacity employs such equilibrium behaviors, because after it reaches its upper limit of capacity, neither its sale position in forward contract nor its bidding strategy can affect market clearing price. Consequently, it commits to higher contract position to get higher market share in the competition before it has fully load. For the producer with larger capacity, it loses interest in signing forward contracts after the smaller one have to generate with full load, because the strategic motivation does not exist. The sale of forward contract lowers the expected profit after its optimal supplies equal to its capacity constraints. Therefore, producer with larger capacity prefers to commit to as small amount of forward contracts as possible.

Finally, in this paper, we also roughly discuss the impact of the option contract in the market. Recently, the option contract seems more and more attractive in electricity market because of its flexibility and its ability to hedge quantity risk. However, we show that the option contract work not as effective as forward contract to mitigate the market power when the spot price is not very high. Since when the spot price is lower than the strike price, the retailers have no obligation to buy any product at strike price from producer. Hence, the option contract has no impact on producers' strategy, so does the market equilibrium.

The paper contributes to the literature from two aspects. First, the major finding in this paper suggest that even without policy obligation of forward contracting, producers still have incentive to sign the future/option contracts, which push down the market clearing closer to the competitive level, so as to introduce more market competition and benefit the social welfare. Second, to our knowledge, this paper is the first one that combines producers' behaviors in both forward sale and spot market bidding with their capacity constraints, which makes the model more realistic. We not only propose a stylized mathematic framework to solve such complicated equilibrium, but also explain the connection between them. It answers the important questions such as how the severe capacity constraints hurt the market competition and why larger size producers to abuse market power by holding up their generation.

The rest of the paper is organized as follows: In Section 2, we analyze producers' strategy if forward contract is allowed and producers have unlimited generation recourses. Section 3 presents the influence of the capacity constraints on producers' behaviors. In section 4, the influence of the option contract is studied. Section 5 includes the discussion and the limitations of our model. We summarize and conclude in section 6.

2. MODEL WITH NO CAPACITY CONSTRAINT

2.1 Model Set-up

Consider N risk-neutral producers, who produce a homogenous perishable good. Their production technology is characterized by a cost function as $c_i(q)\ i=1,..N$, which is the total cost to produce q units of electricity. We assume the cost function is in a quadratic form i.e., $c_i\left(q_i\right)=c_i q_i^2+d_i q_i$. The cost function implies that there is no fixed cost, since $c_i(0)=0$ [11]. Each producer's cost function is public information shared by all producers in the market[12]. Moreover, each producer has unlimited generation capacity, i.e., $\overline{q}_i \to \infty\ \forall\ i=1,2,...,N$.

The real-time demand is represents by the aggregate demand by all retailers in the market, $\tilde{Q}(p,\varepsilon)=\varepsilon-\delta p$. p represents the price and δ characterizes the demand elasticity[13]. The market uncertainty is ε, which is a random drawn from a distribution function, $F(\varepsilon)$ with support $[\underline{\varepsilon},\overline{\varepsilon}]$. We assume that ε, δ and p are positive

---

[11] In electricity production, $c_i(0)\geq 0$ because there are a lot of sunk cost such as warm-up fuel cost.

[12] See Hortacsu and Puller (2005). Their conversation with several market participants in ERCOT suggest that traders have good information on rival's marginal cost because they use similar technology in Texas and public data on fuel efficiency on each generation unit is also available. Moreover, some market participants also purchase large rival plant's real-time production data through a company, Genscape.

[13] We will discuss the assumption of individual retailers and its behavior in the market in the section of discussion later.

Before the real-time demand is realized, producers compete in two stages: contract stage and sport market, illustrated in figure 1.

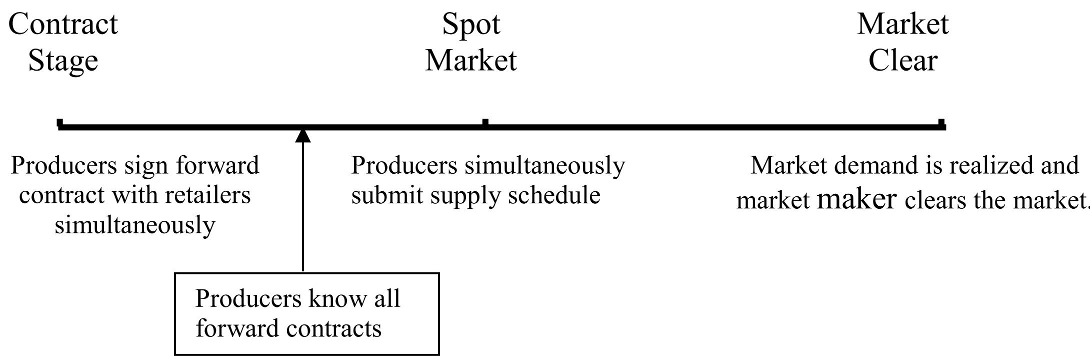


Figure 1: Timeline

In contract stage, we assume that all producers and their counterpart, retailers, simultaneously negotiate for a bilateral contract. producer *i* signs forward contract with retailers, in which producer *i* promises to deliver x_i units of electricity to retailers after the demand is realized. The forward price is f_i [14]. After signing the forward contract, retailers have obligation to buy x_i units of electricity no matter what spot price is. If the spot market clearing price p_{mc} is above f_i, producer *i* will refund $(p_{mc} - f_i)$ to retailers; otherwise, the retail pays $(f_i - p_{mc})$ to producer *i*. After they sign the contract, the forward contract quantity, x_i, is observable to all producers[15]. We assume that there is perfect arbitrage between the contracting stage and the spot market. Therefore, producers will not sign contract for the purpose of exploiting arbitrage profit.

Next, at the real-time spot market, producers involve a uniform price multi-unit auction. There is a market maker, whose only objective is to clear the market. The market maker does not make any profit through market trading. Producer *i* provides its supply

[14] We restrict the forward contract is the fixed order. Otherwise, the equilibrium analysis will be different.
[15] The observability of the forward contract is a key assumption here. We will discuss this assumption in part III.

schedule $q_i(p)$ $i=1,..N$ to the market maker in the spot exchange market, where $q_i^{'}(p)\geq 0$ and $q_i^{"}(p)\leq 0$. The supply schedule, $q_i(p)$, specifies the quantities producer *i* provided at any price level. Thus, producers provide the whole supply curve instead of a fixed quantity.

In the end of the game, the market maker knows the real-time demand and set the market clearing price, p_{mc} to balance the aggregate demand and supply. Each producer produces the quantity $q_i(p_{mc})$ according to market maker's order. In this paper, we are only interested in pure strategy equilibria of supply functions.

2.2 Spot Market

In order to find the equilibrium in spot market, we follow the same procedure as in Klemperere and Meyer (1989). In the spot market, producer *i*'s strategy is a function which maps a price into a level of output, $q^i:[0,\infty)\rightarrow(-\infty,+\infty)$. After real-time market demand is realized, market clearing price p_{mc}, is the one to balance demand and supply, decided by

$$\sum_i q_i\left(p_{mc}(\varepsilon)\right)=\tilde{Q}\left(p_{mc}(\varepsilon)\right) \quad (2.1),$$

and producer *i* is deployed $q_i\left(p_{mc}(\varepsilon)\right)$ units of electricity by market maker.

For every realization of market shock ε , producer *i*'s optimal strategy is to maximize its ex-post profit, as its residual demand is the difference between market demand and the sum of other producers' potential supply, $RD_i(\varepsilon)=\tilde{Q}(p,\varepsilon)-\sum_{j\neq i} q_i(p)$.Therefore, given the forward contract positions for all producers, the equilibria consists of a group of functions $q_i(p)$ such that $q_i(p)$ maximize *i*'s expected profits:

$$\max_{p}\ \mathrm{E}\pi_i = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left\{ \left[\varepsilon - \delta p - \sum_{j \neq i} q_j(p) - x_i \right] p + f_i * x_i - c_i\left(\varepsilon - \delta p - \sum_{j \neq i} q_j(p) \right) \right\} f(\varepsilon)\, d\varepsilon$$

$$\forall i = 1, 2 \ldots N \qquad (2.2)$$

Producer *i's* expected profit is equal to revenue in the spot market, minus production costs and payment transfer of forward contracts.

Given $q_j(p)$, producer *i* fins its profit-maximizing price along its residual demand curve and yield the first order condition

$$\varepsilon - \delta p - \sum_{j \neq i} q_j(p) - x_i + \left[p - c_i'\left(\varepsilon - \delta p - \sum_{j \neq i} q_j(p) \right) \right] \left(-\delta - \sum_{j \neq i} q_j'(p) \right) = 0$$

(2.3)

In order to illustrate the characteristics of producer *i*'s optimal strategy, we can rewrite (2.3) as

$$\frac{p - c_i'(q_i(p))}{p} = \frac{\varepsilon - \delta p - \sum_{j \neq i} q_j(p) - x_i}{\left[\delta + \sum_{j \neq i} q_j'(p) \right] p} = -\frac{RD_i(p) - x_i}{RD_i'(p)p} \quad (2.5)$$

Equation (2.5) can be seen as a makeup expression. The right hand side term of (2.5) is similar to the inversed price elasticity of the residual demand curve producer *i* faces. Therefore, the less the elasticity, the more market power producer *i* has, because the markup in price above marginal cost is larger when producer *i* faces a steeper residual demand curve. As an intuitive consequence, if $\delta + \sum_{j \neq i} q_j'(p) \to \infty$, the residual demand curve is perfectly elastic, then the market clearing price is equal to the marginal cost so that producer *i* has no market power; if $\delta + \sum_{j \neq i} q_j'(p) \to 0$, the results reverses, which most likely occurs in peak time when power demand is very high and power generation resource is tight.

In spot market, the set of first condition of (2.5) for each producer, when written as a system of equations, characterizes equilibrium strategies, $q_i(p)$. However, the computation of equilibrium strategies is not a trivial task. Thus, we now continue to develop our analysis by restricting the functional form of the supply function to a certain kind, in which quantity supply by producers is linear to the price[16].

- The linear supply function equilibrium.

We can show that there exists a group of linear supply schedule satisfies (2.5). Let $q_i(p) = a_i + b_i p$ (2.6), $c_i(q_i) = c_i q_i^2 + d_i q_i$ (2.7), with $b_i, c_i, d_i \geq 0$. Substituting (2.6) and (2.7) into (2.5), we obtains

$$a_i = \frac{x_i - \left(\delta + \sum_{j \neq i} b_j\right) d_i}{1 + 2c_i\left(\delta + \sum_{j \neq i} b_j\right)} \quad (2.8) \qquad b_i = \frac{\left(\delta + \sum_{j \neq i} b_j\right)}{1 + 2c_i\left(\delta + \sum_{j \neq i} b_j\right)} \quad (2.9)$$ [17]

From equation (2.8) and (2.9), we can concludes that

Lemma 1. $\frac{\partial a_i}{\partial x_i}>0$ and $\frac{\partial b_i}{\partial x_i}=0$

Lemma 1 shows that, in linear supply function equilibrium, the position of the forward contract of a producer only affect the intercept of the supply function, but not the slope. Hence, the forward contract position shifts a producers' supply function parallel to the right. It implies that the more forward contract sold by a producer, the more electricity it is willing to provide for the same price level.

Moreover, given any market shock ε and the size of the forward contract, the market clearing price is

---

16 Turnbull (1983), Green and Newbery (1992) show the linear supply function equilibrium is always in the set of solutions.

17 It is easy to show the existence and uniqueness of the solution by substituting (2.6) and (2.7) into (2.4). Therefore, we will not go into details about his.

$$p_{mc}(\varepsilon) = \frac{\varepsilon - \sum_i a_i}{\left[\delta + \sum_i (b_i)\right]} \qquad (2.10)$$

By doing comparative static analysis, the effect of forward contract on the equilibrium in spot market can be characterized in following two propositions.

Lemma 2[18]. $\frac{\partial p}{\partial x_i} < 0 \ \forall i = 1, 2, ..., N$.

Lemma 3. $\frac{\partial q_i}{\partial x_i} > 0$ and $\frac{\partial q_i}{\partial x_j} < 0 \ \forall i \neq j$

First, Lemma 2 proves that the existence of forward contracting can mitigate the market power, since each producer's position of forward contract has negative effect on market clearing price. For a given market shock, ε, producer *i* faces a residual demand curve $RD_i(\varepsilon) = \tilde{Q}(p,\varepsilon) - \sum_{j \neq i} q_i(p)$, and acts as a monopolist. To maximize its profit, producer *i* choose the price, p_1 at the production level where its marginal revenue and marginal cost equals as in Figure 2. Figure 3 illustrates how forward contract affects the strategy chosen by a producer. As shown in Figure 3, after producer *i* signs forward contracts, x_i, producer *i*'s residual demand changes to the one in which x_i units of product should be taken out at every price level. Then the market clearing price becomes p_2, which is lower than p_1.

[18] All proofs are listed in the Appendix.

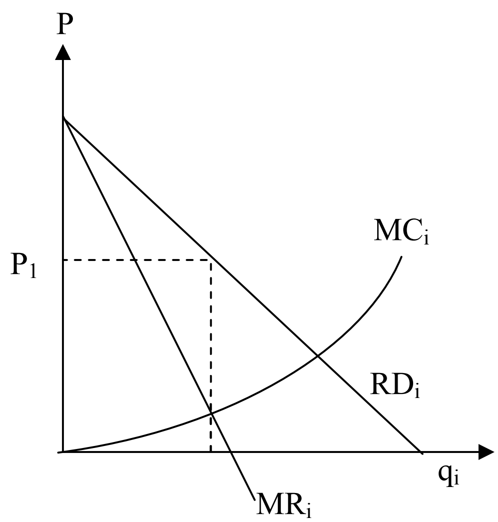


Figure 2

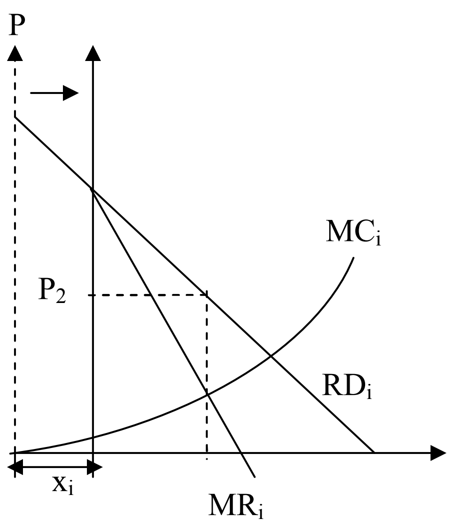


Figure 3

Next, Lemma 3 has very important implication of the strategic interaction among producers. It shows that a producer' positive unit of forward contract with retailers has positive effect on its supply in spot market. When a producer promises to supply more units of electricity in a forward contract, it commits a higher production level even before the bidding occurs. It, hence, will bid more aggressively in terms of quantity in spot market. As a result, it pushes down the market clearing price. Moreover, a producer's position of forward contract has negative effect on all other producers supply plans in spot market, because other producer expects a lower market clearing price after observing a producer's commitment in contracting part. The strategic effect of forward contract is very similar to the effect of forward contract in Cournot competition[19].

2.3 Contract stage

In this stage, all producers choose their positions of forward contracts, $x_i \;\; \forall i = 1, 2, ..., N$ to maximize their expected profits, by knowing the strategic effect of their forward contract on spot market equilibrium,

Since we have assumed that there is no arbitrage between spot market and contracting stage, the hedging incentive for forward contract can be ignored. Thus, the forward contract price will equal to the expected market clearing price, i.e. $f_i = E_i p$. The rationale is that if $f_i > E_i p$, retailers do not want to sign the contract and delay their purchase until spot market opens; however, if $f_i < E_i p$, producers lose money by signing forward contract and would like to sell all products through spot market bidding.

Each producer chooses its position of forward contract so as to maximize its expected profit, which can be formulated as follows:

$$\max_{x_i} \; \mathrm{E}_i \pi = \int_{\varepsilon} \left\{ \left[q_i(p(\vec{x},\varepsilon), x_i) - x_i \right] p(\vec{x},\varepsilon) + f_i * x_i - c_i\left(q_i(p(\vec{x},\varepsilon), x_i)\right) \right\} f(\varepsilon)\, d\varepsilon$$

$$s.t. \; f_i = E_i p$$

(2.11)

[19] See Willems (2005).

After substitute the perfect arbitrage condition, we can see that producer *i*'s expected profit depend indirectly on the sale of forward contract:

$$\max_{x_i}\ \mathrm{E}_i\pi = \int_{\varepsilon} \left\{ q_i(p(\vec{x},\varepsilon),x_i)p(\vec{x},\varepsilon) - c_i\left(q_i(p(\vec{x},\varepsilon),x_i)\right)\right\} f(\varepsilon)\,d\varepsilon \qquad (2.12)$$

Let us take the first derivative of (2.12) with respect to producers' contract sales, the first derivative becomes

$$\int_{\varepsilon} \left\{ \underbrace{\left(q_i - \delta\left(p - c_i'(q_i)\right)\right)\frac{\partial p}{\partial x_i}}_{\text{Part I}} \underbrace{-\left(p - c_i'(q_i)\right)\sum_{j\neq i}\frac{\partial q_j}{\partial x_i}}_{\text{Part II}} \right\} f(\varepsilon)\,d\varepsilon \qquad (2.13)$$

(2.13) contains two parts. In part II of (2.13), $\sum_{j\neq i}\frac{\partial q_j}{\partial x_i}$ is the effect of producer *i*'s forward contract on all other producers, which is negative according to proposition 2. It is easy to show that if a producer's forward contract has no effect on spot market bidding at all, then (2.13) is negative, which follows the proposition 1. Under such circumstance, the optimal strategy for producer *i* is not to commit any position in contract stage. In another word, in absence of the strategic effect for forward selling, producers have no incentive to sign any contract, given the set-up of the game. In (2.13), part I represents producer *i*'s benefit from forward contract that increases its market share, and part II represents producer *i*'s loss from the contract that decreases the market clearing price. The optimal sale of forward contract is the one to balance those two effects.

3. FORWARD CONTRACT WITH CAPACITY CONSTRAINTS

In the previous section, we derive the supply function equilibrium without considering about the capacity constraint. However, in practice, every power provider has such constraint, because of the limited capacity of their producers, or the real-time availability of the generation resource. Therefore, it is very important to know how the capacity limitation affects each firm's strategies and market equilibrium.

To illustrate the effect of capacity contracts, it is very helpful to begin with the example of two-producer case ($\overline{q}_1 \le \overline{q}_2$). Moreover, we also assume that producers use same production technology, i.e. $c_i(q) = cq^2 + dq$ to simplify the computation.

3.1 Spot market

Since they cannot bid beyond their capacity constraints, producers' bidding curves are discontinuous. First, when neither producer reaches its capacity limitation, producers' bidding strategies are the same as the ones without taking into account the presence of the constraints, which is denoted as $q_i^1(p) = a_i^1 + b_i^1 p \ \forall i = 1,2$ [20]from now on. $\{a_i^1, b_i^1\} \ \forall i = 1,2$ is decided by (2.8) and (2.9) respectively . Second, producer 1's supply schedule will be $q_1^2(p) = \overline{q}_1$ after it reaches its capacity cap. Therefore, the residual curve faces by producer 2 is $RD_2 = \tilde{Q}(p,\varepsilon) - \overline{q}_1$. Obviously, the market clearing price is only affected by producer 2 after producer 1 use up all the generation resource. Then before producer 2 reaches its own upper limit of the capacity, the optimal strategy $q_2^2(p) = a_2^2 + b_2^2 p$ is chosen to maximize its profit corresponding to each realization of market shock

$$\max_p \ \pi_i = [\varepsilon - \delta p - \overline{q}_1 - x_2] p + f_2 * x_2 - c_i(\varepsilon - \delta p - \overline{q}_1). \qquad (3.1)$$

Last, after producer 2's capacity also reaches its upper limit, its bidding strategy is $q_i^3(p) = \overline{q}_i \ \forall i = 1,2$.

From the equilibrium supply functions, the price, $\overline{p}_i \ \forall i = 1,2$, at which producer i reaches its capacity limit can then be obtained. Therefore, when producers' bidding is restricted by the upper limit of their capacities, we have

Proposition 3: The spot market equilibrium is

Producer 1

$$q_1(p)=\begin{cases} a_1^1+b_1^1 p & \text{for } p\le \overline{p}_1 \\ \overline{q}_1 & \text{for } p>\overline{p}_1 \end{cases} \qquad (3.2)$$

Producer 2

$$q_2(p)=\begin{cases} a_2^1+b_2^1 p & \text{for } p\le \overline{p}_1 \\ a_2^2+b_2^2 p & \text{for } \overline{p}_1 < p\le \overline{p}_2 \\ \overline{q}_2 & \text{for } p>\overline{p}_2 \end{cases} \qquad (3.3)$$

where a_i^1 and b_i^1 is decide by equation (2.8) and (2.9) respectively,

$a_2^2=\dfrac{x_2-\delta d}{1+2c\delta}$ (3.4), $b_2^2=\dfrac{\delta}{1+2c\delta}$ (3.5), $\overline{p}_1=\dfrac{\varepsilon\text{-}\overline{q}_1-a_2^1}{\delta+b_2^1}$ (3.6) and $\overline{p}_2=\dfrac{\varepsilon\text{-}(\overline{q}_1+\overline{q}_2)}{\delta}$ (3.7)

There are three observations from proposition 3. First, equation (3.4) and (3.5) show that the effect of the forward contract is the same as in Lemma 1. Second, after both producers reach their capacity caps, their forward contracts have nothing to do with their bidding in the next stage. Last, as a consequent, the forward contract does not affect the market clearing price when a producer has already used up its generation resource.

3.2 Contracting stage

In spot market, both producers have kinked supply schedules in the spot market. From equation (3.6) and (3.7) in proposition 3, we know that the cutoff points of prices, $\overline{p}_1$ and $\overline{p}_2$ correspond to the realization of the market shock $\overline{\varepsilon}_1$ and $\overline{\varepsilon}_2$ respectively, where $q_1^1(\overline{p}_1(\overline{\varepsilon}_1))=\overline{q}_1$ and $q_2^2(\overline{p}_2(\overline{\varepsilon}_2))=\overline{q}_2$.

As the same rationale in the case without capacity constraint, the assumption of no arbitrage implies that the forward contract price equals to the unbiased estimator of the market clearing price in the spot market. Each producer chooses the level of its future

---

[20] q_i^k : i=1,2 is the index of the generators; k represents the part of the supply schedule.

sale, $\left(x_i^c \;\; \forall i=1,2\right)$, so as to maximize his expected profit, which can be formulated as follows:

$$\max_{x_i} \; \mathrm{E}\pi_i = \int_{\varepsilon_0}^{\bar{\varepsilon}_1} \left\{ \left[q_i^1\left(p_1(\varepsilon), x_i^c\right) - x_i^c \right] p_1(\varepsilon) + f_i * x_i^c - c\left(q_i^1\left(p_1(\varepsilon), x_i^c\right)\right) \right\} f(\varepsilon) d\varepsilon$$
$$+ \int_{\bar{\varepsilon}_1}^{\bar{\varepsilon}_2} \left\{ \left[q_i^2\left(p_2(\varepsilon), x_i^c\right) - x_i^c \right] p_2(\varepsilon) + f_i * x_i^c - c\left(q_i^2\left(p_2(\varepsilon), x_i^c\right)\right) \right\} f(\varepsilon) d\varepsilon$$
$$+ \int_{\bar{\varepsilon}_2}^{\bar{\varepsilon}} \left\{ \left[q_i^3\left(p_3(\varepsilon), x_i^c\right) - x_i^c \right] p_3(\varepsilon) + f_i * x_i^c - c\left(q_i^3\left(p_2(\varepsilon), x_i^c\right)\right) \right\} f(\varepsilon) d\varepsilon$$
$$s.t \;\; f_i = E_i(P) \qquad \forall i = 1,2$$

(3.8)

3.3 The influence of capacity constraint on forward contract

Proposition 3 illustrates that producers' capacity constraints change the shape of their bidding curve in spot market. In this section, we focus on the effect of the capacity constraint on producers' position of forward contract in this section.

3.3.1 Symmetric case

First of all, let us consider the case with $\bar{q}_1 = \bar{q}_2 = \bar{q} < \infty$. Let x_{un}^* and x_c^* denote the optimal contract when producer with and without capacity constraints respectively. Proposition 4 indicates that when a producer's production has constraint, its sale of forward contract is lower, compared to the case in which it has an unlimited level of production.

Proposition 4. $x_{un} < x_c \;\; \forall i = 1,2$

It seems very intuitive that producers will decrease their forward commitment and also production because they have limited availability of generation resource. We would like to further illustrate that producers sells less forward contracts for strategic reasons.

After simplifying (3.8), producer' expected profit consists two parts.

$$\max_{x} \ \mathrm{E}\pi = \underbrace{\int_{\varepsilon_0}^{\bar{\varepsilon}_1} \left\{ q^1\left(p_1(\varepsilon), x^c\right) p_1(\varepsilon) - c\left(q^1\left(p_1(\varepsilon), x^c\right)\right) \right\} f(\varepsilon) d\varepsilon}_{\text{Part I}} + \underbrace{\int_{\bar{\varepsilon}_1}^{\bar{\varepsilon}} \left\{ \bar{q} p_2(\varepsilon) - c\left(\bar{q}\right) \right\} f(\varepsilon) d\varepsilon}_{\text{Part II}}$$

(3.9)

Part I of (3.9) is the expected profit when the producer's capacity constraint is not binding, which takes the same form as the one when the producer has no capacity constraint. However, part II of (3.9) is the one with binding constraint. It is easy to show that the effect of the forward contract on part II is negative. As we mentioned in section II, the producers sign forward contracts only because their contracts have strategic effect in spot market and they want to get competitive advantage through contracting. If this motivation disappears, the producers will lose money by selling forward contract. After a producer achieves its upper limit of production, neither its position of forward contract nor its bidding strategy affects market clearing price and has strategic effect on the producer's behavior. As a result, in order to lower the loss after its capacity limitation is achieved, a producer has to decrease its forward contract sale.

3.4.2 Asymmetric case

In this section, we assume that $\bar{q}_1 < \bar{q}_2$ The next proposition shows the difference in optimal contract choices because of producers' asymmetric capacity constraints.

Proposition 5. If $\bar{q}_1 < \bar{q}_2$, then $x_2^c \leq x_1^c$.

Proposition 5 shows that producers with larger capacity choose smaller size of forward contract. In order to prove, we compare the optimal contract choices by producers in the case $\bar{q}_1 = \bar{q}_2 = \bar{q}$ and $\bar{q}_1 = \bar{q} < \bar{q}_2$ and find out that their decisions in contract sale is $x_2^c \leq x^c \leq x_1^c$. For producer 2, we can divide its expected profit into three parts:

$$\max_{x_1} \ \mathrm{E}\pi_1 = \underbrace{\int_{\varepsilon_0}^{\hat{\varepsilon}_1} \left\{ q_2^1\left(p_1(\varepsilon), x_2^c\right) p_1(\varepsilon) - c_2\left(q_2^1\left(p_1(\varepsilon), x_2^c\right)\right) \right\} f(\varepsilon) d\varepsilon}_{\text{Part I}} + \underbrace{\int_{\bar{\varepsilon}_2}^{\bar{\varepsilon}} \left\{ \bar{q} p_3(\varepsilon) - c_3\left(\bar{q}_3\right) \right\} f(\varepsilon) d\varepsilon}_{\text{Part II}}$$

$$+ \underbrace{\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_2} \left\{ q_2^2\left(p_2(\varepsilon), x_2^c\right) p_1(\varepsilon) - c_2\left(q_2^1\left(p_2(\varepsilon), x_2^c\right)\right) \right\} f(\varepsilon) d\varepsilon}_{\text{Part III}} \qquad (3.10)$$

In part II and part III of (3.10), producer 2 is not reaching its limitation yet, while producer 1 produces at highest level it can. From producer 2's perspective, it knows that producer 1 will achieve its upper limit first. After that, producer 2 will be a price setter if producer 1 alone cannot cover the market demand, since producer 1's bidding just parallel shift the residual demand curve faced by producer 2 in the market and cannot affect the market clearing price. Producer 2 behaves as a monopolist in the market. Compared to symmetric case $\left(\overline{q}_1 = \overline{q}_2 = \overline{q}\right)$, producer 2 has even less incentive to forward contracts. As producer 2 backs off its position in forward contract sale, producer 1 is better off by selling a little bit more forward contracts than in the symmetric case, so that the market clearing price does not change and producer 1 gets more market share when it has not reach its capacity limit.

3.4.3 Market with N>2 producers

Until this point, we show that the market equilibrium when there are only two producers. If there are more than two producers in the market, the analysis is similar as in two-producer case. We assume that each producer has capacity constraint $\overline{q}_i$ $\forall$i=1,2,...,N , where $\overline{q}_i < \overline{q}_j$ when $i < j$.

In the spot market, there are N>2 cut-off points $\overline{p}_i$ $\forall$i=1,2,...,N which correspondents the capacity constraints $\overline{q}_i$ $\forall$i=1,2,...,N . Up to ith cut-off point $\overline{p}_i$, where the ith producer reaches its upper limit of capacity, its bidding schedule is derived as the same way as there is no capacity constraint. After $\overline{p}_i$, the supply schedules of ith producer and producers who has smaller capacity are $\overline{q}_i$, and consequently, only $N - i$ producers in the market affect the market clearing price. The optimal sale of forward contract is decided the same as in two-producer case, but with more parts because there are N>2 cut-off points . Although we will not go through the details here, we expect that influence of the capacity constraint and market equilibrium is very similar to the ones described in proposition 4 and 5 in the previous sections[21].

4. THE EFFECT OF CALL OPTION CONTRACT ON SPOT MARKET

In US electricity market, option contract is not used as much as forward contract, because of its complexity and relatively higher cost. However, this kind of forward contract calls more and more interest because it has advantage in hedging quantity risk. Since electricity is very expensive to stored, the important issue in risk management is to hedge quantity risk.

After signing the forward contract, producers and retailers are obliged to have payment transfer on a fixed level of power, no matter what the real-tine demand is. For instance, consider the case when market demand is very low so that it is very possible that the market clearing price is lower than the contract one. The retailers who sign the contract will pay producer the difference between the spot and contract prices even when they need not as much power as specified in the contract.

However, in option contract, the quantity of power delivered from producers to retailers depends on the realization of spot price. Specifically, the quantity delivered increases in spot price. In this section, we assume that only financial call option is allowed[22]. At contracting stage, producers can sell call option contract, x_i to retailers at price k_i, where strike price s_i is determined exogenously. If spot price is higher than strike price, then producers return $p - s_i$ to retailer. Otherwise, there is no transfer payment between them. Therefore, the option contract is one-side insurance for retailers to against price rising above strike price, and it is more efficient to hedge quantity risk. The payment of the option is written as:

---

[21] Authors will provide other simulation results by request.

[22] Willems (2005) discusses the difference of financial option contract and physical contract in the electricity market.

$$v_i(p) = \max\{p - s_i, 0\} \quad (4.1)$$

Now, we would like to see how the option contract selling affects producers behavior and market equilibrium. In the spot market, producers still provide their supply schedules without knowledge of the realization of ε .

Giving the position of call option, producers choose their supply functions to maximize their profit corresponding to each realization of market shock as it behaves with forward contract:

$$\underset{p}{Max}\,\pi_i = q_i\left(p_{mc}\right)p_{mc} + \left[k_i - v_i(p_{mc})\right]*x_i - c_i\left(q_i\left(p_{mc}\right)\right) \qquad (4.2)$$

When $p > s_i$, it is easy to show that (4.2) is the same as the one in the model of forward contract. When $p \le s_i$, producers' behavior is independent of the optional contract. Hence, the call option affects the producers' behavior only if the spot price is above the strike price. Proposition 6 describes the spot market equilibrium with call option contract.

Proposition 6: With call option contract, when $p > s_i$, the optimal linear supply function is defined as $q_i = \alpha_i^1 + \beta_i^1 p \ \forall i = 1,2,...,N$, where $\{\alpha_i^1, \beta_i^1\}$ equals $\{a_i, b_i\}$ defined by equation (2.8) and (2.9) respectively. When $p \le s_i$, the optimal linear supply function is $q_i = \alpha_i^2 + \beta_i^2 p \ \forall i = 1,2,...,N$, where

$$\alpha_i^2 = \frac{-\left(\delta + \sum_{j\neq i} b_j\right)d_i}{1 + 2c_i\left(\delta + \sum_{j\neq i} b_j\right)} (4.3) \qquad \text{and} \qquad \beta_i^2 = \beta_i^1 \quad (4.4)$$

Proposition 6 shows that in the spot market, if the market clearing price is above strike price, the bidding strategies for producers are the same as if they sign the forward contracts. The option contract will have the same impact on market equilibrium as described in proposition 1 and 2. However, when the spot price is lower, the producers act like with no contract at all. The conclusions above imply that the option contract can mitigate the market power only when the market clearing price rises beyond the strike price. from this point of view, forward contracts are more efficient than option contract.

5. DISCUSSION

3.1 Observation of the forward contract

In this paper, a key assumption is that producers can observe the forward contract position before spot market opens. Hughes and Kao (1997) discuss the impact of the observability of forward contract. Their model is similar to Allaz (1992) in which spot market stages are Cournot competition. They find that the strategic incentive disappears for risk-neutral producers if forward positions are unobservable. In fact, each producer strictly prefers not to engage in forward contracting. Here, we expect that the conclusion remains the same as in Hughes and Kao (1997) if the contract is not observable.

In most of the US electricity markets, they do not have the mechanism to reveal contract information to all generators in real time. However, they do provide historical information several periods after real-time trading[23]. Moreover, Hortacsu and Puller (2005) propose a method to estimate producers forward contract position. Therefore, the producers can have at least consistent predictors for forward contract either from historical statistics or the estimator from empirical model. The importance of observation of forward contract remains to market designers' consideration about the information releasing issue in market restruction design in the future.

3.1 The forward contract exchange

In this paper, we focus on the producers who produce and sell a particular perishable good, electricity power. Instead of modeling each individual retailer's demand, we simplify the demand side by aggregating the total demand from all retailers. Now, we can have a further look on how each individual retailer behaves in the exchange market, given its forward contract in T=1.

We assume there are K retailers. In the real-time, the retailers' obligation is to meet their customer's demand with very low price elasticity. Hence, the retailers have low ability to grab the market power, compared to the producers. Therefore, we assume that the retailers are price taker in the market. For retailer *k*, its real-time demand denotes as d_k and its contract is y_k, where $\sum_{k=1}^{K} d_k = Q(p)$ and $\sum_{k=1}^{K} y_k = \sum_{i=1}^{N} x_i$. d_k may higher or lower than its contract units, y_k. Hence, in the equilibrium the retailer *k*'s net payment is

$$R_k = p_{mc}\left(d_k - y_k\right) + E\left(p_{mc}\right) y_k$$

R_k consists of two parts. $E\left(p_{mc}\right) y_k$ is retailer *k*'s contract payment. $p_{mc}\left(d_k - y_k\right)$ is net payment in the exchange market. It is positive when retailer *k* has shortage in real-time with its contract quantity, or is negative when retailer *k*'s buys too many contract in contracting stage. From this point of view, the spot exchange market serves as a market-place where retailers exchange their forward contract in real-time to balance their inventory. In another word, the retailers with extra products $\left(\text{i.e. } y_k > d_k\right)$ sell their contracts to the retailers with shortage $\left(\text{i.e. } y_k < d_k\right)$ by market clearing price p_{mc}. The market mechanism is equivalent to a spot forward contract market, which is an efficient vehicle of inventory management for agents with extremely perishable products.

6. CONCLUSION

We studied the essential relationship between producers' forward contract and the bidding strategies in spot market. Our model reveals the strategic incentive for producers to join in forward contract market voluntarily. It shows the presence of forward contract

[23] In Texas, ERCOT publishes all the market information, including each generator's bidding schedule, market clearing price and forward contract six months after the operating day.

can make producers produce more aggressively than without it. Therefore, the sale of forward contract serves as an effective mechanism to mitigate market power. Moreover, although the option contract is better in hedging quantity risk than forward contract, the latter work more efficient to deter the abusing of market power when spot price is not very high. Although market designs may vary, our model addresses a general theoretical framework to explain producers' behaviors in the electricity market. The paper is especially important in providing the market mechanism guideline in introducing more competition.

Chapter 3. Credit-default swap and B2B Exchange Market liquidity

1. INTRODUCTION

This paper proposes a contract-theoretical framework, which integrates two risk management mechanism, the spot market exchange and credit-default swap, into firm's portfolio, when they have demand uncertainties and face a not fully-liquid market..

In an exchange market with limited number of firms, there are two kinds of liquidity risks. The first one is a focus in previous risk management and finance literature, which is how easy firms can trade in the exchange market, when the realization of uncertainty turns out to be different for each firm. The market liquidity can be measured by the degree that market clearing price is affect by a single firm's supply or demand (O'Hara 1995; Kyle 1989).

The second risk is the market failure, which can be considered as an extreme case of market illiquidity. In Lin, Fang and Whinston (2006), they show that even firms with idiosyncratic shocks, the market failure could happen. There is always a chance that the realization of all firms' uncertainty turns out to be the same, i.e. all generators simultaneously face the very high power demand in summer in Texas electricity market. As a result, the exchange market can no longer be a vehicle to help firms hedge their risks, and the aggregate risks appear as a "macro"-like shock. It is significantly possible when the market has limited number of firms. However, the literature to deal with idiosyncratic risk management often ignores the potential damages caused by this kind of market failure, and chooses to trespass it. Some traditional economic or financial researches, like Persaud (2003), study the market failure by simply assuming a perfectly correlated risk faced by all firms in ex-post market. Others assume that there are sufficient numbers of firms with idiosyncratic risk, so that they can apply the law of large numbers to diminish the possibility that ex-post risk appears to be positively correlated. (Lee and Whang, 2002).

However, this kind of market failure is not trivia, especially when firms rely too much on the exchange market to hedge their operational risk. Firms could run into serious liquidation problem because the exchange market is the only hedge platform for them so that firms become vulnerable to this macro-like shock. Moreover, the possibility of such market failure could be large especially when we deviate from the conventional assumption of fat tail distribution of demand risk and consider extreme events as ancillaries (Taleb 2001). Therefore, firms should realize the potential danger of market failure in the risk management decision, even when firms' risks are independent.

Lin, Fang and Whinston (2006) have showed that the exchange market liquidity determines how effectively a B2B exchange market serves as a platform to hedge idiosyncratic operational risk. They argue that the exchange market is fully liquid only with a sufficient number of market participants, so that firms can reduce their operational risks to the largest extent and be the least restricted by their capital structures. However, with only limited number of firms, the market lacks fully liquidity, which causes strategic interaction among firms and strengthens the effect of their capital structures on their production decisions.

In this paper, we illustrate, how a particular financial instrument, credit-default swap, to help firms to temporarily transfer default risks caused by the demand uncertainty and market illiquidity risk to outside investors. In particularly, the financial instrument is a contract between firms and outside investors, in which firms deliver a certain amount of fee to investors in exchange of an insurance offered by investors when firms cannot pay back their loans. As a result, firms can decrease the likelihood of bankruptcy by purchasing such credit derivative. We have a three-period model. In the first period, firms decide how much to invest in production and the optimal amount of credit derivative purchased from a competitive credit-default swap market. In the second period, firms' local demand is realized. Meanwhile, the spot exchange market opens, in which firms with extra products can trade with firms with shortage under a market clearing price.

Finally, if firms have enough profit to cover loan payment, it earns a positive profit in last period; otherwise, they either bankrupt if they did not sign credit-default swap contract, or they get insurance from investors according to the contract signed.

The credit-default swaps are currently used more and more frequently in banking systems, but still a new phenomenon in firms' operational risk management. We will illustrate in more details about the current credit-default swaps market in section 2. This paper proposes this stylized mechanism, which is mostly used in banking system, for firms to share the part of the default risk, which cannot be hedged by the spot exchange market. To our knowledge, it is the first one to study the effect of credit-default swap in the firms' risk management. There several major findings in our paper. First, firms purchase the credit-default swap only when the default is costly. The decision of whether to join the credit-default market is the tradeoff between the bankruptcy cost and the payment for purchasing the credit derivative, if investors in the credit-default swap market are competitive and risk neutral. Secondly, the price for the credit-default swap decreases in the liquidity of the spot exchange market, since firms' default risk decrease. Thirdly, the information availability plays a very important role in credit-swap market. Specifically, when the firms and outsider investors have the exactly same information, then every firm is better off by purchasing the credit-default swap. However, if the firms have private information, "lemon" problem occurs in the credit-default swap market. Under certain condition, firms with lower possibility to default have higher incentive to purchase the credit derivative than the ones with higher possibility to default. Hence, there is potential social cost caused by the costly early default.

The rest of the paper is organized as follows: the next section describes the features of the credit-default swap and reviews the related literature. Section 3 presents the models with and without the credit-default swap market. Section 4 extends the model to consider the correlated interest rate and some other risk-management mechanism. The final section concludes and summarizes.

2. CREDIT-CREDIT SWAP

In financial market, the credit-default swap is an over-the-counter financial contract with payoff contingents on the certain kind credit default condition. International Swaps and Derivatives Association has begun to produce standard document in the trading in credit default in 1991. Although the credit derivatives market remains quite small currently, it grows very rapidly[24].

The Credit Default Swap (CDS) is "a bilateral financial contract in which one counterparty (the Protection Buyer) pays a periodic fee, typically expressed in basis points per annum, paid on the notional amount, in return for a contingent payment by the protection seller following a credit event with respect to a reference entity," where a credit event is the relevant condition, negotiated between counterparties at the inception of the transaction to trigger the contingent payment.[25]

In practice, the credit-default swap is most used by banks to hedge their credit risks, when they provide loan to borrowers. Several studies (see, for example, Pennacchi (1995)) argue that banks are willing to involve in loan sales or credit derivative because of their desire to economize on their regulatory capital. In order to hedge loss caused by the borrower's default, banks sells credit risk (or buy credit protection) to outsiders who receive periodic payment from banks. This is similar as insurance purchase: one party is selling insurance and the other counterparty is buying insurance against the default of the third party.

However, in our paper, our focus is not bank's risk management. Instead, we study how firms can use credit-default swap to hedge their liquidity risk. Hence, firms purchase the credit-default swap from outside investors directly. The incentive for them to make the purchase is to avoid direct or indirect bankruptcy cost. In our model, the bankruptcy cost is that firms will loss opportunity to earn positive expected profit in the future. Among different kinds of credit-default swaps, the credit-default swaps falls into

[24]See Spraos (2001)

the category of a binary credit default swap, in which the payment in the event of a default is a specific dollar amount.[26] Like in most literature, we assume that default probability and interest rates are independent. [27]

The credit-default swap is an emergent financial assistant only when firms are short of cash. With no-arbitrage argument, the cost of the credit derivative is equal to the expected payment of financial assistant from outsiders (Hull, J. and White, A., 2000). This logic suggests that the valuation of the credit derivative critically depends on the default probability of firms. A related literature examines the effect of information asymmetric of the default probability between outsiders and firms on the value of credit derivatives. Duffee and Zhou (2001) show for banks which purchase the credit derivative, the value of the credit-default swap in the adverse selection is different from the one in the moral hazard problem. In the adverse selection problem, the quality of loan is entirely exogenously determined. Therefore, the introduction of the credit derivative market is socially costly. However, the credit derivative market is beneficial to offset the moral hazard problem when the quality of loan is endogenously selected by banks themselves.

3. Benchmark Model: Without Credit-default Swap Market

3.1 Model Set-Up

Before we study the effect of credit derivative purchase, let us examine the benchmark case, in which there is no credit-default swap market available for firms. Consider N identical firms that are monopolies in their local markets, and each has internal asset A. They operate in three periods (0, 1, and 2), which is represented in figure 1.

---

[25] "The J.P. Morgan guide to credit derivatives", with contributions from the RiskMetrics Group, published by Risk, 2002

[26] Some other kinds of credit-default swap often used in risk management are basket credit default swap, contingent credit default swap, dynamic credit default swap etc. The definition of those credit derivative can be seen in Hull, J. and White, A., 2000

[27] "Valuing Credit Default Swaps II. Modeling default correlations" Hull, J. and White, A., 2000

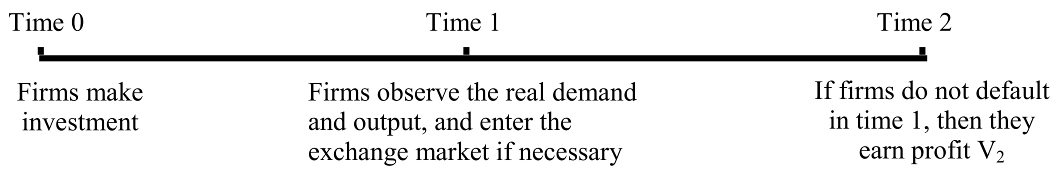


Figure 1: Timeline

In period 0, firms need to invest *I*, in a new technology in order to produce a homogenous, short-lived good (denoted by *y*) in the following two periods. We assume that $I > A$. Therefore, firms have to borrow *I-A* from financial institutes, such as banks, or finance by issuing stocks. Although the capital structure is an important part of firms risk management, it is not our focus here. Therefore, we do not explicitly model firms' capital structure here. Instead, we assume that if firms' profit is less than *B* in period 1 and 2, then they default. There is no depreciation in the model.

In period 1, the new technology incurs production shock: firms obtain either *R* units of products with probability *x*, or zero with probability *1-x*. Meanwhile, firms' local market demand, $Q_i \;\; i \in I = \{1, 2, \cdots, N\}$, is also either *R* or zero. The probability that demand is *R* for firm *i* is β_i, which is an independent random variable drawn from a uniform distribution, $U(\cdot)$ with support $\left[\underline{\beta}, 1\right]$. The local market demand realizes in period 1, and firms sell products at a uniform price, P[28]. The local market demand is primarily served, and firms can trade products in a spot exchange market after that[29]. The more detail discussion on this market remains in the next section.

We denote firm i's expected revenue in period 1 is V_{1i} For simplicity, we assume that if firms do not default in period 1, firms can expect revenue V_{2i} in period 2. V_{2i} is independent of the profit earned in the first period, and the number of firms remains in

[28] This can be considered as contract price.
[29] The firms can be regulated to satisfy their local demand, as power companies in US.

period 2. Next, we also assume that the investment has positive present value, i.e., $x\underline{\beta}(PR+V_2)\geq 2B$. Hence, firms have incentive to invest at period 0.

3.2 The spot exchange market

In period 1, firms' demand and their production levels are realized respectively. There are four possible situations that firms face, which are summarized in Table 1. In case 1 and 4, firms demand and production match while firms have shortage in case 2 and excess product in case 3.

Table 1. The Cases after Demand Realization

	Demand and production	Probability
Case 1	$Q_i = R,\ y_i = R$	$\beta_i x$
Case 2	$Q_i = R,\ y_i = 0$	$(1-\beta_i)x$
Case 3	$Q_i = 0,\ y_i = R$	$\beta_i(1-x)$
Case 4	$Q_i = 0,\ y_i = 0$	$(1-\beta_i)(1-x)$

The spot exchange market opens at period 1. Assume there is no cost to enter the exchange market. Hence, all firms who fall in either case 2 or case 3 are willing to join in the spot exchange market. The market rule is that each firm simultaneously submits its trade plan without knowing other firms local demand and production, and the market-clearing price, *w*, is determined by the intersection of total supply and demand in the market. In the exchange market, the demand correspondence for firms with shortage is

$$D_i(w)=\begin{cases} R & if \quad 0\leq w<P \\ [0,R] & if \quad w=P \\ 0 & if \quad w>P \end{cases} \qquad \forall i\in\{i\mid Q_i=R, y_i=0\} \tag{3.1}$$

The rationale is that since firm *i* can sell product at price *P* in its local market, it wants to buy as much as possible when the price is lower than *P*, and it is indifferent between buying and not buying, when the market-clearing price is *P*, the firm. The firm will stop buying when the price rises above *P*.

Following the similar logic, the supply correspondence for firms with excess products is

$$S_j(w)=\begin{cases} R & if \quad w>0 \\ [0,R] & if \quad w=0 \end{cases} \qquad \forall j\in\left\{ j\,|\,Q_j=0, y_j=R\right\} \tag{3.2}$$

The firm is willing to sell at any positive price because it has already paid the cost of production in period 0. The firm is indifferent toward selling when the market-clearing price is zero. Denote the number of firms who have extra products as m. The market-clearing price, *w,* is defined as

$$w=\begin{cases} P & \text{if} \quad m<N-m \\ [0,P] & \text{if} \quad m=N-m \\ 0 & \text{if} \quad m>N-m \end{cases} \tag{3.3}$$

The equation above shows that the market clearing price is zero when the total units of extra products are more than the total shortage, and it is *P* when there is more shortage. When the units of shortage and excess products are the same, the market clearing can takes any value between zero and *P* with equal possibility, since neither seller or buyer has bargaining power. Hence, the expected market clearing price in this case is $P/2$.

As a result, firm i's revenue, $\tilde{V}_{1i}$, in period 1, is contingent on the real-time demand in all local markets and its realizations, $\tilde{V}_{1i}$, are

$$\tilde{V}_{1i}=\begin{cases} PR & \text{with } t_{1i} \\ \frac{1}{2}PR & \text{with } t_{2i} \\ 0 & \text{with } t_{3i} \end{cases} \tag{3.4}$$

where t_{ki} $\forall k = 1,2,3$ denotes the probability of three different realizations of the profits in period 1. Without loss any generosity, we assume $B \geq \frac{1}{2}PR$, so that firm *i*'s expected profit, π_i, takes form:

$$\pi_i = V_{1i} + (1 - t_{3i})\left(V_{2i} - 2B\right) \tag{3.5}$$

Lemma 1. $\frac{\partial t_{3i}}{\partial \beta_i} < 0$ and $\frac{\partial \pi_i}{\partial \beta_i} > 0$.

Lemma 1 shows that firm *i*'s default probability is decreasing in the probability of high output, and its expected profit is increasing in it. Since firms have the same technology shock and share common information for other firms' local demand risk, only each firm's knowledge of its own local demand risk affects the expected probabilities of default and the expected profit differs. As firms' expected profit monotonically increases in β_i, the proability of high output can represent the type of the firms. In the remaining part of the paper, if $\beta_i > \beta_j$ $\forall i \neq j$, then we say firm *i* is with higher type, compared to firm *j*.

Lemma 2. t_{3i} decreases as *N* increases.

As more firms involves, the trading volume in the market increases, so does the market liquidity. It gets easier for firms to find trading partners in the market. Therefore, firms have less risk to default. Moreover, we also have

Lemma 3. The expected revenue, V_i, increases as *N* increases.

Therefore, as market liquidity increases, the spot exchange market works more efficient as risk sharing mechanism.

However, if with limited *N*, firm *i* has still a certain probability of default at period 1. This default causes a social loss, because the profit has positive NPV. Therefore, the function that the exchange market improves firms' profit and increases social efficiency is limited. There are two possible circumstances under which firms cannot transfer their risk through the spot exchange market. First, since the market is not fully liquid, the realization of the market clearing price may not benefit the firms who join in the exchange. Second, the market may fail, i.e. market clearing price does not exists. For example, when all the firms have zero product and high local market demand, no one sells and market clearing price goes to infinity. As a result, the exchange market alone is not enough for firms to fully hedge firms' default risk. They need extra risk sharing mechanism, especially when the spot exchange market with very volatile price or with high possible of market failure. Hence, we propose a stylized "insurance" market in the following section.

4. WITH CREDIT-DEFAULT SWAP MARKET

4.1. Credit-default swap market.

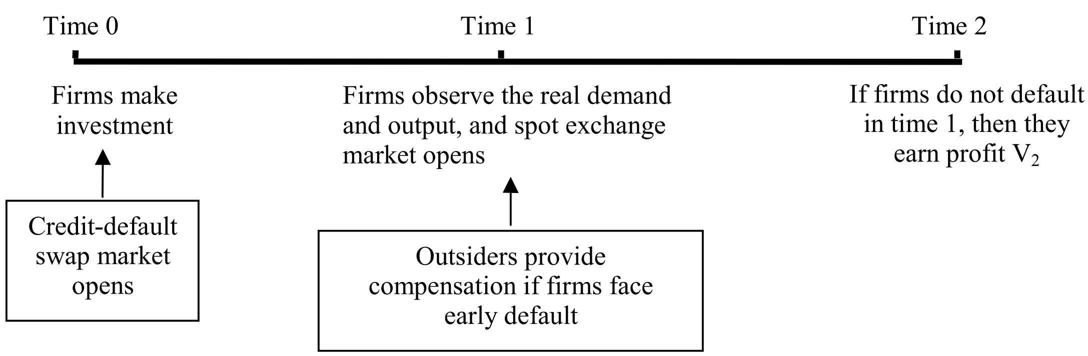


Figure 2: Timeline

We assume that a credit-default swap market is available for firms in period 0. In this market, firms can purchase a credit derivative from competitive risk-neutral outsiders. Like the standard credit-default swap contract used by banks, the credit derivative provides compensation for firms in the end of period 1 if firms' expected profit in period 1, V_1, is less than B. Otherwise, the instrument pays nothing. Since we have $\frac{1}{2}PR \geq B$, firms default only when its revenue in period 1 is zero. Therefore, when firms sign contract with outsiders, they will request that the compensation from the derivative equals to B in order to avoid early default. Firms buy credit-default swap from outsiders with price $T_i\left(B\right)$. For the competitive outsiders, their profits from the credit market are breakeven, because of the no-arbitrage argument. Therefore, the zero-profit condition for competitive outsiders implies that the price is

$$T_i = t_{3i}B \qquad (4.1)$$

Lemma 4. T_i decreases as N increases.

The proof of the lemma 4 follows lemma 2 directly. From outsiders' points of view, since firms have lower probability of bankruptcy as market liquidity increases, the outsiders expect less chance to provide the compensation for firms. From firms' side, when the market liquidity increases, firms have less incentive to buy the insurance because the chance of early default becomes less. Hence, the price charged by the competitive outsiders drops.

From equation (4.1), we know that outsiders' belief of probability of default is critical in pricing credit-default swap. Therefore, information availability to outsiders plays a very important role in this market. In the next two sections, we will discuss about how different information structures affect the credit-derivative pricing and the firms' decision to enter the credit-default swap market.

4.2. Symmetric information

In this section, we assume that each firm knows its own type, β_i, but not others. Moreover, they can truthfully reveal their types to outsiders when they purchase the derivative. Hence, the price for firm i to buy credit derivative from outsider can be written as

$$T_i(YL) = t_{3i}(\beta_i, x)B \qquad (4.2)$$

Then, firm *i*'s expected profit, if it buys derivative, is

$$E_i(V \mid T > 0) = V_{1i} - B + V_{2i} - B \qquad (4.3)$$

By comparing the expected profit with and without buying derivatives, we have the following proposition.

Proposition 1. When outsider knows the true value of β_i, the price for credit-default swap is $T_i(YL) = t_{3i}(\beta_i, x)B, \forall i = 1,2...,N$. Firm i $\forall i = 1,2...,N$ is better off by buying derivative, if $V_{2i} > 2B$. Moreover, firm *i*'s decision of buying derivative is independent of β_i.

Proposition 1 implies that firms will join in credit-default swap market if the expected revenue in the period 2, V_{2i}, is high enough. We notice that this condition set a very strict requirement for firms. It rules out the firms with negative present value project. Moreover, it has even higher requirement such that only firms whose second period revenue can cover all the financial cost, will buy the "insurance". In fact, this condition becomes redundant if firms incur a big enough deadweight loss (For example: bankruptcy cost) when it defaults. Let us denote L_i as this loss. We can show that even in one-period model, firms would like to buy credit derivatives. Their expected profits without buying credit derivatives is

$$E_i(\pi \mid T = 0) = V_{1i} - (1 - t_{3i})B - t_{3i}L_i \qquad (4.4)$$

However, its expected profit if it buys the credit derivative is

$$E_i(\pi \mid T > 0) = V_1 - B \quad (4.5)$$

Hence, firms choose *T>0*, if the deadweight loss is higher than B (i.e., $L_i > B$), even in one-shot game. Therefore, in our model, the second period expected profit, $V_2 - B$, is equivalent to the bankruptcy cost here. In summery, firms have incentive to purchase the credit-default swap only if it occurs high enough direct or indirect bankruptcy cost.

However, since the payment for the credit derivative varies with firms' types, whether firms can credibly reveal their types to outsiders is crucial. Suppose that firm *i* with type β_i tells outsider that its type is $\beta_i^{'}$, which is higher than β_i. From equation (4.1), it is easy to show that $T_i\left(\beta_i\right) > T_i\left(\beta_i^{'}\right)$. By lying about its true type, firm *i* costs less for the same level of insurance from the credit derivative. However, outsiders are worse-off because their expected profit is negative. As a result, if outsiders have no way to audit firms' types, they cannot believe the types revealed by firms, and the equilibrium above does not exist.

4.3. Asymmetric information

In this section, let us exam the case in which we relax the restriction of credible type revelation. A firm normally has more accurate information on its own type than others. Hence, we assume that firm *i* $\left(\forall i = 1, 2..., N\right)$ knows its true type, but other firms and outside investors in the market have no more information on firm *i*'s type, other than the prior distribution. Moreover, firm *i* has no mechanism to reveal its true type to outsiders. Last, when outsiders decide whether to sell credit derivative to firm *i*, they cannot know how many other firms simultaneously pursue the credit derivative in the market[30]. This assumption is actually not very arbitrary. In the credit-default swap market, there are many outside investors. Different firms can purchase credit-default swap with different outsiders. Hence, one outsider may not know how many firms pursue "insurance" from another outsider in the market.

Firm *i*'s information set (denoted as Ω_i) is

$$\Omega_i = \left\{\beta_i, E_i\left(\beta_j\right) = x, \forall j \neq i\right\} \quad (4.6)$$

Outsiders have a prior belief that $\Omega_o = \left\{E_o\left(\beta_i\right) = x, \forall i\right\}$. However, when a firm asks for "insurance", outsiders can update their belief on that particular firm. The intuition is as follows. Suppose that outsiders keep their prior belief on firms' types no matter whether they ask for "insurance". Therefore, the price of the credit derivative is the same for all firms, which is

$$T_i = t_{3i}\left(x\right)B = t(x)B \quad \forall i \quad (4.7)$$

Now, we have a firm *i*, whose probability of high demand is $\beta_i > x$. From lemma 1, we know that $t_{3i}\left(x\right) = t\left(x\right) > t_{3i}\left(\beta_i, x\right)$. Hence, firm *i*'s payment for the "insurance" is higher than the one it pays when it can truthfully reveal its type. Without buying any credit derivative, firm *i*'s expected profit is

$$E_i\left(\pi_i \mid T = 0\right) = V_{1i} - \left(1 - t_{3i}\left(\beta_i, x\right)\right)B + \left(1 - t_{3i}\left(\beta_{3i}, x\right)\right)V_{2i} \quad (4.8)$$

Meanwhile, firm *j*'s expected profit when it purchases the credit derivative is

$$E_i\left(\pi_i \mid T > 0\right) = V_{1i} - B + t_{3i}\left(\beta_i, x\right)B - t\left(x\right)B + V_{2i} \quad (4.9)$$

Hence, firm *i* will purchase the credit derivative only if

$$t_{3i}\left(\beta_i, x\right)V_{2i} > t\left(x\right)B \quad (4.10)$$

According to lemma 1, the right hand side of the inequality above decreases in β_i. Therefore, it is expected that for the firms with high β_i, may not satisfy (4.10) and would not buy credit derivative. Therefore, we can present outsider is updated as

$$\Omega_o = \left\{\beta_i \mid t_{3i}\left(\beta_i, x\right)V_{2i} > t\left(x\right)B \textit{ if } T_i > 0, \textit{ and } E\left(\beta_j\right) = x, \forall j \neq i\right\}, \quad (4.11)$$

and the payment for the credit derivative is

$$T_i = t_{3i}\left(\mathrm{E}\left(\beta_i \mid \Omega_0\right),\ \mathrm{E}\left(\beta_j\right) = x \quad \forall j \neq i\right)B. \quad (4.12)$$

According to the argument above, we can describe the equilibrium in which how firms behave in the credit-default swap market in proposition 2.

Proposition 2. There is a $\hat{\beta}$:

[30] If outsiders can use the number of firms which enter the credit-default swap to update their information set, the market equilibrium will be very complicated.

(1) If $\hat{\beta} \geq 1$, there is a pooling equilibrium such that all firms buy the credit derivatives from outsiders with price $T_i\left(E\left(\beta_i\right)=x,\ \forall i\right)$.

(2) If $\underline{\beta} \leq \hat{\beta} < 1$, there is a separating equilibrium such that only firms with $\underline{\beta} \leq \beta_i \leq \hat{\beta}\ \forall i$ buys the credit derivative from outsiders with price $T_i\left(E\left(\beta_i\right)=\frac{\underline{\beta}+\hat{\beta}}{2}\ \textit{and}\ E\left(\beta_j\right)=x\right)$; firms with $\beta_i > \hat{\beta}\ \forall i$, will not enter the credit-default swap market.

As in proposition 2, either a pooling or separating equilibrium is possible, depending on the model's parameters. In the pooling equilibrium, all firms buy credit derivative and no bankruptcy occurs in the first period. However, in the separating equilibrium, not all firms are willing to pay for this derivative. The reason is that there are two forces to lower the interest of high-type firms to purchase credit-derivative. The first force is that they worry less about early default since they have lower possibility to default no matter in symmetric or asymmetric cases. However, the second force is more important here. When firms cannot reveal their true demand uncertainties to outsiders, they buy the credit derivative with uniform price. Hence, it is too costly for them to buy the derivative, comparing to the case when outsiders know their true type.

4.4. WELFARE ANALYSIS

In the symmetric information case, if we assume that the expected profit in the second period is big enough, i.e. $V_2 > 2B$, firms are always better off if they buy the credit derivative, because it provides insurance to avoid costly default. Hence, all firms in our model join in the credit-default market. The price charged by outsiders varies from one firm to the other, based on their default probability and no firm defaults in the end of period 1. Therefore, in the symmetric information case, the economy reaches the first best. There is no social cost caused by information bias and costly bankruptcy.

In the asymmetric case, however, we can summarize the welfare analysis in the following proposition.

Proposition 3. There is no social cost in the asymmetric case, if $\hat{\beta} \geq 1$; the welfare reduce, if $\hat{\beta} < 1$.

Proposition 3 illustrates that whether asymmetric information causes the welfare reduction depends on the parameters. On one hand, when $\hat{\beta} \geq 1$, all firms buy the insurance so that there is no costly early default. Compared with the symmetric information case, there is only one change in the economy with asymmetric information, which is the amount of "insurance" payment for firms. Hence, an individual firm's profit or a credit buyer's net plus may change. However, it is easy to show that the change of the payment transfer between firms and the credit-default buyers does not altar aggregate social surplus. On the other hand, when $\hat{\beta} < 1$, the firms with their type $\beta_i > \hat{\beta}$ will not purchase the insurance, and they have $t(\beta_i)$ probability to default. Hence, the aggregate social surplus drops by $\sum_{\beta_i > \hat{\beta}} (V_2 - B) t(\beta_i)$, because of costly default from firms with $\beta_i > \hat{\beta}$.

Furthermore, the effect of the number of firms on social welfare is ambiguous.

Proposition 4: $\hat{\beta}$ decreases as N increases

Proposition 4 shows that as more firms involves, the range in which firm buy credit derivatives shrinks. More and more firms find out that it is not worth of purchasing "insurance" if their payment for the "insurance" do not depend on their own types. However, as we shown in Lemma 1, with more firms involved, the spot exchange market becomes more efficient and liquid. Hence, the default risk is lower.

5. Discussion

1. Use loan sale to hedge risk

In this paper, we show that credit-default swap market can help firms to avoid their costly default and increase their expected profits. We have mentioned before, this idea is motivated by that banks used this market to hedge their loan credit risk. In fact, there are many other mechanisms banks usually use to hedge credit risk. A most traditional way is through loan sale. In order to minimize their credit risks, banks can sell a certain fraction of the cash stream from a specific loan to outside investors so that they can guarantee certain level of return on the part of the loan sold. In our model, we can show that if firms can issue one-period loan sale, it can replicate the credit-default swap to hedge firms' default risk. For example, suppose firm *i* sells a fraction, f_i of loan to competitive outsiders in period 0. The buyers receive a fraction of f_i of any cash flows from the loan in the end of period 1. We assume that firms can make committee to sell the factions as assumed in Gorton and Pennachi (1995). Moreover, we also assume there is no arbitrage as in the credit-default market model. Therefore, in the equilibrium, the price for this loan sale firm *i* asks for (denoted as L_i) satisfies

$$L_i = f_i V_{1i} \qquad (5.1)$$

In order to avoid early default, the sale price of the loan sale must be at least equal to *B*. Hence, from equation (5.1), we can conclude that

Lemma 5. With symmetric information between firms and outsiders, firms with $V_2 > 2B$ sells $f_i^* \in \left[\frac{B}{V_{1i}}, 1\right]$ of loan to outsiders with price *B*.

Following the same logic as in the credit-default swap, we can derive an equilibrium which is similar as described in proposition 2 when the firms have more accurate information than outsiders. However, we leave the illustration to readers' own interest and will not dig into more details.

Although loan sale can replicate, at least to some extend, the function of credit-default swap to lower firms' early default risk, there are several obvious advantages for using credit-default swap. First of all, Duffee and Zhou (2001) illustrate that credit-default swap is more flexible to circumvent the "lemons" problem than loan sales. As long as the information asymmetric varies over the life of the loan, credit-default swap works more efficient to transfer the risks than long-term loan sale. They point out that although, a sequence of one-period loans can replace the function of credit derivatives, they cannot replicate credit derivatives perfectly in a broad sense, because of some well-known reasons such as liquidity risks (Diamond,1991) or tax-timing issues.

Secondly, firms prefer to hedge their liquidity risk via credit-swap because it has significant advantage in tax and accounting issues. Credit-swap purchase allows firms to reduce risk without physically removing assets from their balance sheet, so that it does not lead a sale for either tax or accounting purposes. Therefore, it does not affect any risk management decision-making. In banking system, Credit Swaps have been employed more and more in risk-hedge to avoid unintended adverse tax or accounting consequences of otherwise sound risk management decisions. (JP Morgan)

2. Correlated Interest rate

In our model, we assume that the firms are obligated to a fixed financial cost, B, which is not affected by firms' activity in both the spot exchange market and the credit-default swap market. This assumption follows a convention in the literature about the credit-default swap. Although this is not critical in this paper, but deserves some further illustration. In reality, bank usually offers lower interest rate to high quality lenders than to low quality lenders. In our model, we have proved that firms decrease their early-default risks by purchasing credit-default swap. Therefore, they can ask for lower interest rate because their investment is more "secure", compare to the firms without "insurance". However, there is normally no monotonic relationship between interest rate and quality of the investment in banks operation. Duffee and Zhou (2001) explain that the bank's profit merely depends only on that project. Instead, it may correlate with some other part of bank's activity. An important factor to decide the interest rate a bank can charge is the bank's market power in lending to firms. If it is not easy to switch bank, firms have to accept the high interest rate even for high-quality of investment. Moreover, Stiglitz and

Weiss (1981) show that the asymmetric information problem between banks and borrowers contaminate the creditable quality revealing. Therefore, banks may use uniform interest rate for both high and low quality loan. This is essentially related to our model, since how firms can make firms believe that they will buy the credit-default swap after they borrowing the money is a big question.

6. CONCLUSION REMARKS

The demand shock and technology shock may lead the unmatched local market demand and excess supply. As a result, firms face early default, since its revenue cannot cover the contracted financial cost. This risk is essential big for firms who produce perishable goods because of the special characteristics of perishable goods. In Lin, Fang and Whinston (2006), they propose a spot exchange market as a risk sharing mechanism where firms with unmatched demand can trade with firms with shortage. However, whether the market has enough liquidity determines how efficient this spot exchange market works for risk-hedging. It is well accepted that market liquidity is a big problem in most of market. Moreover, whether this market exists is another question. Even if firms have idiosyncratic risk, it has a chance that the realizations of those random factors become the same for all firms, which is similar to "macro"-like shock. Under this circumstance, all firms either have extra product or have shortage so that the market exchange cannot occur.

Therefore, we provide a stylized method for firms to improve their risk management. We build in a role for credit-default swap in firm's risk management portfolio. As a result, firms can get compensation from outside investors when they are exposed to financial distress. In exchange, they agree to pay a certain fee for this "insurance". We show that the credit-default swap contract is affected by information structure in the model. When the information is symmetric between firms and outsiders, all firms are willing to purchase this credit derivative and their prices vary with their expected default probability. However, when firms have superior information to outside investors, outsider investors can only charge a uniform price. As a result, for the group of firms who have lower possibility to default, it is too costly for them to purchase the credit derivative and not join the market. "Lemons" problem leads to social cost, i.e., costly early default.

This paper provides a theoretical basis of introducing new instrument in firms' risk management. We do not claim this is the only way to deal with firm's default risk. However, compared to some other instrument, such as loan sale, it has some obvious advantage in firms' overall management, and it has been proved more flexible to deal with "lemons" problem.

Appendix A: The Proofs for Chapter 1

- **Proof for Lemma 1**

Proof: Firm *i*'s profit function is $\pi_i(q_i;P,c,Q_i)=\min\{Q_i,q_i\}P-cq_i-B_i$

Case 1: If $q_i<\frac{B_i}{P-c}$, then $\pi_i(q_i;P,c,Q_i)=\min\{Q_i,q_i\}P-cq_i-B_i<0$

Case 2: If $q_i\geq\frac{B_i}{P-c}$

Case 2.1. When $q_i<Q_i$, then $\pi_i(q_i;P,c,Q_i)=q_iP-cq_i-B_i>0$

Case 2.2. When $q_i\geq Q_i$, then $\pi_i(q_i;P,c,Q_i)=Q_iP-cq_i-B_i>0$ only if

$Q_i>\hat{Q}_i=\frac{cq_i+B_i}{P}$ QED.

- **Proof of proposition 1**

Proof: Since $F\left(\frac{B_i}{P-c}\right)<1$, $f(x)>0$ and $f^{'}(x)<0$ for all $x\in\left(\frac{B_i}{P-c},+\infty\right)$ and $T(x)=\frac{-f'(x)}{f(x)}$ 31 is non-decreasing as x, then the first order condition (FOC) at $\frac{B_i}{P-c}$ is

$$FOC\left(q_i=\frac{B_i}{P-c}\right)=(P-c)\left(1-F\left(\frac{B_i}{P-c}\right)\right)>0$$

First of all, since $\lim_{q\to\infty}F(q)=1$ and $\lim_{q\to\infty}F\left(\frac{c\overline{q}'+B_i}{P}\right)=1$, it is easy to show that $FOC(q_i\to\infty)\to0$ from below

Secondly, if we can show that second order condition (SOC) only change signs once (from negative to positive), we obtain uniqueness. The second order condition (SOC) is

$$SOC(q_i)=-Pf(q_i)+f(\hat{q}_i)\frac{c^2}{P}=-Pf(q_i)\left[1-\frac{c^2}{P^2}\frac{f\left(\frac{cq_i+B_i}{P}\right)}{f(q_i)}\right]$$

It is easy to show that $SOC\left(\frac{B_i}{P-c}\right) = -\frac{P^2-c^2}{P} f\left(\frac{B_i}{P-c}\right) < 0$ and $\lim_{q_i \to \infty} SOC(q_i) = 0$, Moreover, since $\frac{f\left(\frac{cq_i + B_i}{P}\right)}{f(q_i)}$ increases as q_i, we only need to require $\lim_{q_i \to \infty} \frac{f\left(\frac{cq_i + B_i}{P}\right)}{f(q_i)} > \frac{P^2}{c^2}$.

Since we know that $\lim_{q \to \infty} f(q) = 0$, given any x and $\frac{f(x)}{f(x+\Delta)}$ is a non-decreasing function, we have $\lim_{\substack{x \to \infty \\ \Delta \to \infty}} \frac{f(x)}{f(x+\Delta)} \to \infty$. Hence, we have $\lim_{q_i \to \infty} \frac{f\left(\frac{cq_i + B_i}{P}\right)}{f(q_i)} \to \infty > \frac{P^2}{c^2}$. Therefore, the second order condition only changes signs once, and hence the optimal solution uniquely exists.

- **Proof for proposition 2**

The impact of debt on production is

$$\frac{dq_i}{dB_i} = \frac{cf(\hat{q}_i)}{P^2 f(q_i) - c^2 f(\hat{q}_i)}$$

By using implicit function theorem, we can show that this effect is positive. QED.

- **Proof for Lemma 2**

Proof: Without limited liability, the firm's objection function of the firm is

$$\max_{q_i \geq 0} E\pi(q_i) = P\int_0^{q_i} Qf(Q)\,dQ + P\int_{q_i}^{+\infty} q_i f(Q)\,dQ - cq_i - B_i \;\; \forall\; i = 1, \ldots, N$$

The first order condition is

$$P\left[1 - F(q_i)\right] - c$$

The first order condition in the case of limited liability is

---

[31] It represents the rate of decreasing likelihood.

$$\left\{P\left[1-F\left(q_i\right)\right]-c\right\}+cF\left(\hat{Q}_i\right)$$

Thus $q^{NO} > q^{NL}$, because that $cF\left(\hat{Q}_i\right)$ is strictly positive.

- **Proof for proposition 3:**

  In the exchange market, the market clearing price is decided by

$$w = \begin{cases} P & If \sum_{i=1}^{N} q_i \geq \sum_{i=1}^{N} Q_i \\ 0 & If \sum_{i=1}^{N} q_i < \sum_{i=1}^{N} Q_i \end{cases}$$

For each firm, the cutoff points of bankruptcy is

$$PQ_i + w(q_i - Q_i) - cq_i > B_i \Leftrightarrow Q_i \geq \frac{B_i + (c-w)q_i}{P-w} = \hat{Q}_i$$

Then firms maximize their profit by choosing q_i

Case 1: When $w > c$, the firms is not bankrupt, when

Case 1.1 If $\hat{Q}_i \geq 0$, then we have $q_i \leq \frac{B_i}{w-c}$. The first order condition is

$$\int_{\hat{Q}_i}^{\infty}(w-c)q_i f(Q_i)dQ_i > 0 \Rightarrow q^{FL} = \frac{B_i}{w-c}$$

Case 1.2 If $\hat{Q}_i \leq 0$, then we have $q_i > \frac{B_i}{w-c}$. The first order condition is

$$\int_{0}^{\infty}(w-c)q_i f(Q_i)dQ_i > 0 \Rightarrow q^{FL} \to \infty$$

Comparing among case 1.1 and 1.2, the last one has the highest profit. The firms can produce infinite products when $w \geq c$. Therefore, this is not equilibrium because no firm will buy from the market.

Case 2: When $w < c$, we know that $\hat{Q}_{ii} > 0$. The first order condition is

$$\int_{\hat{Q}_i}^{\infty}(w-c)q_i f(Q_i)dQ_i < 0 \Rightarrow q^{FL} \to 0$$

The firms' optimal production is 0, when $w < c$. Therefore, this is not equilibrium because no firm can sell in the market.

Case 3: when $w = c$, we know that $\hat{Q}_i > 0$. The first order condition is

$$\int_{\hat{Q}_i}^{\infty} (w-c) q_i f(Q_i) dQ_i = \int_{\hat{Q}_i}^{\infty} 0 * f(Q_i) dQ_i = 0 \Rightarrow q_i^* \in [0, \infty]$$

Therefore, the equilibrium market price is $w = c$. Furthermore, in order to have a full liquidity market, the equilibrium production level should satisfy $nq^{FL} = \sum_i Q_i \Rightarrow q^{FL} = \frac{1}{n} \sum_i Q_i = \overline{Q} = E(Q)$. QED.

- **Proof for Lemma 3**

The expected market clearing price (denoted as $E_{Q_i,Q_j} W$) is

$$E_{Q_i,Q_j} W = P \Pr(Q_i + Q_j \geq q_i + q_j) + 0 * \Pr(Q_i + Q_j < q_i + q_j)$$

$$\frac{dE_{Q_i,Q_j} W}{dq_i} = P \frac{d \Pr(Q_i + Q_j \geq q_i + q_j)}{dq_i} < 0 \text{, since } \frac{d \Pr(Q_i + Q_j \geq q_i + q_j)}{dq_i} < 0$$

- **Proof for Lemma 4**

Case1. When $q_i < \frac{B_i}{P-c}$ the expected profit is

$$G\left(q_i; q_j, B_i\right) = \int_0^{q_j + \frac{(P-c)q_i - B_i}{P}} \left[\int_{\frac{cq_i + B_i}{P}}^{q_i + q_j - Q_j} (PQ_i - B_i - cq_i) f(Q_i) dQ_i \right] f\left(Q_j\right) dQ_j$$

Obviously, $\Gamma(Q_i, Q_j) < F(Q_i) = 1$

Case2. When $q_i > \frac{B_i}{P-c}$, the expected function is

$$\begin{aligned} G\left(q_i; q_j, B_i\right) = \int_0^{q_j + \frac{(P-c)q_i - B_i}{P}} & \left\{ \int_{\frac{cq_i + B_i}{P}}^{q_i + q_j - Q_j} (PQ_i - B_i - cq_i) f(Q_i) dQ_i \right\} f(Q_j) Q_j \\ & + (Pq_i - B_i - cq_i) \left[1 - \int_0^{q_j + q_i} F\left(q_i + q_j - Q_j\right) f(Q_j) dQ_j \right] \end{aligned} \quad \forall i = 1, 2$$

$$\Gamma(Q_i, Q_j) = \underbrace{\int_{0_j}^{q_i + q_j} F(\hat{Q}_i) f(Q_j) dQ_j}_{< F(\hat{Q}_i)} - \underbrace{\int_{q_i + q_j - \hat{Q}_i}^{q_i + q_j} \left(\int_{q_i + q_j - Q_j}^{\hat{Q}_i} f(Q_i) dQ_i \right) f(Q_j) dQ_j}_{>0} < F(Q_i) = F(\hat{Q}_i) \text{ QED.}$$

- **Proof for Lemma 5**

Suppose that $q_i < \frac{B_i}{P-c}$. Then we need that

$$q_i + q_j - Q_j > \frac{cq_i + B_i}{P} \Rightarrow q_j > Q_j$$

As we proved before, if the firms' profit can cover the debt cost, the realization of the demand is at least $\hat{Q}_i \ \forall i = 1,2$. Thus

$$q_j > \hat{Q}_j \Rightarrow q_j > \frac{cq_j + B_j}{P} \Rightarrow q_j > \frac{B_j}{P-c} \qquad \text{QED.}$$

- **Proof for proposition 4 (see figure below)**

The first order condition in symmetric case ($q_i > \frac{B_i}{P-c} : q_i = q_j$)

$$FOC_i = e^{-\lambda 2q_i} \left\{ \underbrace{P + 2P\lambda q_i + \frac{1}{2} P\lambda^2 \left(\frac{cq_i + B_i}{\mathrm{P}} \right)^2 - c\lambda \frac{cq_i + B_i}{\mathrm{P}}}_{D} \underbrace{- ce^{\lambda\left(\frac{(2P-c)q_i - B_i}{P}\right)}}_{E} \right\}$$

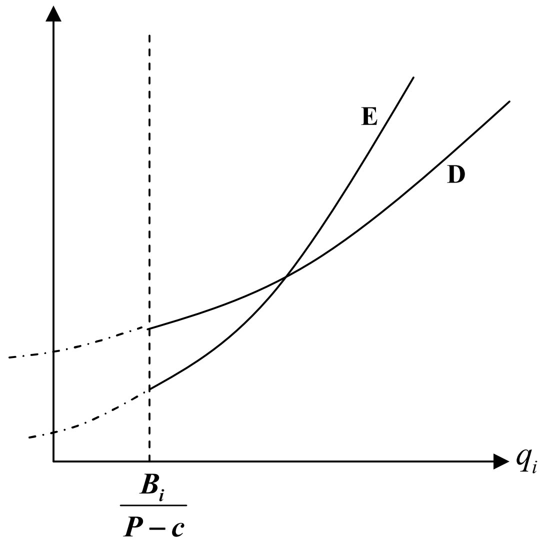
Condition of existing and uniqueness of optimal solution:

(i) It seems almost always if P is not too large and B_i is not too small, we have

$$FOC_i\left(\frac{B_i}{P-c}\right) = P + \lambda\left(2P-c\right)\frac{B_i}{P-c} + \frac{1}{2}P\lambda^2\left(\frac{B_i}{P-c}\right)^2 - ce^{\lambda\left(\frac{B_i}{P-c}\right)} > 0$$

(ii) It is easy to show that there exists a $\overline{B}$, when $B_i > \overline{B}$, E always increases much faster than D, when debt is large enough.

Since the first order condition satisfies the above conditions, it can be equal to zero at most once. Therefore, there exists a unique optimal solution. QED.



- **Proof for Lemma 6**

Proof: If the firm without limited liability, the objective function becomes

$$\max_{q_i > \max\left\{0, \frac{B_i - Pq_j}{P-c}\right\}} G\left(q_i; q_j, B_i\right) = \int_0^{q_j+q_i} \left[\int_0^{q_j+q_i-Q_j} (PQ_i \lambda e^{-\lambda Q_i}) dQ_i + \int_{q_j+q_i-Q_j}^{\infty} (Pq_i \lambda e^{-\lambda Q_i}) dQ_i \right] \lambda e^{-\lambda Q_j} dQ_j$$

$$+ Pq_i \left[1 - \int_0^{q_j+q_i} (1 - e^{-\lambda(q_i+q_j-Q_i)}) \lambda e^{-\lambda Q_j} f(Q_i) \right] - cq_i - B_i$$

The first order condition is

$$(Pe^{-\lambda(q_i+q_j)}) \left[1 + \lambda(q_i+q_j) - \lambda^2 \left(q_i+q_j\right) q_i + \lambda^2 \frac{1}{2} \left(q_i+q_j\right)^2 \right] - c = 0 \quad (*)$$

We can prove that the objective function is concave. Let q^{NLEX} be the optimal solution here.

Consider equation (5.6),

$$\underbrace{(Pe^{-\lambda(q_i+q_j)}) \left[1 + \lambda(q_i+q_j) - \lambda^2 \left(q_i+q_j\right) q_i + \lambda^2 \frac{1}{2} \left(q_i+q_j\right)^2 \right] - c}_{(I)} + \underbrace{e^{-\lambda(q_i+q_j)} \frac{P\lambda^2}{2} \left(\hat{Q}_i + \frac{2c}{P\lambda} \right) \hat{Q}_i + c\left[1 - e^{-\lambda \hat{Q}_i} \right]}_{(II)} = 0$$

q^{EX} is optimal solution in (5.6). Since the second part of (5.6) is positive, therefore, if we substitute q^{EX} into (*), then (*) is negative. It implies that $q^{EX} > q^{NLEX}$. QED.

Appendix B: The Proofs for Chapter 2

- **Proof of Lemma 2:**

Proof: $\frac{\partial p_{mc}}{\partial x_i} = \frac{\partial p_{mc}}{\partial a_i}\frac{\partial a_i}{\partial x_i} + \frac{\partial p_{mc}}{\partial b_i}\frac{\partial b_i}{\partial x_i}$

In Lemma 1, we know that $\frac{\partial a_i}{\partial x_i}>0$ and $\frac{\partial b_i}{\partial x_i}=0$ and from (10), we also know that $\frac{\partial p_{mc}}{\partial a_i} < 0$.

Therefore, $\frac{\partial p_{mc}}{\partial x_i} = \frac{\partial p_{mc}}{\partial a_i}\frac{\partial a_i}{\partial x_i} < 0$

- **Proof of Lemma 3:**

Proof: for any market clearing price, the quantity is deployed for producer i is

$$q_i = a_i + b_i p$$

$$= \frac{x_i - \left[\sum_{j\neq i} b_j + \delta\right] d_i}{1+2c_i\left[\sum_{j\neq i} b_j + \delta\right]} + \frac{\sum_{j\neq i} b_j + \delta}{1+2c_i\left[\sum_{j\neq i} b_j + \delta\right]} * \frac{\varepsilon - \sum_i a_i}{\left[\delta + \sum_i (b_i)\right]}$$

Therefore

$$\frac{\partial q_i}{\partial x_i} = \frac{\partial a_i}{\partial x_i} + b_i\frac{\partial P}{\partial x_i} = \frac{\partial a_i}{\partial x_i} + b_i\frac{\partial a_i}{\partial x_i}\frac{-1}{\delta + \sum_i b_i} = \frac{\partial a_i}{\partial x_i}\frac{\delta + \sum_{j\neq i} b_j}{\delta + \sum_i b_i} < 0$$

$$\frac{\partial q_i}{\partial x_j} = b_i\frac{\partial P}{\partial x_j} < 0$$

- **Proof for Proof 3:**

  - For $p \le \overline{p}_1$ where $\overline{p}_1 = \frac{\varepsilon - \overline{q}_1 - a_2^1}{\delta + b_2^1}$ is decided by $\overline{q}_1 + q_2^1(p) = \varepsilon - \delta p$

    - The symmetric equilibrium supply schedule $\{q_1^1(p), q_2^1(p)\}$ is decided by (4)

- The market clearing price is $p_1 = \frac{\varepsilon - \sum_i a_i}{\left[\delta + \sum_i (b_i)\right]}$

  - For $\overline{p}_2 \geq p > \overline{p}_1$, where $\overline{p}_2 = \frac{\varepsilon\text{-}(\overline{q}_1 + \overline{q}_2)}{\delta}$ is decided by $\overline{q}_1 + \overline{q}_2 = \varepsilon - \delta p$

    - The market clearing price is $p_2 = \frac{\varepsilon - \overline{q}_1 - a_2^1}{\delta + b_2^2}$
    - The supply schedule for firm 1 is $q_1^2(p) = \overline{q}_1$
    - The supply schedule for firm 2 is decided by

      $$\max_p \ \pi_i = [\varepsilon - \delta p - \overline{q}_1 - x_2] p + f_2 * x_2 - c_i(\varepsilon - \delta p - \overline{q}_1) \quad (9)$$

      The first-order condition is

      $$q_2^2(p) - x_i + \left[p - c_i^{'}(q_2^2(p))\right](-\delta) = 0 \quad (10)$$

      Substitute $q_2^2(p) = a_2^2 + b_2^2 p$, we can have

      $$a_2^2 = \frac{x_2 - \delta d_2}{1 + 2c_2\delta}, b_2^2 = \frac{\delta}{1 + 2c_2\delta}$$

  - For $p > \overline{p}_2$, The supply schedule for firm *i* is $q_i^3(p) = \overline{q}_i$

    - The market clearing price is $p_3 = \frac{\varepsilon - \overline{q}_1 - \overline{q}_2}{\delta}$

- **Proof of proposition 4**

1. At T=2
   - For $p \leq \hat{p}$ where $\hat{p} = \frac{\overline{q} - a}{b}$ is decided by $\overline{q} = a + bp$, the symmetric equilibrium supply schedule $\{q_1^1(p) = q_2^1(p) = q^1(p)\}$ is decided by (4)
     - The market clearing price is $p_1 = \frac{\varepsilon - 2a}{2b + \delta}$
   - For $p > \hat{p}$, $q^2(p) = \overline{q}$
     - The market clearing price is $p_2 = \frac{\varepsilon - 2\overline{q}}{\delta}$
2. At T=1
   - The cutoff for ε, which maps $\hat{p}$

- $\hat{\varepsilon}$ is decided by $a + b\hat{p} = \overline{q} \Rightarrow a + b\dfrac{\varepsilon\text{-}2\overline{q}}{\delta} = \overline{q} \Rightarrow \hat{\varepsilon} = \dfrac{(\delta + 2b)\overline{q} - a\delta}{b}$

- For firm i, x is decided by

$$\max_x \; \mathrm{E}\pi = \int_{\varepsilon_0}^{\hat{\varepsilon}} \left\{ \left[q^1\left(p_1(\varepsilon), x\right) - x \right] p_1(\varepsilon) + f * x - c\left(q^1\left(p_1(\varepsilon), x\right)\right)\right\} f(\varepsilon) d\varepsilon$$

$$+ \int_{\hat{\varepsilon}}^{\overline{\varepsilon}} \left\{ \left[\overline{q} - x \right] p_2(\varepsilon) + f * x - c\left(\overline{q}\right)\right\} f(\varepsilon) d\varepsilon$$

$s.t \;\; f = E(P)$

First order condition

$$\int_{\varepsilon_0}^{\hat{\varepsilon}} \left\{ \frac{\partial q^1(\bar{x},\varepsilon)}{\partial x}(p_1(\bar{x},\varepsilon) - 2cq(p_1,x) - d) + q^1 \frac{\partial p_1(\bar{x},\varepsilon)}{\partial x} \right\} f\left(\varepsilon\right) d\varepsilon$$

$$+ \frac{\partial \hat{\varepsilon}}{\partial x}\left\{ \overline{q}p_1(\hat{\varepsilon}) - c\left(\overline{q}\right)\right\} f\left(\hat{\varepsilon}\right) - \frac{\partial \hat{\varepsilon}}{\partial x}\left\{ \overline{q}p_2(\hat{\varepsilon}) - c\left(\overline{q}\right)\right\} f\left(\hat{\varepsilon}\right) = 0$$

$$\Rightarrow \int_{\varepsilon_0}^{\hat{\varepsilon}} \underbrace{\left\{ \frac{\partial q^1(\bar{x},\varepsilon)}{\partial x}(p_1(\bar{x},\varepsilon) - 2cq(p_1,x) - d) + q^1 \frac{\partial p_1(\bar{x},\varepsilon)}{\partial x} \right\}}_{R(\varepsilon)} f\left(\varepsilon\right) d\varepsilon$$

$$+ \underbrace{\frac{\partial \hat{\varepsilon}}{\partial x}}_{<0} \left\{ \underbrace{\overline{q}p_1(\hat{\varepsilon}) - \overline{q}p_2(\hat{\varepsilon})}_{=0} \right\} f\left(\hat{\varepsilon}\right) = 0$$

- One observation is $R\left(\varepsilon\right)$ is increasing in ε

$$Let \;\; R\left(\varepsilon\right) = \left[b_i \frac{\partial p(\bar{x},\varepsilon)}{\partial x_i} + \frac{\partial a_i(p,x_i)}{\partial x_i} \right](p(\bar{x},\varepsilon) - 2c_i q_i(p,x_i) - d_i) + q_i \frac{\partial p(\bar{x},\varepsilon)}{\partial x_i}$$

$$\frac{\partial R\left(\varepsilon\right)}{\partial \varepsilon} = \underbrace{\frac{\partial a_i}{\partial x_i}\frac{\partial p}{\partial \varepsilon}\frac{1}{\delta + \sum_i b_i}}_{>0} \left\{ \left(\delta + b_j\right)\left(1 - 2c_i b_i\right) - b_i \right\}$$

Second order condition

$$\int_{\varepsilon_0}^{\hat{\varepsilon}}\left\{\underbrace{\frac{\partial q^1(\vec{x},\varepsilon)}{\partial x}\left(\frac{\partial p_1(\vec{x},\varepsilon)}{\partial x}-2c\frac{\partial q^1(p,x)}{\partial x}\right)+\frac{\partial q^1(p,x)}{\partial x}\frac{\partial p_1(\vec{x},\varepsilon)}{\partial x}}_{<0}\right\}f(\varepsilon)d\varepsilon$$

$$+\underbrace{\frac{\partial\hat{\varepsilon}}{\partial x}}_{<0}\left\{\underbrace{\frac{\partial q^1(p,x)}{\partial x}(p(\vec{x},\hat{\varepsilon})-2c\overline{q}-d)+\overline{q}\frac{\partial p(\vec{x},\hat{\varepsilon})}{\partial x}}_{R(\hat{\varepsilon})}\right\}f(\hat{\varepsilon})$$

Since $\mathrm{R}(\varepsilon)$ *is* increasing in ε, and $\int_{\varepsilon_0}^{\hat{\varepsilon}} R(\varepsilon)f(\varepsilon)d\varepsilon=0$ (derived from the first order condition), then $\mathrm{R}(\hat{\varepsilon})>0$. As a result, the second order condition<0

- Since $R(\varepsilon)$ is increase in ε, then if we substitute x_{un}^* in the first order condition above, the first order condition is negative. Therefore, x_{un}^* is higher than x_c^*, which is the optimal contracts units with capacity constraint.

- **Proof of Proposition 5**
  - The cutoff for ε, which maps $\hat{p}_1, \hat{p}_2$
    - $\hat{\varepsilon}_1=\dfrac{\left(\sum_i b_i^1+\delta\right)\overline{q}_1+a_2^1b_1^1-\left(\delta+b_2^1\right)a_1^1}{b_1^1}$ is decided by $q_1^1(\hat{p}_1(\varepsilon))=\overline{q}_1$
    - $\hat{\varepsilon}_2=\sum\overline{q}_i+\delta\dfrac{\overline{q}_2-a_2^2}{b_2^2}$ is decided by $q_2^2(\hat{p}_2(\varepsilon))=\overline{q}_2$
  - For firm 1, x_1 is decided by

$$\max_{x_1}\ \mathrm{E}\pi_1=\int_{\varepsilon_0}^{\hat{\varepsilon}_1}\left\{\left[q_1^1\left(p_1(\varepsilon),x_1\right)-x_1\right]p_1(\varepsilon)+f_1*x_1-c_1\left(q_1^1\left(p_1(\varepsilon),x_1\right)\right)\right\}f(\varepsilon)d\varepsilon$$

$$+\int_{\hat{\varepsilon}_1}^{\overline{\varepsilon}}\left\{\left[\overline{q}_1-x_1\right]p_2(\varepsilon)+f_1*x_1-c_1\left(\overline{q}_1\right)\right\}f(\varepsilon)d\varepsilon$$

$$+\int_{\hat{\varepsilon}_2}^{\overline{\varepsilon}}\left\{\left[\overline{q}_1-x_1\right]p_3(\varepsilon)+f_1*x_1-c_1\left(\overline{q}_1\right)\right\}f(\varepsilon)d\varepsilon$$

$s.t\ f_1=E(P)$

The first order condition

$$G_1 = \int_{\varepsilon_0}^{\hat{\varepsilon}_1} \left\{ \frac{\partial q_1^1(\vec{x},\varepsilon)}{\partial x_1} (p_1(\vec{x},\varepsilon) - 2c_1 q_1^1(p,x_1) - d_1) + q_1^1 \frac{\partial p(\vec{x},\varepsilon)}{\partial x_1} \right\}$$

$$+ \frac{\partial \hat{\varepsilon}_1}{\partial x_1} \left[q_1^1(p) p_1 - c\left(q_1^1(p)\right) \right] \Big|_{\varepsilon=\hat{\varepsilon}_1} f(\hat{\varepsilon}_1) - \frac{\partial \hat{\varepsilon}_1}{\partial x_1} \left[\overline{q}_1 p_2 - c(\overline{q}_1) \right] \Big|_{\varepsilon=\hat{\varepsilon}_1} f(\hat{\varepsilon}_1) = 0$$

$$\Rightarrow \int_{\varepsilon_0}^{\hat{\varepsilon}_1} \left\{ \frac{\partial q_1^1(\vec{x},\varepsilon)}{\partial x_1} (p_1(\vec{x},\varepsilon) - 2c_1 q_1^1(p,x_1) - d_1) + q_1^1 \frac{\partial p(\vec{x},\varepsilon)}{\partial x_1} \right\} + \frac{\partial \hat{\varepsilon}_1}{\partial x_1} \left[\overline{q}_1 p_1(\hat{\varepsilon}_1) - \overline{q}_1 p_2(\hat{\varepsilon}_1) \right] f(\hat{\varepsilon}_1) = 0$$

If we substitute x_c^*, which is the optimal forward contract in symmetric case, then we have $a_1^1 = a_2^1$ and $b_1^1 = b_2^1$. Therefore, $\hat{\varepsilon} = \hat{\varepsilon}_1$. As a result, the first order condition

$$G_1\left(x_i^*\right) = \underbrace{\int_{\varepsilon_0}^{\hat{\varepsilon}} R(\varepsilon) f(\varepsilon) d\varepsilon}_{=0} + \underbrace{\frac{\partial \hat{\varepsilon}_1}{\partial x_1}}_{<0} \underbrace{\left[\overline{q}_1 p_1(\hat{\varepsilon}_1) - \overline{q}_1 p_2(\hat{\varepsilon}_1) \right]}_{<0} f(\hat{\varepsilon}_1) > 0$$

- For firm 2, x_2 is decided by

$$\max_{x_2} \ \mathrm{E}\pi_2 = \int_{\varepsilon_0}^{\hat{\varepsilon}_1} \left\{ \left[q_2^1(p_1(\varepsilon), x_2) - x_2 \right] p_1(\varepsilon) + f_2 * x_2 - c_2\left(q_2^1(p_1(\varepsilon), x_2) \right) \right\} f(\varepsilon) d\varepsilon$$

$$+ \int_{\hat{\varepsilon}_2}^{\hat{\varepsilon}_3} \left\{ \left[q_2^2(p_2(\varepsilon), x_2) - x_2 \right] p_2(\varepsilon) + f_2 * x_2 - c_2\left(q_2^2(p_2(\varepsilon), x_2) \right) \right\} f(\varepsilon) d\varepsilon$$

$$+ \int_{\hat{\varepsilon}_3}^{\overline{\varepsilon}} \left\{ \left[\overline{q}_2 - x_2 \right] p_3(\varepsilon) + f_2 * x_2 - c_2(\overline{q}_2) \right\} f(\varepsilon) d\varepsilon$$

$s.t\ f_2 = E(P)$

The first order condition

$$G_2 = \int_{\varepsilon_0}^{\hat{\varepsilon}_1} \left\{ \frac{\partial q_2^1(\vec{x},\varepsilon)}{\partial x_2} (p_1(\vec{x},\varepsilon) - 2c_2 q_2^1(p_1,x_2) - d_2) + q_2^1 \frac{\partial p_1(\vec{x},\varepsilon)}{\partial x_2} \right\} f(\varepsilon) d\varepsilon$$

$$+ \frac{\partial \hat{\varepsilon}_1}{\partial x_2} \left[q_2^1(p_1(\hat{\varepsilon}_1)) p_1(\hat{\varepsilon}_1) - q_2^2(p_2(\hat{\varepsilon}_1)) p_2(\hat{\varepsilon}_1) + c_2\left(q_2^2(p_2)\right) - c_2\left(q_2^1(p_1(\hat{\varepsilon}_1))\right) \right] f(\hat{\varepsilon}_1)$$

$$+ \int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}_2} \left\{ \frac{\partial q_2^2(x_2,\varepsilon)}{\partial x_2} (p_2(x_2,\varepsilon) - 2c_2 q_2^2(p_2,x_2) - d_2) + q_2^2 \frac{\partial p_2(x_2,\varepsilon)}{\partial x_2} \right\} f(\varepsilon) d\varepsilon = 0$$

If we substitute x_c^*, which is the optimal forward contract in symmetric case, then the first order condition becomes

$$G_2\left(x_c^*\right)=\frac{\partial\hat{\varepsilon}_1}{\partial x}\left[\overline{q}_1 p_1\left(\hat{\varepsilon}_1\right)-q_2^2(p_2\left(\hat{\varepsilon}_1\right))p_2\left(\hat{\varepsilon}_1\right)+c_2\left(q_2^2(p_2)\right)-c_2\left(\overline{q}_1\right)\right]f\left(\hat{\varepsilon}_1\right)$$

$$+\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}}\left\{\frac{\partial q_2^2(x,\varepsilon)}{\partial x}(p_2(x,\varepsilon)-2c_2q_2^2(p_2,x)-d_2)+q_2^2\frac{\partial p_2(x,\varepsilon)}{\partial x}\right\}f(\varepsilon)d\varepsilon$$

$$=\underbrace{\frac{\partial\hat{\varepsilon}_1}{\partial x}}_{<0}\underbrace{\left[q_2^1\left(p_1\left(\hat{\varepsilon}_1\right)\right)p_1\left(\hat{\varepsilon}_1\right)-q_2^2(p_2\left(\hat{\varepsilon}_1\right))p_2\left(\hat{\varepsilon}_1\right)+c_2\left(q_2^2(p_2)\right)-c_2\left(q_2^1\left(p_1\left(\hat{\varepsilon}_1\right)\right)\right)\right]}_{=A>0}f\left(\hat{\varepsilon}_1\right)$$

$$+\int_{\hat{\varepsilon}_1}^{\hat{\varepsilon}}\underbrace{\left\{\frac{\partial q_2^2(x,\varepsilon)}{\partial x}(p_2(x,\varepsilon)-2c_2q_2^2(p_2,x)-d_2)+q_2^2\frac{\partial p_2(x,\varepsilon)}{\partial x}\right\}}_{<0}f(\varepsilon)d\varepsilon<0$$

Appendix C: The Proofs for Chapter 3

- **Probabilities Used in The Proofs**
  - **Probability of enter by a firm other than firm i**

    $\Pr ob(\textit{firm j is in the market}) = 2x(1-x)$

  - **There are K out of N firms with extra demand or surplus**

    $\Pr ob(\textit{k out of N firms in the exchange market} \mid \textit{firm i is in the market})$

    $=C_{N-1}^{k-1}\left(2x\left(1-x\right)\right)^{k-1}\left(1-2x\left(1-x\right)\right)^{N-k}$

  - **Give K firms in the market and firm i is in the market too, the firms net product can take two values**

$$\left(y_i - Q_i\right) = \begin{cases} \text{R with } 1/2 \\ \text{-R with } 1/2 \end{cases} \qquad \forall i$$

Let m = number of firms with $\left(y_i\text{-}Q_i\right)>0$ and N-m = number of firms with $\left(y_i\text{-}Q_i\right)\leq 0$

$\Pr ob(\#\textit{of firms in the exchange market with net production} \text{ is } R = m$
$> \#\textit{of firms with net production is} - R = (K-m) \mid \textit{firm i with net production} \text{ is } R)$

$= \Pr ob(m > \frac{K}{2} \mid \textit{firm i with net product} \text{ is R})$

$$= \sum_{m=\mathrm{sub}(K/2)+1}^{K} C_{K-1}^{m-1}\left(1/2\right)^{m-1}\left(1/2\right)^{K-m} = \sum_{m=\mathrm{sub}(K/2)+1}^{K} C_{K-1}^{m-1}\left(1/2\right)^{K-1} = \alpha_1\left(K\right)$$

$\Pr ob(\#\textit{of firms in the exchange market with net production} \text{ is } R = m$
$< \#\textit{of firms with net production is} - R = (K-m) \mid \textit{firm i with net production} \text{ is } R)$

$= \Pr ob(m < \frac{K}{2} \mid \textit{firm i with net product} \text{ is R})$

$$= \sum_{m=1}^{\sup(K/2)-1} C_{K-1}^{m-1}\left(1/2\right)^{m-1}\left(1/2\right)^{K-m} = \sum_{m=1}^{\sup(K/2)-1} C_{K-1}^{m-1}\left(1/2\right)^{K-1} = \alpha_3\left(K\right)$$

$\Pr ob(\# \textit{of firms in the exchange market with net production}$ is $R = m$

$= \# \textit{of firms with net production is} - R = (K - m) \mid \textit{firm i with net product} = R)$

$= \Pr ob(m = \frac{n}{2} \mid \textit{firm i with net product} = R)$

$$=1-\left\{ \sum_{m=\mathrm{sub}(K/2)+1}^{K} \mathrm{C}_{\mathrm{K-1}}^{\mathrm{m-1}} \left(\tfrac{1}{2} \right)^{K-1} + \sum_{m=1}^{\sup(K/2)-1} \mathrm{C}_{\mathrm{K-1}}^{\mathrm{m-1}} \left(\tfrac{1}{2} \right)^{K-1} \right\} = \alpha_2 \left(K \right)$$

$\Pr ob(\# \textit{of firms in the exchange market with net production}$ is $R = m$

$> \# \textit{of firms with net production is} - R = (K - m) \mid \textit{firm i with net production}$ is $-R)$

$= \Pr ob(m > \frac{K}{2} \mid \textit{firm i with net product}$ is -R)$=\alpha_4 = \alpha_3 \left(K \right)$

$\Pr ob(\# \textit{of firms in the exchange market with net production}$ is $R = m$

$< \# \textit{of firms with net production is} - R = (K - m) \mid \textit{firm i with net production}$ is $-R)$

$= \Pr ob(m < \frac{K}{2} \mid \textit{firm i with net product}$ is -R)$=\alpha_6 = \alpha_1 \left(K \right)$

$\Pr ob(\# \textit{of firms in the exchange market with net production}$ is $R = m$

$= \# \textit{of firms with net production is} - R = (K - m) \mid \textit{firm i with net product} = R)$

$= \Pr ob(m = \frac{n}{2} \mid \textit{firm i with net product} = -R)=\alpha_5 = \alpha_2 \left(K \right)$

- **The probability for the realization of revenue for firm i based on firm i's information**

$$\begin{aligned}
&\Pr ob(\pi_i = PR \mid i) \\
&= \Pr ob(y_i = R,\ Q_i = R) \\
&+ \Pr ob(y_i = R,\ Q_i = 0,\ \sum_{i=1}^{N} y_i < \sum_{i=1}^{N} Q_i) + \Pr ob(y_i = 0,\ Q_i = R,\ \sum_{i=1}^{N} y_i > \sum_{i=1}^{N} Q_i) \\
&= x\beta_i + x\left(1-\beta_i\right) \sum_{k=1}^{N} \left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x\left(1-x\right) \right)^{k-1} \left(1 - 2x\left(1-x\right) \right)^{N-k} \alpha_3\left(K\right) \right\} \\
&+ \left(1-x\right)\beta_i \sum_{k=1}^{N} \left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x\left(1-x\right) \right)^{k-1} \left(1 - 2x\left(1-x\right) \right)^{N-k} \alpha_4\left(K\right) \right\}
\end{aligned}$$

$$\Pr ob(\pi_i = \frac{1}{2} PR \mid i)$$

$$= \Pr ob(y_i = R,\ Q_i = 0,\ \sum_{i=1}^{N} y_i = \sum_{i=1}^{N} Q_i) + \Pr ob(y_i = 0,\ Q_i = R,\ \sum_{i=1}^{N} y_i = \sum_{i=1}^{N} Q_i)$$

$$= x(1-\beta_i)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} \alpha_2(K)\right\}$$

$$+(1-x)\beta \sum_{k=1}^{N}\left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} \alpha_5(K)\right\}$$

$$t_3 = \Pr ob(\pi_i = 0 \mid i) = 1 - \Pr ob(\pi_i = \mathrm{PR} \mid i) - \Pr ob(\pi_i = \frac{1}{2} PR \mid i)$$

- **The probability for the realization of revenue for firm i based on outsider's information**

$$\Pr ob(\pi_i = PR \mid i)$$

$$= \Pr ob(y_i = R,\ Q_i = R) + \Pr ob(y_i = R,\ Q_i = 0,\ \sum_{i=1}^{N} y_i < \sum_{i=1}^{N} Q_i)$$

$$+ \Pr ob(y_i = 0,\ Q_i = R,\ \sum_{i=1}^{N} y_i > \sum_{i=1}^{N} Q_i)$$

$$= xE(\beta_i) + x\left(1 - E(\beta_i)\right)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} \alpha_3(K)\right\}$$

$$+(1-x)E(\beta_i)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} \alpha_4(K)\right\}$$

$$\Pr ob(\pi_i = \frac{1}{2} PR \mid i)$$

$$= \Pr ob(y_i = R,\ Q_i = 0,\ \sum_{i=1}^{N} y_i = \sum_{i=1}^{N} Q_i) + \Pr ob(y_i = 0,\ Q_i = R,\ \sum_{i=1}^{N} y_i = \sum_{i=1}^{N} Q_i)$$

$$= x\left(1 - E(\beta_i)\right)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} \alpha_2(K)\right\}$$

$$+(1-x)E(\beta_i)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} \alpha_5(K)\right\}$$

$$E(t_3) = \Pr ob(\pi_i = 0 \mid i) = 1 - \Pr ob(\pi_i = \mathrm{PR} \mid i) - \Pr ob(\pi_i = \frac{1}{2} PR \mid i)$$

- **The Proofs**

- **The Proof for Lemma 1**

$$t_3 = \Pr ob(\pi_i = 0 \mid i) = 1 - \Pr ob(\pi_i = \text{PR} \mid i) - \Pr ob(\pi_i = \frac{1}{2} PR \mid i)$$

$$= 1 - x\beta_i - x(1-\beta_i)\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_3\right\}$$

$$-(1-x)\beta_i\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_4\right\}$$

$$-x(1-\beta_i)\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_2\right\}$$

$$-(1-x)\beta_i\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_5\right\}$$

$$= 1 - x\beta_i - x(1-\beta_i)\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}(\alpha_3+\alpha_2)\right\}$$

$$-(1-x)\beta_i\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}(\alpha_4+\alpha_5)\right\}$$

$$= 1 - x\beta_i - \left[x(1-\beta_i)+(1-x)\beta_i\right]\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}(1-\alpha_1)\right\}$$

$$\Rightarrow \frac{\partial t_{i3}}{\partial \beta_i} = -x - [1-2x]\sum_{k=1}^{N}\left\{C_{N-1}^{k-1}\left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}(1-\alpha_1)\right\}$$

$$= -x - [1-2x]\underbrace{\Pr ob\left(\sum y_i \le \sum Q_i \mid y_i = R, Q_i = 0\right)}_{<1/2} < 0$$

Therefore, $\frac{\partial t_{i3}}{\partial \beta_i} < 0$

Since $\pi_i = V_{1i} + (1-t_{3i})\left(V_{2i} - 2B\right)$, which is decreases in t_{3i}, then $\frac{\partial \pi_i}{\partial \beta_i} > 0$

- **The Proof for Lemma 2**

$$t_3 = \Pr ob(\pi_i = 0 \mid i) = 1 - \Pr ob(\pi_i = \text{PR} \mid i) - \Pr ob(\pi_i = \frac{1}{2}PR \mid i)$$

$$= 1 - x\beta_i - x(1-\beta_i)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_3 \right\}$$

$$-(1-x)\beta_i\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_4 \right\}$$

$$-x(1-\beta_i)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_2 \right\}$$

$$-(1-x)\beta_i\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}\alpha_5 \right\}$$

$$= 1 - x\beta_i - x(1-\beta_i)\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}(\alpha_3+\alpha_2) \right\}$$

$$-(1-x)\beta_i\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \left(2x(1-x)\right)^{k-1}\left(1-2x(1-x)\right)^{N-k}(\alpha_4+\alpha_5) \right\}$$

$$= 1 - x\beta_i - \left[x(1-\beta_i)+(1-x)\beta_i\right]\sum_{k=1}^{N}\left\{ \mathrm{C}_{\text{N-1}}^{\text{k-1}} \Big(\underbrace{2x(1-x)}_{r}\Big)^{k-1}\Big(\underbrace{1-2x(1-x)}_{1-r}\Big)^{N-k}(1-\alpha_1) \right\}$$

$$t_{13}(N+1) - t_{13}(N)$$

$$= -\left[x(1-\beta_i)+(1-x)\beta_i\right]\frac{1}{1-r}$$

$$\underbrace{\sum_{k=1}^{N+1}\left\{ \mathrm{C}_{\text{N}}^{\text{k-1}} (r)^{k-1}(1-r)^{N+1-k}(1-\alpha_1(k))\left(\frac{K-1}{N}-r\right)\right\}}_{B}$$

we can show that $\alpha_1(k+1) > \alpha_1(k)\ \forall k$ is odd and $\alpha_1(k) = \frac{1}{2}\ \forall k$ is even

Moreover, $\alpha_1(1) = 1$ and $\alpha_1(3) = \frac{1}{4}$

$$B = \frac{1}{4}\sum_{k=1}^{N+1}\left\{ \mathrm{C}_{\text{N}}^{\text{k-1}} (r)^{k-1}(1-r)^{N+1-k}\left(\frac{K-1}{N}-r\right)\right\}$$

$$+\sum_{k=1}^{N+1}\left\{ \mathrm{C}_{\text{N}}^{\text{k-1}} (r)^{k-1}(1-r)^{N+1-k}\left(\frac{3}{4}-\alpha_1(k)\right)\left(\frac{K-1}{N}-r\right)\right\}$$

We can prove that $\sum_{k=1}^{N+1}\left\{\mathrm{C}_{\mathrm{N}}^{\mathrm{k-1}}\left(r\right)^{k-1}\left(1-r\right)^{N+1-k}\left(\frac{K-1}{N}-r\right)\right\}=0$

For $\sum_{\mathrm{k=1}}^{\mathrm{N+1}}\left\{\mathrm{C}_{\mathrm{N}}^{\mathrm{k-1}}\left(\mathrm{r}\right)^{\mathrm{k-1}}\left(\mathrm{1-r}\right)^{\mathrm{N+1-k}}\left(\frac{3}{4}\text{-}\alpha_1\left(\mathrm{k}\right)\right)\left(\frac{\mathrm{K-1}}{\mathrm{N}}\text{-r}\right)\right\}$,we can take out $\left(\frac{3}{4}\text{-}\alpha_1\left(5\right)\right)$, then

$$B=\left(\frac{3}{4}\text{-}\alpha_1\left(5\right)\right)\sum_{k=1}^{N+1}\left\{\mathrm{C}_{\mathrm{N}}^{\mathrm{k-1}}\left(r\right)^{k-1}\left(1-r\right)^{N+1-k}\left(\frac{K-1}{N}-r\right)\right\}$$
$$+\sum_{k=1}^{N+1}\left\{\mathrm{C}_{\mathrm{N}}^{\mathrm{k-1}}\left(r\right)^{k-1}\left(1-r\right)^{N+1-k}\left(\alpha_1\left(5\right)-\alpha_1\left(k\right)\right)\left(\frac{K-1}{N}-r\right)\right\}$$

We can continue to take out the coefficient for the term for k is odd until $k<1+Nr$ (negative terms are taken out), then $B>0$

Therefore, $t_{13}\left(N+1\right)-t_{13}\left(N\right)<0$

- **The Proof for Lemma 3**

The proof for Lemma 3 follows the prove in Lemma 3.

- **Proposition 1**

Proof follows directly by comparing equation (4.3) to equation (3.5).

- **Proposition 2.**

Assume there exists a cut-off point, $\hat{\beta}$, such that firm i does not buy derivative if $\beta_i>\hat{\beta}$. Then the outsider's belief of local market risk for the firms ask for credit-default swap is

$$E_o\left(\beta_i\right)=\frac{\underline{\beta}+\hat{\beta}}{2}$$

and probability of default is

$$t_O=1-xE_O\left(\beta_i\right)-x\left(1-E_O\left(\beta_i\right)\right)a-\left(1-x\right)E_O\left(\beta_i\right)r$$

Thus, the firm with local demand risk, $\hat{\beta}$, is indifference between buying credit- default swap and not buying, i.e.

$$E_i\left(V\mid T=0\right)=E_i\left(V\mid T=t_0\left(\hat{\beta}\right)B\right)$$

Therefore, we know that

$$\hat{\beta} \le \frac{\theta \underline{\beta}}{2-\theta} + \frac{2(1-\theta)(1-xh)}{x+h(1-2x)}$$

where $\theta = \dfrac{B}{V_2}$ and $h = \sum_{k=1}^{N} \left\{ \mathrm{C}_{\mathrm{N-1}}^{\mathrm{k-1}} \left(2x(1-x)\right)^{k-1} \left(1-2x(1-x)\right)^{N-k} (\alpha_3 + \alpha_4) \right\}$

- **The Proof for Proposition 3**

When firms can truthfully reveal their type, the social net surplus is

$$W_0 = \underbrace{\sum_i \left[T(\beta_i) - T(\beta_i)\right]}_{\text{insurance provider's surplus}} + \underbrace{\sum_i \left[V_1(\beta_1) + V_2(\beta_2) - B + T(\beta_i) - T(\beta_i)\right]}_{\text{firm's surplus}}$$

$$= \sum_i \left[V_1(\beta_1) + V_2(\beta_2) - B\right]$$

when $\hat{\beta} \ge 1$, then the social net surplus is

$$W\left(\hat{\beta} \ge 1\right) = \underbrace{\sum_i \left[T(x) - T(\beta_i)\right]}_{\text{insurance provider's surplus}} + \underbrace{\sum_i \left[V_1(\beta_1) + V_2(\beta_2) - B + T(\beta_i) - T(x)\right]}_{\text{firm's surplus}}$$

$$= \sum_i \left[V_1(\beta_1) + V_2(\beta_2) - B\right]$$

Compare W_0 and $W\left(\hat{\beta} \ge 1\right)$, there is no social net surplus.

When $\hat{\beta} < 1$, then the social net surplus is

$$W\left(\hat{\beta} < 1\right) = \underbrace{\sum_{i \in \left\{\beta_i < \hat{\beta}\right\}} \left[T(x) - T(\beta_i)\right]}_{\text{insurance provider's surplus}} + \underbrace{\sum_{i \in \left\{\beta_i < \hat{\beta}\right\}} \left[V_1(\beta_1) + V_2(\beta_2) - B + T(\beta_i) - T(x)\right]}_{\text{the net suplus for firms with } \beta_i < \hat{\beta}}$$

$$+ \underbrace{\sum_{i \in \left\{\beta_i > \hat{\beta}\right\}} \left[V_1(\beta_1) + \left(1 - t(\beta_i)\right) V_2(\beta_2) - \left(1 - t(\beta_i)\right) B\right]}_{\text{the net suplus for firms with } \beta_i > \hat{\beta}}$$

$$= \sum_i \left[V_1(\beta_1) + V_2(\beta_2) - B\right] - \sum_{i \in \left\{\beta_i > \hat{\beta}\right\}} \left[t(\beta_i)\left(V_2(\beta_2) - B\right)\right]$$

Compare W_0 and $W\left(\hat{\beta} < 1\right)$, the social net surplus decreases $\sum_{i \in \left\{\beta_i > \hat{\beta}\right\}} \left[t(\beta_i)\left(V_2(\beta_2) - B\right)\right]$.

- **The Proof for Proposition 4**

Proof: $\hat{\beta}$ is increasing in $w = \dfrac{2(1-\theta)(1-xh)}{x+h(1-2x)}$, which varies as N changes. We can rewrite $w = \dfrac{2(1-\theta)}{1+\dfrac{(1-x)(1-xh)}{(1-xh)}}$. We can show that as N increases, h increases as showed in Lemma 2. Moreover, $\hat{\beta}$ rises as h gets bigger. Therefore $\hat{\beta}$ decreases as N increases. QED.

- **The Proof for Lemma 6**

It is easy to show by $L_i = f_i V_{1i} > B$.

References

Allaz, B. 1992. Oligopoly, uncertainty and strategic forward transactions. International. Journal of Industrial Organization. 10(2) 297–308.

Allaz, B., Vila, J. L. 1993. Cournot Competition, Forward Markets and Efficiency. Journal of Economic Theory, 59 1-16.

Baldick, R., Grant, R., Kahn, E. 2004. Theory and Application of Linear Supply Function Equilibrium in Electricity Markets. Journal of Regulatory Economics. 25(2) 143-167.

Borenstein, S. 2001. The trouble with electricity markets and California electricity restructuring disaster. POWER Paper. PWP-081, University of California Energy Institute.

Borenstein, S., Bushnell, J. B., Wolak, F. A. 2002. Measuring Market Inefficiencies in California's Restructured Wholesale Electricity Market. American Economic Review. 92(5), 1376-1405.

Borio, C. 2004. Market distress and vanishing liquidity: anatomy and policy options. BIS working paper.

Brander, J. A., Lewis, T. R. 1986. Oligopoly and Financial Structure: The Limited Liability Effect. American Economic Review. 76 (5), 956-970.

Bushnell, J., Mansur, E., Saravia, C. 2004. Market Structure and Competition: A Cross-Market Analysis of U.S. Electricity Deregulation. Rep. No. CSEM WP-126, University of California Energy Institute.

Diamond, D.W. 1991. Debt maturity Structure and Liquidity Risk, Quarterly Journal of Economics. 106(3) 709-737.

Froot, K., Scharfstein, D., Stein, J. 1993. Risk Management: Coordinating Corporate Investment and Financing Policies. Journal of Finance. 48(5) 1629-1658.

Green, R., Newbery, D. 1992. Competition in the British Electricity Spot Market. Jounrnal of Political Economy. 100(5) 929-953.

Guo, Z, Fang, F., Whinston, A. B. 2004. Supply Chain Information Sharing in a Macro Prediction Market1. Working Paper.

Hortacsu, A., Puller, S. L. 2005. Understanding Strategic Bidding In Restructed Electricity Markets: A Case Study Of ERCOT. Working Paper No. 11123, National Bureau of Economic Research.

Hughes, J. S., Kao J. L. 1997. Strategic forward contracting and observability. International Journal of Industrial Organization. 16 121-133.

Hull, J., White, A. 2000. Valuing Credit Default Swaps I. No counterparty Default Risk. Working paper.

Jensen, M., Meckling, W. 1976. Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure. Journal of Financial Economics. 4 305–360.

Klemperer, P. D., Meyer, M. A. 1989. Supply function equlibria in oligopoly under uncertainty. Econometrica. 57(6) 1243-1277.

Kyle, A. S. 1989. Informed Speculation with Imperfect Competition. Review of Economic Studies. 56(3) 317-355.

Lee, H., Whang, S. 2002. The Impact of the Secondary Market on Supply Chain. Management Science. 48(6) 719-731.

Lin, D., Fang, F., Whinston, A.B. 2006. Capital Structure, Demand Uncertainty and the B2B Exchange Market. Working Paper.

Modigliani, F., Miller, M.H. 1958. Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure. Journal of Financial Economics. 4 261-297.

Newbery, D. M. 1998. Competition, contracts, and entry in the electricity spot market. RAND Journal of Economics. 29(4) 726-749.

O'Hara, M. 1997. Market Microstructure Theory", Blackwell Publishers Ltd.

Persaud, Avinash D. 2003. Liqudity Black Holes. Risk Books, a division of Incisive RWG Ltd.

Pennacchi, G. G., Gorton, G. B. 1995. Banks and Loan Sales Marketing Nonmarketable Assets. Journal Monetary Economics. 35(3) 389-411.

Ross, S. 1977. The determinants of Financial Structure: The Incentive-signaling Approach. Bell Journal of Economics. 8 23-40.

Showalter, D.M. 1995. Oligopoly and financial structure: comment. American Economic Review, 85(3) 16-34.

Spano, M. 2004. Determinants of Hedging and Its Effects on Investment and Debt. Journal of Corporate finance. 10(1) 175-197.

Stiglitz, J. E., Weiss, A. 1981. Credit Rationing in Markets with Imperfect Information. American Economic Review. 71(3) 393-410.

Taleb, N. N. 2005. Fooled by Randomness. Random House.

J.P. Morgan. 1999. The J.P. Morgan Guide to Credit Derivatives: With Contributions from the RiskMetrics Group. The RiskMetrics Group.

Tumbull, S. J. 1983. Choosing Duopoly solutions by Consistent Conjectures and by Uncertianty. Economics Letters. 13 253-258.

Willems, B. 2005. Cournot Competition, Financial Option Markets and Efficiency. Center for the Study of Energy Markets. CSEM WP 139.

Wilson, R. 1979. Auction of shares. Quarterly Journal of Economics. 93(4) 675-689.

Wolak, F. 2003. Measuring Unilateral Market Power in Wholesale Electricity Markets: The California Market 1998 - 2000. Center for the Study of Energy Markets. Paper CSEMWP-114.

Wolak, F. 2001, A Comprehensive Market Power Mitigation Plan for the California Electricity Market.

Vita

Dan Lin was born in Wuhan, China on May 14, 1975, the daughter of Shimin Lin and Xiaomi Guo. After getting her Bachelor Degree in Economics from Huazhong University of Science & Technology in Wuhan, China, she entered University of North Texas in Denton, TX. She received the degree of Master Degree of Economics in July, 1997. In August 2001, she entered the Graduate School of The University of Texas. She is currently working as a senior consultant for Deloitte Tax LLP Dallas office.

Permanent address: 687 E. Royal Ln, Irving, TX 75039

This dissertation was typed by Dan Lin.