

## New redshifts of circumpolar southern galaxies

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Redshifts of nine galaxies south of  $\delta = -60^\circ$  have been determined from ten spectrograms obtained with the Page-Carnegie image-tube spectrograph attached at the Newtonian focus of the Cordoba Observatory 154-cm reflector. The reciprocal dispersions are  $150 \text{ \AA/mm}$  at  $H\alpha$  in the first order and  $78 \text{ \AA/mm}$  at  $\lambda 3727$  in the second order. The derived velocities have mean errors of  $\sim 40 \text{ km sec}^{-1}$ . The velocities are used to discuss group membership. One of the smaller and fainter galaxies, NGC 2915, has the remarkably low corrected velocity  $V_0 = +154 \text{ km sec}^{-1}$  (two plates).

### INTRODUCTION

THE scarcity of radial-velocity data for galaxies at high southern declinations hinders studies of the extragalactic velocity field in the southern hemisphere and, in particular, the search for possible departures from the ideal linear-isotropic expansion law (de Vaucouleurs 1958, 1964, 1966, 1972; de Vaucouleurs and Peters 1969). Data are especially rare for circumpolar objects with  $\delta < -60^\circ$  which are completely inaccessible from northern observatories. Only a few have been previously observed, mainly from Mt. Stromlo (G. and A. de Vaucouleurs 1961, Shobbrook 1966, Ford 1968), Pretoria (Evans 1967, 1969) and Cordoba (Page 1967, Carranza 1967). We report here on velocity determinations of nine southern circumpolar galaxies observed in February 1970 with the Page-Carnegie image-tube nebular spectrograph attached at the Newtonian focus of the 154-cm reflector of the Bosque Alegre Station of Cordoba University Observatory, Argentina. The basic spectrograph, designed by Page, is very similar to the McDonald Observatory B spectrograph (Page 1952), except that the semisolid Schmidt camera is replaced by a special 3-element flat-field Schmidt combination, designed by Bowen, and the detector is a Carnegie image converter (RCA C33011 tube) followed by an Elgeet  $f/1.2$  transfer lens forming the final image on a photographic plate (Page 1967).

The dispersing element is a square 110-mm Bausch and Lomb grating with 400 lines/mm blazed for  $\lambda 6900 \text{ \AA}$  (first order) and  $\lambda 3450 \text{ \AA}$  (second order). The incidence is such that the best parts of the first and second-order spectra are recorded side-by-side on the plate; the first order covers the wavelength range  $4040 < \lambda < 7240 \text{ \AA}$ , the second order, the range  $3650 < \lambda < 4360 \text{ \AA}$ . The reciprocal dispersion on the plate is  $150 \text{ \AA mm}^{-1}$  at  $H\alpha$  in the first order,  $78 \text{ \AA mm}^{-1}$  at  $\lambda 3727$  in the second order. The linear scale along the entrance slit is  $27.5 \text{ arcsec mm}^{-1}$  and  $54 \text{ arcsec mm}^{-1}$  on the plate.

The Bosque Alegre station is far enough from the city of Cordoba ( $\sim 50 \text{ km}$  to the north-east of the observatory) that the night sky is still quite dark in the direction of the South Pole; nevertheless, in addition to the usual airglow lines, the stronger Hg lines are

visible on spectrograms exposed for several hours with the image-tube operated at 19 kv. An unusual feature of the airglow spectrum was the presence of the atmospheric  $[\text{NI}] \lambda 5199$  line on several spectrograms taken after local midnight, although no sign of Aurora australis was visually observed. The intensity of the line was roughly half that of the Na (D) line.

### I. OBSERVATIONS AND MEASUREMENTS

Eleven spectrograms of the brighter galaxies south of  $\delta = -60^\circ$  between  $5^{\text{h}}30$  and  $10^{\text{h}}30$  of right ascension were obtained on preflashed IIA-0 plates with exposures varying from 45 min to 4 h. One plate (NGC 2417) was underexposed and could not be measured. In addition, the Eta Carinae nebula was observed to determine the dispersion and line-curvature constants of the spectrograph. The comparison source was a Pen-Ray Ne-Hg tube with the Neon lines dominating in the red. The projected slit width was  $30\text{--}35 \mu$  corresponding to  $\sim 5 \text{ \AA}$  in the first order and  $\sim 2.5 \text{ \AA}$  in the second order.

The spectrograms were measured with the digitized two-coordinate Grant profile comparator of Kitt Peak National Observatory. The measuring slit varied from 20 to  $30 \mu$  in projected width and 0.188 to 0.313 mm in length, corresponding to 10–17 arc sec in the focal plane of the telescope. Measurements were made at several ordinates along the slit image, depending on the length of the lines. Four sets of readings were taken for the 32 comparison lines. The airglow and Mercury lines crossing the spectrum were measured at 0.3 or 0.5-mm intervals and the galaxy lines at closer intervals and with a shorter slit when the length of the emission lines warranted a more detailed study.

The determination of the line curvature coefficients and dispersion formulae from the numerous bright emission lines in the spectrum of the Eta Carinae nebula followed the procedure previously outlined for the reduction of the McDonald B spectrograms (A. and G. de Vaucouleurs 1967). The apparent velocity of the airglow and Mercury lines on each plate was applied as a zero-point correction to the galaxy lines. The mean correction, varying from plate to plate, ranged from  $-8 \mu$  to  $+28 \mu$ . In addition, a residual mean correction was applied to the first-order spectrum

only varying with wavelength from  $-36 \text{ km sec}^{-1}$  at  $\lambda 5460$  to  $+28 \text{ km sec}^{-1}$  at  $\lambda 6300$ ; an extrapolated value of  $-60 \text{ km sec}^{-1}$  was applied at  $H\beta$ .

The galaxy lines were assigned weights according to intensity and quality and a weighted mean velocity determined for each order. A mean systematic difference of  $+71 \text{ km sec}^{-1}$  between the first and second order was present, but we could find no reason to give preference to one or the other order (the first order has more comparison and airglow lines, the second order more dispersion but is closer to the edge of the field). The weights for the second order were simply multiplied by  $4/3$  to take into account the double dispersion and wavelength dependence of the Doppler shift. The mean errors of lines of different weights showed that the weighting system was consistent (that is,  $W \propto \sigma^{-2}$ ) and that the mean error for unit weight is  $\sigma_1 = 90 \text{ km sec}^{-1}$ . (The same value obtained for the McDonald B spectrograms which had lower dispersion, but better images.) The total weight  $\Sigma W$  of the first and second order is given in Col. (13) of Table I and the internal mean error  $\sigma_1/\sqrt{\Sigma W}$  of the mean velocities in Col. (16). There is only one pair of plates for the same object NGC 2915 (see notes to Table I) and one previously published value, that for NGC 3059 (Page 1967), so that external mean errors cannot be calculated at present.

The redshifts for nine southern galaxies are listed in Table I as follows:

*Column (1)*: NGC or IC number.

*Columns (2) and (3)*: Right ascension and declination for 1950.

*Columns (4) and (5)*: Galactic longitude and latitude  $l, b$ .

*Column (6)*: Revised classification from the "Reference Catalogue of Bright Galaxies" (G. and A. de Vaucouleurs 1964).

*Columns (7) and (8)*: Date and exposure time.

*Columns (9) and (10)*: Slit length and position angle.

*Columns (11) and (12)*: Number of lines measured in emission and in absorption with identifications given in the Notes.

*Column (13)*: Sum of weights assigned to galaxy lines.

*Column (14)*: Observed (heliocentric) mean weighted velocity in  $\text{km sec}^{-1}$ .

*Column (15)*: Corrected (galactocentric) velocity with the  $\Delta V$  values listed in Reference Catalogue.

*Column (16)*: Internal mean error of the velocity  $= 90/\sqrt{\Sigma W}$ .

*Column (17)*: Refers to Notes at the end of the Table. The notes give the identification of the lines measured and discuss possible group membership with nearby bright galaxies.

## II. SOUTHERN GROUPS OF GALAXIES

Short as the list in Table I is, it increases significantly the redshift material for bright galaxies in this poorly

TABLE I. Redshifts of southern galaxies.

Object	RA (1950)	Dec (1950)	$l$	$b$	Type	Date (UT) 1970	Exp.	Slit L p.a.	No. lines Em. Abs.	$\Sigma W$	$V$	$V_0$	m.e.	Notes	
NGC 2082	5 <sup>h</sup> 41 <sup>m</sup> 6	-64°20'	273°8	-31°8	SB(r)b	7 Feb	2 <sup>h</sup> 30 <sup>m</sup>	5' 101°	4 1	3.75	1114	859	47	(1)	
2369	7 16.0	-62 16	273.3	-21.0	SB(s)a	9 Feb	3 30	5 180	3 3	5.16	3306	3026	40	(2)	
2397	7 21.5	-68 54	280.3	-22.6	SB(s)ab	10 Feb	3 30	5 130	3 1	3.50	1363	1092	48	(3)	
2466	7 45.6	-71 17	283.5	-21.5	SAC:	12 Feb	3 32	5 101	3 2	4.16	5171	4902	43	(4)	
2601	8 25.2	-67 57	282.0	-16.9	LA:	8 Feb	4 00	5 101	4 ...	3.33	3244	2965	49	(5)	
2788	9 08.3	-67 44	284.0	-13.5	S:sp.	9 Feb	3 30	5 120	2 2	3.66	1548	1268	47	(6)	
2915	9 26.5	-76 25	292.0	-18.4	SAab:	6 Feb	1 10	5 99	3 ...	2.83		414	154	32	(7)
2915	9 26.5	-76 25	292.0	-18.4	SAab:	7 Feb	0 45	5 101	4 ...	5.16					
3059	9 49.5	-73 41	291.1	-15.4	SB(rs)bc	12 Feb	3 45	5 45	5 2	7.16	1232	967	34	(8)	
IC 2554	10 07.5	-66 48	288.0	-9.0	SB(s)cp	10 Feb	3 35	5 20	4 2	6.50	1377	1099	35	(9)	

### Notes

(1) N 2082—Faint  $H\beta$ ,  $H\alpha$ ,  $[NII]$  6584, and  $[OII]$  3727 in emission along bar. H-line in abs. In N 1947 group, an extension of the Dorado Cloud complex including N 1566, N 1672 groups and several others.

(2) N 2369—Inclined  $H\alpha$ ,  $[NII]$  6584, and  $[OII]$  3727 in emission along major axis. H, K, and D lines in absorption. The large velocity places this galaxy in background of Dorado Cloud in a group possibly including N 2369A and B, N 2417, I 2200, I 2200A.

(3) N 2397—Inclined  $H\alpha$ ,  $[NII]$  6584, and  $[OII]$  3727 in emission along major axis. H line in absorption. Possibly in a group including N 2397A, 2434, N 2442, but not N 2466 (see notes for this object).

(4) N 2466— $H\alpha$ ,  $[NII]$  6584 and  $[OII]$  3727 in emission in

bright central bulge. H and K in absorption. Velocity shows it is in background of above group.

(5) N 2601— $H\beta$ ,  $H\alpha$ ,  $[NII]$  6584,  $[OII]$  3727 in emission in nucleus.

(6) N 2788— $H\alpha$ ,  $[OII]$  3727 in emission along major axis. Possibly in N 3136 group.

(7) N 2915—Faint  $[OIII]$  4959 and 5007,  $H\alpha$  and  $[OII]$  3727 in emission in center. Velocity is mean of two plates: (No. 547:  $+395$ ,  $W=2.83$ , No. 549:  $+425$ ,  $W=5.16$ ). Classification uncertain. Note very low-galactocentric velocity.

(8) N 3059— $H\beta$ ,  $H\alpha$ ,  $[NII]$  6584,  $[SII]$  6717-31,  $[OII]$  3727 in emission along bar. H and K in absorption. Possibly in N 3136 group.

(9) IC 2554— $H\beta$ ,  $H\alpha$ ,  $[NII]$  6584,  $[OII]$  3727 in emission along major axis. D and H lines in absorption. In N 3136 group.

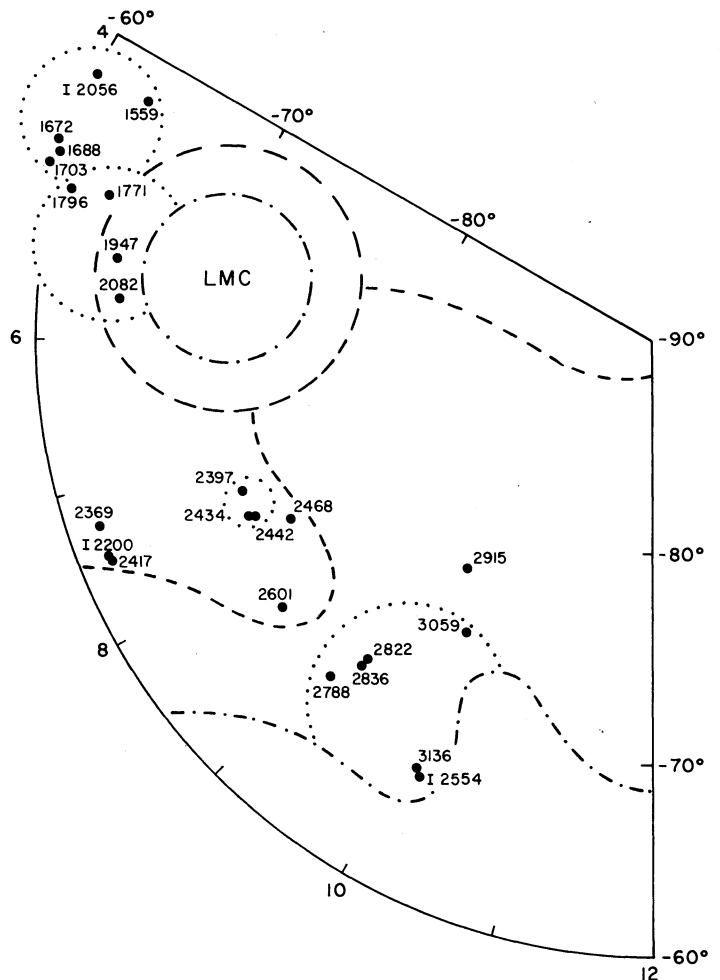


FIG. 1. Groups of southern circumpolar galaxies. The groups discussed in the text are outlined by dotted circles. The dash-dot lines mark the approximate limits of the areas obscured by the Galaxy and the Large Magellanic Cloud; the dash lines mark the edge of the zones of partial obscuration.

populated area of the south polar cap which is covered in part by a great flare of galactic obscuration and in part by the Large Magellanic Cloud (de Vaucouleurs 1956). Only nine galaxies brighter than  $m_H = 13.2$  are listed in the Shapley-Ames catalog between RA  $5^h$  and  $12^h$  south of  $-60^\circ$  (NGC 1796, 1947, 2082, 2369, 2397, 2434, 2442, 3059, 3136); nine other NGC and IC objects are listed in the Reference Catalog (NGC 2417, 2466, 2601, 2788, 2822, 2836, 2915, IC 2200, 2554) to a comparable limit of apparent magnitude or diameter (Fig. 1). We have now velocity data for 14 of these 18 galaxies, nine from this paper, five from Page (1967), both with the Cordoba spectrograph. This degree of completeness compares well with the best observed equivalent areas of the northern sky!

In addition, there is a previous velocity for N 3136 by Evans (1967) at Pretoria, and preliminary values for N 2442 by Carranza (1967) at Cordoba and by Sandage (private communication) at Mt. Stromlo. The

meagre overlap is as follows:

NGC	3059	3136	2442
This paper	967		
Cordoba	940(a)	(810)(a)	(450)(b)
Others		1550 (c)	1122 (d)

(a) Page 1967, (b) Carranza 1967, (c) Evans 1967, (d) Sandage, unpublished.

The preliminary values from Cordoba for N 3136 and N 2442 appear to be in error. The large barred spiral N 2442 forms an apparently interacting pair with the elliptical N 2434 for which Page (1967) finds  $V_0 = +1180$  in good agreement with Sandage's value for N 2442. The latter also agrees with our value  $V_0 = +1092$  for the nearby spiral NGC 2397.

The radial velocity information allows a reexamination of possible membership in galaxy groups previously suggested by the apparent distribution and

similarities of types, diameters and/or magnitudes (de Vaucouleurs 1965, Corwin 1967).

*NGC 1947 group:* NGC 1796, 1947, 2082 (and possibly N 1771) with an average velocity  $\langle V_0 \rangle \simeq +716$  km sec<sup>-1</sup> ( $n=3$  vel.) appear to form a loose group, some 5° in diameter at the north edge of the Large Magellanic Cloud. NGC 1947, a peculiar SO galaxy resembling NGC 5128 (de Vaucouleurs 1956), is the brightest member of the group. This galaxy and N 2082 are seen through the northern outer arm of the LMC whose rich stellar population ( $m > 18$ ) covers both fields. This small group forms an extension toward the south-east of the great, elongated Dorado cloud of bright galaxies which stretches from the Fornax I cluster to the LMC (and possibly beyond, see below). Previously identified galaxy clumpings within this cloud include the NGC 1433, NGC 1566, and NGC 1672 groups (numbered G 21, G 16, and G 22 in de Vaucouleurs 1965) with, respectively, mean velocities  $\langle V_0 \rangle = +665$  ( $n=2$ ),  $+999$  ( $n=5$ ) and  $+1071$  ( $n=2$ ).

NGC 1796 was initially considered as a possible member of the NGC 1672 group, but its lower velocity associates more closely with those of NGC 1947 and 2082. On this basis one might split the NGC 1672 group into two separate groupings, one including N 1559 ( $V_0 = +1108$ ), 1672 ( $V_0 = +1034$ ), 1688, 1783, and possibly I 2056 (no velocities), the other N 1796, 1947, and 2082. On the other hand, the apparent diameters of the three largest galaxies are larger in the N 1672 group for which  $\langle V_0 \rangle = 1071$  than in the N 1947 group for which  $\langle V_0 \rangle = 720$ , leaving the relative distances in doubt.

*NGC 2442 group:* NGC 2397, 2434, 2442 may form a rather compact group (diam  $\sim 2^\circ$ ) with an average velocity  $\langle V_0 \rangle \simeq +1130$  km sec<sup>-1</sup> ( $n=3$ ) on the east side of the Large Magellanic Cloud. This group may, possibly, represent a further extension of the Dorado cloud beyond the LMC, as suggested by the outlines of Region II in the Mt. Stromlo "Reynolds Survey" (de Vaucouleurs 1956), or it may constitute a separate loose association with several other galaxies located still further east and up to the edge of the galactic obscuration belt (see below). The new velocities of several outlying objects in this same general area, including N 2369 ( $V_0 = +3026$ ), N 2466 ( $V_0 = +4902$ ), and N 2601 ( $V_0 = +2965$ ), indicate that they are almost certainly in the background and are not associated with this group.

*NGC 3136 group:* Still further east, several galaxies including N 2788, 3059, 3136, IC 2554 have velocities in the range 950 to 1550 km sec<sup>-1</sup> and may form a loose group, some 10° in diameter with  $\langle V_0 \rangle = +1215$  km sec<sup>-1</sup>. Two other galaxies forming a close pair, N 2822 and N. 2836, may also belong to this group, but no velocities are available.

From the apparent diameters of the largest members of each of the groups discussed above, we derive the following approximate distances and velocity-distance ratios "H":

Group	N 1672	N 1947	N 2442	N 3136
$\Delta$ (Mpc)	17.5 (17.7)	24.7	18.5	13.6
$\langle V_0 \rangle$	1071 (858)	716	1131	1217
"H"	61 (49)	29	61	89
$A_B$ (mag)	0.3	0.4	0.8	1.3

The absorption corrections  $A_B$  listed on the last line are large and rather uncertain for the two groups with the lowest galactic latitudes which are at the edge of the great flare of obscuration covering the south polar cap.

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## Analysis of the neutral hydrogen within 1 kpc of the Sun

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Neutral hydrogen about the LSR is assumed to approximate a plane-parallel layer. Line profiles  $-10^\circ \geq l \geq 250^\circ$  are analyzed using differential motion equations and small nonuniform circular effects are found. Eight latitude positions, symmetric with respect to the plane, yield small velocities ( $< 3$  km/sec) which are found to be functions of  $z$ -position relative to the plane. The result is a "shearing" effect whereby the gas above and below the plane has a slightly different velocity from the gas closer to the plane.

### INTRODUCTION

THIS paper studies the local neutral hydrogen using 21-cm line profiles at galactic latitudes  $b \geq \pm 10^\circ$ . These profiles usually show but one peak, produced by the local gas. A total of 424 such profiles were obtained using the 140-ft telescope at the National Radio Astronomy Observatory in Green Bank, West Virginia. They required a total of 20 hr of observation; one-minute integration and one minute to reach the next position. The positions were at  $5^\circ$  intervals in galactic longitude ( $-10^\circ \leq l \leq 250^\circ$ ) and  $\pm 10^\circ$ ,  $\pm 15^\circ$ ,  $\pm 20^\circ$  and  $\pm 30^\circ$  in galactic latitude.

The receiving instrument was the 100-channel auto-correlation receiver set at 0.625-MHz bandwidth. Thus each line profile was a measure of antenna temperature ( $T_A = \text{ordinate}$ ) as a function of velocity ( $v = \text{abscissa}$ ) where the velocity interval per channel ( $\Delta v$ ) is 1.32 km/sec. A linear base line was hand fitted to each profile and all antenna temperatures were taken relative to it. From this profile, the centroid velocity

$$V = \frac{\sum_i v_i T_{Ai} \Delta v}{\sum_i T_{Ai} \Delta v}$$

was determined for each position. Velocities were taken with respect to the LSR using the standard solar motion of  $20 \text{ km sec}^{-1}$  toward  $\alpha, \delta(1900) = 18^{\text{h}}, +30^\circ$ . Thus the following values of the solar motion were incorporated in the reduction:  $S_0 = 20 \text{ km sec}^{-1}$ ,  $L_0 = 56^\circ.2$ , and  $B_0 = 23^\circ$ . The method employed in these reductions is standard differential rotation kinematics, plotting velocity versus longitude. We will assume (a) a galactic disk in the neighborhood of the Sun which is approximately uniform in thickness, and (b) a some-

what uniform hydrogen density of low-optical depth. From these assumptions all the line profiles taken at a given latitude should correspond to the same distance from the Sun.

### EQUATIONS OF MOTION

The most general equation for the radial velocity due to plane-parallel differential rotation of a region at a distance  $r$  from the Sun is given by the relation (Trumpler and Weaver 1953; Ogorodnikov 1965)

$$V_i = Kr \cos^2 b_i + Ar \cos^2 b_i \sin 2(l_i - h) - V_{\odot i}, \quad (1)$$

where  $l_i$  and  $b_i$  are the respective galactic longitude and latitude positions,  $A$  is Oort's constant, and  $K$  and  $h$  are parameters relating to nonuniform circular motion. The correction for solar motion at the position  $l_i, b_i$  can be written

$$V_{\odot i} = S \cos b_i \cos B \cos(l_i - L) + S \sin b_i \sin B,$$

where  $S, L,$  and  $B$  are the solar values. Since the reduction procedure includes the standard solar correction, this term should be small.

Expanding  $V_{\odot i}$  about  $S_0, L_0$  and  $B_0$ , retaining first order terms, we have

$$V_{\odot i} = V_{\odot i}(S_0, B_0, L_0) + \frac{\partial V_{\odot i}}{\partial S} \Delta S + \frac{\partial V_{\odot i}}{\partial B} \Delta B + \frac{\partial V_{\odot i}}{\partial L} \Delta L.$$

Therefore

$$\begin{aligned} V_i = & Kr \cos^2 b_i + Ar \cos^2 b_i \sin 2(l_i - h) + \sin B_0 \sin b_i \Delta S \\ & + S_0 \sin B_0 \cos b_i \cos(l_i - L_0) \Delta B \\ & - \cos B_0 \cos b_i \cos(l_i - L_0) \Delta S \\ & - S_0 \cos B_0 \sin b_i \Delta B \\ & - S_0 \cos B_0 \cos b_i \sin(l_i - L_0) \Delta L. \quad (2) \end{aligned}$$