The problem of optimization often arises whenever we want to influence a complex system, because we usually want our impact on such a system to be the best it can be in some particular sense. Such complex optimization problems are often accompanied by another problem, however: uncertainty. Specifically, there may be uncertainty in the structure of our mathematical model, uncertainty in various physical constants in our model or uncertainty in the weighting of various conflicting objectives.

The purpose of this dissertation is to develop one particular approach to optimization under uncertainty: parametric nonlinear programming (pNLP). Nonlinear programming is the optimization of a scalar objective function subject to a finite number of equality and inequality constraints over some finite number of variables $u \in \mathbb{R}^n$:

$$\begin{array}{rl} & \min_{u} f(u) \\ s.t. & \\ & h(u) = 0 \\ & g(u) \le 0. \end{array} \tag{1}$$

Parametric nonlinear programming attempts to solve this problem as a function of a finite number of parameters $\alpha \in \mathbb{R}^d$:

$$u^*(\alpha) = \arg\left\{ \begin{array}{rl} & \min_{u} f(u,\alpha) \\ s.t. & \\ & h(u,\alpha) = 0 \\ & g(u,\alpha) \le 0 \end{array} \right\}. \tag{2}$$

This work approaches the pNLP problem numerically with a predictor-corrector (continuation) method. Such methods have been developed for general underdetermined nonlinear equations. Such equations are derived for this problem using the Fritz-John necessary conditions for optimality.

Predictor-corrector methods specifically customized for the single parameter nonlinear programming problem (1-pNLP) have already been developed quite extensively [2, 3], but a comprehensive multi-parameter method has not yet been developed. This work aims to combine some of the 1-pNLP work with Rheinboldt and Brodzik's work on multi-dimensional predictor-correctors in order to allow for global exploration of the mpNLP problem [1, 4]. Specifically, as in the previous work on 1-pNLP, where possible the particular structure of the parametric nonlinear programming problem is exploited to both improve computation times and to yield more optimizationspecific information than a general multi-parametric predictor-corrector algorithm would. The algorithm developed also improves on Brodzik's work by including auto-scaling, step-size adaptation and, most importantly, singularity handling. Further, several verification examples, and several more complex applications are presented and discussed. The applications include implicit optimization model adequacy and multi-objective optimization.

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