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Teaching Functions Through Modeling

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Report

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Abstract

Teaching Functions Through Modeling

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This report discusses topics relating to modeling functions. The pedagogical content knowledge of student teachers and expert teachers and its effect on their ability to teach through modeling is examined. An observed modeling lesson is presented. To conclude, there will be a discussion about the pitfalls of using calculators in modeling and exploration lessons.

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Chapter 1: Introduction

"Why do I have to know this?" is the question that haunts every mathematics teacher across the world. Students have a desire to know the relevance of what they are learning and more rigorous standards require the use of critical thinking skills that procedural mathematics alone will not cultivate. With the increasing pressures from employers and government officials, to have high school graduates "college and career ready," modeling has become a significant piece of secondary mathematics. Even though there is pressure from above for students to apply the mathematics in a conceptual manner to real life scenarios, many teachers are still focusing on procedural mathematics. There must be a way for teachers to balance the procedural with the conceptual so that there will be a significant improvement in the way teachers teach mathematics.

Teaching mathematics through modeling involves a problem or scenario where students must work with collected data. In secondary mathematics the scenarios are set up where there is a functional relationship between two variables, so when students look at the tabular data and the graphical representation they can calculate an equation that represents the data and make certain predictions for various input values not tested. Teachers will often design the questions so that students need to find critical values on the graph or use their equation to explain what those values mean in terms of the scenario.

In Chapter 2 there will be an interview with four teachers conducted by Chinnappan and Thomas. These authors will use their interviews with students and experienced teachers to draw inferences about their schemas. Then, they will consider how those schemas will affect their teaching in turns of modeling. Chinnappan and

Thomas state, "Teachers of mathematics need to have a broad view of mathematics and its learning. They should not be limited to seeing it as primarily skills based, algorithmic subject, nor should they be constrained to thinking in terms of a single representation" [**1**, p. 156]. Because of this view, the authors will speak to the much needed support that novice teachers need during their first few years.

Teachers can be intimidated by the idea of modeling with students. There are specific content requirements that need to be taught and giving students the freedom to explore the mathematics may jeopardize those specific curriculum objectives. In Chapter 3, Chinnappan will observe a classroom where a mathematics lesson employing modeling is taking place. Chinnappan will then analyze the different components of the lesson and look at what made this lesson successful.

In Chapter 4, the discussion will conclude with an overview of a research paper investigating the difficulties that students have when operating a graphing calculator in the context of studying functions. Mitchelmore and Cavanagh state, "The use of graphic calculators can be associated with significant gains" [**3**, p.254]. However, these authors argue that there are several difficulties students have with the operation of the calculators due to a lack of understanding of how the calculator functions. There is a strong push for teachers to use technology with their students so it is very natural for mathematics teachers to use graphing calculators, but it is important to be aware of the problems students might encounter while using a calculator so that teachers can address these issues preemptively.

Chapter 2: Teachers' Role in Modeling

Several studies have been done revealing the connections between the quality of a student's mathematical learning and its significant impact on students' ability to apply this knowledge during problem solving. There is limited knowledge about how teacher subject matter knowledge impacts the quality of learning and about how the role of the teacher knowledge base could drive what students learn.

Chinnappan and Thomas employ a case study to look at the role that teachers' schemas play when choosing what they teach and how they teach it. The aim of this study is to examine the value of the teachers' schemas, by observing both the quantity and the quality of the connections they have made on the topic of functions. Chinnappan and Thomas state, "A key element in the goals that teachers set for their lessons and the structuring of these lessons is their own understanding of both the subject matter and their students" [**1**, p. 151]. For beginning teachers discerning what to teach about a subject and how to teach it can be challenging.

Chinnappan and Thomas chose four student teachers and one expert teacher to observe. Each teacher was interviewed about their thoughts on functions, how they define them and how they teach them. The interviews were set up so that each teacher could speak freely. There were no prompts given so that the researchers could observe the connections each teacher made to subsidiary concepts. In addition to the interviews, Chinnappan and Thomas were also able to visit the expert teacher's classroom and observe two lessons on functions.

The student teachers, though interviewed separately, showed that they thought about functions in similar ways. Their focus was on the graphical representation of a function and they specifically mentioned the vertical line test, even the discussion of inverse functions centered on the graphical definition. One of the student teachers even thought of functions and algebra as two completely different things. A focus on procedural mathematics was revealed as the novice teachers reflected on their own experiences in the secondary mathematics classroom. When it came to the modeling, these teachers struggled to come up with examples of how functions could be used in the real world.

The experienced teacher, Margot, was able to simplify her definition of functions incorporating all representations of a function, Margot stated that the main points students need to realize is that a function is a relationship between two variables, an input and an output. Margot refers to this concept as “one in, one out.” In her interview Margot mentioned the idea of discrete and continuous functions and throughout her response includes many other subsidiary topics. Then, Margot described how she would teach functions and began to explain in detail a modeling problem that the students could experience that would allow them to explore the concepts of different representations of functions and gradients, while using technology. Chippappan and Thomas state, “the transition between representations, preserving the conceptual structure of the mathematics, is a crucial one in her schema” [**1**, p. 160].

Chinnappan and Thomas were able to observe one of the modeling lessons in Margot’s classroom. Her preparation prompt was, “how would you teach linear functions leading to solutions of equations” [**1**, p. 163]. The students were asked to model a human wave like the ones observed during a sporting event. The students lined themselves up and simulated a wave while other students recorded the time it took the line to complete the wave. The lengths of the lines included 3, 7, 10, 11, 14, and 21 students. After their experiment the students used their data to predict how long it would take 5 and 30 people to complete a wave. Then the students had to estimate the number of people in a row that took 5.5 minutes to complete the wave. The students explored the different representations of a function by creating a table, a graph, and a symbolic representation for the function. The students were also able to use their calculators to find a line of best fit for the number of people in the wave versus time to complete the wave graph and checked their original calculations with the graphing calculator.

In conclusion, it is very important that experienced teachers help support first year teachers. Since the novice teachers have not developed a strong pedagogical content knowledge, these teachers depend on the expert teachers to give them advice and help them build lessons that will allow student to gain a conceptual understanding of the mathematics under investigation. New teachers also depend on mentor teachers for guidance in presenting modeling problems. Even though a lesson may be designed to be a great hands-on experience for students, it does not mean that it will be presented in that manner.

Chapter 3: Modeling of an Algebra Problem

In order for this generation of students to be successful in the world job markets of tomorrow it is important that they not only master the traditional procedural mathematics, but that these students can think critically and apply the mathematical concepts to real life problems. Modeling has become an essential instructional component in some of the nontraditional mathematics curriculums and has slowly filtered in to more traditional classrooms as well. In a world of high stakes testing it is imperative that educators have accurate data regarding student success in learning mathematics modeling. This need for understanding the importance of modeling led Chinnappan to investigate its effects on the acquisition of mathematics skills [**2**].

Chinnappan's study had two major goals. The first was to identify essential points of engagement and events during the lesson involving modeling. These points were used to divide the process into five different phases with each phase serving a different purpose in the modeling process. Interpreting the teacher's and students' connections between ideas while working through the modeling problem, was the second goal. Analyzing the assimilation of new ideas and information into students' previous knowledge is key to revealing the effectiveness of modeling lessons.

While reviewing a lesson on solving four step equations of the type,

$$4m + 2 = 2m + 18$$

a student presents the teacher with the question, "Why do we need to learn algebra?" In an effort to answer the student's question and continue the class's study of solving linear equations, the teacher devises a modeling problem. The teacher scaffolds the modeling process for the students walking them through each step of the problem allowing for less structure and more exploration with every step [**2**].

The students are asked to brainstorm ways to recycle the gum found on the underside of their desks. The “Chewing Gum Problem” is not given to the students all at once but emerges through a series of questions and prompts. In order to identify the critical events during the lesson, the lesson was broken down into five phases based on the structural framework for teachers’ mathematical knowledge and modeling of a focus concept. Even though the phases are connected, there is a distinct focus for each [**2**].

Phase one establishes continuity with the previous lesson, this is where the student’s question is posed. During the second phase students are asked to gather data, by counting the pieces of gum under their desks. This helps to establish the real life problem with the purpose of demonstrating the practical uses of algebra. Chinnappan titles phase three “Developing the Context.” This is where the students begin the modeling process. The class estimates the average number of pieces of gum under all desks in the classroom, then in the entire school. This allows the students to develop important connections between the various components of the model. Phase four of the model is where the abstract equations from the day before are linked into the model. The students develop the relations between the variables by writing an equation calculating the profit they could make if they used this gum to create figurines and sell them. Although the teacher does have the class look at the cost equation,

$$P = c\left(\frac{30000}{10}\right) - 2000$$

he misses the opportunity to examine the more general form of the equation,

$$P = c\left(\frac{n}{q}\right) - e$$

where p is the profit, c stands for the selling price of a soldier, n is the number of gums in the school, q represents the number of gums required to make one soldier, and e is the cost of sterilizing the gums. Phase five of the modeling process is the most valuable. This is the point when the connections between the model components and the abstract equation take shape. The students begin to explore the values in the equation substituting different values for the sales price in order to observe the changes to the profit margin. Students then find other ways to increase their profits like reducing the number of gum pieces needed to create each figurine and reducing the number of pieces that are sterilized thus reducing their production costs [**2**].

During the different phases of the lesson, students are asked to process and connect multiple sources of information. This makes the "Chewing Gum Problem" have a *high intrinsic load* for students. That is the modeling process consolidates the content and also calls for students to be active participants in judging the reasonableness of answers and justifying their actions. It also helps to construct a foundation upon which the concepts of formal mathematics can be built. Even though the model itself relies on informal mathematical calculations, it gradually builds to a more general form of the model that supports the advanced mathematical reasoning that is desired [**2**].

The teacher in this scenario shows a great depth of content knowledge. Through prompting and questioning, this teacher was able to guide his Algebra class in applying the mathematics to the model in a meaningful way that allowed students to establish connections between previous knowledge and the current lessons. The interlinking of the abstract and real life concepts also shows the depth of the teacher's pedagogical content knowledge [**2**].

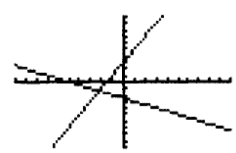
Chinnappan states that there are two key ingredients to the effective learning of algebra, "identify and understanding the relationships among variables," and, "relational understandings need to be fore-grounded in meaningful activities" [**2**, p. 20]. While working through these modeling problems, students are often asked to use graphing calculators of some kind. These calculators are very useful for teaching, through modeling, and exploratory assignments. However, there may be some concerns due to the difficulties that students have in programming them.

Chapter 4: Graphing Calculators

Graphing calculators have become a regular part of the secondary mathematics classroom. Even though there is wide spread usage of these devices, there is also debate about whether they are helping or hindering our students learning. Mitchelmore and Cavanagh gathered research over four areas of concern: scale, accuracy and approximation, linking representations, and representation by pixels. To gather their data, they interviewed twenty five students and asked them to perform eight different tasks, as seen in Figure 1, the list of interview tasks and the corresponding graphing calculator screens. The eight tasks included several problems that students, in algebra 1, normally encounter when doing exploratory work on a graphing calculator [**3**].

1. Draw a sketch of $y = 0.1x^2 + 2x - 4$. You may use the graphics calculator to help you.

2. Explain why the graphs of the lines $y = 2x + 3$ and $y = -0.5x - 2.5$ do not appear at right angles on the screen. What could you do to make the lines look more perpendicular?



3. Use the graphics calculator to find the intersection of $y = 2x - 1.5$ and $y = 3x + 0.8$.

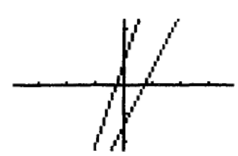
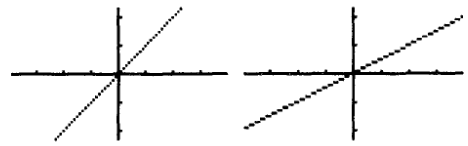


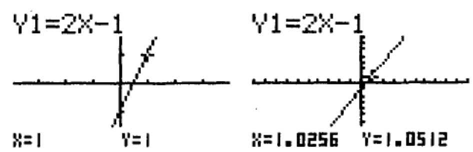
Figure 1: Interview tasks and the given graphing calculator screen [**5**, p. 258]

4. Display the graph of the function $y = 0.75x^2 - 1.455x - 1$ on the graphics calculator. Find the intercept with the positive x-axis and the coordinates of the vertex.

5. Which, if any, of the calculator screens show the line $y = x$?



6. Display the graph of $y = 2x - 1$ on each graphics calculator. Move the cursor to the point $(0, -1)$ and $(1,1)$. What do you notice? Can you explain what has happened?



7. Look at the graph of $y = x^2$. What do you notice about the way that the groups of pixels are arranged? What might this suggest about the gradient of the parabola as you move along the curve?

8. Display the graph of $y = x^2 - 2x + 3$ on the graphics calculator. Can you change the window settings of the calculator so that the graph appears as a horizontal line?

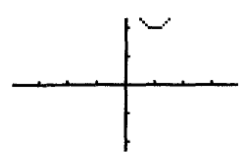


Figure 1: Interview tasks and the given graphing calculator screen [**5**, p. 258]

These twenty five students were interviewed individually for fifty minutes on three separate occasions; the interviews were scheduled every two weeks. The eight tasks were spread over the three interviews and were always administered in the same order. As the students went through each of the tasks, they were asked to think out loud and explain the processes they were using to interpret the information on the graphing calculator [**3**].

Answers to tasks two, five, and eight revealed that the students had very little concept of scale as it applies to graphs. One of the main issues was that students had limited experience dealing with graphs where the axes were not scaled equally. The students' only experience with scale was with scale drawings. This experience only allows for a single ratio resulting in the object maintaining its original shape. However, with graphing calculators this was not the case. Because of their limited understanding of scale, students did not completely understand the "zoom" operation. For instance, the student would zoom in on the vertex of a parabola and would be surprised to see a shape more linear then parabolic. The majority of the students explained zoom as a magnifying glass that never really changed the scale, but allowed for a more detailed view of a part of the graph. Students were also unable to change the spacing for the tick marks using the scale parameter in the window setting [**3**, p. 262].

Task four revealed the students' issues with accuracy and approximation. The most common way students estimated a fractional value was to average the value's on both sides, without any consideration to the actual values proximity to the surrounding values. The students also showed a preference for using integers and one student even remarked, "if it gives you a whole number, it means it's more accurate" [**3**, p. 263]. The students also had a hard time with the concept of irrational numbers represented on a graph since every point is a measurable distance from the origin.

A problem with linking representations of graphs and other representations of functions was also revealed in this study. Throughout the study, students had a problem connecting what they saw on the graphing calculator screen to their general knowledge of functions [**3**, p. 264].

The students consistently recognized the jagged edges on the graph due to the low resolution of the screen. They were even able to explain why the pixel patterns for the graphs of linear and quadratic functions are different. They repeatedly used the picture to answer questions even though the students knew that the coordinates were more accurate and they recognized the inconsistency in where the cursor appeared and the coordinates given. The students did not seem to understand how the calculator creates the graphical images based on the scale. The students had no concept of the graph being composed of pixels or that the graphical representations are limited in their ability to accurately depict graphs. When the students are using the graphs the values that appear on the graphs are merely estimates and not necessarily the actual values [**3**, p. 264].

Using technology though important may provide an extra set of difficulties for students. In an effort to prevent students from making mistakes it is important for teachers of mathematics to be aware of the difficulties a students might encounter when using a graphing calculator. Balancing calculator activities with paper and pencil math is essential to producing students with a well rounded mathematical knowledge and problem solving capabilities.

Chapter 5: Conclusion

Without application the functionality of mathematics is lost. The vast majority of secondary mathematics students will not be theoretical mathematicians, however they will need to know how to think critically and turn procedural knowledge into conceptual knowledge. As teachers, it is our responsibility to ensure that students are given ample opportunity to use the mathematics that they are being taught in creative ways so students will be prepared for life outside of our classrooms.

According to the National Council for Teachers of Mathematics, 9^{th}-12^{th} grade students should be able to identify quantitative relationships when observing a set of data from a given situation and determine the class of functions that might best model the relationships; use symbolic representations, including iterative and recursive forms, to depict the relationships arising from various scenarios; draw reasonable conclusions about a situation being modeled [**4**]. These skills do not come naturally to students. They must be cultivated. There are so many teaching objectives in each course that it can be difficult to make time for these types of lessons, but it is imperative that students leave with the ability to solve these types of modeling problems.

Word problems and other modeling scenarios are usually saved for the end of a lesson so students master the procedural skills then learn to apply these skills to various situations. This means when teachers need to build in extra time for a topic because students are struggling, the modeling problems are minimized or cut out entirely leaving the students to fill in their own gaps in knowledge. This is a great disservice to our students and their future employers who will have to remediate due to their lack of training. A well planned modeling problem can be used throughout a unit to teach a given topic and does not have to be the capstone lesson where students are suddenly asked to step outside of their procedural safety net and apply their content knowledge. Modeling must be incorporated into every aspect of mathematics teaching so that students can appreciate the relevance and gain a conceptual understanding of the mathematics.

Chinnappan and Thomas used interviews to reveal the holes in pedagogical content knowledge that must be filled for beginning teachers, so they will be successful in teaching functions through modeling. These interviews showed the distinct difference in the schemas of student teachers and experienced teachers and the connections they made between the topic functions and its many subsidiary topics. A strong mentor teacher can help a new teacher fill in these gaps in their pedagogical content knowledge so they can facilitate a successful modeling lesson in their own classroom.

An observed modeling lesson is employed by Chinnappan to reveal the crucial components of a modeling lesson. The "Chewing Gum Problem" allowed students the freedom to use previous concepts and apply them in a meaningful way while still addressing the topic at hand. The teacher acted as the facilitator transitioning students from the different representations of a function and guiding them to discover the various impacts of the values in the equation they had created.

Last to be considered was an examination of research on the difficulties students face when operating a graphing calculator. There are many great reasons to use graphing calculators in a classroom. One of the leading arguments is that they allow for student exploration without the time consuming effort of plotting points and drawing graphs by hand. These calculators also allow students to graph a less restricted group of functions. While these arguments hold true, there is no substitute for pencil and paper. Teachers need to be aware and take heed of these areas that make the use of graphing calculators difficult for students at many levels.

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Vita

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This report was typed by the author.