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Abstract. Using an evolutionary strategy algorithm, we optimize the generalized transformer ration of a laser wakefield accelerator. The algorithm tests several realistic pulse shapes by integrating the fluid wakefield differential equation and it converges to the shape that most efficiently produces a strong accelerating gradient while experiencing minimal distortion.

INTRODUCTION

With acceleration fields orders of magnitude greater than standard linear accelerators, laser wakefield accelerators (LWFA's) have drawn considerable attention. While several groups have achieved the generation and characterization of laser-produced wakefields [1-10], the need for multi-parameter optimization has arisen so that LWFA's can become the next generation particle accelerators. The ideal LWFA will make the most efficient use of the laser energy by allowing the laser pulse to propagate with a minimum of distortion so that the wakefield accelerating structure will be as long as possible. While high intensity guiding is being pursued by several groups to overcome the Rayleigh length limitation of LWFA's [11-16] and tailored plasma densities are being considered to overcome the dephasing length limitation [17], it is also necessary to ensure that the accelerating structure remains as stable as possible throughout the interaction length. A standard resonant LWFA with a gaussian pulse produces a portion of a plasma oscillation *during* the pulse that will lead to pulse distortions and to a reduced wake after propagation. The minimization of this distortion is the subject of this work. For the case of a plasma beam based wakefield accelerator, there has been theoretical work done by Bane et al which produced an analytical plasma beam shape to optimize the so-called transformer ratio of the accelerator [18]. The transformer ratio is defined to be the ratio of the maximum accelerating field after the beam to the maximum decelerating field during the beam. Clearly, maximizing the transformer ratio in this case will produce the maximum wakefield after the beam while minimizing the distortion of the beam. Chen et al extended this result to the analogous case of the LWFA [19]. Their optimized result produced no pulse distortion and an arbitrarily large wakefield after the pulse. Unfortunately, the pulse derived by Chen et al is not possible to produce in the lab since it requires a half-delta function and an instantaneous falling edge of the pulse. Neither of these effects is possible with a laser of limited bandwidth. Our approach to this problem for the LWFA is to use an evolutionary strategy computer code to

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"evolve" towards a pulse shape that maximizes a generalized transformer ratio [20]. We perform numeric simulations with realistic limitations built-in (fixed bandwidth lasers and pulse shaping that is achievable with state-of-the-art pulse shapers) and with fully relativistic, 1D fluid codes. This paper is organized as follows: we will first describe the equations we use along with a careful description of the generalized transformer ratio we use; we will then describe our optimization procedure and finally we will discuss the results and conclude.

THEORY

The coupled, 1D fluid equations which govern wakefield production and pulse propagation in a plasma are [21]

$$\frac{\partial^2 a}{\partial \zeta^2} = -\frac{\omega_p^2}{c^2\left(\beta_{ph}^2 - 1\right)} \frac{a}{1+\phi} \tag{1}$$

$$\frac{\partial^2 \phi}{\partial \zeta^2} = \frac{\omega_p^2}{2c^2}\left(\frac{1+a^2}{(1+\phi)^2} - 1\right) \tag{2}$$

where a is the normalized vector potential of the laser, ϕ is the electrostatic potential of the plasma, ζ is the space/time coordinate moving in the frame of the pulse, β_{ph} is given by $(1-v_{ph}/c)^{-1}$, and ω_p is the plasma frequency. We concentrate on the second equation which describes the production of a wakefield from a given pulse envelope $a(\zeta)$. The first equation describes the self-consistent evolution of the pulse as it propagates in the plasma. Our approach is to find a pulse envelope that, while maximizing the wakefield accelerating gradient after the pulse, would experience negligible distortion as it propagates by finding a solution of (2) with minimal plasma disturbance *during* the pulse. We solve for ϕ and then take up to three derivatives: the first gives the accelerating field of the plasma wave, the second gives the plasma density variation (and hence the plasma's refractive index), and the third derivative gives the ζ-dependence of the phase of the pulse. If the laser pulse phase changes with space or time, pulse instabilities arise and grow which diminish the pulse's ability to produce efficient wakefield accelerating structures. It is well known, for example, that the frequency of the pulse will change when it experiences a time-dependent refractive index [22,23]. These new frequencies enable the linear dispersion of the plasma to disrupt the pulse more quickly. Thus by monitoring the derivative of the phase, we can predict which pulses will produce the best results after propagation.

Rather than using the transformer ratio as defined by Chen et al for the LWFA, we parameterize the pulse distortion by considering the weighted effect of all the possible frequency shifts during the pulse. Our "fitness" function is therefore given by the integral of the product of the pulse envelope and the third derivative of the electrostatic potential subtracted from the maximum of the wakefield accelerating field at any point:

$$fitness = w_1 \left.\frac{\partial \phi}{\partial \zeta}\right|_{\max} - w_2 \int a^2(\zeta) \frac{\partial^3 \phi}{\partial \zeta^3} d\zeta \tag{3}$$

where w_i are the weights for each term. The optimum pulse shape will be orthogonal (in the general sense) to the frequency shift term. An addition to the fitness function for numeric sampling reasons is a strong negative weight to the value of a^2 at the first point in the array (ζ_{min}). This ensures that the wakefield does not have an unphysical beginning before the pulse. The pulse shaping is accomplished by varying the spectral phases and amplitudes with the limitations present in real world shapers (e.g. a fixed total bandwidth, a fixed sampling rate etc). It should be noted that temporal shaping of these pulses is generally not possible. When spectral amplitude shaping is considered, the fitness function needs to be augmented to protect against extra nonlinear phase in the amplifier (since most pulse shapers for experiments like that described here are placed before the amplifier). We subtract a term that is essentially the "B-integral" of the amplifier chain for the newly shaped spectral amplitude [24]. The B-integral is affected if there is amplitude shaping of the spectrum if one assumes a fixed total energy from the amplifier. If the peak of the spectrum contains more energy, the B-integral is larger.

Procedure

The genes in our evolutionary strategy are the values of the spectral phase and amplitude of an originally transform limited pulse of fixed duration (set to be resonant with the plasma density). These values mutate at a variable (and evolving) mutation rate to produce the children of each new generation. The children are tested according to the fitness (3) and the best are chosen to mutate into the next generation. This process is repeated until the increase in the fitness from one generation to the next goes to zero (typically 1000 iterations).

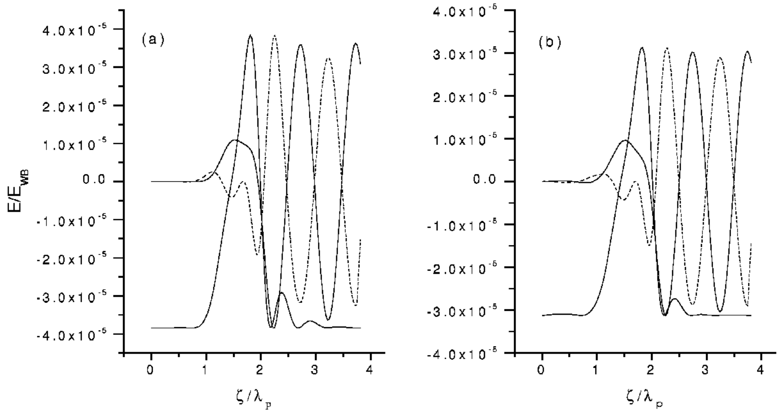

FIGURE 1. Pulse shaping results for a^2_{max}=0.01 for phase-only (a), and amplitude and phase shaping (b). Shown are the pulse envelope (solid), electric field (solid), and the frequency shift (dashed). All plots are normalized to the field which is given in units of the wave-breaking field. The x-axis is the space/time coordinate normalized to the plasma wavelength.

RESULTS

Figure 1a shows the result of phase-only shaping (used for maximum energy throughput and no change in the B-integral) for the case where the maximum a^2 of the original transform limited pulse was 0.01 (in the linear regime [21]). Figure 1b shows the result at the same intensity for both amplitude and phase shaping. In both cases the dominant feature of the pulse is a slowly rising triangle with an abrupt falling edge where the wakefield amplitude dramatically increases. This pulse is reminiscent of the prediction of Chen et al but without the half delta function in front [19]. The best fitness values achieved in the linear regime are nearly twice that of the original transform limited gaussian. The cost of reducing the maximum intensity by lengthening the pulse is overcome in this case by greatly reducing the amount of frequency shifting occurring during the energetic portions of the pulse. It is clear that the greatest amount of frequency shifting occurs during the fast falling edge and hence the evolutionary strategy tries to converge to something with as fast a fall as possible. The frequency shifting is minimized during the pulse because the wakefield produced at the beginning of the pulse (either by a small prepulse or by the first appreciable slope of the envelope – it can be shown that the electron density fluctuations are caused by the gradient of the pulse envelope [21]) is out of phase with the wakefield produced by the main slope of the triangle. Thus the wakefields add destructively until the final falling edge when the electron oscillations attain much larger values. Finally, the largest plasma density gradient for electron acceleration occurs just after the pulse where there is a small postpulse that reduces the wakefield amplitude (i.e. it produces a wake that is out of phase with that of the falling edge of the main pulse). We feel that the postpulse is a limitation of the constraints on the evolutionary strategy as the number of postpulses is reduced when amplitude shaping is allowed.

Figure 2 shows the best result of phase shaping in the nonlinear regime ($a^2_{max} = 0.1$ and 1) [21]. At a^2=0.1 (figure 2a), the result is quite similar to the linear regime in figure 1 but with a pronounced prepulse that sets up the destructive interference inside the pulse mentioned above. As the pulse becomes more and more nonlinear, the best pulse shape begins to converge to the original gaussian. We have found that there is a smooth transition from the triangular shape of the linear regime to the gaussian shape. In the nonlinear regime, the cost of reducing the peak intensity to produce a pulse with less frequency shifting during the pulse is too great. Thus the original transform limited gaussian is the best pulse in the nonlinear regime since it has by definition the greatest peak intensity [24]. We have found that the results of phase-only and amplitude and phase shaping are very similar for $a^2_{max}>0.1$.

DISCUSSION

There are two methods of experimental implementation for this work. The first is to produce the pulse that the simulation converges to and then test it experimentally. There are several pulse shapers (most notably those based on an acousto-optic crystal) that can take a given amplitude and phase mask and produce a desired pulse [25-30].

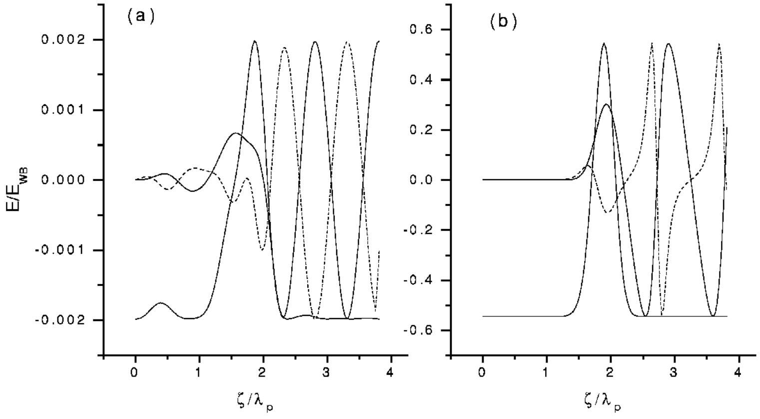

FIGURE 2. Pulse shaping results for a^2_{max}=0.1 (a) and a^2_{max}=1.0 (b). Shown are the pulse envelope (solid), electric field (solid), and the frequency shift (dashed). All plots are normalized to the field which is given in units of the wave-breaking field. The x-axis is the space/time coordinate normalized to the plasma wavelength.

We feel, however, that a more efficient experimental procedure would be to allow all real-world limitations to be taken into account in the evolutionary strategy algorithm. This is accomplished by replacing the numeric integration of equation (2) with an actual measurement of a wakefield produced by a given pulse. Physical realities such as a focusing geometry which is not fully 1D and pulse distorting optics between the pulse shaper and the experiment (most notably the amplifier) can be included in the optimization routine to produce the most efficient LWFA possible with a real laser. Just a few years ago such an experiment would not have been considered because the full results available by integrating equation (2) could not be measured quickly (typically a multi-shot experiment was required that could take hours [4]). Since the evolutionary strategy requires testing up to thousands of different pulse shapes, a single-shot feedback mechanism is necessary for experimental implementation. Such a single-shot diagnostic of wakefields has been proposed and is being tested by the authors [31]. With a fast feedback diagnostic available, the physical implementation of the evolutionary strategy approach should be feasible.

We have determined the optimum, physically realizable laser pulse shape for LWFA. By extending the concept of the transformer ratio and concentrating on the actual distortion of the pulse as it propagates through the plasma we have found pulse shapes that experience minimal frequency shift while still producing large accelerating gradients. In the linear regime of LWFA the optimum shape is an asymmetric triangle with the fast edge on the trailing side while in the nonlinear regime the best pulse is the original transform limited gaussian. We feel that the evolutionary strategy we have employed can be extended to the real-world experiment in order to best optimize LWFA even with non-ideal experimental conditions.

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