The Dissertation Committee for KEREM CAKIRER

certifies that this is the approved version of the following dissertation:

# An Equilibrium Theory of Organizational Forms:A Complementary Market Analysis

Committee:

David S. Sibley, Supervisor

Maxwell S. Stinchcombe, Supervisor

Thomas Wiseman

Randal B. Watson

Barbara Robles

# An Equilibrium Theory of Organizational Forms:A Complementary Market Analysis

by

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TO MY FAMILY

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KEREM CAKIRER

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# An Equilibrium Theory of Organizational Forms:A Complementary Market Analysis

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Supervisor: David S. Sibley

Supervisor: Maxwell S. Stinchcombe

The dissertation develops an equilibrium theory of mergers in a complementary market setting where downstream firms sell a product which must have a compatible variety of products that are supplied by upstream firms. I map each of several different complementary market setting to a merger type, i.e vertical, counter, horizontal or none. I present the conditions under which a downstream firm will prefer integrating with an upstream firm, and conditions under a counter merger of firms occur. The analysis shows that a vertical merger is more likely to occur whenever one of the upstream firm is significantly productive than the other. However, counter integration of firms is also likely whenever the upstream firms are highly productive. Moreover, I present the conditions under which two downstream firms merge and form a new downstream firm. The findings support that the more competitive downstream market is , the more likely a horizontal merger is. In addition, a vertical merger, two vertical mergers and independent ownership can still be the outcome even though downstream competition is high. The results are obtained in a general two downstream firms and two upstream firms market setting that allows efficient compatibility contracts between upstream and downstream producers.

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# Chapter 1

# Introduction

## 1.1 Overview

The mergers and acquisitions literature has examined how a price of a product or market concentration is affected post mergers, such as vertical integration or horizontal merger, to understand the competitive effects not only on the participant firms, but also on firms that are excluded from the merger. A typical assumption in most of the existing literature is that, the type of merger structured by participant firms is exogenous. However, a merger is a type of contract that is agreed by the both participant firms and it is a strategic decision. For example, when a firm decides to acquire a supplier to enhance its product quality, both the firm and the supplier assess the value of the merger before any decision to make. The firms may or may not agree on the merger under the current conditions or future conditions which will arise post merger. Moreover, the merger proposed by a firm affects the excluded firms' ex post incentives to do business with the merged firm. The purpose of this paper is to study how different type of mergers or combination of mergers can be explained in complementary markets when the type of merger is endogenous. In addition, this paper studies the competitive effect(s) of a merger on compatibility and supply decisions.

Particularly, I consider a model where downstream firms sell a product which must have a compatible variety of products that are supplied by an upstream firm. In this setting, downstream firms product does not have any value unless it is supported by a set of compatible products. A downstream firm can integrate or maintain contractual relations with an upstream firm or merge with another downstream firm. The timing of the game is as follows: first, a downstream firm announces a merger decision; after observing the new market structure, an excluded downstream firm can integrate with an upstream firm if a vertical integration occurred in the first stage. Then, each downstream firm that is not integrated with an upstream firm offers a compatibility contract to one of the upstream firms, and based on the contracts upstream firms determine whether to agree; each upstream firm, which agrees on a compatibility contract, then commits on a firm specific research and development investment that can increase the variety of upstream products. Finally firms compete and prices are determined.

If a downstream firm decides to integrate with an upstream supplier, remaining independent downstream firm may offer a contractual relation with the upstream division of the integrated firm even though two firms compete in the downstream market. On the other hand, an integration can also lead to a counter integration which will stiffen the downstream competition. A downstream firm's objective is to decide on a merger type that increases its expected profits. The optimal decision of a downstream firm in the complementary market setting is the focus of this paper.

In order to study this problem, I adopt Heavner's (2004) reduced form profit framework.[1] Heavner shows that, under some conditions, firms may remain inde-

[1] In his reduced form setting, a firm's profit only depends on the quality its product and the

pendently owned because an integrated firm can not commit on supplying a better quality of a product for its downstream competitor if integration is the only alternative for the downstream firm and no counter integration is allowed. If counter merger and a horizontal merger are also allowed, however, a downstream firm faces an additional consequence: an integration may also trigger a counter integration which makes the integration more costly ex ante. Moreover, a horizontal merger may be the best alternative for both of the downstream firms rather than integrating with the upstream firms.

An immediate finding of this paper is that a downstream firm's incentive to integrate with an upstream firm increases as productivity gap between the upstream firms increases. To see this, consider a setting with two downstream firms and a very high productivity upstream firm $U_h$, and very low productivity upstream firm $U_l$. First, the downstream firm $D_1$ decides to integrate with $U_h$, and then the other downstream firm $D_2$ considers whether to offer a compatibility contract to the upstream division of the integrated firm or integrate with $U_l$. If the productivity of $U_h$ is very high, a counter integration decision of $D_2$ is unlikely to change the integration decision of $D_1$. That's why $D_2$ can not be an ex ante threat to $D_1$ in the downstream market. $D_2$'s integration with $U_l$ will produce lower variety of complementary products for $D_2$ than $D_2$'s maintaining contractual relation with $U_h$ post $D_1, U_h$ integration. In contrast, if $D_2$ extends a contract to the upstream division $U_h$ of the integrated firm, $D_2$ will be supplied by more variety of products even though the integrated firm invests less for its downstream competitor. It might be the case that $D_2$ will be a tougher competitor to the downstream division $D_1$ of the integrated firm if it signs a contract with the upstream division $U_h$. In this case, integrated firm either would not agree on a compatibility contract or would invest

quality of competitor's product.

zero for its downstream competitor. Consequently, both the integrated firm's and $D_2$'s are better off. This is valid in a general setting as well.

Following this observation, an integration of a downstream and an upstream firm will be the equilibrium outcome if one of the upstream firms is highly productive than other. I present conditions under which only one of the downstream firms integrates with an upstream firm, conditions under which a counter merger will be observed, and conditions under which downstream and upstream firms are better off with only contractual relations.

This paper contributes to the mergers and acquisitions literature with endogenous merger decision and can be a theoretical basis to the existing literature on mark-up analysis after different types of mergers. Most of the existing literature assumes that an integration or a horizontal merger decision is exogenous or a horizontal merger is not available to a firm.[2] In contrast, endogenous merger decision is analyzed in this paper: downstream firms can choose which firm to acquire prior to any business contracts or price competition. The market structure I study is quite general, and I allow downstream firms to acquire either a complementary good producer or a substitute good producer.

To illustrate the model, I first study endogenous merger decision with a vertical merger option and no counter merger. A downstream firm either can integrate with an upstream firm or can contract with an upstream firm. An integrated downstream firm's profit is contingent upon the compatibility decisions. An integrated firm can acquire an efficient contract from an independent downstream firm. The downstream profits depend on the variety of its complementary product and variety of its downstream competitor's complementary product. The integrated firm can increase its profit by supplying less for its downstream competitor.

[2]Heavner (2004) assumes horizontal mergers away.

A downstream firm and an upstream firm may have conflicting interests in the merger because an upstream firm incurs the investment cost but may have to give up the business of the independent downstream firm. On the other hand, the downstream firm increases its variety of complementary products post integration. Moreover, an integrated downstream firm can solve that its competitor will be supplied by less variety of complementary products if downstream competitor extends a contract to the upstream division of the integrated firm ex post. Therefore, integration decision is a strategic decision for both the downstream firm and upstream firm.

Intuitively, it may seem that a more productive upstream firm always gives a downstream firm a higher incentive to integrate because the downstream firm always prefers more variety of complementary products. However, this kind of thinking is wrong because the downstream firm's incentive to acquire the upstream firm depends on his relative gain from integration rather than its complete gain. As I suggested before, the marginal profit post integration increases as the productivity gap between upstream producers increases.

In this setting, the equilibrium outcome depends on not only the production asymmetries of upstream firms, but also independent downstream firms contractual relations. The following is the reason. If independent upstream firm can produce enough variety of complementary products to independent downstream firm after integration, the upstream division of the integrated firm can not acquire the business of the its downstream competitor. The independent downstream firm can extend a compatibility contract to the less productive upstream firm because upstream division of the integrated firm would invest less for its competitor in order to induce the complementary product variety of its downstream competitor. The upstream firm that is a candidate for an integration can loose the business of the independent downstream firm and loose profits. That's why participant firms may forego an integration. On the other hand, if independent upstream firm can not produce enough variety of complementary products to independent downstream firm after integration, vertical integration can not possibly hurt the participant firms' ex post profits. Again, the upstream division of the integrated firm will invest less for its downstream competitor, but this time the integrated firm still acquires the business of its downstream competitor.

To summarize, the basic model that allows for only a vertical merger has two main findings. First, the downstream firm's incentive to acquire the upstream firm is higher when the production asymmetry is higher, in other words, when the participant upstream firm becomes more and more productive than the excluded one. Second, with compatibility decision of the independent downstream firm, participant parties may forego an integration. An analysis of a more generalized model which includes upstream and downstream price competition is more subtle and complicated because each upstream firm competes and adjusts its downstream firm specific investment in order to attract downstream firms. With an outside option for an upstream firm, the optimal upstream price can be pointed out from the value of the outside option.

In order to analyze a more generalized model which includes a counter merger and horizontal merger, I first provide sufficient conditions under which a downstream firm benefits from a vertical integration. Second, I analyzed that, under the sufficient conditions provided, whether an integration is still the equilibrium outcome whenever counter merge is also possible. I provided the sufficient conditions under which one integration or two integrations will be observed in the market. Third, I analyzed the downstream firms' merger problem. I examined the conditions under a horizontal merger is more beneficial than an integration or two integrations. Finally, I map each of different complementary market setting to a merger type.

## 1.2 A Complementary Market

One of the biggest and fast growing markets in which complementarity exists is the video game console market. In 1984, the home video game market crashed and thus, many hardware and software game manufacturers, such as Apollo, US Games, Spectravision, declared bankruptcy. However, Sega is acquired by investors and re-launched as a software producer in U.S. One year later, Nintendo released its first home video game system. In 1986, Sega decided to launch its Sega Master System and the system was supplied by Sega itself. On the other hand, Atari released its own video game system that features backward compatibility so that Atari could increase the number of titles available to its new video game system.

In the following years, Atari, Nintendo and Sega launched new versions of their home video game systems. In 1994, Sony launched its first home video game system, PlayStation. Sony's Playstation was going to be supported by independent software developers. The corporation agreed to pay a share of the game softwares revenues to independent software suppliers. Consequently, Playstation increased the number software titles which are exclusive to Sony's console Playstation in the console market. At the end of the third business year, Sony Corporation increased its market share to almost 50% in the home video game market. In 2000, Sony launched a backward compatible new home video game system, PS2. The same year video game industry grew 30%. At the end of 2000, Sony Corporation announced that its 50% of profits was generated by PlayStation although only 15% of Sony's total revenue is generated by PlayStation sales. Eventually, the console market consolidated and Atari was acquired by Sega systems.

In 2001, Microsoft decided to launch a new video game system, however, the corporation's main concern was the high variety of game softwares that are exclusive to Sony's PS2. Microsoft was stressed because of the sustainable advantage in the console market Sony's PS2 had. As a market solution, Microsoft adopted a strategy of acquisition of some independently owned software firms. The corporation started to create a portfolio of its own game softwares that are developed under the name of the company.[3] On the other hand, Sega decided to exit the market, nevertheless, the firm resumed producing game softwares for the rival console producers.

Recently, Nintendo, Microsoft and Sony launched their next generation of video game consoles, Wii, XBOX 360 and PS3 respectively. DFC Intelligence's research on game industry reports that there has been a strong sales increase in the video game market over the past few years and there is still plenty of room for growth. The report also indicates that the generation who grew up with Atari and Nintendo is switching to Microsoft and Sony. The report suggests that an industry consolidation, which has not yet occurred as many as predicted, is on its way. A merger an acquisition wave is expected between the biggest game software producers and the biggest home video game console producers.[4]

## 1.3 Related Literature

This paper is related to the literature on mergers & acquisitions, competitive effects of mergers and complementarity & compatibility . First of all, I am presenting a

[3]The recent Halo 2 is a tremendous hit and generated over $300,000,000 revenue.

[4]Nintendo, Sega, EA, Acclaim and Capcom are five biggest software producers. Microsoft, Sony, Nintendo are three biggest home video game producers.

more general framework than the existing literature has. Each of the papers from existing literature considers a special case. In my modeling framework, each of these special cases corresponds to a different set of underlying parameters in the paper. Second, my analysis also related with the studies on competitive effects in given merger types. Finally, this paper is close to the existing literature on competition among complementary products.

The first set of literature related to this work studies compatibility decisions in complementary markets. Matutes & Regibeau (1988) examines a two stage game in which two fully integrated firms make their compatibility decisions before competing in prices. They found that full compatibility is the symmetric perfect Nash equilibrium which leads to higher prices and also increases the variety of systems. This paper assumes the firms are fully integrated. Economides & Salop (1992) analyzes the competition and integration among complementary products that can be combined to create composite goods or systems. The model generalizes the Cournot duopoly complements model and analyzes the equilibrium prices for a variety of organizational and market structures. They solve the Nash equilibrium prices for different types of market structures such as vertical integration, horizontal merger, conglomerate merger. Gandal & Kende and Rob (2000) examines the influence of the different titles of CD on the CD industry. They estimated the elasticity of buying a hardware with respect to CD player prices and the cross elasticity with respect to the variety of CD titles. They showed that the influence of the variety is significant and there is a positive relationship between the profits of a firm and the total variety of the firm's complementary product. They assume that the software industry is competitive and there exists no vertical or horizontal integration in the market. These models examine similar market structures with the one this paper analyzes and focus on influence of complementarities on prices, but the market structures are taken as granted unlike my model which studies the endogenous merger decisions. In each of these papers, certain types of merger and contractual relationship are ruled out by assumption.

A second branch of the literature studies vertical mergers. Beggs(1994) examines competition between groups of firms selling products which are complementary within the group but substitutes across groups, such as components of different computer systems. The paper shows that firms within a group will often prefer to stay as separate companies rather than merge. McAfee(1999) analyzes the reaction of the other input suppliers to vertical integration. In his paper, he examines the price competition in input market post integration and the competitive effects of a vertical merger. The paper shows the input suppliers may reduce the its rival's cost instead of raising the input cost. Chen(2001) shows how the pricing incentive of a downstream producer and the incentive of a competitor in choosing input suppliers is effected by vertical integration. The paper develops an equilibrium theory of vertical merger which can provide a framework in which the competitive effects of vertical mergers are measured and compared. Heavner(2004) examines integrating firms' trading opportunities post integration. The model analyzes the integration decision of a downstream firm if downstream units must commit to suppliers before contracting on the final terms of trade. The paper shows integration can alter the supplier decisions of upstream units. As a result, firms may remain independently owned. In addition to Heavner (2004), the current paper studies the merger decision whenever counter merger and horizontal merger are also possible.

A final related literature studies horizontal mergers and their competitive effects. Farrell & Shapiro(1990) analyze horizontal mergers in Cournot oligopoly.

They looked at the effect of a merger on rivals and consumers. They provide sufficient conditions for profitable mergers to raise welfare. Ziss(1995) sets a duopoly setting which has two upstream and downstream firms. The paper analyzed the downstream merger using linear demand and constant marginal cost. He showed the conditions under which a downstream horizontal merger is not anti-competitive. Economides & Salop (1992) also study the influence of a downstream merger on prices in a complementary market setting.

In the literature, the organizational forms are often taken for granted. The organizational forms that are included in the models can be summarized as

| | Vertical Integration | Horizontal Merger | Contractual Relations |
|---|---|---|---|
| **Matutes & Regibeau (1988)** | Assumes | Assumes Away | Assumes |
| **Economides & Salop (1992)** | Assumes | Assumes | Assumes |
| **Gandal & Rob & Kende (2000)** | Assumes Away | Assumes Away | Assumes |
| **Beggs (1994)** | Assumes | Assumes Away | Assumes |
| **McAfee(1999)** | Assumes | Assumes Away | Assumes |
| **Chen (2001)** | Assumes | Assumes Away | Assumes |
| **Heavner (2004)** | Assumes | Assumes Away | Assumes |
| **Farrell & Shapiro (1990)** | Assumes Away | Assumes | Assumes Away |
| **Ziss(1995)** | Assumes Away | Assumes | Assumes Away |

In contrast to these papers, the current paper identifies the conditions under which vertical merger, horizontal merger or contracting is the only equilibrium organizational form. My paper falls into the same category as Chen(2001) & Heavner(2004). Different than those, the analysis considers a market setting in which downstream and upstream firms produce complementary goods. As RGK(2000) suggested, my paper takes the variety effect into account in merger decision analysis. This paper provides an equilibrium theory of organizational forms which can explain why different types of mergers can be observed. Vertical merger, horizontal merger and contracting can all be organizational forms in equilibrium. This paper can provide the underlying framework for Economides & Salop paper and the merger literature which take the merger types for granted.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 is the analysis of the model with only one vertical merger. Section 4 contains the analysis of endogenous merger decision with counter merger. Section 5 analyzes the merger decision if downstream merger is also included but counter merger is excluded. Section 6 analyzes the merger decision under no restriction, Section 7 discusses and concludes. All proofs are relegated to Appendix.

## 1.4 The Model

The model consists of two downstream firms ($D_1 \& D_2$) which produce a "Base Product" and two upstream firms ($U_1 \& U_2$) which produce "Side Products". Consumers need to buy a base product in order to utilize the side products, i.e. the products are perfect complements in the market. Thus, a downstream firm has to be supported by a variety of a side products which must be compatible with the base product. Each upstream firm develops a variety of a complementary good which will be compatible with a downstream firm's product. The variety depends on a firm specific R&D investment of an upstream firm for a downstream firm, and how efficient the upstream firm's production is.

An upstream firm, which signs a compatibility contract with a downstream firm, will choose a firm specific investment level to supply some variety of goods for its compatible base product. If $U_i$ is compatible with $D_j$ then the upstream firm supplies the variety $v_j$ to $D_j$ where,

$$v_j = \varepsilon_i + \tau(r_{ij})$$

A downstream firm specific variety is determined by three factors: First, downstream firm $D_j$ specific R&D investment $r_{ij}$ of upstream firm $U_i$, second, upstream production technology $\tau(r)$, and third upstream firm $U_i$'s production efficiency $\varepsilon_i$.[5] The amount of variety an upstream firm could supply when the upstream firm has no R&D investment defines the efficiency parameter $\varepsilon$ of the upstream firm. In particular, $\{\varepsilon_i\}_{i=1,2}$ is the parameter space that characterizes different market schemes and generates the different results of the model. The cost of launching $v_j$ upstream product is the level of investment $(r_{ij})$ that the upstream firm $U_i$ invests for its compatible $D_j$.

The technology function $\tau(x)$ satisfies the conditions:

$$\tau(0) = 0,\ \ \frac{\partial \tau}{\partial r} > 0,\ \ \frac{\partial \tau^2}{\partial^2 r} < 0,\ \ \tau^{'}(0) = \infty,\ \ lim_{x \to \infty} \tau^{'}(x) = 0$$

*Variety* can be regarded as the number of different accessories that are available for a downstream product in the market.[6] *Variety* can also have different aspects, such as the quality among the various upstream products or how well the upstream firm's distribution in the market. Another point of view can be that *variety* may be considered as the quality of an upstream firm's product. The model analyzes *variety* as a scalar term, however it could be also represented as a vector.[7]

[5] $\tau(r)$ can be interpreted as the production function which satisfies diminishing marginal return.

[6] For instance, different game titles for Sony's Playstation or different accessories that are available to Apple's IPod.

[7] Variety also can have other attributes such as popularity in the market which has the impact on both the sale performance and the durability of a side product.

Both upstream firms and downstream firms are independently owned prior to any merger decision. Each independently owned firm maximizes its own profit. In case of a merger, the merged entity will have one central management which makes the production decision in order to maximize the merged entity's profit. I am interested in the endogenous merger decision of downstream firms.

One vertical integration, which is $D_1, U_1$ to merge and $D_2, U_2$ to remain independent, will be observed in the market if post integration profit of the integrated firm $U_1^v D_1$ is greater than the sum of independently owned $D_1$'s and $U_1$'s profits before the integration and profit of the integrated firm $U_2^v D_2$ would be less than sum of independently owned $D_2$'s and $U_2$'s profits post integration.

$$\Pi(U_1^v D_1) > \Pi(U_1) + \Pi(D_1) \quad \text{and} \quad \Pi(U_2) + \Pi(D_2) > \Pi(U_2^v D_2)$$

On the other hand, one vertical integration and a counter integration, which is $D_1, U_1$ to merge and $D_2, U_2$ to merge, will be observed in the market if post integration profit of the integrated firm $U_1^v D_1$ is greater than the sum of independently owned $D_1$'s and $U_1$'s profits before the integration and profit of the integrated firm $U_2^v D_2$ would be greater than sum of independently owned $D_2$'s and $U_2$'s profits post integration.

$$\Pi(U_1^v D_1) > \Pi(U_1) + \Pi(D_1) \quad \text{and} \quad \Pi(U_2^v D_2) > \Pi(U_2) + \Pi(D_2) \quad \text{given}$$

Moreover, a horizontal merger of equilibrium, which is $D_1, D_2$ to merge, will be observed in the market if the generated profit after the merger is greater than the sum of independently or integrated firms' profits before the merger.

$$\Pi(D_1^h D_2) > \Pi(D_1) + \Pi(D_2) \quad \text{and}$$
$$\Pi(D_1^h D_2) + \Pi(U_1) + \Pi(U_2) > \Pi(D_1^v U_1) + \Pi(D_2^v U_2) \quad \text{and}$$
$$\Pi(D_1^h D_2) + \Pi(U_1) > \Pi(D_1^v U_1) + \Pi(D_2)$$

Finally, there will be only contractual relations between upstream and downstream firms if a downstream firm would not benefit from both a vertical integration and a horizontal merger. Independently downstream firms will contract with independently owned upstream firms.

I study the endogenous merger decision of a downstream firm by solving the subgame perfect Nash equilibrium in a setting in which the firm must decide to merge or not before any investment or compatibility decisions made in the market. At time zero, $D_1$ has to decide on a merger strategy.[8] The downstream firm can integrate with $U_1$. In case of vertical integration, the integrated firm will produce both a base product and side products that are compatible with its base product. On the other hand, if $D_1$ decides to merge with $D_2$, then the merged firm will be the only base product supplier in the market. In case of independent $D_1$, $D_1$ produces a base product and offers compatibility contract to either of the side product firms to be compatible with its base product.

At time one, $D_2$ may counter integrate with $U_2$ if $D_1$ integrated with $U_1$ at time zero. At time two, each independently owned downstream firm offers a compatibility contract to one supplier. A downstream unit of an integrated firm will

[8] Aaker considered the market strategy of a firm in two parts. According to him, a firm can expand its product market by increasing his market share or a firm can vertically integrate by either forward integration or backward integration. Another common point of view is increasing the customer share.

be supplied by the upstream division of the integrated firm. Upstream firms (or divisions) observe the offers and decide whether to be compatible with the downstream firm or not. At time three, if an upstream firm decides to be compatible with a downstream firm, then the upstream firm will invest on firm specific R&D to establish a variety of the side products for its compatible downstream product. The firm specific R&D investment will determine the variety of the side products. At time four, upstream firms announce and launch the products they have developed. The variety of the side products is going to be observed and firms will compete in the market. The figure illustrates the timing of the model. The left section of the figure shows the timing when there is a horizontal merger. The middle section shows the timing when there is a vertical merger. The right section shows the timing when there is only contractual relations.

This paper examines a setting in which the upstream firms can not alter their supply decision. That is, the investment decision of the upstream firms is a one time decision instead of series of decisions. However, sometimes, the producers increase the variety of a product by adding components which enhances the obsolete upstream products. On the other hand, It is unlikely for firms to forecast the enhancements in the future prior to any investment decision.[9]

Downstream profits are assumed as reduced form functions such that the downstream firm $D_j$'s profit $\Pi(D_j)$ is the equilibrium profit function for a subgame where firms compete on price

$$\Pi_{D_j} = \pi + \alpha v_j - \mu v_i \quad \text{where} \quad \mu \in [0,1] \quad \text{and} \quad \alpha = 1 - \beta$$

[9] The Sony Corporation never expected that Grand Theft Auto, one of Sony's PlayStation title, will be a tremendous hit. Independently owned Electronic Arts, a game developer, launched various extensions of the title which increased the PS2 hit titles in the market

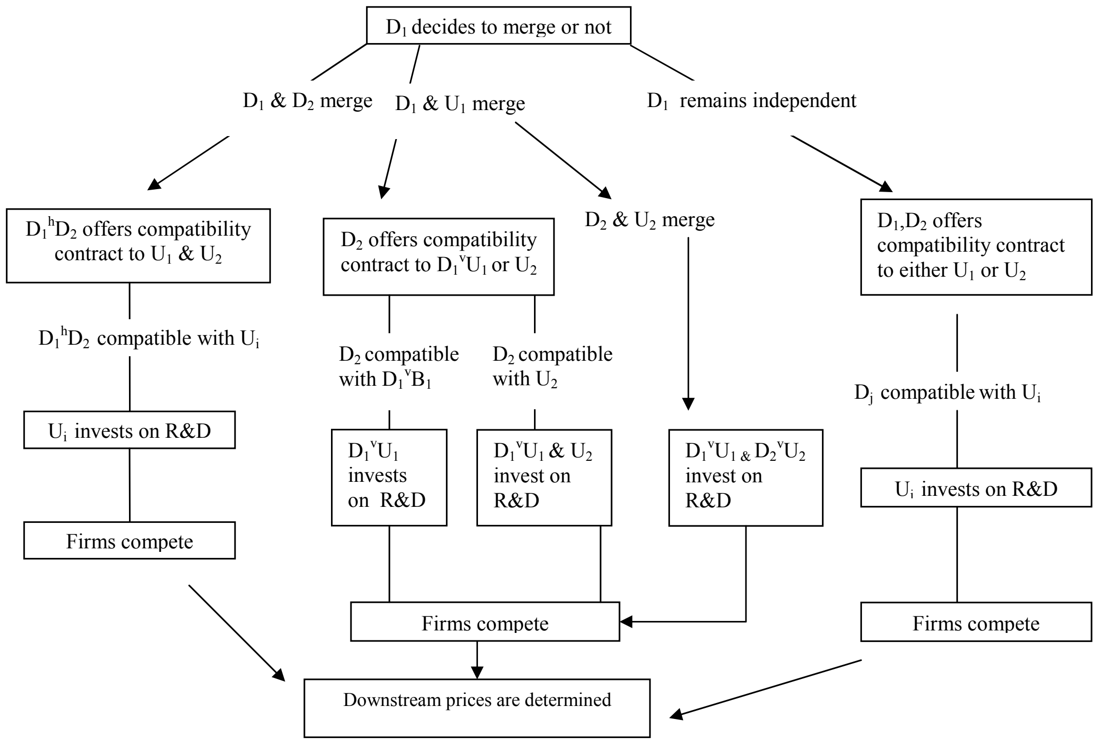


Figure 1.1: Timeline

A compatibility decision between an upstream firm $U_i$ and an downstream firm $D_j$ increases the downstream profits by $v_i$. If $U_i$ and $D_j$ do not integrate, $D_j$ pays a share of its gain, $\beta v_j$, to its compatible upstream firm as a contract fee where $\beta \in [0.1]$. [10] One way to think $\beta$ as just a sharing parameter which is exogenous. The other way is that, without loss of generality, we can assume that the downstream firms have the bargaining power over the upstream firms. $U_1$ and $U_2$ are ex ante identical but ex post different. That's why, downstream producers can hold the upstream suppliers to their opportunity costs. Hence they can have the same opportunity cost to outside opportunities, but different productivity in this market.

The sensitivity $U_i$'s and $D_j$'s revenues to $D_j$' s variety whenever $U_i$ supplies to $D_j$ is measured by the parameter $\beta$. In equilibrium, $\beta$ depends on the outside options of the upstream firms. We can think $\beta$ as a market price per variety for given varieties to cover upstream firms' opportunity cost. In many markets, the expected value of an always available outside option for a firm can be determined. Each firm would have the same expected outside option, especially when they have significantly close production efficiencies. One can do a "local" analysis along the efficiency parameter with same expected outside options from different random draws for their market. In particular, this is true even if $U_1$ and $U_2$ draw different values of $\varepsilon_1$ and $\varepsilon_2$ in their particular market. This paper assumes that downstream firms have all the bargaining power that's why $\beta$ can be determined by the outside option, $\theta$, which is exogenous. [11]

For better exposition purposes, I will assume $\beta \in [\mu, 2\mu]$. However the assumption would not change many results. The assumption suggests that any integrated firm would supply to an independently owned downstream firm. If $\beta < \mu$, then the integrated firm would supply zero to an independently owned downstream firm. However, there may exist $\beta$ and $\mu$ such that $\beta < \mu$. One must be aware

[10] Heavner(2004) suggested that 50/50 bargaining rule implies that upstream and downstream firms share the profit $v_j$. i.e. $\alpha = 1 - \beta = \beta = 1/2$.

[11] In perfectly competitive markets,p=MC

that,there might be a region of $(\varepsilon_1, \varepsilon_2)$ in which no merger activity happens at all. Moreover, the assumption suggests that the price of an upstream firm is not too high that would harm a downstream firm, i.e. $\beta < 2\mu$.[12] That's why I am going to assume $\beta \in [\mu, 2\mu]$ for better exposition of the results.

Downstream profits depend on a fixed term $\pi$, variety of the firm $v_i$ and variety of a downstream competitor $v_j$, and the sensitivity measures $\beta$ and $\mu$. [13]. The exogenous parameter $\mu$ measures the sensitivity of $D_i$'s revenues to $D_j$'s variety.[14]

Reduced form of downstream profit functions come from a pricing game (given varieties) such that each downstream product's demand is

$$Q_i(p_i, p_j) = A + v_i - p_i + \delta(p_j - v_j) \quad i, j = 1, 2$$

where $v_i$ $i = 1, 2$ is the variety of upstream products that are supplied by a contracted upstream firm. The profit of a downstream firm that confirms a compatibility contract with an upstream firm $U_j$is

$$\Pi(D_i; U_j) = (p_i - c_i)Q_i(p_i, p_j) - C(\beta, v_j) \quad i = 1, 2$$

where $c_i$ is the marginal cost of production and $C(\beta, v_j)$ is the cost of acquiring side products from an upstream firm. $C(\beta, v_j) = cv_j$ is the payment that a downstream firm must make to sign a compatibility contract with an upstream firm.

The unique Nash equilibrium exists for the relevant $\delta \in (0, 1)$ and the equi-

[12]The assumption also suggests that outside option of an upstream firm is not high.

[13]$\alpha = 1 - \beta$ could measure the externality effect which is discussed on Gandal & Kende and Rob (2000)

[14]One way that $\mu$ could measure the sensitivity of $D_i$'s revenues to $D_j$'s variety is if the elasticity of demand for $D_i$'s product with respect to price per unit of variety of $D_j$'s product is increasing in $\mu$, i.e. $D_i = A - \frac{p_i}{v_i} + \mu \frac{p_j}{v_j}$

librium prices and profits are,

$$p_i(v_i; v_{-i}) = \frac{A(2+\delta) + (2-\delta^2)v_i - \delta v_{-i}}{4-\delta^2}$$
$$\Pi(D_i; U_j) = (f(v_i, v_{-i}))^2 - cv_i$$

,where

$$f(v_1, v_2) = \frac{A}{2-\delta} + \frac{(2-\delta^2)v_1 - \delta v_2}{4-\delta^2}$$

The downstream profit function is increasing in its variety and decreasing in competitor's variety. As a result, downstream profit functions can be represented by the reduced form functions $\Pi(D_j) = \pi + \alpha v_j - \mu v_i$ that carries the same characteristics. The constant number $\pi$ depends on the demand variables $A$ and $\lambda$. The reduced form enables me to avoid any complex price analysis which has done in the literature quite often.[15]

The profit function of a upstream firm depends on the variety the firm supplies to the market and its R&D cost.

$$\Pi_{U_1} = \Sigma_j \lambda_j (\beta v_j - r_{1j})$$
$$\Pi_{U_2} = \Sigma_j (1-\lambda_j)(\beta v_j - r_{2j}), \quad j = 1, 2$$

$\lambda_j$ is an indicator function which is positive when $U_1$ is compatible with $D_j$ and which is zero when $U_2$ is compatible with $D_j$.[16] $\beta$ is a measure of the marginal upstream profit increase due to a variety effect in the market.

In case of a vertical integration, the integrated firm's profit is the combined

[15] Chen(2001),Ordover,Saloner& Salop(1990)

[16] $D_j$ can be compatible with either $U_1$ or $U_2$

profits of upstream and downstream divisions.

$$\Pi(D_i^v U_j) = \pi + v_i - \mu v_j - r^v + \lambda_j(\beta v_j - r_{ij})$$

$r^v$ is the investment of the upstream division for the downstream division and $\lambda_j$ is an indicator function which is positive when the independent downstream firm is compatible with $D_i^v U_j$ and which is zero when the independent downstream firm is compatible with the independent upstream firm.

In case of a horizontal merger, the downstream reduced form profit function can be obtained from a pricing game (given varieties) such that each downstream product's demand is

$$Q_i(p_i, p_j) = A + v_i - p_i + \delta(p_j - v_j) \quad i, j = 1, 2$$

The profit of the merged downstream firm is

$$\Pi(D_1^h D_2) = p_1 Q_1(p_1, p_2; v_1, v_2) + p_2 Q_2(p_1, p_2; v_1, v_2) - C(\beta, Q_1, Q_2, v_1, v_2)$$

The monopolist will choose $p_1$ and $p_2$ for given varieties such that

$$\frac{\partial \Pi}{\partial p_1} = 0 \quad \text{and} \quad \frac{\partial \Pi}{\partial p_1} = 0$$

The equilibrium prices and quantities for given zero marginal cost can be solved as

$$p_1^h(v_1) = \frac{A}{2(1-\delta)} + \frac{v_1}{2}$$
$$p_2^h(v_2) = \frac{A}{2(1-\delta)} + \frac{v_2}{2}$$
$$Q_1^h(p_1, p_2) = \frac{A + v_1 - \delta v_2}{2}$$
$$Q_2^h(p_1, p_2) = \frac{A + v_2 - \delta v_1}{2}$$

Then,

$$\Pi(D_1, D_2) = p_1^h Q_1^h + p_2^h Q_2^h - C(v_1 + v_2)$$

and that is

$$2(1-\delta)M^2 + (1-\delta)Mv_1 + (1-\delta)Mv_2 + (\frac{v_1 - v_2}{2})^2 - \frac{\delta v_1 v_2}{2} + \frac{v_1 v_2}{4} - cv_1 - cv_2$$

The profit function depends on a constant, variety of $D_1$ and variety of $D_2$. The profit is increasing in both varieties. That's why the profit function can be represented by a reduced form profit function as

$$\Pi_{D_1 D_2} = 2\pi + k + \alpha v_{D_1 D_2}$$

where the exogenous variable $k$ represents an equilibrium profit surplus for a subgame where firms compete on price and $v_{D_1 D_2}$ is the variety of the new downstream firm $D_1^h D_2$. To justify $k$ one can show,

$$p_1^h(v_1)Q_1^h(p_1, p_2) - cv_1 > \Pi(D_1) - cv_1$$
$$p_2^h(v_2)Q_2^h(p_1, p_2) - cv_2 > \Pi(D_2) - cv_2$$

The profits can be represented by the reduced form of profits

$$\Pi(D_1) = \pi + \alpha v_1 - \mu v_2$$
$$\Pi(D_2) = \pi + \alpha v_2 - \mu v_1$$
$$\Pi(D_1, D_2) = 2\pi + k + \alpha(v_1 + v_2)$$

Although the profit surplus $k$ is assumed to be exogenous in the model, k can be found by calculating the equilibrium profit gain. For instance, if the demand for product $i$ is $Q_i(p_i, p_j) = A + v_i - p_i + \delta(p_j - v_j)$ then $k = 2(1-\lambda)M^2 - 2K^2$ where $M = \frac{A}{2(1-\lambda)}$ and $K = (\frac{A}{2-\delta})$ and the remaining other profit surplus due to the variety effect (depending on $(\varepsilon_1, \varepsilon_2)$) is captured by the difference $\mu(v_1 + v_2)$ when $D_1, D_2$ merge.

## 1.5 Discussion

This paper analyzes a value added model of downstream and upstream firms. In the model, the added value is represented by the term variety, $v_j = \varepsilon_i + \tau(r_{ij})$, which is a characteristic of a downstream firm that is supplied by an upstream firm. The added value is modeled with a fixed effect $\varepsilon$, which is specific for each upstream firm, and technology function.[17]. My paper constructs a theory of mergers on Gandal & Rob and Kende (2000). GRK studied a model of fixed effects. However, they take the number of titles as an exogenous value from the data they used. They utilized the number of titles as a proxy to estimate the price and cross price elasticities of the profit function. [18] The fixed effect model controls for the differences between upstream suppliers. A fixed effect model is plausible in many industries in

[17]Heavner (2004) also studies an added value model

[18]$(\Pi = \pi + \alpha_1 v_i + \alpha_{-1} Y$ where $v_i$ is exogenous$)$

which the suppliers (upstream) has the same cost structure but different production capacities (Qualities). Telecommunication industry (AT&T, Lucent Technologies), video game console industry (EA, Konami, Sega) can be some examples. One can argue a downstream and upstream specific fixed effect. Modeling $\varepsilon_{ij}$ instead of $\varepsilon_i$ is not plausible because $\varepsilon_1$ and $\varepsilon_2$ is the production characteristics of upstream firms. The production efficiency of a producer can not depend on any efficient contract. In addition, the parameter space $(\varepsilon_1, \varepsilon_2)$ infers $U_1$'s technological advantage or disadvantage relative to $U_2$. The model has an empirical implication which can be a structural model which estimates profits by using the marginal revenue $v_i$ as a proxy whenever the data contains the number of different titles associated with a downstream product and the supplier's cost in the upstream market. The empirical model can forecast a wave of mergers in complementary markets. The next section starts analyzing the fixed effect model.

# Chapter 2

# Choice of Organizational Form: "Conditional" Equilibria

In this chapter, I analyze the determinants of various mergers and contractual decisions conditional on certain aspects of organizational choice being ascended away. I analyze each of the organizational forms of the literature cited above. First section analyzes the integration decision under the assumptions $D_1$ and $D_2$ can not merge $U_2$. Second section analyzes the integration decision under the assumption that $D_1$ and $D_2$ can also merge but $U_2$ and $D_2$ can not merge.

## 2.1 Vertical Integration Analysis with No Horizontal Merger and No Counter Integration of Firms

This section imposes two restrictions on the model. First, only one downstream firm may integrate with an upstream firm and two downstream firms can not merge. Second, remaining independent downstream and upstream firms can not counter merge. Thus, the section analyzes the integration decision of $D_1$ and $U_1$ in a setting in which $D_2$ and $U_2$ can not integrate as a market reaction. $D_2$ may preserve its contractual relations with either the new integrated firm or remaining upstream firm $U_2$ post integration.

Upstream firms $U_1$ and $U_2$ are not necessarily symmetric. A downstream firm would extend a compatibility contract to an upstream firm if the return of the option is the highest. If $\Pi(D_i; U_j)$ denotes the profit of downstream firm $D_i$ which contracts with upstream firm $U_j$ and $D_i \sim U_j$ denotes that $D_i$ is compatible with $U_j$ which suggests upstream firm $U_j$ will supply a variety of side products to $D_j$'s product, then

$$D_j \sim U_1 \quad if \quad \Pi(D_j; U_1) > \Pi(D_j; U_2)$$
$$D_j \sim U_2 \quad if \quad \Pi(D_j; U_2) > \Pi(D_j; U_1)$$

Before starting the analysis, one must determine a tie breaking rule that should explain which of the independent upstream firms is going to be associated with an independent downstream firm in case the profits are equal. In this case, intuitively, a downstream firm would offer a compatibility contract to the upstream firm that is more efficient. That's why, tie breaking rule favors the more efficient upstream firm.

$$D_j \sim U_1 \quad if \quad \varepsilon_1 \geq \varepsilon_2 \ \& \ \Pi(D_j; U_1) = \Pi(D_j; U_2)$$
$$D_j \sim U_2 \quad if \quad \varepsilon_2 > \varepsilon_1 \ \& \ \Pi(D_j; U_1) = \Pi(D_j; U_2) \ \ , j = 1, 2$$

The integration decision of $D_1$ and $U_1$ depends on two aspects. First, how profitable the new integrated firm can be in both upstream and downstream markets. Second, how the downstream competitor will be supplied post integration. $D_1$ and $U_1$ integrate if the profit of the integrated firm is at least as high as the the sum of $U_1$'s and $D_1$'s profits in case of no integration. An integration can be foregone if the profit of integration is less than the individual profits.

I will follow rules of backward induction to analyze the integration decision under the condition that no counter merger is allowed. The analysis presents the conditions under which an integration is a subgame perfect Nash equilibrium. This subsection also shows the conditions under which firms can renounce vertical integration and maintain only contractual relations.

If each firm decides to have a contractual agreement, each firm is going to maximize its own profit. Under independent ownership, the profits are going to be,

[1]

$$\Pi(D_j;U) = \pi + \alpha v_j - \mu v_{-j} \quad , j = 1,2$$

$$\Pi(U_1) = \lambda_1(\beta v_1 - r_{11}) + \lambda_2(\beta v_2 - r_{12})$$

$$\Pi(U_2) = (1-\lambda_1)(\beta v_1 - r_{21}) + (1-\lambda_2)(\beta v_2 - r_{22}) \quad , (\alpha,\beta) \in \{(0,1)\text{x}(0,1) : \alpha + \beta = 1\}$$

$\lambda_j$ is an indicator function which is determined by the compatibility agreements between the downstream and upstream firms. It is positive if the $D_j$ is compatible with $U_1$ ($\lambda_j = 1$). The downstream firm can offer a contract to only one upstream firm, so the $D_j$ would offer a contract to $U_2$ if the firm did not offer to

[1] $\lambda_1 = 1 \; if \; D_1 \sim U_1, \quad \lambda_1 = 0 \; if \; D_1 \sim U_2, \quad \lambda_2 = 1 \; if \; D_2 \sim U_1, \quad \lambda_2 = 0 \; if \; D_2 \sim U_2$

$U_1(\lambda_j = 0)$. Thus, $U_1$'s goal is to maximize profit. That is,

$$\max_{r_{11}, r_{12}} \quad \lambda_1(\beta(\varepsilon_1 + \tau(r_{11})) - r_{11}) + \lambda_2(\beta(\varepsilon_1 + \tau(r_{12})) - r_{12}) \tag{2.1}$$

The first order conditions imply,

$$\frac{\partial \Pi_{U_1}}{\partial r_{1i}} = \beta \lambda_i \tau^{'}(r_{1i}) - 1 = 0 \tag{2.2}$$

$U_1$'s optimal investment level which maximizes its profit is $r_{1j} = \gamma(\beta^{-1})$ if $\lambda_j$ is positive, where $\gamma(x)$ is the inverse function of the derivative of the investment function, i.e. $\gamma(x) = \tau^{'-1}(x)$.

As a result, the optimal investment $r_{ij}$ depends on two factors: Marginal product of investment and the sensitivity $U_i$'s revenue to $D_j$'s variety. Marginal product of investment, $\tau^{'}(x)$, is a decreasing function and so $\gamma(x)$ is. That's why, the $D_j$'s variety increases as the value of upstream firms' outside option increases because revenue sensitivity parameter $\beta$ increases as the outside option's value increases. If no merger occurs in the market, $U_2$ solves a similar profit maximization problem, and chooses a firm specific investment level $r^*_{2i} = \tau^{'-1}(\beta^{-1})$ whenever $D_i$ is compatible with $U_2$.[2] Any upstream firms agrees on a compatibility contract if the net gain from the contract will be positive. [3]

Moreover, downstream firms' contractual relations must maximize their profits. Thus, an independent downstream firm's main goal is to be supported by as various side products as possible, which necessarily maximizes its profit. In particular, a downstream firm would extend a compatibility contract to the upstream firm

[2]If we allow $\varepsilon_i$ to be negative as well , then an independently owned upstream firm accepts a compatibility contract if $\varepsilon_i \geq \beta^{-1} r^*_{ij} - \tau(r^*_{ij})$

[3]$\Pi(U_i; D_j) > 0$

which can commit to supply the most variety.

At the time, each downstream firm prefers contracting with the upstream firm which has more efficient productivity. Thus, being $\varepsilon_1$ greater than $\varepsilon_2$ leads $D_1$ and $D_2$ to extend a contract to $U_1$; while, $D_1$ and $D_2$ extend a compatibility contract to $U_2$ if $\varepsilon_2$ is greater than $\varepsilon_1$. Moreover, comparing the upstream profits shows that both $U_1$ and $U_2$ prefer accepting as many compatibility contracts as possible.[4] Lemma 1 summarizes the equilibrium compatibility contracts when $U_1$ and $D_1$ are independent.

**Lemma 1.** *Assume that*

*(A1) Upstream firms are independently owned*

*(A2) Downstream firms are independently owned*

*Both downstream firms extend a compatibility contract offer to the more efficient upstream firm. The more efficient upstream firm accepts the downstream firms' contract offers.*

Should no upstream firm and downstream firm merge, downstream producers offer compatibility contract to the more efficient upstream firm. The more efficient upstream firm can supply more variety than the less efficient one which necessarily increases downstream profits. Hence, the only supplier in the market will be the more efficient upstream firm in any equilibrium in which no downstream firm integrate with an upstream firm.

Next, I will analyze the equilibrium compatibility contracts when $U_1$ and $D_1$ are integrated. The integration decision of two firms does not have any affect on the remaining independent upstream firm $U_2$'s gain from any compatibility contracts. That's why, the integration decision of $U_1, D_1$ does not affect $U_2$'s firm specific

[4]If we let $\varepsilon_i < 0$ then $U_i$ may not accept a contract because an independent $U_i$ can not profitably produce.

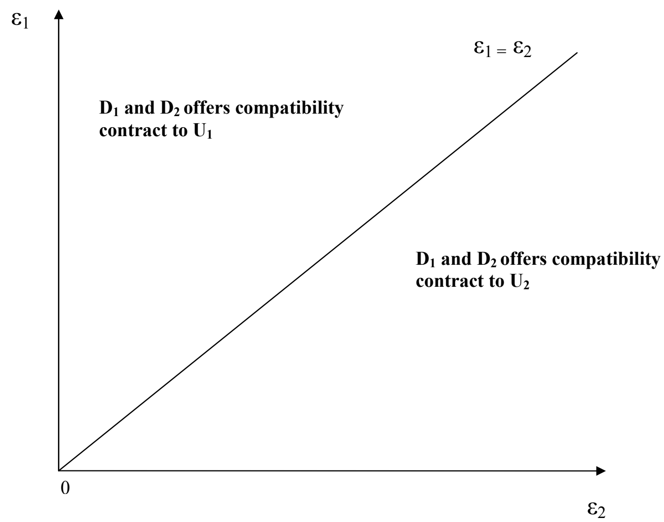


Figure 2.1: Compatibility Contracts under Independent Ownership

optimal R&D investment in case $U_2$ is compatible with the remaining independent downstream firm $D_2$.

Post integration, the upstream division $U_1$ will supply for the downstream division $D_1$ without signing a compatibility contract. Vertical integration of firms has two effects; one direct effect and one indirect effect on the upstream division $U_1$'s incentives. The direct effect is increasing the optimal R&D investment which increases the downstream profit of the integrated firm. Thus, the total variety supplied to its downstream division will be higher post integration. The indirect effect is being reluctant to invest for its downstream competitor $D_2$ in case $D_2$ extends a compatibility contract to the upstream division of the integrated firm. The integrated firm may increase its upstream profits and earn $\beta(\varepsilon_1 + \tau(r_{12}) - r_{12})$.

On the other hand, the integrated firm can loose some of its downstream profit if the upstream division supplies a high variety for $D_2$ and loose $(\mu(\varepsilon_1 + \tau(r_{12})))$. As a result, $U_1$ will have less incentive to supply $D_2$. Consequently, this effect increases $D_2$'s incentives to extend a compatibility contract to the independent upstream firm $U_2$, even if $U_2$ is less efficient than $U_1$

The profit of $U_1^v D_1$ will be

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(r_{11}) - \mu(\varepsilon_2 + \tau(r_{22}^*))) - r_{11} \quad if \quad D_2 \sim U_2$$

$$\Pi(U_1^v D_1; D_2) = \pi + \varepsilon_1 + \tau(r_{11}) + (\beta - \mu)(\varepsilon_1 + \tau(r_{12}^*))) - r_{11} - r_{12} \quad if \quad D_2 \sim U_1^v D_1$$

Since $U_1, D_1$ integration decision has no effect on $U_2$'s gain, $U_2$ invests $r_{22}^* = \gamma(\beta^{-1})$ if $D_2$ extends a compatibility contract to $U_2$. Post integration, the integrated firm $U_1^v D_1$ invests to solve

$$\max_{r_{11}, r_{12}} \quad \pi + \varepsilon_1 + \tau(r_{11}) - r_{11} - (1 - \lambda_2)\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) + \lambda_2((\beta - \mu)(\varepsilon_1 + \tau(r_{12})) - r_{12})$$

$U_1$ invests for $D_2$ if the independent downstream firm extends a contract to the upstream division of the integrated firm and the integrated firm accepts the contract. $U_1$ invests more for $D_1$ post integration, whereas $U_1$ will be more reluctant to invest for $D_2$. Lemma 2 summarizes the equilibrium optimal investments when $U_1, D_1$ integrate.

**Lemma 2.** *Assume that*

*(A1)* $U_1$ *and* $D_1$ *merge*

*(A2) No counter merger*

*(A3)* $\beta > \mu$

*If remaining independent downstream firm* $D_2$ *signs a compatibility contract with the* *upstream division of the integrated firm, then the integrated firm invests* $r_{11} = \gamma(1)$ *and* $r_{12} = \gamma((\beta - \mu)^{-1})$

$D_2$ will be supplied by $U_2$ if the integrated firm does not accept the compatibility contract from $D_2$. In this case, $D_2$ will be supplied by $\varepsilon_2 + \tau(r^*_{22})$. The integrated firm agrees to be the supplier to its downstream competitor if the potential value of the agreement is positive.

$$(\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) - \gamma((\beta - \mu)^{-1}) \geq -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \quad \text{if} \quad \beta > \mu$$
$$(\beta - \mu)\varepsilon_1 \geq -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \quad \text{if} \quad \beta < \mu$$

The integrated firm will supply to its downstream competitor if there is an economic gain. In other words, the integrated firm accepts to produce for $D_2$ if the gain from supplying, which are; a portion of $D_2$'s sales, the competitive effect and the cost of R&D, is higher than the opportunity cost of not supplying, which is the competitive effect when $U_2$ supplies $D_2$. Lemma 3 summarizes the integrated firm's compatibility decision with $D_2$ when $U_1, D_1$ integrate.

**Lemma 3.** *Assume that*

*(A1)* $U_1$ *and* $D_1$ *merge*

*(A2) No counter merger*

*The integrated firm's upstream division* $U_1^v D_1$ *would supply for the remaining independent downstream firm* $D_2$ *if and only if*

$$\varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \geq F_1(\mu, \beta) \qquad if\ \beta > \mu \qquad (2.3)$$

$$\varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \geq F_2(\mu, \beta) \qquad if\ \beta < \mu \qquad (2.4)$$

*,where*

$$F_1(\mu,\beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta-\mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} - \tau(\gamma((\beta-\mu)^{-1}))$$

$$F_2(\mu,\beta) = -\frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu}$$

$$F_2(\mu,\beta) > 0 > F_1(\mu,\beta)$$

That's why, $U_1^v D_1$ always agrees on compatibility contract with $D_1$ whenever $D_2$ extended a contract if $\beta > \mu$. On the other hand, there may still be a positive economic gain, although $\beta < \mu$ in the downstream market so that the integrated firm can agree on a contract with its downstream competitor. However, the integrated firm would have no incentive to invest for $D_2$ since any positive investment would decrease the overall profit. Supplying $D_2$ won't have any affect on $U_1$'s incentives to supply $D_1$. Post integration, upstream division $U_1$ invests more for downstream division $D_1$ even if $U_1^v D_1$ is compatible with $D_2$. In conclusion, the optimal level of $U_1$'s investment for $D_1$ is higher than the optimal level of investment $U_1$ has for the downstream competitor. Consequently, the incentive of the independent downstream firm $D_2$ to be compatible with the integrated firm is reduced due to the integrated firm's unwillingness to invest for its competitor. Thus, $U_1 D_1$ integration increases the likelihood of $D_2$ to be compatible with $U_2$.

Post integration, the profit function of the independent downstream firm $D_2$

is,

If $\beta > \mu$

$\Pi(D_2) = \pi + \alpha(\lambda_2(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) + (1 - \lambda_2)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$

If $\beta < \mu$

$\Pi(D_2) = \pi + \alpha(\lambda_2(\varepsilon_1) + (1 - \lambda_2)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$

$D_2$ extends a compatibility contract to the integrated firm if

$$\Pi(D_2; U_1^v D_1) > \Pi(D_2; U_2)$$

Lemma 4 summarizes the $D_2$'s equilibrium decision to be compatible with $U_1^v D_1$ when $U_1, D_1$ integrate.

**Lemma 4.** *Assume that*

*(A1) $U_1$ and $D_1$ merge*

*(A2) No counter merger*

*The remaining independent downstream firm $D_2$ offers a compatibility contract to upstream division of the integrated firm if and only if*

$$\varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) \qquad \textit{if} \quad \beta > \mu \tag{2.5}$$

$$\varepsilon_1 - \varepsilon_2 \geq G_2(\beta) \qquad \textit{if} \quad \beta < \mu \tag{2.6}$$

*,where*

$$G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta - \mu)^{-1})) \tag{2.7}$$

$$G_2(\beta) = \tau(\gamma(\beta^{-1})) \tag{2.8}$$

*Otherwise, the downstream firm will offer compatibility contract to the independent upstream firm* $U_2$.

$D_2$'s willingness to extend a compatibility contract to $U_1^v D_1$ increases as the upstream division $U_1$'s relative efficiency to $U_2$'s efficiency increases. $D_2$ extends a contract to $U_1^v D_1$ if the $U_1$'s efficiency advantage is adequate. On the other hand, $D_2$ offers a compatibility contract to $U_2$ if $U_2$ is more efficient than $U_1$ or adequate efficient so that $D_2$ will be supplied with a higher variety post integration.

The analysis partitions the $(\varepsilon_1, \varepsilon_2)$ parameter space into three different strategic regions.[5]

$$R_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \geq F_1(\mu, \beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta - \mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta - \mu} - \tau(\gamma((\beta-\mu)^{-1}))\}$$

$$R_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1}))\}$$

$$R_3 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \varepsilon_2 \geq 0\}$$

$$\text{Note that} \quad R_2 \subset R_3 \quad \text{and} \quad R_2 \subset R_1$$

$$\xi_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_2\}$$

$$\xi_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_3 \quad \text{and} \quad (\varepsilon_1, \varepsilon_2) \notin R_2\}$$

$$\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}$$

$\xi_1$ is the first strategic region. If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, then $D_2$ offers a compatibility contract to $U_1^v D_1$ because upstream division $U_1$ has the efficiency superiority. Second strategic region is $\xi_2$. If $(\varepsilon_1, \varepsilon_2) \in \xi_2$, then $D_2$ offers a compatibility contract to

[5] $R_2 \subset R_1$ since $G_1(\beta, \mu) > F_1(\beta, \mu)$. The slope of the line that defines $R_1$ is negative while the slope of the line that defines $R_2$ is one. One can proof that every $(\varepsilon_1, \varepsilon_2)$ which is in $R_3$ also is in $R_2$

$U_2$ even though upstream division $U_1$ has the efficiency superiority because the integrated firm would invest less for its downstream competitor. The last strategic region is $\xi_3$ in which $U_2$'s efficiency is more than $U_1$'s. If $(\varepsilon_1, \varepsilon_2) \in \xi_3$, $D_2$ offers a compatibility contract to $U_2$. $U_1, D_1$'s integration decision does not have any affect on $D_2$'s incentive to be compatible with $U_2$ in $\xi_3$.

Until this part, I have examined the equilibrium compatibility contracts given $U_1$ and $D_1$'s integration decision. The next part will examine the $U_1$'s and $D_1$'s integration decision. For better exposition purposes, the rest of the paper assumes that $\beta > \mu$.

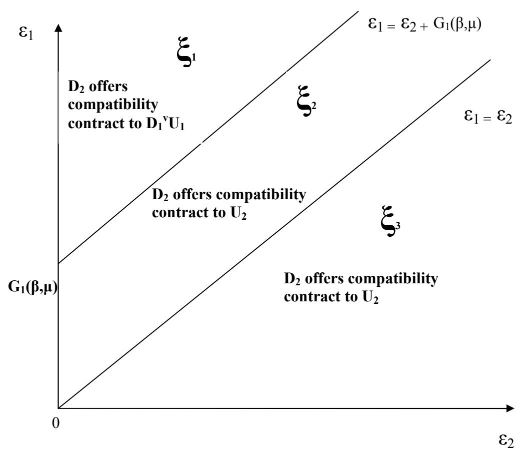


Figure 2.2: Compatibility Contracts if Vertical Integration Occurs with No Counter Merger

If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, in which $U_1$ has an efficiency superiority, then $U_1, D_1$'s integration decision has no affect on $D_2$'s incentives. Both downstream firms will be compatible with $U_1$ when $U_1, D_1$ do not integrate. $D_2$ will be supplied by $U_1$ even when $U_1, D_1$ integrate.

Moreover, if $(\varepsilon_1, \varepsilon_2) \in \xi_2$, both downstream firms will be compatible with $U_1$ when $U_1, D_1$ do not integrate but $D_2$ will be supplied by $U_2$ when $U_1, D_1$ integrate. $U_2$ can attract $D_2$'s business post integration that's why integration decision will change the $D_2$'s incentive to offer a compatibility contract to $U_1$. Should $D_2$ compatible with the upstream division $U_1$, integration harms $D_2$'s profit because $U_1^v D_1$ invests less for $D_2$ post integration. Hence, $D_2$ can switch its first choice supplier when $U_1, D_1$ integrate.

Furthermore, if $(\varepsilon_1, \varepsilon_2) \in \xi_3$, both downstream firms will be compatible with $U_2$ when $U_1, D_1$ do not integrate. As a matter of fact, the willingness of $D_2$'s to extend a compatibility contract to $U_2$ also can not be altered when $U_1, D_1$ integrate. Nevertheless, $U_1^v D_1$ would accept an offer from $D_2$ if $D_2$ offered when $U_1, D_1$ integrate.

The analysis shows that for every $\varepsilon_1 < G_1(\beta, \mu)$ $D_2$ will be compatible with $U_2$ when $U_1, D_1$ integrate. The upstream division $U_1$'s efficiency $\varepsilon_1$ must be at least $G_1(\beta, \mu)$ to attract $D_2$. Note that Lemma 4 suggests that $G_1(\beta, \mu)$ is increasing in $\mu$. Hence, the more competitive downstream market is (higher $\mu$), the more efficient the integrated firm should be in order to alter $D_2$'s incentives. In addition, a higher $\mu$ causes $U_2$ to reduce optimal level of investment that $U_1$ has for $D_2$. That's why, a higher $\mu$ expands the region $\xi_2$ and shrinks the region $\xi_1$.

**Theorem 1.** *Assume that*

*(A1) No counter merger*

*(A2) No horizontal merger of downstream firms*

*If* $(\varepsilon_1, \varepsilon_2) \in \xi_1$, *then* $U_1$ *and* $D_1$ *integrate.* $D_2$ *is supplied by the upstream division* $U_1$.

Lemma 1 states that downstream firms offer compatibility contract to $U_1$ if

$(\varepsilon_1, \varepsilon_2) \in \xi_1$. Lemma 4 states that $D_2$ is supplied by $U_1^v D_1$ when $U_1, D_1$ integrate, if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. Theorem 1 summarizes the firms' strategy when $U_1$ has an efficient superiority over $U_2$. As a result, $U_1, D_1$ integrate under the conditions because of both gain in upstream and downstream markets. $D_2$ extends a contract offer to $U_1^v D_1$ since integration decision of $U_1, D_1$ does not affect $D_2$'s incentives. To summarize, $D_1$ and $U_1$ integrate in $\xi_1$ if no counter merger and no horizontal merger of downstream firms are allowed.

Next, $U_1, D_1$'s integration decision can alter the $D_2$'s motives if $(\varepsilon_1, \varepsilon_2) \in \xi_2$. $U_1$ is more efficient but does not have the superior productivity to attract $D_2$ when $U_1, D_1$ integrate in $\xi_2$. Lemma 4 suggests $D_2$ extends a compatibility contract to $U_2$ if $U_1, D_1$ integrate in $\xi_2$. We define the strategic region $\Lambda_1$ as

$$\Lambda_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \leq X_1(\beta, \mu)\} \quad \text{where}$$

$$X_1(\beta, \mu) = \frac{\tau(\gamma(1)) - \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1})}{\beta - \mu} \quad \text{and}$$

$$\xi_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_3 \quad \& \quad (\varepsilon_1, \varepsilon_2) \notin R_2\}$$

Theorem 2 summarizes the the firms' strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_2$.

**Theorem 2.** *Assume that*

*(A1) No counter merger*

*(A2) No horizontal merger of downstream firms*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_2$,then $U_1$ and $D_1$ merge if and only if $(\varepsilon_1, \varepsilon_2) \in \Lambda_1$. $D_2$ will be supplied by $U_2$. Otherwise, $U_1$ and $D_1$ remains independently owned. $D_1$ and $D_2$ will be supplied by $U_1$.*

The strategic region $\Lambda_1$ defines the region in which the total gain of both

$U_1, D_1$ from integration is sufficiently large. Theorem 2 partitions the region $\xi_2$ into two strategic regions. In the first region , $\Lambda_1 \cap \xi_2$, $U_1$ and $D_1$ integrate even though $D_2$ chooses to be supplied by $U_2$ post integration. The reason is the gain due to $U_1$'s higher investment for $D_1$ after integration being more than $U_1$'s cost of loosing $D_2$'s business. In second region, $U_1$ and $D_1$ forego integration, mostly because it is more costly for $U_1$ not to acquire a contractual relation with $D_2$.

As a result, $U_1, D_1$ integrate $U_2$'s efficiency $\varepsilon_2$ is significantly low. On the other hand, $U_1$ is reluctant to integrate for high values of $\varepsilon_2$ because $D_2$ would switch its complementary good supplier in case of integration. Hence, $U_1, D_1$ integration has two opposite effects on upstream division $U_2$'s profit. Positive effect is the bilateral gain due to higher variety. Negative effect is the loss of a potential business that would be acquired in case $U_1$ remains independent.

To sum up, $U_1, D_1$ always integrate whenever $U_2$ can not provide the required competition to lower the integrated firm's profit. Intuitively, $U_1, D_1$ always integrate if $U_1^v D_1$ has no incentive to accept a compatibility contract from $D_2$. Meanwhile, $U_1$ can forego integration even if $U_1^v D_1$ is willing to supply $D_2$. $D_2$'s supplier decision heavily depends on the efficient asymmetry between upstream suppliers. $U_1^v D_1$ will not be offered a compatibility contract by $D_2$ in $\xi_2$ although $U_1$'s best interest is to supply $D_2$. $U_1, D_1$ may forego integration because of the competitive effect post integration.

The threshold values $(\varepsilon_1, \varepsilon_2)$ which partitions the strategic regions heavily depends on the sensitivity of $D_1$'s profits to $D_2$'s variety (i.e. $\mu$). The more sensitive the profits are, the more efficient $U_1$ should be in order to sustain a contractual relation equilibrium.[6]. Consequently, if $\mu$ is initially large, increasing $\mu$ makes $U_1, D_1$

[6]The slope of the inequality $\varepsilon_1 + \frac{\mu}{\beta-\mu}\varepsilon_2 \leq X_1(\beta,\mu)$ increases as the value $X_1(\beta,\mu)$ if we increase $\mu$

integration more likely.

On the other hand, if $\mu$ is not high, then increasing $\mu$ makes $X_1(\mu, \beta)$ closer to zero, and the region $\Lambda_1 \cap \xi_2$ shrinks.[7] That's why, $U_1, D_1$ integration will be less likely to observe as an equilibrium outcome. In this case, $D_1$ is always eager to integrate, however $U_1$ would not integrate because the opportunity cost of loosing $D_2$'s business is higher than the bilateral gain due to integration. For this reason, there is no general relationship between $\mu$ and integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_2$.

Moreover, $U_1, D_1$ integration decision also depends on how sensitive $U_1's$ profits to $D_1$'s variety (i.e. $\beta$). If $\beta$ is high. $U_1$'s optimal investment and the variety of $D_2$ and increases, that's why $U_1$ will be less willing to integrate. In this case, $U_1, D_1$ integrate if $U_1$'s efficiency is not high so that the loss of $D_2$'s business does not harm $U_1$'s profits significantly.

The last strategic region to analyze is the region in which $U_2$ has the efficiency advantage. $U_1, D_1$'s integration decision do not alter $D_2$' incentives. Lemma 1 states, $D_1$ and $D_2$ extends contracts to $U_2$ when $U_1, D_1$ do not integrate and lemma 4 states that $D_1$ will be compatible with $U_1$ while $D_2$ extends a contract to $U_2$ when $U_1, D_1$ integrate, if $(\varepsilon_1, \varepsilon_2) \in \xi_3$. We define the strategic region $\Lambda_2$ as

$$\Lambda_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \alpha\varepsilon_2 \geq X_2(\alpha)\} \quad \text{where}$$
$$X_2(\alpha) = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1) \quad \text{and}$$
$$\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}$$

Theorem 3 summarizes the firm's strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_3$

**Theorem 3.** *Assume that*

*(A1) No counter merger*

[7] $X_1(\beta, \mu)$ is a decreasing function of $\mu$.

*(A2) No horizontal merger of downstream firms*

*If* $(\varepsilon_1, \varepsilon_2) \in \xi_3$*,then* $U_1$ *and* $D_1$ *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in \Lambda_2$*.* $D_2$ *will be supplied by* $U_2$*. Otherwise,* $U_1$ *and* $D_1$ *remains independently owned.* $D_1$ *and* $D_2$ *will be supplied by* $U_2$*.*

$D_1$'s best interest is always to integrate if $U_1$ has the efficiency advantage (i.e $(\varepsilon_1, \varepsilon_2) \in \xi_1 \cup \xi_2$). If $U_1, D_1$'s decision is not to integrate in the equilibrium, the only reason is the $U_1$'s cost of loosing $D_2$'s business in $\xi_1 \cup \xi_2$. On the other hand, $D_2$'s best interest is not always to integrate if $U_2$ has the efficiency advantage(i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_3$). Not being supplied by the more efficient upstream firm $U_2$ may harm $D_1$'s profits, especially when its downstream competitor $D_2$ will be supplied by $U_2$.

Unlike $D_1$, $D_1$'s best interest may not be always to integrate if $U_1$ has the efficiency advantage (i.e $(\varepsilon_1, \varepsilon_2) \in \xi_1 \cup \xi_2$) because of the cost of integration to $U_1$. However, $D_2$'s best interest is always to integrate if $U_2$ has the efficiency advantage(i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_3$). $U_1$ can supply to any of the downstream firms unless $U_1, D_1$ integrate because both downstream firms extend a contract to $U_2$ under contractual relations.

Theorem 3 partitions $\xi_3$ into two strategic regions. $\Lambda_2$ defines the region in which both $U_1$'s and $D_1$'s best interest is to integrate in $\xi_3$. $U_1, D_1$'s integration causes $U_1$ to invest more for $D_1$, that's why the downstream division $D_1$ may increase profit of $U_1^v D_1$.

Moreover, theorem 3 states that $U_1, D_1$'s integration is more likely when $\beta$is small enough (or $\alpha = 1 - \beta$ is big enough). As $\beta$becomes smaller, the strategic region $\Lambda_2$ shrinks.[8]. We can define the value of the sensitivity of $U_1$'s profits to $D_1$'s variety ($\beta^*$) so that for every $\beta$less tan $\beta^*$, $U_1, D_1$ integration is less likely if $(\varepsilon_1, \varepsilon_2) \in \xi_3$.

[8] $X_2$ is a decreasing function of $\beta$

$\beta^*$ solves the equation $X_2(1-\beta^*) = 0$. An immediate result is; $U_1, D_1$'s integration is more likely if $\beta$is big enough. [9]. That's why there exists a critical efficiency parameter $\varepsilon_2^*$ which necessarily implies $U_1, D_1$'s integration decision.

**Corollary 1.** *Assume that*

*(A1) No counter merger*

*(A2) No horizontal merger of downstream firms*

*If* $\beta < \beta^*$ *and* $(\varepsilon_1, \varepsilon_2) \in \zeta_3$*,* $U_1, D_1$ *integrate if* $\varepsilon_2 < \varepsilon_2^*$*, where* $\varepsilon_2^* = \frac{X_2(1-\alpha)}{\beta-1}$

In conclusion, this section summarizes the firms' strategies under the conditions that no counter merger and no horizontal merger of downstream firms is allowed. The $(\varepsilon_1, \varepsilon_2)$ parameter space is partitioned into five strategic regions in which $U_1, D_1$ integrate or do no integrate in equilibrium. $U_1, D_1$ integration becomes more likely as $\varepsilon_1$ increases whenever $\varepsilon_1 > \varepsilon_2$ and $\varepsilon_1$ is not close to $\varepsilon_2$. In addition, $U_1, D_1$ integration becomes less likely as $\varepsilon_1$ increases whenever $\varepsilon_2 > \varepsilon_1$ and $\varepsilon_1$ is significantly smaller than $\varepsilon_2$. Figure 4 illustrates the predictions of Theorems 1,2 and 3. The next section will analyze $D_1$'s merger decision if a merger of $D_1, D_2$ is possible as well as $U_1, D_1$ integration.

## 2.2 Vertical Integration Analysis with No Counter Integration of Firms

This section removes one of the restrictions on firms' strategies. and imposes only one restriction on the model. The only restriction on the model is that there can be no counter merger of $U_2, D_2$ when $U_1, D_1$ integrate. Thus, this section analyzes the

[9]If $\beta = 1$, then $\Lambda_2$ is characterized by $\varepsilon_1 - \geq X_2(0)$. Then, $\Lambda_2$ is the whole $(\varepsilon_1, \varepsilon_2)$ parameter space since $X_2(0) < 0$

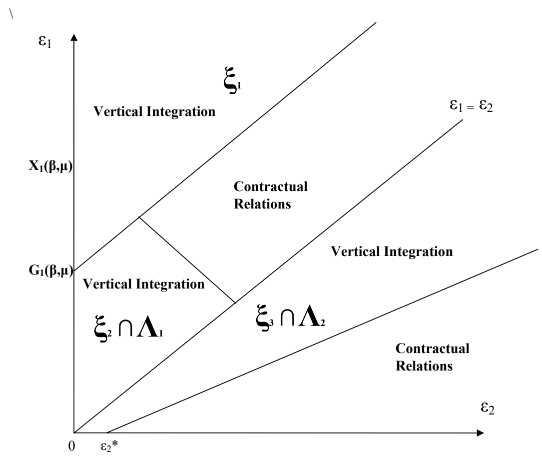


Figure 2.3: $U_1, D_1$ Integration Decision with No Counter Merger & No Horizontal Merger.

merger decision of $D_1$ in a setting that $D_2$ and $U_2$ can not integrate when $U_1, D_1$ can not integrate.

If $D_1, D_2$ merge, the new downstream firm is going to be managed by a central ownership. The firm $D_1^h D_2$ will extend contracts to both $U_1$ and $U_2$ which necessarily maximizes its profits because

$$\Pi(D_1^h D_2; U_1) < \Pi(D_1^h D_2; U_1, U_2) \quad \text{and}$$
$$\Pi(D_1^h D_2; U_2) < \Pi(D_1^h D_2; U_1, U_2)$$

The profit of $D_1^h D_2$ is

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + \alpha v_{D_1^h D_2}$$

Downstream profit depends on the variety of the downstream product. $D_1^h D_2$ will not compete in downstream market that's why any competitive effect $\mu$ will be redundant. $U_1$ and $U_2$ will supply to $D_1^h D_2$. Following lemma summarizes the equilibrium upstream firms' optimal investments when $D_1, D_2$ merge.

**Lemma 5.** *Assume that*

*(A1) No counter merger of* $U_2, D_2$

*(A2)* $D_1, D_2$ *merge*

$D_1^h D_2$ *extend compatibility contracts to both upstream firms. Both upstream firms invest*

$r^* = \gamma(\beta^{-1})$. *The product variety will be* $v_{D_1^h D_2} = \varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1}))$

An independent upstream firm's incentives to invest do not change when $D_1, D_2$ merge. That's why an independent upstream firm invests $\gamma(\beta^{-1})$. Intuitively, a downstream firm would always merge with another downstream firm if $U_1, D_1$ integration can not occur because merger will have a surplus due to no competition in downstream market. However, if an integration with a complementary good producer is also possible , a downstream firm may or may not merge with another downstream firm.

Next, I will analyze $D_1$'s merger decision, i.e merge with $U_1$ or merge with $D_2$, for each of five strategic regions defined in section 2.1.1.

The first strategic region is $\xi_1$. $D_1$ prefers integrating with $U_1$ to maintaining contractual relations if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. That's why, integration is a better option for $D_1$. If $D_1, D_2$ merger is a better option for $D_1$ than $U_1, D_1$ merger whenever $(\varepsilon_1, \varepsilon_2) \in \xi_1$, then $D_1, D_2$ merge. If $U_1, D_1$ merge, $D_2$ will be supplied by $U_1$ post integration in $\xi_2$. $D_1, D_2$ merge if the downstream profits after $D_1, D_2$ merger is higher than the downstream gain after $U_1, D_1$ merger.

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) > \Pi(U_1^v D_1; D_2) + \Pi(D_2; U_1^v D_1)$$

If we define the strategic regions $M_1$ and $HM_1$ as

$$
\begin{aligned}
HM_1 &= \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_1 \cap \xi_1\}, \text{where} \\
M_1 &= \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta}{1-2\mu}\varepsilon_2 \leq H_1(\beta, \mu)\}, \text{and} \\
H_1(\beta, \mu) &= [(1-\mu)(\tau(\gamma((1))) + \tau(\gamma((\beta-\mu)^{-1}))) - (2-\beta)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) - \gamma((\beta-\mu)^{-1}) + k](2\mu
\end{aligned}
$$

Theorem 4 summarizes the firms' strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_1$ and $D_1, D_2$ can merge.

**Theorem 4.** *Assume that*

*(A1) No counter merger of $U_2, D_2$*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, then $D_1$ and $D_2$ merge if and only if $(\varepsilon_1, \varepsilon_2) \in HM_1$. $U_1$ and $U_2$ supply $D_1^h D_2$. Otherwise $D_1$ and $U_1$ integrate. $D_2$ will be supplied by $U_1$.*

$U_1$ has a significant advantage in $\xi_1$. Theorem 4 partitions the strategic region $\xi_1$ into two different strategic regions. The first strategic region is defined by $HM_1$ which is restricted $M_1$ with the set $\xi_1$. $D_1, D_2$ merge in equilibrium if $(\varepsilon_1, \varepsilon_2) \in HM_1$. If $(\varepsilon_1, \varepsilon_2) \notin HM_1$ but $(\varepsilon_1, \varepsilon_2) \in \xi_1$, $D_1$ foregoes the merger with $D_2$ and integrate with $U_1$. The immediate result is that a higher surplus monopoly surplus k makes $D_1, D_2$ merges more likely. In addition, an increase in the magnitude of $U_1$'s efficiency advantage both increases $U_1$'s upstream profits and $D_1$'s gain from leading $D_2$ to be supplied by $U_1^v D_1$ when $U_1, D_1$ integrate. As a result, as the efficiency gap between $U_1$ and $U_2$ increases, $D_1, D_2$ merger becomes less likely. Consequently, for a given $\varepsilon_1$, higher $\varepsilon_2$ makes $D_1, D_2$ merger becomes more likely.

The strategic region $HM_1$ heavily depends on the sign of $1 - 2\mu$ that has two opposite effects on $D_1$'s decision. First, $D_1$'s decision depends on how sensitive upstream profits are to $D_1$'s variety. The more sensitive upstream profits are, the more likely $D_1$ merge with $U_1$. In other words, a smaller $\beta$, which is also a smaller $\mu$ because $\beta > \mu$, makes $D_1, D_2$ merger more likely. That is because $D_1$ profit gain due to the variety decreases especially if $D_1, D_2$ merge. Moreover, $D_1$'s decision depends on the competitive effect $\mu$. A smaller $\mu$ makes $U_1, D_1$ integration more likely because $D_1$ does not face significant competition to offset with a horizontal merger. Also, the profit surplus k would be lower if the downstream market was not competitive pre $D_1, D_2$ merger. Hence, we can not conclude a straight relationship between $D_1$'s decision and the sign of $(1 - 2\mu)$.

Second, $U_1, D_1$ integrate if $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$ in which $U_1$ has efficiency advantage but not adequate to attract $D_2$'s business to the $U_1^v D_1$ when $D_1, D_2$ is not an option. $D_2$ will be supplied by $U_2$ when $U_1, D_1$ integrate. Like in the previous strategic region $\xi_2$, if $D_1, D_2$ merger is a better option for $D_1$ than $U_1, D_1$ merger whenever $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$, then $D_1, D_2$ merge. $D_1, D_2$ merge if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) > \Pi(U_1^v D_1) + \Pi(D_2; U_2)$$

If we define the strategic regions $M_2$ and $HM_2$ as

$$\begin{aligned}
HM_2 &= \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_2 \cap \Lambda_1 \cap \xi_2\}, \text{where} \\
M_2 &= \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \varepsilon_2 \geq H_2(\beta, \mu)\}, \text{and} \\
H_2(\beta, \mu) &= [(1 - \mu)(\tau(\gamma((1)))) - (1 + \mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + \gamma(\beta^{-1}) - k](\mu)^{-1}
\end{aligned}$$

Theorem 5 summarizes the firms' strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$ and $D_1, D_2$ can merge.

**Theorem 5.** *Assume that*

*(A1) No counter merger of* $U_2, D_2$

*If* $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$*, then* $D_1$ *and* $D_2$ *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in HM_2$*.* $U_1$ *and* $U_2$ *supply* $D_1^h D_2$*. Otherwise* $D_1$ *and* $U_1$ *integrate.* $D_2$ *will be supplied by* $U_2$*.*

Although $U_1$ has the efficiency advantage in $\xi_2 \cap \Lambda_1$, $U_1$ can not attract $D_1$ when $U_1, D_1$ integrate. Allowing $D_1$ to merge with $D_2$ may or may not alter $D_1$'s merger decision. Theorem 5 partitions the parameter space $\xi_2 \cap \Lambda_1$ into two different strategic regions.

$D_1, D_2$ merge in equilibrium if $(\varepsilon_1, \varepsilon_2) \in HM_2$ where $HM_2$ is the restricted set $M_2$ with the set $\xi_2 \cap \Lambda_1$. The larger surplus makes $D_1, D_2$ merger more likely, while smaller competitive effect $\mu$ makes $D_1, D_2$ merger less likely due to the similar reasons we explained for theorem 4.

$U_1, D_1$ merge if $(\varepsilon_1, \varepsilon_2) \notin HM_2$ but $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$. The variety of downstream division $D_1$ necessarily increases, although the total variety do not necessarily increase because $D_2$ extends a contract to less efficient upstream firm $U_2$ when $U_1, D_1$ integrate. $D_1$'s best interest would not be a merger with $D_2$ if $v_{D_1^h D_2}$ was not sufficiently large when $D_1, D_2$ merge. In this case, $D_1, D_2$s gain can not compensate $D_1$ and $U_1$'s bilateral opportunity costs due to leaving $U_1, D_1$ integration option out. In addition, $D_1$'s best interest would be to integrate with $U_1$ if $U_1^v D_1$ sustains a competitive advantage over $D_2$ although $D_2$ will be supplied by $U_2$ when $U_1, D_1$ integrate. Furthermore, $U_1, D_1$ merger is more likely for smaller $\mu$ and for smaller surplus $k$.

An immediate corollary summarizes a sufficient condition under which $D_1, D_2$ always merge if $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$. (i.e. $HM_2 = \xi_2 \cap \Lambda_1$)

**Corollary 2.** *Assume that*

*(A1) No counter merger of* $U_2, D_2$

*If* $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$, $HM_2 = \xi_2 \cap \Lambda_1$ *if and only if* $k > H_3(\beta, \mu)$, *where* $H_3(\beta, \mu) = (1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + \gamma(\beta^{-1})$.

$D_1$'s merger decision also depends on the sensitivity of $U_1$'s profits to $D_1$'s variety (i.e $\beta$). There are two opposite effects on profits.

First, a larger $\beta$ gives $U_2$ more to invest more for $D_2$ under contractual relations. $D_2$ will pay a higher fee $\beta v_2$ and $D_2$ will loose downstream profits. A larger $\beta$ makes $D_1, D_2$ merger more likely because $U_2$'s gain will be $D_2$'s loss when $U_1, D_1$ integrate. Second, $D_1^h D_2$ must pay a higher fee , $(1-\beta)v_{D_1^h D_2}$ to be supplied by the upstream firms when $D_1, D_2$ merge that's why a larger $\beta$ makes $D_1, D_2$ merger less likely. Nevertheless, second effect always dominates first effect in markets. Hence, a larger $\beta$ makes $D_1, D_2$ merger more likely. Analytically, a larger $\beta$ decreases the value of $H_3(\beta, \mu)$ which makes $D_1, D_2$ merger more likely for lower values of $k$.

To sum up, theorems 4,and 5 summarize the firms' strategies if $U_1$ has efficiency advantage over $U_2$, and $U_1, D_1$ would integrate if $D_1, D_2$ merger was not an option. Next part analyzes the $D_1$'s decision if $U_1$ has efficiency advantage over $U_2$ and, $U_1, D_1$ would not integrate if $D_1, D_2$ merger was not an option. If we define the strategic regions $M_3$ and $HM_3$ as

$$HM_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in (M_3 \cap \xi_2) \quad \& \quad (\varepsilon_1, \varepsilon_2) \notin \Lambda_1\}, \text{where}$$
$$M_3 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{1-\beta}{\beta + 2\mu - 1}\varepsilon_2 \leq (\beta + 2\mu - 1)^{-1}(k + 2\mu\tau(\gamma(\beta^{-1}))$$

for any $(\beta, \mu)$ where $sgn(1 - \beta - 2\mu) > 0$.

If $(\varepsilon_1, \varepsilon_2) \in \xi_2$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_1$, $D_1$ and $D_2$ would maintain contractual relations with upstream firms. $D_1$ merge with $D_2$ if $D_1$'s and $D_2$'s bilateral gain is positive.

$D_1, D_2$ merge if

$$\Pi(D_1^h D_2; U_1, U_2) < \Pi(D_1; U_1) + \Pi(D_2; U_1)$$

Theorem 6 summarizes the firms' strategies employed if $(\varepsilon_1, \varepsilon_2) \in \xi_2$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_1$.

**Theorem 6.** *Assume that*

*(A1) No counter merger of $U_2, D_2$ is allowed.*

*(A2) $sgn(1 - \beta - 2\mu) > 0$*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_2$ and $(\varepsilon_1, \varepsilon_2) \notin \Lambda_1$, then $D_1$ and $D_2$ merge if and only if $(\varepsilon_1, \varepsilon_2) \in HM_3$. Otherwise firms maintain contractual relations.*

*$D_1$ and $D_2$ merge for every $(\varepsilon_1, \varepsilon_2) \in \xi_2$ and $(\varepsilon_1, \varepsilon_2) \notin \Lambda_1$ if $sgn(1 - \beta - 2\mu) < 0$.*

In this strategic region, $U_1, D_1$ would not integrate if $D_1, D_2$ was not allowed. $D_1$ and $D_2$ extend compatibility contract to $U_1$. Theorem 6 partitions the region as well. $HM_1$, which is the restricted region of $M_1$, is the parameter space in which $D_1, D_2$ merge. Theorem 6 also states that $D_1$'s decision heavily depends on the sign of $1 - \beta - 2\mu$. The results are consistent with the previous results. The smaller competition sensitivity $\mu$ is, the more independent $D_1$'s and $D_2$'s profits will be, that's why $D_1, D_2$ merger would be less tempting for both $D_1$ and $D_2$. Moreover, both $D_1$ and $D_2$ would pay a lower fee $((1 - \beta)v)$ to $U_1$ which necessarily increases independent downstream firms' profits for smaller $\beta$. In addition, the total downstream variety will be lower when $D_1, D_2$ merge which necessarily increases downstream profits under independent ownership. In particular, downstream firms would be better off by just remaining independent because bilateral gain can not compensate both $D_1$ and $D_2$. On the other hand, $D_1, D_2$ always merge if competition sensitivity $\mu$ or variety sensitivity $\beta$ is big enough, although total variety would decrease.

$D_1$'s decision have a strong influence on the market structure. Only more efficient upstream firm $U_1$ supplies to the market under contractual relations, while both $U_1$ and $U_2$ supply when $D_1, D_2$ merge. That's why, $U_1$ always supplies to a downstream firm whenever $U_1$ has the efficiency advantage. $U_2$ can not supply to any downstream firms unless $U_1, D_1$ integrate or $D_1, D_2$ merge whenever $U_1$ has the efficiency advantage. Theorem 6 concludes the $D_1$'s merger decision analysis whenever $U_1$ has the efficiency advantage (i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_1 \cap \xi_2$).

Next, I will analyze $D_1$'s merger decision whenever $U_2$ has the efficiency advantage over $U_1$ (i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_3$). In first section $\xi_3$ is partitioned into two strategic regions. I will analyze $D_1$'s equilibrium merger decision in each of the strategic regions.

First, I examine $D_1$'s merger decision if $(\varepsilon_1, \varepsilon_2) \in \xi_3 \cap \Lambda_2$. In $\xi_3 \cap \Lambda_2$, $U_1, D_1$ would integrate and $D_2$ would be supplied by $U_2$ if $D_1, D_2$ merger was not allowed. $D_1, D_2$ merge if $D_1$ foregoes $U_1, D_1$ integration and $D_2$' best interests is to merge with $D_1$. $D_1, D_2$ merge if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) > \Pi(U_1^v D_1) + \Pi(D_2; U_2)$$

If we define strategic regions $M_4$ and $HM_4$ as

$$HM_4 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_4 \cap \Lambda_2 \cap \xi_2\}, \text{where}$$
$$M_4 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \varepsilon_2 \geq H_2(\beta, \mu)\}, \text{and}$$
$$H_4(\beta, \mu) = [(1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - m + \gamma(\beta^{-1}) - k](\mu)^{-1}$$

Theorem 7 summarizes the $D_1$'s merger decision in equilibrium if $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_2$

**Theorem 7.** *Assume that*

*(A1) No counter merger of* $U_2, D_2$

*If* $(\varepsilon_1, \varepsilon_2) \in \xi_3 \cap \Lambda_2$*, then* $D_1$ *and* $D_2$ *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in HM_4$*.* $U_1$ *and* $U_2$ *supply* $D_1^h D_2$*. Otherwise* $D_1$ *and* $U_1$ *integrate.* $D_2$ *will be supplied by* $U_2$*.*

Strategically, there is no difference between the regions $\xi_3 \cap \Lambda_2$ and $\xi_2 \cap \Lambda_1$. In both regions, $U_1, D_1$ would integrate and $D_2$ will be supplied by $U_2$. The only difference is $U_2$ is more efficient than $U_1$ in $\xi_3 \cap \Lambda_2$. Theorem 7 also partitions $\xi_3 \cap \Lambda_2$ into two strategic regions. $D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_4$. Any implication of theorem 5 is also implied by 7. Moreover, every statement and relation which holds for the strategic region $HM_2$ also holds for strategic region $HM_4$. Consequently, corollary 1 implies an immediate conclusion which is summarized bur corollary 3.

**Corollary 3.** *Assume that*

*(A1) No counter merger of* $U_2, D_2$

*If* $(\varepsilon_1, \varepsilon_2) \in \xi_3 \cap \Lambda_2$*,* $HM_4 = \xi_3 \cap \Lambda_2$ *if and only if* $k > H_3(\beta, \mu)$*, where*

$H_3(\beta, \mu) = (1 - \mu)(\tau(\gamma((1)))) - (1 + \mu)(\tau(\gamma(\beta^{-1}))) - m + \gamma(\beta^{-1})$.

Second, I will analyze $D_1$'s merger decision if $(\varepsilon_1, \varepsilon_2) \in \xi_3$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_2$. In this strategic region, $D_1$ would not integrate with $U_1$ if $D_1, D_2$ merger was not possible and both downstream firms will be supplied by the more efficient upstream firm $U_2$. $D_1, D_2$ merge if the sum of $D_1^h D_2$'s profits are higher than the sum of independently owned $D_1$'s and $D_2$'s profits. $D_1, D_2$ merge if

$$\Pi(D_1^h D_2; U_1, U_2) < \Pi(D_1; U_2) + \Pi(D_2; U_2)$$

If we define the strategic regions $M_5$ and $HM_5$ as

$$HM_5 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_5 \cap \xi_3 \quad \text{and} \quad (\varepsilon_1, \varepsilon_2) \notin \Lambda_2)\}, where$$
$$M_5 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{\beta + 2\mu - 1}{1 - \beta}\varepsilon_2 \geq (\beta - 1)^{-1}(k + 2\mu\tau(\gamma(\beta^{-1})))\}$$

Theorem 8 summarizes $D_1$'s merger decision if $(\varepsilon_1, \varepsilon_2) \in \xi_3$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_2$.

**Theorem 8.** *Assume that*

*(A1) No counter merger of $U_2, D_2$ is allowed*

*(A2) $sgn(1 - \beta - 2\mu) > 0$*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_3$ and $(\varepsilon_1, \varepsilon_2) \notin \Lambda_2$, then $D_1$ and $D_2$ merge if and only if $(\varepsilon_1, \varepsilon_2) \in HM_5$. Otherwise firms maintain contractual relations.*

*$D_1, D_2$ merge for every $(\varepsilon_1, \varepsilon_2) \in \xi_3$ and $(\varepsilon_1, \varepsilon_2) \notin \Lambda_2$ if $sgn(1 - \beta - 2\mu) < 0$.*

Theorem 8 partitions the strategic region into two. $D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_5$ if the efficiency gap is not big enough. That's why downstream firms may agree to merge to earn the surplus $k$ plus the profit due to variety. Although the total variety would be less when $D_1, D_2$ merge, the loss can be compensated by the surplus after the merger.

$D_1$'s decision heavily depends on the sign of $1 - \beta - 2\mu$. In other words, $D_1$'s decision depends on the magnitude of the competition sensitivity and variety sensitivity. Downstream firms get higher returns on variety when $\beta$ is small. Thus, a smaller $\beta$ increases $D_1^h D_2$'s profits. On the other hand, a smaller $\beta$ also increases independent downstream profits as well. A smaller $\beta$ makes $D_1, D_2$ merger less likely because marginal value of $\beta$ is $2\varepsilon_2$ under independent ownership which is higher than marginal value of $\beta$, $\varepsilon_1 + \varepsilon_2$ when $D_1, D_2$ merge in $\xi_3$. Moreover, a bigger $\mu$ makes $D_1, D_2$ merger more likely because; first $D_1$'s and $D_2$'s profits decrease under independent ownership, second $D_1^h D_2$'s bilateral gain is higher if downstream market is more competitive.

In conclusion, $D_1$'s merger decision in equilibrium is analyzed and summarized by theorems through 4, 8 when $D_1, D_2$ merger is allowed as well as $U_1, D_1$ integrate and counter merger of $U_2, D_2$ is not allowed. Figure 5 illustrates the predictions.

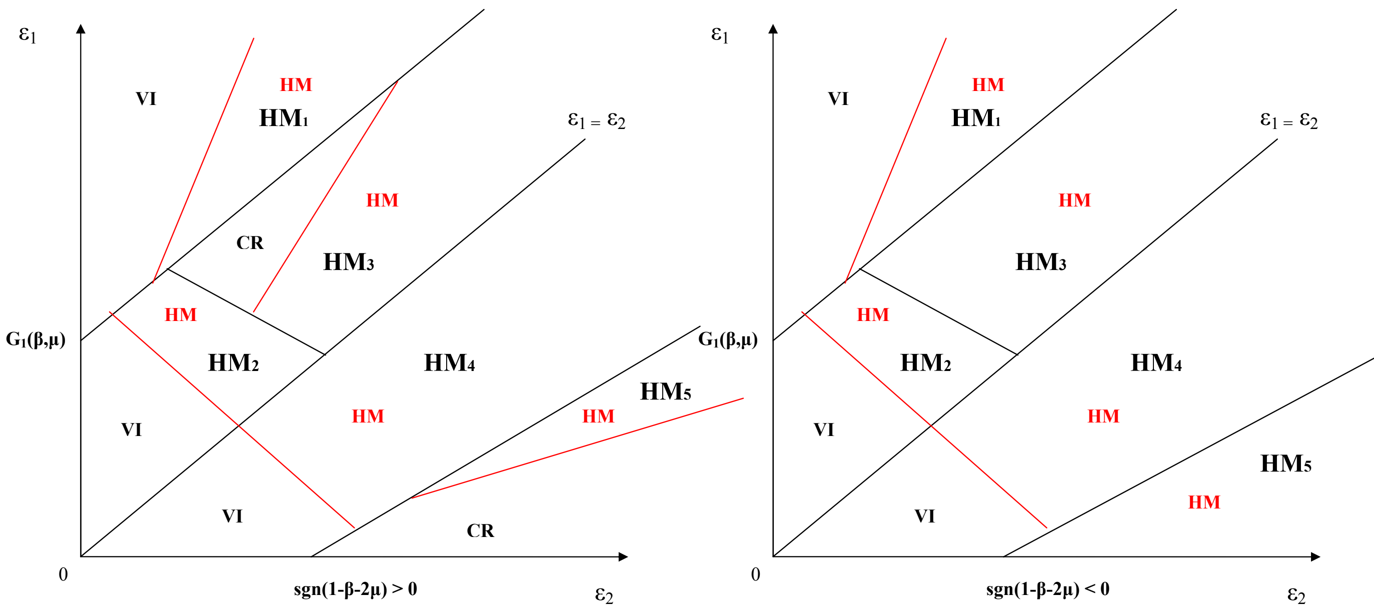


Figure 2.4: $D_1$'s Merger Decision with Horizontal Merger and No Counter Merger

Next chapter analyzes downstream units' unconditional merger decision, i.e. $U_1, D_1$ and counter merger of $U_2, D_2$ and merger $D_1, D_2$ are all allowed. The literature often assumes away a horizontal merger option due to anti-trust regulations, however, considering the huge amount of takeovers and acquisitions, horizontal merger is always an option.[10]

[10] A recent example is a possible merger of two big satellite radio companies, XM & Sirius.

# Chapter 3

# Choice of Organizational Form:"Unconditional" Equilibria

In this chapter, the determinants of various mergers and contractual decisions are analyzed unconditionally. In other words, an integration, a counter integration and a horizontal merger of downstream firms can be the equilibrium organizational form. The ultimate analysis can provide a basis for the literature which take any merger decision and contractual relations as granted. First section analyzes the integration and counter integration decision under the assumption that $D_1$ and $D_2$ can not merge. Second section analyzes the ultimate choice of organizational form.

## 3.1 Integration and Counter Integration Analysis with No Horizontal Merger

In this section, I study $U_1, D_1$'s integration decision if $U_2, D_2$ can counter integrate as a reaction when $U_1, D_1$ integrate. The only restriction this section imposes is that $D_1$ can not choose to merge with $D_2$ in spite of to integrate with $U_1$.

I use backward induction to analyze $D_1$'s and $D_2$'s integration decisions. At time three, each downstream firm pays a fee with the upstream firm from which it was compatible. However, an integrated downstream division is supplied by the integrated upstream division and the joint profit is maximized by a central management.

If $U_1$ and $D_1$ integrate, then an integrated $U_2$ invests to solve

$$\max_{r_2} \quad \pi + \varepsilon_2 + \tau(r_2) - \mu(\varepsilon_1 + \tau(r_1^*)) - r_2$$

The first order conditions imply,

$$\frac{\partial \Pi_{U_2}}{\partial r_2^*} = \beta\tau^{'}(r_2) - 1 = 0$$

$U_2, D_2$ counter integration affects $U_2$'s investment incentives. $U_2$ invests $r_2^* = \gamma(\beta^{-1})$. Lemma 2 states that $U_2$ also invests $r_1^* = \gamma(\beta^{-1})$.

**Lemma 6.** *Assume that*

*(A1)* $U_1$ *and* $D_1$ *merge*

*(A2)* $U_2$ *and* $D_2$ *merge*

*Both upstream divisions invest* $r^* = \gamma(\beta^{-1})$.

I now examine the equilibrium $U_2$ and $D_2$'s counter integration decision when

$U_1$ and $D_1$ are integrated. $U_2, D_2$ counter integrate if and only if the stand alone profits are less than an integrated firm's profits.

$\Pi U_2^v D_2 > \Pi(D_2) + \Pi(U_2)$

$U_1, D_1$ integration affects both $D_2$'s and $U_2$'s gain in the three strategic regions $\xi_1, \xi_2$ and $\xi_3$ differently.

First, I examine the equilibrium $U_2$ and $D_2$'s counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. Lemma 4 states that $D_2$ extends a contract to $U_1^v D_1$ and lemma 3 states that $U_1^v D_1$ accepts the contract if $U_1, D_1$ integrate and $D_2$ remains independent in $\xi_1$. $U_2$ supplies to the market if and only if $D_2$ and $U_2$ integrate in $\xi_1$. That's why $U_2$ always prefers a counter integration. On the other hand, $D_2$'s ex post gain can be negative because it might be better for $D_2$ to be compatible with the more efficient upstream unit $U_1$ which might increase $D_2$'s profits. That's why $D_2$ and $U_2$ might forego counter integration. If we define the strategic regions $C_1$ and $CM_1$ as

$$
\begin{aligned}
&C_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{\varepsilon_2}{1-\beta} \leq S_1(\beta, \mu) \text{ where} \\
&S_1(\beta, \mu) = \frac{\tau(\gamma(1)) - \gamma(1)}{1-\beta} - \tau(\gamma((\beta - \mu)^{-1}))\} \\
&CM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in C_1 \cap \xi_1\}
\end{aligned}
$$

Following lemma summarizes the equilibrium $U_2$ and $D_2$'s counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_1$ when $U_1, D_1$ integrate.

**Lemma 7.** *Assume that*

*(A1) $U_1$ and $D_1$ merge*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, $U_2$ and $D_2$ integrate if and only if $(\varepsilon_1, \varepsilon_2) \in CM_1$*

Lemma 7 states that $U_2$'s gain is not adequate for $D_2$ to integrate with $U_2$ if $(\varepsilon_1, \varepsilon_2)$ is not in the parameter space $CM_1$. $CM_1$ is the strategic regions in which

$U_2, D_2$ counter integrate when $U_1, D_1$ integrate. A larger variety sensitivity $\beta$ makes $U_2, D_2$ counter integration more likely. The main reason is a bugger $U_2, D_2$ bilateral gain when $U_1, D_1$ integrate because an independent $D_2$'s contract fee $\beta v_2$ increases with a larger $\beta$. Lemma 7 also states that there exists always a strategic region $CM_1$ in $\xi_1$.

Second, I examine the equilibrium $U_2$ and $D_2$'s counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_2$ and $(\varepsilon_1, \varepsilon_2) \in \xi_3$. In these two strategic regions, $D_2$ is going to be supplied by $U_2$ even if $D_2$ remains independent. Counter integration with $U_2$ is always a weakly dominant strategy for $D_2$ because investment decision made by a central management necessarily increases the joint profits when $U_1, D_1$ integrate.

$$\Pi(U_2^v D_2) \geq \Pi(D_2; U_2) + \Pi(U_2; D_2) \quad \text{if} \quad (\varepsilon_1, \varepsilon_2) \in (\xi_2 \cup \xi_3)$$

Next lemma summarizes the equilibrium $U_2$ and $D_2$'s counter integration decision if $(\varepsilon_1, \varepsilon_2) \in \xi_2$ and $(\varepsilon_1, \varepsilon_2) \in \xi_3$

**Lemma 8.** *Assume that*

*(A1)* $U_1$ *and* $D_1$ *merge*

$U_2$ *and* $D_2$ *integrate if* $(\varepsilon_1, \varepsilon_2) \in (\xi_2 \cup \xi_3)$

Figure illustrates the equilibrium $U_2$ and $D_2$'s counter integration decision when $U_1, D_1$ integrate.

Up to this point, I have not discussed the equilibrium integration decision of $U_1, D_1$. $D_1$'s decision to integrate can be altered when $U_2, D_2$ counter merger is possible because downstream variety of $D_2$ increases when $U_2, D_2$ integrate.

First, I examine $U_1, D_1$ integration decision when $D_2$ would not integrate with $U_2$ when $U_1, D_1$ integrate. Intuitively, $D_1$'s best interest is to integrate with

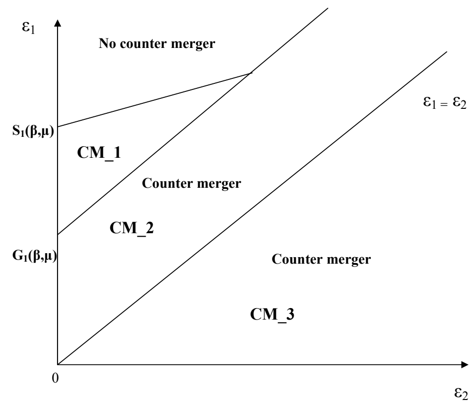


Figure 3.1: $U_2, D_2$ Counter Integration Decision when $U_1, D_1$ Integrate with No Horizontal Merger

$U_1$ because vertical integration would not lead to a counter integration. Theorem 1 claims $U_1, D_1$ integrate if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. In other words, $U_1, D_1's$ bilateral gain is positive in $\xi_1$. In particular, $D_1$'s best choice does not alter if $D_2$ would integrate with $U_2$. Another reason is that the efficiency difference between upstream producers is adequately high so that $D_2$ prefers being independent rather than counter integration. Theorem 9 summarizes the equilibrium integration decision of $U_1, D_1$ if $U_2, D_2$ would not integrate.

**Theorem 9.** *Assume that*

*(A1) No horizontal merger of $D_1, D_2$ is allowed*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_1$ and $(\varepsilon_1, \varepsilon_2) \notin CM_1$ , $U_1$ and $D_1$ integrate and $D_2$ remain independently owned. $D_2$ is supplied by upstream division $U_1$.*

Note that $U_1, D_1$'s integration decision does not have any affect on $D_2$'s incentives. The main reason is the upstream efficiency asymmetry. In this case, $U_2$ can not compete with $U_1$ even though $U_1$ would invest less for $D_2$. As a result, $D_2$ will be supplied by $U_1^v D_1$ in equilibrium.

Second, I examine $U_1, D_1$ integration decision when $D_2$ would not integrate with $U_2$ when $U_1, D_1$ integrate and $U_1$ is more efficient than $U_2$ (i.e., $(\varepsilon_1, \varepsilon_2) \in CM_1 \cup xi_2$). Although $CM_1$ and $\xi_2$ has different strategic implications when $U_2, D_2$ integration is not allowed, $CM_1$ and $\xi_2$ has the same strategic implications when when $U_2, D_2$ integration is allowed. $U_2, D_2$ would counter integrate when $U_1, D_1$ integrate if $U_1$ does not have adequate efficiency advantage even though $U_1$ is more efficient than $U_2$. $U_2, D_2$ counter integration decision can threat the profits of $U_1^v D_1$. Thus, $D_1$ and $U_1$ may forego vertical integration. If we define the strategic regions $TM_1$ and $T_1$ as

$$T_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu \varepsilon_2}{\beta - \mu} \leq S_2(\beta, \mu) \text{ where}$$
$$S_2(\beta, \mu) = \frac{(1-\mu)(\tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))) - \gamma(1) - \beta\tau(\gamma((\beta-\mu)^{-1})) + 2\gamma((\beta-\mu)^{-1})}{\beta - \mu}\}$$
$$TM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_1 \cap (CM_1 \cup \xi_2)\}$$

Theorem 10 summarizes the equilibrium strategies employer by the firms.

**Theorem 10.** *Assume that*

*(A1) No horizontal merger of $D_1, D_2$ is allowed*

*If $(\varepsilon_1, \varepsilon_2) \in CM_1 \cap \xi_2$ , $U_1$ and $D_1$ integrate and $U_2, D_2$ integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_1$.*

*Otherwise, firms remain contractual relations. $D_1$ and $D_2$ is supplied by $U_1$.*

Theorem 10 partitions the strategic region $CM_1 \cap \xi_2$ into two. The first partition is $TM_1$. Even though $U_2, D_2$ counter integration is a threat to $U_1^v D_1$, $U_1, D_1$ integrates in $TM_1$. As a result, two vertical integrations will be observed in the market. On the other hand, both $D_1$'s and $U_1$ loss is higher than a possible bilateral gain from integration whenever $(\varepsilon_1, \varepsilon_2) \in CM_1 \cup \xi_2$ but $(\varepsilon_1, \varepsilon_2) \notin TM_1$.

Consequently, neither of the downstream firms integrates with an upstream firm.

The likelihood of a independent ownership increases as $U_2$'s efficiency increases in $\xi_2$. First reason is that $D_1$ would loose more downstream profits for higher efficiencies of $U_2$ when $U_1, D_1$ and $U_2, D_2$ integrate. Second reason is that an independent $U_1$ would acquire the business of an independent $D_2$. Post integrations, $D_2$ will be supplied by $U_2$. Theorem 10 and theorem 9 state that $U_1, D_1$ never integrate if $U_1$ is significantly efficient but not too efficient so that $D_2$ would not counter integrate. (i.e., $min(G_1(\beta, \mu, S_2(\beta, \mu))) < \varepsilon_1 < S_1(\beta, \mu)$)

Furthermore, $U_1, D_1$ integration decision under possibility of a counter integration depends on how sensitive the profits to variety and downstream competition. $U_1^v D_1$ would loose downstream profits when $U_2, D_2$ if the downstream market is highly competitive, while independent $D_1$ and $U_1$ would not be hurt as much by a high competitive downstream market since $D_2$ would not counter integrate and increase its variety. Thus, a large $\mu$ value makes independent ownership more likely and $U_1, D_1$ and $U_2, D_2$ integrations less likely. In addition, an independent $U_1$ would loose a significant amount of upstream profits if the upstream profits are significantly sensitive to downstream variety. The main reason is that $U_1$ would supply both downstream firms under contractual relations, while $U_1$ would supply only $D_1$ if both downstream firms integrate.[1] Consequently, a higher $\beta$ value makes independent ownership more likely and $U_1, D_1$ and $U_2, D_2$ integrations less likely. In general, the findings support the immediate intuitive sense that it is less likely for an upstream firm to integrate if the price for the upstream service or product is high, upstream firm's market share is significantly high and upstream firm would loose some of its customers after integration.

Third, I examine $U_1, D_1$'s integration decision when $D_2$ would not integrate

[1] If $\beta = 1$, $U_1$ would never integrate with $D_1$ in $\xi_2$

with $U_2$ when $U_1, D_1$ integrate and $U_1$ is less efficient than $U_2$ (i.e., $(\varepsilon_1, \varepsilon_2) xi_3$). $U_2, D_2$ would counter integrate when $U_1, D_1$ integrate especially if $U_1$ $U_1$ is less efficient than $U_2$. $U_2, D_2$ counter integration decision also threats the profits of $U_1^v D_1$. Thus, $D_1$ and $U_1$ may forego vertical integration in $\xi_3$. If we define the strategic regions $TM_2$ and $T_2$ as

$$T_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - (1-\beta)\varepsilon_2 \leq S_3(\beta, \mu) \text{ where}$$

$$S_3(\beta, \mu) = (1-\beta)(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(1)) + \gamma(1)) - \beta\tau(\gamma((\beta-\mu)^{-1}))\}$$

$$\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}$$

$$TM_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_2 \cap \xi_3)\}$$

Theorem 11 summarizes the equilibrium strategies employed by the firms in $\xi_3$.

**Theorem 11.** *Assume that*

*(A1) No horizontal merger of $D_1, D_2$ is allowed*

*If $(\varepsilon_1, \varepsilon_2) \in \xi_3$ , $U_1$ and $D_1$ integrate and $U_2, D_2$ integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_2$. Otherwise, firms remain contractual relations. $D_1$ and $D_2$ is supplied by $U_2$.*

Theorem 11 states that there exits a strategic region $TM_2$ in $\xi_3$ in which both downstream firms integrate with an upstream firm. Theorem 11 also states that there exists a strategic region in $\xi_3$ in which firms maintain their contractual relations and both downstream firms are supplied by the more efficient upstream firm $U_2$.

$D_1$ would be supplied by the more efficient upstream firm $U_2$, whereas $U_1$ would not supply to any of the downstream firms under independent ownership. That's why, $U_1$ is always willing to integrate unlike $D_1$ which may prefer being independent. $D_1$ has no incentive to integrate whenever $U_2$ has a significant efficiency advantage. Consequently, a higher efficiency value $\varepsilon_2$ makes independent ownership more likely and $U_1, D_1$ and $U_2, D_2$ integrations less likely. However, theorem 11 states that $U_1, D_1$ always integrates if $U_2$ is not efficient enough in $\xi_3$ (i.e. $\varepsilon_2 < \frac{S_3(\beta,\mu)}{1-\beta}$) and $U_2, D_2$ counter integrate.

$U_1, D_1$ integration decision under possibility of a counter integration depends on how sensitive the profits to variety and downstream competition in $\xi_3$ similar to the other strategic regions. As the sensitivity measure $\beta$ becomes large, $D_1$ must incur a higher cost to be compatible with $U_2$.[2] Thus, a higher $\beta$ value makes $U_1, D_1$ and $U_2, D_2$ integration more likely and independent ownership less likely. In addition, $D_1$ would know that $D_2$ would be also supplied by $U_2$ if $U_1, D_1$ integrates. A higher competition would hurt the downstream profits of $U_1^v D_1$ more because $U_2^v D_2$ will have a higher variety in $\xi_3$. Consequently, a higher $\mu$ value makes $U_1, D_1$ and $U_2, D_2$ integration less likely and independent ownership more likely.

In conclusion, theorems 9 through 11 summarize the equilibrium integration decision of $U_1, D_1$ and $U_2, D_2$ yet the equilibrium decision is conditional on not allowing $D_1, D_2$ merger. The figure 3.1 illustrates the predictions. The next section examines the unconditional merger decision of downstream firms.

## 3.2 Unconditional Choice of Organizational Forms

Downstream firm's unconditional merger decision is analyzed in this section. I examine the equilibrium decision of $D_1$ under no restrictions; $D_1$ can integrate with $U_1$ and $D_2$ may or may not integrate with $U_2$ or $D_1$ and $D_2$ merge.

There are three different equilibrium organizational forms in the absence of horizontal merger option. Figure 3.1 illustrates the partitions of the parameter

[2] If $\beta = 1$, then $D_1$ does not gain from any variety in the downstream market.

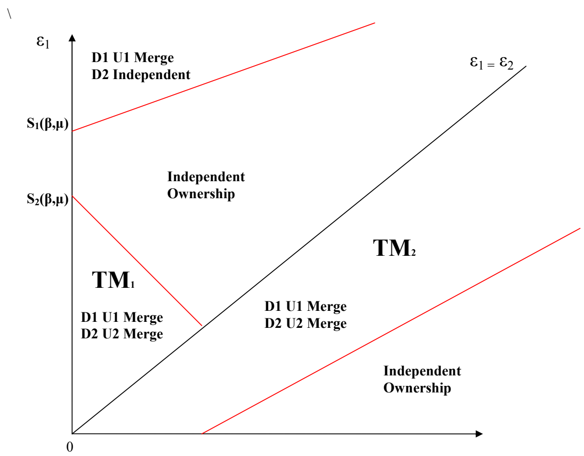


Figure 3.2: Integration Decisions of $U_1, D_1$ and $U_2, D_2$ with No Horizontal Merger

space. $D_1, D_2$ merger option may alter the incentives of $D_1$ and thus the equilibrium outcome may change.

First, I examine $D_1$'s merger decision in the strategic region in which $U_1, D_1$ would integrate and $D_2$ would remain independent and compatible with $U_1^v D_1$ in absence of horizontal merger(i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_1$ and $(\varepsilon_1, \varepsilon_2) \notin CM_1$). Let us first define the strategic region $OM_1$ as $OM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in \xi_1 \quad \text{and} \quad (\varepsilon_1, \varepsilon_2) \notin CM_1\}$. In particular, $D_1, D_2$ merge in $OM_1$ if the bilateral gain due to the horizontal merger is higher than the bilateral gain due to $U_1, D_1$ integration. $D_1, D_2$ merge in $OM_1$ if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) \geq \Pi(U_1^v D_1; D_2) + \Pi(D_2; U_1^v D_1)$$

If we define the strategic regions $AM_1$ and $AM_1$ as

$$M_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta}{1-2\mu}\varepsilon_2 \leq A_1(\beta, \mu)\} \text{ where}$$

$$A_1(\beta, \mu) = [(2-\beta)\tau(\gamma(\beta^{-1})) - (1-\mu)(\tau(\gamma(1)) + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma(\beta^{-1})$$
$$+ \gamma(1) + \gamma((\beta-\mu)^{-1}) + k)](1-2\mu)^{-1}$$
$$AM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_1 \cap OM_1\}$$

Theorem 12 summarizes the firms' strategies employed if $(\varepsilon_1, \varepsilon_2) \in OM_1$.

**Theorem 12.** *If $(\varepsilon_1, \varepsilon_2) \in OM_1$, then $D_1$ and $D_2$ merge if and only if $(\varepsilon_1, \varepsilon_2) \in AM_1$. Otherwise $D_1$ and $U_1$ merge, $D_2$ remain independent and $D_2$ is supplied by $U_1^v D_1$ if $(\varepsilon_1, \varepsilon_2) \in OM_1$*

Theorem 12partitions $OM_1$ into two different strategic regions. $D_1$ alters its integration decision and $D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in OM_1$. On the other hand, $D_1$'s incentives do not alter, especially if $U_1$ is significantly more efficient than $U_2$. The main reason is that the integrated $U_1^v D_1$'s gain from $D_2$'s business increases with $U_1$'s efficiency. Hence, $U_1, D_1$ integration is more likely for high values of $\varepsilon_1$.

In general, $D_1, D_2$ merger is more likely whenever market surplus $k$ is higher. Intuitively, a higher $\beta$ value must increase the likelihood of a vertical merger. $D_1^h D_2$'s gain from downstream variety decreases as the marginal gain decreases because the downstream firm gains only $(1-\beta)$ per variety. Analytically, the strategic region $AM_1$ shrinks as $\beta$ increases. Consequently, a higher $\beta$ value makes $D_1, D_2$ merger. Downstream competition affects the equilibrium $D_1$'s decision in two ways. First, the more competitive the downstream firms are, the more market power $D_1^h D_2$ will have which necessarily increases the surplus $k$. Second, the more competitive the downstream firms are, the more downstream profits $U_1^v D_1$ will loose. Consequently, a higher $\mu$ value makes $D_1, D_2$ merger more likely and $U_1, D_1$ integration less likely.

Second, I examine $D_1$'s merger decision in the strategic region in which $U_1, D_1$ would integrate and $U_2, D_2$ would counter integrate in absence of horizontal merger(i.e. $(\varepsilon_1, \varepsilon_2) \in TM_1 \cup TM_2$. $D_1, D_2$ merge if the total gain is more than the gain in case of two vertical mergers. $D_1, D_2$ merge if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) \geq \Pi(U_1^v D_1; D_2) + \Pi(D_2; U_1^v D_1)$$

If we define the strategic regions $M_2$ and $AM_2$ as

$$\begin{aligned} & M_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \varepsilon_2 \geq A_2(\beta, \mu)\} \text{ where} \\ & A_2(\beta, \mu) = \frac{2((1-\mu)\tau(\gamma(1) - \gamma(1)) - \tau(\gamma((\beta-\mu)^{-1})) + \gamma((\beta-\mu)^{-1})) - k}{\mu} \\ & AM_2\{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_2 \cap (TM_1 \cup TM_2)\} \end{aligned} \tag{3.1}$$

Theorem 13 summarizes the firms' equilibrium strategies in $TM_1 \cup TM_2$

**Theorem 13.** *(A1)* $A_2(\beta, \mu) > 0$
*If* $(\varepsilon_1, \varepsilon_2) \in TM_1 \cup TM_2$, *then* $D_1$ *and* $D_2$ *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in AM_2$.
*Otherwise* $U_1$,$D_1$ *and* $U_2$ *and*$D_2$ *integrate if* $(\varepsilon_1, \varepsilon_2) \in TM_1 \cup TM_2$.
$D_1$ *and* $D_2$ *merge if* $(\varepsilon_1, \varepsilon_2) \in TM_1 \cup TM_2$ *and* $A_2(\beta, \mu) < 0$.

$D_1$ would still choose to integrate with $U_1$ even if $U_2, D_2$ counter integrate in $TM_1 \cup TM_2$ in absence of horizontal merger. Theorem 13 states there exists a strategic region in which $U_1, D_1$ and $U_2, D_2$ still integrate even if $D_2$ can merge with $D_2$. $U_1, D_1$'s bilateral gain is higher as $U_2$'s efficiency is smaller or $U_1$'s efficiency is higher. That's why a smaller $\varepsilon_2$ and a larger $\varepsilon_1$ makes $U_1, D_1$ and $U_2, D_2$ integration more likely and $D_1, D_2$ merger less likely.

Moreover, downstream competition and variety sensitivity affects $D_1$'s equilibrium decision. The likelihood of $D_1, D_2$ merger increases as the upstream profits become more sensitive to downstream variety because independent upstream firms would invest more for $D_1^h D_2$.[3] In addition, a larger $\mu$ value increases the loss due to the downstream competition when $U_1, D_1$ and $U_2, D_2$ integrate. A larger $\mu$ decreases the value of integration, and thus makes $D_1, D_2$ merger more likely and $U_1, D_1$ and $U_2, D_2$ less likely.

Finally, I examine $D_1$'s merger decision in the strategic regions in which $D_1$ would remain independent in absence of horizontal merger(i.e. $\forall(\varepsilon_1, \varepsilon_2) \notin (TM_1 \cup TM_2 \cup OM_1)$. $D_1, D_2$ merge if $D_1^h D_2$'s profit is more than the stand alone profits of each downstream firm. We can define the strategic regions as $NM_1 : \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in \xi_2 \text{ and } (\varepsilon_1, \varepsilon_2) \notin (TM_1 \cup OM_1)\}$ and $NM_2 : \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in \xi_3 \text{ and } (\varepsilon_1, \varepsilon_2) \notin TM_2\}$. Then, $D_1, D_2$ merge if

$$\begin{aligned} \Pi(D_1^h D_2; U_1, U_2) &\geq \Pi(D_1; U_1) + \Pi(D_2; U_1) \quad \text{if} \quad (\varepsilon_1, \varepsilon_2) \in NM_1 \\ \Pi(D_1^h D_2; U_1, U_2) &\geq \Pi(D_1; U_2) + \Pi(D_2; U_2) \quad \text{if} \quad (\varepsilon_1, \varepsilon_2) \in NM_2 \end{aligned} \tag{3.2}$$

$D_1$'s equilibrium integration decision can be mapped to the strategic regions defined

[3] For instance if $\beta = 1$, then independent $U_1$ and $U_2$ would invest $\gamma((1))$ which is the same amount of an integrated upstream firm invests. So, $D_1$ always merge with $D_2$.

by $AM_3, M_3$ and $AM_4, M_4$ where,

$$M_3 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta}{1-\beta-2\mu}\varepsilon_2 \leq A_3(\beta,\mu)\} \text{ where}$$

$$A_3(\beta,\mu) = \frac{k + 2\mu(\tau(\gamma((\beta-\mu)^{-1})))}{1-\beta-2\mu}$$

$$M_4 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta-2\mu}{1-\beta}\varepsilon_2 \geq A_4(\beta,\mu)\} \text{ where}$$

$$A_4(\beta,\mu) = \frac{-(k + 2\mu(\tau(\gamma((\beta-\mu)^{-1}))))}{1-\beta}$$

$$AM_3\{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in (NM_1 \cup M_3)\}$$

$$AM_4\{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in (NM_2 \cup M_4)\}$$

Theorem 14 summarizes firms' equilibrium strategies in $NM_1$.

**Theorem 14.** *(A1)* $sgn(1-\beta-2\mu) > 0$

*If* $(\varepsilon_1, \varepsilon_2) \in NM_1$*, then* $D_1$ *and* $D_2$ *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in AM_3$*.*

*Otherwise firms remain independent and* $D_1$ *and* $D_2$ *is supplied by* $U_1$*.*

$D_1$ *and* $D_2$ *merge if* $(\varepsilon_1, \varepsilon_2) \in NM_1$ *and* $sgn(1-\beta-2\mu) < 0$*.*

Theorem 15 summarizes firms' equilibrium strategies in $NM_2$.

**Theorem 15.** *(A1)* $sgn(1-\beta-2\mu) > 0$

*If* $(\varepsilon_1, \varepsilon_2) \in NM_2$*, then* $D_1$ *and* $D_2$ *merge if and only if* $(\varepsilon_1, \varepsilon_2) \in AM_4$*. Otherwise firms remain independent if* $(\varepsilon_1, \varepsilon_2) \in NM_2$ *and* $D_1$ *and* $D_2$ *is supplied by* $U_2$*.*

$D_1$ *and* $D_2$ *merge if* $(\varepsilon_1, \varepsilon_2) \in NM_2$ *and* $sgn(1-\beta-2\mu) < 0$*.*

The next figure illustrates the unconditional choice of organizational form.

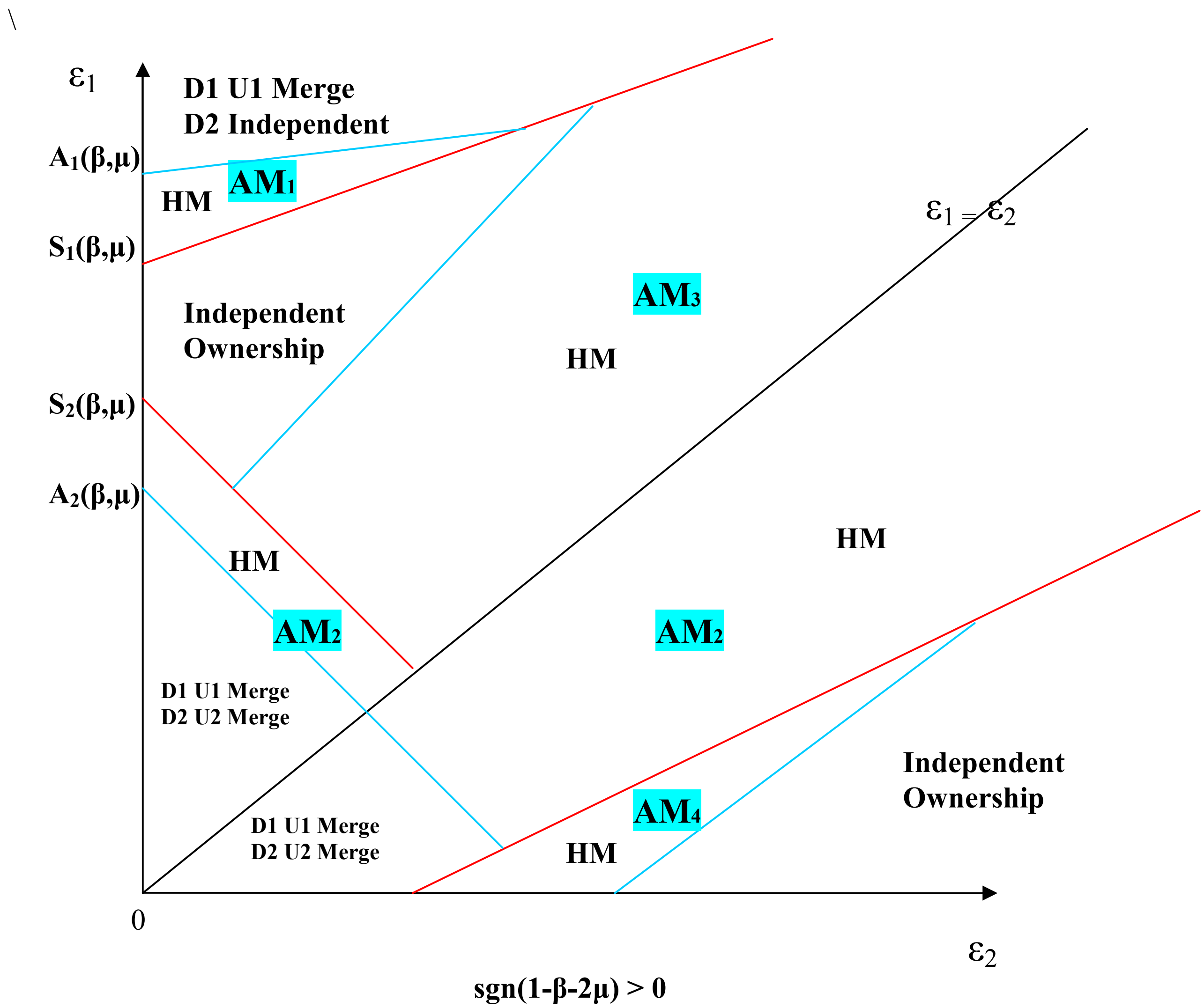


Figure 3.3: Unconditional Choice of Organizational Form

## 3.3 Discussion

The paper analyzes the merger decision, however, it has limited implications on social welfare. First, neither an entry nor an exit is considered in the base product market. Second, there can be possible efficiency gains as a result of vertical integration by eliminating the double mark-up problem such as cost reduction. Moreover, how horizontal merger affects the social welfare is not explicitly considered in the model. Hence, further research is necessary. Nonetheless, the model furnishes the following.

If $U_1$ has high efficiency advantage (i.e $(\varepsilon_1, \varepsilon_2) \in \xi_1$), the compatibility contracts of both downstream producers do not change in case $U_1D_1$ integrates. On the other hand, both upstream firms supply to the market in case $D_1D_2$ merge. The total variety in the market is

$$v_m = v_{U_1D_1} + v_{D_2} = 2\varepsilon_1 + \tau(\gamma(1)) + \tau(\gamma((\beta - \mu)^{-1})))$$

whenever vertical integration occurs. On the other hand, the total variety in the market is

$$v_m = v_{D_1D_2} = \varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1}))$$

whenever horizontal merger occurs. Thus, the horizontal merger strictly decreases the total variety since $(\varepsilon_1, \varepsilon_2) \in \zeta_2$.[4] The total variety is

$$v_m = v_{D_1} + v_{D_2} = 2(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$

when the firms stay independent. The vertical integration, on the other hand, may

[4]The variety is less if and only if $\varepsilon_1 - \varepsilon_2 > \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta - \mu)^{-1})) + \tau(\gamma(\beta^{-1})) - \tau(\gamma(1))$. This is true since $\varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu)$

or may not increase the total variety in the market. The total variety always declines whenever the benefit from an increase in $D_1$'s product variety is more than the loss from a decrease in $D_2$'s product variety in case of vertical integration. That is,

$$\tau(\gamma(1)) - \tau(\gamma(\beta^{-1})) > \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1}))$$

If $U_1$ has inadequate efficiency advantage (i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_2$), vertical integration could make the firms alter their compatibility decisions in the market. The independent downstream firm will be compatible with less efficient side product firm in case vertical integration occurs. Integrated firm's variety will be always $v_{D_1U_1} = \varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1}))$. However, the total variety is

$$v_m = v_{D_1U_1} + v_{D_2} = \varepsilon_1 + \varepsilon_2 + \tau(\gamma(1)) + \tau(\gamma(\beta^{-1}))$$

in case vertical integration occurs and if $(\varepsilon_1, \varepsilon_2) \in \zeta_3/\zeta_2$. Whenever $(\varepsilon_1, \varepsilon_2) \in \zeta_3/\zeta_2$, horizontal merger can not possibly increase the total variety. Thus, the society will not benefit in terms of higher variety from a horizontal merger. Moreover, vertical integration may also decrease the total variety supplied to the market. Vertical integration decreases total variety if $\varepsilon_1 - \varepsilon_2 > \tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))$. However, vertical integration never decreases variety for every $(\varepsilon_1, \varepsilon_2) \in \zeta_3/\zeta_2$ if

$$\tau(\gamma(1)) - \tau(\gamma(\beta^{-1})) > G_1(\beta, \mu)$$

If $U_2$ has efficiency advantage (i.e. $(\varepsilon_1, \varepsilon_2) \in \xi_3$), then the independent downstream firm $D_2$ will never be compatible with an integrated firm. The total variety will be

$$v_m = v_{D_1U_1} + v_{D_2} = \varepsilon_1 + \varepsilon_2 + \tau(\gamma(1)) + \tau(\gamma(\beta^{-1}))$$

The vertical integration can increase the variety of the integrated base product firm, even though it can decrease the total variety in the market. When the firms independently owned, the total variety is

$$v_m = 2(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

Thus, vertical integration decreases total variety whenever $\varepsilon_2 - \varepsilon_1 > \tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))$. Thus, the society is supplied with less variety if $U_2$ has a high efficiency advantage over $U_1$.

The total variety is more likely to decrease post integration when variety competition in the base product market ($\mu$) is high because the integrated firm will be less willing to invest for its downstream competitor. In addition, a higher profit increment of the upstream firms ($\beta$) causes a lower variety post integration because the downstream firms would be supplied with more variety if they remained independent. Vertical integration is more likely to decrease total variety for high values of $\beta$. Hence, the society may not see higher variety in the market after the integration of $D_1U_1$ for high values of $\beta$ and $\mu$.

In conclusion, vertical integration may or may not increase the total variety. Hence, the vertical integration may or may not increase the total welfare. The vertical integration always decreases total variety when the integrated party has high efficiency advantage. That's why, any regulations can be considered whenever the integrated firm has a superiority over the independent upstream supplier. Moreover,

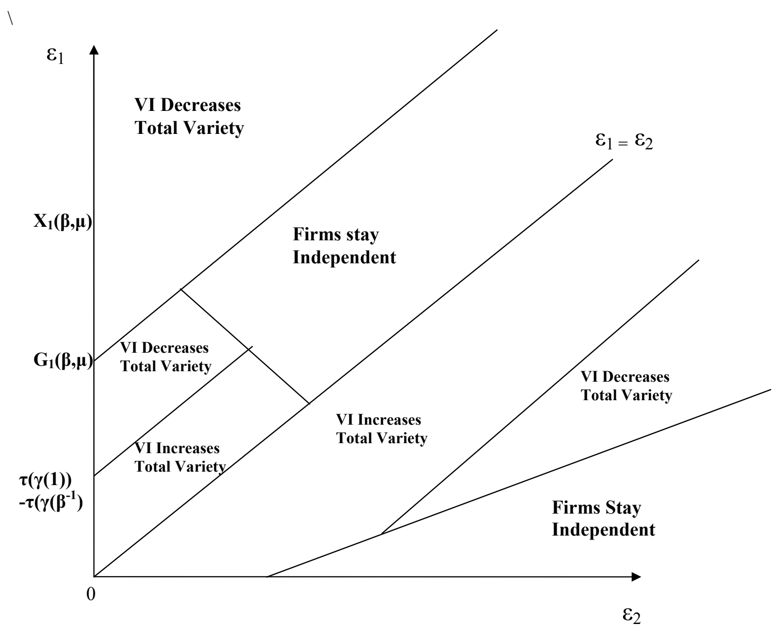


Figure 3.4: Total Variety Change under Vertical Integration

vertical integration can still decrease the total variety supplied when not only the integrated firm has a slight efficiency advantage but also $U_2$ is more efficient. On the other hand, merger of downstream firms always decreases the total variety and tow vertical integration always increases the total variety. In both cases, both upstream firms resume supplying variety to the market.

## 3.4 Conclusion

The goal of this dissertation is to investigate the unconditional choice of an organizational form in complementary markets in a setting such that the firm must commit to a compatibility contract with its complementary good producer. The complementary good producer invests on firm specific R&D. I have identified the conditions under which integration with a complementary good producer or a merger with a substitute good producer is a subgame perfect Nash equilibrium outcome.

In chapter 2, I examine the conditional choice when a counter merger is not allowed. The first section analyzes the integration decision of firms when a horizontal merger is assumed away. Matutes&Regibeau(1998), Beggs(1994), Heavner(2004) are some examples of the literature which assumes the vertical integration option and assumes away a horizontal merger option. The section section builds from the literature and analyzes the case when horizontal merger is also allowed.

In chapter 3, I examine the unconditional choice when a counter merger is also allowed. The first section analyzes the integration and counter integration decision of firms when a horizontal merger is assumed away. Chen(2001), McAfee(1999), Heavner(2004) are some examples of the literature which assumes the vertical integration and counter integration and assumes away a horizontal merger option. The next section builds from the literature and analyzes the case when horizontal merger is also allowed. The chapter presents the model's predictions and claims that each of different organizational forms may be the equilibrium outcome even if there is no restriction on firms. The chapters provides a theory base for Economides & Salop(1992) which assumes different organizational forms.

To be specific, when $U_1$ has a high efficiency superiority($\varepsilon_1 >> \varepsilon_2$), vertical integration will always occur in absence of a horizontal merger option. Vertical integration changes the integrated firm's incentives to invest for its' downstream rival. The independent base product firm will be less apt to be compatible with the integrated firm post integration. Thus, the integration may not occur whenever the efficiency of the integrated firm is inadequate. Moreover, the base product firm may merge with its substitute good producer and forego vertical integration if we introduce horizontal merger as an alternative. Both of the upstream firms can supply variety to the market post merger, although the total variety will be less. Horizontal merger always decreases the total variety in the market. On the other hand, the integration may or may not decrease the total variety.

The model predicts that the horizontal merger is more likely if variety competition is fierce. Moreover, merger with substitute good producer is more likely whenever the rival downstream firm would not be compatible with the integrated firm in case of integration. On the other hand, the higher share the downstream gets from the profits generated by the variety effects, the more likely vertical integration to occur. If the variety competition is not fierce enough($\alpha > 2\mu$), independent ownership can not be sustained whenever $U_1$ has an efficiency advantage. In addition, independent ownership can not be a sustainable market structure whenever $U_2$ has the efficiency advantage and the competition is not fierce enough ($\alpha < 2\mu$). Nevertheless, the firms can stay independent for some efficiency levels even if both vertical integration and horizontal merger are available. In contrast to existing literature which takes the merger decision of the firm for granted, this paper suggests that some type of mergers are more likely than others in complementary markets under the factors such as degree of competition and distribution of the profit increment due to the externality effect.

The theory presented in this dissertation can provide a basis for a possible empirical work on mergers in complementary markets. One can use a structural model to estimate the values of $(\varepsilon_1, \varepsilon_2)$ using the variety of a product as a proxy. The profits also can be estimated as a function of both the firm's variety and the rival's variety. One can observe the different kind of mergers and argue whether the predictions of this paper prevail or not. Further research is needed to test the predictions of the model. Moreover, one can investigate the effect of a pre-existing vertical merger on a horizontal merger and vice versa. Following the idea, one can investigate combination of mergers in complementary markets in a dynamic setting. The model also lacks a complete welfare analysis post merger. One can work work on a model which analyzes the welfare effects when the firm is supported with variety of complementary products using my dissertation as the model's basis.

## Appendix

*Proof of Lemma 1*

If $D_1, D_2, U_1$ and $U_2$ are independent, then their profits are going to be

$$\begin{aligned}
\Pi(U_1) =&(\lambda_1 + \lambda_2)(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1}))\\
\Pi(U_2) =&(2 - \lambda_1 - \lambda_2)(\beta(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1}))\\
\Pi(D_i) =&\pi + \alpha(\lambda_i\varepsilon_1 + (1 - \lambda_i)\varepsilon_2 + \tau(\gamma(\beta^{-1})))\\
&- \mu(\lambda_j\varepsilon_1 + (1 - \lambda_j)\varepsilon_2 + \tau(\gamma(\beta^{-1})))
\end{aligned}$$

If we compare the profits of a downstream firm when $D_i$ is supplied by $U_1$ (i.e. $\lambda_1 = 1$) and $D_i$ is supplied by $U_2$ (i.e. $\lambda_1 = 0$) then,

$$\Pi(D_i; U_i) > \Pi(D_i; U_j) \Leftrightarrow \varepsilon_i > \varepsilon_j$$

A downstream firm maximizes its profit by offering a contract to the upstream firm which is more efficient.

*Proof of Lemma 2*

When $\beta > \mu$,the first order conditions satisfy

$$\frac{\partial \Pi}{\partial r_{11}} = \tau'(r_{11}) - 1 = 0 \quad \Rightarrow r_{11}^{*} = \tau'^{-1}(1)$$
$$\frac{\partial \Pi}{\partial r_{12}} = (\beta - \mu)\tau'(r_{12}) - 1 = 0 \Rightarrow r_{12}^{*} = \tau'^{-1}((\beta - \mu)^{-1})$$

If $\beta > \mu$, the upstream division of the integrated firm $U_1^v D_1$ would invest $r_{11}^{*} = \gamma((1))$ and $r_{12}^{*} = \gamma((\beta - \mu)^{-1})$.

*Proof of Lemma 3*

If $\beta > \mu$, the integrated firm's upstream division $U_1^v D_1$ would supply $D_2$ if and only if

$$\Pi(U_1^v D_1; D_2) \geq \Pi(U_1^v D_1)$$

where

$$\Pi(U_1^v D_1; D_2) = \pi + (\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1)) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) - \gamma((\beta - \mu)^{-1})$$
$$\Pi(U_1^v D_1) = \pi + (\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1)) + -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

Then, $U_1^v D_1$ would supply $D_2$ if and only if

$$\varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \geq F_1(\mu, \beta) = \frac{\gamma((\beta - \mu)^{-1})}{\beta - \mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta - \mu} - \tau(\gamma((\beta - \mu)^{-1})) \; if \; \beta > \mu$$

$$F_1(\mu, \beta) = \frac{\gamma((\beta - \mu)^{-1})}{\beta - \mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta - \mu} - \tau(\gamma((\beta - \mu)^{-1})) < 0 \Leftrightarrow$$
$$\gamma((\beta - \mu)^{-1}) - \mu\tau(\gamma(\beta^{-1})) - (\beta - \mu)(\tau(\gamma((\beta - \mu)^{-1}))) < 0$$

But we know that if $x > 0$ then $\beta\tau(x) > x$, otherwise an upstream firm would invest zero and maximize the variety.Hence,

$$\gamma((\beta-\mu)^{-1}) - \beta\tau(\gamma((\beta-\mu)^{-1})) < 0 \quad \text{and}$$
$$\mu[(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(\beta^{-1}))] < \tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(\beta^{-1})) < 0 \quad \text{then}$$

The two inequalities imply $F_1(\mu,\beta) = \frac{\gamma((\beta-\mu)^{-1})}{\beta-\mu} - \frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} - \tau(\gamma((\beta-\mu)^{-1})) < 0$

If $\beta < \mu$, if the integrated firm decides to supply, the firm maximizes the total gain from accepting the compatibility contract. Thus, the integrated firm's problem is to maximize the gain if integrated firm is compatible with the independent base product firm.

$$\max_{r_{12}} \quad (\beta-\mu)(\varepsilon_1 + \tau(r_{12}^*)) - r_{12}^*$$

Since $\beta - \mu < 0$, the gain is maximized if and only if the firm produces the minimum amount of variety for its rival. Hence,the optimal level of investment for its rival base product firm will be

$$\tau(r_{12}^*) = 0 \Longleftrightarrow r_{12}^* = 0$$

If we plug the optimal level of investment, the integrated firm agrees on a compatibility contract if

$$(\beta-\mu)\varepsilon_1 > -\mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$
$$\varepsilon_1 + \frac{\mu}{\beta-\mu}\varepsilon_2 \leq F_2(\mu,\beta) = -\frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu}$$

such that $F_2(\mu,\beta) = -\frac{\mu\tau(\gamma(\beta^{-1}))}{\beta-\mu} > 0$ since $\beta - \mu < 0$.

*Proof of Lemma 4*

The independent downstream firm offers the compatibility contract to the upstream division of the integrated firm if the gain from the contract is higher than the downstream firm's gain from its outside option. $D_2$ offers a compatibility contract to $U_1^v D_1$ if and only if

$$\Pi(D_2; U_1^v D_1) > \Pi(D_2; U_2)$$

If $\beta > \mu$

$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$
$$\Pi(D_2; U_2) = \pi + ((1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1)))))$$

If $\beta < \mu$

$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$
$$\Pi(D_2; U_2) = \pi + ((1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1)))))$$

Then,$D_2$ offers a compatibility contract to $U_1^v D_1$ if and only if

$$\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1})) > \varepsilon_2 + \tau(\gamma(\beta^{-1})) \qquad \text{if} \quad \beta > \mu$$
$$\varepsilon_1 > \varepsilon_2 + \tau(\gamma(\beta^{-1})) \qquad \text{if} \quad \beta < \mu$$

or

$$\varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) \quad \text{if} \quad \beta > \mu$$
$$\varepsilon_1 - \varepsilon_2 \geq G_2(\beta) \quad \text{if} \quad \beta < \mu 7$$

,where

$$G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta - \mu)^{-1})) \tag{3.3}$$

$$G_2(\beta) = \tau(\gamma(\beta^{-1})) \tag{3.4}$$

,and $\tau(\gamma(\beta^{-1})) > \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta - \mu)^{-1})) > 0$

*Proof of Theorem 1*

The strategic region $\xi_1$ is defined by $\xi_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \varepsilon_2 \geq G_1(\beta, \mu) = \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta - \mu)^{-1}))\}$. If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, lemma 4 states that both independent downstream firms would be supplied by $U_1$ when $U_1, D_1$ do not integrate and $D_2$ would be supplied by $U_1^v D_1$ when $U_1, D_1$ integrate. $U_1 D_1$ integrate if and only if

$$\Pi(U_1^v D_1; D_2) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2)$$

If $U_1$ and $D_1$ remain independently owned then the profits of the firms will be,

$$\Pi(D_1; U_1) = \pi + (1 - \beta)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1; D_1, D_2) = 2(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1}))$$

When $U_1, D_1$ integrate, the profit function of the integrated firm will be,

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) - \gamma(1) - \gamma((\beta - \mu)^{-1})$$

$U_1, D_1$ integrate if the total profit is higher than the sum of the two independent firms' profits.

$$\Pi(U_1^v D_1; D_2) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2) \Leftrightarrow$$

$$\pi + \varepsilon_1 + \tau(\gamma(1)) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta - \mu)^{-1}))) - \gamma(1) - \gamma((\beta - \mu)^{-1})$$

$$\geq \pi + \varepsilon_1 + \tau(\gamma(\beta^{-1})) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - 2\gamma(\beta^{-1}) \tag{3.5}$$

$$\Leftrightarrow \tau(\gamma(1)) + (\beta - \mu)(\tau(\gamma((\beta - \mu)^{-1}))) - 2\tau(\gamma(\beta^{-1})) \geq \gamma((\beta - \mu)^{-1}) + \gamma(1) - 2\gamma(\beta^{-1})$$

Since $U_1^v D_1$ optimizes its profit, it must be true that

$$\tau(\gamma(1)) - \gamma(1) > \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1}) \quad \text{and}$$

$$(\beta - \mu)(\tau(\gamma((\beta - \mu)^{-1}))) - \gamma((\beta - \mu)^{-1}) > (\beta - \mu)\tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1})$$

The inequalities imply that inequality 3.5 holds. Thus, $U_1, D_1$ vertically integrate for $\forall(\beta, \mu)$ and $(\varepsilon_1, \varepsilon_2) \in \xi_1$.

*Proof of Theorem 2*

The strategic region $\xi_2$ is defined by $\xi_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in R_3 \text{ and } (\varepsilon_1, \varepsilon_2) \notin R_2\}$. If $(\varepsilon_1, \varepsilon_2) \in \xi_2$, lemma 4 states that both independent downstream firms would be supplied by $U_1$ when $U_1, D_1$ do not integrate and $D_2$ would be supplied by $U_2$ when $U_1, D_1$ integrate. $U_1 D_1$ integrate if and only if

$$\Pi(U_1^v D_1) \geq \Pi(D_1; U_1) + \Pi(U_1; D_1, D_2)$$

If the firms remain independently owned, the profit functions of the upstream firm and the downstream firm will be

$$\Pi(D_1;U_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1;D_1,D_2) = 2(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1}))$$

If $U_1, D_1$ integrate, the integrated firm will not earn profits surplus by supplying for its downstream rival. Post merger, the profit function of the integrated firm will be,

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

$U_1, D_1$ integrate if and only if there is a positive gain,

$$\Pi(U_1^v D_1) \geq \Pi(D_1;U_1) + \Pi(U_1;D_1,D_2) \Leftrightarrow$$
$$\pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \geq \pi + (1+\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - 2\gamma(\beta^{-1})$$
$$\Leftrightarrow (\beta-\mu)\varepsilon_1 + \mu\varepsilon_2 \leq \tau(\gamma(1))\ \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1}) = X_1^m(\beta)$$

We can define $X_1(\beta,\mu) = \frac{X_1^m(\beta)}{\beta-\mu}$ and $F_1(\beta,\mu) = \frac{F_1^m(\beta,\mu)}{\beta-\mu}$

$$X_1^m(\beta) \geq F^m(\beta,\mu) \Leftrightarrow \tag{3.6}$$
$$\tau(\gamma(1)) - \gamma(1) - (1+\beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1}) \geq \gamma((\beta-\mu)^{-1}) - \mu\tau(\gamma(\beta^{-1})) - (\beta-\mu)\tau(\gamma((\beta-\mu)^{-1}))$$
$$\Leftrightarrow \tau(\gamma(1)) - \gamma(1) - (1+\beta-\mu)\tau(\gamma(\beta^{-1})) + 2\beta\gamma(\beta^{-1}) + (\beta-\mu)\tau(\gamma((\beta-\mu)^{-1})) - \gamma((\beta-\mu)^{-1}) \geq 0$$

We know that the integrated firm optimizes the investment levels for each downstream firm.Thus,

$$\tau(\gamma(1)) - \gamma(1) \geq \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1})$$
$$(\beta - \mu)\tau(\gamma((\beta - \mu)^{-1})) - \gamma((\beta - \mu)^{-1}) \geq (\beta - \mu)\tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1})$$

Rearranging the two inequalities,

$$\tau(\gamma(1)) - \gamma(1) + (\beta - \mu)\tau(\gamma((\beta - \mu)^{-1})) - \gamma((\beta - \mu)^{-1}) \geq (1 + \beta - \mu)\tau(\gamma(\beta^{-1})) - 2\beta\gamma(\beta^{-1})$$
$$\Rightarrow X_1^m(\beta) \geq F^m(\beta, \mu)$$

$$X_1^m(\beta) \geq F^m(\beta, \mu) \Leftrightarrow X_1(\beta, \mu) \geq F_1(\beta, \mu)$$

. As a result, the firms integrate if and only if

$$\varepsilon_1 + \frac{\mu}{\beta - \mu}\varepsilon_2 \leq X_1(\beta, \mu)$$
$$\text{where} \quad X_1(\beta, \mu) = \frac{\tau(\gamma(1)) - \gamma(1) - (1 + \beta)\tau(\gamma(\beta^{-1})) + 2\gamma(\beta^{-1})}{\beta - \mu}$$

,when $(\varepsilon_1, \varepsilon_2) \in \xi_2$.

*Proof of Theorem 3*

The strategic region $\xi_3$ is defined by $\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in \varepsilon_1 - \varepsilon_2 < 0\}$. If $(\varepsilon_1, \varepsilon_2) \in \xi_3$, lemma 4 states that both independent downstream firms would be supplied by $U_2$ when $U_1, D_1$ do not integrate and $D_2$ would be supplied by $U_2$ when $U_1, D_1$ integrate. $U_1 D_1$ integrate if and only if

$$\Pi(U_1^v D_1) \geq \Pi(D_1; U_2)$$

When the firms stay independent, the profit functions of the firms will be

$$\Pi(D_1; U_2) = \pi + (1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

$$\Pi(U_1) = 0$$

If the integration occurs, the profit function of the integrated firm will be,

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

The integration takes place if and only if it is profitable for both parties.

That is, $\Pi(U_1^v D_1) \geq \Pi(D_1; U_2) \Leftrightarrow$

$$\pi + \varepsilon_1 + \tau(\gamma(1)) - \gamma(1) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) \geq \pi + (1-\beta-\mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

$$\Leftrightarrow \varepsilon_1 - (1-\beta)\varepsilon_2 \geq (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1)$$

If we define $X_2(\alpha) = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1)$

Moreover $X_2(\alpha) < G_1(\beta, \mu)$.

$$X_2(\alpha) < G_1(\beta, \mu) \Leftrightarrow$$

$$(1-\beta)(\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) - \gamma(1) < \tau(\gamma(\beta^{-1})) - \tau(\gamma((\beta-\mu)^{-1})))$$

We know that

$$\tau(\gamma(1)) - \gamma(1) > \tau(\gamma((\beta-\mu)^{-1})) - \gamma((\beta-\mu)^{-1}) \quad \text{and}$$

$$\beta\tau(\gamma(\beta^{-1})) > \beta\tau(\gamma((\beta-\mu)^{-1})) > \gamma((\beta-\mu)^{-1}) \quad \text{since} \quad \gamma((\beta-\mu)^{-1}) > 0$$

Rearranging the inequalities, we get $G_1(\beta, \mu) > X_2(\beta)$.

$U_1, D_1$ vertically integrate if $(\varepsilon_1, \varepsilon_2) \in \Lambda_2 \cap \xi_3$ where,

$$\Lambda_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - (1-\beta)\varepsilon_2 \geq X_2(\beta)\}$$
$$X_2(\beta) = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1)$$

*Proof of Corollary 1*

Let $\varepsilon_2^*$ solves

$$\varepsilon_1 = 0$$
$$\varepsilon_1 - (1-\beta)\varepsilon_2^* = (1-\beta)\tau(\gamma(\beta^{-1})) - \tau(\gamma(1)) + \gamma(1) = X_2(\beta)$$

The $\varepsilon_2^* = \frac{X_2(\beta)}{\beta-1}$ solves the equations. By theorem 3, any $\varepsilon_2$ which is less than $\varepsilon_2^*$ lead to $U_1, D_1$ integration.

*Proof of Theorem 4*

$U_1$ has a large efficiency advantage if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. Theorem 1 states that $D_1$ prefers vertical integration to staying independent if $(\varepsilon_1, \varepsilon_2) \in \xi_1$. $D_2$ will be supplied by $U_1^v D_1$ when $U_1, D_1$ integrate. If $U_1, D_1$ integration occurs then the profits of $U_1^v D_1$ and $D_2$ are

$$\Pi(U_1^v D_1; D_2) = \pi + \varepsilon_1 + \tau(\gamma(1)) + (\beta - \mu)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma((\beta-\mu)^{-1}) - \gamma(1)$$
$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$$

If $D_1, D_2$ merger occurs then the profits of $D_1^h D_2$ and $U_1$ are

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1; D_1^h D_2) = \beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})$$

$U_1$ will supply $D_1^h D_2$ when $D_1, D_2$ merge. The gain of $U_1$ is crucial because $U_1, D_1$ integration also depends on $U_1$'s outside option which is supplying $D_1^h D_2$.
Thus, $D_1$ would merge with $D_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) \geq \Pi(U_1^v D_1; D_2) + \Pi(D_2; U_1^v D_1)$$

If we arrange the inequality, $D_1, D_2$ merge in $\xi_1$ if $(1-2\mu)\varepsilon_1 - (1-\beta)\varepsilon_2 \leq (1-\mu)(\tau(\gamma((1))) + \tau(\gamma((\beta-\mu)^{-1}))) - (2-\beta)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) - \gamma((\beta-\mu)^{-1}) + k$.

$D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_1$ where

$$HM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_1 \cap \xi_1\}, \text{where}$$
$$M_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta}{1-2\mu}\varepsilon_2 \leq H_1(\beta, \mu)\}, \text{and}$$
$$H_1(\beta, \mu) = ((1-\mu)(\tau(\gamma((1))) + \tau(\gamma((\beta-\mu)^{-1}))) - (2-\beta)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) - \gamma((\beta-\mu)^{-1}) + k)(2\mu$$

*Proof of Theorem 5*

$U_1$ has an efficiency advantage but not adequate if $(\varepsilon_1, \varepsilon_2) \in \xi_2$. Theorem 2 states that $D_1$ prefers vertical integration to staying independent if $(\varepsilon_1, \varepsilon_2) \in \xi_2 \cap \Lambda_1$. However, $D_2$ will be supplied by $U_2$ when $U_1, D_1$ integrate. If $U_1, D_1$ integration occurs then the profits of $U_1^v D_1$ and $D_2$ are

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \gamma(1)$$

$$\Pi(D_2; U_2) = \pi + (1-\beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$$

If $D_1, D_2$ merger occurs then the profits of $D_1^h D_2$ and $U_1$ are

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$

$$\Pi(U_1; D_1^h D_2) = \beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})$$

Thus, $D_1$ would merge with $D_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) \geq \Pi(U_1^v D_1) + \Pi(D_2; U_2)$$

If we arrange the inequality, $D_1, D_2$ merge in $\xi_2 \cap \Lambda_1$ if $\mu\varepsilon_1 + \mu\varepsilon_2 \geq (1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + \gamma(\beta^{-1}) - k$

$D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_2$ where

$$HM_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_2 \cap \Lambda_1 \cap \xi_2\}, \text{where}$$

$$M_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \varepsilon_2 \geq H_2(\beta, \mu)\}, \text{and}$$

$$H_2(\beta, \mu) = [(1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + x - k](\mu)^{-1}$$

*Proof of Theorem 2*

The conditions in theorem 5 is satisfied for $\forall(\beta, \mu)$ if $H_2 < 0$ because $\varepsilon_1 + \varepsilon_2 > 0$

for $\forall(\varepsilon_1, \varepsilon_2)$. $H_2 < 0 \Leftrightarrow k > (1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + x$.

*Proof of Theorem 6*

Theorem 2 states that $D_1$ and $U_1$ prefer independent ownership to integration if $(\varepsilon_1, \varepsilon_2) \in \xi_2$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_1$, $D_1$ and $D_2$ would maintain contractual relations with upstream firm $U_1$. If $U_1, D_1$ remain independently owned then the profits of $D_1$ and $D_2$ are

$$\Pi(D_1; U_1) = \pi + (1-\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(D_2; U_1) = \pi + (1-\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$

If $D_1, D_2$ merger occurs then the profits of $D_1^h D_2$ is

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1}))) \tag{3.7}$$

Thus, $D_1$ would merge with $D_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) \geq \Pi(D_1; U_1) + \Pi(D_2; U_1)$$

If we arrange the inequality, $D_1, D_2$ merge in $\xi_2$ but not in $\Lambda_1$ if $(\beta + 2\mu - 1)\varepsilon_1 + (1-\beta)\varepsilon_2 \leq (k + 2\mu\tau(\gamma(\beta^{-1}))$

$D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_3$ where

$$HM_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in (M_3 \cap \xi_2) \quad \& \quad (\varepsilon_1, \varepsilon_2) \notin \Lambda_1\}, \text{where}$$
$$M_3 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{1-\beta}{\beta + 2\mu - 1}\varepsilon_2 \leq (\beta + 2\mu - 1)^{-1}(k + 2\mu\tau(\gamma(\beta^{-1}))$$

for any $(\beta, \mu)$ where $sgn(1 - \beta - 2\mu) > 0$.

If $sgn(1 - \beta - 2\mu) < 0$, $D_1, D_2$ merge for $\forall(\varepsilon_1, \varepsilon_2)$ such that $(\varepsilon_1, \varepsilon_2) \in \xi_2$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_1$ because the slope that characterizes the set $M_3$ becomes negative and covers the whole strategic set.

*Proof of Theorem 7*

$U_1$ has an efficiency advantage but not adequate if $(\varepsilon_1, \varepsilon_2) \in \xi_2$. Theorem 3 states that $D_1$ prefers vertical integration to staying independent if $(\varepsilon_1, \varepsilon_2) \in \xi_3 \cap \Lambda_2$. However, $D_2$ will be supplied by $U_2$ when $U_1, D_1$ integrate. If $U_1, D_1$ integration occurs then the profits of $U_1^v D_1$ and $D_2$ are

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma(1)) - \mu(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \gamma(1)$$
$$\Pi(D_2; U_2) = \pi + (1 - \beta)(\varepsilon_2 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma(1)))$$

If $D_1, D_2$ merger occurs then the profits of $D_1^h D_2$ and $U_1$ are

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1 - \beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1; D_1^h D_2) = \beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})$$

Thus, $D_1$ would merge with $D_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) \geq \Pi(U_1^v D_1) + \Pi(D_2; U_2)$$

If we arrange the inequality, $D_1, D_2$ merge in $\xi_2 \cap \Lambda_1$ if $\mu\varepsilon_1 + \mu\varepsilon_2 \geq (1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + \gamma(\beta^{-1}) - k$

$D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_4$ where

$$HM_4 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_4 \cap \Lambda_2 \cap \xi_2\}, \text{where}$$
$$M_4 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \varepsilon_2 \geq H_4(\beta, \mu)\}, \text{and}$$
$$H_4(\beta, \mu) = [(1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + \gamma(\beta^{-1}) - k](\mu)^{-1}$$

*Proof of Corollary 3*

Follows from corollary 2. The conditions in theorem 7 is satisfied for $\forall(\beta, \mu)$ if $H_4 < 0$ because $\varepsilon_1 + \varepsilon_2 > 0$ for $\forall(\varepsilon_1, \varepsilon_2)$. $H_4 < 0 \Leftrightarrow k > (1-\mu)(\tau(\gamma((1)))) - (1+\mu)(\tau(\gamma(\beta^{-1}))) - \gamma((1)) + x$.

*Proof of Theorem 8*

Theorem 2 states that $D_1$ and $U_1$ prefer independent ownership to integration if $(\varepsilon_1, \varepsilon_2) \in \xi_3$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_2$, $D_1$ and $D_2$ would maintain contractual relations with upstream firm $U_2$. If $U_1, D_1$ remain independently owned then the profits of $D_1$ and

$D_2$ are

$$\Pi(D_1; U_1) = \pi + (1 - \beta - \mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(D_2; U_1) = \pi + (1 - \beta - \mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

If $D_1, D_2$ merger occurs then the profits of $D_1^h D_2$ is

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1 - \beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1}))) \tag{3.8}$$

Thus, $D_1$ would merge with $D_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) \geq \Pi(D_1; U_1) + \Pi(D_2; U_1)$$

If we arrange the inequality, $D_1, D_2$ merge in $\xi_2$ but not in $\Lambda_1$ if $(\beta - 1)\varepsilon_1 + (\beta + 2\mu - 1)\varepsilon_2 \leq (k + 2\mu\tau(\gamma(\beta^{-1}))$

$D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in HM_5$ where

$$HM_5 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_5 \cap \xi_3 \quad \text{and} \quad (\varepsilon_1, \varepsilon_2) \notin \Lambda_2)\}, where$$
$$M_5 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{\beta + 2\mu - 1}{1 - \beta}\varepsilon_2 \geq (\beta - 1)^{-1}(k + 2\mu\tau(\gamma(\beta^{-1})))\}$$

If $sgn(1 - \beta - 2\mu) < 0$, $D_1, D_2$ merge for $\forall(\varepsilon_1, \varepsilon_2)$ such that $(\varepsilon_1, \varepsilon_2) \in \xi_3$ but $(\varepsilon_1, \varepsilon_2) \notin \Lambda_2$ because the slope that characterizes the set $M_5$ becomes negative and covers the whole strategic set.

*Proof of Lemma 6*

Follows from lemma 2.

*Proof of Lemma 7*

If $(\varepsilon_1, \varepsilon_2) \in \xi_1$, an independent $D_2$ would be supplied by $U_1^v D_1$ when $U_2, D_2$ do not integrate. $U_2 D_2$ integrate if and only if $\Pi(D_2^v U_2) \geq \Pi(D_2; U_1^v D_1) + \Pi(U_2)$.

The profits of $U_2^v D_2$ when $U_2, D_2$ integrate and $U_2$ and $D_2$ when $U_2, D_2$ do not integrate are

$$\Pi(D_2^v U_2) = \pi + \varepsilon_2 + \tau(\gamma((1))) - \mu(\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1))$$

$$\Pi(D_2; U_1^v D_1) = \pi + (1-\beta)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$

$$\Pi(U_2) = 0$$

$\Pi(D_2^v U_2) \geq \Pi(D_2; U_1^v D_1) + \Pi(U_2)$ if and only if

$$(1-\beta)\varepsilon_1 - \varepsilon_2 \leq \tau(\gamma(1)) - \alpha\tau(\gamma((\beta-\mu)^{-1})) - \gamma(1)$$

Then,$U_2, D_2$ counter integrate when $U_1, D_1$ integrate in $\xi_1$ if $(\varepsilon_1, \varepsilon_2) \in CM_1$ where,

$$C_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{\varepsilon_2}{1-\beta} \leq S_1(\beta, \mu) \text{ where}$$

$$S_1(\beta, \mu) = \frac{\tau(\gamma(1)) - \gamma(1)}{1-\beta} - \tau(\gamma((\beta-\mu)^{-1}))\}$$

$$CM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in C_1 \cap \xi_1\}$$

*Proof of Theorem 9*

If $(\varepsilon_1, \varepsilon_2) \in \xi_1$ and $(\varepsilon_1, \varepsilon_2) \notin CM_1$ , theorem 1 states $U_1, D_1$ integrate if there was no counter merger. In equilibrium, lemma 8 states $U_2, D_2$ would not integrate even if $U_2, D_2$ integration is possible. Hence, $U_1, D_1$ integrate and $D_2$ remain independently owned and is supplied by $U_1^v D_1$ in the equilibrium.

*Proof of Theorem 10*

If $(\varepsilon_1, \varepsilon_2) \in CM_1 \cup \xi_2$, $U_2, D_2$ would counter integrate when $U_1, D_1$ integrate. $D_1$ either integrates with $U_1$ or remains independently owned. $D_1$ remains independently owned if $U_2, D_2$ counter integration decreases $U_1, D_1$'s profits. $D_1$ would be supplied by $U_1$ and $U_1$ would supply both $D_1$ and $D_2$ in $CM_1 \cup \xi_2$ in case of independent ownership. $U_1, D_1$ integrate if

$$\Pi(U_1^v D_1) \geq \Pi(U_1; D_1, D_2) + \Pi(D_1; U_1)$$

$U_1^v D_1$ profit when $U_1, D_1$ integrate and $U_2, D_2$ counter integrate is

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma((1))) - \gamma((1)) - \mu(\varepsilon_2 + \tau(\gamma((1))))$$

$U_1$ and $D_1$'s stand alone profits are

$$\begin{aligned} \Pi(D_1; U_1) &= \pi + (1 - \beta - \mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) \\ \Pi(U_1; D_1, D_2) &= 2(\beta(\varepsilon_1 + \tau(\gamma(\beta^{-1}))) - \gamma(\beta^{-1})) \end{aligned} \tag{3.9}$$

Then $U_1, D_1$ integrate if and only if

$$(\beta-\mu)\varepsilon_1+\mu\varepsilon_2 \leq (1-\mu)(\tau(\gamma(1))-\tau(\gamma(\beta^{-1})))-\gamma(1)-\beta\tau(\gamma((\beta-\mu)^{-1}))+2\gamma((\beta-\mu)^{-1})$$

Rearranging the inequality, $U_1, D_1$ integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_1$ where,

$$T_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \frac{\mu\varepsilon_2}{\beta - \mu} \leq S_2(\beta, \mu) \text{ where}$$
$$S_2(\beta, \mu) = \frac{(1-\mu)(\tau(\gamma(1)) - \tau(\gamma(\beta^{-1}))) - \gamma(1) - \beta\tau(\gamma((\beta-\mu)^{-1})) + 2\gamma((\beta-\mu)^{-1})}{\beta - \mu}\}$$
$$TM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_1 \cap (CM_1 \cup \xi_2)\}$$

*Proof of Theorem 11*

If $(\varepsilon_1, \varepsilon_2) \in \xi_3$, $U_2, D_2$ would counter integrate when $U_1, D_1$ integrate. $D_1$ either integrates with $U_1$ or remains independently owned. $D_1$ remains independently owned if $U_2, D_2$ counter integration decreases $U_1, D_1$'s profits. $D_1$ would be supplied by $U_2$ and $U_1$ would supply neither $D_1$ or $D_2$ in $\xi_3$ in case of independent ownership. $U_1, D_1$ integrate if

$$\Pi(U_1^v D_1) \geq \Pi(U_1) + \Pi(D_1; U_2)$$

$U_1^v D_1$ profit when $U_1, D_1$ integrate and $U_2, D_2$ counter integrate is

$$\Pi(U_1^v D_1) = \pi + \varepsilon_1 + \tau(\gamma((1))) - \gamma((1)) - \mu(\varepsilon_2 + \tau(\gamma((1))))$$

$U_1$ and $D_1$'s stand alone profits are

$$\Pi(D_1; U_2) = \pi + (1 - \beta - \mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1; ) = 0 \tag{3.10}$$

Then $U_1, D_1$ integrate if and only if

$$\varepsilon_1 - (1-\beta)\varepsilon_2 \leq (1-\beta)(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(1)) + \gamma(1)) - \beta\tau(\gamma((\beta-\mu)^{-1}))$$

Rearranging the inequality, $U_1, D_1$ integrate if and only if $(\varepsilon_1, \varepsilon_2) \in TM_2$ where,

$$T_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - (1-\beta)\varepsilon_2 \leq S_3(\beta, \mu)\} \text{ where}$$

$$S_3(\beta, \mu) = (1-\beta)(\tau(\gamma((\beta-\mu)^{-1})) - \tau(\gamma(1)) + \gamma(1)) - \beta\tau(\gamma((\beta-\mu)^{-1}))$$

$$\xi_3 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \notin R_3\}$$

$$TM_2 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in T_2 \cap \xi_3)\}$$

*Proof of Theorem 12*

If $(\varepsilon_1, \varepsilon_2) \in OM_1$, theorem 9 states that $U_1, D_1$ would integrate and $D_2$ would be supplied by $U_1^v D_1$ if $D_1, D_2$ merger is not possible. $D_1, D_2$ merge in equilibrium if the net gain after merger is higher than the net gain after $U_1, D_1$ merger. $D_1, D_2$ merge in $OM_1$ if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) \geq \Pi(U_1^v D_1; D_2) + \Pi(D_2; U_1^v D_1)$$

The firms' payoffs are,

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$

$$\Pi(U_1; D_1^h D_2) = \beta(\varepsilon_1 + \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1}))$$

$$\Pi(U_1^v D_1; D_2) = \pi + (\varepsilon_1 + \tau(\gamma((1)))) - (\mu - \beta)(\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma((1)) - \gamma((\beta-\mu)^{-1})$$

$$\Pi(D_2; U_1^v D_1) = pi + (\varepsilon_1 + \tau(\gamma((\beta-\mu)^{-1}))) - \mu(\varepsilon_1 + \tau(\gamma((1))))$$

When we rearrange the inequality, if $(\varepsilon_1, \varepsilon_2) \in OM_1$, $D_1, D_2$ merge if

$$(1-2\mu)\varepsilon_1 - (1-\beta)\varepsilon_2 \leq (2-\beta)\tau(\gamma(\beta^{-1})) - (1-\mu)(\tau(\gamma((1)))) + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma(\beta^{-1}) + \gamma((1)) + \gamma((\beta-\mu)^{-1}$$

Otherwise $D_1, U_1$ merge and $D_2$ remains independently owned.

Then, $D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in AM_1$ where,

$$M_1 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta}{1-2\mu}\varepsilon_2 \leq A_1(\beta, \mu)\} \text{ where}$$

$$A_1(\beta,\mu) = [(2-\beta)\tau(\gamma(\beta^{-1})) - (1-\mu)(\tau(\gamma(1)) + \tau(\gamma((\beta-\mu)^{-1}))) - \gamma(\beta^{-1})$$
$$+ \gamma(1) + \gamma((\beta-\mu)^{-1}) + k)](1-2\mu)^{-1}$$

$$AM_1 = \{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_1 \cap OM_1\}$$

*Proof of Theorem 13*

If $(\varepsilon_1, \varepsilon_2) \in TM_1 \cup TM_2$, theorem 10 and theorem 11 state that $U_1, D_1$ would integrate and $U_2, D_2$ counter integrate if $D_1, D_2$ merger is not possible. $D_1, D_2$ merge in equilibrium if the net gain after merger is higher than the net gain after $U_1, D_1$

and $U_2, D_2$ merger. $D_1, D_2$ merge in $TM_1 \cup TM_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) + \Pi(U_1; D_1^h D_2) + \Pi(U_2; D_1^h D_2) \geq \Pi(U_1^v D_1) + \Pi(U_2^v D_2)$$

The firms' payoffs are,

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$
$$\Pi(U_1; D_1^h D_2) = \beta(\varepsilon_1 + \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1}))$$
$$\Pi(U_1; D_1^h D_2) = \beta(\varepsilon_2 + \tau(\gamma(\beta^{-1})) - \gamma(\beta^{-1}))$$
$$\Pi(U_1^v D_1) = \pi + (\varepsilon_1 + \tau(\gamma((1)))) - \mu(\varepsilon_2 + \tau(\gamma((1)))) - \gamma((1))$$
$$\Pi(U_2^v D_2) = \pi + (\varepsilon_2 + \tau(\gamma((1)))) - \mu(\varepsilon_1 + \tau(\gamma((1)))) - \gamma((1))$$

Rearranging the inequality, if $(\varepsilon_1, \varepsilon_2) \in TM_1 \cup TM_2$, $D_1, D_2$ merge if

$$\mu(\varepsilon_1 + \varepsilon_2) \geq 2((1-\mu)\tau(\gamma(1) - \gamma(1)) - \tau(\gamma(\beta^{-1})) + \gamma(\beta^{-1})) - k)$$

Otherwise, $U_1, D_1$ and $U_2, D_2$ merge.

Thus, If $((1-\mu)\tau(\gamma(1) - \gamma(1)) - \tau(\gamma(\beta^{-1})) + \gamma(\beta^{-1})) - k) < 0$ then $D_1, D_2$ merge.

$D_1, D_2$ merge if $(\varepsilon_1, \varepsilon_2) \in AM_2$ where,

$$M_2 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 + \varepsilon_2 \geq A_2(\beta, \mu)\} \text{ where}$$
$$A_2(\beta, \mu) = \frac{2((1-\mu)\tau(\gamma(1) - \gamma(1)) - \tau(\gamma((\beta-\mu)^{-1})) + \gamma((\beta-\mu)^{-1})) - k}{\mu}$$
$$AM_2\{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in M_2 \cap (TM_1 \cup TM_2)\}$$

*Proof of Theorem 14*

If $(\varepsilon_1, \varepsilon_2) \in NM_1$, theorem 10 states that $D_1$ would $D_2$ remain independent if $D_1, D_2$ merger is not possible. $D_1, D_2$ merge in equilibrium if the net gain after merger is higher than the net gain of being independently owned. $D_1, D_2$ merge in $NM_1$ if

$$\Pi(D_1^h D_2; U_1, U_2) \geq \Pi(D_1; U_1) + \Pi(D_2; U_1)$$

The firms' payoffs are, The firms' payoffs are,

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$
$$\Pi(D_1; U_1) = \pi + (1-\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(D_2; U_1) = \pi + (1-\beta-\mu)(\varepsilon_1 + \tau(\gamma(\beta^{-1})))$$

When we rearrange the inequality, if $(\varepsilon_1, \varepsilon_2) \in NM_1$, $D_1, D_2$ merge if

$$(1-\beta-2\mu)\varepsilon_1 - (1-\beta)\varepsilon_2 \leq k + 2\mu(\tau(\gamma(\beta^{-1})))$$

Otherwise $D_1$ and $D_2$ remain independently owned. If the sign of $(1-\beta-2\mu)$ is negative then $D_1$ and $D_2$ always merge.

$D_1, D_2$ merge in $NM_1$ if $(\varepsilon_1, \varepsilon_2) \in AM_3$ where,

$$M_3 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta}{1-\beta-2\mu}\varepsilon_2 \leq A_3(\beta, \mu)\} \text{ where}$$
$$A_3(\beta, \mu) = \frac{k + 2\mu(\tau(\gamma((\beta-\mu)^{-1})))}{1-\beta-2\mu}$$
$$AM_3\{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in (NM_1 \cup M_3)\}$$

*Proof of Theorem 15*

If $(\varepsilon_1, \varepsilon_2) \in NM_2$, theorem 11 states that $D_1$ would $D_2$ remain independent if $D_1, D_2$ merger is not possible. $D_1, D_2$ merge in equilibrium if the net gain after merger is higher than the net gain of being independently owned. $D_1, D_2$ merge in $NM_2$ if

$$\Pi(D_1^h D_2; U_1, U_2) \geq \Pi(D_1; U_2) + \Pi(D_2; U_2)$$

The firms' payoffs are, The firms' payoffs are,

$$\Pi(D_1^h D_2; U_1, U_2) = 2\pi + k + (1-\beta)(\varepsilon_1 + \varepsilon_2 + 2\tau(\gamma(\beta^{-1})))$$
$$\Pi(D_1; U_2) = \pi + (1-\beta-\mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$
$$\Pi(D_2; U_2) = \pi + (1-\beta-\mu)(\varepsilon_2 + \tau(\gamma(\beta^{-1})))$$

Rearranging the inequality, if $(\varepsilon_1, \varepsilon_2) \in NM_2$, $D_1, D_2$ merge if

$$(1-\beta)\varepsilon_1 - (1-\beta-2\mu)\varepsilon_2 \leq k + 2\mu(\tau(\gamma(\beta^{-1})))$$

Otherwise $D_1$ and $D_2$ remain independently owned. If the sign of $(1-\beta-2\mu)$ is negative then $D_1$ and $D_2$ always merge.

$D_1, D_2$ merge in $NM_12$ if $(\varepsilon_1, \varepsilon_2) \in AM_4$ where,

$$M_4 = \{(\varepsilon_1, \varepsilon_2) : \varepsilon_1 - \frac{1-\beta-2\mu}{1-\beta}\varepsilon_2 \geq A_4(\beta, \mu)\} \text{ where}$$
$$A_4(\beta, \mu) = \frac{-(k + 2\mu(\tau(\gamma((\beta-\mu)^{-1}))))}{1-\beta}$$
$$AM_4\{(\varepsilon_1, \varepsilon_2) : (\varepsilon_1, \varepsilon_2) \in (NM_2 \cup M_4)\}$$

# Bibliography


1. Aaker D. A. , *Business Development Strategies*, John Wiley & Sons, New York, 1998.

2. Bayus B.L. and Shankar V. , “Network effects and competition: An empirical analysis of the home video game industry,” working paper, 2002.

3. Beggs A.W. , “Mergers and malls,” *The Journal of Industrial Economics*, vol. 42, pp. 419–428, 1994.

4. Besen S. and Farrell J. , “Choosing how to compete:strategies and tactics in standardization,” *Journal of Econonomic Perspectives*, vol. 8, pp. 117–131, 1994.

5. Bolton P. and Whinston M.D. , “Incomplete contracts, vertical integration, and supply assurance,” *Review of Economic Studies*, vol. 60, pp. 902–932, 1993.

6. Bonanno G. and Vickers J. , “Vertical seperation,” *The Journal of Industrial Economics*, vol. 36, pp. 257–265, 1988.

7. Chen Y. , “On vertical mergers and their competitive effects,” *The RAND Journal of Economics*, vol. 32, pp. 667–685, 2001.

8. Church J. and Gandal N. , "Network effects,software provision and standardization," *The Journal of Industrial Economics*, vol. 40, pp. 85–103, 1992.

9. DFC Intelligence , "Game industry research," March 2004.

10. Economides N. , "Desirability of compatibility in the absence of network externalities," *mimeo*, 1989a.

11. Economides N. , "Variable compatibility without network externalities," *mimeo*, 1989b.

12. Economides N. and Salop S.C. , "Competition and integration among complements and network market structure," *The Journal of Industrial Economics*, vol. 40, pp. 105–123, 1992.

13. Farrell J. and Klemperer P. , "Coordination and lock-in:competition with switching costs and network effects," *mimeo*, 2001.

14. Farrell J. and Shapiro C. , "Optimal contracts with lock-in," *The American Economic Review*, vol. 79, pp. 51–68, 1989.

15. Farrell J. and Shapiro C. , "Horizontal mergers:an equilibrium analysis," *The American Economic Review*, vol. 80, pp. 107–126, 1990.

16. Gabaix X. and Laibson D. , "Shrouded attributes and information suppression in competitive markets," *Quarterly Journal of Economics*, vol. Forthcoming, 2005.

17. Gandal N. , "Quantifying the trade impact of compatibility standards and barriers:an industrial organization perspective," mimeo, 2000.

18. Gandal N. , Kende M. , and Rob R. , “The dynamics of technological adoption in hardware/software systems: the case of compact disc players,” *The RAND Journal of Economics*, vol. 31, pp. 43–61, 2000.

19. Grossman S.J. and Hart O.D. , “The costs and benefits of ownership:a theory of vertical and lateral integration,” *Journal of Political Economy*, vol. 94, pp. 691–719, 1986.

20. Heavner D.L. , “Vertical enclosure:vertical integration and the reluctance to purchase from a competitor,” *The Journal of Industrial Economics*, vol. 52, pp. 179–199, 2004.

21. Hemphill J. and Vonartas N. , “U.s. antitrust policy,interface compatibility standards,and information technology,” *working paper*, 2003.

22. Jackson M.O. and Wilkie S. , “Endogeneous games and mechanisms:side payments among players,” *Review of Economic Studies*, vol. Forthcoming, 2005.

23. Klemperer P. , “The competitiveness of markets with switching costs,” *The RAND Journal of Economics*, vol. 18, pp. 138–150, 1987.

24. Lichtman D. , “Property rights in emerging platform technologies,” University of Chicago, mimeo, 2005.

25. Matutes C. and Ragibeau P. , “Mix and match:product compatibility without network externalities,” *The RAND Journal of Economics*, vol. 19, pp. 221–234, 1988.

26. McAfee R.P. , “The effects of vertical integration on competing input suppliers,” The Federal Reserve Bank of Cleveland,mimeo, 1999.

27. Ordover J.A. , Saloner G. , and Salop S.C. , "Equilibrium vertical closure," *The American Economic Review*, vol. 82, pp. 127–142, 1990.

28. Park S. , "Quantative analysis of network externalities in competing technologies:the vcr case," working paper, 2003.

29. Salant S.W. , Switzer S. , and Reynolda R.J. , "Losses from horizontal merger:the effects of an exogeneous change in industry structure on cournot-nash equilibrium," *Quarterly Journal of Economics*, vol. 98, pp. 185–200, 1983.

30. Salinger M. , "The meaning of 'upstream' and 'downstream' and the implications for modelling vertical mergers," *The Journal of Industrial Economics*, vol. 37, pp. 373–387, 1989.

31. Salinger M.A. , "Vertical mergers and market foreclosure," *Quarterly Journal of Economics*, vol. 103, pp. 345–356, 1988.

32. Shapiro K. , "Mergers with differentiated products," *Antitrust*, vol. Spring, 1997.

33. Stole L.A. and Zwiebel J.A. , "Mergers,emloyee hold-up and the scope of the firm:an intrafirm bargaining approach to mergers," *mimeo*, 1998.

34. Ziss S. , "Vertical seperation and horizontal mergers," *The Journal of Industrial Economics*, vol. 43, pp. 63–75, 1995.

# Vita

Kerem Cakirer was born in Istanbul, Turkey on May 2, 1979, the son of Guldan Cakirer and Halim Cakirer. After completing his work at Isik Lisesi, Istanbul, Turkey, in 1996, he entered Bilkent University in Ankara, Turkey. He received the degree of Bachelor of Science in mathematics from Bilkent University in June 2001. In August 2001 he entered the Graduate School of The University of Texas. He received the degree of Master of Science in economics from University of Texas at Austin in May 2003. During the following years he was employed as a teaching assistant at The University of Texas Economics Department. He is currently employed as an assistant lecturer at Texas A&M University Department of Economics. Recently, he published a paper in ICFAI Journal of Mergers and Acquisitions.

Permanent Address: Sukranciftligi Sok. No:4, Da:7
Bakirkoy, 34147 Istanbul TURKEY

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