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Three Essays On Bill-and-Keep Payment Mechanisms Between Communication Networks

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Three Essays On Bill-and-Keep Payment Mechanisms Between Communication Networks

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Dedicated to my parents, JongGeun Lee and ByungSoo Lee

and

my wife, HyunKyung Kim.

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Three Essays On Bill-and-Keep Payment Mechanisms Between Communication Networks

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In many countries, the payment structure in the telecommunications market is based on the Calling Party Pays Principle (CPP), in which only callers pay for calls. Since the CPP model has been generally accepted, little attention has been paid to the fact that call-receivers benefit from the calls without having to pay, except in telecommunications studies in 1970s. This "call externality" has been examined recently in research dealing with the Receiver Pays Principle (RPP). In this paper, I study the interconnection between a regulated fixed-line network and competitive mobile networks. Previous literature says that, under the CPP regime, mobile networks have an incentive to charge monopoly access charges, the profits from which are used to attract their own subscribers. However, taking into consideration the receiver's utility reduces the mobile networks' incentive for above-cost access charges. I consider this phenomenon to be a welfare transfer from fixed-line users to mobile users. I show that the welfare transfer is reversed as mobile networks take into account the receiving-utility of their own subscribers. This reversed welfare effect increases with the size of the receiver's utility. However, the market outcome is still inefficient because the mobile access charges are not sufficiently low given the receiver's utility. These results urge the introduction of a different payment regime into the telecommunications market to incorporate call externalities and remove access market distortions. I show that by introducing a new regime, "Bill-and-Keep", which includes a Receiving Party Pays system and no access charges, efficient allocation can be achieved. Proper meet-points corresponding to receiver's utilities are required for the efficient allocation. Theoretically, if a regulator is able to collect information about the costs of networks and the receiver's utility, an optimal Bill-and-Keep regime can be introduced to the economy. But, because it's nearly impossible to obtain this information, two practical Bill-and-Keep regimes are suggested: Central Office Bill-and-Keep and Meet-Point Bill-and-Keep. They only require information about transport costs, which are relatively easy to detect. Using a example model, I examine the welfare effect of the practical Bill-and-Keep regimes for a range of values for the receiver's utility. I show that for a Bill-and-Keep regime to be superior to a CPP regime the receiver's utility should be fairly large. When receiver's utility is small, a practical Bill-and-Keep regime might not provide better total surplus than a market-driven CPP regime.

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The Role of Receiver's Utility in the Mobile Access Market

1.1 Introduction

The US telecommunications market has an unusual payment structure compared with other telecommunications markets in Asia and Europe. In particular, the fixed rates for local calls and Mobile Party Pays (MPP) system are different from the systems used in other countries. In the US MPP system both callers and receivers pay for the calls, whereas in most European and Asian countries receivers do not pay at all.

In the traditional research on the telecommunications market, the 'Calling Party Pays Principle (CPP)', also known as 'Calling Party Network Pays Principle (CPNP),' is the usual regime.[1] This system assumes that a call gives utility to the calling party but not to the called party. Therefore the caller or calling network has to pay all the cost of a call including the access charge, which compensates the called network. Even though receivers enjoy utility from receiving calls, their utilities are disregarded, ignoring the issue of "call externalities." Since under CPP regime receivers do not pay commensurate with their benefit, there may be a distortion of the market equilibrium, especially in the volume of calls or the usage of telecommunications facilities.

[1] See Laffont, Rey and Tirole (1998*a*) and Armstrong (1998).

This paper expands upon the existing research on the determination of access pricing between fixed-line and mobile networks. Wright (2002*a*) and Armstrong (2002*a*) provide a theoretical explanation for the empirical fact that fixed-to-mobile access charges are much higher than mobile-to-fixed access charges. In the traditional models, the payment structure of the telecommunications market is based on the CPP. It is usually assumed that the mobile market is perfectly competitive, but that each mobile network has a monopoly over delivering calls to its subscribers.[2] As a result, each mobile network sets a monopoly access charge for fixed-to-mobile access. On the other hand, the fixed-line network is assumed to be a regulated monopoly. It is also usually assumed that their retail prices and access charges are regulated, too. In most cases, even if competition in the mobile market for subscribers is intense and mobile networks do not have super-normal profits, the monopoly profits from call termination and the consequent deadweight loss persist and are used to finance subsidized retail tariffs to attract subscribers. Even though they do not emphasize the welfare distribution among agents, one can also see that high access charges increase the welfare transfer from fixed-line users to mobile users.

However, these papers have some limitations. Because Wright (2002*a*) and Armstrong (2002*a*) accept CPP, they neglect the call externality issue. This issue had been raised earlier[3] and forgotten and only recently has been picked up again. In particular, Jeon, Laffont and Tirole (2004) introduce the Receiver Pays Principle (RPP) into the traditional network competition model of Laffont, Rey and Tirole (1998*a*,*b*). This paper does not study the

[2]This service cannot be replaced by other networks if its subscribers do not subscribe to any other networks simultaneously.

[3]See Squire (1973) and Rohlfs (1974).

fixed-mobile access market directly but studies interconnection between two symmetric networks. Furthermore, the paper focuses on receiver's prices instead of access charges so that common access charges are assumed to be given.

In Wright (2002*a*), he analyzes 'receiver cares' in an extended model. He shows that when the receiver's utility is considered by networks, this brings access charges below the monopoly level. If mobile networks take into account the receiving-utility of their own subscribers, which decreases as access charges are increased, they will accept "perceived" termination costs, which are marginal costs discounted by the marginal receiving-utility.

I study the comparative statics of receiver's utility in mobile access markets, which is extended from Wright (2002*a*)'s idea. I set up a traditional interconnection model between two competitive mobile networks and a regulated monopoly fixed-line network under a CPP regime comparable to Wright (2002*a*). All mobile firms can set the retail tariffs and access charges freely but prices under the fixed-line network are regulated. Because this paper focuses on access markets between the fixed-line network and the two mobile networks, mobile-to-mobile and on-net calls are ignored. I introduce the receiver's utility into the model, however the networks do not charge receivers for incoming calls. Evaluation of the model clearly demonstrates the existence of call externalities under the CPP regime. I analyze how the receiver's utility affects the welfare distribution from fixed-line users to mobile users. The main question is how mobile networks respond to the increasing receiving-utility of their own subscribers and how the welfare is distributed among fixed-line users, mobile users and mobile networks.

I show that, in a market equilibrium, as the receiving-utility of mobile users increases, mobile networks decrease their access charges from the traditional monopoly level. This means that the welfare transfer from fixed-line users to mobile users is reversed as the receiving-utility increases. I find that in cases when the receiver's utility is smaller than the caller's utility, the mobile networks continue to set access charges above the "technical" marginal cost but below the traditional monopoly level. Even though the access charges decrease because of the inclusion of receiving-utility, the resulting access charge continues to be inefficiently high, because mobile networks keep the access monopoly. On the other hand, if the regulator does not know how large the receiving-utility of fixed-line users is, the regulatory inefficiency for fixed-line access charges is unavoidable. These two inefficiencies create a necessity for a new payment regime in telecommunications market – one that will incorporate call externalities and remove access market distortions.[4]

1.2 The Model

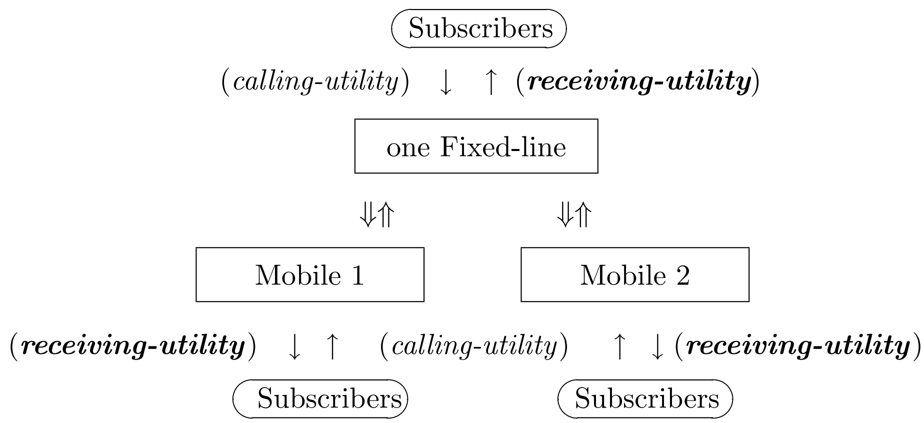
Assume that the telecommunications market has only two kinds of networks: fixed-line and mobile. The fixed-line network is a monopoly and the mobile market has two symmetric and competitive firms.[5] The payment system of the telecommunications market is the CPP system, which means that callers pay for the calls. In the traditional telecommunications models under CPP, callers are assumed to be the only party to benefit from the calls; however, receivers clearly benefit from the calls, too. Note that because under the CPP regime receivers do not pay for their benefits, call externalities arise. To show the call externalities explicitly, I assume that receivers derive utility

[4] See DeGraba (2000) and Atkinson and Barnekov (2000) for new regimes.

[5] This model is based on Wright (2000).

from the calls but do not pay for calls and that they do not hang up on the calls first. Therefore, the callers determine the call length.[6]

Figure 1.1: Interconnection between Fixed-line and Mobile Networks



All calls made from mobile networks are terminated on the fixed-line network, and all calls originating on the fixed-line network are terminated on one of the mobile networks. On-net calls in each network are disregarded[7] and mobile-to-mobile calls are ignored. Since they need access to each other's network to complete their outgoing calls, two-way access markets exist: mobile-to-fixed and fixed-to-mobile.

The costs to complete calls are assumed to be composed of three parts: origination, termination and transport (see Figure 1.2).[8] Origination cost c^o

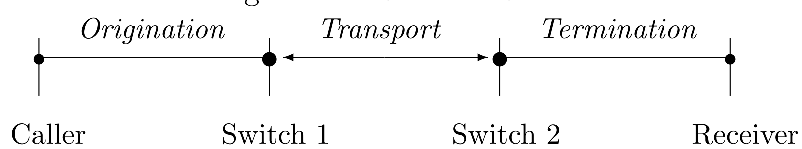
[6]See Jeon et al. (2004) for 'receiver's sovereignty' under Receiver Pays Principle.

[7]See Armstrong (2002*a*) for a model with on-net calls.

[8]For telecommunications study, the concept of marginal cost needs to be cared specially since recently it is generally accepted that the marginal cost of telecommunications service is almost zero. The marginal costs can be thought as "costs of preparation" to provide the telecommunications service rather than costs of providing the service itself because an additional call induces congestion and therefore requires additional capacity. See U.K. Parliament (1999).

is a per-minute cost generated in the mobile network when a subscriber makes a call. The termination cost c^t is a per-minute cost generated in the mobile network when one of its subscribers receives a call. The two mobile networks have symmetric cost structures. The fixed-line network has a different cost structure than the mobile networks. C^O and C^T are, respectively, the origination and termination costs for the fixed-line network. Transport cost τ is a per-minute cost between the two central office switches of the involved networks. Note that the origination or termination cost includes the cost generated in each party's central office switch. The transport cost is covered by the calling network.[9] I assume that the transport cost is the same between the fixed-line network and either mobile network and is symmetric between a fixed-to-mobile call and a mobile-to-fixed call.

Figure 1.2: Costs of Calls



I assume that there aren't any universal fixed costs to reflect a joint and common cost of serving the various customers for each network. But mobile networks have a fixed cost c^f per subscriber, which reflects the monthly cost of connecting the customer to the network as well as billing and servicing the customer. C^F is a fixed cost per subscriber for the fixed-line network.

It is assumed that the access charge of mobile-to-fixed network A is

[9]In some sense, under the CPP regime, the origination cost includes the transport cost. I separate them to compare this regime with the Bill and Keep regime, under which sharing of transport cost becomes an issue.

regulated at termination cost level C^T, as in previous papers.[10]

$$A = C^T \tag{1.1}$$

Each mobile network freely charges access charge a_i to the fixed-line network.

For each network, demand for outgoing calls depends only on the price of an outgoing call and not on the price for incoming calls, which is paid by the customers of the other network. Callers do not care about their calling partners' utilities. Given that each customer has the same marginal willingness to pay for calling and the same utility of receiving calls, every mobile network can offer uniform tariff for the overall use of its services. The fixed-fee F for the fixed-line network is regulated at the fixed cost level, C^F.

$$F = C^F \tag{1.2}$$

The fixed-line network is regulated to set its fixed-to-mobile retail prices proportional to the access charge each mobile firm sets: a_i and a_j. Suppose the fixed-line network offers its subscribers a per-minute usage charge P_i for making calls to mobile network i, which is described by a function of total costs,

$$P_i \equiv g(C^O + \tau + a_i), \quad g' > 0 \tag{1.3}$$

This is obviously true in the case when fixed-to-mobile prices are regulated at variable cost level, $C^O + \tau + a_i$.[11] On the other hand, each mobile network charges its subscribers a monthly fixed-fee f_i and a per-minute charge p_i for making mobile-to-fixed calls. Suppose that once a subscriber has joined a mobile network with usage charge p_i, that subscriber makes $q(p_i)$ minutes of

[10] See Armstrong (2002*a*) and Wright (2000)

[11] See Wright (2002*a*).

outbound calls to a fixed-line user. Each fixed-line network subscriber makes $Q(P_i)$ minute calls to a user of mobile network i.

A fixed set of homogeneous customers subscribe to the telecommunications services. For simplification, I assume that the fixed-line network has N subscribers and mobile networks divide another N subscribers into two groups, N_1 and N_2. This N can be normalized to 1 without loss of generality and then N_1 and N_2 can be understood as market shares s_i and s_j. Notice that full market participation is assumed respectively in the fixed-line market and in the mobile market and, as a result, $s_i + s_j = 1$ always holds. In this framework each market has no network externalities.[12]

Consumer preferences are known to firms. The utility of a consumer with income y joining the fixed-line network is given by

$$y + V_o + \sum_{i=1}^{2} s_i \left\{ U(Q_i) - P_i Q_i + \tilde{U}(q_i) \right\} - F. \qquad (1.4)$$

V_0 is a fixed utility enjoyed by a fixed-line user from subscribing to the fixed-line network.[13] $U(Q_i)$ is the utility derived from making Q_i calls to a user in the mobile network i, and $\tilde{U}(q_i)$ is the utility derived from receiving q_i calls from a user in the mobile network i. All utility functions are assumed to be concave.

The market share of each mobile network, s_i, is determined by network competition for subscribers, which can be explained by the Hotelling product differentiation model.[14] Assume that mobile customers are endowed with a

[12] See Armstrong (2002*a*) for a model with partial market participation and network externalities.

[13] V_0 is assumed to be high enough that all consumers join this network.

[14] The Hotelling model captures the diversity of service characteristics provided by a

value of x, which is drawn from a uniform distribution on the interval $[0, 1]$. Two mobile networks are located at each end point of this unit interval, respectively. A mobile customer with value x has extra disutility from not being able to consume his preferred services,

$$tx^2 \quad \text{and} \quad t(1-x)^2 \tag{1.5}$$

for network 1 and 2 respectively, where t is the parameter that measures how differentiated the networks are. The greater t is, the greater are the switching costs, and the harder it is for one firm to steal away customers from the rival firm by lowering its price. The utility derived by a consumer with income y located at x from joining either mobile network i or j is respectively given by:

$$y + v_0 - tx^2 + u(q_i) - p_i q_i + \tilde{u}(Q_i) - f_i \tag{1.6}$$

$$y + v_0 - t(1-x)^2 + u(q_j) - p_j q_j + \tilde{u}(Q_j) - f_j \tag{1.7}$$

v_0 is a fixed utility, which a mobile network user has from subscribing to a mobile network.[15] $u(q_i)$ is utility from making calls to a fixed-line network user and $\tilde{u}(Q_i)$ is utility from receiving calls from a fixed-line network user. f_i is a fixed-fee which the mobile user has to pay for every billing period. Disregarding fixed utility, define net variable consumer surplus w_i as:

$$w_i \equiv u(q_i) - p_i q_i + \tilde{u}(Q_i) - f_i, \quad i = 1, 2. \tag{1.8}$$

A customer located at x is indifferent between the two mobile networks if

$$w_i - tx^2 = w_j - t(1-x)^2. \tag{1.9}$$

telecommunication network –such as additional service options, customer services and marketing.

[15]The constant v_o ensures that all consumers will always choose to join one of the two mobile networks if it is high enough.

Solving for x, the mobile market shares are, for $i, j = 1, 2$ and $i \neq j$,

$$s_i = \frac{1}{2} + \frac{w_i - w_j}{2t}. \tag{1.10}$$

I assume that each network subscriber benefits from receiving calls. For the fixed-line network subscriber and mobile network subscriber, the utility from receiving calls is assumed to be proportional to the calling-utility of the caller in the corresponding network.[16] This is only for technical simplicity.

$$\tilde{U}(q_i) \equiv \mathcal{B} \cdot u(q_i),\ \mathcal{B} > 0 \tag{1.11}$$

$$\tilde{u}(Q_i) \equiv \beta \cdot U(Q_i),\ \beta > 0 \tag{1.12}$$

Defining the indirect utilities for fixed-to-mobile calls Q_i and mobile-to-fixed calls q_i as:

$$V(P_i) = \max_{Q_i} \big\{U(Q_i) - P_i Q_i\big\}, \quad i = 1, 2 \tag{1.13}$$

$$v(p_i) = \max_{q_i} \big\{u(q_i) - p_i q_i\big\}, \quad i = 1, 2, \tag{1.14}$$

demand functions of $Q(P_i)$ and $q(p_i)$ for the calls per subscriber can be derived. Then,

$$V'(P_i) = -Q(P_i) \quad \text{and} \quad v'(p_i) = -q(p_i) \quad i = 1, 2. \tag{1.15}$$

Disregarding fixed utilities, a consumer's net surplus of belonging to each network is then

$$W \equiv \sum_{i=1}^{2} s_i \left\{V(P_i) + \tilde{U}(q(p_i))\right\} - F \tag{1.16}$$

$$w_i \equiv v(p_i) + \tilde{u}(Q(P_i)) - f_i, \quad i = 1, 2. \tag{1.17}$$

[16] The noise in the utility of receiving a call is introduced in Jeon et al. (2004): $\tilde{U}(q_i) + \epsilon q_i$ and $\tilde{u}(Q_i) + \epsilon Q_i$. In this CPP regime, this noise does not affect an equilibrium since networks do not set the receiver's prices which would be affected by this disturbance term.

where market shares are, for $i = 1, 2$,

$$s_i = \frac{1}{2} + \frac{w_i - w_j}{2t}$$
$$= \frac{1}{2} + \frac{\{v(p_i) + \tilde{u}(Q(P_i)) - v(p_j) - \tilde{u}(Q(P_j))\} + (f_j - f_i)}{2t}. \quad (1.18)$$

Notice that the price of fixed–to-mobile call P_i can affect the mobile market shares because mobile subscribers have utilities from receiving the calls originated in the fixed-line network, $\tilde{u}(Q(P_i))$.

Access charges are chosen first, and then, taking these as given, firms choose their retail tariffs non-cooperatively. Given the access charges (a_i, a_j, A), each mobile firm chooses its tariff, (p_i, f_i) or (p_j, f_j).[17] On the other hand, retail tariffs for the fixed-line network, (P_i, P_j, F) are regulated according to (1.2) and (1.3), given (a_i, a_j, A).

Since the total number of customers N can be normalized to one, all demands and profits can be understood as a per-customer unit: $Q(P_i)$, $q(p_i)$, Π, π_i.

1.3 The Benchmark: Efficient Allocation

I derive the social optimum for future reference. Consider an idealized situation in which a social planner chooses the market shares and the volume of calls. In this symmetric setup, equal market division minimizes the average consumer's disutility from not being able to consume his preferred service.

Let's define $T(s_i)$ as the average consumer's disutility from not being able to consume her preferred service.[18] Using disutilities (1.5), for arbitrary

[17] See Armstrong (1998, p. 552) for this move order.
[18] See Laffont et al. (1998*a*)

s_i, this measure of the distance between preferred and actual brand choice is given by:

$$\begin{aligned} T(s_i) &\equiv t\left[s_i\left(\frac{s_i}{2}\right)^2 + (1-s_i)\left(\frac{1-s_i}{2}\right)^2\right] \\ &= t\left[\frac{s_i^3 + (1-s_i)^3}{4}\right]. \end{aligned} \tag{1.19}$$

$T(s_i)$ is minimized at $s_i = \frac{1}{2}$, which is the symmetric market share.

$$\begin{aligned} \frac{\partial T(s_i)}{\partial s_i} &= \frac{t}{4}\left[3s_i^2 - 3(1-s_i)^2\right] \\ &= \frac{t}{4}\left[-3 + 6s_i\right] = 0 \end{aligned} \tag{1.20}$$

The social planner would choose the volume of calls so as to maximize the social surplus.

$$\max_{Q,q}\ U(Q) + \tilde{U}(q) + u(q) + \tilde{u}(Q) - (C^O + \tau + c^t)Q - (c^o + \tau + C^T)q \tag{1.21}$$

The optimal volumes $\hat{Q}$ and $\hat{q}$ are determined by:

$$U'(\hat{Q}) + \tilde{u}'(\hat{Q}) = C^O + \tau + c^t \tag{1.22}$$

$$u'(\hat{q}) + \tilde{U}'(\hat{q}) = c^o + \tau + C^T \tag{1.23}$$

Since $\tilde{u}'(\cdot) = \beta U'(\cdot)$ and $\tilde{U}'(\cdot) = \mathcal{B} u'(\cdot)$,

$$U'(\hat{Q}) = \frac{C^O + \tau + c^t}{1+\beta} \tag{1.24}$$

$$u'(\hat{q}) = \frac{c^o + \tau + C^T}{1+\mathcal{B}}. \tag{1.25}$$

To implement the optimal outcome, the social planner can use symmetric tariffs so as to achieve equal market division ($s_i = \frac{1}{2}$). Since the volume is determined by the caller, the optimal volume is obtained by choosing:

$$\hat{P} = U'(\hat{Q}) = C^O + \tau + c^t - \tilde{u}'(\hat{Q}) \tag{1.26}$$

$$\hat{p} = u'(\hat{q}) = c^o + \tau + C^T - \tilde{U}'(\hat{q}) \tag{1.27}$$

Under the CPP regime, callers should pay for the origination cost, transport cost and access charges.

$$\hat{P} = C^O + \tau + \hat{a} \tag{1.28}$$

$$\hat{p} = c^o + \tau + \hat{A} \tag{1.29}$$

For optimality, access charges should be given by:

$$\hat{a} = c^t - \tilde{u}'(\hat{Q}) \tag{1.30}$$

$$\hat{A} = C^T - \tilde{U}'(\hat{q}) \tag{1.31}$$

To satisfy the each industry's break-even constraint, F and f should be adjusted.

$$\Pi = (\hat{A} - C^T)\hat{q} + (\hat{P} - C^O - \tau - \hat{a})\hat{Q} + \hat{F} - C^F = 0 \tag{1.32}$$

$$\pi = (\hat{a} - c^t)\hat{Q} + (\hat{p} - c^o - \tau - \hat{A})q + \hat{f} - c^f = 0 \tag{1.33}$$

$$\hat{F} = C^F + \tilde{U}'(\hat{q})\hat{q} \tag{1.34}$$

$$\hat{f} = c^f + \tilde{u}'(\hat{Q})\hat{Q} \tag{1.35}$$

1.4 Market Equilibrium

1.4.1 Equilibrium Retail Tariffs

Let's start with the second stage, in which given (a_i, a_j, A) mobile firms choose their own retail tariffs simultaneously. In the second-stage game, mobile network i sets profit-maximizing tariff, (p_i, f_i), accepting (P_i, P_j) and (p_j, f_j) as given.

$$\max_{p_i,\ f_i} \pi_i = s_i(p_i, p_j, P_i, P_j, f_i, f_j)\Big\{Q(P_i)\left(a_i - c^t\right) \\ +q(p_i)\left(p_i - c^o - \tau - C^T\right) + f_i - c^f\Big\} \quad (1.36)$$

Using equation (1.17), $w_i \equiv v(p_i) + \tilde{u}(Q(P_i)) - f_i$, the optimization problem of each mobile network can be solved with respect to p_i and w_i, instead of p_i and f_i because market share is determined directly by the net surplus.[19]

$$\max_{p_i, w_i} \pi_i = \left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\}\Big\{Q(P_i)\left(a_i - c^t\right) \\ + q(p_i)\left(p_i - c^o - \tau - C^T\right) + v(p_i) + \tilde{u}(Q(P_i)) - w_i - c^f\Big\} \quad (1.37)$$

The first order conditions (FOCs) of each mobile network are the following. For $i, j = 1, 2$ and $i \neq j$,

$$\frac{\partial \pi_i}{\partial p_i} = \left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\}\left\{q(p_i) + \frac{\partial q_i}{\partial p_i}(p_i - c^o - \tau - C^T) - q(p_i)\right\} \\ = s_i \cdot \frac{\partial q_i}{\partial p_i} \cdot (p_i - c^o - \tau - C^T) = 0 \quad (1.38)$$

$$\frac{\partial \pi_i}{\partial w_i} = -\left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\} + \frac{1}{2t}\Big\{Q(P_i)\left(a_i - c^t\right) \\ + q(p_i)\left(p_i - c^o - \tau - C^T\right) + (v(p_i) + \tilde{u}(Q(P_i)) - w_i) - c^f\Big\} \\ = -s_i + \frac{1}{2t}\Big\{Q(P_i)\left(a_i - c^t\right) \\ + q(p_i)\left(p_i - c^o - \tau - C^T\right) + f_i - c^f\Big\} = 0 \quad (1.39)$$

Lemma 1. $\forall\ i,\ \ s_i \neq 0$*: In the mobile market, a cornered-market equilibrium does not exist.*

[19] See Laffont et al. (1998*a*, p. 20).

Proof. See Appendix A. □

Since cornered-market equilibrium does not exist, from the FOC (1.38), the usage price is set at marginal cost according to the two-part tariff problem:

$$p_i^* = c^o + \tau + C^T, \quad i = 1, 2. \tag{1.40}$$

Combining the above result (1.40) with the FOC (1.39), the equilibrium fixed-fee is:

$$f_i^* = c^f + 2t \cdot s_i - Q(P_i^*)\left(a_i - c^t\right), \quad i = 1, 2. \tag{1.41}$$

Furthermore, this is a unique equilibrium for any given (a_i, a_j) and (A, P_i, P_j, F).

Lemma 2. *∃ a unique equilibrium* $(p_i^*, p_j^*, w_i^*, w_j^*)$*, such that* $p_i^* = p_j^* = c^o + \tau + C^T$*, given* $(a_i, a_j; A, P_i, P_j, F)$*: A unique mobile market equilibrium in terms of retail prices and consumer surplus exists, given access charges and fixed-to-mobile retail tariffs. Furthermore, their retail usage prices are symmetric.*

Proof. See Appendix A. □

The results about mobile firms in equations (1.40) and (1.41) are the same as the results of Wright (2000). Equation (1.41) says that for a given market share, as far as access profit increases, increased access revenue results in an equal reduction of fixed-fees. Starting from cost-based access charges, higher access charges yield a bigger "reward" for attracting other subscribers. This "reward" is the access profit that the additional subscriber creates, which creates more aggressive competition for subscribers by mobile firms. If the access charges of both firms are increased simultaneously, equilibrium fixed-fees decrease until access profit is maximized.[20]

[20]See Wright (2000, Proposition 1).

The retail prices of the fixed-line network are regulated by equation (1.3). Since their fixed-fee and access charges are regulated at cost level, the profit is:

$$\Pi = \sum_{i=1}^{2} s_i(P_i^* - C^O - \tau - a_i)Q(P_i^*) \tag{1.42}$$
$$\text{where } P_i^* = g(C^O + \tau + a_i),\ g' > 0.$$

Because the fixed-line network is regulated, they cannot affect the mobile market shares, even though the receiving-utility of mobile users depends on the fixed-to-mobile retail prices.

1.4.2 Equilibrium Access Charges

Now at the first stage, given conditions (1.3), (1.40) and (1.41), each mobile firm chooses its own access charge. The fixed-line network is regulated to set its access charge at the termination cost level C^T.

Mobile network i chooses its access charge a_i, given a_j. Considering the tariffs of the mobile firms determined at the second stage, (1.40) and (1.41), the maximization problem is:

$$\begin{aligned}\max_{a_i}\ \pi_i =& s_i\Big\{Q(P_i^*)\left(a_i - c^t\right) \\ &+ q(p_i^*)\left(p_i^* - c^o - \tau - C^T\right) + f_i^* - c^f\Big\} \\ =& 2t \cdot s_i^2. \end{aligned} \tag{1.43}$$

The equilibrium profits of the mobile firms depend only on its market share, but access charges affect this market share. Using the definition of net consumer surplus (1.17) and the fixed prices determined at the second stage (1.41), consumer surplus is:

$$w_i^* = v(p_i^*) + \tilde{u}(Q(P_i^*)) - c^f - 2t \cdot s_i + Q(P_i^*)(a_i - c^t), \quad i = 1, 2. \tag{1.44}$$

Since $p_i^* = c^o + \tau + C^T$, $v(p_i^*) = v(p_j^*)$. Then, the market share is: for $i, j = 1, 2$ and $i \neq j$,

$$s_i^* = \frac{1}{2} + \frac{\tilde{u}(Q(P_i^*)) + (a_i - c^t)Q(P_i^*) - \tilde{u}(Q(P_j^*)) - (a_j - c^t)Q(P_j^*)}{6t}.$$

Since $P_i^* = g(C^O + \tau + a_i)$ from equation (1.3),

$$\begin{aligned} s_i =& s_i(a_i, a_j) \\ =& \frac{1}{2} + \frac{\tilde{u}(Q(g(C^O + \tau + a_i))) + (a_i - c^t)Q(g(C^O + \tau + a_i))}{6t} \\ &- \frac{\tilde{u}(Q(g(C^O + \tau + a_j))) + (a_j - c^t)Q(g(C^O + \tau + a_j))}{6t}. \end{aligned} \tag{1.45}$$

Using equation (1.43) and (1.45), the optimization problem of each mobile firm at the first stage is:

$$\begin{aligned} \max_{a_i} \ \pi_i =& 2t \cdot s_i(a_i, a_j)^2 \\ =& 2t \left\{ \frac{1}{2} + \frac{\tilde{u}(Q(g(C^O + \tau + a_i))) + (a_i - c^t)Q(g(C^O + \tau + a_i))}{6t} \right. \\ & \left. - \frac{\tilde{u}(Q(g(C^O + \tau + a_j))) + (a_j - c^t)Q(g(C^O + \tau + a_j))}{6t} \right\}^2 . \end{aligned} \tag{1.46}$$

FOC can be derived as the following: From equation (1.12), $\frac{\partial \tilde{u}}{\partial Q_i} = \beta \frac{\partial U}{\partial Q_i} > 0$.

$$\begin{aligned} \frac{\partial \pi_i}{\partial a_i} &= \frac{2}{3} s_i(a_i, a_j) \left\{ Q(P_i) + (a_i - c^t) \frac{\partial Q(P_i)}{\partial P_i} \frac{\partial g}{\partial a_i} + \beta \frac{\partial U}{\partial Q_i} \frac{\partial Q(P_i)}{\partial P_i} \frac{\partial g}{\partial a_i} \right\} \\ &= \frac{2}{3} s_i(a_i, a_j) \left\{ Q(P_i) + \left(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i} \right) \frac{\partial Q(P_i)}{\partial a_i} \right\} = 0, \quad i = 1, 2 \end{aligned} \tag{1.47}$$

where $P_i = g(C^O + \tau + a_i)$. Since $s_i \neq 0$, the access charge should satisfy this condition.

$$Q_i(g(C^O + \tau + a_i)) + \left(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i} \right) \frac{\partial Q_i(g(C^O + \tau + a_i))}{\partial a_i} = 0 \tag{1.48}$$

Furthermore, this is a unique and symmetric equilibrium access charge, $a_i^* = a_j^* = a^*$.

Lemma 3. *Assume $Q(\cdot)$ and $g(\cdot)$ are (almost) linear and $\beta \leq 1$. $\exists$ a unique and symmetric equilibrium in ($a_i^* = a_j^* = a^*$): This equilibrium, in terms of mobile access charges, is unique and symmetric and is characterized by this equation,*

$$Q_i(g(C^O + \tau + a^*)) + \left(a^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right) \frac{\partial Q_i(g(C^O + \tau + a^*))}{\partial a} = 0. \quad (1.49)$$

Proof. a_i^* from equation (1.48) is a dominant strategy for each mobile firm because behavior in the second stage is fixed, as in equations (1.40) and (1.41). So (a_i^*, a_j^*) is a unique market equilibrium and is symmetric because of the symmetrical demand and cost structures. See Appendix A for details. □

From equation (1.49), one can see the negative relation between the equilibrium access charge and β.

Lemma 4. *Assume $Q(\cdot)$ is (almost) linear. Equilibrium mobile access charge a^* is a decreasing function of β ($\frac{\partial a^*}{\partial \beta} < 0$).*

Proof. We know that $\frac{\partial \tilde{u}(Q_i)}{\partial Q_i} = \beta \frac{\partial U(Q_i)}{\partial Q_i}$. By the implicit function theorem, from equation (1.49), $\frac{\partial a^*}{\partial \beta} < 0$ holds, as far as the demand for fixed-to-mobile calls $Q(P)$ is close to linear. Let's say above equation (1.48) is an implicit function of a and β, $FOC(a,\beta)$.

$$\frac{\partial a^*}{\partial \beta} = -\frac{\frac{\partial FOC(a,\beta)}{\partial \beta}}{\frac{\partial FOC(a,\beta)}{\partial a}}$$

$$= -\frac{\frac{\partial U(Q_i)}{\partial Q_i}\frac{\partial Q_i(g(C^O+\tau+a^*))}{\partial a}}{2\frac{\partial Q_i(g(C^O+\tau+a^*))}{\partial a} + \left(a^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right)\frac{\partial^2 Q_i(g(C^O+\tau+a^*))}{\partial a^2}} < 0$$

$$\text{if } \frac{\partial^2 Q_i(g(C^O+\tau+a^*))}{\partial a^2} \simeq 0$$

□

Lemma 5. *Assume $Q(\cdot)$ and $g(\cdot)$ are (almost) linear. If mobile users do not have receiving-utility so that $\beta = 0$, each mobile network sets its access charge at the traditional monopoly level, a^m, which is characterized by:*

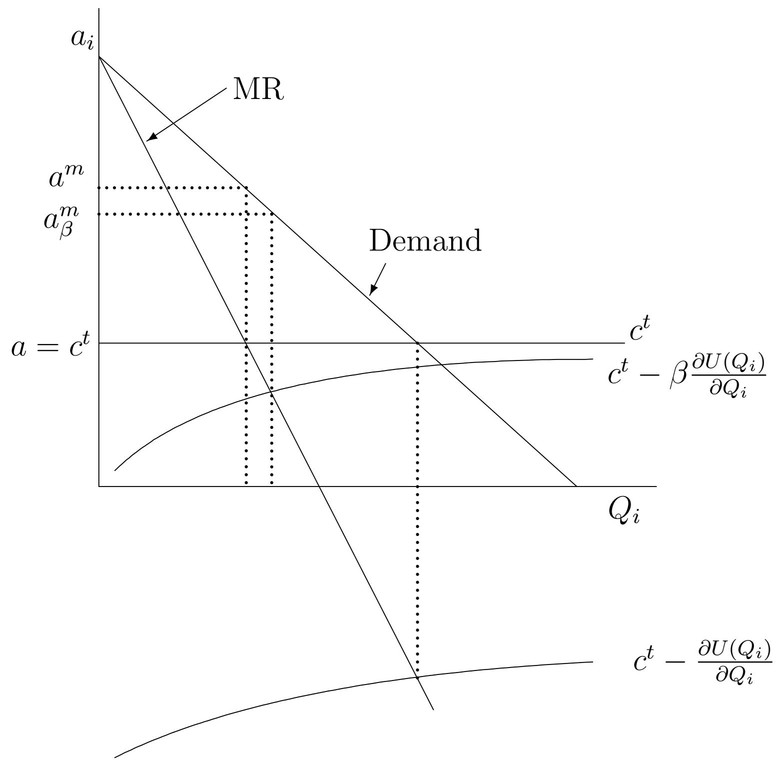
$$Q_i(g(C^O + \tau + a^m)) + \left(a^m - c^t\right)\frac{\partial Q_i(g(C^O + \tau + a^m))}{\partial a} = 0. \qquad (1.50)$$

If mobile users have positive receiving-utility so that ($\beta > 0$), each mobile network sets its access charge below the traditional monopoly level, ($a^* < a^m$).

Proof. See Wright (2002*a*). This is clear from Lemma 4. See Appendix A for details. □

As in Wright (2002*a*), Lemma 5 says that when mobile networks consider the receiving-utility of their subscribers, the mobile access charges are below the traditional monopoly level, a^m, which would be determined without considering the receiving-utility. When the receiving-utility of mobile users is zero, so that $\beta = 0$, the mobile networks set the access charge to the traditional

Figure 1.3: Monopoly Access Market



monopoly level, a^m. As β increases from zero, the mobile networks lower access charges. See Figure 1.3. How does β make access charge lower? If access charge a_i is increased, the fixed-to-mobile retail price P_i is increased according to the rule $g(C^O + \tau + a_i)$. This increased fixed-line retail price reduces the volume of fixed-to-mobile calls $Q(P_i)$. This reduction in call volume leads to the decrease in call-receiving utility of the mobile network subscribers, $\tilde{u}(Q_i)$. This creates a decrease in the welfare of the subscribers, w_i. Eventually, the mobile networks will have some market share loss and profit loss. Networks cannot simply set the traditional monopoly level access charge because of its negative effect on incoming call volume and on the utility of its own subscribers. This effect keeps the access charge below the traditional monopoly level.[21]

From equation (1.49), $c^t - \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}$ can be thought as "perceived marginal costs," which incorporates call externalities. This means that mobile networks accept termination costs discounted by the negative effect of an access charge on its own subscribers' receiving-utility. Note that even though the access charges are lower than the traditional monopoly level, mobile networks still set inefficiently high access charges, creating a new monopoly level that incorporates the receiver's utility. Under the CPP regime, these abnormally high fixed-to-mobile access charges exist and regulatory efforts are necessary to control them.

Proposition 1. *Suppose the regulator sets retail prices for the fixed-line network at the cost level ($P = C^O + \tau + a$) and mobile users have positive receiving-utility ($\beta > 0$). Each mobile network will set its access charge above the termination cost ($a^* > c^t$) if $\beta < 1$. When $\beta = 1$, they set access charges at the cost level, $a^* = c^t$.*

Proof. Suppose each mobile network sets its access charge at the termination cost level ($a = c^t$). From equation (1.48), when $a = c^t$,

$$Q_i(g(C^O + \tau + a)) + \left(\frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right) \frac{\partial Q_i(g(C^O + \tau + a))}{\partial a} = 0$$

where $\frac{\partial \tilde{u}(Q_i)}{\partial Q_i} = \beta \frac{\partial U(Q_i)}{\partial Q_i}$. Then one can say that for mobile firm to set $a = c^t$,

$$\beta = -\frac{Q_i}{\frac{\partial U(Q_i)}{\partial Q_i} \frac{\partial Q_i}{\partial P_i} \frac{\partial g}{\partial a}}.$$

[21]See Wright (2002*a*, pp. 306-307).

Let's call this level of β "β^o." At equilibrium, $\frac{\partial U(Q_i)}{\partial Q_i}\frac{\partial Q_i}{\partial P_i} = \frac{\partial V(P_i)}{\partial P_i} = -Q_i$ by the property of indirect utility from quasi-linear utility. Then if $g(C^O + \tau + a) = C^O + \tau + a$,

$$\beta^o = \frac{1}{\frac{\partial g}{\partial a}} = 1$$

From Lemma 4, a is decreasing function of β. When $\beta < 1$, $a > c^t$. □

According to Proposition 1, when the receiver's utility is smaller than the caller's utility, $\beta < 1$, mobile networks can still set above the "technical cost" (c^t) and have a positive markup on the access market. When the receiving-utility of mobile users is the same as the calling-utility of the caller in the fixed-line network, mobile networks have to set access charges at the technical cost level, $a = c^t$.[22] See Figure 1.3.

1.4.3 Symmetric and Unique Market Equilibrium

Since access charges are determined symmetrically in equilibrium, all other equilibrium prices become symmetric. First, equilibrium fixed-to-mobile retail prices are symmetric.

$$P_i^* = P_j^* = g(C^O + \tau + a^*) = P^* \tag{1.51}$$

The fixed-fee of the fixed-line network is set at the cost level, $F^* = C^T$. From equation (1.45), the equilibrium market share of the mobile market becomes symmetric.

$$s_i^* = s_j^* = \frac{1}{2} = s^* \tag{1.52}$$

[22]If β is big enough such as ($\beta > 1$), the mobile networks may set below-cost access charges to maximize their market share. But these high β cases are considered unusual.

Equilibrium prices for mobile-to-fixed calls are set at cost level and are symmetric from equation (1.40).

$$p_i^* = p_j^* = c^O + \tau + C^T = p^* \tag{1.53}$$

Let's look at the equilibrium fixed-fees of mobile firms, which are derived from equations (1.41) and (1.52).

$$f_i^* = f_j^* = c^f + t - (a^* - c^t)Q(g(C^O + \tau + a^*)) = f^* \tag{1.54}$$

Even though mobile firms have an incentive to attract subscribers by lowering fixed-fees, which are subsidized by positive access profit, an escalation of access charges does not occur.[23] Mobile firms set their access charges below the traditional monopoly level.

Proposition 2. ***(the existence of a unique and symmetric equilibrium)*** *Assume $Q(\cdot)$ and $g(\cdot)$ are (almost) linear and $\beta \leq 1$. There exists a unique and symmetric equilibrium ($p_i^* = p_j^* = p^*, f_i^* = f_j^* = f^*, a_i^* = a_j^* = a^*$) which is characterized by these conditions,*

i) $p^* = c^o + \tau + C^T$.
ii) $f^* = c^f + t - (a^* - c^t)Q(g(C^O + \tau + a^*))$.
iii) $Q(g(C^O + \tau + a^*)) + \left(a^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right) \frac{\partial Q(g(C^O + \tau + a^*))}{\partial a_i} = 0$.

Proof. By Lemmas 1, 2 and 3, there exists a unique and symmetric equilibrium which satisfies conditions i), ii) and iii). □

Because equilibrium mobile retail prices are symmetric, mobile-to-fixed call volumes are symmetric.

$$q_i^* = q_j^* = q(p^*) = q(c^o + \tau + C^T) = q^*$$

[23]See Wright (2002*a*) for the case of escalated access charges.

From equation (1.43) and symmetric market shares, the equilibrium profits of mobile firms are symmetrically determined.

$$\pi_i^* = \pi_j^* = \frac{t}{2} = \pi^* \tag{1.55}$$

The consumer surplus of a mobile user comes from (1.17) and (1.54).

$$\begin{aligned} w_i^* = w_j^* =& v(c^o + \tau + C^T) + \tilde{u}(Q(g(C^O + \tau + a^*))) - c^f - t \\ &+ (a^* - c^t)Q(g(C^O + \tau + a^*)) \\ =& w^* \end{aligned} \tag{1.56}$$

On the other hand, the monopoly fixed-line network is regulated. Since their fixed-fee and access charge are set at the cost level, their equilibrium profit is:

$$\Pi^* = (P^* - C^O - \tau - a^*)Q(P^*) \tag{1.57}$$

since $P_i^* = P_j^* = P^*$. Since the usage price of fixed-to-mobile calls decreases due to low mobile access charges, the level of fixed-to-mobile call volume is greater than it would be without including the receiving-utility of mobile users.

$$Q_i^* = Q_j^* = Q(g(C^O + \tau + a^*)) = Q^* \tag{1.58}$$

A fixed-line network user has the following consumer surplus:

$$W^* = V(g(C^O + \tau + a^*)) + \tilde{U}\big(q(c^o + \tau + C^T)\big) - C^F \tag{1.59}$$

which should be greater than the case that does not account for the receiving-utility of mobile users because the calling volume is greater.

1.5 Welfare Distribution

Before looking at the welfare transfer, it has to be noticed that as β and $\mathcal{B}$ increase, the total utility of each network user increases. As a result, the utility of each consumer can increase in spite of the welfare transfer. In this subsection, I focus on only the change in β, the receiving-utility of mobile users disregarding $\mathcal{B}$, receiving-utility of fixed-line users.

Table 1.1 compares retail tariffs for fixed-line users and mobile users. If $\beta = 0$, mobile networks set the monopoly access charges, a^m. In this case of no receiver's utility, fixed-line users have to pay high retail prices because of high mobile access charges, $P^* = g(C^O + \tau + a^m)$, and the positive access profit is used to subsidize the mobile users, to whom a low fixed-fee, $f^* = c^f + t - Q(P^*)(a^m - c^t)$ is offered. This means that the high access charges increase surplus transfer from fixed-line users to mobile users.

Table 1.1: Welfare Transfer between Fixed-line and Mobile Users

	$\beta = 0\ (a^* = a^m)$	$\beta = 1 (a^* = c^t)$
Fixed-line Users	$P^* = g(C^O + \tau + a^m)$ $F^* = C^F$	$P^* = g(C^O + \tau + c^t)$ $F^* = C^F$
Mobile Users	$p^* = c^o + \tau + C^T$ $f^* = c^f + t - Q(P^*)(a^m - c^t)$	$p^* = c^o + \tau + C^T$ $f^* = c^f + t$

Now, as β increases, this situation changes. Mobile access charges decrease and fixed-line users do not need to pay high mobile access charges. Instead, mobile users are offered a higher fixed-fee. When the receiving-utility is not considered, welfare is transferred from fixed-line users to mobile users. On the other hand, once the receiving-utility is considered, the surplus transfer is reversed from mobile users to fixed-line users. This reversed transfer increases as the receiving-utility becomes larger. Finally, if $\beta = 1$, access charges are set at the cost level, $a^* = c^t$, and mobile access profits disappear, $Q(P^*)(a^* - c^t) = 0$. Fixed-line users pay calling-prices at the technical cost level, $P^* = g(C^O + \tau + c^t)$ and mobile users have to pay a cost level fixed-fee, $f^* = c^f + t$ without getting any subsidy from access profits.

Proposition 3. *Under the CPP regime, when the receiving-utility of mobile users increases (i.e. when β increases), welfare transfer from mobile users to fixed-line users is increased. This is a reversal of the welfare transfer which increases when the receiving-utility of mobile users is not considered.*

Proof. Immediately derived from Lemmas 4 and 5. □

1.6 The Inefficiency of Market Equilibrium

Suppose the regulator sets fixed-to-mobile prices for the fixed-line network at cost level:

$$g(C^O + \tau + a) = C^O + \tau + a. \tag{1.60}$$

Table 1.2 compares the market equilibrium and efficient allocation when β and $\mathcal{B}$ are 0 or 1. First, let's examine the case when $\beta = \mathcal{B} = 0$, which means there is no receiver's utility. In this case, at equilibrium, fixed-to-mobile call volume, Q, is inefficient, but mobile-to-fixed call volume, q, is optimally determined. However, if the regulator does not know the costs of the fixed-line network, efficient mobile-to-fixed call volume cannot be achieved either.

On the other hand, when receiver's utility is equal to caller's utility, $\beta = \mathcal{B} = 1$, both volume, Q and q are not efficient. For any value of receiver's utility, CPP cannot achieve efficient allocation in this market.

Without receiver's utility there do not exist any call externalities. The distortion comes only from the monopoly mobile access market. However,

Table 1.2: Access Market Failure

	$\beta = \mathcal{B} = 0$	$\beta = \mathcal{B} = 1$
Market Equilibrium	$P^* = U'(Q(P^*)) = C^O + \tau + a^m$ $p^* = u'(q(p^*)) = c^o + \tau + C^T$	$P^* = U'(Q(P^*)) = C^O + \tau + c^t$ $p^* = u'(q(p^*)) = c^o + \tau + C^T$
Efficient Allocation	$U'(\hat{Q}) = \frac{C^O+\tau+c^t}{1+\beta} = C^O + \tau + c^t$ $u'(\hat{q}) = \frac{c^o+\tau+C^T}{1+\mathcal{B}} = c^o + \tau + C^T$	$U'(\hat{Q}) = \frac{C^O+\tau+c^t}{1+\beta} = \frac{C^O+\tau+c^t}{2}$ $u'(\hat{q}) = \frac{c^o+\tau+C^T}{1+\mathcal{B}} = \frac{c^o+\tau+C^T}{2}$

as receiver's utility increases, call externalities in fixed-to-mobile calls increase but they are internalized by the mobile networks, which consider the receiving-utility of their own subscribers. So, the distortion in the mobile access market continues to come from the monopoly in the market. See Figure 1.3. When $\beta = 1$, the mobile access charge is determined at $a^* = c^t$, which is much higher than the efficient level $\hat{a} = c^t - \frac{\partial U(Q)}{\partial Q}$.

On the other hand, one more distortion exists in the fixed-line access market. I assume that the fixed-line network is regulated and its access charge is also regulated at the termination cost level, C^T. But to implement the efficient volume of mobile-to-fixed call, q, it is necessary that

$$\hat{A} = C^T - \mathcal{B}u'(\hat{q}).$$

The larger the receiver's utility, the larger the regulatory distortion in mobile-to-fixed call volume. In the case that the receiver's utility is not known to the regulator, it's nearly impossible to correct this regulatory inefficiency.

Because the CPP regime creates inefficiencies in both directions of the access markets, new payment structures for the telecommunications market have been considered to remove these access market distortions and incorporate the call externalities.[24] These proposals recommend removing access

[24]See DeGraba (2000) and Atkinson and Barnekov (2000).

markets and instead, introducing prices for incoming calls.

1.7 Conclusion

Previous literature has shown that high mobile access charges increase the welfare transfer from fixed-line users to mobile users when not considering the receiver's utility. This research focuses on the abnormal mobile-access charge, which the mobile networks receive from the fixed-line network. Mobile networks attract their subscribers by subsidizing costs using the access profits.

If, under the CPP regime, the receiver's utility is taken into account, it decreases the incentive for mobile networks to charge monopoly access charges. When mobile networks take into account the receiving-utility of their own subscribers, they can no longer set monopoly level access charges because the mobile networks must take into account the negative effect of high access charge on their subscribers' receiving-utility. This receiving-utility affects their market share.

In this study, I introduce the receiver's utility into one of the traditional network interconnection models, which has a monopoly fixed-line network and two competitive mobile networks. My focus is on the welfare distribution between fixed-line and mobile users.

I found that, as the receiver's utility increases, the welfare transfer from fixed-line to mobile users is reversed, and finally when the receiver's utility is the same as the caller's utility, the surplus of fixed-line users is no longer exploited by the mobile networks. Instead, the mobile networks must take surplus of mobile users through higher fixed-fee to recover their reduced profit. This market result seems to be efficient for fixed-line users but it's not socially optimal because mobile access charges are still higher than the perceived marginal cost, which incorporates the receiving-utility of mobile users. On the other hand, insofar as the regulator does not know the receiving-utility of fixed-line users, the regulatory inefficiency of the fixed-line access market is unavoidable.

Whether the receiver's utility is high or low, access markets under the CPP regime continue to be inefficient. Because of this, some experts have proposed new payment structures for telecommunications market to incorporate the call externalities and remove access market distortions. I expect that future research will explore the pros and cons of these new payment regimes.

Chapter 2

The Bill-and-Keep Regime in the Fixed-line and Mobile Network Interconnection

2.1 Introduction

In many countries in Asia and Europe, the payment structure of the telecommunications market is based on the Calling Party Pays Principle (CPP), which means that only callers pay for the calls. This principle assumes that a call gives utility to the calling party but not to the called party so that receivers do not pay at all. Since receivers actually enjoy the call, the CPP regime leads to call externalities, which create inefficiencies in the volume of calls and the usage of telecommunications facilities.

When calls are made between two subscribers of different telecommunications networks, the calling network pays an “access charge” to the receiving network. This “access market” is in effect a monopoly, because the caller wants to make a call to a recipient in a different network, but the receiving network is the only one that can deliver the call to the designated receiver.[1] Because of this monopoly property, many countries regulate the access markets between telecommunications networks. However, the European fixed-to-mobile access market is exceptional. In Europe, mobile telecommunications markets

[1]The activity of a receiving network delivering a call to its subscriber is called “call termination.” Therefore, access charges are compensation for the call termination.

have been considered to be competitive and mobile telecommunications companies have been treated differently than fixed-line networks, such as local call companies. In Europe, fixed-to-mobile access charges were not regulated even though mobile-to-fixed access charges were controlled.[2]

Wright (2002*a*) and Armstrong (2002*a*) examine the European access market between fixed-line and mobile networks under the CPP regime. They find that even if competition for subscribers in the mobile market is intense and mobile networks do not have super-normal profits, the monopoly profits from call termination and the consequent deadweight loss persist and are used to finance subsidized retail tariffs to attract subscribers.

Under the CPP regime, it's hard to correct this access market distortion. Wright (2002*a*) shows that if the networks consider the receiving-utility of their own subscribers, the access charges decrease from the monopoly level. However, in the previous chapter I show that this access market distortion does not disappear regardless of the size of the receiving-utility under the CPP. When the receiving-utility is small, the fixed-to-mobile access charges are close to monopoly level. On the other hand, when the receiving-utility is large, call externalities are very large and the access charge problem is reduced.

Recently several researchers have studied other payment structures. Jeon et al. (2004) introduce the Receiver Pays Principle (RPP) into the typical network competition model of Laffont et al. (1998*a*,*b*). This RPP regime introduces receiver's prices to incorporate the call externalities. However, the regime still retains positive access charges. Assuming that access charges are set by regulation or some arrangement between the networks, Jeon, Laffont

[2]Recently, this status has changed because the British telecommunications regulator, Oftel, decided to regulate the fixed-to-mobile access charges in U.K. See Oftel (2003).

and Tirole (2004) show that there exists an equilibrium regardless of whether the receiver's prices are regulated or determined by the market. This paper does not study the fixed-to-mobile access market directly but instead studies the interconnection between two symmetric networks.

DeGraba (2000), in the so-called 'COBAK paper', suggests a new regime to replace the CPP. According to his paper, the Central Office Bill and Keep (COBAK) regime is a unified approach to interconnection pricing. It can be applied to all types of carriers interconnecting with the local circuit-switched network and to all types of traffic passing over to the network. The COBAK proposal consists of two rules: first, the called party's carrier cannot charge the interconnecting carrier to terminate the call. Thus each carrier recovers the cost of the loop and local switch from its own end-users. Second, the calling party's network is responsible for a transport cost – the cost of transporting a call between the calling party's and the called party's central offices. DeGraba insists that the COBAK regime solves or reduces many of the significant problems that plague the existing interconnection regimes, including terminating access monopoly, inefficient end-user prices, and the regulatory arbitrage opportunity.[3]

The COBAK proposal is modified in DeGraba (2003), in which, he suggests a 'Meet-Point Bill and Keep (MBAK).' Unlike the COBAK, the MBAK focuses on symmetric cases and recommends that both networks share transport costs equally. DeGraba (2003) shows that if the caller and the receiver share the value of a minute of call, they can consume the optimal volume when

[3]A typical example of a regulatory arbitrage opportunity is the "ISP(Internet Service Provider) reciprocal compensation" problem. ISPs generally have incoming calls. If a network has an ISP as its customer, the network can get a substantial amount of access revenue under the reciprocal compensation scheme for access charges.

they jointly pay a per minute price equal to the marginal cost of that minute, and each customer pays a proportion of this price equal to the proportion of the value he receives from that minute.[4] A symmetric case is an obvious example. If both parties to a call share the value of a call equally and competing networks have access to the same technology, MBAK maximizes social surplus by charging the cost proportional to the benefit of each party.[5]

Under both of these Bill-and-Keep regimes, the involved networks negotiate and arrange a meet-point between two central offices and they exchange traffic at this point on a Bill-and-Keep basis. If both networks cannot agree on a meet-point negotiation, as a default rule one of the two Bill-and-Keep regimes is applied. Under COBAK, the meet-point should be "the central office of the receiving network," but under MBAK, it should be the "midpoint" of the trunk line between the two central offices.

Because the Bill-and-Keep system requires the involved networks to recover the cost of a call from their own subscriber, it removes the call externalities and problematic access charges by making subscribers pay for receiving as well as making calls. However, neither of these two papers addresses the network competition issue. DeGraba (2000) only gives a description of the COBAK regime and does not provide any specific model. To show the efficiency of the Bill-and-Keep regime, DeGraba (2003) uses a simple interconnection model between two symmetric networks in which carriers set their prices at cost level and the receiver and the caller share the given value of a

[4]This first best outcome occurs when the proportion of call values between both parties to a call is equal to the proportion of marginal costs between both networks. See equation (2.73) at page 53.

[5]When both networks have the same technology, if both networks share the transport cost evenly and each network pays the cost within its own network, the first best outcome occurs.

call. But his paper does not use a typical network competition model such as the one used by Laffont, Rey and Tirole (1998a,b).

In this study, I examine a new setup which combines the Bill-and-Keep idea with a traditional network competition model. I create a traditional interconnection model between two competitive mobile networks and a regulated monopoly fixed-line network. I introduce to the model the Bill-and-Keep regime instead of the CPP regime as a possible way of resolving the above-cost access charges and call externalities. There is no access market, so networks do not need to pay access charges. All mobile firms can set the retail tariffs freely but prices under the fixed-line network are regulated. The Bill-and-Keep requires both the receiving network and the calling network to share call cost, including all transport costs. Because each carrier has to recover the cost of its loop and local switch and part of the transport cost from its own end-users, I allow networks to charge prices for incoming as well as outgoing calls. These prices are the "receiver's price" and the "caller's price." As a result, both parties to a call can decide the call volume.

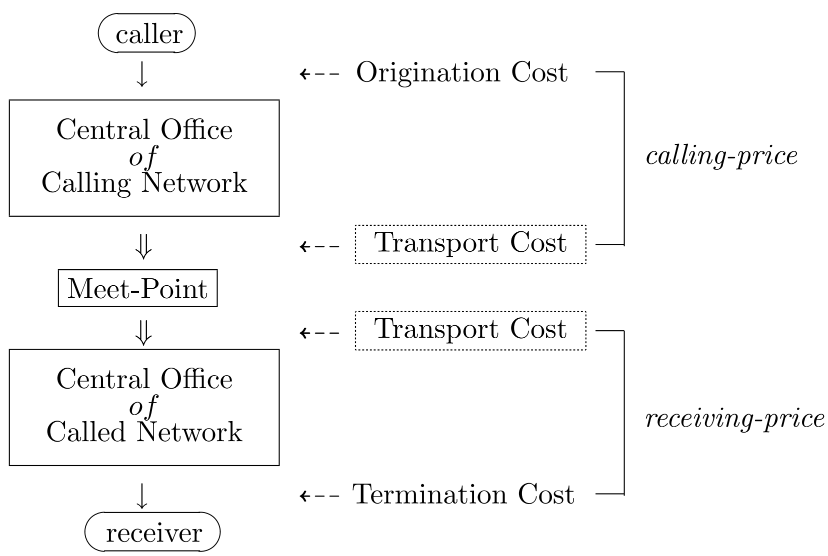
I show that given a meet-point, a market equilibrium exists. Furthermore, I explain that to implement a socially optimal equilibrium, proper meet-points corresponding to the receiver's utility are necessary. In terms of surplus, consumers are the only beneficiaries of the Bill-and-Keep, and the telecommunications networks are not better off in this model.

2.2 The Model

Assume that the telecommunications market has only two kinds of networks: fixed-line and mobile. The fixed-line network is a monopoly and the mobile market has two symmetric and competitive firms. On-net calls for each network and mobile-to-mobile calls are ignored to allow a focus on the interaction between the fixed-line network and the mobile networks.

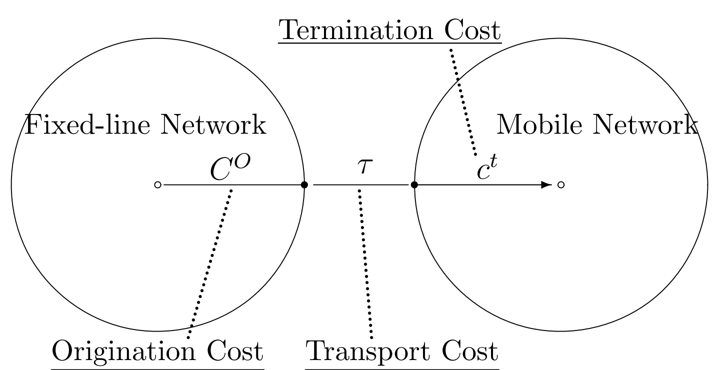
The payment system of the telecommunications market is the Bill-and-Keep system. Assuming that both caller and receiver benefit from the calls, they both have to bear the costs raised in the network to which they subscribe. As a result, it's reasonable to assume that either the receiver or the caller can hang up first. Since both the caller and the receiver pay for a call, the call externalities do not arise. This pricing system weakens the appropriateness of access charges. If the receiving network recovers their cost from their own subscribers, it's hard to accept the access charge paid by the calling network because that is usually compensation for using the receiving network. Therefore, I assume that networks do not pay any access charges.

Figure 2.1: Bill-and-Keep Regime



Let's examine the cost structure of this model. I assume that there aren't any fixed costs to reflect a joint and common cost of serving the various customers for each network. But mobile networks have a fixed cost c^f per subscriber, which reflects the monthly cost of connecting the customer to the network as well as billing and servicing the customer. C^F is a fixed cost per subscriber for the fixed-line network. The costs to complete calls are assumed to be composed of three parts: origination, termination and transport. Origination cost c^o is a per-minute cost generated in the mobile network when a subscriber makes a call. The termination cost c^t is a per-minute cost generated in the mobile network when one of its subscribers receives a call. The two mobile networks have symmetric cost structures. The fixed-line network has a different cost structure than the mobile networks. C^O and C^T are, respectively, the origination and termination costs for the fixed-line network. Transport cost τ is a per-minute cost between the two central office switches of the involved networks. Figure 2.2 shows the costs of a fixed-to-mobile call.

Figure 2.2: Costs of a Fixed-to-Mobile Call



Under the Bill-and-Keep regime, networks exchange traffic on a 'Bill-and-Keep' basis at an arranged meet-point of the trunk line connecting the networks, which means that each network pays part of the cost of operating a single trunk line between the networks. As a result, setting a meet-point can be understood as a cost-allocation between the related networks to maintain a trunk line. I assume that the meet-point between the two central offices is arranged by a regulator or by both networks before the networks choose retail prices, and this does not need to follow the COBAK or MBAK system. I assume the total transport cost is τ between the fixed-line network and either mobile network. It is symmetric for both directions of a call. Mobile network i pays $\theta_i\tau$ of the transport cost when their subscribers make a call to the fixed-line network, and the fixed-line network pays $\Theta_i\tau$ when their subscribers make a call to mobile network i. The receiving network pays the remaining portion of τ. When meet-points are arranged between two central offices in the real world, Θ_i and $\theta_i \in [0, 1]$. However, notice that theoretically Θ_i and θ_i can be negative or greater than 1, especially for optimality.

Given that each customer has the same marginal willingness to pay for making or receiving calls, all mobile networks can offer a uniform tariff for the overall use of its services. The fixed-fee F for the fixed-line network is regulated at the fixed cost level, C^F.

$$F \equiv C^F \tag{2.1}$$

The fixed-line network is regulated such that its retail prices are proportional to the costs, including the allocated transport costs. Suppose the fixed-line network offers its subscribers a per-minute usage charge P_i for making calls to mobile network i and R_i for receiving calls from mobile network i. These are described by functions of the total costs, given meet-points (Θ_i, θ_i).

$$P_i \equiv g(C^O + \Theta_i \tau) \tag{2.2}$$

$$R_i \equiv g(C^T + (1-\theta_i)\tau), \quad g' > 0 \tag{2.3}$$

This is obviously true in the case when two retail prices are regulated at the variable cost level. On the other hand, each mobile network charges its subscribers a monthly fixed-fee f_i and a per-minute charge p_i for making calls to the fixed-line network and r_i for receiving calls from the fixed-line network. Mobile networks choose their own tariffs non-cooperatively. Suppose that once a subscriber has joined a mobile network with usage charge (p_i, r_i), that subscriber wants to make $q(p_i)$ minutes of outbound calls to a fixed-line user and receive $Q(r_i)$ minutes of inbound calls from a fixed-line user for a billing period. Each fixed-line network subscriber wants to make $Q(P_i)$ minutes of outbound calls to a subscriber in mobile network i and wants to receive $q(R_i)$ minutes of inbound calls from a subscriber in mobile network i.

A fixed set of homogeneous customers subscribe to the telecommunications services. For simplification, I assume that the fixed-line network has N subscribers and mobile networks divide another N subscribers into two groups, N_1 and N_2. This N can be normalized to 1 without loss of generality and then N_1 and N_2 can be understood as market shares s_i and s_j. Notice that full market participation is assumed respectively in the fixed-line market and in the mobile market and, as a result, $s_i + s_j = 1$ always holds. In this framework each market has no network externalities.

Consumer preferences are known to firms. Mobile service and fixed-line service each provide different utilities. For example, mobile phone service gives subscribers mobility in addition to the usual utility of calls. I assume that the utility function of fixed-line network subscribers is different from that of mobile network subscribers. Subscribers of each network benefit from outgoing and incoming calls separably. For fixed-line network subscribers and mobile network subscribers, their concave utilities from making calls are:

$$U(Q_i) \text{ and } u(q_i), \quad i = 1, 2$$

where Q_i is the length of fixed-to-mobile calls and q_i is the length of mobile-to-fixed calls between each pair of caller and receiver. Since the receivers have to pay for incoming calls, it is reasonable to assume that the receivers can hang up whenever they want. Both the caller and the receiver have positive probability of hanging up first – so called "caller's sovereignty" and "receiver's sovereignty." As a result, the volume of calls is non-cooperatively decided by callers and receivers. To describe this situation, assume that the marginal utility of receiving a call is subject to noise, expressed by a random variable ϵ.[6] For $i = 1, 2$,

$$\tilde{U}(q_i) + \epsilon q_i \tag{2.4}$$

$$\tilde{u}(Q_i) + \epsilon Q_i \tag{2.5}$$

where ϵ follows the distribution function $\Psi(\cdot)$ with wide enough support $[\underline{\epsilon}, \bar{\epsilon}]$, zero mean and density $\psi(\cdot)$, which is strictly positive for all ϵ in $[\underline{\epsilon}, \bar{\epsilon}]$. Furthermore, I assume that the noise ϵ is identically and independently distributed for each caller-receiver pair. For technical simplicity only, assume that for $i = 1, 2$,

$$\tilde{U}(q_i) = \mathcal{B} \cdot u(q_i)\,, \qquad \mathcal{B} > 0 \tag{2.6}$$

$$\tilde{u}(Q_i) = \beta \cdot U(Q_i)\,, \qquad \beta > 0. \tag{2.7}$$

[6]See Jeon et al. (2004).

$\mathcal{B}$ is an expected ratio of the receiver's utility to caller's utility for a mobile-to-fixed call. Analogously, β is this ratio for a fixed-to-mobile call. Given (P_i, R_i, p_i, r_i) and a realized value of ϵ, unless the receiver hangs up the call first, the caller will continue the call until her marginal utility is equal to the calling price. For $i = 1, 2$,

$$U'(Q_i) = P_i \tag{2.8}$$

$$u'(q_i) = p_i. \tag{2.9}$$

Similarly, unless the caller hangs up the call first, the receiver with noise ϵ will continue the call until her marginal utility is equal to the receiving-price. For $i = 1, 2$,

$$\tilde{U}'(q_i) + \epsilon = R_i \tag{2.10}$$

$$\tilde{u}'(Q_i) + \epsilon = r_i. \tag{2.11}$$

Suppose a subscriber of the fixed-line network calls a subscriber of mobile network i. Given the caller's price P_i, the receiver's price r_i and a realized value of ϵ, if the caller does not hang up the call first, P_i is not greater than her marginal utility U'. In this case, the receiver with noise ϵ will continue the call until her marginal utility is equal to r_i. Namely, the receiver consumes the volume of calls which satisfies above equation (2.11). Let's say this volume quantity Q_i^R. With the volume of calls Q_i^R, the caller has marginal utility of $U'(Q_i^R) = \frac{r_i - \epsilon}{\beta}$ from (2.7).

$$P_i \leq U'(Q_i^R) = \frac{r_i - \epsilon}{\beta} \tag{2.12}$$

Since the P_i is less than or equal to $\frac{r_i - \epsilon}{\beta}$ (low ϵ), the receiver determines the volume of calls, and the consumed volume is $Q_i^R = Q_i(\frac{r_i - \epsilon}{\beta})$, which is the receiver's demand function. On the other hand, if the caller's price P_i is greater than this marginal utility level $U'(Q_i^R) = \frac{r_i - \epsilon}{\beta}$ (high ϵ), the caller determines the volume of calls by (2.8) and the consumed volume of calls is $Q_i^C = Q_i(P_i)$, which is the caller's demand function.

$$\frac{r_i - \epsilon}{\beta} \leq U'(Q_i^C) = P_i \tag{2.13}$$

Therefore, the volume of calls from the fixed-line network to mobile network i is determined by the following equation. For $i = 1, 2$,

$$D(P_i, r_i) = \min\left\{Q_i^C, Q^R\right\} = \min\left\{U'^{-1}(P_i), U'^{-1}(\frac{r_i - \epsilon}{\beta})\right\} \tag{2.14}$$

or

$$D(P_i, r_i) = Q_i(\max\{P_i, \frac{r_i - \epsilon}{\beta}\}). \tag{2.15}$$

A larger ϵ increases the possibility of the caller's sovereignty with realized volume $Q_i(P_i)$, and a smaller ϵ increases the possibility of the receiver's sovereignty with realized volume $Q_i(\frac{r_i - \epsilon}{\beta})$. Similarly, the volume of opposite-direction calls – calls from mobile network i to the fixed-line network – is for $i = 1, 2$,

$$d(p_i, R_i) = q_i(\max\{p_i, \frac{R_i - \epsilon}{\mathcal{B}}\}). \tag{2.16}$$

The above two volumes can be expressed as demand functions:

$$\begin{aligned} D(P_i, r_i) \equiv & \{1 - \Psi(r_i - \beta P_i)\} Q_i(P_i) \\ & + \int_{\underline{\epsilon}}^{r_i - \beta P_i} Q_i(\frac{r_i - \epsilon}{\beta})\psi(\epsilon)d\epsilon \end{aligned} \tag{2.17}$$

$$\begin{aligned} d(p_i, R_i) \equiv & \{1 - \Psi(R_i - \mathcal{B} p_i)\} q_i(p_i) \\ & + \int_{\underline{\epsilon}}^{R_i - \mathcal{B} p_i} q_i(\frac{R_i - \epsilon}{\mathcal{B}})\psi(\epsilon)d\epsilon \end{aligned} \tag{2.18}$$

In equation (2.17), the first term on the right hand side represents the expected demand for fixed-to-mobile calls for large values of ϵ, such that the caller on the fixed-line network is the one that terminates the call. The second term is the expected fixed-to-mobile demand for small values of ϵ, where the receiver on mobile network i hangs up first. The first term on the right hand side of equation (2.18) represents the expected demand for mobile-to-fixed calls for high values of ϵ, where the caller on mobile network i terminates. The second term is expected demand over the range of ϵ where the receiver on the fixed-line network terminates the call.

Similarly the utilities of the fixed-line network subscriber making calls to mobile network i and mobile network i's subscriber making calls to the fixed-line network are, respectively, given by:

$$\begin{aligned}\Omega(P_i, r_i) \equiv & \{1 - \Psi(r_i - \beta P_i)\} U\left(Q_i\left(P_i\right)\right) \\ & + \int_{\underline{\epsilon}}^{r_i - \beta P_i} U\left(Q_i(\frac{r_i - \epsilon}{\beta})\right) \psi(\epsilon) d\epsilon, \quad i = 1, 2 \end{aligned} \qquad (2.19)$$

$$\begin{aligned}\omega(p_i, R_i) \equiv & \{1 - \Psi(R_i - \mathcal{B} p_i)\} u\left(q_i\left(p_i\right)\right) \\ & + \int_{\underline{\epsilon}}^{R_i - \mathcal{B} p_i} u\left(q_i(\frac{R_i - \epsilon}{\mathcal{B}})\right) \psi(\epsilon) d\epsilon, \quad i = 1, 2. \end{aligned} \qquad (2.20)$$

I now define the receiver's utility before payment. The utilities of the fixed-line network subscriber receiving calls from mobile network i and mobile network i's subscriber receiving calls from the fixed-line network are given, respectively, by equations (2.21) and (2.22). For $i = 1, 2$,

$$\begin{aligned}\tilde{\Omega}(p_i, R_i) \equiv & \int_{R_i - \mathcal{B} p_i}^{\bar{\epsilon}} \left\{ \tilde{U}\left(q_i\left(p_i\right)\right) + \epsilon q_i\left(p_i\right) \right\} \psi(\epsilon) d\epsilon \\ & + \int_{\underline{\epsilon}}^{R_i - \mathcal{B} p_i} \left\{ \tilde{U}\left(q_i(\frac{R_i - \epsilon}{\mathcal{B}})\right) + \epsilon q_i(\frac{R_i - \epsilon}{\mathcal{B}}) \right\} \psi(\epsilon) d\epsilon \end{aligned} \qquad (2.21)$$

$$\tilde{\omega}(P_i, r_i) \equiv \int_{r_i - \beta P_i}^{\bar{\epsilon}} \Big\{ \tilde{u}\left(Q_i\left(P_i\right)\right) + \epsilon Q_i\left(P_i\right) \Big\} \psi(\epsilon) d\epsilon$$
$$+ \int_{\underline{\epsilon}}^{r_i - \beta P_i} \left\{ \tilde{u}\Big(Q_i(\frac{r_i - \epsilon}{\beta}) \Big) + \epsilon Q_i(\frac{r_i - \epsilon}{\beta}) \right\} \psi(\epsilon) d\epsilon \qquad (2.22)$$

Assume that all utilities are separably additive. Ignoring fixed utilities, the variable surplus of a fixed-line user is:

$$W \equiv \sum_{i=1}^{2} s_i \Big\{ \Omega(P_i, r_i) - P_i \cdot D(P_i, r_i) \Big\}$$
$$+ \sum_{i=1}^{2} s_i \left\{ \tilde{\Omega}(p_i, R_i) - R_i \cdot d(p_i, R_i) \right\} - F \qquad (2.23)$$

The mobile market share, s_i, is determined by network competition for subscribers, which can be explained by the Hotelling product differentiation model. Assume that mobile network users are endowed with a value of x, which is drawn from a uniform distribution on the interval $[0, 1]$. Two mobile networks are located at each end point of this unit interval, respectively. A mobile user with value x has extra disutility from not being able to consume his preferred services:

$$tx^2 \quad \text{and} \quad t(1-x)^2 \qquad (2.24)$$

for network 1 and 2 respectively. t is the parameter that measures how differentiated the networks are. The greater t is, the greater are the switching costs, and the harder it is for one network to steal away customers from the rival network by lowering its price. The utility derived by a consumer with income y located at x from joining either mobile network i or j is, respectively, given by:

$$y + v_0 - tx^2 + \omega(p_i, R_i) - p_i d(p_i, R_i) + \tilde{\omega}(P_i, r_i) - r_i D_i(P_i, r_i) - f_i \tag{2.25}$$

$$y + v_0 - t(1-x)^2 + \omega(p_j, R_j) - p_j d(p_j, R_j) + \tilde{\omega}(P_j, r_j) - r_j D(P_j, r_j) - f_j \tag{2.26}$$

v_0 is the fixed utility of a mobile network user from subscribing to a mobile network. This constant ensures that all consumers will always choose to join one of the two mobile networks if it's value is high enough. Disregarding fixed utilities, let's define net variable consumer surplus w_i as:

$$\begin{aligned} w_i \equiv & \omega(p_i, R_i) - p_i \cdot d(p_i, R_i) \\ & + \tilde{\omega}(P_i, r_i) - r_i \cdot D(P_i, r_i) - f_i, \quad i = 1, 2. \end{aligned} \tag{2.27}$$

A customer located as x is indifferent between the two mobile networks if

$$w_i - tx^2 = w_j - t(1-x)^2. \tag{2.28}$$

Solving for x, the mobile market shares are, for $i, j = 1, 2$ and $i \neq j$,

$$s_i = \frac{1}{2} + \frac{w_i - w_j}{2t} \quad i = 1, 2 \tag{2.29}$$

The profit functions of the mobile networks and the fixed-line network are:

$$\begin{aligned} \pi_i \equiv & s_i \left(p_i - c^o - \theta_i \tau\right) \cdot d(p_i, R_i) + s_i \left(r_i - c^t - (1 - \Theta_i)\tau\right) \cdot D(P_i, r_i) \\ & + s_i f_i(p_i, r_i) - s_i c^f, \quad i = 1, 2 \end{aligned} \tag{2.30}$$

$$\begin{aligned} \Pi \equiv & \sum_{i=1}^{2} s_i \left\{ \left(P_i - C^O - \Theta_i \tau\right) \cdot D(P_i, r_i) \right. \\ & \left. + \left(R_i - C^T - (1 - \theta_i)\tau\right) \cdot d(p_i, R_i) \right\} \end{aligned} \tag{2.31}$$

2.3 The Benchmark: Efficient Allocation

For future reference, I derive the social optimum. Consider an idealized situation in which a benevolent regulator chooses the market shares and the volume of calls. In this symmetric setup, equal market division minimizes the average consumer's disutility from not being able to consume his preferred service.

Let's define $T(s_i)$ as the average consumer's disutility from not being able to consume her preferred service.[7] Using disutilities (2.24), for arbitrary s_i, this measure of the distance between preferred and actual brand choice is:

$$\begin{aligned} T(s_i) &\equiv t\left[s_i\left(\frac{s_i}{2}\right)^2+(1-s_i)\left(\frac{1-s_i}{2}\right)^2\right] \\ &= t\left[\frac{s_i^3+(1-s_i)^3}{4}\right] \end{aligned} \tag{2.32}$$

$T(s_i)$ is minimized at $s_i=\frac{1}{2}$, which means symmetric market share.

$$\begin{aligned} \frac{\partial T(s_i)}{\partial s_i} &= \frac{t}{4}\left[3s_i^2-3(1-s_i)^2\right] \\ &= \frac{t}{4}\left[-3+6s_i\right]=0 \end{aligned} \tag{2.33}$$

The benevolent regulator would choose the volume of calls so as to maximize

$$\max_{Q,q}\ E\{U(Q)+\tilde{U}(q)+\epsilon q+u(q)+\tilde{u}(Q)+\epsilon Q-(C^O+\tau+c^t)Q-(c^o+\tau+C^T)q\} \tag{2.34}$$

The optimal volume $\hat{Q}$ and $\hat{q}$ are determined by:

$$U'(\hat{Q})+\tilde{u}'(\hat{Q})=C^O+\tau+c^t \tag{2.35}$$

$$u'(\hat{q})+\tilde{U}'(\hat{q})=c^o+\tau+C^T \tag{2.36}$$

[7] See Laffont et al. (1998a)

The sum of the marginal utilities for the caller and receiver should be equal to the marginal cost of a call. Since I assume that $\tilde{u}'(\cdot) = \beta U'(\cdot)$ and $\tilde{U}'(\cdot) = \mathcal{B}u'(\cdot)$,

$$U'(\hat{Q}) = \frac{C^O + \tau + c^t}{1 + \beta} \tag{2.37}$$

$$u'(\hat{q}) = \frac{c^o + \tau + C^T}{1 + \mathcal{B}}. \tag{2.38}$$

2.4 Market Equilibrium

Suppose that the meet-points are arranged symmetrically before the networks compete in their retail market. This symmetry is reasonable because arranged meet-points are highly likely to be detected by the third party, including a potential regulator.[8]

$$\theta_1 = \theta_2 = \theta \tag{2.39}$$

$$\Theta_1 = \Theta_2 = \Theta \tag{2.40}$$

In this case, the fixed-line monopoly network will choose its retail prices by the regulation schedule:

$$P_1 = P_2 = g\left(C^O + \Theta\tau\right) = P \tag{2.41}$$

$$R_2 = R_2 = g\left(C^T + (1 - \theta)\tau\right) = R \tag{2.42}$$

The profit function of a mobile network can be expressed using the definition of net variable consumer surplus, w_i from equation (2.27). Instead

[8] See Atkinson and Barnekov (2001).

of fixed-fee f_i, the mobile firm controls consumer surplus w_i.[9]

$$\pi_i \equiv \left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) \Big\{ (p_i - c^o - \theta\tau) \cdot d(p_i, R) \\ + \big(r_i - c^t - (1-\Theta)\tau\big) \cdot D(P, r_i) \\ + \omega(p_i, R) - p_i \cdot d(p_i, R) + \tilde{\omega}(P, r_i) - r_i \cdot D(P, r_i) \\ - w_i - c^f \Big\}, \quad i = 1, 2 \tag{2.43}$$

The first order conditions (FOCs) are as below. The FOC with respect to f_i is replaced by the FOC with respect to w_i. For $i = 1, 2$,

$$\frac{\partial \pi_i}{\partial p_i} = \left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) \left\{ (-c^o - \theta\tau) \cdot \frac{\partial d(p_i, R)}{\partial p_i} + \frac{\partial \omega(p_i, R)}{\partial p_i} \right\} \\ = \left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) (u'(\cdot) - c^o - \theta\tau) \left\{1 - \Psi(R - \mathcal{B}p_i)\right\} q_i^{'}(p_i) = 0 \tag{2.44}$$

$$\frac{\partial \pi_i}{\partial r_i} = \left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) \left\{ \big(-c^t - (1-\Theta)\tau\big) \cdot \frac{\partial D(P, r_i)}{\partial r_i} + \frac{\partial \tilde{\omega}(P, r_i)}{\partial r_i} \right\} \\ = \frac{1}{\beta}\left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) \cdot \\ \int_{\underline{\epsilon}}^{r_i - \beta P} \left\{ \tilde{u}'(\cdot) + \epsilon - c^t - (1-\Theta)\tau \right\} Q_i'(\frac{r_i - \epsilon}{\beta}) \psi(\epsilon) d\epsilon = 0 \tag{2.45}$$

$$\frac{\partial \pi_i}{\partial w_i} = \frac{1}{2t}\Big\{ (-c^o - \theta\tau) \cdot d(p_i, R) + \big(-c^t - (1-\Theta)\tau\big) \cdot D(P, r_i) \\ + \omega(p_i, R) + \tilde{\omega}(P, r_i) - w_i - c^f \Big\} - \left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) = 0. \tag{2.46}$$

Lemma 6. ***(No Cornered Solution)*** *$\forall i$, $s_i \neq 0$, given $(\theta_1 = \theta_2, \Theta_1 = \Theta_2)$: In the mobile market there does not exist a cornered-market equilibrium when meet-points are symmetric.*

Proof. See Appendix A. □

[9] See Laffont et al. (1998a).

Because there is no cornered solution,

$$s_i = \frac{1}{2} + \frac{w_i - w_j}{2t} \neq 0, \quad i = 1, 2, \ \ i \neq j. \tag{2.47}$$

Because $p_i = u'(\cdot)$ (2.9) and $r_i = \tilde{u}'(\cdot) + \epsilon$ (2.11) in equilibrium, FOCs (2.44) and (2.45) yield a unique symmetric candidate equilibrium:

$$p_1^* = p_2^* = c^o + \theta\tau = p^* \tag{2.48}$$

$$r_1^* = r_2^* = c^t + (1 - \Theta)\tau = r^* \tag{2.49}$$

If the disturbance term ϵ vanishes to zero this can be an equilibrium.[10] By inserting equations (2.48) and (2.49) into FOC (2.46),

$$\begin{aligned} \frac{\partial \pi_i}{\partial w_i} =& \frac{1}{2t}\Big\{\omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - w_i - c^f\Big\} \\ & - \left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right) \\ =& \frac{1}{2t}\left(f_i - c^f\right) - s_i = 0, \qquad i = 1, 2. \end{aligned} \tag{2.50}$$

Then, the equilibrium fixed-fee is:

$$f_i = c^f + 2t \cdot s_i, \quad i = 1, 2 \tag{2.51}$$

The equilibrium consumer surplus for mobile users is:

$$w_i = \omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - c^f - 2t \cdot s_i, \quad i = 1, 2. \tag{2.52}$$

From the above two results (2.51) and (2.52), equilibrium market shares satisfy:

$$s_i^* = \frac{1}{2} + \frac{w_i - w_j}{2t} = \frac{1}{2} - s_i^* + s_j^*, \quad i = 1, 2, \quad i \neq j$$

[10] See Jeon et al. (2004) for details.

Then, equilibrium market share is:

$$s_i^* = \frac{1}{2}, \quad i = 1, 2 \tag{2.53}$$

Now, let's go back to equilibrium fixed-fee and consumer surpluses :

$$f_1^* = f_2^* = c^f + t = f^* \tag{2.54}$$

$$w_1^* = w_2^* = \omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - f^* = w^* \tag{2.55}$$

For future reference, let's define social welfare as the sum of the profits and consumer surpluses of the fixed-line and mobile networks.

$$TS \equiv \sum_{i=1}^{2} \pi_i + \sum_{i=1}^{2} s_i w_i + \Pi + W \tag{2.56}$$

By putting all the solutions into the profit function (2.43), the equilibrium profit level of each mobile network is a "Hotelling profit" – the results of product differentiation.

$$\pi_i^* = \frac{t}{2}, \quad i = 1, 2$$

The equilibrium profit of the perfectly regulated fixed-line network is zero.

$$\Pi^* = 0$$

The mobile network consumer's net surplus is:

$$w^* = \omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - f^*$$

The fixed-line network consumer's net surplus in equilibrium is:

$$W^* = \Big\{\Omega(P, r^*) - P \cdot D(P, r^*)\Big\} + \Big\{\tilde{\Omega}(p^*, R) - R \cdot d(p^*, R)\Big\} - F$$

Then, equilibrium social welfare is:

$$\begin{aligned}TS^* =&t + \Big\{\omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - f^*\Big\} \\ &+ 0 + \Big\{\Omega(P, r^*) + \tilde{\Omega}(p^*, R) - P \cdot D(P, r^*) - R \cdot d(p^*, R) - F\Big\}\end{aligned} \tag{2.57}$$

$$\begin{array}{llll}\text{where} & P = g(C^O + \Theta\tau), & R = g(C^T + (1-\theta)\tau), & F = C^F \\ & p^* = c^o + \theta\tau, & r^* = c^t + (1-\Theta)\tau, & f^* = c^f + t.\end{array}$$

I will explain about social welfare in section 2.6.

Here, I summarize equilibrium prices with a proposition:

Proposition 4. ***(Characterization of Market Equilibrium)*** *There exists an equilibrium in which mobile networks set their receiver's and caller's prices at the cost level, given* $\theta_1 = \theta_2$, $\Theta_1 = \Theta_2$, $P_i = f(C^O + \Theta_i\tau)$, $R_i = f(C^T + (1-\theta_i)\tau)$ *and* $F = C^F$.

i) $p_i^* = c^o + \theta\tau = p_j^*$
ii) $r_i^* = c^t + (1-\Theta)\tau = r_j^*$
iii) $f_i^* = c^f + t = f_j^*$

However, the above market equilibrium does not guarantee an efficient allocation in the presence of non-vanishing noise. For example, in the caller-determined-volume region for mobile-to-fixed calls,

$$u'(q) = p = c^o + \theta\tau \tag{2.58}$$

$$\tilde{U}'(q) + \epsilon \geq R = C^T + (1-\theta)\tau. \tag{2.59}$$

On the other hand, in the receiver-determined-volume region,

$$u'(q) \geq p = c^o + \theta\tau \tag{2.60}$$

$$\tilde{U}'(q) + \epsilon = R = C^T + (1-\theta)\tau. \tag{2.61}$$

In any case, the sum of the marginal utilities almost always exceed the marginal cost. Communications service is provided under the socially optimal level.

$$u'(q) + \tilde{U}'(q) + \epsilon > p + R = c^o + \tau + C^T \tag{2.62}$$

In the next section, I will explain how to implement an efficient allocation by choosing the proper meet-points corresponding to the receivers' utilities, assuming vanishing noise.

2.5 Optimal Meet-Points

In the previous section, mobile firms set their prices at cost level given symmetric meet-points, $\theta_i = \theta_j = \theta$ and $\Theta_i = \Theta_j = \Theta$ and information about the relative size of the receivers' utilities, β and $\mathcal{B}$. Finding the real values of β and $\mathcal{B}$ is an empirical problem and is beyond this paper's scope. Then, what are the socially optimal meet-points, θ and Θ, given β and $\mathcal{B}$?

From the efficient allocation conditions (2.37) and (2.38) in section 2.3,

$$U'(\hat{Q}) = \frac{C^O + \tau + c^t}{1+\beta} \tag{2.63}$$

$$\tilde{U}'(\hat{q}) = \frac{\mathcal{B}}{1+\mathcal{B}}(c^o + \tau + C^T) \tag{2.64}$$

Assuming vanishing noise, the fixed-line network is regulated to set retail prices by rules (2.2) and (2.3), given (Θ, θ). In equilibrium,

$$U'(Q) = P = g(C^O + \Theta\tau) \tag{2.65}$$

$$\tilde{U}'(q) = R = g(C^T + (1-\theta)\tau). \tag{2.66}$$

Suppose the regulator sets the retail prices of the fixed-line network at cost level.

$$g(C^O + \Theta\tau) = C^O + \Theta\tau \tag{2.67}$$

$$g(C^T + (1-\theta)\tau) = C^T + (1-\theta)\tau \tag{2.68}$$

By combining the above equations with efficient allocation conditions (2.63) and (2.64),

$$C^O + \Theta\tau = P = \frac{C^O + \tau + c^t}{(1+\beta)} \tag{2.69}$$

$$C^T + (1-\theta)\tau = R = \frac{\mathcal{B}}{1+\mathcal{B}}(c^o + \tau + C^T). \tag{2.70}$$

Then the optimal meet-points are:

$$\hat{\Theta} = \frac{c^t + \tau - \beta C^O}{(1+\beta)\tau} \tag{2.71}$$

$$\hat{\theta} = \frac{C^T + \tau - \mathcal{B}c^o}{(1+\mathcal{B})\tau} \tag{2.72}$$

Notice that these optimal meet-points can be greater than 1 when β or $\mathcal{B}$ is small enough. For example, examine equation (2.71). Since a small β indicates a small receiver's utility, optimality requires that the caller pay a part of termination costs (c^t) as well as the whole transport cost (τ).

I summarize the above results in this proposition:

Proposition 5. ***(Optimal Meet-Points and Efficient Allocation)*** *When the regulator requires marginal-cost pricing for the monopoly fixed-line network, one can derive optimal meet-points* $(\hat{\Theta}, \hat{\theta})$ *to maximize total surplus (social welfare) as the following:*

$$\hat{\Theta} = \frac{c^t + \tau - \beta C^O}{(1+\beta)\tau}$$

$$\hat{\theta} = \frac{C^T + \tau - \mathcal{B}c^o}{(1+\mathcal{B})\tau}$$

Remark 1. ***(Properties of the Optimal Meet-points)*** *From the optimal meet-points one can know four properties:*

$$
\begin{array}{llll}
i) \ \frac{\partial\hat{\Theta}}{\partial\beta} < 0 & & and \ \ \frac{\partial\hat{\theta}}{\partial\beta} < 0 & \\
ii) \ \frac{\partial\hat{\Theta}}{\partial C^O} < 0 & & and \ \ \frac{\partial\hat{\theta}}{\partial c^o} < 0 & \\
iii) \frac{\partial\hat{\Theta}}{\partial c^t} > 0 & & and \ \ \frac{\partial\hat{\theta}}{\partial C^T} > 0 & \\
iv) \frac{\partial\hat{\Theta}}{\partial\tau} \gtreqless 0 & if \ \beta \gtreqless \frac{c^t}{C^O} & and \ \ \frac{\partial\hat{\theta}}{\partial\tau} \gtreqless 0 & if \ \mathcal{B} \gtreqless \frac{C^T}{c^o}
\end{array}
$$

First, It's natural that the caller's burden decreases as the receiver's utility increases. Second, an increase in origination costs decreases the portion of transport cost ($\hat{\Theta}, \hat{\theta}$) paid by the caller. Third, an increase in termination costs increases the portion of transport cost ($\hat{\Theta}, \hat{\theta}$) paid by the caller. Fourth, the effect of transport cost depends on the relative size of origination and termination costs.

Proposition 5 is consistent with DeGraba (2003). DeGraba's Proposition 1 says that the only pair of prices that yields efficient consumption satisfies this condition,

$$\frac{MU^R}{MU^C} = \frac{MC^R}{MC^C}, \ P^R = MC^R \text{ and } P^C = MC^C \tag{2.73}$$

where superscript R indicates the receiver and C indicates the caller. This condition shows that the ratio of the marginal utilities of the receiver and caller should be the same as the ratio of marginal costs paid by each party as its service price. In my model, the above condition (2.73) can be interpreted as the following: for $i = 1, 2$,

$$\frac{\tilde{u}'(Q)}{U'(Q)} \equiv \beta = \frac{c^t + (1-\Theta)\tau}{C^O + \Theta\tau}, \ r = c^t + (1-\Theta)\tau \text{ and } P = C^O + \Theta\tau \tag{2.74}$$

$$\frac{\tilde{U}'(q)}{u'(q)} \equiv \mathcal{B} = \frac{C^T + (1-\theta)\tau}{c^o + \theta\tau}, \ R = C^T + (1-\theta)\tau \text{ and } \ p = c^o + \theta\tau. \tag{2.75}$$

One can derive the result of Proposition 5 from the above equations.[11] Using a different network model, I obtain a result equivalent to DeGraba (2003).

If the regulator can apply the optimal meet-points corresponding to the receiver's utility, the Bill-and-Keep system yields a socially optimal market equilibrium in which all prices are at cost level and the telecommunications service is provided properly.

2.6 Welfare Implications

In this section, I compare the CPP regime with Bill-and-Keep regime. Let's begin with optimality conditions in terms of retail prices from equations (2.74) and (2.75) :

$$r^* = \beta P \tag{2.76}$$

$$R = \mathcal{B} p^* \tag{2.77}$$

Still assuming vanishing noise, in a socially optimal equilibrium the call volumes are determined such that calling demand and receiving demand are equal:

$$\hat{Q} = Q^C = Q^R \tag{2.78}$$

$$\text{where} \quad Q^C = Q(P) \text{ and } Q^R = Q(\frac{r^*}{\beta})$$

$$\hat{q} = q^C = q^R \tag{2.79}$$

$$\text{where} \quad q^C = q(p^*) \text{ and } q^R = q(\frac{R}{\mathcal{B}})$$

[11] In Jeon et al. (2004), they show optimal access charges equivalent to these optimal meet-points using state-contingent receiving utilities.

Considering the above optimal volumes and vanishing noise, the optimal social welfare level is the following: from equation (2.57),

$$\begin{aligned}\hat{TS}^{BK} =& t + \left\{u(\hat{q}) + \tilde{u}(\hat{Q}) - p^* \cdot \hat{q} - r^* \cdot \hat{Q} - f^*\right\} \\ &+ \left\{U(\hat{Q}) + \tilde{U}(\hat{q})) - P \cdot \hat{Q} - R \cdot \hat{q} - F\right\} \end{aligned} \tag{2.80}$$

where the retail tariffs for the fixed-line network are regulated and retail tariff for mobile networks are market-determined. Note the application of the optimal meet-points as derived in the previous section.

$$\begin{aligned} P =& C^O + \hat{\Theta}\tau \equiv P^{BK} \\ R =& C^T + (1 - \hat{\theta})\tau \equiv R^{BK} \\ F =& C^F \\ p^* =& c^o + \hat{\theta}\tau \equiv p^{BK} \\ r^* =& c^t + (1 - \hat{\Theta})\tau \equiv r^{BK} \\ f^* =& c^f + t \end{aligned}$$

Hereafter, I use superscript "BK" for Bill-and-Keep results.

Now, let's review the market results under the CPP regime. Under the CPP regime, access charges are allowed between interconnecting networks but receiver's prices and meet-points are not allowed. In the previous chapter, I study the same model except that the CPP regime is applied instead of Bill-and-Keep. In that CPP model, the market equilibrium tariffs are as the following:

$$P^{cpp} = C^O + \tau + a^m \tag{2.81}$$

$$F^{cpp} = C^F \tag{2.82}$$

$$p^{cpp} = c^o + \tau + C^T \tag{2.83}$$

$$f^{cpp} = c^o + t - (a^m - c^t)Q(P^{cpp}) \tag{2.84}$$

where a^m is the monopoly level access charge. Then, the equilibrium call volumes are the following:

$$Q^{cpp} = Q(C^O + \tau + a^m) \tag{2.85}$$

$$q^{cpp} = q(c^o + \tau + C^T) \tag{2.86}$$

Therefore, consumer surpluses under CPP are:

$$W^{cpp} = U(Q^{cpp}) + \tilde{U}(q^{cpp}) - P^{cpp} * Q^{cpp} - F^{cpp} \tag{2.87}$$

$$w^{cpp} = u(q^{cpp}) + \tilde{u}(Q^{cpp}) - p^{cpp} * q^{cpp} - f^{cpp} \tag{2.88}$$

The fixed-line network is regulated so that it cannot have any profit, and the mobile networks have only Hotelling profit, $\frac{t}{2}$, under either regime.

$$\Pi^{BK} = \Pi^{cpp} = 0 \tag{2.89}$$

$$\pi^{BK} = \pi^{cpp} = \frac{t}{2} \tag{2.90}$$

By summing up the profits and consumer surpluses, the social welfare level under CPP is the following:

$$\begin{aligned} TS^{cpp} = &t + \left\{u(q^{cpp}) + \tilde{u}(Q^{cpp}) - p^{cpp} \cdot q^{cpp} - f^{cpp}\right\} \\ &+ \left\{U(Q^{cpp}) + \tilde{U}(q^{cpp}) - P^{cpp} \cdot Q^{cpp} - F^{cpp}\right\} \end{aligned} \tag{2.91}$$

The Bill-and-Keep regime can achieve social optimum when the optimal meet-points are arranged. It is therefore clear that the Bill-and-Keep is welfare-superior to CPP. First, the call volume under the Bill-and-Keep regime is larger than the call volume under the CPP regime because $P^{BK} < P^{cpp}$ and $p^{BK} < p^{cpp}$. See Lemma 7.

$$\hat{Q} \equiv Q(P^{BK}) > Q(P^{cpp}) \tag{2.92}$$

$$\hat{q} \equiv q(p^{BK}) > q(p^{cpp}) \tag{2.93}$$

Lemma 7. $\left(P^{BK} < P^{cpp}\right)$ *and* $\left(p^{BK} < p^{cpp}\right)$ *always hold if* $\mathcal{B} > 0$ *and* $\beta > 0$.

Proof. For $\left(P^{BK} < P^{cpp}\right)$ or $\left(C^O + \hat{\Theta}\tau < C^O + \tau + a^m\right)$ to hold, we need $\hat{\Theta} < \frac{\tau + a^m}{\tau}$. Since $a^m \geq c^t$, we need to show just $\hat{\Theta} < \frac{\tau + c^t}{\tau}$. The optimal meet-point is $\hat{\Theta} = \frac{c^t + \tau - \beta C^O}{(1+\beta)\tau}$ from equation (2.71). Then we need $\left(\frac{c^t + \tau - \beta C^O}{(1+\beta)\tau} < \frac{\tau + c^t}{\tau}\right)$ or $0 < \beta(C^O + \tau + c^t)$. This always holds if $\beta > 0$.

Similarly, $\left(p^{BK} < p^{cpp}\right)$ always holds if $\mathcal{B} > 0$. □

Second, the prices paid by the caller and receiver under the Bill-and-Keep regime with optimal meet-points is less than or equal to what callers have to pay under CPP.

$$P^{BK} + r^{BK} = C^O + \tau + c^t < C^O + \tau + a^m = P^{cpp} \tag{2.94}$$

$$p^{BK} + R^{BK} = c^o + \tau + C^T = c^o + \tau + C^T = p^{cpp} \tag{2.95}$$

Proposition 6. ***(Welfare Superiority of Bill-and-Keep)*** *Bill-and-Keep with optimal meet-points is welfare-superior to CPP if the receivers' utilities are positive :*

$$\hat{TS}^{BK} > TS^{cpp} \quad \text{if} \;\; \mathcal{B} > 0 \;\; \text{and} \;\; \beta > 0$$

Proof. See Appendix A. □

Now, let's see who is the beneficiary of the Bill-and-Keep regime. Note that in this model, firms are not better off after introducing Bill-and-Keep. The fixed-line network has no profits and the mobile networks have only Hotelling profits. This means that the additional social welfare generated by the Bill-and-Keep regime goes only to consumers and not to the producers. In other words, only consumer surpluses are enhanced by the regime.

It's not completely clear whether the fixed-line subscribers or the mobile subscribers are greater beneficiaries. But a clue is found from the equilibrium prices under the CPP regime.[12] Under the CPP regime, the mobile users are subsidized by the fixed-line users. Part of what the fixed-line users pay goes to mobile users through the fixed-fee: $(a^m - c^t)Q(P^{cpp})$. By introducing the Bill-and-Keep, the benefit-loss relation between the mobile users and the fixed-line users disappears. This suggests that the benefit of the Bill-and-Keep system might go to fixed-line users rather than to mobile users. It's a reasonable suggestion because often the most distorted parts of an economy experience greater benefits after correcting the distortion.

2.7 Conclusion

Existing literature finds that under the CPP regime, call externalities are ignored and the distortions created by abnormally high access charges is hard to cure, because the regime requires the caller or calling network to pay all the costs of a call. This paper studies the interconnection between a regulated

[12] See equations (2.81) and (2.84).

fixed-line network and competitive mobile networks. I apply the Bill-and-Keep regime into my model following Jeon et al. (2004)'s model which deals with the RPP regime. The RPP regime allows networks to collect a receiver's price from their own subscribers in addition to receiving access charges from the calling networks. Bill-and-Keep is different in that it does not allow access charges but it requires the receiver to pay the termination costs. I show that by introducing a Bill-and-Keep regime, the call externalities and access market distortion can be cured if proper meet-points corresponding to the receivers' utilities are implemented. In my model, because the fixed-line network is perfectly regulated and mobile firms are competitive on their retail market, the benefits of Bill-and-Keep go only to consumers and not to firms.

Even though the Bill-and-Keep regime is shown to bring an efficient allocation in this model, it's difficult for regulators to have complete information about the receivers' utilities. Therefore, some practical Bill-and-Keep policies are suggested assuming the size of the receivers' utilities.[13] I expect that the evaluation of these practical regimes will be explored by future research.

[13] See DeGraba (2000) for COBAK and DeGraba (2003) for MBAK.

Chapter 3

An Examination of Practical Bill-and-Keep Policies between Fixed-line and Mobile Networks

3.1 Introduction

In the telecommunications market, the Calling Party Pays (CPP) regime has two standard problems: a monopoly of access charges and call externalities. Under this regime, the calling network must pay an access charge to the receiving network, which has a monopoly on the ability to deliver calls to its subscribers. This problem must be controlled through complicated regulatory efforts. The call externality arises, because although the receiver is one of two beneficiaries of a call, he does not pay for the call. On the other hand, under a Bill-and-Keep regime, the access market itself is not allowed and the call externality can be eliminated by making both the receiver and the caller share the cost of a call. However, the Bill-and-Keep regime has a meet-point decision problem: how to divide and share the transport cost between the two central offices of the calling and receiving networks. If a regulator has sufficient information about the relative sizes of the utilities of the receiver and the caller as well as information about the cost structures of all the telecommunications networks, an optimal meet-point can be obtained and implemented in the real economy.

Unfortunately, regulators do not have that much information. Because of this, some practical meet-point policies have been suggested: Central Office Bill and Keep (COBAK) and Meet-Point Bill and Keep (MBAK). These programs are "practical" in that they can be implemented in the real economy, because they rely on information that can be easily obtained by regulators. COBAK and MBAK do not require information about consumers' utilities or firms' cost structures; instead, they make assumptions about that information, such as symmetric cost structure between interconnecting networks and an equal distribution of call value between the caller and the receiver. If these assumptions are accepted, deciding the meet-points only requires information about the transport cost, which is relatively easy to detect.

According to Atkinson and Barnekov (2001), telecommunications service carriers have had little difficulty identifying incremental facilities required for transport. Once a clear rule is stated, negotiation between carriers generally produces efficient solutions, and third parties can objectively evaluate which party more closely complies with the rule. Under this model, it is expected that regulators would rarely need to resolve disputes between interconnecting networks.

There are substantial research on the Bill-and-Keep regime. DeGraba (2000) suggests the Central Office Bill and Keep (COBAK) to replace the flawed existing interconnection regimes. In this paper, the so called "COBAK proposal", DeGraba suggests that the calling network or the calling party should pay all the transport costs between the two central offices of the interconnecting networks as well as the origination costs of its own network. He further suggests that the receiving network or the receiving party should pay only the termination costs of its own network.

Wright (2002*b*) criticizes COBAK, because he claims that it fails to apply Ramsey principles to the recovery of joint costs and therefore a Bill-and-Keep regime is not necessarily better than a CPP regime, since the called party may often have a much lower willingness to pay than the calling party.

DeGraba (2003) shows that a Bill-and-Keep regime that equally divides the cost of a call between both parties results in the first-best utilization when customers equally share the benefit of each call and competing carriers have access to the same technology. This is the Meet-Point Bill-and-Keep (MBAK). Atkinson and Barnekov (2000) insist that the mid-point should be the solution of the meet-point decision, if negotiation between both parties does not work. This is exactly the same as the MBAK recommendation. These two papers suggest the mid-point, because they assume that the costs of both networks are symmetric and that the receiver's utility is as much as the caller's utility for a call.

It is the assumption about the size of the receiver's utility that creates the differences between the opposing claims of Wright (2002*b*) and DeGraba (2003). DeGraba (2003) says that, since the receiver obtains almost the utility as the caller, MBAK can be efficient. Wright (2002*b*) claims that since the receiver's utility is close to zero, call externalities are very small and a CPP regime that makes the calling party pay for the whole cost is closer to optimality than Bill-and-Keep.

In the previous chapter, I build a traditional network interconnection model between a fixed-line network and mobile networks in which a Bill-and-Keep regime is applied instead of the CPP without any assumptions about the size of the receiver's utility. I show that the Bill-and-Keep regime can be superior to the CPP if a proper meet-point is chosen corresponding to the given size of the receiver's utility. I derive an optimal meet-point between a fixed-line network and mobile networks.

It is difficult to have information on the receiver's utility and the costs of networks. Furthermore, the costs are very likely to be asymmetric between two different style networks, such as a fixed-line network and a mobile network. In this chapter, I allow asymmetric costs between interconnecting networks[1], and I use a range for the relative size of the receiver's utility to the caller's utility. Using a specific example, I examine whether the practical Bill-and-Keep policies such as COBAK and MBAK yield a higher total surplus than the CPP. Furthermore, I try to evaluate which specific meet-point policy, COBAK or MBAK, is better for the real economy.

First, I show that a practical Bill-and-Keep regime yields a higher total surplus than a market-driven CPP if the receiver's utility is fairly large. Second, I compare performances of COBAK and MBAK and show that COBAK is a safer policy than MBAK, because (1) it provides a larger total surplus for a small receiver's utility, and (2) the performance of a practical Bill-and-Keep approaches that of an optimal Bill-and-Keep as the receiver's utility increases.

3.2 The Calling Party Pays Regime

This model is based on Chapter 1. I add a noise variable in the receiver's utility and derive equilibrium and efficient allocation. This model is used to obtain total surplus for a CPP regime in Section 3.4.

[1] I only use a symmetric cost structure for comparing the symmetric and asymmetric cases.

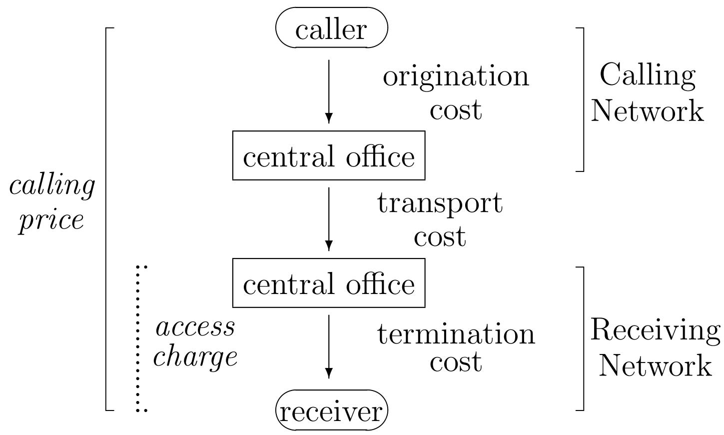
3.2.1 The Model

Assume that the telecommunications market has only two kinds of networks: fixed-line and mobile. The fixed-line network is a monopoly and the mobile market has two symmetric and competitive firms. The payment system of the telecommunications market is the CPP system, which means that callers pay for the calls. See Figure 3.1. In the traditional telecommunications models under CPP, callers are assumed to be the only party to benefit from the calls; however, receivers clearly benefit from the calls, too. Note that because under the CPP regime receivers do not pay for their benefits, call externalities arise. To show the call externalities explicitly, I assume that receivers derive utility from the calls but do not pay for calls and that they do not hang up first. Therefore, the callers determine the call length.

All calls made from mobile networks are terminated on the fixed-line network, and all calls originating on the fixed-line network are terminated on one of the mobile networks. To allow a focus on the interaction between the fixed-line and the mobile networks, on-net calls in each network and mobile-to-mobile calls are ignored. Since they need access to each other's network to complete their outgoing calls, two-way access markets exist: mobile-to-fixed and fixed-to-mobile.

The costs to complete calls are assumed to be composed of three parts: origination, termination and transport. See Figure 3.1. Origination cost c^o is a per-minute cost generated in the mobile network when a subscriber makes a call. The termination cost c^t is a per-minute cost generated in the mobile network when one of its subscribers receives a call. The two mobile networks have symmetric cost structures. The fixed-line network has a different cost structure than the mobile networks. C^O and C^T are, respectively, the orig-

Figure 3.1: Calling Party Pays (CPP) System



ination and termination costs for the fixed-line network. Transport cost τ is a per-minute cost between the two central office switches of the involved networks. Note that the origination or termination cost includes the cost generated in each party's central office switch. The transport cost is covered by the calling network.[2] I assume that the transport cost is the same between the fixed-line network and either mobile network and is symmetric between a fixed-to-mobile call and a mobile-to-fixed call.

I assume that there aren't any universal fixed costs to reflect a joint and common cost of serving the various customers for each network. But mobile networks have a fixed cost c^f per subscriber, which reflects the monthly cost of connecting the customer to the network as well as billing and servicing the customer. C^F is a fixed cost per subscriber for the fixed-line network.

[2]In some sense, under the CPP regime, the origination cost includes the transport cost. I separate them to compare this regime with the Bill-and-Keep regime, under which sharing of transport cost becomes an issue.

It is assumed that the access charge of mobile-to-fixed network A is regulated at termination cost level C^T.

$$A = C^T \tag{3.1}$$

Each mobile network freely charges access charge a_i to the fixed-line network.

For each network, demand for outgoing calls depends only on the price of an outgoing call and not on the price for incoming calls, which is paid by the customers of the other network. Callers do not care about their calling partners' utilities. Given that each customer has the same marginal willingness to pay for calling and the same utility of receiving calls, every mobile network can offer uniform tariff for the overall use of its services. The fixed-fee F for the fixed-line network is regulated at the fixed cost level, C^F.

$$F = C^F \tag{3.2}$$

The fixed-line network is regulated to set its fixed-to-mobile retail prices proportional to the access charge each mobile firm sets: a_i and a_j. Suppose the fixed-line network offers its subscribers a per-minute usage charge P_i for making calls to mobile network i, which is described by a function of total costs,

$$P_i \equiv g(C^O + \tau + a_i), \quad g' > 0 \tag{3.3}$$

This is obviously true in the case when fixed-to-mobile prices are regulated at variable cost level, $C^O + \tau + a_i$. On the other hand, each mobile network charges its subscribers a monthly fixed-fee f_i and a per-minute charge p_i for making mobile-to-fixed calls. Suppose that once a subscriber has joined a mobile network with usage charge p_i, that subscriber makes $q(p_i)$ minutes of outbound calls to a fixed-line user. Each fixed-line network subscriber makes $Q(P_i)$ minute calls to a user of mobile network i.

A fixed set of homogeneous customers subscribe to the telecommunications services. For simplification, I assume that the fixed-line network has N subscribers and mobile networks divide another N subscribers into two groups, N_1 and N_2. This N can be normalized to 1 without loss of generality and then N_1 and N_2 can be understood as market shares s_i and s_j. Notice that full market participation is assumed respectively in the fixed-line market and in the mobile market and, as a result, $s_i + s_j = 1$ always holds. In this framework each market has no network externalities.

Consumer preferences are known to firms. Mobile service and fixed-line service each provide different utilities. For example, mobile phone service gives subscribers mobility in addition to the usual utility of calls. Therefore, I assume that the utility function of fixed-line network subscribers is different from that of mobile network subscribers. Subscribers of each network benefit from outgoing and incoming calls separably. For fixed-line network subscribers and mobile network subscribers, their concave utilities from making calls are:

$$U(Q_i) \text{ and } u(q_i), \quad i = 1, 2$$

where Q_i is the length of fixed-to-mobile calls and q_i is the length of mobile-to-fixed calls between each pair of caller and receiver. To be consistent with the Bill-and-Keep model in the next section, I use a receiver's utility that is subject to noise, expressed by a random variable ϵ.[3] For $i = 1, 2$,

$$\tilde{U}(q_i) + \epsilon q_i \tag{3.4}$$

$$\tilde{u}(Q_i) + \epsilon Q_i \tag{3.5}$$

[3]In this CPP regime, the noise does not affect equilibrium since networks do not set the receiver's prices, which would be affected by this disturbance term.

where ϵ follows the distribution function $\Psi(\cdot)$ with wide enough support $[\underline{\epsilon}, \bar{\epsilon}]$, zero mean and density $\psi(\cdot)$, which is strictly positive for all ϵ in $[\underline{\epsilon}, \bar{\epsilon}]$. Furthermore, I assume that the noise ϵ is identically and independently distributed for each caller-receiver pair. The expected utility from receiving calls is assumed to be proportional to the calling-utility of the caller in the corresponding network. This is only for technical simplicity.

$$\tilde{U}(q_i) \equiv \mathcal{B} \cdot u(q_i),\ \mathcal{B} > 0 \tag{3.6}$$

$$\tilde{u}(Q_i) \equiv \beta \cdot U(Q_i),\ \beta > 0 \tag{3.7}$$

$\mathcal{B}$ is an expected ratio of the receiver's utility to caller's utility for a mobile-to-fixed call. Analogously, β is this ratio for a fixed-to-mobile call.

The utility of a consumer with income y joining the fixed-line network is given by:

$$y + V_o + \sum_{i=1}^{2} s_i \left\{ U(Q_i) - P_i Q_i + \tilde{U}(q_i) + \epsilon q_i \right\} - F. \tag{3.8}$$

V_0 is a fixed utility enjoyed by a fixed-line user from subscribing to the fixed-line network.[4]

The market share of each mobile network, s_i, is determined by network competition for subscribers, which can be explained by the Hotelling product differentiation model. Assume that mobile customers are endowed with a value of x, which is drawn from a uniform distribution on the interval $[0, 1]$. Two mobile networks are located at each end point of this unit interval, respectively. A mobile customer with value x has extra disutility from not being able to consume his preferred services,

$$tx^2 \quad \text{and} \quad t(1-x)^2 \tag{3.9}$$

[4] V_0 is assumed to be high enough that all consumers join this network.

for network 1 and 2 respectively, where t is the parameter that measures how differentiated the networks are. The greater t is, the greater are the switching costs, and the harder it is for one firm to steal away customers from the rival firm by lowering its price. The utility derived by a consumer with income y located at x from joining either mobile network i or j is respectively given by:

$$y + v_0 - tx^2 + u(q_i) - p_i q_i + \tilde{u}(Q_i) + \epsilon Q_i - f_i \tag{3.10}$$

$$y + v_0 - t(1-x)^2 + u(q_j) - p_j q_j + \tilde{u}(Q_j) + \epsilon Q_j - f_j \tag{3.11}$$

v_0 is a fixed utility, which a mobile network user has from subscribing to a mobile network.[5] f_i is a fixed-fee which the mobile user has to pay for every billing period. Disregarding fixed utility, define net variable consumer surplus w_i as:

$$w_i \equiv u(q_i) - p_i q_i + \tilde{u}(Q_i) + \epsilon Q_i - f_i, \quad i = 1, 2. \tag{3.12}$$

A customer located at x is indifferent between the two mobile networks if

$$w_i - tx^2 = w_j - t(1-x)^2.$$

Solving for x, the mobile market shares are, for $i, j = 1, 2$ and $i \neq j$,

$$s_i = \frac{1}{2} + \frac{w_i - w_j}{2t}. \tag{3.13}$$

Defining the indirect utility for fixed-to-mobile calls Q_i and mobile-to-fixed calls q_i as:

$$V(P_i) = \max_{Q_i} \{U(Q_i) - P_i Q_i\}, \quad i = 1, 2 \tag{3.14}$$

$$v(p_i) = \max_{q_i} \{u(q_i) - p_i q_i\}, \quad i = 1, 2, \tag{3.15}$$

[5]The constant v_o ensures that all consumers will always choose to join one of the two mobile networks if it is high enough.

demand functions of $Q(P_i)$ and $q(p_i)$ for the calls per subscriber can be derived. Then,

$$V'(P_i) = -Q(P_i) \quad \text{and} \quad v'(p_i) = -q(p_i) \quad i = 1, 2.$$

Disregarding fixed utility, a consumer's net surplus of belonging to each network is then

$$W \equiv \sum_{i=1}^{2} s_i \left\{ V(P_i) + \tilde{U}(q(p_i)) + \epsilon q(p_i) \right\} - F \tag{3.16}$$

$$w_i \equiv v(p_i) + \tilde{u}(Q(P_i)) + \epsilon Q(P_i) - f_i, \quad i = 1, 2. \tag{3.17}$$

where market shares are, for $i = 1, 2$,

$$\begin{aligned} s_i &= \frac{1}{2} + \frac{w_i - w_j}{2t} \\ &= \frac{1}{2} + \frac{\{v(p_i) + \tilde{u}(Q(P_i)) + \epsilon Q(P_i) - v(p_j) - \tilde{u}(Q(P_j)) + \epsilon Q(P_j)\} + (f_j - f_i)}{2t}. \end{aligned} \tag{3.18}$$

Notice that the price of fixed–to-mobile call P_i can affect the mobile market shares because mobile subscribers have utilities from receiving calls originated in the fixed-line network, $\tilde{u}(Q(P_i)) + \epsilon Q(P_i)$.

Access charges are chosen first, and then, taking these as given, firms choose their retail tariffs non-cooperatively. Given the access charges (a_i, a_j, A), each mobile firm chooses its tariff, (p_i, f_i) or (p_j, f_j). On the other hand, retail tariffs for the fixed-line network, (P_i, P_j, F) are regulated according to (3.2) and (3.3), given (a_i, a_j, A).

Since the total number of customers N can be normalized to one, all demands and profits can be understood as a per-customer unit: $Q(P_i)$, $q(p_i)$, Π, π_i.

3.2.2 The Benchmark: Efficient Allocation

I derive the social optimum for future reference. This efficient allocation is the same under both CPP and Bill-and-Keep regimes, since all utilities and cost structures are the same. Consider an idealized situation in which a social planner chooses the market shares and the volume of calls. In this symmetric setup, equal market division minimizes the average consumer's disutility from not being able to consume his preferred service.

Let's define $T(s_i)$ as the average consumer's disutility from not being able to consume her preferred service. Using disutilities (3.9), for arbitrary s_i, this measure of the distance between preferred and actual brand choice is given by:

$$T(s_i) \equiv t\left[\frac{s_i^3 + (1-s_i)^3}{4}\right]. \tag{3.19}$$

$T(s_i)$ is minimized at $s_i = \frac{1}{2}$, which is the symmetric market share.

$$\frac{\partial T(s_i)}{\partial s_i} = \frac{t}{4}\left[-3 + 6s_i\right] = 0 \tag{3.20}$$

The social planner would choose the volume of calls so as to maximize the expected social surplus.

$$\max_{Q,q}\ E\Big\{U(Q)+\tilde{U}(q)+\epsilon q+u(q)+\tilde{u}(Q)+\epsilon Q-(C^O+\tau+c^t)Q-(c^o+\tau+C^T)q\Big\} \tag{3.21}$$

The optimal volumes $\hat{Q}$ and $\hat{q}$ are determined by:

$$U'(\hat{Q}) + \tilde{u}'(\hat{Q}) = C^O + \tau + c^t \tag{3.22}$$

$$u'(\hat{q}) + \tilde{U}'(\hat{q}) = c^o + \tau + C^T \tag{3.23}$$

Since $\tilde{u}'(\cdot) = \beta U'(\cdot)$ and $\tilde{U}'(\cdot) = \mathcal{B}u'(\cdot)$,

$$U'(\hat{Q}) = \frac{C^O + \tau + c^t}{1+\beta} \tag{3.24}$$

$$u'(\hat{q}) = \frac{c^o + \tau + C^T}{1+\mathcal{B}}. \tag{3.25}$$

3.2.3 Market Equilibrium

Let's start with the second stage, in which given (a_i, a_j, A) mobile firms choose their own retail tariffs simultaneously. In the second-stage game, mobile network i sets expected-profit-maximizing tariff, (p_i, f_i), accepting (P_i, P_j) and (p_j, f_j) as given.

$$\max_{p_i,\ f_i} E[\pi_i] = E\Bigg[s_i(p_i, p_j, P_i, P_j, f_i, f_j)\cdot \\ \Big\{Q(P_i)\left(a_i - c^t\right) + q(p_i)\left(p_i - c^o - \tau - C^T\right) + f_i - c^f\Big\}\Bigg] \tag{3.26}$$

Using equation (3.17), $w_i \equiv v(p_i) + \tilde{u}(Q(P_i)) + \epsilon Q(P_i) - f_i$, the optimization problem of each mobile network can be solved with respect to p_i and w_i, instead of p_i and f_i because market share is determined directly by the net surplus.

$$\max_{p_i, w_i} E[\pi_i] = E\Bigg[\left(\frac{1}{2} + \frac{w_i - w_j}{2t}\right)\Big\{Q(P_i)\left(a_i - c^t\right) + q(p_i)\left(p_i - c^o - \tau - C^T\right) \\ + v(p_i) + \tilde{u}(Q(P_i)) + \epsilon Q(P_i) - w_i - c^f\Big\}\Bigg] \tag{3.27}$$

The first order conditions (FOCs) of each mobile network are the following. For $i, j = 1, 2$ and $i \neq j$,

$$\frac{\partial E[\pi_i]}{\partial p_i} = \left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\}\left\{q(p_i) + \frac{\partial q_i}{\partial p_i}(p_i - c^o - \tau - C^T) - q(p_i)\right\} \\ = s_i \cdot \frac{\partial q_i}{\partial p_i} \cdot (p_i - c^o - \tau - C^T) = 0 \tag{3.28}$$

$$\frac{\partial E[\pi_i]}{\partial w_i} = -\left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\} + \frac{1}{2t}\Big\{Q(P_i)\left(a_i - c^t\right)$$
$$+ q(p_i)\left(p_i - c^o - \tau - C^T\right) + \left(v(p_i) + \tilde{u}(Q(P_i)) - w_i\right) - c^f\Big\}$$
$$= - s_i + \frac{1}{2t}\Big\{Q(P_i)\left(a_i - c^t\right)$$
$$+ q(p_i)\left(p_i - c^o - \tau - C^T\right) + f_i - c^f\Big\} = 0 \qquad (3.29)$$

Since cornered-market equilibrium does not exist[6], from the FOC (3.28), the usage price is set at marginal cost according to the two-part tariff problem:

$$p_i^* = c^o + \tau + C^T, \quad i = 1, 2. \qquad (3.30)$$

Combining the above result (3.30) with the FOC (3.29), the equilibrium fixed-fee is:

$$f_i^* = c^f + 2t \cdot s_i - Q(P_i^*)\left(a_i - c^t\right), \quad i = 1, 2. \qquad (3.31)$$

Furthermore, this is a unique equilibrium for any given (a_i, a_j) and (A, P_i, P_j, F).[7] Note that the noise in receiver's utility does not affect the equilibrium tariffs of mobile networks. From now on, all the results are the same as the case without the noise.

The retail prices of the fixed-line network are regulated by the equation (3.3). Since their fixed-fee and access charges are regulated at cost level, the profit is:

$$\Pi = \sum_{i=1}^{2} s_i(P_i^* - C^O - \tau - a_i)Q(P_i^*) \qquad (3.32)$$
$$\text{where } P_i^* = g(C^O + \tau + a_i),\ g' > 0.$$

[6]See Lemma 1 in Chapter 1 for details.
[7]See Lemma 2 in Chapter 1 for details.

Because the fixed-line network is regulated, they cannot affect the mobile market shares, even though the receiving-utility of mobile users depends on the fixed-to-mobile retail prices.

Now at the first stage, given conditions (3.3), (3.30) and (3.31), each mobile firm chooses its own access charge. The fixed-line network is regulated to set its access charge at the termination cost level C^T.

Mobile network i chooses its access charge a_i, given a_j. Considering the tariffs of the mobile firms determined at the second stage, (3.30) and (3.31), the maximization problem is:

$$\max_{a_i} \ \pi_i = 2t \cdot s_i^2. \tag{3.33}$$

The equilibrium profits of the mobile firms depend only on its market share, but access charges affect this market share. Using the definition of net consumer surplus (3.17) and the fixed prices determined at the second stage (3.31), consumer surplus is:

$$w_i^* = v(p_i^*) + \tilde{u}(Q(P_i^*)) - c^f - 2t \cdot s_i + Q(P_i^*)(a_i - c^t), \quad i = 1, 2. \tag{3.34}$$

Since $p_i^* = c^o + \tau + C^T$, $v(p_i^*) = v(p_j^*)$. Then, the market share is: for $i, j = 1, 2$ and $i \neq j$,

$$s_i^* = \frac{1}{2} + \frac{\tilde{u}(Q(P_i^*)) + (a_i - c^t)Q(P_i^*) - \tilde{u}(Q(P_j^*)) - (a_j - c^t)Q(P_j^*)}{6t}.$$

Since $P_i^* = g(C^O + \tau + a_i)$ from equation (3.3),

$$\begin{aligned} s_i =& s_i(a_i, a_j) \\ =& \frac{1}{2} + \frac{\tilde{u}(Q(g(C^O + \tau + a_i))) + (a_i - c^t)Q(g(C^O + \tau + a_i))}{6t} \\ & - \frac{\tilde{u}(Q(g(C^O + \tau + a_j))) + (a_j - c^t)Q(g(C^O + \tau + a_j))}{6t}. \end{aligned} \tag{3.35}$$

Using equation (3.33) and (3.35), the optimization problem of each mobile firm at the first stage is:

$$\max_{a_i} \ \pi_i = 2t\left\{\frac{1}{2} + \frac{\tilde{u}(Q(g(C^O+\tau+a_i))) + (a_i - c^t)Q(g(C^O+\tau+a_i))}{6t} - \frac{\tilde{u}(Q(g(C^O+\tau+a_j))) + (a_j - c^t)Q(g(C^O+\tau+a_j))}{6t}\right\}^2. \tag{3.36}$$

FOC can be derived as the following: From equation (3.7), $\frac{\partial \tilde{u}}{\partial Q_i} = \beta \frac{\partial U}{\partial Q_i} > 0$.

$$\frac{\partial \pi_i}{\partial a_i} = \frac{2}{3} s_i(a_i, a_j)\left\{Q(P_i) + \left(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right)\frac{\partial Q(P_i)}{\partial a_i}\right\} = 0, \quad i = 1, 2 \tag{3.37}$$

where $P_i = g(C^O + \tau + a_i)$. Since $s_i \neq 0$ and this is a unique and symmetric equilibrium, access charge, $a_i^* = a_j^* = a^*$[8], should satisfy this condition.

$$Q_i(g(C^O+\tau+a^*)) + \left(a^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right)\frac{\partial Q_i(g(C^O+\tau+a^*))}{\partial a} = 0 \tag{3.38}$$

From equation (3.38), $c^t - \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}$ can be thought as "perceived marginal costs," which incorporates call externalities. This means that mobile networks accept termination costs discounted by the negative effect of an access charge on its own subscribers' receiving-utility. Note that even though the access charges are lower than the traditional monopoly level, mobile networks still set inefficiently high access charges, creating a new monopoly level that incorporates the receiver's utility. Under the CPP regime, these abnormally high fixed-to-mobile access charges exist and regulatory efforts are necessary to control them.

[8]See Lemma 3 in Chapter 1 for details.

Since access charges are determined symmetrically in equilibrium, all other equilibrium prices become symmetric. First, equilibrium fixed-to-mobile retail prices are symmetric.

$$P_i^* = P_j^* = g(C^O + \tau + a^*) = P^* \tag{3.39}$$

Note that since the usage price of fixed-to-mobile calls decreases due to low mobile access charges, the level of fixed-to-mobile call volume, $Q(P^*)$, is greater than it would be without including the receiving-utility of mobile users. The fixed-fee of the fixed-line network is set at the cost level, $F^* = C^T$.

From equation (3.35), the equilibrium market share of the mobile market becomes symmetric.

$$s_i^* = s_j^* = \frac{1}{2} = s^* \tag{3.40}$$

Equilibrium prices for mobile-to-fixed calls are set at cost level and are symmetric from equation (3.30).

$$p_i^* = p_j^* = c^O + \tau + C^T = p^* \tag{3.41}$$

Let's look at the equilibrium fixed-fees of mobile firms, which are derived from equations (3.31) and (3.40).

$$f_i^* = f_j^* = c^f + t - (a^* - c^t)Q(g(C^O + \tau + a^*)) = f^* \tag{3.42}$$

In this model, there exists a unique and symmetric equilibrium ($p_i^* = p_j^* = p^*, f_i^* = f_j^* = f^*, a_i^* = a_j^* = a^*$) which is characterized by these conditions,[9]

$$p^* = c^o + \tau + C^T \tag{3.43}$$

$$f^* = c^f + t - (a^* - c^t)Q(g(C^O + \tau + a^*)) \tag{3.44}$$

$$Q(g(C^O + \tau + a^*)) + \left(a^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right)\frac{\partial Q(g(C^O + \tau + a^*))}{\partial a_i} = 0 \tag{3.45}$$

From equation (3.33) and symmetric market shares, the equilibrium profits of mobile firms are symmetrically determined.

$$\pi_i^* = \pi_j^* = \frac{t}{2} = \pi^* \tag{3.46}$$

The consumer surplus of a mobile user comes from (3.17) and (3.42).

$$\begin{aligned} w_i^* = w_j^* =& v(c^o + \tau + C^T) + \tilde{u}(Q(g(C^O + \tau + a^*))) - c^f - t \\ &+ (a^* - c^t)Q(g(C^O + \tau + a^*)) \\ =& w^* \end{aligned} \tag{3.47}$$

On the other hand, the monopoly fixed-line network is regulated. Since their fixed-fee and access charge are set at the cost level, their equilibrium profit is:

$$\Pi^* = (P^* - C^O - \tau - a^*)Q(P^*) + F - C^F \tag{3.48}$$

since $P_i^* = P_j^* = P^*$. If the fixed-line is perfectly regulated, $P^* = g(C^O + \tau + a^*) = C^O + \tau + a^*$, its profit is zero.

A fixed-line network user has the following consumer surplus:

$$W^* = V(g(C^O + \tau + a^*)) + \tilde{U}\big(q(c^o + \tau + C^T)\big) - C^F \tag{3.49}$$

[9]See proposition 2 in Chapter 1 for details.

which should be greater than the case that does not account for the receiving-utility of mobile users because the calling volume is greater.

I define social welfare as sum of all profits and consumer surpluses:

$$TS^* = \pi^* + w^* + \Pi^* + W^* \tag{3.50}$$

3.2.4 Optimal Access Charges

To implement the optimal outcome, the social planner can use symmetric tariffs so as to achieve equal market division ($s_i = \frac{1}{2}$). Since the volume is determined by the caller, from equations (3.22) and (3.23), the optimal volume is obtained by choosing $(\hat{P}, \hat{p})$ such that:

$$\hat{P} = U'(\hat{Q}) = C^O + \tau + c^t - \tilde{u}'(\hat{Q}) \tag{3.51}$$

$$\hat{p} = u'(\hat{q}) = c^o + \tau + C^T - \tilde{U}'(\hat{q}) \tag{3.52}$$

Under the CPP regime, callers should pay for the origination cost, transport cost and access charges.

$$\hat{P} = C^O + \tau + \hat{a} \tag{3.53}$$

$$\hat{p} = c^o + \tau + \hat{A} \tag{3.54}$$

For optimality, access charges should be given by:

$$\hat{a} = c^t - \tilde{u}'(\hat{Q}) \tag{3.55}$$

$$\hat{A} = C^T - \tilde{U}'(\hat{q}) \tag{3.56}$$

To satisfy the each industry's break-even constraint, F and f should be adjusted.

$$\Pi = (\hat{A} - C^T)\hat{q} + (\hat{P} - C^O - \tau - \hat{a})\hat{Q} + \hat{F} - C^F = 0 \tag{3.57}$$

$$\pi = (\hat{a} - c^t)\hat{Q} + (\hat{p} - c^o - \tau - \hat{A})q + \hat{f} - c^f = 0 \tag{3.58}$$

$$\hat{F} = C^F + \tilde{U}'(\hat{q})\hat{q} \tag{3.59}$$

$$\hat{f} = c^f + \tilde{u}'(\hat{Q})\hat{Q} \tag{3.60}$$

3.3 The Bill-and-Keep Regime

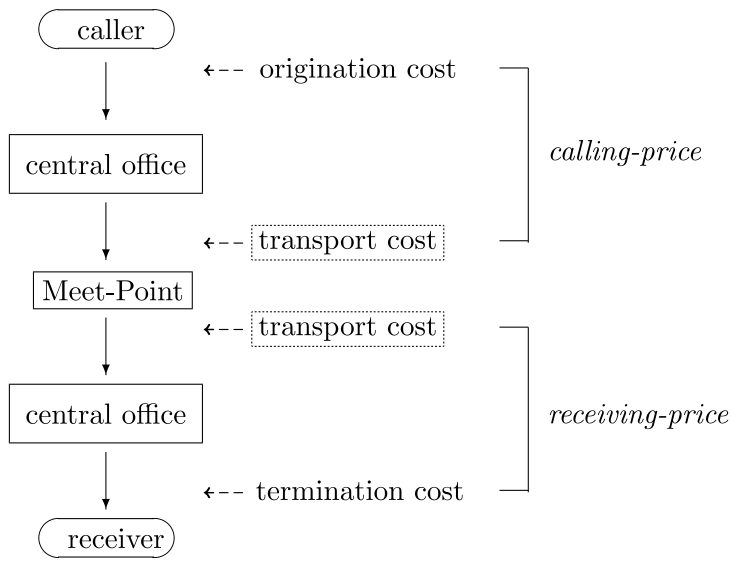
This model is the same as in Chapter 2. Market structure, cost structure and utilities in this model are the same as in the previous CPP model. This model is used to obtain total surplus for the Bill-and-Keep regime in Section 3.4.

3.3.1 The Model

In this model, the payment system of the telecommunications market is the Bill-and-Keep system. Assuming that both caller and receiver benefit from the calls, they both have to bear the costs raised in the network to which they subscribe. As a result, it's reasonable to assume that either the receiver or the caller can hang up first. Since both the caller and the receiver pay for a call, call externalities do not arise. This pricing system weakens the appropriateness of access charges. If the receiving network recovers their cost from their own subscribers, it's hard to accept the access charge paid by the calling network, because that is usually compensation for using the receiving network. Therefore, I assume that networks do not pay any access charges.

Under the Bill-and-Keep regime, networks exchange traffic on a 'Bill-and-Keep' basis at an arranged meet-point of the trunk line connecting the networks, which means that each network pays part of the cost of operating a single trunk line between the networks. As a result, setting a meet-point can be understood as a cost-allocation between the related networks to maintain

Figure 3.2: Bill-and-Keep System



a trunk line. I assume that the meet-point between the two central offices is arranged by a regulator or by both networks before the networks choose retail prices, and this does not need to follow the COBAK or the MBAK system. I assume the total transport cost is τ between the fixed-line network and either mobile network. It is symmetric for both directions of a call. Mobile network i pays $\theta_i\tau$ of the transport cost when their subscribers make a call to the fixed-line network, and the fixed-line network pays $\Theta_i\tau$ when their subscribers make a call to mobile network i. The receiving network pays the remaining portion of τ. When meet-points are arranged between two central offices in the real world, Θ_i and $\theta_i \in [0, 1]$. However, notice that theoretically Θ_i and θ_i can be negative or greater than 1, especially for optimality.

Given that each customer has the same marginal willingness to pay for making or receiving calls, all mobile networks can offer a uniform tariff for the overall use of its services. As in the CPP model, the fixed-line network is regulated. The fixed-fee F for the fixed-line network is regulated at the fixed cost level, C^F.

$$F \equiv C^F$$

The fixed-line network is regulated such that its retail prices are proportional to the costs, including the allocated transport costs. Suppose the fixed-line network offers its subscribers a per-minute usage charge P_i for making calls to mobile network i and R_i for receiving calls from mobile network i. These are described by functions of the total costs, given meet-points (Θ_i, θ_i).

$$P_i \equiv g(C^O + \Theta_i \tau) \tag{3.61}$$

$$R_i \equiv g(C^T + (1 - \theta_i)\tau), \quad g' > 0 \tag{3.62}$$

On the other hand, each mobile network charges its subscribers a monthly fixed-fee f_i and a per-minute charge p_i for making calls to the fixed-line network and r_i for receiving calls from the fixed-line network. Mobile networks choose their own tariffs non-cooperatively. Suppose that once a subscriber has joined a mobile network with usage charge (p_i, r_i), that subscriber wants to make $q(p_i)$ minutes of outbound calls to a fixed-line user and receive $Q(r_i)$ minutes of inbound calls from a fixed-line user for a billing period. Each fixed-line network subscriber wants to make $Q(P_i)$ minutes of outbound calls to a subscriber in mobile network i and wants to receive $q(R_i)$ minutes of inbound calls from a subscriber in mobile network i.

Consumer preferences are the same as in the CPP model. Calling-utilities are:

$$U(Q_i) \text{ and } u(q_i), \quad i = 1, 2$$

Receiving-utilities are:

$$\tilde{U}(q_i) + \epsilon q_i \tag{3.63}$$

$$\tilde{u}(Q_i) + \epsilon Q_i \tag{3.64}$$

where

$$\tilde{U}(q_i) \equiv \mathcal{B} \cdot u(q_i),\ \mathcal{B} > 0 \tag{3.65}$$

$$\tilde{u}(Q_i) \equiv \beta \cdot U(Q_i),\ \beta > 0 \tag{3.66}$$

Since the receivers have to pay for incoming calls in a Bill-and-Keep model, it is reasonable to assume that the receivers can hang up whenever they want. Both the caller and the receiver have positive probability of hanging up first – so called "caller's sovereignty" and "receiver's sovereignty." As a result, the volume of calls is non-cooperatively decided by callers and receivers.

Given (P_i, R_i, p_i, r_i) and a realized value of ϵ, unless the receiver hangs up the call first, the caller will continue the call until her marginal utility is equal to the calling price. For $i = 1, 2$,

$$U'(Q_i) = P_i \tag{3.67}$$

$$u'(q_i) = p_i. \tag{3.68}$$

Similarly, unless the caller hangs up the call first, the receiver with noise ϵ will continue the call until her marginal utility is equal to the receiving-price. For $i = 1, 2$,

$$\tilde{U}'(q_i) + \epsilon = R_i \tag{3.69}$$

$$\tilde{u}'(Q_i) + \epsilon = r_i. \tag{3.70}$$

Suppose a subscriber of the fixed-line network calls a subscriber of mobile network i. Given the caller's price P_i, the receiver's price r_i and a realized value of ϵ, if the caller does not hang up the call first, P_i is not greater than her marginal utility U'. In this case, the receiver with noise ϵ will continue the call until her marginal utility is equal to r_i. Namely, the receiver consumes the volume of calls which satisfies above equation (3.70). Let's say this volume quantity Q_i^R. With the volume of calls Q_i^R, the caller has marginal utility of $U'(Q_i^R) = \frac{r_i - \epsilon}{\beta}$ from (3.66).

$$P_i \leq U'(Q_i^R) = \frac{r_i - \epsilon}{\beta} \tag{3.71}$$

Since P_i is less than or equal to $\frac{r_i - \epsilon}{\beta}$ (low ϵ), the receiver determines the volume of calls, and the consumed volume is $Q_i^R = Q_i(\frac{r_i - \epsilon}{\beta})$, which is the receiver's demand function. On the other hand, if the caller's price P_i is greater than this marginal utility level $U'(Q_i^R) = \frac{r_i - \epsilon}{\beta}$ (high ϵ), the caller determines the volume of calls by (3.67) and the consumed volume of calls is $Q_i^C = Q_i(P_i)$, which is the caller's demand function.

$$\frac{r_i - \epsilon}{\beta} \leq U'(Q_i^C) = P_i \tag{3.72}$$

Therefore, the volume of calls from the fixed-line network to mobile network i is determined by the following equation. For $i = 1, 2$,

$$D(P_i, r_i) = \min\left\{Q_i^C, Q^R\right\} = \min\left\{U'^{-1}(P_i), U'^{-1}(\frac{r_i - \epsilon}{\beta})\right\} \tag{3.73}$$

or

$$D(P_i, r_i) = Q_i(\max\{P_i, \frac{r_i - \epsilon}{\beta}\}). \tag{3.74}$$

A larger ϵ increases the possibility of the caller's sovereignty with realized volume $Q_i\left(P_i\right)$, and a smaller ϵ increases the possibility of the receiver's sovereignty with realized volume $Q_i(\frac{r_i-\epsilon}{\beta})$. Similarly, the volume of opposite-direction calls – calls from mobile network i to the fixed-line network – is for $i = 1, 2$,

$$d(p_i, R_i) = q_i(\max\{p_i, \frac{R_i - \epsilon}{\mathcal{B}}\}). \tag{3.75}$$

The above two volumes can be expressed as demand functions:

$$\begin{aligned} D(P_i, r_i) \equiv & \{1 - \Psi(r_i - \beta P_i)\} Q_i(P_i) \\ & + \int_{\underline{\epsilon}}^{r_i - \beta P_i} Q_i(\frac{r_i - \epsilon}{\beta})\psi(\epsilon)d\epsilon \end{aligned} \tag{3.76}$$

$$\begin{aligned} d(p_i, R_i) \equiv & \{1 - \Psi(R_i - \mathcal{B}p_i)\} q_i(p_i) \\ & + \int_{\underline{\epsilon}}^{R_i - \mathcal{B}p_i} q_i(\frac{R_i - \epsilon}{\mathcal{B}})\psi(\epsilon)d\epsilon \end{aligned} \tag{3.77}$$

In equation (3.76), the first term on the right hand side represents the expected demand for fixed-to-mobile calls for large values of ϵ, such that the caller on the fixed-line network is the one that terminates the call. The second term is the expected fixed-to-mobile demand for small values of ϵ, where the receiver on mobile network i hangs up first. The first term on the right hand side of equation (3.77) represents the expected demand for mobile-to-fixed calls for high values of ϵ, where the caller on mobile network i terminates. The second term is expected demand over the range of ϵ where the receiver on the fixed-line network terminates the call.

Similarly the utilities of the fixed-line network subscriber making calls to mobile network i and mobile network i's subscriber making calls to the fixed-line network are, respectively, given by:

$$\begin{aligned} \Omega(P_i, r_i) \equiv & \{1 - \Psi(r_i - \beta P_i)\} U(Q_i(P_i)) \\ & + \int_{\underline{\epsilon}}^{r_i - \beta P_i} U\Big(Q_i(\frac{r_i - \epsilon}{\beta})\Big)\psi(\epsilon)d\epsilon, \quad i = 1, 2 \end{aligned} \tag{3.78}$$

$$\omega(p_i, R_i) \equiv \{1 - \Psi(R_i - \mathcal{B}p_i)\} \, u\left(q_i\left(p_i\right)\right) + \int_{\underline{\epsilon}}^{R_i - \mathcal{B}p_i} u\left(q_i(\frac{R_i - \epsilon}{\mathcal{B}})\right) \psi(\epsilon) d\epsilon, \quad i = 1, 2. \tag{3.79}$$

I now define the receiver's utility before payment. The utilities of the fixed-line network subscriber receiving calls from mobile network i and mobile network i's subscriber receiving calls from the fixed-line network are given, respectively, by equations (3.80) and (3.81). For $i = 1, 2$,

$$\tilde{\Omega}(p_i, R_i) \equiv \int_{R_i - \mathcal{B}p_i}^{\bar{\epsilon}} \left\{ \tilde{U}\left(q_i\left(p_i\right)\right) + \epsilon q_i\left(p_i\right) \right\} \psi(\epsilon) d\epsilon + \int_{\underline{\epsilon}}^{R_i - \mathcal{B}p_i} \left\{ \tilde{U}\left(q_i(\frac{R_i - \epsilon}{\mathcal{B}})\right) + \epsilon q_i(\frac{R_i - \epsilon}{\mathcal{B}}) \right\} \psi(\epsilon) d\epsilon \tag{3.80}$$

$$\tilde{\omega}(P_i, r_i) \equiv \int_{r_i - \beta P_i}^{\bar{\epsilon}} \left\{ \tilde{u}\left(Q_i\left(P_i\right)\right) + \epsilon Q_i\left(P_i\right) \right\} \psi(\epsilon) d\epsilon + \int_{\underline{\epsilon}}^{r_i - \beta P_i} \left\{ \tilde{u}\left(Q_i(\frac{r_i - \epsilon}{\beta})\right) + \epsilon Q_i(\frac{r_i - \epsilon}{\beta}) \right\} \psi(\epsilon) d\epsilon \tag{3.81}$$

Assume that all utilities are separably additive. Ignoring fixed utilities, the variable surplus of a fixed-line user is[10]:

$$W \equiv \sum_{i=1}^{2} s_i \left\{ \Omega(P_i, r_i) - P_i \cdot D(P_i, r_i) \right\} + \sum_{i=1}^{2} s_i \left\{ \tilde{\Omega}(p_i, R_i) - R_i \cdot d(p_i, R_i) \right\} - F \tag{3.82}$$

As in the CPP model, the mobile market share, s_i, is determined by network competition for subscribers, which can be explained by the Hotelling product differentiation model. The utility derived by a consumer with income

[10] Note that the definitions of surplus in the Bill-and-Keep model are different from the CPP model even though the notations are the same.

y located at x from joining either mobile network i or j is, respectively, given by:

$$y + v_0 - tx^2 + \omega(p_i, R_i) - p_i d(p_i, R_i) + \tilde{\omega}(P_i, r_i) - r_i D_i(P_i, r_i) - f_i \tag{3.83}$$

$$y + v_0 - t(1-x)^2 + \omega(p_j, R_j) - p_j d(p_j, R_j) + \tilde{\omega}(P_j, r_j) - r_j D(P_j, r_j) - f_j \tag{3.84}$$

Disregarding fixed utilities, let's define net variable consumer surplus w_i as:

$$\begin{aligned} w_i \equiv & \omega(p_i, R_i) - p_i \cdot d(p_i, R_i) \\ & + \tilde{\omega}(P_i, r_i) - r_i \cdot D(P_i, r_i) - f_i, \quad i = 1, 2. \end{aligned} \tag{3.85}$$

Even though the definition of w_i is different from the CPP model, the market shares look the same as in the CPP model. For $i, j = 1, 2$ and $i \neq j$,

$$s_i = \frac{1}{2} + \frac{w_i - w_j}{2t} \quad i = 1, 2 \tag{3.86}$$

The profit functions of the mobile networks and the fixed-line network are:

$$\begin{aligned} \pi_i \equiv & s_i \left(p_i - c^o - \theta_i \tau\right) \cdot d(p_i, R_i) + s_i \left(r_i - c^t - (1 - \Theta_i)\tau\right) \cdot D(P_i, r_i) \\ & + s_i f_i(p_i, r_i) - s_i c^f, \quad i = 1, 2 \end{aligned} \tag{3.87}$$

$$\begin{aligned} \Pi \equiv & \sum_{i=1}^{2} s_i \left\{ \left(P_i - C^O - \Theta_i \tau\right) \cdot D(P_i, r_i) \right. \\ & \left. + \left(R_i - C^T - (1 - \theta_i)\tau\right) \cdot d(p_i, R_i) \right\} + F - C^F \end{aligned} \tag{3.88}$$

3.3.2 Market Equilibrium

Suppose that the meet-points are arranged symmetrically before the networks compete in their retail market. This symmetry is reasonable because arranged meet-points are highly likely to be detected by the third party, including a potential regulator.[11]

$$\theta_1 = \theta_2 = \theta \tag{3.89}$$

$$\Theta_1 = \Theta_2 = \Theta \tag{3.90}$$

In this case, the fixed-line monopoly network will choose its retail prices by the regulation schedule:

$$P_1 = P_2 = g\left(C^O + \Theta\tau\right) = P$$
$$R_2 = R_2 = g\left(C^T + (1-\theta)\tau\right) = R$$

On the other hand, each mobile network chooses retail tariff, p_i, r_i and f_i, to maximize the profit. From the profit function (3.87):

$$\max_{p_i, r_i, f_i} \pi_i(p_i, r_i, f_i)$$

In equilibrium, FOCs of the profit maximization problem yield a unique symmetric candidate equilibrium[12]:

$$p_1^* = p_2^* = c^o + \theta\tau = p^* \tag{3.91}$$

$$r_1^* = r_2^* = c^t + (1-\Theta)\tau = r^* \tag{3.92}$$

The equilibrium fixed-fee is:

$$f_i = c^f + 2t \cdot s_i, \quad i = 1, 2 \tag{3.93}$$

Combining the above equilibrium retail tariff and consumer surplus for mobile users (3.85), equilibrium market share is:

$$s_i^* = \frac{1}{2}, \quad i = 1, 2 \tag{3.94}$$

[11] See Atkinson and Barnekov (2001).

[12] See Chapter 2 for details.

Now, let's go back to equilibrium fixed-fee and consumer surpluses:

$$f_1^* = f_2^* = c^f + t = f^* \tag{3.95}$$

$$w_1^* = w_2^* = \omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - f^* = w^* \tag{3.96}$$

By putting all the solutions into the profit function (3.87), the equilibrium profit level of each mobile network is a "Hotelling profit" – the results of product differentiation.

$$\pi_i^* = \frac{t}{2} = \pi^*, \quad i = 1, 2 \tag{3.97}$$

The equilibrium profit of the perfectly regulated fixed-line network is zero.

$$\Pi^* = 0 \tag{3.98}$$

The mobile network consumer's net surplus is:

$$w^* = \omega(p^*, R) + \tilde{\omega}(P, r^*) - p^* \cdot d(p^*, R) - r^* \cdot D(P, r^*) - f^* \tag{3.99}$$

The fixed-line network consumer's net surplus in equilibrium is:

$$W^* = \Big\{\Omega(P, r^*) - P \cdot D(P, r^*)\Big\} + \Big\{\tilde{\Omega}(p^*, R) - R \cdot d(p^*, R)\Big\} - F \tag{3.100}$$

As in the CPP model, social welfare is defined as the sum of the profits and consumer surpluses of the fixed-line and mobile networks. The equilibrium social welfare is:

$$TS^* = \pi^* + w^* + \Pi^* + W^* \tag{3.101}$$

3.3.3 Optimal Meet-Points

In the market equilibrium under a Bill-and-Keep regime mobile firms set their prices at cost level given symmetric meet-points, $\theta_i = \theta_j = \theta$ and $\Theta_i = \Theta_j = \Theta$ and information about the relative size of the receivers' utilities, β and $\mathcal{B}$.

From the efficient allocation conditions (3.24) and (3.25) in subsection 3.2.2,

$$U'(\hat{Q}) = \frac{C^O + \tau + c^t}{1+\beta} \tag{3.102}$$

$$\tilde{U}'(\hat{q}) = \frac{\mathcal{B}}{1+\mathcal{B}}(c^o + \tau + C^T) \tag{3.103}$$

Assuming vanishing noise, the fixed-line network is regulated to set retail prices by (3.61) and (3.62), given (Θ, θ). Now, suppose the regulator sets the retail prices of the fixed-line network at cost level. In equilibrium,

$$g(C^O + \Theta\tau) = C^O + \Theta\tau \tag{3.104}$$

$$g(C^T + (1-\theta)\tau) = C^T + (1-\theta)\tau \tag{3.105}$$

By combining the above equations with efficient allocation conditions (3.102) and (3.103),

$$C^O + \Theta\tau = P = \frac{C^O + \tau + c^t}{(1+\beta)} \tag{3.106}$$

$$C^T + (1-\theta)\tau = R = \frac{\mathcal{B}}{1+\mathcal{B}}(c^o + \tau + C^T). \tag{3.107}$$

Then the optimal meet-points are:

$$\hat{\Theta} = \frac{c^t + \tau - \beta C^O}{(1+\beta)\tau} \tag{3.108}$$

$$\hat{\theta} = \frac{C^T + \tau - \mathcal{B}c^o}{(1+\mathcal{B})\tau} \tag{3.109}$$

Notice that these optimal meet-points can be greater than 1 when β or $\mathcal{B}$ is small enough. For example, examine the first equation (3.108). Since a small β indicates a small receiver's utility, optimality requires that the caller pay a part of termination costs (c^t) as well as the whole transport cost (τ).

If the regulator can apply the optimal meet-points corresponding to the receiver's utility, the Bill-and-Keep system yields a socially optimal market equilibrium in which all prices are at cost level and the telecommunications service is provided properly.

3.4 Practical Bill-and-Keep Policies

In this section, I use a simple model to evaluate two practical Bill-and-Keep policies – COBAK and MBAK – by comparing them with the CPP model. I apply linear demand functions and their indirect utility functions and uniform distribution of receiver's utility to the CPP and Bill-and-Keep models from the previous sections. For the CPP model, I calculate the efficient allocation using optimal access charges as well as market equilibrium. On the other hand, for the Bill-and-Keep model, I calculate market equilibria assuming two meet-point policies: $\Theta = \theta = 1$ and $\Theta = \theta = \frac{1}{2}$, and optimal meet-points.

For the simplicity of analysis, I assume that the relative size of the receiver's utility to the caller's utility is the same for fixed-to-mobile calls and mobile-to-fixed calls: $\beta = \mathcal{B}$. Practical Bill-and-Keep policies are suggested assuming a certain level of β. MBAK assumes a symmetric distribution over the division of benefits between caller and receiver, which means that $E(\beta)$=1. In this study, I do not adhere to any specific level of β to evaluate the two practical Bill-and-Keep policies in a more general environment. I choose multiple points of β between 0 and 1.5 at regular intervals.[13] For each regime, I calculate the equilibrium result, including the total surplus at each level of β.

First, I explain the functions used in this section and for clarity define several concepts of the regimes. Second, I explain the effects of noise and small values of β. Third, I compare practical Bill-and-Keep policies with the optimal Bill-and-Keep system. Fourth, I compare COBAK and MBAK. Finally, I will explain the effect of cost structure.

3.4.1 Example Model and Definitions

In this simulation, I use this linear specification of demand.

Example 1. ***(Linear Demands with Uniformly Distributed Noise)*** *Assume that all demand functions in the CPP model from Section 3.2 and the Bill-and-Keep model in Section 3.3 are linear.*

$$Q(P) = m_0 - P$$
$$q(p) = n_0 - p, \qquad n_0 > m_0 > 0$$

where $n_0 > m_0$ assuming that demand for mobile-to-fixed calls q is greater than demand for fixed-to-mobile calls Q at a price level. When the welfare effect of noise in the receiver's utility is considered, it is important whether demands are big or small for a given support of the noise.[14] *See Lemma 9 in the next subsection.*

[13] I change this $\beta = \mathcal{B}$ from 0.05 to 1.5 by increments of 0.05. This range is reasonable if we accept that the cases of negative receiver's utility are negligibly rare and that it's unlikely that the receiver's utility is significantly higher than the caller's utility. Even if I increase the upper bound of the range, the basic results of this paper do not change.

[14] I use $m_0 = 18$ and $n_0 = 22$ for large demand for a given support $[-b, b]$ of the noise. To show a case of small demand, I use $m_0 = 5.5$ and $n_0 = 6.5$, which correspond to almost minimum level of utilities to guarantee positive call volumes: $D > 0$ and $d > 0, \forall\, \beta \geq 0.05$. See Lemma 10 in the next section.

Since I assume $\mathcal{B} = \beta$, receivers' utilities are:

$$\tilde{u}(Q) + \epsilon Q = \beta U(Q) + \epsilon Q$$
$$\tilde{U}(q) + \epsilon q = \beta u(q) + \epsilon q.$$

The noise in the receiver's utility, ϵ, is assumed to be uniformly distributed on a support $[-b, b]$.[15] *The density function of ϵ is:*

$$\psi(\epsilon) = \begin{cases} \frac{1}{2b} & \textit{for} \quad -b \le \epsilon \le b, \quad b > 0 \\ 0 & \textit{otherwise.} \end{cases}$$

The cost structures are assumed to be asymmetric between the fixed-line network and the mobile networks[16]*: the costs of mobile networks are higher than those of the fixed-line networks and termination costs are higher than origination costs.*

$$c^t > c^o > C^T > C^O$$

Using the above sample model, meet-points which maximize the equilibrium total surplus TS^* in equation (3.101) can be derived in closed forms.

Lemma 8. ***(Adjusted Optimal Meet-Points)*** *Using a linear specification of demand and a uniform distribution with a support $[-b, b]$, one can obtain "adjusted optimal meet-points," which maximize the equilibrium social welfare given the value of the receiver's utility β :*

$$\tilde{\Theta} = \frac{c^t - \beta C^O + \tau}{\tau(1+\beta)} + \frac{b(1-\beta)}{\tau(1+\beta)^2} \tag{3.110}$$

$$\tilde{\theta} = \frac{C^T - \mathcal{B} c^o + \tau}{\tau(1+\mathcal{B})} + \frac{b(1-\mathcal{B})}{\tau(1+\mathcal{B})^2} \tag{3.111}$$

---

[15] $b = 0.4, 0.5, 0.6$ are used to see how the support of noise works in this model. Otherwise, I use $b = 0.4$, which is close to the minimum support of noise for $\beta \in [0, 1.5]$, to guarantee both regions for caller and receiver, meaning that either party has a positive probability of hanging up first.

[16] Here is a typical example of the asymmetric cost structure used in this simulation.
$C^O = 0.07 \quad C^T = 0.08 \quad c^o = 0.12 \quad c^t = 0.13 \quad \tau = 0.1$

Due to the simplicity of the uniform distribution, these can be obtained easily. Under a more complicated distribution, such as normal distribution, it's difficult to obtain them in closed forms. I examine the implication of "adjusted optimal meet-points" in the next subsection, after I explain the effect of noise in receiver's utility on the social welfare under a Bill-and-Keep regime.

To explain my simulation results, I define some concepts which will be used from now on.

Definition 1. ***Optimal Bill-and-Keep*** *means that the optimal meet-points in equations (3.108) and (3.109) are used between interconnecting networks under a Bill-and-Keep regime :* $\hat{\Theta}$ *and* $\hat{\theta}$*. See subsection 3.3.3.*

Definition 2. ***Adjusted Optimal Bill-and-Keep*** *means that the "adjusted optimal meet-points" in equations (3.110) and (3.111) are used between interconnecting networks under a Bill-and-Keep regime:* $\tilde{\Theta}$ *and* $\tilde{\theta}$*. See Lemma 8.*

Definition 3. ***Practical Bill-and-Keep*** *means that a common meet-point is used between interconnecting networks under a Bill-and-Keep regime. "Practical" implies that these meet-points are suggested as implementable policies.*

- ***COBAK*** *means a Bill-and-Keep regime which uses a common meet-point* $\theta^C = \Theta^C = 1$*. All transport costs are paid by the calling network.*
- ***MBAK*** *means a Bill-and-Keep regime which uses a common meet-point* $\theta^M = \Theta^M = \frac{1}{2}$*. Both interconnecting networks divide and pay transport costs equally.*

Definition 4. ***Optimal CPP*** *means that optimal access charges in equations (3.56) and (3.55) are used between interconnecting networks under a CPP regime. See subsection 3.2.4.*

Definition 5. ***Market CPP*** *means that mobile access charges are not regulated nor arranged by the interconnecting networks under a CPP regime. Access charges as well as retail tariffs are determined by each mobile network independently so that mobile access charges are above cost. However, the access charges and retail tariff of the fixed-line network are regulated at cost level. See subsection 3.2.3.*

3.4.2 Noise and Small β

In Chapter 2, the efficiency of optimal Bill-and-Keep is shown theoretically assuming a "vanishing noise" in the receiver's utility. However, in Example 1 with a "non-vanishing noise," the existence of noise affects social welfare under a Bill-and-Keep regime: the noise in receiver's utility affects equilibrium through receiver's price, and equilibrium social welfare depends on the size of support of the noise. Under a CPP regime, since networks do not charge their subscribers for receiving calls, equilibrium does not depend on the size of the support.[17] I summarize the noise effect on social welfare with this lemma:

Lemma 9. ***(Effect of Noise on Social Welfare under Bill-and-Keep)*** *As the noise in the receiver's utility has a bigger support* $[-b, b]$*, social welfare under a Bill-and-Keep regime decreases if* $\beta < 1$ *and increases slightly if* $\beta > 1$*.*

(*i*) *If the size of the demands relative to b is large enough, this noise effect is negligible. See Figure B.1 in B.*

(*ii*) *Even though the size of the demands relative to b is small, the noise effect*

[17]See Section 3.2 for details.

decreases as β increases. But as β is closer to zero, the negative effect of noise increases. See Figure B.2 in B.

See Appendix A for an explanation of Lemma 9. This lemma implies that if the demands are large enough for a given support of noise, the Bill-and-Keep regime should not be affected by the noise effect. Figures B.1 and B.2 in B show total surplus levels of the Optimal Bill-and-Keep for three values of b. It also compares them with Optimal CPP and Market CPP. In Figure B.1, the Optimal Bill-and-Keep keeps its superiority to the Market CPP, and it converges to the Optimal CPP for large demands. In Figure B.2, however, for small demands with high b, the Optimal Bill-and-Keep is inferior to even Market CPP when β is close to zero. Even with optimal meet-points, the Bill-and-Keep can be worse than the CPP in some extreme but possible cases.

Even though there exists a noise effect that hurts the social welfare, it is mainly a "small receiving-utility" or "small β" that makes the Bill-and-Keep worse than the CPP. The noise effect only creates an "additional" negative effect on social welfare.

Lemma 10. ***("Small β" Effect)*** *As β approaches zero, in a symmetric equilibrium under a Bill-and-Keep regime, call volume and total surplus approach to zero, too.*

Proof. See Appendix A. □

According to Figure B.2 in B, even the Optimal Bill-and-Keep cannot avoid the "small β" effect so that social welfare is less than under Market CPP, if β is close to zero. In Chapter 2 I show that with optimal meet-points, a Bill-and-Keep regime is always superior to Market CPP if $\beta > 0$. These results appear to contradict each other. But, in Chapter 2 I assume a vanishing noise and does not take into consideration the existence of noise in equilibrium. On the other hand, this simulation incorporates noise in the model, and the negative effect of noise reduces the optimality of the Optimal Bill-and-Keep.

Now, it's time to go back to the "adjusted optimal meet-points" in Lemma 8. The second term of each equation shows that it is necessary to adjust the relative burdens of the caller and receiver to maximize the total surplus in a noise economy. I summarize two properties of the adjusted optimal meet-points with this lemma:

Lemma 11. ***(Properties of Adjusted Optimal Meet-Points)*** *$\tilde{\Theta}$ and $\tilde{\theta}$ in Lemma 8 have the following properties:*

(i) *If b approaches zero, the "adjusted optimal meet-points" approach the optimal meet-points (3.108) and (3.109), which are obtained by the efficient allocation of a social planner assuming vanishing noise.*

$$\hat{\Theta} = \frac{c^t - \beta C^O + \tau}{\tau(1+\beta)}$$
$$\hat{\theta} = \frac{C^T - \mathcal{B} c^o + \tau}{\tau(1+\mathcal{B})}$$

(ii) *When the receiver's expected utility is smaller than the caller's utility ($\beta < 1$), the support size of the receiver's utility, b, favors the receiver.*[18] *Adjusted optimal meet-points ($\tilde{\Theta}$, $\tilde{\theta}$) require the caller to pay more than at optimal meet-points ($\hat{\Theta}$, $\hat{\theta}$). Conversely, when the receiver's expected utility is greater than the caller's utility ($\beta > 1$), the support size of the*

[18]The size of support for this uniform distribution implies its "variance". For a uniform distribution with a support $[-b, b]$, its variance is $\frac{b^2}{3}$.

receiver's utility, b, favors the caller. $(\tilde{\Theta},\, \tilde{\theta})$ *require the receiver to pay more than* $(\hat{\Theta},\, \hat{\theta})$.

$$
\begin{aligned}
\tilde{\Theta} > \hat{\Theta} \quad &\textit{and} \quad \tilde{\theta} > \hat{\theta} \qquad \textit{if} \quad \beta < 1 \\
\tilde{\Theta} = \hat{\Theta} \quad &\textit{and} \quad \tilde{\theta} = \hat{\theta} \qquad \textit{if} \quad \beta = 1 \\
\tilde{\Theta} < \hat{\Theta} \quad &\textit{and} \quad \tilde{\theta} < \hat{\theta} \qquad \textit{if} \quad \beta > 1
\end{aligned}
$$

For the "adjusted optimal meet-points," there is no noise effect in my simulation. If $\tilde{\Theta}$ and $\tilde{\theta}$ are used under a Bill-and-Keep regime, namely "Adjusted Optimal Bill-and-Keep," the noise effect is mitigated by the adjustment terms in equations (3.110) and (3.111).[19] See Figure B.3 and B.4 in B. Whether the demands are big or small, the Adjusted Optimal Bill-and-Keep is always superior to the Market CPP. Furthermore, the Adjusted Optimal Bill-and-Keep quickly approaches the Optimal CPP as β increases. For future reference, I summarize the relation between the Adjusted Optimal Bill-and-Keep and the CPP with the following proposition.

Proposition 7. ***(Adjusted Optimal Bill-and-Keep and CPP)*** (*i*) *If* $\beta > 0$, *the Adjusted Optimal Bill-and-Keep is superior to the Market CPP.* (*ii*) *The Adjusted Optimal Bill-and-Keep quickly approaches the Optimal CPP as* β *increases, because it recovers the negative welfare effect of noise.*

In my simulation, if adjusted optimal meet-points are used, the superiority of Optimal Bill-and-Keep to CPP is not violated. So, in this paper I use the "adjusted optimal meet-points" $(\tilde{\Theta},\, \tilde{\theta})$ in equations (3.110) and (3.111)

[19]Following the explanation in A, one can say that decreasing calling-surplus is almost compensated by increasing receiving-surplus due to the adjustment term when the adjusted optimal meet-points are used. A simulation using Example 1 supports this argument.

instead of the optimal meet-points ($\hat{\Theta}$, $\hat{\theta}$) in equations (3.108) and (3.109) whenever they are necessary.

3.4.3 Practical Bill-and-Keep versus Optimal Bill-and-Keep

Since regulators do not have sufficient information about the costs of networks, the optimal meet-points are difficult to obtain. However, the transport costs between the two central offices of the calling and the called network are relatively easy to detect by the third party.[20] Due to this, a regulator can divide the transport cost and make both networks pay their own portions. Conceptually a meet-point is a point at which the transport cost is divided. So, practically possible meet-points are normalized to $[0, 1]$. I call these meet-points "practical meet-points." The COBAK suggests a practical meet-point, $\Theta^C = \theta^C = 1$ and the MBAK suggests $\Theta^M = \theta^M = \frac{1}{2}$. In this subsection, I examine whether a Practical Bill-and-Keep can achieve a better social welfare level than a Market CPP. The results will shed light on the applicability of a Bill-and-Keep in the real world.

Lemma 12. ***(Practical Bill-and-Keep and Adjusted Optimal Bill-and-Keep)*** *Practical Bill-and-Keep approaches the Adjusted Optimal Bill-and-Keep as β increases.*

As β increases, the size of the receiving-utility increases : $\tilde{u} = \beta U(Q)$ and $\tilde{U} = \beta u(q)$. This means that the total utility increases as β increases. A practical meet-point cannot be better than the optimal meet-point, but the effects of various meet-points on the social welfare are almost the same when the total utility is sufficiently large. In B, see Figures B.5 and B.6 for

[20]See Atkinson and Barnekov (2001) for the details.

COBAK and Figures B.7 and B.8 for MBAK. As β increases, Practical Bill-and-Keep approaches the Adjusted Bill-and-Keep quicker for the case of large demand than for small demand. The following proposition summarizes the relationships between Practical Bill-and-Keep and CPP.

Proposition 8. ***(Practical Bill-and-Keep and CPP)*** (i) *Practical Bill-and-Keep provides a social welfare closer to Optimal CPP as β increases.* (ii) *Practical Bill-and-Keep gives a better social welfare than Market CPP if β is not too small.*

This is implied by Proposition 7 and Lemma 12. Figure B.5 and B.6 show that COBAK is superior to Market CPP if β is not too small. When demand is small given the support size b, it's more likely that COBAK cannot enhance the social welfare over Market CPP. However, when demand is sufficiently large, COBAK is very likely to be superior to Market CPP. Figure B.7 and B.8 show the same results for MBAK.

3.4.4 COBAK versus MBAK

Often regulators do not have any information about β as well as costs of networks. So, it's not easy to justify an assumed level of β as in MBAK. In this subsection, without assuming a specific value of β, I examine which Practical Bill-and-Keep is the best.

Using Example 1, Figures B.9 and B.10 in B compare how the total surplus changes under MBAK ($\theta^M = \Theta^M = \frac{1}{2}$) and COBAK ($\theta^C = \Theta^C = 1$). While for large demand the difference between MBAK and COBAK is not clear, for small demand the superiority of COBAK to MBAK is clear when β is close to zero. Figures B.11 and B.12 in B illustrate this clearly. They show the welfare gain of the Practical Bill-and-Keep over Market CPP. Especially for values of β close to zero, the superiority of COBAK is clear. As β increases the difference between MBAK and COBAK disappears. The superiority of COBAK for small β is important because there is possibility that a Practical Bill-and-Keep generates a worse social welfare situation than a Market CPP for small β and small demand. To reduce the possibility that a Practical Bill-and-Keep fails to improve the social welfare, COBAK should be chosen over MBAK. This result does not depend on cost structure.[21]

Lemma 13. ***(COBAK vs MBAK)*** *There exists a $\bar{\beta}$ such that if $\beta < \bar{\beta}$, COBAK is more efficient than MBAK and if $\beta > \bar{\beta}$, MBAK is more efficient.*

Proof. See Appendix A. □

Proposition 9. ***(Superiority of COBAK to MBAK)*** *COBAK is a safer Practical Bill-and-Keep than MBAK.*

Proof. By Proposition 8, every Practical Bill-and-Keep approaches the Optimal CPP as β increases. However, by Lemma 13, COBAK is superior to MBAK for small β which is close to zero. So, if a Bill-and-Keep is going to be introduced, it is safer to choose COBAK. □

3.4.5 Symmetric versus Asymmetric Costs

Regulators do not have cost information about the networks, especially origination and termination costs. It is practically easy for a regulator to set a

[21]See Figures B.15 and B.16 to compare the cases of symmetric and asymmetric cost structure between fixed-line and mobile networks. They use Example 2 for asymmetric costs and Example 3 for symmetric costs in the next section. They show the superiority of COBAK for any case.

symmetric meet-point for both call directions – fixed-to-mobile and mobile-to-fixed: $\Theta = \theta$. COBAK and MBAK are examples of just that. These symmetric practical meet-points work better when the cost structure between the interconnecting networks is symmetric. To illustrate that, I use the following two examples:

Example 2. ***(Asymmetric Costs)*** *All other settings are the same as in Example 1 except the following. First, the demands are symmetric:*

$$Q = m_0 - P$$
$$q = n_0 - p, \qquad n_0 = m_0 > 0$$

Second, for each network, origination cost is equal to termination cost.

$$c^o = c^t > C^O = C^T$$

Example 3. ***(Symmetric Costs)*** *All other settings are the same as in Example 2 except the symmetric cost structure between fixed-line and mobile networks:*

$$c^o = c^t = C^O = C^T$$

The only difference between the above two examples is the cost structure between the fixed-line and mobile networks. Figures B.13 and B.14 in B compare symmetric and asymmetric cost structures for COBAK and MBAK using Example 2 and 3. They show a difference in total surplus between a Practical Bill-and-Keep and the Adjusted Optimal Bill-and-Keep for symmetric and asymmetric cost structures. For both COBAK and MBAK, it's obvious that the symmetric cost structure is better than the asymmetric one, but the difference is very small.

Lemma 14. ***(Symmetry and Practical Bill-and-Keep)*** *Suppose the demands for both networks are symmetric* ($n_0 = m_0$) *and for each network the origination cost is the same as the termination cost* ($c^o = c^t = c$, $C^O = C^T = C$). *There exists a* β *for which the symmetric cost structure* ($c = C$) *between the interconnecting networks makes a Practical Bill-and-Keep an optimal meet-point policy.*

Proof. See Appendix A. □

For some values of β, the cost symmetry makes a Practical Bill-and-Keep an optimal meet-point policy but this does not apply for cost asymmetry. However, this result completely depends on the assumptions for $\beta = \mathcal{B}$. If β is different for the two directions of calls, fixed-to-mobile or mobile-to-fixed, symmetry in cost structure cannot benefit a Practical Bill-and-Keep since $\hat{\Theta} \neq \hat{\theta}$. Furthermore, cost structure – symmetry or asymmetry – does not make a big difference in the welfare gain of a Practical Bill-and-Keep over Market CPP. In B, see Figures B.15 and B.16 using Examples 2 and 3. They show the welfare gains of COBAK and MBAK over Market CPP for each cost structure. Even for the case of small demand, cost symmetry does not significantly affect the performance of a Practical Bill-and-Keep.

3.5 Conclusion

In the previous chapter, I show that if regulators have access to information about network costs and the receiver's utility, an optimal Bill-and-Keep regime can be introduced to replace the existing CPP regime to resolve the problematic access markets and call externalities. Since this information is difficult to obtain in the real world, two practical Bill-and-Keep regimes are suggested: COBAK and MBAK. They only use information about transport costs, which is relatively easy to obtain. In this paper, using some example models, I examine the applicability of those practical Bill-and-Keep policies for a reasonable range of receiver's utility.

First, I show that for a Bill-and-Keep regime to be superior to a CPP regime, the receiver's utility should be fairly large. When receiver's utility is small, a practical Bill-and-Keep regime might decrease total surplus over a market-driven CPP. Second, I show that if a Bill-and-Keep regime is introduced, COBAK is a safer policy than MBAK, because the performance of a practical Bill-and-Keep approaches that of an optimal Bill-and-Keep as the receiver's utility increases. For small values of the receiver's utility, COBAK yields a greater total surplus than MBAK.

This paper shows that the performance of a practical Bill-and-Keep regime depends significantly on the size of the receiver's utility. Therefore, future research may include an empirical study to determine how to evaluate the receiver's utility.

Appendices

Appendix A

Proofs

I use a similar method as in Laffont et al. (1998*a*).

Proof of Lemma 1. Suppose that mobile network 1 corners the market, given (P_1, P_2, a_1, a_2). So, $s_1 = 1$ and $s_2 = 0$.

$$\begin{aligned} s_1 &= \frac{1}{2} + \frac{w_1 - w_2}{2t} \\ &= \frac{1}{2} + \frac{\{v(p_1) + \tilde{u}(Q(P_1)) - v(p_2) - \tilde{u}(Q(P_2))\} + \{f_2 - f_1\}}{2t} = 1 \end{aligned}$$

Then,

$$\frac{w_1 - w_2}{2t} = \frac{\{v(p_1) + \tilde{u}(Q(P_1)) - v(p_2) - \tilde{u}(Q(P_2))\} + \{f_2 - f_1\}}{2t} = \frac{1}{2}$$

From the FOCs of mobile firms, (1.38) and (1.39),

$$p_1 = c^o + \tau + C^T$$

$$f_1 = c^f + 2 \cdot t - Q(P_1)(a_1 - c^t)$$

and from equation (1.36),

$$\pi_1 = Q(P_1)(a_1 - c^t) + f_1 - c^f = 2 \cdot t > 0.$$

On the other hand,

$$\pi_2 = 0.$$

i) Suppose $a_1 = a_2$. $P_1 = P_2$ by $P_i = g(C^O + \tau + a_i)$.

$$\frac{w_1 - w_2}{2t} = \frac{\{v(p_1) - v(p_2)\} + \{f_2 - f_1\}}{2t}$$

Since $\pi_2 = 0$, mobile network 2 has an incentive to set its tariff as

$$p_2 = c^o + \tau + C^T$$

$$f_2 = f_1 + \varepsilon, \ \ \varepsilon > 0$$

to get almost half of the market share.

ii) Suppose $a_1 > a_2$. Since $P_1 > P_2$ due to $(g' > 0)$, $\tilde{u}(Q(P_1)) < \tilde{u}(Q(P_2))$. As a result, it's easier for mobile network 2 to obtain positive market share because its subscribers receive more calls (and greater utility) relative to the subscribers of mobile network 1.

iii) Suppose $a_1 < a_2$. Since $P_1 < P_2$, $\tilde{u}(Q(P_1)) > \tilde{u}(Q(P_2))$. If the difference between the two access charges is too large, it might be impossible for mobile network 2 to get any positive market share by changing its retail tariffs. However, at the first stage, mobile network 2 has an incentive to set their access charge as:

$$a_2 = a_1 + \varepsilon, \ \ \varepsilon > 0.$$

Then, mobile network 2 can set retail tariffs to get a positive market share.

From i), ii) and iii), cornered-market equilibrium cannot exist. ■

Proof of Lemma 2. By Lemma 1, only shared-market equilibria can be considered. In such an equilibrium, the marginal-cost pricing condition:

$$p_i = c^o + \tau + C^T, \quad i = 1, 2 \tag{A.1}$$

holds for both mobile networks, given $(a_i, a_j; A, P_i, P_j, F)$. For $i = 1, 2$ and $i \neq j$, the first-order condition with respect to w_i is:

$$\frac{\partial \pi_i}{\partial w_i} = -\left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\} + \frac{1}{2t}\Big\{Q(P_i)\left(a_i - c^t\right) + q(p_i)\left(p_i - c^o - \tau - C^T\right) + \left(v(p_i) + \tilde{u}(Q(P_i)) - w_i\right) - c^f\Big\} = 0. \tag{A.2}$$

Combined with marginal-cost pricing for both networks, for both mobile networks these equations define "pseudo reaction functions", $w_i = w_i^R(w_j)$. For $i = 1, 2$ and $i \neq j$,

$$w_i^R(w_j) = \frac{1}{2}w_j + \frac{1}{2}\Big\{Q(P_i)(a_i - c^t) + v(c^o + \tau + C^T) + \tilde{u}(Q(P_i)) - c^f - t\Big\}$$

The slope of this reaction function is:

$$\frac{dw_i^R}{dw_j} = \frac{1}{2}, \quad i = 1, 2 \text{ and } i \neq j.$$

which is positive and smaller than one. In this case the equilibrium is unique, $(p_i^*, p_j^*, w_i^*, w_j^*)$ such that $p_i^* = p_j^* = c^o + \tau + C^T$. But the equilibrium is not necessarily symmetric due to possible asymmetry in (P_i, P_j) or (a_i, a_j).

Let's now examine the second-order conditions. Given access charges (a_i, a_j, A), retail prices of the fixed-line network (P_i, P_j, F), and network j's strategy $(p_j, w_j \equiv v(p_j) + \tilde{u}(Q(P_j)) - f_j)$, mobile network i's best response is $p_i = c^o + \tau + C^T$, and therefore, keeping w_j and p_j fixed, network i's profit, if it chooses to offer w_i is:

$$\pi_i\left(w_i\right) = \left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\}\Big\{Q(P_i))\left(a_i - c^t\right) + v(c^o + \tau + C^T) + \tilde{u}(Q(P_i)) - w_i - c^f\Big\}.$$

The first order derivative of this function is:

$$\frac{\partial \pi_i(w_i)}{\partial w_i} = -\left\{\frac{1}{2} + \frac{w_i - w_j}{2t}\right\} + \frac{1}{2t}\Big\{Q(P_i)\left(a_i - c^t\right) + v(c^o + \tau + C^T) + \tilde{u}(Q(P_i)) - w_i - c^f\Big\}.$$

The second order derivative of the function is:

$$\frac{\partial^2 \pi_i(w_i)}{\partial w_i^2} = -\frac{1}{2t} - \frac{1}{2t} = -\frac{1}{t} < 0.$$

Therefore mobile network i's profit function is strictly concave, which means that the equilibrium is at least a local maximum.

Therefore, the candidate equilibrium, which must satisfy two conditions, (A.1) and (A.2), is indeed an unique equilibrium, given $(a_i, a_j; A, P_i, P_j, F)$. ■

Proof of Lemma 3. Let's begin with the existence of an equilibrium. It is known that any continuous function defined on a compact set attains a maximum. Assuming that all utility functions and demand functions are continuous, profit function (1.46) is continuous in a_i.

Assume there exists a maximum price, $\bar{P}$ such that $Q(\bar{P}) = 0$. Then, by FOC (1.40), there exists a maximum access charge, $\bar{a}$ such that $\bar{P} = g(C^O + \tau + \bar{a})$. On the other hand, assume that $\beta \leq 1$ so that $a_i^* \geq c^t$.[1]. Therefore a_i is in a compact set, $[c^t, \bar{a}]$.

Since a continuous profit function (1.46) is defined on a compact set, $[c^t, \bar{a}]$, there exists a maximum.

[1] See Proposition 1 for details.

Now, let's prove the uniqueness of the equilibrium. Rewrite equation (1.47), the first order condition of the mobile network at the first stage. For $i = 1, 2$ and $i \neq j$, the first-order condition with respect to a_i is:

$$\begin{aligned}\frac{\partial \pi_i}{\partial a_i} &= \frac{2}{3} s_i(a_i, a_j) \left\{ Q(P_i^*) + (a_i - c^t) \frac{\partial Q(P_i^*)}{\partial P_i} \frac{\partial g}{\partial a_i} + \beta \frac{\partial U(Q_i)}{\partial Q_i} \frac{\partial Q(P_i^*)}{\partial P_i} \frac{\partial g}{\partial a_i} \right\} \\ &= \frac{2}{3} s_i(a_i, a_j) \left\{ Q(P_i^*) + \left(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i} \right) \frac{\partial Q(P_i^*)}{\partial a_i} \right\} = 0, \quad i = 1, 2\end{aligned} \tag{A.3}$$

This defines a reaction function $a_i = a_i^R(a_j)$. By the implicit function theorem,

$$\frac{\partial a_i^R}{\partial a_j} = - \frac{\frac{\partial^2 \pi_i}{\partial a_j \partial a_i}}{\frac{\partial^2 \pi_i}{\partial a_i^2}} = 0$$

since $\frac{\partial^2 \pi_i}{\partial a_i^2} \neq 0$ (see below) and

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial a_j \partial a_i} = - \frac{1}{9t} &\left\{ Q(P_j^*) + \left(a_j - c^t + \frac{\partial \tilde{u}(Q_j)}{\partial Q_j} \right) \frac{\partial Q(P_j^*)}{\partial a_j} \right\} \bullet \\ &\left\{ Q(P_i^*) + \left(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i} \right) \frac{\partial Q(P_i^*)}{\partial a_i} \right\} = 0\end{aligned}$$

due to the first order condition. They do not response to each other's access charges. Both mobile networks try to get as big a market share as possible no matter what the rival networks's access charge is. Therefore, unilaterally each mobile network chooses its own optimal access charge a_i^* which satisfies this condition, for $i = 1, 2$,

$$Q(g(C^O + \tau + a_i^*)) + \left(a_i^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i} \right) \frac{\partial Q(g(C^O + \tau + a_i^*))}{\partial a_i} = 0$$

since $s_i \neq 0$ and $P_i^* = g(C^O + \tau + a_i^*)$. Because the demand function $Q(\cdot)$, utility function $\tilde{u}(\cdot)$, and cost c^t are the same for both networks, their access charges should be symmetric. Namely, $a_i^* = a_j^* = a^*$.

Let's study the second-order condition of mobile network i:

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial a_i^2} =& \frac{1}{9t}\left\{Q(P_i^*) + (a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i})\frac{\partial Q(P_i^*)}{\partial a_i}\right\}^2 \\ &+ \frac{2}{3}s_i(a_i, a_j)\left[2\frac{\partial Q(P_i^*)}{\partial a_i} + \frac{\partial^2 \tilde{u}(Q_i)}{\partial Q_i^2}\left\{\frac{\partial Q(P_i^*)}{\partial a_i}\right\}^2\right. \\ &\left. +(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i})\frac{\partial^2 Q(P_i^*)}{\partial a_i^2}\right] \\ =& \frac{2}{3}s_i(a_i, a_j)\left[2\frac{\partial Q(P_i^*)}{\partial a_i} + \frac{\partial^2 \tilde{u}(Q_i)}{\partial Q_i^2}\left\{\frac{\partial Q(P_i^*)}{\partial a_i}\right\}^2\right. \\ &\left. +(a_i - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i})\frac{\partial^2 Q(P_i^*)}{\partial a_i^2}\right]\end{aligned}$$

due to the first order condition. Let's look at this term:

$$\frac{\partial^2 Q(P_i^*)}{\partial a_i^2} = \frac{\partial^2 Q(P_i^*)}{\partial P_i^2}\left(\frac{\partial g}{\partial a_i}\right)^2 + \frac{\partial Q(P_i^*)}{\partial P_i}\frac{\partial^2 g}{\partial a_i^2}.$$

It's reasonable to say that $\frac{\partial^2 g}{\partial a_i^2} = 0$ because the regulation schedule $g(C^O + \tau + a_i)$ is likely to be linear. If demand function $Q(\cdot)$ is linear or $\frac{\partial^2 Q(P_i^*)}{\partial P_i^2} \cong 0$, namely $\frac{\partial^2 Q(P_i^*)}{\partial a_i^2} \cong 0$,

$$\frac{\partial^2 \pi_i}{\partial a_i^2} = \frac{2}{3}s_i(a_i, a_j)\left[2\frac{\partial Q(P_i^*)}{\partial a_i} + \frac{\partial^2 \tilde{u}(Q_i)}{\partial Q_i^2}\left\{\frac{\partial Q(P_i^*)}{\partial a_i}\right\}^2\right] < 0$$

by the concavity of utility function, $\tilde{u}” = \beta U” < 0$. In this case, mobile network i's profit function is strictly concave, which means that the equilibrium is global maximum.

Therefore, there exists an unique and symmetric equilibrium in terms of access charges, $(a_i^* = a_j^* = a^*)$, which must satisfy this condition,

$$Q(g(C^O + \tau + a^*)) + \left(a^* - c^t + \frac{\partial \tilde{u}(Q_i)}{\partial Q_i}\right)\frac{\partial Q(g(C^O + \tau + a^*))}{\partial a} = 0.$$

With ($a_i^* = a_j^* = a^*$) and equation (1.45), equilibrium market shares are:

$$s_i^* = \frac{1}{2} \text{ for } i = 1, 2.$$

Then, from equation (1.41) and above $s_i^* = \frac{1}{2}$, equilibrium fixed prices become:

$$f_i^* = c^f + t - (a^* - c^t)Q(g(C^O + \tau + a^*)).$$

■

Proof of Lemma 5. Since $s_i \neq 0$, from equation (1.47), the FOC with respect to a_i, mobile network i choose the access charge a_i^* that satisfies this condition,

$$Q(g(C^O+\tau+a_i^*))+(a_i^*-c^t)\frac{\partial Q(g(C^O+\tau+a_i^*))}{\partial a_i}+\frac{\partial \tilde{u}(Q(g(C^O+\tau+a_i^*)))}{\partial a_i}=0 \tag{A.4}$$

where $\frac{\partial \tilde{u}(Q_i)}{\partial a_i} = \frac{\partial \tilde{u}}{\partial Q_i}\frac{\partial Q}{\partial P_i}\frac{\partial g(C^O+\tau+a_i^*)}{\partial a_i}$ is the effect of access charge a_i on the call-reception utility of its own subscribers. Note that:

$$\frac{\partial \tilde{u}}{\partial Q_i} = \beta\frac{\partial U(Q_i)}{\partial Q_i} > 0, \quad (\because \beta > 0),$$
$$\frac{\partial Q}{\partial P_i} < 0 \text{ and}$$
$$\frac{\partial g}{\partial a_i} > 0, \quad (\because g' > 0).$$

Therefore,

$$\frac{\partial \tilde{u}(Q_i)}{\partial a_i} = \frac{\partial \tilde{u}}{\partial Q_i}\frac{\partial Q}{\partial P_i}\frac{\partial g(C^O+\tau+a_i^*)}{\partial a_i} < 0.$$

Suppose $\beta = 0$. Since $\frac{\partial \tilde{u}(Q_i)}{\partial a_i} = 0$, above equation A.4 can be the following. Let's call it $K(a_i)$.

$$K(a_i) \equiv Q(g(C^O+\tau+a_i)) + (a_i - c^t)\frac{\partial Q(g(C^O+\tau+a_i))}{\partial a_i} = 0. \tag{A.5}$$

This equation says that mobile networks choose monopoly level access charge a_i^m when the receiver's utility is not taken into account.

$$K(a_i^m) \equiv Q(a_i^m) + (a_i - c^t)\frac{\partial Q(a_i^m)}{\partial a_i} = 0$$

Let's show that as β increases from zero, mobile network i should decrease its access charge from a_i^m. Let's take the first-order derivative of the above equation with respect to a_i.

$$\frac{\partial K(a_i)}{\partial a_i} = 2\frac{\partial Q(g(C^O + \tau + a_i))}{\partial a_i} + (a_i - c^t)\frac{\partial^2 Q(g(C^O + \tau + a_i))}{\partial a_i^2}$$

Note that:

$$\frac{\partial Q(P_i)}{\partial a_i} = \frac{\partial Q}{\partial P_i}\frac{\partial g}{\partial a_i} < 0$$

$$\frac{\partial^2 Q(g(C^O + \tau + a_i))}{\partial a_i^2} = \frac{\partial^2 Q}{\partial P_i^2}\left(\frac{\partial g}{\partial a_i}\right)^2 + \frac{\partial Q}{\partial P_i}\frac{\partial^2 g}{\partial a_i^2}$$

It's reasonable to say that $\frac{\partial^2 g}{\partial a_i^2} = 0$ because the regulation schedule $g(C^O + \tau + a_i)$ is likely to be linear. Now, only one more condition is necessary to finish the proof. Suppose that demand function $Q(\cdot)$ is linear or $\frac{\partial^2 Q}{\partial P_i^2} \cong 0$. Then, the above term $\frac{\partial^2 Q(P_i)}{\partial a_i^2}$ is zero and $\frac{\partial K(a_i)}{\partial a_i} < 0$.

Comparing equation (A.5) with equation (A.4), one can see that a negative term $\frac{\partial \tilde{u}(Q(g(C^O+\tau+a_i^*)))}{\partial a_i}$ comes in the equation (A.5) to make equation (A.4). Since equation (A.5) is a negative function of a_i, a_i should be decreased to keep the zero condition. Therefore $a_i^* < a_i^m$ should hold. ■

Proof of Lemma 6. Suppose that mobile network 1 corners the market, given symmetric meet-points ($\theta_1 = \theta_2$, $\Theta_1 = \Theta_2$). So, $s_1 = 1$ and $s_2 = 0$. Since the meet-points are symmetric, the retail prices of the monopoly fixed-line network are decided symmetrically. From equations (2.2) and (2.3),

$$P_1 = P_2 = g\left(C^O + \Theta\tau\right) = P$$
$$R_1 = R_2 = g(C^T + (1-\theta)\tau) = R.$$

Using net variable consumer surplus, w_i (2.27) and market share, s_i (2.29), one can derive:

$$\begin{aligned} s_1 =& \frac{1}{2} + \frac{w_1 - w_2}{2t} \\ =& \frac{1}{2} + \frac{\omega(p_1, R) - p_1 \cdot d(p_1, R) + \tilde{\omega}(P, r_1) - r_1 \cdot D(P, r_1) - f_1}{2t} \\ & - \frac{\omega(p_2, R) - p_2 \cdot d(p_2, R) + \tilde{\omega}(P, r_2) - r_2 \cdot D(P, r_2) - f_2}{2t} \\ =& 1. \end{aligned} \tag{A.6}$$

From the FOCs of mobile firms (2.44) and (2.45),

$$\frac{\partial \pi_1}{\partial p_1} = s_1 \left[u'\left(q(p_1)\right) - c^o - \theta\tau\right]\{1 - \Psi(R - \mathcal{B}p_1)\}q'(p_1) = 0$$
$$\frac{\partial \pi_1}{\partial r_1} = \frac{1}{\beta} s_1 \int_{\underline{\epsilon}}^{r_1 - \beta P} \left[\tilde{u}'\left(Q(\frac{r_1 - \epsilon}{\beta})\right) + \epsilon - c^t - (1-\Theta)\tau\right] Q'(\frac{r_1 - \epsilon}{\beta})\psi(\epsilon)d\epsilon = 0.$$

Since ϵ is assumed to have a sufficiently wide support $[\underline{\epsilon}, \bar{\epsilon}]$,

$$\Psi(R - \mathcal{B}p_1) \neq 1.$$

Since $r_1 = \tilde{u}'(\cdot) + \epsilon$ from equation (2.11) and $\tilde{u}'(\cdot) = \beta U'(\cdot)$ from equation (2.7),

$$Q'(\frac{r_1 - \epsilon}{\beta}) = Q'(U'(\cdot)) \neq 0.$$

Considering $p_1 = u'(\cdot)$ (2.9) and $r_1 = \tilde{u}'(\cdot) + \epsilon$ (2.11), mobile network 1 sets its retail prices at the cost level,

$$p_1 = u'(q) = c^o + \theta\tau$$
$$r_1 = \tilde{u}'(Q) + \epsilon = c^t + (1-\Theta)\tau.$$

By putting the above two equations and definition of net surplus w_i (2.27) into the FOC with respect to w_i (2.46), one can find:

$$f_1 = 2t + c^f.$$

From the profit function (2.43), the profit of mobile network 1 is:

$$\pi_1 = 2t$$

On the other hand, since $s_2 = 0$, mobile network 2 has no profit.

$$\pi_2 = 0$$

Now, let's consider the incentives of mobile network 2. From the market share (A.6), one can find:

$$\begin{aligned}\frac{w_1 - w_2}{2t} =& \frac{\omega(p_1, R) - \omega(p_2, R) - p_1 \cdot d(p_1, R) + p_2 \cdot d(p_2, R)}{2t} \\ &+ \frac{\tilde{\omega}(P, r_1) - \tilde{\omega}(P, r_2) - r_1 \cdot D(P, r_1) + r_2 \cdot D(P, r_2)}{2t} \\ &+ \frac{f_2 - f1}{2t} \\ =& \frac{1}{2}. \end{aligned} \tag{A.7}$$

Both mobile networks are assumed to have the same cost structures. Furthermore, their subscribers' utility functions, $\omega(p_i, R)$ and $\tilde{\omega}(P, r_i)$, are the same. They face the same demand functions, $d(p_i, R)$ and $D(P, r_i)$ because of symmetric meet-points and the symmetric retail prices of the fixed-line network.

Consequently the above equation (A.7) can be easily zero by adjusting the tariff of any mobile firm so that s_1 is almost $\frac{1}{2}$. Since $\pi_2 = 0$, mobile network 2 has an incentive to set its tariff to:

$$p_2 = c^o + \theta\tau$$
$$r_2 = c^t + (1 - \Theta)\tau$$

and

$$f_2 = f_1 + \varepsilon, \quad \varepsilon > 0$$

to get slightly less than half of the whole market. Even so, since $f_2 > f_1$ and $s_1 > s_2$, mobile network 1 does not have any incentive to deviate from this state. Therefore, cornered market equilibrium does not exist. ■

Proof of Proposition 6. After removing the common items in both equations (2.80) and (2.91), we must show:

$$\begin{aligned}
&\Big\{u(\hat{q}) + \tilde{u}(\hat{Q}) - p^{BK} \cdot \hat{q} - r^{BK} \cdot \hat{Q}\Big\} \\
&+ \Big\{U(\hat{Q}) + \tilde{U}(\hat{q})) - P^{BK} \cdot \hat{Q} - R^{BK} \cdot \hat{q}\Big\} \\
>&\Big\{u(q^{cpp}) + \tilde{u}(Q^{cpp}) - p^{cpp} \cdot q^{cpp}\Big\} \\
&+ \Big\{U(Q^{cpp}) + \tilde{U}(q^{cpp}) - P^{cpp} \cdot Q^{cpp}\Big\}
\end{aligned} \tag{A.8}$$

or

$$\begin{aligned}
&\Big\{(1+\mathcal{B})u(\hat{q}) - (p^{BK} + R^{BK}) \cdot \hat{q} + (1+\beta)U(\hat{Q}) - (P^{BK} + r^{BK}) \cdot \hat{Q}\Big\} \\
>&\Big\{(1+\mathcal{B})u(q^{cpp}) - p^{cpp} \cdot q^{cpp} + (1+\beta)U(Q^{cpp}) - P^{cpp} \cdot Q^{cpp}\Big\}.
\end{aligned} \tag{A.9}$$

By Lemma 7, we know that if the receivers' utilities are positive ($\mathcal{B} > 0$ and $\beta > 0$),

$$\hat{Q} \equiv Q(P^{BK}) > Q(P^{cpp})$$
$$\hat{q} \equiv q(p^{BK}) > q(p^{cpp})$$

Then, if the following equations hold, $\hat{TS}^{BK} > TS^{cpp}$.

$$P^{BK} + r^{BK} \leq P^{cpp}$$
$$p^{BK} + R^{BK} \leq p^{cpp}$$

Since $a^m > c^t$ if the receivers' utilities are positive[2], we know that:

$$P^{BK} + r^{BK} = C^O + \tau + c^t < C^O + \tau + a^m = P^{cpp}$$
$$p^{BK} + R^{BK} = c^o + \tau + C^T = c^o + \tau + C^T = p^{cpp}$$

Therefore, $\hat{TS}^{BK} > TS^{cpp}$ holds if the receivers' utilities are positive. ■

A Comment on the Noise Effect in Lemma 9

Given a meet-point level, total surplus decreases as the size of the support of noise, $[-b, b]$, increases, if $\beta < 1$. When demands are big enough for a given support of noise, this noise effect can be ignored. Even though demand is small for a given support of noise, the noise effect is not significant for large values of β. However, for small values of β and small demand levels, the negative effect of noise is significant. On the other hand, total surplus increases very slightly as b increases, if $\beta > 1$. This is derived from the properties of the utility functions which I assume in the Bill-and-Keep model: only the receiver's utility is subject to noise and the caller's utility is not. So, I want to focus on how utilities change as the support of noise changes.

In this model, the networks' profits do not depend on the support of the noise. The fixed-line network has no profit due to complete regulation and the mobile networks have only Hotelling profit because of the competition for subscribers. So, I therefore disregard the producer surpluses and focus on the consumer surpluses. Furthermore, because the case of fixed-to-mobile calls is symmetric with the case of mobile-to-fixed calls, I only explain the fixed-to-mobile calls. In subsection 3.3.1, consumer surpluses from making

[2]See Chapter 1.

and receiving calls are the following:

$$\Omega(P,r) - PD(P,r)$$
$$\tilde{\omega}(P,r) - rD(P,r)$$

At first, disregarding payments, let's examine utilities. The utilities of the callers and receiver are:

$$\Omega(P,r) = \{1 - \Psi(r - \beta P)\}U(Q(P)) + \int_{\underline{\epsilon}}^{r-\beta P} U\left(Q(\frac{r-\epsilon}{\beta})\right)\psi(\epsilon)d\epsilon$$
$$\tilde{\omega}(P,r) = \int_{r-\beta P}^{\bar{\epsilon}} \{\tilde{u}(Q(P)) + \epsilon Q(P)\}\,\psi(\epsilon)d\epsilon$$
$$+ \int_{\underline{\epsilon}}^{r-\beta P} \left\{\tilde{u}\left(Q(\frac{r-\epsilon}{\beta})\right) + \epsilon Q(\frac{r-\epsilon}{\beta})\right\}\psi(\epsilon)d\epsilon$$

For each equation, the first term is for a caller-region with large value of noise, in which the caller determines call volume. The second term is for a receiver-region with small value of noise, in which the receiver determines call volume.

Let's begin with a small support, $[\underline{\epsilon}, \bar{\epsilon}] = [-b_0, b_0]$. When ϵ is realized at the upper bound ($\epsilon = b_0$), call volume is highly likely to be determined by the caller since the receiver's marginal utility is high. Let's say this caller-determined volume is $Q(b_0)$:

$$Q(b_0) \equiv Q(P)$$

With $\epsilon = b_0$, the caller's and receiver's utilities are:

$$U(Q(P)) \qquad \text{and} \qquad \tilde{u}(Q(P)) + b_0 Q(P)$$

When ϵ is realized at the lower bound ($\epsilon = -b_0$), call volume is likely to be determined by the receiver, since the receiver's marginal utility is low. Let's say this receiver-determined volume is $Q(-b_0)$:

$$Q(-b_0) \equiv Q(\frac{r+b_0}{\beta})$$

With $\epsilon = -b_0$, the caller's and receiver's utilities are:

$$U(Q(\frac{r+b_0}{\beta})) \qquad \text{and} \qquad \tilde{u}(Q(\frac{r+b_0}{\beta})) - b_0 Q(\frac{r+b_0}{\beta})$$

Now, the support becomes the larger one, $[\underline{\epsilon}, \bar{\epsilon}] = [-b_1, b_1], \quad b_1 > b_0$. Even if ϵ is realized at the upper bound ($\epsilon = b_1$), caller-determined volume $Q(b_1)$ does not change:

$$Q(b_1) \equiv Q(P) \equiv Q(b_0)$$

Neither does the caller's utility:

$$U(Q(P)) = U(Q(b_1)) = U(Q(b_0))$$

But the receiver's utility is affected, because it includes ϵ. It increases since $b_1 > b_0$:

$$\tilde{u}(Q(P)) + b_1 Q(P) > \tilde{u}(Q(P)) + b_0 Q(P) \tag{A.10}$$

If ϵ is realized at the lower bound ($\epsilon = -b_1$), the receiver-determined volume $Q(-b_1)$ is smaller than before:

$$Q(-b_1) \equiv Q(\frac{r+b_1}{\beta}) < Q(\frac{r+b_0}{\beta}) \equiv Q(-b_0)$$

So is the caller's utility:

$$U(Q(-b_1)) = U(Q(\frac{r+b_1}{\beta})) < U(Q(\frac{r+b_0}{\beta})) = U(Q(-b_0)) \tag{A.11}$$

The receiver's utility is somewhat complicated.

$$\tilde{u}(q(\frac{r+b_1}{\beta})) - b_1 q(\frac{r+b_1}{\beta}) \tag{A.12}$$

Note that in the second term, b_1 is bigger than before but $q(\frac{r+b_1}{\beta})$ is smaller than before. Since the change in the first term, $\tilde{u}(q(\frac{r+b_1}{\beta}))$, is likely to be greater than the change in the second term, $b_1 q(\frac{r+b_1}{\beta})$, the receiver's utility decreases as b increases, but the negative term mitigates the decrease.

Summing up the above results: calling-utility Ω decreases as b increases because the upper bound of it does not change and the lower bound of it decreases as b increases. Receiving-utility $\tilde{\omega}$ increases with b, because the upper bound of it increases with b and the lower bound of it is likely to decrease slightly.

Now, let's see the change in the two payments: calling-payment and receiving-payment. By the above logic, the call volume $D(P, r)$ decreases as b increases. Prices are determined at cost level given specific meet-points. So, the payment amounts for making and receiving calls also decreases as b increases. Apparently, receiving-surplus, $\tilde{\omega}(P, r) - rD(P, r)$, increases with b. On the other hand, decreasing calling-utility is very likely to be dominant in the effect of b even though decreasing calling-payment reduces the effect. So, calling-surplus $\Omega(P, r) - PD(P, r)$ decreases as b increases.

For this linear specification of demand, the change in utility level is likely to be greater than the change in quantity level. So, the decrease in calling-utility is dominant in the whole effect of b on total surplus.

However, this is only when β is small. See equation (A.11). If β is big, the change in calling-utility cannot be dominant since the noise effect is discounted by a large β. On the other hand, increasing the receiving-utility has a larger effect as β increases. See equation (A.10). Remember that $\tilde{u} = \beta U$. β indicates the size of the receiving-utility. So, at some level of β, the effect of increasing the receiving-utility begins to dominate the decreasing calling-utility.

With a linear specification of demand and uniform distribution in Example 1, one can confirm the above arguments. If β is less than one, the receiving-utility slightly increases with b, but the increase is dominated by decreasing calling-utility. The change in payments cannot be larger than the change in utilities. So the total surplus decreases as b increases, if $\beta < 1$. However, the relative size of this negative effect of b decreases as β increases. With $\beta > 1$, the b has a slight positive effect on the total surplus.

Using a normal distribution, one can see that the noise effect is weaker since more weight is put around the center of the support. However, the basic results of a model using uniform distribution are not changed using normal distribution.

Proof of Lemma 10. In a symmetric equilibrium of the Bill-and-Keep model, the profits of all networks do not depend on the support of the noise. So, I consider only the effect of β on consumer surpluses. Because the case of fixed-to-mobile calls is symmetric with the case of mobile-to-fixed calls, I only explain the fixed-to-mobile calls. In subsection 3.3.1, the consumer surplus of caller and receiver are:

$$\Omega(P,r) - PD(P,r) \tag{A.13}$$

$$\tilde{\omega}(P,r) - rD(P,r) \tag{A.14}$$

The utilities and call volumes are determined after the noise is determined. Let's examine the demands, first.

$$\begin{aligned} D(P,r) \equiv & \{1 - \Psi(r - \beta P)\} Q(P) \\ & + \int_{\underline{\epsilon}}^{r-\beta P} Q(\frac{r-\epsilon}{\beta})\psi(\epsilon)d\epsilon \end{aligned} \tag{A.15}$$

In the caller-region with large noise, the call volume is:

$$Q(P)$$

In the receiver-region with small noise, the call volume is:

$$Q(\frac{r-\epsilon}{\beta})$$

As β decreases and approaches zero, this receiver-region volume approaches zero, too. It is irrelevant whether the value of the noise is high or low. Since a smaller β increases the probability of the receiver-region, one can say that the call demand $D(P,r)$ goes to zero as β goes to zero.

The utilities are slightly different but fundamentally the same.

$$\Omega(P,r) = \{1-\Psi(r-\beta P)\}U(Q(P)) + \int_{\underline{\epsilon}}^{r-\beta P} U\left(Q(\frac{r-\epsilon}{\beta})\right)\psi(\epsilon)d\epsilon \quad \text{(A.16)}$$

$$\begin{aligned}\tilde{\omega}(P,r) = &\int_{r-\beta P}^{\bar{\epsilon}} \{\tilde{u}(Q(P)) + \epsilon Q(P)\}\,\psi(\epsilon)d\epsilon \\ &+ \int_{\underline{\epsilon}}^{r-\beta P} \left\{\tilde{u}\left(Q(\frac{r-\epsilon}{\beta})\right) + \epsilon Q(\frac{r-\epsilon}{\beta})\right\}\psi(\epsilon)d\epsilon \quad \text{(A.17)}\end{aligned}$$

In the caller-region, the utilities of the caller and the receiver are:

$$U(Q(P))$$

$$\beta U(Q(P)) + \epsilon Q(P)$$

As β approaches zero, the receiving-utility approaches zero, too. On the other hand, in the receiver-region, the utilities of the caller and the receiver are:

$$U(Q(\frac{r-\epsilon}{\beta}))$$

$$\beta U(Q(\frac{r-\epsilon}{\beta})) + \epsilon Q(\frac{r-\epsilon}{\beta})$$

The utilities of both the caller and the receiver approach zero as β approaches zero. Combining the results of both regions, one can say that utilities of both the caller and the receiver, Ω and $\tilde{\omega}$ go to zero as β goes to zero. ∎

Proof of Lemma 13. Before assuming $\beta = \mathcal{B}$, the optimal meet-points are:

$$\hat{\Theta} = \frac{c^t + \tau - \beta C^O}{(1+\beta)\tau}$$
$$\hat{\theta} = \frac{C^T + \tau - \mathcal{B} c^o}{(1+\mathcal{B})\tau}$$

Note that the optimal meet-point for each direction of calls – fixed-to-mobile and mobile-to-fixed – might not be the same: $\hat{\Theta} \neq \hat{\theta}$.

One knows that the optimal meet-points are decreasing functions of β or $\mathcal{B}$

$$\frac{\partial \hat{\Theta}}{\partial \beta} < 0$$
$$\frac{\partial \hat{\theta}}{\partial \beta} < 0$$

One can analyze the market for each direction of calls separately, since they are not correlated in this model.[3] So, as β (or $\mathcal{B}$) increases from 0, first $\Theta = 1$ ($\theta = 1$) becomes the optimal meet-point for a smaller β ($\mathcal{B}$) and then $\Theta = \frac{1}{2}$ ($\theta = \frac{1}{2}$) becomes so for a bigger β ($\mathcal{B}$) for the fixed-to-mobile call market (for the mobile-to-fixed call market). Let's confirm it a different way.

If $\beta = 0$ and $\mathcal{B} = 0$, the optimal meet-points are greater than the

[3] See Chapter 2.

meet-point suggested by COBAK:

$$\hat{\Theta}_{\beta=0} = \frac{c^t + \tau}{\tau} > 1$$
$$\hat{\theta}_{\mathcal{B}=0} = \frac{C^T + \tau}{\tau} > 1$$

Using the formula of optimal meet-points, one can obtain the values of $\beta_{\theta=1}$ and $\mathcal{B}_{\Theta=1}$ which make the COBAK policy ($\theta = \Theta = 1$) an optimal policy.

$$\beta_{\Theta=1} \equiv \frac{c^t}{C^O + \tau}$$
$$\mathcal{B}_{\theta=1} \equiv \frac{C^T}{c^o + \tau}$$

On the other hand, for the MBAK policy ($\theta = \Theta = \frac{1}{2}$) to be an optimal policy, β and $\mathcal{B}$ should be:

$$\beta_{\Theta=\frac{1}{2}} \equiv \frac{c^t + \frac{1}{2}\tau}{C^O + \frac{1}{2}\tau}$$
$$\mathcal{B}_{\theta=\frac{1}{2}} \equiv \frac{C^T + \frac{1}{2}\tau}{c^o + \frac{1}{2}\tau}$$

One can see that:

$$\beta_{\Theta=1} < \beta_{\Theta=\frac{1}{2}}$$
$$\mathcal{B}_{\theta=1} < \mathcal{B}_{\theta=\frac{1}{2}}$$

This result reveals that as β (or $\mathcal{B}$) increases from 0, first COBAK provides the optimal surplus for a smaller β ($\mathcal{B}$) and then MBAK provides the optimal surplus with for a greater β ($\mathcal{B}$) for the fixed-to-mobile call market (for the mobile-to-fixed call market).

Now let's assume $\beta = \mathcal{B}$. Combining the two markets for both directions, the same relation between COBAK and MBAK clearly holds if $C^O = c^o$

and $C^T = c^t$ since $\hat{\Theta} = \hat{\theta}$. If the costs are different for fixed-line and mobile network, neither of COBAK nor MBAK can be an optimal meet-point policy for specific level of β. However, despite that, one can at least say that there exists a value $\bar{\beta} \in [\beta_{\Theta=1}, \beta_{\Theta=\frac{1}{2}}]$ that satisfies these conditions:

$$\begin{aligned} TS^{COBAK} &> TS^{MBAK} \text{ if } \beta < \bar{\beta} \\ TS^{COBAK} &= TS^{MBAK} \text{ if } \beta = \bar{\beta} \\ TS^{COBAK} &< TS^{MBAK} \text{ if } \beta > \bar{\beta} \end{aligned} \tag{A.18}$$

where TS means total surplus. ■

Proof of Lemma 14. Suppose each network faces the same demand and the origination cost is equal to the termination cost for each network.

$$n_0 = m_0$$
$$c^o = c^t = c \quad \text{and} \quad C^O = C^T = C$$

Given a level of β, the optimal meet-points for both direction of calls are:

$$\hat{\Theta} = \frac{(c - \beta C)}{(1+\beta)\tau} + \frac{1}{(1+\beta)}$$
$$\hat{\theta} = \frac{(C - \beta c)}{(1+\beta)\tau} + \frac{1}{(1+\beta)}$$

If the costs are symmetric, namely $c = C$, the optimal meet-points are symmetric.

$$\hat{\Theta} = \hat{\theta}$$

In this case, a Practical Bill-and-Keep can be an optimal policy for some value of β, because it requires symmetric meet-points. For example, COBAK requires:

$$\Theta^C = \theta^C = 1$$

There exists a value of β which makes COBAK an optimal policy.

$$\Theta^C = \theta^C = 1 = \frac{(C - \beta C)}{(1+\beta)\tau} + \frac{1}{(1+\beta)}$$

The β is:

$$\beta^C = \frac{C}{C+\tau}$$

By continuity, around this β^C, COBAK can be an optimal policy.

On the other hand, if the costs are not symmetric, namely $c \neq C$, optimal meet-points cannot be symmetric.

$$\hat{\Theta} \neq \hat{\theta}$$

In this case, no Practical Bill-and-Keep would be an optimal policy because they require symmetric meet-points.

Only for a symmetric cost structure can a Practical Bill-and-Keep be the optimal policy. ■

Appendix B

Graphs

Figure B.1: Optimal Bill-and-Keep and Support of Noise

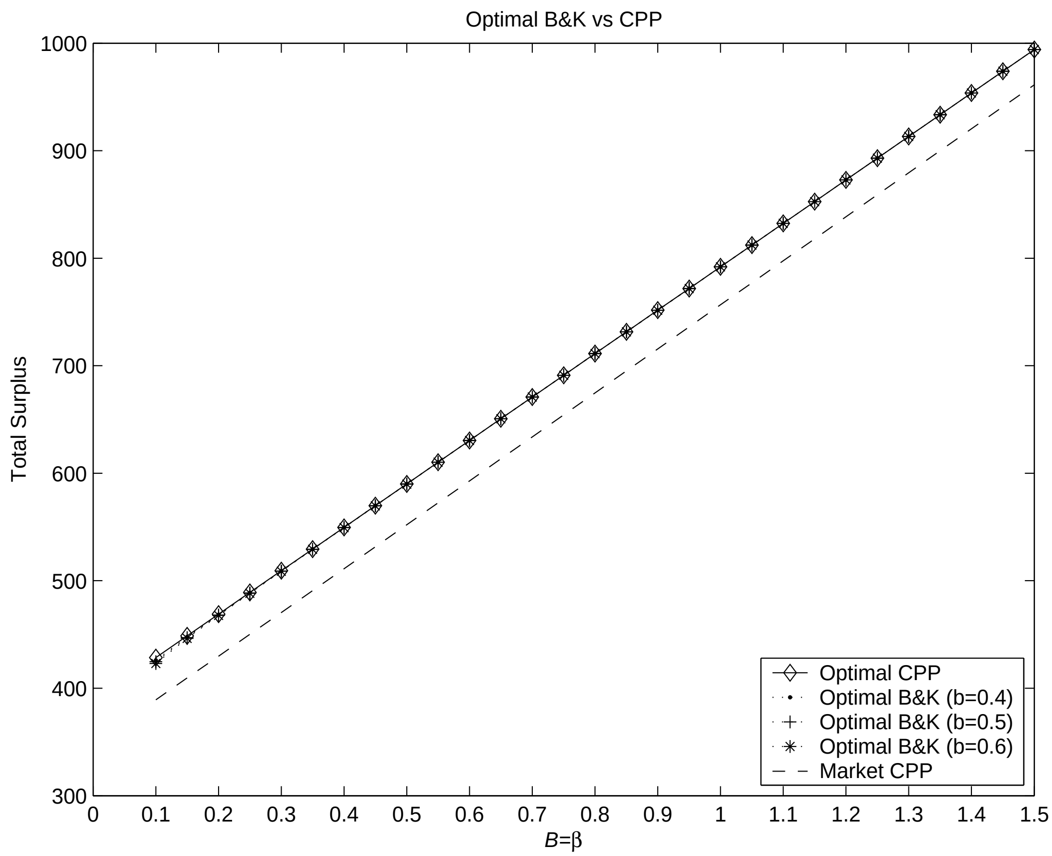


Figure B.2: Optimal Bill-and-Keep and Support of Noise (Small Demand)

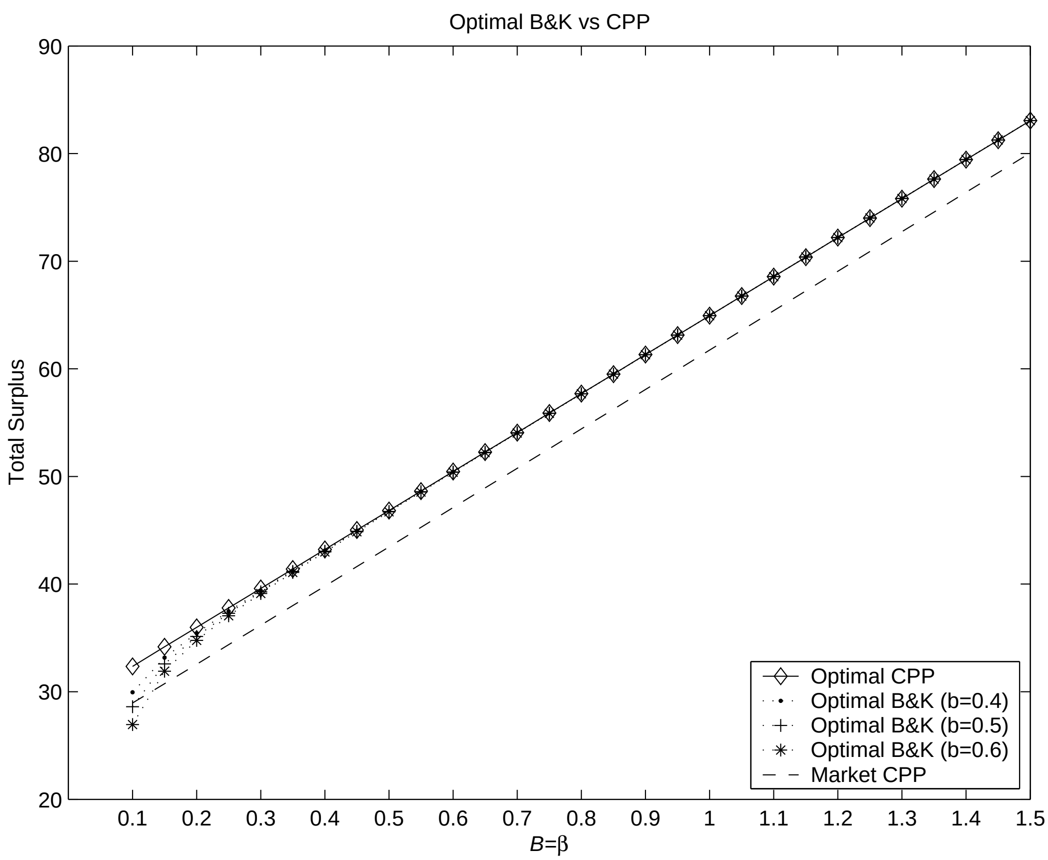


Figure B.3: Adjusted Optimal Bill-and-Keep and Support of Noise

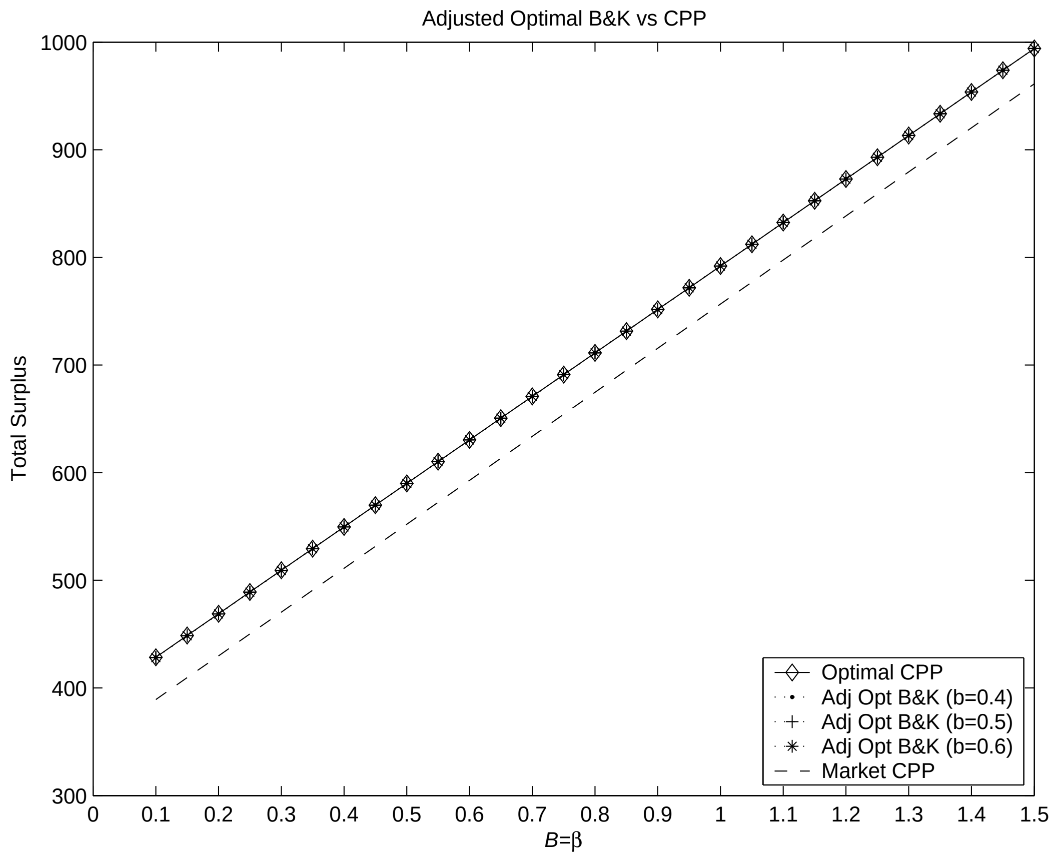


Figure B.4: Adjusted Optimal Bill-and-Keep and Support of Noise (Small Demand)

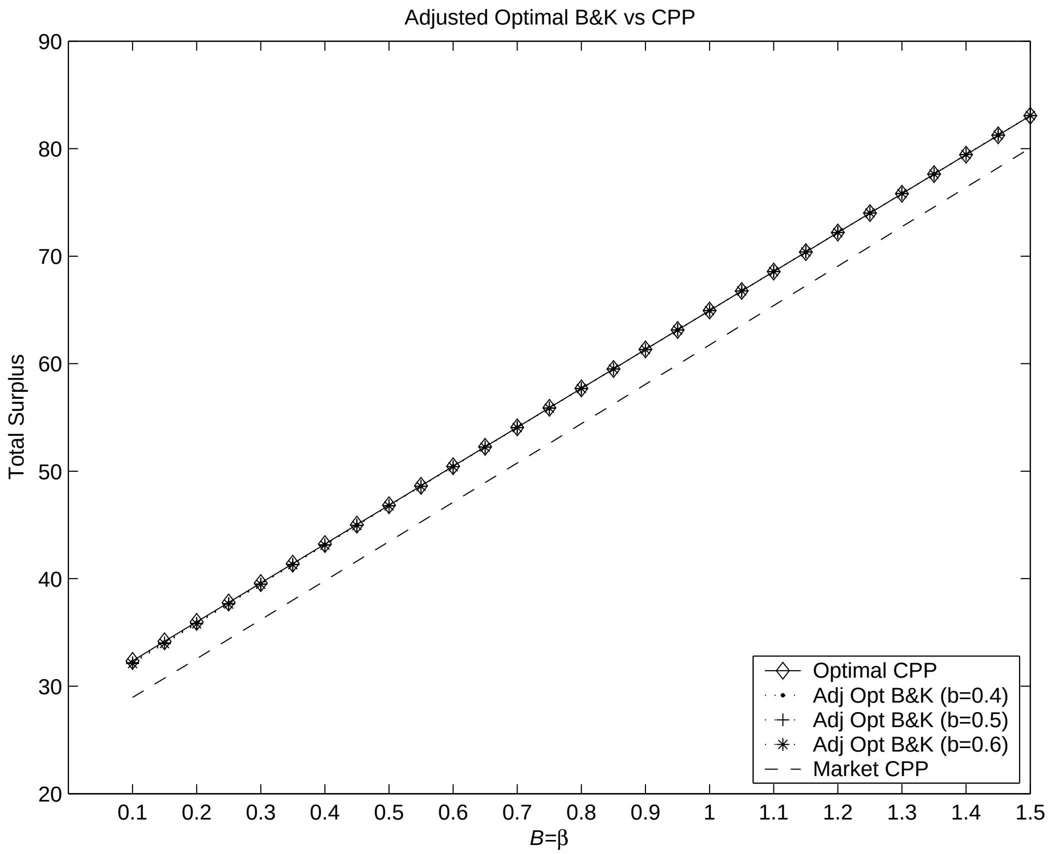


Figure B.5: COBAK and Support of Noise

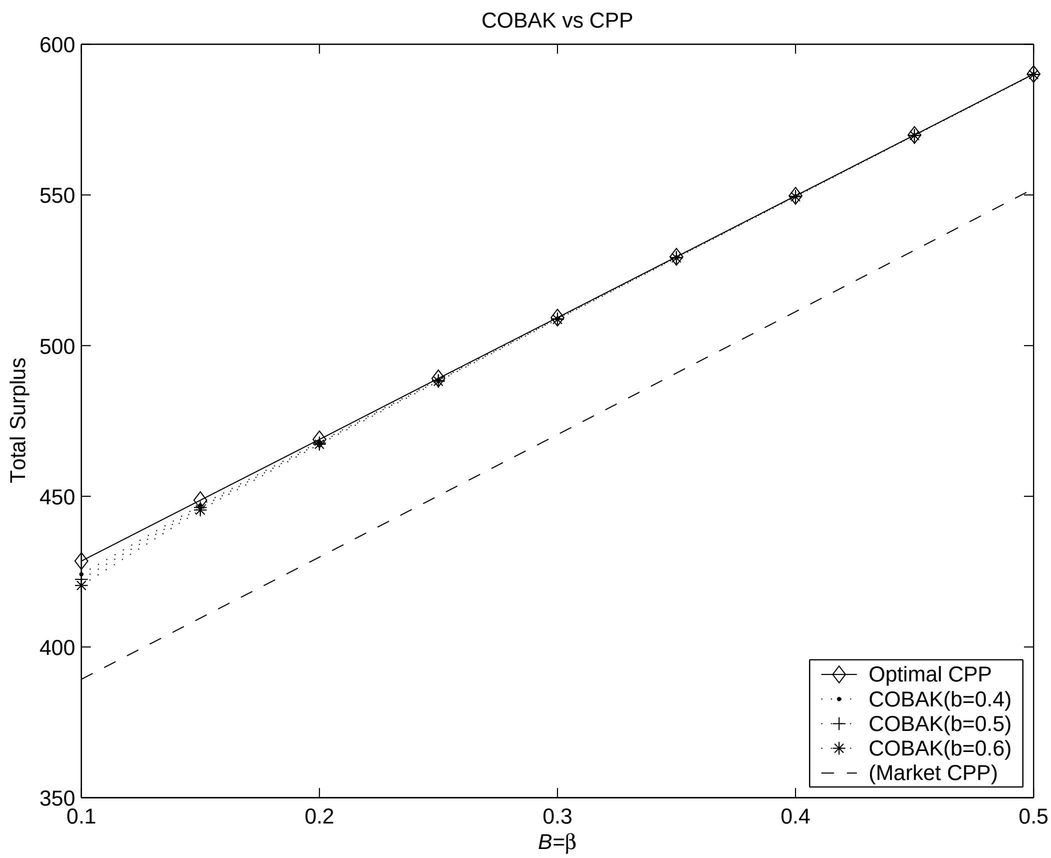


Figure B.6: COBAK and Support of Noise (Small Demand)

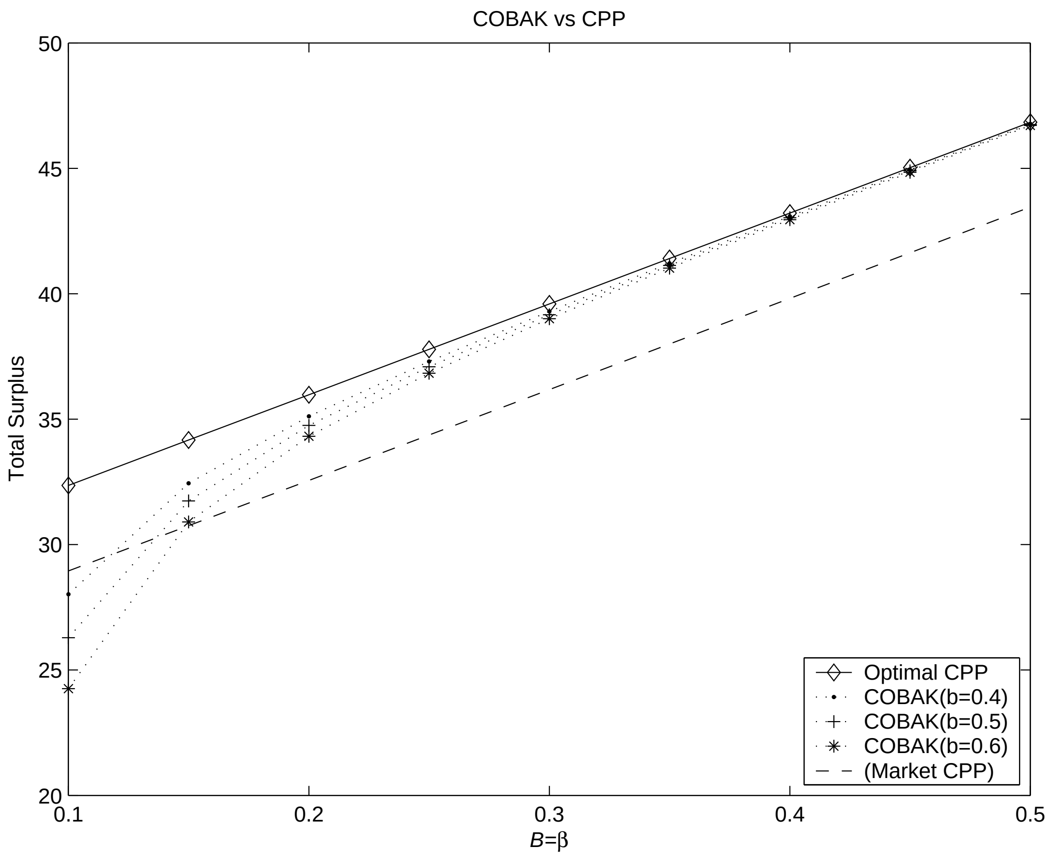


Figure B.7: MBAK and Support of Noise

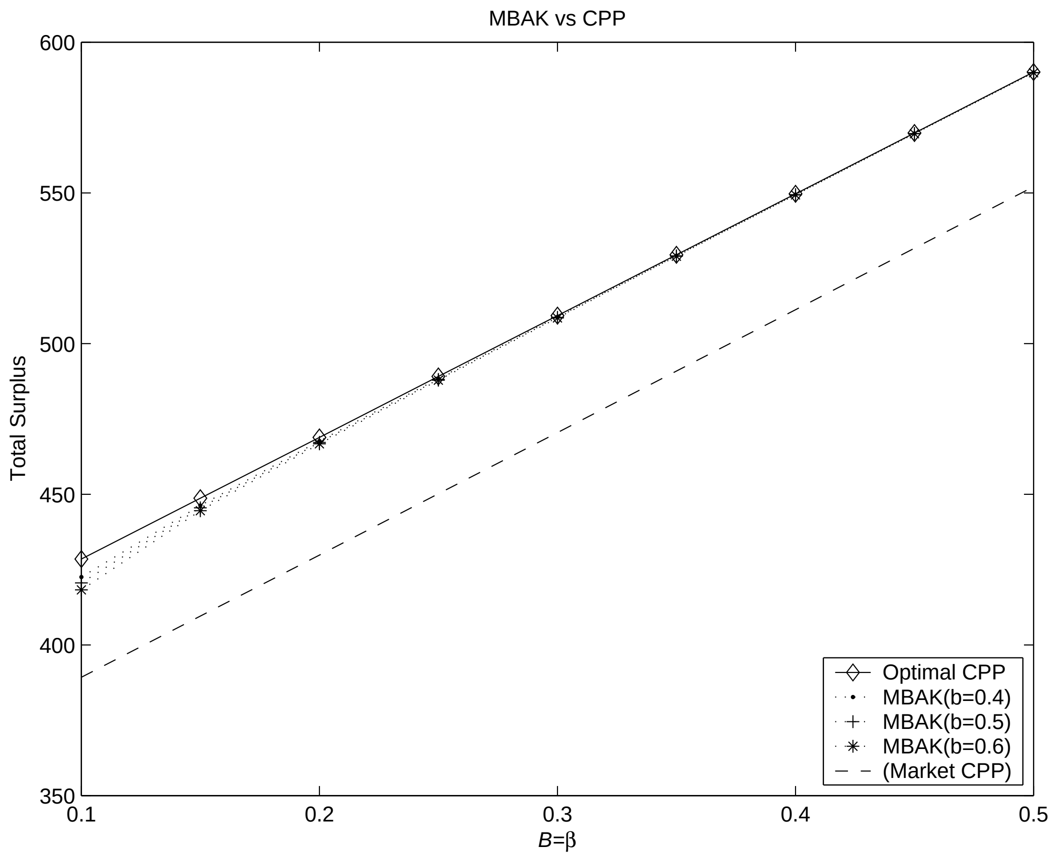


Figure B.8: MBAK and Support of Noise (Small Demand)

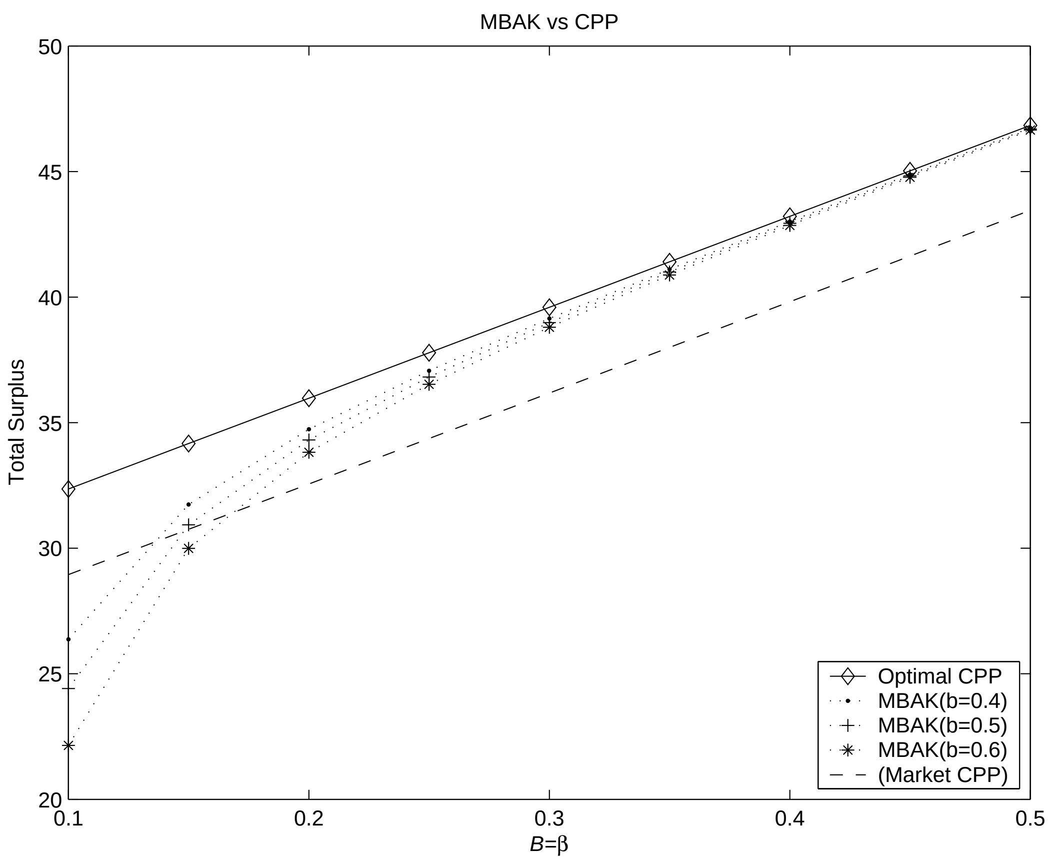


Figure B.9: Practical Bill-and-Keep and Market CPP

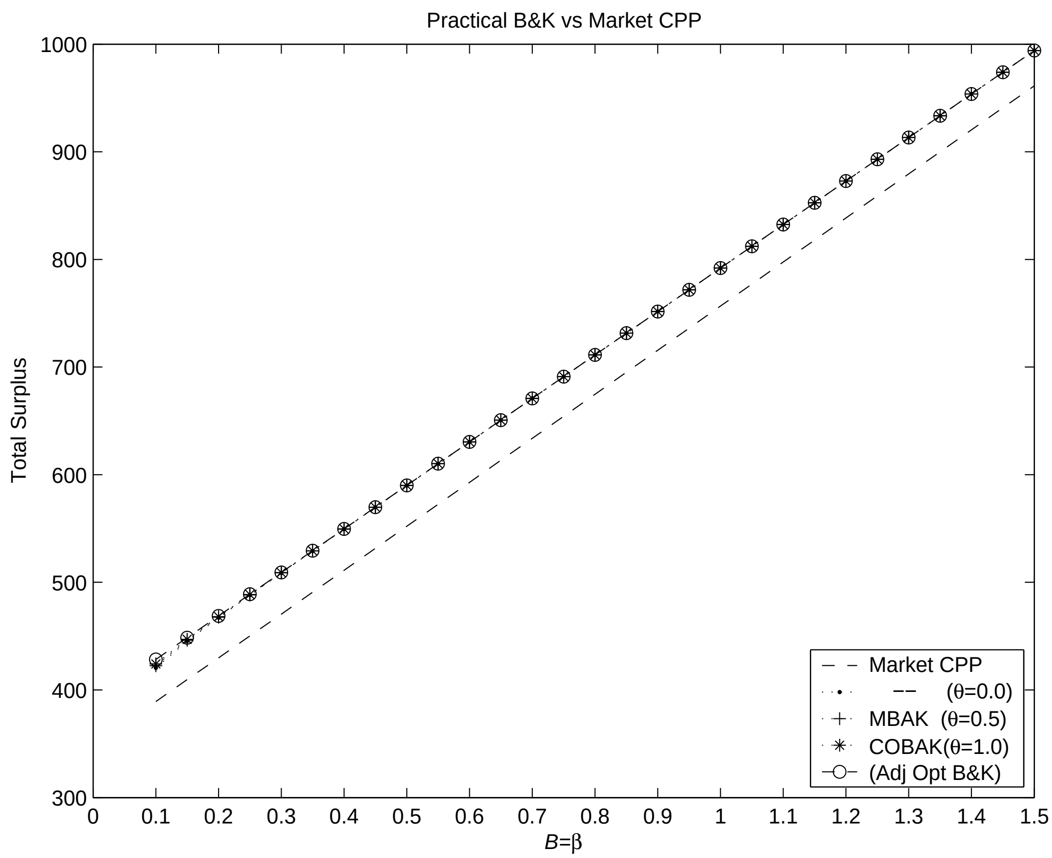


Figure B.10: Practical Bill-and-Keep and Market CPP (Small Demand)

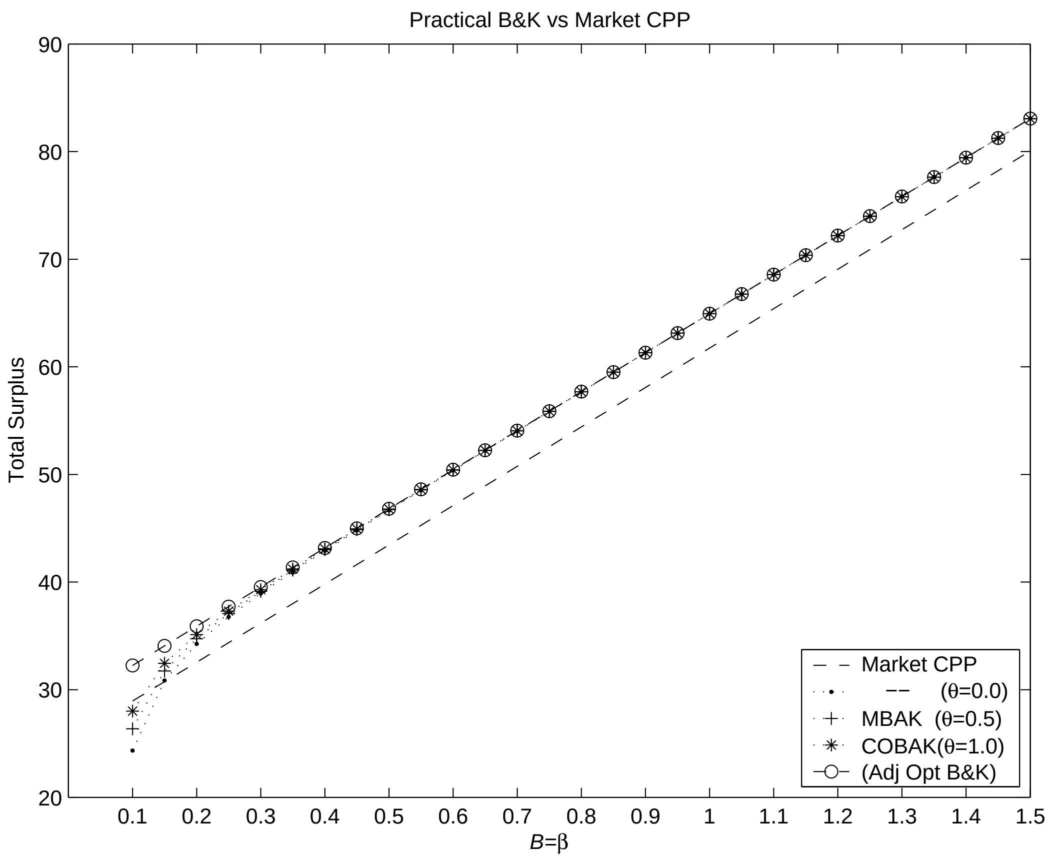


Figure B.11: Welfare Gain of Practical Bill-and-Keep

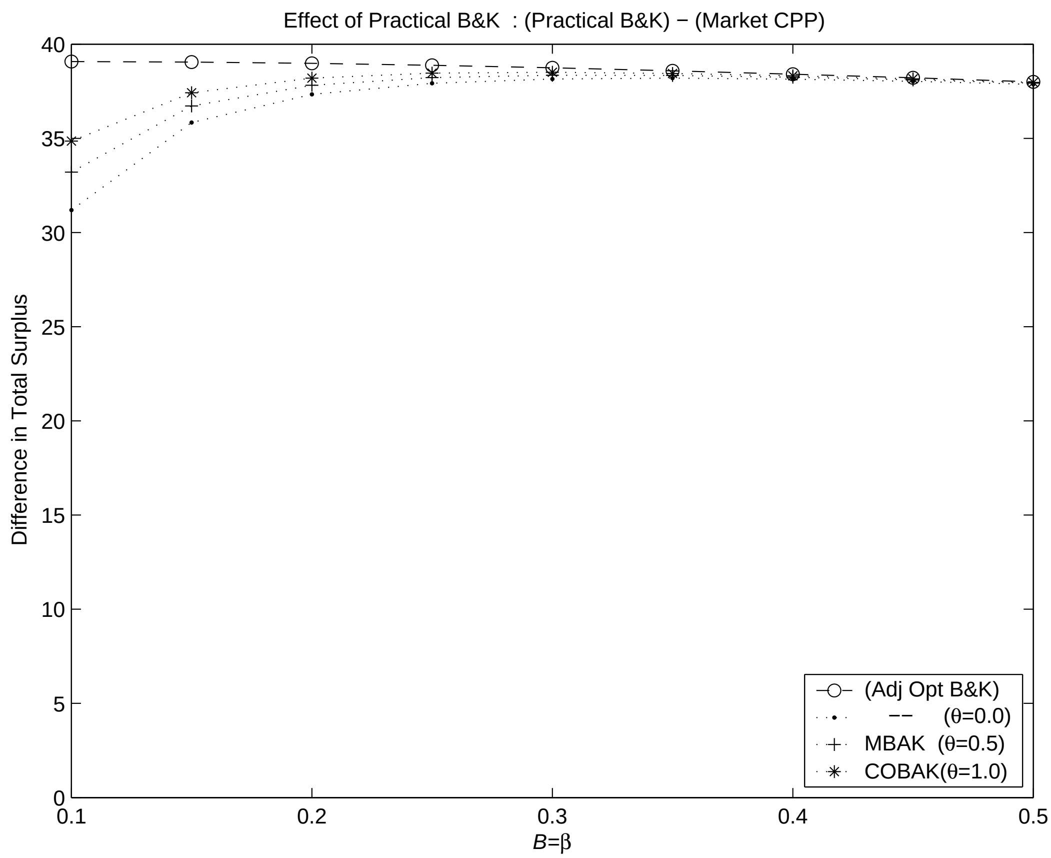


Figure B.12: Welfare Gain of Practical Bill-and-Keep (Small Demand)

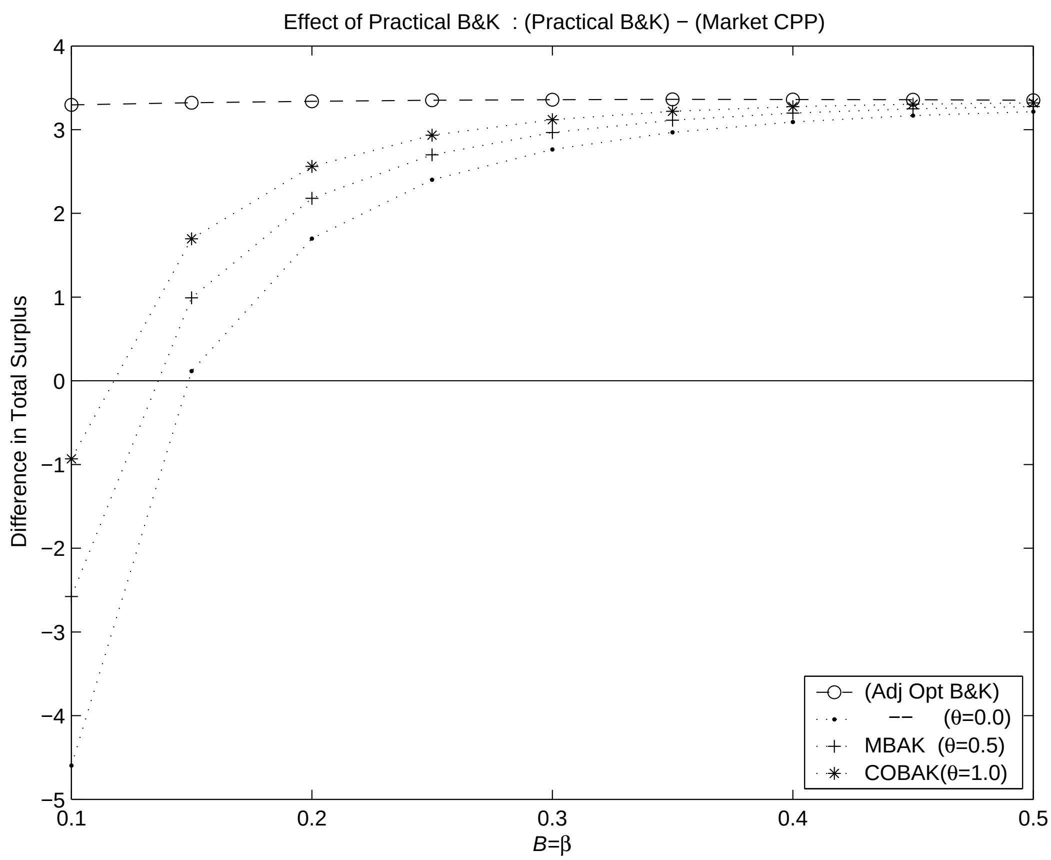


Figure B.13: Symmetric vs Asymmetric Costs : Difference from Optimality

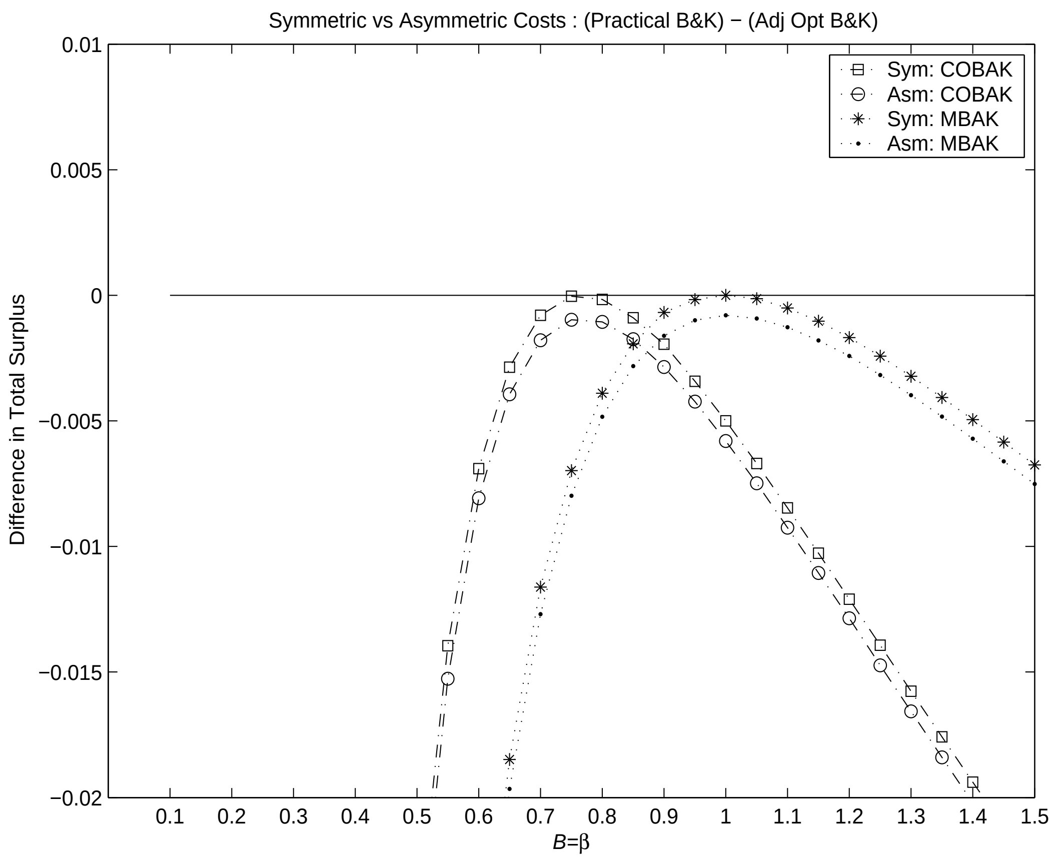


Figure B.14: Symmetric vs Asymmetric Costs : Difference from Optimality (Small Demand)

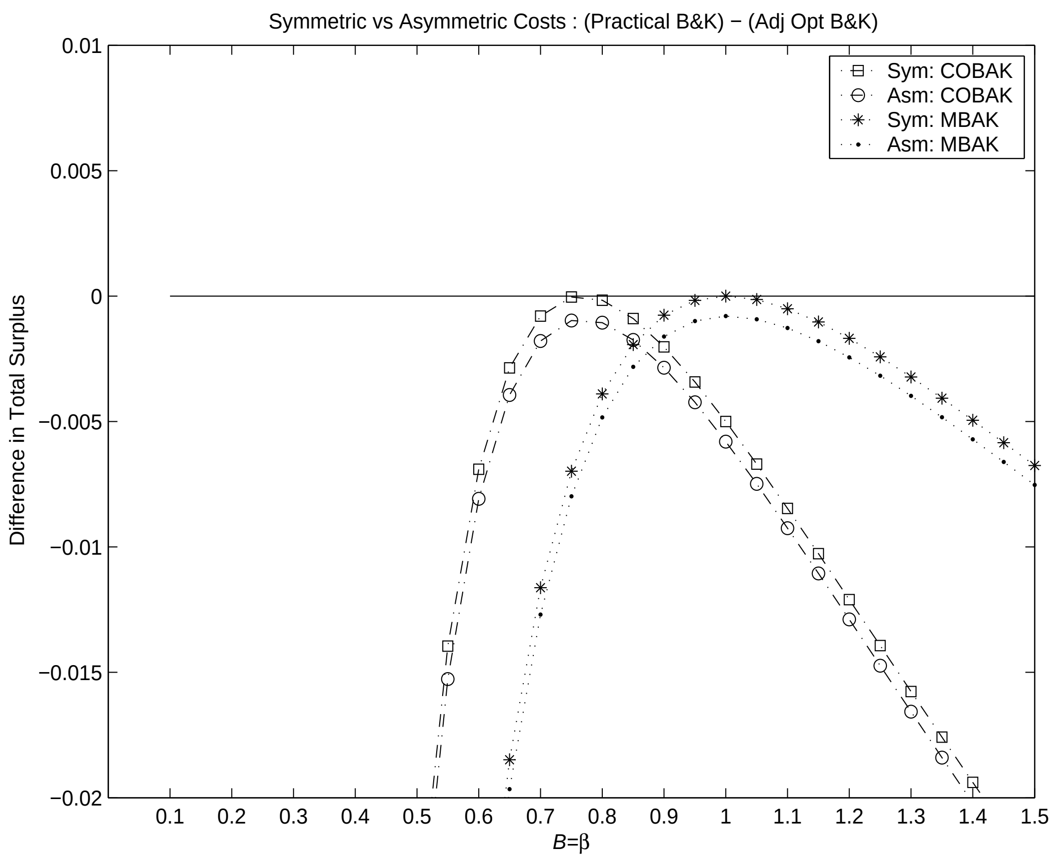


Figure B.15: Symmetric vs Asymmetric Costs : Difference from Market CPP

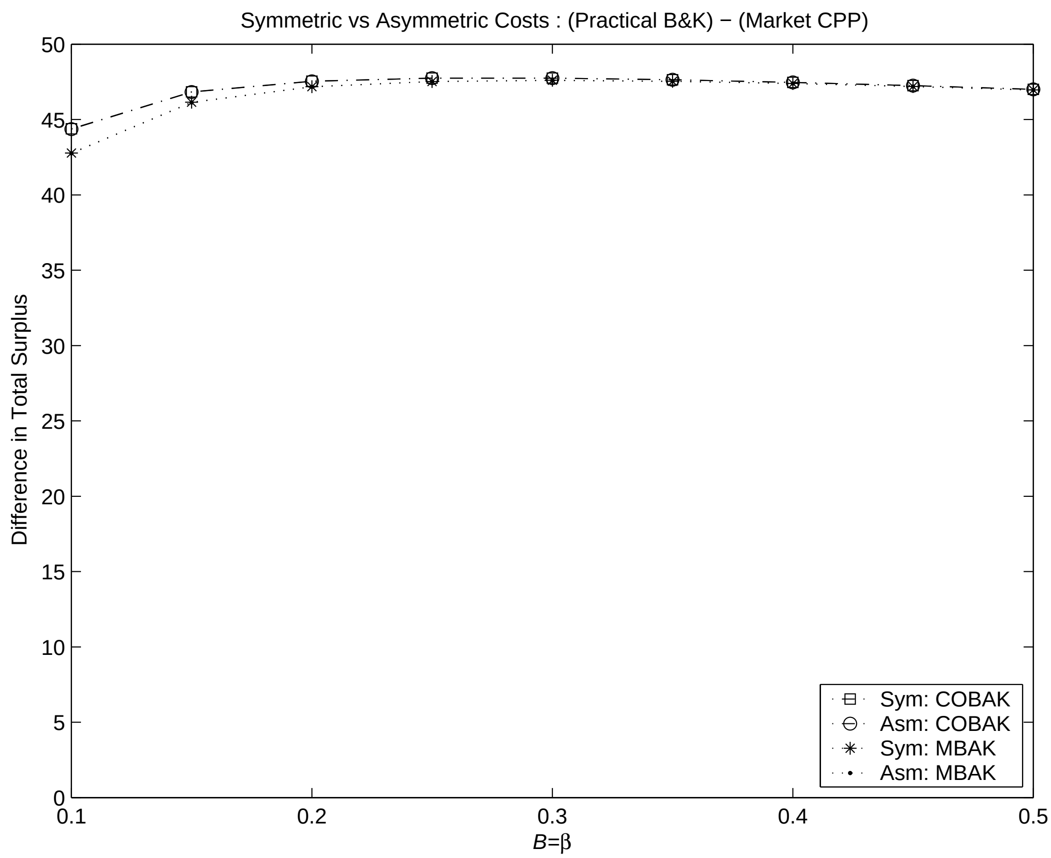
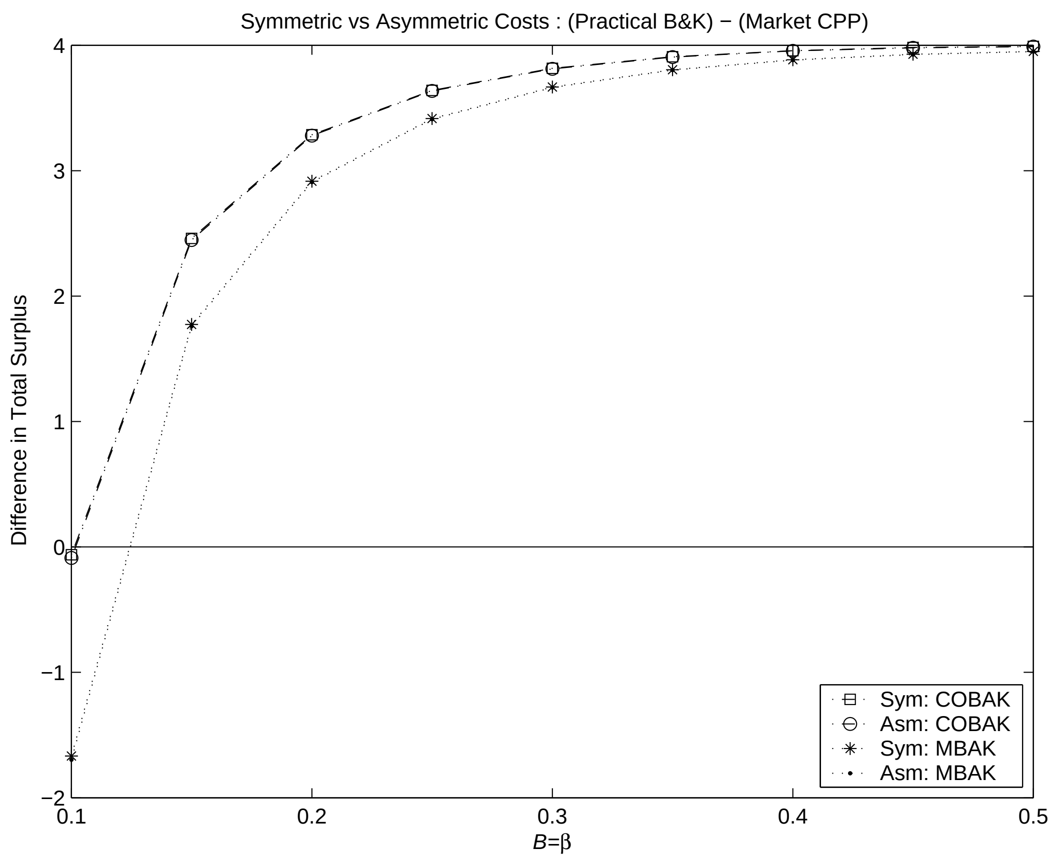


Figure B.16: Symmetric vs Asymmetric Costs : Difference from Market CPP (Small Demand)



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