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Bargaining, Searching and Price Dispersion in Consumption Good Markets

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Bargaining, Searching and Price Dispersion

in Consumption Good Markets

by

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Dedication

This thesis is dedicated to my mother who has supported me all the way since the beginning of my study.

This thesis is dedicated to my husband who has been a great source of motivation and inspiration.

This thesis is also dedicated to my daughter, who has already brought so much joy into my life, even though she has not been born yet.

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Bargaining, Searching and Price Dispersion

in Consumption Good Markets

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In consumption goods markets, we observe both bargaining and searching. However, in this literature, very little work has been done to incorporate both features into one model. This study addresses this problem.

In my first chapter, I add a bargaining parameter to a traditional sequential search model and solve for the new equilibrium in this set-up. Then, I do some comparative statics, changing the distribution of the bargaining parameter to see what happens to the equilibrium. Finally, I use the model to explain two seemingly contradicting empirical works in the literature of discrimination in the auto market. Ayres and Siegelman (1995), using data they collected from a controlled experiment, found that the initial offers for the minorities are higher. Yet Goldberg (1996), using consumer expenditure survey data (CES), reported that there is no significant difference between the final prices for minorities and non-minorities.

My model reconciles these two results and shows that if minorities have a more dispersed bargaining parameter distribution and if the final transaction prices are the same at the mean level, then the initial offer distribution for the minorities first-order stochastically dominates that for the non-minorities.

In my second chapter, I investigate how the bargaining process affects firms' offer distribution and thus the final price distribution. Based on Varian (1980), I add a bargaining parameter into the model, and solve for the new equilibrium in this set up. Then, I do some comparative statics, changing the distribution of the bargaining parameter to see what would happen to the equilibrium. This model yields the same results as the first chapter.

In the third chapter, I applied my theoretical model to the automobile market, and empirically test the model. I used CES data, and my findings support the theoretical model. The minority dummies are not significant in determining the mean level of consumers' bargaining ability distribution, but are significantly positive in determining the variance of the distribution.

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Bargaining, Searching and Price Dispersion in Consumption Goods Markets

Chapter One Abstract

In consumption goods markets, we observe both bargaining and searching. However, in this literature, very little work has been done to incorporate both features into one model. This study addresses this problem.

In this paper, we add a bargaining parameter to a traditional sequential search model and solve for the new equilibrium in this set-up. Then, we do some comparative statics, changing the distribution of the bargaining parameter to see what happens to the equilibrium. Finally, we use the model to explain two seemingly contradicting empirical works in the literature of discrimination in the auto market. Ayres and Siegelman (1995), using data they collected from a controlled experiment, found that the initial offers for the minorities are higher. Yet Goldberg (1996), using consumer expenditure survey data (CES), reported that there is no significant difference between the final prices for minorities and non-minorities. A similar situation also exists in the housing market: minority consumers face higher initial offers but end up with similar transaction prices.

Our model reconciles these two results and shows that if minorities have a more dispersed bargaining parameter distribution than non-minorities and if the final transaction prices are the same at the mean level, then the initial offer distribution for the minorities first-order stochastically dominates that for the non-minorities.

Section 1 – 1. Motivation and Literature Review

In consumption goods markets, price dispersion is very common. It has long been noticed that even for homogenous goods, the prices in the market may be dispersed because of information problem. When consumers do not know all the prices in the market, and have to search for lower prices at some searching cost, the price setting game is no longer a Bertrand or Cournot competition. It involves a more complicated payoff system, and leads to a more complicated price distribution in the market.

Stigler (1961) is the first in the literature to study people's searching behaviors. He studied consumers' behaviors in searching for a lower price as an example of information acquisition with searching costs. He assumed that there was price dispersions in the market, and stated that the consumers would search to the extent that the expected marginal return equal to the searching cost. He did not solve for the sellers' price dispersion. Instead, he only gave some intuition on the distribution in the market. Based on Stigler's work, economists began to pay attention to information problem and searching behaviors.

Based on Stigler (1961), soon there were models of sequential search with/without perfect recall, and models of simultaneous search. It has been shown that in the game of sequential search, no matter whether the search is with or without recall, consumers' optimal searching rule[1] is to continue to search if price is higher than the reservation price, and to stop searching otherwise, where the reservation price is determined implicitly by the price distribution in the market and consumers' searching costs. In cases of simultaneous search, on the other hand, consumers decide on the number of draws by equaling marginal cost of search to expected marginal benefits.

Rothschild (1973) pointed out that Stigler (1961) was just a partial equilibrium model, in

[1] Please refer to, say, McCall (1965) for a discussion of optimal stopping rule.

that it only considered the demand side of the market, and did not consider sellers' price setting behaviors. And Diamond (1971) found that, if all consumers have positive searching costs, in homogeneous goods market, the unique NE would be that all firms charge the (same) monopolistic price, and hence, there is no price dispersion.

The uniqueness can be proved by contradiction. If there exists an equilibrium where the lowest price is lower than the monopoly price, then ε plus the lowest price will lead to higher profit than the lowest price. This is because all consumers have positive searching cost, and if they reject the price that is ε higher than the lowest price, the best they can expect is the lowest price minus the searching cost. If ε is low enough, no consumer would reject. On the other hand, it is not possible for the highest price to be higher than monopoly price, because such a price will not only lower the profits made from each consumer but also deter consumers from buying. With the highest price not higher than monopoly price and lowest price not less than monopoly price, the only equilibrium case is that all firms charge the monopoly price.

In 1970s and 1980s, there was a large literature modeling consumers' searching behaviors and price dispersion, like Braverman (1980), Salop and Stiglitz (1977), Varian (1980), and etc. Varian (1980) assumed that some consumers were uninformed and others were informed, and it was exogenously determined whether a consumer was informed or not. The informed consumers would buy at the lowest price, while the uninformed ones would randomly pick up a store. Under these assumptions, he proved that there would be price dispersion and solved the price distribution out explicitly. He also made a free entry assumption and solved for the long-run zero-profit equilibrium price distribution.

Salop and Stiglitz (1977) modeled a scenario, in which, if a consumer buys information at some cost, he would be perfectly informed of all the prices and identities in the market. Therefore, if the unique equilibrium price in the market was so high, it would be profitable for some sellers to charge lower prices, knowing that consumers would rationally buy information. And in the equilibrium, there would be price dispersion, and consumers would be indifferent between buying information and not buying.

Stahl (1989) built up a model in which a proportion of consumers were "shoppers" who like to shop (gathering information) and hence were fully informed. In that paper, Stahl explicitly solved for the equilibrium price dispersion on the market. And he also linked his result to Bertrand's competition result when all consumers were shoppers and there was no information problem; and to Diamond (1971) local monopoly result when no consumers were shoppers. Stahl (1996) was a complementary work to Stahl (1989). In this latter work, Stahl assumed that consumers were heterogeneous in searching cost. With this small twist, the model became exponentially complicated, and he could not solve explicitly for equilibrium price distribution. In stead, he only discussed when equilibrium would exist, and gave some characteristics of the equilibrium.

All these works shared a similar idea that, if part/all of consumers have all/partial information, there will be price dispersion. Varian (1980) assumed that it was exogenously decided that which consumers were informed and which ones were not, while Salop and Stiglitz (1977) and Stahl (1989) endogenize the information acquisition decision and the let consumers choose between being informed or not. Actually, Varian (1980) can be viewed as a special case for the searching models, or the searching models can be viewed as an extension to Varian (1980). For example, Varian (1980) can be obtained from Stahl (1989) by assuming that the searching cost for *non-shoppers* was very high so that $\overline{p} = p^*$ in the market, where p^* was the monopoly price. Therefore, *non-shoppers* in Stahl (1989) would always choose not to search and become the "*Uninformed*" in Varian (1980).

All these works assumed that consumers were searching for lower transaction prices which were set by the firms. For most consumption goods markets in the U.S., this assumption is true. But for some durable goods markets, say, automobile market and housing market, consumers not only search for lower prices, but also bargain to push the price even lower. Back in our hometown in China, people not only bargain for durable goods like automobile, they also bargain for electronic devices, clothes, and even for vegetables. In these cases where bargaining exists, the transaction price may not be the original price set by the sellers. Knowing that they would get lower prices, consumers' searching behaviors may change, and sellers' price setting behaviors may change accordingly. Therefore, the price dispersion in the market would be different. If we only focus on transaction prices or initial offers, we are surely missing something important. In this paper, we would like to extend Stahl (1989) to include this aspect of the world.

There is a literature modeling both searching and bargaining in the labor market. It has long been noticed that when one party of negotiation has an outside option, he has better position in bargaining for better outcomes. Therefore, this literature mainly focuses on how searching affects the bargaining results. For example, Carpenter and Rudisill (2003) documented two cases of labor-management bargaining with outside options with similar negotiation structures but totally different results. They found that, overall, searching of one party triggers concessions from the other party of the negotiation. Wolinsky (1986) set up a model in which both firms and workers searched for better match in the labor market. And when they got one, they entered the bargaining stage. They could also search for outside options. If they got one, it would give them better bargaining position.

Also, as we study the literatures in housing market, we find that there are amount of literatures working on both information acquisition and bargaining in housing market, for example, Gan and Zhang (2006), Burdett, Shi and Wright (2001), and etc. These works assumed that the houses in the market are of heterogeneous characteristics and people searching in the market are of heterogeneous preferences. With similar set up as in labor market literatures, they concentrated on market thickness and the quality of matching between buyers and houses.

This paper differs from these literatures in that we study consumption goods markets, which differ from labor market in two aspects. First of all, in labor market, it is very natural to assume that both firms and workers search. But in a consumption good market, as pointed out by Stigler (1961), it is "empirically unimportant" for sellers to search. True, we have Cutco that would knock at your door to sell you a knife. However, in most cases, we do not have such luxury. Some would also argue that sellers would advertise their goods, which can be viewed in some extend as "searching". Although there is a big chunk of literature studying advertisement, the effects of advertisement and how it affects consumers' behavior are still not clear. Therefore, we would want to ignore the sellers' searching through advertising here.

What's more, our work differs from labor market searching-bargaining literature in that, in labor market, there are many sellers and the product is heterogeneous, and the quality of "match" between firms and workers matters. Firms are not searching for the highest productivity. A firm in need of an IT person to maintain a database does not have to have an Oracle expert. Similarly, workers are not searching only for highest salary. Rather, both sides are searching for the best "match". On the other hand, in the consumption goods markets, at least the new product is homogeneous. A Beetle from dealer A is the same as a Beetle from dealer B.

We will focus on how the bargaining results would affect consumers' searching behaviors and firms' price setting behaviors, which would in turn lead to different price dispersion in the consumption goods markets. And the paper proceeds as follows. In Section 1 – 2, we set up the model. In Section 1 – 3, we solve for the initial offer equilibrium and final price equilibrium. In Section 1 – 4, we will do some comparative static analysis and try to explain two seemingly controversial empirical results in the literature of discrimination in automobile market. Section 1 – 5 concludes the paper.

Section 1 – 2, Model Set Up

1 – 2 – 1, Preliminary Assumptions

In the supply side of the model, there are $N \geq 2$ firms selling a homogenous good. We will assume that the firms are homogenous in the sense that their marginal costs are constant and the same. And thus, we will further normalize their marginal costs to be zero, $mc_f = 0$, $\forall f = 1, 2,\ldots, N$. The firms may be different in the initial offers they are setting. And let p_f denote offer from firm f, $f = 1, 2,\ldots, N$. The N firms maximize their profits by setting offers simultaneously. And let $F(p)$ denote the Nash Equilibrium initial offer distribution, and let $\underline{p}$ and $\overline{p}$ be the lower and upper bound of support of $F(.)$.

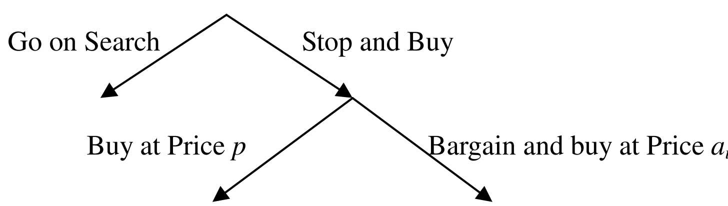
To formally define firms' profit functions, we need to examine consumer behavior first. On the demand side of the model, there are M consumers each with unit demand. The utility from consumption is assumed to be the same for all consumers, $u_i = 1$, $\forall\, i = 1,\ldots, M$. Consumers differ in their searching costs and bargaining parameters.

For the searching cost difference, we will assume that proportion $\mu \in (0,1)$ of consumers have searching costs being zero. And the other $1 - \mu$ proportion of consumers have searching costs $c > 0$. Following Stahl (1989), we will call them "*shoppers*" and *"non-shoppers"* respectively. As to the searching rule, we will assume sequential search with perfect recall. Following the traditional rule in searching literature, we will also assume that the consumers enjoy the first draw with no cost. As to the information set of the searching game, we will assume that the initial offer distribution in the market $F(p)$ is common knowledge, and so is the number of firms, N, and the costs of firms, mc.

So far, everything is the same as in traditional searching models. Now we will give the consumers the ability to bargain. In this work, we will mainly focus on how bargaining affects consumer behavior and firm behavior, and which in turn affect the offer price and transaction price distribution. Instead of giving a detailed model on bargaining process, we will assume that, somehow, the consumer i is endowed exogenously with a bargaining ability a_i which is identically independently distributed across all the consumers following a continuous distribution $G(a)$ with support $[\underline{a}, \overline{a}]$.

We will assume that the consumer, when facing an initial offer p, first chooses between buying from the firm and continuing to search. If he chooses to buy from the firm, he can then choose to bargain or not to bargain. If he does not bargain, the final price is p. And if he bargains, the final price will be a_i.[2] Please refer to Figure 1 – 1 for an illustration.

Figure 1 – 1, Consumers' Decision Tree



Then, it is easy to see that consumer i will bargain if and only if $a_i < p$. Therefore, the final price $Y(p,a_i)$ is defined as $Y(p,a_i) = \min\{ p,a_i \}$.

To ensure a closed-form solution to the model, the bargaining structure is very simple, and yet it still captures some features of bargaining. First of all, the final price is a non-decreasing and concave function of p. And it is very intuitive that, ceteris paribus, a higher initial offer would lead to a non-lower final price. The intuition for the concavity comes from the observation that, for some very low initial offer p, few consumers can bargain for even lower final price. In this case, $Y = p$. Yet for higher initial offer, there is room for bargaining, and thus $Y < p$.

[2] This assumption simplifies the model; otherwise, there would be no closed-form solution. We will discuss the alternative form later in Section 2-2.

Secondly, the final price is also a non-decreasing and concave function of a_i. That is, the higher the bargaining ability, the lower the a_i is, and thus the lower the final price. In that sense, a_i is best understood as bargaining "inability". The concavity comes from the observation that, for consumer with lower a_i, meaning higher bargaining ability, bargaining is more likely to happen, which induces a final price less than *p,* while as a_i increases, consumer *i* would more likely to buy the item without bargaining, and hence induces a final price *p*, and as a_i increases even further, the final price is no higher than *p*. As to the functional form how a_i affects the final price, it also depends on how one would define the bargaining ability and its scale. The limitation of the assumption is that there is no interaction term of initial offer and bargaining ability.

1 – 2 – 2, Consumer Behavior

Consumers differ in their bargaining ability as well as searching ability. For bargaining ability, we will assume that each consumer is endowed with a bargaining ability a_i exogenously, and a_i follows a continuous distribution $G(a)$ in $[\underline{a}, \overline{a}]$. Without loss of generality, we will assume $\overline{a} \leq 1$. Since $u_i = 1 \ \forall i$, it is reasonable that a consumer can at least bargain for a price lower than his utility level. For searching ability, we will assume that there are two types of consumers in terms of their searching costs, *shopper*s and *non-shoppers*, and we need to consider their searching rule separately.

For the *shoppers*, the searching rule is relatively simple. When facing an offer higher than the lowest possible offer, they would just reject the offer and keep searching since with perfect recall they can come back anyway. The problem is how they would behave when facing a "lowest possible" offer.

If we assume that they will stop and buy from that firm, there would be a pure strategy Nash Equilibrium where every firm sets the monopoly price, which is just the Diamond (1971) result. This is because the "lowest possible" offer is the "lowest" one in equilibrium beliefs, which is, in turn, consistent with the equilibrium behaviors of firms. And if all firms set monopoly price, and hence all *shoppers* have "rational beliefs" that there is only one monopoly price outside there in the market, they will stop and buy from the very first firm offering the monopoly price. And even if a firm sets lower price, no one would know it. As one can see, in this way, still no one has full information, and hence there is still no price dispersion. Therefore, following Stahl (1989), we will assume that *shoppers* will always search for all firms before buying.

When there is more than one firm offering the same initial offer, we will assume that the *shoppers* would randomly pick one. Since all these initial offers will lead to the same final price, *shoppers* are indifferent among them.

However, there is another case in which all initial offers lead to the same final price. It is when all offers are higher than the *shopper's* a_i, that is, $a_i \leq min\left\{p_f\right\}_{f=1}^{N}$. In this case, should the *shoppers* randomly pick one firm (call this option (1)) or should they buy from the lowest initial offer (option (2))?

Intuitively, option (1) would be more likely to "encourage" firms to set higher initial offers, because, even if a firm sets higher offer, in case of option (1), with some probability, it loses part of *shoppers*, but in case of option (2), with the same probability, it loses all *shoppers*. Therefore, intuitively, we would expect the resulting equilibrium offer distribution for option (1) to have higher expectation, lower variance, and to be more likely to collapse to $\overline{p}$.

We will adopt option (2) for two reasons. First, option (2) sounds more plausible. As a game model, when consumers are indifferent among all the offers, a "random draw" sounds better. But consider the real world. If one is to shop for an automobile and has got quotes from all dealers in the city, which dealer he would go back and bargain for a better price? Ceteris paribus, it is more likely that he would go back to the store with lowest quote, because that one seems easier to bargain with. Also, recall that we could have assumed the final price be (strictly) increasing in initial offer *p;* in that case consumers would for sure buy from the lowest offer.

The second reason is that option (2) is much easier to solve, and only under some rigid assumptions, will there exist equilibrium under option (1). Therefore, from now on, we will hold on to *Assumption S* as following, and assume that shoppers buy from lowest offers only.[3]

Assumption S[4]: Shoppers search every firm, and buy from the firm with lowest price. If there is more than one firm setting the lowest price, then shoppers randomly select one from among those setting the lowest price.

Next we consider *non-shoppers'* searching behavior. Given the equilibrium offer distribution, $F(p)$, *non-shoppers*' optimal searching rule is to continue searching if $p >$ *reservation price*, to stop searching if $p <$ *reservation price*, and either way if $p =$ *reservation price.* So the problem becomes how to define the *reservation price*. In addressing this question, we will first solve for the reservation price for a traditional searching model without bargaining, where final price equals to the initial offer $Y(p) = p$, if accepted. Then we will come back and solve for the *reservation price* for the *non-shoppers* in this model.

If this searching game is an infinite game[5], the standard way to solve for the reservation price would be to write down the Bellman equation first, and, at the reservation level, the utility of rejecting equals to the utility of accepting, which implicitly define the reservation price. But since this is a finite searching game[6], we need to solve by

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[3] We will discuss some intuitive inferences for option (1) from our results later.
[4] *S* stands for "shoppers".
[5] A searching game is an infinite game either because there are infinite firms in the market, or because we assume sampling with replacement.
[6] This is because we have finite number of firms, $N < \infty$.

backward induction. And it can be shown that the reservation price r is defined implicitly from equation (1 – 1) below. Please refer to Appendix 1 – 1 for details.

$$\int_{\underline{p}}^{r} F(x)\,dx = c \qquad (1-1)$$

Note that r is defined the same for all periods in equation (1-2-1) above. This result is actually proved by Kohn & Shavell (1974). They showed that in a static searching set up[7] as in this study, reservation price is independent of number of firms not sampled yet, so long as there is at least one more firm to sample. The key points that lead to this result are that (1) when consumer rejects the offer, his expected payoff would be the same, no matter what offer he rejects; and (2) with perfect recall, he can always go back and buy at the lowest price.

Also note that the searching rule is independent of u_i. This is because we have a constant searching cost rather than constant discount rate. In the later case, the actually cost of search again is proportion to the utility from the good, and it is not surprising that the utility level would enter the reservation price function.

Now that we have solved for reservation price for traditional searching model without bargaining, we are coming back and solve for *reservation price* for the *non-shoppers* in this model. Let r denote the reservation price from tradition searching model; i.e. r solves equation (1 – 1).

Proposition 1 – 2 – 1: reservation price = r if $r \leq a_i$; otherwise, non-shoppers will stop upon any offer and decide to bargain or not accordingly.

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[7] A dynamic set up, in contrast to the static set up, involves a learning process while searching. It is either because consumers are not sure about the price distribution on the market, or because they know "exact" price distribution. "Exact", in the sense that consumers know what prices are really charged in market. What they do not know is just which firm charges which price. Therefore, when they get quotes from firms, their remained price distribution actually changes from period to period. Stahl (1996) discusses the pro and con of this set up relative to a Nash equilibrium set up in which consumers only know the equilibrium price distribution.

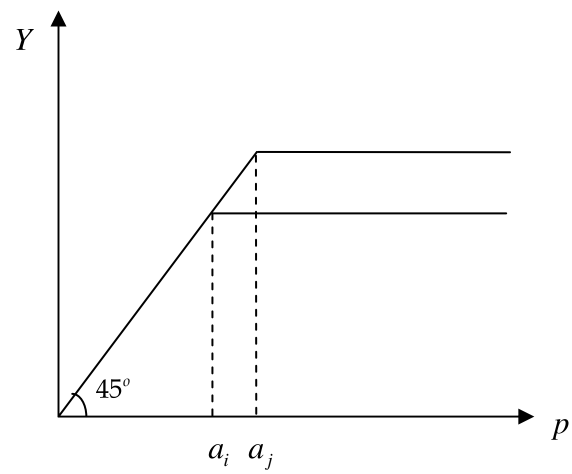
In the case where non-shoppers will stop upon any offer, for convenience, we will define *reservation price* = ∞**.** Please refer to Appendix 1 – 2 for the proof.

Note that for any non-shoppers whose $a_i > r$, their *reservation prices* are the same, independent of a_i. And those whose $a_i < r$ will stop at any offer. Therefore, it is easy to see that the searching behavior of *non-shoppers* is independent of a_i. The bargaining abilities only work as a filter, indicating which i has a *reservation price* being r and which has an infinite *reservation price*. Therefore, only one *reservation price*, r, is to be computed, which makes the searching part of the model easy to solve.

This convenience comes from the structure of the final price $Y(p, a_i) = \min\{p, a_i\}$[8]. Note first that the *non-shoppers* are actually searching for a lower final price, Y. For $p \geq a_i$, the final price will always be a_i, independent of p. Therefore, *non-shopper* i will be indifferent among all offers where $p \geq a_i$, and if he stops at p, he will stop at any offer where $p \geq a_i$. Otherwise, the final price Y is strictly increasing in p, then there are never two initial offers that lead to the same situation for the *non-shoppers*. Considering that different a_i will yield different Y, this set up will lead to the result that has different *reservation prices* for different bargaining abilities. And if *reservation prices* are indexed by i, the equilibrium will consist of two distributions: firms' initial offer distribution $F(p)$ and consumers' *reservation price* distribution, with firms' profit maximization behaviors mapping the *reservation price* distribution onto the *initial offer* distribution; and consumers' searching behaviors mapping the *initial offer* distribution onto the *reservation price* distribution. This situation has been discussed in Stahl (1995), and there was no closed-form solution generated, because the system was too complicated to solve. And Stahl only discussed when there existed equilibrium, and gave some properties for the equilibrium scenarios. Therefore, to ensure a closed-form solution, in this paper, we assume Y independent of p if bargaining happens.

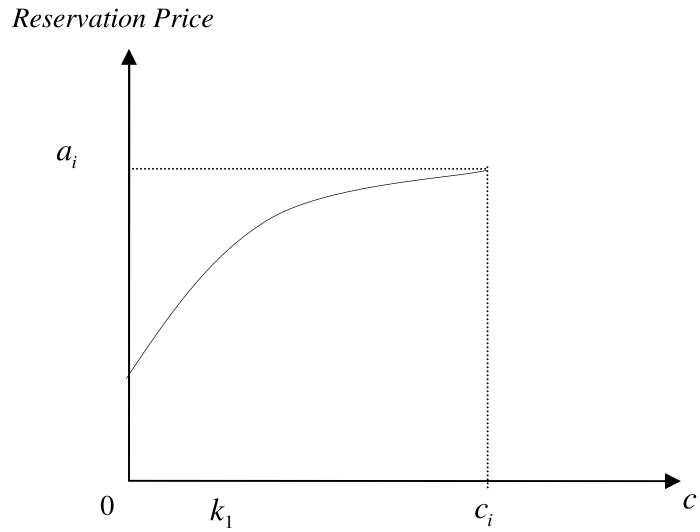
[8] Please refer to Figure 1 – 2 for an illustration of the final price as a function of initial offers for different levels of bargaining ability.

Figure 1 – 2, Final Price as a Function of Initial offer for Different Levels of Bargaining Ability



Before summarizing this section, let's take a look at the effect of searching cost on *non-shoppers'* reservation price. It is easy to check from equation (1 – 1) that r is strictly increasing in c, and thus *reservation price* also increases in c. But up to some point, *reservation price* would jump to ∞. This point is when *reservation price* = $r = a_i$. Define $c_i(a_i)$ as $c_i \equiv \int_{\underline{p}}^{a_i} F(x)\,dx$. The relationship of *reservation price* and searching cost can be viewed from Figure 1 – 3.

Figure 1 – 3, The Effect of Searching Cost on Non-Shoppers' Reservation Price



Summary of Consumers Searching Behaviors. *Shoppers* will search all firms and buy from the firm with the lowest offer. And if there is more than one firm offering $p = min\{p_f\}_{f=1}^{N}$, *shoppers* will randomly pick one. *Non-shoppers* will stop upon receiving any offer lower than or equal to their *reservation prices*, and continue searching otherwise[9]. *Reservation price* = r if $r \leq a_i$, where r solves $\int_{\underline{p}}^{r} F(x)\,dx = c$ (1 – 1); otherwise, *reservation price* = ∞.

[9] Actually, at reservation price, non-shoppers are indifferent from accepting and rejecting. And you will see that this specification does not change the equilibrium result.

Section 1 – 3, Solve for Symmetric Nash Equilibrium

In this section, we would solve for equilibrium initial offer distribution and final price distribution. To simplify the problem, we will focus on symmetric solutions.

1 – 3 – 1, Equilibrium Analysis

Proposition 1 – 3 – 1, There is no pure strategy Nash Equilibrium.

The idea is that if all firms would set the same initial offer $p > 0$, there would be profitable deviation by setting an offer a little bit lower than p to attract all the *shoppers;* and if $p = 0$, there would be profitable deviation by setting a positive offer to take profits from some of the *non-shoppers*. This proposition is proved in Appendix 1 – 3.

Let $\underline{p}$ and $\overline{p}$ be the lower and upper bound of support of $F(.)$. By the argument as in *Proposition 1 – 3 – 1,* $\underline{p} > 0$ and there is no mass point in $\left[\underline{p}, \overline{p}\right]$, because if there exists some mass point, one could profitably deviate by pricing a little bit lower.

Let $G(a)$ denote the population distribution of bargaining, and let $\underline{a}$ and $\overline{a}$ denote the lower and upper bound of this distribution.

Proposition 1 – 3 – 2, If $F(p)$ is a Nash Equilibrium initial offer distribution, then the upper bound of this distribution is $\overline{p} = min\{\overline{a}, r\}$.

Proposition 1 – 3 – 2 can be proved in three steps. First of all, it is easy to see that no firm can profitably deviate by choosing offers higher than r. Secondly, the expected payoff of $\overline{p}$ is strictly increasing on $\left[0, \overline{a}\right)$. Thirdly, $\overline{p} > \overline{a}$ can not be an equilibrium scenario. Please refer to Appendix 1 – 4 for detailed proof.

Proposition 1 – 3 – 3, There is no gap on $\left[\underline{p},\overline{p}\right]$.

Please refer to Appendix 1 – 5 for the proof.

1 – 3 – 2, Equilibrium Initial Offer Distribution

To summarize Section 1 – 3 – 1, the initial offer follows a continuous and atomless distribution $F(p)$ on $\left[\underline{p},\overline{p}\right]$, with $\underline{p} < \overline{p} = min\{r,\overline{a}\}$. In this section, we will solve for $F(p)$. First of all, we will list the firms' expected payoffs for all p in the support of $F(p)$. Secondly, we will calculate $\underline{p}$ and $F(p)$ as a function of $\overline{p}$. And finally, we will solve for $\overline{p}$.

1 – 3 – 2 – 1, Expected Payoffs

Let $E\pi(p)$ denote the expected payoff for initial offer p. If $p = \overline{p}$, only *non-shoppers* will stop at the firm. Therefore, the expected profit is

$$E\pi(\overline{p}) = (1-\mu)\frac{M}{N}\left[\int_{\underline{a}}^{\overline{p}} ag(a)\,da + \int_{\overline{p}}^{\overline{a}} \overline{p}g(a)\,da\right] \qquad (1-2)$$

If the firm sets $p = \underline{p}$, all *shoppers* would come and some lucky *non-shoppers* would also stop there, and hence the expected profit is

$$E\pi(\underline{p}) = \left[\mu M + (1-\mu)\frac{M}{N}\right]\left\{\int_{\underline{a}}^{\underline{p}} ag(a)\,da + \int_{\underline{p}}^{\overline{a}} \underline{p}g(a)\,da\right\} \qquad (1-3)$$

If the firm sets initial offer, $p \in \left(\underline{p},\overline{p}\right)$, the probability that it has the lowest initial offer would be $[1-F(p)]^{N-1}$. Therefore, only with probability $[1-F(p)]^{N-1}$ will *shoppers* buy from the firm, while with probability $1/N$ will a *non-shopper* visit the firm. Therefore, a firm with initial offer being $p \in \left(\underline{p},\overline{p}\right)$ has expected profit as below.

$$E\pi(p)=\left\{[1-F(p)]^{N-1}\mu M+(1-\mu)\frac{M}{N}\right\}\left[\int_{\underline{a}}^{p}ag(a)\,da+\int_{p}^{\overline{a}}pg(a)\,da\right] \qquad (1-4)$$

1 – 3 – 2 – 2, Equilibrium Distribution as a Function of $\overline{p}$

For the initial offer distribution to be a Nash Equilibrium, all *p* in the support of *F*(*p*) should yield the same level of expected payoff. Therefore, $\underline{p}$ and *F*(*p*) can be solved as a function of $\overline{p}$ by equaling the expected payoffs given above.

Equating (1 – 2) and (1 – 3), $\underline{p}(\overline{p})$ can be defined implicitly as in equation (1 – 5) below.

$$\int_{\underline{a}}^{\underline{p}}adG(a)+\underline{p}\left[1-G(\underline{p})\right]=\frac{1-\mu}{\mu N+(1-\mu)}\left[\int_{\underline{a}}^{\overline{p}}ag(a)\,da+\int_{\overline{p}}^{\overline{a}}\overline{p}g(a)\,da\right] \qquad (1-5)$$

Similarly, we can solve for *F*(*p*) in terms of $\overline{p}$ from equation (1 – 2) and (1 – 4):

$$F(p;\overline{p})=1-\left\{\frac{(1-\mu)}{\mu N}\varphi(p)\right\}^{\frac{1}{N-1}} \text{ where } \varphi(p)=\frac{\int_{\underline{a}}^{\overline{p}}ag(a)\,da+\int_{\overline{p}}^{\overline{a}}\overline{p}g(a)\,da}{\int_{\underline{a}}^{p}ag(a)\,da+\int_{p}^{\overline{a}}pg(a)\,da}-1 \qquad (1-6)$$

1 – 3 – 2 – 3, Solve for $\overline{p}$

Now comes the question of how to solve for $\overline{p}$. Recall that $\overline{p}=min\{r,\overline{a}\}$. If $r>\overline{a}$, $\overline{p}=\overline{a}$. If $r\le\overline{a}$, $\overline{p}$ = *r*. $\underline{p}$ and *F*(*p*) are then defined accordingly. So, it is important to find out whether *r* is less than or greater than $\overline{a}$. Recall that *r* is defined implicitly from $\int_{\underline{p}}^{r}F(x)\,dx=c$ (1 – 1). Also remember that *F*(*p*) and $\underline{p}$ are both functions of $\overline{p}$. Therefore, $\overline{p}$ = *r* can be solved from equation (1 – 7) below.

$$\int_{\underline{p}(\overline{p}=r)}^{r}F(x;\overline{p}=r)\,dx=c \qquad (1-7)$$

First, we prove that the left hand side of equation (1 – 7) is strictly increasing in *r*[10] on $\left(\underline{p},\overline{a}\right)$. That is, each *c* corresponds to a unique *r* in $\left(\underline{p},\overline{a}\right)$, and vice versa; and the higher the *c* is, the higher the corresponding *r*. Therefore, whether *r* is less than or greater than $\overline{a}$ is equivalent to whether *c* is less than or greater than some critical point. If *c* is large enough, and there is no solution in $\left(\underline{p},\overline{a}\right)$, in which case we will define $\overline{p}=\overline{a}$; otherwise, *r* will be lower than $\overline{a}$, and $\overline{p}=r$.

1 – 3 – 2 – 4, A Critical *c*, $\overline{c}$

Let $\overline{c}$ be the critical point of searching cost *c* where $r\left(\overline{c}\right)=\overline{a}$, and since *r*(*c*) is strictly increasing, by substituting *r* = $\overline{a}$ into equation (1 – 7), we define $\overline{c}$ as in equation (1 – 8) below. On the right hand side of the equal sign "=", we substitute $F\left(x;\overline{p}=\overline{a}\right)$ using equation (1 – 6) above.

$$\overline{c} \equiv \int_{\underline{p}\left(\overline{p}=\overline{a}\right)}^{\overline{a}} F\left(x;\overline{p}=\overline{a}\right)dx = \int_{\underline{p}\left(\overline{p}=\overline{a}\right)}^{\overline{a}} \left\{1-\left\{\frac{\left(1-\mu\right)}{\mu N}\varphi\left(p\right)\right\}^{\frac{1}{N-1}}\right\}dp \qquad (1-8)$$

$$where \ \varphi\left(p\right)=\frac{E\left(a\right)}{\int_{\underline{a}}^{p} ag\left(a\right)da+\int_{p}^{\overline{a}} pg\left(a\right)da}-1$$

It is worth pointing out that $\overline{a}$ is actually the initial offer a monopolist would charge. As discussed above, if $c \geq \overline{c}$, $\overline{p}=\overline{a}$; otherwise, $\overline{p}<\overline{a}$. Therefore, $\overline{c}$ can be viewed as a constraint in firms' profit maximization problem, imposed by consumers' searching behavior. If $c < \overline{c}$, consumers are likely to search, the constraint is binding, and $\overline{p}$ is solved from equation (1 – 7); otherwise, consumers do not search that much, the constraint is not binding, and $\overline{p}$ is the monopoly offer.

[10] Appendix 1 – 6 proves of the claim that *the left hand side of equation* (*1 – 7*) *is strictly increasing in r in* $\left(\underline{p},\overline{a}\right)$.

It is very easy to see that if $\bar{c}$ is high, the constraint is more likely to be binding; and if $\bar{c}$ is low, it is less likely to be binding. The level of $\bar{c}$ is important. We want to check equation (1 – 8) to see which factor(s) affect(s) $\bar{c}$.

First of all, *G*(*a*) affects $\bar{c}$. Changes in *G*(*a*) will lead to changes in *F*(*p*) and $\underline{p}$, and as *G*(*a*) changes, $\bar{a}$ may change, too. But the effect of *G*(*a*) is undetermined. It depends on how *G*(*a*) changes.

Secondly, μ affects $\bar{c}$. It can be shown that as μ increases, (1) $F\left(x;\bar{p}=\bar{a}\right)$ increases, or the integrand of equation (1 – 7) increases, and (2) $\underline{p}$ decreases, or the integration range broadens. Therefore, as μ increases, $\bar{c}$ increases, and the constraint is more likely to be binding. Recall that μ is the proportion of *shoppers*. If there are more shoppers in the market, the information problem is mild, and hence, firms cannot take much advantage.

Thirdly, *N* also affects $\bar{c}$. Same as μ, as N increases, (1) $F\left(x;\bar{p}=\bar{a}\right)$ increases, and (2) $\underline{p}$ decreases. Therefore, as *N* increases, $\bar{c}$ increases, and the constraint is more likely to be binding. In this case, the tighter constraint comes from the more severe competition in the market.

1 – 3 – 2 – 5, Another Critical *c*, $\underline{c}$

Another threshold value of searching cost of interest is the one that leads to $\bar{p}=\underline{a}$. Following the same argument as in Section 1 – 3 – 2 – 4, $\underline{c}$ is defined by substituting *r* = $\underline{a}$ into equation (1 – 7).

$$\underline{c} \equiv \int_{\underline{p}\left(\bar{p}=\underline{a}\right)}^{\underline{a}} F\left(x;\bar{p}=\underline{a}\right)dx = \int_{\frac{1-\mu}{1+(N-1)\mu}\underline{a}}^{\underline{a}} \left\{1-\left\{\frac{(1-\mu)}{\mu N p}\left[\underline{a}-p\right]\right\}^{\frac{1}{N-1}}\right\}dp \qquad (1-9)$$

Also as discussed above, if searching cost c does not exceed $\underline{c}$, then $\overline{p} = r \le \underline{a}$, which means that all initial offers would be lower than the best final price a consumer can bargain for, $\underline{a}$. Therefore, in this case, there is actually no bargaining. All *non-shoppers* would accept the very first offer they get, and all *shoppers* would shop around and buy at the lowest price. This is exactly Stahl (1989) result. This is because, in this economy, all consumers, *shoppers* and *non-shoppers*, have better searching ability than bargaining ability (lower c and higher a's.). And when all consumers select (implicitly) to exercise searching ability, firms are confined to lower prices and hence lower profits. Since this scenario yields exactly the same results as in traditional searching models, we will assume away this case and concentrate on the case where $c \ge \underline{c}$

<u>Summary for Section 1 – 3 – 2.</u> There are three possible ranges for $\overline{p}$, $\overline{p} \le \underline{a}$, $\underline{a} < \overline{p} < \overline{a}$ and $\overline{p} = \overline{a}$, corresponding to three ranges for searching cost c, $c \le \underline{c}$, $\underline{c} < c < \overline{c}$, and $\overline{c} \le c$. When $\overline{p} \le \underline{a}$, the results would be all the same as in Stahl (1989). In this paper, we will focus on the later two cases and assume that it is always the case where $c \ge \underline{c}$. When $\underline{c} < c < \overline{c}$, $\overline{p} = r < \overline{a}$ is defined in equation (1 – 8). When $\overline{c} \le c$, $\overline{p} = \overline{a}$. After solving for $\overline{p}$, $F(p)$ and $\underline{p}$ can then be solve from equation (1 – 6) and (1 – 7).

1 – 3 – 3, Equilibrium Final Price Distribution

For the case where $c \ge \underline{c}$, bargaining happens. Therefore, the final price distribution would be different from the initial offer distribution. Let $H(y)$ denote the final price distribution. $H(y)$ can be expressed as in equation (1 – 10) below.

$$\begin{aligned} H(y) &= Pr(Y \le y) = Pr\left(min\{p, a_i\} \le y\right) \\ &= \mu\left[G(y) + (1 - G(y))Pr\left(min\{p_f\}_{f=1}^{N} \le y\right)\right] + (1-\mu)\left[G(y) + (1 - G(y))Pr(p \le y)\right] \\ &= G(y) + (1 - G(y))\left\{\mu\left(1 - [1 - F(y)]^N\right) + (1-\mu)F(y)\right\} \end{aligned}$$

(1 – 10)

Note that the final price distribution above can be viewed as a weighted average of *1* and the final offer consumers accept, with $G(y)$ being the weight. In $\left(\underline{p}, \underline{a}\right]$, $H(y)$ is the same as the distribution of final offers consumers (including both *shoppers* and *non-shoppers*) stop at. And as y goes up, $G(y)$ increases, and the cumulative density function $H(y)$ is going faster to 1 than the cumulative density function of consumers' final offers.

It looks difficult to solve for the expected final price from $E(y) = \int z dH(z)$. Yet please note that the expected amount of money paid by consumers should be equal to the expected profits for firms. Therefore, we can calculate the expected final price as below.

$$E(y) = \frac{N}{M} E(\pi) = \frac{N}{M} E\pi(\overline{p}) = (1-\mu)\left[\int_{\underline{a}}^{\overline{p}} a g(a)\, da + \int_{\overline{p}}^{\overline{a}} \overline{p} g(a)\, da\right] \qquad (1-11)$$

For the case where $\underline{a} < \overline{p} < \overline{a}$, $E(y)$ = $(1-\mu)\left[\int_{\underline{a}}^{\overline{p}} a dG(a) + \int_{\overline{p}}^{\overline{a}} \overline{p} dG(a)\right]$ < $(1-\mu)E(a)$, and for the case where $\overline{p} = \overline{a}$, the expected final price now is $E(y) = (1-\mu)E(a)$.

As discussed in Section 1 – 3 – 2, $\overline{p} = \overline{a}$ is the case where the constraint from consumers' searching behavior is not binding. It can be seen that, in this case, only μ and the expectation of the bargaining ability enters firms' expected profit function/expected final price function. *Non-shoppers*' searching behavior, on the other hand, does not affect $E(y)$. This is because, in this case, *non-shoppers*' searching cost c is high, imposing no constraint on firms' profit maximization problem. Only the existence of *shoppers* and consumers' bargaining behavior would lower firms' profit.

For the case where $\overline{p} < \overline{a}$, *non-shoppers*' searching constraint is binding. Therefore, firms' profit in this case is lower, and it can be seen that r, or reservation price, enters firms' expected profit function.

Section 1 – 4, Comparative Static

The purpose of this paper is to see how bargaining would change the initial offer distribution and final price distribution in the market. In Section 1 – 3, we solved for the equilibrium, and in this section, we will do a comparative static study, and see how would changes in $G(a)$ affect $F(p)$ and $H(y)$. More specifically, we will assume $G(a)$ changes from $G^c(a)$ to $G^d(a)$, so that $G^c(a)$ second order stochastically dominates $G^d(a)$[11]. Therefore, we know that $E^c(a) = E^d(a)$, and $G^d(a)$ looks more dispersed than $G^c(a)$. For example, $G^d(a)$ can be achieved by applying a mean-preserving spread process from $G^c(a)$. We want to examine what would happen to the equilibrium after the change. For simplicity, we will assume that both $G^c(a)$ and $G^d(a)$ are defined in $[\underline{a}, \overline{a}]$ = [0,1]. We will still use the notation $[\underline{a}, \overline{a}]$ so as to make the explanation more clear. What's more, given this assumption, we will always be in the case where $\underline{a} \le \underline{p}$; that is, there exist some good bargainers who can bargain for lower prices at any possible initial offer level. Then we do not need to consider the lower part of $F(p)$.

In this whole section, we will use superscript "*c*" to denote the original bargaining ability distribution and whatever associated with the original distribution, and use "*d*" to denote the new bargaining ability distribution and whatever associated with the new distribution. For example, we will use $F^c(p)$ to denote the original initial offer distribution, and use $F^d(p)$ to denote the new initial offer distribution.

Also, in this section, we will assume that $\underline{c} \le c$, because, otherwise, the equilibrium would be all the same as in traditional searching model without bargaining. And one can refer to Stahl (1989) for more detailed discussion of the equilibrium properties.

[11] The superscript "*c*" stands for "*centered*" and "*d*" stands for "*dispersed*". We are assuming "*c*" second order stochastically dominates "*d*", so case "*d*" is more dispersed.

1 – 4 – 1, Equilibrium^c vs. Equilibrium^d

Recall that, when solving for the equilibrium, the most important point is to solve for $\overline{c}$, so as to examine whether $c > \overline{c}$ or not. Therefore, as $G(a)$ changes from $G^c(a)$ to $G^d(a)$, the first question to ask is whether $\overline{c}$ increases or decreases. From equation (1 – 7), a change in $G(a)$ affects $\overline{c}$ through $\underline{p}$, and $F(p)$. As $G(a)$ becomes more dispersed, $\underline{p}$ will increase and $F(p)$ will decrease. As $\underline{p}$ increases, the range to integrate shrinks, and as $F(p)$ decreases, the integrand decreases. Therefore, it is easy to see that $\overline{c}$ will decrease[12], or, $\overline{c}^d < \overline{c}^c$, and thus c can fall into three ranges. (1) $\overline{c}^d < \overline{c}^c < c$, so $\overline{p}^c = \overline{a}$ and $\overline{p}^d = \overline{a}$; (2) $\overline{c}^d < c < \overline{c}^c$, so $\overline{p}^c = r^c < \overline{a}$ and $\overline{p}^d = \overline{a}$; and (3) $c < \overline{c}^d < \overline{c}^c$, so $\overline{p}^c = r^c < \overline{a}$ and $\overline{p}^d = r^d < \overline{a}$.

For case (1) and case (2), following the argument as in Appendix 1 – 7, it can be shown that $F^d(p)$ first order stochastically dominates $F^c(p)$. That is, the initial offers for group "*d*" are always higher than those for group "*c*". To explain the intuition behind this result, first take a look at how bargaining and searching affect firms' offer setting behaviors, respectively. If there is only bargaining and no searching, firms' best response is to set "monopoly offer" $\overline{a}$, because consumers would always stop and bargain, and there is no cost of raising offer. If there is only searching and no bargaining, firms' will have to set price carefully, because a high price will stop the consumers from buying. Therefore, it can be seen that it is consumers' searching behavior that provides some incentive for firms to lower their offers.

Then, let us go back to case (1) and case (2). In case (1), for both group "*c*" and group "*d*", the searching constraint is not binding, meaning, in neither case will *non-shoppers* search in equilibirum. In case (2), as $G(a)$ goes from $G^c(a)$ to $G^d(a)$, the searching

[12] Please refer to Appendix 1 – 7 for detailed proof.

constraint is actually loosened. Firms no longer have incentive to lower offers for the reason of *non-shoppers'* searching. Therefore, in both cases, offers for group "*d*" are higher.

As to the final price, in case (1), at the mean level, $E^c(Y) = E^d(Y)$; and for case (2), $E^c(Y) < E^d(Y)$. It is easy to prove. Recall the expectation of the final price is defined as

$$E(Y) = \frac{N}{M} E(\pi) = \frac{N}{M} E\pi(\bar{p}) = (1-\mu)\left[\int_{\underline{a}}^{\bar{p}} ag(a)\,da + \int_{\bar{p}}^{\bar{a}} \bar{p}g(a)\,da\right].$$

For case (1), $\bar{p}^c = \bar{a}$, $\bar{p}^d = \bar{a}$, and hence $E^c(Y) = (1-\mu)E^c(a) = (1-\mu)E^d(a) = E^d(Y)$, while for case (2), $E^c(Y) < (1-\mu)E^c(a) = (1-\mu)E^d(a) = E^d(Y)$. It is because, in case (1), the *reservation pric*e constraint is not binding for both $G^c(a)$ and $G^d(a)$. Firms can increase their profits the same way. Yet for case (2), the constraint is binding for $G^c(a)$ while loosen for $G^d(a)$. Therefore, firms can get higher profits from $G^d(a)$ case.

In the scenario of case (3), however, it is not necessary that there is first order stochastic dominance result. Because for both $G^c(a)$ and $G^d(a)$, the *reservation price* constraints are binding, and hence, it is not clear whether firms can get higher or lower profits. If the firms can get higher profits, the first order stochastic dominance result still holds[13], yet if the firms' expected profit actually goes down, the result may not hold[14].

To summarize Section 1 – 4 – 1

Proposition 1 – 4 – 1, for two bargaining ability distribution $G^c(a)$ and $G^d(a)$, if $G^c(a)$ second order stochastically dominates $G^d(a)$, and the expected final prices satisfy $E^d(Y) \geq E^c(Y)$, then it must be true that $F^d(p)$ first order stochastically dominates $F^c(p)$.

[13] Please refer to Appendix 1 – 8 for detailed proof.
[14] Please refer to Section 1 – 4 – 2 for a counter example.

1 – 4 – 2, Examples

In this section, we make up two examples. In the first example, we assume a truncated normal distribution, and simulate $F(p)$, $H(y)$, and some other important variables. In the second example, we make up a case where $F^{d}\left(p\right)$ does not first order stochastically dominate $F^{c}\left(p\right)$.

1 – 4 – 2 – 1, Example 1

Suppose that a follows a truncated normal distribution[15]. Taking a normal distribution $N\left(\mu_{a},\sigma_{a}^{2}\right)$, $G(a)$ is truncated at $\underline{a}=0$ from below and $\overline{a}=1$ from above. Please refer to Figure 1 – 4 for an illustration of the distribution. Therefore, when σ_a increases from σ_{a}^{c} to σ_{a}^{d}, and $\mu_{a}^{c}=\mu_{a}^{d}=0.5$, $G^{c}\left(a\right)$ second order stochastically dominates $G^{d}\left(a\right)$.

[15] We assume a truncated normal distribution because we want to confine the bargaining ability in [0,1].

Figure 1 – 4, Truncated Normal Distribution

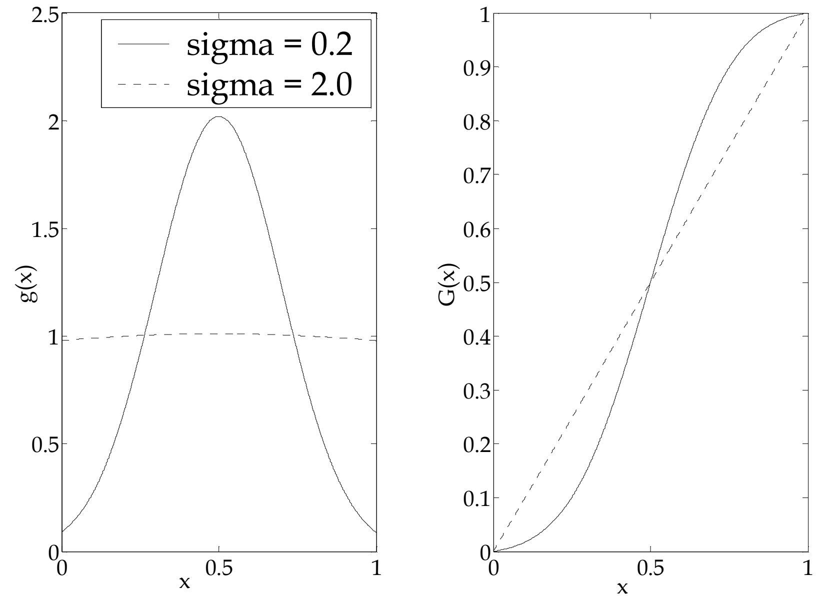
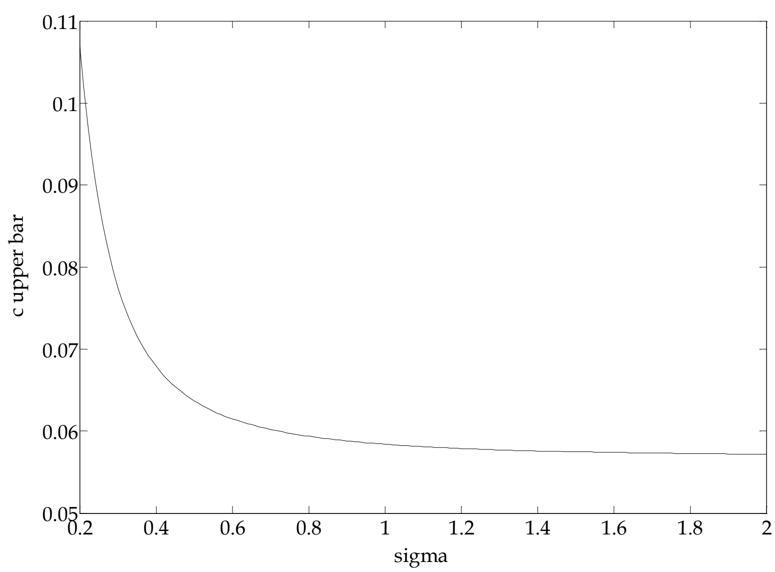


Figure 1 – 5, $\bar{c}\left(\sigma_a\right)$



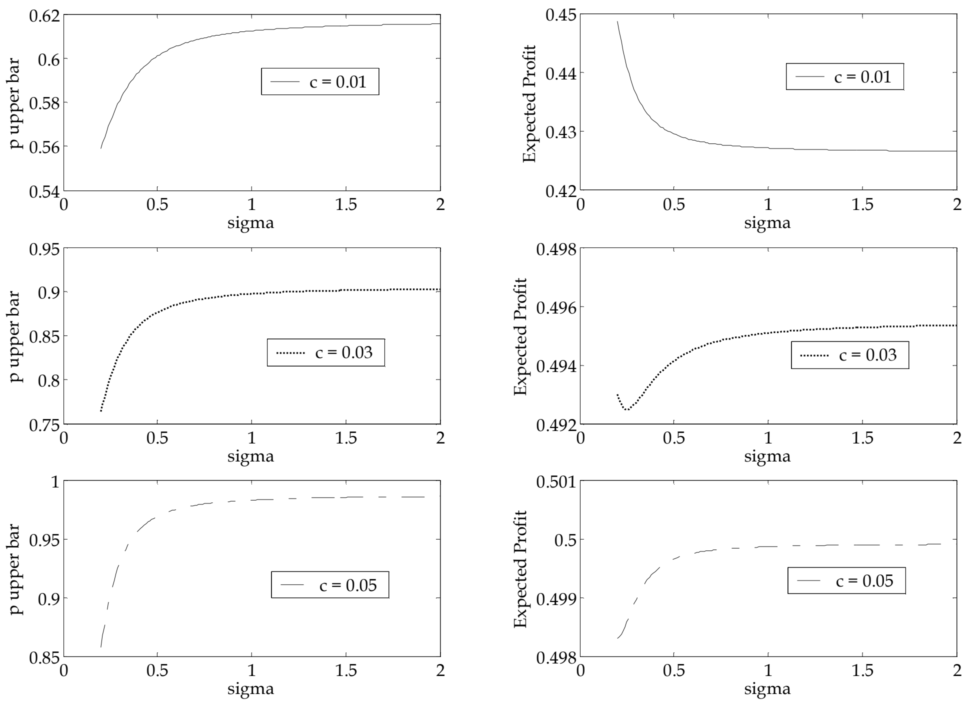
Then we simulate $\bar{c}$ as a function of σ_a when the other parameters are $N = 10$, $\mu_a = 0.5$, $\mu = 0.01$[16]. According to equation (1-3-7), we plot $\bar{c}(\sigma_a)$ in Figure 1 – 5. It can be seen from the graph above that, as $G(a)$ becomes more dispersed, $\bar{c}$ goes down.

As $G(a)$ goes from $G^c(a)$ to $G^d(a)$, it has been proven that $F^d(p)$ first order stochastically dominates $F^c(p)$, so, we would expect that when σ_a increases, $\bar{p}$ goes up. Yet the expectation of final price does not necessarily go up. From the analysis in Section 1 – 3 – 2, we know that as σ_a increases, (1) if *non-shoppers'* searching constraint is never binding, that is, if c is so high that $c > \bar{c}^c > \bar{c}^d$, then the expected final price will keep unchanged; (2) if *non-shoppers*' searching constraint becomes loosen, that is $\bar{c}^c > c > \bar{c}^d$, the expected final price will go up; and (3) if c is so low that the constraint is always binding, then how the expected final price changes depends on shapes of $G(a)$.

In Figure 1 – 6, we plot $\bar{p}$ and $E(Y)$ as a function of σ_a for three different levels of searching cost, $c = 0.01$, $c = 0.03$, and $c = 0.05$. It is not surprising to see that $\bar{p}(\sigma_a)$ goes up in σ_a. As for $E(Y;\sigma_a)$, it can be seen that sometimes $E(Y)$ increases along with σ_a, and sometimes, it decreases. Back to Figure 1 – 5, we can see that as σ_a goes from 0.2 to 2, $\bar{c}$ is always higher than 0.05. Therefore, for all the 3 cases we plot in Figure 1 – 6, the *non-shoppers'* searching constraint is binding, and hence how the expected final price changes is uncertain.

[16] If not specified otherwise, we will stick to this assumption in this example.

Figure 1 – 6, $\overline{p}\left(\sigma_a\right)$ and $E(Y;\sigma_a)$



Finally, we simulate the initial offer distribution and final price distribution as in Figure 1 – 7 still with N=10, μ_a = 0.5 and μ = 0.01. What's more, we assume that c = 0.06. Therefore, when σ_a = 0.2, $c < \overline{c}$ = 0.1069, and hence $\overline{p} = r$ = 0.8919; and when σ_a = 1.0, $c > \overline{c}$ = 0.0584, and hence $\overline{p} = \overline{a}$ = 1. This figure represents case (2) in Section 1 – 4 – 1, where when $G(a)$ becomes more dispersed, the *reservation price* constraint is loosen. Note that the kink in probability density function of final price distribution comes from $\underline{p}$.

Figure 1 – 7, $F(p)$ and $H(y)$

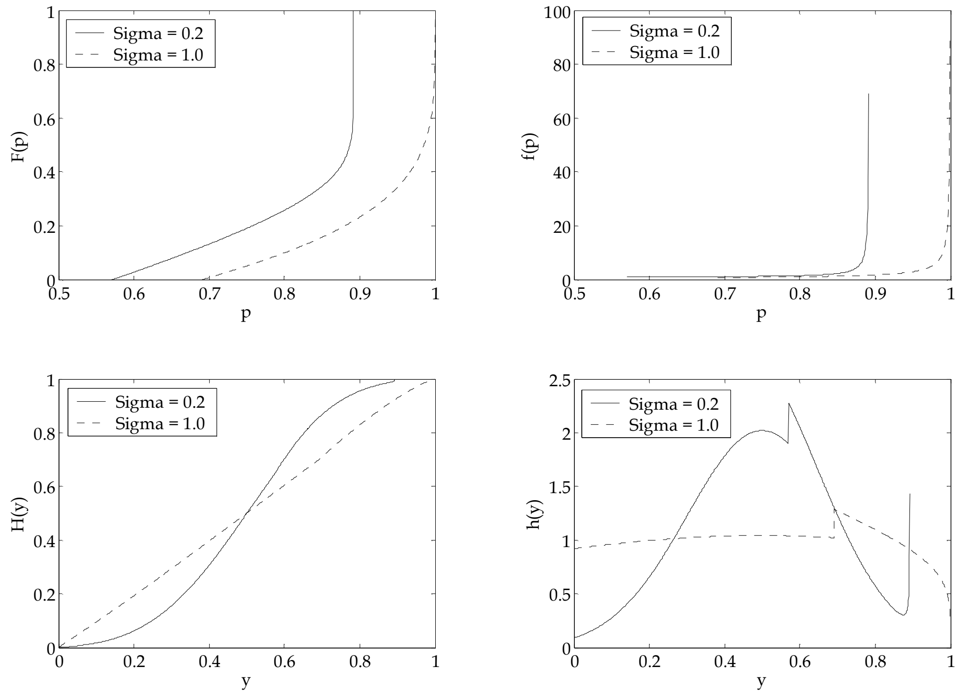


Figure 1 – 8 plots case (3) in section 1 – 4 – 1, where for both $G^{d}(a)$ and $G^{c}(a)$, the *non-shoppers'* searching constraint is binding. For this group of figures, c = 0.05. Figure 1 – 9 plots case (1) in Section 1 – 4 – 1, where the *non-shoppers'* searching constraint is never binding, and we assume c = 0.12.

Figure 1 – 8, *F*(*p*) and *H*(*y*)

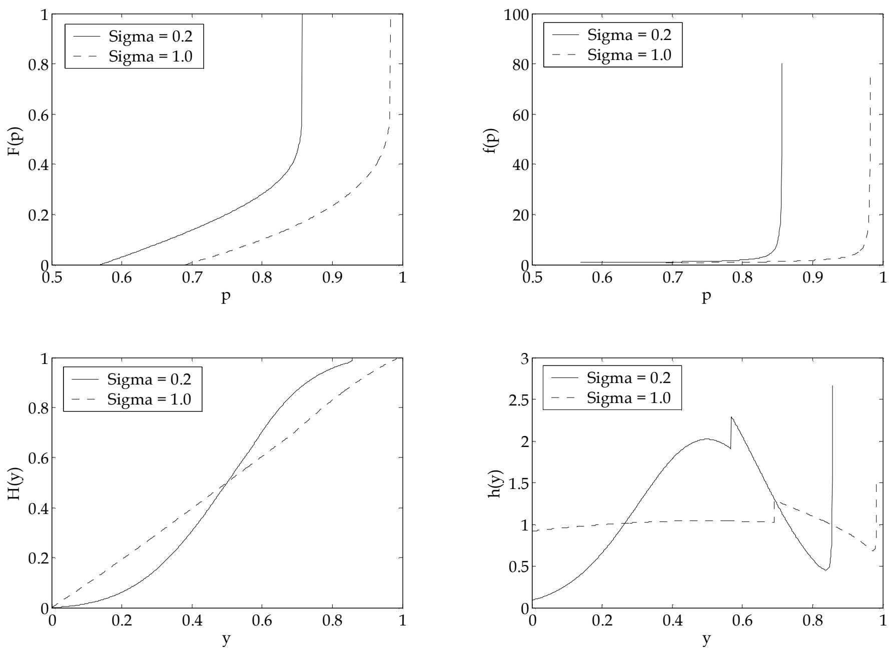
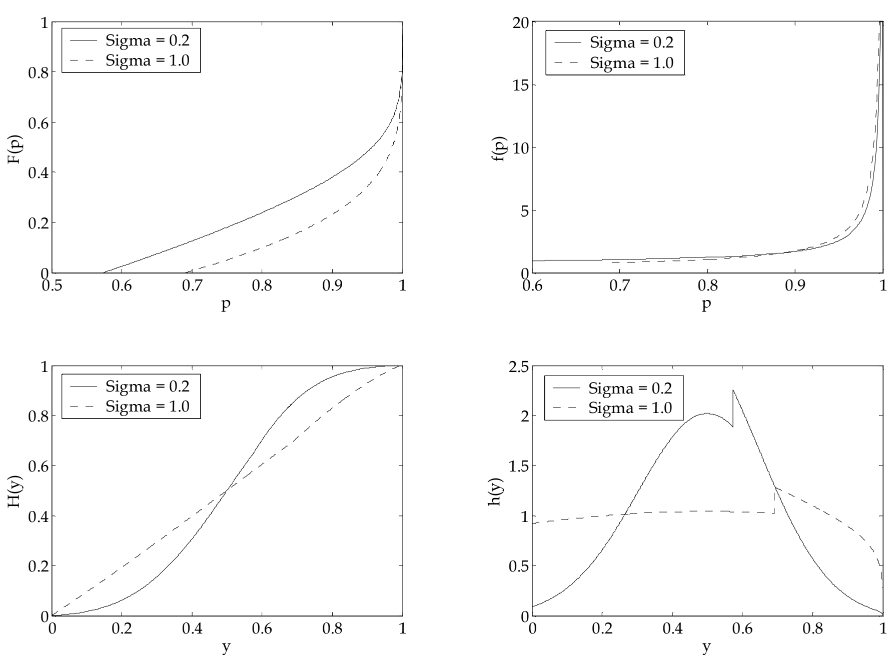


Figure 1 – 9, *F*(*p*) *and H*(*y*)



1 – 4 – 2 – 2, Example 2

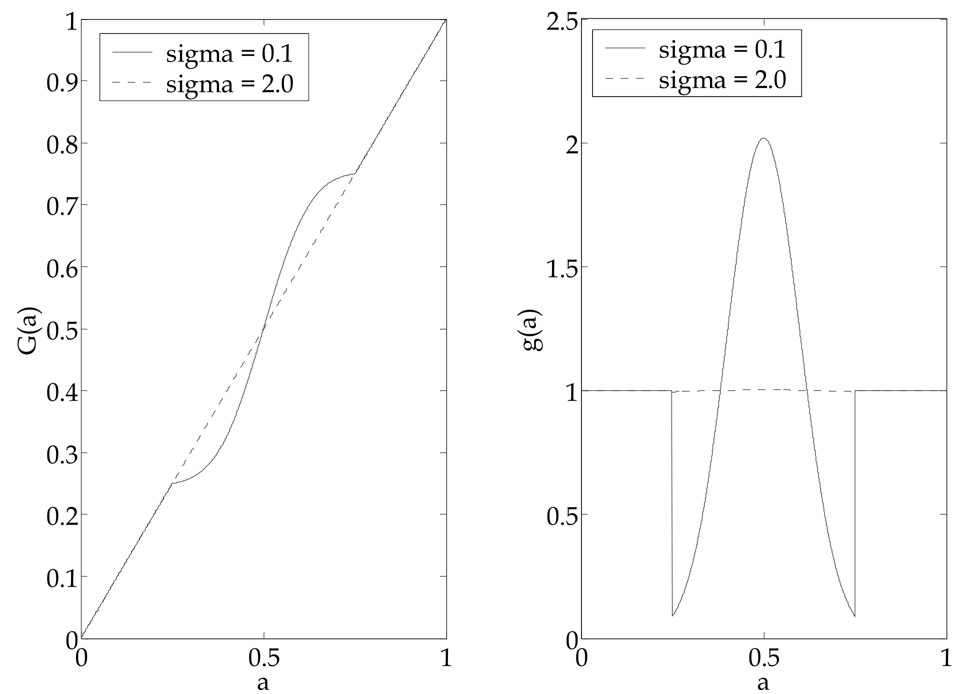
In this example we give a counter example in which there exist some area where $F^c(p) < F^d(p)$. It has been proven in Section 1 – 4 – 1 that this scenario is only possible in case (3), where for both $G^d(a)$ and $G^c(a)$ cases, the non-shoppers' searching constraint is binding. Furthermore, as proved in Proposition 1 – 4 – 1, only when $E^d(Y) < E^c(Y)$, is this scenario possible. Therefore, we will concentrate on case (3) and try to find a case where $E^d(Y) < E^c(Y)$.

Suppose that $G(a)$ is defined in three parts as following.

$$G(a) = \begin{cases} a & \text{if } a \in [0, 0.25] \\ \Phi(a) + 0.25 & \text{if } a \in [0.25, 0.75] \\ a & \text{if } a \in [0.75, 1] \end{cases}$$

where $\Phi(a)$ is a truncated normal distribution which takes a normal distribution $N(\mu_a, \sigma_a^2)$ and truncates it from above at 0.75 and below from 0.25. Please refer to Figure 1 – 10 for an illustration for the distribution.

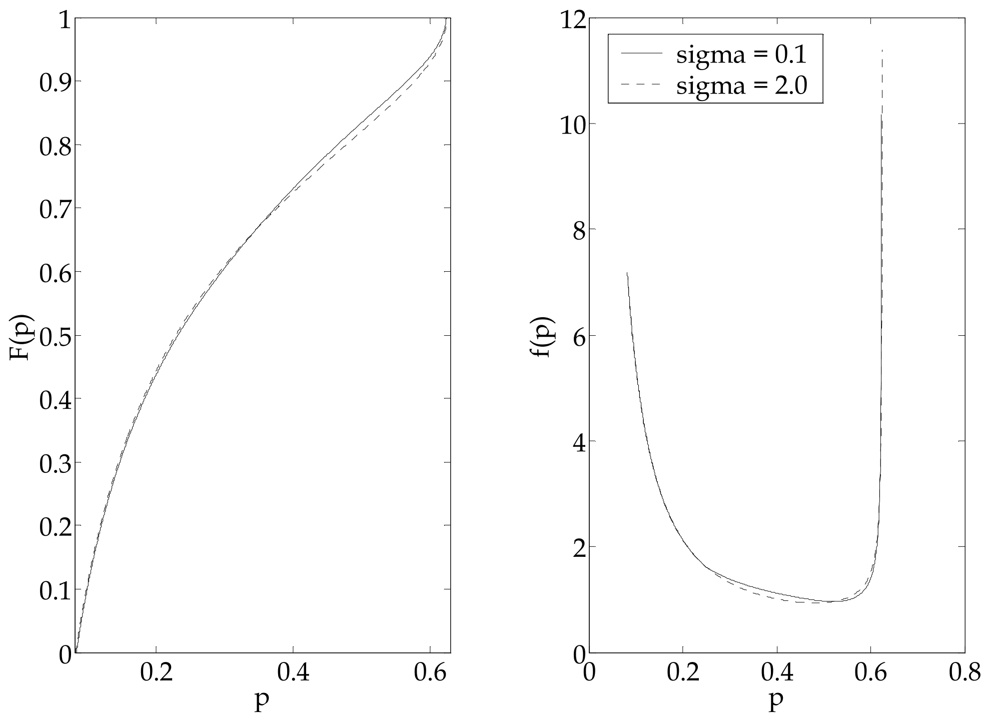
Figure 1 – 10, *G*(*a*) *and g*(*a*)



It can be seen from the figure that in the first part and the last part, the two distributions $G^d(a)$ and $G^c(a)$ coincide, while in the middle of the distribution, $G^c(a)$ exhibits second order stochastic dominance to $G^d(a)$. It is worth pointing out that if *p* is in the first part of the distribution, for either $G^d(a)$ or $G^c(a)$ case, the expected final price would be the same. If $\overline{p}^c$ and $\overline{p}^d$ lie in the area [0.25, 0.75], it might be possible that $E^d(Y) < E^c(Y)$. If both $\underline{p}^c$ and $\underline{p}^d$ are less than 0.25, in case of $E^d(Y) < E^c(Y)$, it must be true that $\underline{p}^c > \underline{p}^d$. Therefore, in [$\underline{p}^c$, $\underline{p}^d$], $F^c(p) < F^d(p)$.

We assume that *a* follows the distributions plot above, *N*=3, $\mu = 0.6$, and *c* = 0.34. As a result, $\overline{c}^c = 0.6368$, $\overline{c}^d = 0.6342$, both greater than *c* = 0.34. And hence, we are in case (3) as in Section 1 – 4 – 1. Also, we have $\overline{p}^c = 0.6235$, $\overline{p}^d = 0.6248$, both in the range [0.25, 0.75], and $\underline{p}^{c} = 0.0825$, $\underline{p}^{d} = 0.0814$, both below 0.25. Note that $\underline{p}^{c} > \underline{p}^{d}$, and it must be true that in [$\underline{p}^{c}$, $\underline{p}^{d}$], $F^{c}(p) < F^{d}(p)$. Also, as can be seen from Figure 1 – 11, there is no first order stochastic dominance result. The figure does not show it very well, because the difference is too small. But if you examine it carefully, the two cumulative density functions actually intersect with one another.

Figure 1 – 11, *F*(*p*) and *f*(*p*)



1 – 4 – 3, Discrimination in Automobile Market

Automobile market is one market in which both search and bargain happen. In the empirical literature of discrimination in auto market, two entirely distinctive and contradicting conclusions were reached regarding to the price discrimination. To state more precisely, the two papers, Goldberg(1996) and Ayres and Siegelman (1995), argued on whether there exist any differences in dealers' pricing behaviors toward minorities and non-minorities.

Ayres and Siegelman (1995), using controlled experiment data[17], found that the initial offers for the minorities are higher. They designed the experiment carefully, controlling many biographic characteristics relevant to dealers' pricing behaviors, such as education, age, attractiveness, cars owned, financial characters, and even place of residence. The participants were trained to give similar answers to all questions asked, and they were ignorant of the purpose of the research.

Ayres and Siegelman exploited the panel structure of their data to control the audit-specific errors. They regressed absolute profits and percentage profits[18] on race and gender dummies, and found very large and significant parameters for the race dummies in all regressions. The results were robust for both parametric method and non-parametric method.

Goldberg (1996), using consumer expenditure survey data (CES), reported that there is no significant difference between the final prices for minorities and non-minorities. In this JPE paper, Goldberg used "discount" for each transaction as dependent variable[19]. She controlled all of the biographic characteristics, automobile features, as well as the interactions between the two. Neither the dummy variable for minority nor the interactions between minority dummy and other independent variables were significant.

She tried to explain her finding as a result of sample selection bias since it is possible that

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[17] Ayres and Siegelman referred to their data as "controlled experiment" data, yet the "experiment" they were talking about is not what we do in lab nowadays. In their research, they hired some people, including both minorities and non-minorities, trained them to behave the same, and sent them to dealers to ask for quotes for the cars. For a detailed description of the factors they "controlled", please refer to their AER paper.

[18] Ayres and Siegelman have four dependent variables. They estimate the cost for each vehicle model first and then use their estimation to construct "initial dollar profit"=initial offer-cost, "final dollar profit"=final offer-cost, and percentage profit for both cases, with respect to cost. And by "final offer", they mean that the last offer the testers get for the first visit.

19 She has absolute discount, that is the difference between list price and the transaction price, and relative discount, relative to list prices.

some minorities might drop out of the market due to the discrimination against them. To account for such sample selection bias, she designed a two-stage model. In the first stage, she ran a Probit model estimating the probability that a household would buy a new vehicle, a used vehicle, and no vehicles at all. In the second stage, she re-estimated her previous model. The improved model reported no significant changes. The dummy for minority was still not significant.

Figure 1 – 12, Goldberg's Quantile Regression Results[20]

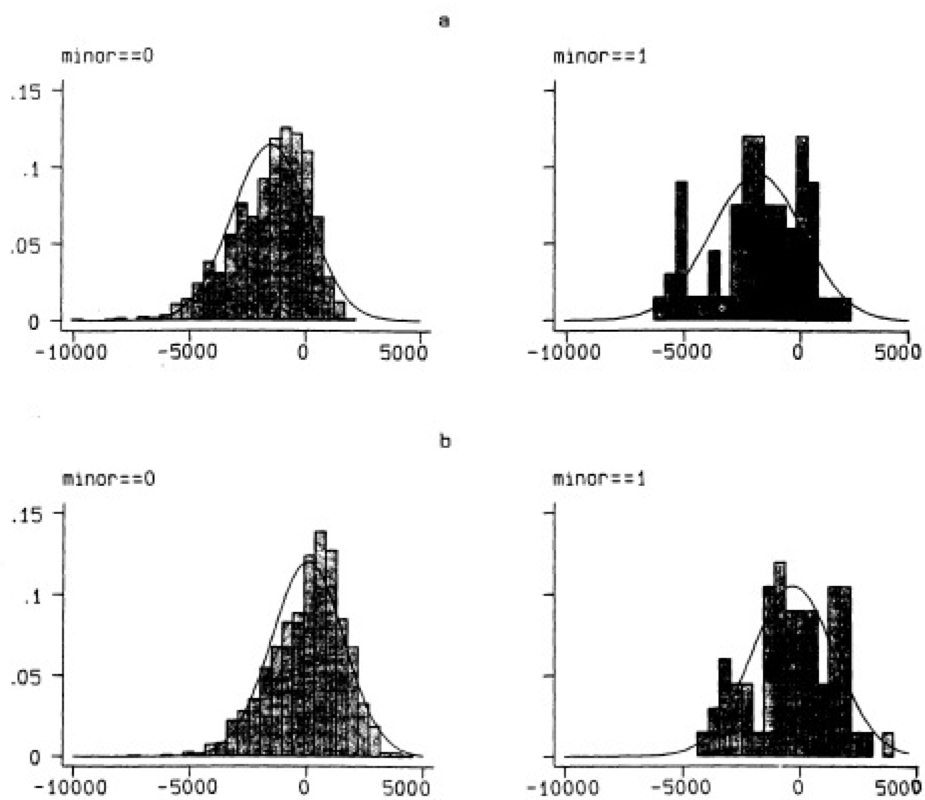

FIG. 1.—Empirical discount distribution: *a*, raw data; *b*, residuals from OLS regression.

Goldberg attributed the inconsistencies between her findings and Ayres and Siegelman's findings to the different distributions of reserve prices for minorities and non-minorities. She ran a set of quantile regressions. Please refer to Figure 1 – 12 for her results. She

[20] This is Figure 1 from Goldberg (1996).

found that there were more minorities receiving lower discounts than non-minorities on the left tail of the distribution, and more minorities receiving higher discounts on the right tail of the distribution. Goldberg tried to explain her findings, hypothesizing that the reserve price distribution of minorities was more dispersed than that of non-minorities. Then the dealers, knowing the distributions for the minorities and the non-minorities, would offer higher initial prices to the minorities, expecting for higher profits. On the other hand, since the final prices depend on the reserve prices, the minorities get a more spread out distribution than the non-minorities, but the mean of the final prices for the two groups can be similar.

However, Goldberg did not set up a theoretical model, and it is not very clear why more dispersed reservation price distribution would lead to higher initial offer. The reservation price is endogenously determined. If higher initial offers can be an equilibrium scenario, then we must observe reservation price distribution shift higher accordingly, and to support such an equilibrium, we need consumer's searching costs to be higher. These do not seem likely from the figures above.

These two seeming contradicting empirical results are exactly what have been predicted from our model, i.e., higher initial offers and similar expected final price exist simultaneously. Imagine the case where minorities have more dispersed bargaining ability than non-minorities, yet at the mean level, their bargaining are of similar effectiveness. Denote minority's bargaining ability distribution to be $G^d(a)$, and non-minority's to be $G^c(a)$, we have $G^c(a)$ second order stochastic dominates $G^d(a)$. Following the argument in Section 1 – 4 – 1, if constraint from searching is not very strong, firms would charge higher initial offers to minorities. This is because, for the firms, the distribution of profits from non-minorities is more dispersed, and hence, the firms would set higher price to cover their possible loss. Also, it can be explained that, since some non-minorities are not good at bargaining, firms would like to raise the offers to take advantages from them. And as for those minorities who are good at bargaining, they can always bargain for lower prices. Either way, we will observe the "discriminative" offers in the auto market. And on the other hand, the final prices would stay similar for the two groups at the mean level.

Why would minorities have more dispersed bargaining ability? It might be explained using a learning process[21]. Suppose minorities and non-minorities only socialize within their own groups. Through their interaction with other members in their group, they learn the newest bargaining results in the group, which they use as a reference in their own bargaining. This "reference" effect may diminish as time goes by. Therefore, a person in the smaller group, or a minority, will have less chance to learn, and hence the bargaining ability distribution for the minorities is more dispersed.

[21]We thank Professor Randal Watson for providing this explanation.

Section 1 – 5, Conclusion and Further Research

In the consumption goods markets, we can observe both bargaining and searching. However, in this literature, very little work has been done to incorporate both features into one model. We address this question. Also, we try to use our model to explain two seemingly contradictory empirical works in the literature of discrimination in the auto market.

This work still has some problems. The major one is the model of consumers' bargaining behaviors. We do not have a detailed model to account for the bargaining process. Also, the final price function seems too simple.

In the literature of bargaining, Nash bargaining and Rubinstein alternative offer game both have the result that final output be determined only by utility form of the two sides of bargainers. For Nash bargaining game, it is the concavity level of the utility function that leads to different bargaining result. And for Rubinstein bargaining game, it is the patience level that is critical. Neither can be nested into our model. If we assume Nash bargaining game, we need to specify consumers' utility function, which is concave. And, of course, the concavity of the utility function would affect the searching behavior of consumers. And if we assume Rubinstein alternative offer game, we are facing the inconsistency of searching and bargaining model. In bargaining part, we assume cost to take the form of fixed discount rate and in searching part, we assume that the cost takes the form of fixed amount. And if we want to keep consistence and assume that, in the searching part of the model, the cost also takes the form of fixed discount rate, then we will have r_i as a function of a_i, and it will be extremely difficult to solve for a close form solution, as can be seen from Stahl (1996). However, it is still possible to get some properties of the equilibrium here. We think this is a good direction to try.

Another possible direction is to assume bargaining result as a function of initial offer.

Again, we will face the problem of a distribution of reservation prices. Such an assumption can be justified as an observation from real world. And we are actually more interested in this set up.

Another issue is that we assume consumers' bargaining abilities to be independent of searching cost. In real world, we would expect a person who is relatively keen on searching to have a better position in bargaining. In our paper, this means that the □ proportion of *shoppers* should have different and lower bargaining ability distribution than the *non-shoppers*. If that is true, we would expect the firms to set higher initial offers, now that their profits from *shoppers* are even lowered. Also, if the bargaining ability distribution for the shoppers and the non-shoppers would change in a same way[22], then we would still expect the first stochastic dominance result as in Proposition 1 – 4 – 1.

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[22] Say, if *non-shoppers'* bargaining ability distribution $G^d(a)$ is obtained from a mean preserving spread from $G^c(a)$, then *shoppers'* bargaining ability distribution is changed following the same mean preserving spread process.

Chapter 2

Bargaining and Price Dispersion in Consumption Goods Markets

Chapter Two Abstract

In the literature of discrimination in the auto market, there are two seemingly contradicting empirical works. Ayres and Siegelman (1995), using controlled experiment data, found that the initial offers for the minorities are higher. Yet Goldberg (1996), using consumer expenditure survey data (CES), reported that there is no significant difference between the final prices for minorities and non-minorities. An important difference between these two works is that Ayres and Siegelmean (1995) used the prices before bargaining, and Goldberg (1996) used the prices after bargaining. Therefore, we construct a model with bargaining in this paper to reconcile these two seemingly contradicting results.

Most people have experiences in bargaining. Knowing that there will be bargaining, firms' offer setting behaviors are different, which, in turn, lead to different final prices in the market. In this study, we investigate how the bargaining process affects firms' offer distribution and thus the final price distribution. Based on Varian (1980), we add a bargaining parameter into the model, and solve for the new equilibrium in this set up. Then, we do some comparative statics, changing the distribution of the bargaining parameter to see what would happen to the equilibrium.

Our model shows that, the two contradicting results can be explained by different bargaining abilities of minorities and non-minorities. If minorities have a more dispersed bargaining parameter distribution than non-minorities, in equilibrium we will observe that minorities face an offer distribution that first order stochastically dominates the non-minorities'. The final transaction prices for minorities and non-minorities, on the other hand, will be at the same mean level, but the final price distribution for the minorities is more dispersed on the right tail, comparing to that for the non-minorities.

Section 2 – 1, Motivation and Literature Review

In the empirical literature of discrimination in auto market, two entirely distinctive and contradicting conclusions were reached regarding to the price discrimination. To state more precisely, the two papers, Goldberg (1996) and Ayres and Siegelman (1995), argued on whether there exist any differences in dealers' pricing behaviors toward minorities and non-minorities.

Ayres and Siegelman (1995), using controlled experiment data[23], found that the initial offers for the minorities are higher. They designed the experiment carefully, controlling many biographic characteristics relevant to dealers' pricing behaviors, such as education, age, attractiveness, cars owned, financial characters, and even place of residence. The participants were trained to give uniform answers to all questions asked, and they were ignorant of the purpose of the research.

Ayres and Siegelman exploited the panel structure of their data to control the audit-specific errors. They regressed absolute profits and percentage profits[24] on race and gender dummies, and found very large and significant parameters for the race dummies in all regressions. The results were robust for both parametric method and non-parametric method.

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[24] Ayres and Siegelman have four dependent variables. They estimate the cost for each vehicle model first and then use their estimation to construct "initial dollar profit"=initial offer-cost, "final dollar profit"=final offer-cost, and percentage profit for both cases, with respect to cost. And by "final offer", they mean that the last offer the testers get for the first visit.

this JPE paper, Goldberg used "discount" for each transaction as dependent variable[25]. She controlled all of the biographic characteristics, automobile features, as well as the interaction terms between the two. Neither the dummy variable for minority nor the interaction terms were significant.

She tried to explain her finding as a result of sample selection bias since it is possible that some minorities might drop out of the market due to the discrimination against them. To account for such sample selection bias, she designed a two-stage model. In the first stage, she ran a Probit model estimating the probability that a household would buy a new vehicle, a used vehicle, and no vehicles at all. In the second stage, she re-estimated her previous model. The improved model reported no significant changes. The dummy for minority was still not significant in either step.

Goldberg also ran a set of quantile regressions. She found that minorities at the very left tail of the distribution received lower discounts than non-minorities, and minorities at the very right tail of the distribution received higher discounts.

Goldberg pointed out that the inconsistencies between her findings and Ayres and Siegelman's findings might be resulted from the different distributions of reserve prices for minorities and non-minorities. However, Goldberg did not set up a theoretical model, and it is not very clear why more dispersed reservation price distribution would lead to higher initial offer. From the literature of searching and information economics, the reservation price is endogenously determined[26]. If higher initial offers can be an equilibrium scenario, then we must either observe reservation price distribution shift higher accordingly[27], or have lower searching cost. If we have lower searching cost,

[25] She has absolute discount, that is the difference between list price and the transaction price, and relative discount, relative to list prices.
[26] Please refer to, say, Kohn and Shavell (1974) and Stahl (1989) for examples.
[27] In a sequential searching game, consumers' best strategy is to stop at a price lower than his reservation price and to go on and search otherwise. The reservation price, r, is defined from the equation below. Please refer to McCall (1965) for detailed discussion.

however, firms' high offer can not be supported as an equilibrium scenario; and if consumers' reservation prices shift up, the transaction price would shift up as a result, contradicting with what were found from Goldberg's paper.

Actually, the difference between these two works is that Ayres and Siegelman used initial offer, and Goldberg used transaction price. Between these two prices, we have bargaining process. Bargaining is important in consumption goods markets. Here in America, we can observe bargaining in some durable goods markets, like in automobile market and in housing market. Bargaining is more common out of America. Back in my hometown in China, people even bargain for a T-Shirt. Also, it is intuitive that bargaining will affect price distribution in the market. Because of consumers' bargaining behaviors, the transaction prices are not the same as the initial offers. Knowing that they would get lower prices, consumers' behaviors may change, and sellers' price setting behaviors may change accordingly. Therefore, the price dispersion in the market would be different. If we only focus on transaction prices or initial offers, we are surely missing something important.

In this paper, we would like to model how bargaining would affect the price dispersion in the market. In the consumption good markets, price dispersion is very common. And it has long been noticed that even for homogenous goods, the prices in the market may not be homogenous because of information problem. Consumers do not know all the prices in the market, or not all of consumers know all the prices in the market, and hence, firms can take advantage from consumers' ignorance of information. In this case, the price setting game is no longer a Bertrand or Cournot competition. It involves more complicated payoff system, and leads to more complicated price distribution in the market.

$$\int_{-\infty}^{r} F(x)\,dx = c$$

where $F(x)$ is firms' offer distribution, r is the reservation price and c is consumers' searching cost.

Stigler (1961) is the first one in the literature to study information problem and price dispersion in the market. In this paper, Stigler studied two examples of information acquisition. In the first example, people obtained information through searching, which was costly. It could be that consumers search for a lower price, and it could also be that sellers search for a higher bid. After searching, some/all people became informed, partially informed, or uninformed at all. The second example was advertisement, through which, consumers became informed or stayed uninformed. As Stigler pointed out, the ignorance in the market would lead to price dispersion.

Stigler's work exploited a new area in information economics. Yet, soon, it had been noticed that Stigler's model was a partial equilibrium, "partial" in the sense that it only considered consumers' equilibrium behavior, and did not allow firms to make their best response. As shown in Diamond (1970), if all consumers incur strictly positive cost in information acquisition, the unique equilibrium scenario would be that all firms set the monopoly price. That is, there would be no price dispersion, and hence consumers do not have to worry about information acquisition.

In 1970s and 1980s, there were large amount of literatures modeling consumers' information problem and price dispersion in the market, like Braverman (1980), Salop and Stiglitz (1977), Varian (1980), Stahl (1989) and etc. Varian (1980) assumed that some consumers were uninformed and the others were informed, and it was exogenously determined whether a consumer was informed or not. The informed consumers would buy at the lowest price, while the uninformed ones would randomly pick up a store. Under these assumptions, he proved that there would be price dispersion and solved the price distribution out explicitly. He also made a free entry assumption and solved for the long-run zero-profit equilibrium price distribution.

Salop and Stiglitz (1977) modeled a scenario, in which, if a consumer buys information at some cost, he would be perfectly informed of all the prices and identities in the market.

Therefore, if the unique equilibrium price in the market was so high, it would be profitable for some sellers to charge lower prices, knowing that consumers would rationally buy information. And in the equilibrium, there would be price dispersion, and consumers would be indifferent between buying information and not buying.

Stahl (1989) built up a model in which a proportion of consumers were “shoppers” who like to shop (gathering information) and hence were fully informed. In that paper, Stahl explicitly solved for the equilibrium price dispersion on the market. And he also linked his result to Bertrand competition result, when all consumers were shoppers and there was no information problem, and to Diamond (1971) local monopoly result, when no consumers were shoppers. Stahl (1996) was a complementary work to Stahl (1989). In this latter work, Stahl assumed that consumers were heterogeneous in searching cost. With this small twist, the model became exponentially complicated, and he could not solve explicitly for equilibrium price distribution. In stead, he only discussed when equilibrium would exist, and gave some characteristics of the equilibrium.

All these works shared a similar idea that, if part/all of the consumers have all/partial information, there will be price dispersion. Varian (1980) assumed that it was exogenously decided that who were informed and who were not, while Salop and Stiglitz (1977) and Stahl (1989) let consumers choose between being informed or not implicitly. Actually, Varian (1980) can be viewed as a special case for the searching models, or the searching models can be viewed as an extension to Varian (1980). For example, Varian (1980) can be obtained from Stahl (1989) by assuming that the searching cost for *non-shoppers* was very high so that $\overline{p} = p^*$ in the market, where p^* was the monopoly price. Therefore, *non-shoppers* in Stahl (1989) would always choose not to search and become the “*Uninformed*” in Varian (1980).

In this paper, we would extend Varian (1980) to include bargaining. The reason why we build our model based on Varian (1980) is that, first of all, we believe that Varian (1980) captures the basic relationship of information acquisition and price dispersion. Also, Varian (1980) assumed that the information is exogenously assigned, which simplifies the set up and enables us to concentrate more on the effect of bargaining. One can introduce searching by making similar assumptions as in Stahl (1989) or even Stahl (1996), but as we can see later, it will be very difficult to get a closed form solution.

There are actually some literatures modeling both information problem and bargaining in the labor market. It has long been noticed that when one part of negotiation has an outside option, he has better position in bargaining for better outcomes. Therefore, these literatures mainly focus on how the information acquisition would affect the bargaining results. For example, Carpenter and Rudisill (2003) documented two cases of labor-management bargaining with outside options with similar negotiation structures but totally different results. They found that, overall, information acquisition of one part triggers concessions from the counterpart of the negotiation.

Also, there are literatures of more general model on information, bargaining and price dispersion. For example, Wolinsky (1986) set up a model in which both firms and workers searched for better match in the labor market. And when they got one, they entered the bargaining stage. They could also search for outside options. If they got one, it would give them better bargaining position.

What's more, as we study the literatures in housing market, we find that there are amount of literatures working on both information acquisition and bargaining in housing market. These literatures, with similar set up as in labor market literatures, assume that the houses in the market are heterogeneous, and concentrate on market thickness and the quality of matching between buyers and houses[28].

This paper differs from these literatures in that we study consumption good markets,

[28] Please refer to Gan and Zhang (2006) as an example.

which differ from labor market in two aspects. First of all, in labor market, it is very natural to assume that both firms and workers search. But in a consumption good market, as pointed out by Stigler (1961), it is "empirically unimportant" for sellers to search. True, we have Cutco that would knock at your door to sell you a knife. However, in most cases, we do not have such luxury. Some would also argue that sellers would advertise their goods, which can be viewed in some extend as "searching". Although there is a big chunk of literature studying advertisement, the effects of advertisement and how it affects consumers' behavior are still not clear. Therefore, we would want to ignore the sellers' searching through advertising here.

What's more, our work differs from labor market and housing market searching-bargaining literature in that, in those markets, there are many sellers and the product is heterogeneous, and the quality of "match" between firms and workers matters. Take labor market for an example. Firms are not searching for the highest productivity. A firm in need of an IT person to maintain a database does not have to have an Oracle expert. Similarly, workers are not searching only for highest salary. Rather, both sides are searching for the best "match". On the other hand, in the consumption good market, at least the new product is homogeneous. A Beetle from dealer A is the same as a Beetle from dealer B.

As a summary, our work is an extension based on Varian (1980), and we will focus on how bargaining affects price dispersion in the market, including initial offer distribution and final price distribution. The paper processes as following. In Section 2 – 2, we set up the model. In Section 2 – 3, we solve for the equilibrium initial offer distribution and final price distribution. In Section 2 – 4, we will do some comparative static analysis and try to explain the two contradicting empirical results in the literature of discrimination in automobile market. And Section 2 – 5 concludes the paper.

Section 2 – 2, Model Set Up

In the supply side of the model, there are $N \geq 2$ firms selling a homogenous good. We will assume that the firms are homogenous in the sense that their marginal costs are constant and the same. And thus, we will further normalize their marginal costs to be zero, $mc_f = 0$ $\forall f = 1, 2,\ldots, N$. The firms may be different in the initial offers they are setting. And let p_f denote offer from firm f, $f = 1, 2,\ldots, N$. The N firms maximize their profits by setting offers simultaneously. Let $F(p)$ denote the sequential Nash Equilibrium initial offer distribution.

To formally define firms' payoff functions, we need to examine consumers' behaviors first. On the demand side of the model, there are M consumers with unit demand. The utility from consumption is assumed to be the same for all consumers, and we further normalize it to be one, i.e., $u_i = 1 \, \forall \, i = 1,\ldots, M$. Consumers differ in their information possession and bargaining abilities.

Following Varian (1980), we will assume that, for the information possession, proportion $\mu \in (0,1)$ of consumers is fully informed, and we'll call them "*informed consumers*". They know all the prices in the market, and hence would buy from the lowest price offered. And the other $1 - \mu$ proportion of consumers has no information at all, and we will call them "*Uninformed consumers*". They will randomly pick up a firm and buy from it.

Then, we will give the consumers an ability to bargain. In this work, we will mainly focus on how bargaining results affect the initial offer distribution and transaction price distribution. Therefore, instead of giving a detailed model on the bargaining process, we will just assume that, somehow, the consumer i is endowed with a bargaining ability a_i, and through bargaining, reach a final price $Y(p,a_i)$, where p is the initial offer the consumer gets. a_i follows a continuous distribution $G(a)$ in the support $[\underline{a},\overline{a}]$, and is identically independently distributed across all consumers.

We will assume that $Y(p,a_i)$ is a non-decreasing and concave function of p. The intuition for the non-decreasing property comes from the observation that, in general, by offering a higher initial offer, firms expect a higher final price. The intuition for the concavity comes from the observation that, for some very low initial offer p, few consumers can bargain for even lower final price. In this case, $Y = p$. Yet for higher initial offer, there is room for bargaining, and thus $Y < p$. For technical convenience, we would also assume that $Y(p,a_i)$ is continuous in p.

Note that the effect of the initial offer p on the final price is diminishing as p increases. We will further assume that there exists an initial offer p^*, so high that $\forall\, p \geq p^*$, $Y(p,a_i) = Y(p^*,a_i)$. With this assumption, the final price would not go all the way up, and p^* can be viewed as the monopoly price a firm can set. Think of p^* as consumers' utility from the good, $U_i = 1$. Actually, knowing that $U_i = 1\ \forall\, i$, firms would not set offers higher than 1, and even if they set their offers higher than 1, they would expect bargaining, and the bargaining result should not differ from setting an offer of 1. We assume p^* rather than 1 only to allow for the probability that there can be such an initial offer level less than 1.

We will also assume that $Y(p,a_i)$ is an non-decreasing and concave function of a_i. Consumer with higher a_i will end up with a higher final price, in which sense, a_i is better understood as bargaining "inability". The concavity comes from the observation that, for the consumer with lower a_i (a better bargainer), bargaining is more likely to happen, which induces a final price less than p; while as a_i increases, consumer i (who is not a good bargainer) would more likely to buy the item without bargaining, and hence induces a final price p, and as a_i increase even further, the final price is no higher than p. Therefore, we are in the situation that final price increases along with a_i, and up to some point final price keeps unchanged as a_i increases, which can be viewed as a concave function. Also, for technical convenience, we would assume that $Y(p,a_i)$ is continuous in

a_i.

The assumptions of the concavity and the monotony have nothing to do with the scale of the bargaining ability. Also, because we do not have a detailed model for bargaining process, the scale of the bargaining ability is not defined. For example, if $Y(p,a) = a^{1/2}p$, and let $a' = a^2$, then we can have $Y(p,a) = a^{1/4}p$, another final price function. This is very much like the utility function. We then want to impose another assumption on the final price function, so as to determine the scale. We want to impose this restriction at the highest initial offer level, p^*. Assumer that at this "very high" initial offer p^*, $Y(p^*,a_i) = a_i$. Also, it is easy to see that $Y\left(p^*,\overline{a}\right)=\overline{a}$ must be less than or equal to 1, which is consumers' utility from the good.

One more thing worth noticing is that, the offer higher than the lowest offer may also lead to the lowest final price, since we only assume that $Y(p,a_i)$ increases in p, not strictly increases in p, as a result of which, it is possible that different initial offers will induce the same lowest final price. Our question is, among all the offers that lead to the same lowest final price, should the *informed consumers* randomly pick up one (option (1)) or should they buy from the lowest initial offer (option (2))?

Intuitively, option (1) would more likely to "encourage" firms to set higher initial offers, because, even if they set higher offers, they still enjoy equal chance to be selected by those *informed consumers*. Or, let's put it this way. If a firm sets higher offer, in case of option (1), with some probability, it loses part of *informed consumers*, but in case of option (2), with the same probability, it loses all *informed consumers*. Therefore, intuitively, we would expect the resulting equilibrium offer distribution for option (1) to have higher expectation, lower variance, and to be more likely to collapse to $\overline{p}$, which is the highest offer in the market.

In this paper, we will adopt option (2) for two reasons. First of all, option (2) sounds more plausible. As a Game model, when consumers are indifferent among all the offers, a "random draw" sounds better. But consider the real world. If one is to shop for an automobile and has quotes from all dealers in the city, which dealer he would go and bargain for a better price? Ceteris paribus, it is more likely that he would go to the store with lowest quote, because that one seems easier to bargain with.

The second reason is that option (2) is much easier to solve, while only under some rigid assumptions, will there exist equilibrium under option (1). Therefore, from now on, we will hold on to *Assumption I* as following, and assume that shoppers buy from lowest offers only.

Assumption I[29]: Informed consumers buy from the firm with lowest initial offer. If there is more than one firm setting the lowest price, then informed consumers randomly select one from among those setting the lowest price.

To summarize consumers' behavior, each consumer is labeled as *Uninformed/Informed* in one dimension and bargaining "inability" a_i in the other dimension. *Uninformed* consumers would buy from a randomly picked store at $Y(p,a_i)$, where p is the initial offer he gets; while *informed* consumers would buy from the lowest offer at $Y(p,a_i)$, where p is the lowest initial offer in the market.

Facing a market described above, N firms will set initial offers simultaneously to maximize their expected profits, which is the production of expected revenue at offer p and the probability that a consumer would buy from the firm. Let $R(p)$ denote the expected revenue from a randomly picked consumer stopping at the store, given initial offer being p, and $R(p) = E[Y(p,a_i)]$. It follows from the properties of $Y(p,a_i)$ that $R(p)$ is a continuous, non-decreasing and concave function of p, and we would make further assumption that $R(p)$ is strictly increasing up to p^*. Note that, by assuming $R(p)$ "strictly

[29] *I* stands for "*informed consumers*".

increasing in p" instead of "non-decreasing in p", we only exclude the probability that two initial offers may lead to the same final price level for all consumers. This is not a strong assumption and it's very intuitive. Most importantly, it saves us a lot of technical work.

Therefore, to summarize the analysis above, (1) $R(p)$ is strictly increasing on [0, p^*], and is constant for any $p \geq p^*$; (2) $R(p)$ is a concave function of p; and (3) $R\left(p^*\right) = E\left(a\right)$.

The simplest functional form that complies with the assumptions above may be $Y(p;a_i) = \min\{p,a_i\}$[30]. In this set up, a_i can be viewed as the final price consumer i can achieve if she chooses to bargain. Note that no matter what initial offer she gets, she can always ensure herself a final price no higher than a_i. If p is low enough, consumer i will accept the offer and buy at price p, otherwise, she will bargain and buy at a_i. The limitation of this set up is that there would be no interaction term of initial offer and bargaining ability. In this case, $R\left(p\right) = \int_{\underline{a}}^{\overline{a}} min\left\{p,a\right\} g\left(a\right) da = \int_{\underline{a}}^{p} ag\left(a\right) da + p\left[1 - G\left(p\right)\right]$. $R(p)$ is a continuous, strictly increasing and concave function in $\left[0,\overline{a}\right]$, and is constant for any $p \geq \overline{a}$. In the numerical example, we will assume $Y(p;a_i) = \min\{p,a_i\}$.

[30] Check: $Y(p;a_i) = \min\{p,a_i\}$ is continuous, non-decreasing and concave in p, and is continuous, non-decreasing and concave in a_i

Section 2 – 3, Solve for Symmetric Nash Equilibrium

In this section, we would solve for equilibrium initial offer distribution and final price distribution. To simplify the problem, we will focus on symmetric solutions.

2 – 3 – 1, Equilibrium Analysis

Proposition 2 – 3 – 1, there is no pure strategy Nash Equilibrium.

The idea is that if all firms would set the same initial offer p, there would be profitable deviation by setting an offer a little bit lower than p to attract all the *informed consumers*. And if p is zero, there would be profitable deviation by setting a positive offer to take profits from some of the *uninformed consumers*. This proposition is proved in Appendix 2 – 1.

Let $\underline{p}$ and $\overline{p}$ be the lower bound and upper bound of support of $F(.)$, respectively. Follow the same argument as in *Proposition 2 – 3 – 1,* it is easy to see that there is no mass point on $\left[\underline{p},\overline{p}\right]$, because if there exist some mass point, one would have profitable deviation by setting offer a little bit lower.

Recall that $\forall p \ge p^*, Y\left(p,a_i\right)=Y\left(p^*,a_i\right)\forall a_i$. That is, offers higher than p^* will always yield the same expected revenue as $R(p^*)$. Therefore, loosely speaking, setting offer $p > p^*$ is equivalent to setting offer $p = p^*$. It follows from *Proposition 2 – 3 – 1* that $\underline{p} < p^*$.

Proposition 2 – 3 – 2, If there exists Nash equilibrium, then in the equilibrium it must be the case that $\overline{p} = p^*$***.***

The proof for *Proposition 2 – 3 – 2* follows two steps. First of all, it can be shown that the highest possible initial offer is no less than p^*. Secondly, $\overline{p} > p^*$ is not an equilibrium scenario either. Please refer to Appendix 2 – 2 for detailed proof.

Proposition 2 – 3 – 3, There is no gap on $\left[\underline{p}, \overline{p}\right]$.

Please refer to Appendix 2 – 3 for the proof.

2 – 3 – 2, Equilibrium Initial Offer Distribution

To summarize section 2 – 3 – 1, the initial offer follows a continuous and atomless distribution $F(p)$ on $\left[\underline{p}, \overline{p}\right]$, with $\underline{p} < \overline{p} = p^*$. In this section, we will solve for $F(p)$. First of all, we will list the firms' expected payoffs for all $p \in \left[\underline{p}, \overline{p}\right]$. Secondly, we will calculate $\underline{p}$ and $F(p)$ as a function of $\overline{p} = p^*$, and hence solve for equilibrium initial offer distribution.

Let $E\pi(p)$ denote the expected payoff for initial offer p. If $p = \overline{p}$, only *uninformed consumers* will buy from the firm. Therefore, the expected profit is

$$E\pi(\overline{p}) = (1-\mu)\frac{M}{N}\left[\int_{\underline{a}}^{\overline{a}} Y\left(\overline{p}, a\right) da\right] = (1-\mu)\frac{M}{N} R\left(\overline{p}\right) = (1-\mu)\frac{M}{N} R\left(p^*\right) \quad (2-1)$$

If the firm sets $p = \underline{p}$, all *informed consumers* would come and buy from it and some lucky *uninformed consumers* would also buy there, and hence the expected profit is

$$E\pi\left(\underline{p}\right) = \left[\mu M + (1-\mu)\frac{M}{N}\right]\int_{\underline{a}}^{\overline{a}} Y\left(\underline{p}, a\right) g\left(a\right) da = \left[\mu M + (1-\mu)\frac{M}{N}\right] R\left(\underline{p}\right) \quad (2-2)$$

If the firm sets initial offer $p \in \left(\underline{p}, \overline{p}\right)$, the probability that it has the lowest initial offer would be $[1-F(p)]^{N-1}$. Therefore, only with probability $[1-F(p)]^{N-1}$, will *informed consumers* buy from the firm, while with probability $1/N$, will an *uninformed consumer* visit the firm. Therefore, a firm with initial offer being $p \in \left(\underline{p}, \overline{p}\right)$ has expected profit as below.

$$
\begin{aligned}
E\pi(p) &= \left\{[1-F(p)]^{N-1}\mu M + (1-\mu)\frac{M}{N}\right\}\int_{\underline{a}}^{\overline{a}} Y(p,a)\,dG(a) \\
&= \left\{[1-F(p)]^{N-1}\mu M + (1-\mu)\frac{M}{N}\right\}R(p)
\end{aligned}
\qquad (2-3)
$$

For the initial offer distribution to be a NE, all *p* in the support of *F*(*p*) should yield the same level of expected payoff. Therefore, $\underline{p}$ and *F*(*p*) can be solved as a function of $\overline{p} = p^*$ by equaling the expected payoffs given above.

$\underline{p}$ can be solved from $R(\underline{p}) = \dfrac{1-\mu}{N\mu+1-\mu}R(p^*)$ (2 – 4)

F(p) can be expressed as $F(p) = 1 - \left[\dfrac{1-\mu}{N\mu}\left(\dfrac{R(p^*)}{R(p)} - 1\right)\right]^{\frac{1}{N-1}}$ (2 – 5)

Please notice that the initial offer distribution expression is exactly the same as the price distribution in Stahl (1989). In Stahl (1989), consumers could choose between being informed and uninformed by comparing benefit from searching and searching cost, and the decision was endogenously made; while in our setup, informed and uninformed are exogenously given. Searching cost in Stahl (1989) served as a factor that limited the profitability of firms. Yet it had no effects on the structure of the distribution. The structure of the distribution is determined by the interaction among firms, which is the same in our work, in Stahl (1989), and in Varian (1980).

2 – 3 – 3, Equilibrium Final Price Distribution

Equilibrium final price distribution differs from initial offer distribution because of the bargaining process. Since different initial offer may lead to different final price, the *informed consumer* and the *uninformed consumers* should be examined separately. For *uninformed consumers*, the probability that she will get an offer no greater than p is just the probability that a firm would set an initial offer no greater than p, which is $F(p)$. For *informed consumer*, on the other hand, the probability that she will stop at initial offer no greater than p is the probability that the lowest offer is no greater than p, which is $\Pr\left(min\left\{p_f\right\}_{f=1}^{N} \leq p\right) = 1 - [1 - F(p)]^N$. Therefore, let $B(p)$ denote the probability that a randomly selected consumer will stop at an initial offer no greater than p, and $B(p) = \mu*\{1 - [1 - F(p)]^N\} + (1 - \mu)*F(p)$. Then, the probability density function $b(p)$ is $b(p) = B'(p) = \{\mu N* [1 - F(p)]^{N-1} + (1 - \mu)\}*f(p)$.

Let $H(y)$ denote the final cumulative price distribution. $H(y)$ can then be expressed as

$$\begin{aligned} H(y) &= \iint_{Y(p,a)\leq y} dB(p)\ dG(a) \\ &= \iint_{Y(p,a)\leq y} \left\{\mu N\left[1-F(p)\right]^{N-1} + (1-\mu)\right\} dF(p)\ dG(a) \end{aligned} \qquad (2-6)$$

As the bargaining ability a goes from $\underline{a}$ to $\overline{a}$, and initial offer p goes from $\underline{p}$ to $\overline{p}$, the final price $Y(p,a)$ will go from $\underline{y} = Y\left(\underline{p},\underline{a}\right)$ to $\overline{y} = Y\left(\overline{p},\overline{a}\right) = Y\left(p^*,\overline{a}\right) = \overline{a}$. Therefore, the support for $H(y)$ is $\left[Y\left(\underline{p},\underline{a}\right),\overline{a}\right]$.

To specify the integration area $Y(p,a) \leq y$ clearly, we want to define the highest initial offer $p \in \left[\underline{p},\overline{p}\right]$ as a function of y and a, such that given any bargaining ability $a \in \left[\underline{a},\overline{a}\right]$, the final price will be less than or equal to y. Let $\eta(y;a)$ denote such a function. To formally define $\eta(y;a)$, two things are to be noticed. The first is that, for some combination of a and y, it is possible that even the lowest initial offer $\underline{p}$ could not satisfy the condition $Y(p,a) \leq y$, say like the case when $y = \underline{y}$, yet the bargaining ability a is very high. In this case, we will define $\eta(y;a) = \underline{p}$, since $\underline{p}$ is achieved with zero probability anyway. Our second concern is that, for some other combinations of *y* and *a*, it is possible that all initial offer $p \in \left[\underline{p}, \overline{p} \right]$ can satisfy the condition $Y\left(p,a\right) \leq y$, and even an impossible high initial offer $p > \overline{p}$ will yield $Y\left(p,a\right) \leq y$. In this case, we will define $\eta\left(y;a\right) = \overline{p}$. All in all, $\eta(y;a)$ can be expressed as in equation (2-3-7) below.

$$\eta\left(y;a\right) = \begin{cases} max\left\{p \mid p \in \left[\underline{p}, \overline{p}\right], Y\left(p;a\right) \leq y\right\} & if \left\{p \mid p \in \left[\underline{p}, \overline{p}\right], Y\left(p;a\right) \leq y\right\} \neq \varnothing \\ \underline{p} & otherwise \end{cases} \qquad (2-7)$$

$\eta(y;a)$ is non-decreasing in *y*, and is non-increasing in *a*. This is because that the final price *y* is non-decreasing in initial offer *p* and bargaining ability *a*. Therefore, keeping the bargaining ability *a* constant, to achieve a lower *y*, the highest possible initial offer has to be of no higher; and given a higher bargaining parameter *a*, to achieve the same level of final price *y*, the highest possible initial offer has to be of no higher.

Also, it is worth to point out that $\eta(y;a)$ may not be continuous, because $Y(p,a)$ is not strictly increasing in *p*.

Using function $\eta(y;a)$, we can rewrite the cumulative density function for the final price.

$$\begin{aligned} H\left(y\right) &= \int_{\underline{a}}^{\overline{a}} \left(\int_{\underline{p}}^{\eta\left(y;a\right)} \left\{ \mu N \left[1 - F\left(p\right)\right]^{N-1} + \left(1-\mu\right) \right\} f\left(p\right) dp \right) dG\left(a\right) \\ &= \int_{\underline{a}}^{\overline{a}} \left(\int_{\underline{p}}^{\eta\left(y;a\right)} b\left(p\right) dp \right) dG\left(a\right) \qquad (2-8) \\ &= \int_{\underline{a}}^{\overline{a}} B\left[\eta\left(y;a\right)\right] dG\left(a\right) \end{aligned}$$

Note that in equation (2-3-8) above, we integral over the initial offer *p* first, from $\underline{p}$, to $\eta(y;a)$, the highest possible initial offer that will lead to a final price less than or equal to *y*. When no such an initial offer exists, $\eta\left(y;a\right) = \underline{p}$, and the integrant will be zero, meaning the probability being zero; when all initial offer will lead to final price less than or equal to *y*, $\eta\left(y;a\right) = \overline{p}$, and the integrant will be one.

Similarly, if we want to integral out of bargaining ability, a, first, we can define $\tau(y;p)$ to be the highest bargaining ability parameter $a \in [\underline{a}, \overline{a}]$ that leads to a final price of y, given initial offer p. Also, follow the argument for $\tau(y;p)$, we will define $\tau(y;p) = \underline{a}$, when it is impossible to find a bargaining ability a to achieve a final price lower than or equal to y; and we will define $\tau(y;p) = \overline{a}$, when any bargaining ability a will satisfy $Y(p,a) \le y$. Then we can express $\tau(y;p)$ out as in equation (2-3-9) below.

$$\tau(y;p) = \begin{cases} \max\{a \mid a \in [\underline{a}, \overline{a}], Y(a;p) \le y\} & if \{a \mid a \in [\underline{a}, \overline{a}], Y(a;p) \le y\} \ne \varnothing \\ \underline{a} & otherwise \end{cases} \qquad (2-9)$$

Similarly, it is easy to see that $\tau(y;p)$ is non-decreasing in y and non-increasing in p. Also $\tau(y;p)$ may not be continuous. Given $\tau(y;p)$, the final price distribution function $H(y)$ can be rewritten as following.

$$H(y) = \int_{\underline{p}}^{\overline{p}} \left(\int_{\underline{a}}^{\tau(y;p)} g(a)\,da \right) b(p)\,dp = \int_{\underline{p}}^{\overline{p}} G\left[\tau(y;p)\right] b(p)\,dp \qquad (2-10)$$

Equation (2-3-8) can be interpreted as weighted average of bargaining abilities, where the weight is the probability of getting an initial offer lower than $\eta(y;a)$, that is, $B[\eta(y;a)]$. Similarly, equation (2-3-10) can be interpreted as weighted average of accepted initial offers, where the weight is the probability of having a bargaining ability lower than $\tau(y;p)$, that is, $G[\tau(y;p)]$.

Now that we have the cumulative density function for the final price, we can solve for the expectation of the final price $E(Y)$.

$$\begin{aligned} E(Y) &= \int z\,dH(z) = \overline{y} - \int_{\underline{p}}^{\overline{p}} b(p) \left(\int_{\underline{y}}^{\overline{y}} G[\tau(z,p)]\,dz \right) dp \\ &= \int z\,dH(z) = \overline{y} - \int_{\underline{a}}^{\overline{a}} g(a) \left(\int_{\underline{y}}^{\overline{y}} B[\eta(z,a)]\,dz \right) da \end{aligned} \qquad (2-11)$$

It looks difficult to solve for the expected final price from equation above. Yet please note that the expected amount of money paid by consumers should be equal to the expected profits for firms. Therefore, we can calculate the expected final price as below.

$$\begin{aligned} E(y) &= \frac{N}{M} E\pi = \frac{N}{M} E\pi(\overline{p}) = \frac{NM}{MN}(1-\mu)R(p^*) \\ &= (1-\mu)\int_{\underline{a}}^{\overline{a}} Y(p^*;a)\,dG(a) = (1-\mu)E(a) \end{aligned} \qquad (2-12)$$

The second equation follows from the fact that all the initial price $p \in [\underline{p}, \overline{p}]$ yields the same expected profit, $E\pi = E\pi(p) \forall p \in [\underline{p}, \overline{p}]$; and the third and fourth equation follows from equation (2-3-1).

Section 2 – 4, Comparative Static and Racial Discrimination in the Automobile Market

The purpose of this paper is to see how bargaining would change the initial offer distribution and final price distribution in the market, and use the model to explain the two seemingly contradicting results in the literature of racial discrimination in the automobile market, namely, higher initial offers for minorities and similar transaction prices for minorities and non-minorities. In section 2 – 3, we solved for the equilibrium, and in this section, we will try to use the model to explain the empirical findings in the automobile market.

If the differences in initial offer distribution and final price distribution are induced because of different bargaining ability of minorities and non-minorities, the question is how different bargaining ability distributions for minorities and non-minorities will lead to the initial offer distributions and final price distributions as observed from the real world. First of all, it is easy to see that, to have the same expected transaction price, the two distributions need to have the same mean level, because the expected transaction price $E(Y) = (1-\mu)*E(a)$, and hence we know that the difference lies in the higher moment.

In this section, we will do a comparative static analysis and assume that $G(a)$ changes from $G^c(a)$ to $G^d(a)$, so that $G^c(a)$ second order stochastically dominates $G^d(a)$, and $E^c(a) = E^d(a)$. We want to examine what would happen to the equilibrium distributions to see if we can have the results as observed in real world.

In this whole section, we will use "*c*", to denote the original bargaining ability distribution and whatever associated with the original distribution, and use "*d*", to denote the new bargaining ability distribution and whatever associated with the new distribution[31]. For example, we will use $F^c(p)$ to denote the original initial offer distribution, and use $F^d(p)$ to denote the new initial offer distribution.

2 – 4 – 1, Comparative Static

First of all, let's examine the initial offer distribution. When facing a more dispersed distribution of bargaining ability, $G^d(a)$, firms may want to set higher initial offers so as to take advantage from those *uninformed consumers* who are not very good at bargaining. As to the *uninformed consumers* who are good at bargaining, since they would not reject the offer anyway, there is no incentive for firms to lower offers for them. Therefore, we are expecting higher initial offers for $G^d(a)$ than for $G^c(a)$.

Proposition 2 – 4 – 1, suppose $G^c(a)$ second order stochastically dominates $G^d(a)$, and $E^c(a) = E^d(a)$, then we will have $F^d(p)$ first order stochastically dominates $F^c(p)$.

Please refer to Appendix 2 – 4 for the proof.

Although we have high initial offers, the expected profits for firms when facing $G^d(a)$ stay the same. This is because $E\pi = (1-\mu)\ R(p^*)$, where $R(p^*)$ is determined only from the expectation of bargaining abilities, $E(a)$, and $E^c(a) = E^d(a)$. Therefore, it follows from equation (2-3-10), $E(y) = (N/M) * E\pi = (1-\mu) * R(p^*)$ that the expected final price for $G^d(a)$ and $G^c(a)$ would be the same, or $E^c(y) = E^d(y)$.

How would bargaining ability distribution affect final price distribution? Recall that the cumulative density function of final price $H(y)$ can be interpreted as weighted average of bargaining ability. Given $G^c(a)$ second order stochastically dominates $G^d(a)$, we may expect to have $H^c(y)$ second order stochastically dominates $H^d(y)$, too. Yet it is not always

[31] The "*c*" stands for "*centered*" and "*d*" stands for "*dispersed*". We are assuming "*c*" second order stochastically dominates "*d*", so case "*d*" is more dispersed.

true.

Recall that the probability that a consumer would buy at an offer lower than p is $B(p)=\mu\left\{1-\left[1-F(p)\right]^{N-1}\right\}+(1-\mu)F(p)$. Given that $F^d(p)$ first order stochastically dominates $F^c(p)$, it is easy to see that $B^c(p)\geq B^d(p)\forall p\in\left[\underline{p},\overline{p}\right]$. That is, comparing with consumers from $G^c(a)$, a consumer out of $G^d(a)$ is less likely to have an offer lower than p.

We assume that all consumers have the same final price function $Y(p,a)$. Therefore, given $B^c(p)\geq B^d(p)$, consumers from $G^d(a)$ are tend to have higher final price than consumers from $G^c(a)$ with the same level of bargaining ability.

Since $G^d(a)$ is more dispersed than $G^c(a)$, there are more consumers from $G^d(a)$ who have lower a, but still they face higher offers than consumers from $G^c(a)$. All in all, the effect is ambiguous. On the other hand, there are more consumers from $G^d(a)$ who have higher a, and they also face with higher offers. Therefore, for the final price distribution, we are certain to have a fatter right tail for the case $G^d(a)$ than $G^c(a)$.

The paragraphs above explain the intuition, and still we need proof. First of all, we need to show that it is not necessary to have the second order stochastic dominance result. Following the intuition analysis above, we will focus on the left tail of final price distribution for a counter example.

For second order stochastic dominance, we need to have, for any x in the support, $T(x)=\int_{-\infty}^{x}\left[H^d(y)-H^c(y)\right]dt\geq 0$. Yet it can be shown that sometimes we have $\underline{y}^c<\underline{y}^d$. This is because (1) $\underline{p}^c\leq\underline{p}^d$ [32] and (2) $Y(p,a)$ is increases in p. Therefore, if

[32] Please refer to Appendix 2 – 5 for detailed proof.

$\underline{a}^c = \underline{a}^d = \underline{a}$, we will have $\underline{y}^c = Y\left(\underline{p}^c, \underline{a}\right) \leq \underline{y}^d = Y\left(\underline{p}^d, \underline{a}\right)$, and sometimes we can have strict inequality $\underline{y}^c < \underline{y}^d$.

If it is the case where $\underline{y}^c < \underline{y}^d$, there exists *t* greater than $\underline{y}^c$ and less than or equal to $\underline{y}^d$, such that $H^c(t) > 0$ and $H^d(t) = 0$, which in turn leads to $T(t) < 0$, where *T*(.) is defined above in the last paragraph, contradicting with the definition of second order stochastic dominance. The second example in the next section describes some cases where $\underline{y}^c < \underline{y}^d$.

Secondly, we need to proof the claim that $h^d(y)$ has a fatter right tail than $h^c(y)$. Please refer to Appendix 2 – 6 for detailed proof.

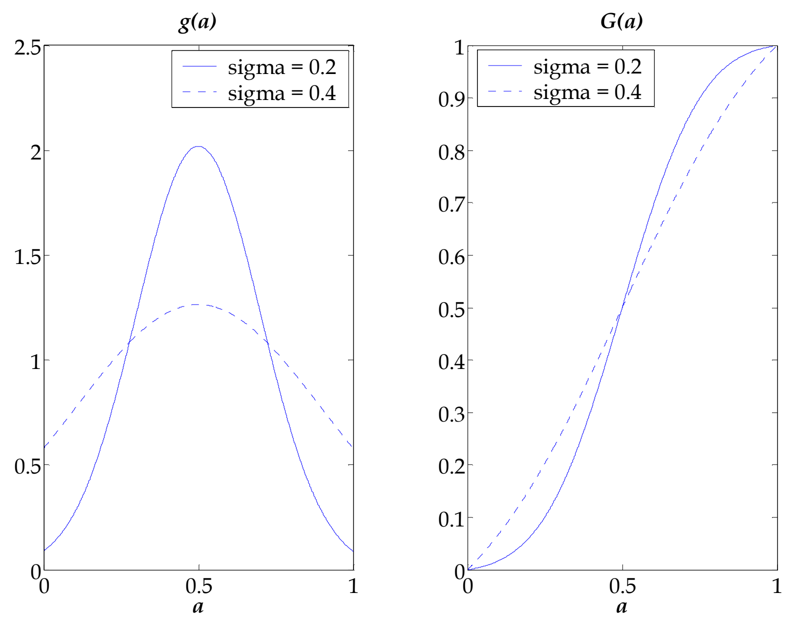
2 – 4 – 2, Examples

In this section, we cook up two examples. The first example is the case where we have $H^c(y)$ second order stochastically dominates $H^d(y)$, while the second one is the case where $H^c(y)$ does not second order stochastically dominates $H^d(y)$.

2 – 4 – 2 – 1, Example 1

In the first example, we assume that bargaining ability *a* follows a truncated normal distribution. Taking the original normal distribution $N\left(\mu_a, \sigma_a^2\right)$, *G*(*a*) is truncated at 0 from below and 1 from above. Please refer to Figure 2 – 1 below for an illustration of the distribution. We assume that $\mu_a^c = \mu_a^d = 0.5$. Therefore, when σ_a increases from σ_a^c to σ_a^d, $G^d(a)$ can be obtained from a mean preserving spread process from $G^c(a)$.

Figure 2 – 1, Truncated Normal Distribution

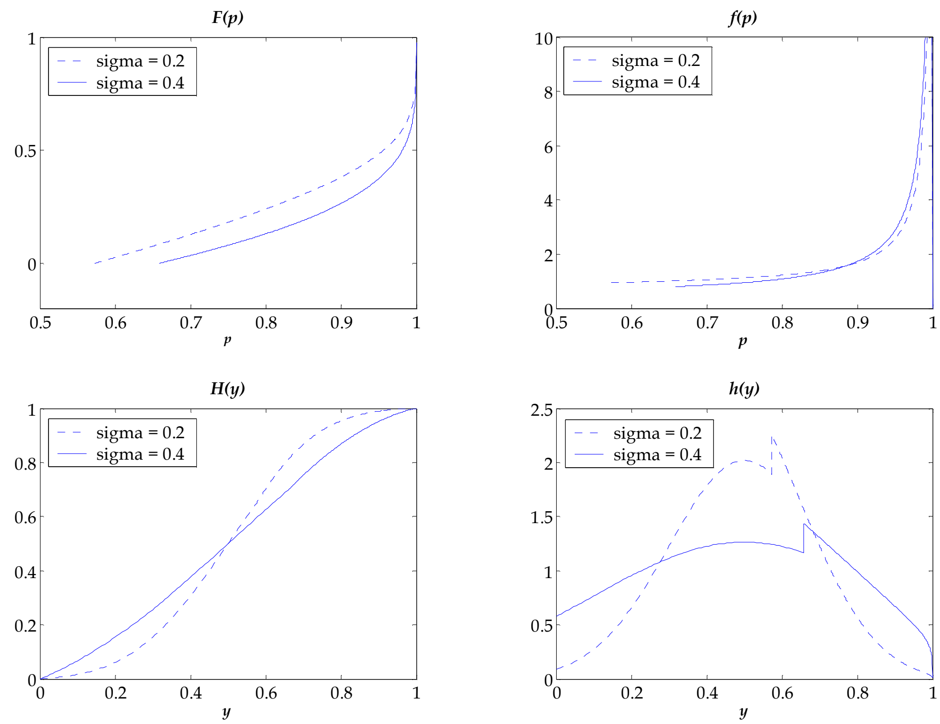


Also, we will assume that the final price $Y = min\{p, a_i\}$, where p is the offer from which consumer i buys the product, and a is her bargaining ability parameter. As has been discussed in section 2 – 2, a_i can be viewed as the final price consumer i can achieve if she chooses to bargain, and if she chooses not to bargain, she can always buy at initial offer p.

Under the set up above, $R(p) = \int_{\underline{a}}^{\overline{a}} min\{p,a\} g(a) da = \int_{\underline{a}}^{p} ag(a) da + \int_{p}^{\overline{a}} pg(a) da$, and hence, $R'(p) = 1 - G(p)$. It is also quite obvious that the lowest p at which level $R(p)$ obtains its maximum is $p^* = \overline{a}$, and for any $p < p^* = \overline{a}$, we will have $R(p)$ strictly increasing in p, or, $R'(p) > 0$.

Then, we simulate the initial offer distribution as in Figure 2 – 2 below. We assume N = 10, $\mu = 0.01$, $\mu_a^c = \mu_a^d = 0.5$, $\sigma_a^c = 0.2$ and $\sigma_a^d = 0.4$.

Figure 2 – 2, *F*(*p*) *and H*(*y*)



The $F(p)$ and $f(p)$ figures are similar to Figure 2 in Varian (1980) and Figure 1 in Stahl (1989), in that all have a steep increase in their right tail. This represents the case when firms set very high offers to ensure profits from *uninformed consumers*. It is reasonable that firms would be more likely to charge a high initial offer than to charge a median initial offer. When a firm charge median initial offer, the probability that it will attract the *informed consumers* is higher than when it charges a higher initial offer, yet the difference is not that much. A firm needs to lower its offer a lot to have a significant increase in the probability of having the lowest offer, especially when the number of firms is high.

From Varian and Stahl, we can also observe high $f(p)$ for very low p. This represents the case where firms charge low prices to attract *informed consumers*. In our case, however, because the proportion of the *informed consumers* is low here, $\mu = 0.01$, the probability that firms charge lower offers is not high. It is very intuitive, since the firms do not have much incentive to lower offers for only 1% of consumers.

H(*y*) and *h*(*y*) depict the cumulative density function and probability density function of final price, respectively. From the figures above, it can be seen that $H^c(y)$ is lower than $H^d(y)$ when *y* is small, and as *y* increases, $H^c(y)$ exceeds $H^d(y)$ gradually. $H^c(y)$ and $H^d(y)$ intersects at $\underline{y} = \underline{a} = 0$, y^m in the middle, and $\overline{y} = \overline{p} = \overline{a} = 1$. Therefore, we have $H^d(y) - H^c(y) \geq 0 \forall y \in [0, y^m]$, and $H^d(y) - H^c(y) \leq 0 \forall y \in [y^m, 1]$.

Recall that $E^c(y) = E^d(y)$. Therefore, given $\overline{y}^c = \overline{y}^d = 1$, it can be shown that $\int_{\underline{y}}^{\overline{y}} [H^d(y) - H^c(y)] dy = [\overline{y} - E^d(y)] - [\overline{y} - E^c(y)] = 0$. Also, we know that $\int_{\underline{y}}^{\underline{y}} [H^d(y) - H^c(y)] dy = 0$. Let $T(x) = \int_{\underline{y}}^{x} [H^d(y) - H^c(y)] dy$. Along with the results from last paragraph, it is easy to conclude that *T*(0) = *T*(1) = 0, and $T(x) > 0 \forall x \in (0,1)$. Or $T(x) \geq 0 \forall x \in [0,1]$, satisfying the definition of second order stochastic dominance. Therefore, in this setup, we have $H^c(y)$ second order stochastically dominates $H^d(y)$.

2 – 4 – 4 – 2, Example 2

Secondly, we cook up some examples where $\underline{y}^c < \underline{y}^d$.

Suppose that we have $G^c(a)$ and $G^d(a)$ also following truncated normal distribution as described in Example 1, except that now the support for $G^c(a)$ and $G^d(a)$ is [0.2,0.8]. Therefore, now we take the original normal distribution *N*(0.5,*sigma*), and truncated it at 0.2 from below and 0.8 from above. Then, as *sigma* increases, we will have *G*(*a*) changes from $G^c(a)$ to $G^d(a)$, such that $G^c(a)$ second order stochastically dominates $G^d(a)$.

For the final price function, we assume that $Y(p,a) = min\{p^{1/2}a^{1/2}, a\}$. It can be checked that $Y(p,a)$ satisfies all the assumptions. Table 2 – 1 gives the lowest final price $\underline{y}^c$ and $\underline{y}^d$ for different combinations of parameters. In the table, *miu* is the proportion of *informed consumers,* μ, and N is the number of firms. It can be seen from the table that for the case where μ = 0.1, N = 10 and where μ = 0.01, N =100, $\underline{y}$ goes up as $G(.)$ becomes more dispersed. This illustrates that if $G^c(a)$ second order stochastically dominates $G^d(a)$, we will have $\underline{y}^c < \underline{y}^d$. Therefore, for y such that $\underline{y}^c < y \leq \underline{y}^d$, $H^c(t) > 0$ and $H^d(t) = 0$, which means that $T(t) = \int_{-\infty}^{t}\left[H^d(y) - H^c(y)\right]dt < 0$, contradicting with the definition of second order stochastic dominance.

Table 2 – 1, $\underline{p}$ and $\underline{y}$ for Different Combination of Parameters

sigma	$\mu = 0.01$; N = 10		$\mu = 0.1$; N = 10		$\mu = 0.01$; N = 100	
	p_low	y_low	p_low	y_low	p_low	y_low
0.1	0.42840	0.20000	0.11333	0.15055	0.12497	0.15810
0.2	0.45947	0.20000	0.11488	0.15158	0.12666	0.15916
0.3	0.47100	0.20000	0.11541	0.15193	0.12723	0.15952
0.4	0.47558	0.20000	0.11562	0.15207	0.12746	0.15966
0.5	0.47780	0.20000	0.11572	0.15213	0.12757	0.15973
0.6	0.47903	0.20000	0.11578	0.15217	0.12763	0.15977
0.7	0.47978	0.20000	0.11581	0.15219	0.12766	0.15979
0.8	0.48027	0.20000	0.11584	0.15221	0.12769	0.15980
0.9	0.48060	0.20000	0.11585	0.15222	0.12770	0.15981
1.0	0.48084	0.20000	0.11586	0.15222	0.12771	0.15982

2 – 4 – 3, Discrimination in Automobile Market

Automobile market is one market in which bargain happens. When we incorporate bargaining, our model predicts exactly the same things as observed by Goldberg(1996) and Ayres and Siegelman (1995), namely, higher initial offers for the minorities and similar transaction prices. Suppose that minorities have more dispersed bargaining ability than non-minorities, yet at the mean level, their bargaining are of similar effectiveness. Denote minority's bargaining ability distribution to be $G^d(a)$, and non-minority's to be $G^c(a)$, we have $G^c(a)$ second order stochastically dominates $G^d(a)$. And follow the argument in section 2 – 4 – 1, firms would charge higher initial offers to minorities than non-minorities.

This is because the firms know that even if their offers are high, *uninformed consumers* would buy from them. Since some non-minorities are not good at bargaining, firms would like to higher the offers to take advantages from them. And as for those minorities who are good at bargaining, they can always bargain for lower prices. This explains why we observe the "discriminative" offers in the auto market.

Of course, firms could not increase offer without limit. They have incentive to lower their offers to attract *informed consumers*, and it is *informed consumer* proportion that dictates how profitable the firms can be. If the proportion is the same for minorities and non-minorities, the firms' profits from the two groups would be the same, and, as expected profits for the firms are the same as expected final price paid by consumers, the final prices would stay similar for the two groups at the mean level.

What's more, our model fits into this case especially well, when considering the set up of Ayres and Siegelman (1995)'s experiment. In that experiment, Ayres and Siegelman trained their testers to use the same bargaining strategy, and hence we would suppose that they had the same level of bargaining ability. As shown in section 2 – 4 – 1, if minorities' bargaining ability distribution is more dispersed, we will have initial offer higher for minorities, and since for both groups, the final price function is the same, we would expect a higher final price for a minority consumer than non-minority consumer with the same level of bargaining ability. In Ayres and Siegelman's experiment, we can observe that, as bargaining process went on and on, minorities consistently had higher offers than non-minorities, exactly as what our model predicts.

One remaining question is why minorities would have more dispersed bargaining ability? Goldberg (1996) claimed that, whatever reason that led to firms' discriminative behaviors, it could not be explained by household characteristics available in the CES dataset, because she could not find such relationships in her regression. One potential explanation is "learning"[33]. Suppose minorities and non-minorities only socialize within their own groups. Through their interaction with other members in their group, they learn the newest bargaining results in the group, which they use as a reference in their own bargaining. This "reference" effect may diminish as time goes by. Therefore, a person in the smaller group, or a minority, will have less chance to learn, and hence the bargaining ability distribution for the minorities is more dispersed.

[33] We thank Professor Randal Watson for providing this explanation.

Section 2 – 5, Conclusion and Further Research

In some consumption good markets, people bargain for lower price. Knowing that there will be bargaining, firms' offer setting behaviors are different, which, in turn, lead to different final prices in the market. We, here in this study, are to study how would the bargaining process affects firms' offer distribution and thus the final price distribution. Also, we try to use our model to explain two seemingly contradicting empirical works in the literature of discrimination in the auto market.

This work still has some problems, providing room for further work. First of all, we assume that the information possession is exogenously given, so that we can concentrate on the bargaining part of the model. Such an assumption, however, is not very realistic. It will be good if we can introduce searching into the model.

Secondly, we do not have a detailed model to account for the bargaining process, and it is possible to incorporate Nash bargaining game or Rubinstein alternative offering game into this model.

Chapter 3

Bargaining, Searching and Price Dispersion

---- Evidences from the Automobile Market

Chapter Three Abstract

The automobile market is a market where we can observe both bargaining and searching. In the first chapter of my thesis, I included both features into one model to have a better understanding of the pricing behaviors in the market. In this chapter, I applied my theoretical model to the automobile market, and empirically test the model.

In the empirical literature of discrimination in automobile market, there is a debate on whether there exist any differences in dealers' pricing behaviors toward minorities and non-minorities. If my theoretical model is valid, it should be true that the mean level of the bargaining ability distribution for the minorities would be similar as that for non-minorities, but the distribution would be more dispersed for the minorities. In this paper, I used CES data, and my findings support the theoretical model. The minority dummies are not significant in determining the mean level of consumers' bargaining ability distribution, but are significantly positive in determining the variance of the distribution.

Section 3 – 1, Introduction

In the empirical literature of discrimination in auto market, two entirely distinctive and contradicting conclusions were reached regarding to the price discrimination. To state more precisely, the two papers, Goldberg(1996) and Ayres and Siegelman (1995), argued on whether there exist any differences in dealers' pricing behaviors toward minorities and non-minorities.

Ayres and Siegelman (1995), using "controlled experiment" data[34], found that the initial offers for the minorities are higher. They designed the experiment carefully, controlling many biographic characteristics relevant to dealers' pricing behaviors, such as education, age, attractiveness, cars owned, financial characters, and even place of residence. The participants were trained to give similar answers to all questions asked, and they were ignorant of the purpose of the research.

Ayres and Siegelman exploited the panel structure of their data to control the audit-specific errors. They regressed absolute profits and percentage profits[35] on race and gender dummies, and found very large and significant parameters for the race dummies in all regressions. The results were robust for both parametric method and non-parametric method.

---

[34] Ayres and Siegelman referred to their data as "controlled experiment" data, yet the "experiment" they were talking about is not what we do in lab nowadays. In their research, they hired some people, including both minorities and non-minorities, trained them to behave the same, and sent them to the dealers to ask for quotes for the cars. For a detailed description of the factors they "controlled", please refer to their AER paper.

[35] Ayres and Siegelman have four dependent variables. They estimate the cost for each vehicle model first and then use their estimation to construct "initial dollar profit"=initial offer-cost, "final dollar profit"=final offer-cost, and percentage profit for both cases, with respect to cost. And by "final offer", they mean that the last offer the testers get for the first visit.

Goldberg (1996), using consumer expenditure survey data (CES), reported that there is no significant difference between the final prices for minorities and non-minorities. In this JPE paper, Goldberg used "discount" for each transaction as dependent variable[36]. She controlled all biographic characteristics and automobile features that were available from the CES data, as well as the interactions between the two groups. Neither the dummy variable for minority nor the interactions between minority dummy and other independent variables were significant.

Although there was no significant difference between the transaction prices for minorities and non-minorities, one could not conclude that there was no discrimination on the automobile market. It was very possible that some minorities found that they were discriminated after searching for a period of time, and had to drop out of the market. Therefore, the transaction prices for minorities we observed were from those who suffered less or no discrimination. To account for such sample selection bias, Goldberg designed a two-stage model. In the first stage, she ran a Probit model estimating the probability that a household would buy a new vehicle, a used vehicle, and no vehicles at all. In the second stage, she re-estimated her previous model. The improved model reported no significant changes. The dummy for minority was still not significant, for either the Pribit model or the revised OLS model.

Goldberg attributed the inconsistencies between her findings and Ayres and Siegelman's findings to the different distributions of reserve prices for minorities and non-minorities. She ran a set of quantile regressions. Please refer to Figure 3 – 1 for her results. She found that there were more minorities receiving lower discounts than non-minorities on the left tail of the distribution, and more minorities receiving higher discounts on the right tail of the distribution. Goldberg tried to explain her findings, hypothesizing that the reserve price distribution of minorities was more dispersed than that of non-minorities.

[36] She has absolute discount, that is the difference between list price and the transaction price, and relative discount, which is defined as (transaction price – list pirce)/(list price).

Then the dealers, knowing the distributions for the minorities and the non-minorities, would offer higher initial prices to the minorities, expecting higher profits. On the other hand, since the final prices depend on the reserve prices, the minorities get a more spread out distribution than the non-minorities, but the mean of the final prices for the two groups can be similar.

Figure 3 – 1, Goldberg's Quantile Regression Results [37]

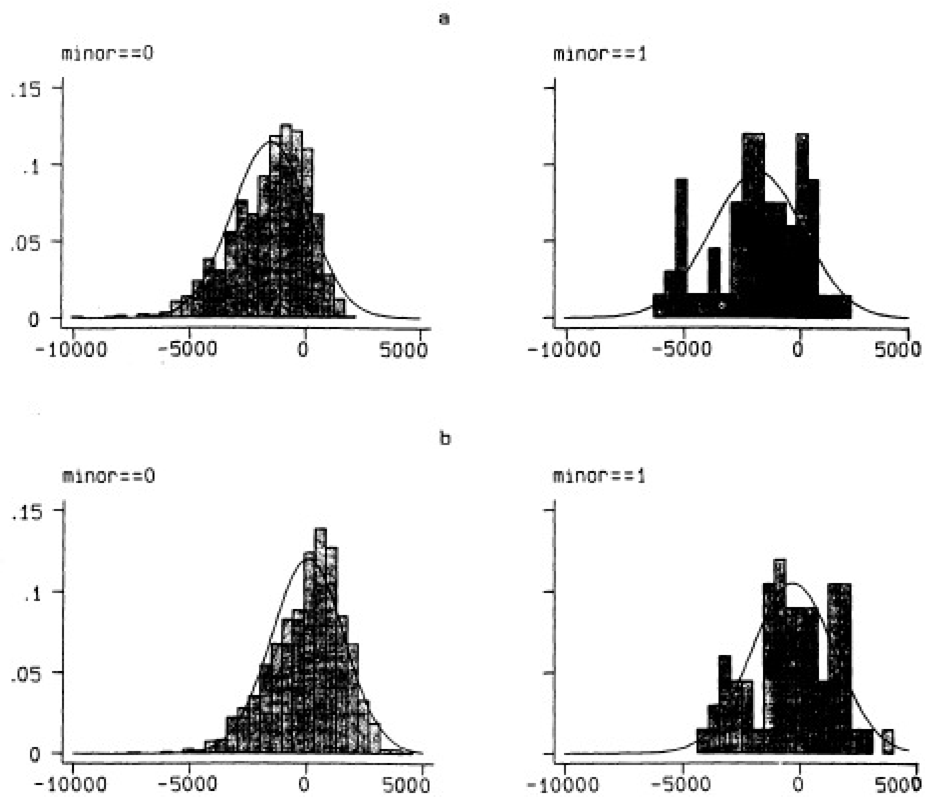


FIG. 1.—Empirical discount distribution: *a*, raw data; *b*, residuals from OLS regression.

However, Goldberg did not set up a theoretical model, and it is not clear why more dispersed reservation price distribution would lead to higher initial offer. The reservation price is endogenously determined. If higher initial offers can be an equilibrium scenario, then we must observe reservation price distribution shift higher accordingly, and to support such an equilibrium, we need consumer's searching costs to be higher. These do

[37] This is Figure 1 from Goldberg (1996).

not seem likely from the figures above.

In Chapter One of my thesis, I built up a model, where a consumer can search for a dealer with an acceptable initial offer, and then bargain for an even lower transaction price. I derived the equilibrium initial offer distribution and the final price distribution, as functions of consumers' bargaining ability distribution, number of dealers and the proportion of shoppers[38] in the market. Then, I used the theoretic model to explain the two seemingly contradicting results in the empirical literatures concerning the automobile market, and showed that, for two groups of consumer, C (Centered) and D (Dispersed), if the bargaining ability distribution for group C second order stochastically dominates the distribution for group D, we can observe similar transaction prices at the mean level and high initial offers for group D. Therefore, the contradicting results are explained, if the assumption is true that the bargaining ability distribution for minorities is of the same mean but more dispersed than that for non-minorities.

In this Chapter, I empirically tested this hypothesis. I found that at the mean level, there is no significant difference between the bargaining ability distributions for minorities and non-minorities, while the variance of the bargaining ability distribution for minorities is indeed significantly higher than that for non-minorities, whatever the information assumption is made.

Section 3 – 2 of this paper re-iterates the assumptions and the results of the theoretical model in the Chapter One of my thesis, and builds up an empirical model; Section 3 – 3 introduces the data I used in this work and the construction of some of the variables; Section 3 – 4 provides some descriptive analysis of the data; Section 3 – 5 presents the regression results; Section 3 – 6 simulates the initial offer distribution and final price levels for several scenarios; and Section 3 – 7 concludes the paper.

[38] A shopper is a consumer whose searching cost is zero, or, to put it more intuitively, a shopper is a consumer who enjoys to shop/search.

Section 3 – 2, The Model

3 – 2 – 1, Theoretical Model

Assume that there are $N \geq 2$ firms selling a homogenous good, and the firms are homogeneous in the sense that their marginal costs are constant and the same. Therefore, we can further normalize their marginal costs to be zero. The N firms maximize their profits by setting offers simultaneously.

On the demand side of the model, there are M consumers each with unit demand. The utility from consumption is assumed to be the same for all consumers, $u_i = 1, \ \forall i = 1,\ldots, M$. Consumers differ in their searching costs and bargaining parameters. For the searching cost difference, assume that proportion $\mu \in (0,1)$ of consumers have searching costs being zero, and are called "shoppers", because they enjoy shopping. The other 1 - μ proportion of consumers have searching costs $c > 0$, and are called "non-shoppers". For the bargaining ability difference, on the other hand, assume that, somehow, the consumer i is endowed exogenously with a bargaining ability a_i which is identically independently distributed across all the consumers following a continuous distribution $G(a)$ with support $\left[\underline{a}, \overline{a}\right]$.

For the bargaining process, assume that the consumer, when facing an initial offer p, first chooses between buying from the firm and continuing to search. If he chooses to buy from the firm, he can then choose to bargain or not to bargain. If he does not bargain, the final price is p. And if he bargains, the final price will be a_i.[39]

As to the information set of the searching game, assume that the initial offer distribution in the market, $F(p)$, is common knowledge, and so is the number of firms, N, and the costs of firms.

[39] This assumption simplifies the model; otherwise, there would be no closed-form solution.

For this model, there is no pure strategy equilibrium, but a unique symmetric mixed strategy equilibrium in which the firms randomly choose an initial offer according to a continuous and atomless distribution defined on $\left[\underline{p}, \overline{p}\right]$, and the equilibrium initial offer distribution, $F(p)$, is as followed.

$$F(p;\overline{p}) = 1 - \left\{\frac{(1-\mu)}{\mu N}\varphi(p)\right\}^{\frac{1}{N-1}} \quad where \ \varphi(p) = \frac{\int_{\underline{a}}^{\overline{p}} ag(a)da + \int_{\overline{p}}^{\overline{a}} \overline{p}g(a)da}{\int_{\underline{a}}^{p} ag(a)da + \int_{p}^{\overline{a}} pg(a)da} - 1 \qquad (3-1)$$

Where N is the number of firms in the market;

μ is the proportion of shoppers in the market;

$g(.)$ is the probability desnsity function of consumers' bargaining ability distribution;

$\underline{a}$ and $\overline{a}$ are the upper and lower bound of consumers' bargaining ability;

And $\overline{p}$ is the upper bound for $F(p)$, where

$$\overline{p} = min\left\{ r, \overline{a} \,\middle|\, \int_{\underline{p}(\overline{p}=r)}^{r} F(x;\overline{p}=r)\,dx = c \right\}$$

Then, the cumulative density function of equilibrium final price distribution is as followed.

$$H(y) = G(y) + (1 - G(y))\left\{\mu\left(1 - \left[1 - F(y)\right]^N\right) + (1-\mu)F(y)\right\} \qquad (3-2)$$

Where $G(.)$ is the cumulative density function of consumers' bargaining ability distribution.

3 – 2 – 2, Empirical Model

The goal of this empirical work is to apply my theoretical model to explain the seemingly contradicting empirical results in the automobile market, one from Goldberg (1996) and the other from Ayres and Siegelman (1995). Ayres and Siegelman (1995) used the audit data they collected that contains initial offers; and Goldberg (1996) used the CES data that contains final prices. The ideal way to apply my theoretical model is to have both initial offer information from the audit data from Ayres and Siegelman (1995) and the final price data from Goldberg (1996), and solve for the bargaining ability distributions for both minorities and non-minorities that maximize the predicted probability of initial offer, $f(p)$, and the predicted probability of final price, $h(y)$, to see whether the bargaining ability distribution for minorities is of the same mean but more dispersed than that for non-minorities. If it is true, my theoretical model can be a valid potential explanation, otherwise, the hypothesis can be rejected.

However, I do not have the audit data that Ayres and Siegelman used. I only have the CES data Goldberg used, which gives the transaction prices for different car models and the corresponding household characteristics. Therefore, I will solve for the bargaining ability distribution as a function of consumers' characteristics, which best matches with the predicted probability of final price, $h(y)$. That is, I will maximize the log-likelihood function, $\sum_{i=1}^{Nobs} \ln\left(h\left(p_i\right)\right)$, where $h\left(p_i\right)$ is the probability that the i^{th} consumer ends up with a transaction price p_i.

As listed in Section 3 – 2 – 1, there are four groups of unknown parameters in the model, the parameters involved in the bargaining ability distributions, $g(.)$,the highest initial offer, $\overline{p}$, the number of firms in the market, N, and the proportion of shoppers, μ. To parametrically estimate the model, first of all, I would assume that the consumers' bargaining ability follows a normal distribution.

Assumption 1. Suppose that the consumers' bargaining ability follow a normal distribution with mean u and variance σ^2, where both u and σ^2 are linear functions of household's characteristics.

$$u = X_1\beta$$

$$\sigma^2 = X_2\gamma$$

A dummy for minorities will be included in both X_1 and X_2. Therefore, if the dummy variable that enters X_1 is not significant while the dummy in X_2 is, it shows that my theoretical model can be a potential explanation of the empirical results in the automobile market.

Secondly, without the searching cost, c, $\overline{p}$ could not be identified. I will use the highest price observed in the market as a proxy for it. Therefore, my second assumption is as follows.

Assumption 2. Suppose that the highest initial offer $\overline{p}$ is known to be the highest price observed in the dataset.

This assumption has three important implications. First of all, it assumes that the highest price observed in the dataset is the highest price in the market[40]. While it is for sure that the highest price observed is lower than the highest price possible, hopefully, the difference is not too much.

Secondly, this statement assumes that the highest price is the same for both minorities and non-minorities. Suppose that the minorities and non-minorities have the same bargaining ability distribution, a smaller $\overline{p}$ will affect the estimation the same, which means that I would not be able to find any significant difference between the two distributions from the estimation. Suppose that, on the other hand, minorities do have a more dispersed bargaining ability distribution, a lower but similar $\overline{p}$ would lead to smaller difference between the two distributions. Therefore, if, under this assumption, I still could find significant difference between the two distributions, it means that the difference could be even higher.

[40] I actually rounded up the highest price observed to the second digit after the decimal point, and use that as $\overline{p}$

Thirdly, this statement assumes that the highest initial offer equals the highest final price, which means that there is always someone in the market who would not bargain. This may not seem reasonable at the first glance, but please note that the "initial offer" discussed here agrees with the price level in Ayres and Siegelman (1995). In Ayres and Siegelman's work, consumers were asked to make some preliminary bargain, where they could take either "split-the-difference" strategy or "fixed-concession" strategy. At the end of this preliminary bargaining, the last offers were recorded, too. Ayres and Siegelman used this "last" offer as their dependent variable, and still found that the offers for minorities are significantly higher than those for non-minorities. Ayres and Sigelman's "experiments" involved some tentative bargaining behaviors. The time was limited, and the methods were limited. Therefore, it can be viewed as some routine talk between the buyers and the dealers, so as to get a more serious initial offer. Based on this offer, a good bargainer can achieve even lower price.

As mentioned before, in this work, I will focus on the difference between the results from Ayres and Siegelman and Goldberg, and I will take Ayres and Siegelman's prices as initial offers and Goldberg's price as final prices. Therefore, it could be imagined that some consumers would only go through the routine talk, lower the price a litter bit, and stop there, and by my definition, these were the consumers who would "not bargain". Also, some other consumers would go from this offer and bargain for even lower price, and lead to the difference between Goldberg (1996) and Ayres and Siegelman (1995).

Recall that the final price distribution can be written as following.

$$
\begin{aligned}
H(p) &= G(p) + (1 - G(p))\left\{\mu\left(1 - \left[1 - F(p)\right]^N\right) + (1-\mu)F(p)\right\} \\
&= 1 - (1 - G(p))\left\{\frac{(1-\mu)}{\mu N}\varphi(p)\right\}^{\frac{1}{N-1}}(1-\mu)\left\{\frac{\varphi(p)}{N}+1\right\}
\end{aligned}
\tag{3 – 3}
$$

where,

- $F(p;\overline{p}) = 1 - \left\{\frac{(1-\mu)}{\mu N}\varphi(p)\right\}^{\frac{1}{N-1}}$ is the cumulative density function of the initial offer distribution;
- $f(p;\overline{p}) = \frac{1}{N-1}\left\{\frac{(1-\mu)}{\mu N}\varphi(p)\right\}^{\frac{1}{N-1}}\frac{\varphi'(p)}{\varphi(p)}$ is the probability density function of the initial offer distribution;
- $\varphi(p) = \frac{\int_{\underline{a}}^{\overline{p}} ag(a)\,da + \int_{\overline{p}}^{\overline{a}} \overline{p}g(a)\,da}{\int_{\underline{a}}^{p} ag(a)\,da + \int_{p}^{\overline{a}} pg(a)\,da} - 1$
- $g(.)$ is the bargaining ability distribution.

Therefore, the log-likelihood function can be derived as the following.

$$
\begin{aligned}
\ln\left[h(p_i)\right] &= \frac{1}{N-1}\ln\left[\frac{(1-\mu)}{\mu N}\right] + \ln(1-\mu) + \frac{1}{N-1}\ln\left[\varphi(p_i)\right] \\
&+ \ln\left\{g(p_i)\left\{\frac{\varphi(p_i)}{N}+1\right\} - (1 - G(p_i))\frac{\varphi'(p_i)}{\varphi(p_i)}\frac{1}{N-1}\left[1+\varphi(p_i)\right]\right\}
\end{aligned}
\tag{3 – 4}
$$

Note that the log-likelihood $\ln\left[h(p_i)\right]$ is strictly decreasing in μ on [0,1], and is also strictly decreasing in N on $(0,+\infty)$. Therefore, when maximizing $\sum_{i=1}^{Nobs}\ln\left(h(p_i)\right)$, μ and N can not be identified. This is because that these two variables can only affect the initial offer distribution, and when an initial offer is given, the final price is determined solely from the bargaining process. Therefore, these two variables can only be identified from the initial offer data. With only final price data, I can only assume some scenarios of

μ and N and try to estimate the model under these assumptions. Also note that μ does not affect the estimation of the parameters of the probability density function of consumers' bargaining ability distribution. Therefore, to estimate the model, I only need to assume the value of N.

To summarize this section, the log-likelihood function given the assumptions above can be expressed as following.

$$
\begin{aligned}
&\frac{1}{N-1}\sum_{i=1}^{Nobs}\ln\left[\varphi(p_i)\right]+\sum_{i=1}^{Nobs}\ln\left(\begin{array}{l} g(p_i)\left[\dfrac{\varphi(p_i)}{N}+1\right] \\ -\left(1-G(p_i)\right)\dfrac{\varphi'(p_i)}{\varphi(p_i)}\dfrac{1}{N-1}\left[1+\varphi(p_i)\right]\end{array}\right) \\
&=\frac{1}{N-1}\sum_{i=1}^{Nobs}\ln\left(\int_{p_i}^{\bar{p}}(a-p_i)\,g(a)\,da\right) \\
&-\frac{1}{N-1}\sum_{i=1}^{Nobs}\ln\left(\int_{\underline{a}}^{p_i}ag(a)\,da+\int_{p_i}^{\bar{a}}p_i g(a)\,da\right) \\
&+\sum_{i=1}^{Nobs}\ln\left\{\begin{array}{l} g(p_i)\left[\dfrac{1}{N}\dfrac{\int_{\underline{a}}^{\bar{p}}ag(a)\,da}{\int_{\underline{a}}^{p_i}ag(a)\,da+\int_{p_i}^{\bar{a}}p_i g(a)\,da}+\dfrac{N-1}{N}\right] \\ -\dfrac{1}{N-1}\dfrac{\left(1-G(p_i)\right)^2\left[\int_{\underline{a}}^{\bar{p}}ag(a)\,da\right]^2}{\left[\int_{p_i}^{\bar{p}}(a-p_i)\,g(a)\,da\right]\left[\int_{\underline{a}}^{p_i}ag(a)\,da+\int_{p_i}^{\bar{a}}p_i g(a)\,da\right]^2}\end{array}\right\}
\end{aligned}
\qquad (3-5)
$$

Section 3 – 3, Data

To be comparable with Goldberg's work, I tried to use the same dataset whenever it is possible. Goldberg (1996) used data from Consumer Expenditure Survey (CES)[41], which provided information on household characteristics, new vehicle purchase, and old vehicle characteristic; accompanied by the Automotive News Market Data Book (ANMDB), which provided suggested retail prices for new cars on the market. Her dataset was from 1983 to 1987, and had about 1300 observations.

I also used data from CES and ANMDB. But my dataset is from 1984 to 1988, one year less than hers, because I could not find the ANMDB data for the year 1983.

The CES datasets are divided by the contents. For each quarter, there are the "FMLY" file, containing information about the household characteristics, the "MEMB" file, containing information about the characteristics and personal income of each household member, the "MTAB" file, containing detailed expenditure information, and the "ITAB" file, containing the detailed household income information. Also, for each year, there are "OVB" file, containing information about the vehicle characteristics, "OVC" file, containing information about the vehicles that were disposed, and the files about other miscellaneous information.

For this paper, I first got all the households who bought new vehicles in the year interviewed from "OVB" data, and obtained their household characteristics from the "FMLY" data. I dropped all the households with missing information for the variables

[41] The Consumer Expenditure Survey data is collected by the Bureau of Labor Statistics. Each quarter, about 5000 households are interviewed. 4 groups of questions are asked, including (1) the consumer unit characteristics and income; (2) member characteristics and income; (3) detailed expenditures; and (4) detailed income. The dataset takes the format of a pseudo – panel, in that 75% of the households would be interviewed again in the quarter that followed, and the other 25% of the households would be dropped and replaced by new households that randomly selected from all over the country. The sample is selected carefully so as to better represent the U.S. population. The dataset is used to construct the Consumer Price Index.

needed. Please refer to Table 3 – 1 for a list of the variables used in the paper and their descriptions. Also, I dropped all the households who bought new vehicles for gifts, whose vehicles were used for business, or who bought vehicles from a source other than dealers. Finally, I was left with 1125 observations. Please refer to Table 3 – 2 for the list of number of observations from the CES dataset.

Table 3 – 1, Variable List

File Type	Variable Name	Description
FMLY	NEWID	The unique household ID number, used to match with OVB/OVC data
FMLY	AGE	Age of the head of household
FMLY	URBAN	= 1 if the household lives in urban area
FMLY	EDUC	Years of education of the head of household
FMLY	FAMSIZE	Family size
FMLY	JOB	Status of job -- head of household
FMLY	OCCU	Occupancy of the head of household
FMLY	Marital	The marital status of the head of the household
FMLY	IncBT	Income before tax
FMLY	MONTH	Month of the interview
FMLY	YEAR	Year of the interview
FMLY	RACE	Race of the head of household
FMLY	REGION	4 regions: Mid West, South, Northeast, and West
FMLY	SEX	Gender of the head of household
FMLY	SMSASTAT	= 1 if the household lives in SMSA area
OVB	QYEAR	Quarter and year of the interview
OVB	NEWID	The unique household ID number
OVB	VEHYEAR	Year of the vehicle
OVB	MKMDLY	Maker and model of the vehicle
OVB	CYLQ	How many cylinders?
OVB	AUTOTRAN	= 1 if the vehicle has auto transition
OVB	PWRSTEER	= 1 if the vehicle has power steering
OVB	PWRBRAKE	= 1 if the vehicle has power brake
OVB	AC	= 1 if the vehicle has air conditioner
OVB	RADIO	= 1 if the vehicle has radio
OVB	DIESEL	= 1 if the vehicle use diesel
OVB	VEHBZ	= 1 if the vehicle is used for business
OVB	VEHNEW	= 1 if the vehicle is brand new
OVB	VEHSOURCE	Wher the vehicle is bought from

Table 3 – 1, Variable List (Continured)

File Type	Variable Name	Description
OVB	VEHOWN	Whether the vehicle is bought for own use or a gift
OVB	VEHPURMO	Which month is the vehicle bought?
OVB	VEHPURYR	Which year is the vehicle bought?
OVB	FINANCE	= 1 if the household borrowed money to buy the vehicle
OVB	TRADIN	= 1 if the transaction involve a trade-in of an old vehicle
OVB	TRADEX	Trade-in price for the old vehicle
OVB	NETPURX	Net purchase price for the new vehicle
OVB	DOWNPAY	Down payment
OVB	CRSOURCE	Source of credit
OVB	BORROW	How much money is borrowed
OVC	NEWID	The unique household ID number
OVC	MKMDLY	Maker and model of the vehicle

Table 3 – 2, Number of Households in the Dataset

	Q1	Q2	Q3	Q4	Whole Year (non-repeat)	# Obs w/ no info missing	# Obs who bought new car(s)
1984	5171	5120	5085	5224	10368	8590	287
1985	5236	5181	5069	5319	10437	8677	269
1986	4007	5814	5774	5869	11233	9264	265
1987	5905	5874	5776	5998	11656	9714	304
Total					43694	36245	1125

To define the "final price" is a little bit tricky. The CES dataset provided the transaction prices. For different vehicles, however, it needs to take into account the vehicle characteristic differences. $30,000 for a Honda Civic is too high a price, but a $30,000 Mercedes is a good bargain. Following Goldberg, I used the relative discount as a proxy for the uniform final price.

$$Relative\ Discount = \frac{Final\ Price - Listing\ Price}{Listing\ Price} \quad (3-6)$$

The idea is that, if the listing price equals $\alpha \times Dealer\ Cost$, and α is a constant across all makers and all models, the relative discount measures how much the household spends, controlling for the vehicle characteristics and the cost. To calculate this "final price", the listing prices are needed.

The Automotive News Market Data Book (ANMDB) provides information on the listing prices for different vehicle models. The problem, however, is to match the vehicle models from the CES data to the vehicle models from ANMDB. On ANMDB, detailed information about the vehicle is given, while in CES data, only some general information. Say, for example, the CES data had a household who bought a 1986 Honda Civic, with auto transmission, auto brake, auto steering system, no AC and no radio, but then it did not show whether it was Civic CRX, Civic SI, or Civic Wagon Van, nor did it tell whether the car was a 2 door model or a 4 door model, not to mention any special upgrades such as 4 wheel drive and etc.

My solution is to take all information I have, and come up with a best guess. Take Ford Escort as an example. If the vehicle had an air conditioner, it could not be an Escort Pony, because for that model, AC is not available. If it did not have a power steering system, it could not be Escort LX, GT, or EXP, because for those models, the power steering system is free. Therefore, it must be an Escort L.

In most of the cases, however, there would be more than one model that could meet the criteria. I would then calculate the weighted average listing price, using the sale numbers as the weight. There are also some other special cases, where none of the car models could meet the criteria. Then I had to decide which criterion to believe and which not. My order goes like this: from the most reliable to the least reliable, name of the model, whether it had an auto transmission system, whether it had a radio, whether it had an air conditioner, whether it had a power steering system, whether it had a power braking system, and whether it used diesel.

Also, there are some cases where there is no listing price for the model. For example, a household claimed that they bought a new 1984 Mercury Monarch, but then according to www.wikipedia.com, Mercury stopped producing this model in 1980. For this case, I had to drop this observation from the sample, because I do not have the ANMDB for the year 1980. If the household claimed that they bought a 1987 model, which stopped producing in 1985, I would change the year of the observation into 1985 and use the 1985 listing price for the vehicle.

Finally, ANMDB did not provide listing prices for all models available in the market, and I had to drop these models from my sample. After this last round of data cleaning, I was left with 856 observations.

For these 856 observations, I calculated the Relative Discount, as defined in (3-1), used the highest one as $\overline{p}$, and dropped this observation from the sample. So, finally, my sample has 843 observations.

Please refer to Table 3 – 3 below for a list of the variables used in the MLE. As stated in Section 3 – 2 Assumption 1, X_1 includes variables that affect the mean of the consumers' bargaining ability distribution, and X_2 includes variables that affect the variance of the consumers' bargaining ability distribution.

There is one last note for the variables "FirstBuyer" and "BrLoyalty". They were constructed from the households' owned vehicle information (from "OVB"), and the information on the vehicles that were disposed (from "OVC").

Table 3 – 3, Variables in the MLE

Variable Name	Description	Matrix
Chrysler	= 1 if the maker is Chrysler	X_1
Ford	= 1 if the maker is Ford	X_1
GM	= 1 if the maker is GM	X_1
JapanAll	= 1 if the maker is a Japanese company	X_1
D1985	= 1 if the vehicle is a 1985 model	X_1
D1986	= 1 if the vehicle is a 1986 model	X_1
D1987	= 1 if the vehicle is a 1987 model	X_1
V6	= 1 if the vehicle has 6 cylinder	X_1
V8	= 1 if the vehicle has 8 cylinder	X_1
AutoTran	= 1 if the vehicle has auto transmission	X_1
PwrSteer	= 1 if the vehicle has a power steering system	X_1
PwrBrake	= 1 if the vehicle has a power braking system	X_1
Radio	= 1 if the vehicle has a radio	X_1
AC	= 1 if the vehicle has an air conditioner	X_1
Diesel	= 1 if the vehicle uses diesel	X_1
TradIn	= 1 if the transaction involve a trade-in of an old vehicle	X_1
DLFinance	= 1 if the source of finance is the dealer	X_1

Table 3 – 3, Variables in the MLE (Continued)

Variable Name	Description	Matrix	
Female	= 1 if the head of the household is a female	X_1	X_2
Black	= 1 if the head of the household is a black	X_1	X_2
OtherRace	= 1 if the head of the household is not a white or a black	X_1	X_2
HighSchGrad	= 1 if the head of the household has a high school degree	X_1	X_2
ColGrad	= 1 if the head of the household has a college degree	X_1	X_2
WhiteCollar	= 1 if the head of the household has a white collar job	X_1	X_2
Retired	= 1 if the head of the household has been retired	X_1	X_2
Age	The age of the head of the household	X_1	X_2
Age2	The square of the age of the head of the household	X_1	X_2
Married	= 1 if the head of the household is married	X_1	X_2
IncBT	Income before tax	X_1	X_2
UrbanNE	= 1 if the household lives in the urban area of northeast	X_1	X_2
UrbanMW	= 1 if the household lives in the urban area of midwest	X_1	X_2
UrbanS	= 1 if the household lives in the urban area of south	X_1	X_2
UrbanW	= 1 if the household lives in the urban area of west	X_1	X_2
FirstBuyer	= 1 if the household had not had any cars before this transaction	X_1	X_2
BrLoyalty	= 1 if the household had had (a) vehicle(s) from the same maker before	X_1	X_2

Section 3 – 4, Descriptive Analysis

Before moving forward to list the result of the estimation, let's look at some descriptive statistics of the data.

Table 3 – 4 below lists the mean and standard deviation of the dependent variable, discount rate.

Table 3 – 4, Descriptive Statistics of the Discount Rate

	White		Black		Other Race	
	Male	Female	Male	Female	Male	Female
Number of Observations	581	168	38	27	32	10
Average Discount Rate	-0.065	-0.071	-0.017	-0.107	-0.062	-0.080
Standard Deviation of the Discount Rate	*0.289*	*0.280*	*0.361*	*0.260*	*0.212*	*0.312*

According to the data, the black males paid highest prices, and the black females paid lowest prices for their cars. The differences, however, are very small across all six groups, comparing to the standard deviations numbers.

Table 3 – 5, Descriptive Statistics of the Vehicle Characteristics

	White		Black		Other Race	
	Male	Female	Male	Female	Male	Female
Chrysler	8.6%	10.7%	5.3%	14.8%	9.4%	6.1%
Ford	17.4%	18.5%	26.3%	14.8%	15.6%	11.6%
GM	37.3%	38.1%	39.5%	37.0%	31.3%	22.9%
Japan Car	27.5%	27.4%	15.8%	25.9%	43.8%	17.5%
Europe Car	9.1%	5.4%	13.2%	7.4%	0.0%	4.4%
V6	26.0%	26.2%	23.7%	25.9%	12.5%	14.3%
V8	18.9%	13.7%	18.4%	3.7%	15.6%	8.8%
Auto Transmission	69.2%	61.9%	89.5%	85.2%	65.6%	46.4%
Power Sterring	86.4%	76.8%	86.8%	88.9%	81.3%	52.5%
Power Brake	88.1%	83.3%	92.1%	88.9%	81.3%	54.2%
Radio	95.0%	90.5%	94.7%	96.3%	93.8%	71.3%
Air Conditioner	80.6%	73.2%	92.1%	88.9%	78.1%	51.6%
Diesel Engine	0.7%	0.6%	0.0%	0.0%	0.0%	0.2%
Trade In	54.0%	47.6%	42.1%	51.9%	25.0%	20.0%
Dealer Finance	25.6%	38.7%	39.5%	29.6%	9.4%	30.0%

Table 3 – 5 above lists the vehicle characteristics across the 6 different groups. The data shows, first of all, that there are not much difference between white people's and black people's preferences over the brands of the vehicles. Secondly, the black people did seem to be more likely to buy a car with more options, as evidenced from higher rate of auto-transmission and air conditioner for black people. Therefore, it is obvious that the vehicle options should be controlled in the estimation process. Otherwise, the black dummy might be biased at least in estimating the mean value of the bargaining ability. Thirdly, black people and white people did not seem to behave differently in financing the purchases. The percentages of trade-in and dealer finance are similar for the blacks and the whites.

Table 3 – 6 below lists the household characteristics across the 6 different groups.

Table 3 – 6, Descriptive Statistics of the Household Characteristics

	White		Black		Other Race	
	Male	Female	Male	Female	Male	Female
Age	42.29 (*13.44*)	37.55 (*14.59*)	44.03 (*13.43*)	38.59 (*13.43*)	39.81 (*11.06*)	33.40 (*11.70*)
Income Before Tax	35,777 (*24,506*)	24,871 (*19,017*)	33,854 (*17,537*)	26,422 (*19,563*)	34,362 (*28,472*)	37,429 (*36,057*)
High School	94.8%	97.6%	89.5%	96.3%	93.8%	100.0%
High School Graduate	88.8%	89.3%	76.3%	92.6%	84.4%	100.0%
College Graduate	31.8%	29.2%	10.5%	25.9%	34.4%	50.0%
Graduate School	15.0%	11.9%	2.6%	11.1%	15.6%	20.0%
Married	86.4%	22.0%	89.5%	22.2%	87.5%	20.0%
White Collar Job	60.1%	77.4%	39.5%	81.5%	53.1%	90.0%
Retired	6.4%	3.6%	10.5%	3.7%	0.0%	0.0%
SMSANE	20.3%	20.2%	15.8%	14.8%	12.5%	10.0%
SMSAMW	22.9%	29.2%	23.7%	18.5%	15.6%	0.0%
SMSAS	24.6%	20.8%	39.5%	51.9%	15.6%	20.0%
SMSAW	16.9%	19.0%	13.2%	3.7%	56.3%	70.0%
First Buyer	48.5%	69.0%	57.9%	77.8%	50.0%	60.0%
Brand Loyalty	8.8%	6.0%	7.9%	3.7%	6.3%	0.0%

* Note: the numbers in brackets are standard deviations of the corresponding variables.

Firstly, it can be observed that the 6 sub-samples do not differ much in age, income and martial status. The black males earned a little bit less, but not very much comparing to its standard deviation. Secondly, the education levels are similar across the 6 sub-samples except for the black males, who seemed to get the least education among the whole sample. Therefore, it is not very surprised to find that only 39.5% of the black males got white collar jobs, far less than for the other 5 sub-samples. Thirdly, different races did tend to live in different places. The black households were likely to be in the south, and the other races were likely to be in the west. Fourthly, a black household were slightly more likely to be a first buyer[42]. Lastly, the black households and the white households do not differ much in being a brand loyalty[43] buyer.

[42] I defined the "first buyer" to be a household who owned no vehicles before they bought the new car.
[43] I defined the "brand loyalty" to be a household who owned a vehicle of the same brand as the new car they bought.

Section 3 – 5, Regression Results

As mentioned in Section 3 – 2, I used the maximum likelihood method to estimate the model. For the parameter N, the number of dealers in the market, which is not identifiable but useful, I will assume 2 scenarios: $N = 20$ and $N = 30$. In January 1984, the Market Data Book lists the number of dealers operating in the market. After adjusting for multiple franchises and inter-corporate duals, there were 20,841 "net dealers" for new U.S. makes of passenger cars, 10,096 for new imported cars, and 19,345 for new truck[44]. Therefore, overall, the number of dealers would be about 35,000. As stated in the government internet of the United States, the geographic area of the U.S. is 9,826,639 square kilometers. Suppose that all these dealers were distributed uniformly. Also suppose that a consumer would like to travel up to 30 miles (=48 kilometers) to shop for a new car. Therefore, the area he covers is 3.14 * 48 * 48 = 7,235 square kilometers, which, on average, would contain 7,235 * 35,000 / 9,826,639 = 26 dealers. Most of the consumer, though, would like to go even further. Therefore, the number of dealers might be higher. On the other hand, some consumers might have some preference restrictions, although they might be willing to go further, they would face a smaller N.

Another change I made is for the dependent variable, the relative discount rate as defined in (3 – 6). I add a constant to this variable, so as to make all dependent variables to be positive. This is because that, in my theoretical model, it is assumed by default that the profit the dealers make should be positive. This change will affect the constant term in matrix X_1 only. Therefore, when viewing the regression result, please ignore the constant level of the matrix X_1.

Table 3 – 7 and Table 3 – 8 show the regression results for the scenarios $N = 20$, and $N = 30$. Matrix X_1 includes all the variables that affect the mean level of the consumers' bargaining ability distribution, and Matrix X_2 includes all the variables that affect the

[44] Note that the dealers for trucks could overlap with the dealers for passenger cars, since the Market Data Book only controlled for possible "multiple franchising".

variance of the consumers' bargaining ability distribution. It can be seen that the estimation results for the $N = 20$ scenario and the $N = 30$ scenario do not differ a lot. The variables that are significant in $N = 20$ case are significant in $N = 30$ case, though not vice versa. Also, the signs of the significant variables are the same across the two cases.

First of all, it can be seen from Table 3 – 7 that all of the vehicle characteristics, such as whether it has auto transmission, have positive signs, indicating an increase in the mean of the consumers' bargaining ability distribution. Recall that in the model, the bargaining ability is actually the bargaining inability. Actually, in the model, the bargaining ability can be interpreted as the final price a consumer can bargain for. Although I have controlled for the option prices[45] in the listing prices, some of the variables are statistically significant. Therefore, it is very intuitive that that it is harder to bargain for a vehicle with more options, like auto-transmission, power steering system and etc.

Secondly, trade-in an old car increased the mean of the bargaining ability, or, to be more intuitive, increased the final prices. This result is consistent with the findings in the literature. Actually, as mentioned by Goldberg (1985), the April 1985 issue of Consumer Reports recommends that, when discussing the price for a new vehicle, a buyer should keep his old car out of discussion, so as to bargain for a lower price easier.

Thirdly, if a consumer chose to use the loan from the dealer, it was easier for the consumer to get extra discount. This is very intuitive, since the dealers can always make additional profits by setting higher interest rates, or from the cuts they receive as agents for the loan providers.

Fourthly, look at the group of variables that capture the households' financial abilities, the education dummies, the WhiteCollar dummy (indicating whether the head of a household has a white collar job) and the Retired dummy (indicating whether the head of a

[45] The listing price is defined as the listing price for the vehicle + $\sum_i Dummy_i \times Price_i$. Where Dummy_i equals to one if option *i* is selected, and the price_i is the corresponding price for option *i*.

household is retired). The education dummies are all positive and the ColGrad is significant. The WhiteCollar is also significant in $N = 30$ case, and the t-value for the $N = 20$ case is 1.76, a little bit shy of being significant. The Retired dummy is negative, though not significant. It seemed that the households with better financial situation were not as good in bargaining. One potential reason is that the opportunity costs of these groups of people are higher, and would not choose the time-consuming price negotiations. I like this argument. It fits my model better. Another potential reason is that these people are those who would buy luxurious vehicles, vans and SUVs, and these vehicles are not likely to give discounts.

Fifthly, the consumer who was brand loyalty was less likely to bargain for a lower price. This might because that such consumers are less price elastic, and the dealers would then want to take more advantage of it. The FirstBuyer dummy is positive, and significant in $N = 30$ case, meaning, a first buyer is likely to be a bad bargainer. This might because a first time buyer is new to the market, and does not have enough information or experience to bargain for a better deal. This result, however, is not consistent with the findings in the literature, which showed that the first buyers are likely to get extra discounts.

Sixthly, the regional dummies are of small scale, and only SMSANE and SMSAW are significant in the $N = 30$ case, indicating that the differences between SMSA area and rural area were not very big.

Last but not least, there is no significant difference between the mean levels of the bargaining ability distribution for white people, black people and other races, and there is no significant difference for males and females.

When examining Table 3 – 8, the most important results are that the Black dummy is positive and very significant, and so is the Female dummy. This confirms the hypothesis that the bargaining ability distribution for the minorities is of the same mean level, but more dispersed than that for the non-minorities.

Why would minorities have more dispersed bargaining ability? It might be explained using a learning process[46]. Suppose minorities and non-minorities only socialize within their own groups. Through their interaction with other members in their group, they learn the newest bargaining results in the group, which they use as a reference in their own bargaining. This "reference" effect may diminish as time goes by. Therefore, a person in the smaller group, or a minority, will have less chance to learn, and hence the bargaining ability distribution for the minorities is more dispersed.

Following the explanation above, a variable that is significantly positive in matrix X_2 serves as a factor that limits people's information intake, or it may simply indicate that people in such a group behave very differently when bargaining. On the other hand, a negative significant variable may indicate a group of consumers who have better information. The most prominent terms include Retired, OtherRace and FirstBuyer, besides the two minority dummies Female and Black. All have negative signs. The retired people have time to socialize more. Also, they tend to have more information through their experiences. As for the significantly negative OtherRace and FirstBuyer, my only explanation is that, within the group, they might behave similarly.

[46] I thank Professor Randal Watson for providing this explanation.

Table 3 – 7, Regression Results I – Matrix X_1

Variable Name	N = 20			N = 30		
	Coefficient	Std. Dev	T - Test	Coefficient	Std. Dev	T - Test
C	0.5076	0.1396	3.6349	0.6745	0.0760	8.8731
Chrysler	0.2687	0.0568	4.7310	0.0934	0.0271	3.4521
Ford	-0.0601	0.0461	-1.3037	-0.0878	0.0209	-4.1988
GM	0.0541	0.0462	1.1700	-0.0490	0.0205	-2.3924
JapanAll	0.0296	0.0447	0.6628	-0.0036	0.0206	-0.1751
D1985	0.0316	0.0306	1.0349	-0.0179	0.0140	-1.2734
D1986	-0.0487	0.0272	-1.7875	0.0060	0.0155	0.3836
D1987	-0.0288	0.0284	-1.0144	-0.0341	0.0149	-2.2901
V6	0.2081	0.0260	8.0153	0.1201	0.0133	9.0544
V8	0.2378	0.0298	7.9745	0.1691	0.0175	9.6832
AutoTran	0.0906	0.0260	3.4806	0.0481	0.0128	3.7471
PwrSteer	0.0849	0.0381	2.2273	0.0418	0.0204	2.0477
PwrBrake	0.1225	0.0316	3.8822	0.1261	0.0205	6.1604
Radio	0.0107	0.0550	0.1945	0.1078	0.0245	4.3993
AC	0.0354	0.0239	1.4794	-0.0016	0.0141	-0.1169
Diesel	0.3105	0.2025	1.5331	0.0414	0.0807	0.5139
TradIn	0.0472	0.0208	2.2734	0.0822	0.0101	8.0947
DLFinance	-0.0426	0.0210	-2.0251	-0.0327	0.0121	-2.7063
Female	-0.0427	0.0360	-1.1858	0.0340	0.0257	1.3250
Black	0.0246	0.0414	0.5938	-0.0131	0.0392	-0.3343
HighSchGrad	0.0311	0.0521	0.5970	-0.0125	0.0282	-0.4441
ColGrad	0.1608	0.0464	3.4685	0.0748	0.0151	4.9541
WhiteCollar	0.0522	0.0297	1.7564	0.0497	0.0164	3.0322
Retired	-0.0042	0.0526	-0.0802	-0.0249	0.0369	-0.6748
Age	0.0150	0.4967	0.0301	0.0067	0.3353	0.0201
Age2	0.0292	0.5772	0.0506	-0.1795	0.3831	-0.4685
OtherRace	0.0466	0.0488	0.9548	-0.0349	0.0263	-1.3239
SMSANE	0.0328	0.0581	0.5658	0.0707	0.0228	3.1044
SMSAMW	-0.0393	0.0505	-0.7778	-0.0361	0.0217	-1.6634
SMSAS	-0.0457	0.0525	-0.8702	0.0049	0.0216	0.2272
SMSAW	-0.0309	0.0580	-0.5331	0.0575	0.0240	2.3993
FirstBuyer	0.0046	0.0335	0.1383	0.0480	0.0154	3.1230
BrLoyalty	0.1287	0.0601	2.1423	0.0490	0.0238	2.0607
IncBT	0.0346	0.0688	0.5024	0.1041	0.0262	3.9767
Married	0.0150	0.0430	0.3499	0.0188	0.0205	0.9205

Table 3 – 8, Regression Results II – Matrix X_2

Variable Name	N = 20			N = 30		
	Coefficient	Std. Dev	T - Test	Coefficient	Std. Dev	T - Test
C	0.5631	0.0834	6.7506	0.2473	0.0412	6.0073
Age	-0.0835	0.3551	-0.2352	-0.0188	0.1977	-0.0949
Age2	-0.1364	0.3853	-0.3539	0.1229	0.2248	0.5466
HighSchGrad	-0.0084	0.0352	-0.2393	-0.0107	0.0212	-0.5061
ColGrad	0.0625	0.0437	1.4306	-0.0126	0.0091	-1.3874
WhiteCollar	-0.0235	0.0292	-0.8047	-0.0384	0.0113	-3.4057
Retired	-0.0924	0.0398	-2.3218	-0.0734	0.0214	-3.4384
Female	0.1905	0.0392	4.8575	0.0865	0.0168	5.1610
Black	0.1144	0.0254	4.5037	0.0650	0.0298	2.1850
OtherRace	-0.1136	0.0442	-2.5677	-0.0452	0.0173	-2.6111
SMSANE	-0.1165	0.0610	-1.9093	0.0036	0.0151	0.2383
SMSAMW	-0.0855	0.0560	-1.5256	0.0336	0.0147	2.2850
SMSAS	-0.0878	0.0636	-1.3800	0.0113	0.0149	0.7588
SMSAW	0.0385	0.0679	0.5670	0.0365	0.0161	2.2689
FirstBuyer	-0.0853	0.0301	-2.8362	-0.0411	0.0095	-4.3295
BrLoyalty	-0.0208	0.0485	-0.4294	-0.0228	0.0141	-1.6188
IncBT	-0.0327	0.0470	-0.6946	-0.0962	0.0172	-5.5923
Married	-0.0167	0.0343	-0.4860	0.0181	0.0131	1.3831

Section 3 – 6, Simulate Initial Offer Distribution

With the estimation in Section 3 – 5, I can simulate the initial offer distribution, given the vehicle information and household information. In this section, I try to simulate the initial offer distribution following Ayres and Siegelman (1995)'s specification. In my simulation, I will simulate the scenarios of 4 cases, where $N = 20$ or 30 and $\mu = 0.1$ or 0.2.

Vehicle characteristics: Ayres and Siegelman (1995) did not specify the brand and the model of the vehicles in their study. They only mentioned that their testers were randomly assigned to 153 dealerships bargaining for 9 car models. Table 3 – 9 below shows the vehicle characteristics in my sample. It can be seen that the most common vehicle brand is GM. Therefore, I choose to simulate the case of GM.

Ayres and Siegelman (1995) was published in 1995. They did not mention when the study was conducted, but it would not be too early. I choose 1987, which, in my sample, is the closest to the year the paper was published.

As for the other vehicle characteristics, in my sample, it is common that a vehicle had auto-transmission, power steering system, power brake, radio and air conditioner. It is not so common that a vehicle had diesel engine. I will follow the common cases.

Similarly, I will assume the case that the consumer chose to trade in his/her old car, but he/she would not finance the purchase using the loan provided by the dealer.

Table 3 – 9, Number of Vehicles Bought by Each Group of Consumers

	White		Black		Other		Whole Sample
	Male	Female	Male	Female	Male	Female	
Chrysler	50	18	2	4	3	0	77
Ford	99	31	10	4	5	3	152
GM	211	62	15	9	10	1	308
JapanAll	160	45	6	7	14	6	238
EuropeAll	52	9	5	2	0	0	68
D1984	132	37	3	5	7	2	186
D1985	134	28	8	3	6	1	180
D1986	163	49	15	10	9	4	250
D1987	143	51	12	8	10	3	227
AutoTran	394	102	34	22	21	6	579
PwrSteering	493	127	33	23	26	8	710
PwrBrake	503	138	35	23	26	8	733
Radio	543	149	36	25	30	8	791
AC	459	122	35	23	25	6	670
Diesel	4	1	0	0	0	0	5
Trade In	309	78	16	13	8	2	426
Dealer Finance	146	64	15	8	3	3	239
Total	572	165	38	26	32	10	843

Household characteristics: Ayres and Siegelman (1995)'s testers were between 28 and 32 years old, had 3 to 4 years of postsecondary education. They were of average attractiveness. They drove to the dealer in similar rented cars and wore similar sportsware. If asked, they would give similar addresses, which were from a prosperous Chicago neighborhood. Therefore, I would simulate the case where the household lives in SMSAMW area, of age 30, has finished high school education, and is not first buyer.

As for the other household characteristics, I choose to follow the majorities of my sample. Please refer to Table 3 – 6 in Section 3 – 4 for the average household characteristics by racial groups. According to the information provided, I will assume that an average White/Black consumer has a white collar job and has not retired, whose income before tax is 33,000, and who does not have brand loyalty. For male consumer, I assume he is married; while for female consumer, I assume she is single. Table 3 – 10 below lists the normal distribution parameters of the bargaining ability distributions for the four samples.

Table 3 – 10, the Bargaining Ability Distribution Parameters for the Four Sub-Samples

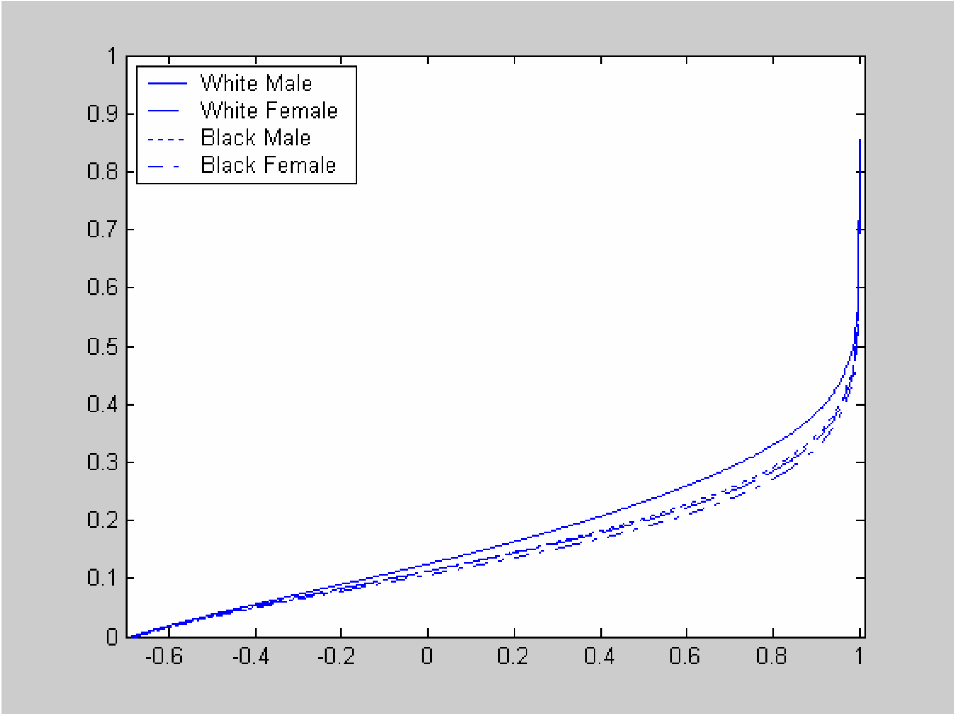
		White		Black	
		Male	Female	Male	Female
N=20	Mean	1.0017	0.9440	1.0262	0.9685
	Standard Deviation	0.3810	0.5881	0.4953	0.7025
N=30	Mean	1.0358	1.0510	1.0227	1.0379
	Standard Deviation	0.2235	0.2919	0.2885	0.3570

Note: the values listed above are the fitted values of the regression results. Therefore, the mean level is the predicted relative discount rate plus a constant, as mentioned in the Section 3 – 5.

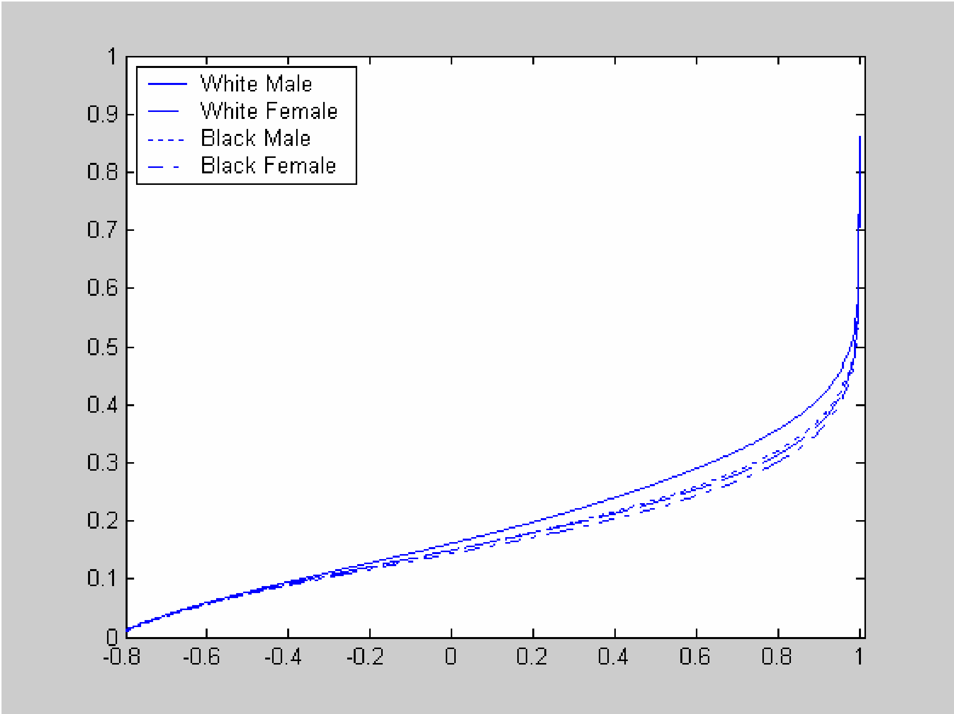
Figure 3 – 2 and Figure 3 – 3 plot the cdf and pdf of initial offer distribution for white male, white female, black male and black female. It is very difficult to tell what happens from Figure 1 and 2, since all four curves are tangled together. Therefore, I enlarge 3 spots of the cdf of initial offer distribution for the case $N = 20$ and $\mu = 0.2$, as in Figure 3 – 4.

Figure 3 – 2, Cumulative Density Function of the Initial Offer Distribution

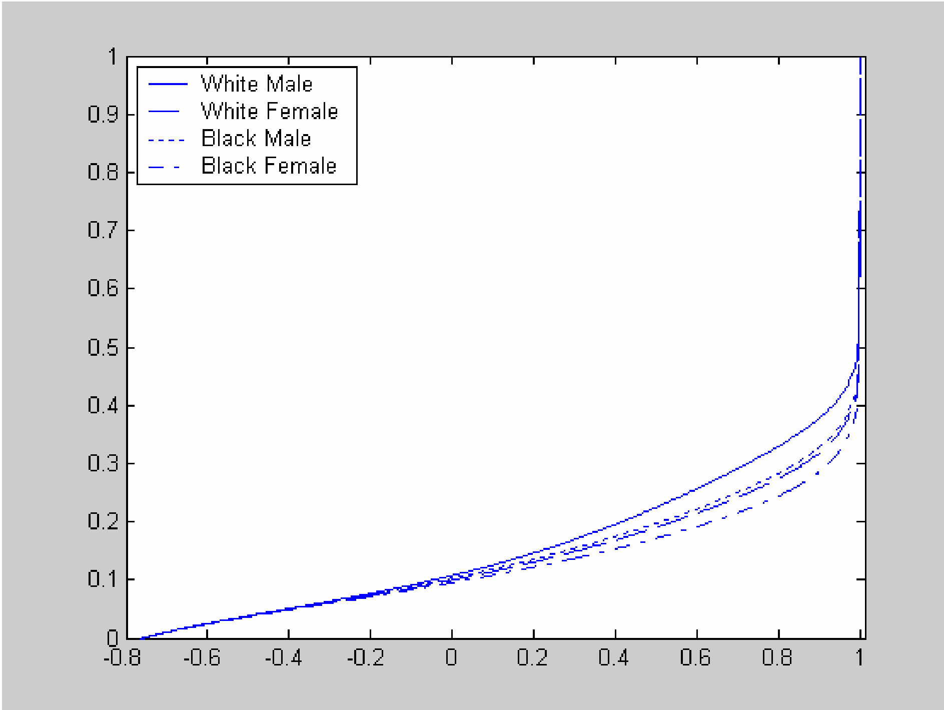
3 – 2 – 1, N = 20, μ = 0.1



3 – 2 – 2, N = 20, μ = 0.2



3 – 2 – 3, $N = 30$, $\mu = 0.1$



3 – 2 – 4, $N = 30$, $\mu = 0.2$

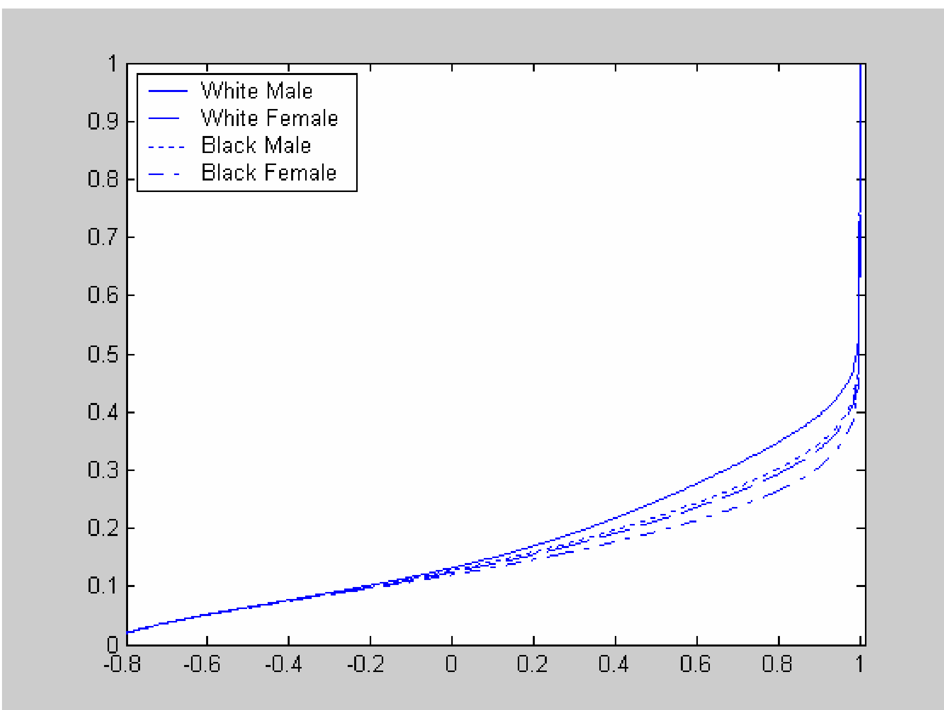
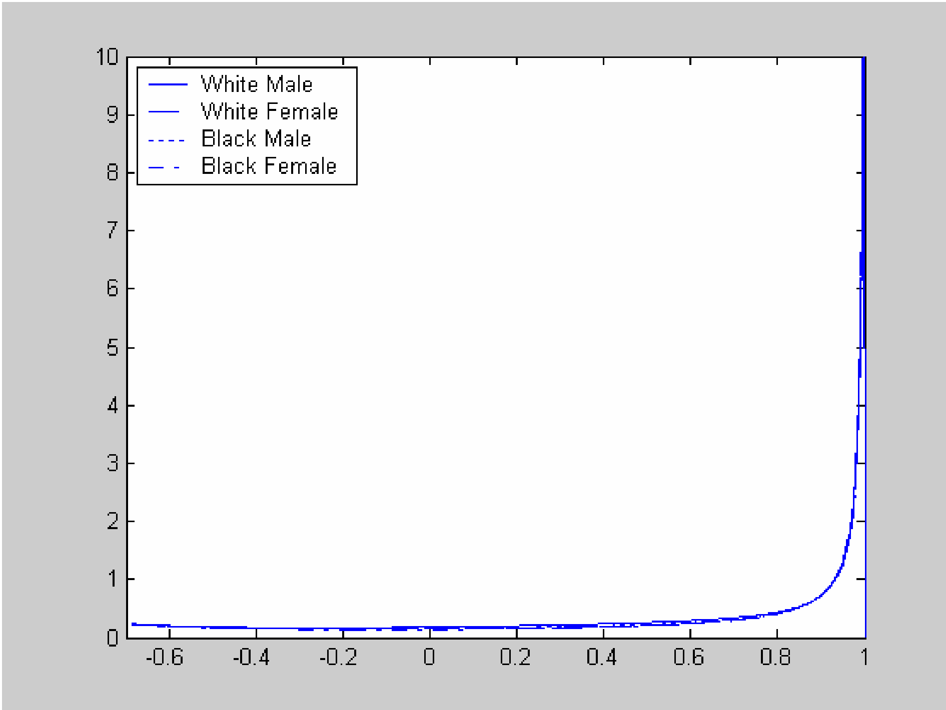
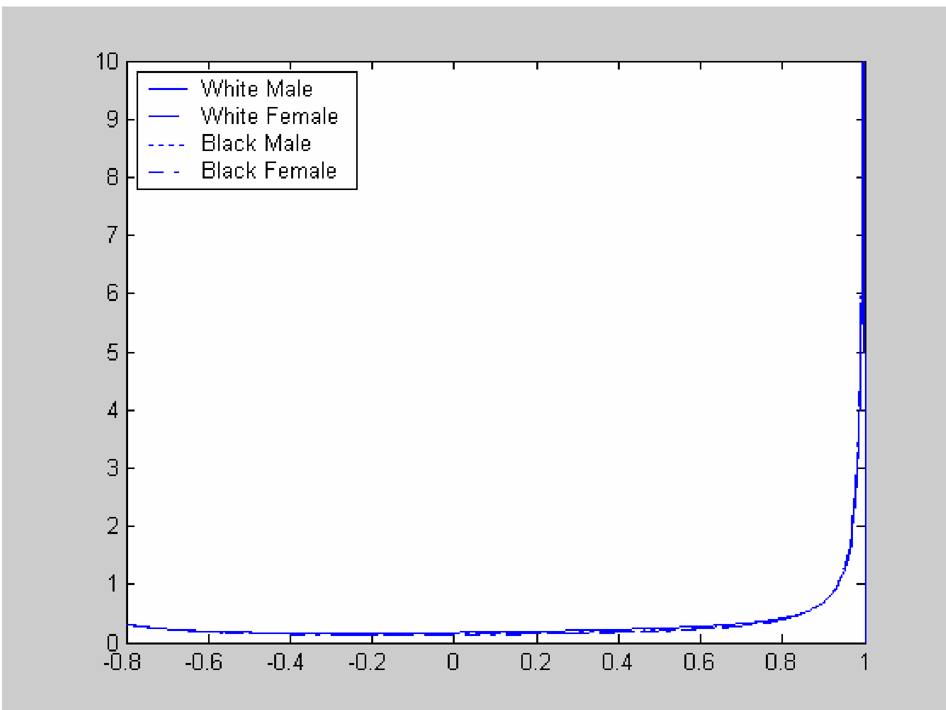


Figure 3 – 3, Probability Density Function of the Initial Offer Distribution

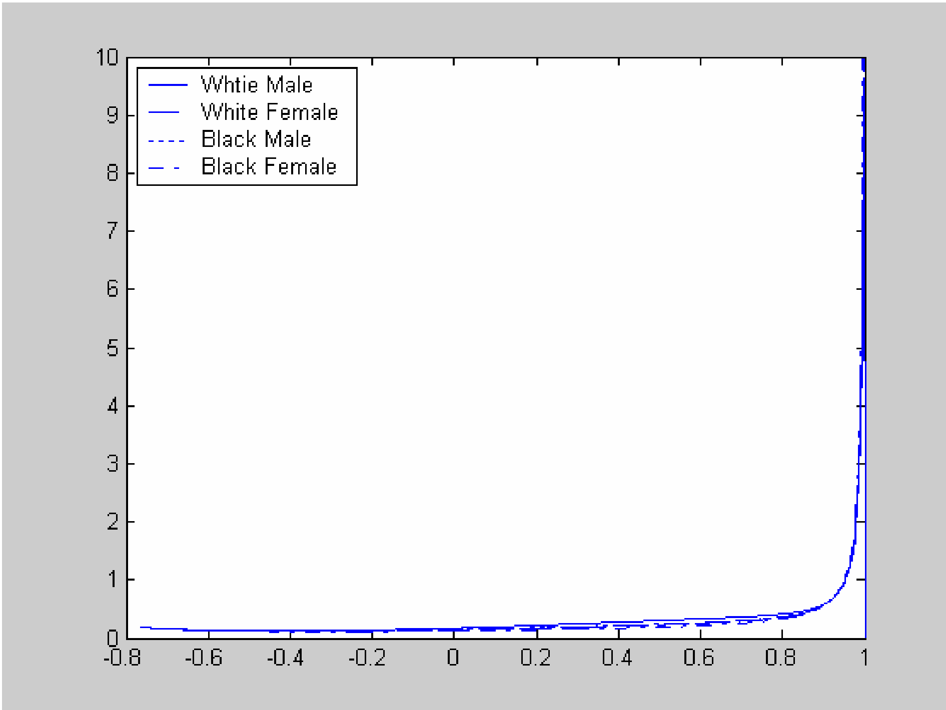
3 – 3 – 1, $N = 20$, $\mu = 0.1$



3 – 3 – 2, $N = 20$, $\mu = 0.2$



3 – 3 – 3, N = 30, μ = 0.1



3 – 3 – 4, N = 30, μ = 0.2

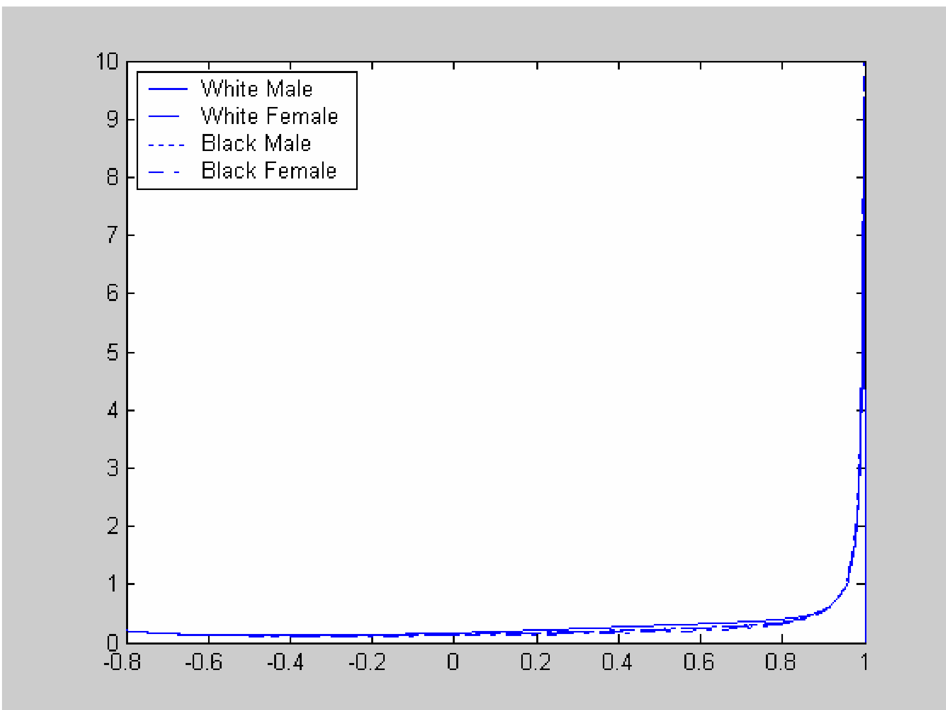
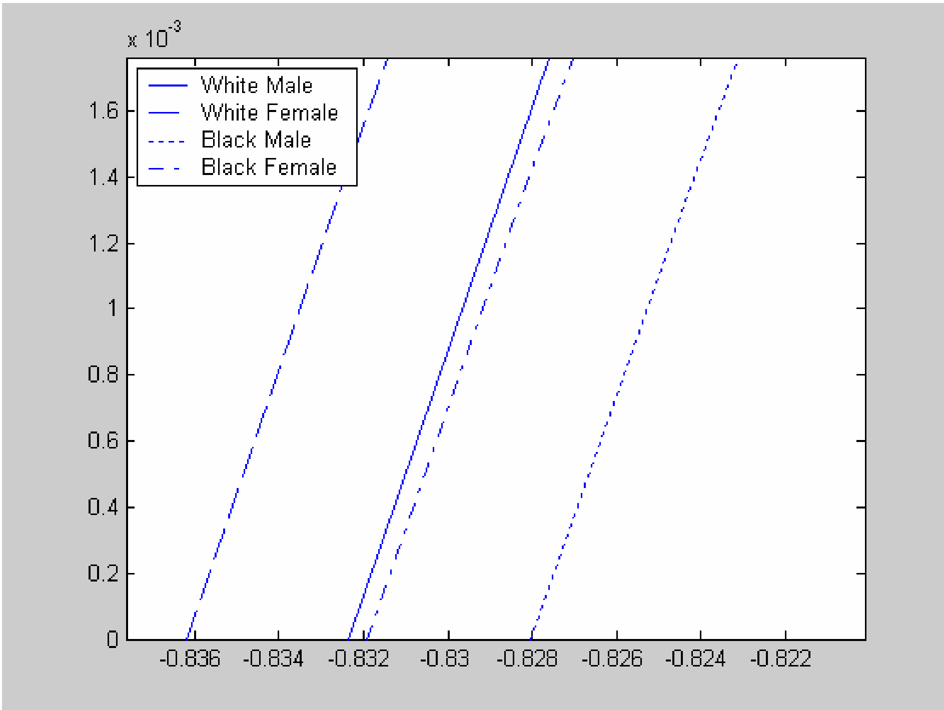
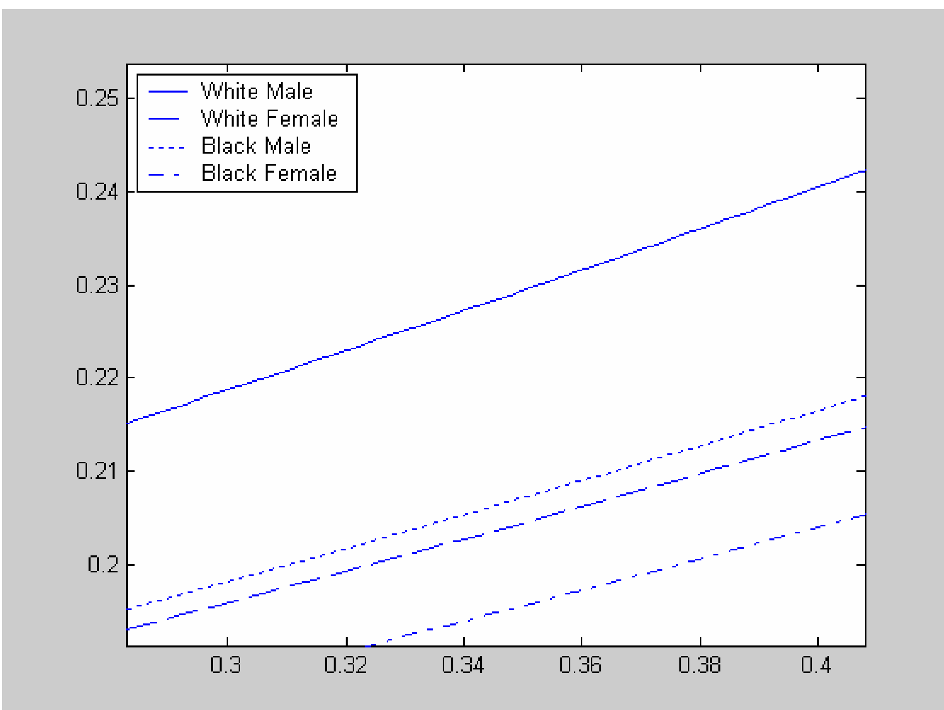


Figure 3 – 4, Enlarged Spots for F(p) [$N = 20$, $\mu = 0.2$]

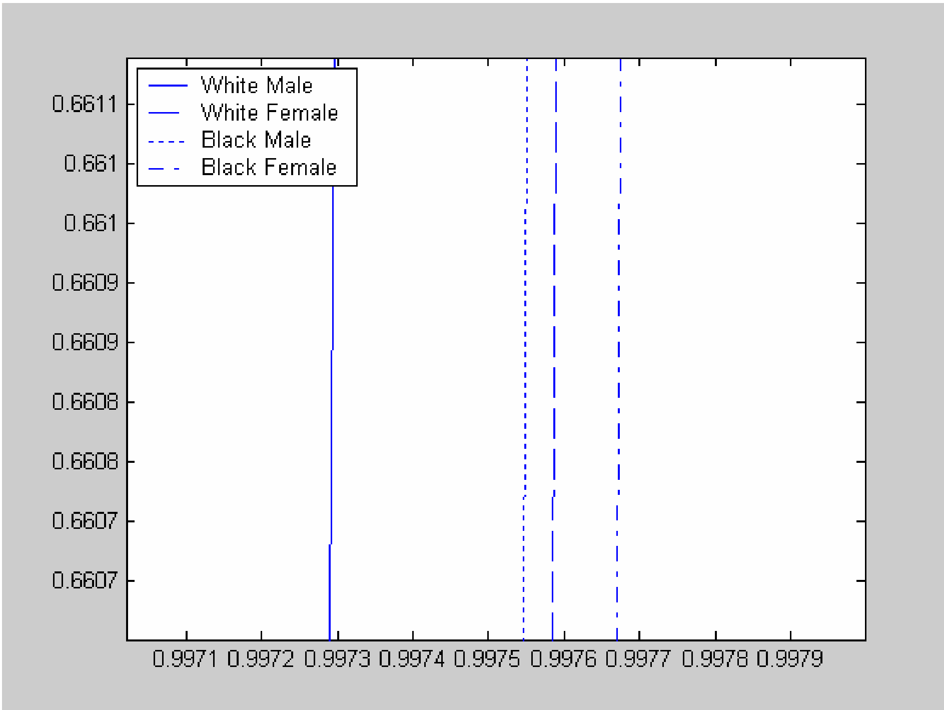
3 – 4 – 1



3 – 4 – 2



3 – 4 – 3



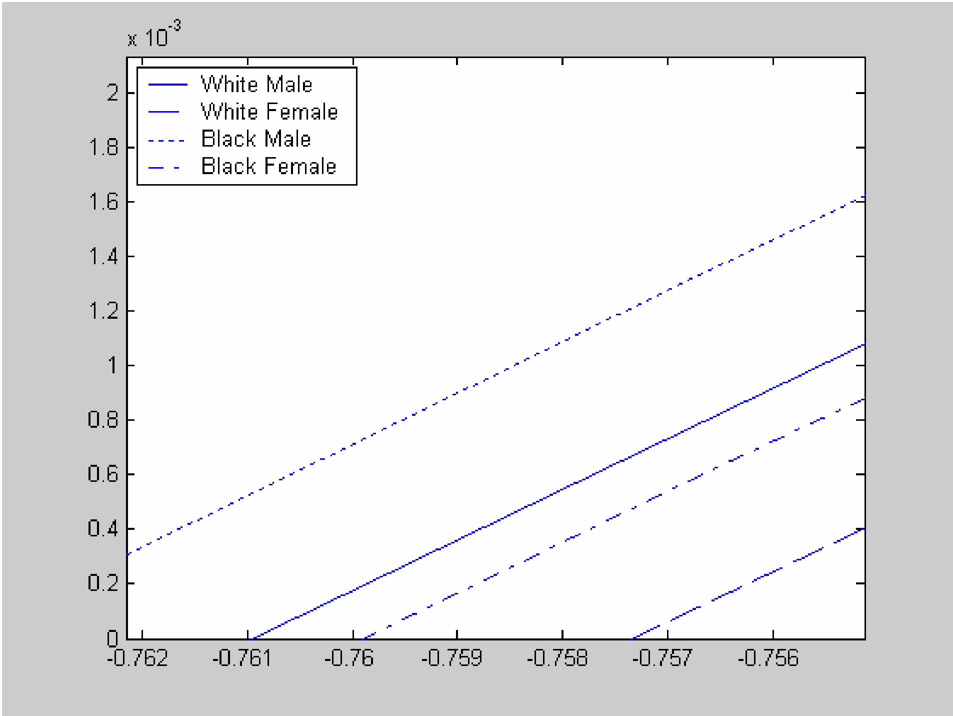
From the enlarged cdf figures, it is clearly shown that, from the lower bound of the initial offer distribution (Figure 3 – 4 – 1) to the upper bound of the distribution (Figure 3 – 4 – 3), the cumulative density function curve of the white male is always above that of the black male, and the cdf curve of the white female is always above that of the black female, which means that the initial offer distributions for the white consumers first order stochastic dominate those for the black consumers.

However, since the expectation of the mean of the initial offer distribution is not the same for the blacks and the whites, the first order stochastic condition does not always hold. Figure 3 – 5 – 1 to Figure 3 – 5 – 3 below is the case where $N = 30$ and $\mu = 0.1$. Figure 4 – 1 shows that, from the very beginning, the cdf curve of the black male is above that of the white male and the curve of the black female is above the white female. Soon, the curves for the white consumers catch up and go on top of the curves for the blacks

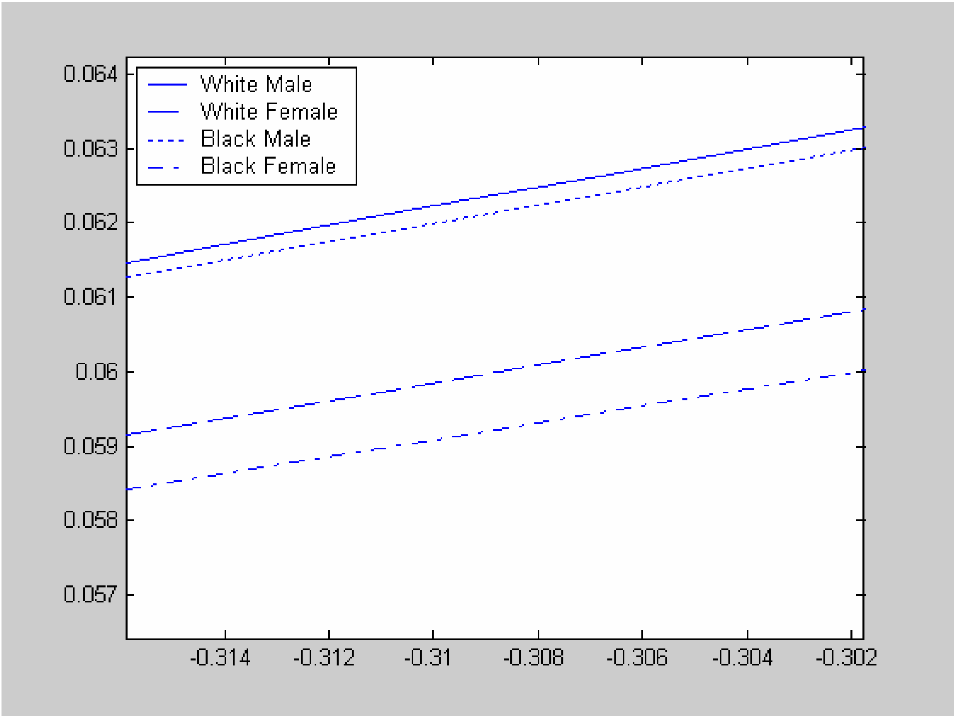
(Figure 3 – 5 – 2). Later on, the curves for black male exceeds that for white male, again, while the curve for white female always on top of that for black female (Figure 3 – 5 – 3).

Figure 3 – 5, Enlarged Spots for F(p) [N = 30, μ = 0.1]

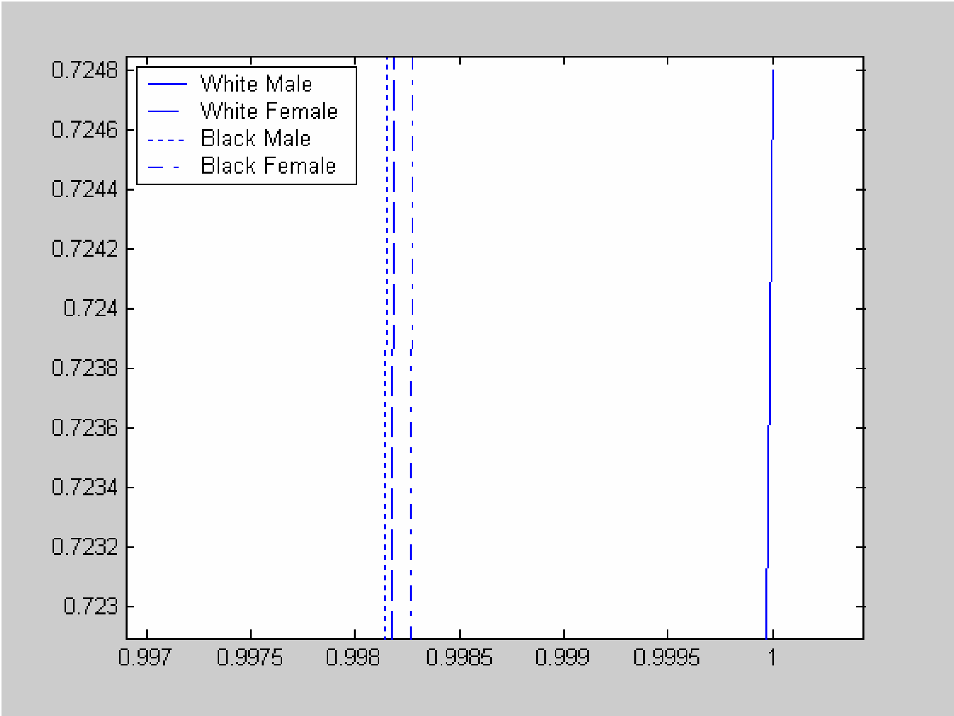
3 – 5 – 1



3 – 5 – 2


White Male
White Female
Black Male
Black Female
0.064
0.063
0.062
0.061
0.06
0.059
0.058
0.057
-0.314
-0.312
-0.31
-0.308
-0.306
-0.304
-0.302

3 – 5 – 3


White Male
White Female
Black Male
Black Female
0.7248
0.7246
0.7244
0.7242
0.724
0.7238
0.7236
0.7234
0.7232
0.723
0.997
0.9975
0.998
0.9985
0.999
0.9995
1

Now that I've simulated the initial offer distribution for the represent customers from the four groups, it is easy to calculate the expectation of the initial offer that each represent customer would get.

Table 3 – 11, Average Initial Offer
for the Black and White Consumers

	White		Black	
	Male	Female	Male	Female
$N = 20$, $\mu = 0.1$	0.5070	0.5351	0.5394	0.5556
$N = 20$, $\mu = 0.2$	0.4170	0.4383	0.4435	0.4557
$N = 30$, $\mu = 0.1$	0.5074	0.5285	0.5331	0.5453
$N = 30$, $\mu = 0.2$	0.4257	0.4405	0.4458	0.4543
Average	0.4643	0.4856	0.4905	0.5027

* Note that these are the average discount ratios.

By simple algebra, black males are offered 2.6% higher than white males, white females are offered 2.1% higher than white males, and black females are offered 3.8% higher. Ayres and Siegelman (1995) used several different estimation methods to estimate their model, and they found that the offers for black males were higher than those for white males by 8.4% to 9.8% of the listed price; the offers for white females were higher than those for white males by 0.9% to 1.7% of the listed price; and the offers for black females were higher by 2.7% to 4.5% of the listed price. From my simulation, the most close case is when N = 30 and μ = 0.2, where the differences are 2.1%, 1.47% and 2.86%, respectively. While the difference between white male and black male is far less than Ayres and Siegelman's result, the other two numbers are within the range.

Table 3 – 12 below shows the expected final prices for the white consumers and the black consumers. The difference between any two groups of consumers is about 0.15%. If the price of the vehicle is $12,000, which is about the average price in my sample, then the difference is less than $18, almost negligible. Therefore, I do not think any significant difference can be found through regression, which is consistent with Goldberg's finding.

Table 3 – 12, Expected Final Prices

for the Black and White Consumers

	White		Black	
	Male	Female	Male	Female
$N = 20$, $\mu = 0.1$	0.0451	0.0433	0.0459	0.0443
$N = 20$, $\mu = 0.2$	0.0401	0.0385	0.0408	0.0394
$N = 30$, $\mu = 0.1$	0.0300	0.0288	0.0306	0.0295
$N = 30$, $\mu = 0.2$	0.0267	0.0256	0.0272	0.0262
Average	0.0355	0.0340	0.0361	0.0348

* Note that these are the average discount ratios.

Section 3 – 7, Conclusion

The automobile market is a market where we can observe both bargaining and searching. In the first chapter of my thesis, I included both features into one model to have a better understanding of the pricing behaviors in the market. In this chapter, I applied my theoretical model to the automobile market, and empirically test the model.

In the empirical literature of discrimination in automobile market, there is a debate on whether there exist any differences in dealers' pricing behaviors toward minorities and non-minorities. If my theoretical model is valid, it should be true that the mean level of the bargaining ability distribution for the minorities would be similar as that for non-minorities, but the distribution would be more dispersed for the minorities. In this paper, I used CES data, and my findings support the theoretical model. The minority dummies are not significant in determining the mean level of consumers' bargaining ability distribution, but are significantly positive in determining the variance of the distribution.

Appendices

Appendix 1 – 1, solve the traditional searching rule by backward induction

If a *non-shopper* has sampled the N^{th} firm (the last firm), should he accept this offer? Let $p^{(x)}$ denote the lowest offer he gets in hand after sampling x firms, x=1,…, N. If the current offer $p \leq p^{(N\text{-}1)}$, he will accept the N^{th} offer; otherwise, he will reject and go back to buy from the lowest offer. Therefore, the reservation price at period N would be $r^{(N)} = p^{(N\text{-}1)}$.

Consider the case where a *non-shopper* has sampled N-1 firms, with best offer $p^{(N\text{-}1)}$ at hand. Should he go on and sample the N^{th} firm?

Payoff of sampling again is $u_i - \int_{\underline{p}}^{p^{(N-1)}} x dF(x) - \int_{p^{(N-1)}}^{\overline{p}} p^{(N-1)} dF(x) - c$, while payoff of buying at $p^{(N\text{-}1)}$ is u_i - $p^{(N\text{-}1)}$. At $r^{(N\text{-}1)}$ we have *payoff of rejecting = payoff of accepting*. And hence $r^{(N\text{-}1)}$ is defined implicitly in equation (A1 – 1) below.

$$\begin{gathered} u_i - \int_{\underline{p}}^{r^{(N-1)}} x dF(x) - \int_{r^{(N-1)}}^{\overline{p}} r^{(N-1)} dF(x) - c = u_i - r^{(N-1)} \\ \int_{\underline{p}}^{r^{(N-1)}} F(x) dx = c \end{gathered} \qquad \text{(A1 – 1 – 1)}$$

It is easy to show that $\frac{\partial}{\partial r^{(N-1)}} \int_{\underline{p}}^{r^{(N-1)}} F(x) dx = F\left(r^{(N-1)}\right) > 0$. Therefore, equation (A1 – 1 – 1) has a unique solution or no solution. Also note that the expected payoff for sampling again would be u_i - $r^{(N\text{-}1)}$, no matter what offer he rejects.

And then, let's go back to check the case where a *non-shopper* has sampled N-2 firms, and has a best offer $p^{(N\text{-}2)}$ at hand. $r^{(N\text{-}2)}$ is defined in equation (A1 – 1 – 2) below.

$$
\begin{aligned}
& u_i - c - \int_{\underline{p}}^{r^{(N-1)}} x dF(x) - \int_{r^{(N-1)}}^{\overline{p}} r^{(N-1)} dF(x) = u_i - r^{(N-2)} \\
\Leftrightarrow & -\int_{\underline{p}}^{r^{(N-1)}} F(x)\,dx - \int_{\underline{p}}^{r^{(N-1)}} x dF(x) - \int_{r^{(N-1)}}^{\overline{p}} r^{(N-1)} dF(x) = -r^{(N-2)} \\
\Leftrightarrow & -\int_{\underline{p}}^{r^{(N-1)}} F(x)\,dx - r^{(N-1)} F\left[r^{(N-1)}\right] + \int_{\underline{p}}^{r^{(N-1)}} F(x)\,dx - r^{(N-1)}\left[1 - F\left(r^{(N-1)}\right)\right] = -r^{(N-2)} \\
\Leftrightarrow & \; r^{(N-2)} = r^{(N-1)} \\
\Leftrightarrow & \int_{\underline{p}}^{r^{(N-2)}} F(x)\,dx = c
\end{aligned}
$$

(A1 – 1 – 2)

And it is easy to go on and show that $r^{(x)} = r \;\forall x = 1, \ldots, N-1$, where r solves equation (1 – 1), $\int_{\underline{p}}^{r} F(x)\,dx = c$.

Appendix 1 – 2 , want to show

Proposition 1 – 2 – 1: reservation price = r if r ≤ a_i, otherwise, non-shoppers will stop upon any offer and decide on bargain or not accordingly, that is, there does not exist reservation price.

Proof

First of all, for the consumers, the final cost is actually $Y(p) = \min\{p,a_i\}$.

(1) Want to show *"reservation price" = r if r ≤ a_i.*

Let's solve for the *reservation price* for period *N*-1.

Payoff of sampling again is $u_i - \int_{\underline{p}}^{p} min\{x,a_i\} dF(x) - \int_{p}^{\overline{p}} min\{p,a_i\} dF(x) - c$, while payoff of sampling again is $u_i - \min\{p,a_i\}$. At *reservation price* $\hat{r}$, we should have *payoff of accepting = payoff of rejecting*.

$$u_i - \int_{\underline{p}}^{\hat{r}} min\{x,a_i\} dF(x) - \int_{\hat{r}}^{\overline{p}} min\{\hat{r},a_i\} dF(x) - c = u_i - min\{\hat{r},a_i\}.$$

$$\text{Or } \int_{\underline{p}}^{\hat{r}} (min\{\hat{r},a_i\} - min\{x,a_i\}) dF(x) = c. \qquad \text{(A1 – 2 – 1)}$$

If *reservation price* $\hat{r}$, $\hat{r} \le a_i$, solves equation (A1-2-1) above, then $\hat{r}$ also satisfies $\int_{\underline{p}}^{\hat{r}} (\hat{r} - x) dF(x) = c$, or $\int_{\underline{p}}^{\hat{r}} F(x)\,dx = c$, which is the same as equation (1 – 1) where *r* is defined. Since the solution to (1-2-1) is unique, $\hat{r} = r$. It follows from Kohn & Shavell (1974)'s result, *reservation price = r* $\forall$ *period i* = 1,…,*N*.

(2) Want to show if $r > a_i$**,** the *non-shoppers* will stop upon any offer, and thus reservation price does not exist.

Note first that left hand side of equation (1 – 1) is strictly increasing in *r*. Therefore, if $r > a_i$, we must have $\int_{\underline{p}}^{p} F(x)\,dx \le \int_{\underline{p}}^{a_i} F(x)\,dx < c \;\; \forall p \in [\underline{p}, a_i]$.

In period N-1, with a best offer p at hand, payoff of accepting is $u_i - \min\{p,a_i\}$, while payoff of rejecting is $u_i - \int_{\underline{p}}^{p} min\{x,a_i\}dF(x) - \int_{p}^{\overline{p}} min\{p,a_i\}dF(x) - c$. The difference of the two is

$$\begin{aligned}
&\int_{\underline{p}}^{p} min\{x,a_i\}dF(x) + \int_{p}^{\overline{p}} min\{p,a_i\}dF(x) - min\{p,a_i\} + c \\
&= \int_{\underline{p}}^{p} \left[min\{x,a_i\} - min\{p,a_i\}\right]dF(x) + c \\
&= \begin{cases} \int_{\underline{p}}^{a_i} (x-a_i)dF(x) + c = -\int_{\underline{p}}^{a_i} F(x)dx + c > 0 & if\ \ a_i \le p \\ \int_{\underline{p}}^{p} (x-p)dF(x) + c = -\int_{\underline{p}}^{p} F(x)dx + c > 0 & if\ \ a_i > p \end{cases}
\end{aligned}$$

That is, no matter what offer the *non-shopper* gets, he would be strictly better off if he stop searching and buy from the offer (with or without bargaining).

Appendix 1 – 3, want to show

Proposition 1 – 3 – 1, There is no pure strategy Nash Equilibrium.

Proof: prove by contradiction.

Suppose there exists a pure strategy NE in which all firms set offer = p.

For a specific firm, payoff of setting offer = p is

$$E\pi(p) = (M/N)*E[\pi\min\{p,a_i\}+(1-\pi)\min\{p,a_i\}]$$

While the payoff of setting offer a little bit lower than p, say, $p' = p - \pi$, where $\pi > 0$, is

$$E\pi(p)= (M/N)\ (1-\pi)*E(\min\{p',a_i\})+M\pi E(\min\{p',a_i\})$$

Then we have,

$$E\pi(p)-E\pi(p')$$
$$=\frac{M}{N}(1-\mu)E\left[min\{p,a_i\}-min\{p',a_i\}\right]+\frac{M}{N}\mu min\{p,a_i\}-M\mu min\{p',a_i\}$$

If $p > \bar{a}$, setting p' where $\bar{a} < p' < p$ will always yields higher expected payoff.

If $p<\bar{a}$, $\frac{N}{M}\left[E\pi(p)-E\pi(p')\right]=\varepsilon-(N-1)\mu(p-\varepsilon)=\left[(N-1)\mu+1\right]\varepsilon-(N-1)\mu p$.

We can always find an ε, such that $E\pi(p)-E\pi(p')<0$.

If $p = 0$, setting offer higher than p will always gains you some positive profit from *non-shoppers,* better than stick to $p = 0$.

Appendix 1 – 4, want to show

Proposition 1 – 3 – 2, If F(p) is a Nash Equilibrium initial offer distribution, then the upper bound of this distribution, $\bar{p} = min\{\bar{a}, r\}$**.**

Proof

First of all, it is easy to see that no firm would set the initial offer higher than *r*, because otherwise, no one would buy from the firm.

Secondly, if $r > \bar{a}$, it must be true that $\bar{p} \geq \bar{a}$. Suppose $\bar{p} < \bar{a}$. If a firm sets offer $\bar{p}$, $E\pi(\bar{p}) = (1-\mu)(M/N)*E(\min\{\bar{p}, a_i\}) = \frac{M}{N}(1-\mu)\left[\int_{\underline{a}}^{\bar{p}} adG(a) + \int_{\bar{p}}^{\bar{a}} adG(a)\right]$, since only *non-shopper* would buy from the firm. But setting offer $\bar{a}$ would yield expected payoff being $E\pi(\bar{a}) = (1-\mu)(M/N)*E(a)$. This is because $\bar{a} < r$, and hence *non-shoppers* enter this firm would also buy from it. Therefore we have $E\pi(\bar{a}) > E\pi(\bar{p})$, which contradicts $\bar{p}$ being a best response of firms.

Thirdly, we know that setting offer higher than $\bar{a}$ will yield the same expected profit made from the consumer who stops, but then setting offer equal to $\bar{a}$ provides a little bit higher probability to be the lowest offer and to attract all the *shoppers*. Therefore, payoff of setting $\bar{a}$ is higher than that of setting $\bar{p}$, contradictory with $\bar{p}$ being a best response of the firms.

Appendix 1 – 5, want to show

Proposition 1 – 3 – 3, There is no gap on $\left[\underline{p}, \overline{p}\right]$.

Proof

Suppose there exists a gap on $\left[p_1, p_2\right]$, where $\underline{p} \le p_1 < p_2 \le \overline{p} \le a$. Then it must be true that $F\left(p_1\right) = F\left(p_2\right)$. And the expected payoffs of p_1 and p_2 are as following.

$$E\pi\left(p_1\right) = \left\{\frac{M}{N}(1-\mu) + M\mu\left[1-F\left(p_1\right)\right]^{N-1}\right\}\left[\int_{\underline{a}}^{p_1} a dG(a) + \int_{p_1}^{\overline{a}} p_1 dG(a)\right]$$

$$E\pi\left(p_2\right) = \left\{\frac{M}{N}(1-\mu) + M\mu\left[1-F\left(p_2\right)\right]^{N-1}\right\}\left[\int_{\underline{a}}^{p_2} a dG(a) + \int_{p_2}^{\overline{a}} p_2 dG(a)\right]$$

Given $F\left(p_1\right) = F\left(p_2\right)$ and $p_1 < p_2$, we know that $E\pi\left(p_1\right) < E\pi\left(p_2\right)$, contradicting with the fact that p_1 and p_2 are both firms' best responses.

Appendix 1 – 6, want to show

The left hand side of equation (1-3-6) is strictly increasing in r on $\left(\underline{p},\bar{a}\right)$.

$$\frac{d}{dr}\int_{\underline{p}(\bar{p}=r)}^{r} F\left(x;\bar{p}=r\right)dx$$

$$=F\left(r;\bar{p}=r\right)-F\left(\underline{p}\left(\bar{p}=r\right);\bar{p}=r\right)\frac{d}{dr}\underline{p}\left(\bar{p}=r\right)+\int_{\underline{p}(\bar{p}=r)}^{r}\frac{d}{dr}F\left(x;\bar{p}=r\right)dx$$

If $\underline{p}\leq\underline{a}$,

$$\frac{d}{dr}\int_{\underline{p}(\bar{p}=r)}^{r} F\left(x;\bar{p}=r\right)dx$$

$$=1+\int_{\underline{p}(\bar{p}=r)}^{\underline{a}}\frac{d}{dr}\left\{1-\left\{\frac{(1-\mu)}{\mu Np}\left[\int_{\underline{a}}^{r}ag\left(a\right)da+\int_{r}^{\bar{a}}rg\left(a\right)da-p\right]\right\}^{\frac{1}{N-1}}\right\}dp$$

$$+\int_{\underline{a}}^{r}\frac{d}{dr}\left\{1-\left\{\frac{(1-\mu)}{\mu N}\varphi\left(p\right)\right\}^{\frac{1}{N-1}}\right\}dp$$

$$\text{where } \varphi\left(p\right)=\frac{\int_{\underline{a}}^{r}ag\left(a\right)da+\int_{r}^{\bar{a}}rg\left(a\right)da}{\int_{\underline{a}}^{p}ag\left(a\right)da+\int_{p}^{\bar{a}}pg\left(a\right)da}-1$$

$$=1+\int_{\underline{p}(\bar{p}=r)}^{\underline{a}}-f\left(p\right)\left[1-G\left(r\right)\right]dp$$

$$+\int_{\underline{a}}^{r}-f\left(p\right)\frac{1-G\left(r\right)}{1-G\left(p\right)}\frac{\int_{\underline{a}}^{p}ag\left(a\right)da+\int_{p}^{\bar{a}}pg\left(a\right)da}{\int_{\underline{a}}^{r}ag\left(a\right)da+\int_{r}^{\bar{a}}rg\left(a\right)da}dp$$

$$=\int_{\underline{p}(\bar{p}=r)}^{\underline{a}}\left\{f\left(p\right)-f\left(p\right)\left[1-G\left(r\right)\right]\right\}dp$$

$$+\int_{\underline{a}}^{r}\left\{f\left(p\right)-f\left(p\right)\frac{1-G\left(r\right)}{1-G\left(p\right)}\frac{\int_{\underline{a}}^{p}ag\left(a\right)da+\int_{p}^{\bar{a}}pg\left(a\right)da}{\int_{\underline{a}}^{r}ag\left(a\right)da+\int_{r}^{\bar{a}}rg\left(a\right)da}\right\}dp$$

$$= \int_{\underline{p}(\overline{p}=r)}^{\underline{a}} f(p)G(r)dp +$$

$$\int_{\underline{a}}^{r} \frac{\left[1-G(p)\right]\left[\int_{\underline{a}}^{r} adG(a)+r\left[1-G(r)\right]\right]-\left[1-G(r)\right]\left[\int_{\underline{a}}^{p} adG(a)+p\left[1-G(p)\right]\right]}{\left[1-G(p)\right]\left[\int_{\underline{a}}^{r} adG(a)+r\left[1-G(r)\right]\right]} dF(p)$$

It is easy to see that the first part is greater than zero.

As to the second part, since $G(p) \leq G(r) \, \forall \, p \in \left[\underline{p}.\overline{p}\right]$, we have $1\text{-}G(r) \leq 1\text{-}G(p)$. Also notice that $\int_{\underline{a}}^{p} ag(a)da + \int_{p}^{\overline{a}} pg(a)da \leq \int_{\underline{a}}^{r} ag(a)da + \int_{r}^{\overline{a}} rg(a)da$. Therefore, the second term should be greater than zero, too.

If $\underline{p} > \underline{a}$,

$$\frac{d}{dr}\int_{\underline{p}(\overline{p}=r)}^{r} F(x;\overline{p}=r)dx$$

$$=1+\int_{\underline{p}(\overline{p}=r)}^{r} \frac{d}{dr}\left\{1-\left\{\frac{(1-\mu)}{\mu N}\varphi(p)\right\}^{\frac{1}{N-1}}\right\}dp$$

$$\text{where } \varphi(p)=\frac{\int_{\underline{a}}^{r} ag(a)da+\int_{r}^{\overline{a}} rg(a)da}{\int_{\underline{a}}^{p} ag(a)da+\int_{p}^{\overline{a}} pg(a)da}-1$$

$$=1+\int_{\underline{p}(\overline{p}=r)}^{r} -f(p)\frac{1-G(r)}{1-G(p)}\frac{\int_{\underline{a}}^{p} ag(a)da+\int_{p}^{\overline{a}} pg(a)da}{\int_{\underline{a}}^{r} ag(a)da+\int_{r}^{\overline{a}} rg(a)da}dp$$

$$=\int_{\underline{p}(\overline{p}=r)}^{r} \left\{f(p)-f(p)\frac{1-G(r)}{1-G(p)}\frac{\int_{\underline{a}}^{p} ag(a)da+\int_{p}^{\overline{a}} pg(a)da}{\int_{\underline{a}}^{r} ag(a)da+\int_{r}^{\overline{a}} rg(a)da}\right\}dp$$

$$=\int_{\underline{p}(\overline{p}=r)}^{r} \frac{\left[1-G(p)\right]\left[\int_{\underline{a}}^{r} adG(a)+r\left(1-G(r)\right)\right]-\left[1-G(r)\right]\left[\int_{\underline{a}}^{p} adG(a)+p\left(1-G(p)\right)\right]}{\left[1-G(p)\right]\left[\int_{\underline{a}}^{r} adG(a)+r\left(1-G(r)\right)\right]}dF(p)$$

And as argued above, this is greater than zero.

Appendix 1 – 7, want to show,

As $G(a)$ becomes more dispersed, $\underline{p}\left(\overline{p}=\overline{a}\right)$ will increase and $F(p)$ will decrease. As $\underline{p}$ increases, the range to integrate shrinks, and as $F(p)$ decreases, the integrand decreases. Therefore, it is easy to see that $\overline{c}$ will decrease.

Proof

I. As $G(a)$ goes from $G^c(a)$ to $G^d(a)$, $\underline{p}\left(\overline{p}=\overline{a}\right)$ will increase.

Recall that $\underline{p}$ is defined from

$$\int_{\underline{a}}^{\underline{p}} min\left\{a,\underline{p}\right\}dG(a)=\frac{1-\mu}{\mu N+(1-\mu)}\left[\int_{\underline{a}}^{\overline{p}} adG(a)+\int_{\overline{p}}^{\overline{a}} \overline{p}dG(a)\right] \qquad (1-5)$$

Then, $$\int_{\underline{a}}^{\underline{p}(\overline{p}=\overline{a})} min\left\{a,\underline{p}\left(\overline{p}=\overline{a}\right)\right\}dG(a)=\frac{1-\mu}{\mu N+(1-\mu)}E(a) \qquad (A1-7-1)$$

As G(a) goes from $G^c(a)$ to $G^d(a)$, the right hand side of equation (A1-7-1) does not change. Yet note that the integrand on the left hand side is min{$\underline{p}$,a}, a concave function of $\underline{p}$. Therefore, if $\underline{p}$ keeps unchanged, the left hand side will be less than the right hand side when we take integration with respect to $G^d(a)$[47].

$$\int_{\underline{a}}^{\underline{p}^c} min\left\{\underline{p}^c,a\right\}dG^d(a)<\int_{\underline{a}}^{\underline{p}^c} min\left\{\underline{p}^c,a\right\}dG^c(a)=rhs \qquad (A1-7-2)$$

Also, the left hand side is strictly increasing in $\underline{p}$. Thus, we need a higher $\underline{p}$ so as to achieve the equality again.

II. As $G(a)$ goes from $G^c(a)$ to $G^d(a)$, $F(p)$ decreases.

Recall that $F(p)$ is defined from

[47] Proposition for Second Order Stochastic Dominance: Letting U^1 be the set of all increasing, concave functions on [a, b], then F second order stochastically dominates G if and only if

$$\int_a^b u(x)dF(x)\geq\int_a^b u(x)dG(x) \quad \forall u\in U^1$$

$$F(p;\overline{p}) = 1 - \left\{ \frac{(1-\mu)}{\mu N} \varphi(p) \right\}^{\frac{1}{N-1}} \ where \ \varphi(p) = \frac{\int_{\underline{a}}^{\overline{p}} ag(a)\,da + \int_{\overline{p}}^{\overline{a}} \overline{p}g(a)\,da}{\int_{\underline{a}}^{p} ag(a)\,da + \int_{p}^{\overline{a}} pg(a)\,da} - 1 \quad (1-6)$$

Then $F(p;\overline{p}=\overline{a})$ is defined as following.

$$F(p;\overline{p}=\overline{a}) = 1 - \left\{ \frac{(1-\mu)}{\mu N} \varphi(p) \right\}^{\frac{1}{N-1}}$$

$$where \ \varphi(p) = \frac{E(a)}{\int_{\underline{a}}^{p} ag(a)\,da + \int_{p}^{\overline{a}} pg(a)\,da} - 1 \qquad (A1-7-3)$$

Follow the same argument as above, as $G(a)$ goes from $G^c(a)$ to $G^d(a)$ the denominator of $\varphi(p)$ decreases, and hence $F(p)$ will decreases.

III. As $\underline{p}$ increases, the range to integrate shrinks, and as $F(p)$ decreases, the integrand decreases. Therefore, it is easy to see that $\overline{c}$ will decrease. This is self-evident.

Appendix 1 – 8, want to show

Proposition 1 – 4 – 1, For two bargaining ability distribution $G^c(a)$ and $G^d(a)$, if $G^c(a)$ second order stochastically dominates $G^d(a)$, and the expected final prices satisfy $E^d(Y) \geq E^c(Y)$, then it must be true that $F^d(p)$ first order stochastically dominates $F^c(p)$.

Proof

Recall that $F(p)$ is defined in function (1-3-5).

$$F(p;\overline{p}) = 1 - \left\{ \frac{(1-\mu)}{\mu N} \varphi(p) \right\}^{\frac{1}{N-1}} \; where \; \varphi(p) = \frac{\int_{\underline{a}}^{\overline{p}} ag(a)\,da + \int_{\overline{p}}^{\overline{a}} \overline{p}g(a)\,da}{\int_{\underline{a}}^{p} ag(a)\,da + \int_{p}^{\overline{a}} pg(a)\,da} - 1 \quad (1-6)$$

If $E^d(Y) \geq E^c(Y)$, since $E(y) = (1-\mu)\left[\int_{\underline{a}}^{\overline{p}} ag(a)\,da + \int_{\overline{p}}^{\overline{a}} \overline{p}g(a)\,da\right]$, we have

$$\int_{\underline{a}}^{\overline{p}^d} ag^d(a)\,da + \int_{\overline{p}^d}^{\overline{a}} \overline{p}^d g^d(a)\,da \geq \int_{\underline{a}}^{\overline{p}^c} ag^c(a)\,da + \int_{\overline{p}^c}^{\overline{a}} \overline{p}^c g^c(a)\,da \qquad (A1-8-1)$$

$\int_{\underline{a}}^{p} ag(a)\,da + \int_{p}^{\overline{a}} pg(a)\,da = \int_{\underline{a}}^{\overline{a}} min\{a,p\}\,g(a)\,da$, where $\min\{a,p\}$ is concave in a.

$$\left.\begin{array}{c} G^c(a) \; SOSD \; G^d(a) \\ min\{a,p\} \; concave \, in \, a \end{array}\right\} \Rightarrow \int_{\underline{a}}^{\overline{a}} min\{a,p\}\,dG^c(a) \geq \int_{\underline{a}}^{\overline{a}} min\{a,p\}\,dG^d(a) \quad (A1-8-2)$$

The derivation of (A1-8-2) follows from the proposition of second order stochastic dominance as stated in footnote 22.

Combine equation (A1-8-1) and equation (A1-8-2), we will have $\varphi^c(p) \leq \varphi^d(p)$. Therefore, $F^c(p;\overline{p}=\overline{a}) \geq F^d(p;\overline{p}=\overline{a})$ or, $F^d(p;\overline{p}=\overline{a})$ first order stochastically dominates $F^c(p;\overline{p}=\overline{a})$.

Appendix 2 – 1, want to show

Proposition 2 – 3 – 1, there is no pure strategy Nash Equilibrium.

Proof: prove by contradiction.

Suppose there exists a pure strategy Nash Equilibrium in which all firms set offer = *p,* s.t., $R(p) > 0$. Then for a specific firm, payoff of setting offer = *p* is

$$E\pi(p) = \frac{M}{N} E_{a_i}\left[Y(p, a_i)\right] = \frac{M}{N} R(p),$$

while the payoff of setting offer a little bit lower than *p*, say, $p' = p - \varepsilon$, where $\varepsilon > 0$ is $E\pi(p') = \frac{M}{N}(1-\mu)R(p') + M\mu R(p')$. Then we have,

$$\begin{aligned} E\pi(p) - E\pi(p') &= \frac{M}{N}(1-\mu)\left[R(p) - R(p')\right] + \frac{M}{N}\mu R(p) - M\mu R(p') \\ &= \frac{M}{N}\left[R(p) - R(p')\right] - \frac{(N-1)M\mu}{N} R(p') \end{aligned}$$

Recall that $R(p)$ is continuous and strictly increasing in *p*. Given continuity and $R(p) > 0$, $\exists \varepsilon$ small enough s.t., $E\pi(p) - E\pi(p') < 0$. Contradiction

Suppose there exists a pure strategy NE in which all firms set offer = *p*, s.t., $R(p) = 0$. Then for a specific firm, payoff of setting offer = *p* is zero, while the payoff of setting offer a little bit higher than *p* is $E\pi(p') = \frac{M}{N}(1-\mu)R(p')$. Since $R(p)$ is continuous and strictly increasing in *p*, and $R(p) = 0$, we know that $R(p') > 0$, and hence $E\pi(p') > 0 = E\pi(p)$. Contradiction

For any possible *p*, firms always have incentive to deviate from the equilibrium strategy. Therefore, there is no pure strategy Nash Equilibrium.

Appendix 2 – 2, want to show

Proposition 2 – 3 – 2, If there exists Nash equilibrium, then in the equilibrium it must be the case that $\bar{p} = p^*$.

The proof for *Proposition 2 – 3 – 2* follows two steps. First of all, it can be shown that the highest possible initial offer is no less than p^*. Secondly, $\bar{p} > p^*$ is not an equilibrium scenario, either.

Step 1, prove by contradiction. Suppose $\bar{p} < p^*$. The payoffs for $\bar{p}$ and p^* are as below.

$$E\pi(\bar{p}) = \frac{M}{N}(1-\mu)R(\bar{p})$$

$$E\pi(p^*) = \frac{M}{N}(1-\mu)R(p^*)$$

Because $\bar{p} < p^*$, and $R(p)$ strictly increases in p on $[0,p^*]$, we have $R(\bar{p}) < R(p^*)$, and hence $E\pi(\bar{p}) < E\pi(p^*)$, contradicting with $\bar{p}$ belonging to the set of firms' best responses.

Step 2, suppose we have an equilibrium scenario where $\bar{p} > p^*$. The payoff of setting initial offer $\bar{p}$ and p^* are as below.

$$E\pi(\bar{p}) = \frac{M}{N}(1-\mu)R(\bar{p}) = \frac{M}{N}(1-\mu)R(p^*)$$

$$E\pi(p^*) = \frac{M}{N}(1-\mu)R(p^*) + M\mu R(p^*)\left[1-F(p^*)\right]^{N-1}$$

Note that

$$\bar{p} > p^* \Rightarrow F(p^*) < 1 \Rightarrow M\mu R(p^*)\left[1-F(p^*)\right]^{N-1} > 0 \Rightarrow E\pi(p^*) > E\pi(\bar{p}),$$

contradicting with $\bar{p}$ belonging to the set of firms' best responses.

Therefore, we must have the upper bound of the equilibrium initial offer distribution $\bar{p}$ equals to p^*.

Appendix 2 – 3, want to show

Proposition 2 – 3 – 3, There is no gap on $\left[\underline{p}, \overline{p}\right]$**.**

Proof

Suppose there exists a gap on $[p_1, p_2]$, where $\underline{p} \le p_1 < p_2 \le \overline{p} = p^*$. Then it must be true that $F(p_1) = F(p_2)$. And the expected payoffs of p_1 and p_2 are as following.

$$E\pi(p_1) = \left\{\frac{M}{N}(1-\mu) + M\mu\left[1 - F(p_1)\right]^{N-1}\right\} R(p_1)$$

$$E\pi(p_2) = \left\{\frac{M}{N}(1-\mu) + M\mu\left[1 - F(p_2)\right]^{N-1}\right\} R(p_2)$$

Given $p_1 < p_2$, we know that $R(p_1) < R(p_2)$, and hence $E\pi(p_1) < E\pi(p_2)$. Contradiction.

Appendix 2 – 4, want to show

Proposition 2 – 4 – 1, suppose $G^c(a)$ second order stochastically dominates $G^d(a)$, and $E^c(a) = E^d(a)$, then we will have $F^d(p)$ first order stochastically dominates $F^c(p)$.

Proof

Recall that $F(p) = 1 - \left[\frac{1-\mu}{N\mu}\left(\frac{R(p^*)}{R(p)} - 1\right)\right]^{\frac{1}{N-1}}$, where, $R(p) = \int_{\underline{a}}^{\overline{a}} Y(p,a)\, g(a)\, da$

and $R(p^*) = E(a)$. Also recall that $Y(p,a)$ is concave in a, and hence given that $G^c(a)$ second order stochastically dominates $G^d(a)$, it must be true that

$R^c(p) = \int_{\underline{a}}^{\overline{a}} Y(p,a)\, g^c(a)\, da \geq R^d(p) = \int_{\underline{a}}^{\overline{a}} Y(p,a)\, g^d(a)\, da$, "=" if $Y(p,a)$ is linear in a $\forall a \in [\underline{a}, \overline{a}]$.

We can write $F^c(p)$ and $F^d(p)$ as following.

$$\hat{F}(p) = 1 - \left[\frac{1-\mu}{N\mu}\left(\frac{\hat{R}(p^*)}{\hat{R}(p)} - 1\right)\right]^{\frac{1}{N-1}} = 1 - \left[\frac{1-\mu}{N\mu}\left(\frac{\hat{E}(a)}{\hat{R}(p)} - 1\right)\right]^{\frac{1}{N-1}}$$

$$\tilde{F}(p) = 1 - \left[\frac{1-\mu}{N\mu}\left(\frac{\tilde{R}(p^*)}{\tilde{R}(p)} - 1\right)\right]^{\frac{1}{N-1}} = 1 - \left[\frac{1-\mu}{N\mu}\left(\frac{\tilde{E}(a)}{\tilde{R}(p)} - 1\right)\right]^{\frac{1}{N-1}}.$$

It can be easy seen from above that $F^c(p) \geq F^d(p) \forall p \in [\underline{p}, \overline{p}]$. That is, $F^d(p)$ second order stochastically dominates $F(p)$.

Appendix 2 – 5, want to show

$\underline{p}^c \leq \underline{p}^d$

Proof

Recall that $\underline{p}$ is defined from function (2 – 4), $R\left(\underline{p}\right)=\frac{1-\mu}{N\mu+1-\mu}R\left(p^*\right)$. Also recall that $R^c\left(p^*\right)=E^c\left(a\right)=E^d\left(a\right)=R^d\left(p^*\right)$, and hence $R\left(\underline{p}^c\right)=R\left(\underline{p}^d\right)$.

Since $R\left(p\right)=\int_{\underline{a}}^{\bar{a}}Y\left(p,a\right)g\left(a\right)da$, where *Y(p,a)* is concave in *a*, we know that $R^c\left(p\right)\geq R^d\left(p\right)\forall p$. Finally, recall that *R(p)* increases in *p*. Therefore, to obtain $R^c\left(\underline{p}^c\right)$, we need a *p* higher than $\underline{p}^c$, that is $\underline{p}^c \leq \underline{p}^d$.

Appendix 2 – 6, want to show

$\exists \boldsymbol{y}' < \overline{\boldsymbol{y}}$

$$s.t.\ \boldsymbol{T}(\boldsymbol{y}) = \int_{\underline{y}}^{y} \left[\boldsymbol{H}^d(\boldsymbol{x}) - \boldsymbol{H}^c(\boldsymbol{x})\right] \boldsymbol{dx} \geq 0\ \forall \boldsymbol{y} \geq \boldsymbol{y}',$$

That is, $\forall \boldsymbol{y} \geq \boldsymbol{y}'$***, second order stochastic dominance property holds.***

Proof

1, $G^c(a)$ second order stochastically dominates $G^d(a)$

$\Rightarrow \exists a'$ such that $g^d(a) \geq g^c(a)\ \ \forall a \geq a'$.

2, 1 - $H(y)$ = Pr (final price higher than y)

$$1 - H(y) = \int_{\alpha(y,\overline{p})}^{\overline{a}} \left\{ \int_{\kappa(y,a)}^{\overline{p}} b(p)\,dp \right\} g(a)\,da,$$

Where $\alpha(y,p)$ is the lowest a, such that, given offer p, the price is y.

$\kappa(y,a)$ is the lowest p, such that, given bargaining ability a, the final price is y.

$\beta(p)$ is the probability that a consumer would buy at offer p.

$$1 - H(y) = \int_{\alpha(y,\overline{p})}^{\overline{a}} \left[1 - B(\kappa(y,a))\right] g(a)\,da$$

3, $F^d(p)$ first order stochastically dominates $F^c(p)$

$\Rightarrow F^c(p) \geq F^d(p)\ \ \forall p\ \ \Rightarrow B^c(p) \geq B^d(p)\ \ \forall p$

$\Rightarrow 1 - B^c(\kappa(y,a)) \leq 1 - B^d(\kappa(y,a))$

4, Combine 1 and 3, $\left[1 - B^c(\kappa(y,a))\right] g^c(a) \leq \left[1 - B^d(\kappa(y,a))\right] g^d(a)\ \ \forall a \geq a'$

$\Rightarrow H^c(y) \geq H^d(y)\ \ \forall y$ such that $\alpha(y,\overline{p}) \geq a'$

$\Rightarrow H^c(y) \geq H^d(y)\ \ \forall y$ such that $y \geq a'$

5, $$\left.\begin{array}{r} E(y) = \int_{\underline{y}}^{\overline{y}} y h(y)\,dy = \overline{y} - \int_{\underline{y}}^{\overline{y}} H(y)\,dy \\ E^c(y) = E^d(y) \end{array}\right\} \Rightarrow \overline{y}^c - \int_{\underline{y}^c}^{\overline{y}^c} H^c(y)\,dy = \overline{y}^d - \int_{\underline{y}^d}^{\overline{y}^d} H^d(y)\,dy$$

$$\Rightarrow T(y) = \int_{-\infty}^{\overline{y}^d} \left[H^d(x) - H^c(x)\right] dx \geq 0\ \ \forall y \geq a'$$

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