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A Static Model for Predicting Disrupted Network Behavior

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A Static Model for Predicting Disrupted Network Behavior

by

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Thesis

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science in Engineering

The University of Texas at Austin

August 2010

Acknowledgements

I would like to thank Dr. Waller for helping me to develop and refine this model at every stage. I would also like to thank Dr. Bradley Love for providing the perceived travel time data and providing important insights and suggestions for this thesis.

Additionally, I would like to thank Dr. Avinash Unnikrishnan for providing some of the framework for this model and helping to improve the framework. I would also like to thank all of the other graduate students in our department who have supported and helped me through this entire process.

Finally, I would like to thank my parents and siblings for their love and encouragement at every stage of my education.

Abstract

A Static Model for Predicting Disrupted Network Behavior

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The University of Texas at Austin, 2010

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This thesis compares actual and perceived travel times and presents a model for predicting traffic flows when there is a network disruption. The goal of this research is to demonstrate the necessity of accounting for possible differences in travel time perception and actual travel times, and also to show trends in how the route choices change based on the transformation of the perceived travel times. A pilot test was done to determine actual travel time perceptions, and the results provided the foundation for the tests presented in this thesis and the model framework. The model is separated into three phases: equilibrium assignment, link travel time transform, and logit assignment. The transform of the link travel times is best represented by an inverse cumulative Normal distribution, and the corresponding values provide quantifiable measure of the severity of a traffic network disruption. The methodology is presented and applied to two test networks to demonstrate the resulting route choice patterns. Both networks are tested for three severity levels and three levels of demand.

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Chapter 1

Introduction

One of the most critical unexplored areas in transportation network modeling is the behavior of traffic in situations where general equilibrium concepts cannot be applied. The drivers' route choices inevitably change when there is a disruption to the traffic network which causes a high degree of unfamiliarity with travel time distributions. This work will compare perceived and measured travel time distributions and present a novel framework for predicting driver reactions to network disruptions.

1.1 Motivation

The majority of static route choice models currently in use assume that drivers perceive the time required to travel on a given link with a high degree of accuracy. The value of utility assigned to each link is based on these perceived travel times, but very little work has been done to determine if network users' perceived travel time distributions are accurate. Past studies have shown that people correctly predict event time distributions in some cases, but the same studies demonstrate that there is a high degree of error in the participants' predictions of the time distributions for some events [26; 32]. To ensure the highest degree of accuracy in calculating the change in perceived travel times due to various levels of network disruption, this work will test the perceived travel time distributions for three incident severity levels. These values can then be used to calculate the change in perceived travel times for different road conditions.

The nature of any traffic disruption from rain, to lane closures for construction, to catastrophic road failures, is that there is a lack of familiarity and a higher degree of uncertainty regarding travel times. In the past, traditional models using stochastic travel times with different error variable distributions have been used to attempt to model this behavior. While these models do account for the uncertainty inherent in network disruptions, they do not provide a clear numerical framework to prove the distributions of the error variables. Additionally, they do not alter the average travel time used based on how users will likely inflate or deflate the average travel times based on the network conditions. The model developed in this thesis will provide a framework for transforming the travel times based on the severity of the disaster. Moreover, this model framework will be used to design more advanced network assignment models in future extensions of this research. The model formulation will be outlined and discussed in more detail in Chapter 3.

A small set of pilot data from survey questions regarding drivers' travel time perceptions based on various network conditions was collected to inform the hypotheses being used and tested in this model. The preliminary results showed that in a situation with unfamiliar traffic conditions, as with a network disruption, users apply a transform on the expected travel times. Additionally, the pilot data demonstrated that these transforms penalize roads with higher travel time volatilities. The tests assumed that subjects were familiar with actual travel time distributions and transformed the travel times based on those distributions. This initial data set provided the framework to create the test outlined in Section 3.1 of this thesis and the transforms tested and selected as described in Section 3.3.

The closest that most existing research has come to thoroughly studying and modeling the effects of traffic disruptions derived from observed behavior is a group of papers written after the collapse of the I-35 W Bridge in Minnesota in 2007. These works employed various methods for estimating the impact of the bridge collapse on long-term traffic patterns in the area. The studies primarily focused on the way that traffic re-equilibrated in the days and weeks after the collapse. This provided some insight into the long-term behaviors of drivers after a large traffic disruption, but there was no data collected regarding immediate reactions of drivers as they heard about the collapse. The model created in this work, on the other hand, will examine the initial driver reactions. Proposed extensions of this work will model not only the initial driver route choices but also the adaptive choices over time. The data and models developed from the bridge collapse can be used to inform and potentially confirm some of the results obtained by these future works [54; 55].

An overview of the model framework is shown in Figure 1.1, and a more detailed explanation of each of the proposed stages is presented in Section 1.2.

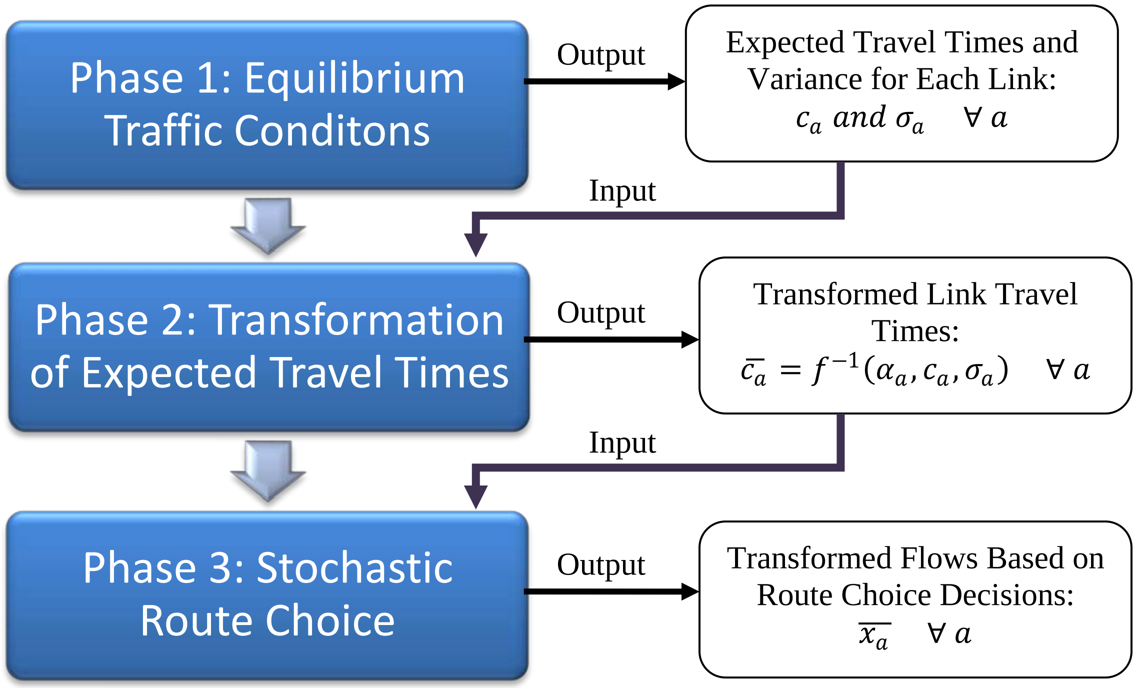


Figure 1.1: Model Framework

1.2 Modeling Framework

As shown above, the model is comprised of three phases, each of which alters the link flows or travel times in some way. The equilibrium operating conditions are found, in this work, using the Deterministic User Equilibrium model. There are many existing solution methods to decrease the required computational time available in the existing UE literature. The two primary methods that will be implemented in this thesis are Dijkstra's shortest path algorithm and the Frank-Wolfe line-search algorithm [16; 18]. Although Dijkstra's is not always the most efficient solution method, the existing DUE code being implemented applies Dijkstra's algorithm to solve the shortest path [16]. The Frank-Wolfe algorithm is a sufficiently efficient line-search algorithm for the small test networks used in this thesis, and it reaches convergence in reasonable time [18].

Phase 2 of the model transforms the travel times from the equilibrium assignment performed in Phase 1. Although the UE model only provides the expected travel time, field data from freeway and non-freeway roads can be used to estimate a value of the standard deviation of the distribution based on the expected travel time. This phase calculates the transformed travel times using the value of an inverse cumulative distribution selected based on the perceived travel times and the level of disruption (α), which is also evaluated based on the travel time perceptions. The average over all perceived travel times is transformed to determine the average travel time for each incident severity level. Several possible distributions are tested in order to determine the optimal transform.

Once this transform is found, the new travel times can be calculated using the incident severity level as the probability and the equilibrium travel time and volatility for each link in the inverse cumulative distribution chosen. The disruption levels are assigned relative to average travel conditions, so the average over all possible conditions is $\alpha = 0.5$. The three incidents tested are sunny, rainy, and train derailed causing a chemical spill. At $\alpha = 1$, it is assumed that there is a catastrophic annihilation of the entire traffic network. Once the distribution and incident levels are calculated, the link travel times can be transformed and used as input for Phase 3.

The third and final phase of the model uses logit assignment to find new path flows and the resulting link flows. These changes to the flows in the network will provide insight into driver behavior in traffic networks with various levels of disruptions. The travel times and potential error in the perceived average travel times are used to calculate the probability of choosing a given path from an origin to a destination, and these probabilities are multiplied by the OD flows to determine the path flows. The change in the path flows illustrates trends in the route choices as they relate to the incident severity and the level of congestion in the network. This comprehensive methodology provides a novel framework for predicting and understanding user path choice in uncertain environments.

1.3 Organization

The next chapter in this thesis includes a review of the relevant literature in this area of research. Chapter 3 describes the model formulation and solution method. Chapter 4 of this thesis presents and discusses the results of the travel time perception tests and the network assignment. This chapter also includes a discussion of some relevant assumptions made and the effect of both network congestion level and incident severity on the final network assignment. The final chapter provides some conclusions drawn from this work and a discussion of future research directions in the area of driver behavior in traffic disruptions.

Chapter 2

Literature Review

This research is comprised of three primary phases: (a) equilibrium assignment, (b) travel time transform, and (c) logit assignment. To view this work in the context of similar research, it is important to look at three crucial areas: (a) travel time distributions, (b) route choice modeling, and (c) network assignment under uncertainty.

2.1 Travel Time Distribution

There is an increasing interest in the area of travel time uncertainty based on traffic flow volume fluctuations and possible disruptions. The majority of these methods, however, require a probability distribution for the travel times. Because of these requirements, there has been an increasing move toward gathering travel time data using various ITS detectors and Bluetooth and GPS technology [48]. The data availability, however, is extremely limited, especially for non-freeway roads. This is due primarily to the fact that freeways and major arterials are considered of prime importance for measuring travel time to determine which methods are most suitable for alleviating congestion [21]. As a consequence of the lack of travel time data for non-freeway links, most studies requiring this data implement statistical techniques to estimate these distributions. Additionally, there is no clear consensus on the optimal data travel time collection method. Therefore, the incorporation of data from multiple types of sensors (Bluetooth, GPS, ITS sensors, etc.) may increase the reliability of the results [33; 46]. In this work, arterial and freeway travel time data was collected Bluetooth and traffic sensors, so true distributions can be used instead of estimates.

Aside from the problem of data collection, there is also a need to classify and understand the of incidents that cause traffic disruptions. These can range from inclement weather to chemical spills and other catastrophic events. The first step in this area of research is to determine which factors affect the frequency and severity of some incidents. A clear relationship between accident rates and roadway geometry, traffic volume, and weather was found by Karlaftis and Golias (2002) and Golob and Recker (2003). Karlaftis and Golias used hierarchal tree-based regression to show that geometric design and pavement variables have the greatest affect on incident frequency [31]. Golob and Recker, on the other hand, used both linear and nonlinear multivariate statistical analyses to demonstrate that when weather and lighting conditions are accounted for, traffic volume has a greater influence on accident frequency than speed [25].

It is also imperative to formulate an analytical representation of incident severity. This was done by Wirasinge in 1978 using shock-wave analysis. Morales' 1986 work, on the other hand, created an analytical tool for predicting the impacts of planned traffic disruptions such as lane closures. Stochastic methods were used by Sheu et al. (2001) and Fu and Rilett (2007) to predict the impacts of planned and unplanned incidents and to determine optimal methods for minimizing those impacts. Boyles and Waller (2007) also created an analytical framework to predict incident duration including using Monte Carlo sampling when the complexity prevents analytical analysis. These works implement various different methods for estimating incident duration, including linear regression

[20], Poisson regression [30], nonparametric regression [39], hazard-based models [38], decision trees [47] and Bayesian methods [40; 7]. Instead of estimating incident duration using these methods, Golob et al. (1987) and Garib et al. (1997) utilized field data to estimate incident delay. A combination of the previously referenced methods will be used in this work. Field data is used to estimate variance of travel times on freeway and non-freeway roads based on average travel times, and data from psychological tests will be used to determine the level of inflation or deflation of travel times based on the incident severity. Analytical modeling methods will then be used to determine the impact of these travel time changes on traffic flows. Voruganti et al. (2009) studied the impact of weather, roadway geometry, regional characteristics, and travel time reliability. They established that in cases where the drivers are likely unfamiliar with traffic conditions (such as with incidents) roadway geometry does significantly impact travel time distributions [50].

Another vital consideration is the calculation of perceived travel time distributions to compare to actual travel time distributions. Lewandowski et al. (2009) and Griffiths and Tenenbaum (2006) provided a framework for calculating how people perceive time distributions of various events such as life spans and movie run times. These works also demonstrated that people's perceptions generally match the true time distributions when the tests are constructed using the framework outlined by Lewandowski et al. (2009) and Griffiths and Tenenbaum (2006). These works provided the general structure for the testing of travel time perceptions done for this project, and they give some insight into the possible errors in perceptions as compared to actual event time distributions [26; 32].

Another consideration in modeling route choice decisions in traffic disruptions is the change in people's behavior due to stress. Some research has found that stress can lead to problems in judgment and risk-avoiding behavior [17; 22]. In the context of route choice, this would seem to indicate that in stressful situations, such as disruptions to the traffic network, users would be more likely to choose a route with lower travel time volatility. Other studies done in this area, however, have found that stress may not in fact lead to any increase in risk-avoiding behavior [27]. This is one of the principles that will be explored through modeling the route choices in this work.

This review demonstrates that there are significant gaps in the research related to travel time distributions and the impacts of incidents on those distributions. This thesis will further the literature by collecting and analyzing travel time data for both freeway and non-freeway roads. Additionally, this research will compare actual travel time distributions to perceived travel time distributions to determine whether the use of field data is sufficient in estimating perceptions of travel times.

2.2 Route Choice Modeling

The general route choice model utility function describes how users value a path based on certain characteristics and establishes a rule for selecting a path based on those assigned values. In the most simplistic case, this is achieved by using a deterministic utility maximization model assuming that the network users have perfect information regarding link cost and foresight. The first two static assignment methods were developed by Wardrop in 1952 and have since been used and extended in many different works. These two methods are User Equilibrium (UE) and System Optimal (SO), which assign flow with the objective of minimizing the travel time for each user and the total system travel time respectively [51]. Phase I of this work implements the static User Equilibrium model to find the equilibrium traffic flows under average conditions. In the case explored here, the limitations of assuming perfect information are not critical due to the assumption that users are familiar with the roadways and the average conditions. The purpose of this work is to determine the reaction to unusual conditions, and the error and uncertainty are accounted for in Phase 2 and Phase 3.

The UE and SO models both rely on the assumption that the network users have perfect and complete information regarding network conditions. The recognition of the limitations of the perfect information assumption led to a new area of research in probabilistic route choice models. These models assume that users will minimize their perceived costs when given a set of route choice options. The next set of models developed on this principle created what is known as the Stochastic User Equilibrium (SUE) framework, which assumes that costs are perceived differently by different users [14; 15]. The logit assignment method proposed in Dial (1971) is used in Phase 3 of this work to redistribute traffic between the paths based on the transformation of the equilibrium costs and the distribution of that transformation. This algorithm is based on the principle of Random Utility Maximization (RUM) where users select the route which provides them with the highest utility. The RUM framework also includes an error term to account for unknown factors which could affect the utility and non-uniform perception of the utility [41]. Dial's logit assignment algorithm follows a Multinomial Logit (MNL) model framework, which assumes that the error terms have a Gumbel distribution and are Independent and Identically Distributed (IID), and the paths exhibit the Independence from Irrelevant Alternatives (IIA) property. The assumption of Gumbel distribution on the error terms creates a simple analytical format, which contributes to the widespread use of the MNL framework in practice, but the IID and IIA properties can be limiting and unrealistic in practice. There has been some research into implementing the RUM framework without the MNL limitations on the error terms. The C-logit model alters the definition of utility, accounts for problems with overlapping paths (paths that share links), and allows for error terms that are not necessarily IID [10; 11]. The Path-Size logit model includes a size variable in the utility equations and removes the issues in past formulations with overlapping paths [4].

Another class of models has also emerged to capture some of the same characteristics of the MNL model without its limitations. The Multinomial Probit model shares many characteristics with the MNL model, but it assumes that the error terms are normally distributed [14]. McFadden's (1978) Generalized Extreme Value (GEV) theorem can be applied to the Link-Nested Logit (LNL) model to create a closed-form solution with error term correlations. The Paired Combinatorial Logit (PCL) model [12], has a closed-form solution using the GEV theorem as well and was successfully applied to the route choice problem by Prashker and Bekhor (1998) and Gliebe et al. (1999). The GEV theorem is also applied to the Error Components Logit (ECL) model to create a closed-form solution with correlations by adapting the Logit Kernel model to the route choice problem ([5], [35], and [3]).

There are also models such as Mirchandi and Soroush's 1987 work that created route choice models specifically for situations of recurrent congestion. These models use expected utility theory, which does not account for travel time variance. This property makes the expected utility theory models unsuitable for larger traffic disruptions since the travel time distribution changes when the disruption is significant. Avineri and Prashker (2003 and 2004) adapted prospect theory and cumulative prospect theory for route choice modeling. These models are founded on the principle that users are more likely to accept higher risks to avoid losses than to take advantage of possible gains ([1] and [2]).

There is clearly a great deal of literature related to route choice preferences, but there is insufficient data regarding people's perception and understanding of travel time uncertainty. The uncertainty and non-uniformity of travel time distributions increases significantly in situations where traffic is disrupted. This thesis will attempt to quantify and study the users' perceptions of travel times under different conditions from survey testing.

2.3 Network Assignment Under Uncertainty

Network assignment under uncertainty generally deals with travel time or behavioral uncertainty (stochastic network assignment), demand and capacity uncertainty, and online equilibrium models. This work will only utilize stochastic network assignment as it does not include any exploration of demand or capacity uncertainty, and the online equilibrium models will not be included at this stage in the research. As previously noted, Daganzo and Sheffi (1977) developed a stochastic network assignment model to account for uncertainty in travel time perception. This perceived travel time can be quantified as the actual experienced travel time with an error term added. This error term has some probability distribution, as discussed previously, and the MNL models assume a Gumbel distribution for this error term. Dial (1971) developed the STOCH algorithm to perform the network loading phase of the stochastic network assignment, while the focal portion of the model can be solved in reasonable time for most networks using the Method of Successive Averages or the Frank Wolfe algorithm [15; 18].

There is another category of SUE models that uses probit-based methods. Sheffi et al. (1982) used simulation techniques to solve the probit-based SUE models, and Daganzo (1979) developed a solution algorithm that relies on path enumeration and numerical integration. Maher (1997), on the other hand, produced a heuristic using Markovian routing, which precludes the need to enumerate all possible paths. There are several other works that use stochastic network assignment models to study route choice and flow dynamics by including the SUE model in a dynamic framework [29; 8; 52; 9; 28].

After the collapse of the 1-35 bridge over the Mississippi River in Minnesota in 2007 there were a few projects aimed at exploring how such a large disruption affected traffic flow patterns in the Minnesota area. Xie and Levinson (2008) studied the effect of the bridge collapse on traffic flow in order to estimate the monetary value of the additional travel time incurred by drivers who generally used the bridge. This work did examine the impacts of a large traffic disruption, but the study was limited to long-term effects. The change in travel patterns immediately after the collapse, which is the type of problem examined in this thesis, was not dealt with [54]. A different approach was taken by Zhu et al. (2008) where travelers were asked to record their route choices in the days after the collapse. This data can provide some insight into the changing travel behavior as drivers learn more about the new route travel times, but there is still the shortcoming of requiring an adaptive/learning behavior. This work does not provide any insight into the immediate changes in traffic flow following the disaster but rather the adaptive changes in the following days as flows re-equilibrated [55].

2.4 Contributions

This work will provide a novel approach for predicting the route choices of individuals driving at various levels of traffic disruptions. The first contribution of this work is an analysis of perceived travel time distributions and a comparison of perceived and actual distributions as calculated from field data. The second critical contribution is a novel framework for forecasting changes in route choices at different levels of network disruptions. Although Phase 1 and Phase 3 implement existing network modeling frameworks, the transformation of the travel times based on driver perception provides a new and more accurate method for predicting driver behavior and flows in different traffic network conditions. Future extensions will include more complex route choice models in Phase 1 and Phase 3, but this first work provides the initial results to demonstrate the effect of these travel time changes on the link flows.

Chapter 3

Model Formulation

There are two separate goals of this research: (a) to compare the actual and perceived travel time distributions and (b) to develop a static model that predicts the change in traffic flows as a result of a network disruption. The model is separated into three phases. The first phase is the equilibrium assignment. The second phase is the transformation of the link travel times based on each link's equilibrium travel time and volatility. The third phase is the stochastic route choice phase where traffic is assigned to the network based on the transformed link costs. The experimental setup for determining the perceived travel times and each of the phases of the model are discussed in more detail below.

3.1 Travel Time Perception Tests

As noted in the literature review, the format of the travel time perception experiment was based on previous psychological tests which successfully showed that people's perceptions of event time distributions are comparable to the actual distributions [26; 32]. The participants were all members of the University of Texas at Austin community, which allows for a higher degree of familiarity with local road conditions and some uniformity in expectations of average travel times. Each participant was assigned a road type (major highway or regular road with stoplights) and condition (sunny, rainy, or derailed train and chemical spill on the road). Given these conditions, the question below was posed to the participant.

X is commuting from x's home to work in Austin during rush hour. Traveling on a [road type], the total distance from x's home to x's office is y miles. [condition]. X has been commuting for z minutes, how much longer do you think the trip will take?

There were 20 possible names (represented by the x's), and they were ordered randomly for each participant. Providing a different traveler name for each trip allows the participant to differentiate between the trips so that their previous answers have less inherent influence on their current answers. The commute distance (y) provided was between 14.5 and 15.5 miles with an average of 15 miles. The initial value of z, the current time on the road, provided was either 10 or 20 minutes for each participant. Subsequent values of the current time were calculated using a Sequential Monte Carlo procedure where the current time provided is chosen from a uniform distribution between 0 and the total time from the participant's previous answer. This sampling method forms a Markov chain, which has been shown to quickly converge to provide the participant's true perceived distribution [32].

3.2 Phase 1 – Equilibrium Assignment

Phase 1 of this model is simply the Deterministic User Equilibrium (UE) assignment of flows. The traditional equilibrium formulation as outlined by Sheffi (1981) is presented below.

$$\min_x \sum_a \int_0^{x_a} c_a(\omega)\, d\omega$$

Subject to:

$$\sum_k h_k^{r,s} = q_{rs}, \qquad \forall\, r, s$$

$$h_k^{r,s} \geq 0 \qquad \forall\, k, r, s$$

$$x_a = \sum_r \sum_s \sum_k h_k^{r,s}\, \delta_{a,k}^{r,s} \qquad \forall\, a$$

In this formulation, $q_{r,s}$ is the Origin to Destination (OD) demand from origin r to destination s, and $h_k^{r,s}$ is the flow on path k from origin r to destination s. Additionally, $\delta_{a,k}^{r,s}$ is a binary variable equal to 1 if path k from origin r to destination s uses link a and 0 otherwise, x_a is the flow on link a, and c_a is the cost of link a, which is simply equal to the travel time in this model [43].

This UE problem is solved using Dijkstra's shortest path algorithm and the Frank-Wolfe convex combination algorithm. Both perform reasonably well for the small networks tested in this thesis, and the algorithm implemented is outlined below.

Shortest Path:

Step 1: Initialize costs to $c_a = c_a(0)\ \forall a$. Perform all-or-nothing assignment using $c_a(0)$ to get x_a^1. Set counter: n = 1.

Step 2: Update the costs based on these new flows so that $c_a = c_a(x_a^n), \forall a$.

Step 3: Perform all-or-nothing assignment of flows again using $c_a = c_a(x_a^n)$. This assignment results in the flows y_a^n.

Step 4: Use Frank-Wolfe algorithm to find a γ_n that solves $\min_{0\leq\gamma\leq1} \sum_a \int_0^{(1-\gamma_n)x_a^n+\gamma_n y_a^n} c_a(\omega)\ d\omega$ [18]

Step 5: Set $x_a^{n+1} = (1-\gamma_n)x_a^n + \gamma_n y_a^n\ \forall a$

Step 6: If convergence criterion is met, stop. If convergence criterion is not met, set n = n+1 and go to Step 2 [16].

This solution method provides the equilibrium costs and flows for the average case [16; 18]. Once the equilibrium flows and costs have been calculated, the costs are transformed based on the travel time volatility and the incident severity in Phase 2.

3.3 Phase 2 – Travel Time Transformation

Phase 2 of the model transforms the perceived costs account for the severity of the incident on each link. The transformation of the link costs is done in this case with the inverse cumulative Normal distribution. This transform was chosen based on the perceived travel times for each incident severity level. The inverse cumulative Normal distribution most closely matches the transformed travel times for each severity level. The sum of the squares of the difference between the transformed average travel time and the average travel time for each incident level from the test data is approximately equal to 7.95. To put this value in perspective, Table 3.1 includes the values of the sum of the errors squared for the 5 distributions with the best fit.

Table 3.1: Transform Distribution Fit Results

Distribution Type	Sum of Squared Errors for Freeway	Sum of Squared Errors for Arterial	Total Sum of Squared Errors
Normal	3.0597	4.8855	7.9452
Levy	9.5040	16.9034	26.4074
Nakagami	14.8913	21.0065	35.8978
Exponential	18.1467	18.6892	36.8359
Gumbel	28.7890	24.5192	53.3082

As Table 3.1 illustrates, the inverse cumulative Normal distribution provides a significantly closer fit to the actual transformation of the perceived travel times. This value was considerably lower than the sum of squared errors for all other distributions tested. This transform optimization also provided the appropriate values for the severity levels, represented by α in this model. The optimization indicated that the severity levels should be given as $\alpha = 0.45$ for sunny, $\alpha = 0.59$ for rainy, and $\alpha = 0.91$ for a chemical spill. The value of α for the sunny conditions is less than 0.5 because the average is assumed to be the average travel time over all observed conditions. The equilibrium flows calculated in Phase 1 are assumed to be the average flows on that link, corresponding to the severity level of 0.5.

The other variable necessary to calculate the transformation of the travel time based on the inverse cumulative Normal distribution is the standard deviation of the travel time. The standard deviation represents the volatility of the travel time for the Normal distribution. If future research indicates that another distribution may provide a more accurate transformation, the σ variable used in the notation as volatility would be either a variable other than standard deviation or a vector of volatility-related variables.

Because of this notation, the same formulation described below can be applied to distributions other than Normal if required at a later stage in the research.

The formal expression of the link cost transformation is included below.

$$\bar{c_a} = f^{-1}(\alpha_a, c_a, \sigma_a)$$

In this formula, $\bar{c_a}$ is the transformed link time, and $f^{-1}(\alpha_a, c_a, \sigma_a)$ is the inverse cumulative function of the Normal distribution with α_a corresponding to the probability in the inverse cumulative calculation and c_a representing the equilibrium travel time found in Stage 1. The values of σ_a are estimated for each link based on a relationship between the average travel times and the volatility variable(s) in the field data distributions. The σ_a value for the Normal distribution is simply the standard deviation of the link travel time.

The general relationship between the mean and standard deviation of travel times from field data provided an approximate method for estimating the standard deviation of travel time on a link given the average travel time. The sensitivity of the transformation to the possible errors in the estimation of the standard deviation was deemed relatively insignificant for this model. Additionally, there is a clear difference between the variability of the freeway and non-freeway links, so the standard deviation was estimated differently for each link-type in the formulation.

The values of σ_a are estimated for each link based on an approximate relationship between the average travel times and the volatility variables in the field data distributions. Figure 3.1 illustrates the relationship between the mean and standard deviation of the field data for freeway and arterial roads. The σ_a value for the Normal distribution is equal to the standard deviation.

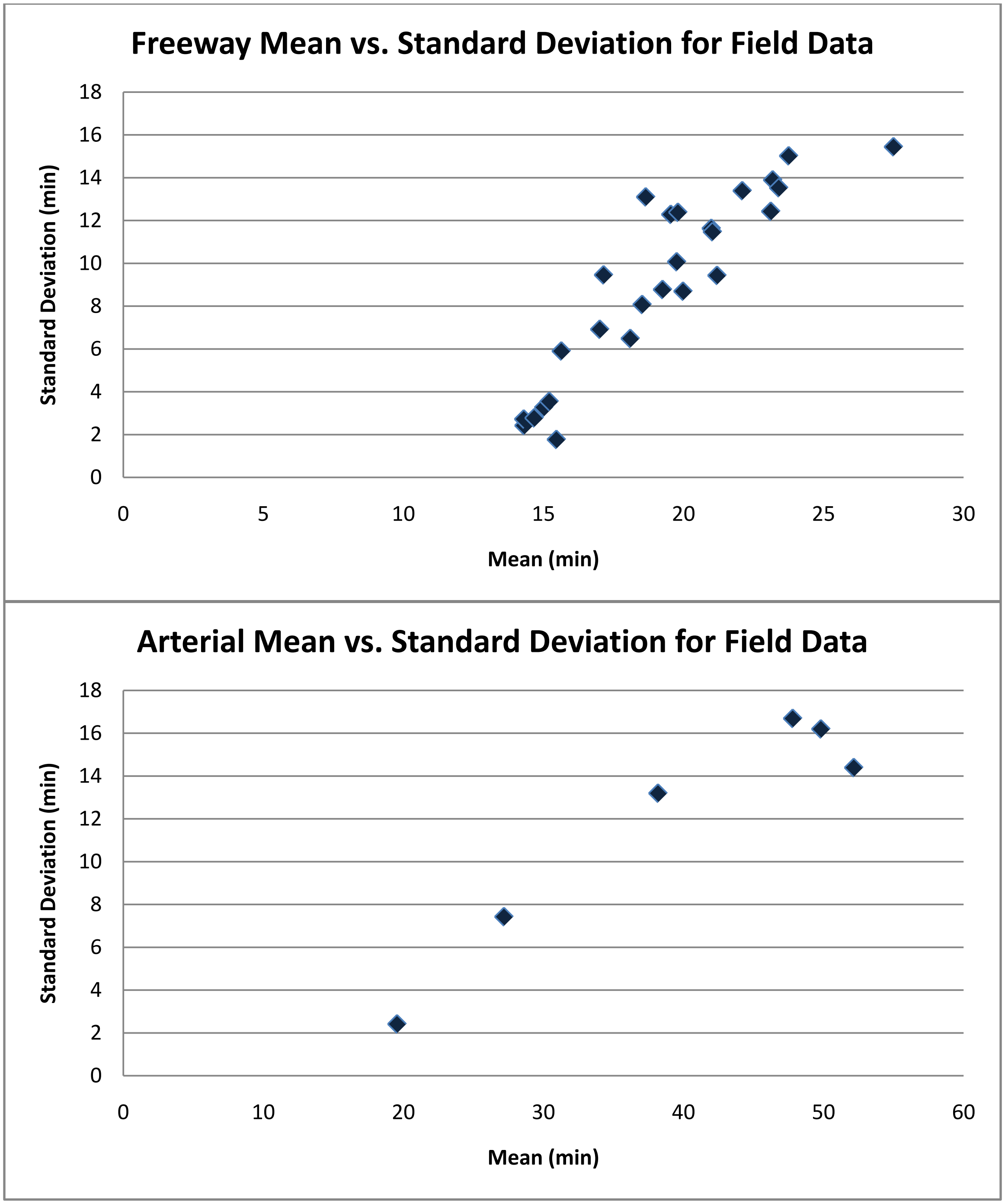


Figure 3.1: Mean vs. Standard Deviation Graphs for Freeway and Arterial Field Data

3.4 Phase 3 – Stochastic Route Choice

The logit assignment problem has been discussed in the literature review, and it is applied in Phase 3 to redistribute the traffic based on the level of dispersion across paths. The logit assignment problem calculates the probability that a given path from an origin to a destination will be selected. That probability is then multiplied by the demand for that OD pair, and this gives the value of the path flow. The STOCH algorithm developed by Dial can be applied to the problem at this stage to eliminate the need for enumerating all paths before performing the probability calculations. This is especially useful when dealing with larger networks with many links and many possible paths. The equation for the path flow is shown below [15].

$$h_k^{r,s} = q_{r,s} * \frac{e^{\left(-\theta t_k^{r,s}\right)}}{\sum_k e^{\left(-\theta t_k^{r,s}\right)}}$$

In the above equation $h_k^{r,s}$ is the flow on path k from origin r to destination s, $q_{r,s}$ is the total demand from origin r to destination s, θ is the scaling parameter indicating the level of accuracy of the travel time perceptions, and $t_k^{r,s}$ is the travel time on path k from origin r to destination s [15]. As noted by Sheffi (1981), the scaling parameter (θ) represents the dispersion of flow across links, and it is inversely proportional to the standard error of the distribution of the perceived travel times. Because of this property, the scaling parameters used in this work were set to 0.1 for low demand, 0.05 for intermediate demand levels, and 0.02 for high demand levels. This was assumed to provide a relatively accurate approximation of the standard error since the travel times, and, therefore, volatilities are higher for higher demands [43].

Given a sample network, these three phases can be implemented to determine how link flows would change for a given level of disruption. The discussion of those implementations and descriptions of the sample networks used in the calculations are included in the next chapter.

Chapter 4

Results and Discussion

The modeling framework described in Chapter 3 can be implemented on many sample networks. The results discussed in Section 4.2 and Section 4.3 are from two relatively small sample networks. These networks have one freeway-based path and one arterial-based path. The larger network also has a path that allows for mixed use of freeways and arterials. The purpose of using such simplistic paths is to demonstrate the change in usage of freeways (which have a lower average travel time and higher volatility) versus arterials (which have a higher average travel time and lower volatility) in disrupted traffic situations. The comparison of the perceived and actual travel time distributions is also presented in Section 4.1 The results are presented and discussed in the remainder of this chapter.

4.1 Travel Time Distributions

The perceived average travel time for both freeway and arterial links for all incident levels did not follow the patterns that might be expected. Table 4.1 includes the mean and standard deviation of the perceived travel times for the average and each level of disruption tested. In general, freeway links have lower travel times and greater travel time variability for a given travel time than arterial links of the same length, as shown in Figure 3.1. This field data demonstrates the expected mean-standard deviation relationships for average conditions, and not all of the conditions tested followed these expected trends. Some possible reasons for these discrepancies are included in the discussion below Table 4.1.

Table 4.1: Statistical Data for Perceived Travel Time Distributions

	Freeway		Arterial	
Level of Disruption	Mean (min)	Standard Deviation (min)	Mean (min)	Standard Deviation (min)
$\alpha = 0.5$	35.74	14.76	39.59	11.68
$\alpha = 0.45$	33.23	12.39	38.82	15.65
$\alpha = 0.59$	40.28	16.76	40.93	11.51
$\alpha = 0.91$	57.17	32.12	54.02	15.94

The average travel time was greater for the regular road with stop lights, as expected, and the corresponding standard deviation was greater for the freeway. This result conforms to expected trends because arterials have lower speeds and more stops, so they would most likely have higher travel times. Freeways, on the other hand, have higher speeds, but the travel time on a highway is generally more variable, leading to a higher standard deviation. The only two results that are unexpected are the standard deviation of the arterial link for sunny weather and the average travel time for arterials in a chemical spill. The high value of the standard deviation on the arterial travel time distribution for sunny road conditions is most likely an error due to the sample size. Larger samples may provide a lower standard deviation for this road type and severity. There may also be an issue in the way the question was posed, leading to this error. If larger samples have this same issue, the phrasing of the question or the seed values of current time on the road may be changed to attempt to resolve this issue. The reason for the lower travel time on the arterial due to the chemical spill may be an unexpected trend in actual perception. People may anticipate less congestion on arterial links, which would lead to a lower expected travel time. The travel time distributions were also compared to the field data collected for arterial and freeway links.

The perceived travel time distributions were calculated based on the results of 87 participants. There were 44 subjects for the arterial distribution and 43 subjects for the freeway distribution. For the freeway values, any total travel times less than or equal to 11 minutes were removed. These results were considered unreasonable travel time predictions given that they would require speeds of over 80 mph to complete the provided trip. For the arterial results, observations with a total time of less than 18 minutes were removed, as they indicate a speed of greater than 50 mph. The first three results for each participant were also excluded from the calculations due to the fact that the Sequential Monte Carlo procedure used requires a few iterations to converge to a set of values that correctly represent the perceived distribution [26; 32]. Figure 4.1 illustrates the cumulative probability distributions of the test data for both freeway and arterials as compared to field data for both link types. All of the field data is scaled for a 15 mile road segment to make it comparable to the test data.

The field data was taken from a combination of Bluetooth and sensor travel time data. The Bluetooth data included travel times for freeways and arterials in Maryland, Virginia, and Pennsylvania for November 2009 provided by Dr. Stanley Young at the University of Maryland. The data for the Mopac (Loop 1 in Austin, TX) was sensor data provided by TTI (Texas Transportation Institute) for both peak and off-peak travel times in April 2008. The distribution of the test data was also calculated by weighting the data from each incident severity level founded on the predicted frequency of the event occurring. The values were weighted assuming that the conditions are sunny slightly less than 2/3 of the time and rainy-level severity approximately 1/3 of the time. The weights applied to the chemical spill data also assumed that an incident of this level of severity happens once a year. It rains significantly less than 1/3 of the time in Austin, but the weighting accounts for other conditions of rain and higer-level severity. To accurately account for the distribution, many other incidents should be included, but the rain data had to be weighted more to account for the fact that only rain, sun, and chemical spill conditions were tested.

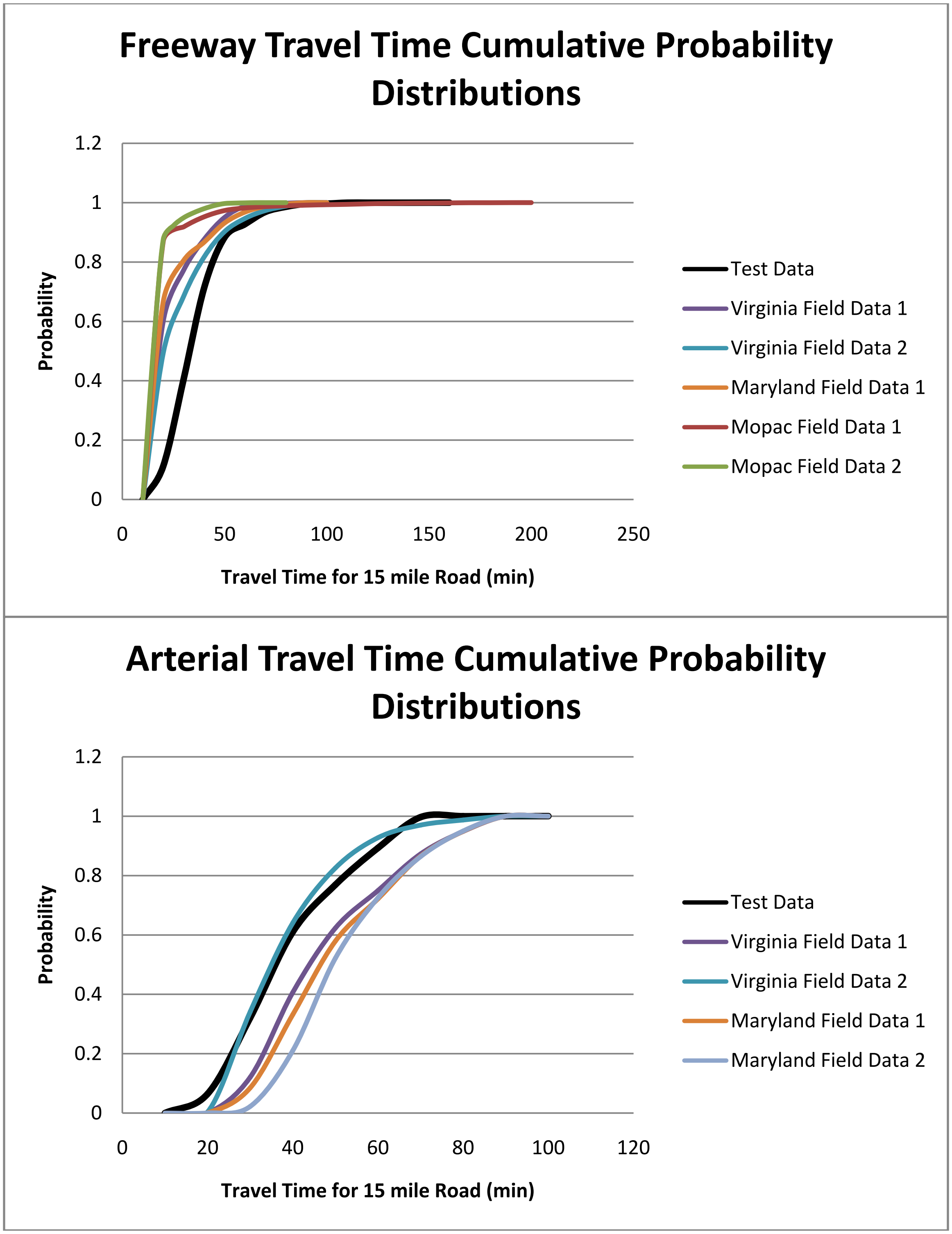
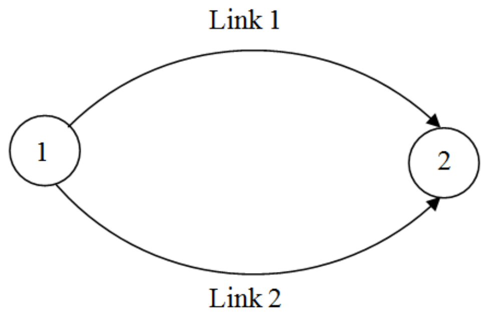


Figure 4.1: Comparison of Actual and Perceived Travel Time Distributions

Based on the graphs shown in Figure 4.1, it is clear that the test distributions match the actual travel time distributions quite closely. Although the distributions are not an exact fit to the field data, the general shape and location are approximate matches. While the perceived travel time distributions are similar to the actual travel time distributions, the differences demonstrate the necessity of accounting for the perceived travel time when predicting driver behavior rather than simply relying on the measured travel time distribution. This model can also be extremely useful for predicting traffic flows is situations that rarely occur. There are some situations, such as bridge collapses and chemical spills that rarely occur, and existing models could not be used due to the lack of familiarity with travel time distributions for such instances.

4.2 Sample Network 1

Figure 4.2 below includes a diagram of the first sample network and link costs used to test the effects of the incidents on traffic flow.



(a)

	Link Cost Function
Link 1	$c_1 = 11.25\left[1 + 0.15\left({x_1}/{9{,}600}\right)^4\right]$
Link 2	$c_2 = 20\left[1 + 0.15\left({x_2}/{3{,}800}\right)^4\right]$

(b)

Figure 4.2: a) Sample Network 1 b) Sample Network 1 Link Costs

Link 1 in the sample network is designed as a 4-lane freeway-type link, and Link 2 is a 2-lane arterial link. This basic network was chosen to illustrate the way the predicted flow shifts between freeway and arterial links as a result of the travel time transformations and the dispersion from the logit assignment. The tests were performed for three levels of demand to determine how the effects of the incident change with flow volume. The demand levels are 20,000 vehicles, 25,000 vehicles, and 40,000 vehicles. Figure 4.3 includes graphs illustrating the percentage change in flow on the freeway and arterial paths for each severity level and demand level. The percentages are calculated based on the difference between the predicted values and the logit assignment flows without the transformation from Phase 2. The difference in these values illustrates the necessity of transforming the travel times to accurately predict the changes in traffic flow for incidents of different severities.

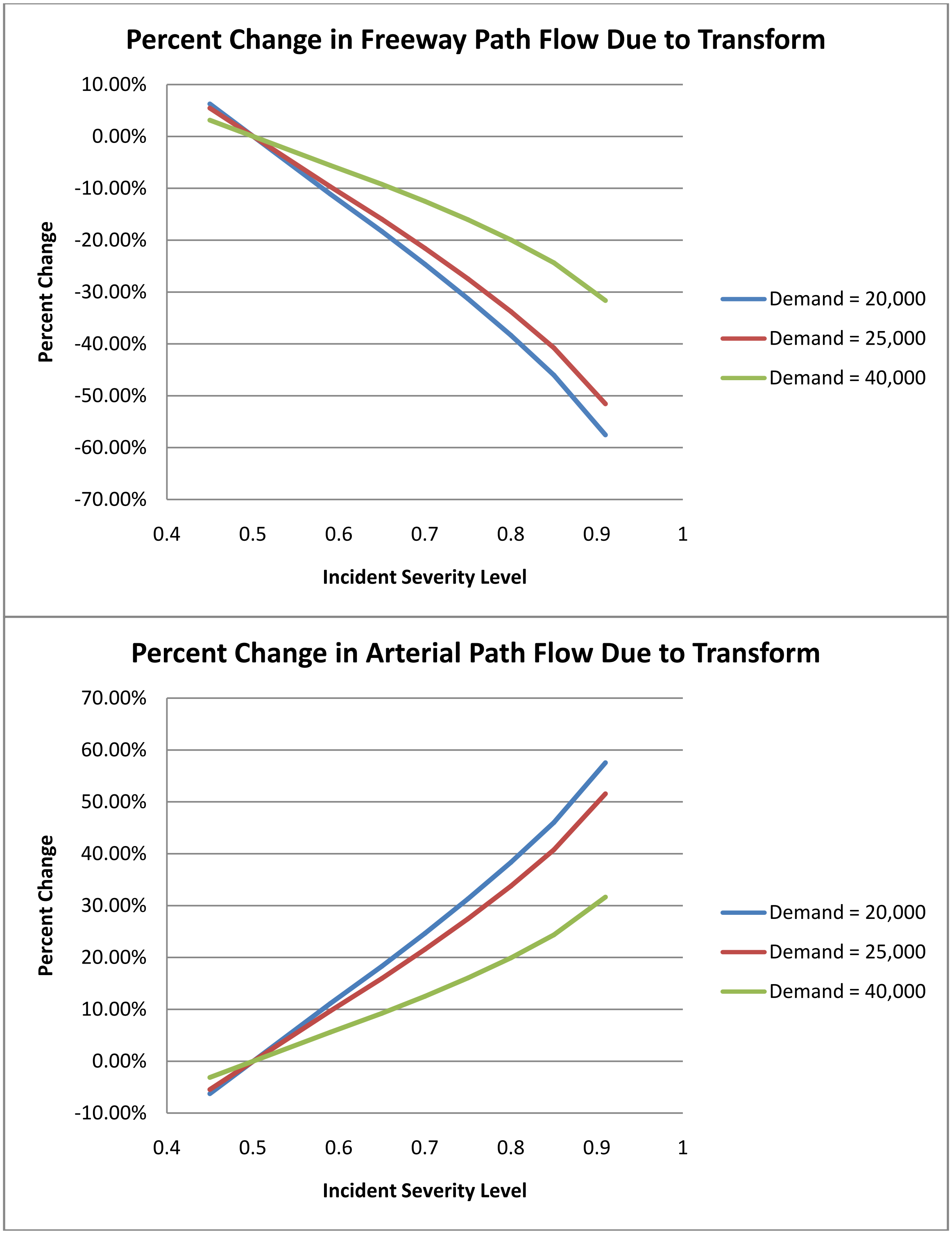


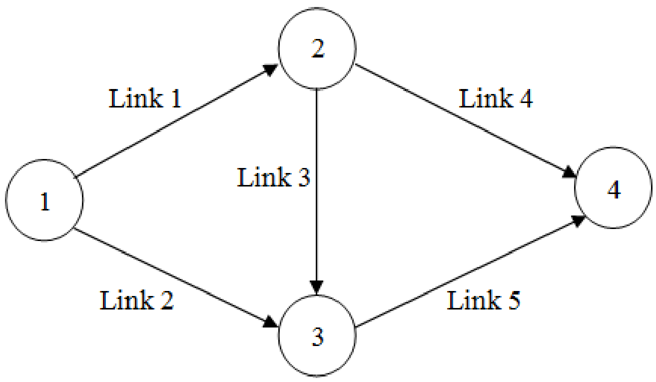
Figure 4.3: Percent Change in Path Flows for Sample Network 1

The results illustrate that the shift of flow from freeway to arterial paths is greater for lower demand levels. This is due to the fact that at lower demand levels, more of the equilibrium flow is on the freeway. Therefore, a higher percentage of the flow can shift from the freeway to the arterial. The percent change between the base and predicted flow values is greater for lower demands. This result indicates that the incidents have a greater effect on the flow distribution at lower levels of congestion, where there is less flow on the arterial path at equilibrium.

For the sunny weather ($\alpha = 0.45$) the percent change in flow is positive for freeways and negative for arterials. This demonstrates the importance of using perceived travel times to predict the actual path choices even in sunny weather. The shift in flow increases significantly as the severity level increases, for all levels of demand. This result indicates that as the travel time uncertainty increases, users gravitate toward arterial paths. This may be due in part to people's tendency to avoid high-risk options in unfamiliar situations with a high degree of uncertainty [17; 22]. The other reason for the increased shift of flow from the freeway to the arterial path as the severity increases is the transform. Due to the higher volatility of travel times on freeway links, drivers' expected travel times increase more for freeway links. The results also indicate that as the network becomes more congested, the effect of the incident severity is diminished. As the demand level increases, the slope of the percent change as the incident severity level increases is less steep. This trend indicates that higher congestion levels decrease the effect of the incident severity on the route choices.

4.3 Sample Network 2

Figure 4.4 illustrates the second sample network used to test the effects of the incidents on traffic flow. The link cost formulae are also included in Figure 4.4.



(a)

	Link Cost Function
Link 1	$c_1 = 7.5\left[1 + 0.15\left(x_1/9{,}600\right)^4\right]$
Link 2	$c_2 = 13.3\left[1 + 0.15\left(x_2/3{,}800\right)^4\right]$
Link 3	$c_3 = 2\left[1 + 0.15\left(x_3/3{,}800\right)^4\right]$
Link 4	$c_4 = 3.75\left[1 + 0.15\left(x_4/7{,}200\right)^4\right]$
Link 5	$c_5 = 6.67\left[1 + 0.15\left(x_5/3{,}800\right)^4\right]$

(b)

Figure 4.4: a) Sample Network 2 b) Sample Network 2 Link Costs

Link 1 in the sample network is designed as a 4-lane freeway-type link, and Link 2 is a 2-lane arterial link, Link 3 is a two-lane arterial link, Link 4 is a 3-lane freeway link, and Link 5 is a 2-lane arterial link. Testing was also done on this slightly-larger network in order to determine whether the transforms would make drivers more likely to choose a freeway-only or arterial-only path or whether they might utilize a path that traverses one freeway link and then uses two arterial links. The tests were run for demand levels of 20,000 vehicles, 25,000 vehicles and 40,000 vehicles. This network can provide more insight into the effects of congestion levels and incident severity on route choice decisions. Figure 4.5 includes graphical representations of the percent change in flow (relative to logit assignment with no transformation) on each type of path for each level of demand and severity.

The results in Figure 4.5 illustrate the changes in flow on freeway, arterial and mixed paths as a result of different levels of demand and incident severity. For the freeway-only and arterial-only paths, the lowest congestion level demonstrated almost no change from the base (no transform) flows to the predicted flows for the sunny conditions. Additionally, the percent change in the highest demand level increases more slowly than the middle demand level of 25,000 vehicles for the arterial and mixed paths. This result demonstrates the importance of estimating the demand on a network before generating the flow predictions.

One of the most notable differences between the results of Sample Network 1 and Sample Network 2 are that in the larger network the slope of the line representing the change in percentage change is much more curved. On the mixed path the slope changes after the incident level of 0.59 (rain). This is most noticeable for the higher two demand levels where the predicted flows are only slightly smaller than the base (no transform) flows for values of incident less than approximately 0.6. This indicates that for higher demand levels the path using both freeway and arterial links is used significantly less as the severity of the incident increases. This seems to suggest that for higher incident severity, which corresponds to higher uncertainty and unfamiliarity with the travel times, drivers are more likely to choose a path that uses only one type of link. All of these results provide important insight into the change in route choices for different types of network disruptions.

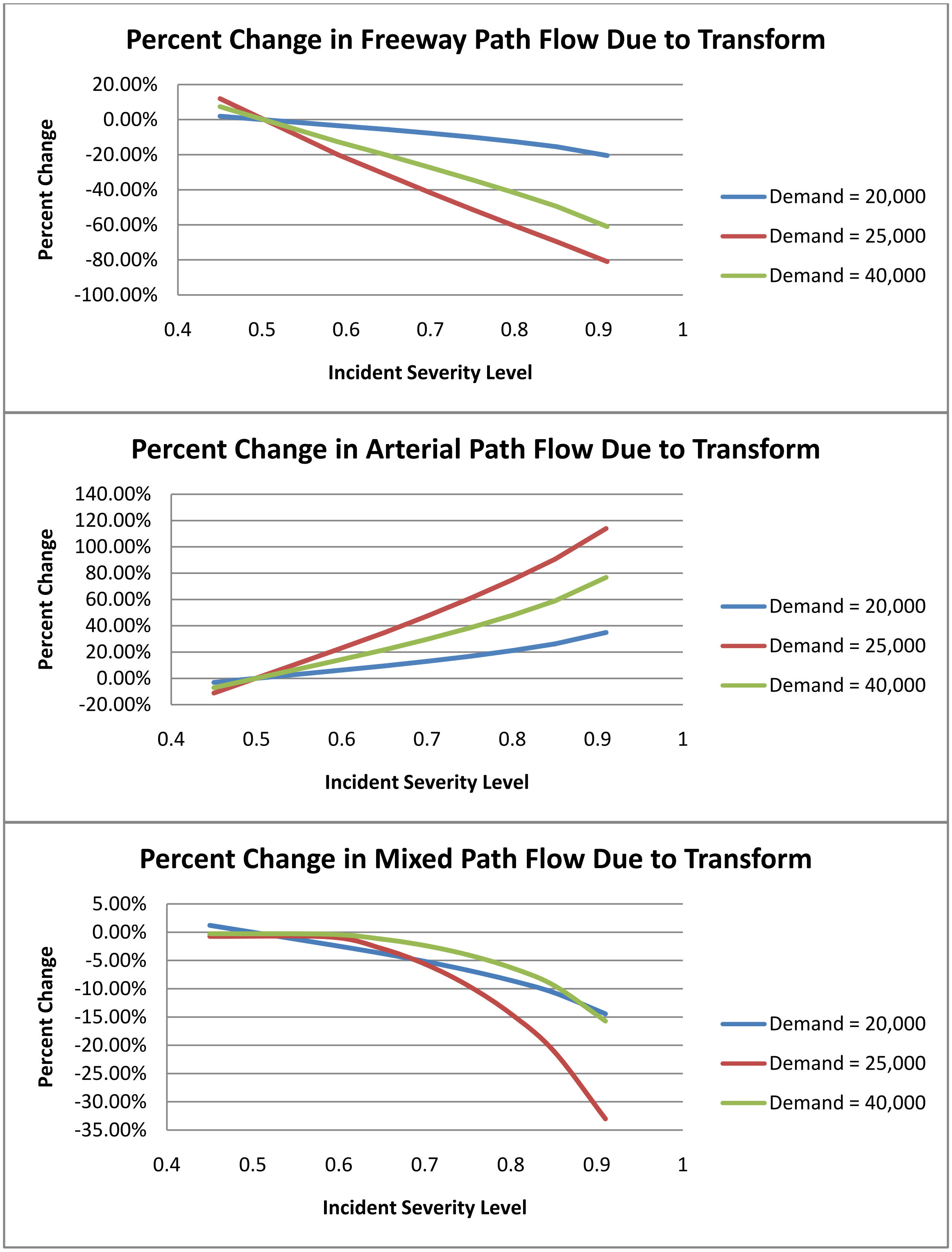


Figure 4.5: Percent Change in Path Flow for Sample Network 2

Chapter 5

Conclusions

This work has provided a comparison of actual and perceived travel time distributions and a framework for modeling the changes in static traffic flow due to various levels of network disruptions. The current literature has explored traffic networks under various levels of uncertainty including travel time and demand uncertainty. There are also methods that account for errors and uncertainty in driver's perception of travel times [48; 33; 46]. There is a limited amount of literature dealing with driver reactions to network disruptions and travel time perception, and the existing works deal primarily with the long-term route choice changes rather than immediate and short-term effects of the disruption [54; 55]. The model developed in this thesis provides a framework for predicting the short-term effects of a network disruption on driver behavior. While other models assume that drivers already have some knowledge of the travel time distributions, this model instead uses actual calculated travel time perceptions for three different levels of network disruption. By providing a framework for transforming the expected travel times and incorporating uncertainty into those travel times, driver choices in network disruptions can be more accurately predicted.

This thesis has outlined and tested a new model for predicting driver behavior in the case of a disruption to the traffic network. The data from the travel time perception tests indicates that while perceived travel time distributions are close to the actual travel time distributions observed, there is some degree of error in the way people perceive travel time distributions. Due to the assumption of most network models that people's route choices are based on their perception of travel times, the error in these distributions indicates that it is important to account for the actual perception of travel times instead of the measured travel times.

The results of the traffic flow predictions on both test networks provide some insight into route choice behavior in uncertain network conditions. As the severity of the incident increases, drivers are more likely to choose arterial paths. The results also suggest that users are less likely to select a route that traverses both freeway and arterial links for higher levels of incident severity. The results also demonstrate the importance of including the travel time transformation phase in the model. Without the transformation, the predicted flow on freeway paths would most likely be too high, and the predicted flows on arterial paths would be too low. The results also indicate that the predicted flow distributions are dependent on the congestion level, so the demand must be estimated before the predictions are performed to achieve the most accurate results. These changes in the path flow provide some intuition as to the way that route choice decisions change when the network experiences some level of disruption.

There are many extensions of this work already being planned. One of the next steps in this area is to incorporate Unnikrishnan's User Equilibrium with Recourse to model the travel times. This formulation would serve as an initial step toward incorporating some degree of recourse into the model [49]. Other possible formulations to incorporate into this modeling framework are online shortest path calculations and adaptive route choice. The final goal is to provide a model that performs dynamic adaptive modeling of traffic in situations with network disruptions. This dynamic model will explore how the traffic flow patterns change over time as a result of a disruption and how drivers route choices may be altered as they learn more about the actual travel times en-route. This can provide a framework for dealing with traffic disruptions of any level from rainy weather to a catastrophic road closure. This could change the way that local government agencies respond to network disruptions. This thesis and future extensions will provide new route choice models to enhance the understanding of how users make route choice decisions in uncertain environments.

Appendix A

Notation

c_a − Equilibrium travel time over all observed realizations of α on link a, also cost

$\overline{c_a}$ − Transformed travel time on link a

x_a − Equilibrium flow on link a

$\overline{x_a}$ − Predicted flow on link a after model is complete

$h_k^{r,s}$ − Equilibrium flow on path k from origin r to destination s

$\overline{h_k^{r,s}}$ − Predicted flow on path k from origin r to destination s after model is complete

$t_k^{r,s}$ − Travel time on path k from origin r to destination s function on link a

$q_{r,s}$ − Demand from origin r to destination s

$\delta_{a,k}^{r,s}$ − Binary variable equal to 1 if path k from origin r to destination s uses link a, 0 otherwise

α_a − Incident severity level on link a (0.45 to 1)

σ_a − Vector of volatility variables for link a (standard deviation in this formulation)

$f^{-1}(\alpha_a, c_a, \sigma_a)$ − Travel time transform function, in this formulation = inverse cumulative of Normal distribution

θ − Scaling parameter used in logit assignment

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Vita

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