The Report Committee for Andy Chi-chung Tang

Certifies that this is the approved version of the following report:

Pythagoras at the Smithy:

Science and Rhetoric from Antiquity to the Early Modern Period

APPROVED BY

SUPERVISING COMMITTEE:

Supervisor: ______________________________

Luisa Nardini

____

Lorenzo Candelaria

Pythagoras at the Smithy:

Science and Rhetoric from Antiquity to the Early Modern Period

by

Andy Chi–chung Tang, BA, BSChem

Report

Presented to the Faculty of the Graduate School

of the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Music

The University of Texas at Austin

May 2011

Acknowledgements

This report is a culmination of three years of work. I never dreamed it would turn out quite the way it did when I first started, and I could not have finished this document without the support of certain people. I want to thank both Andrew Dell'Antonio and Luisa Nardini for introducing me to musicology, Luisa Nardini for her clarity of thought, Lorenzo Candelaria for his wise words and guidance, Tim Moore for his precision of language and mentoring, and my family and friends whose ears I talked off from time to time with my questions and musings. Thank you all.

Pythagoras at the Smithy:

Science and Rhetoric from Antiquity to the Early Modern Period

by

Andy Chi–chung Tang, M. Music

The University of Texas, 2012

SUPERVISOR: Luisa Nardini

It has been said that Pythagoras discovered the perfect musical intervals by chance when he heard sounds of hammers striking an anvil at a nearby smithy. The sounds corresponded to the same intervals Pythagoras had been studying. He experimented with various instruments and apparatus to confirm what he heard. Math, and in particular, numbers are connected to music, he concluded. The discovery of musical intervals and the icon of the musical blacksmith have been familiar tropes in history, referenced in literary, musical, and visual arts. Countless authors since Antiquity have written about the story of the discovery, most often found in theoretical texts about music.

However, modern scholarship has judged the narrative as a myth and a fabrication. Its refutation of the story is peculiar because modern scholarship has failed to disprove the nature of Pythagoras's discovery with valid physical explanations. This report examines the structural elements of the story and traces its evolution since Antiquity to the early modern period to explain how an author interprets the narrative and why modern scholarship has deemed it a legend. The case studies of Nicomachus of Gerasa, Claudius Ptolemy, Boethius, and Marin Mersenne reveal not only how the story about Pythagoras's discovery functions for each author, but also how the alterations in each version uncover an author's views on music.

Table of Contents

THERE IS GEOMETRY IN THE HUMMING OF THE STRINGS.

THERE IS MUSIC IN THE SPACING OF THE SPHERES.

—PYTHAGORAS, IN GUY MURCHIE'S *MUSIC OF THE SPHERES*[1]

It has been said that Pythagoras, the 6th century BCE philosopher, discovered the perfect musical intervals. He was passing by a blacksmiths' workshop when he heard by chance the sounds of hammers hitting an anvil. They produced pitches that corresponded to the same musical intervals Pythagoras had been studying. He analyzed what he heard and concluded that numerical values of weight correlate with musical sounds, in particular, the perfect fourth, fifth, and octave intervals Pythagoras had sought could be constructed this way. To confirm the discovery, he built an apparatus of weights hung up by string that produce sounds of musical intervals. Pythagoras extended his testing to other instruments like reed pipes, jars, and the monochord. All of Pythagoras's experimentation validated his discovery. Math, and, in particular, numbers are connected to music, he concluded.

The discovery of musical intervals and the icon of the musical blacksmith have been familiar tropes in history, referenced in literary, musical, and visual arts. Countless authors since Antiquity have written about the story of the discovery. Iconography in medieval manuscripts shows Pythagoras striking musical objects. Renaissance composer Antoine Busnoys crafted his motet *In hydraulis* with the cantus firmus consisting of three-note segments that leap by Pythagorean intervals,

[1] This quote is found in the front matter of Guy Murchie's 1967 book, *Music of the Spheres: The Material Universe–from Atom to Quasar, Simply Explained*. It is a two-volume book that describes

an appropriate musical setting for its anonymously authored text about Pythagoras's discovery. Enlightenment scientist Isaac Newton defended his formulation of universal gravitation against rival Robert Hooke by invoking the authority of the earlier discovery of Pythagorean intervals. Baroque composer George Frederic Handel was likely inspired by the story to write "The Harmonious Blacksmith," the popular name for the last movement of Suite Vol. 1 No. 5 for harpsichord. Alongside Pythagorean attributions like the theorem and the music of the spheres, this story with its quantitative description is the philosopher's legacy.

Despite the familiarity the discovery has had in history, connecting what is essentially number theory to music is not perfectly clear. Generally, the discovery relates the physical formation of the perfect intervals based on whole numbers. The details of the discovery are confusing though. What the story does not tell exactly is how musical pitch and number are related to each other. In fact, explanation in the story is brief and unsatisfactory, and the understanding of the narrative's conclusion is usually taken for granted. These deficiencies often label Pythagoras's discovery as a legend, fable, and myth; they suggest a negative connotation. To avoid tainting one's opinion, I will refer to Pythagoras's discovery with more neutral terms like "story" or "narrative." A thorough examination of the story's elements is in Chapter 1: "The Story of Pythagoras Discovering the Musical Intervals."

Unique interpretations of the story in modern times have been less than in previous historical periods. The trend has resulted in fewer artistic and literary works and more attention to the academic or educational study of the story as an example of the importance numerology in Pythagoreanism. Thus, the recent view has been frozen in time. One case in point is Weiss and Taruskin's 2007 anthology *Music in the Western World: A History in Documents*.[2] Their presentation of ancient music combines the Pythagorean story with the mythology of Orpheus's musical abilities and the comparisons of Platonic and Aristotelian musical doctrines. Weiss and Taruskin frame the story as part of Pythagorean musical tradition, connecting it to the music of the spheres and the primacy of number in philosophical study. The editors judge the story as an ancient legend, arguing that the relationship of pitch and number applies to string tension, not string length or weight of hammers. No proof is needed, Weiss and Taruskin claim, since the story as described is flawed. The explanation is presumptive, as if it is unnecessary to consider potential musical properties of weight. One might think of sizes of related instruments. For example, larger cymbals sound lower than smaller ones.

Pythagoras's discovery of the musical intervals, some may consider, is another esoteric attribution like his ability for bilocation or his distaste for beans. It is not as tangible and elegant in its explanation as the Pythagorean theorem. The difficulty in comprehending the technical aspects of the story may be a problem of language. The story is originally from the *Enchiridion harmonicês* (*Manual of*

---

[2] The approach of the editors is to introduce musical thought throughout history with primary sources. The target audience is undergraduate music majors and non-majors. Weiss and Taruskin assume no prior knowledge of music in their selections and commentary. They make the readings as accessible as possible, writing an introductory paragraph for context. No selections contain musical notation. The readings are self-contained, to allow a customizable reading list and not to draw a continuous narrative of music history. See Weiss, Piero, and Richard Taruskin. *Music in the Western World: A History in Documents*. New York: Schirmer Books, 1984, xiii-xiv.

Harmonics), by the 2nd century CE Greek writer, Nicomachus of Gerasa. His is the earliest recorded account, though Pythagoras precedes Nicomachus by 600 years. The text is highly compressed and abrupt probably because the author dictated this treatise while traveling. Nicomachus's language, as discussed in Chapter 2: "Nicomachus and the First Affirmation," shapes later interpretations, including translations.

Financial and legal constraints may limit the reproduction of more accurate translations in a profit-driven textbook market. Weiss and Taruskin uses the stilted but royalty-free in the public domain Thomas Stanley's translation in John Hawkins's 1776 book *A General History of the Science and Practice of History*.[3] Though a stilted work, Weiss and Taruskin select this text because it is royalty-free in the public domain.[4] Nicomachus has not translated as often as other authors. Alternatives are rare, but subsequent (and more recent) translations are in Flora Levin's *The Manual of Harmonics of Nicomachus the Pythagorean* and Andrew Barker's *Greek Musical Writings: Volume 2, Harmonic and Acoustic Theory* are clearer but likely have higher royalties. Thus, reproductions of their texts are not frequently cited and the English translation most often seen lacks the additional finesse or

[3] Hawkins, John. *A General History of the Science and Practice of Music*. New York: Dover Publications, 1963, 9–10

[4] For example, Weiss and Taruskin secured Calvin Bower's permission to reprint an excerpt of his translation of Boethius from his 1967 PhD dissertation for one of the anthology's readings on the Middle Ages. The editors could have picked, instead, Bower's 1989 book for a revised and clearer translation of the same text, but dealing with publishers and copyright issues might have been less preferable to Weiss and Taruskin. See Weiss and Taruskin 28n.

precision of Barker's or Levin's renditions. What is already an abstruse story is complicated more by the translation.[5]

Another barometer of modern scholarship is *The Cambridge History of Western Theory* edited by Thomas Christensen. Part of the beginning to Calvin Bower's Chapter 5: "The transmission of ancient music theory into the Middle Ages" is devoted to story of Pythagoras's discovery, relating the importance of number in musical thought.[6] Bower offers Boethius's use of the story as a connection between Antiquity and the Middle Ages. After providing a translation of Boethius's text on the narrative, Bower immediately claims "the empirical data offered in the myth is wholly specious, for hammers of comparable weights would not sound the musical intervals presented in the story" to justify his assessment. His citations and footnotes do not immediately explain what makes elements of the story so objectionable, but they lead ultimately to a 17th century French scholar named Marin Mersenne. His works on acoustics and string instruments, *Harmonie universelle* and *Questions harmoniques* , are the last texts that evaluate Pythagoras's discovery critically.[7] Since Mersenne, by and large, opinion of the story judges it as a fabrication, initiating the trend in modern reception.

---

[5] Barker, Andrew. *Greek Musical Writings: Volume II, Harmonic and Acoustic Theory*. Cambridge University Press, 1989. Levin, Flora. *The Manual of Harmonics of Nicomachus the Pythagorean*. Grand Rapids, MI: Phanes Press, 1994.
[6] Christensen, Thomas Street, ed. *The Cambridge History of Western Music Theory*. Cambridge University Press, 2002, 142–143.
[7] Mersenne, Marin. *Questions harmoniques*. F. Frommann, 1972. Mersenne, Marin. *Harmonie Universelle, Contenant La Théorie Et La Pratique de La Musique.* 3 vols. Paris, 1963. These are two facsimile editions of Mersenne's 1634 and 1636 treatises, respectively.

As I discuss in Chapter 3: "The Rejection of Ptolemy and Mersenne," Mersenne offers no proof to discredit the discovery; instead, he contrasts the veracity of his experiments with string against the dubious nature of narrative. Mersenne's work marks the most recent critical interpretation of the story. As a formidable authority on acoustics even beyond the early modern period, Mersenne solidifies subsequent reception. Modern scholarship often, if not consistently, repeats Mersenne's assessment and derives a physical proof to extend his repudiation, and thus refute any possibility[8]. Mersenne was not the first to reject the story. His conclusion replicates Claudius Ptolemy's 1500 earlier. A contemporary critic of Nicomachus, Ptolemy, in his musical treatise *Harmonics*, argues against a Pythagorean mindset and advocates a theory based on the monochord.[9] He abandons the story as rubbish, since it constructs music based on values of weight.

Affirmation of the discovery is recapped in Chapter 4: "Affirmation Revisited: Boethius and the Quadrivium" with the philosopher Anicius Manlius Severinus Boëthius—known commonly as Boethius—implicitly endorsing the story. Boethius expands on Nicomachus's writings and Pythagorean thought in his

[8] For example, see the following: Barker 257, n. 47; Barbera, André. *The Euclidean Division of the Canon: Greek and Latin Sources: New Critical Texts and Translations on Facing Pages, with an Introduction, Annotations, and Indices Verborum and Nominum Et Rerum*. Lincoln, NE: University of Nebraska Press, 1991, 60, n. 158; Levin (1994) 96, n. 14; Mathiesen, Thomas. *Apollo's Lyre: Greek Music and Music Theory in Antiquity and the Middle Ages*. Lincoln, NE: University of Nebraska Press, 1999, 399, n. 77; Ptolemy. *Harmonics*. Edited by Jon Solomon. Leiden: Brill, 2000, 25, n. 130, 135; Creese, David. *The Monochord in Ancient Greek Harmonic Science*. Cambridge University Press, 2010, 82, n.5. These sources cite Burkert, Walter. *Lore and Science in Ancient Pythagoreanism*. Translated by Edwin Minar. Harvard University Press, 1972, 374–378. Burkert then cites Mersenne *Questions*. 1634, 166. These citations illustrate bibliographic bloat and the ultimate dependency of a single source, whose conclusions themselves have not been reconfirmed.

[9] Ptolemy, ed. I. Düring. *'Αρμονικά*. Gothenburg, 1930.

conceptualization of music for his medieval and Renaissance audiences. Featured in his tome, *De institutione musica*, the story about Pythagoras's discovery elegantly links his concepts of music to number. Although not a Pythagorean himself, Boethius uses Pythagorean ideals to formulate his fundamentals of music.

Boethius, Nicomachus, Ptolemy, and Mersenne serve as case studies in the propagation and the evolution of the story's reception throughout history. Each version carries an attitude about the story's possibility. This calculates into each author's use for the musical discovery; anyone's engagement of the story might construct a different idea for what its purpose is. Thus, each author shapes the story to match with his understanding and expectations, in some cases defying the laws of physics in their explanations of the story.

> A new scientific truth does not triumph by convincing its
> opponents and making them see the light,
> but rather because its opponents eventually die,
> and a new generation grows up that is familiar with it.
> —Max Planck, *Scientific Autobiography and Other Papers*[10]

William Stahl, in his commentary of Vincent of Beauvais's use of Macrobius's *Commentary on the Dream of Scipio*, observed how "the account of Pythagoras's discovery was recorded by so many authors, Greek and Latin, in substantially the same wording, as to have become common property."[11] His observation about common property is on the mark. Reading through even a couple of versions, one would find this correlation among authors. Indeed, the uncanny similarities reflect a persistence of memory for Pythagoras's discovery, but details of the discovery and the use of formulaic wording vary among versions. The wording among versions is not a marker of similarity as much as the idea the words expressed. While an author emphasizes and alters aspects of the story to align his intentions for the story, the components of accounts about Pythagoras's discovery remain surprisingly consistent.

[10] Planck, Max. *Scientific Autobiography, and Other Papers*. New York: Philosophical Library, 1949, 33–34.
[11] Macrobius, Ambrosius Aurelius Theodosius. *Commentary on the Dream of Scipio*. Translated by William Harris Stahl. New York: Columbia University Press, 1952, 45.

Table 1.1. Components of the Story

Section	Component				
Motivation	**1a.**	Author's context	*often followed by*	**1b.**	Pythagoras's motivation
Discovery	**2.**	At the smithy			
Analysis	**3a.**	Examination	*sometimes followed by*	**3b.**	Rejection of fifth hammer
	4a.	Results	*sometimes followed by*	**4b.**	Homeward
Confirmation	**5.**	The apparatus			
	6.	Other instruments			
	7.	Summary			

As seen in Table 1.1, comparisons may be made among a representative sample of different versions. The story is structured broadly in four sections: A. Pythagoras's motivation, B. discovery at the smithy, C. analysis of the discovery, and D. confirmation. Each section is varied in some of its components by the differences in the ordering, addition, or subtraction of events. The beginning of the story usually starts a new chapter in a text. (1a) Often authors, especially those with fuller renditions, contextualize the story with a preface bridging the previous chapter's topic to the discovery. (1b) The story properly begins with Pythagoras's desire to quantify musical intervals. If this section is omitted, then authors begin the story directly with the discovery. (2) The core of the story is the section in which Pythagoras happens upon the smithy and hears the sound of striking hammers. It is the section most consistently found in instances of the story (One notable exception is Ptolemy's criticism of Nicomachus and the apparatus.).

Hearing sounds of hammer by chance, Pythagoras enters the workshop to investigate the hammers firsthand. (3a) Pythagoras examines the hammers individually, noting which pairs of weights produce which consonant interval.

(3b) In some versions, Pythagoras rejects a fifth hammer for being dissonant with the other four. (4a) After sufficient testing, he concludes that the proportions of the hammers' weight correlate to the consonances. (4b) He heads home afterward.

To be sure of the results, Pythagoras extends his testing of the intervallic relationships to other instruments and devices. (5) He builds an apparatus of weights equivalent to the hammers and string to reproduce what he heard. If Pythagoras already makes the connection before returning home, then the apparatus of weights and string serves as another device to verify. (6) With it, Pythagoras experiments on reed pipes, containers, string instruments, the monochord, and assorted instruments. (7) The essentials of the story, whatever they may be to a particular author, are once again restated to relate to the author's text.

Not all seven components are necessary for the story, and neither is the sequence of events. Some features must be present though. Foremost, (2) Pythagoras must be at the smithy for the story to exist. Without it, (1b) Pythagoras's

Table 1.2. Comparison of Authors

Author	Date	1a	1b	2	3a	3b	4a	4b	5	6	7
Nicomachus	2nd century CE		X	X	X		X	X	X	X	X
Ptolemy	2nd century CE				X						
Iamblichus[1]	3rd century CE		X	X	X		X		X	X	X
Macrobius[3]	5th century CE	X		X	X		X		X	X	X
Boethius[1]	6th century CE	X	X	X	X	X	X		X	X	X
Florentius[4]	1485–1492	X		X	X		X		X	X	X
Gaffurio[2]	1492	X	X	X	X	X	X			X	
Galilei, Vincenzo	1570–80s				X						
Mersenne, Marin	1634–1637				X						

[1] Primarily citing Nicomachus. [2] Primarily citing Boethius. Gaffurio's conclusion is different from his story preface. [3] The author frames his motivation (1a) as Pythagoras's. [4] Primarily citing Macrobius.

motivation is meaningless and unnecessary. Any reference to Pythagoras is, otherwise, cursory. The smithy is the source of the consonant sounds Pythagoras has been studying.

Table 1.2 provides a list of authors and the components each one uses. The table is a representative sample of the multitude of versions throughout history. The similarities in structure and, ultimately, the precise working, can determine distinctive threads of lineage. Nicomachus's is the first version, which directly influences Boethius's and Iamblichus's, then Boethius's influences Gaffurio's. In a related thread, Macrobius's version, borrowing from Boethius and Porphyry, influences Florentius's. The primordial source for Pythagoras's discovery rests with the philosopher and his followers though. The story has been embedded in Western history and lore so deeply that it is perceived as nearly timeless.

Details of the components aside, the story exists in two modes of time: explicative and episodic. The episodic mode describes Pythagoras at a particular setting. There are only two, the smithy (story component 2) and home (4b). There, he performs actions, such as inspecting hammers or constructing an apparatus. The explicative mode consists of all the remaining components. They are more variable than the components in episodic time, changing according to author. Pythagoras does not necessarily take a physical action, and instead, an author characterizes Pythagoras's state and relates reasons for this story before Pythagoras happens upon the smithy.

Narrative markers delineate the transitions between the timelessness of the explicative mode and the action-driven events of the episodic mode. After laying out Pythagoras's motivation, the typical expression "one day" signals the start of indefinite time and sets in motion the physical actions of Pythagoras in the story. He is deep in thought and pondering musical intervals. He is walking by a smithy hearing musical sounds. The next and last narrative marker is when Pythagoras heads home so as to confirm his analysis. In a few authors, Pythagoras never explicitly leaves the workshop to return home, which, consequently, submerges the story back into an explicative mode of time from the first section of the story.

The hurried shifts in the aspects of time may represent an earlier fusion of narrative structures. It is conceivable that the story consisted originally of the episodic sections: Pythagoras at the smithy and Pythagoras at home. Thus, the narrative markers may be relicts of an earlier form. The accretion of the explicative sections contextualizes Pythagoras's actions, which results in the form we have from Nicomachus's first transmission. While it is very probable that the story has existed since Pythagoras's time, the components of the story and any changes during the 400-year span from Pythagoras's to Nicomachus's are speculative.

In general, Pythagoras's analysis of his experiment is the most variable section. In some versions, the object of the analysis is the hammer. In others, it is the apparatus. In some, Pythagoras encounters four hammers, in others, five. In the case of five hammers, Pythagoras always rejects the fifth, being dissonant with the other hammers. Boethius was the first to add this hammer. For the four accepted hammers, the analysis is methodical. Pythagoras individually identifies the musical intervals for every pair of weight values: the octave, fifth, fourth, and second. Pythagoras correlates the consonances to the proportions of hammers' weights while still at the smithy. In some cases, the analysis shifts homeward, where he makes the connection between weight and pitch after further experimentation.

The Apparatus

The construction of the apparatus, as first described by Nicomachus, is ambiguous, and its source of sound is vague. To produce sound, either the weights hanging from string are struck, like the hammers, or the strings stretched by the hanging weights are plucked. Since the apparatus follows the hammers in the story, it is likely that Nicomachus intended for the struck weights to be the producers of sound, not the strings. While his text implies that the strings are struck, later authors, especially those citing him, ensure that the weights are the source of sound.

Nicomachus's account is an interesting exception to the order of events. In his account, Pythagoras, after having examined the hammers at the smithy, returns home to recreate the intervals with the apparatus. His analysis is based on the apparatus, rather than the hammers. Most authors reproducing the story after Nicomachus have Pythagoras examine the hammers and relegate the apparatus as confirmation. However, a consequence of this unusual ordering has been the frequent conflation of Pythagoras's examination of hammers and the apparatus. Worse is the fact that Nicomachus's is the first recorded account.

Primacy of his version in modern scholarship has clouded the scientific principles Pythagoras lays out. The foremost difficulty in interpreting and understanding the story has been determining the source of sounds in each section of the story. Pythagoras encounters sound in three instances: the sound of the hammers at the smithy, the apparatus Pythagoras builds, and any confirming experiments afterward. Modern commentaries tend to conflate the first two, since it would be possible for the apparatus to sound from its weights struck or from the strings stretched by the weights. The confusion is unsurprising, since later in the story Pythagoras does play on string instruments or sometimes the monochord to confirm his results. The apparatus acts like a bridge in the story, relating two different potentials for musical sounds, one by weights, the other by strings.

Indeed, there is a relationship between the mass of an object and the resonant pitch it produces. Large bells, for example, sound lower than smaller bells. The exact relationship among particular objects can be complicated though. A longer string vibrates at lower frequency than a short string. This correlation is formalized as its frequency being the square of its tension (which also may be stated in terms of length). Bells, though, being more of a three-dimensional object than idealized as a one-dimensional string, can have a multitude of vibrational modes. A bell's momentary deformation is non-uniform and asymmetric when it vibrates. It is quite difficult to model the frequency in a simple formula. Whatever kind or shape of weights or hammers used must be idealized as well to simplify an analysis of its resonant frequency.

Affirmation, Repudiation, Refutation

For the Pythagoreans, the story, indeed, finds a correlation between mass objects and its resonance, but correlation does not imply causation. Aside from the fact that such a relationship exists, its precise details between the mass and resonance is never addressed. It is a simple qualitative relationship that is difficult to quantify. Misunderstandings about text in translation or about the nature of physics disrupt legitimate consideration of the story. The case study about Nicomachus in Chapter 2 shows how abrupt his text confuses comprehension. In Chapter 3, the discussion about reception since Marin Mersenne's account explores the fallacies of physical explanations to disprove the story. In the end, each author's point of view may be classified as one of three possible conclusions: (1) affirmation, (2) repudiation, and (3) refutation.

(1) *affirmation*: This conclusion typically reproduce the story in the manner that Nicomachus wrote, using all three major sections. The explanation of the musical intervals is the proof. Use of the story usually confirms a related point an author is making around where the story is placed in his text.

(2) *repudiation*: Rather than prove or disprove the story, authors simply discard it because its serves as a foil to the point they are making. Otherwise, the story has no productive function for them.

(3) *refutation*: Authors attempt to disprove the story by attacking the physics as described. They argue against the default position of the possibility for the story's events. The story can never be proven but can only be disproven.

Historically, authors who affirm the story are in Antiquity or the Middle Ages, in a time when Pythagorean thought was still regarded as an authority. As the Renaissance and the early modern period develop, Pythagorean sensibilities and the contents of the story are questioned. The abnormality of the discovery at the smithy seems to contradict the continuity classical mechanics provides in characterizing natural events. Breaking with ancient and medieval tradition and attention to universality in description during the Scientific Revolution of the 16th and 17th centuries inhibited acceptance of the story as a physical possibility. Authors in the late modern period have attached themselves to the conclusions of the early modern period and have expanded them to include explanations that disprove the story with those same physical laws.

This overview of the story, its structure, and general reception provides the context in the following case studies: Nicomachus as the first to record the story and its subsequent influence, Ptolemy and Marin Mersenne as authors who have repudiated the story and the modern reception of refutation, and Boethius in laying his foundations of the quadrivium with the story as its linchpin. Each version provides a unique function for the authors and, in part, explains why the story has been such a common trope in western art and historiography.

Traduttore, traditore
(Translator, traitor)

—Italian expression

The first of three case studies involves the late Antiquity writer Nicomachus of Gerasa and his account of Pythagoras's discovery. His version of the story is the oldest extant account, but there might have been earlier exemplars. It establishes a baseline for interpretation not just for our discussion but also directly for writers after him, like Ptolemy and Boethius. He essentially affirms the story as part of his construction of music according to Pythagorean ideals. He wrote treatises, which were widely circulated in his time, not only on music theory, but also on mathematics and Pythagorean philosophy in the 2nd century CE. The fame Nicomachus experienced is, however, a stark contrast to the contemporary study of this philosopher. Of the nine works he likely authored, only two survive in complete form today: one treatise on arithmetic, the other on music titled *Enchiridion harmonicês* (*Manual of Harmonics*).[12] While the dearth of his extant output is one of the hindrances in studying Nicomachus, the lack of canonization of those works in collections of classical literature such as the Loeb Classical Library, Oxford Classical Texts, and the Bibliotheca Teubneriana in English and the Collection Budé in French

[12] The critical edition I use is Jan, Karl von. *Musici Scriptores Graeci: Aristoteles, Euclides, Nicomachus, Bacchius, Gaudentius, Alypius Et Melodiarum Veterum Quidquid Exstat*. Hildesheim: G. Olm, 1962. Nicomachus's account of the discovery is found in pp. 245–248. Some English translations include Thomas Stanley's in Hawkins (1776), Barker (1989), and Levin (1994).

has also exacerbated the lack of familiarity. Consequently, all these difficulties culminate with few scholars noting any contemporary impact of Nicomachus.

Structure of *Enchiridion*

Nicomachus's work has not been canonized, but his influence appears in other writers like Ptolemy and Iamblichus, whose works have been better studied. In particular, Nicomachus first recorded the account of Pythagoras at the smithy in *Enchiridion*.[13] It is but a small section of the treatise, which, in general, describes music in thoroughly Pythagorean terms. The pervasiveness of Pythagorean thought is most evident with the pre-Socratic philosopher featured as a protagonist in Nicomachus's explanations of music and mathematics.[14] Understanding Nicomachus's prose in *Enchiridion* is troublesome; the writing and arguments are highly compressed, fragmentary, with many phrases open to interpretation.

The structure of *Enchiridion* is a bit clearer though. The trajectory of its twelve chapters is linear and advances thematically. *Enchiridion* transitions from the dedication and reference to the later explanation of intervallic and harmonic relationships at the beginning to the end with the construction and division of the octave in the three Greek musical genera. The chapters are typically at most only a few pages of printed text. Because the explanations are so brief, it is sometimes frustrating to understand. Nicomachus assumes steps in his derivations for brevity

[13] Levin (1994) 95.
[14] Indeed, Pythagoras in *Manual* is not unlike the role of Socrates in many of Plato's dialogues.

at the expense of clarity. He apologizes for this in his introduction to the work, since he had to dictate the text while traveling.[15]

While it may seem unusual that a story set at a metal workshop should be included in a work devoted to the theoretical discussions of music and mathematics, the explanation of the story is as technical as other sections of the treatise. The story's function for Nicomachus is relevant to the treatise's progression. It comprises the entire sixth chapter, which incidentally marks the middle point of the twelve chapters of text.[16]

Chapter 4 describes, in ever Pythagorean terms, how numbers dictate musical pitches and their underlying characteristics. Chapter 5 attributes to Pythagoras the addition of the eighth string to the 7-string lyre to allow an octave-capable lyre, as well as discussing what intervallic relationships result from the addition. Most striking is the ratios that may be formed: the hemiolic (3 : 2), which happens to create the interval of a fifth; the epitritic (4 : 3) for the fourth; the epidoigic (9 : 8) for the second; and what brought about this discussion in the first

[15] Nicomachus. *Manual*, I.
[16] Further investigation might reveal whether more numerological or intervallic coincidences occur. For example, if each chapter represented a semitone in Pythagorean tuning (based on the 3:2 ratio Pythagorean perfect fifth), and if the first and last chapter represented the coinciding octave pitch, then the sixth and seventh chapters might represent the Pythagorean comma found in this tuning system. Stacking a five consecutive perfect fifths up from the first chapter to the sixth chapter would produce a pitch whose intervallic distance is the Pythagorean comma from a pitch produced by stacking five consecutive perfect fifths down from the twelfth chapter to the seventh. In other words, it is the gap between two pitches that would otherwise be enharmonically equivalent in 12-tone equal temperament, representing the tritone.

place, the duple (2 : 1) for the octave.[17] Consequently, Pythagoras added the octave-capable string after stumbling upon the aforementioned ratios at the blacksmith's!

Stylistically, the story appears as a momentary diversion in the middle of a substantial discussion of mathematics and music theory. This intermezzo is, instead, quite deceiving: It begins with a seemingly innocuous anecdote of Pythagoras visiting the smithy for what purpose is initially unclear. Nicomachus quickly delves back into the thick of things when the observant Pythagoras hears the fateful intervals and attempts to recreate them with a homemade apparatus and with different kinds of instruments. Pythagoras confirms the validity of these intervals through his observation of correlations, at the smithy and at home. In the last lines of the chapter, Pythagoras fills out the rest of a scale already established by the intervals of the second, fourth, fifth, and octave with diatonic insertions.

Nicomachus, then, explains in the seventh chapter how exactly the octave may be divided according to the diatonic genus, recalling that the diatonic additions in Pythagoras's experimentation. That is, of the three genera, the diatonic one may be defined as a tetrachord, or four pitches that span the interval of a fourth, in which the first two pitches are separated by a semitone, the next two by a tone, and the last pairing by a tone. Relating to the previously 7-stringed lyre, a heptachord, or seven pitches that span a seventh, consists of two tetrachords stacked consecutively. Both tetrachords share a common note. The octochord, which corresponds to the 8-string

[17] I transliterate the names of these intervals from Greek, instead of giving the English equivalent, since the pitches formed by these intervals also take the same Greek names.

lyre credited to Pythagoras, may be similarly constructed but with two disjunct tetrachords, that is, with a whole tone separating the two tetrachords, or it may be made with a tetrachord and a pentachord with a common note. Thus, being constructed from the diatonic genus, the different pitch constructions contain the intervals previously mentioned.

Chapters 4–7 illustrate how integrated the idea of intervallic relationships and their manifestations are, from their basis in pure numbers to their construction of the tetrachord and octochord. Generally throughout the book, Pythagoras frequently appears as a sort of protagonist or progenitor of these musical ideas. *Enchiridion* is almost as much a discussion of harmonics as much as it is a recollection of Pythagoras's transactions and accomplishments. It is interesting how the story has, in a sense, taken a life of its own and outlasted the fame of Nicomachus himself.

Nicomachus's story of the discovery is often repeated in later literature without reference to his treatise at large. It is as if authors pluck the story and insert in their own text without differentiating their source. There is no evidence that this story existed before Nicomachus, but with the familiarity of lore about Pythagoras, Nicomachus is simply the first to record this story. It might not matter in any case whether this account is historical, just as much as whether the historical Socrates actually gave his speeches in Plato's dialogues. The story exists more as a timeless moment, that nevertheless, illustrates a mathematical and physical basis for music. That Nicomachus was the first to report it may not matter. While casting Pythagoras as *Enchiridion*'s primary protagonist imbues an aura of authority to the text, it is not merely a token invocation. Pythagoras is the actor through which Nicomachus illustrates the technical content.

With the constant cavalcade of personal feats and scientific discoveries attributed to Pythagoras well before Nicomachus, it is difficult to establish that the account at the smithy ever happened. Pythagoras is so far removed in time from Nicomachus that little was known by Nicomachus about Pythagoras's historical figure. The distance of time and the weight of lore blur the lines between myth and reality. Nicomachus may be a self-declared Pythagorean, but the span of 600 years in between these two figures makes a direct connection more difficult to establish. Instead, the story may be seen as mythical parable, much like Galileo and the Tower of Pisa experiment or Newton and the apple. Fragments of truth and fantasy permeate and hybridize the story into a unique creation, thus, make the understanding difficult.

Structure of the Story

After a brief introduction that transitions the previous chapter about the formation of intervals into this one, the account may generally be divided into three sections that are defined by the ways in which Pythagoras approaches the musical intervals: (1) by hearing them incidentally at the blacksmith with hammers, (2) by experimenting with weights and strings to recreate what he heard, and (3) by applying the results of his investigation onto other instruments to recreate the intervals.

The narrative indicator, “one day” (ποτὲ), marks the first section; that is, in a generalized sense of time, Pythagoras is walking by a metal workshop. The second marker is when Pythagoras leaves the smithy to go home (πρὸς ἑαυτὸν ἀπηλλάγη). The third is after finishing his testing on the apparatus and his hands and mind are tired from the work (τυλώσας δὲ καὶ τὴν χεῖρα καὶ τὴν ἀκοὴν πρὸς τὰ ἐξαρτήματα καὶ βεβαιώσας πρὸς αὐτὰ τὸν τῶν σχέσεων λόγον, μετέθηκεν εὐμηχάνως. . .).

Thus, two difficulties challenge the understanding of Nicomachus’s account of Pythagoras at the smithy: (1) the diction: critical vocabulary and phrasing, pronouns and their antecedents, and the resulting ambiguity of description, all of which consequently impact (2) the physicality of the story itself for those reading Nicomachus and the later writers who take after him.

The problems with diction expose Nicomachus’s writing style in the larger work of *Enchiridion*. Some of this is likely the result of writing, or rather, dictating this book to his scribe while traveling.[18] Some of his claims come off as lacking detail or are unsubstantiated, in part because Nicomachus planned to reserve the proofs for a more comprehensive treatise. His *Eisagoge musicês* (*Introduction to Music*), which may have existed at some point, is no longer extant. Although the title of the work, *Enchiridion*, literally *Handbook* or *Manual*, reflects incidentally the work as a

[18] Levin (1994) 34–35.

reference text, the brevity of his writing should not betray the depth of implications and complexity of the musical and mathematical concepts.

The delineation of the story, therefore, is important to mark exactly the context in which Pythagoras interacts with the intervals. Levin quips that this chapter of *Enchiridion* may be the easiest to follow: the anecdote is a refreshing digression from the largely technical prose, but understanding it is a subject of relativity.[19] The writing even at this point is no less troublesome than before.

Understanding the Story

(i) *Introduction and the Smithy*

XI. Πῶς οἱ ἀριθμητικοὶ τῶν φθόγγων λόγοι ηὑρέθησαν.

Τὴν δὲ κατ' ἀριθμὸν ποσότητα ταύτην ἤτε διὰ τεσσάρων χορδῶν ἀπόστασις ἤτε διὰ πέντε καὶ ἡ κατ' ἀμφοτέρων σύνοδον διὰ πασῶν λεγομένη καὶ ὁ προσκείμενος μεταξὺ τῶν δύο τετραχόρδων τόνος τρόπῳ τινὶ τοιούτῳ ὑπὸ τοῦ Πυθαγόρου καταληφθέντι ἔχειν ἐβεβαιοῦτο.

XI. How the mathematical proportions of pitches were discovered.

The interval of notes consisting of that which is a fourth, that which is a fifth, and a union of both (the fourth and fifth), which is an octave, as well as a tone situated between two tetrachords, was established to have such a numerical quantity by Pythagoras with a certain method that he discovered.

Even before Pythagoras is at the smithy, Nicomachus provides context and motive, recalling the content of the previous chapter to blend with that of this chapter. Musical intervals are essentially made of numbers, and it was Pythagoras who derived their quantities. This statement is by nature Pythagorean; any physical

[19] Levin (1994) 86.

phenomenon, such as music, may be construed by quantitative means. Note that Nicomachus uses a figure of speech for what has been translated as interval. Literally, the phrase *chordê apostasis* (χορδῶν ἀπόστασις) would be "distance of strings" or "distance of pitches." The difference is based on the word *chordê*. Levin and Barker translate it as "interval" too; indeed, it is an elegant paraphrase of two possible literal translations.

The ambiguity results from the last chapter, which describes the strings of the lyre and the resulting distance of their pitches, or intervals. If Nicomachus intends for *chordê* to means pitch in this phrase, then it is possible that it is pitch elsewhere. His cleverness in prose, though, confuses the understanding, and the double-entendre propagates in the chapter. While Levin and Barker are consistent to render *chordê* as "interval" throughout the chapter, we will see this issue of translation arise when Pythagoras returns home from the blacksmith's. After this introduction, Pythagoras enters the story:

ἐν φροντίδι ποτὲ καὶ διαλογισμῷ συντεταμένῳ ὑπάρχων, εἰ ἄρα δύναιτο τῇ ἀκοῇ βοήθειάν τινα ὀργανικὴν ἐπινοῆσαι παγίαν καὶ ἀπαρα λόγιστον, οἵαν ἡ μὲν ὄψις διὰ τοῦ διαβήτου καὶ διὰ τοῦ κανόνος ἢ καὶ διὰ τῆς διόπτρας ἔχει, ἡ δ' ἀφὴ διὰ τοῦ ζυγοῦ ἢ διὰ τῆς τῶν μέτρων ἐπινοίας, παρά τι χ α λ κ ο τ υ π ε ῖ ο ν περιπατῶν ἔκ τινος δαιμονίου συν τυχίας ἐπήκουσε ῥαιστήρων σίδηρον ἐπ' ἄκμονι ῥαιόντων καὶ τοὺς ἤχους παραμὶξ πρὸς ἀλλήλους συμφωνοτάτους ἀποδιδόντων πλὴν μιᾶς συζυγίας·

One day being in thought and intense debate whether it would be possible to conceive of any instrumental tool for the ear, something reliable and accurate, much like vision has through the compass and the ruler or the protractor, and touch through the beam balance or a system of measurements, passing by the blacksmith by some divine chance, [Pythagoras] overheard hammers striking iron upon an anvil and rendering sounds mixed in a combination of most harmonious sounds except one pairing:

ἐπεγίνωσκε δ’ ἐν αὐτοῖς τὴν δὲ διὰ πασῶν καὶ τὴν διὰ πέντε καὶ τὴν διὰ τεσσάρων συνῳδίαν. τὴν δὲ μεταξύτητα τῆς τε διὰ τεσσάρων καὶ τῆς διὰ πέντε ἀσύμφωνον μὲν ἐώρα αὐτὴν καθ’ ἑαυτὴν, συμπληρωτικὴν δὲ ἄλλως τῆς ἐν αὐτοῖς μείζονος. ἄσμενος δὴ ὡς κατὰ θεὸν ἀνυομένης αὐτῷ τῆς προθέσεως εἰσέδραμεν εἰς τὸ χαλκεῖον καὶ ποικίλαις πείραις παρὰ τὸν ἐν τοῖς ῥαιστῆρσιν ὄγκον εὐρὼν τὴν διαφορὰν τοῦ ἤχου, ἀλλ’ οὐ παρὰ τὴν τῶν ῥαιόντων βίαν οὐδὲ παρὰ τὰ σχήματα τῶν σφυρῶν οὐδὲ παρὰ τὴν τοῦ ἐλαυνομένου σιδήρου μετάθεσιν, σηκώματα ἀκριβῶς ἐκλαβὼν καὶ ῥοπὰς ἰσαιτάτας τῶν ῥαιστήρων πρὸς ἑαυτὸν ἀπηλλάγη.

He recognized in these sounds the consonance of the octave, the fifth, and the fourth. He observed that the interval between both the fourth and fifth was dissonant in itself, but that it was complementary to the greater (interval) of these intervals. Happy, therefore, as if his purpose was accomplished god willing, he ran into the smithy, and having found by various experiments that the different of the sound was according to the weight in/of the hammers, but not according to the force of the blows nor the dimensions of the hammers nor according to the iron being forged, having accurately measured weights and masses weighed out identical to the hammers, he left for home.

The story itself begins with this section of text. From the onset, Nicomachus specifies when this story takes place. The enclitic particle ποτὲ meaning “one day” indicates that the story is shifting the temporal aspect to a timeless mode, much like with fairy tales or the Gospel’s parables beginning (“At that time. . .”). The story is also set in the past, given by the first sentence’s aorist tense of the main verb “he overheard” (ἐπήκουσε). The story exists in a generalized timeless world of the past, one in which Pythagoras the actor lives though not necessarily in a historical way.[20]

It is clear that Nicomachus through his character Pythagoras is seeking a way to measure the phenomenon and sensation of sound much in the same way it is possible to quantify the distance of things seen or the weight of things touched. It

[20] Discussion of one day (ποτὲ) as a device in legendry is found in Chapter 5.

happens that hearing hammers striking the anvil at a metal workshop is the key to finding this standardized tool. It is the weight of the hammers itself that creates the sounds Pythagoras heard. The ways in which the hammers strike the anvil or how differently they might be shaped are not factors.[21] In this first section of the story, hammers create sounds, but to reproduce what he heard at home, Pythagoras measures weights equal to those of the hammers. The specific, interval-creating strings Nicomachus explains about in earlier chapters of *Enchiridion* are never mentioned in this section of the story. Rather, the story's first section is about discovery based on Pythagoras's serendipitous amble by the smithy.

(ii) *At Home to Build an Apparatus: Experimentation and Analysis*

However, when Pythagoras returns home, which signals the second section of the story, strings in general reenter as part of the apparatus Pythagoras builds to recreate the sounds of the blacksmith's workshop:

καὶ ἀπό τινος ἐνὸς πασσάλου διὰ γώνων ἐμπεπηγότος τοῖς τοίχοις, ἵνα μὴ κἀκ τούτου διαφορά τις ὑποφαίνηται ἢ ὅλως ὑπονοῆται πασσάλων ἰδιαζόντων παραλλαγή, ἀπαρτήσας τέσσαρας χ ο ρ δ ὰ ς ὀμοΰλους καὶ ἰσοκώλους, ἰσοπαχεῖς τε καὶ ἰσοστρόφους ἐκάστην ἐφ' ἐκάστης ἐξήρτησεν,

And having planted in the walls of his house from any one rod across the ceiling, in order that any difference might not be observed from this or, in short, that a difference in the individual stake not be suspected, having hung up four strings of the same material, of equal number of strands, of equal thickness, and of equal twist, he attached each string after another,

[21] By dimensions of the hammers, I presume that there is generally the same or similar design for all the hammers used that would not influence the sounds generated upon striking the anvil.

ὁλκὴν προσδήσας ἐκ τοῦ κάτωθεν μέρους. τὰ δὲ μήκη τῶν χορδῶν μηχανησάμενος ἐκ παντὸς ἰσαίτατα, εἶτα κρούων ἀνὰ δύο ἅμα χορδὰς ἐναλλὰξ συμφωνίας εὕρισκε τὰς προλεχθείσας, ἄλλην ἐν ἄλλῃ συζυγίᾳ.	having suspended a weight from the bottom part of each string. Having prepared the lengths of the strings nothing but exactly equal, next alternately striking two ***strings*** at the same time upwards, he discovered consonances mentioned before, a different consonance for each other pairing (emphasis added).

The apparatus consists only of a rod upon which the four strings are attached, from which hang the weights, one string per weight. Nicomachus is careful to detail the attributes of the string; they're equal in all aspects. He does this to emphasize the relationship between weight and the ratios that form the musical intervals. How Nicomachus describes the process for Pythagoras discovering the intervals is not apparent. In particular, the author does not reveal what part of the apparatus even produces the sounds of the hammered intervals.

The crucial phrase that best describes the production of sound from the apparatus is "εἶτα κρούων ἀνὰ δύο ἅμα χορδὰς ἐναλλὰξ." Stanley and Levin both describe the process as Pythagoras striking two strings at the same time.[22] Barker renders it similarly, that he is plucking both strings, one at a time.[23] With them, there is no disagreement that the string is what is acted on to effect sound because of the word *chordas* (χορδὰς). In that construction, "strings," is the object of the verb form *krouôn* (κρούων). However, the difference between the two translations is the

[22] Hawkins 9, Levin (1994) 84.
[23] Barker 257.

verb form *krouôn*, which has as its definitions either striking (e.g. an object, like a weight) or plucking (e.g. a string).

The phrase by itself might not suggest any issue of whether it is string or weight making the sound. The surrounding text, however, might suggest otherwise. Nicomachus explains that the strings are completely equal in all characteristics. The stake used to tie the strings, only a sentence earlier, is described in terms that suggest no influence or function on the production of sound, “that no difference might be observed” (ἵνα μὴ κάκ τούτου διαφορά τις ὑποφαίνηται). While the dimensions of the apparatus are given in general terms, the qualitative description is certain. Nicomachus goes through such an effort to describe the equality of the strings and the neutrality of the stake that he nullifies any potential role the stake and strings might have to produce sound.

It would also be unusual to strike or pluck the strings in the motion described: "from the bottom up" or "in an upward direction" (ἀνὰ δύο ἅμα χορδὰς). Barker and Levin are silent for the upward directional adverb ἀνὰ. Recalling the placement of the apparatus might be useful. The stake is fixed on the wall from the ceiling, so the apparatus is likely hanging to some extent over Pythagoras. To make sound, striking the apparatus would mean hitting the hanging weight first before reaching the string. This seems more intuitive than moving past the weight to pluck the string in an upward motion. Nicomachus might be poetic here when he uses "the strings," referring to them by synecdoche: not that each the string itself is struck but that each string-weight complex as part of the apparatus is struck, with the weight actually being hit.

This confusion is also exacerbated by the noun *chordê*, which already was ambiguous at the introduction of the sixth chapter, translating to either pitch or string. The etymology of the figurative meaning, strings, comes from Greek music theory, which uses the names of the strings on a lyre to refer to pitch. A similar situation arises in Latin, which in large part owes its vocabulary to the Greek writings. The word *tonus* may mean "that which is stretched," "a pitch," or "an interval." The word *fides* can mean "a stringed instrument" in the plural, or "a [single] string" or "pitch" in the singular. We will continue to see in the story the double entendre sense that *chordê* has.

What Nicomachus intends as the source of sound in the apparatus is unclear. The text reads like string is plucked to produce sound. The apparatus melds weights that correlate with the hammers and string in the apparatus. This bridge combines two methods of experimentation, one from the smithy and one with strings. Nicomachus expands importance of the apparatus into a nearly discrete section of its own. Unlike the later accounts (except for Iamblichus's heavy borrowing), Nicomachus is the only one to emphasize the role of the apparatus so intensely. Authors after him acknowledge, instead, the supremacy of the hammers rather than the apparatus in the equivalent section of analysis in the story.

After Pythagoras constructs the apparatus, the subsections A-D describe how Pythagoras analyzes each of musical intervals by the pair of weights that create them:[24]

A. τὴν μὲν γὰρ ὑπὸ τοῦ μεγίστου ἐξαρτήματος τεινομένην πρὸς τὴν ὑπὸ τοῦ μικροτάτου διὰ πασῶν φθεγγομένην κατελάμβανεν. ἦν δὲ ἡ μὲν δώδεκά τινων ὁλκῶν, ἡ δὲ ἕξ. ἐν διπλασίῳ δὴ λόγῳ ἀπέφαινε τὴν διὰ πασῶν, ὅπερ καὶ αὐτὰ τὰ βάρη ὑπέφαινε.

B. τὴν δ' αὖ μεγίστην πρὸς τὴν παρὰ τὴν μικροτάτην ⌊οὖσαν ὀκτὼ ὁλκῶν⌋ διὰ πέντε συμφωνοῦσαν, ἔνθεν ταύτην ἀπέφαινεν ἐν ἡμιολίῳ λόγῳ, ἐν ᾧπερ καὶ αἱ ὁλκαὶ ὑπῆρχον πρὸς ἀλλήλας·

C. πρὸς δὲ τὴν μεθ' ἑαυτὴν μὲν τῷ βάρει, τῶν δὲ λοιπῶν μείζονα, ἐννέα σταθμῶν ὑπάρχουσαν, τὴν διὰ τεσσάρων, ἀναλόγως τοῖς βρίθεσι.

A. He understood that which is stretched by the greatest weight with that which made the sound through the least [created] the interval of the octave. The former weight was of some twelve units, the latter of six. Consequently, the apparatus produced the sound of the octave in a double ratio (of weight), as the weights themselves were revealing.

B. The apparatus sounded at the interval of a fifth from, again, the greatest weight with the one next to the smallest (being eight units of weight), from which confirmed this interval of a fifth in a hemiolic ratio (3:2), the ratio in which the weights were also next to one another.

C. The one next to it in weight [the greatest, 12, but] being greater than the rest, namely, nine units of weight, [made the sound of] the fourth, being proportional in weight.[1]

[24] The subsection headings A–D do not originally appear in Jan, but I have included them here to delineate what part of the text describes which of the intervals.

D. καὶ ταύτην δὴ ἐπίτριτον ἄντικρυς κατελαμβάνετο, ἡμιολίαν τὴν αὐτὴν φύσει ὑπάρχουσαν τῆς μικροτάτης, (τὰ γὰρ ἐννέα πρὸς τὰ ἓξ οὕτως ἔχει,) ὅνπερ τρόπον ἡ παρὰ τὴν μικρὰν ἡ ὀκτὼ πρὸς μὲν τὴν τὰ ἓξ ἔχουσαν ἐν ἐπιτρίτῳ ἦν, πρὸς δὲ τὴν τὰ δώδεκα ἐν ἡμιολίῳ.

τὸ ἄρα μεταξὺ τῆς διὰ πέντε καὶ τῆς διὰ τεσσάρων τουτέστιν ᾧ ὑπερέχει ἡ διὰ πέντε τῆς διὰ τεσσάρων, ἐβεβαιοῦτο ἐν ἐπογδόῳ λόγῳ ὑπάρχειν, ἐν ᾧπερ τὰ ἐννέα πρὸς τὰ ὀκτώ. ἑκατέρως τε ἡ διὰ πασῶν σύστημα ἠλέγχετο τῆς διὰ πέντε καὶ διὰ τεσσάρων ἐν συναφῇ, ὡς ὁ διπλάσιος λόγος ἤτοι ἡμιο λίου τε καὶ ἐπιτρίτου, οἷον δώδεκα ὀκτὼ ἕξ, ἢ ἀναστρόφως τῆς διὰ τεσσάρων καὶ διὰ πέντε, ὡς τὸ διπλάσιον ἐπιτρίτου τε καὶ ἡμιολίου, οἷον δώδεκα ἐννέα ἓξ ἐν τάξει τοιαύτῃ.

D. And it was understood that this 9 unit weight clearly made the epitritic ratio (4:3) [with 12 units of weight mentioned earlier], that it also formed the hemiolic ratio by nature with the smallest weight [the 6 unit weight] (for the nine unit weight with the six unit weight thus has [this ratio]), in the same way that the one next to the small one, namely the 8 units was in an epitritic ratio with the one carrying 6 units, but in a hemiolic ratio with one carrying 12.

He established that that between the fifth and fourth, that is, in which the fifth surpasses the fourth, is in epogdoic ratio (9:8), in which it is 9 units with 8 units. Either way, the octave proved a system of the fifth and the fourth in conjunction, since the double ratio surely is [a system] of the hemiolic and epitritic ratios, using, for example, the numbers 12, 8, and 6. Or, conversely, a system of the fourth and the fifth, since the double ratio is made of the epitrite and the hemiolic ratios, using, for example, the numbers 12, 9, and 6 in such an order.

In this section, Nicomachus explains how the musical intervals are created.

Assignments to the weights are given immediately in the analysis, rather than at the smithy. For the octave, there is a 2 : 1 ratio of the units of weight used. In Pythagoras's apparatus, it is 12 units of weight to 6. For the interval of a fifth, a 3 : 2 ratio is used with the weights of 9 and 6. For the fourth, it is a 4 : 3 ratio, using weights of 8 and 6. And for the second, also known as the difference between the fourth and fifth, the ratio consists of 9 : 8 with weights of 9 and 8. It is interesting to note that the values are only whole numbers, though the ratios themselves could be mathematically reduced. It may be easier to keep these whole numbers consistent to trace how the units of weight correspond to the values that make up the ratios. Otherwise, it would add another layer of complexity to take the ratio of the weights and reduce them when concluding what exactly makes a particular interval. For example, 9 : 8 would be 1.125 : 1. This confusing story would only become more confusing with the ratio itself expressed with a nested decimal fraction in the ratio's antecedent (i.e. 1.125).[25] The goal is to express the musical intervals as rational numbers, fractions composed of integers, thus the choice for whole numbers.

The fact that specific units for weight or mass are not provided, like the pound and the kilogram nowadays or the drachma and the libra in Antiquity, indicates Nicomachus's more conceptual or universal approach to explaining the musical intervals. In the same way when making a homemade cleaning solution of vinegar and water, the ratio of the two substances is usually given in parts, rather than specific units to allow scalability of the cleaning solution. Thus, we could expect the same from this story: the weights given are not explicit in the units since the ratios themselves should hold true to produce the intervals.

[25] We will see later in Chapter 4: "Affirmation Revisited: Boethius and the Quadrivium" how Boethius is initially silent on actual values for the weights themselves and how he articulates the ratios, such that he constructs something of a logic puzzle when describing the ratios without providing example values for the weights. Later, though, Boethius provides an example set anyway, using the same numbers that Nicomachus cites.

Though this section of the story describes Pythagoras forms the intervals, the manner in which he does this is obscure. What is perhaps most troubling is the first sentence in subsection A of the text, which explains how the apparatus creates the interval of an octave: "τὴν μὲν γὰρ ὑπὸ τοῦ μεγίστου ἐξαρτήματος τεινομένην πρὸς τὴν ὑπὸ τοῦ μικροτάτου διὰ πασῶν φθεγγομένην κατελάμβανεν." Here, Nicomachus writes only the singular feminine article τὴν (the first of the two in this sentence) without a clear corresponding noun as the accusative subject of an indirect statement with the verb "He understood (that)" (κατελάμβανεν). What is it that Pythagoras understood? Barker and Levin logically assume both instances of τὴν with χορδήν, which was featured in the previous sentence, so "Pythagoras understood that the string stretched by the greatest weight with the least sounded the interval of an octave." Stanley is circumspect, translating it as "that which was stretched. . ." In both sets of translations, τὴν connects with the participle τεινομένην, which here means "stretched," and the next participle, φθεγγομένην, "sounded" is rendered as a verb.

It is unusual to consider τὴν τεινομένην as a verbal adjective, while changing the part of speech of φθεγγομένην, itself a participle, to a verb. If *chordên* is assumed for both instances of the article τὴν, it would properly read as "the string which is stretched by the greatest weight. . .the string sounding through the least weight." In this case, Nicomachus is simultaneously indicating that string is stretched by the greatest weight and is making sound through the agency of the least weight. This does not necessarily make the rendering any clearer. Pythagoras, moments before, had heard sounds of hammers striking an anvil, but now produces sounds from strings rather than weights.

Recalling the double meaning of *chordê*, it could also be "the string stretched by the greatest weight. . .the pitch sounded through the least weight." By chance, it is also possible that τεινομένην here might mean "referring to, belonging to," so "Pythagoras understood that that which refers to the greatest weight with that which makes a sound by the least weight [creates] the interval of the octave." Again, Nicomachus's prose is ambiguous. The issue is what is making the sound. If the generalized string-weight complex is interpreted, confusion may somewhat be avoided, but there is not a clean way to read it.

Barker, Levin, and Stanley maintain the primacy of the string over weight throughout the rest of the chapter, but the blurring of how string and weight might generate sound, in fact, continues through Nicomachus's description of what all intervals are created by any combination of two weights. If string carrying the weight produces sound as described in subsection A, weight is now directly addressed as the source of sound in subsection C: "The one next to it in weight [the greatest, 12, but] being greater than the rest, namely, nine units of weight, [made the sound of] the fourth, being proportional in weight" (πρὸς δὲ τὴν μεθ' ἑαυτὴν μὲν τῷ βάρει, τῶν δὲ λοιπῶν μείζονα, ἐννέα σταθμῶν ὑπάρχουσαν, τὴν διὰ τεσσάρων, ἀναλόγως τοῖς βρίθεσι). This could be a case of a metonym in which weight is actually the string. It is ineffectual because of the ambiguity. The issue with τὴν arises again, but in this case, it must be translated with a noun meaning weight since all the relationships described here are in the context of the proportionality of weights.

There might be incongruence in translation for Barker, Levy, and Stanley, if Nicomachus first focuses on the strings in the apparatus's construction but then on the weights in the analysis of the intervals. In any case, he cannot abandon the notion of string as producer of sound, if he never intended to make this point in the first place. Nicomachus, in fact, summarizes which units of weight make what intervals and how a set of intervals may add together to form a larger interval. That is, he connects the value of the weights to ratios, then ratio to musical intervals. While string is mentioned in the formation of intervals, one might interpret this as part of the procedure, but not as a factor in the analysis. That is, strings may contribute to the construction of intervals, but they themselves are likely not the source of sound. From this, their importance cannot necessarily be implied.

Nicomachus's effort to deemphasize the role of string in a section devoted to experimentation and analysis discourages transparency of understanding. From grammatical and physical standpoints, interpreting that strings are the source of the sounds from the apparatus would just heighten the awkwardness of the story. As written, it must be the strings that produce sound in Nicomachus's version of the apparatus, even though Pythagoras heard sounds from mass objects just earlier at the smithy. Nicomachus blurs the source of sound in the apparatus to accentuate the connection between pure number and music. Regardless of how an object resonates, proportionality to its mass is maintained; Nicomachus achieves what he designs.

(iii) *Applying the Results Elsewhere*

When Pythagoras finishes his experimentation on the apparatus, he applies what he learns from it to different instruments:

τυλώσας δὲ καὶ τὴν χεῖρα καὶ τὴν ἀκοὴν πρὸς τὰ ἐξαρτήματα καὶ βεβαιώσας πρὸς αὐτὰ τὸν τῶν σχέσεων λόγον, μετέθηκεν εὐμηχάνως τὴν μὲν τῶν χορδῶν κοινὴν ἀπόδεσιν τὴν ἐκ τοῦ διαγωνίου πασσάλου εἰς τὸν τοῦ ὀργάνου βατῆρα, ὃν χορδότονον ὠνόμαζε, τὴν δὲ ποσὴν ἐπίτασιν ἀναλόγως τοῖς βάρεσιν εἰς τὴν τῶν κολλάβων ἄνωθεν σύμμετρον περιστροφήν.	Having worn out his hand and hearing on the weights, and having confirmed the ratio of their relationships relative to those weights, he skillfully transferred the common tying-points of the strings from the diagonal rod onto the bridge of his instrument, which he called a chordotonos, and he transferred proportionately a certain amount of tension as the weights had on the strings onto the twisting of the tuning pegs from the top.
ἐπιβάθρᾳ τε ταύτῃ χρώμενος καὶ οἷον ἀνεξαπατήτῳ γνώμονι εἰς ποικίλα ὄργανα τὴν πεῖραν λοιπὸν ἐξέτεινε, λεκίδων τε κροῦσιν καὶ αὐλοὺς καὶ σύριγγας καὶ μονόχορδα καὶ τρίγωνα καὶ τὰ παραπλήσια, καὶ σύμφωνον εὕρισκεν ἐν ἅπασι καὶ ἀπαράλλακτον τὴν δι' ἀριθμοῦ κατάληψιν.	Using this as both a basis and as a sort of infallible benchmark, he went on to extend the experiment onto assorted instruments, [including] the striking pots, auloi, syringes, monochords, trigona, and those similar to them. He discovered, in all, this understanding to be consistent and indistinguishable through number.
ὀνομάσας δὲ ὑπ ά τ η ν μὲν τὸν τοῦ ἓξ ἀριθμοῦ κοινωνοῦντα φθόγγον, μ έ σ η ν δὲ τὸν τοῦ ὀκτὼ, ἐπίτριτον αὐτοῦ τυγχάνοντα, π α ρ α μ έ σ η ν δὲ τὸν τοῦ ἐννέα, τόνῳ τοῦ μέσου ὀξύτερον καὶ δὴ καὶ ἐπόγδοον, ν ή τ η ν δὲ τὸν τοῦ δώδεκα, καὶ τὴς μεταξύτητας κατὰ τὸ διατονικὸν γένος συναναπληρώσας φθόγγοις ἀναλόγοις	Having called the hypate a pitch associated with the number 6, the mese with the number 8, which itself happens to the epitritic ratio with the hypate; the paramese with the number 9, which is higher than the mese by a tone, and is in epidogoic ratio with the mese; the nete with the number 12, and having filled up the gaps according to the diatonic genus with pitches of analogous ratios,

οὕτως τὴν ὀκτάχορδον ἀριθμοῖς συμφώνοις ὑπέταξε, διπλασίῳ ἡμιολίῳ ἐπιτρίτῳ καὶ τῇ τούτων διαφορᾷ ἐπογδόῳ.	he thus made the octochord as a subset to consonant numbers, that is, the double ratio, the hemiolic ratio, epitritic ratio, and, the difference between the last two, the epidoigic ratio.

The third and last section of the story is marked by when Pythagoras tires himself of testing the apparatus. It also marks the difference in what produces sound at this point in the story. In the first and second sections, hammers and weights were responsible for sound, but now the strings, namely their tension from their attachment to the stake and stretched by the weights, play the role. This is the significant shift in the story. Pythagoras transfers the tension of each of the four strings onto a string instrument, the chordotonos. It is distinct that the chordotonos operates from string length and tension, not stretching of weights.[26] The manner in which he does this is a little more detailed than in the constructing and operation of the apparatus earlier. The key term during this process is ἀναλόγως. Levin translates literal with "analogous," Stanley with "proportionably," but Barker with "the same." It is clear that some concept of proportionality is at hand.

We might interpret some analogous relationship to correlate from the ratios obtained from the apparatus to the ratio of string tension applied to the other instruments. Certainly this is not a one-to-one relationship, given by the difference in producing the octave. That is, a tension ratio of 2 : 1 does not produce an octave,

[26] The chordotonos is another name for a monochord.

since, in fact, the ratio needs to be 4 : 1.[27] However, this may not even be a linear relationship from ratios of weight to ratios of string tension, but some other defined proportional relationship.[28] The relationship may, instead, consist of exponential numbers with the same base. That is, string tension is proportional to the interval by a power; $(3/2)^n$ for a fifth and $(2/1)^n$ for an octave, in which n is a positive integer.

Many, though, have interpreted this as equating the discussed ratios and string tension to pitch. This does not automatically prove Nicomachus's account as false. Rather, it has been the conflation of the second and third sections of the story. There are two separate "experiments," one with the apparatus just to recreate the intervals from the smithy, and the second with various instruments to confirm. The former uses weights and strings to generate sound, while the latter only with tension and the plucking of strings. There is, instead, a gulf of understanding between the last two sections of the story. Nicomachus does not describe in precise enough detail how Pythagoras applies his analysis of sound from the second section of the story to the third section in which the findings are applied to other instruments. It almost seems like Pythagoras is able to perform the events of the story in an arbitrary manner just because Nicomachus writes it so. Nicomachus might wish his audience to be satisfied by this alone, but any critical review quickly reveals the awkward and, at times, contradictory description and text of the story.

---

[27] The general relationship between pitch and tension is such that pitch is a square root of tension. This finding is credited to Mersenne. Chapter 3: "The Rejection of Ptolemy and Mersenne" discusses, in part, Mersenne's discovery of this relationship to the story about Pythagoras.
[28] That is, the relationship could be governed by a mathematical function.

The Aftermath

The difficulty of Nicomachus's story is evident by the number of possibilities in translating his text. It would be easy to understand, then, why many scholars have considered this account to be false. The conceptual complexity is astonishing for such a brief story. The vibrations that generate sound come from two sources, the striking of weights and hammers and the plucking of strings. Pythagoras builds an apparatus that is frankly hard (but not necessarily impossible) to imagine. The generated sounds correspond to some pitch, of which a pair forms a musical interval. The hammers and weights first create these intervals, and Nicomachus, by nebulous analogy, connects string tension as capable of forming the same intervals. These observations somehow also apply to different types of instruments. The level of detail in the prose required for precise intelligibility is simply not there, and an amount of assumptions must be made to complete these logical lacunae.

Unless it is verbatim, it is uncommon to find a version of the story that maintains the story as a whole. The distinct sections of the story that progresses from one to the next, instead, separate into individual stories: Pythagoras discovers the intervals by striking an anvil with hammers, by hitting weights (or bells in some versions), or by plucking strings. The introduction that motivates Pythagoras to experiment for these intervals becomes lost. All these things do not necessarily encourage a positive interpretation, since sections that exist alone make Pythagoras seem more like a mythical character, with him begging the question. What was a confusing unified whole now seems like disjointed vignettes of a tale.

The difficulty with understanding the story about Pythagoras at the smithy is largely from the difficulty Nicomachus poses in his prose. The nuanced, confusing, and even contradictory elements of the story do not hinder the lack of clarity, but what is certain is the structure of the story. Nicomachus designates the sixth chapter of *Enchiridion* as an anecdotal but technical account to describe the musical intervals discussed in the previous and following chapters. Within the sixth chapter, Nicomachus provides an introduction for the purpose before beginning the story. The story itself has three sections, based on how the protagonist Pythagoras interacts with the musical intervals: (1) discovery at the smithy with striking hammers, (2) experimentation and analysis at home with an apparatus of string and weight, (3) transferal of the results of the apparatus onto string-based and other instruments.

It is the conflation of the sections of the narrative that has altered the trajectory of interpretation, in particular with the second and third sections. Nicomachus is not clear in the second section about how exactly Pythagoras constructs and uses his apparatus of string and weights. Scholarship has struggled to interpret this peculiar section, usually by combining it with the third section, which is based on strings. Since the second and third sections have string in common, commentaries of the story, like those of Barker and Levin, generalize their interpretation to span the whole sixth chapter. There is a lack of distinction between the sections and what, indeed, generates sound at certain points of the story. It may be unsurprising why so many who come across this story judge it impossible. The story itself is confusing and the interpretation of the story becomes confusing as well.

As a Pythagorean, it is reasonable to assume that Nicomachus believes in the possibility of the story. *Enchiridion* is a Pythagorean text and its inclusion implicitly affirms the musical discovery. Nicomachus renders the story in as great of detail as being the first recorded version. It is not quite an anecdote or a mere attribution. His theoretical explanation works out in reality as described; there is a linear relationship with a given string length and its pitch, all other factors being equal. The apparatus connects this relationship to the hammers, albeit in a clumsy way. The story is useful for Nicomachus; thus, he affirms the story.

A criticism of Nicomachus's text is also one of the first recorded instances of an interpretation, from Ptolemy. It may explain why in history there is this typical disconnect between how sound is generated in the story about Pythagoras and whether the story could be possible in reality. Ptolemy's reading of the story may be one of the most consequential; he is likely one of the first to comment on the story, which shapes the way in which others after him tend to view it. His lasting reputation from astronomy and geography, unlike that of Nicomachus, no doubt engenders dismissing the story. Marin Mersenne, a French polymath 1500 years later, who picks up Ptolemy's work, reinforces the prevailing notion of impossibility. Nicomachus represents the period in Antiquity in which the story is very favorable as true in some aspects; Ptolemy exemplifies a rare contradiction in ancient thought and Mersenne the mark of a new era.

Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actiones in se mutuo semper esse aequales et in partes contrarias dirigi
(To every action there is always an opposite and equal reaction: or the actions of two bodies upon each other are always equal and directed in contrary parts.)

—Isaac Newton's Third Law of Motion[29]

Most interpretations of the story at the smithy overwhelmingly give attention to strings as the source of sound for the intervals, like the reception of Nicomachus's version in his *Enchiridion harmonicês* (*Manual of Harmonics*) in the previous chapter. This emphasis seems strange since reference to the strings as sources of sound first appears in the second section of the story, when Pythagoras builds his apparatus of string and weights, the latter to replace the hammers he heard at the smithy. It is only in the third section that strings take a prominent role, when Pythagoras transfers his findings from the apparatus onto string instruments. Focusing on string diminishes the role of weights and hammers in the narrative. It may explain, in part, why strings plays a preponderant role instead of weight and how using string influences one's evaluation of Pythagoras's discovery toward refusing its validity.

In particular, two historical figures act as bookends for this interpretation based on string: Claudius Ptolemy, the famous 2nd century astronomer and scientist, and the 17th century French polymath and music theorist, Marin Mersenne. What

[29] Newton, Isaac. *Philosophiae naturalis principia mathematica*. 1687, 13.

ties them together is their investigations into the relationship between string tension and the pitch generated from a plucked string. Ptolemy, as one of the first to interpret Nicomachus's account, observed this connection in his work *Harmonics*.[30] However, beyond a mere description, he did not conclude a quantitative relationship. Only 1500 years later did Mersenne in his *Questions harmoniques* complete Ptolemy's work with a formula relating string tension and pitch.[31] Consequently, their attention to string tension has set the tone for which the story has been interpreted, while notions of weight have withered as secondary to string.

Ptolemy: Testing Nicomachus

Little is known about Ptolemy's life except that, based on the references found in his writings, he was likely a younger contemporary of Nicomachus. None of Ptolemy's work is found in Nicomachus', while the reverse is true, as for instance in the *Harmonics*. This treatise is Pythagorean in nature in that it describes the mathematical quality of musical intervals.[32] Unlike Nicomachus and other Pythagoreans using a theoretical methodology, Ptolemy seeks a new understanding of the intervals on the basis of empirical observation.[33] Before proceeding to his formulation of the intervals, Ptolemy in Book I of *Harmonics* criticizes the work of

---

[30] Solomon 24, n. 129.
[31] Levin, Flora R. "Πληγή and Τάσις in the Harmonika of Klaudios Ptolemaios." *Hermes* 108, no. 2 (January 1, 1980): 205–206.
[32] Solomon x. While primarily a treatise on musical ideas, *Harmonics* encompasses a vast range of topics including mathematics, physics, astronomy, and philosophy.
[33] Barker (1989) 270.

others without, however, citing any names or titles. Instead, he summarizes relevant passages and follows them with his critique.

While it was not uncommon in ancient practice merely to condense or quote others' arguments, the degree of absolute silence on names and titles is striking. Since he is able to disguise any accusations of direct attacks on these writers, Ptolemy is particularly biting in his analyses. His central strategy effectively focuses only on "wrong" ideas themselves, while juxtaposing the "correct" viewpoints that are associated with Ptolemy's own name.[34] Thus, in a way, he delegitimizes their claims by refusing any acknowledgement of authorship.

Ptolemy summarizes only the second part of the story, the one on the apparatus of string and weights (Chapter 8 of Book I). Having discussed the definition of the consonant intervals in the previous chapter, the treatise progresses to how one may create these intervals with a monochord. Ptolemy reasons that the monochord has greater precision than instruments such as the auloi and syrinxes. In a sly reference to Nicomachus, Ptolemy also cites the imprecision of weights attached to strings for creating intervals. He declares that any attempts to demonstrate the intervals by the last method would only attract criticism. Incidentally, he promptly provides his own criticism.

Ptolemy is right to note that it would be problematic to apply consistent air pressure in testing wind instruments, but the difficulties he cites with keeping

[34] Barker, Andrew. *Scientific Method in Ptolemy's Harmonics*. Cambridge ; New York: Cambridge University Press, 2000, 145.

strings invariable is not as clear, since strings stretched by a different weight no longer have the same properties. That is, an increasing tension that a string experiences lengthens it and makes it thinner, which in turn produces an even higher pitch. Even controlling these variables and assuming an equal length of string, each string would not produce the predicted pitch as a correlation to the weights that stretch it. Thus, the ratio of weights would not produce the expected intervals.[35]

It is clear from Ptolemy's explanation that, by using Pythagoras's supposed apparatus, sound comes from the plucking of the strings and not from the striking of the weights. He notes that the ratio of weights to the ratio of musical intervals is not one-to-one and seems inconsistent. To determine this, he must have tested the attributes of string like its thickness, density, and length by isolating one as a variable and making the rest as constant as possible in his experiments.[36] While it would be relatively easy to replicate the same length of string in general with a ruler or canon, Ptolemy would likely have had trouble measuring the thickness of string with any analytical precision without a standard tool like a caliper.[37]

However, the greatest difficulty in measuring these attributes would be that for tension. For Ptolemy, the ability to measure tension precisely would be nearly impossible, unless one begged the question or in actuality "tuned" the string (likely

[35] Solomon 24–25.
[36] Levin (1980) 223.
[37] Barker (2000) 51.

in this case by adjusting the weight) to match pitch.[38] The latter of which would in essence be cheating and nevertheless destroy the ratio that two weights would establish. No standard tool, such as a weighing scale or a spring scale, to quantify tension in an absolute sense existed at that point.[39] A viable option for Ptolemy would have been the beam balance, which can only measure mass relative to another. In any case, Ptolemy does not indicate exactly what his methodology is for his observations and results up to this point.

As for the quantitative relationship of pitch to tension, Ptolemy, indeed, recognizes the nonlinearity of his observations. Pitch seems to be higher at an increasing rate when more tension (or weight) is applied to a string. However, it may be a logical stretch to consider that he was mathematically approaching the formula, that the value for the pitch of a string in terms of frequency is a square root of the tension applied to the string. His experiment in itself is a description and does not prove a calculation.[40] We are left to speculate his technique that lead to his conclusions, just like we have had to speculate the physicality description of Nicomachus's apparatus and process for Pythagoras's discovery.

Only later in the chapter does Ptolemy provide a detailed explanation of his experimentation, that is, with the monochord. Nevertheless, it connects the purpose

---

[38] Barker (1989) 258, n. 51. Barker (2000) 51.
[39] Strictly speaking, a scale measures the tension experienced by the scale from the downward force due to gravity, while a balance measures mass, the amount of matter. the here does exist the beam balance, which requires weight to be placed on the other side of the beam to equalize the
[40] Levin (1980) 205 surmises Ptolemy's experiments here as approaching Mersenne's mathematical relationship later. Barbera (1980) 309–313), on the other hand, is circumspect at the possibility of Ptolemy recognizing the quantitative value of his experiments. See Barbera (1980) 309–313.

of disproving Nicomachus. Instead of reasoning that the purpose of the monochord comes from the relationship of weights and strings, Ptolemy argues from first principles that the monochord itself is the foundation for hearing these intervals and describes its construction the monochord.

Ptolemy's transmission of the story is unique since, of the ancient writers, only he seems to have reviewed Nicomachus through experimentation. A representative list of writers that includes Gaudentius, Iamblichus, Censorinus, Macrobius, Chalcidius, Diogenes Laertius, and Boethius, on the other hand, either transmit the story verbatim from Nicomachus or paraphrase him. However, rather than determining the validity of the whole story, Ptolemy only evaluates a part of Nicomachus's story and ignores the critical role that hammers and weights have. It might be that Ptolemy discards this notion, knowing fully the improbability of hammers striking an anvil as well as to produce different pitches. He quickly mentions that one should encounter difficulties similar to strings stretched by weights, unfortunately, without qualifying them.[41]

It is possible to assume that he did test hammers without any appreciable results. However, if we are to believe that Ptolemy was certainly aware of the idea of the methodical nature of the scientific method, then it is unusual for him to state that his results with the hammers are inconclusive with providing greater detail.[42] It

[41] Solomon 25, n. 135.

[42] The basis for Barker's *Scientific Method in Ptolemy's Harmonics* rests on the idea that Ptolemy was aware of the process of scientific method when writing *Harmonics*.

is likely instead that the hammers are mentioned for thoroughness, though unlikely that Ptolemy himself recreated the first part of Nicomachus's story.

There might be reason to doubt Ptolemy's assertions about the difficulty of producing the intervals either by string or hammer though. After all, Ptolemy is invested to emphasize the importance of strings and not that of hammers, when he refers to Nicomachus's story in a chapter of Harmonics, in which he asserts string and the monochord should be the basis of understanding musical intervals. Ptolemy presents himself as a scientist and, in particular, an empirical Pythagorean; he believes that numbers govern natural phenomenon, which may be directly observable and reproducible. Accordingly, Barker presents Ptolemy as one who is acutely aware of the intricacies of argument in investigating harmonics. While Ptolemy lays out his evidence in a systematic fashion, Barker notes the hasty procedure in which Ptolemy describes predecessors' work. The Greek scientist may be acting more rhetorical than rational at those points.[43] Thus, it is unusual when Ptolemy's methodology seems to falter or his consistency in describing a process wavers, in this case, his brief reference to hammers.

The way Ptolemy approaches Harmonics may be connected to the way he also explores astronomy, especially in the *Syntaxis*, since both the conceptual methodology and the mathematical relationships and patterns are relevant for both music and astronomy.[44] Incidentally, R.R. Newton who deconstructed Ptolemy's

---

[43] Barker (2000) 4.
[44] Barker (2000) 3.

analysis of astronomical phenomena has challenged Ptolemy's credibility in astronomy. According to Newton, nearly all, if not all, of Ptolemy's own reported observations were likely fabricated to fit the conclusions Ptolemy desired. Data he attributed to predecessors was also falsified. This does not consider the numerous theoretical errors that pervade the *Syntaxis* and contradict the observations Ptolemy purports.[45]

It is shocking to consider the possibility of Ptolemy as a fraud like Newton does, but the latter's methodology for determining his scientific crime is sound: astronomical phenomena tend to be periodic and predictable once enough data is known. For example, predictions of the occurrences of lunar eclipses may be based on the saros cycle, the 18-year period between repetitions of solar and lunar eclipses. Ptolemy was apparently aware of this cycle among other formulae for astronomical phenomena.[46] Newton determined the flagrant inconsistencies by deriving what dates and observable conditions Ptolemy could have made at his location from known periodicity of the events Ptolemy recorded. The deviations from what Ptolemy should have seen and what he recorded are beyond statistical probability, which paints Ptolemy in one of two unsavory conclusions, either terribly incompetent or a fraud. The foundation of the scientific method is based on the trust of unbiased data and observation without any intentional or grossly

[45] Newton, Robert R. *The Crime of Claudius Ptolemy*. Baltimore: Johns Hopkins University Press, 1977, 378.

[46] Syntaxis VI.2 in Ptolemy. *Claudii Ptolemaei Opera Quae Exstant Omnia*. Vol. 1. 3 vols. Bibliotheca Scriptorum Graecorum Et Romanorum Teubneriana. Lipsiae: in aedibus B. G. Teubneri, 1898.

negligent action that might distort the process or results. If Ptolemy manipulated and fabricated the data in the *Syntaxis* to match his theory, then we must consider a similar possibility in *Harmonics*: Ptolemy might not have not tested the strings or hammers in a verifiable, reproducible manner, if at all, to assert the importance of the monochord over a Nicomachean apparatus of strings and weights for constructing musical intervals.

Mersenne: Testing Pythagoras

It is consequential then that Ptolemy interprets the story in this way, since the next scholar to examine aspects of this story so critically is Mersenne. The seemingly vast gulf of time between Ptolemy and Mersenne—roughly 1500 years—is bridged by Ptolemy's persistent dominance over astronomy, mathematics, and, in general, intellectual thought. Before eventually converting to the Copernican heliocentric model, Mersenne was a firm believer of Ptolemaic geocentricism. However, this was not without debate. He examined both sides on their scientific merit as well as their alignment to Catholic thought. Part of his *Quaestiones celeberrimae in Genesim* examined the relationship between science and religion, and, in particular, whether heavenly spheres were composed of solids or liquids. In Question VII, Mersenne deliberates a text about the heavenly spheres by Adam Tanner, who cites a myriad of authors, one of which includes Ptolemy. In another section of *Quaestiones*, Mersenne quotes Philip Lansberg's *Progymnasmatum astronomiae restitutae liber unus*, which Mersenne argues against Ptolemy's reasoning against the earth's diurnal rotation, the apparent motion of the stars around the Earth.[47]

In any case, without a doubt, Mersenne's work on properties of string and string instruments in *Questions harmoniques* harkens back to Ptolemy's *Harmonics*. Whether or not he was completely aware of Ptolemy's works themselves, Mersenne completes the mathematical proportionality of pitch to string tension. Mersenne was at least aware of the story about Pythagoras, directly citing him instead of another writer like Nicomachus, Ptolemy, or Boethius. He recalls the story briefly in Corallaries 1 and 2 of *Questions harmoniques*:[48]

(i) *Questions harmoniques, Corallary 1*

> One must not be surprised by the fact that several doubt the truth of the principles of Music, since the manner in which one holds that Pythagoras used to invent it is very wrong, because if all the principles and conclusions of this science are as hardly truthful as what one tells about of the hammers, which he used to find the ratio of the consonances, is a certain thing, that what it teaches is false, especially since the different hammers whose magnitudes or weights are in the same ratio which 12 : 9 : 8 : 6 {the numerical ratios} do not make the octave, the fifth, and the fourth, when one strikes them on an anvil, as anyone is able to experiment by noting the sound of the said hammers, which one will judge in unison rather than the octave, the fifth, and the fourth. Which will happen similarly if their lengths, or their areas keep the preceding ratios. But I will treat this subject more fully in another place, where one will see the proportion, which the anvils, or the hammers ought to have to make all kinds of consonances, or dissonances, when one strikes them or when they are attached to string.[49]

---

[47] Hine, William L. "Mersenne and Copernicanism." *Isis* 64, no. 1 (March 1, 1973): 23.
[48] Mersenne (1634) 166–7.
[49] This refers to Corollary 2.

(ii) *Questions harmoniques, Corallary 2*

> Those who hold the opinion of the Pythagoreans on the topic of these hammers, say that he took them in the preceding proportion, and that he attached them to four strings of equal size and length, and that [the strings] made the three consonances of which I spoke: that which is again very false because the slightest hammer must only weigh less three pounds, to make the string descend to the octave of [that which] supports the hammer of 12 pounds, as I will demonstrate by explaining the proportion of sound, made by the strings stretched by the hammers of Pythagoras, and what ratios must have all kinds of weights to make all kinds of sound and consonances or dissonances.

Not only had the story by the 1600s most likely grown to become its own entity, separate from any later author who transmitted the story, but it also was considered false, given Mersenne's upfront conclusion at the start of the corollary. Instead of distinguishing the three original sections of Nicomachus's version, Mersenne largely condenses the story to the action of the hammers on the anvil and the apparatus Pythagoras built. After providing the familiar whole number values for the weights of the hammers, Mersenne, evoking Ptolemy's conclusion, stresses that hammers upon an anvil do not produce musical intervals, but rather the same sound. From this, the prominence of *le Grand Père* Mersenne himself might settle the question about the hammers, weights, and anvil. However, there is no indication that he tested that part of the story, even though it is apparent that Mersenne tests strings (since this writing is in a work that deals with string and string instruments). As with Ptolemy, if Mersenne did actually experiment with hammers and weights, he does not provide the details.

Unlike Ptolemy, though, Mersenne has primarily the distinction of being the last commentator of having conducted experiments with this story. As a result, a lineage of contemporary scholars have seized upon this citation as evidence for the impossibility of the story as a whole.[50] This is troublesome since credibility is given to Mersenne alone as authority for discounting the story without any physical explanation why parts or the whole of the story do not function.

Defining the Impossible is Impossible

One of the few explicit explanations may be found in William Guthrie's *A History of Greek Philosophy*, in which he briefly deconstructs the role of the hammer, weights, and anvils, writing:[51] "These stories are repeated by several authors, but cannot be true. Beating a piece of iron on an anvil with hammers of different weights produce little or no difference in the pitch of the sounds." Levin builds upon Guthrie, suggesting two fundamental flaws concerning the generation of sound in Nicomachus's account: tension and percussion. Tension, experienced by the strings, in the apparatus from the second section of the story is assigned the source of the sound, since sound from percussive means has been defeated. It is the assertion that four hammers striking the same anvil would give the same sound and, in general, any struck object makes sound, while the striking object does not.[52] Instead, to make the four different sounds at the smithy, Pythagoras would have needed four

[50] See note 7.
[51] Guthrie Guthrie, W. K. C. *A History of Greek Philosophy*. Vol. 1. 5 vols. Cambridge University Press, 1971, 223. Incidentally, Levin (1994) 97 cites Guthrie to explain why the story is not possible.
[52] Levin (1994) 92.

different anvils.[53] This principle, of course, does not align with what Nicomachus reports in the first section, that the different hammers striking the anvil (with a piece of iron in between hammer and anvil) each create a sound. If this relationship does not hold true in the first section, then the second section, which concerns the apparatus of string and weights, would fail as well, making the story completely moot.

The difficulty with these assertions is the lack of empirical testing or, more importantly, the theory support. That is, the physical description of the generation of sound through the hammers and anvil from Guthrie via Levin are not consistent with fundamental principles in physics. To illustrate, when one hits a drum with a stick, the drum loudly sounds. When the stick should hit the rim of the drum, one might hear the stick better. Both objects make a sound, but the sound depends on where the stick hits the drum. The point in this case is not that striking different parts of an object produce sounds from different sources. Instead, it is that two objects simultaneously make two sounds, one from each object. Accordingly, these observations contradict Guthrie and Levin's of only the struck object sounding.

In general, it takes two objects colliding to generate sound from both objects. Note that I write "colliding" without distinguishing between what is striking and what is struck. The frame of reference does not matter; an observer moving along the direction and speed of the striking object would see the struck object as the one

[53] Guthrie 223.

striking now.[54] At the point of contact, both colliding objects experience simultaneously an equal magnitude of force, however, in opposite directions. This is, by definition, Newton's Third Law of Motion. Otherwise, the striking object according to Guthrie and Levin would pass through the struck object or push the object along the same path that the striking object takes, in which case no sound could even result.

We may model the collision of these two objects as inelastic, one in which the total kinetic energy before the collision is not equal to the total kinetic energy afterwards. The difference in energy before and after the collision is due to the instantaneous deformation of the objects.[55] They dissipate this energy in the form of sound. That is, both objects are set into vibration, which effects sound waves. The anvil will give a certain pitch, as well as each hammer that strikes it. We may hear simultaneously the combination of the pitch of the anvil and that of a hammer, but in sum, there are four different pitches.

One might suggest how a hammer might be giving sound. We are not certain about the design of the hammer, other than it has acoustic properties. If the hammers act like a tuning fork or a hand bell, hitting an anvil, one would hear both anvil and hammer. For four different hammers, then, there would be four different pitches. Thus, the physical description given by Guthrie and Levin for why only an

[54] Imagine traveling in a vehicle approaching a landmark. It would appear that the landmark is coming ever closer toward the vehicle. An observer standing outside the vehicle would observe it moving toward the landmark. Another at the landmark would see the vehicle coming at him.

[55] This is in contrast to elastic collisions, which assume a rigid body for the objects, such that no deformation or loss of kinetic energy results.

anvil would sound if struck fundamentally violates Newton's Third Law of Motion and the consequential relationships with momentum and sound.

A Null Hypothesis

The tendency to concentrate on the strings alone as the representative part of the story about Pythagoras at the smithy understandably comes from Ptolemy's interpretation. Whether is it right or wrong, Ptolemy's is one of the few critical descriptions of the story that does not simply repeat or reconstitute what Nicomachus ultimately wrote. Ptolemy concludes that the story is not possible based on his observation of the strings under tension from weights. However, he hedges on the possibility that hammers could be producing sound as well.

What is troubling is the possibility that Ptolemy might not have been as empirical and ethical as he claims as a scientist. The doubt surrounding Ptolemy's theoretical errors and fabricated data in the *Syntaxis* does not foster confidence in his assertions and recorded observations on the difficulty in producing musical intervals with strings or weights alone. He could be spouting pure rhetoric or advocating from a theoretical standpoint, both of which does not align with Ptolemy's supposed methodology of verifiable experimentation. Mersenne finishes, in a sense, Ptolemy's analysis of the strings, giving the formula for the pitch as a square root of string tension. Like Ptolemy's, Mersenne's description about the role of the hammers is vague, while at the same time it is more definite in stating that it is not possible. No explanation is given, but contemporary scholars find it well enough to cite Mersenne as the prominent authority who disproves the story.

While Ptolemy and Mersenne plainly dismiss the possibility of the hammers and weights' roles, a litany of scholars assert likewise without proof, citing Mersenne as authority. However, Guthrie and Levin attempt a physical rationalization of the impossibility of the hammers sounding upon the anvil, that only the anvil might produce a sound when struck. Guthrie and Levin's reasoning, consequently, is not possible because their proof violates the very laws of physics. Hammers striking the anvil represent an inelastic collision of two objects, which results in sound from both hammer and anvil. In fact, they argue from a null hypothesis, that hammers and weights have nothing to do with creating musical intervals in the story. The null hypothesis must be assumed true until proven false. However, since the explanation to disprove the null hypothesis is not valid, the null itself is not disproven. Their proposed alternative hypothesis, that strings are responsible for the intervals in the story, is not necessarily a hypothesis that is contrary to the null. It is more so alternative in name and bears little relationship to the null hypothesis. It is a logical fallacy to associate these two hypotheses, since the null hypothesis has not been disproven even though the alternative hypothesis is a likely valid case.

What we have is a story that is seemingly proven false by the authority of Ptolemy and especially Mersenne, even though they provide no proof of contradiction to physical laws. The bibliographic bloat for depending on Mersenne, in a sense, hurts the reception of this story, because of the lack of a critical review of the story. The story is, instead, assumed as a literary figment and a physical impossibility. I do not deny that it is possible that the story truly is the work of Pythagorean imagination. Alternatively, I assert my own null hypothesis that aspects of the story are possible and invite others to disprove it. We will see, in the next chapter, how Boethius, like Ptolemy and Mersenne, uses the story for his own purposes. However, Boethius accepts the premise of the null hypothesis, instead of rejecting it.

Different Look, Same Great Taste!

—product label

When something is 'new and improved!' which is it? If it's new,

then there has never been anything before it.

If it's an improvement,

then there must have been something before it.

—attributed to George Carlin

We revisit the possibility of affirmation, having discussed Ptolemy and Mersenne's repudiation of the musical discovery in the previous chapter. The story and its Pythagorean understanding of musical sounds do not align with Ptolemy and Mersenne's conception of music. The monochord is supreme to them, not supposed musical hammers or an apparatus of weights and string. While Ptolemy and Mersenne's interpretation propagates on its own, another thread surfaces at the boundaries of Antiquity and the Middle Ages whose influence persists through the Renaissance. Anicius Manlius Severinus Boëthius (480–524 CE)—commonly called Boethius—was a prolific author and commentator of philosophical, pedagogical, and theological writings in the late Western Roman Empire. Some have considered him a transitional figure: the last of the Romans and the first of the medieval scholastics.[56]

[56] Though the chronology is also correct, strictly speaking, Renaissance humanist Lorenzo Valla in *Elegentiae* calls Boethius a transitional figure more for his criticism of Boethius's Latin prose style than as a historical marker. Valla was highly critical of what he perceived as the butchered elegance of Latin beginning with Boethius compared to the beauty of Classical literature before Boethius. To

His writings on music contribute to what Charles Atkinson describes as the critical nexus: the interweaving of tone-systems, modes, notation, and other musical topics.[57] Boethius's training in Greek granted him access to Aristotelian, Platonic, Ptolemaic, Pythagorean and other philosophical treatises, many of which he translates for the first time into Latin, the lingua franca after Greek. These translated and commented texts form the basis for much of medieval knowledge about Pythagorean and, in general, ancient thought. It is through this that Boethius interacts with the story about Pythagoras's discovery of the musical intervals. The extent of Boethius's interaction with the story is greater than how Ptolemy and Mersenne engage it. He recounts the story fully with his own variation on it. This chapter will show how Boethius understands and manipulates the story for the benefit of the quadrivium, and thus, affirms a belief in the possibility of the story.

Sources for Boethius's Work

One of Boethius's best remembered contributions has been his organization of the quadrivium, a pedagogical grouping of arithmetic, music, geometry, and astronomy. It is an idea that dates as early as Plato. In Book VII of *The Republic*,

---

Valla, Boethius represents the devolution of Classical Latin. Boethius's prose style is discussed later in this chapter, as proof of his understanding of Pythagoras's discovery. For a review of humanist evaluations of Boethius and especially his *Consolatio* in the Renaissance, see Grafton, Anthony. "Epilogue: Boethius in the Renaissance" in *Boethius: His Life, Thought and Influence*. Oxford: Blackwell, 1981, 410–415; Nauta, Lodi. "A Humanist Reading of Boethius's *Consolatio Philosophiae*: The Commentary by Murmellius and Agricola (1514)," *Between Demonstration and Imagination. Essays in the History of Science and Philosophy Presented to John D. North*, eds. Lodi Nauta and Arjo Vanderjagt, Leiden: Brill, 1999, 313–338; King, Peter. "Boethius: The First of the Scholastics." *Carmina Philosophiae* 16 (2007): 23–50.

[57] Atkinson, Charles M. *The Critical Nexus: Tone-System, Mode, and Notation in Early Medieval Music*. AMS Studies in Music. Oxford: Oxford University Press, 2009, 4.

Socrates and Glaucon discuss the role of each subject separately. Although they are not intimately tied together themselves, the subjects form part of the prescribed education for philosophers. [58] While the quadrivium itself has not been understood as a unitary concept of four subjects, the 5th century CE scholar Proclus formalized the four subjects as a pair of pairs, one about quantity and the other about magnitude. Quantity is concerned with itself and with its relationships with other quantities. Thus, arithmetic pertains to quantity per se, while music is about the relationships among quantities. Magnitude involves the subjects considered stationary and in motion; thus, geometry is magnitude at rest and unchanging, while astronomy is magnitude in motion.[59] While Proclus divided the concept of the quadrivium, it is not until Boethius that the term quadrivium appears. He is likely to have coined the term himself.[60]

Accepting the dual division of the quadrivium, Boethius wrote four separate treatises, one for each subject, in two sets. His two extant works on the quadrivium are on music and mathematics, *De institutione musica* and *De institutitione arithmetica* (commonly known as *De musica* and *De arithmetica*, respectively). He addresses the four subjects as the quadrivium explicitly in his first work on the quadrivium, *De arithmetica 1.1.* By Boethius' time, a number of philosophical approaches (e.g. Platonism, Aristotelian, Pythagorean) would be competing for his

[58] Plato, *The Republic* VII.522a–528e.
[59] Proclus, *A Commentary on the First Book of Euclid's Elements* XII.35–36.
[60] Boethius. *Fundamentals of Music*. Edited by Claude V Palisca. Translated by Calvin M Bower. New Haven: Yale University Press, 1989, xix. This text is a translation and commentary of Boethius's *De musica*; I will refer to it as Bower (1989). See *De arithmetica* 1.1 for Boethius's explanation of the quadrivium.

attention. The basis for his sources ultimately is Pythagorean texts. He adopts a very Pythagorean tone in most of the texts, depending heavily on 2nd century CE Pythagorean writer Nicomachus' *Enchiridion* and its now lost sequel, *Eisagoge musicês*. His preference for Nicomachus underscores the authority the writer and Pythagorean thought have still in Boethius's time. To a much lesser extent, he also uses other writers such as Ptolemy and Aristoxenus.[61]

At first glance, one might consider that his works on the quadrivium are derivatives of the earlier Pythagorean and ancient writings. Boethius's works are, in a basic sense, a translation of chief sources of ancient musical and mathematical thought, from Greek to Latin for the first time. However, beyond the act of translation, Boethius's greater motivation is to compile these writings as the essential basis for knowledge in the quadrivium. Boethius expounds on many of these sources, while expanding and contracting the arguments to present the concepts in an understandable manner.[62] His goal is to propagate this knowledge of the quadrivium.

The lore of Pythagoras radiating in the background obfuscates critical examination of Boethius's sources. To complicate matters, Boethius rarely cites the text or author. However, Boethius borrows not only the structure of a text he cites but also the phrasing. Bower has traced a majority of source material for *De musica*

---

[61] For an in-depth analysis of Boethius's sources for *De institutione musica*, compared with *De institutione arithmetica*, see Bower, Calvin M. "Boethius and Nicomachus: An Essay Concerning the Sources of De Institutione Musica." *Vivarium* XVI (1978): 1–45.

[62] Bower (1978) 45. It is another issue how successful Boethius is at making the material accessible though. See later for discussion of Boethius's prose style.

to Nicomachus's *Enchiridion*, including the story about the musical discovery. Boethius maintains the tripartite structure and striking elements from *Enchiridion*.[63] Although Boethius lifts much of his material, Bower and Edmiston separately conclude that Boethius's *De musica* is not merely a translated Latin facsimile of *Enchiridion* but a reinterpretation of, largely Pythagorean, musical thought, including the discovery of the intervals. Boethius's account is a unique reproduction of the story and differs in aspects such as the order of events and the point at which Pythagoras analyzes his discovery. Accounting for these differences may explain how Boethius utilizes the story in *De musica*. There are three possibilities for them: Boethius could be reading from an updated version of the story in Nicomachus's *Eisagoge musicês* (*Introduction to Music*), not *Enchiridion*. He could be reading from mediated sources chronically between him and Nicomachus. Or, he could be changing the story himself.

(1) *Updated Version*

Nicomachus's supposed later work completes and revises topics missing or truncated in *Enchiridion*. Thus, Boethius's account of the story would be more reflective of *Eisagoge musicês* as its sources, and the parts that differ from *Enchiridion*, Nicomachus's only extant work, would be the result of Nicomachus's updates to his text rather than Boethius's alterations. Bower argues that elements of *Eisagoge musicês* can be found in *De musica*, which he

[63] The story tends to be related fully and with noticeably similar wording and structure. See Table 1.2 for list of other versions of the story.

considers largely a blend of *Enchiridion* and *Eisagoge musicês* with any influence of lore. In fact, Nicomachus lists the topics he wishes to fill up in a sequel work throughout *Enchiridion*, such as a fuller discussion of the music of the spheres and the division of the monochord. Since sections of *De musica* address the very topics Nicomachus promised to treat in the later work, Bower asserts that Boethius likely had a copy of the now missing *Eisagoge musicês*. However, the story is not mentioned in Nicomachus's list.[64]

(2) *Mediated Sources*

If Boethius has access to Nicomachus's *Enchiridion*, mediated sources would no longer be mediators. With sufficient prior knowledge, mediation does not affect a line of transmission. Boethius would be free to consider all his sources parallel to one another. In Boethius's time, the aura of Pythagoras was still a powerful presence. Lore and attributions to Pythagoras like his alleged musical discovery would compete with textual sources. Even if Boethius was aware of the chronological provenance of sources after Nicomachus, he only selects certain aspects that differ from Nicomachus, likely for the clarity of understanding. This mode of interpolation still makes Boethius's version unique as a pastiche of versions dating from Nicomachus's time.

[64] Nicomachus *Enchiridion* 1, 3, 9, 11, 12.

(3) *Boethius's Changes*

> This possibility is related to (2) mediated sources, but rather than choosing from others' versions, Boethius alters the story to his preference, again, likely for the clarity of understanding. Boethius probably has an astute comprehension of the material he writes about and, in particular, the objective of the story. He does not merely translate Nicomachus and other ancient writers, as much as he tends to expand the text when clarification is necessary and contracts it when unnecessary.[65] Thus, the differences in the story are because of Boethius's own alterations and commentary.

The first possibility is unlikely. Nicomachus has little reason to reengage the story in *Eisagoge musicês* when he does not consider a topic undeveloped or omitted from *Enchiridion*. The second possibility is feasible as well, but, because Boethius so overwhelmingly cites Nicomachus, mediation is ineffective if it exists. At the same time, Pythagorean lore is competing against documented sources for Boethius's attention. I am in favor of the third as the predominant possibility. Following Bower's suggestion, any differences in the versions of the story are reasonably Boethius's for the purposes of clarifying the content. Boethius conceivably takes the account of the discovery for granted, as a story of common knowledge both documented and in lore. His version integrates into his construction of the

---

[65] Bower (1978) 45 and Edmiston, Jean. "Boethius on Pythagorean Music." *The Music Review* 35, no. 3–4 (November 1974): 179.

quadrivium, to make a defensible argument for this pedagogical regime.[66] That is, narrative of Pythagoras's discovery is a linchpin that unites arithmetic and music, the parts of the quadrivium that concern quantity.

References to Pythagoras and Boethius's Affirmation

Boethius intended for *De musica* and *De arithmetica* to be read together to emphasize the connections within the realm of numerical quantity. He uses number to construct the numerical part of the quadrivium. Number is a central hallmark of Pythagorean thought, to which Boethius subscribes for a quantitative basis for music.[67] With Boethius's coverage of Pythagorean thought and reading from Nicomachus, it should not be surprising that Boethius would encounter the story about Pythagoras's discovery. Boethius devotes Chapters 10-11 in Book I of *De musica* to the narrative.

The story is one of four direct references to Pythagoras in *De musica*. The other three are the introduction of Pythagoras as a contemplative night owl in 1.1, the allusion that Pythagoras (under the pseudonym Lycoan, a name itself based etymologically on the nocturnal wolf) added the eighth string to the lyre in 1.20, and the identification of Pythagoras as an indisputable, all-knowing man in 1.33. It may seem unusual that Boethius would include these mythical elements in a tome devoted to the transmission of fundamental knowledge. Accordingly, Bower

[66] Bower (1978) 2.
[67] See Boethius, *De musica* 1.3 for text; Bower (1989) xix–xx for further explanation.

categorizes these references as mythical parables that permeate *De musica* but otherwise do not interfere with the purpose of the work.

Three of the references are only short attributions to Pythagoras though. They briefly point out something about Pythagoras or something he did: A person could be nocturnal and pensive. A person could be considered sagacious. One could allege a person's actions, such as adding a string to the lyre. Regardless, what is said about Pythagoras here is not exceedingly fantastic and not particularly mythical. The story, on the other hand, is a lengthy and highly technical account. While the discovery does attribute something to Pythagoras, the story details how Pythagoras derives the intervals from ratios. Bower qualifies this seeming exception among the four references not as something scientific or empirical, but as an allegory.[68] It would seem unusual, though, for Boethius to include this story when it is not based on scientific reality. His goal is to disseminate knowledge of the quadrivium. The fact that Boethius includes the story in *De musica* signifies his affirmation of the events in the story. It is not necessarily hypothetical for Boethius; instead, the story is an empirical basis to hear pitches from hammers, which correspond to numerical musical intervals.

Indeed, Boethius interacts with the story in contrast to Ptolemy and Mersenne. The latter two were dismissive of it, arguing that the study of music lies foremost with strings. For Ptolemy, the story does not align with his focus on the monochord as the fount of music theory. For Mersenne, it is a convenient foil to his

[68] Bower (1989) xxi.

analysis of the attributes of strings, and, in particular, the factors that influence the pitch of a plucked string. Both merely refer to elements of the story: Ptolemy, alluding to Nicomachus, alleges the imprecision weights have for creating musical intervals; Mersenne supposes that the weight of hammers and pitch do not correlate. Both merely refer to the elements of the story: Ptolemy by the imprecision weights have for creating musical intervals in an allusion to Nicomachus himself; Mersenne by Pythagoras himself and the supposition of the hammers (granted that by Mersenne's time the story is just an often told anecdote). The story does nothing to advance their arguments. Thus, they repudiate the story. Boethius, on the other hand, recalls the story more fully. The act of reproducing the story in such an unabridged way affirms Boethius's belief in the possibility of the story or, at least, in its pedagogical usefulness. It is not to say, however, that he merely reproduces the story. Boethius's version is not a facsimile of Nicomachus. It retains similar elements and structure as his predecessor, while being distinct in its presentation.

The story's placement in *De musica* is also suggestive of its purpose for Boethius. The progression of topics in Book I is quite expansive, but its organization is logical and elegant. Bower categorizes Book I into six sections: (1) an introduction to musical topics and terms, and how musical phenomena may be calculated through numbers; (2) the functions of reason and the senses, and how Pythagoras discovered the intervals; (3) how human voices and hearing work; and (4) the discussion of intervals and their relationship to each other; (5) integration of the intervals into various scalar systems, (6) an analysis of consonance.[69] The story shares a conceptual midpoint in *De musica*. It bridges the definitions of music and the mechanics of those definitions.

Motivation and the Story: Chapters 9-11, Book I, *De musica*

Before the story in Chapters 10 and 11, the second section begins with Chapter 9, laying out the superiority of reason over the senses in perception. Boethius does not deny the role the senses have in perception. For him, they are, of course, essential to understand musical sounds. Without the senses, one cannot perceive sound as a physical phenomenon. However, without reason, one cannot fully comprehend sound. This is not necessarily a novel concept as Boethius is essentially advocating for an analytical and predominantly Pythagorean mode of listening over the purely perceptual Aristoxenian model by digressing into a story about Pythagoras. To be clear, Boethius is not a Pythagorean (though he leans heavily as one); he is not an Aristoxenian. Boethius blends material from other sources like Ptolemy, Plato, and Aristotle to illustrate his points. They include the story of Pythagoras's discovery. It is the tie that binds with music and arithmetic: Pythagoras first hears what he perceives are musical intervals, but only with analysis does he confirm and characterize them.

[69] Bower (1989) xix–xxxi.

After the preceding chapter that explains how his preference for reason over the senses, Boethius introduces the story in Chapter 10, with a reminder of reason's role over the senses:

X. Quemadmodum Pythagoras proportiones consonantiarum investigaverit.	X. How Pythagoras investigated the proportions of consonances
Haec igitur maxime causa fuit, cur relicto aurium iudicio Pythagoras ad regularum momenta migraverit, qui nullis humanis auribus credens, quae partim natura, partim etiam extrinsecus accidentibus permutantur, partim ipsis variantur aetatibus, nullis etiam deditus instrumentis, penes quae saepe multa varietas atque inconstantia nasceretur, dum nunc quidem si nervos velis aspicere vel aer umidior pulsus obtunderet vel siccior excitaret vel magnitudo chordae graviorem redderet sonum vel acumen subtilior tenuaret vel alio quodam modo statum prioris constantiae permutaret, et cum idem esset in ceteris instrumentis, omnia haec inconsulta minimaeque aestimans fidei diuque aestuans inquirebat, quanam ratione firmiter et constanter consonantiarum momenta perdisceret.	Thus, this was especially the reason why after forgoing the ears' perceptions Pythagoras turned to the principles of rules. He did not trust human ears, which are subject to change in part by nature, in part by external circumstances, in part are altered by one's age. And he was not attached to any instruments, among which often much variation and inconsistency arise. For if you were wish to wish to examine strings, more humid air would flatten the vibrations or drier air would sharpen them; the size of a string would either give a lower sound or a thinner string would raise the pitch, or in some other way, it would completely change the previous consistency [of sounds]. In addition, since the same thing would happen in other instruments, [Pythagoras] determined that all these were unreliable and not at all to be trusted. He was fervently seeking or a long time a way in which he would confidently and thoroughly understand the principles of consonances.

Like Nicomachus, Boethius breaks from his arguments and introduces the story with a transition. Unlike Nicomachus's introduction, Boethius is connecting the story not necessarily to the intervals themselves, but to the larger picture that studying music analytically is more preferable than one's sense perception. Boethius interjects with a caution against studying musical phenomenon with the senses or with temperamental instruments before Pythagoras appears in the text. A dependable method for studying music rests with a story about Pythagoras:

Cum interea divino quodam nutu praeteriens fabrorum officinas pulsos malleos exaudit ex diversis sonis unam quodam modo concinentiam personare. Ita igitur ad id, quod diu inquirebat, adtonitus accesit ad opus diuque considerans arbitratus est diversitatem sonorum ferientium vires efficere, atque ut id apertius conliqueret, mutare inter se malleos imperavit. Sed sonorum proprietas non in hominum lacertis haerebat, sed mutatos malleos comitabatur.	Meanwhile, when, by certain divine will, passing by the workshops of the blacksmiths, Pythagoras clearly heard that beaten hammers somehow produced a single consonance from different sounds. Thus, in this way, in regards to what he was searching for for a long time, he, stunned, approached the activity, and pondering for a while, thought that the force of the beating (hammers) created an assortment of sounds. To be certain about it more clearly, he directed that the hammers change hands. But the nature of the sounds was dependent not on the muscles of men; instead it followed the exchanged hammers.

Boethius, like Nicomachus and ποτὲ (one day), begins the story with a narrative marker, *interea*, which sets the action of *De musica* at this point in another reality from Boethius's own discussion. Pythagoras, by chance, hears hammers at a smithy making the sounds of the perfect intervals. The duration of Pythagoras's visit to the smithy is lengthier; Boethius describes more of his actions there to focus on the role of the hammers. Pythagoras studies the hammers and notes their relationships among each other:

Ubi id igitur animadvertit, malleorum pondus examinat, et cum quinque essent forte mallei,

A. dupli reperti sunt pondere, qui sibi secundum diapason consonantiam respondebant.

B. Eundem etiam, qui duplus esset alio, sesquitertium alterius conprehendit, ad quem scilicet diatessaron sonabat.

C. Ad alium vero quendam, qui eidem diapente consonantia iungebatur, eundem superioris duplum repperit esse sesqualterum.

D. Duo vero hi, ad quos superior duplex sesquitertius et sesqualter esse probatus est, ad se invicem sesquioctavam proportionem perpensi sunt custodire.

E. Quintus vero est reiectus, qui cunctis erat inconsonans.

Thus, when he notices this, he examines the weight of the hammers, and since there happened to be five hammers,

A. those which together were resounding [according to] a diapason consonance were found to be twice in weight [i.e. one hammer [12 units] was twice in weight to another [6 units], together which sounded [according to] a diapason consonance.].

B. He understood that this same one [12 units], which was twice the weight of another was four-thirds (a sesquitertian) the weight of another [9 units], in relation to which, of course, sounded the diatessaron consonance.

C. He found that the same hammer, the one twice the weight of the one agove was in a ratio of three-halves (a sesquialter) to a certain other hammer, which was joined to it in a diapente consonance.

D. These two hammers [the 9 and 8], to which the hammer weighing twice as much as the one mentioned before was proven to be in sesquitertian ratio and sesquialter ratio, were in turn assessed carefully to maintain a sesquioctave ratio (nine-eighths) in relation to one another.

E. But the fifth hammer, which was dissonant with all the hammers, was rejected.

In contrast to Nicomachus, Boethius adds a superfluous fifth hammer that is rejected outright for its dissonance to the other hammers. It is an odd addition, since this fifth hammer only produces dissonances with the other hammers. The sesquioctave (9 : 8) is a major second and should be considered a dissonance, but Boethius must mention this as natural consequence of the 9-lbs and 8-lbs hammers, which also create consonance with other hammers. This fifth hammer must produce an interval unlike the perfect ones and the second. Indeed, the entire focus is on the principles of consonance, especially in a Pythagorean system; Boethius discards anything that generates dissonance for the story.[70] After this remark, Boethius never mentions the fifth hammer again. It would be speculation to estimate the value of the fifth hammer. It likely is not a whole or even rational number, but something irrational, which would violate the Pythagorean basis using countable numbers.

He explains the relationships of the relevant four hammers circuitously by not providing example values of the hammers. This section reads more like a logic puzzle, in which the values are indefinite at this point of the story. Boethius's reasoning is sound, but his approach to describing the relationships of hammers is cumbersome. Boethius is emphasizing as much as possible the pure relationship that any two weights might have to produce a particular consonance. Only in a

[70] A discussion of the pictorial representations of the story, especially the hammers, is found in Munxelhaus, Barbara. *Pythagoras Musicus*. ; Bd. 19. Bonn-Bad Godesberg: Verlag fur Systematische Musikwissenschaft, 1976, 42–45. She cites an image from Canterbury MS 1 1.3, 12, fol. 61^{V}, in which Pythagoras has at his feet the rejected fifth hammer. A book-length discussion of this fifth hammer and, broadly, the role and representation of dissonance in the historical reception of Pythagorean thought may be found in Heller-Roazen, Daniel. *The Fifth Hammer: Pythagoras and the Disharmony of the World*. New York: Zone, 2011.

summary of Pythagoras's findings does Boethius provide the values of the hammers' weights:

Cum igitur ante Pythagoram consonantiae musicae partim diapason partim diapente partim diatessaron, quae est consonantia minima, vocarentur, primus Pythagoras hoc modo repperit, qua proportione sibimet haec sonorum concordia iungeretur. Et ut sit clarius quod dictum est, sint verbi gratia malleorum quattuor pondera, quae subter scriptis numeris contineantur: XII. VIIII. VIII. VI.	So, although the consonances of music before Pythagoras were called sometimes diapason, sometimes diapente, sometimes diatessaron, which is the smallest consonance, Pythagoras was first to discover in this way by what ratio this consonance of sounds was joined with one another. And so that what has been said may be clearer, for example, let there be four weights of the hammers, which would be comprised of the following numbers: 12, 9, 8, 6.

The set of numbers here are the same as the natural numbers Nicomachus cites for the weights of the apparatus. They are not arbitrary; they are related to the Pythagorean tetractys. The first quaternary numbers of the tetractys {1, 2, 3, 4} is the simplified set to construct the consonant intervals, e.g. 3 : 2 is a perfect fifth. The set correlates to the intervals too as compound intervals, e.g. 3 : 1 is a perfect twelfth or a compound fifth.[71] The solution set {12, 9, 8, 6}, expressed in musical ratios, simplifies to the first quaternary, e.g. 8 : 6 becomes 4 : 3, a perfect fourth.[72] Based on his observations, Pythagoras is declared to have discovered how consonant

[71] 4:1 is the consonant double octave.
[72] Münxelhaus (1976) 41–42; Mathiesen (1999) 425.

intervals are formed. Boethius next formalizes what the ratios each pairing of the four remaining hammers make:

Hi igitur mallei, qui XII et VI ponderibus vergebant, diapason in duplo concinentiam personabant. Malleus vero XII ponderum ad malleum VIIII et malleus VIII ponderum ad malleum VI ponderum secundum epitritam proportionem diatessaron consonantia iungebatur. VIIII vero ponderum ad VI et XII ad VIII diapente consonantiam permiscebant. VIIII vero ad VIII in sesquioctava proportione resonabant tonum.

Therefore, these hammers, which weighed 12 lbs and 6 lbs, sounded the diapason consonance in a double [ratio of weight]. The hammer of 12 lbs in relation to that of 9 lbs and the hammer of 8 lbs in relation to that of 6 lbs are joined by the diatessaron consonance according to the epitritic ratio (four-thirds). And the one of 9 lbs in relation to that of 6 lbs and the one of 12 lbs in relation to that of 8 lbs produce together the diapente consonance. And the 9-lbs hammer in relation to the 8-lbs one sounded a tone in the sesquioctave ratio.

Boethius adds a unit of weight, pounds (*pondus*), to the values of the hammers. This differs from Nicomachus, who kept the magnitude of the values without specifying units. Perhaps Boethius does this to give a realistic flair, since the hammers are much more existent in the story. In any case of units of weight, the ratio of weight values would be maintained regardless of the kind of units, for example, modern pounds and kilograms or ancient talents and oboloi. The summary of each pairing of hammers and the corresponding musical intervals is clear. For example, 12 lbs and 6 lbs make a ratio of 2 : 1, the same as the diapason octave.

The length of the discussion in Chapter 10 permits space for another chapter to conclude the story. Chapter 11 begins with Pythagoras going home after thoroughly examining the hammers in the previous chapter to test his revelation on other instruments:

XI. Quibus modis variae a Pythagora proportiones consonantiarum perpensae sint.

Hinc igitur domum reversus varia examinatione perpendit, an in his proportionibus ratio symphoniarum tota consisteret. Nunc quidem aequa pondera nervis aptans eorumque consonantias aure diiudicans, nunc vero in longitudine calamorum duplicitatem medietatemque restituens ceterasque proportiones aptans integerrimam fidem diversa experientia capiebat.

Saepe etiam pro mensurarum modo cyathos aequorum ponderum acetabulis inmittens; saepe ipsa quoque acetabula diversis formata ponderibus virga vel aerea ferreave percutiens nihil sese diversum invenisse laetatus est. Hinc etiam ductus longitudinem crassitudinemque chordarum ut examinaret adgressus est. Itaque invenit regulam, de qua posterius loquemur, quae ex re vocabulum sumpsit, non quod regula sit lignea, per quam magnitudines chordarum sonumque metimur, sed quod regula quaedam sit huiusmodi inspectio fixa firmaque, ut nullum inquirentem dubio fallat indicio.

XI. In what ways the various proportions of consonances were measured out by Pythagoras

From here, therefore, having returned home, Pythagoras pondered by various testing, whether the ratio of consonances would remain completely in these proportions. First, he fit equivalent weights to strings and determined their consonances by ear. Next, he established in the length of reed pipes the same double and the mean [i.e. the double being the octave and the mean being the intervals in between the octave, the fourth and the fifth] and fitting the other proportions, he was realizing a most complete confirmation by various experiments.

As a method of measurement, he poured ladles of equivalent weights to cups; and he struck the cups themselves shaped by their different weights with either a copper or iron stick, he was glad to have discovered that nothing was different. Also, he, led from this point, turned to examine the length and thickness of strings. And so he found a measure (about which we speak later), which got its name from the object, not because the ruler is made of wood, through which we measure the sizes and sounds of strings and sounds; but because there is this certain ruler may be a fixed and firm investigation, so that it may not deceive any investigation with false evidence.

Pythagoras builds an apparatus of string and weights equivalent to those of the hammers. In addition to reed pipes, Pythagoras finds that the relationships in the apparatus are the same as the ones he found with the hammers. For further confirmation, he also tests vessels, filling them with liquid of equivalent weight and striking them with various kinds of sticks. Boethius refers to *regula* or the methodology for measuring sounds as a review of the necessity for a methodology in the story's introduction. After the story ends, Boethius devotes the next chapters to the nature of voices and hearing, setting up the discussion of the intervals later.

Concordance with Nicomachus

Boethius retells the story as his own, and not as a blind translation or repetition of lore. In general, he expands his text when the argument is scarce and condenses when plentiful. Here, he magnifies the role of the hammers and how the sounds they produce make musical intervals. The expansion suggests that the story might have been troublesome to understand for him. Thus, he fleshes out the story, an overall increase to Nicomachus's compressed text. A synopsis of the events in both versions is found in Table 4.1. The differences between the two are noticeable. One chapter of *Enchiridion* contains the entire story, while Boethius uses two in *De musica*. The sequence of events generally correlates with Nicomachus. The core of the story—Pythagoras hearing the intervals at the smithy— remains intact with very similar wording. The principal difference between Nicomachus's and Boethius's accounts is at what point of the story Pythagoras analyzes how sound is produced.

Table 4.1. Synopsis of Nicomachus's and Boethius's Versions

Plot	Setting	Nicomachus, *Enchiridion*	Setting	Boethius, Book I, *De institutione musica*
Motivation		**Chapter 6** • Pythagoras seeks numerical description of the perfect intervals		**Chapter 10** • Not trusting ears or instruments, Pythagoras seeks the basis of consonant intervals.
	At the Smithy	• Pythagoras contemplates a reliable tool for the ear.		• Boethius claims the unpredictability of strings and instruments, because of the fluctuating ambient conditions.
Discovery		• He happens upon the smithy and recognizes the sounds of the hammers as musical intervals. • He physically examines the hammers and goes home.	**At the Smithy**	• He happens upon the smithy and recognizes the sounds of the hammers as musical intervals. • He physically examines five hammers.
Analysis	**At Home**	• Pythagoras builds an apparatus to hang up weights equivalent to the hammers. • He tests the intervallic relationships among the weights, during which they're each assigned one of the values: 12, 9, 8, 6. • The relationships hold in the weights. Nicomachus summarizes Pythagoras's findings.		• He tests the intervallic relationships among the hammers. • The weights are labeled on the hammers with 12, 9, 8, 6. • Boethius summarizes Pythagoras' findings.
Confirmation		• Pythagoras applies his findings to pots, pipe and string instruments and discovers that the same relationships correlate.	**At Home**	**Chapter 11** • Pythagoras builds an apparatus to hang up weights equivalent to hammers. The relationships hold.
Conclusion		• The intervals are rational numbers and fill the notes of the octochord.		• Pythagoras has a found a reliable method.

See Table 1.1 for the components of the story and Table 1.2 for a comparison of authors' versions.

For Nicomachus, it is only after the smithy at home. For Boethius, it is while Pythagoras is still at the smithy. This emphasizes the role of the hammers over anything else as the basis for the musical intervals.

The apparatus of weights and strings still appears, but Boethius's preference for the hammers makes it redundant. Boethius repurposes the apparatus as another verifying instrument like the liquid vessels and the reed pipes, and not as the proof of concept. There is nothing about tension in the apparatus. Pythagoras, here, plays the weights and not the strings. Boethius is so unambiguous at the metal workshop of the relationship of the hammers and intervals that it is likely the weights that make sound and unlikely strings. In fact, Boethius's introduction to the story in Chapter 10 cautions against the reliability of strings. Nothing about tension is ever mentioned. At best, Boethius indirectly refers to the effects ambient conditions have on pitch because of altered tension. The diminution of the apparatus focuses the story on the hammers. The apparatus is relegated to the role of instruments that confirm Pythagoras's discovery. Its lack of importance is marked by the fact that the hammers at the smithy have the entire chapter to itself.

It is clear that Boethius understands the story. Boethius expands Nicomachus's awfully condensed text to explain it better. He reinforces the sense of discovery and diminishes the expectation of a foregone conclusion. Boethius does not list which certain intervals Pythagoras is investigating, only that he is interested in consonance. He takes great pains to discuss the relationship of the hammers and musical intervals strictly using their proportionality, delaying to provide a sample set of numbers that conform to premises. Boethius is not only demonstrating his knowledge but is also challenging his audience to calculate possible weights.[73] Only when Boethius summarizes the intervals created does he provide a solution. The stiffness from maintaining the logic puzzle until the end of the analysis makes the understanding more confusing than beneficial.[74] For Boethius, the discovery is the linking of the pitches Pythgaoras hears to the numbers on the hammers. Because he comprehends the story, Boethius is able to conceptualize the relationships of the hammers without a sample set.

His understanding is augmented by demonstrating his prowess in word play, mixing etymological or transferred meanings of weight and measurement, like the verbs:

(1) *pendere*: *perpensae sint* [Friedlein 198.10]–literally, the two hammers were weighed to maintain a sesquioctave ratio; or figuratively, they were assessed or measured; *perpensi sunt* [Friedlein 197.21]–Pythagoras measures the intervals by "weighing" them;

(2) *vergere*: *vergebant*–etymologically, they were inclined downward or were situated. Boethius uses it figuratively: "they were weighed";

[73] There is a possibility Boethius is essentially describing a diagram of the set in the vein of Elizabeth Mellon's recent unpublished article "The Picture of Sound: "Reading the diagrams of Boethius's De institutione musica" The pure relationships Boethius describes might be a prescription to the reader to imagine or draw out the ratios before he gives a solution set of numbers that fit the ratios.

[74] It contrasts with Nicomachus and other authors who provide the numbers when the relationships are first described.

and the nouns:

(1) *momentum*: literally, movement and motion; but as a trope, weight or principle. Boethius means the latter but alludes to the former.[75]

(2) *regula*: *regula* [196.9, 198.23] is at first ambiguous, whether literally as a measuring stick or, more abstractly, methodology. Boethius explains his double entendre in 198.23 by referencing the ruler [198.25] and a methodology for measurement [198.26]. He translates it from ruler (κανών), playing on the fact that it is also a metonym for monochord, a primary tool for determining intervals and made of wood like a measuring stick.[76]

Nicomachus, on the other hand, uses etymologically linked words for discovery, like "They were discovered" (ηὑρέθησαν [Jan 245.18]), "he discovered" (literally, "he seized with his mind," καταληφθέντι [Jan 245.23]), "he discovered" (εὕρισκεν [Jan 248.17]). Boethius inconspicuously emphasizes weight and the quantification of measurements in a clever albeit clunky manner. The ambiguity shades the meaning since context could support either the etymological meaning or trope. It does not evoke the elegance and clarity of Lucretius and Cicero, but Boethius's writing is neither poetry nor oration. The material is technical, and, although his diction may be cumbersome, Boethius is writing (however, ironically) with intelligibility in

[75] Bower (1989) 17, n. 69.
[76] Bower (1989) 17, n. 67; 19, n. 74.

mind. It is still the case that the alterations and additions to the story, the logic puzzle, and word play construct a unique version, which conforms to Boethius's intention to unite mathematics and music though numbers.

Boethius's version of the story is a distinct retelling. Even though Boethius is certainly indebted to Nicomachus as his primary source, the story in *De musica* is not a facsimile. The structure and elements of the story remain the same, but Boethius alters and augments the critical moment of Pythagoras analyzing the hammers to emphasize their role. Their weight is the basis for formulating the ratios of musical intervals. The analysis of the discovery is transferred to the smithy where the hammers are from Pythagoras's home where the apparatus of weights and strings are in Nicomachus's *Manual*. It is clear that Boethius understands the story.

He shapes the story for his purposes and integrates it into the first book of *De musica*. Its inclusion in *De musica* asserts Boethius's belief in the possibility of the story, which contrasts with Ptolemy and Mersenne's repudiation. Boethius affirms the story; it has value for him. Boethius's affirmation is built on the premise of the sound of weights correlating to pitches that, in pairs, correspond to intervals. There is no indication that Boethius doubts this premise, but he does not present the story as if it were a challenge in the story's possibility. His negligence shows his affirmation; otherwise, uncertainty would disrupt the dynamics of Boethius's presentation of mathematics and music. The story is an essential part of *De institutione musica* for Boethius; it is part of what Boethius believes are the fundamentals of music. The story acts as the linchpin between arithmetic and music, the subjects of numerical quantity. It reflects the nature of the Atkinson's critical nexus, the blending of musical topics, inseparable and fundamental. Boethius is able to justify the construction of the quadrivium as an interconnected concept and a developed pedagogy.

And if you listen very hard, the tune will come to you at last.

—Led Zeppelin, "Stairway to Heaven"

After examining the appearance and use of the musical discovery in the Nicomachus's original account, Ptolemy's criticism of Nicomachus, Mersenne's criticism of Pythagoras, Boethius's endorsement, one of the prime questions that remains is what to consider Pythagoras's discovery. Is it a historical tale? Is it a pedagogical narrative? Is it legend? Is it myth? I have been careful not to label the story with charged words like legend, myth, or fable throughout this text so as not to taint the reading of the story. It is ambiguous and conjectural whether the historical Pythagoras discovered musical intervals as described. The actions within the story, though, do not necessarily need to be false as well. A story, in general, may have true elements while seeming like fiction. Indeed, the details of Pythagoras's discovery do not invite a sense of possibility. The level of refutability, or the possibility of contradicting Pythagoras's observations and his results, is high. Pythagoras is a mythical and mystical character; the explanation of his discovery is incomplete and unfulfilling. The story of the smithy as a whole or by its sections individually reads quite well as legend. It fits into many of the characteristics one finds in a legend. Folklorist Elliott Oring broadly defines legend, first, as a narrative that is reported as truth. This is a necessary condition of a legend. Second, a legend may contain some degree of interpretation and inference left to the audience after the legend presents a set of facts. This condition is not necessary but frequently occurs in a legend.[77]

The story in its full form has typically been presented as truth. Implied inference completes the gaps in the details of the story. There are a number of rhetorical devices—framed by the Aristotelian art of persuasion—that suggests the story of Pythagoras has a legendary status: (1) authority of the source, (2) framing, (3) narrative detail, (4) story logic, and (5) pathos as "conforming to the cognitive. . . expectations of [the] audience."[78]

(1) *Authority of the Source*: Pythagoras is cited as the discoverer of the musical intervals, whose fame by itself lends authority to the claim. This is usually integrated with the rest of an author's text. Pythagoras appears as the source in many of Nicomachus's explanations of musical and mathematical phenomena elsewhere in *Enchiridion*. In the end of Book I, *De musica*, Boethius alludes to the unquestionable wisdom of Pythagoras. Newton invokes him as ancient authority in his proof of universal gravitation.

(2) *Framing*: The discovery is set in an episodic sense of time; the events happened at some point in the fuzzy past. A chronology of events is possible, but dating the chronology itself is impossible. For example, the formulaic

---

[77] Oring, Elliott. "Legendry and the Rhetoric of Truth." *The Journal of American Folklore* 121, no. 480 (April 1, 2008): 129.
[78] Oring 131–132, 140–141, 147–150, 157.

narrative markers ποτὲ (one day. . .) in Nicomachus and interea (meanwhile) in Boethius are present.[79] That is, meanwhile one day, Pythagoras is walking by the blacksmith while thinking of a way to measure musical sounds. Compare this with the typical "once upon a time" used in fairy tales.

(3) *Lack of Narrative Detail*: The amount of detail for how Pythagoras discovers the intervals is insufficient. He hears them at the smithy and notices that pitch correlates with the weight of each hammer. Weight of the hammers alone determine pitch, but the dimensions of the apparatus he constructs is unclear as well as how he produces the same pitches on the apparatus heard from the work shop. It is also unclear as to how Pythagoras, thus satisfied of his work on the apparatus, applies his results onto instruments.

(4) *Story Logic*: Beyond Pythagoras's desire to find a tool to measure pitches, there is no certain motivation for transferring his results from the apparatus onto instruments other than with the pure curiosity of seeing whether the same results may be found. Authors relate Pythagoras's intentions to their own as an articulation of their text's arguments. For example, Nicomachus desires to justify his explanations of intervals and music theory in general earlier in *Enchiridion* with a story that constructs the intervals.

[79] Jan 245, line 6. Friedlein, 197, line 3.

(5) *Pathos*: Since the musical story is a technical trope, it is nearly always found in works about music, mathematics or Pythagoras. The audience has already been primed to understand the topics' interconnections. Thus, when Nicomachus presents a story about Pythagoras and musical ratios in the middle of *Enchiridion*, the audience is much more likely to receive the story as legitimate. We should note the highly technical aspects of *Enchiridion* before and after the sixth chapter, compared to the more narrative aspects of the sixth chapter. The story in Macrobius is a manifestation of music of the spheres; the intervals Pythagoras investigates are the same ones that govern the heavens.

Without context of the subject matter surrounding the story in each author's work, (aside from the pathos, which itself does require context), these elements of legendry strongly suggest that the story is, in fact, a legend. This may explain why so many interpreting this story might dismiss or accept Nicomachus's account outright without criticism. The writers who largely repeat this story verbatim tend to understand it as being genuine, while those who examine the story in whatever critical capacity view it as false.

A synergistic result occurs from the interaction of the believers and nonbelievers. The story survives history because believers propagate it faithfully with the mystically elegant but vague details of Pythagoras's discovery. The nonbelievers, in turn, latch onto the fantastic narrative elements. They prejudge their perspective for this story as just another tale in which a magical Pythagoras achieves the impossible. There is a common gap in explaining whether the story is possible in reality, primarily because of how sound is produced in the story. Indeed, this story has been transformed. Although Nicomachus was first to record this story in his musical treatise *Enchiridion*, he is no longer known as its original author. The narrative becomes legendary and exists in its own right. Mersenne refers to Pythagoras directly and not to any previous author who wrote on the story. Pythagoras's discovery gains a life of its own and becomes a legend itself. Pythagoras visits the smithy on his own accord, without anyone directing him.

We still see the effects the legendary status of the story in a recent publication by science historian Alberto Martínez. His book, *Science Secrets*, examines a series of science myths commonly found in history from Pythagoras to Einstein. The premise is that each myth is ultimately false, either based on no conclusive historical documentation or verifiable scientific observation. Martínez artfully marshals his evidence for contradictions in the accounts themselves and provides the details for why a myth might have been inflated from a miniscule truth. In many cases, the context for each myth is highly nuanced. With reference to Pythagoras, Martínez deconstructs the myth of Isaac Newton and the apple falling on his head. Newton, while formulating a general theory for gravity, sought to reinforce his claim of first discovery against the simultaneous work of his rival Robert Hooke by citing Pythagoras as ancient authority. In particular, Newton correlated that Pythagoras experimenting with weights and strings must have conceived of the concept of universal gravitation first. Pythagoras and his followers must have discovered that musical ratios relate to a harmony of the spheres, which Newton interpreted as a concept of universal gravity. Newton's invocation is more rhetoric than technical support though. By connecting himself to the ancients, Newton, thus, stands himself on the shoulders of giants.[80]

Martínez assumes that the story about Pythagoras is false. He refers to the usual conclusion that ratio of string tension to pitch does not correlate one-to-one with the ratio of weight to pitch. Although that is a true statement, Martínez fails to explain the mechanics of pitch generated from weights and that from strings. He does not indicate why the story is necessarily false, which begs the question. It is ironic, then, that Martínez dismisses the story about Pythagoras using a commonly held assertion, while disproving the myth of Newton and the falling apple.

The Decline of Pythagoreanism

The authors discussed in this text have are all considered theorists, writing about music conceptually. There is little to no mention of performance practice. This congruity aside, authors' positions may be based on their historical period. A majority of authors from Antiquity and the Middle Ages view the story favorably. The dominance of Pythagorean thought may explain this trend. The essence of Pythagoreanism is the pervading numerology in its approaches to philosophy and

[80] Martínez, Alberto A. *Science Secrets: The Truth About Darwin's Finches, Einstein's Wife, and Other Myths*. Pittsburgh: University of Pittsburgh Press, 2011, 55–57.

science. Nicomachus subscribes to this as a self-declared Pythagorean in his only surviving complete treatise, *Manual of Harmonics*. His influence on Boethius is evident; much of Book I of Boethius's *De institutione musica* is structured, if not borrowed, from Nicomachus's work. Boethius himself is not a Pythagorean, but his approach to music is Pythagorean in nature. Number is prime.

Ptolemy is the notable exception. In his *Harmonics*, he promotes the monochord as the tool of choice for studying intervals. Ptolemy dismisses any notions of utility for Nicomachus's apparatus. His strong preference for the monochord compels his rejection of the story, rather than addressing the refutability of Pythagoras's discovery. Ptolemy blazes his own trail using aspects of Pythagorean and Aristoxenian musical thought to construct a blended, but unique interpretation of musical sounds. It is suspicious that Ptolemy does not explain why the apparatus is insufficient for studying music, while being thorough in other topics in *Harmonics*. The doubts about the authenticity his data in his *Syntaxis* are troubling for interpreting his view on Pythagoras's discovery. Ptolemy obfuscates the discussion by omitting the reasons for the apparatus's failure while promoting the positive aspects of the monochord. Regardless on how Ptolemy studies musical intervals, the role of numbers to construct them is still evident.

Up until the Renaissance, the music all of these writers discuss is only with a purity of theory and lacks any taint of performance practice. Like Pythagoras's discovery, music of the spheres is premised on the ratio of numbers. It is an inaudible but harmonic conception of celestial bodies, like the planets and the Sun.

It was originally a Pythagorean notion, but persisted through the end of the Renaissance. The story is more tangible than that though, with hammers, anvil, strings, weights, and other physical objects. Unlike the music of the spheres, one should be able to hear musical sounds, in whatever capacity, from the vibrations of those objects. However, authors writing these theoretical tests never seem to actualize the events of the story to replicate about what they write. How do we know something works in reality without realizing it?

The Renaissance marks a shift in the reception of the story. It is increasingly viewed as insubstantial among the growth empiricism and the scientific method. Pythagorean theorizing and thought experiments fall out of favor, since no direct observation can be made in this way. Boethius himself becomes repudiated as an authority; Pythagorean numerology as the agent of authority is no longer sufficient to explain natural phenomenon. In a sense, Aristoxenian thought finally prevails over Pythagoreanism. To Aristoxenus and his followers, the ear is the ultimate tool for listening, not the monochord or any device Pythagoras may have used to define musical sounds. The investigation of phenomena guides theory in contrast to a conceptualization of them that may or may not have any relevance in practice.

A case in point is acoustic work of lutenist and theorist Vincenzo Galilei (1520s–1591).[81] The father of Galileo studied under the venerable theorist Gioseffo Zarlino (1517–1590), who may be best known for introducing third and sixth

[81] Palisca, Claude, "Scientific Empiricism in Musical Thought," in Rhys, Hedley Howell. *Seventeenth Century Science and Arts*. The William J. Cooper Foundation Lectures, Swarthmore College, 1960. Princeton, N. J.: Princeton University Press, 1961. 91–137.

intervals as consonances. The addition of the third interval to create a triad with the root and the fifth was not only permissible, but preferred in tuning. Zarlino was a Pythagorean in disguise. His belief in the perfection of intervals and the Pythagorean tetractys {1, 2, 3, 4} guided his construction of music theory: numbers are key to musical pitch. To improve tuning with triads, he added two numbers {5, 6} to the tetractys to expand it into a senarius {1, 2, 3, 4, 5, 6}. With these new numbers, the ratio 5:4 created the major third interval, 6:5 the minor third, and 5:3 the major sixth. The exception is the minor sixth from the ratio 8:5, which Zarlino conditionally allowed.[82]

Zarlino introduced Galilei to the concept of musical numbers and other ancient musical writings in their study of acoustics. Coinicidentally, Galilei encountered the writings of Aristoxenus in his annotations to Zarlino's *Le istitutioni harmonische*. Troubled by the discrepancies among the ancient and medieval theorists, Galilei sought Girolamo Mei, a Florentine humanist, who reportedly read all available Greek and Latin musical writings of the time. Mei, himself a empiricist Aristoxenian, converted Galilei's view to his, in opposition to his former teacher, Zarlino, who by chance was Mei's rival.[83] With Mei, Galilei realizes that the fluidity of pitches on a viola violates the discreteness of musical numbers. Intervals consist of

---

[82] Zarlino, Gioseffo. *Le istitutioni harmoniche. A Facsimile of the 1558 Venice ed*. Monuments of Music and Music Literature in Facsimile. Second Series, Music Literature 1. New York: Broude Bros, 1965, xiv–xvi, I.15.25–56.

[83] Mei, Girolamo. *Letters on Ancient and Modern Music to Vincenzo Galilei and Giovanni Bardi; a Study with Annotated Texts*. Musicological Studies and Documents no. 3. n.p.: American Institute of Musicology, 1960. Mei's correspondence to Galilei reveals his predilection for Aristoxenian musical thought.

“minimal particles, almost like atoms,” so that a collection of infinitesimal, discrete units creates continuity.[84] The senarius is insufficient to explain these intervals.

Galilei analyzes the acoustic properties of different materials like weights, strings, and pipes. He does not interact with Pythagoras himself in his experiments, but challenges the concepts of Pythagoreanism. He concludes that the length of string matched one-to-one with its resonance frequency, For string tension or weights attached to strings, the frequency, Galilei found, is the square of the applied tension. For cavity resonators, like in organ pipes, the frequency is a cube of the volume of air inside the cavity. Based on these conclusions, Galilei dismisses the story about Pythagoras’s discovery in his last published work. He assumes that Pythagoras meant a linear relationship between values of weight in the apparatus and the produced interval, which does not correspond to his findings on string tension.[85] What Galilei fails to account for is the ambiguous relationship that the story describes. There is some relationship, but every instance of the story in history never specifies it as linear, exponential, or otherwise. Galilei is the closest in his empirical observations to realize the exact connections of mass objects and their resonance frequency, but his assumption of a linear relationship misleads his conclusion. Nevertheless, Galilei is representative of the turn away from

---

[84] Galilei, Vincenzo, “Discorso particulare intorno alla diversità delle forme del Diapason,” Florence, Bibl. Naz. Cent., Mss. Galileiani, Ant. a Galileo, Vol. III, fol. 54v. For the Italian text and an English translation, see Palisca, Claude, ed. *The Florentine Camerata: Documentary Studies and Translations*. Music Theory Translation Series. New Haven: Yale University Press, 1989, 180–197. For additional commentary, see also Galilei, Vincenzo, ed. Claude V. Palisca. *Dialogue on Ancient and Modern Music*. Yale University Press, 2003.

[85] Galilei, Vincenzo, “Discorso. . .delle forme del Diapason,” III, fol. 47v–49r. See Palisca, *The Florentine Camerata*, 185–187 for commentary.

Pythagorean numerology toward Aristoxenian empiricism, and in general, of modernity and the Scientific Revolution of the 16th and 17th centuries. He refutes the story based on his experimentation with string and instruments, the former of which Mersenne later confirmed. Instead of accepting the tradition of lore and the past, Galilei rejects his teacher Zarlino's doctrine of numbers in preference to a practical and observable construction of music.

"What's the Frequency, Kenneth?": Harmonic vs. Proportional

The question that ultimately remains is what the relationship is between mass objects like the hammers and its resonance frequency. The frequency of a plucked string based on its length is a one-to-one relationship. The square of the string tension generates the same frequency. Vincenzo Galilei discovered these two formulas for string; Marin Mersenne confirmed them later. Galilei also determined the resonance frequency for cavity resonators, which is the cube of the volume of air contained in the cavity. The resonance frequency of the hammers and the weights remains unsolved. Depictions of the apparatus in manuscripts typically show the weights as bell-shaped objects. The resonance frequency of a bell is especially difficult to model, but the vibrations of a conical warping of a plate (for example, a cymbal or a gong) may approximate it. The vibration of a free-standing bar (like one on a glockenspiel) can model the vibration of hammers. A tuning fork would also be an analog to a hammer. The physical dimensions, volume, and type of material are the prime determinants of frequency here.

Accordingly, the frequencies of the longitudinal vibrations of a bar are harmonic. That is, overtone frequencies are based on integer multiples of the fundamental frequency.[86] They are not harmonic, though, for the transverse vibrations and with the ratios 1 : 2.70 : 5.40 : 8.93, and so on.[87] The frequencies of the vibrations of a bell are also nonharmonic. These values are not whole or integer numbers, but rational numbers. Pythagorean numerology cannot account for rational numbers. What Galilei failed to realize from his experiments was not that the relationship between weight and pitch was false, but that it can be nonharmonic. Rational numbers provide the continuity Galilei sought when he heard bending of pitch on a viola. Thus, if a vibrating cone represents the weights of the story's apparatus and a vibrating bar for a hammer, there is a definite relationship between the mass of an object and its resonant frequency. The relationship is not always harmonic and expressible in whole numbers, but it is still proportional, just as Nicomachus originally related.

What the Pythagoreans and believers of the story failed to realize was that the relationship was nonharmonic. The Pythagorean tetractys and Zarlino's senarius do not accurately describe the musical sounds of hammers or weights. It is ironic that Pythagoreanism's unified theory of whole numbers for musical thought, cosmological, and philosophy collapses on the nonharmonic nature of certain objects' resonance. In light of this, Boethius's conception of the quadrivium is based

[86] Heuven, Engelbert Wiegman van. *Acoustical Measurements on Church-Bells and Carillons*. 's-Gravenhage: Gebroeders Van Cleef, 1949.

[87] Bishop, R. E. D., and D. C. Johnson. *The Mechanics of Vibration*. Reissue. Cambridge University Press, 2011.

on nonharmonic natural of rational numbers in the relationship of weight and pitch. He did not intend this, but he also does not necessarily exclude the any possibility of rational numbers. They are just not his preferred set of numbers. His linchpin still holds the pedagogical regime together.

The modern reception of Pythagoras has been cautious to credit his supposed accomplishments, seeking to dismiss or disprove them. The apprehension is understandable; the weight of lore and mythical attribution undoubtedly bears down on any study of the ancient philosopher. Latching onto Mersenne's dismissal of the story, modern commentators themselves have violated physical laws to disprove the physicality of generating musical intervals. Newton's Third Law of Motion describes how force acts between a pair of objects, and not on a single object, in equal magnitude but opposite directions. The conservation of momentum applies to all colliding bodies. Their loss of kinetic energy after impact generates sound energy. The spectrum of frequency of sound is unique for different hammers hitting an anvil because of their differences in weight. Frame of reference does not matter in classical mechanics, so whether the anvil strikes the hammer or the hammer strikes the anvil does not matter. The force of their collision is equal and opposite of each other. Modern consensus, that the Pythagorean basis for intervals from mass in inaccurate, is correct, but the reasoning to debunk the story is also false. The trend to disprove or reject the story has been a recent but static phenomenon. Since Galilei, there have been no reports of physically testing these assertions repeated in modern scholarship. It is ironic to note the reliance on description and logical argument to refute the Pythagorean story rather than replicating Galilei's work to confirm his conclusions.

The modern commentators are as indicative as any of the other authors in history in their intentions for the story about Pythagoras's discovery of the musical intervals. The purpose has mostly been expressed rhetorically. At one level, it is the evaluation of musical sounds with a numerical basis. Its technical nature shades the author's intentions when integrating the story in his text. At another level, the purpose shapes each version of the story as well as the authors constructing it. The lore and perennial fame of Pythagoras are too attractive for those who follow or denigrate his ideas ignore to the ancient philosopher. Whether legend or truth, Pythagoras's discovery, ultimately, is an instrument itself. It has been continually manipulated to meet the needs of an author. Affirmation, repudiation, and refutation are not just for the story itself but also for each author's conception of Pythagoras.

Bibliography

Atkinson, Charles M. *The Critical Nexus: Tone-System, Mode, and Notation in Early Medieval Music*. AMS Studies in Music. Oxford: Oxford University Press, 2009.

Barbera, André, ed. *The Euclidean Division of the Canon: Greek and Latin Sources: New Critical Texts and Translations on Facing Pages, with an Introduction, Annotations, and Indices Verborum and Nominum Et Rerum*. Greek and Latin Music Theory. Lincoln: University of Nebraska Press, 1991.

Barker, Andrew. *Greek Musical Writings: Volume II, Harmonic and Acoustic Theory*. Cambridge: Cambridge University Press, 1989.

———. *Scientific Method in Ptolemy's Harmonics*. Cambridge: Cambridge University Press, 2000.

Bishop, R. E. D., and D. C. Johnson. *The Mechanics of Vibration*. 2011 Reissue. Cambridge: Cambridge University Press, 1960.

Boethius. *Anicii Manlii Torquati Severini Boetii De Institutione Arithmetica Libri Duo: De Institutione Musica Libri Quinque. Accedit Geometria Quae Fertur Boetii*. 1966 Reprint. Frankfurt: Minerva, 1867.

———. *Fundamentals of Music*. Edited by Calvin M Bower. New Haven: Yale University Press, 1989.

Bower, Calvin M. "Boethius and Nicomachus: An Essay Concerning the Sources of De Institutione Musica." *Vivarium* 16 (1978): 1–45.

Burkert, Walter. *Lore and Science in Ancient Pythagoreanism*. Translated by Edwin Minar. Cambridge: Harvard University Press, 1972.

———. *Weisheit Und Wissenschaft; Studien Zu Pythagoras, Philolaos Und Platon*. Nuremberg: H. Carl, 1962.

Christensen, Thomas Street, ed. *The Cambridge History of Western Music Theory*. Cambridge: Cambridge University Press, 2002.

Creese, David. *The Monochord in Ancient Greek Harmonic Science*. Cambridge Classical Studies. Cambridge: Cambridge University Press, 2010.

Düring, Ingemar, ed. *Ptolemaios Und Porphyrios Über Die Musik*. 1980 Reissue. Ancient Philosophy. New York: Garland, 1980.

Edmiston, Jean. "Boethius and Pythagorean Music." *The Music Review* 35 (1974): 179–184.

Florentius. *Book on Music*. Edited by Bonnie J Blackburn and Leofranc Holford-Strevens. The I Tatti Renaissance Library 43. Cambridge: Harvard University Press, 2010.

Gaffurius, Franchinus. *Theorica Musice*. New York: Broude Bros, 1967.

Galilei, Vincenzo. *Dialogue on Ancient and Modern Music*. New Haven: Yale University Press, 2003.

Grafton, Anthony. "Epilogue: Boethius in the Renaissance." In *Boethius: His Life, Thought and Influence*, edited by Margaret T Gibson, 410–415. Oxford: Blackwell, 1981.

Guthrie, W. K. C. *A History of Greek Philosophy*. Vol. 1. 5 vols. Cambridge: Cambridge University Press, 1971.

Hawkins, John. *A General History of the Science and Practice of Music*. 2 vols. 1963 Reprint. New York: Dover Publications, 1853.

Heller-Roazen, Daniel. *The Fifth Hammer: Pythagoras and the Disharmony of the World*. New York: Zone, 2011.

Heuven, Engelbert Wiegman van. *Acoustical Measurements on Church-Bells and Carillons*. The Hague: Gebroeders Van Cleef, 1949.

Hine, William L. "Mersenne and Copernicanism." *Isis* 64, no. 1 (March 1, 1973): 18–32.

Jan, Karl von, ed. *Musici Scriptores Graeci: Aristoteles, Euclides, Nicomachus, Bacchius, Gaudentius, Alypius Et Melodiarum Veterum Quidquid Exstat*. 1962 Reprint. Hildesheim: Georg Olms, 1899.

King, Peter. "Boethius: The First of the Scholastics." *Carmina Philosophiae* 16 (2007): 23–50.

Levin, Flora R. "Πληγή and Τάσις in the Harmonika of Klaudios Ptolemaios." *Hermes* 108, no. 2 (January 1, 1980): 205–229.

Levin, Flora. *The Manual of Harmonics of Nicomachus the Pythagorean*. Grand Rapids: Phanes Press, 1994.

Macrobius, Ambrosius Aurelius Theodosius. *Ambrosii Theodosii Macrobii Saturnalia Apparatu Critico Instruxit, In Somnium Scipionis Commentarios Selecta Varietate Lectionis Ornavit*. Leipzig: Bibliotheca Teubneriana, 1963.

———. *Commentary on the Dream of Scipio*. Translated by William Harris Stahl. New York: Columbia University Press, 1952.

Martínez, Alberto A. *Science Secrets: The Truth About Darwin's Finches, Einstein's Wife, and Other Myths*. Pittsburgh: University of Pittsburgh Press, 2011.

Mathiesen, Thomas. *Apollo's Lyre: Greek Music and Music Theory in Antiquity and the Middle Ages*. Publications of the Center for the History of Music Theory and Literature 2. Lincoln: University of Nebraska Press, 1999.

Mei, Girolamo. *Letters on Ancient and Modern Music to Vincenzo Galilei and Giovanni Bardi; a Study with Annotated Texts*. Musicological Studies and Documents 3. American Institute of Musicology, 1960.

Mersenne, Marin. *Questions Harmoniques*. Stuttgart: F. Frommann, 1972.

Münxelhaus, Barbara. *Pythagoras Musicus: Zur Rezeption Der Pythagoreischen Musiktheorie Als Quadrivialer Wissenschaft Im Lateinischen Mittelalter*. Orpheus-Schriftenreihe Zu Grundfragen Der Musik 19. Bonn-Bad Godesberg: Verlag für Systematische Musikwissenschaft, 1976.

Murchie, Guy. *Music of the Spheres: The Material Universe, from Atom to Quasar, Simply Explained*. New York: Dover Publications, 1967.

Nauta, Lodi. "A Humanist Reading of Boethius's Consolatio Philosophiae: The Commentary by Murmellius and Agricola (1514)." *Brills Studies in Intellectual History* 96 (1999): 313–338.

Newton, Isaac. *Philosophiae naturalis principia mathematica*, 1687.

Newton, Robert R. *The Crime of Claudius Ptolemy*. Baltimore: Johns Hopkins University Press, 1977.

Oring, Elliott. “Legendry and the Rhetoric of Truth.” *The Journal of American Folklore* 121, no. 480 (April 1, 2008): 127–166.

Palisca, Claude V. “Scientific Empiricism in Musical Thought.” In *Seventeenth Century Science and Arts*, 91–137. The William J. Cooper Foundation Lectures, Swarthmore College, 1960. Princeton: Princeton University Press, 1961.

Planck, Max. *Scientific Autobiography and Other Papers*. New York: Philosophical Library, 1949.

Plato. *Platonis Opera*. 3 vols. Scriptorum Classicorum Bibliotheca Oxoniensis. Oxford: Oxford University Press, 1995.

Proclus. *Procli Diadochi in Primum Euclidis Elementorum Librum Commentarii*. Edited by Gottfried Friedlein. 1967 Reprint. Bibliotheca Scriptorum Graecorum Et Romanorum Teubneriana.[S. G.]. Hildesheim: Georg Olms, 1873.

The Florentine Camerata: Documentary Studies and Translations. New Haven: Yale University Press, 1989.

Weiss, Piero, and Richard Taruskin. *Music in the Western World: A History in Documents*. New York: Schirmer Books, 2008.

Zarlino, Gioseffo. *Le istitutioni harmoniche*. Monuments of Music and Music Literature in Facsimile. Second Series, Music Literature 1. New York: Broude Bros, 1965.