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Three Essays on Capital Adjustment, Reallocation and Aggregate Productivity

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Three Essays on Capital Adjustment, Reallocation and Aggregate Productivity

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Three Essays on Capital Adjustment, Reallocation and Aggregate Productivity

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This dissertation consists of three chapters. Chapter one estimates the capital adjustment costs at the plant level in a model with entry and exit. We find that the estimated variance of plant-specific productivity shock is larger than that obtained from balanced panel estimation. Estimation using the unbalanced panel generates a larger irreversibility cost, a smaller disruption cost, and a smaller convex cost, all compared with the estimates by Cooper and Haltiwanger (2006). In chapter two, we study how much of the aggregate productivity changes can be accounted for by the capital reallocation. We also study the impact of capital reallocation on the productivity dispersion across firms. We find that capital reallocations accounts for roughly 12 percent of the labor productivity growth in the late 1980s in the U.S. The dispersions of both labor productivity and capital productivity are reduced as the reallocation activity increases. When the economy-wide technology has a positive

change, the reallocation increases temporarily then drops to its original level. After a short transition, the economy settles down with an increased labor productivity. Chapter three further studies the quantitative role of reallocation, entry and exit in the growth of aggregate productivity. We find that, without including in the model the forces that drive the entry and exit changes, the model economy has a modest increase in the aggregate productivity as a result of decrease in the fixed reallocation cost.

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Chapter 1

Plant Entry and Exit and Capital Adjustment Cost

1.1 Introduction

Production and cost functions are fundamental components in economic models. Estimating these functions is crucial and a necessary step to study productivity and investment dynamics. It is also indispensable for analyzing the effects of policy and regulation changes on the firm's production and investment. For instance, to study how the investment tax credit policy affects investment by firms and plants, one first needs to have an estimate of investment cost.

This paper estimates capital adjustment cost functions in a dynamic programming model that allows plant entry and exit. This extends Cooper and Haltiwanger (2006) where the authors estimate capital adjustment costs on a balanced panel of the large U.S. plants. We also estimate the production function. Using the simulated method of moments estimation based on the unbalanced panel data of plants, we provide a unified approach to estimating simultaneously the production function and capital adjustment cost parameters. The estimation involves moment matching but requires intensive computation.

Cooper and Haltiwanger (2006) estimate the structural parameters of both non-convex and convex components of capital adjustment costs to explain the stylized facts of investment dynamics of LRD plants. They find that non-convex and convex components of capital adjustment costs together fit best the LRD investment moments. Two potential problems arise from Cooper and Haltiwanger's estimation. First, there exists potential selection bias due to excluding plants that exit from and enter into the industry in the sample period. The productivity shock estimated from the balanced panel is biased. The idiosyncratic shock of plants in the balanced panel is truncated from the left tail of the distribution and causes upward bias. This is intuitive since drawing a low productivity shock is one source driving the plant exit. Related with productivity shock truncation, plant entry and exit decisions are correlated with the investment costs. It is observed that the investment pattern of newly entered and exiting plants differs from that of continuing plants, as documented by Becker etal (2005).

Secondly, Cooper and Haltiwanger estimate the production function as a first step separately from their estimation of adjustment cost parameters. Their estimate of production curvature is potentially biased due to excluding entry and exit. The authors use generalized method of moments to estimate the quasi-difference equation of production function. The serial correlation of productivity shock does not cause bias in the differencing equation. Excluding the endogenous exit choice from their estimation may cause selection bias, which is addressed in Olley and Pakes (1996).

The estimation bias of production function is well recognized. Olley and Pakes (1996) and Levinsohn and Petrin (2003) use semi-parametric steps to correct the selection bias and the simultaneity bias. These authors base their estimation methods on the structural model of the plant's problem. Olley and Pakes (1996) find that omitting the endogenous exit decision causes sample selection bias when estimating the production function. The capital coefficient doubles and the labor coefficient drops by about 20 percent when replacing the balanced panel data with an unbalanced one to estimate the production function of telecommunication equipment plants. The resulting industrial productivity derived from the balanced panel is upward biased, because exiting plants tend to have lower productivity than continuing plants.

Others use the dynamic GMM methods to address the bias problem of serially correlated unobservable productivity shocks or plant heterogeneity (see Arellano and Bover 1995, and Blundell and Bond 1999).

One problem with Olley and Pakes (1996) is that the invertibility condition of policy function does not hold for observations with zero investment. This can be a serious problem if the data sample has a large amount of observations of zero investment, for example, in the data used by Bergoeing, Hernando and Repetto (2005) where 40 percent of observations have zero investment.[1]

[1]Pakes (1994) shows that for invertibility condition to hold, it suffices if capital adjustment cost is non-increasing in capital for a fixed investment level. When zero investment presents, policy function is not invertible.

It is important that, even if Cooper and Haltiwanger had used the Olley-Pakes estimation procedure to estimate the production function, the resulting estimates of adjustment costs would be still potentially biased because zero investment observations need to be thrown away. Omitting zero investment would cause more serious bias of adjustment cost parameters simply because zero investment is one observed fact that motivates Cooper and Haltiwanger to estimate non-convex adjustment costs.

Therefore, three identification issues of estimation arise, namely the simultaneity problem, the selection problem and the invertibility problem. Simultaneity problem arises because the observed inputs (e.g., capital) are correlated with productivity shocks. Also, the unobserved productivity is serially correlated. The selection problem arises when estimation regressions are run on a balanced panel data in which only survived plants are included. The invertibility problem appears because the policy function is not monotonic if there is inaction.

Incorporating all the potential bias problems in one econometric model seems to be a daunting job. It is natural to use the simulated method of moments in hoping to understand these problems. Our unified procedure is as follows: Given initial values of parameters, we simulate the production, investment and exit decisions of a panel of plants. We compute a set of moments from both the full panel and the balanced panel. We use the minimal distance method to match the simulated moments with moments calculated from the plant level data. A subset of moments is obtained from Cooper and

Haltiwanger's estimation of the production function.

Entrants and exiting plants are usually much smaller than continuing plants in the U.S. data. In addition to the econometric issues in the model, comparing estimates from balanced and unbalanced panels may indicate the differential investment decisions made by large and small plants.

Our estimation results show that moving from a balanced panel to an unbalanced panel, the parameter values of capital adjustment costs change. In the model with the fixed cost, the full panel estimation gives larger values of adjustment costs, while in the model with the disruption cost the estimates of irreversibility and quadratic cost are larger and the estimate of disruption cost is smaller than those obtained in balanced panel estimation. In the full-panel estimation, the standard deviation of productivity shock is much larger compared with the balanced panel. The production concavity is larger in all estimations with entry and exit.

In addition, we find that the relative magnitude of different components of adjustment costs are important for plant turnover. The estimates based on the balanced panel can not match plant turnover moments well. Our estimation results show that the total adjustment cost may not change much compared with estimates on the balanced panel, but the component relative magnitudes have important implications for plant entry and exit.

Before presenting our model and estimation, it is noted that our estimation does not exactly correct the bias problems discussed above in the sense that we do not provide an improved estimation technique based on the same model of Cooper and Haltiwanger (2006). Rather, our parameter space is different from theirs. The comparison between our estimates and their estimates makes sense only if we realize that we also estimate other parameters while we estimate cost parameters

1.2 Plant Turnover Facts

LRD is a sub-sample of the Annual Survey Manufactures (ASM). Our full data sample is ASM, hence moments for the full panel should come from it. In this section we provide some facts about investment and entry/exit of ASM plants.

ASM provides annual plant data between two economic censuses. The plants are selected from economic census. Among 55,000 plants in ASM, about 28,000 are selected with certainty, these plants are large with more than 250 employees. About 16,000 plants are non-mail surveys, accounting for only 2 percent of shipments, these small plants are selected randomly, and probability of being selected is positively correlated with size. Many of these are new entrants. The ASM sample rotation is every five years with only large establishments sampled with certainty across panels. As such, data for small establishments are typically left-censored in the first year of a five-year ASM panel and right-censored in the last.

Becker, Haltiwanger, Jarmin, Klimek and Wilson (2005) document the zero investment and investment spikes of ASM plants. Figure 1.1 is replicated

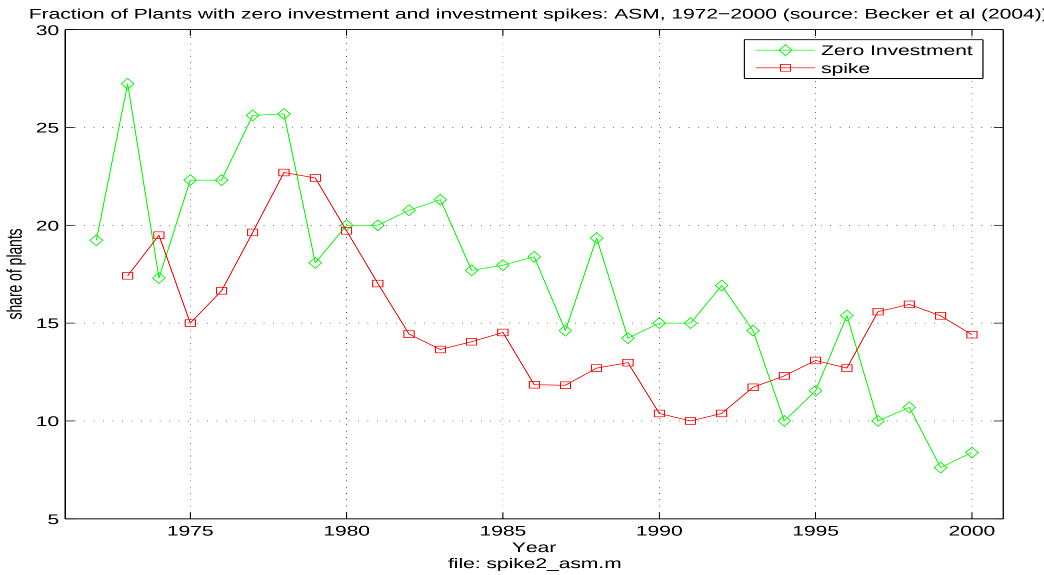

Figure 1.1: Zero Investment and Investment Spikes in ASM

from their paper. The average annual spike rate is around 0.164, and the average annual rate of zero-investment is 0.203. Becker etal (2005) also show that plants of five years or older are less likely to have zero investment, but more likely to have investment spikes.

The LRD plant investment has three features documented in Cooper and Haltiwanger (2006). To repeat, the three facts are: 8.1% of plant-year observations have investment rate near zero; 18.6% of observations have investment rate larger than 20% and 1.8% of observations have investment rate less than -20%; Serial correlation of investment rates is 0.058, and correlation of profit shock and investment rate is 0.143. In addition, 10.4% of observations have negative investment. These moments are for the balanced panel of plants.

The annual entry rate and exit rate, measured as job creation of new plants and job destruction of exiting plants, are respectively 1.44 percent and 2.52 percent of total annual employment in the U.S. manufacturing sector. According to Davis, Haltiwanger and Schuh (1996), startups account for 15.5 percent of annual job creation, while shutdowns account for 22.9 percent of annual job destruction.

Entry and exit rates in any given period vary across industries. Dunne, Roberts and Samuelson (1988, 1989) and Dunne and Roberts (1998) present some of investment, entry and exit facts of U.S. plants. Dunne, Roberts and Samuelson (1988) report that on average 79.6% of all firms exit within ten years. For firms sampled in 1963 census, the exit rate measured by output share of exited firms over total output of all firms is 0.326 between 1963 and 1977, and 0.387 between 1963 and 1982. Most exits occurred by 1972.[2] For the same 1963 census firms, the cumulative exit rate measured by the number of exited firms over number of firms in 1963 cohort is 0.741 in 1977, and 0.815 in 1982. That is, 81.5% firms in 1963 census exited by 1982. These numbers are calculated as the average of across 387 4-digit SIC industries. In their 1989 paper, the same authors document that totally 36.3% of plants exited in

[2]These numbers may be not accurate, because the total number of firms changes in census cohort, hence total output also changes. In addition, these numbers do not account for the firms who close plants while the firm still exists. Finally, these numbers are sum of exit rates in each five-year census, where the exit rate in each census is the average exit rate over 387 4-digit SIC industries.

the 1967, 1972, 1977, and 1982 Census of Manufactures. This percentage is calculated based on five-year census where new plants enter in each census.

Gort, Jensen, and Lee (2002) document the plant turnover using the ASM data. Their finding is shown in the following table.

Survival Ratios of New U.S. Manufacturing Plants: 1967-1997

Cohorts	New Plants	1972	1977	1982	1987	1992	1997
1967	42,246	0.52	0.39	0.31	0.24	0.19	0.15
		(0.52)	(0.75)	(0.79)	(0.77)	(0.79)	(0.79)
1972	53,526	1.00	0.53	0.39	0.28	0.22	0.18
			(0.53)	(0.74)	(0.72)	(0.79)	(0.82)
1977	56,897		1.00	0.49	0.34	0.25	0.20
				(0.49)	(0.69)	(0.74)	(0.80)
1982	52,860			1.00	0.49	0.34	0.27
					(0.49)	(0.69)	(0.79)
1987	46,035				1.00	0.55	0.40
						(0.55)	(0.73)
1992	59,872					1.00	0.55
							(0.55)

In this table, very small plants are excluded for which information is derived from administrative records rather than from census responses. Entry year is the census year in which each plant's record first appears. Survival ratio is the ratio for each cohort of the number of plants operating in successive census years to number of new plants in the first year of the cohort. Ratios in parentheses are survival ratios of surviving plants form each cohorts to the number surviving in the preceding year.

Figure 1.2 shows the hazard rates of plants.

Entrants and exiting plants are usually small, capital of new plants that

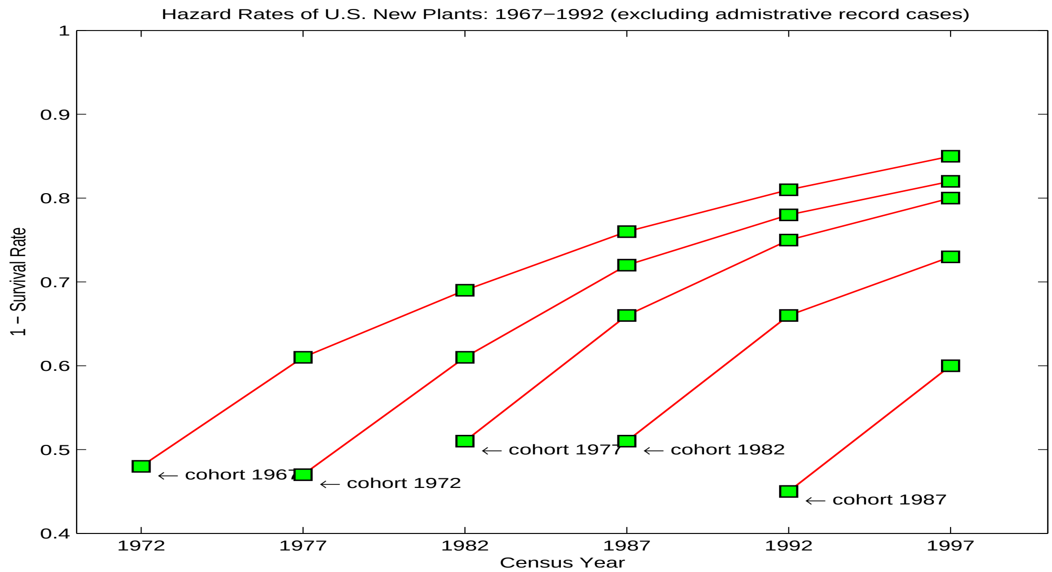

Figure 1.2: ASM: Hazard Rates of New Plants, 1967-1992.

exited within 20 years accounts for 50 percent of cohort's total capital. Plant turnover is tremendous. More than 80 percent of new plants exit the industry within 20 years.

In our model, we are interested in the capital adjustment. The plant's labor decision and we do not have labor decision in plants, it is necessary that entry and exit rates are measured in capital. The only source of capital entry and exit rates is Becker, Haltiwanger, Jarmin, Klimek and Wilson (2004). They use a measure similar to that used for job reallocation by Davis, Haltiwanger and Schuh (1996) and calculate the capital creation and destruction rates. They find that the annual capital destruction rate is below 1% between mid-1970s and 1998. This is very underestimated due to excluding first-year data of each ASM panel. The share of capital destruction accounted for by exit exceeds 20% in a typical year. Based on these authors, we set the entry rate to be 0.012, and the exit rate to be 0.015. The entry rate and exit rate is defined by these authors as the capital creation of entrants and capital destruction of the exiting plants.

We also imputed the exit rate of capital from existing literature. Davis, Haltiwanger and Schuh (1996) document that, during 1973-1988, in the U.S. manufacturing sector, shutdowns account for 22.9 percent of annual job destruction, and startups account for 15.5 percent of annual job creation. In this period, the average annual job creation rate by startups is 1.44%, and the average annual job destruction rate by shutdowns is 2.52%. This imputation is very rough, so we use the Becker etal (2006) data.

Abowd, Haltiwanger, Jarmin, Lane, Lengermann, McCue, McKinney, and Sandusky (2004) use Census of Manufactures 1997 micro data, and obtain that the capital intensity (log of the capital stock per worker employed on March 12.) has mean = 4.23, std.dev.= 1.201. Capital is book value in Census of Manufactures. This data seems to be at the firm level.

Bernard and Jensen (2004) use the LRD data in period 1987-1997 and find that the mean of log capital/labor ratio is 3.185 for surviving plants, and 2.888 for deaths. Here capital is the book value, labor is number of workers employed. Their LRD sample has 172,536 surviving plants and 63,556 plants exited over five years, so the exit rates in terms of plant number is 0.27. Hence, approximately the mean capital intensity of all plants is 3.105. Using Bernard and Jensen's probit estimation on plant survival, we predict that the average exit probability of a plant is 0.0525. Use the data from Bernard and Jensen (2004), roughly the average annual capital exit rate is 2%. [3]

We did not find data on overhead capital cost in LRD. But Davis and Haltiwanger (1991) and Barlevy (2004) document that non-production workers account for about 20% of the labor force during the post-war period. Suppose that labor accounts for two-thirds of total output, this suggests an overhead labor cost of 13% of output.

1.3 An Extended Model

Our theoretic model is an extended version of Cooper and Haltiwanger (2006). The model environment is the same as it is in Cooper and Haltiwanger, assuming that the demand for plant product is isoelastic, and the market has a monopolistic competition. Plants are independent establishments in making investment and exit decisions.

The plant's production requires the overhead inputs $\bar{k}$ and $\bar{l}$ (see Chatterjee and Cooper 1993 and Cooper 2004). The overhead cost creates a form of increasing returns. Plants can not survive until the capital stock and em-

[3] From Haltiwanger's personal website on job flows (file RTMBD.DAT), the average job destruction of deaths is 2.52%. Then the average capital exit rate is $2.52e^{2.888-3.185} = 1.87\%$. If the total population (survivors plus exiters) is used as base, the average capital exit rate is roughly $2.52e^{2.888-3.105} = 2.03\%$. However, this calculation is problematic because the measures of capital and labor in Bernard and Jensen (2004) are different from those in Davis, Haltiwanger and Schuh (1996).

ployment exceed minimal requirements.[4] Assume that the plant's technology is Cobb-Douglas, $y = s(k - \bar{k})^{1-\alpha_L}(l - \bar{l})^{\alpha_L}$ where s is technology shock. [5] The current period profit function is $\pi(s, k) = \max_l p(y)y - wl$. The resulted (indirect) profit function is $\pi(a, k) = a(k - \bar{k})^{\theta}$ where $\theta = \frac{(1-\alpha_L)(1+\xi)}{1-\alpha_L(1+\xi)}$ with ξ the inverse of price elasticity of demand.[6] The productivity shock a is a function of technology shock and wage. So, productivity shock captures all factors that affect profitability except capital, it can include shocks to demand, wage, technology, imperfect competition, etc. The productivity shock a contains parameters including θ. Given the very complicated function form of a, it is difficult to estimate ξ and α_L, so we have simplified a and assume that $a \in A$ is an exogenous shock. It is the only source of uncertainty in each plant, and has a conditional distribution $F(a', a)$. The shock a is identically and independently distributed cross sections.

[4] In the Enterprise Statistics: 1992, the "[A]uxiliaries are defined as as establishments whose employees are primarily engaged in general and business administration; management; research, development, and testing; warehousing; electronic data processing; and other supporting services performed centrally for other establishments of the same company rather than for other companies or the general public." Overall, in 1992 manufacturing sector the employment by auxiliaries is about 12.2% of firms' total employment for those firms with one or more auxiliary establishments. Auxiliaries can serve for more than one plants owned by the same firm.

[5] Overhead capital is assumed to be fixed. One interesting alternative of $\bar{k}$ is to allow it to depend on plant capital stock k_i, say a proportion of k_i. The it can also be interpreted as the capital utilization rate.

[6] Assume the plant's inverse demand is $p(y) = y^{\xi}$ with $\xi < 0$. To solve the plant's profit-maximization problem, the first-order necessary condition is $l - \bar{l} = \left(\frac{\alpha_L(1+\xi)(sk^{1-\alpha_L})^{1+\xi}}{w}\right)^{\frac{1}{1-\alpha_L(1+\xi)}}$. Plug this back to profit function, one gets $\pi(a, k) = w^{\frac{-\alpha_L(1+\xi)}{1-\alpha_L(1+\xi)}} \cdot \frac{1-\alpha_L(1+\xi)}{\alpha_L(1+\xi)} \cdot [\alpha_L(1+\xi)]^{\frac{1}{1-\alpha_L(1+\xi)}} \cdot s^{\frac{\theta}{1-\alpha_L}}(k - \bar{k})^{\theta} = C \cdot s^{\frac{\theta}{1-\alpha_L}}(k - \bar{k})^{\theta} = a(k - \bar{k})^{\theta}$, where a is function of s and wage w, and $\theta = \frac{(1-\alpha_L)(1+\xi)}{1-\alpha_L(1+\xi)}$.

Later, the actual function form used for estimation is $\pi(a,k) = ak^{\theta} - c_f\bar{k}$, where c_f is a parameter and $\bar{k}$ is the steady state industry average capital level having the same value for all plants. It is noted that $\bar{k}$ can change with the structural parameters.

Individual plants in each period decide whether to invest a positive, negative or zero amount of capital, or to exit. If the plant exits, the sell-off value of plant is $p_s k$. If the plant takes no action in investment, $k' = (1-\delta)k$. If the plant invests or dis-invests, then $k' = (1-\delta)k + I$ with I being the investment level. The cost of capital adjustment is $c(a, I, k)$.

The plant's value function is:

$$v(a,k) = \max\{p_s k, v^n(a,k), v^i(a,k)\},$$

where on the right hand side, the first term is sell-off value, the second term is the value if the plant takes no investment action, and the third term is the value if plant invests or dis-invests. The value of inaction is $v^n(a,k) = \pi(a,k) + \beta\int_{a'\in A} v(a',(1-\delta)k)dF(a',a)$. The value of action is $v^i(a,k) = \max_{k'} \pi(a,k) - I - c(a,I,k) + \beta\int_{a'\in A} v(a',k')dF(a',a)$. We follow Cooper and Haltiwanger in specifying the cost of investment and disinvestment, as follows

$$I + c(a,I,k) = \left\{ \begin{array}{ll} 0, & \text{if} \quad I = 0; \\ (1-\lambda)\pi(a,k) + Fk + I + \frac{\gamma I^2}{2k}, & \text{if} \quad I > 0; \\ (1-\lambda)\pi(a,k) + Fk - p_s R + \frac{\gamma R^2}{2k}, & \text{if} \quad I = R < 0. \end{array} \right\}$$

The only asymmetry between positive investment and negative investment is that the irreversibility $p_s < 1$. The first component of investment cost is disruption, $(1-\lambda)$. Fixed cost F is independent of individual investment level.

The capital adjustment cost has three properties. First, $c(a, I, k)$ is not differentiable with respect to I and k at $I = 0$. Second, $c(a, I, k)$ is convex in I, but not in k. Third, $c(a, I, k)$ is asymmetric for $I > 0$ and $I < 0$. It is these specifications that create nonlinear relationship between optimal investment $I(a, k)$ and the unobserved profit shock a. As we will see later, these specifications of adjustment cost violate the assumption for invertibility condition of investment policy function $I(a, k)$.

We now show that there exists a cut-off value $\underline{a}$ given a fixed k such that if $a < \underline{a}$, the plant exits. First note that $v(a, k)$ is increasing in a for a given k. Let $\chi = 0$ if the plant exits. Let $v^s(a, k) = \max\{v^n(a, k), v^i(a, k)\}$, where v^n is the second term in value function, and v^i is the last term in plant's value function. By assumptions on profit function and productivity shocks, the plant's value $v(a, k)$ is non-decreasing in a and k, so is $v^s(a, k)$. The plant will exit the industry if $v^s(a, k) \leq p_s k$, or $\frac{v^s(a,k)}{k} \leq p_s$. It is clear that there exists a function $\underline{a}(k)$ that is non-increasing in k, such that the exit condition holds if $a \leq \underline{a}(k)$.

Similarly, given any value of a, there exists a $\underline{k}$ which depends on a, such that $\chi = 0$ if $k > \underline{k}(a)$. But this is true only when $F = 0$. A large plant can choose to exit if it draws a very low productivity shock. However, given the presence of overhead capital, small plants also will exit if its productivity is low. Exit is driven by both the scrap value and the overhead capital.

It should be emphasized that the plant sell-off value is the used capital price. An alternative setup is that the sell-off value is not necessarily the same as the capital selling price. For instance, a plant exiting through acquisitions may be sold in its entirety at a price that is much higher than if the plant sells capital in parts.

Entry

At the beginning of each period, there exist a large number of potential entrants who decide whether to enter the industry. Assume that the distribution of shocks to entrants' initial productivity is $G(\epsilon)$, which is the asymptotic distribution of productivity shock of all plants. This assumption is consistent with the empirical studies showing that the productivity of entrants is roughly equal to the average productivity of incumbents. Entrants are no more productive than average incumbents. In the stationary path, cross-section distribution of the productivity shock $G(\varepsilon)$ is invariant, and can be practically obtained from iterations on the Markov transition matrix of productivity shocks.

Before entry, potential entrants must make a start-up investment k in order to enter the industry, the investment cost is one unit of capital at entry. The total cost of entry is then ϕk with ϕ known to each potential entrant when making the entry decision. We assume that ϕ for each plant is independently and identically distributed with cumulative distribution $U(\phi)$. In simulation, we estimate the moments of ϕ to match the capital entry rates in LRD, as well as entry rate measured in number of plants.

The potential entrant observes the entry cost, makes the entry decision, and chooses k before it draws the productivity shock. It takes one period to enter into the industry. The value of entrant at the entry period is

$$v^e(\cdot) = \max_k \beta \int v(a,k)G(d\epsilon) - \phi k,$$

where $v(a,k)$ is a plant's value function at the first period of staying in industry. A plant will enter the industry if $v^e \geq 0$ and the optimal capital at entry satisfies $k > 0$.

The restriction on the entry cost is $\phi > p_s$. This condition prevents the entrant from making profits by exiting right after entry (without any production action). This implies that we need to have $\int v(a,k)G(d\epsilon) > p_s k$. If the expected value at entry is lower than sell-off value, the plant is not allowed to enter, or practically, the plant is counted as an entrant in estimation.

The entrant does not observe its shock realization unless it enters the industry. This assumption is motivated by the fact that the first-five year survival rate of new entrants in Annual Survey of Manufactures (ASM) is only around 0.5. If entrants observe their shock before entry, then the first-five year exit rate should be small.

The assumption of entry cost distribution allows the size distribution of entrant's starting capital. Given any capital level to be chosen, the value of entry v^e is monotonically decreasing in ϕ. There exists a cut-off value $\hat{\phi}$, for which $v^e = 0$. Only those who draw a cost lower than $\hat{\phi}$ will enter. If the entry cost is the same for all potential entrants, then the entry capital is same for all entrants, as does not match the fact in the data.

It is noted that in this model plant entry has no effects on plant productivity shock, nor entry costs depend on the entry rate. An alternative model could be to allow that entry will drive down the profitability. For instance we could assume that the profit function is $\pi(a, k, e) = h(e)ak^{\alpha}$, where $h(e)$ is increasing with the total size of entry e. This is left for future study.

Incorporating entry in the model complicates the structural estimation. It is an important component of the model for two reasons. First, we do not have data moments (e.g. exit rates) for sample without entry. All the literature that deals with LRD data has both entry and exit simultaneously. Second, without entry we need to simulate a short panel of plants to match the LRD data used in Cooper and Haltiwanger (2006), but this is only plausible if we have accurate data on initial distribution of plant size measured in capital. In addition, by adding entry to the model, we are able to estimate the parameters of aggregate shocks, as is not possible in short panel.

1.4 Estimation Issues

Our goal is to estimate the production function and capital adjustment costs simultaneously. This section discusses for our purpose the problems of using Olley and Pakes (1996) or Cooper and Haltiwanger (2006).

The three-step procedure in Olley and Pakes (1996) is valid only if investment policy $I(a,k)$ is monotonic and non-decreasing in a for each k, hence it is invertible with respect to a. This requires that the capital adjustment cost is convex and differentiable in I and $\frac{\partial c^2(k',k)}{\partial I \partial k} < 0$.[7] With fixed component in the adjustment cost, $c(a,I,k)$ is not differentiable at $I=0$ and is not necessarily convex depending upon $(1-\lambda)$. For each k, the presence of fixed cost independent of investment level creates region of a in which the optimal policy is $I=0$. Suppose the plant now has two choice: to invest or not. the value difference between investing and not investing is

$$\Delta v(a,k) = \beta \int_{a' \in A} [v(a',k') - v(a',(1-\delta)k)] dF(a',a) - I - (1-\lambda)\pi(a,k) - Fk - \frac{\gamma I^2}{2k}.$$

Consider the case of $\lambda = 1$ (no disruption), the first term on the right hand side is non-decreasing in a by $\frac{\partial v(a,k)}{\partial k} > 0$ and the assumption on $F(a',a)$. It is clear that only a is larger than some $\widetilde{a}(k)$ that the plant has positive investment. This is true if $\lambda < 1$ as is more costly to invest.

Even one can invert the policy function for positive investments, one needs to throw away zero investment observations to replicate the Olley-Pakes procedure, but doing this would create the selection problem for estimating investment cost parameters.

Cooper and Haltiwanger (2006) estimate the adjustment cost parameters in a balanced panel of plants in which only survived plants are selected.

[7] Other assumptions for monotonicity are: profit function is supermodular, i.e., $\frac{\partial \pi(a,k)}{\partial k}$ is increasing in a; and $\frac{\partial c(k',k_1)}{\partial I} < \frac{\partial c(k',k_2)}{\partial I}$ for $k_1 > k_2$. Then the value function $v(a,k)$ is also supermodular. In addition, the distribution of a' conditional on a is continuous and integrable. See Pakes (1994) for details.

They use simulated method of moments to estimate the adjustment cost parameters by matching simulated moments and the moments obtained from LRD. Their estimation requires a first step of estimating the production function. The authors do this by GMM on first-difference equation. Blundell and Bond (2000) use the similar estimation procedure. The GMM approach gives the consistent estimation of the production function by taking account of serially correlated profit shock (simultaneity problem). The limitation of this estimation is the selection problem due to excluding exited plants. If the selection criteria is likely associated with future profits, then the profit or productivity of plants are likely to systematically differ among the survivors. The Appendix A shows the bias caused by the selection problem. The bias using a balanced panel data is caused by selecting the subsets of a and k. The effect of capital selection is unclear, depending on the magnitudes of parameters. The productivity selection causes the estimates of θ and serial correlation of the productivity shock to be upward biased.

Empirical researches show that a large amount of exited plants have lower productivity before they exit than the continuing plants. The productivity is truncated at $\underline{a}(k)$ by ignoring the exit, as changes the behavior of plant investment. With truncated productivity distribution, a plant is now not as "cautious" as if the productivity is not truncated in making investment. It expects that its productivity draw can not be lower than $\underline{a}(k)$, hence it may invest more than it should be. Consequently, we may see more positive investments for the continuing plants.

Truncated distribution of productivity shock causes selection bias of adjustment cost parameters because the plant's exit decision depends not only on productivity shocks, but also on the magnitude of capital adjustment costs. A bad draw of profitability shock may drive the plant to disinvest, but if the plant incurs high adjustment cost of selling, the plant may choose either to exit or not to sell. On the other hand, the endogeneity of investment decisions leads to a positive correlation between the investment and the unobserved productivity. If the adjustment cost is convex, this correlation is stronger than if the adjustment cost is non-convex. It is possible that when the plant draws a high productivity shock but unable to invest due to non-convex adjustment cost, it might exit instead of staying in the industry because high productivity could imply that the economy is in boom hence the sell-off value is high.

1.5 Simulation

In this section, we simulate the investment and entry/exit model presented in previous section We present two simulation results using two different set of parameters: one from Cooper and Haltiwanger (2006), the other from some preliminary estimation results. The purpose is to understand, given a set of parameter values, how the moments change because of entry and exit. This is necessary since the high non-linearity of the model makes less straightforward the relations between investment adjustment and entry/exit. This exercise also shows the computation part for the structural estimation in the next section.

Given a set of parameter values, the model is simulated in three steps.

First, we solve for the incumbent's dynamic decisions, and simulate a panel of plants who are incumbents in the initial periods. Following Cooper and Haltiwanger, the profit shock is decomposed into two components. The idiosyncratic shock follows the AR(1) process $\epsilon_{it} = \rho_\epsilon \epsilon_{it-1} + \eta_{it}$, and the aggregate shock also follows an AR(1) process $b_t = \rho_b b_{t-1} + u_t$.

The second step is to simulate entrants in each period. We first randomly draw the entry cost according to its distribution $U(\phi)$. The number of draws in each period equals to the number of plants in the initial period. Solving the entrant's problem gives the optimal capital level at entry corresponding to each entry cost and number of entrants relative to total number of plants. Then for each entrant the idiosyncratic profit shock is drawn from the invariant distribution of idiosyncratic shock. Each entrant thus enters the industry with a profit shock and an optimal amount of starting capital.

The last step is to compute the moments. To mimic the LRD plants, we obtain 16 panels, each one with 17 periods corresponding the time period of the LRD plants used by Cooper and Haltiwanger. We compute the moments for each panel, then take the means of 10 set of moments.

The two sets of simulation differ only in the values of capital adjustment cost parameters. We compare how the capital adjust cost values affect investment moments given that the rest of parameters are from our full model estimation. Table 3 summarizes the parameter values used in our simulation. Other common parameters are from Cooper and Haltiwanger (2006), $\beta = 0.95, \delta = 0.069$.

Table 3. Parameter values for simulation

	λ	γ	p_s	α	ρ_ϵ	σ_ϵ	ρ_b	σ_b	$\underline{\phi}$	$\bar{\phi}$	c_f
CH	.796	.153	.981	.592							
Model	.813	.234	.950	.405	.919	1.07	.225	.686	1.15	1.40	.684

The first simulation uses the full set of parameter estimates obtained in the next section, the results are shown in Table 4 column Result 1. The second simulation is the same as the first one except that the capital adjustment cost parameter values are the estimates by Cooper and Haltiwanger (2006), the results are shown in Table 4 column Result 2.

Table 4. Simulation results (moments with * are for balanced panel).

Moments	Data	Result 1	Result 2
Annual inaction rate	.203	0.793	0.786
Inaction rate ($\|i\| < 0.01$)*	.081	0.82	0.81
Annual positive spike ($i > 0.2$)	.164	0.139	0.133
Positive spike*	.186	0.150	0.143
Fraction of ($i < 0$)*	.104	0.206	0.214
Negative spike ($i < -0.2$)*	.018	0.030	0.047
Corr(i_t, i_{t-1})*	.058	0.158	0.128
Corr(a, i)*	.143	0.225	0.231
Mean exit rate	.015	0.0146	0.010
Mean entry rate	.012	0.015	0.013
Production curvature*	-	0.515	0.436
Plant AR(1) coeff.*	-	0.849	0.868
Plant shock Std.Dev.*	-	0.570	0.590
Agg. AR(1) coeff.*	-	0.471	0.565
Agg. shock Std.Dev.*	-	0.921	1.054
5-year hazard	0.47	0.270	0.26
10-year hazard	0.61	0.373	0.36
15-year hazard	0.72	0.48	0.43

The quadratic cost and irreversibility cost are larger in Result 1 than in Result 2, but the disruption cost is smaller in the first simulation.

Under the two sets of adjustment costs, the investment moments (inaction and spikes) do not have significant difference. But when quadratic cost is larger, there is less negative investment and larger serial correlation of investment. Surprisingly, as the quadratic cost is lower and the irreversibility is lower, the exit rate is also lower.

One important finding from this exercise is that the change of total adjustment cost may not change investment moments significantly, but changing magnitude of different components of adjustment costs can have significant impacts on plant turnover and correlations between investment demand and productivity shock.

1.6 Estimation

Our estimation is to use the simulated method of moments to estimate all parameters by matching a large set of investment moments. Using simulated method of moments has several advantages. First, it is straightforward to implement. The difficulty is that simulating a panel of plants in order to generate moments can be very time consuming. Second, we do not need to get access to the LRD data which is not publicly available. The only data required in our estimation are the moments to be matched, as can be obtained from Cooper and Haltiwanger (2006) and existing literature that uses the LRD data. Finally, by simulating plants we are free to compute any simulated moments.

The parameters to be estimated are $(F, p_s, \gamma, \rho_\epsilon, \sigma_\epsilon, \rho_b, \sigma_b, \alpha, \underline{\phi}, \bar{\phi}, c_f)$ and $(\lambda, p_s, \gamma, \rho_\epsilon, \sigma_\epsilon \rho_b, \sigma_b, \alpha, \underline{\phi}, \bar{\phi}, c_f)$, corresponding two alternative specifications of adjustment costs. We choose the moments as following:

1. Serial correlation of investment rates, correlation coefficient of profit shocks and investment rates, positive spike rate, and negative spike rate, all for balanced panel. These moments are used by Cooper and Haltiwanger, and informative for adjustment costs.

2. Parameters in the AR(1) process of profit shocks for survived plants and parameters of the aggregate AR(1) process, and production function

curvature. These moments are estimated parameters in Cooper and Haltiwanger with the balanced LRD data. These moments are intended such that survivors in our simulated full panel match plants in balanced LRD sample.

3. Average annual exit rate and entry rate, both measured in capital. We follow the definitions from Becker etal (2005).

4. Average annual positive investment spike rate, and hazard rates. These moments are from Becker etal (2005) and Gort etal (2002).

The serial correlation of investment, correlation of investment and profit shock, the positive spike and the negative spike are informative for the adjustment cost parameters as shown in Cooper and Haltiwanger (2005). Implied in plants' dynamic optimization problem, non-convex and convex costs together cause the low serial correlation of investment, and make the investment less sensitive to profit shock. Part of these low correlations is through inaction and spikes created by the adjustment costs.

The production curvature, standard deviation of profit shocks, and serial correlation of shocks are chosen so that the simulated balanced panel should be an accurate replication of the LRD plants.

The average exit rate measured in capital reflects the proportion of capital exited out of total industrial capital stock. In the model, low productivity is not the only driving force of exit. Capital adjustment costs and fixed production cost also affect the plant's exit decision.

To estimate entry costs and overhead cost, we use the average annual entry rate and exit rate, but we cannot use these moments to pin down the distribution of variable entry cost. We then use the plant hazard rates to capture the fact that more than 80 percent of new plants exit within 20 years.

To estimate the AR(1) processes for plant productivity shock and the aggregate shock for the full panel, we first use as the moments the parameter values of same processes in LRD balanced panel. To do that we make sure that the full panel AR(1) processes are able to generate the same processes for balanced data that match with the LRD balanced panel. Secondly, we recognize that these processes are one of driving sources that drives plant entry and exit, thus relative size moments and hazard rates can be used to pin down to full panel AR(1) processes.

It is also necessary to include inaction and investment spikes for the full panel, we are able to find the positive spike rates from Becker etal (2005). Unfortunately, we do not have other investment moments from ASM.

One important thing is that we have assumed that the productivity draw of entrants is from the ergodic distribution of full panel. Averagely, entrants are as productive as incumbents.

The estimation starts with a set of parameter values. We first fix a random draw of initial distribution of productivity shocks and initial capital for some plants as incumbents at the initial period. Then in each period, we randomly draw the productivity shocks for entrants. An unbalanced panel of plants is then simulated. From the full simulated panel we compute entry and exit rates, relative size, hazard rates and full-sample positive spike rate. We keep only the balanced panel to compute or estimate the moments for balanced panel.

The length of simulated panel is 51 periods, three times of the sample period of LRD used by Cooper and Haltiwanger. To overcome the dependency on initial conditions, we simulate 2 panels, each with 360 periods. From each long panel, we obtain 7 short panels, each having 51 periods. We use the last 5 panels to compute the moments, then take the average. The final values of the simulated moments are the average of the two sets of moments.

The length of simulated panel is constrained by the computer memory. At maximum, in each period the number of entrants equals to the number of initial incumbents. In the process of estimation, when the cost parameter values are small, the unbalanced panel can be explosively large.

The estimation is conducted in several stages. The first stage of estimation uses the identity matrix as the variance covariance matrix of moment conditions. The second stages uses the variance covariance matrix of moment conditions estimated using the first-stage parameter estimates, and so on. We stop when the parameter estimates do not change much between two stages. Practically, we use three stages of estimation.

1.6.1 Case I: $F > 0, \lambda = 1$

Our first estimation gives results when the fixed adjustment cost is positive while assuming there is no disruption cost. Table 5 gives the estimated parameter values. Values in parenthesis are the standard errors of coefficients. The adjustment cost parameter estimates from Cooper and Haltiwanger are shown in the bottom row.

Table 5. Estimation when $F > 0, \lambda = 1$

F	γ	p_s	α	ρ_ϵ	σ_ϵ	ρ_b	σ_b	$\underline{\phi}$	$\bar{\phi}$	c_f
.012	.267	.977	.381	.754	.473	.414	.119	.095	4.22	.184
(.134)	(.037)	(.159)	(.026)	(.007)	(.044)	(.271)	(.061)	(.080)	(2.90)	(.044)
.039	.049	.975	CH							

Moments

	$\text{cor}(i, i_{-1})$	$\text{cor}(i, a)$	S_+^{CH}	S_-^{CH}	α^{CH}	ρ_ϵ^{CH}	σ_ϵ^{CH}	ρ_b^{CH}	σ_b^{CH}
data	.058	.143	.186	.018	.592	.885	.64	.76	.08
model	.145	.520	.253	.0002	.210	.783	.387	.96	.47
	exit	entry	spike^+	H_1	H_2	H_3			
data	.015	.012	.164	.47	.61	.72			
model	.068	.069	.267	.23	.23	.24			

The moment statistic is large, and the over-identification test is rejected as in most dynamic estimations.

From Table 5, we see that the quadratic component is much larger than that obtained by Cooper and Haltiwanger, while the fixed component in our estimation is much smaller. This indicates that small plants that live for short periods should have more positive investment spikes and more inactions. Indeed, the positive spike for the full panel (.267) is larger than the positive spike of balanced panel (.253). Qualitatively, this result does not match the plant level data facts. Becker etal (2006) show that five-year continuing plants are more likely to have investment spikes and less likely to have zero investment. The positive spike in the full data is .164 while it is .186 for large and survived plants.

The estimated curvature for profit function is .381, smaller than the estimated value in balanced panel. As shown in Appendix A, it is very likely that the estimate in balanced panel is upward biased because of truncation.

The truncation problem of the productivity shock distribution can be examined with the standard deviation of plant specific shocks. In the balanced panel, this standard deviation is 0.387 while our estimation of this parameter for full panel is 0.47. Intuitively, the productivity shock dispersion is a main source driving the exit. In balanced panel, the distribution of shock is truncated, very low shock draws are eliminated from unbalanced panel. So, in a model without exit, plants are less cautious in decisions on investment because plants do not consider the bad shocks. While in the model based on unbalanced data, plants are more cautious in deciding investment because they may draw a very low profit shock that induces them to exit. So, switching from balanced to unbalanced panel, plant's inaction rate, investment rate, and spike will change. Truncated shock distribution on balanced panel helps explain why in the full panel the capital adjustment cost is larger. Without worrying about drawing a very bad shock that induces to exit, plants in a balanced model may make more sanguine decisions by over investing or over dis-investing. Thus the estimated adjustment costs are smaller compared with the model allowing for exit.

The estimated fixed overhead cost .095 is very small. It is less than 1 percent of the simulated median capital level.

1.6.2 Case II: $F = 0, \lambda < 1$

In Cooper and Haltiwanger (2006), this specification is estimated a little differently from the case with $F > 0$ because the disruption cost is proportional to the profit. Since the shock parameters are also estimated in our model, the estimation procedure is the same as it is in the previous section.

Parameter estimates

λ	γ	p_s	α	ρ_ϵ	σ_ϵ	ρ_b	σ_b	$\underline{\phi}$	$\bar{\phi}$	c_f
.881	.278	.941	.686	.922	.706	.674	.067	1.18	8.86	.485
(.0004)	(.046)	(.021)	(.019)	(.001)	(.012)	(.278)	(.047)	(.843)	(2.43)	(.061)
.796	.153	.981	CH							

Moments

	cor(i, i_{-1})	cor(i, a)	S_+^{CH}	S_-^{CH}	α^{CH}	ρ_ϵ^{CH}	σ_ϵ^{CH}	ρ_b^{CH}	σ_b^{CH}
data	.058	.143	.186	.018	.592	.885	.640	.76	.08
model	.270	.465	.191	.002	.232	.844	.315	.99	.89
	exit	entry	spike$^+$	H_1	H_2	H_3			
data	.015	.012	.164	.47	.61	.72			
model	.060	.007	.175	.88	.91	.94			

A comparison with the estimates based on the balanced panel shows that in full estimation, both the irreversibility and the quadratic cost are larger while the disruption cost is smaller. The simulated average adjustment cost is about 14 percent of plant investment. However, it is not certain whether the differences in irreversibility and disruption parameters are significant.

This estimation matches the investment moments better than the previous one. The positive spike for the balanced panel is larger than the same moment for the full panel, which is consistent with the finding by Becker etal (2006).

The obtained serial correlation of investment and the correlation coefficient of shock and investment are larger than those in Cooper and Haltiwanger (2006). This could be because our estimate of quadratic cost is larger. In addition, the profit function curvature is larger than that estimated in the balanced panel, and the standard deviation of the plant specific shocks is larger in the full panel.

The estimated aggregate shock process has a smaller serial correlation coefficient and smaller standard deviation than those estimates in the balanced panel. To understand these estimates, we simulate the model and obtain the serial correlation and standard deviation of the aggregate investment rate, which is defined as the total investment divided by the total capital. We find that the serial correlation is 0.36 and the standard deviation is 0.045. The serial correlation of aggregate investment rate in postwar U.S. is 0.70 and the standard deviation of the aggregate investment rate is 0.008. Our estimates of the aggregate shock process create a much larger standard deviation than the U.S. data.

1.7 Problems and Extensions

In this section, we discuss some problems that arise from our estimation. First, in both models, the estimated zero investment rate for both full panel and balanced panel are extremely large, compared with the LRD and ASM plants. Becker etal (2005) document that the average annual inaction rate for the ASM plants is .203. The inaction rate in the LRD data used by Cooper and Haltiwanger is .081, which is consistent with the finding by Becker etal (2005) that the survived large plants tend to be less likely to have zero investment. In all the estimation results, including those by Cooper and Haltiwanger (2006), the estimated inaction rates are high. This may indicate that the model itself does not capture the zero investment.

We can examine this issue by adding inaction rates to the moment set to be matched. We implemented this exercise for the case with $\lambda < 1$. We find that including inaction rates affects the estimation results. Further estimation will be conducted later.

The second problem is that the estimates of the standard deviation of the aggregate shock are very different between the two alternative specifications. They are also both different from the U.S. data. Potentially, this might be caused by the fact that we have used short panels when computing the moments. Such a short time period may not be sufficient to estimate the aggregate shock parameters. More importantly, the moments to be matched do not include the aggregate moments. It should be informative for estimating the aggregate shock process if we add moments like the serial correlation and the standard deviation of the aggregate investment

Finally, the entry and exit in the model impose particular assumptions. Mainly, we have assumed that the entry cost is heterogeneous and drawn from a uniform distribution, the sell-off value equals to capital selling price, and the fixed overhead cost is one source that drives exit. It is necessary to examine whether the estimation results are sensitive to alternative assumptions.

Chapter 2

Capital Reallocation and Its Effects on Aggregate Productivity

2.1 Introduction

Existing empirical studies find that reallocation of labor and capital among establishments is an important source of aggregate productivity changes. As surveyed in Bartelsman and Doms (2000), productivity dispersion is large among establishments; this dispersion is persistent over time and a large portion of aggregate productivity growth is attributable to resource reallocation. Studies that use the plant level U.S. manufacturing sector data show that about 25 percent of productivity increases during the 1980s resulted from resource reallocation from less to more productive plants. The change in output share (which is related to reallocation) has positive effects on industry level productivity. The net industry productivity growth rate would be negative if there was no share effect between 1972-1977, and at least 20 percent of productivity growth in 1980s is due to reallocation.[1]

This paper studies the role of capital reallocation in aggregate productivity. Capital reallocation, i.e., reallocation of productive capital across firms

[1] See Bartelsman and Dhrymes (1998) and Baily, Hulten and Campbell (1992).

has been increasing in the U.S. It is an important component of reallocation activities. On average, the total value of reallocation (mainly through merger and acquisitions) in the U.S. is about 3-4 percent of the GDP. In 1999, the total value of capital reallocation increased substantially to more than 15 percent of the GDP. Using Compustat, we find that the annual average reallocated capital to capital stock ratio is 8.7 percent. Ramey and Shapiro (1998), Jovanovic and Rousseau (2004) and Eisfeldt and Rampini (2005) are among the first to study capital reallocation.

In this paper we investigate how much of aggregate productivity change can be accounted for by capital reallocation. We also examine the impact of capital reallocation on productivity dispersion over firms.

In the model economy, a homogeneous good is produced by an infinitely large number of firms that are heterogeneous in idiosyncratic management ability. Production technology displays decreasing returns to scale in capital and labor. Each firm decides between buying capital—from the representative household—and buying capital from other firms. We call capital purchased from the household unbundled capital, and capital purchased from other firms bundled capital. Bundling is endogenously determined by capital adjustment costs and the price of unbundled capital relative to bundled capital.

Differences in investment costs of unbundled and bundled capital drive heterogeneous firm behaviors. Firms with low production shocks tend to sell capital and firms with high production shocks tend to buy capital. This drives capital to flow from less productive to more productive firms. The reallocated capital experiences productivity increase in the sense that this part of capital is used for production with the buyer's production technology.

We measure the aggregate productivity with the average of firm level output-to-labor ratios and the average firm level output-to-capital ratios, since labor and capital are the only physical inputs for production and total factor productivity is exogenously specified. Aggregate productivity can change with the firm's joint distribution of management ability shock and firm size. And this distribution change is a consequence of investment and reallocation decisions by firms.

We estimate the cost parameters of investment, then use them and other parameters to calibrate the model to the U.S. firm data. Our baseline results can match the investment and reallocation moments. Using the calibrated model, we conduct two policy experiments to study the recent two waves of mergers and acquisitions in the U.S. In the first, we find that when the average of firm level reallocation-to-capital ratios increases by 50 percent, the average output per unit of labor increases almost 2 percent. Meanwhile, the average output per unit of capital is not affected. This result shows that capital reallocation explains roughly 15 percent of aggregate productivity growth in the late 1980s. In addition, we find that increased reallocation helps enlarge labor productivity dispersion across firms by about 10 percent.

In the second experiment, we find that reallocation increases temporarily then drops to original level, as the aggregate technology changes. After a transition period, the economy settles down with increased labor productivity

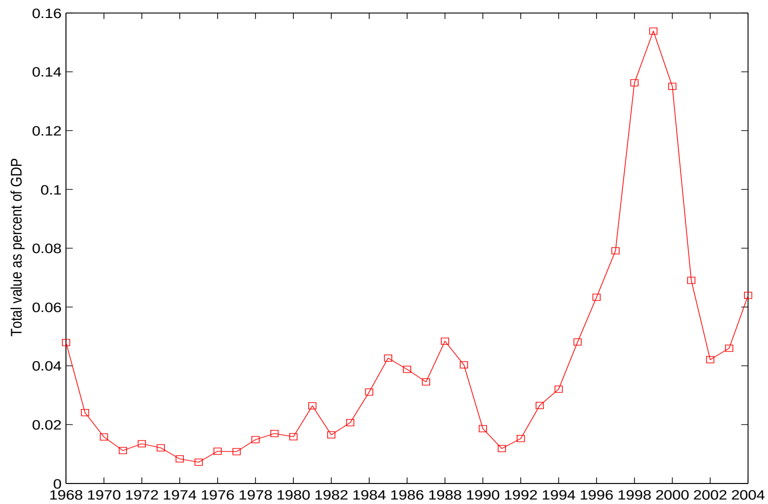

Figure 2.1: Mergers and Acquisitions as a percentage of the GDP

in the new stationary equilibrium.

2.2 Empirical Evidence

Capital reallocation through mergers and acquisitions has been increasing since the 1960s in the U.S. In 1999, the total value of reallocation reached a historical record, 15.4 percent of the GDP as shown in Figure 2.1. The annual reallocated capital accounts for 8.7 percent of total capital stock between 1971-2004.

In industrial organization and finance literature, a large number of studies address the motives of reallocation and the consequence of reallocation on industry competition. One of the questions is whether reallocation improves firms' or plants' productivity, and whether it is the less productive or more productive firms and plants that are more possible to be reallocated.[2] Lichtenberg and Siegel (1987) use the data consisting of the large U.S. plants (with employment more than 250) from LRD to study the effects of ownership changes of manufacturing plants on productivity. About 21 percent of the plants in their sample changed ownership at least once over a ten-year period. They find that plants with lower total factor productivity are more likely to be sold. The authors attribute the low productivity to bad random matches between plants and their parent firms (owners). Plants experienced with ownership changes have higher productivity growth in years after transfer transactions. The authors find that, for transferred plants the average productivity residual three years before the transaction is -3.5 and that three years after the transaction is -2.7, an increase of 23 percent. In contrary, McGuckin and Nguyen (1995) use the LRD food manufacturing industry data and find that the labor productivity of plants with ownership changes was about 20 percent higher than the industry average at the time of transaction, but for large plants that have 250 or more workers the acquired plants tend to have low productivity. Moreover, plants that experience ownership changes gain productivity during 5-9 years following transaction.

Maksimovic and Phillips (2001) recently study ownership changes of plants, divisions and whole firms in LRD data from 1974-1992. On average, 3.89 percent of establishments change ownership annually. They find that plants or whole firms that are transacted experience significant productivity

[2] See Nguyen (1998) for the manufacturing plant ownership change data in the U.S.

gains. The buyers tend to have higher total factor productivity and tend to be larger than the sellers. In addition, 56.8 percent of transactions occurs within the same industries of three-digit SIC codes.

The empirical studies can be summarized as follows:

1. Plants with lower productivity are more likely to be transferred than plants with higher productivity. The probability of ownership change declines as plant size increases.

2. Plants of buying firms are usually larger and more productive than plants of selling firms.

3. Plants with ownership changes experience significant productivity growth. This is shown in studies by McGuckin and Nguyen (1955a), Lichtenberg and Siegel (1992), Baldwin and Gorecki (1991), and Maksimovic and Phillips (2001), among others.

4. Following the ownership change, the acquiring firm's existing plant does not show significant productivity change.

These stylized facts are consistent with other empirical research on the role of reallocation in aggregate productivity changes (for example, Baily, Hulten and Campbell 1992, and Foster,Hatiwanger and Krizan 2001).

2.3 The Model

The model economy is composed of one representative household and a continuum of firms with unity measure. One homogeneous good is used for investment and consumption. The markets for investment and consumption are both perfectly competitive. In each period, the firm makes three decisions: on investment in capital, on employment, and on capital purchased from other firms. The household in each period makes decisions about consumption, investment, and labor supply.

2.3.1 Firm

The production unit is firm. We abstract from the firm organizational issues by assuming that each firm is operated by a manager who maximizes the firm's profit.[3] We index the manager quality using ε. It is interpreted as the way the production is organized and how technology is chosen. With the same amount of capital and labor, managers with higher ε produce more. A firm uses management ability, labor l and capital k to produce. Production function is $f(k, l, \varepsilon) = z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu}$, where $\alpha \in (0, 1)$ and $\nu \in (0, 1)$. The decreasing return to scale implies that variable profit is positive in equilibrium, and it also implies that firm size is finite and bounded above.

[3] In reality firms own plants. Here, we implicitly assume that all plants owned by the same firm have the same management shock which is firm specific because all plants of a firm are all under the same management. Under this assumption, if the firm owns $n > 1$ plants, and its production function is of CES type, that is, $y = z\varepsilon\left[\sum_i^n f(k_i, l_i)^{\gamma}\right]^{\frac{\nu}{\gamma}}$, then the optimal sizes of all plants should be all equal to each other given that plants pay the same capital price and the same wage rate. It can be shown that the optimal number of plants n is undetermined. This is because the optimal capital is $k^* = \left[\Delta n^{1-\frac{\nu}{\gamma}}\right]^{\frac{1}{\nu-1}}$ with Δ being a constant including shock. The optimal labor demand is $l^* = \frac{r\alpha}{w(1-\alpha)}k^*$. If we use two-stage optimization, we can plug the optimal k^* and l^* into the firm's problem, then it is straightforward to find that the optimal n is undetermined. This indicates that the firm can own many small plants or a few large plants to achieve the same level of profits.

Aggregate technology level is z, which is constant.[4] We use it for comparative statics in later sessions.

With the above firm production technology, we ignore the management as an agency problem by implicitly assuming that the management maximizes the firm's profit. Agency problem can be serious in reallocation activities because in some cases the selling firm's managers can be offered a large amount of compensation in order to sell the business under their control.

Assumption 1: Firm specific shock ε follows an AR(1) process, $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_t$. The innovation term η_t is identically and independently distributed, with $\eta_t \sim N(0, \sigma_\eta^2)$.

Besides employment, in each period a firm decides on the optimal amount of its capital stock to change. The firm can sell (buy) capital to (from) the output market or the reallocation market. In the reallocation market, the firm buys capital from another firm. Let I_n denote the capital investment in the output market, and I_a denote the capital investment in the reallocation market. Following Jovanovic and Rousseau (2002), if the firm buys capital in the output market, the adjustment cost of capital $h_1(I_n, k)$ is convex. If it buys capital from the reallocation market, the firm pays p_a per unit of capital. Price p_a is the capital price relative to output price. The adjustment cost for capital acquired from the reallocation market is $h_2(I_a, k)$ which is also convex.

[4] In another ongoing paper, I extend the current model by allowing aggregate uncertainty to study the cyclicality of reallocation and its effects on productivity dynamics.

In addition, firms that participate in the reallocation market pay a fixed cost Φk. We make the following assumption on convex adjustment costs.

Assumption 2: $\frac{\partial h_1(I_n,k)}{\partial I_n}|_{I_n=I} > \frac{\partial h_2(I_a,k)}{\partial I_a}|_{I_a=I}$.

From one investment level, the additional cost of one additional unit investment in unbundled capital is larger than the additional cost of one additional unit investment in bundled capital. This assumption implies that without fixed cost, the firm always buys or sells capital in the reallocation market.

It is noted that buying capital and selling capital incur the same adjustment cost, and adjusting employment is costless. With these assumptions on investment costs, if the demand for investment is small, the firm buys capital only from the output market because in doing so the firm does not pay the fixed cost. As the demand increases, the adjustment cost of investing in unbundled capital increases faster than the adjustment cost if the firm buys bundled capital. After some threshold of investment level, it is cheaper for the firm to buy a fraction of capital from the reallocation market. So, we also refer to the reallocation market as the bundled capital market. The intuition is that assembling new (unbundled) capital, and training workers to use it, is more costly than if the firm buys bundled capital. However, because of fixed cost of searching for bundled capital, the firm would rather buy unbundled capital when its demand for investment is low. As the demand for investment grows, the assembling cost and training cost are too large for the firm to invest only in unbundled capital, thus the firm will switch some investment to bundled capital because with bundled capital the firm pays less for assembling capital and training workers. On the other hand, if the firm sells a large amount of capital it is more costly to disassemble it to sell it in the output market, then it is optimal to sell it in the bundled capital market.

The firm's recursive problem is

$$v(\varepsilon, k) = \max_{\{I_n, I_a, l\}} z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu} - wl - I_n - p_a I_a - h_1(I_n, k) - h_2(I_a, k) - \Phi k 1_{\{I_a \neq 0\}} + \frac{1}{1+r} E v(k', \varepsilon')$$

subject to $k' = (1-\delta)k + I_n + I_a$. The fixed cost $\Phi = 0$ if $I_a = 0$. In this problem, $(1+r)$ is the gross interest rate, endogenously determined in equilibrium.

Optimal Investment

Let the investment rate be $i = I/k$, and bundled capital investment rate be $i_a = I_a/k$. Given a total investment level, the firm's choice of splitting between I_a and I_n is determined by a static cost minimization. Let $\tilde{i}$ be the investment rate where the firm is indifferent between choosing $i_a = 0$ and $i_a \neq 0$. Suppose $h_1(I_n, k) = \frac{\gamma_n}{2}\frac{I_n^2}{k}$ and $h_2(I_a, k) = \frac{\gamma_a}{2}\frac{I_a^2}{k}$ with $\gamma_a < \gamma_n$. At $\tilde{i}$, the unit cost of investing new capital only equals to the unit cost of investing both types of capital,

$$\tilde{i} + \frac{\gamma_n}{2}\tilde{i}^2 = \min_{i_a}\{\tilde{i} - i_a + p_a i_a + \frac{\gamma_n}{2}(\tilde{i} - i_a)^2 + \frac{\gamma_a}{2} i_a^2 + \Phi\}.$$

If the firm chooses both types of investment, the optimal choice of i_a is $i_a^* = \frac{\gamma_n}{\gamma_n+\gamma_a} i + \frac{1-p_a}{\gamma_n+\gamma_a}$. As i increases, both the left and right hand sides are increasing, but the right hand side increases slower as i increases since $\gamma_a < \gamma_n$. When $i < \tilde{i}$, left hand side is smaller than the right hand side. When $i > \tilde{i}$, the left hand side becomes larger. The threshold investment level $\tilde{i}$ can be obtained by solving the above equation after obtaining the optimal i_a. Because the fixed cost is proportional to the firm's capital stock, the threshold value $\tilde{i}$ is determined by only cost parameters, it is independent of the firm's state variables. If the fixed cost is Φ, instead Φk, then the absolute value of $\tilde{i}$ is decreasing in k.

When adjustment costs are quadratic and under Assumption 2, it is never optimal for the firm to choose $I_n = 0$.

In this setup, the firm is allowed to buy (sell) capital in one market and sell (buy) in the other market. It can be shown that when $p_a < 1 - \gamma_a\sqrt{\frac{2\Phi}{\gamma_n+\gamma_a}}$, the firm chooses $i_a > 0$ and $i_n < 0$. When $p_a > 1 + \gamma_a\sqrt{\frac{2\Phi}{\gamma_n+\gamma_a}}$, the firm chooses $i_a < 0$ and $i_n > 0$. See Appendix C for details.

Figure 2.2 illustrates the firm's tradeoff between unbundled capital and bundled capital. Each curve shows the total expenditure of investment per unit of current capital stock (investment plus adjustment costs per unit of capital stock). To draw this graph, we choose $\gamma_n = 0.758$, $\gamma_a = 0.474$, $\Phi = 0.059$ and $p_a = 1.10$. Figure 2.3 shows an example of the optimal split between investments in unbundled and bundled capital.

We have used capital adjustment cost and participation cost to summarize the frictions in capital market, following Cooper and Haltiwanger (2006).

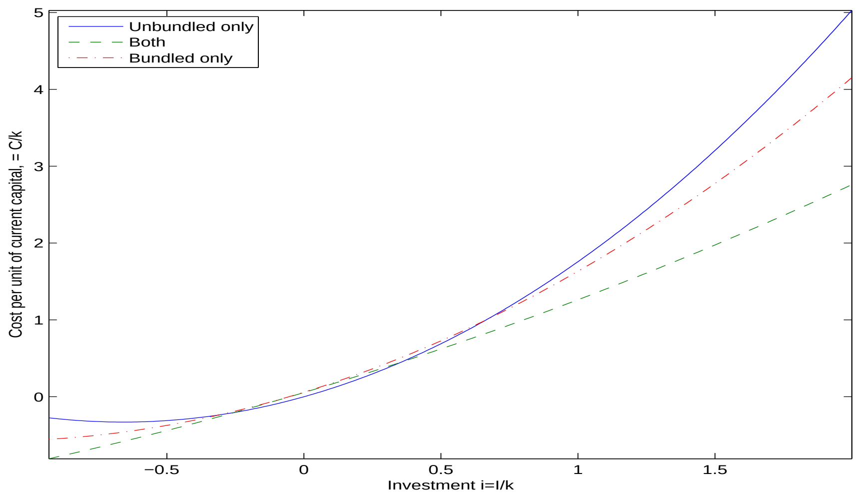

Figure 2.2: Comparing Costs of Investment

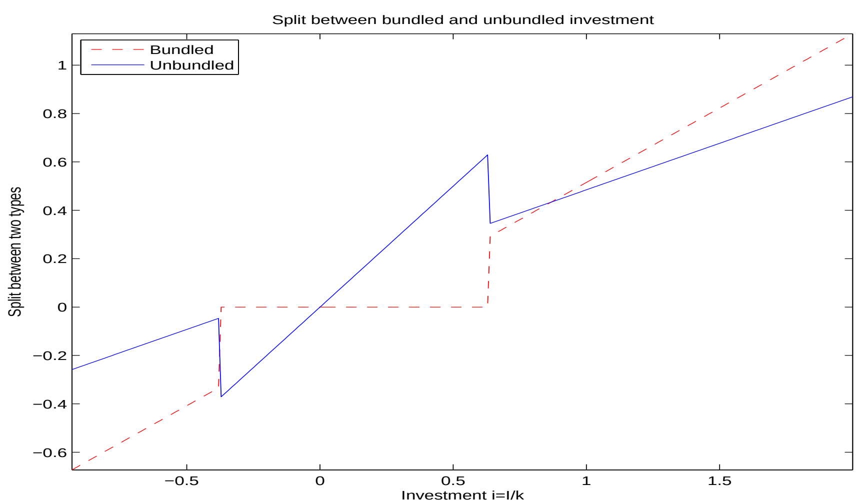

Figure 2.3: Optimal split between bundled and unbundled capital

Alternative modeling of frictions could be the firm's financial constraint, information asymmetry and adverse selection in bundled capital market. Adverse selection due to asymmetric information seems to be relevant in bundled capital market. Empirical evidence shows that a significant portion of capital reallocation are not successful in improving productivity. As Kaplan (2000) points out, the extent to which the buyers understand the target before acquisition is an important factor affecting the performance of acquired capital. Acquisition failure can be because the buyers do not have sufficient information on the target. House and Leahy (2004) presents an sS model of used car where the agent's adjustment costs of car purchase and selling arise from adverse selection problem in used car market. Eisfeldt (2004) considers a model similar to House and Leahy (2004) but focuses on interaction of liquidity and adverse selection in equity market. We can interpret the fixed cost as it arises from the adverse selection problem or financial frictions. Actually, our specification of investment costs summarize all possible frictions so that our model is general enough to allow for frictions other than information and financial market.

2.3.2 Firm Distribution

Let the firm's policy function be $k'(\varepsilon, k)$. The distribution of firms over (ε, k) can be summarized by the probability measure μ defined on $\mathcal{S}$, where $\mathcal{S}$ is the σ-field generated by the open subsets of product space $(\mathcal{E}, \mathcal{K})$. The evolution of firm distribution is

$$\mu'(\varepsilon', k') = \int_{\mathbb{S}} \pi(\varepsilon', \varepsilon) 1_{k'=k'(\varepsilon,k)} d\mu(\varepsilon, k),$$

where $\pi(\varepsilon', \varepsilon)$ is the distribution of ε' conditional on ε.

2.3.3 Reallocation Market

Let $\tilde{\varepsilon}$ and $\tilde{k}$ be the firm's state in which the firm's investment is such that $i(\tilde{\varepsilon}, \tilde{k}) = \tilde{i}$. The measure of capital being sold is

$$\Lambda(\widetilde{\varepsilon}, \widetilde{k}, p_a) = \int_{\mathbb{S}} 1_{\{i_a(\varepsilon,k)<0\}} I_a(\varepsilon, k) d\mu$$

. It is negative and a decreasing function of price p_a. The measure of capital being purchased is $\Psi(\widetilde{\varepsilon}, \widetilde{k}, p_a) = \int_{\mathbb{S}} 1_{\{i_a(\varepsilon,k)>0\}} I_a(\varepsilon, k) d\mu$ which is positive and decreasing in p_a. In equilibrium, two measures sum to zero.

In the model, we assume that the production technology is not embodied in capital. If technology is embodied in capital, the price of bundled capital would be determined differently. In that case, the older the capital, the more outdated the technology is, hence less attractive in the market. Then firms with older capital tend to sell more capital in unbundled capital market than firms with relatively newer capital. This may drive up the bundled capital price.[5]

[5] I thank Jon Willis for pointing this out.

2.3.4 Household

The economy has one representative household with preference

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \xi L_t]$$

where c_t is consumption and L_t is the fraction of individuals employed. This preference is used in Hansen (1985) and Rogerson (1988). Since there is no aggregate uncertainty, household's optimization is deterministic. The household owns all firms. In each period, the household decides on optimal consumption, labor supply and investment in firm shares. Let $w(\mu)$ be the wage rate relative to output price, let $dQ(\varepsilon, k, \mu)$ be household's portfolio of the one-period shares of firms with ε and k. Also let $\rho(\varepsilon, k, \mu)$ be the share price of all firms with ε and k. The household buys firm portfolios at the beginning of each period. Household's recursive optimization problem is

$$W(Q, \mu) = \max_{c,L,Q'} [u(c, L) + \beta W(Q', \mu')]$$

subject to

$$c + \int_{\mathbb{S}} \rho(\varepsilon', k') dQ'(\varepsilon', k', \mu) \leq w(\mu)L + \int_{\mathbb{S}} v(\varepsilon, k) dQ(\varepsilon, k, \mu).$$

Here $v(\varepsilon, k)$ is the sum of dividend and price of firm (ε, k). On the left hand side of budget constraint, $\rho(\varepsilon', k', \mu)$ is the price of the firm that enters next period with ε' and k'. In stationary equilibrium, the number of firms with ε' is certain and given by the invariant distribution of ε. In this sense, household's portfolio $dQ(\varepsilon, k, \mu)$ is risk-free.

Let the shadow price of firm's output be p. It is the Lagrange multiplier in the household's dynamic problem. The first order conditions are $u_1(c, L) = p$, $w = -\frac{u_2(c,L)}{u_1(c,L)}$, and $\rho(\varepsilon', k') = \beta \frac{p'}{p} v(\varepsilon', k')$.

2.3.5 Recursive Equilibrium

A recursive equilibrium is defined as a set of functions

$$(w, p, r, p_a, \rho, v, L, K', W, C, L^s, Q, \mu)$$

such that household and firms maximize their expected values, and markets for reallocation, asset, labor and output clear:

1. Taking prices and μ as given, v solves firm's Bellman equation, and $l(\varepsilon, k, \mu)$and $k'(\varepsilon, k, \mu)$ are the firm's policy functions for labor and capital, for all $(\varepsilon, k) \in \mathcal{S}$. The aggregate future capital is $K'(\mu) = \int_{\mu \in \mathcal{S}} k'(\varepsilon, k, \mu) d\mu$. The aggregate labor demand is $L(\mu) = \int_{\mu \in \mathcal{S}} l(\varepsilon, k, \mu) d\mu$.

2. Taking prices and μ as given, W satisfies the household's problem. (C, L^s) are the associated policy functions for households.

3. Reallocation market clears: p_a solves $\Psi(\widetilde{\varepsilon}, \widetilde{k}, \mu) + \Lambda(\widetilde{\varepsilon}, \widetilde{k}, \mu) = 0$.

4. Asset market clears: $Q'(\varepsilon', k') = \mu'(\varepsilon', k')$ for each $(\varepsilon', k') \in \mathcal{S}$.

5. Labor market clears: $L^s(\mu) = L(\mu)$.

6. Aggregate consumption equals to the total output net of total investments and related costs,

$$C(\mu) = \int_{\mathbb{S}} [z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu} - k'(\varepsilon, k, \mu) + (1-\delta)k - H(I_a(\varepsilon, k), I_n(\varepsilon, k))]\, d\mu.$$

7. Firm distribution, $\mu' = \mathcal{U}(\mu)$, is induced by $k'(\varepsilon, k, \mu)$ and exogenous shock process for ε. That is, $\mu'(\varepsilon', k') = \int_{\mathbb{S}} \pi(\varepsilon', \varepsilon) 1_{k'=k'(\varepsilon,k)} d\mu(\varepsilon, k)$.

We now characterize the stationary equilibrium. Household's first-order necessary conditions are $-\frac{u_2(c,L)}{u_1(c,L)} = w(\mu)$ and $p = u_1(c, L)$. In equilibrium, we require that $\frac{1}{1+r} = \frac{\beta u_1(c',L')}{u_1(c,L)}$ or $1 = \frac{p}{p'} = \beta(1+r)$. So interest rate is $\frac{1}{\beta} - 1$. The interpretation of these necessary conditions are standard. The household chooses optimal consumption the equalize the market values of marginal utility between two periods. Within period, the household's marginal rate of substitution of consumption and leisure equals to relative price of labor.

2.3.6 Model Solution

We assume that utility of consumption takes the logarithm form. then the equilibrium conditions satisfy $p(z, \mu) = \frac{1}{C}$ and $w(z, \mu) = \xi C$. From the equilibrium condition with respect to output, we know that $\beta(1+r) = \frac{p}{p'}$. The equilibrium can be computed by solving the Bellman equation that combines the firm's dynamic problem and the household's first-order conditions. Plug household's optimal decision rules and condition $\beta(1+r) = 1$ into the firm's problem, the re-formulated recursive problem is obtained as following[6]

$$\begin{aligned} v(\varepsilon,k,\mu) &= \max_{\{k',I_a,I_n,l\}} \{[z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu} - wl - k' + (1-\delta)k + (1-p_a(\mu))I_a - \\ &\quad h_1(I_n,k) - h_2(I_a,k) - \Phi k 1_{\{I_a\neq 0\}}] + \beta E v(\varepsilon',k',\mu')\}. \end{aligned} \tag{2.1}$$

For given prices, each firm with (ε, k) make decisions on labor and investments. Let $f(k,l) = (k^{1-\alpha}l^{\alpha})^{\nu}$. In this problem, the optimal labor decision of a typical firm is determined by

$$\nu z\varepsilon f(k,l)^{\nu-1}\frac{\partial f(k,l)}{\partial l} = w(\mu).$$

The optimal investment decision is

$$k' = K'(\varepsilon,k,\mu), \quad \text{and} \quad I_a = \begin{cases} I_a(\varepsilon,k,\mu), & \text{if } K'(\cdot) \geq K'^{*}; \\ 0, & \text{otherwise.} \end{cases}$$

$K'^{*}(\varepsilon,k,\mu)$ is the threshold value of future capital at which the firm is indifferent between participating in the reallocation market and not.

Let the convex adjustment cost be $h_1(I_n,k) = \frac{\gamma_n}{2}\frac{I_n^2}{k}$ and $h_2(I_a,k) = \frac{\gamma_a}{2}\frac{I_a^2}{k}$ with $0 < \gamma_a < \gamma_n$. Conditional on that the firm does not participate in reallocation market, the optimality condition is

$$1 + \gamma_n \cdot i_n = \beta\frac{\partial E v(\varepsilon',k',\mu')}{\partial k'} \quad \text{and} \quad i_a = 0.$$

This says that the firm's optimal investment rate i_n is determined when the total current marginal cost of investment equals discounted and marginal expected value of that investment. For firms that participate in the reallocation

[6] Actually, if we plug the household's first-order conditions into the firm's problem, we would have that the firm's value in nominal term is $V = p \cdot v$. But since in stationary equilibrium, p is constant, so solving for V is same as solving for v.

market, the optimal investment decision rule is same as above for new capital, and the optimal decision rule for acquired capital is

$$p_a + \gamma_a \cdot i_a = \beta \frac{\partial E v(\varepsilon', k', \mu')}{\partial k'}.$$

After having paid the fixed cost of participating in the reallocation market, the firm's optimal strategy is to equalize the marginal costs of two types of investment, that is $1 + \gamma_n \cdot i_n = p_a + \gamma_a \cdot i_a$. This gives the optimal split between i_a and i_n, $i_a^* = \frac{\gamma_n}{\gamma_n + \gamma_a} \cdot i + \frac{1 - p_a}{\gamma_n + \gamma_a}$.

As we showed earlier, it can be optimal for firms to buy unbundled capital while selling bundled capital in equilibrium. This does not constitute arbitrage since the firm that does so is unable to buy an infinitely large amount of new capital and sell it in the reallocation market due to the cost structure in both markets.

Given the firm distribution and optimal decisions on capital and labor, the market clearing conditions used to determine equilibrium output price, wage and bundled capital price are determined as

$$C = \int_{\mathbb{S}} [z\varepsilon f(k, L(\varepsilon, k, \mu))^{\nu} - K'(\varepsilon, k, \mu) + (1 - \delta)k + I_a(\varepsilon, k, \mu) - H_{\text{adj}}]\, d\mu.$$

$$L^s = \int L(\varepsilon, k, \mu) d\mu.$$

$$\Psi(\widetilde{\varepsilon}, \widetilde{k}, \mu) = \Lambda(\widetilde{\varepsilon}, \widetilde{k}, \mu).$$

2.4 Calibration

We calibrate the model to the U.S. data on investment and reallocation, then we study the two recent U.S. merger waves using the calibrated model to investigate the role of reallocation. The household discount factor is $\beta = 0.9615$ to reflect a 4 percent annual interest rate. The decreasing return to scale is set to be $\nu = 0.8507$. This value is derived from the production curvature estimated in Cooper and Haltiwanger (2006).[7] The Cobb-Douglas technology parameter is $\alpha = 0.7454$, which implies that the labor share in production is 0.64 as in Prescott (1986). Given this the output-to-capital ratio is about 0.22. The capital depreciation rate is chosen as $\delta = 0.069$ as in Cooper and Haltiwanger (2006). The share of leisure in utility is $\xi = 0.94$ so that around 80 percent of the population works in steady state.

The first-order autocorrelation process of the firm specific shock is from Cooper and Haltiwanger (2006), $\rho_\varepsilon = 0.885$, $\sigma_\eta = 0.30$, where the AR(1) equation is $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_t$. These values were estimated using the U.S. plant data, which provided a sample consisting of mainly large plants, most of which are owned by publicly held firms. The aggregate technology is normalized to 1.

Jovanovic and Rousseau (2002) use a similar capital adjustment cost specification. However they do not estimate these costs. Since we cannot

[7] Cooper and Haltiwanger estimate that the production curvature is 0.592. Under our assumption, production function has decreasing return to scale. If $\alpha = 0.7454$, then we get $\nu = 0.8507$.

find these cost parameter values in the existing literature, we estimate the parameters γ_n, γ_a and Φ using the baseline model. First, we present the estimation results.

2.4.1 Adjustment Cost Estimation

We use the simulated method of moments to estimate the capital adjustment cost parameters and the fixed cost parameter. The estimation method is borrowed from Cooper and Haltiwanger (2006). Our estimation however, differs as it is based on the general equilibrium model with prices being endogenous. The estimation procedure consists of an inner loop and and outer loop. In the inner loop, equilibrium prices are solved given cost parameters, and the outer loop searches for parameter values to match moments. The moments we match in estimation are the average annual investment rate, the average annual reallocation rate, the proportion of firms with positive spikes, the proportion of firms with inaction, and the proportion of firms that participate in the reallocation market. We use the firm data Compustat to calculate the data moments.[8] More details on the data moments can be found in Appendix B.

The estimated parameter values are $\gamma_n = 0.758$, $\gamma_a = 0.474$ and $\Phi = 0.0585$. The estimate of γ_n is within a reasonable range of the estimates of Cooper and Haltiwanger (2006) and Hall (2004). The estimate of γ_a is close

[8]The Compustat data is downloaded from WRDS website at Wharton School, to which the University of Texas at Austin is a subscriber.

to that implied in the Q theory of mergers by Jovanovic and Rousseau (2002). These two authors estimate that the coefficient value for Q is 1.916, which implies that the quadratic adjustment cost in the bundled capital market is 0.52.

2.4.2 Baseline Results

The following table summarizes all parameter values we use for calibration.

β	ξ	α	ν	δ	ρ_ε	σ_u	γ_n	γ_a	Φ
0.9615	0.94	0.745	0.8507	0.069	0.885	0.30	0.758	0.474	0.059

To compute the model, we first approximate the AR(1) process of firm shocks using the Markov chain. We choose 19 grid points for the transition matrix of the shock process. We then discretize the capital state with 1000 grid points.

The equilibrium is computed in two steps. The first step is to find equilibrium prices given firm distribution. Given output price p, we first find equilibrium price p_a. Possible corner solutions for bundled capital investment are dealt with by allowing firms to choose zero I_a. Next, given equilibrium p_a, we find the equilibrium price p in the output market, and we iterate this until the excess demands in both markets are close to zero. The second step is to obtain the firm distribution over (ε, k). We impose the stationary equilibrium condition, which dictates an invariant firm distribution in equilibrium. This invariant distribution is computed by iterating on the equilibrium condition 7 as in Aiyagari (1994).[9] The convergence is made when both markets' excess demands are less than 10^{-5}.

We then match the computed moments to the U.S. firm level data. The investment and reallocation moments in data are computed for the manufacturing industries in Compustat. Appendix B gives a summary of the data set and the procedures of computing data moments. The investment rate is the average of firm level values of unbundled capital investment to capital ratio. The reallocation rate is the average of firm level values of bundled capital investment to capital ratio. The positive spike rate is the average of annual proportions of firms with unbundled investment greater than 20 percent. Inaction is the average of annual proportions of firms with unbundled investment less than 1 percent. Participation rate is the average of annual proportions of firms who have nonzero bundled capital investment. The model moments are computed analogous to data moments. The following table summarizes our baseline results.

Investment and Reallocation

	Inv.rate	Spike^{+}	Spike^{-}	Inaction	Cap.sale/Cap.	Acqui./Cap.	Participation
Data	.18	.23	-	.031	.014	.087	.24
Model	.16	.30	.03	.037	.047	.077	.15

[9]An alternative method to compute the equilibrium is by simulation. Given an initial firm distribution, we can simulate a panel of a large number of firms. In each simulation period, we need to solve for equilibrium prices to determine each firm's investment decisions. The invariant firm distribution can be obtained when the distributions from two consecutive periods are very close to each other.

In equilibrium, no firm buys capital in one market while selling capital in the other market. The baseline results show that our model can match the long-term data moments well.

We also compute the aggregate productivity measures. The output per unit of capital is computed as the sum of weighted outputs divided by the sum of weighted capital. Output per unit of labor and the output-to-investment ratio are computed in the same way. Cost per unit of output, on the other hand, is calculated as the total cost in the economy divided by the total output.

Output and Prices in Model

	Output/Cap.	Output/Labor	Inv.cost/output	Price	p_a
Model	0.69	2.89	0.053	0.513	1.10

The investment cost to output ratio is the total investment and reallocation costs divided by total output. Total cost accounts for 5 percent of total output, which is consistent with estimates based on plant level data by Cooper and Haltiwanger (2006).

The equilibrium price p_a is 1.10, indicating that the price of bundled capital is higher than that of unbundled capital. The absolute level of equilibrium price of bundled capital is about 0.56, while the price of new capital is 0.51. This seems to be consistent with the U.S. data. *Mergerstat Review* documents that the average merger premium (computed as (offer price - stock market price)/stock market price) in the U.S. is about 50%, which indicates that p_a is 1.5. That is, price in reallocation market is 1.5 times the firm's value per unit of capital. In our model, the price p_a is for the bundled capital, not the firm value. So, the equilibrium price p_a is not directly comparable with the average merger premium, but it does reflect the fact that the bundled capital can be more expensive than new capital.

2.4.3 Fixed Cost and Reallocation

In the late 1980s, the U.S. experienced a wave of mergers and acquisitions. The annual total value of all acquisitions reached 4.84 percent of GDP in 1988 from 3.11 percent in 1984 (Holmstrom and Kaplan 2001). In the mid- and late-1980s, a large proportion of acquisitions was leveraged buyout with investment groups and managers using borrowing to buy back firm shares. Nearly half of the U.S. corporations received takeover offers in this period. After this wave of acquisitions, some firms went private. Meanwhile, from 1985 to 1989, the aggregate labor productivity, aggregate capital productivity and 4-factor TFP grew by 12.3 percent, 6.0 percent and 2.0 percent respectively. These productivity measures are weighted averages of 4-digit SIC industries. We use the real shipment value share of each industry for the weight.

In this period, the average investment rate declined and the reallocation rate increased as shown in the following table.

Investment, Reallocation and Productivity 1985-1989[10]

[10]Investment and reallocation rates are computed from Compustat. These are the unweighted mean over firm level values. Productivity measures are computed using NBER-CES Manufacturing Industry database. The productivity measures are weighted average of 458 4-digit SIC industries.

Year	Inv./Cap.	Acq./Cap.	Output/worker	Output/Cap.	TFP
1985	.21	.074	213.2	2.56	.98
1986	.21	.146	222.5	2.55	.97
1987	.19	.108	232.7	2.75	1.00
1988	.18	.098	238.7	2.78	1.01
1989	.17	.097	239.5	2.72	1.00

Source: Author's calculation from Compustat and NBER-CES Manufacturing Industry Database.

The reallocation rate rose from .074 in 1985 to an annual average of 12 percent in 1986-1988, representing an increase of 58 percent. Many firms became private and were delisted from Compustat in this period, and thus the reallocation rate increase is very likely to be underestimated.[11]

The capital reallocation in the late-1980s was mainly because the stock market price was low relative to the cost of building new capacity. Expanding through takeovers of the capital of other firms appeared to be cheaper than building up new plants. Jensen (1993) takes the view that acquisitions in the 1980s are reaction of capital market to corporate mismanagement of conglomerates. In addition, the antitrust law was less strictly enforced than before.

It seems that fixed cost decreases help explain this wave of acquisitions in the U.S. From 1985 to 1989, the 4-factor TFP increased only 2 percent while the output per worker increased 12 percent. Our model can generate labor and

[11]We plot the firm size distributions in 1985 and 1989, and find that the size distribution in capital shifted to the left side.

capital productivity gains arising from capital reallocation that is consistent with the facts during this period. We take the year 1985 as the base year with low reallocation, while the period 1986-1988 represents the years with high reallocation. We then compare the two steady states with different fixed cost values. In order to examine the effects of reallocation, we do not change the aggregate technology, but let only the fixed cost decrease from $\Phi = .065$ to $\Phi = .028$.[12]

	Inv.rate	Acqui/Cap	Output/Cap	Output/Labor
$\Phi = .065$	.152	.067	.384	2.73
$\Phi = .028$	.142	.102	.377	2.77
	$h(\cdot)$/output	Price	p_b	
$\Phi = .065$	.049	.54	1.10	
$\Phi = .028$	.048	.53	1.06	

When the fixed cost drops, the reallocation rate increases by 50 percent and the participation rate rises from 10 percent to 30 percent. The productivity gain is positive but small relative to that in data. The output per unit of labor increases by 1.5 percent and the output per unit of capital shows essentially no change. Comparing this result with the data, we see that increased capital reallocation activities increase aggregate productivity. The small change in output per unit of capital can be explained as follows. The production technology displays decreasing returns to scale, thus the output to input ratio is decreasing as the firms size increases. When the volume of

[12] We could have estimated the cost parameter values with simulated method of moments since we can compute the investment and reallocation moments for each year. But we need to assume that the cost parameter values can vary over time.

reallocation increases, even the firms with higher management ability become larger, the productivity measures can become smaller. On the other hand, firms that sell off their capital experience productivity gains as this represents a downsizing. In addition, the decreased fixed cost has a price effect, which influences the firm's decision on the split between unbundled capital and bundled capital. The absolute price of bundled capital declines from 0.54 to 0.53. Because of this price drop, more firms buy more the bundled capital in the new equilibrium. Some firms may reduce investment in unbundled capital and switch some investment to bundled capital. The consequence of all of these factors is that the aggregate investment rate of unbundled capital decreases slightly.

As the reallocation rate increases, the standard deviation of output-to-capital ratio increases by 12 percent, and the standard deviation of output-to-labor ratio decreases by 4 percent. The firm size is also changed as more firms participate in the reallocation market. Figure 2.4 shows the distribution change in the two stationary equilibria. As reallocation activities increase, low productivity firms tend to sell more capital and high productivity firms tend to buy more capital from the reallocation market. Therefore, the firm's capital size is positively correlated with firm specific shocks. Since we do not allow exit by merger, more firms are getting smaller as reallocation increases while other firms grow to larger sizes.

In summary, we have shown that the reallocation of capital can account for 12 percent of labor productivity gain between 1985-1989.

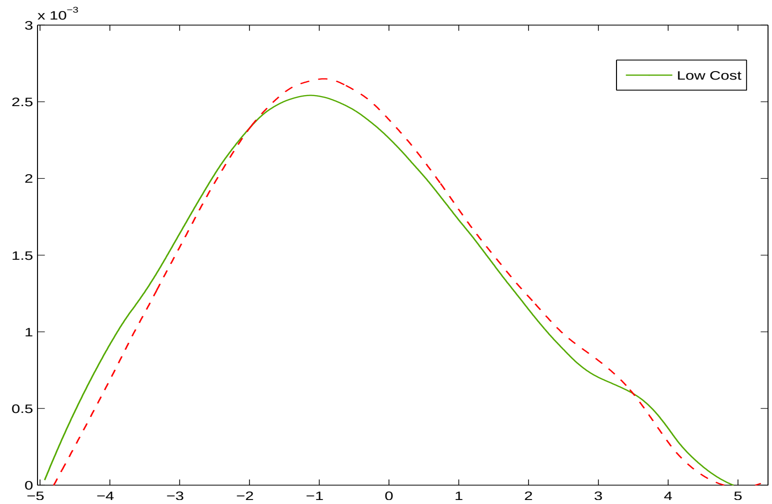

Figure 2.4: Fitted Firm Size Distribution change in Log(k)

Figure D.1 in the Appendix D shows the transitional dynamics of aggregate variables as the fixed participation cost drops permanently.

2.4.4 Aggregate Technology and Reallocation

In this section, we study the most recent wave of mergers and acquisitions in the late 1990s. In this period, the U.S. experienced high economic growth, accompanied by the IT change. The volume of reallocation reached an historical record of 15 percent of GDP in 1999. Unlike the merger wave in the late 1980s, in this later wave, 58 percent of transactions were financed with stocks (Andrade, Mitchell and Stafford, 2001).

Investment, Reallocation and Productivity 1996-2000[13]

[13]Productivity measures are from BLS. All values are indexed with 100 in 1996. TFP is

Year	Inv./Cap.	Acq./Cap.	Output/hour	Output/Cap.	TFP
1996	0.22	0.14	100.0	100.0	100.0
1997	0.21	0.18	103.8	101.4	103.1
1998	0.19	0.25	108.9	101.7	105.7
1999	0.16	0.17	114.0	101.7	108.7
2000	0.21	0.13	118.3	101.0	111.3

Source: Author's calculation from Compustat and BLS productivity tables.

This table shows that from 1996 to 2000, TFP increased by 11.3 percent, capital productivity increased by 1 percent and labor productivity increased by 18.3 percent.

We are interested in what drives the reallocation increase and whether reallocation changes contribute to the increase of the aggregate output-labor ratio. As shown in Section 1, the threshold investment rate at which the firm is indifferent between $I_b = 0$ and $I_b \neq 0$ is independent of the state variables. But the demands for investment and labor tend to increase because of increased z. We would expect that the wage rate will go up, and the output price goes down. The increased capital demand implies that more firms tend to buy capital from the reallocation market and fewer firms sell capital to the reallocation market, so p_b should go up. This is true in the following experiment. However, in the long run the increased z has no effects on reallocation.

In the two computed stationary equilibria, we let the aggregate technology levels be $z = 0.9476$ and $z = 1.0553$ respectively, thus implying an 11.3

the five factor total productivity.

percent increase. The fixed cost in both cases is the same as in the baseline model. The following table summarizes the results.

Effects of Aggregate Technology on Reallocation and Productivity

	Inv.rate	Acqui./Cap.	Output/Cap.	Output/Labor
$z = .947$	.15	.075	.376	2.65
$z = 1.055$	.15	.075	.376	3.04
	cost/output	Price	p_b	
$z = .947$	.052	.560	1.096	
$z = 1.055$	.052	.487	1.096	

The result shows that a permanent change in the aggregate technology has no impact on investment and reallocation in the long run. Instead, the output per unit of labor increases by 13.5 percent driven by the technology change. The output price decreases by 10 percent, which is exactly the same magnitude as the percentage change of aggregate technology. In addition, the aggregate labor demand exhibits no changes despite the wage rate increase. The distribution of firm size measured in capital shift to the right as show in Figure 2.5.

This exercise indicates that the aggregate technology might not be the source of reallocation. However, if the aggregate technology change does alter reallocation activities in the economy, it should deliver some effects through changing the dispersion of firm specific shocks. The correlation between aggregate technology and firm specific shocks should then be positive. Of course, if

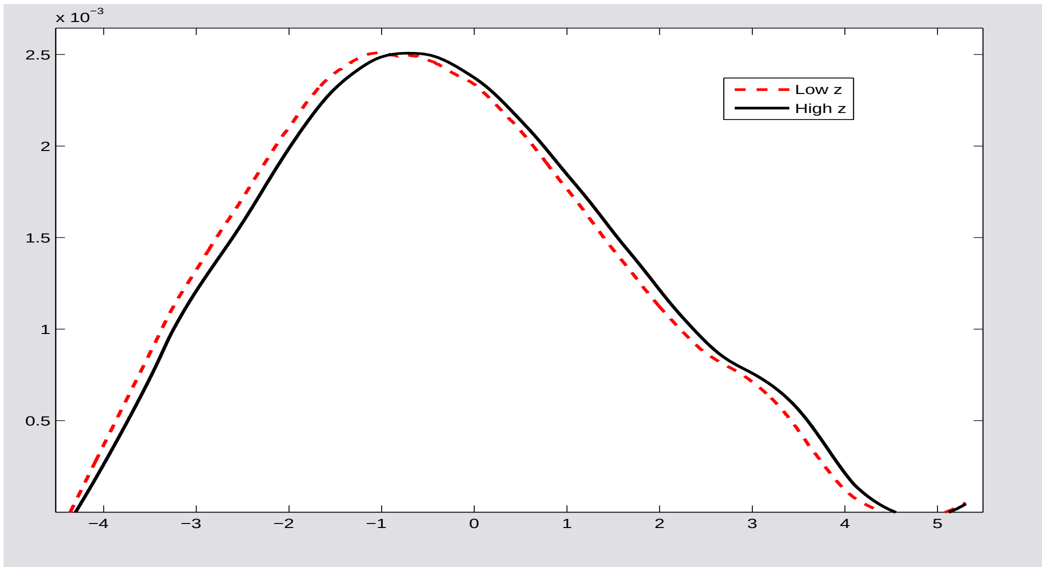

Figure 2.5: Fitted Size Distribution ($\log(k)$)

adjusting labor is costly or the capital adjustment costs are correlated with the aggregate technology through disruptions on production, then the aggregate technology change can also have effects on reallocation.

To further demonstrate the short-run behavior of our model economy, we also obtain the transition path from the low steady state to the high steady state. Figure D.2 in Appendix D shows the transitions of moments and prices. When the unanticipated change to the aggregate technology occurs, The marginal products of both labor and capital increase suddenly. The firm tends to increase the amount of investment and reallocation. The investment rate and reallocation rate increase to 25 percent and 10 percent respectively from the initial steady state. This causes both prices to increase. As the prices become higher, the firm's demand for investment decreases. In addition, since adjusting capital is costly while adjusting labor is costless, the firm continues to increase the demand for labor, which drives the wage rate up and output price down.

Jovanovic and Rousseau (2004) study a mechanism of reallocation. In their model, when new technology is created the firm can reorganize its production internally through investment and employment. If a firm is not able to reorganize, it has an option of being acquired as a way of exit, by other firms that succeed in reorganization. We extend our model to incorporate this mechanism into the firm's decision. Though firms do not exit in our model, they do respond differently to the capital price change that is a result of the heterogeneity among firms adopting new technology. [14]

In this extension, initially all firms have the same aggregate technology z_1. When a new technology z_2 arises, some firms are able to adopt z_2, but others cannot. After the arrival of technology z_2, we assume the transition of the aggregate technology as follows

$$\pi_z = \begin{bmatrix} .20 & .80 \\ .02 & .98 \end{bmatrix}.$$

In the steady state, 98 percent of firms ultimately adopt the technology z_2. We let $z_1 = 0.936$ and $z_2 = 1.068$. Again we compare the two steady-state equilibria, with all firms first having technology z_1. In the second equilibrium, the aggregate technology evolves with transition π_z.

[14] I thank Raphael Solomon for suggesting this extension.

Effects of Aggregate Technology: Extension

	Inv.rate	Acqui./Cap.	Output/Cap.	Output/Labor
z_1 only	.15	.0718	.383	2.52
z_1 and z_2	.15	.0733	.381	3.01
	cost/output	Price	p_b	
z_1 only	.049	.588	1.095	
z_1 and z_2	.050	.492	1.095	

When the economy converges to the new steady state where 2 percent of firms are left to use the old technology z_1, the aggregate investment in new capital does not change, while the acquisition rate increases by 6 percent. Consequently, the 14 percent change in aggregate technology causes a 2 percent increase in acquisition and 19.4 percent increase in labor productivity. Figure 2.6 shows the capital distribution change between the two steady states. It

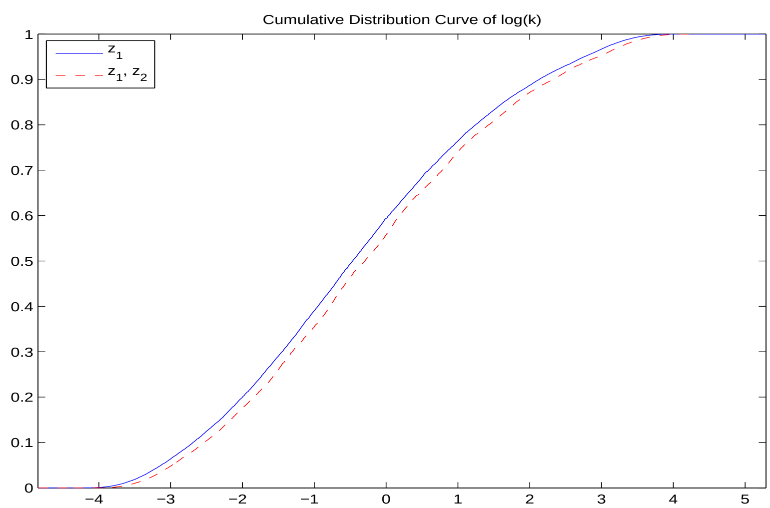

Figure 2.6: Cumulative Distribution Curve of Log(k)

is clear that the labor productivity change is driven by both the aggregate technology changes and the related firm's investment. We now decompose the aggregate labor productivity change into two components: one arising from aggregate technology change, and the other arising from firm distribution change due to technology changes. Let the joint distribution of firms be $\mu_z(z, \varepsilon, k)$. The aggregate labor productivity is defined as

$$y_l = \int [\log z + \log \varepsilon + \nu(1-\alpha) \log k + (\nu\alpha - 1) \log l] d\mu_z(z, \varepsilon, k).$$

It is noted that we use $\frac{f(\varepsilon,k,l)}{l}$ for labor productivity in all previous computations, while now we use the logarithm of this measure. With this new definition, the aggregate labor productivity increases from .924 to 1.1022, a change of .178. The contribution of labor to the labor productivity increases from 1.0678 to 1.0817, a change of .014. Thus, close to 8 percent of productivity change is contributed by labor components. The capital component increases by .0155, from -.0778 to -.0423. Thus, the capital component contributes 8.7 percent to the aggregate labor productivity increase.

The aggregate technology changes result in capital reallocation decision changes. From Figure 2.6, the firm's capital distribution shifts to the right, indicating that there are more larger firms. This firm size distributional change contributes to productivity increase in addition to the direct effect of aggregate technology change.

2.5 Related Research

To my best understanding, our model is the first quantitative one to investigate the effects of capital reallocation on aggregate productivity. Our model has similar features to Jovanovic and Rousseau (2002). Motivated by the fact that in nearly 70 percent of mergers and acquisitions in the U.S. the buyer's Tobin's Q value is larger than the seller's Tobin's Q value, Jovanovic and Rousseau (2002) use the Q theory to explain why some firms buy others. The authors regress bundled investment rate on firm's Q values and find that the Q value and the bundled investment rate are significantly and positively correlated. Jovanovic and Rousseau (2004) study the waves of mergers in the U.S. They use new technology adoption to explain merger waves. In their model, when new technology is invented, firms can adopt it and adjust their production in order to use the new technology. But it may be too costly for some firms to adopt the new technology, so these firms may exit or may be sold to firms that succeed in adopting the new technology. Our model differs from Jovanovic and Rousseau (2004), because what drives reallocation in our model is the heterogeneity in management ability across firms. But we also study the effects of the aggregate technology change.

In addition, Ramey and Shapiro (1998) document the magnitude of capital reallocation. Our definition of capital reallocation is different from theirs. Ours follows Jovanovic and Rousseau (2002) and Eisfeldt and Rampini (2006).

Our model emphasizes the role of adjustment costs in firm investment in a general equilibrium model. We have estimated the cost parameters in such a model which is, to my knowledge, the first to estimate adjustment cost parameters by taking price as endogenous. Cooper and Haltiwanger (2006) estimate the factor adjustment cost parameters using the plant level data. We have borrowed some of their estimation results.

Several recent papers study the general equilibrium implications of non-convex adjustment costs (Khan and Thomas 2003,2006, among others). The primary finding is that investment spikes are not eliminated although in the household they tend to smooth consumption. But, our main focus is the effects of reallocation on productivity measures, rather than investment dynamics.

2.6 Conclusion and Future Research

We set up a general equilibrium model to quantify the effects of capital reallocation on aggregate productivity measures. Our calibration results show that the model can match the investment and reallocation moments in the U.S. Our model helps explain the source of aggregate productivity gain in the late 1980s. However, the change in aggregate technology only temporarily changes the level of reallocation before reaching a new steady state.

Several features are missing from the baseline model. We now discuss extensions to the model.

First, alternative specifications of adjustment costs should be experimented to examine whether our model results are sensitive to different assumptions on the capital adjustment cost. Cooper and Haltiwanger (2006) find that production disruption cost, quadratic cost, and irreversibility together best fit the investment moments. In the current model, the fixed cost is proportional to capital stock, making the threshold investment rate to participate in the reallocation market independent of the firm's state. It is interesting to check whether our results are robust by adding disruption cost to the model.

Secondly, our model does not allow entry and exit. In Compustat, on average about 1-2 percent firms exit per year, and 60 percent of exit is due to reallocation. If exit is added to the firm's choice in the current model, firms that are forced to stay now can choose to exit by selling the whole firm in the reallocation market. Since the selling of capital decision is negatively correlated with firm shocks, exit would truncate the firm distribution on the left tail. It is expected that allowing for exit would strengthen our results on the effects of reallocation.

Finally, we do not consider the agency problems explicitly in the model. Incentive problems of managers and information asymmetry in the reallocation market can be added to the model. However, it is necessary to distinguish between the magnitudes of capital adjustment costs and costs associated with informational frictions.

Chapter 3

Reallocation, Entry, Exit and Aggregate Productivity

3.1 Introduction

Empirical evidence surveyed in Chapter 1 shows that entry and exit are important components of reallocation that can contribute to the productivity changes. In Compustat, between 1986-2004, on average 4 percent of firms exited from the data, of which 60 percent is due to mergers and acquisitions. The share of capital of exited firms is 1.3 percent of the total capital in an average year. At plant level, Becker etal (2005) find that the capital destruction rate by exited plants is roughly 1.3 percent. Foster etal (2001) find that entering establishments averagely are more productive than exited ones.

In this chapter, we ask how much productivity change can be accounted for by entry, exit and capital reallocation. To address this question, we extend the model of reallocation of previous chapter by adding entry and exit. In the extended model, the average productivity shock of entrants is the same as that of the incumbents. Firms can exit through selling the whole firm to other firms. There is also an exogenous probability of sudden death when the firm is forced to scrap its capital. Low-productivity firms now can choose to exit, thus shifting the productivity shock distribution to the right. Consequently, the aggregate labor productivity tend to increase compared with the model that does not allow the firm to exit. We find that the aggregate labor productivity increases slightly as the reallocation fixed cost drops. The entry and exit processes are not affected by the decrease in reallocation cost. This suggests that in order for the entry and exit to play a role in affecting the aggregate productivity, we need to find forces that drive the entry and exit dynamics.

3.2 The Model

3.2.1 Firm

The model assumptions follow Chapter 2. There are two additional assumptions. First, the firm can choose to sell the whole firm to other firms on the reallocation market, hence the sell-off value is $p_b k$. In addition, there is a probability π_d that an incumbent shuts down and exits the industry. If the firm exits through the sudden death, the sell-off value is a proportion of capital market value, $\theta p_u k$. Besides, firms pay a fixed cost for staying in the industry. Given the production function specified in the previous chapter, the firm's static labor choice is $l^* = \left(\frac{\alpha_2 z \varepsilon}{w}\right)^{\frac{1}{1-\alpha_2}} k^{\frac{\alpha_1}{1-\alpha_2}}$ where $\alpha_1 = (1-\alpha)\nu$ and $\alpha_2 = \alpha\nu$. The firm's profit net of the labor cost is

$$R(\varepsilon, k) = \frac{1-\alpha_2}{\alpha_2} w l^*.$$

Let $v(\varepsilon, k)$ be the value of an incumbent firm in any period, $v^e(\varepsilon, k)$ be the firm value if it exits, and $v^s(\varepsilon, k)$ be the value if the firm stays.

The incumbent firm's recursive problem is

$$v(\varepsilon, k) = \max\{v^e(\varepsilon, k), v^s(\varepsilon, k)\}$$

where

$$v^e(\varepsilon, k) = p_b k$$

and

$$\begin{aligned} v^s(\varepsilon, k) &= \max_{\{I_u, I_b\}} R(\varepsilon, k) - c_f - I_u - p_b I_b - h_1(I_u, k) - h_2(I_b, k) - \Phi k \cdot 1_{\{I_b \neq 0\}} \\ &+ \frac{1}{1+r}\{(1-\pi_d) E v(k', \varepsilon') + \pi_d \theta k'\} \end{aligned}$$

subject to $k' = (1-\delta)k + I_u + I_b$. The fixed cost $\Phi = 0$ if $I_b = 0$. The gross interest rate $(1+r)$ is endogenously determined in equilibrium. It is noted that here we have used the real values, so p'_u does not show up in the last term of v^s.

With this specification, the exiting firm does not pay any adjustment cost. It is the incumbents who pay the capital adjustment costs of acquired firms. In addition, the exogenous exit probability π_d is the same for all firms. This assumption appears to be strong because the empirical evidence shows that the exiting firms on average have lower productivity than the survivors. We maintain this assumption as a benchmark.

Entry

A continuum of potential entrants exits and decides whether to enter the industry or not. It takes two periods to finish the entry. In the entry period, the entrant makes the start-up investment decision before it draws the idiosyncratic shock realization from the distribution $\Omega(\varepsilon)$. We assume that $\Omega(\varepsilon)$ is independent and identical across firms and over time. The entrant pays a cost $(\psi k + c_e)$ and decides the start-up capital level by solving the following problem

$$v^0(k_0) = \max_k \int_{\varepsilon \in \mathcal{E}} \frac{1}{1+r} v(\varepsilon, (1-\delta)k) d\Omega(\varepsilon) - \psi k - c_e.$$

In the entry period, the firm does not produce. After entry, the firm draws its realization of productivity shock from $\Omega(\varepsilon)$ and starts production and investment. It is important that in equilibrium we should have $v^s(\varepsilon, (1-\delta)k_0) \geq p_b(1-\delta)k_0$ for all ε. This inequality in equilibrium prevents the entrant from making profits by exiting right after the entry (without any production and investment). This condition implies that $[\frac{1}{1+r}p_b(1-\delta)-\psi]k_0 \leq c_e$, which is satisfied if either capital has a certain lower bound or the entry cost parameter satisfies $\psi \geq \frac{1}{1+r}p_b(1-\delta)$.

We assume that the entrant buys capital from the unbundled capital market. This is mainly to simplify the model. It is more realistic to allow the firm to be built from the existing firms rather than from the scratch. The free-entry condition implies that $v^0(k_0) \leq 0$. When $v^0(k_0) < 0$, there is zero entry. In the rest of the paper, we look at the case with positive entry, i.e. $v^0(k_0) = 0$.

The entrants' productivity shocks follow the uniform distribution, with the same support as the invariant distribution of the incumbent's productivity shock. The incumbents' invariant distribution of productivity shock is log-normal. Hence there are more firms with high productivity shocks among entrants than incumbents.

3.2.2 Household

The economy has one representative household with preference

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \xi L_t]$$

where c_t is consumption and L_t is the fraction of individuals employed. This preference is used in Hansen (1985) and Rogerson (1988). Since there is no aggregate uncertainty, household's optimization is deterministic. The household owns all firms. In each period, the household chooses the optimal consumption, the labor supply and investment in firm shares. Let $w(\mu)$ be the wage rate relative to output price, let $dQ(\varepsilon, k, \mu)$ be household's portfolio of the one-period shares of firms with ε and k. Also let $\rho(\varepsilon, k, \mu)$ be the share price of all firms with ε and k. The household buys the firm portfolios at the beginning of each period. Household's recursive optimization problem is

$$W(Q, \mu) = \max_{c,L,Q'} [u(c, L) + \beta W(Q', \mu')]$$

subject to

$$c + \int_{\mathbb{S}} \rho(\varepsilon', k') dQ'(\varepsilon', k', \mu') \leq w(\mu)L + \int_{\mathbb{S}} v(\varepsilon, k) dQ(\varepsilon, k, \mu).$$

On the left hand side of the budget constraint, $\rho(\varepsilon', k', \mu)$ is the price of the firm that enters next period with ε' and k'. In a stationary equilibrium, the number of firms with ε' is certain and given by the invariant distribution of ε. In this sense, household's portfolio $dQ(\varepsilon, k, \mu)$ is risk-free.

Let the shadow price of firm's output be p_u. It is the Lagrange multiplier in the household's dynamic problem. The first order conditions are $u_1(c, L) = p$, $w = -\frac{u_2(c,L)}{u_1(c,L)}$, and $\rho(\varepsilon', k') = \beta \frac{p'}{p} v(\varepsilon', k')$.

3.3 Equilibrium Analysis

We focus on the stationary equilibrium with positive entry and exit.

3.3.1 Firm Distribution

Let the firm's policy function be $k' = g(\varepsilon, k) \in \mathcal{K}$. The firm distribution over (ε, k) can be summarized by the probability measure μ defined on $\mathcal{S}$, where $\mathcal{S}$ is the σ-field generated by the open subsets of product space $(\mathcal{E}, \mathcal{K})$. Let M be the mass of entering capital. The evolution of firm distribution is

$$\mu'(\varepsilon', k') = (1 - \pi_d) \int_{\mathcal{S}} \pi(\varepsilon', \varepsilon) 1_{\{k' = g(\varepsilon, k)\}} d\mu(\varepsilon, k) + 1_{\{k_0 \in \mathcal{K}\}} M \Omega(\varepsilon'),$$

where $\pi(\varepsilon', \varepsilon)$ is the transition matrix of ε. The term $1_{\{k_0 \in \mathcal{K}\}}$ is a vector of zeros, except that where $k = k_0$ it is one. This evolution can be expressed as $\mu'(\varepsilon', k') = \Gamma(\mu(\varepsilon, k))$. On the right hand side, the first term is the conditional distribution of firms that stay in the industry. The second term is the distribution of entrants on $\mathcal{S}$. Let $P(\varepsilon', k' | \varepsilon, k) = \pi(\varepsilon', d\varepsilon) 1_{\{k' = g(\varepsilon, k)\}}$. It is the transition matrix of the state (ε, k). Then $P(\varepsilon', k' | \varepsilon, k) \mu(\varepsilon, k) = \int_{\mathcal{S}} P(\varepsilon', k' | \varepsilon, k) d\mu(\varepsilon, k)$.

If the invariant distribution exists, then we have

$$\mu(\varepsilon, k) = [I_0 - (1-\pi_d)P]^{-1} \cdot (M 1_{\{k_0 \in \mathcal{K}\}} \Omega(\varepsilon)) \tag{3.1}$$

where I_0 is the identity matrix.

3.3.2 Labor and Output Markets

Now let the household preference be $u(c, L) = \log c - \xi L$. From the household's problem, the aggregate consumption is $C = \frac{1}{p}$. Not surprisingly for the quasi-linear preference, the optimal aggregate consumption is independent of the non-labor revenue. In addition, the wage rate is $w = \frac{\xi}{p}$. In the stationary equilibrium, the household budget constraint and the first-order conditions give the optimal labor supply as follows

$$L^s = \frac{1}{\xi} + \frac{(\beta - 1)}{w} \int_{\mathcal{S}} v(\varepsilon, k) d\mu(\varepsilon, k)$$

where we have used the equilibrium conditions for the asset market (see the equilibrium definition below). Use the household's first-order optimality condition $w = \frac{\xi}{p}$ and define $V(\varepsilon, k) = p \cdot v(\varepsilon, k)$, the labor supply can be written

$$L^s = \frac{1}{\xi} \left[1 + (\beta - 1) \int_{\mathcal{S}} V(\varepsilon, k) d\mu(\varepsilon, k) \right].$$

The optimal labor decision by the continuing and entering firms is $l(\varepsilon, k)$ as shown in section 2.1. The aggregate demand for labor is

$$L^d(\varepsilon, k, \mu) = \int_{\mathcal{S}} l(\varepsilon, k) d\mu(\varepsilon, k).$$

It is not clear whether one can use the labor market clearing condition to obtain the entry mass M. Both L^d and L^s are determined not only by M, but also by p_u. Moreover, price p_u is affected by the size of the entry mass. In computing the model, we guess the value of M then verify whether the resulting equilibrium labor demand and supply equal.

The net aggregate output that the household can use for consumption in each period is computed as the weighted output level of all the firms. In the stationary equilibrium, the continuing firm has the following real revenue

$$\Pi(\varepsilon,k)=z\varepsilon(k^{1-\alpha}l^{\alpha})^{\nu}-c_f-I_u(\varepsilon,k)-p_b I_b(\varepsilon,k)-h_1(I_u(\varepsilon,k),k)-h_2(I_b(\varepsilon,k),k).$$

If the firm exits, the sell-off value is $p_b k$ or θk, dependent upon how the firm exits. It is noted that in equilibrium the aggregate net investment in bundled capital is zero. The reallocation market affects consumption only because it is costly to participate in the reallocation market. The labor income is also canceled out in the budget constraint condition. The net aggregate output is the output of all the firms net of investment and related costs by incumbent firms and entrants.

$$C^s(\mu)=(1-\pi_d)\int_{\mathcal{S}}[(\Pi(\varepsilon,k)-p_b k)1_{\{g(\varepsilon,k)\in\mathcal{K}\}}+p_b k]d\mu(\varepsilon,k)+\pi_d\int_{\mathcal{S}}\theta k d\mu(\varepsilon,k)-M(\psi k_0+c_e).$$

The first two terms on the right hand side are the aggregate output (revenue) of continuing firms and exiting firms, the second term is the total cost of entry.

3.3.3 Reallocation Market

Chapter 2 shows that there exists a threshold value $\tilde{i}$ at which the firm is indifferent between choosing $I_b = 0$ or not. Given the price p_b and if the firm participates in the reallocation market, we can use the firm's optimal split decision conditions to find that the firm (ε, k) buys bundled capital if its total investment rate $i(\varepsilon, k) > \tilde{i_2}$, and it sells bundled capital if $i(\varepsilon, k) < \tilde{i_1}$, where $\tilde{i_1} = \frac{1}{\gamma_n}(p_b - 1 - \sqrt{2\Phi(\gamma_n + \gamma_a)})$ and $\tilde{i_2} = \frac{1}{\gamma_n}(p_b - 1 + \sqrt{2\Phi(\gamma_n + \gamma_a)})$. As the price p_b increases, the firm tends to buy less or sell more bundled capital.

The measure of capital being sold is

$$\Lambda(p_b) = (1 - \pi_d)\left(\int_{\mathcal{S}} 1_{\{I_b(\varepsilon,k)<0\}} I_b(\varepsilon, k) d\mu(\varepsilon, k) - \int_{\mathcal{S}} 1_{\{g(\varepsilon,k)\notin\mathcal{K}\}} k d\mu(\varepsilon, k)\right).$$

The first term is the total amount of capital being sold by the continuing firms. The second term on the right side is the total amount of capital sold in the reallocation market by the exiting firms. $\Lambda(p_b)$ is negative and a decreasing function of price p_b. The measure of capital being purchased is

$$\Psi(p_b) = (1 - \pi_d) \int_{\mathcal{S}} 1_{\{I_b(\varepsilon,k)>0\}} I_b(\varepsilon, k) d\mu(\varepsilon, k)$$

which is positive and decreasing in p_b. In equilibrium, two measures sum to zero.

3.3.4 Recursive Equilibrium

A stationary recursive equilibrium is defined as a set of functions

$$\left(w, p, r, p_b, \rho, v, L^d, K', W, C, L^s, Q, M, \mu\right)$$

such that household and firms maximize their expected values, and markets for reallocation, asset, labor and output clear:

1. Given prices, v solves firm's Bellman equation, and $l(\varepsilon, k, \mu)$ and $k'(\varepsilon, k, \mu)$ are the firm's policy functions for labor and capital, for all $(\varepsilon, k) \in \mathcal{S}$. The aggregate future capital is $K'(\mu) = \int_{\mu \in \mathcal{S}} k'(\varepsilon, k, \mu) d\mu$.

2. Given prices, W satisfies the household's problem. (C, L^s) are the associated policy functions.

3. The reallocation market clears: p_b solves $\Psi(p_b) + \Lambda(p_b) = 0$.

4. The asset market clears: $Q(\varepsilon, k) = \mu(\varepsilon, k)$ for all $(\varepsilon, k) \in \mathcal{S}$.

5. The labor market clears: $L^s(\mu) = L^d(\mu)$.

6. The output market clears: $C = C^s(\mu)$.

7. The firm distribution is given by equation (3.1).

8. Free-entry condition is satisfied, i.e. $v^0(k_0) = 0$.

To repeat, the household's first-order necessary conditions are $-\frac{u_2(c,L)}{u_1(c,L)} = w(\mu)$ and $p = u_1(c, L)$. In equilibrium, we have $\frac{1}{1+r} = \frac{\beta u_1(c',L')}{u_1(c,L)}$ or $1 = \frac{p}{p'} = \beta(1 + r)$. So the interest rate is $\frac{1}{\beta} - 1$. The interpretation of these necessary conditions are standard. The household chooses the optimal consumption to equalize the market values of marginal utility between two periods. Within period, the household's marginal rate of substitution of consumption and leisure equals to the wage rate.

Existence of Stationary Equilibria

For the existence of invariant distribution $\mu(\varepsilon, k)$, see Stokey and Lucas (1989) and Hopenhayn and Prescott (1992). These authors prove the existence of stationary equilibrium without the aggregate uncertainty.

3.4 Model Solution

The model solution is in many ways similar to the method in Chapter 2. To facilitate the calibration below, here we repeat the computation procedure.

The equilibrium can be solved by formulating a new Bellman equation that combines the household's first-order conditions and the firm's dynamic programming equation. As defined earlier, let $V = p_u \cdot v$. Plug the household's optimal decision rules and condition $\beta(1+r) = 1$ into the firm's problem, the re-formulated recursive problem is obtained as following

$$V(\varepsilon, k, \mu) = \max\{V^e(\varepsilon, k, \mu), V^s(\varepsilon, k, \mu)\}$$

where

$$V^e(\varepsilon, k, \mu) = p_u(\mu) p_b(\mu) k$$

and

$$\begin{aligned} V^s(\varepsilon, k, \mu) &= \max_{\{I_u, I_b\}} p_u(\mu)[R(\varepsilon, k, \mu) - c_f - I_u - p_b(\mu) I_b - h_1(I_u, k) - h_2(I_b, k) \\ &\quad - \Phi k \cdot 1_{\{I_b \neq 0\}}] + \beta[(1-\pi_d) EV(\varepsilon', k', \mu') + \pi_d \theta p_u'(\mu') k'] \end{aligned}$$

We use the following procedure to compute the model. We first guess the value of entry mass M. Then we compute the model as in Chapter 2.

We then obtain the aggregate labor supply and aggregate labor demand. If they are equal, we stop, otherwise, we change the value of M and repeat. The computed equilibrium may not be the only one.

3.5 Calibration

We calibrate the model to the U.S. data on investment and reallocation, then we study the two recent U.S. merger waves using the calibrated model to investigate the role of reallocation. The household discount factor is $\beta = 0.9615$ to reflect a 4 percent annual interest rate. The decreasing return to scale is set to be $\nu = 0.8507$. This value is derived from the production curvature estimated in Cooper and Haltiwanger (2006).[1] The Cobb-Douglas technology parameter is $\alpha = 0.7454$, which implies that the labor share in production is 0.64 as in Prescott (1986). Given this value the output-to-capital ratio is about 0.22. The capital depreciation rate is chosen as $\delta = 0.069$ following Cooper and Haltiwanger (2006). The share of leisure in utility is $\xi = 0.94$ so that around 80 percent population works in steady state.

The first-order autocorrelation process of the firm specific shock is from Cooper and Haltiwanger (2006), $\rho_\varepsilon = 0.885$, $\sigma_\eta = 0.30$, where the AR(1) equation is $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \eta_t$. These values were estimated using the U.S. plant data, which provided a sample consisting of mainly large plants, most of which

[1] Cooper and Haltiwanger estimate that the production curvature is 0.592. Under our assumption, production function has decreasing return to scale. If $\alpha = 0.7454$, then we get $\nu = 0.8507$.

are owned by publicly held firms. The aggregate technology is normalized to 1. The estimated parameter values are $\gamma_n = 0.758$, $\gamma_a = 0.474$ and $\Phi = 0.0585$, which are estimated in the previous chapter.

We choose $\theta = 0.75$, which is close to the selling price of displaced capital for plants shutting down as in Ramey and Shapiro (2001). We set $c_f = 0$ in the benchmark calibration. The exit probability is set $\pi_d = 0.02$. In Compustat, from 1986 to 2004, on average 4 percent firms exited from the data set each year. In these 4 percent exited firms, 60 percent are due to mergers and acquisition. The entry cost parameter is $\psi = 1.237$ such that the entry rate measured in capital is approximately 1 percent.

3.5.1 Baseline Results

The equilibrium is computed in the following steps:

1. Fix a value of M.

2. Given p_u and p_b, solve the firm's recursive problem. Corner solutions for bundled capital investment are taken care of by allowing firms to choose $I_b = 0$.

3. Find the optimal capital choice at entry.

4. Compute the stationary distribution.

5. Compute the aggregate output and the aggregate bundled capital investment. Check the market clearing conditions. If both the output market

and the reallocation market have zero excess demands, then stop. Otherwise, go back step 2.

6. Check whether the labor market clears. If yes, M is the right value. If not, then change M and repeat from step 2.

Practically, the entry mass M can be found in a few rounds of iteration. The following table summaries some moments of the baseline calibration.

Investment and Reallocation

	Inv.rate	Spike$^+$	Spike$^-$	Inaction	Cap.sale/Cap.	Acqui./Cap.	Participation
Data	.18	.23	-	.031	.014	.087	.240
Model	.14	.286	.096	.038	.057	.089	.152

In equilibrium, no firm buys capital in one market while selling capital in the other market. The baseline results show that our model can match the long-term data moments well.

Output and Prices in Model

	Output/Cap.	Output/Labor	Output/Inv.	Inv.cost/output	Price	p_b
Model	0.42	2.68	8.90	0.058	0.55	1.055

3.5.2 Fixed Cost and Reallocation

In this section, we compare the two steady states with different fixed cost values to examine the quantitative contribution of reallocation. In order to examine the effects of reallocation, we let the fixed cost decrease from $\Phi = .065$ to $\Phi = .028$. This exercise largely follows Chapter 2.

	Inv.rate	Acqui./Cap.	Output/Cap.	Output/Labor	Inv./Output	Inv.cost/output	Price	p_b
$\Phi = .065$	.141	.085	0.425	2.66	0.279	.058	.55	1.054
$\Phi = .028$	.140	.113	.0.421	2.69	0.307	.055	.55	1.055

As the reallocation cost is reduced, the reallocation market participation rate increases from 14 percent to 29 percent. The reallocation rate also increases. The labor productivity improves but only by 1 percent. In addition, when $\phi = .065$, the capital exit rate is about 5.6 percent and 2.1 percent is due to exogenous exit probability, while when $\phi = .028$ the exit rate is 4.92 percent and 2.1 percent of exit is forced by the exogenous probability.

These numbers imply some interesting effects of reallocation cost decrease. As this cost drops, the firm exit through reallocation actually decreases which is surprising since it is now less costly to both buy and sell capital in the reallocation market. There should be increased reallocation activities in the economy. The above numbers tell that the increased reallocation occurs among the firms that stay in the industry. Due to cost decrease, it is more profitable for the firm to stay than to exit, hence the exit through reallocation drops. This in turn dampens the productivity improvement because now fewer firms exit.

It is not surprising that the exit rate through the sudden death does not change since the reallocation cost does not affect the exogenous exit process. Figure 3.5.2 shows that when the reallocation cost drops, the size distribution does not shift much. On the contrary, when cost is low there are more small firms staying in the economy.

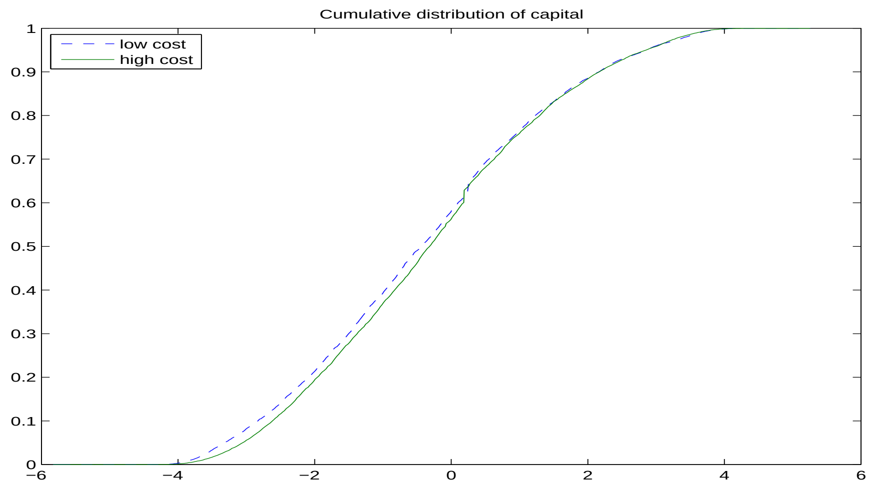

Figure 3.1: Cumulative Distribution of Capital

Adding entry and exit to the model does not change the magnitude of productivity improvement when the reallocation cost decreases. The main reason is that decrease in the reallocation cost does not affect the entry and exit processes. On the contrary, the exit through reallocation decreases. In order for the entry and exit to play a role in affecting the aggregate productivity, we need to find forces that drive the change of entry and exit.

3.6 Concluding Remarks

We have shown that increased capital reallocation has modest impact on the aggregate productivity changes. In our model, this is so because decrease in the reallocation cost does not affect the entry and exit processes. It actually helps reduce the firm exit through reallocation.

There are several directions that would lead to improve the quantitative results of this model. First, we need to find other sources that drive the changes in entry and exit. Because firms with lower productivity tend to exit first, increased exit would contribute to the productivity gain. Second, the exit process can be changed so that less productive firms are more probable to exit. This is left for the future research.

Appendices

Appendix A

Selection Bias of Using Balanced Panel

Assume η_t follows the normal distribution $N(0, \sigma^2)$. Let $\chi_t = 1$ indicates the plant survival. From the quasi-difference equation of production function, the expected profit in period t is then

$$E[\pi_t|\pi_{t-1}, k_{t-1}, \chi_t = 1] = \rho_\varepsilon \pi_{t-1} + \theta k_t - \rho_\varepsilon k_{t-1} + b_t - \rho_\varepsilon b_{t-1} + E[\eta_t|\pi_{t-1}, k_{t-1}, \chi_t = 1].$$

The plant survives if $v^e(k_t) < v^s(a_t, k_t, \eta_t; \Psi)$, where v^s is plant's value if it continues to stay, and $\Psi = (F, \gamma, p_s, \theta)$. Solving this inequality and knowing that $v^s = \pi(a, k, \eta) + \int_\eta \int_{a \in A} v(a', k', \eta) dF(a', a) dN(\eta)$, we get $\eta_t > h(a_t, k_t, \Psi, \Phi)$ for $\chi_t = 1$, where $h(\cdot)$ is the cutoff value of η_t below which the plant exits.

The conditional expectation is

$$E[\eta_t|\eta_t > h(a_t, k_t)] = \frac{\int_{\eta_t > h(a_t, k_t)} \eta_t dN(\eta_t)}{1 - N(h(a_t, k_t))} = g(a_t, k_t).$$

The expectation of profit equation is then

$$E[\pi_t|\pi_{t-1}, k_{t-1}] = \rho_\varepsilon \pi_{t-1} + \theta k_t - \rho_\varepsilon \theta k_{t-1} + b_t - \rho_\varepsilon b_{t-1} + g(a_t, k_t).$$

Selecting plants with higher productivity can cause upward biased estimates of θ and ρ.

It can be shown that

$$\frac{\partial g(\cdot)}{\partial k_t} = n(h(a,k))\frac{\partial h(\cdot)}{\partial k_t}\frac{\int_{\eta_t > h(a_t,k_t)}[\eta_t - h(a_t,k_t)]dN(\eta_t)}{[1 - N(h(a_t,k_t))]^2},$$

where $n(\eta_t)$ is the density function of η.

Now look at the bias caused by the selection of k. Estimating the profit function without correcting for $h(\cdot)$ would give biased results. If $\frac{\partial h(a_t,k_t)}{\partial k_t} < 0$, estimates of θ would be downward biased. Small plants would be more productive.

Suppose that

$$v^s(a_t,k_t,\eta_t;\Psi) = \pi(a_t,k_t,\eta_t) - Fk_t - I_t - \frac{\gamma}{2}\frac{I_t^2}{k_t} + \beta\int_\eta\int_{a\in A} v(a',k',\eta)dF(a',a)dN(\eta)$$

and $v^e(k_t) = p_s k_t + \Phi$, then the plant stays in if

$$\pi(a_t,k_t,\eta_t) + \beta\int_\eta\int_{a\in A} v(a',k',\eta)dF(a',a)dN(\eta) > \Phi + (F+p_s)k_t + I_t + \frac{\gamma}{2}\frac{I_t^2}{k_t}.$$

Let $\pi(a_t,k_t,,\eta_t) = e^{\eta_t}a_t k^\theta$, then

$$\frac{\partial h(\cdot)}{\partial k_t} = [\Phi + (F+p_s)k + I + \frac{\gamma I^2}{2k} - \beta\int_\eta\int_{a\in A} v(a',k',\eta)dF(a',a)dN(\eta)]^{-1}\cdot(F+p_s-\frac{\gamma I^2}{2k^2}) - \frac{\theta}{k}.$$

The sign of this derivative is ambiguous, depending upon the parameters. So, it is unclear whether the estimate of θ is upward biased or downward biased.

Appendix B

Data on Reallocation and Productivity

Reallocation

The manufacturing U.S. data is from Compustat. We choose periods between 1971 and 2004 for computing moments because the reallocation variable in Compustat is only available during this period. Adding the mineral industries and construction industries only changes moment values very slightly. All dollar values are deflated using CPI (with 100 in 1996). Firms with capital value less than 0.5 million dollars are removed. Capital is the period beginning level of data item 7. Investment is the expenditure on plant, property and equipment (data item 128). Capital sale is dollar value of sold plant, property and equipment (data item 107). Acquisition is total expenses of acquiring plant, property and equipment from other firms (data item 129). Employment is the total number of employees (data item 29).

Moments for Manufacturing Industries

Investment and Reallocation Moments

	Inv.rate	Spike	Inaction	Cap.sale/Cap.	Acqui./Cap.	Participation
Micro	.181	.23	.031	.014	.087	.24
Macro	.114	.23	.031	.012	.075	.24

Micro moments are computed as the averages of firm level values. Investment rate is the average of firm investment rates. Spike is the proportion of firm-year observations with investment rate larger than 20 percent. Inaction is the proportion of firm/year observations with investment rate less than 1 percent. Participation is the proportion of firm-year observations that has non-zero acquired capital value.

Macro moments are computed as the averages of annual moments. Investment rate is calculated as the average annual investment rate which is the yearly total investment of all firms divided by yearly total capital stock of all firms.

Investment distribution

std dev	skewness
0.34	12.1

Investment distribution moments are computed as the averages of annual values.

Firm Size Distribution

	mean	std dev	skewness	kurtosis
Capital	1.88 (.356)	2.22 (.160)	0.49 (.068)	2.80 (.110)
Employment	2.74 (.292)	1.92 (.180)	0.25 (.053)	2.72 (.123)

Capital is the logarithm of demeaned data item 7. First we divide firm's capital by the yearly industry average capital, then take the logarithm.

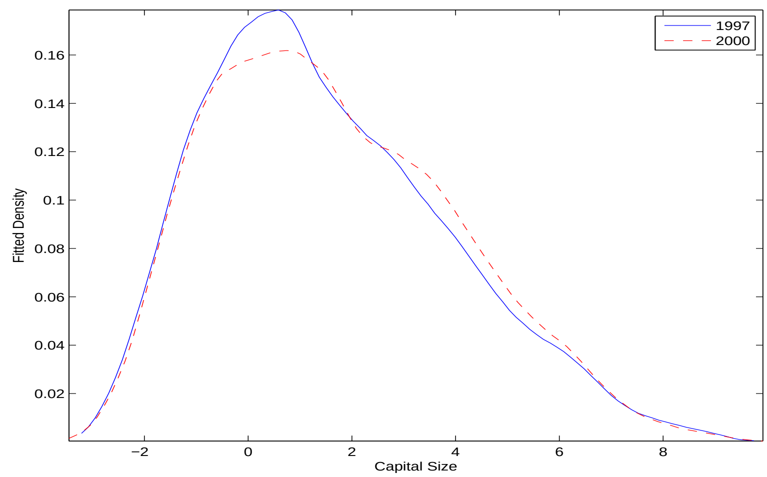

Figure B.1: The U.S. Firm Distribution in Capital

Employment is computed in the same way. All moments are calculated as the averages of yearly values during 1971-2004. Values in brackets are standard deviations of moments. The fitted distribution curves in selected years are in the following graphs.

Moments for Mineral, Construction and Manufacturing

The following moments are computed by choosing all firms in the mineral, construction and manufacturing industries.

Moments: investment and reallocation rates

	Inv.rate	Spike	Inaction	Cap.sale/Cap.	Acqui./Cap.	Participation
Micro	.180	.25	.036	.016	.088	.22
Macro	.116	.25	.036	.012	.078	.22

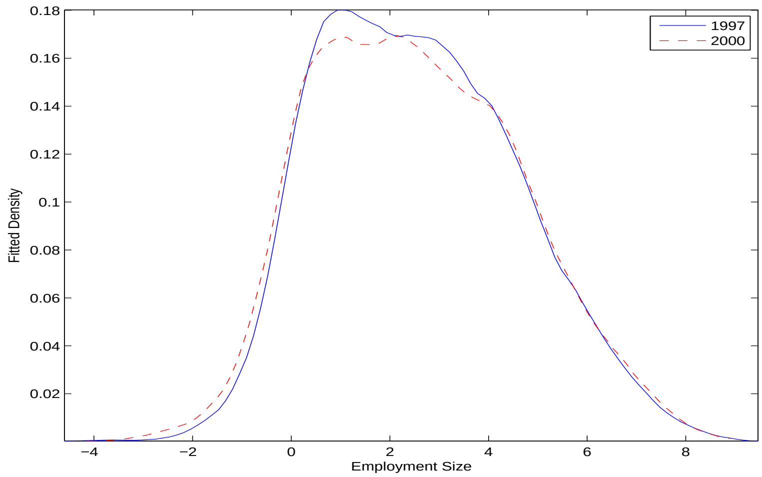

Figure B.2: The U.S. Firm Distribution in Employment

Investment distribution

std dev	skewness
0.37	13.72

Firm Size Distribution

	mean	std dev	skewness	kurtosis
Capital	1.95 (.336)	2.21 (.160)	0.45 (.066)	2.75 (.137)
Employment	2.64 (.325)	2.04 (.221)	0.052 (.130)	2.90 (.192)

Productivity

We use the NBER-CES industry data to compute productivity moments for years between 1971-1996. The data are on 4-digit SIC industry level. The aggregate productivity measures are mean productivity measures weighted by share of industry shipment in total shipment. The standard deviation of productivity measures is also weighted by shipment values.

The productivity measures between 1997-2001 are computed from the BLS productivity website at http://www.bls.gov/.

Appendix C

When does the firm choose $I_a = 0$?

If the firm chooses to invest in both types of capital, the optimal investment rate in bundled capital is

$$i_a = \frac{1}{\gamma_n + \gamma_a}(\gamma_n i + 1 - p_a). \tag{C.1}$$

When is $I_a = 0$ optimal?

The firm chooses $I_a = 0$ if

$$i + \frac{\gamma_n}{2} i^2 \leq \min\{i - i_a + p_a i_a + \frac{\gamma_n}{2}(i - i_a)^2 + \frac{\gamma_a}{2} i_a^2 + \Phi, p_a i + \frac{\gamma_a}{2} i^2 + \Phi\}.$$

Let $\tilde{i}$ be the total investment rate at which the firm is indifferent between $I_a = 0$ and $(I_n \neq 0, I_a \neq 0)$, then $\tilde{i} = \frac{1}{\gamma_n}(p_a - 1 \pm \sqrt{2\Phi(\gamma_n + \gamma_a)})$. That is, when

$$i \in \left(\frac{1}{\gamma_n}(p_a - 1 - \sqrt{2\Phi(\gamma_n + \gamma_a)}), \frac{1}{\gamma_n}(p_a - 1 + \sqrt{2\Phi(\gamma_n + \gamma_a)})\right), \tag{C.2}$$

the firm chooses $I_a = 0$, investing in unbundled capital only. Next we check conditions under which the firm does not choose $I_n = 0$ over $I_a = 0$. We compare the costs of choosing $I_n = 0$ and $I_a = 0$.

If the firm chooses $I_a = 0$, then we have $p_a i + \frac{\gamma_a}{2} i^2 + \Phi > i + \frac{\gamma_n}{2} i^2$. Then if

$$i \in \left(\frac{1}{\gamma_n - \gamma_a} (p_a - 1 - \sqrt{(p_a - 1)^2 + 2\Phi(\gamma_n - \gamma_a)}), \frac{1}{\gamma_n - \gamma_a} (p_a - 1 + \sqrt{(p_a - 1)^2 + 2\Phi(\gamma_n - \gamma_a)}) \right), \tag{C.3}$$

the firm would choose $I_a = 0$ over $I_n = 0$.

Therefore, when investment rate is in the intersection of the above two sets (C.2) and (C.3), the firm will choose $I_a = 0$, investing in unbundled capital only. From inequality (C.4) (see below), we know that the cost of $I_n = 0$ is always larger than the cost of $I_n \neq 0, I_a \neq 0$. So the condition (C.3) is redundant.

Proof of Proposition 1

For the proposition to hold, the investment rate must satisfy

$$p_a i + \frac{\gamma_a}{2} i^2 + \Phi < \min\{i - i_a + p_a i_a + \frac{\gamma_n}{2}(i - i_a)^2 + \frac{\gamma_a}{2} i_a^2 + \Phi, i + \frac{\gamma_n}{2} i^2\}.$$

First, we need to find the range of investment rate at which the firm chooses $I_n = 0$ over investing in both types of capital. The difference between the two cost functions of $I_n = 0$ and $(I_n \neq 0, I_a \neq 0)$ is

$$\begin{aligned} &= p_a i + \frac{\gamma_a}{2} i^2 - [i - i_a + p_a i_a + \frac{\gamma_n}{2}(i - i_a)^2 + \frac{\gamma_a}{2} i_a^2] \\ &= \frac{1}{\gamma_n + \gamma_a} (\gamma_a i - 1 + p_a)^2 \geq 0, \end{aligned} \tag{C.4}$$

where in the second line equation (C.1) is used. This positive difference of investment costs between the two cases implies that for any investment rate the firm never chooses $I_n = 0$ over $(I_n \neq 0, I_a \neq 0)$.

Next, from the above algebra we know that the firm will choose $I_n = 0$ over $I_a = 0$ if investment rate i is located in the complement of (C.3). This implies that the firm must also choose $(I_n \neq 0, I_a \neq 0)$ over $I_a = 0$ because the set (C.3) is larger than set (C.2). But inequality (C.4) implies that the firm always choose $(I_n \neq 0, I_a \neq 0)$ over $I_n = 0$.

Therefore, the firm will never choose $I_n = 0$.

It is noted that the above proof is also valid if the fixed cost is Φ, instead Φk. However, this two forms of fixed cost induces different cut-off values $\tilde{i}$ at which the firm is indifferent between $I_a = 0$ and not.

When does the firm choose to buy capital in one market and to sell it in another market?

From above, we know that the optimal $i_a = \frac{1}{\gamma_n+\gamma_a}(\gamma_n i + 1 - p_a)$. So, if $i_a > 0$, then $i > \frac{-1+p_a}{\gamma_n}$. If $i_n < 0$, then $i < \frac{1-p_a}{\gamma_a}$. The two inequalities hold together only if $p_a < 1$. Meanwhile, the investment rate must be located in the complement of (C.2). It can be shown that it is possible that $i_a > 0$ and $i_n < 0$ only when $i > \frac{1}{\gamma_n}(p_a - 1 + \sqrt{2\Phi(\gamma_n+\gamma_a)})$. For i to be not located in an empty set, we require that $\frac{1-p_a}{\gamma_a} > \frac{1}{\gamma_n}(p_a - 1 + \sqrt{2\Phi(\gamma_n+\gamma_a)})$. Hence the proposition holds, and i must be larger than $\frac{1}{\gamma_n}(p_a - 1 + \sqrt{2\Phi(\gamma_n+\gamma_a)})$.

In a similar way, we can show that if $p_a > 1 + \gamma_a\sqrt{\frac{2\Phi}{\gamma_n+\gamma_a}}$, the firm chooses $i_a < 0$ and $i_n > 0$. At this time, the investment rate must be lower than $\frac{1}{\gamma_n}(p_a - 1 - \sqrt{2\Phi(\gamma_n+\gamma_a)})$.

Appendix D

Transition Dynamics

The transition path is computed as follows, starting with the stationary equilibrium for $\phi = 0.065$. Given this initial steady state, change the the cost value to 0.028, then find the equilibrium prices. After obtaining the prices, the firm's transition matrix from initial steady state to the next period is then obtained. Thus the next period's firm distribution will be known. With this distribution, the firms enter into the text period. These steps repeat until the prices and moments in the last period are very close to the corresponding values computed from the stationary equilibrium with the low cost.

The transition path in the second experiment is obtained similarly as the previous one. But we allow the aggregate technology to change gradually in two steps.

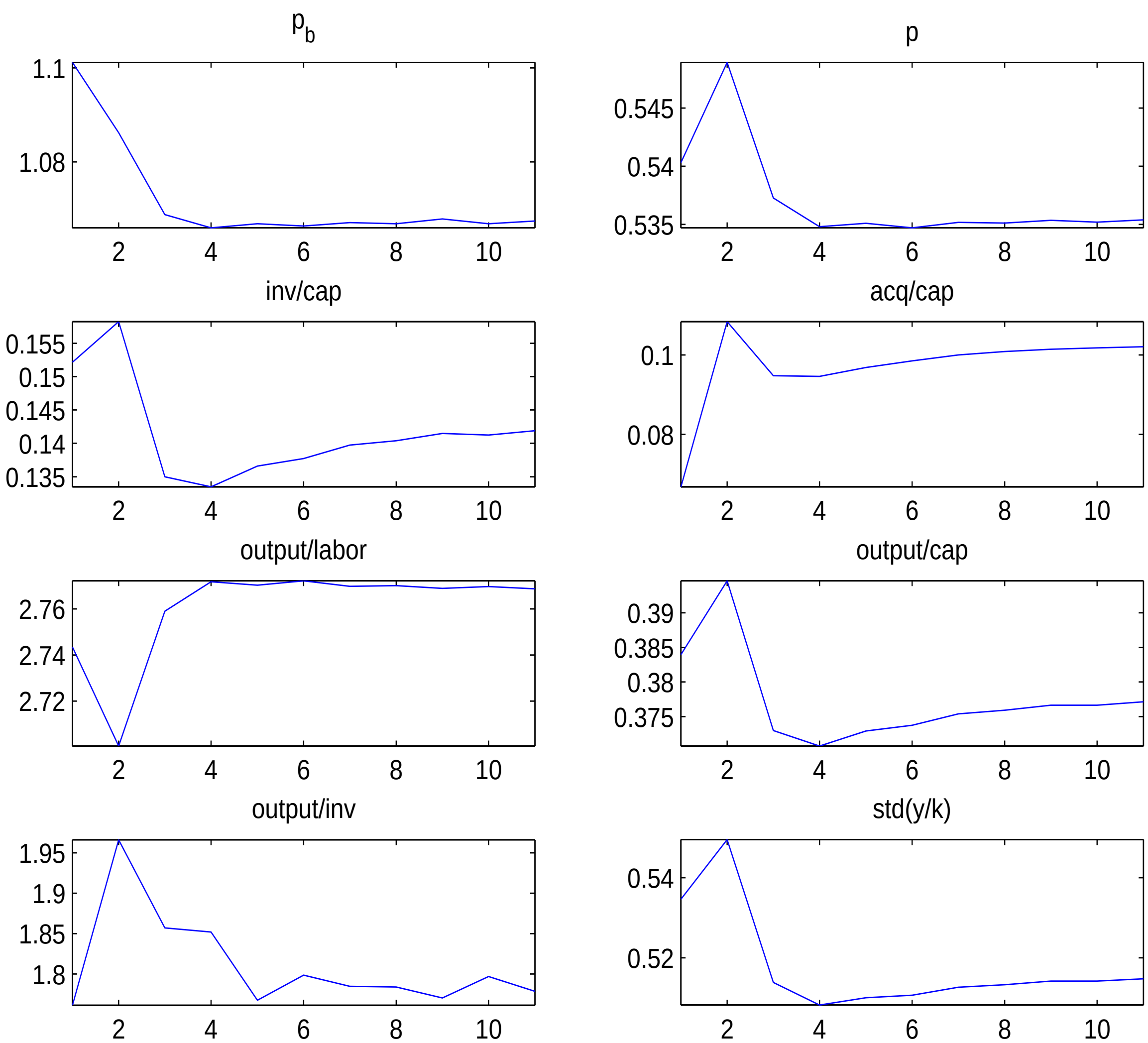

Figure D.1: Transition path from $\Phi = 0.065$ to $\Phi = 0.028$

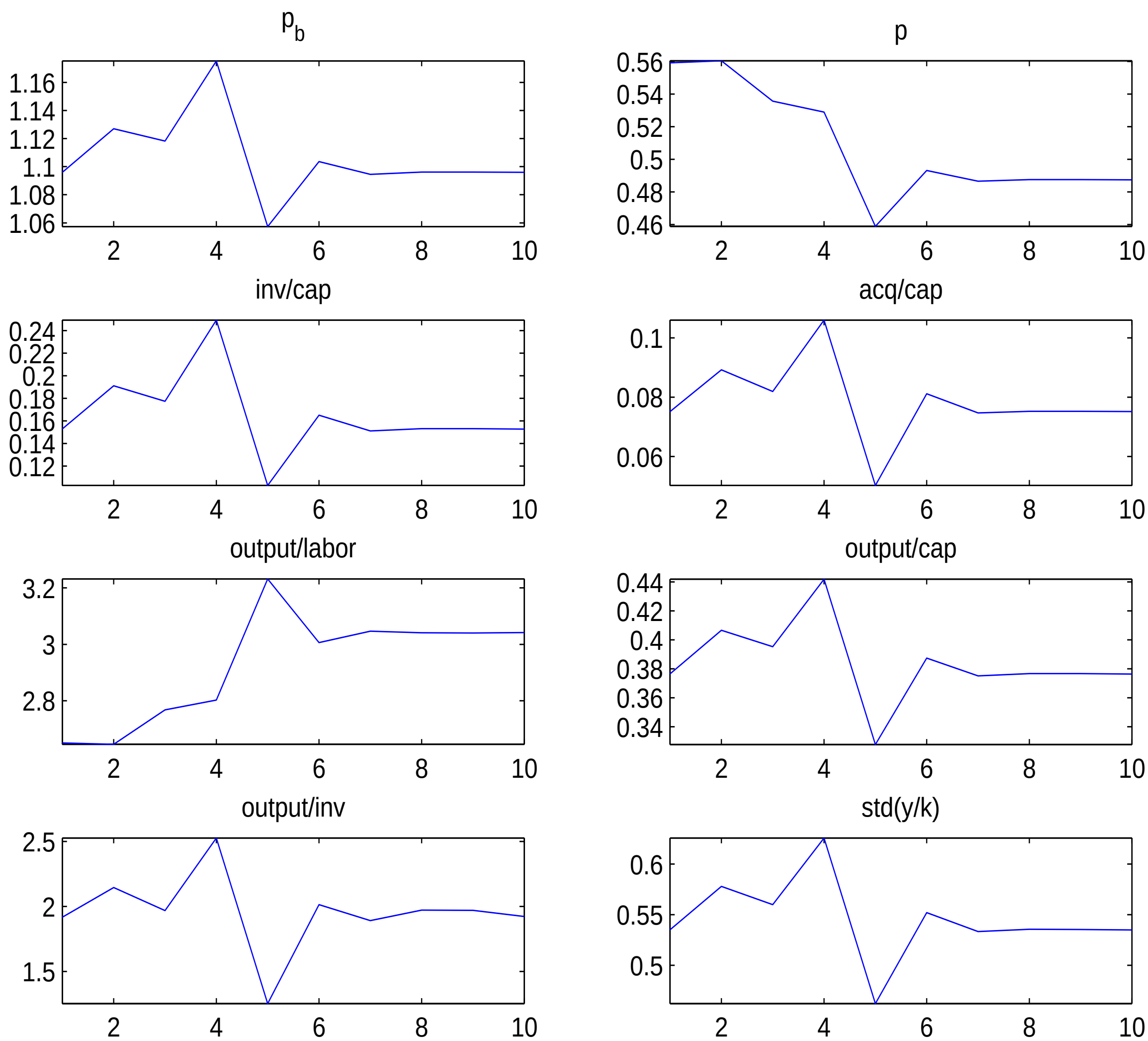

Figure D.2: Transition path from $z = 0.947$ (t=2) to $z = 0.973$ (t=3) to $z = 1.055$ (after t=3)

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Vita

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