SHAPE AND TOPOLOGY STRUCTURAL REDESIGN BY LARGE ADMISSIBLE PERTURBATIONS

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ABSTRACT

The ultimate goal in concurrent engineering of structures is to achieve simultaneously in the design stage the following objectives: (1) A shape that performs its *function*, conforms with the boundary conditions, and can support the external loads. (2) A product with structural *integrity*, i.e. with stress levels remaining below acceptable limits. (3) A product with acceptable *performance,* e.g. modal dynamics, i.e. with natural frequencies and mode shapes that do not amplify external dynamic loads; and static, i.e. acceptable deflection. (4) A composite *microstructure* that can optimally satisfy the above topology/ shape, load, and performance constraints. (5) A microstructure *fabrication* process that efficiently produces the above optimal structure. The purpose of our ONR funded project is to address the complete problem in concurrent structural design by further developing the LargE Admissible Perturbations (LEAP) theory which is being developed at the University of Michigan since 1983, and combining it with micromechanics constitutive equations. At the fabrication end, the Selective Laser Sintering (SLS) process will be simulated so that the SLS variables are defined as the final product of the concurrent structural design optimization process. LEAP theory -- as implemented in Code RESTRUCT (REdesign of STRUCTures) -- produces the final design without trial and error or repeated Finite Element Analyses (FEAs), thus, shortening the redesign process and contributing to rapid prototyping.

1. CONCURRENT STRUCTURAL DESIGN FOR MANUFACTURING

Solid Freeform Fabrication (SFF) techniques enable designers new flexibilities for designing structural components [6]. Microengineered materials can provide seemingly endless possibilities from which a designer might make a selection. In order to realize fully the potential of SFF processes and speed up the rapid prototyping cycle, the structural design should not be decoupled from the material design and the fabrication process. The goal of our research project is to develop structural design methodology which links the macrostructural design with the microengineering of materials and with the material fabrication process. Equivalently, the engineering problem addressed in our research is posed as follows: Given a particular function, what shape should the components have, what should it be made of, and how should it be made? The above concurrent design problem for freeform solids is shown schematically in Figure 1 which indicates the linkage of macrostructural properties and performance with microscale material characteristics and the fabrication process. A global optimization of material and geometry (topology, shape, and size) must address the issues of production efficiency and quality of the product.

As shown in Figure 1, the design process requires establishing the spatial distribution of the material. Thus, the material stiffness which is represented by the elastic modulus E, would be a function of position $E(\bar{x})$. Similarly, the density and strength of the material must be established; $\rho(\bar{x})$ and $\sigma(\bar{x})$ respectively. These parameters are related to the microstructure of the layered material and are described by – for example – porosity $P(\bar{x})$, crack distribution $D(\bar{x})$, and the Lamé elastic constants of material $\lambda(\bar{x})$ and $\mu(\bar{x})$.

In this project, the fabrication process we focus on is Selective Laser Sintering [7, 12]. There has been some work in establishing the effects of process variables such as binder and primary phase volume content on the composite strength. Particle size has also been studied with regard to its influence on strength. These functional relationships are essentially first order approximations which will be developed further during the course of the project. Further analyses will be performed to establish the interrelationship of such parameters on the macrostructure's stiffness, density and strength, etc.

These relations will be employed within the framework of LargE Admissible Perturbations theory to find optimal structural designs. Because the functional relations between the fabrication process, the microstructure, and the macroscale structural parameters are incorporated in the design methodology, an overall optimum design would be established. Accordingly, the five objectives of concurrent engineering of structures are listed in the Abstract and are abbreviated below for future reference:

(1) *Function* = function, boundary conditions, load support
(2) *Integrity* = structural integrity, strength limits
(3) *Performance* = static and modal dynamics performance
(4) *Microstructure* = material properties
(5) *Fabrication* = fabrication process

Several aspects of the concurrent structural design problem have been addressed in design optimization methods [9, 14] and inverse design methodologies that allow for small [13] or large [1-5, 8, 10, 11] structural changes.

2. LARGE ADMISSIBLE PERTURBATIONS APPROACH TO REDESIGN

Several problems in structural analysis and design (see Figure 2) – including the problem of redesign or inverse design – can be cast as two-state problems. State S1 is the initial state which is known and for which all required finite element analyses (modal dynamics, static buckling, etc.) have been performed. It is assumed that State S1 has undesirable characteristics or performance and should be improved to satisfy the designer's specifications. State S2 is the objective unknown state modeled by the same finite element grid but defined by different design variables.

The methodology we have been developing since 1983 for solving two-state problems is based on the LargE Admissible Perturbations theory. This methodology provides several advantages over trial and error, sensitivity methods, iterative techniques, and methods requiring repeated finite element runs. The basic features of our methodology are listed below.

(i) Primarily Two-State Theory is not an optimization methodology. It just uses NPSOL to find a solution to several optimization problems appearing in the process. Two-State Theory is a universal formulation and solution methodology for a plethora of analysis, design, redesign, model correlation (calibration), model reduction, reliability analysis, and monitoring problems related to the entire life of a structure from conception to dismantling. The Large Admissible Perturbations approach to redesign has two parts. The first part PAR (Perturbation Approach to Redesign) is the formulation of a two-state problem. The second uses a large admissible perturbations algorithm to solve the problem.

(ii) The Perturbation Approach to Redesign (PAR) develops the general perturbation equations by relating the two states S1 (known) and S2 (unknown). These are highly nonlinear equations – with implicit and explicit dependence on redesign variables. Nevertheless, they are equations somebody can work with as opposed to dealing only with numerical computations. The generality of the applicability of this methodology is most obvious in the solution it has provided to structural reliability problems where other theories (Stochastic Finite Elements, Structural Systems Reliability, and Response Surface Approach) cannot further progress due to lack of equations describing the failure state.

Some of the general perturbation equations developed in references [1-5, 10, 11] are provided below where unprimed and primed symbols correspond to states S1, S2, respectively. For modal dynamics, where ω is neutral frequency, $\{\psi\}$ is normal mode; k, m are stiffness and mass matrices; and α_e's are fractional change variables, we have:

$$\sum_{e=1}^{p}\left(\{\psi'\}_i^T[k_e]\{\psi'\}_i - \omega_i'^2\{\psi'\}_i^T[m_e]\{\psi'\}_i\right)\alpha_e = \omega_i'^2\{\psi'\}_i^T[m]\{\psi'\}_i - \{\psi'\}_i^T[k]\{\psi'\}_i \ ,$$

$$\sum_{e=1}^{p}\{\psi'\}_j^T[k_e]\{\psi'\}_i\alpha_e = -\{\psi'\}_j^T[k]\{\psi'\}_i \ ,$$

$$\sum_{e=1}^{p}\{\psi'\}_j^T[m_e]\{\psi'\}_i\alpha_e = -\{\psi'\}_j^T[m]\{\psi'\}_i \ ,$$

for $i = 1,2,\ldots,n,\ j = i+1, i+2,\ldots,n$ [1, 2] . For static deflections u ,

$$u_i' = \sum_{m=1}^{n}\left\{\frac{\phi_{im}' A_m}{B_m + \sum_{e=1}^{P} C_{me}\alpha_e}\right\} \ ,$$

where $A_m = \sum_{j=1}^{n}\phi_{jm}' f_j$, $B_m = \{\psi'\}_m^T[k]\{\psi'\}_m$,

$C_{me} = \{\psi'\}_m^T[k_e]\{\psi'\}_m$, and φ_{jm} represents modal amplitudes.

For static stresses σ ,

$$\sigma_k' = \sum_{i=1}^{g} S_{ki}\sum_{m=1}^{n}\left\{\frac{\phi_{im}' A_m}{B_m + \sum_{e=1}^{p} C_{m_e}\alpha_e}\right\}(1+\alpha_h) \ .$$

For buckling loads P , and modes $\{\psi_b\}$

$$\sum_{e=1}^{p}\{\psi_b'\}_i^T\left([k_{c_e}] - P_i'[k_{\sigma_{0e}}]\right)\{\psi_b'\}_i\alpha_e = \{\psi_b'\}_i^T\left(P_i'[k_{\sigma_0}] - [k_c]\right)\{\psi_b'\}_i \ ,$$

$$\sum_{e=1}^{p}\{\psi_b'\}_j^T[k_{c_e}]\{\psi_b'\}_i\alpha_e = -\{\psi_b'\}_j^T[k_c]\{\psi_b'\}_i \ ,$$

$$\sum_{e=1}^{p}\{\psi_b'\}_j^T[k_{\sigma_{0e}}]\{\psi_b'\}_i\alpha_e = -\{\psi_b'\}_j^T[k_{\sigma_0}]\{\psi_b'\}_i \ ,$$

for $i = 1,2,...,n$, $j=i + 1,i + 2,...,n$, where $[k_c] = [k_o] - [k_{\sigma F}]$, $k_{\sigma F}$ includes the body force, and $[k_\sigma] = - P_i[k_{\sigma o}] - [k_{\sigma F}]$.

(iii) The LEAP (LargE Admissible Perturbations) algorithms can solve the general perturbation equations for large changes (about 100%) of both the design variables and structure's specified response. LEAP algorithms are not limited by the 7% increments in sensitivity methods.

(iv) LEAP algorithms do not require repeated FEAs. In fact, for changes of the order of 100% no FEA other than that of the original known structural state S1 is needed.

(v) Code RESTRUCT performs routinely shape optimization as part of all the problems it can solve – see (i) above. The advantage here is that it keeps track of changes in elemental matrices so that the structure produced at the end is real. Some model correlation methods change the mass and stiffness global matrices and do not result in a real structure.

(vi) Code RESTRUCT has been developed for several complex structures; e.g. stiffened plates where the neutral axis and connectivity of stiffness to plate are affected during the redesign process.

(vii) Topology optimization has been achieved by RESTRUCT -- as presented in this paper -- in a very short period of time for 3-D bodies by introducing a brick finite element. The method is equivalent to those developed in [9, 14]. Further, it has all the advantages of large changes, no iterations, and no repeated finite element analyses.

(viii) It postprocesses data of a widely available finite element code MSC/NASTRAN. That is, structural state S1 is analyzed by FEM and RESTRUCT postprocesses these results to produce state S2 from its specifications.

The first step in achieving concurrent design including the five objectives listed in Section 1, is to develop Two-State Theory to formulate and solve (by a LEAP algorithm) the topology/shape optimization problem for concurrent structural integrity, modal dynamics, static deflection, and stress constraints (objectives for state S2). This encompasses objectives (1)-(3) (function, integrity, performance) of concurrent design and can be achieved by: (a) Introducing solid finite elements in code RESTRUCT; producing stiffness and mass, three-dimensional distributions to achieve the optimal topology and material properties; (b) Developing a LEAP algorithm to find the objective state S2 subject to modal dynamic constraints; (c) Developing the LEAP algorithm further to add static deflection constraints; (d) Finalizing the LEAP algorithm to include stress constraints. Actually, since 1983 LEAP algorithms have been developed to solve two-state problems for various finite elements and single or multiple modal dynamics, static deflection and stress redesign objectives. This paper and the following numerical applications provide the first step towards solution of the concurrent design problem defined above for solids.

3. NUMERICAL APPLICATIONS

Development of LEAP algorithms for a new redesign problem usually takes one year. An algorithm is considered fully developed when it can be used to redesign a structure for 100% changes in redesign objectives and large changes in the redesign variables, handle about 100 redesign variables, several simultaneous modal dynamics, static deflection and stress objectives, and about 1000 finite element degrees of freedom. Further development requires only more computational time. Such fully developed algorithms can produce redesign with about 3% error without trial and error or repeated FEAs.

The results presented in this section represent the first attempt to develop a LEAP algorithm for three-dimensional topology redesign for static and modal dynamics objectives. Thus, the error is still above the desired accuracy level of 3%. Improvement of the algorithm will be based on advanced formulation in the prediction phase based on nonlinear approximations of the general perturbation equations, as well as identification of appropriate extracted modes, admissibility conditions, and selection of redesign variables.

Several numerical applications are presented in Tables 1, 2 and 3 on the redesign of the cantilever beam in Figure 4 and the cantilever plate in Figure 5. In this early stage of development of the LEAP algorithm for solid elements, single frequency redesign is very accurate (cases b1. and d5.). Redesign for static displacement, or simultaneous static displacement and modal dynamics objectives requires further algorithmic development.

Figure 6 shows a simple two-dimensional topology redesign problem which has been used in the literature ([9] and [14]) as a bench mark problem. The results produced by Code RESTRUCT which implements the LEAP algorithm for topology optimization of solid elements are consistent with those published in the literature. The starting structure (state S1) of a solid plate is redesigned to become a multiply connected stucture at state S2.

CONCLUDING REMARKS

Presently, the large admissible perturbations methodology is capable of solving relatively simple three-dimensional redesign problems for static deflection and modal dynamics objectives. After fully developing the topology redesign algorithm for 3-D problems, the large admissible perturbations methodology will be developed further to address concurrent design problems for manufacturing including micromechanics constitutive equations and modeling of the SLS fabrication process.

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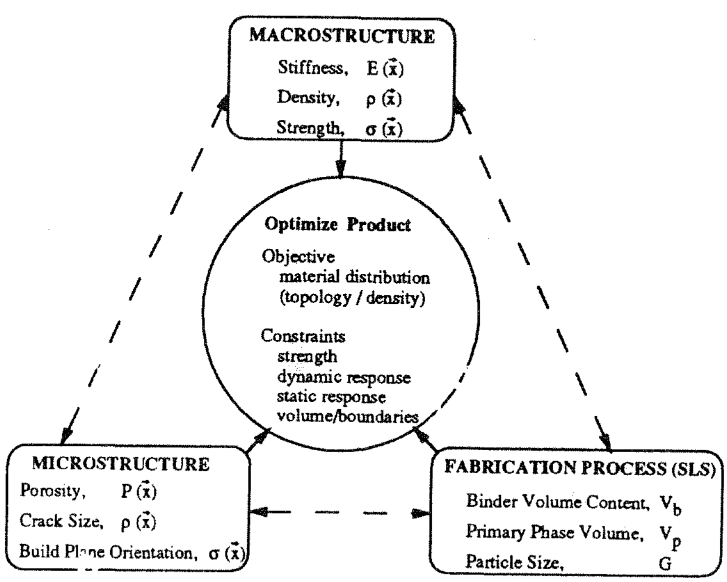


Figure 1. Concurrent design of freeform solids for structural integrity, topology, and microstructure fabrication

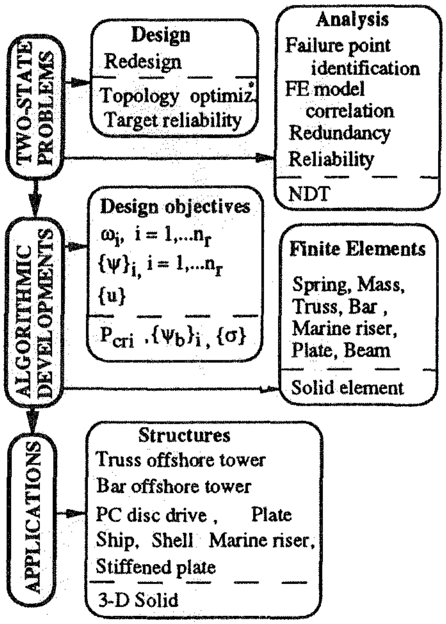


Figure 2. Status of large admissible perturbations theory for solution of two-state problems in structural analysis and design.
* Items below dashed lines are being developed.

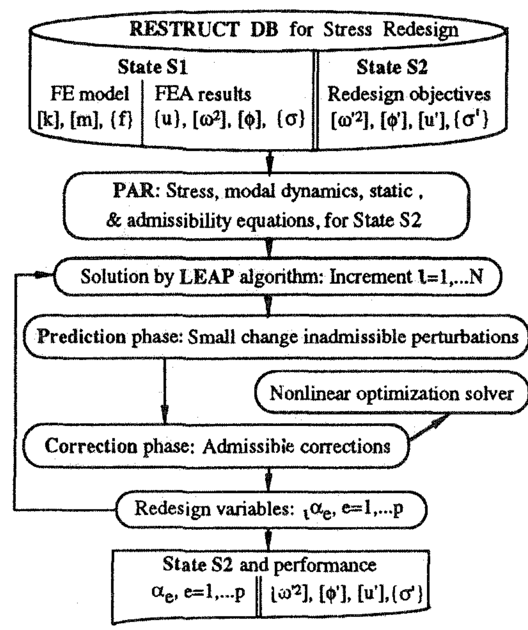
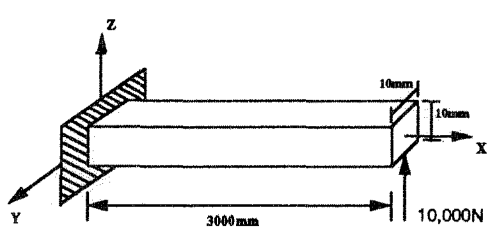


Figure 3. Stress redesign by a large admissible perturbations algorithm

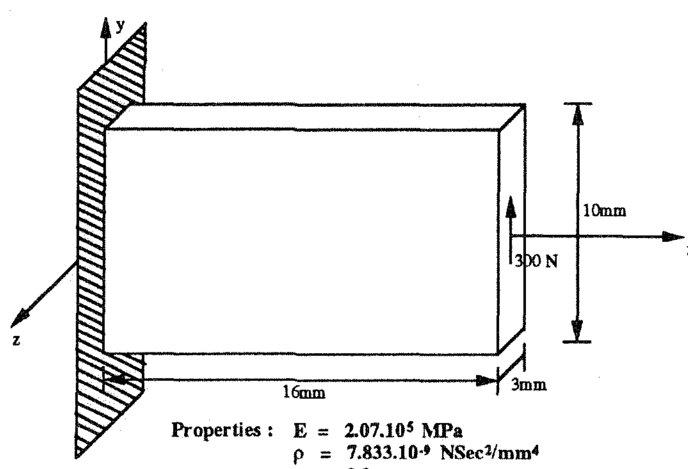


Properties : E = 2.07.10^5 MPa
ρ = 7.833.10^{-9} NSec^2/mm^4
ν = 0.3

Response : f_1 = 9.2778 Hz
w_{42} = w_{82} = 51.6851 mm

Model #a has 10 Hexa elements
Model #b has 30 Hexa elements

Figure 4. 3-D model for redesign of cantilever beam



Properties : E = 2.07.10^5 MPa
ρ = 7.833.10^{-9} NSec^2/mm^4
ν = 0.3

Response : f_1 = 9702.04 Hz
v_{289} = v_{102} = 0.01 mm

Model #c has 40 Hexa elements
Model #d has 160 Hexa elements

Figure 5. Solid element model of cantilever plate

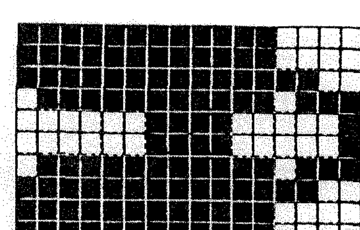


Figure 6. Cantilever plate : elimination of solid elements with small strain energy (case d4)

Table 1. Redesign of Cantilever Beam

Case #	Redesign Goal			Reanalysis			Error (%)		
	u'_{42}/u_{42}	u'_{82}/u_{82}	ω'^2_1/ω^2_1	u'_{42}/u_{42}	u'_{82}/u_{82}	ω'^2_1/ω^2_1	u'_{42}/u_{42}	u'_{82}/u_{82}	ω'^2_1/ω^2_1
a1.	0.499	-----	1.068	0.503	-----	1.048	-0.802	-----	1.873
a2.	0.500	-----	-----	0.503	-----	-----	0.600	-----	-----
b1.	-----	-----	3.000	-----	-----	3.000	-----	-----	0.000
b2.	-----	0.500	2.000	-----	0.497	2.233	-----	0.600	-11.65

Case #a1 and a2: Modeled by 10 solid elements; 10 redesign variables (p) for K
Case #b1 and b2: Modeled by 30 solid elements; p = 20 for M in case b_1; p = 20 for K in case b_2
Case #b2: Use of non-linear static prediction equation and NP solver

Table 2. Redesign of Cantilever Plate with 40 Hexa Elements

Case #	Redesign Goal		Reanalysis		Error (%)		p(1)	n_r(2)	n_a	%(3)	incr.
	u'_{53}/u_{53}	ω'^2_1/ω^2_1	u'_{53}/u_{53}	ω'^2_1/ω^2_1	u'_{53}/u_{53}	ω'^2_1/ω^2_1					
c1.	0.750	-----	0.820	-----	-9.333	-----	40	8	0	7	4
c2.	0.750	1.500	0.830	1.529	-10.667	-1.933	40	8	0	7	6
c3.	0.750	-----	0.810	-----	-8.000	-----	40	20	2	7	4
c4.	0.750	1.500	0.820	1.520	-9.333	-1.333	40	20	4	7	6
c5.	0.750	-----	0.797	-----	-6.267	-----	24	20	2	4	9
c6.	0.750	1.500	0.810	1.527	-8.000	-1.800	48	20	0	4	11
c7.	0.750	-----	0.783	-----	-4.400	-----	24	20	2	7	5

Case #c1-c6 calculated using linear static prediction eqn. with QP solver
Case #c7 calculated using non-linear static prediction eqn. with NP solver
(1) p: in all cases except c6, all redesign variables are for K
(2) n_r = number of extracted nodes
(3) % = incremental change to reach redesign goal

Table 3. Redesign of Cantilever Plate with 160 Hexa Elements

Case #	Redesign Goal		Reanalysis		Error (%)		p	n_r	%	incr.
	u'_{289}/u_{289}	ω'^2_1/ω^2_1	u'_{289}/u_{289}	ω'^2_1/ω^2_1	u'_{289}/u_{289}	ω'^2_1/ω^2_1				
d1.	0.755	-----	0.880	-----	-16.556	-----	80	20	7	4
d2.	0.755	1.500	0.880	1.479	-16.556	-1.400	80	20	7	6
d3.	0.755	-----	0.839	-----	-11.126	-----	80	20	7	4
d4.	0.755	-----	0.833	-----	-10.331	-----	80	20	4	7
d5.	-----	1.500	-----	1.500	-----	0.000	80	20	4	7

Case #d1.-d2. calculated using linear static prediction eqn. with QP solver
Case #d3.-d5. calculated using non-linear static prediction eqn. with NP solver