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# Strategic Behavior Analysis in Electricity Markets

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# Strategic Behavior Analysis in Electricity Markets

by

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To my LORD and family

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**Strategic Behavior Analysis in Electricity Markets**

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Strategic behaviors in electricity markets are analyzed. Three related topics are investigated. The first topic is a research about the NE search algorithm for complex non-cooperative games in electricity markets with transmission constraints. Hybrid co-evolutionary programming is suggested and simulated for complex examples. The second topic is an analysis about the competing pricing mechanisms of uniform and pay-as-bid pricing in an electricity market. We prove that for a two-player static game the Nash Equilibrium under pay-as-bid pricing will yield less total revenue in expectation than under uniform pricing when demand is inelastic. The third topic is to address a market power mitigation issue of the current Texas electricity market by limiting Transmission Congestion Right (TCR) ownership. The strategic coordination of inter zonal scheduling and balancing market manipulation is analyzed.A market power measurement algorithm useful to determine the proper level of TCR ownership limitation is suggested.

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# Chapter 1

# Introduction

Research on competitive and deregulated electricity markets has been very active since the introduction of restructuring in the British electricity market. Most of these research activities focus on the market design and evaluation of the designed market. When considering a new market design and evaluating a designed market, the most important concern will be the potential flaws and performance of the market structure. The strategic behaviors of market participants in a weak structure can greatly hamper the positive functioning of a competitive electricity market. The result of bad functioning can damage suppliers and buyers seriously. We have already experienced an extreme case of the deregulated electricity market's bad functioning in the California crisis.

The modeling and analyzing of the market participant or player's strategic behavior is the very core in electricity market research. From a social perspective we need to analyze a market's structural robustness when it faces strategic behavior or market power. From an individual perspective, the suppliers and buyers need to analyze the strategic behavior of the other players for a better profit or hedging.

In this dissertation three research topics related to strategic behavior modeling in electricity markets are investigated.

In chapter 2, we study NE search algorithms for games with local optima, where profit maximizing individual players face a profit function with local optima. We prove that any conventional iterative NE search algorithms applied to games with local optima can misidentify NE by following a local optimization path. We introduce the concept of "local NE trap" that attracts the conventional iterative NE search algorithms based on local optimization tools. The failure of NE search by the iterative algorithm is exemplified by a conceptual game having local optima. In order to overcome the problem of the conventional NE search, we apply a parallel and global search algorithm called co-evolutionary programming. By introducing the concept of "best rival matching and fine tuning," we develop an enhanced version of co-evolutionary programming, which we call "hybrid co-evolutionary programming". In order to test the effectiveness and performance of conventional NE search algorithms, co-evolutionary programming, and hybrid co-evolutionary programming, we apply them to a numerical example and two electricity market examples with transmission constraints.

In chapter 3 strategic behaviors under the competing pricing mechanisms of uniform and pay-as-bid pricing in an electricity market are analyzed. We deal with the issue of uniform pricing and pay-as-bid pricing in electricity markets from the viewpoint of the effect of strategic bidding and market power in the short term. In order to address the core gaming issue in the auction pricing mechanism in electricity market, we use a specific two player auction game model having a big player with market power and a strategic small player. We first analyze the two-player auction game by a qualitative model to show the core reasoning of the gaming under each auction pricing mechanism. We show that the profit maximizing bidding strategy can be dependent on the auction pricing mechanism. Then we analyze the gaming in terms of bid price itself and compare the expected revenue under each pricing mechanism. “Revenue inequivalence” under the two price mechanisms is explained and proved based on the concept of the static mixed strategy Nash Equilibrium (NE). To confirm the theoretical result, we simulate the model using the Lemke and Howson algorithm and compare the expected total revenues from the given NEs under both pricing mechanisms. We also extend our model to an elastic demand case and compare the total expected market cleared demands under both pricing mechanisms.

In chapter 4 a market power mitigation issue of the current Texas electricity market by limiting Transmission Congestion Right(TCR) ownership is addressed. We model the Independent System Operator ( ISO) balancing zonal market manipulation in conjunction with the day ahead strategic scheduling. We introduce the strategic coordination of inter zonal schedule and ISO real time balancing market manipulation in the current Electric Reliability Council of Texas (ERCOT) market setup. We analyze the strategic coordination in a two zone market model and extend our analysis to general flow gate base market models. We derive a market power exercising condition for general flow gate based market models and implement the market power measurement algorithm based on the general market power exercising conditions. In order to show the market power measurement algorithm’s practicality we simulate ERCOT 2003 market model.

# Chapter 2

# Hybrid Co-evolutionary Programming for Nash Equilibrium Search in Games with Local Optima

## 2.1 Chapter introduction

Nash Equilibrium (NE) is an essential concept of game theory and market power analysis. The concept has been used to understand the strategic actions of multiple players in a deterministic gaming environment. It is also very useful for studying the potential performance of a market structure before the market is introduced. For these reasons, NE has been a research focus in microeconomics and industrial organization [1, 2].

Techniques for searching for a NE of a specific game are usually based on the definition of NE. That is, search for a point which satisfies every player's optimizing condition given the other players' choices. In a simple two player normal form game, we can find NE as the intersection of two players' best response curves. This manual approach can also be applied to NE search in a general game by drawing best response curves and searching for an intersection [3]. The analytical approach of solving the optimality conditions of all players is useful when the solution is obtained by simple mathematical manipulations. For example, a Cournot solution for symmetric oligopoly players can be easily obtained by solving simultaneously the first order condition for maximizing each player's profit [1].

Lemke and Howson introduced a Linear Complementarity Problem (LCP) for solving NE for bimatrix games [4]. Stoft applied the algorithm to a two-player game in an electricity market [5]. Although the algorithm is mathematically well proven, the algorithm is limited to the bi-matrix game.

Iterative NE search algorithms that use repeated individual profit maximization have been applied to more complex games [6, 7]. Hobbs *et al.* formulated the individual profit maximization in a transmission constrained electricity market as a Mathematical Program with Equilibrium Constraints (MPEC) [6]. By solving each player's MPEC in turn, they seek a pure strategy NE in the two-player gaming model. Weber and Overbye formulated the individual profit maximization as a two level optimization problem [7].

Besides classical NE definition based algorithms, the evolutionary game and adaptive learning based algorithms have also been applied to seek the outcomes of complex games [8, 9, 10, 11, 12, 13, 14]. Evolutionary game theory is based on the player's evolutionary learning process [15, 16]. Price applied Genetic Algorithm (GA) based co-evolutionary programming to the study of a simple electricity pool market model [8]. He showed the potential role of co-evolutionary programming as a modeling tool for general market study through several simulation examples. Cau and Anderson tested the co-evolutionary approach on a two-player electricity pool market game [9]. Dawid derived the stability of GA dynamics in economic models and applied the theoretical results to the study of a double auction market [10]. Riechmann showed that the canonical GA based economic learning processes corresponded to evolutionary games and noted that the learning process results in "near Nash Equilibrium" [11]. Pollack studied co-evolutionary programming enhancement through the advanced selection method [12].

Adaptive learning multi-agent simulation also has been studied for market analysis [13, 14]. Nicolaisen *et al.* applied a modified Roth-Erev algorithm to the study of a discriminatory pricing auction for an electricity market [13]. Bunn and Oliveira performed an agent-based simulation for the New Electricity Trading Arrangements of the England and Wales electricity market by using autonomous agents [14].

In this chapter we study NE search algorithms for games with local optima, where profit maximizing individual players face a profit function with local optima. We prove that any conventional iterative NE search algorithms applied to games with local optima can misidentify NE by following a local optimization path. A major contribution of this chpater is the introduction of an enhanced algorithm to overcome this problem.

We introduce the concept of "local NE trap" that attracts the conventional iterative NE search algorithms based on local optimization tools. We explicitly define NE, local NE and local NE trap in games with local optima. We describe the general framework of the conventional iterative NE search algorithms and prove that any local NE including a "local NE trap" can be the solution of the conventional iterative NE search methods. The failure of NE search by the iterative algorithm is exemplified by a conceptual game having local optima.

In order to overcome the problem of the conventional NE search, we apply a parallel and global search algorithm called co-evolutionary programming [8]. We describe the algorithmic steps of co-evolutionary programming to solve games with profit maximizing players. By introducing the concept of "best rival matching and fine tuning," we develop an enhanced version of co-evolutionary programming, which we call "hybrid co-evolutionary programming".

In order to test the effectiveness and performance of conventional NE search algorithms, co-evolutionary programming, and hybrid co-evolutionary programming, we apply them to a numerical example and two electricity market examples with transmission constraints. Transmission constraints in electricity markets can cause individual profit functions with local optima [7]. A transmission constrained electricity market model is not only a good mathematical example with a complex game structure but also an important model for market power analysis related to the restructured electricity industry [17, 18]. We simulate a two player, two bus model from [7] and a three player three, bus model in [3].

## 2.2 Local NE trap and iterative NE search

### 2.2.1 Local NE trap

When we consider a general multi-player game, the game can be described by each player's strategy choice and the resulting profit (also called the payoff). In an $N$ player game, when player $i$ chooses his pure strategy $x_i \in X_i$ and given pure strategy choices of others $x_{-i} = (x_1...x_{i-1}, x_{i+1}...x_N) \in X_{-i}$, he receives a profit of $\pi_i(x_i, x_{-i})$. Pure strategy Nash Equilibrium (NE) is a point where no player can obtain a higher profit by unilateral movement. That is, any profit maximizing player will not deviate from NE. We can define Nash Equilibrium for a general $N$-player game by:

**Definition 1 (Nash Equilibrium)** $x^* = (x_1^*, ..., x_i^*, ..., x_N^*)$ *is a NE if:*

$$\forall i, \forall x_i \in X_i, \pi_i(x_i^*, x_{-i}^*) \geq \pi_i(x_i, x_{-i}^*). \tag{2.1}$$

When the profit function of player $i$, $\pi_i(x_i, x_{-i})$, is strictly concave with respect to $x_i$ for each $i$ and each fixed $x_{-i}$, we have a unique NE [2]. However, in the case of transmission constrained electricity markets, the profit function is not concave due to the transmission constraints and the non-concave profit function can cause multiple local optima [7]. In those games with local optima there can exist points that satisfy the NE condition in (2.1) locally. We define these points as local NE:

**Definition 2 (Local NE)** $x^* = (x_1^*, ..., x_i^*, ..., x_N^*)$ *is a local NE if:*

$$\exists \epsilon > 0 \text{ such that } \forall i, \forall x_i \in B_i^\epsilon(x_i^*),$$

$$\pi_i(x_i^*, x_{-i}^*) \geq \pi_i(x_i, x_{-i}^*), \tag{2.2}$$

*where* $B_i^\epsilon(\hat{x}_i) = \{x_i \in X_i | \|x_i - \hat{x}_i\| < \epsilon\}$.

Each NE satisfying definition 1 is also a local NE satisfying definition 2. However, there are local NE that do not satisfy the conditions in definition 1 for a NE. Apparently these local NE are not NE of the game. We call these points "local NE traps":

**Definition 3 (Local NE trap)** $x^* = (x_1^*, ..., x_i^*, ..., x_N^*)$ *is a local NE trap if:*

$$\begin{aligned} \exists \epsilon > 0 \text{ such that} \quad & \forall i, \forall x_i \in B_i^\epsilon(x_i^*), \\ & \pi_i(x_i^*, x_{-i}^*) \geq \pi_i(x_i, x_{-i}^*)), \\ \text{but } \exists i \text{ such that} \quad & \exists x_i^{**} \in X_i, \\ & \pi(x_i^{**}, x_{-i}^*) > \pi_i(x_i^*, x_{-i}^*). \end{aligned} \tag{2.3}$$

We illustrate a conceptual example of NE and local NE trap in a two-player game in figure 2.1.

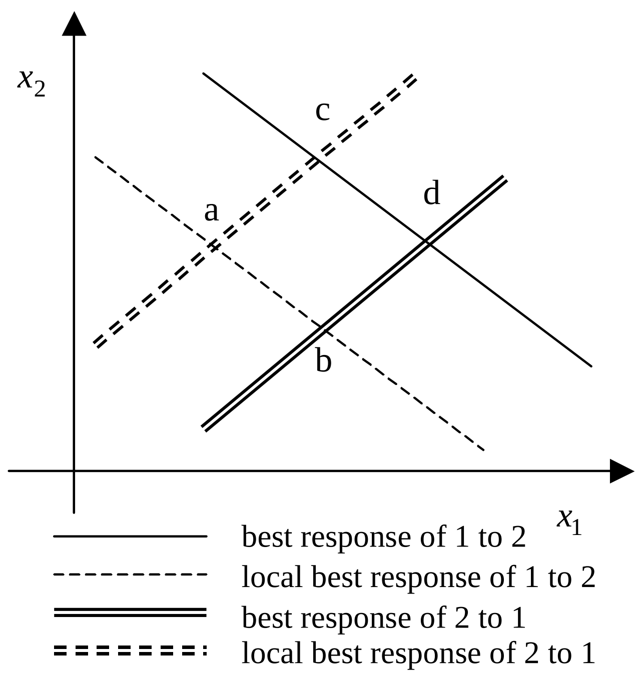



Figure 2.1: NE {d} and NE trap {a,b,c}.

"Best responses" and "local best responses" are illustrated in figure 2.1. "Best response" means each player's profit maximizer $x_i^*$ given the other player's strategic choice $x_{-i}$. "Local best response" means the local profit maximizer when the profit function has local optima. The points {a,b,c,d} in figure 2.1 satisfy the local NE condition in (2.2) but only {d} satisfies the NE condition in (2.1). The points {a,b,c} are local NE traps satisfying (2.3). Our numerical simulation example shows a similar form to the best response function in figure 2.1.

Figure 2.2 illustrates another game case where a local NE trap can cause a problem. This example shows that even when there is no pure strategy NE, there can exist a local NE trap. The discontinuity of the best response plane can cause there to be no intersection of the players' best responses. However, because of the existence of the local best response curve, a local NE can still exist. The point {e} is the intersection of player 1's local best response and player 2's best response. The point {e} is a local NE but there is no NE. We find this type of game in our transmission constrained electricity market simulation.

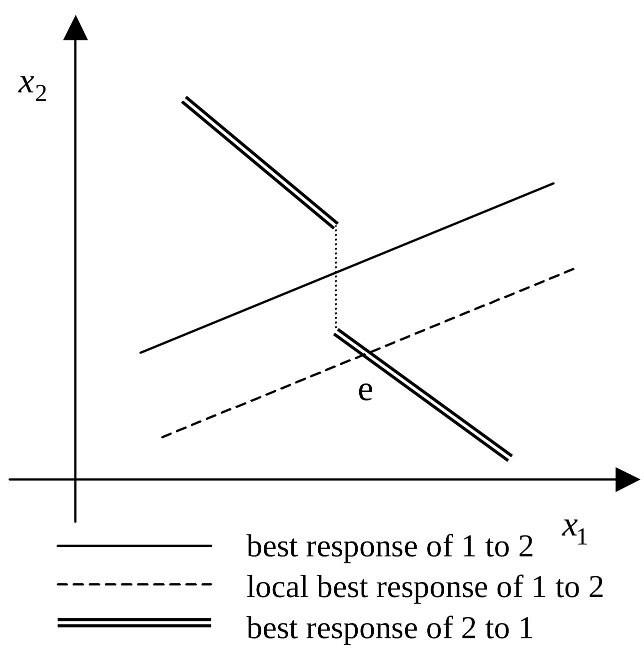



Figure 2.2: No pure strategy NE but a local NE trap {e}.

### 2.2.2 Model of iterative NE search algorithms

Iterative NE search algorithms perform iterative individual profit maximization by turns until no player changes its strategy choice or a termination condition is met. The conceptual framework of iterative NE search algorithms is given by:

1. Initialize each player's strategy choice set.
2. Given other players' decisions, solve each player's profit maximization problem. Do this step for all players.
3. Repeat step 2 until the NE condition is satisfied or the maximum iteration number is reached.

The conceptual framework matches the hypothetical process of finding a pure strategy Nash Equilibrium for a perfect information game.

However, when we consider the implementation of the conceptual procedure, iterative NE search algorithms may not exactly match the NE condition. The searching ability is limited by the local search capability of the implemented iterative NE search algorithms. Individual profit maximization is the main engine of iterative NE search algorithms and is usually implemented by a local optimization algorithm [6, 7]. For example, Penalty Interior Point Algorithm is applied in [6] and the modified Newton-step is used in [7]. The algorithm level procedure of iterative NE search algorithms will be summarized by:

**[Iterative NE search algorithm]**

1. Set up basic parameters including size of local search space $\epsilon > 0$ and termination tolerance criterion $\zeta > 0$.

2. For a general $N$ player game, set the initial strategy set $x^0 = \{x_1^0, ..., x_N^0\}$.

3. Individual profit maximization by turns.

   For $i$=1:$N$

   $x_i^k = \arg\max_{x_i \in B_i^{\epsilon}(x_i^{k-1})} \pi_i(..., x_{i-1}^k, x_i, x_{i+1}^{k-1}, ...)$

   End;

4. Termination condition check.

   If $\sum_{i=1}^{N} \|x_i^k - x_i^{k-1}\| \leq \zeta$ then

   $x^* = x^k$

   Else if $k = k_{max}$ then

   Solution could not be obtained within $k_{max}$ iterations

   Else

   go to step 2

   End;

A variant of the algorithm above is to perform step 3 in parallel for each player simultaneously using the most recent values of the strategies for the other players.

By connecting the concept of local NE to iterative NE search algorithms, we draw a proposition:

**Proposition 1** *Every local NE, including any local NE trap, can be the solution of any conventional iterative NE search algorithm.*

*Proof:* For any local NE $x^*$, initialize $x^0 = x^*$ for the iterative NE search algorithm. By step 3, we have $x_i^1 = \arg\max_{x_i \in B_i^{\epsilon}(x_i^0)} \pi_i(..., x_{i-1}^1, x_i, x_{i+1}^0, ...)$. By definition of local NE, $\arg\max_{x_i \in B_i^{\epsilon}(x_i^0)} \pi_i(..., x_{i-1}^1, x_i, x_{i+1}^0, ...) = x_i^*$. After step 2, we have $x^1 = x^*$. Because $\sum_{i=1}^{N} \|x_i^1 - x_i^0\| = \sum_{i=1}^{N} \|x_i^* - x_i^*\| = 0 \leq \zeta$ at step 3, it terminates the procedure and yields the local NE $x^*$ as the solution. Any local NE $x^*$ can be the solution of an iterative NE search algorithm. ■

Proposition 1 also holds for the variant of the algorithm where step 3 is performed in parallel for all players.

### 2.2.3 Example of NE trap and iterative NE search algorithm

From proposition 1, we know that any local NE, including a local NE trap, could be the solution of an iterative NE search algorithm. We illustrate the action of local NE search algorithms by a prisoner's dilemma model and an augmented version of it.

Although our electricity market applications involve a continuous space, the prisoner's dilemma example will use a discrete strategy space labelled by integers. By a local move in this space we mean a move where the label changes by no more than one unit. The discrete strategy space simplifies the discussion. The example can easily be expanded to a continuous problem by, for example, interpolating continuous payoff functions through the specified values.

Figure 2.3 is an example of prisoner's dilemma games. Each player, A and B, can choose either 0 or 1 as a strategy. The entries in the matrix show values of (profit for A, profit for B) given the corresponding strategy choices by A and B. Here we know that the pure strategy NE is the point where the best response of each player meets together (A=1, B=1). In this simple case, the iterative local optimization method yields the NE from any starting point.

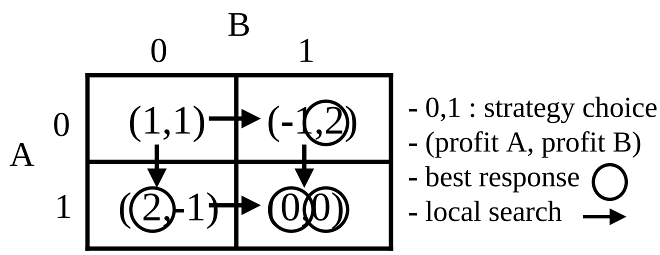



Figure 2.3: Prisoner's dilemma.

In this game, we define a local move to be one where the choice of a player changes by one unit. The arrows represent the action of local optimization. Wherever we start, once we follow the arrows, we will get to the NE. For example, when starting from (A=0, B=0), player A moves to (A=1) following the local optimizing path. Given (A=1, B=0), player B will choose (B=1), then player A cannot improve. NE (A=1, B=1) is obtained by iterative local optimization scheme in this case. (The same result would hold if both players move simultaneously.)

When we extend this model by adding more strategy choices for player A, we can create a game with local optima. In figure 2.4 we illustrate the augmented prisoner's dilemma game having local optima. In the extended model, B still chooses either 0 or 1, but A chooses either 0, 1, 2, or 3 as its strategy. By a local optimum for A in this extended model we mean a choice that has higher payoff for A than any adjacent choice. Moreover, local search involves a change in the strategy of at most 1 unit.

Given B=1, player A has two local optima of A=0 and A=3. In this case we can easily notice that we have no pure strategy NE by checking the best responses of players. But there exists a local NE trap (A=3, B=1) due to the multiple local

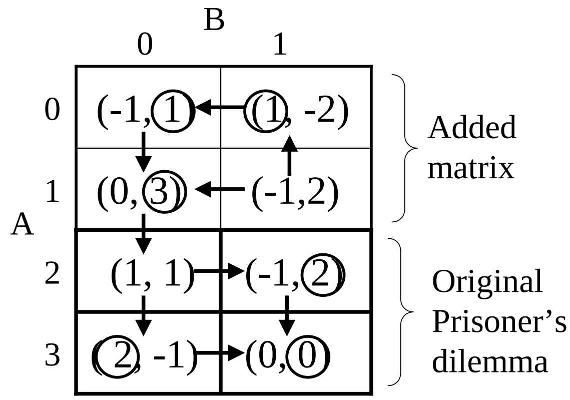



Figure 2.4: Augmented prisoner's dilemma.

maxima of player A's profit function given player B's choice (B=1). Suppose that we start from (A=0, B=0). Player A moves to (A=3) by the local optimizing path. Given (A=3) player B will move to (B=1) following the local optimization path. At (A=3, B=1) none of the players improve using local search. The algorithm will terminate at this iteration, yielding (A=3, B=1). However, because A can improve its payoff by unilaterally choosing (A=0), we therefore note that (A=3, B=1) is not an NE.

## 2.3 Hybrid co-evolutionary programming

### 2.3.1 Co-evolutionary programming

Genetic algorithm based co-evolutionary programming is a parallel and global approach to identify NE [8]. The population of strategies of each player will be represented by the genetic codes. Each population of players will follow the rule of evolution where the fittest survives under the given environment. During the simulation of co-evolution process, the individual strategy in each player's population will survive or die by the results of each tournament.

Each generation of the genetic algorithm based co-evolutionary programming consists of two parts: the random matching and the evolutionary process of each player. The random matching process assigns a profit to every strategy of the population based on the results of a tournament with other strategies of other populations. After random matching, each population evolves by genetic operators.

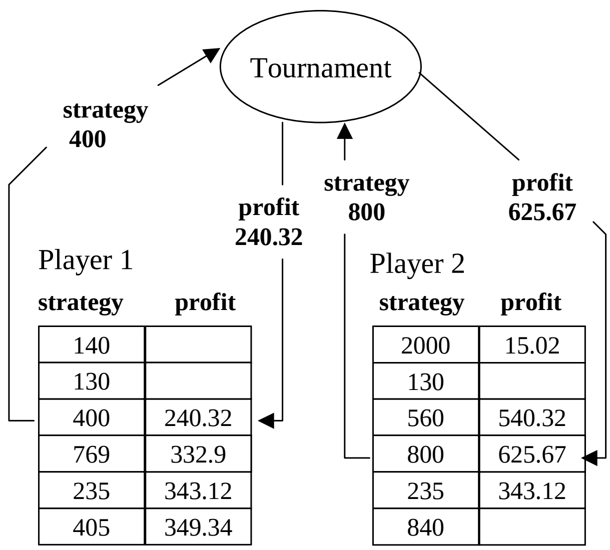



Figure 2.5: Random matching process.

As an example of a two-player game, the random matching process is illustrated in figure 2.5. A randomly chosen strategy from each player's population will be assigned the profit values decided by the results of the tournament among those strategies. In figure 2.5, the strategy of 400 for player 1 and the strategy of 800 for player 2 were chosen for the tournament and these strategies are assigned the resulting payoff.

After each strategy is assigned a profit value through the random matching process, we apply the evolutionary operators: selection, crossover, and mutation [19, 20]. By applying the evolutionary operators, we will get a new set of populations. Figure 2.6 illustrates co-evolutionary step where each population is evolved through the evolutionary operators.

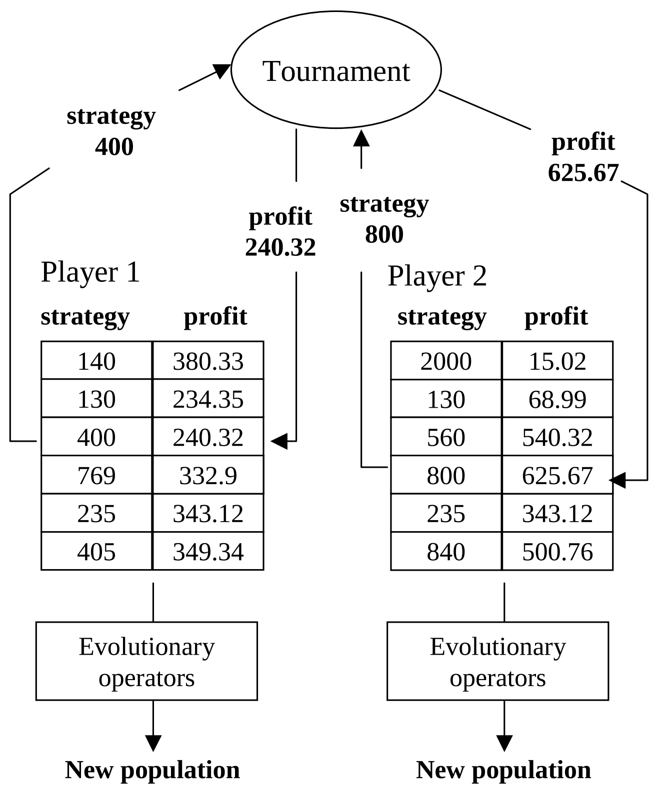



Figure 2.6: Co-evolution.

The whole process of genetic algorithm based co-evolutionary programming can be summarized into steps below.

**[Co-evolutionary programming steps]**

1. Set the basic parameters of maximum generation number, population size, crossover rate, and mutation rate.

2. Initialize the strategy population of each player.

3. Choose one strategy from the strategy population of each player randomly from amongst the strategies that have not already been assigned profits. Input the strategy information to the tournament. The result of the tournament will decide profit values for these chosen strategies.

4. Repeat step 3 until every strategy is assigned a profit value.

5. Apply the evolutionary operators to each strategy population. Keep the best strategy of the current generation alive (elitism).

6. Repeat steps 3, 4 and 5 until the maximum generation number is reached.

### 2.3.2 Hybrid co-evolutionary programming

In genetic algorithms for optimization, researchers have used hybrid techniques for speeding up the convergence and fine tuning control variables [19, 20]. Similarly, we apply the hybrid technique to the simple co-evolutionary programming. We incorporate a local hill climbing algorithm to the basic co-evolutionary programming to improve its convergence. In order to apply the hybrid technique to the co-evolutionary programming, we suggest the concept of the "best rival matching and fine tuning," which means that the chosen individual will be matched against the best strategy of the other populations in the current generation. The algorithmic steps of "best rival matching and fine tuning" are summarized below. The steps are illustrated in figure 2.7.

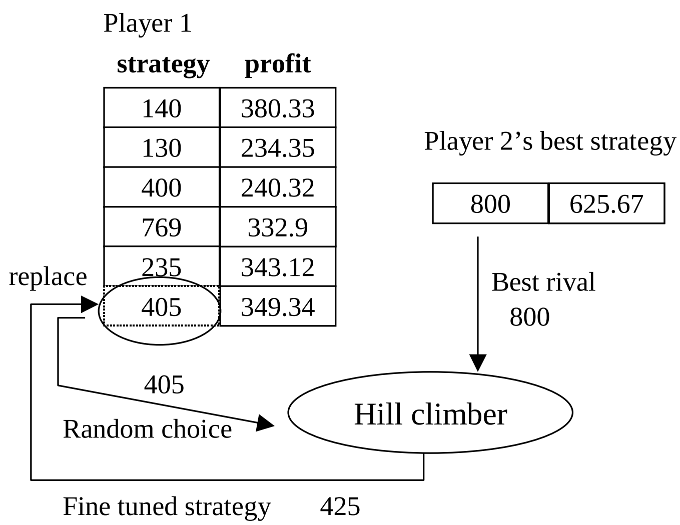



Figure 2.7: Best rival matching and fine tuning.

**[Best rival matching and fine tuning]**

1. Randomly choose a strategy from the strategy population of a player.

2. Take the best strategies of all other rivals from the previous generation.

3. Apply a local hill climber for fine tuning of the chosen strategy of the player.

4. Repeat steps 1-3 as many times as set by the best rival matching rate. For example, with a best rival matching rate of 0.2, twenty percent of strategies in each population will be chosen.

5. Repeat step 1-4 for every other player in turn.

The whole process of hybrid co-evolutionary programming including steps of "best rival matching and fine tuning" can be summarized in the steps below.

**[Hybrid co-evolutionary programming]**

1. Set the basic parameters of co-evolutionary programming and hill climbing rate.

2. Initialize the strategy population of each player randomly.

3. Perform random matching and co-evolution. (Simple co-evolutionary programming)

4. Apply the "best rival matching and fine tuning." (Hybrid co-evolutionary programming)

5. Repeat steps 3 and 4 until the maximum generation number is reached.

## 2.4 Simulation with a numerical example

### 2.4.1 Game configuration

We use a simple numerical example with multiple local NE in order to compare the conventional NE search algorithms, simple co-evolutionary programming and hybrid co-evolutionary programming. We developed a numerical game example with local optima by modifying the non-concave objective function in the section 2 of Part I in [20]. The profit functions for player A and player B are defined by:

$$\pi_A(x_A, x_B) = 21 + x_A \sin(pi \times x_A) + x_A x_B \sin(pi \times x_B)$$
$$\pi_B(x_A, x_B) = 21 + x_A x_B \sin(pi \times x_A) + x_B \sin(pi \times x_B), \tag{2.4}$$

where $pi = 3.1415$. The profit matrices of the two players are illustrated in figure 2.8.

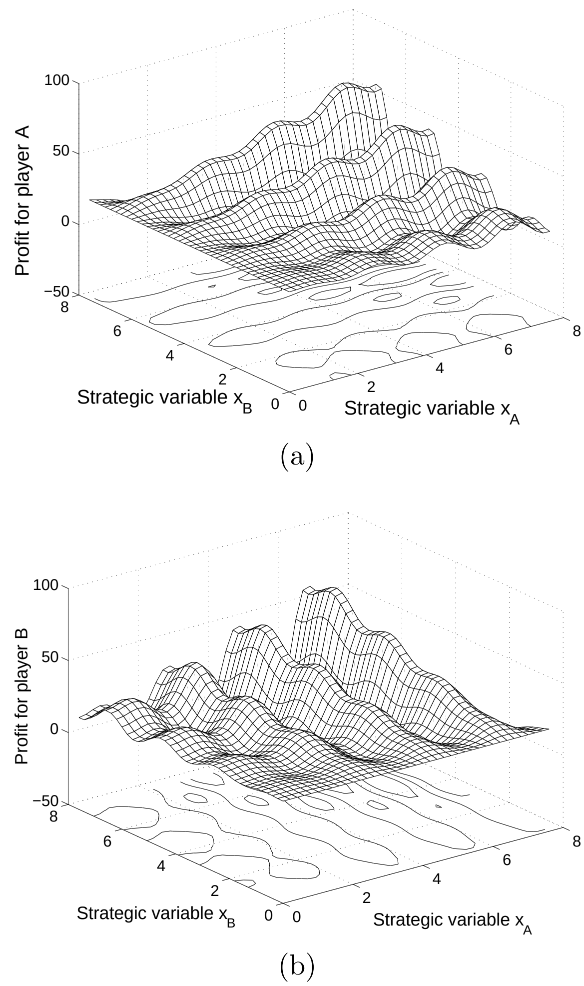



Figure 2.8: (a) Player A profit matrix (b) Player B profit matrix.

When player $A$ chooses $x_A$ as his strategic variable and given player $B$'s strategic variable $x_B$, the profit of player A will be given by $\pi_A$ in (2.4). Player $B$ will get the profit $\pi_B$ given player $A$'s choice of $x_A$. For the simulation, we restricted the strategic choice of player $A$ and $B$ between 0 and 7.5.

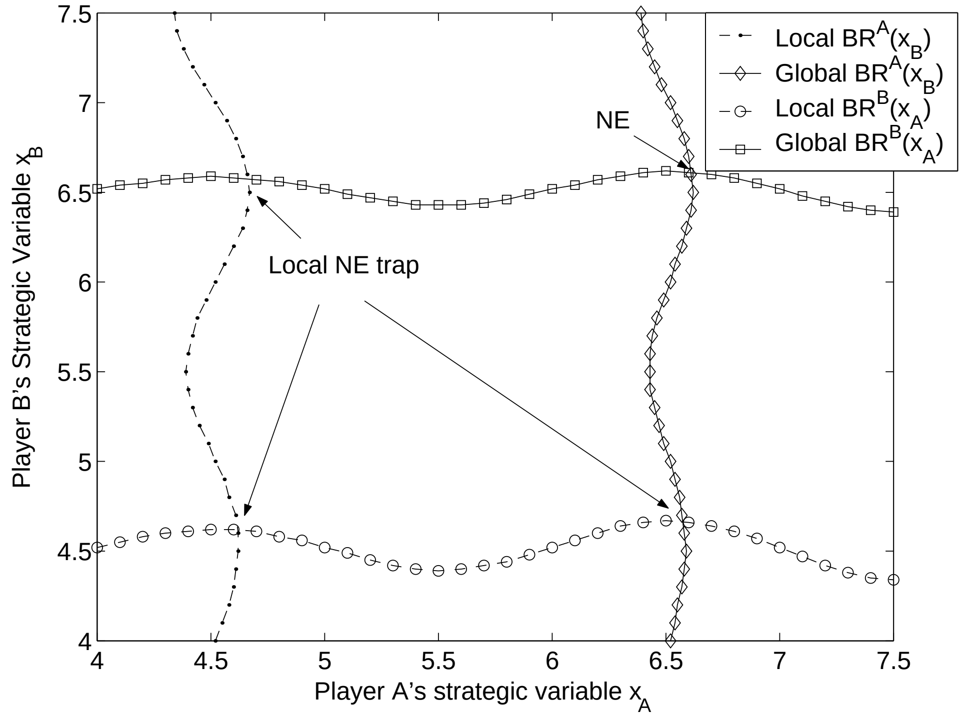



Figure 2.9: NE and local NE trap of the numerical game example

Figure 2.9 illustrates global best responses and local best responses to the other player's strategic variable near NE. The NE and the multiple local NE traps can be identified by checking the intersections of best response functions. In this situation, following proposition 1, conventional NE search algorithms will yield multiple solutions including NE and local NE trap without being able to differentiate the NE from the local NE traps.

### 2.4.2 Local iterative search

We tested the conventional local iterative NE search method by using a local optimizing software "*fmincon*" in the MATLAB Optimization Toolbox 2.0 [21]. Four different initial points of (4, 3), (5, 1), (6, 7) and (7, 3) are used for the simulation. Table 2.1 illustrates the simulation results.

Table 2.1: Iterative NE search result of the simple numerical example

| Initial points | Solution | Result |
|---|---|---|
| (4, 3) | (6.58, 4.66) | local NE trap |
| (5, 1) | (0.25, 0.92) | local NE trap |
| (6, 7) | **(6.61, 6.61)** | **NE** |
| (7, 3) | (6.54, 2.8) | local NE trap |

The solution column in table 2.1 shows the final outputs of the local iterative search from four different initial points. Outputs of local iterative search include NE and local NE trap without differentiating between them. By relying on the local iterative search we can misjudge the game result as having multiple NE. The last column in table 2.1 shows the real identities of the solutions. The NE **(6.61, 6.61)** was obtained only by starting at the point (6, 7) which is near the NE. Other initial points yielded local NE traps that satisfy the condition of a local NE but do not satisfy the condition for being a NE.

### 2.4.3 Co-evolutionary programming

For the simple co-evolutionary programming simulation, the real number coded genetic algorithm engine was used [19, 20]. Table 2.2 illustrates the genetic algorithm parameters that were used. The simple co-evolutionary process is illustrated by plotting the mode of each population at each generation in figure 2.10.

Table 2.2: GA parameters for simple co-evolutionary programming

| Parameter name | Parameter value |
|---|---|
| Maximum generation number | 500 |
| Population size | 30 |
| Crossover rate | 0.5 |
| Mutation rate | 0.02 |

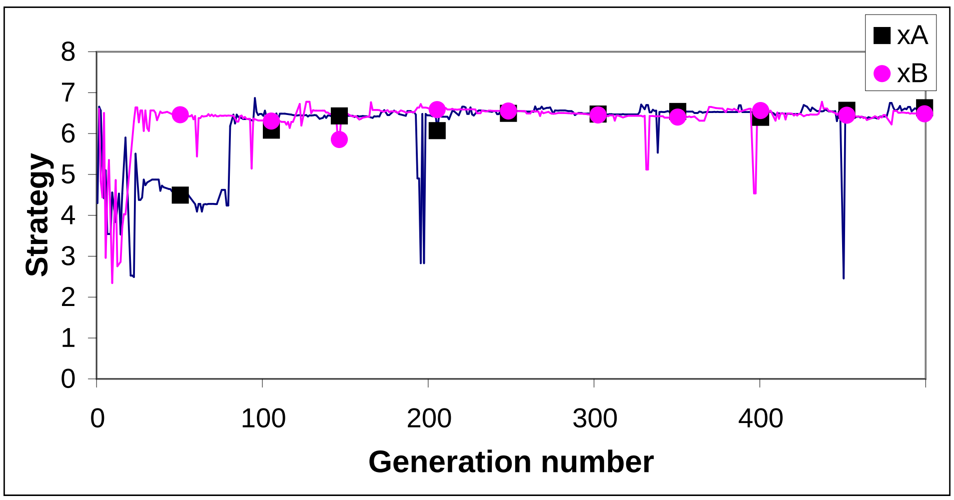



Figure 2.10: Mode of strategic variables during the simple co-evolutionary process for a numerical example.

The strategic choices of both players hover around the NE but do not converge. We only get some rough information about the NE. The simple co-evolutionary programming could not identify the NE exactly, but figure 2.10 does suggest the presence of a NE.

### 2.4.4 Hybrid co-evolutionary programming

We tested hybrid co-evolutionary programming by using the parameters given in table 2.3.

Table 2.3: GA parameters for hybrid co-evolutionary programming

| Parameter name | Parameter value |
|---|---|
| Maximum generation number | 500 |
| Population size | 30 |
| Crossover rate | 0.5 |
| Mutation rate | 0.02 |
| Hill climbing rate | 0.2 |

The same parameter values as in the simple co-evolutionary programming are used except for the addition of the hill climbing rate. In order to avoid over-optimization in early stage and over-randomizing, it is important to choose a proper level of the hill climbing rate. The rates from 0.2 to 0.3 generally worked well, but the choice of values for this parameter presumably depends on the application. Moreover, we have not tried to find the optimum values of any of the parameters for either the simple or hybrid co-evolutionary programming algorithms.

The mode of each population during the hybrid co-evolutionary process is illustrated in figure 2.11. The hybrid co-evolutionary programming identified the NE (6.61, 6.61) successfully.

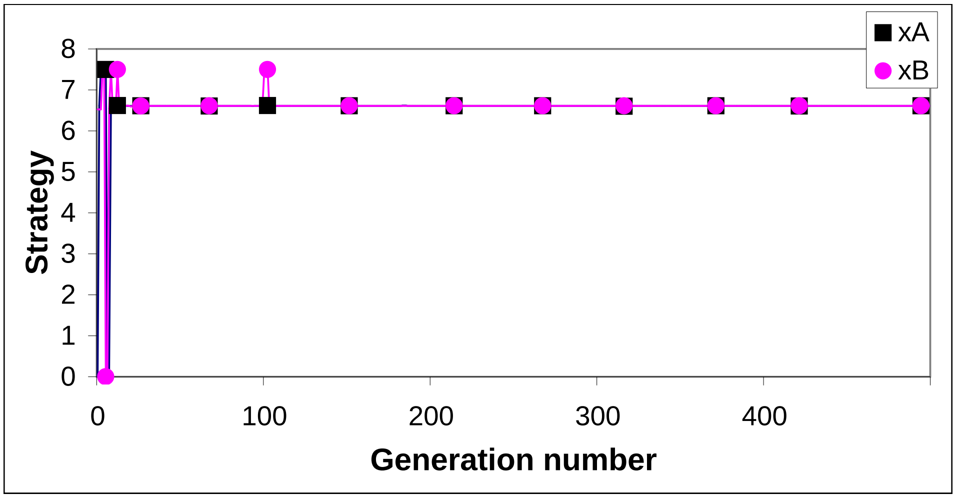



Figure 2.11: Mode of strategic variables during the hybrid co-evolutionary process for a numerical example

## 2.5 Simulation of transmission constrained electricity markets

The transmission constrained electricity market is a good example of complex games that have local optima and discontinuous best response functions [3, 6, 7, 17]. The local optima and discontinuous best response functions are due to the transmission constraints. In particular, in the presence of transmission constraints, a player has two different functional forms for its profit, corresponding to the two cases of binding and non-binding transmission constraints, respectively. For each case, the profit can have local optima.

We applied conventional NE search algorithms, simple co-evolutionary programming, and hybrid co-evolutionary algorithm to a two bus and a three bus Cournot electricity market model. These models include a case where there is a unique NE and a case where there is no pure strategy NE [3, 17].

Electricity market models are typically formulated as two-level problems. At one level, a market clearing function solves for prices, given the bids by the players. This is an optimization problem. At the second level, each player chooses its bid to maximize its profit. We seek the NE in the second level decisions, given the implicit solution of the market clearing function. Further details about such problems is presented in [3, 6, 7].

### 2.5.1 Two bus Cournot model

#### Game configuration

We modified the two bus model in [7] into a Cournot game. The modified two bus model is qualitatively similar to the California model in [17]. We assumed a generator $G$ and load $L$ at each bus and transmission limit $T^{\max}$. Players 1 and 2 submit bids to generate quantities $s_1$ and $s_2$, respectively. We assume quadratic cost functions $C_1$ and $C_2$ for players 1 and 2, respectively. Demand is represented by demand benefit functions $B_1$ and $B_2$, respectively. The two bus model is depicted in figure 2.12, with the form of the cost and demand functions also specified in the figure.

In order to analyze the Cournot strategy of each player, we define each player's profit by:

$$\begin{aligned}\pi_1(s_1, s_2) &= \lambda_1^* s_1 - C_1(s_1), \\ \pi_2(s_1, s_2) &= \lambda_2^* s_2 - C_2(s_2). \end{aligned} \tag{2.5}$$

Each player's profit maximizing behavior is to solve two level optimization problem where the prices in the profit equations in (2.5) are determined by the market

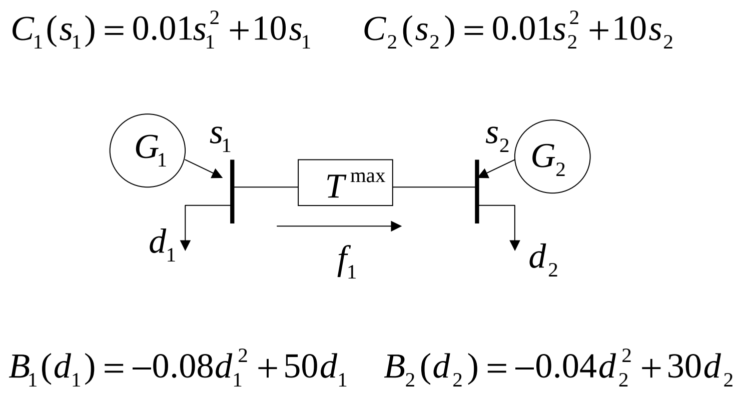



Figure 2.12: Two bus Cournot model.

clearing result. Locational marginal prices $\lambda_1^*, \lambda_2^*$ are determined by the Lagrange multipliers of the locational energy balance equality condition in the market clearing problem in (2.6) [22]:

$$\begin{aligned} \max_{d_1,d_2,f_1} \; & B_1(d_1) + B_2(d_2), \\ \text{such that } & d_1 - s_1 + f_1 = 0, \\ & d_2 - s_2 - f_1 = 0, \\ & -T^{\max} \leq f_1 \leq T^{\max}. \end{aligned} \tag{2.6}$$

That is, $\lambda_1^*$ and $\lambda_2^*$ are the Lagrange multipliers on the constraints $d_1 - s_1 + f_1 = 0$ and $d_2 - s_2 - f_1 = 0$, respectively.

We simulated the two bus Cournot game under two different levels of transmission limit $T^{\max} = 80$ MW and $T^{\max} = 55$ MW. We discretized the Cournot quantity to be an integer multiple of 1 MW between 0 MW and 250 MW.

**Case 1 :** $T^{\max} = 80$ **MW**

We first examine the relatively large transmission limit case of $T^{\max} = 80$ MW. This case results in a unique NE [17]. We depict the global best response functions of this case in figure 2.13. In this case there is a unique NE determined by the intersection of global best response functions.

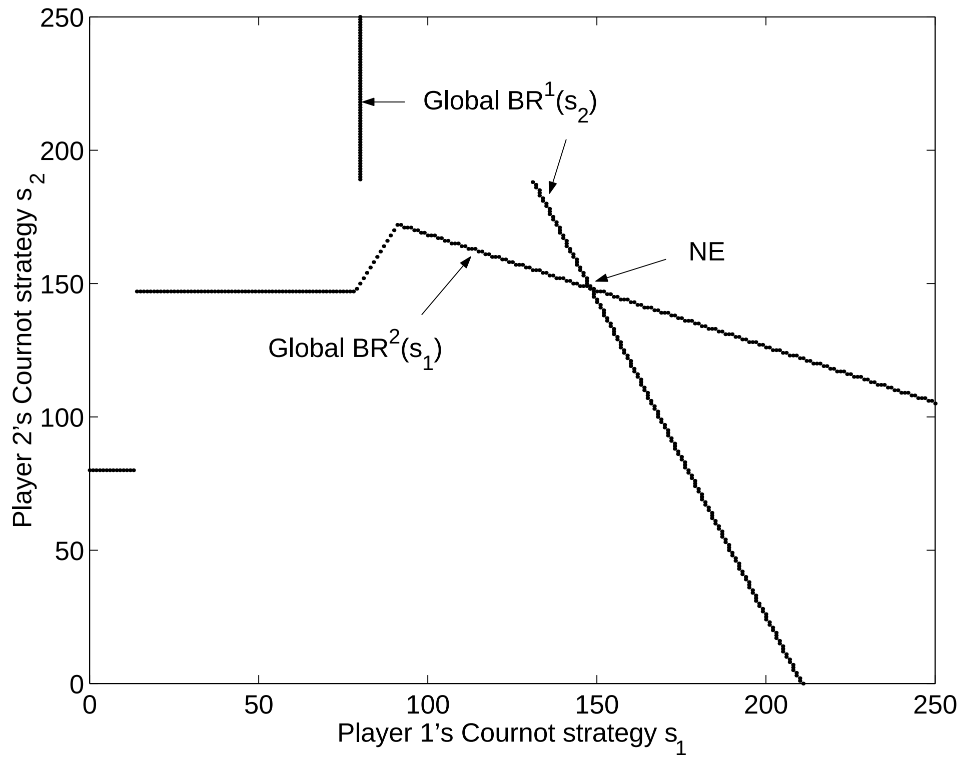



Figure 2.13: Global best response functions in two bus model with 80 MW transmission limit

Each player can have additional local best response to the other player's strategic choice. We numerically searched all local best responses in turn. Addition of the local best response functions into the global best functions can show the possible local NE trap problem. Figure 2.14 shows the global and local best response functions of the two bus Cournot game with 80 MW transmission limit. Figure 2.14 shows there is no local NE trap in this case because there is no intersection of the best response curves, except at the NE.

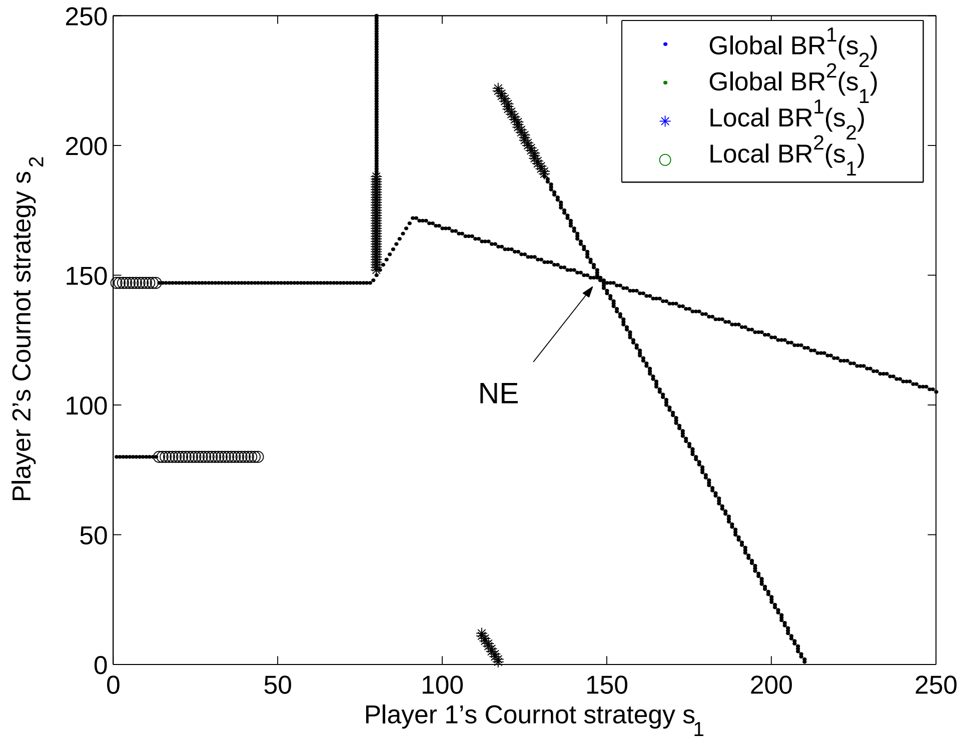



Figure 2.14: Global and local best response functions in two bus model with 80 MW transmission limit

**Local iterative search** We adopted the steepest descent algorithm with line search as the local optimization tool. In order to get an integer solution, we searched over the neighborhood integers of the obtained continuous solution from the steepest descent algorithm. We randomly chose four initial points. The simulation results are summarized in the table 2.4. The local iterative NE search yields only one solution of (148,148) but we cannot be sure that this is the unique NE without

Table 2.4: Iterative NE search result of the two bus Cournot model with 80 MW transmission limit

| Initial points (MW) | Solution (MW) | Result |
|---|---|---|
| (50, 140) | (148, 148) | NE |
| (50, 30) | (148, 148) | NE |
| (100, 120) | (148, 148) | NE |
| (160, 40) | (148, 148) | NE |

further information. Figure 2.14 confirms that the solution is the unique NE in this case; however, in general it takes considerable computational effort to construct the global best response curves and find their intersection [3].

**Co-evolutionary programming** For simple co-evolutionary programming, we used the same parameters as in table 2.2. The simple co-evolutionary programming could not identify the NE because of poor convergence performance. Figure 2.15 shows the simple co-evolution process.

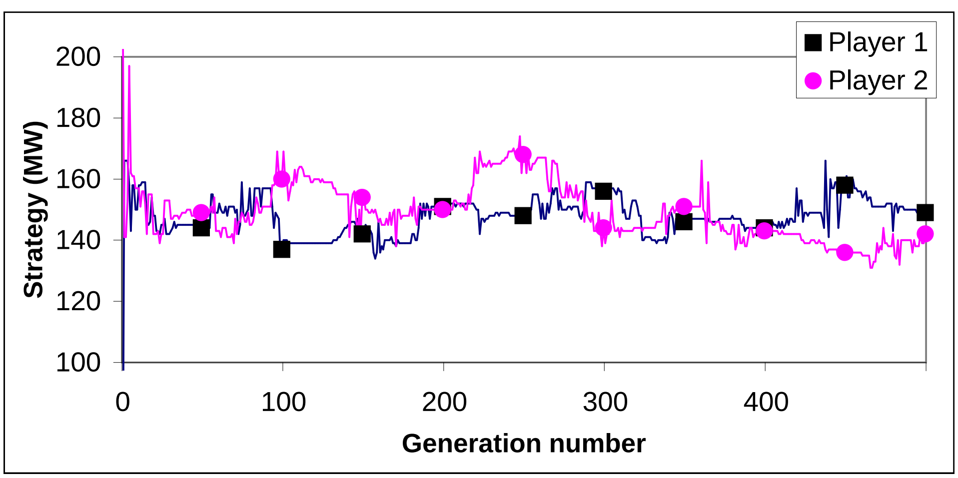



Figure 2.15: Mode of Cournot strategic variables during the simple co-evolutionary process for 2 bus model with 80 MW limit.

**Hybrid co-evolutionary programming** For hybrid co-evolutionary programming, we used the same parameters as in table 2.3. Hybrid co-evolutionary programming successfully identified the NE of (148 MW, 148 MW). We illustrate the hybrid co-evolution process in figure 2.16. It converges to the NE strategies. Again, additional information is required to confirm that the NE has been found; however, the smooth approach is suggestive that the algorithm is successful in finding the NE.

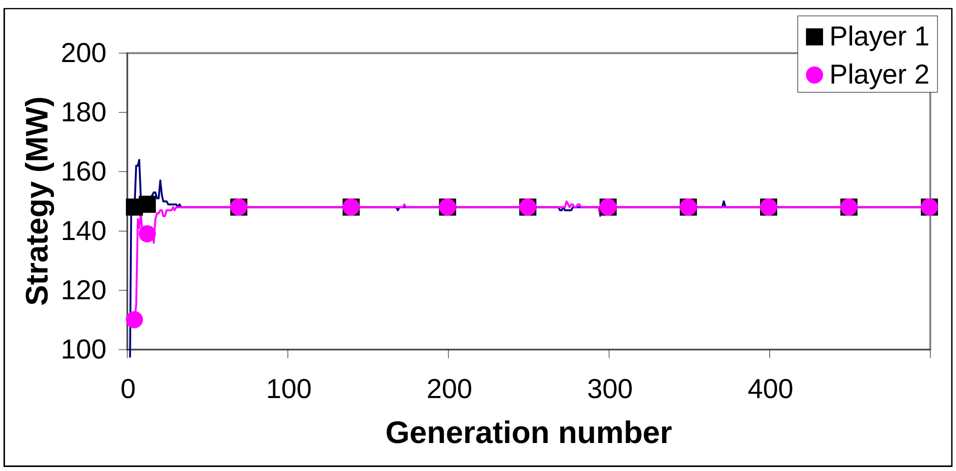



Figure 2.16: Mode of Cournot strategic variables during the hybrid co-evolutionary process for 2 bus model with 80 MW limit.

**Case 2 : $T^{\max} = 55$ MW**

We simulated the tighter transmission limit of 55 MW. The Cournot game with tighter transmission limit cases can result in no unique NE [17]. We depict the global best response functions of this case in figure 2.17. In this case there is no unique NE since there is no intersection of the global best response functions.

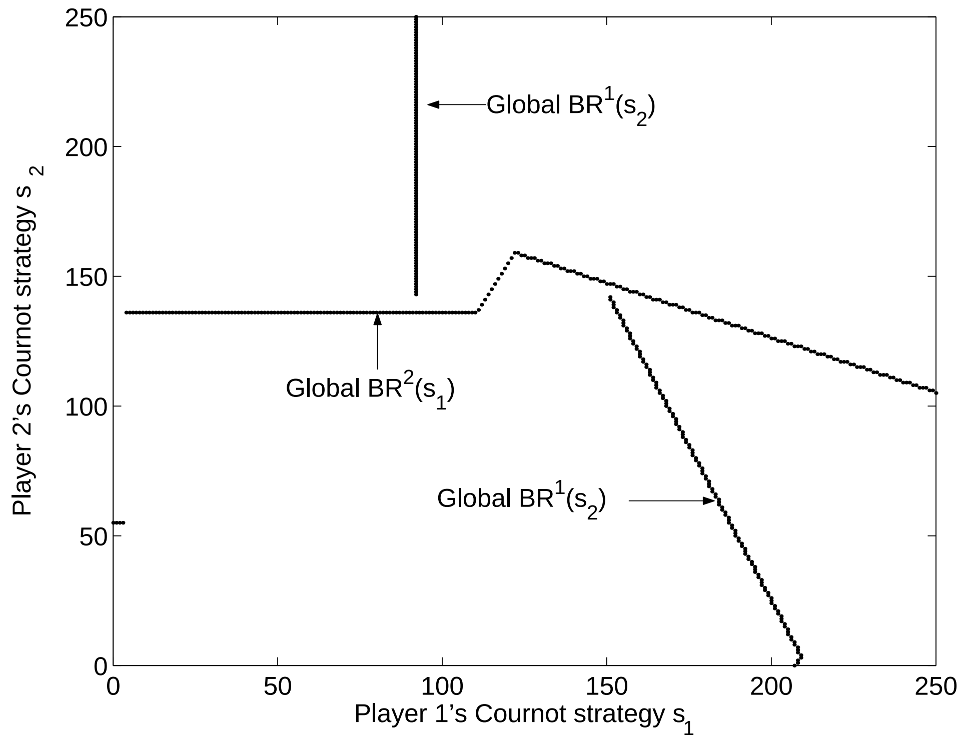



Figure 2.17: Global best response functions in two bus model with 80 MW transmission limit

In order to check the existence of local NE trap, we added the local best response functions to figure 2.17. Figure 2.18 shows two local NE traps as the intersections of local best response functions.

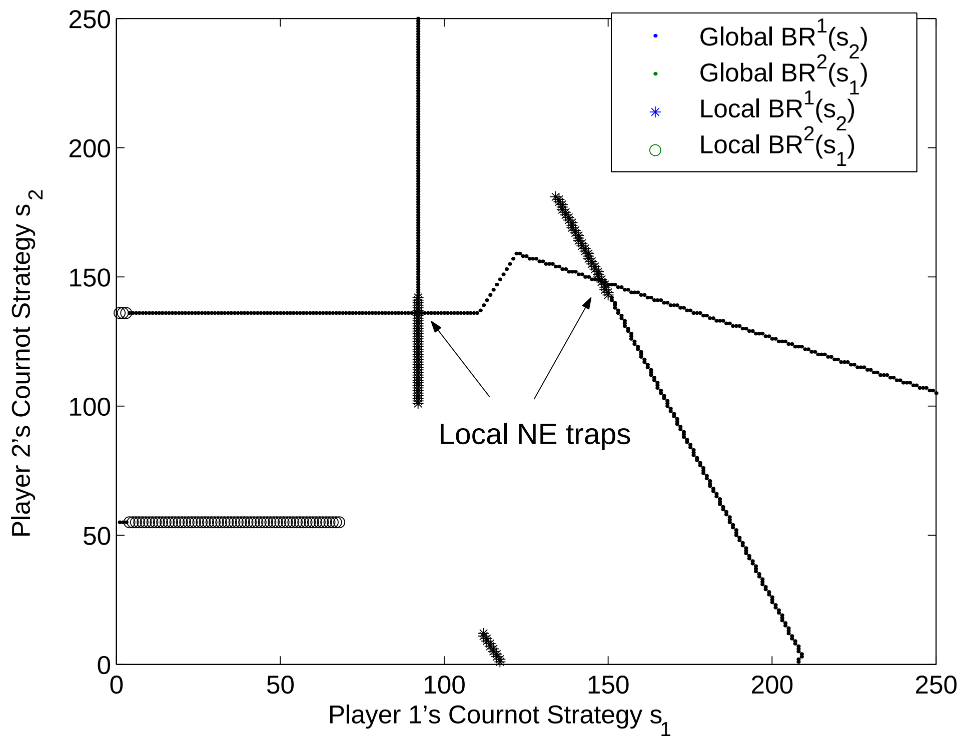



Figure 2.18: Global and local best response functions in two bus model with 80 MW transmission limit

**Local iterative search** We used the same four starting points as previously for the iterative NE search. The simulation result for the 55MW transmission limit case is summarized in table 2.5.

Table 2.5: Iterative NE search result of the two bus Cournot model with 80$MW$ transmission limit

| Initial points (MW) | Solution (MW) | Result |
|---|---|---|
| (50, 140) | (92, 136) | local NE trap |
| (50, 30) | (148, 148) | local NE trap |
| (100, 120) | (92, 136) | local NE trap |
| (160, 40) | (148, 148) | local NE trap |

The iterative NE search yielded two solutions of (92 MW, 136MW) and (148 MW, 148 MW). If we rely on the iterative NE search, we misjudge these two solutions as two NE. Figure 2.18 shows that these are local NE traps.

**Co-evolutionary programming** Figure 2.19 illustrates the simulation result of simple co-evolutionary programming for the 55 MW limit case. Like figure 2.15, simple co-evolutionary program shows the poor convergence in this case too.

**Hybrid co-evolutionary programming** Hybrid co-evolutionary programming does not converge in this case. It suggests that there is no pure strategy NE in the case of 55 MW limit case as shown in figure 2.20. Hybrid co-evolutionary programming is not caught in the local NE traps in this example.

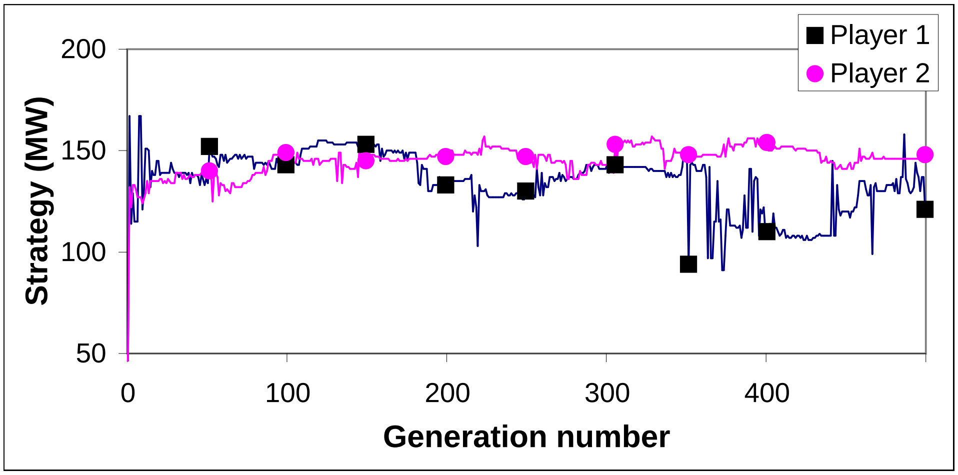



Figure 2.19: Mode of Cournot strategic variables during the simple co-evolutionary process for 2 bus model with 55 MW limit.

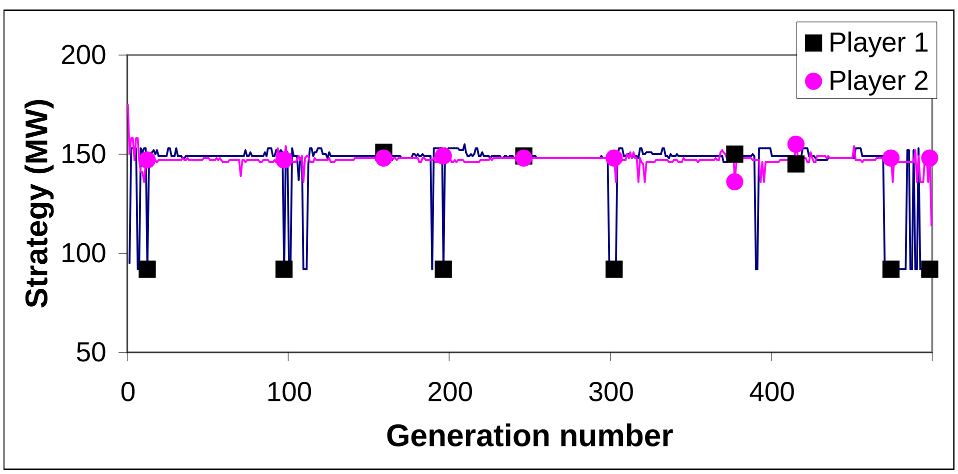



Figure 2.20: Mode of Cournot strategic variables during the hybrid co-evolutionary process for 2 bus model with 55 MW limit.

### 2.5.2 Three bus Cournot model

#### Game configuration

For a more complex case, we simulated the 3 bus, 3 player Cournot electricity market model in [3]. Instead of using the graphical approach of [3], we apply the local iterative search method and co-evolutionary programming.

The three-bus network is depicted in figure 2.21. We use the same supply cost and demand benefit data as in [3]. We consider two transmission limit cases of 100 MW and 66 MW. As reported in [3], there is a unique pure strategy NE in the 100 MW constraint case but no pure strategy equilibrium exists at the constraint level of 66 MW.

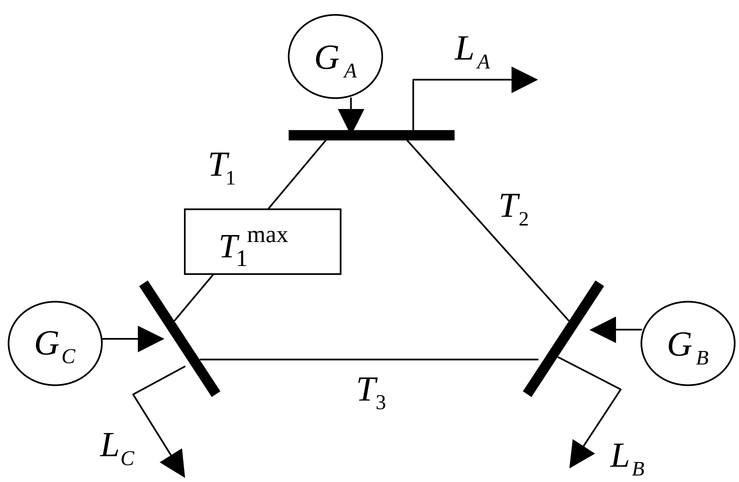



Figure 2.21: 3 bus transmission network

Three players $A$, $B$ and $C$ maximize their profit by adjusting their Cournot strategic variable $s_A$, $s_B$ and $s_C$. The profit functions of each player are analogous to those in (2.5). The prices for energy sold are given by the locational marginal prices $\lambda_A$, $\lambda_B$ and $\lambda_C$ that can be determined by solving the transmission constrained market clearing mechanism. For a three bus model, the transmission lines form an electrical loop and the market clearing mechanism includes “DC power flow” equations [22]. The mathematical model of the three bus market clearing mechanism is defined by:

$$\begin{aligned} \max_{\mathbf{d}} \quad & B_A(\mathbf{d}_A) + B_B(\mathbf{d}_B) + B_C(\mathbf{d}_C) \\ Const. \quad & \mathbf{Y}'\boldsymbol{\theta} = \mathbf{P} \\ & P_{AC} \le T_1^{max} \\ & P_{CA} \le T_1^{max}. \end{aligned} \tag{2.7}$$

The DC power flow equations are specified by (2.8) when we set the $C$ bus as the “reference bus” [22]:

$$\mathbf{Y}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \\ -1 & -1 \end{pmatrix}, \boldsymbol{\theta} = \begin{pmatrix} \theta_A \\ \theta_B \end{pmatrix}, \mathbf{P} = \begin{pmatrix} s_A - d_A \\ s_B - d_B \\ s_C - d_C \end{pmatrix}. \tag{2.8}$$

The power flows on T1, denoted by $P_{AC}$ and $P_{CA}$, are equal to $\theta_A$ and $-\theta_A$, respectively. Given supply bid quantities $s_A, s_B, s_C$, the market clearing mechanism will generate the demand quantities $d_A, d_B, d_C$ and the nodal marginal prices $\lambda_A, \lambda_B, \lambda_c$ corresponding to the Lagrange multipliers of the power flow equations [22].

From [3], we know that a unique pure strategy NE exists in the case of 100 MW constraints on T1 but that no pure strategy NE exists in the case of 66 MW constraint. The NE with 100 MW constraint is $s_A = 1106, s_B = 1046, s_C = 995$.

**Local iterative search**

Because of the non-smoothness of profit functions, we applied the bundle method as a local optimization engine for iterative NE search [23]. We restricted the searching space to an integer plane to avoid local chattering. In order to get the integer solution, we used same technique as in the two bus case. We choose the perfect competition output (1465, 1,257, 1256) as the initial point. Figures 2.22 and 2.23 show the results of the 100 MW and 66 MW constraints cases.

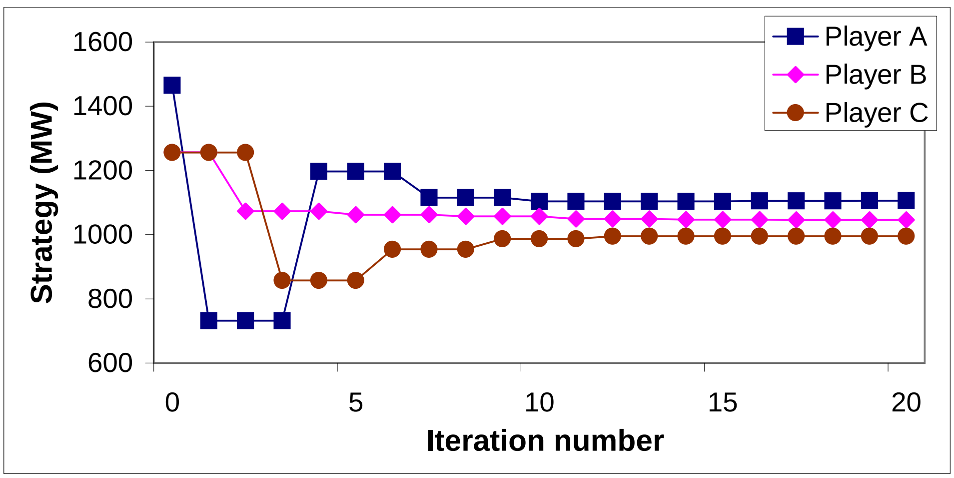



Figure 2.22: Iterative bundle method for 3 bus Cournot model with 100 MW constraint case

Figure 2.22 shows that from both starting points the strategic bids converged to the NE (1106, 1046, 995) for the 100 MW constraint case. Figure 2.23 shows that in the case of 66 MW constraint, the algorithm also converges to the same point, which is a local NE trap. Although no NE exists in 66 MW constraint case, the iterative method misled us as if there were the same unique NE as in the 100 MW case.

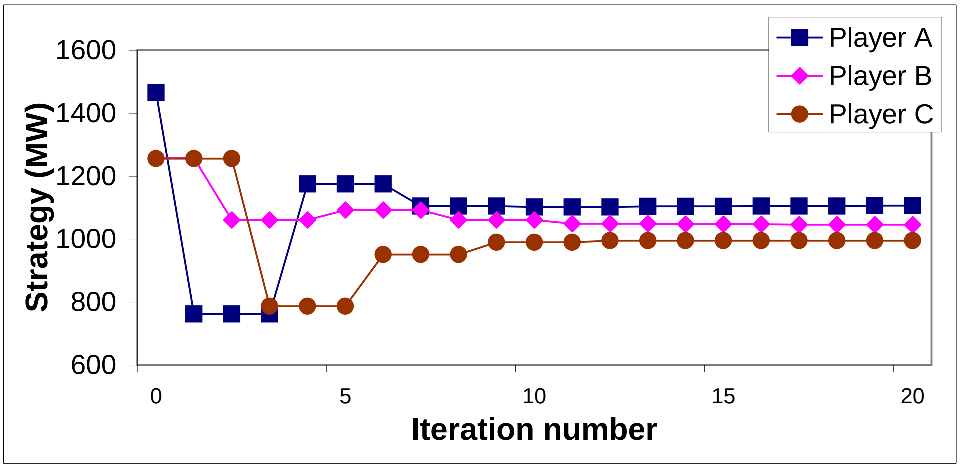



Figure 2.23: Iterative bundle method for 3 bus Cournot model with 66 MW constraint case

#### Co-evolutionary programming

For the simple co-evolutionary simulation without hill climbing, we set the maximum generation number, population size, crossover rate, and mutation rate as 1000, 50, 0.5, and 0.02 respectively. We illustrated the mode for every population at each generation in figure 2.24 and figure 2.25.

The figures show that the simple co-evolutionary programming [8] fails in identifying NE for both the 100 MW and 66 MW constraint cases. We tried various choices of parameters to see if the simple co-evolutionary programming would converge; however, we could not find such a parameter set.

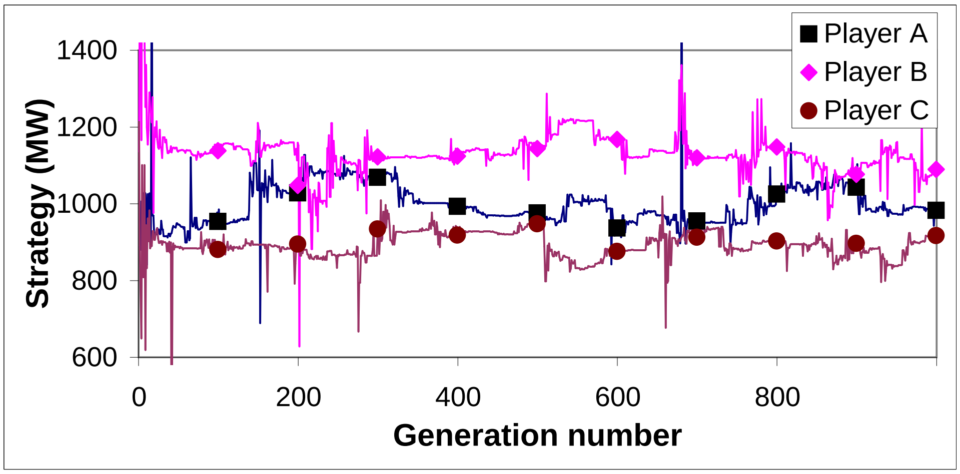



Figure 2.24: Mode of Cournot strategic variables during the simple co-evolutionary process for 3 bus model with 100 MW limit

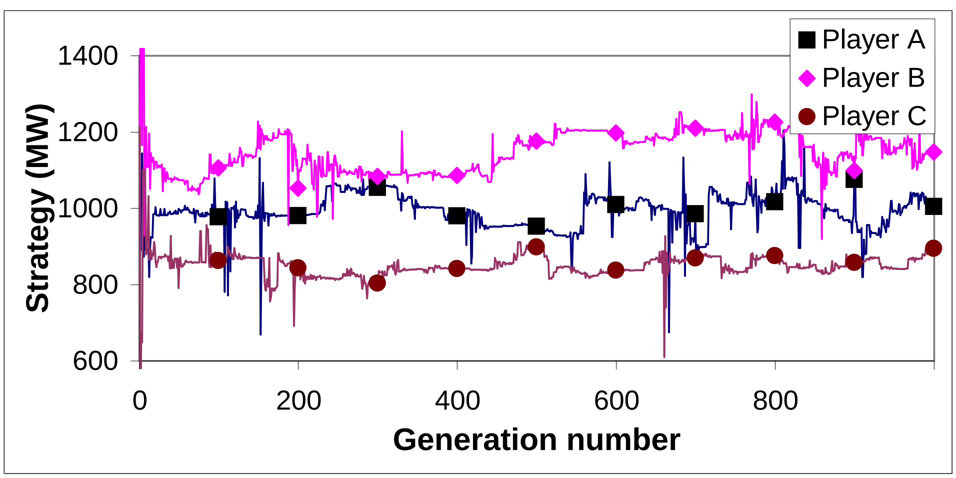



Figure 2.25: Mode of Cournot strategic variables during the simple co-evolutionary process for 3 bus model with 66 MW limit

### Hybrid co-evolutionary programming

For the hybrid co-evolutionary simulation, The maximum generation number, population size, crossover rate, mutation rate, and hill climbing rate are set as 500, 30, 0.5, 0.02, and 0.3 respectively. The bundle method was applied for the hill climbing. Despite the smaller maximum generation number and smaller population size than used in the simple co-evolutionary programming, the hybrid co-evolutionary programming could successfully identify the NE. The mode for every population at each generation is shown in figure 2.26 and 2.27.

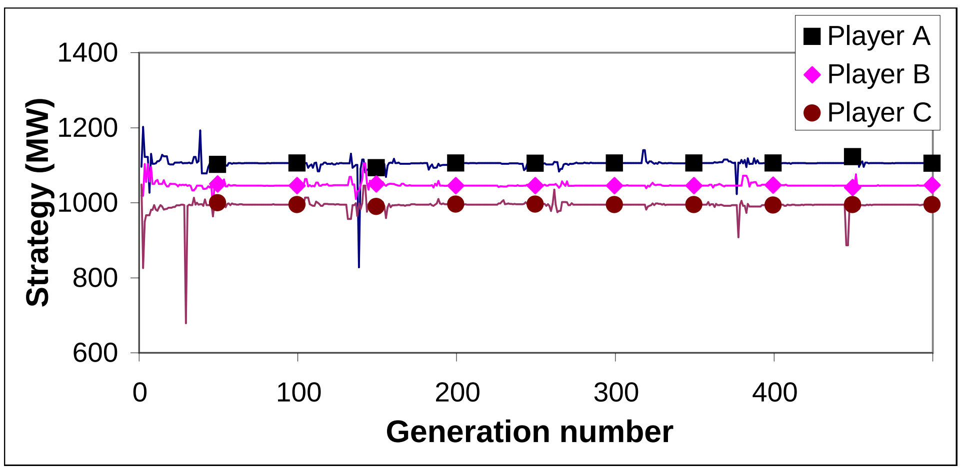



Figure 2.26: Mode of Cournot strategic variables during the hybrid co-evolutionary process for 3 bus model with 100 MW limit

In the 100 MW limit case, it shows a quite stable quantity bid set corresponding to the NE. The deviation from the NE due to the mutation intrusions was restabilized through the hybrid co-evolutionary process. In the case of the 66 MW constraint the evolutionary process shows unstable fluctuations. Hybrid co-evolutionary programming is not caught in the local NE trap in the example.

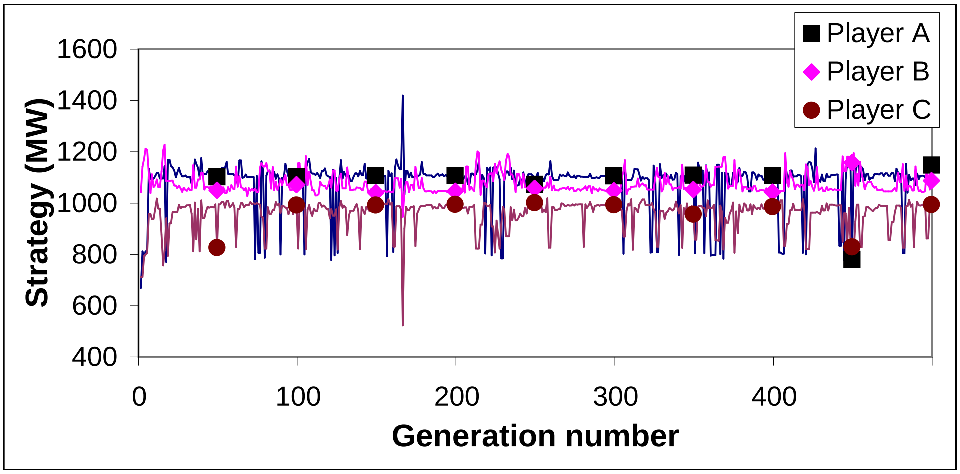



Figure 2.27: Mode of Cournot strategic variables during the hybrid co-evolutionary process for 3 bus model with 66 MW limit

## 2.6 Chapter conclusion

We studied NE search algorithms for games with local optima. We illustrated the problematic issue of "local NE trap". We proved that any iterative NE search algorithms based on local optimization cannot differentiate between the real NE and the "local NE traps." We illustrated possible cases of misjudgement by conventional iterative NE search algorithms.

Co-evolutionary programming, a parallel and global search algorithm, is applied to overcome the "local NE trap" problem. We suggested an enhancement of co-evolutionary programming, which we call hybrid co-evolutionary programming, to solve games with local optima.

We tested the conventional iterative NE search algorithm, simple co-evolutionary programming, and hybrid co-evolutionary programming through a simple numerical example and two transmission constrained electricity market examples, including a two bus, two player Cournot game and a three bus, three player Cournot game. Simulation results showed that hybrid co-evolutionary programming successfully identified the NE when other algorithms failed. In all cases studied, hybrid co-evolutionary programming converged to the NE, if it existed, and did not converge to any local NE traps. In future work, we plan to investigate the properties of hybrid co-evolutionary programming further to confirm that it can reliably differentiate between NE and local NE traps.

# Chapter 3

# Short Term Electricity Market Auction Game Analysis: Uniform and Pay-as-bid pricing

## 3.1 Chapter introduction

Game theory and auction theory have proven useful to understand markets based on auctions including electricity markets and especially in modeling auction participants, auction types, and potential market performance [1, 2, 24]. In a single unit auction, Vickrey proved that "English" and "Dutch" type auctions will yield the same expected revenue under the assumptions of risk neutral participants and privately known value drawn from a common distribution [25]. Vickrey's result is embodied in the "Revenue Equivalence Theorem" (RET) [24].

The RET also applies to multi-unit auctions when no buyer wants more than one of the $k$ available objects [24]. The RET does not, however, apply to general multi-unit auctions, which is a more appropriate model for an electricity market.

For general multi-unit auctions, the effect of auction type on outcome has been a research focus [26, 27]. Wilson showed that there are "seemingly collusive"

Nash Equilibria in a uniform pricing auction for shares [26]. Back and Zender compared Nash Equilibria under uniform pricing and discriminatory pricing in the US Treasury auction [27]. They also argued that uniform pricing could yield collusive equilibria and that the seller's expected revenue would be smaller than in the equilibria under discriminatory pricing [27].

References [24, 25, 26, 27] describe "buyers' auctions" where there are several strategic bidders offering to buy. In contrast, electricity markets are typically organized as "sellers' auctions" with strategic sellers. In the rest of this chapter, we will focus on "sellers' auctions" as models of electricity markets.

In electricity markets, a huge volume of commodities are being traded or will be traded through multi-unit auctions. The economic effects of trading in electricity markets have been very serious, as have been observed in the California crisis and British reform [28, 29].

The choice between uniform and pay-as-bid pricing for electricity auctions has been one of most important issues in newly deregulated electricity markets [30, 31, 32, 33, 34]. Federico and Rahman studied the perfect competition and monopoly case under both uniform and discriminatory (pay-as-bid) pricing in an electricity auction [30]. They noted the trade-off between efficiency and consumer surplus. Fabra set up a two-player auction game and considers it in a static and a dynamic setting [31]. She noted that in the static setting the profit level of suppliers will be greater under uniform pricing and in the dynamic setting uniform pricing can facilitate collusion. Vázquez, Rivier, and Pérez-Arriaga noted that an entry barrier for small players could exist under pay-as-bid auction and that it can be harmful in the long run [33].

Wolfram illustrated a simple electricity auction example to explain the differences between discriminatory and uniform pricing auctions [32]. She criticized the UK reform, which involved a switch from uniform pricing to discriminatory pricing, illustrating with an example where there is equal revenue between the uniform pricing and discriminatory pricing cases. The blue ribbon panel report on a proposed switch from uniform pricing to pay-as-bid pricing in California concluded that the expectation of a price drop due to the switch was dubious and that the switch could discourage increased competition in the long run [34].

In this chapter, we deal with the issue of uniform pricing and pay-as-bid pricing in electricity markets from the viewpoint of the effect of strategic bidding and market power in the short term. We ignore long run effects in this study.

The auction model in this chapter follows the basic framework of the sealed-bid multiple-unit auction in [35]. In order to address the core gaming issue in the auction pricing mechanism in electricity market, we use a specific two player auction game model having a big player with market power and a strategic small player. The illustrated simple example has a similar configuration to that examined in [32] but it contains a potential structural problem of market power. We show that the potential structural problem can yield a different conclusion to that in [32].

We first analyze the two-player auction game to show the core reasoning of each player under each auction pricing mechanism. We show that the profit maximizing bidding strategy can be dependent on the auction pricing mechanism. Then we analyze the gaming in terms of bid price itself and compare the expected revenue under each pricing mechanism. "Revenue inequivalence" under the two price mechanisms is explained and proved based on the concept of the static mixed strategy Nash Equilibrium (NE).

To confirm the theoretical result, we simulated the model using the Lemke and Howson algorithm [4, 5] and obtained the mixed strategy Nash Equilibria for the example. We compared the expected total revenues from the given NEs under both pricing mechanisms. We also extended our model to an elastic demand case and compared the total expected market cleared demands under both pricing mechanisms using the Lemke and Howson algorithm.

## 3.2 Auction model and assumptions

We follow the basic framework of the sealed-bid multi-unit auction model for the electricity market [35]. We assume a uniform tradable energy unit size (for example, 50MW) for the auction. We consider a sellers' auction model given fixed demands as in usual electricity pools. As mentioned in the introduction, typical auction theories model buyers' auctions [24, 25, 26, 27]. However most of the analysis still holds for a sellers' auction under similar assumptions. A seller owning energy unit $G_i$ having production cost $c_i$ sets a bid price $p_i$ for that unit. Bid winners will be determined by the ordered price bid stack given the demand level. The general multi-unit auction in the electricity market given $N$ units of demand is illustrated in figure 3.1. The bid prices are shown as solid bars, potentially set higher than the costs of the corresponding units of production.

We define two pricing mechanisms for multi-unit auctions in electricity markets: uniform and pay-as-bid pricing.

**Definition 4** *[Uniform pricing] Under the uniform pricing structure, the marginal bid block sets the uniform market clearing price $p^*$. An individual seller's profit $\pi$ is*

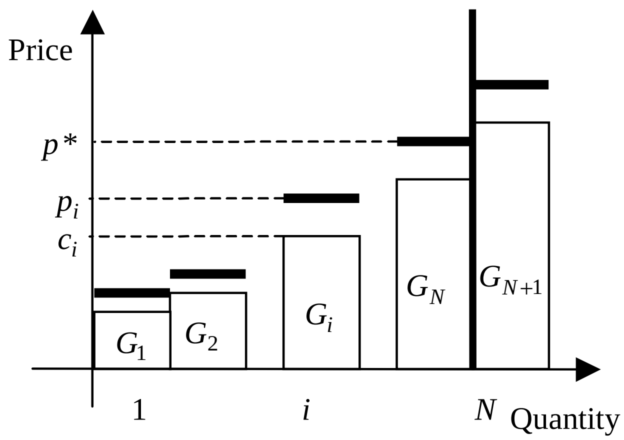



Figure 3.1: Multi-unit auction in the electricity market.

*defined as* $\pi = Kp^* - \sum_{j=1}^{K} c_{i(j)}$ *where the seller wins* $K$ *blocks of bid, namely blocks* $i(1), \ldots, i(K)$*. The total revenue under uniform pricing is defined by* $TR_{uni} = Np^*$.

**Definition 5** *[Pay-as-bid pricing] Under the pay-as-bid pricing structure, every winning unit* $i$ *gets the price* $p_i$ *as its income. An individual seller's profit is defined as* $\pi = \sum_{j=1}^{K}(p_{i(j)} - c_{i(j)})$ *when the seller wins* $K$ *blocks of bid, namely blocks* $i(1), \ldots, i(K)$*. The total revenue under pay-as-bid pricing is defined by* $TR_{pay} = \sum_{j=1}^{N} p_{i(j)}$.

If we assume that each seller bids only one energy block then the RET holds between both pricing mechanisms [24]. To illustrate the case where each seller has only one energy block, let us consider a perfect information case and suppose that all costs and prices are a multiple of a minimum bid increment or decrement size $\epsilon$. Assume that the blocks are ordered so that $c_1 < c_2 < ... < c_{N+1}$. Under uniform pricing the marginal cost seller will set the market clearing price as $p^* = c_{N+1} - \epsilon$. The rest of the sellers who own $G_i, i < N$ will behave as price takers. Under the pay-as-bid pricing, every seller who owns $G_i, i < N$ will maximize its profit by bidding at the price of $p_i = c_{N+1} - \epsilon$. Under both auction mechanisms the total revenue $TR_{uni} = TR_{pay} = N(c_{N+1} - \epsilon)$.

Similar reasoning applies to a multi-unit auction example described by Wolfram [32]. However the reasoning above does not seem to hold when there are sellers with market power [30, 31]. Since the exercise of market power was the impetus for considering a change from uniform to pay-as-bid in both California and England and Wales, it is important to represent market power. Different pricing mechanisms can potentially result in different equilibrium results in the presence of market power.

To consider the market power issue, we consider an oligopolistic auction game. In order to analyze the market power problem under uniform and pay-as-bid pricing electricity auctions, we set up a simple two player game model. We assume a big player with market power and a small player. Both players will behave strategically to maximize their profits. Demand level and cost functions of each player are assumed to be available publicly. In electricity markets these assumptions can be justified by widespread knowledge of information about the heat rate of each unit and accurate forecasts of demand.

For a clearer analytical understanding of the situation, we take the simplest configuration as illustrated in figure 3.2. Player A owns two energy units $G_{A1}, G_{A2}$ with cost $c_{A1}$, $c_{A2}$, respectively. Player B owns only one unit $G_{B1}$ costing $c_{B1}$, with $c_{A1} < c_{B1} < c_{A2}$. We assume two units of demand with a price cap in the market of $p_{cap}$. Each player will simultaneously bid prices $p_{A1}, p_{B1}, p_{A2}$ with all prices no larger than $p_{cap}$. As a tie-breaking rule, we assume that player A is declared the winner if both A and B declare equal bids that are both marginal. (Other assumptions

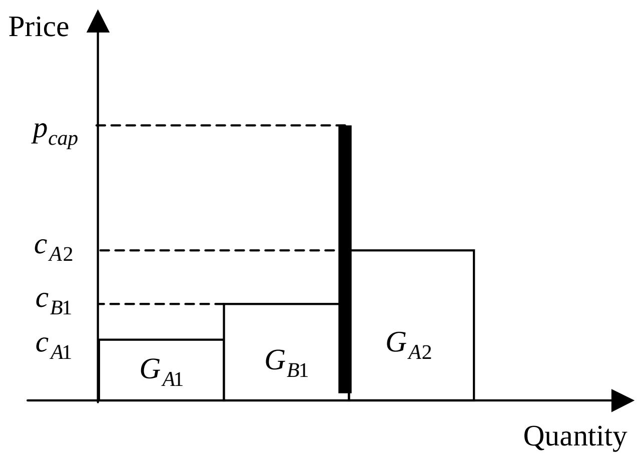



Figure 3.2: A two-player game configuration.

for tie-breaking lead to essentially the same results although the analysis differs in details.)

## 3.3 Gaming under Uniform and Pay-as-bid pricing

### 3.3.1 Strategic bid price classification

In order to describe the strategic behaviors of player A and B in the two player model, we classify the bid strategies of player A and B. We divide player A's price bid actions into three categorized bid price set in table 3.1.

"W (Withholding)" represents player A exercising extreme market power. By withholding $G_{A2}$ and raising the price of $G_{A1}$ to the extreme $p_{cap}$, player A tries to maximize his profit.

"U (Undercutting)" represents another strategic action of player A based on the belief that the undercutting player B's bid price is more profitable. The price

Table 3.1: Player A's categorized bid price set.

| Strategy ($X_A$) | Range of bid price $p_A$ |
|---|---|
| W (Withholding) | $p_W = p_{cap}$ |
| U (Undercutting) | $1/2(p_{cap} + c_{A2}) \leq p_U < p_{cap}$ |
| T (Timid) | $c_{A2} \leq p_T < 1/2(p_{cap} + c_{A2})$ |

range of $p_U$ is determined through the profit comparison with "Withholding":

$$\pi_{Undercutting} > \pi_{Withholding}$$

$$2p_U - c_{A2} - c_{A1} \geq p_{cap} - c_{A1}. \tag{3.1}$$

From (3.1), we have:

$$p_U \geq 1/2(p_{cap} + c_{A2}). \tag{3.2}$$

"T (Timid)" represent the remaining range of prices that A can bid.

Player B has two options. He can take the conservative bid by bidding $p_B < 1/2(p_{cap} + c_{A2})$ (and to maximize profits under this condition would choose the largest bid that is less than $1/2(p_{cap} + c_{A2})$.) We call this "S (Safe)." A second option is for player B to take the risk of being undercut. We call this "Risky." The categorized bid price set of player B are summarized into table 3.2.

Table 3.2: Player B's categorized bid price set.

| Strategy ($X_B$) | Range of bid price $p_B$ |
|---|---|
| S (Safe) | $p_S < 1/2(p_{cap} + c_{A2})$ |
| R (Risky) | $1/2(p_{cap} + c_{A2}) \leq p_R \leq p_{cap} - \epsilon$ |

### 3.3.2 Auction game under uniform pricing

We analyze the auction game under uniform pricing based on the classification in section 3.3.1. Under uniform pricing the resulting profits of the two players are summarized in table 3.3 and table 3.4 respectively. We use the notation $u(.)$ for a Boolean function such that $u(\text{True}) = 1$ and $u(\text{False}) = 0$. Each entry $\pi_k^{X_A X_B}$ in tables 3.3 and 3.4 represents the player $k$'s profit when player A and B chooses strategy $X_A \in \mathcal{X}_A$ and $X_B \in \mathcal{X}_B$ respectively.

Table 3.3: Player A's profits under uniform pricing.

| Case | Value |
|---|---|
| $\pi_A^{TS}$ | $u(p_T > p_S)(p_T - c_{A1}) + u(p_T \leq p_S)(2p_T - c_{A1} - c_{A2})$ |
| $\pi_A^{TR}$ | $2p_T - c_{A1} - c_{A2}$ |
| $\pi_A^{US}$ | $p_U - c_{A1}$ |
| $\pi_A^{UR}$ | $u(p_U > p_R)(p_U - c_{A1}) + u(p_U \leq p_R)(2p_U - c_{A1} - c_{A2})$ |
| $\pi_A^{WS}$ | $p_W - c_{A1}$ |
| $\pi_A^{WR}$ | $p_W - c_{A1}$ |

Table 3.4: Player B's profits under uniform pricing.

| Case | Value |
|---|---|
| $\pi_B^{TS}$ | $u(p_T > p_S)(p_T - c_{B1})$ |
| $\pi_B^{TR}$ | $0$ |
| $\pi_B^{US}$ | $p_U - c_{B1}$ |
| $\pi_B^{UR}$ | $u(p_U > p_R)(p_U - c_{B1})$ |
| $\pi_B^{WS}$ | $p_W - c_{B1}$ |
| $\pi_B^{WR}$ | $p_W - c_{B1}$ |

From table 3.3 and 3.4 we can analyze each player's best response to the other player's strategic choice. Table 3.5 shows player A's best response given player B's strategic choice $X_B$ under uniform pricing. Table 3.6 shows player B's best response given player A's strategic choice $X_A$ under uniform pricing.

Table 3.5: Player A's best response under uniform pricing.

| $X_B$ | Player A's best response |
|---|---|
| S | W |
| R | $U_{p_U<p_R}=\{p_U\|p_U<p_R\} \subset U$ |

Table 3.6: Player B's best response under uniform pricing.

| $X_A$ | Player B's best response |
|---|---|
| T | $S_{p_S<p_T}=\{p_S\|p_S<p_T\} \subset S$ |
| U | S and $R_{p_R<p_U}=\{p_R\|p_R<p_U\} \subset R$ |
| W | S and R |

By aggregating the results of best response analysis in tables 3.5 and 3.6, we can get the information about the Nash Equilibrium for the auction game. When a player's best response is conditionally limited (for example, when $X_B$ = R, player A's best response is $X_A$ = U only if the price $p_U$ is less then $p_R$), we call this a "conditional" best response hereafter. Using the best responses in tables 3.5 and 3.6 we illustrate the normal form game result in figure 3.3.

A pure strategy Nash Equilibrium is determined by the intersection of best responses. The choice set ($X_A$=W, $X_B$=S) is a NE. We can represent the NE by the range of player A and B's bid prices:

$$\{(p_A^*, p_B^*)|p_A^* = p_{cap},\ p_B^* < 1/2(p_{cap} + c_{A2})\}. \quad (3.3)$$

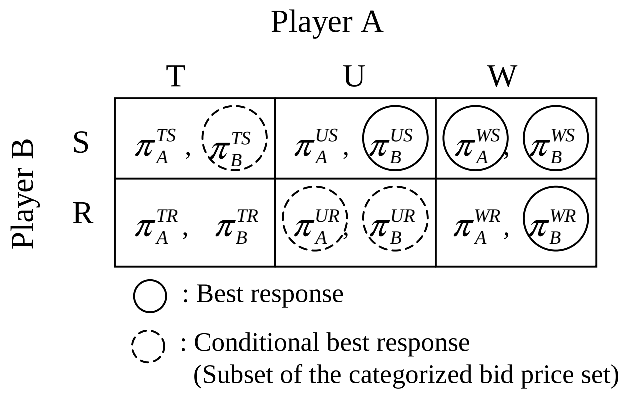



Figure 3.3: Best responses in the two-player auction game under the uniform pricing mechanism.

The intersection of "conditional" best responses yields no pure strategy NE because each player will change their bid price in turn repeatedly.

### 3.3.3 Auction game under pay-as-bid pricing

We analyze the auction game under pay-as-bid pricing by similar steps to that for unform pricing. Table 3.7 and 3.8 show player A and B's profits in possible cases, respectively.

Table 3.7: Player A's profits under pay-as-bid pricing.

| Case | Value |
|---|---|
| $\pi_A^{TS}$ | $u(p_T > p_S)(p_T - c_{A1}) + u(p_T \leq p_S)(2p_T - c_{A1} - c_{A2})$ |
| $\pi_A^{TR}$ | $2p_T - c_{A1} - c_{A2}$ |
| $\pi_A^{US}$ | $p_U - c_{A1}$ |
| $\pi_A^{UR}$ | $u(p_U > p_R)(p_U - c_{A1}) + u(p_U \leq p_R)(2p_U - c_{A1} - c_{A2})$ |
| $\pi_A^{WS}$ | $p_W - c_{A1}$ |
| $\pi_A^{WR}$ | $p_W - c_{A1}$ |

Table 3.8: Player B's profits under pay-as-bid pricing.

| Case | Value |
|---|---|
| $\pi_B^{TS}$ | $u(p_T > p_S)(p_S - c_{B1})$ |
| $\pi_B^{TR}$ | 0 |
| $\pi_B^{US}$ | $p_S - c_{B1}$ |
| $\pi_B^{UR}$ | $u(p_U > p_R)(p_R - c_{B1})$ |
| $\pi_B^{WS}$ | $p_S - c_{B1}$ |
| $\pi_B^{WR}$ | $p_R - c_{B1}$ |

Player A's profit for each case under pay-as-bid pricing is same as under uniform pricing. Player B's profit for each case under is changed since he is not paid the uniform market clearing price anymore.

From table 3.7 and 3.8 we can analyze each player's best response to the other player's strategic choice. Table 3.9 shows player A's best response given player B's strategic choice $X_B$ under pay-as-bid pricing. Table 3.10 shows player B's best response given player A's strategic choice $X_A$ under pay-as-bid pricing.

Table 3.9: Player A's best response under pay-as-bid pricing.

| $X_B$ | Player A's best response |
|---|---|
| S | W |
| R | $U_{p_U<p_R} = \{p_U \mid p_U < p_R\} \subset U$ |

By aggregating the results of best response analysis, we can get the information about the Nash Equilibrium for the auction game. We illustrate the results in figure 3.4.

In this case, we have no pure strategy NE. However there will still exist a

Table 3.10: Player B's best response under pay-as-bid pricing.

| $X_A$ | Player B's best response |
|---|---|
| T | $S_{p_S<p_T} = \{p_S \| p_S < p_T\} \subset S$ |
| U | $R_{p_R<p_U} = \{p_R \| p_R < p_U\} \subset R$ |
| W | R |

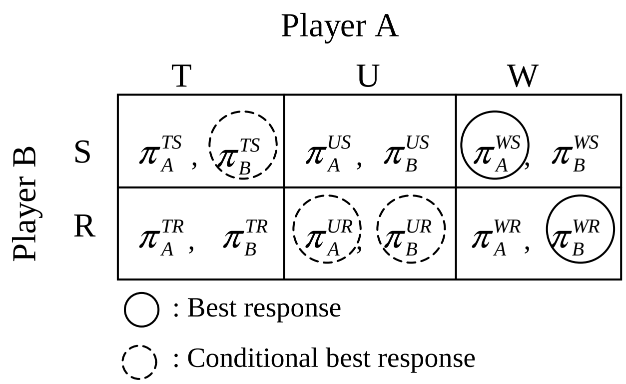



Figure 3.4: Best responses in the two-player auction game under the pay-as-bid pricing mechanism.

mixed strategy NE. By eliminating the player A's dominated strategy "Timid" and player B's dominated strategy "Safe", we get the set of "rationalizable" strategies for each player in (3.4) [2]:

$$P_A = \{p_A | 1/2(p_{cap} + c_{A2}) \leq p_A \leq p_{cap}\},$$
$$P_B = \{p_B | 1/2(p_{cap} + c_{A2}) \leq p_B \leq p_{cap} - \epsilon\}. \tag{3.4}$$

Consider probability mass functions $\sigma_A, \sigma_B$ over the set of rationalizable strategies

$P_A, P_B$. The probability mass function $\sigma_A$ satisfies:

$$\sum_{p_A \in P_A} \sigma_A(p_A) = 1; \forall p_A \in P_A,\ \sigma_A(p_A) \geq 0,$$
$$\sum_{p_B \in P_B} \sigma_B(p_B) = 1; \forall p_B \in P_B,\ \sigma_B(p_B) \geq 0, \tag{3.5}$$

with an analogous condition for $\sigma_B$. The expected profits for player A and B are defined by:

$$\pi_A(\sigma_A, \sigma_B) = \sum_{p_A \in P_A} \sigma_A(p_A) \left\{ \sum_{p_B \in P_B} \sigma_B(p_B) \pi_A(p_A, p_B) \right\},$$
$$\pi_B(\sigma_A, \sigma_B) = \sum_{p_B \in P_B} \sigma_B(p_B) \left\{ \sum_{p_A \in P_A} \sigma_A(p_A) \pi_B(p_A, p_B) \right\}. \tag{3.6}$$

For $\sigma_A$ and $\sigma_B$ satisfying (3.5), the conditions for probability mass functions $\sigma_A^*$ and $\sigma_B^*$ to be a mixed strategy NE under pay-as-bid pricing are:

$$\forall \sigma_A, \pi_A(\sigma_A^*, \sigma_B^*) \geq \pi_A(\sigma_A, \sigma_B^*),$$
$$\forall \sigma_B, \pi_B(\sigma_A^*, \sigma_B^*) \geq \pi_B(\sigma_A^*, \sigma_B). \tag{3.7}$$

### 3.3.4 Revenue comparison between uniform and pay-as-bid pricing

We compare the total revenues under uniform and pay-as-bid pricing. We consider the expected total revenue when market players set NE strategy as their strategic choices. By checking the NE under each pricing mechanism, we have:

**Proposition 2** *For the two player multi-auction game with market power described in section 3.2, the expected total revenue under the uniform pricing auction is larger than under the pay-as-bid pricing auction.*

*Proof:* From (3.3), we know the marginal bid is always $p_{cap}$ at any NE under the uniform pricing auction. From definition 4 the expected total revenue at NE under uniform pricing auction is:

$$TR^*_{uni} = TR_{uni}(p^*_A, p^*_B) = 2p_{cap}. \tag{3.8}$$

From (3.7) and definition 5, the expected total revenue under pay-as-bid pricing auction is represented by:

$$\begin{aligned} TR^*_{pay} &= \sum_{P_A} \sigma^*_A(p_A) \left\{ \sum_{P_B} \sigma^*_B(p_B) TR_{pay}(p_A, p_B) \right\}, \\ &= \sum_{P_B} \sigma^*_B(p_B) \left\{ \sum_{P_A} \sigma^*_A(p_A) TR_{pay}(p_A, p_B) \right\}. \end{aligned} \tag{3.9}$$

Because there is no pure strategy NE, $\sigma^*_A(p_{cap}) < 1$, so since $TR_{pay}(p_A, p_B) < TR_{pay}(p_{cap}, p_B)$, we have that:

$$\sum_{P_A} \sigma^*_A(p_A) TR_{pay}(p_A, p_B) < TR_{pay}(p_{cap}, p_B). \tag{3.10}$$

Similarly $\sigma^*_B(p_{cap} - \epsilon) < 1$ and so since, $\forall p_B \in P_B \setminus p_{cap} - \epsilon$, $TR_{pay}(p_{cap}, p_B) < TR_{pay}(p_{cap}, p_{cap} - \epsilon)$,

$$\sum_{P_B} \sigma^*_B(p_B) TR_{pay}(p_{cap}, p_B) < TR_{pay}(p_{cap}, p_{cap} - \epsilon). \tag{3.11}$$

From (3.10) and (3.11), we have:

$$\begin{aligned} TR^*_{pay} &< TR(p_{cap}, p_{cap} - \epsilon) \\ &= 2p_{cap} - \epsilon < 2p_{cap} = TR^*_{uni}. \end{aligned} \tag{3.12}$$

■

We will see in 3.4 that $TR^*_{pay}$ can be significantly smaller than $TR^*_{uni}$.

## 3.4 Simulation

In order to confirm our analytical results, we simulate the short term electricity market auction game with inelastic and elastic demand case. For a quantitative result we use bid prices directly instead of categorized bid price sets in 3.3. We apply the Lemke and Howson algorithm [4, 5] to deal with the mixed strategy NE (The pure strategy NE is a special case of the mixed strategy NE). The simulation steps follow:

1. construct the bid price vector $P_A$ and $P_B$ with the increment size $\epsilon$,
2. define vectors $\mathbf{x}$ and $\mathbf{y}$ for the probability masses $\sigma_A$ and $\sigma_B$ in (3.5), respectively,
3. generate the profit matrix $\Pi_A$ and $\Pi_B$ given the game environment,
4. represent the mixed strategy NE in (3.7) by:

$$\mathbf{x}^{*T}\Pi_A\mathbf{y}^* \geq \mathbf{x}^T\Pi_A\mathbf{y}^*, \forall \mathbf{x} \in R^m : \sum_{i=1}^{m} x_i = 1, \mathbf{x} \geq 0,$$
$$\mathbf{x}^{*T}\Pi_B\mathbf{y}^* \geq \mathbf{x}^{*T}\Pi_B\mathbf{y}, \forall \mathbf{y} \in R^n : \sum_{j=1}^{n} y_j = 1, \mathbf{y} \geq 0. \tag{3.13}$$

5. convert conditions in (3.13) into a linear complementarity problem (LCP) and solve by a complementary pivot algorithm [4].

### 3.4.1 Inelastic demand case

We simulated the two player auction game analyzed in 3.3. We set $c_{A1}$, $c_{B1}$, and $c_{A2}$ as \$10, \$15, and \$20, respectively. The price cap was set as $p_{cap}$ = \$60. The strategic bid vectors for player A and B are $P_A = \{p_A | \$20 \leq p_A \leq \$60\}$ and

$P_B = \{p_B | \$20 \leq p_B \leq \$60\}$. The bid increment or decrement size was set as $\epsilon = \$1$. The structure of $\Pi_A$ and $\Pi_B$ is illustrated in figure 3.5.

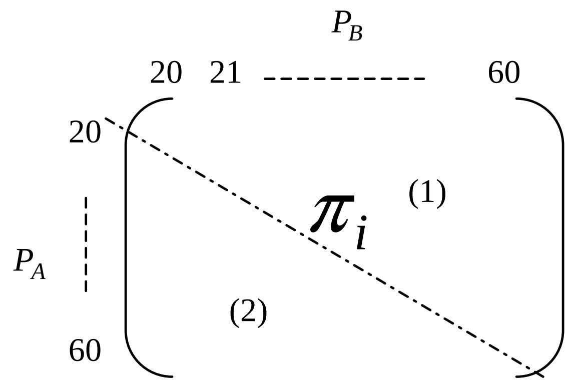



Figure 3.5: Profit matrix structure in the inelastic demand case. Region (1) is when $p_A \leq p_B$ and Region (2) is when $p_A > p_B$.

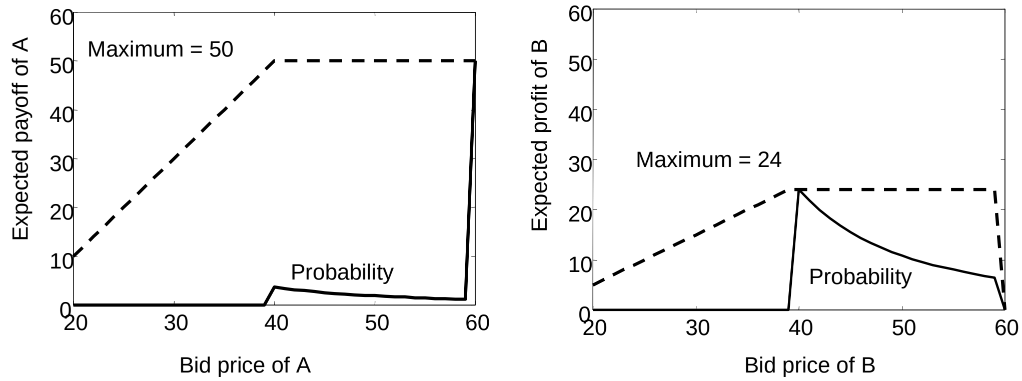
The elements of $\Pi_A$ and $\Pi_B$ are determined by table 3.11.

Table 3.11: Profits in the inelastic demand case.

| Region | Uniform | | Pay-as-bid | |
|---|---|---|---|---|
| | $\pi_A$ (\$) | $\pi_B$ (\$) | $\pi_A$ (\$) | $\pi_B$ (\$) |
| (1) | $2p_A - 30$ | 0 | $2p_A - 30$ | 0 |
| (2) | $p_A - 10$ | $p_A - 15$ | $p_A - 10$ | $p_B - 15$ |

The NE solution obtained by the algorithm under uniform pricing was a pure strategy NE of $p_A = 60$ and $p_B = 15$ consistent with the analysis in section 3. The expected total revenue is $TR^*_{uni} = 120$.

The NE solution obtained under pay-as-bid pricing was a mixed strategy NE of $\mathbf{x}^*$ and $\mathbf{y}^*$, which is represented by figure 3.6. The expected total revenue was $TR^*_{pay} = \mathbf{x}^{*T}\mathbf{T}R_{pay}\mathbf{y}^* = 100.17$. Consistent with our theoretical analysis in section




(a) (b)

Figure 3.6: Probability distribution on mixed strategy NE in the inelastic demand case (a) Player A (b) Player B. The solid curves represent the scaled probability distribution, while the dashed curves represent the expected profits for the corresponding bid, given the distribution of the other player's bid.

3, $TR^*_{pay}$ is less than $TR^*_{uni}$.

We tried several different values of $\epsilon$ to see if there is any serious sensitivity to $\epsilon$. The resulting expected total revenues are illustrated in table 3.12. The NE and expected total revenue were not very sensitive to $\epsilon$ and confirm that $TR^*_{pay}$ is significantly less than $TR^*_{uni}$ for this example.

Table 3.12: Expected total revenues with different bid increment sizes.

| $\epsilon$ | $TR^*_{uni}$ (\$) | $TR^*_{pay}$ (\$) |
|---|---|---|
| 0.1 | 120 | 100.98 |
| 0.5 | 120 | 100.62 |
| 1 | 120 | 100.17 |
| 2 | 120 | 100.95 |

### 3.4.2 Elastic demand case

The theoretical development in 3.3 assumes inelastic demand, unit size generation blocks, and an enforced price cap. Potentially, proposition 2 depends critically on these assumptions. To test the extensibility of the proposition 2 to an elastic demand case, we set up and extended the model by incorporating demand willingness-to-pay and different sized generation blocks.

We assume three demand blocks $W_1$, $W_2$ and $W_3$ that have different willingness-to-pay and size. The demand blocks are assumed to be bid in at the willingness-to-pay. We assume two strategic suppliers, players A and B. Player A owns two blocks, $G_{A1}$ and $G_{A2}$, while player B owns one block, $G_{B1}$. The supply is bid strategically. Supply that is cleared in the auction is paid either the market clearing price or its bid depending on the auction design. The configuration for demand and supply shown in table 3.13.

Table 3.13: Demand and supply for the elastic demand case.

| Willingness-to-pay | | | Cost | | |
|---|---|---|---|---|---|
| demand | price ($/MW) | size (MW) | supply | cost ($/MW) | size (MW) |
| $W_1$ | 60 | 100 | $G_{A1}$ | 10 | 100 |
| $W_2$ | 50 | 30 | $G_{B1}$ | 15 | 80 |
| $W_3$ | 35 | 20 | $G_{A2}$ | 20 | 80 |

The stacked supply bid and demand blocks determine the market clearing price and quantity. The price cap was set as $p_{cap} = \$60$. The strategic bid vectors for player A and B are $P_A = \{p_A | \$20 \leq p_A \leq \$60\}$ for $G_{A1}$ and $G_{A2}$ and $P_B =$

$\{p_B | \$20 \leq p_B \leq \$60\}$ for $G_{B1}$, respectively. The bid increment or decrement size was set as $\epsilon = \$1$. In the elastic demand case the structure of $\Pi_A$ and $\Pi_B$ is illustrated in figure 3.7.

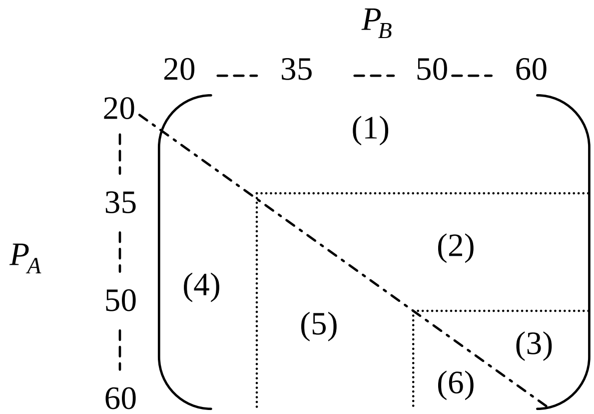



Figure 3.7: Profit matrix structure in the elastic demand case. The regions represent the bid price ranges of: (1) $p_A \leq \$35$, $p_A \leq p_B$, (2) $\$35 < p_A \leq \$50$, $p_A \leq p_B$, (3) $\$50 < p_A \leq \$60$, $p_A \leq p_B$, (4) $p_B < p_A \leq \$35$, (5) $p_B < p_A \leq \$50$ and (6) $p_B < p_A \leq \$60$.

In table 3.14 and table 3.15 we illustrate the profit of player A and B in each region under the uniform and pay-as-bid pricing, respectively.

Table 3.14: Profits in the elastic demand case under uniform pricing .

| Region | $\pi_A$ (\$) | $\pi_B$ (\$) |
|---|---|---|
| (1) | $150p_A - 2000$ | 0 |
| (2) | $130p_A - 1600$ | 0 |
| (3) | $100p_A - 1000$ | 0 |
| (4) | $70p_A - 700$ | $80p_A - 1200$ |
| (5) | $50p_A - 500$ | $80p_A - 1200$ |
| (6) | $20p_A - 200$ | $80p_A - 1200$ |

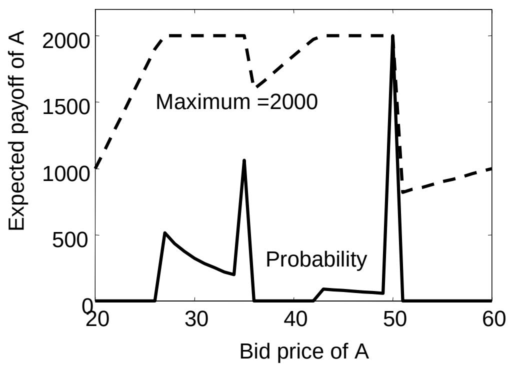
Table 3.15: Profits in the elastic demand case under pay-as-bid pricing .

| Region | $\pi_A$ (\$) | $\pi_B$ (\$) |
|---|---|---|
| (1) | $150p_A - 2000$ | 0 |
| (2) | $130p_A - 1600$ | 0 |
| (3) | $100p_A - 1000$ | 0 |
| (4) | $70p_A - 700$ | $80p_B - 1200$ |
| (5) | $50p_A - 500$ | $80p_B - 1200$ |
| (6) | $20p_A - 200$ | $80p_B - 1200$ |

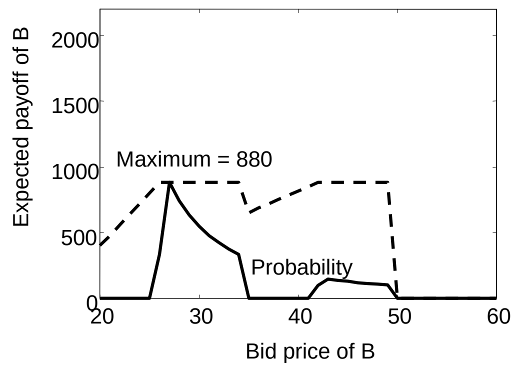
Under uniform pricing, a pure strategy NE of ($p_A = 50$, $p_B = 15$) was obtained through the algorithm resulting in a uniform clearing price of \$50. Although there are other pure strategy NEs and a mixed strategy NE, they all result in the same market clearing price and demand as ($p_A = 50$, $p_B = 15$). The expected market cleared demand $TD^*_{uni}$ was 130 MW out of the total bid demand 150 MW. The uniform market clearing price was \$50 yielding an expected total revenue of $TR_{uni} = \$6500$.

Under pay-as-bid pricing, a mixed strategy NE is obtained. Figure 3.8 represents the NE probability distribution over each player's strategies. The expected market cleared demand is $TD^*_{pay} = 141.85$ MW out of the total bid demand 150 MW and the total expected revenue of $TR^*_{pay} = \$4726.2$.

Under pay-as-bid pricing, the expected market cleared demand is more and the expected total revenue is less than under the uniform pricing. Table 3.16 shows the results with different bid increment sizes $\epsilon$. Again, the results are not heavily dependent on the choice of $\epsilon$, $TR^*_{pay}$ is significantly lower than $TR^*_{uni}$, and more demand is cleared under pay-as-bid than uniform pricing.




(a)




(b)

Figure 3.8: Probability distribution on mixed strategy NE in the elastic demand case (a) Player A (b) Player B. The solid curves represent the scaled probability distribution, while the dashed curves represent the profits for the corresponding bid, given the distribution of the other player's bid.

Table 3.16: Expected total revenues and market cleared demand with different bid increment sizes.

| $\epsilon$ | Uniform | | Pay-as-bid | |
|---|---|---|---|---|
| | $TR^*_{uni}$ (\$) | $TD^*_{uni}$ (MW) | $TR^*_{pay}$ (\$) | $TD^*_{pay}$ (MW) |
| 0.1 | 6500 | 130 | 4769 | 141.5 |
| 0.5 | 6500 | 130 | 4760.4 | 141.48 |
| 1 | 6500 | 130 | 4726.2 | 141.85 |
| 2 | 6500 | 130 | 4716.9 | 141.2 |

## 3.5 Chapter conclusion

We analyzed a two-player static auction game that has a big player with market power and small player. We proved that the total expected revenue for the NEs under uniform pricing and pay-as-bid pricing were not equivalent. The total payment of consumers would be smaller under pay-as-bid pricing for the two-player game; however, under pay-as-bid the equilibrium is a mixed strategy equilibrium, which may be undesirable from an operational point of view. We illustrated the simulation results of the model. We also extended the model to an elastic demand case and showed that pay-as-bid pricing also led to a larger expected total demand being served.

# Chapter 4

# Market Power Mitigation by Limiting Transmission Congestion Right in ERCOT market

## 4.1 Chapter introduction

ERCOT began its operation as the ISO for the new deregulated Texas electricity market in July, 2001. In ERCOT, bilateral contracts between consumers and suppliers cover the day ahead energy market similarly to the New Electricity Trading Arrangement (NETA) adopted for the England and Wales electricity market. ERCOT manages ancillary service market and real time balancing market for the congestion management and the real time balancing deviation. In ERCOT, Commercially Significant Constraint (CSC) is used to represent the zonal flow gate interface. The congestion cost to relieve the congestion caused by the aggregated day ahead schedule was uplifted until the implementation of congestion cost "direct assignment" in 2002.

The mechanism change from the uplift to the direct assignment was prompted by the great volume of congestion costs during the uplift period in the hope that the change would prevent irresponsible scheduling and strategic inter zonal scheduling. In parallel with the congestion cost direct assignment implementation, a hedging method for the incurred cost had to be provided for market participants. Transmission Congestion Right (TCR) as a form of Financial Transmission Right (FTR) was adopted as the institutional hedging method. Regarding the market power issues of TCR [36, 37, 38], Texas Public Utility Commission (PUC) decided to restrict the TCR ownership to 25% of total available TCR for each CSC.

In this chapter we analyze the current TCR setup with transmission ownership limitation in ERCOT market. We adopt Joskow and Tirole's modelling framework for transmission rights [36] and develop it further to deal with the details of strategic day ahead resource scheduling and the real time balancing market manipulation in ERCOT. Joskow and Tirole [36] modelled the zonal market price as being determined by the demand function in that zone. We model zonal balancing market power in conjunction with the day ahead strategic scheduling.

Our focus is how a market situation, including available competitive resources at the balancing market and bilateral contracts, affects the strategic player's market power exercise. We model the strategic player's profit structure following current ERCOT market setup [39]. Using the introduced strategic profit structure we analyze the relationship between TCR ownership limitation and the condition for exercise of market power given the market situation.

Another contribution of this chapter is to suggest a market power measurement algorithm based on our finding the relationship between TCR limitation and the condition for exercise of market power. We extend our analysis on the two zone market model prevalently used in analytical studies for electricity market [36, 37, 38, 17] to general flow gate base market models [40]. We derive a market power exercising condition for general flow gate base market models and implement the market power measurement algorithm based on the general market power exercising conditions.

In order to show market power measurement algorithm's practicality we simulate ERCOT 2003 market model consisting of four zones and three CSCs. We apply the market power measurement to two big players separately located in two high price import zones for multiple demand level and various TCR ownership limits.

## 4.2 Market model and assumption

We model a two zone ERCOT type market model focusing on the day ahead scheduling mechanism and real time balancing mechanism for congestion management [39].

### 4.2.1 Model structure

We first introduce a two-zone (*South* and *North*) model with one CSC. We characterize *South* as a zone with low demand and abundant supply and *North* as having high demand and tight supply. *North* and *South* zone are assumed the import and export zone respectively in nature. Market players at both zones are:

- $G_{S0}$: aggregated competitive player in *South* with abundant resource,
- $G_{N0}$: aggregated competitive player in *North* with the limited capacity $q_{N0}^{max}$,
- $G_{N1}$: strategic player in *North* with limited capacity $q_{N1}^{max}$.

For CSC congestion management we assume the parameters of [39]:

- $SF_{S,SN}$: $CSC_{SN}$ shift factor for $South$ injection,
- $SF_{N,SN}$: $CSC_{SN}$ shift factor for $North$ injection,
- $I_{SN}^{max}$: $CSC_{SN}$ flow limit.

### 4.2.2 Base schedule

Each player submits the load and resource day ahead schedule for an operation hour to ISO. Without strategic resource scheduling, the submitted schedule would be the production cost minimizing resource schedule for the contracted load. We call this the "base schedule". The base resource schedule in our two zone model can be illustrated by figure 4.1.

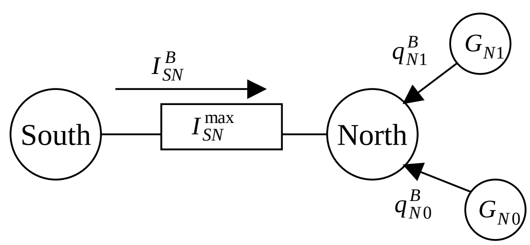



Figure 4.1: Base bilateral schedule.

The variables in figure 4.1 are:

- $q_{N0}^{B}$: $G_{N0}$'s base bilateral resource schedule,
- $q_{N1}^{B}$: $G_{N1}$'s base bilateral resource schedule,
- $I_{SN}^{B}$: aggregated inter-zonal flow based on the base bilateral schedule.

### 4.2.3 ISO balancing market

When a day ahead schedule for an operation hour does not meet all the real time demand or if congestion happens, the real time ISO balancing market should solve the problem. Because we focus on the strategic resource scheduling related with congestion, we assume the aggregated day ahead schedule meets the real time demand for an operating hour. We do not consider any balancing error.

When the day ahead schedule brings congestion over $CSC_{SN}$, that is, $I_{SN}^{B} > I_{SN}^{max}$, the "overflow" $I_{SN}^{O} = I_{SN}^{B}$ - $I_{SN}^{max}$ should be eliminated by "counter flow" through ISO balancing market. The counter flow $I_{SN}^{C}$= - $I_{SN}^{O}$ is deployed through balancing up in North ($q_{N}^{U}$) and balancing down in South ($q_{S}^{D}$). When we assume that there is no loss, $q_{N}^{U} = q_{S}^{D}$,

$$I_{SN}^{C} = q_{N}^{U} SF_{N,SN} - q_{S}^{D} SF_{S,SN} = q_{N}^{U} SF_{NS,SN}, \tag{4.1}$$

where $SF_{NS,SN}$=$SF_{N,SN}$-$SF_{S,SN}$ is the $CSC_{SN}$ shift factor difference for *North* injection and *South* withdrawal. Zonal market clearing price $MCP_N$ and $MCP_S$ are determined based on locational marginal pricing [22].

In order to model the bidding action in the balancing market, we use a simple block bidding approach. That is, each player bids a pair ($p_{ij}^{bid}$, $q_{ij}^{bid}$). Given $q_{N}^{U}$, the strategic player $G_{N1}$'s bidding action and the following market clearing result in the *North* zone can be mathematically defined as a function:

$$(MCP_N, q_{N1}^{W}) = RM(p_{N1}^{bid}, q_{N1}^{bid}; q_{N}^{U}), \tag{4.2}$$

where $q_{N1}^{W}$ is $G_{N1}$'s winning bid quantity.

We assume that there is no physical withholding in the ISO balancing market. Each player bids its whole residual resource into the ISO balancing market. That is, $q_{ij}^{bid} = q_{ij}^{R} = q_{ij}^{max}$ - $q_{ij}^{B}$. To model market power we instead allow economic withholding by bidding at high price. We assume that there is a price cap $p^{cap}$ for the ISO balancing market.

### 4.2.4 Congestion cost and TCR

When ERCOT market first opened in July 2001, the congestion costs were charged to market participants on the basis of the load ratio share. The indirect congestion cost assignment is believed to be an important driving factor for the high congestion cost during that period. For that reason, ERCOT started applying the direct congestion cost assignment from January 2002.

The direct assigned congestion cost is determined by each participant's scheduled flow between zones and balancing market clearing price difference between zones. If player $G_{N1}$ scheduled an import $q_{N1,SN}$ from *South* to *North* zone, the impact schedule flow $I_{N1,SN}^{S}$ over the $CSC_{SN}$ will be determined through the flow gate model as:

$$I_{N1,SN}^{S} = q_{N1,SN}^{S} \times SF_{SN,SN}. \tag{4.3}$$

For the two zone model, the shadow price $SP_{SN}$ over the $CSC_{SN}$ can be easily calculated by:

$$SP_{SN} = (MCP_N - MCP_S)/SF_{SN,SN}. \tag{4.4}$$

Then the direct assigned congestion cost for $G_{N1}$ will be calculated by multiplying the CSC impact schedule flow $I_{N1,SN}^{S}$ in (4.3) and the shadow price $SP_{SN}$ in (4.4).

$$
\begin{aligned}
CC_{N1,SN} &= I^S_{N1,SN} \times SP_{SN} \\
&= q^S_{N1,SN} \times (MCP_N - MCP_S).
\end{aligned} \tag{4.5}
$$

ERCOT implemented the financial transmission congestion right as a hedging tool for the direct assigned congestion cost. If player $G_{N1}$ has the TCR ownership of $t_{N1,SN}$ MW over the $CSC_{SN}$, then he would be paid the TCR income as:

$$
TCR^{IN}_{N1,SN} = t_{N1,SN} \times SP_{SN}. \tag{4.6}
$$

If he arranges $t_{N1,SN} = I^S_{N1,SN}$, the congestion cost will be perfectly hedged.

TCR ownership is purchased as the form of strip (for example, a monthly strip consists of 24 hours $\times$ 30 days) from annual and monthly auctions. We assume that strategic players purchase TCR ownership at a low price from a TCR auction ahead of the real time interval and treat it as a sunk cost. We deal with an issue related to TCR purchase cost in the appendix.

The TCR ownership is limited under PUCT rules to:

$$
t_{N1,SN} \leq t^{max}_{SN} = \%\, limit \times I^{max}_{SN}. \tag{4.7}
$$

We investigate the implications of ownership limits.

## 4.3 Strategic coordination modelling

### 4.3.1 Basic insight

Because $North$ zone has tight supply and high demand, the strategic player in $North$ might have market power. He might exercise market power to maximize his profit by a strategic coordination. The strategic coordination is that he coordinates his bilateral schedule and real time balancing bid to maximize his profit. The steps of strategic coordination can be summarized as:

1. Instead of the "base schedule" a strategic player submits a strategic schedule causing congestion through strategic import schedule.

2. A strategic player can manipulate the balancing market when there is not enough competitive resource to clear the congestion.

3. When the income from TCR payment and balancing market is enough to cover the congestion cost by the strategic scheduling, the strategic player will get higher profit than in the "base schedule".

### 4.3.2 Strategic congestion scheduling

$G_{N1}$'s strategic variable for strategic congestion scheduling is:

- $q^S_{N1,SN}$: strategic import schedule.

The $G_{N1}$'s self production changes from $q^B_{N1}$ to $q^B_{N1} - q^S_{N1,SN}$. The aggregated inter-zonal flow changes from $I^B_{SN}$ to $I^B_{SN} + I^S_{N1,SN}$. The overflow caused by $q^S_{N1,SN}$ is:

$$I^O_{SN}(q^S_{N1,SN}) = I^B_{SN} + I^S_{N1,SN} - I^{max}_{SN}. \tag{4.8}$$

### 4.3.3 Balancing market manipulation

The counter flow $I_{SN}^{C}(q_{N1,SN}^{S})$ = -$I_{SN}^{O}$ should be deployed through ISO balancing market. The up balancing needed in $North$ is:

$$\begin{aligned} q_N^U(q_{N1,SN}^S) &= I_{SN}^C(q_{N1,SN}^S)/SF_{NS,SN} = I_{SN}^O/SF_{SN,SN} \\ &= (I_{SN}^B + I_{N1,SN}^S - I_{SN}^{max})/SF_{SN,SN} \end{aligned} \quad (4.9)$$

When the needed balancing up $q_N^U(q_{N1,SN}^S)$ is larger than the real time residual competitive resource $q_{N0}^R = q_{N0}^{max} - q_{N0}^B$ in $North$, $G_{N1}$ can control the market clearing price in north zone. Figure 4.2 illustrates this situation.

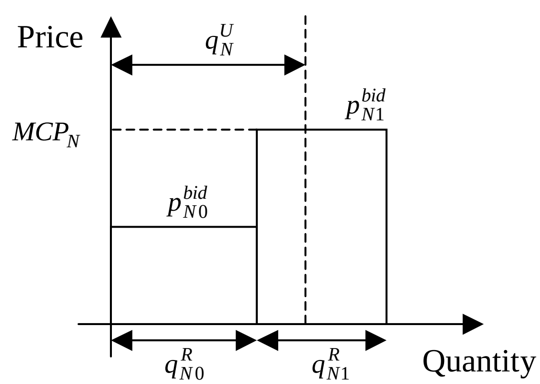



Figure 4.2: Real time balancing market clearing at $North$ zone

Because we assume there is no physical withholding, $G_{N1}$'s strategic variable in ISO balancing market is:

- $p_{N1}^{bid}$: balancing market bid price.

In figure 4.2 $MCP_N$ is directly controllable by $p_{N1}^{bid}$. When $G_{N1}$ maximizes its balancing market income by raising $p_{N1}^{bid}$ up to $p_{cap}$, the market clearing result will be:

$$q_{N1}^{W} = q_{N}^{U}(q_{N1,SN}^{S}) - q_{N0}^{R}, \tag{4.10}$$

$$MCP_N = p^{cap}. \tag{4.11}$$

### 4.3.4 Settlement and profit structure

$G_{N1}$'s profit based on the "base schedule" is represented by:

$$\pi_{N1}^{base} = p_{N1}^{B} \times q_{N1}^{B} - c_{N1}(q_{N1}^{B}) \tag{4.12}$$

The rearranged "strategic schedule" changes $G_{N1}$'s profit structure from (4.12) to another including other costs and incomes. $G_{N1}$ reduces its production by $q_{N1,SN}^{S}$ and purchase $q_{N1,SN}^{S}$ from *South* at the price of $p_{N1}^{import}$. Balancing market revenue, congestion cost and TCR income will be added. The changed parts are:

- changed production cost: $c_{N1}(q_{N1}^{B} - q_{N1,SN}^{S} + q_{N1}^{W})$
- import cost: $p_{N1}^{import} \times q_{N1,SN}^{S}$ ( $p_{N1}^{import}$ is the import price )
- balancing market revenue: $q_{N1}^{W} \times MCP_N$
- congestion cost: $I_{N1,SN}^{S} \times SP_{SN}$
- TCR income: $t_{N1,SN} \times SP_{SN}$.

The cost of acquiring TCR ownership is not included since we assumed that it is relatively cheap and sunk.

$G_{N1}$'s profit based on the "strategic schedule" become:

$$\begin{aligned}\pi_{N1}^{strategic} &= p_{N1}^{B} \times q_{N1}^{B} - c_{N1}(q_{N1}^{B} - q_{N1,SN}^{S} + q_{N1}^{W}) \\ &\quad - p_{N1}^{import} \times q_{N1,SN}^{S} + q_{N1}^{W} \times MCP_{N} \\ &\quad - I_{N1,SN}^{S} \times SP_{SN} + t_{N1,SN} \times SP_{SN} \end{aligned} \tag{4.13}$$

The final strategic choice of $G_{N1}$ for maximizing his strategic profit is:

- $t_{N1,SN}$: amount of TCR ownership.

$G_{N1}$'s TCR ownership sensitivity for the strategic profit in (4.13) is:

$$\frac{\partial \pi_{N1}^{strategic}}{\partial t_{N1,SN}} = SP_{SN} > 0, \tag{4.14}$$

since the congestion occurs at the direction from *South* to *North*. Then, $G_{N1}$'s profit maximizing choice should be:

$$t_{N1,SN} = t_{SN}^{max} \tag{4.15}$$

Applying (4.10), (4.11) and (4.15) to (4.13), we have:

$$\begin{aligned}\pi_{N1}^{strategic} &= p_{N1}^{B} \times q_{N1}^{B} - c_{N1}(q_{N1}^{B} - q_{N1,SN}^{S} + q_{N}^{U} - q_{N0}^{R}) \\ &\quad - p_{N1}^{import} \times q_{N1,SN}^{S} + (q_{N}^{U} - q_{N0}^{R}) \times p^{cap} \\ &\quad - I_{N1,SN}^{S} \times (p^{cap} - MCP_{S})/SF_{SN,SN} \\ &\quad + t_{SN}^{max} \times (p^{cap} - MCP_{S})/SF_{SN,SN}. \end{aligned} \tag{4.16}$$

## 4.4 Market power measurement and mitigation

### 4.4.1 Conditions for exercising market power

In our model, the conditions for market power can be specified by the two conditions below.

**Condition 1** *[Balancing market manipulability] The needed balancing up* $q_N^U$ *to eliminate the intended congestion overflow should be larger than the competitive residual resource available in the balancing market. That is,*

$$q_N^U(q_{N1}^{SN}) > q_{N0}^R \tag{4.17}$$

**Condition 2** *[Profitability] The strategic profit should be larger than the base profit.*

$$\pi_{N1}^{strategic} > \pi_{N1}^{base} \tag{4.18}$$

### 4.4.2 Market power measurement index for two zone model

We derive a flow based market power measurement index set for our two zone model from the condition 1 and condition 2.

**Proposition 3** *[Flow based market power measurement index] The condition 1 and 2 is approximately equivalent to the flow based conditions below:*

$$I_{N1,SN} > I_{SN}^{max} + I_{N0,SN}^C - I_{SN}^B, \tag{4.19}$$

$$t_{SN}^{max} > I_{SN}^{max} + I_{N0,SN}^C - I_{SN}^B, \tag{4.20}$$

*where* $I_{N0,SN}^C = q_{N0}^R \times SF_{SN,SN}$. *The approximate equivalency is from the further assumptions that (1) marginal cost of* $N_1$ *between* $q_{N1}^B$ *and* $q_{N1}^B - q_{N1,SN}^S + q_N^U - q_{N0}^R$ *is approximated to a constant level, (2)* $P_{N1}^{import} \approx MCP_S$ *and (3)* $p^{cap} \gg mc_{N1}$ *and* $p^{cap} \gg MCP_S$

*Proof:*

The equivalency of the condition 1 and the condition in (4.19) can be easily proved. When we multiply $SF_{SN,SN}$ to (4.17), we have:

$$I_{SN}^O > I_{N0,SN}^C. \tag{4.21}$$

By (4.8) we can develop (4.21) to the condition in (4.19)

From (4.12) and (4.16) we have:

$$\begin{aligned}
&\pi_{N1}^{strategic} - \pi_{N1}^{base} \\
&= c_{N1}(q_{N1}^{B}) - c_{N1}(q_{N1}^{B} - q_{N1,SN}^{S} + q_{N}^{U} - q_{N0}^{R}) \\
&- p_{N1}^{import} \times q_{N1,SN}^{S} + (q_{N}^{U} - q_{N0}^{R}) \times p^{cap} \\
&- I_{N1,SN}^{S} \times (p^{cap} - MCP_{S})/SF_{SN,SN} \\
&+ t_{SN}^{max} \times (p^{cap} - MCP_{S})/SF_{SN,SN}. \qquad (4.22)
\end{aligned}$$

We approximate the production cost difference by:

$$\begin{aligned}
&c_{N1}(q_{N1}^{B}) - c_{N1}(q_{N1}^{B} - q_{N1,SN}^{S} + q_{N}^{U} - q_{N0}^{R}) \\
\approx\ &(q_{N1,SN}^{S} - q_{N}^{U} + q_{N0}^{R}) \times mc_{N1}(q_{N1}^{B} - \triangle q/2), \qquad (4.23)
\end{aligned}$$

where $\triangle q = q_{N1}^{SN} - q_{N}^{U} + q_{N0}^{R}$.

If we apply (4.23) to (4.22) and multiply $SF_{SN,SN}$ to that, we have:

$$\begin{aligned}
&SF_{SN,SN} \times (\pi_{N1}^{strategic} - \pi_{N1}^{base}) \\
&= (-I_{N1,SN}^{S} + I_{SN}^{O} - I_{N0,SN}^{C}) \times (p^{cap} - mc_{N1}) \\
&-(p_{N1}^{import} - MCP_{S}) \times I_{N1,SN}^{S} \\
&+ t_{SN}^{max} \times (p^{cap} - MCP_{S}), \qquad (4.24)
\end{aligned}$$

where $mc_{N1}{=}mc_{N1}(q_{N1}^{B} - \triangle q/2)$.

Because we assume that *South* is a competitive zone, we can approximate $p_{N1}^{import} \approx MCP_S$. We apply this to (4.24), we have:

$$SF_{SN,SN} \times (\pi_{N1}^{strategic} - \pi_{N1}^{base})$$
$$= (-I_{N1,SN}^{S} + I_{SN}^{O} - I_{N0,SN}^{C}) \times (p^{cap} - mc_{N1})$$
$$+t_{SN}^{max} \times (p^{cap} - MCP_S). \tag{4.25}$$

When we apply the condition 2 to (4.25) and further approximate it through the fact that $p^{cap} \gg mc_{N1}$ and $p^{cap} \gg MCP_S$, we have:

$$t_{SN}^{max} > I_{N1,SN}^{S} - I_{SN}^{O} + I_{N0,SN}^{C} \tag{4.26}$$

When we develop (4.26) through (4.8), we have the condition in (4.20).

■

### 4.4.3 Extension to general flow gate market models

In the case of two zone market, for example, California market [17], if there were apply the same market design as in ERCOT, the market power exercising condition can be determined by (4.19) and (4.20). However in multi-zone network type markets, for example, ERCOT, we cannot apply the conditions for our two zone model directly. For multi-zone network markets we define general flow base conditions based on our analysis of the two zone market model.

From (4.21) we know the overflow should be larger than the counter flow. In multi-zone network models overflow can occur on multiple CSCs at the same time since the strategic player's strategic import schedule impacts multiple CSCs. By the same reason the counter flow can be deployed from multiple competitive resources in multiple zones. Here we define the first condition for exercising market power in a multi-zone network model:

**Condition 3** *[Balancing market manipulability in multi-zone network models]* *The overflow set $I^O(Q^S)$ from the strategic player's strategic import set $Q^S$ should not be cleared by the counterflow set $I^C(Q^R_O)$ from the residual competitive resource set $Q^R_O$.*

When we multiply $SF_{SN,SN}$ to (4.10), we have:

$$I^W_{N1} = I^O_{SN} - I^C_{N0,SN}, \tag{4.27}$$

where $I^W_{N1} = q^W_{N1} SF_{SN,SN}$. When we apply (4.27) to (4.26), we have:

$$t^{max}_{SN} > I_{N1,SN} - I^W_{N1,SN} \tag{4.28}$$

From (4.28) we know that to exercise market power the maximum TCR ownership over the CSC should be larger than the amount after subtracting the winning bid flow from the strategic schedule flow. In the case of two zone model, the amount of $I_{N1,SN} - I^W_{N1,SN}$ is consistent as we see in (4.20). However in the case of multi-zone network models the relationship between the winning bid flow and the strategic schedule flow cannot be defined easily. For multi-zone network models we define more conservative profit condition below.

**Condition 4** *[Profitability in multi-zone network models]* *For every $CSC_k$, the strategic schedule flow $I^S_k(Q^S)$ by the strategic import set $Q^S$ should be less than the maximum TCR ownership $t^{max}_k$ in order to yield a positive profit.*

From the condition 3 and condition 4, we derive the market power measurement algorithm for multi zone network models.

**[Market power measurement algorithm]**

1. Define the basic market data (CSC limits, shift factors, competitive resource data, strategic player data)

2. Determine the "base schedule". If we assume a competitive bilateral market, solve a zonal Optimal Power Flow (OPF) based on marginal costs.

3. Maximize the overflow while the strategic schedule flow $I_k^S(Q^S)$ by the strategic import set $Q^S$ should be less than the maximum TCR ownership $t_k^{max}$ for ever $CSC_k$.

$$\max_{Q^S} \sum I_k^O \text{ constrained by } I^S < T^{max}. \tag{4.29}$$

4. If the overflow is not cleared by the competitive resource counterflow, then the market is potentially vulnerable to market power condition. To test the overflow is cleared by the competitive resource counterflow, use the inequality condition below:

$$\min_{Q_0^R} \sum max(I_k^O - I_k^C, 0) > 0. \tag{4.30}$$

   We do not consider the economic dispatch since our concern is only the overflow can be cleared by the residual competitive resources. If the inequality condition in (4.30) is satisfied, then the market is vulnerable to market power.

## 4.5 ERCOT market simulation

### 4.5.1 2003 ERCOT congestion management zones

Every year ERCOT studies the load flow cases to determine the next year's congestion management zones. In 2003 ERCOT market has four zones ($West$, $North$, $Houston$ and $South$) and three CSCs ($CSC_{WN}$, $CSC_{SN}$ and $CSC_{SH}$) [41]. We define the zone indication set and CSC indication set as $\mathcal{Z} = \{W, N, H, S\}$ and $\mathcal{F} = \{WN, SN, SH\}$ respectively. Inter zonal schedule impact on each CSC is determined by the ERCOT posted shift factors. In our simulation we use ERCOT 2003 annual shift factors in table 4.1.

Table 4.1: ERCOT 2003 annual shift factors. Source: www.ercot.com

| Zone | $CSC_{WN}$ | $CSC_{SN}$ | $CSC_{SH}$ |
|---|---|---|---|
| $West$ | 0.428484450 | 0.024988360 | 0.015584690 |
| $North$ | 0.007655688 | 0.005107820 | -0.003440607 |
| $Houston$ | 0.017975560 | 0.202391595 | -0.179045116 |
| $South$ | 0.031717639 | 0.397075141 | 0.189228026 |

Table 4.2 shows the CSC limit of each CSC.

Table 4.2: ERCOT 2003 annual CSC limits (MW). Source: www.ercot.com

| | $CSC_{WN}$ | $CSC_{SN}$ | $CSC_{SH}$ |
|---|---|---|---|
| CSC limit | 989 | 754 | 870 |

### 4.5.2 2003 ERCOT demand and supply

We use 2003 ERCOT zonal peak demand projection in table 4.3 from the annual shift factor document [41]. We also apply 90, 80 and 70 percent of the peak demand to our simulation.

Table 4.3: ERCOT 2003 zonal demand (MW) in the summer peak base load flow. Source: www.ercot.com

| | *West* | *North* | *Houston* | *South* |
|---|---|---|---|---|
| Peak demand | 3943 | 25095 | 19167 | 16123 |

For the supply model in our simulation, we use a roughly estimated supply resource in each zone. We use linear supply function for our simulation. As strategic players we chose two big suppliers ($G_{N1}$ and $G_{H1}$) in *North* and *Houston* respectively. Each zone has a competitive resource of $G_{i0}$. Table 4.4 shows each player's zonal supply function.

Table 4.4: Estimated ERCOT 2003 supply (MW)

| | | Supply function | | |
|---|---|---|---|---|
| Zone | Player | Slope ($\alpha_{ij}$) | Intercept ($\beta_{ij}$) | Max |
| West | $G_{W0}$ | 0.005831265 | 23.48262798 | 5,909 |
| North | $G_{N0}$ | 0.006579241 | 2.002403581 | 8,452 |
| North | $G_{N1}$ | 0.003314834 | 1.679567836 | 17,713 |
| Houston | $G_{H0}$ | 0.006934854 | 20.99985577 | 5,349 |
| Houston | $G_{H1}$ | 0.00438033 | 11.29391173 | 12,495 |
| South | $G_{S0}$ | 0.00219359 | 5.984843523 | 22,823 |

The study results can be affected by the data set used. Because we use a roughly estimated supply data, the study result may not be very realistic.

### 4.5.3 Base schedule

The second step in our market power measurement algorithm is to determine the base schedule. We assume that the forward bilateral market is competitive for the purposes of evaluating the forward bilateral market. The base bilateral schedule is obtained through the zonal OPF:

$$\begin{aligned} \min_{Q^B} \sum_{q_{ij}^B \in Q^B} c_{ij}(q_{ij}^B), \\ \text{const. } \mathbf{0} \le Q^B \le Q^{max}, \\ q_{W0}^B - d_W = q_W^B, \\ q_{N0}^B + q_{N1}^B - d_N = q_N^B, \\ q_{H0}^B + q_{H1}^B - d_H = q_H^B, \\ q_{S0}^B - d_S = q_S^B, \\ \sum_{q_{ij} \in Q^B} q_{ij}^B - \sum_{d_i \in D} d_i = 0, \\ I^B \le I^{max}, \end{aligned} \tag{4.31}$$

where $Q^B$ is the set of each player's bilateral schedule, $Q^{max}$ is the maximum capacity in table 4.4, $c_{ij}$ is each player's cost function:

$$c_{ij} = \alpha_{ij} q_{ij}^B / 2 + \beta_{ij} q_{ij}^B \tag{4.32}$$

defined by table 4.4, $D$ is the set of zonal demand in table 4.3, $I^{max}$ is the set of each CSC limit in table 4.2, and $I_B$ is the set of each CSC's aggregated inter zonal flow $I_{ii'}^B$ equal to:

$$q_W^B SF_{W,ii'} + q_N^B SF_{N,ii'} + q_H^B SF_{H,ii'} + q_S^B SF_{S,ii'}. \tag{4.33}$$

Solving (4.31) we get each player's base schedule and the inter zonal flow on every CSC. Table 4.5 shows each player's bilateral schedule for each demand level (peak, 90%, 80%, 70%).

Table 4.5: Base bilateral schedule (MW)

| | Peak | 90% | 80% | 70% |
|---|---|---|---|---|
| $q^B_{W0}$ | 5169 | 4350 | 3592 | 2834 |
| $q^B_{N0}$ | 8452 | 7217 | 6531 | 5846 |
| $q^B_{N1}$ | 17713 | 14421 | 13061 | 11701 |
| $q^B_{H0}$ | 5349 | 4951 | 4182 | 3412 |
| $q^B_{H1}$ | 12495 | 10055 | 8836 | 7618 |
| $q^B_{S0}$ | 22823 | 16902 | 15261 | 13619 |

Table 4.6 shows each CSC's inter zonal flow for the base schedule in each demand level. The flows on $CSC_{SH}$ are at limits for each demand. The flows on the other CSCs are below limits for each demand.

Table 4.6: Base schedule inter zonal flow (MW)

| | Peak | 90% | 80% | 70% |
|---|---|---|---|---|
| $I^B_{WN}$ | 547 | 371 | 217 | 63 |
| $I^B_{SN}$ | 470 | 510 | 478 | 445 |
| $I^B_{SH}$ | 870 | 870 | 870 | 870 |

### 4.5.4 Market power measurement for North player

The first and second step in our market power measurement algorithm in 4.4.3 are the common steps for alternative strategic players. The third step of market power measurement for the North zone strategic player $G_{N1}$ is to solve the overflow maximization problem in (4.29) given the base schedule in 4.5 and 4.6. $Q^S$ for $G_{N1}$ in (4.29) is $\{q^S_{WN}, q^S_{HN}, q^S_{SN}\}$. Table 4.7 shows the solutions of $G_{N1}$'s overflow maximization problem for significant scenarios for the market power issue.

Table 4.7: Results of $G_{N1}$'s overflow maximization

| | | $Q^S$ (MW) | | | $I^O$ (MW) | | |
|---|---|---|---|---|---|---|---|
| Demand | $T^{max}$ | $q^S_{WN}$ | $q^S_{HN}$ | $q^S_{SN}$ | $I^O_{WN}$ | $I^O_{SN}$ | $I^O_{SH}$ |
| Peak | 100% | 0 | 0 | 1,924 | 0 | 470 | 371 |
| 90% | 100% | 0 | 0 | 1,924 | 0 | 510 | 371 |
| 80% | 100% | 0 | 0 | 1,924 | 0 | 478 | 371 |
| 70% | 100% | 0 | 0 | 1,924 | 0 | 445 | 371 |

The *South* to *North* inter zonal schedule $q^S_{SN}$ is used for overflow maximization. Given the scenarios of strategic schedules in table 4.7 we apply the fourth step in our algorithm. In the market power measurement case of $G_{N1}$ all other players' resources are considered as the competitive resource. That is, $Q^R_0 = \{q^R_{W0}, q^R_{N0}, q^R_{H0}, q^R_{H1}, q^R_{S0}\}$. We solve the overflow clearing problem in (4.30) given the overflow in table 4.7. The balancing up and down limit of each competitive resource is given by $q^{R_{max}}_{ij}$=$q^{max}_{ij}$ - $q^{B'}_{ij}$ and $q^{R_{min}}_{ij}$=$q^{B'}_{ij}$ respectively, where $B'$ represents the strategic schedule reflected bilateral schedule from table 4.5 and 4.7.

Table 4.8 shows each zonal balancing up, down and the sum of overflow $\sum I^{O'}_k$ after applying the overflow clearing in (4.30).

From table 4.8 we find that $G_{N1}$ will have no chance of market power exercising by itself although 100 % transmission ownership is allowed since overflow is zero in each case.

Table 4.8: Results after applying the fourth step for $G_{N1}$'s case

| Demand | $T^{max}$ | Balancing Energy West | North | Houston | South | $\sum I_k^{O'}$ |
|---|---|---|---|---|---|---|
| Peak | 100% | 549 | 381 | 598 | -1528 | 0 |
| 90% | 100% | 672 | 438 | 460 | -1570 | 0 |
| 80% | 100% | 570 | 381 | 598 | -1549 | 0 |
| 70% | 100% | 482 | 381 | 598 | -1461 | 0 |

### 4.5.5 Market power measurement for Houston player

We apply the third step in our algorithm for $G_{H1}$. $Q^S$ for $G_{H1}$ in (4.29) is $\{q^S_{WH}, q^S_{NH}, q^S_{SH}\}$. Table 4.9 shows the solutions of $G_{H1}$'s overflow maximization problem for significant scenarios for the market power issue.

Table 4.9: Results of $G_{H1}$'s overflow maximization

| | | $Q^S$ (MW) | | | $I^O$ (MW) | | |
|---|---|---|---|---|---|---|---|
| Demand | $T^{max}$ | $q^S_{WH}$ | $q^S_{NH}$ | $q^S_{SH}$ | $I^O_{WN}$ | $I^O_{SN}$ | $I^O_{SH}$ |
| Peak | 50% | 0 | 0 | 1,181 | 0 | 0 | 435 |
| Peak | 75% | 0 | 0 | 1,772 | 0 | 61 | 652 |
| 90% | 100% | 0 | 0 | 2,362 | 0 | 216 | 870 |
| 80% | 100% | 0 | 0 | 2,362 | 0 | 184 | 870 |
| 70% | 100% | 0 | 0 | 2,362 | 0 | 151 | 870 |

The *South* to *Houston* inter zonal schedule $q^S_{SH}$ is used for overflow maximization in this case. Given the scenarios of strategic schedules in table 4.9 we apply the fourth step in our algorithm. In the market power measurement case of $G_{H1}$, $Q^R_0 = \{q^R_{W0}, q^R_{N0}, q^R_{N1}, q^R_{H0}, q^R_{S0}\}$. We solve the overflow clearing problem in (4.30) given the overflow in table 4.9. Table 4.10 shows each zonal balancing up and down and the result of overflow after applying the counterflow by (4.30) for each scenario.

Table 4.10: Results after applying the fourth step for $G_{H1}$'s case

| Demand | $T^{max}$ | Balancing Energy | | | | $\sum I_k^{O'}$ |
|---|---|---|---|---|---|---|
| | | West | North | Houston | South | |
| Peak | 50% | 740 | 1591 | 0 | -2331 | 0 |
| Peak | 75% | 740 | 2288 | 0 | -3028 | 83 |
| 90% | 100% | 1071 | 2790 | 398 | -4259 | 0 |
| 80% | 100% | 1424 | 1002 | 1167 | -3593 | 0 |
| 70% | 100% | 515 | 349 | 1937 | -2801 | 0 |

From table 4.10 we find that $G_{H1}$ can exercise its market power by itself if 75% transmission ownership limit is allowed during the peak demand period. On the other hand, a regulatory body can put 50% transmission ownership limitation to prevent market power exercise by $G_{H1}$ during the peak demand.

### 4.5.6 Market power measurement for collusive case

We also considered the collusive cases of $G_{N1}$ and $G_{H1}$. Table 4.11 shows the combined effects of the two players' overflow maximization.

The *South* to *Houston* inter zonal schedule $q^S_{SH}$ by $G_{H1}$ and the *South* to *North* inter zonal schedule $q^S_{SN}$ by $G_{N1}$ are considered at the same time. Given the scenarios of strategic schedules in table 4.11 we apply the fourth step in our algorithm. In this case $Q^R_0 = \{q^R_{W0}, q^R_{N0}, q^R_{H0}, q^R_{S0}\}$. We solve the overflow clearing problem in (4.30) given the overflow in table 4.11. Table 4.12 shows each zonal balancing up and down and the overflow after applying for (4.30) for each scenario.

Table 4.11: Results of overflow maximization in the collusive case

| | | $Q^S$ (MW) | | $I^O$ (MW) | | |
|---|---|---|---|---|---|---|
| Demand | $T^{max}$ | $q^S_{SN}$ | $q^S_{SH}$ | $I^O_{WN}$ | $I^O_{SN}$ | $I^O_{SH}$ |
| Peak | 15% | 289 | 354 | 0 | 0 | 186 |
| Peak | 20% | 385 | 472 | 0 | 0 | 248 |
| 90% | 25% | 481 | 591 | 0 | 19 | 310 |
| 90% | 50% | 962 | 1,181 | 0 | 323 | 620 |
| 80% | 75% | 1,443 | 1,772 | 0 | 626 | 930 |
| 80% | 100% | 1,924 | 2,362 | 0 | 930 | 1241 |
| 70% | 100% | 1,924 | 2,362 | 0 | 915 | 1241 |

Table 4.12: Results after applying the fourth step for the collusive case

| | | Balancing Energy | | | | |
|---|---|---|---|---|---|---|
| Demand | $T^{max}$ | West | North | Houston | South | $\sum I^{O'}_k$ |
| Peak | 15% | 670 | 362 | 0 | -1032 | 0 |
| Peak | 20% | 740 | 485 | 0 | -1225 | 26 |
| 90% | 25% | 493 | 406 | 398 | -1296 | 0 |
| 90% | 50% | 1103 | 1235 | 398 | -2736 | 44 |
| 80% | 75% | 752 | 1921 | 1167 | -3840 | 0 |
| 80% | 100% | 1072 | 1921 | 1167 | -4160 | 255 |
| 70% | 100% | 146 | 2606 | 1937 | -4688 | 0 |

From table 4.12 we find that regulatory body can prevent the ERCOT market being exposed to the collusive market power exercise setting the transmission ownership limit 15%, 25% and 75% for the peak demand period, 90% and 80%, respectively. For the 70% peak demand, the collusive players can not find any chance of market power exercise in any scenario.

## 4.6 Chapter conclusion

We introduced the strategic coordination of inter zonal scheduling and ISO realtime balancing market manipulation in ERCOT market. We modelled a two zone model for analytical purposes following the basic structure of ERCOT market including bilateral schedule, ISO balancing market for congestion management and the settlement of congestion cost and TCR. We modelled the strategic player's resultant profit structure from its strategic coordination for the two zone model. We derived the relationship between TCR ownership limitation and market power exercise conditions from the strategic player's profit structure.

We suggested a market power measurement algorithm for general flow gate market models extending the analytical result from the two zone model to general flow gate market models. We simulated ERCOT 2003 market using the algorithm. We tested multiple demand level scenarios with two big players. We showed the levels of TCR ownership limitation to mitigate market power for each player and collusive case during the multiple demand levels.

# Chapter 5

# Conclusion

Three research topics are studied in chapter 2, 3 and 4, respectively:

1. Hybrid Co-evolutionary Programming for Nash Equilibrium Search in Games with Local Optima,

2. Short Term Electricity Market Auction Game Analysis: Uniform and Pay-as-bid pricing,

3. Market Power Mitigation by Limiting Transmission Congestion Right in ERCOT market.

In chapter 2 we addressed conventional Nash Equilibrium search algorithms in games with local optima can misidentify NE by following local optimization path. We proved that any iterative NE search algorithms based on local optimization cannot differentiate real NE and "local NE trap". Co-evolutionary programming, a parallel and global search algorithm, was applied to overcome this problem. In order to enhance the poor convergence of simple co-evolutionary programming, hybrid co-evolutionary programming was suggested. The conventional NE algorithms, simple co-evolutionary programming and hybrid co-evolutionary algorithms are tested through a simple numerical example and transmission constrained electricity market examples.

In chapter 3 we analyzed the competing pricing mechanisms of uniform and pay-as-bid pricing in an electricity market. Game theory and auction theory were adopted to analyze the strategic behavior of a big player and a small player in an short term auction game. We proved that for a two-player static game the Nash Equilibrium under pay-as-bid pricing would yield less total revenue in expectation than under uniform pricing when demand was inelastic. To confirm this theoretical result we simulated the model using a mixed strategy NE solver. We also extended the model to an elastic demand case and showed that pay-as-bid pricing also led to a larger expected total demand being served when demand was elastic.

In chapter 4 we addressed a market power mitigation issue of the current Texas electricity market by limiting TCR (Transmission Congestion Right) ownership. We analyzed the strategic coordination of inter zonal scheduling and balancing market manipulation. We found the relationship between the level of TCR ownership limitation and the strategic player's market power exercise condition. Extending the two zone model for the analysis to general flow gate market models we suggested a market power measurement algorithm useful to determine the proper level of TCR ownership limitation given a market scenario. We showed the practicality of the algorithm simulating ERCOT (Electricity Reliability Council of Texas) 2003 market.

Future research topics are:

**Application** The suggested hybrid co-evolutionary programming in chapter 2 can be applicable to other complex form games.

**Alternative algorithm** As an alternative approach of chapter 2, a heuristic global optimum search algorithm [42] will be tried to overcome "NE traps" in games with local optima.

**Model extension** The basic framework in chapter 3 can be extended to examine the two competing pricing mechanism in electricity markets with transmission constraints.

**Model generalization** We conjecture that the main reasoning and intuition in chapter 3 might be applied to general multi-player auction games with more than two players. Further analytic work and complex game simulation might be needed.

**Regulatory usage** The market power measurement algorithm in chapter 4 can be applied to ERCOT market for the purpose of regulatory usage. More realistic data can be used.

# Appendix A

# Example of TCR ownership purchase cost impact

If a strategic player can purchase TCR ownership only near the real time and does not treat it as a sunk cost, then the strategic player may include the purchase cost in his profit consideration for the target interval. In this case, the TCR ownership purchase cost might affect the player's strategy. In order to show the effect of purchase cost on the strategic player's behavior, we set up an example. We use the two node model from 4.2. We use the same base model structure as in section 4.2.1. North player will consider the exercise of market power given following values:

- bilateral contract: $q^B_{N1} = 500$ MW, $p^B_{N1} = 30$\$/MW
- cost function: $c_{N1}(q) = 0.005q^2 + 25q$
- transmission: $I^{max}_{SN} = 200$ MW, $SF_{SN} = 1.0$, $I^B_{SN} = 200MW$
- import price: $p^{import}_{N1} = 25$ \$/MW (abundant resource at the price of 25 \$/MW in South)

- competitive resource: $q_{N0}^{R}$= 50 MW
- strategic schedule: $q_{N1,SN}^{S}$= 60 MW
- balancing market: $q_{N1}^{W}$ = $q_{N0}^{R}$ - $q_{N1,SN}^{S}$ = 10 MW, $MCP_N = P_{cap} = 1000$ \$/MW, $MCP_S = p_{N1}^{import} = 25$ \$/MW
- TCR ownership limit: $t_{N1,SN} = 100$ MW (limited 50% of $I_{SN}^{max}$)
- TCR purchase cost: $\eta_{N1,SN}$

The profit with "base schedule" $\pi_{N1}^{base}$ can be calculated by (4.12). When the player includes the TCR purchase cost $\eta_{N1,SN}$, the profit with "strategic schedule" in (4.13) $\pi_{N1}^{stragegic}$ is changed to:

$$\begin{aligned}
\pi_{N1}^{strategic} &= p_{N1}^{B} \times q_{N1}^{B} - c_{N1}(q_{N1}^{B} - q_{N1,SN}^{S} + q_{N1}^{W}) \\
&- p_{N1}^{import} \times q_{N1,SN}^{S} + q_{N1}^{W} \times MCP_N \\
&- I_{N1,SN}^{S} \times SP_{SN} + t_{N1,SN} \times SP_{SN} \\
&- t_{N1,SN} \times \eta_{N1,SN} \qquad \text{(A.1)}
\end{aligned}$$

The profits of both cases for varying TCR purchase costs are illustrated in table A.1.

When the TCR price is relatively cheap, the profit with "strategic scheduling" is much larger than the profit with "base schedule". When the TCR price is higher than 485 \$/MW, the strategic player will lose incentive for exercise of market power. 485 \$/MW is a very expensive price. The bilateral price of energy is only 25 \$/MW in South and 30 \$/MW in North. However in reality, market TCR price will not

Table A.1: Profits with varying TCR purchase costs

| $\eta_{N1,SN}$ (\$/MW) | $\pi_{N1}^{base}$ (\$) | $\pi_{N1}^{stragegic}$ (\$) |
|---|---|---|
| 5 | 1250 | 49737.5 |
| 10 | 1250 | 49237.5 |
| 15 | 1250 | 48737.5 |
| 20 | 1250 | 48237.5 |
| 480 | 1250 | 2237.5 |
| 485 | 1250 | 1737.5 |
| 490 | 1250 | 1237.5 |
| 495 | 1250 | 737.5 |

go up to that high level unless everybody in the market is sure of an extremely high $MCP_N$. For example, the market clearing prices for 2003 TCR annual auction are \$0.18/MW for West-North, \$1.7/MW for South-North and \$3.10/MW South-Houston, respectively [41]. If the TCR price is really high, the strategic player will not purchase TCR and will not exercise market power [36]. Consequently the value of TCR will drop.

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