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Essays on Dynamic Contracts: Allocation of Ambiguity and Delegation

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Essays on Dynamic Contracts: Allocation of Ambiguity and Delegation

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DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2016

Dedicated to Kolkata, my city.

Acknowledgments

First of all, I would like to thank my advisor, Maxwell Stinchcombe for his invaluable guidance and support, without which this journey would not have been possible. He has been a great mentor, teacher and a constant source of inspiration. With him as my advisor, my graduate life has been an enriching experience.

I would like to thank Dale Stahl for his thorough guidance, encouragement, and advice in both professional and personal sphere.

I want to take this opportunity to express my gratitude to Svetlana Boyarchenko and Caroline Thomas for their kind help in perfecting my job market preparation. Thanks to Andres Almazan, V. Bhaskar, Mark Feldman, David Sibley, and Thomas Wiseman for their insightful comments and feedback. I also wish to thank Vivian Goldman-Leffler for being the most helpful and friendly graduate coordinator.

I would like to thank Piin-Hueih Chiang, Can Cui, Urmee Khan, Anna Klis, Xi Qin, Liangfei Qiu, Ina Taneva, Ahmet Ali Taskin, Megan Thomas, and the UT Economics theory group for the intellectual as well as social fellowship. I am also grateful to all my friends, especially the ones in Austin: Shrijata Chattopadhyay, Anamika Dubey, and Debarati Kundu, who were always there to cheer for my success, or to provide a shoulder to cry on.

I wish to thank my school, Rama Krishna Sarada Mission Sister Nivedita Girls' School, Kolkata, India for inspiring me to dream. I have been fortunate to be taught by exceptional teachers in school and college who encouraged me in my pursuit of an academic career and above all, inspired me to be a good human being.

Finally, I would like to thank my parents, whose unconditional love and unrelenting faith in me kept me going in the moments of self-doubt. I want to share my success with my sister Satarupa Bhattacharjee, who, despite being the younger one, was always the wiser. A special thanks to my husband, Dr. Shraman Banerjee, whose academic and emotional support sustained me throughout the years.

Essays on Dynamic Contracts:
Allocation of Ambiguity and Delegation

Publication No. ____________________

Swagata Bhattacharjee, Ph.D.
The University of Texas at Austin, 2016

Supervisor: Maxwell B. Stinchcombe

My dissertation studies the design of contracts in different contexts. It contains two theoretical investigations about contracting under ambiguity: in the context of research partnerships and venture capital financing; and an experimental study to examine delegation of decision rights within organizations.

The first chapter studies contract design for innovation under ambiguity. Outsourcing of research is a large and growing trend in knowledge-intensive industries such as the biotechnology and software industries. I model innovation as an ambiguous stochastic process and assume that the commercial firms and research labs differ in their attitude towards ambiguity. I characterize the optimal sequence of short-term contracts and examine how the features of this contract facilitate ambiguity sharing: the dynamic moral hazard problem is mitigated under ambiguity; experimentation stops earlier than is socially optimal; the project may be liquidated even after being granted a patent. I find that redesigning the patent law can not implement the Policymaker's desired optimum.

The second chapter analyzes venture capital investment under ambiguity. A central feature of venture capital financing is the extensive use of control rights as an instrument. In this chapter, I present a model of venture capital financing where investment is allowed to depend on an intermediate ambiguous signal. I show how the presence of ambiguity explains the allocation of control rights if the investor is more ambiguity averse than the entrepreneur.

In the third chapter, I discuss how delegation of decision rights can be used as a signal of trust that can be reciprocated by cooperation. First, I theoretically show that in a principal-agent framework, using delegation as a signal is the only Perfect Bayesian Equilibrium that survives forward induction criterion. Then I use experimental methods to test this theoretical prediction. I find that the players do not use delegation very often, thus the forward induction logic is not supported by the observed data. However, once the players are given information about the past sessions, they choose the forward induction equilibrium more often. This suggests that information affects the formation of beliefs and equilibrium selection in Bayesian games.

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Chapter 1

Contracting for Innovation Under Ambiguity

1.1 Introduction

Outsourcing of research is a growing and prevalent trend in knowledge intensive sectors (*e.g.* Biotechnology, Information Technology, and Software sectors). In these industries, big commercial firms often outsource their research to smaller research oriented firms. These inter-organizational research alliances are generally voluntary agreements between firms involving exchange, sharing or co-development of products, technologies, or services, and play an important role in organizing R&D in the innovation-intensive industries. For example, in Biotechnology sector, 650 new alliances formed in 2006 alone, with related financial commitments of over $90 billion [56]. During 1996-2007, the industry-university strategic partnerships alone resulted in $457.1 billion worth of patented innovations [133]. In Pharmaceutical industry, more than 70% of the U.S. companies are involved in research partnerships, and each year on average 25% of the 26bn industry-financed R&D is invested in research alliances [1]. Information technology sector, accounting for 37% of all strategic research partnerships, registered 254 technology agreements in the year 1996 alone([120], [82]). This chapter studies these research partnerships and evaluates them as modes of organizing research.

In the context of innovation, the projects in question are unique in nature. So, sufficient amount of data from very similar situations are generally not available to form a reliable estimate of the true profitability of the project. Thus, it is often difficult to form a unique single-valued probability measure about the profitability. Such situations can be modeled as "Knightian Uncertainty," or, "Ambiguity," using Knight's definition [103]:

> "The practical difference between the two categories, risk and uncertainty, is that in the former the distribution of the outcome in a group of instances is known (either through calculation a priori or from statistics of past experience), while in the case of uncertainty this is not true, the reason being in general that it is impossible to form a group of instances, because the situation dealt with is in a high degree unique."

In innovation contexts, then, we can assume that the researching entities know only a partial description of the underlying probability distribution associated with the choices. Here innovation is modelled as a stochastic ambiguous process, with the research labs, specialized in dealing with ambiguity are less ambiguity averse than the commercial firms. The strategic partnerships between the commercial firms and the research firms aim to exploit the gains from this specialization to deal with ambiguity.

Given the importance of research alliances in innovation-based industries as demonstrated above, it is important to examine how these alliances optimally organize R&D. To this end, this chapter provides a theoretical model to analyze the strategic partnerships carrying out innovation in ambiguous environment. The main focus is on the dynamic contracts that govern these partnerships.

The questions that we can address in the present framework are: what is the optimal sequence of short term contracts governing innovation in these strategic partnerships? How does the optimal investment in the project evolve over time? When does the research alliance stop experimenting? Assuming that the Policymaker is a risk and ambiguity neutral entity and cares only for the payoffs the project generates, we analyze how the Policymaker sets the Patent Law. Then, the natural question is: how does the optimal contractual outcome in the strategic partnerships compare to the Policymaker's desired optimal outcome? Also, is it possible to re-design the patent laws so as to implement the Policymaker's desired optima?

We consider a dynamic principal-agent framework to address these questions. In particular, we examine a sequence of short term contracts where the contractees differ in their attitude towards ambiguity.

We characterize the optimal sequence of short term contracts conducting the innovation, and show how the contractual terms facilitate ambiguity sharing. However, the contractual optimal outcome diverges from the desired outcome by the Policymaker: the strategic alliance stops experimenting earlier than the Policymaker deems optimal, sometimes liquidates the project even after being granted a patent, also invests less in the project. We can show that it is never possible to implement the Policymaker's optima by restructuring the patent law.

The chapter is organized as follows. In the next section I include some examples of strategic partnerships. The following section reviews the existing body of literature related to the questions addressed in this chapter. Section 4 develops the model and analyzes Policymaker's Optimum. In Section 5 I characterize the contractual optimum. Section 6 provides a comparison between the contractual outcome and the Policymaker's optima and discusses the policy implications of the results of this chapter. In section 7 I consider some generalizations and robustness checks of the model. Section 8 reflects on the general implications of the results. The last section summarizes the findings of this chapter and concludes.

1.2 Motivating Examples

The contracts within the research partnerships take a special form: they are generally of short duration, designed to overcome the problems that may arise in inter-organizational collaborations and use a mix of explicit (legally enforceable) and implicit (legally unenforceable, *e.g.* , allocation of decision rights, property rights, etc.) terms (Gilson et al., 2003). In this subsection, we will study a contracts governing a research partnership. From this case study, we make note of the contractual features, so that in the theoretical model, we can retain these properties and show how they help organizing research in this context.

1.2.1 *Example 1: Warner- Lambert-Ligand agreement*

Let us examine the "Warner- Lambert-Ligand agreement" (September 1, 1999): a research, development, and license agreement between Warner-Lambert, a large pharmaceutical company, and Ligand Pharmaceuticals, a much smaller biotech company.

The Warner-Lambert-Ligand partnership was engaged in directed research to discover and design small-molecule compounds that act through the estrogen receptors, to develop those compounds into pharmaceutical products, and to take those products through the FDA approval process and through commercialization [140]. They started off with almost 10,000 compounds, out of which only 250 compounds reached the pre-clinical stage[1]. During the research stage, Ligand engaged in directed

[1]During the drug-development process, the initial screening of compounds and pre-clinical work takes, on average, three to six years. During that period, the number of compounds under consid-

research, with Warner-Lambert providing the bulk of the funding[2]. The research stage consisted of three periods with duration of fifteen months to three years, after each of the periods Warner-Lambert had the option of unilaterally abandoning the project with little or no direct cost.

Once a successful compound was identified, the project moved from the research to the development stage, and regulatory and market experience became more important. The cost of the project, all of which will be borne by Warner-Lambert, also increased exponentially. As a result, both responsibility and decision making shifted to Warner-Lambert, who had the option to develop the project[3].

The gap between contract formation and the appearance of a marketable drug was more than a decade. So, Ligand's compensation was carefully structured. First, it was paid for some fraction of the resources assigned to the task. Second, the agreement established a number of specific *milestones*, and, upon reaching each

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eration is winnowed from 5,000-10,000 down to a quite small number through scientific and animal testing. At that point, an application for an Investigational New Drug is filed with the FDA. If the FDA approves, the drug can move to clinical testing on humans. Clinical testing takes another six to seven years. If the drug surmounts these hurdles, the sponsoring company submits a New Drug Application (NDA) with supporting documentation. FDA review of the NDA can take another six months to two years. If the FDA approves, the drug can be brought to market. Estimates are that out of 5,000 to 10,000 compounds, only 250 enter pre-clinical testing, and only about twenty percent of drugs that begin phase one testing are ultimately approved by the FDA. Only upon approval does the pharmaceutical company discover whether the drug will be successful commercially.

[2] If the project ultimately succeeds, only a small fraction of costs would be associated with the research phase. The major costs of bringing a drug to market are incurred in the later stages, in which the manufacturer must prove efficacy and safety through clinical studies in the FDA approval process.

[3] In the contract, Warner-Lambert promises to "use diligent efforts to pursue the Clinical Development and commercialization of each Collaboration Lead Compound at its own expense;" however, it "shall have the sole discretion to determine (a) which Products to develop or market or to continue to develop or market, (b) which Products to seek regulatory approval for, and (c) when and where and how and on what terms and conditions, to market such Products in the Territory."

milestone, Ligand received an additional payment. Finally, after the research produced marketable products, Ligand received royalty payments on sales. However, if Warner-Lambert chose to abort the project at any time, they retained the property rights.

This example illustrates the unique features of a typical contract governing a strategic alliance that operates in an innovation-intensive industry. Our model retains these features as well.

1.2.2 *Modelling the Dynamic Contracts:*

- *Short Term Contracting:* In Warner-Lambert-Ligand agreement, each contracting phase lasted for fifteen months up to three years, whereas the partnership lasted for more than a decade. Likewise, many of the collaborative R&D ventures are governed by short term contracts, with the contracting terms being renegotiated after every contracting phase. This chapter studies the optimal sequence of short term contracts with the contractees having no commitment power.

- *Rich forms of collaborating:* The Warner-Lambert agreement, containing rich braiding of explicit and implicit terms, shows that often the contracts governing innovation process are quite complex in structure. On one hand there is an elaborate description of the payments under various possible contingencies (*e.g.* , the milestone bonuses, the royalty rate), which are legally enforceable. On the other hand, the contract specifies the control rights and property rights, which gives unilateral decision power to one of the contracting parties. To mimic this interesting blend of explicit and implicit contracting terms, the present model assumes a contract structure containing both the state contingent payment structure and the movement of unilateral decision power.

- *Learning about the Project's Prospects:* The project started off with almost 10000 possible candidates for the molecule to be developed into a commercial drug. Only through a series of experiments the true potential of the project is learned. At each contracting phase, Ligand conducts a series of experiments on a particular subset of molecules, at the end of which a report summarizes the results: if there is a molecule fit to be taken to the clinical trials. The present model considers innovation as a learning process, where at the end of each period, a binary signal is publicly realized which contains information about the true state of the project.

- *Moral Hazard:* In the R&D conducted by Warner-Lambert partnership, the public signal depends on the resources devoted to the project. For example, if Ligand does not carry out the experiments using the expensive laboratory testing procedure, and instead, to save time and money, uses some cheaper and unreliable methods of testing, then it is unlikely that they will find a molecule suitable for clinical trial among the subset of molecules to be tested at that period. This possible diversion of resources to cross-subsidize other projects or used for personal gain underlines the existing moral hazard concern in this context. Since Warner-Lambert cannot perfectly monitor Ligand's activity, such cross-subsidization possibility gives rise to potential moral hazard problem in the contractual relationship.

In the dynamic relationship between the two firms, the moral hazard problem is more severe. Apart from the one-time gain by diverting resources, the researching party can also appropriate a dynamic gain from diversion. Once Ligand diverts resources, the test results turn out to be negative. Observing this public signal, Warner-Lambert's perception about the project's profitability changes accordingly. However, Ligand, who privately observed its own action, disregards this signal as it contains no information. Thus, following a diversion of resources, the learning paths for the two firms diverge. Warner-Lambert, who could not observe the diversion, updates its beliefs about the project's prospects differently than Ligand. Hence Ligand evaluates the next period's contracting terms using a different, and more optimistic, belief. This gives rise to a further incentive to cheat and is referred to in the literature as the "dynamic moral hazard" problem. In this model we consider dynamic contracting environment, so dynamic moral hazard problem arises here.

- *Innovation as an Ambiguous Process:* Finally, we discuss why the innovation activity carried out in Warner-Lambert agreement can be considered an ambiguous, rather than risky process.

  In the strategic partnership between Warner-Lambert and Ligand, the research could have ended in one of the three possible ways:

  (a) They could have found a molecule which passes all the clinical trials and is found fit to be developed into a drug. This can be modeled as the case when the true state (or, profitability) of the project is "*Good.*"

(b) They could have failed to find a suitable molecule even after testing all the candidate molecules. This case can be modeled as the true state being "*Bad.*"

(c) Apart from these two states, the research could have ended in finding a molecule which is capable to work through the estrogen receptors, but, given the state of the present pharmaceutical technology, can not be developed into a drug. If the research finds such a molecule, it is not presently known if in the future the pharmaceutical technology will ever improve and the molecule can be developed into a drug. So, in this case, even after conducting the decade-long research, we stumble upon an "Open question." We model this case as a new epistemic state and call it "*Unknowable*" or "*Amalgamated,*" because if the research ends up here, the true profitability of the project is simply not known.

We follow the ambiguity framework developed in [54], which shows that this new state captures the idea of ambiguity. It can be considered as an alternative interpretation of the multiple prior model. Appendix B contains the preliminaries of this framework.

In the present model, the binary signal observed at each contracting term reveals information about the true state, which can be "*Good,*" "*Bad,*" or, "*Unknowable.*" For example, if at any period, Ligand finds that a molecule among the ones being tested is suitable for conducting clinical trial, that may indicate that it is more likely that the true state is "*Good*" or "*Unknowable,*" rather than "*Bad.*" We also assume that Ligand, being a research firm, prefers this "*Unknowable*" state more than Warner-Lambert. For Ligand, this presents an opportunity to work on developing new pharmaceutical technology which might earn them revenue in future, but for Warner-Lambert, reaching the "*Unknowable*" state does not generate any immediate payoff.

Let us look at another example to illustrate the interpretation of ambiguity we will deal with in this chapter.

1.2.3 *Example 2: Cancer Genome Anatomy Project (CGAP)*

Cancer Genome Sequencing refers to the laboratory method of characterization and identification of genetic sequencing of cancer cells. Funded in 1997, the Cancer Genome Anatomy Project (CGAP) published their first Cancer Genome Sequencing report in 2003, which enables identification and characterization of all the genetic and epigenetic mutational changes that happen in the process of tumorigenesis. Before the CGS, such an exhaustive list of all possible variants of cancer cells was not available, thus different variants and subtypes of cancer were not identified (Cancer Genome Sequencing Report, 2003).

Now, let us consider a Biotechnology research venture aiming to find a medicine to treat Acute Myeloid Leukemia (AML), a particular type of cancer, before this CGS report was made available . The CGS identified several new subtypes of variants of carcinogenic mutational changes associated with AML. Before CGS, then, the research could have ended in one of the three states:

(a) The research venture could have found a medicine which can treat one of the already identified subtype of carcinogenic cells, which can be considered as the case when the true state (or, profitability) of the project is "*Good*."

(b) The project could have ended in discovering that the medicine is not even biologically active on the epigenetic mutational changes. This case can be identified with the true state being "*Bad*."

(c) The research could have found a medicine which is biologically active, but can not treat any identified variant of cancer. However, it could have been possible that there are epigenetic changes in cancer cells which are not yet identified (before CGS), and the medicine might be useful to treat those not-yet-identified variants. This state can be considered as the "*Unknowable*" or "*Amalgamated*" state, where the true profitability of the research venture is yet unknown[4].

Thus, from the two examples, it can be seen that in the innovation-intensive sectors, we can consider a new epistemic state: "Unknowable," which captures the idea that the true probability distribution associated with the choices may not be completely known, so innovation can be considered to be an ambiguous process. This chapter provides a model of how these research alliances operate under ambiguity and examines the contractual structures that govern these inter-organizational research partnerships.

Specifically, we consider innovation to be an ambiguous process where investing in research every period generates informative signals which enable the research-

[4]Indeed, much later, after the CGS report was available, targeted drugs like vemurafenib (ZELBORAF®) were discovered (approved by the Food and Drug Administration (FDA) in 2011) for the treatment of some specific mutation in the BRAF gene as detected by an FDA-approved test using CGS.

ing parties to learn about the true nature of the project. This process is organized in a research alliance through a sequence of short term contracts with both explicit and implicit contracting terms, which take care of the existing moral hazard problem. In this set up, we characterize the optimal contract, analyze its properties, and show how this research alliances fail to implement the Policymaker's desired optima.

1.3 Related Literature

This chapter is primarily related to the literature discussing *Optimal Contracts for Innovation.* It is most closely related to the seminal work by Bergemann and Hege ([16],[17]), which characterize the optimal contract for experimentation under risk. These two papers model innovation as a risky stochastic optimal stopping time problem, where an entrepreneur and a capitalist invest funds every period to learn about the project's true profitability and if the project succeeds, the game ends immediately. In this framework, the authors document the potential dynamic moral hazard problem and how it makes the funding conditions more stringent in the earlier rounds. In their setting, they find the possibility of in-equilibrium delay of funding (in a finite horizon version of the game) and in the infinite horizon, they find that the investment volume may increase over time. Hörner and Samuelson [95] examine a similar framework of experimentation and characterize all possible equilibria.

There are two significant differences between these papers and ours. Firstly, here we consider innovation as an ambiguous process, rather than a risky one. Thus, the central problem of this chapter is the characterization of the optimal contract in presence of ambiguity. We show that the introduction of ambiguity and the different attitudes towards ambiguity among the contractees alleviate the dynamic moral hazard problem, preventing in-equilibrium delay in funding in the finite horizon case, and in the infinite horizon this leads to a monotonically decreasing level of investment. Also, in the current chapter we model innovation as a two stage game, where at the first stage, in each period the firms experiment to observe an informative binary signal, and depending on the signal realization, may enter the development stage, where the true quality of the project is finally revealed. This modelling framework with non-conclusive signals gives rise to a positive option value of waiting and changes the optimal contract structure. It illustrates the role of patent laws, which enables us to analyze the role of government policies in innovation.

Bonatti and Horner [26], and Campbell *et al.* [32] study experimentation in teams with unobservable actions and they also find the possibility of delay. In a two period model with the principal having the commitment power, Manso [113], and Ederer and Manso [55], show that the contracts that foster experimentation greatly differ from the standard pay-for-performance contracts. Halac *et al.* [84] examine long term contracting for experimentation with moral hazard and adverse selection, and show that the optimal contract implements low effort from the low ability agent. Adrian and Westerfield [2] develop a model in which the principal and the agent disagree about the resolution of uncertainties and show that this disagreement risk sharing leads to an endogenous regime shift. He *et al.* [88] introduce uncertainty in the seminal work by Holmstrom and Milgrom [92], and show that the optimal contract displays a front-loading pattern. Optimal contracting for experimentation under moral hazard or adverse selection concerns has been studied in a growing body of literature ([118], [76], [70]).

In contrast, this chapter studies innovation under ambiguity and in an infinite horizon stopping time problem, characterizes the dynamic contract organizing the research activities.

This chapter is part of the literature examining the impact of *ambiguity in the contracting* environment. Similar to this study, the paper by Besanko, Tong and

Wu [21] considers delegated experimentation under ambiguity. However, while their paper examines the adverse selection problem in the experimentation context and using maximum likelihood updating, shows the optimality of a pooling contract in a perfect objectivist equilibrium, here I focus on the moral hazard problem.

In a static context, Lopomo, Rigotti and Shannon [110] examine the moral hazard problem under ambiguity and show how simple contract structures turn out to be optimal. In a static general equilibrium framework, Amarante *et al.* [6] discuss the effects of ambiguity and heterogeneous belief among the decision makers and the entrepreneur. Rigotii *et al.* [122] characterize the diffusion profile of a new technology under ambiguity. Byun [31] characterizes the optimal incentive scheme for innovation in a static game. In contrast, we analyze ambiguity in a dynamic environment and using dynamically consistent Bayesian updating, we show how the optimal contract structure facilitates ambiguity sharing.

There is also a growing strand of literature that analyzes *dynamic contracts* and mechanism design problem and illustrates the importance of dynamic agency costs. This chapter, discussing the dynamic agency cost under ambiguity, is related to that strand of literature as well. Bergemann and Pavan [18] contain a detailed survey of this literature. The importance of dynamic agency cost has been well documented in literature using both the continuous time framework ([50], [126], [51], [25], [67]) and discrete time models ([24], [23], [106]). In this chapter, we analyze the dynamic agency cost arising from the diversion of resources by the researcher and show that the presence of ambiguity and difference in attitude towards ambiguity among the contracting parties alleviate the dynamic moral hazard problem.

Following Gilboa and Schmeidler's seminal work on ambiguity [72], multiple prior models of ambiguity have been applied to various dynamic decision making contexts. However, with multiple priors, prior-by-prior updating of belief using Bayes rule usually leads to dynamic inconsistency. There are different approaches to modelling ambiguity averse preferences in a dynamically consistent way. Some papers take the approach that deals with recursive extensions (*e.g.* , [58], [111], [102]), others posit dynamic inconsistency and adopt assumptions, such as backward induction or naive ignorance of the inconsistency, to pin down behavior (e.g. Siniscalchi, 2008) , yet another approach uses non-consequentialist updating rules ([112])[5]. In this chapter, we use the ambiguity framework developed in [54], which characterizes a vNM approach to ambiguity and uses Bayes rule to obtain dynamically consistent updating of beliefs. Thus, this chapter fits in the literature of *decision making with ambiguity* in a dynamic framework.

Apart from these strands of literature, there is a vast body of literature in Economics, Management, Law and Organization design that discusses the *strategic partnerships*, their governance structure, and the role of government policies in innovation. Gilson, Sabel and Scott [74] analyze the specific features of the strategic partnerships and underline the importance of Knightian uncertainty in innovation context. Baker *et al.* [10] show how all possible governance structures may emerge in such contexts. Van de Ven [137] discusses how the management of innovation can overcome the problems associated with the innovation process. Lerner and Malmendier [107] show how incomplete contracts can be used as the optimal contractual

[5] For a more complete survey, refer to [61].

design to solve the problem of moral hazard in Biotechnology research partnerships. Hagedoorn *et al.* [83] underline the importance of research partnerships and suggest that the patent granting authority should be aware of the benefits and shortcomings of these partnerships in conducting R&D. A vast body of literature discuss various related issues in the context of different industries ([81], [82], [120], [133], [123]). This chapter, analyzing the research alliances from a theoretical point of view and showing how the observed contract structure optimally organizes innovation, fits in this strand of literature as well.

1.4 Model and Analysis

In this section I first set up the model, characterize the Policymaker's optimum and then examine the contractual equilibrium.

1.4.1 General Set-up

States: The innovation activity is centered around a project, success of which depends on the true state or true profitability of the project: $\theta \in \Theta$. The true state is not known; moreover, it is not possible to form a single probabilistic assessment about it. In a multiple prior setting,

$$\Theta = \{Good, Bad\}$$
$$\Pr(\theta = Good) = [r_0, s_0] \quad ; 0 \leq r_0 < s_0 \leq 1.$$

Using the framework of ambiguity developed in [54] (described in greater detail in Appendix B of this chapter), we observe that the interval $[r_0, s_0]$ has a unique representation as a convex combination of extreme sets given by $\Theta' = \{Good, Bad, Unknowable\}$, where the new epistemic state "*Unknowable*" is motivated in Section 2.

Thus, each $[r_0, s_0]$ is represented as:

$$[r_0, s_0] = r_0[1, 1] + (1 - s_0)[0, 0] + (s_0 - r_0)[0, 1]$$

The state *Unknowable* is represented as $[0, 1]$, the state at which the decision maker knows only that the probability of $\theta = Good$ is someplace between 0 and 1.

Thus, in this framework, we can alternatively represent this set-valued prior by a three state expected utility model, where the true state of the project lies in Θ' :

$$\begin{aligned}
\Theta' &= \{Good, Bad, Unknowable\} \\
\Pr(\theta = Good) &= r_0 \\
\Pr(\theta = Bad) &= 1 - s_0 \\
\Pr(\theta = Unknowable) &= s_0 - r_0 \\
0 \leq r_0 &< s_0 \leq 1.
\end{aligned}$$

That is, with probability r_0, at the end it will be revealed that the project is profitable, with probability $1-s_0$ it will be revealed that the project is not profitable, but with probability $s_0 - r_0$, the true profitability of the project will turn out to be "*Unknowable*," or, Not Yet Known, depending on the current state of technology and knowledge. Notice that $s_0 - r_0$ captures the idea that the decision maker knows only a partial description about the underlying distribution; if $r_0 = s_0$ then we are back to the "risky" context.

If the payoff for $\theta = Good$ is u_G, for $\theta = Bad$ is $u_B < u_G$, then the payoff associated with the new state $\theta = Unknowable$ is computed as:

$$u(\theta = Unknowable) = \frac{1}{2}(u_G + u_B) - \frac{v}{2}(u_G - u_B);$$

where the ambiguity aversion parameter v captures the attitude towards ambiguity. $v > 0$ refers to the decision maker being ambiguity averse. The higher v

is, the more the decision maker dislikes the state $\theta = Unknowable$, hence can be considered as more ambiguity averse. Here, I assume $v \in (0, 1)$.

Innovation Time line: To finish the project, one must go through two distinct stages:

1. *Experimentation stage:* At this stage of innovation, at every period t, some fund $K_t \in [0, \overline{K}] \subset [0, 1)$ is invested in the project and at the end of the period an informative signal S_t is realized. The signal is binary: $S_t \in \{s_H, s_L\}$, with the distribution to be specified below. Only if the signal is "high enough," *i.e.* it surpasses the quality threshold determined by the patent-granting authority, the project is allowed to move to the next stage: the Development stage. This threshold can be interpreted as the Patent Law or the FDA approval criterion. If the signal fails to clear the threshold, the researching authorities may continue experimenting (move to period $t + 1$ in the experimentation stage), or abandon the project forever (gross return$= 0$).

2. *Development stage:* If the signal is high enough to clear the patenting threshold, the project is enters the Development stage. Here, the researcher(s) can choose to develop the project by making a fixed investment of the amount $I > 0$, after which the true state will be revealed. If the true state is $\theta = Good$, the project yields a return of $R > I$, otherwise the gross return is 0. However, instead of investing I to reveal the true state, the researching authority may want to liquidate the project as well, collecting a liquidation value of $L > 0$.

The general time line is represented in Figure 1.1.

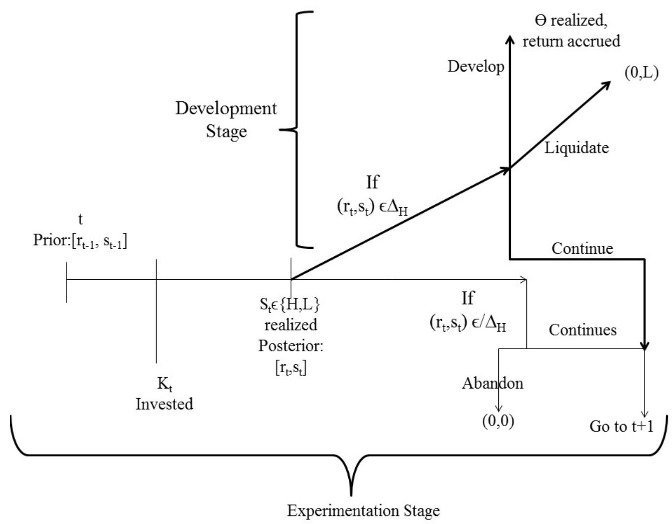

Figure 1.1: Innovation Process

Signal Structure: The signal structure assumed throughout this chapter is given below. At any period t, the signal is conditionally independent and jointly distributed with the state $\theta \in \Theta'$.

At any period t, investment flow increases signal precision.

$$\begin{aligned}
\Pr(S_t = s_H|\theta = G) &= \lambda_G(K_t) \\
\Pr(S_t = s_H|\theta = U) &= \lambda_U(K_t) \\
\Pr(S_t = s_H|\theta = B) &= \lambda_B(K_t) \qquad (1.1)
\end{aligned}$$

The parametric restrictions we impose on the signal structure are:

$\boldsymbol{Assumption 1:}$

$$1 > \lambda_G(K_t) > \lambda_U(K_t) > \lambda_B(K_t) \geq 0 \ \ \forall K_t \in [0, \overline{K}]$$

***Assumption*2 :**

$$\lambda'_G(K_t) > \lambda'_U(K_t) > \lambda'_B(K_t) \ \ \forall K_t \in [0, \overline{K}]$$

***Assumption* 3 :**

$$\frac{\lambda_G(K_t)}{1-\lambda_G(K_t)} > \frac{\lambda_U(K_t)}{1-\lambda_U(K_t)} > \frac{\lambda_B(K_t)}{1-\lambda_B(K_t)} \ \ \forall K_t \in [0, \overline{K}] \qquad \text{(MLRP)}$$

While the first assumption ensures that $\lambda_\theta(K_t)$ is a valid probability measure defined on Θ', the second assumption states that higher investment increases the signal precision. The third assumption is called the Monotone Likelihood Ratio Property and is defined as follows:

Definition 1.4.1 (Monotone Likelihood Ratio Property)**.** The signal structure satisfies Monotone Likelihood Ratio Property (MLRP) if the probability of observing $S_t = s_H$ relative to that of observing $S_t = s_L$ is increasing in the true state, when the states are ordered *Good* $\succ$ *Unknowable* $\succ$ *Bad*. Mathematically, it is captured by equation MLRP.

Now, the conditional distribution associated with this binary signal is characterized below:

Signal Structure

S_t	s_H	s_L	
$\theta = G\ (1,1)$	$r_{t-1}\lambda_G(K_t)$	$r_{t-1}(1-\lambda_G(K_t))$	r_{t-1}
$\theta = Unknowable\ (0,1)$	$(s_{t-1}-r_{t-1})\lambda_U(K_t)$	$(s_{t-1}-r_{t-1})(1-\lambda_U(K_t))$	$s_{t-1}-r_{t-1}$
$\theta = B(0,0)$	$(1-s_{t-1})\lambda_B(K_t)$	$(1-s_{t-1})(1-\lambda_B(K_t))$	$1-s_{t-1}$
	μ_t	$1-\mu_t$	1

So that,

$$\begin{aligned}\Pr(S_t = s_H) &= \mu_t(K_t) \\ &= r_{t-1}\lambda_G(K_t) + (s_{t-1} - r_{t-1})\lambda_U(K_t) + (1 - s_{t-1})\lambda_B(K_t)\end{aligned}$$

After observing the binary signal, at the end of each period, the beliefs are updated using Bayes Law.

After observing a high signal $S_t = s_H$, the updated posterior puts weight on the three states as follows:

$$\begin{aligned}\Pr(\theta = G|S_t = s_H) &= \frac{r_{t-1}\lambda_G}{\mu_t} = r_t^H \\ \Pr(\theta = B|S_t = s_H) &= \frac{(1 - s_{t-1})\lambda_B}{\mu_t} = 1 - s_t^H \\ \Pr(\theta = U|S_t = s_H) &= \frac{(s_{t-1} - r_{t-1})\lambda_B}{\mu_t} = s_t^H - r_t^H\end{aligned}$$

Thus, in the multiple prior interpretation, the set valued posterior after observing a high signal $S_t = s_H$ is:

$$\Pr(\theta = G)|_{S_t = s_H} = [r_t^H, s_t^H] = \left[\frac{r_{t-1}\lambda_G(K_t)}{\mu_t}, 1 - \frac{(1 - s_{t-1})\lambda_B(K_t)}{\mu_t}\right]$$

Similarly, after $S_t = s_L$, posterior becomes:

$$\Pr(\theta = G)|_{S_t = s_L} = [r_t^L, s_t^L] = \left[\frac{r_{t-1}(1 - \lambda_G(K_t))}{1 - \mu_t}, 1 - \frac{(1 - s_{t-1})(1 - \lambda_B(K_t))}{1 - \mu_t}\right]$$

To save on notation, let us define the average of the posterior belief as the posterior mean:

$$\textit{posterior mean} = \frac{r_t + s_t}{2} = p_t$$

and the average spread of the posterior belief as the posterior ambiguity:

$$\textit{posterior ambiguity} = \frac{s_t - r_t}{2} = q_t$$

Note that, by $MLRP$, after observing $S_t = s_H$, posterior mean p_t increases and posterior ambiguity q_t decreases; and after $S_t = s_L$, p_t decreases and q_t increases.

Intuitively, the signals can be thought of as random draws from a Bernoulli distribution:

$$S_t \sim Bernoulli(\lambda_G(K_t)) \quad \text{if } \theta = Good$$
$$S_t \sim Bernoulli(\lambda_U(K_t)) \quad \text{if } \theta = Unknowable$$
$$S_t \sim Bernoulli(\lambda_B(K_t)) \quad \text{if } \theta = Bad$$

Then, after observing each binary signal, the decision maker updates his belief about the true parameter. The following graph (Figure 1.2) depicting 30 simulations of signals for each of the three true states (with parameters: $\lambda_G = 0.7, \lambda_U = 0.5, \lambda_B = 0.1, \overline{K} = 1$) shows how repeated sampling for a long time eventually reveals the state, as the posterior converges to one of the states with almost certainty. However, due to the positive cost of experimenting, it is not optimal to experiment forever. Then the problem for the decision maker becomes an optimal stopping problem: the decision maker has to follow an optimal rule about when to stop experimenting, depending on the observed sequence of signals.

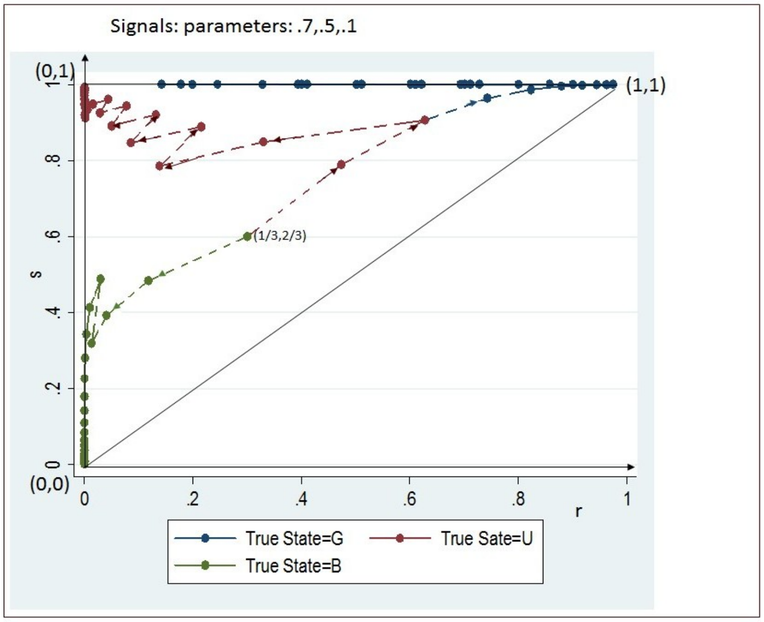

Figure 1.2: Evolution of Beliefs for 30 Consecutive Signals

In the main body of this chapter, we will assume linear signal structure, *i.e.*

$$\begin{aligned}
\Pr(S_t = s_H|\theta = G) &= \lambda_G(K_t) = \lambda_G K_t \\
\Pr(S_t = s_H|\theta = U) &= \lambda_U(K_t) = \lambda_U K_t \\
\Pr(S_t = s_H|\theta = B) &= \lambda_B(K_t) = \lambda_B K_t \qquad (1.2)
\end{aligned}$$

with

$$\begin{aligned}
\Pr(S_t = s_H) &= \mu_t = K_t\lambda_t \\
&= K_t\underbrace{[r_{t-1}\lambda_G + (1-s_{t-1})\lambda_B + (s_{t-1}-r_{t-1})\lambda_U]}_{\lambda_t} \qquad (1.3)
\end{aligned}$$

In section 8, we discuss the case with general non-linear signal structure and show that qualitatively the results hold in that case.

The next subsection discusses how the patent law is set, depending on the signal structure described above.

1.4.2 Patent Law

Assume that the patent law is set by the Policymaker (the patent-granting authority, or the regulatory agency), who is a risk and ambiguity neutral entity. The Policymaker values the "open questions," or the "*Unknowable*" state more than the commercial firms do, hence is less ambiguity averse (for simplification, I assume ambiguity neutrality). Assume that the Policymaker cares only for the payoffs generated from the project[6]. The Policymaker sets the patent law to reflect his own desired outcome: the "Policymaker's Optimum," or, the "*Risk and Ambiguity Neutral Optimum* (*RAN Optimum*)."

After observing the signal at the end of each period, the Policymaker chooses whether to develop ($a_t^{RAN} = Dev$), or to liquidate the project ($a_t^{RAN} = Liq$), or to continue experimenting further ($a_t^{RAN} = Continue$).

The payoffs associated with the actions are:

Payoffs

	a_t^{RAN} =Dev	a_t^{RAN} =Liq
$\theta = Good$	$R - I$	L
$\theta = Bad$	$-I$	L
$\theta = Unknowable$	$\frac{1}{2}R - I$	L

[6] It might be argued that it is more natural to assume that the Policymaker would internalize the positive externalities the project might generate as well. However, to make the comparison between the contractual outcome and the outcome desired by the Policymaker, here I do not consider the externalities. In Section 5, I discuss how including the externalities make the contractual outcome diverge further from the risk and ambiguity neutral benchmark outcome.

Thus, after observing a signal at period t, with the updated posterior $[r_t, s_t]$, the expected payoff to the Policymaker from choosing action $a_t^{RAN} = Dev$ is:

$$\begin{aligned}Eu_t^{RAN}(a_t^{RAN} &= Dev, (r_t, s_t)) \\ &= r_t(R - I) + (1 - s_t)(-I) + (s_t - r_t)(\frac{1}{2}R - I) \\ &= \frac{r_t + s_t}{2}R - I = p_t R - I\end{aligned}$$

The expected payoff from choosing $a_t^{RAN} = Liq$ is L.

The Policymaker's optimal stopping rule identifies the regions of posterior beliefs where it is optimal to stop experimenting and develop the project: Δ_H, and the region where it is optimal to stop experimenting and liquidate the project: Δ_S. Then, at the beginning of each period, the problem can be formulated recursively using the optimality equation or Bellman equation:

$$\begin{aligned}V_t^{RAN}(r_{t-1}, s_{t-1}) = \max_{\Delta_H, \Delta_S, K_t^{RAN}} & \Pr_t((r_t, s_t) \in \Delta_H)(p_t R - I) + \Pr_t((r_t, s_t) \in \Delta_S)L - K_t \\ & + \delta E_t V_{t+1}^{RAN}(r_t, s_t) \qquad (RAN)\end{aligned}$$

where the regions Δ_H, Δ_S are defined as follows:

$$\Delta_H = \{(r_t, s_t) \in \mathbb{K}_{\Delta[0,1]} |\ a_t^{RAN} = Dev\}$$

$$\Delta_S = \{(r_t, s_t) \in \mathbb{K}_{\Delta[0,1]} |\ a_t^{RAN} = Liq\}$$

Lemma 1.4.1. *There exists a unique solution to the RAN optimization problem.*

Proof. The proof involves showing that the optimality equation satisfies the Blackwell sufficiency conditions, hence is a contraction. Then a direct use of the Contraction Mapping Theorem gives the existence and uniqueness of the result. Details in Appendix A. □

Now, let us examine the optimal stopping rule. After observing the signal, based on the updated posterior $[r_t, s_t]$, the expected payoff is:

$$\max\{p_t R - I, L, \delta E_t V_{t+1}^{RAN}(r_t, s_t)\}$$

In order to solve for the RAN optima, let us define:

$$\begin{aligned} F_j(r_t, s_t) = &\text{ based on } [r_t, s_t], \text{ the maximum expected value if experimentation stops at } j \\ = E_t &\left[\delta^{j-t} \max\{p_j R - I, L\} - \sum_{s=t}^{j-1} \delta^{s-t} K_s\right] \end{aligned} \tag{1.4}$$

Define:

$$A_t = \{F_t > (F_{t+1} | (r_t, s_t)\} \quad t = 1, 2, ..$$

we show that A_t s form a monotone sequence.

Lemma 1.4.2. *If $F_t(r_t, s_t) \geq F_{t+1}(r_t, s_t)$, then $F_{t+1}(r_t, s_t) \geq F_{t+2}(r_t, s_t)$, i.e. $A_1 \subset A_2 \subset .. \cup_1^\infty A_n$, hence the region where stopping immediately is optimal forms a monotone sequence.*

Proof. In Appendix A. □

Then, the "One-stop ahead" rule is optimal, *i.e.* , if stopping the experimentation process today is better than continuing experimenting for exactly one more period, then it is always optimal to stop today ([37]). Using that, we obtain the optimal stopping rule, given in the next proposition.

Proposition 1.4.3. *The RAN optima, or, the "Policymaker's Optimum" is given by the stopping rule*

$$a_t^{RAN}(r_t, s_t) = \begin{cases} Dev & if \ (r_t, s_t) \in \Delta_H \\ Liq & if \ (r_t, s_t) \in \Delta_S \\ Continue & otherwise \end{cases}$$

where the optimal stopping thresholds are:

$$\Delta_H : = \{(r_t, s_t) | \beta_{H1} r_t + \beta_{H2} s_t \geq \beta_{H3}\};$$

$$\Delta_S : = \{(r_t, s_t) | \beta_{S1} r_t + \beta_{S2} s_t < \beta_{S3}\}$$

The stopping time is:

$$T_{RAN} = \inf\{t | (r_t, s_t) \in \Delta_H \cup (r_t, s_t) \in \Delta_S\}$$

Also, the project receives full funding in every period it is continued.

$$K_t = \overline{K} \qquad \forall t \leq T_{RAN}$$

Proof. In Appendix A. □

Thus, the Policymaker's value from this innovation project becomes:

$$V_0^S = E_0 \left[\sum_{t=1}^{T_S} \delta^{t-1} \left(\Pr_t((r_t, s_t) \in \Delta_H)(p_t R - I) + \Pr_t((r_t, s_t) \in \Delta_S) L - \overline{K} \right) \right] \quad (1.5)$$

The Policymaker sets the region Δ_H as the patent threshold. According to the patent law, the project has to clear this threshold in order to be granted a patent.

Only after the patent is granted, the property rights are recognized; hence the project can be liquidated for a positive liquidation value $L > 0$[7].

The patent law threshold is depicted in the Figure 1.3.

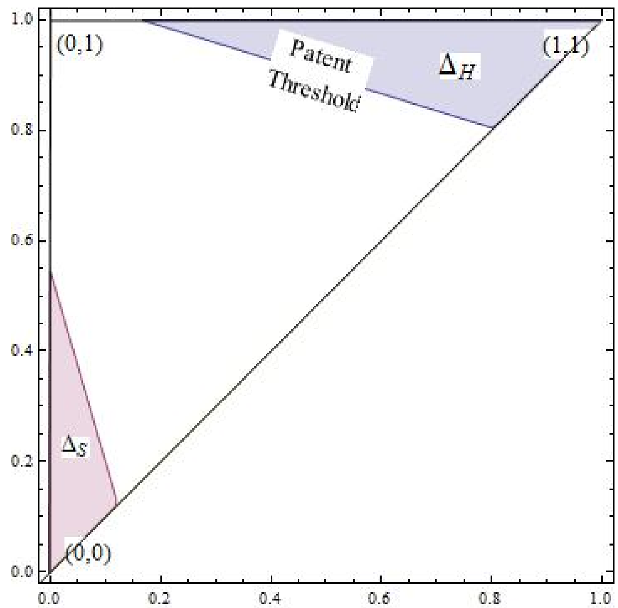

Figure 1.3: Policymaker's Optimum and the Patent Law

Note that, once the posterior belief $[r_t, s_t] \in \Delta_H$, so that the project is granted a patent, according to the RAN optima, it is optimal to stop experimenting and develop the project. However, we will see in the next sections that the contractual outcome between an ambiguity neutral research lab and a ambiguity averse commercial firm may differ from this RAN optimal stopping rule.

[7]The patent law mandates that before clearing the patenting threshold, the project is not worth any positive value. This loss of value associated with the patent law reflects the social cost of granting monopoly power to the patent owners.

1.5 Contractual Outcome

Given the patent law set by the Policymaker, now let us focus on the contractual problem. The two parties forming the research alliance are: a big commercial firm (henceforth CF) and the smaller research-oriented firm or research lab (henceforth RL). Both the parties are risk-neutral and initially share a common prior about the true profitability of the project:

$$\Pr(\theta = Good) = [r_0, s_0]; \quad 0 \leq r_0 < s_0 \leq 1.$$

$$[r_0, s_0] \notin \Delta_H$$

RL owns the project, but is liquidity constrained, so CF funds the project. At the experimentation phase, RL conducts the research activities, but after the project moves to the development phase, CF takes over the clinical trial and/or commercialization process ("development of the project").

The two parties, however, differ in their attitude towards ambiguity. RL likes the "open questions," or the "*Unknowable*" state more than the commercial firm, so is less ambiguity averse than CF. It can be justified by arguing that identifying open questions can open up the avenue of further research and help RL, or, "learning by doing" might add to the existing knowledge base of RL, whereas the commercial firm, which cares only for current profits, dislikes this state more, because the project does not yield a stream of payoffs if the true state is "*Unknowable*." To simplify, we assume that RL is ambiguity neutral while CF is ambiguity averse[8].

[8]In section 5, I discuss how the ability to write a contract on the knowledge generated from the research can change the ambiguity attitude of the two firms.

Now, let us describe the contracting time line, as captured in the figures 1.4 and 1.5 below. At the beginning of each period t, RL makes a take-it-or-leave-it offer to CF specifying

(a) $x_{t:}$: the proportional share of the final return RL receives, if the project is developed till the end

(b) b_t : the bonus that RL gets once the project clears the threshold, *i.e.* , is granted a Patent, and,

(c) K_t : amount of investment to be disbursed in the t^{th} period[9].

CF accepts or rejects the offer. If accepted, the funds are disbursed and then RL privately decides whether to invest the fund or divert it for personal benefit (or cross-subsidization). At the end of the period, the signal S_t is publicly realized and beliefs are accordingly updated. If the signal is high enough, *i.e.* , $[r_t, s_t] \in \Delta_H$, then the project is allowed to move to the Development Stage. In the Development stage, CF unilaterally decides whether to continue developing the product, liquidate the project, or keep experimenting further. If the project is continued till the end, after investing the fixed amount I, the true state θ is realized and returns accrue to the contracting parties. If the project is liquidated, CF appropriates the property rights, therefore obtains the liquidation value $L > 0$.

If the signal is not high enough , i. e. , $[r_t, s_t] \notin \Delta_H$, then CF decides whether

[9]Here, it is assumed that the research lab owns the project and faces a competetive market of commercial firms for that project, hence enjoys all the bargaining power. In real life, such contexts feature multiple commercial firms as well as research labs, so in any contracting environment, no party enjoys the full extent of the bargaining power. However, this assumption, while simplifying the calculations, does not qualitatively change the results.

to continue experimenting at period $t+1$ with updated beliefs, or to abandon the project, earning a return of 0 forever. The time line is depicted in the two figures below.

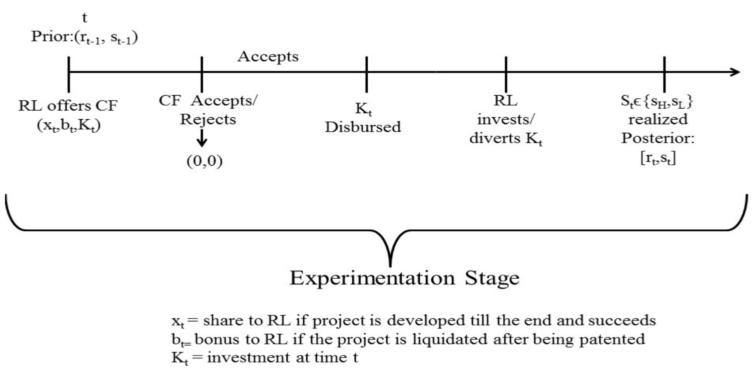

Figure 1.4: Contracting Time Line: Experimentation Stage

After observing the signal, with posterior$[r_t, s_t]$, the expected payoffs for the contracting parties are:

Payoffs of RL

	$a(CF)$ =Dev	$a(CF)$ =Liq
$\theta = Good$	Rx_t	b_t
$\theta = Bad$	0	b_t
$\theta = Unknowable$	$\frac{1}{2}Rx_t$	b_t

Payoffs of CF

	$a(CF)$ =Dev	$a(CF)$ =Liq
$\theta = Good$	$R(1-x_t) - I$	$L - b_t$
$\theta = Bad$	$-I$	$L - b_t$
$\theta = Unknowable$	$\frac{1}{2}R(1-x_t)(1-v) - I$	$L - b_t$

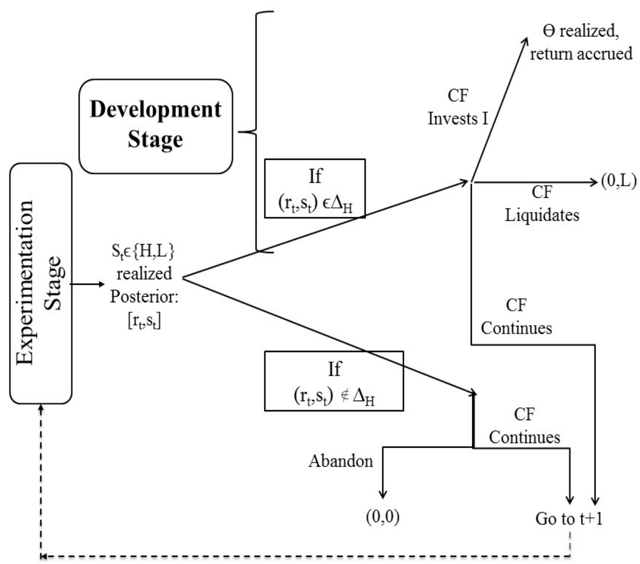

Figure 1.5: Contracting Time Line: Development Stage

Thus, the expected payoffs:

$$CF:$$

$$Eu(CF)(a(CF)) = Dev, (r_t, s_t)) = (p_t - vq_t)R(1 - x_t) - I$$

$$Eu(CF)(a(CF)) = Liq, (r_t, s_t)) = L - b_t$$

$$RL:$$

$$Eu(RL)(a(CF)) = Dev, (r_t, s_t)) = p_t R x_t$$

$$Eu(RL)(a(CF)) = Liq, (r_t, s_t)) = b_t$$

The contracting parties do not have the power to commit to a long term contract. Then, RL, who has the full bargaining power in this model, always offers a contract that ensures CF only the minimum payment required to keep investing, so

CF always breaks even. After observing $[r_t, s_t] \in \Delta_H$, CF obtains a payoff of $p_t R(1-x_t) - I$ if he develops the project, $L - b_t$ if he liquidates, and an expected payoff of 0 from future experimentation. Clearly, CF always chooses to stop experimentation as soon as $[r_t, s_t] \in \Delta_H$[10]. Thus, at any period t, if the observed signal induces a posterior belief higher than the patenting threshold, CF never continues experimentation.

Before discussing the infinite horizon model, let us first analyze the two period contracting game, which will illustrate the intuitions behind the main results of this chapter. The findings from this two period example are readily extendable to the finite horizon contracting problem, and they will provide the intuitive understanding about the model in the general infinite horizon setting.

1.5.1 Two Period Example

In this example, the project is exogenously terminated after $t = 2$. Let us first describe the problem, then using backward induction, we will analyze the optimal contract.

If the project is continued till $t = 2$, at the beginning of the last period, RL chooses the contractual term considering $CF's$ optimal action choice after the signal clears the patent threshold: $a(CF)|_{[r_2,s_2]\in\Delta_{H.}} \in \{Dev, Liq\}$.

At $t = 2$, the state variables on the equilibrium path are $[r_1, s_1]$, the updated

[10]If we relax the assumption that RL has limited liability, then RL can make a payment to CF in order to continue experimenting even after clearing the patenting threshold. I discuss this case in section 6 and show that qualitatively the results do not change.

belief after observing last period's signal. RL solves:

$$V_2(r_1, s_1) = \max_{a(CF)} \{V_2^{Dev}, V_2^{Liq}\}$$

where

$$V_2^{Dev} = \begin{array}{c} RL's \text{ expected payoff from period 2 if, given the contractual terms,} \\ \text{CF develops the product after reaching } \Delta_{H.}(a(CF) = Dev) \end{array}$$

$$V_2^{Liq} = \begin{array}{c} RL's \text{ expected payoff from period 2 if, given the contractual terms,} \\ \text{CF liquidates the product after reaching } \Delta_{H.}(a(CF) = Liq) \end{array}$$

Now,

$$V_2^{Dev} = \max_{x_2, b_2, K_2} \Pr((r_2, s_2) \in \Delta_H)[Rp_2 x_2]$$

$$\Pr((r_2, s_2) \in \Delta_H)[Rp_2|_{(r_2,s_2)\in\Delta_H} x_2] \geq K_2 \quad (IC_2, Dev(RL))$$

$$\Pr((r_2, s_2) \in \Delta_H)[R(p_2 - vq_2)|_{(r_2,s_2)\in\Delta_H}(1 - x_2) - I] \geq K_2 \quad (PC_2, Dev(CF))$$

$$R(p_2 - vq_2)|_{(r_2,s_2)\in\Delta_H}(1 - x_2) - I \geq L - b_2 \quad (IC_2, Dev(CF))$$

$$x_2 \in [0, 1]; b_2 \geq 0; K_2 \in [0, \overline{K}]$$

And,

$$V_2^{Liq} = \max_{x_2, b_2, K_2} \Pr((r_2, s_2) \in \Delta_H)[b_2] \tag{1.6}$$

$$\Pr((r_2, s_2) \in \Delta_H)[b_2] \geq K_2 \quad IC_2, Liq(RL)$$

$$\Pr((r_2, s_2) \in \Delta_H)[L - b_2] \geq K_2 \quad (PC_2, Liq(CF))$$

$$R(p_2 - vq_2)|_{(r_2,s_2)\in\Delta_H}(1 - x_2) - I \leq L - b_2 \quad (IC_2, Liq(CF))$$

$$x_2 \in [0, 1]; b_2 \geq 0; K_2 \in [0, \overline{K}]$$

Let us take a closer look at the constraint set. The first constraint is the standard incentive compatibility constraint for RL, which ensures that the expected payoff for RL at $t = 2$ has to be greater than or equal to the static gain that RL might enjoy by diverting the investment, thereby implementing no diversion on the equilibrium path. Notice that, in this setting, if any partial diversion is beneficial, so is the full diversion, that is why it is sufficient to consider the incentive constraint only for the full diversion case. The second constraint is the participation constraint for CF, guaranteeing CF an expected return to cover the investment cost. Without loss of generality, $CF's$ outside option is normalized to 0. The last constraint shows that after the signal realization, it is sequentially optimal for CF to develop the project in the first case and liquidate in the second.

Solving the problem, we get three regions of posterior belief: Δ_D, Δ_L , such that

$$\Delta_D = \{(r_t, s_t) \in \Delta_H | \ a(CF)|_{\Delta_H} = Dev$$

i.e. , CF chooses to develop the project once being granted a patent

Remark 1.5.1 (Ambiguity Sharing). Observe that, as v increases, *i.e.* , CF becomes more ambiguity averse, the share he receives, $1 - x_2$, goes up. Thus, the contractual payment rule effectively shares ambiguity.

Remark 1.5.2 ((Evolution of Share)). As experimentation continues, the contracting parties grow more pessimistic as posterior belief declines. The share CF demands goes up accordingly over time to compensate.

Thus, RL solves:

$$V_2 = \lambda_2 \max_{K_2^{Dev}, K_2^{Liq}} \left\{ K_2^{Dev} p_2 \left(R - \frac{1}{p_2 - vq_2} \left(I + \frac{1}{\lambda_2} \right) \right), K_2^{Liq} \left(L - \frac{1}{\lambda_2} \right) \right\} \tag{1.7}$$

subject to the constraint:

$$\begin{aligned} K_2^{Dev} p_2 \left(R - \frac{1}{p_2 - vq_2} \left(I + \frac{1}{\lambda_2} \right) \right) &\geq \frac{K_2^{Dev}}{\lambda_2} \quad \text{if } (r_2, s_2) \in \Delta_D \\ K_2^{Liq} \left(L - \frac{1}{\lambda_2} \right) &\geq \frac{K_2^{Liq}}{\lambda_2} \quad \text{if } (r_2, s_2) \in \Delta_L \end{aligned}$$

So, $K_2^{Dev} = \overline{K}$, and $K_2^{Liq} = \overline{K}$, if

$$\max \left\{ p_2 \left(R - \frac{1}{p_2 - vq_2} \left(I + \frac{1}{\lambda_2} \right) \right), L - \frac{1}{\lambda_2} \right\} \geq \frac{1}{\lambda_2} \tag{1.8}$$

If this condition is satisfied, the expected value to RL from $t = 2$ is:

$$V_2(r_1, s_1) = \lambda_2 \overline{K} \left\{ p_2 \left(R - \frac{1}{p_2 - vq_2} \left(I + \frac{1}{\lambda_2} \right) \right), \left(L - \frac{1}{\lambda_2} \right) \right\} \tag{1.9}$$

The regions where the project is developed till the end, and where it is liquidated are identified as:

$$\Delta_D = \left\{ (r_2, s_2) \in \Delta_H |\; p_2 \left(R - \frac{1}{p_2 - vq_2} \left(I + \frac{1}{\lambda_2} \right) \right) \geq \left(L - \frac{1}{\lambda_2} \right) \right\} \tag{1.10}$$

$$\Delta_L = \left\{ (r_2, s_2) \in \Delta_H |\; p_2 \left(R - \frac{1}{p_2 - vq_2} \left(I + \frac{1}{\lambda_2} \right) \right) < \left(L - \frac{1}{\lambda_2} \right) \right\} \tag{1.11}$$

Remark 1.5.3 (Patent Troll). Observe that, in the absence of ambiguity, or, if both the parties were ambiguity neutral ($v = 0$), then

$$p_2R - I > L \quad \forall (r_t, s_t) \in \Delta_H,$$

so, $\Delta_L = \phi$.

In ambiguous context, however, there exists v_m such that for $v \in (v_m, 1)$ [11], $\Delta_L = \Delta_H \backslash \Delta_D \neq \phi$.

This region resembles Patent Troll[12] behavior, where even after being granted a patent, the research alliance liquidates the project. Patent troll happens because of the ambiguity aversion of CF, who acts more pessimistically after observing each low signal. So, even if the posterior ensures that a risk and ambiguity neutral entity would optimally choose to develop the project, CF decides to liquidate.

Now, let us go one step backward at $t = 1$.

At $t = 1$, RL solves:

$$V_1(r_0, s_0) = \max_{a(CF)} \{V_1^{Dev}, V_1^{Liq}\}$$

[11] $v_m = \frac{(r_0+s_0)((r_0+s_0)R-L-I)\lambda_0}{\lambda_0[(s_0-r_0)((r_0+s_0)R-L]+1)}$

[12] Technically, the term "patent troll" refers to the entities which obtain and enforce patent rights but do not manufacture products or supply services based upon the patent in question, thus engaging in economic rent-seeking.

where

$$
\begin{aligned}
V_1^{Dev} = \max_{x_1, b_1, K_1} & K_1\lambda_1[Rp_1x_1] + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& K_1\lambda_1[Rp_1x_1] + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& \geq K_1 + \delta E_1V_2(r_1, s_1; r_0, s_0) && (IC_1(RL)) \\
& K_1\lambda_1[R(p_1 - vq_1)(1 - x_1) - I] \geq K_1 && (PC_1(CF)) \\
& R(p_1 - vq_1)(1 - x_1) - I \geq L - b_1 && (IC_1(CF)) \\
& x_1 \in [0, 1]; b_1 \geq 0; K_1 \in [0, \overline{K}]
\end{aligned}
$$

And,

$$
\begin{aligned}
V_1^{Liq} = \max_{x_1, b_1, K_1} & K_1\lambda_1b_1 + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& K_1\lambda_1b_1 + \delta(1 - K_1\lambda_1)E_1V_2(r_1, s_1) \\
& \geq K_1 + \delta E_1V_2(r_1, s_1; r_0, s_0) && (IC_1, Liq(RL)) \\
& K_1\lambda_1[L - b_1] \geq K_1 && (PC_1, Liq(CF)) \\
& R(p_1 - vq_1)(1 - x_1) - I \leq L - b_1 && (IC_1, Liq(CF)) \\
& x_1 \in [0, 1]; b_1 \geq 0; K_1 \in [0, \overline{K}]
\end{aligned}
$$

In period 1, compared to the problem at $t = 2$, the participation constraint for CF remains same with the corresponding posterior belief at $t = 1$; however the incentive constraint for RL requires a closer look. The incentive constraints $(IC(RL))$ and $(IC(RL))$ highlight the two sources of gain from cheating: the static gain and the dynamic gain. The static gain is similar as in the second period, stemming from the benefit RL derives by diverting the investment amount (K_1), so the IC at $t = 1$ has to ensure that $RL's$ expected payoff from $t = 1$ has to be greater than the investment. However, there is a dynamic gain from cheating as well, captured by the dynamic cheating value: which arises from the fact that following a diversion of funds at $t = 1$, the posterior belief of RL and CF diverge. Because of the diversion, the signal S_1 is always s_L, observing which CF is prompted to update his belief to $[r_1, s_1]|_{S_1=s_L}$, with posterior mean p_1 and ambiguity q_1. The next period's contract will then be based on this public belief $[r_1, s_1]$. However, RL has perfectly observed his own action, so even after the low signal he does not update his belief and evaluates the future contracting terms using his private belief $[r_0, s_0]$. This constitutes the dynamic agency cost:

$$\begin{aligned}
DAC_2 &= \delta[V_2(cheat) - V_2(no\ cheat)] \\
&= \delta[E_1V_2(r_1, s_1; r_0, s_0) - (1 - K_1\lambda_1)E_1V_2(r_1, s_1)] \\
&= \delta \begin{cases} \left[\frac{\lambda_1 p_1}{\lambda_2 p_2} - (1 - K_1\lambda_1)\right] V_2(r_1, s_1) & \text{if } (r_2, s_2) \in \Delta_D \\ \left[\frac{\lambda_1}{\lambda_2} - (1 - K_1\lambda_1)\right] V_2(r_1, s_1) & \text{if } (r_2, s_2) \in \Delta_L \end{cases} \\
&> 0
\end{aligned}$$

Under some parametric conditions, the dynamic agency cost leads to delay in funding as well, so that it is optimal for the project to receive funding at $t = 2$ but no contract with positive funding satisfies both the participation and moral hazard constraints. Let us analyze all possible cases separately to see the region of posteriors where in-equilibirum delay might occur.

Case 1: $(r_1, s_1) \in \Delta_D$ *and* $(r_2, s_2) \in \Delta_D$:

With $\delta = 0$, delay never occurs, since:

$$\lambda_1 \left(p_1 R - \frac{p_1}{(p_1 - vq_1)\lambda_1} - \frac{Ip_1}{p_1 - vq_1} \right) \geq \lambda_2 \left(p_2 R - \frac{p_2}{(p_2 - vq_2)\lambda_2} - \frac{Ip_2}{p_2 - vq_2} \right) \geq 1$$

However, if $\delta > 0$, dynamic moral hazard makes funding the project at $t = 1$ more difficult than at $t = 2$. As a result, in-equilibrium delay happens if

$$\frac{1 + \lambda_1 p_1 \left(\frac{1}{(p_1 - vq_1)\lambda_1} + \frac{I}{p_1 - vq_1} \right) - \delta \left(\lambda_1 p_1 - (1 - \mu_1)\lambda_2 p_2 \left(\frac{1}{(p_2 - vq_2)\lambda_2} - \frac{I}{p_2 - vq_2} \right) \right)}{\lambda_1 p_1 - \delta \left(\lambda_1 p_1 - (1 - \mu_1)\lambda_2 p_2 \right)}$$
$$> R \geq \frac{1 + \lambda_2 p_2 \left(\frac{1}{(p_2 - vq_2)\lambda_2} + \frac{I}{p_2 - vq_2} \right)}{\lambda_2 p_2} \tag{1.12}$$

The possibility of in-equilibrium delay due to dynamic agency cost is well documented in the literature of dynamic contracts ([16], [26]). In this chapter, we find that in the presence of ambiguity, the commercial firm's ambiguity aversion reins in this dynamic moral hazard problem. Intuitively, CF, being ambiguity averse, becomes much more cautious and pessimistic after each low signal. So, following a low signal, CF has to be guaranteed a greater share of the final return in order to keep investing. This ambiguity sharing agreement disciplines RL and lowers his dynamic expected value from cheating (DAC_2) which, in turn, eases the funding constraint at $t = 1$ and possibility of in-equilibrium delay falls.

The next proposition summarizes the finding that, in this two period context, under *Case 1,* the dynamic value of cheating decreases with v and in-equilibrium delay happens for a smaller range of R , and, in fact if $v \geq \widetilde{v}$, where $\widetilde{v} \in (0, 1)$ is characterized below, then delay in funding does not happen on the equilibrium path.

Proposition 1.5.1. *For discount rate* $\delta \leq \overline{\delta}$, $\exists \widetilde{v} \in (0,1)$, *such that* $\forall v \geq \widetilde{v}$, *in-equilibrium delay never happens in the basic two period model.*

Proof. In Appendix A. $\square$

Also, in this case, if funding condition is met at $t = 1$, full funding is disbursed, because of the linearity of signal structure.

In the next two cases, there is no possibility of in-equilibirum delay.

Case 2: $(r_1, s_1) \in \Delta_D$ *and* $(r_2, s_2) \in \Delta_L$:

Here, funding at $t = 2$ requires

$$L - \frac{2}{\lambda_2} \geq 0 \tag{1.13}$$

Now, at $t = 1$,

$$\begin{aligned}
&p_1 \left[R - \frac{1}{p_1 - vq_1} \left(I + \frac{1}{\lambda_1} \right) \right] - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1\lambda_2) \right) \left(L - \frac{2}{\lambda_2} \right) \\
&> \left(L - \frac{2}{\lambda_2} \right) \left[1 - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1\lambda_2) \right) \right] \\
&\geq 0
\end{aligned} \tag{1.14}$$

so, full funding is always available at $t = 1$ is always met if 1.13 is satisfied.

Similarly, in *Case 3:* $(r_1, s_1) \in \Delta_D$ *and* $(r_2, s_2) \in \Delta_L$,

since

$$\begin{aligned}
&L - \frac{2}{\lambda_1} - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1\lambda_2) \right) \left(L - \frac{2}{\lambda_2} \right) \\
&> \left(L - \frac{2}{\lambda_2} \right) \left[1 - \left(\frac{\lambda_1}{\lambda_2} - (1 - K_1\lambda_2) \right) \right] \\
&\geq 0
\end{aligned}$$

there is no possibility of in-equilibrium delay.

The contractual terms at $t = 1$ are otherwise similar to those at $t = 2$.

Thus, from analyzing this two period problem, we observe that

Remark 1.5.4 (Result 1:). With ambiguity averse CF and ambiguity neutral RL, dynamic moral hazard problem is alleviated. As a result, under some parametric restrictions, in-equilibirium delay does not happen.

Remark 1.5.5 (Result 2:). The research alliance may liquidate the project even after being granted a patent.

1.5.2 Infinite Horizon Model

In this section we analyze the infinite horizon sequential contracting game between CF and RL and derive the equilibrium contractual outcome. Let us first formally define the equilibrium.

At any period t, the observable, or, public history consists of the past contracts offered, the past realizations of signals and $CF's$ decision whether to develop, liquidate or continue the project. Potentially, this public history can be different than the private history of RL, who observes his own decision to divert the fund as well.

Formally, let H_t^P denote the set of all possible public histories up to, but not including, period t. Each element $h_t^P \in H_t^P$ contains

(a) the past contractual terms: $\{x_j, b_j, K_j\}_{j=1}^{t-1}$

(b) past strategic choices of CF to accept or reject the contract offered at each period: $\{\zeta_j\}_{j=1}^{t-1}$ ($\zeta_t = 1$ if CF accepts an offer at period t, 0 otherwise)

(c) past realized values of the signals: $\{S_j\}_{j=1}^{t-1}$

(d) past strategic choices of CF after observing the signal realizations at every period: $\{a(CF)\}_{j=1}^{t-1}$.

In contrast, the set of possible private histories is denoted by H_t, where each element $h_t \in H_t$, in addition to h_t^P, contains $\{d_j\}_{j=1}^{t-1}$, the past realizations of the strategic choices of RL whether to divert the fund ($d_t = 1$ if the fund is invested in period t and 0 if diverted).

The true history leads to the posterior belief formed by RL at the beginning of period $t : [r_{t-1}, s_{t-1}] : H_t \to \mathbb{K}_{\Delta_{[0,1]}}$. In consequence, CF also has a belief about the true history, captured by the belief about the true posterior formed by CF : $[r'_{t-1}, s'_{t-1}] : H_t^P \times D'_t \to \mathbb{K}_{\Delta_{[0,1]}}$, which depends on the public history as well as the belief CF has about $RL's$ past investment behavior: $\{d'_j\}_{j=1}^{t-1}$. D'_t contains the set of all beliefs $\{d'_j\}_{j=1}^{t-1}$.

Then, a contract (x_t, b_t, K_t) by RL is a mapping from the true history H_t into the sharing rule x_t , bonus rule b_t and investment flow K_t.

$$x_t : H_t \to [0, 1]$$

$$b_t : H_t \to \mathbb{R}_+$$

$$K_t : H_t \to [0, \overline{K}] \subset [0, 1]$$

A decision rule by CF whether to accept or reject the contract is then a mapping from the perceived history: $\{x_j, b_j, K_j, \zeta_j, a(CF), d'_j\}_{j=1}^{t-1}$, and the contract proposed, into a binary decision to reject or accept the contract:

$$\zeta_t : H_t^P \times [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] \to \{0,1\}$$

An investment policy by RL is:

$$d_t : H_t \times [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] \times \{0,1\} \to \{0,1\}$$

A decision rule by CF after observing the signal at the end of period t is a mapping from the public history, contractual terms, perceived belief about diversion strategy of RL given the incentives provided by the contract, and the realized signal $S_t \in \{s_H, s_L\}$ into the choice to develop, liquidate, continue, or abandon the project at the end of period t.

$$a(CF) : H_t^P \times [0,1] \times \mathbb{R}_+ \times [0, \overline{K}] \times \{0,1\} \times \mathbb{K}_{\Delta_{[0,1]}} \to \{Dev, Liq, Abandon, Cont\}$$

In this model, we are in a Markovian world, because all the payoff relevant history can be captured by the four state variables: $(r_{t-1}, s_{t-1}, r'_{t-1}, s'_{t-1})$: the true posterior belief held by $RL : [r_{t-1}, s_{t-1}]$ and the belief of CF about the true posterior:$[r'_{t-1}, s'_{t-1}]$. In this context, let us define the suitable Markov equilibrium concept.

Definition 1.5.1 (Markov Sequential Equilibrium)**.** A Markov sequential equilibrium is a sequential equilibrium $\{x_t, b_t, K_t, \zeta_t, a(CF), d_t\}_{t=1}^{\infty}$, if

$$
\begin{array}{rcc}
(r_{t-1}, s_{t-1})(h_t) = (r_{t-1}, s_{t-1})(\hat{h}_t) & \implies & \begin{array}{c} x_t(h_t) = x_t(\hat{h}_t) \\ b_t(h_t) = b_t(\hat{h}_t) \\ K_t(h_t) = K_t(\hat{h}_t) \end{array} \\
\left.\begin{array}{c} (r'_{t-1}, s'_{t-1})(h_t^P) = (r'_{t-1}, s'_{t-1})(\hat{h}_t^P) \\ (x_t, b_t, K_t) = (\hat{x}_t, \hat{b}_t, \hat{K}_t) \end{array}\right\} & \implies & \zeta_t(h_t^P, x_t, b_t, K_t) = \zeta_t(\hat{h}_t^P, \hat{x}_t, \hat{b}_t, \hat{K}_t) \\
\left.\begin{array}{c} (r_{t-1}, s_{t-1})(h_t) = (r_{t-1}, s_{t-1})(\hat{h}_t) \\ (x_t, b_t, K_t) = (\hat{x}_t, \hat{b}_t, \hat{K}_t) \\ \zeta_t = \hat{\zeta}_t \end{array}\right\} & \implies & d_t(h_t, x_t, b_t, K_t, \zeta_t) = d_t(\hat{h}_t, \hat{x}_t, \hat{b}_t, \hat{K}_t, \hat{\zeta}_t) \\
\left.\begin{array}{c} (r_{t-1}, s_{t-1})(h_t) = (r_{t-1}, s_{t-1})(\hat{h}_t) \\ (x_t, b_t, K_t) = (\hat{x}_t, \hat{b}_t, \hat{K}_t) \\ \zeta_t = \hat{\zeta}_t \\ d_t = \hat{d}_t \end{array}\right\} & \implies & a(CF)(h_t^P, x_t, b_t, K_t, \zeta_t, d_t) = a(\hat{C}F)
\end{array}
$$

$$
\forall h_t \in H_t; \forall h_t^P \in H_t^P; \forall \hat{h}_t \in \hat{H}_t; \forall \hat{h}_t^P \in \hat{H}_t^P; \forall (x_t, b_t, K_t), (\hat{x}_t, \hat{b}_t, \hat{K}_t); \forall \zeta_t, \hat{\zeta}_t, \forall d_t, \hat{d}_t
$$

The Markovian sequential equilibrium ensures that the continuation strategies are time consistent and identical after any history with identical updated true posterior belief $[r_{t-1}, s_{t-1}]$ and $CF's$ belief about the posterior: $[r'_{t-1}, s'_{t-1}]$. It imposes that on the equilibrium path CF has the true belief given the incentives, i. e., on the equilibrium path $[r_{t-1}, s_{t-1}] = [r'_{t-1}, s'_{t-1}]$, but allows for the possibility of divergence of posterior beliefs off the equilibrium path.

The stopping regions are defined as before:

$$
\Delta_D = \{(r_t, s_t) \in \Delta_H |\ a(CF) = Dev\}
$$

$$
\Delta_L = \{(r_t, s_t) \in \Delta_H |\ a(CF) = Liq\}
$$

$$
\Delta_S^C = \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} |\ a(CF) = Abandon\}
$$

Now, at every period t, RL solves:

$$V_t(r_{t-1}, s_{t-1}) = \begin{array}{c} \max_{\Delta_D, \Delta_L, \Delta_S^C, (x_t, b_t, K_t) \in \mathbb{C}_t} \Pr_t((r_t, s_t) \in \Delta_D) p_t R x_t + \Pr_t((r_t, s_t) \in \Delta_L) b_t \\ +\delta(1 - \Pr_t((r_t, s_t) \in \Delta_D) - \Pr_t((r_t, s_t) \in \Delta_L) - \Pr_t((r_t, s_t) \in \Delta_S^C)) E_t V_{t+1}(r_t, s_t) \end{array} \tag{1.15}$$

where the contract space $\mathbb{C}_t$ is given by:

$$\mathbb{C}_t = \{(x_t, b_t, K_t) \in [0,1] \times \mathbb{R}_+ \times [0, \overline{K}]|$$

$$\begin{aligned} &\Pr_t((r_t, s_t) \in \Delta_D)(p_t R x_t) + \Pr_t((r_t, s_t) \in \Delta_L) b_t \\ &+\delta(1 - \Pr_t((r_t, s_t) \in \Delta_D) - \Pr_t((r_t, s_t) \in \Delta_L) - \Pr_t((r_t, s_t) \in \Delta_S^C)) E_t V_{t+1}(r_t, s_t) \\ &\geq K_t + \delta E V_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) \end{aligned} \tag{$IC_t^R L$}$$

$$\begin{aligned} &\Pr_t((r_t, s_t) \in \Delta_D)[(p_t - v q_t) R(1 - x_t) - I] + \Pr_t((r_t, s_t) \in \Delta_L)(L - b_t) \\ &\geq K_t \end{aligned} \tag{$PC_t^C F$}$$

$$\begin{aligned} &\text{if } (r_t, s_t) \in \Delta_D, (p_t - v q_t) R(1 - x_t) - I \geq L - b_t \\ &\text{if } (r_t, s_t) \in \Delta_L, (p_t - v q_t) R(1 - x_t) - I \quad < L - b_t \end{aligned} \tag{$IC_t^C F$}$$

Now, by the same logic as in the two period example, we observe that the experimentation stops the first time $(r_t, s_t) \in \Delta_H$. Thus, the problem can be simplified as:

$$\begin{aligned} V_t(r_{t-1}, s_{t-1}) = \max_{\Delta_D, \Delta_L, \Delta_S^C, (x_t, b_t, K_t) \in \mathbb{C}_t} & \mu_t \mathbf{1}_t((r_t, s_t) \in \Delta_D)(p_t R x_t) + \mu_t \mathbf{1}_t((r_t, s_t) \in \Delta_L) b_t \\ & + \delta(1 - \mu_t) E_t V_{t+1}(r_t, s_t) \end{aligned}$$

where the contract space $\mathbb{C}_t$ is:

$$\mathbb{C}_t = \{(x_t, b_t, K_t) \in [0,1] \times \mathbb{R}_+ \times [0, \overline{K}]|$$

$$if\ \ (r_t, s_t) \in \Delta_D$$

$$\mu_t p_t R x_t + (1-\mu_t)\delta E V_{t+1}(r_t, s_t)$$

$$\geq K_t + \delta E V_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) \qquad (IC_t^R L(Dev))$$

$$\mu_t[(p_t - vq_t)R(1-x_t) - I] \geq K_t \qquad (PC_t^C F(Dev))$$

$$(p_t - vq_t)R(1-x_t) - I \geq L - b_t \qquad (IC_t^C F(Dev))$$

$$if\ \ (r_t, s_t) \in \Delta_L$$

$$\mu_t b_t + (1-\mu_t)\delta E V_{t+1}(r_t, s_t)$$

$$\geq K_t + \delta E V_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) \qquad (IC_t^R L(Liq))$$

$$\mu_t[L - b_t] \geq K_t \qquad (PC_t^C F(Liq))$$

$$L - b_t \geq (p_t - vq_t)R(1-x_t) - I \qquad (IC_t^C F(Liq))$$

$$if\ \ (r_t, s_t) \in \Delta_S^C$$

$$E_t V_{t+1}(r_t, s_t) = 0$$

Lemma 1.5.2. *There exists a unique Markov sequential equilibrium in the dynamic contracting game.*

Proof. Similar to Lemma 1, the Bellman equation satisfies monotonicity and discounting properties with the discount factor $\delta(1-\mu)$, hence is a contraction mapping by Blackwell's sufficiency conditions (Theorem 3.3 in [132]) . Then, by contracting mapping theorem (Theorem 3.2 in [132]), it has a unique solution. □

Now let us find the optimal contracting terms.

At every period, by the same logic as in the two period example, the participation constraint for CF holds as an equality, so

$$
\begin{aligned}
&if\ (r_t, s_t) \in \Delta_D \\
&\qquad x_t = 1 - \frac{1}{R(p_t - vq_t)}\left(I + \frac{1}{\lambda_t}\right); \\
&\qquad b_t \geq L - \frac{1}{\lambda_t}
\end{aligned}
\tag{1.16}
$$

and

$$
\begin{aligned}
&if\ (r_t, s_t) \in \Delta_L \\
&\qquad b_t = L - \frac{1}{\lambda_t} \\
&\qquad x_t \geq 1 - \frac{1}{R(p_t - vq_t)}\left(I + \frac{1}{\lambda_t}\right);
\end{aligned}
\tag{1.17}
$$

From 1.16, we can observe how the contracting terms facilitate ambiguity sharing among the ambiguity neutral RL and ambiguity averse CF.

Then, the optimal stopping regions are[13] given by the following proposition.

Proposition 1.5.3. *The strategic alliances develop the project after being granted patent if* $(r_t, s_t) \in \Delta_D$*, liquidate the project after being patented if* $(r_t, s_t) \in \Delta_L$*, and*

[13] Note that due to the linearity of the signal structures, the stopping decision does not depend on the investment amount at the last period.

abandon the project forever if $(r_t, s_t) \in \Delta_S^C$*, where*

$$\Delta_D = \left\{ (r_t, s_t) \in \Delta_H | \; \left[p_t R - \frac{p_t}{(p_t - v q_t)} \left(I + \frac{1}{\lambda_t} \right) \right] \geq L - \frac{1}{\lambda_t} \right\}$$
$$\Delta_L = \left\{ (r_t, s_t) \in \Delta_H | \; \left[p_t R - \frac{p_t}{(p_t - v q_t)} \left(I + \frac{1}{\lambda_t} \right) \right] < L - \frac{1}{\lambda_t} \right\}$$
$$\Delta_S^C = \left\{ (r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} | \; L < \frac{2}{\lambda_t} \right\}$$

Let T *be the optimal stopping time:*

$$T := \inf\{t | (r_t, s_t) \in \Delta_H \cup (r_t, s_t) \in \Delta_S^C\}$$

Proof. In Appendix A. □

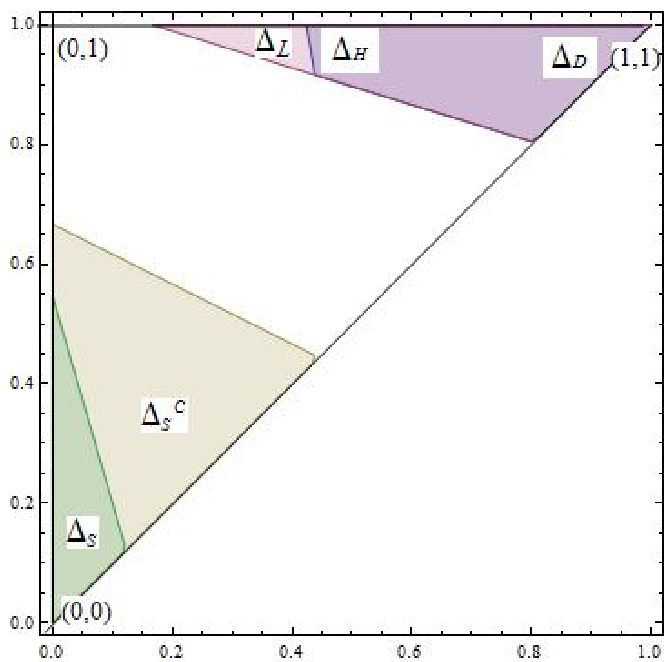

Figure 1.6: Contractual Equilibrium

Now we will turn to the funding pattern. We need to characterize the optimal investment schedule to answer the questions:

a) is it possible that the project will obtain full funding till the end, i.e. till the time the posterior $(r_t, s_t) \in \Delta_S^C$,

b) if full funding is not available at all times, how does the funding flow evolve over time?

To examine the funding flow, first let us look at the incentive constraint RL faces at any t.

If $(r_t, s_t) \in \Delta_D$, the dynamic incentive constraint is:

$$\mu_t p_t R x_t + (1-\mu_t)\delta EV_{t+1}(r_t, s_t) \geq K_t + \delta EV_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t)$$

Substituting for the optimal share x_t from 1.16, rewrite it as:

$$\mu_t p_t \left(R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right) + (1-\mu_t)\delta EV_{t+1}(r_t, s_t)$$
$$\geq K_t + \delta EV_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t)$$

Now, the dynamic expected payoff to be collected by RL in future periods following a diversion can be expressed as:

$$EV_{t+1}(r_{t-1}, s_{t-1}, r_t, s_t) = \frac{\lambda_{t-1}p_{t-1}}{\lambda_t p_t} EV_{t+1}(r_t, s_t)$$

So, the dynamic IC can be rewritten as:

$$\mu_t p_t \left(R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right) - K_t \geq \delta \left[\frac{\lambda_{t-1}p_{t-1}}{\lambda_t p_t} - (1-\mu_t)\right] EV_{t+1}(r_t, s_t) \quad (1.18)$$

where the RHS captures the dynamic agency cost.

Similarly, if $(r_t, s_t) \in \Delta_L$, the dynamic incentive constraint can be rewritten as:

$$\mu_t \left(L - \frac{1}{\lambda_t} \right) - K_t \geq \delta \left[\frac{\lambda_{t-1}}{\lambda_t} - (1 - \mu_t) \right] EV_{t+1}(r_t, s_t) \tag{1.19}$$

and it does not depend on $CF's$ ambiguity aversion.

We show that under a sufficient condition on the initial parameters, the project will never receive full funding till the end. In that case, there will be a switching point, captured by a range of posterior beliefs such that if the posterior belief lies below that locus then full funding is no longer available. Then, we show that for the range of posteriors where full funding is not available, the funding volume decreases with posterior belief over time. Also, as CF becomes more ambiguity averse, the dynamic moral hazard problem is alleviated, resulting in a longer horizon of full funding. The investment pattern is characterized by the following proposition.

Proposition 1.5.4. *The project does not receive full funding till the end if:*

$$\lambda_0 < \frac{2 - \delta}{L \left(1 - \frac{\delta}{2} \right)} \tag{1.20}$$

If 1.20 holds, then there is a region of posterior beliefs Δ_F where the project does not receive full funding:

$$\Delta_F := \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} \backslash \Delta_S^C | \text{ the region of posterior beliefs where full funding is not available}\} \tag{1.21}$$

Then, there exists a δ_L such that *a) If* $\frac{\lambda_0 L-2}{\lambda_0\left(\frac{L}{2}+\overline{K}\right)-1} \leq \delta \leq \delta_L, \Delta_D \cap \Delta_F = \phi$*; so full funding is available for all* $(r_t, s_t) \in \Delta_D$.

b) If $1 > \delta \geq \delta_L, \Delta_D \cap \Delta_F \neq \phi$*; the project does not receive full funding for all* $(r_t, s_t) \in \Delta_D$*. In this case, as* v *increases, the project receives full funding for a longer time horizon, i.e.* $\Delta_D \backslash \Delta_F$ *expands.*

After full funding stops, investment volume monotonically decreases over time.

Proof. In Appendix A. □

From the proposition 1.5.4, we observe how the different components of the model interact with each other to determine the investment level.

1. Discount factor (δ) : For higher discount factor, $\delta \geq \delta_L$ full funding horizon shrinks. There exists a range of posteriors for which the project is developed after being patented, still only restricted funding is available. This is intuitive because as RL becomes more patient, he values the future gains more, so the dynamic moral hazard problem is more severe and the incentive constraint is more difficult to hold. As a result, only partial funding is available for a large range of posterior belief.

2. Prior belief (r_0, s_0) : If the prior belief that the true state $\theta = Good$ is high, i.e. initially the belief about the profitability of the project is favorable enough, the project can receive full funding till the end.

3. Ambiguity aversion coefficient (v): If CF is more ambiguity averse (v increases) the dynamic moral hazard problem is alleviated. The intuition is similar to the two period example. CF, being ambiguity averse, becomes much more cautious and pessimistic after each low signal. So, following a low signal, CF has to be guaranteed a greater share of the final return in order to keep investing. Thus, the contractual terms sharing ambiguity also discipline RL and lower his dynamic expected value from cheating which, in turn, eases the funding constraint towards the beginning. Thus, if the project receives full funding till $\Phi_D(r_t, s_t) = 0$, as v increases, full funding horizon increases. After the project stops receiving full funding, the investment flow is monotonically decreasing over time. This result is in contrast with the result in [17], where it is possible to have monotonically increasing investment pattern over time due to the severity of the dynamic agency problem.

1.6 Policy Recommendations

In this section, we will compare the equilibrium outcome of the strategic partnerships to the Policymaker's optima derived in section 3.2. Notice that in the contractual scenario, there are three possible sources of deviation from the RAN outcome, i.e. the risk and ambiguity neutral Policymaker's preferred outcome. Firstly, the static and dynamic moral hazard can potentially distort the incentives and make it harder for the project to obtain funding at every period, thereby creating a divergence from the optima the Policymaker intends to implement. Also, the presence of ambiguity and $CF's$ ambiguity aversion creates a divergence in preferences among the strategic alliance and the Policymaker, thus contributing to the difference from the RAN optima. Lastly, the short term contracting and lack of commitment can result in the contractual outcome being different that the RAN optima. Let us first examine how the two outcomes differ and then we will analyze the effects of each of these possible sources of inefficiencies.

The Policymaker's optimal value from the project is given by:

$$V_0^S = E_0\left[\sum_{t=1}^{T_S}\delta^{t-1}\left(\Pr_t((r_t,s_t)\in\Delta_H)(p_tR-I)+\Pr_t((r_t,s_t)\in\Delta_S)L-\overline{K}\right)\right] \quad (1.22)$$

whereas the Policymaker's value from the project carried out by the strategic partnership is given by:

$$V_0^{SC} = E_0\left[\begin{array}{c}\sum_{t=1}^{T}\delta^{t-1}[\Pr_t((r_t,s_t)\in\Delta_D)(p_tR-I)+\Pr_t((r_t,s_t)\in\Delta_L)L \\ -(1-\Pr_t((r_t,s_t)\in\Delta_F))\overline{K}-\Pr_t((r_t,s_t)\in\Delta_F)K_t]\end{array}\right] \quad (1.23)$$

The contractual outcome diverges from the Policymaker's outcome in three ways:

(a) *Patent Troll:* If the posterior belief $(r_t, s_t) \in \Delta_L \subset \Delta_H$, the risk and ambiguity neutral Policymaker finds it optimal to develop the product, but because of $CF's$ ambiguity aversion $v > 0$, the strategic partnership liquidates the product even after being granted patent. So, every time the posterior lies in this region, there is a loss of value $p_t R - I - L > 0$ to the Policymaker. This loss is attributed to the difference in ambiguity attitude of the Policymaker and CF.

(b) *Less experimentation:* The Policymaker optimally stops experimentation and abandons the project as soon as the posterior belief enters Δ_S, while the research alliance abandons it when the posterior lies in Δ_S^C. Algebraically, it can be shown that $\Delta_S \subset \Delta_S^C$, so the research alliance abandons the project for a larger range of posterior beliefs, compared to the Policymaker. This result is due to the short termism, lack of commitment power of the research alliance, and the moral hazard problem.

(c) *Partial Funding:* The Policymaker optimally invests the maximal funding in the project till the end, whereas the research partnership, if the prior belief is not too high (if 1.20 is not satisfied), does not receive full funding till the end. The lower investment flow is driven by the static and dynamic moral hazard problem, which makes the incentive constraints harder to satisfy. However, as we have noted in Proposition 1.5.4, dynamic moral hazard problem is alleviated as v goes up, causing the project to receive maximal funding for a longer time horizon.

The next proposition captures how the equilibrium contractual outcome diverges from the Policymaker's optimal outcome.

Proposition 1.6.1. *Compared to the Policymaker's optima, the equilibrium contracts governing the research alliances result in (a) liquidation of the project even after being patented, (b) less experimentation, and (c) lower investment flow.*

Proof. In Appendix A. □

The following figure (Figure 1.7) illustrates the difference between the two outcomes.

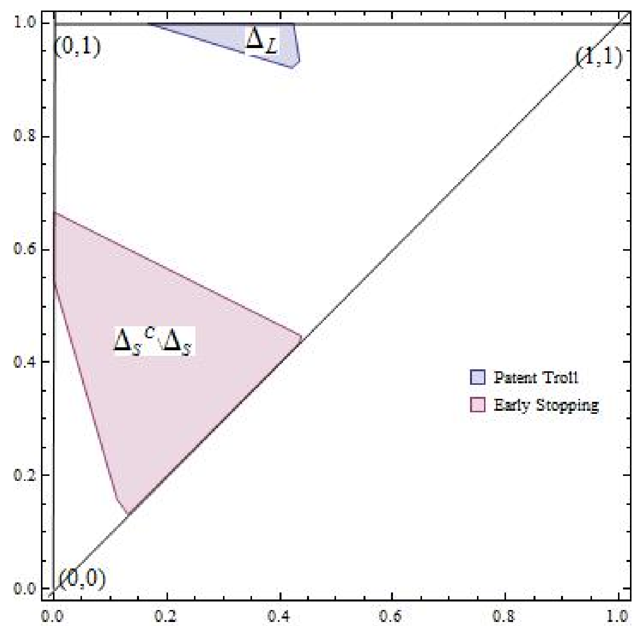

Figure 1.7: Two Sources of Welfare Loss

Given that the contracts governing the strategic partnerships fail to implement the Policymaker's optima, next we examine if the Policymaker can restructure the patent law in order to implement its desired optima. Specifically, if the patent law is designed to internalize the possible response from the research alliances, is it possible to alleviate the three sources of inefficiency discussed above? Analyzing the effects of changing the patent law, we find that if the patent law is made stricter, i.e. Δ_H is set at a higher level, it will shrink Δ_L, so it is less likely that the project will be liquidated after being granted patent. However, this would lower the incentive to experiment as well, because $\Pr_t((r_t, s_t) \in \Delta_H)$ decreases, causing the research alliance to abandon the project even earlier (for a larger range of posteriors) than before. In fact, setting $\Delta_H = \Delta_D$ eliminates the possibility of patent troll, but increases the range of posteriors for which the project is abandoned forever; i. e. , Δ_S^C expands.

On the other hand, if the patent policy is relaxed, that boosts the incentive to invest in the project, increasing $\Pr_t((r_t, s_t) \in \Delta_H)$ at every period, and results in longer experimentation and higher level of investment. However, it also results in an expansion of Δ_L, so patent troll problem becomes more severe. Thus, changing the patent law can never fully implement the Policymaker's optima and eliminate all three sources of efficiency. If initially Δ_L is large, i.e. patent troll is a severe problem to start off with, then making the patent law more stringent benefits the Policymaker more, whereas if the inefficient stopping proves to be a more severe concern, then relaxing the patent policy would be beneficial. So, depending on the initial parameter values, the patent policy should be redesigned to consider the possible effects on the innovation conducted in the strategic alliances.

1.7 Generalizations

First, we will discuss how the model behaves under a few possible extensions and alternative assumptions.

Non-linear signal

In this model, we used the simplifying assumption of linearity in the signal structure. This resulted in the Policymaker's optima characterized by full funding at all times.

With a more general signal structure satisfying only the Assumptions 1-3, we can characterize the optimal contractual outcome as well as the Policymaker's optima using similar technique. Instead of full funding, the optimal outcomes are characterized by a partial investment flow that decreases over time for the Policymaker as well as the strategic partnership. The regions Δ_D, Δ_L, and Δ_S^C can be characterized likewise. The main results qualitatively stays the same:

(a) $\Delta_L \neq \phi$, so Patent Troll happens if posterior lies in Δ_L.

(b) Optimal funding in strategic alliances decreases with time. As v increases, dynamic moral hazard is alleviated.

(c) Restructuring the patent law can not implement the Policymaker's outcome.

It is also interesting to examine a more general signal structure instead of the binary signal discussed in this chapter. Indeed, in some real life contexts, the information flow that arrives at each period of experimentation can not be encoded into a simple binary signal. For example, assuming a continuous signal structure will generalize the model and consequently change the optimal contract structure.

No Limited Liability of RL

In the present model, the research lab is assumed to be liquidity constrained, thus always requires non-negative payment in each period. However, in many real life scenario, the research based firms, though smaller in comparison to the commercial giants, can afford to put forth some investment, in the form of collateral , in order to continue experimentation even after clearing the patent thresholds.

Under this assumption, experimentation continues even after clearing the patenting threshold, the patent troll region shrinks, and the alliance experiments longer.

Long Term Contracts

In some situations, firms can attain commitment power through brand reputations, press releases and a variety of other ways. If the contracting parties can commit to long term relations, the participation constraint of CF will not have to be met in every period, so intertemporal transfer of payments will be possible. This relaxes the funding condition at every period and results in longer experimentation. In this case, experimentation may continue even after being granted a patent and the patent troll region shrinks. It is interesting to compare the optimal outcome in long term contracting with the one in this chapter and analyze the effect of commitment.

Partially Observable Signal

In many scenario, the informative signal is not publicly revealed. Sometimes, the financing firm hires experts to evaluate the reports given by the research firm, whose evaluation criteria varies from the research firm. It is also possible that the results from the experimentation can be mis-reported. In these cases, the assumption that the signal at each period is publicly observed breaks down. A very interesting question will be to characterize the contract under this partial observability and possible mis-reporting of the signals, using a mechanism design approach to this contracting environment.

1.8 Discussion

In the innovation intensive industries, we observe that research partnership is increasingly becoming an important mode of organizing research. The results from this chapter suggest that the policy making organizations should recognize the fact and be aware of how the innovation activity conducted in the research alliances is affected by the patent policy. Using the predictions from the theoretical model, we observe that relaxing the patent criteria is likely to result in longer experimentation, but at the same time the possibility of patent troll like cases increases; whereas if the patent law is made more stringent then the patented projects are more likely to be developed, but the research alliances stop experimenting inefficiently early. This result suggests that studying the present state of the industry, the patent authority should decide on the patent criterion.

Also, comparing the optimal contractual outcome and the Policymaker's optima, we can see that it is never possible to implement the Policymaker's optima. As the contextual ambiguity associated project increases, the divergence of the contractual outcome and the desired outcome increases. This suggests that the projects with high level of ambiguity can not be satisfactorily organized by research partnerships. Indeed, there can be projects, which the Policymaker deems profitable enough to invest in, that can be never funded in a research partnership. In innovative industries, the concern about important innovations not being carried out has long been voiced (Clayton Christensen, ITExpo, 2011). The industry's Internal Rate of Return Criterion and lack of foresight, are often blamed for not investing in innovative technologies.

This suggests a potential role of a regulatory body or the "State" as an entrepreneur. State intervention in innovation in the form of funding programs for smaller research oriented firms can support innovation organized in research firms. State programs for Small and Medium Enterprises (SMEs) and New Biotechnology Firms (NBFs) like Small Business Innovation Research (SBIR), 1982, Small Business Technology Transfer (STTR), 1992 have been able to fund numerous ventures by smaller research firms and touted as success(SBIR/STTR Impact Report, 2012). In the US, 57% of "basic research" is supported through Federal funding (2008) (source: NSF report, 2008). Programs such as these, providing funds to the research oriented smaller firms, lead to the development of the projects not otherwise funded (Mazzucato, 2013).

Another mode of organizing innovation when the research alliances can not efficiently carry it out is direct state initiative. There are several examples where State as an entrepreneur has participated in innovation and led to successful development of projects. In UK, Medical Research Council (MRC), funded by the Department for Business, Innovation and Skills (BIS) has been leading the Pharmaceutical innovation and was behind the development of monoclonal antibodies, widely used in Pharmaceutical industry since then. In the US, National Institute of Health (NIH) has been key funding source for research in Biotechnology, spending $30.9 bn in 2012 alone. Another example of State's entrepreneurial venture is National Nanotechnology Initiative (NNI), which, funded in 2000, strives to engage in cutting edge research in Nanotechnology. According to the famous adage by Polanyi (1944):

> "The road to the free market was opened and kept open by an enor-

mous increase in continuous, centrally organized and controlled interventionism."

1.9 Summary and Conclusion

Research alliances are responsible for a major share of innovation activity in the research-intensive industries. The innovation processes they undertake is often characterized by ambiguity rather than risk. Given the prevalence of these research alliances in these sectors, it is important to examine the optimal research outcome that is generated in these R&D partnerships, understand the strategic incentives of the contracting parties and how these interact to shape the optimal choices, and to evaluate the research alliance as a mode of organizing research in the ambiguous environment. This chapter provides a theoretical model to analyze these partnerships and compare it to the optimal outcome that a risk and ambiguity neutral Policymaker wants to implement.

In this chapter, we consider a dynamic principal-agent model with moral hazard where the contracting parties differ in their attitude towards ambiguity. The contractees use short term contracts to organize innovation in the research alliances. To model the ambiguous preference, I follow a dynamically consistent framework of ambiguity that uses Bayes rule to consistently update ambiguous belief. We focus on Markov sequential equilibrium to characterize the optimal contract in this model of strategic experimentation with moral hazard.

Analyzing the optimal sequence of short term contracts, we find that the contractual terms facilitate ambiguity sharing and thus prevents in-equilibrium delay. The investment flow that the project receives decreases over time. We have shown that the Policymaker's optimal outcome can never be implemented in the research alliances. This leads us to suggest policy recommendations regarding the patent law.

Apart from the different extensions and robustness issues mentioned in the previous section, this research can open up the path of further research on strategic partnerships. It will be interesting to study multi-lateral strategic partnerships in the innovation-based industries as networks and examine the optimal network structure that emerges under ambiguity with different parametric assumptions. Also, analyzing different patent policies in this context under ambiguity constitutes another interesting direction for future research.

Chapter 2

Venture Capital Investment Under Ambiguity

2.1 Introduction

Venture capital plays an important role in the financing of start-ups firms with potentially high-reward projects. It provides the young firms with a source of independent, professionally managed, dedicated pool of capital. This mode of financing has seen rapid growth since the 1970s. In the first quarter of 2013, 863 new deals were signed by the U.S. based venture capitalists, and the total fresh investment in that quarter amounted to $5.9 billion, 21% of U.S. GDP [115].

Venture capital (hereafter, "VC") financing is more prominent in Software, Biotechnology, and Clean Technology sector and 40% of the entire investment by the VCs is devoted to these sectors. Now, in these sectors, the projects in question are often unique in nature. So, sufficient amount of data from very similar situations are generally not available to form a precise estimate of the true profitability of the project. Thus, it is often difficult to form a unique single-valued probability measure about the profitability. Such situations can be modeled as "Knightian Uncertainty," or, "Ambiguity," using Knight's [103] definition.

In VC financing contexts, then, it can be assumed that only a partial description of the underlying probability distribution associated with the choices is known.

I capture this ambiguity by the assumption that the investors and entrepreneurs involved in VC financing have multiple priors about the true distribution. However, the entrepreneurs specialize in dealing with this ambiguity, so they are less ambiguity averse than the venture capitalists. In this chapter, I show that the presence of ambiguity and the difference in ambiguity attitude explains the allocation of control rights, which is a salient feature observed in VC financing.

One of the most important and well-discussed features of VC financing is the use of control rights. The financial contracts between the venture capitalists and entrepreneurs put a lot of stress on the allocation of control rights. Control rights typically include voting right, liquidation right, rights to choose the management team , etc., liquidation right being arguably the most crucial [5]. The commonly observed patterns regarding control rights found in venture capital financing are:

(A) Control rights are frequently contingent on the observable measures of financial and non-financial performance;

(B) If the project performs poorly, the VCs obtain full control. As the performance improves, the entrepreneur retains / obtains more control rights. If the company performs very well, the VCs retain their cash flow rights, but relinquish most of their control and liquidation rights [99].

While the role of control rights has been extensively discussed in the existing literature in Economics and Corporate Finance, only a few of them consider the dynamic structure of venture capital financing and the staged infusion of capital observed in this context. In VC financing context, the investment volume is often contingent on the observable performance measures. The existing static theories of control rights allocation which do not account for this staged infusion of capital can not fully explain the movement of control rights. In contrast, here I present a two-stage contracting model with staged financing, and show that the allocation of control rights can be explained by the presence of ambiguity and the difference in ambiguity attitude between the VC and the entrepreneur.

Formally, in this chapter I analyze the financing problem of a new venture characterized by

(i) two stages of investment (the “start-up” or “seed” stage, and “expansion” stage) with a public signal arriving in between the financing stages, and

(ii) an ambiguity-neutral entrepreneur (E) and an ambiguity-averse investor (VC), who share a common set-valued prior about the venture.

VC and E sign a two-period contract at the beginning specifying the funds to be invested in both stages and the future earnings[1]. However, the contract structure is inherently incomplete; it is not possible to specify a-priori the liquidation contingencies as a function of the intermediate signal.

In this environment I show that the differential attitude towards ambiguity may create a hold-up problem for the venture capitalist. The entrepreneur, being ambiguity neutral and protected by limited liability, always wants to continue the project. However, if the realized signal turns out to be “bad,” then after observing it

[1]Note that no revenue is generated before the end of the two stages and investment is mostly in intangible assets, with small liquidation value. Hence, bank-like financial contracts are not feasible.

the investor may want to liquidate the project. This gives rise to an agency conflict. Then, under entrepreneurial control, the investor may face a hold-up problem and is more reluctant to invest initially. In such a scenario, if the investor is very ambiguity averse, the entrepreneur is better off relinquishing control rights to the investor. Thus, this model explains the features of VC investment using the assumption that VCs operate in an environment of ambiguity rather than risk.

The famous success stories of venture capital financing are consistent with the results of our model. During 1970-80, Apple Inc. went through three rounds of venture capital financing, starting with \$.5m investment in round 1, with the investor having almost 50% ownership. In round 2, investment increased to \$.7m, and in round 3 in increased further to \$2.3m. FedEx also benefitted from venture capital financing ($1973-76$), where round 1 raised \$12m with the venture capitalist having 37.5% control rights but after an unfavorable performance signal, investment in round 2 decreased to \$6m, with the investor acquiring 84.5% ownership [124]. The prominence of control rights and the shift of capital flow are common in these success stories. In this chapter I attempt to explain these features, under the assumption of ambiguity.

The chapter is organized as follows. In the remaining part of this section, we will discuss some stylized facts of VC financing and how the model attempts to capture these features. In the next subsection we will examine a case study which motivates our framework. Section 2 contains a review of the existing literature. In Section 3 presents the model and the results. Section 4 analyzes the results and suggests some possible extensions. The concluding section summarizes the findings.

2.1.1 Features of Venture Capital Financing

- *Stage Financing:*

  In VC financing, the infusion of capital occurs in stages, matching investment decisions based on the information that arrives over time. In between these financing rounds, the venture capitalist monitors the short-term performance indicators to gather more information about the potential of the venture. This feature, according to many, is the "*most potent control mechanism*" of venture capital financing [124]. Typically, we observe that the capital flow is lower in the initial round of investment and later it may increase or decrease depending on the performnace measures.

  To capture this, we have two-period ("seed stage" and "expansion stage") model with the VC and the entrepreneur signing a contract at the beginning of the two stages. Capital flow is chosen ex-ante contingent on the interim signal. The results of the current model exhibits the evolution of the capital flow which is consistent with the observed phenomenon.

- *Monitoring:*

  The venture capitalists often identify important areas to monitor at the beginning of a project and engage in information collection and monitoring once the project is under way. This monitoring helps them to avoid any potential moral hazard problems and gives them an accurate idea of the intermediate performance measures of the project. This is why in this model we assume that the investment decisions are publicly observable and verifiable, so there is no moral hazard concern. The interim performance measure, modelled as a continuous signal, is also considered to be publicly observable in our framework.

- *Control Rights Allocation:*

  VC financing is characterized by separation of cash flow rights and control rights. In this mode of financing, control rights are not associated with assets; they are used as an independent instrument aimed to complement the cash flow rights. The allocation of control rights is frequently contingent on the observable measures of financial and non-financial performance. If the project does not show signs of success, the VCs obtain full control. As the project performs well, the entrepreneur is able to retain and obtain more control rights. This feature strongly suggests that despite the prevalence of contingent contracting, VC financing contracts are inherently incomplete.

  In this chapter, I seek to explain this movement of control rights. In order to do so, I use the incomplete contracting approach and assume that control rights can not be specified ex-ante. I consider liquidation rights as the only aspect of control rights. This theoretical abstraction simplifies the contracting environment and helps us to understand the interaction of the allocation of control rights and the contextual ambiguity.

- *Ambiguity in VC Financing:*

  VC financing operates in an environment where the true probability distribution governing the innovation process is only partially known. To understand the presence of contextual ambiguity, let us examine the following real life example of VC financing.

2.1.2 *Motivating Example: Eli Lily and Capital Funds Portfolio*

Biotehnology is one of the major industries which depend heavily on VC financing. In 2010, in this sector the venture capitalists contributed $3.7 billion in 460 deals.

Eli Lily, a leading Biotechnology firm, has raised funds from the venture capital firm Capital Funds Portfolio for carrying out research in finding a medicine for Alzheimer's disease. The lead compound to be tested is LY2062430 (solanezumab) which is a biologic entity that binds to soluble monomeric forms of amyloid ß (Ab) after it is produced. LY2062430 is being studied for its potential to slow the progression of Alzheimer's disease. This disease affects the patients in two ways: it destroys the patients' *cognitive* ability, and at the same time it also impairs the *functionality* of the patients' brain and other organs. Any drug claiming to "cure" Alzheimer's disease has to show significant improvement in delaying *cognitive* and *functional* endpoint. While testing for cognitive improvement by carefully designed large scale trials yields more reliable results, functional improvement can not be tested reliably given the current designs. Lily has carried out two large scale trials in Phase II and in the pooled secondary sample, has registered a significant improvement in delaying the cognitive endpoint, but the functional improvement has not been much tested. So, after their entire research on this drug solanezumab, the project may end in one of the three ways:

(a) A new technique to test the functional improvement is invented before Phase III is completed and the drug solanezumab is shown to significantly improve functionality. In this case the true state or profitability of this project is definitely "*Good*."

(b) A new technique is invented to test the functional improvement which conclusively shows that this particular drug has no effect on functionality of the patients' brains. This case can be identified with the true state being "*Bad*."

(c) The medicine can produce results showing a significant cognitive improvement but inconclusive evidence about the functional end-point delay. No new technique to test the functional improvement is invented, and in absence of enough indication that the drug improves functionality, FDA does not grant permission to go ahead in research. Now, at this state. it is possible that the reason behind this confusing results is the lack of a carefully designed medical test to assess functional improvement. It is possible that in future, once this test is devised, this medicine may prove to cure Alzheimer's disease. But at the same time, it is also possible that this drug has no effect in functional improvement associated with Alzheimer's disease[2]. This state can be considered as the "*Unknowable*" or "*Amalgamated*" state, where the true profitability of the research venture is yet unknown.

This new epistemic state, where the causal interpretations are not fully understood, captures the idea of ambiguity in this framework of ambiguity by Dumav and Stinchcombe [54]. As we will discuss in Section 3, it can be considered as an

[2]Since most of the scientists claim that functional improvement is the key to curing Alzheimer's disease, this will be a bad news for the project.

alternative interpretation of the multiple prior model, where the range of prior beliefs reflect the initial belief that the true state will turn out to be "*Unknowable.*"

In the context of this example, the two players: Eli Lily (E) and Capital Funds (VC) start the venture containing multiple stages. Even after ending up at the "Unknowable" state, Eli Lily learns about the disease in general, which may help them in a future project on this same disease. So, Eli Lily like this "Unknowable" state more than Capital Funds Portfolio does. Hence, we can consider Lily as less ambiguity averse than Capital Funds.

After each Phase, the observed data is examined and tested if there is a significant difference compared to a placebo. The standardized reduction in cognitive and functional end-point delay serves as a signal. This signal, drawn from a continuous distribution, induces posterior belief about the true state. So, after the start up stage, the signals are revealed, but the uncertainty prevails. Only after the expansion stage the true state is realized and returns accrue. The theoretical model presented here attempts to capture this environment and explain the allocation of control rights in such a venture operating under ambiguity.

The contract, signed at the beginning of the two stages, specify (a) the cash flow rights once the true state is finally observed, (b) the investment volume in stage 1 and stage 2 (which depends on the interim results), and (c) the liquidation rights.

Now, the ambiguity-averse investor, unlike the ambiguity-neutral Lily, is more *pessimistic* about the first round results. If the results from the first round turn out to be bad according to the investor (his net return from continuing investment is negative), he may want to opt out of this project before funding another round. However, Eli Lily, protected with limited liability and attaching a higher value to the "Unknowable" state, most often does not agree to give up. This results in a hold-up problem, which potentially leads to some viable projects not to be financed.

To solve this problem, at the beginning, the parties can allocate control rights, which gives the authority to liquidate the project after the first round. Clearly, if the venture capitalist holds the control rights, he quits the project after observing a "bad" result. Under certain conditions, relinquishing control rights to the venture capitalist actually makes the scientist better off in ex-ante expected terms.

2.2 Literature Review

There is an extensive literature discussing control theories in financial contracts. The incomplete contracts approach, pioneered by Hart and Moore [86], Aghion and Bolton [4] develop the theory of control rights based on incompleteness of contracts. In spirit, our model is the closest to the chapter by Aghion and Bolton [4]. They show the optimality of state-contingent allocation of control rights under contractual incompleteness. They consider a one-period investment scheme with fixed investment requirement, where uncertainty ("risk") is resolved in an interim stage before taking action. In their framework, the risk-neutral principals face hold-up issues because of non-verifiability of action. In this context they show that, if renegotiation is allowed, then state-contingent control allocation achieves the first best outcome. In contrast, in the current chapter I introduce ambiguity as the source of agency conflict. In this chapter I assume that uncertainty about the true state persists till the end of the game. Also, this model relaxes the fixed investment requirement and considers stage financing, where investment is allowed to depend on the performance indicators of previous round. In such an environment with contingent capital flows, agency conflict can arise only through the difference in ambiguity attitude. Thus, this chapter attempts to explain control rights in a dynamic contracting environment under ambiguity.

Incomplete contracts are used in various studies to explain control rights. Berglof [19], Hellman and Puri [89], and Gebhardt and Schmidt [69] interpret control right as the right to replace the management. With assumptions similar to Aghion and Bolton [4], they show that allocation of control right can be used to solve the moral hazard problem that arises due to non-verifiability of action[3]. In the chapter, I do not consider moral hazard. Also, I use a unidimensional control right, which is interpreted as liquidation right instead of considering the right to chose the management So, in my model the agency conflict occurs regarding liquidation decision.

Secondly, this chapter is related to the literature on the financing of innovation (*e.g.* , [53], [16], [17]). These papers consider infinte-horizon investment problems characterized by moral hazard, where the principal (venture capitalist) chooses the optimal stopping time. They find that short term contract results in premature stopping, and the venture capitalist invests the maximum amount as long as the project continues. In this model, in contrast, I consider a finite horizon investment scheme under ambiguity and our central focus is to explain the allocation of control rights in venture capital financing.

A growing body of literature studies venture capital investments using empirical methodology. Papers like [99], [100], [12], [19], [77], [79], [98], [124] provide important evidence about the working of the VCs and how they add value. They also examine the contractual structures adopted in VC financing. These papers thus motivate our framework and identifies the importance of control rights in VC financing. Specifically, Kaplan and Stromberg [99] examine how well the current theoretical work on VC financing fits the observed phenomena and identifies the caveats in the theoretical predictions. They find that while the incomplete contract approach (pio-

[3]For an extensive review, see [135], [15] and [5].

neered by [4], [86]) seems to be most consistent with the observed regularities, there is a need for a unifying theory to explain control rights in a dynamic environment with contingent capital flows. In this chapter, we have a two period framework and seek to explain how control rights are allocated in the contractual setting. Our results largely support the empirical findings: the optimal evolution of the capital flows and control rights match the real life data.

This chapter is also related to the strand of literature on optimal security design in venture capital financing (*e.g.* , [121], [129], [44], [34]). These papers focus on the incentive properties of the conditional allocation of cash flow rights and assess the performance of different financial instruments like debt, equity and convertibles in situations characterized with both-sided moral hazard. This chapter talks about compensations in a much more abstracted way, but we discuss how the optimal contract derived in our model can be implemented using financial instruments. The sharing rule in our optimal contract resembles the commonly observed financial mechanisms such as preferred stock options.

Lastly, following Gilboa and Schmeidler's seminal work on ambiguity [72], multiple prior models of ambiguity have been applied to various dynamic decision making contexts. However, with multiple priors, prior-by-prior updating of belief using Bayes rule usually leads to dynamic inconsistency. There are different approaches to modelling ambiguity averse preferences in dynamic setting. Some papers take the approach that deals with recursive extensions (*e.g.* , [58], [111], [102]), others posit dynamic inconsistency and adopt assumptions, such as backward induction or naive ignorance of the inconsistency, to pin down behavior (e.g. Siniscalchi, 2008) , yet another approach uses non-consequentialist updating rules (Machina [112])[4]. In this chapter, we use the ambiguity framework developed in Dumav and Stinchcombe [54], which characterizes a vNM approach to ambiguity and uses Bayes rule to obtain dynamically consistent updating of beliefs. Thus, this chapter fits in the literature of *decision making with ambiguity* in a dynamic framework.

The most important contribution of this chapter is the introduction of ambiguity in the dynamic investment environment operated by venture capitals and analyzing its role as a source of agency conflict and allocation of control rights. The model incorporates the aspects of stage financing and obtains an optimal investment path which is consistent with the empirical findings.

[4]For a more complete survey, refer to [61].

2.3 Model and Analysis

This section describes the ingredients of our model of venture capital finance. In the first part, I will lay out the model: describe the agents, the time-line, the environment and the nature of agency conflict. In the next subsection I will use an example of binary signal structure to illustrate the effect of ambiguity on the allocation of control rights. Next I will present the results for the general model.

- *States:*

  The innovation activity is centered around a project, success of which depends on the true state or true profitability of the project: $\theta \in \Theta$. The true state is not known; moreover, it is not possible to form a single probabilistic assessment about it. In a multiple prior setting,

  $$\Theta = \{Good, Bad\}$$
  $$\Pr(\theta = Good) = [a, b] \quad ; 0 \leq a < b \leq 1.$$

  Using the framework of ambiguity developed in [54] (described in greater detail in Appendix A), we observe that the interval $[a, b]$ has a unique representation as a convex combination of extreme sets given by $\Theta' = \{Good, Bad, Unknowable\}$, where the new epistemic state "*Unknowable*" is motivated in the previous examples.

  $$[a.b] = a[1, 1] + (1 - b)[0, 0] + (b - a)[0, 1]$$

  The state *Unknowable* can be represented as $[0, 1]$, the state at which one knows only that the probability of $\theta = Good$ is someplace between 0 and 1.

Thus, in this framework, we can alternatively represent this set-valued prior by a three state expected utility model, where the true state of the project lies in Θ' :

$$
\begin{aligned}
\Theta' &= \{Good, Bad, Unknowable\} \\
\Pr(\theta = Good) &= a \\
\Pr(\theta = Bad) &= 1 - b \\
\Pr(\theta = Unknowable) &= b - a \\
0 \leq a &< b \leq 1.
\end{aligned}
$$

That is, with probability a, at the end it will be revealed that the project is profitable, with probability $1 - b$ it will be revealed that the project is not profitable, but with probability $b - a$, the true profitability of the project will turn out to be "*Unknowable*," or, Not Yet Known, depending on the current state of technology and knowledge.

If the payoff for $\theta = Good$ is u_G, for $\theta = Bad$ is $u_B < u_G$, then the payoff associated with the new state $\theta = Unknowable$ is computed as:

$$u(\theta = Unknowable) = \frac{1}{2}(u_G + u_B) - \frac{v}{2}(u_G - u_B);$$

where the ambiguity aversion parameter v captures the attitude towards ambiguity. $v = 0$ implies ambiguity neutrality; $v > 0$ indicates that the decision maker is ambiguity averse, or, dislikes the state $\theta = Unknowable$. The higher v is, the more the decision maker dislikes the state $\theta = Unknowable$, hence can be considered as more ambiguity averse. Here, I assume $v \in (0, 1)$.

- *Players:*

  There are two decision makers: an entrepreneur (hereafter "E") and a venture capitalist (hereafter "VC") making choices in a dynamic investment scenario. E owns a project, so has the full bargaining power[5]. Both the parties start off with the same prior about the true state as described above. However, VC is more ambiguity-averse than E. For simplification, assume that E is ambiguity neutral whereas VC is ambiguity averse with the ambiguity aversion parameter as $v \in (0, 1]$.

- *Time-line:*

  The investment project takes two periods to complete: the "seed stage" and the "expansion stage." The return from the investment is stochastic; it depends on the investment amounts as well as the true profitability of the project: θ.

  After the first round of investment, an informative signal is publicly revealed. Due to the irreversible nature of the investment, the contract has to be signed at the beginning. This long term contract specifies

  (a) K_1 : the amount to be invested in period 1 or the start up stage,

  (b) $K_2(\cdot)$: the amount to be invested in period 2 or the expansion stage, as a function of the intermediate signal,

  (c) t : the monetary transfer to E after the final return has been realized, and

  (d) $\alpha \in \{0, 1\}$: the allocation of "control rights" or "liquidation rights."

[5]This is a simplifying assumption. Considering a Nash bargaining solution with both the parties having intermediate bargaining power qualitatively does not change the results.

Here we assume contractual incompleteness, because of which the liquidation rights can not be pre-specified. The party who holds the control rights decides whether to liquidate the project after the public signal is realized.

The time-line is as follows. The entrepreneur offers the investor a contract. The venture capitalist can accept or reject this offer. If he rejects the offer, he will receive his outside option worth $L > 0$. If the offer is accepted, the project gets underway and K_1 amount is invested. At the end of the first period, an informative signal is publicly revealed. After observing the signal, the party endowed with control rights chooses whether to continue or liquidate the project. Liquidation yields a return of 0[6]. If the project is continued to the second period, the pre-determined amount $K_2(\cdot)$ is invested depending on the signal. At the end of the second period, the true state θ is revealed, final return π is realized and the profits accrue according to the contract.

The time-line is given below (Figure 2.3):

- *Signal Structure:*

  After the first round of investment, an informative signal R is publicly revealed. This signal can be thought of as an independent random draw from a distribution whose parameter depends on the true state θ. Real life examples of such signals include the first stage profits, market research, or the seed stage results.

  Thus, the signal R induces a posterior distribution over the true state

$$\Pr(\theta = Good|\ R) = [r, s]$$

[6]The results remain same if we assume a positive liquidation value.

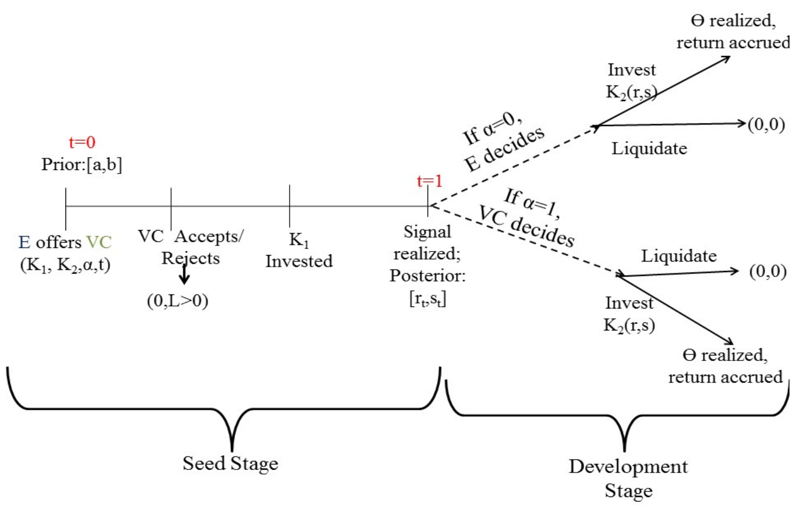

Figure 2.1: Time Line

where the posterior is drawn from a non-atomic continuous distribution $Q(r, s; \theta)$ such that

$$\iint_{(r,s)\in K_{\Delta(0,1)}} (r, s)dQ(r, s; \theta) = (a, b)$$

For notational simplicity, we use:

$$\frac{r+s}{2} = p = posterior\ mean$$
$$\frac{s-r}{2} = q = posterior\ ambiguity$$

and

$$\frac{a+b}{2} = p_0 = prior\ mean$$
$$\frac{b-a}{2} = q_0 = prior\ ambiguity$$

Right now we do not put any further assumption on the signal distribution.

- *Investment Project:*

  The project yields a positive return according to a known investment function if $\theta = Good$, but if the state is *Bad* then the project fails, yielding a normalized return of 0. The final return in the event of success is given by $\Pi(K_1, K_2)$. Let us put some regularity conditions on the functional form of π.

  Assumption 1: The return function is strictly increasing and strictly concave in both the arguments. Moreover, it is additively separable in K_1 and K_2. *i.e.*

  ,

  $$\Pi(K_1, K_2) = \pi(K_1) + \pi(K_2);$$

  where $\pi^{''} > \pi' > 0 > \pi''$ for all K_1, K_2.

  Also, there exist $\overline{K_1} > 0$ and $\overline{K_2} > 0$ such that

  $$\lim_{K_1 \to \overline{K_1}} \pi'(K_1) = 1$$
  $$\lim_{K_2 \to \overline{K_2}} \pi'(K_2) = 1 \tag{2.1}$$

  So there is a maximum efficient scale of investment[7]; thus $K_1, K_2 \in [0, \max\{\overline{K_1}, \overline{K_2}\}]$.

  Assumption 2: The return function satisfies the Inada conditions, *i.e.*

  $$\lim_{K_i \to 0} \pi(K_i) \to \infty, \lim_{K_i \to \infty} \pi(K_i) = 0 \forall i = 1, 2 \tag{2.2}$$

  Assumption 3: The return functions satisfy:

  $$E_{t=0}[\pi(K_1) + \pi(K_2(a, b)) - K_1 - K_2(a, b)] > L \tag{2.3}$$

[7]In a single agent decision problem with observable states, the optimal investment amounts would be $\overline{K_1}$ and $\overline{K_2}$.

The separability in the arguments is a simplifying assumption and relaxing this will not qualitatively change the analysis. Assumption 2.3 puts some restrictions on the slope and curvature of the return function. The third assumption can be viewed as a viability condition, which says that even if the intermediate signal adds no new information, the expected return from the project under the optimal policy will always be greater than the outside option.

- *Contract*

The entrepreneur owns the project but has initially no wealth and seeks to obtain external funds to realize the project. Financing is available from a competitive market of venture capitalists, which is represented in the model by a single investor who can only accept or reject the take-it-or-leave-it offer made by the entrepreneur[8].

The set of ex-ante contracts includes all contracts specifying:

i) a control rights allocation:

An important feature of this contract is its inherent incompleteness, which is often observed empirically. In the relevant body of literature it is often observed that in the venture capital financing liquidation decision is the most frequent source of conflict[9]. In this model. the "control rights" is assumed to be unidimensional and identified with the liquidation rights. I define the

[8]Assuming a different distribution of bargaining power yields qualitatively similar results. The assumption that the entrepreneur has all the bargaining power exacerbates the possible hold-up problem faced by the venture capitalists.

[9]Kaplan and Stromberg (2003)

measure of control as:

$$\alpha \in \{0, 1\}$$

where $\alpha = 0$ indicates E holding the control rights and $\alpha = 1$ implies VC holds the control rights[10].

ii) a compensation schedule for the entrepreneur:

I adopt the convention that the venture capitalist is the residual claimant and the entrepreneur is compensated with a monetary transfer (t) once the true state is revealed. Because of the entrepreneur's limited liability, $t(\theta) \geq 0$. Clearly, the optimal contract always specifies $t(\theta = Bad) = 0, t(\theta = Good) > 0$. Abusing the notation, I denote $t(\theta = Good) = t$.

In the three-state interpretation, thus, the payoff of E after observing the signal is:

Payoffs of E

	Continue	Liquidate
$\theta = Good$	t	0
$\theta = Bad$	0	0
$\theta = Unknowable$	$\frac{1}{2}t$	0

Similarly, the payoff for the VC after observing the signal R can be captured by:

Payoffs of VC

	Continue	Liquidate
$\theta = Good$	$\Pi(K_1, K_2(r, s)) - K_1 - K_2(r, s) - t$	0
$\theta = Bad$	$-K_1 - K_2(r, s)$	0
$\theta = Unknowable$	$\frac{1-v}{2}[\Pi(K_1, K_2(r, s)) - t] - K_1 - K_2(r, s)$	0

[10] I do not consider joint control rights.

iii) investment schedule:

The contract specifies the amount to be invested in period 1: K_1 and the investment schedule upon observing the signal with posterior $[r, s]$: $K_2(r, s)$. This captures the flavor of stage financing, which is a unique feature of venture capital investments [99].

2.3.1 Binary Signal Example

First let us consider an example with binary signal structure with specific parametric environment, which will illustrate the effect of ambiguity on the allocation of control rights. While all other features of the model remain the same, this example considers a signal structure given by:

$$R \in \{H, L\} \;; \Pr(R = H) = \mu.$$

Starting with the prior $\Pr(\theta = \mathit{Good}) = [a, b]$, the posterior after observing R becomes:

$$\Pr(\theta = \mathit{Good}|R = H) = [r_H, s_H]$$
$$\Pr(\theta = \mathit{Good}|R = H) = [r_L, s_L]$$

Assume: $\frac{r_H + s_H}{2} > \frac{r_L + s_L}{2}$ and $\frac{s_H - r_H}{2} < \frac{s_L - r_L}{2}$.

Let us also assume a functional form of the return function, for the ease of solving explicitly.

$$\pi(K) = K^{1/2}$$

At the beginning, the contract specifies $(t, K_1, K_2(\cdot), \alpha)$.

Thus, ex-ante payoff of E can be calculated as:

$$\begin{aligned}
V_E &= \mu\left[r_H t + (1 - s_H)0 + (s_H - r_H)\frac{t}{2}\right] \\
&+ (1-\mu)\left[r_L t + (1 - s_L)0 + (s_L - r_L)\frac{t}{2}\right] \\
&= \mu\frac{r_H + s_H}{2}t + (1-\mu)\frac{r_L + s_L}{2}t \\
&= \frac{a+b}{2}t = p_0 t
\end{aligned} \tag{2.4}$$

Similarly, the ex-ante payoff for VC :

$$\begin{aligned}
V_{VC} = \mu\frac{r_H + s_H - v(s_H - r_H)}{2}&\begin{bmatrix} \Pi(K_1, K_2(r,s)) \\ -K_1 - K_2(r_H, s_H) - t \end{bmatrix} \\
+(1-\mu)\frac{r_L + s_L - v(s_L - r_L)}{2}&\begin{bmatrix} \Pi(K_1, K_2(r,s)) \\ -K_1 - K_2(r_L, s_L) - t \end{bmatrix} \\
&= (p_0 - vq_0)[\sqrt{K_1} - t] \\
&+\mu[(p_H - vq_H)\sqrt{K_{2H}} - K_{2H}] \\
&+(1-\mu)[(p_L - vq_L)\sqrt{K_{2L}} - K_{2L}]
\end{aligned} \tag{2.5}$$

If E holds the control rights, *i.e.* , $\alpha = 0$, E solves:

$$P_{\alpha=0}\begin{cases}
\max_{(t,K_1,K_2^H,K_2^L)}[p_0 t] \\
st \quad (p_0 - vq_0)[\sqrt{K_1} - t] \\
+\mu[(p_H - vq_H)\sqrt{K_{2H}} - K_{2H}] \\
+(1-\mu)[(p_L - vq_L)\sqrt{K_{2L}} - K_{2L}] \quad \geq L \\
t \geq 0 \\
K_1, K_2^H, K_2^L \in [0, \max\{\overline{K_1}, \overline{K_2}\}]
\end{cases} \tag{2.6}$$

Where the constraints are: (a) participation constraint for VC, and (b) limited liability condition for E. First note that since E has full bargaining power, the participation constraint for VC will always hold as an equality. So, the problem can alternatively be written as:

$$\max_{(K_1 K_{2H}, K_{2L})} p_0\sqrt{K_1} + \frac{p_0}{p_0 - vq_0}\left(\begin{array}{c} \mu(p_H - vq_H)\sqrt{K_{2H}} - \mu K_{2H} \\ +(1-\mu)(p_L - vq_L)\sqrt{K_{2L}} - (1-\mu)K_{2L} \\ -K_1 - L \end{array}\right) \tag{2.7}$$

We use first order approach to solve for the optima. Ignoring the non-negativity constraints, the FOCs take the form:

$$\frac{1}{2\sqrt{K_1}} = \frac{1}{p_0 - vq_0}$$
$$\frac{1}{2\sqrt{K_{2H}}} = \frac{1}{p_H - vq_H}$$
$$\frac{1}{2\sqrt{K_{2L}}} = \frac{1}{p_L - vq_L}$$

The optimal contract with $\alpha = 0$ then satisfies this set of FOCs and leaves the investor with zero net payoff. Optimal contract becomes:

$$\begin{aligned}
K_1 &= \frac{(p_0 - vq_0)^2}{4} \\
K_{2H} &= \frac{(p_H - vq_H)^2}{4} \\
K_{2L} &= \frac{(p_L - vq_L)^2}{4} \\
t &= \frac{(p_0 - vq_0)}{4} - \frac{L}{(p_0 - vq_0)} \\
&+ \frac{1}{8(p_0 - vq_0)} \left[\mu(p_H - vq_H)^2 + (1-\mu)(p_L - vq_L)^2\right]
\end{aligned}$$

From this optimal contract, we observe:

Remark 2.3.1. **Observation 1:** $K_{2H} > K_1 > K_{2L}$: Investment flow increases after a high signal and decreases after a low signal.

Observation 2: t decreases as v increases, *i.e.* , as VC becomes more ambiguity averse, he is compensated by a higher share of future profit. This shows that in this contractual environment, ambiguity sharing takes place[11].

Under this contractual agreement, after observing the signal, the net expected return (excluding the sunk cost K_1) for the investor from continuing is:

$$g^v_{VC}(r_i, s_i) = (p_i - vq_i)\left(\sqrt{K_1} + \sqrt{K_{2i}} - t\right) - K_{2i} \qquad (2.8)$$
$$\forall i = H, L$$

[11] $\frac{\partial t}{\partial v} < 0$ because $\frac{\partial}{\partial v} \frac{\left[\mu(p_H - vq_H)^2 + (1-\mu)(p_L - vq_L)^2\right]}{8(p_0 - vq_0)} < 0$

If this net expected return is negative, the investor faces a hold up problem after observing the signal, because he would want to quit but it is always weakly optimal for E to continue investing in the project because $t \geq 0$.

In this example,

$$g^v_{VC}(r_i, s_i) = \frac{(p_0 - vq_0)(p_i - vq_i)}{4} + \frac{(p_i - vq_i)^2}{4} + \frac{L(p_i - vq_i)}{(p_0 - vq_0)} - \frac{(p_i - vq_i)}{8(p_0 - vq_0)}\left[\mu(p_H - vq_H)^2 + (1-\mu)(p_L - vq_L)^2\right]$$

Next Lemma shows that $g^v_{VC}(r_i, s_i)$ is always non-negative if $v = 0$ (*i.e.* , under no ambiguity). So, under risk, agency conflict does not arise[12]. Here, the only source of agency problem is the contracting parties' differential attitude towards ambiguity. Under ambiguity, with $v > 0$, agency conflict can arise. If VC is sufficiently ambiguity averse, i.e. $v \geq \overline{v}$ for a $\overline{v} \in (0, 1]$, E would want to continue the project but VC is better off liquidating. In that case, control rights will be important to decide whether the project is continued till the end.

Lemma 2.3.1. $g^v_{VC}(r_i, s_i) \geq 0 \ \forall i = H, L$ *if* $v = 0$.

There exists a $\overline{v} \in (0, 1]$ *such that for* $v \geq \overline{v}$, $g^v_{VC}(r_L, s_L) < 0$.

Proof. In the Appendix B. □

[12]This is a consequence of the assumption that both the agents are risk neutral. There will be non-empty conflict regions if the venture capitalist is more risk-averse than the entrepreneur. However, conventionally, the investor, having more wealth than the entrepreneur, is assumed to be less risk-averse.

Using the following set of parameters

$$[a,b] = \left[\frac{1}{3}, \frac{2}{3}\right]; \mu = \frac{1}{2}; L = \frac{1}{50}$$
$$[r_L, s_L] = \left[0, \frac{1}{3}\right]; [r_H, s_H] = \left[\frac{2}{3}, 1\right]$$

The next figure (Figure 2.2) show that $g_{VC}(H) \geq 0$ for all $v \geq 0$, whereas $g_{VC}(L) < 0$ for $v \geq \overline{v} = .91$.

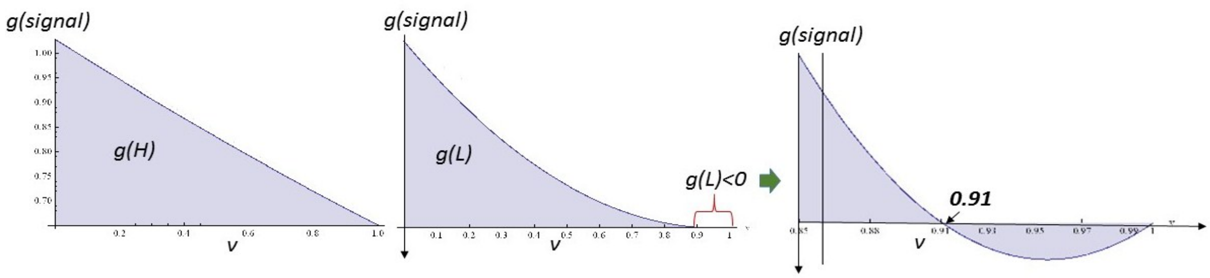

Figure 2.2: Net Return from $t = 2$: Binary Signal Example

The ex-ante expected value to E under $\alpha = 0$:

$$\begin{aligned} V_{\alpha=0} &= \frac{p_0(p_0 - vq_0)}{4} \\ &+ \frac{p_0[\mu(p_H - vq_H)^2 + (1-\mu)(p_L - vq_L)^2]}{4} - \frac{Lp_0}{p_0 - vq_0} \end{aligned}$$

Now, if VC holds the control rights, *i.e.* , $\alpha = 1$, after observing $R = Low$, he will abandon the project. So, under $\alpha = 1$, E solves:

$$P_{\alpha=1} \left\{ \begin{array}{c} \max_{(t_1, K_{11}, K_{21}^H)} [\mu[(p_H - vq_H)t_1] \\ st \quad \mu[(p_H - vq_H)\left(\sqrt{K_{11}} + \sqrt{K_{2H1}} - t_1\right) - K_{2H1}] \\ -K_{11}] \quad \geq L \\ t_1 \geq 0 \\ K_{11}, K_{21}^H \in [0, \max\{\overline{K_1}, \overline{K_2}\}] \end{array} \right.$$

Now, solving the FOCs similarly, we obtain:

$$K_{11} = \frac{\mu^2(p_H - vq_H)^2}{4} < K_1$$
$$K_{2H1} = \frac{(p_H - vq_H)^2}{4} = K_{2H}$$
$$t_1 = \frac{(p_H - vq_H)}{2} - \frac{L}{\mu(p_H - vq_H)}$$

Comparing the two solutions, we observe the following patterns of capital flow:

Observation 3: Under VC control, investment levels are non-increasing compared to E control.

Observation 4: Given the parameter values,

$$t_1 \gtreqless t \Leftrightarrow L(p_H - vq_H) \gtreqless \mu(p_H - vq_H)^2 + (1-\mu)(p_L - vq_L)^2$$

A sufficient condition for $t_1 > t$ is:

$$v > \frac{p_0 - Lp_H}{q_0 - Lq_H} = v_0$$

For the given parametric example, $v_0 = 0.61$.

Define the optimal ex-ante expected payoff of E from both these problems as:

$$V_E(\alpha = 0) = p_0 t$$

$$V_E(\alpha = 1) = \mu p_H t_1$$

If $v > v_0$, there is a trade-off in determining the optimal way to allocate the control rights. On one hand, ex-ante success probability decreases. However, on the other hand, the compensation in the event of success is also higher. We show in the following proposition that if VC is very ambiguity averse in nature, the effect of increase in t outweighs the possible liquidation loss and $V_E(\alpha = 1) > V_E(\alpha = 0)$. Comparing the ex-ante expected values of E under entrepreneur-control and VC control, we reach the following proposition. It identifies the range of ambiguity aversion of VC for which E obtains higher ex-ante expected payoff under $\alpha = 1$ than under own control, and hence it is optimal for her to relinquish control rights to VC.

Proposition 2.3.2. *There exists* $\widetilde{v} \in (0, 1)$ *such that, if* $v \geq \widetilde{v}$, *then* $V_E(\alpha = 1) \geq V_E(\alpha = 0)$, *so it is optimal for* E *to relinquish control rights to* VC.

In the numerical example discussed above, for all $v \geq \max\{\bar{v} = 0.91, v_0 = 0.61\}$, $V_E(\alpha = 1) \geq V_E(\alpha = 0)$, as shown in Figure 2.3.1.

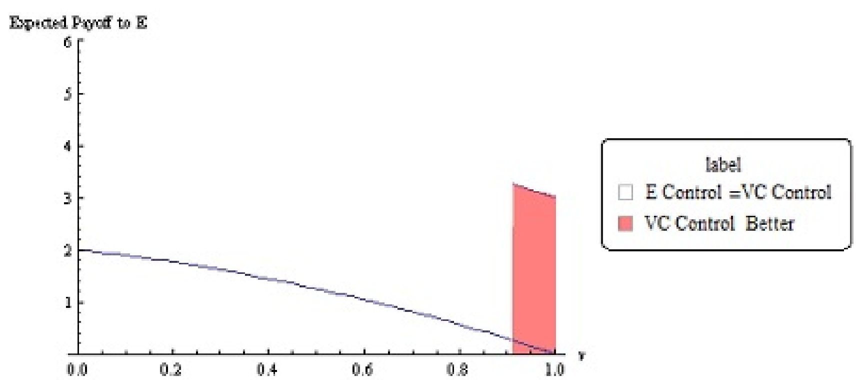

Figure 2.3: Ex-ante Expected Payoff for E

Even without any information asymmetry or hidden actions, allocation of ambiguity thus provides an explanation for the importance and use of control rights allocation observed in VC investment context.

In the next section we look at the general signal structure and characterize the optimal contract structure in that environment. We show that the similar intuition holds in the general case as well and thus ambiguity aversion can explain allocation of control rights.

2.3.2 General Signal Structure

In this section we will look at a signal structure where the signal is drawn from a non-atomic continuous distribution function. We model the posterior belief $[r, s]$ after observing the signal as following a continuous distribution $Q(r, s)$, such that the average of the posterior is always the prior:

$$\int_{(r,s)\in K_{\Delta(0,1)}} r dQ(r,s) = a;$$
$$\int_{(r,s)\in K_{\Delta(0,1)}} s dQ(r,s) = b.$$

If $\alpha = 0$, E solves the following problem:

$$P_{\alpha=0}\left\{\begin{array}{ll} & \max_{(t,K_1,K_2)} \int_{(r,s)\in K_{\Delta(0,1)}} \left(\frac{r+s}{2}\right) t dQ(r,s) \\ s.t. & \int_{(r,s)\in K_{\Delta(0,1)}} \left\{\begin{array}{c} \left(\frac{r+s-(s-r)v}{2}\right)(\Pi(K_1, K_2(r,s)) - t) \\ -K_1 - K_2(r,s) \end{array}\right\} dQ(r,s) \geq L \\ & t \geq 0, \overline{K_1} > K_1 > 0, \overline{K_2} > K_2 > 0 \end{array}\right. \tag{2.9}$$

Using the notations $p = \frac{r+s}{2}, q = \frac{r-s}{2}$; rewrite the problem as:

$$P_{\alpha=0}\left\{\begin{array}{ll} & \max_{(t,K_1,K_2)} p_0 t \\ s.t. & \int_{(r,s)\in K_{\Delta(0,1)}} \left\{\begin{array}{c} (p - vq)(\Pi(K_1, K_2(r,s)) - t) \\ -K_1 - K_2(r,s) \end{array}\right\} dQ(r,s) = L \\ & t \geq 0, \overline{K_1} > K_1 > 0, \overline{K_2} > K_2 > 0 \end{array}\right.$$

Now, because of $E's$ full bargaining power, the participation constraint will hold as an equality. Then,

$$t = \frac{1}{(p_0 - vq_0)} \left[\begin{array}{c} (p_0 - vq_0)\,\pi(K_1) + \int (p - vq)\,\pi(K_2(r,s))dQ(r,s) \\ -L - K_1 - \int K_2(r,s)dQ(r,s) \end{array} \right]$$

So, ignoring the non-negativity constraints[13], the problem becomes:

$$P_{\alpha=0} \left\{ \max_{(t,K_1,K_2)} \frac{p_0}{(p_0 - vq_0)} \left[\begin{array}{c} (p_0 - vq_0)\,\pi(K_1) + \int (p - vq)\,\pi(K_2(r,s))dQ(r,s) \\ -L - K_1 - \int K_2(r,s)dQ(r,s) \end{array} \right] \right. \tag{2.10}$$

First assume interior solutions to the program and later we will check that it is indeed the case. Optimal contract under entrepreneur control is given by $(t^*, K_1^*, K_2^*(\cdot), \alpha = 0)$ satisfying the necessary and sufficient conditions:

$$\pi^{'}(K_1^*) = \frac{1}{(p_0 - vq_0)} \tag{2.11}$$

$$\pi^{'}(K_2^*(r,s)) = \frac{1}{(p - vq)} \tag{2.12}$$

$$= \frac{1}{(s + r) - v(s - r)} \quad \forall (r,s) \tag{2.13}$$

$$t = \frac{1}{(p_0 - vq_0)} \left[\begin{array}{c} (p_0 - vq_0)\,\pi(K_1^*) + \int (p - vq)\,\pi(K_2^*)dQ(r,s) \\ -L - K_1^* - \int K_2^* dQ(r,s) \end{array} \right] \tag{2.14}$$

Due to the concavity of the objective function, the SOCs are always satisfied.

Note: From the FOCs, $\pi^{'}(K_1^*), \pi^{'}(K_2^*(r,s)) > 1$ at every (r,s). So, we have $\overline{K_1} > K_1, \overline{K_2} > K_2$.

[13] Later we check that the optimum derived using FOCs is indeed an interior optimum.

Denote the optimal value of this contract for the entrepreneur as:

$$V_{\alpha=0} = \frac{p_0}{(p_0 - vq_0)} \left[\begin{array}{c} (p_0 - vq_0)\,\pi(K_1^*) + \int\limits_{(r,s)\in K_{\Delta(0,1)}} (p - vq)\,\pi(K_2^*) dQ(r,s) \\ -L - K_1^* - \int\limits_{(r,s)\in K_{\Delta(0,1)}} K_2^* dQ(r,s) \end{array} \right] \tag{2.15}$$

Properties of the optimal contract under entrepreneur control:

The optimal contract under entrepreneur control is given by $(t^*, K_1^*, K_2^*, \alpha = 0)$, that satisfies the FOCs and leaves the investor with zero net utility. It is instructive to examine the properties of this optimal contract by analyzing the FOCs.

First, consider the range of posteriors for which investment goes up after observing the signal. These signal realizations can be termed as "good" signals. If VC were ambiguity neutral, if the signal induced a posterior mean $p > p_0$, then investment would go up. With ambiguity aversion, though, investment depends on the ambiguity-adjusted posterior mean: $p - vq$. We find that the higher the ambiguity-adjusted posterior mean, the higher the second period investment is.

$$\begin{aligned} \frac{r + s - (s - r)v}{2} &> \frac{a + b - v(b - a)}{2} \\ &\Longleftrightarrow K_2^*(r,s) > K_1 \end{aligned}$$

For "balanced" ambiguity attitude ($v < 1$) , after observing the signal, a sufficient condition for $K_2(r,s) > K_1$ is $p - q > p_0 - q_0$. In the following figure we show how the region of "good" signals depends on the parameter v.

Since the return function satisfies Inada conditions, as the posterior approaches $(0,0)$, $K_2 \to 0$. So, for all signal realizations, second period's optimal investment is positive. Also, the FOCs show that investment volume in both the periods: K_1^* and K_2^* decrease as VC becomes more ambiguity averse. Moreover, K_2^* is a convex function of $p - vq$. This result is captured in the next proposition.

Proposition 2.3.3. *As the VC becomes more ambiguity-averse, investment volume K_1^* and $K_2^*(r,s)$ decrease $\forall(r,s)$. $K_2^*(r,s)$ and $\frac{r+s-v(s-r)}{2}\pi(K_2^*(\cdot)) - K_2^*$ are convex in $\frac{r+s-(s-r)v}{2}$.*

Proof. From the FOCs,

$$\Psi(r,s) \equiv \pi^{'}(K_2^*(r,s))\,(p - vq) - 1 = 0$$

By Implicit Function Theorem,

$$\begin{aligned}
\frac{\partial K_2^*}{\partial v} &= -\frac{\frac{\partial \Psi(r,s)}{\partial v}}{\frac{\partial \Psi(r,s)}{\partial K_2^*}} \\
&= -\frac{-q\pi^{'}(K_2^*(r,s))}{\pi^{''}(K_2^*(r,s))(p - vq)} \\
&< 0
\end{aligned}$$

because $\pi^{''}(K_2^*(r,s)) < 0$ by concavity assumption. Similarly, we can show that $\frac{\partial K_1^*}{\partial v} < 0$. And

$$\begin{aligned}
\frac{\partial K_2^*}{\partial(p - vq)} &= -\frac{\pi^{'}(K_2^*)}{\frac{r+s-v(s-r)}{2}\pi''(K_2^*)} \\
&> 0 \\
\frac{\partial^2 K_2^*}{\partial(p - vq)^2} &= \frac{\left(\pi^{'}(K_2^*)\right)^2}{\left(\frac{r+s-v(s-r)}{2}\right)^2 (\pi''(K_2^*))^3}\left[\frac{\pi^{''}(K_2^*)^2}{\pi'(K_2^*)} - \pi^{'''}(K_2^*)\right] \\
&\geq 0
\end{aligned}$$

Hence K_2^* is convex in the ambiguity-adjusted mean: $p - vq$. The net return from investing in period 2= $NR_2 = \frac{r+s-v(s-r)}{2}\pi(K_2^*(\cdot)) - K_2^*$. By Envelope Theorem,

$$\frac{\partial NR_2}{\partial(p - vq)} = \pi(K_2^*) > 0$$
$$\frac{\partial^2 NR_2}{\partial(p - vq)^2} = \pi'(K_2^*) > 0$$

□

Second, we examine how $E's$ compensation depends on $VC's$ ambiguity aversion v. We find that under the given assumptions, as v increases, t decreases. Hence, similar to the Binary environment, we observe that VC and E share ambiguity through the financial contracting. The next proposition captures this.

Proposition 2.3.4. *As the venture capitalist becomes more ambiguity averse, he is compensated by a higher future cash flow. i.e. as v increases, t^* falls.*

$$\frac{\partial t^*}{\partial v} < 0$$

Proof. In Appendix B. □

Now, since E is protected by limited liability ($t \geq 0$), she is always weakly better off continuing the project till the end. However, for VC, after observing the signal, if the expected return net of investment is less than the sunk first period investment, it is better to liquidate the project. Let us define this net return as in the Binary Example:

$$\begin{aligned} g^v_{VC}(r,s) &= (p - vq)\left(\pi(K_1^*) + \pi(K_2^*) - t^*\right) - K_2^* \\ &= (p-vq)(\pi(K_1^*) + \pi(K_2^*)) \\ &- \frac{p - vq}{(p_0 - vq_0)}\left[\begin{array}{c} (p_0 - vq_0)\,\pi(K_1^*) + \int (p - vq)\,\pi(K_2^*) dQ(r,s) \\ -L - K_1^* - \int K_2^* dQ(r,s) \end{array}\right] \end{aligned}$$

Thus, under entrepreneur control, there exists a range of posteriors where the venture capitalist and the entrepreneur have conflicting interests regarding the liquidation of the project. In this region the hold-up problem faced by the investor becomes apparent. Define this region as CZ (Conflict Zone):

$$CZ(v) = \left\{(r,s) \in K_{\Delta(0,1)} \mid g^v_{VC}(r,s) < 0\right\} \tag{2.16}$$

Proposition 2.3.5. *If VC is ambiguity neutral, agency conflict does not arise. i.e. if $v = 0$, $CZ(v = 0) = \phi$. $\exists \bar{v} \in (0,1)$, such that as $v > \bar{v}$, $CZ(v) \neq \phi$. As v increases, $CZ(v)$ expands.*

Proof. In Appendix B. □

For a simulated example with the prior and the liquidation value as in the binary example and with uniformly distributed signals, we find that

$$\bar{v} = .4351$$

In the simulated example with $v = 0.5$, we identify the conflict zone in the simplex, shown in Figure 2.4:

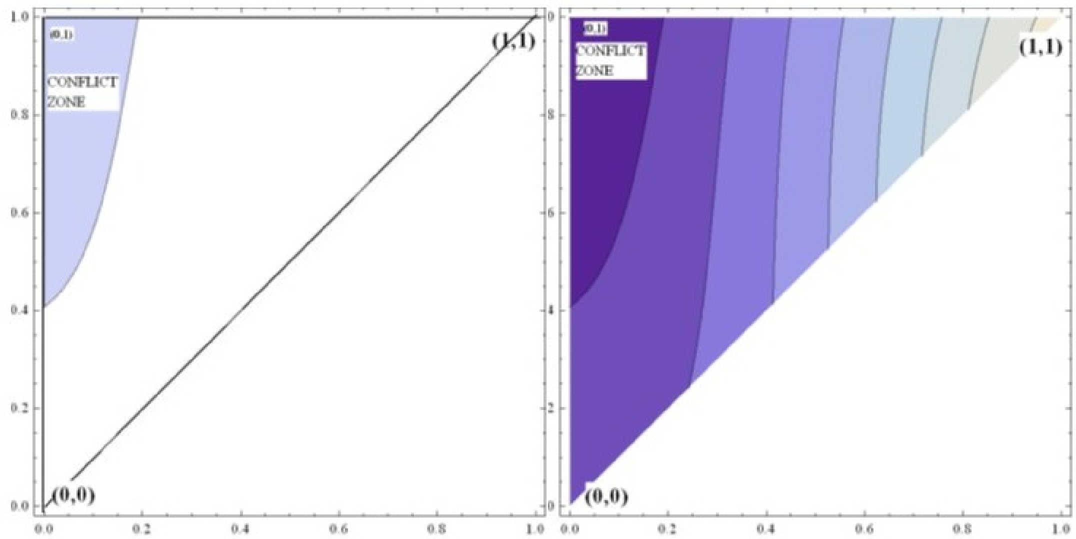

Figure 2.4: Conflict Zone

If the observed signal realizations fall in this region CZ, agency conflict arises. VC wants to liquidate the project but E wants to continue. In this region, control rights become relevant.

Next we examine the case with VC control: $\alpha = 1$.

First we characterize the optimal contract under $\alpha = 1$ and show that in the region where in equilibrium the project is continued till the end, the expected payoff of the entrepreneur is higher. Thus, if VC is sufficiently ambiguity averse, relinquishing control rights to the investor will be optimal for E.

Under VC control, let the optimal contract be denoted as $(\widetilde{t}, \widetilde{K_1}, \widetilde{K_2}, \alpha = 1)$. Now, the venture capitalist will continue as long as his expected payoff from continuation is higher than the sunk cost of first period investment. As before, denote the set of posteriors where the investor decides to liquidate at the optimal contract as CZ, where

$$CZ = \{(r,s) \in K_{\Delta(0,1)} | g^v_{VC}(\widetilde{t}, \widetilde{K_1}, \widetilde{K_2}) < 0\} \tag{2.17}$$

And let

$$\begin{aligned} \Delta_0 &:= K_{\Delta(0,1)} \backslash CZ \\ &= \{(r,s) \in K_{\Delta(0,1)} | g^v_{VC}(\widetilde{t}, \widetilde{K_1}, \widetilde{K_2}) \geq 0\} \end{aligned} \tag{2.18}$$

denote the set of posteriors for which the venture capitalist decides to continue the project till the end.

Under investor control, E solves the following problem:

$$P_{\alpha=1} \left\{ \begin{array}{cc} & \max_{(t,K_1,K_2)} \int\limits_{\{(r,s)\in\Delta_0} ptdQ(r,s) \\ s.t. & \int\limits_{(r,s)\in\Delta_0} \left\{ \begin{array}{c} (p-vq)\left(\pi(K_1)+K_2(r,s)-t\right) \\ -K_1 - K_2(r,s) \end{array} \right\} dQ(r,s) \geq L \\ & t \geq 0, \overline{K_1} > K_1 > 0, \overline{K_2} > K_2 > 0 \end{array} \right. \tag{2.19}$$

Denote:

$$\begin{aligned} \int\limits_{(r,s)\in\Delta_0} \frac{r+s}{2} dQ = x \\ \int\limits_{(r,s)\in\Delta_0} \frac{s-r}{2} dQ = y \end{aligned}$$

Since the participation constraint binds, we have:

$$t = \frac{1}{(x-vy)} \begin{bmatrix} (x-vy)\pi(K_1) \\ + \int\limits_{(r,s)\in\Delta_0} [(p-vq)\pi(K_2(r,s)) - K_2(r,s)]\, dQ(r,s) \\ -K_1 - L \end{bmatrix}$$

Rewriting the problem:

$$P_{\alpha=1} \left\{ \max_{(K_1,K_2)} \frac{x}{(x-vy)} \begin{bmatrix} (x-vy)\pi(K_1) \\ + \int\limits_{(r,s)\in\Delta_0} [(p-vq)\pi(K_2(r,s)) - K_2(r,s)]\, dQ(r,s) \\ -K_1 - L \end{bmatrix} \right. \tag{2.20}$$

The necessary and sufficient conditions that characterize the optimal contract under entrepreneur control are given by the FOCs.

$$\pi^{'}(\widetilde{K_1}) = \frac{1}{(x-vy)} \tag{2.21}$$

$$\begin{aligned} \pi^{'}(\widetilde{K_2}(r,s)) &= \frac{1}{(p-vq)} \\ &= \frac{1}{(\frac{r+s}{2} - v\frac{s-r}{2})} \quad \forall (r,s) \in \Delta_0 \end{aligned} \tag{2.22}$$

$$\widetilde{t} = \frac{1}{(x-vy)} \begin{bmatrix} (x-vy)\pi(\widetilde{K_1}) \\ + \int\limits_{(r,s)\in\Delta_0} \left[(p-vq)\pi(\widetilde{K_2}(r,s)) - \widetilde{K_2}\right] dQ(r,s) \\ -\widetilde{K_1} - L \end{bmatrix} \tag{2.23}$$

Let us denote the maximized value of the objective function as:

$$V_{\alpha=1} = \widetilde{t}x \tag{2.24}$$

Note that:

$$\int_{\Delta_0} r dQ < a; \int_{\Delta_0} s dQ < b$$

Also, Δ_0 is the region where the venture capitalist decides to continue the project till the end. Now, $g^v_{VC}(r,s) < 0$ for high values of $\frac{(s-r)}{2}$. As a result, the continuation region Δ_0 contains posteriors with lower ambiguity:

$$x = \int_{\Delta_0} \frac{r+s}{2} dQ < p_0$$
$$y = \int_{\frac{s-r}{2} < \frac{1}{v}[\varsigma(L,Q)]} \frac{s-r}{2} dQ < q_0$$

Since the venture capitalist no longer faces the hold up problem, the participation constraint is now relaxed. E can exploit it to extract higher share of the final return. Next proposition shows that compared to the optimal contract under entrepreneur control, K_1 decreases; second period's investment stays the same in the continuation region; but $E's$ compensation increases. So, there is a trade-off between the loss of early liquidation and the higher share of the return if the project succeeds. The next proposition shows that as VC becomes sufficiently ambiguity averse, the prospect of obtaining higher share dominates the loss of liquidation and the ex-ante expected payoff for E under VC control is greater than that under E control.

Proposition 2.3.6. *Under VC Control,*

$$\widetilde{K_1} < K_1^*; \widetilde{K_2(r,s)} < K_2^*(r,s) \forall (r,s) \in \Delta_0;$$
$$\widetilde{t} > t^* \textit{ for } v > \overline{v}$$

Also, $\exists v^* > \overline{v}$, $v^* \in (0,1)$, *such that* $\forall v > v^*, x\widetilde{t} > p_0 t^*$, *hence the entrepreneur optimally relinquishes control rights to the venture capitalist.*

Proof. In Appendix B. □

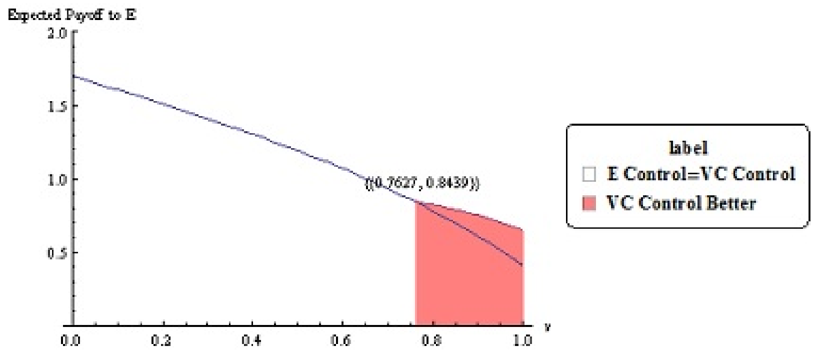

Figure 2.5: Range of v for which $V_{\alpha=1} > V_{\alpha=0}$

In the figure above (Figure 2.3.2) in a simulated example we identify the range of v for which relinquishing control rights is optimal for E. For the example, $v^* = 0.762$.

The intuition behind the result is straightforward. Since the venture capitalist is more ambiguity-averse, under some signal realizations he wants to abandon the project. Then, for the entrepreneur, holding on to the control right is more expensive. Even though the entrepreneur does not want to abandon the project, the investor is ready to forego future earnings in order to obtain control rights. If the investor is very ambiguity averse, then he values the control rights even more. Proposition 2.3.6 shows that for high enough v, it is possible that the higher future earnings for E outweighs the expected loss from abandoning the project under some signal realizations.

2.4 Discussion

Let us discuss some possible extensions of the model and how the results will change under these generalizations.

No Limited Liability:

If E does not have limited liability, under "low" realizations of the signal, E can compensate VC for the loss. The project will be continued for a larger set of signal realizations. However, there may still be a range of $v \in (0, 1)$ such that if the ambiguity aversion of VC lies in that range, it will be too expensive for E to retain control. In this case, similar to the results derived here, E will find it optimal to relinquish control rights. Thus, relaxing this assumption will not qualitatively change the results.

Multiple Stages of Financing:

Usually there can be more than two financing rounds in VC financing. If we consider more financing stages, then using the same intuition as used in this chapter, we can explain the movement of control rights over time. The results will be consistent to the observed data: after high signal the control rights will move to the entrepreneur, but even after that if the next period's signal turns out to be low, investor may obtain more control.

Moral Hazard:

If VC can not fully monitor if the disbursed funds are fully invested in the project, then it can lead to possible moral hazard concern. This will provide another rationale to the allocation of control rights. Since this can confound the effect of ambiguity alone, we choose not to include this aspect.

Asymmetric Information:

If E does not perfectly observe $VC's$ ambiguity aversion, it may give rise to asymmetric information. Then E will offer menu of contracts to screen the VCs according to their preference. It will be interesting to see if a separating equilibrium exists in this scenario and how this affects control rights allocation.

Apart from these possible generalizations, there are various other questions that can be analyzed using this framework. The issue of control rights in VC context is a broad and important question and we can look at the other aspects of VC financing as well.

One broad area of research examines the optimal security design in VC context. We can generalize this model to analyze how different security instruments can be used to implement the control rights allocation and cash flows.

While liquidation rights is the most prominent aspect of control rights, it will also be interesting to include other dimensions of control and see how the presence of ambiguity affects them.

2.5 Summary and Conclusion

This chapter sheds light on venture capital contracting and shows that under ambiguity, allocation of liquidity rights can be used as an instrument to share ambiguity and mitigate the hold up problem the investor may face. I show that if the investor is very ambiguity averse, it is optimal for the entrepreneur to relinquish control rights.

In these two chapters, I have examined different contractual contexts under ambiguity. The framework of ambiguity [54] can be used to analyze various real life scenario where the causal interpretations are only partially known. Important issues such as the environmental policies to fight the climate change, the patent policies in innovation-based industries with competition constitute interesting directions for future research.

Chapter 3

Delegation as a Signal to Sustain Cooperation

3.1 Introduction

Inter-organizational delegation is a very important issue in economics. In an organizational relationship between a principal and an agent, we often observe decisions being delegated. Existing literature proposes a number of explanations for why decisions are delegated. Some of the common explanations include: the delegate might have lower opportunity costs, be better informed or equipped with more adequate skills. In this paper we explore another potentially important role of delegation, as a signalling device to facilitate cooperation.

First, using a theoretical model, I show that even if the agent does not have superior skill or information about the project, the principal can delegate a task to him, in order to facilitate cooperation at a later stage. The central idea is as follows: consider an inter-organizational relationship between an agent and a principal, where there are two separate tasks to perform. An example of such a relationship is the organization relation between a doctor (principal) and a nurse (agent). The first task (in the example, routine check-up of the patient) is a simple one and any one of the doctor and the nurse can do it, while the second task, the surgery, requires cooperation from both of them. The nurse may be trustworthy or not, but the doctor does not know his true type, she only has access to a private signal about the nurse's type. The doctor would benefit from cooperating with a trustworthy nurse but not an untrustworthy nurse. A trustworthy nurse would like to help the doctor but only if the doctor also helps the nurse, but an untrustworthy nurse would shirk. If the doctor can delegate the first task (*i.e.* , let the nurse go through the check up routine) , this Bayesian game has two equilibria which can be Pareto-ranked. In one of the equilibria, delegation can be used to signal the doctor's belief. If the doctor believes the nurse to be trustworthy, she can delegate a task to the nurse to signal her trust. Observing this signal, the trustworthy nurse will infer that the doctor must have a higher belief about her trustworthiness, and is likely to cooperate in the next task. In this equilibrium, delegation can bring about cooperation in the second task[1]. The forward induction argument predicts that this equilibrium with delegation as a signaling device will be chosen. However, because of the presence of multiple equilibria in this game, whether this equilibrium is actually chosen by decision makers is an empirical question.

Therefore I take the next step: in a controlled laboratory experiment I observe how subjects make decisions in such a game theoretic situation. From the experimental data we can test the theoretical predictions and examine how equilibrium is selected in this Bayesian game. Through the experiments, we can simulate the exact environment postulated in the theoretical model, so the generated data can be analyzed to test the theory. Thus, this experimental study can provide an explanation

[1]In a similar argument, using Intuitive Criterion, Herold [91] shows that contractual incompleteness may arise to signal principal's trust.

of the observed phenomenon of inter-organizational delegation using a theoretical prediction as well as empirical data. This paper also adds to the relatively new area of experimental studies that deal with the principal-agent relationship.

At the same time this experimental study can complement another strand of literature. In Bayesian games characterized by some uncertainty about an agent's type, theoretical models suffer from the phenomenon of multiplicity of equilibria. It is important to know how economic agents choose between these equilibria to obtain unique theoretical prediction. This paper studies a Bayesian game with two equilibria which can be Pareto-ranked. So, the data on participants' choices in this game can add to the literature of equilibrium selection. Also, the standard theories of Bayesian games define equilibria consistent with different beliefs, but do not explain how economic agents actually form their beliefs. In this paper, I conduct several sessions where I provide some information about past sessions to the participants, and the results from these sessions suggest that the participants who received this information behave significantly differently than the ones who did not. Hence, the results indicate that the formation of belief depends on information. Thus, this study not only sheds light on the issue of equilibrium selection, but also seeks to identify the role of information in equilibrium selection. The results provide fresh insight and can be used in future to formulate behavioral models to show how belief formation in Bayesian games depends on the information environment.

From the experimental data I find that the subjects do not choose to delegate very often; the Pareto inferior equilibria is chosen most of the times. However, when the subjects are informed about the choices made in previous experimental sessions, they choose delegation as a signal of trust statistically more often than without such historical information.

The next section describes how the paper is related to the existing literature. In section 3, I describe the formal theoretical model and state the theoretical predictions. In section 4, I describe in detail the experimental design, along with the description of the sessions. Sections 5 contains the analysis of the results. In section 6 I discuss the significance of the results and suggest possible explanations of the observed behavior. Section 7 contains some concluding remarks.

3.2 Literature Review

This paper is related to the strand of literature in economics that discusses the reasons behind delegation. Bolton and Dewatripont [27] summarize the explanations of delegation in principal-agent framework as discussed in the existing literature, which include superior skill of the delegate or asymmetric information about the task to be performed. Schelling [128] showed that delegation can also act as a commitment device. Delegation of control rights is often discussed as an important tool to provide incentives in the incomplete contract framework [3]. In the theoretical and experimental literature a number of papers (*e.g.* , [14], [117]) attribute the choice to delegate on the principal's desire to shift the responsibility to the delegate. In particular, in dictator games, they find that delegates are often punished less severely. Hence, it can be shown that delegation can be used to shirk responsibility of an action. Vetter [138] shows how in a political scenario delegation for anticipated rewards can be used as an alternative to corruption. While these reasons do play crucial roles in many real life delegation decisions, this paper proposes an alternative explanation of delegation, where delegation can be used as a signal to facilitate cooperation later. Here, I do not assume that the agent has superior skills or information about the task. The experimental design followed in this paper makes it possible to isolate all other factors, as there is only one asymmetry of information in this framework: the principal does not observe the agent's trustworthiness. In this context, the experimental results examine if subjects use delegation of decision rights in order to achieve cooperation. The results of this experimental study is thus aimed at complementing the existing literature on delegation.

In a broader way, this experimental study is part of the growing experimental literature on equilibrium selection in signalling games. Bayesian games generally suffer from multiplicity of equilibria. To obtain predictive power, different refinements have been suggested theoretically. Mainly following Kohlberg and Mertens' [104] concept of stability, these refinements pick equilibria from the set of Perfect Bayesian Equilibria which satisfy the stability criteria; hence they are more likely to be chosen by a decision maker. In this paper, the theoretical prediction of the use of delegation as a signal is consistent with a forward induction argument.

These refinements of Nash equilibria refine the beliefs of players about the strategies selected by their opponents. However, since beliefs are inherently unobservable, we need to validate these solution concepts using the observed play of decision makers in a laboratory experimental environment. The laboratory results can provide important insights to complement the theoretical debate about which refinement is the most appropriate one. Since the early days of experimental economics, various studies have presented mixed evidence on the predictive power of the refinements (for an exhaustive review of these works, see [47] and [66]). Brandts and Holt [28] found that in a signaling game with multiple equilibria, the Pareto dominant Nash equilibrium is often chosen, which supports the Intuitive Criterion; however, after gaining experience with different partners in a series of these signaling games, behavior closer to the unintuitive equilibrium outcome is observed. Such mixed predictions require us to further investigate the out-of-equilibrium adjustment process. Cooper *et al.* [39] found evidence supporting forward induction argument in coordination games, but only when the equilibrium chosen by the forward induction refinement coincides with the Pareto dominant equilibrium. In the battle of sexes game, forward induction is shown to be effective along with a focal point argument [42]. Another study [41] found that preplay communication can increase the predictive power of forward induction and solve the coordination failure problem. In general, it is found that the outcomes are often game-specific (see [13], [29]) and a small change in the parameter value can change the outcome even when the play followed equilibrium prediction before [75] and even a small payoff asymmetry may lead to coordination failure [46][2]. This paper adds to this body of literature by investigating the predictive power of forward induction and suggesting how informational environment can play a role in the formation of out-of-equilibrium path beliefs. I find that the majority of the subjects choose the Pareto dominated equilibrium rather than the Pareto superior one supported by the forward induction argument. However, this refinement performs better if the subjects are informed about past sessions. Thus, this paper sheds light on the issue of equilibrium selection in signalling games.

[2]For an extensive review, see [125].

3.3 Theoretical Model

Consider a principal agent relationship: a principal (he) and an agent (she) are engaged in a project that involves two separate tasks where monetary transfers are not allowed. The first task requires effort from only one of the players, the principal can do it himself or delegate it to the agent, while the second task involves simultaneous choice of effort by both the principal and the agent where efforts are complementary in nature. This second task represented by a coordination game with two Nash equilibria: one of which Pareto dominates the other. If the players coordinate on the Pareto dominant equilibrium, we call it "cooperation" in this context.

In this model, the agent does not have any superior skill or knowledge relevant to the first task compared to the principal. The only information asymmetry is about the agent's "type": she is either "Biased" (B) or "Unbiased" (U), which is privately observed by the agent. A biased, or, untrustworthy agent does not care about the project's success whereas the unbiased or trustworthy agent has preferences completely aligned with the principal. The proportion of unbiased agents in the economy is known to be $\mu \in (0, 1)$. Let us describe the timeline of the game:

- At the beginning, Nature moves and chooses the agent's type $\theta \in \{U, B\}$. The principal can not observe the true type, he gets a private binary signal

$s \in \{H, L\}$ about θ. The signal structure is given by:

$$\Pr(s = H|\theta = U) = p_U$$
$$\Pr(s = H|\theta = B) = p_B$$

Assumption 1: The signal structure satisfies Monotone Likelihood Ratio Property (MLRP), *i.e.* , $p_U > p_B$.

Thus the posterior belief about the true type becomes

$$\begin{aligned}
\mu_H &= \Pr(\theta = U|s = H) = \frac{\mu p_U}{\mu p_U + (1-\mu)p_B} \\
\mu_L &= \Pr(\theta = U|s = L) = \frac{\mu(1-p_U)}{\mu(1-p_U) + (1-\mu)(1-p_B)}; \\
&\Rightarrow \mu_H > \mu > \mu_L
\end{aligned}$$

For conducting the experiments, I use a set of parameters to simulate the signal structure and the tasks. Here I state the theoretical results in terms of these parameters.

The following parameters define the signal structure:

$$\begin{aligned}
\mu = \frac{1}{2}, \mu_H = \frac{3}{5}, \mu_L = \frac{3}{7}; \\
p_U = \frac{1}{2}, p_B = \frac{1}{3}
\end{aligned}$$

Task 1: The principal can either perform Task 1 herself or delegate it to the agent. Formally, in this task the active player chooses effort $e_1 \in \{0, 1\}$. The payoffs of the players from this task are:

$$u_{P,1}(e_1 = 1) = 2 = u_{U,1}(e_1 = 1);$$

$$u_{B,1}(e_1 = 1) = 0$$

$$u_{P,1}(e_1 = 0) = 1 = u_{U,1}(e_1 = 0)$$

$$u_{B,1}(e_1 = 0) = 1$$

Thus, the unbiased agent's preferences are closely aligned with the principal's, unlike the biased agent. Given a choice, the principal and the Unbiased agent would choose $e_1 = 1$ but the Biased agent would choose $e_1 = 0$.

The effort choice in this task is not observable before the completion of task 2.

- *Task 2:* After task 1, both the principal and the agent have to choose efforts simultaneously to complete task 2, where efforts are complementary in nature. Task 2 involves simultaneous choice of effort $e_{2P}, e_{2A} \in \{0, 1\}$, which yields payoff according to the following 2x2 matrix.

  If the agent is Unbiased, the game becomes a coordination game:

P\AU	1	0
1	$(9, 9)$	$(1, 5)$
0	$(5, 1)$	$(5, 5)$

  If, however, the agent is biased, the game becomes:

P\AB	1	0
1	$(9, 1)$	$(1, 5)$
0	$(5, 1)$	$(5, 5)$

Thus, a Biased agent always has a dominant action in Task 2: to choose $e^B_{2A} = 0$, whereas if the Unbiased agent and principal chose with complete information, the coordination game will have two pure strategy Nash Equilibria: $(e_{2P}, e_{2A}^U) = (1, 1)$ and $(e_{2P}, e_{2A}^U) = (0, 0)$, with the former Pareto dominating the latter.

Total payoff of a player is the sum of his/ her payoffs obtained from both the tasks.

Note that, in the second task, the complementarity of effort choices implies that if the agent is unbiased then the principal would want him to choose higher effort in task 2. The unbiased agent's effort choice in task 2 in turn depends on his belief about the principal's "trust" in him (formally, belief about the principal's posterior after receiving the private signal). Thus, if delegating the first task can serve as a signalling device, then the principal with a more favorable signal could use it to induce higher effort from the unbiased agent in task 2. I look for Perfect Bayesian Equilibria that in this context.

Definition 3.3.1 (Perfect Bayesian Equilibrium)**.** Consider a strategy profile for all players: the principal, the Biased and the Unbiased agent; as well as beliefs about the other players' types at all information sets (after observing Delegation and after observing No Delegation). This strategy profile and belief system form a *Perfect Bayesian Equilibrium (PBE)* if:

(1) *sequential rationality*—at each information set, each player's strategy specifies optimal actions, given her beliefs and the strategies of the other players, and

(2) *consistent beliefs*—given the strategy profile, the beliefs are consistent with Bayes' rule whenever possible.

Definition 3.3.2 (Forward Induction (van Damme, 1988))**.** A PBE satisfies Forward

Induction if the following property is satisfied. In a generic 2 player game in which player i chooses between an outside option or to play a game G of which a unique and viable equilibrium e^* yields the player more than the outside option, only the outcome in which player i plays G and then e^* is played is plausible.

Then, in the signaling game described above, the pure strategy PBE are:

Proposition 3.3.1. *If the prior belief is such that*

$$p_U < \frac{5}{9}, \mu_H > \frac{5}{9} > \mu_L$$

then there exist two pure strategy Perfect Bayesian Equilibria:

(A) a separating equilibrium: *principal with a high private signal chooses to delegate Task 1, and then chooses high effort in the coordination game, and the principal with low signal does not delegate the task 1 and chooses low effort in the coordination game; Unbiased agent chooses High effort in Task 2 whenever he is delegated Task 1 and chooses low effort in Task 2 whenever not delegated; Biased agent always chooses low effort in Task 2.*

(B) a pooling equilibrium: *Both high and low signal principals choose not to delegate; subsequently in Task 2, both the principal and the agent always choose low effort, so cooperation fails to occur.*

Under the parametric restriction, the separating equilibrium is the unique PBE satisfying the forward induction refinement.

Proof. Let us define the Unbiased Agent's belief as:

$$\alpha_i^U = \Pr(P \text{ got a High Signal}| \text{ own type}, P \text{ chose } i);$$
$$i = \{Delegate, No\ Delegate\}$$

The strategies are:

$$for \text{ Principal:}$$
$$\sigma_{2j} = \Pr(P \text{ chooses Task 2 effort=1}|\text{Signal=} j)$$
$$\sigma_{Dj} = \Pr(P \text{ chooses to Delegate}|\text{Signal=} j)$$
$$j = \{High, Low\}$$
$$for \text{ Unbiased Agent:}$$
$$\sigma_U^i = \Pr(A \text{ chooses Task 2 effort=1}|P \text{ chose } i)$$
$$i = \{Delegate, No\ Delegate\}$$

Then, a pooling PBE is given by:

$$for \text{ P:}$$
$$(\sigma_{2H} = \sigma_{2L} = 0; \sigma_{DH} = \sigma_{DL} = 0)$$
$$for \text{ Unbiased A:}$$
$$(\alpha_D^U < p_U, \alpha_{ND}^U = p_U; \sigma_U^{ND} = \sigma_U^D = 0)$$

For all parameter range, such a PBE exists. A separating equilibrium is given by:

$$for\ \text{P:}$$

$$(\sigma_{2H} = 1, \sigma_{2L} = 0; \sigma_{DH} = 1, \sigma_{DL} = 0)$$

$$for\ \text{Unbiased A:}$$

$$(\alpha_D^U = 1, \alpha_{ND}^U = 0; \sigma_U^{ND} = 0, \sigma_U^D = 1)$$

Given $(\alpha_D^U = 1, \alpha_{ND}^U = 0; \sigma_{2H} = 1, \sigma_{2L} = 0)$, the ex-ante expected value of P with a private signal $j \in \{H, L\}$: Then,

$$V_j(D) \gtreqless V_j(ND)$$
$$\Leftrightarrow \mu_j \gtreqless \frac{5}{9}$$

For $\mu_H \geq \frac{5}{9} > \mu_L$, $\sigma_{DH} = 1$ and $\sigma_{DL} = 0$. So, the only off the equilibrium belief consistent with the forward induction argument is:

$$\alpha_D^U = 1, \alpha_{ND}^U = 0$$

Thus, this separating equilibrium satisfies Forward Induction refinement. It is easy to see that off equilibrium belief $\alpha_D^U < p_U$ is never consistent with Forward Induction refinement, so the pooling PBE does not satisfy this refinement. $\square$

The experiment is intended to test Proposition 3.3.1, and reveal if decision to delegate can be considered as a signalling device to facilitate cooperation, and how the decision to delegate depends on information.

3.4 Experimental Design

To examine the delegation behavior of subjects and which equilibrium is chosen in the principal-agent game, I have conducted eight experimental sessions, where a total of 174 subjects participated, creating a dataset with 2784 observations. Four of the sessions feature the sequential game discussed above (I call this *Treatment NH*), and four sessions were conducted where the subjects were given information about the behavioral trends observed in a past session (I call this *Treatment H*).

Each experimental session consisted of two parts: in the first part the players sequentially played Task 1 and Task 2, but the principals did not have the option of delegation. So, in this part, the two tasks can be treated independently; hence this part can be treated as the "Control Group." Part Two gave the principals the option to delegate the first task, and thus can be treated as the "Treatment Group." Below I describe the specific features of the experimental design followed in this study.

- **Within Subjects Design:** In this experiment, I use the *"within subjects"* design, where the same subject pool serves both as the Control Group and the Treatment Group. This helps us increase the number of observations at a lower budget. It also reduces the error variance due to individual fixed effects since there are more observations for each participant.

- **Role Switching:** So that all the subjects are aware of the incentives faced by both the roles, I use "role switching" in the design. At the beginning of each experimental session, every participant randomly receives a role: either a principal or an agent with equal probability. After that, at the beginning of each round the role switches, *i.e.* , if an individual is assigned as a principal in round one, he/she will be an agent in round two, and so on.

- **Random and Anonymous Matching:** To implement the static nature of the theoretical model, I use random and anonymous matching among the participants in different roles in every round.

- **Risk Neutrality:** To simulate the theoretical set up, I conduct lotteries to pay the subjects in order to impose risk neutrality. I follow the approach proposed by Walker, Smith and Cox [139] and use their finding that risk neutrality can be induced in subjects' decisions by paying them in lotteries on money that are linear in the outcome probabilities.

- **Fair Payment Scheme:** The payment scheme is designed to be fair and efficient. While conducting the lottery, the computer takes care of the roles and types the subject was assigned and adjusts the probability of winning accordingly. At the lottery, for each participant, the computer randomly draws an integer between 0 and the maximum payoff points that subject could have earned, given the roles and types that he/she was assigned to in each round. This ensures fairness of the lottery.

The experiment sessions are conducted in the Computer laboratory in the Economics Department of UT Austin. For the baseline treatment (*Treatment NH*), there are three sessions with 24 participants each and one with 20 participants;

for *Treatment H*, two sessions have 22 participants, one has 20 and the other has 18 participants participating. zTree software [64] is used to design the interface and record the participants' responses. At the beginning of a session, each participant is assigned a random subject number generated by the computer. The experimental instructions are then given verbally to the participants and a copy of the instructions are also distributed among them. At the beginning of each round, every participant receives a role: either a principal or an agent, with role switching in every round. Then, the agents are randomly assigned as biased or unbiased types (with equal probability) and randomly and anonymously matched to the subjects assigned as principals in that round. The principals do not observe the type of the agent he/she is matched to in that round, but receive a randomly generated signal sent by the computer. The matching and signalling structures remain the same throughout the session. To avoid any positive or negative connotations, I call the types Green (for Unbiased) and Red (for Biased); the signals as Lime (high) and Pink (low).

Since the game consists of multiple tasks, it is imperative that the subjects are trained in each of these tasks and have sufficient experience with them before playing the sequential game. So, at the beginning, the players face the two tasks separately. Stage One of Part One features four rounds of task 2, where in each round the matched pair of a principal and an agent play the coordination game described above. After that, instructions about task 1 are given and a short quiz is conducted to ensure the subjects' understanding of the task. Stage Two of Part One features six rounds of the entire game, where each matched pair of a principal and an agent will play task 1 and task 2 sequentially, but without the option to delegate task 1. Thus, the data generated from Stage Two of Part One can be used as the data from the Control Group. Part Two consists of ten rounds of the entire game, with the principals having the option to delegate task 1 to the agents; thus this stage provides the data from the Treatment Group. Henceforth, I use the terms

(a) *Part One Group:* to denote the Control Group in each session,

(b) *Part Two Group:* to denote the Treatment Group in each session,

(c) *Treatment NH sessions:* to denote the sessions where no historical information was given (as described above), and

(d) *Treatment H sessions:* to denote the sessions where historical information were given (as described next).

Apart from conducting four sessions with no historical information, I also conduct five sessions with historical information given to the subjects. In these sessions, termed as the *Treatment H sessions*, the Part One Group is conducted similar to the *Treatment NH sessions.* However, before Part Two, the subjects are given information about

(a) the proportion of principals who chose high effort after delegating Task 1 and after not delegating, and

(b) the proportion of Red (Biased) and Green (Unbiased) agents who chose high effort after being delegated and after not being delegated.

In the first session with *Treatment H ,* the information given was from the previous *Treatment NH session.* The next *Treatment H sessions* were conducted using information from the last *Treatment H session.* Hence, in the first *Treatment H* *session*, the subjects were informed about behavioral trends of others who, in turn, were not given any information; whereas in the next Treatment H sessions, subjects observed data generated from a session where historical information was given. This may lead to inconsistency problems, so to maintain consistency, I do not use the data from the first *Treatment H session.*

For the payment scheme, I use lotteries to implement risk-neutrality of the players. In each round, depending on the choices made by a participant and the matched partner, the participants were awarded payoff points specified in the theoretical model. At the end of a session, two lotteries were conducted. In Lottery One, a random integer was drawn by the computer from the interval of 0 to the maximum number of points a participant could have earned in Part One, given his/her roles and types. If the actual points earned was greater than the random integer, the participant got $15, otherwise $2. In Lottery Two, a random integer was drawn by the computer from the interval of 0 to the maximum number of points a participant could have earned in Part Two and if the actual points earned was greater than that random integer, the participant was rewarded $15, otherwise he/she got $4. The detailed set of Instructions used to conduct the experimental sessions is attached in Appendix

Hypotheses:

In the baseline treatment (*Treatment NH*), I examine the data observed to see if the subjects' choices are consistent with any of the equilibrium predictions of the theoretical game, and if the subjects indeed use delegation as a signaling device. Here I state the hypothesis, later on we will see if the results support these hypothesis.

Firstly, the Part One Group (observations from subjects playing the entire game without the delegation option) serves as a benchmark. In absence of any connection between the two tasks, from the proportion of cooperation, I get a benchmark about the cooperation behavior of the subject pool. The Part Two Group data will then shed light on the equilibrium selection behavior.

A *Hypothesis NH (Part One Group):* In the Part One Group of the baseline treatment, in Task 1 the principal will chose high effort and in Task 2, $(e_{2P}, e_{2A}) = (0, 0)$ will be played irrespective of the principal's signal or the agent's types, so the outcome will be consistent with the Pareto inferior outcome $(5, 5)$.

B *Hypothesis NH (Part Two Group):* In the Part Two Group of the baseline treatment, the separating equilibrium will be chosen, where the high signal principal will delegate Task 1 and achieve cooperation in Task 2 if matched with an Unbiased agent.

This hypothesis can be broken into several components:

B1 The principal with a high signal more frequently chooses to delegate the task 1 than the principal with a low signal.

B2 After delegating task 1 to the matched agent, the principal is more likely to choose high effort in task 2 than when not delegating.

B3 After observing delegation by the principal, the matched Unbiased agent chooses high effort more often than after observing no delegation.

In the sessions where the subjects are given information about the past session (*Treatment H*), I test if there is a statistically significant difference in the equilibrium selection behavior. In those sessions, in addition to testing the above hypothesis, I test the following hypothesis as well:

C *Hypothesis H:* In the Part Two Group in *Treatment H sessions* , the separating equilibrium is played more often than in *Treatment NH sessions.* Also, delegation is more frequently observed with *Treatment H.*

3.5 Results

This section describes the results analyzing the data from the eight experimental sessions.

***Treatment NH*: Part One Group**

We need to closely examine the results from the Part One Group, with 552 observations. Apart from showing if the subjects' play conforms to any equilibrium behavior, the results also shed light on the natural cooperative tendency in the subject pool. For each of the observations, besides presenting the proportions, I also conduct t-tests to test the relevant hypothesis and present the t-statistics in the parentheses.

1. *Observation 1:* The Unbiased agents choose high effort in Task 2 significantly more often (t-stat: -8.3757). The following table reports the total number and proportion of occasions where the agent chose high effort.

Type\Task 2 Effort	High	Low	Total
Unbiased	68 (41.72%)	95 (58.28%)	163
Biased	2 (1.77%)	111 (98.23%)	113
Total	70	206	276

2. *Observation 2:* The principals choose high effort in Task 1 (which is the dominant strategy) almost always (t-stat: 6.6641), indicating the consistency of behavior in the subject pool.

	Task 1 Effort: High	Task 1 Effort: Low	Total
	20 (7.25%)	256 (92.75%)	276

3. *Observation 3:* The principals choose low effort in Task 2 if they receive low signal. They choose high effort in Task 2 significantly more often if the private signal is high (t-stat: -6.3011).

Signal\Task 2 Effort	High	Low	Total
High	50 (45.87%)	59 (54.13%)	109
Low	23 (13.77%)	144 (86.23%)	167
Total	73	203	276

4. The outcome $(5, 5)$ is chosen significantly more often than the outcome $(9, 9)$ in Task 2.

Equilibrium Chosen	Frequency	Percent
Outcome $(9, 9)$	22	7.97%
Outcome $(5, 5)$	203	73.55%
Total Play	276	100%

Also, the subjects' behavior mostly conforms to an equilibrium prediction; only 18% of the times the behavior observed is different than predicted by an equilibrium. Together, these four observations show support for Hypothesis A.

The following table (Table 3.1) shows the results of a t-test to check if the outcome $(9, 9)$ is chosen significantly less often than the outcome $(5, 5)$ in Task 2 and the evidence suggests that the majority of the participants chose the Outcome $(5, 5)$ in the Part One Group data.

Finally, I ran basic logistic regressions to understand the factors that affect the Task 2 effort choices by the principals and agents. In particular, I examine if there is any subject-specific, session-specific or period-specific fixed effect on the

Table 3.1: Outcome $(5, 5)$ Chosen More Frequently in Part One

Two-sample test with equal variance					
Group	Obs	Mean	Std Er	95% Conf. Interval	
Outcome $(9, 9)$	276	0.797101	0.0168825	0.0475575	0.1118628
Outcome $(5, 5)$	276	0.785507	0.0209859	0.3664887	0.787867
Combined	552	.4076087	.0209339	.3664887	.4487287
diff		$t = -21.0114$	d.f.= 550		
Ho: diff= 0					
Ha: diff < 0		Ha: diff != 0		Ha: diff >0	
$Pr(T < t) = 0.0000$		$Pr(\|T\| > \|t\|) = 0.0000$		$Pr(T > t) = 1.0000$	

choice of Task 2 effort. The following table (Figure 3.1) summarizes the findings. The principals' choice of Task 2 effort depends only on the private signal, while the agents' choice depends on the type and the period. As the session proceeds, the agents become pessimistic about cooperation possibilities and choose low efforts increasingly often, but the effect is not statistically significant at 5% level. Overall, these results support Hypothesis A1.

3.5.1 ***Treatment NH*****: Part Two Group**

From the data collected from the Part Two Groups in the baseline treatment sessions, analyzing the 920 observations, I observe the following trends. As before, the t-statistics are reported within parentheses.

1. Principals after observing high (*i.e.* , Lime) signal delegate more often than after low (i.e. Red) signal (t-stat: -4.1037). Thus, the private belief about

	Principal b/se	Agent b/se
Task 2 Effort		
signal	1.708***	
	(0.30)	
Subject	0.011	- 0.024
	(0.02)	(0.02)
session	-0.234	-0.056
	(0.14)	(0.15)
Period	-0.044	-0.213*
	(0.09)	(0.09)
Type		3.720***
		(0.74)
Constant	1.397	-0.847
	(1.91)	(2.15)

* p<0.05, ** p<0.01, *** p<0.001

Figure 3.1: Task 2 Effort Choices in the Part One Group of *Treatment NH*

the matched agent's type influences the delegation decision, as posited in Hypothesis B1.

Signal\Delegation	Delegate	No Delegate	Total
High	55 (28.65%)	137 (71.35%)	192
Low	36 (13.43%)	232 (86.57%)	268
Total	91	369	460

2. After Delegation, principals more often follow with high effort choice in Task

2 (t-stat: −5.0013). This supports Hypothesis B2.

Delegation\Task 2 Effort	High	Low	Total
After Delegation	33 (36.26%)	58 (63.74%)	91
After No Delegation	52 (14.09%)	317 (85.91%)	369
Total	85	375	460

3. After observing Delegation, Unbiased agents are more likely to respond by choosing High Effort in task 2, as posited in Hypothesis B3 (t-stat: −3.3962).

Delegation\Task 2 Effort	High	Low	Total
After Delegation	24 (42.11%)	33 (57.89%)	57
After No Delegation	35 (20%)	140 (80%)	175
Total	59	173	232

Biased agents almost never choose high effort.

Delegation\Task 2 Effort	High	Low	Total
After Delegation	0 (0%)	34 (100%)	34
After No Delegation	2 (1.03%)	192 (98.23%)	194
Total	2	226	228

4. After a delegation occurs, the proportion of plays choosing (High, High) in Task 2 is significantly greater than after no delegation. The following table

shows that after delegation it is ten times more likely to end up at $(9,9)$ in Task 2.

Delegation\Task 2 Outcome	Task 2 payoff : $(9,9)$	Total
After Delegation	11 (12.09%)	69
After No Delegation	6 (1.63%)	323
Total	17	392

A logistic regression attempts to explain the delegation decision and the Task 2 effort choice. The results[3] are described in the following table (Figure 3.2):

From the table, we observe that the agent's effort choice significantly depends on her type and also whether he was delegated. Also, as the sessions proceeds, he chooses high effort less often. For the principal, delegation decision depends only on the signal, though the variable "period" has a dampening effect (not significant at 5% level). The principal's Task 2 effort choice significantly depends on her own delegation decision and private signal.

Equilibrium Selection

Next we test which equilibrium is selected more often in the observed play. First, note that the Pareto-inferior PBE and the PBE satisfying Forward Induction Refinement both predict a similar outcome if the Principal observes a low signal:

[3] I drop the variables "session"and "subject", which were insignificant at 5% level.

	Agent b/se	Principal b/se	Delegation b/se
If Principal Delegated	1.001**	0.966***	
	(0.33)	(0.28)	
Type	3.541***		
	(0.73)		
Period	-0.121*	0.028	-0.057
	(0.06)	(0.05)	(0.04)
signal		1.686***	0.984***
		(0.28)	(0.24)
Constant	-2.626	-1.393	1.435
	(2.12)	(1.70)	(1.56)

* $p<0.05$, ** $p<0.01$, *** $p<0.001$

Figure 3.2: Task 2 Effort Choices in Part Two Group with *Treatment NH*

both equilibria predict that the Principal will not delegate Task 1 and subsequently choose low effort in Task 2, and the matched Agent will respond by choosing low effort in Task 2. So, we examine the proportion of times each of the equilibria is chosen separately for each signal realization and put higher emphasis on the behavior observed after a High signal is observed.

If a High Signal is observed, the Forward Induction equilibrium (termed as *"FI"* hereafter) is chosen significantly less often than the Pareto-inferior PBE (*"PBE"* hereafter). The next table summarizes the proportions of plays conforming to the two respective equilibrium predictions.

Equilibrium Chosen \ Signal	High	Low	Total
FI	19 (9.90%)	212 (79.10%)	231
PBE	105 (54.69%)	212 (79.10%)	317
Total Equilibrium Play	124 (64.58%)	212 (79.10%)	336(73.04%)

We conduct a t-test to test Hypothesis B and find that the FI equilibrium is chosen significantly less often (Table 3.2).

Table 3.2: FI is not Chosen Frequently

Two-sample test with equal variance					
Group	Obs	Mean	Std Er	95% Conf. Interval	
PBE	192	.546875	.0360194	.4758281	.6179219
FI	192	.0989583	.0216064	.0563406	.1415761
Combined	384	.3229167	.0238928	.2759392	.3698941
diff		$t = 10.6640$	d.f. = 382		
Ho: diff= 0					
Ha: diff $<$ 0		Ha: diff != 0		Ha: diff $>$ 0	
$Pr(T < t) = 1.0000$		$Pr(\|T\| > \|t\|) = 0.0000$		$Pr(T > t) = 0.0000$	

Since t-tests use the normality assumption, I also use a non-parametric test, viz. Mann-Whitney U test and obtain similar results (z-stat: 7.72, significant at 1% level). Combining the observations with High and Low signal realizations, we observe that FI is chosen 50.21% of the times, while PBE is chosen 68.9% of the times and the difference is statistically significant at 1% level (t-stat: 5.88).

This result clearly shows that the proportion of plays conforming to the Pareto dominated PBE is significantly greater than the proportion conforming to the PBE that satisfies the forward induction criterion. This result contradicts Hypothesis B.

Also, the proportion of plays conforming to an equilibrium prediction is also significantly lower (only 64.58%, as shown in the above table) compared to the same if a Low signal is observed (79.10%). A t-stat shows that difference is significant (Table 3.3).

Table 3.3: Equilibrium Play Observed More Often with Low Signal

Two-sample test with unequal variance					
Group	Obs	Mean	Std Er	95% Conf. Interval	
Low Signal	268	.7910448	.0248812	.7420564	.8400331
High Signal	192	.6458333	.0346057	.5775749	.7140917
Combined	460	.7304348	.0207117	.6897332	.7711363
diff		$t = 3.4070$	d.f. = 368.979		
Ho: diff= 0					
Ha: diff < 0			Ha: diff != 0		Ha: diff > 0
$Pr(T < t) = 0.9996$			$Pr(\|T\| > \|t\|) = 0.0007$		$Pr(T > t) = 0.0004$

To sum up the results from this treatment, we observe that:

(a) The observed play mostly conforms to a PBE.

(b) After a delegation decision, the choices made by the principal and the agent supports the theoretical prediction of forward induction.

(c) However, the PBE that survives the forward induction criterion is seldom chosen. Principals do not delegate often. The Pareto inferior PBE is chosen significantly more frequently, indicating that forward induction fails to predict the outcome in this context. To explain these results, I use the next set of treatments to check if history has any impact on the decisions and belief formation.

3.5.2 *Treatment H*

The question that I address in this section is: how does the delegation choice depend on the information given to the participants? I use the data from the last four sessions (I will call them History sessions, or *Treatment H*) containing 1312 observations. In each session, before Part Two, the participants were given summary statistics about the past History session[4]. Analyzing this data, I examine if this additional information affects the decision making of the subjects and equilibrium selection in general. The observations from these four sessions are listed below:

1. *Observation 1:* The data from the Part One Group in *Treatment H sessions* is similar to the Part One Group data observed in *Treatment NH sessions.*

   The Unbiased agents choose Task 2 effort in a similar way (t-statistic for comparing the Task 2 effort between *Treatment NH* and *Treatment H* is 1.63, insignificant at 10% level), similar for the Biased agents (t-stat: -0.7303). The principals choose Task 2 effort similarly (for low-signal principals, t-stat: 1.53, for high-signal, t-stat: 1.35). The cooperation achieved in Task 2 is also similar (t-stat: 0.31). This is not surprising, given that the Part One Group was not given any additional information. For the Part Two Group, we need to ex-

[4]In all of these sessions, the historical information given was from the last session conducted with similar informational environment. For the sake of consistency, I do not use the first session where the data given was from a session which was conducted without history.

amine the results more closely. The tables (3.10.1) and the logistic regressions (Figure 3.5)are given in Appendix A.

2. *Observation 2:* The principals who observe high signals delegate more often in *Treatment H* than in *Treatment NH* (t-stat: -2.75).

Treatment\Delegation	Delegation	No Delegation	Total
History	65 (42.76%)	87 (57.24%)	152
No History	55 (28.65%)	137 (71.35%)	192
Total	120	224	344

The result of the t-test is shown in the following table (Table 3.4). Here, we test if the proportion of principals who delegate after observing high signal is different between *Treatment NH sessions* and *Treatment H sessions*. The test finds clear evidence of a significant difference in delegation behavior across treatments.

Since t-tests use the normality assumption, I also use a non-parametric test, viz. Mann-Whitney test to check if the proportion of delegation choices is significantly different in *Treatment H*, and these results are also similar to the t-test, as shown in the following table (Table 3.5).

3. *Observation 3:* The proportion of times the observed play conforms to the forward induction equilibrium is significantly higher in *Treatment H* compared

Table 3.4: Higher Delegation Frequency with History

Two-sample test with equal variance					
Group	Obs	Mean	Std Er	95% Conf. Interval	
Treatment NH	192	0.2864583	.0327133	.2219327	.350984
Treatment H	152	.4276316	.040261	.348084	.5071792
Combined	344	.3488372	.0257341	.2982207	.3994537
diff		$t = -2.7503$	d.f. = 342		
Ho: diff= 0					
Ha: diff < 0		Ha: diff != 0		Ha: diff > 0	
$Pr(T < t) = 0.0031$		$Pr(\|T\| > \|t\|) = 0.0063$		$Pr(T > t) = 0.9969$	

to *Treatment NH.* i.e. the separating equilibrium with delegation as a way to achieve cooperation is chosen more frequently in *Treatment H.* The following table captures the number (and proportion) of times the forward induction equilibrium (*FI*) and the Pareto-dominated PBE (*PBE*) is chosen in *Treatment H.* As discussed before, we put more emphasis on the results for the observations with High signal realization, since for Low signal, the two equilibrium predictions converge.

Equilibrium Selection After High Signal

Equilibrium Outcome\Treatment	H	NH	Total
FI	29 (19.08%)	19 (9.90%)	48
PBE	79 (51.97%)	105 (54.69%)	184
Total No of Equilibrium Plays	108 (71.05%)	124 (64.58%)	232 (67.44%)

Using t-test we examine if the frequency of choosing the respective equilibrium depends on the information given. While we find that the frequency of choosing the Pareto dominated PBE does not significantly vary from *Treatment H* to *Treatment*

Table 3.5: Treatment H vs NH: Mann-Whitney Test

Two-sample Wilcoxon rank-sum (Mann-Whitney) test			
History	Obs.	Rank-sum	Expected
Without History	192	31060	33120
With History	152	28280	26220
Combined	344	59340	59340
unadjusted variance	839040.00		
adjustment for ties	-267271.84		
adjusted variance	571768.16		
$H_0 : d(NH) - d(H)$			
$z = -2.724$			
$\Pr ob > \|z\|$	$= 0.0064$		

NH (t-stat:0.4999, statistically insignificant), for the *FI*, the treatment matters, as shown next (Table 3.6).

Overall frequencies (for both High and Low signal realizations) are given below:

Equilibrium Selection

Equilibrium Outcome\Treatment	H	NH	Total
FI	244 (59.51%)	231 (50.22%)	475 (54.6%)
PBE	294 (71.71%)	317 (68.91%)	611(70.23%)
Total No of Equilibrium Plays	323 (78.78%)	336 (73.04%)	659 (75.75%)

For PBE, we check that the treatment does not significantly affect the proportion of plays conforming to this Pareto-dominated equilibrium (both t-test and Mann-Whitney test findings agree; t-stat: $-.8991$). For the Forward Induction equilibrium, however, History matters. The following table (Table 3.7) shows the results of the Mann-Whitney test to check if the proportion of play selecting the separating

Table 3.6: FI Chosen More Often With History

Two-sample test with equal variance					
Group	Obs	Mean	Std Er	95% Conf. Interval	
Treatment NH	192	.0989583	.0216064	.0563406	.1415761
Treatment H	152	.1907895	.0319757	.127612	.2539669
Combined	344	.1395349	.0187094	.1027352	.1763346
diff		$t = -2.4553$	d.f. = 342		
Ho: diff= 0					
Ha: diff < 0		Ha: diff != 0		Ha: diff > 0	
$Pr(T < t) = 0.0073$		$Pr(\|T\| > \|t\|) = 0.0146$		$Pr(T > t) = 0.9927$	

equilibrium is significantly different in *Treatment H*, and I do find support in the result.

We also observe that the proportion of plays conforming to an equilibrium prediction is significantly different in Treatment H (mean: 78.78%) vs in Treatment NH (mean: 73.04%) at 5% level (t-stat: -1.97). These results indicate that the given information about past session affects belief formation and is more conducive to forward induction reasoning.

1. The following table (Figure 1) shows the logistic regression results to see what factors affect the Task 2 effort choices and delegation decisions in *Treatment H*. As predicted in the theoretical model, the Green Agent's effort choice significantly depends on whether he is delegated Task 1; the Principal's effort choice depends on own delegation decision and private signal whereas her delegation

Table 3.7: FI Chosen More Often: Mann-Whitney Test

Two-sample Wilcoxon rank-sum (Mann-Whitney) test			
History	Obs.	Rank-sum	Expected
Without History	460	191565	200330
With History	410	187320	178555
Combined	870	378885	378885
unadjusted variance	13689217		
adjustment for ties	-3509103		
adjusted variance	10180114		
$H_0 : d(NH) - d(H)$			
$z = -2.747$			
$\Pr ob > \|z\|$	$= 0.0060$		

decision depends on private signal.

	Agent	Principal	Del
	b/se	b/se	b/se
main			
If Delegated	3.377***		
	(0.45)		
Subject	0.054	-0.010	0.049
	(0.04)	(0.02)	(0.03)
session	-0.066	-0.039	-0.018
	(0.15)	(0.08)	(0.12)
Period	-0.033	0.017	0.013
	(0.08)	(0.04)	(0.06)
signal		1.527***	1.594***
		(0.24)	(0.35)
If Delegates		3.484***	
		(0.36)	
Constant	-1.234	-1.117	-4.062
	(3.69)	(2.10)	(3.00)

* p<0.05, ** p<0.01, *** p<0.001

Figure 3.3: Task 2 Effort Choice in Treatment H

3.6 Discussion

Analyzing the results from last section, we can clearly see that:

(a) In general, the participants choose the pooling PBE. Delegation is not used often and later in Task 2 (Low, Low) effort choice is observed. Thus, the forward induction logic breaks down here.

(b) In *Treatment H* when subjects are given information about the past session, participants increasingly choose the separating equilibrium. The effect of information on the frequency of choosing the other PBE, however, is not significant. This suggests that this additional information helps the participants to form their belief about how the other participants will play.

In this section, I will discuss these two central features of the results.

- *On forward induction:* The results indicate that the forward induction reasoning is unlikely to be empirically valid in this context. This finding is consistent with the existing studies ([40], [80]) which discuss the limitations of forward induction reasoning. It has been found that specially in cooperation games with multiple Pareto ranked equilibria, forward induction refinement does not have much predictive power. Forward induction relies essentially on the common belief of rationality assumption. So, if the players are unsure of other players' rationality, they can choose the "safe" option of playing low effort and this can lead to the observed results.

- *On the Importance of Information:* In *Treatment H*, the information about the past session is shown to increase the proportion of cooperation. To investigate the effects of information, we notice that

(a) The proportion of times the separating equilibrium[5] was chosen after observing a High signal does not significantly differ across sessions under *Treatment H*. As the following figure (Figure 3.6) shows, the proportions of equilibrium play for both the equilibria do not exhibit any trend over time.

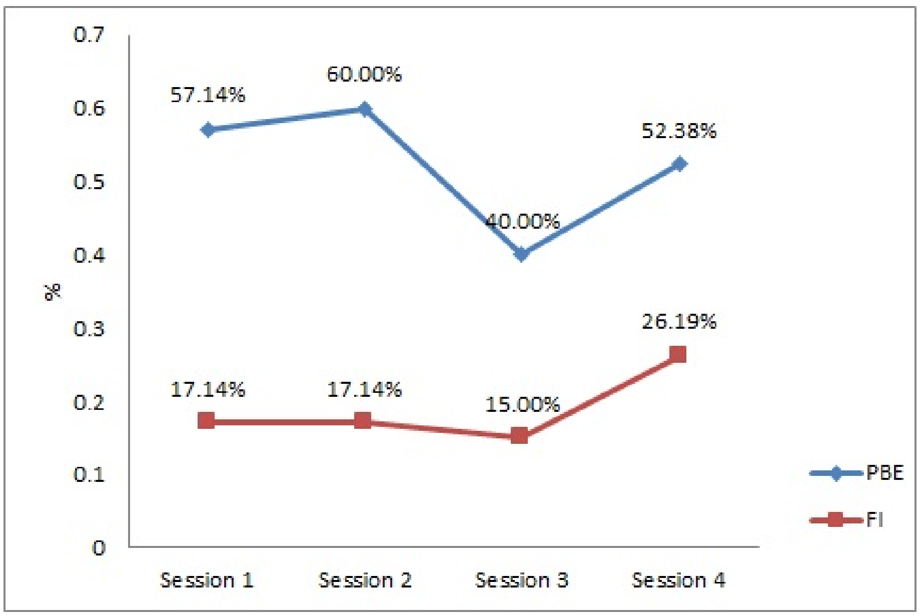

Figure 3.4: Equilibrium Selection by Session in Treatment H for High Signal

Also, as noted before in the logistic regression explaining the principals' effort choice in *Treatment H* (Figure 1), the variable "session" is not affecting the choice significantly.

[5]As defined before, by "Coordination", I refer to the outcome where principals delegate and then in Task 2 end up with (High, High) effort choice.

Clearly, the proportion of plays conforming to either of the equilibria does not show any significant cumulative growth pattern over the sessions. Given that each subsequent session was given data from a previous session which already had historical information, this lack of pattern is all the more stark. These results suggest that the effect of information on cooperation behavior can not be explained by the given information itself; rather the availability of information is what creates a significant difference.

So, we offer the conclusion that the equilibrium selection and belief formation depend on the informational environment of the game. In this particular Bayesian game, the information about past play increased the predictive power of forward induction refinement. These results thus stress the need of a fully formulated behavioral model of equilibrium selection in Bayesian games.

3.7 Conclusion

In this study I have shown that theoretically it is possible to explain the delegation phenomenon in various real life contexts as a signal of trust in order to achieve cooperation in a later phase. However, the experimental data show that the subjects do not often choose this equilibrium. However, providing more information about past play increases the proportion of subjects choosing this equilibrium, hence using delegation to achieve cooperation.

On one hand, this paper sheds light on the determinants of cooperation in many real life scenarios. In inter-organization partnerships, it is often crucial to sustain cooperation among the employer and the employee in order to enhance the value of the relationship. This study shows how the use of delegation can be used to signal the employer's trust in the employee's devotion and bring about cooperation. It also underlines the importance of factors like the workplace environment and past information in forming new employee's belief and consequently in equilibrium selection.

On the other hand, this study provides fresh evidence on equilibrium selection in a Bayesian game. The results suggest that to understand the issue of equilibrium selection, we need a better model of how beliefs are formed and how these beliefs depend on historical information.

Appendices

3.8 Appendix A: Ambiguity Framework

Denote the space of consequences as $\mathcal{X}$, which is a separable metric space with a topology that can be given by a metric making it complete. Let $C_b(\mathcal{X})$ denote the set of bounded, continuous functions on $\mathcal{X}$ with the supnorm topology, and $\Delta(\mathcal{X})$ be a weak* closed and separable, convex subset of the dual space of $C_b(\mathcal{X})$. Let $\mathbb{K}_{\Delta(\mathcal{X})}$ be the set of non-empty, compact, convex subsets of $\Delta(\mathcal{X})$ with the Hausdorff metric.

Then, a weak* continuous rational preference relation on $\mathbb{K}_{\Delta(\mathcal{X})}$ is a complete, transitive relation, $\succeq$, such that for all $B \in \mathbb{K}_{\Delta(\mathcal{X})}$, the sets $\{A : A \succ B\}$ and $\{B : B \succ A\}$ are open. The continuous linear preferences satisfy the Independence axiom given below.

Axiom 1. *(Independence) For all $A, B, C \in \boldsymbol{K}_{\Delta(\mathcal{X})}$, and all $\beta \in (0,1)$, $A \succeq B$ if and only if $\beta A + (1-\beta)C \succeq \beta B + (1-\beta)C$.*

Then, the representation theorem shows that a continuous rational preference relation on $\mathbb{K}_{\Delta(\mathcal{X})}$ satisfies Axiom 1 if and only if it can be represented by a continuous linear functional.

Theorem 3.8.1 (Representation Theorem: Dumav and Stinchcombe, 2013)**.** *A continuous rational preference relation on $\mathbb{K}_{\Delta(\mathcal{X})}$ satisfies Axiom 1 if and only if it can be represented by a continuous linear functional $L : \mathbb{K}_{\Delta(\mathcal{X})} \to \mathbb{R}$.*

Using this representation theorem, we can define the value of ambiguous information analogous to the risky case.

In a risky case, for an expected utility maximizing decision maker, the information they will have when making a decision can be encoded in a posterior distribution, $\beta \in \Delta(\mathcal{X})$. The value of β is

$$V_u(\beta) = \max_{a \in A} \int u(a,x) d\beta(x), \quad where \;\; u : A \times X \to \mathbb{R}.$$

In risky case, a prior is a point $p \in \Delta(\mathcal{X})$, and an information structure is a dilation of p, that is, a distribution, $Q \in \Delta(\Delta(X))$, such that

$$\int \beta dQ(\beta) = p.$$

The value of the information structure is given by

$$V_u(Q) := \int_{\Delta(\mathcal{X})} V_u(\beta) dQ(\beta)$$

An information structure Q dominates Q' if for all u, $V_u(Q) \geq V_u(Q')$.

Analogously, for vNM utility maximizing decision maker facing an ambiguous problem, the information they will have when making a decision can be encoded in a set of posterior distributions, $B \in \mathbb{K}_{\Delta(\mathcal{X})}$.

The value of B is

$$V_U(B) = \max_{a \in A} U(\delta_a \times B)$$

where $U : A \times \mathbb{K}_{\Delta(\mathcal{X})} \to \mathbb{R}$ is a continuous linear functional on compact convex subsets of $\Delta(A \times \mathcal{X})$ of the form $\delta_a \times B$ (where δ_a is point mass on a).

A set-valued prior is a set $A \in \mathbb{K}_{\Delta(\mathcal{X})}$,and an information structure is a distribution, $Q \in \Delta(\mathbb{K}_{\Delta(\mathcal{X})})$, such that

$$\int_{\mathbb{K}_{\Delta(\mathcal{X})}} B dQ(B) = A.$$

Then, the value of the information structure Q is given by

$$V_U(Q) := \int_{\mathbb{K}_{\Delta(\mathcal{X})}} V_U(B) dQ(B).$$

As above, an information structure Q dominates Q' if for all U, $V_U(Q) \geq V_U(Q')$.

This framework follows the standard Bayesian approach and models information structures as dilations. By contrast, previous work has limited the class of priors, A, and then studied a special class of dilations of each $p \in A$. The set of A for which this can be done is non-generic in both the measure theoretic and the topological sense, and the problems that one can consider are limited to ones in which the decision maker will learn only that the true value belong to some $E \subset \mathcal{X}$.

In this approach, A is expressed as a convex combination of/integral of B's in $\mathbb{K}_{\Delta(\mathcal{X})}$, and this is what makes the problem tractable and brings about dynamic consistency.

In a two-consequence case which will be considered in this chapter, this approach simplifies to representing preferences as linear functionals in a simplex. If $\mathcal{X} = \{Good, Bad\}$, then $\mathbb{K}_{\Delta(\mathcal{X})}$ is the class of non-empty closed, convex subsets of the probabilities represented as a simplex:

$$\mathbb{K}_{\Delta(\mathbb{X})} = \{[p-r, p+r] : 0 \leq p-r \leq p+r \leq 1\}.$$

In this case, continuous linear functionals on the convex sets of probabilities must be of the form

$$U([a, b]) = u_1 a + u_2 b$$

for $u_{1,} u_2 \in \mathbb{R}$.

Rewriting $[a, b]$ as $[p-r, p+r]$, where $p = \frac{a+b}{2}$ and $q = \frac{b-a}{2}$ yields

$$U([p-r, p+r]) = (u_1 + u_2)p - (u_1 - u_2)r = p - vr$$

with $v = u_1 - u_2$ measuring the trade-off between risk and ambiguity, $v > 0$ represents ambiguity averse attitude.

Graphically, a set-valued prior $[a, b]$ can be represented as a point in the simplex T with three vertices, $(0, 0)$ representing *Bad* state, $(1, 1)$ representing *Good* state and the new epistemic state "*Unknowable*" represented by the vertex $(0, 1)$. Each $[a, b]$ has a unique representation as

$$(a, b) = w_{1,1}(1, 1) + w_{0,1}(0, 1) + (1 - w_{1,1} - w_{0,1})(0, 0)$$

solving,

$$w_{1,1} = a, w_{0,1} = b - a, w_{0,0} = 1 - b.$$

Thus, the prior $[a, b]$ assigns weight a on $(1, 1)$, $1 - b$ on $(0, 0)$ and $b - a$ on the state $(0, 1)$, i. e. , according to the decision maker, the evidence is thoroughly inconclusive with probability $(b - a)$.

In this setting, a signal is a dilation of the prior which enables Bayesian updating of the weights on each vertex of T. For example, if a binary signal $s \in \{s_1, s_2\}$, $\Pr(s = s_1|Good) = \eta_{1,1}$; $\Pr(s = s_1|Bad) = \eta_{0,0}$ and $\Pr(s = s_1|Unknowable) = \eta_{0,1}$, then the decision maker with prior $[a, b]$ updates his prior after observing s_1 as follows:

$$\begin{aligned}
\Pr(Good|s_1) &= \frac{\eta_{1,1}a}{\eta_{1,1}a + \eta_{0,0}(1-b) + \eta_{0,1}(b-a)} \\
\Pr(Bad|s_1) &= \frac{\eta_{0,0}(1-b)}{\eta_{1,1}a + \eta_{0,0}(1-b) + \eta_{0,1}(b-a)} \\
\Pr(Unknowable|s_1) &= \frac{\eta_{0,1}(b-a)}{\eta_{1,1}a + \eta_{0,0}(1-b) + \eta_{0,1}(b-a)}
\end{aligned}$$

Hence, posterior

$$[a', b']|_{s=s_1} = \left[\frac{\eta_{1,1}a}{\eta_{1,1}a + \eta_{0,0}(1-b) + \eta_{0,1}(b-a)}, 1 - \frac{\eta_{0,0}(1-b)}{\eta_{1,1}a + \eta_{0,0}(1-b) + \eta_{0,1}(b-a)}\right]$$

In this chapter we use this framework to model ambiguous decision making in the innovation process.

3.9 Appendix B: Proofs from Chapter 1 and 2

Proof of Lemma 1. The Policymaker's problem is recursively written as:

$$\begin{aligned} V^{RAN}(r,s) = \max_{\Delta_H,\Delta_S,K^{RAN}} & (\Pr((r',s') \in \Delta_H)(pR - I) + \Pr((r',s') \in \Delta_S)L - K) \\ & + \delta E V^{RAN}(r',s') \end{aligned}$$

We can define the operator $T : \boldsymbol{C}(\mathbb{K}_{\Delta_{[0,1]}}) \to \boldsymbol{C}(\mathbb{K}_{\Delta_{[0,1]}})$ as:

$$\begin{aligned} T(V^{RAN}) = \max_{\Delta_H,\Delta_S,K^{RAN}} & (\Pr((r',s') \in \Delta_H)(pR - I) + \Pr((r',s') \in \Delta_S)L - K) \\ & + \delta E V^{RAN}(r',s') \end{aligned}$$

As $V^{RAN}(r,s) \leq V^1(r,s)\ \forall (r,s) \in \mathbb{K}_{\Delta_{[0,1]}}$, $T(V^{RAN}) \leq T(V^1)$ for all $(r,s) \in \mathbb{K}_{\Delta_{[0,1]}}$ as well. Also, the discount factor $\delta \in (0,1)$ ensures that

$$[T(V^{RAN} + a)](r,s) \leq T(V^{RAN})(r,s) + \delta a$$

for all $V^{RAN}, a \geq 0, (r,s) \in \mathbb{K}_{\Delta_{[0,1]}}$. By Theorem 3.3 in [132], T satisfies Blackwell's sufficiency conditions: monotonicity and discounting, so it is a contraction. Then, by Contraction Mapping Theorem (Theorem 3.2 in [132]), T has exactly one fixed point V^{RAN} that solves the Policymaker's problem. □

Proof of Lemma 2. Suppose $F_t(r_t,s_t) \geq F_{t+1}(r_t,s_t)$. Consider if $(r_t,s_t) \in \Delta_H$.

$$F_t(r_t,s_t) = p_t R - I$$

If $(r_{t+1},s_{t+1}) \in \Delta_H$ for both $S_t = s_H$ and $S_t = s_L$, then for all j,

$$F_{t+j}(r_t,s_t) = \delta^j \left[p_t R - I - \sum_{s=t+1}^{t+j} K_s \right] \leq F_t$$

so the result follows. If $(r_{t+1}, s_{t+1})|_{S_t=s_L} \in \Delta_S$ and $(r_{t+1}, s_{t+1})|_{S_t=s_H} \in \Delta_H$,

$$\begin{aligned} F_{t+1}(r_t, s_t) &= \delta[\mu_{t+1}(p_{t+1}R - I) + (1-\mu_{t+1})L - K_{t+1}] \\ &\leq p_t R - I \\ &\iff (1-\delta)\mu_{t+1}(p_{t+1|H}R - I) \\ &\geq \delta(1-\mu_{t+1})[L + p_{t+1|L}R - I] - K_{t+1}] \qquad (3.1) \end{aligned}$$

then,

$$F_{t+2}(r_t, s_t) = E_t\left[\delta^2 \max\{p_{t+2}R - I, L\} - \delta^2 K_{t+2} - \delta K_{t+1}\right]$$

Thus,

$$\begin{aligned} &F_{t+2}(r_t, s_t) - F_{t+1}(r_t, s_t) \\ &= E_t\left[\delta^2 \max\{p_{t+2}R - I, L\} - \delta^2 K_{t+2}\right] - \delta[\mu_{t+1}(p_{t+1}R - I) + (1-\mu_{t+1})L] \\ &\leq \delta \left[\begin{array}{c} \delta\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) + 2\delta(1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I) \\ +\delta(1-\mu_{t+1})(1-\mu_{t+2})L \\ -[\mu_{t+1}(p_{t+1}R - I) + (1-\mu_{t+1})L] \end{array}\right] - \delta^2 K_{t+2} \\ &= \delta \left[\begin{array}{c} \delta\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) + 2\delta(1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I) \\ +\delta(1-\mu_{t+1})(1-\mu_{t+2})L \\ -\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) - (1-\mu_{t+2})\mu_{t+1}(p_{t+2}|_{LH}R - I) \\ -(1-\mu_{t+1})L \end{array}\right] - \delta^2 K_{t+2} \\ &= \delta \left[\begin{array}{c} (1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I)(2\delta - 1) \\ -(1-\delta)\mu_{t+2}\mu_{t+1}(p_{t+2}|_{HH}R - I) \\ -(1-\mu_{t+1})(1-\mu_{t+2})(1-\delta)L \end{array}\right] - \delta^2 K_{t+2} \\ &= \delta \left[\begin{array}{c} (1-\mu_{t+1})\mu_{t+2}(p_{t+2}|_{LH}R - I)\delta \\ -(1-\delta)\mu_{t+1}(p_{t+1}|_H R - I) \\ -(1-\mu_{t+1})(1-\mu_{t+2})(1-\delta)L \end{array}\right] - \delta^2 K_{t+2} \\ &\leq \delta \left[\begin{array}{c} -\delta(1-\mu_{t+1})((p_{t+1}|_H - p_{t+2}|_{LH})R - I) \\ -L(1-\delta)(1-\mu_{t+1})(1-\mu_{t+2}) \\ -K_{t+1} \end{array}\right] - \delta^2 K_{t+2} \\ &\qquad \text{(using 3.1)} \end{aligned}$$

Similarly, we can prove for $(r_t, s_t) \in \Delta_L$. $\square$

Proof of Proposition 1. By Lemma 2, $A_t s$ form a monotone sequence, by Theorem 3.3 from [37], the "One-stop ahead" rule is optimal, i.e. if stopping the experimentation process today is better than continuing experimenting for exactly one more period, then it is always optimal to stop today . Then, the optimal stopping rules are found by equating F_t and F_{t+1}. If $p_t R - I \geq L$,

$$F_t(r_t, s_t) = F_{t+1}(r_t, s_t)$$

yields the equation:

$$\beta_{H1} r_t + \beta_{H2} s_t = \beta_{H3}$$

and if $p_t R - I < L$, we obtain:

$$\beta_{S1} r_t + \beta_{S2} s_t = \beta_{S3}$$

where:

$$\beta_{H1} = R[1 - \delta(2\lambda_G - \lambda_U)] + \delta 2\overline{K}(I + L)(\lambda_G - \lambda_U)$$

$$\beta_{H2} = R[1 - \delta\lambda_U] + \delta 2\overline{K}(I + L)(\lambda_U - \lambda_B)$$

$$\beta_{H3} = 2I + 2\overline{K}\delta(1 - \lambda_B(I + L))$$

$$\beta_{S1} = \delta[R(2\lambda_G - \lambda_U) - 2\overline{K}(I + L)(\lambda_G - \lambda_U)]$$

$$\beta_{S2} = \delta[R\lambda_U - 2\overline{K}(I + L)(\lambda_U - \lambda_B)]$$

$$\beta_{S3} = 2L(1 - \delta) + 2\overline{K}\delta\lambda_B(I + L)$$

$\square$

Proof of Proposition 2. Using a few lemmata leads us to the main result of the two period example, captured in Proposition 2. Let us, for the sake of brevity, define:

$$T_1 = \frac{1+\lambda_1 p_1\left(\frac{1}{(p_1-vq_1)\lambda_1}+\frac{I}{p_1-vq_1}\right)-\delta\left(\lambda_1 p_1-(1-\mu_1)\lambda_2 p_2\left(\frac{1}{(p_2-vq_2)\lambda_2}-\frac{I}{p_2-vq_2}\right)\right)}{\lambda_1 p_1-\delta\left(\lambda_1 p_1-(1-\mu_1)\lambda_2 p_2\right)}$$
$$T_2 = \frac{1+\lambda_2 p_2\left(\frac{1}{(p_2-vq_2)\lambda_2}+\frac{I}{p_2-vq_2}\right)}{\lambda_2 p_2}$$

The first step identifies the values of ambiguity aversion coefficient v for which $(T_1 - T_2)$ decreases with v.

Lemma 3.9.1. *If the discount factor is not too high, $\delta \leq \overline{\delta} < 1$, for all $v \in [0,1]$, as v increases, $T_1 - T_2$ falls, where $\overline{\delta}$ is given by:*

$$\overline{\delta} = 1 - \left(\frac{p_2 - vq_2}{p_1 - vq_1}\right)^2 \frac{q_1}{q_2}\left(\frac{\frac{1}{\lambda_1}+I}{\frac{1}{\lambda_2}+I}\right)$$

The proof follows directly from taking derivatives. Next, we show that if CF is ambiguity neutral, then there is a possibility of delay.

Lemma 3.9.2. *For $v = 0$, i. e. , if the principal is ambiguity neutral, then*

$$T_1 > T_2$$

So, in equilibrium delay happens whenever $T_1 > R \geq T_2$.

Proof. If $v = 0$,

$$T_1 = \frac{2+\lambda_1 I-\delta\left(\lambda_1 p_1-(1-\mu_1)\lambda_2 I\right)}{\lambda_1 p_1-\delta\left(\lambda_1 p_1-(1-\mu_1)\lambda_2 p_2\right)}$$
$$T_2 = \frac{2+\lambda_2 I}{\lambda_2 p_2}$$

Hence,

$$
\begin{aligned}
T_1 - T_2 &= \frac{[I\lambda_2 p_2 \lambda_1 \delta + \lambda_1 p_1(\delta - (1-\delta)\lambda_2 I)]}{(\lambda_1 p_1 - \delta\,(\lambda_1 p_1 - (1-\mu_1)\lambda_2 p_2))(\lambda_2 p_2)} \\
&\geq 0
\end{aligned}
$$

□

Next, we prove the existence of a threshold value of $v = \widetilde{v}$ for which delay does not happen.

Lemma 3.9.3. *There exists* $\widetilde{v} \in (0,1)$ *for which* $T_1 = T_2$.

Proof.

$$
T_1 - T_2 = \frac{1}{(p_2 - vq_2)(p_1 - vq_1)} \left[\begin{array}{c} (p_2 - vq_2)\left(\frac{1}{\lambda_1} + I\right) \\ -(1-\delta)(p_1 - vq_1)\left(\frac{1}{\lambda_2} + I\right) \\ -(p_1 - vq_1)(p_2 - vq_2)\{\lambda_1 p_1(1-\delta) \\ +\lambda_2 p_2[\delta\lambda_1 p_1 + \delta(1-\mu_1) - 1]\} \end{array} \right]
$$

For $v = 1$,

$$
T_1 - T_2|_{v=1} = \frac{1}{(p_2 - q_2)(p_1 - q_1)} \left[\begin{array}{c} (p_2 - q_2)\left(\frac{1}{\lambda_1} + I\right) \\ -(1-\delta)(p_1 - q_1)\left(\frac{1}{\lambda_2} + I\right) \\ -(p_1 - q_1)(p_2 - q_2)\{\lambda_1 p_1(1-\delta) \\ +\lambda_2 p_2[\delta\lambda_1 p_1 + \delta(1-\mu_1) - 1]\} \end{array} \right]
$$

Now,

$$
(p_2 - q_2)\left(\frac{1}{\lambda_1} + I\right) - (1-\delta)(p_1 - q_1)\left(\frac{1}{\lambda_2} + I\right) \leq 0
$$

$$
\Leftrightarrow \delta \leq 1 - \frac{p_2 - q_2}{p_1 - q_1}\frac{\frac{1}{\lambda_1} + I}{\frac{1}{\lambda_2} + I} \tag{3.2}
$$

And

$$\lambda_1 p_1(1-\delta) + \lambda_2 p_2[\delta\lambda_1 p_1 + \delta(1-\mu_1) - 1]$$
$$= (1-\delta)(\lambda_1 p_1 - \lambda_2 p_2) + \lambda_2 p_2 \delta[\lambda_1 p_1 - \mu_1] > 0$$

Since

$$1 - \frac{p_2 - q_2}{p_1 - q_1}\frac{\frac{1}{\lambda_1} + I}{\frac{1}{\lambda_2} + I} > \overline{\delta} = 1 - \left(\frac{p_2 - q_2}{p_1 - q_1}\right)^2 \frac{q_1}{q_2}\left(\frac{\frac{1}{\lambda_1} + I}{\frac{1}{\lambda_2} + I}\right),$$
$$\forall \delta \leq \overline{\delta}, T_1 - T_2|_{v=1} < 0$$

So, $T_1 - T_2$ continuous in v and it decreases as v increases, $T_1 - T_2|_{v=0} > 0$ and $T_1 - T_2|_{v=1} < 0$, hence there must exist a $\widetilde{v} \in (0,1)$, for which $T_1 = T_2$. □

Proof of Proposition 3. Since CF stops experimenting the first time the posterior crosses the patenting threshold, RL only chooses the contract to offer depending on whether developing the project after being patented is more beneficial than liquidating. Thus, whenever $RL's$ expected payoff if CF develops the product: $\mu_t p_t \left[R - \frac{\left(I + \frac{1}{\lambda_t}\right)}{p_t - vq_t}\right]$ is greater than the expected payoff if CF liquidates: $L - \frac{1}{\lambda_t}$, he chooses

$$x_t = 1 - \frac{\left(I + \frac{1}{\lambda_t}\right)}{R(p_t - vq_t)}, b_t \geq L - \frac{1}{\lambda_t}$$

and the reverse otherwise. This gives us Δ_D, Δ_L. The project is abandoned when no contract satisfying both the incentive constraint for RL and the participation constraint for CF can be offered. Combining both the constraints, it is most difficult to hold if $(r_t, s_t) \in \Delta_L$:

$$L - \frac{1}{\lambda_t} \geq \frac{1}{\lambda_t}$$

So, the project is abandoned if

$$(r_t, s_t) \in \Delta_S^C = \{(r_t, s_t) | L < \frac{2}{\lambda_t}\}$$

□

□

Proof of Proposition 1.5.4. The first lemma finds the sufficient conditions under which the project receives full funding till the end.

Lemma 3.9.4. *Sufficient condition for the project to obtain full funding till the end is:*

$$\lambda_0 \geq \frac{2-\delta}{L\left(1-\frac{\delta}{2}\right)}$$

□

Let us look at the last period T, after which the project is abandoned forever. At T^{th} period, the incentive constraint binds:

$$\mu_T \left[L - \frac{1}{\lambda_T}\right] = \frac{1}{\lambda_T}$$

So,

$$EV_T(r_{T-1}, s_{T-1}) = K_T$$

At the penultimate period, the dynamic IC is:

$$\mu_{T-1}(L - \frac{1}{\lambda_{T-1}}) - K_{T-1} \geq \delta \left[\frac{\lambda_{T-1}}{\lambda_T} - (1 - \mu_{T-1})\right] EV_T(r_{T-1}, s_{T-1})$$

$$\iff \mu_{T-1}(L - \frac{1}{\lambda_{T-1}}) - K_{T-1} \geq \delta \left[\frac{\lambda_{T-1}}{\lambda_T} - (1 - \mu_{T-1})\right] K_T$$

Clearly, this incentive constraint is most difficult to satisfy if $K_T = \overline{K}$. Thus, the project receives full funding till the end if:

$$\delta \leq \frac{\lambda_{T-1}L - 2}{\lambda_{T-1}\left(\frac{L}{2} + \overline{K}\right) - 1}$$

The sufficient condition becomes:

$$\lambda_0 \geq \frac{2-\delta}{L\left(1-\frac{\delta}{2}\right)} \tag{4}$$

If 1.5.4 is violated, the project may not receive full funding till the end. Then, we want to characterize the switching point, i. e. the posterior beliefs for which the investment flow switches from full funding to partial funding. To characterize the equilibrium switching point, we derive the difference equation for $CF's$ funding decision, provided the IC_t^{RL} is binding under restricted funding.

Denote $\Delta_F =$ the region of posterior belief where the project does not receive full funding.

There are two cases: one when the switching point lies in the region of posterior beliefs where after being patented, the project is liquidated, i.e. $\Delta_F \cap \Delta_D = \phi$; and the other when at the switching point, after being granted a patent, the project is developed till the end, i.e. $\Delta_F \cap \Delta_D \neq \phi$.

First, let us focus on the case where at the switching point after being granted patent it is optimal to liquidate the project.

Lemma 3.9.5. *If $\Delta_F \cap \Delta_D = \phi$, then the switching point can be given as a quadratic equation in (r_t, s_t) :*

$$\Phi_L(r_t, s_t) = \gamma_{L1}r_t^2 + \gamma_{L2}s_t^2 + \gamma_{L3}r_ts_t + \gamma_{L1} = 0 \tag{3.3}$$

and

$$\Delta_F := \{(r_t, s_t) | \Phi_L(r_t, s_t) < 0\}$$
$$= \textit{ the region of posteriors where the project does not receive full funding.}$$

Proof. The expected value of RL along the equilibrium path can be represented as:

$$EV_t(r_{t-1}, s_{t-1}) = \mu_t(L - \frac{1}{\lambda_t}) + \delta(1 - \mu_t)EV_{t+1}(r_t, s_t) \tag{3.4}$$

Now, with restricted funding, IC_t^{RL} binds on the equilibrium path, so:

$$\mu_t(L - \frac{1}{\lambda_t}) - K_t = \delta \left[\frac{\lambda_t}{\lambda_{t+1}} - (1 - \mu_t) \right] EV_{t+1}(r_t, s_t)$$

□

Using the Bayesian updating:

$$\lambda_{t+1} = \frac{(\lambda_G - \lambda_U)r_{t-1}(1 - K_t\lambda_t) + 1 - \lambda_t K_t - (1 - s_{t-1})(1 - \lambda_B K_t)(\lambda_U - \lambda_B)}{1 - \lambda_t K_t}$$
$$= \frac{A_t - B_t K_t}{1 - \mu_t}$$

where A_t and B_t are expressions involving r_{t-1} and s_{t-1} and do not depend on K_t.

$$EV_{t+1}(r_t, s_t) = \frac{\left(\mu_t(L - \frac{1}{\lambda_t}) - K_t\right)(A_t - B_t K_t)}{\delta(1 - \mu_t)\left[\lambda_t - (1 - \mu_t)(A_t - B_t K_t)\right]} \tag{3.5}$$
$$= h_L(K_t)$$

Taking derivatives, it can be shown that

$$\frac{\partial h_L}{\partial K_t} \leq 0 \tag{3.6}$$

Substituting 3.5 into 3.4, we obtain:

$$\begin{aligned}
EV_t(r_{t-1}, s_{t-1}) &= \mu_t(L - \frac{1}{\lambda_t}) + \delta(1-\mu_t)h_L(K_t) \\
&= \mu_t(L - \frac{1}{\lambda_t}) + \frac{\left(\mu_t(L - \frac{1}{\lambda_t}) - K_t\right)(A_t - B_t K_t)}{\lambda_t - (1-\mu_t)(A_t - B_t K_t)}
\end{aligned}$$

Moving it one period forward, an alternative expression for $EV_{t+1}(r_t, s_t)$ is found:

$$\begin{aligned}
EV_{t+1}(r_t, s_t) &= \mu_{t+1}(L - \frac{1}{\lambda_{t+1}}) \\
&+ \frac{\left(\mu_{t+1}(L - \frac{1}{\lambda_{t+1}}) - K_{t+1}\right)(A_{t+1} - B_{t+1}K_{t+1})}{\lambda_{t+1} - (1-\mu_{t+1})(A_{t+1} - B_{t+1}K_{t+1})} \\
&= g_L(K_t, K_{t+1})
\end{aligned} \tag{3.7}$$

where it can be shown that

$$\frac{\partial g_L}{\partial K_t} \geq 0, \frac{\partial g_L}{\partial K_{t+1}} \leq 0.$$

Then, the difference equation with restricted funding is obtained by equating 3.4 and 3.7:

$$g_L(K_t, K_{t+1}) = h_L(K_t) \tag{3.8}$$

By Implicit function theorem,

$$\begin{aligned}
\frac{dK_{t+1}}{dK_t} &= -\frac{\frac{\partial g_L}{\partial K_t} - \frac{\partial h_L}{\partial K_t}}{\frac{\partial g_L}{\partial K_{t+1}}} \\
&\geq 0
\end{aligned}$$

Thus, the difference equation 3.8 expresses K_{t+1} as an increasing function of K_t. This ensures the existence of a fixed point of the equation 3.8 at the full funding level, denoted by:

$$\mu_{t+1}(L-\frac{1}{\lambda_{t+1}})+\frac{\left(\mu_{t+1}(L-\frac{1}{\lambda_{t+1}})-\overline{K}\right)\left(A_{t+1}-B_{t+1}\overline{K}\right)}{\lambda_{t+1}-(1-\mu_{t+1})(A_{t+1}-B_{t+1}\overline{K})}$$
$$=\frac{\left(\mu_t(L-\frac{1}{\lambda_t})-\overline{K}\right)\left(A_t-B_t\overline{K}\right)}{\delta(1-\mu_t)\left[\lambda_t-(1-\mu_t)(A_t-B_t\overline{K})\right]}$$

which can be succinctly rewritten as the quadratic equation:

$$\Phi_L(r_t,s_t)=\gamma_{L1}r_t^2+\gamma_{L2}s_t^2+\gamma_{L3}r_ts_t+\gamma_{L1}=0$$

This denotes the switching point. Δ_F is the area below the switching point:

$$\Delta_F:=\{(r_t,s_t)\in\mathbb{K}_{\Delta_{[0,1]}}\backslash\Delta_S^C|\Phi_L(r_t,s_t)\leq 0\}$$

Next lemma establishes that the switching point given by 3.3 lies above the stopping threshold, i.e. $\Delta_S^C\subset\Delta_F$, and also at the switching point the project is liquidates after obtaining patent, i.e. $\Delta_F\cap\Delta_D=\phi$.

Lemma 3.9.6. *The switching point locus always lies above the optimal stopping threshold, i. e.* $\Delta_S^C\subset\Delta_F$.

There exists a δ_L *such that, if* $\frac{\lambda_0 L-2}{\lambda_0\left(\frac{L}{2}+\overline{K}\right)-1}\leq\delta\leq\delta_L$*, at the switching point the project is liquidated after obtaining patent, i. e.* $\Delta_F\cap\Delta_D=\phi$.

Proof. we show that at the last period, the posterior belief lies below the switching point, which will show that $\Delta_S^C\subset\Delta_F$. At $t=T$, $L=\frac{2}{\lambda_T}$. Plugging this in 3.3, it is shown that, if the sufficiency condition does not hold,

$$\begin{aligned}\Phi_L(r_T, s_T) &= \frac{\left(\frac{2}{L}(1+\overline{K})-2\right)\left(A_T - B_T\overline{K}\right)}{\frac{2}{L}-(1-\frac{2\overline{K}}{L})(A_T - B_T\overline{K})} \\ &- \frac{\left(A_{T-1}-B_{T-1}\overline{K}\right)}{\delta(1-\frac{2\overline{K}}{L})\left[\frac{2}{L}-(1-\frac{2\overline{K}}{L})(A_{T-1}-B_{T-1}\overline{K})\right]} \\ &< 0\end{aligned}$$

Similarly, the boundary of Δ_D is given by the locus where CF is indifferent between developing the product and liquidating after being granted a patent (say, at time $t = t_D$):

$$p_t\left[R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right] = L - \frac{1}{\lambda_t}$$

plugging it in 3.3, we can show that

$$\Phi_L(r_{t_D}, s_{t_D}) \geq 0$$

if

$$\begin{aligned}\delta &\leq \delta_L \\ &= \frac{\left(A_{t_D} - B_{t_D}\overline{K}\right)}{\lambda_{t_D}\left[\lambda_{t_D} - (1-\lambda_{t_D}\overline{K})(A_{t_D+1} - B_{t_D+1}\overline{K})\right]}.\end{aligned}$$

Now, we consider the second case: where at the switching point the project will be developed after being granted a patent. □

Lemma 3.9.7. *If $\Delta_F \cap \Delta_D \neq \phi$, then the switching point can be given as a quadratic equation in (r_t, s_t) :*

$$\Phi_D(r_t, s_t) = \gamma_{D1}r_t^2 + \gamma_{D2}s_t^2 + \gamma_{D3}r_ts_t + \gamma_{D1} = 0 \quad (3.9)$$

and

$$\Delta_F := \{(r_t, s_t) | \Phi_D(r_t, s_t) < 0\}$$
$$= \textit{ the region of posteriors where the project does not receive full funding.}$$

The expected value of RL along the equilibrium path can be represented as:

$$EV_t(r_{t-1}, s_{t-1}) = \mu_t p_t \left[R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right] + \delta(1-\mu_t)EV_{t+1}(r_t, s_t)$$

Now, with restricted funding, IC_t^{RL} binds on the equilibrium path, so:

$$\mu_t p_t \left[R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right] - K_t = \delta\left[\frac{\lambda_t p_t}{\lambda_{t+1}p_{t+1}} - (1-\mu_t)\right] EV_{t+1}(r_t, s_t) \quad (3.10)$$

Using the expression for $\lambda_{t+1}p_{t+1}$:

$$\begin{aligned}\lambda_{t+1}p_{t+1} &= \frac{\lambda_G - \lambda_U}{\lambda_G + \lambda_B - 2\lambda_U}\lambda_t + \left(\lambda_U - 2\frac{\lambda_G - \lambda_U}{\lambda_G + \lambda_B - 2\lambda_U}\right)p_t \\ &= F\lambda_t + Gp_t\end{aligned}$$

we can rewrite 3.10 as:

$$\begin{aligned}EV_{t+1}(r_t, s_t) &= \frac{\mu_t p_t \left[R - \frac{1}{p_t - vq_t}\left(I + \frac{1}{\lambda_t}\right)\right] - K_t}{\delta\left[\frac{\lambda_t p_t}{F\lambda_t + Gp_t} - (1-\mu_t)\right]} \\ &= h_D(K_t)\end{aligned}$$

with

$$\frac{\partial h_D}{\partial K_t} \leq 0$$

Using the similar technique as in the case of deriving Δ_F, we obtain the difference equation with restricted funding as:

$$g_D(K_t, K_{t+1}) = h_D(K_t) \quad (3.11)$$

where

$$
\begin{aligned}
EV_{t+1}(r_t, s_t) &= \mu_{t+1} p_{t+1} \left[R - \frac{1}{p_{t+1} - v q_{t+1}} \left(I + \frac{1}{\lambda_{t+1}} \right) \right] \\
&+ \frac{\mu_{t+1} p_{t+1} \left[R - \frac{1}{p_{t+1} - v q_{t+1}} \left(I + \frac{1}{\lambda_{t+1}} \right) \right] - K_{t+1}}{\delta \left[\frac{\lambda_{t+1} p_{t+1}}{F \lambda_{t+1} + G p_{t+1}} - (1 - \mu_{t+1}) \right]} \\
&= g_D(K_t, K_{t+1}) \qquad (3.12)
\end{aligned}
$$

with

$$\frac{\partial g_D}{\partial K_t} \geq 0, \frac{\partial g_D}{\partial K_{t+1}} \leq 0.$$

Then, by Implicit function theorem,

$$
\begin{aligned}
\frac{dK_{t+1}}{dK_t} &= -\frac{\frac{\partial g_D}{\partial K_t} - \frac{\partial h_D}{\partial K_t}}{\frac{\partial g_D}{\partial K_{t+1}}} \\
&\geq 0
\end{aligned}
$$

Thus, the difference equation 3.11 expresses K_{t+1} as an increasing function of K_t. The fixed point can be written as the quadratic equation:

$$\Phi_D(r_t, s_t) = \gamma_{D1} r_t^2 + \gamma_{D2} s_t^2 + \gamma_{D3} r_t s_t + \gamma_{D1} = 0 \qquad (3.13)$$

This denotes the switching point. Also, denote the area below the switching point as:

$$\Delta_F := \{(r_t, s_t) \in \mathbb{K}_{\Delta_{[0,1]}} \backslash \Delta_S^C | \Phi_D(r_t, s_t) \leq 0\}$$

For $\delta > \delta_L, \Delta_S^C \subset \Delta_F$, and at the switching point the project is developed after obtaining patent.

Next lemma shows that $\Delta_D \backslash \Delta_F$ shrinks as v increases, i. e. as CF becomes more ambiguity averse, the project receives full funding for longer horizon under the case where at the switching point the project would be developed if granted a patent.

Lemma 3.9.8. *If $\Delta_F \cap \Delta_D \neq \phi$, then $\Delta_D \backslash \Delta_F$ shrinks as v increases.*

Proof. The switching point 3.13 is given as:

$$\Phi_D(r_t, s_t) = g_D(K_t, K_{t+1}) - h_D(K_t) = 0$$

Taking derivative with respect to v, it can be shown that

$$\begin{aligned}
\frac{\partial \Phi_D}{\partial v} &= \frac{\mu_t p_t \left[\frac{q_t}{p_t - v q_t}\left(I + \frac{1}{\lambda_t}\right)\right]}{\delta\left[\frac{\lambda_t p_t}{F\lambda_t + G p_t} - (1-\mu_t)\right]} \\
&- \mu_{t+1} p_{t+1}\left[-\frac{q_{t+1}}{p_{t+1} - v q_{t+1}}\left(I + \frac{1}{\lambda_{t+1}}\right)\right]\left[1 + \frac{1}{\delta\left[\frac{\lambda_{t+1} p_{t+1}}{F\lambda_{t+1} + G p_{t+1}} - (1-\mu_{t+1})\right]}\right] \\
&> 0
\end{aligned}$$

Thus, as v increases, the project receives full funding for a longer time if $\delta > \delta_L$. □

Now, as v increases, the dynamic moral hazard decreases in the region where the project will be developed if patented. Thus, in the region $\Delta_D \backslash \Delta_F$, the project always receives full funding, and in the region Δ_F, investment gradually declines. This completes the proof of the proposition 1.5.4.

Proof of Proposition 5. Since $\Delta_L \neq \phi$, the project is liquidated even after being patented in that region. The optimal stopping region for the Policymaker is:

$$\Delta_S = \{(r_t, s_t) | \beta_{S1} r_t + \beta_{S2} s_t < \beta_{S3}\}$$

where:

$$\beta_{S1} = \delta[R(2\lambda_G - \lambda_U) - 2\overline{K}(I+L)(\lambda_G - \lambda_U)]$$

$$\beta_{S2} = \delta[R\lambda_U - 2\overline{K}(I+L)(\lambda_U - \lambda_B)]$$

$$\beta_{S3} = 2L(1-\delta) + 2\overline{K}\delta\lambda_B(I+L)$$

For the partnership, the analogous region is:

$$\Delta_S^C = \{(r_t, s_t) | \lambda_t < \frac{2}{L}\}$$

At $r_t = s_t$, we can see the point on $\beta_{S1}r_t + \beta_{S2}s_t = \beta_{S3}$ is $r_S = s_S = \frac{L(1-\delta)+\overline{K}(I+L)\delta\lambda_B}{\delta R\lambda_G - \delta\overline{K}(I+L)(\lambda_G-\lambda_B)}$ and the point on $\lambda_t = \frac{2}{L}$ is $r_S^C = s_S^C = \frac{\frac{2}{L}-\lambda_B}{\lambda_G-\lambda_B}$. Even for $\delta = 1$, since $R > I$, it is always the case that (r_S^C, s_S^C) lies to the right of (r_S, s_S). Thus, $\Delta_S \subset \Delta_S^C$. Also, we have already established in Proposition 1.5.4 that the project may not obtain full funding till the end, unlike the case with the Policymaker. □

Proof of Proposition 2.3.4.

$$\begin{aligned}
\frac{\partial t^*}{\partial v} = &\frac{\partial}{\partial v}\left[\frac{-(L + K_1^* + K_2^*)}{(p_0 - vq_0)}\right] \\
&+ \frac{\partial}{\partial v}\left[\frac{1}{(p_0 - vq_0)}\int (p - vq)\,\pi(K_2^*)dQ(r,s)\right] \\
= &\frac{\partial}{\partial v}\left[\frac{-(L + K_1^* + K_2^*)}{(p_0 - vq_0)}\right] \\
&+ \int \frac{\partial}{\partial v}\left[\frac{(p - vq)}{(p_0 - vq_0)}\right]\pi(K_2^*)dQ(r,s)
\end{aligned}$$

Now,

$$\frac{\partial}{\partial v}\left[\frac{(p - vq)}{(p_0 - vq_0)}\right] = \frac{1}{(p_0 - vq_0)^2}\left[(b-a)(s+r) - (s-r)(b+a)\right]$$

Hence,

$$\begin{aligned}
&\frac{b-a}{b+a} \gtreqless \frac{s-r}{s+r} \\
\Leftrightarrow &\frac{\partial}{\partial v}\left[\frac{(p-vq)}{(p_0-vq_0)}\right] \gtreqless 0
\end{aligned}$$

$$\begin{aligned}
&\int \frac{\partial}{\partial v}\left[\frac{(p-vq)}{(p_0-vq_0)}\right]\pi(K_2^*)dQ(r,s) \\
&\leq \int_{\frac{b-a}{b+a}>\frac{s-r}{s+r}} \bar{\pi}\left[(b-a)\,(s+r)-(s-r)(b+a)\right]dQ \\
&+\int_{\frac{b-a}{b+a}<\frac{s-r}{s+r}} 0.\left[(b-a)\,(s+r)-(s-r)(b+a)\right]dQ \\
&=0
\end{aligned}$$

□

Proof of 2.3.5. Under risk, $r=s=p, v=0$. So, under risk,

$$g_{VC}^{v=0}(p)=p\left(\pi(K_1^*)+\pi(K_2^*)-t^*\right)-K_2^*$$

$$\begin{aligned}
\frac{\partial g_{VC}^{v=0}}{\partial p} &= \pi(K_1^*)+\pi(K_2^*)-t^* \\
&+\frac{p}{p_0}\int \pi(K_2^*)dQ \\
&=\pi(K_2^*)+\frac{K_1^*+L+\int K_2^*dQ}{p_0} \\
&+\frac{p}{p_0}\int \pi(K_2^*)dQ-\frac{1}{p_0}\int p\pi(K_2^*)dQ
\end{aligned}$$

Now, since $\pi(K_2^*)$ is concave in $p-vq$, by Jensen's inequality,

$$\int p\pi(K_2^*)dQ \leq p_0\pi(p_0)$$

Thus,

$$\frac{p}{p_0}\int \pi(K_2^*)dQ - \frac{1}{p_0}\int p\pi(K_2^*)dQ$$
$$\geq \frac{p}{p_0}\int \pi(K_2^*)dQ - \pi(K_2^*(p_0))$$

So,

$$\frac{\partial g_{VC}^{v=0}}{\partial p} > 0 \text{ for all } p \in [0,1]$$

As $(r,s) \to (0,0)$, $g_{VC}^{v=0}(p=0) \to 0$. Thus, $g_{VC}^{v=0}(p) > 0$ for all $(r,s) \in K_{\Delta(0,1)}$. Now, for $v > 0$, we note that:

$$\frac{\partial g_{VC}^{v}}{\partial s} = (1-v)\left[\pi(K_1^*) + \pi(K_2^*) - t^* + \frac{p - vq}{p_0 - vq_0}\int \pi(K_2^*)dQ\right]$$

Around $(r,s) = (0,0)$,

$$\frac{\partial g_{VC}^{v}}{\partial s}|_{(r,s)=(0,0)} = (1-v)\left[\pi(K_1^*) - t^*\right]$$
$$\leq 0$$
$$\iff v \geq \bar{v}$$

At $(r,s) = (0,0), g_{VC}^{v} \to 0$, so from $(r,s) = (0,0)$, as we move along the $s-$axis, $g_{VC}^{v} < 0$. Clearly, as v increases, g_{VC}^{v} decreases, so $CZ(v)$ expands. □

Proof of 2.3.6. From the two problems, E's share of the return under $\alpha = 0:$

$$t^* = \frac{1}{(p_0 - vq_0)}\left[(p_0 - vq_0)\pi(K_1^*) - K_1^*\right]$$
$$+ \int \left(\frac{(p - vq)}{(p_0 - vq_0)}\pi(K_2^*) - \frac{K_2^*}{(p_0 - vq_0)}\right) dQ - \frac{L}{(p_0 - vq_0)}$$

And, E's share of the return under $\alpha = 1$:

$$\tilde{t} = \frac{1}{(x-vy)}\left[(x-vy)\pi(\tilde{K}_1) - \tilde{K}_1\right] + \int\left(\frac{(p-vq)}{(x-vy)}\pi(K_2^*) - \frac{K_2^*}{(x-vy)}\right)dQ - \frac{L}{(x-vy)}$$

since $K_2^* = \tilde{K}_2$. Now,

$$x\tilde{t} - p_0 t^* \equiv C_2 v^2 + C_1 v + C_0 \tag{3.14}$$

where, we find that

$$C_1 \geq 0$$
$$C_2, C_0 < 0$$
$$C_2 \geq -\frac{C_1}{2}$$

Using Descartes' Rule, we know that the equation 3.14 has two real positive roots and no real negative root. At $v = 0$, $x\tilde{t} - p_0 t^* < 0$. At $v = 1$, $x\tilde{t} - p_0 t^* > 0$ and

$$\frac{\partial}{\partial v}[x\tilde{t} - p_0 t^*] = 2C_2 v + C_1 > 0 \text{ for all } v > 0$$

Thus, there exists at least one real positive root in the interval $(0, 1)$. Call that root as v^*. Clearly, as $v > v^*$, $x\tilde{t} - p_0 t^* \geq 0$. □

3.10 Appendix C: Appendix of Chapter 3

3.10.1 Appendix C.1: Results From *Treatment H*

Here, I display the detailed results from the *Treatment H* sessions.

Behavior Trends of *Treatment H* (Part Two Group):

1. High signal principals delegate more often than low signal principals (t-stat: -6.8925).

Signal\Delegation	Delegate	No Delegate	Total
High	65 (42.76%)	87 (57.24%)	152
Low	36 (13.95%)	222 (86.05%)	258
Total	91	369	460

2. After Delegation, principals more often follow it up with high effort choice in Task 2 (t-stat: -17.8545).

Delegation\Task 2 Effort	High	Low	Total
After Delegation	67 (66.34%)	34 (33.66%)	101
After No Delegation	15 (4.85%)	294 (95.15%)	309
Total	82	328	410

3. After observing Delegation, Unbiased agents are more likely to respond by choosing High Effort in task 2 (t-stat: -11.2993).

Delegation\Task 2 Effort	High	Low	Total
After Delegation	37 (62.71%)	22 (37.29%)	59
After No Delegation	9 (5.96%)	142 (94.04%)	151
Total	46	164	210

Biased agents never choose high effort.

Delegation\Task 2 Effort	High	Low	Total
After Delegation	0 (0%)	42 (100%)	42
After No Delegation	0 (0%)	158 (100%)	158
Total	0	200	200

4. *Hypothesis H (Treatment):* The cooperation rate is higher with delegation.

Delegation\Task 2 Outcome	Task 2: $(9, 9)$	Task 2: $(5, 5)$	Total
After Delegation	28 (27.72%)	34 (33.66%)	62
After No Delegation	0 (0%)	294 (95.15%)	294
Total	28	328	356

3.10.2 Appendix C.2: Instructions for Experimental Sessions

Instructions (PI's Copy)

Comments and explanations of actions have been included in italics.

Part One

Thank you for participating in this experiment on economic decision making. Please pay attention to this instruction and also the accompanying slides. If you follow these instructions carefully and make careful decisions you might earn a considerable amount of money which will be paid to you in cash and in private at the end of the experiment.

(show them wads of cash)

The experiment will consist of two parts and last about one and a half hours. The amount of money you make will depend on the decisions you and all other participants make during the experiment.

Your computer will assign you an ID number, and at the end of the session you will be given an envelope with that ID number on it containing your monetary earnings. The person handing you your envelope will not know how much money is in the envelope. Thus, absolute anonymity and privacy will be maintained.

Please remain silent during the experiment. If you have any questions, or need assistance of any kind, raise your hand; one of the experiment administrators will come to you and you may whisper your question to him. Please do not talk, laugh, or exclaim out loud. We expect and appreciate your adherence to these rules.

You will be making choices using the computer mouse and keyboard. You may reposition the mouse pad so it is comfortable for you. Do NOT click the mouse buttons until told to do so.

(Please look up at the first slide)

This experiment will consist of two Parts, in each Part there will be several Stages. Each stage will feature a decision problem, which you will face for several "rounds." At the beginning of each stage, instructions about that stage will be given verbally and also will be displayed on the screen in front of the room. A copy of the instructions for Stage One of Part One are already handed out to you, for each stage fresh instructions will be distributed.

Throughout the experiment, at the beginning of each round, you will be assigned one of the two roles: PRINCIPAL or AGENT. You will be assigned to a role randomly at the beginning of the experiment. After that, in each round, the roles will be switched, *i.e.* , if you are a PRINCIPAL in round 1, you will be an

AGENT in round 2 and so on. There will be an equal number of PRINCIPALS and AGENTS in each round. At the beginning of each round, each participant will be randomly and anonymously matched with another participant of the other role, thus a matched pair will stay matched for at most one round.

The AGENTS can be one of two types: GREEN or RED. The AGENT's type will be randomly assigned at the beginning of EVERY round.

AGENT's type will be GREEN or RED with equal probability in every round, *i.e.* , with probability (1/2) it will be GREEN, with probability 1/2 it will be RED. The AGENT will be informed of his or her type at the beginning of each round, but the PRINCIPAL will not know the type of the AGENT he or she is matched to.

However, the PRINCIPAL will privately observe a signal about his/her matched AGENT'S type. This signal is randomly drawn by the COMPUTER; AGENTS have no control over it, and will not be able to observe it.

(Next slide shows the signals distribution.)

The signal can be LIME or PINK. On average, for 1 out of 2 GREEN AGENTs, a LIME signal is observed, and for 2 out of 3 RED AGENTS a PINK signal is observed.

For example, if there are 24 participants in a session, in each round 12 of them are assigned as PRINCIPALS and the other 12 as AGENTS. Out of the 12 AGENTS in each round, on average 1/2 (or 6) of them will be GREEN and 6 will be RED. Out of the 6 GREEN agents, on average a LIME signal will be sent to the PRINCIPAL for 3 AGENTs, and a PINK signal will be sent for the other 3 of the

6 GREEN AGENTs. Look now at the RED column: out of the 6 RED agents, on average a LIME signal is sent for 2 of the 6, and a PINK signal is sent for the other 4 RED AGENTs.

So, in any round, if you are a PRINCIPAL and observe a LIME signal, it means that your matched AGENT is GREEN with probability 3/(3+2)=3/5, or 60%. If you are a PRINCIPAL and observe a PINK signal, it means that your matched AGENT is RED with probability 4/(4+3)=4/7 or 57.1%. This matching and signalling structure will be followed throughout the experiment.

	GREEN	RED	
LIME	3	2	5
PINK	3	4	7
	6	6	12

(please look up at the next slide)

In each round, depending on the decisions you and the participant matched to you make, you will earn some payoff points.

(next slide discusses how your cash rewards from Part One will be calculated.)

The computer will calculate the sum of payoff points you earned from all the rounds in Part One. Also, in each round, given the role and type assigned to you in that round, there is a maximum number of payoff points that you can earn. The computer will keep track of these maximum payoff points for each participant. The sum of your earned payoff points relative to the sum of maximum payoff points you could earn will determine your cash rewards for Part One as follows.

At the end of the experimental session, for each participant the computer will draw a random integer between 0 and the maximum number of points the participant can get in Part One, given the assigned roles and types in each round. If your earned payoff points total is greater than that random integer, you will win a prize of $15, otherwise you will receive $2 from Part One. A similar lottery will be conducted for Part Two, to be discussed later.

(Please look up at the next slide)

Stage One

In Part One of this experiment, there will be two Tasks or decision problems. To gain experience, we will first start with a decision task which we will call Task 2.

In each round, the PRINCIPAL and the AGENT of a matched pair will make a choice in the following scenario. There are two possible choices: X and Y. You will not know your matched participant's choice until after you make your own choice, and the participant matched to you will not know your choice until after he or she has made it. In other words, you both make your decisions simultaneously without knowing the choice that the other person is making.

(next slide shows the payoff table)

The payoff consequences depend on the choice the PRINCIPAL and the AGENT make, and the AGENT's type.

	PRINCIPAL picks X	PRINCIPAL picks Y
GREEN AGENT picks X	9 9	5 1
GREEN AGENT picks Y	1 5	5 5

	PRINCIPAL picks X	PRINCIPAL picks Y
RED AGENT picks X	9 1	5 1
RED AGENT picks Y	1 5	5 5

You must choose either "X" or "Y" by clicking on your choice displayed above in the game table. The left table is for GREEN agents, so if a GREEN AGENT chooses "X" and the matched PRINCIPAL chooses "X" *(point with laser)*, each receives 9 payoff points, as indicated in the upper left cell. In each cell the lower left corner entry (which is colored according to the AGENT's type) is the payoff for the AGENT and the upper-right corner black entry is for the PRINCIPAL. If a GREEN AGENT chooses "X" and the matched PRINCIPAL chooses "Y," the AGENT receives 1 points and the PRINCIPAL receives 5 points (upper right-hand cell). If the GREEN AGENT chooses "Y" and the PRINCIPAL chooses "X," the AGENT receives 5 points and the PRINCIPAL receives 1 points (lower left-hand cell). If the GREEN AGENT and the PRINCIPAL both choose "Y," each receives 5 points (lower right-hand cell). Similarly, if the AGENT is RED, the AGENT's and the matched PRINCIPAL's payoff consequences are given by table on the right. For example, if a RED AGENT chooses "X" and the matched PRINCIPAL chooses "X," the AGENT receives 1 points and PRINCIPAL receives 9 points.

However, remember that a PRINCIPAL does not know the matched AGENT's type before making a choice. (point with laser) The PRINCIPAL will only receive a LIME or a PINK signal.

A PRINCIPAL who receives a LIME signal, knows only that with 60% (3/5) probability the AGENT is GREEN and the relevant payoff table is the one on the LEFT, and with 40% (2/5) probability the AGENT is RED and the relevant payoffs is the one on the RIGHT.

A PRINCIPAL who receives a PINK signal knows only that the AGENT is RED with 57.1% (4/7) probability and the relevant payoff table is the one on the RIGHT, and with 42.9% (3/7) probability, the AGENT is GREEN and the relevant payoff table is the one on the LEFT.

After all the participants have entered a valid choice, the AGENT's type and the choices made by you and the participant you were matched with for this round will be displayed on your monitor along with the resulting payoff points you earned in this round.

Before we begin, we will have a short quiz. Please turn to the next page and answer the short questions. We will discuss the answers in five minutes.

(quiz.

while they do quiz, the screen with payoff tables displayed.

change slide after quiz.)

Anyone needs more time to finish the quiz?

Okay, now we will discuss the answers to the Quiz. *(please look up at the next slide)*

Answer to Quiz:

1. You are assigned as a GREEN type AGENT in a particular round and randomly and anonymously matched with a PRINCIPAL. If you choose X and the PRINCIPAL chooses Y, what will be your payoff in this round?

Ans: 1.

(change slide)

Since you are assigned as a GREEN AGENT, the payoff table on the left is relevant to you. If you pick X, the green shaded cells give the possible payoffs. The PRINCIPAL chooses Y, which gives the grey shaded cells. The resulting payoffs are displayed in the dark shaded cell and YOUR payoffs are on the left corner.

(change slide)

2. You are assigned as a PRINCIPAL in a particular round and randomly and anonymously matched with an AGENT. You observe a LIME signal in this round. If you choose X, what are the possible payoffs you can get?

Ans: 9 or 1.

If the matched AGENT is GREEN and picks X, you get 9. If the matched AGENT is RED and picks X, you get 9. If the matched AGENT picks Y, you get 1 irrespective of which Type the AGENT is.

(slide change)

In the table, since the PRINCIPAL chooses X, the blue shaded cells give possible payoffs, but the PRINCIPAL does not know which table is relevant. Since he has received LIME signal, AGENT is GREEN and the left table is relevant with 60% probability. So all four payoffs that are possible are: 9, 1, 9 and 1.

(change slide)

3. In Task 2, what is the maximum payoff you can expect to earn if you are assigned as:

a. GREEN AGENT: Ans: 9 (if you and the matched PRINCIPAL both choose X)

b. PRINCIPAL matched to a GREEN AGENT: Ans: 9 (both PRINCIPAL and AGENT choose X)

c. RED AGENT: Ans: 5 (if you choose Y, no matter what the PRINCIPAL chooses)

d. PRINCIPAL matched to a RED AGENT: Ans: 9 (you and RED AGENT choose X)

(change slide and keep it at T2 table)

We will now begin interaction with the computers. If you have any questions before we begin the experiment, please RAISE YOUR HAND and a moderator will be with you shortly.

We will now begin the experiment. Please pay attention to your monitor and click the mouse when prompted to do so. Please click on the Continue button on each screen after you have read the information and/or made the choice. There are four rounds in this stage, once we have finished all the rounds, I will direct your attention to the screen in the front of the room again for the instructions for Stage Two.

Stage Two

Before starting Stage Two, we will discuss Task 1. Task 1 involves one of each matched pair (either the PRINCIPAL or the AGENT) choosing LEFT or RIGHT, where the payoff points each participant gets are given by this table:

Choice:	LEFT	RIGHT
PRINCIPAL gets	1	2
GREEN type AGENT gets	1	2
RED type AGENT gets	1	0

Please look at your computer screen and take the quiz on this task.

(quiz on personal computer screen)

(slide change after done with quiz)

We will now begin Stage Two of Part One, which contains six rounds. In this stage, you will do Task 1 and Task 2 sequentially. The sequence of actions is as follows:

- You will be assigned as PRINCIPAL or AGENT, with roles switching in every round as before. The AGENTs will receive their types (GREEN or RED) and the PRINCIPALs will not know the types but observe PINK or LIME signals. The matching and signalling will be exactly same as before.

- First, each matched pair will do Task 1. In this stage, the PRINCIPALs will be choosing LEFT or RIGHT and the AGENTs will have to wait for the PRINCIPAL to make the decision. The payoff points are as before. AGENTs will observe the PRINCIPAL's choice only after the entire round is completed.
- After completing Task 1, you will do Task 2 with the participant you are matched with. Task 2 is identical to what you did in Stage One. In each pair, both of you will simultaneously choose X or Y, as in Stage One. The instructions for AGENTs and PRINCIPALs will be displayed on your monitor.

Please turn to your monitors now.

(blank displayed while they play.)

Part Two

We are about to begin Part Two of the experiment. This part will consist of only one stage, which will contain ten rounds.

In each round, depending on the decisions you and the participant matched to you make, you will earn some payoff points. The computer will calculate the sum of payoff points you earned from all the rounds in Part Two. Also, in each round, given the role and type assigned to you in that round, there is a maximum number of payoff points that you can earn. The computer will keep track of these maximum payoff points as well. The sum of your earned payoff points relative to the sum of maximum payoff points you could earn in Part Two will determine your cash rewards for Part Two as follows.

At the end of the experimental session, for each participant the computer will draw a random integer between 0 and the maximum number of points the participant can get in Part Two, given the assigned roles and types in each round. If your earned total payoff points is greater than that random integer, you will win a prize of $15, otherwise you will receive $4 from Part Two.

((please look up at the next slide)

Stage One

In Stage One of Part Two, you will do Task 1 and Task 2 sequentially. The sequence of actions is as follows:

You will be assigned as PRINCIPAL or AGENT, with roles switching in every round as before. The AGENTs will receive their Types (GREEN or RED) and the PRINCIPALs will not know the Types but observe PINK or LIME signals. The matching and signalling will be exactly same as before.

(please look up to the next slide)

First you will do Task 1 with the participant you are matched with in this round. In this task, as before, the possible choices are LEFT or RIGHT, but there is one important difference.

If you are a PRINCIPAL in a round, you can choose whether to delegate the task to your matched AGENT, *i.e.* , let him/her choose between LEFT or RIGHT. If you are an AGENT, you will observe if your matched PRINCIPAL has chosen to delegate the task to you. If the PRINCIPAL does NOT delegate, he/she will be making the choice on his/her own. If the PRINCIPAL DELEGATES the task, the matched AGENT will be choosing. The payoff consequences are given as before.

(slide change)

If the PRINCIPAL delegates Task 1, AGENT's choices will not be visible to the PRINCIPAL right after Task 1, but only after the completion of Task 2. After the entire round is completed, the choice made in the tasks, consequent payoffs and AGENT's type will be revealed.

(please look up to the next slide)

After completing Task 1, each matched pair will do Task 2 as before. Both of you will simultaneously choose X or Y.

(slide change and keep it blank)

Now, please turn to your computer to make choices in this Part. The instructions for AGENTs and PRINCIPALs and the payoffs will be displayed on your monitors.

After the ten rounds of this stage, the COMPUTER will conduct the lotteries for the two Parts to determine your cash rewards.

Please turn to your monitors now.

(later)

Please complete the questionnaire displayed on your screen. To preserve your privacy, type xxx when asked for name; do not write your own name. While you give us your valuable feedback, we will be putting your winning amounts in the respective envelopes. Please fill out the receipt with your winning amount as well. Thanks for participating in this experiment!

	Principal b/se	Agent b/se
Task 2 effort		
signal	1.893***	
	(0.37)	
Subject	-0.018	0.013
	(0.03)	(0.03)
session	0.094	0.000
	(0.12)	(0.12)
Period	-0.026	-0.124
	(0.10)	(0.11)
Type		2.679***
		(0.55)
Constant	-4.146	-2.267
	(3.01)	(3.12)

* p<0.05, ** p<0.01, *** p<0.001

Figure 3.5: Task 2 Effort choice in the Part One Group of *Treatment H*

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