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No. 2027: May 10, 1920

The Texas Mathematics Teachers' Bulletin

Volume V, Number 3

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

University of Texas Bulletin

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The Texas Mathematics Teachers' Bulletin

Volume V, Number 3

Edited by

J. W. CALHOUN,
Associate Professor of Applied Mathematics,

and

H. J. ETTLINGER,
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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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LOOKING FORWARD

This number of the Texas Mathematics Teachers' Bulletin closes Volume Five. During the past five years this Bulletin has been sent to all the teachers of mathematics in Texas whose addresses could be obtained. It has had but one reason for existence—to aid in giving a better grade of instruction in mathematics in the schools of Texas. It has had but one standard—to be intelligible and helpful to the teachers of secondary mathematics. The writer has been one of its editors since its inception but claims no disproportionate share in the making of the publication what it has been. In fact, he has not been its most frequent contributor.

He has had the fullest cooperation of his fellow editors and of his colleagues both in the University of Texas and in the high schools of the state. Some excellent articles have been contributed by teachers in other states.

Whether the Bulletin has justified its existence during the past five years the writer can not say. What effect it has had on the teaching of mathematics in Texas he has no means of knowing with any sort of accuracy. He does not even know to what extent the mathematics teachers of the state desire its continuation. He would welcome information on that point. He would be exceedingly glad if every one who reads this would write him a note stating frankly his views as to the usefulness of the Bulletin, the desirability of continuing it, and such suggestions as he may care to make for its improvement in case it is continued.

The present school year is about spent. Little can now be done to retrive its failures or to correct its mistakes. It will be profitable to look back only long enough to see what there has been in the past year that we should like to avoid and what that we should like to retain. For the rest we should look forward. We live in a time when all things are moving. We must move with them. We must go forward or backward. There is no standing in one place now.

Teachers, of all people, should be forward looking people. Teachers deal not with the past or even the present generation but with the succeeding one. We are making the citizens of tomorrow. Tomorrow will not be like yesterday. Hence we must turn our faces forward.

In this forward movement mathematics is included. Its place in the course of study is being studied and it ought to be. Its content is being scrutinized, and that is proper. Its relation to life is being investigated and that, too, is fitting. No other subject has been able to maintain so secure a place in education for so long a time as has mathematics. There has been good reason for this. It will continue to be a basic subject because it is a prerequisite to so many forms of human thought and indispensable in so many fields of activity and investigation.

But this does not mean that mathematics has nothing to fear and that its friends and devotees need give themselves no concern as to its future. It means that in the case of a subject so fundamental there is good reason why those who love it and who understand it should take the lead in looking out for its future. That does not mean that we should have a Society for the Prevention of the Abolition of Mathematics. Mathematics can not be abolished. It can not be removed from its place in the scheme of things educational. But the content of courses offered in various branches can be greatly altered and in some cases with great improvement. It is at this junction that the friends of mathematics can do a service to education. There is much demand, some of it proper, for a showing of utility on the part of any subject given a place in the course of study. There is much insistence on what is called "vitalizing" subjects—mathematics along with other things. This is not bad, it is good. Let us vitalize mathematics but let us be sure that after the vitalizing process it is still mathematics. This is not impossible, though it is not easy. But it must be done by people who have *both mathematics* and *vitality*. Mathematics can never be vitalized by people having only one. And unfortunately much of the talk on this subject is done by people whose vitality may be great but whose mathematics is assuredly wabbly.

Now what can we do? Let us set this summer to study the question. Let us go at it whole heartedly. There is a national committee at present at work on requirements in secondary mathematics. It has on it some able men. A preliminary report of this committee is printed in this Bulletin. We can begin by reading this report.

We each can and should make a systematized effort during the coming summer to do the following things:

1. Learn more about the subject of mathematics.
2. Learn some new facts as to its applications.
3. Acquire some information about the modern methods of teaching its various branches.

The means available for doing these things are:

1. Attending summer school.
2. Making a critical examination of several texts in the various subjects, making written comments on the good and the bad features.
3. Reading some of the many books on the teaching of the various branches.
4. Subscribing for and studying some of the journals on mathematics, its teaching, and its applications.
5. Securing and studying the vavious mathematical bulletins issued by the Bureau of Education at Washington, D. C. These may be had for the asking.
6. Some personal cerebration. This method is very painful.

If we will try the plan of looking forward, of studying our subject, of improving our scholarship, of really trying to do something besides assigning lessons daily and hearing them recited the next day, we can do a great deal for our subject, more for ourselves and most of all for those whom we teach.

J. W. CALHOUN,
University of Texas. Austin, Texas.

A PROPOSED STATISTICAL STUDY OF HIGH SCHOOL GRADES

Statistical studies of college grades and of normal school grades have been made at the University of Texas by candidates for the M.A. degree, majoring in mathematics. Professor William D. Baten, M.A., of the Grubbs Vocational College, studied certain accessible records of grades of students of the University of Texas to determine the correlation between grades in mathematics and grades in other subjects. Professor James M. Bledsoe, M.A., of the East Texas State Normal College, made a somewhat similar study of normal school grades. An adequate description of the results of these studies can not be given in a few words. But the coefficients of correlation obtained indicated a high degree of correlation between mathematics and other subjects. The student who does good work in mathematics will probably do good work in his other courses—this is one of the more simple of the conclusions reached.

It appears possible to conduct in the fall of 1920—perhaps, with a view to publication—a study of the relation of high school grades to college grades in mathematics. All freshmen taking mathematics will be studying algebra in the fall term—according to present arrangements—and the first few lessons of college algebra review in some measure elementary algebra. A written test given about the second week of the fall term would reveal a student's knowledge of elementary algebra.

The following interesting questions arise:

1. Do students trained in large high schools do better work on the average than students trained in small high schools? The former have usually the better equipment, but this advantage may be offset by the close contact of teacher and student in the smaller schools.

2. With what degree of success do Texas high schools, considered individually, prepare students for college mathematics?

3. Have the students who fail in freshmen mathematics

received high grades or low grades, in general, in their preparatory mathematics?

4. Is a student who receives a barely passing grade in one course of mathematics prepared to take the next higher course in mathematics? Should he not repeat the first course to determine whether the low grade was to some extent accidental or whether it represents the limit of his intellectual performance?

In this connection it may be mentioned that for the degree of B.A., the University of Texas requires the successful completion of twenty courses properly chosen, with the special requirement that the student must make in his last ten courses a grade higher than the mere passing grade. Thus if a student should barely pass the entire twenty courses, he would be required to repeat half of the work in order to get the degree of B.A. *In other words, a student can not go from the College of Arts into the Graduate School of the University of Texas by passing the required twenty courses with all grades low.*

Furthermore, at the University of Texas a student who passes in mathematics with low grades is not encouraged to pursue further his study of mathematics. Usually he realizes, himself, that an attempt to step higher would mean failure. A general recognition of the undesirability of permitting a low grade student to attempt the higher work, without first repeating the lower work, would go far toward raising standards both in high schools and in colleges.

The psychological tests, introduced into the University of Texas last year, may be utilized ultimately.

No sensible business man would keep in his employ, year after year, a man whose work continued to be submediocre. Not even as an apprentice would such a man be of value. The school is the training place of the youth who have the ambition and the aptitude to prepare for certain forms of life-work. At a time when economy and efficiency are to be emphasized is it proper that large numbers of submediocre students should be encouraged to remain in school?

But more than ever before, the bright student, the student with ability should be encouraged to continue his study. The need of teachers, of properly trained teachers, is very great. The world, too, needs trained minds in every field to combat with the ever-increasing complexity of the problems of modern civilization. The teacher, by proper discrimination in giving grades, and by personal influence, can be of inestimable service in encouraging the right kind of student to continue his work.

EDWARD L. DODD.

THE TEACHING OF ELEMENTARY ALGEBRA

By elementary algebra, we will mean what is encompased in a year's course in mathematics in the first year of the high school curriculum, which ordinarily follows the work in arithmetic. To introduce the subject in such a manner that it will link with the previous mathematical studies of the student, the teacher must bear in mind the kind of problems which the student has been solving. To maintain the relationship of mathematics with the world of reality of time and space may be readily accomplished for this student by holding before him the problems which he has already met and solved. To proceed from the known to the unknown is to build on a sure foundation.

The atmosphere of numbers should be kept before the student at all times and the symbols used should at no time overshadow the numbers back of the processes and ideas. There is no greater obstacle to a real understanding of the principles and methods of algebra than excessive drill in juggling long aggregation of letters and symbols, during which the inwardness of the processes has been lost.

It is very paralyzing to the mind of the average student to find on the first page of his algebra the expression

$$a+b$$

with the statement that this represents the process of addition. He can not understand why the addition is not then performed. He has been thinking entirely in terms of numbers and such language is absolutely foreign to him. Instead he should be introduced to the symbolic language of algebra as a vehicle for expressing concisely the numerical facts with which he is already familiar. It is true that most modern language teachers use at the present time what is known as the "direct method," viz.: from the very beginning the student without the medium of a vocabulary and without the formal study of grammar speaks and reads the foreign language. It is essential, however, in presenting the elements of algebra to bridge the gap between the vernacular and the foreign tongue.

To further bind the ideas and principles of algebra with those of every day experience and life, it is important to use whatever historical and biographical material we may have at our disposal. To inspire the breath of life into the algebra course, there is no better opportunity than to recount the achievements of the men who by their contributions have made the science what it is today.

But there is another side that the beginner can appreciate aside from the applications of his subject and that is the relationship which mathematical facts have with each other. To develop a consciousness of the upbuilding of a rational structure of ideas and principles should be the aim of every teacher. This requires a sufficient amount of practice in the uses of methods and ample opportunity should be afforded to develop skill of this kind.

The course opens with the use of letters and symbols as a shorthand method of translating and formulating. The pupil has solved many problems in mensuration of which we select the following:

The area of a rectangle is equal to the product of the length by the width.

This may be written in somewhat more concise form as

$$\text{Area} = \text{length} \times \text{width}.$$

The simplification of using a letter to represent a word leads immediately to

$$A = l \times w.$$

The student from the very beginning should have ample practice in not only translating sentences like this into letters and symbols, but he should be given an opportunity to formulate simple problems. The teacher frequently meets students who can solve equations readily, but can not formulate a problem which required the translation of words into symbols. The realization that these letters represent not only words but numbers and that the formula thus written down expresses a relationship between quantities comes without great difficulty.

The objects of algebra are letters representing numbers or quantities. How are these to be grouped or combined? The answer is according to certain rules. Every game of cards or athletics, designates the materials of that game, and in addition prescribes the rules of combination. The four fundamental processes as illustrated by arithmetic processes and the direct connection may be established by concrete problems. As an example of addition, we may cite

$$S=C+G$$

where S is selling price, C is cost and G is gain. As an example of multiplication we may cite

$$I=P\times r$$

where I is interest, p is principal and r is rate.

The subject of factoring, usually so formal and meaningless, can be interpreted in terms of the formula, as a means of simplifying computation. A very good illustration is afforded by the example

$$a^2-b^2=(a+b)\ (a-b).$$

This formula is of considerable advantage in multiplying 97 by 103 as $(100-3)\ (100+3)$.

Algebraic fractions are most easily taught by introducing numerical fractions.

In solving equations of the first degree, it is important to point out that every correct result is not an answer and the only test is substitution into the original equation. Furthermore, the mysterious process of transposition should be banished and the process explained in terms of the operation if addition or subtraction of the *same* number on *both* sides.

In involution and evolution, the nature of the irrational can be put within the reach of the student. That the irrational number is approximated to by as near a rational number as we choose to take is a fact that is to be noted while it may not be proved with a rigorous demonstration. That is exactly what we do with the incommensurable number π. We substitute for π, the rational number 3, or 3.1 or 3.14 or 3 1/7, or 3.1416, and our choice is determined by the accuracy desired in our result.

A complete list of the topics to be treated would include theory of exponents, simultaneous linear equations and radicals. Whatever be the content of the course, these three elements must be kept in sight, (1) the atmosphere of number, (2) the development of power to apply principles, (3) the practice in using symbols and operations.

H. J. ETTLINGER.

MATHEMATICS IN THE SUMMER SCHOOL

The following courses in mathematics are offered in the summer session of 1920. Teachers who wish to continue their mathematics studies this summer will be interested in planning their work. It is also desirable to call the attention of students graduating from the high school to the fact that they can begin their University work this summer.

FIRST TERM

1a. PLANE TRIGONOMETRY.

This course will cover the subjects of trigonometric functions of angles, identities, solution of all sorts of triangles, inverse functions, circular measure, and logarithms. The arithmetic side of the subject will be emphasized. There will be much problem solving.

(Two Sections 1a1 and 1a2.)

Professor ROEVER and Adjunct Professor ETTLINGER.

1b. ALGEBRA.

This course will assume a knowledge of the amount of algebra usually covered by a good high school, but will review some of the topics of the high-school course. Especial stress will be laid on quadratic expressions and equations, the graph, logarithms, progressions, determinants, and the binomial theorem.

(Two Sections 1b1 and 1b2.)

Instructors HORTON and BATCHELDER.

1c. INTRODUCTION TO ANALYTIC GEOMETRY.

This course will be devoted to a brief consideration of Cartesian co-ordinates, plotting curves from their equations, the analytic geometry of the straight line and of the circle.

Prerequisite: Trigonometry.

Professor HEDRICK.

1d. ANALYTIC GEOMETRY.

This course, to which 1c is a prerequisite, will make a very brief review of the straight line and the circle, and will be mainly devoted to a consideration of the analytic geometry of the parabola, the ellipse, and the hyperbola.

Professor BENEDICT.

3a. A. CALCULUS.

In this course the fundamental principles of calculus will be considered. The work given will be equivalent to that of the fall term of Pure Mathematics 3 or to that of Applied Mathematics 209a.

Prerequisite: Three-thirds of Mathematics 1 or of Applied Mathematics 1 (exclusive of solid geometry) or Mathematics 215.

Professor BENEDICT.

102. A. FUNDAMENTALS IN ELEMENTARY MATHEMATICS.

For students interested in the philosophy or the pedagogy of mathematics. The significance of the number-concept; some important proofs in algebra; brief survey of the foundations of geometry; geometrical construction; de Moivre's theorem; and trigonometry.

Prerequisite: Two courses in mathematics.

Adjunct Professor ETTLINGER.

104. ALGEBRA.

This course will include complex numbers, the elementary theory of equations, the solution of higher equations, symmetric functions, etc. Equivalent to the fall term of Mathematics 205.

Prerequisite: Three terms of Mathematics 1, including 1b.

Instructor BATCHELDER.

110. A. DESCRIPTIVE GEOMETRY WITH APPLICATIONS.

Of interest to engineering students as well as to students in the College of Arts.

Professor ROEVER.

111. A. SECOND YEAR CALCULUS.

Professor HEDRICK.

115. SOLID ANALYTIC GEOMETRY.

Lines and planes in space, surfaces of revolution, conicoids.

Prerequisite: Mathematics 1d or 215.

Instructor HORTON.

128. THE MATHEMATICS OF LIFE INSURANCE.

Compound interest; mortality tables; premiums for insurance, annuities, and pensions; loading, reserves, cash surrender values, paid-up insurance and extended insurance.

Prerequisite: Three-thirds of Mathematics 1 (exclusive of Solid Geometry) or Mathematics 109.

Associate Professor DODD.

129. A. LEAST SQUARES.

Probability and the theory of errors. The adjustment of series of measurements and of numerical data containing inconsistencies. The standard deviation and the probable error. Probable errors of sums, products, and other functions of measured quantities. Applications to natural science, engineering, artillery fire, and social science.

Prerequisite: Mathematics 3 or Applied Mathematics 209.

Associate Professor DODD.

ADVANCED MATHEMATICS.

Any advanced course in Mathematics will be given during the Summer terms provided application for this course is made by a reasonable number of students who are prepared to take it. Students desiring courses which are not listed are requested to write to Professor Benedict.

SECOND TERM

1a. PLANE TRIGONOMETRY.

As in the first term.

Instructor BURNAM.

1c. INTRODUCTION TO ANALYTIC GEOMETRY.

As in the first term.

Instructor DECHERD.

1d. ANALYTIC GEOMETRY.

As in the first term.

Adjunct Professor RIDER.

1x. SOLID GEOMETRY.

This course will cover the matter usually covered in a course in solid geometry. An attempt will be made to instill sound ideas as to the nature of a geometrical proof. Attention will be called to the fundamentals of the science. Many original exercises and numerical problems will be solved. Some of the applications of the subject will be pointed out.

Instructor BURNAM.

105. ALGEBRA.

A continuation of Mathematics 104, including such topics as exponential and logarithmic functions, determinants, permutations and combinations. Equivalent to the winter term of Mathematics 205.
Prerequisite: Mathematics 104 or 205a.

Instructor DECHERD.

3b. A. CALCULUS.

The work of this course is based upon a knowledge of the calculus given in Mathematics 3a, and is a continuation of that course. It is equivalent to the winter term of Mathematics 3.

Adjunct Professor RIDER.

3c. A. CALCULUS.

This course corresponds to the work given in the spring term of Mathematics 3, or to that of Applied Mathematics 209b.

Associate Professor RICE.

117. A. ANALYTICAL MECHANICS.

The Elements of Mechanics will be given as an application of Differential and Integral Calculus.

Associate Professor RICE

ADVANCED MATHEMATICS.

Any advanced course in Mathematics will be given during the Summer terms provided application for this course is made by a reasonable number of students who are prepared to take it. Students desiring courses which are not listed are requested to write to Professor Benedict.

MATHEMATICS (APPLIED)

Certain of the courses in pure mathematics correspond so closely to certain applied mathematics courses that they may be registered for as follows by candidates for engineering degrees:

111. SOLID GEOMETRY.

The same as Pure Mathematics 1x.

1a. TRIGONOMETRY.

The same as Pure Mathematics 1a.

1b. ALGEBRA.

The same as Pure Mathematics 1b.

1c. INTRODUCTION TO ANALYTICS.

The same as Pure Mathematics 1c.

104. ANALYTIC GEOMETRY.

The same as Pure Mathematics 1d.

209a. CALCULUS.

The same as Pure Mathematics 3a.

209b. CALCULUS.

The same as Pure Mathematics 3c.

Pure Mathematics 1c and 1d will be allowed to count as Pure Mathematics 215, provided Pure Mathematics 115 is passed with a grade of at least C.

THE REORGANIZATION OF THE FIRST COURSES IN SECONDARY SCHOOL MATHEMATICS

A PRELIMINARY REPORT BY THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS

This preliminary report is issued by the national committee to serve as a basis for study and discussion. It is hoped that wide-spread consideration by organizations, committees, local groups, and individuals will result in suggestions which may be incorporated in a final report to be issued by the national committee.

I. *The Scope of the Report.*—The report is intended to suggest the most desirable mathematical training, in addition to the usual arithmetic, which a pupil should be able to secure by the end of the tenth school year. With the continued growth of the junior high school movement, it is believed that some of the work here outlined will be found appropriate for inclusion in the curriculum of the seventh and eighth school years. The present report, however, is intended to apply primarily to the first two years of the standard four-year high school.[1] A two-year period has been chosen for the scope of this preliminary report because of the fact that, at the present stage of the discussion, agreement as to what should constitute the work of the ninth and tenth years separately may be difficult to secure.

No attempt is made to consider questions relative to specific sequence of topics or to methods of presentation.

II. *Principles of Organization.*—The following two principles are made the basis of the discussion:

(1) *The primary purposes of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better appreciation of the progress of civilization and a better understanding of life and of the universe about us, and to develop those habits of thinking which will make these powers effective in the life of the individual.*

(2) *The courses in each year should be so planned as to*

give the pupil the most valuable mathematical information and training which he is capable of receiving in that year, with little reference to the courses which he may or may not take in succeeding years.

All topics, processes, and drill in technique which do not directly contribute to the development of the powers mentioned should be eliminated from the curriculum. It is recognized that in the earlier periods of instruction the strictly logical organization of subject matter is of less importance than the acquisition, on the part of the pupil, of experience as to facts and methods of attack on significant problems, of the power to see relations, and of training in accurate thinking in terms of such relations. Care must be taken, however, through the dominance of the course by certain general ideas that it does not become a collection of isolated and unrelated details.

Continued emphasis throughout the course must be placed on the development of power in applying ideas, processes, and principles to concrete problems rather than to the acquisition of mere facility or skill in manipulation. The excessive emphasis now commonly placed on maniplation is one of the main obstacles to intelligent progress. On the side of algebra, the ability to understand its language and to use it intelligently, the ability to analyze a problem, to formulate it mathematically, and to interpret the result must be dominent aims. *Drill in algebraic manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent mathematics.* It must be conceived throughout as a means to an end, not as an end in itself. Within these limits, skill in algebraic manipulation is important, and drill in this subject should be extended far enough to enable students to carry out the fundamentally essential processes accurately and expeditiously.

On the side of geometry, it is felt that the work in formal demonstrative geometry must be preceded by a reasonable amount of informal work of an intuitive, experimental, and constructive character. Such work is of great value in itself; it is needed also to provide the necessary familiarity with geometric ideas, forms, and relations, on the basis of which alone intelligent appreciation of formal demonstrative work is feasible.

III. *General Ideas.*—The one great idea which is sufficient in its scope to unify the course is that of the *functional relation.* The concept of a variable and of the dependence of one variable upon another is of fundamental importance for everyone. It is true that the general and abstract form of these concepts can become significant to the pupil only as a result of very considerable mathematical experience and training. There is nothing in either concept, however, which prevents the presentation of specific concrete examples and illustrations of dependence even in the early parts of the course. Means to this end will be found in connection with the tabulation of data, and the study of the formula and of the graph and of their uses.

The primary and underlying principle of the course, particularly in connection with algebra and trigonometry, should be the notion of relationship between variables, including the methods of determining and expressing such relationship. The notion of relationship is fundamental both in algebra and in geometry. The teacher should have it constantly in mind, and the pupil's advancement should be consciously directed along the lines which will present first one and then another of the details upon which finally the formation of the general concept of funtionality depends.

The general ideas which appear more explicitly in the course and under the dominance of one or another of which all details should be brought are: (1) The formula, (2) graphic representation, (3) the equation, (4) measurement and computation, (5) congruence and similarity, (6) demonstration. These are considered in more detail in a later section of the report.

IV. *Problems.*—As already indicated, much of the emphasis now generally placed on the formal exercise should be shifted to the "concrete" or "verbal" problem. The selection of problem material is, therefore, of the highest importance.

The demand for "practical" problems should be fully met in so far as the maturity and previous experience of the pupil will permit. But above all, the problems must be "real" to the pupil, must connect with his ordinary thoughts, must be within the world of his experience and interest. "The educational utility of problems is not to be measured by their commercial or scientific value, but by their degree of reality for the pupils. They must exemplify those leading ideas, which it is desired to impart, and they must do so through media which are real to those under instruction. The reality is found in the students; the utility in their acquisition of principles."[1]

The solution of problems should offer opportunity throughout the course for considerable arithmetical and computational work. The conception of algebra as an extension of arithmetic should be made significant both in numerical applications and in elucidating algebraic principles. Emphasis should be placed on the use of common sense and judgment in computing from approximate data, especially with regard to the number of significant figures retained, and on the necessity for checking the result. The use of tables to facilitate computation (such as tables of squares and square roots, interest, trigonometric functions, etc.) should be encouraged.

V. *Topics to Be Included:*

1. *The formula—its meaning and use—*
   - *a.* As a concise language.
   - *b.* As a shorthand rule for computation.
   - *c.* As a general solution.
   - *d.* As an expression of the dependence of one variable on other variables.
2. *The graph and graphic representations in general—their construction and interpretation—*
   - *a.* As a method of representing facts (statistical, etc.).

b. *As a method of representing dependence.*
c. As a method of solving problems.

3. *Positive and negative numbers—their meaning and use—*
   a. As expressing both magnitude and one of two opposite directions or senses.
   b. Their graphic representation.
   c. The fundamental operations applied to them.
4. *The equation—its use in solving problems—*
   a. The linear and the quadratic equation in one unknown; their solutions and applications.
   b. Equations in two variables: (i) As expressing a functional relation, with numerous concrete illustrations; (ii) As making possible the determination of the unknowns.
   c. Ratio, proportion, variation.

The pupil should gain a clear working knowledge of the idea of ratio and of its uses, and of what is meant by saying that a variable is proportional to another variable. Proportion is thus conceived of as a special instance of variation and of the functional relationship of the form $y=kx$. Some technical terms, such as "inversely prooprtional to," and "mean proportional," should be given, but the usual theorems on proportion (alternation, inversion, composition, and division) should be omitted. The subject of variation may receive considerable emphasis, in view of the usefulness of the ideas and training involved, and of the wealth of significant and easy problem material.

The consideration of simultaneous equations in two unknows should be confined to a pair of linear equations and possibly to a pair consisting of one linear and one quadratic equation, the latter possibility depending largely on the availability of significant problem material leading to such equations. Equations of higher degree than the second and in more than two unknows should be omitted from the work of the first two years.

5. *Algebraic technique.* *a.* The fundamental operations.

Their connection with the rules of arithmetic should be clearly brought out and made to illuminate arithmetical processes. Drill in these operations should be limited strictly in accordance with the principle mentioned in section II above. In particular, nests of parentheses should be avoided, and multiplication and division need not involve much, if anything, beyond monomial and binomial multipliers, divisors, and quotients.

b. Factoring. The only cases that need be considered are:

i. Monomial factors;
ii. The difference of two squares;
iii. The square of a binomial;
iv. Trinomials of the second degree that can be easily factored by trial.

c. Fractions. Here again the intimate connection with the corresponding processes of arithmetic should be made clear and should serve to illuminate such processes. The founr fundamental operations with fractions should be considered only in connection with simple cases, and should be applied constantly throughout the course to gain the necessary accuracy and facility. The most difficult complex fractions which will be needed will only contain numerical fractions in numerator and denominator.

d. Exponents and radicals. The laws for positive integral exponents should be included. The consideration of radicals should be confined to the simplification of expressions the form $\sqrt{a^2b}$ and $\sqrt{a/b}$ and to the numerical evaluation of simple expressions involving the radical sign. Extracting the square roots of numbers[1] should be included, but extracting the square roots of polynomials should certainly be omitted.

6. *Intuitional geometry.*

a. The direct measurement of distances and angles by means of a linear scale and protractor. The approximate character of measurement. The degree of precision as expressed by the number of significant figures.

b. Indirect measurement by means of drawings to scale. Uses of square ruled paper.

c. Areas of the square, rectangle, parallelogram, triangle, and trapezoid, circumference, and area of a circle, surfaces and volumes of solids of corresponding importance.

d. Practice in numerical computation with due regard to the relation between the precision of the data and significant figures in the result.

e. Simple geometric constructions with ruler and compasses, T square and triangle.

f. Symmetry, axial and central.

g. Familiarity with such forms as the equilateral triangle, the 30°–60° right triangle, and the isosceles right triangle; a knowledge of such facts as those concerning the angle-sum for the triangle and the Pythagorean relation.

h. Simple cases of geometrical loci in the plane and in space.

It should be stated that the above list contains topics such as direct measurement and the intuitional consideration of areas which, while appropriate for fairly extended treatment in the seventh grade, should not receive such extended treatment if the ninth grade ogers the first opportunity for their consideration. They are here included as an indication of the desirable preliminary foundation for which provision should be made in the earlier grades. Other topics, such as practice in numerical computation and the intelligent use of significant figures, should receive consideration throughout the course as opportunity offers, both in the algebraic and geometric work, rather than be treated as a separate topic.

The work in intuitional geometry should make the pupil familiar with the elementary ideas concerning geometric forms in the plane and in space with respect to shape, size, and position. It should, moreover, be carefully planned so as to bring out geometric relations and logical connections. Before the end of this intuitional work the pupil should have very definitely begun to make inferences and draw valid conclusions from the relations discovered. In other words, this informal work in geometry should be so organized as to make it a gradual approach to, and provide a foundation for, the subsequent work in formal demonstrative geometry.

7. *Numerical trigonometry.*

a. Definition of sine, cosine, and tangent;

b. Their elementary properties as functions;

c. Their use in solving problems involving right triangles:

d. The use of table of these functions.

The introduction of the elementary notions of trigonometry into the earlier courses in mathematics has not been as general in the United States as in foreign countries. Among the reasons for the early introduction of this topic are its great practical usefulness for many citizens; the insight it gives into the nature of mathematical methods, particularly those concerned with indirect measurement; the rôle that mathematics plays in the life of the world; the fact that it is not difficult and that it offers wide opportunity for concrete and significant application; and the interest it arouses in the pupils.

In accordance with both fundamental principles formulated at the beginning of this report, and especially in accordance with the second, numerical trigonometry must take precedence in the selection of subject matter over all of the more advanced topics of elementary and intermediate algebra and over much of the work in plane geometry that is now customarily taught.

8. *Demonstrative geometry.*—The principal purposes of the instruction in this subject are: To make the student familiar with the great basal propositions and their applications, to develop understanding and appreciation of a deductive proof and the ability to use this method of reasoning where it is applicable, and to form the habits of precise and succinct statement and of the logical organization of ideas. Enough time should be spent on this subject adequately to accomplish these purposes.

Many of the geometric facts previously inferred intuitionally may be used as the basis on which the demonstrative work is built. This is not intended to preclude the possibility of giving at a later time vigorous proofs of some of the facts inferred intuitionally. It should be noted that from the strictly logical point of view the attempt to reduce to a minimum the list of axioms, postulates, or assumptions is not at all necessary, and from a pedagogical point of view such an attempt is very undesirable. It is necessary, however, that those propositions which are to be used as the basis of subsequent formal proofs be explicitly listed and their logical significance recognized.

It is believed that a more frequent use of the idea of motion in the demonstration of theorems is desirable, both from the point of view of gaining greater insight and of saving time.

The following is a suggested list of topics under which the work in demonstrative geometry may be organized.

a. Congruent triangles, perpendicular bisectors, angle bisectors;
b. Subtended arcs, angles, and chords in circles;
c. Parallel lines and related angles, parallelograms;
d. Angle sum for triangle and polygon;
e. Seconts and tangents to circles with related angles, polygons;
f. Similar triangles, similar figures;
g. Areas; numerical computation of lengths and areas, based on geometric theorems already established.

Under these topics constructions, loci, areas, and other exercises are to be included.

It is recommended that the formal theory of limits and incommensurable cases be omitted, but that the ideas of limit and incommensurable magnitudes receive informal treatment.

If the great basal theorems are selected and effectively organized into a logical system, a considerable reduction (from 30 to 40 per cent) can be made in the number of theorems given either in the Harvard list or in the Report of the Committee of Fifteen. In this connection we may suggest that more attention than is now customary may profitably be given to those methods of treatment which make consistent use of the idea of motion (already referred to), continuity (the tangent as the limit of a chord, etc.), symmetry, and the dependence of one geometric magnitude on another.

9. *History and biography.*—To be used throughout the course for the purpose of emphasizing the idea that mathematics has been, and is, a growing science.

10. *Optional topics.*—Some schools have been able to cover satisfactorily the work suggested in sections 1 to 9 before the end of the tenth grade. The committee looks with favor on the efforts, in such schools, to introduce earlier than is now customary certain topics and processes which are closely related to modern needs, such as the meaning and use of fractional and negative exponents, the use of logarithms and of other simple tables, the use of the slide rule, simple work in arithmetic and geometric progressions (on account of their importance in finance and in scientific thinking), simple problems involving combinations, permutations and probability (on account of their frequent occurrence in daily life and their thought-provoking qualities), the elementary ideas concerning statistics, and the like.

Some of the topics above listed will be thought by many teachers to belong more properly in later elective courses. Others will, however, find it feasible and desirable to introduce some work at least of this character into the ninth and tenth grades, especially if their pupils have had previous training in geometry and algebra in the seventh and eighth grades.

VI. *Topics to Be Omitted.*—In addition to the large amount of drill in algebraic technique already referred to, the following topics should, in accordance with our basic principles, be excluded from the work of the first two years:

Highest common factor and lowest common multiple, except the simplest cases involved in the addition of simple fractions.

The theorems on proportion relating to alternation, inversion, composition, and division.

Literal equations, except such as appear in common formulas, such as may be necessary in the derivation of formulas, the discussion of geometric facts, or to show how needless computation may be avoided.

Radicals, except as indicated in a previous section.

Extraction of square roots of polynomials.

Cube root.

Theory of exponents.

Simultaneous equations in more than two unknows.

Pairs of simultaneous quadratic equations.

The theory of quadratic equations (remainder and factor theorems, etc.).

The binomial theorem.

Arithmetic and geometric progressions.

Theory of imaginary and complex numbers.

Radical equations, except such as arise in dealing with elementary formulas.

All equations of degree highe rthan the second.

VII. *Required Courses—Orzanization of the course for the ninth grade—Distribution of time.*—Most, if not all, of the material suggested in this report is of sufficient importance for every educated person to warrant making it a requirement for all pupils. In view of the fact that large numbers of students, either because they leave school or because the further study in mathematics is not required in the school they are attending, do not study mathematics beyond the ninth grade, it is felt that, i naccordance with principle 2 in section II above, the first year's work should make the student acquainted with as broad a foundation of mathematical training as is consistent with sound scholarship. In particular, the course should contain both algebra and geometry, with at least an indication of the nature of a geometric demonstration.

It is therefore suggested that in the first year (ninth grade) about two-thirds of the time be devoted to the most useful parts of algebra, the numerical topics of intuitional geometry and the beginnings of trigonometry, and about one-third of the time to geometry, including both the necessary informal introduction (if this has not been provided for in earlier grades) and the first part of demonstrative geometry.

The second year's program would then cover further work in algebra, demonstrative geometry, and trigonometry. If the student has had a satisfactory course in intuitional geometry before the ninth grade, he may find it possible to cover a minimum course in demonstrative geometry, giving the great basal theorems and constructions, together with exercises fairly well in the 90 periods constituting a half year's work. If he had not had such earlier instruction a full year should probably be devoted to geometry, both informal and demonstrative, which may, however, as indicated, be divided between the ninth and tenth grades. Additional courses should be offered in the third and fourth years of the secondary school to enable students who so desire to continue their study of mathematics.[1]

VIII. *Questions for Discussion and Report.*—It is urged that the present preliminary report be made the basis of discussion and that statements containing the results of such discussion be sent to the National Committee on Mathematical Requirements (J. W. Young, chairman, Hanover, N. H.). This committee will give them careful consideration in its future deliberations.

The following questions are suggested:

1. Do you approve the general principles and aims underlying the report? If not, in what particulars do you disapprove, and why?

2. Do you approve the specific selection of content? What topics, if any, would you omit? What topics, if any, would you add? Why?

3. What should constitute the work of the ninth grade alone, and why?

4. How much time should be devoted to the demonstrative geometry, (*a*) if only one year of mathematics is required? (*b*) if two years are required?

5. What are the principal obstacles in the way of putting such recommendations into effect in your locality?

6. Do you believe that such reorganization as here suggested would result in economy of time?

7. Information as to schools which are now giving courses approximating the program here suggested or departing substantially from customary practice (a year of formal algebra and a year of demonstrative geometry) would be very welcome.

CORRESPONDENCE

We publish in full a letter from Professor Young, chairman of the National Committee on Mathematical Requirements. This information should help to clear up two troublesome points in the preliminary report. The final report of the committee is also published in full in this number of the *Bulletin.*

MY DEAR PROFESSOR ETTLINGER:

I have just received through Mr. Underwood and at my request, a copy of the Texas Mathematics Teachers' Bulletin (Volume 5, No. 2) containing your article on a recent report issued by the National Committee.

We are very grateful for the publicity given our work in this way and also for the sympathetic comment which you have made on the report.

It is probably true that there will be wide differences of opinion concerning the proposal of making the functional relation fundamental in the organization and teaching of a first year high school course. The difficulties in the way of doing this are admittedly great and yet the desirability of centering courses in mathematics about this relation is so important and so desirable that every effort should be made to overcome the difficulties and minimize the dangers. A good deal of the opposition to this part of our proposals rests on a misunderstanding or lack of understanding of precisely what is meant. You will be interested to know that we have in preparation a monograph on the subject of the "Function Concept in Secondary School Education." This monograph is being prepared for the committee by Professor E. R. Hedrick of the University of Missouri. Its purpose will be to explain in detail just how the functional relation can be made fundamental in a high school course. It is my belief and hope that the appearance of this monograph will dissipate a good deal of the opposition. Suffice it to say at this time that the committee does not have in mind any formal or systematic instruction in the concept as such or in the notation that is usually associated with it; but is primarily concerned with emphasizing throughout all the courses in mathematics the notion of dependence. This is a notion already in the minds of pupils and is of such educational importance that every effort should be made to develop it, to make it more precise and more serviceable.

Your quotation of one of our principles that "the course in each year should be so planned, etc.," makes it clear that you were quoting from a preliminary draft of our report. That particular principle has been changed so as to read towards the end "with little reference to the courses which he may or may not take in succeeding years" instead of "without reference." It is the conviction of the National Committee that a course planned in accordance with this principle will lay a sure and desirable foundation for future work. I assume that you have received a copy of the report as finally issued for us by the U. S. Bureau of Education as Secondary School Circular, No. 5.

Very sincerely yours,

J. W. YOUNG.

THE STRAIGHT EDGE

OBSERVATIONS OF A. N. OPTIMIST

Texas will not fall down on the public schools.

* * * * * * * * * *

Texas is going to treat her teachers right.

* * * * * * * * * *

She is going to stand by Public Education.

* * * * * * * * * *

She may be slow but she is sure—and just.

* * * * * * * * * *

Those who train her citizens are going to be paid.

* * * * * * * * * *

They are going to be well paid.

* * * * * * * * * *

Texas will then demand well trained teachers.

* * * * * * * * * *

The time to get ready is now.

* * * * * * * * * *

I am going to be one of those who get ready.

* * * * * * * * * *

I am going to be one of those who use vacation to get forward.

* * * * * * * * * *

I am going to stay with my profession.

* * * * * * * * * *

(And he did and Texas did.)