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A Simulation to Evaluate the Ability of Nonmetric Multidimensional Scaling to Recover the Underlying Structure of Data Under Conditions of Error, Method of Selection, and Percent of Missing Pairs

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A Simulation to Evaluate the Ability of Nonmetric Multidimensional Scaling to Recover the Underlying Structure of Data Under Conditions of Error, Method of Selection, and Percent of Missing Pairs

by

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A Simulation to Evaluate the Ability of Nonmetric Multidimensional Scaling to Recover the Underlying Structure of Data Under Conditions of Error, Method of Selection, and Percent of Missing Pairs

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A simulation was conducted to evaluate the ability of nonmetric MDS to recover the true structure of the data under conditions of proportion of missing pairs of dissimilarities, method of selection of missing pairs, and data with and without error. The percent of pairs missing in the matrix of observations had an effect on the ability of nonmetric ALSCAL to recover the true structure of the data. The results showed that with .10 missing pairs and with .20 missing pairs the recovery was excellent. With .30 missing pairs, recovery was good. With .40 missing pairs, and .50 missing pairs recovery was poor, and solutions had

degenerate configurations with .80 missing pairs and .90 missing pairs. Method of missing and amount of error did not have an effect on either of two measures of recovery used: Correlations between recovered and true coordinates (CC) and the index of metric determinacy (M). Values of STRESS and values of RSQ obtained from the algorithm run in nonmetric ALSCAL SPSS did not represent the true recovery of the underlying structure. Ninety percent of STRESS values were good or excellent and one hundred percent of RSQ values were strong and significant even in the case of degenerate solutions. The true measures of recovery correlated poorly with the apparent measures of recovery.

Therefore, it appears that values of STRESS and RSQ while informative with low levels of missing, are misleading when percent of missing pairs reach .30 or more. Conversely, scatter plots of monotonic transformation were excellent predictors of the quality of the solution at all levels of missing pairs. Researchers should view the apparent measures of fit obtained in the SPSS nonmetric MDS output with reservation and examine the plots of monotonic transformation to evaluate the quality of the nonmetric MDS solution.

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CHAPTER I

Introduction

A picture is worth a thousand numbers (Young, 1985)

Multidimensional scaling (MDS) is a data reduction method used to analyze complex phenomena and graphically display them based on an understandable and parsimonious model. In a typical study subjects are presented with stimuli and asked to make preference choices that are treated as distance like data and referred to as proximities. In the solution, the stimuli are represented by points in a multidimensional space and are arranged so that there are greater distances among the pairs of stimuli that are most dissimilar (Young & Harris, 1997). The placement of the stimuli in space can lead to hypotheses about dimensionality and it may result in insight about the hidden structure of the underlying constructs in an objective way (Berven & Scofield, 1982). The structure is presented visually and it can be used to test theory or to suggest areas of research (Shepard, 1974).

MDS has a practical advantage for theory building in that it reduces a large set of variables to a smaller number of underlying dimensions by examining the degree of similarity among the variables. When the construct being studied is of a psychological nature -- and often not directly observable -- assumptions of normality about the data may not be realistic. MDS allows the researcher to treat responses to comparisons between elements made by observers as ordinal measures of the similarity of variables to obtain a set of coordinates that can be used to guide, clarify, and interpret a construct (Gnanadesikan, 1977). Conversely, psychological attributes are not necessarily limited to subjective evaluation; they can be observed and recorded as well. Objective measures can also be used in MDS as long as they represent the amount of difference between objects or events. For example, the events could represent the amount of time that people interact with each other in a group or the recorded percentage of time members of the different parties vote for an issue in congress (Young & Harris, 1997).

A pervasive problem with MDS occurs when the number of input stimuli presented is large. Individuals may become tired, confused, or frustrated when approaching the task. If the participants perceive the task as tedious, the resulting fatigue or boredom may lead to faulty data (Coxon, 1982). Also, ranking may become more difficult because of the cognitive complexity created by the increasing number of objects. Respondent guessing or error may compromise the quality of the data, while lack of response leads to incomplete rank orders (DeSarbo, Young, & Rangaswamy, 1997). Researchers have conducted Monte Carlo studies (i.e., Spence & Domoney, 1974) to test various designs that can be used to reduce the number of stimuli presented to participants in a MDS task. Some of the suggestions made by these researchers involve complex schemes to control the proportion of items.

Even with these limitations, MDS permits the researcher to expose unknown properties of a set of stimuli making it a valuable alternative to null hypothesis testing when the goal may be to discover psychological dimensions underlying the data (Gnanadesikan, 1977; Weinberg, 1991).

Many disciplines have used MDS to analyze information and generate theory. Psychologists have used it in a variety of contexts such as in the study of mental organization of people with schizophrenia (Catalano, 1999; Padula, Conoley, & Garbin, 1998), to examine theories of emotions (Shalif, 1988), or visual processing information in non-human subjects (Blough, 1997). Cognitive psychologists have used MDS to map structural representations of knowledge (Gonzalvo, Cañas, & Bajo, 1994), and school psychologists have used it to examine perceptions of cultural differences (Frisby, 1996). Psycholinguistic similarity data has also been used to analyze semantic structure in speech perception (Shepard, 1988).

In education, MDS is used to produce representations of students' structural knowledge. The representation of the structure that the students use to organize knowledge, ideas, or principles is called a cognitive map.

Cognitive maps can be used to evaluate structural knowledge. They can also be used by instructors to teach the relationships between the elements of complex ideas and to enhance students' comprehension. After defining the knowledge domain, a teacher selects a sample of principles, ideas, or key terms to be used as stimuli. All possible pairs between the selected ideas are then constructed. Each pair is given a rating to represent how well they relate to each other thus converting the judgments into proximity measures. The matrix of proximities is analyzed using MDS. The MDS analysis produces a map with highly related concepts placed close together and unrelated concepts placed further apart. This visual array of points in space represents a cognitive map of the conceptual domain (Diekhoff & Wigginton, 1982).

Cognitive maps can be used for class discussions. For example Diekhoff and Diekhoff (1982) conducted an experiment in a general psychology class with 69 students. Test scores of students that were taught using cognitive maps showed gains in understanding over students in a class that had not used cognitive mapping. They concluded that use of the cognitive maps forced students to think at the structural level and therefore it was useful tool (Diekhoff & Diekhoff, 1982).

Concept maps based on MDS are used to examine student progress and task mastery. Kealy (2001) compared the maps of five groups of students periodically to study the effect of collaborative learning during a six-week graduate course. Streveler, Miller, and Boyd (2001) used MDS to analyze the cognitive representations that students had formed about chemical engineering design. They presented students in a capstone course with 32 terms central to their discipline. Their responses were analyzed using SAS programming. The resulting map was evaluated by the design instructor who determined the areas of knowledge that appeared to be mastered by students (i.e., economic analysis) and areas that were not conceptually understood by them (i.e., operating heuristics). These results led the instructors to develop instructional changes such as new modules to be added to the curriculum and new exercises to be practiced in current modules. The researchers recommended the use of MDS as a technique for classroom assessment in such way that results can be used as feedback to modify instruction in order to correct areas of deficiency.

In the field of psychometrics, MDS is used not just as a tool for scale development, but also to investigate the validity of existing scales; for example, Johnston (1995) examined the underlying structure of the Rokeach Value Survey (RVS) and uncovered two dimensions; in doing so he also found that perceptual differences did not appear to exist for gender, but instead the differences appeared to be linked to developmental level.

In rehabilitation research similarity data has been used to explore and generate hypothesis using MDS (Berven & Scofield, 1982), and to examine assumptions of the characteristics, skills, and support services that foster parents believe are needed in order to be successful (Brown & Calder, 2000). Rhodes and Stern (1995) used MDS to categorize sexual harassment behaviors – publicness and traditionality – that appear to be perceived differently by individuals.

In the field of marketing research survey data have been analyzed with MDS to reveal market structure (Carroll & Green, 1997), consumer preferences (Green, Carmone, & Smith, 1989), and product positioning (DeSarbo, Young, & Rangaswamy, 1997). MDS graphic capabilities have been widely used to present the results of product research, to map brand preferences (Cooper, 1983), and as a tool for planning and evaluation (Hare, 1999; Trochim, 1989).

Presidential campaigns have been analyzed to identify voter's attitudes, as well as to discover what events influenced their decisions (Barnett, 1981; Shikiar, 1976). Archeologists have successfully utilized frequency data to map ancient sites in the absence of geographical data (Myers, 1998).

The problems of large data sets have been acknowledged and the practical usefulness of MDS can be seen in the previous examples of research in psychology, business, marketing, and other sciences. Advances in the speed, capacity, and availability of computers have contributed to areas of research that allow the simulation of data sets (i.e., Monte Carlo methods) and resampling techniques that do not require assumptions of distribution of the data (i.e., bootstrapping). However, simulation studies in MDS, also heavily dependent on computers, have not received the same level of attention.

This study was designed to investigate the effect of incomplete designs on the ability of MDS to recover the true structure of the data matrix. Figure 1 shows the first 18 cities in the US served by contract airmail (CAM) that were selected to create a matrix of points with known structure (Wells, 1994)

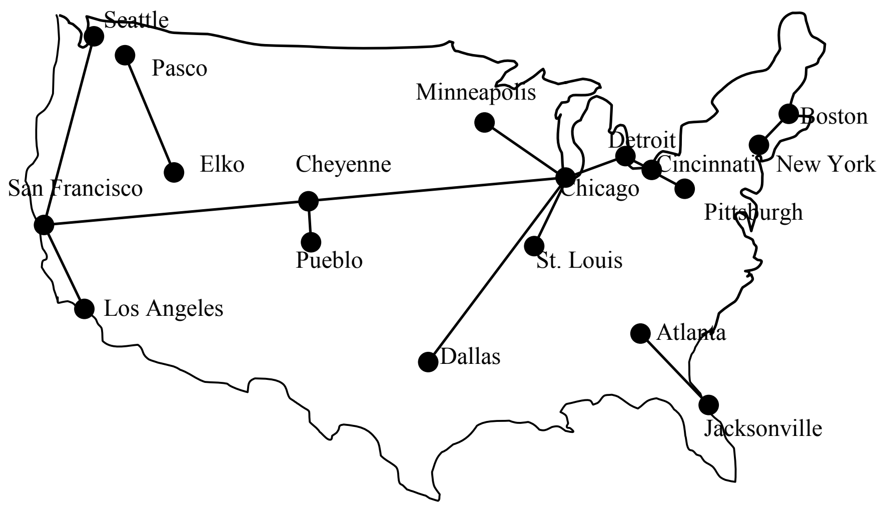


Figure 1.

Map of first cities served by contract airmail (Adapted from Wells, 1994).

The distances "as the crow flies" were obtained using an online service (http://www.indo.com/distance/) that uses US Census data and a Xerox Map Server that provides mapping capabilities to plot two places in the US. This set of real data points with known proximities and coordinates served as the source for the sampling designs using a resampling technique. A list of the cities is presented in appendix A.

Data may be missing at random because a participant lacks the required knowledge about one or more of the stimuli. On the other hand, a researcher may choose a limited subset of pairs to make the task more manageable. Both cases will be simulated to determine the proportion of pairs that can be missing without losing the ability of the algorithm to recover the true underlying structure of the data matrix.

This study will focus on two issues that have practical application in the use of MDS. First, participants may not have knowledge about one or more of the stimuli. They could guess or could choose a no-answer. Guessing only adds error to the data set. Allowing no-responses appears to be a better choice. However, the researcher has no prior knowledge or control over of the number of times an item is missing. The repeated sampling simulation will be designed to randomly select pairs missing without limitations of equal proportion of items represented.

Second, the quality of the data may also be compromised when respondents become tired or bored when the number of pairs is large. An often used solution is to limit the number of pairs presented to the participants. The second design will simulate the presentation of pre-selected number of pairs.

CHAPTER 2

Literature Review

Historical Background of Multidimensional Scaling

The development of methods to study internal processes in experimental psychology was spearheaded by an interest to apply physical measurement to psychological phenomena. It was this goal that led to the beginning of experimental psychology by Wundt in 1874 at the University of Leipzig. In an effort to make psychology more like physiology, he and others like Fetchner introduced techniques to measure the internal perceptual world by observing external events that included asking subjects to judge differences between stimuli (Boring, 1950). Psychophysical methods were defined as techniques used to measure physical attributes of the world in terms of their psychological values. These techniques involved measurements that lie along the physical dimension -- i.e., physical weight -- or the psychophysical dimension -- i.e., judgment of heaviness. In the physical world, there is a correspondence between what we are measuring and the numbers that are assigned, while in psychophysics the link between the physical stimuli and human judgments of magnitude is perception.

Fetchner formalized the method of paired comparisons in 1876 (Kantowitz, Roediguer, & Elmes, 1995). He wanted to model the relationship between brain activity and sensation when an individual was exposed to external stimuli. He recognized that some concepts, happiness for example, did not have a physical dimension; therefore, it could not be verified by direct measurement. In his method all that was required was that one stimulus could be ranked in relation to the other member of the pair. He theorized that even if human beings may not be capable of accurately detecting absolute magnitudes of the stimuli they are able to perceive that there are differences among pairs (Bock & Jones, 1968). The concept of the "just noticeable difference" or jnd came from the desire to identify the smallest difference that individuals can perceive between pairs of stimuli (Kantowitz, Roediguer, & Elmes, 1995).

In the late 1920s and early 1930s L. L. Thurstone contributed to the fields of psychophysics and psychological scaling as he became interested in examining social psychological issues. For example, he studied the effects that moving and talking pictures had on the attitudes of people and, in doing so he proposed mathematical models for comparative and categorical judgments of stimuli. Psychophysical measures were not appropriate to measure the underlying psychological construct, hence he developed scales to measure a single trait that assumed a unidimensional continuum. In his representation of the data, the stimuli were mapped as points along a psychological scale. In Thurstone scaling, data were collected by comparing pairs of stimuli and estimating their distances relative to each other. Also, Thurstone made the assumption that the distribution of differences between stimuli *i* and *j* taken on different presentations was normal (Bock & Jones, 1968).

Richardson (1938) proposed that subjective judgments of similarity resulted in a psychological measure that was analogous to a geometric model in an underlying metric space. If it is assumed that dissimilarities vary directly with distances, then coordinates for the points can be estimated based on the measures of dissimilarity between the stimuli. This was called the "estimation problem." Young and Householder (1938) continued this idea and proposed three theorems to obtain a map of the data using distances between the points instead of the coordinates. A centroid for all the points was obtained by way of double centering -- setting the mean of elements in each row and column equal to zero and subtracting the grand mean from the row and column means. Principal components was then used to obtain coordinate estimates for the stimuli. Finally, an additive constant was used to convert the dissimilarities to a scale with a zero origin (Gregson, 1975).

It was not until 1958 that the term "multidimensional scaling" was introduced by Torgerson. Among other things he analyzed the perceptions of customers to a new line of silverware patterns in the late 1950s and in doing so he may have been the first to use MDS in marketing research (Carrolll & Green, 1997). He made the assumption that judges were responding to more than one dimension of the stimuli that was presented to them. He suggested a Euclidean model, based on the Young and Householder theorems, which assumed equal interval data and no measurement error. This model, known as Classical MDS (CMDS), required a complete symmetric matrix with no missing values and quantitative measures that represented dissimilarities (Young, 1985). This method is no longer used because these restrictions have been shown to be unnecessary by Attneave and others (Baird & Noma, 1978).

Attneave (1950) relaxed the CMDS approach and proposed a non-Euclidean model. In a series of experiments he questioned the correctness of the additive constant and arrived at a metric model with interval data known as "City block" or "Manhattan Metric." The term Manhattan metric makes reference to cities that are laid out in a rectilinear grid plan -- "one walks a total of ten blocks to get from 33rd Street and 7th Avenue to 42nd Street and 8th Avenue -- 9 blocks north and 1 block west." (Attneave, 1950, p. 549).

The additive constant is used to transform values measured on a ratio scale to an interval scale by adding the smallest possible constant integer that will not permit negative distances. Messick & Abelson (1956) have proposed methods for solving what had been labeled as the "additive constant problem." Computer programs use an iterative process to arrive at the smallest additive constant that will produce a scale that conforms to a Euclidean space.

Nishisato (1978) identified two breakthroughs in the history of MDS. The first was Shepard's non-metric MDS procedure. The second breakthrough came when Takane, Young, and DeLeeuw (1977) proposed a non-metric approach that only required the assumption that dissimilarities be monotonically related to the distances. In Forrest Young's view, the first major step was the development of metric MDS. Shepard contributed to the next advancement when he proposed a more intuitive method. Then, in 1964 Kruskal developed an algorithm to judge the degree of conformity to monotonicity.

The algorithm developed by Kruskal generalized Shepard's model beyond Euclidean spaces, making his model nonmetric, more general, and less restrictive than metric MDS. In nonmetric MDS responses only need to be on an ordinal scale, but the solution is transformed to an equal interval scale. In his paper Kruskal (1964) acknowledged that it is not always possible or even desirable to observe all the dissimilarities and he accommodated missing data in the computational method without "loss of elegance" (p.116). He also defined a measure of how closely the distances fit the monotonic transformed dissimilarities. This measure, known as STRESS formula one, is actually an index of "badness of fit."

It took about one hundred years to move from systematic, paired observations of expert's perceptions of judgment to models of MDS that required only rank order information to obtain a dimensional space without making the assumption that variability follows a normal distribution. With the improvements in accessibility and speed of computers applications of MDS have become more commonplace and research more promising (Carrol & Green, 1997).

Data Sets Used in Multidimensional Scaling Simulations

A variety of data sets have been used in simulation studies where the underlying dimensions are known and the goal is to verify the ability of MDS to recover the true structure. The first nonmetric multidimensional scaling study using paired comparisons was conducted in 1957 at Bell Lab by Ernest Rothkopf. He analyzed the errors that people make when using Morse code symbols. Rothkopf presented 598 subjects with pairs of 36 Morse codes signals -- letters and numbers -- as stimuli. Participants were presented with one symbol in one ear while they heard a symbol in the other ear. Their task was to judge if the stimuli were the same or different. The entries in the 36 x 36 data matrix consisted of the percentage of time that the participants made an error. He considered the percentage of time that stimuli was confused as a measure of similarity between pairs. MDS analysis revealed two dimensions: the number of components -- total numbers of dots and dashes -- was ordered in a vertical arrangement, and the symbol composition -- ratio of dots to number of dashes -- was ordered on a horizontal arrangement. The point configuration can be seen in Kruskal and Wish (1978, p.16).

Shepard (1962) utilized data that Ekman collected in 1954 to factor analyze the color wheel. For the original factor analysis 31 participants were asked to rate the similarity of 14 colors on a 5-point scale. The factor analysis generated 5 factors -- violet, blue, green, yellow, and red. Shepard hypothesized that a two dimensional solution was more intuitively representative of the color wheel. Using the same correlation matrix he applied MDS techniques and obtained a solution in which the position of the colors corresponded to a circle with hues arranged in consonance with the color wheel (Gnanadesikan, 1977).

Geographical maps have been used frequently in demonstrations of MDS because the "true" structure is known and objective, we know what to expect and therefore can judge how well the algorithm can recover that structure (i.e., Schiffman, Reynolds, & Young, 1981). If cities are the stimuli, then the mileage between them are the dissimilarities -- the higher the mileage the more dissimilarity. Young demonstrated that if the scale properties of ratio distances between cities are ignored by converting them to ordered data the nonmetric solution by ALSCAL is practically identical for both cases (See Appendix B).

MDS Missing Data Research

In a study of incomplete designs for nonmetric MDS, Spence and Domoney (1974) proposed that large sets of stimuli escalate into an undesirable number of judgment pairs to be presented in a multidimensional task. Subject fatigue and boredom may result in disinterest and error, and some of the information may be redundant. They reviewed the effect that different proportions and different patterns of missing data have on the ability of MDS to recover the true structure of the data.

In the two part study they used Monte Carlo techniques to generate incomplete data. A single incomplete matrix representing one subject was used to avoid multiple subjects judging subsets of the data that can then be combined to generate a complete set. A constraint was imposed that each point had to be compared to at least another (connectedness). Another constrain was that all points were paired the same number of times (balance). The dependent variable was r (d, d_i); the correlation between the true distances generated for the study and the distances recovered using TORSCA-9.

Both studies used three configurations. In the first configuration the points were generated randomly inside a sphere with radius equal to 1.0. In the second configuration the points were generated randomly inside the unit sphere, but points closer than 0.9 to the center were not used, thus creating a spheroid configuration. Finally, four clusters of equal numbers of points were randomly generated so that the overall size was similar to the other two configurations. In addition, three levels of error were introduced: zero ($\sigma = 0.0$), low ($\sigma = 0.15$) and high ($\sigma = 0.30$).

In study one, three matrices of distances among 32 points were generated using the previously described spherical, spheroid, and cluster configurations.

Dissimilarities were deleted using one of two proportions -- 1/3 and 2/3 -- and one of four methods -- random, overlapping cliques, cyclic I, and cyclic II. In the random design the desired proportion was deleted using a random number generator. The overlapping cliques method was based on a design suggested by Torgerson in 1958. The method resulted in two matrices with 19 stimuli and a 6x6 overlap in the 1/3 missing condition; there were seven matrices with eight stimuli and a 4 x 4 overlap in the 2/3 missing condition. The other two methods also used submatrices that were connected and that satisfied the requirement that each stimulus appeared the same number of times. They used three replication in part one.

Results from study one showed that error and method of deletion had an effect on the ability of TORSCA-9 to recover the original structure of the data matrices. However, when holding either error or percent deleted constant, the choice of design did not appear to be important. The random design performed just as well as any of the other designs, with the overlapping cliques being the worst. Recovery with 1/3 deleted was good, but with 2/3 deletion recovery was not good unless error was very low.

In part two configurations and error levels were the same as in part one, this time using 40 and 48 points. Only the random design was used since it appeared to be the most efficient after the previous study. Levels of proportion deleted ranged from 0% to 80% in nine intervals of 10%. Only one replication was used for each of the 162 combinations. Recovery was better with the higher number of stimuli and also with lower error rates.

Overall, Spence and Domoney (1974) recommended the use of random designs and assumed that with a large number of stimuli there was a high probability that the designs would be connected. It was also suggested that this results might not apply to smaller data sets.

MacCallum (1978) conducted a simulation with 30 stimuli and three dimensions. He considered the case of replicated MDS. If different subjects receive a different set of random stimuli pairs, then it would become highly probable that there would be complete information about all possible pairs available for analysis. However, if all persons are missing the same matrix elements, then some pairs may never be available for analysis. This became the same (S) versus different (D) condition. In addition he manipulated the number of replications (10 and 20) and the proportion missing (.20, .40, and .60). As expected, he found that as proportion missing increases the recovery indexes deteriorate. Number of replications did not have an effect. However, when size of sample was small and the proportion of missing pairs large the D condition had better recovery indexes. He used three measures suggested by earlier research (MacCallum & Cornelius, 1977) to evaluate the accuracy of the recovered structure.

In the MacCallum and Cornelious (1977) research the focus was on recovery of true dissimilarities from the observed data under conditions of varied amounts of stimuli, respondents, dimensions, and random error. They considered measures of recovery obtained from respondents' dissimilarity pairs as indexes of apparent fit, and measures of recovery derived from the true data as indexes of true fit. Measure of apparent fit was evaluated using SSTRESS obtained from an early version of ALSCAL. The formula they used was

$$SSTRESS = \frac{\sum_{i=1}^{n}\sum_{j=2}^{p}\sum_{k=1}^{j-1}\left(t_i^o\left(O_{ijk}^2\right)-\hat{d}_{ijk}^{\,2}\right)^2}{\sum_{i-1}^{n}\sum_{j=2}^{p}\sum_{k=1}^{j-1}\hat{d}_{ijk}^{\,4}}$$

in which t_i^O is the monotonic transformation for respondent *i*; O_{ijk} is the observed measurement for respondent *i* on pair *jk*; and $\hat{d}_{ijk}$ is the recovered distance between stimulus *i* and *j* for respondent *i*.. This is not the same formula for SSTRESS used in the ALSCAL algorithm in SPSS.

Index of metric determinacy

Measure of fit to the true distances was evaluated using the "index of metric determinacy" (McCallum & Cornelius, 1977, p. 409) developed by Young in 1970. This index, labeled M, is simply the squared correlation between the distances recovered by the ALSCAL algorithm and the true distances for all pair of stimulus. This correlation is then squared. The formula for this index is as follows:

$$M = \left\{ \frac{\sum_{j=2}^{p} \sum_{k=1}^{j-1} \left(d_{jk} - \bar{d} \right) \left(\hat{d}_{jk} - \bar{\hat{d}} \right)}{\sqrt{\left[\sum_{j=2}^{p} \sum_{k=1}^{j-1} \left(d_{jk} - \bar{d} \right)^2 \right] \left[\sum_{j=2}^{p} \sum_{k=1}^{j-1} \left(\hat{d}_{jk} - \bar{\hat{d}} \right)^2 \right]}} \right\}^2 \tag{1}$$

In this formula d_{jk} is the true distance between stimuli j and k; $\bar{d}$ is the mean of the true distances across all paired comparisons; $\hat{d}_{jk}$ is the recovered distance between stimulus *j* and stimulus *k*; and $\bar{\hat{d}}_{jk}$ is the mean of the recovered distances across all pairs

A second measure of true fit used by McCallum and Cornelius (1977) was the root of the sum of the differences between the true and the recovered projections of the coordinates.

$$\delta = \sqrt{\sum_{j=1}^{p} \sum_{t=1}^{r} \frac{\left(b_{jt} - \hat{b}_{jt}\right)^2}{pr}}$$

In this formula, $\hat{b}_{jt}$ is the recovered coordinate projection to the X and Y axis for stimulus j and dimension t, and b_{jt} is the true coordinate projection to the X and Y axis for stimulus j and dimension t. If the recovered coordinates fitted the true coordinates perfectly the sum of the differences would equal zero. Since the ALSCAL solution is scaled to have a mean of zero and a sum of squares equal to p, all the solutions are on the same scale.

Their results showed that error had a significant and strong effect on the all the measures, but number of respondents did not. The researchers also correlated all of the dependent variables. The correlation between the index of the differences in distances between pairs of stimuli (M) and the index of differences in the coordinate projections was .77. The correlation between M and SSTRESS was -.62, however, the correlation between SSTRESS and the coordinate projection differences was only .36. They concluded that these measures may address different aspects of the recovery, but also that SSTRESS may not be a good indicator of goodness of fit.

Problems and Limitations of Multidimensional Scaling

Some of the limitations of MDS that make its use less appealing to researchers are the lack of significance tests, the large number of pairs of stimuli that is presented to subjects, and the effects of order of presentation of stimuli.

Lack of significance tests

Since the information required to obtain the dimensional space is of only rank order, nonmetric MDS techniques do not require assumptions that variability follows a normal distribution. Therefore, there are no statistical significance tests available. There are some measures of fit, like STRESS, but they are only descriptive of how well the recovered data fits the input matrix.

Number of stimuli

A serious problem with MDS is that as the stimuli become large the number of paired comparisons that are presented to the participants increases dramatically. According to Spence (1983), when the number of stimuli reaches 60 it is unrealistic to present a person with all of the possible paired comparisons (n = 1770); MacCallum, (1978) considers as few as 20 stimuli large enough (n = 190) to suggest the need for incomplete designs.

The formula for calculating the number of comparisons is $I(I-1)/2$. For nine stimuli the number of pairs is 36 and it increases rapidly to 153 for 18 stimuli.

Approaches to presenting subsets of the data such as overlapping cliques and cyclic designs have been reviewed. Results from Monte Carlo studies indicate that random selection is almost as effective as more complex designs used in the recovery of the true structure of the data (Spence, 1983; Spence & Domoney, 1974; Spector & Rivizzigno, 1983). However, one problem with these studies has been that the number of simulations was limited to one to five simulations.

Order of Presentation

Another problem with MDS, which has been noted by researchers, is that rating of pairwise data is prone to response set (Cronbach, 1946; Rorer, 1965). Robert Ross (1939) developed a method for ordering pairs that is widely used.

The Ross Matrix. To avoid problems of primacy and recency effects in paired-comparison data collection, researchers have used a technique proposed by Ross (1939) to systematically arrange all possible paired comparisons. A matrix is created where all the possible pairs are arranged so that each stimulus is preceded and followed by each other stimulus the same number of times. The number of rows and columns in the matrix are calculated according to the following formulas: If there is an odd number of stimuli, the number of rows is $(n + 1)/2$, and the number of columns is $(n-1)$. For an even number (i.e., n = 18) the table is developed for $(n+1)$ stimuli to get the ordering, and then all pairs that include $(n+1)$ are deleted. In this paper this number will be referred as n_R.

Therefore, for 18 stimuli with $n_R = 19$ the Ross matrix has 10 rows and 18 columns. After the pairs are formed according to the Ross table, pairs in even-numbered columns that have the same stimulus number are deleted, and for pairs in odd-columns that have the same stimulus number the variable number is replaced by the number 1 (See Appendix C). A list of pairs is created by going down each column. Appendix C also presents the arrangement for 18 stimuli. It should be noted that each stimulus is represented at least once for any consecutive $(n_R + 1)/2$ possible pairs.

Types of data used in MDS

The measure obtained from the comparisons between pairs of stimuli is called a proximity. Input data for MDS analysis may consist of one or more proximity matrices. Each element in the matrix represents the amount of similarity or dissimilarity between each pair. If dissimilarities are used, large numbers will represent large dissimilarity and small numbers will mean not much dissimilarity. That is, the more dissimilar a pair of items is judged to be, the greater the distance between the points that represent them (Coxon, 1982; Gnanadesikan, 1977). A variety of techniques can be used to obtain proximity measures that indicate the degree of relationship between every pair of variables within the set. Information about pairs can be collected in many ways including judgments of similarities, correlations, and frequency of co-occurrence (Berven & Scofield, 1982; Weinberg, 1991).

Judgments of similarities.

Paired comparisons are used in metric and non-metric MDS. However, metric MDS requires the assumption that the input data are at least interval, while nonmetric MDS only requires that the data be ranked. In direct similarity judgments participants may be presented with a pair of objects and asked to state the similarity among them using a rating scale such as

Highly similar				Highly dissimilar
1	2	3	4	5

These direct measures can be objective (i.e., number of accidents) or subjective (i.e., emotional distress) depending on how the task is presented to the respondents (Diekhoff, 1992).

Correlations

In another technique profile similarity measures are used, such as correlation coefficients. Higher correlations indicate more similarity, and lower correlations indicate less similarity between pairs of stimuli (Sneath & Sokal, 1973). Correlations have values that range from –1.00 to + 1.00. Negative correlations are indicative of less similarity between the points and positive correlations are indicative of more similarity (Hanneman, 1998). Pearson correlations are useful because they provide information about the strength of the perceptions of proximity. They also can be presented as objective or subjective measures (Diekhoff, 1992).

Frequency of co-occurrence or confusion

Finally, data may represent the probability that two stimuli arise together. Measures can be obtained by asking the subject to sort the stimuli. The number of times that two stimuli appear in the same category is counted. Then the frequency of co-occurrence is converted into proportions (Rosenberg, 1982). The proportions are entered in the matrix that is submitted to the ALSCAL algoritm.

Data Functions

Assumptions about measurement models are rarely met in real data sets (Coxon, 1982). In nonmetric MDS all that is necessary to represent the data is the rank order of the entries in the matrix that is used to arrive at a solution. Ordering the stimuli re-scales the data into a set of values that can be represented in Euclidean space. Therefore, knowing the rank order is sufficient to give a solution. As stated earlier, even if the data are only ordinal, the MDS solution is metric. By definition, a measure is a metric if it satisfies the following properties for all points I and J: positivity, reflexive minimality, symmetry, and triangle inequality (Coxon, 1982).

Positivity

The distance between two stimuli is greater than zero. There are no negative distances. Therefore

$$\mathrm{d}(x_i, x_j) \geq 0;$$

the distance from *i* to *j* is equal to or larger than zero for all points *i* and *j*.

Reflexive minimality

The distance between *i* and *j* is zero if and only if the two points coincide

$$d(x_i, x_j) = 0 \qquad i.f.f. \quad i = j;$$

Symmetry

The distance from *i* to *j* is the same as the distance from *i* to *j* for all points *i* and *j*

$$d(x_i, x_j) = d(x_j, x_i)$$

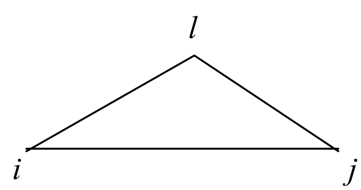
Triangle inequality

The sum of two sides of the triangle is more than the third side. It can only be equal if point *l* lies on the line *ij*

$$d(x_i, x_j) \le d(x_i, x_l) + d(x_l, x_j) \qquad \text{for all points } i, j, l$$

If *l* lies off line *ij*, then

$$d(x_i, x_j) < d(x_i, x_l) + d(x_l, x_j);$$



If C lies on line AB, then

$$d(x_i, x_j) = d(x_i, x_l) + d(x_l, x_j)$$

● -------------- ● ------------- ●
i *l* *j*

If a distance does not fulfill this properties it is not a metric.

Distance Functions

The metric that is used depends on the purpose of the measurement. The general formula for the distance function is called the Minkowski metric,

$$d_{ij} = \left[\sum_{a=1}^{r} \left| x_{ia} - x_{ja} \right|^{c} \right]^{\frac{1}{c}} \quad , \quad 1 \leq a \leq \infty$$

If c = 1, then the Minkowski metric reduces to the city block or Manhattan metric;

$$d_{ij} = \sum \left| x_{ia} - x_{ja} \right|$$

if c = 2, it reduces to the Euclidean metric.

$$d_{ij} = \left[\sum_{a=1}^{r} \left(x_{ia} - x_{ja} \right)^{2} \right]^{\frac{1}{2}}$$

As c approaches infinity, it becomes the supremum metric (Coxon, 1982).

$$d_{ij} = \max_{a} \left| x_{ia} - x_{ja} \right|$$

As stated earlier the term Manhattan metric makes reference to distances measured between two points in cities that are laid out in a rectilinear grid plan (Attneave, 1950) as opposed to going from point I to point J as the crow flies. According to Gregson (1975) "homogeneous stimuli are compared by an Euclidian rule, whereas a city block model better represents simple stimuli that vary on perceptually distinct dimensions" (p. 109). In the city block metric psychological dimensions are in an additive space (Attneave 1950).

Incomplete Data in MDS

There is a variety of ways in which researchers may encounter incomplete data in MDS. Data may be missing by chance or by choice. Coxon (1982) posed reservations about how much missing data can be tolerated before a solution becomes unstable, and he questioned which is more dangerous: random or systematic loss of information about one or several points. Either all the stimuli are familiar to all subjects or the number of stimuli should be reduced. Another possibility is that researchers may simply choose to reduce the number of judgments by not rating all possible pairs.

A formula has been suggested for estimating the minimum number of subjects that are needed to evaluate each pair when not all pairs are presented to each subject (McCullum, 1979; Spence & Domoney, 1974).

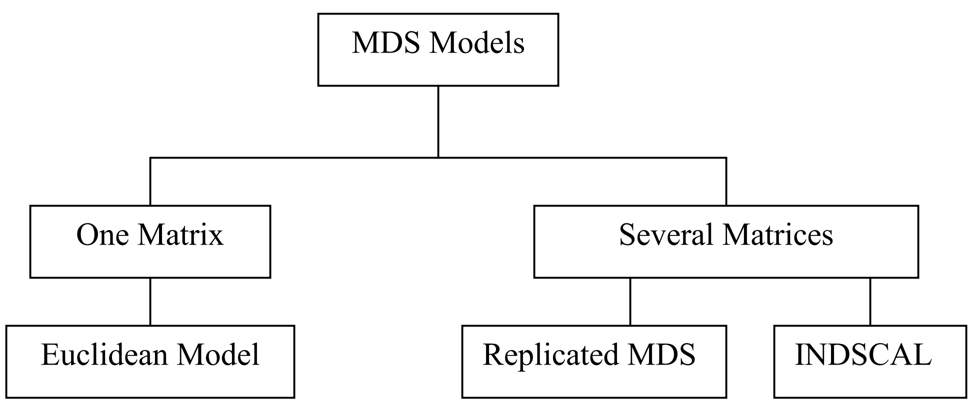
$$M = 40K^* / (I - 1)$$

where M = number of times pair is presented, K = number of dimensions, and I = number of stimuli.

Types of MDS

In psychology, researchers often study variables that are latent or subjectively defined. Measurement of these variables cannot be observed directly because they only exist in people's minds (Young & Hamer, 1987). Measurements can be obtained by eliciting the dissimilarities between pairs of stimuli and then analyzed using MDS to represent each geometrically as points in space.

MDS models can be summarized as follows



The algorithm that is used to locate the points in Euclidean space and uncover the structure of the underlying constructs depends on the type of data and the number of matrices being analyzed. For example, in classical nonmetric MDS there is only one matrix of proximities to be analyzed using a Euclidean model (Young, 1985) while weighted MDS makes the assumption that each individual places a different weight on each of the dimensions; therefore, several matrices are analyzed and the solution maps both the stimuli and the individuals. Weighted MDS, also known as INSDCAL, is a Euclidean model that assumes variation in the way that subjects weight each of the dimensions. It analyzes several matrices that contain either ordinal (non-metric) or interval (metric) data.

<u>The Metric MDS Model.</u>

In metric MDS at least an interval scale is used for the distances between points. The distances are fitted to the dissimilarities using a least squares method. In the equation

$$l\{S\} = D^2 + E$$

S is the “linear transformation of the dissimilarities” (Young & Harris, 1997, p. 126), In the linear transformation the intercept will be zero if the data is at a ratio level, but it can be nonzero is the data is measured on an interval scale.

The slope is positive because the data represents dissimilarities. *D* represents the distances, which are a function of the coordinates, and *E* is a matrix of residual errors. The goal of metric MDS is to calculate the coordinates so as to minimize the sum of the squares of *E*.

<u>The nonmetric MDS model.</u>

In nonmetric MDS the data only need to be ordinal. The goal of the nonmetric model of MDS is to maintain a one to one correspondence between the rank order of the dissimilarities and the rank order of the distances among the stimuli.

In the Kruskal (1964) model $\underline{\Delta}$ is a matrix of dissimilarities, $\underline{\mathrm{D}}$ is a matrix of distances, and $\underline{\widetilde{\mathrm{D}}}$ is a matrix of disparities. Disparities refer to the monotonically transformed data.

$$\underset{nn}{\underline{\Delta}} = \underset{nn}{\underline{\widetilde{\mathrm{D}}}} \cong \underset{nn}{\underline{\mathrm{D}}} = g\left(\underset{nr}{\underline{\mathrm{X}}}\right)$$

The responses of participants to the pair dissimilarity task are entered in a symmetric matrix of proximities ($\underset{nn}{\underline{\Delta}}$).

$$\underset{nn}{\underline{\Delta}} = \begin{bmatrix} dii & . & . & dij \\ & . & . & . \\ & & . & . \\ & & & djj \end{bmatrix}$$

The next step involves creating a starting configuration matrix of coordinates for the given number of dimensions. A principal components analysis generates rational starting values; but the computer program can be superseded if there is a desired order.

$$\underset{nr}{\underline{\mathrm{X}}} = \begin{bmatrix} x_{11} & . & x_{1r} \\ & . & . \\ & & x_{nr} \end{bmatrix}$$

Euclidian distances are then calculated from the coordinate information for all pairs to generate a matrix of distances corresponding to the rank.

$$\underset{nn}{\underline{\mathrm{D}}} = \begin{bmatrix} d_{ii} & . & . & d_{ij} \\ & . & . & . \\ & & . & . \\ & & & d_{jj} \end{bmatrix}$$

Distances are calculated using a Euclidean model

$$d_{ijk} = \sqrt{\sum \left(x_{ir} - x_{jr}\right)^2}$$

where x_{ir} is the coordinate of stimulus i on dimension r, x_{jr} is the coordinate of stimulus j on dimension r.

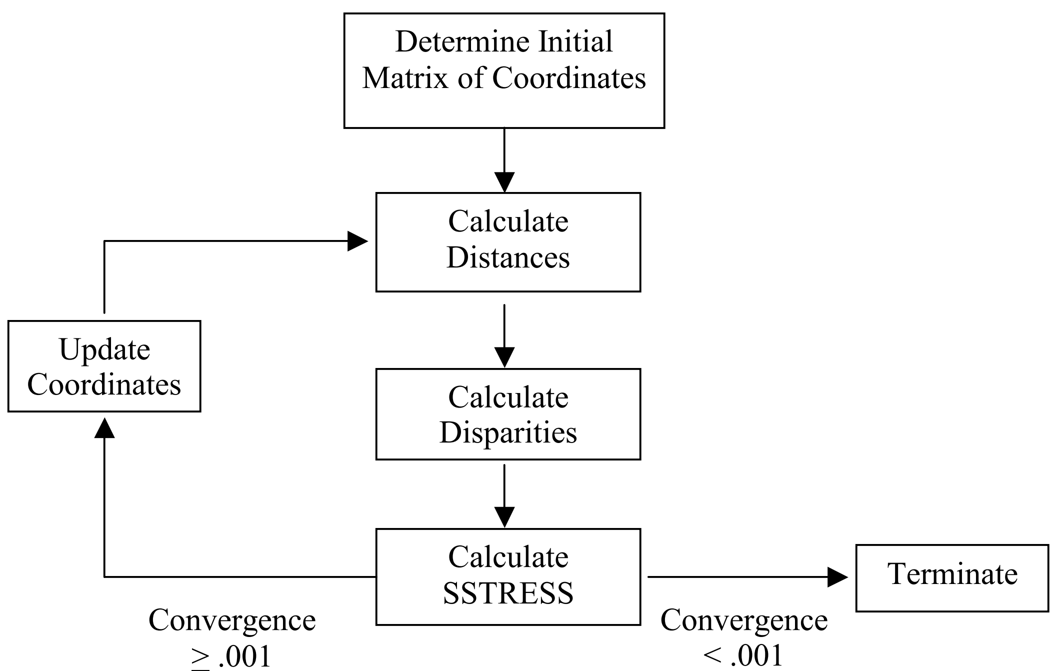
The distances must be monotonically related to the original rank of the dissimilarities. A new set of values, called disparities, are calculated so that their rank order will be as close as possible to the rank order of the distances. In the monotonic transformation the only information that is preserved is rank order. Disparities are estimated to satisfy the constraint that for all points

$$\delta_{ij} < \delta_{il} \Rightarrow \tilde{d}_{ij} < \tilde{d}_{il}$$

In SPSS ALSCAL the disparity matrix is compared to the distance matrix using the squared stress (SSTRESS) formula one developed by Takane, Young, and deLeeuw (SPSS, 1997), where d_{ijk} = Euclidian distance, d^*_{ijk} = disparities. The formula used by the ALSCAL algorithm is (SPSS, 1997)

$$\text{SSTRESS} = \left[\frac{1}{m} \sum_{k=1}^{m} \left[\frac{\sum_i \sum_j \left(d_{ijk}^2 - d_{ijk}^{*2} \right)^2}{\sum_i \sum_j d_{ijk}^{*4}} \right] \right]^{1/2}$$

Steps are repeated using the values from the preceding set of coordinates. The new value of SSTRESS is compared with the value from the previous iteration. This process is repeated until the value of SSTRESS does not change significantly. The following chart shows the nonmetric ALSCAL algorithm model



Measures of fit

In non-metric MDS, measures that evaluate how well the stimulus coordinates account for the proximity data are descriptive. Two measures are used to assess how well the solution fits the original matrix: STRESS and RSQ. The goal is to minimize the former and maximize the latter.

STRESS

STRESS is the square root of the sum of the squared deviations, between the distances and the disparities; it is a measure of conformity to monotonicity. STRESS is thought of as a measure of badness-of-fit since higher values indicate worse fit (Gnanadesikan, 1977).

The formula is referred to as Kruskal formula 1. It is different from the SSTRESS formula used during the iterations in ALSCAL SPSS.

$$\text{STRESS} = \sqrt{\frac{\sum_{i<j}\left(d_{ij} - \tilde{d}_{ij}\right)^2}{\sum_{i<j} d_{ij}^2}} \tag{2}$$

where d_{ij} are the distances, and $\tilde{d}_{ij}$ are the disparities.

Since stress has no known distribution it is an index rather than a statistic. The goal is to minimize its value, but a perfect fit value of zero will be obtained only if the number of dimensions equals one less than the number of objects or in the case of degenerate solutions. Stress will approach 1.0 as the data are less accounted for by the solution (Davison, 1992). Kruskal (1964) proposed rules of thumb for evaluating STRESS given that the following conditions exist: (1) ordinal scaling, (2) half matrix data without diagonal, (3) no ties in the data, and (4) a single matrix. STRESS of .11 to .20 suggests poor fit, .06 to .10 fair fit, .03 to .05 good fit, and .025 or less excellent fit. The number of dimensions can also be evaluated by examining the "elbow" in a plot of STRESS versus number of dimensions similarly to what is done in factor analysis.

Two other guidelines to evaluate STRESS were derived through empirical research by MacCallum (1978) and by Sturrock and Rocha (2000). MacCallum developed a formula for calculating the values of STRESS that would be obtained when the data is random. Given that D is the number of dimensions, N is the number of variables, and the coefficients are a0 = -524.25, a1 = 33.80, a2 = -2.54, a3 = -307.26, and a4 = 588.35 the expected value of stress for random data is

$$E(STRESS) = a0 + a1 * D + a2 * N + a3 * \ln D + a4 * \sqrt{\ln N} = 0.284$$

That is, for 18 stimuli in two dimensions the expected value of Stress is 0.284. Values higher than this represent data sets of points that have a random relationship.

Sturrock and Rocha (2000) conducted a computer simulation that generated 587,200 random matrices to arrive to a table of stress values for one to ten dimensions and four to one hundred objects. According to their table, for 18 objects and 2 dimensions, values of STRESS below 0.263 represent a matrix with objects that are structurally related ($p<. 01$).

R-Squared

R-squared (RSQ) is the squared simple correlation between the corresponding distances in the solution space and the monotonically scaled disparities.

$$\text{RSQ} = r^2_{d,\bar{d}}$$

R-squared can be evaluated for statistical significance. Small values of STRESS suggest a good fit of the output measures to the original data, while small values of R-square show a poor relationship between the distances and the disparities. Its value equals one if the coordinates account for the data perfectly and decreases toward zero for lesser fit. For 153 pairs generated from 18 stimuli the critical value at $p<. 01$ for r is .208 ($r^2 = .043$).

Computer programs

A variety of self-standing programs have been written over time to evaluate proximity measures. Alternating Least Squares Scaling (ALSCAL) was originally written at the L. L. Thurstone Psychometric Laboratory by Forrest

Young and colleagues as a self-standing FORTRAN program to perform MDS analysis. It is now incorporated with PROC MDS in SAS, and is used as a data reduction procedure in SPSS (Young &Lewyckyj, 1979).

Other programs are also available. KYST, MDPREF, PREFMAP, and INDSCAL are available from Bell Laboratories and can be accessed through NETLIB (http://www.netlib.org/) and other Internet sites.

CHAPTER 3

Method

The purpose of this study was to evaluate the effect of missing values on the ability of nonmetric ALSCAL to recover the true underlying structure of a set of data. A 18 x 18 matrix of city distances was used as the true dissimilarities (TD). A second 18 x 18 matrix of dissimilarities (DE) was created by adding random error to the original data set to represent a respondent overestimating or underestimating pairs of dissimilarities. Using a repeated sampling technique missing values were selected either systematically (S) or randomly (R). In the first condition (S) the number of missing pairs was increased using multiples of $(n_R+1)/2 = 10$. In this condition each stimulus appeared the same number of times. In the second condition (R) missing pairs were selected randomly in proportions that corresponded to number missing in the first condition. The number of stimuli (18) and the number of dimensions (2) were set constant. Table 1 summarizes the 2 x 2 x 8 conditions. Each condition was simulated 100 times.

Source of Data

Cities have been used often in MDS research because they have a known two-dimensional structure and both the mileage between them and the point coordinates are easy to determine using objective measures. The source of data for this study was the first contract airmail (CAM) routes across the United States (Wells, 1993). The distances as the crow flies among the 18 cities were obtained from an Internet service that employs U. S. Census Bureau data and Xerox PARC mapping to generate mileage as well latitude and longitude coordinates (http://www.indo.com/distance/). The mean distance for the 153 pairs was 1199.84 miles with a standard deviation of 678.92. The minimum distance was 96 and the maximum distance was 2708. A map of the true location of the cities generated by SPSS ALSCAL using the complete matrix is shown in appendix B.

A second data set was created by adding random error to the city distances to represent overestimates or underestimates of dissimilarities. A random value was generated from a normal distribution with a mean of zero and standard deviation of 100 and added to each of 153 pairs. The added random error ranged from –248 to +296. The resulting added error had a mean of -0.16 and a standard deviation of 100.01.

Design

The easy accessibility to computers has allowed development of methods and procedures that make use of iterations to obtain a solution. It has also spearheaded the use of data simulations to observe how a particular procedure performs by generating random samples and empirically assessing if the solution fits the expected results (Nunnally & Bernstein, 1994). Bootstrapping was developed as a resampling technique that did not require distributional assumptions. In this technique an observation is removed from the original data repeatedly with replacement to generate the chosen number of new samples of the same size (Weinberg, Carroll, & Cohen, 1984).

SPSS syntax was used to design a program that randomly selected a proportion of entries in the original matrix and set them to missing. A macro routine was included to repeat the process 100 times (see appendix C). In a separate program a random starting point was selected and followed by a selected number of consecutive entries from the Ross table. It was also included in a macro routine to repeat it 100 times. Each of the matrices generated was analyzed using nonmetric ALSCAL. Model conditions are shown in appendix F.

Variables

The number of stimuli was 18, which yielded 153 paired comparisons. Two dimensions and 100 replications were used in each condition. The independent variables were the source of data – without error (TD) and with error added (DE) – the method of selection of missing pairs – random (R) and systematic (S) – and the proportion of missing pairs (eight levels). The dependent variables were obtained from SPSS output, from subjective evaluation of geometrical plots, from comparisons between the original and the recovered coordinates, and from comparisons between the original and the recovered distances.

Independent variables

Two data sets were used 1) matrix of true dissimilarities (TD) and 2) matrix of dissimilarities with random error added (DE). For each data set the second independent variable had two conditions 1) randomly missing pairs (R), and 2) systematically missing pairs (S). The third independent variable was the proportion of pairs missing in the data matrix.

The number of pairs missing was increased by multiples of $(n_R + 1)/2$ for the systematic missing design (8 conditions). When using the list of all possible comparisons generated by the Ross matrix each pair appeared at least once in any $(n_R + 1)/2$ pairs sequence. Using multiples determined the number of times that each stimulus appeared equally in the sequence. The least number of times that each stimulus was selected to appear was once. Selecting 10 pairs resulted in 93 percent or 143 pairs missing. The most number of times a stimulus was selected to appear was fourteen, which represented nine percent or 13 pairs missing.

For the random case eight levels of proportions were selected – from .10 to .90 – to approximate the number missing at each level already selected in the systematic condition. Table 1 shows the design with the 2 x 2 x 8 conditions. Each was replicated 100 times and the results analyzed using nonmetric ALSCAL in SPSS.

Dependent variables

The dependent variables used to evaluate how well nonmetric ALSCAL recovered the original matrix of proximities were STRESS (Kruskal formula 1), RSQ, the correlation between the original coordinates and the recovered coordinates (CC), and the squared correlation between the original distances and the recovered distances which is referred to as index of metric determinacy or M (McCallum & Cornelius, 1977).

Kruskal STRESS and RSQ were obtained from SPSS output. The product moment coefficient of correlation between the recovered coordinates and the original coordinates was calculated for each replication and labeled CC. Finally the square correlation between the true and the recovered distances were calculated using the formula employed by McCallum and Cornelius (1977) and presented as formula 1 earlier in this paper.

Data Analysis

Results from the nonmetric ALSCAL simulations were analyzed using univariate analysis of variance (ANOVA) to assess the effects of error, method of deletion, amount missing, and interaction. Plots of STRESS against percent missing were used to identify at which point the values deteriorated rapidly. The correlations between the original and the recovered coordinates and the squared correlation between the actual distances and the recovered distances were evaluated for statistical significance. Finally, as suggested by McCallum and Cornelius (1977), the correlations between all of the dependent variables were reviewed to ascertain the performance of each of the measures. It should be noted that STRESS and RSQ are indexes that measure apparent fit, while CC and M measure true fit.

CHAPTER 4

Results

Two sets of data – true dissimilarities (TD) and dissimilarities plus random error (DE) – were used to evaluate the ability of non-metric MDS to recover the true underlying structure of the stimuli under conditions of random missing pairs of stimuli (R) and systematic missing pairs of stimuli (S). Eight levels of missing data were simulated 100 times for each condition. Table 1 shows the percent of missing and actual number missing for the 153 pairs of stimuli on each of the 2 x 2 x 8 conditions.

Evaluation of MDS solutions is traditionally conducted by inspecting the plot of stimuli in geometrical space and the values for STRESS and RSQ. Additionally Shepard diagrams are examined to see how well the recovered solution fits the model. These measures assess how well the solution fits the data that was entered in the model; however, the true structure may remain unknown. To measure the accuracy of the position of points in the two dimensional space the true coordinates were correlated with the corresponding recovered coordinates (CC) for each of the 18 stimuli. The index of metric determinacy (M) was calculated to measure how well each of the 153 dissimilarities correlate with the corresponding recovered dissimilarities.

Results are presented for each of the dependent variables on each data set and each method of selecting missing pairs. Maps and scatter plots for the complete data set and for a representative solution on each of the levels of missing pairs will be shown in the figures section. Descriptive statistics and analysis of variance will be executed for STRESS, RSQ, coordinate correlations (CC), and index of determinacy (M).

Plots of Stimuli in Two Dimensional Space

The tenth simulation in each level of missing data was selected to represent the geometrical solution for that level. Plots of the points in the geometrical space generated by nonmetric ALSCAL for both data sets and both methods of missing data are presented in the figures section.

The two-dimensional plots for each of the complete data sets are shown in figure 2. Although the points are not arranged in the exact position for each of the two data sets (zero error and random error added), the stimuli formed comparable configuration groupings in the geometrical space.

Data Set Without Error and Random Selection of Missing

Plots from the true dissimilarities data set (TD) with random selection of missing (R) are presented in figure 3. Examination of the plots for TD and R revealed that as the percent missing increased, points overlapped and tended to spread in a more circular arrangement indicative of degeneracy. This arrangement started to appear with .40 (L4) missing and it was most noticeable for the .80 (L7) and .90 (L8) missing. Also it can be seen that categorization of stimuli in the wrong dimension appeared at .40 (L4) missing and increased with percent of pairs missing.

Data Set With Added Error and Random Selection of Missing Pairs

Plots of the points in the geometrical space generated by nonmetric ALSCAL for the dissimilarities plus random error data set (DE) with random selection (R) of missing pairs are presented in figure 4. Displacement of stimuli appeared in all the levels of missing pairs except for .10 (L1) and .30 (L3). Point overlap appeared with the .60 (L6) level and grew worse as the percent of missing pairs increased.

Data Set Without Error and Systematic Selection of Missing Pairs

Missing pairs at level 1, level 2, level 3, and level 4 produced geometrical plots that had very small deviations from the plot without missing data. Misplacement of points increasingly leading to degenerate solutions were present at level 5, level 6, level 7, and level 8 (see figure 5).

Data Set With Added Error and Systematic Selection of Missing Pairs

Figure 6 shows that when missing pairs were selected systematically and there was random error in the data set, plots generated with missing pairs of stimuli at level 1, level 2, and level 3 showed minor deviations from the data set without missing data. A small amount of movement towards the wrong dimension appeared with level 4. The solution degenerated rapidly with increases in missing pairs at level 5, level 6, level 7, and level 8.

Summary of Plots of Stimuli in Two Dimensional Space

Deviations from the placement of points in the two dimensional space generated from the complete data set appeared earlier for the data with error and random selection of missing (L2). With systematic missing and dissimilarities without error, movement of points did not appear until missing level 5. Overall, data recovery appeared to perform better when each stimulus was compared to other stimuli the same number of times as opposed to randomly selecting pairs, and also when the data did not contain error.

Best recovery was for systematic (S) selection with dissimilarities without error (TD), followed by systematic (S) selection with dissimilarities with added random error (DE), random (R) selection with dissimilarities without error (TD), and finally random (R) selection with dissimilarities with added random error (DE).

Scatter Plots

Scatter plots of monotonic transformation, also known as Shepard diagrams, for both data sets and both methods of missing are presented in the figures section. The tenth simulation for each level of missing was selected to represent the solution for that level. The points are plotted with the raw data on the horizontal axis versus the disparities on the vertical axis (Norusis, 1997). A negative distance in the horizontal axis represents unknown information because of missing data. Disparities for the missing pairs of stimuli are stacked vertically on the negative value. A smooth line signifies a non-degenerate solution while a series of steps may suggest a degenerate transformation.

The plots of monotonic transformation for the complete data sets for both data without and with added random error are displayed in figure 7. It can be seen that the monotonic transformation for the data set with random error added was not as smooth as the plot for the data without error. The small steps in the plot of monotonic transformation characterize the perceptual errors that had been incorporated in the complete data set.

Data Set Without Error and Random Selection of Missing

Plots of linear fit for the data set without error (TD) and random selection of missing pairs of stimuli (R) can be seen in figure 8. The transformation plots were fairly smooth for L1, L2, L3, and L4 missing. Horizontal steps, suggesting possible degenerate transformations, appeared with the L5 of missing pairs and increased at each level to the L8 missing pairs.

Data Set With Added Error and Random Selection of Missing

Plots of linear fit for the dissimilarities with random error added data set (DE) and random selection of missing stimuli (R) are shown in figure 9. There were small steps apparent in the scatter plot for the complete data set and in level 1 of missing pairs. The small series of steps continued to increase for level 2, and level 3 of missing pairs. Deviations from the linear fit were more severe for L5, L6, L7, and L8 missing pairs.

Data Set Without Error and Systematic Selection of Missing

The plots for the transformations of the data set without error and systematic selection of pairs were very smooth for L1, L2, L3, and L4 (see figure 10). More severe steps appeared at level 5. Plots for levels 6, 7, and 8 strongly suggested degenerate solutions.

Data Set With Added Error and Systematic Selection of Missing

The plots for the transformations of the data set with random error added using systematic selection of missing pairs are shown in figure 11. Plots for levels 1 and 2 of missing pairs showed small steps in the line. Steps became marked at L3 and very obvious at L6, L7, and L8.

Summary of Plots of Monotonic Transformation.

Data without error produced the best plots of monotonic transformation. The line was very smooth for L1, L2, and L3 with systematic selection of missing pairs and L1 and L2 with random selection of missing pairs. With random error added the best transformations were for the systematic selection at L1 and L2.

STRESS

Table 2 shows the evaluation suggestions proposed by Kruskall (1964), by McCallum (1978), and by Sturrock and Rocha (2000). Interaction plots of mean STRESS values generated by non-metric ALSCAL for each condition are shown in the figures section. For the complete data set without error (TD), the value of STRESS was .004, which represents excellent fit. The complete data set of dissimilarities with random error added (DE) had a STRESS value of .05567, which is considered good. Both values are suggestive of structurally related data under guidelines proposed by McCallum (1978) and Sturrock and Rocha (2000). Table 3 shows a summary of the percent of cases that achieved excellent, good, fair, or poor STRESS values for each of the 2 x 2 x 8 conditions (also see figure 13). Only one value of STRESS suggested poor fit – level 8 on the systematic data condition of missing pairs for dissimilarities with random error added. Overall, approximately 35 percent of the values of STRESS were excellent, approximately 55 percent of the values of STRESS were good, and approximately 10 percent of the values of STRESS were fair.

Table 4 shows the mean values of STRESS for the 100 replications on each cell, and Figure 13 shows the plot of mean values by level of missing pairs. Univariate analysis of variance showed significant differences for level of missing pairs ($\underline{F}$ = 254.94, $\underline{p}$ < .001), data set (F = 3337.38, p< .001), level by data (F = 284.86, p< .001), and level by method (F = 27.45, p< .001), but not for method (F = 3.384, p < .066), data by method (F = 1.366, p < .243), or level by data by method (F = 1.46, p< .177). Table 5 summarizes the between subjects effects for each condition and table 6 shows the overall post hoc Tukey HSD.

Data Set Without Error and Random Selection of Missing

The range of values for STRESS was from .00448 to .0418. The value for .10 (L1) missing was .00448, for .20 (L2) missing it was .00549, for .30 (L3) missing it was .00866, and then it increased rapidly for .40 (L4) missing (.0211). All of those values of STRESS can be considered as excellent even under the rules of thumb that Kruskall (1964) proposed to judge badness of fit. The values deteriorated rapidly to .0383 for .50 (L5) missing, and to .0418 for .60 (L6) missing and then it appeared to improve at .80 (L7) missing (.0296) and .90 (L8) missing (.0254). The last group of STRESS values are considered good under Kruskall rules of thumb, and all of the them would represent data that is structurally related according to the research conducted by MacCallum (1978) and Sturrock and Rocha (2000).

Univariate ANOVA results showed a significant difference between the mean values of STRESS at different levels of percent missing ($\underline{F} = 185.7099$, $\underline{p} < .001$) with a large effect size ($\underline{\eta}^2 = .621$). Post hoc analysis using the Tukey HSD procedure (see table 7) showed no differences between percentages of missing pairs at .10 (L1), .20 (L2), and .30 (L3); no difference between .40 (L4) and .90 (L8); no difference between .50 (L5) and .60 (L6); and no difference between .80 (L7) and .90 (L8). Elbows in the plot of mean S-STRESS against percent missing in figure 13 illustrate that the worst levels of stress were for .50 (L5) and .60 (L6) missing, followed by .80 (L7) and .90 (L8) missing, .40 (L4) missing, and the best STRESS values were for .10 (L1), .20 (L2), and .30 (L3) missing. The worst levels are considered good values for stress (Kruskall, 1964) while all the others are considered to be excellent values of stress.

Histograms of the STRESS values for the TD and R condition on each of the 100 simulations are presented in figure 14. No remarkable results were apparent other than, as missing increased, the distribution became either skewed or nearly uniformly spread. Values for the .10 missing had a distribution that appeared to be close to normal.

Data Set With Added Error and Random Selection of Missing

The mean values of STRESS on each of the missing conditions were slightly lower than the corresponding values of stress for the complete data set and all of them are considered good. For .10 missing (L1) the mean STRESS was .05131, it decreased to .04782 for .20 (L2), .04635 for .30 (L3), and .04625 for .40 (L4). It increased to .05086 for .50 (L5) and then, as missing increased, it improved to .483 for .60 (L6), .03045 for .80 (L7), and .02537 for .90 (L8).

The univariate ANOVA for mean STRESS values showed that there was a strong ($\eta^2 = .466$) significant difference ($F = 98.793$, $p < .001$) between the levels of percent missing. Post-hoc Tukey HSD (see table 8) showed that the mean STRESS value for .90 (L8) was significantly lower than at any of the other levels of missing followed by .80 (L7). The highest mean STRESS values were for .10 (L1), and .50 (L5) missing; those values were not statistically different from each other.

Histograms of the STRESS values for the DE and R conditions on each of the 100 simulations are presented in figure 15. No systematic trends were observed for this condition.

Data Set Without Error and Systematic Selection of Missing Pairs

The mean stress values using systematic selection of dissimilarities from the true data set ranged from .00409 to .0435. The values for levels 1 to 4 are considered excellent and the values for levels 5 to 8 are considered good. The mean stress values were .00409, .00462, .00628, .0139, .0406, .0435, .0288, and .0351 for L1, L2, L3, L4, L5, L6, L7, and L8 respectively (see table 4).

Results of univariate ANOVA produced a strong ($\underline{\eta}^2 = .65$) significant difference for level of missing as a factor, $\underline{F} = 210.032$, (7, 792), $\underline{p} < .001$. Post hoc Tukey HSD (see table 9) showed that there were no differences between levels 1, 2, and 3 or between levels 5 and 6. It can be seen in figure 12 that missing pairs in levels from L1 to L3 produced the best stress values while L5 and L6 had the worst stress values.

Histograms for each of the levels of missing pairs are shown in Figure 16. No systematic trends were observed for this condition other than the lower levels of missing pairs appeared to have a negative skew.

Data Set With Added Error and Systematic Selection of Missing

The range of STRESS values was from .0325 to .0561. The means for levels 1 to 8 were .0519, .0459, .0397, .0393, .0561, .0520, .0325, and .0386 respectively (see table 4). All the values of mean stress are considered good except for level 5, which is considered fair by Kruskall rules of thumb.

ANOVA showed significant effect for levels of missing pairs ($\underline{F} = 58.256$, $p < .001$) with $\underline{\eta}^2 = .340$. It can be seen from figure 12 and Post Hoc Tukey HSD (see table 11) that the stress value for level 7 was significantly lower than all the others and levels 1, 5, and 6 were no different from each other but also had the highest mean values of stress.

Summary of Analysis of Stress Results

Each of the conditions showed significant differences in the values of mean STRESS for level of missing pairs. The lower levels performed differently for each data set. The differences decreased at the higher levels of missing data. Results were similar for method used to select the missing pairs. For all the conditions levels 5 and 6 had the highest values of stress and then it decreased when the number of missing pairs increased (levels 7 and 8).

With true dissimilarities, there were no differences in the lowest three levels of missing. The line formed an elbow at level 3 and rapidly increased up to level 6, and then it changed to lower values of stress. With the data set with random error added to the dissimilarities, the behavior of the stress values was more irregular. The differences for mean STRESS values between data sets were larger at the lower levels and disappeared at the higher levels of missing (see figure 13).

R-Square

Values of RSQ for the complete data set were .99993 for the true dissimilarities data matrix (TD) and .98646 for the dissimilarities plus random error (DE) data matrix. Both of those values are significant and suggest that the two dimensional model fit the respective data sets. Summary of descriptive statistics for each of eight levels of missing, two data sets and two methods of selection of missing are reported in table 11. Analysis of variance showed significant effects for level of missing pairs, data set, level by data, level by method, and level by data by method but not for method or for interaction of data by method. Summary for tests of between-subjects effects are presented in table 12. Plot of mean RSQ by level of missing pairs is shown in figure 18.

Data Set Without Error and Random Selection of Missing

Values of RSQ ranged from .991313 in the .60 (L6) missing to .9999122 in the .10 (L1) missing condition. All of the RSQ were significant ($p < .01$).

Significant differences between the mean values of RSQ for the different levels of missing resulted from the univariate ANOVA ($F = 102.008$, $p < .01$) with $\eta^2 = .474$ (large effect). Post hoc analysis of the results using Tukey HSD showed that the values of RSQ for.60 (L6) and .50 (L5) were not different from each other, but were significantly lower than all the other levels; the mean RSQ values increased for .80 (L7) missing, then for .90 (L8) and .40 (L4), and finally the highest mean RSQ values were for .30 (L3), .20 (L2), and .10 (L1) missing. These three levels were not different from each other (see table 14 and figure 18).

Histograms of the 100 samples generated by the simulation for TD and R are shown in figure 19. There are no remarkable results other than an increase in skewness up to .40 missing and then the distribution looks more evenly spread.

Data Set With Added Error and Random Selection of Missing

Range of RSQ mean values was from .987787 for .50 missing to .9970827 for .90 missing. All RSQ were significant ($p < .01$). Univariate ANOVA showed a significant difference ($F = 72.505$, $p < .001$) with a large effect ($\eta^2 = .391$) for levels of missing. Post hoc Tukey HSD identified five homogeneous subsets (see table 15). The subset with the highest mean RSQ was for .90 (L8) missing pairs, followed by .80 (L7) missing pairs. The lowest mean values were in the subset that included .50 (L5), .60 (L6), and .10 (L1) missing (see figure 18).

Histograms for the mean RSQ samples generated by the simulation using DE and R are in figure 20. Inspection of the distributions for each level of missing data did not reveal any consistencies or trends.

Data Set Without Error and Systematic Selection of Missing Pairs

The highest value of RSQ was .9999288 for L1; the lowest was .9912089 for L5. All RSQ values in this condition were significant.

Univariate ANOVA showed a strong ($\eta^2 = .545$) and significant ($F = 135.756$, $p < .00$) effect for levels of missing and post hoc Tukey HSD identified 5 homogeneous subsets that were significantly different of each other. The best RSQ values were for L1, L2, and L3; the worst RSQ values were for L6 and L5 (See table 15 and figure 18). Histograms for the RSQ values under TD and S conditions for the 100 repetitions generated in the simulation are presented in figure 20.

Data Set With Added Error and Systematic Selection of Missing Pairs

The highest and lowest mean values of RSQ were .99538 for L8 and .98557 for L5 (see table 11). Every one of the values for RSQ in this condition were significant ($p < .01$). Univariate ANOVA showed a significant effect for level of missing pairs, with $F = 65.620$ $p < .001$ and $\eta^2 = .367$. Post hoc Tukey HSD showed that the lowest average values of RSQ were for L5 and L6; the highest were for L7 and L8 (see table 16 and figure 18).

Histograms of RSQ values at each level of missing pairs for 100 replications are shown in figure 22. It can be seen that the values of RSQ for L4 and L8 have more positively skewed distributions than the other levels.

Summary of Analysis of RSQ Results

One hundred percent of values of RSQ were significant ($p < .01$). The estimates for the dissimilarities without error produced better values of RSQ at the lower levels of missing pairs. Effects for method of deletion followed the same trend for the data of true dissimilarities, but it behaved more erratically with the dissimilarities with error added. Differences in RSQ were larger at the lower levels of missing pairs and became very close at higher levels of missing pairs. Values of RSQ dropped at L5 and L6, and then increased at L7 and L8.

Fit Between True and Recovered Coordinates

Summary of descriptive statistics for each level of missing pairs on two data sets and two methods of missing pairs is presented in table 18. The range of values for the mean correlations between coordinates was from .1332998 (L8, ED, S) to .9998565 (L1, TD, S). Only levels 6, 7, and 8 had mean correlation values that were not significant (see Figure 24). A summary of the number of significant correlations for one hundred replications in each condition is presented in table 19.

ANOVA results showed that there was no effect for either method or error (see table 20). There was a large effect for level of missing (η^2 = .522) and a small effect for level by method (η^2 = .02). Post hoc Tukey HSD results are shown in table 21.

Data Set Without Error and Random Selection of Missing Pairs

Correlation values were highest at level 1 (.10) and dropped continually with each increase in missing pairs to level 8 (.90). One hundred percent of the replications measuring the correlation between the true and the recovered coordinates for levels 1 to 3 (.10 to .40) were significant at $p < .01$. At level 4 (.40) 99 percent were significant at $p < .01$, at level 5 (.50) three percent were significant at $p < .01$, and 91 percent were significant ($p < .01$) at level 6 (.60). Table 18 illustrates that levels 7 (.80) and 8 (.90) of missing data did not have significant results 40 and 49 percent of the time respectively (see figure 19).

ANOVA results showed a large effect ($\eta^2 = .529$) for level of missing pairs ($F = 127.159$, $p < .001$). Tukey HSD post hoc results are summarized in table 12. Levels 1 to 4 (.10 to .40) were not significantly different from each other, there was some overlap with the subset for levels 4 and 5 (.40 and .50), correlations dropped significantly again for level 6 (.60) and for subset for levels 7 (.80) and 8 (.90).

Data Set With Added Error and Random Selection of Missing Pairs

Mean correlations between true and recovered coordinates were from .992 for .10 missing pairs (L1) to .137 for .90 percent missing pairs (L8). One hundred percent of the replications for levels 1 to 4 and 97 percent at level 5, had significant correlation values at $p < .01$. Levels 7 (.80), and 8 (.90) had 66 percent and 35 percent respectively at $p < .01$, and 28 percent and 53 percent respectively with no significant correlations (see table 18 and figure 22).

ANOVA results showed significant effects ($\underline{\eta}^2 = .522$) for level of missing data ($\underline{F} = 130.75$, $\underline{p} < .001$). Post hoc Tukey HSD results are summarized in table 13. Levels 1 to 4 (.10 to .40) were not significantly different; the correlations between true and recovered coordinates were high. Mean correlation values dropped significantly for each successive level from 5 (.50) to 8 (.90).

Data Set Without Error and Systematic Selection of Missing Pairs

Levels 1 thru 6 had significant ($p < .01$) mean correlations between true and recovered coordinates (see table 9 and figure 24). Values ranged from .1401 (L8) to .9999 (L1). Correlations for 100 percent of the replications in levels 1 thru 4 were significant at $\underline{p} < .01$, and at level 5 one percent of the results were not significant.

ANOVA results of correlations between true and recovered coordinates showed a significant effect ($\underline{\eta}^2 = .532$) for level of missing ($\underline{F} = 128.385$, $p < .001$). Summary of post hoc Tukey HSD results are shown in table 14. It can be seen that there were no differences between levels 1 thru 5; there was a drop in the value of the correlations for subset grouping levels 6 and 7, and again a significant drop for level 8.

Data Set with Added Error and Systematic Selection of Missing Pairs

The average correlations between the original and the recovered dissimilarities were high for levels 1 thru 4, and dropped significantly for each succeeding level. Mean correlation was not significant for level 8. One hundred percent of the mean correlation values for levels 1 thru 4 and 99 percent in level 5 were significant at $p < .01$.

ANOVA results showed a large effect ($\underline{\eta}^2 = .527$) for level of missing ($\underline{F} = 125.926$, $\underline{p} < .001$). Tukey HSD post hoc analysis (see table 15) showed that levels 1 thru 4 were not different from each other and had significant higher correlations than the other levels. Level 8 had the lowest mean correlation.

Summary of Analysis of Correlations between Coordinates

Correlations between the true and recovered coordinates showed that levels 1, 2, 3, and 4 had the highest values and were not significantly different from each other. Values deteriorated significantly for each successive level of missing (see figure 24). The true data set with systematic selection tolerated a larger percent missing.

Index of Metric Determinacy

The fit between true and recovered distances was evaluated by calculating the index of metric determinacy used by Young (1970) and labeled as M.

Summary of descriptive statistics for two levels of error, two modes of selection and 8 levels of missing pairs is presented in table 20. Analysis of variance shows that there was a significant and strong effect for percent missing and a significant but small effect for error. There was no significant effect for method. There were significant but small effects for level by error and for level by method (see table 21). Plot of mean values of M at each level of missing by error and method is shown in Figure 24. In all conditions mean values of M decreased as level of missing pairs increased.

Data Set Without Error and Random Selection of Missing Pairs

The mean squared correlation between actual and recovered dissimilarities ranged from .9996 for level 1 to .0905 at level 8. Analysis of variance between levels was significant (F = 1831.78, p < .001) with a strong effect (η^2 = .942). Post hoc Tukey HSD (see table 22) revealed no differences between levels 1 to 3 and overlap with level 4. Values of M became statistically lower at each successive level from 5 to 8.

Data Set With Added Error and Random Selection of Missing Pairs

Mean values of M in this condition ranged from .0844 in level 8 to .9844 in level 1. Analysis of variance in this condition showed a significant (F = 1831.78, p < .00) and strong effect (η^2 = .942) for level of missing. Post hoc

Tukey HSD showed that there was no significant difference between levels 1, 2, and 3, and no difference between levels 3 and 4. Mean values of M dropped significantly for each successive increment in missing (see table 23).

Data Set Without Error and Systematic Selection of Missing Pairs

Range of mean values of M was from .0544 at level 8 to .9997 at level 1. Analysis of variance resulted in a strong effect ($\underline{\eta}^2 = .958$) with $\underline{F} = 2560.57$, $p < .00$ Post hoc Tukey HSD showed that in this condition there were no differences between levels 1, 2, 3, and 4. Each successive level of missing data had significantly lower indexes of determinacy (see table 24).

Data Set With Added Error and Systematic Selection of Missing Pairs

Mean values of M ranged from .026 at level 8 to .9846 at level 1. Analysis of variance showed a significant ($\underline{F} = 4104.17$, $\underline{p} < .00$) and strong effect ($\underline{\eta}^2 = .973$) for level of missing. Post hoc Tukey HSD identified five subsets (see table 25). M values for levels 1, 2, 3, and 4 were not different. At every consecutive level of missing pairs, M values dropped significantly.

Summary of Analysis of Index of Metric Determinacy

There were no statistical differences in the values of the squared correlation between the actual and the apparent dissimilarities for levels 1, 2, 3 and 4 in all conditions. For both data sets using systematic selection of missing level 4 had squared correlations as high as the first three levels. The interaction plot of M mean values against level of missing (figure 24) displayed a noticeable change in slope at level 4 and again at level 6.

Correlations Between Dependent Variables

The correlations between the dependent variables used in this study are in summarized in Table 33. Since high values of STRESS indicate poor fit, it would be expected that the correlations with other measures of fit would be negative. STRESS and RSQ measure how well the recovered dissimilarities fit the entered matrix. CC and M measure how well the recovered configuration fits the true structure.

As expected there was a strong negative correlation between overall values of STRESS and RSQ ($\underline{r} = -.940$). The correlations maintained high values at all levels of missing with a small reduction at level 8 ($\underline{r} = .820$) where approximately 90 percent of the cells in the dissimilarities matrix are set to missing.

The overall correlation between CC and STRESS was significant but small ($\underline{r} = -.102$, $p < .01$). When broken down by level of missing the values ranged from +. 001 to -.958. At level 1 the correlation was strong ($\underline{r} = -.958$); it dropped markedly for levels 2 ($\underline{r} =-.330$) to 5 ($\underline{r} = -.178$) and it practically disappeared for levels 6 ($r = -.099$) to 9 ($\underline{r} = .001$).

The correlations between CC and RSQ behaved similarly, but the overall correlation was not statistically significant suggesting the placement of the points in space was more unpredictable than the distances between them. The correlations ranged from +. 957 (L1) to +. 012 (L8). The value was high for level 1 with only 10 percent missing, dropped for levels 2 ($\underline{r}$ = .370) and 3 ($\underline{r}$ = .419), became small at levels 4 ($\underline{r}$ = .179) and 5 ($\underline{r}$ = .184) and practically disappeared for levels 7 ($\underline{r}$ = .068) and 8 ($\underline{r}$ = -.012).

The overall correlation between M and STRESS was significant but small. Levels 1 and 2 had strong correlations ($\underline{r}$ = -.969; $\underline{r}$ = -.928). The correlations were not as strong for level 3 ($\underline{r}$ = -.696) and level 5 ($\underline{r}$ = -.533), dropped to $\underline{r}$ = -.320 and $\underline{r}$ = -.205 for levels 4 and 6 respectively, and practically disappeared for level 7 ($\underline{r}$ = -.092) and level 8 ($\underline{r}$ = -.084).

The correlations between M and RSQ also followed a similar pattern. Very high correlations for levels 1 ($\underline{r}$ = .968) and 2 ($\underline{r}$ = .939), strong correlations for levels 3 ($\underline{r}$ = .757) and 5 ($\underline{r}$ = .537), smaller values for levels 4 ($\underline{r}$ = .368), 6 ($\underline{r}$ = .190), and 7 ($\underline{r}$ = .103), and a small and not significant correlation at level 8 ($\underline{r}$ = .037).

The correlations between the two measures of relationship between the true and the perceived structure varied as a function of missing. The overall correlation between M and CC was .709. At level 1 the correlation was very strong ($\underline{r}$ = .992). For levels 2 to 5 the correlations ranged from .267 to .603, but not in a consistent manner. Levels 6 ($\underline{r}$ = .162), 7 ($\underline{r}$ = .149), and 8 ($\underline{r}$ = .108) had significant but small correlations.

CHAPTER 5

Summary and Discussion

The number of pairs presented in a typical multidimensional scaling task increases rapidly with the number of stimuli that is being mapped. The large number of dissimilarity judgments presented to individuals in a MDS task is a drawback to the widespread use of this technique. As individuals form judgments about an increasing number of dissimilarity pairs they may become tired, they may perceive the task as boring, they may encounter unfamiliar information, or they may find the task overwhelmingly complex (Spence & Domoney, 1974; McCallum, 1978). Reducing the amount of information that is presented to participants or allowing respondents to select only the information that is known to them would help to improve the practicability of the task.

Forced answers may add error to the data (DeSarbo, Young, & Rangaswamy, 1997); therefore, allowing a participant to omit pairs may produce more accurate information matrices. On the other hand a point cannot be located in space if information about it does not appear in the comparison task. Some researchers have suggested that pairs should appear the same number of times (Spence & Domoney, 1974).

This study arose from the desire to investigate what proportion of the dissimilarity judgments can be missing before the algorithm fails to return an accurate solution. A data set with known structure and without error was selected. A second data matrix was created by adding random error to the dissimilarity measures. This second data set may better represent an individual's performance when making dissimilarity judgments about pairs of stimuli. Pairs were deleted from the matrices using one of two methods. Systematic selection represented a researcher selecting a number pairs from the complete set. The design involved the use of the Ross matrix. In any sequence of $(n_R - 1) / 2$ pairs each stimulus appears once. Multiples of the sequence were used to select equally appearing numbers of stimuli. The numbers were transformed to percentages and rounded. The same percentages were then used to generate random missing pairs within the data matrix. Eight levels of missing pairs were generated in this study (see table 1).

Measures of fit used by researchers to evaluate how well the structure recovered by nonmetric ALSCAL fits the data are STRESS and RSQ. STRESS is a measure of correlation between the squared distances and the squared disparities (Davison, 1992), while RSQ is the squared correlation between the raw data and the distances (Norusis, 1997). These two measures are obtained from the data entered in the model. Because there is no assumption that the truth is known, they can be considered as apparent measures of fit (Sherman, 1972). Additionally, SPSS displays plots of the points in geometrical space and scatter plots of monotonic transformations.

Conversely, the correlations between the true and the recovered coordinates (CC) and the squared correlation between the true and recovered distances (M) are measures of how much the data entered in the model deviates from the truth. The CC is a measure of displacement of the points in the dimensional space, while M is a measure of the distortion of dissimilarities between the stimuli. These last two measures cannot be obtained in absence of knowledge of the true properties of the construct being considered. Therefore they are actual measures of recovery (Sherman, 1972).

Comparisons of the two dimensional maps of the points in space using data generated under the various conditions in the simulation with the maps generated from the true and complete set of points showed that there were less amount of point displacement with data that did not contain error and when the method of selection was systematic (see figures 2 to 6). With true data and systematic missing of paired stimuli plot recovery was good with as much as 50 percent of the pairs missing. But errors in placement appeared as early as with 20 percent missing when the data contained error and selection of missing pairs was random. With error and systematic missing of dissimilarity pairs, recovery was good with as much as 30 percent of the pairs missing. Overall small misconfigurations emerged when 30 percent of the comparisons were missing;

errors were more obvious when 40 percent of the pairs were missing, and became severe after 60 percent of the pairs were missing.

The behavior of the positioning of points in the two dimensional maps was clearly reflected in the plots of monotonic transformations (see figures 8 to 11). The data set without error produced smoother lines for both methods of selection of missing pairs. Even for the complete data sets, the line in the scatter plot was closer to a straight line for the data without error (see figure 7). For data without error, strong steps were evident with 50 percent of the pairs missing. Steps in the scatter plots closely followed the displacement of points observed in the two dimensional maps.

Values of STRESS were obtained from the nonmetric MDS SPSS output. High values reflect poor fit, values closer to zero reflect good fit between the observed data and the recovered solution. Researchers are warned about very low values of STRESS that may result from degenerate solutions. This effect was observed in this simulation. Only one value out of the 3,200 replications was considered poor. Ninety percent of the values were good or excellent.

STRESS had the best values when the data set had no error. The largest values came from the data set with error added. In both cases, the method used to select the missing pairs did not have an effect on the quality of structure recovery. The differences disappeared when the level of missing was very high. A very unusual pattern was observed (see figure 13). STRESS values started to deteriorate at 40 percent of pairs missing, increased for 50 percent and 60 percent missing, but then they improved when 80 percent and 90 percent of the dissimilarities were not entered in the model. Those results can be attributed to the degenerate solutions that occurred at high levels of missing.

At the lower levels of missing pairs STRESS had better values when the data set had no error and selection of missing pairs was systematic. Values of STRESS became closer when the number of missing pairs reached 50 percent. This apparent improvement denotes the importance of examining the plots in geometrical space as well as the Shepard diagrams to look for evidence of degenerativity in the solution.

Spence and Domoney (1974) investigated the effect of missing pairs on the STRESS index. The present study agrees with their results. They found that recovery was good when they deleted 1/3 of the data and not as good when they deleted 2/3 of the data. However, they did not simulate conditions at the highest levels of missing.

The plot of mean scores of RSQ was similar to the plots of mean STRESS values (see figure 18). RSQ decreased noticeably for 50 percent and 60 percent of missing pairs and then it improved at 80 percent and 90 percent of missing data. The values of RSQ were strong and significant ($p <. 01$) for each of the 100 replications in each condition. RSQ was lower for the data with random error added but the method of selection did not have an effect. With a lower percentage of missing pairs, values of RSQ were better with the true data set, but it was practically the same at 80 percent and 90 percent of missing pairs.

The strong values of STRESS and RSQ obtained on each of the conditions attested to the robustness of the algorithm used to analyze the data. But it also pointed out to the dangers of interpreting these measures of fit without examination of the maps and the scatter plots also easily available in SPSS. It was observed that point arrangement that suggested degenerate solutions started to appear on the two dimensional plots for increases in percent missing over .40, and was obvious at .80 and .90 missing. Review of plots of monotonic transformation was highly indicative of poor solutions that were obtained at higher levels of missing.

As stated earlier STRESS and RSQ are only descriptive measures of how well the recovered data fits the observed data. They do not provide information about the true configuration of the points used as stimuli. True recovery can be better estimated from measures that allow comparisons between the recovered data and the true data.

The correlations between the coordinates in the true and the recovered configurations (CC) revealed that neither the method of selection nor the presence of error made a difference in the ability of nonmetric ALSCAL to recover the true configuration of the data (see figure 23). The correlations were high for up to 40 percent missing, declined for 50 percent and 60 percent of missing pairs, and dropped noticeably for 80 percent and 90 percent of missing pairs.

Spence and Domoney (1974) also used a measure of coordinate recovery. It revealed that amount of error and method of deletion had an overall effect, but when they controlled for method they found that random selection performed just as well. The present study does not replicate a similar effect.

The index of metric determinacy (M) followed the same pattern as CC (see figure 24). True recovery of the data configuration was good with as much as 40 percent of the dissimilarities missing it dropped for 50 percent and 60 percent of missing pairs, and deteriorated noticeably for 80 percent and 90 percent of missing pairs. Error in the dissimilarity measures or method of selection of the missing pairs did not affect the measures of true recovery of the distances between points in the underlying configuration.

MacCallum (1978) used M and a measure of recovery of the true coordinates for their simulation of replicated MDS. Although number of replications had no effect, the proportion of missing data resulted in increases of deterioration of STRESS values. That was not the case in the present study. However, they only used .20, .40, and .60 as levels of missing pairs.

Their results on the effect of proportion of missing appear to be supported by this simulation. What has not been observed in previous studies is the surprising behavior of the measures of fit generated by nonmetric ALSCAL when the proportion of missing rises to excessive levels.

The mean values on each dependent variable obtained from the 100 replications in each condition is summarized in table 34. In the apparent measures of recovery small values of STRESS and high values of RSQ represent good fit. For the actual measures of recovery high values of CC as well as high values of M are indicative of good recovery.

All the dependent variables used in this simulation were correlated to compare the performance of the apparent and the true measures of fit. Since high values of CC and M are indicative of good recovery while the opposite is true of STRESS values, negative correlations with STRESS were anticipated. Compared to the results obtained by MacCallum and Cornelious (1977) only the relationship between M and CC was similar; the relationships of M and CC with STRESS were much lower in the present simulation.

The relationship between STRESS and RSQ behaved as expected in a nonmetric MDS analysis. The correlation was strong and negative ($r = -.940$). The relationships of apparent measures of fit with measures of true recovery were not as strong. However, it was also observed that at the lower levels of missing the correlations between apparent and true fit measures were stronger, suggesting that indexes of fit calculated by the algorithm in SPSS can be valuable under appropriate conditions. That is, no more than 20 percent missing if perceptual error is present.

The worst relationship was between M and STRESS. The correlation was -.06, much lower than -.62 in the MacCallum and Cornelius simulation. The relationship between CC and STRESS also was poor (-.102) in the present study; however, it was much better that the positive correlation (.36) which was obtained in the previously mentioned study.

The correlations between the true measures of recovery, CC and M, although good were not very high. The value was .709 in the present study, close to .77 in the McCallum and Cornelius simulation.

It would be unreasonable to expect that respondents in a multidimensional task would not commit perceptual errors that would generate either over estimations or under estimations of the differences between pairs of stimuli. Therefore, it appears from the results of this simulation that the data matrix should not miss more than 20 percent of the dissimilarities to produce a very good representation of the true structure of the data. Some small amount of point misconfiguration is to be expected when the number of paired comparisons missing reaches 30 percent. Recovery deteriorates rapidly when 40 percent or more of the dissimilarity measures are missing from the observations matrix

Limitations and Recommendations for Further Research

This study used one matrix of true dissimilarities and one matrix of dissimilarities with random error added. Random selection of missing pairs has been utilized by others researchers, but use of the Ross matrix as a means of systematic selection of paired stimuli to be presented to participants in an MDS task has not been investigated before. Although missing 80 percent or 90 percent of the data may appear unreasonable in a real comparison MDS task it revealed some surprising results in this simulation. Other conditions can be varied in future research such as levels of error, number of dimensions, or type of MDS algorithm used to analyze the data. Some of the limitations apparent in this research and recommendations for future research are as follow:

- This study investigated nonmetric ALSCAL with a single matrix. The case of replicated MDS should be investigated under similar conditions. The number of replications should be varied and systematic bias should be included.
- Only one level of random error was introduced in this study. Performance under varied levels of random as well as systematic error should be investigated at all levels of missing. Additionally, introduction of proportional rather than additive error should be investigated.
- The number of stimuli was set to 18 in the present simulation. It would be informative to vary the number of points in the dimensional space. It

would also be of interest to discover if a different selection of cities would produce similar results.

- Although this study did not investigate the stress values proposed by McCallum (1978), and by Sturrock and Rocha (2000) it appears that they are too high. Their research related to data that is structurally connected. That condition may not be violated when data is missing and error is added to the dissimilarity judgments. In the present study, it appears that the data retained the underlying structure even when most of the paired comparisons were not observed in the model.

Conclusions

The quality of the recovered structure was affected by the percent of paired comparisons missing, but not as much by perceptual error or method of selection. It appears safe to tolerate as much as 20 percent of the observations to be missing in the dissimilarities matrix even when random error is apparent since all the measures of recovery displayed excellent results. Small errors can be expected with as much as 30 percent missing. The solution becomes more unreliable when proportion of pairs missing reaches 40 percent.

Presentation of stimuli pairs by systematic selection appeared to produce better STRESS and RSQ values. But the measures of true fit discounted those results. In this study values of STRESS and RSQ did not reflect the quality of recovery observed in the two dimensional maps and it appears that the visual inspection of the plots of monotonic transformations are an indispensable tool in evaluating the results of nonmetric MDS.

One of the most remarkable outcomes of this study was the ability of nonmetric ALSCAL to return a solution even under extreme conditions of missing pairs in the observations matrix submitted for analysis. However, values of STRESS and RSQ were not reflective of the quality of the recovered structure as displayed in the two dimensional configuration. Finally, It was observed that the use of the Ross matrix could be a practical and efficient method that can be used by practitioners to select a desired number of paired comparisons thus reducing the need to expose an individual to all possible pairs generated in the task.

TABLES

Table 1

Levels of Simulation by Condition

Level	Number of times each stimulus is compared $(n_R + 1)/2$	Percent missing	Number missing	p-value in Bernoulli function	Percent missing	Median Number missing
	True Dissimilarities (TD)					
	Systematic (S)			Random (R)		
1	14	.09	13	.90	.10	15
2	12	.22	33	.80	.20	32
3	10	.34	53	.70	.30	47
4	9	.41	63	.60	.40	62
5	8	.48	73	.50	.50	76
6	6	.60	93	.40	.65	92
7	3	.80	123	.20	.80	122.5
8	1	.93	143	.10	.90	138
	Dissimilarities plus Random Error (DE)					
	Systematic (S)			Random (R)		
1	14	.09	13	.90	.10	15
2	12	.22	33	.80	.20	32
3	10	.34	53	.70	.30	47
4	9	.41	63	.60	.40	62
5	8	.48	73	.50	.50	76
6	6	.60	93	.40	.65	92
7	3	.80	123	.20	.80	122.5
8	1	.93	143	.10	.90	138

Table 2

Values of STRESS that suggest fit

	STRESS
Kruskall	
Excellent	< .025
Good	.03 to .05
Fair	.06 to .10
Poor	.11 to .20
McCallum	< .284
Sturrock & Rocha	< .263

Table 3

Number of Cases in Each Kruskall Category of Badness of Fit in 100 Replications

		TD		DE	
Level	Badness of Fit	R	S	R	S
1	Excellent	100	100	0	0
	Good	0	0	93	87
	Fair	0	0	7	13
	Poor	0	0	0	0
2	Excellent	100	100	0	0
	Good	0	0	97	100
	Fair	0	0	3	0
	Poor	0	0	0	0
3	Excellent	96	100	0	0
	Good	4	0	89	100
	Fair	0	0	11	0
	Poor	0	0	0	0
4	Excellent	70	90	0	0
	Good	26	0	80	94
	Fair	4	10	20	6
	Poor	0	0	0	0

Table 3 (continued)

		TD		DE	
Level	Badness of Fit	R	S	R	S
5	Excellent	23	36	1	0
	Good	62	30	61	44
	Fair	15	34	38	56
	Poor	0	0	0	0
6	Excellent	5	6	1	0
	Good	87	84	73	64
	Fair	8	10	26	36
	Poor	0	0	0	0
7	Excellent	37	40	38	27
	Good	61	60	59	72
	Fair	2	0	3	1
	Poor	0	0	0	0
8	Excellent	52	26	53	17
	Good	48	68	47	73
	Fair	0	6	0	9
	Poor	0	0	0	1

Table 4

Descriptive Statistics for 100 Replications of STRESS for Two levels of Error, Two Methods of Selection and Eight Levels of Missing

	True Dissimilarities				Dissimilarities Plus Error			
	Random		Systematic		Random		Systematic	
Level	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	.00448	.00108	.00409	.00066	.0513	.00286	.0519	.0024
2	.00549	.00134	.00462	.00116	.0478	.00459	.0459	.0029
3	.00866	.00715	.00628	.00139	.0464	.00831	.0397	.0026
4	.02110	.01560	.01390	.01580	.0463	.01050	.0393	.0097
5	.03830	.01540	.04060	.02080	.0509	.01220	.0561	.0153
6	.04180	.01010	.04350	.00920	.0483	.01100	.0520	.0104
7	.02960	.01050	.02880	.00854	.0304	.01120	.0325	.0099
8	.02540	.01280	.03510	.01500	.0254	.01320	.0386	.0195

Table 5

Analysis of Variance for STRESS with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Source		F	η^2	p
Model	31	236.231	.698	.000
Level	7	254.943	.360	.000
Data	1	3337.381	.513	.000
Method	1	3.384	.001	.066
Level by Data	7	284.863	.386	.000
Level by Method	7	27.451	.057	.000
Data by Method	1	1.366	.243	.243
Level by Data by Method	7	1.460	.003	.177

Note: **Bold** face indicates significance at $p < .01$

Table 6

Post Hoc Tukey HSD Results for STRESS with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
1 (.10)	2 (.20)	-.001014	1.000
	3 (.30)	-.004184	.162
	4 (.40)	**-.016670**	.000
	5 (.50)	**-.033840**	.000
	6 (.60)	**-.037290**	.000
	7 (.80)	**-.025150**	.000
	8 (.90)	**-.020880**	.000
2 (.20)	1 (.10)	.001014	1.000
	3 (.30)	-.003170	1.000
	4 (.40)	**-.015650**	.000
	5 (.50)	**-.032830**	.000
	6 (.60)	**-.036280**	.000
	7 (.80)	**-.024140**	.000
	8 (.90)	**-.019870**	.000
3 (.30)	1 (.10)	.001840	.162
	2 (.20)	.003170	1.000
	4 (.40)	**-.012480**	.000
	5 (.50)	**-.029660**	.000
	6 (.60)	**-.033110**	.000
	7 (.80)	**-.020970**	.000
	8 (.90)	**-.016700**	.000
4 (.40)	1 (.10)	**.016670**	.000
	2 (.20)	**.015650**	.000
	3 (.30)	**.012480**	.000
	5 (.50)	**-.017170**	.000
	6 (.60)	**-.020630**	.000
	7 (.80)	**-.008482**	.000
	8 (.90)	-.004212	.153

Table 6 (continued)

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
5 (.50)	1 (.10)	**.033840**	.000
	2 (.20)	**.032830**	.000
	3 (.30)	**.029660**	.000
	4 (.40)	**.017170**	.000
	6 (.60)	-.003454	.632
	7 (.80)	**.008690**	.000
	8 (.90)	**.012960**	.000
6 (.60)	1 (.10)	**.003729**	.000
	2 (.20)	**.036280**	.000
	3 (.30)	**.033110**	.000
	4 (.40)	**.020630**	.000
	5 (.50)	.003454	.632
	7 (.80)	**.012140**	.000
	8 (.90)	**.016410**	.000
7 (.80)	1 (.10)	**.025150**	.000
	2 (.20)	**.024140**	.000
	3 (.30)	**.020970**	.000
	4 (.40)	**.008482**	.000
	5 (.50)	**-.008690**	.000
	6 (.60)	**-.012140**	.000
	8 (.90)	.004270	.136
8 (.90)	1 (.10)	**.020880**	.000
	2 (.20)	**.019870**	.000
	3 (.30)	**.016700**	.000
	4 (.40)	.004212	.153
	5 (.50)	**-.012960**	.000
	6 (.60)	**-.016410**	.000
	7 (.80)	-.004270	.036

Note: **Bold** face indicates significance at $p < .01$

Table 7

STRESS Means for Groups in Tukey HSD Homogeneity Subtests for TD and R

		Subset			
Level	N	1	2	3	4
1	100	.0045			
2	100	.0055			
3	100	.0087			
4	100		.0211		
5	100		.0254	.0254	
6	100			.0296	
7	100				.0383
8	100				.0418

Table 8

STRESS Means for Groups in Tukey HSD Homogeneity Subtests for DE and R

Level	N	Subset 1	Subset 2	Subset 3	Subset 4
8	100	.0254			
7	100		.0304		
4	100			.0463	
3	100			.0464	
2	100			.0478	.0478
6	100			.0483	.0483
5	100				.0509
1	100				.0513

Table 9

STRESS Means for Groups in Tukey HSD Homogeneity Subtests for TD and S

		Subset				
Level	N	1	2	3	4	5
1	100	.00409				
2	100	.00462				
3	100	.00628				
4	100		.0139			
7	100			.0288		
8	100				.0351	
5	100					.0406
6	100					.0435

Table 10

STRESS Means for Groups in Tukey HSD Homogeneity Subtests for DE and S

		Subset			
Level	N	1	2	3	4
7	100	.0325			
8	100		.0386		
4	100		.0393		
3	100		.0397		
2	100			.0459	
1	100				.0519
6	100				.0520
5	100				.0561

Table 11

Descriptive Statistics for 100 Replications of RSQ for Two levels of Error, Two Methods of Selection and Eight Levels of Missing

	True Dissimilarities				Dissimilarities Plus Random Error			
	Random		Systematic		Random		Systematic	
Level	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	.99991	.00004	.99993	.00002	.98850	.00126	.98819	.00108
2	.99987	.00007	.99990	.00006	.99001	.00191	.99074	.00167
3	.99948	.00150	.99983	.00008	.99049	.00348	.99311	.00086
4	.99713	.00422	.99829	.00411	.99028	.00458	.99291	.00420
5	.99258	.00537	.99168	.00614	.98779	.00544	.98557	.00689
6	.99131	.00422	.99121	.00310	.98835	.00539	.98700	.00480
7	.99494	.00363	.99508	.00277	.99463	.00400	.99390	.00330
8	.99696	.00256	.99653	.00215	.99708	.00242	.99538	.00725

Table 12

Analysis of Variance for RSQ with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Source	df	F	η^2	p
Model	31	139.309	.577	.000
LEVEL	7	218.840	1.000	.000
DATA	1	1883.084	.326	.000
METHOD	1	.001	.000	.976
LEVEL by DATA	7	114.943	.203	.000
LEVEL by METHOD	7	10.594	.023	.000
DATA by METHOD	1	.088	.000	.767
LEVEL by DATA by METHOD	7	3.538	.001	.001

Note: **Bold** face indicates significance at $p < .01$

Table 13

Post Hoc Tukey HSD Results for RSQ with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
1 (.10)	2 (.20)	**-.000999**	.003
	3 (.30)	**-.001593**	.000
	4 (.40)	-.000519	.491
	5 (.50)	**.004729**	.000
	6 (.60)	**.004666**	.000
	7 (.80)	-.000505	.528
	8 (.90)	**-.002354**	.000
2 (.20)	1 (.10)	**.000999**	.003
	3 (.30)	-.000593	.310
	4 (.40)	-.000481	.591
	5 (.50)	**-.005729**	.000
	6 (.60)	**-.005667**	.000
	7 (.80)	-.000495	.555
	8 (.90)	**-.001354**	.000
3 (.30)	1 (.10)	**.001593**	.000
	2 (.20)	.000593	.310
	4 (.40)	**-.001074**	.001
	5 (.50)	**-.006322**	.000
	6 (.60)	**-.006259**	.000
	7 (.80)	**-1.088000**	.000
	8 (.90)	**-.000761**	.000
4 (.40)	1 (.10)	.000519	.491
	2 (.20)	.000481	.591
	3 (.30)	**.001074**	.001
	5 (.50)	**-.005247**	.000
	6 (.60)	**-.005185**	.000
	7 (.80)	-.000014	1.000
	8 (.90)	**-.001835**	.000

Table 13 (continued)

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
5 (.50)	1 (.10)	**.004729**	.000
	2 (.20)	**.005729**	.000
	3 (.30)	**.006322**	.000
	4 (.40)	**.005247**	.000
	6 (.60)	-.000063	1.000
	7 (.80)	**.005234**	.000
	8 (.90)	**.007083**	.000
6 (.60)	1 (.10)	**.004667**	.000
	2 (.20)	**.005666**	.000
	3 (.30)	**.006259**	.000
	4 (.40)	**.005185**	.000
	5 (.50)	.000064	1.000
	7 (.80)	**.005171**	.000
	8 (.90)	**.007020**	.000
7 (.80)	1 (.10)	.000051	.528
	2 (.20)	-.000495	.555
	3 (.30)	**-.001088**	.001
	4 (.40)	-.000014	1.000
	5 (.50)	**.005234**	.000
	6 (.60)	**.005171**	.000
	8 (.90)	**-.001849**	.000
8 (.90)	1 (.10)	**.002354**	.000
	2 (.20)	**.001354**	.000
	3 (.30)	.000076	.070
	4 (.40)	**.001835**	.000
	5 (.50)	**.007083**	.000
	6 (.60)	**.007020**	.000
	7 (.80)	.001849	.000

Note: **Bold** face indicates significance at $p < .01$

Table 14

RSQ Means for Groups in Tukey HSD Homogeneity Subtests for TD and R

		Subset			
Level	N	1	2	3	4
6	100	.99131			
5	100	.99258			
7	100		.99494		
8	100			.99696	
4	100			.99713	
3	100				.99948
2	100				.99987
1	100				.99991

Table 15

RSQ Means for Groups in Tukey HSD Homogeneity Subtests for DE and R

		Subset				
Level	N	1	2	3	4	5
5	100	.98779				
6	100	.98884				
1	100	.98885	.98850			
2	100		.99001	.99001		
4	100			.99028		
3	100			.99049		
7	100				.99463	
8	100					.99708

Table 16

RSQ Means for Groups in Tukey HSD Homogeneity Subtests for TD and S

		Subset				
Level	N	1	2	3	4	5
6	100	.99121				
5	100	.99168				
7	100		.99508			
8	100			.99653		
4	100				.99829	
3	100					.99983
2	100					.99991
1	100					.99993

Table 17

RSQ Means for Groups in Tukey HSD Homogeneity Subtests for DE and S

		Subset				
Level	N	1	2	3	4	5
5	100	.98557				
6	100	.98700	.986700			
1	100		.98819			
2	100			.99074		
4	100				.99291	
3	100				.99311	
7	100				.99390	.99390
8	100					.99538

Table 18

Descriptive Statistics for Correlations Between True Coordinates and Recovered Coordinates of 100 Replications

	True Dissimilarities				Dissimilarities Plus Error			
	Random		Systematic		Random		Systematic	
Level	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	**.999791**	.00017	**.999857**	.00013	**.992315**	.00171	**.992258**	.00154
2	**.999427**	.00038	**.999536**	.00029	**.979002**	.04998	**.990744**	.00307
3	**.993631**	.02629	**.998755**	.00066	**.962529**	.06530	**.985363**	.02424
4	**.939208**	.20759	**.987008**	.03287	**.926557**	.08429	**.962196**	.06122
5	**.840147**	.31683	**.870883**	.19212	**.804210**	.30971	**.849871**	.18427
6	**.667308**	.48099	**.444739**	.69297	**.671348**	.42064	**.440220**	.68456
7	.206287	.52729	**.351735**	.40601	**.335457**	.45898	**.352692**	.43321
8	.154768	.40913	.140123	.31667	.136666	.38666	.133299	.27768

Note: **Bold** face indicates significance at $p < .01$

Table 19

Number of Correlations Between True and Recovered Coordinates That Are Significant in 100 Replications

		TD		DE	
Level	Significance	R	S	R	S
1	.01	100	100	100	100
	.05	0	0	0	0
	Not Significant	0	0	0	0
2	.01	100	100	100	100
	.05	0	0	0	0
	Not Significant	0	0	0	0
3	.01	100	100	100	100
	.05	0	0	0	0
	Not Significant	0	0	0	0
4	.01	99	100	100	100
	.05	0	0	0	0
	Not Significant	1	0	0	0
5	.01	97	99	97	99
	.05	0	0	0	0
	Not Significant	3	1	3	1
6	.01	91	76	93	76
	.05	0	0	0	0
	Not Significant	9	24	7	24
7	.01	55	67	66	68
	.05	5	3	6	5
	Not Significant	40	30	28	27
8	.01	39	12	35	9
	.05	12	26	12	19
	Not Significant	49	62	53	72

Table 20

Analysis of Variance for Correlations Between True and Recovered Coordinates on Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Source	df	F	η^2	p
Model	31	113.785	.527	.000
LEVEL	7	492.971	.521	.000
DATA	1	.102	.000	.750
METHOD	1	.497	.000	.521
LEVEL by ERROR	7	.917	.002	.491
LEVEL by METHOD	7	9.269	.020	.000
ERROR by METHOD	1	.450	.000	.503
LEVEL by ERROR by METHOD	7	.759	.001	.759

Note: **Bold** face indicates significance at p < .01

Table 21

Post Hoc Tukey HSD Results for Coordinate Correlations with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
1 (.10)	2 (.20)	.003878	1.000
	3 (.30)	.010990	1.000
	4 (.40)	.042310	.518
	5 (.50)	**.154777**	.000
	6 (.60)	**.440151**	.000
	7 (.80)	**.684512**	.000
	8 (.90)	**.854841**	.000
2 (.20)	1 (.10)	-.003878	1.000
	3 (.30)	.007117	1.000
	4 (.40)	.038430	.641
	5 (.50)	**.150899**	.000
	6 (.60)	**.436273**	.000
	7 (.80)	**.680634**	.000
	8 (.90)	**.850963**	.000
3 (.30)	1 (.10)	-.010990	1.000
	2 (.20)	-.007117	1.000
	4 (.40)	.031131	.838
	5 (.50)	**.143783**	.000
	6 (.60)	**.429156**	.000
	7 (.80)	**.673517**	.000
	8 (.90)	**.843846**	.000
4 (.40)	1 (.10)	-.042310	.518
	2 (.20)	-.038430	.641
	3 (.30)	-.031310	.838
	5 (.50)	**.112468**	.000
	6 (.60)	**.397842**	.000
	7 (.80)	**.642203**	.000
	8 (.90)	**.812532**	.000

Table 21 (continued)

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
5 (.50)	1 (.10)	**-.154777**	.000
	2 (.20)	**-.150899**	.000
	3 (.30)	**-.143783**	.000
	4 (.40)	**-.112469**	.000
	6 (.60)	**.285374**	.000
	7 (.80)	**.529735**	.000
	8 (.90)	**.700064**	.000
6 (.60)	1 (.10)	**-.440151**	.000
	2 (.20)	**-.436273**	.000
	3 (.30)	**-.429156**	.000
	4 (.40)	**-.397843**	.000
	5 (.50)	**-.285374**	.000
	7 (.80)	**.244361**	.000
	8 (.90)	**.414690**	.000
7 (.80)	1 (.10)	**-.684512**	.000
	2 (.20)	**-.680634**	.000
	3 (.30)	**-.673517**	.000
	4 (.40)	**-.642203**	.000
	5 (.50)	**-.529735**	.000
	6 (.60)	**-.244361**	.000
	8 (.90)	**.170329**	.000
8 (.90)	1 (.10)	**-.854841**	.000
	2 (.20)	**-.850963**	.000
	3 (.30)	**-.843846**	.000
	4 (.40)	**-.812532**	.000
	5 (.50)	**-.700064**	.000
	6 (.60)	**-.414690**	.000
	7 (.80)	**-.170329**	.000

Note: **Bold** face indicates significance at $p < .01$

Table 22

Coordinates Correlation Means for Groups in Tukey HSD Homogeneity Subtests for TD and R

		Subset			
Level	N	1	2	3	4
8	100	.154768			
7	100	.206287			
6	100		.667308		
5	100			0840147	
4	100			.939208	.939208
3	100				.993631
2	100				.999427
1	100				.999791

Table 23

Coordinates Correlation Means for Groups in Tukey HSD Homogeneity Subtests for DE and R

		Subset				
Level	N	1	2	3	4	5
8	100	.154768				
7	100		.335457			
6	100			.671348		
5	100				.804210	
4	100				.926573	.926573
3	100					.962529
2	100					.979002
1	100					.992315

Table 24

Coordinates Correlation Means for Groups in Tukey HSD Homogeneity Subtests for TD and S

		Subset		
Level	N	1	2	3
8	100	.140123		
7	100		.351735	
6	100		.444739	
5	100			.870883
4	100			.987008
3	100			.998755
2	100			.999535
1	100			.999857

Table 25

Coordinates Correlation Means for Groups in Tukey HSD Homogeneity Subtests for DE and S

		Subset			
Level	N	1	2	3	4
8	100	.133299			
7	100		.352692		
6	100		.440220		
5	100			.849871	
4	100			.962196	.962196
3	100				.985326
2	100				.990744
1	100				.992258

Table 26

Descriptive Statistics for M

	True Dissimilarities				Dissimilarities Plus Random Error			
	Random		Systematic		Random		Systematic	
Level	Mean	SD	Mean	SD	Mean	SD	Mean	SD
1	.99955	.00030	.99968	.00025	.98442	.00279	.98460	.00270
2	.99879	.00069	.99910	.00052	.98060	.00695	.98073	.00481
3	.99374	.01802	.99734	.00139	.96631	.02727	.97420	.00610
4	.96451	.05747	.98333	.03105	.93731	.05845	.96044	.02573
5	.87741	.08075	.88489	.07777	.85179	.07814	.69581	.07390
6	.74650	.08542	.76915	.06916	.72190	.10065	.74496	.06595
7	.27156	.16761	.28976	.12844	.27642	.15574	.29903	.12200
8	.09050	.08538	.05440	.11957	.08440	.09536	.02600	.03656

Table 27

Analysis of Variance for M with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Source	f	F	η^2	p
Model	31	2157.923	.955	.000
Level	7	9537.204	.955	.000
Data	1	48.106	.015	.000
Method	1	2.353	.001	.125
Level by Data	8	2.238	.005	.029
Level by Method	7	9.478	.021	.000
Data by Method	1	.020	.000	.888
Level by Data by Method	7	.383	.001	.913

Note: **Bold** face indicates significance at $p < .01$

Table 28

Post Hoc Tukey HSD Results for M with Two Levels of Error, Two Methods of Selection, and Eight Levels of Missing

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
1 (.10)	2 (.20)	.002258	1.000
	3 (.30)	.009166	.654
	4 (.40)	**.031417**	.000
	5 (.50)	**.123642**	.000
	6 (.60)	**.246438**	.000
	7 (.80)	**.707860**	.000
	8 (.90)	**.928257**	.000
2 (.20)	1 (.10)	-.002258	1.000
	3 (.30)	.006907	.892
	4 (.40)	**.029159**	.000
	5 (.50)	**.121384**	.000
	6 (.60)	**.244179**	.000
	7 (.80)	**.705602**	.000
	8 (.90)	**.925999**	.000
3 (.30)	1 (.10)	-.009166	.654
	2 (.20)	-.006907	.892
	4 (.40)	**.022252**	.001
	5 (.50)	**.114476**	.000
	6 (.60)	**.237272**	.000
	7 (.80)	**.698695**	.000
	8 (.90)	**.919092**	.000
4 (.40)	1 (.10)	**-.031417**	.000
	2 (.20)	**-.029159**	.000
	3 (.30)	**-.022516**	.001
	5 (.50)	**.092225**	.000
	6 (.60)	**.215021**	.000
	7 (.80)	**.676443**	.000
	8 (.90)	**.896840**	.000

Table 28 (continued)

Level (Percent Missing) (I)	Level (Percent Missing) (J)	Mean Difference (I-J)	p
5 (.50)	1 (.10)	**-.123642**	.000
	2 (.20)	**-.121384**	.000
	3 (.30)	**-.114476**	.000
	4 (.40)	**-.092225**	.000
	6 (.60)	**.122796**	.000
	7 (.80)	**.584218**	.000
	8 (.90)	**.804616**	.000
6 (.60)	1 (.10)	**-.246438**	.000
	2 (.20)	**-.244179**	.000
	3 (.30)	**-.237272**	.000
	4 (.40)	**-.215021**	.000
	5 (.50)	**-.122796**	.000
	7 (.80)	**.461422**	.000
	8 (.90)	**.681820**	.000
7 (.80)	1 (.10)	**-.707860**	.000
	2 (.20)	**-.705602**	.000
	3 (.30)	**-.698695**	.000
	4 (.40)	**-.676443**	.000
	5 (.50)	**-.584218**	.000
	6 (.60)	**-.461422**	.000
	8 (.90)	**.220397**	.000
8 (.90)	1 (.10)	**-.928257**	.000
	2 (.20)	**-.925999**	.000
	3 (.30)	**-.919092**	.000
	4 (.40)	**-.896840**	.000
	5 (.50)	**-.804616**	.000
	6 (.60)	**-.681820**	.000
	7 (.80)	**-.220397**	.000

Note: **Bold** face indicates significance at $p < .01$

Table 29

M Means for Groups in Tukey HSD Homogeneity Subtests for TD and R

	Subset					
Level	1	2	3	4	5	6
8	.09050					
7		.27160				
6			.74650			
5				.87741		
4					.96451	
3					.99375	.99375
2					.99875	.99879
1						.99955

Table 30

M Means for Groups in Tukey HSD Homogeneity Subtests for DE and R

	Subset					
Level	1	2	3	4	5	6
8	.0844					
7		.2764				
6			.7219			
5				.8519		
4					.9343	
3					.9663	.9663
2						.9806
1						.9844

Table 31

M Means for Groups in Tukey HSD Homogeneity Subtests for TD and S

		Subset				
Level	N	1	2	3	4	5
8	100	.054400				
7	100		.289763			
6	100			.769150		
5	100				.884894	
4	100					.983326
3	100					.997338
2	100					.999102
1	100					.999685

Table 32

M Means for Groups in Tukey HSD Homogeneity Subtests for DE and S

		Subset				
Level	N	1	2	3	4	5
8	100	.026000				
7	100		.299032			
6	100			.744957		
5	100				.859505	
4	100					.960441
3	100					.974200
2	100					.980734
1	100					.984600

Table 33

Correlations Between Dependent Variables

	STRESS	RSQ	CC
RSQ	-.940**		
L1	-.998**		
L2	-.994**		
L3	-.978**		
L4	-.961**		
L5	-.972**		
L6	-.969**		
L7	-.972**		
L8	-.820**		
CC	-.102**	.018	
L1	-.958**	.957**	
L2	-.330**	.370**	
L3	-.365**	.419**	
L4	-.144**	.179**	
L5	-.178**	.184**	
L6	-.099*	.087	
L7	.072	.068	
L8	.001	-.012	
M	-.062**	-.067**	.709**
L1	-.969**	.968**	.992**
L2	-.928**	.939**	.443**
L3	-.696**	.757**	.603**
L4	-.320**	.368**	.318**
L5	-.533**	.537**	.267**
L6	-.205**	.190**	.162**
L7	-.092	.103*	.149**
L8	-.084	.037	.108*

** Correlation is significant at the .01 level
* Correlation is significant at the .05 level

Table 34.

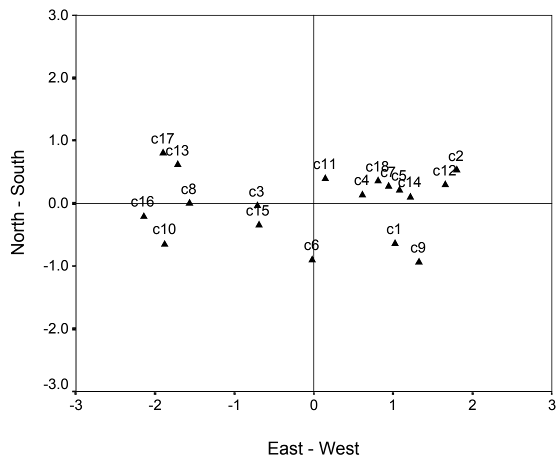
Summary of Mean Values on Each Dependent Variable by Level of Missing Pairs

	Apparent Measures of Recovery							
	STRESS				RSQ			
	TD		DE		TD		DE	
	R	S	R	S	R	S	R	S
L1	.005	.004	.051	.051	.999	.999	.989	.988
L2	.006	.005	.048	.048	.999	.999	.990	.990
L3	.009	.006	.009	.040	.999	.999	.990	.993
L4	.021	.014	.021	.039	.997	.998	.990	.992
L5	.038	.041	.038	.056	.993	.991	.988	.986
L6	.042	.044	.042	.052	.991	.991	.988	.987
L7	.030	.029	.030	.033	.995	.995	.995	.994
L8	.025	.035	.025	.039	.997	.997	.997	.995

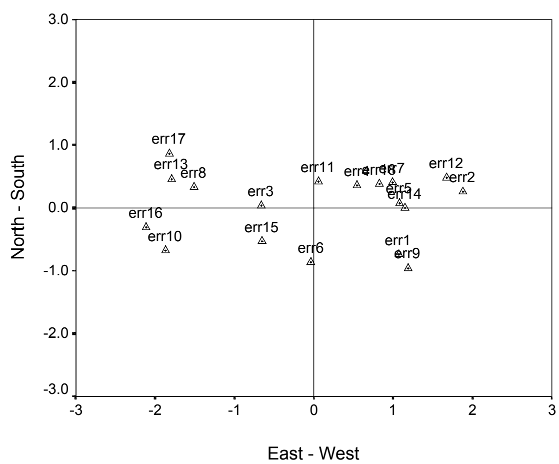
Table 34 (continued)

	Actual Measures of Recovery							
	CC				M			
	TD		DE		TD		DE	
	R	S	R	S	R	S	R	S
L1	.9997	.9998	.9923	.9922	.9995	.9996	.9844	.9846
L2	.9994	.9995	.9790	.9907	.9987	.9991	.9806	.9807
L3	.9936	.9987	.9625	.9853	.9937	.9973	.9663	.9742
L4	.9392	.9870	.9265	.9621	.9645	.9833	.9373	.9604
L5	.8401	.8708	.8042	.8498	.8774	.8848	.8517	.6958
L6	.6673	.4447	.6713	.4402	.7465	.7691	.7219	.7449
L7	.2062	.3517	.3354	.3526	.2715	.2897	.2764	.2990
L8	.1547	.1401	.1366	.1332	.0905	.0544	.0844	.0260

FIGURES

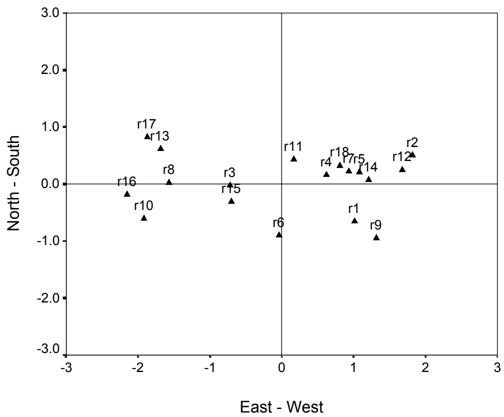


TD and R with no missing

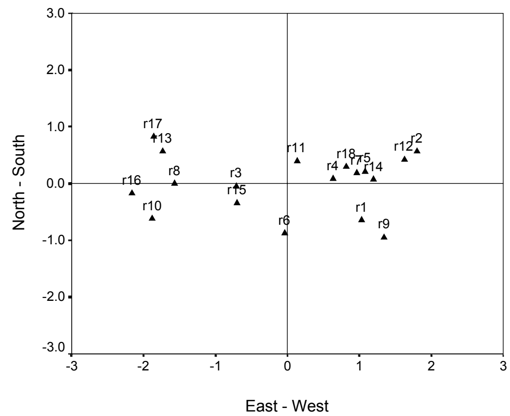


DE and R with no missing

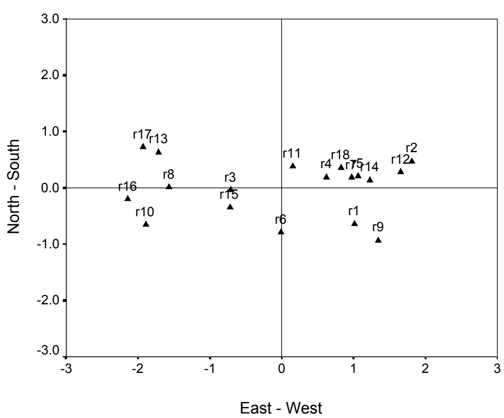
Figure 2. Plots of stimuli from true dissimilarities (TD) and dissimilarities with random error added (DE) without missing pairs.



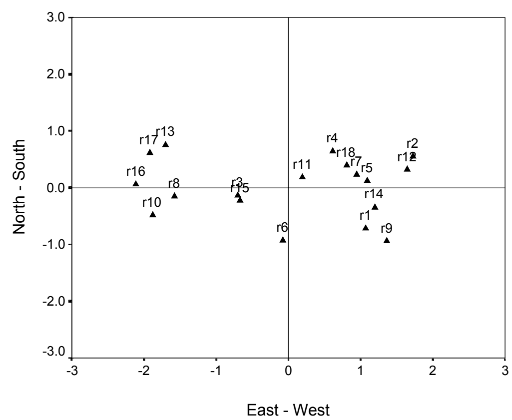
TD and R with .10 missing (L1)



TD and R with .20 missing (L2)

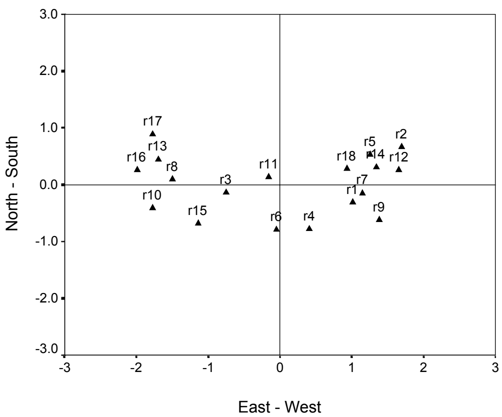


TD and R with .30 missing (L3)

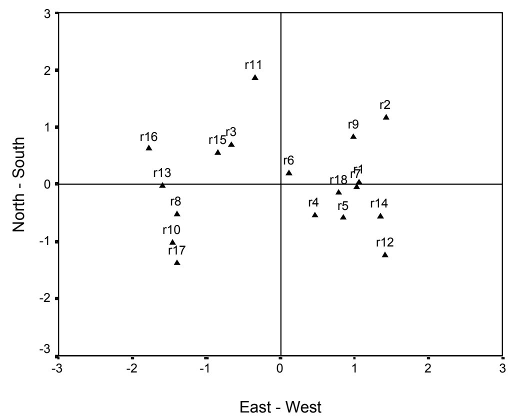


TD and R with .40 missing (L4)

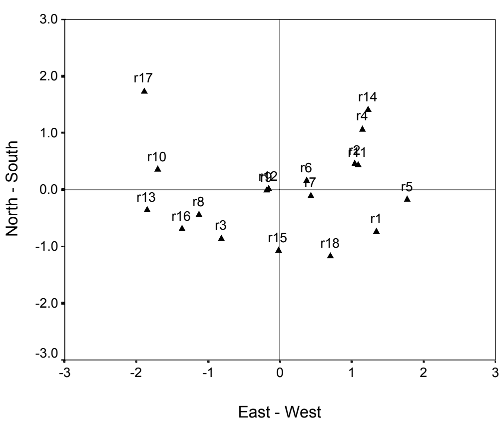
Figure 3. Plots of stimuli from true dissimilarities (TD) with random selection (R) of stimuli for eight levels of missing.



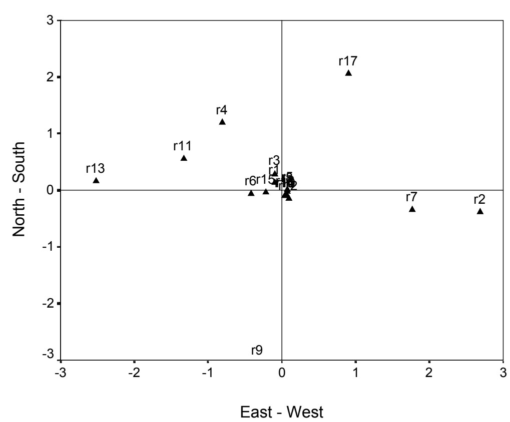
TD and R with .50 missing (L5)



TD and R with .60 missing (L6)

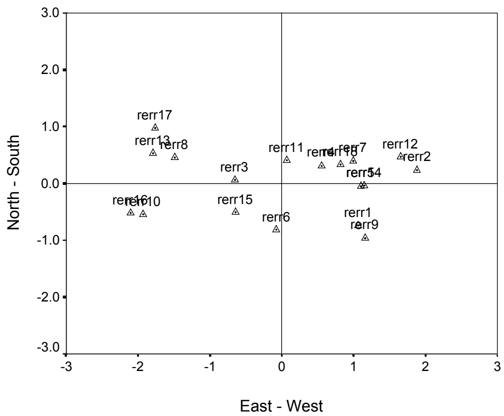


TD and R with .80 missing (L7)

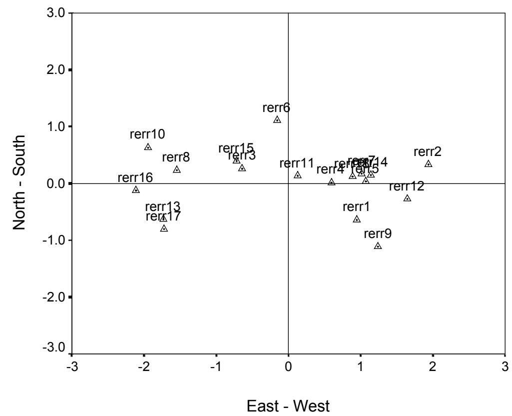


TD and R with .90 missing (L8)

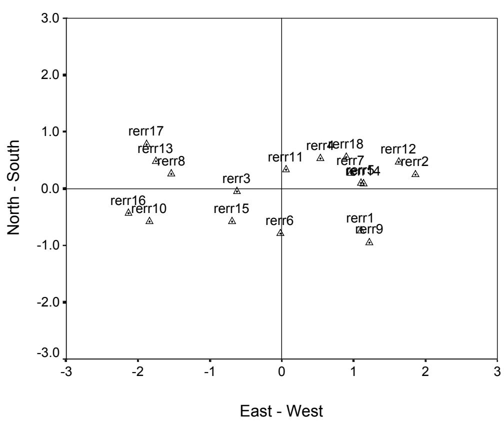
Figure 3 (continued).



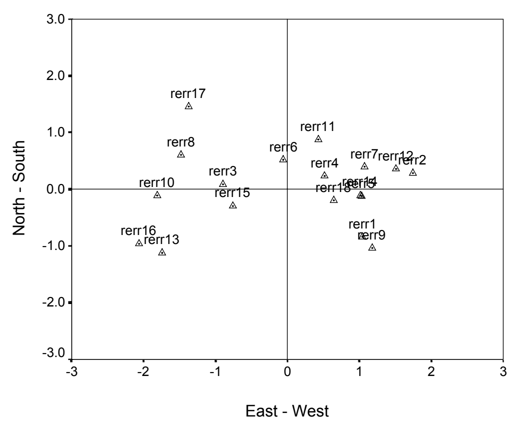
DE and R with .10 missing (L1)



DE and R with .20 missing (L2)

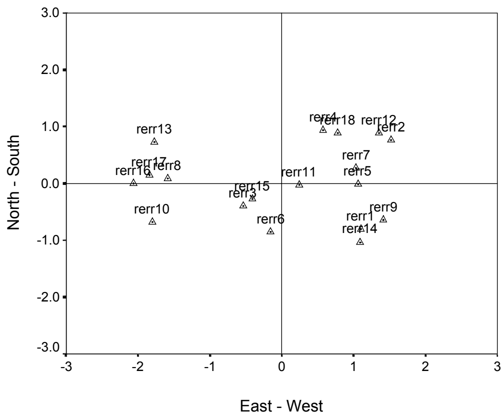


DE and R with .30 missing (L3)

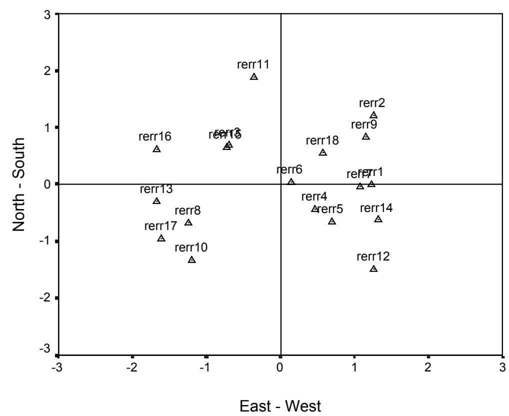


DE and R with .40 random (L4)

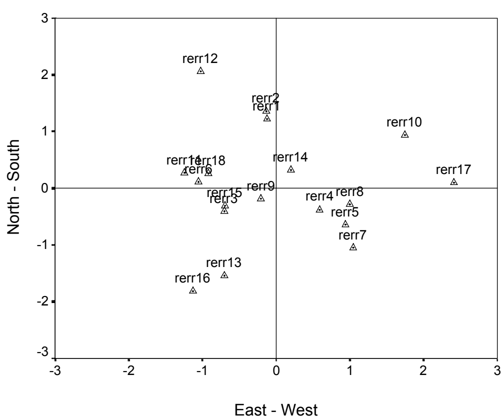
Figure 4. Pots of stimuli from dissimilarities plus error (DE) with random (R) selection for eight levels of missing.



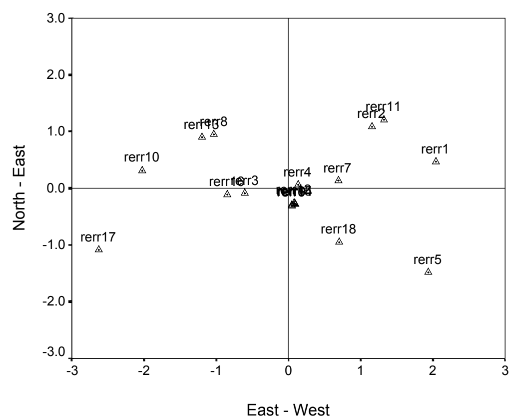
DE and R with .50 random (L5)



DE and R with .60 random (L6)

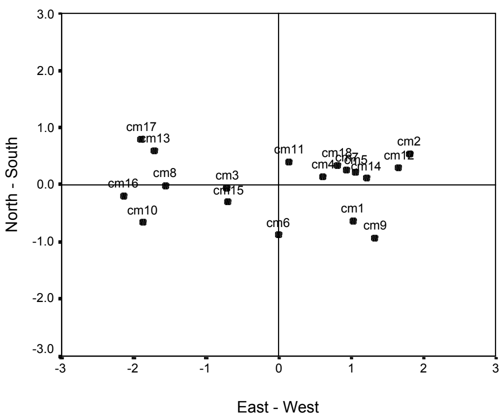


DE and R with .80 missing (L7)

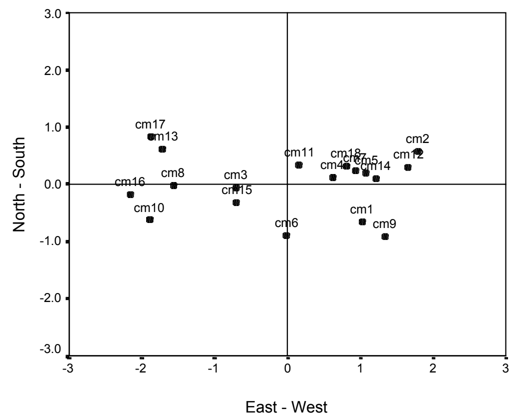


DE and R with .90 missing (L8)

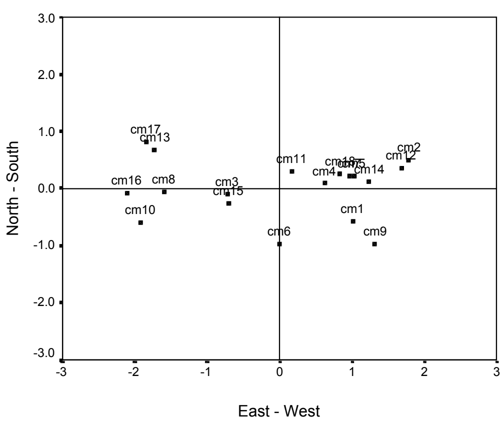
Figure 4 (continued).



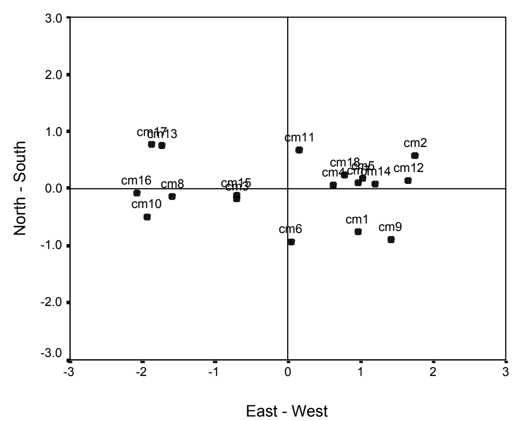
TD and S level 1



TD and S Level 2

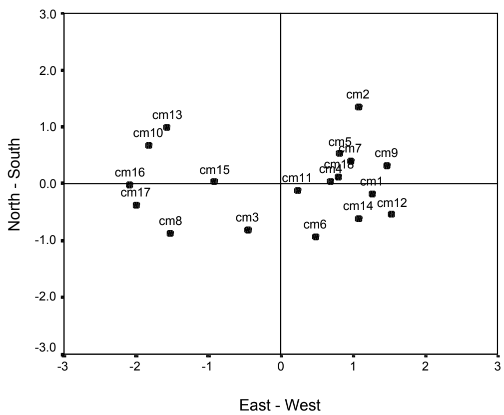


TD and S Level 3

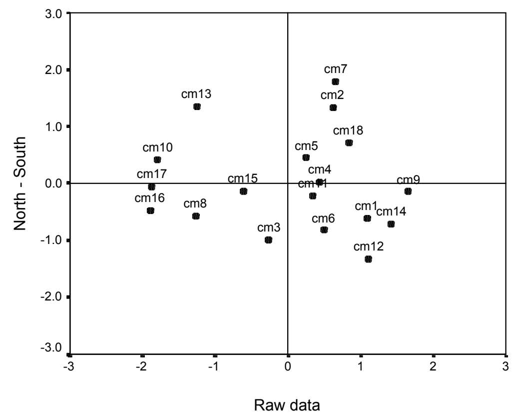


TD and S Level 4

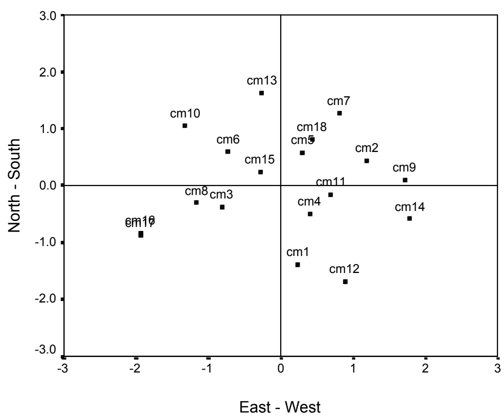
Figure 5. Plots of stimuli from true data set (TD) with systematic (S) selection for eight levels of missing.



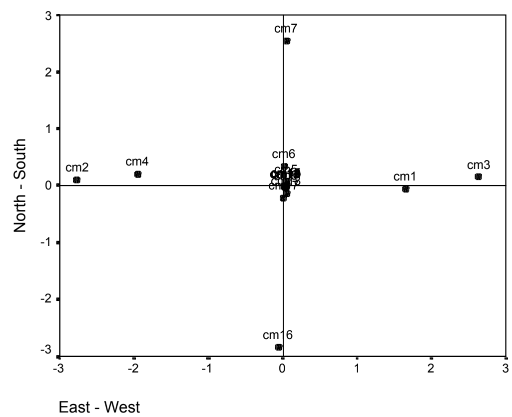
TD and S Level 5



TD and S Level 6



TD and S Level 7



TD and S Level 8

Figure 5 (continued).

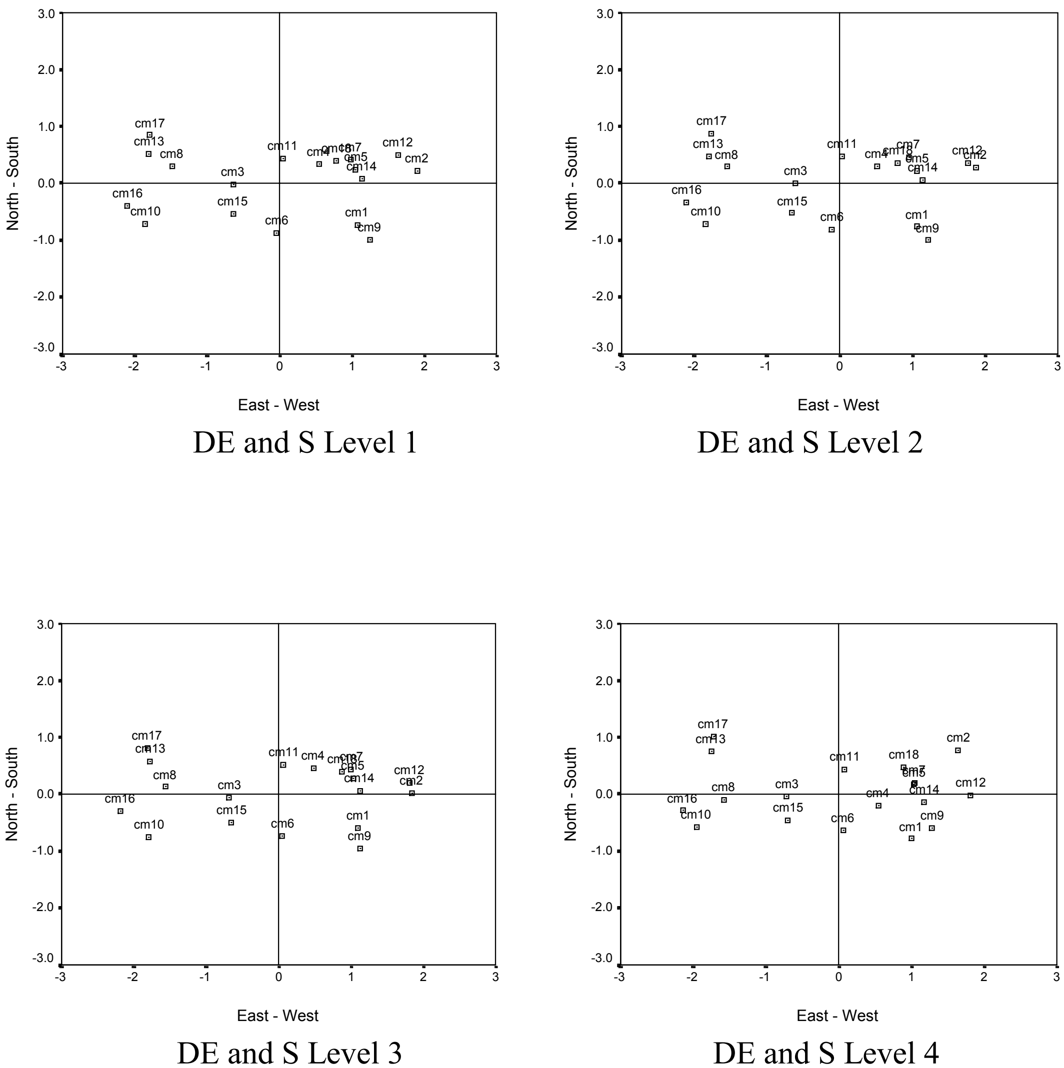
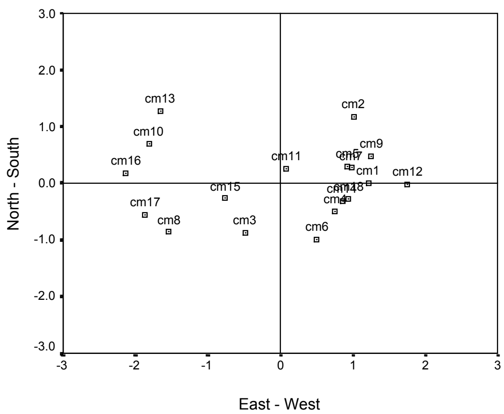
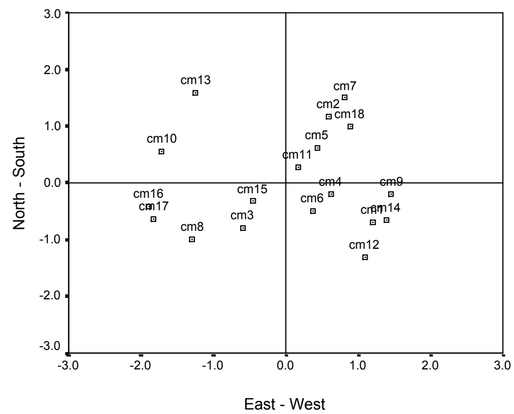


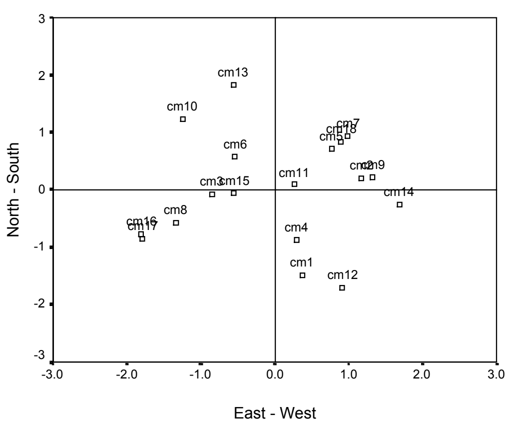
Figure 6. Plots of stimuli from data set of dissimilarities with added error (TD) and systematic (S) selection of eight levels of missing.



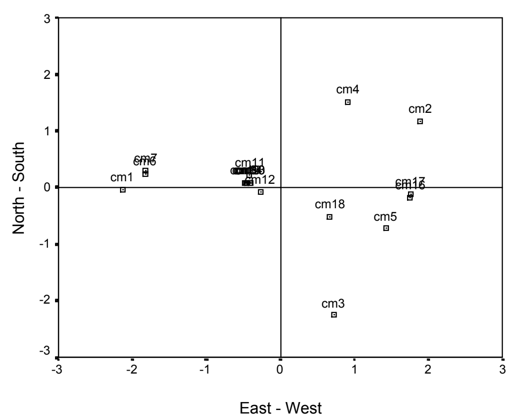
DE and S Level 5



DE and S Level 6

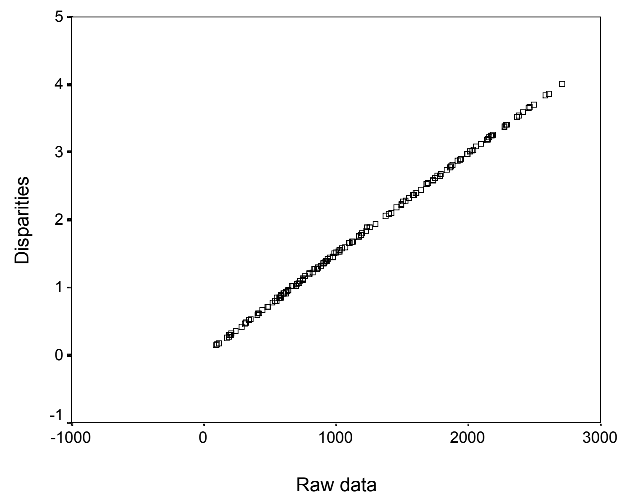


DE and S Level 7

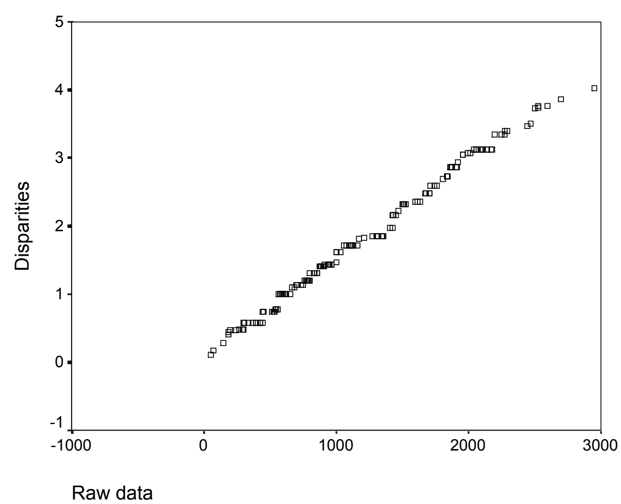


DE and S Level 8

Figure 6 (continued).

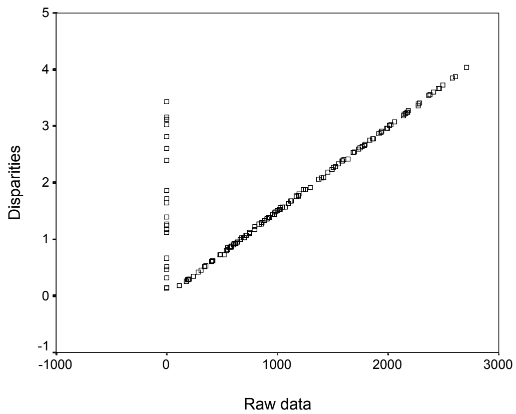


TD with no missing

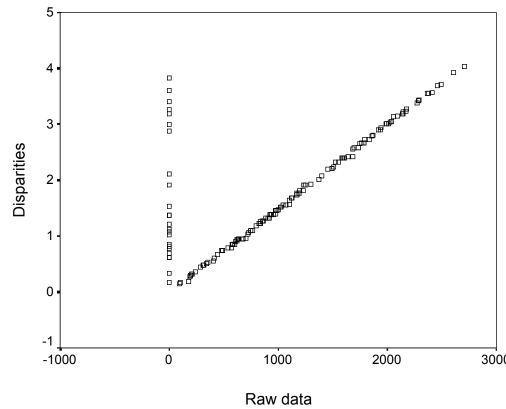


DE with no missing

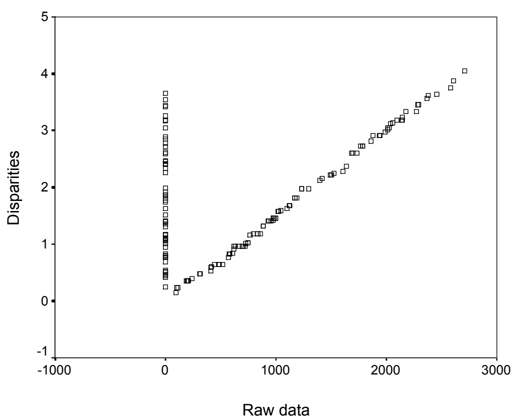
Figure 7. Plots of monotonic transformation with true data (TD) and data with error added (DE) with no missing.



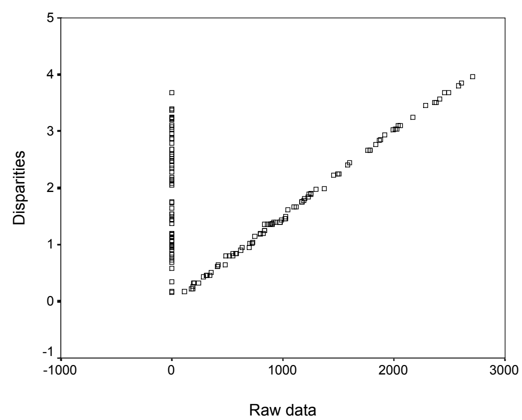
TD and R with .10 missing (L1)



TD and R with.20 missing (L2)

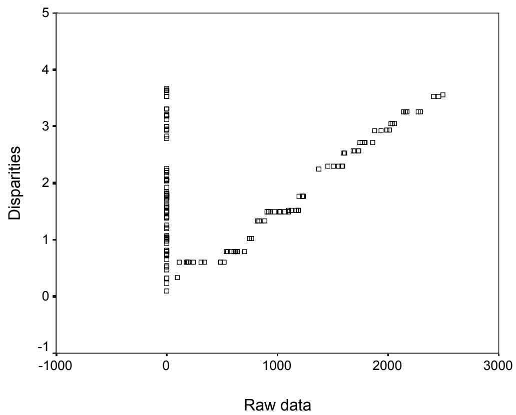


TD and R with.30 missing (L3)

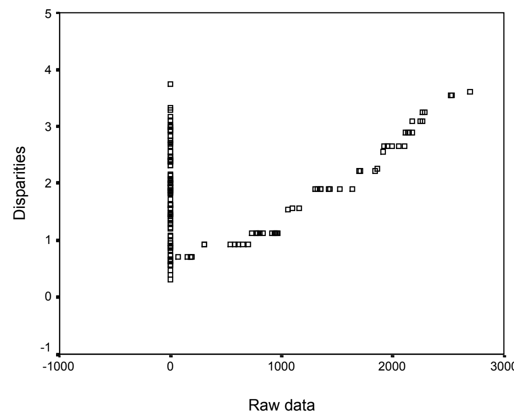


TD and R with.40 missing (L4)

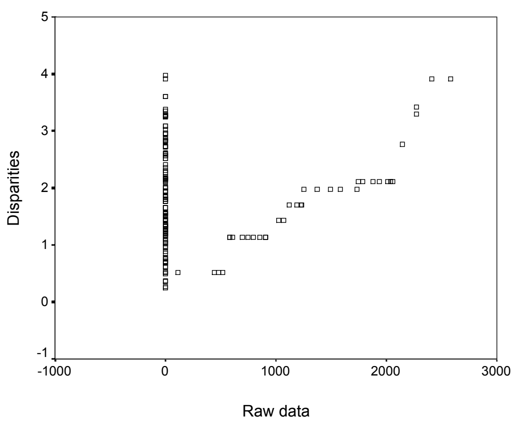
<u>Figure 8.</u> Plots of monotonic transformation with true data (TD) and random (R) selection for 8 levels of missing.



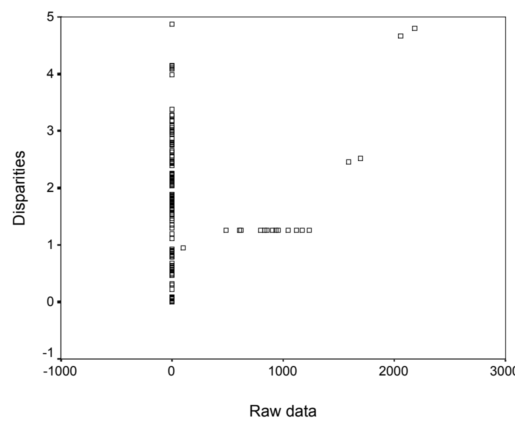
TD and R with .50 missing (L5)



TD and R with .60 missing (L6)

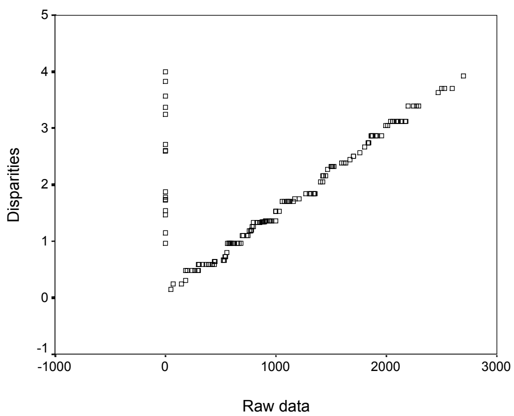


TD and R with .80 missing (L7)

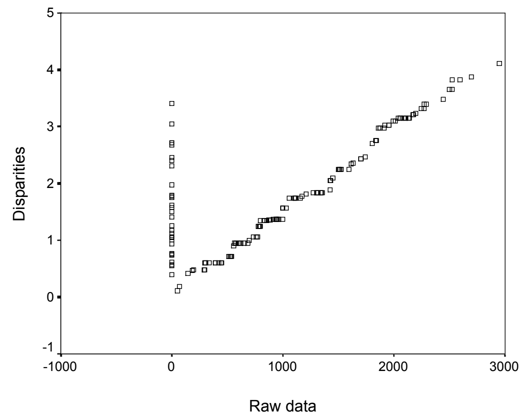


TD and R with .90 missing (L8)

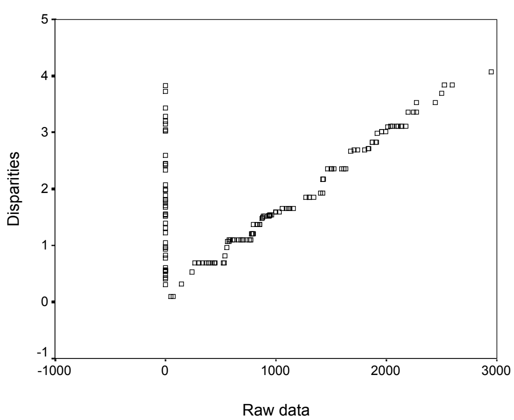
Figure 8 (continued).



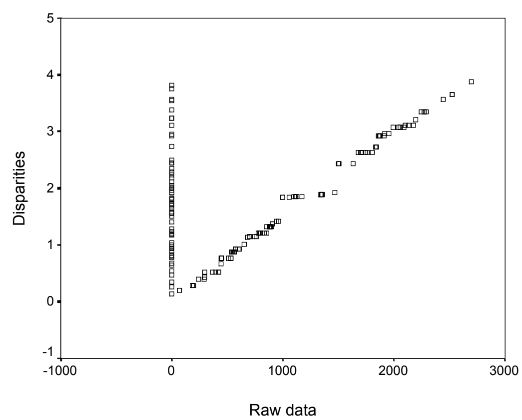
DE and R with .10 missing (L1)



DE and R with .20 missing (L2)

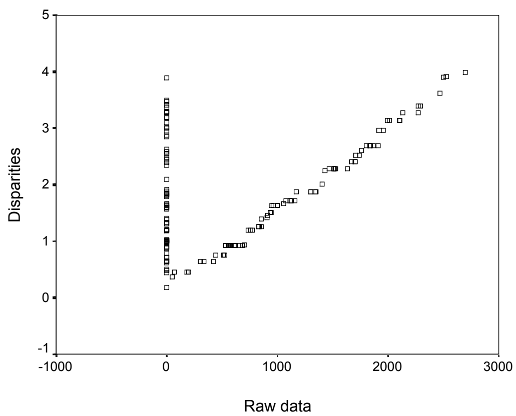


DE and R with .30 missing (L3)

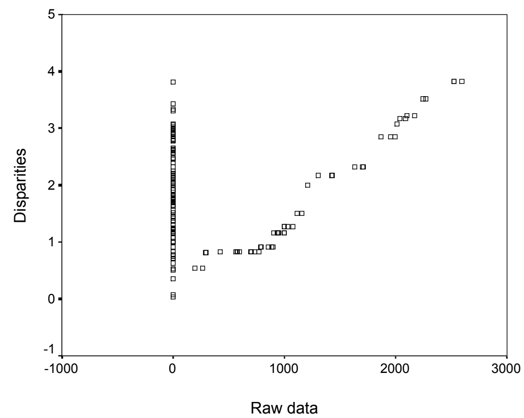


DE and R with .40 missing (L4)

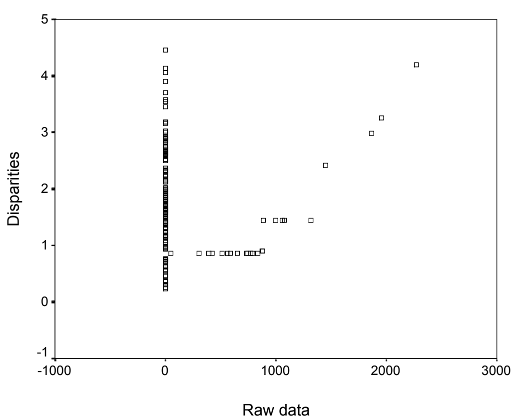
Figure 9. Plots of monotonic transformation with data plus error (DE) and random (R) missing.



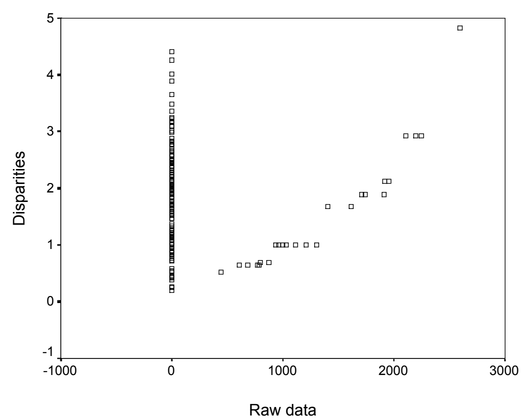
DE and R with .50 missing (L5)



DE and R with .60 missing (L6)

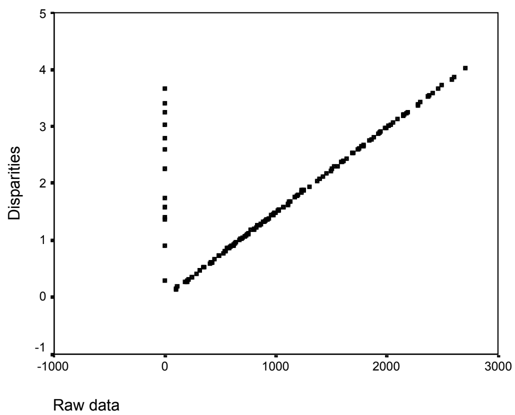


DE and R with .80 missing (L7)

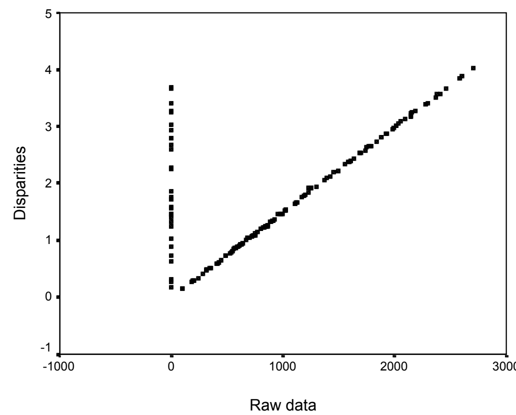


DE and R with .90 missing (L8)

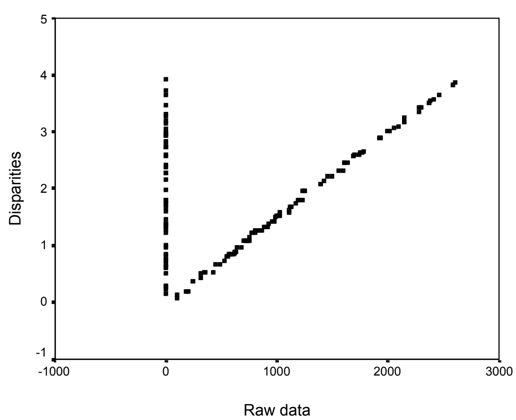
Figure 9 (continued).



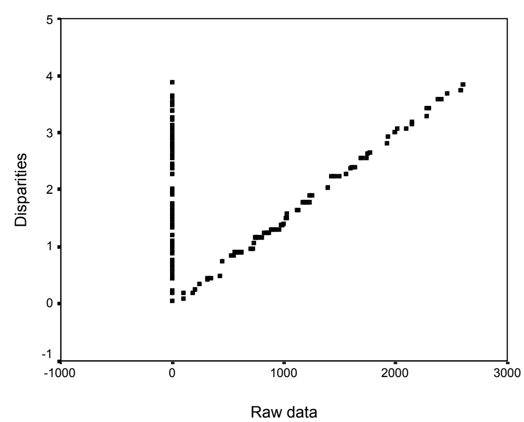
TD and S level 1



TD and S level 2

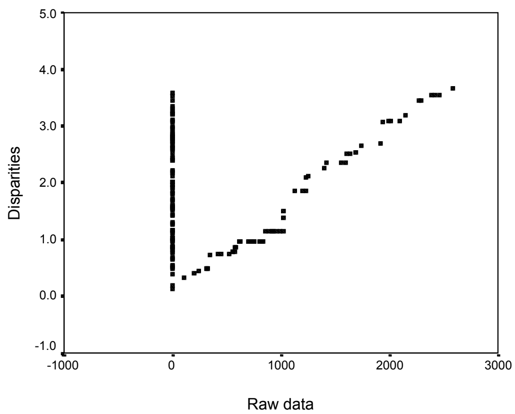


TD and S Level 3

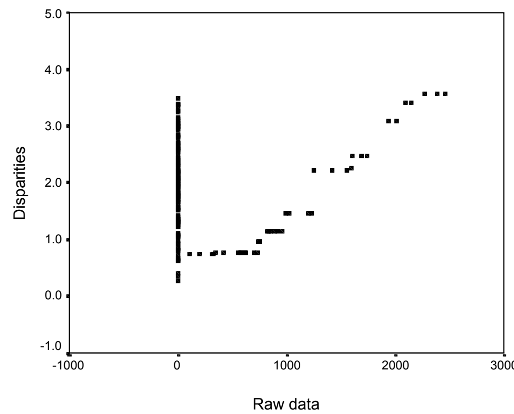


TD and S Level 4

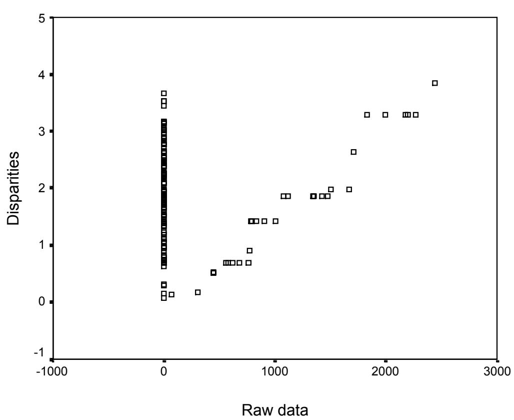
Figure 10. Plots of monotonic transformation with true data (TD) and systematic (S) selection of eight levels of missing.



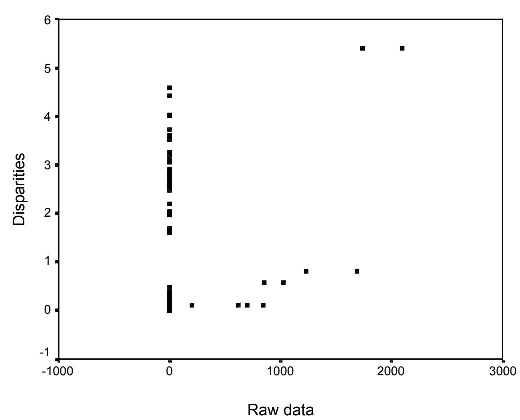
TD and S Level 5



TD and S Level 6

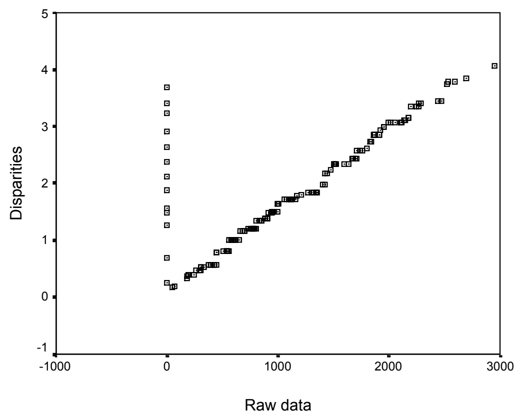


TD and S Level 7

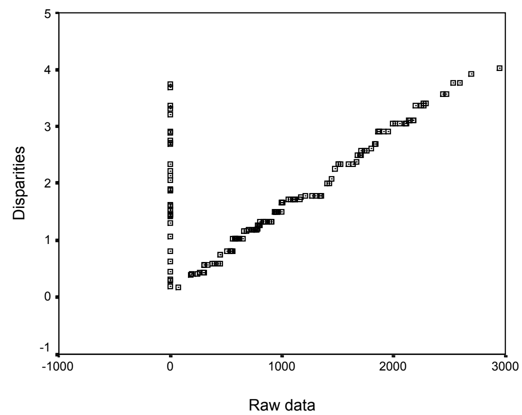


TD and S Level 8

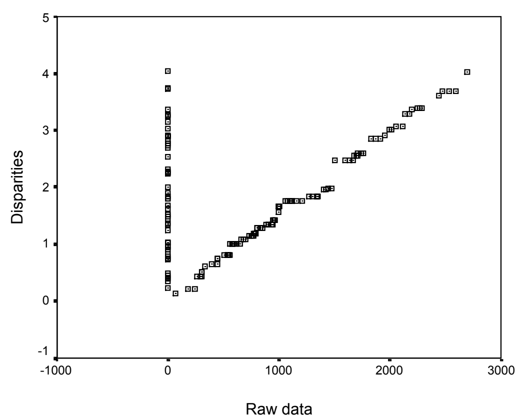
Figure 10 (continued).



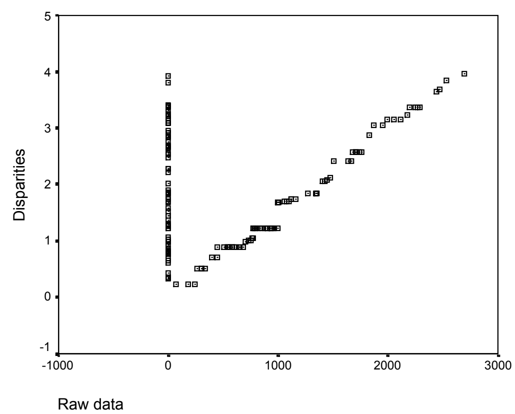
DE and S Level 1



DE and S Level 2

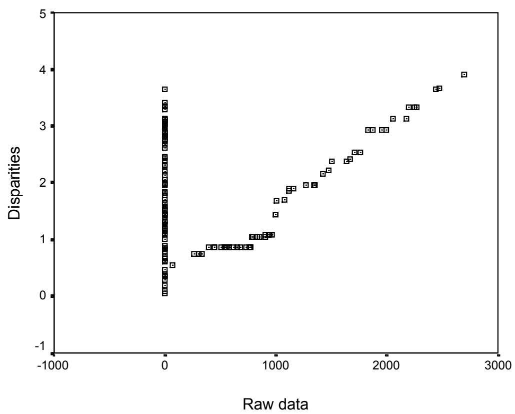


DE and S Level 3

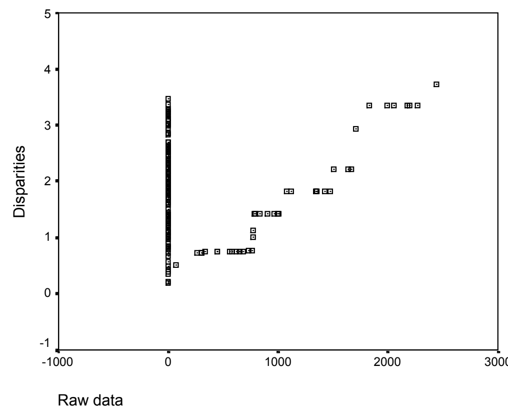


DE and S Level 4

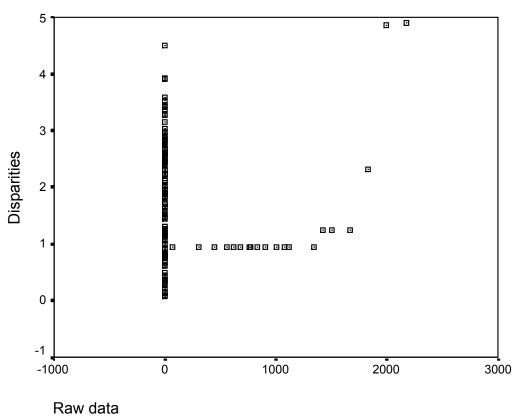
Figure 11. Plots of monotonic transformation with data set of dissimilarities with added error (DE) and systematic (S) selection for each of eight levels of missing.



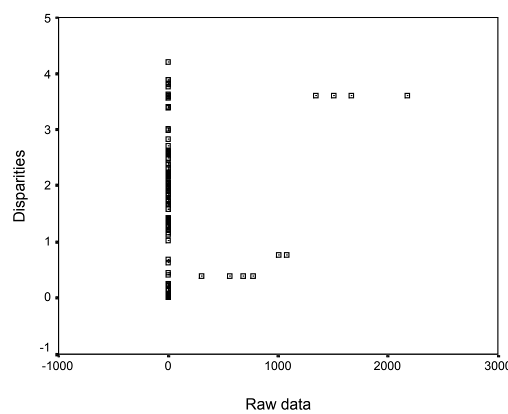
DE and S Level 5



DE and S Level 6



DE and S Level 7



DE and S Level 8

Figure 11 (continued).

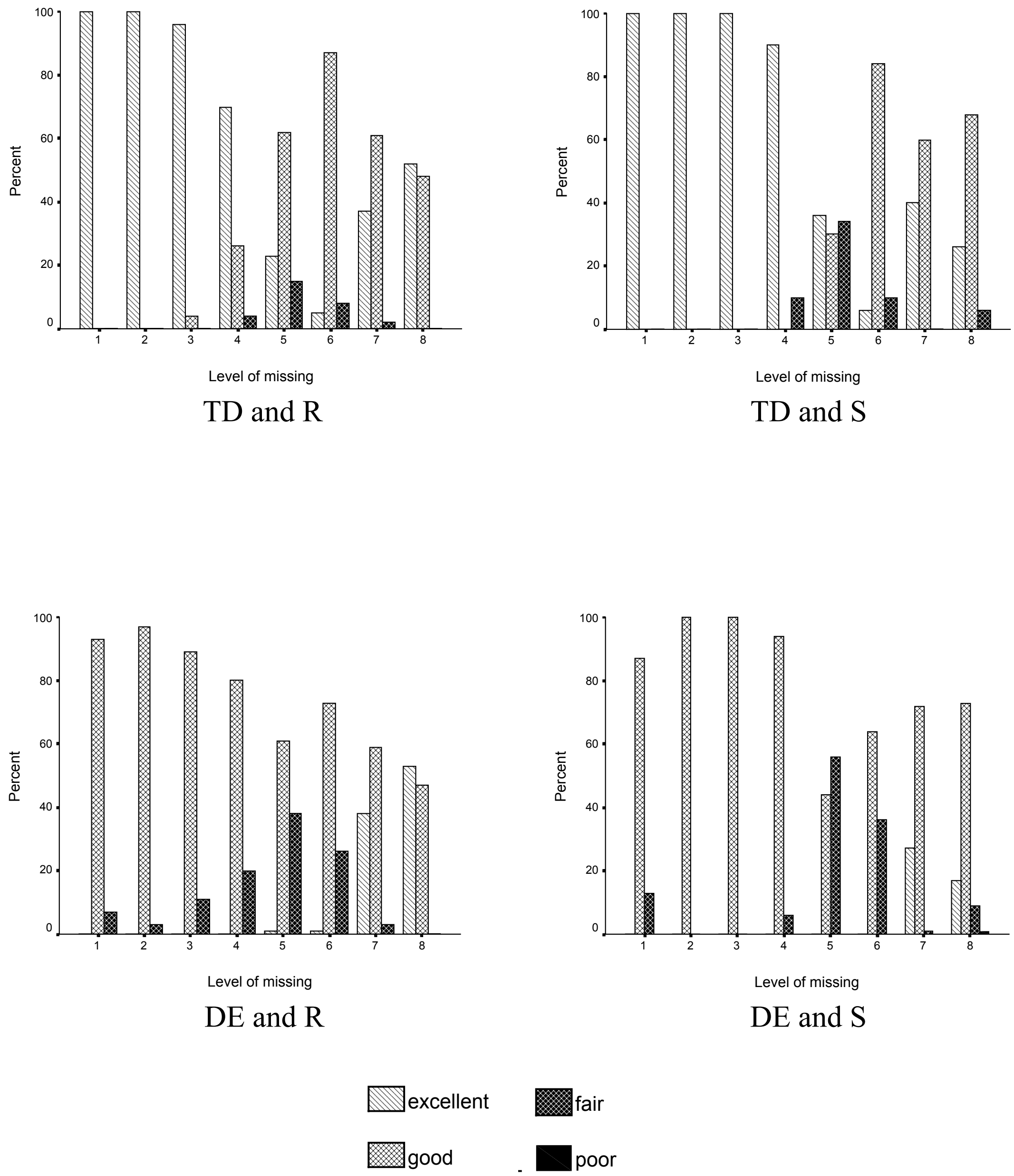
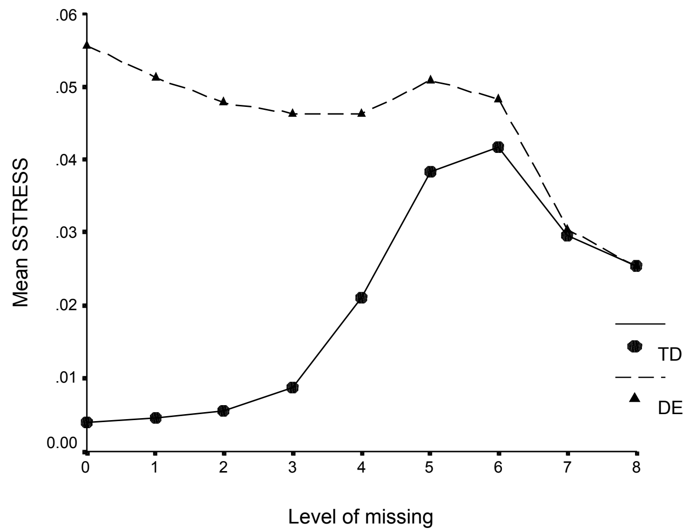
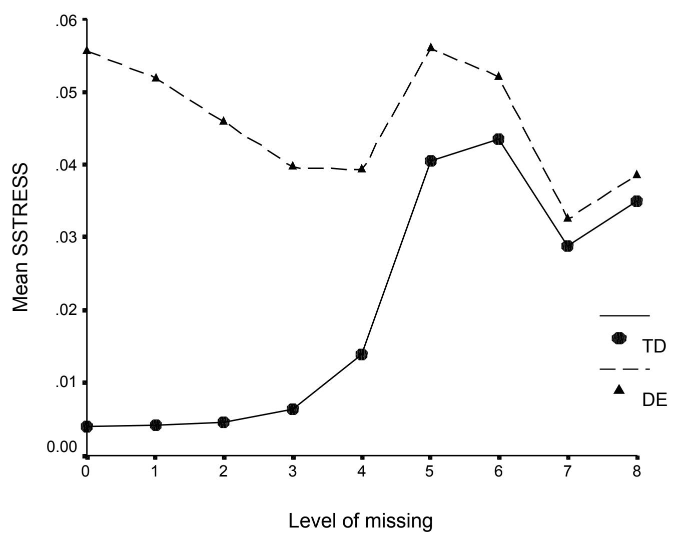


Figure 12. Number of excellent to poor cases of STRESS on each level of missing.

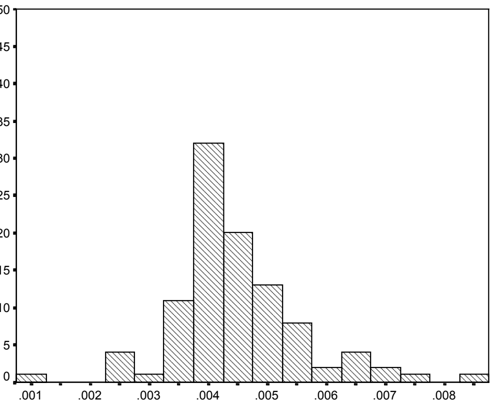


Random selection

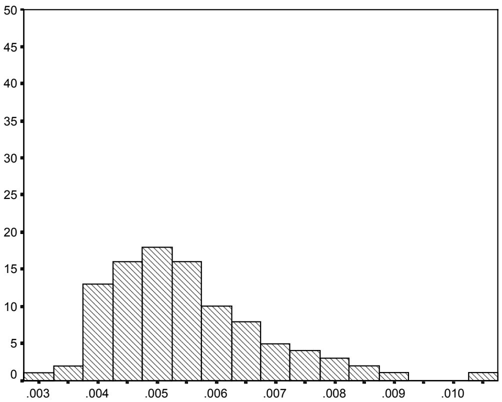


Systematic selection

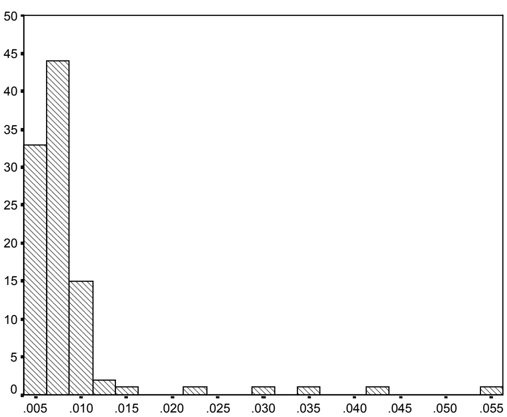
Figure 13. Mean STRESS against level of missing.



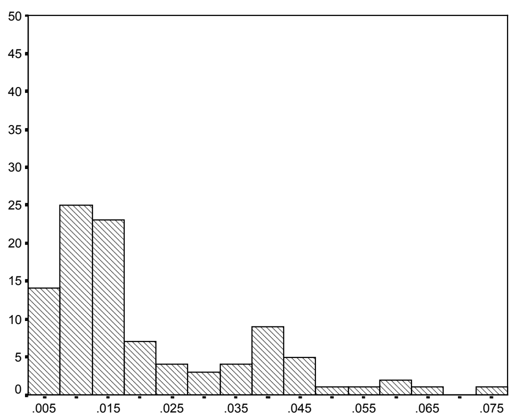
STRESS with TD, R, and L1



STRESS with TD, R, and L2

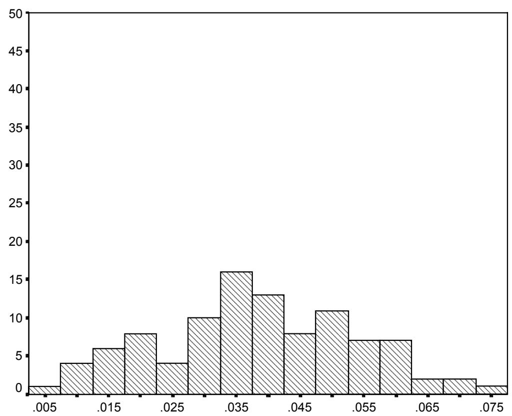


STRESS with TD,R, and L3

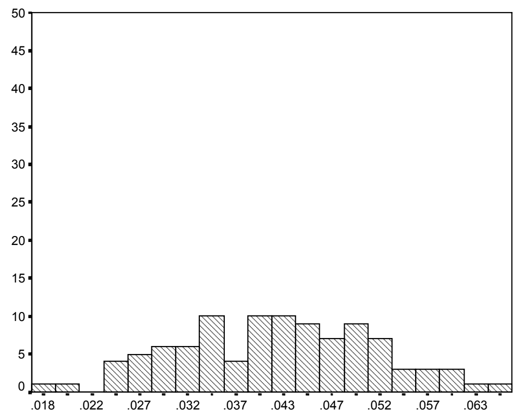


STRESS with TD, R, and L4

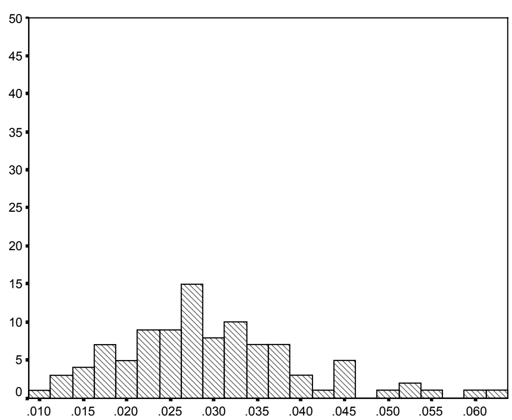
<u>Figure 14</u>. Histograms of Stress values from 100 simulations from true data (TD) and random missing (R) under different percent missing.



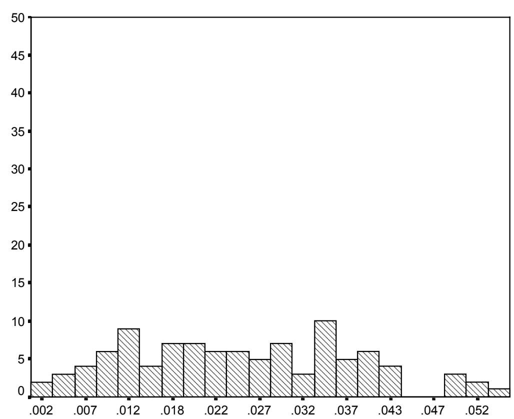
STRESS with TD, R, and L5



STRESS with TD, R, and L6

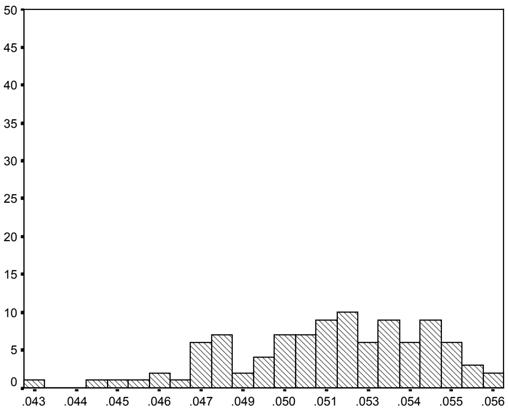


STRESS with TD, R, and L7

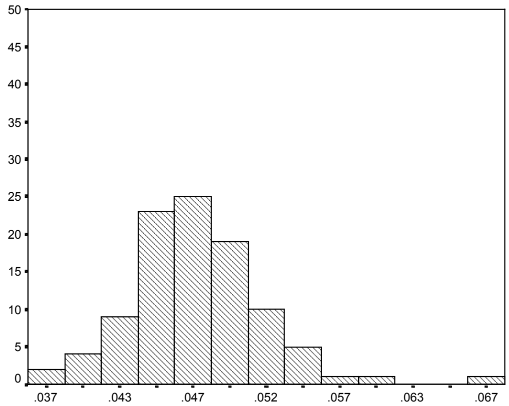


STRESS with TD, R, and L8

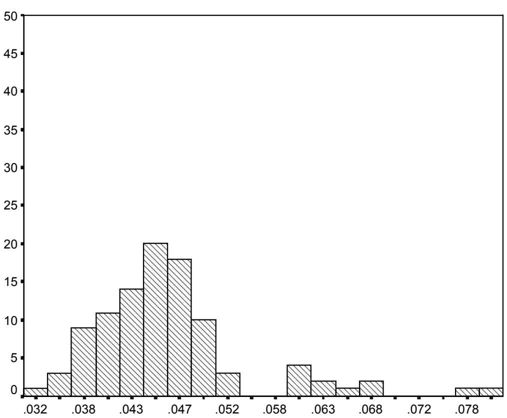
Figure 14 (continued).



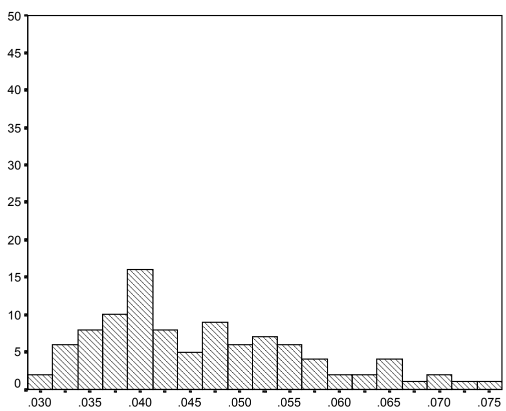
STRESS with DE, R, and L1



STRESS with DE, R, and L2

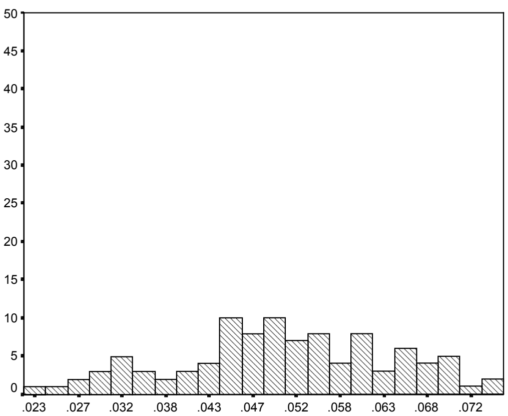


STRESS with DE, R, and L3

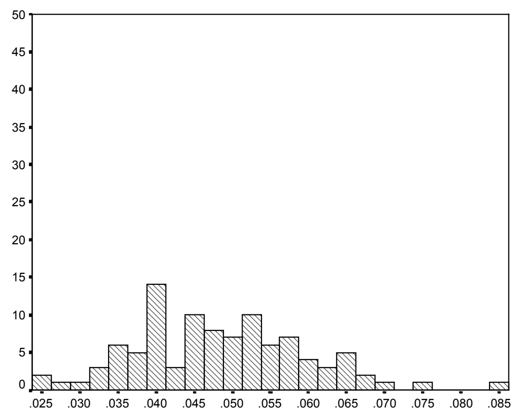


STRESS with DE, R, and L4

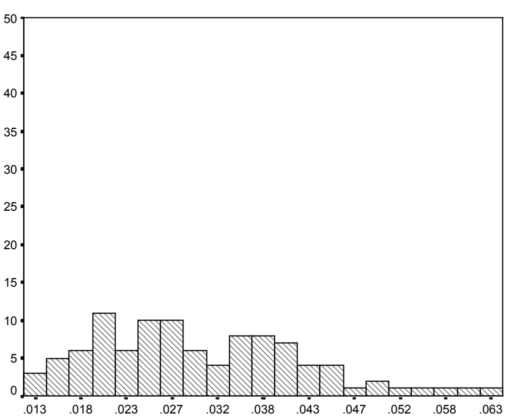
Figure 15. Histograms of Stress values from 100 simulations from data with error (DE) and random missing (R) under different percent missing.



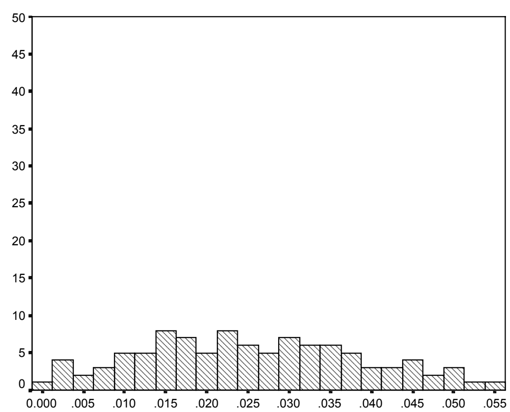
STRESS with DE, R, and L5



STRESS with DE, R, and L6

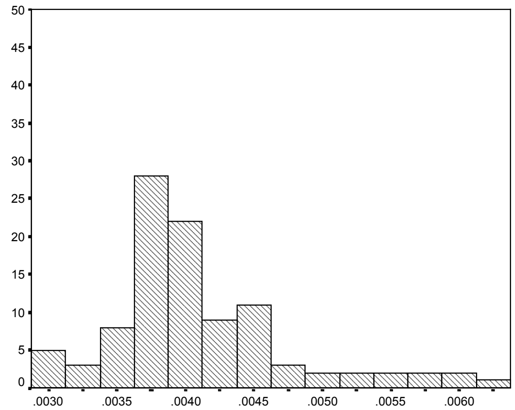


STRESS with DE, R, and L7

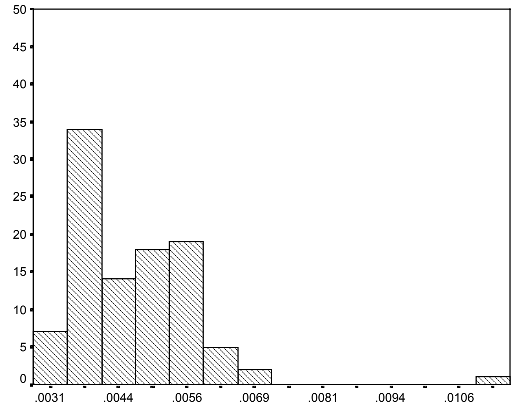


STRESS with DE, R, and L8

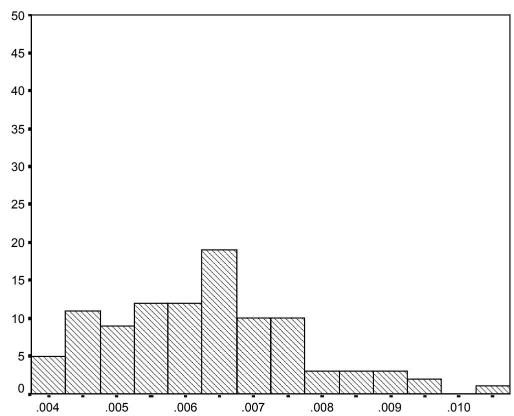
Figure 15 (continued).



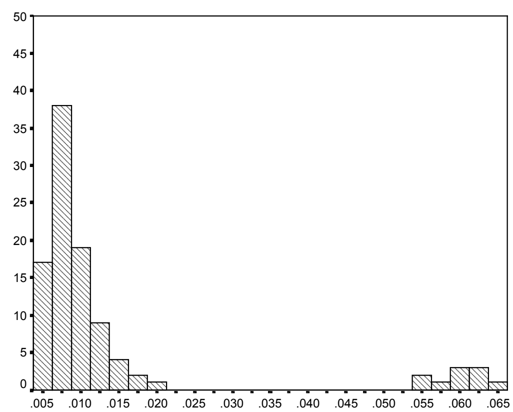
STRESS with TD, S, and L1



STRESS with TD, S, and L2

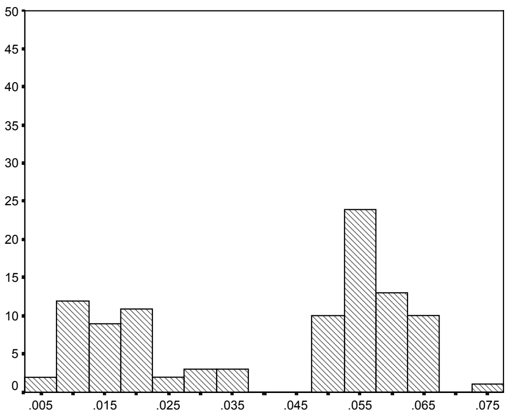


STRESS with TD, S, and L3

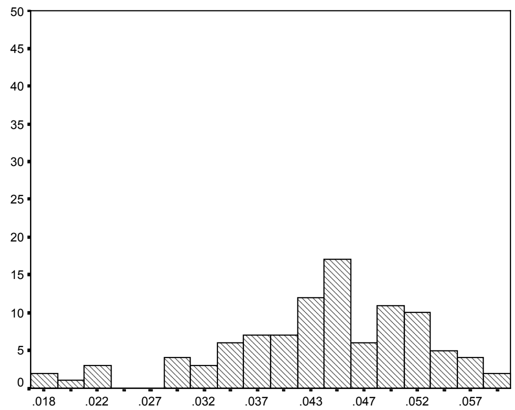


STRESS with TD, S, and L4

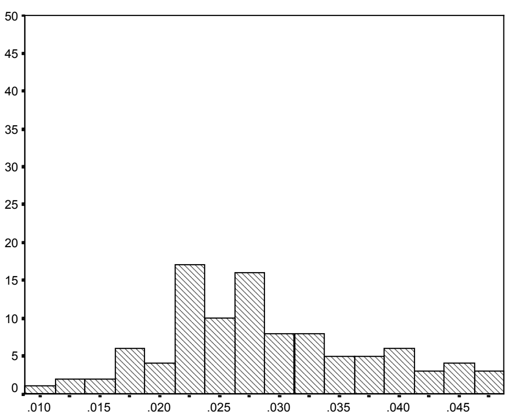
Figure 16. Histograms of Stress values from 100 simulations from true dissimilarities (TD) and systematic missing (S) for different number of missing.



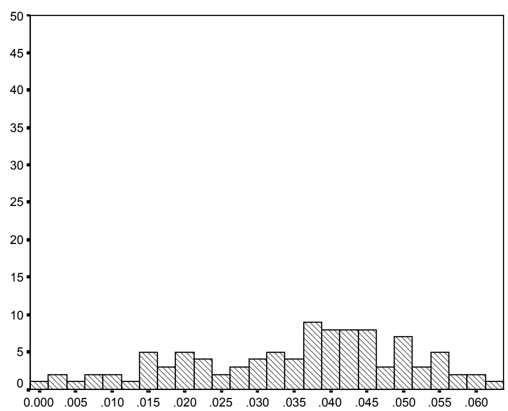
STRESS with TD, S, and L5



STRESS with TD, S, and L6

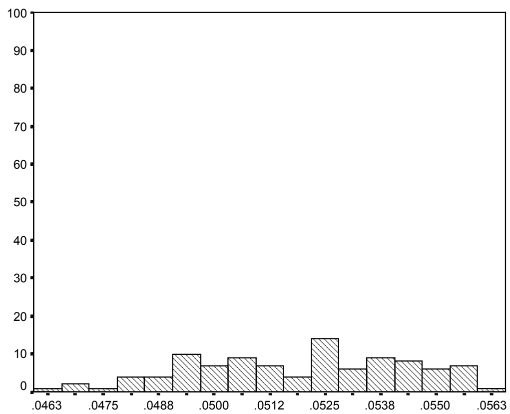


STRESS with TD, S, and L7

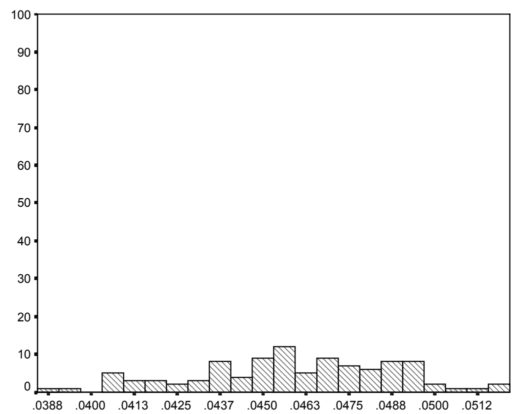


STRESS with TD, S, and L8

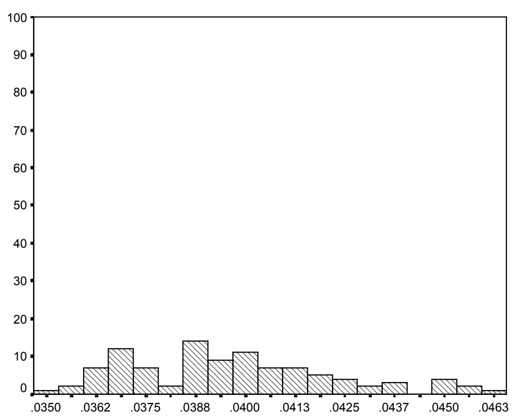
Figure 16 (continued)



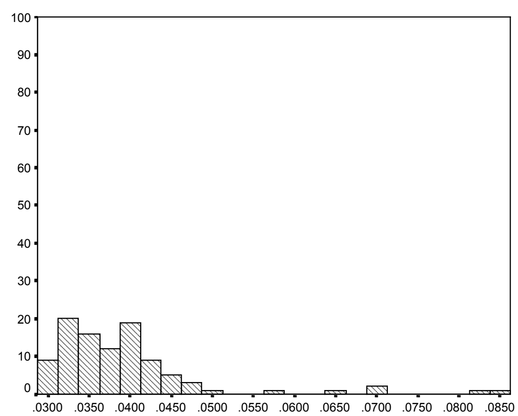
STRESS with DE, S, and L1



STRESS with DE, S, and L2

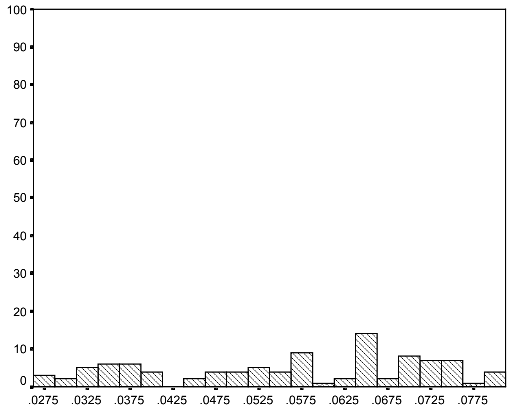


STRESS with DE, S, and L3

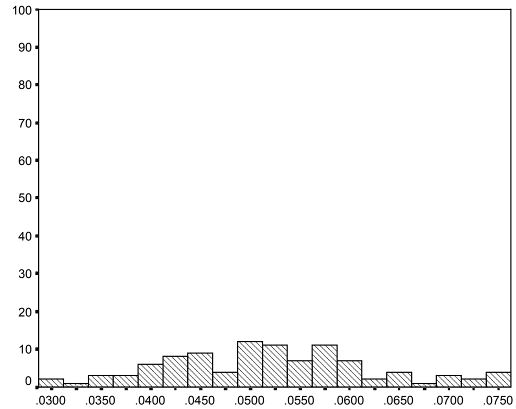


STRESS with DE, S, and L4

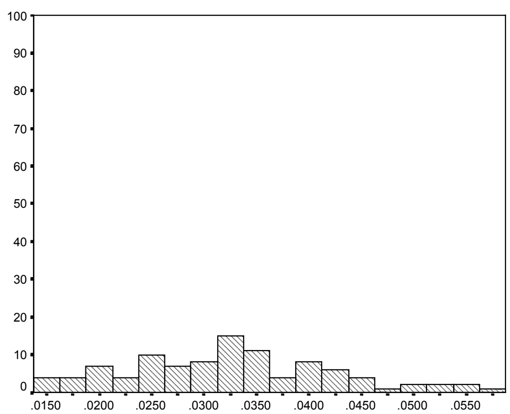
Figure 17. Histograms of STRESS values for data plus error (DE) and systematic missing (S)



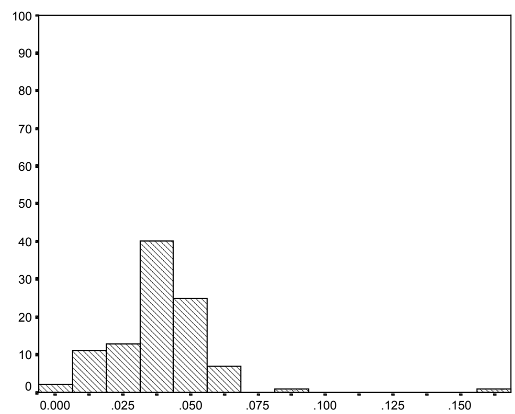
STRESS with DE, S, and L5



STRESS with DE, S, and L6

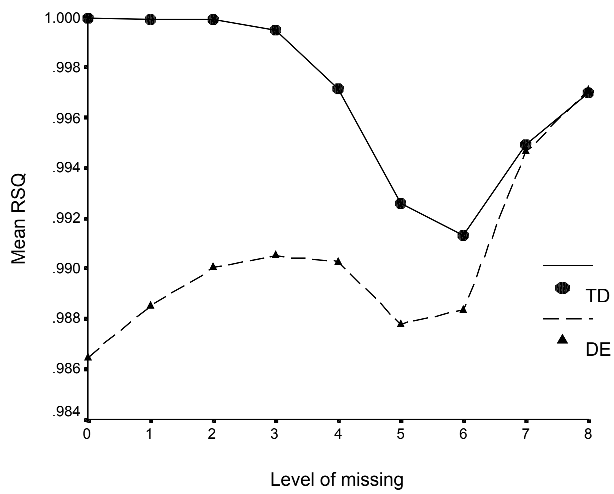


STRESS with DE, S, and L7

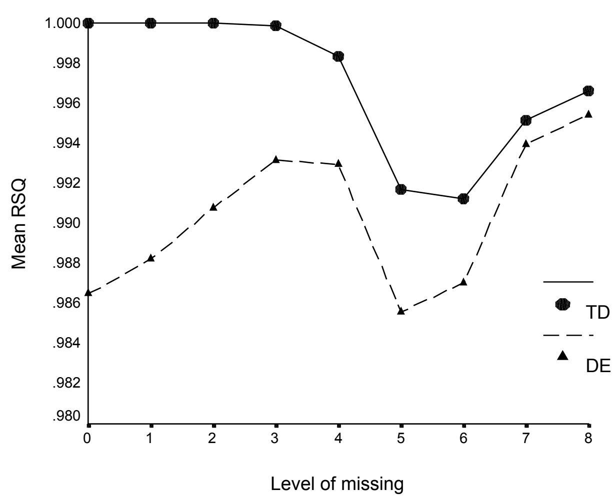


STRESS with DE, S, and L8

Figure 17 (continued).

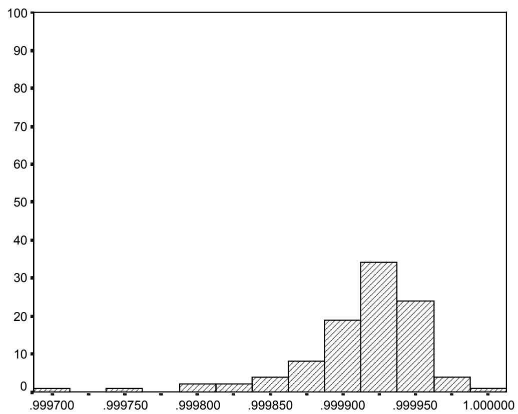


Random selection

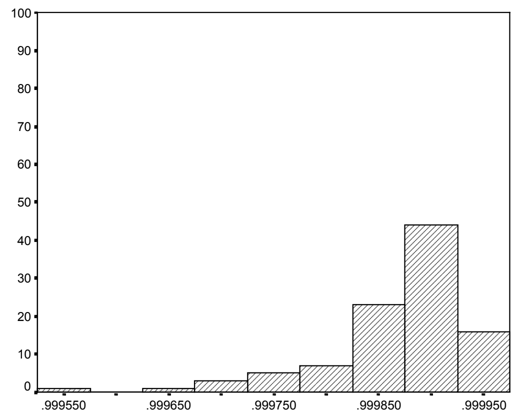


Systematic Selection

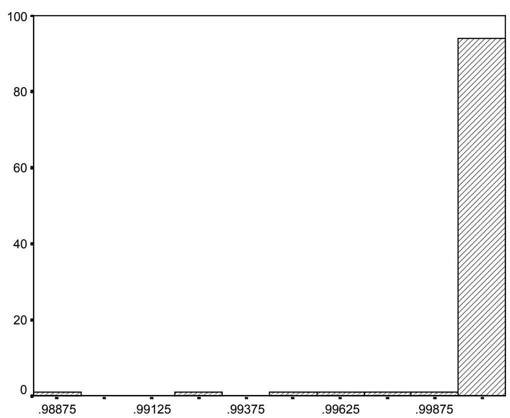
Figure 18. Mean RSQ Against Level of Missing



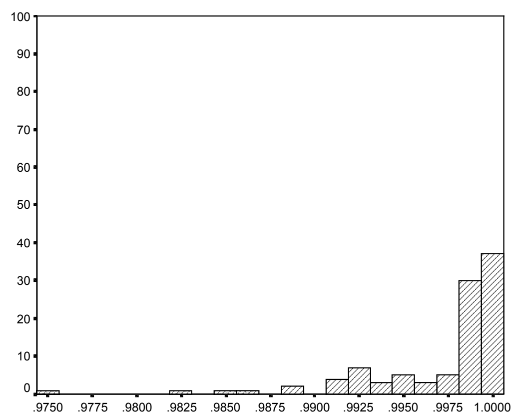
RSQ with TD, R, and L1



RSQ with TD, R, and L2

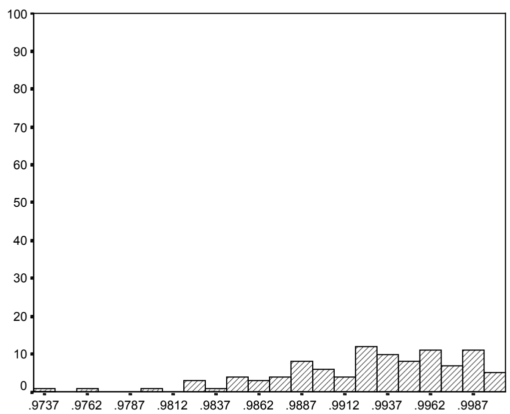


RSQ with TD, R, and L3

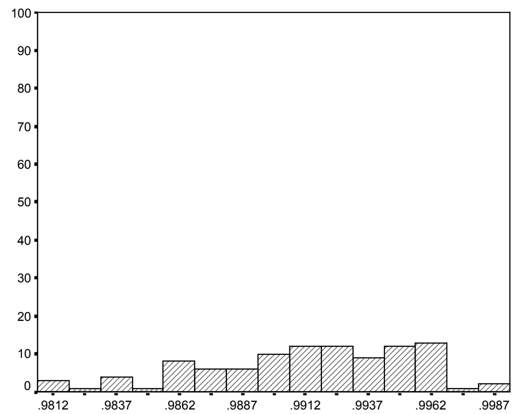


RSQ with TD, R, and L4

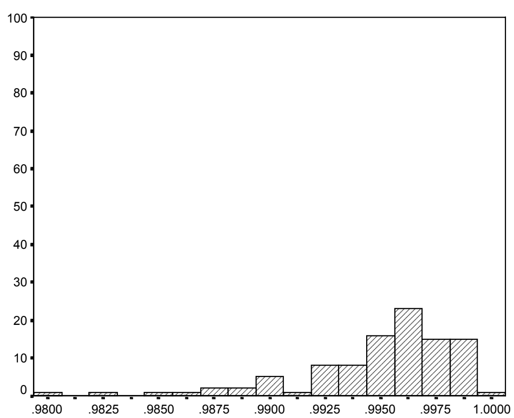
Figure 19. Histograms of SRQ values from 100 simulations from true data (TD) and random missing (R) under different percent missing.



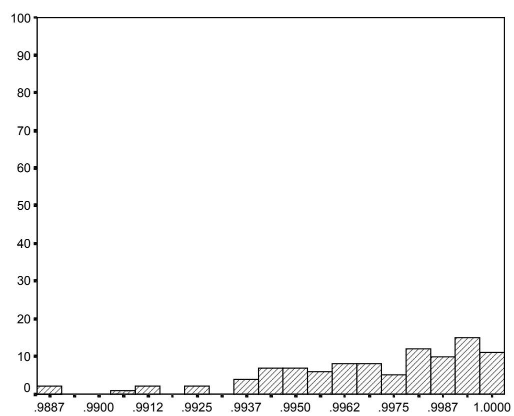
RSQ with TD, R, and L5



RSQ with TD, R, and L6

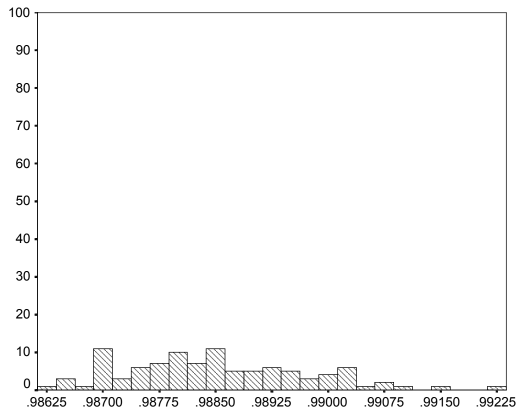


RSQ with TD, R, and L7

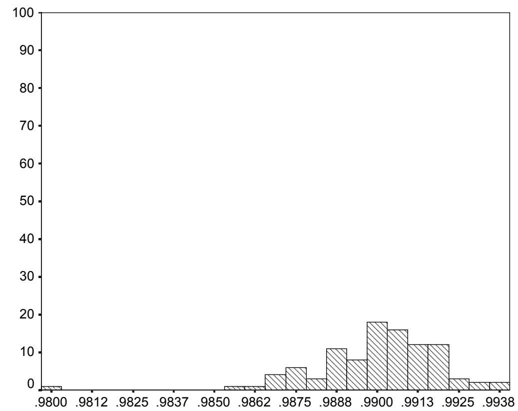


RSQ with TD, R, and L8

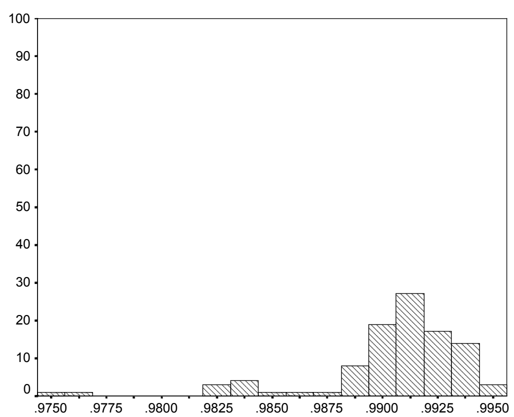
Figure 19 (continued).



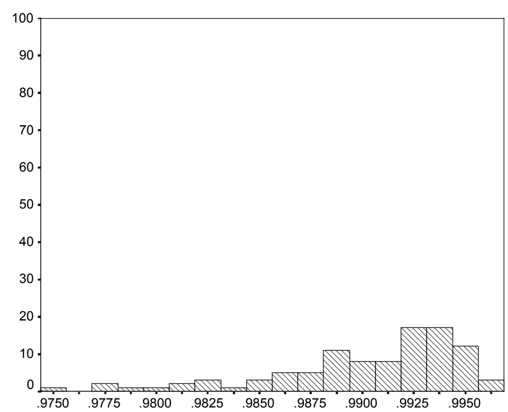
RSQ with DE, R, and L1



RSQ with DE, R, and L2

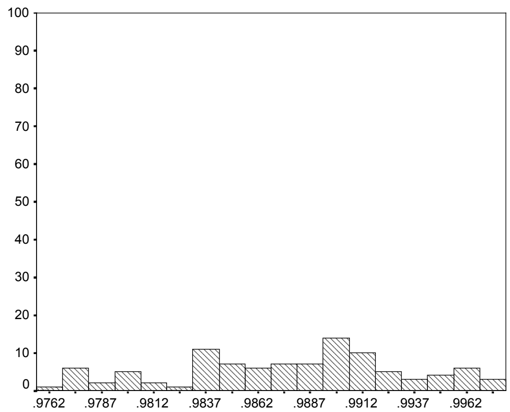


RSQ with DE, R, and L3

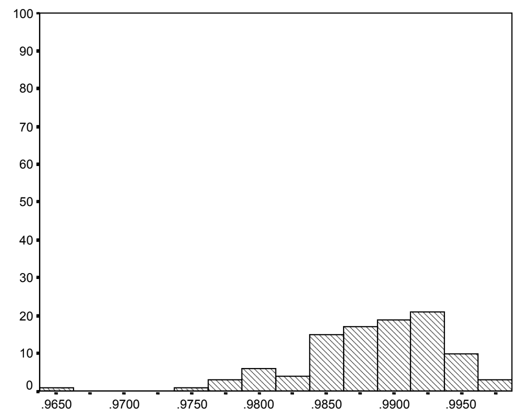


RSQ with DE, R, and L4

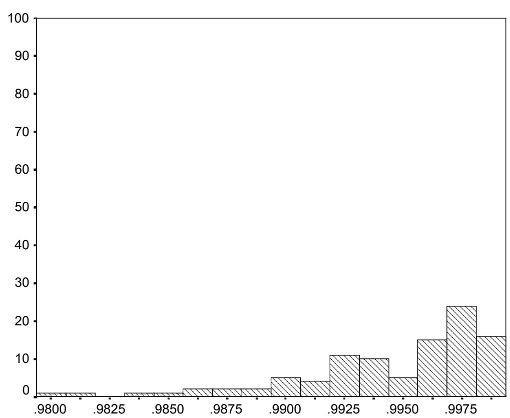
Figure 20. Histograms of RSQ values from 100 simulations from dissimilarities plus error (DE) and random selection (R) under different percent missing.



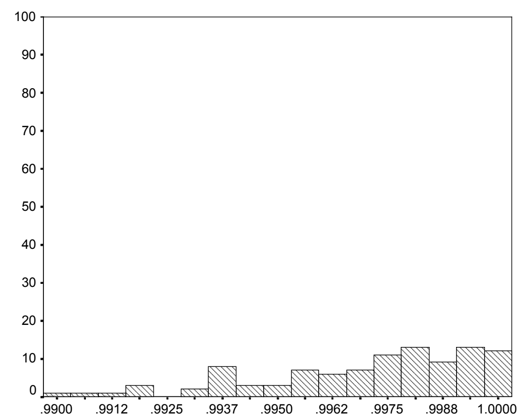
RSQ with DE, R, and L5



RSQ with DE, R, and L6

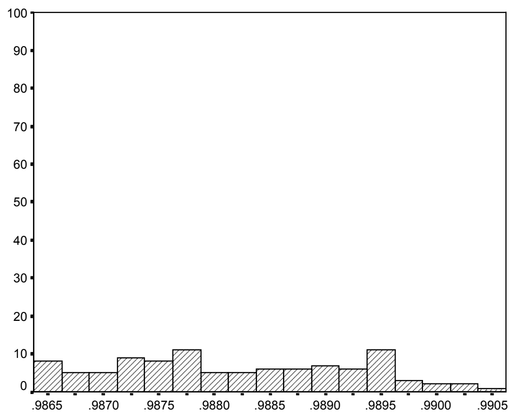


RSQ with DE, R, and L7

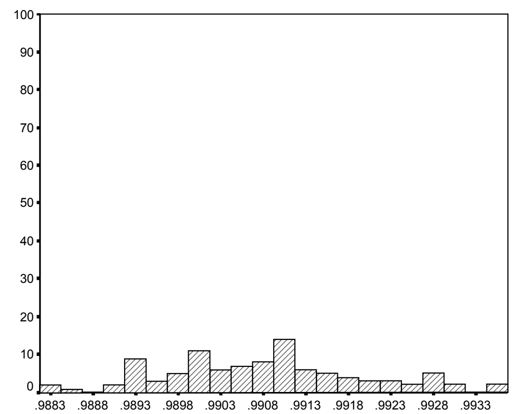


RSQ with DE, R, and L8

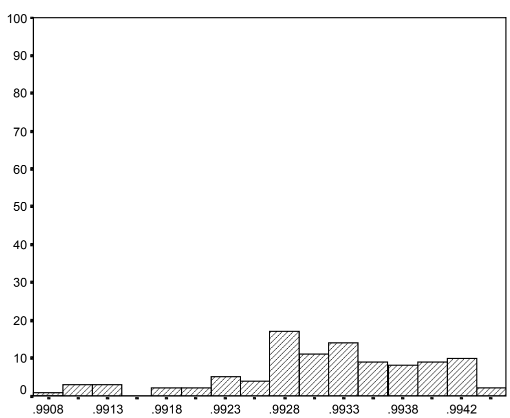
<u>Figure 20</u> (continued).



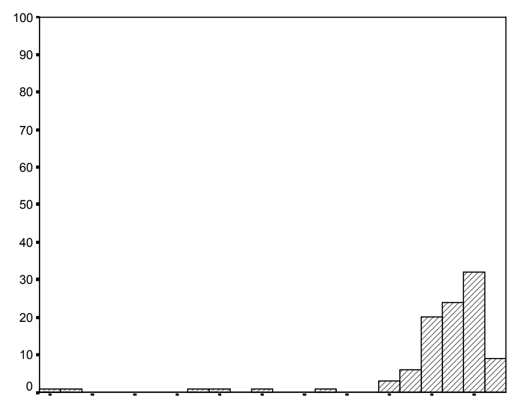
RSQ with DE and S for L1



RSQ with DE and S for L2

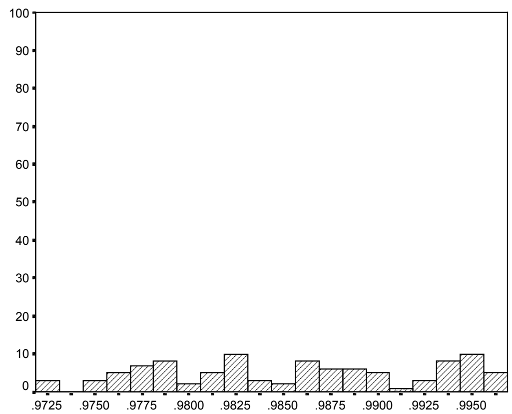


RSQ with DE and S for L3

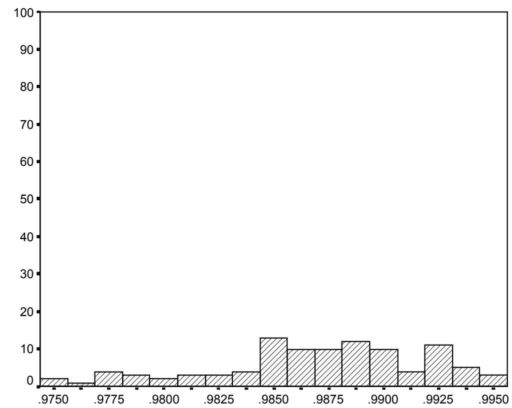


RSQ with DE and S for L4

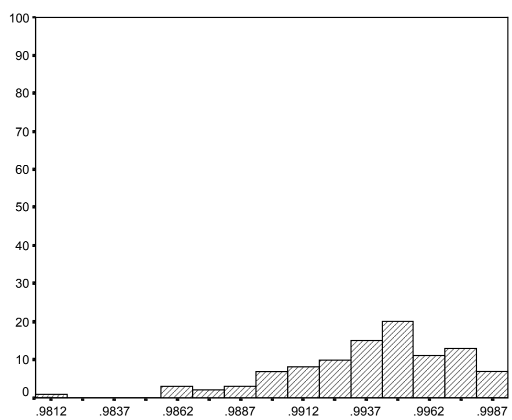
Figure 21. Histogram of RSQ values for 100 replications of eight levels of missing for Dissimilarities with added error and Systematic selection of missing.



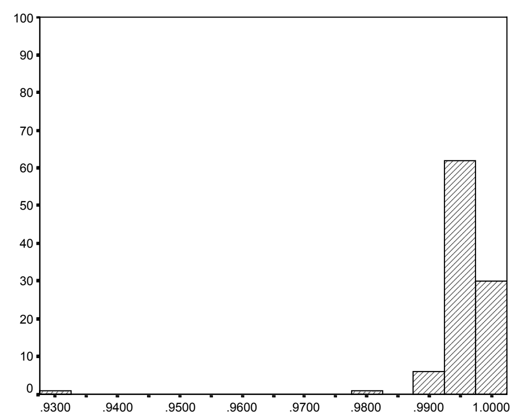
RSQ with DE and S for L5



RSQ with DE and S for L6



RSQ with DE and S for L7



RSQ with DE and S for L8

Figure 21 (continued)

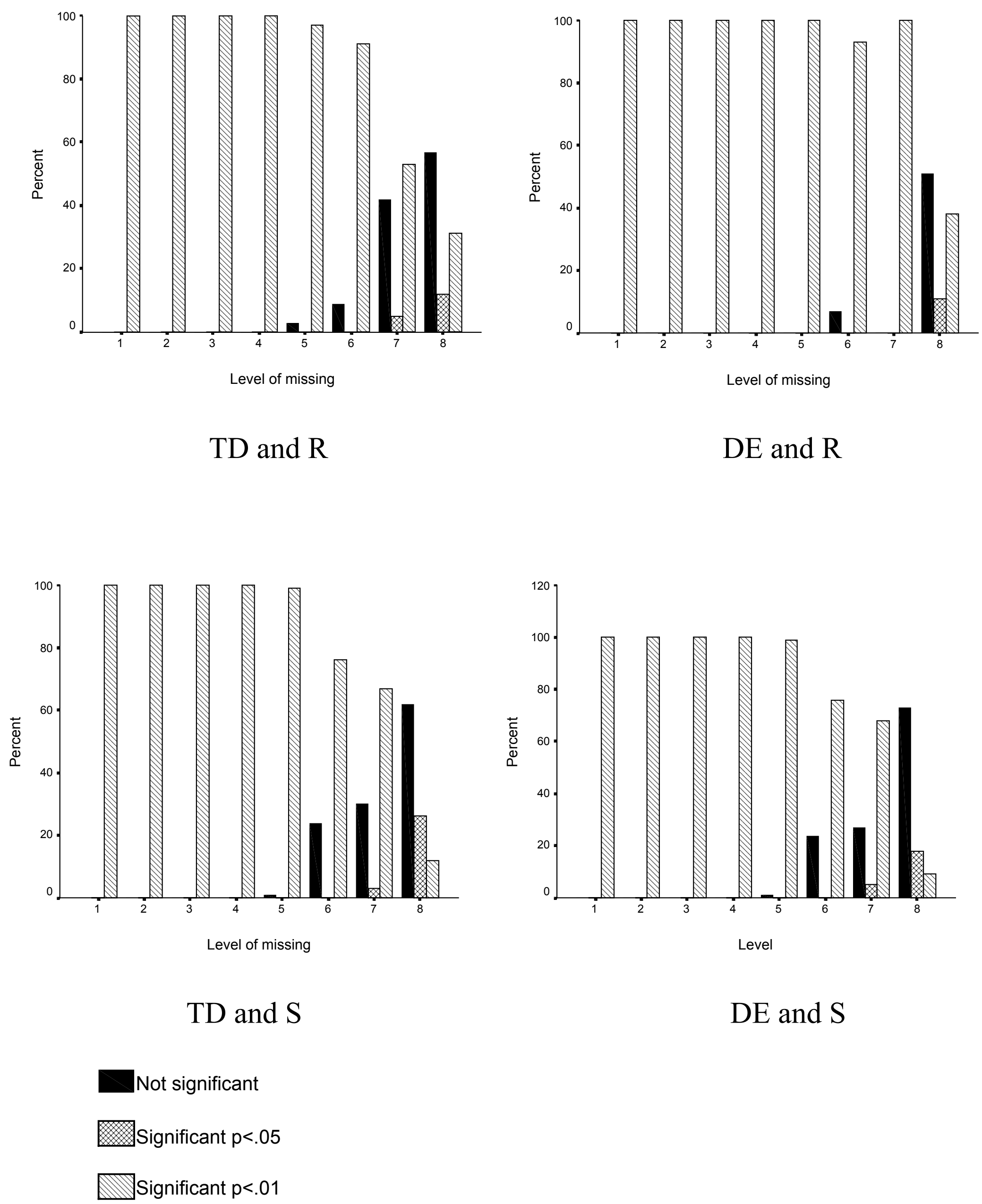
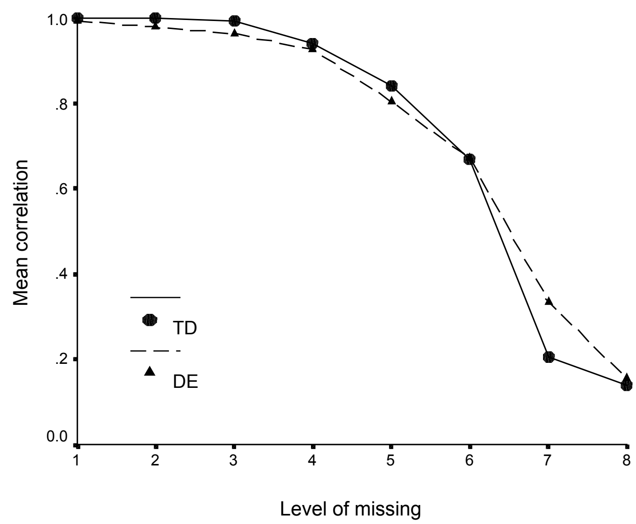
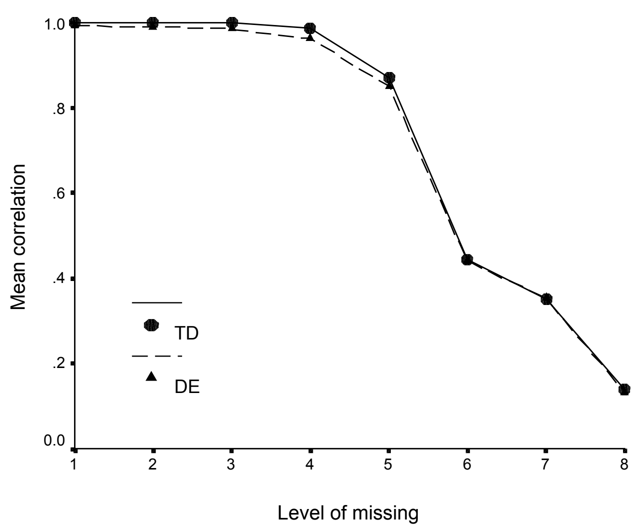


Figure 22. Percent of significant correlations between true and recovered coordinates for 100 replications.

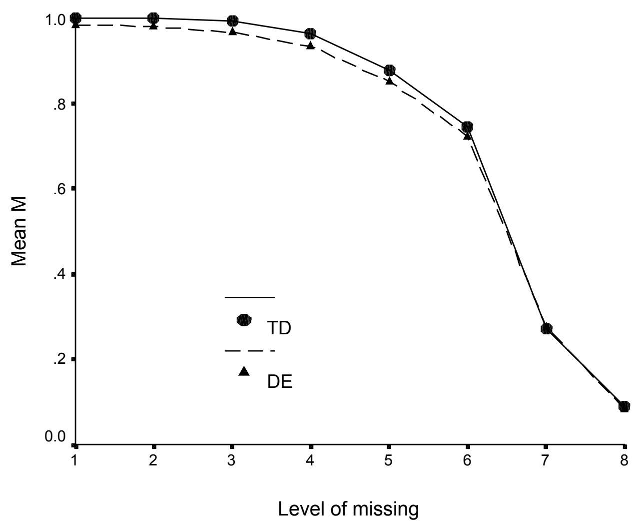


Random Selection

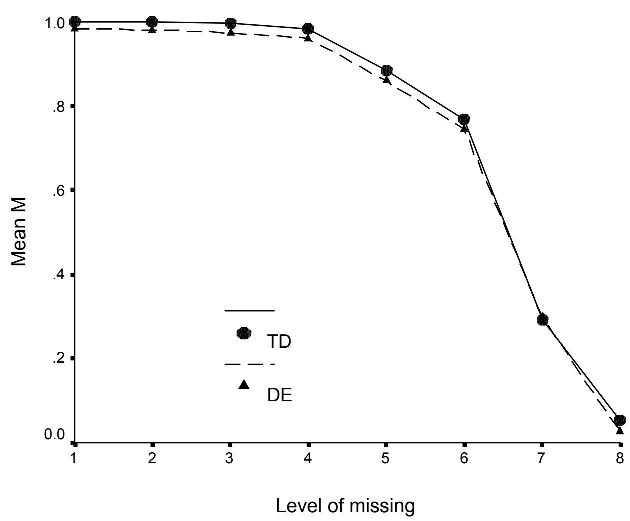


Systematic Selection

<u>Figure 23</u>. Mean Correlations Between True and Recovered Coordinates.



Random Selection



Systematic Selection

<u>Figure 24</u>. Mean M values against level of missing

APPENDIXES

Appendix A

List of Cities

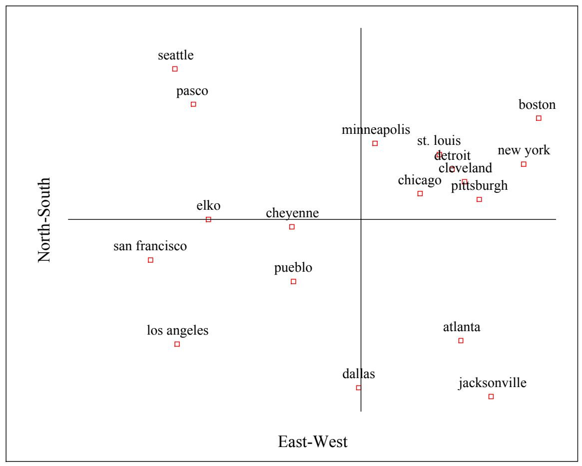
List of cities used as stimuli to generate dissimilarities for the simulation

1. Atlanta
2. Boston
3. Cheyenne
4. Chicago
5. Cleveland
6. Dallas
7. Detroit
8. Elko
9. Jacksonville
10. Los Angeles
11. Minneapolis
12. New York
13. Pasco
14. Pittsburgh
15. Pueblo
16. San Francisco
17. Seattle
18. St Louis

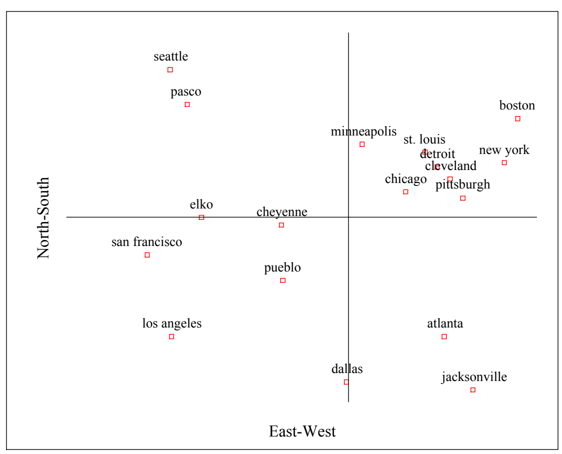
Appendix B

Plots of Stimulus Configurations

Comparison of stimulus configuration from the same proximity data matrix using metric and non metric ALSCAL



Output from SPSS with distances treated as ratio data



Output from SPSS with distances treated as ordinal data

Appendix C

Ross Matrix

Ross Matrix for 18 Stimuli

Column Row	I	II	III	IV	V	VI
1	1-2	2-3	1-3	3-4	1-4	4-5
2	~~3-n~~	~~n-4~~	4-2	2-5	5-3	3-6
3	4-(n-1)	(n-1)-5	5-n	n-6	6-2	2-7
4	5-(n-2)	(n-2)-6	6-(n-1)	(n-1)-7	7-n	n-8
5	6-(n-3)	(n-3)-7	7-(n-2)	(n-2)-8	8-(n-1)	(n-1)-9
6	7-(n-4)	(n-4)-8	8-(n-3)	(n-3)-9	9-(n-2)	(n-2)-10
7	8-(n-5)	(n-5)-9	9-(n--4)	(n-4)-10	10-(n-3)	(n-3)-11
8	9-(n-6)	(n-6)-10	10-(n-5)	(n-5)-11	11-(n-4)	(n-4)-12
9	10-(n-7)	(n-7)-11	11-(n-6)	(n-6)-12	12-(n-5)	(n-5)-13
10	11-(n-8)	(n-8)-12	12-(n-7)	(n-7)-13	13-(n-6)	(n-6)-14

<u>Ross Matrix for 18 Stimuli (continued)</u>

Column Row	VII	VIII	IX	X	XI	XII
1	1-5	5-6	1-6	6-7	1-7	7-8
2	6-4	4-7	7-5	5-8	8-6	6-9
3	7-3	3-8	8-4	4-9	9-5	5-10
4	8-2	2-9	9-3	3-10	10-4	4-11
5	9-n	n-10	10-2	2-11	11-3	3-12
6	10-(n-1)	(n-1)-11	11-n	n-12	12-2	2-13
7	11-(n-2)	(n-2)-12	12- (n-1)	(n-1)-13	13-n	n-14
8	12-(n-3)	(n-3)-13	13-(n-2)	(n-2)-14	14-(n-1)	(n-1)-15
9	13-n-4)	(n-4)-14	14-(n-3)	(n-3)-15	15-(n-2)	(n-2)-16
10	14-(n-5)	(n-5)-15	15-(n-4)	(n-4)-16	16-(n-3)	(n-3)-17

Ross Matrix for 18 Stimuli (continued)

Column Row	XIII	XIV	XV	XVI	XVII	XVIII
1	1-8	8-9	1-9	9-10	1-10	10-11
2	9-7	7-10	10-8	8-11	11-9	9-12
3	10-6	6-11	11-7	7-12	12-8	8-13
4	11-5	5-12	12-6	6-13	13-7	7-14
5	12-4	4-13	13-5	5-14	14-6	6-15
6	13-3	3-14	14-4	4-15	15-5	5-16
7	14-2	2-15	15-3	3-16	16-4	4-17
8	15-n	n-16	16-2	2-17	17-3	3-18
9	16-(n-1)	(n-1)-17	17-n	n-18	18-2	~~2-19~~
10	17-(n-2)	(n-2)-18	18-(n-1)	(n-1)-19	19-n	~~n-20~~

List of ordered pairs of 18 cities

1.	1 – 2	18.	1 - 3	35.	1 – 4
2.	4 – 18	19.	4 – 2	36.	5 – 3
3.	5 – 17	20.	6 – 18	37.	6 – 2
4.	6 – 16	21.	7 – 17	38.	8 – 18
5.	7 – 15	22.	8 – 16	39.	9 - 17
6.	8 – 14	23.	9 – 15	40.	10 – 16
7.	9 – 13	24.	10 – 14	41.	11 – 15
8.	10 – 12	25.	11 – 13	42.	12 – 14
9.	11 - 1	26.	12 - 1	43.	13 - 1
10.	2 – 3	27.	3 – 4	44.	4 – 5
11.	18 – 5	28.	2 – 5	45.	3 – 6
12.	17 – 6	29.	18 – 7	46.	2 – 7
13.	16 – 7	30.	17 – 8	47.	18 – 9
14.	15 – 8	31.	16 – 9	48.	17 – 10
15.	14 – 9	32.	15 – 10	49.	16 – 11
16.	13 – 10	33.	14 – 11	50.	15 – 12
17.	12-11	34.	13 – 12	51.	14 – 13

List of ordered pairs of 18 cities (continued)

52.	1 – 5	70.	7 – 5	88.	9 – 5
53.	6 – 4	71.	8 – 4	89.	10 – 4
54.	7 – 3	72.	9 – 3	90.	11 – 3
55.	8 – 2	73.	10 – 2	91.	12 – 2
56.	10 – 18	74.	12 – 18	92.	14 – 18
57.	11 – 17	75.	13 - 17	93.	15 - 17
58.	12 – 16	76.	14 – 16	94.	16 - 1
59.	13 - 15	77.	15 – 1	95.	7 – 8
60.	14 – 1	78.	6 – 7	96.	6 – 9
61.	5 – 6	79.	5 – 8	97.	5 – 10
62.	4 – 7	80.	4 - 9	98.	4 – 11
63.	3 – 8	81.	3 – 10	99.	3 – 12
64.	2 – 9	82.	2 – 11	100.	2 – 13
65.	18 – 11	83.	18 – 13	101.	18 – 15
66.	17 – 12	84.	17 - 14	102.	17 – 16
67.	16 – 13	85.	16 – 15	103.	1 – 8
68.	15 – 14	86.	1 – 7	104.	9 – 7
69.	1 – 6	87.	8 – 6	105.	10 – 6

List of ordered pairs (continued)

106. 11 – 5
107. 12 – 4
108. 13 – 3
109. 14 – 2
110. 16 – 18
111. 17 - 1
112. 8 – 9
113. 7 – 10
114. 6 – 11
115. 5 – 12
116. 4 – 13
117. 3 – 14
118. 2 - 15
119. 18 – 17
120. 1 – 9
121. 10 – 8
122. 11 – 7
123. 12 – 6
124. 13 – 5
125. 14 – 4
126. 15 – 3
127. 16 – 2
128. 18 - 1
129. 9 – 10
130. 8 – 11
131. 7 – 12
132. 6 – 13
133. 5 – 14
134. 4 – 15
135. 3 – 16
136. 2 – 17
137. 1 - 10
138. 11 – 9
139. 12 – 8
140. 13 – 7
141. 14 – 6
142. 15 – 5
143. 16 – 4
144. 17 –3
145. 18 – 2
146. 10 – 11
147. 9 – 12
148. 8 – 13
149. 7 – 14
150. 6 – 15
151. 5 – 16
152. 4 – 17
153. 3 – 18

Appendix D

SPSS Syntax

Sample SPSS Syntax to generate random stimuli

```
GET
FILE='C:\My Documents\DISSERTATION\SPSS\SQUARE MATRIX.sav'.
EXECUTE.

DEFINE loopme(arg1 = !TOKENS(1)/arg2 = !TOKENS(1)/arg3 = !TOKENS(1))

!DO !i= !arg1 !to !arg2

do repeat       x=c1 to c18 /
                y = r1 to r18.
. compute y = x*RV.BERNOULLI(!arg3).
end repeat.
execute.

ALSCAL
        VARIABLES=R1 R2 R3 R4 R5 R6 R7 R8 R9 R10 R11 R12 R13 R14
R15 R16 R17 R18
        /SHAPE=SYMETRIC
        /LEVEL=ORDINAL (UNTIE)
        /CONDITION=MATRIX
        /MODEL=EUCLID
        /CRITERIA=CONVERGE(.001) STRESSMIN(.005) ITER(30)
        CUTOFF(0.1) DIMENS(2,2).

!DOEND
!ENDDEFINE.

loopme arg1 = 1 arg2 = 100 arg3 = .50.
```

Sample syntax for systematic missing

```
DEFINE loopme(arg1 = !TOKENS(1) / arg2 = !TOKENS(1) / arg3=!TOKEN(1)
/ arg4 = !TOKENS(1) / arg5 = !TOKENS(1) / arg6 = !TOKENS(1)).

!DO !i= !arg1 !to !arg2.

!let !n=!LENGTH(!CONCAT(!BLANK(!arg3),!BLANK(!arg4))).
!let !m=!LENGTH(!CONCAT(!BLANK(!arg5),!BLANK(!arg6))).


RECODE r1 r2 r3 r4 r5 r6 r7 r8 r9 r10 r11 r12 r13 r14 r15 r16 r17 r18  (!arg3 thru
!n =1) (!arg5 thru !m = 1) (ELSE = 0)   INTO  mr1  mr2 mr3 mr4 mr5 mr6 mr7
mr8 mr9 mr10 mr11 mr12 mr13 mr14 mr15 mr16 mr17 mr18.

EXECUTE .

 do repeat       x=c1 to c18 /
                 y = mr1 to mr18/
                 z = cm1 to cm18.

. compute z = x*y.
end repeat.
execute.

ALSCAL
  VARIABLES= cm1 cm2 cm3 cm4 cm5 cm6 cm7 cm8 cm9 cm10 cm11 cm12
cm13 cm14 cm15 cm16 cm17 cm18
  /SHAPE=SYMMETRIC
  /LEVEL=ORDINAL (UNTIE)
  /CONDITION=MATRIX
  /MODEL=EUCLID
  /CRITERIA=CONVERGE(.001) STRESSMIN(.005) ITER(30) CUTOFF(0.1)
DIMENS(2,2) .

!DOEND
!ENDDEFINE.

loopme arg1 = 1 arg2 =1  arg3=102 arg4= 90
                         arg5 = 1 arg6 = 37.
EXECUTE .
```

Appendix E

SPSS ALSCAL Model

ALSCAL Model

Conditionality	Matrix
Data Cutoff at	.000000

Model Options-

Model	Euclid
Maximum Dimensionality	2
Minimum Dimensionality	2
Negative Weights	Not Permitted

Output Options-

Job Option Header	Printed
Data Matrices	Printed
Configurations and Transformations .	Plotted
Output Dataset	Not Created
Initial Stimulus Coordinates . . .	Computed

Algorithmic Options-

Maximum Iterations	30
Convergence Criterion	.00100
Minimum S-stress	.00500
Missing Data Estimated by	Ulbounds
Tiestore	153

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