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INTEGRATED NETWORK-BASED MODELS FOR EVALUATING AND OPTIMIZING THE IMPACT OF ELECTRIC VEHICLES ON THE TRANSPORTATION SYSTEM

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INTEGRATED NETWORK-BASED MODELS FOR EVALUATING AND OPTIMIZING THE IMPACT OF ELECTRIC VEHICLES ON THE TRANSPORTATION SYSTEM

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Dedication

To my parents

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Integrated Network-Based Models for Evaluating and Optimizing the Impact of Electric Vehicles on the Transportation System

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The University of Texas at Austin, 2012

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The adoption of plug-in electric vehicles (PEV) requires research for models and algorithms tracing the vehicle assignment incorporating PEVs in the transportation network so that the traffic pattern can be more precisely and accurately predicted. To attain this goal, this dissertation is concerned with developing new formulations for modeling travelling behavior of electric vehicle drivers in a mixed flow traffic network environment. Much of the work in this dissertation is motivated by the special features of PEVs (such as range limitation, requirement of long electricity-recharging time, etc.), and the lack of tools of understanding PEV drivers traveling behavior and learning the impacts of charging infrastructure supply and policy on the network traffic pattern.

The essential issues addressed in this dissertation are: (1) modeling the spatial choice behavior of electric vehicle drivers and analyzing the impacts from electricity-charging speed and price; (2) modeling the temporal and spatial choices behavior of electric vehicle drivers and analyzing the impacts of electric vehicle range and penetration rate; (3) and designing the optimal charging infrastructure investments and policy in the perspective of revenue management. Stochastic traffic assignment that can take into account for charging cost and charging time is first examined. Further, a quasi-dynamic stochastic user equilibrium model for combined choices of departure time,

duration of stay and route, which integrates the nested-Logit discrete choice model, is formulated as a variational inequality problem. An extension from this equilibrium model is the network design model to determine an optimal charging infrastructure capacity and pricing. The objective is to maximize revenue subject to equilibrium constraints that explicitly consider the electric vehicle drivers' combined choices behavior.

The proposed models and algorithms are tested on small to middle size transportation networks. Extensive numerical experiments are conducted to assess the performance of the models. The research results contain the author's initiative insights of network equilibrium models accounting for PEVs impacted by different scenarios of charging infrastructure supply, electric vehicles characteristics and penetration rates. The analytical tools developed in this dissertation, and the resulting insights obtained should offer an important first step to areas of travel demand modeling and policy making incorporating PEVs.

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Chapter 1 Introduction and Background

1.1 Motivation

The world's dependence on oil has created numerous negative impacts on both the environment and economy either directly or indirectly. For example, transportation activities are a dominant factor for the carbon emissions. In addition, the high dependence on such non-renewable resource results in facing energy shortages in the near future. More seriously, in the foreseeable future, climate change will likely become more obvious and petroleum prices will continuously climb. These problems/issues have caused a growing interest in reducing the petroleum dependence and diversifying the energy consumption. Reforms to transportation sector can significantly reduce these negative impacts caused by excessive petroleum consumption.

The transportation sector accounts for 70% of U.S. petroleum consumption and 27% of U.S. greenhouse gas emissions. With motor vehicles consuming such a large quantity of petroleum, the requirement for alternative fuel vehicles is being more demanding. A variety of fuel alternatives are available and innovations to battery technology are making electric vehicles a viable option. Plug-in electric vehicles (PEVs), which include battery electric vehicles (BEVs) and plug-in hybrid vehicles (PHEVs), have received tremendous attention in recent energy policy discussions because they produce no exhaust or emissions and relieve our current heavy dependence on petroleum. BEVs use a battery to store the electrical energy that powers the motor and is recharged by plugging a cord into an electric power source.

1.1.1 Perspective of plug-in electric vehicles

The growth of the PEVs market and the PEVs traveler behavior will mostly be affected by the following two facts: the government support (e.g. incentives), and the charging infrastructures (e.g., availability, capacity, pricing, charging speed).

First, the U.S. federal government highlighted electricity as a promising alternative to gasoline for transportation sector in the future (Ashtiani et al. 2011) and proposed that PEVs capture a significant share (5%-15%) of the market over the next 15 to 20 years (Edward 2011). The federal government also established a goal of putting one million PEVs on the road by 2015. In addition, federal, state and local government implemented a series of policies and incentives to encourage the penetration of vehicle electrifications. The national academies report that there will be 13 to 40 million PEV out of 300 million total vehicles on the U.S. roads by 2030 under different scenarios (John 2010).

Second, with the expending ownership of the electric vehicles (EV), incentives and grants (such as electric vehicle charging equipment tax credit and electric vehicle battery and infrastructure tax exemptions) have become available for the installation of electric vehicle charging outlet in a house or housing unit in many states (e.g., Arizona, Georgia, Washington). The need for widely distributed publicly accessible power points is also growing, and the public charging stations used to power the electric vehicles have been installed in many states over the country (Figure 1-1). California has more than 500 charging stations installed, Texas and Florida both have 201-300 charging stations installed, and states such as Washington, Origen, Minnesota, Illinois, and Maryland also have more than 100 charging stations installed up until year 2012. It is expected that some projects will pave the way for deploying charging infrastructure across the country by 2013 (2012).

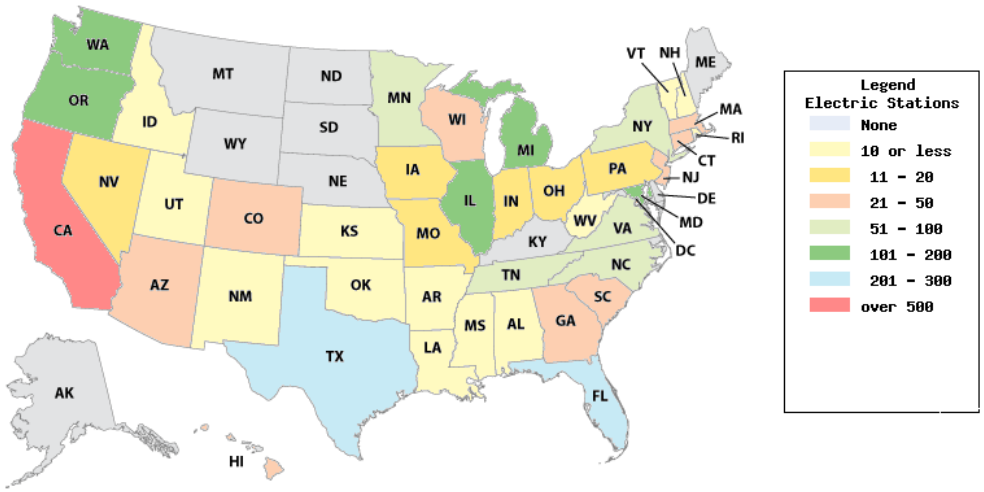

Figure 1-1: Density of Electric Charging Stations in the United States[1]

Two problems inhibit the installation of a PEV charging infrastructure. Firstly, companies who initiate and perform the installation of charging infrastructure want to optimize the funding usage and maximize their profits by providing the public charging stations. Secondly, charging PEV batteries presents a high load on the power gird network. However, this can be mitigated by reducing PEV charging during peak electricity usage periods. Additional issues will arise once a charging infrastructure is available. Deciding the capacity of the charging stations and the time of day pricing for EV charging are crucial for scheduling the appropriate load on the electrical grid and optimizing the charging infrastructure's revenue. This research addresses these two problems by studying the development of network design models for charging infrastructures capacity and pricing strategies at different time of the day.

1 Figure source: Website of U.S. Department of Energy. http://www.afdc.energy.gov/afdc/fuels/electricity_locations.html

1.1.2 Potentials of plug-in electric vehicles

One of the goals of employing PEVs is to optimize the interactions between the multiple systems, such as transportation systems and energy systems. Figure 1-2 shows the interaction between travel choices and the power system. For example, PEVs demands of recharging in a certain zone will impacts the setup of the number of charging infrastructure at that zone. In turn, the PEV demands to a zone are influenced by the charging infrastructure availability and recharging price at that zone. Another instance is that if the charging station at a destination (e.g. whole foods, Wal-Mart) has incentives (such as coupons, vouchers), this might stimulate the demands to the destination.

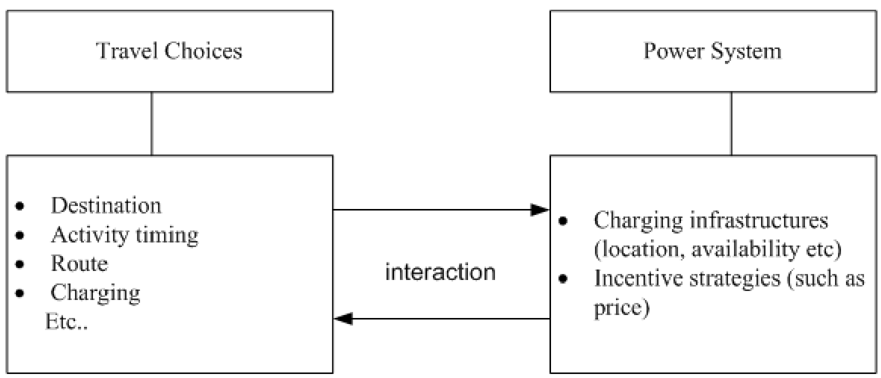

Figure 1-2: Travel Choices and Power Supply Interactions.

To facilitate potential interactions, it is important to investigate the PEVs travelers' response to the transportation network conditions and the charging facility supplies. Such response may include different travel choices, such as destination choice, departure time choice, route choices, charging choice, etc. Investigating the temporal and spatial traveling and charging behavior of PEV users under different charging schemes is desirable for learning the new traffic pattern. Knowing PEVs travelers' responses and behaviors will, in turn, help decision makers and agencies to set transportation policies and charging infrastructure supply and incentives.

1.1.3 Characteristics of battery electric vehicles

In this dissertation research, the study focuses on the adoption of BEVs, where the models and algorithms can also be applied to incorporating general PEVs. Although BEVs have many benefits on improving environments and reducing gas usage, they are facing some concerns as well.

First, an average BEV can travel a shorter distance before recharging than the distance a gasoline vehicle (GV) can travel before refueling. On the other hand, given the lack of enough charging infrastructures, BEV drivers often worry about becoming stranded far from a charging station, a phenomenon called “range anxiety.” Unlike the gas stations, the charging infrastructures for electric vehicles are typically located at parking places such as street parking, parking lots, and home garages. These parking places with charging infrastructures are typically origins and destinations of vehicle trips. This is because, under the current battery-charging technology, the charging time for electric vehicles is usually very long. A passenger car with a driving range of 40~60 miles will require 6~8 hours for a full charge at 220V, and around 20 hours at 110V (Hamilton 2011). Therefore, it is less likely that a BEV motorist stops at a charging station during his/her journey for vehicle charging only. With the proposed “battery swap” services at the charging stations, quickly replacing the battery seems reasonable. However, currently there are no battery swap stations built since it requires a large amount of expensive batteries.

The comprehensive adoption of PEVs will have substantial impacts on the existing transportation network models both temporally and spatially, but we lack the models for predicting such impacts at this early stage of adopting electric vehicle, The overarching theme of this work is the need to build models for quantifying the BEV travelers' behavior and for predicting the new traffic patterns with some BEVs penetration on the road network.

Overall, the need for this dissertation research on building various models for studying the transportation networks with PEVs considerations is required to match the government policies and incentives, the market driven, and the future convergence goal. The dissertation studies both the travel behaviors of PEV users and the charging incentives and policy. The goal of the dissertation is to develop models and algorithms tracing the vehicle assignment in the transportation network account for PEVs considerations.

1.2 RESEARCH PROBLEMS AND CONTRIBUTIONS

Three primary problems are examined in this dissertation: the route choice behavior of BEV users, the joint choices of departure time choice, duration of stay choice and route choice behavior of BEV users, and the pricing and capacity design for charging infrastructure. Each problem is raised from currently pressing need of research after adoption of BEVs, for which analytical tools are absent.

The route choice behavior of BEV users is considered in this dissertation primarily on account of the special characteristic of BEVs. Existing traffic assignment models could not account for the BEVs in completion. For BEV traffic assignments, the vehicle range is restricted, the charging infrastructure availability is limited, and the charging speed of most charging stations is confined. As consequence, the primary elements that are taken into account for BEV users and are considered in this dissertation for the route choice behavior are the travel time costs, path length, charging speed, and electricity-charging price. This dissertation explores answers to the questions (such as spatial choices behaviors of BEV users and algorithms to solve the models regarding to such behaviors) that are posed before the joint choices analysis proceeds. It is shown in the chapter 3, a path-based algorithm for stochastic user equilibrium with BEV charging behavior is developed, which will be used for examine the BEVs spatial flow patterns.

The joint choices problem is studied with regards to the temporal choices in addition to the spatial choices. The temporal choices considered in this research are referring to the departure time and duration of stay at destination. A combined time-dependent joint choices model is presented to identify the traveling behavior of PEV users on a network level with mix flows. The problem examined in this part is not only timely and important on their own, but also can be mingled within the consideration of the network design problem.

On top of the joint-choices equilibrium model, a continuous network design problem of charging infrastructure capacity and electricity-charging pricing is discussed and solved. The objective of the problem tackled here is to optimize the revenue of building and operating the charging facilities. The revenue management is not necessarily the only objective but in the view of the author that analytical tools produced in this part are useful for other objectives as well. This is because this part of the research lays a foundation by developing a model and its solution algorithm for the continuous network design problem.

The contribution of this dissertation research, thus, is to explore the above mentioned problems and develop modeling techniques to facilitate quantitative analysis. In summary, the main contribution of this study includes the following:

- New formulations for modeling the BEVs travelers' route choice behavior based on the stochastic user equilibrium traffic assignment models.

- Algorithms to solve the path-based network equilibrium models based on disaggregated simplicial decomposition and lagrangian relaxation with enumeration
- Multi-class quasi-dynamic model formulations for joint choices of departure time, duration of stay and route choice for BEVs and gasoline vehicles, and its solution algorithm.
- Continuous network design model for deciding charging infrastructure capacity and electricity-charging pricing and its solution algorithm.
- Quantitative analysis of temporal and spatial traffic flow patterns with varied BEV penetration rates, BEV range, and charging infrastructures' characters.

1.3 DISSERTATION OVERVIEW

This chapter has presented the background, the basic motivation and research problems of this dissertation, and the objectives to be achieved. Section 1.1 states the motivation of this research. According to the motivation and challenges, the objectives/contributions of this research are given in section 1.2. The rest of this dissertation is organized as follows.

Chapter 2. A detailed and critical review of the relevant literature is conducted. It gives review of stochastic user equilibrium traffic assignment, solution algorithms of SUE models, the discrete choice formulations, as well as combined and integrated models for joint travel choices.

Chapter 3. Development of two new models of the stochastic user equilibrium traffic assignment problems for BEVs. The new traffic flow patterns within the context of BEVs on the road network are determined and analyzed. New route cost functions are defined. In order to study the impacts of charging on the BEVs users' route choice, the electricity-charging cost and the charging time penalty cost are considered separately in the stochastic user equilibrium assignment models. Solution algorithms for solving the developed models are presented. Again, the traffic flow patterns are examined to test whether changes in charging price and charging speed leads to pressing impacts.

Chapter 4. Combined travel choices modeling. The network travel pattern, overall, will be affected by introducing BEVs and their charging behavior. Given fixed charging price and charging capacity, the travelers' choices are be studied, including the activity time choices and route choices. The travelers' joint choices are considered hierarchical and modeled using the nested logit structure. A quasi-dynamic time-dependent choice process is presented and an equivalent variational inequality model is built to account for the multi-class, time-dependent combined choices. This chapter presents formulation, solution algorithm, and numerical experiments that illustrate use of the methodology to evaluate various scenarios of BEVs penetration and BEV range improvements

Chapter 5. Continuous network design of charging infrastructure. The design of the charging infrastructure capacity and the electricity-charging price is captured by an optimization model with variational inequality equilibrium constraint. The optimal capacity and pricing is determined in terms of revenue generation. The travel choices model developed in Chapter 4 is presented as the lower level equilibrium problem. A sensitivity analysis based optimization approach is derived and implemented to solve the proposed model.

Chapter 6. concludes the dissertation, summaries the key contributions, the results from the analysis, and suggests the future research directions.

Chapter 2 Literature Review

The research contained herein draws from existing research in the area of discrete choice models, user equilibrium models, stochastic traffic assignment models and algorithms to implementing them, joint choices models, and network design models with particular interests in continuous network design models. The remainder of this chapter is devoted to reviewing the literature techniques in these areas that are relevant to the research work.

2.1 Discrete Choice Models for Travel Choices

Random utility models for the route choice play a central role in vehicles assignment to road networks. The utility maximization rule states that an individual selects the alternative from his/her set of available alternatives that maximizes his or her utility. Let $U(\cdot)$ be the function representing the utility of an alternative, x, y are the alternatives from the available alternative choice set C. Mathematically, the random utility rules can be described as follows

$$U(x) \geq U(y) \Rightarrow x \succ y \in C \tag{2.1}$$

Where $x \succ y$ means the alternative x is preferred to the alternative y. That is, if the utility of alternative i is greater than or equal to the utility of all other alternatives in the choice set, the alternative x is preferred over all other alternatives and is selected.

The discrete choice models have been widely adopted to analyze and predict individual decisions on a finite set of discrete alternatives (Ben-Akiva and Lerman 1985). The discrete choice models have very good fit because, in transportation behavior modeling, the analysis of travel behavior is usually disaggregated and the choice set in traveling decisions are usually discrete and finite (such as routes, destinations, mode, departure time, etc.).

In the discrete choice models, the preference of an alternative is based on the utility measure associated with that alternative. However the decision-maker may not have the complete information on the utility of the each alternative due to different sources of uncertainties, such as unobserved alternative attributes, measurement errors etc.(Manski, 1977). In such case, the utility is modeled as a random variable and the choice model gives a chosen probability of each alternative in the choice set in an aggregated manner. Mathematically, this probabilistic choice theory is represented as follows,

$$U_x = V_x + \varepsilon_x \tag{2.2}$$

Where U_x is the true utility of the alternative x to a decision maker, V_x is the observable portion of the utility, and ε_x is the portion of the utility that is random. Based on the different distribution of the random term, two typical discrete choice model families, namely Logit family and Probit family, are widely used in the traffic assignment context (Ben-Akiva and Bierlaire 2003; Cascetta 2009; Sheffi 1985).

2.1.1, Multinomial Logit model

The Logit models are models where the random variables are introduced by the Gumbel distributions. There are specific assumptions that lead to the Multinomial Logit Model (MNL). The assumptions are: the error components are Gumbel distributed; the error components are identically and independently distributed (i.i.d) across alternatives; and the error components are identically and independently distributed across individuals (Koppelman and Bhat 2006).

The MNL models assign the choice probabilities of each alternative as a function of the systematic portion of the utility of all the alternatives. Given that the random term

ε_x in Equation (2.2) is i.i.d. Gumble distributed, the mathematical expression of the probability of choosing an alternative x from a set of alternatives C is as follows,

$$\Pr(x) = \frac{\exp(V_x)}{\sum_{y \in C} \exp(V_y)}, \qquad \forall x \in C \tag{2.3}$$

As stated in Equation (2.2), V_x is the systematic component of the utility of the alternative x. The exponential function of V_x is positive and monotonically increasing. This implies that the probability of choosing an alternative increases monotonically with the increase of its systematic utility and decreases with the increase of the systematic utility of the other alternatives in the choice set. Noted that the variance of the random term ε_x is

$$\mathrm{var}(\varepsilon_x) = \frac{\pi}{6\mu^2} \tag{2.4}$$

Where μ is the scale parameter which determines the variance of the Gumble distribution.

The MNL models for travel choices were widely used and adopted by many researchers because of its simple mathematical form, ease of estimation and interpretation. They has been applied in the areas of mode choice (Koppelman and Bhat 2006), mode choice and departure time choice (C. Bhat 1998a; H. Huang, J and Lam 2003), departure time and route choice (Ben-Akiva and Bierlaire 2003), etc.

2.1.2, Nested-Logit model

MNL choice models sometimes lead to unreasonable results for a correlated choice set. For example, in the context of route choice; the MNL does not properly address the overlaps in different routes. There are renewed interests to improve the Logit-based models. Some recent models in the Logit family relax restrictions on the covariance structure and still maintain the tractability. Different models from the extended and modified logit models based on different assumptions concerning the structure of the random term of alternative utilities are derived (Bekhor and Prashker 1999; Ben-Akiva and Bierlaire 1999; Cascetta et al. 1996; A. Chen et al. 2003a; Hoogendoorn-Lanser et al. 2005; Ramming 2001; Vovsha and Bekhor 1998). They can all be used to address the correlations between alternatives. Among them, the Nested Logit (NL) model, is the simplest and most widely used. The NL model is characterized by nesting subsets of similar alternatives. The NL model normally has a tree structure as shown in Figure 2-1. The Figure shows a two-level nested structure, there are four alternatives in the upper level nest and a number of alternatives in the lower level nest (McFadden and Train 2000; Williams 1977).

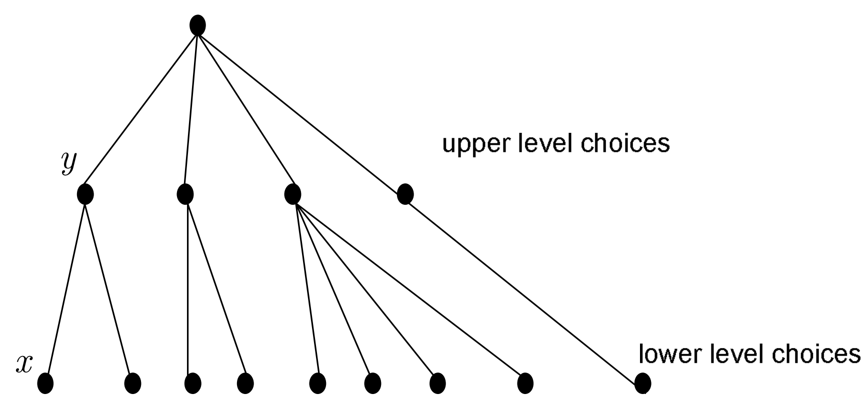

Figure 2-1: Two-Level Nest Structure

At the lower level in the NL model, the alternative's utility depends on the utilities as subsequent level. This utility is called conditional utility. Let x represents an alternative in the lower level nest and y represents the alternative superior to x in the upper level nest.

Let $U_{x/y}$ represent the conditional utility of a lower level choice x. Providing that upper-level choice y has already been made, $U_{x/y}$ is equal to the sum of lower-level choice systematic utility, V_x, the combined utility of first and second-level choice, V_{xy}, and the random utility terms ε_{xy},

$$U_{x/y} = V_x + V_{xy} + \varepsilon_{xy} \tag{2.5}$$

With this utility structure, according to (Oppenheim 1995), the conditional probability of selecting the lower level alternative x, given that the upper-level alternative y has already been chosen, equals to Eq. (2.6); while the unconditional probability that the upper level alternative y is chosen equals to Eq.(2.7)

$$\Pr(x / y) = \frac{\exp\{\beta_x (V_x + V_{xy})\}}{\sum_{x'} \exp\{\beta_{x'} (V_{x'} + V_{x'y})\}} \tag{2.6}$$

$$\Pr(y) = \frac{\exp\{\beta_y (V_y + W_y)\}}{\sum_{y'} \exp\{\beta_{y'} (V_{y'} + W_{y'})\}} \tag{2.7}$$

where the term W_y represents the expected indirect utility of the choice of (x, y). The probability of choosing the alternatives x can be obtained by multiplying the conditional probability of the alternative x by the marginal probability of choosing alternative y, as follows:

$$\Pr(x) = \Pr(x / y) \times \Pr(y) \tag{2.8}$$

By the utility theory, an individual would choose the alternative with the maximum utility (Eq. 2.9). The expected utility receives from utility maximizing choices is given by Eq. 2.10. W_y is computed from the log of the sum of the exponents of the nested utilities, commonly referred to as the “logsum” variable. This is how the utilities are propagated upward in the NL model.

$$W_y = E_\varepsilon\{\max_x(U_{x/y})\} \tag{2.9}$$

$$W_y = \frac{1}{\beta_x}\ln\sum_x \exp\{\beta_x(V_x + V_{xy})\} = \frac{1}{\beta_x}\ln\sum_x \exp\{\beta_x(U_{x/y})\} \tag{2.10}$$

The logsum parameter β_x (also called the "dissimilarity parameter", "nesting coefficient"), is a function of the correlation between unobserved components for alternatives in the nest. This parameter characterizes the degree of dissimilarity between the pair of alternatives. Noted that $0 \le \beta_x \le 1$ needs to be satisfied to be consistency with random utility maximization principles. The values of the parameter β_x are related to the variances of random component ε_{xy}; the relationship is $\sigma^2 = \pi^2 / 6\beta_x^2$, where σ is the variance of the Gumbel distributed random term ε_{xy}.

2.2 STOCHASTIC USER EQUILIBRIUM AND ALGORITHMS

2.2.1, user equilibrium models

According to Wardrop's first principle, the user equilibrium (UE) condition in transportation networks is reached when no travelers can improve their travel time by unilaterally changing the routes (Sheffi 1985; Wardrop 1952). The UE models are the most widely used route choice models in the transportation planning process. The deterministic UE traffic assignment assumes that the actual travel costs are known to all travelers in the network; therefore, all travelers choose the route that minimizes their own travel costs. The UE flow pattern can be found by solving the following Beckmann equivalent optimization problem (2.11)-(2.14) (Beckmann et al. 1956).

$$\min_x . z(x) = \sum_a \int_0^{x_a} t_a(x)dx \tag{2.11}$$

Subject to

$$\sum_k f_k^{rs} = q^{rs}, \qquad \forall r,s \tag{2.12}$$

$$f_k^{rs} \geq 0, \qquad \forall r,s,k \tag{2.13}$$

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs}, \qquad \forall a \tag{2.14}$$

Where $\mathbf{x}$ represents the link flow vector and x_a represents the flow on link a. $t_a(\cdot)$ is the link performance function (e.g. travel time) as a function of the link flow. It is usually assumed positive and increasing. q^{rs} is the trip rate between origin r and destination s, and f_k^{rs} is the flow on path k between Origin-Destination (O-D) pair (r,s). The objective function is the sum over all arcs of the integrals of the link performance function. Eq. 2.12 is the flow conservation constraint at origins and destinations. Eq. 2.13 is the non-negative constraint of the total flow on all paths. Eq. 2.14 is the link-path incidence relationship constraints, where $\delta_{a,k}^{rs}$ is defined as follows:

$$\delta_{a,k}^{rs} = \begin{cases} 1 & \text{If link } a \text{ is on path } k \text{ of O-D pair } (r,s) \\ 0 & \text{Otherwise} \end{cases} \tag{2.15}$$

2.2.2, stochastic user equilibrium models

The assumptions of perfect knowledge of the network travel costs and thus to accurately identify the minimum travel time are rather unrealistic since travelers do not always end up picking the minimum travel cost route due to their imperfect perceptions. The stochastic user equilibrium (SUE) traffic assignment models are considered as a realistic generalization of the deterministic UE traffic assignment models because they relax the presumptions that every traveler in the network has accurate perceptions on the travel cost by introducing a random term to represent differences in the travelers' perception of travel cost. Let $c_k^{perceived}$ be the perceived travel cost of route k, c_k^{real} be the real travel cost on route k, and ε_k be the random errors. $c_k^{perceived} := c_k^{real} + \varepsilon_k$. A SUE is defined as *no motorists can improve his or her perceived travel cost by unilaterally* *changing the routes* (Daganzo and Sheffi 1977; Sheffi 1985). The SUE conditions are satisfied by the following optimization problem (Sheffi 1985).

$$\min_{x} z(x) = -\sum_{rs} q^{rs} E[\min_{k \in \mathrm{K}rs}\{c_k^{rs}\} \mid \mathbf{c}^{rs}(\mathbf{x})] + \sum_{a} x_a t_a(x_a) - \sum_{a} \int_0^{x_a} t_a(\omega) d\omega \quad (2.16)$$

where $\mathbf{c}^{rs}$ is a vector of the perceived path cost with element of $\{c_k^{rs}\}$. As the equation shown in (Sheffi, 1985), the first term in the objective function is the expected perceived travel time function $S_{rs}[\mathbf{c}^{rs}(\mathbf{x})] = E[\min_{k \in \mathrm{K}rs}\{c_k^{rs}\} \mid \mathbf{c}^{rs}(\mathbf{x})]$. It has been proved that the function $S_{rs}[\mathbf{c}^{rs}(\mathbf{x})]$ is concave with respect to $\mathbf{c}^{rs}$ (Sheffi 1985), and the following equation holds,

$$\frac{\partial S_{rs}(\mathbf{c}^{rs})}{\partial c_k^{rs}} = \mathbf{P}_k^{rs} \quad (2.17)$$

Where $\mathbf{P}_k^{rs}$ is the probability vector of choosing any route k between r and s .The above formulation is the link-based unconstrained optimization formulation for the general SUE problem developed by Sheffi and Powell (Sheffi and Powell 1982). Because the SUE traffic assignment models are considered more behavioral realistic, they have attracted more attention from many researchers in the traffic assignment area.

Summarized by (Lo and Chen 2000a), traffic assignment formulations typically follow four approaches, namely, mathematical program (Fisk 1980; Janson 1991), nonlinear complementarity problem (H. Aashtiani 1979a), variational inequality formulation (S. Dafermos 1980; Nagurney 1993), and fixed point problem (K. Zhang and Mahmassani 2008). There are linkages and equivalence conditions among these approaches. In general , existing literatures for SUE traffic assignment are based on three types of formulations: (i) optimization-based general models for both Logit and Probit SUE models (Sheffi and Powell 1982) (ii) the Logit SUE model and its equivalent mathematical formulations by Fisk (Fisk 1980) (iii) Fixed point formulations and nonlinear complimentary formulations with gap functions for general models (Daganzo 1983; Lo and Chen 2000b).

The logit SUE formulation developed by (Fisk 1980) uses path flow variables. The following equations (2.18)-(2.20) give Fisk's path-based optimization formulations of the logit-based SUE problem.

$$\text{(Fisk's)} \qquad \min_{f_k^{rs}} z = \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw \tag{2.18}$$

Subject to

$$\sum_{k\in \mathrm{K}} f_k^{rs} = q^{rs}, \quad \forall r,s \tag{2.19}$$

$$f_k^{rs} \geq 0, \qquad \forall k,r,s \tag{2.20}$$

where an entropy term is added to the objective function of Beckmann's equation (2.11) (Beckmann et al. 1956).

2.2.3, solution algorithms for stochastic user equilibrium

One classification method for the algorithms solving the deterministic UE and SUE traffic assignment problem is based on how the link flows are aggregated. The link flows could be aggregated from three different types of flows: path flows, origin-based link flows, and destination-based link flows (Ahuja et al. 1993a). This classifies the algorithms into three categories: link-based algorithms, path-based algorithms, and origin-based or destination-based algorithms (Lee et al. 2010).

Handful solution methods have been developed so far for the deterministic UE traffic assignment problems for all of the above three types of algorithms. Some novel literatures on link-based Frank-Wolfe algorithms are (Michael Florian et al. 1987; LeBlanc et al. 1975; Masao 1984). Origin-based approach has received more attractions after introduced by (Bar-Gera and Boyce 2003) (for a summary of some of those early algorithms see (Michael Florian and Hearn 1995; Patriksson 1994)). After the first convergent path-based feasible descent direction method was proposed by Gibert (Gibert 1968), Larsson and Patrikson (Larsson and Patriksson 1992) designed the Disaggregate Simplicial Decomposition (DSD) method. Later (Jayakrishnan et al. 1994) proposed a path-based gradient projection method.

The link-based and origin-based algorithms have the advantage of avoiding path enumerations and route storage, which, subsequently is more computational efficient and inexpensive, especially for a large networks where the number of routes is much more than the number of links. However, there is resurgence in the interest of reformulating or solving the traffic assignment with path flows directly since in some problems the route flow results are necessary (Lo and Chen 2000a). For instance, when the route costs are non-additive (i.e. there are route specific costs), it is not possible to solve the problem with link flows alone (Agdeppa et al. 2007; Gabriel and Bernstein 1997).

Unlike deterministic UE traffic assignment problem, only a few algorithms have been emerged for solving the logit-based SUE traffic assignment models (Bell 1995a; Damberg et al. 1996; David 2006; H.-J. Huang and Bell 1998; Lee et al. 2010; M. J. Maher 1992; M. J. Maher and Hughes 1997; M. Maher 1998; M. Maher et al. 2005; K. Zhang and Mahmassani 2008). Although the logit-based route choice model has a closed-form probability expression based on the path flow variables, most solution algorithms for the above SUE formulations are link-based algorithms. Among the link-based solution algorithms, the Dial's STOCH (Dial 1971) approach and its variants for stochastic network loading are, perhaps, the best known algorithms that simultaneously assign flows to the efficient links on reasonable paths in the network. The link efficiency is represented by likelihood and link weight, which is detailed in (Sheffi 1985). (Sheffi and Powell 1982) proposed the link-based method of successive average (MSA) with a predetermined step size sequence. In link-based algorithms, although routes are generated in each iteration of the column generation procedure, the algorithms does not store or makes direct use of the routes gnerated. Motivated by Bar-Gera's origin-based algorithm for deterministic UE traffic assignment, (Lee et al. 2010) developed an origin-based algorithm for solving the logit-based SUE traffic assignment problem based on Akamatsu's model (Akamatsu 1996). Overall, there are limited algorithms that explicitly make use of the path flows when solving the SUE traffic assignment problems.

Despite its several weakness, the logit-based SUE traffic assignment with the equivalent mathematical programming formulation (Fisk 1980) is the most widely used model because of its simplicity. Although Fisk's mathematical formulation is represented using the path flow variable, many algorithms solving this SUE traffic assignment models use link flow variables instead of path flow variables. Although there has led to increased focus on path-based algorithms for deterministic UE, the path-based algorithms available for SUE models are very limited. However, the investigation on path-based algorithms is necessary and fulfilling. Because models resulting in explicit route flows allow more realistic behavioral assumptions and path-based algorithms provide higher accuracy for equilibrium solutions.

Some of the path-based SUE traffic assignment models can be solved using path enumeration and column generation technics (Anthony Chen et al. 2003b). The explicit path enumeration technics assigns probabilities onto the pre-selected paths, which is called selective-explicit enumeration approach (Cascetta et al. 2002). This pre-selection of path is reasonable since motorists do not consider or perceive all alternatives in a transportation network. The perceived paths (pre-selected paths) in path enumeration approaches are obtained as those satisfying some rules (Ben-Akiva et al. 1984b; Cascetta et al. 1997; Cascetta et al. 2002) since the exhaustive enumerating of all paths in the network is computationally impossible. Some other path-based column generation techniques for SUE traffic assignment models do not store the paths generated during running the algorithm (Bell 1995b; Bell et al. 1997; Jayakrishnan et al. 1994). Based on the disaggregated simplicial decomposition (DSD) algorithm proposed by (Larsson and Patriksson 1992), (Damberg et al. 1996) extended this algorithm for the solutions of the SUE traffic assignment problem. It is the first work to present an algorithm that provides route flows explicitly for the SUE traffic assignment. The algorithm alternatives between two main phases: a restricted master problem phase where the equilibrium of path flows are solved within a restricted path set and a sub-problem phase where new routes are generated and augmented into the restricted path set. Huang and Bell (H.-J. Huang and Bell 1998) presented an efficient approach to generate all non-cyclic paths to solve SUE traffic assignment problem by applying the method of successive averages. The number of paths decreases greatly by the generation techniques, which results in reduction of the computational burden. Meanwhile, the generation of the perceived path set is very important, and plentiful work has been done on the choice set modeling for generating reasonable and realistic choice set within a satisfied computation burden (Cascetta and Papola 2009; Damberg et al. 1996)

2.3 EQUILIBRIUM TRAVEL DEMAND PROBLEMS

The network equilibrium models are useful tools for long term transportation planning. In order to address the drawback of inconsistency between different choices and to model more than one choice dimension in travelers' behavior, many researchers proposed network equilibrium models which combine different choices together. Such combined choices include the mode and routes choices, destination choices and route choices, departure time and route choices, etc. Many researchers proposed network equilibrium models for the combined choices (García-Ródenas and Marín 2005; Oppenheim 1995).

(Sheffi 1985) presented a hypernetwork approach to accommodate the joint travel choices. The hypernetwork consists of hyperlinks for trip generation, mode choice, destination choice and the route choice (Figure 2-2). The network includes two origins, 3 destinations. Node 1 and 2 are connected to destination 4 and 5 by transit links (dashed line). In addition, the destinations are connected to origin-specific dummy destination nodes.

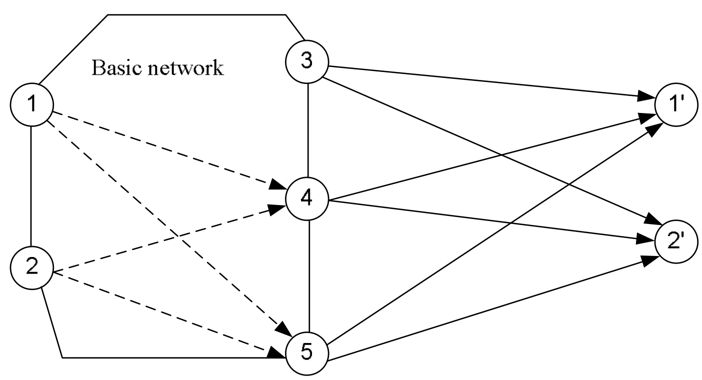

Figure 2-2: Hypernetwork Representing Joint Mode Split/Trip Distribution/Traffic Assignment Problem (Sheffi, 1985 (Sheffi 1985))

2.3.1, mathematical programming models

For many years, formulating the combined choices into a mathematical programming problem is quite attractive because of the computational efficiency. The combined choices have been proposed for dozens of years on different levels of choice combinations. The combined choices model in this dissertation research is not a mathematical programming model, however, some innovation research works in the mathematical models are presented here as for the review completeness. (Michael Florian 1977) and (Evans 1976) combined the trip distribution and traffic assignment into one model and solved the two step choices simultaneously to obtain more consistent results (Brake et al. 2007; Michael Florian et al. 1975). Evans (1976) extended the convex optimization formulation by introducing origin and/or destination constraints and she also presented an efficient convergent algorithm for solving the model (Brake et al. 2007). (Michael Florian and Nguyen 1978) extended the trip distribution and assignment models with two modes which had unrelated travel time. (T Abrahamsson and Lundqvist 1999a) developed a convex optimization problem combining trip distribution, mode choice and traffic assignment models with a set of hierarchical choices, where the transit and auto travel times are independent (Martin 1998). (William H. K. Lam and Huang 1992) and (Boyce and Bar-Gera 2001) formulated multimode, multiclass network equilibrium models. (William H. K. Lam and Huang 1992) incorporate trip distribution with the user-equilibrium assignment for multiclass transportation networks. Convex programming was used after converting the link cost functions into symmetric forms; while modified algorithms based from Frank-Wolfe's and Evans' were applied to solve the problem (Diana and Dessouky 2004).

2.3.2, fixed-point models

The mathematical programming sometimes cannot be used to deal with multiple user classes; the interacting modes will cause asymmetric link cost functions that cannot be solved using mathematical formulation. Thus, in combined models, the equilibrium conditions are usually considered as a system of equations and inequalities (Aldaihani and Dessouky 2003), which have an intuitive behavior interpretation. Examples of such models are the nonlinear complementarily problem (H. Z. Aashtiani 1979b) and the fixed-point problem (Bar-Gera and Boyce 2003, 2006; Cantarella 1997; Lin et al. 2008; Martínez and Henríquez 2007).

(Cantarella 1997) presented a fixed-point formulation of a multi-modal, multi-user equilibrium assignment with elastic demand. (Daganzo 1983), in addition, allowed stochastic link costs when modeling multimodal transportation network equilibrium problems. The SUE path flows can also be expressed as the solution of a fixed-point model defined on the feasible path flow set (Cascetta 2009).

$$\mathbf{f}^{rs*} = q^{rs}\mathbf{p}^{rs}(-\Delta_{rs}^{T} c(\sum_{rs} \Delta_{rs}\mathbf{f}^{rs*})) \qquad (2.21)$$

where $\mathbf{f}^{rs*}$ is the path flow vector with element $\{f_k^{rs}\}$, Δ_{rs} is the link-path incident vector with element $\{\delta_{a,k}^{rs}\}$.

(Bar-Gera and Boyce 2003) presented a fixed-point model for the general combined travel demand and network assignment problem. The authors also proposed an origin-based algorithm for solving the combined models. Later, they discussed the algorithm's ability to handle non-convex combined models (Bar-Gera and Boyce 2006). They demonstrated that a proper choice of step size in the method of successive averages for fixed-point problems can solve the non-convex combined models efficiently and precisely with an origin-based algorithm (Bar-Gera and Boyce 2006). (Lin et al. 2008) developed a fixed-point formulation and explored practical integration issues for combining activity-based travel demand modeling approaches and dynamic traffic assignment.

2.3.3, variational inequality models

Besides the fixed-point models, variational inequality (VI) is widely used for the combined choices (Chang and Yu 1996; Stella Dafermos 1982; Friesz and Mookherjee 2006; Guo et al. 2010), especially for the time-dependent combined travel choices (Daniele et al. 1998). The VI problem is defined as follows.

Let $F : D \subset \Re^n \rightarrow \Re^n$ be a vector-valued and continuous mapping on a nonempty, closed and convex set. The VI problem is to find a vector $\mathbf{x}^* \in D$ such that

$$< F(\mathbf{x}^*), \mathbf{x} - \mathbf{x}^* > \geq 0, \qquad \forall \mathbf{x} \in D \tag{2.22}$$

where $< a, b >= a^T b$. The existence theorem for the VI model states that: If D is compact convex set and $F(\mathbf{x})$ is continuous on D, the VI problem has at least one solution $\mathbf{x}^*$. Furthermore, the uniqueness theorem states that: if $F(\mathbf{x})$ is strictly monotone on D, then, the solution is unique if there exists one.

(Ran and Boyce 1996) proposed a path-based VI formulation for dynamic stochastic models. (Liu et al. 2002) presented a VI model over a stochastic network for dynamic stochastic models to capture the travelers' decision making among discrete choices in a probabilistic environment.

Yang et al. (1998) proposed a space-time expended network (STEN) for the departure time and route choice in a queuing network with elastic demand to determine the optimal variable congestion tolls. (Friesz et al. 1993) first formulated an infinite-dimensional VI model for the combined choices of departure time and route choice without providing the solution approaches. Later, (Wie et al. 1995) developed a discretized VI formulation for simultaneous route and departure time choice equilibrium problem. In addition, they presented a heuristic algorithm but with no convergence established.

(Zhou et al. 2007) developed a VI model and a heuristic procedure to describe and to solve the combined mode, departure time and route choices in multimodal urban transportation network. (X. Zhang 2007) considered simultaneous departure time and route choices using VI formulation. (M. Florian et al. 2002) formulated a VI formulation for a multi-class multi-mode variable demand equilibrium, in which the joint choices model was a hierarchical Logit function. (Wu and Lam 2003) proposed a network equilibrium model that predicts the mode choice and route choice simultaneously in the VI form. (William H.K. Lam et al. 2006) proposed a time-dependent network equilibrium VI formulation for simultaneously departure time, route, parking location and parking duration choice in deterministic user equilibrium.

2.4 Bi-level models for continuous network design problem

The continuous network design problem (CNDP) is an important problem in transportation planning, with a vast volume of literature published on this topic (Friesz 1993; Wang and Lo 2010). The CNDP problems are typically formulated as bi-level programs or a mathematical programming with equilibrium constraints (MPEC). An MPEC problem can be formulated as a generalized bi-level programming model as follows (Luo et al. 1996).

$$\min_{x} .F(x,y) \tag{2.23}$$

subject to

$$x \in X \tag{2.24}$$

where y is optimal for

$$\min_{y} .f(y,x) \tag{2.25}$$

subject to

$$y \in Y(x) \tag{2.26}$$

The objective $\min_{x\in X} F(x,y)$ is referred to as the upper-level problem, and $\min_{y\in Y(x)} f(y,x)$ is referred to as the lower-level problem for a fixed x. When this lower-level problem is formulated as a VI problem, it is usually referred to as MPEC problem.

The models are generally non-convex due to the traffic assignment equilibrium conditions and non-linear travel time function. Many algorithms have been proposed to solve the CDNP formulations, including, e.g., Hooke–Jeeves algorithm proposed by (Abdulaal and LeBlanc 1979), sensitivity analysis based heuristic algorithms (Friesz et. al., 1990; Cho 1988; Yang 1995, 1997), equilibrium decomposed optimization heuristic (Suwansirikul et. al. 1987), gradient-based methods (Chiou 2005). (H. Yang and Bell 1998) comprehensively surveyed on the models and algorithms known and adopted by the transportation field for solving network design problems.

Among all solution approaches, sensitivity analysis based (SAB) method has been developed as an important approach for optimization problems in transportation systems, especially for the network design problems (usually bi-level programming problems). Such applications include a variety of optimal pricing, network design and traffic control problems in traffic networks (Bell and Iida 1997; Luo et al. 1996; Miyagi and Suzuki 1996; H. Yang 1997; H. Yang et al. 2001). A detained and comprehensive review on models and algorithms for road network design can be found in (H. Yang and Bell 1998).

Chapter 3 Stochastic Traffic Assignment for Battery Electric Vehicles

3.1 INTRODUCTION

As mentioned in the first chapter, the electric vehicles (EVs) go a lot shorter distance between recharges than a conventional gasoline vehicle (GVs) between refueling. Because of the recharging requirements discussed above, we are wondering how these charging activities will influence their travel behaviors. Such influences may come from two factors: the charging cost and the charging time. Our initial focus is given to route choice behaviors. These factors, call for new route choice models for BEV drivers. This chapter aims to investigate the impacts of charging cost and charging time on the individual route choice behavior and network flow pattern under the stochastic user equilibrium conditions.

A driver's generalized total cost typically consists of travel time and operating cost. The motorist will choose a route that minimizes his/her generalized travel cost for the journey. Comparing to GV drivers, the BEV drivers will encounter the same non-monetary cost (time spent undertaking the journey), but different out-of-pocket costs (operating cost, charging time costs, etc.). The operating cost for gasoline vehicle uses is basically the gas cost, while the operating cost for BEV motorists is mainly the electricity cost. It has been mentioned earlier that the difference between recharging/refueling infrastructure availability and recharging/refueling time for BEVs and GVs. Gas stations are available almost everywhere and the gas prices between different gas stations in a daily commute region are comparable; while in a traffic network, the number of electricity charging stations are limited at present as well as in the foreseeable future. These charging stations are mostly located at origin and destination nodes, for example, homes, offices, schools, shopping centers, etc. In addition, the charging price at home and at a public charging station could be very different. For example, most of the BEV motorists are likely to charge their vehicles during night at home because of the low electricity cost at night and because of the convenience; while the electricity price is typically much higher at public charging stations because of the commercial purpose. Thus, unlike GV motorists, the BEV motorists will also consider where to charge their vehicles in addition to their route choice in order to complete their journey.

Traditionally, in the route choice models, when the operating cost of gasoline vehicles are considered, this operating cost (mainly referring to the gas cost) is decomposed onto the links constituting the chosen route. Usually, this operating cost can be calculated by multiplying the link length and the per unit length gas cost ($/mile) on the link level. However, for BEV drivers, the electricity costs at different places are very different, as we discussed above, which leads to the operating cost is a mix of different electricity-charging costs, depending on how long the trip is and when and where the BEV is charged. Therefore, simply decomposing the electricity costs to links by using an average per-unit-length electricity cost is not reasonable. As a result, the operating cost for BEVs should be calculated with taking into account the different charging costs at different charging places.

In addition, the charging time of a BEV at its destination influences the duration of stay. For instance, let us assume the motorist expects to spend 2 hours at a destination. But, in order to get back to the origin, the required charging time for his/her vehicle exceeds 2 hours, which means that this driver will have to stay longer until the minimum electricity storage in his/her BEV battery is reached. The battery charging time of a BEV depends on the amount of electricity it needs, and this amount of electricity it needs depends on how much electricity is left and how much more electricity is needed for the next trip. The route chosen for a BEV driver directly determines the electricity consumption (and how much electricity remains) and becomes a factor of the recharging time. If we consider a simple origin-based trip chain, such as a origin-destination-origin round trip, the route chosen for the origin-destination trip obviously determines whether the vehicle needs to be recharged at the destination and how long the recharging time is if it is needed. Therefore, it is better to taking the charging time into account when building the route choice model for BEV drivers.

The purpose of the research in this chapter is to develop a stochastic network equilibrium model that can also take into account for the above discussed (i) charging cost and (ii) the charging time for BEV drivers. This research result contains our initiative insights of an equilibrium route choice model for BEV motorists impacted by the charging price and speed. The concern of "range anxiety "for BEVs is not addressed here since most daily commuters will have a very good estimation on their travels, and will not go for any trip that is beyond the vehicle's driving range.

This chapter is structured as follows. . Section 3.1 presents the assumptions, definitions and problem statement of the stochastic user equilibrium assignment problem for EVs. The next section discusses the problem settings and assumptions, followed by the model formulations for incorporating charging cost and charging time as travel impedances, respectively in section 3.3. Two logit-based stochastic traffic assignment models are built to study the route choice behavior of BEV drivers in a network with charging infrastructures located at origins and destinations. Section 3.4 presents a solution algorithm to solve the proposed models and section 3.5 gives the result of the numerical tests for the developed models.

3.2 Problem statement and assumptions

3.2.1, Problem setups

We consider a traffic network $G=(N,A)$, where N is a finite set of all nodes and A is a finite set of all directed arcs. Let R be the set of origins and S be the set of destinations, then (r,s) represents an origin-destination (O-D) pair, where $r \in R$ and $s \in S$. K_{rs} is the set of routes between O-D pair (r,s) and $\widehat{K}_{rs}$ is the subset of routes between O-D pair (r,s). Other notation and variables are summarized as follows.

Parameters:

d_a	length/distance of link a
d_k^{rs}	length/distance of route k between O-D pair (r,s)
t^{rs}	expected activity duration for drivers between O-D pair (r,s)
$\delta_{a,k}^{rs}$	link-path indicator for link a and route k between O-D pair (r,s)
e_h	travel time-equivalent cost per unit distance for electricity charging at origin h
e_s	travel time-equivalent cost per unit distance for electricity charging at destination s
q^{rs}	O-D trip rates between O-D pair (r,s)
λ^{rs}	Lagrangian multiplier associated with flow-demand reservation equation
θ	dispersion parameter, which is a positive scaling factor related to the variance of the perceived travel costs of BEV drivers
σ	charging time needed for per unit distance traveled

Functions

t_a	travel time cost functions on link a
e_k^{rs}	charging cost for BEV drivers using route k between O-D pair (r,s)
tc_k^{rs}	charging time needed for drivers using route k between O-D pair (r,s)

ctp_k^{rs} charging time penalty for travelers using route *k* between O-D pair (r,s)

pe_k^{rs} charging time penalty cost for travelers using route *k* between O-D pair (r,s)

Variables

f_k^{rs} path flow on path/route k between O-D pair (r,s), where path flow pattern $\mathbf{f} = [f_k^{rs}]$

x_a Link flow on link a, where link flow pattern $\mathbf{x} = [x_a]$

U_k utility of route k

V_k systematic utility of route k

ε_k random error term of route k

U_k^{rs} utility of route k between O-D pair (r,s)

V_k^{rs} systematic utility of route k between O-D pair (r,s)

ε_k^{rs} random error term of route k between Od pair (r,s)

C_k^{rs} generalized route cost of choosing route k between O-D pair (r,s)

p_k^{rs} probability of choosing route k between O-D pair (r,s)

The OD demands for the entire analysis period, the duration distributions of motorist staying at destinations, electricity-charging prices at destinations (e.g., workplaces, schools, or shopping malls) and at all origins (i.e., homes) are assumed to be known a priori. For modeling static network equilibrium problems, the analysis period is typically referred to as a specific steady-state time-of-day period, such as morning peak hours, afternoon peak hours, or midday off-peak hours, among others. In particular, the analysis period of interest in this study is the morning peak period and all travel demands are home-to-work commuting trips. As we will discuss below, different time-of-day periods may imply different electricity-charging behaviors of BEV drivers.

The route choice behaviors is specified by the random utility maximization framework, that is, each driver chooses a route that maximizes his/her perceived utility. The choice probability of each route *k* between an O-D pair (r,s) can be determined as follows,

$$p_k^{rs} = p_k^{rs}(\mathbf{f}) = \text{prob}[U_k^{rs}(\mathbf{f}) = \max_{k'}\{U_{k'}^{rs}(\mathbf{f})\}] \tag{3.1}$$

[probability] [maximum utility amount all alternatives]

The SUE network flow is obtained by applying both the equilibrium principle and random utility theory to congested networks. The perceived travel utility on route k between an O-D pair is described as $U_k = V_k + \varepsilon_k$, where V_k is the deterministic travel utility (actual travel cost) at a given flow pattern and ε_k is a random component. Depending on the probability distributions chosen for this random term, different models are obtained (Sheffi 1985). We used a logit- based SUE model, where the random term is i.i.d Gumble variables.

3.2.2, Charging logic of EV users and assumptions

Assumption 1 (Cost-minimization charging behavior). We assume that the electricity-charging cost at origins (i.e., homes) is cheaper than destinations (i.e., workplaces). Whenever possible, all BEV drivers prefer to charge their vehicles at home garages as much as possible. We also assume that all BEV drivers fully charged their vehicles at origins, or charge their vehicles at destinations at least as much as electricity energy into the battery that is sufficient for their return trips to origins, before departing home. If a full charge at origins is not sufficient to support the entire round trips, BEV drivers will charge their batteries at destinations. Given that the electricity-charging cost at origins is cheaper, however, they will not charge more electricity energy than what is needed for their return trips. Under this assumption, it is readily known that the "whether to charge" behavior at any destination will depend on the electricity consumption along the trip or the trip distance between the origin and the destination.

Assumption 2 (Existence of feasible paths). We assume that for any O-D pair with a positive demand rate, there are a number of feasible routes, the length of which is less than or equal to the "effective" distance limit of BEVs.

The effective distance limit is defined as the maximum driving distance that a BEV can achieve after a full charge under ideal driving conditions minus a "safety" or "buffer" range. The consideration of the safety range is necessary since extra electricity energy will more or less consumed under varying, imperfect driving conditions. For a round trip or a home-based trip chain (e.g., a home-workplace-home trip), if the distance of the home-workplace trip is greater than some threshold distance, the BEV driver has to charge their vehicles at the destination for his/her workplace-home trip; if the home-workplace distance is less than the threshold distance, the BEV driver is not required to charge his/her vehicle at the destination. The threshold distance is named the threshold distance for recharging requirement, which, as we will discuss in the next section, is related to the effective distance limit defined above. The network equilibrium models presented in this paper are constructed on the basis of an alternative mixed travel cost structure determined by the threshold distance for recharging requirement.

Assumption 3 (Equivalence of departure and return trip lengths). In a single origin-based trip chain, for example, the home-work-home tri chain, it is reasonable to assume that a driver will choose a route for his/her work-to-home trip parallel to the route for his/her home-to-work trip. The distances of the home-workplace trip and workplace-home trip are equivalent. With this assumption, it is readily known that the threshold distance for recharging requirement is the half of the effective distance limit.

The charging formulations in this chapter are based on the above deterministic setting of the charging behavior, which could be shown in the following figure.

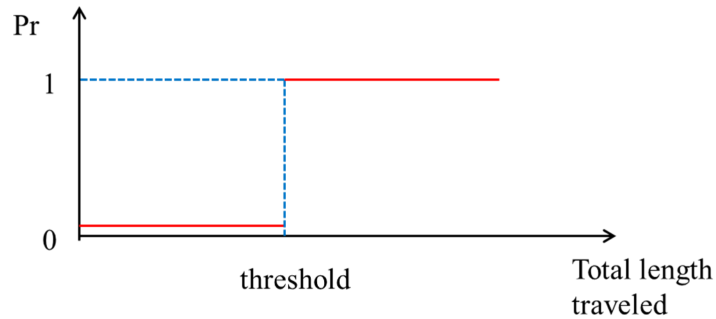

Figure 3-1: A Simple Deterministic Setting of the Charging Logic

3.3 MODEL FORMULATIONS

3.3.1, SUE with charging cost

(Daganzo and Sheffi 1977) defined the stochastic user equilibrium conditions as: *No user can reduce his/her perceived travel time by unilaterally changing route.* Based on this definition, we first give the definition of stochastic user equilibrium conditions for electric vehicle users:

Definition 3.1: EVSUE

No traveler in the network believes he can improve sum of his travel cost and charging cost by unilaterally changing routes.

3.3.1.1 charging cost function

Based on the problem settings and assumptions stated in the previous section, we now construct the charging cost functions for BEV drivers as follows: For a BEV choosing path k between O-D pair (r,s), if $D/2 < d_k^{rs} < D$, where $d_k^{rs} = \sum_a d_a \delta_{a,k}^{rs}$, the BEV will need to be recharged for at least $(2d_k^{rs} - D)$ amount of electricity (in terms of length/distance) for powering the return trip to the origin.

For modeling convenience, the electricity-charging cost is represented by the time-equivalent travel cost. If a BEV driver can complete the whole trip origin-destination-origin (e.g., home-work-home) without recharging at the destination (e.g., work), the total electricity-charging cost for completion the round trip for the driver will be $2e_h \sum_a d_a \delta_{a,k}^{rs}$. However, if the driver needs to recharge the vehicle at the destination, the electricity-charging cost incurred is $e_s(2d_k^{rs} - D)$. Thus, if recharging at the destination is required, the total electricity-charging cost for a BEV driver is $e_s(2d_k^{rs} - D) + e_h D$ for a trip chain.

From assumption 3, we conclude that the electricity-charging cost for the origin-destination (e.g., home-workplace) trip is half of the above total electricity-charging cost derived, as shown in E.q. (3.2). This can be further illustrated by the Fig.3-2, where the piece-wise solid line represents the electricity-charging cost, which is a function of the distance traveled.

In overall, depending on the above settings, the charging cost for a BEV driver choosing route k between O-D pair (r,s) is as follows,

$$e_k^{rs} = \begin{cases} e_h \sum_a d_a \delta_{a,k}^{rs}, & \sum_a d_a \delta_{a,k}^{rs} \le \dfrac{D}{2} \\ \dfrac{1}{2}[e_s \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) + e_h D], & \dfrac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D \end{cases} \qquad \forall k,a,r,s \qquad (3.2)$$

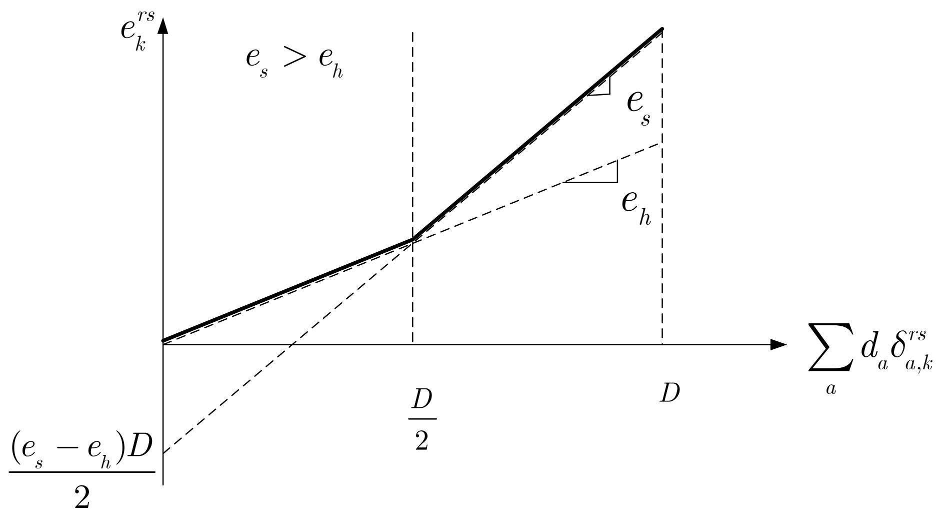

Figure 3-2: Electricity-Charging Cost Function

The dash line with slop e_h represents the first part of the charging cost function, which is based on the home charging cost rate. The solid line gives the charging cost function, which is based on both the home and the destination charging cost rate. The slope of these two dash lines gives the per unit distance charging cost at home and destinations respectively. The intersection of the dash line with slop e_s and the y axis gives the difference of destination and home charging cost for half of the vehicle range distance. The charging cost function derived in above is the maximum of the two dash lines, which can also be in the form of

$$e_k^{rs} = \max\{\underbrace{e_h \sum_a d_a \delta_{a,k}^{rs}}_{\text{no destination charging}}, \underbrace{e_s \sum_a d_a \delta_{a,k}^{rs} + \frac{e_h - e_s}{2} D]}_{\text{with destination charging}}\}, \qquad \sum_a d_a \delta_{a,k}^{rs} \le D \tag{3.3}$$

Let T_k^{rs} be the travel time cost of choosing route k between O-D pair (r,s) in an congested network. The route generalized cost is the summation of the travel time cost (T_k^{rs}) and the charging cost (e_k^{rs}) for BEV travelers:

$$C_k^{rs} = T_k^{rs} + e_k^{rs}, \qquad \forall k, r, s \tag{3.4}$$

In logit-based choice model, the random error vector follows a Gumble distribution. The systematic utility function is defined in Eq. (3.5), and the total utility function is defined in Eq.(3.6) and also follows a i.i.d Gumble distribution.

$$V_k^{rs} = -\theta C_k^{rs}, \qquad \forall k, rs \tag{3.5}$$

$$U_k^{rs} = V_k^{rs} + \varepsilon_k^{rs}, \qquad \forall k, rs \tag{3.6}$$

Eq. (3.1) gives the probability of choosing an alternative route k, in the context of the utility defined as in Eq. (3.5) and Eq. (3.6). The probability of choosing alternative k between O-D pair (r, s), which is also the stochastic user equilibrium conditions, is as follows:

$$p_k^{rs} = \frac{f_k^{rs}}{q^{rs}} = \text{prob}[C_k^{rs} + \varepsilon_k^{rs} \le C_{k'}^{rs} + \varepsilon_{k'}^{rs}, \forall k' \ne k], \quad \forall k, k', r, s \tag{3.7}$$

3.3.1.2 Mathematical formulations for EVSUE

Given the electricity-charging cost the route choice behavior of BEV drivers should be different from the GV drivers. We have already discuss the reasons why they may behave differently in the previous sections. We build a new SUE traffic assignment model here, where one is concerned with the electricity-charging cost. The model is established on the behavioral basis of minimization of the sum of travel time and operating cost (i.e., charging cost). The model implies the origin-destination-origin trip chain as the basic travel analysis unit. Given Assumption 3, however, we only need to pay attention to the network flows consisting of either origin-destination trips or destination-origin trips and accordingly form trip-based network equilibrium models. In this model, we assume the BEV motorists between an O-D pair choose the route that minimizes his perceived travel time cost and electricity-charging cost. We claim that

BEV motorists have random perceptions on travel times but have the accurate perceptions on distances. Travel times are dependent on traffic flows and affected by many disturbing factors, such as traffic conditions, accidents, weather conditions, light conditions, and so on. Different travelers may experience different travel times in a time-varying traffic network. The perceived distance of a route among all drivers can be very accurate since the distance of a route is fixed. We use the SUE traffic assignment to model the network equilibrium, not only because it can accommodate the stochastic perception of travel times, but also because its solution gives a unique route flow pattern, which is required for calculating the electricity-charging cost and the extra charging time.

The mathematical formulation for electric vehicle SUE (EVSUE) traffic assignment is developed Fisk's logit-based SUE traffic assignment. (Fisk 1980). The model was developed using the path flow variable f_k^{rs}, and the problem was proven to be a strictly convex minimization problem:

Based on Fisk's formulation, our mathematical formulation for stochastic traffic assignment is developed as

(EVSUE)

$$\begin{aligned}\min_{f_k^{rs}} z_1 = & \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw \\ & + \sum_{rs}\sum_{k} f_k^{rs} \cdot \max\{e_s\sum_{a} d_a\delta_{a,k}^{rs} - \frac{e_s - e_h}{2}D, e_h\sum_{a} d_a\delta_{a,k}^{rs}\}\end{aligned} \tag{3.8}$$

subject to

$$\sum_{k\in \mathrm{K}} f_k^{rs} = q^{rs}, \quad \forall r,s \tag{3.9}$$

$$f_k^{rs} \geq 0, \qquad \forall k,r,s \tag{3.10}$$

where

$$x_a = \sum_{rs}\sum_{k} f_k^{rs}\delta_{a,k}^{rs}, \qquad \forall a,k,rs \tag{3.11}$$

An optimization problem is a convex minimization problem if it is a problem of minimizing convex functions over convex sets (Boyd and Vandenberghe 2004). The following statement shows that the mathematical formulations for EVSUE problem Eq.(3.8)-Eq. (3.11) is a convex minimization problem.

Proof: The feasible solution set defined in Eq. (3.9) and Eq. (3.10) is convex set since it is the same as that in the Fisk's formulation, Eq. (2.19) and Eq. (2.20). Therefore, we only need to show that the objective function is convex function. The first two terms in the objective function $\frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw$ are convex since it is the same as those in the Fisk's formulation's objective function. Now we check with the last term in our objective function (3.8),

$$\sum_{rs}\sum_{k} f_k^{rs} \cdot \max\{e_s \sum_{a} d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_{a} d_a \delta_{a,k}^{rs}\}$$

In this term, the variable is f_k^{rs} , and the max term is a piecewise linear function of the path length , but is independent of the variable f_k^{rs} . Therefore, it is a linear function of the variable f_k^{rs}, which is a convex function. Above all, the objective function is convex. Therefore, the mathematical formulation (EVSUE) developed is a convex minimization problem. □

Next, we show that the optimality conditions of the above formulation are equivalent to the logit-based choice model.

Proof. Construct the Lagrangian of the optima in the feasible region defined by Eq. (3.9) and Eq. (3.10).

$$
\begin{aligned}
L(f_k^{rs}) = & \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs} \ln f_k^{rs} + \int_0^{x_a} t_a(w)dw \\
& + \sum_{rs}\sum_{f} f_k^{rs} \max\{e_s\sum_a d_a\delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h\sum_a d_a\delta_{a,k}^{rs}\} \\
& + \lambda^{rs}(\sum_k f_k^{rs} - q^{rs})
\end{aligned} \tag{3.12}
$$

where λ^{rs} is the lagrange multiplier associated with Eq. (3.9). Equating the partial derivatives of the Lagrangian function L to zero will give the conditions of the stationary point of the path flow variable f_k^{rs}. The partial derivatives of the Lagrangian function L ensures that the path flow variable is positive, because $\partial L(f_k^{rs}) / \partial f_k^{rs} = \infty$ when $f_k^{rs} = 0$. Therefore, a solution will only be valid if all components of the stationary points of the feasible region are strictly positive. By setting the derivatives of L to zero, we get

$$
\begin{aligned}
\frac{\partial L(f_k^{rs})}{\partial f_k^{rs}} = & \frac{1}{\theta}(\ln f_k^{rs} + 1) + \sum_a t_a(x_a)\delta_{a,k}^{rs} \\
& + \max\{e_s\sum_a d_a\delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h\sum_a d_a\delta_{a,k}^{rs}\} + \lambda^{rs} = 0
\end{aligned} \tag{3.13}
$$

Using $\frac{\partial L(f_k^{rs})}{\partial f_k^{rs}} = 0$, we can get

$$
\begin{aligned}
f_k^{rs} = & \exp\{-\theta[\sum_a t_a(x_a)\delta_{a,k}^{rs} \\
& + \max\{e_s\sum_a d_a\delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h\sum_a d_a\delta_{a,k}^{rs}\} \\
& + \lambda^{rs}] - 1\}, \qquad \forall k, r, s
\end{aligned} \tag{3.14}
$$

Using the O-D demand conservation constraint, we obtain,

$$
\begin{aligned}
p_k^{rs} &= \frac{f_k^{rs}}{q^{rs}} \\
&= \frac{\exp\{-\theta(\sum_a t_a(x_a)\delta_{a,k}^{rs} + \max\{e_s \sum_a d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_a d_a \delta_{a,k}^{rs}\})\}}{\sum_{l \in \Re} \exp\{-\theta(\sum_a t_a(x_a)\delta_{a,l}^{rs} + \max\{e_s \sum_a d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_a d_a \delta_{a,k}^{rs}\})\}} \\
&= \frac{\exp(-\theta C_k^{rs})}{\sum_{k'} \exp(-\theta C_{k'}^{rs})}, \qquad \forall k, r, s
\end{aligned} \tag{3.15}
$$

Finally, we get,

$$
f_k^{rs} * = q^{rs} \cdot \frac{\exp(-\theta C_k^{rs})}{\sum_{k'} \exp(-\theta C_{k'}^{rs})}, \qquad \forall k, r, s \tag{3.16}
$$

where the generalized route cost is a sum of travel time cost and electricity-charging cost.

$$
\begin{aligned}
C_k^{rs} &= \sum_a t_a(x_a)\delta_{a,k}^{rs} \\
&\quad + \max\{e_s \sum_a d_a \delta_{a,k}^{rs} - \frac{e_s - e_h}{2} D, e_h \sum_a d_a \delta_{a,k}^{rs}\}, \forall k, r, s
\end{aligned} \tag{3.17}
$$

Since the right hand side of Eq. (3.16) is strictly positive for all routes between all O-D pairs, it follows form Fisk's formula that the higher cost path are expected to be less traveled than lower cost paths. $f_k^{rs} *$ is a valid stationary point of Lagrangian function L and consequently of z_1 subject to the feasible region defined by Eq. (3.9) and Eq. (3.10). $f_k^{rs} *$ produces a strict local minimum of z_1 and since the feasible region is convex, it is also the unique minimum of z_1 in the feasible region. □

3.3.2, SUE with charging time

In the context of BEVs, the charging cost would be a significant part for route chosen behavior, since the charging time is usually long, and the charging time depends on the route length. The previous section presents a model formulation for EVSUE traffic assignment, where the piecewise electricity-charging cost functions are included into the total perceived cost for BEV drivers. This section, based on the previous one, gives another model, which takes into account the destination activity duration.

We then construct a cost function for representing the charging time in the travel impedance. As discussed in the previous section, the BEV drivers may have an expected activity duration of stay at their destinations (e.g., the working time of 8 hours) and the required recharging time of some BEVs may result in an extra time of stay for the drivers at destinations in addition to the expected stay duration. In this case, a penalty cost for this extra time is imposed in addition to the route travel cost to the drivers. Herein, we construct a function to calculate this extra time of stay.

Let t^{rs} be the expected activity duration for the BEV drivers at destination. We compare the required charging time tc_k^{rs} with this expected activity duration t^{rs}. Only when the charging time exceed the expected activity duration, will the charging time become impacting the motorists' activity duration. We name the cost for the difference between charging time needed for the BEV users and their expected duration of stay at destination as the *charging time penalty cost*. Therefore, we define another SUE for BEV travelers by taking into account this duration of stay at destination, which is the electric vehicle stochastic user equilibrium with charging time penalty (EVSUETP).

Definition 3.2: EVSUETP

No traveler in the network believes he can improve the sum of his travel cost and charging time penalty cost by unilaterally changing routes.

3.3.2.1 charging time penalty cost function

Again, a same trip typed is considered in this model as in the previous one with charging cost functions. The charging time needed tc_k^{rs} is a linear function of total distance traveled $(2d_k^{rs} - D)$, therefore, we have $tc_k^{rs} \propto (2d_k^{rs} - D)$. The relationship among the desired activity duration, the charging time needed, and the charging time penalty (ctp_k^{rs}) is as follows,

$$tc_k^{rs} - t^{rs} = ctp_k^{rs}, \text{ if } \quad tc_k^{rs} > t^{rs} \tag{3.18}$$

Let σ be the electricity-charging rate, in the unit of charging time /unit distance, the total charging time needed is $\sigma(2d_k^{rs} - D)$.

Since the charging time penalty cost is positive if and only if the required charging time is great than the expected activity duration, this charging time penalty is also a piecewise linear function as Eq. (3.19): The extra charging time cost is also illustrated in Fig.3-3, where the solid line represents the charging time penalty cost for choosing route k between O-D pair (r,s).

$$pe_k^{rs} = \begin{cases} 0 \quad , & \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) \le t^{rs}, \dfrac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D, \\ 0 \quad , & \sum_a d_a \delta_{a,k}^{rs} \le \dfrac{D}{2}, \\ \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs}, & \begin{cases} \dfrac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D, \\ \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) > t^{rs} \end{cases} \end{cases} \tag{3.19}$$

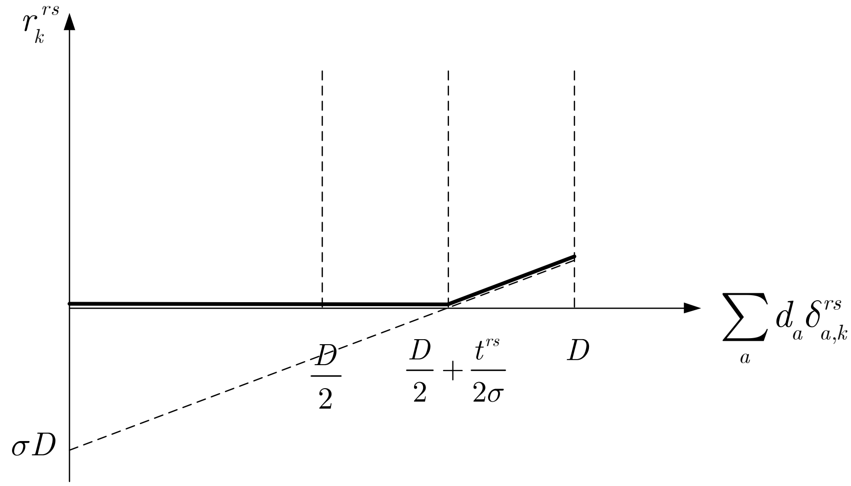

Figure 3-3: Charging Time Functions

3.3.2.2 Mathematical formulations for EVSUETP

Similarly to the mathematical formulations for EVSUE developed in previous section, the logit-based EVSUETP is developed in this section. Given the charging time penalty cost, the route choice behavior of BEV driver is studied. The model is concerned with the charging time penalty cost, and it emphasizes minimizing the sum of travel time and charging time (at destinations). The model also implies the origin-destination-origin trip chain as the basic travel analysis unit. It is assumed that the BEV motorists between an O-D pair choose the route that minimizes the sum of his perceived travel time cost and the extra charging time cost. The SUE traffic assignment modeling approach is used since the route flow pattern is required for calculating the charging time. These two models could be wrapped together; however, either of the two models has its own value. Thet EVSUETP could be formulation as follows,

(EVSUETP)

$$\min_{f_k^{rs}} z_2 = \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw + \sum_{rs}\sum_{k} f_k^{rs}\cdot[\max\{\sigma(2\sum_{a} d_a \delta_{a,k}^{rs} - D), t^{rs}\} - t^{rs}] \tag{3.20}$$

subject to

$$\sum_{k\in \mathrm{K}} f_k^{rs} = q^{rs}, \quad \forall r,s \tag{3.21}$$

$$f_k^{rs} \geq 0, \qquad \forall k,r,s \tag{3.22}$$

where

$$x_a = \sum_{rs}\sum_{k} f_k^{rs}\delta_{a,k}^{rs}, \qquad \forall a,k,rs \tag{3.23}$$

The t_k^{rs} term in the objective function is a constant for a certain BEV traveler, thus it could be moved into the max bracket, which will not impact the optimal solutions. This could also be seen from the Figure (3-3), as the solid line gives the charging time penalty. Then the objective function is:

$$\min_{f_k^{rs}} z_3 = \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw + \sum_{rs}\sum_{k} f_k^{rs}\cdot[\sigma(2\sum_{a} d_a \delta_{a,k}^{rs} - D) - t^{rs}]^{+} \tag{3.24}$$

where $[\cdot]^{+}$ means the function term equals to zero when the inside function has a negative value. This simple transformation reduces the complexity in the solution algorithm, by reducing the number of constrained shortest path we need to solve at each subproblem iteration. In the original objective function, we need to solve three constrained shortest path problem, while with the help of Figure (3-3), in the transformed objective function, we need to solve two constrained shortest path problem. More details will be discussed in the next section.

Convexity: The above EVSUETP problem is also convex minimization problem, as is similar to the discussion for the EVSUE model, with the same feasible convex set and convex objective function. □

The equivalency of the above EVSUETP mathematical formulation for the electric vehicles assignment with charging time penalty costs to the logit-based discrete choice model is checked by the first order optimality conditions of its Lagrangian . Let λ^{rs} be the Lagrangian multiplier associated with the constraint Eq.(3.22), we construct the Lagrangian,

$$L(f_k^{rs}) = z_3(f_k^{rs}) + \lambda^{rs}(\sum_k f_k^{rs} - q^{rs}) \tag{3.25}$$

Equating the partial derivatives on the path flows of the Lagrangian equation to zero will give the conditions that a stationary point f_k^{rs} of formulation Eq. (3.24)-Eq. (3.25) must satisfy. In addition, the partial derivatives also restrict the path flow as strictly positive. By setting the partial derivate to zero, we have

$$\begin{aligned} \frac{\partial L(f_k^{rs})}{\partial f_k^{rs}} &= \frac{1}{\theta}(\ln f_k^{rs} + 1) + \sum_a t_a(x_a)\delta_{a,k}^{rs} \\ &+ [\sigma(2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs}]^+ + \lambda^{rs} = 0, \forall k, r, s \end{aligned} \tag{3.26}$$

This leads to

$$\begin{aligned} f_k^{rs*} = \exp\{-\theta[\sum_a t_a(x_a)\delta_{a,k}^{rs} + [\sigma(2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs}]^+ + \lambda^{rs}] \\ -1\}, \qquad \forall k, r, s \end{aligned} \tag{3.27}$$

The probability of choosing route k, which is also the ratio of path flow on k to the total trip rates between an O-D pair, is

$$p_k^{rs*} = \frac{f_k^{rs*}}{q^{rs}}$$
$$= \frac{\exp\{-\theta(\sum_a t_a(x_a)\delta_{a,k}^{rs} + [\sigma(2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs}]^+)\}}{\sum_{l \in K} \exp\{-\theta(\sum_a t_a(x_a)\delta_{a,l}^{rs} + [\sigma(2\sum_a d_a \delta_{a,l}^{rs} - D) - t^{rs}]^+)\}} \tag{3.28}$$

where the total measured cost for choosing path k between O-D pair (r,s) is,

$$C_k^{rs} = \sum_a t_a(x_a)\delta_{a,k}^{rs} + [\sigma(2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs}]^+ \tag{3.29}$$

The above equations shows that the path flows are strictly positive, and optimal path flow $f_k^{rs}*$ is a valid stationary point of equation (3.25) and (3.26), subject to constraint (3.23) and (3.24). In addition, the total measured cost (3.30) is a summation of travel time cost and charging time penalty cost. Equation (3.29) and (3.30) show that the optimal solution satisfies the EVSUETP equilibrium conditions.

3.4 SOLUTION ALGORITHM

Both the electricity-charging cost and charging time penalty cost for a BEV motorist are calculated based on the length of his/her chosen route. We need to find the routes and the path flow pattern in order to obtain the generalized total travel cost of all BEV drivers. Therefore, a path-based algorithm for solving the above SUE problems is preferred. Among all possible choices, the simplicial decomposition method has been shown to be an efficient tool for equilibrium network flows.

3.4.1, Disaggregate Simplicial Decomposition

(Larsson and Patriksson 1992) first developed a modified simplicial decomposition method - the disaggregated simplicial decomposition (DSD), to solve the deterministic traffic assignment problem. They gave a disaggregated representation of the feasible solutions for convex problems over the Cartesian product sets.

For solving the traffic assignment problem, the DSD algorithm works at each iteration on a the subset of all routes between an O-D pair (restricted set) instead of on the complete set of all feasible routes between an O-D pair by solving a restricted master phase and a subproblem (column generation) phase iteratively. Following Larsson and Patriksson, the DSD algorithms works as follows (Larsson and Patriksson 1992):

"*Suppose that a disaggregated master problem, defined by a restricted set of routes,* $\widehat{K}_{rs}$*, has been solved. Given this solution, the shortest routes are calculated for all commodities. These sets* $\widehat{K}_{rs}$ *are augmented by the routes not contained in the sets already, and the procedure is repeated.*"

Based on this idea, (Damberg et al. 1996) extended the DSD algorithms for logit-based SUE traffic assignment problem. In the restricted master problem phase, a path flow pattern is obtained over the subset of routes $\widehat{K_{rs}}$, between each O-D pair, by directly applying the logit probability function. In the subproblem phase, new routes are generated and augmented into the restricted master set. The algorithm works iteratively between the master problem and subproblem phases to solve the SUE traffic assignment problem. Let $\mathbf{f^i}$ be a given feasible route flows vector at iteration i, the corresponding route costs could be calculated. Then an auxiliary route flow vector $\mathbf{h^i}$ is defined by

$$h_k^{rs}(i) = q^{rs} p_k^{rs}(i), \qquad \forall k \in \widehat{K}_{rs}, rs \tag{3.30}$$

If the vector $\mathbf{f^i} - \mathbf{h^i}$ is nonzero, a descent direction with respect to $z(\mathbf{f})$ is defined. A new solution $\mathbf{f^{i+1}}$ could be obtained by searching in the descent direction. The process is repeated until convergent, which means the master problem is solved. This algorithm is applied in this research with some adjustments in the column generation phase to solve the alternative SUE problems for BEVs. In particular, in the column generation phase, for solving either EVSUE or EVSUETP, we need to solve constrained shortest path (CSP) problems to find the augmenting routes.

The details of the algorithms applied to solve the two mathematical models developed in this chapter are discussed in the following sections respectively.

3.4.2, DSD for EVSUE

The total number of routes for each O-D pair can be very large; however, it is reasonable to assume that in practice a small portion of them will carry most of the demand. Therefore, column generation method, ideally, becomes a most used method to generated utilized routes, and the error made by assuming that the remaining routes have zero flow is small. The DSD algorithm works in a master problem phase while combined with a column generation phase for generating utilized routes.

3.4.2.1 Initialization

The initialized subset of the restricted master problem could be generated in one of the following two ways:

i. Calculated the shortest path for each O-D pair based on the free flow cost of the network, which could also be considered as the running the algorithm by going to the column generation phase (subproblem phase) first.

ii. As suggested by Damberg (1996), run a few steps of an incremental assignment algorithm (see Sheffi, 1985). The incremental assignment is summarized as following:

Step 0. Divide the entire trip distribution matrix q into smaller part matrices $\{q^1, q^2, \ldots q^m\}$, and the sum of all the part matrices should be equal to the actual trip distribution matrix, i.e. $q = q^1 + q^2 + \ldots + q^m$.

Set counter n=1, link flow is zero

Step 1. Based on the current link flow, calculate the current link costs, assign q^n part matrix using all-or-nothing assignment technique. Store the link volumes obtained from the all-or-nothing assignment.

Step 2. Update the link flows, n=n+1, go to Step 1.

Termination: when n=m

3.4.2.2 The restricted master problem phase

As presented in the previous section, the master problem is solved over a restricted subset of routes between each O-D pair, $\widehat{K}_{rs}$, by using a descent approach. The descent approach is presented as following:

Let $f_k^{rs}(i)$ be a given feasible route flow on route k between O-D pair (r,s) at iteration i. Calculate the resulted route costs. Let $C_k^{rs}(i) = C_k^{rs}(f_k^{rs}(i))$, for all routes in the subset $\widehat{K}_{rs}$, define an auxiliary route flow $h_k^{rs}(i)$ by the following equation:

$$h_k^{rs}(i) = q^{rs} p_k^{rs}(i) = q^{rs} \frac{\exp\{-\theta C_k^{rs}(f_k^{rs}(i))\}}{\sum_k \exp\{-\theta C_k^{rs}(f_k^{rs}(i))\}}, \quad \forall k \in \widehat{K}_{rs}, r, s \qquad (3.31)$$

Let $\mathbf{h}(i)$ be a vector of auxiliary path flows with element $\{h_k^{rs}(i)\}$ and $\mathbf{f}(i)$ be a vector of the current path flows with element $\{f_k^{rs}(i)\}$. If the vector $\mathbf{h}(i) - \mathbf{f}(i)$ is not zero, then it defines a descent direction with respect to objective function (e.g. z_1) In the direction of $\mathbf{h}(i) - \mathbf{f}(i)$, a line search is made, which results in a new solution vector $\mathbf{f}(i+1)$, and this process is repeated. The restricted master problem is solved when $\mathbf{h}(i) - \mathbf{f}(i) < \zeta$, where ζ is the tolerance. That is, the SUE conditions (e.g. Eq. (3.15)) are satisfied for the restriction of the original problem to the subset of the total set of routes. The motivation behind the above scheme is detailed in (Damberg et al. 1996; Larsson and Patriksson 1992).

After the above restricted master problem is solved on the subset of the routes, a column generation method is used to generate the augmenting route to the subset $\widehat{K}_{rs}$, i.e. solving the subproblem phase.

3.4.2.3 The subproblem-column generation phase

One method to generate route is through the solution of the shortest path problem based on the *measured* cost. By saying measured travel cost, it means, when generating the shortest path, the random components of the travel costs is ignored temporarily. The approach is motivated when the random components are less significant, which is usually the case in daily commute in urban network. In this situation, the value of the parameter θ is large and drivers are sensitive to the travel costs. This is usually true when the congestion level is high.

In our problem, for solving EVSUE, two constraint shortest path problem needs to be solved in order to generate the augmenting route into the restricted master problem routes set. The two constrained shortest path (CSP) problems, [EVSUE-Sub1] and [EVSUE-Sub2], are as follows:

Let x_{uv} be a binary variable, where x_{uv} equals to 1 if the link $(u,v)=a$ is on a given feasible path and equals to 0 if the link $(u,v)=a$ is not on the given feasible path. The two CSP subproblems for EVSUE are:

[EVSUE-Sub1]

$$z_1^{sub1} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv}x_{uv} + e_s \sum_{(u,v)\in A} d_{uv}x_{uv} - \frac{1}{2}(e_s - e_h)D \qquad (3.32)$$

subject to

$$\sum_{\{v|(u,v)\in A\}} x_{uv} - \sum_{\{u|(u,v)\in A\}} x_{uv} = \begin{cases} 1 & u = r \\ 0 & u \in N - \{r,s\} \\ -1 & u = s \end{cases} \tag{3.33}$$

$$\frac{1}{2}D < \sum_{(u,v)\in A} d_{uv} x_{uv} \le D \tag{3.34}$$

$$x_{uv} = \{0,1\}, \qquad \forall (u,v) \in A \tag{3.35}$$

[EVSUE-Sub2]

$$z_1^{sub2} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + e_h \sum_{(u,v)\in A} d_{uv} x_{uv} \tag{3.36}$$

subject to

$$\sum_{\{v|(u,v)\in A\}} x_{uv} - \sum_{\{u|(u,v)\in A\}} x_{uv} = \begin{cases} 1 & u = r \\ 0 & u \in N - \{r,s\} \\ -1 & u = s \end{cases} \tag{3.37}$$

$$\sum_{(u,v)\in A} d_{uv} x_{uv} \le \frac{D}{2} \tag{3.38}$$

$$x_{uv} = \{0,1\}, \qquad \forall (u,v) \in A \tag{3.39}$$

In this column generation phase, the above two CSP problems need to be solved to find the augmenting path. The objective values from these two CSP problems, z_1^{sub1} and z_1^{sub2}, are compared. If $z_1^{sub1} < z_1^{sub2}$, the resulted shortest path from [EVSUE-Sub1] will be chosen and added into the restricted master problem set, otherwise, chose the constrained shortest path generated by [EVSUE-Sub2]. Referring to Assumption 2, the feasibility of at least one of the subproblems is assured, since the feasible sets of [EVSUE-Sub1] and [EVSUE-Sub2] are complementary sets in the region $[0, D]$. Furthermore, if $\sum_{(u,v)\in A} d_{uv} x_{uv} \le \frac{D}{2}$ is feasible to both [EVSUE-Sub1] and [EVSUE-

Sub2], z_1^{sub2} is always less than z_1^{sub1} because $e_h < e_s$. Due to this results, the constraint (3.35) for [EVSUE-Sub1] could be simplified to $\sum_{(u,v)\in A} d_{uv} x_{uv} \leq D$.

3.4.2.4 Lagrangian relaxation and enumeration for constrained shortest path[2]

The CSP problem is known to be NP-complete in the weak sense.

[CSP] (Ahuja et al. 1993b)

$$z = \min_{\mathbf{x}} \mathbf{cx} \tag{3.40}$$

subject to

$$A\mathbf{x} = \mathbf{b} \tag{3.41}$$

$$F\mathbf{x} \leq \mathbf{g} \tag{3.42}$$

$$\mathbf{x} = \{0,1\} \tag{3.43}$$

where the Eq. (3.43) are the side constraints, and the structure leads to binary solutions without explicit constraint $\mathbf{x} \leq 1$.

A number of different methods have been developed to solve the above CSP problem, and exact methods appearing in the literature can be divided into three categories: those based on the kth-shortest path algorithm, dynamic programming, and Lagrangian relaxation.

A number of different methods have been developed to solve the CSP problem, and methods appearing in the literature can be divided into three categories: those based on the k-shortest path algorithm, dynamic programming, and Lagrangian relaxation. Algorithms based on dynamic programming used node-labeling schemes have been

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[2] This section and the based algorithm is summarized from a variety of sources, namely Ahuja,et. al. (1993), Carlyle et. al. (2008), Carlyle and Wood (2005), Dumitrescu and Boland (2003), Handler and Zang (1980).

considered as an effective one for weight constrained shortest path problem, such as the work by Desrosiers et. al. (Desrochers and Soumis 1988; Desrosiers et al. 1995; Dumitrescu and Boland 2003; Jaumard et al. 1996; Jiang et al. 2012). Recently, new techniques based on Lagrangian relaxation are proposed, such as the Lagrangian relaxation plus enumeration (LRE) method (Carlyle et al. 2008). A new algorithm for enumerating the near-shortest path (NSPs) (Carlyle and Wood 2005) was developed, and is more appropriate in the context of LRE for the CSP problem than "k-shortest paths" which is used (Handler and Zang 1980). This algorithm was compared with the label-setting algorithm of (Dumitrescu and Boland 2003), which suggests that it is more computational efficient in solving the single-resource-constrained shortest path problem, which is in our case. In this research we adopted the algorithm developed by Carlyle et. al. (Carlyle et al. 2008), with slightly modification in solving the Lagrangian relaxation formulation.

Noted that, the CSP problem Eq. (3.41) – Eq. (3.44) could be easily solved as a shortest path problem with modified length values, if the constraint Eq. (3.43) could be relaxed, as shown in Eq. (3.45). Using the standard theory of Lagrangian relaxation, the relaxed shortest path (RSP) problem could be written as

[SPLR]

$$z \geq L(\lambda) = \min_{\mathbf{x}} \mathbf{cx} + \lambda(F\mathbf{x} - \mathbf{g}) \tag{3.44}$$

Subject to (3.42) and (3.44).

where $\boldsymbol{\lambda}$ is the Lagrangian multiplier vector for the constraint Eq. (3.43).

The Lagrangian lower bound is

$$\underline{z} = \max_{\lambda \geq 0} L(\lambda) = \max_{\lambda \geq 0}[\min_{\mathbf{x}}(\mathbf{c} + \lambda F)\mathbf{x} - \lambda \mathbf{g})] \tag{3.45}$$

subject to (3.42) and (3.44).

The outer maximization over $\boldsymbol{\lambda}$ was solved via bisection search in (Carlyle et al. 2008), which is sufficient for single side constraint, but we will solve it using subgradient optimization, which is more general.

Suppose $L(\boldsymbol{\lambda})$ has a unique solution $\mathbf{x'}$ and is differentiable. Then the solution $\mathbf{x'}$ remains optimal for small change of $\boldsymbol{\lambda}$, and let the small change be $\alpha(F\mathbf{x'}-\mathbf{g})$, which is the direction and α is the step size. For Lagrangian multiplier updating, the following notations are defined:

λ^{k+1} $\max\{\lambda^k + \alpha_k(Fx^k - g), 0\}$

λ^0 initial choice of the Lagrangian multiplier

x^k solution to the Lagrangian subproblem when $\lambda = \lambda^k$

α_k step length at the *k*th iteration

The choice of step size is important for convergence of the solution, and one simple example is just to set $\alpha_k = \frac{1}{k}$. However, a reasonable and popular way for selecting the step length is

$$\alpha_k = \frac{\mu^{k*}[\bar{z} - L(\lambda^k)]}{| Fx^k - g |^2} \tag{3.46}$$

Where $\bar{z}$ is the upper bound on the optimal objective function z^*, and μ^k is a scalar between 0 and 2. One can start with μ^k=2 and then reduce μ^k until failing to find a better solution.

Often, in process of optimizing $L(\lambda)$, if F is non-negative, a feasible solution $\hat{\mathbf{x}}$ could be found, and the upper bound can be compute $\bar{z} \equiv \mathbf{c}\hat{\mathbf{x}}$. Given this upper bound, and a good vector $\boldsymbol{\lambda}$, the problem of identifying $\mathbf{x}^*$ is a straightforward enumeration:

Theorem 3.1 (Carlyle et al. 2008). *Let* $\widehat{X}(\lambda, \bar{z})$ *denote the set of feasible solutions to RSP such that* $(\mathbf{c} + \lambda F)\mathbf{x} - \lambda\mathbf{g} \leq \bar{z}$. *Then* $\mathbf{x}^* \in \widehat{X}(\lambda, \bar{z})$. *That is, an optimal solution* $\mathbf{x}^*$ *to CSP can be identified by enumerating* $\widehat{X}(\lambda, \bar{z})$ *and selecting* $\mathbf{x}^* \in \underset{\mathbf{x} \in \widehat{X}(\lambda, \bar{z}) | F\mathbf{x} \leq \mathbf{g}}{\arg\min} \; \mathbf{cx}$.

The proof to the above theorem could be found in (Carlyle et al. 2008). It was shown only $\lambda \geq 0$ is required to satisfy Theorem 3.1 and an near-optimal λ for the RSP can exponentially reduce the computational workload by reducing the size of $\widehat{X}(\lambda, \bar{z})$.

Note that Theorem 3.1 implies a complete enumeration of each path, represented by $\hat{\mathbf{x}}$, satisfying $(\mathbf{c} + \lambda F)\hat{\mathbf{x}} - \lambda\mathbf{g} \leq \bar{z}$. In other words, if $\mathbf{x}_\lambda{}^*$ solves the shortest path problem given the modified link-length vector $\mathbf{c} + \lambda F$, and $L(\lambda) \equiv (\mathbf{c} + \lambda F)\hat{\mathbf{x}}_\lambda{}^* - \lambda\mathbf{g}$, then the CSP is solved by enumerating all path $\hat{\mathbf{x}}$ such that $L(\lambda) \leq (\mathbf{c} + \lambda F)\hat{\mathbf{x}} - \lambda\mathbf{g} \leq \bar{z}$. In turn , this means that given the modified link-length vector $\mathbf{c} + \lambda F$, all the ϵ-optimal path for $\epsilon \equiv \bar{z} - L(\lambda)$ need to be identified. (Carlyle and Wood 2005) presented a new near-shortest path (NSP) algorithm to identify all the ϵ-optimal paths, which is more natural for *k*-shortest paths algorithm in this problem setting. In the new NSP algorithm, all paths that are within ϵ units of being shortest for a pre-specified $\epsilon \geq 0$ are enumerated in an efficient manner (see (Carlyle and Wood 2005) for the outline and pseudo-code of the NSP algorithm).

In overall, the adopted Lagrangian relaxation plus enumeration algorithm for solving the CSP problem, which is the algorithm for solving the subproblems of our DSD algorithm, is summarized as following:

1. Reformulate the original CSP problem into a shortest path Lagrangian relaxation problem.
2. Use subgradient optimization method to optimize the Lagrangian lower bound $L(\lambda)$ to find a “good” Lagrangian multiplier vector $\boldsymbol{\lambda}$.

3. Identify a feasible path $\hat{\mathbf{x}}$ and an upper bound on z^*, let the upper bound be $\bar{z}$, then $\bar{z} \equiv \mathbf{c}\hat{\mathbf{x}}$.
4. Let the O-D pair be (r,s), use the NSP algorithm to enumerate all path $\hat{\mathbf{x}}$ such that $(\mathbf{c}+\lambda F)\hat{\mathbf{x}} - \lambda\mathbf{g} \le \bar{z}$
   a. Use the modified link-length $\mathbf{c}(\lambda) = \mathbf{c} + \lambda F$, calculate the all-to-one shortest-paths, from any node v in the network to the destination node s, and name the corresponding minimum distance from v to s as the minimum *Lagrangian distance*, noted as $c_l(v)$.
   b. Calculate the corresponding distance (which is the actual link length in our problem) $d(v)$ from node $\forall v \in V$ to destination s.
   c. Perform a standard path-enumeration algorithm with modifications. Suppose we have a current $r \to u$ subpath, where r is the origin, $u \in V$:
      i. this subpath is extended along the link (u,v) if and only if (i) the length of the current subpath $l_\lambda(u)$, plus the modified link-length $c_{us}(\lambda)$, plus $c_l(v)$, does not exceed the definition of the "near-shortest", i.e. $l_\lambda(u) + c_{uv}(\lambda) + c_l(v) \le \bar{z} - \zeta$, where the "near-shortest" path is defined by a solution within units of the optimality rather than an exact solution.(ii) the path des not loop back on itself.
      ii. Use the side constraints to reduce the amount of enumeration by not violating the side constraints. That is, in our problem, if the current true length of the subpath $r \to u$, is noted as $l(u)$, do not extend the path if $l(u) + d_{a=(u,v)} + c_l(v) > g$. In

addition, do not extend a subpath when it would lead to violation of the inequality $\mathbf{cx} < \bar{z}$.

iii. Update the current solution $\hat{\mathbf{x}}$, and the upper bound, $\bar{z} = \mathbf{c}\hat{\mathbf{x}}$ whenever a better solution is detected.

5. The best solution, $\hat{\mathbf{x}}$ identified during this process, is optimal.

So far, the algorithm for soling the CSP problem is presented. This subproblem phase generate the CSP, that could be augmented to the master problem phase. The whole DSD algorithm will be terminate if the change in the path flows between two consecutive restricted master problem less than the tolerance.

3.4.3, DSD for EVSUETP

The initialization and restricted master problem phase for solving the EVSUETP problem is the same as those for solving the EVSUE problem. The algorithms are presented in the previous sections. For the column generation, which is the subproblem phase, the Lagrangian relaxation and enumeration is applied to solve CSP problems. For EVSUETP, different CSP problems need to be solved for the subproblem phase.

3.4.3.1 The subproblem-column generation phase

For solving EVSUETP, using the intuitive objective function we developed earlier, Eq. (3.21) will result in solving three constrained shortest path problem as the subproblem phase, as follows.

[EVSUETP-Sub1]

$$z_2^{sub1} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + t^{rs} \tag{3.47}$$

subject to

$$\sum_{\{v|(u,v)\in A\}} x_{uv} - \sum_{\{u|(u,v)\in A\}} x_{uv} = \begin{cases} 1 & u = r \\ 0 & u \in N - \{r,s\} \\ -1 & u = s \end{cases} \tag{3.48}$$

$$\frac{1}{2} D < \sum_{(u,v)\in A} d_{uv} x_{uv} \leq D \tag{3.49}$$

$$x_{uv} = \{0,1\}, \qquad \forall (u,v) \in A \tag{3.50}$$

[EVSUETP-Sub2]

$$z_2^{sub2} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + t^{rs} \tag{3.51}$$

Subject to Eq. (3.49), Eq. (3.51) and

$$\sum_{(u,v)\in A} d_{uv} x_{uv} \leq \frac{D}{2} \tag{3.52}$$

[EVSUETP-Sub3]

$$z_2^{sub3} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + \sigma(2 \sum_{a=(u,v)\in A} d_a x_{uv} - D) \tag{3.53}$$

Subject to Eq. (3.49)-Eq. (3.51) .

The above three CSP problems need to be solved separately. In the case when all three CSP problems are feasible, the result of the [EVSUETP-Sub1] and [EVSUETP-Sub3] needs to be compared first. The one with the larger objective value is picked and further compared with [EVSUETP-Sub2], finally, the one with smaller objective value gives the augmenting path. In other words, the augmenting path is obtained by the subproblem with the objective value $\min\{z_2^{sub2}, \max\{z_2^{sub1}, z_2^{sub3}\}\}$ If any of the subproblems is not feasible, just compare the remaining ones based on the above logic. It is guaranteed that at least one of the above three subproblems will be feasible, as discussed in section 3.4.2.3.

As discussed in section 3.3.2.2, the alternative objective function Eq. (3.25), is developed to reduce the number of constraint shortest path problems that need to be solved in the subproblem phase. We present here the two CSP problems that need to be solved, which give the same augmenting route into the restricted master problem routes set as the above three CSP problems, namely, [EVSUETP-Sub4] and [EVSUETP-Sub5].

[EVSUETP-Sub4]

$$z_3^{sub4} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} \tag{3.54}$$

subject to E.q (3.49) and

$$\sum_{(u,v)\in A} d_{uv} x_{uv} \leq \frac{D}{2} + \frac{t_m^{rs}}{2\sigma} \tag{3.55}$$

$$x_{uv} = \{0,1\}, \qquad \forall (u,v) \in A \tag{3.56}$$

[EVSUETP-Sub5]

$$z_3^{sub5} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + \sigma[(2 \sum_{a=(u,v)\in A} d_a x_{uv} - D) - t^{rs}] \tag{3.57}$$

Subject to Eq. (3.49), Eq.(3.56) and

$$\frac{D}{2} + \frac{t_m^{rs}}{2\sigma} < \sum_{(u,v)\in A} d_{uv} x_{uv} \leq D \tag{3.58}$$

The above two CSP problems need to be solved, and the objective values from these two CSP problems are compared to determine the augmenting path. If $z_3^{sub4} < z_3^{sub5}$, the resulted CSP from [EVSUETP-Sub4] will be chosen and augmented to the restricted master problem set, otherwise, augment the CSP generated by [EVSUETP-Sub5].

3.4.4, Algorithm synthesis

A modified DSD algorithm for solving both EVSUE and EVSUETP would be as follows,

Step 0. Initialization

Find an initial subset of routes $\widehat{K}_{rs}$ for each O-D pair (r,s)

Compute and initial route $\mathbf{f}_0^0$ (in the case of using shortest path for initialization, assigning all trips onto the shortest path between each O-D pair)

Set $i = 0$

Step 1. Restricted master problem phase

Set $j = 0$.

Let $\mathbf{f^j} = \mathbf{f_0^i}$

Repeat the following until the stop criteria is met

Compute the total cost of choosing route k at *j*th iteration, $C_k^{rs}(\mathbf{f^j})$ for all routes in the subset $\widehat{K}_{rs}$

Compute the auxiliary route flow $\mathbf{h}^j$ according to the formulation

$$\mathbf{h}^j = q^{rs} \frac{\exp\{-\theta C_k^{rs}(\mathbf{f}^j)\}}{\sum_{l \in \widehat{\mathrm{K}}_{rs}} \exp\{-\theta C_l^{rs}(\mathbf{f}^j)\}}$$

Stop if $|\,\mathbf{f}^j - \mathbf{h}^j\,| < \varepsilon$.

Otherwise, find the step size t_j , and let the new point be

$$\mathbf{f}^{j+1} = \mathbf{f}^j + t_j(\mathbf{h}^j - \mathbf{f}^j)$$

Set $j := j+1$.

Output $\mathbf{f^j}$

Step 2. Subproblem phase (column generations)

For EVSUE problem, using the LRE to solve two CSP [EVSUE-Sub1] and [EVSUE-Sub2], get objective value and binary vector as $(z_1^{sub1}*, \mathbf{x}_1*)$ and $(z_1^{sub2}*, \mathbf{x}_2*)$ respectively.

If both [EVSUE-Sub1] and [EVSUE-Sub2] are feasible, and $z_1^{sub1}* \leq z_1^{sub2}*$, output $\mathbf{x}_1*$; if $z_1^{sub1}* > z_1^{sub2}*$, output $\mathbf{x}_2*$.

If [EVSUE-Sub1] is infeasible, then output $\mathbf{x}_2*$

If [EVSUE-Sub2] is infeasible, then output $\mathbf{x}_1*$

For EVSUETP problem, using the LRE to solve two CSP [EVSUETP-Sub4] and [EVSUETP-Sub5], get objective value and binary vector as $(z_3^{sub4}*, \mathbf{x}_1*)$ and $(z_3^{sub5}*, \mathbf{x}_2*)$ respectively.

If both [EVSUETP-Sub4] and [EVSUETP-Sub5] are feasible, and $z_3^{sub4}* \leq z_3^{sub5}*$, output $\mathbf{x}_1*$; if $z_3^{sub4}* > z_3^{sub5}*$, output $\mathbf{x}_2*$.

If [EVSUETP-Sub4] is infeasible, then output $\mathbf{x}_2*$

If [EVSUETP-Sub5] is infeasible, then output $\mathbf{x}_1*$

Let $\mathbf{f}^{i+1} = \mathbf{f}^i$

Set $i := i + 1$

Go to step 1.

Note in the proposed algorithm, we pick up the constrained shortest path as the to-be-added path in the column generation phase at each iteration. The iteration terminates when there is no new shortest path to be added to the master problem. The results are approximations and the accuracy has not been rigorously investigated. The algorithm may have the short come that it terminates too early without having enough routes in the solution space. Herein, we propose two methods to compensate the short come. Given that our numerical experiments show that the proposed algorithm is able to obtain satisfactory solutions. The two improvement methods are supplemented here for the completeness of the solution procedure.

The first supplementary approach is to find the constrained k-shortest path instead of the constrained shortest path in the column generation phase. These *k*-shortest paths will be augmented into the route set in the restricted master problem phase. Obviously, the accuracy of the results depends on the value of *k*. This *k* could be an increasing constant at every iteration, until a suitable value of obtained. For example, let $k = 1$, which means to find the constrained shortest path, if this path has already been included in the master problem set, increase $k = 2$, check if there is new path in this 2 shortest path to be augmented in the master problem set, if yes, add it, if not, increase $k = 3, 4, 5$ … until new path is find or a certain termination condition is reached.

The second approach is to solve the traffic assignment problem using an exact link-based algorithm instead to obtain the link-flow pattern. This link-flow pattern can be converted into the path-flow pattern by the following approximation method. That is to find the *k*-shortest paths between each O-D pair and the use the logit function derived in this paper to assign O-D trip flows on the *k* paths in terms of the equilibrium travel costs. A sufficient large value of *k* should be used so that the generated path flow pattern can closely approximate and the link flow pattern. The second approach is preferred to the first one in terms of computational efficiency.

3.5 NUMERICAL ANALYSIS

This section first describes the example network used in this numerical example, and then analyzes the flow patterns with different charging cost parameters both at home and at destination for the BEVs assignment model. Also, the flow patterns and duration of stay at trip end for the EVSUEPT assignment models will be studied.

3.5.1, Example networks

Two networks are used in the numerical experiments. The numerical experiments are conducted to examine the influence of electricity-charging cost on the BEVs drivers for choosing path, to study the impacts of using BEVs on the duration of stay at destination, and to gain insights on the spatial flow pattern in the traffic network as well.

The first network is a four-node, four-route network with single O-D pair (A→D), depicted in Figure 3-4. It will be used to illustrate the charging costs impacts on the BEV travelers' path choice, the impacts on duration of stay, and path flow patterns of using BEVs in a trivial uncongested manner. The second is the Sioux Falls network, depicted in Figure 3-5. It will be used in analyzing the impacts under different scenarios of electricity-charging prices and speeds.

The first network is a small uncongested network, with (travel cost, link length) marked on each link in Fig 3-4. There are 4 paths from node A to node D, path #1 (A→B→D), path #2 (A→B→C→D), path #3 (A→C→B→D) and path #4 (A→C→D). The total trip demand between A and D is 100. The BEVs have an effective range of 120 miles.

The Sioux Falls network has 24 zones, 24 nodes, 76 links and 528 O-D pairs. The travel cost is calculated by using the BPR function:

$$t_a(x_a) = t_0 \cdot [1 + 0.15 \times (\frac{x_a}{C})^4] \tag{3.59}$$

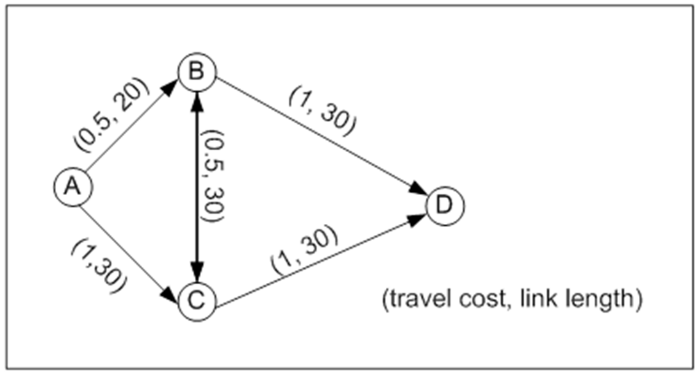

Figure 3-4: A Trivial Sample Network

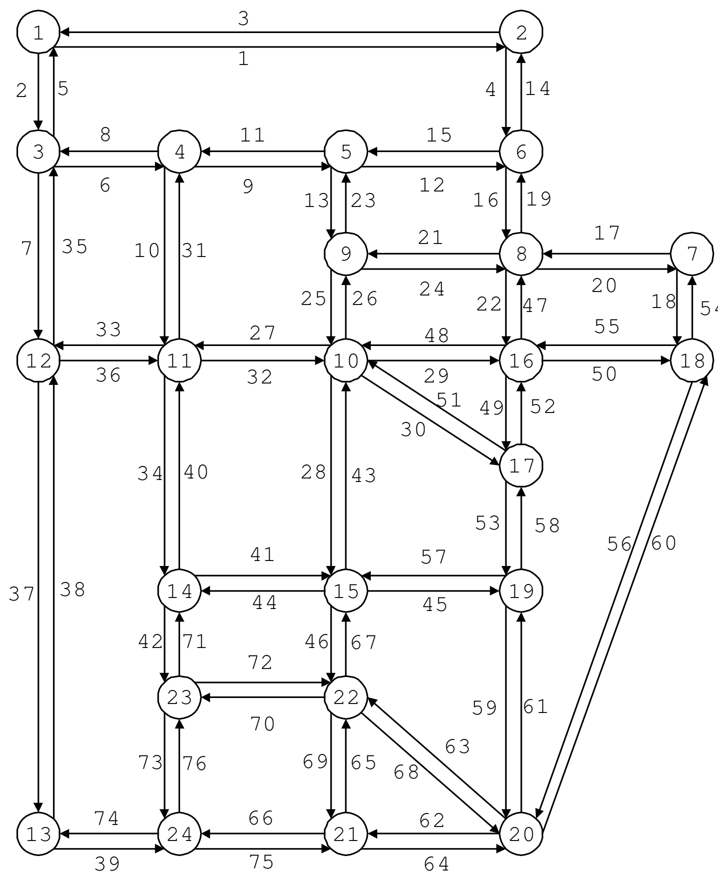

Figure 3-5: Sioux Falls Test Network

For the first trivial network, the charging cost is calculated by using the threshold for charging at 60 miles, which is half of the vehicle's effective range. The per unit distance charging cost at home is set as $e_h = 0.01$ units, and the per unit distance charging cost at destination is set as $e_s = 0.03$ units. The example network is solved by assigning probabilities of using on each utilized route (we let all four routes in this network be utilized). The total cost is the summation travel cost on a route and the charging cost for choosing the route. After the probabilities are calculated, the path flows could be obtained by assigning the O-D trips onto each path. Table 3-1 shows the traffic patterns with and without considering the electricity-charging cost. Column 2 gives the total travel cost on each route, and column 3 is the path length of the routes. The charging cost shown in column 4 is a result of the piecewise function. We can see that electricity-charging cost at destination is zero if the path length if not greater than 60. The total cost in column 5 is the summation of the travel cost and the charging cost. Column 6 shows the path flow pattern without considering the charging cost, which is a result from the traditional SUE, and column 7 shows the path flow pattern by taking the charging costs into account. The difference between column 6 and column 7 makes the charging costs a reasonable consideration. The results shows that although the travel costs for route # 1 and route # 3 are the same, the paths using link B-C is less attractive sine taking this link will increase the path length which increases the charging cost. Therefore, more BEV drivers will choose the route #1 and route #4. The decreases of flows on link B-C and C-B can easily been seen in Figure 3-6.

path #		travel cost	path length	charging cost	total cost	path flow (non-EV)	path flows (EV)
1	A-B-D	1.5	50	0.5	2	38.7455619	55.9164
2	A-B-C-D	2	80	1.8	3.8	23.50037122	9.2429
3	A-C-B-D	2.5	90	2.1	4.6	14.25369566	4.1531
4	A-C-D	2	60	0.6	2.6	23.50037122	30.6876

Table 3-1 Path Flow Patterns for the Trivial Network

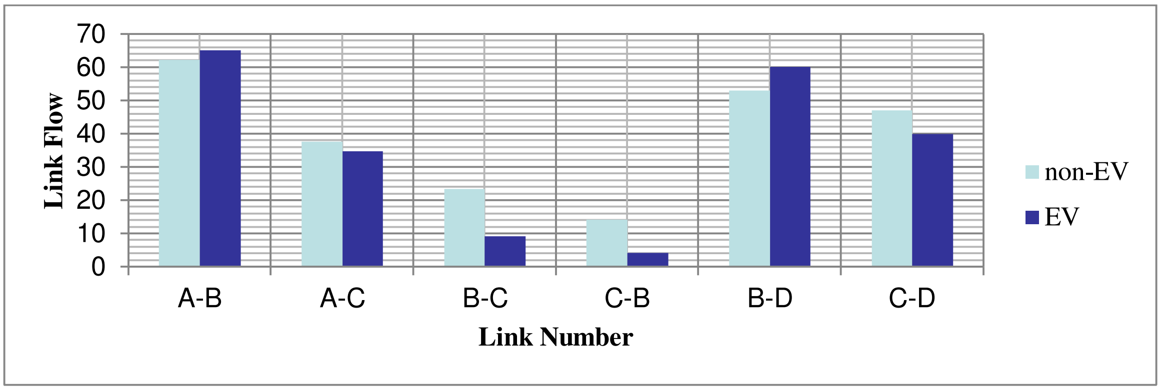

Figure 3-6: Link Flow Patterns

3.5.2, Network flow pattern

We first compare the BEV flow pattern and the GV flow pattern in the traffic network, where the GV flow pattern is produced by solving the following SUE problem.

$$\min_{f_k^{rs}} z_4 = \frac{1}{\theta}\sum_{rs}\sum_{k} f_k^{rs}(\ln f_k^{rs}) + \sum_{a}\int_0^{x_a} t_a(w)dw + \sum_{a} g\cdot d_a \cdot x_a \qquad (3.60)$$

subject to Eq (3.9)-(3.11).

where g is the travel time-equivalent fuel cost per unit distance for GVs, and let us set $g = 4e_h$. In this particular numerical test, we set $e_h = 0.08, g = 0.32$, and $e_s = 0.16$. The BEV traffic assignment is conducted using the model in Problem EVSUE. This network flow pattern is studied on the Sioux Falls network. Table 3-2 gives the link flow pattern for GVs and the link flow pattern for BEVs with the above discussed gas and electricity-charging prices. It can be seen that the link flow pattern differs a lot on some of the links, for example Link 22, 24, 49,56, etc. With the gas price settled as 4 times the home electricity-charging price and 2 times the destination charging price, the BEV drivers tends to use longer routes than GV drivers since the operating cost is much smaller. However, it should be noted that this choices also depend on the network congested level, the charging speeds, the BEV range etc. If the charging speed is not too slow (which means the charging time cost for BEV drivers is not very high) and the BEV range is long enough for most routes (which means the routes choice set for BEV drivers is similar to that of the GV drivers), we can conclude that with $e_h < e_s < g$, BEV drivers tends to choose the longer path. In such case the total vehicle miles traveled (VMT) for BEVs are longer than that of GVs, while the total cost (total travel time cost and the operating cost) for BEVs is less than that of GVs. For this numerical test, the VMT for GVs is 2060103.12 and the VMT for BEVs is 2096140.893, which is greater than GVs. The total cost for GVs drivers is 1996555.293 and the total cost for BEVs drivers is 1527119.117, which is less than that of GVs drivers.

Table 3-2 Link Flow Pattern of GVs and BEVs and System Performance

Link_ID	1	2	3	4	5	6	7
GV	1583.091	2500.909	1583.091	2750.091	2500.909	3676.097	2135.189
BEV	1455.862	2628.138	1455.899	2622.862	2628.101	3964.663	2554.395
Link_ID	8	9	10	11	12	13	14
GV	3551.097	5428.62	2935.729	5303.621	3232.596	4051.366	2750.091
BEV	3973.584	5822.77	3380.53	5489.602	4660.328	4213.398	2622.899
Link_ID	15	16	17	18	19	20	21
GV	3107.596	5529.913	2762.027	5201.759	5404.913	2762.027	468.3867
BEV	4340.902	5714.344	3138.155	5092.433	5394.955	3122.627	643.3024
Link_ID	22	23	24	25	26	27	28
GV	5998.665	4051.366	468.3867	8107.659	8149.659	9637.856	5660.797
BEV	4908.069	4199.657	1259.781	8188.643	8833.379	9854.077	6735.008
Link_ID	29	30	31	32	33	34	35
GV	11732.41	312.656	2977.729	9429.856	5237.35	7122.507	2260.189
BEV	9756.237	416.4767	3764.618	9266.424	4661.967	6547.056	2545.437
Link_ID	36	37	38	39	40	41	42
GV	5112.35	4832.621	4873.621	4339.462	7122.507	3443.88	4503.028
BEV	4618.859	4674.372	4663.306	4720.795	6427.599	3219.728	3874.309
Link_ID	43	44	45	46	47	48	49
GV	5702.797	3443.88	8871.933	8826.511	5873.665	11898.41	12305.02
BEV	7108.747	3296.248	7219.916	8536.531	3956.673	10600.97	10536.46
Link_ID	50	51	52	53	54	55	56
GV	4274.175	312.656	12305.02	10612.37	5201.759	4315.175	2652.788
BEV	4882.63	388.3919	10241.62	8939.849	5107.961	5070.802	3987.735
Link_ID	57	58	59	60	61	62	63
GV	8871.933	10612.37	3930.688	2652.788	3930.688	1845.746	1829.177
BEV	7323.179	8616.931	4168.728	4150.436	3949.074	2717.511	2446.846
Link_ID	64	65	66	67	68	69	70
GV	1804.746	5153.896	5065.92	8826.511	1829.177	5153.896	3719.632
BEV	2669.308	4557.513	5307.921	8841.526	2397.097	4656.783	3350.631
Link_ID	71	72	73	74	75	76	
GV	4503.028	3719.632	2799.82	4339.462	5024.92	2799.82	
BEV	3678.333	3705.147	2705.402	4668.729	5160.447	2863.941	

3.5.3, Impacts of charging price

According to the assumption that the range of the vehicle would be enough for at least the one way trip, the we manipulate the link lengths in the network, and let the longest shortest path (in terms of path length only), noted as D_s, be approximately $D_s = 50$ distance units (miles), and set the vehicle range accordingly, as $\kappa \cdot D_s$, where κ is a multiplier, $\kappa = \{1, 1.1, 1.2, \ldots, 2\}$.

3.5.3.1 Sensitivities to electricity-charging prices at home

In this numerical test, we fixed the electricity-charging price at the destination, and change the per unit distance electricity-charging price at home for BEV drivers. The per unit distance electricity-charging price at destination is set to $e_s = 0.1$ units, which is close enough to the real world case. However, this price could be varying in a large range due to different cities, or different destinations. The full tests results are shown in the Appendix A.

From the Table A-1, we note that not the entire link flow pattern has significant changes. For instance, link 61, link 59, link 51, link 43 have minor changes with the changes of e_h. However, some of the link flows have been impacted by the changes in e_h in a more significant way (up to 15% changes in link flows). The link flow changes are divided into 5 ranges, [0, 3%) ,[3%, 6%), [6%, 9%), [9%, 12%), [12%, 15%), the number of links, out of the total 76 links, that have the link flow changes following in each range are counted. The following table and figure (Figure 3-7) shows these statistical counts of number of links.

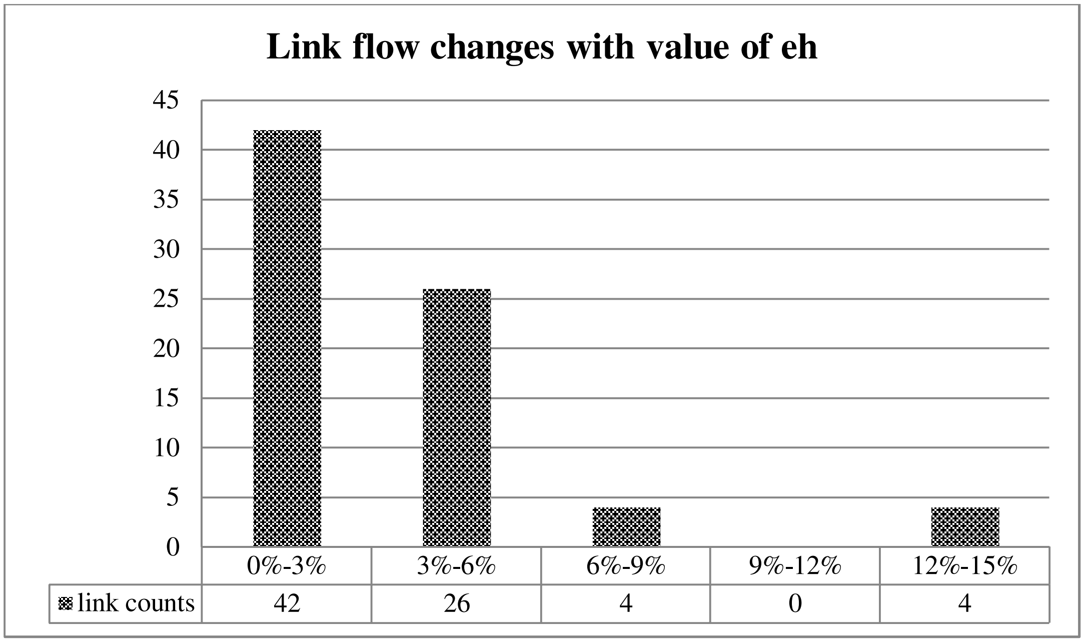

Figure 3-7: Counts in Each Range of Link Flow Changes

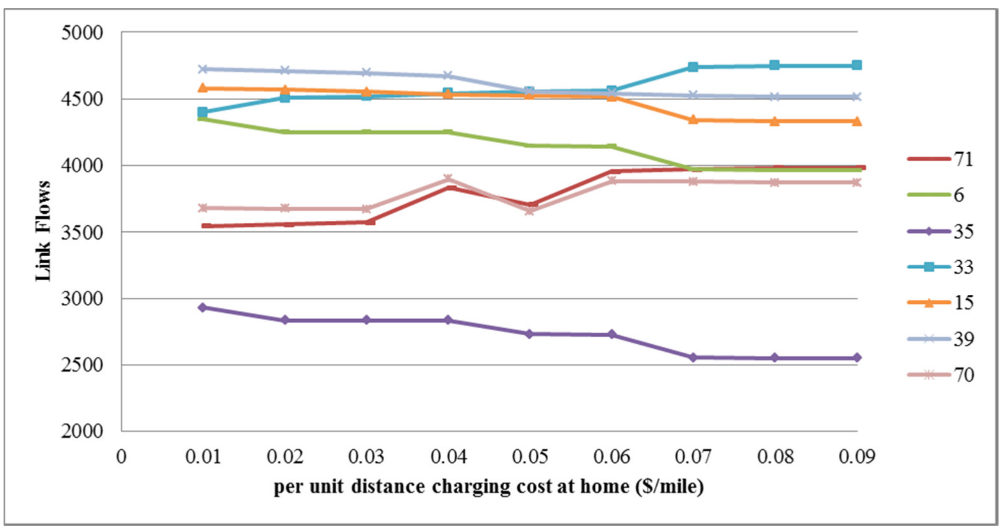

Figure 3-8: Link Flow Changes According to e_h

Figure 3-7 shows that, most of the links have flow changes less than 6%, which is not very significant. However, this number really depends on how BEV drivers value the electricity-charging cost comparing with the travel time cost. In addition, we set the destination charging cost at a fixed value, 0.1, and the charging cost at home is lower than that, which makes the electricity-charging cost not expensive. It is nature to anticipate that when the electricity-charging cost at destination and at home both increase at the same time, the percentage of links that have a higher link flow changes will increase, but with the similar changing trends.

Figure 3-8 shows some selected links with notable changes in the link flows. Some of the link flows decrease (link 6, link 15, link 33, link 39)) with the increase of the electricity-charging cost; while some link flow increases (link 35, link 70, link 71). In both case, we can see that the most link flow changes have a sudden drop or a sudden increase, for example, flows on link 39 is near stable when the charging cost at home is less than 0.04, but with more significant decrease when the cost reaches 0.05. In addition, flows on link 71 increase suddenly when charging cost at home increase from 0.03 to 0.04. This phenomenon is reasonable because of the piecewise charging cost functions. By noticing such phenomenon and thus acquiring the "turning point" will provide policy makers a tool for setting the value of electricity-charging price at home given fixed charging cost at destinations.

3.5.3.2 Sensitivities to electricity-charging price at destination

In addition to the above section, we also study the electricity-charging price at destination given a fixed electricity-charging price at home, which is more realistic, since the electricity-charging price at destinations in public charging infrastructure are usually within control and the electricity-charging price at home is highly typically related to the electricity cost for residents.

The electricity-charging price per unit distance at home is fixed to be consistent to the real world, which is 0.08. The per unit distance electricity-charging price at destinations with public charging infrastructure is set from 0.1 to 0.5. The full tests results are shown in the Appendix A.

From the Table A-2, we that link flow patterns changes more than that in the previous case (section 3.5.3.1). The maximum change in link flow is up to 55%, which is significantly high. The link flow changes are divided into 11 ranges, form 0% to 55% with 5% interval for each range. The number of links that have the link flow changes falling in each range is counted. The following table and figure (Figure 3-9) shows these statistical counts of number of links. There are about 38% of the total links that have link flow changes more than 10%, and 16 links out of 76 total links have link flow changes more than 15%. This changes is due to the significant increase in e_s. Unlike in the previous section, where home electricity-charging price could not be too high, the destination electricity-charging price could be much higher than that at home. Therefore, by controlling the destination electricity-charging price, the network flow patterns of BEV users could be very different. The destination electricity-charging price is more changeable, such as giving coupon for travelers, charge for time-vary costs, etc. Although not all the link flows in the network are sensitive to the charging cost (e.g. this can be seen from both Fig. 3-7 and Fig. 3-9), the model still proved us a reference tool for controlling some of the link flows by changing the electricity-charging price.

From Fig. 3-10, the 'sudden drop' and 'sudden increase' of link flows according to e_s is more obvious. For instance, flows on link 56 and link 50 drop dramatically when the electricity-charging price at destination increases from 0.14 to 0.18. The flows on link 23 increase a lot when the electricity-charging price at destination increases from 0.18 to 0.22. In both cases, after the significant decrease or increase, the link flows are more stable and seem not very sensitive to the changes of electricity-charging price at destination any more.

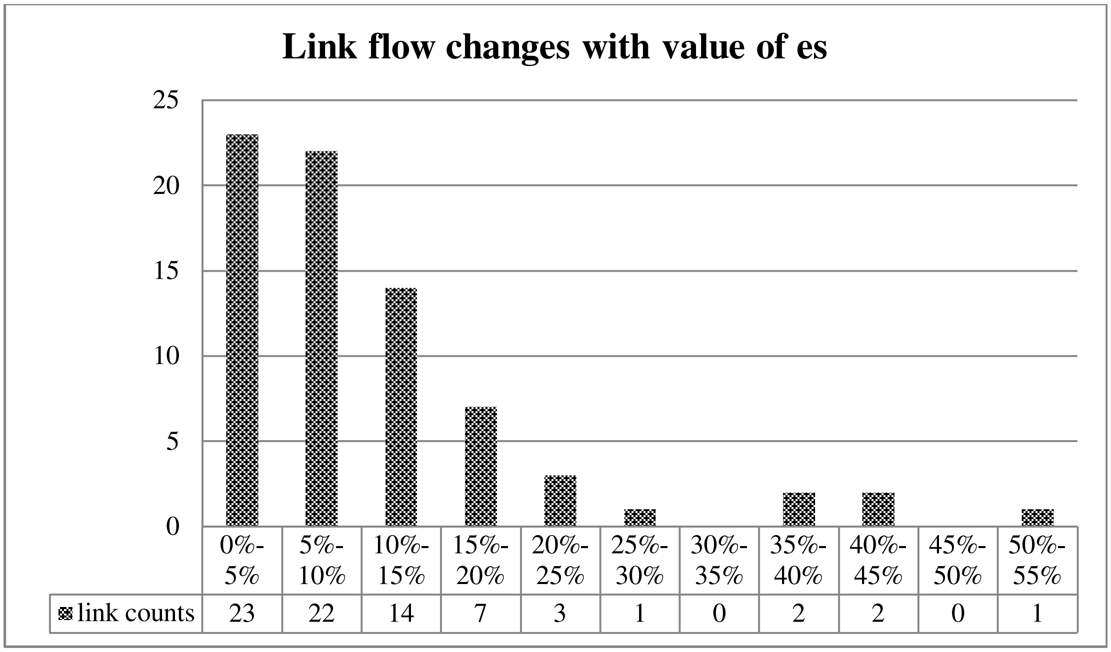

Figure 3-9: Counts in Each Range of Link Flow Changes

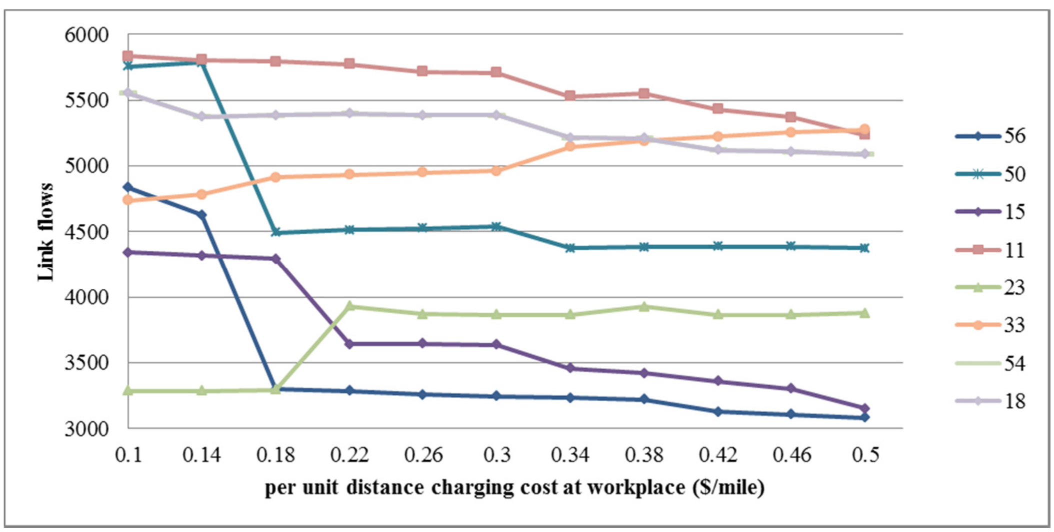

Figure 3-10: Link Flow Changes According to e_s

3.5.4, Impacts of charging speed

This section investigates the impacts on the duration of stay for BEV drivers between all O-D pairs under different charging speed. The charging speed is represented by the parameter σ in the formulation, which is the charging time needed for per unit distance traveled. Therefore, the larger σ is, the slower the charging speed is and vice versa. We set σ from 0.05 to 0.3, that is, for instance, for a BEV with effective range of 60 miles (the manufacturing claimed range is higher than the effective range), the charging time needed ranges from 3 hours to 18 hours depends on the charging infrastructure installed. This is close to the realistic cases, where level I (110V), level II (220V) and quick charging infrastructures provide different charging speeds. Given the expected expected duration of stay for BEV drivers between each O-D pair, we examine the resulted duration of stay for them under different charging speeds. In our numerical test, the data for expected duration of stay, t^{rs}, is randomly generated and follows a "Gaussian" distribution over the time period range [0,9]. This time period range [0,9] is discretized and divided into 18 intervals, that is, the duration of stay for all drivers are grouped into 0~0.5hrs, 0.5~1hrs, 1~1.5hrs, 1.5~2hrs….The numbers of drivers falling into each duration of stay period interval are counted, and the results are represented using curves, instead of scatters by curve fitting method (see Figure 5). The solid line is the Gaussin fitting of the expected duration of stay curve. The dash lines are the Gaussin fitting curve of resulted duration of stay under different charging speeds. The Gaussian fitting method is used because the original data of expected duration of stay are generated using Gaussian distribution.

It indicates that when charging speed decreases, the peak value of the duration of stay shifted to the right. This also means the average duration of stay is longer. It is obvious when we compare $\sigma = 0.05$ and $\sigma = 0.3$. With $\sigma = 0.05$, the duration stay only have minor changes. However, when $\sigma = 0.3$, the portion of drivers staying at the destination between 0~5 hours decreases and the percentage of drivers staying between 5~9 hours increases. It is logical because when the charging speed is slow, the drivers spend more time at the destination to wait until their vehicle finishes the recharging. It is worth noting that, not all drivers need to charge their vehicles at the destination. Also, drivers will shift their routes accordingly to reduce the extra time of stay at destination. Therefore, the percentage change is within a reasonable range.

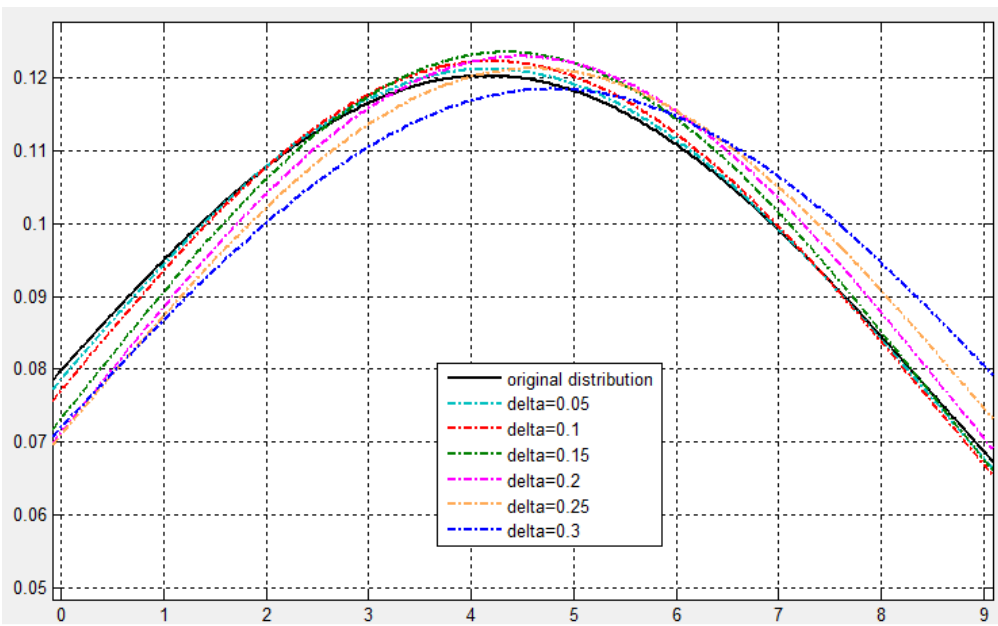

Figure 3-11: Duration of Stay under Different Charging Speed

3.5.5, Impacts on the system performance

Table 3-3 and table 3-4 gives the total vehicle miles traveled (VMT), system total costs, total electricity-charging cost, as well as total charging time penalty cost with different e_s, e_h, and charging speed. The VMT decreases with the increase of electricity-charging cost both at home and at destination. The VMT also decreases with the decreases of charging speed. This is reasonable since the charging cost and charging time penalty cost both depends on the total miles traveled.

Table 3-4 also indicates that when the charging speed increases, say, σ decreases from $\sigma = 0.3$ to $\sigma = 0.025$, the total charging time penalty cost decreases up to 99.5%. Figure 3-12 shows that this decrease is not linear and the decreasing rate is not a constant. With the charging speed increases at a constant rate, the total charging time penalty cost decrease is slower. This is obvious when $\sigma < 0.1$, where the total charging time penalty cost decreases only a little.

e_h=0.08	e_s =0.14	e_s =0.18	e_s =0.22	e_s =0.26	e_s =0.3
VMT	2112225	2091081	2083888	2081759	2080880
System total cost	1502448	1505707	1508466	1510594	1512443
total charging cost	172052.1	172288.7	173635.5	175322	177106.9
e_h=0.08	e_s =0.34	e_s =0.38	e_s =0.42	e_s =0.46	e_s =0.5
VMT	2074810	2073604	2071293	2069819	2067119
System total cost	1514594	1516280	1518189	1519692	1521553
total charging cost	177476.4	178850.4	179952.1	181218	182347.2
e_s=0.1	e_h=0.01	e_h =0.02	e_h =0.03	e_h =0.04	e_h =0.05
VMT	2125765	2124476	2123214	2121978	2120772
System total cost	1356064	1376702	1397327	1417938	1438535
total charging cost	26109.73	46789.73	67448.03	88085.46	108702.9
e_s=0.1	e_h =0.06	e_h =0.07	e_h =0.08	e_h =0.09	
VMT	2119596	2118452	2117341	2116263	
System total cost	1459121	1479694	1500255	1520806	
total charging cost	129301.2	149881.3	170444.2	190990.8	

Table 3-3 VMT (mile) and Costs (time equivalent) with e_s and e_h

σ	0.3	0.275	0.25	0.225	0.2	0.175
VMT	2142278	2144333	2147381	2148391	2149531	2150579
System total cost	5991499	5979825	5968843	5959687	5950975	5943489
total charging time penalty cost	68697.51	57015.91	46026.97	36870.27	28139.93	20647.47
σ	0.15	0.125	0.1	0.075	0.05	0.025
VMT	2152604	2152681	2153887	2153283	2154559	2155807
System total cost	5936422	5931110	5926819	5924662	5923923	5923251
total charging time penalty cost	13578.37	8266.617	3974.474	1837.442	1072.765	382.2585

Table 3-4 VMT (mile) and Costs (time equivalent) with Charging Speed Changes

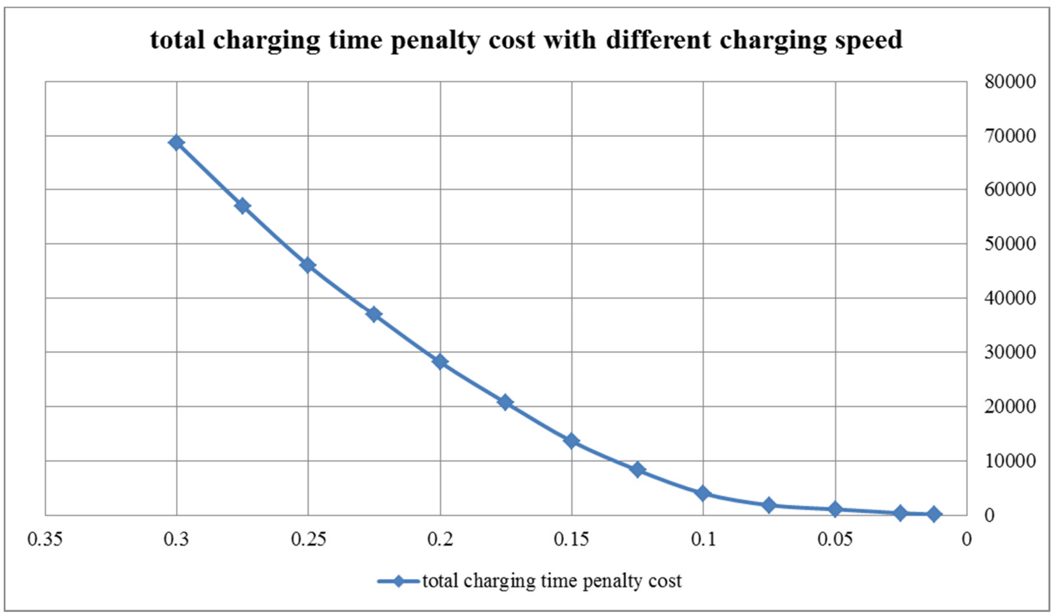

Figure 3-12: Total Charging Time Penalty Cost

3.6 SUMMARY

This chapter developed two Logit-based SUE traffic assignment mathematical models for BEVs, which can account for the electricity-charging cost and charging time respectively. The objective functions of the model formulations contain special piecewise linear functions terms in both models. The piecewise linear functions are functions of the travel distance. This special feature requires a path flow patterns as solution results. Therefore, a path-based solution algorithm is chosen and the DSD algorithm is adopted. The algorithm works iteratively between a restricted master phase and a column generation phase. In this modeling context, the column generation phase involves two CSP problems for both models. The solutions for these two CSP problems are compared in order to find the augmenting path to the restricted master problem.

Numerical tests are given for evaluating the developed models and investigating the impacts of electricity-charging price and charging speed. The tests show that the models developed are able to capture the origin/destination based electricity-charging price changes. Also, the models could paint a rough relationship between flow patterns and charging speed at the destinations. The impacts of electricity-charging price and charging speed on the network level are investigated. The spatial flow patterns are influenced by the electricity-charging price and charging speed. Their impacts on total system performance are also studied, such as system wide various costs and vehicle miles traveled.

The research in this chapter aims to be a preliminary study for evaluating the impacts of introducing electric vehicles on the transportation system. Those impacts are considered to be from the electricity-charging costs and the charging time in this research. For a more realistic model, traffic flows should be a combined one with both gasoline vehicles and electric vehicles and this will be studied in the next chapter. Nonetheless, for modeling simplicity, in this chapter, we focus on the BEVs only. Moreover, the models could be easily extended to incorporate both types of vehicles. In this next chapter, more realistic cases such as the combined choices for electric vehicle drivers with different electric vehicle penetration rates, where both spatial impacts and temporal impacts will be studied.

Chapter 4: Time –Dependent Combined Travel Choices for Electric Vehicles Users

4.1 INTRODUCTION

The network flow pattern, overall, might be affected by introducing BEVs onto the road and by the BEVs drivers' charging behavior. In this chapter, the investigation of the subsequent travelers' choices is carried out in terms of the activity time choices, the charging choices and the route choices. A multi-class joint choices network problem is studied.

The activity time herein is comprised of two components: the departure time choice and the duration of stay choice. We assume that motorists have a preferred time window for their arrivals at the destination and they will encounter a schedule delay cost if arriving at the destination out of their preferred time window. In addition, as presented in the previous chapter, motorists will also have an expected duration of stay at their destinations. During different time of the day, the traffic network conditions are different in terms of congestion level. Therefore, different departure times of motorists will result in different travel time on a certain route. The departure time and the network conditions together will affect the motorists' arrival time at destinations. Moreover, as discussed in the previous chapter, the electricity-charging time at destinations plays a role in the duration of stay for the BEVs drivers.

The previous chapter investigates a static SUE traffic assignment problem for BEVs. It has been concluded that the battery-charging time and as well as the electricity-charging price at destinations may affect the duration of stay of motorists at destinations and their route choices. In this chapter, a time-dependent network equilibrium problem will be studied with different user class. The user class is defined by the duration of stay, purpose of trips and the types of vehicles used. Both the temporal and spatial choice behaviors will be examined. A quasi-dynamic combined choices model on the network level is built. By saying quasi-dynamic, it means that in the model developed, the travel demand in each time interval is in steady-state equilibrium. The connection between successive time intervals is represented by the charging stations occupancy that is carried over to the next interval, which is similar to the problem setting in (Bifulco 1993). A combined choice of departure time, duration of stay and route is considered in this chapter. The departure time and duration of stay choices belong to the temporal choice and the route choice belongs to the spatial choice. The decision-making process for each motorist is assumed to follow a two-level hierarchical choice structure (Figure 4-1). In the upper-level, the motorists make choices on the departure time and the expected duration of stay at destination before traveling. In the lower-level, the motorists choose the perceived cheapest route directing to the destination. This hierarchical structure is modeled using a nested-Logit model, where all the choices are considered.

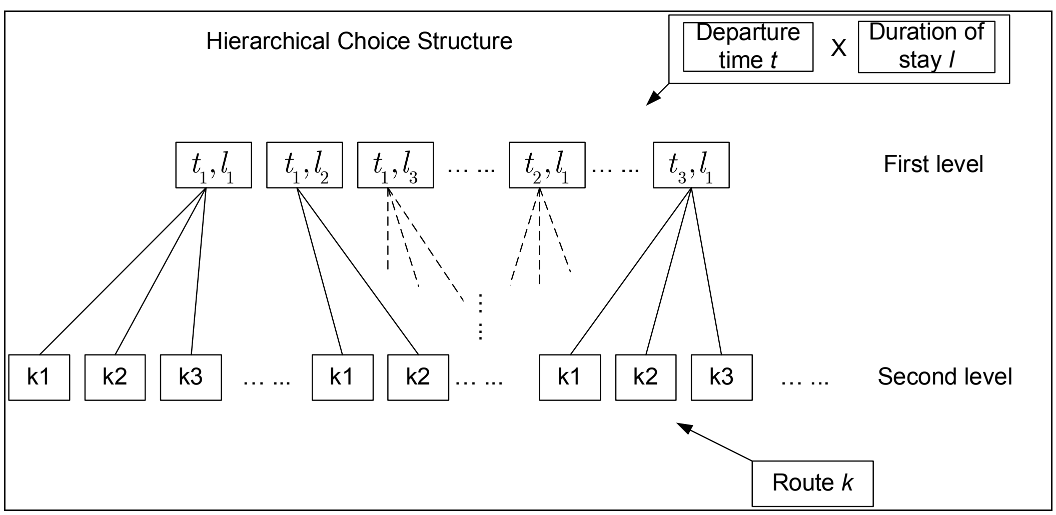

Figure 4-1: Combined Choices.

This chapter is structured as follows. Section 4.2 presents the definitions and problem statement of the time-dependent combined travel choices for BEV drivers. Section 4.3 presents a Virational Inequality (VI) formulation for the combined choices model. The optimality conditions are examined for achieving the Logit choice structure. Section 4.4 gives a solution algorithm for solving the VI model presented. Finally, Sensitivity analysis of travel patterns is discussed in the numerical experiments.

4.2 PROBLEM STATEMENT

4.2.1, The combined choices sets

The departure time choice set is predefined as a set of discrete time intervals. Hypothesizing the whole day splitting into several periods, let the whole study period be $[0,\overline{T}]$. This whole study period is divided into several equal time intervals $t \in T = \{0,1,...,\overline{T}\}$. It is assumed that $\overline{T}$ is large enough for all travelers to complete their journeys.

The duration of stay choice set is also a set of discrete time durations $l \in L = \{l_1, l_2, ...\}$. However, the feasible choices of durations of stay in this set depend on the route choice BEV drivers (Figure 4-2).

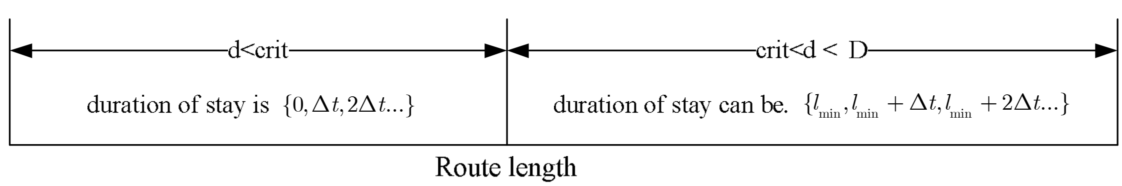

Figure 4-2: Duration of Stay Choice Set

For example, let l be the duration of stay for a BEV driver travelling from an origin to a destination. If the route the user chose has a length less than the “threshold” (as discussed in the previous chapter: users will charge their vehicle if the distance they have traversed is beyond this threshold) for charging, then the duration of stay will be a set of time duration from zero, that is $l \in L = \{0, \Delta t, 2\Delta t ...\}$. The Δt is a time interval, within which a steady-state equilibrium is assumed. Otherwise, if the BEV drivers need to charge their vehicles at the destination, the minimum duration of stay choice in the choice set is not zero. Specifically, the minimum duration of stay of a motorist is the minimum electricity-charging time for his/her BEV at destination. This electricity-charging time is related to the length of the path the motorist took. The electricity-charging time should be long enough for the BEV to get enough electricity for the motorist to go back to the origin. Let $l_{\min}$ be the minimum electricity-charging time for the BEV driver, then the duration of stay for the driver will be $l = L = \{l_{\min}, l_{\min} + \Delta t, l_{\min} + 2\Delta t ...\}$.

The route choices are probabilistic choices over a subset of all routes between each O-D pair, which is obtained from a Logit SUE traffic assignment as discussed in the previous chapter. In addition, the choice of charging station can also be accommodated into the route choices (see Figure 4-3). For example, let $p = \{p_1, p_2, p_3, p_4\}$ be charging locations (which are usually located at a parking garage/surface), and let index k be a route from an origin r to a destination s, then the generic choice (p,k) is a (charging location, route) alternative in the study area. As is presented in Figure 4-3, a dummy destination s' is adjacent to all the charging locations available at destination s. An "extended network" can be used to represent the charging location/route choice. In this case, the alternative (p,k) discussed above corresponds to an extended path, made up of the original path k and a sequence of links corresponding to the choice of charging location p. Thus, the generic alternative (p,k) described could be represented on the extended network by a path, say k instead (Bifulco 1993).

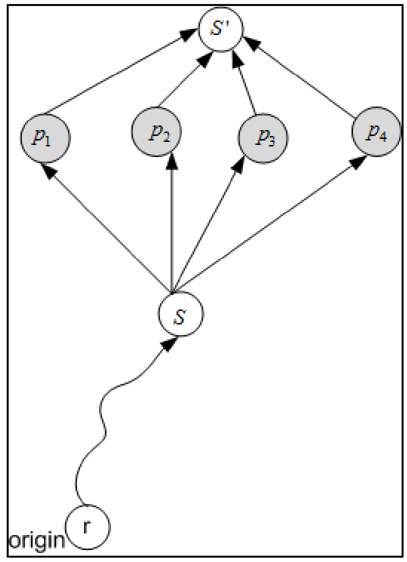

Figure 4-3: Extended Network for Charging Location and Route Choice

4.2.2, Problem setups

As stated in Chapter 3, we consider a traffic network $G = (N, A)$, where N is a finite set of all nodes and A is a finite set of all directed arcs. Let B be the set of vehicle types, let J be the set trip purposes, and let L be the set of durations of stay for motorists at destinations, then $M = \{m : (b, j, l) \in B \times J \times L\}$ represents the set of motorist class, where "×" denotes the Cartesian product. Let k be the generic choice of route (or route/charging station combination) for BEV drivers. Let K_c^{rs} be the subset of route between O-D pair (r, s) by taken which the BEV drivers need to recharge their vehicles at destination s. In addition to the notations stated in the section 3.2.1, we have the following setups:

Parameters

$q^{rs,b,j}$ total demand of motorists of vehicle type b with trip purpose j traveling between O-D pair (r, s)

Functions

$T_k^{rs,m}(t)$	the in-vehicle travel time cost for motorists between O-D pair (r,s), for the travelling purpose j, using vehicle type b, departure at interval t stay at destination of duration of l, via generic route k. Alternatively represented by $T_k^{rs,b,j,l}(t)$
$z^{rs,m}(t)$	waiting/congestion costs at charging for user class m, arrival at interval t at charging at destination s.
$e_k^{rs,m}$	charging cost for BEV drivers using route k between O-D pair (r,s) for user class m.
ctp_k^{rs}	charging time penalty cost for motorists using route k between O-D pair (r,s) for traveler class m.
$SD_k^{rs,m}(t)$	schedule delay costs for motorists of class m between O-D pair (r,s), via route k, arrival at time interval *t.*

Variables

$f_k^{rs,m}(t)$	vehicle flows on route k of user class m, between O-D pair (r,s), departure at time interval t.
$q^{rs,m}(t)$	the portion of demand using vehicle type b that departs at interval t and stays at destination for duration of l between O-D pair (r,s). Alternatively represented as $q^{rs,b,j,l}(t)$.
$C^{rs,m}$	the systematic disutility component (measured in time unit) for motorists class m departing at interval t between O-D pair (r,s).
$G^{rs,m}(t)$	perceived disutility for motorists class m departing at time interval t between O-D pair (r,s).
$\varepsilon^{rs,m}(t)$	random residual for the perceived disutility $G^{rs,m}(t)$.

$C_k^{rs,m}(t)$ the systematic disutility component (measured in time unit) for motorists class m departing at time interval t via generic route k between O-D pair (r,s).

$G_k^{rs,m}(t)$ perceived disutility for motorists class m departing at time interval t via generic route k between O-D pair (r,s).

$\varepsilon_k^{rs,m}(t)$ random residual for the perceived disutility $G_k^{rs,m}(t)$.

$P^{rs,b,j}(t,l)$ the probability of choosing departure time and duration of stay alternative (t,l) for traveling between O-D pair (r,s) for travelers using type b vehicles and purpose of travel j.

$P^{rs,m}(t)$ an alternative representation of $P^{rs,b,j}(t,l)$, where m is defined by b,j,l.

$P^{rs,b,j}(k/t,l)$ the conditional probability of choosing path k after choosing departure time and duration of stay alternative (t,l) between O-D pair (r,s) for travelers using type b vehicles and purpose of travel j. Alternatively represented as $P_k^{rs,m}(t)$.

The proposed formulation for the travel choices is a generalization of the classical random utility model (Cascetta 2009; Sheffi 1985). The choices of departure time, duration of stay at destination, route/charging is determined by a nested-Logit (NL) model. The Logit model has been adopted in many research for investigating many choices such as mode choices, destination choices, route choices, departure time choices and the joint/combined choices (T. Abrahamsson and Lundqvist 1999b; Ben-Akiva et al. 1984a; C. R. Bhat 1998b; M. Florian et al. 2002; H.J. et al. 2003; T. Zhang et al. 2011)

The O-D trip demand data for all purposes of travel and all vehicle types are considered given in the model. The disutility for user class m departing at interval t and though generic route k between O-D pair (r,s) is the summation of the following cost components:

- On board cost $T_k^{rs,m}(t)$ / $T_k^{rs,j,l}(t)$
- Charging station waiting/congestion costs $z^{rs,m}(t)$
- Charging cost $e_k^{rs,m}$
- Charging time cost $ctp_k^{rs,m}$
- Schedule delay cost $SD_k^{rs,m}(t)$

Noted, the charging cost and the charging time cost in the total cost functions are not time-dependent. The measured travel disutility, $C_k^{rs,m}(t)$, is formulated as the summation of the travel time cost from origin to the destination, the charging station waiting/congestion cost, and the charging cost, the charging time cost, and the schedule delay cost. The perceived travel disutility, $G_k^{rs,m}(t)$ is summation of the measured travel disutility and the random residual $\varepsilon_k^{rs,m}(t)$, where the random residual $\varepsilon_k^{rs,m}(t)$ is i.i.d Gumbel distributed. We have the total travel disutility of choosing route k, departing at time interval t and staying for a duration of l as follows,

$$\begin{aligned} C_k^{rs,m}(t) &= \alpha_1^m T_k^{rs,m}(t) + \alpha_2^m z^{rs,m}(t + T_k^{rs,m}(t)) \\ &+ \alpha_3^m e_k^{rs.m} + \alpha_4^m ctp_k^{rs,m} \\ &+ \alpha_5^m SD^{rs,m}(t + T_k^{rs,m}(t) + z_s^{rs,m}(t + T_k^{rs,m}(t))) \end{aligned} \tag{4.1}$$

Where, $\alpha_1^m, \alpha_2^m, \ldots \alpha_5^m$ are the coefficients of the different cost components, measuring the disutility changes with per unit change of that cost components in this disutility function. For example, α_1^m is the disutility change with one unit of change in the on board travel cost by class m users, which can also be explained as the disutility of a unit on board travel cost as perceived by class m users.

4.3 MATHEMATICAL FORMULATIONS

4.3.1, Nested-Logit model

As shown in Figure 4-1, the combined choices follow a hierarchical structure, where the motorists choose departure time and duration of stay before taking the journey, and then they choose the route. The perceived utility of choosing a combination of departure time and duration of stay is shown in Eq. (4.2) and the perceived utility of choosing an alternative of route k of user class m departing at time interval t is shown in Eq. (4.3)

$$G^{rs,m}(t) = C^{rs,m}(t) + \varepsilon^{rs,m}(t) \tag{4.2}$$

$$G_k^{rs,m}(t) = C_k^{rs,m}(t) + \varepsilon_k^{rs,m}(t) \tag{4.3}$$

Let W_{lt} represents the expected utility of choosing a (departure time, duration of stay) alternative. This expected utility could be calculated from the subsequent choice - route choices. A motorist's expected utility from an alternative of departure time and duration of stay can be defined as

$$W_{lt} = \frac{1}{\theta} \ln \sum_k \exp\{-\theta C_k^{rs,m}(t)\} \tag{4.4}$$

The probability of motorist driving vehicle type b, for the purpose of traveling j, departing at time interval t and staying at destination for the duration of l, between O-D pair (r,s) as follows

$$P^{rs,m}(t) = \frac{\exp\{\beta W_{lt}\}}{\sum_{(l,t)} \exp\{\beta W_{lt}\}} \tag{4.5}$$

where the values of parameters β reflects the accuracy of drivers' perception of the variation of the disutility of the (departure time, duration of stay) alternative. The portion of motorists demand with trip purpose j between O-D pair (r,s) using vehicle type b departing at time t and staying at the destination for l, can be calculated as follows,

$$q^{rs,b,j,l}(t) = q^{rs,m}(t) = q^{rs,b,j} \cdot P^{rs,m}(t) \tag{4.6}$$

The conditional probability of user class m traveling between O-D pair (r,s) at time interval t and staying at the destination for duration l via generic route k, can be given by the following equation

$$P_k^{rs,m}(t) = \frac{\exp\{-\theta C_k^{rs,m}(t)\}}{\sum_{k'} \exp\{-\theta C_{k'}^{rs,m}(t)\}} \tag{4.7}$$

The trip demand that uses route k, which is the route flow of user class m departing at time interval t between O-D pair (r,s), is as follows,

$$f_k^{rs,m}(t) = q^{rs,m}(t) \cdot P_k^{rs,m}(t) \tag{4.8}$$

The above equations (Eq. 4.5-4.8) give the NL model for the travelers' departure time, duration of stay and route choices.

4.3.2, Cost functions

This section presents the different cost functions involved in the model. The travel time cost through a route is the summation of the travel time cost on different links constituting the route. Different user classes experience the same travel time on the same link at the same time. The travel time cost function through a route, which is also referred as the on board cost in this research, is shown as follows,

$$T_k^{rs,m}(t) = \sum_a t_a(x_a(t))\delta_{a,k}^{r,s} \tag{4.9}$$

As the model in Chapter 3, where the origin-destination-origin trip chain is considered as the basic travel analysis unit. The BEV users need to charge their vehicle at the destination if necessary. This necessary condition for recharging at destinations has been discussed in previous chapter. The charging cost $e_k^{rs,m}$ and the charging time penalty cost were already discussed in the previous chapter. Both of these two costs are functions of the path length. Here, the charging cost has at destination s includes an additional term, i.e. e_0 (see Eq. 4.10). This e_0 could be considered as the charging/parking access fee for BEVs since the charging infrastructures are usually located in the parking lot or parking garage. This term e_0 adds the flexibility of defining the charging cost, and $e_0 = 0$ means there is only the electricity-charging cost. The charging time penalty cost function developed in the previous chapter is presented in Eq. (4.11).

$$e_k^{rs} = \begin{cases} e_h \sum_a d_a \delta_{a,k}^{rs}, & \sum_a d_a \delta_{a,k}^{rs} \le \frac{D}{2} \\ \frac{1}{2}[e_s \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) + e_h D] + e_0, & \frac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D \end{cases} \quad \forall k, a, r, s \tag{4.10}$$

$$pe_k^{rs} = \begin{cases} 0 \ , & \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) \le t^{rs}, \frac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D, \\ 0 \ , & \sum_a d_a \delta_{a,k}^{rs} \le \frac{D}{2}, \\ \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) - t^{rs}, & \begin{cases} \frac{D}{2} < \sum_a d_a \delta_{a,k}^{rs} \le D, \\ \sigma \cdot (2\sum_a d_a \delta_{a,k}^{rs} - D) > t^{rs} \end{cases} \end{cases} \tag{4.11}$$

The number of charging infrastructures is finite at each destination and this may cause congestion or waiting for the BEVs that need to recharge. The waiting or searching time for a free charging spot is considered as waiting/congestion cost for BEV drivers. It should be noted that only those BEV drivers who need to recharge their vehicles will encounter this cost. Herein, a soft capacity is considered for the charging infrastructures and a well-define the charging waiting/congestion time cost function is employed, which is the Bureau of public roads (BPR) function (W. Lam et al. 1999). It is a function of the current number of BEVs that recharge at the destination charging infrastructure when a new BEV comes for waiting. The charging waiting /congestion cost at time interval t is $z^{rs,m}(t)$ as follows,

$$z^{rs,m}(t + T_k^{rs,m}(t)) = z_s^{0,m} + 0.31 \times (\frac{D_s(t + T_k^{rs,m}(t))}{C_s})^{4.03} \qquad (4.12)$$

Where $z_s^{0,m}$ is the access time for a BEV driver who needs to recharge his/her vehicle when there are no other BEVs using charging infrastructures at the same destination, so called the *free-flow charging access time*. C_s is the number of charging infrastructures at the destination s (which is the capacity of the charging infrastructure). $D_s(t)$ is the number of existing BEVs that are recharging at time interval t. For a motorist departing at time interval t traveling between O-D pair (r,s) via route k, his/her arrival time at the destination's charging station is $t + T_k^{rs,m}(t)$. The number of existing BEVs recharging electricity at destination s at time interval t equals the total number of BEVs recharging electricity that arrives before time interval t (also represented as the accumulating recharging arrival flow $A_s^m(t)$) minus the total number of BEVs recharging electricity that leaves before time interval t (also represented as the accumulating recharging departure flow $B_s^m(t)$),

$$D_s^m(t) = A_s^m(t) - B_s^m(t) \qquad (4.13)$$

where the accumulation of arrival flow $A_s^m(t)$ and the accumulation of departure flow $B_s^m(t)$ at destination s are shown as follows,

$$A_s^m(t) = \sum_{r:(r,s)} \sum_{\xi-1}^{t-1} \sum_{tt:tt+T_k^{rs,m}(t)=\xi} \sum_{k \in K_c^{rs}} f_k^{rs,m}(tt) \tag{4.14}$$

$$B_s^m(t) = \sum_{r:(r,s)} \sum_{\xi-1}^{t-1-l} \sum_{tt:tt+T_k^{rs,m}(t)=\xi} \sum_{k \in K_c^{rs}} f_k^{rs,m}(tt) \tag{4.15}$$

As discussed in the introduction, all motorists have their own preferred time of arrival at destinations. For example, motorists going to work need to be arriving at work at a required time or a required time window. Let the preferred time of arrival at destination s for user class m traveling between O-D pair (r,s) be represented by $[t_{rs}^{m,lb}, t_{rs}^{m,ub}]$. We assume either early or late arrival at the destination result in a cost, which is named as the schedule delay cost. Within the context of (Lu and Mahmassani 2008), the schedule delay cost could be defined as a piecewise linear cost function (H. Yang and Meng 1998).

$$SD(t) = \begin{cases} \lambda_e^m (t_{rs}^{m,lb} - t), & t_{rs}^{m,lb} > t \\ 0, & t_{rs}^{m,lb} \le t \le t_{rs}^{m,ub} \\ \lambda_l^m (t - t_{rs}^{m,ub}), & t_{rs}^{m,ub} < t \end{cases} \tag{4.16}$$

Where λ_e^m is the value of early schedule delay and λ_l^m is the value of late schedule delay for user class m. This schedule delay cost function assumes that if travelers arrive at destination within the time window, there is no schedule delay cost.

4.3.3, Variational Inequality formulation

The joint choices problem is the simultaneous prediction of departure time choice, duration of stay choice and the route choice for each user class between each O-D pair. As discussed in the previous sections, the departure time and the duration of stay choice are represented using the Logit model and the route choices is also a Logit SUE traffic assignment problem. These three choices form a NL model with a hieratical (two-level) structure (see section 4.3.1).

In this section, an equivalent variational inequality (VI) formulation of the joint choices problem is given. The feasible set, Ω, of all the constraints that are associated with the joint travel choices is defined by the following equations (4.17)-(4.20), where $q^{rs,m}(t) = q^{rs,b,j,l}(t)$,

$$\sum_{k} f_k^{rs,m}(t) = q^{rs,m}(t), \quad \forall rs, m, l, t \tag{4.17}$$

$$\sum_{(l,t)} q^{rs,b,j,l}(t) = q^{rs,b,j}, \quad \forall rs, b, j \tag{4.18}$$

$$f_k^{rs,m}(t) > 0, \quad \forall rs, m, l, t, k \tag{4.19}$$

$$q^{rs,m}(t) \geq 0, \quad \forall rs, m, t, k \tag{4.20}$$

Equation (4.17) is the route flow conservation constraint. Equation (4.18) is the departure time and duration of stay choice demand conservation constraint. Constraints (4.19) and (4.20) are the nonnegative and positive requirements of route flows, departure time and duration of stay demands.

In the next section, the equivalent VI formulation of the quasi-dynamic model will be adopted to capture all the components of the proposed Logit models in an integrated form, with the description of the VI formulation first, followed by the proof of equivalence to the choice models

$$\begin{aligned} &\sum_{rs}\sum_{m}\sum_{t}\sum_{k} C_k^{rs,m^*}(t)\cdot[f_k^{rs,m}(t) - f_k^{rs,m^*}(t)] \\ &\quad + \sum_{rs}\sum_{m}\sum_{t}\sum_{k}\frac{1}{\theta}\ln\frac{f_k^{rs,m^*}(t)}{q^{rs,m^*}(t)}[f_k^{rs,m}(t) - f_k^{rs,m^*}(t)] \\ &\quad + \sum_{rs}\sum_{m}\sum_{t}\frac{1}{\beta}\ln q^{rs,m^*}(t)[q^{rs,m}(t) - q^{rs,m^*}(t)] \geq 0 \end{aligned} \tag{4.21}$$

subject to the feasible space $(f_k^{rs,m}(t), q^{rs,m}(t)) \in \Omega$. For the model to be internally consistent, $\theta \geq \beta > 0$ must hold (S. Dafermos 1980).

4.3.4, The analysis of the VI model

In this section, we prove that the developed VI formulation is equivalent to the SUE traffic assignment and simultaneously satisfies the Logit choice model of departure time and duration of stay. The Karush–Kuhn–Tucker (KKT) conditions of the VI formulation is derived and analyzed. It is shown that the KKT conditions recover the Logit model components.

Let $L_t^{rs,m}$ be the dual variable associated with the constraint (4.17), and let $L^{rs,b,j}$ be the dual variable associated with the constraint (4.18). let $\lambda_{t,k}^{rs,m}$ be the dual variable associated with the constraint (4.19) and (4.20). The KKT conditions are as follows,

$$f_k^{rs,m}(t): \qquad C_k^{rs,m}(t) + \frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)} - L_t^{rs,m} - \lambda_{t,k}^{rs,m} = 0 \tag{4.22}$$

$$q^{rs,m}(t): \qquad \frac{1}{\beta}\ln q^{rs,m}(t) - L^{rs,b,j} + L_t^{rs,m} = 0 \tag{4.23}$$

The complementarity conditions

$$\lambda_{t,k}^{rs,m} \cdot f_k^{rs,m}(t) = 0 \tag{4.24}$$

$$\lambda_{t,k}^{rs,m} \geq 0 \tag{4.25}$$

The above equations (4.27)-(4.30) combined with the primal feasible conditions Ω (4.22)-(4.25) together is the KKT conditions of the VI formulation. Next the equivalence of the VI to the equilibrium conditions and the joint choice conditions are proved.

From the complementarity conditions we know that if $f_k^{rs,m}(t) \neq 0$ then $\lambda_{t,k}^{rs,m} = 0$. For SUE traffic assignment problem, we know that the route flow $f_k^{rs,m}(t) > 0$, therefore, we get the following transformation from Eq.(4.22),

$$\frac{1}{\theta} \ln \frac{f_k^{rs,m}(t)}{q^{rs,m}(t)} = -C_k^{rs,m}(t) + L_t^{rs,m} \tag{4.26}$$

$$\Rightarrow \frac{f_k^{rs,m}(t)}{q^{rs,m}(t)} = \exp(-\theta C_k^{rs,m}(t) + \theta L_t^{rs,m}) \tag{4.27}$$

Sum both sides of the above equation over all routes k , together with the route flow conservation constraint (4.17), we get the following equations,

$$\begin{aligned} \sum_k \frac{f_k^{rs,m}(t)}{q^{rs,m}(t)} &= \sum_k \exp(-\theta C_k^{rs,m}(t) + \theta L_t^{rs,m}) \\ &= \exp(-\theta L_t^{rs,m}) \cdot \sum_k \exp(-\theta C_k^{rs,m}(t)) = 1 \end{aligned} \tag{4.28}$$

$$\Rightarrow \exp(\theta L_t^{rs,m}) = \frac{1}{\sum_k \exp(-\theta C_k^{rs,m}(t))} \tag{4.29}$$

Therefore, we get the route choice model which follows the Logit choice model as shown in equation (4.7) and (4.8).

$$f_k^{rs,m}(t) = q^{rs,m}(t) \cdot \frac{\exp\{-\theta C_k^{rs,m}(t)\}}{\sum_{k'} \exp\{-\theta C_{k'}^{rs,m}(t)\}} \tag{4.30}$$

From Eq.(4.23), we have

$$q^{rs,m}(t) = \exp(\beta L^{rs,b,j} - \beta L_t^{rs,m}) \tag{4.31}$$

Sum both sides of the Eq. (4.31) over all combination of departure time t and duration of stay l , we have

$$\sum_{(t,l)} q^{rs,m}(t) = \exp(\beta L^{rs,b,j}) \cdot \sum_{(t,l)} \exp(-\beta L_t^{rs,m}) \tag{4.32}$$

Divide Eq. (4.31) over Eq. (4.32) on both sides, we have the following equation,

$$\frac{q^{rs,m}(t)}{\sum_{(t,l)} q^{rs,m}(t)} = \frac{\exp(\beta L^{rs,b,j}) \cdot \exp(-\beta L_t^{rs,m})}{\exp(\beta L^{rs,b,j}) \cdot \sum_{(t,l)} \exp(-\beta L_t^{rs,m})} \tag{4.33}$$

From constraint (4.18), and the above constraint (4.33), we get the following departure time choice and duration of stay choice model, presented by the Logit choice model,

$$q^{rs,m}(t) = q^{rs,b,j} \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)} \exp(-\beta L_t^{rs,m})} \tag{4.34}$$

By recalling from the section (2.1.2), in the hierarchical Logit structure, the expected upper-level disutility is obtained from utility of lower-level choices, as shown in Eq. (2.9) and (2.10). Therefore, the costs of choosing an alternative of departure time t and duration of stay l is the propagated cost from the route choices, in the logsum format, as follows:

$$L_t^{rs,m} = -\frac{1}{\theta} \ln \sum_k \exp(-\theta C_k^{rs,m}(t)) \tag{4.35}$$

Therefore, the proposed VI formulation does lead to the joint choices of departure time, duration of stay and the stochastic route choice. That is, the hierarchical Logit model representing the joint probabilities of simultaneous choices is equivalent to the VI formulation (4.21) subject to Ω.

Since the feasible set of the VI model is defined by linear constraints, positive and nonnegative constraint, it is compact set. In addition, all the functions in the VI formulation are continuous; therefore, we know that there is at least one solution to the VI model.

4.4 SOLUTION ALGORITHM

A solution algorithm for the above derived VI formulation is developed in this section. The algorithm is developed based on the Gauss-Seidel decomposition approach (Equilibration Algorithm), where the network assignment for given demands is computed in one block and the departure time and duration of stay choice model is solved in another block. Since the choices are inter-related in our model, these two blocks are solved iteratively.

The demand matrices with specified vehicle type, departure time, duration of stay and purpose of travel between O-D pair are computed at every successive iteration by using the method of successive averages (MSA). The departure time choice set is predefined. The feasible duration of stay choice set for BEVs is updated at every successive iteration based on the utilized route length and the vehicle flows. The diagram of the solution approach is shown in Fig. 4-4.

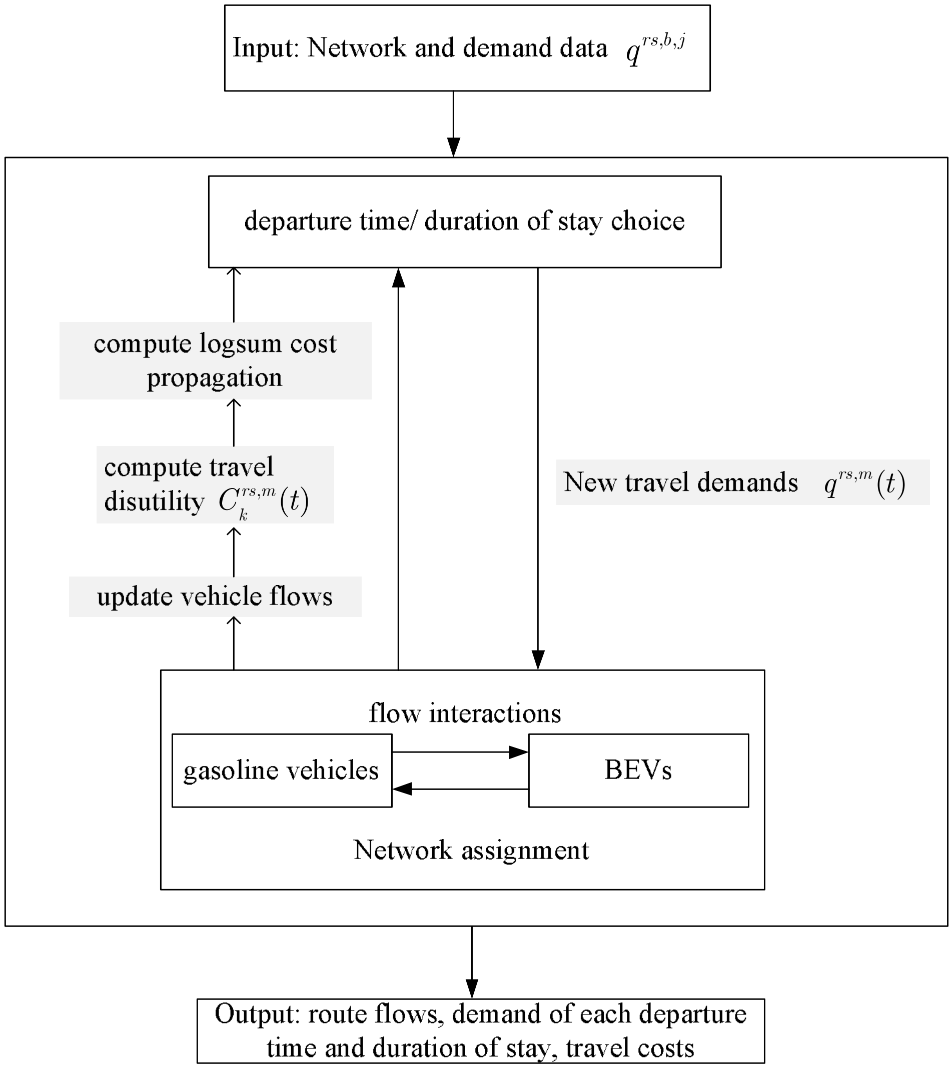

Figure 4-4: Solution Approach

Although there are multi-classes of travelers in the network, they are all auto users experiencing the same travel time on the network. We assume the motorists are homogenous on value of time and perception variants. In this chapter, two distinct types of vehicles are considered, the GVs and the BEVs. For BEV drivers, both the electricity-charging cost and the charging time penalty cost are considered. The network assignment block for both types of vehicles requires the solution for the following VI problem.

$$\sum_{rs}\sum_{m}\sum_{t}\sum_{k} C_k^{rs,m^*}(t)\cdot[f_k^{rs,m}(t)-f_k^{rs,m^*}(t)] + \sum_{rs}\sum_{m}\sum_{t}\sum_{k}\frac{1}{\theta}\ln\frac{f_k^{rs,m^*}(t)}{q^{rs,m^*}(t)}[f_k^{rs,m}(t)-f_k^{rs,m^*}(t)]\geq 0 \tag{4.36}$$

Subject to (4.17) and (4.19)

Next we prove that the solution of the above VI model (4.36) with constraint (4.17) and (4.19) is equivalent to the SUE condition (4.30).

First of all, the Lagrangian function for the above VI problem (4.36) can be written as follows,

$$L(f,\lambda)=\sum_{rs}\sum_{m}\sum_{t}\sum_{k} C_k^{rs,m}(t)+\frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)} \sum_{rs}\sum_{m}\sum_{t} L_t^{rs,m}[q^{rs,m}(t)-\sum_{k} f_k^{rs,m}(t)] \tag{4.37}$$

Where $L_t^{rs,m}$ is the Lagrangian multiplier associated with conservation constraint (4.17). $\boldsymbol{L}$ is the vector with elements $\{L_t^{rs,m}\}$. The set of KKT conditions for the VI problem can be expressed as follows,

$$f_k^{rs,m}(t)[C_k^{rs,m}(t)+\frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}-L_t^{rs,m}]=0, \qquad \forall rs,k,m,t \tag{4.38}$$

$$C_k^{rs,m}(t)+\frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}-L_t^{rs,m}\geq 0, \qquad \forall rs,k,m,t \tag{4.39}$$

$$f_k^{rs,m}(t)\geq 0, \qquad \forall rs,k,m,t \tag{4.40}$$

The Lagrangian function requires $f_k^{rs,m}(t)\neq 0$, otherwise, it will be meaningless, therefore, $f_k^{rs,m}(t)>0$ must hold. The KKT conditions can be simplified as follows,

$$C_k^{rs,m}(t)+\frac{1}{\theta}\ln\frac{f_k^{rs,m}(t)}{q^{rs,m}(t)}-L_t^{rs,m}=0, \qquad \forall rs,k,m,t \tag{4.41}$$

According to the above conditions (4.41), the rout flows of user class m, departed at time interval t between O-D pair (r,s) can be deduced as follows,

$$f_k^{rs,m}(t) = q^{rs,m}(t) \cdot \exp(\theta L_t^{rs,m} - \theta C_k^{rs,m}(t)), \quad \forall rs,k,m,t \qquad (4.42)$$

Equations (4.17) and (4.42) lead to

$$L_t^{rs,m} = -\frac{1}{\theta}\ln[\sum_k \exp(-\theta C_k^{rs,m}(t))], \quad \forall rs,m,t \qquad (4.43)$$

It can be seen that the Lagrangian multiplier $L_t^{rs,m}$ is the expected minimum perceived route travel time cost of user class m for departing at time interval t between O-D pair (r,s). Substituting (4.43) into (4.42), we can obtain the following Logit-based route choice model,

$$f_k^{rs,m}(t) = q^{rs,m}(t) \cdot \frac{\exp(-\theta C_k^{rs,m}(t))}{\sum_{k'} \exp(-\theta C_k^{rs,m}(t))}, \quad \forall rs,k,m,t \qquad (4.44)$$

Eq. (4.44) indicates that the motorists of user class m, departed at time interval t between O-D pair (r,s) choose their travel route by a Logit model, which is consistent with the our SUE condition.

The DSD algorithm stated in the previous chapter is used to solve the above network assignment model. In the column generation phase of the DSD algorithm, at least two paths are generated to the restricted master problem set. One is the shortest path for the GVs drivers, where the cost is only the travel time cost. The other is the preferred path for BEV drivers, where both the electricity-charging cost and the charging time penalty cost are considered into the total travel cost of choosing a route. This augmenting route is generated based on the following three CSP problems.

[CSP1]

$$z^1 = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + e_s \sum_{(u,v)\in A} d_{uv} x_{uv} - \frac{1}{2}(e_s - e_h)D \tag{4.45}$$

$$z^{1'} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + e_s \sum_{(u,v)\in A} d_{uv} x_{uv} \tag{4.46}$$

subject to

$$\frac{1}{2}D < \sum_{(u,v)\in A} d_{uv} x_{uv} \le \frac{D}{2} + \frac{t^{rs}}{2\sigma} \tag{4.47}$$

$$\sum_{\{v|(u,v)\in A\}} x_{uv} - \sum_{\{u|(u,v)\in A\}} x_{uv} = \begin{cases} 1 & u = r \\ 0 & u \in N - \{r,s\} \\ -1 & u = s \end{cases} \tag{4.48}$$

$$x_{uv} = \{0,1\}, \qquad \forall (u,v) \in A \tag{4.49}$$

[CSP2]

$$z^2 = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + e_h \sum_{(u,v)\in A} d_{uv} x_{uv} \tag{4.50}$$

subject to Eq.(4.48) , Eq. (4.49) and,

$$\sum_{(u,v)\in A} d_{uv} x_{uv} \le \frac{D}{2} \tag{4.51}$$

[CSP3]

$$\begin{aligned} z^3 = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + e_s \sum_{(u,v)\in A} d_{uv} x_{uv} - \frac{1}{2}(e_s - e_h)D \\ + \sigma[(2 \sum_{(u,v)\in A} d_{uv} x_{uv} - D) - t^{rs}] \end{aligned} \tag{4.52}$$

$$z^{3'} = \min_{x_{uv}} \sum_{(u,v)\in A} t_{uv} x_{uv} + e_s \sum_{(u,v)\in A} d_{uv} x_{uv} + \sigma(2 \sum_{(u,v)\in A} d_{uv} x_{uv} - D) \tag{4.53}$$

subject to Eq.(4.48) , Eq. (4.49) and

$$\frac{D}{2} + \frac{t^{rs}}{2\sigma} < \sum_{(u,v)\in A} d_{uv} x_{uv} \le D \tag{4.54}$$

There are three CSP problems shown above. In some of the CSP problems, the objective functions could be rewrite by eliminating the constant terms (e.g.CSP1 and CSP3). After rewrite those objective functions, the three CSP problems are: CSP1 (Eq. 4.46-4.49), CSP2 (Eq. 4.48-4.51), CSP3 (Eq. 4.53-4.54, 4.48-4.59). These three CSP problems are solved separately, and their objective values of the original objective function z^1, z^2, z^3 are compared. The solution route with the least objective values among $\{z^1, z^2, z^3\}$ is selected and augmented to the restricted master problem set.

In overall, the above discussed DSD could be used to solve the route choices (i.e. the lower-level choice of the hieratical structure). The upper-level choice and the lower-level choice consisting the VI formulation (4.21) are solved in two sequential phase iteratively. The step-by-step procedure is for solving the joint choices problem is described in the following:

Step 0. Initialization.

Set $i = 1$, given the demand data, choose an initial O-D travel pattern $q^{rs,m}(t)$.

Step 1. DSD algorithm for SUE assignment (refer to the previous chapter for more details).

Step 1.1. Find initial subset of routes, compute the initial route flow. Set $j = 1$

Step 1.2. Restricted master problem phase. Assign $q^{rs,m}(t)$ to the subset of routes between O-D pair (r,s). Obtain the travel time cost $T_k^{rs,m}(t)$, and route flows $f_k^{rs,m}$

Step 1.3. Column generation phase. Find the augmenting paths obtained from the shortest path (in terms of travel time) and from solving the CSP1 ~CSP3 problems above.

Step 1.4. At convergence, compute the electricity-charging cost $e_k^{rs,m}$, charging time penalty cost $ctp_k^{rs,m}$. Compute the charging/parking congestion cost, the schedule delay cost, and finally the travel disutility $C_k^{rs,m}(t)$.

Step 2. In accordance to the above travel disutility, calculate the logsum cost $L_t^{rs,m}$ Update the feasible duration of stay set.

Step 2.1. Perform the departure time and duration of stay choice according to the Logit choice model, get the auxiliary O-D travel pattern $q'^{rs,m(i)}(t)$.

Step 2.2. Update the O-D travel demand using MSA

$$q^{rs,m(i+1)}(t) = q^{rs,m(i)}(t) + \frac{1}{i}[q'^{rs,m(i)}(t) - q^{rs,m(i)}(t)]$$

Step 3. Convergence check. If the equilibrium condition for the joint choice of departure time and duration of stay is reached, then terminate and output the solution; otherwise, set $i = i + 1$, and go to Step 1.

The proposed algorithm to the solution of the VI problem consists of two sequential phases. Although the convergence of proposed algorithm has not been rigorously investigated, the numerical experiments next section show that this algorithm is able to obtain a satisfactory solution.

4.5 NUMERICAL EXPERIMENTS

This section first describes the test network used in the numerical experiments in section 4.5.1. The solution quality is checked in terms of convergence and feasibility in section 4.5.2. Finally, the analysis of the impacts of different BEV ranges and different BEV penetration rates in the test network are presented in section 4.5.3 and 4.5.4 respectively.

4.5.1, test network

The numerical experiments are conducted on the following network to examine the convergence pattern, the impact of BEV ranges, and the impact of the BEVs penetration rates. The network is shown in Figure 4-5.

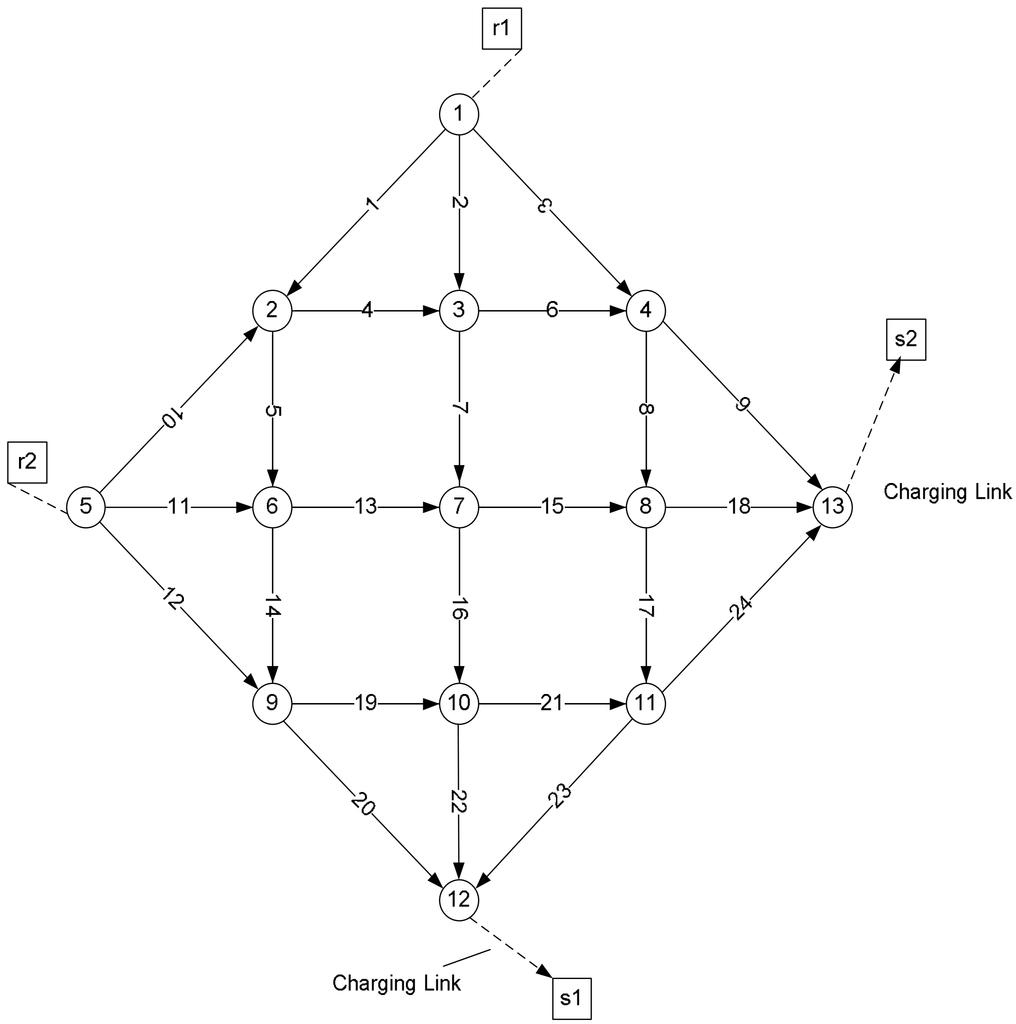

Figure 4-5: Test Network

In this test network, there are two O-D pairs $(1,12)$ and $(5,13)$, 13 nodes, and 24 road links. The dummy links between node 12 and $s1$, and between node 13 and $s2$

represent the charging links, where BEVs get recharging for electricity. In this numerical test, one charging station is considered associated with each destination. The parameters of all the link travel time functions are given in Appendix B, Table B-1.

The free-flow charging access time $z_s^{0,m}$ is 0.1h for both charging locations at destination $s1$ and $s2$. The charging capacities at both destinations are set to be 80% of the total BEVs that are traveling to that destination.

Both the GVs and BEVs are considered in this numerical test, therefore, it is a mixed flow environment. There are a total demand of 10,000 vehicles (including both BEVs and GVs) between O-D pair $(1,12)$ and a total demand of 7,000 vehicles between O-D pair $(5,13)$.

As stated in the previous sections, the GVs drivers do not encounter the electricity-charging cost or the charging time penalty cost or the charging waiting/congestion cost. Instead GVs drivers encounter operating cost. This operating cost in the numerical example is considered as the fuel cost. The per unit distance operating cost for GV drivers is set as of $g = 0.2$ units and the operating cost for GV drivers is a function of the path length, i.e., $g\sum_a d_a \delta_{a,k}^{rs,GV}$. The weighting parameter of GV drivers for the operating cost is represented using α_6^{GV}. The travel time-equivalent electricity-charging cost at home is set as $e_h = 0.05$, and the travel time-equivalent electricity-charging cost at both two destinations are assumed to be $e_s = 0.15$, the access fee for charging is set to be $e_0 = 1$. The coefficient for electricity-charging speed is set to be $\sigma = 0.03$.

The study period runs from 6:00 am to 7:00 pm and it is assumed that all travelers finished their journey within this study period. The whole period is divided into hourly intervals, and there are 13 intervals in total. Both GV drivers and BEV drivers are further categorized into classes according to different durations of stay, which last for 1 hour, 2 hours, or 3hours. Two types of trips are considered, namely, type A and type B. For all motorists between O-D pair $(1,12)$, type A trip is considered. For all motorists between O-D pair $(5,13)$, type B trip is considered. The perception coefficient θ and β are assumed to be the same for motorists between the same O-D pair for the same type of trip purpose. The perception coefficient for all motorists between O-D pair $(1,12)$ for trip purpose A is θ_1 and β_1. The perception coefficient for all motorists between O-D pair $(5,13)$ for trip purpose B is θ_2 and β_2.The number of classes in the example is twelve (2 vehicle types × 3 duration of stay × 2 types of trip purpose=12). The scheduled arrival time window for motorists of trip purpose A between O-D pair (1,12) is $[t_{(1,12)}^{1,lb}, t_{(1,12)}^{1,ub}]$ and the parameters used for the schedule delay cost function are λ_e^1, λ_l^1. The scheduled arrival time window for motorists of trip purpose B between O-D pair (5,13) is $[t_{(5,13)}^{2,lb}, t_{(5,13)}^{2,ub}]$ and the parameters used for the schedule delay cost function are λ_e^2, λ_l^2. Weight parameter values of different cost components for GVs and BEVs in the total cost functions are shown in Table 4-2. The parameters used for all classes are summarized in Appendix B, TableB-2.

4.5.2, solution quality and convergence analysis

First of all, the solution quality is checked on the tested network (Fig. 4.5). A relative gap value is defined to measure the effectiveness of the solutions, which measures how close is the output approaching the equilibrium conditions.

4.5.2.1 Measurement values

The solution effectiveness is measured as the absolute difference between the number of assigned path flows and the probabilistic path flows. That is the gap value between the solution and the equilibrium condition. For the SUE traffic assignment, the measured gap value, G_1 is defined as follows,

$$G_1 = | f_k^{rs,m}(t) - q^{rs,m}(t) \frac{\exp(-\theta C_k^{rs,m}(t))}{\sum_{k'} \exp(-\theta C_{k'}^{rs,m}(t))} | \tag{4.55}$$

If the value G_1 is zero for every path flow, the solution obtained satisfies the SUE conditions (Eq. 4.30). We use the following the total of all relative gap values to measure the closeness of solution results to the equilibrium conditions

$$M_1 = \frac{\sum_{rs}\sum_{m}\sum_{t}\sum_{k} | f_k^{rs,m}(t) - q^{rs,m}(t) \frac{\exp(-\theta C_k^{rs,m}(t))}{\sum_{k'} \exp(-\theta C_{k'}^{rs,m}(t))} |}{\sum_{rs}\sum_{m}\sum_{t}\sum_{k} f_k^{rs,m}(t)} \tag{4.56}$$

For the departure time and duration of stay choice, the absolute difference between the solution demand trip rates and the expected trip rates, G_2 is defined as follows,

$$G_2 = | q^{rs,m}(t) - q^{rs,b,j} \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)} \exp(-\beta L_t^{rs,m})} | \tag{4.57}$$

If the above value is zero for all feasible departure time and duration of stay choice combination, the solution obtained satisfies the Logit choice conditions (Eq. 4.34). We use the following the total of all relative gap values to measure the departure time and duration of stay choice solution qualities.

$$M_2 = \frac{\sum_{rs}\sum_{m}\sum_{t} | q^{rs,m}(t) - q^{rs,b,j} \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)} \exp(-\beta L_t^{rs,m})} |}{\sum_{rs}\sum_{m}\sum_{t} q^{rs,m}(t)} \tag{4.58}$$

Both of the total relative gap values M_1 and M_2 provide a clue of the solution quality. At termination, we obtain both M_1 and M_2 value at around $10^{-5} \sim 10^{-3}$ for all the numerical tests.

4.5.2.2 Convergence to the equilibrium conditions

The convergence pattern of the solution is presented in this section. The convergence criterion is defined by the total relative gap value M_1 for the inner loop of SUE traffic assignment and by the total relative gap value M_2 for the outer loop of departure time and duration of stay choice.

As the algorithm presented, the joint choices are given by the NL model. Therefore, the Logit-based equilibrium for all joint choice is always satisfied.

Figures4-6 and 4-7 show the convergence patterns of the inner loop for SUE traffic assignment and outer loop for trip demands in the example network. The unit of the M-value is the number of vehicles. It can be seen that the algorithm achieve good convergence patterns for both level of choices.

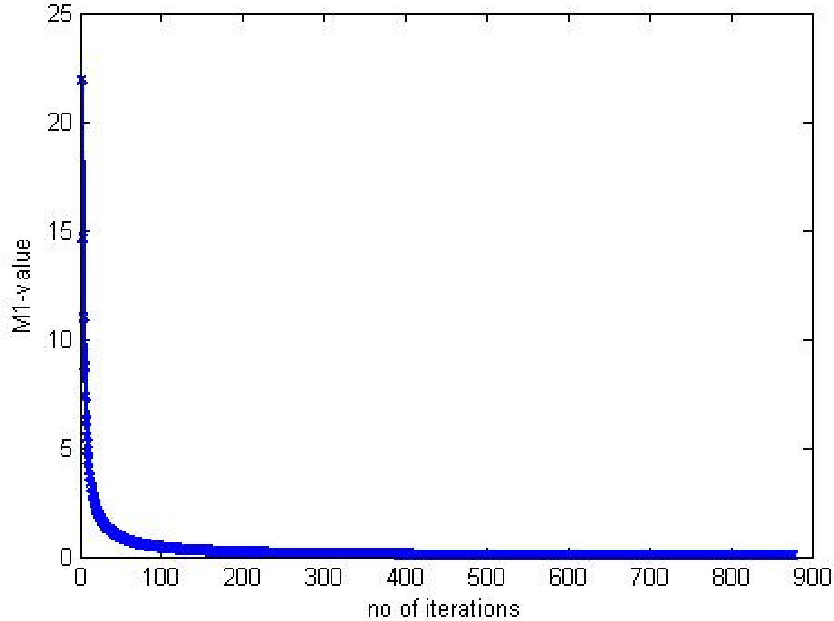

Figure 4-6: Convergence Pattern for the Stochastic User Equilibrium

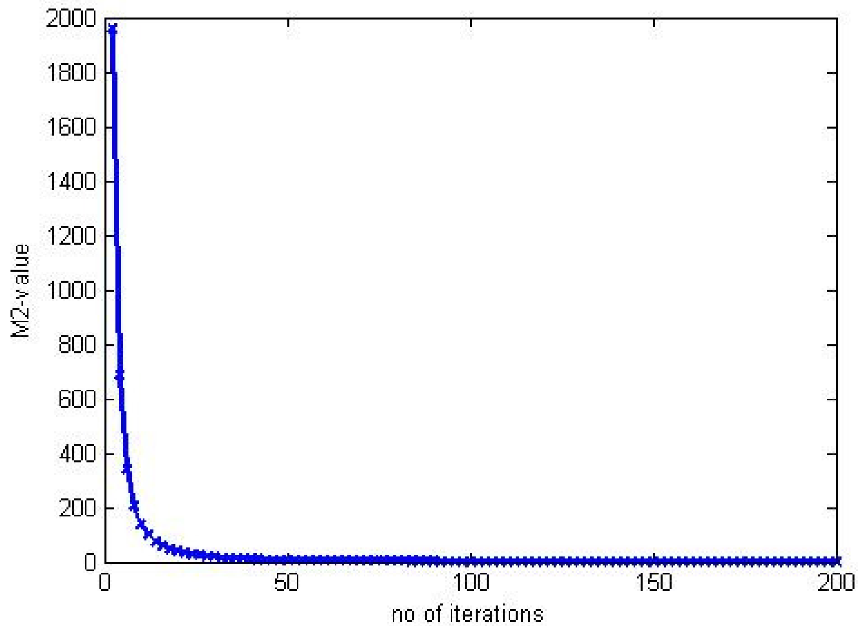

Figure 4-7: Convergence Pattern for The O-D Trip Rates

4.5.2.3 Solution quality

Since there are two O-D pairs in the test network, one shortest path is found between each O-D pair. Between these two shortest paths, we name the longer one as *longer shortest path*. It is found that the longer shortest path is 68 miles in the example network. For the numerical experiments in this section, we set the BEV effective range at 1.5 times the longer shortest path length, which is 102 miles. This ensures that all O-D pairs can be reached by BEV drivers. The BEV penetration rate is set at 30%, which means there are 3,000 BEVs between O-D pair $(1,12)$ and 2,100 BEVs between O-D pair $(5,13)$. While there are 7,000 GVs between O-D pair $(1,12)$ and 4,900 GVs between O-D pair (5.13).

The departure demands of BEV drivers for each O-D pair, for each time slice departure and duration of stay are shown in the following Table 4-1. The BEV effective vehicle range is 102 miles, while the length of the shortest path between O-D pair (5,13) is 68 miles, which means all BEVs drivers between this O-D pair needs to charge their vehicles at the destination $s2$. The charging speed parameter σ is again assumed to be 0.03, which means the recharging of BEVs between this O-D needs at least $(68 \times 2 - 102) \times 0.03 = 1.02$ hrs. Therefore, as shown in Table 4-1, between O-D pair (5,13), there is no demand at any time slices for staying at destination for one hour or less for all BEV drivers. The demands for l =1h, l =2h, and l =3h between O-D pair (1,12) for BEVs are 872.3496 , 1039.567, and 1088.0826 respectively. The demands for l =1h, l =2h, and l =3h between O-D pair (5,13) for BEVs are 0, 1027.9349, and 1072.065 respectively. This indicates that BEV drivers tend to stay longer at the destination. The reason for this might be one or more of the following facts: (1) the BEVs drivers needs to wait until the necessary electricity-recharging finished; (2) the BEVs drivers needs to wait for an unoccupied charging infrastructure, where the congestion in charging stations exists.

Figure 4-8 presents the solution trip demands at different time of day with 2-hour duration of stay for BEV drivers and GV drivers. Both Table 4-1 and Figure 4-8 suggest that most of the motorists between O-D pair (1,12), both BEVs and GVs drivers, depart at the intervals between 9:00am~2:00pm, which is reasonable since all motorists try to reach their destination within the time window to reduce their schedule delay costs. Most of the motorists between O-D pair (5,13) depart between 6:00am~10:00 am to satisfy their expected time window. Overall, the trip demands at each departure time interval and each duration of stay obtained from the developed model consistent with expectations relative to the joint choices.

Duration of stay			l =1hour		l =2 hours		l =3 hours	
Departure Time			(1,12)	(5,13)	(1,12)	(5,13)	(1,12)	(5,13)
6:00	~	7:00	21.68	0.00	22.75	176.07	23.09	181.89
7:00	~	8:00	38.09	0.00	39.96	275.51	40.56	284.05
8:00	~	9:00	69.63	0.00	72.76	294.10	73.77	309.37
9:00	~	10:00	121.40	0.00	126.91	193.76	128.69	204.71
10:00	~	11:00	129.57	0.00	144.85	63.21	149.57	65.76
11:00	~	12:00	145.58	0.00	184.83	18.08	196.01	18.81
12:00	~	13:00	139.61	0.00	184.71	5.16	197.85	5.36
13:00	~	14:00	125.71	0.00	173.98	1.46	187.87	1.52
14:00	~	15:00	62.03	0.00	68.71	0.41	70.79	0.43
15:00	~	16:00	15.52	0.00	16.46	0.12	16.77	0.12
16:00	~	17:00	2.90	0.00	3.07	0.03	3.13	0.03
17:00	~	18:00	0.54	0.00	0.57	0.01	0.00	0.00
18:00	~	19:00	0.10	0.00	0.00	0.00	0.00	0.00

Table 4-1 Departure Rate for BEVs

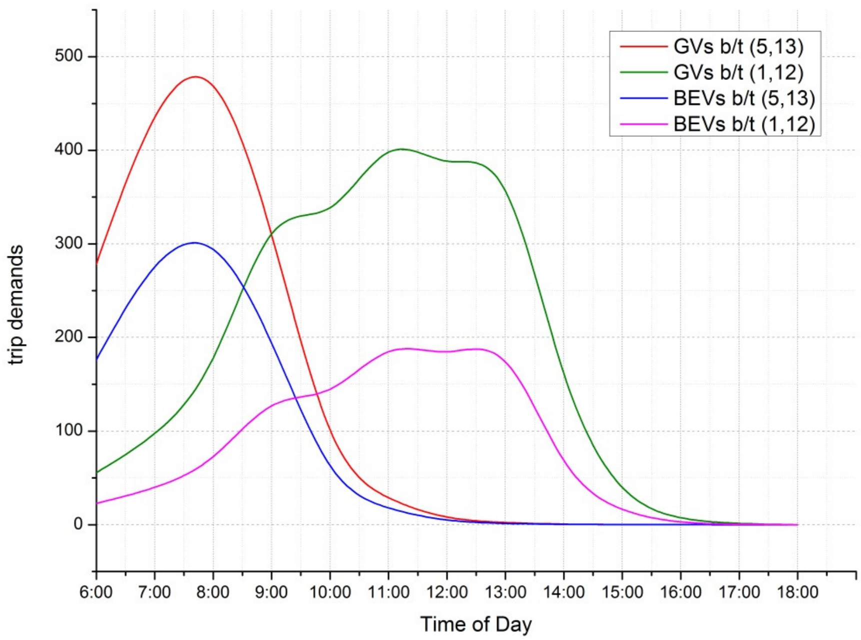

Figure 4-8: Trip Demands for 2-Hour Stay at Different Time of Day

The following Figure 4-9 and 4-10 presents the total number of accumulative arrivals, departures and accumulation of BEVs that recharging at charging locations at different time of the day. The accumulation for charging is the vertical distance between the cumulative arrival and cumulative departure curves. The capacity of the charging location at destination node 12 is 2400 and is 1680 at destination node 13. The maximum recharging flow at destination 12 is 590.3221 and is 1627.611 at destination 13. The number of BEV recharging flows is related to link length and the utilized paths' length between each O-D pair. Thus, although the BEV trip demand between O-D (1,12) is higher than that of O-D (5,13), the total number of recharging flows between (1,12) is much less than that between (5.13). The charging facility started to be occupied at around 12:00pm and the peak occupied time is at around 3:00pm at destination 12; while the peak occupation at charging facility at destination 13 is between 9:00am~1:00pm. The behavior of the arrivals, departures and charging is consistent with our expectations in terms of time of the day and duration of stay.

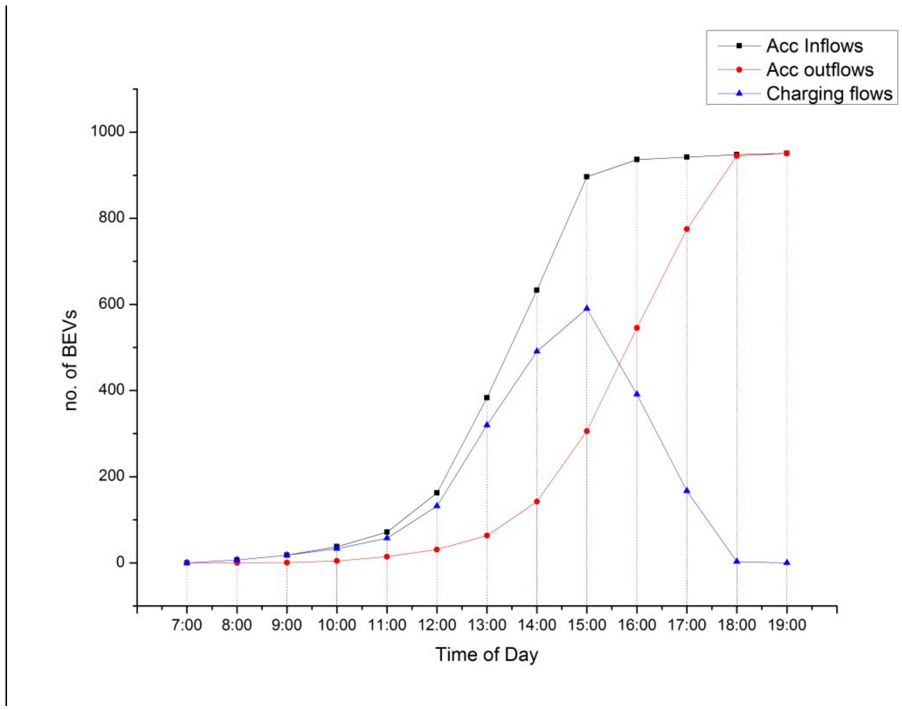

Figure 4-9: Charging Facility Arrival, Departure, and Accumulation at 12

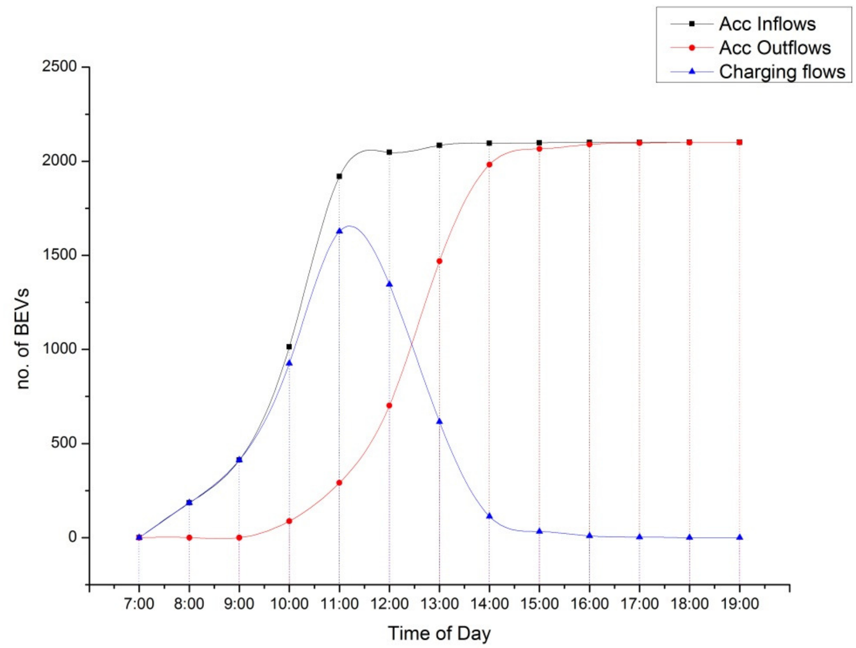

Figure 4-10: Charging Facility Arrival, Departure, and Accumulation at 13

The above results of the combined demand of departure time and duration of stay, as well as the behavior of accumulative arrivals, departures and charging curves at the charging facility verifies that the proposed model can indeed generate a network equilibrium solution of the joint choices.

4.5.3, impacts of the vehicle range

With the development of battery technologies, we might expect that the BEV batteries could hold more electricity. This will result in larger effective range of the BEVs. In this numerical test, the BEVs are set to have various effective ranges, which are used to study the impacts of improving BEV battery technologies.

Recall the definition of the longer shortest path in the previous section, which is the longest path among all the shortest paths between all O-D pairs. The BEV efficient range is set ρ times the longest shortest path, where ρ increases from 1.2 to 2.3. The BEVs penetration rate in this numerical test is fixed at 30% between each O-D pair.

4.5.3.1 Trip demand for different durations of stay

Following table 4-2 gives the BEV trip demands for the durations of stay with different effective ranges. The effective ranges of BEVs (shown in the second column of the table) are set from 81.6 miles to 156.4 miles. The 3rd~6th columns show the total trip demands of BEV travelers between each O-D pair for 1-hour, 2-hour, and 3-hour durations of stay respectively.

Noted that, at the tested charging speed $\sigma = 0.03$, the BEVs trip demands between O-D (5,13) departing at any time for 1-hour duration of stay are zeros when BEV effective range is below 102 miles. The BEV trip demands between O-D (5,13) for 1-hour duration of stay increase when the BEV effective range is improved (above 108.8 miles). In addition, it can be observed from the table that the number of BEV drivers staying at destinations for 3 hours decreases with increasing BEV ranges; and the number of BEV drivers staying at destinations for 1 hour increases with increasing BEV ranges. The number of BEV drivers staying at destination for 2 hours increases first and then decreases with increasing BEV ranges. This indicates that when the BEV range is limited to a low value, BEV trips distributed diversely over different durations of stay (1h, 2h, 3h) since the duration of stay are restricted by the range; while when the BEV range is large enough, BEV trips distributed more evenly over different durations of stay. These intuitive results indicate that the model's fidelity for capturing the duration of stay feasibility and the duration of stay choice for BEV drivers.

Duration of stay		l=1 hour		l=2 hours		l=3 hours	
ρ	Range	(1,12)	(5,13)	(1,12)	(5,13)	(1,12)	(5,13)
1.2	81.6	875.07	0.00	989.38	1018.77	1135.55	1081.23
1.3	88.4	872.25	0.00	1014.43	1022.76	1113.31	1077.24
1.4	95.2	872.35	0.00	1039.57	1027.93	1088.08	1072.07
1.5	102.0	874.82	0.00	1044.46	1034.60	1080.72	1065.40
1.6	108.8	882.54	673.19	1050.21	708.69	1067.25	718.13
1.7	115.6	899.55	672.94	1050.48	713.53	1049.96	713.53
1.8	122.4	932.42	676.48	1034.04	711.76	1033.54	711.76
1.9	129.2	962.21	681.09	1019.13	709.46	1018.65	709.45
2.0	136.0	972.57	687.54	1013.95	706.23	1013.48	706.23
2.1	142.8	988.29	694.81	1006.08	702.60	1005.63	702.59
2.2	149.6	1000.20	700.00	1000.12	700.00	999.68	700.00
2.3	156.4	1000.20	700.00	1000.12	700.00	999.68	700.00

Table 4-2 Total Demands of BEVs for Different Durations of Stay and Vehicle Ranges

Figures 4-11 depicts the above trip demand rates with different BEVs ranges between O-D (1,12). It can be observed from the figure that when the BEV range is less than 102 miles, the demands for 1-hour stay is almost stable and very low; while the demand for 2-hour stay increases and demand for 3-hour stay decreases. This suggests that with the BEV range increase (but still below 102 miles), BEV drivers switch from 3-hour stay to 2-hour stay. It can also be observed that when the BEV range continuous increases (but still below 115.6 miles), both the 1-hour and 2-hour stay demands increase while the 3-hour stay demand decreases. After the BEV range increases beyond 115.6 miles, both the 2-hour and 3-hour stay demand decreases while the 1-hour stay demand continues increasing. These indicates that the BEV drivers switch from 2-hour and 3-hour stay to 1-hour stay. From this figure, the effective range "points" for switching duration of stay could be observed at 102 and 115.6. The model is capable to capture the turning point of the switching behaviors.

Second, it can be observed from the figure that if the BEV range is not limited, the demand of BEV drivers for each duration of stay is evenly divided. This is due to the travel cost functions defined in the model. In the defined cost function, if there is no range limitation, none of the cost components a function of the duration of stay. We can expect that when there is a cost component settled as a function of the duration of stay (for example the vehicles are charged for the duration of parking), the number of travelers for different duration of stay will not be equal any more.

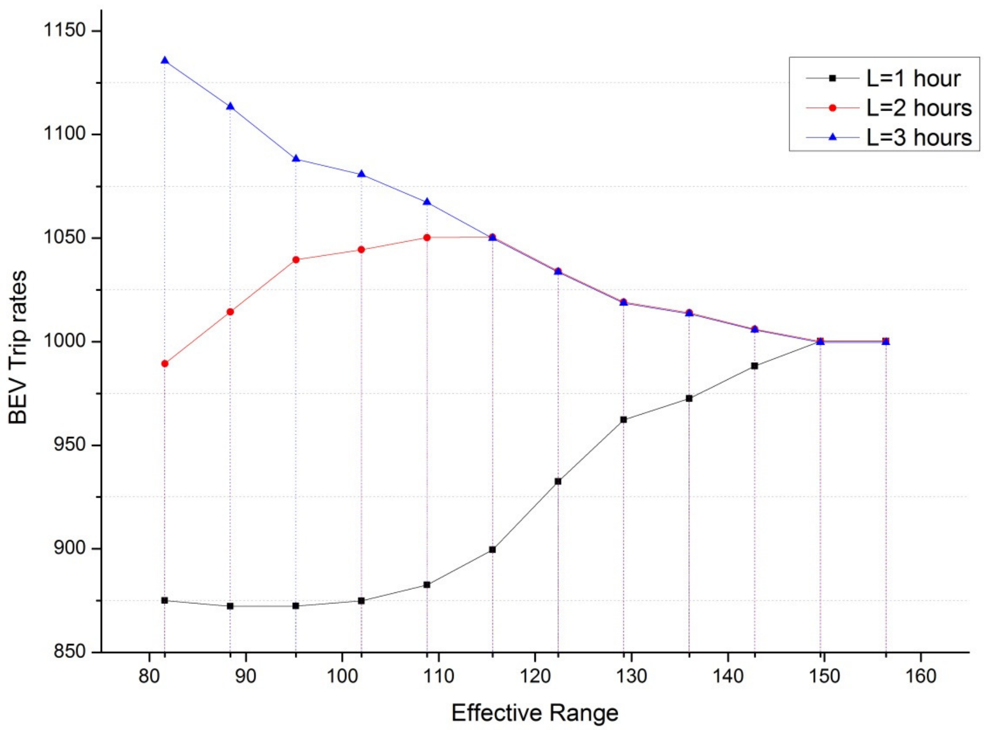

Figure 4-11: Total BEVs Trip Demands Between (1,12) for Different Duration of Stay

4.5.3.2 Impact on road networks

Table 4-3 summarizes the total system performance with different BEV effective ranges, including total system travel time cost (TSTT), total vehicle miles traveled (TVMT), total system charging cost (TTCC), and total system charging time (TTCT).

The table shows that both TTST and TTCC decrease with the BEV range increase. This is intuitive because with the BEV range increase, the BEV drivers are less restricted by the length they can traverse. Therefore, the BEV drivers can pursue faster routes in terms of travel time. The increase of BEV ranges will either reduce the number of recharging BEVs at destinations or reduce the amount of recharging electricity for BEVs at destinations. Consequently, the total charging costs for the BEV drivers decrease. Moreover, the TVMT increases with the increase of BEV range. This is due to the fact that BEV drivers might be able to pursuing faster but longer routes. The total charging time penalty for all BEV drivers decreases to zero after the BEV range increase to and beyond 108.8 miles. That is because the charge time needed for all BEV drivers at destination is less than 1 hour.

Ranges	TSTT	TVMT	TTCC	TTCT
81.6	29366.8	999537	5655.46	584.028
88.4	29083	1002897	5299.94	392.453
95.2	28785.2	1006548	4938.18	203.337
102	28524.7	1009967	4569.87	17.9211
108.8	28336	1012476	4364.73	0.00
115.6	28034.2	1016517	4000.46	0.00
122.4	27776.4	1019850	3673.19	0.00
129.2	27529.9	1023148	3344	0.00
136	27336.9	1025921	3163.32	0.00
142.8	27098	1029382	2894.19	0.00
149.6	26834.2	1033038	2818.39	0.00
156.4	26598	1035958	2759.14	0.00

Table 4-3 System Performance

#	Path	Free Flow Travel Time Cost	Length
1	1-5-14-20	0.85	90
2	2-7-16-22	0.9	40
3	3-8-17-23	0.85	80
4	10-4-6-9	0.8	90
5	11-13-15-18	0.8	70
6	12-19-21-24	0.9	68

Table 4-4 Selected Paths Attributes

Six paths are selected for investigating the path flows results. These paths between each O-D pair, these paths are summarized in the above table 4-4. The free flow travel time and the path lengths are shown in column 3~4. The second column is the associated links for each path. The path flow results for BEVs and GVs are shown in Appendix B, Table B-3 and B-4 respectively.

Figure 4-12 depicts the selected BEV path flows changes with increased BEV ranges. Figure 4-13 depicts that for GV path flows. The three selected paths between O-D pair (1,12) are shown. The BEV path flows on path #1 and path #3 increase while that on path #2 decrease. This is because the travel time on path #2 is greater than those on the other two paths. The GV path flows follow an opposite behavior. This indicates that the BEV ranges not only affects the BEV drivers' route choice behavior but also affects the GV drivers' route choice behavior indirectly. Such indirect influence might not be very big but it is expected. The GVs drivers' route choice behavior is affected by two factors: the travel time costs and the operating costs. When the BEV drivers switch to path #1 and path #3, the travel time cost on these two paths might increase because of congestions. Consequently, some GVs drivers might switch to path#2 since it has lowest operating costs and since the GVs operating cost is greater than that of the BEVs.

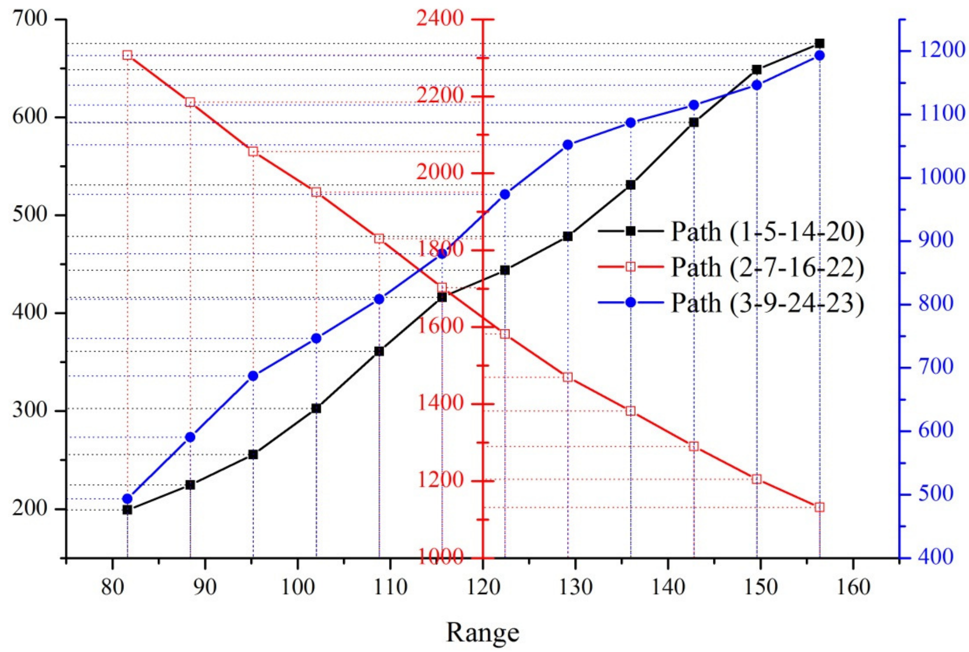

Figure 4-12: BEVs Path Flows Changes with Increasing BEV Range

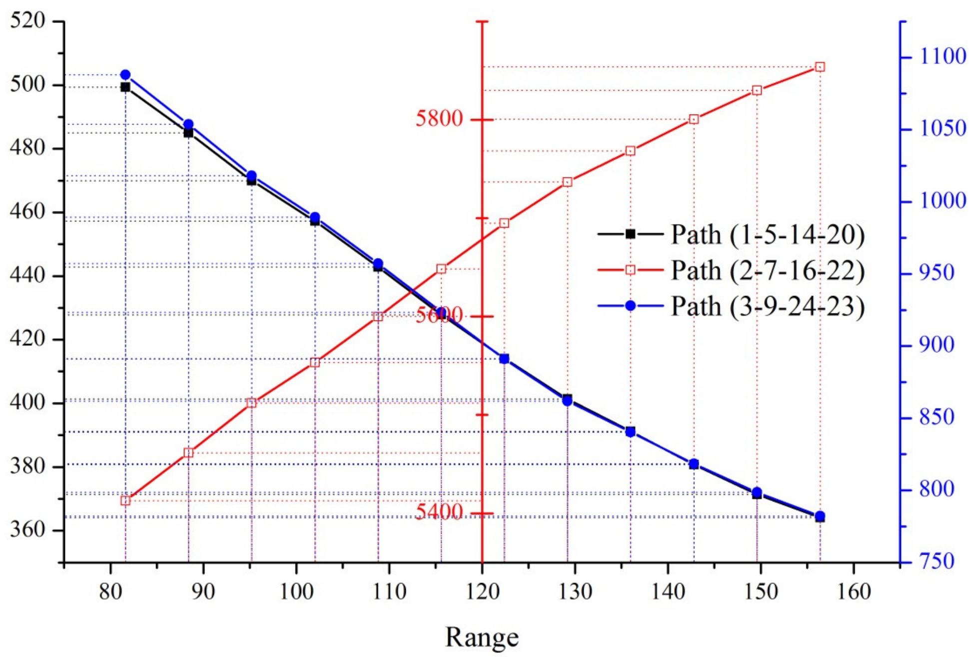

Figure 4-13: GVs Path Flows Changes with Increasing BEV Range

4.5.3.3 Impact on charging behavior

The following two figures, Figure 4-14 depicts the number of accumulative BEV arrivals for recharging and accumulative departures from recharging at different time of day at destination node 13 with different BEV effective range. Figure 4-15 depicts the number of recharging BEVs at different time of day (the complete results are shown in Appendix B Table B-5).

It can be observed from Figure 4-14 that when the BEV range is improved the BEV drivers are able to leave the charging stations earlier. This can be seen when the range is improved from 81.6 miles to 108.8 miles: while the accumulative arrivals do not change very much, the accumulative departure curve switches towards the left. This indicates that the charging time is shortened in general. Also, the peak recharging demands drops from around 1,630 vehicles to 1,404 vehicles when the BEV range increase from 81.6 miles to 108.8 miles (Fig. 4-15).

In addition, the total demand of recharging BEVs drops when the BEV range increases, which is intuitive. This can be seen from both Figure 4-14 and 4-15. For instance, once the BEV effective range reaches 136 miles, the total demand for recharging drops from around 2,100 vehicles to around 1,200 vehicles, comparing to the range of 108.8 miles (Figure 4-14).

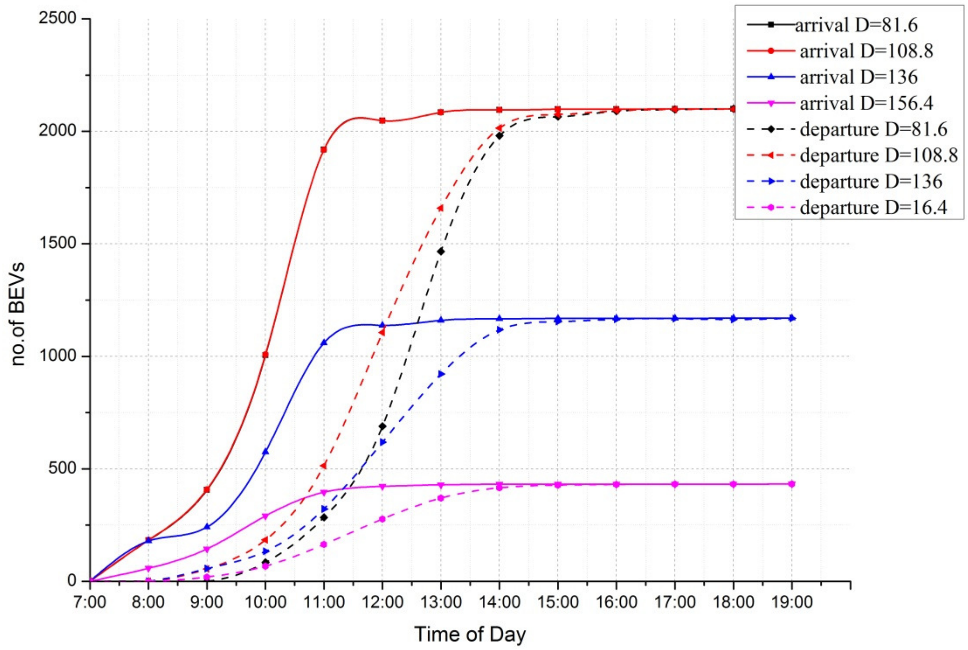

Figure 4-14: Charging Demand at Different Time of Day with Different BEV Ranges

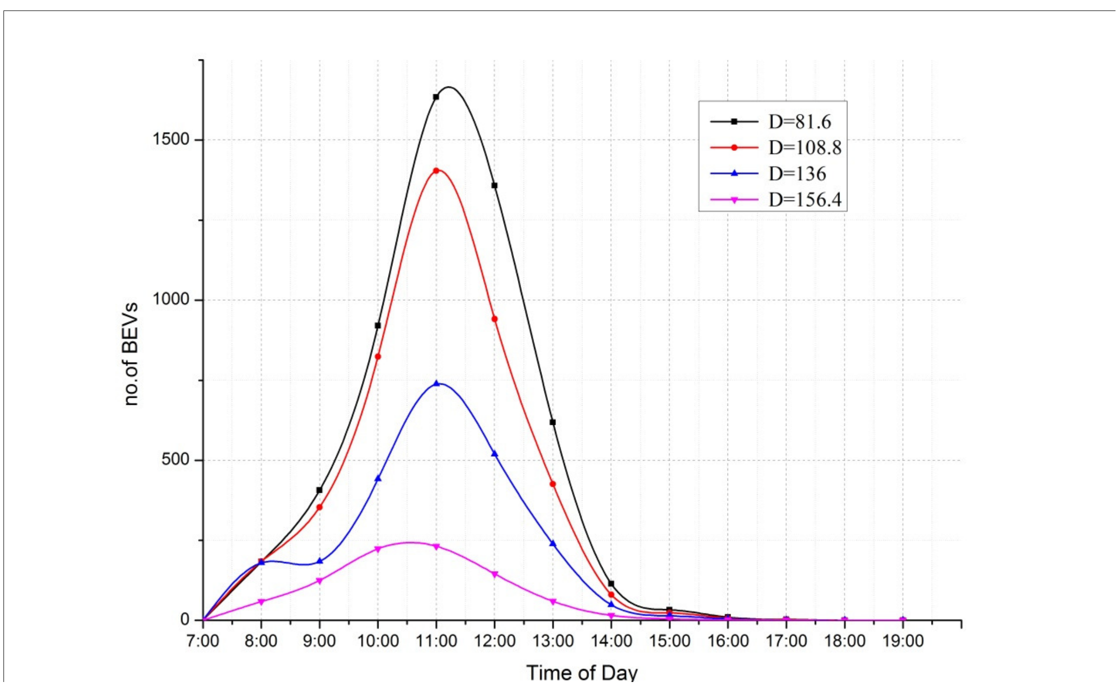

Figure 4-15: Charging Demand at Different Time of Day with Different BEV Ranges

4.5.4, impacts of penetration rate

The portion of BEVs on the road network affects the importance of studying the BEVs users' behavior. If there is only a small portion of BEVs on the road, it is expect that current road network conditions and trips rates at different time of day remains. However, if the portion of BEVs on the road is relatively large, studying the BEVs users' behavior adds value to the research.

The future BEV penetration rate is influenced by the market conditions, the improvement of BEV technologies, and the policy incentives, thus it is uncertain. Therefore, it is necessary to study different scenarios with different penetration rates. In the numerical example, scenarios with the BEV penetration rates ranging from 5% to 100% of the total vehicles on the road are studied. The scenarios with BEV sharing more than 30% of the market are quite ambitious and unlikely. However, the purpose of these scenarios is to gain a more complete knowledge of the impacts from BEV market share. The BEV effective range is fixed at $\rho = 1.5$, i.e. $D = 102$. Other parameters remain the same (see Appendix, Table B-2).The demands of BEVs and GVs with different BEV penetration rate scenarios are shown in the following table 4-5.

O	D	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
1	12	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
5	13	350	700	1050	1400	1750	2100	2450	2800	3150	3500
O	D	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
1	12	5500	6000	6500	7000	7500	8000	8500	9000	9500	10000
5	13	3850	4200	4550	4900	5250	5600	5950	6300	6650	7000

Table 4-5 Trip Table of BEVs with Different Penetration Rate

The results of the BEVs and GVs trip demands at different time of day are collected (see Appendix B, Table B-6~Table B-9). We note that, in the test results, the different penetration rates of BEVs have minor impacts on the departure time of travelers. However, this might subject to change when the charging facility capacity is limited at the destination. In this example, the charging facility capacity is quite large, and neither of the two charging locations is fully occupied at any time. Therefore, the charging locations are not congested, which have minor impacts on the departure time choice of BEVs drivers. Actually, the BEV drivers and GV drivers between the same O-D pair, for the same purpose of travel, with the same arrival time window, may depart from the origins at different time intervals to avoid the congestion at charging if the charging facility capacity is very limited.

Figure 4-16 depicts the TSTT and TVMT with different BEV penetration rates. It can be observed that the total length traveled by all motorists increases with more BEVs on the road network. But the increase is not significant. This might because the per unit distance operating cost (fuel costs) for GV drivers is greater than that for BEV drivers. Therefore, BEV drivers can choose longer path; however, this subject to the BEV range as well. In addition, the TSTT decreases with the increase of BEV penetration rates at first and then increases. The decrease is in TSTT is expected, as explained above, since BEV drivers have smaller operating cost (which is the charging cost here) than GV drivers that they can choose routes with longer length but shorter travel time. The increase of TSTT after the BEV penetration rates is greater than 80% might due to the congestions on those utilized routes by BEV drivers. These results are consistent of that presented in section 3.5.2. The TSTT and TVMT results are displayed in Appendix B, Table B-10~Table B-11 respectively.

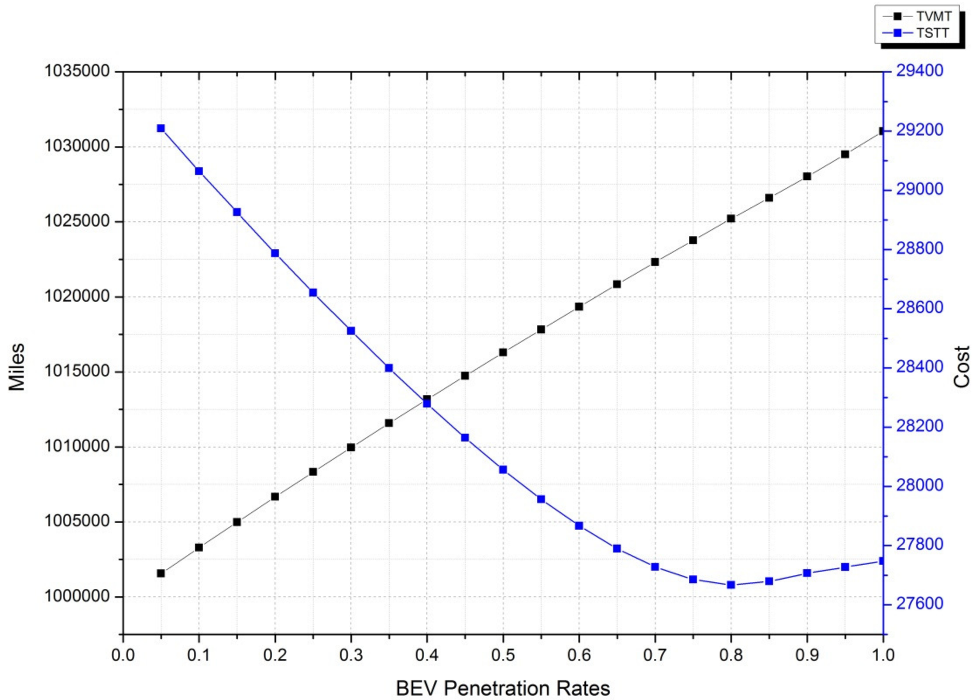

Figure 4-16: TSTT and TVMT with Different BEV Penetration Rates

4.6 SUMMARY

New opportunities exist for studying the traveling behaviors of motorists and for investigating the impacts from introducing BEVs on to the road networks. Tractable modeling techniques are critical for developing realistic and effective travel behaviors models. The model formulation and solution approach presented in this paper provide a means for accounting for BEV drivers' behavior in the time-dependent, multi-class, combined travel choices network equilibrium. Special consideration of BEV drivers including the recharging cost and electricity charging time are included in the model. The problem is formulated as a variational inequality model where the joint choices are proved to follow the NL structure.

The main contribution of this work is in formulating the joint travel choices behavior of BEV drivers and in formulating a multi-class time-dependent joint choices model for mixed traffic flows, including both BEVs and GVs. Further, the formulation was solved using a decomposition technique where the two-level travel choices are solved iteratively.

The subsequent numerical analysis shows the demand matrix at different time of the day and the recharging flows at destinations' charging locations. The results suggest that most of the motorists depart according to their time window. It was found that the duration of the stay choice for BEV drivers at destination is restricted by the vehicle range and the charging speed. The numerical solution result verifies that the proposed model can indeed generate a time-dependent network equilibrium solution. In addition, the numerical experiments aim to show the effects of various BEV effective ranges and different BEV penetration scenarios. The proposed modeling approach can be used to reveal and study the complex temporal and spatial interactions between BEV traffic pattern and BEV effective range, between BEV traffic pattern and BEV penetration rates, and between BEV and GV traffic pattern.

Chapter 5: Charging Facility Capacity and Pricing Design

5.1, INTRODUCTION

In response to the employing of PEVs, charging infrastructure is required for serving existing PEVs and for boosting the penetration rate for future PEVs. In general, the pricing is one of the most efficient ways to manage travel demand (Verhoef et al. 1995). In the sense of managing travel demand, the supply and pricing of electricity-charging infrastructure might be similar to that of the parking supply and pricing, (see (Garcia and Marín 2002; Li et al. 2007) for examples). Particularly, the pricing of the electricity-charging at destination together with the infrastructure supply may influence the PEV drivers' choice of routes, duration of stay and departure time. In addition, policies on charging supply and pricing might potentially impact the total transportation system performance and the market share of PEVs.

The equilibrium model built in the previous chapter is able to answer the questions of route choice, departure time choice and duration of stay at destination choice with given electricity-charging pricing and charging infrastructure supply. Based on the drivers' responses to the charging pricing and the supply, this chapter builds a model to determine the optimal pricing and supply with application in revenue management. The decisions to be made are the charging infrastructure supply and the time-dependent pricing within a day. The model proposed is situated in tactical planning and this type of optimization belongs to the well-known continuous network design problem (CNDP).

This chapter of the dissertation proposes a model to assist the planner in deciding the optimal charging infrastructure supply and pricing of the electricity-charging to optimize the revenue. The mathematical formulation in the form of mathematical program with equilibrium constraints (MPEC) is developed in this chapter. A solution approach based on the sensitivity analysis method is proposed. At the end, computational results on two test networks are presented to demonstrate the application of the proposed solution approach.

The chapter is structured as follows: section 5.2 reviews some of the existing work on sensitivity analysis and its application in solving optimization problems. Section 5.3 proposed the MPEC model and section 5.4 present the sensitivity analysis based approach for solving the proposed MPEC model. Section 5.5 briefly states the genetic algorithm (GA) which could alternatively solve the proposed model. The results from the sensitivity analysis based (SAB) method and from GA are compared. The last section presents illustration examples.

5.2, SENSITIVITY ANALYSIS IN TRAFFIC ASSIGNMENT

Sensitivity analysis in traffic network equilibrium problems is typically concerned with the implicit relationship between the solution of the equilibrium flows and the changes in the input to the problem.

A variety of problem formulation techniques have been used for the purpose of sensitivity analysis. A well-known method for sensitivity analysis was developed by (Tobin and Friesz 1988) for the Wardrop equilibrium modeling of traffic networks. Sensitivity analysis of the Wardrop equilibrium model based on Tobin and Friesz's work has been extended to a number of generalizations. Yang (1997) developed and applied the sensitivity analysis for DUE model with elastic travel demand for solving congestion pricing and network design problems. Other extensions for such sensitivity analysis method including a DUE with steady state queuing (H. Yang 1995) and with randomly distributed values of time (Leurent 1998). Overall, existing work in the literature on traffic assignment sensitivity analysis derives almost exclusively from applications and extensions of Tobin and Friesz's results (Cho et al. 2001; Patriksson and Rockafellar

2003); However, such DUE-style model receives many critics in the sense of non-uniqueness of DUE path flows, which poses difficulty. Later Tobin and Friesz demonstrated a restricted formulation of the DUE problem to overcome this difficulty, but additional conditions need to be hold in the restricted formulation. In addition, some methods of sensitivity analysis for general variational inequality problems were also developed and applied to the DUE model on traffic networks (S. Dafermos 1988; kyparisis 1987; Noor 2009; Qiu and Magnanti 1989).

Since the SUE models are known to give rise to unique path flows under mild conditions (Sheffi 1985). Bell and Iida (1997) studied the logit-based SUE assignment model, and showed the derivatives of sensitivity expressions. (Davis 1994) proposed an exact computational algorithms for sensitivity analysis of the logit SUE model and applied to network design problem for finding local optimal. Other efficient algorithms based on sensitivity analysis of Logit SUE model were developed later for traffic networks with fixed and elastic travel demands (Gentile and Papola 2001; H. Huang, J et al. 2001; Ying and Miyagi 2001; Ying et al. 2001). (Ying and Yang 2005) presented a computational method for sensitivity analysis for SUE with both auto and physically separate transit network. The algorithm was applied to the optimal pricing problem in a combined mode network.

(Clark and Watling 2002) provided a formulation of sensitivity analysis method deal with Probit SUE models and later showed a variety of applications including the network design problems (Clark and Watling 2006). (Connors et al. 2007) derived the sensitivity expression for a formulation of multiple user class, variable demand Probit SUE and applied it on a network design problem. The Probit SUE network design problem they presented was solved using a gradient-based approach utilizing the sensitivity expressions derived.

Besides the sensitivity analysis of traffic assignment models, (C. Yang and Chen 2009) presented the sensitivity analysis method for combined travel demand models formulated as a non-linear programming problem based on Oppenheim's work (Oppenheim 1995). In addition, some applications such as Paradox analysis, access control, destination choice analysis and error analysis were presented in their work. (Abou Zeid and Chabin 2003) derived the sensitivity expression of equilibrium flows to congestion pricing in a dynamic traffic networks with combined choices of routes and departure time.

5.3, NETWORK DESIGN MODEL FORMULATION

The charging facility design problem herein is to determine the capacity and pricing strategy. The problem is formulated by using MPEC, where at upper-level, the revenue of the charging facility is maximized and at the lower-level the motorists react to the charging capacity and pricing. At the lower-level, the motorists take decisions following their own criteria and in the end a time-dependent equilibrium condition is reached.

This model assumes that the location of charging facilities and topological design of the road network are known and fixed. In addition to the notations in Chapter 3 and Chapter 4, we have the following notations:

s	destination index;
	also represent the charging facility link at that destination
$\tilde{k}$	route index, where the charging facility link s is contained in the route, i.e. $s \in \tilde{k}, \forall s, \tilde{k}$, $\tilde{k} \in \widetilde{K} \subseteq K$
k, y	route index of all routes, $k, y \in K$,
ρ_s	cost of per unit capacity at destination s

φ	scaling factors converting the cost (management and financing costs) of charging capacity to travel time units
$E_s(\cdot)$	a function of charging capacity at destination s, cost function of managing and financing the charging infrastructure at destination s
$e_k^{rs.m}(t)$	charging cost for PEV travelers using route k between O-D pair (r,s) for user class m ,arriving at charging facility at time t
$P^{rs,b,j}(k,t,l)$	the joint probability of choosing departure time and duration of stay alternative (t,l) and the path k for traveling between O-D pair (r,s) using type b vehicles and for the purpose of travel j

Recall that the disutility (measured in time unit) for user class m departing at time interval t and though route k between O-D pair (r,s) is a summation of the cost components:

1) on board cost (travel time cost on route) $T_k^{rs,m}(t)$;

2) charging waiting/congestion cost $z_s^{rs,m}(t+T_k^{rs,m}(t))$;

3) charging cost $e_k^{rs.m}(t+T_k^{rs,m}(t))$;

4) charging time cost $ctp_k^{rs,m}$;

5) the schedule delay cost $SD^{rs,m}(t+T_k^{rs,m}(t)+z_s^{rs,m}(t+T_k^{rs,m}(t)))$

In this chapter, the electricity-charging cost $e_k^{rs,m}(t)$ is time dependent and it is defined as follows:

$$e_k^{rs,m}(t)=\begin{cases} e_h\sum_a d_a\delta_{a,k}^{rs} & \sum_a d_a\delta_{a,k}^{rs}\le\frac{D}{2} \\ \frac{1}{2}[e_s(t)\cdot(2\sum_a d_a\delta_{a,k}^{rs}-D)+e_hD], & \frac{D}{2}<\sum_a d_a\delta_{a,k}^{rs}\le D \end{cases} \tag{5.1}$$

The measured travel disutility, $C_k^{rs,m}(t)$, is formulated as the summation of the above cost components:

$$
\begin{aligned}
C_k^{rs,m}(t) = {} & \alpha_1^m T_k^{rs,m}(t) + \alpha_2^m z_s^{rs,m}(t + T_k^{rs,m}(t)) \\
& + \alpha_3^m e_k^{rs.m}(t + T_k^{rs,m}(t) + z_s^{rs,m}(t + T_k^{rs,m}(t))) \\
& + \alpha_4^m ctp_k^{rs,m} + \alpha_5^m SD^{rs,m}(t + T_k^{rs,m}(t) + z_s^{rs,m}(t + T_k^{rs,m}(t)))
\end{aligned}
\tag{5.2}
$$

5.3.1, the lower level problem

The lower level problem in the network design problem is the problem stated in the previous chapter, but with time-dependent electricity-charging costs. The equivalent VI formulation to the hierarchical choice structure for motorists was presented in Chapter 4 (E.q. 4.21). The feasible set Ω, of all the constraints that are associated with the joint travel choice was defined by equations (4.17)-(4.20).

5.3.2, the upper level optimization problem

The objective of the upper-level problem in this research is to optimize the revenue of the electricity-charging facility. This revenue is defined as the difference between charging facility incomes and costs, where the income is generated from the electricity-charging by PEV drivers, and the cost is from the investment of building the charging facility (financing cost) and the management cost.

Both the income and the cost are considered on a daily basis. Let the income be noted as I in the daily period, raised by electricity-charging. Let the cost (E) be the sum of the daily management cost and the daily financing cost of the charging facility, which is a function of the charging facility capacity.

The income from electricity-charging is the summation of all the electricity that all PEVs recharged at the charging facilities, and can be mathematically represented as follows,

$$I = \sum_{rs}\sum_{m}\sum_{\tilde{k}}\sum_{t} e_s(t_{\tilde{k}}^{rs,m}(t)) \cdot (2\sum_{a} d_a \delta_{a,\tilde{k}}^{rs} - D) \cdot f_{\tilde{k}}^{rs,m}(t) \tag{5.3}$$

where $\tilde{k}$ is the route which contains the charging facility link at destination s; $t_{\tilde{k}}^{rs,m}(t)$ is the arrival time at charging facility using the route $\tilde{k}$ between O-D pair (r,s) by user class m when departing from origin at time interval t. Therefore, $t_{\tilde{k}}^{rs,m}(t)$ could be calculated as,

$$t_{\tilde{k}}^{rs,m}(t) = t + T_{\tilde{k}}^{rs,m}(t) \tag{5.4}$$

Let the cost function $E_s(\cdot)$ be a linear function of the charging capacity C_s. The cost can be calculated as follows,

$$E = \sum_{s} E_s(C_s) = \sum_{s} \rho_s C_s \tag{5.5}$$

The revenue is thus, $R = I - E$, that is,

$$R = \sum_{rs}\sum_{m}\sum_{\tilde{k}}\sum_{t} e_s(t_{\tilde{k}}^{rs,m}(t)) \cdot (2\sum_{a} d_a \delta_{a,\tilde{k}}^{rs} - D) \cdot f_{\tilde{k}}^{rs,m}(t) - \varphi \cdot \sum_{s} E_s(C_s) \tag{5.6}$$

Where φ is the scaling factor converting the management cost and financing cost to the time unites. In this way, the income and the cost are in the same unit.

5.3.3, the MPEC problem

Overall, the upper-level problem together with the lower-level problem constitutes the bi-level network design problem. It aims to maximize the revenue while subjects to the lower-level equilibrium constraint. The MPEC problem is summarized as follows:

(A)

$$\max_{C_s, e_s(t)} R \tag{5.7}$$

Subject to VI equation (4.21) and constraint (4.17)~(4.20) and,

$$C_{\min} \le C_s \le C_{\max}, \qquad \forall \mathrm{s} \tag{5.8}$$

$$e_{\min} \le e_s(t) \le e_{\max}, \qquad \forall s \tag{5.9}$$

where the constraint (5.8) is the minimum and maximum capacity restriction constraint. Constraint (5.9) is imposed for a proper range of electricity-charging price, which can also be regarded as relevant to electric grid power supply. For example, if the electricity-charging pricing at a very low point, it may attract too much electricity demand at a destination, which might overloads the local transformer. To be in line with our assumption, the electricity-charging price at destination needs to be at least equal to or higher than the home electricity-charging price.

5.4, SOLUTION APPROACH

Solution algorithms for bi-level optimization problems where the traffic equilibrium arises as a lower-level problem can be developed by conducting a sensitivity analysis (H. Yang 1997). In this dissertation, a sensitivity analysis based solution strategy is adapted to search for the local optimum of the MPEC model developed. The sensitivity analysis expression is used to find the descent-gradients of the objective function.

5.4.1, gradient based solution approach

The upper-level problem, that is, the objective function with the pricing and capacity constraints, consists of nonlinear and implicit functions with decision variable vectors $\mathbf{C}$ and $\mathbf{e}$, where $\mathbf{C} = \{C_s, \forall s\}$ and $\mathbf{e} = \{e_s(t), \forall s, t\}$. Let the decisions variable vector be $\mathbf{x} = (\mathbf{C}, \mathbf{e})$, the gradient based method uses the iteration:

$$\mathbf{x}^{(k+1)} = P_X[\mathbf{x}^{(k)} + \gamma_k \mathbf{g}^{(k)}] \tag{5.10}$$

where $\mathbf{x}^{(k)}$ is the kth iteration solution, $\mathbf{g}^{(k)}$ is the gradients of the objective function R (Eq. 5.7) at $\mathbf{x}^{(k)}$, and $\gamma_k > 0$ is the kth step size. $P_X[y]$ is the projection of a vector y on the set X. By the Projection Theorem, the projection of any vector y on X exists. When X is closed and convex, the projection is unique (Nedich 2006). In the upper-level problem, the constraints are boxes, and the X is a box set $X = \{x \mid a_i \leq x \leq b_i, \forall i\}$. The projection on X can be decompose into projections per coordinate, and the components of the projection $P_X[x]$ vector are given by

$$[P_X[x]]_i = \begin{cases} a_i & \text{if } x_i < a_i \\ x_i & \text{if } a_i \leq x_i \leq b_i \\ b_i & \text{if } x_i > b_i \end{cases} \tag{5.11}$$

Noted that, the objective function R is continuous over feasible set X, which ensures ∂R is nonempty for every $x \in X$. As a maximization problem, at each iteration of the method, we take a step in the direction of a positive gradient. In order to maintain feasibility at each iteration, the step size needs to be limited be the upper level problem constraints.

The gradient of objective function R is a column vector, that is $\mathbf{g}$, and it can be calculated directly as follows,

$$\mathbf{g} = [g_1, g_2, \ldots, g_n] = \left[\frac{\partial R}{\partial e_1(1)}, \frac{\partial R}{\partial e_2(1)}, \ldots, \frac{\partial R}{\partial e_1(2)}, \frac{\partial R}{\partial e_2(2)}, \ldots, \frac{\partial R}{\partial C_1}, \frac{\partial R}{\partial C_2}, \ldots\right]^T \tag{5.12}$$

where,

$$\begin{aligned} \frac{\partial R}{\partial e_s(t)} = & \sum_r \sum_m \sum_{t':t'+T_{\tilde{k}}^{rs,m}(t)=t} \sum_{\tilde{k}} (2\sum_a d_a \delta_{a,\tilde{k}}^{rs} - D) f_{\tilde{k}}^{rs,m} * (t') \\ & + \sum_r \sum_k \sum_{t':t'+T_{\tilde{k}}^{rs,m}(t)=t} e_s(t)(2\sum_a d_a \delta_{a,\tilde{k}}^{rs} - D) \nabla_{e_s(t)} f_{\tilde{k}}^{rs,m}(t') \end{aligned} \tag{5.13}$$

$$\frac{\partial R}{\partial C_s} = \sum_m \sum_{t':t'+T_{\tilde{k}}^{rs,m}(t)=t} \sum_{\tilde{k}} e_s(t)(2\sum_a d_a \delta_{a,\tilde{k}}^{rs} - D) \nabla_{C_s} f_{\tilde{k}}^{rs,m}(t') - \varphi \rho_s \tag{5.14}$$

where $f_{\tilde{k}}^{rs,m*}(t')$ is the equilibrium path flows at current solution of vector $\mathbf{C}$ and $\mathbf{e}$, and this equilibrium solution is a function of vector $\mathbf{C}$ and $\mathbf{e}$, that is $f_{\tilde{k}}^{rs,m*}(t') = f_{\tilde{k}}^{rs,m*}(t')(\mathbf{C},\mathbf{e})$.

The solution approach solves a local approximation at each iteration. The gradient can be obtained on the basis of the derivatives of electricity-charging costs, path flows and operating costs with respect to the charging infrastructure supply and electricity-charging pricing, which is shown in the next section.

5.4.2, derivatives with respect to $\mathbf{C}$ and $\mathbf{e}$

In this section, the derivatives of all variables with respect to the charging facility capacity supply and electricity-charging pricing are presented. From the electricity-charging cost function (5.1) and the definition of the operating cost function of the charging facility (5.5), it is easy to see that $\nabla_{\mathbf{e}} E_s(\mathbf{C}^*) = 0$, and this term is neglected in the formulas of (5.14).

Next, the gradient expressions, $\nabla_{\mathbf{C}} f_{\tilde{k}}^{rs,m}(t)(\mathbf{C}^*,\mathbf{e}^*)$, $\nabla_{\mathbf{e}} f_{\tilde{k}}^{rs,m}(t)(\mathbf{C}^*,\mathbf{e}^*)$, and $\nabla_{\mathbf{C}} E_s(\mathbf{C}^*)(\mathbf{C}-\mathbf{C}^*)$ are derived. The gradients can be obtained by adopting the method that was proposed in (Tobin and Friesz 1988; Viti et al. 2003; H. Yang 1995; Zeid 2003).

The flow on path $\tilde{k}$ between O-D pair (r,s) is a function of the disutilities of all choice alternatives of path between O-D pair (r,s) and departure time and duration of stay alternatives (t,l), that is,

$$\nabla_{C_s} f_{\tilde{k}}^{rs,m}(t) = \sum_{y} \sum_{(t',l')} \frac{\partial f_{\tilde{k}}^{rs,m}(t)}{\partial C_y^{rs,m}(t')} \cdot \frac{\partial C_y^{rs,m}(t')}{\partial C_s} \tag{5.15}$$

The second term of the right hand side of the above equation (5.15) is as follows,

$$\frac{\partial C_y^{rs,m}(t')}{\partial C_s} = \begin{cases} \dfrac{\partial z_s^{rs,m}(t' + T_y^{rs,m}(t'))}{\partial C_s}, & \text{if } s \in y \\ 0 & \text{otherwise.} \end{cases} \tag{5.16}$$

The first tem of the right hand side of the above equation (5.15) can be derived as follows using our Logit model of route choices and departure time and duration of stay choices.

$$\frac{\partial f_{\tilde{k}}^{rs,m}(t)}{\partial C_y^{rs,m}(t')} = \frac{\partial (q^{rs} \cdot P^{rs,b,j}(\tilde{k}, t, l))}{\partial C_y^{rs,m}(t')} = q^{rs} \frac{\partial (P^{rs,b,j}(\tilde{k}, t, l))}{\partial C_y^{rs,m}(t')} \tag{5.17}$$

The parameter q^{rs} is the total trip rates between O-D pair (r, s) and is given and fixed. Therefore, it can be taken out of the partial derivatives. In chapter 4, it was shown that the probability of choosing alternative (t, l) is

$$P^{rs,b,j}(t, l) = \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t',l')} \exp(-\beta L_{t'}^{rs,m'})} \tag{5.18}$$

And the probability of choosing alternative k conditional on the choices of alternative (t, l) (Eq. 4.12) is

$$P^{rs,b,j}(k / t, l) = \frac{\exp(-\theta C_k^{rs,m}(t))}{\sum_{k'} \exp(-\theta C_{k'}^{rs,m}(t))} \tag{5.19}$$

Therefore, we get the joint probability of alternative of route, departure time and duration of stay (k, t, l) as

$$\begin{aligned} P^{rs,b,j}(k, t, l) &= P^{rs,b,j}(k / t, l) P^{rs,b,j}(t, l) \\ &= \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t',l')} \exp(-\beta L_{t'}^{rs,m'})} \frac{\exp(-\theta C_k^{rs,m}(t))}{\sum_{k'} \exp(-\theta C_{k'}^{rs,m}(t))} \end{aligned} \tag{5.20}$$

We have shown the way the cost propagated different levels of the Logit model, which satisfies Eq. (4.40), that is in Eq. (5.20) the term $\sum_{k'} \exp(-\theta C_{k'}^{rs,m}(t))$ satisfy the following,

$$\sum_{k'} \exp(-\theta C_{k'}^{rs,m}(t)) = \exp(-\theta L_t^{rs,m}) \tag{5.21}$$

Substitute Eq. (5.21) into Eq. (5.20),

$$\begin{aligned} P^{rs,b,j}(k,t,l) &= \frac{\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)} \exp(-\beta L_t^{rs,m})} \frac{\exp(-\theta C_k^{rs,m}(t))}{\exp(-\theta L_t^{rs,m})} \\ &= \frac{\exp((\theta-\beta) L_t^{rs,m} - \theta C_k^{rs,m}(t))}{\sum_{(t,l)} \exp(-\beta L_t^{rs,m})} \end{aligned} \tag{5.22}$$

Next, we show the derivations of the term $\dfrac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')}$ in Eq. (5.17). The derivations are developed and conducted based on different cases as discussed in (Zeid 2003),

- **Case I** $y = \tilde{k}, (t,l) = (t',l')$, that is,

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} = \frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m}(t)} \tag{5.23}$$

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m}(t)} =$$

$$\frac{(-\theta+(\theta-\beta)\dfrac{\partial L_t^{rs,m}}{\partial C_{\tilde{k}}^{rs,m}(t)})\cdot\exp((\theta-\beta)L_t^{rs,m}-\theta C_{\tilde{k}}^{rs,m}(t))\cdot\sum_{(t,l)}\exp(-\beta L_t^{rs,m})}{[\sum_{(t,l)}\exp(-\beta L_t^{rs,m})]^2} \quad (5.24)$$

$$-\frac{\exp((\theta-\beta)L_t^{rs,m}-\theta C_{\tilde{k}}^{rs,m}(t))\cdot\exp(-\beta L_t^{rs,m})\cdot(-\beta\dfrac{\partial L_t^{rs,m}}{C_{\tilde{k}}^{rs,m}(t)})}{[\sum_{(t,l)}\exp(-\beta L_t^{rs,m})]^2}$$

From Eq. (4.40) and (5.19) we have,

$$\frac{\partial L_t^{rs,m}}{\partial C_{\tilde{k}}^{rs,m}(t')} = \begin{cases} -\dfrac{1}{\theta}\dfrac{-\theta\cdot\exp(-\theta C_{\tilde{k}}^{rs,m}(t))}{\sum_k \exp(-\theta C_k^{rs,m}(t))} = P^{rs,b,j}(\tilde{k}\,/\,t,l), \ \text{if}(t,l)=(t',l') \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{o.w.} \end{cases} \quad (5.25)$$

Substitute (5.24) into Eq. (5.23), we get

$$\begin{aligned} \frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m}(t)} &= [-\theta+(\theta-\beta)P^{rs,b,j}(\tilde{k}\,/\,t,l)]P^{rs,b,j}(\tilde{k},t,l) \\ &\quad - P^{rs,b,j}(\tilde{k},t,l)\frac{-\beta P^{rs,b,j}(\tilde{k}\,/\,t,l)\exp(-\beta L_t^{rs,m})}{\sum_{(t,l)}\exp(-\beta L_t^{rs,m})} \\ &=[-\theta+(\theta-\beta)P^{rs,b,j}(\tilde{k}\,/\,t,l)]P^{rs,b,j}(\tilde{k},t,l) \\ &\quad - P^{rs,b,j}(\tilde{k},t,l)(-\beta)P^{rs,b,j}(\tilde{k}\,/\,t,l)P^{rs,b,j}(t,l) \end{aligned} \quad (5.26)$$

Since,

$$P^{rs,b,j}(\tilde{k},t,l) = P^{rs,b,j}(\tilde{k}\,/\,t,l)P^{rs,b,j}(t,l) \quad (5.27)$$

Therefore,

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m}(t)} = [-\theta + (\theta-\beta)P^{rs,b,j}(\tilde{k}\ /\ t,l) + \beta P^{rs,b,j}(\tilde{k},t,l)]P^{rs,b,j}(\tilde{k},t,l) \quad (5.28)$$

- **Case II** $y \neq \tilde{k},(t,l) = (t',l')$, that is,

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{y}^{rs,m}(t')} = \frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{y}^{rs,m}(t)} \quad (5.29)$$

$$\begin{aligned} &\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{y}^{rs,m}(t)} = \\ &\frac{(\theta-\beta)\dfrac{\partial L_t^{rs,m}}{\partial C_y^{rs,m}(t)} \cdot \exp((\theta-\beta)L_t^{rs,m} - \theta C_{\tilde{k}}^{rs,m}(t)) \cdot \sum_{(t,l)} \exp(-\beta L_t^{rs,m})}{[\sum_{(t,l)} \exp(-\beta L_t^{rs,m})]^2} \\ &- \frac{\exp((\theta-\beta)L_t^{rs,m} - \theta C_{\tilde{k}}^{rs,m}(t)) \cdot \exp(-\beta L_t^{rs,m}) \cdot (-\beta \dfrac{\partial L_t^{rs,m}}{C_y^{rs,m}(t)})}{[\sum_{(t,l)} \exp(-\beta L_t^{rs,m})]^2} \end{aligned} \quad (5.30)$$

Substitute Eq. (5.21) and (5.24) to Eq. (5.30), we have

$$\begin{aligned} &\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{y}^{rs,m}(t)} = \\ &(\theta-\beta)P^{rs,b,j}(y\ /\ t,l)P^{rs,b,j}(\tilde{k},t,l) - P^{rs,b,j}(\tilde{k},t,l)(-\beta)P^{rs,b,j}(y\ /\ t,l)P^{rs,b,j}(t,l) \\ &= [(\theta-\beta) + \beta P^{rs,b,j}(t,l)]P^{rs,b,j}(y\ /\ t,l)P^{rs,b,j}(\tilde{k},t,l) \end{aligned} \quad (5.31)$$

- **Case III** $y = \tilde{k},(t,l) \neq (t',l')$, that is,

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{y}^{rs,m}(t')} = \frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m}(t')} \quad (5.32)$$

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m'}(t')} = \frac{0\cdot\exp((\theta-\beta)L_t^{rs,m}-\theta C_{\tilde{k}}^{rs,m}(t))\cdot\sum_{(t,l)}\exp(-\beta L_t^{rs,m})}{[\sum_{(t,l)}\exp(-\beta L_t^{rs,m})]^2} - \frac{\exp((\theta-\beta)L_t^{rs,m}-\theta C_{\tilde{k}}^{rs,m}(t))\cdot\exp(-\beta L_{t'}^{rs,m})\cdot(-\beta\frac{\partial L_{t'}^{rs,m}}{C_{\tilde{k}}^{rs,m}(t')})}{[\sum_{(t,l)}\exp(-\beta L_t^{rs,m})]^2} \tag{5.33}$$

Substitute Eq. (5.21) and (5.29) to Eq. (5.33)

$$\begin{aligned}\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{\tilde{k}}^{rs,m'}(t')} &= \beta P^{rs,b,j}(\tilde{k},t,l)P^{rs,b,j}(\tilde{k}\,/\,t',l')P^{rs,b,j}(t',l') \\ &= \beta P^{rs,b,j}(\tilde{k},t,l)P^{rs,b,j}(\tilde{k},t',l')\end{aligned} \tag{5.34}$$

- **Case IV** $y \neq \tilde{k},(t,l)\neq(t',l')$, that is,

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} = \frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} \tag{5.35}$$

$$\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} = \frac{0\cdot\exp((\theta-\beta)L_t^{rs,m}-\theta C_{\tilde{k}}^{rs,m}(t))\cdot\sum_{(t,l)}\exp(-\beta L_t^{rs,m})}{[\sum_{(t,l)}\exp(-\beta L_t^{rs,m})]^2} - \frac{\exp((\theta-\beta)L_t^{rs,m}-\theta C_{\tilde{k}}^{rs,m}(t))\cdot\exp(-\beta L_{t'}^{rs,m})\cdot(-\beta\frac{\partial L_{t'}^{rs,m}}{C_y^{rs,m}(t')})}{[\sum_{(t,l)}\exp(-\beta L_t^{rs,m})]^2} \tag{5.36}$$

Substitute Eq. (5.21) and (5.24) to Eq. (5.36)

$$\frac{\partial (P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} = \beta P^{rs,b,j}(\tilde{k},t,l) P^{rs,b,j}(t',l') P^{rs,b,j}(y / t',l')$$
$$= \beta P^{rs,b,j}(\tilde{k},t,l) P^{rs,b,j}(y',t',l') \tag{5.37}$$

In summary, we have

$$\frac{\partial (P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} =$$
$$\begin{cases} [-\theta + (\theta - \beta) P^{rs,b,j}(\tilde{k} / t,l) + \beta P^{rs,b,j}(\tilde{k},t,l)] P^{rs,b,j}(\tilde{k},t,l), & \text{case I} \\ [(\theta - \beta) + \beta P^{rs,b,j}(t,l)] P^{rs,b,j}(y / t,l) P^{rs,b,j}(\tilde{k},t,l), & \text{case II} \\ \beta P^{rs,b,j}(\tilde{k},t,l) P^{rs,b,j}(\tilde{k},t',l'), & \text{case III} \\ \beta P^{rs,b,j}(\tilde{k},t,l) P^{rs,b,j}(y',t',l'), & \text{case IV} \end{cases} \tag{5.38}$$

Substitute (5.38) into Eq. (5.17), $\dfrac{\partial f_{\tilde{k}}^{rs,m}(t)}{\partial C_y^{rs,m}(t')}$ could be obtained, together with (5.16), we can get the gradient expression of $\nabla_{C_s} f_{\tilde{k}}^{rs,m}(t)$,

$$\nabla_{C_s} f_{\tilde{k}}^{rs,m}(t) = \sum_y \sum_{(t',l')} q^{rs} \frac{\partial (P^{rs,b,j}(\tilde{k},t,l))}{\partial C_y^{rs,m}(t')} \cdot \frac{\partial z_s^{rs,m}(t' + T_y^{rs,m}(t'))}{\partial C_s} \tag{5.39}$$

The above gradient expression is subject to $s \in y$, otherwise, when $s \notin y$, $\nabla_C f_{\tilde{k}}^{rs,m}(t) = 0$

Similarly, we can derive the gradient expression of $\nabla_e f_{\tilde{k}}^{rs,m}(t)$. Again, the flow on path $\tilde{k}$ between O-D pair (r,s) is a function of the disutilities of all choice alternatives of path between O-D pair (r,s) and departure time and duration of stay (t,l), that is

$$\nabla_{\mathbf{e}} f_{\tilde{k}}^{rs,m}(t) = \sum_y \sum_{(t',l')} \frac{\partial f_{\tilde{k}}^{rs,m}(t)}{\partial C_y^{rs,m}(t')} \cdot \frac{\partial C_y^{rs,m}(t')}{\partial e_s(t_{\tilde{k}}^{rs,m}(t))} \tag{5.40}$$

Where the term $\dfrac{\partial f_{\tilde{k}}^{rs,m}(t)}{\partial C_{y}^{rs,m}(t')}$ has been derived in the above equations (5.17) and (5.39). The following equation gives the expression of $\dfrac{\partial C_{y}^{rs,m}(t')}{\partial e_{s}(t_{\tilde{k}}^{rs,m}(t))}$. We assume that travelers of the same class m between the same O-D pair (r,s) taking the same route $\tilde{k}$ and charging at the destination s could reach the destination's charging facility at the same time $t_{\tilde{k}}^{rs,m}$ only if they depart from the origin at the same time interval. This assumption is intuitively reasonable, thus, we have the following expression,

$$\begin{aligned}\frac{\partial C_{y}^{rs,m}(t')}{\partial e_{s}(t_{\tilde{k}}^{rs,m}(t))} &= \frac{\partial C_{y}^{rs,m}(t')}{\partial e_{y}^{rs}(t_{\tilde{k}}^{rs,m}(t'))}\frac{\partial e_{y}^{rs}(t_{\tilde{k}}^{rs,m}(t'))}{\partial e_{s}(t_{\tilde{k}}^{rs,m}(t))} \\ &= \begin{cases}\alpha_{3}^{m}\times 1, & \text{if } s\in y,(t,l)=(t',l') \\ 0, & \text{o.w.}\end{cases}\end{aligned} \tag{5.41}$$

Substitute (5.38) and (5.41) into Eq. (5.40), the gradient expression of $\nabla_{\mathbf{e}} f_{\tilde{k}}^{rs,m}(t)$ when $s\in y$ and $(t,l)=(t',l')$, is as follows

$$\nabla_{\mathbf{e}} f_{\tilde{k}}^{rs,m}(t) = \sum_{y}\sum_{(t',l')}\alpha_{3}^{m}\cdot q^{rs}\cdot\frac{\partial(P^{rs,b,j}(\tilde{k},t,l))}{\partial C_{y}^{rs,m}(t')} \tag{5.42}$$

In other cases $\nabla_{\mathbf{e}} f_{\tilde{k}}^{rs,m}(t)=0$.

The last derivative $\nabla_{\mathbf{C}} E_{s}(\mathbf{C}^{*})$ can be simply derived as follows,

$$\nabla_{C_s} E_{s}(C_{s}) = \nabla_{C_s}\rho_{s} C_{s} = \rho_{s} \tag{5.43}$$

The gradient expressions of the objective functions are shown in Eq. (5.39), (5.42) and (5.43). To be expended to detail, the upper-level problem is convex in the feasible space of variables $\mathbf{C}$ and $\mathbf{e}$. The constraints for the upper-level problem are only box constraints Eq. (5.13) and (5.14). This upper-level problem can be solved using the gradient based method elaborated in the next section.

5.4.3, gradient-based method for the upper-level problem

Using the sensitivity analysis expressions, the algorithm solves, at each iteration, an approximation of the objective function, subject to the box constraint. The step by step gradient based algorithm for solving the upper-level problem is as follows:

Step 0. Initialization. Let $\mathbf{x}^{(0)} \in X$, , set $k = 0$.

Step 1. Computing. Terminate if $\mathbf{x}^{(k)}$ satisfies a stopping criterion

Otherwise, compute the gradient $\mathbf{g}$ (E.q. 5.12).

Step 2. Computing. Compute the step size γ_k. The step size can be obtained by a Polynomial interpolation (e.g. quadratic interpolation)

Step 3. Updating. Update the solution

$$\mathbf{x}^{(k+1)} = P_X[\mathbf{x}^{(k)} + \gamma_k \mathbf{g}^{(k)}] \tag{5.44}$$

Step 4. set $k = k + 1$ and go to Step 1.

5.4.4, Algorithm for solving the bi-level problem

The upper-level problem of optimizing the revenue can be solved using the gradient based method presented above. The algorithm of solving the lower-level VI formulation was given in the previous chapter. Up to this end, the bi-level program can be solved as follows, where the step-by-step procedure of the solution algorithm is described, and the flowchart of the algorithm is given in Fig. (5.1).

Step 1 Initialize the charging infrastructure capacity at each destination $\mathbf{C}^{(k)} = \{C_s^k\}$ and the charging pricing at each charging infrastructure and time interval $\mathbf{e}^{(k)} = \{e_s^k(t)\}$. Set $k = 0$.

Step 2 Solve the lower-level time-dependent stochastic network equilibrium problem for the above given $\mathbf{C}^{(k)}$ and $\mathbf{e}^{(k)}$ using the decomposition algorithm developed in the previous chapter, obtained the equilibrium solutions $\mathbf{f}$ and $\mathbf{q}$.

Step 3 Calculate the derivative as shown in section (5.4.2), find the gradient expressions, $\nabla_C f_{\tilde{k}}^{rs,m}(t)(\mathbf{C}^*, \mathbf{e}^*)$, $\nabla_e f_{\tilde{k}}^{rs,m}(t)(\mathbf{C}^*, \mathbf{e}^*)$, and $\nabla_C E_s(\mathbf{C}^*)(\mathbf{C} - \mathbf{C}^*)$

Step 4 Use the above derivatives to get the gradient of the objective function. Solve the upper-level problem use the gradient based method (section 5.4.3). Obtain the updated charging infrastructure capacity at each destination $\mathbf{C}^{(k)}$ and the charging pricing at each time interval $\mathbf{e}^{(k)}$.

Step 5 Check the stop criterion, if satisfied, then stop and get the output. Otherwise, set $k = k + 1$ and go to Step 2.

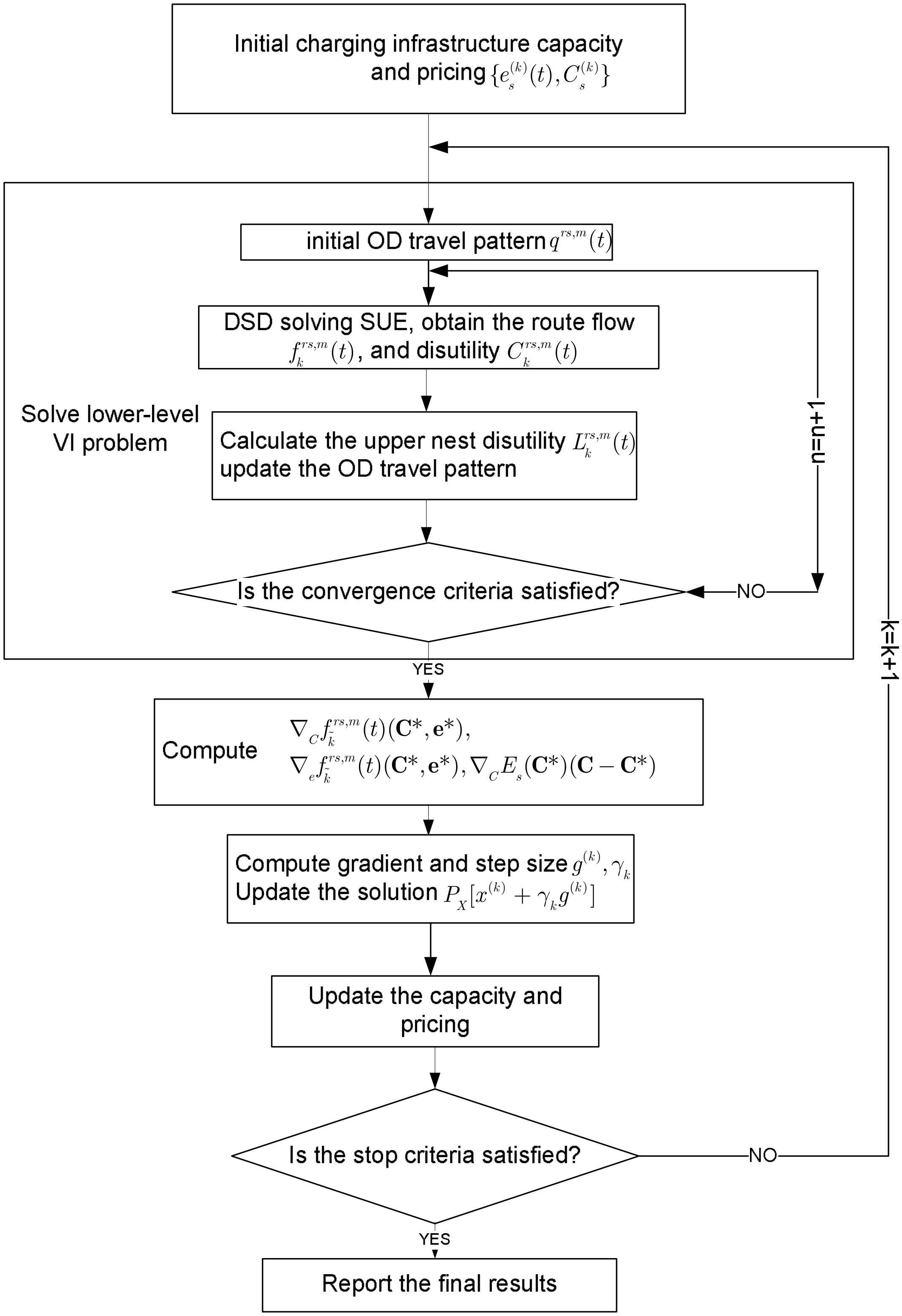
Initial charging infrastructure capacity and pricing $\{e_s^{(k)}(t), C_s^{(k)}\}$
initial OD travel pattern $q^{rs,m}(t)$
DSD solving SUE, obtain the route flow $f_k^{rs,m}(t)$, and disutility $C_k^{rs,m}(t)$
Solve lower-level VI problem
Calculate the upper nest disutility $L_k^{rs,m}(t)$ update the OD travel pattern
n=n+1
Is the convergence criteria satisfied?
NO
YES
k=k+1
Compute $\nabla_C f_{\tilde{k}}^{rs,m}(t)(\mathbf{C}^*, \mathbf{e}^*)$, $\nabla_e f_{\tilde{k}}^{rs,m}(t)(\mathbf{C}^*, \mathbf{e}^*), \nabla_C E_s(\mathbf{C}^*)(\mathbf{C} - \mathbf{C}^*)$
Compute gradient and step size $g^{(k)}, \gamma_k$
Update the solution $P_X[x^{(k)} + \gamma_k g^{(k)}]$
Update the capacity and pricing
Is the stop criteria satisfied?
NO
YES
Report the final results

Figure 5-1: Flowchart of Algorithm

5.5, ILLUSTRATION EXAMPLE

The model and solution method presented in the previous sections are implemented on two sample networks to gain insights and to test the model's and solution approach's capabilities. The two networks are the grid network and the Nguyen-Dupuis network. This section gives the descriptions of the transportation networks used, the analysis of the results, and the evaluation of the results comparing to GA.

The motivation is that for the first example network, the SAB method is evaluated. The main results of the proposed MPEC model are demonstrated on a middle sized network (Fig. 5-3), the network of Nguyen-Dupuis (Nguyen and Dupuis 1984).

5.5.1, Network Data

The presented model was solved on two sample networks. Both of the two networks are well-established networks and have been widely used in the literatures. The first one is a grid network (Y. W. Xu et al. 1999), with 9 nodes and 12 links and one O-D pair (the attributes of the network are described in Appendix C, Table C-1). The link IDs are shown on the links in the figure. The link length, free flow speed and the capacity of the links are shown in each column of the Table C-1. The example on the grid network (Fig 5-2) has one O-D pair (1, 9) with trip demand of 5,000, including 2,500 GVs and 2,500 BEVs. The capacity of the charging infrastructure is constrained to [50, 1500], and the electricity-charging pricing of at destination (node 9) at different time of the day is within the range of [0.05, 5].

The second network is the Nguyen-Dupuis network (Nguyen and Dupuis 1984). The network topology is shown in Fig. 5-3, where the link numbers are marked on the links. The network has 13 nodes and 18 links with four O-D pairs. The O-D pairs and the total trip demands (including both BEVs and GVs) between each O-D pair are given in Table 5-1, where BEVs penetration rate ranges from 11.75%~33.33% between different O-D pairs. The network attributes of this network is shown in Appendix C, Table C-2. The capacity of the charging infrastructure is constrained to [10, 900], and the electricity-charging pricing of at destination (node 2 and node 3) at different time of the day is within the range of [0.05, 5].

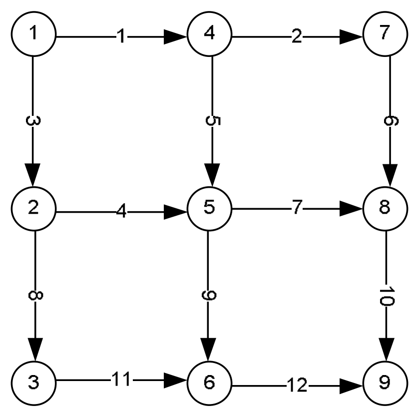

Figure 5-2: A Grid Network Example

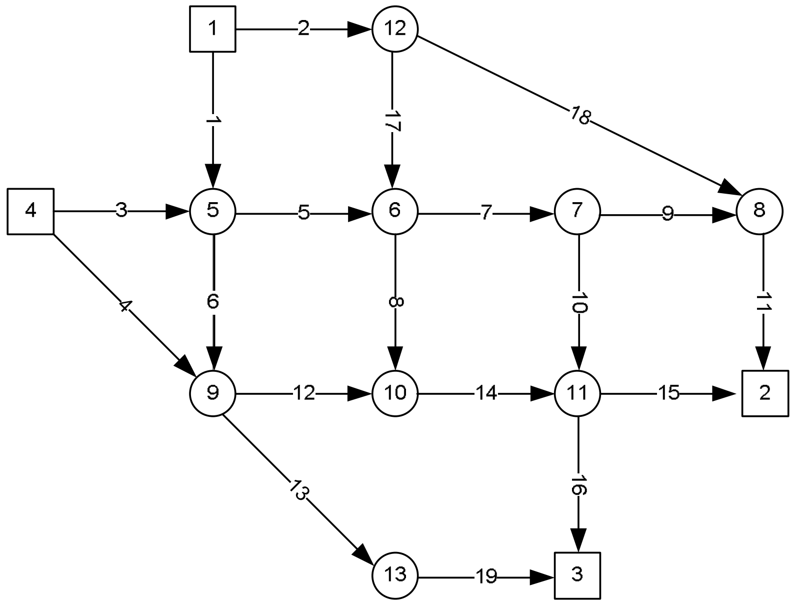

Figure 5-3: Nguyen-Dupuis Test Network

	GVs		BEVs	
	2	3	2	3
1	1300	1800	600	300
4	1400	1500	700	200

Table 5-1 Demand Matrix for Nguyen-Dupuis

5.5.2, The grid network

In this network example, the time period studied is between 6:00am~7:00pm. The time window of arrival for all drivers is between [1:00pm, 3:00pm]. The time interval is 1 hour.

5.5.2.1 The results

Since the network is small, it is able to list all the path flow results in the solution. There are six different possible paths between O-D pair (1, 9) (see Table 5-2). The lengths of these paths are shown in the third column (Table 5-2). The results show that, there are only 5 paths for BEVs, while 6 paths for GVs. This is because the path 1-2-6-10 has a total length of 122, which is beyond the BEV effective range. Therefore, this path (1-2-6-10) is not feasible for BEV drivers. The path flow results depend very much on the parameter settled for each cost component in the total cost function (that is $\alpha_1^m, \alpha_2^m, \alpha_3^m \ldots$). The path flow results for BEVs and GVs are shown in Table 5-3 and Table 5-4 respectively.

O-D	Path#	Length	Path
(1-9)	1	122	1-2-6-10
	2	65	1-5-7-10
	3	80	1-5-9-12
	4	59	3-4-7-10
	5	74	3-4-9-12
	6	76	3-8-11-12

Table 5-2 O-D Pair and Paths of the Grid Network

path	6:00~7:00	7:00~8:00	8:00~9:00	9:00 ~10:00	10:00 ~11:00	11:00 ~12:00	12:00 ~13:00
2	10.1728	22.1342	94.8685	62.8989	66.9487	60.0514	59.9737
3	2.0886	1.8954	2.0177	0.3831	1.0901	0.5889	0.7392
4	18.5765	52.5453	287.3591	237.3197	218.3452	201.8556	177.347
5	4.1885	5.3928	9.8253	3.6663	5.3282	3.5689	4.3352
6	3.3374	3.8241	5.7266	1.7131	3.2081	1.9956	2.4437
path	13:00~ 14:00	14:00~ 15:00	15:00~ 16:00	16:00~ 17:00	17:00~ 18:00	18:00~ 19:00	
2	45.7100	50.6703	43.0564	21.1110	9.1044	9.1047	
3	0.3735	1.3625	0.7805	0.5031	0.4669	3.2501	
4	176.0713	184.5666	161.0143	80.5163	25.3885	14.1348	
5	2.2730	5.7106	4.0701	2.3709	1.6199	5.1891	
6	1.2651	3.5274	2.3067	1.3748	1.0397	4.3137	

Table 5-3 Path Flows of BEVs at Different Time of the Day

path	6:00~7:00	7:00~8:00	8:00~9:00	9:00~ 10:00	10:00~ 11:00	11:00~ 12:00	12:00~ 13:00
1	0.3722	1.3755	7.8553	25.0939	28.4132	30.6832	33.3796
2	1.1363	4.1998	27.2525	63.9581	71.5988	79.8431	82.8855
3	0.8540	3.1551	15.0008	49.6495	95.6396	97.5488	96.8301
4	1.2751	4.7133	28.2066	51.0779	56.4490	57.2375	53.3911
5	0.9583	3.5408	18.6919	64.2601	75.4029	80.6828	86.4617
6	0.9266	3.4236	17.1600	58.4972	83.3999	87.6265	91.2725
path	13:00~ 14:00	14:00~ 15:00	15:00~ 16:00	16:00~ 17:00	17:00~ 18:00	18:00~ 19:00	
1	26.3747	13.8791	6.3582	2.1638	0.5733	0.1710	
2	69.1804	40.8663	21.2136	7.7818	2.0703	0.6175	
3	92.7285	45.8803	17.5225	5.3409	1.4069	0.4196	
4	51.4734	36.1975	22.5745	8.9920	2.4023	0.7165	
5	68.9943	40.6387	18.6467	6.1716	1.6326	0.4869	
6	77.3742	42.1427	17.9496	5.7179	1.5095	0.4502	

Table 5-4 Path Flows of GVs at Different Time of the Day

Table 5-3 and Table 5-4 give the path flows obtained at different time of the day respectively. In the numerical illustration, the weight of the electricity-charging cost, the charging time cost, or the charging waiting cost might be too high that there are very few BEVs on the path (1-5-9-12), path (3-4-9-12), or path (3-8-11-12) comparing to the flows on path(1-5-7-10) and path(3-4-7-10). As expected, the GV flows are indirectly influenced by the charging cost and the supply of the charging infrastructures. Those paths (path#3, #5, #6) with very limited BEV flows have more GV flows. Noted, for neither BEVs drivers nor GV drivers is path#1 a preferred path because of its length (The GV drivers' fuel cost also depends on the route length).

The result of the SAB method is displayed in Table 5-5. For example $e(1)$=0.05 means at the first time interval, which is (6:00am~7:00am), the electricity-charging price at destination (node 9) for all BEVs is 0.05. The optimal capacity of the charging infrastructure at destination (node 9) is 100.085 at the local optimal solution. The maximized revenue obtained is 19853.955. At each iteration, the solution moves closer to the optimum. After 4 quadratic approximation iterations, it results in an optimal point.

$e(1)$	$e(2)$	$e(3)$	$e(4)$	$e(5)$	$e(6)$	$e(7)$
0.8877	1.4704	2.1916	3.2545	2.9607	3.2393	3.0571
$e(8)$	$e(9)$	$e(10)$	$e(11)$	$e(12)$	$e(13)$	Capacity
3.4228	2.5105	2.5687	2.2630	1.7454	0.4518	100.085

Table 5-5 Results from the SAB Method

Convergence is assumed when the absolute change in the decision variables is less than 10^{-3}. The gradients of the revenue to the decision variables at each iteration (including the initial iteration) are displayed in Table 5-6. There are 14 decision variables in total, including 13 electricity-charging pricing variables and 1 capacity variable. The gradients at the 14 variables at shown in each row respectively. For example, the second row, i.e., g_1 row, represents the revenue gradients at $e(1)$. The last row, i.e. g_{14} row, represents the revenue gradients at the current capacity C_9. It can be seen from the last row that, within a range of the initial solution of the charging infrastructure capacity (100), the revenue is not very sensitive to the capacity around a local optimal (e.g. at iteration 0, gradient is -0.22913) comparing to the electricity-charging pricing (e.g. at time interval 5, gradient is 72.1231). But it should be noted that, this also depends on the value of parameter ρ_s and φ.

It is shown from Fig5-4 that the gradients reduce very fast at each iteration. At the solution of iteration 3, the gradients surface almost flat to zero.

Iteration	0	1	2	3	4
g_1	1.2126	9.8382	3.3688	0.1058	0.0889
g_2	4.1466	26.7490	0.6874	0.4584	0.1747
g_3	16.0726	27.7622	1.3025	0.9603	-0.1800
g_4	72.3644	1.5431	2.0037	0.0326	-0.0013
g_5	72.1231	1.4012	1.9300	0.0536	0.0000
g_6	72.1229	1.4013	1.9146	0.0241	-0.0006
g_7	69.1698	1.6615	2.0545	-0.0093	0.0040
g_8	61.5592	2.4111	2.7935	-0.1376	-0.0144
g_9	49.8231	4.2799	4.0383	-0.3025	-0.1316
g_{10}	26.6582	13.2419	4.0402	-0.1624	-0.0829
g_{11}	7.0966	31.6005	0.5867	0.4085	0.1317
g_{12}	1.9690	15.9209	2.0426	0.3300	0.1798
g_{13}	0.6670	4.8134	4.8445	0.0248	0.0241
g_{14}	-0.22913	-0.14968	-0.05	-0.0253	-0.00499

Table 5-6 Revenue Gradients

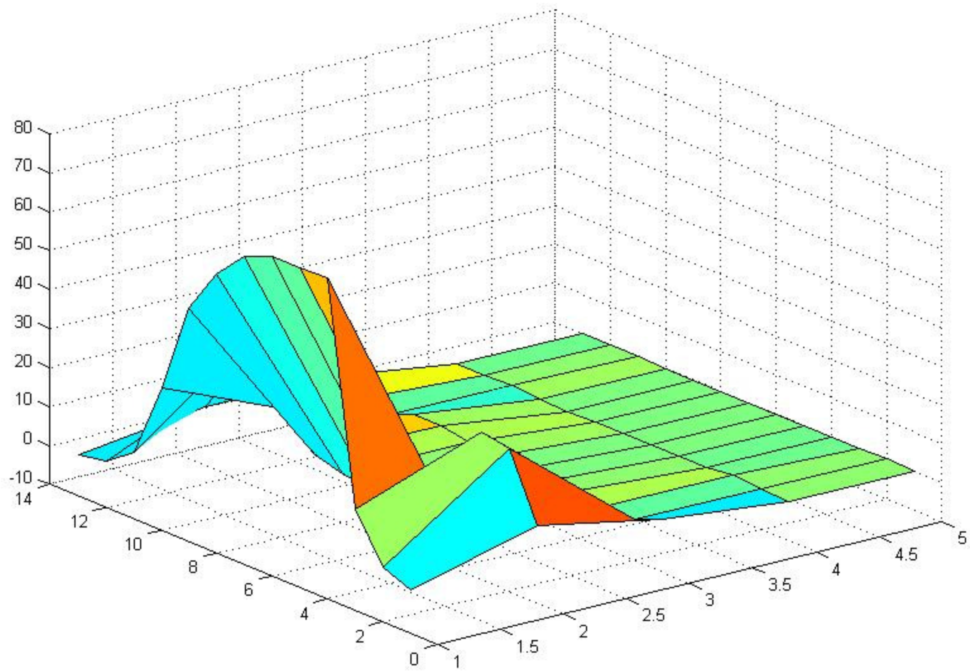

Figure 5-4: Revenue Gradients

5.5.2.2 Comparing the results with genetic algorithm

The sensitivity analysis based strategy for solving the bi-level problem is an optimization based heuristic method. The computational efficiency is achieved using the polynomial interpolation (the quadratic approximation) of finding the best solution at each iteration. Due to the non-convexity of the lower level problem, the method searches along the decent direction leading to local optima. In addition, the final solution depends very much on the initial solution given. Therefore, in this dissertation research, genetic algorithm (GA) is also briefly presented, which reduces the convergence to local optima solutions and increases the possibility of achieving a global optima point. However, the genetic algorithm is not the focus of this chapter. It will be briefly discussed in this section. The purpose of presenting the GA is for comparison with the results obtained from sensitivity analysis based approach to see if the local optimal obtained from SAB method is reasonable.

Although the solutions founded by most of the meta-heuristics are not proved to be optimal solutions, for the past two decades, it was shown from the experiences that meta-heuristics could find the optimal solution or a very good solution rapidly and effectively. This is particularly true for the combinatorial problems, such as network design problem (Gallo et al. 2010; Poorzahedy and Rouhani 2007). One of the well applied meta-heuristic algorithms is GA. It was shown in many literatures that GA outperforms the other two algorithms in their test problems (Karoonsoontawong and Waller 2006; Salhi and Drezner 2002; T. Xu et al. 2009a; T. Xu et al. 2009b).

The notation used to describe a GA applied to our MEPC model is as follows: It works starting from an initial *population*. Each string of the population is a *chromosome*, which is a vector of *gene* of the electricity-charging pricing and charging infrastructure capacity value. For example, if there are 3 pricing variables and 2 capacity variables, a chromosome is a vector of length 5, with the first 3 gene representing the 3 pricing variables and the last 2 gene representing the 2 capacity variables. Each generation a new population is evaluated and operated to generate the next one based on the *fitness function* value. The fitness function value here is the objective value, that is, the revenue. For a thorough discussion of GA approaches, see Goldberg (1989) and Deb (2002). The GA results were obtained by utilizing the MATLAB GA optimization toolbox (R2008b).

The parameter using in the code is informed by pervious work by (Jeon et al. 2006) and (Duthie 2008). Their results suggest a mutation rate of zero and crossover rate of 0.85. The results from the GA algorithm for the grid network are shown in the following table. (Table 5-7). It can be seen that the optimal capacity obtained from the GA is 222.809, which is much different from the one from the SAB method (100.085).

This may indicate that multiple local optima exist. The maximum revenue achieved is 20545.571, which is slightly higher than the results obtained from the SAB method (19853.955). Therefore, in the point of maximizing revenue, the results obtained from GA method might be "better". However, from the practical point, in the results of SAB might be better since the capacity needed is much less than that of GA method. In addition, the SAB method is efficient in terms of computation burden and time. Unlike GA, which solves the lower level equilibrium hundreds of times depending on the number of populations in each generation and the number of generations, the SAB method solves the lower level equilibrium twice for each quadratic approximation iteration. For the grid network, for example, the convergence achieved after 4 quadratic approximation iterations; therefore, it solves the lower level problem 8 times to obtain the result.

$e(1)$	$e(2)$	$e(3)$	$e(4)$	$e(5)$	$e(6)$	$e(7)$
0.348	0.305	0.691	0.638	0.935	0.828	0.914
$e(8)$	$e(9)$	$e(10)$	$e(11)$	$e(12)$	$e(13)$	Capacity
0.857	0.868	0.861	0.951	0.993	0.369	222.810

Table 5-7 Results from the GA

5.5.3, The Nguyen-Dupuis network

In this network example, the time period studied is between 6:00am~7:00pm. The time window of arrival for all drivers is between [12:30pm, 3:30pm]. The time interval is 1 hour. The numerical experiment is designed to illustrate the application of the network model developed in this chapter.

The objective is to make a plan of enhanced capacity and the policies of electricity-charging fares. The original infrastructure capacities are 10 at each destination.

In this situation, $C_{\min} = 10$. The budget for the charging infrastructure only allows for a an additional capacity of 890, which means $C_{\max} = 900$. The electricity-charging pricing is greater than the home charging, where $e_{\min} = 0.5$ The policy allows a maximum electricity-charging price of 5, which means $e_{\max} = 5$.

The results of electricity-charging pricing at charging infrastructure at destination node 2 and node 3 are shown in the following figure (Figure 5-5,). The optimal charging infrastructure capacities are 29.97 and 19.96 at destination 2 and 3 respectively. The total revenue from these charging infrastructures is 6100.298. Figure 5-5 shows that the highest pricing occurs between 9:00am and 3:00pm. This might be because this interval corresponds to the traffic rush hours. During this hour, the BEV drivers may not be that sensitive to the electricity-charging pricing since they tend to arrive at destination within the time window. Therefore, increasing the electricity-charging price at the rush hour will increase the revenue without a big loss of recharging demand.

In general, the electricity-charging pricing at destination 3 is less than that at destination 2. This might indicate that the electricity-charging pricing at destination is relative to the total charging demand at this destination. The total number of BEVs heading to destination 3 is less than that heading to destination 2 (1300 v.s. 500). When the electricity-charging price goes higher, the BEV drivers might be able to switch to a path which does not require charging at destination. Since the BEV demand to destination 3 is small, this switching will not increase a lot the travel time cost through those shorter paths, and such increase of travel time cost might be still lower than the increase of charging cost. In such case, the electricity-charging price could not be set too high. While the BEV demand to destination 2 is large, the switching to shorter paths will increase a lot the travel time cost on these paths. Such increase may be higher than the increase in charging costs; therefore, the BEV drivers tend to stay on the longer paths even when the electricity-charging price increases.

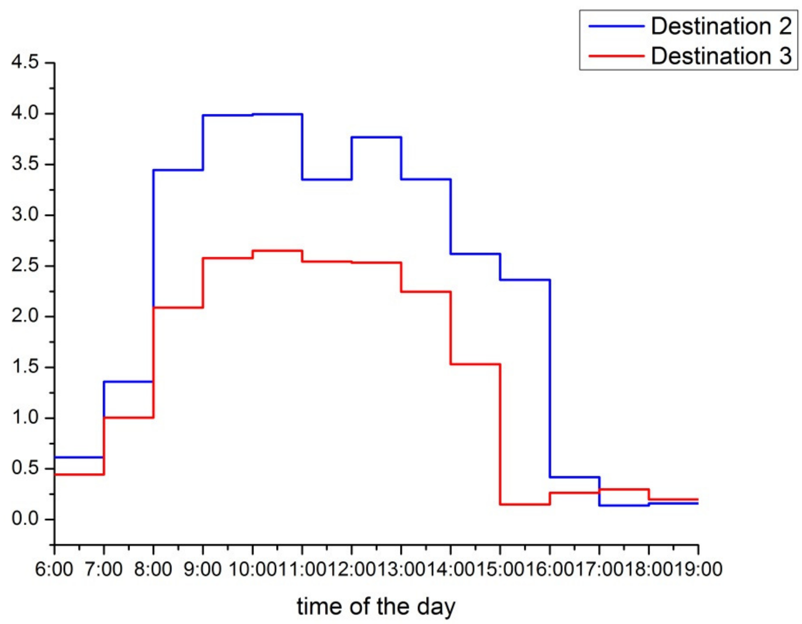

Figure 5-5: Electricity-Charging Pricing at Different Time of Day

5.6, SUMMARY

In this chapter, a new continuous equilibrium network design problem is presented to decide the electricity-charging pricing policy and capacity of the charging infrastructure at a tactical planning level. It is network design problem with time-dependent multi-class equilibrium constraints. The problem application itself is new: it is the first time that the design of time-dependent electricity-charging pricing for plug-in electricity vehicles is conducted.

The mathematical formulation with equilibrium constraints have been developed and been evaluated computationally. A sensitivity analysis based optimization method is used to find a local optimal of the problem. The derivatives of the gradients expressions are developed. The solution approach tackles the searching for step size by using the quadratic interpolation, which is a very efficient. It also deals with the box constrained variables by means of projection of the variables into the feasible set.

The computational issues about the convergence and efficiency were addressed and the results are evaluated by comparing with genetic algorithm. The illustration examples also show the application of the model for the capacity enhancement of charging infrastructure and time-of day electricity-charging fare policy for BEVs. Finally, the analyses of the results are given.

Chapter 6 Conclusion and Future Work

6.1 CONCLUSIONS

The dissertation explored the possible future traffic patterns with PEVs by studying the travel behavior of PEV drivers. An extension was given to an optimal design of charging infrastructure capacity and pricing. A comprehensive study on different travel choices problems for PEV drivers has been presented. Different modeling approaches such as mathematical programming models and variational inequality models were developed to facilitate the study. The problems covered included stochastic user equilibrium traffic assignment (EVSUE and EVSUETP), time-dependent joint choices problem, and continuous network design problem.

The first part of the dissertation focused on developing techniques for modeling stochastic traffic assignment for BEV drivers. Specifically, two separate models accounting for charging cost and charging time penalty cost were developed to capture the route choice behavior of BEV drivers. The proposed models require a path-based algorithm which is able to show the utilized routes by the motorists. Therefore, an algorithm based on the simplicial decomposition method was adopted. By the limitation of the BEV effective range and the recharging infrastructure's availability, the utilized routes are constrained by the total length. Thus, a constrained shortest path problem was solved at each iteration using the lagrangian relaxation with enumeration method within the simplicial decomposition algorithm. The proposed models and solution algorithm were followed by the numerical analysis. It was found that the link flows pattern vary due to the BEV drivers' behavior influenced by the charging price at home or at destination. Practically, it is more reasonable to regulate and control the charging price at destination and the impacts of this price on the BEV drivers' behavior is larger. It was also found that, the higher charging price at home and/or destination, the lower the total vehicle miles traveled would be. The numerical results also indicated the relationship between the electricity-charging speed at destination and the durations of stay of BEV drivers. On average, the duration of stay would rise when the charging speed decreases. But this deviation is not constant, as some drivers might not have to charge at the destination or might take other alternative routes to reduce extra time caused. When the charging speed increases, the total charging time penalty decreases initially very fast but slow down gradually. Based on the results of the numerical analysis, the developed model and algorithm were very well verified for the impact study of the charging price and the charging speed on BEV users' behavior and subsequently on the network flow.

The investigation was carried out further by taking into account BEV drivers' behavior in the time-dependent combined travel choices. The second part of the dissertation developed a variational inequality formulation concentrated on modeling the quasi-dynamic mixed flow equilibrium. Proof of the formulation's equivalence to a two-level nested-Logit choice structure was conducted, where the departure time and duration of stay belongs to the first level and the route choice belongs to the second. This combined travel choice modeling was built on the basis of EVSUE and EVSUETP, where the charging cost and charging time penalty for BEV drivers were considered simultaneously. In this study, a decomposition algorithm was proposed to solve the problem iteratively. As demonstrated, the proposed model formulation and solution approach could capture the feasible duration of stay choice set for BEV drivers, the electricity-charging cost of BEV drivers at destination, and the recharging behavior of the BEV drivers at destination. It was found that the temporal and spatial choice of BEV drivers could be influenced by the BEV effective range and the penetration rates, and these impacts would indirectly change the GV drivers' behavior.

The last part of the dissertation extended the above time-dependent multi-class combined choices equilibrium with an application to a network design problem. In particular, this study proposed a methodology to design an optimal capacity and time-dependent pricing strategy for the charging infrastructure. The problem was formulated as a mathematical program with equilibrium constraints. The sensitivity analysis based method for solving the network design problem showed its efficiency to find a local optimal solution.

6.2 FUTURE WORK

The dissertation work of modeling the PEV drivers' behavior into transportation network equilibrium problems opens several directions for the future work. This section outlines a few directions as follows:

The charging cost function and charging time penalty cost function used in this study is only suitable for BEV drivers. However, the modeling framework developed in this study can be adapted to other classes motorists than BEV drivers, such as plug-in hybrid vehicle drivers, as long as the motorists' operating cost is a function of the path attributes (e.g. path length) that cannot be directly decomposed onto links. Additional cost components for traveling may be incorporated.

There is a need to develop efficient solution algorithms to solve the path-based stochastic user equilibrium. The algorithm used in this dissertation is an approximation method that limited routes are generated as utilized routes. Ideas could be stimulate from those efficient algorithms used in deterministic user equilibrium, such as gradient projection method. The algorithm for solving the stochastic traffic assignment model runs through the algorithms for solving the variational inequality formulation and thus for solving the lower-level problem of the network design problem. Therefore, an efficient algorithm for the stochastic traffic assignment will improve the computational burden and improve the solution quality at every part of this study.

The cost components of the PEV drivers considered in this study use a single trip chain, i.e. a round trip as the study base, while one direction of the round trip is considered. To some extent, it could be categorized as a trip-based model. In the real word, the travel behavior is much complicated and a more complicated trip chain might be considered instead.

In modeling the combined travel choices behavior, the demand elasticity could be taken into account. In addition, other travel choices, such as destination choices and mode choice could be incorporated. The study of demand elasticity and destination choice is meaningful. For example, the PEV drivers might switch their destinations according to the availability of charging infrastructure at destinations or change their travel plan regarding to the charging pricing. The equilibrium model with demand elasticity could be used as a lower-level problem in the network design problem studied in this dissertation, which would make the charging pricing and capacity design problem more realistic.

The study only presented one case of extending the combined travel choices equilibrium. However, the equilibrium model can be used as a network analysis tool for a rich set of network design problems, such as location-allocation of charging infrastructure and time-dependent capacity of charging infrastructure. The network design problem can be to a discrete one other than a continuous one.

Appendix A

Table A-1 Link Flow Patterns for Different Charging Cost at Home ($\kappa = 1.2$, $e_s = 0.1$)

link	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9
2	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1
3	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9	1582.9
4	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9
5	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1	2501.1
6	4347.1	4249.1	4249.1	4249.3	4147.2	4142.6	3971.9	3964.9	3964.9
7	2931.0	2837.6	2833.2	2833.1	2731.2	2726.4	2555.7	2548.8	2548.8
8	4347.1	4253.7	4249.3	4249.3	4147.3	4142.6	3971.9	3964.9	3964.9
9	6169.9	6060.9	6050.0	6029.2	6018.9	6009.2	5833.6	5821.8	5821.8
10	2823.8	2814.9	2806.1	2789.2	2883.1	2880.0	2877.0	2874.2	2874.2
11	6169.9	6065.5	6050.2	6029.2	6019.1	6009.2	5833.6	5821.8	5821.8
12	4579.5	4564.8	4554.7	4535.4	4525.8	4516.7	4341.5	4330.3	4330.3
13	3368.8	3276.8	3278.4	3281.6	3283.3	3285.1	3287.0	3288.9	3288.9
14	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9	2749.9
15	4579.5	4569.4	4554.9	4535.4	4525.9	4516.7	4341.6	4330.3	4330.3
16	7026.0	7012.7	7004.1	6978.4	6965.7	6953.4	6775.1	6760.7	6760.7
17	3434.1	3437.4	3436.4	3435.2	3435.0	3435.0	3269.0	3266.8	3266.8
18	5400.3	5397.6	5399.6	5394.9	5392.7	5390.5	5554.6	5555.1	5555.1
19	7026.1	7017.3	7004.3	6978.4	6965.8	6953.4	6775.1	6760.7	6760.7
20	3434.1	3432.8	3436.3	3435.2	3435.0	3435.0	3269.0	3266.8	3266.8
21	415.1	416.6	418.0	421.0	422.6	424.1	425.7	427.3	427.3
22	6040.2	6048.9	6057.1	6072.4	6079.2	6085.9	6091.9	6097.6	6097.6
23	3368.8	3276.8	3278.4	3281.6	3283.3	3285.1	3287.0	3288.9	3288.9
24	415.1	416.6	418.0	421.0	422.6	424.1	425.7	427.3	427.3
25	7291.3	7384.7	7384.7	7384.9	7385.1	7385.3	7385.6	7385.9	7385.9
26	7333.3	7426.7	7426.7	7426.9	7427.1	7427.3	7427.6	7427.9	7427.9
27	8239.8	8353.8	8373.9	8413.8	8440.3	8459.3	8644.1	8665.0	8665.0
28	6033.0	6029.0	6025.4	6019.9	6025.0	6023.5	6022.5	6022.2	6022.2
29	10583.4	10584.2	10584.3	10582.4	10580.7	10577.9	10740.7	10739.1	10739.1
30	339.8	338.9	338.0	336.3	335.5	334.8	334.1	333.5	333.5
31	2865.8	2856.8	2848.1	2831.2	2925.1	2922.0	2919.0	2916.2	2916.2
32	8156.8	8270.8	8291.1	8330.8	8357.5	8376.3	8561.1	8582.0	8582.0
33	4400.9	4506.2	4517.8	4540.5	4551.3	4561.7	4737.8	4749.9	4749.9

34	6195.2	6216.7	6238.0	6279.9	6395.2	6420.1	6444.4	6468.0	6468.0
35	2931.0	2833.0	2833.0	2833.1	2731.0	2726.4	2555.7	2548.8	2548.8
36	4400.9	4506.2	4518.0	4540.5	4551.4	4561.7	4737.8	4749.9	4749.9
37	5175.0	5164.7	5150.0	5130.5	5019.2	5005.6	4992.4	4979.9	4979.9
38	5216.0	5201.1	5191.0	5171.5	5060.2	5046.6	5033.4	5020.9	5020.9
39	4721.3	4709.4	4693.0	4670.4	4557.6	4542.5	4528.0	4514.2	4514.2
40	6195.2	6216.7	6238.0	6279.9	6395.2	6420.1	6444.4	6468.0	6468.0
41	3534.5	3541.7	3548.8	3324.6	3568.3	3341.6	3349.5	3357.0	3357.0
42	3546.3	3559.1	3571.9	3834.8	3704.8	3954.8	3969.5	3983.9	3983.9
43	6075.0	6071.0	6067.4	6061.9	6067.0	6065.5	6064.5	6064.2	6064.2
44	3534.5	3541.7	3548.8	3324.6	3568.3	3341.6	3349.5	3357.0	3357.0
45	8118.8	8137.3	8155.8	8193.0	8211.6	8230.3	8249.2	8268.1	8268.1
46	9242.3	9227.2	9211.9	8943.5	9172.7	8924.8	8912.0	8899.4	8899.4
47	6040.2	6048.9	6057.3	6072.4	6079.3	6085.9	6091.9	6097.6	6097.6
48	10624.3	10625.1	10625.1	10623.4	10621.5	10618.9	10781.7	10780.1	10780.1
49	10127.4	10150.7	10172.8	10213.7	10232.5	10250.2	10267.0	10282.8	10282.8
50	5653.6	5646.5	5638.8	5621.5	5612.0	5601.7	5757.0	5748.1	5748.1
51	339.8	338.9	338.0	336.3	335.5	334.8	334.1	333.5	333.5
52	10127.4	10150.7	10172.8	10213.7	10232.5	10250.2	10267.0	10282.8	10282.8
53	8453.4	8476.3	8498.1	8538.3	8556.8	8574.3	8590.8	8606.4	8606.4
54	5400.3	5402.2	5399.7	5394.9	5392.7	5390.5	5554.6	5555.1	5555.1
55	5694.5	5687.4	5679.7	5662.5	5653.0	5642.7	5798.0	5789.1	5789.1
56	5033.3	5002.4	4976.8	4918.6	4890.4	4862.7	4835.5	4808.8	4808.8
57	8118.8	8137.3	8155.8	8193.0	8211.6	8230.3	8249.2	8268.1	8268.1
58	8453.4	8476.3	8498.1	8538.3	8556.8	8574.3	8590.8	8606.4	8606.4
59	2474.4	2470.6	2467.8	2465.7	2466.2	2467.7	2470.3	2473.8	2473.8
60	5033.3	5006.9	4976.8	4918.6	4890.4	4862.7	4835.5	4808.8	4808.8
61	2474.4	2470.6	2467.8	2465.7	2466.2	2467.7	2470.3	2473.8	2473.8
62	1955.9	1951.8	1952.4	1944.8	1941.2	1937.6	1934.2	1930.8	1930.8
63	2300.6	2295.1	2289.8	2280.2	2275.8	2271.7	2267.8	2264.1	2264.1
64	1914.9	1915.4	1911.4	1903.8	1900.2	1896.6	1893.2	1889.8	1889.8
65	5507.2	5499.4	5491.6	5476.6	5476.5	5469.5	5462.9	5456.6	5456.6
66	5546.2	5534.2	5527.0	5504.4	5486.5	5476.3	5466.4	5457.1	5457.1
67	9242.3	9227.2	9211.9	8943.5	9172.7	8924.8	8912.0	8899.4	8899.4
68	2300.6	2295.1	2289.8	2280.2	2275.8	2271.7	2267.8	2264.1	2264.1
69	5507.2	5499.4	5491.6	5476.6	5476.5	5469.5	5462.9	5456.6	5456.6
70	3680.7	3676.0	3671.2	3899.1	3656.9	3884.7	3877.5	3870.3	3870.3
71	3546.3	3559.1	3571.9	3834.8	3704.8	3954.8	3969.5	3983.9	3983.9

72	3680.7	3676.0	3671.2	3899.1	3656.9	3884.7	3877.5	3870.3	3870.3
73	2377.0	2377.0	2377.0	2377.0	2471.9	2476.7	2481.4	2485.9	2485.9
74	4721.3	4704.8	4693.0	4670.4	4557.6	4542.5	4528.0	4514.2	4514.2
75	5505.2	5497.8	5486.0	5463.4	5445.5	5435.3	5425.4	5416.1	5416.1
76	2377.0	2377.0	2377.0	2377.0	2471.9	2476.7	2481.4	2485.9	2485.9

Table A-2 Link Flow Patterns for Different Charging Cost at Destination ($\kappa = 1.2$, $e_h = 0.08$)

Link	0.1	0.14	0.18	0.22	0.26	0.3	0.34	0.38	0.42	0.46	0.5
1	1582.9	1582.9	1583.0	1583.0	1583.0	1583.1	1583.1	1583.1	1583.2	1583.2	1583.2
2	2501.1	2501.1	2501.0	2501.0	2501.0	2500.9	2500.9	2500.9	2500.8	2500.8	2500.8
3	1582.9	1582.9	1583.0	1583.0	1583.0	1583.1	1583.1	1583.1	1583.2	1583.2	1583.2
4	2749.9	2749.9	2750.0	2750.0	2750.0	2750.1	2750.1	2750.1	2750.2	2750.2	2750.2
5	2501.1	2501.1	2501.0	2501.0	2501.0	2500.9	2500.9	2500.9	2500.8	2500.8	2500.8
6	3971.9	3934.4	3887.2	3864.3	3846.1	3832.1	3635.8	3595.0	3560.1	3531.6	3508.7
7	2555.7	2518.3	2471.1	2448.3	2430.2	2402.7	2219.9	2179.1	2144.2	2115.8	2093.0
8	3971.9	3934.4	3887.2	3864.3	3846.1	3818.6	3635.8	3595.0	3560.1	3531.6	3508.7
9	5833.6	5807.6	5792.5	5712.0	5716.1	5707.3	5588.5	5488.8	5431.3	5370.9	5344.3
10	2877.0	2890.1	2907.8	2974.4	2998.2	3018.5	2961.3	3026.5	3051.7	3033.4	2927.5
11	5833.6	5807.6	5792.5	5773.3	5716.1	5707.3	5527.2	5550.1	5431.3	5370.9	5233.2
12	4341.5	4316.4	4294.1	3639.2	3644.1	3636.1	3456.8	3419.2	3360.8	3301.0	3262.9
13	3287.0	3286.2	3293.4	3867.7	3866.9	3866.1	3926.7	3864.6	3865.5	3864.9	3876.6
14	2749.9	2749.9	2750.0	2750.0	2750.0	2750.1	2750.1	2750.1	2750.2	2750.2	2750.2
15	4341.6	4316.4	4294.1	3639.2	3644.1	3636.1	3456.8	3419.2	3360.8	3301.0	3151.5
16	6775.1	6745.9	6720.1	6062.3	6026.4	6011.2	5825.0	5781.2	5717.2	5650.0	5609.1
17	3269.0	3034.3	2999.6	2970.6	2927.3	2905.1	3057.2	3042.2	2943.7	2924.0	2893.6
18	5554.6	5374.4	5387.7	5398.3	5386.0	5387.1	5215.4	5212.4	5121.8	5107.9	5089.7
19	6775.1	6745.9	6720.1	6062.3	6026.4	6011.2	5825.0	5781.2	5717.2	5650.0	5497.8
20	3269.0	3034.3	2999.6	2970.6	2927.3	2905.1	3057.2	3042.2	2943.6	2924.0	2897.3
21	425.7	425.7	425.9	425.9	426.0	426.0	596.4	596.4	601.7	601.9	603.9
22	6091.9	6297.2	6305.9	5677.2	5684.5	5691.4	5872.1	5912.8	6006.9	6067.9	6201.1
23	3287.0	3286.2	3293.4	3929.1	3866.9	3866.1	3865.3	3925.9	3865.5	3864.9	3876.8
24	425.7	425.7	425.9	425.9	426.0	426.0	596.4	596.4	601.7	601.9	600.8

25	7385.6	7384.8	7392.2	7966.6	7965.9	7965.1	7855.3	7793.2	7799.4	7799.0	7812.7
26	7427.6	7426.8	7434.2	8069.9	8007.9	8007.1	7836.0	7896.5	7841.4	7841.0	7851.9
27	8644.1	8705.5	8745.3	8766.8	8930.4	8943.3	9127.2	9168.9	9207.1	9236.6	9372.4
28	6022.5	6053.7	6179.3	6190.9	6057.1	6056.3	6055.5	6054.8	6054.0	6053.3	6064.9
29	10740.7	10769.6	10690.9	11259.2	11276.1	11288.5	11363.4	11343.2	11377.8	11407.3	11781.9
30	334.1	334.7	334.9	350.9	350.9	351.4	351.4	351.9	351.9	351.9	351.9
31	2919.0	2932.1	2949.8	2955.0	3040.2	3047.0	3064.7	3007.2	3093.7	3075.4	3080.6
32	8561.1	8622.5	8662.3	8683.8	8847.4	8860.3	9044.2	9085.9	9124.1	9153.6	9526.8
33	4737.8	4781.1	4911.5	4932.9	4949.8	4962.7	5146.6	5188.4	5224.2	5253.4	5277.0
34	6444.4	6439.3	6547.6	6614.1	6771.1	6791.4	6734.2	6799.4	6827.0	6808.9	6815.3
35	2555.7	2518.3	2471.1	2448.3	2430.2	2416.2	2219.9	2179.1	2144.2	2115.8	2093.0
36	4737.8	4781.1	4911.5	4932.9	4949.8	4962.7	5146.6	5188.4	5224.2	5253.4	5277.0
37	4992.4	4969.4	4845.1	4843.0	4841.3	4826.3	4827.2	4828.0	4828.8	4829.5	4830.2
38	5033.4	5010.4	4886.1	4884.0	4882.3	4880.9	4868.2	4869.0	4869.8	4870.5	4871.2
39	4528.0	4502.0	4372.5	4367.9	4363.8	4346.6	4345.5	4344.5	4343.6	4342.8	4342.0
40	6444.4	6439.3	6547.6	6552.8	6771.1	6777.9	6795.5	6738.1	6827.0	6808.9	7163.8
41	3349.5	3325.3	3416.6	3478.7	3479.5	3480.3	3419.7	3481.8	3504.8	3540.2	3195.0
42	3969.5	3988.4	4005.4	4009.8	4166.0	4185.5	4189.0	4192.1	4194.9	4198.6	4200.5
43	6064.5	6095.7	6221.3	6232.9	6099.1	6098.3	6097.5	6096.8	6096.0	6095.3	6107.3
44	3349.5	3325.3	3416.6	3417.4	3479.5	3480.3	3481.1	3420.5	3504.8	3540.2	3544.6
45	8249.2	8453.8	8565.9	8647.3	8654.5	8661.3	8606.3	8673.5	8762.6	8804.1	8476.9
46	8912.0	9096.2	9007.7	9015.4	8889.6	8896.5	8902.8	8908.7	8975.7	8982.5	8989.1
47	6091.9	6297.2	6305.9	5677.2	5684.5	5691.4	5872.1	5912.8	6006.9	6067.9	6096.5
48	10781.7	10811.1	10731.9	11361.5	11317.1	11329.5	11343.1	11445.5	11418.8	11448.2	11582.2
49	10267.0	10472.9	11691.3	11609.9	11617.7	11625.6	11693.8	11639.6	11707.8	11743.8	11751.0
50	5757.0	5784.0	4492.5	4511.9	4527.0	4538.2	4375.9	4381.6	4385.3	4387.9	4375.3
51	334.1	334.2	334.9	350.9	350.9	351.4	351.4	352.0	352.0	352.0	352.0
52	10267.0	10473.4	11691.3	11671.2	11617.7	11625.6	11632.5	11700.8	11707.7	11743.7	11386.7

53	8590.8	8796.7	10014.4	9949.1	9956.9	9964.3	10032.5	9977.5	10045.7	10081.7	10088.9
54	5554.6	5374.4	5387.7	5398.3	5386.0	5387.1	5215.4	5212.4	5121.8	5107.9	5086.1
55	5798.0	5825.0	4533.5	4552.9	4568.0	4579.2	4416.9	4422.6	4426.3	4428.9	4435.1
56	4835.5	4625.4	3297.1	3285.7	3256.0	3243.7	3232.3	3221.8	3125.9	3107.9	3082.0
57	8249.2	8453.8	8565.9	8585.9	8654.5	8661.3	8667.6	8612.2	8762.6	8804.1	8826.8
58	8590.8	8796.7	10014.4	10010.4	9956.9	9964.3	9971.1	10038.9	10045.7	10081.7	9724.7
59	2470.3	2469.1	3782.6	3782.1	3781.5	3781.0	3780.5	3780.0	3804.4	3810.2	3840.3
60	4835.5	4625.4	3297.1	3285.7	3256.0	3243.7	3232.3	3221.8	3125.9	3107.9	3097.2
61	2470.3	2469.1	3782.6	3782.1	3781.5	3781.0	3780.5	3780.0	3804.4	3810.2	3826.0
62	1934.2	1932.0	1930.2	1928.6	1907.9	1903.9	1900.2	1896.6	1831.9	1827.3	1822.9
63	2267.8	2061.0	2051.2	2042.0	2033.5	2025.7	2018.5	2012.0	2006.1	1999.4	1994.6
64	1893.2	1891.0	1889.2	1887.6	1866.9	1862.9	1859.2	1855.6	1790.9	1786.3	1781.9
65	5462.9	5442.3	5345.7	5345.7	5212.7	5212.7	5212.7	5212.7	5274.1	5275.8	5277.6
66	5466.4	5457.3	5343.0	5341.3	5174.0	5170.0	5166.3	5162.7	5159.4	5156.5	5153.9
67	8912.0	9096.2	9007.7	9015.4	8889.6	8896.5	8902.8	8908.7	8975.7	8982.5	8989.4
68	2267.8	2061.0	2051.2	2042.0	2033.5	2025.7	2018.5	2012.0	2006.1	1999.4	1995.5
69	5462.9	5442.3	5345.7	5345.7	5212.7	5212.7	5212.7	5212.7	5274.1	5275.8	5277.6
70	3877.5	3875.4	3873.6	3872.2	3871.0	3870.0	3869.2	3868.6	3868.2	3866.6	3866.6
71	3969.5	3988.4	4005.4	4009.8	4166.0	4172.0	4189.0	4192.1	4194.9	4198.6	4199.3
72	3877.5	3875.4	3873.6	3872.2	3871.0	3870.0	3869.2	3868.6	3868.2	3866.6	3867.8
73	2481.4	2498.3	2513.5	2516.5	2671.5	2690.0	2692.7	2695.2	2697.6	2699.7	2701.6
74	4528.0	4502.0	4372.5	4367.9	4363.8	4360.1	4345.5	4344.5	4343.6	4342.8	4342.0
75	5425.4	5416.3	5302.0	5300.3	5133.0	5129.0	5125.3	5121.7	5118.4	5115.5	5112.9
76	2481.4	2498.3	2513.5	2516.5	2671.5	2676.5	2692.7	2695.2	2697.6	2699.7	2701.6

Appendix B

Table B-1 Parameters of Link Travel Time and Length for Test Network

Link ID	Link Tails	Link Heads	free flow time	length	Capacity
1	1	2	0.25	25	800
2	1	3	0.2	10	700
3	1	4	0.25	25	800
4	2	3	0.2	20	700
5	2	6	0.2	20	500
6	3	4	0.2	20	600
7	3	7	0.3	10	500
8	4	8	0.2	20	500
9	4	13	0.2	25	650
10	5	2	0.2	25	800
11	5	6	0.2	20	800
12	5	9	0.2	13	800
13	6	7	0.3	15	500
14	6	9	0.3	20	500
15	7	8	0.2	15	600
16	7	10	0.2	5	600
17	8	11	0.3	20	500
18	8	13	0.1	20	650
19	9	10	0.3	20	500
20	9	12	0.1	25	700
21	10	11	0.2	20	600
22	10	12	0.2	15	650
23	11	12	0.1	15	700
24	11	13	0.2	15	650

Table B-2 Parameters for Numerical Experiments in Chapter 4

$\alpha_1^{BEV} = 1$	$\alpha_1^{GV} = 1$	$\theta_1 = 0.8$
$\alpha_2^{BEV} = 1.4$	$\alpha_2^{GV} = \alpha_3^{GV} = \alpha_4^{GV} = 0$	$\beta_1 = 0.4$
$\alpha_3^{BEV} = 0.5$	$\alpha_6^{GV} = 0.5$	$\theta_2 = 0.7$
$\alpha_4^{BEV} = 2$	$\alpha_5^{GV} = 2.8$	$\beta_2 = 0.3$
$\alpha_5^{BEV} = 2.8$	$t_{(1,12)}^{1,lb} = 12:00pm$	$t_{(5,13)}^{2,lb} = 8:30am$
$\lambda_e^m = 0.5, \lambda_l^m = 1.5, \forall m$	$t_{(1,12)}^{1,ub} = 2:00pm$	$t_{(5,13)}^{2,ub} = 9:30am$

Table B-3 Path Flow Results for BEVs

Range	1	2	3	4	5	6
81.6	199.216	2307.06	493.722	244.421	931.483	924.096
88.4	224.537	2184.75	590.714	260.256	922.799	916.945
95.2	255.75	2056.73	687.524	280.559	911.687	907.754
102	302.596	1950.77	746.638	306.322	897.625	896.053
108.8	361.071	1830.26	808.671	260.166	922.874	916.96
115.6	416.44	1703.22	880.341	286.394	908.525	905.081
122.4	443.921	1582	974.079	296.943	902.766	900.291
129.2	478.41	1469.68	1051.91	310.516	895.366	894.118
136	531.129	1381.73	1087.14	315.88	853.731	930.389
142.8	594.877	1290.45	1114.67	344.749	904.018	851.233
149.6	648.537	1205.01	1146.45	394.576	876.486	828.937
156.4	675.322	1131.62	1193.06	432.523	855.964	811.513

Table B-4 Path Flow Results for GVs

	1	2	3	4	5	6
81.6	499.326	5412.7	1087.97	631.382	2105.59	2163.03
88.4	484.993	5461.36	1053.65	630.261	2104.55	2165.19
95.2	469.992	5511.92	1018.09	628.821	2103.23	2167.94
102	457.297	5553.31	989.393	626.988	2101.58	2171.43
108.8	442.873	5599.92	957.209	630.254	2104.46	2165.29
115.6	427.876	5648.64	923.483	628.419	2102.73	2168.85
122.4	414.043	5695.01	890.95	627.659	2102.07	2170.28
129.2	401.348	5736.85	861.8	626.682	2101.21	2172.11
136	391.242	5768.49	840.265	628.472	2111.52	2160
142.8	380.77	5800.73	818.504	622.615	2091.45	2185.93
149.6	371.399	5829.89	798.713	619.261	2088.76	2191.98
156.4	364.075	5853.73	782.196	617.064	2087.5	2195.43

Table B-5 Charging Demand at Destination Node 13 at Different Time of Day with Different BEV Ranges

Time of Day	81.6	88.4	95.2	102	108.8	115.6	122.4	129.2	136	142.8	149.6	156.4
6:00 ~ 7:00	0	0	0	0	0	0	0	0	0	0	0	0
7:00 ~ 8:00	4.41	5.42633	6.59489	7.60829	8.84037	10.2409	11.6894	13.1662	14.4014	15.7433	17.0613	18.2398
8:00 ~ 9:00	11.82	14.4733	17.4799	19.9558	22.8941	26.1032	29.1737	32.32	35.1585	38.167	41.1674	44.014
9:00 ~ 10:00	22.77	27.5947	33.0685	37.6757	43.0906	48.984	54.6548	60.4796	65.8219	71.4605	77.1003	82.5102
10:00 ~ 11:00	40.50	48.476	57.3194	64.5943	72.9577	81.8432	90.1586	98.6311	106.492	114.721	123.008	131.119
11:00 ~ 12:00	94.96	112.622	131.652	147.054	164.513	182.822	199.883	216.584	231.307	246.43	261.286	275.496
12:00 ~ 13:00	237.68	278.045	319.76	352.463	388.449	424.878	457.519	487.858	513.707	539.348	564.188	588.223
13:00 ~ 14:00	377.56	434.395	490.946	533.816	579.488	623.641	660.063	694.035	725.818	756.702	786.497	815.698
14:00 ~ 15:00	469.12	530.433	590.322	636.636	684.963	730.654	767.229	801.303	834.621	866.613	897.346	927.603
15:00 ~ 16:00	322.57	357.707	391.056	415.576	439.789	461.082	475.725	490.817	509.803	527.281	544.981	564.592
16:00 ~ 17:00	147.88	157.557	166.978	175.939	184.127	191.278	197.109	203.253	211.547	219.122	226.943	235.925
17:00 ~ 18:00	1.43	2.1194	2.90787	3.39701	4.00149	4.6163	5.03359	5.4604	5.88492	6.31565	6.75166	7.19303
18:00 ~ 19:00	0.01	0.0138	0.02075	0.03129	0.04667	0.06857	0.09939	0.12998	0.14874	0.17107	0.19174	0.20477

Table B-6 BEV Trip Demands at Different Time of Day under Different BEV Penetration Rate (1,12)

Time			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
6:00	~	7:00	11.177	22.234	33.109	43.984	54.674	65.241	75.684	86.005	96.203	106.280
7:00	~	8:00	19.648	39.076	58.172	77.268	96.030	114.569	132.886	150.983	168.861	186.523
8:00	~	9:00	36.157	71.653	106.230	140.807	174.524	207.702	240.364	272.528	304.214	335.439
9:00	~	10:00	60.462	121.444	183.173	244.901	307.345	370.242	433.575	497.328	561.483	626.022
10:00	~	11:00	69.170	138.772	209.000	279.228	350.055	421.264	492.843	564.777	637.054	709.661
11:00	~	12:00	88.195	176.692	265.653	354.614	444.050	533.815	623.914	714.353	805.140	896.277
12:00	~	13:00	88.631	177.264	265.907	354.551	443.215	531.903	620.621	709.379	798.185	887.049
13:00	~	14:00	83.696	167.181	250.350	333.519	416.379	499.037	581.499	663.772	745.861	827.775
14:00	~	15:00	33.066	66.191	99.377	132.563	165.784	199.013	232.236	265.441	298.615	331.747
15:00	~	16:00	8.087	16.089	23.961	31.834	39.575	47.227	54.791	62.268	69.656	76.957
16:00	~	17:00	1.510	3.003	4.472	5.941	7.385	8.812	10.223	11.617	12.995	14.356
17:00	~	18:00	0.185	0.367	0.547	0.727	0.903	1.078	1.250	1.421	1.589	1.756
18:00	~	19:00	0.017	0.033	0.049	0.066	0.081	0.097	0.113	0.128	0.143	0.158
Time			0.550	0.600	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
6:00	~	7:00	116.236	126.074	135.794	145.399	154.891	164.270	173.540	182.703	191.772	200.794
7:00	~	8:00	203.970	221.206	238.234	255.057	271.678	288.102	304.333	320.374	336.250	352.043
8:00	~	9:00	366.217	396.564	426.495	456.022	485.158	513.915	542.306	570.341	598.070	625.651
9:00	~	10:00	690.928	756.184	821.772	887.675	953.877	1020.362	1087.114	1154.117	1221.377	1288.974
10:00	~	11:00	782.583	855.807	929.320	1003.108	1077.160	1151.461	1226.001	1300.765	1375.776	1451.174
11:00	~	12:00	987.777	1079.642	1171.878	1264.489	1357.480	1450.854	1544.615	1638.765	1733.057	1826.389
12:00	~	13:00	975.978	1064.983	1154.072	1243.254	1332.536	1421.927	1511.434	1601.064	1690.872	1781.083
13:00	~	14:00	909.519	991.102	1072.530	1153.812	1234.954	1315.966	1396.853	1477.623	1558.332	1639.209
14:00	~	15:00	364.824	397.836	430.771	463.618	496.368	529.010	561.534	593.934	626.221	658.490
15:00	~	16:00	84.172	91.301	98.345	105.305	112.184	118.981	125.700	132.340	138.911	145.449
16:00	~	17:00	15.701	17.031	18.344	19.642	20.924	22.191	23.444	24.682	25.907	27.126
17:00	~	18:00	1.920	2.083	2.244	2.402	2.559	2.714	2.867	3.019	3.169	3.318
18:00	~	19:00	0.173	0.188	0.202	0.217	0.231	0.245	0.259	0.272	0.286	0.299

Table B-7 BEV Trip Demands at Different Time of Day under Different BEV Penetration Rate (5,13)

Time			5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
6:00	~	7:00	59.557	119.069	178.534	237.999	297.474	357.031	416.767	476.815	537.358	598.627
7:00	~	8:00	92.941	185.864	278.792	371.719	464.740	557.943	651.475	745.547	840.443	936.528
8:00	~	9:00	100.589	201.252	302.056	402.860	503.883	605.147	706.769	808.899	911.729	1015.490
9:00	~	10:00	66.570	133.153	199.752	266.351	332.957	399.560	466.145	532.693	599.171	665.520
10:00	~	11:00	21.724	43.431	65.039	86.647	107.961	128.825	148.925	167.828	184.968	199.641
11:00	~	12:00	6.176	12.348	18.505	24.662	30.781	36.847	42.823	48.660	54.298	59.659
12:00	~	13:00	1.752	3.503	5.253	7.002	8.752	10.504	12.260	14.023	15.797	17.588
13:00	~	14:00	0.497	0.994	1.490	1.986	2.483	2.980	3.479	3.980	4.486	4.997
14:00	~	15:00	0.141	0.282	0.423	0.563	0.704	0.845	0.987	1.129	1.272	1.418
15:00	~	16:00	0.040	0.080	0.120	0.160	0.200	0.240	0.280	0.320	0.361	0.402
16:00	~	17:00	0.011	0.023	0.034	0.045	0.057	0.068	0.079	0.091	0.102	0.114
17:00	~	18:00	0.002	0.003	0.005	0.006	0.008	0.010	0.011	0.013	0.014	0.016
18:00	~	19:00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Time			55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
6:00	~	7:00	660.912	724.560	789.962	857.541	927.703	1000.778	1076.932	1158.783	1247.353	1342.009
7:00	~	8:00	1034.262	1134.207	1237.030	1343.504	1454.502	1571.013	1694.243	1816.636	1936.912	2060.458
8:00	~	9:00	1120.442	1226.867	1335.045	1445.215	1557.519	1671.934	1788.177	1908.406	2032.020	2156.853
9:00	~	10:00	731.638	797.345	862.337	926.107	987.846	1046.304	1099.620	1147.888	1189.319	1219.345
10:00	~	11:00	211.014	218.158	220.116	216.033	205.355	188.083	165.022	138.571	111.311	85.238
11:00	~	12:00	64.659	69.204	73.202	76.566	79.219	81.105	82.182	82.637	82.506	81.805
12:00	~	13:00	19.404	21.252	23.143	25.089	27.100	29.186	31.352	33.671	36.162	38.807
13:00	~	14:00	5.517	6.048	6.592	7.155	7.737	8.343	8.972	9.646	10.370	11.139
14:00	~	15:00	1.565	1.715	1.870	2.029	2.195	2.366	2.545	2.736	2.942	3.160
15:00	~	16:00	0.444	0.487	0.530	0.576	0.623	0.671	0.722	0.776	0.834	0.896
16:00	~	17:00	0.126	0.138	0.150	0.163	0.177	0.190	0.205	0.220	0.237	0.254
17:00	~	18:00	0.018	0.019	0.021	0.023	0.025	0.027	0.029	0.031	0.033	0.036
18:00	~	19:00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table B-8 GV Trip Demands at Different Time of Day under Different BEV Penetration Rate (1,12)

Time	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
6:00 ~ 7:00	231.156	217.486	204.059	190.633	177.457	164.450	151.614	138.949	126.457	114.135
7:00 ~ 8:00	406.459	382.328	358.651	334.973	311.757	288.852	266.258	243.976	222.005	200.343
8:00 ~ 9:00	750.432	703.206	657.630	612.055	567.999	524.883	482.660	441.287	400.728	360.947
9:00 ~ 10:00	1251.131	1189.348	1126.717	1064.085	1000.649	936.704	872.272	807.377	742.038	676.278
10:00 ~ 11:00	1365.096	1297.188	1228.476	1159.764	1090.287	1020.321	949.889	879.007	807.695	735.971
11:00 ~ 12:00	1606.445	1525.853	1444.633	1363.413	1281.553	1199.270	1116.550	1033.392	949.792	865.740
12:00 ~ 13:00	1576.463	1495.079	1413.469	1331.858	1250.019	1168.023	1085.864	1003.537	921.035	838.352
13:00 ~ 14:00	1454.195	1377.171	1300.257	1223.343	1146.545	1069.821	993.170	916.585	840.061	763.592
14:00 ~ 15:00	657.466	623.062	588.499	553.936	519.250	484.505	449.716	414.900	380.071	345.245
15:00 ~ 16:00	165.965	156.168	146.542	136.916	127.465	118.134	108.922	99.832	90.864	82.016
16:00 ~ 17:00	30.983	29.151	27.352	25.553	23.788	22.045	20.324	18.627	16.953	15.301
17:00 ~ 18:00	3.850	3.622	3.398	3.175	2.956	2.739	2.525	2.314	2.106	1.901
18:00 ~ 19:00	0.359	0.338	0.317	0.296	0.275	0.255	0.235	0.216	0.196	0.177
Time	55%	60%	65%	70%	75%	80%	85%	90%	95%	100%
6:00 ~ 7:00	101.984	90.002	78.187	66.539	55.054	43.731	32.567	21.559	10.705	0.000
7:00 ~ 8:00	178.989	157.939	137.190	116.737	96.578	76.707	57.119	37.809	18.772	0.000
8:00 ~ 9:00	321.914	283.598	245.974	209.016	172.701	137.004	101.906	67.386	33.426	0.000
9:00 ~ 10:00	610.118	543.579	476.681	409.445	341.890	274.037	205.904	137.508	68.870	0.000
10:00 ~ 11:00	663.854	591.361	518.511	445.321	371.809	297.993	223.888	149.510	74.878	0.000
11:00 ~ 12:00	781.240	696.286	610.873	525.000	438.663	351.863	264.596	176.864	88.648	0.000
12:00 ~ 13:00	755.480	672.411	589.138	505.655	421.955	338.032	253.880	169.494	84.871	0.000
13:00 ~ 14:00	687.168	610.781	534.423	458.085	381.758	305.433	229.103	152.759	76.396	0.000
14:00 ~ 15:00	310.436	275.657	240.921	206.243	171.633	137.104	102.665	68.329	34.106	0.000
15:00 ~ 16:00	73.289	64.683	56.196	47.826	39.574	31.436	23.412	15.499	7.697	0.000
16:00 ~ 17:00	13.672	12.066	10.482	8.921	7.381	5.863	4.366	2.891	1.435	0.000
17:00 ~ 18:00	1.699	1.499	1.302	1.108	0.917	0.728	0.543	0.359	0.178	0.000
18:00 ~ 19:00	0.158	0.140	0.121	0.103	0.085	0.068	0.051	0.033	0.017	0.000

Table B-9 GV Trip Demands at Different Time of Day under Different BEV Penetration Rate (5,13)

Time	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
6:00 ~ 7:00	1134.525	1074.380	1014.310	954.240	894.251	834.322	774.458	714.667	654.953	595.318
7:00 ~ 8:00	1770.708	1677.323	1583.977	1490.630	1397.329	1304.067	1210.852	1117.693	1024.596	931.569
8:00 ~ 9:00	1907.515	1807.792	1707.946	1608.100	1508.109	1407.999	1307.746	1207.325	1106.713	1005.891
9:00 ~ 10:00	1260.874	1194.636	1128.378	1062.119	995.845	929.567	863.290	797.022	730.770	664.545
10:00 ~ 11:00	412.645	390.804	368.984	347.165	325.369	303.592	281.837	260.105	238.399	216.720
11:00 ~ 12:00	117.316	111.106	104.901	98.697	92.499	86.306	80.118	73.936	67.760	61.590
12:00 ~ 13:00	33.278	31.516	29.756	27.996	26.238	24.481	22.726	20.973	19.221	17.470
13:00 ~ 14:00	9.439	8.940	8.440	7.941	7.443	6.944	6.446	5.949	5.452	4.956
14:00 ~ 15:00	2.678	2.536	2.394	2.253	2.111	1.970	1.829	1.687	1.546	1.406
15:00 ~ 16:00	0.759	0.719	0.679	0.639	0.599	0.559	0.519	0.479	0.439	0.399
16:00 ~ 17:00	0.215	0.204	0.193	0.181	0.170	0.158	0.147	0.136	0.124	0.113
17:00 ~ 18:00	0.041	0.039	0.036	0.034	0.032	0.030	0.028	0.026	0.024	0.021
18:00 ~ 19:00	0.006	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.003	0.003
Time	0.550	0.600	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
6:00 ~ 7:00	535.761	476.275	416.840	357.428	298.002	238.515	178.931	119.528	60.003	0.000
7:00 ~ 8:00	838.614	745.728	652.900	560.111	467.326	374.496	281.551	187.411	93.179	0.000
8:00 ~ 9:00	904.851	803.604	702.184	600.661	499.148	397.810	296.871	196.930	97.948	0.000
9:00 ~ 10:00	598.354	532.206	466.106	400.049	334.015	267.950	201.749	135.459	68.449	0.000
10:00 ~ 11:00	195.065	173.429	151.802	130.172	108.522	86.838	65.116	43.465	21.793	0.000
11:00 ~ 12:00	55.426	49.267	43.111	36.957	30.801	24.641	18.474	12.330	6.182	0.000
12:00 ~ 13:00	15.722	13.975	12.229	10.483	8.737	6.990	5.240	3.497	1.754	0.000
13:00 ~ 14:00	4.460	3.964	3.469	2.974	2.478	1.983	1.486	0.992	0.497	0.000
14:00 ~ 15:00	1.265	1.124	0.984	0.843	0.703	0.562	0.422	0.281	0.141	0.000
15:00 ~ 16:00	0.359	0.319	0.279	0.239	0.199	0.160	0.120	0.080	0.040	0.000
16:00 ~ 17:00	0.102	0.090	0.079	0.068	0.057	0.045	0.034	0.023	0.011	0.000
17:00 ~ 18:00	0.019	0.017	0.015	0.013	0.011	0.009	0.006	0.004	0.002	0.000
18:00 ~ 19:00	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000

5%	10%	15%	20%	25%
29209	29064.5	28925.9	28787.3	28654.1
30%	35%	40%	45%	50%
28525	28399.4	28278.9	28164.1	28056
55%	60%	65%	70%	75%
27956	27866.6	27789.5	27727.8	27685.2
80%	85%	90%	95%	100%
27667	27679.1	27706.9	27727.5	27747.9

Table B-10 Total System Travel Time Cost with Different BEV Penetration Rates

5%	10%	15%	20%	25%
1E+06	1003289	1016291	1006675	1008333
30%	35%	40%	45%	50%
1E+06	1011579	1013170	1014740	1016291
55%	60%	65%	70%	75%
1E+06	1019337	1020835	1022314	1023773
80%	85%	90%	95%	100%
1E+06	1026600	1028024	1029504	1031046

Table B-11 Total Vehicle Miles Travelled with Different BEV Penetration Rates

Appendix C

Link	length	f_0	capacity
1	20	60	210
2	20	60	210
3	11	60	210
4	18	60	210
5	15	60	210
6	59	60	210
7	7	60	210
8	18	60	210
9	20	60	210
10	23	60	210
11	22	60	210
12	25	60	210

Table C-1 Attributes of the Grid Network Links

Link	From	To	Length	f0	Capacity
1	1	5	20.52	35	220
2	1	12	26.40	35	220
3	4	5	26.40	35	220
4	4	9	35.20	35	220
5	5	6	8.80	35	220
6	5	9	26.40	35	220
7	6	7	14.63	35	220
8	6	10	38.12	35	220
9	7	8	14.69	35	220
10	7	11	26.40	35	220
11	8	2	26.40	35	220
12	9	10	29.32	35	220
13	9	13	26.40	35	220
14	10	11	17.60	35	220
15	11	2	26.40	35	220
16	11	3	23.49	35	220
17	12	6	20.52	35	220
18	12	8	41.09	35	220
19	13	3	32.29	35	220

Table C-2 Nguyen-Dupuis Test Network Attributes

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