The Dissertation Committee for Hideaki Miwa certifies that
this is the approved version of the following dissertation:

Adaptive Output Feedback Controllers for a Class
of Nonlinear Mechanical Systems

Committee:

Maruthi R. Akella, Supervisor

Robert H. Bishop

David G. Hull

Cesar A. Ocampo

Joe Qin

Adaptive Output Feedback Controllers for a Class of Nonlinear Mechanical Systems

by

Hideaki Miwa, B.S., M.S.

Dissertation

Presented to the Faculty of the Graduate School
of the University of Texas at Austin
in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

The University of Texas at Austin
August 2002

Dedication

To the advancement of science and technology.

and

To my dearest family,

Yukie,

Tomoyasu,

and M.Naritaka

Acknowledgments

I would like to express my sincere thanks to my advisor, Professor Maruthi R. Akella. He not only led me to the field of nonlinear and adaptive control theory but helped me to develop philosophical way of thinking in my mind with his extensive and comprehensive knowledge. I am convinced that this way of thinking will help me tackle all the situations in the rest of my life. It was also a distinct honor for me to be his first doctoral student here at the University of Texas at Austin.

I also devote my thanks to all my committee members, Professor David G. Hull, Professor Robert H. Bishop, Professor Ceasar A. Ocampo and Professor Joe Qin. They improved my knowledge to its current level through their insightful lectures, seminars and classes. My academic life was supported by several faculty members and department staff. Thus, I would like to thank all of them, especially, Ms. Nita Pollard and Ms. Sherry Powers.

Finally, I thank Japan Air Self-Defense Force and Japan Defense Agency, the financial supporter throughout my academic program here at The University of Texas at Austin.

Adaptive Output Feedback Controllers for a Class
of Nonlinear Mechanical Systems

Publication No. _________

Hideaki Miwa, Ph.D.
The University of Texas at Austin, 2002

Supervisor: Maruthi R. Akella

Even from the early days of adaptive control theory, it has been a primary target for several researchers to guarantee global stability using as limited assumptions as possible. Currently, there exist applicable theories for output feedback adaptive control with which we can often guarantee only semi-global stability. When compared to the corresponding non-adaptive (deterministic) case, these solutions need several extra assumptions in the synthesis of the adaptive controller. In this dissertation, we introduce the definition of a specific class of nonlinear systems, which can be guaranteed global asymptotical stability. This is one of the main result of this dissertation. We named this class as "Passivity Based Globally Stabilizable Systems via Adaptive Output feedback(PBGSS/AOF)". During the arguments, we show how to construct passivity based adaptive controller.

As examples of actual systems, spacecraft attitude control problem and n degree of freedom(DOF) robot arm problem are chosen. For each case, The method to construct a controller and an estimator is shown with its stability proof and its effectiveness is displayed with numerical simulation results.

Table of Contents

List of Figures

Chapter 1

Introduction

In general engineering fields, we have to deal with numerous mechanical systems. Here, the phrase "mechanical systems" is generic, which describes systems whose governing equations are derived from classically analytical dynamics approaches. As Greenwood says in [23], kinematics and dynamics are integral parts of mechanics and therefore, they have been the primary motivation to study dynamical systems and their properties. In this dissertation, we adopt the following definition for mechanical systems as follows.

Definition 1.1 (Mechanical Systems). The dynamics of an n-degree of freedom satisfies the following well-known Euler-Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{q}}}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right) - \frac{\partial \mathcal{L}}{\partial \boldsymbol{q}}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{Q} \tag{1.1}$$

where, $\boldsymbol{q}(t)$ and $\dot{\boldsymbol{q}}(t)$ are practical position and velocity respectively and $\mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the scalar Lagrangian function defined by,

$$\mathcal{L}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \triangleq \mathcal{T}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - \mathcal{V}(\boldsymbol{q}) \tag{1.2}$$

Naturally, $\mathcal{T}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the total kinetic energy and $\mathcal{V}(\boldsymbol{q})$ denotes the potential energy function. Also, $\boldsymbol{Q}$ expresses the external force into the system
We refer to this class of systems as *Mechanical Systems*. (Note that this is a relatively narrow sense definition for mechanical systems.)

Interesting feedback control problems always arise when we try to make these mechanical systems execute desired motions. Since the dawn of modern control theory, it has been the ultimate objective researchers in this field to establish a control design method applicable to any type of mechanical systems in order to guarantee global stability using least amount of a priori information and simplifying assumptions. Although several researchers have developed numerous remarkably sophisticated control theories such as H^{∞} robust optimal control [14], sliding mode control [15] etc., this ultimate objective has not been reached yet. Some of the reasons for that may be stated as follows:

1. It is very difficult (if not impossible) to ignore or cancel the effects of unknown (uncertain) parameters in the controller structure;

2. Even if we achieve the overall control objective when the full state measurements are available, there exists no generally available method to extend these solutions for the case of output feedback when the complete state measurement is unavailable/impractical. However, more often than not, they are severely handicapped in the presence of unknown parameters except when one can formulate adaptive control solutions. This is the case because non-adaptive robust control methods require prior availability of bounds on all unknown/uncertain parameters leading to the requirements of additional assumptions. Furthermore, the requirement of robustness often

leads to degradation of both closed loop stability and performance.

In order to achieve the overall objective of tracking desired motions, we seek a control methodology which does not violate the two points stated above.

1.1 The Adaptive Control

As far as we can dig back into the literatures, The word “Adaptive Control” was introduced by Drenick and Shahbender in 1957. It is defined as “the last parts of a series of three stages in the development of control system” by Bellman in 1960’s. (The first stage is defined as the control in deterministic case and the stochastic case is that for the second stage.) During 1960’s and 1970’s, the achievements of adaptive control can be broadly classified into three fields.

1. Extremum adaptation;

2. Sensitivity models;

3. Adaptive methods based on Lyapunov’s theory.

According to the reference of Narendra [42], the objective of extremum adaptation has been adjustment of the parameters of a plant after determining their direct effects on the overall system performance index of that. This technique was popularly well accepted by many practitioners of adaptive control of that time due to its simplicity, applicability to nonlinear systems and the aspect that it did not need explicit identification of the plant parameters.
Referring the words of Cruz [33], compared to the extremum methods, in the sensitivity methods, researchers required more information about the target plant to

be controlled. When we assume that the structure of a system is known but its parameters are unknown, the sensitivity functions of associated signals in the system can be obtained via "sensitivity model." Thus, when such sensitivity functions are available online, the parameters can be adjusted for optimal performance. This is the basic concept behind sensitivity methods and it has been currently inherited to modern H^∞ optimal control and even μ synthesis techniques. Point of time, sensitivity methods fundamentally treat the adaptive system to be linear with slowly time-varying coefficients.

The Lyapunov's direct method, which is currently one of the most popular methods did not receive much attention during 1970's because of certain mathematical difficulties. However, discovery of the well known "Barbalat's lemma" in 1970's led to the development of general stable adaptive control methods using Lyapunov's direct methods. (Currently it is slightly extended for a type of systems by Tao [54].) Consequently, it led to the current prosperity of adaptive control theories and contributed towards the development of the robust adaptive control, multivariable adaptive control and several associated fields in 1980's.

From the history of adaptive control, we know that adaptive controllers can be broadly categorized as follows:

1. Model Reference Adaptive Control (MRAC) and Self Tuning Controller (STC)

2. Direct and indirect adaptive control

Before giving a spotlight to the purpose of this dissertation, we would need to review briefly the adaptive control methodologies mentioned above.

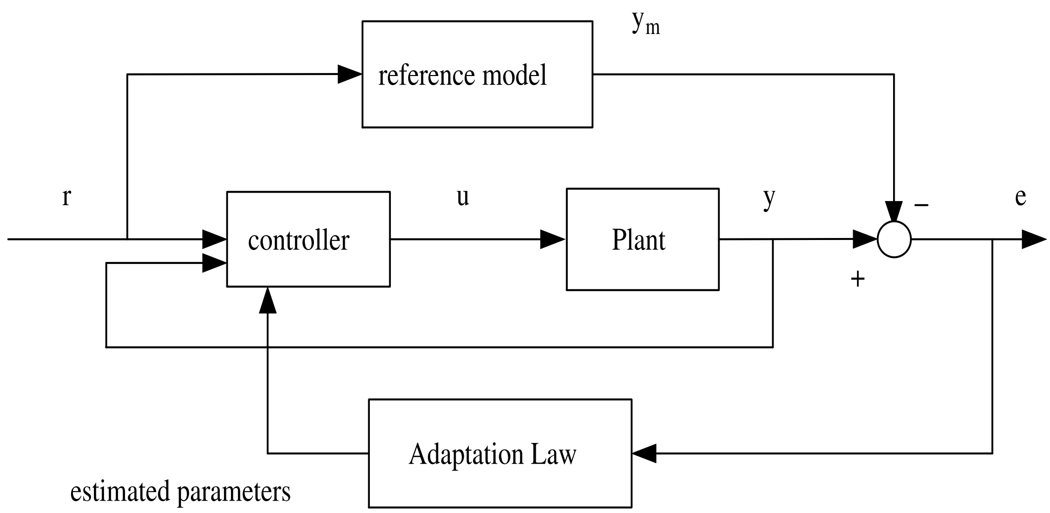

Figure 1.1: MARC Control Scheme

1.1.1 Model Reference Adaptive Control

Fig. 1.1 shows the general frame work of Model Reference Adaptive Control (MRAC). In the MRAC scheme, the elements of an overall system are made up of four important sub-components.

The "plant" is assumed to have a known structure but parameters can be unknown. For example, in a linear dynamical system, this means that we know the number of poles and zeros in the system, although we do not know their exact locations. This also implies that in the nonlinear system we know the structure of dynamic equations except for some linearly appearing constant or slowly time varying unknown parameters.
The "reference model" is used to specify the desired response characteristics of the adaptive control system. It provides the ideal plant response which the adapta-

tion law must seek when it adjusts the parameters. The questions of choosing the reference model is one of the important aspects of adaptive control system design and any acceptable choice must essentially satisfy two requirements. Primarily, it has to reflect the specific closed-loop performance requirements such as rise time, settling time, peak overshoots and frequency domain characteristics; secondly, the desired behavior should be achievable for the adaptive control systems; in other words, there are already several inherited constraints on the structure of reference model due to the assumed structure of actual plant model.

The "controller" is frequently parameterized by a number of adjustable parameters. The controller should have perfect tracking capacity to allow the possibility of tracking convergence when we try to track the above reference model. This yields that when the plant parameters are not known, the adaptive mechanism will adjust the controller parameters so that perfect tracking is asymptotically achieved. If the control law is linear with respect to controller parameters, it is said to be "linearly parameterized" and existing adaptive control schemes usually require this linear parametrization of the controller to obtain adaptive mechanism.

The "adaptive law" is the adaptive mechanism which is used to adjust the parameters in the control law. In MRAC, the adaptive law seeks parameters such that the response of the plant with adaptive controller behaves as the same as that of the reference trajectory. Naturally, the primary difference between the conventional control and the adaptive control lies on the existence of this function. The main issue of adaptation design is to synthesize an adaptation mechanism which guarantees that the overall system remains stable and the tracking error converges to zero.

Note that both in MRAC, the controller parameters are computed from the

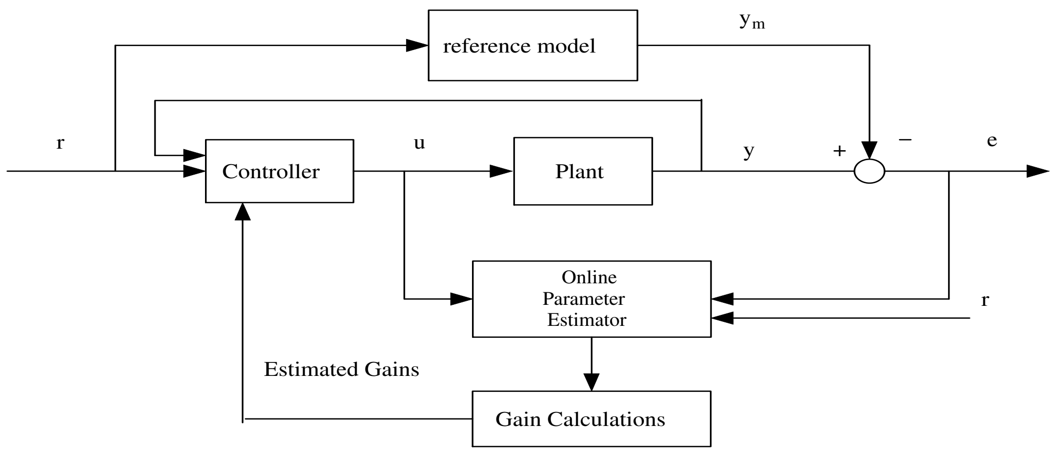

Figure 1.2: Indirect Adaptive Control Scheme

estimates of the plant parameters as if they were the true values of the actual plant parameters. This idea is frequently called the "certainty equivalence principle."

1.1.2 Indirect Adaptive Control

As mentioned above, an adaptive controller is formed by combining an online parameter estimator, which provides estimates of unknown parameters at each instant, with a control law that is motivated from the known parameters case. The way to estimate the parameters is combined with the control law gives rise to two different approaches; In the first approach, the plant parameters are estimated online and used to calculate the controller parameters. This approach is normally called "Indirect adaptive control" and Fig.1.2 shows that framework.

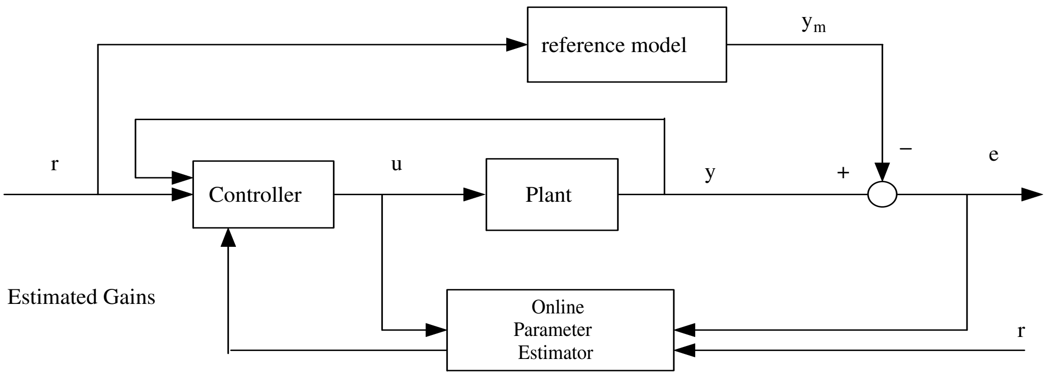

Figure 1.3: Direct Adaptive Control Scheme

1.1.3 Direct Adaptive Control

In the second approach, the plant model is "re"parameterized in terms of the controller parameters that are estimated directly without calculated by the estimates of actual plant parameters. This approach is normally called "Indirect adaptive control" and Fig.1.3 shows that framework. The typical difference between these two approaches are the presence of calculator for control parameters. Direct adaptive scheme is slightly more complicate due to the presence of it.

1.2 The Concept of Passivity

On the other hand of developing adaptive control theory, passivity properties of a system have received much attention to the development of new control schemes that utilize measurement/output feedback instead of the restrictive sate feedback assumption. Its basic concept and the relationship between the passivity and the stability have been already introduced in late 1950's by Youla et.al [12]. Roughly speaking, the concept of passivity is that a system which holds passivity cannot store more energy than what is externally supplied. Mathematically, this property within the system is defined by next.

Definition 1.2 (Passivity in Mechanical Systems). If a mechanical system (1.1) with $\boldsymbol{Q} = \mathcal{M}^T \boldsymbol{u}$ ($\mathcal{M}$ is a constant matrix and $\boldsymbol{u}$ is a torque vector.) can define an operator $\Sigma : \boldsymbol{u} \rightarrow \mathcal{M}^T \dot{\boldsymbol{q}}$ such that

$$\boldsymbol{u} \cdot \mathcal{M}^T \dot{\boldsymbol{q}}|_{t=T} \geq \mathcal{H}(\boldsymbol{q}(T), \dot{\boldsymbol{q}}(T)) - \mathcal{H}(\boldsymbol{q}(T), \dot{\boldsymbol{q}}(T)) \quad \text{for all } T \geq 0 \tag{1.3}$$

where, an operator $(\cdot)$ is an inner product which can be defined arbitrarily, and $\mathcal{H}$ is a total stored energy function of the system.
Then, we can say that *the system holds passivity* and the operator Σ is called *a passive map* between $\boldsymbol{u}$ and $\mathcal{M}^T \dot{\boldsymbol{q}}$.

In the later chapter, we also use the another definition of passivity by Slotine [51].

Lemma 1.1 (Slotine). If the time derivative of $\mathcal{H}(\boldsymbol{q(t)}, \dot{\boldsymbol{q}}(t))$ is expressed as,

$$\dot{\mathcal{H}}(\boldsymbol{q(t)}, \dot{\boldsymbol{q}}(t)) = \boldsymbol{u} \cdot \boldsymbol{y} - g(t) \tag{1.4}$$

and $\mathcal{H}(\boldsymbol{q(t)}, \dot{\boldsymbol{q}}(t))$ is lower bounded and $g(t) \geq 0$.

Then the system from $\boldsymbol{u}$ to $\boldsymbol{y}$ is passive. [$g(t)$ is called "passive map."]

The reasons why this property is much focused upon by many researchers are as follows.

1. Passivity is invariant under negative feedback interconnection of two passive systems.

2. Consider two interconnected passive systems. If the energy created by one subsystem is dissipated by the other, then the closed loop system is stable

3. Passivity is independent of the full state measurement if one of the subsystem is a controller.

Before introducing the true mathematical fashion of these concepts, The concepts of $\mathcal{L}_2$ and $\mathcal{L}_{2e}$ space and $\mathcal{L}_2$ stability must be prepared to be utilizable.

Definition 1.3 ($\mathcal{L}_2$ and $\mathcal{L}_{2e}$ space).

$$\mathcal{L}_2 \triangleq \{\boldsymbol{x} | \|\boldsymbol{x}\|^2_{\mathcal{L}_2} \triangleq \int_0^\infty \|\boldsymbol{x}\|^2 dt < \infty\} \tag{1.5}$$

$$\mathcal{L}_{2e} \triangleq \{\boldsymbol{x} | \|\boldsymbol{x}\|^2_{\mathcal{L}_2} \triangleq \int_0^T \|\boldsymbol{x}\|^2 dt < \infty, \forall T\} \tag{1.6}$$

Note: This norm is induced from a vector inner product such that,

$$\boldsymbol{u} \cdot \boldsymbol{y}|_{\mathcal{L}_{2,2e}} \triangleq \int_0^{\infty,T} \boldsymbol{u}^T \boldsymbol{y} dt \tag{1.7}$$

Definition 1.4 ($\mathcal{L}_2$ stability). Σ is said to be $\mathcal{L}_2$ stable if there exists a positive constant γ s.t. for every initial condition $\boldsymbol{x}_0$, there exists a finite constant $\beta(\boldsymbol{x}_0)$ s.t.,

$$\|\boldsymbol{y}\|_{\mathcal{L}_{2e}} \leq \gamma \|\boldsymbol{u}\|_{\mathcal{L}_{2e}} + \beta(\boldsymbol{x}_0) \tag{1.8}$$

Here, we are ready to introduce the mathematical meanings of the passivity properties.

Property 1.1 (Invariance of passivity). Consider the general input-output system shown in Fig.1.4. (We denote Σ_1 and Σ_2 as passive map corresponding to systems 1 and 2 respectively.) If Σ_1 and Σ_2 are both passive, then assuming $\boldsymbol{u} \triangleq (u_1, u_2)$ and $\boldsymbol{y} \triangleq (y_1, y_2)$, a new mapping $\Sigma : \boldsymbol{u} \rightarrow \boldsymbol{y}$ is also passive.

Property 1.2 (Stability of Passive Systems). Assume Σ_1 and Σ_2 are passive, which means that there exist constants $\alpha_{i1},\ \alpha_{i2},\ \alpha_{o1},\ \alpha_{o2},\ \beta_1,\ \beta_2$ such that,

$$e_1 \cdot y_1|_{\mathcal{L}_{2e}} \geq \alpha_{i1} \|e_1\|^2_{\mathcal{L}_{2e}} + \alpha_{o1} \|y_1\|^2_{\mathcal{L}_{2e}} + \beta_1 \tag{1.9}$$

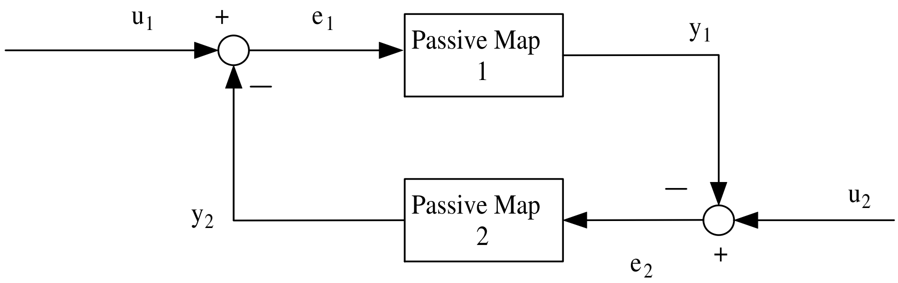

Figure 1.4: Interconnection of two passive systems

$$e_2 \cdot y_2|_{\mathcal{L}_{2e}} \geq \alpha_{i2}\|e_1\|^2_{\mathcal{L}_{2e}} + \alpha_{o2}\|y_2\|^2_{\mathcal{L}_{2e}} + \beta_2 \tag{1.10}$$

and $\alpha_{i1} + \alpha_{o2} > 0, \ \alpha_{i2} + \alpha_{o1} > 0$ holds for all $T \geq 0$.

If $e_1, e_2 \in \mathcal{L}_{2e}$, ($\mathcal{L}_{2e}$ is extended $\mathcal{L}_2$ space) then, Σ is $\mathcal{L}_2$ stable.

These definitions and properties were developed starting from the late 1970's through the mid 1980's. Passivity based controller was introduced by Ortega and Spong [48]. From this result, we have two basic steps to design a controller for mechanical systems; the first is the *energy shaping* stage, in which we change potential energy of the system to have a global and unique minimum at the favored/desired equilibrium state. In the second *damping injection* stage, we create the dissipation term to guarantee asymptotic stability. This same logic flow is always applied to accomplish many objectives when we try to design passivity based controllers. We will also follow these same steps in this dissertation.

1.3 Motivation for Adaptive Output Feedback Control

Existing adaptive control theories are, in general, are limited by the following factors:

1. As introduced in MRAC scheme, the controller in deterministic case must be linear parameterizable with respect to unknown parameters in order to be extended to adaptive controller.

2. Unknown parameters in plant must be constants or "very slowly time varying" values.

Furthermore, we have to mention two more inferior aspects of adaptive control.

3. Adaptive control typically guarantees only state convergence and convergence of parameter estimates to their true values happens only in rare and restricted case, depending on persistence of excitation conditions on the input to the reference input. In other cases, parameter convergence does not happen. In any case, the parameter estimates fluctuate significantly leading to poor transient performance.

4. These methods are not easy to extend to the adaptive output feedback control case. In the real world, there exist a lot of constraints on measurement of full state vector in actual systems. For example, there are some cases we cannot measure angular velocity of a robot arm joint due to the lack of cost or space limitation.

In this dissertation, our target is to address the fourth aspect for a class of mechanical systems by taking advantage of the property of passivity, since passivity properties help us relax requirements on the full state feedback. Simultaneously, we also try to guarantee global asymptotic stability via adaptive output feedback. The key issues are follows:

1. Adaptive controllers are designed based on "certainty equivalence principle." Thus, the region of attraction (convergence) is the same as that in the deterministic case when all the plant parameters are completely known.

2. A traditional method to update unknown parameter estimates in adaptive control scheme is to construct differential equations dynamics for the parameter estimates. In almost all the cases, the differential equation update mechanisms include all the state signals which causes problems when only output signals are available for feedback.

In order to handle the first item, we define a certain class of nonlinear systems which are "globally" stabilizable via output feedback and describe the properties of this class. For the second one, we introduce the two techniques to construct "feasible" adaptive update laws that are implementable with output feedback. By resolving these main issues for the chosen class of mechanical systems, we construct adaptive controllers which guarantee global stability

Later chapters are organized as follows. First of all, we formulate a particular class of nonlinear systems, that are globally stabilizable via output feedback for both the deterministic case and adaptive case. As representative examples for this class of systems, spacecraft attitude tracking problem and robot arm tracking problem are also mentioned. At the beginning of each chapter of examples, we review historical development of spacecraft attitude tracking control and robot arm tracking control. Finally, this dissertation is summarized in the conclusions chapter.

Chapter 2

An Adaptively Output Stabilizable Class of Nonlinear Systems

As shown in previous literatures ([51], [23]) , both the attitude tracking problem of spacecraft and the desired trajectory tracking problem of robot manipulators have very similar dynamics and there exist even more similar dynamics, when a dynamics is constructed by Lagrangian. This means that we can define this class as one of the adaptive output feedback stabilizable class. The purpose of this chapter is to define this class as "Passivity Based Globally Stabilizable System via Adaptive Output Feedback " (PBGSS/AOF). This chapter also defines the condition that this class of nonlinear systems must possess to be included in this class. We also mention the relationship between this class and so called "passivity property."

2.1 Passivity Based Globally Stabilizable Systems via Output Feedback

First, General system dynamics are defined that can be stabilized via output feedback with no uncertainty present.

2.1.1 Definition

Defining a general system state $\boldsymbol{\psi}(\in \mathcal{R}^{3n}) = [\boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}, \boldsymbol{\psi}_2]^T(\boldsymbol{\psi}_{11,12,2} \in \mathcal{R}^n)$, that dynamics must be expressed as follows. (Note:We assume that functions which depends only on time are all bounded.)

$$\dot{\boldsymbol{\psi}}_{11} = \boldsymbol{A}_1(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) + \boldsymbol{B}_1(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12})\boldsymbol{\psi}_2 \tag{2.1}$$

$$\dot{\boldsymbol{\psi}}_{12} = \boldsymbol{A}_2(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) + \boldsymbol{B}_2(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12})\boldsymbol{\psi}_2 \tag{2.2}$$

$$\dot{\boldsymbol{\psi}}_2 = \boldsymbol{D}^{-1}(t, \boldsymbol{\psi}_{11,12})(\boldsymbol{F}(t, \boldsymbol{\psi}_{11,12}, \boldsymbol{\psi}_2) + \boldsymbol{u}) \tag{2.3}$$

where, $\boldsymbol{D}(\in \mathcal{R}^{n\times n}) = \boldsymbol{D}^T$ is a symmetric positive definite matrix and $\boldsymbol{F} \in \mathcal{R}^n$ is a nonlinear function.

Remark 2.1. In typically practical systems, each sub-state has the following meaning.

$$\begin{aligned} \boldsymbol{\psi}_{11} &: \text{Kinematics (Position Variable)} \\ \boldsymbol{\psi}_{12} &: \text{Filter (Observer) States} \\ \boldsymbol{\psi}_2 &: \text{Velocity states} \end{aligned}$$

Also, $\boldsymbol{A}_{1,2} \in \mathcal{R}^{n\times n}$ each has a next property.

Assumption 2.1. A vector function $\boldsymbol{A} \triangleq [\boldsymbol{A}_1,\ \boldsymbol{A}_2]^T$ can be summarized via next expression.

$$\begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{bmatrix} = \underbrace{\left[\begin{pmatrix} \boldsymbol{A}_{ca} & \boldsymbol{0}_{n\times n} \\ \boldsymbol{0}_{n\times n} & \boldsymbol{A}_{cb} \end{pmatrix} + \boldsymbol{N}(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) \right]}_{\boldsymbol{A}_c} \cdot \underbrace{\begin{bmatrix} \boldsymbol{f}_1(t, \boldsymbol{\psi}_{11}) \\ \boldsymbol{f}_2(t, \boldsymbol{\psi}_{12}) \end{bmatrix}}_{\boldsymbol{f}(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12})} \tag{2.4}$$

where, each $\boldsymbol{A}_{ca,cb} \in \mathcal{R}^{n\times n}$ is a constant Hurwitz matrix and $\boldsymbol{N}(\cdot) \in \mathcal{R}^{2n\times 2n}$ is a zero matrix or a skew symmetric matrix. $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ must be bounded with respect to their arguments and $\boldsymbol{f}_1 = 0$ must imply $\boldsymbol{\psi}_{11} = 0$ and $\boldsymbol{f}_2 = 0$ must also imply $\boldsymbol{\psi}_{12}$ as well. Summarizing these assumptions yields,

$$\boldsymbol{f} = 0 \Rightarrow \boldsymbol{\psi}_{11},\ \boldsymbol{\psi}_{12} = 0 \tag{2.5}$$

We are now trying to construct a general class of PBGSS/AOF and assume that only measurable state is $\boldsymbol{\psi}_{11}$. Here, we have to mention about a condition to make $\boldsymbol{\psi}_{12}$ feasible.

Assumption 2.2. If the matrix $\boldsymbol{B}_1$ in (2.1) must be full rank for $\forall \boldsymbol{\psi}_{11},\ \boldsymbol{\psi}_{12},\ t$ and

$$\boldsymbol{B}_2 \cdot \boldsymbol{B}_1^{-1} = \boldsymbol{k} \tag{2.6}$$

where, $\boldsymbol{k} \in \mathcal{R}^{n\times n}$ is a nonsingular constant matrix, then, we can construct a feasible differential equation to calculate $\boldsymbol{\psi}_{12}$ that does not involve $\boldsymbol{\psi}_2$ terms in contrast to (2.2).

Remark 2.2. Typically, we design the filter dynamics (2.2). Hence it is not too restrictive to assume that this proposition holds. As an example, we can always select $\boldsymbol{B}_1 = \boldsymbol{B}_2$ and that satisfy (2.6).

Proof. From (2.1), we have

$$\boldsymbol{\psi}_2 = \boldsymbol{B}_1^{-1}(\dot{\boldsymbol{\psi}}_{11} - \boldsymbol{A}_1) \tag{2.7}$$

Thus, substituting (2.7) into (2.2) yields,

$$\begin{aligned}\dot{\boldsymbol{\psi}}_{12} &= \boldsymbol{A}_2 + \boldsymbol{B}_2\boldsymbol{B}_1^{-1}(\dot{\boldsymbol{\psi}}_{11} - \boldsymbol{A}_1) \\ &= \boldsymbol{A}_2 + \boldsymbol{k}(\dot{\boldsymbol{\psi}}_{11} - \boldsymbol{A}_1) \end{aligned} \tag{2.8}$$

When we define $\boldsymbol{\vartheta} \triangleq \boldsymbol{\psi}_{12} - \boldsymbol{k}\boldsymbol{\psi}_{11}$, we can rewrite (2.8) as,

$$\dot{\boldsymbol{\vartheta}} = \boldsymbol{A}_2(t, \boldsymbol{\psi}_{11}, \boldsymbol{k}\boldsymbol{\psi}_{11} + \boldsymbol{\vartheta}) - \boldsymbol{k} \cdot \boldsymbol{A}_1(t, \boldsymbol{\psi}_{11}, \boldsymbol{k}\boldsymbol{\psi}_{11} + \boldsymbol{\vartheta}) \tag{2.9}$$

This does not depend on unmeasured state any more, thus this is feasible differential equation. Naturally, $\boldsymbol{\psi}_{12}$, which is our requiring state, is obtained by

$$\boldsymbol{\psi}_{12} = \boldsymbol{k}\boldsymbol{\psi}_{11} + \boldsymbol{\vartheta} \tag{2.10}$$

$\square$

Before introducing a class of nonlinear system, which is stabilizable via output feedback, we need one more assumption.

Assumption 2.3. A scalar function $\boldsymbol{G}(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12})$, which is defined by next (We summarize $[\boldsymbol{\psi}_{11}^T, \boldsymbol{\psi}_{12}^T]^T = \boldsymbol{\psi}_1$),

$$\boldsymbol{G}(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) \triangleq \int_0^t \boldsymbol{f}^T \boldsymbol{P} \dot{\boldsymbol{\psi}}_1 dt \tag{2.11}$$

where, $\boldsymbol{P}$ has the next partition,

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{P}_a & \boldsymbol{0}_{n\times n} \\ \boldsymbol{0}_{n\times n} & \boldsymbol{P}_b \end{pmatrix} \tag{2.12}$$

and each symmetric positive definite matrix $\boldsymbol{P}_a$ and $\boldsymbol{P}_b$ satisfies the next Lyapunov equations,

$$\begin{aligned}
\boldsymbol{A}_{ca}^T \boldsymbol{P}_a + \boldsymbol{P}_a \boldsymbol{A}_{ca} &= -\boldsymbol{Q}_a (= \boldsymbol{Q}_a^T > 0) && (2.13)\\
\boldsymbol{A}_{cb}^T \boldsymbol{P}_b + \boldsymbol{P}_b \boldsymbol{A}_{cb} &= -\boldsymbol{Q}_b (= \boldsymbol{Q}_b^T > 0) && (2.14)
\end{aligned}$$

must be a mapping s.t.

$$\boldsymbol{G} : \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n \rightarrow \mathcal{R}^+ \tag{2.15}$$

and also $\boldsymbol{G}$ must be radially unbounded w.r.t $\boldsymbol{\psi}_{11}$ and $\boldsymbol{\psi}_{12}$.

Here, we are ready to introduce a class of "Passivity Based Globally Stabilizable System via Output Feedback." (PBGSS/OF)

Proposition 2.1. The system expressed as (2.1), (2.2) and (2.3) is PBGSS/OF if and only if the next condition holds. (This is the sufficient condition.)

$$\boldsymbol{\psi}_2^T \boldsymbol{F}(t, \boldsymbol{\psi}_1, \boldsymbol{\psi}_2) = \boldsymbol{\psi}_2^T \boldsymbol{g}(t, \boldsymbol{\psi}_1, \boldsymbol{\psi}_2) - \frac{1}{2} \boldsymbol{x}_2^T \dot{D} \boldsymbol{x}_2 \tag{2.16}$$

where, $\boldsymbol{g}$ can be written in terms of

$$\boldsymbol{g} = -\boldsymbol{g}_1(t, \boldsymbol{\psi}_1)\boldsymbol{\psi}_2 + \boldsymbol{g}_2(t, \boldsymbol{\psi}_1) \tag{2.17}$$

and $\boldsymbol{g}_1$ holds,

$$-\boldsymbol{\psi}_2^T \boldsymbol{g}_1 \boldsymbol{\psi}_2 \leq 0 \tag{2.18}$$

2.1.2 Stability and Controllability Proof

The controllability (closed loop stability) of this system is shown by the existence of a certain control structure.

Theorem 2.1. *Consider the system* (2.1), (2.2) *and* (2.3). *The control input* $\boldsymbol{u}$ *described as, (Arguments of functions are omitted for simplification.)*

$$\boldsymbol{u} = -\boldsymbol{B}_1^T \boldsymbol{P}_a \boldsymbol{f}_1 - \boldsymbol{B}_2^T \boldsymbol{P}_b \boldsymbol{f}_2 - \boldsymbol{g}_2 \tag{2.19}$$

guarantees global stability for the system; i.e,

$$\lim_{t \to \infty} [\boldsymbol{\psi}_1, \boldsymbol{\psi}_2] = 0$$

Proof. From above properties, we choose the Lyapunov function candidate as,

$$V = \boldsymbol{G}(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) + \frac{1}{2} \boldsymbol{\psi}_2^T \boldsymbol{D} \boldsymbol{\psi}_2 \tag{2.21}$$

When we take a time derivative of this Lyapunov function, it yields, ($\boldsymbol{Q} = diag[\boldsymbol{Q}_a, \boldsymbol{Q}_b] > 0$)

$$\begin{aligned} \dot{V} &= \boldsymbol{f}^T \boldsymbol{P} \dot{\boldsymbol{\psi}}_1 + \boldsymbol{\psi}_2^T [\boldsymbol{F} + \boldsymbol{u}] \\ &= -\boldsymbol{f}^T \boldsymbol{Q} \boldsymbol{f} + \boldsymbol{\psi}_2^T [-\boldsymbol{g}_1 \boldsymbol{\psi}_2 + \boldsymbol{g}_2 + \boldsymbol{B}_1^T \boldsymbol{P}_a \boldsymbol{f}_1 + \boldsymbol{B}_2^T \boldsymbol{P}_b \boldsymbol{f}_2 + \boldsymbol{u}] \\ &\leq -\boldsymbol{f}^T \boldsymbol{Q} \boldsymbol{f} - \boldsymbol{\psi}_2^T \boldsymbol{g}_1 \boldsymbol{\psi}_2 \\ &\leq 0 \end{aligned} \tag{2.22}$$

We use the control input (2.19) in the third step. Then, we have, $\boldsymbol{G} \in \mathcal{L}_\infty, \boldsymbol{\psi}_2 \in \mathcal{L}_\infty$. $\boldsymbol{G}$ is radially unbounded from definition, thus, $\boldsymbol{\psi}_1 \in \mathcal{L}_\infty$ and this implies $\boldsymbol{f}, \dot{\boldsymbol{f}} \in \mathcal{L}_\infty$. From the dynamics of overall system (2.1), (2.2) and (2.3), $\dot{\boldsymbol{\psi}}_1,\ \dot{\boldsymbol{\psi}}_2 \in$

$\mathcal{L}_\infty$. Thus, by Barbalat's lemma, we get $\boldsymbol{\psi}_1 \to 0$ as $t \to \infty$. Substituting these result into (2.1) or (2.2) and considering the assumption 2.6 yield $\boldsymbol{\psi}_2 \to 0$ as $t \to \infty$.

$\square$

The relationship between the condition (2.16) and the system passivity can be mentioned as follows.

Theorem 2.2. *The condition* (2.16) *holds if a system holds passivity between* $\boldsymbol{\psi}_2$ *and* $\boldsymbol{u} + \boldsymbol{g}$ *and also between* $\boldsymbol{\psi}_2$ *and* $\boldsymbol{B}^T\boldsymbol{f}$ *(*$\boldsymbol{B} \triangleq diag[\boldsymbol{B}_1, \boldsymbol{B}_2]$*).*

Proof. From the original Lyapunov function (2.21), choose two sub-classes of Lyapunov function V_1, V_2 as,

$$V_1 = \frac{1}{2}\boldsymbol{\psi}_2^T D \boldsymbol{\psi}_2 \tag{2.23}$$

$$V_2 = \boldsymbol{G}(t, \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) \tag{2.24}$$

their time derivatives are

$$\dot{V}_1 = \boldsymbol{\psi}_2^T[\boldsymbol{g} + \boldsymbol{u}] \tag{2.25}$$

$$\dot{V}_2 = \boldsymbol{\psi}_2^T[\boldsymbol{B}^T\boldsymbol{f}] - \boldsymbol{f}^T\boldsymbol{Q}\boldsymbol{f} \tag{2.26}$$

where, (2.16) are taken advantage of. These are the definition of passivity itself by Slotine [51] which has been already introduced in chapter 1. We can also show the passivity based on the original definition of passivity. By integrating (2.25) and (2.26) from 0 to T (time), we have,

$$V_1(T) - V_1(0) = \int_0^T \boldsymbol{\psi}_2^T[\boldsymbol{g} + \boldsymbol{u}]dt \tag{2.27}$$

$$V_2(T) - V_2(0) \leq \int_0^T \boldsymbol{\psi}_2^T[\boldsymbol{B}^T\boldsymbol{f}]dt \tag{2.28}$$

These are obviously the originally same definition of passivity in chapter 1. Thus, if (2.16) holds, the claimed passivity holds. □

Remark 2.3. If $\boldsymbol{F}(t, \boldsymbol{\psi}_{11,12}, \boldsymbol{\psi}_2) + \boldsymbol{u} - \boldsymbol{g}_2(\boldsymbol{h}_{g2}(t), \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) = 0$ when $\boldsymbol{\psi}_{12},\ \boldsymbol{\psi}_2 = 0$ (not required $\boldsymbol{\psi}_{11} = 0$), then the condition for $\boldsymbol{A}_c$ in (2.4) can be relaxed as at least $\boldsymbol{A}_{cb}$ is a Hurwitz and $\boldsymbol{A}_{ca}$ can be permitted to be a zero matrix. (At this time, $\boldsymbol{P}_a$ does not have to satisfy (2.13) and it can be chosen to be arbitrarily symmetric positive definite matrix.)

Proof. In order to simply the arguments, we assume $\boldsymbol{A}_{ca} = 0_{n\times n}$. When we use the same Lyapunov function (2.21) and take a time derivative of it, we have

$$\begin{aligned}\dot{V} &= -\boldsymbol{f}_2^T\boldsymbol{Q}_b\boldsymbol{f}_2 - \boldsymbol{\psi}_2^T\boldsymbol{g}_1\boldsymbol{\psi}_2 \\ &\leq 0\end{aligned} \tag{2.29}$$

Thus, we can conclude $\boldsymbol{\psi}_{12},\ \boldsymbol{\psi}_2 \to 0$ as $t \to \infty$ with the same procedure of Theorem.1. Also we have $\boldsymbol{\psi}_1,\ \boldsymbol{\psi}_2,\ \dot{\boldsymbol{\psi}}_1,\ \dot{\boldsymbol{\psi}}_2 \in \mathcal{L}_\infty$. When we substitute (2.19) into (2.3) and take a time derivative of this equation, we have,

$$\begin{aligned}\dot{\boldsymbol{D}}(\boldsymbol{h}_{dD}(t), \boldsymbol{\psi}_1,\ \boldsymbol{\psi}_2,\ \dot{\boldsymbol{\psi}}_1,\ \dot{\boldsymbol{\psi}}_2)\dot{\boldsymbol{\psi}}_2 + \boldsymbol{D}\ddot{\boldsymbol{\psi}}_2 &= \frac{d}{dt}\left[\boldsymbol{F}(\boldsymbol{h}_F(t), \boldsymbol{\psi}_{11,12}, \boldsymbol{\psi}_2)\right. \\ &\quad -\boldsymbol{g}_2(\boldsymbol{h}_{g2}(t), \boldsymbol{\psi}_{11}, \boldsymbol{\psi}_{12}) - \boldsymbol{B}_1^T\boldsymbol{P}_a\boldsymbol{f}_1 \\ &\quad \left.-\boldsymbol{B}_2^T\boldsymbol{P}_b\boldsymbol{f}_2\right] \\ &\triangleq \boldsymbol{H}(\boldsymbol{h}_H(t), \boldsymbol{\psi}_1,\ \boldsymbol{\psi}_2,\ \dot{\boldsymbol{\psi}}_1,\ \dot{\boldsymbol{\psi}}_2)\end{aligned} \tag{2.30}$$

The first term of LHS of (2.30) and RHS are bounded because they are continuous function and all their arguments are bounded. Also $\boldsymbol{D}$ is bounded below from its definition, thus, this conclude $\ddot{\boldsymbol{\psi}}_2 \in \mathcal{L}_\infty$. By using Barbalat's lemma recursively, $\boldsymbol{\psi}_2 \to 0$ and $\dot{\boldsymbol{\psi}}_2,\ \ddot{\boldsymbol{\psi}}_2 \in \mathcal{L}_\infty$ imply $\dot{\boldsymbol{\psi}}_2 \to 0$ as $t \to \infty$. Consequently, we have $\boldsymbol{\psi}_{12},\ \boldsymbol{\psi}_2,\ \dot{\boldsymbol{\psi}}_2 \to 0$ as $t \to \infty$. When we substitute this result into (2.3) with (2.19) again, LHS of (2.3) goes to zero as time goes to infinity. However, there remains only one term $-\boldsymbol{B}_1^T \boldsymbol{P}_a \boldsymbol{f}_1$ in RHS. Hence,

$$-\boldsymbol{B}_1^T \boldsymbol{P}_a \boldsymbol{f}_1 \to 0 \quad as \quad t \to \infty \tag{2.31}$$

From the Assumption.2.1, it happens only when $\boldsymbol{f}_1 \to 0$. Thus, this automatically implies $\boldsymbol{\psi}_{11} \to 0$ as $t \to \infty$.

$\square$

Remark 2.4. With the same reason as above, if $\boldsymbol{F} - \boldsymbol{g}_2 = 0$ when $\boldsymbol{\psi}_{11},\ \boldsymbol{\psi}_2 = 0$ (not required $\boldsymbol{\psi}_{12} = 0$), then the condition for $\boldsymbol{A}_c$ in (2.4) can be relaxed as at least $\boldsymbol{A}_{ca}$ is a Hurwitz and $\boldsymbol{A}_{cb}$ can be permitted to have zero poles.

2.2 Passivity Based Globally Stabilizable System via Adaptive Output Feedback

2.2.1 Definition

Theorem 2.3. *Let us assume $\boldsymbol{A}_{1,2}$ are completely known. If $\boldsymbol{g}_2$ are linear with respect to unknown parameters, then, the system is "Passivity Based Globally Stabilizable system via Adaptive Output Feedback." (PBGSS/AOF)*

2.2.2 Stability and Controllability Proof

Proof. We denote $\boldsymbol{g}_2^*$ as $\boldsymbol{g}_2$ with unknown parameters.

Choose Lyapunov function as,

$$V_a = V + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \tilde{\boldsymbol{\theta}} \tag{2.32}$$

When we take a time derivative of V_a, we have

$$\begin{aligned} \dot{V}_a \ \leq \ & -\boldsymbol{f}^T \boldsymbol{Q} \boldsymbol{f} + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}} \\ & + \boldsymbol{\psi}_2^T [-\boldsymbol{g}_1 \boldsymbol{\psi}_2 + \boldsymbol{g}_2^* + \boldsymbol{B^*}_1^T \boldsymbol{P}_a \boldsymbol{f}_1 + \boldsymbol{B^*}_2^T \boldsymbol{P}_a \boldsymbol{f}_2 + \boldsymbol{u}] \end{aligned} \tag{2.33}$$

Here, we choose the control input as

$$\boldsymbol{u} = -\hat{\boldsymbol{g}}_2 - \boldsymbol{B}_1^T \boldsymbol{P}_a \boldsymbol{f}_1 - \boldsymbol{B}_2^T \boldsymbol{P}_a \boldsymbol{f}_2 \tag{2.34}$$

where, $\hat{\boldsymbol{g}}_2$ means estimated values of $\boldsymbol{g}_2^*$.

By putting this control input into (2.33), V_a is going to be,

$$\begin{aligned} \dot{V}_a \ \leq \ & -\boldsymbol{f}^T \boldsymbol{Q} \boldsymbol{f} - \boldsymbol{\psi}_2^T \boldsymbol{g}_1 \boldsymbol{\psi}_2 + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}} \\ & + \boldsymbol{\psi}_2^T \tilde{\boldsymbol{g}}_2 \end{aligned} \tag{2.35}$$

where, $\tilde{\boldsymbol{g}}_2 \triangleq \boldsymbol{g}_2^* - \hat{\boldsymbol{g}}_2$. As we mention above, all the unknown parameters are included in this system linearly, thus, it can be parameterized by $\tilde{\boldsymbol{\theta}}$, i.e,

$$\tilde{\boldsymbol{g}}_2 = \boldsymbol{W}(t, \boldsymbol{\psi}_1) \tilde{\boldsymbol{\theta}} \tag{2.36}$$

Hence, $\dot{V}_a$ can be summarized as,

$$\dot{V}_a \leq -\boldsymbol{f}^T \boldsymbol{Q} \boldsymbol{f} - \boldsymbol{\psi}_2^T \boldsymbol{g}_1 \boldsymbol{\psi}_2 + \tilde{\boldsymbol{\theta}}^T [\Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}} + \boldsymbol{W}^T \boldsymbol{\psi}_2] \tag{2.37}$$

Finally, we can choose $\dot{\tilde{\boldsymbol{\theta}}}$ as

$$\dot{\tilde{\boldsymbol{\theta}}} = -\dot{\hat{\boldsymbol{\theta}}} = -\Gamma \boldsymbol{W}^T \boldsymbol{\psi}_2 \tag{2.38}$$

in order to show that,

$$\dot{V}_a = -\boldsymbol{f}^T \boldsymbol{Q} \boldsymbol{f} - \boldsymbol{\psi}_2^T \boldsymbol{g}_1 \boldsymbol{\psi}_2 \leq 0 \tag{2.39}$$

Thus, by processing the same steps in the previous cases, we can show the stability of $\boldsymbol{\psi}_{1,2}$.
We have to mention about the property of (2.38). This differential update law seems to be unfeasible due to the presence of $\boldsymbol{\psi}_2$, however, we can apply two techniques to make this update law feasible; one is called decomposition technique and the other is called integration technique. With these techniques, we can construct $\hat{\boldsymbol{\theta}}$ itself which does not include $\boldsymbol{\psi}_2$ any more. It depends on the system which technique we can use to construct a feasible update equation.

2.2.3 Feasible Update Law

Lemma 2.1. If $\boldsymbol{W}^T \boldsymbol{B}_1^{-1} \cdot [\boldsymbol{I}_{n\times n}, 0_{n\times n}]$ has the next decomposition,

$$\boldsymbol{W}^T(t, \boldsymbol{\psi}_1) \boldsymbol{B}_1^{-1}(t, \boldsymbol{\psi}_1) = \boldsymbol{\phi}(t) \cdot \boldsymbol{w}(\boldsymbol{\psi}_1) \tag{2.40}$$

then, (2.38) can be calculated by the next equation.

$$\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}_1(t) + \hat{\boldsymbol{\theta}}(0) + \boldsymbol{\phi}(t) \int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \boldsymbol{w}(\boldsymbol{\xi}) d\boldsymbol{\xi} - \int_0^t \dot{\boldsymbol{\phi}}(\tau) \int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \boldsymbol{w}(\boldsymbol{\xi}) d\boldsymbol{\xi} d\tau \tag{2.41}$$

where, $\hat{\boldsymbol{\theta}}_1(t)$ is an output from the next differential equation.

$$\dot{\hat{\boldsymbol{\theta}}}_1(t) = -\Gamma \boldsymbol{W}^T \boldsymbol{B}_1^{-1} \boldsymbol{A}_1 \tag{2.42}$$

Proof. Substituting (2.7) into (2.38), we have

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}(t) &= \Gamma \boldsymbol{W}^T \boldsymbol{B}_1^{-1} (\dot{\boldsymbol{\psi}}_{11} - \boldsymbol{A}_1) \\ &= -\Gamma \boldsymbol{W}^T \boldsymbol{B}_1^{-1} \boldsymbol{A}_1 + \Gamma \boldsymbol{W}^T \boldsymbol{B}_1^{-1} \dot{\boldsymbol{\psi}}_{11} \\ &\triangleq \dot{\hat{\boldsymbol{\theta}}}_1(t) + \dot{\hat{\boldsymbol{\theta}}}_2(t) \end{aligned} \tag{2.43}$$

Naturally, $\dot{\hat{\boldsymbol{\theta}}}_1$ is a feasible part and we try to make $\dot{\hat{\boldsymbol{\theta}}}_2$ feasible. Considering $\boldsymbol{\psi}_{11} = [\boldsymbol{I}_{n\times n}, 0_{n\times n}] \cdot \boldsymbol{\psi}_1$, $\dot{\hat{\boldsymbol{\theta}}}_2$ will be,

$$\dot{\hat{\boldsymbol{\theta}}}_2(t) = \Gamma \boldsymbol{W}^T \boldsymbol{B}_1^{-1} [\boldsymbol{I}_{n\times n}, 0_{n\times n}] \dot{\boldsymbol{\psi}}_1 \tag{2.44}$$

When we substitute (2.40) into (2.44), we have

$$\dot{\hat{\boldsymbol{\theta}}}_2(t) = \Gamma \boldsymbol{\phi}(t) \cdot \boldsymbol{w}(\boldsymbol{\psi}_1) \dot{\boldsymbol{\psi}}_1 \tag{2.45}$$

Thus, applying integration by parts to (2.44) directly and combining $\hat{\boldsymbol{\theta}}_1$ give us the final expression (2.41).

□

If, this decomposition property does not hold, we also have the following tool.

Lemma 2.2. The vector function $\hat{\boldsymbol{\theta}}$, which is defined by next

$$\hat{\boldsymbol{\theta}}(t) \triangleq \boldsymbol{\Gamma} \int_0^t \boldsymbol{W}^T(\tau, \boldsymbol{\psi}_1) \dot{\boldsymbol{\psi}}_2 d\tau \tag{2.46}$$

where, $\mathbf{\Gamma}$ is any arbitrarily positive definite matrix, can be calculated without using $\boldsymbol{\psi}_2$ by the following expression.

$$\begin{aligned}\mathbf{\Gamma}\int_0^t \boldsymbol{W}^T(\tau,\boldsymbol{\psi}_1)\boldsymbol{\psi}_2 d\tau &= \hat{\boldsymbol{\theta}}_1(t) + \mathbf{\Gamma}\boldsymbol{H}(t,\boldsymbol{\psi}_1) \\ &\quad -\mathbf{\Gamma}\int_0^t \int_{\boldsymbol{\psi}_{10}}^{\boldsymbol{\psi}_1} \boldsymbol{W}_t^T(\tau,\boldsymbol{\epsilon})d\boldsymbol{\epsilon}d\tau \end{aligned} \tag{2.47}$$

where, $\boldsymbol{H}(t,\boldsymbol{\psi})$ is defined as following.

$$\boldsymbol{H}(t,\boldsymbol{\psi}_1) \triangleq \int_{\boldsymbol{\psi}_{10}}^{\boldsymbol{\psi}_1} \boldsymbol{W}^T(t,\boldsymbol{\epsilon})\boldsymbol{B}_1^{-1}(t,\boldsymbol{\epsilon})[\boldsymbol{I}_{n\times n}, 0_{n\times n}]d\boldsymbol{\epsilon} \tag{2.48}$$

and $\hat{\boldsymbol{\theta}}_1$ is an output from the same differential equation (2.42). Also, subscript "t" of $\boldsymbol{W}$ in (2.47) means partial derivative with respect to time.

Proof. As the same in decomposable case, substituting (2.7) into (2.38) yields,

$$\begin{aligned}\dot{\hat{\boldsymbol{\theta}}}(t) &= -\Gamma\boldsymbol{W}^T\boldsymbol{B}_1^{-1}\boldsymbol{A}_1 + \Gamma\boldsymbol{W}^T\boldsymbol{B}_1^{-1}\dot{\boldsymbol{\psi}}_{11} \\ &\triangleq \dot{\hat{\boldsymbol{\theta}}}_1(t) + \dot{\hat{\boldsymbol{\theta}}}_2(t)\end{aligned} \tag{2.49}$$

As shown before, the first part of (2.49) is feasible and the second part is a target to make feasible.

Let us consider time derivative of vector function $\boldsymbol{H}(\boldsymbol{h}_H(t),\boldsymbol{\psi}_1)$, which is,

$$\frac{d}{dt}\boldsymbol{H}(t,\boldsymbol{\psi}) = \frac{\partial}{\partial t}\boldsymbol{H}(t,\boldsymbol{\psi}) + \frac{\partial}{\partial \boldsymbol{\psi}}\boldsymbol{H}(t,\boldsymbol{\psi})\cdot\dot{\boldsymbol{\psi}} \tag{2.50}$$

When we integrate this expression with respect to time, we will get

$$\boldsymbol{H}(t,\boldsymbol{\psi}_1) = \int_0^t \frac{\partial}{\partial \tau}\boldsymbol{H}(\tau,\boldsymbol{\psi}_1)d\tau + \int_0^t \boldsymbol{W}(\tau,\boldsymbol{\psi}_1)\dot{\boldsymbol{\psi}}_1 d\tau \tag{2.51}$$

The second term of (2.51) is nothing but our $\hat{\boldsymbol{\theta}}_2$ itself. Thus, $\hat{\boldsymbol{\theta}}_2$ can be calculated as shown next.

$$
\begin{aligned}
\hat{\boldsymbol{\theta}}_2 &= \boldsymbol{\Gamma}\boldsymbol{H}(t,\boldsymbol{\psi}_1) - \boldsymbol{\Gamma}\int_0^t \frac{\partial}{\partial\tau}\boldsymbol{H}(\tau,\boldsymbol{\psi}_1)d\tau \\
&= \boldsymbol{\Gamma}\boldsymbol{H}(t,\boldsymbol{\psi}_1) - \boldsymbol{\Gamma}\int_0^t \frac{\partial}{\partial\tau}\int_{\boldsymbol{\psi}_{10}}^{\boldsymbol{\psi}_1}\boldsymbol{W}(\tau,\epsilon)d\boldsymbol{\epsilon}d\tau \\
&= \boldsymbol{\Gamma}\boldsymbol{H}(t,\boldsymbol{\psi}_1) - \boldsymbol{\Gamma}\int_0^t \int_{\boldsymbol{\psi}_{10}}^{\boldsymbol{\psi}_1}\frac{\partial}{\partial\tau}\boldsymbol{W}(\tau,\epsilon)d\boldsymbol{\epsilon}d\tau \\
&= \boldsymbol{\Gamma}\boldsymbol{H}(t,\boldsymbol{\psi}_1) - \boldsymbol{\Gamma}\int_0^t \int_{\boldsymbol{\psi}_{10}}^{\boldsymbol{\psi}_1}\boldsymbol{W}_\tau(\tau,\epsilon)d\boldsymbol{\epsilon}d\tau
\end{aligned}
\tag{2.52}
$$

From the definition of $\boldsymbol{H}(t)$ and the nature of $\boldsymbol{W}(t,\boldsymbol{\psi}_1)$, (2.52) is not dependent on $\dot{\boldsymbol{\psi}}_1$ any longer.

□

According to this new definition of a class of nonlinear systems, we solve the spacecraft attitude tracking problem first, and reference trajectory tracking problem of robot manipulators secondly.

Chapter 3

Spacecraft Attitude Tracking Problem

3.1 Introduction

Spacecraft attitude tracking control has been researched for many years and its adaptive problems with full state feedback has been successfully solved by Junkins.et.al [32]. On the other hand, only during the past decade, great progress has been achieved in the field of spacecraft attitude control without using angular velocity measurements. When no inertia uncertainty is present, we have already had a lot of interesting output feedback solutions for both attitude regulation and tracking. Especially, when we focus on the passivity based formalisim, the history began with Lizarralde and Wen's achievements [17] for a regulation problem of spacecrat attitude control. Furthermore, Tsiotras [46] extended this result and took advantage of certain passivity properties inherent to this problem to formulate a dynamic controller for attitude regulation, in which the kinematics are expressed in terms of the Modified Rodrigues Parameters (MRPs). For the tracking case, Caccavale and Villani [16] provide a solution with guaranteed local exponential stability by adopting the nonminimal set of quaternions for kinematics and constructing a model based observer to estimate the angular velocity. Recently, Akella [40] extended these results by developing an angular velocity free controller formulation using a Lyapunov construction that guarantees global asymptotic stability. An important feature within the results of both Caccavale and Villani [16] and Akella [40] is that the control input torque has a linear dependence on the inertia matrix, suggesting the applicability of Model Reference Adaptive Control (MRAC) techniques for the unknown inertia matrix case.

For this class of problems, it must however be noted that there are only very few and strongly limited solutions within the literatures. One of the latest and practical solution is based on the "complete observability assumption" [8], [41], [13]. However, in this framework, we need the following assumptions

1. Upper and lower bounds of unknown parameters

2. Upper bounds of measured signals and their time derivative signals.

These assumptions cause requirements of much costs and time for designing a controller and it is interesting and important research to get rid of these assumptions from the designing process of adaptive control scheme.

Thus, our purpose in this example is to formulate an adaptive output feedback controller with as few a priori knowledge as possible. Actually, as shown later, we try to formulate a controller which can guarantee global stability with no extra assumption on the system.

3.2 Using Modified Rodrigues Parameters (MRPs)

3.2.1 Problem Formulation

As it is well known, Euler's rotational equation is formulated as follows.

$$\mathbf{I}\dot{\boldsymbol{\omega}} \; = -S(\boldsymbol{\omega})\mathbf{I}\boldsymbol{\omega} + \mathbf{u} \tag{3.1}$$

where, $\boldsymbol{\omega} \in \mathcal{R}^3$ is angular velocity of spacecraft, $\mathbf{I} = \mathbf{I}^T \in \mathcal{R}^{3\times 3}$ is the inertia matrix of a spacecraft and $\mathbf{u} \in \mathcal{R}^3$ is an external torque input. $S(\cdot)$ denotes the skew-symmetric matrix to perform the vector cross product.

In order to construct the kinematic equation, we take advantage of the modified Rodrigues parameters (MRPs), which is defined as the next equation.

$$\boldsymbol{\sigma} = \hat{\boldsymbol{e}} \tan \frac{\Phi}{4} \tag{3.2}$$

$\boldsymbol{\sigma} \in \mathcal{R}^3$ denotes MRP vector, $\hat{\mathbf{e}} \in \mathcal{R}^3$ and $\Phi \in \mathcal{R}$ represent the principal rotation axis and principal rotation angle respectively. With using this MRPs, the kinematic equation of the spacecraft attitude dyamics is defined by next.

$$\dot{\boldsymbol{\sigma}} = \frac{1}{4} B(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{3.3}$$

The function $B(\sigma)$ is given by the following expression.

$$B(\boldsymbol{\sigma}) = (1 - \boldsymbol{\sigma}^T\boldsymbol{\sigma})I_{3\times 3} + 2S(\boldsymbol{\sigma}) + 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T \tag{3.4}$$

with $I_{3\times3}$ being the 3×3 identity matrix.

In order to create the whole system dynamics, let us introduce several reference frames. $\mathbf{N}$, $\mathbf{B}$ and $\mathbf{C}$ denote the inertial frame, a body fixed frame and the commanded motion frame respectively. Also $\hat{\mathbf{n}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ represent the unit vector triads in each frame. These three vectors are mutually related with,

$$\begin{aligned} \hat{\mathbf{b}} &= C(\boldsymbol{\sigma})\hat{\mathbf{n}} \\ \hat{\mathbf{c}} &= C(\boldsymbol{\sigma}_c)\hat{\mathbf{n}} \\ \hat{\mathbf{b}} &= C(\mathbf{s})\hat{\mathbf{c}} \end{aligned} \tag{3.5}$$

$C(\cdot)$ is the direction cosine matrix and $\boldsymbol{\sigma}_c$ is the commanded MRPs in $\mathbf{C}$. (Namely, $\boldsymbol{\omega}_c$ and $\dot{\boldsymbol{\omega}}_c$ represent the commanded angular velocity and accelaration in $\mathbf{C}$.) It is possible to show the relationship between the direction cosine matrix and MRPs by the next equation.

$$C(\boldsymbol{\sigma}) = I_{3\times3} - 4\frac{1-\sigma^2}{(1+\sigma^2)^2}S(\boldsymbol{\sigma}) + \frac{8}{(1+\sigma^2)^2}S^2(\boldsymbol{\sigma}) \tag{3.6}$$

and $C(\mathbf{s})$ is defined by

$$C(\mathbf{s}) = C(\boldsymbol{\sigma})C^T(\boldsymbol{\sigma}_c) \tag{3.7}$$

In our adaptive output feedback problem, we make use of this $\mathbf{s}$ to simplify attitude tracking dynamics. Defining angular velocity error $\boldsymbol{\delta\omega} \triangleq \boldsymbol{\omega} - \boldsymbol{\omega}_c^B$ and the next relationship with respect to $\boldsymbol{\omega}_c^B$

$$\boldsymbol{\omega}_c^B = C(\mathbf{s})\boldsymbol{\omega}_c \tag{3.8}$$

$$\dot{\boldsymbol{\omega}}_c^B = C(\mathbf{s})\dot{\boldsymbol{\omega}}_c - S(\boldsymbol{\omega})C(\mathbf{s})\boldsymbol{\omega}_c \tag{3.9}$$

deliver the next open loop dynamics of a spacecraft attitude dyamics with respect to $\mathbf{s}$ and $\boldsymbol{\delta\omega}$.

$$\begin{aligned} \dot{\mathbf{s}} &= \frac{1}{4}B(\mathbf{s})\boldsymbol{\delta\omega} && (3.10) \\ \dot{\boldsymbol{\delta\omega}} &= -S(\boldsymbol{\omega})\mathbf{I}\boldsymbol{\omega} + \mathbf{u} - \mathbf{I}[C(\mathbf{s})\dot{\boldsymbol{\omega}}_c - S(\boldsymbol{\omega})C(\mathbf{s})\boldsymbol{\omega}_c] && (3.11) \end{aligned}$$

Without proof, here we show the theorem to guarantee the global stability for the system (3.10) and (3.11) with no angular velocity.

Theorem 3.1. *Let us consider the system* (3.10) *and* (3.11)*. If we adopt the control torque input* $\mathbf{u}$ *as*

$$\mathbf{u} = -\frac{1}{4}B^T(\mathbf{s})\mathbf{s} - \frac{1}{4}B^T P(\mathbf{s} + A_m\mathbf{z}) + \mathbf{I}C(\mathbf{s})\dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B)\mathbf{I}\boldsymbol{\omega}_c^B \tag{3.12}$$

then the closed loop system is globally asymptotically stable. Where A_m *is any Hurwitz matrix and* $P = P^T$ *is positive definite matrix which satisfies next Lyapunov equation*

$$A_m^T P + PA_m = -Q(= Q^T > 0) \tag{3.13}$$

and $\mathbf{z}$ *is the filtered output, which dynamics is defined as,*

$$\dot{\mathbf{z}} = A_m\mathbf{z} + \mathbf{s} \tag{3.14}$$

Remark 3.1. The overall system (3.10), (3.11) and (3.14) is PBGSS/OF. Corresponding expression in general formulation as in chapter 3, $\boldsymbol{\psi}_{11} \rightarrow \boldsymbol{s}$, $\boldsymbol{\psi}_{12} \rightarrow \dot{\boldsymbol{z}}$, $\boldsymbol{\psi}_2 \rightarrow \boldsymbol{\delta\omega}$. Also in this case, matrix A in chapter 3 has the next form.

$$\boldsymbol{A}_c = \begin{pmatrix} \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \boldsymbol{A}_m \end{pmatrix} \tag{3.15}$$

As shown in the sequel, We can relax the condition for A which is mentioned in chapter 3 for this case.

3.2.2 Adaptive Output Feedback Controller

Now we are ready to discuss about the adaptive controller for the system (3.10) and (3.11). We will try to estimate six entries in an inertia matrix, which is,

$$\boldsymbol{\theta}^* \triangleq \begin{bmatrix} I_{11}^* & I_{12}^* & I_{13}^* & I_{22}^* & I_{23}^* & I_{33}^* \end{bmatrix}^T \tag{3.16}$$

Let us summarize the main result as a theorem.

Theorem 3.2. *Consider the system* (3.10) *and*(3.11) *again with no information of inertia matrix. If we adopt the next control structure and adaptive update law ,*

$$\mathbf{u} = -\frac{1}{4}B^T(\mathbf{s})\mathbf{s} - \frac{1}{4}B^T P(\mathbf{s} + A_m\mathbf{z}) + \mathbf{I}(t)C(\mathbf{s})\dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B)\mathbf{I}(t)\boldsymbol{\omega}_c^B \tag{3.17}$$

$$\hat{\boldsymbol{\theta}}(t) = \Gamma\hat{\boldsymbol{\theta}}(0) + \Gamma\sum_{i=1}^{9}\hat{\boldsymbol{\theta}}_i(t) \tag{3.18}$$

$$\hat{\boldsymbol{\theta}}_i(t) = \phi_i(t)\int_0^{\mathbf{s}}\mathbf{w}_i(\mathbf{s})d\mathbf{s} - \int_0^t \frac{d}{d\tau}\phi_i(\tau)\int_0^{\mathbf{s}}\mathbf{w}_i(\mathbf{s})d\mathbf{s}d\tau \tag{3.19}$$

where, $\Gamma = \Gamma^T$ *is an arbitrarily positive definite matrix and* $\mathbf{w}_i(\mathbf{s}) \in \mathcal{R}^{6\times 3}$ *and* $\phi_i(t) \in \mathcal{R}$ *are defined later.* $\boldsymbol{z}$ *is the same filtered output from* (3.14). *Then, we can gurantee the globally asymptotical stability for the system.*

3.2.3 Stability and Controllability Proof

Before showing stability and controllability, we have to show the relationship between "feasible" update law of unknown parameters and "unfeasible" differential update law. Note that the feasible update law (3.18) and (3.19) are derived from decomposition technique in chapter 3.

Remark 3.2. As mentioned later, the feasible adaptive update law (3.18) and (3.19) is completely equivalent to,

$$\dot{\hat{\boldsymbol{\theta}}} = \Gamma \mathbf{W}_d^T(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)\boldsymbol{\delta\omega} \tag{3.20}$$

where $\mathbf{W}_d^T(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B) \in \mathcal{R}^{6\times 3}$ is described in the stability proof, then, we use this differential expression for the stability proof and show the equivalency of these two update laws in the following section.

Proof. Let us construct the next Lyapunov function candidate.

$$V = \frac{1}{2}\boldsymbol{\delta\omega}^T \mathbf{I}^* \boldsymbol{\delta\omega} + \underbrace{\frac{1}{2}\mathbf{s}^T\mathbf{s} + \frac{1}{2}(\mathbf{s} + A_m\mathbf{z})^T P(\mathbf{s} + A_m\mathbf{z})}_{\boldsymbol{G}} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{3.21}$$

When we take the time derivative of (3.21), it is going to be

$$\begin{aligned} \dot{V} &= \boldsymbol{\delta\omega}^T \mathbf{I}^* \dot{\boldsymbol{\delta\omega}} + \mathbf{s}^T\dot{\mathbf{s}} + \frac{1}{2}(\mathbf{s} + A_m\mathbf{z})^T P(\dot{\mathbf{s}} + A_m\dot{\mathbf{z}}) \\ &\quad + \frac{1}{2}(\dot{\mathbf{s}} + A_m\dot{\mathbf{z}})^T P(\mathbf{s} + A_m\mathbf{z}) + \tilde{\boldsymbol{\theta}}^T\Gamma^{-1}\dot{\tilde{\boldsymbol{\theta}}} \\ &= \boldsymbol{\delta\omega}^T[\mathbf{u} + \frac{1}{4}B^T(\mathbf{s})\mathbf{s} + \frac{1}{4}B^T P(\mathbf{s} + A_m\mathbf{z}) - \mathbf{I}^* C(\mathbf{s})\dot{\boldsymbol{\omega}}_c \\ &\quad - S(\boldsymbol{\omega})\mathbf{I}^*\boldsymbol{\omega} + \mathbf{I}^* S(\boldsymbol{\omega})\boldsymbol{\omega}_c^B] \\ &\quad + \frac{1}{2}(\mathbf{s} + A_m\mathbf{z})^T(A_m^T P + P A_m)(\dot{\mathbf{s}} + A_m\dot{\mathbf{z}}) + \tilde{\boldsymbol{\theta}}^T\Gamma^{-1}\dot{\tilde{\boldsymbol{\theta}}} \end{aligned} \tag{3.22}$$

By virtue of (3.13) and (3.17), (3.22) will end up with

$$\begin{aligned}\dot{V} &= \boldsymbol{\delta\omega}^T[S(\boldsymbol{\omega}_c^B)\mathbf{I}^*\boldsymbol{\omega}_c^B - S(\boldsymbol{\omega})\mathbf{I}^*\boldsymbol{\omega} + \mathbf{I}^*S(\boldsymbol{\omega})\boldsymbol{\omega}_c^B] \\ &\quad -\frac{1}{2}\dot{\mathbf{z}}^TQ\dot{\mathbf{z}} + \boldsymbol{\delta\omega}^T[\tilde{\mathbf{I}}(t)C((s))\dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B)\tilde{\mathbf{I}}(t)\boldsymbol{\omega}_c^B] + \tilde{\boldsymbol{\theta}}^T\Gamma^{-1}\dot{\tilde{\boldsymbol{\theta}}} \qquad (3.23)\end{aligned}$$

As mentioned in [40], the first part of (3.23) can be shown to be cancelled and the second term is stable term. The third term is linear with respect to each entry of inertia parameter and can be parameterized with using $\tilde{\boldsymbol{\theta}}(t)$ like,

$$\boldsymbol{\delta\omega}^T[\tilde{\mathbf{I}}(t)C((s))\dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B)\tilde{\mathbf{I}}(t)\boldsymbol{\omega}_c^B] = \boldsymbol{\delta\omega}^T\mathbf{W}_d(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)\tilde{\theta}(t) \qquad (3.24)$$

Noting that from the nature of (3.24), $\mathbf{W}_d(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)$ can divided into two parts like,

$$\mathbf{W}_d(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B) = \mathbf{W}_{d1}(\mathbf{s}, \dot{\boldsymbol{\omega}}_c) + \mathbf{W}_{d2}(\boldsymbol{\omega}_c^B) \qquad (3.25)$$

$\mathbf{W}_{d1}$ and $\mathbf{W}_{d2}$ can be calculated as follows.

$$\mathbf{W}_{d1}(\mathbf{s}, \dot{\boldsymbol{\omega}}_c) = \begin{bmatrix} cw_1 & cw_2 & cw_3 & 0 & 0 & 0 \\ 0 & cw_1 & 0 & cw_2 & cw_3 & 0 \\ 0 & 0 & cw_1 & 0 & cw_2 & cw_3 \end{bmatrix} \qquad (3.26)$$

where, scalar functions cw_1, cw_2 and cw_3 are defind by

$$cw_1 = c_{11}\dot{\omega}_{c1} + c_{12}\dot{\omega}_{c2} + c_{13}\dot{\omega}_{c3} \qquad (3.27)$$

$$cw_2 = c_{21}\dot{\omega}_{c1} + c_{22}\dot{\omega}_{c2} + c_{23}\dot{\omega}_{c3} \qquad (3.28)$$

$$cw_3 = c_{31}\dot{\omega}_{c1} + c_{32}\dot{\omega}_{c2} + c_{33}\dot{\omega}_{c3} \qquad (3.29)$$

with letting c_{ij} be ith row and jth column element of direction cosine matrix.

Here is the $\mathbf{W}_{d2}$.

$$\mathbf{W}_{d2}(\boldsymbol{\omega}_c^B) = \begin{bmatrix} 0 & -(\omega_{c1}^B)^2 & \omega_{c1}^B\omega_{c2}^B & -\omega_{c1}^B\omega_{c2}^B & -\omega_{c1}^B\omega_{c3}^B+\omega_{c1}^B\omega_{c2}^B & \omega_{c2}^B\omega_{c3}^B \\ (\omega_{c1}^B)^2 & \omega_{c1}^B\omega_{c2}^B & 0 & 0 & -\omega_{c2}^B\omega_{c3}^B & -(\omega_{c2}^B)^2 \\ -\omega_{c1}^B\omega_{c2}^B & -(\omega_{c2}^B)^2+\omega_{c1}^B\omega_{c3}^B & -\omega_{c2}^B\omega_{c3}^B & \omega_{c2}^B\omega_{c3}^B & (\omega_{c3}^B)^2 & 0 \end{bmatrix} \tag{3.30}$$

Then, we can construct adaptive update law using (3.23) and fourth term in (3.23).

$$\dot{\hat{\boldsymbol{\theta}}}(t) = -\dot{\tilde{\boldsymbol{\theta}}} = \Gamma \mathbf{W}_d^T(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)\boldsymbol{\delta\omega} \tag{3.31}$$

Finally, adopting (3.20), (3.23) becomes

$$\dot{V} = -\frac{1}{2}\dot{\mathbf{z}}^T Q \dot{\mathbf{z}} \leq 0 \tag{3.32}$$

Thus, $\boldsymbol{\delta\omega}$, $\mathbf{s}$, $\mathbf{z}$, $\dot{\mathbf{z}}$, $\tilde{\theta} \in \mathcal{L}_\infty$. It is reasonable to assume that $\boldsymbol{\sigma}_c$, $\boldsymbol{\omega}_c$ and its higher derivatives are all bounded. Then, by (3.20), $\tilde{\boldsymbol{\theta}}(t) \in \mathcal{L}_\infty$. By integrating (3.32), $\dot{\mathbf{z}} \in \mathcal{L}_2$ can be easily shown. From (3.14), we get

$$\ddot{\mathbf{z}} = A_m \dot{\mathbf{z}} + \dot{\mathbf{s}} \tag{3.33}$$

By the previous bounds and Eq.(3.10), we can conclude $\ddot{\boldsymbol{z}} \in \mathcal{L}_\infty$, thus, $\dot{\boldsymbol{z}} \to 0$ as $t \to \infty$. Using the same process above implies $\frac{d^3}{dt^3}\boldsymbol{z} \in \mathcal{L}_\infty$. Hence, by recursive Barbalat's lemma, $\ddot{\boldsymbol{z}} \to 0$ as $t \to \infty$. Substituting these two results to (3.33), we have $\dot{\boldsymbol{s}} \to 0$ as $t \to \infty$. This automatically implies $\boldsymbol{\delta\omega} \to 0$ as $t \to \infty$ by Eq.(3.10) (Note that $B(\boldsymbol{s})$ is a non-singular matrix.). Eq.(3.11) can be differentiated to be taken advantage of to show $\ddot{\boldsymbol{\delta\omega}} \in \mathcal{L}_\infty$. Thus, by recursive Barbalat's lemma again,

$$\boldsymbol{\delta\omega} \to 0,\ \dot{\boldsymbol{\delta\omega}} \in \mathcal{L}_\infty,\ \ddot{\boldsymbol{\delta\omega}} \in \mathcal{L}_\infty \ \Rightarrow\ \dot{\boldsymbol{\delta\omega}} \to\ 0\ \ as\ \ t \to \infty \tag{3.34}$$

This final result can be used to show that$\boldsymbol{s} \to\ 0$ as $t \to \infty$ by (3.11) with (3.17). Consequently, by Eq.(3.14),

$$\boldsymbol{s} \to\ 0,\ \dot{\boldsymbol{z}} \to 0 \Rightarrow\ \boldsymbol{z}\ \to 0\ \ as\ \ t \to \infty \tag{3.35}$$

When we summarized above, we have,

$$\lim_{t\to\infty}[\boldsymbol{\delta\omega}, \boldsymbol{s}, \boldsymbol{z}] = 0 \tag{3.36}$$

as required.

3.2.4 Proof of Equivalence Between Update Laws

The adaptive update law (3.20) can not be implemented directly because of the lack of information $\boldsymbol{\delta\omega}$. Here, we try to create practically implementable update law.

Proof. From the (3.10), we can describe $\boldsymbol{\delta\omega}$ as a function of $\mathbf{s}$ and $\dot{\mathbf{s}}$.

$$\boldsymbol{\delta\omega} = 4B^{-1}(\mathbf{s})\dot{\mathbf{s}} \tag{3.37}$$

Substituting (3.37) into (3.20), we get a new differential update law.

$$\dot{\hat{\boldsymbol{\theta}}}(t) = 4\Gamma\mathbf{W}_d^T(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)B^{-1}(\mathbf{s})\dot{\mathbf{s}} \tag{3.38}$$

Its integrated expression is the same as,

$$\hat{\boldsymbol{\theta}}(t) = \Gamma\hat{\boldsymbol{\theta}}(0) + 4\Gamma\int_0^t \mathbf{W}_d^T(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)B^{-1}(\mathbf{s})\dot{\mathbf{s}}\mathrm{d}\tau \tag{3.39}$$

(3.39) is still dependent on $\dot{\mathbf{s}}$. However, (3.39) can be converted to executable form by the next technique. Before introducing a certain technique, we change the notation of $\mathbf{W}_d$. As described in (3.8), $\boldsymbol{\omega}_c^B$ is a function of $\mathbf{s}$ and $\boldsymbol{\omega}_c$. Then, $\mathbf{W}_d$ is a function of $\mathbf{s}$, $\dot{\boldsymbol{\omega}}_c$ and $\boldsymbol{\omega}_c^B$, i.e,

$$\mathbf{W}_d = \mathbf{W}_d(\mathbf{s}, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c) \tag{3.40}$$

Furthermore,this new $\mathbf{W}_d$ can be divided into nine parts, i.e,

$$\mathbf{W}_d^T(\mathbf{s}, \omega_c, \dot{\omega}_c)B^{-1}(\mathbf{s}) = \sum_{i=1}^{9} \beta_{di} \tag{3.41}$$

where, each β_{di} is described as,

$$\beta_{di} = \begin{cases} \dot{\omega}_{ci}\mathbf{w}_i(\mathbf{s}) & for \quad i = 1,2,3 \\ \omega_{ci}^2\mathbf{w}_i(\mathbf{s}) & for \quad i = 4,5,6 \\ \omega_{c1}\omega_{c2}\mathbf{w}_7(\mathbf{s}) & for \quad i = 7 \\ \omega_{c1}\omega_{c3}\mathbf{w}_8(\mathbf{s}) & for \quad i = 8 \\ \omega_{c2}\omega_{c3}\mathbf{w}_9(\mathbf{s}) & for \quad i = 9 \end{cases} \tag{3.42}$$

Naturally, each $\mathbf{w}_i(\mathbf{s}) \in \mathcal{R}^{6\times 3}$ is a function only by $\mathbf{s}$ calculated as follows.

For $i = 1,2,3$, $\mathbf{w}_i(\mathbf{s})$ have the next form.

$$\mathbf{w}_i^T(\mathbf{s}) = B^{-T}(\mathbf{s})\gamma_i \tag{3.43}$$

where, each $\gamma_i \in \mathcal{R}^{3\times 6}$ is defined as follows with letting $c_{ij}(\mathbf{s})$ be each entry of direction cosine matrix with respect to $\mathbf{s}$.

$$\gamma_1 = \begin{bmatrix} c_{11}(\mathbf{s}) & c_{21}(\mathbf{s}) & c_{31}(\mathbf{s}) & 0 & 0 & 0 \\ 0 & c_{11}(\mathbf{s}) & 0 & c_{21}(\mathbf{s}) & c_{31}(\mathbf{s}) & 0 \\ 0 & 0 & c_{11}(\mathbf{s}) & 0 & c_{21}(\mathbf{s}) & c_{31}(\mathbf{s}) \end{bmatrix} \quad (3.44)$$

$$\gamma_2 = \begin{bmatrix} c_{12}(\mathbf{s}) & c_{22}(\mathbf{s}) & c_{32}(\mathbf{s}) & 0 & 0 & 0 \\ 0 & c_{12}(\mathbf{s}) & 0 & c_{22}(\mathbf{s}) & c_{32}(\mathbf{s}) & 0 \\ 0 & 0 & c_{12}(\mathbf{s}) & 0 & c_{22}(\mathbf{s}) & c_{32}(\mathbf{s}) \end{bmatrix} \quad (3.45)$$

$$\gamma_3 = \begin{bmatrix} c_{13}(\mathbf{s}) & c_{23}(\mathbf{s}) & c_{33}(\mathbf{s}) & 0 & 0 & 0 \\ 0 & c_{13}(\mathbf{s}) & 0 & c_{23}(\mathbf{s}) & c_{33}(\mathbf{s}) & 0 \\ 0 & 0 & c_{13}(\mathbf{s}) & 0 & c_{23}(\mathbf{s}) & c_{33}(\mathbf{s}) \end{bmatrix} \quad (3.46)$$

For i=4 to 9, $\mathbf{w}_i(\mathbf{s})$ have the same form as (3.43). However, the structure of each γ_i is different from those of the cases $i = 1, 2, 3$. Here are the exact definitions of them.

$$\gamma_i = \begin{bmatrix} 0 & -\gamma_{i1} & \gamma_{i2} & -\gamma_{i2} & \gamma_{i3} & \gamma_{i4} \\ \gamma_{i1} & \gamma_{i2} & 0 & 0 & -\gamma_{i4} & -\gamma_{i5} \\ -\gamma_{i2} & -\gamma_{i3} & -\gamma_{i4} & \gamma_{i4} & \gamma_{i5} & 0 \end{bmatrix} \quad (3.47)$$

where, each γ_{i1} to γ_{i5} is defined as follows.

$$\begin{aligned} \gamma_{41} &= c_{11}^2(\mathbf{s}) & \gamma_{51} &= c_{12}^2(\mathbf{s}) \\ \gamma_{42} &= c_{11}(\mathbf{s})c_{12}(\mathbf{s}) & \gamma_{52} &= c_{12}(\mathbf{s})c_{22}(\mathbf{s}) \\ \gamma_{43} &= c_{21}^2(\mathbf{s}) - c_{11}(\mathbf{s})c_{31}(\mathbf{s}) & \gamma_{53} &= c_{22}^2(\mathbf{s}) - c_{12}(\mathbf{s})c_{32}(\mathbf{s}) \\ \gamma_{44} &= c_{21}(\mathbf{s})c_{31}(\mathbf{s}) & \gamma_{54} &= c_{22}(\mathbf{s})c_{32}(\mathbf{s}) \\ \gamma_{45} &= c_{31}^2(\mathbf{s}) & \gamma_{55} &= c_{32}^2(\mathbf{s}) \end{aligned}$$

$$\begin{aligned}
\gamma_{61} &= c_{13}^2(\mathbf{s}) \\
\gamma_{62} &= c_{13}(\mathbf{s})c_{23}(\mathbf{s}) \\
\gamma_{63} &= c_{23}^2(\mathbf{s}) - c_{13}(\mathbf{s})c_{33}(\mathbf{s}) \\
\gamma_{64} &= c_{23}(\mathbf{s})c_{33}(\mathbf{s}) \\
\gamma_{65} &= c_{33}^2(\mathbf{s})
\end{aligned}$$

$$\begin{aligned}
\gamma_{71} &= 2c_{11}(\mathbf{s})c_{12}(\mathbf{s}) \\
\gamma_{72} &= c_{11}(\mathbf{s})c_{22}(\mathbf{s}) + c_{12}(\mathbf{s})c_{21}(\mathbf{s}) \\
\gamma_{73} &= 2c_{21}(\mathbf{s})c_{22}(\mathbf{s}) - (c_{11}(\mathbf{s})c_{32}(\mathbf{s}) + c_{12}(\mathbf{s})c_{31}(\mathbf{s})) \\
\gamma_{74} &= c_{21}(\mathbf{s})c_{32}(\mathbf{s}) + c_{22}(\mathbf{s})c_{31}(\mathbf{s}) \\
\gamma_{75} &= 2c_{31}(\mathbf{s})c_{32}(\mathbf{s})
\end{aligned}$$

$$\begin{aligned}
\gamma_{81} &= 2c_{11}(\mathbf{s})c_{13}(\mathbf{s}) \\
\gamma_{82} &= c_{11}(\mathbf{s})c_{23}(\mathbf{s}) + c_{13}(\mathbf{s})c_{21}(\mathbf{s}) \\
\gamma_{83} &= 2c_{21}(\mathbf{s})c_{23}(\mathbf{s}) - (c_{11}(\mathbf{s})c_{33}(\mathbf{s}) + c_{13}(\mathbf{s})c_{31}(\mathbf{s})) \\
\gamma_{84} &= c_{21}(\mathbf{s})c_{33}(\mathbf{s}) + c_{23}(\mathbf{s})c_{31}(\mathbf{s}) \\
\gamma_{85} &= 2c_{31}(\mathbf{s})c_{33}(\mathbf{s})
\end{aligned}$$

$$
\begin{aligned}
\gamma_{91} &= 2c_{12}(\mathbf{s})c_{13}(\mathbf{s}) \\
\gamma_{92} &= c_{12}(\mathbf{s})c_{23}(\mathbf{s}) + c_{13}(\mathbf{s})c_{21}(\mathbf{s}) \\
\gamma_{93} &= 2c_{22}(\mathbf{s})c_{23}(\mathbf{s}) - (c_{12}(\mathbf{s})c_{33}(\mathbf{s}) + c_{13}(\mathbf{s})c_{32}(\mathbf{s})) \\
\gamma_{94} &= c_{22}(\mathbf{s})c_{33}(\mathbf{s}) + c_{23}(\mathbf{s})c_{32}(\mathbf{s}) \\
\gamma_{95} &= 2c_{32}(\mathbf{s})c_{33}(\mathbf{s})
\end{aligned}
$$

Using these $\mathbf{w}_i$, $\hat{\boldsymbol{\theta}}$ can be implemented as follows.

$$
\hat{\boldsymbol{\theta}}(t) = \Gamma\hat{\boldsymbol{\theta}}(0) + \Gamma\sum_{i=1}^{9}\hat{\boldsymbol{\theta}}_i(t) \tag{3.48}
$$

Each $\hat{\boldsymbol{\theta}}_i(t)$ can be implemented by the next technique.

Here is a property of integral.

$$
\frac{d}{dt}\int_0^t \phi_i(t)\int_0^{\mathbf{s}} \mathbf{w}_i(\mathbf{s})d\mathbf{s}dt = \int_0^t \frac{d}{dt}\phi_i(t)\int_0^{\mathbf{s}} \mathbf{w}_i(\mathbf{s})d\mathbf{s}dt + \int_0^t \phi_i(t)\mathbf{w}_i(\mathbf{s})\dot{\mathbf{s}}dt \tag{3.49}
$$

where $\phi_i(t)$ is a coefficient scalar function of each $\mathbf{w}_i$.

By (3.49), $\hat{\boldsymbol{\theta}}_i(t)$ can be realized by the next expression.

$$
\begin{aligned}
\hat{\boldsymbol{\theta}}_i(t) &= \int_0^t \phi_i(t)\mathbf{w}_i(\mathbf{s})\dot{\mathbf{s}}dt \\
&= \phi_i(t)\int_0^{\mathbf{s}} \mathbf{w}_i(\mathbf{s})d\mathbf{s} - \int_0^t [\frac{d}{dt}\phi_i(t)\int_0^{\mathbf{s}} \mathbf{w}_i(\mathbf{s})d\mathbf{s}]dt
\end{aligned} \tag{3.50}
$$

This is the continuous time expression of the estimator realization. In (3.50), there is no dependence on the unmeasured signals. Thus this update law can be feasible to estimate the inertia parameter instead of (3.20)

3.2.5 Numerical Example

In order to show the performance of proposed adaptive control structure, we show the simulation result of tracking a certain reference trajectory, which is the same as one in [41].

$$\boldsymbol{\sigma}_c(t) \triangleq \kappa(t)\tan(\Phi_c/4) \tag{3.51}$$

with $\kappa(t) = [0.5\cos(0.2t), 0.5\sin(0.2t), \sqrt{3}/2]^T$ and $\Phi_c = \pi$.

$$I = \begin{pmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{pmatrix}$$

ALL initial conditions are set like these.

$$\begin{aligned} \mathbf{s}(0) &= [\ 0.5 \quad 0 \quad \sqrt{3}/2\ ]^T \\ \boldsymbol{\delta\omega}(0) &= [\ 0.0 \quad 0.0 \quad 0.0\ ]^T \\ \mathbf{z}(0) &= [\ 0.4 \quad 0.005 \quad 0.7\ ]^T \\ \hat{\theta}(0) &= [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]^T \end{aligned}$$

After some simulation, we also set $A_m = -0.5 I_{3\times3}$ and $P = diag([5, 16, 16])$ (subsequently $Q = -diag([2.5, 8, 8])$). Adaptive gain matrix Γ is chosen to be,

$$\Gamma = diag([1000, 50, 50, 1000, 50, 1000])$$

Fig.(3.1), Fig.(3.2), Fig.(3.3) and Fig.(3.4) show MRPs tracking error vector, angular velocity tracking error, estimated parameters and control torques respectively. As shown in Figs. 3.1,3.2, MRP vector $\mathbf{s}$ and angular velocity error $\delta\omega$ are asymptotically stable. However, as seen in Fig.3.3, we can not get parameter

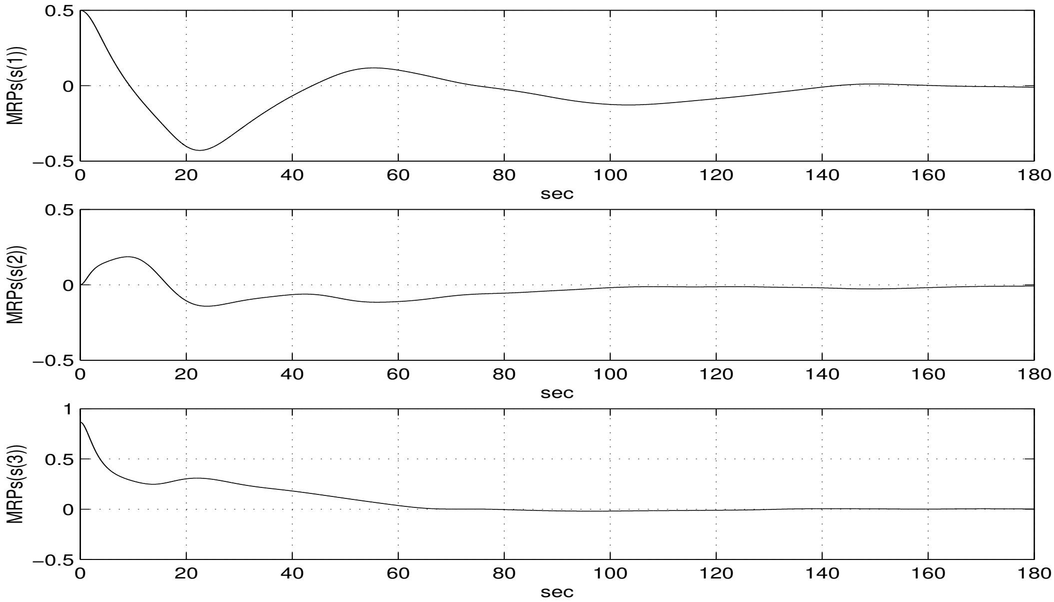

Figure 3.1: Position tracking error with respect to MRPs

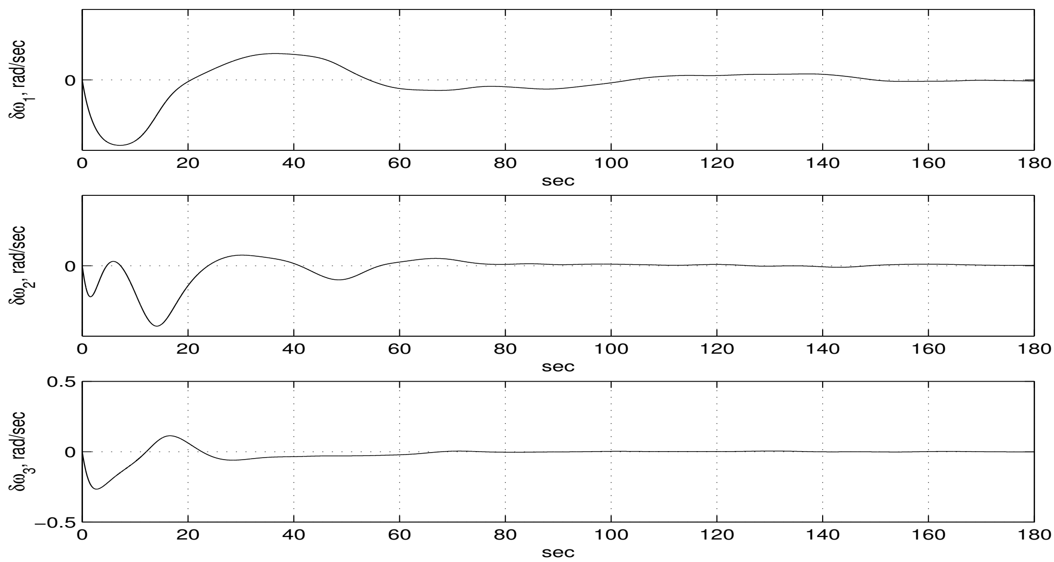

Figure 3.2: Angular velocity tracking error

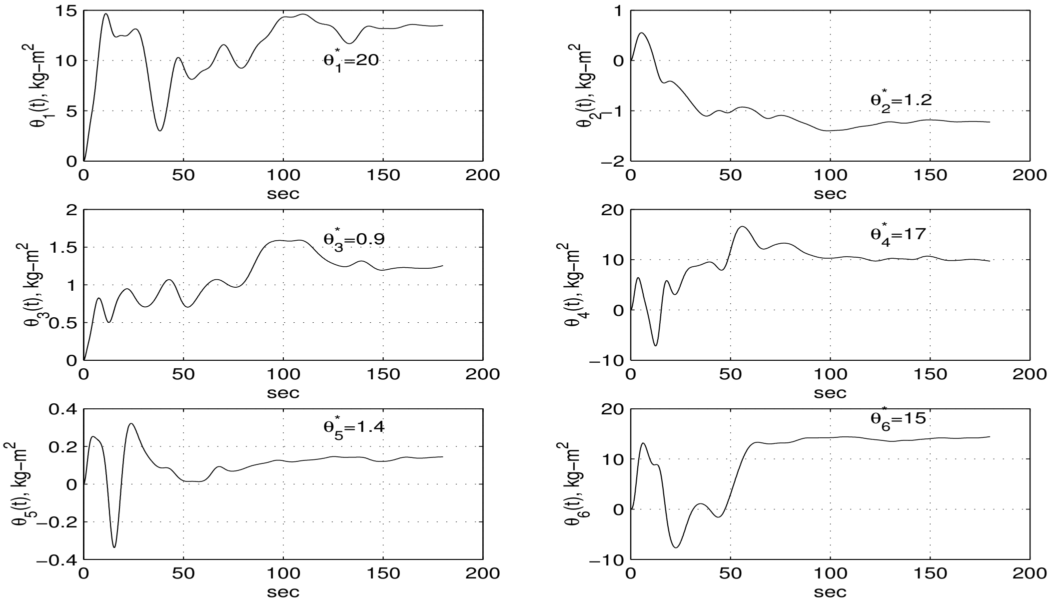

Figure 3.3: Estimated Parameters

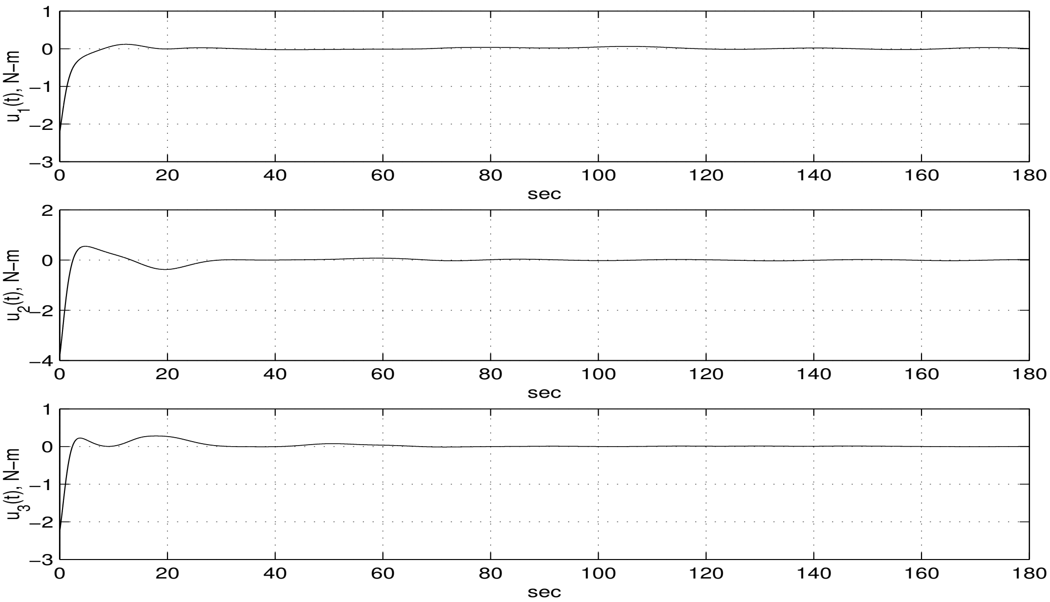

Figure 3.4: Control torques

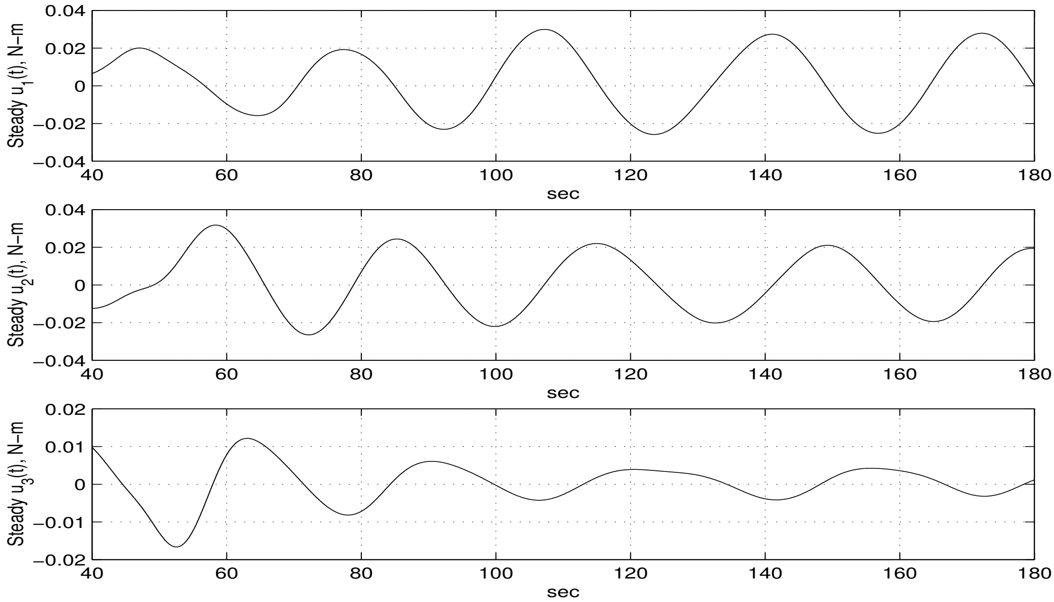

Figure 3.5: Control torques during steady states.

convergence for estimated values. This is due to violation of persistent exitation condition. Control torques in Fig.(3.4) seem to converge to origin, however, actual torques are oscillatory to keep the reference trajectory as shown in Fig.(3.5).

3.3 Using Unit Quaternions

3.3.1 Problem Formulation

As it is well known, the unit quaternion for the attitude representation of spacecraft is defined as,

$$\boldsymbol{q} = \hat{\boldsymbol{a}} \sin \frac{\Phi}{2} \tag{3.54}$$

$$q_0 = \cos \frac{\Phi}{2} \tag{3.55}$$

where,$[\boldsymbol{q}, q_0] \in \mathcal{R}^4$ denotes unit quaternion vector, $\hat{\mathbf{a}} \in \mathcal{R}^3$ and $\Phi \in \mathcal{R}$ represent the principal rotation axis and principal rotation angle respectively. Using this unit quaternion and Euler's rotational equation, the kinematic equation of the spacecraft attitude dynamics and motion of equation are defined by next.

$$\dot{\boldsymbol{q}} = \frac{1}{2}\left(S(\boldsymbol{q})\boldsymbol{\omega} + q_0\boldsymbol{\omega}\right) \tag{3.56}$$

$$\dot{q}_0 = -\frac{1}{2}\boldsymbol{q}^T\boldsymbol{\omega} \tag{3.57}$$

$$\mathbf{I}\dot{\boldsymbol{\omega}} = -S(\boldsymbol{\omega})\mathbf{I}\boldsymbol{\omega} + \mathbf{u} \tag{3.58}$$

where, $\boldsymbol{\omega} \in \mathcal{R}^3$ is angular velocity of spacecraft, $\mathbf{I} = \mathbf{I}^T \in \mathcal{R}^{3\times3}$ is the inertia matrix of a spacecraft and $\mathbf{u} \in \mathcal{R}^3$ is an external torque input. $S(\cdot)$ denotes the skew-symmetric matrix to perform the vector cross product.
unit quaternion is unit vector because of holding the following property.

$$\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1 \tag{3.59}$$

Furthermore, the rotation matrix is expressed via quaternion as follows.

$$C(\boldsymbol{q}) \triangleq (q_0^2 - \boldsymbol{q}^T\boldsymbol{q})I_{3\times3} + 2\boldsymbol{q}\boldsymbol{q}^T - 2q_0S(\boldsymbol{q}) \tag{3.60}$$

In order to create the whole system dynamics for a tracking problem, let us introduce several reference frames. **N**, **B** and **C** denote the inertial frame, a body fixed frame and the commanded motion frame respectively. Also $\hat{\mathbf{n}}$, $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ represent the unit vector triads in each frame. These three vectors are mutually related with,

$$
\begin{aligned}
\hat{\mathbf{b}} &= C(\boldsymbol{q}, q0)\hat{\mathbf{n}} \\
\hat{\mathbf{c}} &= C(\boldsymbol{q}_c, q_{c0})\hat{\mathbf{n}} \qquad (3.61) \\
\hat{\mathbf{b}} &= C(\mathbf{e}, e_0)\hat{\mathbf{c}}
\end{aligned}
$$

$C(\cdot)$ is the direction cosine matrix and $\boldsymbol{q}_c$ is the quaternion in commanded frame $\mathbf{C}$. (Namely, $\boldsymbol{q}_c$ and $\dot{\boldsymbol{q}}_c$ represent the commanded angular velocity and acceleration in $\mathbf{C}$.) It is possible to show the relationship between the direction cosine matrix and unit quaternion by the next equation.

$$
C(\boldsymbol{q}, q_{c0}) = (q_0^2 - \boldsymbol{q}^T\boldsymbol{q})I_{3\times 3} + 2\boldsymbol{q}\boldsymbol{q}^T - 2q_0 S(\boldsymbol{q}) \qquad (3.62)
$$

and $C(\mathbf{e})$ is defined by

$$
C(\mathbf{e}, e0) = C(\boldsymbol{q}, q_0)C^T(\boldsymbol{q}_c, q_{c0}) \qquad (3.63)
$$

As in the case using MRPs, we make use of this $\mathbf{e}$ to simplify attitude tracking dynamics. Defining angular velocity error $\boldsymbol{\delta\omega} \triangleq \boldsymbol{\omega} - \boldsymbol{\omega}_c^B$ and the next relationship with respect to $\boldsymbol{\omega}_c^B$

$$
\boldsymbol{\omega}_c^B = C(\mathbf{e}, e_0)\boldsymbol{\omega}_c \qquad (3.64)
$$

$$
\dot{\boldsymbol{\omega}}_c^B = C(\mathbf{e}, e_0)\dot{\boldsymbol{\omega}}_c - S(\boldsymbol{\omega})C(\mathbf{e}, e_0)\boldsymbol{\omega}_c \qquad (3.65)
$$

deliver the next open loop dynamics of a spacecraft attitude dynamics with respect to $\mathbf{e}$ and $\boldsymbol{\delta\omega}$.

$$
\begin{aligned}
\dot{\mathbf{e}} &= \frac{1}{2}\left(S(\boldsymbol{e}) + e_0 I_{3\times 3}\right)\boldsymbol{\delta\omega} && (3.66)\\
\dot{e}_0 &= -\frac{1}{2}\boldsymbol{e}^T\boldsymbol{\delta\omega} && (3.67)\\
\dot{\boldsymbol{\delta\omega}} &= -S(\boldsymbol{\omega})\mathbf{I}\boldsymbol{\omega} + \mathbf{u} - \mathbf{I}[C(\mathbf{e}, e_0)\dot{\boldsymbol{\omega}}_c - S(\boldsymbol{\omega})C(\mathbf{e}, e_0)\boldsymbol{\omega}_c] && (3.68)
\end{aligned}
$$

We show the theorem to guarantee the global stability for the system (3.66) and (3.68) with no angular velocity.

Theorem 3.3. *Let us consider the system* (3.66) *and* (3.68). *If we adopt the control torque input* $\mathbf{u}$ *as*

$$
\mathbf{u} = \frac{1}{2}\left(S(\boldsymbol{e}) - e_0 I_{3\times 3}\right)\left(P(A_m\boldsymbol{z} + \boldsymbol{e}) + \boldsymbol{e}\right) + \mathbf{I}C(\mathbf{s})\dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B)\mathbf{I}\boldsymbol{\omega}_c^B \qquad (3.69)
$$

then the closed loop system is globally asymptotically stable. Where A_m *is any Hurwitz matrix and* $P = P^T$ *is positive definite matrix which satisfies next Lyapunov equation*

$$
A_m^T P + PA_m = -Q(= Q^T > 0) \qquad (3.70)
$$

and $\mathbf{z}$ *is the filtered output, which dynamics is defined as,*

$$
\dot{\mathbf{z}} = A_m\mathbf{z} + \mathbf{e} \qquad (3.71)
$$

Remark 3.3. The overall system (3.66), (3.68) and (3.71) is PBGSS/AOF. Corresponding expression in general formulation as in chapter 3, $\boldsymbol{\psi}_{11} \rightarrow [\boldsymbol{e}, e_0]^T$,

$\boldsymbol{\psi}_{12} \rightarrow \dot{\boldsymbol{z}}$, $\boldsymbol{\psi}_2 \rightarrow \boldsymbol{\delta\omega}$. Also in this case, matrix A in chapter 3 has the next form.

$$\boldsymbol{A}_c = \begin{pmatrix} \mathbf{0}_{n\times n} & \mathbf{0}_{n\times n} \\ \mathbf{0}_{n\times n} & \boldsymbol{A}_m \end{pmatrix} \tag{3.72}$$

As shown in the sequel, We can relax the condition for A which is mentioned in chapter 3 for this case.

Proof. Let us construct the next Lyapunov function candidate.

$$V = \frac{1}{2}\boldsymbol{\delta\omega}^T \mathbf{I}\boldsymbol{\delta\omega} + \frac{1}{2}\mathbf{e}^T\mathbf{e} + \frac{1}{2}(\mathbf{e} + A_m\mathbf{z})^T P(\mathbf{e} + A_m\mathbf{z}) \tag{3.73}$$

When we take the time derivative of (3.73), it is going to be

$$\begin{aligned} \dot{V} &= \boldsymbol{\delta\omega}^T \mathbf{I}\dot{\boldsymbol{\delta\omega}} + \mathbf{e}^T\dot{\mathbf{e}} + \frac{1}{2}(\mathbf{e} + A_m\mathbf{z})^T P(\dot{\mathbf{e}} + A_m\dot{\mathbf{z}}) \\ &\quad + \frac{1}{2}(\dot{\mathbf{e}} + A_m\dot{\mathbf{z}})^T P(\mathbf{e} + A_m\mathbf{z}) \\ &= \boldsymbol{\delta\omega}^T[\mathbf{u} + \frac{1}{2}T^T(\mathbf{e}, e_0)\mathbf{e} + \frac{1}{2}T^T P(\mathbf{e} + A_m\mathbf{z}) - \mathbf{I}C(\mathbf{e}, e_0)\dot{\boldsymbol{\omega}}_c \\ &\quad - S(\boldsymbol{\omega})\mathbf{I}\boldsymbol{\omega} + \mathbf{I}S(\boldsymbol{\omega})\boldsymbol{\omega}_c^B] \\ &\quad + \frac{1}{2}(\mathbf{e} + A_m\mathbf{z})^T(A_m^T P + PA_m)(\dot{\mathbf{e}} + A_m\dot{\mathbf{z}}) \end{aligned} \tag{3.74}$$

where,

$$T(\boldsymbol{e}, e_0) \triangleq (S(\boldsymbol{e}) + e_0 I_{3\times 3}) \tag{3.75}$$

are used to simplify the arguments. By virtue of (3.70) and (3.69), (3.74) will end up with

$$
\begin{aligned}
\dot{V} &= \boldsymbol{\delta\omega}^T[S(\boldsymbol{\omega}_c^B)\mathbf{I}\boldsymbol{\omega}_c^B - S(\boldsymbol{\omega})\mathbf{I}\boldsymbol{\omega} + \mathbf{I}S(\boldsymbol{\omega})\boldsymbol{\omega}_c^B] \\
&\quad -\frac{1}{2}\dot{\mathbf{z}}^T Q\dot{\mathbf{z}} + \boldsymbol{\delta\omega}^T[\tilde{\mathbf{I}}(t)C((e), e_0)\dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B)\tilde{\mathbf{I}}(t)\boldsymbol{\omega}_c^B] \qquad (3.76)
\end{aligned}
$$

As mentioned in [40], the first part of (3.76) can be shown to be cancelled and the second term is stable term. Finally,

$$
\dot{V} = -\frac{1}{2}\dot{\mathbf{z}}^T Q\dot{\mathbf{z}} \leq 0 \qquad (3.77)
$$

Thus, $\boldsymbol{\delta\omega},\ \mathbf{e},\ \mathbf{z},\ \dot{\mathbf{z}},\ \tilde{\theta} \in \mathcal{L}_\infty$. It is reasonable to assume that $\boldsymbol{q}_c$, $\dot{\boldsymbol{q}}_c$ and its higher derivatives are all bounded. Then, by integrating (3.77), $\dot{\mathbf{z}} \in \mathcal{L}_2$ can be easily shown. From (3.71), we get

$$
\ddot{\mathbf{z}} = A_m\dot{\mathbf{z}} + \dot{\mathbf{e}} \qquad (3.78)
$$

By the previous bounds and Eq.(3.66), we can conclude $\ddot{\boldsymbol{z}} \in \mathcal{L}_\infty$, thus, $\dot{\boldsymbol{z}} \to 0$ as $t \to \infty$. Using the same process above implies $\frac{d^3}{dt^3}\boldsymbol{z} \in \mathcal{L}_\infty$. Hence, by recursive Barbalat's lemma, $\ddot{\boldsymbol{z}} \to 0$ as $t \to \infty$. Substituting these two results into (3.78), we have $\dot{\boldsymbol{e}} \to 0$ as $t \to \infty$. This automatically implies $\boldsymbol{\delta\omega} \to 0$ as $t \to \infty$ by Eq.(3.66) (Note that $T(\boldsymbol{e}, e_0)$ is a non-singular matrix.). Eq.(3.68) can be differentiated to be taken advantage of to show $\ddot{\boldsymbol{\delta\omega}} \in \mathcal{L}_\infty$. Thus, recursive Barbalat's lemma again,

$$
\boldsymbol{\delta\omega} \to 0,\ \dot{\boldsymbol{\delta\omega}} \in \mathcal{L}_\infty,\ \ddot{\boldsymbol{\delta\omega}} \in \mathcal{L}_\infty \ \Rightarrow\ \dot{\boldsymbol{\delta\omega}} \to\ 0 \ \ as \ \ t \to \infty \qquad (3.79)
$$

This final result can be used to show that$\boldsymbol{e} \to\ 0$ as $t \to \infty$ by (3.68) with (3.69). Consequently, by Eq.(3.71),

$$\boldsymbol{s} \to \ 0, \ \dot{\boldsymbol{z}} \to 0 \Rightarrow \ \boldsymbol{z} \ \to 0 \ \ as \ \ t \to \infty \tag{3.80}$$

When we summarized above, we have,

$$\lim_{t\to\infty} [\boldsymbol{\delta\omega}, \boldsymbol{e}, \boldsymbol{z}] = 0 \tag{3.81}$$

as required.

□

Also, by calculating decomposition matrix for this case as in chapter 3, we can easily extend this control input to adaptive case.

3.3.2 Adaptive Output Feedback Controller

Now we are ready to discuss about the adaptive controller for the system (3.66) (3.67)and (3.68). We will try to estimate six entries in an inertia matrix, which is,

$$\boldsymbol{\theta}^* \triangleq \begin{bmatrix} I_{11}^* & I_{12}^* & I_{13}^* & I_{22}^* & I_{23}^* & I_{33}^* \end{bmatrix}^T \tag{3.82}$$

Let us summarize the main result as a theorem.

Theorem 3.4. *Consider the system* (3.66),(3.67) *and*(3.68) *again with no information of inertia matrix. If we adopt the next control structure and adaptive update law ,*

$$\mathbf{u} = \frac{1}{2} T^T \left(P(A_m \boldsymbol{z} + \boldsymbol{e}) + \boldsymbol{e} \right) + \hat{\mathbf{I}}(t) C(\mathbf{e}, e_0) \dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B) \hat{\mathbf{I}}(t) \boldsymbol{\omega}_c^B \tag{3.83}$$

$$\hat{\boldsymbol{\theta}}(t) = \Gamma\hat{\boldsymbol{\theta}}(0) + \Gamma\sum_{i=1}^{9}\hat{\boldsymbol{\theta}}_i(t) \tag{3.84}$$

$$\hat{\boldsymbol{\theta}}_i(t) = \phi_i(t)\int_0^{\boldsymbol{\epsilon}} \mathbf{w}_i(\boldsymbol{\xi})d\boldsymbol{\xi} - \int_0^t \frac{d}{dt}\phi_i(t)\int_0^{\boldsymbol{\epsilon}} \mathbf{w}_i(\boldsymbol{\xi})d\boldsymbol{\xi}dt \tag{3.85}$$

where, $\Gamma = \Gamma^T$ *is an arbitrarily positive definite matrix and* $\mathbf{w}_i(\boldsymbol{\xi}) \in \mathcal{R}^{6\times 3}$ *and* $\phi_i(t) \in \mathcal{R}$ *are defined later.* $\boldsymbol{z}$ *is a filtered output from* (3.71)*. Also note that* $\boldsymbol{\epsilon}$ *is a combined state as* $\boldsymbol{\epsilon} = [\boldsymbol{e}, e_0]^T$*. Then, we can guarantee the globally asymptotical stability for the system.*

3.3.3 Stability and Controllability Proof

As the same as MPRs case, the feasible adaptive update law (3.84) and (3.85) is completely equivalent to,

$$\dot{\hat{\boldsymbol{\theta}}} = \Gamma\mathbf{W}_d^T(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)\boldsymbol{\delta\omega} \tag{3.86}$$

where $\mathbf{W}_d^T(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B) \in \mathcal{R}^{6\times 3}$ is described in the stability proof, then, we use this differential expression for the stability proof and show the equivalency of these two update laws in the following section.

Proof. Let us construct the next Lyapunov function candidate.

$$V = \frac{1}{2}\boldsymbol{\delta\omega}^T\mathbf{I}^*\boldsymbol{\delta\omega} + \underbrace{\frac{1}{2}\mathbf{e}^T\mathbf{e} + \frac{1}{2}(\mathbf{e} + A_m\mathbf{z})^T P(\mathbf{e} + A_m\mathbf{z})}_{\boldsymbol{G}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\Gamma^{-1}\tilde{\boldsymbol{\theta}} \tag{3.87}$$

When we take the time derivative of (3.87), it is going to be

$$
\begin{aligned}
\dot{V} &= \boldsymbol{\delta\omega}^T \mathbf{I}^* \dot{\boldsymbol{\delta\omega}} + \mathbf{e}^T \dot{\mathbf{e}} + \frac{1}{2}(\mathbf{e} + A_m \mathbf{z})^T P (\dot{\mathbf{e}} + A_m \dot{\mathbf{z}}) \\
&\quad + \frac{1}{2}(\dot{\mathbf{e}} + A_m \dot{\mathbf{z}})^T P (\mathbf{e} + A_m \mathbf{z}) + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}} \\
&= \boldsymbol{\delta\omega}^T [\mathbf{u} + \frac{1}{2} T^T(\mathbf{e}, e_0)\mathbf{e} + \frac{1}{2} T^T P (\mathbf{e} + A_m \mathbf{z}) - \mathbf{I}^* C(\mathbf{e}, e_0) \dot{\boldsymbol{\omega}}_c \\
&\quad - S(\boldsymbol{\omega}) \mathbf{I}^* \boldsymbol{\omega} + \mathbf{I}^* S(\boldsymbol{\omega}) \boldsymbol{\omega}_c^B] \\
&\quad + \frac{1}{2}(\mathbf{e} + A_m \mathbf{z})^T (A_m^T P + P A_m)(\dot{\mathbf{e}} + A_m \dot{\mathbf{z}}) + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}}
\end{aligned}
\tag{3.88}
$$

By virtue of (3.70) and (3.83), (3.88) will end up with

$$
\begin{aligned}
\dot{V} &= \boldsymbol{\delta\omega}^T [S(\boldsymbol{\omega}_c^B) \mathbf{I}^* \boldsymbol{\omega}_c^B - S(\boldsymbol{\omega}) \mathbf{I}^* \boldsymbol{\omega} + \mathbf{I}^* S(\boldsymbol{\omega}) \boldsymbol{\omega}_c^B] \\
&\quad - \frac{1}{2} \dot{\mathbf{z}}^T Q \dot{\mathbf{z}} + \boldsymbol{\delta\omega}^T [\tilde{\mathbf{I}}(t) C((e), e_0) \dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B) \tilde{\mathbf{I}}(t) \boldsymbol{\omega}_c^B] + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \dot{\tilde{\boldsymbol{\theta}}}
\end{aligned}
\tag{3.89}
$$

As mentioned in [40], the first part of (3.89) can be shown to be cancelled and the second term is stable term. The third term is linear with respect to each entry of inertia parameter and can be parameterized with using $\tilde{\boldsymbol{\theta}}(t)$ like,

$$
\boldsymbol{\delta\omega}^T [\tilde{\mathbf{I}}(t) C(\mathbf{e}, e_0) \dot{\boldsymbol{\omega}}_c + S(\boldsymbol{\omega}_c^B) \tilde{\mathbf{I}}(t) \boldsymbol{\omega}_c^B] = \boldsymbol{\delta\omega}^T \mathbf{W}_d(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B) \tilde{\boldsymbol{\theta}}(t) \tag{3.90}
$$

Noting that from the nature of (3.90), $\mathbf{W}_d(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)$ can divided into two parts like,

$$
\mathbf{W}_d(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B) = \mathbf{W}_{d1}(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c) + \mathbf{W}_{d2}(\boldsymbol{\omega}_c^B) \tag{3.91}
$$

$\mathbf{W}_{d1}$ and $\mathbf{W}_{d2}$ can be calculated as follows.

$$\mathbf{W}_{d1}(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c) = \begin{bmatrix} cw_1 & cw_2 & cw_3 & 0 & 0 & 0 \\ 0 & cw_1 & 0 & cw_2 & cw_3 & 0 \\ 0 & 0 & cw_1 & 0 & cw_2 & cw_3 \end{bmatrix} \tag{3.92}$$

where, scalar functions cw_1, cw_2 and cw_3 are defined by

$$cw_1 = c_{11}\dot{\omega}_{c1} + c_{12}\dot{\omega}_{c2} + c_{13}\dot{\omega}_{c3} \tag{3.93}$$

$$cw_2 = c_{21}\dot{\omega}_{c1} + c_{22}\dot{\omega}_{c2} + c_{23}\dot{\omega}_{c3} \tag{3.94}$$

$$cw_3 = c_{31}\dot{\omega}_{c1} + c_{32}\dot{\omega}_{c2} + c_{33}\dot{\omega}_{c3} \tag{3.95}$$

with letting c_{ij} be ith row and jth column element of direction cosine matrix. Here is the $\mathbf{W}_{d2}$.

$$\mathbf{W}_{d2}(\boldsymbol{\omega}_c^B) = \begin{bmatrix} 0 & -(\omega_{c1}^B)^2 & \omega_{c1}^B\omega_{c2}^B & -\omega_{c1}^B\omega_{c2}^B & -\omega_{c1}^B\omega_{c3}^B+\omega_{c1}^B\omega_{c2}^B & \omega_{c2}^B\omega_{c3}^B \\ (\omega_{c1}^B)^2 & \omega_{c1}^B\omega_{c2}^B & 0 & 0 & -\omega_{c2}^B\omega_{c3}^B & -(\omega_{c2}^B)^2 \\ -\omega_{c1}^B\omega_{c2}^B & -(\omega_{c2}^B)^2+\omega_{c1}^B\omega_{c3}^B & -\omega_{c2}^B\omega_{c3}^B & \omega_{c2}^B\omega_{c3}^B & (\omega_{c3}^B)^2 & 0 \end{bmatrix} \tag{3.96}$$

Then, we can construct adaptive update law using (3.90) and fourth term in (3.89).

$$\dot{\hat{\boldsymbol{\theta}}}(t) = -\dot{\tilde{\boldsymbol{\theta}}} = \Gamma \mathbf{W}_d^T(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)\boldsymbol{\delta\omega} \tag{3.97}$$

Finally, adopting (3.86), (3.89) becomes

$$\dot{V} = -\frac{1}{2}\dot{\mathbf{z}}^T Q \dot{\mathbf{z}} \leq 0 \tag{3.98}$$

Thus, $\boldsymbol{\delta\omega},\ \mathbf{e},\ \mathbf{z},\ \dot{\mathbf{z}},\ \tilde{\boldsymbol{\theta}} \in \mathcal{L}_\infty$. It is reasonable to assume that $\boldsymbol{\sigma}_c$, $\boldsymbol{\omega}_c$ and its higher derivatives are all bounded. Then, by (3.86), $\tilde{\boldsymbol{\theta}}(t) \in \mathcal{L}_\infty$. By integrating (3.98), $\dot{\mathbf{z}} \in \mathcal{L}_2$ can be easily shown. Finally, by applying Barlalat's lemma recursively from $\dot{\mathbf{z}}$ to $\ddot{\mathbf{z}}$, $\dot{\mathbf{e}}$, $\dot{\boldsymbol{\delta\omega}}$, we can show that $\mathbf{e} \to 0$ and $\boldsymbol{\delta\omega} \to 0$. $\mathbf{z} \to 0$ follows by using (3.71).

3.3.4 Proof of Equivalence for Update Laws

The adaptive update law (3.86) can not be implemented directly because of the lack of information $\boldsymbol{\delta\omega}$. Here, we try to create practically implementable update law. From the (3.66), we can describe $\boldsymbol{\delta\omega}$ as a function of $\mathbf{e}$ and $\dot{\mathbf{e}}$.

$$\boldsymbol{\delta\omega} = 2T^{-1}(\mathbf{e}, e_0)\dot{\mathbf{e}} \tag{3.99}$$

Substituting (3.99) into (3.86), we get a new differential update law.

$$\dot{\hat{\boldsymbol{\theta}}}(t) = 2\Gamma \mathbf{W}_d^T(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)T^{-1}(\mathbf{e}, e_0)\dot{\mathbf{e}} \tag{3.100}$$

Its integrated expression is the same as,

$$\hat{\boldsymbol{\theta}}(t) = \Gamma\hat{\boldsymbol{\theta}}(0) + 2\Gamma \int_0^t \mathbf{W}_d^T(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c^B)T^{-1}(\mathbf{e}, e_0)\dot{\mathbf{e}}\mathrm{d}\tau \tag{3.101}$$

(3.101) is still dependent on $\dot{\mathbf{e}}$. However, (3.101) can be converted to executable form by the next technique. Before introducing a certain technique, we change the notation of $\mathbf{W}_d$. As described in (3.64), $\boldsymbol{\omega}_c^B$ is a function of $\mathbf{e}$ and $\boldsymbol{\omega}_c$. Then, $\mathbf{W}_d$ is a function of $\mathbf{e}$, $\dot{\boldsymbol{\omega}}_c$ and $\boldsymbol{\omega}_c^B$, i.e,

$$\mathbf{W}_d = \mathbf{W}_d(\mathbf{e}, e_0, \dot{\boldsymbol{\omega}}_c, \boldsymbol{\omega}_c) \tag{3.102}$$

Furthermore,this new $\mathbf{W}_d$ can be divided into nine parts, i.e,

$$\mathbf{W}_d^T(\mathbf{e}, e_0, \omega_c, \dot{\omega}_c)T^{-1}(\mathbf{e}, e_0) = \sum_{i=1}^{9} \beta_{di} \tag{3.103}$$

where, each β_{di} is described as,

$$\beta_{di} = \begin{cases} \dot{\omega}_{ci}\mathbf{w}_i(\mathbf{e}, e_0) & for \quad i = 1, 2, 3 \\ \omega_{ci}^2\mathbf{w}_i(\mathbf{e}, e_0) & for \quad i = 4, 5, 6 \\ \omega_{c1}\omega_{c2}\mathbf{w}_7(\mathbf{e}, e_0) & for \quad i = 7 \\ \omega_{c1}\omega_{c3}\mathbf{w}_8(\mathbf{e}, e_0) & for \quad i = 8 \\ \omega_{c2}\omega_{c3}\mathbf{w}_9(\mathbf{e}, e_0) & for \quad i = 9 \end{cases} \tag{3.104}$$

Naturally, each $\mathbf{w}_i(\mathbf{e}, e_0) \in \mathcal{R}^{6\times3}$ is a function only by $\mathbf{e}$ and e_0. The exact expression of each $\mathbf{w}_i$ is follow.

For $i = 1, 2, 3$, $\mathbf{w}_i(\mathbf{e})$ have the next form.

$$\mathbf{w}_i^T(\mathbf{e}, e_0) = T^{-T}(\mathbf{e}, e_0)\gamma_i \tag{3.105}$$

where, each $\gamma_i \in \mathcal{R}^{3\times6}$ is defined as follows with letting $c_{ij}(\mathbf{e})$ be each entry of direction cosine matrix with respect to $\mathbf{e}$.

$$\gamma_1 = \begin{bmatrix} c_{11}(\boldsymbol{\epsilon}) & c_{21}(\boldsymbol{\epsilon}) & c_{31}(\boldsymbol{\epsilon}) & 0 & 0 & 0 \\ 0 & c_{11}(\boldsymbol{\epsilon}) & 0 & c_{21}(\boldsymbol{\epsilon}) & c_{31}(\boldsymbol{\epsilon}) & 0 \\ 0 & 0 & c_{11}(\boldsymbol{\epsilon}) & 0 & c_{21}(\boldsymbol{\epsilon}) & c_{31}(\boldsymbol{\epsilon}) \end{bmatrix} \tag{3.106}$$

$$\gamma_2 = \begin{bmatrix} c_{12}(\boldsymbol{\epsilon}) & c_{22}(\boldsymbol{\epsilon}) & c_{32}(\boldsymbol{\epsilon}) & 0 & 0 & 0 \\ 0 & c_{12}(\boldsymbol{\epsilon}) & 0 & c_{22}(\boldsymbol{\epsilon}) & c_{32}(\boldsymbol{\epsilon}) & 0 \\ 0 & 0 & c_{12}(\boldsymbol{\epsilon}) & 0 & c_{22}(\boldsymbol{\epsilon}) & c_{32}(\boldsymbol{\epsilon}) \end{bmatrix} \tag{3.107}$$

$$\gamma_3 = \begin{bmatrix} c_{13}(\boldsymbol{\epsilon}) & c_{23}(\boldsymbol{\epsilon}) & c_{33}(\boldsymbol{\epsilon}) & 0 & 0 & 0 \\ 0 & c_{13}(\boldsymbol{\epsilon}) & 0 & c_{23}(\boldsymbol{\epsilon}) & c_{33}(\boldsymbol{\epsilon}) & 0 \\ 0 & 0 & c_{13}(\boldsymbol{\epsilon}) & 0 & c_{23}(\boldsymbol{\epsilon}) & c_{33}(\boldsymbol{\epsilon}) \end{bmatrix} \tag{3.108}$$

where, a combined state $\boldsymbol{\epsilon}$ is used for simplicity. For i=4 to 9, $\mathbf{w}_i(\boldsymbol{\epsilon})$ have the same form as (3.105). However, the structure of each γ_i is different from those of the cases $i = 1, 2, 3$. Here are the exact definitions of them.

$$\gamma_i = \begin{bmatrix} 0 & -\gamma_{i1} & \gamma_{i2} & -\gamma_{i2} & \gamma_{i3} & \gamma_{i4} \\ \gamma_{i1} & \gamma_{i2} & 0 & 0 & -\gamma_{i4} & -\gamma_{i5} \\ -\gamma_{i2} & -\gamma_{i3} & -\gamma_{i4} & \gamma_{i4} & \gamma_{i5} & 0 \end{bmatrix} \tag{3.109}$$

where, each γ_{i1} to γ_{i5} is defined as follows.

$$\begin{aligned} \gamma_{41} &= c_{11}^2(\boldsymbol{\epsilon}) & \gamma_{51} &= c_{12}^2(\boldsymbol{\epsilon}) \\ \gamma_{42} &= c_{11}(\boldsymbol{\epsilon})c_{12}(\boldsymbol{\epsilon}) & \gamma_{52} &= c_{12}(\boldsymbol{\epsilon})c_{22}(\boldsymbol{\epsilon}) \\ \gamma_{43} &= c_{21}^2(\boldsymbol{\epsilon}) - c_{11}(\boldsymbol{\epsilon})c_{31}(\mathbf{s}) & \gamma_{53} &= c_{22}^2(\boldsymbol{\epsilon}) - c_{12}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon}) \\ \gamma_{44} &= c_{21}(\boldsymbol{\epsilon})c_{31}(\boldsymbol{\epsilon}) & \gamma_{54} &= c_{22}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon}) \\ \gamma_{45} &= c_{31}^2(\boldsymbol{\epsilon}) & \gamma_{55} &= c_{32}^2(\boldsymbol{\epsilon}) \end{aligned}$$

$$\begin{aligned} \gamma_{61} &= c_{13}^2(\boldsymbol{\epsilon}) \\ \gamma_{62} &= c_{13}(\boldsymbol{\epsilon})c_{23}(\boldsymbol{\epsilon}) \\ \gamma_{63} &= c_{23}^2(\boldsymbol{\epsilon}) - c_{13}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon}) \\ \gamma_{64} &= c_{23}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon}) \\ \gamma_{65} &= c_{33}^2(\boldsymbol{\epsilon}) \end{aligned}$$

$$
\begin{aligned}
\gamma_{71} &= 2c_{11}(\boldsymbol{\epsilon})c_{12}(\boldsymbol{\epsilon}) \\
\gamma_{72} &= c_{11}(\boldsymbol{\epsilon})c_{22}(\boldsymbol{\epsilon}) + c_{12}(\boldsymbol{\epsilon})c_{21}(\boldsymbol{\epsilon}) \\
\gamma_{73} &= 2c_{21}(\boldsymbol{\epsilon})c_{22}(\boldsymbol{\epsilon}) - (c_{11}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon}) + c_{12}(\boldsymbol{\epsilon})c_{31}(\boldsymbol{\epsilon})) \\
\gamma_{74} &= c_{21}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon}) + c_{22}(\boldsymbol{\epsilon})c_{31}(\boldsymbol{\epsilon}) \\
\gamma_{75} &= 2c_{31}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon})
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{81} &= 2c_{11}(\boldsymbol{\epsilon})c_{13}(\boldsymbol{\epsilon}) \\
\gamma_{82} &= c_{11}(\boldsymbol{\epsilon})c_{23}(\boldsymbol{\epsilon}) + c_{13}(\boldsymbol{\epsilon})c_{21}(\boldsymbol{\epsilon}) \\
\gamma_{83} &= 2c_{21}(\boldsymbol{\epsilon})c_{23}(\boldsymbol{\epsilon}) - (c_{11}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon}) + c_{13}(\boldsymbol{\epsilon})c_{31}(\boldsymbol{\epsilon})) \\
\gamma_{84} &= c_{21}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon}) + c_{23}(\boldsymbol{\epsilon})c_{31}(\boldsymbol{\epsilon}) \\
\gamma_{85} &= 2c_{31}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon})
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{91} &= 2c_{12}(\boldsymbol{\epsilon})c_{13}(\boldsymbol{\epsilon}) \\
\gamma_{92} &= c_{12}(\boldsymbol{\epsilon})c_{23}(\boldsymbol{\epsilon}) + c_{13}(\boldsymbol{\epsilon})c_{21}(\boldsymbol{\epsilon}) \\
\gamma_{93} &= 2c_{22}(\boldsymbol{\epsilon})c_{23}(\boldsymbol{\epsilon}) - (c_{12}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon}) + c_{13}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon})) \\
\gamma_{94} &= c_{22}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon}) + c_{23}(\boldsymbol{\epsilon})c_{32}(\boldsymbol{\epsilon}) \\
\gamma_{95} &= 2c_{32}(\boldsymbol{\epsilon})c_{33}(\boldsymbol{\epsilon})
\end{aligned}
$$

Using these $\mathbf{w}_i$, $\hat{\theta}$ can be implemented as follows.

$$\hat{\theta}(t) = \Gamma\hat{\theta}(0) + \Gamma\sum_{i=1}^{9}\hat{\theta}_i(t) \tag{3.110}$$

Each $\hat{\boldsymbol{\theta}}_i(t)$ can be implemented by the next technique.

Here is a property of integral.

$$\frac{d}{dt}\int_0^t \phi_i(\tau)\int_0^{\boldsymbol{\epsilon}} \mathbf{w}_i(\boldsymbol{\xi})d\boldsymbol{\xi}d\tau = \int_0^t \frac{d}{d\tau}\phi_i(\tau)\int_0^{\boldsymbol{\epsilon}} \mathbf{w}_i(\boldsymbol{\xi})d\boldsymbol{\xi}d\tau + \int_0^t \phi_i(\tau)\mathbf{w}_i(\mathbf{e}, e_0)\dot{\mathbf{e}}d\tau \tag{3.111}$$

where $\phi_i(t)$ is a coefficient scalar function of each $\mathbf{w}_i$.

By (3.111), $\hat{\theta}_i(t)$ can be realized by the next expression.

$$\begin{aligned} \hat{\boldsymbol{\theta}}_i(t) &= \int_0^t \phi_i(\tau)\mathbf{w}_i(\mathbf{e}, e_0)\dot{\mathbf{e}}d\tau \\ &= \phi_i(t)\int_0^{\boldsymbol{\epsilon}} \mathbf{w}_i(\boldsymbol{\xi})d\boldsymbol{\xi} - \int_0^t [\frac{d}{d\tau}\phi_i(\tau)\int_0^{\boldsymbol{\epsilon}} \mathbf{w}_i(\boldsymbol{\xi})d\boldsymbol{\xi}]d\tau \end{aligned} \tag{3.112}$$

This is the continuous time expression of the estimator realization. In (3.112), there is no dependence on the unmeasured signals. Thus this update law can be feasible to estimate the inertia parameter instead of (3.86)

3.3.5 Numerical Example

In order to show the performance of proposed adaptive control structure, we show the simulation result of tracking a certain reference trajectory, which is the same as one in MRPs case.

$$\boldsymbol{q}_c(t) \triangleq \kappa(t)\tan(\Phi_c/4) \tag{3.113}$$

with $\kappa(t) = [0.5\cos(0.2t), 0.5\sin(0.2t), \sqrt{3}/2]^T$ and $\Phi_c = \pi$.

$$I = \begin{pmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{pmatrix}$$

ALL initial conditions are set like these.

$$\begin{aligned} \mathbf{e}(0) &= [\ 0.7906 \quad 0 \quad 0.6124\ ]^T \\ e_0(0) &= 0 \\ \boldsymbol{\delta\omega}(0) &= [\ 0.0 \quad 0.0 \quad 0.0\ ]^T \\ \mathbf{z}(0) &= [\ 0.4 \quad 0.005 \quad 0.7\ ]^T \\ \hat{\theta}(0) &= [\ 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\ ]^T \end{aligned}$$

After some simulation, we also set $A_m = -0.5I_{3\times 3}$ and $P = diag([5, 16, 16])$ (subsequently $Q = -diag([2.5, 8, 8])$). Adaptive gain matrix Γ is chosen to be,

$$\Gamma = diag([1000, 50, 50, 1000, 50, 1000])$$

Fig.3.6, Fig.3.7, Fig.3.8 and Fig.3.9 show quaternions tracking error vector, angular velocity tracking error, estimated parameters and control torques respectively. As shown in Figs.3.6, 3.7, quaternions vector $\mathbf{e}$ and angular velocity error $\boldsymbol{\delta\omega}$ are asymptotically stable. However, as seen in Fig.3.8, we can not get parameter convergence for estimated values either. This is also due to violation of persistent excitation condition. Control torques in Fig.3.9 seem to converge to origin, however, actual torques are oscillatory to keep the reference trajectory as shown in Fig.3.10.

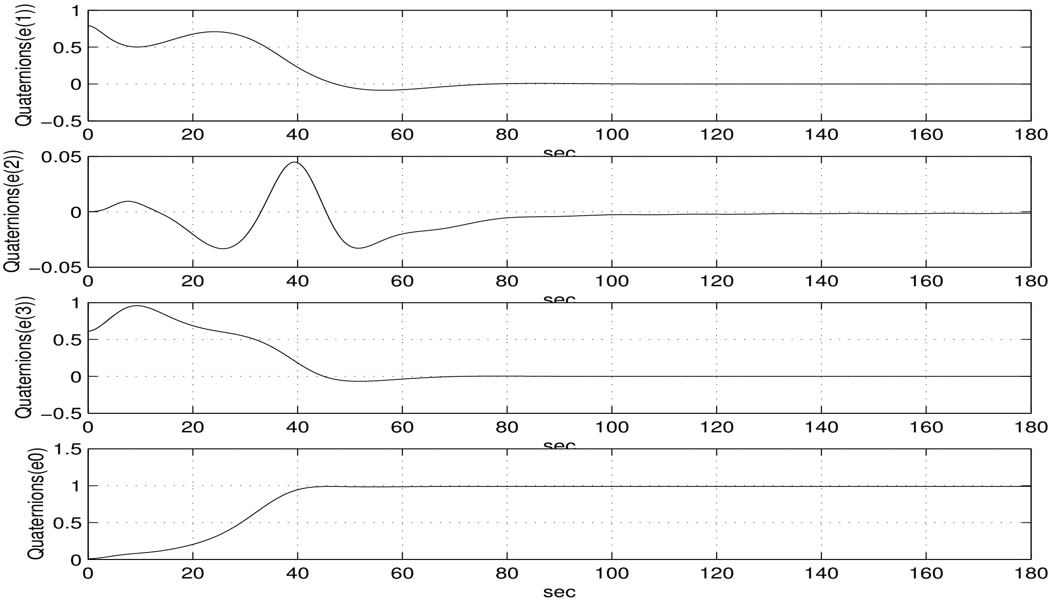

Figure 3.6: Position tracking error with respect to MRPs

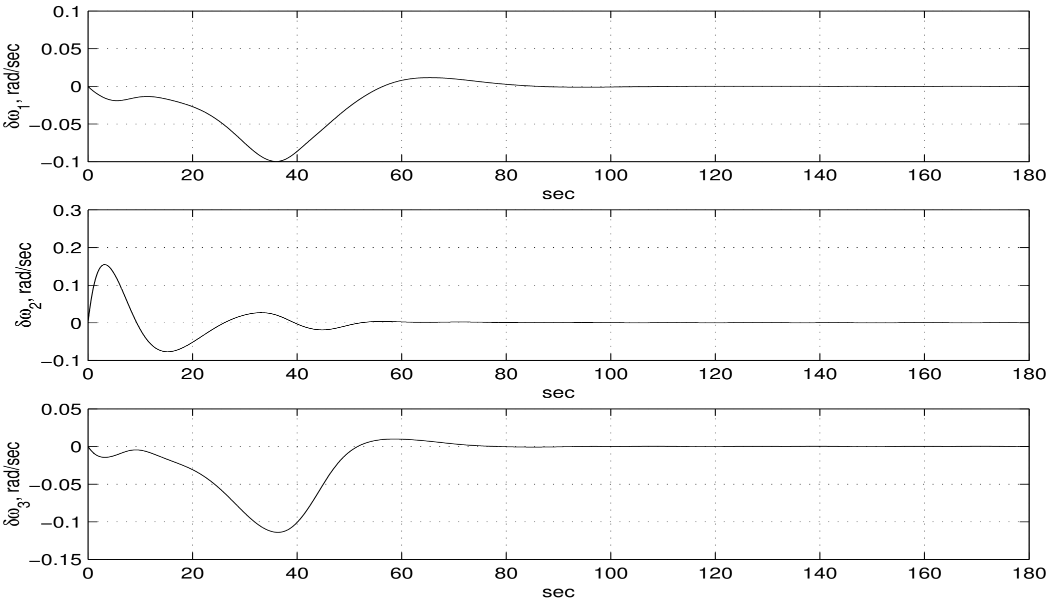

Figure 3.7: Angular velocity tracking error

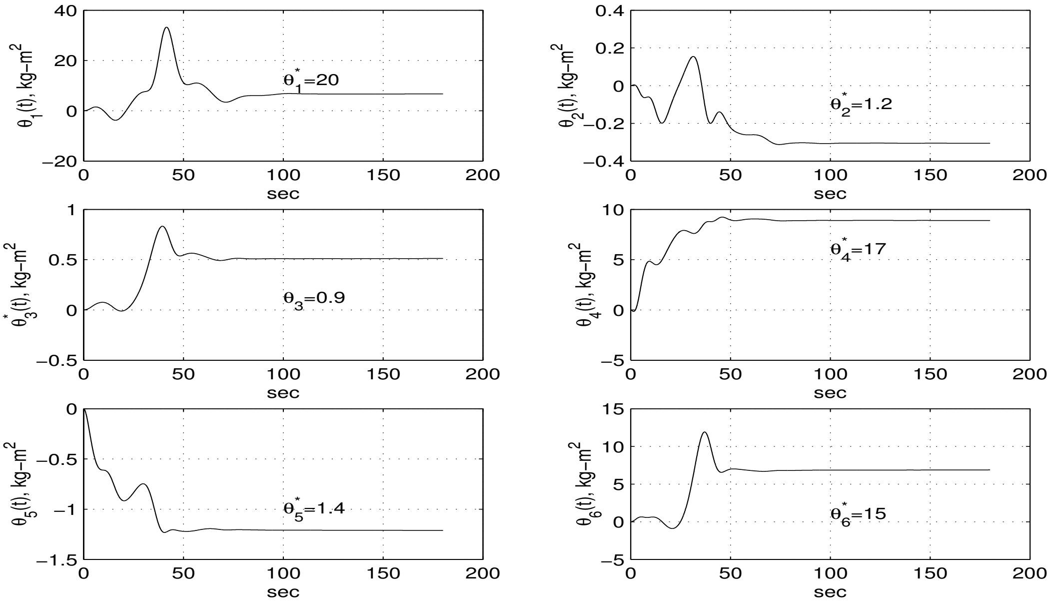

Figure 3.8: Estimated Parameters

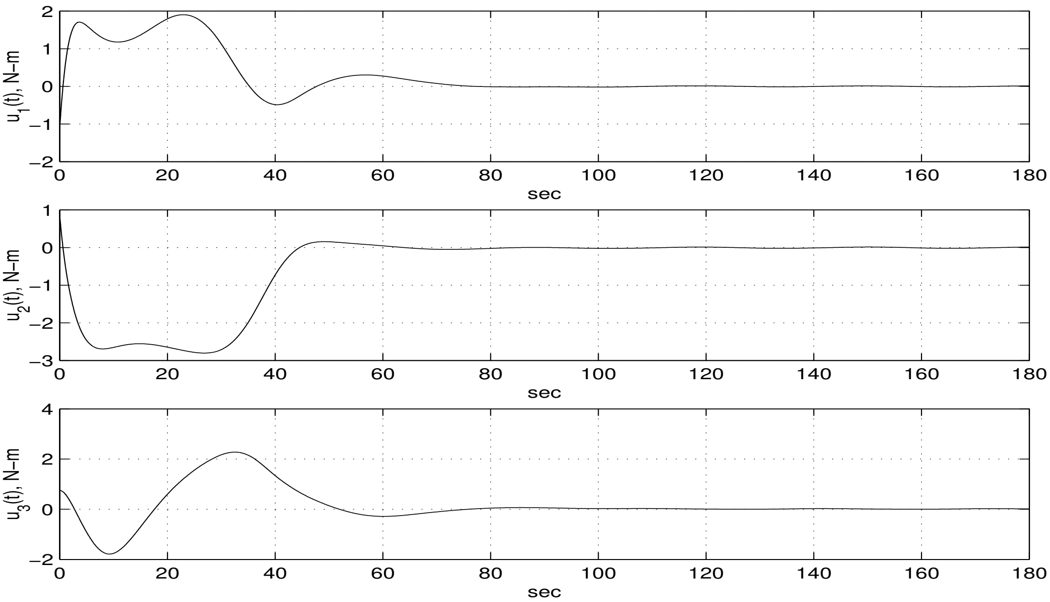

Figure 3.9: Control torques

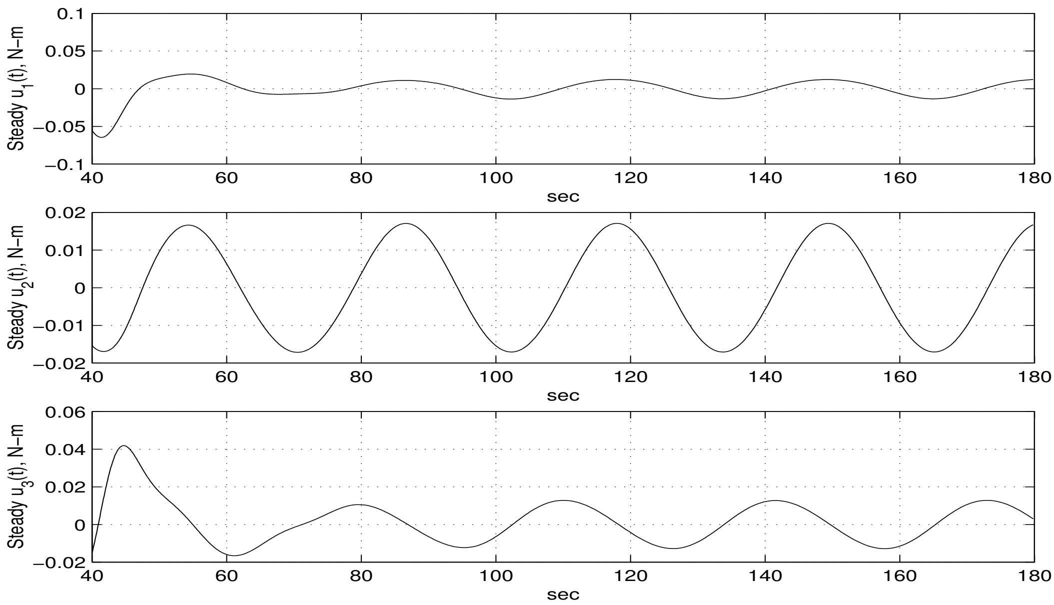

Figure 3.10: Control torques during steady states.

3.4 Using Other Kinematics

As introduced in [27], there are also several kinematic equations for attitude representation of spacecraft like Euler-Rodrigues (Gibbs) Parameters and they are also useful to construct these types of controllers. However, these representations have singular points between 0 *deg* and 360 *deg*. Thus, if we need to use these representation, we have to consider these singular points carefully and required to define a "shadow" sets to transform the singular point to actual attitude.

Chapter 4

Robot Arm Trajectory Tracking Problem

4.1 Introduction

As well known, a dynamics of robot manipulator is one of a typical Euler-Lagrange system. Euler-Lagrange method is frequently used to derive motion of equations in many engineering fields and derived dynamics are frequently expressed as nonlinear differential equations with respect to their state variables. For the full state feedback control case of this dynamics, Many useful solutions have already existed and they are applied to practical robot like industrial robot and manipulators in space and deep ocean ([37], [50]). However, the history of full state adaptive case has begun with the research of Sadegh and Horowitz [43]. They have successfully formulated how to solve the full state adaptive control problem for general robot manipulator in regulation case and this is extended to many types of controllers (e.g. [45]) even for the tracking problem. However, in the output feedback case (only link position measurements are available.), there is no general solutions for the tracking problem without any extra assumptions, although several attempts were tried to solve this problem. For examples, using quaternions was attempted by Yuan [34], Funda [31], Chou [30] and Jain [2]. Kosuge [37] and Wen [35] tried to use the torque sensors in stead of velocity sensors. Attempt to decouple each link motion were performed by Arai [24], Fardanesh [9] and Liu [22] and this technique is known to be partially useful to adaptive case when we only require semi-global stability by the results of Liu [39], Zhu [55] and Shishkin [53]. These whole stories tells us that we need a certain type of observers or filters to estimate unmeasured link velocity signals. For this problem, there exists lots of remarkable approaches in deterministic cases (no uncertainty presents.). For example, in the robotics fields, Nicosia and Tomei [52] try to construct a model based observer to estimate unmeasured joint angular velocity and guarantee semi-global stability. Non-model based observer was also formulated by Wit [11]. In all these great results, Nijimeijer and Berghuis' results [25] were passivity based and gave much influences on later researches like [26], [36], [10], [38]. When the purpose of control is just a regulation problem, Ortega et al. [49] use (low-pass)filtered variable of output signals instead of velocity signals and guarantee semi-global stability. Also, Laib [4] succeeds in guaranteeing semi-global stability with inertia uncertainty and actuator saturation recently.

Tracking case of this problem is also solved by many researchers. Loria [5] treats a robot arm which has only one joint with using special type of filtered variables from output and also Loria and Nijimeijer [7] take advantage of the same type of filter and guarantee semi-global stability for higher degree of freedom case. Here, we should remark the fact that most of these approaches use not only the generalized inertia matrix and centrifugal matrix but also their inverse matrix, normed value, upper and lower bounds and so on. This causes the difficulty to extend these results to adaptive case directly. Only under several a priori knowledge, Zhang et al. [19], [18] and Pagilla and Tomizuka [44] propose a certain type of controller for semi-global stability, however, they required following a priori knowledge.

1. Upper and/or lower bounds of generalized inertia matrix

2. Upper bound of Corioris and centrifugal matrix with respect to the norm of angular velocity vector

3. Upper bound of reference trajectory

4. In addition to above, Upper bounds on generalized inertia matrix and Corioris and centrifugal term must be expressed by certain type of function with respect to measurable states.

4.1.1 Details of History in Global Stability

The purpose of this dissertation is not only to treat uncertainties of the systems, but also to guarantee global stability. As shown in the above historical developments, there exists only a few solutions for global stability.
In the case of regulation problem, globally stability is achieved by Kelly et al. [47] using (high-pass) filtered variables of output information. On the other hand, the first solution which treat global stability for tracking case is Loria's [5]. The author considers an Euler Lagrange system (robot arm) whose degree of freedom is just one and guarantee the global stability. In this approach, the author suggest a way to construct a controller and an observer and simultaneously the lower

bounds on the controller and filter gains as functions of system parameters and reference trajectory norms. Separately from above solution, Burkov [29] showed that it is possible to create a controller which is based on a linear observer for tracking a trajectory form any initial condition. However, explicit (lower) bounds on gains of controller and observer were not presented by the author and only existence of the gains were shown.

After the results, Lefeber [1] introduced a new approach to switch two control laws, one of which is a set point control law and the other of which is a local output feedback control law. Naturally, this control law is discontinuous and the switching time is dependent on the bounds on unmeasured velocity signals. Thus, this approach can not guarantee global stability in the true sense.

Recently, Besancon [20] gave a very simple alternative methods compared with Loria's [5]. This controller is one of PD type controllers and designed on nonlinear coordination change. This controller has a remarkable property such that the controller grows up its gains at most linearly in the state variables. Thus, in the practical sense, this is much superior to the results of Loria's [5].

In general n-degree of freedom case, however, there is no established method in order to guarantee global stability with no a priori knowledge. Only possibility of existence of output feedback controller is proposed by Besancon [21] checking the existence of suitable coordinate transformation of original states. As a state of the art results, Loria [6] shows the explicit existence of such a (nonlinear) state transformation. However, an adaptive case of this problem is still an open problem. Zhang [18] show the global stability in the sense that tracking errors approaches to origin asymptotically in adaptive cases. This approach uses some special type of observer to estimate unmeasured velocity signals and the initial condition of this observer can not be taken arbitrarily. Thus, in the true sense [6], this is not a solution of global stability. However, in the practical sense, it is adequate to guarantee global stability for error signals from the practical point of view.

4.2 Problem Formulation

As well known, The original dynamics of an n degree of freedom robot arm is formulated by Lagragian, which is

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{x_2} \\ \mathbf{M}(\mathbf{x_1})\dot{\mathbf{x}}_2 + \mathbf{C}(\mathbf{x_1}, \mathbf{x_2})\mathbf{x_2} + \mathbf{g}(\mathbf{x_1}) + \boldsymbol{F}_d\mathbf{x_2} &= \boldsymbol{\tau} \end{aligned} \tag{4.2}$$

where $\mathbf{x_1} \in \mathcal{R}^n, \mathbf{x_2} \in \mathcal{R}^n$ are the generalized position and velocity vectors respectively, $\mathbf{M} \in \mathcal{R}^{n\times n}$ is the generalized inertia matrix, $\mathbf{C} \in \mathcal{R}^{n\times n}$ is the matrix due to Coriolis and centrifugal forces, $\mathbf{g} \in \mathcal{R}^n$ is the gravitational force term and $\boldsymbol{\tau} \in \mathcal{R}^n$ is the each control torque in each joint. $\boldsymbol{F}_d\mathbf{x_2}$ is a viscus friction term, thus, $\boldsymbol{F}_d$ is a known, symmetric positive definite matrix. This system has the next properties.

Property 4.1. The generalized inertia matrix is positive definite and upper and lower bounded by some positive constants σ_m and σ_M, i.e,

$$\sigma_m \boldsymbol{I} \leq \mathbf{M}(\mathbf{x_1}) \leq \sigma_M \boldsymbol{I} \tag{4.3}$$

Property 4.2. $\dot{\mathbf{M}}(\mathbf{x_1}) - 2\mathbf{C}(\mathbf{x_1}, \mathbf{x_2})$ is the skew symmetric.

Property 4.3. The matrix $\mathbf{C}(\mathbf{x_1}, \mathbf{x_2})$ caused by Coriolis and centrifugal force is bounded by the norm of its second argument and able to interchange with any vector when it is combined by some vectors, i.e,

$$\|\mathbf{C}(\mathbf{x_1}, \mathbf{x_2})\| \leq c_m \|\mathbf{x_2}\| \tag{4.4}$$

$$\mathbf{C}(\mathbf{x_1}, \mathbf{x_2})\boldsymbol{y} = \mathbf{C}(\mathbf{x_1}, \boldsymbol{y})\mathbf{x_2} \tag{4.5}$$

Our purpose is to formulate the passivity based controller which guarantee the global stability for both deterministic case and adaptive case of tracking problem. It is natural to assume that there exists a certain bounds on reference trajectory signals. Especially, we need the bound for the velocity reference trajectory, i.e,

Property 4.4.

$$\|\mathbf{x_{2d}}\| \leq v_M \tag{4.6}$$

4.3 New Tracking Dynamics of A Robot Arm

First of all, we should note that direct parametrization is not useful to construct tracking dynamics in robotics case. The dynamics through direct parametrization of tracking error can be described as,

$$\begin{aligned} \dot{\boldsymbol{e}}_1 &= \boldsymbol{e}_2 \\ \mathbf{M}(\mathbf{x_1})\ddot{e}_2 &= \boldsymbol{\tau} - \mathbf{C}(\mathbf{x_1}, \boldsymbol{e}_2 + \mathbf{x_{2d}})(\boldsymbol{e}_2 + \mathbf{x_{2d}}) - \mathbf{g}(\mathbf{x_1}) - \boldsymbol{F}_d\mathbf{x_2} \end{aligned}$$

where, $\boldsymbol{e}_1 \triangleq \mathbf{x_1} - \mathbf{x_{1d}}$ and $\boldsymbol{e}_2 \triangleq \mathbf{x_2} - \mathbf{x_{2d}}$.

When we choose the Lyapunov function as,

$$V = \frac{1}{2}\boldsymbol{e}_2^T \mathbf{M}(\mathbf{x_1})\boldsymbol{e}_2 + \frac{1}{2}\boldsymbol{e}_1^T \boldsymbol{e}_1$$

then $\dot{V}$ includes $\boldsymbol{e}_2^T \mathbf{C}(\mathbf{x_1}, \mathbf{x_{2d}})\boldsymbol{e}_2$, which is quadratic and neither positive or negative definite. We cannot handle this term within PBGSS/OF or/AOF framework. Thus, as in the previous literature [19], we take advantage of a filtered error-like variables $\boldsymbol{\eta} \in \mathcal{R}^n$, which has the next dynamics.

$$\boldsymbol{\eta} = \dot{\boldsymbol{e}} + \mathrm{Tanh}(\boldsymbol{e}) + \mathrm{Tanh}(\boldsymbol{e}_f) \tag{4.8}$$

where, $\boldsymbol{e}$ is a position tracking error defined by $\boldsymbol{e} \triangleq \mathbf{x_{1d}} - \mathbf{x_1}$, Tanh($\cdot$)is defined by next.

$$\mathrm{Tanh}(\boldsymbol{\xi}) = [\tanh(\xi_1), \tanh(\xi_2), \cdots, \tanh(\xi_n)]^T \tag{4.9}$$

Also, $\boldsymbol{e}_f \in \mathcal{R}^n$ is an auxiliary filter variable which is defined to have the next dynamics. ($\boldsymbol{e}_f(0) = 0$)

$$\dot{\boldsymbol{e}}_f = -\mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e}) - k\mathrm{Cosh}^2(\boldsymbol{e}_f)\boldsymbol{\eta} \tag{4.10}$$

A positive scalar constant k is to be determined in the stability proof later and a matrix function Cosh is defined by the following expression.

$$\mathrm{Cosh}(\boldsymbol{\xi}) = diag[\tanh(\xi_1), \tanh(\xi_2), \cdots, \tanh(\xi_n)] \tag{4.11}$$

We have to note that we can not use $\boldsymbol{\eta}$ in an actual control torque due to the presence of $\dot{\boldsymbol{e}}$ in (4.8), however, Tanh($\boldsymbol{e}_f$) can be used by [19]. When we define $y_i = \tanh e_i$, this y_i can be calculated by the following differential equation.

$$
\begin{aligned}
\dot{p}_i &= -(1-(p_i-ke_i)^2)(p_i-ke_i-\tanh(e_i)) \\
&\quad -k(\tanh(e_i)+p_i-ke_i), \quad p_i(0)=ke_i(0) \\
y_i &= p_i-ke_i
\end{aligned} \tag{4.12}
$$

Thus, we use the expressions (4.8) and (4.10) only in stability proof.

Now we are ready to reconstruct the system dynamics with respect to $\boldsymbol{\eta}$. Taking the time derivative of (4.8), multiplying both side of this equation $\mathbf{M}(\mathbf{x_1})$ and substituting (4.2) yields,

$$
\begin{aligned}
\mathbf{M}(\mathbf{x_1})\dot{\boldsymbol{\eta}} &= \mathbf{M}(\mathbf{x_1})\dot{\mathrm{x}}_{2d}+\mathbf{C}(\mathbf{x_1},\mathbf{x_2})\mathbf{x_2}+\mathbf{g}(\mathbf{x_1})+\boldsymbol{F}_d\mathbf{x_2}-\boldsymbol{\tau} \\
&\quad +\mathbf{M}(\mathbf{x_1})\mathrm{Cosh}^{-2}(\boldsymbol{e})\dot{\boldsymbol{e}}+\mathbf{M}(\mathbf{x_1})\mathrm{Cosh}^{-2}(\boldsymbol{e}_f)\dot{\boldsymbol{e}}_f
\end{aligned} \tag{4.13}
$$

Here, we can utilize (4.8) and (4.10) again and summarize (4.13).

$$
\mathbf{M}(\mathbf{x_1})\dot{\boldsymbol{\eta}} = -C(\mathbf{x_1},\mathbf{x_2})\boldsymbol{\eta}-k\mathbf{M}(\mathbf{x_1})\boldsymbol{\eta}+\boldsymbol{\chi}-\boldsymbol{\tau} \tag{4.14}
$$

where, $\boldsymbol{\chi}\in\mathcal{R}^n$ are defined as follows.

$$
\begin{aligned}
\boldsymbol{\chi} \;=\;& \mathbf{M}(\mathbf{x_1})\mathrm{Cosh}^{-2}(\boldsymbol{e})(\boldsymbol{\eta} - \mathrm{Tanh}(\boldsymbol{e}_f) - \mathrm{Tanh}(\boldsymbol{e})) \\
& + \mathbf{M}(\mathbf{x_1})\mathrm{Cosh}^{-2}(\boldsymbol{e}_f)(-\mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e})) \\
& + \mathbf{C}(\mathbf{x_1}, \mathbf{x_{2d}} + \mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e}))(\mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e})) \\
& + \mathbf{C}(\mathbf{x_1}, \mathbf{x_{2d}})(\mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e})) \\
& - \mathbf{C}(\mathbf{x_1}, \mathbf{x_{2d}} + \mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e}))\boldsymbol{\eta} \\
& + \mathbf{M}(\mathbf{x_1})\dot{\mathrm{x}}_{2d} + \mathbf{C}(\mathbf{x_1}, \mathbf{x_{2d}})\mathbf{x_{2d}} + \mathbf{g}(\mathbf{x_1}) \\
& + \boldsymbol{F}_d(\mathrm{Tanh}(\boldsymbol{e}_f) + \mathrm{Tanh}(\boldsymbol{e}) - \boldsymbol{\eta} + \mathbf{x_{2d}}) \qquad (4.15)
\end{aligned}
$$

and

$$
\begin{aligned}
\boldsymbol{\chi}_1 \;\triangleq\;& (\mathbf{M}(\mathbf{x_1})\mathrm{Cosh}^{-2}(\boldsymbol{e}) - \mathbf{C}(\mathbf{x_1}, \mathbf{x_{2d}} + \mathrm{Tanh}(\boldsymbol{e}_f) \\
& + \mathrm{Tanh}\boldsymbol{e}) - \boldsymbol{F}_d)\boldsymbol{\eta} \qquad (4.16) \\
\boldsymbol{\chi}_2 \;\triangleq\;& \boldsymbol{\chi} - \boldsymbol{\chi}_1 \qquad (4.17)
\end{aligned}
$$

Here, we define a combined state $\boldsymbol{X}$ as

$$
\boldsymbol{X} = [\boldsymbol{\eta}^T, \mathrm{Tanh}^T(\boldsymbol{e}), \mathrm{Tanh}^T(\boldsymbol{e}_f)]^T \qquad (4.18)
$$

Our final target is to stabilize this $\boldsymbol{X}$ with a certain control law.

4.4 Deterministic Case

Now, we are ready to introduce one of our main result.

Theorem 4.1. *Consider the system* (4.14) *with* (4.8) *and* (4.10)*. If we adopt the next control torque to the system,*

$$\boldsymbol{\tau} = \mathrm{Tanh}(\boldsymbol{e}) - k\mathrm{Cosh}^2(\boldsymbol{e}_f)\mathrm{Tanh}(\boldsymbol{e}_f) + \boldsymbol{\chi}_2 \tag{4.19}$$

Then, the closed loop system of (4.14) *is globally asymptotically stable.*

Remark 4.1. The overall system (4.8), (4.10) and (4.14) is PBGSS/OF. Corresponding expression in general formulation as in chapter 3, $\boldsymbol{\psi}_{11} \rightarrow \mathrm{Tanh}(\boldsymbol{e})$, $\boldsymbol{\psi}_{12} \rightarrow \mathrm{Tanh}(\boldsymbol{e}_f)$, $\boldsymbol{\psi}_2 \rightarrow \boldsymbol{\eta}$. Also in this case, matrix A in chapter 3 has the next form.

$$\boldsymbol{A}_c = \begin{pmatrix} -\boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times n} \\ \boldsymbol{0}_{n\times n} & -\boldsymbol{I}_{n\times n} \end{pmatrix} \tag{4.20}$$

Proof. Let us choose the Lyapunov function V as,

$$V = \underbrace{\sum_i^n \ln(\cosh(e_i)) + \sum_i^n \ln(\cosh(e_{fi}))}_{\boldsymbol{G}} + \frac{1}{2}\boldsymbol{\eta}^T\mathbf{M}(\mathbf{x_1})\boldsymbol{\eta} \tag{4.21}$$

When we take a time derivative of this Lyapunov function, it yields,

$$\begin{aligned} \dot{V} &= \mathrm{Tanh}^T(\boldsymbol{e})\dot{\boldsymbol{e}} + \mathrm{Tanh}^T(\boldsymbol{e}_f)\dot{\boldsymbol{e}}_f + \boldsymbol{\eta}^T\mathbf{M}(\mathbf{x_1})\dot{\boldsymbol{\eta}} \\ &\quad + \frac{1}{2}\boldsymbol{\eta}^T\dot{\mathbf{M}}(\mathbf{x_1},\mathbf{x_2})\boldsymbol{\eta} \end{aligned} \tag{4.22}$$

When we use the property 4.2 ,substitute (4.8) and (4.10) and adopt the control torque (4.19), $\dot{V}$ is going to be,

$$\dot{V} \leq -\|\text{Tanh}(\boldsymbol{e})\|^2 - \|\text{Tanh}(\boldsymbol{e}_f)\|^2 + \boldsymbol{\eta}^T[-k\mathbf{M}(\mathbf{x_1})\boldsymbol{\eta} + \boldsymbol{\chi}_1] \tag{4.23}$$

We construct an upper bound of the third term in (4.23) with properties 4.1, 4.3 and 4.4. This makes $\dot{V}$ as

$$\begin{aligned} \dot{V} \;\leq\; & -\|\text{Tanh}(\boldsymbol{e})\|^2 - \|\text{Tanh}(\boldsymbol{e}_f)\|^2 - \underline{\lambda}(\boldsymbol{F}_d))\|\boldsymbol{\eta}\|^2 \\ & - \underbrace{(k\sigma_m - \sigma_M - c_m(v_M + 2\sqrt{n}))}_{k_1}\|\boldsymbol{\eta}\|^2 \end{aligned} \tag{4.24}$$

where, $\underline{\lambda}(\cdot)$ means the smallest eigenvalue. Thus, by choosing k as,

$$k > \frac{\sigma_M + c_m(v_M + 2\sqrt{n})}{\sigma_m} \tag{4.25}$$

We may ensure $k_1 > 0$ and thus finally get,

$$\dot{V} \leq -\|\text{Tanh}(\boldsymbol{e})\|^2 - \|\text{Tanh}(\boldsymbol{e}_f)\|^2 - k_1\|\boldsymbol{\eta}\|^2 \leq 0 \tag{4.26}$$

where, k_1 is some positive constant.

Thus, we get the bounds on $\sum_i^n \ln(\cosh(e_i))$, $\sum_i^n \ln(\cosh(e_{fi}))$, $\boldsymbol{\eta}$, i.e,

$$\sum_i^n \ln(\cosh(e_i)) \in \mathcal{L}_\infty, \quad \sum_i^n \ln(\cosh(e_{fi})) \in \mathcal{L}_\infty, \quad \boldsymbol{\eta} \in \mathcal{L}_\infty \tag{4.27}$$

Due to the nature of $\ln(\cdot)$ and $\cosh(\cdot)$, the first two bounds in (4.27) imply $\boldsymbol{e} \in \mathcal{L}_\infty$ and $\boldsymbol{e}_f \in \mathcal{L}_\infty$. These bounds also imply $\dot{\boldsymbol{e}} \in \mathcal{L}_\infty$ and $\dot{\boldsymbol{e}_f} \in \mathcal{L}_\infty$ considering the dynamics (4.8) and (4.10). These all bounds can be summarized as follows.

$$\boldsymbol{X} \in \mathcal{L}_\infty, \quad \dot{\boldsymbol{X}} \in \mathcal{L}_\infty \tag{4.28}$$

From (4.26), we also have

$$\boldsymbol{X} \in \mathcal{L}_2 \tag{4.29}$$

Then, by using Barbalat's lemma, we can conclude $\boldsymbol{X} \to 0$ as $t \to \infty$. This automatically implies $\boldsymbol{e}, \boldsymbol{e}_f \to 0$ as $t \to \infty$

□

4.5 Proof of Equivalence between Update Laws

We extend theorem 4.1 to adaptive case without any additional assumption. Before that, we introduce an important lemma, which we always encounter when we try to construct feasible adaptive update law in our controller design scheme.

Lemma 4.1. The vector function $\hat{\boldsymbol{\theta}}$, which is defined by next

$$\hat{\boldsymbol{\theta}} \triangleq \boldsymbol{\Gamma} \int_0^t \boldsymbol{W}(\tau, \boldsymbol{\psi})\dot{\boldsymbol{\psi}} d\tau \tag{4.30}$$

where, $\boldsymbol{\Gamma}$ is any arbitrarily positive definite matrix, can be calculated without using $\dot{\boldsymbol{\psi}}$ by the following expression.

$$\boldsymbol{\Gamma} \int_0^t \boldsymbol{W}(\tau, \boldsymbol{\psi})\dot{\boldsymbol{\psi}} d\tau = \boldsymbol{\Gamma}\boldsymbol{H}(t, \boldsymbol{\psi}) - \boldsymbol{\Gamma} \int_0^t \int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \boldsymbol{W}_t(\tau, \boldsymbol{\epsilon}) d\boldsymbol{\epsilon} d\tau \tag{4.31}$$

where, $\boldsymbol{H}(t, \boldsymbol{\psi})$ is defined as following.

$$\boldsymbol{H}(t, \boldsymbol{\psi}) \triangleq \int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \boldsymbol{W}(t, \boldsymbol{\epsilon}) d\boldsymbol{\epsilon} \tag{4.32}$$

and subscript "t" of $\boldsymbol{W}$ in (4.31) means partial derivative with respect to time.

Proof. Let us consider time derivative of vector function $\boldsymbol{H}(t, \boldsymbol{\psi})$, which is,

$$\frac{d}{dt}\boldsymbol{H}(t, \boldsymbol{\psi}) = \frac{\partial}{\partial t}\boldsymbol{H}(t, \boldsymbol{\psi}) + \frac{\partial}{\partial \boldsymbol{\psi}}\boldsymbol{H}(t, \boldsymbol{\psi}) \cdot \dot{\boldsymbol{\psi}} \tag{4.33}$$

When we integrate this expression with respect to time, we will get

$$\boldsymbol{H}(t, \boldsymbol{\psi}) = \int_0^t \frac{\partial}{\partial \tau}\boldsymbol{H}(\tau, \boldsymbol{\psi})d\tau + \int_0^t \boldsymbol{W}(\tau, \boldsymbol{\psi})\dot{\boldsymbol{\psi}}d\tau \tag{4.34}$$

The second term of (4.34) is nothing but our $\hat{\boldsymbol{\theta}}$ itself. Thus, $\hat{\boldsymbol{\theta}}$ can be calculated as shown next.

$$\begin{aligned}
\hat{\boldsymbol{\theta}} &= \boldsymbol{\Gamma}\boldsymbol{H}(t, \boldsymbol{\psi}) - \boldsymbol{\Gamma}\int_0^t \frac{\partial}{\partial \tau}\boldsymbol{H}(\tau, \boldsymbol{\psi})d\tau \\
&= \boldsymbol{\Gamma}\boldsymbol{H}(t, \boldsymbol{\psi}) - \boldsymbol{\Gamma}\int_0^t \frac{\partial}{\partial \tau}\int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \boldsymbol{W}(\tau, \epsilon)d\boldsymbol{\epsilon}d\tau \\
&= \boldsymbol{\Gamma}\boldsymbol{H}(t, \boldsymbol{\psi}) - \boldsymbol{\Gamma}\int_0^t \int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \frac{\partial}{\partial \tau}\boldsymbol{W}(\tau, \epsilon)d\boldsymbol{\epsilon}d\tau \\
&= \boldsymbol{\Gamma}\boldsymbol{H}(t, \boldsymbol{\psi}) - \boldsymbol{\Gamma}\int_0^t \int_{\boldsymbol{\psi}_0}^{\boldsymbol{\psi}} \boldsymbol{W}_\tau(\tau, \epsilon)d\boldsymbol{\epsilon}d\tau
\end{aligned} \tag{4.35}$$

From the definition of $\boldsymbol{H}(t)$ and the nature of $\boldsymbol{W}(t, \boldsymbol{\psi})$, (4.35) is not dependent on $\dot{\boldsymbol{\psi}}$ any longer.

□

Remark 4.2. The calculation result of $\hat{\boldsymbol{\theta}}$ is numerically equivalent to the output from the next differential equation.

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma} \boldsymbol{W}(t, \boldsymbol{\psi}) \dot{\boldsymbol{\psi}} \tag{4.36}$$

Thus, we will use this expression only in the following stability proof. However, actual control is performed by adopting (4.35).

Remark 4.3. The most typical difference between (4.35) and the previous literatures (e.g. [19] and [44]) is the dependence of $\boldsymbol{W}$ on the measured states. In the previous literature, their $\boldsymbol{W}$ only depend on the reference trajectory signals in order to use "integration by parts" directly. Our results shows we do not have to choose such a $\boldsymbol{W}$ to execute "integration by parts."

4.6 Adaptive Output Feedback Controller and Stability Proof

At this point, we have completely prepared to show our second main result.

Theorem 4.2. *Consider the system* (4.13) *again with presenting parameter uncertainty. we assume that there are "m" numbers of unknown parameters in* $\mathbf{M}$*,* $\mathbf{C}$*,* $\boldsymbol{g}$ *and* $\boldsymbol{F}_d$ *and they are summarized as a vector* $\boldsymbol{\theta}^*$*, i.e,*

$$\boldsymbol{\theta}^* = [p_1^*, p_2^*, \cdots, p_m^*]^T \tag{4.37}$$

Note that $\hat{\boldsymbol{\theta}}$ *is an estimater of this* $\boldsymbol{\theta}^*$*.*

If we adopt the next control torque to the system,

$$\boldsymbol{\tau} = \mathrm{Tanh}(\boldsymbol{e}) - k\mathrm{Cosh}^2(\boldsymbol{e}_f)\mathrm{Tanh}(\boldsymbol{e}_f) + \hat{\boldsymbol{\chi}}_2 \tag{4.38}$$

where, $\hat{\boldsymbol{\chi}}_2$ and $\hat{\mathbf{Y}}_2$ are their estimated values and each unknown parameters are simultaneously estimated by (4.31) *to be reflected to estimated values $\hat{\mathbf{M}}$, $\hat{\mathbf{C}}$, $\hat{\boldsymbol{g}}$, $\hat{\boldsymbol{F}}_d$. (Actual definition of $\boldsymbol{W}(t, \boldsymbol{\psi})$ will appear in the stability proof.)*

Then, the closed loop system of (4.14) *is globally asymptotically stable.*

Proof. Let us choose the Lyapunov function V_a as,

$$V_a = \underbrace{\sum_i^n \ln(\cosh(e_i)) + \sum_i^n \ln(\cosh(e_{fi}))}_{\boldsymbol{G}} + \frac{1}{2}\boldsymbol{\eta}^T\mathbf{M}^*(\mathbf{x_1})\boldsymbol{\eta} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}} \tag{4.39}$$

where, superscript "*" means "unknown." Again, when we take a time derivative of this Lyapunov function and use the property 4.2, it yields,

$$\begin{aligned} \dot{V}_a &= \mathrm{Tanh}^T(\boldsymbol{e})\dot{\boldsymbol{e}} + \mathrm{Tanh}^T(\boldsymbol{e}_f)\dot{\boldsymbol{e}}_f + \boldsymbol{\eta}^T\mathbf{M}(\mathbf{x_1})\dot{\boldsymbol{\eta}} \\ &\quad + \frac{1}{2}\boldsymbol{\eta}^T\dot{\mathbf{M}}(\mathbf{x_1}, \mathbf{x_2})\boldsymbol{\eta} + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\theta}}} \end{aligned} \tag{4.40}$$

when we adopt the control torque (4.38), apply the property 4.1 to 4.4 and take the same procedures in the deterministic case, it renders us,

$$\begin{aligned} \dot{V}_a &\le -\|\mathrm{Tanh}(\boldsymbol{e})\|^2 - \|\mathrm{Tanh}\boldsymbol{e}_f\|^2 - k_1\|\boldsymbol{\eta}\|^2 \\ &\quad + \boldsymbol{\eta}^T\tilde{\boldsymbol{\chi}}_2 + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\theta}}} \end{aligned} \tag{4.41}$$

The first three terms of (4.41) can be negative definite with suitable k_1 in (4.25). Now, all we have to do is to cancel the third term by using "unfeasible" update law (4.36). We note that $\boldsymbol{\chi}_2$ is linear with respect to their unknown parameters. Thus, by using $\tilde{\boldsymbol{\theta}}$, the third term of (4.41) can be parameterized as follows.

$$\boldsymbol{\eta}^T \tilde{\boldsymbol{\chi}_2} = \boldsymbol{\eta}^T \boldsymbol{W}(t, \boldsymbol{\psi}) \tilde{\boldsymbol{\theta}} \tag{4.42}$$

where, $\boldsymbol{\psi} \triangleq [\boldsymbol{e}^T, \boldsymbol{e}_f^T]^T$ and the regressor matrix $\boldsymbol{W} \in \mathcal{R}^{m \times n}$ only include reference trajectory and position error signals. When we substitute (4.8), we can construct the next update law.

$$\begin{aligned}
\dot{\hat{\boldsymbol{\theta}}} &= \boldsymbol{\Gamma} \boldsymbol{W}^T(t, \boldsymbol{\psi})[\dot{\boldsymbol{e}} + \text{Tanh}(\boldsymbol{e}) + \text{Tanh}(\boldsymbol{e}_f)] \\
&= \boldsymbol{\Gamma} \boldsymbol{W}_d^T(t, \boldsymbol{\psi}) \dot{\boldsymbol{\psi}} + \boldsymbol{\Gamma} \boldsymbol{W}^T(t, \boldsymbol{\psi})[\text{Tanh}(\boldsymbol{e}) + \text{Tanh}(\boldsymbol{e}_f)]
\end{aligned} \tag{4.43}$$

where $\boldsymbol{W}_d^T(t, \boldsymbol{\psi}) \triangleq \boldsymbol{W}^T \cdot [I_{n \times n}, 0_{n \times n}]$. We can use (4.36) in order to make feasible update law for the first part of (4.43) and the last part of (4.43) is feasible itself.

Above all, we finally get

$$\dot{V} \leq -\|\text{Tanh}(\boldsymbol{e})\|^2 - \|\text{Tanh}\boldsymbol{e}_f\|^2 - k_1 \|\boldsymbol{\eta}\|^2 \tag{4.44}$$

Thus, using Barbalat's Lemma again , we can guarantee global stability of $\boldsymbol{X}$ even in the case of adaptive case.

□

4.7 Numerical Example

In order to show the effectiveness of our design scheme, we show the numerical simulation result. We take the second order (DOF) system as an example. For the two joint arm robot, generalized inertia matrix $\mathbf{M}(\mathbf{x_1})$ and centrifugal term $\mathbf{C}(\mathbf{x_1}, \mathbf{x_2})$ are described as follows.

$$\mathbf{M}(\mathbf{x_1}) = \begin{bmatrix} p_1 + 2p_3\cos(x_{12}) & p_2 + p_3\cos(x_{12}) \\ p_2 + p_3\cos(x_{12}) & p_2 \end{bmatrix} \tag{4.45}$$

$$\mathbf{C}(\mathbf{x_1}, \mathbf{x_2}) = p_3\sin(x_{12}) \begin{bmatrix} -x_{22} & -x_{21} - x_{22} \\ x_{21} & 0 \end{bmatrix} \tag{4.46}$$

Here, p_1,p_2 and p_3 are constant parameters and chosen to be 3.6, 0.15 and 0.2 (kgm^2) respectively. (These are totally the totally same in [44].) We assume that the robot is located in horizontal plain, thus, $\mathbf{g} = 0$. Also, the friction term is defined by next.

$$\boldsymbol{F}_d(\mathbf{x_1}, \mathbf{x_2}) = \begin{bmatrix} fd_1 & 0 \\ 0 & fd_2 \end{bmatrix} \tag{4.47}$$

and the coefficients of friction are set to be $fd_1 = 5.3$ and $fd_2 = 1.1$.

Reference trajectories are also chosen to be,

$$\mathbf{x_{1d1}} = 1.57\sin(2t)(1 - \exp^{-0.05t^3}) \tag{4.48}$$

$$\mathbf{x_{1d2}} = 1.2\sin(3t)(1 - \exp^{-0.05t^3}) \tag{4.49}$$

All the initial conditions are set zero except for the next signals in deterministic case.

$$\begin{aligned} \boldsymbol{e}_0 &= [0.2, \ -0.2]^T && (4.50) \\ \mathrm{Tanh}(\boldsymbol{e}_{f0}) &= k \cdot \boldsymbol{e}_0 && (4.51) \end{aligned}$$

where, k must be a positive number greater than 1.5, thus, we choose $k = 5$.

Also, in adaptive case, we choose relatively large initial conditions such that,

$$\boldsymbol{e}_0 = [0.5, \ -0.5]^T \tag{4.52}$$

in order to make clear the difference of adaptive control torques.

Fig.4.1, 4.2 and 4.3 show the simulation result of deterministic case. We can choose any arbitrarily large k to guarantee the stability, however, it cases to increase the initial torques. Thus, it is better to choose k as small as possible.

Fig.4.4 to 4.8 show the result of adaptive case. In this case, the persistent excitation seems to be held because parameter convergence is achieved. We also show the control torques in Fig.4.6, in which red line shows control torques and blue line shows that of deterministic case. Initially, adaptive controller try to adapt the system, thus, the torques seems to hesitate. However, after 5 seconds, it almost adapt to the system and the two torques are almost the same.

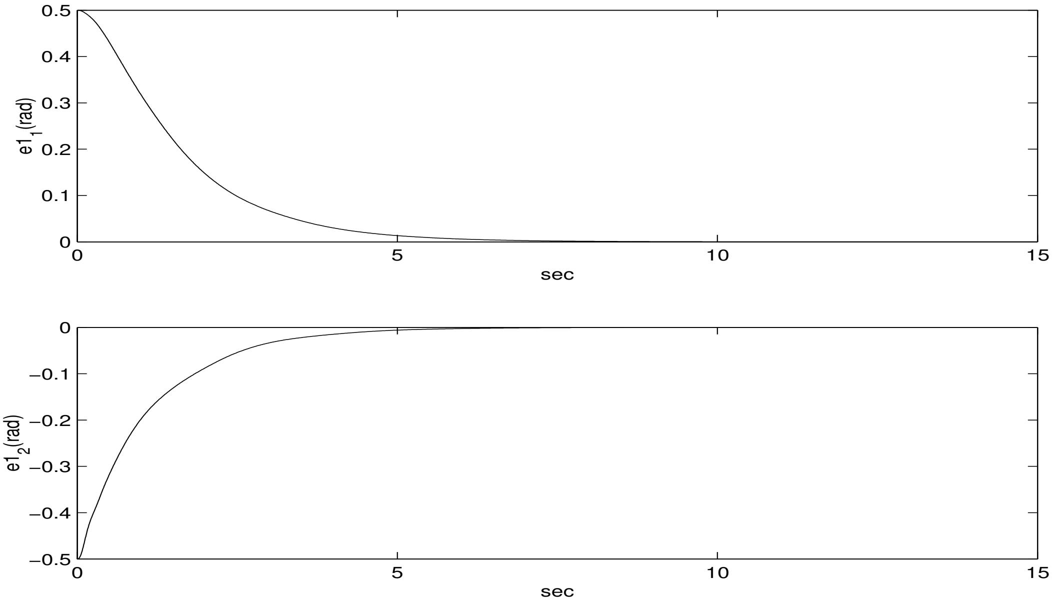

Figure 4.1: Position tracking error in deterministic case

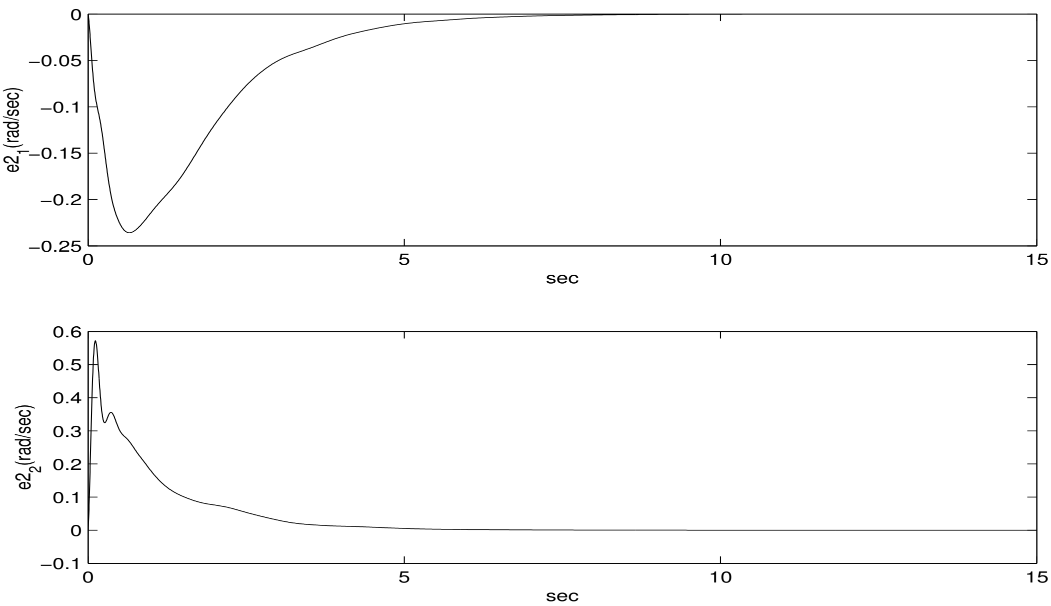

Figure 4.2: Angular velocity tracking error in deterministic case

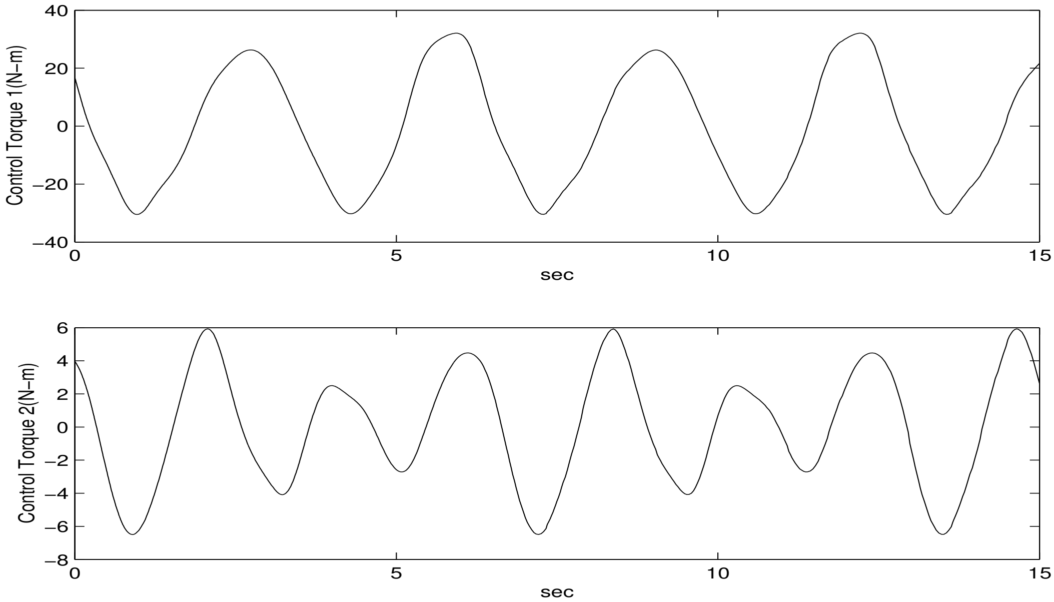

Figure 4.3: Control torques in deterministic case

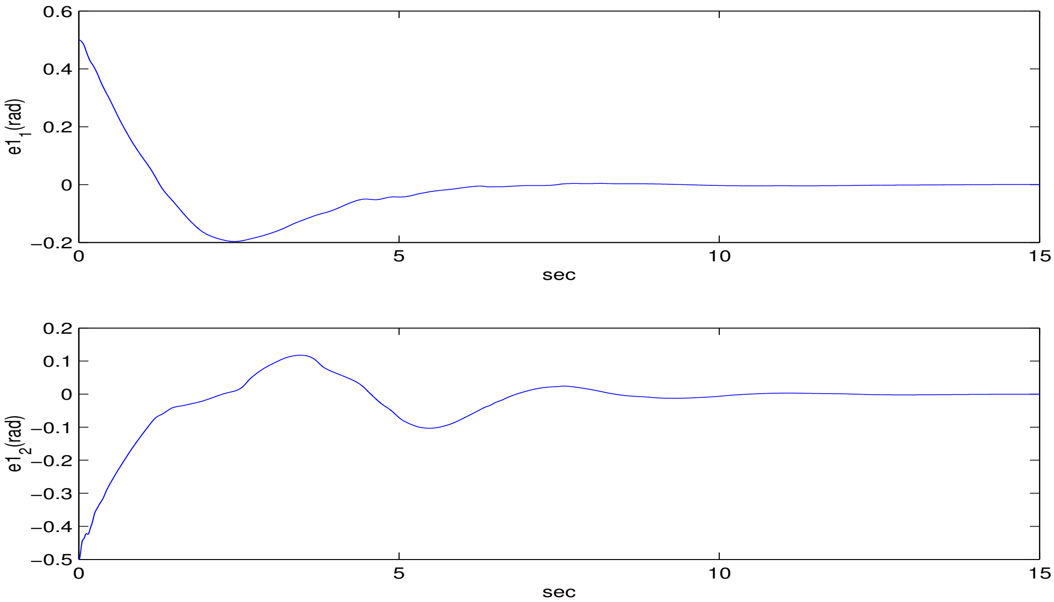

Figure 4.4: Position tracking error in adaptive case

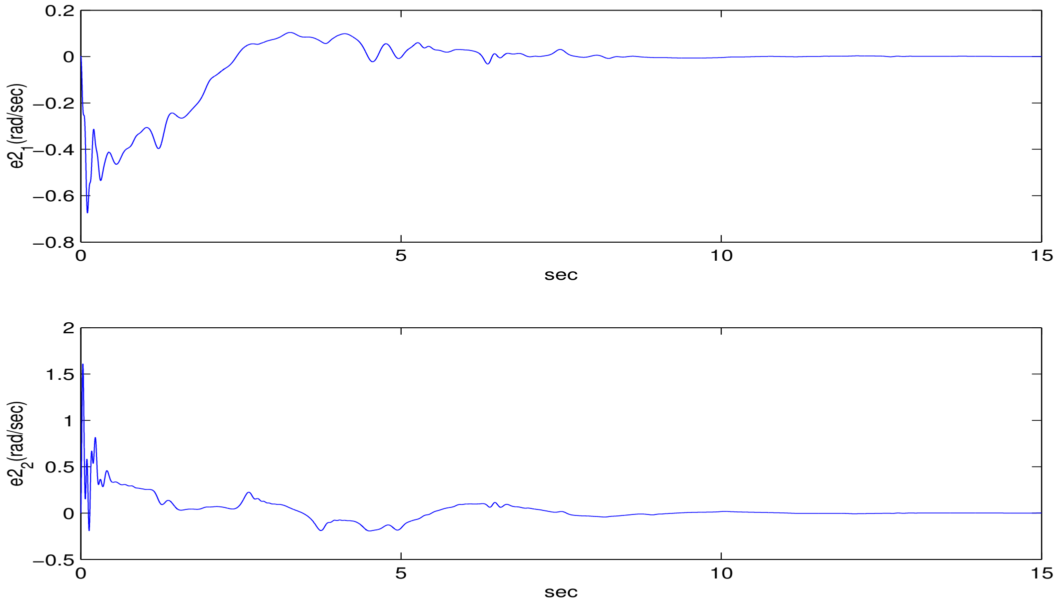

Figure 4.5: Angular velocity tracking error in adaptive case

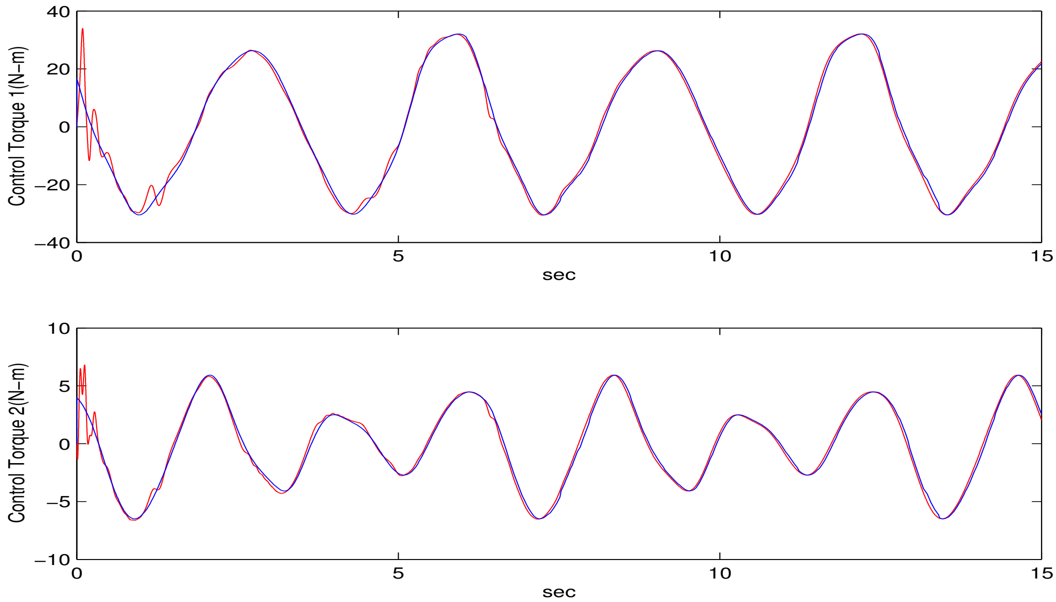

Figure 4.6: Control torques in adaptive case

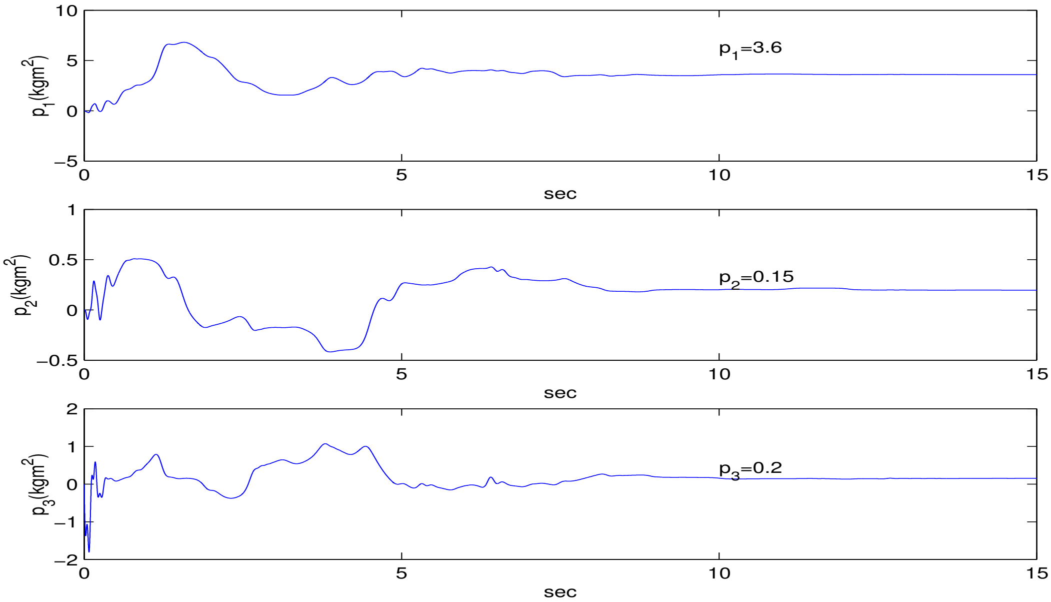

Figure 4.7: Inertia Parameter Estimates in adaptive case

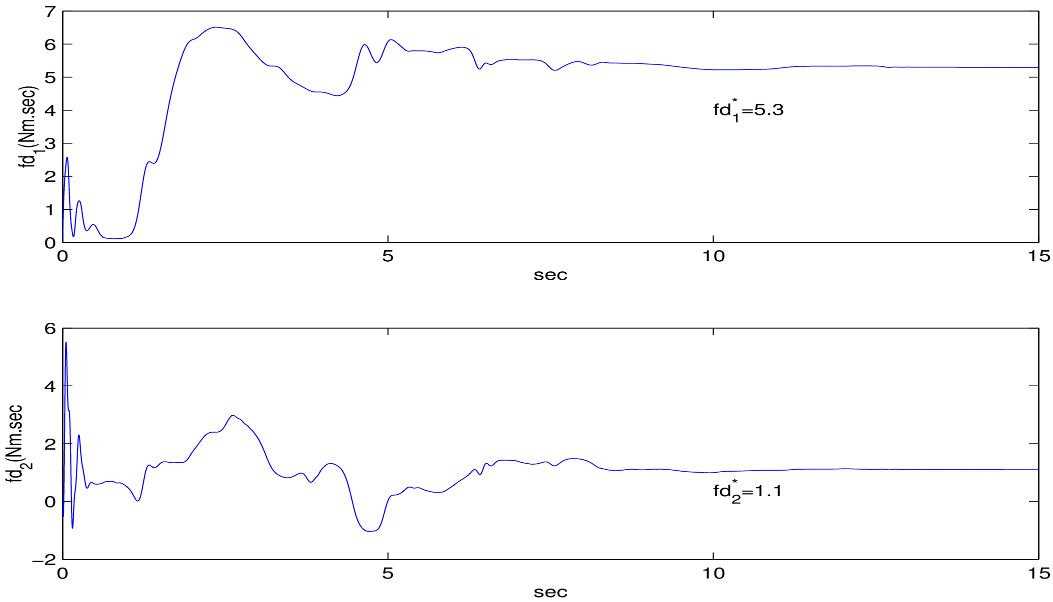

Figure 4.8: Friction Coefficients Estimates in adaptive case

Chapter 5

Conclusions

5.1 Summary of Results

In this dissertation, we have addressed the problem of adaptive output feedback control for a special class of nonlinear systems. We expect this study to successfully contribute to the field of nonlinear control in the following ways:

1. Formal definition of a class of nonlinear systems that are controllable via adaptive output feedback.

2. Introduction of novel methods that implement feasible adaptive update laws in the presence of partial state (output) measurements.

3. Prototype examples for rigid spacecraft motion and robot manipulator are analyzed to discuss the applicability and impact of the proposed methodology.

Following is a brief summary of the major aspects within each of contributed achievements.

5.1.1 Definition of PBGSS/AOF

In this dissertation, we defined a class of nonlinear systems that can be stabilized with adaptive output feedback. We summarize this class as "Passivity Based Globally Stabilizable Systems via Adaptive Output Feedback. (PBGSS/AOF) " Within this definition, we indicated the following important aspects:

- We explicitly characterized the structure (or dynamics) and the properties of this class. This automatically provides us sufficiency criteria to determine whether a system is PBGSS/AOF. One of the most important properties within this class of systems must hold is that the dynamics of all the measurable signals must be linear with respect to the unmeasured states.

- We also show the relationship between this class and the property of passivity. In previous arguments, we stated that passivity is a sufficient condition for a system to be included in PBGSS/OF (or AOF). Further exploration is required to discover regarding the necessity of this condition.

- Especially in the adaptive case, we showed that if unknown parameters exist linearly in the deterministic controller of a system, then the same controller can be directly extended to the adaptive case according to the "certainty equivalence principle." However, if unknown parameters exist non-linearly, we can not apply our method and there is no general approach for such a case. Under specific conditions, Kojic [3] has shown the way to adapt for non-linearly parameterized systems. The applicability of these techniques to PBGSS/AOF needs to be investigated in the future.

5.1.2 Implementation of Feasible Adaptive Update Laws

During the definition of PBGSS/AOF, we also introduced two ways to construct a feasible update law from an unfeasible differential equation update law. Either method can be chosen and both are suitable for our chosen target systems. However, the most important condition that the unfeasible (differential equation) update law must hold is that it should be linear with respect to unmeasured states. Under this condition, the following implementation techniques have been proposed:

- If the regressor matrix has a decomposition property, "integration by parts" can be directly made use for the purpose of constructing feasible update laws.

- If the regressor matrix cannot be decomposed, it becomes necessary to construct an integrated function from the regressor matrix with respect to the measured states, followed by integration by parts to obtain available update laws.

- Both techniques do not adversely affect to the stability of the overall system. Even when a higher order system holds the decomposition property of the regressor matrix, it may be efficient to use the alternate technique to reduce the analysis time involved in evaluating the decomposition matrices.

5.1.3 Actual Examples

As prototypical system examples, we choose the spacecraft attitude tracking problem and reference trajectory tracking problem of robot manipulators. Our achievements in the case of each system are listed as follows.

Spacecraft Attitude Tracking Problems

- Simple filtered output signals are useful in lieu of unmeasured angular velocity signals.
- No additional assumptions on the system parameters or reference trajectory in order to guarantee global stability in the adaptive case.
- The above results do not change no matter how the kinematic equation for attitude representation is chosen. However, as always, it is necessary to be cautious concerning singularities in the representation of rotational motion.
- In this dissertation, actuator constraints were not considered. Naturally in the actual system, actuator constraints (constraint of maximum torques) do exist. In order to solve this problem, further exploration concerning gain selection is required.

Trajectory Tracking Problem of Robot Manipulators

- Using previous results of Wong [41] and Teel [8], a certain type of observer was formulated to estimate the unmeasured joint angle rate variables.
- We needed three assumptions on the system in order to guarantee global stability and to determine the gains of the above observer. Most recent adaptive solution by Wong [41] require seven assumptions to set up an adaptive controller. Hence, in the sense that less assumption required, our methodology is superior to that results.

5.2 Future Work

In this dissertation, we do not investigate several aspects of this class of problems. Thus, future work on these types of problems will be required to be focused on the following.

5.2.1 Actuator Constraints

As shown in Loria [7], under certain assumption, it is possible to guarantee semi-global stability for robot manipulator problems and there may be a possibility of guaranteeing global stability with actuator constraints. However, adaptive output feedback control methodology for this problem has not been developed. Due to the result of Laib [4], we can guarantee global stability for set-point regulation problems and this technique can prove to be fruitful in extending to the tracking case.

5.2.2 Noisy Measurements

In this dissertation, we assume perfect measurements and complete absence of noise. In real applications, there always exists some type of noise along with effects of un-modeled dynamics. In our proposed scheme, unknown parameters are estimated by adaptive update law and there is always the possibility of the parameter drift, divergence, bursting and other catastrophic effects due to the presence of un-modeled phenomena. It will also be important to explore and try to extend our formulation with dead-zone, parameter projection and other basic techniques to eliminate these undesirable effects. (e.g. Ioannou [28])

5.2.3 Structure of the Filter or Observer

In the definition of PBGSS/AOF, we do not determine the structure of the filter dynamics. Actually, we use a simple first order filter for the spacecraft problem and a certain high-gain observer for the robotics case. These filter and observer dynamics certainly satisfy the property of PBGSS/AOF. However, we do not investigate the explicit synthesis of such a filter or an observer. The selection of this filtered output is intimately dependent on the structure of the control input. Especially, if the control input holds to the so called "reduction property," it is related to the regulation part of the input. This fact is automatically reflected in the structure of a Lyapunov function as well. Hence, in order to investigate the issue of construction of the filter or observer, it is essential to survey the intrinsic relationships between the required filtered output, control input and the underlying Lyapunov function.

5.2.4 Persistency in Excitation

As mentioned in the results of each chapter, we do not guarantee the presence of "persistent excitation." This means that we cannot guarantee the parameter convergence within this framework. Eventually, it is possible to obtain parameter convergence in the robotics case due to the time varying inertia matrix and centrifugal terms.

5.2.5 Transient Performance

As introduced in chapter 1, one of weak properties of adaptive control is that the transient fluctuation cannot be controlled. This characteristic is also inherited in our adaptive control scheme. In order to restrict the transient response, the general methodology to restrict the migration width of estimation errors (e.g. $\tilde{\boldsymbol{\theta}}(t)$) must be developed in our adaptive control design procedure.

5.2.6 Nonlinearly Appearing Parameters

In the numerical example of a robotics case, we try to estimate $p_1,\ p_2,\ p_3$ in our control structure and it has succeeded due to their linear presence in the original dynamics of robot manipulator. However, they do not have physical meanings. Actually, these unknown parameters are described by (unknown) physical values as follows.

$$\begin{aligned} p_1 &= m_1 r_1^2 + m_2(r_1^2 + r_2^2) \\ p_2 &= m_2 r_2^2 \\ p_3 &= m_2 r_1 r_2 \end{aligned} \tag{5.1}$$

where each parameter has the next physical meanings.

m_1:weight of the first joint of manipulator

m_2:weight of the second joint of manipulator

r_1 :length of the first joint of manipulator

r_2 :length of the second joint of manipulator

As shown above, unknown parameters are present nonlinearly in the sense of $m_1,\ m_2,\ r_1$ and r_2, thus, it is impossible to estimate these physical values in our adaptive scheme. One way to estimate these practical values is to try Kojic's framework [3] to our design scheme.

Bibliography

[1] A.A.J.Lefeber. (adaptive) control of chaotic and robot systems via bounded feedback control. *Master's thesis, University of Twente, Enschede, The Netherlands*, 1996.

[2] A.Jain and G.Rodriguez. An analysis of the kinematics and dynamics of underactuated manipulators. *IEEE Trans. on Robotics and Automation*, 9:411–422, 1993.

[3] A.Kojic and A.M.Annaswamy. Adaptive control of nonlinearly parameterized systems with a triangular structure. *Automatica*, 38:115–123, 2002.

[4] A.Laib. Adaptive output regulation of robot manipulators under actuator constraints. *IEEE Trans. on Robotics and Automation*, 16:29–35, 2000.

[5] A.Loria. Global tracking control of one degree of freedom euler-lagrange systems without velocity measurements. *European Journal of Control*, 2:144–151, 1996.

[6] A.Loria. Position feedback global tracking control of el systems: A state transformation approach. *IEEE Trans. on Automatic Control*, 47:841–847, 2002.

[7] A.Loria and H.Nijimeijer. Bounded output feedback tracking control of fully actuated euler-lagrange systems. *Systems & Control Letters*, 33:151–161, 1998.

[8] A.R.Teel and L.Praly. Tools for semiglobal stabilization by partial state and output feedback. *SIAM Journal of Control and Optimization*, 33:1443–1488, 1995.

[9] B.Fardanesh and J.Rastegar. A new model-based controller for robot manipulators using trajectory pattern inverse dynamics. *IEEE Trans. on Robotics and Automation*, 8:279–285, 1992.

[10] B.Siciliano and L.Villani. A passivity-based approach to force regulation and motion control of robot manipulators. 32:443–447, 1996.

[11] N.Fixot C. Canudas de Wit and K.J.Astrom. Trajectory tracking in robot manipulators via nonlinear estimated state feedback. *IEEE Trans. on Robotics and Automation*, 8:138–144, 1992.

[12] L.Castriota D.C. Youla and H.Carlin. Bounded real scattering matrices and the foundations of linear passive networks. *IRE Tran. Circ. Theory*, 4, 1959.

[13] B.T.Costic D.M.Dawson, M.S.de Queiroz and V. Kapila. Quaternion-based adaptive attitude tracking controller without velocity measurements. *AIAA Journal of Guidance, Control and Dynamics*, 24:1214–1222, 2001.

[14] John C. Doyle. *Essential of Robust Control.* Prentice Hall, Upper Saddle River, New Jersey 07458, 1998.

[15] Christopher Edwards and Sarah K. Spurgeon. *Sliding Mode Control: Theory and Applications*. Taylor & Francis, 1900 Frost Road, Suite 101, Bristol, PA 19007, 1998.

[16] F.Caccavale and L.Villani. Output feedback control for attitude tracking. *Systems & Control letters*, 38:91–98, 1999.

[17] F.Lizarralde and J.T.Wen. Attitude control without angular velocity measurements: A passivity approach. *IEEE Trans. on Automatic Control*, 41:468–472, 1996.

[18] M.Queiroz F.Zhang, D.Dawson and W.Dixon. Global adaptive output feedback tracking control of robot manipulators. *IEEE Trans. on Automatic Control*.

[19] M.Queiroz F.Zhang, D.Dawson and W.Dixon. Global adaptive output feedback tracking control of robot manipulators. In *Proceedings of 36th IEEE Conference on Decision and Control,San Diego,CA*, 1997.

[20] G.Besancon. Simple global output feedback tracking control of one-degree-of-freedom euler-lagrange systems. 1998.

[21] G.Besancon. Global output feedback control for a class of lagrangian systems. *Automatica*, 36:1915–1921, 2002.

[22] G.Liu and A.A.Goldenberg. Robust control of robot manipulators based on dynamics decomposition. *IEEE Trans. on Robotics and Automation*, 13:783–789, 1997.

[23] Donald T. Greenwood. *Principles of Dynamics.* Prentice Hall, Englewood Cliffs, New Jersey 07632, 1988.

[24] H.Arai and S.Tachi. Position control of a manipulator with passive joints using dynamic coupling. *IEEE Trans. on Robotics and Automation*, 7:528–534, 1991.

[25] H.Berghuis and H.Nijimeijer. A passivity approach to controller-observer design for robots. *IEEE Trans. on Robotics and Automation*, 9:740–754, 1993.

[26] H.Berghuis and H.Nijimeijer. Robust control of robots via linear estimated state feedback. *IEEE Trans. on Automatic Control*, 39:2159–2162, 1994.

[27] Peter C. Hughes. *Spacecraft Attitude Dynamics.* A Wiley-Interscience Publication, 605 Third Avenue, New York 10158, 1986.

[28] Petros A. Ioannou and Jing Sun. *Robust Adaptive Control.* PTR Prentice Hall, Upper Saddle River, NJ 07458.

[29] I.V.Burkov. Stabilization of a natural mechanical system without measuring its velocities with application to the control of a rigid body. *Journal of Applied Mathimatics and Mechanics*, 62:853–862, 1998.

[30] J.C.K.Chou. Quaternion kinematic and dynamic differential equations. *IEEE Trans. on Robotics and Automation*, 8:53–63, 1992.

[31] R.H.Taylor J.Funda and R.P. Paul. On homogeneous transforms, quaternions and computational efficiency. *IEEE Trans. on Robotics and Automation*, 6:382–388, 1990.

[32] M.R.Akella J.L.Junkins and R.D.Robinett. Nonlinear adapitve control of sapcecraft maneuvers. *AIAA Journal of Guidance,Control and Dynamics*, 20:1104–1110, 1996.

[33] J.B Cruz Jr. *System Sensitivity Analysis*. Dowden, Huchinson & Ross, Stroudsburg, PA 19007, 1973.

[34] J.S.C.Yuan. Closed-loop manipulator control using quaternion feedback. *IEEE Trans. on Robotics and Automation*, 4:434–440, 1988.

[35] J.T.Wen and S.Murphy. Stability analysis of position and force control for robot arms. *IEEE Trans. on Automatic Control*, 36:365–370, 1991.

[36] K.Kaneko and R.Horowitz. Repetitive and adaptive control of robot manipulators with velocity estimation. *IEEE Trans. on Robotics and Automation*, 13:204–217, 1997.

[37] H.Takeuchi K.Kosuge and K.Furuta. Motion control of a robot arm using joint torque sensors. *IEEE Trans. on Robotics and Automation*, 6:258–263, 1990.

[38] M.Erlic and W.S.Lu. A reduced order adaptive velocity observer for manipulator control. *IEEE Trans. on Robotics and Automation*, 11:293–303, 1995.

[39] M.Liu. Decentralized control of robot manipulators:nonliear and adaptive approaches. *IEEE Trans. on Automatic Control*, 44:357–363, 1999.

[40] M.R.Akella. Rigid body attitude tracking without angular velocity feedback. *Systems & Control letters*, 42:321–326, 2001.

[41] H.Wong M.S.de Queiroz and V. Kapila. Adaptive tracking control using synthesized velocity from attitude measurements. *Automatica*, 37:947–953, 2001.

[42] Kumpati S. Narendra. Parameter adaptive control - the end......or the beginning ? *IEEE Conf. on Decision and Control*, 1994.

[43] N.Sadegh and R.Horowitz. Stability and robustness analysis of a class of adaptive controllers for robotic manipulators. *International Journal of Robotics Research*, 9:74–92, 1990.

[44] P.R. Pagilla and M.Tomizuka. An adaptive output feedback controller for robot arms. *Automatica*, 37:983–995, 2001.

[45] P.Tomei. Adaptive pd controller for robot manipulators. *IEEE Trans. on Robotics and Automation*, 7:565–570, 1991.

[46] P.Tsiotras. Further passivity results for the attitude control problem. *IEEE Trans. on Automatic Control*, 43:1597–1600, 1998.

[47] A.Ailon R.Kelly, R.Ortega and A. Loria. Global regulation of flexible joint robots using approximate differentiation. *IEEE Trans. on Automatic Control*, 39, 1994.

[48] R.Ortega and M.Spong. Adaptive motion control of rigid robots: A tutorial. *Automatica*, 25:877–888, 1989.

[49] A.Loria R.Ortega and R.Kelly. A semiglobally stable outout feedback $\mathbf{PI^2D}$ regulator for robot manipulators. *IEEE Trans. on Automatic Control*, 40:100–104, 1995.

[50] S.Lin. Dynamics of the manipulator with closed chains. *IEEE Trans. on Robotics and Automation*, 6:496–501, 1990.

[51] Jean-Jacques E. Slotine and Weiping Li. *Applied Nonlinear Control*. Prentice Hall, Englewood Cliffs, New Jersey 07632, 1991.

[52] S.Nicosia and P.Tomei. Robot control by using only joint position measurements. *IEEE Trans. on Automatic Control*, 35:1058–1061, 1990.

[53] D.Hill S.Shishkin, R.Ortega and A.Loria. On output feedback stabilization of euler-lagrange systems with nondissipative forces. *Systems & Control Letters*, 27:315–324, 1996.

[54] Gang Tao. Simple alternative to the barbalat lemma. *IEEE Trans. on Automatic Control*, 42, 1997.

[55] W.Zhu and J.De Shutter. Adaptive control of mixed rigid/flexible joint robot manipulators based on virtual decomposition. *IEEE Trans. on Robotics and Automation*, 15:310–317, 1999.

VITA

Hideaki Miwa was born in Nagasaki, Japan on May 12, 1969 as the first son of Akito Miwa and Toshie Miwa. After completing his work at Kaisei High School, Nagasaki, Japan with the top student honor in the dean's list, he got admitted to The Tokyo Institute of Technology and studied there from 1988 to 1994. From that school, He obtained Bachelor of Engineering and Master of Engineering, in 1992 and 1994 respectively. After graduation, he has served as a military officer at Japan Air Self-Defense Force. His current rank is Captain.

Permanent Address: 459-15 Kabashima Nomozaki-cho Nishisonoki-gun,
Nagasaki, Japan, 851-0535

This dissertation was typed by the author.