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Teaching Mathematics and the Problems of Practice: Understanding Situations and Teacher Reasoning Through Teacher Perspectives

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Teaching Mathematics and the Problems of Practice: Understanding Situations and Teacher Reasoning Through Teacher Perspectives

by

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DEDICATION

To my grandfather, who always inspired me to love education

and was an educator himself in every sense of the word:

Edwin Earl Plowman

1911–2004

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We see what we understand rather than understand what we see.
(Labinowicz, 1985, p. 7)

Many people have supported me these last 7 years, from my family to friends to my colleagues at school and work. I am very grateful for all of the support I have been given and hope to find the right words to convey my gratitude. Through the writing of this dissertation and other writing I have learned that saying what you mean to say is not easy! So I want to say how much I appreciate all of you. There are layers of support that have been helpful to me including my family, friends, and colleagues. It seems impossible to convey the depth of this support and its meaning for me in a few pages, but I am going to try.

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PREFACE

This dissertation is arranged as a series of standalone articles. Section 1, titled: "Teachers as Problem Solvers: Studies of the Pedagogical Problem Solving Teachers Do While Teaching Mathematics" is a review of the literature. Section 2, titled: "Points of Interest: Predicaments Recognized by Teachers as They Implement Inquiry-Based Practices for Teaching Fractions", is a research article which embodies all of the research components. Section 3, titled: "The Challenge of Interpreting Students' Invented Representations: Teaching Predicaments as Learning Opportunities for Teachers" is intended as a practitioner article highlighting just one particular aspect of the research.

Teaching Mathematics and the Problems of Practice: Understanding Situations and Teacher Reasoning Through Teacher Perspectives

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In this study, 4 teachers were asked to identify classroom-teaching situations that they "wondered" about. Each teacher was using an inquiry-based, National Science Foundation funded curriculum (Investigations in Data Number and Space or Connected Mathematics*)* to teach fractions. Results showed that teachers' problems of practice centered on interactions in which they struggled to understand students' strategies, both invented and school based. Though difficult, the teachers strove to find ways to support student thinking and instructional intentions of inquiry-based mathematics practices rather than resorting to more didactic approaches. Teachers recognized and valued children's construction and use of representations for fractions that were often in the form of area models, and teachers wanted to find ways to interpret these strategies from the children's point of view. Teachers in this study often perceived themselves as "stuck" rather then empowered because they did not

have the strategies for teaching needed to support these novel uses of models and often unexpected strategies.

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SECTION 1

TEACHERS AS PROBLEM SOLVERS: STUDIES OF THE PEDAGOGICAL PROBLEM SOLVING TEACHERS DO WHILE TEACHING MATHEMATICS

PROLOGUE

In the spring of 2000, I observed and video-taped one teacher's lessons on fractions over a period of 5 weeks, and then I chose some interactions from the lessons that I believed were problematic in some way (Junk, 2000). I used these clips with her in a series of interviews as a way to support reflection on her interactions with her students. Then, I analyzed the interviews to examine the effectiveness of video to stimulate recall and its usefulness to help teachers reflect on their practice. In the process of choosing the parts of the lesson to view during each of the interviews, I seemed to be saying, "Here is what I viewed as a problem for you; what was going on here?"

For one particular interaction, I believed that the teacher, Ms. Andre, had missed several opportunities to support a student's thinking because she had not listened to what the student said. As a result of misunderstanding the student's strategy, she intervened and heavily scaffolded the child's work. She led him to a correct solution, but on a different path than he had begun. I thought that by viewing the tape she could reflect on her actions, explain her thinking about the interaction, and possibly reconsider what the student did. What actually happened was that she had little to say about this particular interaction; in fact, she barely recalled it at all.

As a result, she chose to talk about something else loosely related to what was on the clip. Thus, I concluded that the video clip did not support reflection. Later, I began to think that the reason the video clip did not encourage reflection was because the situation was *not an issue for the teacher*; especially in the way I saw it.

This experience led me to question the method I was using to research teachers' thought processes. Perhaps what we know about teacher thinking is incomplete because we have not used methods to tap into what the teacher thinks about the problems of practice.

THE PROBLEM

There are differences in how researchers conceive teaching and how teachers conceive their practice. These differences have been reported in the literature as a gap between theory and practice (Stigler & Hiebert, 1999) or as a mismatch between beliefs and practice (Spillane, 1999). The responsibility for the gap has often been attributed to the teacher's lack of knowledge or beliefs (Ball, 1996; Ma, 1999). Instead of making assumptions on how to fix the gap, we need clearer and more complete understanding about the ways teachers perceive their problems of practice. If teaching is about problem solving, then teachers are problem solvers and have particular problems to solve. Of all of these considerations, the role of the teacher's perspective is a key component. We can add substantially to what we understand about the work of teachers and how to support them by including their perspectives.

TEACHING AS PROBLEM SOLVING

Research from the last half century often has conceptualized teaching as the application of certain effective teaching behaviors to achieve certain curricular goals (e.g., Good & Grouws, 1997; Leinhardt, 1985). In the *effective teaching* conception, studying teaching is a matter of describing effective teaching behaviors, with the idea that these behaviors produce student learning. This conception projects an idealized image that the problems of teaching are predictable; teachers may infer from this view that the problems of teaching can be solved by closely following textbook lessons and applying ready-made solutions. Consequently, the teachers' role is characterized as a person who applies solutions to problems that someone else has solved (Lave, 1996; Taplin & Chan, 2001). This image is supported by a kind of teaching in which the conception of learning is simply a matter of mastering skills and concepts through transfer of knowledge (Carpenter, 1988). "It reduces teaching to narrowly specific prescriptions for what should be transplanted into the heads of kids" (Lave, p. 158).

On the other hand, when conceptions about teaching are aligned with theories of teaching and learning that support children's construction of knowledge, teaching cannot be as closely prescribed or scripted, because the teacher needs to respond dynamically to students' developing understandings about mathematics (Lampert, 2001; Sherin, 1998; Simon, 1999). To begin to think about teaching as problem solving or *teaching as inquiry*, definitions of mathematical problem solving can be used to describe what teachers must do (Carpenter, 1988). The National Council of

Teachers of Mathematics (2000) defined problem solving as "engaging in a task for which the solution method in not known in advance" (p. 52). Further,

> In order to find a solution, students must draw on their knowledge, and through this process they will often develop new mathematical understandings. Solving problems is not only a major goal of learning mathematics but also a major means of doing so. (p. 52)

Replace key words with *teaching* and *teachers* and this definition of problem solving reads as follows: In order to find a solution, *teachers* must draw on their knowledge, and through this process they will often develop new understandings *about teaching*. Solving problems is not only a major goal of learning *about teaching,* but also a major means of doing so.

Teachers as Problem Solvers

The previous section described two ways of conceptualizing teaching. The second conception, teaching as inquiry*,* holds a view the teaching process cannot be prescribed completely because teachers need to be able to adapt to the situations as they arise (Franke, Carpenter, Levi, & Fennema 2001; Lampert, 1998a; Sherin 2002b) Moreover, this view takes into account that teachers are constructing knowledge as they solve their problems of practice (Ball & Bass, 2000; Carpenter, 1988; Franke & Carpenter, 1998; Lampert, 2001; Simon, 1995). In the teaching as inquiry view, the teacher is the kind of problem solver who cannot simply solve problems by applying solutions, can resolve problems of practice, which are unpredictable, and tailor responses according to the nuances of the particulars of the problem (Lampert, 1997; Taplin & Chan, 2001).

Problems of Practice

During mathematics lessons teachers have to solve pedagogical problems, such as how to probe student thinking, and then must decide what problem to address next (Ball & Cohen, 1999; Hiebert et al., 1997; Steinberg, Empson, & Carpenter, 2004; van Zee & Minstrell, 1997). In this vein, listing explicit teaching actions may not be as helpful, because lists of actions do not reveal the reasons teachers chose the action or the subtle characteristics of the problem that needed attention. Because teachers' problematic situations are not entirely predictable, teachers' reasoning about the problems needs to be explained (Darling-Hammond, 1996). When researchers pay attention to particular types of problems, such as how they are able to attend to student thinking, make ideas public, and build on students' strategies for solving problems (Ball, 1996; Featherstone, Smith, Beasley, Corbin, & Shank, 1995), they gain insight into how teachers think and reason about their practice.

Teacher's Perspective

Methods of research that include any of these components address the teacher's perspective in some way. At the very least, the methodology includes some aspect of classroom interactions either by directly observing lessons (e.g., Featherstone et al., 1995) or by recreating classroom scenarios (e.g., Empson & Junk, 2004b). For almost all of the studies, the problems of practice are conceptualized from the researcher's point of view. Carpenter (1988) suggested that one way to better understand teacher's problem solving in practice is to conceive teachers' problems of practice as we conceive problem solving in general. This view highlights an important issue relating to the importance of the teacher's perspective. Research on children's thinking and problem solving with whole-number operations has shown that children can solve problems using previous informal knowledge, with little assistance from the teacher. Like all people, teachers construct knowledge: "They interpret them [the problems of practice] in terms of their own constructs and adapt them to fit the situation as they perceive it" (Carpenter, p. 190).

RESEARCH ON PROBLEMS OF PRACTICE

Studies have been conducted to explicate teachers' problems of practice either directly or indirectly, by focusing in classroom interactions. When researchers investigate teacher knowledge, they identify the situation the knowledge is intended to address. So, although the identification of the pedagogical problem is implicit, research on teacher knowledge can be analyzed to identify categories of problems of practice that have been the focus of these studies. In the same way, research on teacher thinking and reasoning as well as studies of teacher reflection can be analyzed in terms of problems of practice. Other studies have conducted research on the problems of practice by trying to understand them as the result of beliefs, knowledge, dispositions, and teacher thinking and reasoning (Noddings, 1992). These problems of practice have been labeled using a variety of terms, including concerns or classroom issues (Walen & Williams, 2000), dilemmas (Ball, 1993; Lampert, 1997;

Windschitl, 2002), predicaments (Burbles & Hansen, 1997), complex decision-making situations (O'Connor, 2001), or creative tensions (Simon, Tzur, Heinz, Kinzel, & Schwan Smith, 2000).

In my search for studies on teacher's pedagogical problems, I found a lot of theoretically based arguments in favor of studying teaching as problem solving (e.g., Bransford, Brown, & Cocking, 2000; Carpenter, 1988; Windschitl, 2002) and few studies that actually set out to identify what situations are actually problematic for teachers during inquiry-based mathematics lessons. I limited my search to studies involving classroom-based issues in mathematics and reviewed over 34 research studies. Of those studies, a little less than half involved non-novice, non-expert teachers, and only four of those included some kind of analysis of teacher-chosen problems of practice (Kazemi & Franke, 2001; Sherin, 1998, 2002a; Walen & Williams, 2000).

Currently a growing body of research recognizes the importance of the teacher's role as decision maker and pedagogical problem solver. In this review, I pay special attention to those research programs that conceptualize teaching as a matter of problem solving in some way. Most of the research programs studied teachers who were engaging in some sort of reform-based practices, though some of the older studies involved teachers who were using more traditional methods (e.g., Leinhardt 1985). In some of those studies researchers designed situations in which they studied how teachers use knowledge to solve identified problems (e.g., O'Connor 2001), and in others the problems of practice are identified as a result of the study (e.g., Featherstone et al., 1995). To illuminate how these teachers' pedagogical problems were associated with the teacher's perspective, I answered questions concerning three methodological factors:

1. What kinds of teachers were studied?
2. What kinds of problems were identified?
3. How were the problems identified, or whose problem is it?

What Kinds of Teachers Were Studied?

Richard Elmore (1996) pointed out that success in changing core patterns in teaching, such as the way students and teachers interact, happens in a maximum of 25% of classrooms. If we desire a large-scale change in the way mathematics is viewed, what is needed in research on math education is a better understanding of what is happening in the remaining 75% of the classrooms and "workable theories about how human beings learn to do things differently" (Elmore, p. 24). The commitment of school systems to adopt curricula designed to support reform means that the needs of all teachers have to be addressed, not just of the smaller proportion of teachers who historically have been able to change core elements of their practice. Ms Andre, the teacher previously discussed, was one of many teachers who were implementing the Investigations (TERC, 1995–1998) curriculum through a district wide adoption. Unlike many teachers who choose to utilize reform materials, this teacher was not an innovator as described by Elmore. She was competent teacher but was using the reform materials because the district required it. I believe Ms Andre is representative of the kind of teacher in the 75% of classrooms Elmore was writing about.

Researchers have utilized novice or expert teachers as subjects and compared the two. Expert teacher studies often are of just one teacher, and several self-studies on teaching have involved the researcher-teacher type of expert. If the participants are teachers from all levels or of average competence or experience, the study most likely focused on the effectiveness of particular innovations.

Expert–Novice Studies

Historically, studies that compared the experiences of novice and/or expert teachers were designed primarily to explicate the qualities of an effective teacher (Fenstermacher, 1994). The research on effective teaching aims to identify concise sets of competent behaviors that teachers use to produce successful student outcomes (e.g., Good & Grouws, 1977; Leinhardt, 1985, 1987, 1989; Leinhardt, Putnam, Stein, & Baxter, 1991; Leinhardt & Smith, 1985). These studies and similar ones used contrasting behaviors between novice and expert teachers for particular purposes. One of the purposes is the expectation that the results of the research will serve as exemplars of expert teacher behavior (e.g., Good & Grouws 1977). Fenstermacher reported, " Researchers in this category [the effective teaching approach] do not see themselves as studying teacher knowledge so much as they perceive themselves producing knowledge about teaching" (p. 7).

Lienhardt and Smith (1985) eloquently described expertise in mathematics instruction by constructing maps that detailed teaching behaviors of 4 expert and 4 novice teachers during their lessons. The initial study compared the teachers to determine the relationship between knowledge of fractions, their teaching behaviors, and student outcomes. Their findings showed differences between experts and novices aligned with previous studies. Experts, who had higher student outcomes, had more complex subject-matter knowledge than novices. This complexity was defined as the number of multiple connections competent teachers had between mathematical concepts. A second analysis of the expert teachers found differences between individual, expert teachers' "explanation behavior," that is, how they were able to communicate important concepts about fractions during instruction. Lienhardt and Smith reasoned that because 2 of the teachers held better connections between procedures and concepts and understood these connections, they were better able to provide good, conceptually based explanations to their students. This second analysis hinted at how teachers are able to cope with the particular pedagogical problem of explaining the mathematics in a conceptually connected way.

Additionally, Leinhardt and Smith's (1995) recognition of the differences between the experts' knowledge of teaching fractions and the impact on their classroom actions compels researchers to think differently about assigning teachers into expert categories. Another problem with using experts as exemplars is that nonexperts may sense that those expert teaching behaviors are too far removed from their own experiences and abilities (Clandinin & Connelly 1992; Fenstermacher, 1994).

A recent comparison study was designed to examine the problem of implementing curricula compared novice teachers to veteran teachers (Sherin & Drake, 2002). Sherin and Drake's purpose was to explicate the kinds of decisions teachers make when they implement curricula that is designed to elicit children's thinking and that is grounded in constructivist-based learning theory. Observations and interviews of 4 teachers showed different ways of implementing the curricula. The way teachers implemented the curricula was framed by their previous experience, but like the Leinhardt and Smith (1995) study, the researchers found important differences between the veteran teachers. For example, when the veteran teachers made adjustments to the lessons, one teacher added components with the goal of deepening the children's opportunities to understand the concepts, but the other veteran teacher modified the lesson to make the activities simpler to do.

Expert Teacher Studies

One kind of teacher who is often the subject of studies on teacher's practice is the teacher who is considered an expert (Jacobson & Lehrer, 2000; O'Connor, 2001; Rittenhouse, 1998; van Zee & Minstrell, 1997; Yackel, 2002). Recent studies of expert teachers have provided researchers opportunities to study the problems of practice that arise as a result of implementing reform-based practices. O'Connor chose to study such an expert because she knew there would be an opportunity to observe how a teacher facilitated discussions that supported student thinking. Lampert (2001), Heaton (2000), and Simon (1995) are examples of experts in the teacher-researcher category. Lampert (2001) explained the problems of practice from the standpoint of her role as a classroom teacher. In her book, *Teaching Problems and the Problems of Teaching*, she explained in great detail the nuances of the work of implementing inquiry-based mathematics instruction. Lampert claimed that teaching *is* problem solving. She explained that a teacher's work is complex; the descriptions of her practice are rich in detail and clearly show how Lampert's understanding of the content and its relationship to children's thinking supports the problems of practice she chooses to pay attention to and how she copes with these problems. Because Lampert is an extraordinary teacher and also a researcher, her conceptions and descriptions of teaching as problem solving may not match the character of the ordinary, nonresearcher's problems of practice (Simon, 1999). Even so, Lampert's work is useful for thinking about teaching, because it explains the problems of practice from the teacher's point of view.

Novice Teachers' Problems of Practice

Research on novice teachers often centers on how to make problems of practice visible and available during their induction phase to improve ways they learn about teaching (Mewborn, 2000; Taplin & Chan 2001). Borko et al. (2000) observed the effects of a preservice education program designed to support teachers' practice by providing them experiences with the teaching problem concerning the design and implementation of tasks related to issues of proof and by conducting discussions around the tasks. They found that the teacher they observed was able to use what she learned in her university class to construct meaningful, conceptually based tasks about proof. However, she struggled to understand how to conduct meaningful discourse about the task during the lessons.

Other Kinds of Teachers

Studies that have investigated teachers who are neither novices nor experts often also examine the effectiveness of inservice programs or interventions (Empson & Junk 2004b; Featherstone et al., 1995; Knapp & Peterson 1995; Peterson, Carpenter, & Fennema, 1989; Sherin, 2002a; Walen & Williams, 2000). They also can examine how teachers understand the subject matter (Ma, 1999). Generally, the teachers who are involved have a range of experience and expertise, although they may have limited experience with the intervention being studied. As a result, this type of research reveals a range of implementation practices, with a range of ways teachers cope with the problems of practice they encounter. For example, in the initial cognitively guided instruction teacher knowledge studies (Carpenter, Fennema, Peterson, & Carey, 1988; Peterson et al., 1989), the problem of practice the researchers were interested in was how teachers use knowledge to inform their instruction. They found that when teachers know more about their students' strategies, the students of those teachers have higher achievement.

What Kinds of Problems Were Studied?

The types of problems teacher face are dependent on the approach to teaching (Smith, 1997; Taplin & Chan, 2001). Problem-based, inquiry approaches to teaching mathematics have teaching problems particular to the approach, in contrast to those encountered when teachers conceive of their teaching as a telling kind of practice. Smith pointed out that this is a challenge for teachers who are faced with changing from one kind of practice to another. He noted that these issues are of concern for teachers as they challenge their sense of efficacy.

The four components of teaching Smith (1997) wrote about can be framed in terms of teaching activities: (a) choosing and implementing tasks; (b) predicting, understanding, and responding to student reasoning; (c) generating and directing discourse; and (d) judicious telling. All of these teaching activities have the potential to become pedagogical problems as they involve a direct conflict between conceptions of teaching mathematics as telling and teaching for understanding. Additionally, Smith's list demonstrates the importance of the teacher's role as the primary problem solver during the act of teaching.

The following descriptions of research focus on the categories of activities from Smith's (1997) characterization and the problems that have been identified in that category. For all categories, the problems of practice are identified through observations of lessons or by analyzing teachers' reflective journals or responses during interviews (these aspects are detailed in the following section). In addition to Smith's categories, I added a fifth one to emphasize that some researchers wanted to investigate how particular math topics related to teacher's use of knowledge to solve teaching problems. I focus on the problems of practice identified that are associated with specific problems related to the math topic, such as how teachers use understanding children's thinking for addition and subtraction, or how teachers facilitate children's understanding about fractions or geometry to address issues that arise in teaching.

At this point I would like to justify my selection of the research studies reviewed. Many researchers' goals in these studies centered on what knowledge (or beliefs) teachers have, and/or how teachers use their knowledge. My claim is that the particular knowledge or beliefs are the focus of such studies because the researchers assume they are needed to solve particular problems of practice. Teachers' actions are in response to the problem being addressed, and their actions are based on their use of knowledge. Ultimately, the importance of understanding teachers' knowledge from any angle is to build understanding about how teachers solve their problems of practice.

Choosing and Implementing Tasks

Choosing tasks that are relevant and engaging to students (Featherstone et al., 1995; McNair, 1998; Sherin & Drake, 2002; Simon, 1995; Windschitl, 2002) is a problem of concern, and researchers are interested in understanding how teachers deal with the problem of task choosing that is relevant to curriculum development and use. Sherin and Drake (2002) were curious about how teachers interpreted and used the tasks prescribed for them in an inquiry-based curriculum. The problem of practice can be described as what to do when the task seems too hard or too easy for the students. Teachers in this study made changes to the task in different ways to solve this problem. Novice teachers more often wondered if the task would work, but taught the lesson as it was outlined anyway. The veteran teachers usually made changes to tasks they believed would not work for some reason or another during planning stages. Interestingly, there was a distinct difference in the way the veteran teachers changed the tasks. One teacher often believed the conceptually based tasks would be too hard, so she adjusted the tasks to make them easier. The other veteran teacher added to or refined some of the same tasks to make them more interesting or to extend the activity.

Predicting, Understanding, and Responding to Student Reasoning

Understanding children's work or thinking (e.g., Kazemi & Franke, 2001; Sherin, 1998) and responding to children's nonstandard strategies (Empson & Junk, 2004b; Jacobs & Ambrose, 2003) are issues often at the forefront of teaching interventions meant to increase teacher's knowledge about children's mathematics. These studies judge the effectiveness or levels of implementation of the intervention by observing how teachers are able to cope with the difficult task of interpreting what children know and its relationship to the mathematics to be learned.

Generating and Directing Discourse

How to engage students in mathematical discussion (Borko et al., 2000; O'Connor, 2001; Rittenhouse, 1998; van Zee & Minstrell, 1997; Yackel, 2002) or to each other in groups (Walen & Williams, 2000) is a popular type of situation to researchers. How teachers manage to facilitate rich mathematical discussions (e.g., O'Connor 2001; Sherin 2002a, 2002b) presents a variety of particular pedagogical problems to solve, such as understanding what students say in the context of whole-group discussion and finding ways to coordinate students' strategies in group discussion so that key mathematics concepts are learned.

O'Connor (2001) utilized the context of fractions to study the emergence of mathematical ideas and one teacher's struggle to manage class' discussions to support students' ideas. By choosing to closely examine discussion, O'Connor implicitly claimed that this type of situation is one problem of practice that needs to be addressed. The tasks that a teacher has to address are finding out what methods students have, making sense of those methods and assessing their validity. Then teachers need to effectively juxtapose the students' methods so that productive discussion will ensue. In particular, O'Connor wrote about a set of lessons involving discussion about fractions and decimals. The vignette featured one teacher who was part of a larger study involving a curriculum intervention designed to support students' understanding of mathematics. The problem encountered by the teacher during a whole-class discussion began when a child asked, "Can any fraction be turned into a decimal?" (p. 152). O'Connor described how the teacher deftly balanced telling what she knew about the question and eliciting other students' ideas about the answer to the question. O'Connor stated that the reason this teacher was able to facilitate the discussion is that she had familiarity with the content and the possible directions and positions students might take. In the context of conducting group discussion, O'Connor pointed to the complexity of the teacher's decision-making as she coped with the problem of facilitating a meaningful discussion.

Judicious Telling

When researchers notice that teachers tell students how to solve problems, it is usually in the context of comparing teachers who have a more sophisticated conception of teaching for understanding to those teachers who do not have that conception (Franke et al., 2001; Kazemi & Stipek, 2001). Jacobs and Ambrose (2003) studied how teachers responded to students' strategies during one-on-one interviews. They found that teachers progressed from direct teacher actions to less directive actions as they gained more experience with and discussed children's strategies for solving word problems. Jacobs and Ambrose reported that they believed these problem-solving interviews would serve as a context for teachers to develop questioning expertise. The teachers participated in choosing and viewing their video-taped interviews; however, there was no analysis of how the teachers responded to their own teaching. Perhaps they were able to recognize their own struggles to support children's thinking without being directive, which enabled them to develop better questioning strategies, but it was not evident in the results reported in the paper.

Problems Related to the Math Topic

In an example presented earlier, O'Connor (2001) was able to talk about the mechanisms involved in rich discussion because she also paid attention to the content of fractions. This way she was able to analyze the role of content in supporting the teachers' abilities to cope with the problem of conducting discussion. However, the strategy of balancing children's ideas with the teacher's to facilitate discussion can be applied to a number of math topics, not just fractions.

Carpenter (1988) stated, "In order to effect comprehensive change in problem solving instruction, research must ultimately address the complexity of problem solving within the mathematics curriculum as well as the complexity of classroom instruction" (p. 188). The problems of practice directly associated with particular math topics such as whole-number operations, fractions, and geometry have been investigated in several research studies. Some investigated the specific kind of knowledge used to solve the problem of understanding and interpreting children's work; others showed that specific content knowledge supports teacher's facility to conduct discussions.

Cognitively Guided Instruction

In the context of instruction centered on problem solving, the overarching problem of practice is how to elicit and build on children's informal thinking. The initial cognitively guided instruction study about teachers (Carpenter et al., 1988) found that the teachers' increased knowledge of children's strategies for solving whole-number problems had positive effects on students' achievement, and the teachers who knew more about their students strategies produced even better student achievement (Peterson et al., 1989). Researchers speculated that this particular kind knowledge about addition and subtraction aided teachers' ability to support the development of students' understanding (Peterson et al., 1989). Later studies found that teachers who were able to understand children's thinking espoused beliefs that matched their classroom practices and were able to incorporate what they learned and fundamentally to change the core of their practice (Knapp & Peterson, 1995). Teachers were able to facilitate children's problem solving because they could elicit children's thinking about their strategies, ask specific questions about their students' work, focus on the students' answers, and listen to students' explanations for the answers. Less knowledgeable teachers in the cognitively guided instruction study helped children solve math problems by telling the children directly what to do.

In a later study related to these findings, Jacobson and Lehrer (2000) designed teaching intervention to investigate the effect of specific content knowledge about geometry. All 4 teachers in their study participated in professional development based on children's thinking about whole-number operations (cognitively guided instruction). Two of the teachers participated in an additional professional development to learn about children's thinking in geometry. Jacobson and Lehrer found that teachers' facility to conduct group discussions about geometry was enhanced when they knew more about children's thinking in geometry. These teachers' discussions were richer; teachers could sustain rich discussions about geometry with their students. As a result, their students knew more about geometry at the end of the unit. The tasks involved in productively conducting discussion entailed making connections that "refined, elaborated and extended students' thinking about space" (Jacobson & Lehrer, p. 86).

Whose Problem Is It?

One example (Sherin 2002) of research shows how a teacher can directly identify a problem in teaching and how the researchers' response to investigate that problem can lead to a deeper insight on teachers' problems of practice. The problem is characterized as balancing discussions between a focus on mathematical processes and concepts were directly identified by the teacher, not the researcher. As I have alluded to before, this is rare in research on teacher pedagogical problem solving.

Involving teachers in the design of research so that their perspectives can be centralized includes a range of teacher perspectives. Fenstermacher (1994) described the contrast in the kind of knowledge gained from these differing levels of teacher perspective as "the knowledge that teachers generate as a result of their experience as teachers" and "the knowledge of teaching that is generated by those who specialize in research on teaching" (p. 3).

The range of inclusion of the teacher's perspective is illustrated in Figure 1.1.

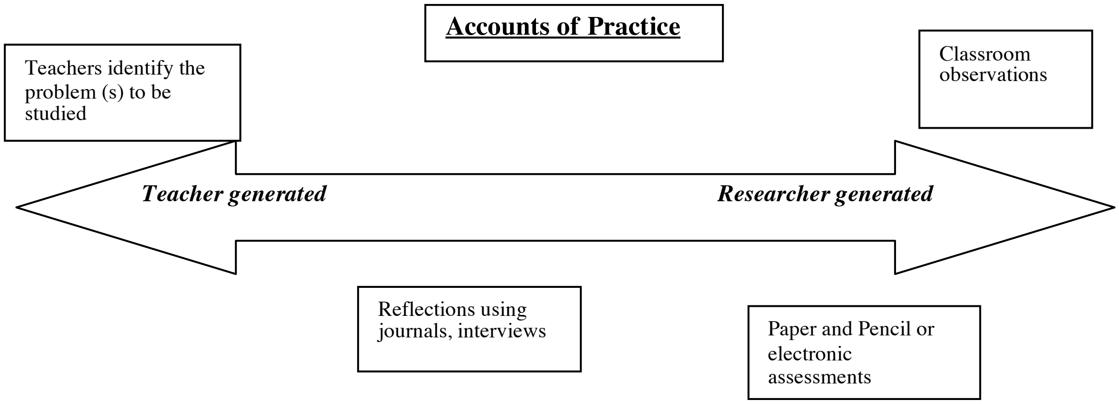

Figure 1.1. Teacher-generated and researcher-generated methods of data collection and accounts of practice.

Inside the arrow are the extremes described by Fenstermacher (1994). Outside the arrow are examples of the kind of data that can be used to analyze problems of practice that align with the range of perspectives. Researchers often use more than one kind of data, and the kinds of data can be used to different degrees. To the extreme left in Figure 1.1 are classroom observations. They are considered researcher generated is because in the extreme case the data are collected by observing one or two lessons, and the lessons are analyzed without further input from the teacher. Moving to the right are paper-and-pencil and electronic assessments. Sometimes these assessments are in addition to classroom observations, but in any case the teacher is given the opportunity to express his or her perspective by choosing from a choice of responses or, in some cases, by providing a written explanation. These assessments are usually constructed so that the questions take the form of teaching scenarios. Further to the right I include more open-ended types of data collection, such as when teachers keep a journal or when teachers are interviewed about their practices. Depending on how the interviews and journals are used, the degree of inclusion of the teacher's perspective varies. For example, in the case of Ms. Andre, data were collected via classroom observations and a follow-up interview. The protocol for the interview directed the teacher to respond to interactions that the researcher had chosen. Other types of interviews and journals encourage the teacher to write or talk about whatever issue they choose.

Classroom Observations

Researchers who detail teaching as a result of close analysis of a particular set of lessons, like O'Connor (2001), have an empathetic view of the teachers work (e.g., Jacobson & Lehrer 2000; Rittenhouse, 1998). Throughout her report, O'Connor aligned herself with what she supposed the teacher might be thinking and the decisions she faced as she facilitated a discussion on fractions. Although many of the studies reviewed depicted a close collaboration between researcher and teacher, the choice of the problem focus and subsequent analysis of the issues involved in coping with the problem of practice was primarily researcher driven.

Paper-and-Pencil or Electronic Assessments

The researchers' perspective in paper-and-pencil or electronic assessments of teacher problem solving is reflected in the kinds of situations researchers choose for the teachers to respond to (Ambrose, Clement, Philipp, & Chauvot, 2004; Taplin & Chan 2001). For example, the IMAP project (Ambrose et al., 2004) involved a Web-based assessment in which preservice teachers responded to a variety of situations having to do with children's thinking about mathematics. The gist of the types of pedagogical problems was determined by the researcher's knowledge of situations involving issues that would provoke teacher thinking. The teachers' perspectives were accessed through the analysis of answers; teachers also had opportunities to explain in essay form the reasons for their responses.

Reflections Using Journals or Interviews

Reflective journaling as a type of data collection has been a popular form of gathering information on how preservice teachers conceive problems of practice (Borko et al., 2000; Mewborn, 2000). In Walen and Williams' (2000) study, they asked teachers to respond in journals to four case study vignettes about problems in teaching mathematics. Teachers studied and discussed the vignettes in small groups. They were encouraged to talk about whatever they noticed about the teaching problems and then were asked to write about them in their journals. Walen and Williams found that the teachers could identify personally with the problems characterized in the vignettes and in addition were able to formulate rich discussion with their peers about how to solve the problems.

Interviews have been conducted to explore teacher thinking about children's nonstandard solutions to problems (Empson & Junk, 2004b; Peterson et al., 1989) and to find out how teachers reason about their subject matter (Ma, 1999). Most of these types of interviews are guided by problems of practice the researcher knows have a foundation in theories about learning and teaching. Interviews also have been used to evaluate teacher practices after an intervention (Featherstone et al., 1995; Knapp & Peterson, 1995) and often are used as supplements to observations (e.g., van Zee & Minstrell, 1997).

Teachers Identify the Problem to Be Studied

The least common method of studying teachers' problems of practice is the type in which teachers identify the problematic situations. There are self-reports from teacher-researchers (e.g., Ball, 1993; Heaton, 2000; Lampert, 2001), but I could only find four examples of research on teacher practices in which the participants were of an intermediate level of expertise and the analysis included teachers' identification of problems of practice.

For example, during 10 meetings of a video club, Sherin's (1998, 2002b) teachers were allowed to focus on the issues that they wanted to. The video club was designed as an after-school professional development in which teachers observed each other's lessons and discussed what they observed. Two of the 4 teachers took turns having their lessons video-taped. Then the teacher and a researcher would meet to preview the tape and select a portion to share with the video club. During the first few discussions about the teaching they observed, the teaching problems the teachers identified were not the types of teaching problems Sherin wanted them to focus on. When Sherin suggested they pay attention to student thinking, the teachers responded superficially (if at all) to the issue and returned to a focus on teacher actions. Sherin let the teachers take the lead and allowed the discussions during the video clubs to proceed as the teachers wished. Gradually, the teachers began to take on interest in student actions, at first focusing on how to correct student misconceptions. Over time the teachers became more interested in the students' conceptions about mathematics. This study suggests that when teachers are given the opportunity, they do not always focus on the same topics or problems that a researcher would.

Sherin's (2002a, 2002b) research of one teacher's struggle to manage group discussion is an example of research on a problem directly identified by the teacher involved as the subject of the study. In a separate study unrelated to the video-club study above, Sherin effectively collaborated with a teacher after he identified a particular issue that perturbed him. The teacher identified this particular teaching problem in his journal, "Today I was forced to consider an interesting issue. The issue is, 'Do I sacrifice some . . . content in order to foster discussions in class?'" Sherin examined teacher problem solving by focusing on the tensions involved in managing mathematical discourse. Sherin (2002a) characterized the issue as one in which the teacher has "to facilitate class discussions in which student ideas were at the center and in which mathematics was discussed in a deep and meaningful way" (p. 206). She described the teacher's struggle as a balancing act in which the teacher's task is to create an environment where both doing and talking about mathematics is valued. To cope with this problem, the teacher needed to manage competing goals of focusing on *process* of mathematical discourse and the *content* of mathematical discourse.

Sherin (2002a) decided that this problem needed more investigation. She collected and analyzed 68 classroom observations in which discussion played a key element across several math topics traditionally taught in eighth grade. Patterns emerged that showed shifting but equal emphases on process and content, until the students began to study algebra. At that point the teacher's discussions became more traditional: The teacher told students what he wanted them to know and showed them what to do and how to do it. He believed that algebra needed a highly structured approach and explained that he needed to emphasize content over discussion because of pressures from parents and the school to get the students ready for algebra the following year.

Another example of allowing teachers to take the lead in discussions about teaching problems is found in Kazemi and Franke's (2001) work. These researchers designed their professional development with the express purpose of provoking inquiry into practice through discussion of children's work. Teachers were involved in learning about children's problem solving and were asked to bring samples of the children's work to the meetings. The teachers' perspective was preserved in the freedom teachers had to choose the child whose work was interesting to them and to choose the topic regarding what they had noticed about the work. Like teachers in Sherin's (2002a) study, teachers did not notice details in student work that could provide them clues about student thinking. This was prompted by the fact that much of the work brought to the meetings did not have much detail. This lack of detail prompted the teachers to probe their students' thinking more in class, which in turn gave the teachers opportunities to talk about student work in more detail at the meetings. The researchers predetermined the problem of practice, understanding children's work. However, the teachers identified the specific problems of practices entailed in understanding children's work. They realized that they were not probing children's thinking, and that when children expressed their thinking they did not understand the strategies.

> By struggling to make sense and detail their students thinking, the teachers' participation developed the intellectual practices of the group. As the year progressed, within the context of the workgroup, teachers questioned their roles in the classroom, analyzed and interpreted their students thinking further, and some began to ask harder questions about how to help students develop increasingly sophisticated mathematical understandings" (Franke &Kazemi, 2001, p. 36).

DISCUSSION

Researchers have made progress in the study of teaching as a problem-solving practice. Studies about teaching have shown that teachers have to engage in complex teaching problems that cannot be solved with ready-made solutions. Researchers can be more aware of how the methods they choose to study teaching constrain or enable the degree to which the teachers' perspective is recognized by paying attention to (a) the type of teachers invited to participate, (b) the type of problems studied, and (c) the way the problems are chosen. These factors lend support to the complexity of teaching, and at the same time provide a venue to make claims about teaching and the problems of practice that can narrow the gap between research and practice.

This review of the research revealed need for a closer and more frequent focus on what ordinary, nonexpert teachers find problematic in teaching practice. Teachers' problems of practice have the potential to inform research on the effects and progress of reform efforts (Ball, 1993; Burbles & Hanson 1997). The identification of problems of practice rarely comes directly from the teachers' own words. By default, researchers studying how knowledge is used in practice have identified an array of pedagogical problems teachers encounter and have analyzed how teachers deal with these problems.

While researchers are using methods that tap into the teacher's perspectives, for the most part, the problems of practice central to the studies are selected by the researchers. A common problem of practice that researchers study is how teachers facilitate discussions. This particular problem is most often studied during lessons taught by a teacher that is considered an expert. Conducting productive discussion involves the teachers' ability to understand children's strategies as well as how to integrate other children's strategies and content issues into the conversations. On the other hand, there are not as many studies found in this review that study teachers' problems of practice as they interact with individual children, with the exception of Jacobs and Ambrose (2002) who studied how teachers responded to children's thinking during interviews.

Additionally, teachers' concerns about when to tell children certain information and when to hold back information (i.e. judicious telling) were rare in these studies. Smith's idea that teachers' struggles arise as a result of the need to reconcile older, more directive type of teaching approaches with more open-ended types of teaching approaches is most evident in this category of problems. When teachers encounter problems involved in conducting productive discussions the concern about what to tell and what not too tell seems to be an issue, though not directly acknowledged as a particular problem.

These two findings in particular demonstrate examples of how the concerns of researcher and the concerns of teachers may not match up. Understanding how to interpret and build on children's ideas while interacting with individual children and how to make decisions involving judicious telling may be the problems that are closest to how non-expert, ordinary teachers perceive their practice. I do not claim that one concern is more important then the other, but that addressing concerns from the teachers' perspective may provide opportunities to describe teaching and teachers' problems of practice that are more representative of what actually happens in classrooms, thus lessening the gap between research and practice.

To teachers, understanding what happens during lessons may matter most. The quest to understand the resources teachers need to solve the central problems of their work should be focused on classroom situations (Noddings, 1992). The most direct method of including the teacher's perspective on teaching problems is to allow the teacher to identify the problems of practice (e.g., Sherin, 2002a), although it is rarely done.

IMPLICATIONS FOR FURTHER STUDY

I propose that researchers have not concentrated enough on nonexpert, ordinary teachers. Evidence shows the deficits inherent in novice teachers' practices, and research has produced frameworks and categories of effective teaching behaviors gained from studying experts. However, not have enough studies describe the teaching practice of those in between (Brown, 1992; Noddings, 1992).

Additionally, studies have been conducted to explicate teachers' problems of practice with a focus on classroom interactions, but in many, the problematic situations are identified and described from the researcher's point of view (Noddings, 1992; Simon, 1999). Researchers who make a concerted effort to understand the teachers' point of view can provide a better understanding of teaching practices that perhaps can lesson the tension between theory and practice.

Teachers who adopt practices for teaching mathematics as recommended by reform face particular challenges. Reform practices position teachers as having to solve problems that cannot be preplanned. However, teachers who are successful at this more complex kind of practice are open to novel situations and prepared to rethink teaching on a continual basis (Franke et al., 2001). At this stage, teachers' voices are more important than ever.

Finally, the insights gained from tapping into teachers' perspectives using methods that allow teachers to identify problems of practice of their own concern have implications for professional development. Teachers may be better able to incorporate what they learn in professional development settings if the discussions are centered on their concerns (Walen & Williams, 2000). Teachers may feel more empowered by being allowed to focus on their concerns first, as in Sherin's (2002b) video club. When teachers are directed to notice other concerns such as those that researchers identify, they can see connections between their initially identified problems and the broader concerns of researchers.

SECTION 2

POINTS OF INTEREST: PREDICAMENTS RECOGNIZED BY TEACHERS AS THEY IMPLEMENT INQUIRY-BASED PRACTICES FOR TEACHING FRACTIONS

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Points of Interest: Predicaments Recognized by Teachers as They Implement Inquiry-Based Practices for Teaching Fractions

INTRODUCTION

Recent studies have identified many of the difficulties for teachers in implementing inquiry-based instruction (Empson & Junk 2004a; Floden 1997; Hiebert & Carpenter, 1998; Lampert, 2001; Sherin 1997). Throughout this paper I refer to *inquiry-based instruction*, meaning instruction in which teachers are expected to pay attention to the knowledge learners bring to instruction and to monitor students' conceptions during instruction. Inquiry-based instruction is usually supported by instructional methods that encourage students to express their ideas about the subject through construction of solutions and having opportunities to share these ideas, extend them and revise the ideas (Bransford, Brown, & Cocking, 2000). These accounts of difficulties in implementing inquiry-based instruction are usually reported from the researcher's point of view, rather than the teacher's (Walen & Williams 2000; Windschitl, 2002). But ignoring the teachers' perspective may cause a gap between what we think we know about problems of practice and the in-depth understanding we need to support teachers.

The goal of this study was to describe and categorize the kinds of problems teachers experienced as they implemented problem-centered instruction. I asked teachers to identify situations during fraction lessons that were problematic in some way. I wanted to find out what problems of practice these teachers identified and how they reasoned about them. Analyses of teacher-chosen interactions, teachers' descriptions of the problems, and their thinking about a subset of the interactions provided a profound sense of the complexity involved in implementing inquiry-based practices as well as an appreciation of teachers' commitment to grapple with perplexing and sometimes frustrating situations.

RATIONALE

I wanted to be able to provide findings about an ordinary set of teachers who were willing to implement new practices when asked to do so. In the past 20–30 years several researchers have studied the difficult task of implementing inquiry-based mathematics (Koehler & Grouws, 1992). The teachers in these studies often represented the extremes, either novice or expert (Ball, 2000; Lampert, 2001; Leinhardt, 1989; Taplin & Chan, 2001; Yackel 2002). However, I believe that this "nonexpert but willing" type of teacher is more representative of many teachers who currently are learning to implement inquiry-based practices through the adoption of new kinds of curricula. Because most teachers are neither novices nor experts, it is crucial to develop more complete descriptions of how these teachers frame their teaching.

Given that whole schools and more often entire districts have chosen to adopt inquiry-based programs, many teachers are in the midst of learning to teach mathematics in new ways (Empson & Junk, 2004a; Floden, 1997; Simon, 1995). Relatively unexplored are details of teachers' problems of practice and the way teachers engage with these problems during lessons designed to elicit children's mathematics (Ball, 1996; Ball & Cohen, 1999; Kilpatrick, Swafford, & Findell, 2001). Researchers must recognize the practical concerns of this key group of teachers to gain a more widespread use of teaching approaches known to facilitate children's deeper understanding of mathematics (Kilpatrick et al.). We can learn more about teaching mathematics and what it means for teachers to implement inquiry practices if we understand what situations teachers find problematic. Rich, detailed descriptions of the problems of practice as teachers see them can provide better knowledge of teachers' interpretation of practice to the current body of research on teaching (Koehler & Grouws, 1992; Lampert, 2001). Insights gained studying the problems of practice from the teachers' perspectives may help explain differences in teachers' learning experiences; reveal the impact of curricular changes; and provide understanding of how teachers interpret, respond to, and add to their knowledge base for teaching (Simon, 1999).

CONCEPTUAL FRAMEWORK

Teaching often has been considered a problem-solving task (Ball & Bass, 2000; Burbles & Hansen, 1997; Carpenter, 1988; Dewey, 1933). How researchers conceptualize teaching greatly influences the way research on teaching is conducted as well as how teaching is interpreted. Descriptions of the nature of teaching and of the problem solving teachers must do have included a range of features. Particularly, researchers should consider descriptions of the teachers' role, the types of teaching problems teachers need to solve, and how the teacher's perspective is included (Koehler & Grouws, 1992).

Conceptions of Teaching: Teaching as Problem Solving

Research from the last half century often has conceptualized teaching as the application of certain effective teaching behaviors to achieve certain curricular goals. In this case, providing descriptions of teaching is a matter of describing effective teaching behaviors (Lave, 1996): "It reduces teaching to narrowly specific prescriptions for what should be transplanted into the heads of kids" (p. 158). This conception unintentionally may project an image of teaching that depicts the problems of teaching as predictable. Further, teachers may interpret this view that teaching problems can be solved by closely following textbook lessons and applying ready-made solutions. This image of teaching supports the belief that learning is a matter of mastering skills and concepts through transfer of knowledge (Carpenter, 1988). However, the teachers' perspective is ignored by assuming the teacher acts as a technician (Schon 1988) and as a person who applies solutions to problems that someone else has solved (Lave, 1996; Taplin & Chan, 2001).

When conceptions about teaching are aligned with theories that support children's construction of knowledge, teaching cannot be as closely prescribed or scripted, because the teacher needs to respond dynamically to students' developing understandings about mathematics (Lampert, 2001; Sherin, 1998; Simon, 1999). When researchers adopt this conception, they need to consider the problems teachers choose to solve and how they reason about those problems. Researchers who privilege teachers' perspectives gain knowledge of teaching that reflects more genuine, accurate accounts of the problems of practice (Simon, 1999).

To begin to think about teaching as problem solving in this particular way, definitions of mathematical problem solving can be used to describe what teachers must do (Carpenter, 1988). The National Council of Teachers of Mathematics (2000) standards defined problem solving as "engaging in a task for which the solution method in not known in advance" (p. 52). Further,

> In order to find a solution, students must draw on their knowledge, and through this process they will often develop new mathematical understandings. Solving problems is not only a major goal of learning mathematics but also a major means of doing so. (p. 52)

Replace key words with teaching and teachers and this definition of problem solving reads: In order to find a solution, *teachers* must draw on their knowledge, and through this process they will often develop new understandings *about teaching*. Solving problems is not only a major goal of learning *about teaching,* but also a major means of doing so.

Teachers as Problem Solvers

The previous section described two ways of conceptualizing teaching. The first conception, the *effective teaching conception,* came about during the process-product research of the 1960s and 1970s. A second conception, *teaching as inquiry,* holds a view that teachers need to understand the teaching process and that teachers need to accept the unpredictable nature of teaching. In the second case, in addition to studying and learning about teaching behaviors, the process itself must be understood.

Both conceptions of teaching assume teachers are problem solvers; however, the effective teaching conception assumes that teaching problems and their solutions can be scripted. Cochran-Smith and Lytle (1999) described this conception as “knowledge-for-practice” in which the teachers’ roles are framed as “knowledge users, not generators” (p. 257). This leads to the idea that if researchers can just compile enough examples of expert teaching, then they can simply pass the information to teachers through classes, textbooks, and school curricula.

For example, Lienhardt (1989) reported about expertise in teaching mathematics. She described effective teaching in this way: “Expert teachers used students’ work and remarks to frame and teach concepts; expert mathematics teachers’ lessons were characterized as when teachers presented multiple representations, gave clear explanations and had minimal student confusion during lessons” (p.269). In this depiction of teaching it is easy to see that if teachers were to have difficulty with “multiple representations,” they might simply access this knowledge in a textbook listing suggestions for multiple representations. In addition, Lienhardt’s study described teachers who use a more didactic form of teaching, one that assumes learning occurs through the transfer of information from teacher to student. Under this guise, the practice of teaching may be oversimplified, which results in some educators assuming that teachers need simply to adopt expert behaviors to become expert teachers (Darling-Hammond, 1996).

The second conception, *teaching as inquiry*, frames teaching as problem solving based on constructivist theories of learning (Ball & Bass, 2000; Carpenter, 1988; Lampert, 2001). The teachers' tasks in the teaching as inquiry view are of a different nature. Teachers have to solve problems such as how to probe student thinking and decide what problem to give next (Ball & Cohen, 1999; Hiebert et al., 1997; Steinberg, Empson, & Carpenter, 2004; van Zee & Minstrell, 1997). In this vein, identifying explicit teaching behaviors may not be as helpful because teachers' problematic situations are not predictable; instead, teachers' reasoning about the problems can be explained (Darling-Hammond, 1996). So, researchers must pay attention to teachers' thinking and decision making as they solve particular types of problems, such as how to attend to student thinking, make ideas public, and build on students' strategies for solving problems (Ball, 1996; Featherstone, Smith, Beasley, Corbin, & Shank, 1995).

Researchers also have acknowledged the importance of recognizing the teacher's role in framing these problems (Carpenter, 1988; Cochran-Smith & Lytle, 1999; Simon, 1999). Cochran-Smith and Lytle described that from the "knowledge-*in*-practice" perspective, "*professionals [teachers] pose and construct problems* out of the uncertainty and complexity of practice situations and that they make sense of situations by connecting them to previous ones and to a variety of other information" (p. 273, emphasis added). Also, from the "knowledge-*of*-practice" perspective, teachers are described as problem solvers: "Teachers across the professional life span from very new to very experienced *make problematic their own knowledge* and practice as well as the knowledge and practice of others" (p. 274, emphasis added). These two descriptions of how teachers use their knowledge place emphasis on the teachers' role in solving problems of practice through their own inquiry.

Types of Teaching Problems

What situations have researchers identified as problematic? Some researchers have categorized teaching problems in terms of the teaching context, such as solving problems during planning, during lessons, or about assessing children's work (Fennema, Franke, Carpenter, & Carey, 1993; Putnam & Borko, 2000). Others have detailed the problems as decision-making points. Ball and Bass (2000) proposed thinking of the problems of practice in terms of *core activities*. The authors listed the core activities in problem solving terms: "Figuring out what students know; choosing and managing representations of mathematical ideas; appraising, selecting, and modifying textbooks; deciding among alternative courses of action; and steering productive discussion" (p. 453).

Situations in which teachers interact with children's ideas about mathematics have been recognized implicitly as an especially powerful kind of context for teacher problem solving (Franke et al., 1998; Jacobs & Ambrose, 2003; Kazemi & Stipek, 2001; Knapp & Petersen, 1995). Often the students in these studies are allowed to devise their own ways of solving problems because of the curriculum design (e.g., Investigations, Connected Mathematic Project, and others) or because of an approach to presenting problems that encourages children to invent strategies to solve them, such as Cognitively Guided Instruction. Within the activity of understanding and responding to children's thinking, teachers inevitably deal with children's mathematical ideas with which the teachers have little previous experience (Ball &Cohen, 1999; Carpenter, 1988; Sherin, 1997; Taplin & Chan, 2001).

Ball and Bass (2000) also claimed,

> Teachers need also *need to puzzle about the mathematics in an unanticipated idea or formulation proposed by a student*, to analyze a textbook presentation, to decide the numerical parameters of a problem, to make up homework exercises, and to consider the relative value of two different representations in the face of a particular mathematical issue. (p. 453, emphasis added)

Lampert (1997) asserted that teaching is about practice that "reveals vividly the mathematical reasoning involved in choosing and using particular representations, in managing complex classroom discussion, and in designing a problem or figuring out how to formulate a good question" (p. 150).

Ball and Bass (2000) also pointed out the particular problem solving that teachers must be able to do. They described teaching with a situation in which a child has produced a solution that surprises and perplexes the teacher. In this case the teacher must understand the student's strategy mathematically and then decide how respond both mathematically and in a way that is pedagogically consistent with an inquiry-based approach. Ball and Bass claimed that this situation is "the kind of mathematical problem solving in which teachers regularly engage" (p.89).

Schön (1988) explicitly recognized that when teachers teach, they might encounter a situation in which previous experience does not work as they expect:

> A familiar routine produces an unexpected result; and error stubbornly resists correction; or, although the usual actions produce the usual outcomes, we find

> something odd about them because, for some reason, we have begun to look at them in a new way (p. 26).

Consequently, the teacher has to reconsider what he or she knows about the situation (knowing-in-action) and either deal with it as it arises (reflection-in-action) or think about it later (reflection-on-action). Schön claimed the activity of reflection-in-action has a critical function in teaching because it gives rise to on-the-spot (Lampert, 1997) experiments in which previous understandings about situations are tested. This is the particular type of problem solving that frames the teachers' problem situations addressed in this study.

Choosing and Framing the Problem to Solve: Whose Perspective?

Whereas these problems certainly comprise important situations in teaching, how they are described and categorized by researchers may idealize the situations. This kind of treatment may not lead to the kind of support teachers need to improve practice (Darling-Hammond, 1996; Schön, 1988). The underlying assumption that teachers experience these situations as choice-making opportunities may be inaccurate. Consequently, the descriptions may miss nuances between the kinds of problems that are most salient for teachers and the disposition teachers have toward them.

The descriptions in the last section of types of problems illuminate teaching as problem solving and point to certain types of situations as key opportunities to solve problems. These studies and others (e.g., Ma, 1999; Shulman, 1987) study teaching and the problems of practice primarily according to the researcher's perspective

(Simon, 1999; Kennedy, Ball, & McDiarmid, 1993). However, listing particular problems of practice from the researcher's point of view may be incomplete, because the problems identified may not reflect authentically situations that are perceived as problems by teachers. Hiebert, Gallimore, and Stigler (2002) pointed out that the distinctions made by researchers between the types of knowledge teachers need to solve their teaching problems do not reflect the integrated nature of how teachers experience their problems of practice. They characterized the teachers' knowledge as one organized around problems of practice:

> Whereas researchers often are interested in making distinctions among types of knowledge, practitioners often are interested in making connections. Researchers have identified many kinds of teacher knowledge-content knowledge, pedagogical knowledge, and pedagogical content knowledge (Shulman, 1986). There is also knowledge of students—what they know and how they learn. In practitioner knowledge, all these types of knowledge are intertwined, organized not according to the type but according to the problem the knowledge is intended to address. (p. 6)

One method researchers have used to address this divide between the researcher and teacher perspectives is to place the researcher in the role of teacher (Heaton, 2000; Lampert, 2001; Simon, 1999). In *Teaching Problems and the Problems of Teaching*, Lampert (2001) told the story of her own teaching experiences, focusing carefully on the way she made decisions about the problems of practice she encountered as she implemented problem-based instruction. However, this type of account has its own limits for generalizing about all teachers as her descriptions and reflections of the problematic situations were dependent on her extraordinary ability to integrate a researcher's perspective with her perspective as a practicing teacher. Lampert admitted this problem as a limitation, as did Ball (1996).

I propose that if teachers do agree on what the problems of practice are, they may not have the same reasons for seeing the situation as problematic as researchers do. Because problem solving entails the problem-solver's recognition of a problem to be solved (Charles & Silver, 1988), problems in teaching should be described in terms of the teacher's immediate experience. Describing problems of practice as observable situations or in the form of teacher's reasoning presents these problems through the "eye of the beholder." It is necessary to prioritize the teacher's perspective, because the problems of practice depend on the teacher's role in choosing and framing each problem as it occurs.

Problems of Practice as Dilemmas and Predicaments

Careful consideration of how teachers view their problematic situations can help ensure focus on the teacher's point of view. Two ways the problems of practice have been conceptualized are as dilemmas or as predicaments. In Lampert's (2001) work, she often began the description of a problem of teaching with a juxtaposition of two or more goals. Then she described how she attempted to achieve a balance between the goals. For example, one problematic situation is teaching while leading a whole-class discussion. Lampert (2001) wrote, "As I interact with the whole class at once, I need to maintain overall coherence while drawing different kinds of individuals into a common experience of the content" (p. 143). In her experience teaching problems present themselves as dilemmas.

Dilemmas

Beyond a simplified and common definition of a dilemma as a difficult, unsatisfactory choice between two or more alternatives, dilemmas are framed by a tension between what is and what could be. Many researchers have characterized teachers' decision-making tasks as dilemmas (Burbles & Hansen, 1997; Hiebert et al., 1996; Lampert, 1997; Windschitl, 2003). Burbles and Hansen defined a dilemma as one of a dual perspective, "of seeing at the same time the possibilities and limits, the gains and the costs, the hopes and the disappointments, of any human endeavor" (p. 66). Windschitl (2003) claimed, "Dilemmas are aspects of teachers' intellectual and lived experiences that prevent theoretical ideals of constructivism from being realized in practice in school settings" (p. 132).

While some teachers experience the problems of practice as dilemmas, like Lampert (2001), these assumptions rest on what teachers are able to consider in the context of teaching. Conceiving these situations as dilemmas assumes that teachers make conscious decisions between clearly defined but competing solutions while teaching. Not all teachers have the background and predisposition to be able to consider their problems of practice in this highly idealized way. Additionally, this characterization of teaching problems as dilemmas may influence the researcher's interpretation of the problem and subsequent conclusions about teachers' thinking, leading to claims that are more in the researcher's head than the teacher's.

Predicaments

Perhaps the reality for most teachers is that their problems of practice feel more like predicaments (Burbles & Hansen, 1997). “A predicament is a problematic situation seen in terms of a difficult decision and implies that one does not know what to do and is considering it rationally” (*American Heritage Dictionary,* 1976, p. 1031-1032). Unlike dilemmas, predicaments are not conceived as a conflict of choices, but as a feeling of *not knowing what to do*. Indeed, during lessons where teachers allow children to use a range of strategies to solve problems, teachers often face the challenge of interpreting their students’ work, particularly when the students’ constructions may be completely unfamiliar to them. This challenge is one example of how teachers experience their teaching problems as predicaments, since they may have few if any experiences dealing with situations like these.

Why does it matter if a problem of practice is characterized as a dilemma or a predicament? Both dilemmas and predicaments are situations in which the teacher has no immediate solution to a problem. Both dilemmas and predicaments have uncertain outcomes. However, as I have argued, many teachers in the throes of implementing new practices experience their situation differently than researchers do. In other words, some teachers may not have the disposition to consider competing solutions, but simply may feel that the situation is not meeting their expectations. The distinction between dilemma and predicament captures this difference.

For example, both teachers and researchers have recognized the importance and the difficult nature of interpreting children’s work. The researcher may view the problem of interpreting children's thinking as one of knowing certain mathematics and making decisions about what to do next. If the problem yields an array of acceptable but imperfect solutions, the teacher may feel it is a matter of making a hard choice with certain trade-offs, thus it is a dilemma. However, if the teacher has a limited array of approaches, the teacher simply feels "stuck" or "puzzled" by the problematic situation. Thus, this is a different sort of problem, and the situation may feel more like a predicament. Finally, the teachers' experiential sense of the problem differs from the idealized version of a problem as a dilemma, especially if they are relatively inexperienced in dealing with children's thinking. Consequently, understanding the teacher's sense of the problem as a predicament rather than a dilemma may provide understanding for why certain problems may be relevant.

Nonetheless, predicaments such as these are beneficial for teachers. As Burbles and Hansen (1997) explained in their book, *Teaching and its Predicaments,*

> Predicaments compel people to reconceive their circumstances and what they can realistically accomplish. Predicaments require compromise and trade-offs. People can always elect to sidestep predicaments, but that course of action usually means abandoning human possibilities rather than creating new ones. (p. 9)

RESEARCH QUESTIONS

This study was designed to answer two particular questions:

1. What are the problems teachers identify as they teach children using reform curricula?

2. What do teachers think about these problems?

I wanted to address these particular questions because I felt they would centralize the teachers' perspective and would provide insight to what teachers notice as problematic. Understanding what "nonexpert, but willing" teachers see as problematic and how they think about the situations can lead to better support for teachers implementing reform practices.

I supposed that given the opportunity, teachers would identify some of the same issues that researchers would. After working with teachers for a number of years in a variety of settings, I saw that teachers often were concerned with similar issues, but explained the reasons for them in a different way than I would have. On the other hand, I worried that opening up the research in this way might only reveal teachers' frustrations with more general problems such as discipline or might be limited to sociomathematical issues such as how to get students to share strategies or listen to each other. I hoped that providing teachers more control over what we talked about in the interview would elicit a more engaged type of responses, increasing the validity of my claims. I also expected that teachers would feel some tension within their interactions with children, since the belief survey showed a mix of beliefs about children's thinking.

METHODS

Participants

Four teachers were chosen from a group of 16 teachers who were part of a larger study involving professional development on conducting discussions with children about fractions. Ms. Marks taught third grade, Ms Edwards and Ms Ingram taught fifth grade, and Ms. McDonald taught sixth grade. All taught in high-need inner city schools. None of the teachers in the sample had more than 5 years of experience with inquiry-based curricula. All 4 teachers actively participated in the after-school professional development sessions. This sample represented teachers who were willing to implement new practices but were not completely developed in using inquiry-based approaches.

As part of a larger project (Empson & Drake, in progress) the teachers in the study completed a Web-based survey developed by the Integrated Mathematics and Pedagogy project (Ambrose, Clement, Philipp, & Chauvot, 2004). Beliefs assessed were associated with teaching mathematics through problem solving and building on children's thinking. For this study, the results were coded by another researcher according to the coding guidelines from that project (see LoPresto, in progress). Teachers' responses were coded from 0 to 3, with 0 indicating no evidence of the belief and 3 indicating strong evidence of the belief. The outcome of the analysis of the 4 teachers' beliefs showed some evidence for all the beliefs in most of the categories, but each teacher reported various strong to weak beliefs. In other words, none of the teachers showed strong beliefs in all categories, but no teacher was coded as weak in all the categories.

The belief survey results substantiated the claim that the teachers were interested in inquiry-oriented math practices but had not fully invested their teaching methods to inquiry-based approaches. The survey results showed that all 4 of the teachers had mixed beliefs on the nature of learning and knowing mathematics. In other words, they were not exactly sure how to teach in this way, but they believed children should learn mathematics differently and with understanding.

Data Collection

The teachers' lessons (about one hour each) were videotaped 7 to 10 times each. In all of the videotaped lessons, teachers used the curriculum, Investigations in Data Number and Space (TERC, 1995–1998) or Connected Mathematics (Fey, Fitzgerald, Friel, Lappan, & Phillips, 1997; Lapan, Fey, Fitzgerald, Friel, & Phillips, 1996), and occasionally, problems developed by district personnel. Both of these curricula are designed to engage children in mathematics through exploring topics in depth. "Students are encouraged to reason mathematically, develop problem-solving strategies, and represent their thinking using models, diagrams, and graphs" (TERC, n.d., para. 1).

After each lesson teachers answered a postlesson reflection survey (Appendix A) designed to find out what interactions during the lesson provoked their thinking in some way. The survey simply asked,

> Did anything happen today during the lesson that caused you to stop and wonder what to do? Who was involved and what was it about? How did you deal with the situation? (Describe however many situations of this kind you experienced today.)

In their responses, teachers identified one or two portions of the lessons where they perceived a problem for one reason or another. In all, there were 44 identified interactions from the observations. At the conclusion of the unit, these teacher responses and corresponding video clips were analyzed for themes, and three clips were used in the postunit interview (Appendix B) with each of the teachers. The interview questions were designed to probe the teachers' thinking about the interaction to get a deeper understanding of how teachers reasoned about their problems of practice.

Analysis

I analyzed the postlesson reflection survey and the corresponding video clips of the interactions to identify emergent themes that could help describe the problems of practice. This analysis identified particular features of the situations. These features described the teaching situation and the responsive nature of the interactions. Also, these features were themes that occurred across all 4 teachers' identified situations, and the relationship between the features pointed to patterns in how the teachers perceived their problems of practice. Data from the interviews provided important information about how teachers viewed their predicaments, and the teachers explained why they chose particular interactions as problematic.

After summarizing and in some cases transcribing the data, I used the database program, File Maker Pro®, to organize information from the postlesson reflection, video clips, and interviews. All primary data sets were included in the database. Identified situations were sorted by type to check for consistency within each feature and to provide a way to look for links between the features. For instance, if the situation concerned a student strategy, it was added to the other situations that had student strategy as a feature. Then, items within the category of student strategy were analyzed again into subcategories. Those subcategories then were analyzed in terms of their relationship to teachers' supportive actions during the situations.

Postlesson Reflection

The teachers' written postlesson reflections were analyzed in a first pass to see if there was enough pertinent information to find the interaction to which the teacher referred. In some cases the response was general, such as "I hate it when the kids are not ready," which did not refer to a specific child and made no reference to mathematics. If there was only general information as in the example, the summary of the entire lesson was paired with the teacher's comment. All of these were later categorized as problems with management or the curriculum. Most of the teachers' comments mentioned a specific interaction, in which case the interaction was found and summarized. Later the summaries along with the postlesson reflection were analyzed for trends. These trends became the problem features, and the tapes were reviewed if needed.

Lesson Interactions

Each lesson was summarized in terms of subsets of interactions identified from the postlesson reflection data. A total of 35 lessons produced 44 identified interactions. Six of the written responses from the postlesson reflection were too general to identify a specific interaction on the video. A first pass noted themes that were identified as significant. As each of the remaining 38 interactions was summarized or transcribed, the problem the children were working on was noted as well as other important details such as how the teacher responded, whether or not the student was using a strategy learned in instruction, and how the child represented the strategy.

Postunit Interviews

Each interview was conducted using a semistructured protocol that allowed probing the teacher's thinking about the problem. Within each interview three video clips representative of the themes within the identified problems from teacher were shown as a method of stimulated recall. All interviews and accompanying video clips were transcribed. Trends within and between teachers showed common themes in teachers' perspectives about their situations, what resources they may have used to support themselves during the interaction, and what additional knowledge they would have liked to have had during the interactions.

Inter-Rater Reliability

Three sets of codes were used to describe the features of the situations. All of the sets were subject to reliability tests. Three graduate students from the Science and Math Education program agreed to code these data. One third of the situations were randomly assigned in groups of six to each coder. The first set of codes describing the type of situation was in agreement at 71%. The differences were discussed and definitions refined, which resulted in a 94% agreement. The second set of codes meant to identify the mathematical context of children's work was in agreement 100%, and the third set of codes describing the teachers' responsive actions was in agreement for 85% of the codes. Discussion of the definition and additional viewing of the videotapes and reflection data reconciled the disagreements about math context and increased the reliability to 94%.

FINDINGS

In this section I report on the features of teachers' identified problems of practice. Data from the written postlesson reflection, video-taped interactions, and postunit interview were combined to make claims about the nature of the predicaments. Describing the situations teachers wondered about in terms of their features and considering possible relationships between features provided a way to make preliminary claims about what kinds of situation give rise to predicaments. Initial analyses of the postlesson reflection and the postunit interview revealed that the teachers felt they were in a predicament of sorts, rather than in a dilemma in which they could consider options and consequences. During the postunit interview, teachers revealed how they reasoned about their situations and why they experienced them as predicaments.

What Did Teachers Wonder About?

In this section I introduce and describe the features of the situations teachers wondered about. These descriptions provide a sense of the situations and circumstances in which a teaching problem might arise for teachers. Although I am not claiming that teachers necessarily will experience a problem when they are in these situations, some circumstances appeared more likely to cause a problem for teachers than others. (I will return to this point in the discussion of teachers reasoning about the situations.) Situations were analyzed in terms of three characteristics.

1. I found that the *types of situations* could be categorized in terms of the activities that were prominent in during lessons. A small portion of situations concerned management problems or curriculum-implementation problems. Most of the situations concerned children's work that included issues of children's thinking and their strategies.

2. Within *children's work*, issues of validity, and children's use of informal mathematics framed teachers' problems. A surprising number of situations had to do with children's use of area representations of fractions.

3. Teachers sometimes wondered about the quality of their *responses* to children's work. The teacher sometimes directly referred to how they were able to

respond as a problem (e.g., “I did not want to tell too much”). For other situations, the teacher’s response was a particular type that could be identified through observing the interaction. Teachers responded to students’ strategies with supporting actions, reflecting a range of responsiveness to student thinking. Analyzing the situations in associated with the response features added insight to why these situations may have been problematic for the teachers. In some cases, they wondered about their ability to respond, but in others the level responsiveness seemed to be a result of their predicament.

Types of Situations

I categorized the situations by their main features having to do with (a) managing students’ attention, (b) implementing tasks, and (c) understanding and responding to students’ work (see Table 2.1). When the problem concerned students’ work, these situations could be subdivided into three subcategories: solutions based on (a) a valid, teacher-introduced strategy (consisting primarily of strategies that were alternatives to standard algorithms), (b) a valid student-created strategy, or (c) an invalid strategy (see Table 2.2).

Table 2.1

Types of Situations

Type of situation	Definition	Number of situations (44)
Student's attention	The teacher expresses difficulty with student's behavior	4
Implementing tasks	The teacher expresses difficulty with the task as suggested by the text.	7
Student work	The teacher expresses difficulty or her comments are associated with interaction with a student and the student's strategy	33

Table 2.2

Features of Student Work (n=33)

Valid teacher-introduced (alternative) approaches	Strategies introduced by the teacher, usually as a result of suggested activities in the curricula. None of these strategies were algorithms.	8
Valid student-created approaches	The strategy is introduced by the child, and leads to or could have lead to a correct solution to the math problem	9
Invalid approaches	These approaches could not have or do not lead to valid solutions.	16

Managing Students' Attention

Teachers wondered what to do when their students did not pay attention. Four situations concerned management of students. In these situations students were not participating in the lesson for some reason. Three of these situations concerned one particular child's inability to pay attention during math lessons. The teacher was particularly aware of this child, because she was having difficulty in math but often refused to participate in the lesson. In the fourth situation, the class was distracted and restless and so had they problems paying attention during whole-group discussion.

Implementing Tasks and Managing Materials

Seven situations concerned difficulties with implementing the task. Sometimes the teachers were frustrated when the materials were lost or hard to keep track of, and at other times they questioned productivity of the tasks suggested by the curriculum. In the Investigations curriculum lessons are often linked to particular student-created manipulatives that are used throughout the week. Teachers believed that having these manipulatives was crucial to being able to do the activities from day to day. For instance, sets of lessons in the fifth-grade book are designed to help children understand the relationships between fractions and percent. The children use a percent strip that has 100 sections to shade the given fraction and name the percent equivalent (see Figure 2.1). If children misplace the strip, they no longer have the materials necessary to participate in the lesson. Ms. Ingram said, “I had to stop and wonder what to do when half the class couldn’t locate their equivalence strips.”

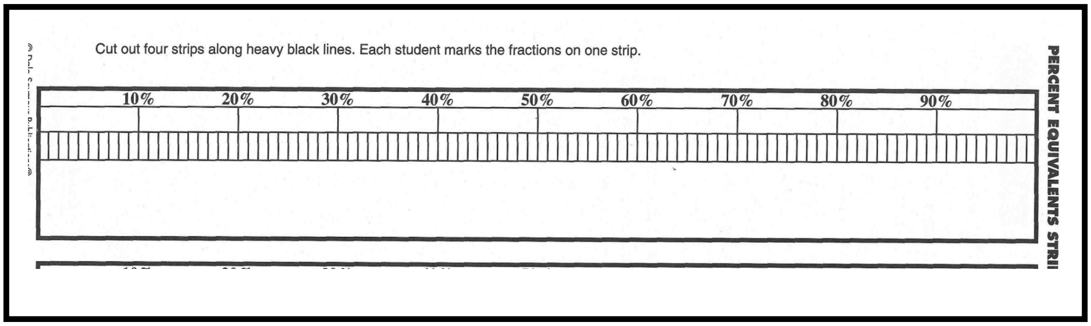
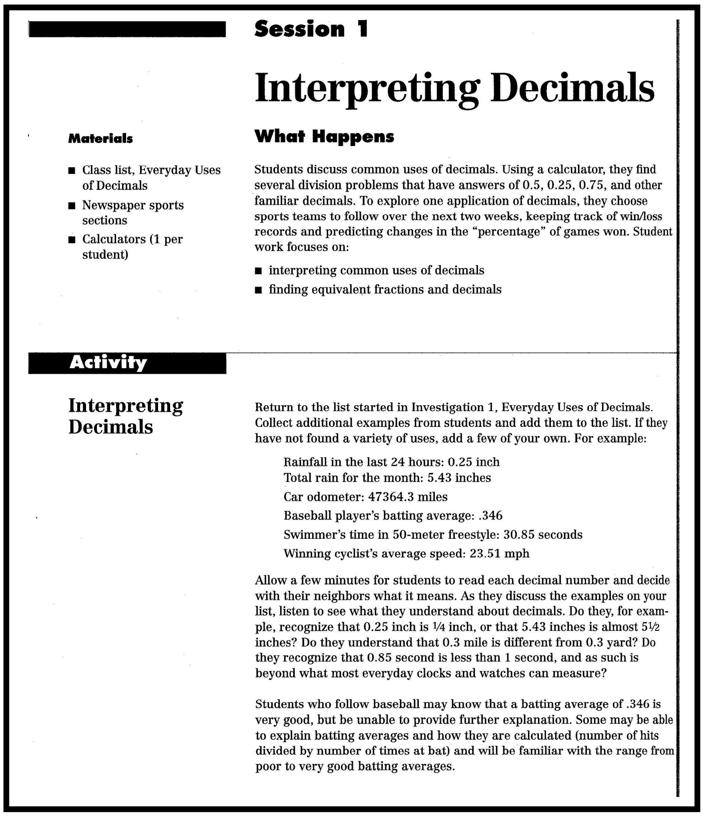


Figure 2.1. Equivalence strip.

Sometimes teachers thought the task was unproductive because the design did not support the mathematics concept for one reason or another. For instance, in the third-grade class, Ms. Marks recognized that children using the fraction cards to make one whole could have made something that looked like a whole but was not actually mathematically equivalent. The problem, as she saw it, was when children folded and cut the fraction cards, reconstructing the pieces into a whole depended on the preciseness of the cuts. This legitimate concern led her to propose a different way for doing the activity next time.

> I wasn't quite sure what to do when working with some of the groups when they weren't grouping their fraction pieces into wholes. It's a little bit difficult because different kids cut the different pieces, so they aren't exact and so it makes it less exact, but I think they did pretty well understanding the concept although they all did not make wholes with the pieces they won. Next time, I would have them bring their math journals with them so they can record their fractions that add together to equal one whole and then have a closing discussion with the tables then the whole class to check what fractions they added together to make a whole. (Ms. Marks, postlesson reflection)

Sometimes the productivity of task was in question because the context was not relevant to the children. In a fifth-grade lesson designed to help children see uses of decimals in the real world, students got caught up in the unfamiliar activities used to illustrate uses of decimals (see Figure 2.2). Ms. Edwards had to stop the lesson because the students were confused about what the examples meant. Consequently, she spent a large amount of time during the lesson explaining to kids what these activities on the list were rather than exploring decimals.



Session 1

Interpreting Decimals

Materials

- Class list, Everyday Uses of Decimals
- Newspaper sports sections
- Calculators (1 per student)

What Happens

Students discuss common uses of decimals. Using a calculator, they find several division problems that have answers of 0.5, 0.25, 0.75, and other familiar decimals. To explore one application of decimals, they choose sports teams to follow over the next two weeks, keeping track of win/loss records and predicting changes in the "percentage" of games won. Student work focuses on:

- interpreting common uses of decimals
- finding equivalent fractions and decimals

Activity

Interpreting Decimals

Return to the list started in Investigation 1, Everyday Uses of Decimals. Collect additional examples from students and add them to the list. If they have not found a variety of uses, add a few of your own. For example:

Rainfall in the last 24 hours: 0.25 inch
Total rain for the month: 5.43 inches
Car odometer: 47364.3 miles
Baseball player's batting average: .346
Swimmer's time in 50-meter freestyle: 30.85 seconds
Winning cyclist's average speed: 23.51 mph

Allow a few minutes for students to read each decimal number and decide with their neighbors what it means. As they discuss the examples on your list, listen to see what they understand about decimals. Do they, for example, recognize that 0.25 inch is 1/4 inch, or that 5.43 inches is almost 5 1/2 inches? Do they understand that 0.3 mile is different from 0.3 yard? Do they recognize that 0.85 second is less than 1 second, and as such is beyond what most everyday clocks and watches can measure?

Students who follow baseball may know that a batting average of .346 is very good, but be unable to provide further explanation. Some may be able to explain batting averages and how they are calculated (number of hits divided by number of times at bat) and will be familiar with the range from poor to very good batting averages.

Figure 2.2. Interpreting decimals.

Student Work

The remaining 33 situations concerned students' work. In these cases, students used strategies to solve problems that were (a) valid, teacher-introduced alternative approaches: (b) valid, student-created approaches: and (c) invalid approaches. In an effort to discourage dependence on standard algorithms, the Investigations series and the Connected Mathematics curricula suggest alternative methods of solving problems presented in the lessons. These alternative methods are designed so children can build conceptually based understanding of the content. For instance, instead of teaching children to convert fractions to percents using division (i.e. divide denominator into numerator), fifth graders are encouraged to use what they know about common fractions and to use those relationships to find other fractions. In addition, manipulatives like the 100 grid and percent strips are used to make area representations of percents.

Another method that decreases the necessity of teaching algorithms is to allow children to invent their own strategies. I wanted to make a distinction here between student-invented and teacher-introduced alternative approaches because the nature of situations seemed to have important differences. I found that when circumstances involved a teacher-introduced alternative approach, teachers were surprised about how some students used the alternative approaches; teachers sometimes had only a partial understanding about how these strategies worked. However, because the teachers had introduced the alternative strategy to the class, when children struggled to use them, teachers focused on the execution of the strategy rather than on understanding the children's thinking. On the other hand, when students invented a strategy that was novel to the teacher, their predicament was more about how to make sense of the approach; thus, teachers often focused on the child's thinking about the strategy.

Valid, Alternative, Teacher-Introduced Approaches

Strategies in which students used approaches introduced in school that were not standard algorithms are alternative, teacher-introduced approaches. Both of the curricula used by the teachers in this study introduced alternative strategies. These alternatives are suggested through the use a variety of models for fractions and the absence of standard algorithms. In these situations (7 out of 8) the students had at least a partial solution to the math problem when the teacher began to interact with them. Teachers often were helping children use the strategies they introduced, and often the teachers were faced with new facets of the strategy they had not considered.

For example, Ms. Edwards encouraged an alternative approach suggested in the teacher's guide for finding percent equivalents for common fractions using a 100 grid. This approach entailed counting out the number of squares on a 100 grid equal to the denominator and then shading the amount indicated in the numerator. This is repeated until the squares are accounted for. Figure 2.3 below shows what the solution looked like when Mario and Diego used this method to solve the problem 3/8 = ?/100.

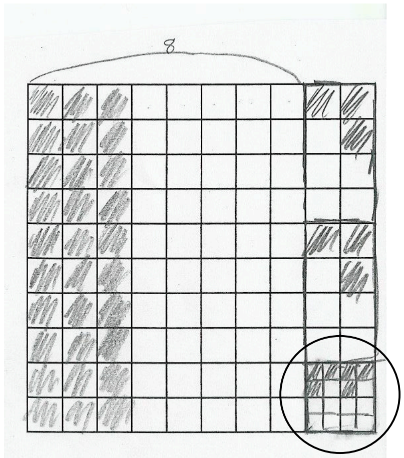


Figure 2.3. Mario’s solution for 3/8 = ?/100.

The children had already begun the shading when Ms. Edwards checked in on their progress. During the interaction she watched as the students shaded in 3 out of 8 tens. Then they chose 8 squares, shaded in 3 and another 8 squares, and shaded in 3. Finally they had just 4 squares (100ths) left. They decided to divide each square into 4 parts. When the two students finished shading in all the parts representing 3 out of 8 as in Figure 2.3, the students along with Ms. Edwards struggled to name the last 4 partitioned, shaded squares (see the circled portion in Figure 2.3) as a fraction of 100. I believe the approach became problematic because the denominator was not a factor of 100, and to shade in 3 out of 8 in the last iteration, partial squares should be shaded, not 3 out of 8 whole squares.

Together, they decided that the shaded portion was 6/8 squares instead of 1 and 1/2 squares. The logic here is that they had portioned 2 shares into 4ths to make 8 partitions, and then shaded 3 of 8 sections, twice. As a result, they incorrectly calculated the percent equivalent to 3/8 by adding 30 + 6 + 6/8 to get 36 6/8. At that point Ms. Edwards questioned the reasonableness of the 6/8 part of the answer, asking, “Is this right? This can’t be right.” After the lesson she wrote what she wondered about: “Mario and Diego dividing 3/8—the last four squares were divided into 4ths each. We redrew and had a hard time thinking of the parts as not wholes” (Ms. Edwards, postlesson reflection). Her attention for this particular case was drawn to executing the strategy and finishing the solution. Rather than a focus on children’s thinking, her comment indicated that she was focused on learning how to use the strategy for herself.

Valid, Student-Created Approaches

Nine predicaments involved valid, student-created approaches. For most of these situations the student had solved the problem and was explaining the strategy to the class or to the teacher. Teachers recognized that the students had correct answers but wondered about the child’s thinking and puzzled about how a child could devise such strategies. Student-created approaches are a result of the child’s construction, so the teacher’s primary resource for understanding it is the child’s thinking. In contrast to student-created approaches, alternative teacher-introduced approaches were used by children as a result of instruction, so the teacher expected herself to understand the approach. On the other hand, student-created approaches are strategies not directly introduced by the teacher, so the teacher is not expected to understand it from an execution standpoint. Instead, teachers can focus on how the student understood the strategy.

For example, Ricky had explained that two same-sized but different-shaped amounts were the same, saying, “But half of this one is half of this one,” and pointing to sections shaded in Figure 2.4. Ms. Marks wrote,

> A few times I wasn’t understanding what a student [Ricky] was trying to describe and that made it frustrating because I found myself losing sight of the original question and what I was trying to get all the students see or understand. Ricky was explaining his thoughts and I got confused.

Later in a private conversation, she told me that she did not know that a third grader would be able to solve a problem without drawing and just could use his own logic like he did. In fact, she confessed that she had not thought of comparing the two areas as halves of halves, but now understood what Ricky might have been thinking.

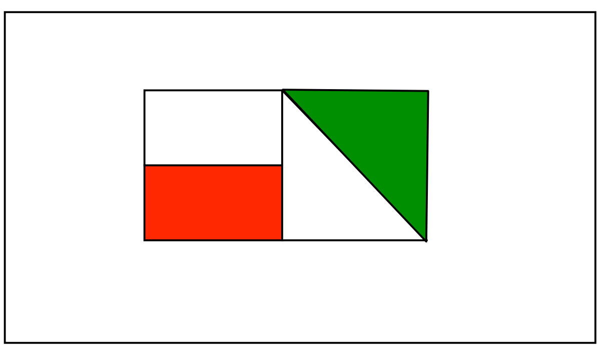


Figure 2.4. Different-shaped fourths.

Invalid Approaches

Sixteen situations involved a student using a strategy that was invalid (see Table 2.2), and thus the teacher perceived the student as struggling. In some of these cases children were inventing a strategy that could not have led to a productive answer. In other cases, the children were applying a previously taught strategy incorrectly; sometimes these were standard algorithms, and at other times they were alternative approaches.

Student-created, invalid strategies. In the following example Ms. McDonald approached Reggie, who had solved a problem involving ordering fractions. The problem the children were solving was to put four basketball players' shooting ratios in order from worst shooter to best shooter: Naomi, 19 out of 25; Bobbie, 8 out of 10; Kate, 36 out of 50; and Olympia, 16 out of 20. Who has a better chance of making the next shot? Reggie made a drawing representing each child's free throw attempts as a fraction bar (see Figure 2.5). Children could have used percentages to compare the four players' shooting accuracy, but Reggie compared the four ratios using fraction bars. This had not been suggested by the curriculum or by the teacher.

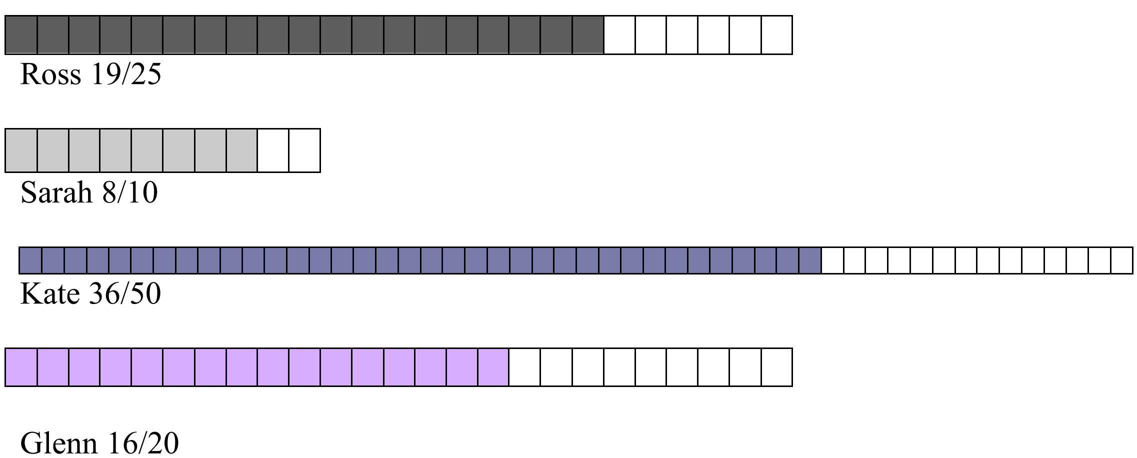


Figure 2.5. Reggie's solution to ratio problem.

Ms. McDonald recognized the inaccuracy of the student's drawing would lead to an incorrect answer but had little success in leading him out of it. She wrote, "Reggie was trying to draw a comparison of 4 fractions and was not succeeding b/c they were not divided into equal parts." She said that the comparison did not work because the divisions of each bar were not even; however, the actual problem with the drawing was that the bars are not the same size. After working with him for a while she understood that Reggie had made each ratio as a fraction of shots out of total attempts, but was unable to show him why his comparison was flawed. She finally told Reggie to go over to another student for help. However, that student had successfully solved the problem using a completely different strategy.

Using rules without understanding. Sometimes these students' invalid approaches were the result of overgeneralizing a rule that the teacher had provided for them. For example, Juan was comparing the numbers 0.078 and 0.708. Juan said he chose 0.078 as the larger number by comparing the two digits farthest to the right. The rule overgeneralized by the student in this case was "the further out to the right of the decimal point, the smaller the number." Ms. McDonald wrote, "Juan on 'b.'—putting in order. Confused by the thousands place—it is bothering me because his logic is that the farther out the number—it is automatically smaller."

Sometimes teachers discouraged the use of procedures and rules unless the student could show they understood them. Ms. Edwards struggled to understand why her fifth-grade students continued to use the long division algorithm, when the assignment clearly was designed to elicit alternative strategies. She dealt with the situation by asking her students to explain what the long division algorithm meant:

> Everyone (automatic) goes to long division or multiplication strategies [windows] that aren't as helpful for understanding the problem. Frank—long division [he] can't explain ("that's what my dad said"). [I] talked to him in the hall about teaching and showing and understanding. Sometimes parents can tell and show us but if it doesn't make sense if we can't explain it. This means we should find another way that we can explain so we can show understanding.

Whether the student was using a student-created or alternative, teacher-introduced strategy or an algorithm, when the strategy was invalid, teachers expressed their view of the situation as one in which they could not understand where the confusion on the part of the child arose. Often they resorted to telling the student more directly what to do or to abandon the strategy altogether.

Children's Use of Area Representations for Fractions as a Special Area of Difficulty

Within all types of student work, the children often used area representations as models for fractions. This feature is particular to the topic of fractions because many of lessons encouraged drawing or using manipulatives as area representations to solve fraction problems. Children's use of these models was a feature of 18 situations. Sometimes the representation was centered on using the model the text recommended, such as clocks, pattern blocks, and fraction bars (alternative, teacher-introduced approaches). Other times the representation was the student's own construction (student-created approaches). Teachers could depend on the representations in addition to the child's explanation to gain understanding of the child’s thinking. However, children used representations in unexpected ways, and sometimes the drawings were unclear.

For example, Mario’s answer for the amount of pizza needed for 12 children to each have ¾ of a pizza to eat was 47 slices. When Ms. Edwards asked him how he arrived at his answer he referred to his drawing in the margin of the worksheet, which showed 12 connected squares, each divided into 4 parts (see Figure 2.6). Mario was not able to explain his drawing in detail for Ms. Edwards, but he did say he knew the number of slices. Still, the drawing was difficult to decipher, and since he did not answer Ms. Edwards’s questions clearly, she had to rely on the picture to interpret his thinking. In an interview she said, “I knew that with this (pointing to picture) he had all the slices there and he couldn’t explain—he couldn’t—I couldn’t get it out of him or I wasn’t understanding his reasons for how he got that many slices.” Despite this difficulty, the drawing provided a venue for communication between student and teacher and gave Ms. Edwards an opportunity to ask Mario specific questions about his thinking.

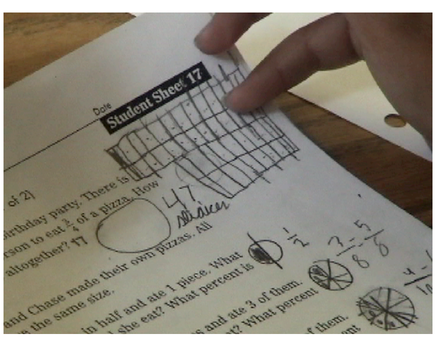


Figure 2.6. Mario’s solution for ¾ x 12.

Additionally, teachers realized that if they could find out how the drawing was constructed, they would understand the strategy better. Mario's teacher wished she had known the history of the strategy, "And he wasn't really able to tell me if I gave him the chance. To explain he was just saying that there were 47 slices . . . and I didn't know how that picture came to be." In a fractional model, one finished picture is the result of several steps, each representing a different aspect of what a child understands. Had Ms. Edwards recognized the drawing was of 12 pizzas cut into 4ths during the lesson, she might have understood Mario's initial answer of 47 as the result of a miscount of 12 times 4 slices and thus been able to support Mario's thinking using his own picture. Perhaps, Mario drew 12 squares first. Then he might have drawn the partitions dividing each square into 4ths. Finally, he could have counted all the slices, making a miscount and saying 47 instead of 48. Knowing "how that picture came to be" could have provided the information about Mario's thinking that Ms. Edwards needed to better support his understanding of the problem.

How Did the Situations Impact the Way Teachers Responded to Students?

Sometimes teachers wondered about how they should respond to their students. Most of this concern related to the teachers desire to support children's thinking, without completely taking over the interaction. Also, teacher's responses arose as an important feature of the situations that involved student work since it seemed that certain types of situations were associated with particular forms of support. Given that these teachers expressed beliefs that children could devise their own strategies and that understanding is as important as or more important than learning procedures, it is not unexpected that they would wonder about their ability to support children's thinking.

Levels of Response

Teachers can respond to students with actions that support children's thinking, ranging from more directive to less directive. The situations were coded according to the teacher's actions using a framework developed by Jacobs and Ambrose (2003) to get a sense of the responsive features of the each situation (see Table 2.3). Appendix C shows the full framework.

Table 2.3

Teacher–Student Interactions

Type of teacher actions	Definition	Number of situations (33)
Directive	Teacher heavily assists. Mostly directive actions like telling students step by step what to do.	11
Observational	Students share their strategy, and the teacher takes few actions, accepts the strategy, and moves on.	5
Exploratory	The teacher asks questions about the students' strategy but does not get a full understanding of the child's strategy.	12
Responsive	Teachers probe children's thinking and get full explanations, such that students' responses can be extended.	6

Directive: Sometimes teachers just told students what to do. The 11 interactions dealing with student work were coded in this study as directive. Interestingly, some of these began with exploration of the child's thinking, but then the teacher would heavily scaffold the child to another way of solving the problem. In most of these interactions the child had an invalid strategy.

For instance, Maria could not begin to solve 50/450 = ?/100. Maria was working on a word problem that stated, "What percent off is a 50 cent discount on a $4.50 item?" Ms. McDonald encouraged Maria to think of the invented strategy she had just used to solve 75/300 = ?/100. However, whereas the number choices of the first problem (.75/3.00 = ?/100) made it easy to build up the equivalence since the reasoning could be: 300 ÷ 3 = 100, so it must be 25% because 3 x 25 is 75. This build-up strategy is difficult to apply to 50/450 = ?/100, since 450 is not divisible by 100. In response to Ms. McDonald's questioning, Maria first said that the answer was 50/100; then she offered the answer as 50/400. From that point on Ms. McDonald's actions became more directive as she heavily scaffolded Maria's work. In the end Ms. McDonald told Maria and her partner, Angel, how to use the algorithm for finding percent (50 ÷ 450). When they struggled to execute the division, she directed them to use the calculator.

> Ms. McDonald: What's 50 divided by 450? Enter it just the way I say it.
> (Angel is putting numbers into the calculator.)
> Maria: Ohhh (shows her the number on the calculator: .11111111).
> Angel: 11
> Ms. McDonald: what 11?
> Maria: cents.
> Ms. McDonald: So is it reasonable to say that 50 cents for 450 that I would save about 11 cents per dollar?
> Students: Yes.
> Ms. McDonald: So what is 11 cents times?

(They then figured what you would have to multiply 11 by [~4.5] to get 50 cents for the discounted rate.)

Observational: Sometimes teachers just listened to their students. There were few interactions of this type, maybe because during classroom interactions, teachers are expected to do something other than observe. Nonetheless, in five of the interactions the teacher's actions could be coded as observational. For example, Ms. Marks listened as Sondra told how she knew that the two 4ths were the same even though they were not the same size (see Figure 2.4). After she explained, Ms. Marks told her that was an interesting observation and said nothing else. She wrote what she wondered about: "Sondra seemed to have made a good point but I really didn't understand. Sometimes it's hard for me to branch discussions from what the kids share because I don't fully understand. Yikes!"

Exploratory and responsive: Teachers probed students' thinking. Teachers often asked questions about students' thinking, but sometimes the questions were more general (e.g., "How did you get your answer?" and "Why do you think that?"). Twelve situations were coded as exploratory. Despite the attention to children's work, teachers were not able to get children to explain their thinking.

Sometimes teachers were able to pose questions specific to children's work and gain clarity about children's thinking. Additionally, in some of these situations teachers were able to extend what children did by posing a related task. Jacobs and Ambrose (2003) described these responsive interactions as having a conversational quality, a "back-and-forth" interchange between the teacher and child. Only six interactions were coded as responsive. During the interaction in the transcript below, Ms. Ingram elicited details about a child's invented valid strategy to find out how he was able to reason about it. Daniel was explaining his solution for a problem comparing 7/8 to 5/6. He had drawn two fraction circles on the board to represent each amount (see Figure 2.7).

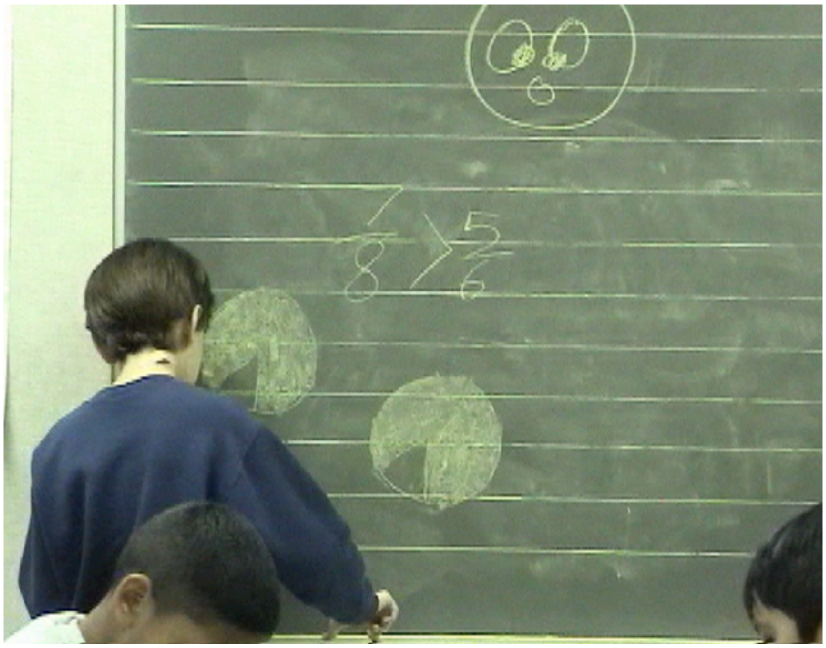


Figure 2.7. Daniel's solution to 5/6 ? 7/8.

Daniel: 7/8 is greater than 5/6.
Ms. Ingram: And why is that? I mean, you've got a—well-- you've given us two diagrams. To me they look exactly the same. Watch. Shakia, you are not paying attention.
Daniel: This is a smaller space than this one (pointing to the unshaded portion of each circle).
Ms. Ingram: How do you know? Why? Why is that smaller? Huh? 'Cause you could see it?
Daniel: 'Cause I could see it.
Ms. Ingram: Why is it has that circle the one that represents 7/8, has it been divided more than the other one? 5/6? (Daniel nods yes). So why is 7/8 greater?
Daniel: It's got more shaded

A few minutes later, Ms. Ingram extended Daniel's thinking by pushing him to be more specific about the parts of his drawing. She asked him to label each unshaded portion, and then he was able to explain that 1/8 is less than 1/6, so 7/8 is more than 5/6.

Relationship of Responses to Other Features of the Situations

The majority of interactions identified as predicaments concerning valid strategies could be coded as exploratory or responsive (see Table 2.4). Invalid approaches seemed to invoke the least supportive responses, because often the teachers reverted to more direct teaching actions that led the students to a correct solution.

Table 2.4

Student Strategies and Types of Responses

Strategy:	Valid	Invalid	Totals
	(17)	(16)	(33)
Response Type:			
Directive	3	8	11
Observational	4	1	5
Exploratory	6	6	12
Responsive	4	1	5

I found that teachers' responses during their interactions were linked to the mathematical context of the children's work. When the predicaments included a children's area representation as a part of their strategy, the teachers were able to be more responsive to children's thinking than for other types of mathematical contexts. In fact, of the 18 interactions that featured a child's representation, only 3 were coded as directive (see Table 2.5).

Table 2.5

Area Representations and Types of Responses

Strategy:	Valid (12)	Invalid (6)	Totals (18)
Response Type:			
Directive	2	1	3
Observational	1	1	2
Exploratory	5	4	9
Responsive	4	0	4

The data also revealed trends within each teacher's coded responses (see Table 2.6). For example, Ms. McDonald had only one instance in which she was able to explore children's thinking. Most of her responses to children were coded as directive. On the other hand, Ms. Edwards was able to explore children's thinking in over half of her predicaments.

Table 2.6

Individual Teachers' Responses to Student Work

Teacher:	Ms. Marks (11)		Ms. Ingram (12)		Ms. Edwards (10)		Ms. McDonald (11)	
Strategy:	Valid	Invalid	Valid	Invalid	Valid	Invalid	Valid	Invalid
Response Type:								
Directive	2	0	1	0	0	1	0	7
Observational	2	0	0	0	0	0	2	1
Exploratory	1	1	1	2	4	2	0	1
Responsive	1	0	3	1	0	0	0	0

Of those 11 directive interactions, 7 were Ms. McDonald's situations. In fact, Ms. McDonald rarely reacted to children's work by taking actions that showed a close attention to the children's work, with the exception of Reggie's solution as described earlier. In that particular predicament, Ms. McDonald was able to ask Reggie's specific questions about the strategy perhaps because it was a set of fractions represented by a drawing. In other interactions in which the invalid approach did not involve an invented model, this particular teacher resorted to heavily scaffolding the child to a correct answer.

How Did Teachers Reason About These Situations?

On the first and subsequent passes through all 44 postlesson reflections and the interviews, I was struck by the similar way teachers felt about these situations. The teachers expressed feelings of discomfort and confusion. They rarely talked or wrote about choosing between one course and another; instead, their feelings indicated that they were in the middle of a predicament. As they reviewed videotapes of their chosen interactions, they talked about reasons why they wondered about them. Teachers expressed ideas about these problematic situations that reflect the nature of inquiry math teaching: Children are solving problems and offering ideas for solutions (Bransford et al., 2000). The comments also reflected the development of a constructivist view of the teacher's role, which was to support students to make connections and build understanding of mathematics (Simon, 1995).

Predicaments not Dilemmas

Heaton (2000), a teacher-researcher, reported feelings of discomfort as she implemented inquiry mathematics for the first time. She described the challenge as

> Loosening my hold on rules and procedures, while searching for some deeper conceptual meaning. Being uncertain was unsettling. I knew a way to stop mucking about with the addition of fractions and move on: [Tell them that you do not change the denominator]. . . . Throughout this series of lessons, I made decisions about what to do next based on my best guess at the time about what was going on with students' understandings of the mathematics, as I struggled to figure out what it was I wanted to them to learn. (p. 130)

These feelings described by Heaton of being uncertain or stuck, the trial-and-error based decision making, and the challenge of finding out how to support children’s ideas were also the ways teachers in this study described their problematic situations. These are not the descriptions I would have expected if the teachers had conceptualized their situations as dilemmas, since they did not talk about competing choices. Instead, the nature of the situations and the way the teachers referred to them as not knowing what to do revealed teachers’ confusion. I believe it is important to make the distinction between predicament and dilemma; if it is a dilemma, one assumes the teacher has enough knowledge about teaching mathematics to consider viable options, but if the situation is a predicament, the teacher may have only a partial understanding of the situation and has to take some sort of action and hope it works.

Feeling Stuck

In the postlesson reflection, teachers wrote that they felt kind of “stuck,” “confused,” and “surprised” with their situation. The teachers’ descriptions reflected feelings that were distinctly uncomfortable or unpleasant. They described the problem as feeling unsure of what to do next and as having “a hard time thinking.” They used phrases including the following:

1. “I got confused.”
2. “I was skeptical and hesitant.”
3. “It [student’s logic] is bothering me.”
4. “I was startled to see it [the strategy].”
5. “I was frustrated.”

6. “I was pretty blank at that point.”

7. “I wasn’t sure how to work with Kevin.”

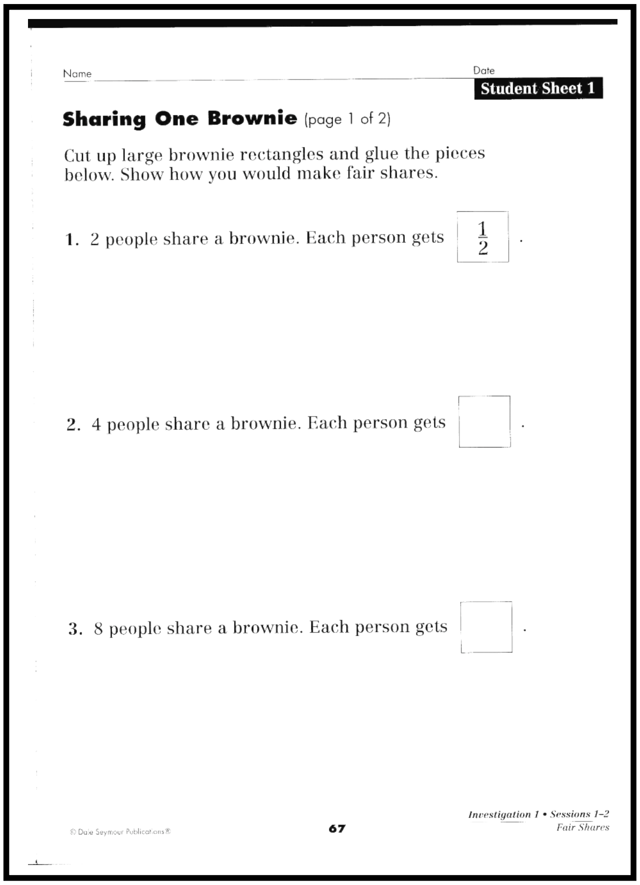
8. “[It was] like herding cats!”

9. “Again some odd clues from students’ behavior as well as their thinking.”

The teachers’ remarks did not reflect a tension about making choices, simply that they were in the middle of problems that needed their immediate attention. Thus these problems were predicaments.

Choosing an Action and Hoping it Works

Teachers sometimes felt stuck or uncomfortable when they did not know what to do and as a result had to try different actions to appeal to students reasoning and to assist them in their problem solving. Ms. Marks identified a situation in which a student was struggling to solve equal sharing problems (see Figure 2.8).


Name Date
Student Sheet 1

Sharing One Brownie (page 1 of 2)

Cut up large brownie rectangles and glue the pieces below. Show how you would make fair shares.

1. 2 people share a brownie. Each person gets $\boxed{\frac{1}{2}}$.

2. 4 people share a brownie. Each person gets $\boxed{}$.

3. 8 people share a brownie. Each person gets $\boxed{}$.

© Dale Seymour Publications® 67 *Investigation 1 • Sessions 1–2* *Fair Shares*

Figure 2.8. Sharing one brownie.

Here, Kevin shared 1 brownie for 4 people by tearing off parts of the paper brownie and distributing the pieces. After he dealt out 4 pieces he had some left, and so he continued to tear pieces. When Ms. Marks saw that her student was not solving the problem of 4 children share 1 brownie as she expected (i.e.cut the brownie in half then in half again), she applied a teaching strategy that had worked with another student earlier that day: They acted out the situation with the other children at the table. This action did not work. Therefore, she appealed to the student’s sense of fairness by asking if his division of the brownie would be fair to everyone. Kevin thought it would be fair since everyone was going to get the same number of pieces. In reviewing the video clip, it was not clear if he intended on redistributing the rest of the brownie. His first distribution had 4 pieces of approximately the same size, and he had begun to tear the remaining piece when Ms. Marks intervened. Finally, she asked Kevin to look at how he solved 2 people share one brownie for number 1 in which he had a line drawn in the middle of the paper brownie. She referred to his solution for one half and assisted him to use the strategy of repeated halving to solve the next two problems (4 share 1 and 8 share 1). After the lesson, she wrote,

> I wasn't sure how to work with Kevin after I realized he wasn't cutting pieces for # 2 on the Fair Shares Worksheet (student sheet 1). I decided after a couple of *trial and errors* to use # 1 as a guidepost. (This same situation occurred w/ # 4 (1/8) and with Pablo (1/3). (emphasis added)

The Challenge of Supporting Children's Ideas

The teachers' feelings of being in a predicament or feeling "stuck" did not deter the teachers' appreciation for children's ideas. Because they valued children's thinking but had difficulty building on children's ideas, this conflict set up the situations as predicaments. For example, Ms. Edwards wanted to support Mario's solution, though she did not understand his reasons for using his strategy. In this solution, Mario had the grid shaded into 8 sections of 12 and ½ squares each (see Figure 2.9). This was different from the strategy that the class had been using, and Ms. Edwards struggled to understand how his solution could have been constructed without knowledge of the equivalency beforehand.

> Mario had great strategy, but I tried to figure out his thoughts before he knew 1/8 = 12 1/2 and divided his grid into 8 parts w/ 12 1/2 in each. I wanted other [students] to see relationship [between shading strategies] (some did)—We'll continue on Monday.

During class she extensively probed Mario's thinking but was not able to find out how he arrived at his answer. Another student wanted to erase Mario's solution and show how to get the answer by sectioning off 8 squares and shading 1 until the grid was used up, but Ms. Edwards stopped him and insisted that Mario's solution be saved. Near the end of the interaction she asked the class to compare two different representations for 1/8 of 100 squares which included Mario's and the "one out of eight" solution. Even though this situation was difficult for her, Ms. Edwards valued his thinking and knew his solution could be used to make important connections between representations as part of a class discussion.

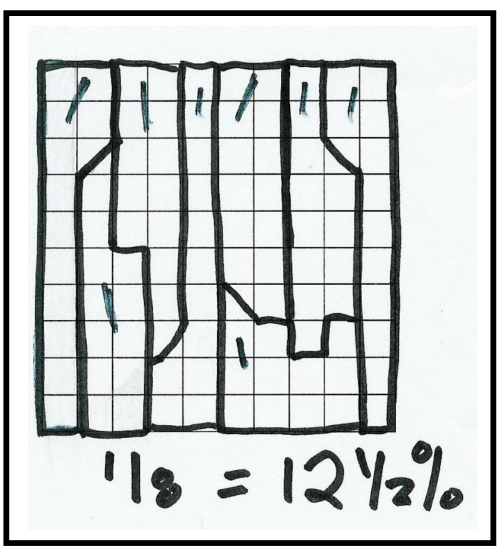


Figure 2.9. Mario's solution.

The Teachers Explain Their Predicaments

The postunit interview also allowed the teachers to explain why they felt stuck. Specifically, they wanted to know more about the thinking behind students' construction of area models and about children's thinking in general. Often they wanted to understand better the mathematics within children's strategies for themselves. They also wanted to know how to support students without directly telling them how to solve the problem.

Understanding Children's Thinking

All of the teachers expressed a need for more information on children's thinking, both specific children and children's thinking in general. Ms. Edwards explained her thinking about her interaction with Mario and how she wanted more information about how he and his partner knew what to do: "I mean, they had the 8 sections, but I was confused on how they knew to all of a sudden start and do 12- 10 and group 2 and cut one in a half." Ms. Marks reflected that the reason she was confused about the students' work was because she did not understand their thinking about the problem: "The only thing I could think of, 'cause I did not know why they thought that too, was that they each added half. They each added a cookie maybe. But I don't know that that is true."

Supporting Students' Thinking

Teachers also worried that they would not be able to sustain the interaction without resorting to directly telling a child how to solve a problem. After reviewing a clip in which a student used an unexpected strategy, Ms. Marks said, "Well, I was thinking that I wasn't aware that I was going to be hit with the situation that he wasn't going to cut the pieces evenly." As we talked about the interaction and explored what we saw the student do, Ms. Marks framed the problem this way:

> I think the main thing I was trying to get him to do is, it is hard for me to get him to come to the conclusion that they needed to be even without me just saying, "Look, they have got to be even." Like I really just wanted to take the paper and just cut it and show . . . him how to do it, but I was trying to make it to where he would come to the conclusion on his own. That was like my struggle.

Interpreting Children's Mathematics

Previous research has claimed that elementary teachers' knowledge of mathematics is not adequate (Floden, 1997; Ma, 1999). Although this may be true, it is also true that teachers who demonstrate a high level of mathematical knowledge do not necessarily have the best student achievement (Begle, 1979). The teachers in this study recognized that they needed a certain kind of information about children's mathematics.

I refer again to the predicament with Ms. Edwards's student Mario, who used a partitioning strategy to make 1/8 of 100, as an example of teachers' needing more information about the strategies children may construct to solve problems. Ms.

Edwards could only think about dividing a 100 grid to make the equivalent to 1/8 using a certain method. She said,

> I would think they would have grouped 8 colored one, grouped 8 colored one, OR divided it into the drawing? The way I would have thought they would have done it would have been to look for 8 of them, get and identify one, look for another 8 identify one.

If she had been able to think about dividing up the 100 grid by making 8 partitions (this would have involved some estimation on Mario's part), she might have understood how he arrived at a solution.

Summary

The analyses of teacher-chosen predicaments suggested that teachers were willing to push for children to construct the mathematics even though they might feel uncomfortable. Teachers felt "stuck" when the children did not respond to the teachers' actions as expected; teachers might not understand why children understand and solve problems in the ways that they do. In most cases, the teachers in this study did not express their problems of practice as dilemmas. Dilemmas are situations in which the choices are not easy to make because both have both good and bad consequences. The "distinct, uncomfortable feeling" teachers had during the situations they wondered about may come from several sources that are related to certain features of the situations. The kinds of actions teachers take appeared to be related to particular features of children's work, especially to the task of interpreting children's models and drawings for fractions.

DISCUSSION

I began this study with two questions: (a) What are the problems teachers identify as they teach children using reform curricula, and (b) what do teachers think about these problems? Teachers identified situations with particular features, mostly having to do with the difficult task of interpreting students' work and supporting their thinking. In turn, teachers' responses were associated with the features of children's work, especially to children's ideas about fractions expressed though models and drawings. Teachers feel "stuck" when the children do not solve problems as expected, and may not understand why children understand and solve problems in the ways that they do, even when they use strategies introduced by the teacher.

In most situations, the teachers in this study did not perceive their problems of practice as dilemmas, as they have been described in other research (Ball, 1993; Burbles & Hansen, 1997; Hiebert et al., 1996; Windschitl, 2002). Instead, the teachers expressed feelings of confusion they had during the situations they wondered about. However, the analyses of teacher-chosen predicaments suggest that teachers are willing to push for children to construct the mathematics even though they may feel uncomfortable. Additionally, teachers in this study recognized that they needed more knowledge about strategies and children's thinking as a result of grappling with their predicaments.

Why Are Certain Situations Predicaments for Teachers?

Many researchers that investigate teacher practice choose to investigate the nature of whole class discussions and how teachers coordinate a focus on children's ideas and the mathematics to be learned (e.g. Ball & Bass, 2000; O'Connor, 2001; Sherin, 2002). Very few of the predicaments identified in this study concerned these kinds of issues. Instead, teachers' predicaments arose as they struggled to understand the student's explanation of or thinking about the strategy. In these situations teachers felt instantly confronted with the task of making sense of children's invented strategies. However, teachers did not understand these unexpected strategies.

Sometimes teachers struggled to understand students' drawings. However, the teaching task of interpreting children's pictures seemed to support teacher's reasoning about students' thinking, and they were better able to take supportive actions when students used area models. One conclusion that might be made is that given a mathematical feature that includes a drawing, teachers are able to use what they know, at whatever level, to inquire closely about children's thinking. Within this particular context, teachers asked specific questions about what they observed and often learned something new about the mathematics of fractions as well as important information about the student's thinking. It is significant that when teachers are given the opportunity to identify situations that puzzle them, they identify situations that have been recognized by researchers. Ball and Bass (2000) pointed out that this kind of pedagogical problem solving is important for teachers to learn about inquiry-based practices.

Teachers feel “stuck” when children do not react as expected to the teachers’ actions. The teacher seems to feel bound to solving the problem situation right there. In this study, a teacher never suggested that the student’s confusion could be “cleared up” by working through more problems in future lessons or by discussing the various solutions with the whole class.

Teachers also wondered about the curriculum. They believed that the design of the tasks was supposed to support children’s understanding of concepts, but when they believed that task was not working, they did not know how to make adjustments during lessons. Ms. Marks noticed that making fraction cards was physically difficult for some of her students and recognized that the mistakes children made in cutting them could lead to unproductive work. Ms. Ingram was not convinced that using percent strips was the best way to link fractions to percents. Ms. Edwards worried about the relevance of curriculum examples as a way to familiarize students with decimals. The predicament arose in part because teachers felt bound to the lessons as they were written, and the district guidelines intended to support implementation had little flexibility. Like some of the teachers in Sherin and Drake’s (2000) study on curriculum adaptation, changing curricular activities was not an option that teachers took during the lessons, even though they expressed that they wanted to do something else.

Like Floden’s (1997) concern that teachers are asked to teach what they do not understand, teachers in this study struggled to fully understand teacher-introduced strategies (outlined in the curricular materials). Teachers who are confronted with the issue of supporting struggling students may feel pressured to use more direct strategies, which contradict their developing beliefs about teaching and understanding mathematics. Teachers with limited experience in teaching for understanding may respond to students with directive actions if they do not have a working knowledge of the array of teaching strategies available that maintain reform goals. For instance, one way teachers can support children is to allow the child to abandon the problem or offer an easier problem (Jacobs & Ambrose, 2003). In the postunit interviews, teachers rarely noted that a students' confusion or their own could be resolved through future work on similar problems or through whole-group discussion about the problem the child was having difficulty solving.

Teacher's Responses in Situations Concerning Student Work

Like the teachers in previous studies on teacher–student interactions, few of the teachers' predicaments involved extending students' responses (Fraivillig, Murphy, & Fuson, 1999; Jacobs & Ambrose 2003). The teachers' situations rarely included a struggle to extend student thinking. Extending students' thinking just was not a problem for them. About one third of the predicaments included situations in which teachers took directive actions, and in another third teachers' actions were exploratory. I wondered if teachers were choosing the situation because of their perceived ability to respond. Perhaps the situations were chosen for other features such as the use of an invalid strategy. I conjecture that when the interaction was directive, this quality of the interaction revealed how the teacher responded to the predicament, not the reason they wondered about it. I also noticed that when teachers responded with actions at the exploratory level, often the reason they chose that situation as a predicament was that they felt ineffective about their response. Whereas in several situations a teacher wondered about the quality of her responses during exploratory interactions, in no instances in which a teacher whose response was coded as directive directly worried about being too directive.

Additionally, this study showed evidence that when the features of the predicament were student work that included an area representation, teachers were more likely to explore children's thinking during the interaction. (Recall that only 3 of the 18 had directive actions; 13 had either exploratory or responsive teacher actions.) Constructing, evaluating, and revising models to represent mathematical ideas and processes support children's learning (Lehrer & Schauble, 2000; Lesh, Landau, & Hamilton, 1983) and have been reported as a focus of teacher's instructional problems (Ball, 1993; Lampert, 2001). These types of predicaments may be the kind of situations in which teachers can test ways of responding, explore children's mathematics, and begin to consider new ways to solve math problems and support children's thinking.

One of the most significant findings in this study is that teachers identified problematic situations in which the mathematical and pedagogical concerns are in areas that are important for learners. Studies on children's learning have shown that when teachers listen to children's ideas and encourage the use of invented strategies, they have better conceptual understandings of the mathematics (Carpenter, Fennema,

& Franke, 1996; Kazemi & Stipek, 2001). Tapping into the teachers' perspective adds the dimension of understanding how teachers reason about their difficulties in teaching inquiry-based mathematics. Additionally, listening to children's thinking has been shown to contribute significantly to teacher's learning (Cobb, Wood, & Yackel, 1990; Steinberg et al., 2004; Stigler & Hiebert, 1999).

As Burbles and Hansen (1997) claimed, predicaments are inevitable in teaching. For most situations the teachers appreciated their predicaments as learning opportunities. During the postunit interview, I asked Ms. Marks to compare her previous year's implementation of the Investigations curriculum to the present year's instruction. Just before this exchange, we had been discussing her predicament in which she was surprised to discover a students' particular way of thinking about a fraction problem.

> Interviewer: Oh, so you didn't do this lesson last year? This specific one?
> Ms Marks: Oh I did, but I didn't, it seemed like last year I only had 13 kids, and so we would do everything pretty much whole group on the carpet, and they would give ideas, and so the kid, if there was a child who didn't see that—
> Interviewer: Ohh.
> Ms Marks: I would not have picked up on it. As much as I would have this year. Because it is more in small group and with you guys [the University of Texas math group], I have been really—
> Interviewer: Going around—
> Ms. Marks: —Looking at them more closely.
> Interviewer: So you are thinking that last year that someone might have done the same thing, but since you were on the carpet just throwing out ideas, you think you wouldn't have seen that?
> Ms. Marks: Um hum. And then I would have just changed it [the child's ideas] already instead of [letting them] exploring it themselves.

In the past 30 years or so as new teaching strategies have been introduced, the practice of teaching and what counts as teacher knowledge have been the focus of education research. Pedagogical problems that teachers face when they implement new teaching practices have provided important insights to teaching (Ball & Bass, 2000; Putnam & Borko, 2000). Rather than conceptualizing teaching as a process-product system where inputs produce outputs as a way to capture teaching, a host of researchers have proposed to conceive teaching as problem solving (Ball, 1996; Featherstone et al., 1995; Hiebert et al., 1996; Lampert, 2001; Sherin, 1997). Solving problems always has been a part of teaching practice, but in the context of reform the solutions to the problem are not as straightforward as they are within didactic methods of teaching mathematics.

> Constructivism in practice includes the ambiguities, contradictions, and compromises that are a part of implementing constructivist instruction—it presents a highly problematized view that takes into account the tensions that characterize reform teaching in general and teaching for understanding in particular. (Windschitl, 2002, p. 132)

CONCLUSION

I believe that the features of these predicaments show that teachers take notice of important, key issues in teaching for understanding, even though each teacher experiences implementation as a slightly unique set of problems. However, all of the teachers in this study were receptive to alternative ideas for teaching fractions, even though some problematic situations were difficult to resolve. Mostly teachers believed the trade-offs between (a) implementing inquiry-based mathematics and the possibility of confusion and (b) using more direct methods and not having the opportunity to see what children really understand were worth it. They also demonstrated the self-realization that they needed a deeper understanding of children's thinking and alternate strategies.

Methods of investigating teachers' problems of practice that appreciate these tensions and are supportive of the teacher's point of view can provide essential perspectives, leading to more authentic forms of descriptions of teaching as well as improved professional development. Curricula and professional development may support teachers' awareness of the expectations of inquiry learning and may provide some guidance on how to enact these expectations. However, teachers need support beyond the curricular materials in order to cope with these predicaments.

There is more work to be done to understand how teachers are interpreting mathematics teaching in the spirit of reform. Researchers have identified a range of problems of practice (Ball & Cohen, 1999; Empson & Junk, 2004a; Floden, 1997; Hiebert et al., 1996; Kazemi & Stipek, 2001; Lampert, 2001; Sherin, 1997; Windschitl, 2002). However, more needs to be known about ordinary teachers' problems of practice during the act of teaching. The research reported in this paper provokes more questions that can be explored in the service of supporting teachers such as these. How can predicaments be used to structure professional development experiences? Analyses of the teachers' predicaments show that teachers are willing to explore student's thinking in the topic of fractions, especially if there are tangible representations on which both the student and teacher can focus attention. What other kinds of mathematical contexts of student work encourage teachers to explore student thinking? How do teachers from different curricular programs predicaments compare?

In conclusion, all of these questions should be addressed with a close attention to the teachers' perspectives. Moreover, the participants need to be of a variety of teaching experiences and expertise to ensure that authentic accounts of ordinary teachers' practices are being studied and described.

SECTION 3

THE CHALLENGE OF INTERPRETING STUDENTS' INVENTED REPRESENTATIONS: TEACHING PREDICAMENTS AS LEARNING OPPORTUNITIES FOR TEACHERS

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The Challenge of Interpreting Students' Invented Representations: Teaching Predicaments as Learning Opportunities for Teachers

In classrooms where teachers allow children to use a range of strategies to solve problems, teachers often face the challenge of interpreting their students' work, particularly when the students' constructions are unfamiliar to them. Indeed, the National Council of Teachers of Mathematics (NCTM, 2000) standards call on teachers "to be able to adjust and take advantage of opportunities to move lessons in unanticipated directions" (p. 15). Additionally,

> Effective teaching requires deciding what aspects of a task to highlight, how to organize and orchestrate the work of students, what questions to ask students having varied levels of expertise, and how to support students without taking over the process of thinking for them. (p. 19)

Under this call for changes in their role, teachers find themselves in a predicament of sorts when students invent strategies to solve problems that are novel, incomplete, or incorrect, since those strategies are most likely to be unpredictable.

When teachers pose interesting problems, and predicaments arise, how can these predicaments be a positive experience? How can these predicaments inform educators about understanding students and the subject matter? Even though teachers' situations are particular to their classrooms, their students, and the topics they teach, are certain situations predicaments for many teachers?

PREDICAMENTS AS OPPORTUNITIES FOR LEARNING

Instructional dilemmas have been documented as a mechanism for change (Ball, 1993; Dewey, 1964; Lampert, 1998b; Sherin, 1996). We also know that teachers learn when they use standards-based materials (Empson & Junk, 2004b; Featherstone, Smith, Beasley, Corbin, & Shank, 1995; Hull, 2000; Remillard, 2000; Sherin, 1996). How teachers learn as they implement inquiry-based practices is not well understood. One way to understand teachers' learning from classroom practices better is to study the predicaments that teachers face as they transform their direct teaching methods to less direct, student-centered practices.

"A *predicament* is a problematic situation seen in terms of a difficult decision and implies that one does not know what to do and is considering it rationally" (*American Heritage Dictionary*, 1976, pp. 1031-1032). However uncomfortable this kind of situation sounds, I argue in this paper that predicaments have important and positive consequences for teachers. Challenges such as these are opportunities for teachers to learn as they teach. As Burbles and Hansen (1997) explained in their book, *Teaching and its Predicaments,*

> Predicaments compel people to reconceive their circumstances and what they can realistically accomplish. Predicaments require compromise and trade-offs. People can always elect to sidestep predicaments, but that course of action usually means abandoning human possibilities rather than creating new ones. (p. 9)

TAKING ON A PREDICAMENT

Ms. Marks[1] was working with a struggling student on a problem involving how much of a brownie one person would get if the brownie were shared equally among four people. All of the children in the class had paper brownies to cut or draw on to show the equal shares. One student, Kevin, carefully "eyeballed" a section big enough for one person and cut a corner from the rectangular brownie. He did this three more times, yielding four pieces of roughly the same size. However, he still had some of the paper brownie left. Ms. Marks was puzzled by this approach. She decided to restate the problem for him, and the two worked together to get four equal shares by repeatedly halving the paper. As she reflected on the predicament later she commented,

> I think the main thing I was trying to get him to do is it is hard for me to get him to come to the conclusion that they [the brownies] needed to be even without me just saying, "Look, they have got to be even!" Like I really just wanted to cut the paper and show him how to do it, but I was trying to make it to where he would come to the conclusion on his own. That was my struggle.

When asked if she thought the struggle was worth it, she said that in the previous year she and her class

> Would do everything pretty much whole group on the carpet, and they would give ideas and so if there was a kid who didn't see that [how to equally share the brownies], I would not have picked up on it as much as I would have this year.

When asked which way worked better, she said, "I think the way with Kevin worked better with him cutting." Essentially, the predicament allowed her to see better how

[1] The names of all the teachers and students are pseudonyms.

Kevin understood fourths, and Ms. Marks believed that he understood the problem better in the end.

Ms. Marks was one of four elementary teachers who were observed multiple times during lessons that involved fraction, ratios, and percents. After each observation, teachers took a few minutes to reflect on the lesson. These reflections had two parts. The first part encouraged them to think about the lesson as a whole. The second was designed to help teachers write about their predicaments. Teachers were asked two questions for reflection:

1. What well in the lesson?

2. Did anything happen today during the lesson that caused you to stop and wonder what to do? Who was involved and what was it about? How did you deal with the situation?

The third- to sixth-grade teachers had teaching experience that ranged from 2 years to 10 years. Their experience using a reform mathematics text ranged from 1 to 5 years (i.e., *Investigations in Data, Number and Space,* TERC, 1995–1998, or *Connected Mathematics,* Fey et al., 1997).

A COMMON CONCERN

As I collected their reflections, I noticed that these teachers' predicaments shared a common characteristic. This characteristic involved the interpretation of children's representational drawings for fractions as they solved problems involving rational numbers (23 out of 41 reflections stated this characteristic). Often these interactions involved students who struggled, but had enough understanding to begin working on the problem. Their representations often confused the teachers and thus were hard to interpret. That the students in these teachers' classrooms had the freedom to invent the drawings in the first place is a testament to the teachers' willingness to let students solve the problems in ways that made sense to them. The teachers' choice to reflect on these types of predicaments highlights the tension between showing or telling children how to solve problems and supporting without "taking over the process for them" (NCTM, 2000, p. 19).

All of the predicaments discussed in this article are situations that teachers chose to focus on, and they share important characteristics. These characteristics demonstrate that teachers are concerned about the way children think about fractions. The teachers respect children's ideas and but are puzzled by children's use of part–whole representations to solve problems.

A NOVEL STRATEGY

For example, Ms. Ingram, a fifth-grade teacher from the project, struggled to interpret Daniel's strategy for comparing 7/8 and 5/6 (see Figure 3.1). Daniel had drawn his solution on the chalkboard as part of a group of children who were sharing their solutions. Daniel explained why he believed 7/8 to be larger than 5/6. The interchange between the teacher and Daniel went like this:

> Daniel: 7/8 is greater that 5/6.
> Ms. Ingram: And why is that? I mean you've got two diagrams. To me they look exactly the same.

Daniel: This [pointing to the unshaded portion in the 7/8 pie] is a smaller space than this [pointing to the unshaded portion in the 5/6 pie].

Ms. Ingram: How do you know? Why? Why is that smaller? Huh? 'Cause you could see it?

Daniel: 'Cause I could see it

Ms. Ingram: Why is it—Has that circle, the one that represents 7/8, has it been divided more than the other one? 5/6? [Daniel nods yes.] So why is 7/8 greater?

Daniel: It's got more shaded—

(At this point Ms. Ingram is searching for another question, so the researcher in the room suggests that Ms. Ingram ask Daniel about the unshaded piece)

Ms. Ingram: What size is the missing piece in 7/8?

Daniel: This one is smaller than that one [pointing to 1/8 unshaded space and then 1/6 unshaded space].

Ms. Ingram: Exactly what fraction is it, exactly what fraction?

(Then Daniel labels his drawing so that the unshaded spaces have fractions 1/6 and 1/8, respectively).

Until this point the teacher had never considered comparing fractions using their "missing pieces". This approach is actually a method of comparing the complement, e.g. 7/8 is more than 5/6 because it is closer to a whole. She was unsure of how to respond to Daniel and did not understand why he drew the diagrams in the first place. In the postlesson reflection she wrote,

> When Daniel was showing how he compared 5/6 and 7/8 (which is larger?), the pictures he drew seemed suddenly so inappropriate. Even though he figured it out (the picture didn't help him that he drew). . . . And of course 1/6 as a larger portion than 1/8 confused many students when put in the context of which is larger, 5/6 or 7/8? I was pretty blank at that point—pictures weren't helping.

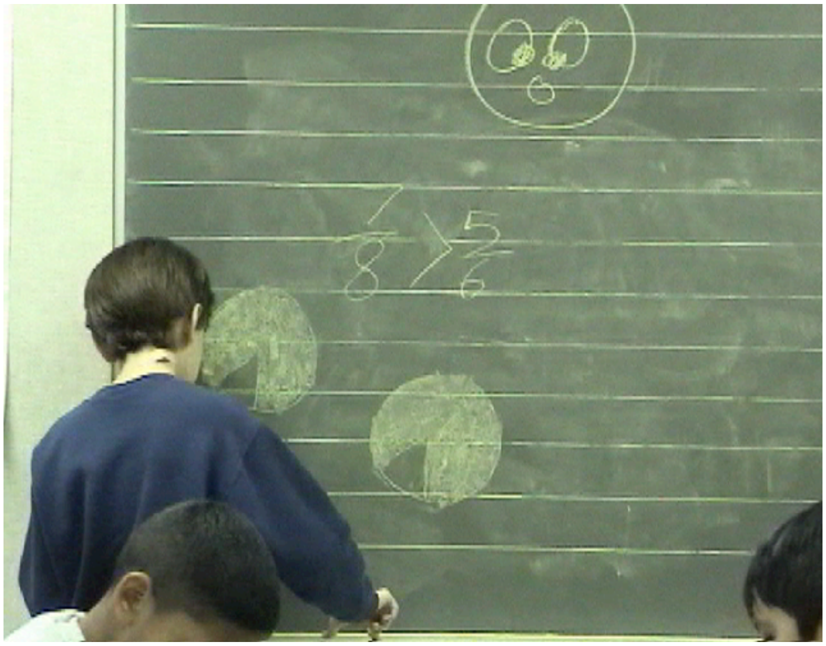

Figure 3.1. Daniel's strategy: The missing piece.

APPRECIATING THE PREDICAMENT

When teachers puzzle over what their students do and say, they can learn to appreciate these predicaments as opportunities for growth. The case of interpreting students' representations for fractions provides opportunities for teachers to build understanding about students' conception of fractions, deepen their own understanding of fractions, and consider alternative teaching moves and their impacts on student learning.

Explore the Strategy

First, teachers might explore the strategy before making any kind of judgment or assessment of its validity. Often teachers are caught up in the fast pace of their lessons and may miss important elements of a student's construction if they jump right in with an assessment of right or wrong. For example, in Daniel's strategy for the problem above, the act of partitioning the two circles allowed him to evaluate the relative size of the two fractions based on how their complements were related.

Mathematical Value

Second, it is important to explore the mathematical value of the strategy, whether or not the child executed it in a way to arrive at the correct answer. What is the big idea behind the strategy? Will it work for other problems/numbers, and what are its limits? This exploration gives the teacher the opportunity to explore the concepts for herself. In the case of Daniel's complementary strategy, Ms. Ingram was able to consider the inverse relationship between the relative sizes of the complements and the relative sizes of the fractions being compared: 1/6 is greater than 1/8 and that means that 5/6 is less than 7/8. This idea opened the possibility for deeper understanding but also created a second predicament for Ms. Ingram: how to deal with the whole class's understanding of the concept.

Respond to the Situation

This leads to a third opportunity for learning from predicaments: Teachers can explore the range of responses possible for the situation. Additionally, each response can be evaluated for its impact on student learning. Ms. Ink's responses to Daniel at the time were to question him further about his method, to assist him in stating more explicitly what was happening in his picture. She did this by instructing him to label the "missing piece" and then he was able to say 1/6 is larger than 1/8 so that means 5/6 is smaller than 7/8, and by eliciting reactions from the class about his solution. Also, the children were able to compare Daniel's strategy to three other strategies presented for the same problem. Ms. Ingram could have reacted in a range of ways, with a range of consequences to student learning. If she had insisted on comparing the fractions by their shaded areas she would have projected an implicit message that area comparisons are what she preferred and she would have derailed a logically based comparison method. That response would have undermined Daniel's motivation to solve problems using his own ideas.

MORE EXAMPLES OF PREDICAMENTS

To further appreciate the predicament of understanding the representations children invent when they solve problems involving rational numbers, try interpreting the solutions presented by different students below in Figures 3.2–3.4.

1. First explore the strategy (how do you think the student was thinking?).

2. Then consider the mathematical value (could the strategy be viable?).

3. Finally, propose two or three ways a teacher could respond to the situation and the impact each response could have on students' learning. This activity can be enhanced if you are able to discuss these with a colleague.

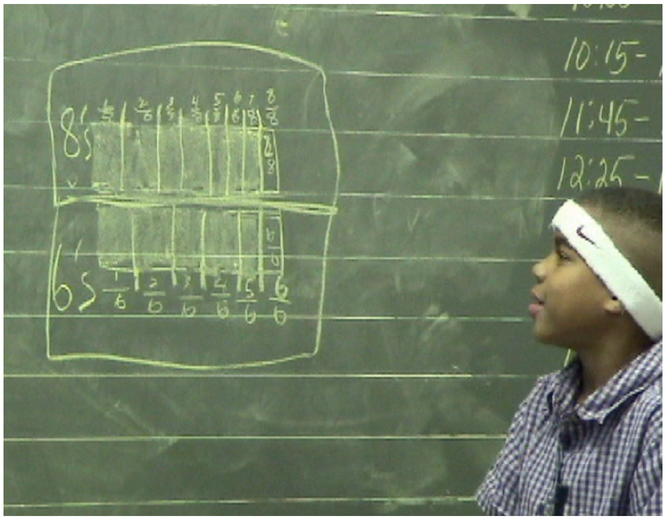

Figure 3.2. Donald's strategy: They're the same! The problem asks, "Which is bigger, 5/6 or 7/8?" Donald says, "I tried to draw my rectangle even but I wasn't able to. So I measured with my fingers. What I think is that they are the same."

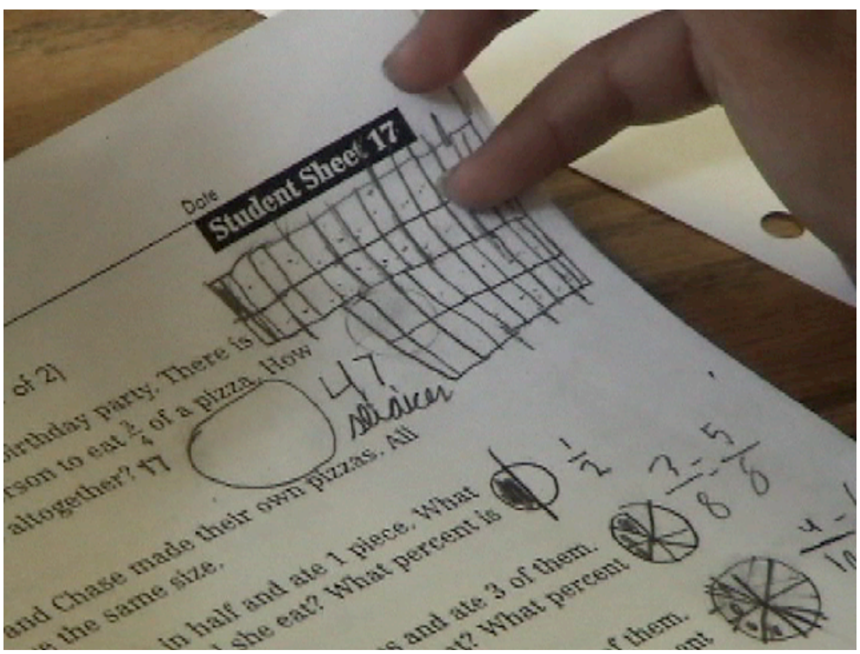

Figure 3.3. Mario's strategy: 47. The problem asks, "There are 12 children at a birthday party. There is enough pizza for each person to eat 3/4 of a pizza. How many pizzas are there altogether?" (From Investigations, Name That Portion, by TERC, 1995-1998). Mario tells the teacher there are 47, and he has difficulty explaining his thinking to the teacher beyond his representation.

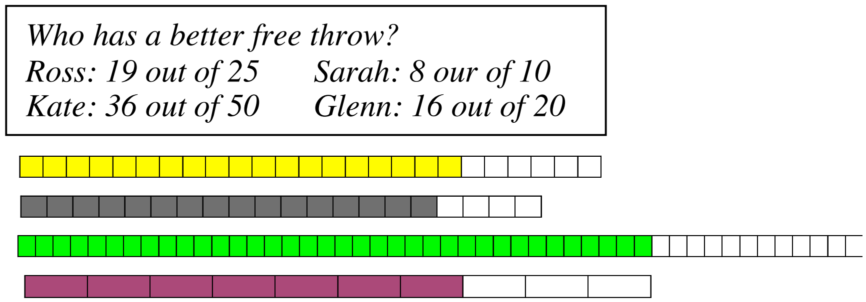

Figure 3.4. Reggie's strategy: Kate is better. Reggie drew a picture by hand like the drawing here. He believed that Kate had the better free throw because his drawing "shows it."

EXPLORING PREDICAMENTS OF YOUR OWN

Many of the reform texts encourage students to use a variety of representations to show their thinking. Often students construct representations in unexpected ways to justify their thinking. Just as often, the supporting materials in these texts do not provide examples of what these students do, so teachers do not have opportunities to learn about them ahead of time. Understandably, knowing all the possibilities is impossible and impractical anyway. Teachers may want to reflect on their lessons as these teachers did to understand where their predicaments arise and to use them to understand their own thinking about teaching mathematics.

Teaching mathematics through methods that promote children's construction of solutions has predicaments that are particular to the content, the problem, and the teacher's personal approach to teaching mathematics. This article is not meant to provide a quick fix to problematic situations but to provide insights to a different orientation to teaching. Predicaments are inherent and inevitable in the work of teaching, but also rich with opportunities to learn about children's thinking and the practice of teaching mathematics.

APPENDICES

APPENDIX A

POSTLESSON REFLECTION SURVEY

Teacher's name:________________________ Grade: ______
Date:____________________________
Lesson: ________________

What went well today in your lesson?

Did anything happen today during the lesson that caused you to stop and wonder what to do? Who was involved and what was it about? How did you deal with the situation? (Describe however many situations of this kind you experienced today.)

APPENDIX B

POSTUNIT INTERVIEW

Name of teacher: Grade level: Date of interview:

Introduction: In this interview I am interested in your thinking about discussions you have with your students about mathematics. There are no right or wrong answers. My goal is to get behind your thinking and decision making when it comes to dealing with student thinking in your classrooms. I will take notes as you talk and also audio-record the interview today. Since we last met I have reviewed the lessons I videotaped along with the situations you identified as important to you. In this interview I will ask you more questions about one or two of the clips we have already viewed together, plus we will look at a clip you have chosen or I have chosen that we have not looked at before. After we watch each clip, I will ask you some questions and probe your thinking about the clip we watched. If something comes to mind that you want to tell me about, but I did not ask, please tell me. After the interview if you are thinking about the lesson and the clips and you want to add something to the interview, you can call or e-mail me. With your permission I would like to call you or e-mail you if I have more questions about your thinking.

1. Show each video clip after saying: You wrote, “Quote what the teacher wrote in the postlesson reflection.” Tell me more of your thinking about this situation.

2. OR As I watched the tapes I found this interaction with ___ interesting. Can you tell me more about what was going on here?
3. OR As I thought about what you said about this interaction with ____, I wanted to know more about____ (may include what teacher said, additional probes to get at clarification, or deeper understanding of the situation).

What do you think teachers need to know in order to teach mathematics well?

As you think about teaching elementary mathematics what have you learned that has helped you deal best with students thinking about mathematics?

What is the most important thing you have learned about teaching fractions (or decimals, percents)?

Looking back, what would you do differently when you teach this same unit next year? Why?

APPENDIX C

FRAMEWORK FOR TEACHER–STUDENT INTERACTIONS

Understanding teacher–student interactions		**Directive interactions**	**Observational interactions**	**Exploratory interactions**	**Responsive interactions**
Supporting the child during problem solving	Make sure the child understands the problem.	May repeat problem	May repeat problem	May repeat, explain, embellish, and personalize problem	May repeat, explain, embellish, and personalize problem and ask child to articulate the problem
	Change the problem when necessary.	Rarely adjusts problems	Rare but may change numbers or context	May change numbers or context	May change problem type, numbers, or context
	Explore what the child already has done.	Rarely probes except sometimes when answer is wrong	May do minimal probing with general questions	Tries to probe but may not get a complete or accurate explanation	Elicits details of the child's strategy and encourages reflection on how that strategy is linked to the story
	Remind the child that other strategies are possible.	May jump in quickly and tell the child how to solve the problem	Generally moves on without encouraging other strategies	May suggest that a child should solve the problem another way	May provide general or specific suggestions based on what the child has done
Clarifying the teacher's understanding of the child's strategy		May request strategy explanation	Generally requests explanation when strategy is unclear but accepts any response	Generally requests explanation when strategy is unclear and pushes for explanation, but it may fall short	Requests and pushes for full explanation of child's actual strategy
Extending the child's thinking	Promote reflection on the strategy just completed	Rarely encourages reflection	Rarely encourages reflection	May ask minimal questions to encourage clear articulation and reflection	May push child to clearly articulate and reflect on his/her strategy and representations
	Encourage the child to explore multiple strategies and their connections	Rarely asks for a second strategy	Rarely asks for a second strategy	May ask for a second strategy but limited requests for strategy comparisons	May encourage more sophisticated or efficient strategies and promote strategy comparisons
	Connect the child's thinking to symbolic notation	Rarely requests links between strategy and symbolic notation; may ask for a number sentence	Rarely requests links between strategy and symbolic notation but may ask for a number sentence	Rarely requests links between strategy and symbolic notation but may ask for a number sentence	May elicit symbolic links to child's strategy and encourage reflection on multiple representations
	Generate follow-up problems linked to the problem just completed	Rarely links problems	Rarely links problems	Rare but may provide a follow-up problem	May link subsequent problems conceptually

Note. Source: Jacobs and Ambrose (2003).

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