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The Impact of a Mathematics Workshop on Mathematics Teachers' Knowledge and Skills

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The Impact of a Mathematics Workshop on Mathematics Teachers' Knowledge and Skills

by

Samantha Elizabeth Bos

Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

May 2022

Acknowledgements

This dissertation and my entire doctoral journey would have been possible with the support of so many people. First and foremost, I cannot adequately express how grateful I am for the support and guidance of my advisor, Dr. Sarah Powell. Your infinite patience and faith in me have helped me become a better researcher, better teacher, and overall better person than when I first started this program. You have provided me with so many opportunities to learn what is most important to me, and I appreciate your willingness to let me pursue my passions in research and learning. I would also like to express my gratitude to Dr. Schnakenberg, Dr. Pfannenstiel, Dr. Doabler, and Dr. Clemens for serving on my dissertation committee. I greatly appreciate your time and thoughtful feedback in helping me develop the best version of this study that I could.

I am so grateful to the amazing people that I have met while at UT Austin, including incredible colleagues, fellow students, and professional staff at the Meadows Center. I have too many people to thank for their mentorship and kind words of advice and support during my doctoral studies than what can fit on these pages, but I want to acknowledge the many incredible people who have helped me on this journey.

Finally, I would like to acknowledge the love and support of my family. Without the early childhood support of my grandparents, the never-ending love and acceptance of my parents and siblings, and the constant willingness to make things better by my husband, I would not be the person that I am now.

I dedicate this work to my former colleagues and students in San Antonio. You helped me become a teacher first, a part of me that I promise to never forget.

Abstract

The Impact of a Mathematics Workshop on Mathematics Teachers' Knowledge and Skills

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The University of Texas at Austin, 2022

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Teaching mathematics can be challenging and requires depth and breadth of both content and pedagogical knowledge unique to the subject. Teaching students experiencing mathematical difficulties (MD) requires additional knowledge and skills related to implementing data-driven instructional practices. Professional development has been shown to be an effective method of improving teacher knowledge and skills related to both mathematics and data-driven practices. In a systematic review of the literature on mathematics PD, however, I found variability in the most effective structures and methods for delivering PD.

This study examined the effects of a 9-week intensive summer PD regarding the use of data-based individualization within mathematics classrooms on teachers' mathematical knowledge for teaching, self-efficacy, and instructional practices. A total of 48 initial participants were randomly assigned to complete the summer PD either as part of an Asynchronous group, a Synchronous/Independent group, or a Synchronous/PLC group. Participants completed the National Center on Intensive Intervention (NCII) *Mathematics*

Intervention Course over the course of the 9 weeks, completing approximately 40 hours of materials and activities. ANOVA tests yielded significantly positive results showing participants improved on the *Mathematics Teaching Self-Efficacy* measure as well as the *Teachers Instructional Practices (TIP)* measure. Results also showed positive but not significant improvements on Mathematical Knowledge for Teaching (MKT) as well as s *Teacher's Sense of Efficacy Scale*. Although not significantly different, participants in the Synchronous groups (i.e., Synchronous/Independent and Synchronous/PLC) saw greater improvements than participants in the Asynchronous group on all measures, and participants in the Synchronous/PLC group saw greater improvements than participants in the Non-PLC groups (i.e., Asynchronous group and Synchronous/Independent group) on all measures. Results of thematic analysis of semi-structured interviews also suggested that participants valued the PD for resources, accountability, and a boost to their confidence, although experiences qualitatively differed across groups. These findings provide initial support for the use of the NCII *Mathematics Intervention Course* as the foundation for future PD and the inclusion of peer support and accountability structures for online PD. Future research might investigate further the most effective ways of supporting participants in completing the NCII *Mathematics Intervention Course* during the school year or with observations and coaching support to ensure long-lasting instructional change.

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Chapter 1: Introduction

Teaching mathematics requires a depth of knowledge regarding both mathematical concepts, as well as specific strategies to effectively communicate these mathematical concepts to students of various ages (Ball et al., 2005). Even in a general education classroom, with students who do not experience mathematics difficulty, effective mathematics teachers must be skilled at presenting concepts in ways that are understandable and meaningful to students (Hill et al., 2008). It is estimated that within the United States, approximately 5 to 7% of students have a mathematical disability (Devine et al., 2018; Morsanyi et al., 2018), and an additional 40 to 60% of students may struggle with mathematics (National Assessment of Educational Progress, NAEP, 2020). Many students experiencing mathematical difficulties (MD), have immature understandings or misconceptions of mathematical concepts and procedures (Gersten et al., 2007). Therefore, to teach students experiencing MD, teachers must be well equipped to address procedural as well as conceptual gaps in students' mathematical understandings.

Teachers who are responsible for addressing these gaps in student understandings, typically interventionists or special education teachers, however, may lack the necessary mathematical training to acquire the depth of mathematical knowledge for teaching (MKT) to adequately address students' needs (Ball et al., 2005). In addition, beyond having the necessary MKT for addressing the needs of students experiencing MD, interventionists and special education teachers oftentimes need to be proficient in additional skills, including collecting and analyzing specific data necessary to target the needs of students experiencing MD (Bruhn et al., 2019; Stecker et al., 2005). With only finite time during the school year, and many students experiencing MD benefiting from extra time or practice to master mathematical topics (Fuchs et al., 2017), interventionists and special education teachers often need to make deliberate decisions regarding which topics to emphasize and target. Therefore, in addition to possessing the necessary MKT to adequately address students' needs, many interventionists and special education teachers need to also be competent in a variety of additional specialized skills to ensure that they are addressing students' needs in the most efficient and effective way possible.

With these significant asks of many mathematical interventionists and special education teachers, teachers who do not feel confident in their mathematical knowledge may not feel adequately prepared when they leave their teacher preparation programs to teach mathematics (Ekstam et al., 2017). Teachers, therefore, are dependent on developing their knowledge and skills after they graduate through a variety of professional development (PD) opportunities at the district, state, or even federal level. According to a burgeoning field of research, PD has the potential to positively impact teacher outcomes, including developing their knowledge and skills (Darling-Hammond, et al., 2017). When conducted effectively, PD has been shown to positively impact both teachers' knowledge and skills regarding mathematics as well as students' mathematical knowledge and skills (Scher & O'Reilly, 2009).

Despite positive findings that PD can positively affect both teacher and student mathematics outcomes, there is less research regarding the most effective means of delivering PD, especially to teachers of students experiencing MD. Most research on PD is focused on individual programs and how to implement these programs, but PD is inherently complex and multifaceted. Less is known about the specific features of PD that lead to the greatest teacher outcomes (Scher & O'Reilly, 2009). PD can be costly and cumbersome to implement depending on the features of the session, and many districts may see the price of PD as a barrier to implementing effective PD to improve teacher outcomes. Determining the most efficient and cost-effective methods to deliver PD can therefore potentially positively impact a significant number of teachers and districts.

In addition, interventionists or special education teachers are tasked with teaching mathematics as well as applying the data-driven skills (e.g., data-based individualization; DBI) unique to the field of special education. DBI is central to providing mathematics intervention to students experiencing MD (Schumacher et al., 2017) as the framework combines instruction with assessment and decision making. Teachers may require specialized PD to target their specific needs to providing intervention for students experiencing MD. Little work has been done in this field. Faulkner and Cain (2013) demonstrated that special education teachers can significantly increase their mathematical content knowledge when they participate in effective PD, and Bruhn et al. (2019) demonstrated that special education teachers can develop knowledge and skills related to implementing DBI when they are supported. However, less is known about PD that addresses both teachers' MKT as well as their knowledge and skills in implementing DBI for students experiencing MD.

Statement of Purpose

The purpose of this study is to examine the effect of a mathematics PD developed to address both teacher mathematics knowledge and instruction as well as implementation of DBI. In this study, I will investigate the effect the National Center on Intensive Intervention *Mathematics Intervention Course*, a research-based PD. I will use a randomized control trial to test the effects of the PD on interventionist and special education teachers' knowledge, skills, self-efficacy, and practices in implementing DBI in mathematics classrooms with students experiencing MD. Additionally, I will investigate whether the format of the PD, including with peer-support or through asynchronous coursework, will impact teachers' outcomes.

Primary Research Questions

1. What is the impact of an intensive online summer mathematics PD on the outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) of in-service SPED teachers who teach mathematics to students experiencing mathematics difficulties (MD)?
2. Do participants who participate in weekly synchronous meetings (i.e., Synchronous/Independent and Synchronous/PLC groups) when completing an online summer PD demonstrate improved outcomes (i.e., MKT, self-efficacy, and

teacher instructional practices) compared to peers who asynchronously complete (i.e., Asynchronous group) the online summer PD?

3. Do participants who receive peer support through PLCs during the online summer PD (i.e., Synchronous/PLC group) demonstrate improved outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) compared to peers who do not receive such peer support (i.e., Synchronous/Independent and Asynchronous groups) during the online summer PD?

Chapter 2: Review of Literature

Compared to many of their international peers in developed nations, students in the United States consistently underperform in mathematics (PISA, 2020; Provasnik et al., 2016). National standards reflect similar patterns of underperformance: in 2019, only 41% of fourth-grade, 34% of eighth-grade, and 25% of 12th-grade students demonstrated proficiency in critical mathematics content (National Assessment of Educational Progress, NAEP, 2020). The patterns of underperformance become even more pronounced when examining students with disabilities. In 2019, 17% of fourth-grade, 9% of eighth-grade, and 7% of 12th-grade students with a disability demonstrated proficiency on the national exam (NAEP, 2020).

To address the needs of students with disabilities, many schools across the United States have adopted Multi-Tiered Systems of Support (MTSS) or Response-to-Intervention (RTI) systems (Fuchs et al., 2012; Sailor et al., 2021). In these systems, students who do not respond to instruction provided in general education classes are provided with more intensive interventions, often in smaller groups with more targeted instruction (Sailor et al., 2021; Schumacher et al., 2017). Within these intensive interventions, data-based individualization (DBI, National Center on Intensive Intervention [NCII], n.d.) is often utilized and includes systematic evaluation and modification of instruction based on the needs of students with intensive needs. When implemented properly, the DBI framework of providing intensive intervention has been shown to improve student outcomes in the academic areas of reading, mathematics, and writing or spelling (Jung et al., 2018; Powell et al., in press). The DBI framework can be challenging for teachers to implement with fidelity (Mason et al., 2019), and there are many potential barriers in place in implementing DBI, including a lack of PD opportunities that focus on data literacy and DBI practices (Datnow & Hubbard, 2016). Therefore, the purpose of this study is to examine the impact of an intensive summer professional development (PD) developed to increase teachers' knowledge, skills, and confidence in implementing the DBI framework in mathematics classrooms.

In this chapter, I explain the roles of PD, professional learning communities (PLCs), and asynchronous learning in improving teachers' knowledge and skills. In addition, I address the role of teachers' mathematical knowledge for teaching (MKT), self-efficacy, and instructional practices.

ROLE OF PROFESSIONAL DEVELOPMENT IN IMPROVING TEACHER KNOWLEDGE AND SKILLS

PD is a broad title for a variety of learning structures, workshops, trainings, or educational classes teachers complete every year to develop their knowledge and skills. Examples of PD can include teacher training or workshops, coaching sessions, online or asynchronous classes or opportunities to reflect on practice, or in-person PLC activities (e.g., Avineri, 2016; Birkhead et al., 2017; Feuerborn et al., 2009; Miller, 2017). PD can focus on teachers' knowledge, build skills related to pedagogy, or be focused on building other school or district-goals such as implementing a school-wide protocol or system (e.g., DeJaeghere & Cao, 2009). With such a wide variety of formats and topics provided by a variety of organizations and individuals, it is little wonder that there is substantial variability in the effectiveness of PD (Scher & O'Reilly, 2009; Yoon et al., 2007). However, consensus is building regarding the importance of high-quality PD as a means to increasing teacher learning to increase student outcomes (Darling-Hammond, et al., 2017).

Components of High-Quality PD

There is a growing research base regarding critical components of PD that are necessary to improve teachers' knowledge and skills to the degree that students are positively impacted (Borko, 2004; Darling-Hammond et al., 2017). A widely-used framework proposed by Desimone (2009) synthesizes the findings of the field and recommends five core components of all high-quality PD: (a) *content focus*, (b) *active learning*, (c) *coherence*, (d) *duration*, and (e) *collective participation*.

Content focus refers to the need to target teachers' own conceptual knowledge of the subject they teach to ensure they are prepared to teach that same content to students. Focusing specifically on content, rather than behaviors or skills has been tied to greater teacher and student outcomes (Kennedy, 1998). PD focused on building teachers' content knowledge is associated with an increase in both teachers' knowledge and skills, which in turn is associated with positive changes in teachers' practices (Garet et al., 2001). PD with a *content focus* has also resulted in greater teacher outcomes than PDs focused on pedagogy alone (Marra et al., 2009). To see teacher success however, it is important for

PD to integrate an engaging *content focus* with sustainable practices that lead to long-term integration of knowledge and skills (Borko et al., 2010).

Active learning is rooted in adult learning theory and includes opportunities for teachers to be active participants in the creation of new knowledge, rather than passive receptacles of knowledge (Darling-Hammond et al., 2009; Trotter, 2006). Examples of *active learning* include observing or being observed by others and receiving feedback, examining student work, planning instruction, presenting, or solving mathematics problems (Garet et al., 2001). Teachers need *active learning* opportunities to develop the conceptual depth of knowledge and flexibility in thinking necessary to effectively help students learn (Borko, 2004). When PD utilizes *active learning* structures, teachers are better able to learn and understand the materials and master the skills presented in the PD.

PD that aligns with, or has greater *coherence* with, teachers' state, district, school, and personal goals have greater impact upon teacher change (Copur-Gencturk, et al., 2019). Specifically, high-quality PD seeks to provide knowledge and strategies teachers can easily integrate into their current practices without having to create additional work that conflicts with the school, district, or state priorities. Various educational experts recommend tying PD content and strategies to state and district expectations for teachers to ensure a high degree of *coherence* between the PD and established expectations (Marrongelle et al., 2013). These state and district standards should align with research-based practices, and focusing on how to best implement these standards allows teachers to more effectively and efficiently implement best practices.

Extensive research has sought to determine the tipping point of necessary *duration* to ensure effective PD. Research recommendations include meeting a minimum of as few as 20 hours (Desimone, 2009), while a synthesis of rigorous PD suggests teachers require extensive time, up to 49 hours, to see measurable results (Yoon et al., 2007). Despite being the focus of many studies and syntheses, there is no consensus regarding the minimum number of contact hours needed to improve teacher or student outcomes, nor is there enough data to suggest a certain saturation point in which more contact hours fail to lead to teacher improvement. Instead, it may not be the specific number of hours that is critical to teacher success but instead what activities and content is focused upon during those PD hours that shifts teacher success (Copur-Gencturk, et al., 2019). For example, teachers actively engaged in examining student work saw greater gains than teachers spending comparable time solving problems or being lectured (Copur-Gencturk, et al., 2019). Perhaps more important than the number of contact hours is the total length of the PD, measured in days, weeks, months, or years (Copur-Gencturk, et al., 2019). PD that engages teachers in long lasting and continued practices have a greater impact on teachers' outcomes, including long term implementation of strategies (Fixsen et al., 2005).

When teachers participate with their colleagues at the same school, across similar grades or content, they have greater opportunities to engage in specific and meaningful interactions that address their needs. When teachers implement new strategies or programs as part of a larger whole-school implementation, they are more successful when supported by other teacher collaboration (Akiba & Liang, 2016). Teachers who are able to connect with their peers and create support groups to develop knowledge together have more opportunities to engage in meaningful activities outside of the PD setting and implement the strategy or practice. Even though there is less research regarding the impact of *collective participation* across different schools, it is still critical teachers develop networks to help them implement new knowledge they acquire at PD. Therefore, *collective participation* can also include teachers of the same grade band or content working together at PD sessions and developing peer support groups.

PD Focused on the Teaching of Mathematics

Within mathematics PD (i.e., PD focused on how teachers should teach mathematics), similar to the general field of PD, there are a variety of structures and topics researchers have examined, including the effect of summer training institutes (Zwiep & Benken, 2013), master's courses (Copur-Gencturk & Lubienski, 2013), online lesson studies (Lewis & Perry, 2014), and district in-service trainings with continued support throughout the school year (Polly et al., 2013). Additionally, mathematics PD cover a multitude of topics including specific content areas such as fractions (Jayanthi et al., 2017), algebra (Jacobs et al., 2007), and geometry (Wright, 2015) as well as teacher-skills identified by theory as important for student-learning, including leading mathematical discussions (Herbel-Eisenmann et al., 2013), and including higher-level questioning (Yenmez et al., 2017).

Even though some mathematical PD has been shown to positively impact student and teacher outcomes, most of the typical PD sessions that teachers attend do not lead to positive student achievement (Blank & de las Alas, 2009). Most mathematics teachers are not receiving the intensive content-focused PD necessary to increase their own content knowledge (Birman et al., 2007). More work is needed to determine the most effective structures, supports, and content necessary to productively increase both teacher outcomes and students' mathematical outcomes.

Role of Professional Learning Communities (PLCs) in Improving Teacher Knowledge and Skills

PLCs are structured groups of teachers focused on developing teachers' practices for the benefit of students, and when conducted well, provide a space and opportunity for professionals to share, evaluate, and reflect up on their practices in a collaborative, learning-oriented way (Stoll et al., 2006). Although typically conducted within a school, PLCs can also exist across schools and involve in-person or online collaborations, so long as teachers are willing to engage in critical conversations about improving their practice and knowledge. PLCs have been viewed as opportunities for teachers to develop their professional learning in ways that both validate teachers as learners in ways that are feasible and accessible for many schools that may lack the financial resources or professional contacts to have access to external coaches, which may be six to twelve times more expensive for school districts compared to other PD opportunities (Knight et al., 2018). Unfortunately, many PLCs are poorly implemented, lack support or training for teachers regarding effective means of evaluating and improving practice, or function as PLCs in name only without practices in place to improve teaching for the benefit of students (DuFour, 2007).

Despite a small empirical research base, PLCs are popular in many schools internationally (Stoll & Louis, 2007), and have been shown to positively impact teacher and school outcomes (Helman & Rosheim, 2016; Vescio et al., 2008). When well-supported, and implemented with a high degree of quality, PLCs can improve teachers' practice and improve student achievement (Darling-Hammond et al., 2017). Teachers participating in PLCs can see similar gains in teacher outcomes compared to teachers who received one-on-one coaching (Early et al., 2017). Quantitative studies that have examined the impact of PLCs on student achievement have largely reported small but significant positive effects (Lomos et al., 2011). In addition, because PLCs often bring together teachers from multiple disciplines in a school and focus on all students learning, PLCs may be beneficial structures for special education teachers to work with their general education peers to support students with learning disabilities (Blanton & Perez, 2011). Just as PLCs can be aligned to support and refine MTSS and RTI frameworks in schools (Helman & Rosheim, 2016), PLCs can also be a tool in supporting teachers implementing DBI.

Within PLC research, most studies examine the impact either on general teacher skills such planning lessons (Andrews & Lewis, 2002) or attitudes towards teaching, including self-efficacy (Zonoubi et al., 2017). What little research has been conducted on the impact of teachers' academic-specific skills, including mathematics-specific skills are largely positive. For example, after participating in a pilot study examining the impact of PLCs, four South African teachers improved the quality of mathematical instruction, including developing students' conceptual understandings and increasing student discourse (Chauraya & Brodie, 2017). In the full-scale project of the same pilot PD through PLCs, teachers developed greater diagnostic competence, referring to their ability to correctly identify the source of a students' misconception and correct it, a key skill in mathematics instruction (Brodie et al., 2018). Although the field is still thin, there is potential for PLCs to positively impact mathematics teachers or special education teachers teaching mathematics.

Role of Asynchronous Learning in Improving Teacher Knowledge and Skills

In-person and synchronous PD can be costly or cumbersome for districts to implement on a regular basis (Knight, et al., 2018). As the quality of streaming services and learning platforms has increased, many districts and states are examining whether asynchronous PD can result in the same positive teacher gains as in-person or synchronous PD. The on-going global pandemic due to COVID-19 has also generated the need to examine asynchronous PD in greater depth.

Recent research suggests that multimedia, or asynchronous PD, can positively impact learning of new strategies and knowledge to the same degree as in-person learning (Means et al., 2009). For example, Garet et al. (2011) demonstrated that large-scale mathematics PD conducted online can improve teachers' mathematical content knowledge and some teaching behaviors. Although structurally, asynchronous PD will not be able to include all of the five components of PD recommended by Desimone (2009), especially regarding collective participation and the collaboration many teachers find at in-person PD, Schumaker et al. (2020) saw that teachers who participated in online PD had the same positive results as those attending in-person PD, including significant growth in teachers' knowledge, planning, satisfaction as well as student learning.

A Synthesis of Mathematics PD Including PLCs and Asynchronous Learning

In this section, I describe a recent synthesis I conducted (Bos, 2022) that ties PD together with the components of using PLCs and implementing PD in asynchronous formats. I wanted to examine the effects of mathematics PD on teachers' content knowledge and the impact of the critical components recommended by Desimone (2009). The goal of the synthesis was to capture and evaluate the types of mathematics PD researchers are conducting, and whether various features impacted teachers' learning. In addition, if studies included student-level effects, those results were also evaluated, but not required to be included in the synthesis.

Search Process and Criteria for Inclusion

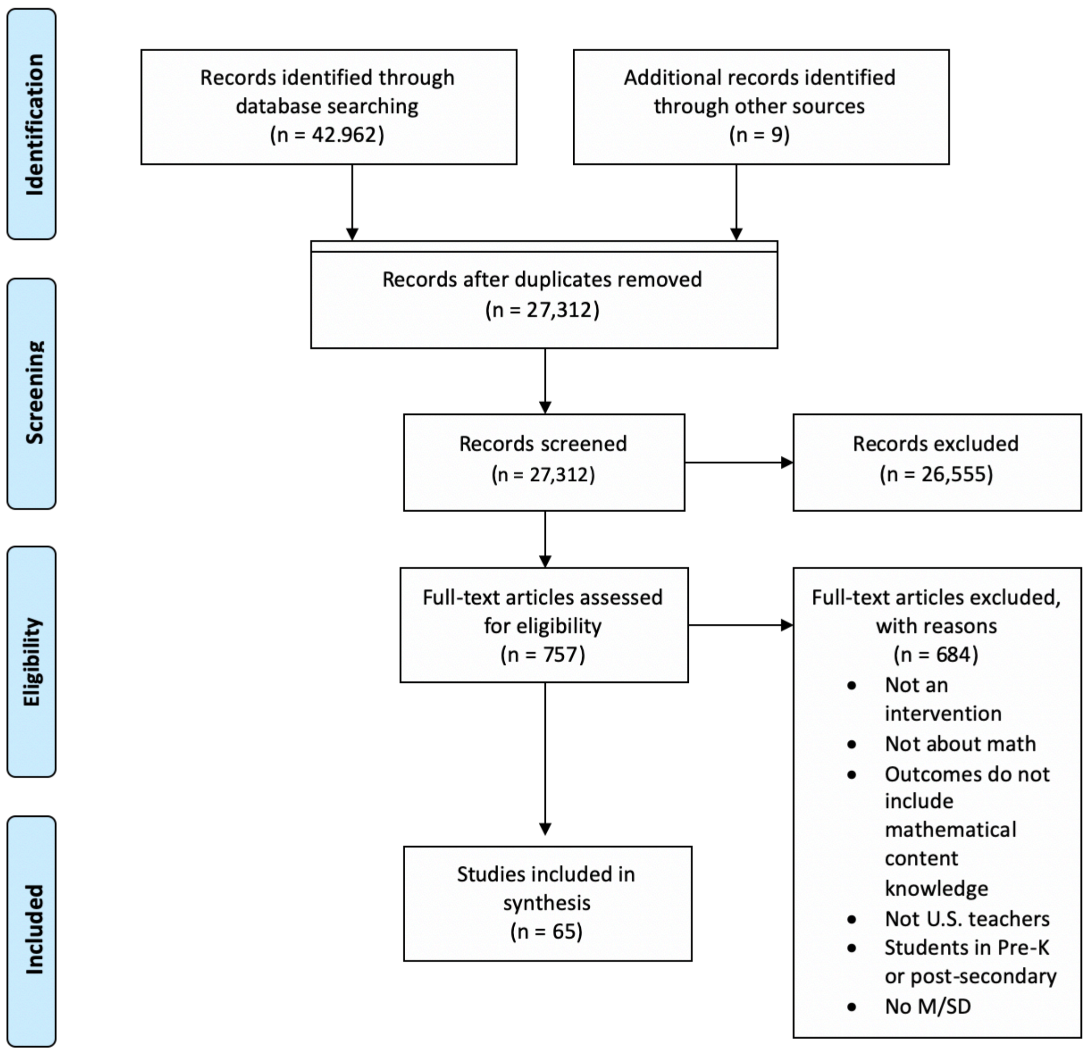
The following steps were taken to locate studies that met the inclusion criteria. First, I conducted a computer search of four electronic databases: *Education Source,* *Education Administration Abstracts, Education Resource Information Center (ERIC),* and *PsycINFO.* The search included both peer-reviewed journals and gray literature including dissertations. I did not limit the search by any date as professional development has been a topic of interest for decades, but there is little definitive knowledge on the most effective strategies to improve teacher outcomes. The first line of the electronic search field included the following search terms: teacher OR educator. The second line included: coach* OR mentor* OR professional development OR professional learning OR career development OR teacher development OR professional education OR training OR "teacher improvement" OR "instructional improvement." The third line included: math* OR geometry OR algebra OR calculus OR numeracy OR arithmetic* OR computation OR fractions OR multiplication OR division OR manipulatives OR word problem OR schema OR decimals.

This electronic search yielded 39,697 studies, including journal articles and other gray literature. Additionally, I completed a second search using the database *Dissertation and Thesis Global* and the same search terms to ensure I included all dissertations and theses, resulting in an additional 3,265 studies. After removing 15,657 duplicate studies, I screened the abstracts of the remaining 27,962 to identify studies that met inclusion criteria for this review. During the first-round screening, I eliminated studies that did not include PD in-service for U.S. mathematics teachers or did not include teacher mathematical knowledge as an outcome.

I also conducted a comprehensive hand search of *Education and Treatment of Children*, *Exceptional Children, Intervention in School and Clinic, Journal for Research in Mathematics Education, Journal of Mathematics Teacher Education, Journal of Staff Development, Journal of Special Education Remedial and Special Education, Learning Disabilities Quarterly, Learning Disabilities Research and Practice, Learning Professional, Mathematics Teacher Educator, North American Chapter of the International Group for the Psychology of Mathematics Education, Professional Development in Education,* and *Teaching Education and Special Education.* I reviewed all journals in their entirety as there was no date limitations set for this synthesis. After conducting the hand search, I reviewed studies from top funding agencies including *Institute of Education Sciences (IES)*, and I reviewed *Google Scholar* by searching my original search terms and reviewing the first 10 results pages of titles for additional articles. Additionally, I conducted a search of reference lists of relevant studies, literature reviews, syntheses, and meta-analyses in PD for mathematics teachers (e.g., Kraft et al., 2018; Kretlow & Bartholomew, 2010; Yeon, 2018; Yoon et al., 2007) as well as a forward search of included studies using *Google Scholar*. I identified an additional 9 articles meeting inclusion criteria.

After screening the abstracts, I selected a total of 757 articles for further review. I read the full text of all the 757 articles to determine whether they met the inclusion criteria for this review. Figure 2.1 displays a PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) diagram (Moher et al., 2009) describing the search process. I identified 73 initial studies for inclusion in this synthesis. Upon further review, eight studies included duplicate data and analysis of the outcomes of interest. I combined these duplicate studies together, resulting in 65 total studies in this synthesis.

Figure 2.1 PRISMA Diagram



To be included in my synthesis, the study had to meet the following criteria:

1. The study included U.S. in-service teachers who taught at least one mathematics class in Kindergarten through Grade 12. I included studies that included pre-service teachers, pre-Kindergarten teachers, or teachers from other countries if the data was disaggregated to allow for detailed analysis of only teachers from the U.S., only in-service teachers, or only K-12 teachers. Additionally, if studies included mathematics and science teachers, I included the studies that only reported the results for the mathematics teachers separately from science teachers.
2. The authors implemented a mathematics professional development (PD) program and measured teachers' mathematical knowledge after the PD. I defined "professional development" as any form of instructional programming provided about mathematics. PD included in-district and out-of-district institutes or workshops, college courses, online workshops, study groups, or resource centers. Additionally, I included PD programs that targeted STEM (Science, Technology, Engineering, and Mathematics) outcomes only if mathematics outcomes were reported separately from other STEM outcomes.
3. The authors utilized a randomized control trial design, a quasi-experimental design, or a single-case design.
4. The authors reported means or data that could be used to calculate means, or data necessary to conduct a visual analysis and calculate percentage of nonoverlapping data (PND) for single-case designs.
5. The study was published in English.

Coding Procedures

I developed a coding manual and a code sheet to extract relevant information from the studies meeting inclusion criteria. For each study, I extracted the following information: study information, study quality, methodological characteristics, participant characteristics, intervention characteristics, and outcome measures.

Study Information

I coded variables related to the study including: year of publication, state or region in which the study took place, and the type of publication (e.g., peer-reviewed journal, dissertation, conference brief).

Study Quality

I rated the quality of each study using the indicators and guidelines provided by the Council for the Exceptional Children Standards for Classifying Evidence-Based Practices (Cook et al., 2015). According to the guidelines provided by Cook et al. (2015), I coded eight quality indicators as either meeting the necessary criteria (1) or not meeting the necessary criteria (0). Quality indictors included descriptions of the: (a) context and setting (1 point); (b) participants (2 points); (c) intervention agent (2 points); (d) description of practice (2 points); (e) implementation fidelity (3 points); (f) internal validity (6 points); (g) outcome measures and dependent variables (6 points for group design studies, 5 points for single-case design studies); and (h) data analysis (2 points for group design studies, 1 point for single-case design studies). I calculated a rating for each study by dividing the sum of points by 24 (for group design studies) or 22 (for single-case design studies).

Methodological Characteristics

Variables related to the methodological characteristics of the studies included: research design (i.e., randomized control trial, quasi-experimental, or single-case design) including whether the study included a control or comparison group; total sample size, sample size of the treatment and control groups, group equivalence, attrition of the total sample size, and attrition of the treatment and control groups.

Participant Characteristics

I included information regarding both the teacher participants as well as student participants, if students were included in the study. Teacher characteristics included: teachers' role as either a general education or special education teacher, the grade level(s) taught, the subject taught, teachers' gender, ethnicity, educational level, and teaching experience including previous mathematics PD. Student-level information included: the total sample size, students' ages or grades, special education status, ethnicity, and the socioeconomic status of the school.

Intervention Characteristics

In coding for information regarding intervention characteristics, I included categories identified by Copur-Gencturk et al. (2019) and Garet et al. (2001) to capture the impact of PD upon mathematical knowledge for teaching that align with Desimone (2009) recommendations for high-quality PD. These characteristics included five core features of PD: (a) *content focus*, (b) *active learning*, (c) *coherence*, (d) *duration*, and (e) *active learning*. Descriptions of (a) *content focus* included six dimensions: curricular content knowledge, knowledge of student mathematics thinking, knowledge of mathematics teaching, general pedagogy, other content, and general mathematics content taught in the program. Dimensions of (b) *active learning* included five activities that have been shown to actively engage teachers' in developing knowledge: observing others or being observed; planning classroom implementation; examining student work; presenting leading, and writing; and solving mathematics problems. PD programs could exhibit three different forms of (c) *coherence*: coherence with state/district standards, coherence with teachers' own goals for their PD; or coherence with teachers' practice. One program could include more than one form of *coherence*. (d) *Duration* of programs included the number of contact hours, the frequency of meetings or sessions, and the total time span of the program. The fifth core feature, (e) *collective participation* referred to whether teachers from the same school participated in the program together.

In addition to the core features of PD, I also coded characteristics of interest, including whether the PD sessions were held in person, online, or both; if coaching was included as part of the program; and if the programs compensated or incentivized teachers for participating.

Outcome Measures

To be included in my synthesis, all studies needed to have reported at least one measure of teacher mathematics knowledge. Many studies included other measures such as teacher skills, attitudes, or self-efficacy growth as result of the program. I only recorded measures of teacher mathematics knowledge as well as student achievement (if included in the study). Studies did not need to have measured student achievement to be included in this synthesis. I coded both teacher and student outcomes for the same characteristics. Features of interest included: the measure's content, administration procedures, reliability, validity, and developer. If a measure included multiple components or multiple subtests, I included all data. For the purpose of calculating effect sizes, I also coded statistically important data such as the sample sizes reported in statistical analyses, group means, standard deviations, percentage of nonoverlapping data (PND) for single-case design studies, significant effects, and effect sizes. An unbiased estimate of delta was calculated for each statistic using the available data provided in each study.

Coding Reliability

I initially coded all studies as the first rater, and a second rater, who was a doctoral student in special education, independently coded 16 studies (25%) that I selected using a random number generator. I trained the second rater using the code sheet, and the second rater reached 90% agreement on practice articles before she coded the articles in this study. Interrater agreement was calculated as: [agreements ÷ (agreements + disagreements) × 100]. We discussed any coding discrepancies to determine the final code to be used in the analyses.

Results

I identified a total of 65 studies for this synthesis (see Table 2.1 for study characteristics). The majority of studies reviewed were published in the past 10 years (k = 50, 77%) even though the search was not limited by any date; one study was published in 1966. The location of the studies aligned to the Regional Educational Laboratory's (REL) geographical guidelines. Studies took place throughout the U.S. with many studies taking place in the Southeast (k = 10, 15%), West (k = 8, 12%) and Midwest (k = 8, 12%). Of the 65 studies in this synthesis, one study (2%) included both a single case design and a group design; the remaining 64 studies utilized group designs only. Fifty-one of these group design studies (80%) included quasi-experimental designs, and 13 studies (20%) included randomized control trials.

Table 2.1: Summary of Studies Characteristics

Characteristic	n	%
Publication Year		
1960-1969	1	1.5
1970-1979	0	0
1980-1989	0	0
1990-1999	3	4.6
2000-2009	10	15.4
2010-2020	50	76.9
Not Reported	1	1.5
Publication type		
Journal article	43	66.2
Dissertation	13	20.0
Conference paper	4	6.2
Government report	4	6.2
Not Reported	1	1.5
Study location		
Appalachia	3	4.6
Central	1	1.5
Mid-Atlantic	1	1.5
Midwest	8	12.3
Northeast & Islands	5	7.7
Northwest	2	3.1

Table 2.1: Summary of Studies Characteristics Cont.

Pacific	0	0
Southeast	10	15.4
Southwest	4	6.2
West	8	12.3
Online	5	7.7
Multiple regions	1	1.5
Not reported	17	26.2
Grade level		
Elementary (K-5)	29	44.6
Elementary & Middle school	3	4.6
Middle school (6-8)	15	23.1
Middle school & High school	7	10.8
High school (9-12)	3	4.6
K-12	7	10.8
Not reported	1	1.5
Research design		
RCT	13	20.0
Quasi-experimental	51	78.5
SCD	1	1.5
Sample size (teachers)		
1-10	3	4.6
11-25	15	23.1
26-50	10	15.4
50-100	19	29.2
100-200	7	10.8
200-500	8	12.3
500-1000	1	1.5
>1000	2	3.1
Sample size (students)		
1-500	2	14.2
500-1000	0	0
1001-2000	3	21.4
2001-3000	1	7.1
3001-4000	3	21.4
4000-5000	1	7.1
5000+	1	7.1
NR	3	21.4

Because of variability in grade-level groupings, I categorized studies as targeting elementary, middle, or high school teachers, and I created categories for studies that involved both elementary and middle school teachers, middle school and high school teachers, or teachers throughout Grades K-12. A total of 39 (60%) studies included elementary teachers, 32 (49%) studies included middle school teachers, and 17 (26%)

studies included high school teachers.

Sample sizes for both teachers and students varied greatly. Three studies (5%) included 10 or fewer teachers in the study, while 2 studies (3%) included more than 1,000 teachers in the sample size. Despite several studies with very large teacher sample sizes, 47 studies (72%) included 100 or fewer teachers in the study. Fourteen studies included student-level analyses, and many of these studies included large sample sizes with 6 studies (64%) including more than 1,000 students. Table 2.2 displays details about each of the 65 studies.

Table 2.2: Study Components

Authors	Participants		Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components					Effect size			Study Quality
	Teachers	Students							content focus	active learning	coherence	duration	collective participation	Teachers		Students	
														Pre- to Posttest Effect	Treatment Effect		
Avineri (2016)	196		Elementary	*Fraction Foundations* Fraction-focused MOOC	QED	Online	(1 semester)			✓	✓	✓		Pre < Post *Cohort 1* $\hat{\delta}$ = 0.24 *Cohort 2* $\hat{\delta}$ = 0.34			0.46
Bailey (2010)	30		2-3	Training sessions	QED	In-person	(3 years)		✓	✓	✓	✓	✓	Pre < Post $\hat{\delta}$ = 1.47			0.46
Bell et al. (2010)	308		2-6	*Developing Mathematical Ideas* National professional development	QED	In-person	21 hours (1 intensive week or 1 semester)		✓	✓		✓		(M.C.) Pre < Post $\hat{\delta}$ = 0.27 (O.E.) Pre < post $\hat{\delta}$ = 0.64	(M.C.) Ctl = Tx $\hat{\delta}$ =-0.02 (O.E.) Ctl < Tx $\hat{\delta}$ = 0.63		0.83
Birkhead et al. (2017)	54		2-9	Professional development	QED	In-person	(NR)		✓	✓	✓			Pre < Post $\hat{\delta}$ = 0.30			0.58
Boston (2013)	19		Middle & high school	*Enhancing Secondary Mathematical Teacher Preparation (ESP) Project* Workshop	QED	In-person	(1 year)		✓	✓	✓	✓		Pre < Post $\hat{\delta}$ = 0.69	Ctl < Tx $\hat{\delta}$ = 2.22		0.58
Brendefur et al. (2013)	94		K-6	DMT PD	QED	In-person	(3 years)		✓			✓		(N.O.) Pre < Post $\hat{\delta}$ = 1.09 (M.S.) Pre < Post $\hat{\delta}$ = 1.18 (P.S.) Pre < Post $\hat{\delta}$ = 1.38			0.50
Cady & Rearden (2009)	8		Middle school	Online courses	QED	Online	(1 ½ years)		✓	✓	✓	✓	✓	(N.C.) Pre = Post $\hat{\delta}$ = 0.12 (Algebra) Pre = Post $\hat{\delta}$ = 0.11			0.42

(G.M.)
Pre = Post
$\hat{\delta}$ = 0.28

Table 2.2: Study Components Cont.

Authors	Participants		Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components					Effect size			Study Quality
	Teachers	Students							content focus	active learning	coherence	duration	collective participation	Teachers		Students	
														Pre- to Posttest Effect	Treatment Effect		
Carey et al. (2008)	91		7-8	Facilitated cohort	RCT	Online	40 hours (10 weeks)		✓	✓		✓	✓	Facilitated Pre = Post[a] Self-Paced Pre = Post[a]			0.67
Carney et al. (2019)	4304		K-12	*Mathematical Thinking for Instruction (MTI)* Mathematics course	QED	In-person	45 hours (1 week or 6-10 weeks)		✓			✓	✓	(K-3) Pre < Post (4-8) Pre < Post (6-12) Pre < Post			0.71
Carney et al. (2016)	3933		K-12	*Mathematical Thinking for Instruction (MTI)* Mathematics course	QED	In-person	45 hours (1 week)		✓			✓	✓	Cohort 1 Pre < Post $\hat{\delta}$ = 0.76 Cohort 2 Pre < Post $\hat{\delta}$ = 0.59 Cohort 3 Pre < Post $\hat{\delta}$ = 0.74			0.58
Carpenter (2017)	48		3-5	*Urban Math Institute* Mathematics Summer Institute	QED	In-person	(1 year)	Included coaching	✓	✓	✓	✓		Ctl < Tx			0.71
Copur-Gencturk & Lubienski (2013)	24		Elementary & middle school	Mathematics content and mathematics methods course	QED	Hybrid	(1 semester)		✓	✓	✓	✓		Hybrid (LMT) Pre < Post $\hat{\delta}$ = 1.71 (DTAMS) Pre < Post $\hat{\delta}$ = 0.50 Math Course (LMT) Pre = Post $\hat{\delta}$ = -0.12 (DTAMS) Pre = Post $\hat{\delta}$ = 0.28 Hybrid + Math Course (LMT) Pre < Post (DTAMS)			0.67

Table 2.2: Study Components Cont.

Pre < Post $\hat{\delta}$ = 0.60

Authors	Participants: Teachers	Participants: Students	Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components: content focus	active learning	coherence	duration	collective participation	Effect size: Teachers: Pre- to Posttest Effect	Teachers: Treatment Effect	Students	Study Quality
Courtney (2018)	5	93	Middle and high school	Teacher educator embedding	QED	Hybrid	(NR)	Included coaching	✓	✓	✓		✓	Pre < Post $\hat{\delta}$ = 1.71		NR	0.46
Creek (2017)	20		6-8	Content-based PD	QED	In-person	4 hours (7 weeks)		✓	✓		✓		Pre < Post $\hat{\delta}$ = 0.55			0.46
Dash et al. (2012)	79	1438	5	Online PD	RCT	Online	70 hours (1 ½ years)		✓	✓		✓		Pre < Post $\hat{\delta}$ = 0.70	Ctl < Tx $\hat{\delta}$ = 0.77	Ctl = Tx $\hat{\delta}$ = 1.42	0.71
Duncan et al. (20XX)	96	1500	4-5	*Math For All* Intensive professional development	RCT	In-person	550 hours (10 months)	Included Coaching	✓	✓	✓	✓		Pre < Post	Ctl = Tx $\hat{\delta}$ = 0.65	NWEA Ctl = Tx State Test Ctl > Tx	0.79
Evans (2011)	42		Secondary	*New York City Teaching Fellows (NYCTF)* Alternative certification program	QED	In-person	(NR)					✓		Pre < Post $\hat{\delta}$ = 0.91			0.46
Faulkner & Cain (2013)	146		K-12	*North Carolina Foundations Mathematical Training* Professional development module	QED	In-person	40 hours (8-10 weeks)					✓		Pre < Post $\hat{\delta}$ = 0.57	Ctl = Tx[a]		0.63
Feuerborn et al. (2009)	31		Middle-level	Institutes	QED	In-person	(1 week)		✓			✓		Institute 1 Pre < Post $\hat{\delta}$ = 4.14 Institute 2 Pre < Post $\hat{\delta}$ = 1.79			0.58
Garet et al. (2016)	165	1697	4	*Intel Math, Mathematics Learning Communities, Video Feedback Cycle* Content-intensive professional development	RCT	In-person	93 hours (1 year)	Included coaching		✓		✓	✓		Ctl < Tx $\hat{\delta}$ = 0.54	Ctl = Tx[a]	0.83
Garet et al. (2011)	89	2132	7	*America's Choice* and *Pearson Achievement Solutions* National PD	RCT	In-person	114 hours (2 years)	Included coaching		✓	✓	✓			Ctl = Tx $\hat{\delta}$ = 0.05	Ctl = Tx	0.83

Table 2.2: Study Components Cont.

Authors	Participants: Teachers	Participants: Students	Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components: content focus	active learning	coherence	duration	collective participation	Effect size: Teachers: Pre- to Posttest Effect	Teachers: Treatment Effect	Students	Study Quality
Gerber et al. (2011)	95		3-5	Workshop session	QED	In-person	(3 years)		✓	✓	✓	✓	✓	Pre < Post $\hat{\delta}$ = 0.46			0.42
Greabell & Phillips (1990)	18		Elementary	*Summer Mathematics Institute* Inservice	QED	In-person	60 hours (1 month)					✓		Pre < Post			0.42
Griffin et al. (2017)	23	312	3-5	*Prime Online* Online workshop	QED	Online	(1 year)		✓	✓	✓	✓	✓	(CKT-M (CK)) Pre = Post $\hat{\delta}$ = 0.39 (CKT-M (KS)) Pre = Post $\hat{\delta}$ = -0.10		Pre= Post	0.67
Grunow (1998)	20		5-12	Professional Development Institute	QED	Hybrid	(1 year)		✓	✓	✓	✓	✓	(C.T.) Pre < Post $\hat{\delta}$ = 0.67 (K.R.) Pre < Post $\hat{\delta}$ = 0.75 (A.C.) Pre < Post $\hat{\delta}$ = 0.67			0.50
Harris et al. (2011)	73		Middle school	*West Texas Middle School Math Partnership* Summer Courses	QED	In-person	(2 week)			✓		✓	✓		Ctl = Tx[a]		0.50
Hill & Ball (2004)	398		Elementary	*California's Mathematical Professional Development Institutes (MPDs)* Summer Institutes	QED	In-person	40-120 hours (1-3 weeks)		✓	✓		✓		Pre < Post $\hat{\delta}$ = 0.76			0.54
Jacob et al. (2017)	105	1523	4-5	*Math Solutions* Commercial professional development	RCT	In-person	40 hours (1 year)		✓	✓	✓	✓	✓		(N.O.) Ctl < Tx $\hat{\delta}$ = 0.31 (Geometry) Ctl = Tx $\hat{\delta}$ = 0.21	Study Test Ctl = Tx State Test Ctl = Tx	0.96
Jacobs et al. (2007)	180	3735	1-5	Professional Development	RCT	In-person	16.5 hours (10 months)	Included coaching	✓	✓	✓	✓	✓		Ctl = Tx	Ctl < Tx	0.625

Table 2.2: Study Components Cont.

Authors	Participants		Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components					Effect size			Study Quality
	Teachers	Students							content focus	active learning	coherence	duration	collective participation	Teachers		Students	
														Pre- to Posttest Effect	Treatment Effect		
Jayanthi et al. (2017)	264	4204	4	*Developing Mathematical Ideas (DMI): Making Meaning for Operations in the Domains of Whole Numbers and Fractions* Module	RCT	In-person	24 hours (4 months)		✓	✓		✓		Pre < Post $\hat{\delta}$ = 1.84	Ctl = Tx $\hat{\delta}$ = 0.31	Ctl = Tx	0.96
Jiang et al. (2015)	64		Middle & high school	*Dynamic Geometry (DG)* Interactive software	RCT	In-person	(1 year)			✓	✓	✓		Pre < Post $\hat{\delta}$ = 0.50	Ctl = Tx	Ctl < Tx	0.71
Jones et al. (2009)	65		4-6	Workshop	QED	In-person	(NR)		✓					Year 1 (C. K.) Pre < Post $\hat{\delta}$ = 1.92 (P. K.) Pre < Post $\hat{\delta}$ = 2.07 Year 2 (C.K.) Pre < Post $\hat{\delta}$ = 1.96 (P.K.) Pre < Post $\hat{\delta}$ = 1.61			0.20
Koellner & Jacobs (2015)	62	67849	Middle school	*Problem-Solving Cycle Model* Workshops	QED	In-person	(NR)			✓	✓			Pre < Post $\hat{\delta}$ = 0.34		NR	0.54
Kutaka et al. (2017)	218		K-3	*Primarily Math* Mathematical courses	QED	In-person	80 hours (14 months)		✓	✓	✓	✓	✓		(N.O.) Ctl < Tx (Geometry) Ctl = Tx (P.F.A.) Ctl = Tx	Ctl < Tx	0.79

Table 2.2: Study Components Cont.

Authors	Participants: Teachers	Participants: Students	Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components: content focus	active learning	coherence	duration	collective participation	Effect size: Teachers (Pre- to Posttest Effect)	Effect size: Students (Treatment Effect)		Study Quality
Lewis & Perry (2014)	213		2-5	Lesson study with resources	RCT	In-person	(3 months)		✓	✓	✓	✓	✓	(E.P.) Pre = Post (F.N.) Pre < Post $\hat{\delta}$ = 0.56 (W.) Pre < Post $\hat{\delta}$ = 0.32 (M.E.) Pre < Post $\hat{\delta}$ = 0.60 (U.F.) Pre < Post $\hat{\delta}$ = 0.33 (L.R.) Pre < Post $\hat{\delta}$ = 0.85 (C.) Pre = Post	(E.P.) Ctl = Tx $\hat{\delta}$= -0.18 (F.N.) Ctl < Tx $\hat{\delta}$ = 0.89 (Whole) Ctl = Tx $\hat{\delta}$ = 0.18 (M.E.) Ctl < Tx $\hat{\delta}$ = 0.18 (U.F.) Ctl < Tx $\hat{\delta}$ = 0.31 (L.R.) Ctl < Tx $\hat{\delta}$ = 1.15 (Circle) Ctl = Tx $\hat{\delta}$= -0.46		0.75
Luebeck et al. (2017)	58		Middle school	*Standards-based Teaching Renewing Educators Across Montana (STREAM)* Online modules with support	QED	Hybrid	120 hours (6-8 months)		✓	✓	✓	✓		Year 1 Pre < Post $\hat{\delta}$ = 1.24 Year 2 Pre < Post $\hat{\delta}$ = 0.48			0.58
McCartney (2013)	15		3-5	Professional Development	QED	In-person	8 hours (4 weeks)		✓		✓	✓	✓	Pre < Post	Ctl = Tx $\hat{\delta}$ = 0.91		0.71
McCoy (2016)	27		Elementary	*Common Core Mathematics for Elementary Teachers* Hands-on course	QED	In-person	18 hours (6 weeks)		✓	✓	✓	✓	✓	Pre < Post $\hat{\delta}$ = 1.52			0.58
Middleton et al. (2011)	35	1509	High school	Intensive year-long PD	QED	In-person	135 hours (1 year)		✓	✓	✓	✓		Pre < Post	Ctl < Tx $\hat{\delta}$ = 2.04	NR	0.63

Table 2.2: Study Components Cont.

Authors	Participants		Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components					Effect size			Study Quality
	Teachers	Students							content focus	active learning	coherence	duration	collective participation	Teachers		Students	
														Pre- to Posttest Effect	Treatment Effect		
Miller (2017)	14		Middle school	Workshops	QED	Hybrid	54-162 hours (1-3 years)	Included Coaching	✓	✓	✓	✓		Pre < Post $\hat{\delta}$ = 0.57			0.58
Murphy (2002)	16		6-8	*MathStar Summer Institute* Professional development	QED	Hybrid	(4-7 months)		✓	✓	✓	✓		Pre < Post $\hat{\delta}$ = 1.55			0.5
Patel et al. (2012)	54		6-8	*Connect Mathematics Project (CMP)* Curriculum training	QED	In-person	40 hours (1 week)			✓		✓		Pre < Post $\hat{\delta}$ = 0.54			0.5
Polly et al. (2017)	15	566	Kindergarten	*Content Development to Teach Investigations (CoDE-I)* Professional development	QED	In-person	70 hours (10 month)		✓	✓	✓	✓	✓	System 1 Pre = Post $\hat{\delta}$ =0.11 System 2 Pre = Post $\hat{\delta}$ = 0.47			0.38
Polly et al. (2015)	291	3293	1-5	*Content Development to Teach Investigations (CoDE-I)* Professional development	QED	In-person	80 hours (1 year)		✓		✓	✓	✓	Pre < Post $\hat{\delta}$ = 0.43		Pre< Post	0.58
Polly et al. (2013)	28		Elementary	Professional Development Program	QED	In-person	84 hours (13 months)		✓	✓	✓	✓	✓	Pre < Post $\hat{\delta}$ = 1.22			0.33
Ribeiro (2009)	31		K-6	Professional development with peers and pre-service teachers at a university setting	QED	In-person	23 hours (11 weeks)			✓		✓		Group 1 & 2 Pre < Post $\hat{\delta}$ = 1.62			0.67
Russell et al. (2009)a	81		1-5	*Building a System of Tens* Online course with support	RCT	Online	24 hours (8 weeks)		✓			✓		Online Pre < Post Face to Face Pre < Post			0.67
Russell et al. (2009)b	231		7-8	*Building Algebraic Thinking in the Middle Grades*	RCT	Online	32 hours (8 weeks)		✓	✓		✓		Function Pre = Post Patterns Pre < Post			0.67

Table 2.2: Study Components Cont.

Online course with support

Authors	Participants		Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components					Effect size			Study Quality
	Teachers	Students							content focus	active learning	coherence	duration	collective participation	Teachers		Students	
														Pre- to Posttest Effect	Treatment Effect		
Schoen et al. (2019)	275		Middle school & high school	*Institute for Early Secondary Statistics and Probability* Statistics institute	RCT	Hybrid	80 hours (2 weeks)		✓			✓			Ctl < Tx		0.83
Seago (2013)	127	266	5-10	*Learning and Teaching Geometry* Geometry modules	QED	In-person	30 hours (1 year)		✓	✓		✓			Ctl < Tx $\hat{\delta}$ = 0.40	Ctl < Tx	0.71
Siebers (2012)	68		4-8	*Project Making Mathematics Matter (PM3)* Intensive program	QED	In-person	125 hours (1 year)	Included Coaching	✓	✓	✓	✓	✓		Ctl < Tx $\hat{\delta}$ = 1.02	Ctl < Tx	0.79
Silverman (2011)	54		Middle school	*Online Asynchronous Collaboration (OAC)* Graduate course	QED	Online	(10 weeks)					✓		Pre < Post $\hat{\delta}$ = 0.98			0.46
Silverman (2017)	76		Middle and/or secondary	*Online Asynchronous Collaboration (OAC)* Graduate course	QED	Online	(10 weeks)		✓			✓		Pre < Post $\hat{\delta}$ = 0.42			0.38
Swafford et al. (1997)	49		4-8	*LINCS* Intervention program	QED	In-person	64 hours (1 year)		✓	✓	✓	✓		Pre < Post $\hat{\delta}$ = 0.77			0.5
Swars et al. (2018)	32		Elementary	K-5 Mathematics Endorsement Program	QED	In-person	150 hours (9 months)	Included coaching	✓	✓	✓	✓		Pre < Post $\hat{\delta}$ =1.23			0.46
Vega (2015)	15		High school	Professional Development	QED	In-person	(3 years)		✓	✓	✓	✓	✓	Pre < Post $\hat{\delta}$ = 0.63			0.58
Walker (2012)	24		Elementary	*Appleton Area School District (AASD) Math Institute* Professional development	QED	In-person	39 hours (7 months)		✓	✓	✓	✓	✓	Pre < Post $\hat{\delta}$ =0.64			0.54

Table 2.2: Study Components Cont.

Authors	Participants		Grade levels teachers supported	Intervention	Design	Format	Contact Hours (Duration)	Coaching	Critical Components					Effect size			Study Quality
	Teachers	Students							content focus	active learning	coherence	duration	collective participation	Teachers		Students	
														Pre- to Posttest Effect	Treatment Effect		
Walters & Ogut (2018)	798		Elementary	*Developing Teaching Expertise @ Mathematics (Dev-TE@M)* Mathematical modules	QED	Hybrids	45 hours (NR)		✓	✓	✓	✓		Module 1 Pre < Post $\hat{\delta}$ = 0.48 Module 2 Pre < Post $\hat{\delta}$ =0.46 Module 3 Pre < Post $\hat{\delta}$ = 0.53			0.50
Wang et al. (2013)	185	5070	Elementary	*Content Development to Teach Investigations (CoDE-I)*	QED	In-person	72 hours (1year)		✓	✓	✓	✓	✓	System 1 Pre < Post $\hat{\delta}$ = 0.23 System 2 Pre = Post $\hat{\delta}$ = 0.13		Pre = Post*	0.50
Wasserman (2014)	12		Elementary middle and secondary schools	*Algebraic Reasoning and Patterns* Mathematics course	QED	In-person	6-8 hours (3-4 days)					✓		Pre < Post			0.25
Weber et al. (2015)	20		Elementary	*Modeling Instruction* Master's course	QED	In-person	(15 weeks)		✓	✓		✓		Pre < Post $\hat{\delta}$ = 0.53			0.46
White et al. (2013)	50		5-9	*Math Teachers Circle (MTC)* Professional development workshops	QED	In-person	48-54 hours (1 year)		✓		✓	✓	✓	(N.C.) Pre < Pos $\hat{\delta}$ = 0.44 (Geometry) Pre = Post $\hat{\delta}$ = 0.07			0.54
Whitman (1966)	22		Elementary	Workshop in Mathematics	QED	In-person	30 hours (3 weeks)					✓		Pre < Post $\hat{\delta}$ = 1.85			0.50
Wright (2015)	6		High school	Professional Development	SSCD, QED	Hybrid	32 hours (2 weeks)		✓	✓	✓	✓		Pre < Post $\hat{\delta}$ = 0.96			0.75
Zwiep & Benken (2013)	51		4-9	Professional Development	QED	In-person	80 hours (1 year)		✓	✓		✓	✓	Pre < Post $\hat{\delta}$ = 0.42			0.58

Note. All studies with sufficient statistical analysis include a calculated estimate of delta ($\hat{\delta}$) as an effect size. Studies that do not include an estimate of delta did not report sufficient data. M.C. = multiple choice; O.E. = open ended; N.O. = Number and Operations; M.G. = Measurement and Geometry; P.S. = Probability and Statistics; N.C. = Number and Computation; G.M. = Geometry and Measurement; LMT = Learning Mathematics for Teaching; DTAMS = Diagnostic Teacher Assessments in Mathematics and Science; NWEA = Northwest Education Association; CKT-M (CS) = Content Knowledge for Teaching (Knowledge of Content); CKT-M (KS) = Content Knowledge for Teaching (Knowledge of Students and Content); C.T. = Concepts and Terminology; K.R. = Knowledge of the Relationships Among Concepts; A.C. = Ability to Communicate; C.K. = content knowledge; P.K. = pedagogical knowledge; P.F.A = Patterns, Functions, and Algebra; E.Q = Equal Parts; F.N. = Fraction as Numbers; M.E. = Math Errors; U.F. = Unit Fraction; L.R. = Linear Representation; M. G. = Measurement and Geometry; P.S. = Probability and Statistics.

[a] Reflects studies with mixed results.

Mathematics PD Impact on Teacher Content Knowledge

To be included in my synthesis, studies had to include outcomes measuring teacher mathematical knowledge, including conceptual knowledge, procedural knowledge or MKT. Of the 65 studies included in this synthesis, the majority (89%) reported teacher knowledge growth on at least one measure after participation in PD, with 45 studies (69%) reporting significant growth in mathematical content knowledge. The effect sizes (ES) were calculated as the difference between the groups' means divided by the pooled standard deviation. A total of 86 posttest effect sizes were calculated from 55 studies. I was unable to calculate effect sizes for 10 studies due to insufficient data. Effect sizes ranged from -0.46 to 4.14. A total of 16 studies reported including all five critical components. Estimates of these studies ranged from -0.46 to 1.52. The results are presented by critical components included in each study (i.e. content focus, active learning, coherence, duration, and collective participation). As many studies included several components, there is substantial overlap in the effect sizes calculated.

As described earlier in this Chapter 2, Desimone (2009) recommended all PD include five critical components: *content focus*, *active learning*, *coherence*, *duration*, and *collective participation*. As described by Copur-Gencturk et al. (2019), each of the critical components included several dimensions. In this synthesis, I categorized each study as reporting a critical component if the study reported including over half of the unique dimensions specified by Garet et al. (2001) and Copur-Gencturk et al. (2019). For example, *active learning* included five dimensions: observations, problem-solving, student work, planning classroom implementation, and presenting in professional settings. If a study included at least three of the five dimensions, I recorded that study as addressing the critical component of *active learning*. All studies in this synthesis reported including at least one critical component.

Content focus

Fifty-one studies (78%) reported at least three of the five dimensions of PD *content focus*. *Content focuses* included curricular content knowledge, knowledge of students' mathematics thinking, knowledge of mathematics teaching, general pedagogy, or some other form of mathematical knowledge (e.g., inquiry-based practices or problem-solving strategies). Estimates of studies that reported including a content focus ranged from -0.46 to 4.14, similar to the larger sample size.

Active learning

Regarding *active learning*, 48 studies (74%) reported including at least three of the activities listed above (i.e., observations, problem-solving, student work, planning classroom implementation, and presenting in professional settings). Estimates of studies that reported including active learning strategies or components ranged from -0.46 to 2.04.

Coherence

Of the 65 studies, 37 studies (57%) reported including two of the three forms of *coherence*, either coherence with state or district standards, coherence with teachers' own goals, or coherence with teachers' later practice). Estimates of studies that reported coherence with teachers' needs and situations ranged from -0.46 to 2.04.

Duration

Duration was most often reported across studies with 61 studies (94 %) reporting either the length of the PD, the number of contact hours, or both. Estimates of studies that reported duration data ranged from -0.46 to 4.14, similar to the larger sample size.

Collective participation, 6.h6

Finally, only 26 studies (40%) reported *collective participation* of the teachers, referring to teachers attending the PD with colleagues from their school or at the same grade level. Estimates of studies that reported a focus on collective participation ranged from -0.46 to 1.71.

Impact of Mathematics PD on Student Outcomes

Of the 65 studies in this synthesis, 14 studies included analysis on student outcomes, and the results varied with 4 studies reporting positive results, 3 studies reporting mixed results, and 7 studies reporting null effects.

Analysis

The five critical components Desimone (2009) recommended for PD are meant to provide teachers with meaningful opportunities to develop lasting knowledge and habits of implementing evidence-based strategies in their classroom. While not yet empirically tested, the guidelines recommended by Desimone (2009) provide a theoretical framework for researchers to position their programs.

Studies widely reported *duration*, *content focus*, and *active learning*, with 94% studies reporting *duration* data, 78% reporting a *content focus*, and 74% reporting *active* *learning* dimensions. All three of these components have a large research base supporting their importance in engaging adult learners (Wei et al., 2009). It is noteworthy so many studies in this synthesis took place over several months or even years. It is still common for teachers to receive single-day PD with no follow up support (Rotermund et al., 2017), and there is a building consensus that these abbreviated forms of PD are often ineffective at improving teacher outcomes (Darling-Hammond et al., 2009). It is heartening the shortest PD reported in this synthesis lasted at least 3 days, with most studies reporting much greater duration, signaling a greater recognition among researchers that high-quality PD requires a significant time commitment. In a similar way, it is notable so many PDs included *content focus* and *active learning* components, moving away from the lecture-only skills-driven PD which has been shown to be less effective (Darling-Hammond et al., 2009).

Author teams were less likely to report aligning PD for greater *coherence* with teachers' and districts' goals. Similarly, author teams less frequently included teachers from the same schools, encouraging *collective participation*. Even though these two components support teachers in implementing strategies that align with district requirements and provide team support so teachers do not have act in isolation, there is a smaller research base to support either of these components (Akiba & Liang, 2016). In addition, as *coherence* and *collective participation* may not seem as relevant to researchers, author teams may have implemented these components without reporting them. As the research base develops to bring these two components to the same forefront as *duration*, *content focus*, and *active learning* currently occupy, it is hopeful that more studies will report including *coherence* and *collective participation* as well.

Across many of the studies, there was no clear pattern of certain critical components being included in effective PDs. No one critical component was found in a greater number of studies that reported positive results. Instead, many of the components were as likely to be found in ineffective PD as they were in effective PD. This may be an artifact of the way that I chose to code for critical components, and a fine-grain analysis of the data might reveal that certain dimensions of each component (e.g., being observed, or participating with school-based colleagues) have a greater impact on teacher outcomes than other dimensions within the same critical component. However, this lack of clarity also may suggest that more empirical work needs to done determining the effect of each component, and it will be necessary for researchers to provide detailed reports of their PD in order for this analysis to take place.

Within the 65 studies included in this synthesis, only 14 studies reported student-level outcomes, and these outcomes largely reflect the inconclusiveness found in the field regarding the impact of PD on student achievement (Yoon et al., 2007). With only 29% of these studies reporting positive student growth, this synthesis suggests most PDs do not yet impact teachers' learning to the degree that students are positively impacted. In addition, while null teacher effects generally led to null student-level effects, there was no clear pattern between teacher-level growth and student-level growth. The lack of consistent impact of teacher growth on student growth aligns with other research that highlights many factors contribute to student achievement, and teacher knowledge is just one piece of the puzzle (Borko & Whitman, 2008). While teacher content knowledge is a potential target for PDs to develop, these findings suggest that additional studies should continue to explore other teacher-level factors that contribute to student learning that PDs can potentially impact as well.

A lack of detailed PD component descriptions, coupled with a lack of theory driving research investigations, lead to thin and variable findings. Even studies that report positive teacher growth do little to contribute to the larger question of how can we improve all teachers' mathematical content knowledge. Instead, the current field resembles a grab bag of different models and strategies for improving teachers' knowledge with few answers regarding the most effective and efficient methods for addressing how to best support teacher learning that results in greater student knowledge.

To interpret the superficially positive results as conclusive is problematic, but examining the increasing volume of studies that measure teachers' content knowledge is encouraging. As more is learned about the impact of teachers' mathematical content knowledge, specifically teachers' MKT (Ball et al., 2008), it may become easier to develop PDs that support long-lasting teacher growth and meaningful student impact. In the same way, as more studies include detailed descriptions of the critical components recommended by Desimone (2009), it will become easier to evaluate the true impact of each of these components upon teacher and student learning.

Teacher Outcomes after Participating in PD including PLCs and Asynchronous Learning

The results from my synthesis of PD (Bos, 2022) point to the need to understand the impact of PD in greater detail. In this section, I review the impact of teachers participating in PD on different types of teacher outcomes, including MKT, self-efficacy, and instructional practices. The teacher outcomes reviewed include outcomes that I examined in my synthesis of PD as well as additional outcomes that research suggests impact student achievement.

Importance of Teachers Mathematical Knowledge for Teaching (MKT)

MKT is a specific construct that reflects a teachers' unique knowledge base required for teaching. Unlike the general content knowledge an accountant or engineer needs to solve mathematical problems, MKT reflects a teachers' need to be able address students' questions of why certain mathematical procedures work, identify student misconceptions, and be able to choose the most effective model or tool to teach a mathematical concept or procedure (Ball et al., 2008). Both deep conceptual mathematical knowledge and pedagogical knowledge are needed for teachers to be highly effective at improving students' conceptual understandings (Hill, Ball, et al., 2008). Teachers who display both higher mathematical content knowledge and pedagogical content knowledge, on average, display greater quality of mathematics instruction (Hill, Blunk, et al., 2008). In addition, teachers with greater MKT positively impact students (Hill et al., 2005).

Despite the importance of developing teachers' MKT, research has shown that it is difficult to substantially increase teachers mathematical content knowledge and MKT (Phelps et al., 2016). In addition, there may be an MKT threshold that teachers must possess before their practice will change (Santagata et al., 2011), suggesting that teachers with low MKT or low mathematical content knowledge may be at a disadvantage for learning new concepts and skills. Therefore, it is critical that high-quality PD include MKT both as a targeted outcome of the training but also a possible moderator to explain any differences in gains between different groups of teachers. The MKT assessment developed by Deborah Ball, Heather Hill, and colleagues (2008) continues to be a popular measure of mathematics PD by researchers interested in teachers' mathematical knowledge and ability to teach students effectively (e.g., Avineri, 2016; Bell et al., 2010; Birkhead et al., 2017; Carney et al., 2019; Carpenter et al., 2017; Copur-Gencturk & Lubienski, 2013; Gerber et al., 2011; Harris et al., 2011; Jacob et al., 2017).

Importance of Teachers' Self-Efficacy

A teachers' self-efficacy has been tied to many favorable teacher-level outcomes, such as lower burnout and higher job satisfaction (Granziera & Perera, 2019) as well as favorable classroom outcomes, such as better classroom management (Holzberger et al., 2013), and also greater student achievement (Zee & Koomen, 2016). Teachers with higher self-efficacy believe that they have more control in their classroom and is better able to teach their students effectively. Additionally, teachers with high self-efficacy will believe they can hold students to higher expectations and will be able to effectively teach even students who are struggling. Together, these beliefs and judgments impact the decisions teachers make including investing time and effort in their students' learning (Woolfolk Hoy et al., 2009). Additionally, teachers with higher self-efficacy are oftentimes more organized and have better classroom management skills (Künsting et al., 2016), leading to higher overall instructional quality and greater quantity of time spent on-task, learning necessary concepts.

Within the field of mathematics, especially elementary education, some teachers report feeling anxious and not comfortable teaching mathematics (Beilock et al., 2010; Bryant, 2009). A teacher with lower self-efficacy, including lower self-efficacy in mathematics, can negatively impact students' attitudes and outcomes in mathematics (Beilock et al., 2010; Ramirez et al., 2018). On the other hand, however, teachers who report greater mathematics self-efficacy positively impact students' perceptions of the class as well as students' mathematics achievement scores (Perera & John, 2020). Teachers with higher mathematics self-efficacy scores seem to create more positive classroom environments for their students, leading students to have both more positive attitudes towards mathematics as well as higher scores. Students who report having higher self-perceptions of mathematics as well as positive attitudes towards mathematics often have higher mathematic achievement, irrespective of the teacher (Perera & John, 2020), but it is critical that teachers can also positively impact students' perceptions and outcomes.

Importance of Teachers' Instructional Practices

In addition to internal qualities of a teacher, including self-efficacy and MKT, the classroom instruction teachers deliver are critically important to the success of students in learning mathematics. Examples of effective, evidence-based instructional strategies include the use of explicit instruction (Dennis et al., 2016; Ennis & Losinski, 2019; Fuchs et al., 2021; Kroesbergen & Van Luit, 2013; Marita & Hord, 2017; Misquitta, 2011; Zheng et al., 2013), use of multiple representations (Bouck, et al., 2018; Fuchs et al., 2021; Hwang et al., 2019; Peltier et al., 2019), and using concise mathematical language (Fuchs et al., 2021; Hughes et al., 2016). Explicit instruction includes the use of engaging modeling, scaffolded practice, and many opportunities to engage learners and provide specific feedback (Archer & Hughes, 2016; Doabler et al., 2015). The use of multiple representations, also known as the concrete-representational-abstract (CRA) framework engages students in developing conceptual mathematical knowledge by using concrete tools and visual representations (e.g., pictures, graphs, or drawings) (Ainsworth, 2006). Throughout all mathematical instruction, the use of concise and precise mathematical language helps students efficiently develop the necessary procedural and conceptual understandings needed to be successful (Riccomini et al., 2015). Even though a strong sense of self-efficacy and MKT would contribute to a teachers' ability to deliver effective explicit instruction using both multiple representations and concise mathematical language, the ability to implement the instructional practices require unique skills and knowledge beyond self-efficacy and MKT.

Students experiencing MD often do not respond sufficiently to general education or Tier 1 instruction and need additional, more intensive evidence-based instruction (Fuchs et al., 2017). The interventionists who deliver the intensive intervention, often known as Tier 2 or Tier 3 instruction, therefore need to be equipped with strategies that meet the need of the students they are teaching. Teachers who deliver intensive intervention should have the knowledge and skills to effectively implement evidence-based strategies including explicit instruction, multiple representations, and concise mathematical language.

In addition to being able to effectively implement evidence-based instructional strategies, intervention teachers should be able to employ additional skills necessary to identify and target the specific mathematical weaknesses of students of students experiencing MD to more effectively deliver necessary instruction. DBI relies on a teacher's ability to choose and evaluate assessments, progress monitor students' academic trajectories, and meaningfully interpret and act upon data collected. Implementing DBI therefore requires an additional set of knowledge and skills beyond instructional practices (Bruhn et al., 2019). Teachers who are knowledgeable and able to successfully implement the DBI framework in their classroom see higher student growth than teachers who do not utilize a DBI framework or implement a DBI framework poorly (Fuchs et al., 2014).

How Teacher Outcomes Relate to Student Outcomes

Teacher effects contribute significantly to the potential success of students across all academic fields, including mathematics (Perera & John, 2020). As part of the analysis on the necessary components of effective PD, Desimone (2009) suggests that to achieve meaningful student achievement, there must first be changes in teacher knowledge which will then lead to a change in classroom practice. In mathematics, teachers' specific MKT, their self-efficacy, and their instructional practices have all been tied to greater student achievement (Hill et al., 2005; Perera & John, 2020). Additionally, students experiencing

MD require more intensive instruction and therefore require teachers who are able to implement effective instruction confidently and knowledgeably in addition to skillfully implement extra supportive strategies, such as DBI. This study therefore addresses teachers' self-efficacy, MKT, and knowledge and ability to implement evidence-based instructional strategies within a DBI framework.

CONCLUSION

In this Chapter 2, I reviewed how PD can be used to improve teacher knowledge and skills. I introduced the five core components of high-quality PD as discussed by Desimone (i.e., *content focus*, *active learning*, *coherence*, *duration*, and *collective participation,* 2009). Then, I described how PLCs and asynchronous learning may also contribute to improved teacher knowledge and skills. Next, I provided an overview of a recent synthesis I conducted (Bos, 2022) in which I examined mathematics-focused PD and the impact of the five components of high-quality PD. Finally, I reviewed teacher outcomes, which research suggests may lead to greater student achievement.

I reviewed PD, PLCs, and asynchronous learning as it relates to the mathematics teaching of teachers because these three elements are core within my proposed study. The purpose of my study is to examine the effects of targeted, high-quality mathematics PD to determine its impact on teachers' internal qualities, including their self-efficacy and MKT, as well as their understanding and comfort with implementing evidence-based strategies within a DBI framework.

Chapter 3: Methods

In this study, I examined the effect of an intensive online summer mathematics PD (i.e., National Center on Intensive Intervention [NCII] *Mathematics Intervention Course*) on teachers' knowledge and skills in supporting their students experiencing MD. In addition, I analyzed the impact of the inclusion of peer support or asynchronous learning opportunities during the PD on teachers' knowledge and skills in implementing the content and strategies described in the PD.

Research Questions

1. What is the impact of an intensive online summer mathematics PD on the outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) of in-service SPED teachers who teach mathematics to students experiencing mathematics difficulties (MD)?
2. Do participants who participate in weekly synchronous meetings (i.e., Synchronous/Independent and Synchronous/PLC groups) when completing an online summer PD demonstrate improved outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) compared to peers who asynchronously complete (i.e., Asynchronous group) the online summer PD?
3. Do participants who receive peer support through PLCs during the online summer PD (i.e., Synchronous/PLC group) demonstrate improved outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) compared to peers who do

not receive such peer support (i.e., Synchronous/Independent and Asynchronous groups) during the online summer PD?

Participants

Participants were recruited from schools throughout the state of Texas. Participants included teachers (i.e., interventionists, special education teachers, general education teachers) who provided mathematics support to students experiencing MD in Grades K-12 settings as well as school and district administrators who provided support to mathematics and special education teachers. This support occurred in the general education classrooms as well as in specialized settings, including tutoring centers or Tier 2 and Tier 3 classrooms. The unifier for participants was that they all provided mathematics support to students experiencing MD or they provided support to teachers of students experiencing MD.

Participants were initially recruited via connections through colleges and universities in Texas, social media postings (i.e., Twitter), and word of mouth between teachers. A total of 60 teachers completed an initial interest survey, after which they were sent a link containing the consent form and pre-assessment. A total of 48 participants completed both the consent form and pre-assessment. Three participants removed themselves from the course after randomization occurred but before the PD began, resulting in 45 participants completing at least a portion of the PD. A total of 35 participants completed the posttest at the conclusion of the study.

I used What Works Clearinghouse (WWC, Institute of Education Sciences, 2020) guidelines to assess differential and overall attrition. The overall attrition was 27.08%. Attrition was 18.75% for the Asynchronous group, 25.00% for the Synchronous/ Independent group, and 37.50% for the Synchronous/PLC group. Table 3.1 presents the demographic characteristics (i.e., average years teaching, race/ethnicity, gender) of the participants in this study in each treatment group. Figure 3.1 presents a Consort diagram.

Table 3.1: Demographic Characteristics of Participants

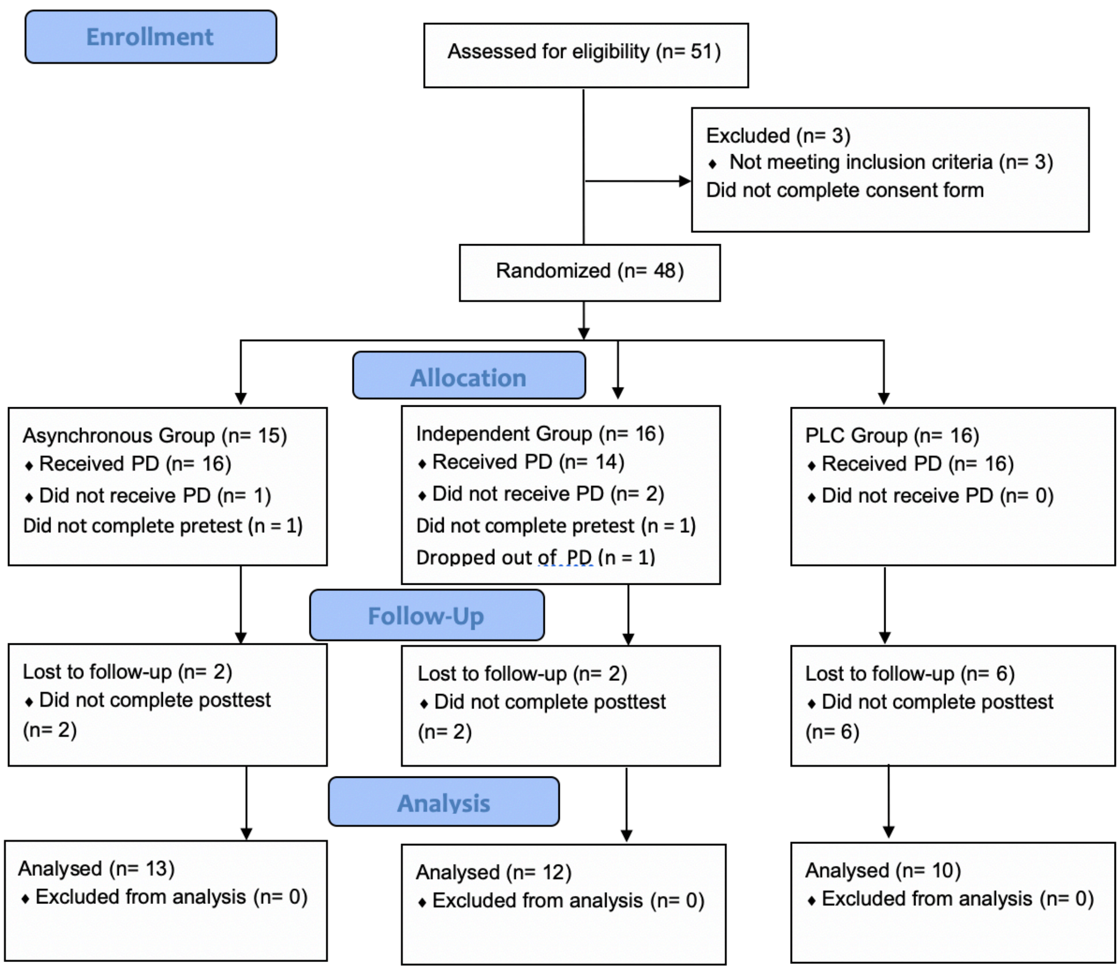
Characteristic	Asynchronous (*n* = 13)	Synchronous/ Independent (*n* = 12)	Synchronous/ PLC (*n* = 10)
Gender (*n* = 35)			
Female	12	10	10
Male	1	1	0
Prefer Not to Say		1	
Race/Ethnicity (*n* = 35)			
Asian American/Pacific Islander	0	1	1
Black/African American	1	1	0
Hispanic/Latino American	1	1	1
Native American	0	0	1
White/European American	10	8	6
Multiracial/ 2 or more races	0	0	1
Other	0	0	0
Prefer not to respond	1	1	0
Age (*n* = 35)			
20-29	1	1	3
30-39	3	1	2
40-49	2	8	5
50-59	2	1	0
60 years or older	5	1	0
Level of Education (*n* = 35)			
Bachelor’s Degree	11	12	9
Master’s Degree	9	7	5
Post Master’s Degree	2	1	2

Table 3.1: Demographic Characteristics of Participants Cont.

Characteristic	Asynchronous (n = 13)	Synchronous/ Independent (n = 12)	Synchronous/ PLC (n = 10)
Instructional Role (n = 35)			
Classroom teacher	5	5	4
Special education teacher	3	1	2
Mathematics coach	1	0	2
Interventionist	1	0	0
Other	3	6	2
Certification			
General education: Elementary	9	8	8
General education: Middle school	6	6	3
General education: High school	1	3	2
Special education	4	5	4
Mathematics	4	4	3
Average number of years at current position (n = 31)	8.00	6.56	3.90
Average number of years teaching (n = 34)	13.54	13.36	8.60
Average number of years teaching in current school (n = 29)	6.75	5.13	3.67
Average number of years teaching in a mathematics classroom (n = 32)	11.55	11.27	6.90
Average number of years teaching special education mathematics (n = 24)	8.30	10.57	5.43
Average number of undergraduate general education mathematics methods courses completed (n = 29)	4.55	2.00	2.00
Average number of undergraduate special education mathematics methods courses completed (n = 15)	2.78	2.00	3.33
Average number of undergraduate mathematics courses completed (n = 30)	4.40	4.55	2.89
Average number of graduate general education mathematics methods courses completed (n = 18)	3.83	1.50	2.75

Average number of graduate special education mathematics methods courses completed (*n* = 12)	1.50	1.25	0.25
Average number of graduate mathematics courses completed (*n* = 13)	3.00	6.00	0.00

Figure 3.1: Consort diagram of participants



Setting

All synchronous summer PD sessions took place remotely via Microsoft Teams. Participants included educators from various regions in Texas as well as one educator from Canada and one educator from Ecuador. Therefore, there was significant variability in the districts that participants served. As a state in 2020, 61.6% of the Texas population identified as White (22.3% Non-Hispanic White and 39.3% Hispanic White), 12.2% as Black or African American, 5.4% Asian, 1.0% Native American and Alaskan Native, 0.1% Pacific Islander, 13.6% some other race, and 17.6% two or more races.

Professional Development Provider and Coach

I (Samantha Bos) provided the PD to all participating teachers. I am a doctoral candidate in Learning Disabilities and Behavioral Disorders within the Department of Special Education at The University of Texas at Austin. I have a Master's degree in Teaching, a Strategic Instruction Model Professional Developer certification, and four years teaching experience in elementary, middle school, and special education settings.

RESEARCH DESIGN

The study implemented a randomized control trial, considered to the be the gold standard of empirical research (Shadish et al., 2002) to capture an unbiased estimate of the effect of the *NCII Mathematics Course* with additional supports on the participants' ability to effectively teach mathematics to students experiencing MD. Participants were randomized prior to the start of the *NCII Mathematics Course* PD to one of three groups. The three groups included: (a) Asynchronous, (b) Synchronous/Independent, and (c)

Synchronous/PLC. Using a randomization function on Excel, the participants were randomly assigned to complete the *NCII Mathematics Course* either without synchronous meetings (i.e., Asynchronous Group), alone with weekly synchronous meetings (i.e., Synchronous/Independent Group), or with a synchronous cohort (i.e., Synchronous/PLC Group). Overall, I assigned 16 participants to each group. Figure 3.2 provides a brief description of each condition.

Figure 3.2: Group assignment

	No Peer Collaboration during *NCII Mathematics Course*	Peer Collaboration during *NCII Mathematics Course*
Asynchronous Participation Only	Asynchronous Group (n = 16)	
Synchronous Learning Sessions	Synchronous/ Independent Group (n = 16)	Synchronous/PLC Group (n = 16)

MATERIALS AND PROCEDURES

Recruitment

The study was approved by the Institutional Review Board (IRB) at The University of Texas at Austin. To recruit eligible participants to join the study, I promoted the opportunity via social media, including Facebook and Twitter. I also connected with local and major universities as well as school districts across the state of Texas to disseminate information about the study to their former graduates and current employees, respectively. Promotional information included a brief summary of the goals and purpose of the *NCII Mathematics Course*, potential benefits and compensation for participating in the study, as well as eligibility requirements.

To be eligible to participate in this study, candidates had to be mathematics teachers, mathematics instructional coaches, or mathematics specialists for a school or district. Candidates had to be in a position in which they either worked directly with students experiencing MD or with mathematics teachers who provided support to students experiencing MD the following school year (i.e., 2021-2022). University officials who worked with pre-service teachers or school officials who served only in administrative roles did not qualify to participate in this study. Candidates who were interested in the study completed a brief form, acknowledging their availability for the duration of the study and their commitment to completing all requirements throughout the study duration. Upon completing the study, participants received up to 40 CPE hours as well as a $50 gift card.

Participant Consent

Qualified candidates who indicated a commitment to completing the study were asked to complete a consent form. As part of the consent form, participants were informed in detail regarding (a) the purpose of the study, (b) what each participant would be asked to do, (c) risks involved in the study, (d) possible benefits of participating, (e) compensation information, and (f) confidentiality and privacy protections.

Materials

I provided all participants access to all materials associated with the *NCII Mathematics Course* materials including videos, activities, and related resources. All course materials are available for free online (https://intensiveintervention.org/intensive-intervention-math-course) and also were housed in a shared Canvas site which all participants were able to access and complete activities and contribute to the discussion board. The *NCII Mathematics Course* includes eight modules covering topics including: (1) developing a scope and sequence, (2) progress monitoring, (3) selecting and evaluating evidence-based practices, (4) instructional delivery, (5) instructional strategies, (6) whole-number content, (7) rational-number content, and (8) data-based individualization. During the summer PD, participants completed all eight modules, including all activities and discussion topics.

Treatment Groups

Asynchronous Group

The first treatment group, identified as the Asynchronous Group, completed the materials available as part of the *NCII Mathematics Course*, but they did not attend the weekly meetings expected of the Synchronous/Independent and Synchronous/PLC groups. The Asynchronous Group was considered function as the Business as Usual (BaU) Group because if teachers access the *NCII Mathematics Course* on their own, they would be completing the course in a similar way to the Asynchronous Group. When NCII developed the *NCII Mathematics Course*, they designed all materials (i.e., videos and activities) to be accessed in an asynchronous manner. Participants in this group were independently responsible for completing all of the activities, discussions, and reflections available on Canvas before the end of the study. Participants were encouraged at the outset to complete one module every week and a reminder email was sent out at the midway point of the summer session, but no other contact was made with the participants unless they contacted me with questions about the materials or content.

Synchronous/Independent Group

The second treatment group, identified as the Synchronous/Independent group, completed the *NCII Mathematics Course* materials in a 9-week summer PD. Participants were responsible for watching assigned videos and completing activities prior to weekly meetings with the instructor and fellow participants. During the weekly meetings, participants participated in activities designed to increase their MKT and deepen their understandings of the concepts covered in each module. Example activities included analyzing examples of student work, solving problems similar to those presented in each module, and planning lessons for the fall semester. Participants were randomly assigned to different small groups for each session, and all materials and discussion boards were available to all participants. During each session, I presented new material, organized and facilitated small group discussions, introduced activities or reflections that participants completed in their small groups, and answered questions as they arose throughout the meeting. Small group discussions and activities were interspersed with whole-group activities and discussions throughout each session.

Synchronous/PLC Group

The third treatment group, identified as the Synchronous/PLC group, also completed the *NCII Mathematics Course*. Similar to the Synchronous/Independent group, participants in the Synchronous/PLC group were expected to complete all assigned videos and activities prior to weekly meetings. During the weekly meetings, participants were expected to participate in activities including analyzing examples of student work, solving problems similar to those presented in each module, and planning lessons for the fall semester. Participants in this condition were assigned to a small cohort (i.e., 3-4 participants) prior to the first meeting. Members of each small cohort conducted all activities with only members of their group and had access to a unique discussion board limited to only the small cohort members. Participants in each small cohort were encouraged to share their contact with each other and complete activities together. During each session, I introduced any necessary logistical information at the start of the session. I then introduced the activities and reflection questions for small group discussions and then facilitated small group discussions. Unlike sessions for the Synchronous/ Independent group, the Synchronous/PLC group only met as a whole group at the beginning and end of each session to clarify questions. There were no whole-group discussions or activities, and instead participants spent 35-45 minutes of each session in their assigned small group breakout session.

Mathematics Course

The *NCII Mathematics Course* includes materials created by the National Center on Intensive Intervention for the online *Intensive Intervention in Mathematics Course,* which comprises eight modules. Within each module, there are three parts. Each part comes with a separate video to watch and activities to complete. Parts range from 10 minutes to 65 minutes.

1) Developing a Scope and Sequence for Intensive Intervention
    a. Why is mathematics intensive intervention important?
    b. What mathematical content for students need to master across kindergarten through eighth grade?
    c. How do you identify mathematical content for intensive intervention and how to sequence intervention content?
2) Mathematics Progress Monitoring and Determine Response
    a. What are different types of assessments used to monitor student progress in mathematics within DBI?
    b. How do you administer progress monitoring measures with fidelity?
    c. How do you interpret progress monitoring scores?
3) Selecting and Evaluating Evidence-Based Practices in Mathematics
    a. What are the forms of evidence-based practices in intensive intervention?
    b. Where do you locate evidence-based practices?
    c. How do you determine the instructional platform for intensive intervention?
4) Intensive Mathematics Intervention: Instructional Delivery
    a. How do you use explicit instruction within intensive intervention?

    b. How should multiple representations be used within intensive intervention?
    c. How do you attend to language within intensive intervention?
5) Intensive Mathematics Intervention: Instructional Strategies
    a. How do you build fact fluency within intensive intervention?
    b. How do you incorporate effective problem-solving strategies within intensive intervention?
    c. How do you incorporate a motivational component within intensive intervention?
6) Whole-Number Content for Intensive Intervention
    a. What whole-number core concepts should be emphasized in intensive intervention?
    b. What whole-number procedures should be emphasized in intensive intervention?
    c. What does DBI look like with intensive interventions that focus on conceptual and procedural understanding of whole numbers?
7) Rational-Number Content for Intensive Intervention
    a. What rational-number core concepts should be emphasized in intensive intervention?
    b. What rational-number procedures should be emphasized in intensive intervention?

    c. What does DBI look like with intensive interventions that focus on conceptual and procedural understanding of rational numbers?

8) Data-based Individualization for Intensive Mathematics Intervention
    a. How do you implement intensive mathematics interventions with fidelity?
    b. How do you make adaptations within DBI?
    c. How does all of this come together within a DBI framework?

Each module includes informational three parts with three separate videos participants were expected to watch before weekly meetings. Additional pre-meeting activities included knowledge-building assignments, application opportunities, and journal reflections. See Figure 3.3 for the workbook checklist or Module 1 which contains the instructions and assignments participants were expected to compete on their own time. I housed all materials, participant activities, and discussion boards on a Canvas that were accessible to all participants.

Figure 3.3: Module 1 Checklist

Module 1 Checklist

The purpose of this Activity Workbook is to help organize content for this Module. You will do some Activities on your own to help you engage with and think about the content. You will not be required to submit your responses for those activities. There are other activities, however, that you will submit online and apply in your classroom. The activities that you must submit before completing this Module are listed in the "Online" column below.

Section	Assignment	To Be Completed In Activity Workbook	To Be Completed Online	To Be Completed With Coach
Intro	Video		❑ Watch Module 1 Introduction Video Presentation	
Part 1	Video		❑ Watch Module 1 Part 1 Video Presentation	
	Activity 1	❑ Examine NAEP Data		
	Journal		❑ Journal Entry: *Provide Rationale for Intensive Interventions in Math*	
Part 2	Video		❑ Watch Module 1 Part 2 Video Presentation	
	Activity 2	❑ Put Operations Standards in Order		
	Activity 3	❑ Put Problem-Solving Standards in Order		
	Activity 4	❑ Determine Skill Gaps		
	Discussion		❑ Discussion Board: *Ponder Upcoming Lessons* ❑ Write Your Response ❑ Respond to 2 Others	
Part 3	Video		❑ Watch Module 1 Part 3 Video Presentation	
	Activity 5	❑ Determine Skills Needed to Successfully Solve Problems		
	Activity 6	❑ Determine Intervention Needs for a Student		
Next Steps	Video		❑ Watch Module 1 Closing Video Presentation	
	Classroom Application			❑ Identify Foundational Skills Needed in Your Classroom

I developed the virtual synchronous meeting sessions to align with the findings from the PD studies reported in Chapter 2. In other words, the sessions included the five critical components recommended by Desimone (2009) to the degree that each component was feasible in a virtual leaning setting, including content focus, active learning, coherence, duration, and collective participation. The primary focus of the *NCII Mathematics Course* is to develop teachers content knowledge regarding skills necessary to be an effective mathematics special education teacher. Therefore, I included active learning structures, including opportunities to observe and be observed by others and receive feedback, examine student work, plan instruction, present solutions or reflections, and solve mathematics problems (Garet et al., 2001). Although PD that aligns with teachers' state, district, school, and personal goals have a greater impact upon teacher change (Copur-Gencturk et al., 2019), it was difficult to ensure that this PD, developed for a national audience, aligned with the unique district goals of each teacher participant. To address this challenge, the PD included opportunities for teachers to reflect on how the NCII module content aligned with their district and school goals, and ensure that teachers had opportunities to make the connection between the material presented in the *NCII Mathematics Course* for a national audience and their local standards and needs.

To ensure that participants engaged in mathematics learning for a sufficient duration, the summer PD took place over the course of 9 weeks to ensure that teachers had ample time to process the information presented, which is necessary for teacher growth (Copur-Gencturk, et al., 2019). In addition to synchronous 1-hour weekly meetings for participants in the Synchronous/Independent and Synchronous/PLC groups, all participants were also expected to complete approximately 4-5 hours of activities between the meetings. In total, participants were engaged for approximately 40-45 hours of mathematical learning during the summer PD. Finally, the critical component of collective participation, one of the most difficult to address in an online PD, was addressed by including purposeful opportunities for participants to engage in team-building exercises. It is likely that participants in the Synchronous/Independent and Asynchronous Groups did not develop the same connection to their peers as their participants in the Synchronous/PLC Group, due to the continued randomization of activity partners throughout the *NCII Mathematics Course* and lack of direct meeting with fellow participants, respectively.

To ensure that each of the 5 components were clearly delineated in each module and session, I mapped the activities and lessons in the *NCII Mathematics Course* on to the specific components of each content focus, active learning, coherence, duration, and collective participation according to Soine and Lumpe (2014). Figures 3.4 to 3.7, shown below, are the Active Learning Components, Focus on Content Knowledge Components, Coherence with Teachers' Need and Circumstances, and Collective Participation components mapped across the *NCII Mathematics Course* and summer PD sessions.

Figure 3.4: Active Learning Components

	Analyze student work	Create instructional materials for use in classroom	Complete the work or problems that the students would be doing in class	Write learning objectives	Adapt curriculum to match state learning standards	Reflect on the effectiveness of a lesson	Write assessments to match state learning standards	Observe videos of classroom instruction	Practice a new skill under simulated conditions	Make a presentation to colleagues	Reflect on new learning in a journal	Participate in a coaching cycle	Discus articles from an educational journal or book
Session 1													
Module 1 and Module 6 Part 1	✓		✓					✓	✓		✓		
Session 2													✓
Module 2 and Module 6 Part 2			✓					✓	✓	✓	✓		
Session 3				✓									
Module 3 and Module 6 Part 3								✓	✓	✓	✓		
Session 4							✓						
Module 4 and Module 7 Part 1	✓					✓		✓	✓		✓		
Session 5		✓											
Module 5 and Module 7 Part 2		✓	✓					✓	✓		✓		
Session 6			✓										
Module 8 and Module 7 Part 3						✓		✓	✓	✓	✓		
Session 7											✓		
School Plan Development											✓		
Session 8										✓			

Figure 3.5: Focus on Content Knowledge Components

	Gain a deeper understanding of the subjects taught	Learn content by teaching it to students more about the content by teaching it to students	Raise expectations for student performance because teacher understands the content more	Become more confident in ability to answer student questions about a topic	Become more interested in the subject	Change the way they thought about the subject	Learn more about the content on their own	Use the teacher resources provided in the curriculum to learn more about the content	Learn how to recognize and address common student misconceptions	Develop skills to connect students' new learning to prior learning and experiences	Become more confident in their ability to know the next step needed to take to deepen students' conceptual	Expand understanding of how students learn particular subjects	Learn ways to use data to assess student learning needs	Learn strategies to help students monitor their own understanding
Session 1														
Module 1 and Module 6 Part 1	✓		✓	✓	✓		✓	✓			✓		✓	
Session 2			✓			✓						✓		
Module 2 and Module 6 Part 2	✓		✓	✓			✓				✓	✓	✓	
Session 3			✓								✓	✓	✓	
Module 3 and Module 6 Part 3	✓		✓	✓			✓				✓	✓		
Session 4	✓			✓						✓	✓			
Module 4 and Module 7 Part 1	✓		✓	✓			✓	✓	✓	✓	✓	✓		
Session 5			✓	✓								✓		
Module 5 and Module 7 Part 2	✓		✓	✓			✓		✓	✓	✓	✓		✓
Session 6	✓		✓	✓			✓		✓		✓	✓		✓
Module 8 and Module 7 Part 3	✓						✓							
Session 7	✓			✓									✓	
School Plan Development							✓				✓		✓	
Session 8											✓		✓	

Figure 3.6: Coherence with Teachers' Needs and Circumstances

	Designed to build upon each other as the year progressed	Planned based on analysis of student data	Aligned with our school's mission and vision	Part of a coherent program for teacher growth	Aligned with the district's learning improvement goals	Designed to support state or district assessments	Focused on improving student learning	Designed to support state performance expectations or grade level expectations	Spread evenly throughout the school year (duration)
Sesssion 1					✓				
Module 1 and Module 6 Part 1								✓	
Session 2				✓					
Module 2 and Module 6 Part 2						✓			
Session 3						✓		✓	
Module 3 and Module 6 Part 3									
Session 4						✓			
Module 4 and Module 7 Part 1									
Session 5									
Module 5 and Module 7 Part 2									
Session 6									
Module 8 and Module 7 Part 3									
Session 7									
School Plan Development			✓		✓		✓	✓	
Session 8									

Figure 3.7: Collective Participation

	Collaborate with grade level colleagues to improve student learning	Spend time building trusting relationships with my colleagues	Collaborate with teachers at other schools in my district	Create norms for effective social interaction with myleagues	Attend professional development activities with a team from my school	Collaborate with colleagues to design flexible groups based on student need	Share effective instructional strategies with colleagues	Co-teach lessons	Learn effective ways to collaborate to improve student learning	Collaborated with the teachers in the grade level below me	Am a member of a professional learning community	Felt a sense of collective responsibility for improve student performance	Observed colleagues and provided feedback	Follow norms to maximize group effectiveness	Collaborate with the teachers in the grade level above me	Was encouraged by my colleagues to grow professionally
Session 1	✓	✓		✓						✓	✓			✓	✓	✓
Module 1 and Module 6 Part 1	✓						✓			✓					✓	✓
Session 2	✓						✓			✓				✓	✓	✓
Module 2 and Module 6 Part 2							✓						✓	✓		✓
Session 3	✓						✓			✓				✓	✓	
Module 3 and Module 6 Part 3							✓						✓	✓		
Session 4	✓						✓			✓				✓	✓	
Module 4 and Module 7 Part 1							✓						✓	✓		
Session 5	✓						✓			✓				✓	✓	
Module 5 and Module 7 Part 2							✓						✓			
Session 6	✓						✓			✓					✓	
Module 8 and Module 7 Part 3													✓	✓		✓
Session 7	✓						✓			✓					✓	
Session 7 Hmwk																
Session 8	✓								✓	✓			✓	✓	✓	✓

Session 1

I designed all sessions for the Synchronous/Independent and Synchronous/PLC groups only. The Asynchronous group did not have access to the activities and discussion questions presented in the weekly synchronous sessions. The purpose of Session 1 was to establish group norms, introduce participants, and introduce protocols for the PD. To develop a sense of community and collective participation, I asked all participants to introduce themselves and describe their teaching situation. In addition to answering questions about the content, grade, and location of the current teaching position, participants also shared their goals or purpose for taking the course to establish a coherence with the *NCII Mathematics Course* instruction and their own learning. The session included a wrap up sequence in which I presented the work that participants needed to complete by the following session and then allowed time for participants to ask questions and reflect on their learning for this session.

Session 2

The purpose of Session 2 was to review and reflect on the materials of Module 1 and Module 6 Part 1. The material from Module 1 established the importance of intensive intervention in addressing students' needs as well as appropriate measures and assessments to capture student growth. In addition to the material presented in Module 1 (Developing a Scope and Sequence), the participants also reviewed whole number computation concepts, introduced in Module 6 Part 1 (Whole-Number Content). Participants engaged in reflection activities that drew in the activities assigned to be completed prior to the start of the sessions, allowing for participants to relate the material to their own experience, thus developing a greater coherence between the course materials and the participants' own expectations and experiences. In addition, participants discussed their plans to include the learned material in future lessons. Finally, the session included the wrap up sequence in which I presented the work that participants needed to complete by the following session and then allowed time for participants to ask questions and reflect on their learning for the session.

Session 3

The purpose of Session 3 was to review and reflect on the materials of Module 2 (Progress Monitoring) and Module 6 Part 2 (Whole-Number Content). The material from

Module 2 included information on progress monitoring and determining responses within the DBI model. Participants also reviewed whole-number procedures including critical concepts such regrouping and different algorithms for solving computation problems. Similar to previous sessions, participants engaged in activities and discussions designed to link the knowledge and procedures highlighted in the *NCII Mathematics Course* to the participants' current and future practice. The session concluded with the same wrap up sequence as previous sessions.

Session 4

The purpose of Session 4 was to review and reflect on the materials of Module 3 (Selecting and Evaluating Evidence-Based Practices) and Module 6 Part 3 (Whole-Number Content). The material from Module 3 included information on selecting and evaluating evidence-based practices within mathematics. Participants also reviewed how to develop conceptual and procedural understandings of whole numbers within the DBI framework. Similar to previous sessions, participants engaged in activities and discussions designed to link the knowledge and procedures highlighted in the *NCII Mathematics Course* to the participants' current and future practice. The session concluded with the same wrap up sequence as previous sessions.

Session 5

The purpose of Session 5 was to review and reflect on the materials of Module 4 (Instructional Delivery) and Module 7 Part 1 (Rational-Number Content). The material from Module 4 included information on how to effectively deliver mathematical content, including explicit instruction, the inclusion of multiple representations, and formal mathematical language. Participants also reviewed rational number concepts including fractions and decimals. Similar to previous sessions, participants engaged in activities and discussions designed to link the knowledge and procedures highlighted in the *NCII Mathematics Course* to the participants' current and future practice. The session concluded with the wrap up sequence.

Session 6

The purpose of Session 6 was to review and reflect on the materials of Module 5 (Instructional Strategies) and Module 7 Part 2 (Rational-Number Content). The material from Module 5 included information regarding key instructional strategies to embed within sessions, including fluency practice, word problem-solving strategies, and a motivation component. Participants also reviewed rational number procedures including multiplying and dividing fractions. Similar to previous sessions, participants engaged in activities and discussions designed to link the knowledge and procedures highlighted in the *NCII Mathematics Course* to the participants' current and future practice. The session concluded with the same wrap up sequence as previous sessions.

Session 7

The purpose of Session 7 was to review and reflect on the materials of Module 8 (Data-Based Individualization) and Module 7 Part 3 (Rational-Number Content). The material from Module 8 included implementing the DBI framework in a mathematics classroom with fidelity, and problem-solving strategies to adapt the instructional platform. Participants also reviewed how to develop conceptual and procedural understandings of rational numbers within the DBI model. Similar to previous sessions, participants engaged in activities and discussions designed to link the knowledge and procedures highlighted in the *NCII Mathematics Course* to the participants' current and future practice. The session concluded with the same wrap up sequence as previous sessions.

Session 8

The purpose of Session 8 was to provide an opportunity for participants to reflect on the information presented throughout the *NCII Mathematics Course* and develop a plan for implementing the new skills and knowledge into their classroom teaching when they returned to schools in the fall semester of the following year (i.e., 2021-2022). During this session, participants presented a school success plan they were asked to develop on their own. Participants presented their plans, including potential challenges, resources, and benchmarks for implementing DBI at their own schools, to small groups. These small groups were designed to provide feedback and brainstorm strategies to address these challenges and develop a collective framework for proactively addressing issues that may arise in the fall. The session concluded with the wrap up sequence developed for previous sessions, but additionally I presented the follow-up steps of the program including posttests. Finally, I allowed time for participants to ask questions and reflect on their learning for the session and the *NCII Mathematics Course.*

DATA COLLECTION AND MEASURES

To answer the primary research questions, I assessed teacher outcomes on measures of teacher knowledge and skills, including: a) Mathematical Knowledge for Teaching (MKT, Ball et al., 2008), b) *Teacher's Sense of Efficacy Scale* (*TSES*, Tschannen-Moran & Woolfolk Hoy, 2001), c) *Mathematics Teaching Self-Efficacy* (Perera & John, 2020), and d) *Teacher's Instructional Practices* (*TIP*) survey (Ketterlin-Geller et al., 2018). These measures were administered to teacher participants within two weeks prior to the start of the summer PD*,* within 2 weeks of the end of the summer PD. In addition to these quantitative measures of teacher-outcomes, I also collected qualitative data to capture a comprehensive evaluation of the impact of the *NCII Mathematics Course* on teacher outcomes in the form of exit interviews. I designed the exit interview questions to explore the potential benefits of the PD with the goal of seeking convergence and triangulation with the quantitative data (Greene, 2007). The measures are described in detail below.

Mathematical Knowledge for Teaching

The Mathematical Knowledge for Teaching *(*MKT) assessment is an electronic multiple-choice assessment designed as part of the Learning Mathematical Teaching project (LMT, Hill et al., 2007). The LMT project developed a series of MKT assessments to align with the grade level and mathematical topics taught by various teachers. For the purpose of this study, I chose the *Elementary Number Concepts and Operations – Content Knowledge (EL_NCOP-CK)* and the *Middle School Patterns Function and Algebra – Content Knowledge (MS_PFA-CK)* assessments developed by the LMT project. The assessments together include 35 questions, but 12 questions have multiple parts. In total there are 70 problems.

To access the MKT assessments, participants provided their email addresses to register for the PD and received an email with a link to the MKT assessment website (https://az1.qualtrics.com/apps/harvard-tkas/accesscode) as well as an access code to complete the assessment. Participants generated a website-specific password and ID so the program was able to later link their pretest and posttest data. Participants completed the online assessment, which takes approximately 1 hour to complete. The assessment was not timed, and if needed, participants could stop the assessment and return to it at a later time.

To set up the MKT assessment for the study, I completed the necessary steps required by the website and LMT project to assess teachers' MKT knowledge at two points in the study. The program randomly assigned each participant to versions A or B of the assessment at each testing point to reduce testing bias. At the completion of the study, I was notified when all participants completed the final MKT assessment, and the program provided raw data as well as a statistical analysis report, providing analyses using calibrated MKT IRT scores.

The internal reliability of the *MKT Elementary Number Concepts and Operations – Content Knowledge (EL_NCOP-CK)* assessment as reported by the authors ranged from 0.72 to 0.74, and the internal reliability of the *MKT Middle School Patterns Function and Algebra – Content Knowledge (MS_PFA-CK)* assessment ranged from 0.77 to 0.78. Reliability data was based off of a national sample of approximately 650 elementary mathematics teachers (Hill, 2008). To address the validity of the assessments, the original researchers who designed the MKT assessments conducted cognitive interviews and observed classroom instruction to assess whether high scores on the measure were associated with teachers' reasoning and classroom practices. Classroom instruction was evaluated using a rubric assessing a) use of mathematical language, b) representation of mathematical ideas, c) linkages between classroom task (i.e., what kids are doing) and important mathematical ideas, d) facility in listening to children's' mathematical productions, and e) computational or other mathematical errors (Hill et al., n.d.). Researchers found a .75-.76 correlation between the classroom instruction and the measure. Additionally, the content of the assessment measures was mapped to NCTM standards, and during the development of the measures, mathematicians were interviewed to ensure high validity (Ball et al., 2005). When scoring the MKT assessment, an IRT scale score is generated, meaning that participants who score below the average number of correct answers will receive a negative score. The MKT assessments developed by the LMT project have been utilized in several studies examining the impact of PD on teachers' understanding of how to teach mathematics effectively (e.g., Avineri, 2016; Bell et al., 2010; Birkhead et al., 2017; Carney et al., 2019; Carpenter et al., 2017; Copur-Gencturk & Lubienski, 2013; Gerber et al., 2011; Harris et al., 2011; Jacob et al., 2017).

Examples of released items from the MKT assessments are included as Figure 3.8, shown below (Hill et al., 2004).

Figure 3.8: Released items from the Mathematical Knowledge for Teaching (MKT) assessments

18. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e., a (b + c) = ab + ac]? (Mark APPLIES, DOES NOT APPLY, or I'M NOT SURE for each.)

	Applies	Does not apply	I'm not sure
a) Adding $\frac{3}{4}+\frac{5}{4}$	1	2	3
b) Solving 2x – 5 = 8 for x	1	2	3
c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$	1	2	3
d) Adding 34 + 25 using this method: $\begin{array}{r} 34 \\ \underline{+25} \\ 59 \end{array}$	1	2	3

19. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes <u>closest</u> to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

c) They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

d) They're equivalent because what you do to one side you must always do to the other.

e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

Teacher's Sense of Efficacy Scales

The *Teacher's Sense of Efficacy Scale* (*TSES*, Tschannen-Moran & Woolfolk Hoy, 2001) is a survey with 24-items on a 9-point Likert scale that measures teachers' general self-efficacy across academic areas. The survey contained three subconstructs: Instructional Strategies (IS), Classroom Management (CM), and Student Engagement (SE). The test-retest reliability for the 24-item scale was 0.94, and construct validity was assessed by measuring the correlations between the TSES and other existing measures of teacher efficacy and found to have high correlations with many other established self-efficacy measures (Tschannen-Moran & Woolfolk Hoy, 2001). The assessment was not timed, and participants completed the assessment as part of a longer multi-component survey using Qualtrics. Figure 3.9 includes the complete 24-item survey, shown below.

Figure 3.9: Teacher's Sense of Efficacy Scale (TSES)

Teacher Beliefs - TSES

This questionnaire is designed to help us gain a better understanding of the kinds of things that create challenges for teachers. Your answers are confidential.

<u>Directions:</u> Please indicate your opinion about each of the questions below by marking any one of the nine responses in the columns on the right side, ranging from (1) "None at all" to (9) "A Great Deal" as each represents a degree on the continuum.

Please respond to each of the questions by considering the combination of your *current* ability, resources, and opportunity to do each of the following in your present position.

	None at all		Very Little		Some Degree		Quite A Bit		A Great Deal
1. How much can you do to get through to the most difficult students?	1	2	3	4	5	6	7	8	9
2. How much can you do to help your students think critically?	1	2	3	4	5	6	7	8	9
3. How much can you do to control disruptive behavior in the classroom?	1	2	3	4	5	6	7	8	9
4. How much can you do to motivate students who show low interest in school work?	1	2	3	4	5	6	7	8	9
5. To what extent can you make your expectations clear about student behavior?	1	2	3	4	5	6	7	8	9
6. How much can you do to get students to believe they can do well in school work?	1	2	3	4	5	6	7	8	9
7. How well can you respond to difficult questions from your students?	1	2	3	4	5	6	7	8	9
8. How well can you establish routines to keep activities running smoothly?	1	2	3	4	5	6	7	8	9
9. How much can you do to help your students value learning?	1	2	3	4	5	6	7	8	9
10. How much can you gauge student comprehension of what you have taught?	1	2	3	4	5	6	7	8	9
11. To what extent can you craft good questions for your students?	1	2	3	4	5	6	7	8	9
12. How much can you do to foster student creativity?	1	2	3	4	5	6	7	8	9
13. How much can you do to get children to follow classroom rules?	1	2	3	4	5	6	7	8	9
14. How much can you do to improve the understanding of a student who is failing?	1	2	3	4	5	6	7	8	9
15. How much can you do to calm a student who is disruptive or noisy?	1	2	3	4	5	6	7	8	9
16. How well can you establish a classroom management system with each group of students?	1	2	3	4	5	6	7	8	9
17. How much can you do to adjust your lessons to the proper level for individual students?	1	2	3	4	5	6	7	8	9
18. How much can you use a variety of assessment strategies?	1	2	3	4	5	6	7	8	9
19. How well can you keep a few problem students from ruining an entire lesson?	1	2	3	4	5	6	7	8	9
20. To what extent can you provide an alternative explanation or example when students are confused?	1	2	3	4	5	6	7	8	9
21. How well can you respond to defiant students?	1	2	3	4	5	6	7	8	9
22. How much can you assist families in helping their children do well in school?	1	2	3	4	5	6	7	8	9
23. How well can you implement alternative strategies in your classroom?	1	2	3	4	5	6	7	8	9
24. How well can you provide appropriate challenges for very capable students?	1	2	3	4	5	6	7	8	9

The *Mathematics Teaching Self-Efficacy* measure examined teachers' self-efficacy for teaching mathematics (Perera & John, 2020) The measure was created using items for the TIMSS 2015 Teacher Questionnaire Mathematics, which was developed including responses from 300,000 students, their parents, teachers, and school principals from a variety of international schools (Martin et al., 2016). The measure included 9 items on a four-point Likert-type scale, ranging from 1 (Low) to 4 (Very High). Cronbach's alpha for the measure was α = 0.911 (Perera & John, 2020). The assessment was not timed, and participants completed the assessment as part of a longer multi-component survey using Qualtrics. Figure 3.10 includes the complete 9-item survey, shown below.

Figure 3.10: Mathematics Teaching Self-Efficacy Survey

Item	Item Description
ATBM02A	Inspiring students to learn mathematics
ATBM02B	Showing students a variety of problem solving strategies
ATBM02C	Providing challenging tasks for the highest achieving students
ATBM02D	Adapting my teaching to engage students' interests
ATBM02E	Helping students appreciate the value of learning mathematics
ATBM02F	Assessing student comprehension of mathematics
ATBM02G	Improving the understanding of struggling students
ATBM02H	Making mathematics relevant to students
ATBM02I	Developing students' higher-order thinking skills.

Teacher Instructional Practices

The *Teacher Instructional Practices (TIP)* survey measured teachers' perceptions of their espoused and enacted instruction and assessment practices (Ketterlin-Geller et al., 2018). The *TIP* survey included 27 items on a 4-point or 6-point Likert scale and measured three areas of teacher perception and practice: DBI content knowledge, instructional practices, and assessment practices. For each prompt, teachers read a prompt and rated their perception of the importance of the practice, their understanding of the practice, their confidence in implementing the practice, and the frequency in which they used the practice. The assessment was not timed, and participants completed the assessment as part of a longer multi-component survey using Qualtrics. Cronbach's alpha for each section of the *TIP* was α = .84 for DBI construct, α = .96 for instructional practices, and α = .93 for assessment practices (Powell et al., in press). Figure 3.11 includes example items from the survey, shown below.

Figure 3.11: Sample questions from the *Teacher Instructional Practices* (*TIP*) survey

Data Based Individualization

	Frequency 0: Less often than 1 time per month 1: 1 time per month 2: 2-3 times per month 3: 1 time per week 4: 2-3 times per week 5: Everyday 0 1 2 3 4 5	Understanding of the practice 0: I don't know much about it 1: I know a little bit about it 2: I know some details about it 3: I know a lot about it 0 1 2 3	Importance of practice 0: Not very important 1: Somewhat important 2: Important 3: Very important 0 1 2 3	Confidence in implementing the practice 0: Not very confident 1: Somewhat confident 2: Confident 3: Very confident 0 1 2 3
Administer (weekly) measures of progress monitering	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Analyze progress monitoring data every (4) weeks	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Make instructional adaptations based on progress monitoring data	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○

	Frequency 0: Less often than 1 time per month 1: 1 time per month 2: 2-3 times per month 3: 1 time per week 4: 2-3 times per week 5: Everyday 0 1 2 3 4 5	Understanding of the practice 0: I don't know much about it 1: I know a little bit about it 2: I know some details about it 3: I know a lot about it 0 1 2 3	Importance of practice 0: Not very important 1: Somewhat important 2: Important 3: Very important 0 1 2 3	Confidence in implementing the practice 0: Not very confident 1: Somewhat confident 2: Confident 3: Very confident 0 1 2 3
Explicitly model mathematics concepts and procedures	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Provide guided practice opportunities (i.e. teacher and students working together; students working together)	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Provide independent practice opportunities	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Use planned examples	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Use multiple representations (concrete, representational, and abstract), linking back to concepts or procedures	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Use precise mathematical vocabulary and mathematics terminology	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Use questioning strategies that elicit a variety of student responses (why, when, how)	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○
Require students to respond frequently	○ ○ ○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○	○ ○ ○ ○

Exit Interviews

I conducted semi-structured interviews designed to be conversational (Fylan, 2005) with participants regarding their experiences in the summer PD within 2 weeks of the conclusion of the PD. Semi-structured interviews provide enough structure to allow for themes to develop across participants, but they also allowed me to ask follow-up questions that prompted the participants to reflect more deeply on their responses. I asked all participants about their experience with the content presented in the, as well as additional questions that aligned with the specific format of the participants' group. Example questions included:

> Can you describe any strategies that you used prior to this PD that have helped you be successful in implementing DBI at your school? (All)
>
> Do you foresee yourself reaching out to any of the other participants for support when you implement DBI in the fall? Do you foresee any participants reaching out to you for support in the fall? (Synchronous/Independent and Synchronous/ PLC Group)
>
> How would you describe your strategy for completing this course by the deadline (e.g., setting goals, completing modules when convenient, etc.)? (Asynchronous)

I interviewed each willing participant individually via Zoom, recording the video unless a participant requested that the video was not recorded, in which case only the audio was recorded. I converted all recordings to transcripts and then analyzed the interviews for recurring themes, following the six steps recommended by Kiger and Varpio (2020), that is: 1) become familiar with the data set, 2) generate initial codes, 3) search for themes, 4) review themes, 5) define and name themes, and 6) produce the report. I employed an inductive approach to analyze the data, meaning that I developed codes from the data. During the initial coding, I examined the data for responses that reflected participants' skills or attitudes related to mathematics content or teaching, their confidence in teaching or teaching mathematics, and their confidence, knowledge, or attitudes related to data-based individualization and evidence-based practices. Through iterative coding procedures, I developed themes addressing the perceived benefits of the PD.

DATA ANALYSIS

Quantitative data were collected via Qualtrics surveys prior to the beginning of the summer PD and at the conclusion of the summer PD. Data were collected regarding participants' Mathematical Knowledge for Teaching (MKT, Ball et al., 2008), *Teacher's Sense of Efficacy Scale* (*TSES*, Tschannen-Moran & Woolfolk Hoy, 2001), *Mathematics Teaching Self-Efficacy* (Perera & John, 2020), and *Teacher's Instructional Practices (TIP,* Ketterlin-Geller et al., 2018). Data were exported to R for data analysis. Data were analyzed using a series of ANOVA tests, and F statistics are presented along with eta-squared for each test as a measure of effect size. Qualitative data include interviews conducted with 13 teachers at the conclusion of the summer PD. I transcribed these interviews transcribed using Otter.ai and analyzed the transcripts using Dedoose.

Implementation Fidelity

To capture a comprehensive evaluation of the implementation fidelity, I assessed multiple components of fidelity throughout the *NCII Mathematics Course*. I assessed each section of the study on four dimensions of fidelity, including adherence, quality of instruction, dosage, and participant engagement (Dane & Schneider, 1998).

Fidelity of the NCII Mathematics Course

I recorded all *NCII Mathematics Course* sessions and saved all Chat discussions. I developed a fidelity checklist based on the work of Desimone (2009) to measure the critical components of PD. To measure the adherence of implementor, the protocol evaluates whether the required elements and procedures are included throughout each session (e.g., examining student work, building relationships to establish long-term networks, tying purpose of the PD to district, school, and personal goals) using a yes/not present criteria. To measure quality of instruction and capture the logistical and atmospheric elements of the PD that fell outside of the bounds of the critical elements of PD (Desimone, 2009), I created a second protocol and codebook. Example of quality of instruction components include: preparedness, provide clear directions, responds to participant inquiries, engages participants actively in the session, and develops a learning environment conducive to adult learning. The quality on instruction checklist includes a Likert scale ranging from highest to lowest quality. I trained independent coders to evaluate quality of instruction based on a continuum of the extent to which the components are demonstrated during each session, ranging from consistently demonstrating the components (i.e., high quality), to inconsistently demonstrating the components (i.e., average quality), to rarely demonstrating the components (i.e., lowest quality). To capture the dosage of each session, I captured the number of opportunities for participants to participate in the PD Planning Guide (see Figure 3.12), and I recorded attendance for every session. In addition, I recorded the number of instructional minutes across all video recordings.

Figure 3.12: Sample PD Planning Guide

Content	Time	Who - Independent Group	Synchronous/ PLC Group	Critical Component/ Learning Structure	Materials Needed/ Logistics to Consider
Course Introduction & Orientation	2-3 min	Sam	Sam	Orient to Module 1	
Part 1: Review Data Presented	1-2 min	Sam	PLC - Part 1 (12 min)	Review graphs and data presented in Part 1 of Module 1	
Part 1: Chat	5 min	Participants – Whole Group		In the chat box or unmute: How did this speak to your own experiences?	
Part 1: Activity Reflection	6 min	Participants – Breakout Rooms		For Journal Entry, address questions: Do you have to be a cheerleader for mathematics DBI? How will you convince others of the need?	
Part 2: Review Data & Objectives	2 min	Sam	PLC - Part 2 (12 min)	Provide review of key content from Part 2 of Module 1	
Part 2: Organize Skills Jamboard Activity	5 min	Participants – Whole Group		Ask participants to complete the following prompt on Jamboard: What resources can be used to determine an appropriate score and sequence?	Will need additional resources in case teachers do not have a strong background here; may also include out of state resources since all teachers will be from TX

To measure participant engagement, I developed a third fidelity protocol and codebook. The protocol evaluates the level of participant engagement throughout the session and the type of comments or questions participants offer throughout the presentation. Using a timed sampling protocol, the independent coders assessed the participants every five minutes. The coders noted whether participants had an opportunity to speak or react during that five-minute interval and noted the category of question or comment (e.g., social, DBI-related, mathematics-related, practice-related).

Adherence, quality of instruction, dosage, and participant engagement fidelity data were assessed at the end of the *NCII Mathematics Course*. I trained two coders to conduct fidelity ratings on 30% of the *NCII Mathematics Course* sessions. Because I was the primary implementor of the PD, I trained the two coders using example PDs found online to ensure a 95% interrater reliability before the coders assessed the summer PD sessions. Interrater reliability was calculated as the number of agreements divided by the number of agreements plus disagreements. Figure 3.13, shown below, displays the Adherence Checklist.

Figure 3.13: Adherence Fidelity Checklist

Task	Is Evidence of Task Apparent?
Content Focus: The primary focus of the PD is to develop participants' content knowledge (in this case, DBI, MKT, and use of evidence-based practices)	
Includes content or activities related to building content knowledge regarding the data-based individualization (DBI) framework	Yes ❑ Not Present ❑
Includes content or activities related to building content knowledge regarding mathematical knowledge for teaching (MKT) including mathematics content, students' thinking about math, or strategies to teach math	Yes ❑ Not Present ❑
Includes content or activities related to building content knowledge regarding use of evidence-based practices including explicit instruction, the use of multiple representations, or precise mathematics language	Yes ❑ Not Present ❑
Active Learning: Participants are actively engaged in learning new content and skill related to teaching mathematics	
I. provides time and task related to examining student work	Yes ❑ Not Present ❑
I. provides time and task related to planning instruction	Yes ❑ Not Present ❑
I. provides time and task related to solving mathematics problems	Yes ❑ Not Present ❑
I. provides time and task related to observing others or being observed teaching mathematics	Yes ❑ Not Present ❑
I. provides time and task related to writing, leading a discussion, or presenting knowledge to an outside audience	Yes ❑ Not Present ❑
Coherence: The purpose of the PD is related to participants' district, school, or personal goals	
Participants have an opportunity or task of relating the goals of the PD to their districts' goals, curriculum, or expectations	Yes ❑ Not Present ❑
Participants have an opportunity or task of relating the goals of the PD to their schools' goals, curriculum, or expectations	Yes ❑ Not Present ❑
Participants have an opportunity or task of relating the goals of the PD to their own goals, curriculum, or expectations	Yes ❑ Not Present ❑
Duration: Participants should have sufficient time and exposure during the PD to develop new knowledge and skills	
Session duration is sufficient to develop new knowledge on the stated focus of session	Yes ❑ Not Present ❑
Sessions are spaced sufficiently to allow for participants to reflect on new knowledge or skills over a sufficient period of time	Yes ❑ Not Present ❑
Collective Participation: Time and tasks are provided for participants to develop networks of support with other participants	
Structures or opportunities are present for participants to connect with other participants	Yes ❑ Not Present ❑
Participants are encouraged to continue to reach out and utilize other participants as resources after the completion of the PD	Yes ❑ Not Present ❑

* I. refers to Instructor(s) who are delivering the professional development

Finally, to assess treatment differentiation, I evaluated a random selection of publicly available recorded mathematics PD offered to teachers around the state of Texas. I assessed the alternative PDs using the same protocols I developed for the *NCII Mathematics Course*, including on measures of adherence, quality, dosage, and participant engagement. I trained a second coder using 20% of the selected PDs to ensure 95% interrater reliability before we continued to code the remaining PD sessions independently.

Implementation fidelity is still being evaluated by the trained coders. Implementation fidelity data will be included in the manuscript submitted for publication.

Chapter 4: Results

I examined the effects of an intensive online summer mathematics PD (i.e., National Center on Intensive Intervention (NCII) *Mathematics Intervention Course*) on teachers' knowledge and skills in supporting their students experiencing MD. I conducted quantitative analysis, including analysis of variance (ANOVA) to measure the impact of the PD across three conditions over time. I conducted additional qualitative analysis, including thematic analysis, to corroborate themes found in the quantitative data regarding the perceived benefits of the summer PD. The research questions and hypotheses are included below.

Data Analysis

Quantitative data were collected via Qualtrics surveys prior to the beginning of the summer PD and at the conclusion of the summer PD. Data were collected regarding participants' Mathematical Knowledge for Teaching (MKT, Ball et al., 2008), *Teacher's Sense of Efficacy Scale* (*TSES*, Tschannen-Moran & Woolfolk Hoy, 2001), *Mathematics Teaching Self-Efficacy* (Perera & John, 2020), and *Teacher's Instructional Practices (TIP,* Ketterlin-Geller et al., 2018). Data were exported to R for data analysis. Data were analyzed using a series of ANOVA tests, and F statistics are presented along with eta-squared for each test as a measure of effect size. Descriptive statistics were calculated for all measures; pre-test, posttest, and standard deviations are reported by condition in Table 4.1.

Qualitative data include interviews conducted with 13 teachers at the conclusion of the summer PD. I transcribed these interviews transcribed using Otter.ai and analyzed the transcripts using Dedoose. Central themes are presented in Figure 4.1.

Figure 4.1: Overarching themes identified in the interview analysis.

Evaluating ANOVA Assumptions

Before analyzing the quantitative data, I first evaluated the data to ensure it met the basic requirements for a repeated measures ANOVA test, including a) independence of observations, b) normal distribution of groups, and c) sphericity, or equality of variance between groups.

As participants were randomly assigned to one of the three groups, it can be assumed that observations were independent of each other. Six participants indicated that they could only participate if they were placed in the Asynchronous group; therefore, I conducted a series of t-tests to determine if there were any substantial differences across measures during the pretest between these two groups. Analysis of independent *t* tests resulted in no significant differences between the two groups on the following measures given at pretest: *MKT Elementary Rational Numbers – Content Knowledge* [$t(15) = 0.25$, $p > .05$], : *MKT Middle School Rational Numbers – Content Knowledge* [$t(15) = 0.06$, $p > .05$], *Teacher's Sense of Efficacy Scale* [$t(15) = 0.574$, $p > .05$], *Mathematics Teaching Self-Efficacy* [$t(15) = 0.353$, $p > .05$], *Teacher's Instructional Practices* [$t(15) = 0.094$, $p > .05$], and the researcher-developed measure of intensive intervention knowledge [$t(15) = 0.331$, $p > .05$]. Therefore, the data of the participants who self-selected into the Asynchronous group were included in the analysis of this study.

Additionally, I evaluated the data for the potential of extreme outliers that could unduly influence the outcomes. All outcome measures were graphed using a box plot method and extreme outliers that fell above or below three times the interquartile range were identified by the *identify_outliers* program in R. There were no extreme outliers on any measure across all three data configurations for the three research questions.

In order to evaluate the assumption of normality, I conducted a Shapiro-Wilk test on each outcome measure. Evaluating each measure across both time points, there were cases that failed the Shapiro-Wilk test for normality, specifically the Synchronous/Independent group on the *TSES* general self-efficacy pretest and the

Synchronous/PLC group on the *TIP* posttest measure. Regarding the *TSES* pretest data, according to a visual inspection of the box plots, the data appears to fall primarily within the scope of normality, see Figure 4.2. However, regarding the Synchronous/PLC posttest data on the *TIP*, a visual inspection of the data, seen on Figure 4.3, suggests non-normality of the data. Therefore, any conclusions drawn regarding ANOVA results should be made cautiously.

Figure 4.2: Normality distributions of all groups on *TSES* data

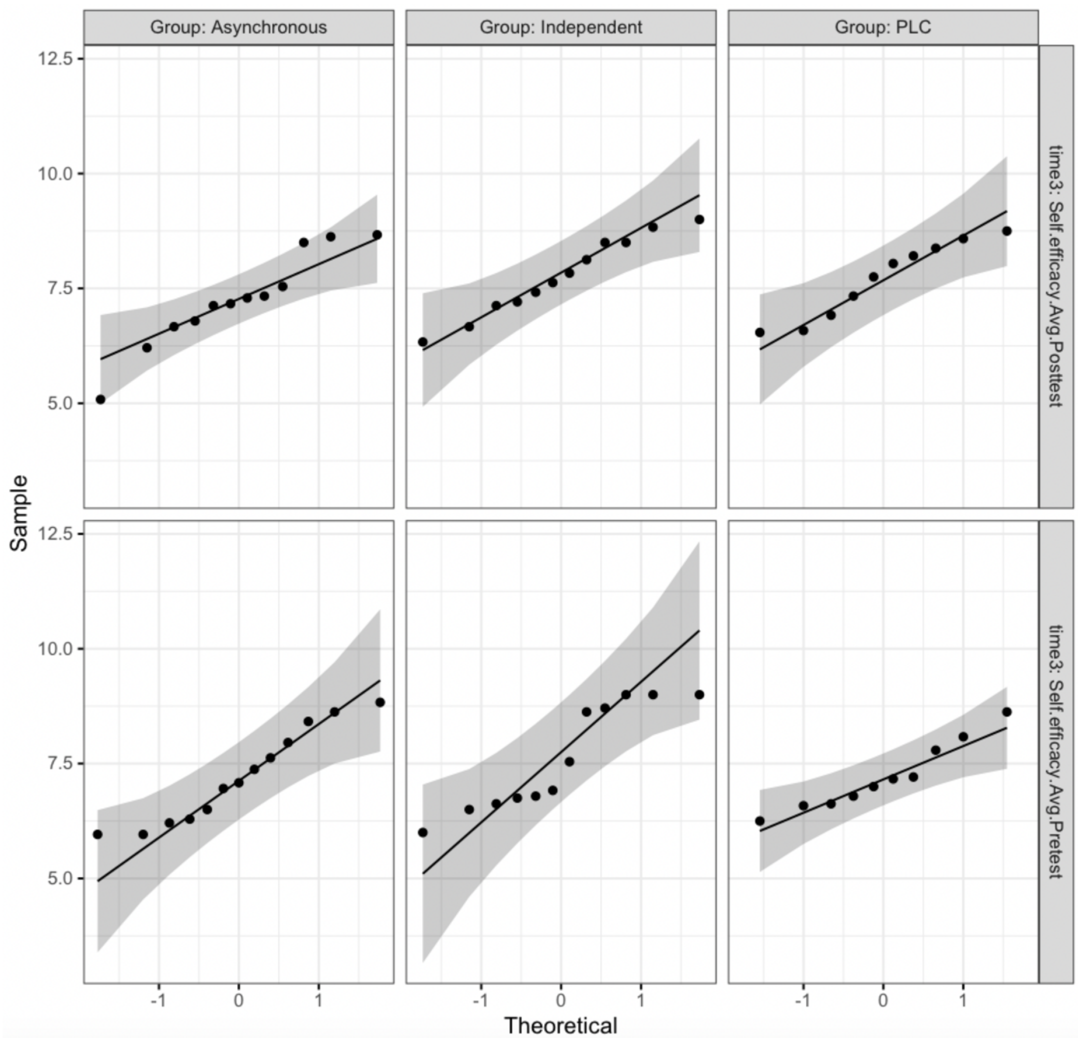
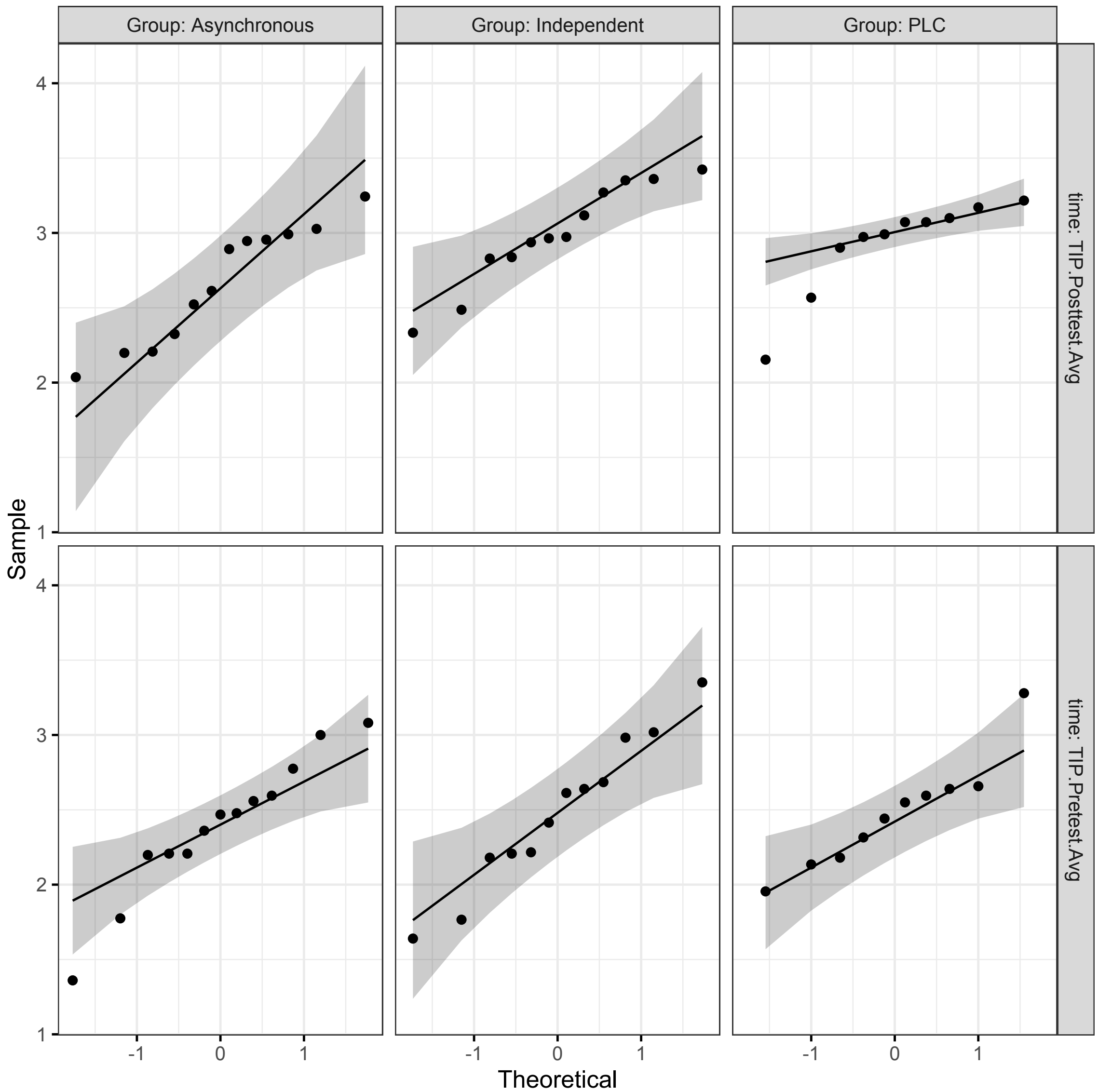


Figure 4.3: Normality distributions of all groups on *TIP* data



To evaluate the assumption of sphericity, I used the Mauchly's test to evaluate whether the variance between groups was equal. Using the *anova_test* function in R, the assumption of sphericity was automatically checked and the Greenhouse-Geisser sphericity correction was applied to any factors violating the sphericity assumption.

Analyses of Pretest Data

Analysis of variance (ANOVA) tests resulted in no significant differences between the three treatment groups on the following measures given at pretest: *MKT Elementary Rational Numbers – Content Knowledge* [F (2, 43) = 1.196, $p > .05$], : *MKT Middle School Rational Numbers – Content Knowledge* [F (2, 40) = 0.835, $p > .05$], *Teacher's Sense of Efficacy Scale* [F (2, 46) = 0.755, $p > .05$], *Mathematics Teaching Self-Efficacy* [F (2, 46) = 2.62, $p > .05$], *Teacher's Instructional Practices* [F (2, 46) = 0.154, $p > .05$], and the researcher-developed measure of intensive intervention knowledge [F (2, 46) = 2.406, $p > .05$].

RESULTS: RESEARCH QUESTIONS

I examined the effects of the summer PD via five ANOVA tests on each outcome measure. Main effects examined the overall impact of the PD on participant outcomes. Simple main effects and interaction effects examined the differences between groups. Additional qualitative data analyses provide greater insight into participants' percept

Table 4.1: Pretest and posttest means and standard deviations for outcome measures

Outcome Measure	Pretest *M* (*SD*)						Posttest *M* (*SD*)					
	Asynchronous (BaU)		Synchronous/ Independent		Synchronous/ PLC		Asynchronous (BaU)		Synchronous/ Independent		Synchronous/ PLC	
MKT Elementary	0.01	(0.96)	0.46	(0.74)	0.15	(0.77)	0.25	(1.00)	0.59	(0.77)	0.53	(0.69)
MKT Middle School	-0.72	(1.30)	-0.25	(1.00)	-0.59	(0.69)	-0.82	(0.92)	-0.35	(1.16)	-0.32	(0.96)
Self-efficacy	7.21	(0.90)	7.52	(1.22)	7.11	(0.82)	7.25	(1.04)	7.76	(0.85)	0.71	(0.82)
Mathematics self-efficacy	2.63	(0.55)	3.03	(0.74)	2.58	(0.51)	3.07	(0.48)	3.27	(0.62)	3.41	(0.49)
TIP	2.46	(0.46)	2.38	(0.50)	2.45	(0.43)	2.66	(0.40)	2.99	(0.34)	2.92	(0.33)
Intensive intervention	24.06	(6.18)	25.06	(5.42)	28.44	(6.26)						

Research Question 1:

What is the impact of an intensive online summer mathematics PD on the outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) of in-service SPED teachers who teach mathematics to students experiencing MD?

H_1: Participation in an intensive online summer mathematics PD is positively related to outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) for in-service teachers and officials who support students experiencing MD.

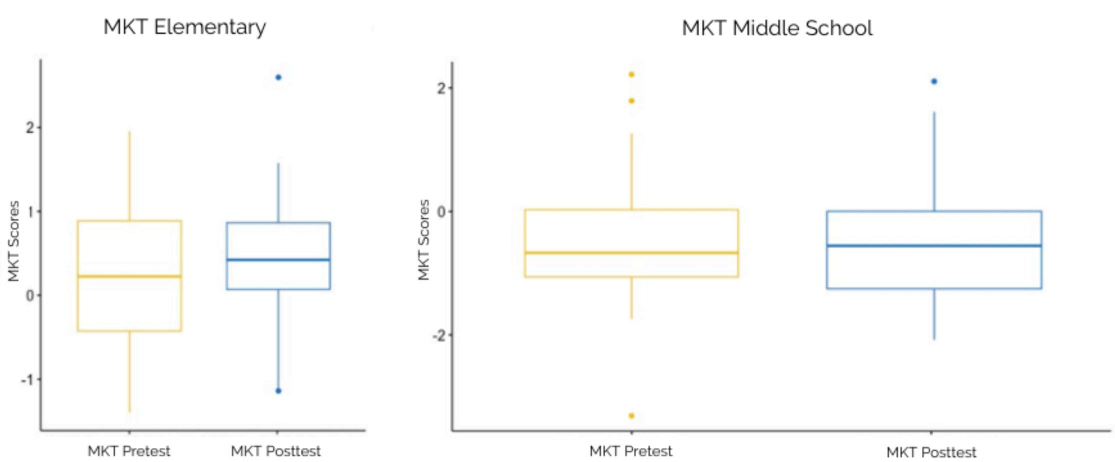
H_0: Participation in an intensive online summer mathematics PD is not positively related to outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) for in-service teachers and officials who support students experiencing MD.

As part of the intensive online summer mathematics PD, all participants completed the *NCII Mathematics Course*, focused on developing teachers' ability to support their students who struggle with mathematics. The *NCII Mathematics Course* included materials to support teachers in implementing evidence-based mathematics instruction and data-based decision making. To measure the impact of the intervention, participants completed five measures before and following the summer PD to measure the impact of the *NCII Mathematics Course* on teachers' MKT, self-efficacy, and instructional practices. To determine the overall impact of the intervention, an ANOVA test examined if there were significant main effects for the impact of time across all groups.

Across all groups, there was no significant improvement in participants' MKT, measured by the *MKT Elementary Rational Numbers – Content Knowledge* [F (1, 31) =

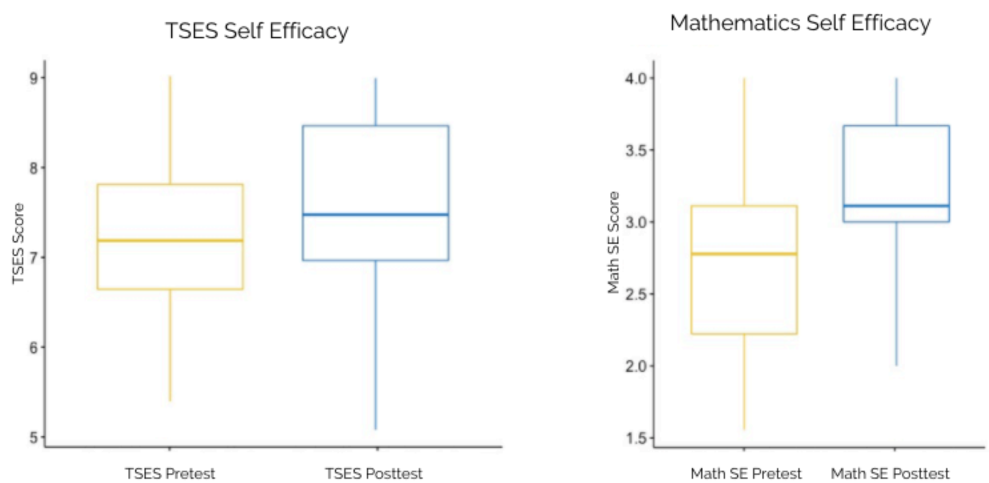
1.754, $p = .196$, $\eta 2 = 0.015$] or the *MKT Middle School Rational Numbers – Content Knowledge* [F (1, 31) = 0.001, $p = .977$, $\eta 2 = 0.000$]. Although there were no significant main effects for time, all three groups improved their scores on *MKT Elementary Rational Numbers – Content Knowledge*. On the *MKT Middle School Rational Numbers – Content Knowledge* test, only the Synchronous/PLC group saw an increase in their mean scores. See Figure 4.4 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.4: Comparison boxplots of aggregated group data of MKT pretest and posttest



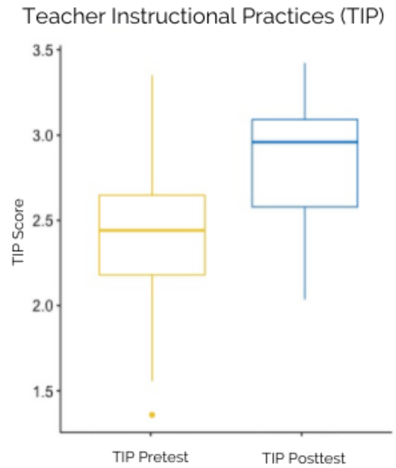
Across all groups, there was no significant improvement in participants' self-efficacy, measured by the *TSES*, a general measure of teacher self-efficacy [F (1, 33) = 1.827, $p = .186$, $\eta 2 = 0.012$] but there was a significant increase in participants' *Mathematics Teaching Self-Efficacy* [F (1, 33) = 23.915, $p = .00003$, $\eta 2 = 0.14$]. Post-hoc analyses with a Bonferroni adjustment revealed that the pairwise differences, between time points, were significantly different ($p < 0.05$). Although there were no significant main effects for time on the general measure of self-efficacy, all three groups improved their scores on the *TSES*. See Figure 4.5 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.5: Comparison boxplots of aggregated group data of *TSES* and *Mathematics Teaching Self-Efficacy* pretest and posttest



Across all groups, there was a significant increase in participant's confidence, expected frequency, and understanding of evidence-based practices, measured by the *TIP*, the teachers' instructional practices measure [F (1, 33) = 24.821, p = .00002, $\eta 2$ = 0.21]. See Figure 4.6 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.6: Comparison boxplots of aggregated group data of *TIP* pretest and posttest



All 13 participants who were interviewed spoke about the benefits of attending the summer PD. Although participants identified a variety of personal and professional rewards for attending the summer PD, I found they clustered under three overarching themes: resources, accountability, and confidence. Figure 4.1 shows the overlap of key themes that emerged from my analysis of the interview data. In all of the 13 interviews, participants noted at least once that they benefitted or expected to benefit from resources acquired through the summer PD. The majority of participants listed videos, readings, and websites provided through the *NCII Mathematics Course* as the highlight of the PD. Participants expressed excitement about being able to return to the materials throughout the school year and share the resources with their colleagues and supervisors.

In addition to the access to resources, 11 participants noted that they benefitted from the accountability structures put in place throughout the summer PD. The *NCII Mathematics Course* are freely available year-round, meaning that anyone has the ability to complete the course on their own without needing to attend a structured PD. The participants noted, however that without the deadlines imposed by the summer PD, they would have struggled to complete the work on their own. One participant in the Synchronous/Independent group noted: “I do really well, with having deadlines and goals. So having the meeting as my goal of having everything done was really helpful.”

Finally, 12 participants spoke of either their new confidence in teaching mathematics or feeling validation in knowing they were already teaching evidence-based strategies. One participant, a high school special education teacher, noted that she took the course because she struggled with mathematics as a student and felt that she still had gaps in her own knowledge of how to best teach her students. She felt the summer PD provided her with resources to help with those gaps. A first-year special education teacher remarked on the confidence-boost she received when discovering that she was already implementing evidence-based strategies: “A lot of the things in the PD were best practices that we were already doing and that my school already encouraged. But it felt good to be like, Oh, this is research-based, this is what we’re supposed to be doing.” Other participants remarked they felt the summer PD helped them refine their knowledge of mathematics concepts or ways to use data in supporting students.

Research Question 2:

Do participants who participate in weekly synchronous meetings (i.e., Synchronous/Independent and Synchronous/PLC groups) when completing an online summer PD demonstrate improved outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) compared to peers who asynchronously complete (i.e., Asynchronous group) the online summer PD?

H_1: Participation in weekly synchronous meetings as part of an online summer PD is positively related to outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) for in-service teachers and officials who support students experiencing MD.

H_0: Participation in weekly synchronous meetings as part of an online summer PD is not positively related to outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) for in-service teachers and officials who support students experiencing MD.

Participants were randomly assigned to complete the online summer PD as part of an Asynchronous (BaU) group, a group that met weekly but completed assignments independently (i.e., Synchronous/Intendent group), or a group that met weekly and completed assignments as part of the Synchronous/PLC group. To determine the impact of participating in weekly synchronous meetings, an ANOVA test examined if there were significant interactions effects of the effects of time across different groups. For the purpose of this research question, the data from the Synchronous/Independent group and the Synchronous/PLC group were aggregated as a Synchronous group compared to the Asynchronous group data. Descriptive statistics of aggregated data were calculated for all measures; pre-test, posttest, and standard deviations are reported by condition in Table 4.2.

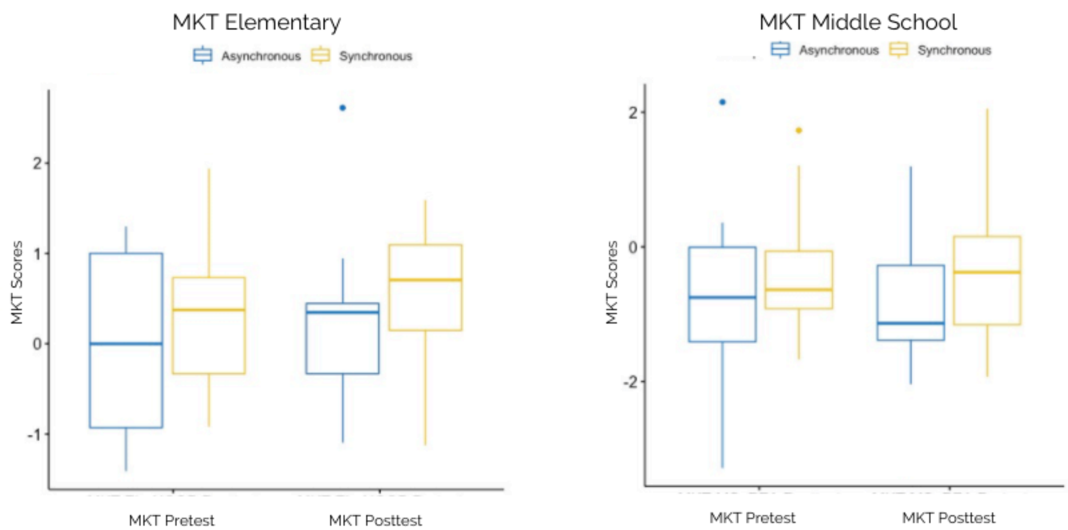
Table 4.2: Pretest and posttest means and standard deviations for outcome measures of Asynchronous group and aggregated Synchronous groups

Outcome Measure	Pretest M (*SD*)				Posttest M (*SD*)				Difference in Means	
	Asynchronous (BaU)		Synchronous		Asynchronous (BaU)		Synchronous		Asynchronous (BaU)	Synchronous
MKT Elementary	0.01	(0.96)	0.30	(0.76)	0.25	(1.00)	0.56	(0.72)	0.246	0.266
MKT Middle School	-0.72	(1.30)	-0.42	(0.86)	-0.82	(0.92)	-0.34	(1.05)	-0.094	0.085
Self-efficacy	7.21	(0.90)	7.31	(1.04)	7.25	(1.04)	7.74	(0.82)	0.037	0.425
Mathematics self-efficacy	2.63	(0.55)	2.81	(0.67)	3.07	(0.48)	3.33	(0.56)	0.447	0.528
TIP	2.46	(0.46)	2.41	(0.46)	2.66	(0.40)	2.96	(0.33)	0.202	0.547

Note: Synchronous group data consists of Synchronous/Independent group data and Synchronous/PLC data aggregated

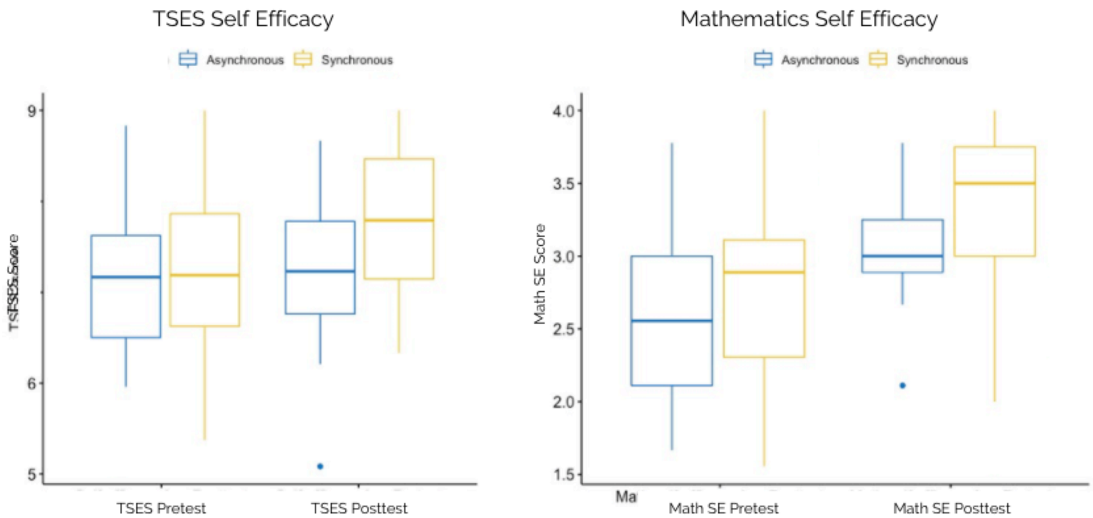
Between the Asynchronous (BaU) and Synchronous groups (i.e., Synchronous/ Independent group and Synchronous/PLC group), there was no significant difference in participant's growth of MKT, measured by the *MKT Elementary Rational Numbers – Content Knowledge* [F (1, 46) = 0.051, p = .823] or the *MKT Middle School Rational Numbers – Content Knowledge* [F (1, 46) = 0.021, p = .887]. Although there were no significant interaction effects for time between the two groups, the Synchronous group had slightly greater growth on both the *MKT Elementary Rational Numbers – Content Knowledge* and the *MKT Middle School Rational Numbers – Content Knowledge*. See Figure 4.7 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.7: Comparison boxplots of Asynchronous and Synchronous group data of MKT pretest and posttest



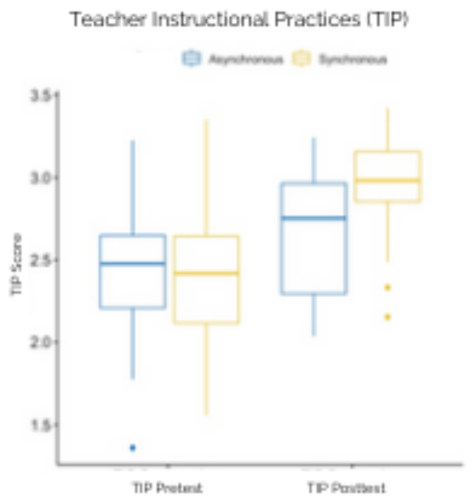
Between the Asynchronous (BaU) and Synchronous groups (i.e., Synchronous/ Independent group and Synchronous/PLC group), there was no significant difference in participant's growth of self-efficacy, measured by the general self-efficacy *TSES* measure [F (1, 46) = 0.777, p = .385] or the *Mathematics Teaching Self-Efficacy* [F (1, 46) = 0.001, p = .975]. Although there were no significant interaction effects for time between the two groups, the Synchronous group had slightly greater growth on both measures of self-efficacy. See Figure 4.8 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.8: Comparison boxplots of Asynchronous and Synchronous group data of *TSES* and *Mathematics Teaching Self-Efficacy* pretest and posttest



Between the Asynchronous (BaU) and Synchronous groups (i.e., Synchronous/ Independent group and Synchronous/PLC group), there was no significant difference in participant's growth in confidence, expected frequency, and understanding of evidence-based practices, measured by the *TIP*, [F (1, 46) = 1.044, p = .314] Although there were no significant interaction effects for time between the two groups, the Synchronous group had slightly greater growth on the *TIP*. See Figure 4.9 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.9: Comparison boxplots of Asynchronous and Synchronous group data of *TIP* pretest and posttest



When reviewing the key themes that arose from the participant interviews, there were notable differences in the reflections of the Asynchronous participants compared to the participants in either the Synchronous groups (i.e., Synchronous/Independent and Synchronous/PLC group). In addition to the *NCII Mathematics Course* materials, 9 of the 10 participants in the Synchronous groups remarked on the peer support and discussions throughout the summer PD as a key resource and source of accountability. As one participant noted: "I possibly would have dropped out had I not had the meetings to go to… [The meetings] were absolutely huge and completing as much of the courses as I did." One participant from the Asynchronous group reflected: "I would never, ever again, take this asynchronously. This was a very difficult course, to take asynchronously… It was a difficult thing to be in a vacuum, and not have any way to test out what you're thinking or see if you're on the mark. So I don't recommend it that way." The benefit of having or missing the support that came from the weekly synchronous meetings was a key theme across all but one of the interviews.

Research Question 3:

Do participants who receive peer support through PLCs during the online summer PD demonstrate improved outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) than their peers who do not receive such peer support (i.e., Synchronous/Independent and Asynchronous groups) during the online summer PD?

H_1: Participation in PLCs as part of an online summer PD is positively related to outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) for in-service teachers and officials who support students experiencing MD.

H_0: Participation in PLCs as part of an online summer PD is not positively related to outcomes (i.e., MKT, self-efficacy, and teacher instructional practices) for in-service teachers and officials who support students experiencing MD.

Participants were randomly assigned to complete the online summer PD as part of an Asynchronous (BaU) group, a group that met weekly but completed assignments independently (i.e., Synchronous/Independent group), or a group that met weekly and completed assignments as part of a Synchronous/PLC group. To determine the impact of participating in a PLC, an ANOVA test examined if there were significant interactions effects of the effects of time across different groups. For the purpose of this research question, the data from the Asynchronous (BaU) group and the Synchronous/Independent group were aggregated as a Non-PLC group compared to the Synchronous/PLC group data. Descriptive statistics of aggregated data were calculated for all measures; pre-test, posttest, and standard deviations are reported by condition in Table 4.3.

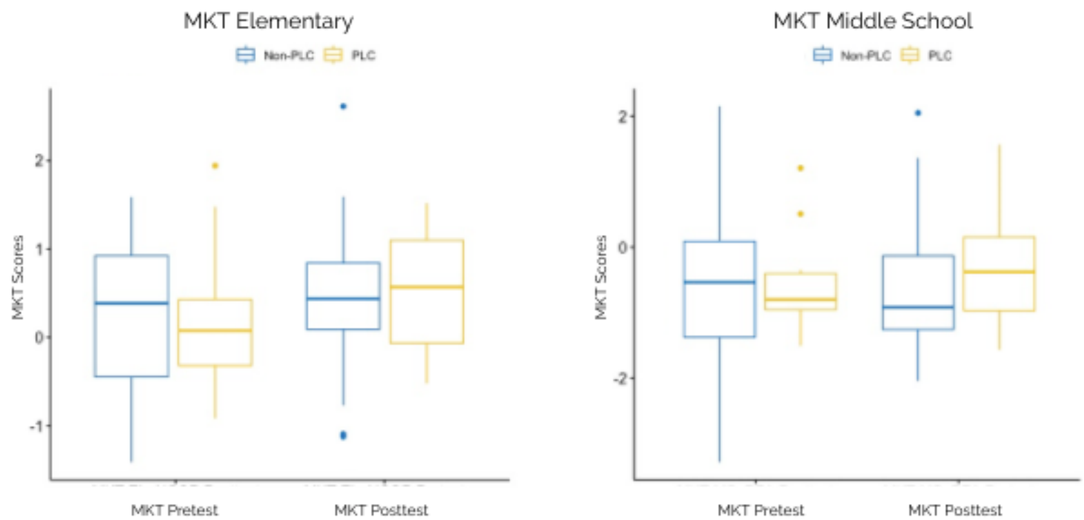
Table 4.3: Pretest and posttest means and standard deviations for outcome measures for Non-PLC and PLC groups

Outcome Measure	Pretest *M* (*SD*)				Posttest *M* (*SD*)				Difference in Means	
	Non-PLC		PLC		Non-PLC		PLC		Non-PLC	PLC
MKT Elementary	0.23	(0.89)	0.15	(0.77)	0.43	(0.89)	0.53	(0.69)	0.20	0.38
MKT Middle School	-0.47	(1.15)	-0.59	(0.69)	-0.57	(1.06)	-0.32	(0.96)	-0.10	0.27
Self-efficacy	7.36	(1.06)	7.11	(0.82)	7.51	(0.97)	7.71	(0.82)	0.15	0.60
Mathematics self-efficacy	2.82	(0.67)	2.58	(0.51)	3.17	(0.55)	3.41	(0.49)	0.35	0.83
TIP	2.42	(0.47)	2.45	(0.43)	2.83	(0.40)	2.92	(0.33)	0.41	0.47

Note: Non-PLC group data consists of Asynchronous group data and Synchronous/Independent group data aggregated

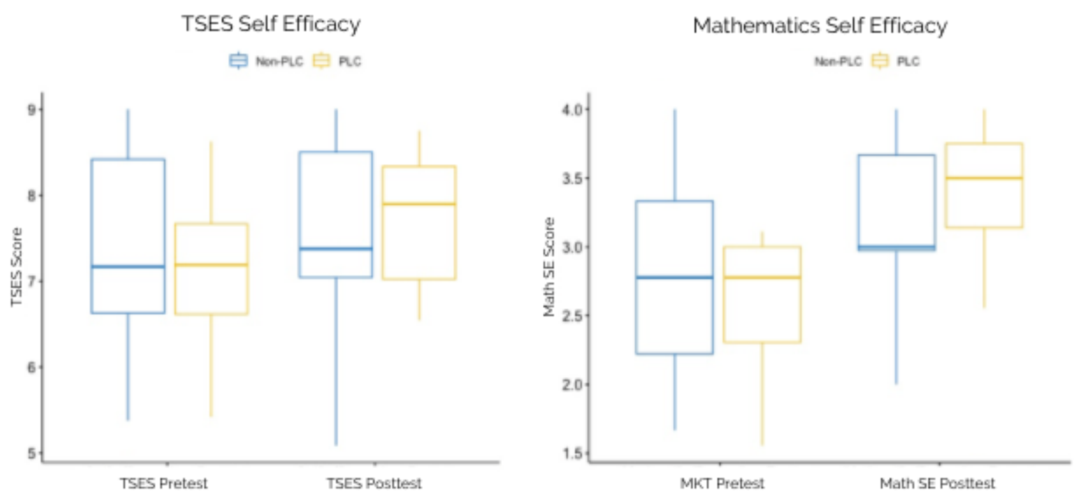
Between the Non-PLC (i.e., Asynchronous group and Synchronous/Independent group), and Synchronous/PLC groups there was no significant difference in participant's growth of MKT, measured by the *MKT Elementary Rational Numbers – Content Knowledge* [F (1, 46) = 0.017, p = .897] or the *MKT Middle School Rational Numbers – Content Knowledge* [F (1, 46) = 0.258, p = .616]. Although there were no significant interaction effects for time between the two groups, the Synchronous/PLC group had slightly greater growth on both the *MKT Elementary Rational Numbers – Content Knowledge* and the *MKT Middle School Rational Numbers – Content Knowledge*. See Figure 4.10 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.10: Comparison boxplots of Non-PLC and PLC group data of MKT pretest and posttest



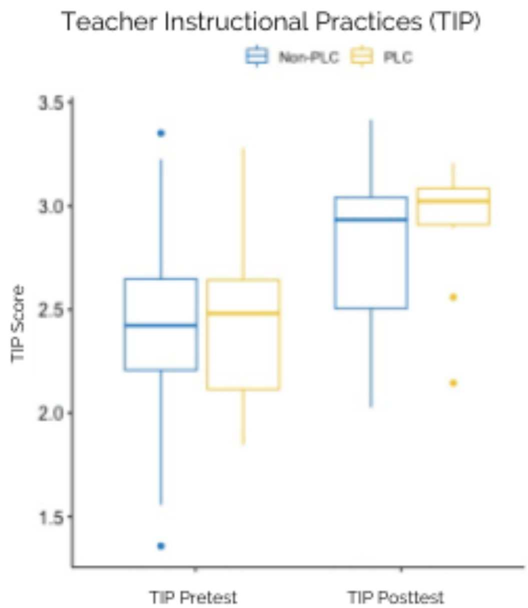
Between the Non-PLC (i.e., Asynchronous group and Synchronous/Independent group) and Synchronous/PLC groups, there was no significant difference in participant's growth of self-efficacy, measured by the general self-efficacy *TSES* measure [F (1, 46) = 1.585, p = .217] or the *Mathematics Teaching Self-Efficacy* [F (1, 46) = 3.927, p = .0562. Although there were no significant interaction effects for time between the two groups, the PLC group had slightly greater growth on both measures of self-efficacy. See Figure 4.11 for a comparison of the aggregated groups data at pretest and posttest.

Figure 4.11: Comparison boxplots of Non-PLC and PLC group data of *TSES* and *Mathematics Teaching Self-Efficacy* pretest and posttest



Between the Non-PLC (i.e., Asynchronous group and Synchronous/Independent group) and Synchronous/PLC groups there was no significant difference in participant's growth in confidence, expected frequency, and understanding of evidence-based practices, measured by the TIP, [F (1, 46) = 104, p = .747] Although there were no significant interaction effects for time between the two groups, the Synchronous/PLC group had slightly greater growth on the *TIP*. See Figure 4.12 for a comparison of the aggregated groups' data at pretest and posttest

Figure 4.12: Comparison boxplots of Non-PLC and PLC group data of *TIP* pretest and posttest



Although less pronounced than the differences in responses between the Asynchronous (BaU) and Synchronous groups (i.e., Synchronous/Independent and Synchronous/PLC groups), there were noteworthy differences between the participants in the Synchronous/PLC group and those in Non-PLC groups (i.e., Asynchronous and Synchronous/Independent groups). As noted earlier, participants in both the Synchronous groups felt that weekly meetings motivated them to complete assignments. Members in both the Synchronous/Independent and Synchronous/PLC groups cited the conversations as valuable and beneficial, but members of the Synchronous/Independent group were less likely to cite a member of their group as a resource. Members of the Synchronous/PLC group were more likely to list a specific person they could picture themselves reaching out to if they had questions in the future.

Members of the Synchronous/PLC group discussed the group membership overall in greater depth than members of the Synchronous/Independent group when discussing components of the PD they felt should remain the same in future iterations of the PD. Additionally, members of the Synchronous groups were not informed that the structure of the two different groups differed, but one member of the Synchronous/PLC group reflected that the consistency of her group was a major factor of the PD: “Given this was a self-selected summer thing, getting to know new people week after week would have been crazy. And something I would not have enjoyed. So having a small consistent group was really nice.” Overall, even though both Synchronous groups spoke to the benefit of having weekly groups, the Synchronous/PLC group spoke in greater depth and with greater enthusiasm than participants in the Synchronous/Independent group.

Chapter 5: Discussion

Teaching mathematics to students experiencing MD can be challenging for many teachers. Teaching mathematics effectively to any student requires a sophisticated depth of both pedagogical and mathematical content knowledge (Ball et al., 2005; Hill et al., 2008). Students experiencing MD often have immature understandings or misconceptions regarding mathematical procedures or concepts (Gersten et al., 2007), meaning teachers will need to be equipped with additional strategies to meet these students' needs. One such strategy, using DBI has been shown to improve student outcomes when implemented well (Jung et al., 2018; Powell et al., in press). However, the DBI framework is also challenging for many teachers to implement with fidelity (Mason et al., 2019) and there are many barriers preventing teachers from effectively implementing the DBI framework, including a lack of PD focused on the framework and data literacy (Datnow & Hubbard, 2016).

When conducted effectively, PD has been shown to improve both teachers' mathematical knowledge and skills (Scher & O'Reilly, 2009) as well as their knowledge of DBI (Bruhn et al., 2019). Although the effects of PD targeting teachers' mathematical knowledge and DBI knowledge have both been explored separately, there is little research on the impact of a PD that targets both teachers' mathematical knowledge as well as their knowledge and skills related to DBI. Additionally, although high quality PD has been shown to increase teacher learning to the degree that student learning also improves (Darling-Hammond et al., 2017), there is less known about what key features of PD lead to the greatest teacher improvements.

This study examined the effects of an intensive online summer PD on the MKT, self-efficacy, and instructional practices of teachers and other participants who support students experiencing MD. Texas teachers, school leaders, and district officials who directly supported teachers or students participated in the summer PD; 48 participants enrolled in the PD and were randomly assigned to complete the PD in one of three groups: 1) in an Asynchronous group, 2) in a Synchronous/Independent group, or 3) a Synchronous/PLC group. In all, 35 participants completed the posttest at the conclusion of the PD. The PD utilized the National Center on Intensive Intervention (NCII) *Mathematics Intervention Course* which included 8 modules that participants completed over the course of 9 weeks. The results of the study are discussed below.

FINDINGS RELATED TO RESEARCH QUESTION 1

The first research question addressed the effect of an intensive online summer mathematics PD on teacher-level outcomes, including their MKT, self-efficacy, and instructional practices. To assess growth in MKT, participants completed the *MKT Elementary Rational Numbers – Content Knowledge* as well as the *MKT Middle School Rational Numbers – Content Knowledge*. To assess growth in self-efficacy, participants completed both the *TSES*, a general measure of teacher self-efficacy as well as the *Mathematics Teaching Self-Efficacy*, designed to target mathematics teachers. Finally, to assess participants' confidence, understanding, and likelihood of using evidence-based practices, participants completed the *TIP* (Ketterlin-Geller et al., 2018).

I hypothesized that participants would benefit from the intensive online summer mathematics PD. The general findings of this study partially support this hypothesis, but many of the results are not conclusive. When I analyzed all three groups' data as an aggregated single group, participants significantly improved their scores on the *Mathematics Teaching Self-Efficacy*, as well as the *TIP* ($p < 0.05$). Additionally, across all groups, participants improved their scores on the *MKT Elementary Rational Numbers – Content Knowledge* and the *TSES*, but not to the degree of statistical significance. On the *MKT Middle School Rational Numbers – Content Knowledge*, only the Synchronous/PLC group improved their mean scores from pretest to posttest, again not to a significant degree.

These findings align with the larger research conversation regarding changing teacher outcomes in that participants improved their scores on measures of instructional understanding and some measures of self-efficacy but not MKT. Previous research has demonstrated that PD that targets teachers espoused knowledge or discrete skills and measures the impact of those same knowledge or skills typically have higher effect sizes than PD that targets more generalized knowledge or complex content, such as MKT (Blank & de las Alas, 2009). Additionally, the *TIP* asks participants to reflect on their own frequency, understanding, valuation, and confidence in implementing evidence-based instructional and assessment practices, including components of DBI. As there was no direct observation of participant behavior before or after the summer PD, there is no data available to support whether participants changed their instructional or assessment practices after participating in the summer PD. Therefore, although a change in espoused beliefs of knowledge is often the first step in changing teacher behaviors (Desimone, 2009), these results represent only the first step in changing teacher instructional and assessment practices.

Regarding participants' outcomes on measures of self-efficacy, it is notable that there was a significant improvement in participants' mathematics self-efficacy but not their general self-efficacy as a teacher. Both general self-efficacy and mathematical self-efficacy have been associated with greater student achievement (Perera & John, 2020; Zee & Koomen, 2016). This difference may reflect that the construct of mathematical self-efficacy is an important component of mathematical learning, but it is able to exist separate from general pedagogical self-efficacy. For example, questions on the *Mathematics Teaching Self-Efficacy* measure (Perera & John, 2020) asked participants to rate their confidence in "showing students a variety of problem-solving strategies," or "making mathematics relevant to students," two skills that could likely be achieved even if participants struggled with questions of general self-efficacy on the *TSES* (Tschannen-Moran & Woolfolk Hoy, 200) such as confidence in their ability to "control disruptive behavior in the classroom" or "respond to defiant students."

The lack of significant growth on the *TSES* may reflect the difficulty in improving general self-efficacy, even though all three groups saw an improvement in the *TSES* scores after participating in the summer PD. Another contributing factor may be the high degree of general self-efficacy prior to the summer PD, while mathematics self-efficacy was more variable across the groups. Even though the *NCII Mathematics Course* provided evidence-based assessment and instructional practices that if implemented well would likely lead to favorable outcomes and improved overall instructional quality, the focus on the summer PD was not general pedagogical practices, and therefore this slight misalignment may have led to the different results between the two measures. That significant improvement of participants on the measure of mathematics self-efficacy is also promising, suggesting that participants did not need to significantly improve their scores on the MKT measures to feel more confident about their ability to teach mathematics to their students.

The lack of significant results on either measure of MKT is also unsurprising due to the difficulty in improving teachers MKT (Phelps et al., 2016). The purpose of the *NCII Mathematics Course* included best practices within a mathematical classroom, but only two of the eight modules included explicit instruction on how to teach mathematical concepts. Additionally, the included material and activities included foundational mathematical concepts, more likely to be taught in elementary grades than middle school grades. Therefore, the improvement of participants' *MKT Elementary Rational Numbers – Content Knowledge* demonstrates the promise of the *NCII Mathematics Course* as the potential core of future PD that can significantly improve participants' *MKT Elementary Rational Numbers – Content Knowledge*. More must be done, however, to determine the most effective components of PD to align with the *NCII Mathematics Course* in producing a meaningful and impactful PD.

In addition to the quantitative results which painted an overall positive picture of the impact of the PD, the data from qualitative interviews corroborated the findings that the summer PD had a positive impact on the participants. Participants in the summer PD expressed satisfaction in the overall PD experience and were able to describe both short-term and long-term benefits to their own teaching. Three dominant themes arose across the participants' interviews. Participants from all three groups spoke of the resources, the accountability, and the confidence they gained as a result of participating in the summer PD.

One of the most popular themes across all participants interviewed included the benefit of having access to the *NCII Mathematics Course* and additional resources from the summer PD. Participants commented most often on the benefit of having the videos and readings available to them throughout the summer and the effectiveness of information communicated via these mediums. Although not explicitly addressed, many of the participants who spoke of the benefits of the videos and readings mentioned the benefit of being able to re-watch videos or return to readings later, two activities that require additional time and flexibility that would not be feasible in a typical one-day isolated PD session. The flexibility to access (and re-access) the materials of the summer PD were balanced by the sense of accountability that participants felt to complete the requirements by set deadlines. Therefore, the summer PD balanced to some degree the autonomy that supports adult learning (Bei et al., 2019) with the accountability structures that support effective and timely change (Hill et al., 2021).

In addition to the themes of resources and accountability, participants also spoke more obliquely about the confidence they gained from participating in the summer PD. Many school teachers, both new and experienced, spoke of the sense of validation they felt when they saw their current school practices being touted as effective practices by the *NCII Mathematics Course*. In addition to appreciating the access to resources for their own classroom, many participants also expressed excitement about sharing the resources and practices with their colleagues and peers. Although not intended as part of the summer PD, these participants were becoming instructional resources at their schools, taking leadership roles in disseminating the information to others, another key aspect of making lasting adult change (Desimone, 2009). Together the sense of accountability participants felt in completing the summer PD, tied to their new confidence and access to meaningful resources all put the participants in a position to make lasting change in their instructional practices.

FINDINGS RELATED TO RESEARCH QUESTION 2

The second research question addressed the differential effect of participating in a group that participating in weekly synchronous meetings on participant outcomes including MKT, self-efficacy, and instructional practices. I hypothesized that participants would benefit from participating in weekly synchronous meetings where they would have opportunities to receive feedback and discuss course materials with peers. To answer this research question, I aggregated the Synchronous/Independent and Synchronous/PLC groups into one Synchronous group and compared the growth of that Synchronous group to the growth of the Asynchronous (BaU) group using an ANOVA test. The findings of this study partially support this hypothesis, but none of the quantitative results are significant, suggesting that the results are not conclusive and should be interpreted cautiously. Qualitative data provide a more comprehensive view of the potential differences between the two groups.

Across all measures, including measures of MKT (i.e., *MKT Elementary Rational Numbers – Content Knowledge* and *MKT Middle School Rational Numbers – Content Knowledge*), measures of self-efficacy (i.e., *TSES* and *Mathematics Teaching Self-Efficacy*) as well as measures of instructional practices (i.e., *TIP*) there was no significant differences between the growth of the Synchronous group compared to the Asynchronous (BaU) group. Although not significant, the Synchronous group did improve to a slightly greater degree on all measures compared to the Asynchronous group.

Likely a major contributing factor to the lack of significant results was the small sample size of each group. Power analysis prior to the study suggested that at least 20 participants were needed in each group to achieve significant results. The initial pool of 48 participants was already slightly too small according to power analysis, and with 27% attrition and a final pool of only 35 posttests, it was unlikely that any results would reach statistical significance. The issues of attrition and motivation to complete the summer PD are notable challenges for voluntary PD and are addressed below.

Interestingly, although the quantitative analyses suggest that there were perhaps minor or insignificant differences between the Asynchronous (BaU) and the Synchronous groups, the qualitative analyses from the participants' interviews suggested that participants experienced the summer PD in very different ways depending on whether they had access to the weekly meetings. Two participants in the Asynchronous (BaU) group agreed to be interviewed, and both spoke at length about the difficulty of completing the *NCII Mathematics Course* without feedback or opportunities to process the information with others. Even though one participant had originally requested the Asynchronous group, saying that her participation in the summer PD was contingent on her ability to complete the materials at her own pace, in the interview she reflected: "I think I would have liked it synchronously better…I would have loved to have had the meetings." The other participant who participated in the Asynchronous group noted that she would never take the course asynchronously again and that her main motivation in completing the course was the anxiety that it caused her to have the course unfinished and hanging over her.

Compared to the Asynchronous (BaU) group, participants in the Synchronous groups touted the weekly meetings as a key motivator for completing the materials week-to-week. Of the 11 participants who participated in a Synchronous group and agreed to be interviewed, 9 participants shared that the weekly meetings provided the necessary accountability to stay on track and complete the course materials. Many participants reflected that without that peer pressure, they likely would have struggled to pace themselves and complete all the activities and readings in the *NCII Mathematics Course*. Additionally, participants of the Asynchronous (BaU) asked fewer questions of me as the facilitator, which only two participants reaching out via email to ask about technical requirements of the PD. Participants of the Synchronous groups regularly asked questions during the sessions regarding conceptual understanding, access to resources, and sought out both my opinion as well as their peers' opinions on a variety of topics.

For the purpose of this study, all participants, regardless of whether they had completed all components of the *NCII Mathematics Course* were asked to complete the posttest in an effort to measure the effectiveness of summer PD structure. In theory, the success or failure of certain PD structures of each group would lead more or fewer participants to complete all components of the course, which would then impact their posttest scores. Reviewing data from the Canvas site where the activities were housed and where participants were asked to upload their completed activities reflects different patterns of completion across the groups. Of the 16 members of the Asynchronous (BaU) group, eight participants (50%) completed less than one full module of the *NCII Mathematics Course.* Of the 32 members of the Synchronous groups, eight participants (25%) completed less than one full module of the *NCII Mathematics Course.* Interestingly, of those eight Synchronous participants who completed less than a full module, four attended three or more meetings and participated in whole group and small group discussions throughout these meetings.

Together, the quantitative results, qualitative interview analyses, and the participation data from Canvas suggest that there are meaningful benefits in expecting participants to attend regular synchronous meetings. These findings align with the high-quality component of collective participation (Desimone, 2009). Although participants were not able to participate with their school or district colleagues, which is recommended to maximize participants *coherence* and utilize longer-standing support groups, the participants who felt that they had a peer group to answer to were more likely to stay engaged and motivated to complete the *NCII Mathematics Course*. For the purposes of this PD, in which participants were volunteering to commit a substantial amount of time and mental resources for the relatively minor incentives of CPE credit hours and $50, the peer pressure seemed to be the major force of accountability and motivation for many participants to continue to participate in the summer PD. As noted earlier, these forces alone (i.e., minor incentives and peer pressure) were not strong enough for many participants, leading to a high overall attrition. Peer pressure and the benefits of social accountability seemingly can only carry a PD so far.

FINDINGS RELATED TO RESEARCH QUESTION 3

The third research question addressed the differential effect of participating in a PLC on participant outcomes including MKT, self-efficacy, and instructional practices. I hypothesized that participants would benefit from participating in PLCs in which they would meet with the same group of peers consistently throughout the summer PD. To answer this research question, I aggregated the Asynchronous and the Synchronous/ Independent groups into one Non-PLC group and compared the growth of that Non-PLC group to the growth of the Synchronous/PLC group using an ANOVA test. The findings of this study partially support this hypothesis, but none of the quantitative results are significant, suggesting that the results are not conclusive and should be interpreted cautiously.

Across all measures, including measures of MKT (i.e., *MKT Elementary Rational Numbers – Content Knowledge* and *MKT Middle School Rational Numbers – Content Knowledge*), measures of self-efficacy (i.e., *TSES* and *Mathematics Teaching Self-Efficacy*) as well as measures of instructional practices (i.e., *TIP*) there was no significant differences between the growth of the Non-PLC group compared to the PLC group. Although not significant, the PLC group did improve to a slightly greater degree on all measures compared to the Non-PLC group. Similar to the results of the second research question, it is likely that the small sample size of each group contributed to the lack of significant differences between the groups.

The qualitative analyses of interview data reflected subtle differences between the Synchronous/PLC and certain participants of the Non-PLC group. As noted above, there were significant differences between the Asynchronous group and the Synchronous groups regarding the impact of weekly meetings as a source of motivation and accountability. That same stark comparison existed when the responses of the Synchronous/PLC group were compared to the Asynchronous group. Therefore, for the purpose of this research question, I chose to hone in on the differences between the Synchronous/PLC group and the Synchronous/Independent group to assess if there was any additional value in participating in the summer PD with the same small cohort. Perhaps the greatest difference between the Synchronous/PLC group and the Synchronous/Independent group was that Synchronous/PLC participants cited specific individuals from their cohort that they foresaw as a future resource, while participants in the Synchronous/Independent group expressed appreciation for their PD colleagues but did not foresee reaching out to them after the conclusion of the summer PD. For many participants in the Synchronous/PLC group, their peers had become a source of support, providing both the peer pressure to encourage accountability as well as a source of information and potential help at a later time.

It is important to note that the PLC groups in this study were structured to increase the camaraderie between participants but they lacked many of the important features of more traditional PLCs. Key features of effective PLCs should consist of a team of educators dedicated to using student-level data and peer support to be critically evaluative of their own practices with the ultimate goal of improving student outcomes (DuFour, 2007). Ideally, those teachers would share the same students to increase the *coherence* and *collective participation* of the PLC on teacher practice (Desimone, 2009). The *NCII Mathematics Course* is well structured to meet the needs of a PLC as many of the activities require application of the concepts taught in the videos and readings to participants' own students, but not all features could be utilized during the summer PD.

One critical shortcoming of the summer PD that many participants brought up in their interviews was the difficulty in completing the assignments that called for student data during the summer months. Participants from all three groups, commented on the difficulty of applying concepts without having current student data accessible. Even though all of the participants had at least one year of experience in the classroom, many expressed that it was difficult to remember specifics about students or create hypothetical data on students they might be expected to teach the following school year. Research regarding both PLC as well as general PD agree that teachers are highly motivated by student outcomes and it is more common to see lasting changes in teacher practice and beliefs when positive student outcomes support those changes (Popp & Goldman, 2016). When asked if they would have preferred to complete the *NCII Mathematics Course* during the school year, the same participants who noted the difficulty of completing the activities without student data also noted that time restrictions likely would have made it difficult if not impossible to complete the *NCII Mathematics Course* in a 9-week timeframe similar to the summer PD. These seeming contradictions in participant feedback reflect the challenges in creating PD that incorporate the best practices of PLCs (i.e., using student data, including school partners, providing feedback on authentic teaching) during the summer, the time that many participants felt was the only time they had the mental bandwidth and time to devote to their own improvement.

One interesting case study that organically arose due to the randomization of participants offers a snapshot into the challenges and potential benefits of trying to create a PLC over the summer. Four teachers from the same school signed up to complete the summer PD without realizing their peers had also committed to complete the PD. Due to the randomization of participants, one of the four teachers was assigned to the Asynchronous (BaU) group, one teacher was assigned to the Synchronous/Independent and two teachers were assigned to the Synchronous/PLC group. Once teachers were randomly assigned to the Synchronous/PLC group, I created small groups of participants that I felt shared commonalities (e.g., similar grade or setting). Therefore, I assigned the two teachers in the Synchronous/PLC group to the same small group PLC. Of the four participants, the teacher in the Asynchronous (BaU) group completed less than 1 full module and never completed the posttest. The three teachers in the Synchronous groups all completed the PD and improved on at least one measure.

The one teacher who completed the course in the Synchronous/Independent group also agreed to be interviewed, and she reflected that she felt the summer PD would have been more meaningful if she had been able to complete the course with her school colleagues (whom she found out after starting the course had also signed up for the PD). She shared that the teachers had created a folder just for their school-based peers and were communicating about ways to incorporate materials and concepts from the *NCII Mathematics Course* in the upcoming school year. Without prompting, these teachers created their own PLC, independent of the summer PD. Although it is difficult to say without follow-up observation data, it is likely that the teachers at this school, who created their own school-based PLC will have an easier time sustaining the practices that were introduced in the summer PD and *NCII Mathematics Course.*

FUTURE RESEARCH

The research questions of the study focused on the growth of the participants across different groups when provided with structures and incentives to complete the freely available *NCII Mathematics Course.* The research questions that I did not ask but which are also of interest are the ways in which participants interacted and participated in this course. There are opportunities to analyze the data collected for this study in a variety of ways that have the potential to answer questions about the type of participant for whom this PD was successful, the ways in which participants engaged with the *NCII*

Mathematics Course materials, as well as the types of conversations that participants had while in small group and whole group synchronous discussions. As the field of PD is so complex, there are so many opportunities to refine our thinking about the best methods of communicating concepts to participants and encouraging lasting, meaningful change.

Beyond the data collected in this study, a substantial line of research could develop the most effective means of developing high quality and impactful PD. Researchers could investigate ways of creating engaging and retaining participants for voluntary PD or investigate the effectiveness of mandatory PD compared to voluntary PD. As noted earlier, there was substantial attrition during this summer PD, but little enthusiasm for a PD scheduled during the school year. More research is needed to determine the best time to offer (or require) PD so that it is both applicable as well as not overwhelming, noting that teachers of various experience levels may require more or less support and/or time to process and incorporate knowledge and skills from PD. Using the freely available *NCII Mathematics Course* materials, one could present the same materials in a variety of different formats, including district-mandated, voluntary, PLC-driven, over the course of few or several months, with or without a coach, and so on. Ultimately, once a method of delivering PD to achieve meaningful teacher-outcomes has been achieved, researchers should then refine PD to ensure that meaningful student outcomes are also achieved and sustained.

Implications for Research

As an initial study, this study showed some promise for the potential benefits of the *NCII Mathematics Course* as the basis for a meaningful and impactful PD. However, the limitations of the study also reflect larger issues in general with the field of PD. For example, a considerable challenge to this PD was the high attrition across groups, and the low completion rates of the Asynchronous (BaU) group. Researchers should consider ways to encourage long-term buy-in for PD, especially for participants who may enter the PD with lower levels of mathematical content knowledge and who require more support. Examples may include pairing participants to form asynchronous support groups, ensuring that PD providers regularly reach out to participants to clarify questions or confusions, and ensuring that PD providers are regularly providing feedback to participants so participants do not feel isolated. Additionally, researchers should consider the need to develop a greater range of measures to assess teacher knowledge and skills gained in PD sessions, especially PD that targets special education teachers and supporting students with MD. Between the quantitative results and the qualitative feedback from the participants, researchers should consider the different experiences of the Asynchronous and Synchronous participants. Although not quantitatively significant, the results of this study do point to the difficulty of completing PD sessions completely asynchronously. Researchers who are therefore looking to conduct completely asynchronous PD or trainings should be cautious.

Implications for Practice

This is an initial study examining the effectiveness of the *NCII Mathematics Course* across a variety of virtual PD settings with a small sample size and a lack of significant results; therefore, findings should be interpreted cautiously. PD providers should be cautious about providing completely asynchronous PD opportunities that require substantial time commitments and do not have opportunities for feedback or connections to PD providers or other participants. Instead PD providers should consider ways in which they can build in opportunities for participants to connect with others taking the same PD, preferably with colleagues at the same school or district. Alternatively, PD providers should consider building in motivational incentives for completing substantial PD if not required by participants' schools or districts.

Based on the findings from this study and research on high quality PD (Desimone, 2009), PD providers should provide PD opportunities that include *active learning* opportunities including activities that utilize student data and give participants ways to practice and receive feedback on their knowledge and skills they learn through the PD. Additionally, based on the feedback from participants, PD providers should consider the ways they are supporting teachers' autonomy both during and after the conclusion of the PD. To support teachers' autonomy during the PD, providers should consider allowing for flexibility when possible and should provide participants with prior access to resources so they can access the materials repeatedly if necessary. Additionally, PD providers should consider making resources and support staff available for participants to access at a later date to encourage long term implementation and change in practice.

When possible, teachers and practitioners should complete PD with colleagues at their school or institution, preferably when they have access to student data. If it is not possible or feasible to complete PD with a work colleague, PD participants should reach out and establish connections with other participants in the PD, with whom they can work together to hold each other accountable for completing the work. The results of the study suggest that it is important for participants to feel some sort of connection, either with the PD provider or their peers to continue to stay engaged with long-term PD.

LIMITATIONS

The findings of this study are limited in several ways. First and foremost, I worked as the deliverer of the summer PD, the interviewer of participants, and the researcher analyzing both quantitative and qualitative results. For the qualitative results especially, it is important to note that the interviews and data were not blind, that participants answering questions about the effectiveness of the summer PD were talking to me, the person who delivered their PD, which likely influenced their answers. Because the *NCII Mathematics Course* was developed externally, there is a level of separation between me as a researcher and evaluator of the program and the program itself, but likely the elements of the summer PD that I created to present the *NCII Mathematics Course* impacted participants' experience with the course. Therefore, more research using the same *NCII Mathematics Course* or the structures that I built to deliver the summer PD would bolster the results of this study.

Additionally, generalizability of the findings of this study should be interpreted with caution for several reasons. The current study reports the findings from an underpowered sample size and the majority of the results were not significant. The initial sample size was small, but considerable attrition across all three groups resulted in a final sample size that was underpowered and impacted the results of this study. In addition, according to Canvas data, a significant number of participants did not complete all activities required as part of the *NCII Mathematics Course*. Together, the significant attrition and lower completion rates (or low dosage) likely mean that enough participants did not receive the full dosage they needed to see significant results across groups. The results of this study may have been different if the sample size was larger and participants completed all of the required components of the PD. Future studies with larger sample sizes would may be able to utilize more sophisticated statistical analysis that allows for a more comprehensive impact of the PD; in the same way, alternative forms of qualitative analysis have the potential to reveal a more robust understanding of the way that participants interacted with and within the PD structures. Future research should also consider ways to encourage all participants to complete all components of the *NCII Mathematics Course* and reduce attrition.

Although a popular measure used within the field of mathematics PD, the MKT assessments may not have been an appropriate measure for this PD. First, the measure is recommended to be used with large sample sizes, preferably samples of 60 or more participants, and this study had significantly fewer participants. Additionally, in the *NCII Mathematics Course,* only two of the eight modules directly focus on teaching mathematics. Therefore, the likelihood of seeing effect sizes was unlikely. A better measure would have more closely measured teachers' ability to implement strategies that target students with MD. Unfortunately, there currently is not a validated measure that captures specifically teachers' ability to implement strategies designed to improve mathematics outcomes for students with MD. The creation of a specialized measure would be a great asset to the field and may have served as a better measure for this study. In spite of these limitations, the results from this study suggest there is promise in further investigating the most effective methods of delivering PD to improve participants' knowledge of mathematics teaching and DBI implementation.

SUMMARY

This study examined the effects of an intensive summer PD utilizing the *NCII Mathematics Course* on the MKT, self-efficacy, and instructional practices of in-service teachers and officials who support students with MD. A series of ANOVA tests yielded largely positive but not significant results suggested the summer PD improved participant outcomes. Additional qualitative analysis in the form of thematic analysis of semi-structured interviews suggested that participants found the summer PD provided them with accountability, resources, and confidence. There is some evidence to suggest that participants who completed the summer PD in the Synchronous/PLC group experienced the greatest improvement and found the PD to be the most meaningful, compared to their peers in other groups. Future research might further investigate the most effective method and structures needed to improve participant outcomes, especially MKT.

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* Signifies study is included in synthesis.

**Signifies study data is a duplicate of other study data included in synthesis.