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Development and Application of Capacitance-Resistive Models to Water/CO₂ Floods

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Development and Application of Capacitance-Resistive Models to Water/CO₂ Floods

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Dedication

To God, the Lord, for all the blessing received.

To Whom It May Concern!

To the memory of my cousin Nasrolah, his wife Sara and their daughter Neda.

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Quick evaluation of reservoir performance is a main concern in decision making. Time-consuming input data preparation and computing, along with data uncertainty tend to inhibit the use of numerical reservoir simulators. New analytical solutions are developed for capacitance-resistive models (CRMs) as fast predictive techniques, and their application in history-matching, optimization, and evaluating reservoir uncertainty for water/CO₂ floods are demonstrated. Because the CRM circumvents reservoir geologic modeling and saturation-matching issues, and only uses injection/production rate and bottomhole pressure data, it lends itself to rapid and frequent reservoir performance evaluation.

This study presents analytical solutions for the continuity equation using superposition in time and space for three different reservoir-control volumes: 1) entire field volume, 2) volume drained by each producer, and 3) drainage volume between an injector/producer pair. These analytical solutions allow rapid estimation of the CRM unknown parameters: the interwell connectivity and production response time constant. The calibrated model is then combined with oil fractional-flow models for water/CO₂

floods to match the oil production history. Thereafter, the CRM is used for prediction, optimization, flood performance evaluation, and reservoir uncertainty quantification. Reservoir uncertainty quantification is directly obtained from several equiprobable history-matched solutions (EPHMS) of the CRM. We validated CRM's capabilities with numerical flow-simulation results and tested its applicability in several field case studies involving water/ CO_2 floods.

Development and application of fast, simple and yet powerful analytic tools, like CRMs that only rely on injection and production data, enable rapid reservoir performance evaluation with an acceptable accuracy. Field engineers can quickly obtain significant insights about flood efficiency by estimating interwell connectivities and use the CRM to manage and optimize real time reservoir performance. Frequent usage of the CRM enables evaluation of numerous sets of the EPHMS and consequently quantification of reservoir uncertainty. The EPHMS sets provide good sampling domains and reasonable guidelines for selecting appropriate input data for full-field numerical modeling by evaluating the range and proper combination of uncertain reservoir parameters. Significant engineering and computing time can be saved by limiting numerical simulation input data to the EPHMS sets obtained from the CRMs.

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Chapter 1: INTRODUCTION

Evaluating reservoir characteristics and predicting future production efficiently and economically have always been among the most challenging tasks for petroleum engineers. The engineer must choose the method for predicting performance, a predictive model (PM), while considering the availability of time and resources, the reliability, quality and uncertainty of the basic data available, and the ultimate application of his/her prediction.

All the predictive models available to reservoir engineers range in complexity from those that provide an estimate only of ultimate recovery to those that use comprehensive reservoir simulation models that are capable of predicting both reservoir and individual well performances. However, the time requirements and costs for simulation are directly proportional to the complexity of the technique used. In some instances, a simple estimate of ultimate recovery may be sufficient and, in fact, it may be the only reliable estimate possible because of data limitations. However, in most cases, more detailed projections will be required to evaluate the economic potential of the proposed project, and a method must be used that will allow the estimation of future production rate and oil recovery as functions of time.

1.1- PROBLEM STATEMENT

Comprehensive numerical reservoir simulation is a regular practice in all major reservoir management development decisions. Although numerical simulations are widely used, there are still major limitations and restrictions in their application especially for decision making under uncertainty. Limited time and available resources, inevitable uncertainty of input data and time-consuming preparation, computing and analyzing of the results for all different possible scenarios, even with today's fast computers, still restrict use of full-scale comprehensive reservoir simulation.

Numerical reservoir simulators need a large amount of reservoir data, engineering time and operating costs; thus, in many situations running full-scale numerical simulations does not meet the economical requirement and the time limit restrictions of the project. On the other hand, because of the intrinsic uncertainty of input data and non-uniqueness of parameters combination, different scenarios that satisfy the reservoir model (production history) should be considered to confidently and accurately evaluate the impacts of uncertainty in the input data on the objective function. To make an ideal decision in ideal petroleum engineering uncertainty analysis, numerous realizations of reservoir properties and consequently numerous numerical reservoir simulation runs are required to evaluate the stochastic nature of the outcomes; nevertheless, in real practice, this objective can rarely be fulfilled. Therefore, in real petroleum engineering uncertainty analysis the optimal number of realizations of reservoir properties and consequently the optimal number of reservoir simulation runs should be selected to capture the stochastic nature of outcome reasonably.

Achieving high accuracy for a reservoir model is generally in a direct relation to the amount of time, data, and resources that are available in the project. The more the amount of the resources and time, the higher will be the accuracy. For example, in an ideal numerical reservoir simulation the relationship between the level of accuracy and effort, and the number of grid blocks can be schematically presented as in Fig. 1-1.



Figure 1-1: Hypothetical curves for accuracy, effort and accuracy to effort ratio as objective function in a simple homogeneous reservoir as a function of number of grid blocks.

In this example, we define effort as the required simulation cost and accuracy as a measure to compare the similarity of simulation prediction and the observation, and the objective function as the accuracy divided by the effort. Maximum value for objective function shows the optimal approach that is the optimal number of the grid blocks. It is possible that either the optimum point does not fulfill the desired accuracy of the project, or time and resources limitation of the project prevent us from reaching the optimum number of numerical simulation runs.

The desired accuracy of modeling can be increased if, before performing the numerical simulations, classical or fundamental reservoir engineering methods such as tank models are used. Classical reservoir engineering techniques can assist in obtaining a good estimate and understanding of different scenarios and reasonable ranges of reservoir parameters that satisfy past, present and future reservoir performance. Unfortunately, numerical reservoir simulations have often been used without conducting such simple assessments of simple predictive models that can provide significant information in a short period of time with minimum effort. Information gained from predictive methods can decrease the range of input uncertainty and guide comprehensive numerical simulations. Therefore, a combination of preliminary-analytical tools as simple simulators and full-scale numerical simulators can optimize the number of runs and simulating time in a project.

1.2- RESEARCH OBJECTIVES

The capability of evaluating reservoir performance accurately in a short period of time is the main concern in decision making. Among known traditional reservoir engineering methods, predictive models normally use material or energy balance equations on a simple geometry to evaluate reservoir performance and characteristics. Due to their simple approach, these models are very fast and inexpensive. Thus, with a minimum of reservoir data, e. g. only injection and production data, and a small investment in computing and engineering time, it will be possible to have a preliminary evaluation, prediction of reservoir characteristics and future production before running numerical simulators. In this study, based on continuity equation we develop capacitance resistive models (CRM) as our predictive models to rapidly history-match and estimate future reservoir performance.

Even in PMs, considering all scenarios for the input data might not be feasible. Stochastic response surface method (SRSM) can be used to reduce the number of simulations in the PMs. Therefore, a combination of improved predictive models, sampling techniques and numerical simulators can be used to optimize the accuracy that can be achieved with available reservoir data, computing, engineering time and operating costs. On the other hand, the accuracy of a reservoir model is related to the simplified assumptions that are made for reservoir properties. For example, considering an incompressible reservoir is a common assumption to simplify the PMs; hence, the accuracy can be improved if compressibility is considered in predictive models.

In this work, we develop and conduct quick, cost-effective and reliable predictive techniques and introduce an algorithm to narrow down the wide range of uncertainty of major reservoir properties while losing only a small fraction of the stochastic outcome accuracy. The CRM is a predictive technique that is developed and used in this work. Since the CRM only requires injection-production rate data, it can be used widely in many field application.

Insights gained from performing CRM are used to evaluate reservoir operating conditions and flood efficiency during production history. Then the CRMs are used to predict and optimize future reservoir performance. Furthermore, the CRMs' results are used to narrow down the range of some of the parameters that are involved in predictive model from a wide to a narrow discontinuous range. This is achieved by performing many runs of the predictive model that provide the best production history-match. Therefore, we can limit and minimize the number of full-scale numerical runs by restricting selection of reservoir properties for comprehensive reservoir simulation to those that satisfy production history in CRMs. Consequently, we shorten the overall number of comprehensive numerical simulation, simulating time and analysis by mixing comprehensive numerical simulation runs with the CRM.

The CRMs are capable of matching production history for an entire field, a group of wells or individual well, and can predict and optimize a field, a group of wells, or individual well production. We match production history by the CRM and back calculate some of major reservoir parameters, such as original oil in place, N, original water in place, W and residual saturations, within an acceptable range for error calculation. By history-matching, we find several likely sets of reservoir parameters that are equiprobable history-matched solutions (EPHMS). The none-uniqueness of possible matches generate discrete sets of internally linked reservoir parameters. By internally linked we mean that for a given value of any back calculated parameter such as water in place ,W, other parameters such as original oil in place, N, and residual saturations can not freely change and are known. These vectors or sets of reservoir properties form a sampling domain that not only satisfy the production history but also provide probability distribution functions for major reservoir properties.

The internally linked sets of reservoir parameters create a sampling domain, which relates certain values of parameters together and does not allow random selection of all the variables. This biased sampling strategy is superior to any random sampling technique by inhibiting sampling of unreasonable combination of reservoir parameters. We used and developed predictive techniques prior to or during full field numerical simulations to reduce the overall simulation, analysis and project time. We maintain or even improve the accuracy in the prediction by screening and confining common input between predictive models and numerical simulations.

In general, the following steps are taken in this work to evaluate reservoir performance and confine reservoir uncertainty in a short time accurately:

1. Develop a set of quick, cost-effective and reliable predictive models (PM), the capacitance resistive models (CRM). CRMs use the minimum amount of reservoir

data, injection and production data, to predict and optimize reservoir performance for two major hydrocarbon recovery processes of water and CO₂ floods.

2. Optimize the number of runs of numerical reservoir simulators. This is achieved by generating probability distribution functions for the main reservoir parameters, such as residual saturations and initial water and hydrocarbons in place, and mobility ratio through predictive models. Then we use these to narrow down the range of parameters and their reasonable and possible combination as inputs for numerical reservoir simulators.

Chapter two presents a literature survey and our approach in confining reservoir uncertainty by the CRM. First, some of common techniques for efficiently handling reservoir uncertainty in numerical reservoir simulations are presented. Then, existing predictive techniques for different flooding agents such as water, polymer, carbon dioxide (CO₂), and chemical are reviewed and applications of predictive models as fast simulators in enhancing reservoir uncertainty management are discussed. Lastly, an effective guided sampling technique based on application of developed CRMs is introduced.

In chapter three, we present a background of the CRMs and then the details and developments of CRM's analytical solutions by superposition in time and space. Analytical solutions by superposition in time are developed based on two different projections of injection rate: 1) step wise and 2) linear injection rate variation, for three different reservoir control volumes: 1) entire filed, 2) a single producer and 3) control volume between each injector-producer pair. Furthermore, by considering series of tanks

between each injector-producer pair CRMs are modified and CRM-Block analytical solutions solution based on superposition in space are presented.

Since CRMs predict total liquid production we must have an estimate of oil fractional-flow as a function of time. Therefore, chapter four reviews and presents some of the oil fractional-flow models for immiscible waterflood and miscible CO_2 floods. Based on these fractional-flow models we will be able to estimate oil production for a producer, a group of producers or the entire field. Oil fractional-flow models for waterflood are either based on Buckley and Leverett fractional-flow theory or empirical models. For a CO_2 flood, we have developed oil fractional-flow models based on logistic equation, which can be applied to any tertiary recovery.

Chapter five demonstrates CRMs and oil fractional-flow models validation and application in history-matching, prediction and optimization of reservoir performance. Numerical simulation of synthetic case studies and field case studies are used to validate CRMs and demonstrate their application in history-matching and relating CRM model parameters to the reservoir characteristics and optimization. Optimization is performed by reallocating the injected water to maximize oil production rate.

In chapter six the CRM applications for field cases are presented. The CRMs have been used to match the production history for several field cases such as: 1) Reinecke, 2) Malongo 3) South Wasson Clear Fork, (SWCF), 4) UP-Ford 5) Rosneft 6) Lobito, 7) Seminole 8) McElroy. Four of these case studies are presented in chapter six: Reinecke, SWCF, UP-Ford and McElroy. Except for McElroy, which is a CO₂ pilot flood, the other case studies are waterfloods. Chapter seven demonstrates the use of the CRM in evaluating and confining reservoir uncertainty to both synthetic and field examples by creating an internally linked sampling domain for uncertain reservoir parameters. The CRMs as fast, simple and yet powerful and independent predictive techniques are repetitively used to match the production history. As a result, numerous sets of independent equiprobable historymatched solutions (EPHMS) are obtained by the CRM. These EPHMS provide probability distribution functions for major reservoir properties, such as the original oil and water in place, and residual oil and water saturations. Internally linked sets of reservoir parameters create a good sampling domain in which groups of uncertain reservoir parameters are selected dependently. Significant engineering and computing time can be saved by limiting numerical simulation input data to the EPHMS sets.

Lastly, chapter eight summarizes, concludes and presents recommendations for future work.

Chapter 2: LITERATURE SURVEY

Uncertainty and sensitivity analysis are commonly performed by using statistical sampling and stochastic modeling approaches to investigate the impact of uncertain reservoir parameters on reservoir model prediction. Ideally, numerous sets of equally viable samples of different uncertain parameters are selected, and then reservoir flow simulation is performed for these equally probable scenarios to capture the uncertainty of an objective function such as oil recovery or cumulative oil production. In reality, time and resources are limited to evaluate reservoir performance for all possible scenarios. Therefore, reservoir engineers have used different techniques, such as improved sampling techniques and rapid reservoir model estimators, that we call predictive models (PM), to avoid intensive computational effort in capturing the uncertainty range of their reservoir performance prediction.

In this chapter, first, some of common techniques for efficiently handling reservoir uncertainty in numerical reservoir simulations are presented. Then, existing predictive techniques for different flooding agents such as water, polymer, carbon dioxide, and chemical are reviewed. Next, applications of predictive models as fast simulators in enhancing reservoir uncertainty management are discussed. Finally, an effective guided sampling technique based on application of a developed predictive technique, the capacitance-resistive model (CRM), is introduced. In our approach, we first find equally probable sets of CRM parameters that satisfy the production history and use these equally viable sets of reservoir parameters to quantify the uncertain range of reservoir parameters and desired objective function uncertainty. This approach limits

selection of uncertain reservoir parameters to those values gained from the CRM equiprobable history-matched solutions (EPHMS).

2.1- MANAGING RESERVOIR UNCERTAINTY

In general, we can categorize techniques that manage reservoir uncertainty efficiently into two major categories: 1) techniques that focus either on improving sampling methodology to take representative samples, or on reducing the number of samples to minimize the number of submitted runs for reservoir simulation, and 2) techniques to speed up the reservoir simulation models. In this work, both of these techniques are implemented. We use the CRM to evaluate reservoir performance quickly and, based on numerous sets of the EPHMS obtained from CRM, we use effective sampling techniques to efficiently mange reservoir uncertainty.

2.1.1- Uncertainty Management by Sampling Technique

Techniques focused on sampling methods of uncertain variables are mainly based on reducing the number of samples or using a method to select representative samples that can still map the impact of data uncertainty on the reservoir model prediction. In this category, fall Monte Carlo (MC) and Latin Hypercube (LHC) sampling techniques as direct method to map the uncertainty of input data to output results. Sensitivity analysis based on the applications of response surface and experimental design as well as the use of scaling groups are other techniques in this category that indirectly reduce the effort to quantify uncertainty in prediction. Normally, a reservoir simulation model deals with several uncertain variables simultaneously. If we have N_{UV} uncertain variables and within each uncertain variable, N_{PS} possible selections exist, the total number of equally probable scenarios, N_{EPS} , is

$$N_{EPS} = \prod_{i=1}^{N_{UV}} N_{PS,i}$$
(2-1)

As a simple example, for only five uncertain variables each of which can freely have 10 values, 10⁵ statistically probable scenarios exist. Unless we are dealing with a very simple calculation or rapid estimation, it is impossible to quantify the sensitivity of the output completely and accurately.

Traditionally, MC sampling has been used extensively in the reservoir engineering literature to capture the impact of uncertain parameters on simple objective functions (Evers and Jennings, 1973; James, 1997; Liu and Oliver, 2005). Since samples are independently selected in the MC method, it is possible to reselect a selected sample, or values that are very close to previously selected ones. For a fixed number of samples, the LHC sampling method covers the range of each uncertain variable better than MC sampling and we avoid taking multiple samples. Thus the LHC sampling provides more representative samples of an uncertain variable compared to the MC sampling. Figure 2-1 shows two realization of 10 and 50 randomly selected samples of normalized reservoir area and thickness based on the MC and LHC sampling methods. As shown in these figures LHC samples are distributed better and cover a wider range of values for some of the uncertain variables, while some of the MC samples for both realizations are identical and do not add much variation in prediction of reservoir volume.



0.3

0.2

0.1

0

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Area

٠

1

1

Figure 2-1: Comparison between spatial distribution of 10 and 50 randomly selected samples of normalized area and pay thickness between MC and LHC sampling techniques.

1

0.3

0.2

0.1

0

0

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Area

Experimental design (ED) and response surface (RS) methods are other techniques that significantly enhance the quantification of output uncertainty and improve the optimization for sophisticated reservoir models. In general, in ED and RS based on a few values of uncertain model parameters, especially the extreme values, a polynomial

correlation between input data and reservoir model output is generated. Then, a timeconsuming reservoir model is replaced by the polynomial response surface. This polynomial response surface is used as a fast evaluator of reservoir model response to the variation of uncertain variables. An optimized design can easily be identified by locating the maximum or the minimum of the response surface.

Experimental design has been used in many reservoir engineering analysis from estimation of reserves (Cheong and Gupta, 2004) to performance prediction (Chu, 1990; Aanonsen, 1995), and uncertainty modeling and sensitivity analysis (Peake et al., 2005). Normally in experimental design, the values of an uncertain variable are limited to its base or lower/upper bound values. However, the number of samples within an uncertain variable and the number of uncertain variables, N_{uv} might be small but the number of equally probable combinations of uncertain variables can quickly become very large. For example, for two- and three-level full factorial design there are 2^{Nuv} and 3^{Nuv} equally probable combinations of uncertain variables to be considered, respectively. In general, the number of equally probable scenarios and consequently the quality of response surface varies based on the type of design (Yeten et al., 2005). For further reduction of the number of samples and consequently number of reservoir simulation runs, full factorial design are normally replaced by fractional factorial design such as Plackett-Burman design, central composite design, D-optimal design and space filling design, Yeten et al., (2005).

Besides techniques that focus on reducing the number of samples, application of dimensionless groups directly decreases the number of variables involved; therefore, instead of perturbing uncertain variables independently one can focus on dimensionless group variations. Chewaroungroaj et al. (2000) applied a combination of scaling groups experimental design and response surface to reduce the effort in estimating uncertainty in

hydrocarbon recovery. Different scaling groups might depend on and share one or more uncertain variables, consequently selecting a value for one of the scaling groups can limit the range of the other scaling groups. Therefore, selecting proper design that allows flexible selection within the range of scaling group is a key to combining scaling groups and experimental design. In general, reducing the number of uncertain variables by grouping uncertain variables into dimensionless groups and using experimental design reduces the number of required simulation runs significantly.

2.1.2- Reservoir Model Speed up Techniques

Rapid prediction of future recovery and reservoir performance under uncertainty provides the basis for efficient economic evaluation of the profitability of a proposed project. In these techniques, application of a fast reservoir model is the focal point of a rapid projection of input uncertainty to objective function for different scenarios. The most common approaches are either to use a coarse grid numerical simulation of the reservoir model as a fast simulation (Ballin, 1992, 1993) or to use predictive models to enhance uncertainty management (Guevara, 1997). Application of any rapid reservoir simulation model such as streamline simulation or material balance models can enhance uncertainty management and decision making. Fast simulations, referred to as predictive models (PM), can be performed in combination with comprehensive numerical simulations. Predictive models can capture the impact of some of the uncertain variables quickly and therefore reduce the time and effort needed to investigate the impact of uncertain variables on hydrocarbon recovery.

Many methods such as empirical correlations, screening guides, predictive models, and numerical simulations are used for estimating the future performance of a reservoir. These methods are important for flooding procedure decision making, especially at the end of primary recovery. During flooding, a flooding agent such as water or CO_2 is injected into a reservoir to obtain additional hydrocarbons recovery after the reservoir has approached its economically productive limit by primary recovery. Among these methods, predictive models with a minimum of reservoir data and small investment in computing and engineering time are appropriate for preliminary economic analysis and screening especially after primary recovery.

2.2- PREDICTIVE MODELS

Predictive models consider most of the phenomena that affect flood performance such as cross flow, heterogeneity, aerial sweep and injectivity. These are their advantages over screening guides, empirical correlations and analytical methods. The advantages over numerical simulators are that they remain simple, fast and inexpensive. Simplifying procedures are commonly used in PM's to keep these models fast and capable for analyzing three dimensional problems. For instance, by considering vertical equilibrium (VE) in PMs one can generate pseudo functions for relative permeability that reflects vertical heterogeneity in reservoirs and simplifies the flow equation by reducing the vertical dimension.

Based on different types of flooding agent, PMs can be categorized as water, miscible and immiscible gas, and chemical flooding predictive models. A series of PMs were developed for U.S. Department of Energy and National Petroleum Council (NPC) in 1984 for investigating the potential for enhanced oil recovery: PM for polymer flooding (Jones, 1984), PM for micellar-polymer flooding (Paul et al., 1982), PM for CO₂ miscible flooding (Paul et al., 1984), PM for srteamdrive performance (Aydelotte and Pope, 1983), and in-situ Combustion PM (Ray and Munoz, 1986).

2.2.1- Water Flooding Predictive Models

Water flood prediction methods can be categorized into empirical correlations, simple analytical methods, and numerical models. There are several fundamental techniques for calculating the performance of a waterflood. The Buckley and Leverett (1942) frontal advance theory and a subsequent extension of it by Welge (1952) are simple methods for calculating fractional-flow and recovery performance after water breakthrough in a linear reservoir segment with homogenous properties. An example of the Welge technique was presented by Craig (1993). Stiles (1949) and Dykstra-Parsons (1950) developed simple methods for application in stratified homogenous reservoirs. The Stiles method is based on the assumption of a piston-like displacement in a linear bed with a specific permeability, and that the rate of advance of flood front is proportional to the permeability of the bed. The Dykstra-Parsons' method includes changing fluid mobilities rather than an assumption of equal mobility for displacing and displaced fluids.

Craig described and compared the classic prediction methods that were presented before 1971, and included recommendations for selecting the appropriate waterflood prediction technique to obtain the desired results. The methods that were considered were categorized into five groups, which may be summarized as follows: (1) reservoir heterogeneity, (2) areal sweep methods, (3) displacement mechanism, (4) numerical methods, and (5) empirical approaches (Thomas et al., 1992). Craig compared the capabilities of each method to the capabilities of the "perfect method," in which the calculation procedures would allow consideration of all relevant fluid-flow, well-pattern, and heterogeneity effects. In practice, portions of reservoir are never contacted by injected water, therefore, a truly linear displacement is never used in water flood operation and it is necessary to consider the areal sweep efficiency to make better estimates of reservoir performance. Muskat (1949) presents a comprehensive review of his early discussions of the steady-state flow capacity of various pattern that should be considered in predicting flooding performance.

Waterflooding has been recognized as an accepted operation for increasing oil recovery since 1950. The fields do not always perform as predicted regardless of the method that is used to estimate future performance. This is true for many reasons, including (1) an incorrect or inadequate description of the reservoir rock, fluid, and water/oil flow properties, (2) a prediction technique that does not have the capability to consider all the factors that affect waterflood performance, and (3) the fact that there is always a question of the reliability of the estimates of the interwell character of the reservoir rock and the vertical and horizontal variations that exist in reservoir rock and fluid properties.

2.2.2- Chemical Flooding (Polymer Flooding)

In general, chemical flooding refers to isothermal EOR process to recover oil by reducing the mobility of the displacing agent and/or lowering the oil/water interfacial tension (Lake, 1989). The displacement by water soluble polymers is referred to as polymer flooding. In polymer flooding, a small amount of polymer is added to thicken brine. The volumetric and displacement efficiency increases with the reduction of the mobility ratio, which results in additional oil recovery. The classical solution by Buckley and Leverett (1942) can be used to describe the effect of mobility lowering on displacement efficiency.

Jones (1984) in his stratified water/polymer PM, considers vertical heterogeneity through the vertical equilibrium theory, and areal sweep through a well pattern. In his model for polymer flooding, he includes permeability reduction, adsorption, viscous
fingering of drive water into polymer slug, and viscosity, all as a function of polymer concentration. The resulting output consists of cumulative produced volumes and producing rates as a function of time for oil water and gas. Jones' predictive technique is based on incompressible oil-water flow in a stratified model for only a five-spot pattern. Due to the VE assumption, his model does not have the ability to account for noncommunicating layers, but it includes initial gas saturation and different layers of permeability by using Dykstra-Parsons permeability variation.

2.2.3- Miscible and Immiscible Gas Flooding (CO₂)

In 1888, the importance of gas injection in increasing the oil recovery was recognized by Dinsmoor (Smith, 1975). Generally, gas is a compatible fluid with both reservoir rock and reservoir oil, and normally up to 30% additional recovery is obtained after gas displacement. Based on the displacement mechanism the gas displacement will be miscible or immiscible. Normally the efficiency of miscible displacement is higher than that of immiscible displacement. The reservoir engineer has to recognize which method is most suitable for the reservoir type, the fluid properties, the quantity and the quality of the displacing agent and economic environment. The greatest disadvantage of every gas miscibility method is the low viscosity of the displacing agent that can result in fingering and early breakthrough. Therefore, same form of mobility control is a basic requirement.

There are several calculation methods, which deal with miscible gas flooding for oil recovery, depending on 1) whether the displacement is a vertical or horizontal, 2) whether the solvent injection takes place continuously or in the form of a slug, 3) whether the realization of the EOR technology process is a secondary or tertiary process. These displacement calculations can be categorized as empirical/analytical methods, streamtube and streamline modeling, and numerical modeling. Horizontal and vertical displacements are two major categories of empirical/analytical methods. The CO_2 flooding characteristic applications were summarized by Brock and Brayan (1989) depending on whether the process is miscible or immiscible, the whole reservoir or only a part of it is flooded and the same well is used for injection and production.

2.3- ENHANCING UNCERTAINTY MANAGEMENT BY PREDICTIVE MODELS

Matching the production history is one of the major tasks of reservoir engineer must perform prior to any prediction of reservoir performance. Uncertainty in reservoir data makes history-matching a tedious task and the non unique nature of the match creates uncertainty and mismatch in the future predictions. Based on different equally viable matches of production history, the uncertainty in future performance projection can be captured. As mentioned before, running numerical reservoir simulations for a large number of realizations of uncertain variables is expensive and time consuming. Many predictive models have been developed for predicting reservoir performance. Combining predictive models with comprehensive reservoir simulators has the potential to optimize the prediction process efficiently and economically.

Ballin et al. (1993) treated a coarse grid representation of their reservoir model as fast simulator (FS) or predictive model. Next, they used MC sensitivity analysis of the FS to screen the input data for a field scale fine grid representation, comprehensive simulation (CS). Then, correlated performance of coarse and fine grid simulations were used to obtain the performance projections of all the realizations.

Guevara (1997) presents a screening technique to reduce the many sets of stochastic parameters to find the approximate net present value (NPV) cumulative distributions function. In his study, he used Johns' polymer flood predictive model, UTSTREAM, and Vertical Implicit Program (VIP) simulator. Guevara categorized reservoir parameters into two major groups: primary and secondary variables. Primary variables are those that exist in both the PM and numerical simulation and their impact on objective function can be captured by either PM or numerical simulation. Secondary variables are other uncertain variables whose impact on an objective function can only be quantified by numerical simulation. A combination of full MC sensitivity analysis of primary variables by fast PM and secondary variables by comprehensive numerical simulation was used by Guevara to capture the impact of uncertain reservoir parameters on net present value estimation.

We have developed the CRM as our PM and used it to match production history. Based on the matches obtained by the CRMs, one can generate probability distribution functions (PDF) or cumulative distribution functions (CDF) of major reservoir parameters such as water/oil in place or water/oil residual saturation. Since these equiprobable history-matched solutions (EPHMS) are related, randomly selected sets of these solutions should be used to define numerical simulation input data. We use numerous sets of EPHMS obtained from the CRM to determine reservoir uncertainty and narrow down the continuous range of uncertain parameters to discrete sampling domains.

In the following chapters, details of the CRM developments and its application in optimizing reservoir performance are presented. In addition, we demonstrate the use of the CRM in evaluating and confining reservoir uncertainty to both synthetic and field examples by creating an internally linked sampling domain for uncertain reservoir parameters.

Chapter 3: CAPACITANCE RESISTIVE MODEL (CRM)

In general, the capacitance resistive model (CRM) relies upon signal-processing techniques in which injection rates are treated as input signals and total production rates are the reservoir response or output signals. The name CRM is selected for this model because of its analogy to a resistor-capacitor (RC) circuit (Thompson, 2006). A production rate response to a step-change in injection rate is analogous to voltage measurement of a capacitor in a parallel RC circuit where the battery potential is equivalent to the injection signal.

The interwell connectivity and response delay constitute the CRM unknown parameters. Therefore, for a multiwell system, CRM's parameters represent the connectivity between each injector-producer pair based on the historical injection and production data.



Figure 3-1: Schematic representation of the impact of an injection rate signal on total production response for an arbitrary reservoir control volume in capacitance resistive model.

The CRM is developed as a spreadsheet-based predictive model. This enhances CRM's application for a reservoir engineer in his/her real-time field performance analysis and optimization. The main motivation behind developing the CRM is its speed and capability to match production history and predict production rate based on injection and production rate and, if available, bottomhole pressure (BHP) data.

In this chapter, we first present a background of the CRM and then the details of the CRMs development and analytical solutions for three different reservoir control volumes: 1) the entire field, 2) a single producer and 3) the volume between an injectorproducer pair. Analytical solutions for each CRM are developed based on superposition in time and space. Compared to the previous numerical solution of the CRM developed by Yousef et al. (2006) and Liang et al. (2007), the analytical solutions developed in this work enhance the CRMs setup and application especially for large field studies.

Since the CRMs only provide an estimation of the total production rates, in chapter four we present several oil fractional-flow models for secondary and tertiary recovery to combine with the CRMs total production estimation to evaluate oil production rate as a function of time.

3.1- CRM BACKGROUND AND DEVELOPMENT

The CRM's parameters, connectivities and time constants are evaluated based on injection and production history. Once the model parameters are estimated, performance predictions can be made with the fitted model parameters. In this regard, CRM may also be viewed as a nonlinear multivariate regression analysis tool which accounts for compressibility and fluid flow in the reservoir based on time constant (Yousef et al. 2006). Unlike a grid-based numerical-simulation approach, the CRM models the

reservoir flow behavior in accord with interactions (connectivities) between well-pairs, which is a measure of the reservoir permeability.

In some respects, the CRM may be construed as analogous to a streamline approach. The connectivity between each injector-producer pair is analogous to the relative number of streamlines of an injector that support a producer. Streamline simulations (Gupta, 2007) have gained considerable popularity because of their fast computational speed. We think the CRM can further speed up the overall study time by providing clues about the integrity of rate data, injector-producer connectivity, and fluid influx.

From injection/production data, Albertoni and Lake (2003) used a linear multivariate regression technique with diffusivity filters to predict the total fluid production of a well based on injection rates. In a continuation of Albertoni's work (2002), Gentil (2005) explained the physical meaning of multivariate-regression-analysis constants by expressing the connectivity constant solely as a function of transmissibility. Yousef et al. (2006) showed the improved capability of extracting reservoir properties from injection and production data by introducing the capacitance model in which the diffusivity filter is replaced by a time constant. The capacitance model considers the effects of compressibility, pore volume, and productivity index in nonlinear multivariate regression by introducing a time constant to characterize the time delay of the injection signal at the producers. Therefore, connectivity indices and time constants can represent reservoir and fluid properties between injectors and producers.

This work introduces analytical solutions for the continuity equation, which is the fundamental differential equation of the CRMs. There are solutions based on two different projections of stepwise variation of injection rate (SVIR) and linear variation of

the injection rate (LVIR), as a result of discrete nature of injection rate measurements, and three different reservoir control volumes. The three different control volumes are

- Drainage volume of the entire field, or a tank representation of the field, CRMT,
- Drainage volume of each producer, or a tank representation of each producer, CRMP,
- Drainage volume between each injector/producer pair, or a tank representation of the volume between each injector and producer pair, CRMIP.

In the analytical solutions for CRMs, SVIR or LVIR are considered with the effects of linear variation of bottomhole pressure (LVBHP) at the producers between consecutive data points. Furthermore, by considering a series of tanks between each injector/producer pair, CRM solutions are modified and CRM-Block analytical solutions based on superposition in time and space are developed.

3.2- CRMT, SINGLE TANK REPRESENTATION OF A FIELD BY THE CRM

A reservoir may be represented as a single tank if one pseudo producer and one pseudo injector, respectively, represent all producers and injectors in the field, as shown in Fig. 3-2. Therefore, in a tank representation of a field by the CRM denoted as CRMT, or in a field with only one producer and one injector, the material balance leads to the following equation for production rate of q(t) and injection rate of i(t) (details presented in Appendix A):

$$c_t V_p \frac{\mathrm{d}\overline{p}}{\mathrm{d}t} = i(t) - q(t) \tag{3-1}$$

where c_t is the total compressibility, V_p is the reservoir pore volume and \overline{p} is the average reservoir pressure. Based on the definition of productivity index *J* (Walsh and Lake 2003) the total production rate in reservoir volumes, q(t), is

$$q(t) = J(\overline{p} - p_{wf}) \tag{3-2}$$

Elimination of the average reservoir pressure from Eqs. 3-1 and 3-2, as presented by Yousef (2006), leads to the fundamental first-order ordinary differential equation for the CRM as

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} + \frac{1}{\tau}q(t) = \frac{1}{\tau}i(t) - J\frac{\mathrm{d}p_{wf}}{\mathrm{d}t}$$
(3-3)

where J is assumed to be constant and the time constant, τ , is defined as

$$\tau = \frac{c_l V_p}{J} \tag{3-4}$$

and has units of time.



Figure 3-2: Schematic representation of a field with one injector and one producer, the CRMT.

To represent a field or a group of wells with CRMT, all the injection and production rates should be added and represented as $i_F(t)$ and $q_F(t)$. Consequently, the

time constant, τ , will be field-time constant, $\tau_{F_{r}}$ which yields field-average properties. The general solution for Eq. 3-3 is as follows:

$$q(t) = Ce^{\frac{-t}{\tau}} + e^{\frac{-t}{\tau}} \int_{\xi=t_0}^{\xi=t_0} e^{\frac{\xi}{\tau}} \left[\frac{1}{\tau} i(\xi) - J \frac{\mathrm{d}p_{wf}}{\mathrm{d}\xi} \right] \mathrm{d}\xi$$
(3-5)

By applying an initial condition at time t_0 , the constant C can be evaluated as

$$C = q(t_0)e^{\frac{t_0}{\tau}}$$
(3-6)

Therefore, the particular solution for Eq. 3-3, as was presented by Yousef et al. (2006), can be written as

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + e^{\frac{-t}{\tau}} \int_{\xi=t_0}^{\xi=t} \frac{1}{\tau} e^{\frac{\xi}{\tau}} i(\xi) d\xi - e^{\frac{-t}{\tau}} \int_{\xi=t_0}^{\xi=t} J e^{\frac{\xi}{\tau}} \frac{dp_{wf}}{d\xi} d\xi \quad ; t > t_0$$
(3-7)

The output signal, q(t), is composed of three elements on the right of Eq. 3-7. Changes in rate at the producer are comprised of primary depletion, the injection input signal, and the changing of the BHP at the producer. Yousef et al. (2006) expands upon this point in detail. For a better understanding of the impact of primary and secondary production on the CRM refer to Appendix B. If the injection rate is zero and producer's BHP is kept constant this solution simplifies to an exponential decline solution (see Walsh and Lake, 2003) where time constant will be the decay rate.

By integrating by parts, Eq.3-7 can be written as

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + e^{\frac{-t}{\tau}}\int_{\xi=t_0}^{\xi=t_0} \frac{1}{\tau}e^{\frac{\xi}{\tau}}i(\xi)dt - e^{\frac{-t}{\tau}}\left[Je^{\frac{\xi}{\tau}}p_{wf}(\xi)\right]_{\xi=t_0}^{\xi=t} + e^{\frac{-t}{\tau}}\int_{\xi=t_0}^{\xi=t} \frac{J}{\tau}e^{\frac{\xi}{\tau}}p_{wf}(\xi)d\xi \qquad (3-8)$$

Assuming a constant productivity index, J, and time constant, τ , gives:

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + \frac{1}{\tau}e^{\frac{-t}{\tau}}\int_{\xi=t_0}^{\xi=\tau} e^{\frac{\xi}{\tau}}i(\xi)d\xi$$

$$-J\left[p_{wf}(t) - e^{-(\frac{t-t_0}{\tau})}p_{wf}(t_0)\right] + \frac{J}{\tau}e^{\frac{-t}{\tau}}\int_{\xi=t_0}^{\xi=\tau} e^{\frac{\xi}{\tau}}p_{wf}(\xi)d\xi$$
(3-9)

Yousef (2006) discretized the integrals in Eq. 3-9 over the entire production history to find the model parameters by considering *m* equal discretizations of interval Δn as

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + \frac{1}{\tau}e^{\frac{-t}{\tau}}\sum_{m=1}^{n}e^{\frac{m\Delta n}{\tau}}i(m)\Delta n$$

- $J\left[p_{wf}(t) - e^{-(\frac{t-t_0}{\tau})}p_{wf}(t_0)\right] + \frac{J}{\tau}e^{\frac{-t}{\tau}}\sum_{m=1}^{n}e^{\frac{m\Delta n}{\tau}}p_{wf}(m)\Delta n$ (3-10)

Integrals in Eq. 3-9 can be evaluated analytically for any consecutive injection and BHP data points; an analytical solution at the end of each time interval can be used as initial condition for the next time interval. Therefore, by superposition in time we can analytically evaluate production rate at any time.

3.2.1- CRMT Superposition in Time Solution

Instead of numerically integrating Eq. 3-9, we use superposition in time to find an analytical solution. We start with Eq. 3-7, the particular solution of Eq. 3-3. Based on an assumed variation of injection rates between two consecutive injection data points, two

forms of solutions are discussed: 1) SLVR, a step change of injection rate from $I(t_{k-1})$ to $I(t_k)$, and 2) LVIR, a linear change of injection rate between two measurements $i(t_{k-1})$ to $i(t_k)$. In these cases, the fixed injection rate is shown as I(t) and the variable injection rate is i(t). For both injection scenarios, for CRMT in this section and for CRMP and CRMIP in other sections, we present the final analytical solution by superposition in time.

To enhance the discretization based on possibly varying time intervals between the data points and for the purpose of setting up the Microsoft Excel spreadsheet, we start with Eq. 3-7 and, by integrating the second term by parts, rewrite it as

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + e^{\frac{-t}{\tau}} \left[e^{\frac{\xi}{\tau}}i(\xi) \right]_{\xi=t_0}^{\xi=t} - e^{\frac{-t}{\tau}} \int_{\xi=t_0}^{\xi=t} e^{\frac{\xi}{\tau}} \frac{di(\xi)}{d\xi} d\xi - e^{\frac{-t}{\tau}} \int_{\xi=t_0}^{\xi=t} Je^{\frac{t}{\tau}} \frac{dp_{wf}}{d\xi} d\xi$$
(3-11)

or

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + \left[i(t) - e^{-(\frac{t-t_0}{\tau})}i(t_0)\right] - e^{\frac{-t}{\tau}}\int_{\xi=t_0}^{\xi=t} e^{\frac{\xi}{\tau}}\frac{\mathrm{d}i(\xi)}{\mathrm{d}\xi}\mathrm{d}\xi - e^{\frac{-t}{\tau}}\int_{\xi=t_0}^{\xi=t} Je^{\frac{\xi}{\tau}}\frac{\mathrm{d}p_{wf}}{\mathrm{d}\xi}\mathrm{d}\xi \qquad (3-12)$$

We use this form of solution and, based on the discrete nature of the injection rate and producer's BHP data, integrate last two terms of Eq 3-12 for each time interval between two consecutive injection rate and producer's BHP data points. Derivative of injection rate and producer's BHP with respect to time are directly calculated for any two consecutive data points.

A simplified form of the solution can be obtained if the injection rate and producer's BHP are kept constant between two consecutive data points, last two terms in Eq 3-12 will be zero and results in the following solution:

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + I\left[1 - e^{-(\frac{t-t_0}{\tau})}\right]$$
(3-13)

where *I* represent a fixed injection rate from time t_0 to *t*. For $q(t_0)=0$, the relative production response to a step change of injection rate based on dimensionless time is as in Fig. 3-3. As shown in this figure the production reaches 50% of its ultimate rate after $t_D=t_{0D}+0.69$. In an RC circuit, the time that takes for capacitor voltage or resistor current to rise from 10% to 90% of final value is defined as the risetime (Thompson, 2006) as

$$\tau_{\scriptscriptstyle R} = 2.2\tau \tag{3-14}$$



Figure 3-3: Production response to a step change of injection rate vs. dimensionless time in CRM.

Therefore, an injection signal can be detected at a producer if the injection signal has enough magnitude and duration, and its impact on the production rate response is considerably greater than production rate measurement error. Note that at $t_D=t_{0D}+3$ the production rate reaches to 95% of its final value if the injection rate is the only support for the production.

The time constant, τ , is based on reservoir characteristics. A small τ means either a small pore volume and compressibility or a large productivity index. A large τ can be either a large reservoir with small compressibility or a small reservoir with high compressibility, or a very low permeability.

3.2.1.1- CRMT Solution for Series of SVIR

For series of SVIR, $i(\Delta t_k) = I^{(k)}$, and LVBHP, as shown in Figs. 3-4 and 3-5, if we assume constant productivity index during the time interval Δt_k , Eq. 3-12 can be integrated from time t_{k-1} to t_k , and written as

$$q(t_{k}) = q(t_{k-1})e^{-(\frac{\Delta t_{k}}{\tau})} + \left(1 - e^{-(\frac{\Delta t_{k}}{\tau})}\right)\left[I^{(k)} - J\tau \frac{\Delta p_{w_{f}}^{(k)}}{\Delta t_{i}}\right]$$
(3-15)

Eq. 3-15 is derived for one time interval of constant injection rate of $I^{(k)}$. Therefore, for time series of SVIR, as shown in Fig. 3-4 and LVBHP, Fig. 3-5, we can estimate the total production rate at the end of each time interval, Δt_{k-1} , and use this production rate as the initial production rate for the next time interval, Δt_k .



Figure 3-4: Stepwise change of injection rate schedule from time t_0 to t_n .



Figure 3-5: Piecewise linear producer bottom hole pressure change schedule from time t_0 to t_n .

Therefore, from Eq. 3-15 we can write at the end of time interval Δt_n :

$$q(t_n) = q(t_{n-1})e^{\frac{-\Delta t_n}{\tau}} + \left(1 - e^{\frac{-\Delta t_n}{\tau}}\right) \left[I^{(n)} - J\tau \frac{\Delta p_{wf}^{(n)}}{\Delta t_n}\right]$$
(3-16)

Replacing $q(t_{n-l})$ from the previous time interval solution and repeating this process for all time intervals from t_0 to t_n gives the superposition in time solution, as shown in Appendix C:

$$q(t_n) = q(t_0)e^{-(\frac{t_n - t_0}{\tau})} + \sum_{k=1}^n \left\{ \left(1 - e^{\frac{-\Delta t_k}{\tau}}\right) \left[I^{(k)} - J\tau \frac{\Delta p_{_{vf}}^{(k)}}{\Delta t_k}\right] e^{-(\frac{t_n - t_k}{\tau})} \right\}$$
(3-17)

Eq. 3-17 is the general solution for one injector and one producer model in which the injection rate variations are stepwise and the producer's BHP variations are linear between each consecutive data points, as are shown in Figs. 3-4 and 3-5, respectively. Δt_k in Eq. 3-17 is the difference between t_k and t_{k-1} , and $q(t_0)$ is the total production rate at the end of primary recovery. Figure 3-6 shows the CRMT production response, based on Eq. 3-17, to six intervals of SVIR for three different time constants of 10, 20 and 50 days while the producer's BHP is kept constant.



Figure 3-6: CRM production rate estimate for a synthetic case of one injector and one producer with six stepwise injection rate changes for time constants of 10, 20 and 50 days.

3.2.1.1- CRMT Solutions for Series of LVIR

If we assume a LVIR, as well as LVBHP, between two consecutive data points as shown in Figs. 3-7 and 3-5, and assume a constant productivity index during the time interval $\Delta t_k = t_k - t_{k-1}$, Eq. 3-13 can be integrated from time t_{k-1} to t_k as

$$q(t_{k}) = q(t_{k-1})e^{-(\frac{\Delta t_{k}}{\tau})} + \left[i(t_{k}) - e^{-(\frac{\Delta t_{k}}{\tau})}i(t_{k-1})\right] - \left(\frac{i(t_{k}) - i(t_{k-1})}{t_{k} - t_{k-1}}\right)e^{-\frac{t_{k}}{\tau}}\int_{\xi=t_{k-1}}^{\xi=t_{k}}e^{\frac{\xi}{\tau}}d\xi - J\left(\frac{p_{wf}(t_{k}) - p_{wf}(t_{k-1})}{t_{k} - t_{k-1}}\right)e^{-\frac{t_{k}}{\tau}}\int_{\xi=t_{k-1}}^{\xi=t_{k}}e^{\frac{\xi}{\tau}}d\xi$$
(3-18)

By rearranging Eq. 3-18, the production rate at time t_k can be written as

$$q(t_{k}) = q(t_{k-1})e^{-(\frac{\Delta t_{k}}{\tau})} + \left[i(t_{k}) - e^{-(\frac{\Delta t_{k}}{\tau})}i(t_{k-1})\right] -\tau \left(1 - e^{-(\frac{\Delta t_{k}}{\tau})}\right) \left[\left(\frac{i(t_{k}) - i(t_{k-1})}{t_{k} - t_{k-1}}\right) + J\left(\frac{p_{wf}(t_{k}) - p_{wf}(t_{k-1})}{t_{k} - t_{k-1}}\right)\right]$$
(3-19)

or

$$q(t_{k}) = q(t_{k-1})e^{-(\frac{\Delta t_{k}}{\tau})} + \left[i(t_{k}) - e^{-(\frac{\Delta t_{k}}{\tau})}i(t_{k-1})\right] - \tau \left(1 - e^{-(\frac{\Delta t_{k}}{\tau})}\right) \left[\alpha_{k} + J\alpha_{k}\right]$$
(3-20)

where α_k and α'_k are known and equal to the slope between two consecutive injection rate and producer's BHP data points for any time interval of Δt_k respectively.



Figure 3-7: Linear variation of injection rate between data points from time t_0 to t_n .

Equation 3-19 is developed for only one time interval, Δt_k , of LVIR and LVBHP. For a series of LVIR and LVBHP as shown in Figs. 3-7 and 3-5 and a constant τ , Eq. 3-19 estimates the total production rate at the end of any time interval, Δt_k . This production rate is the initial value for the next time interval, Δt_{k+1} . Therefore, from Eq. 3-19 we can write at the end of time interval Δt_n :

$$q(t_{n}) = q(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau})} + \left[i(t_{n}) - e^{-(\frac{t_{n}-t_{0}}{\tau})}i(t_{0})\right] -\tau \sum_{k=1}^{n} \left\{ e^{-(\frac{t_{n}-t_{k}}{\tau})} \left(1 - e^{-\frac{\Delta t_{k}}{\tau}}\right) \left[\frac{\Delta i^{(k)}}{\Delta t_{k}} + J\frac{\Delta p_{w_{t}}^{(k)}}{\Delta t_{k}}\right] \right\}$$
(3-21)

where $\Delta i^{(k)}$ and $\Delta p_{vf}^{(k)}$ represent a change in the injection rate and bottom hole pressure for any time interval of t_{k-1} to t_k .

Eq. 3-21 is the general solution for a case of one injector and one producer in which both the injection rate variations and producer's BHP variations are assumed to be linear between each consecutive data points, as shown in Figs. 3-7 and 3-5.

Figure 3-8 shows the CRMT production response to six intervals of LVIR for three different time constants of 10, 20 and 50 days.



Figure 3-8: CRM production rate estimate for a synthetic case of one injector and one producer with six piecewise linear injection rate changes for time constants of 10, 20 and 50 days.

As Figs. 3-6 and 3-8 show, a large value of the time constant diminishes the injection signal and, for a very large value of τ , the injection can not affect the production rate. On the other hand, the smaller the time constant, the higher the

sensitivity of the production response to the injection signals, and the closer the CRM to the multivariate linear regression response.

3.2.2- CRMT Field Application

In a tank representation for a group of wells or the entire field, CRMT, Eqs. 3-17 and 3-21 can be used with some modifications. If more than one producer exists, the variation of BHP of individual wells can not be accounted for in estimating CRMT parameters. Consequently in Eqs. 3-17 and 3-21 BHP variation terms must be eliminated. On the other hand, if a portion of the field injection is maintained in the reservoir one must modify the field injection rate by a factor of f_F . Therefore, the CRMT equations for any time interval of Δt_k for step changes of injection rates and considering that only part of the field injection is maintained in the reservoir can be written as

$$q_{F}(t_{k}) = q_{F}(t_{k-1})e^{\frac{-\Delta t_{k}}{\tau_{F}}} + f_{F}I_{F}^{(k)}\left(1 - e^{\frac{-\Delta t_{k}}{\tau_{F}}}\right)$$
(3-22)

where subscript *F* is used to represent field parameters and f_F represents the fraction of the field injection that is directed toward the producers at steady-state ($\Delta t \rightarrow \infty$); f_F can vary from zero, an indication of no contribution, to one, an indication of full contribution from injectors in the field production.

If there is any other source of support beside injectors, such as an aquifer influx, e_w , Eq. 3-22 should be modified to preserve material balance as

$$q_F(t_k) = q_F(t_{k-1})e^{\frac{-\Delta t_k}{\tau_F}} + (e_w^{(k)} + f_F^{(k)}I_F^{(k)})\left(1 - e^{\frac{-\Delta t_k}{\tau_F}}\right)$$
(3-23)

where e_w indicates the flux into the reservoir from any other source beside the known injectors. Even if there is no estimate of the amount of support from other sources, a new fitting parameter can be added to account for the unknown sources of support. Eq. 3-17 can be modified as

$$q_F(t_n) = q_F(t_0) \left(e^{-\frac{(t_n - t_0)}{\tau_F}} \right) + \sum_{k=1}^n \left[(e_w^{(k)} + f_F^{(k)} I_F^{(k)}) e^{-\frac{(t_n - t_k)}{\tau_F}} \left(1 - e^{\frac{-\Delta t_k}{\tau_F}} \right) \right]$$
(3-24)

All the model unknowns can change as a function of time especially if any major change occurs in the reservoir, but as a simplifying assumption we can assume f_i and e_w are the same for all time intervals.

The field time constant, τ_F , can be modified as a function of time based on the number of active producers in a reservoir. Changes in the number of active producers translates as increase or decrease in the reservoir productivity index in CRMT. As a result, if new producers are added within the same reservoir compartment, the field production rate increases, but the field time constant decreases by the ratio of currently active producers to previously active producers in the field. On the other hand, if some of the field producers are shut down, the field production rate will decrease but the field time constant will increase, which results in a longer depletion period.

For two intervals of production with a different number of active producers, the field time constant as well as the production rate at the beginning of second time interval should be modified based on the number of active producers as

$$\tau_{F,nP} = -\frac{m}{n} \tau_{F,mP} = \eta_{mn} \tau_{F,mP}$$
(3-25)

$$q_{F,nP}(t_0) = \frac{n}{m} q_{F,mP}(t) = \frac{1}{\eta_{mn}} q_{F,mP}(t)$$
(3-26)

where, *m* and *n* are the number of active producers in the first and second time interval, respectively. η_{mn} is the modification factor for the time constant if the number of active producers is changed from *m* to *n*. The production rate only at the beginning of the second interval is multiplied by $1/\eta_{mn}$ to account for an increase or decrease of the production rate.

Figure 3-9 shows a simple example of three intervals of one, two and four active producers in a reservoir with a constant pore volume and the impact of the number of active producers, on the field total production rate. Field production rate is doubled while the field time constant is cut in half, faster decline, after third and sixth month of production as the number of active producers are doubled.



Figure 3-9: CRMT production rate estimate for a synthetic case of three intervals of one, two and four active producers in a reservoir.

For a very large number of active producers the time constant goes to zero and the field production immediately reaches to its final value. The final value of the production rate is controlled either by the total reservoir volume expansion, because of reservoir depletion, or by the field total injection rate.

As a simple hypothetical example Fig. 3-10 shows six intervals of constant injection rate, observed production data points as well as estimated production rates from the CRM in a system of one injector and one producer. To generate the data in this example, we imposed a series of six different fixed injection rates, a fixed time constant of three days, and a Gaussian random error with a range of 100 RB/D to the observed production rates. Using Eq. 3-17 we evaluated a value of 3.15 days for the time constant by minimizing the relative error between production data points and the CRM estimations.



Figure 3-10: CRM production rate estimate for a synthetic case of one injector and one producer with 50 RB/D production rate measurement absolute errors.

CRMT solutions are developed for a system of one injector and one producer, but for field application solutions of multiwall systems must be developed. For a multiwell system of N_{inj} injectors and N_{pro} producers, $N_{inj} \times N_{pro}$ communications between injectors and producers exist, which we refer to as connectivities, f_{ij} 's. Figure 3-11 shows a schematic of communications between injector *i* with all the producers, and producer *j* with all the injectors in a field. The size of the circles and triangles represent the relative average production and injection rates of the wells, and the thickness of the arrows is a measure of relative connectivities between injector-producer pair.



Figure 3-11: Schematic representation of N_{inj} injectors supporting producer *j* and N_{pro} producers supported by injector *i*.

For any arbitrary reservoir control volume, the CRM production response can be evaluated by replacing the injection and production rates in the Eq. 3-1 by those of the arbitrary control volume. In the following, the CRM is developed for multiwell systems by considering two other control volumes: a) a capacitance-resistive model for a producer drainage control volume, CRMP, and b) a capacitance resistive model for injectorproducer pair control volumes, CRMIP. Compared to the CRMT, in which the entire reservoir volume is represented as a single tank with only one time constant, the number of tanks, and thus the number of time constants, increases in CRMP to the number of producers, and in CRMIP to the number of injector-producer pairs.

3.3- CRMP: PRODUCER BASE REPRESENTATION OF THE CRM

For a control volume around a producer, Fig. 3-12, from the continuity equation we can develop CRMP governing differential equation that represents in-situ volumetric balance over the effective pore volume of a producer. Liang et al. (2007) presented the governing differential equation for this capacitance model by

$$\frac{\mathrm{d}q_{j}(t)}{\mathrm{d}t} + \frac{1}{\tau_{j}}q_{j}(t) = \frac{1}{\tau_{j}}\sum_{i=1}^{N_{inj}}f_{ij}i_{i}(t) - J_{j}\frac{\mathrm{d}p_{wf,j}}{\mathrm{d}t}$$
(3-27)

where τ_j is producer j 's time constant,

$$\tau_j = \left(\frac{c_i V_p}{J}\right)_j \tag{3-28}$$

and pore volume, V_p , total compressibility, c_t , and productivity index, J, are producer j parameters in its effective area; the f_{ij} term, connectivity, represent the steady-state fraction of the rate of injector i flowing toward producer j.

$$f_{ij} = \frac{q_{ij}(t)}{i_i(t)}$$
(3-29)

Note that the sum of connectivities for any injector is less than or equal to one and f_{ij} are positive values. These limiting constraint should be satisfied when the CRMP parameters are evaluated.



Figure 3-12: Schematic representation of the control volume of producer *j*, CRMP.

Earlier, Liang et al. (2007) presented the particular solution of Eq. 3-27 by neglecting the effect of producer's BHP variation. However, solution for Eq. 3-27 with BHP variations can be written as

$$q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + e^{\frac{-t}{\tau_{j}}}\int_{\xi=t_{0}}^{\xi=t}e^{\frac{\xi}{\tau_{j}}}\frac{1}{\tau_{j}}\sum_{i=1}^{N_{inj}}f_{ij}i_{i}(\xi)d\xi - e^{\frac{-t}{\tau_{j}}}\int_{\xi=t_{0}}^{\xi=t}e^{\frac{\xi}{\tau_{j}}}J_{j}\frac{dp_{wf,j}}{d\xi}d\xi$$
(3-31)

Liang et al. (2007) used numerical integration to evaluate the integrals of the injection rates in Eq. 3-31 while they neglected the variation of producer BHP. We develop two analytical forms of Eq. 3-31 by analytical integration with superposition in time to enhance CRMP application based on the discrete nature of both injection rate and BHP data.

3.3.1- CRMP Superposition in Time Solutions

As CRMT, two analytical forms of solution for Eq. 3-31 can be derived for 1) a linear variation of BHP, but stepwise changes in injection rate, and 2) a linear variation of both injection rate and BHP during consecutive time intervals. Integrating Eq. 3-31 by parts leads to the following:

$$q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + \sum_{i=1}^{N_{inj}} \left[f_{ij}\left(i_{i}(t) - e^{-(\frac{t-t_{0}}{\tau_{j}})}i_{i}(t_{0})\right) \right] - e^{\frac{-t}{\tau_{j}}} \int_{\xi=t_{0}}^{\xi=t} e^{\frac{\xi}{\tau_{j}}} \left(\sum_{i=1}^{N_{inj}} f_{ij} \frac{\mathrm{d}i_{i}(\xi)}{\mathrm{d}\xi} + J_{j} \frac{\mathrm{d}p_{wf,j}}{\mathrm{d}\xi} \right) \mathrm{d}\xi$$
(3-32)

Note that Eq. 3-32 for a case of fixed injection rate, I_i , and constant BHP for producer *j* from t_0 to *t* simplifies to

$$q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + \sum_{i=1}^{N_{inj}} \left[f_{ij}I_{i}\right] \left(1 - e^{-(\frac{t-t_{0}}{\tau_{j}})}\right)$$
(3-33)

3.3.1.1- CRMP solution for series of SVIR

By assuming a constant productivity index, fixed injection rates for all injectors, $i_i(t) = I_i$, and a linear bottom hole pressure drop for producer *j*, from time t_0 to *t*, we can integrate Eq. 3-32 as

$$q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + \left(1 - e^{-(\frac{t-t_{0}}{\tau_{j}})}\right)\left[\sum_{i=1}^{N_{inj}} \left[f_{ij}I_{i}\right] - J_{j}\tau_{j}\frac{\Delta p_{wf,j}}{\Delta t}\right]$$
(3-34)

For a case of series of SVIR during time interval Δt_k , $i_i(\Delta t_k) = I_i^{(k)}$, for all the injectors, Fig. 3-4, and a constant productivity index, and series of LVBHP for producer *j*, Fig. 3-5, from Eq. 3-34 we can write at the end of time interval Δt_n :

$$q_{j}(t_{n}) = q_{j}(t_{n-1})e^{\frac{-\Delta t_{n}}{\tau_{j}}} + \left(1 - e^{\frac{-\Delta t_{n}}{\tau_{j}}}\right)\left[\sum_{i=1}^{N_{inj}} \left[f_{ij}I_{i}^{(n)}\right] - J_{j}\tau_{j}\frac{\Delta p_{ij}^{(n)}}{\Delta t_{n}}\right]$$
(3-35)

Assuming f_{ij} 's and τ_j are constant in all time intervals of Δt_k , and replacing $q(t_{n-1})$ in Eq. 3-35 from the previous time step solution and repeating this process for all time intervals from t_0 , to t_n we obtain:

$$q_{j}(t_{n}) = q_{j}(t_{0}) \left(e^{-(\frac{t_{n}-t_{0}}{\tau_{j}})} \right) + \sum_{k=1}^{n} \left\{ e^{-(\frac{t_{n}-t_{k}}{\tau_{j}})} \left(1 - e^{\frac{-\Delta t_{k}}{\tau_{j}}} \right) \left[\sum_{i=1}^{N_{inj}} \left[f_{ij} I_{i}^{(k)} \right] - J_{j} \tau_{j} \frac{\Delta p_{ij}^{(k)}}{\Delta t_{k}} \right] \right\}$$
(3-36)
(for $j = 1, 2, ..., N_{pro}$)

Eq. 3-36 is the general form of solution for SVIR, Fig. 3-4, and LVBHP, Fig. 3-5, for the CRMP. This equation simplifies to CRMT solution, Eq. 3-17, if only one injector exists.

3.3.1.2- CRMP solution for series of LVIR

For a LVIR, a constant productivity index, and a LVBHP during time interval t_0 to *t*, we can integrate and write Eq. 3-32 as

$$q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + \sum_{i=1}^{N_{inj}} \left[f_{ij}\left(i_{i}(t) - e^{-(\frac{t-t_{0}}{\tau_{j}})}i_{i}(t_{0})\right) \right] -\tau_{j}\left(1 - e^{-(\frac{t-t_{0}}{\tau_{j}})}\right) \left[\sum_{i=1}^{N_{inj}} f_{ij}\frac{\Delta i_{i}}{\Delta t} + J_{j}\frac{\Delta p_{wf,j}}{\Delta t} \right]$$
(3-37)

Equation 3-37 is developed for one time interval of linear injection rate and BHP changes. Therefore, for a series of linear injection rate and BHP changes, shown in Fig. 3-7 and Fig. 3-5, we can use Eq. 3-37 to estimate the total production rate at the end of each time interval of Δt_k and use this production rate as the initial value for next time interval, Δt_{k+1} . Therefore, at the end of time interval Δt_n we obtain:

$$q_{j}(t_{n}) = q_{j}(t_{n-1})e^{-(\frac{\Delta t_{n}}{\tau_{j}})} + \sum_{l=1}^{N_{inj}} \left[f_{ij}\left(i_{i}(t_{n}) - e^{-(\frac{\Delta t_{n}}{\tau_{j}})}i_{i}(t_{n-1})\right) \right] -\tau_{j}\left(1 - e^{-(\frac{\Delta t_{n}}{\tau_{j}})}\right) \left[\sum_{l=1}^{N_{inj}} f_{ij}\frac{\Delta t_{i}^{n}}{\Delta t_{n}} + J_{j}\frac{\Delta p_{uf,j}^{n}}{\Delta t_{n}} \right]$$
(3-38)

Assuming that f_{ij} 's and τ_j remain constant in all time intervals of Δt_k , and replacing $q(t_{n-1})$ in Eq. 3-38 from the previous time step solution and repeating this process for all the time intervals from t_0 , to t_n we get:

$$q_{j}(t_{n}) = q_{j}(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau_{j}})} + \sum_{i=1}^{N_{inj}} \left[f_{ij}\left(i_{i}(t_{n}) - e^{-(\frac{t_{n}-t_{0}}{\tau_{j}})}i_{i}(t_{0})\right) \right] \\ - \sum_{k=1}^{n} \left\{ \tau_{j}e^{-(\frac{t_{n}-t_{k}}{\tau_{j}})} \left(1 - e^{-(\frac{\Delta t_{k}}{\tau_{j}})}\right) \left[\sum_{i=1}^{N_{inj}} f_{ij}\frac{\Delta i_{i}^{(k)}}{\Delta t_{k}} + J_{j}\frac{\Delta p_{wf,j}^{(k)}}{\Delta t_{k}} \right] \right\}$$
(3-39)
(for $j = 1, 2, ..., N_{pro}$)

Equation 3-39 is the general form of the solution in CRMP for calculating the total production rate of producer *j*, for LVIR and LVBHP shown in Fig. 3-7 and Fig. 3-5.

3.4- CRMIP, INJECTOR-PRODUCER BASE REPRESENTATION OF THE CRM

In the CRMIP we consider the affected pore volume of any pair of injector/producer, *ij*, shown in Fig. 3-13. We can modify Eq. 3-3 to develop the CRMIP

governing differential equation that represents in-situ volumetric balance over the effective pore volume of any pair of injector/producer. The CRMIP governing differential equation was stated implicitly by Yousef et al. (2006) as

$$\frac{\mathrm{d}q_{ij}(t)}{\mathrm{d}t} + \frac{1}{\tau_{ij}}q_{ij}(t) = \frac{1}{\tau_{ij}}f_{ij}i_i(t) - J_{ij}\frac{\mathrm{d}p_{wf,j}}{\mathrm{d}t}$$
(3-40)
(for $i = 1, 2, ..., N_{inj}$ and $j = 1, 2, ..., N_{pro}$)

where time constant, τ_{ij} , is defined as

$$\tau_{ij} = \left(\frac{c_i V_P}{J}\right)_{ij} \tag{3-41}$$

and pore volume, V_p , total compressibility, c_t , and productivity index, J, are associated with the control volume between injector i and producer j pair. As for the CRMP, f_{ij} is the steady-state fraction of injection rate of injector i directed to producer j. Compared to the CRMT and the CRMP, in which we have only one time constants, one initial production, and productivity index, in the CRMIP there are $N_{inj} \times N_{pro}$ time constants, τ_{ij} 's, and $q_{ij}(0)$'s and J_{ij} 's.



Figure 3-13: Schematic representation of control volume between each injector/producer pair.

The solution of Eq. 3-40 results in the portion of the rate of producer *j* from the control volume between injector *i* and producer *j*. The particular solution of Eq. 3-40 has the same form as a system of one injector and one producer, as Eq. 3-12, in which q(t), τ , and i(t) are replaced by $q_{ij}(t)$, τ_{ij} , and $f_{ij}i(t)$ correspondingly. Therefore, for the control volume between injector *i* and producer *j* we can write:

$$q_{ij}(t) = q_{ij}(t_0)e^{-(\frac{t-t_0}{\tau_{ij}})} + f_{ij}\left[i_i(t) - e^{-(\frac{t-t_0}{\tau_{ij}})}i_i(t_0)\right] \\ - e^{\frac{-t}{\tau_{ij}}}\int_{\xi=t_0}^{\xi=t} e^{\frac{\xi}{\tau_{ij}}}\left(\frac{di_{ij}(\xi)}{d\xi} - J_{ij}\frac{dp_{wf,j}}{d\xi}\right)d\xi$$
(for $i = 1, 2, ..., N_{inj}$ and $j = 1, 2, ..., N_{pro}$)
$$(3-42)$$

The production rate for producer *j* is simply calculated by summing the contribution from all the injectors; therefore,

$$q_{j}(t) = \sum_{i=1}^{N_{inj}} q_{ij}(t) \qquad (j = 1, 2, ..., N_{pro})$$
(3-43)

Yousef et al. (2006) initially summed Eq. 3-40 over all the injectors and presented the following equation for the production rate of producer *j* in a multiwell system:

$$q_{j}(t) = -\sum_{i=1}^{N_{inj}} \tau_{ij} \frac{\mathrm{d}q_{ij}(t)}{\mathrm{d}t} + \sum_{i=1}^{N_{inj}} f_{ij}i_{i}(t) - \frac{\mathrm{d}p_{wf,j}}{\mathrm{d}t} \sum_{i=1}^{N_{inj}} \tau_{ij}J_{ij}$$
(for $j = 1, 2, ..., N_{pro}$)
(3-44)

Eq. 3-44 shows that the solution has three distinct parts: the first term shows the effect of primary production, the second is the impact of the injection rate of different injectors and the third term is the effect of producer BHP variation. To simplify Eq. 3-44, Yousef

et al. (2006) assumed equal τ_{ij} for both the primary production and the BHP terms. This assumption weakens the impact of different control volume and productivity index between injector-producer pairs on the total production.

A solution equivalent to that of Yousef et al. (2006) can be obtained if we replace the first term in Eq. 3-44 by an exponential decline. Then modify the second term by a filter to enforce a shift in the injection signal as it was proposed initially by Albertoni (2002) and replace the last term by producer pore volume by using τ_{ii} definition:

$$q_{j}(t) = \sum_{i=1}^{N_{inj}} q_{ij}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{ij}}} + \sum_{i=1}^{N_{inj}} f_{ij}i^{*}_{i}(t) - \frac{\mathrm{d}p_{wf,j}}{\mathrm{d}t}(c_{i}V_{P})_{j}$$
(for $j = 1, 2, ..., N_{pro}$)
(3-45)

In the following, we introduce a straightforward approach to find the solution for CRMIP production rate by superposition in time and space.

3.4.1- CRMIP Superposition in Time and Space Solution

To find the producer j rate, we first solve for the flow rate associated with each injector/producer pair, q_{ij} through superposition in time for all time intervals of different injection rates and BHP variations; then apply superposition in space to find the flow rate associated with each producer q_j , by summing up contributions from each injector, Eq. 3-43. As with the CRMT and the CRMP, SVIR and LVIR approaches are presented to obtain analytical solutions for the CRMIP.

3.4.1.1- CRMIP Solution for series of SVIR

For fixed injection rate $i(\Delta t_k) = I_i^{(k)}$, and a linear BHP variation during time interval Δt_k , Figs 3-4 and 3-5, a simpler form of solution can be obtained from Eq. 3-42 by replacing $i_i(t)$ and $i_i(t_0)$ by $I_i^{(k)}$ or directly from Eq. 3-15 we can obtain:

$$q_{ij}(t_k) = q_{ij}(t_{k-1})e^{\frac{-\Delta t_k}{\tau_{ij}}} + \left(1 - e^{\frac{-\Delta t_k}{\tau_{ij}}}\right) \left[f_{ij}I_i^{(k)} - J_{ij}\tau_{ij}\frac{\Delta p_{ij}^{(k)}}{\Delta t_k}\right]$$
(3-46)

where $I_i^{(k)}$ and $\Delta p_{wf,j}^{(k)}$ represent the rate of injector *i* and changes in BHP of producer *j* during time interval t_{k-1} to t_k , respectively. We apply superposition in time for a time series, from t_0 to t_n , by assuming a constant productivity index during any time interval of Δt_k . to find q_{ij} at the end of time t_n as

$$q_{ij}(t_{n}) = q_{ij}(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} + \sum_{k=1}^{n} \left\{ \left(1 - e^{\frac{-\Delta t_{k}}{\tau_{ij}}}\right) \left[f_{ij}I^{(k)} - J_{ij}\tau_{ij}\frac{\Delta p_{wf,j}^{(k)}}{\Delta t_{k}} \right] e^{-(\frac{t_{n}-t_{k}}{\tau_{ij}})} \right\}$$
(3-47)

Thereafter, $q_i(t_n)$ can be calculated by considering each injector's contribution as

$$q_{j}(t_{n}) = \sum_{i=1}^{N_{inj}} q_{ij}(t_{n}) = \sum_{i=1}^{N_{inj}} q_{ij}(t_{0}) e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} + \sum_{i=1}^{N_{inj}} \left\{ \sum_{k=1}^{n} \left[\left(1 - e^{\frac{-\Delta t_{k}}{\tau_{ij}}} \right) \left(f_{ij} I_{i}^{(k)} - J_{ij} \tau_{ij} \frac{\Delta p_{vj}^{(k)}}{\Delta t_{k}} \right) e^{-(\frac{t_{n}-t_{k}}{\tau_{ij}})} \right] \right\}$$
(3-48)

Eq. 3-48 is the general solution for CRMIP by considering SVIR and LVBHP of producer *j* between each consecutive production data point as shown in Figs. 3-4 and 3-5.

3.4.1.2- CRMIP Solution for Series of LVIR

If we assume a linear change between two consecutive injection rate and producer's BHP during time interval Δt_k (t_{k-1} to t_k), as shown in Figs. 3-7 and 3-5, by using Eq. 3-42 we can write:

$$q_{ij}(t_{k}) = q_{ij}(t_{k-1})e^{-(\frac{\Delta t_{k}}{\tau_{ij}})} + f_{ij}\left[i_{i}(t_{k}) - e^{-(\frac{\Delta t_{k}}{\tau_{ij}})}i_{i}(t_{k-1})\right] -\tau_{ij}\left(1 - e^{-(\frac{\Delta t_{k}}{\tau_{ij}})}\right)\left[f_{ij}\left(\frac{\Delta i_{i}^{(k)}}{\Delta t_{k}}\right) + J_{ij}\left(\frac{\Delta p_{wf,j}^{(k)}}{\Delta t_{k}}\right)\right]$$
(3-49)
for $(i = 1, 2, ..., N_{inj}), (j = 1, 2, ..., N_{pro})$ and $(k = 1, 2, ..., n),$

where $\Delta i_i^{(k)}$ and $\Delta p_{_{wf,j}}^{(k)}$ represent change in the rate of injector *i* and BHP of producer *j*, during time interval t_{k-1} to t_k , respectively. For a time series of data points, by superposition in time and assuming a constant productivity index during any time interval of Δt_k , at the end of time interval Δt_n , q_{ij} can be calculated by:

$$q_{ij}(t_{n}) = q_{ij}(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} + f_{ij}\left[i_{i}(t_{n}) - e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})}i_{i}(t_{0})\right] -\tau_{ij}\sum_{k=1}^{n}\left\{e^{-(\frac{t_{n}-t_{k}}{\tau_{ij}})}\left(1 - e^{\frac{-\Delta t_{k}}{\tau_{ij}}}\right)\left[f_{ij}\frac{\Delta i_{ij}^{(k)}}{\Delta t_{k}} + J\frac{\Delta p_{vf,j}^{(k)}}{\Delta t_{k}}\right]\right\}$$
(3-50)

Therefore, $q_j(t_n)$ can be calculated by considering each injector's contribution as

$$q_{j}(t_{n}) = \sum_{i=1}^{N_{inj}} q_{ij}(t_{n}) = \sum_{i=1}^{N_{inj}} q_{ij}(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} + \sum_{i=1}^{N_{inj}} f_{ij}\left[i_{i}(t_{n}) - e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})}i_{i}(t_{0})\right] \\ - \sum_{i=1}^{N_{inj}} \left\{\tau_{ij}\sum_{k=1}^{n} \left\{e^{-(\frac{t_{n}-t_{k}}{\tau_{ij}})}\left(1 - e^{\frac{-\Delta t_{k}}{\tau_{ij}}}\right)\left[\frac{f_{ij}\Delta i_{i}^{(k)}}{\Delta t_{k}} + J\frac{\Delta p_{w_{i},j}^{(k)}}{\Delta t_{k}}\right]\right\}\right\}$$
(3-51)

Equation 3-51 is the general form of solution for CRMIP by considering LVIR and LVBHP between any consecutive production data point as shown in Figs. 3-7 and 3-5.

3.5- CRM-BLOCK, BLOCK REFINEMENT REPRESENTATION OF CRMS

The CRMIP considers only one control volume, one tank, with one time constant of τ_{ij} , between injector *i* and producer *j*. This configuration enforces the assumption of an immediate response of the pressure signal generated from injector *i* at producer *j*. If we consider a series of M_{ij} tanks connecting injector *i* to producer *j* as shown in Fig. 3-14, the CRM solution at any time will account for the pressure delay and can estimate the flow rate in/out of any grid block between injector-producer pairs.

We derive the CRM-Block solution at the producer by applying superposition in space to capture the impact of the injection rate at the last grid block which is equal to the production rate of producer *j*.



Figure 3-14: Block refinement representation between injector *i* and producer *j*, CRM-Block.

The total production rate in CRMT for a system of one injector and one producer without producer's bottomhole pressure variation is:

$$q(t) = q(t_0)e^{-(\frac{t-t_0}{\tau})} + i(t)(1 - e^{-(\frac{t-t_0}{\tau})})$$
(3-52)

For simplicity, first we assume that the flow rate at time t_0 to be zero, for all the blocks between injector and producer. From Eq. 3-52, the flow rate out of the first block after time *t* can be written as

$$q_1(t) = i(t)(1 - e^{-(\frac{t-t_0}{\tau_1})})$$
(3-53)

where τ_i is the time constant of the first block between injector *i* and producer *j*. For the second block:

$$q_{2}(t) = q_{1}(t)(1 - e^{-(\frac{t-t_{0}}{\tau_{2}})}) = i(t)(1 - e^{-(\frac{t-t_{0}}{\tau_{1}})})(1 - e^{-(\frac{t-t_{0}}{\tau_{2}})})$$
(3-54)

The flow rate out of block *l* can be written as

$$q_{k}(t) = q_{k-1}(t)(1 - e^{-(\frac{t-t_{0}}{\tau_{l}})}) = i(t)\prod_{b=1}^{l} (1 - e^{-(\frac{t-t_{0}}{\tau_{b}})})$$
(3-55)

The production rate at the producer is equal to the flow rate out of last block;

$$q(t) = q_M(t) = q_{M-1}(t)(1 - e^{-(\frac{t-t_0}{\tau_M})}) = i(t)\prod_{b=1}^M (1 - e^{-(\frac{t-t_0}{\tau_b})})$$
(3-56)

If all the blocks between injector and producer have equal time-constants, τ_b , then Eq. 3-56 is simplified to

$$q(t) = i(t) \left(1 - e^{-(\frac{t-t_0}{\tau_b})} \right)^M$$
(3-57)

This solution returns the CRM tank model solution, CRMT, if only one block is considered between injector and producer, M = 1. Modification of Eq. 3-56 for multiwell system CRMIP-Block without the effect of primary recovery can be written by replacing the injection rate, i(t), by a fraction of the injection rate of injector *i* which is contributing in the production rate of producer *j*, $f_{ij}i_i(t)$ as

$$q_{ij}(t) = q_{ij,M_{ij}}(t) = q_{ij,(M_{ij}-1)}(t)(1 - e^{\frac{-(t-t_0)}{\tau_{ij,M_{ij}}}}) = f_{ij}i_i(t)\prod_{b=1}^{M_{ij}}(1 - e^{\frac{-(t-t_0)}{\tau_{ij,b}}})$$
(3-58)

where M_{ij} is the number of blocks between injector *i* and producer *j*. Thus, the production rate at producer *j* can be calculated by

$$q_{j}(t) = \sum_{i=1}^{N_{inj}} q_{ij}(t) = \sum_{i=1}^{N_{inj}} q_{ij,M}(t) = \sum_{i=1}^{N_{inj}} q_{ij,M_{ij}-1}(t)(1 - e^{\frac{-(t-t_{0})}{\tau_{ij,M_{ij}}}})$$

$$= \sum_{i=1}^{N_{inj}} \left[f_{ij}\dot{i}_{i}(t) \prod_{b=1}^{M_{ij}} (1 - e^{\frac{-(t-t_{0})}{\tau_{ij,b}}}) \right]$$
(3-59)

For equal block time constants, τ_{ij}^* , between injectors and producers Eqs. 3-58 and 3-59 simplify to:

$$q_{ij}(t) = f_{ij}i_i(t) \left(1 - e^{\frac{-t}{\tau_{ij}}}\right)^{M_{ij}}$$
(3-60)

and

$$q_{j}(t) = \sum_{i=1}^{N_{inj}} q_{ij}(t) = \sum_{i=1}^{N_{inj}} \left[f_{ij}i_{i}(t) \left(1 - e^{\frac{-t}{\tau_{ij}^{*}}} \right)^{M_{ij}} \right]$$
(3-61)

The number of blocks between injector i and producer j, M_{ij} , which can be a fitting parameter also, can be different for each injector-producer pair. The CRMIP-Block solution simplifies to the CRMIP solution if we consider only one block between any

injector-producer pair, $M_{ij} = 1$. At time t_0 all production rates, primary production rate, for all the blocks are assumed to be zero to simplify the derivation of Eqs. 3-58 and 3-59. In the following we include the primary production term in CRM-Block development.

3.5.1- Primary Production Term in the CRM-Block

The primary production flow rate, the first term in Eq. 3-52, out of block l in Fig. 3-14, after time t is:

$$q_{l}(t) = q_{l}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{l}}} + q_{l-1}(t)(1-e^{\frac{-(t-t_{0})}{\tau_{l}}})$$

$$= q_{l}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{l}}} + \sum_{b=1}^{l-1}\left\{q_{b}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{b}}}\prod_{a=1}^{l-b}(1-e^{\frac{-t}{\tau_{a}}})\right\} + i(t)\prod_{b=1}^{l}(1-e^{\frac{-t}{\tau_{b}}})$$
(3-62)

Therefore, the production rate at the producer is equal to the flow rate out of block M as

$$q_{M}(t) = q_{M}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{M}}} + q_{M-1}(t)(1-e^{\frac{-(t-t_{0})}{\tau_{M}}})$$

$$= q_{M}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{M}}} + \sum_{b=1}^{M-1} \left\{ q_{b}(t_{0})e^{\frac{-(t-t_{0})}{\tau_{b}}} \prod_{a=1}^{M-b} (1-e^{\frac{-(t-t_{0})}{\tau_{a}}}) \right\} + i(t)\prod_{b=1}^{M} (1-e^{\frac{-t}{\tau_{b}}})$$
(3-63)

We can modify Eq. 3-63 for a multiwell version to get the CRMIP-Block solution with the effect of primary recovery by replacing the injection rate, i(t), with a fraction of the injection rate of injector *i* that is contributing in the production rate of producer *j*, $f_{ij}i_i(t)$. We can write the following equation for CRMIP-Block without considering the variation of producer bottomhole pressure.

$$q_{ij}(t) = q_{M_{ij}}(t_0)e^{\frac{-(t-t_0)}{\tau_{M_{ij}}}} + \sum_{b=1}^{M_{ij}-1} \left\{ q_{ij,b}(t_0)e^{\frac{-(t-t_0)}{\tau_{ij,b}}} \prod_{a=1}^{M_{ij}-b} (1-e^{\frac{-(t-t_0)}{\tau_{ij,a}}}) \right\} + f_{ij}i_i(t)\prod_{b=1}^{M_{ij}} (1-e^{\frac{-(t-t_0)}{\tau_{ij,b}}})$$
(3-64)
Total production of producer *j* can be calculated by summing up q_{ij} to account for all the injectors' contribution as

$$q_{j}(t) = \sum_{i=1}^{N_{inj}} q_{ij}(t) = \sum_{i=1}^{N_{inj}} q_{M_{ij}}(t_{0}) e^{\frac{-(t-t_{0})}{\tau_{ij,M_{ij}}}} + \sum_{i=1}^{N_{inj}} \sum_{b=1}^{M_{ij}-1} \left\{ q_{ij,b}(t_{0}) e^{\frac{-(t-t_{0})}{\tau_{ij,b}}} \prod_{a=1}^{M_{ij}-b} (1-e^{\frac{-(t-t_{0})}{\tau_{ij,a}}}) \right\} + \sum_{i=1}^{N_{inj}} f_{ij}i_{i}(t) \prod_{b=1}^{M_{ij}} (1-e^{\frac{-(t-t_{0})}{\tau_{ij,b}}})$$
(3-65)

Eqs. 3-64 and 3-65 simplify to the following for equal block time constants, τ_{ij}^* , between injectors *i* and producers *j*:

$$q_{ij}(t) = \sum_{b=1}^{M_{ij}} \left[q_{ij,b}(t_0) e^{\frac{-(t-t_0)}{\tau_{ij}^*}} \left(1 - e^{\frac{-(t-t_0)}{\tau_{ij}^*}} \right)^{M_{ij}-b} \right] + f_{ij} i_i(t) \left(1 - e^{\frac{-(t-t_0)}{\tau_{ij}^*}} \right)^{M_{ij}}$$
(3-66)

And

$$q_{j}(t) = \sum_{i=1}^{N_{inj}} \sum_{b=1}^{M_{ij}} q_{ij,b}(t_{0}) e^{\frac{-(t-t_{0})}{\tau_{ij}^{*}}} \left(1 - e^{\frac{-(t-t_{0})}{\tau_{ij}^{*}}} \right)^{M_{ij}-b} + \sum_{i=1}^{N_{inj}} f_{ij}i_{i}(t) \left(1 - e^{\frac{-(t-t_{0})}{\tau_{ij}^{*}}} \right)^{M_{ij}}$$
(3-67)

The major difference between the CRMs with and without block refinement consideration between injector-producer pairs is reflected in the shape of the production response. Since in the CRMIP only one cell with uniform characteristic exists between injector i and producer j, the production response as a result of the pressure wave breakthrough is immediately observed at the producer and the slope of the production response decreases monotonically in respect to time. On the contrary, in the CRMIP-Block, the delay of the production response is captured as a result of block modification which leads to the exact superposition in space solution for CRMIP. The slope of the

production response for CRMIP-Block first increases, before the pressure wave reaches the producing well, and then follows the same projection as CRMIP without block refinement.

We recommend application of CRMIP-Block especially for cases for which there is a lag between production response to an injection signal such as

- **a.** Distant injector-producer pairs
- **b.** Low permeability reservoirs for which the impact of the injection signal is not instantaneously captured at the producers.
- **c.** For field case studies with a high frequency of injection production data.
- **d.** Treating the injectors and producers wellbore as two tanks with small time constant in series with the reservoir with a large time constant.

Figure 3-15 compares the shape of the CRM production response, with and without block refinement, between an injector and a producer for a fixed injection rate. In this figure, the CRM-10Block with ten blocks of equal time constant of three days is considered as observed production response and the equivalent time constants for other CRM-Blocks are evaluated to minimize their difference with CRM-10Blocks. Any block refinement with more than 10 blocks can completely match the CRM-10Blocks, while the fewer the number of the blocks the larger is the mismatch. Figure 3-16 shows a log-log linear relationship between the equivalent block time constant and the number of blocks for cases that production response has the minimum difference with CRM-10Blocks model with time constant of three days.



Figure 3-15: Comparison between CRM with and without block refinement between an injector and producer.



Figure 3-16: Equivalent time constant as a function of the number of block with equal time constant between injector-producer pair.

3.6- CRM PARAMETER EVALUATION

During the course of history-matching the CRMs' parameters should be evaluated. For each CRM model, the number of parameters varies. The number of model parameters increases from CRMT to CRMP, and from CRMP to CRMIP. Based upon the nonlinear form of the CRM response, as presented in Fig. 3-3, to evaluate τ accurately few production measurements from t_{0D} to $t_D \le t_{0D}$ +3 should be available. However, sufficient data points ensure proper estimation of model parameters.

To match the total production history for a pattern of N_{inj} injectors and N_{pro} producers by the CRMT as shown in Eq. 3-22, one needs to evaluate only three parameters: the field production rate at time t_0 , $q(t_0)$, a field-time constant, τ_{Field} , and the fraction of field injection which is confined in the field f_{inj} . In contrast, the CRMP has $N_{inj}+3$ model parameters for each producer: f_{1j} , f_{2j} , ..., $f_{Ninj,j}$, τ_j , $q_j(t_0)$ and J_j . Therefore, to use the CRMP in a field one must evaluate $N_{pro}(N_{inj}+3)$ model unknowns. In CRMIP, for each injector-producer pair, four model parameters exist: $q_{ij}(t_0)$, τ_{ij} , f_{ij} , and J_{ij} . Therefore, in CRMIP there are $4 \times N_{inj} \times N_{pro}$ model parameters. In other words, $4 \times N_{inj} \times N_{pro}$ is the minimum number of data points required for this model. If the BHP at the producers is kept constant, the model parameters decreases to $N_{pro}(N_{inj}+2)$ for CRMP, and to $3 \times N_{inj} \times N_{pro}$ for CRMIP. Table 3-1 summarize the list and the number of the unknown parameters in different CRMs.

Based upon the nonlinear form of the CRM and considering measurement error in production data, we recommend that the number of production data points be at least four times the number of CRM unknowns. This rule of thumb roughly ensures solution quality.

In all the CRMs unknowns are evaluated by minimizing the difference between CRMs' estimation and the field rate measurements. For CRMT, the objective is to

minimize the absolute error for total field production prediction. In CRMP and CRMIP, the average absolute error for each of the producers can be evaluated and the sum of these errors becomes the objective function. In a balanced waterflood, we recommend matching production history simultaneously for all producers. Therefore, instead of minimizing the production estimation error for one producer at a time, one should minimize the error over the entire field production. In this exercise, the sum of the fractions of injection rate of any injector that is going toward producers should be less than or equal to one. It should be mentioned that all the model parameters are positive and real values.

$$\sum_{i=1}^{N_{inj}} f_{ij} \le 1$$
(3-68)

	CRMT	CRMP	CRMIP
Unknowns	$q_F(t_0), f_F, \tau_F$	$q_j(t_0)$'s, f_{ij} 's, τ_j 's, and J_j 's	$q_{ij}(t_0)$'s, f_{ij} 's, τ_{ij} 's and J_{ij} 's
No. of Unknowns without BHP data	3	$N_{pro} \times (N_{inj}+2)$	$3 imes N_{inj} imes N_{pro}$
No. of Unknowns with BHP data	3	$N_{pro} \times (N_{ijn} + 3)$	$4 imes N_{inj} imes N_{pro}$

Table 3-1: Comparison between number of unknowns in the CRMs.

3.7- COMPARISON BETWEEN CRM AND MULTIVARIATE LINEAR REGRESSION

The CRMs are general forms of nonlinear regression of the production rate based on changing injection rate. For an incompressible fluid the injection signal reaches the producers simultaneously, therefore, we can replace in CRM $\tau = 0$. As a result, the CRM solution simplifies to a multivariate linear regression (MLR) form; for example, the rate of producer *j* in CRMP has the following form:

$$q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + \left(1 - e^{-(\frac{t-t_{0}}{\tau_{j}})}\right)\left[\sum_{i=1}^{N_{i}} \left[f_{ij}I_{i}\right] - J_{j}\tau_{j}\frac{\Delta p_{wf,j}}{\Delta t}\right]$$
(3-69)

which simplifies, if the producer's BHP is constant, to:

$$q_{j} = \sum_{i=1}^{N_{inj}} f_{ij} i_{i}(t)$$
(3-70)

This equation has the same form of the production response at steady-state, which can be evaluated as time approaches infinity in Eq. 3-69. For a balanced system, the sum of f_{ij} 's should be one for MLR model as well as CRMs.

Enforcing the MLR to a slightly compressible or compressible reservoir can result in negative f_{ij} 's. Figure 3-8 shows that it is possible to encounter negative correlation between injection and production rate for fields with a large time constant. This negative correlation causes multivariate linear regression to have unrealistic negative connectivities.

3.8- SUMMARY

Analytical solutions for the continuity equation based on superposition in time and space were developed for three different reservoir control volumes: 1) CRMT, the entire field volume, 2) CRMP, the drainage volume of a producer and 3) CRMIP, the control volume between injector-producer pairs. These solutions are obtained based on stepwise or linear variation of injection rate and linear variation of the producers' bottomhole pressures projections. Furthermore, by considering a series of tanks between each injector/producer pair, CRM solutions were modified and CRM-Block analytical solutions based on superposition in time and space were developed.

Analytical solutions facilitate CRMs' application for rapid assessment at different levels of a field study, from a single well, to a group of wells, and to an entire field. The CRM's analytical solutions in conjunction with the physical meaning of its parameters, capability to discern reservoir connectivities, flexibility in taking variable timesteps, simplicity, and speed are major advantages over those presented previously, Table 3-2. A summary of the solution of different CRMs for step variation of injection rates are presented in Table 3-3. To facilitate the application of the developed CRMs a spreadsheet based tool was created in Microsoft Excel and automated using Visual Basic macros (CRM-Generator). The user interface for this tool is shown in Appendix D.

Compared Criteria	Yousef et al. (2005)	Liang et al. (2007)	This Work	
Analytical solution for only one change of injection rate and bottomhole pressure for total liquid production	CRMIP	CRMP	All CRM's	
CRMIP solution approach	Summation of CRM differential equations of each injector- producer pair is solved	n/a	q_{ij} are evaluated individually and then their summation generates stable q_j 's	
Solution for injection rate fluctuations for total liquid solution	Numerical solution	Numerical solution	Analytical solution with superposition in time	
Analytical solution for both injection rate and BHP fluctuations for total liquid production	n/a	n/a	Analytical solution with superposition in time for all CRM's	
Oil production optimization	n/a	Based on maximizing net present value	Based on reallocation of fixed field-injection rate	
Model validation examples	Variable injection rates and fixed BHP	Variable injection rates and fixed BHP	Variable injection rates and variable BHP	
Timestep increments	fixed	fixed	variable	
Block refinement	n/a	n/a	CRMT and CRMIP	

Table 3-2: Comparison between previously developed CRMs.

Model	Differential equation.	One time interval solution for a step change in injection rate	Series of step variation of injection rate (SVIR) solution	
CRMT	$\frac{\mathrm{d}q(t)}{\mathrm{d}t} + \frac{1}{\tau}q(t) = \frac{1}{\tau}i(t) - J\frac{\mathrm{d}p_{wf}}{\mathrm{d}t}$ $\tau = \frac{c_t V_P}{J}$	$q(t_k) = q(t_{k-1})e^{-(\frac{\Delta t_k}{\tau})} + \left(1 - e^{-(\frac{\Delta t_k}{\tau})}\right)\left[I^{(k)} - J\tau \frac{\Delta p_{wf}^{(k)}}{\Delta t_i}\right]$	$q(t_n) = q(t_0)e^{-\left(\frac{t_n - t_0}{\tau}\right)} + \sum_{k=1}^n \left\{ \left(1 - e^{\frac{-\Delta t_k}{\tau}}\right) \left[I^{(k)} - J\tau \frac{\Delta p_{w_f}^{(k)}}{\Delta t_k}\right] e^{-\left(\frac{t_n - t_k}{\tau}\right)} \right\}$	
CRMP	$\frac{\mathrm{d}q_{j}(t)}{\mathrm{d}t} + \frac{1}{\tau_{j}}q_{j}(t) = \frac{1}{\tau_{j}}\sum_{i=1}^{N_{inj}}f_{ij}i_{i}(t) - J_{j}\frac{\mathrm{d}p_{wf,j}}{\mathrm{d}t}$ (for $j = 1, 2,, N_{pro}$) $\tau_{j} = \left(\frac{c_{i}V_{p}}{J}\right)_{j}$	$q_{j}(t_{k}) = q_{j}(t_{k-1})e^{\frac{-\Delta t_{k}}{\tau_{j}}} + \left(1 - e^{\frac{-\Delta t_{k}}{\tau_{j}}}\right) \left[\sum_{i=1}^{N_{inj}} \left[f_{ij}I_{i}^{(k)}\right] - J_{j}\tau_{j}\frac{\Delta p_{v(j,j)}^{(k)}}{\Delta t_{k}}\right]$	$q_{j}(t_{n}) = q_{j}(t_{0}) \left(e^{-(\frac{t_{n}-t_{0}}{\tau_{j}})} \right) + \sum_{k=1}^{n} \left\{ e^{-(\frac{t_{n}-t_{k}}{\tau_{j}})} \left(1 - e^{\frac{-\Delta t_{k}}{\tau_{j}}} \right) \left[\sum_{i=1}^{N_{inj}} \left[f_{ij}I_{i}^{(k)} \right] - J_{j}\tau_{j} \frac{\Delta p_{ij}^{(k)}}{\Delta t_{k}} \right] \right\}$	
CRMIP	$\frac{\mathrm{d}q_{ij}(t)}{\mathrm{d}t} + \frac{1}{\tau_{ij}}q_{ij}(t) = \frac{1}{\tau_{ij}}f_{ij}i_i(t) - J_{ij}\frac{\mathrm{d}p_{wf,j}}{\mathrm{d}t}$ (for $i = 1, 2,, N_{inj}$ and $j = 1, 2,, N_{pro}$) $\tau_{ij} = \left(\frac{c_t V_P}{J}\right)_{ij}$	$q_{ij}(t_k) = q_{ij}(t_{k-1})e^{\frac{-\Delta t_k}{\tau_{ij}}} + \left(1 - e^{\frac{-\Delta t_k}{\tau_{ij}}}\right) \left[f_{ij}I_i^{(k)} - J_{ij}\tau_{ij}\frac{\Delta p_{ij}^{(k)}}{\Delta t_k}\right]$	$q_{ij}(t_{n}) = q_{ij}(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} + \sum_{k=1}^{n} \left\{ \left(1 - e^{\frac{-\Delta t_{k}}{\tau_{ij}}}\right) \left[f_{ij}I^{(k)} - J_{ij}\tau_{ij} \frac{\Delta p_{v(j,j)}^{(k)}}{\Delta t_{k}} \right] e^{-(\frac{t_{n}-t_{k}}{\tau_{ij}})} \right\}$	
CRM- Block with equal τ	Same as CRMIP, solved on each block for M_{ij} number of blocks between injector <i>i</i> and producer <i>j</i>	$ q_{ij}(t) = \sum_{b=1}^{M_{ij}} \left[q_{ij,b}(t_0) e^{\frac{-(t-t_0)}{\tau_{ij}}} \left(1 - e^{\frac{-(t-t_0)}{\tau_{ij}}} \right)^{M_{ij}-b} \right] $ + $f_{ij} i_i(t) \left(1 - e^{\frac{-(t-t_0)}{\tau_{ij}}} \right)^{M_{ij}} $	Not available	

Table 3-3: A brief summary of different capacitance resistive model differential equations and solutions.

Chapter 4: OIL FRACTIONAL-FLOW MODELS

Application of the developed capacitance resistive models (CRM) in the previous chapter led to an estimation of the total liquid production; thus one must evaluate the fraction of oil production as a function of time to predict the oil production rate. Our focus in this chapter is to develop fractional-flow models to evaluate oil fractional-flow as a function of time for continuous immiscible water and miscible CO_2 floods.

Throughout the production history of a field, the oil cut varies over time due to changes in recovery processes, as shown in Fig. 4-1. During primary recovery, the field production rate, which mainly consists of oil, decreases and the oil fractional-flow is nominally equal to one. During secondary recovery, such as waterflooding, the reservoir oil saturation decreases with time. Ideally before the breakthrough of the flooding agent, only oil is produced and the oil fractional-flow is at its maximum value. However, as soon as the flooding agent breaks through at the producers the oil cut decreases monotonically and the reservoir oil saturation can ultimately reduce to the residual oil saturation of secondary recovery, *S*_{orw}. Unless the secondary recovery process is followed by a tertiary recovery process eventually the oil cut will go to zero. During tertiary recovery, such as CO₂ flooding, the residual oil saturation partially or totally is mobilized and normally the cut increases to a peak and then declines to zero. If the field residual oil saturation reduces to zero after tertiary recovery, the cumulative oil production, *N*_P, at the end of tertiary recovery should be equal to the original oil in place.

Total production rate during secondary or tertiary recoveries are obtained easily by CRMs. Then, combining total production estimation with an oil fractional-flow model enables us to match the oil production history and estimate the oil production rate for secondary and tertiary recoveries for a producer, a group of producers, or the entire field. We use historical oil production rate or oil cut to find oil fractional-flow model parameters, and then use these models to predict future reservoir performance.



Figure 4-1: Schematic oil cut as a function of time during primary, secondary and tertiary recoveries.

4.1- IMMISCIBLE OIL FRACTIONAL-FLOW MODEL

During an immiscible flood such as water flooding, the flooding agent replaces recoverable reservoir oil by contacting and displacing it. In general, the oil cut decreases monotonically from one to zero during secondary recovery. Immiscible oil fractionalflow models are either based on saturation front propagation or empirical fractional-flow models as it is discussed here.

4.1.1- Buckley-Leverett Based Fractional Flow Models, (BLBFFM)

Based on Buckley-Leverett fractional-flow model we predict the waterflood oil fractional-flow as a function of saturation. Leverett (1941) introduced a fractional-flow model in an immiscible two phase of water and oil system:

$$f_{w} = \frac{1 + \frac{kk_{ro}A}{q\mu_{o}} \left(\frac{\partial P_{c}}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \frac{k_{ro}\mu_{w}}{k_{rw}\mu_{o}}} = \frac{1 + \frac{kk_{ro}(S)A}{q\mu_{o}} \left(\frac{\partial P_{c}}{\partial x} - \Delta\rho g \sin\alpha\right)}{1 + \frac{k_{ro}(S)\mu_{w}}{k_{rw}(S)\mu_{o}}}$$
(4-1)

Eq. 4-1 implicitly contains time as variation of saturations in relative permeability terms. Based on different form of relative permeability curves, and considering the effect of other controlling parameters such as gravity and capillary pressures we can determine oil fractional-flow models as a function of normalized water saturation. Normalized water saturation is defined as

$$S(t) = \frac{S_w(t) - S_{wr}}{1 - S_{or} - S_{wr}}$$
(4-2)

where $S_w(t)$ is the reservoir water saturation and is a function of time. By determining values of the initial and residual water and oil saturations, we can use water injection and oil/water production data to determine the normalized average water saturation in the reservoir over time. The average water saturation in the reservoir changes by the rate of water accumulation in the reservoir for each production interval:

$$\overline{S}_{w}(t_{n}) = \overline{S}_{w}(t_{n-1}) + \frac{\left(i(t_{n}) - q_{w}(t_{n})\right)}{V_{P}}\Delta t_{n}$$

$$(4-3)$$

The last step to evaluate the oil/water fractional-flow is to determine the relative permeability of oil and water as a function of saturation. Several empirical relationship based on laboratory measurements are obtained for relative permeability as a function of saturation. Therefore to estimate oil/water fractional-flow, the following parameters must be known:

- a) Residual water saturation, S_{wr} ,
- b) Residual oil saturation, *S*_{or},
- c) Pore volume, $V_{P_{i}}$
- d) Relative permeability parameters.

These parameters can be determined by history-matching.

By neglecting capillary pressure for horizontal reservoirs and using power-law model for relative permeability curves (Brooks and Corey, 1964) Eq. 4-1 can be modified to present the oil fractional-flow as

BLBFFM(
$$M_o, m, n$$
): $f_o(S) = 1 - \left[1 + \frac{(1-S)^m}{M_o S^n}\right]^{-1}$ (4-4)

where *m* and *n* are relative-permeability exponents, and M_o is the endpoint mobility ratio:

$$M_o = \frac{k_{rw}^o \mu_o}{k_{ro}^o \mu_w}$$
(4-5)

End-point mobility ratio, M_o , and the relative-permeability curve exponents, m and n, are the unknowns in the Buckley-Leverett based fractional-flow model (BLBFFM), Eq. 4-4, designated by BLBFFM(M_o , m, n). Note that V_P , S_{wr} and S_{or} are also Eq. 4-4 model parameters and can be evaluated during history-matching which makes the total number of unknowns to be evaluated in a BLBFFM to be six unknowns: V_P , S_{wr} , S_{or} , M_o , m and n.

We consider two major forms of the BLBFFM by considering linear (m = n = 1) or nonlinear relative permeability curves ($m \neq 1, n \neq 1$). In the BLBFFMs, there are limits on some of the unknown parameters that facilitate the fitting. For instance, *S* should be between zero and one. Note that the buoyancy term or the effect of the capillary pressure can also be added to the BLBFFM if needed.

As mentioned before we find sets of equiprobable history-match solutions (EPHMSs) that satisfy oil and total production history. These sets of reservoir parameters can be evaluated efficiently by BLBFFMs and used to confine the uncertainty associated with reservoir parameters. Probability/cumulative distribution functions (PDF/CDF) can be determined for model parameters and related reservoir characteristics can be evaluated by sets of EPHMS.

4.1.2- Empirical Models

Estimation of saturations as a function of time can be hard to evaluate for multiwell system; therefore, we use an empirical oil fractional-flow model especially when the objective is reservoir performance estimation and optimization rather than reservoir characteristics evaluation. Different empirical fractional-flow models that have less calculation effort, can replace the traditional BLBFFM. Some of these fractional-flow models are presented in Table 4-1; *a*, *b*, *c*, α , and β are model constants.

The fractional-flow models presented in Table 4-1 are either a function of cumulative oil production, N_p , or the average reservoir water saturation, \overline{S}_w , which are unknowns during a forward estimation. Therefore, one must estimate either of these two variables prior to any estimation of field performance. The average reservoir saturation can be written as a function of cumulative oil production as

$$\overline{S}_{w} = S_{wi} + (1 - S_{wi}) \frac{N_{P}}{N}$$
(4-6)

where S_{wi} is the initial water saturation and N is the original oil in place.

	Developed by	Oil Fractional-flow Model	Equation Number
1	Timmermann (1971)	$f_o = \left(1 + 10^{-(a+bN_P)}\right)^{-1}$	Eq. (4-7)
2	Makszimov (1959)	$f_o = \left(1 + ba^{N_P} \ln a\right)^{-1}$	Eq. (4-8)
3	Craft-Hawkins (1959); Ershagi-Omoregie (1978)	$f_o = \left(1 + ae^{bS_w}\right)^{-1}$	Eq. (4-9)
4	Kazakov (1976)	$f_o = \frac{(1 - bN_P)^2}{(a - 2bN_P)(1 - bN_P) + bN_P(a - bN_P)}$	Eq. (4-10)
5	Gunkel Marsal-Philip (1968)	$f_o = \left(1 + a + (1 + cN_P)be^{cN_P}\right)$	Eq. (4-11)
6	Gentil (2005)	$f_o(t) = \frac{1}{1 + F_{wo}} = \frac{1}{1 + \alpha W_i^{\beta}}$	Eq. (4-12)

Table 4-1: Some of empirical oil fractional-flow models (for details see Papay 2003)

Gentil (2005) introduced an empirical power-law fractional-flow model (EPLFFM) to estimate the water-oil ratio and, consequently, the oil fractional-flow as a function of cumulative water injection, Eq. 4-12. Liang et al. (2007) used this approach to predict the oil production rate, which considers a power-law relationship between the instantaneous water-oil ratio, F_{wo} , and cumulative water injected, W_i . In Eq. 4-12, the constants α and β can be evaluated from the oil production history for any producer, group of producers, or the entire field. After evaluating these constants, Eq. 4-12 is used for oil-production estimation. For a system of one injector and one producer or as in the CRMT, Eq. 4-12 can be applied directly. Therefore, we can match the field oil-production history by combining Eq. 4-12 with Eq. 3-17, which gives

$$q_o(t) = \frac{q(t)}{1 + \alpha_F W_i^{\beta_F}} \tag{4-13}$$

For the CRMT, the oil production for the field can be written as

$$q_{o}(t) = \frac{q(t_{0})e^{-(\frac{t_{n}-t_{0}}{\tau})} + \sum_{k=1}^{n} \left\{ \left(1 - e^{\frac{-\Delta t_{k}}{\tau}}\right) \left[I^{k} - J\tau \frac{\Delta p_{w^{f}}^{k}}{\Delta t_{k}}\right] e^{-(\frac{t_{n}-t_{k}}{\tau})} \right\} - \frac{1 + \alpha W_{i}^{\beta}}{1 + \alpha W_{i}^{\beta}}$$

$$(4-14)$$

Applying modifications to the cumulative water injection term in Eq. 4-12, we can extend its application to other CRMs. For instance, the fractional-oil flow for producer j for CRMP can be written as

$$f_{o,j}(t) = \left[1 + \alpha_j \left(\int_{t=t_0}^{t=t} \left(\sum_{i=1}^{N_i} f_{ij} i_i(\xi)\right) \mathrm{d}\xi\right)^{\beta_j}\right]^{-1}$$
(4-15)

and for CRMIP as

$$f_{o,j}(t) = \left[1 + \sum_{i=1}^{N_i} \alpha_{ij} \left(\int_{t=t_0}^{t=t} f_{ij} i_i(\xi) \mathrm{d}\xi\right)^{\beta_{ij}}\right]^{-1}$$
(4-16)

Note that Eq. 4-15 has only two fitting parameters for each producer, α_j and β_j , but Eq. 4-16 has two fitting parameters, α_{ij} and β_{ij} , for each injector included in the model. We can match the oil production history by combining any of the fractional-flow calculations with the total production of each producer or entire field production. After evaluating the constants, the prediction of future performance of the field for any injection rate as inputs and oil production rates as outputs becomes feasible. The cumulative water injected, W_i , for any producer in the field can be evaluated by cumulative total production of CRM to account for response delay in the injection signal. Therefore, Eqs. 4-15 and 4-16 can be written as

$$f_{o,j}(t) = \left[1 + \alpha_j \left(\int_{t=t_0}^{t=t} q_j(\xi) \mathrm{d}\xi\right)^{\beta_j}\right]^{-1}$$
(4-17)

For CRMP, and for CRMIP as

$$f_{o,j}(t) = \left[1 + \sum_{i=1}^{N_i} \alpha_{ij} \left(\int_{t=t_0}^{t=t} q_{ij}(\xi) d\xi\right)^{\beta_{ij}}\right]^{-1}$$
(4-18)

4.2- CO₂ MISCIBLE OIL FRACTIONAL-FLOW MODEL

During a miscible flood as a tertiary recovery mechanism, additional oil recovery is obtained mainly by mobilizing the residual or trapped oil in the reservoir. The more contacted oil by the miscible flood agent, the higher the ultimate recovery. Ideally, all the remaining oil should be recovered after miscible front breakthrough at the producers, and the residual oil saturation should decrease to zero. The oil production rate normally increases as a function of time as more trapped/residual oil is mobilized and the oil bank breaks through. Finally, the oil production reaches its maximum and then decline begins and eventually the oil rate reaches zero as either miscibility is lost or the residual/trapped oil supply vanishes, Fig. 4-1. Immiscible floods need a fractional-flow model for CO_2 -oil miscible flood as a function of time to split the share of oil production rate from the CRM's total production rate estimation. We use two approaches, 1) K-factor method or Koval Model and 2) an empirical model based on the logistic equation.

4.2.1- Koval Method/ K-factor

The Koval model or the K-factor method is one of the earliest approaches in modeling oil/solvent fractional-flow by modifying the Buckley and Leverett method for immiscible displacement for a miscible flood. In his approach Koval (1963) modified the ratio of oil to solvent viscosities, μ_o/μ_s , in the Leverett (1941) fractional-flow equation by effective viscosity, *E*, for oil-solvent mixing zone based on Blackwell et al., (1959) data using the following correlation, Eq. 4-19.

$$E = \left[0.78 + 0.22 \left(\frac{\mu_o}{\mu_s}\right)^{0.25}\right]^4 \tag{4-19}$$

By neglecting the effect of gravity, and capillary pressure in the Leverett fractional-flow model, Eq. 4-1, the Koval modified solvent fractional-flow model as

$$f_s = \frac{1}{1 + \frac{1 - S_s}{S_s} \frac{1}{K}}$$
(4-20)

where S_s is the solvent saturation and K=EH. The heterogeneity factor, H is a measure of media heterogeneity. If a rock is homogenous then H=1; if the media is heterogeneous then H>1. Koval (1963) presented solvent fractional-flow model at/after the breakthrough

as a function of pore volume injected, V_{Pi} , for K=1.5 to K=10 based on the following expression:

$$f_s = \frac{K - (K/V_{P_i})^{0.5}}{K - 1} \tag{4-21}$$

The pore volume injected at the time of solvent breakthrough is 1/K, and solvent fractional-flow will be one when the pore volume injected is equal to K. Based on the Koval Model, the cumulative oil production, as a function of injected pore volume and Koval factor is

$$N_P(t) = N_P(V_{P_i}) = \frac{2(K/V_{P_i})^{0.5} - 1 - V_{P_i}}{K - 1}$$
(4-22)

4.2.2- Logistic Equation Based Fractional-flow Model (LEBFFMs)

Production of any exhaustible resource as a function of time can be modeled by a bell shaped function. Hubbert (1956, 1962), based on rates of increasing oil exploration and production, and decreasing of oil reserves, estimated that United State oil production would peak around 1972; the actual peak year was 1970. Hubbert used the logistic equation (LE) to capture the bell shape behavior of oil production in his analysis. The LE that Hubbert used was originally introduced by Verhulst in 1838.

$$\frac{\mathrm{d}N_P}{\mathrm{d}t} = rN_P(1 - \frac{N_P}{N}) \tag{4-23}$$

where N_P is the amount of discovered/produced oil and N is the total amount of oil (discovered and not discovered) in place, and r is the intrinsic growth or decline rate. Analytical solution for this equation has the following exponential form:

$$N_{P}(t) = \frac{N}{1 + \exp[r(t_{peak} - t)]}$$
(4-24)

which represents the amount of oil produced from the reserve. At peak time, half of the reserve is produced. Oil rate as a function of time can be calculated simply from Eq, 4-23 by replacing $N_P(t)$ from Eq. 4-24.

$$q_{o}(t) = \frac{dN_{P}}{dt} = \frac{rN \exp[r(t_{peak} - t)]}{\left(1 + \exp[r(t_{peak} - t)]\right)^{2}}$$
(4-25)

Later on Deffeyes (2001) added production of Alaska and offshore oil fields to U.S. oil production and showed that a Gaussian equation provides a better fit to U.S. oil production compared to the logistic curve model. A Gaussian model for oil production rate can be defined with three parameters:

$$q_o(t) = q_o(t_{peak}) \exp\left(\frac{(t - t_{peak})^2}{2\sigma^2}\right)$$
(4-26)

where $q_o(t_{peak})$ is the maximum production peak value, t_{peak} is the time of maximum production, and σ is the standard deviation of the production curve. Integration Eq. 4-26 over the productivity period provides the cumulative oil production as a function of time:

$$N_{P}(t) = \int_{t=t_{0}}^{t} q_{o}(\xi) d\xi = q_{o}(t_{peak}) \int_{t=t_{0}}^{t} \exp\left(\frac{(\xi - t_{peak})^{2}}{2\sigma^{2}}\right) d\xi = V_{P}\left(S_{oi} - \overline{S}_{o}(t)\right)$$
(4-27)

where V_P is the reservoir pore volume, and S_{oi} is the initial reservoir oil saturation and $S_o(t)$ is the average reservoir saturation at time *t*.

Equation 4-26 is symmetric, but oil production rate during tertiary recovery is normally asymmetric. Therefore, an asymmetric Hubbert form of Eq. 4-26 can be defined by modifying the standard deviation as a function of time:

$$\sigma(t) = \sigma_{dec} - \frac{\sigma_{dec} - \sigma_{inc}}{1 + \exp(\kappa(1 - t_{peak}))}$$
(4-28)

where σ_{inc} and σ_{dec} represent the standard deviation of increase and decay of the oil production rate, respectively, as was presented by Brandt (2007).

The logistic equation provides a reasonable and fundamental approach for estimating the oil rate as a function of time for a known reservoir with finite recoverable oil in place. We use the logistic equation to develop an oil fractional-flow model for CO_2 flooding. In general, the LE approach can be applied in predicting the oil production for any type of enhanced oil recovery (EOR) processes.

4.2.2.1- Generalized Logistic Equation

The logistic equation was introduced by Verhulst in 1838 to model the population growth and has been used extensively in biology (Bertalanffy 1938, Richards 1959). Tsoularis and Wallace (2002) proposed a general form for logistic growth equation that incorporates all previously reported of logistic curves as special cases. They adopted the term generalized logistic equation (GLE) from Nelder (1961) for the following equation:

$$LE(\alpha,\beta,\gamma): \qquad \frac{\mathrm{d}N(t)}{\mathrm{d}t} = r\left(N(t)\right)^{\alpha} \left[1 - \left(\frac{N(t)}{K}\right)^{\beta}\right]^{\gamma} \tag{4-29}$$

where α , β , and γ are positive real numbers; r is the intrinsic growth or decline rate and K is the carrying capacity and N is the population size at time t. The population will ultimately reach its carrying capacity, K, when time, t, goes to infinity.

The LE(α , β , γ) does not in general admit an analytic solution, but special cases of analytical solutions are reported in the literature, Table 4-2. See Tsoularis and Wallace (2002) for more details and analytic form of time and rate of growth at the inflection point and the maximum growth rate. For instance exponential growth (r > 0) or decline (r < 0) are achieved if $\alpha = 1$ and $\beta = \gamma = 0$, represented by LE(1, 0, 0):

$$LE(1,0,0): \qquad \frac{dN}{dt} = rN \tag{4-30}$$

which has the following analytical solution:

$$N(t) = N_0 e^{rt} (4-31)$$

Analytical solutions for special cases of the LE, presented in Table 4-2, can be adapted to determine the oil rate as a function of time. Ultimate recovery of an oil reservoir can be simply modeled by LE if we replace the population size, N(t), with cumulative oil production, $N_P(t)$ and replace carrying capacity, K, with the recoverable oil in place, *ROIP*. As a result, the generalized form of the LE for the cumulative oil production from an exhaustible source can be written as

$$q_{o}(t) = \frac{\mathrm{d}N_{P}(t)}{\mathrm{d}t} = r\left(N_{P}(t)\right)^{\alpha} \left[1 - \left(\frac{N_{P}(t)}{ROIP}\right)^{\beta}\right]^{\gamma}$$
(4-32)

Different forms of Eq. 4-32 can be implemented to model the oil rate as a function of time for any recovery processes. For those LE with analytic solutions, the oil rate can be simply expressed as a function of time by replacing cumulative oil production, $N_P(t)$, in Eq. 4-32.

	Logistic equation	Equation form	Analytic solution	Introduced by
1	LE (1,0,0)	$\frac{\mathrm{d}N}{\mathrm{d}t} = rN$	$N(t) = N_0 e^{rt}$	
2	LE(1,1,1)	$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1 - \frac{N}{K}\right)$	$N(t) = \frac{K}{(K / N_0 - 1)e^{-rt} + 1}$	Verhulst (1838)
3	$LE(\frac{2}{3},\frac{1}{3},1)$	$\frac{\mathrm{d}N}{\mathrm{d}t} = rN^{\frac{2}{3}} \left[1 - \left(\frac{N}{K}\right)^{\frac{1}{3}} \right]$	$N(t) = K \left[1 - \left(1 - \left(\frac{N_0}{K} \right)^{\frac{1}{3}} \right) e^{-\left(rt/3K^{\frac{1}{3}} \right)} \right]$	Von Bertalanffy (1938)
4	LE(1, β, 1)	$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left[1 - \left(\frac{N}{K}\right)^{\beta}\right]$	$N(t) = \frac{N_0 K}{\left[N_0^{\ \beta} + \left(K^{\ \beta} - N_0^{\ \beta}\right)e^{-\beta rt}\right]^{1/\beta}}$	Richard (1959)
5	LE(1-1/ <i>n</i> ,1,1+1/ <i>n</i>)	$\frac{\mathrm{d}N}{\mathrm{d}t} = rN^{1-\frac{1}{n}} \left[1 - \frac{N}{K}\right]^{1+\frac{1}{n}}; r = n(K/b)^{1/n}$	$N(t) = \frac{K(t+a)^n}{b+(t+a)^n}$	Blumberg (1968)
6	LE(1+ β (1- γ), β , γ) for 1+1/ β > γ ; β , γ >0	$\frac{\mathrm{d}N}{\mathrm{d}t} = rN^{1+\beta(1-\gamma)} \left[1 - \left(\frac{N}{K}\right)^{\beta}\right]^{\gamma}$	$N(t) = K \left[1 + \left[(\gamma - 1)\beta r K^{\beta(1-\gamma)} t + \left[(K/N_0)^{\beta} - 1 \right]^{1-\gamma} \right]^{1/(1-\gamma)} \right]^{1/\beta}$	Turner et al. (1976)
7	Generalized Gompetz Logistic Equation	$\frac{dN}{dt} = rN \left[\ln\left(\frac{K}{N}\right) \right]^{\gamma}$	$N(t) = K \exp\left[-\left\{(\gamma - 1)rt + \left[\ln\left(K / N_0\right)\right]^{1 - \gamma}\right\}^{1/(1 - \gamma)}\right]$ $N(t) = K\left(\frac{N_0}{K}\right)^{e^{-n}} for \qquad \gamma = 1$	Turner et al. (1976)

Table 4-2: A summary of special forms of the logistic equations and their analytic solutions suitable for oil production rate estimation.

Note: To adapt these solutions for oil rate estimation, replace N with N_p and K with Recoverable Oil in Place (ROIP).

4.3- SUMMARY

In this chapter, different forms of the fractional-flow model for miscible and immiscible flood were presented. We use the empirical power-law fractional-flow model (EPLFFM) as our preferred fractional-flow model in the course of history-matching and optimization of waterfloods, presented in Chapters 5 and 6. The Leverett based fractional-flow models (BLBFFM) are suitable for uncertainty evaluation. Chapter 7 presents application of these fractional-flow models. The logistic equation based fractional-flow models (LEBFFM) have the flexibility to be applied for any flooding processes. We use LEBFFM for miscible floods.

Chapter 5: SYNTHETIC CASE STUDIES, HISTORY-MATCHING AND OPTIMIZATION

In this chapter validation and application of capacitance resistive models (CRMs) for history-matching and their application for oil production optimization are demonstrated based on numerical simulation results. Production and injection data from numerical simulators such as Eclipse and CMG are treated as field data and the CRMs and oil fractional-flow models are used to match historical total as well as oil production rates. Then based on an optimization objective the injection rates are adjusted to optimize reservoir performance. Figure 5-1 shows the workflow for history-matching and optimization/prediction by the CRMs.



Figure 5-1: Workflow for the CRM application in history-matching and optimization.

We demonstrate the capability of the CRMs in capturing interwell connectivities and history-matching of the total and oil production rates for five case studies, and perform oil production optimization for two of these. We present applications of the CRM by combining its results with an empirical power law oil-fractional-flow model (EPLFFM) for water and with the logistic equation fractional-flow model (LEFFM) for carbon dioxide (CO₂) floods. The EPLFFM, introduced by Gentil (2005) and developed by Liang et al. (2007) for the CRMs, allows the maximization of oil production rates by reallocating water amongst the injectors.

Since fields are normally operated at maximum injection capacity, the most practical and best injection signal to be introduced in a reservoir is to shut in different injectors at different time intervals. This strategy introduces a unique injection pulse in the reservoir, which ensures the reliability of the CRM parameters evaluation. Therefore, for most of the following simulated case studies, unique injection shut in pulses (UISPs) are introduced deliberately for calibrating the CRMs in the numerical simulation models.

5.1- CRM INITIALIZATION, HISTORY-MATCH, AND OPTIMIZATION

Initializing the CRM parameters correctly lowers the work load of historymatching. For case studies in the chapter, first the tank model representations of the CRM, CRMT, are performed to obtain general field information. The CRMT are used to identify the existence of other sources of supports, such as an aquifer or out of pattern injectors. The time constant obtained from the CRMT is used to initialize the time constants for the producer control volume base CRM, CRMP or injector-producer control volume base CRM, CRMIP. The connectivities, f_{ij} 's, can also be initialized based on the inverse of the distance between different producers and the injector,

$$f_{ij}^{*} = \frac{1/d_{ij}}{\sum_{j=1}^{N_{pro}} 1/d_{ij}}$$
(5-1)

where d_{ij} is the distance between injector *i* and producer *j*. For very close well-pair, d_{ij} is very small, which causes f_{ij}^* to approach unity; for very far producers d_{ij} is very large and gives an estimation of zero for f_{ij}^* .

Minimizing the difference between the CRMs' response and the simulated production rate is the objective function during the course of history-matching. Historymatching for both total and oil production rates are performed by minimizing either the percentage of average relative error (ARE) or the mean of square errors (MSE) between historical numerical simulation data and the CRMs' estimations. The ARE is

$$ARE = \frac{\sum_{n=1}^{N_{data}} \left| \frac{q_{obs} - q_{est}}{q_{obs}} \right|}{N_{data}} \times 100$$
(5-2)

and the MSE is

$$MSE = \frac{\sum_{n=1}^{N_{data}} (q_{obs} - q_{est})^2}{N_{data}}$$
(5-3)

where q_{obs} and q_{est} represent the observed and estimated flow rates and N_{data} is the number of rate data points.

The CRMs' production responses have an exponential form; therefore, the nonlinear gradient base solvers in Microsoft Excel (GRG solver) or the general algebraic modeling system (GAMS) software are used to minimize these errors during history-matching. Solver in Excel 2003 can handle up to 200 unknowns for a nonlinear problem which makes it suitable for case studies with a few wells, which results in few fitting

parameters. Problems with more than 200 unknowns can be solved with the premium solver platform (see <u>www.solver.com</u> for details) in Excel. Compared to the solver in Excel, GAMS is designed for handling large scale modeling problems and applications (see <u>www.gams.com</u> for details).

Different objective functions can be specified for optimizing future reservoir performances. Some of objective functions:

- 1- Maximizing cumulative field oil production for a fixed time interval, such as a year, by reallocating field injection while maintaining the same total injection rate in the field (applied in the work).
- 2- Minimizing cumulative field water production over a specific time domain by reallocating field injection while maintaining the same total injection rate in the field.
- 3- Maximizing future net present value by considering the cost of injection and water disposal (Liang et al., 2007).
- 4- Maintaining a specific field oil production rate while minimizing field cumulative non-hydrocarbon phase production.
- 5- Maximizing fluid storage in a reservoir e.g., CO₂ sequestration.

As indicated in this work, we use the first optimization objective function for two of the simulated case studies.

5.2- SIMULATED CASE STUDIES

Each of the simulated case studies or synfields has different characteristics and is selected to test the CRMs capabilities in different aspects of its applications. These case studies are: 1) a sector model of peripheral water injection, Synfield-1, 2) an 18 acre

seven-spot pattern, Synfield-2, 3) a 40 acre five-spot pattern, Synfield-3, 4) the streak case, and 5) the MESL case. For the first three examples the UISPs are used to calibrate the CRM. And for the last two cases, rich injection signals with frequent fluctuations are used to calibrate the CRMs.

5.2.1- Synfield-1: Sector Model of a Peripheral Water Injection

In the peripheral case study, UISPs are applied to calibrate the CRMs' parameters and correlation between injector-producer well-pair distances and connectivities are obtained. These correlations can be used to evaluate new producer connectivities based on its relative location to field injectors.

5.2.1.1- General Information

This example is a sector model of peripheral water injection, which consists of 16 injectors and 32 producers, all vertical wells. The model has $35 \times 57 \times 5$ gridblocks in the x, y, and z directions. Average reservoir properties are given in Table 5-1. Injectors are located in the eastern and western sides of a homogenous reservoir with porosity of 0.18 and horizontal and vertical permeabilities of 40 and 4 md, respectively. All of the wells are vertical and perforated in all of the layers. Producers are on a constant bottomhole pressure constraint. To account for injection losses and supports out of the pattern, other injectors and producers are considered in the numerical simulation model in the northern and southern regions of the sector model but are excluded from the CRM history-match. Figure 5-2 shows the well location map and the region considered in the CRM evaluation for this case study.

Parameter	Value
Number of gridblocks	$N_x=35, N_y=57, N_z=5$
Gridblock size, ft	$\Delta x = 80, \Delta y = 40, \Delta z = 12$
Oil viscosity, cp	2.0
Water viscosity, cp	0.5

Table 5-1: Reservoir and fluid properties for Synfield-1.

Except for the time that an injector is deliberately shut in to introduce a UISP into the field to calibrate the CRM, all the injectors contribute equally with an injection rate of 1000 RB/D. Shut in periods for the injectors are 10 days and no two injectors are shut in at the same time. Figure 5-3 shows the total injection and production rates, as well as the UISP associated with each injector considered in the sector model. Out of sector injectors cause the total production to fluctuate at late time and also reduces the total injection rate.



Figure 5-2: Well location map for 16 injectors, I1 to I16 and 32 producers, P1 to P32 in the sector model of a peripheral waterflood, Synfield-1.



Figure 5-3: Injection and total production rates in the sector model, USIPs from different injectors are indicated.

5.2.1.2- History-matching

The CRMT and CRMP total and oil production rate matches are presented in Figs. 5-4 and 5-5. The production history up to 600 days of production is considered to calibrate both CRMT and CRMP to avoid the impact of out of pattern injectors. Both the CRMT and CRMP match the total and oil production rates satisfactorily. The time constant obtained from the CRMT is 2.27 days and the relative errors for both the CRMT and the CRMT total production rate history-match are less than 1%. The obtained time constant for the CRMT along with the initial estimation of the f_{ij} in Eq. 5-1, are used as the initial value for the CRMP history-match.

The oil production estimation has about 9% of relative error. The oil rate matches are obtained by using empirical power-law fractional-flow model, EPLFFM(α, β) as described in Chapter 4:

EPLFFM(
$$\alpha,\beta$$
): $q_o(t) = \frac{q(t)}{1 + \alpha W_i^{\beta}}$ (4-19)

The reliability of this oil fractional-flow model depends on the linearity of the logarithm of the water-oil ratio, F_{wo} , and logarithm of cumulative water injected, W_i .



Figure 5-4: The CRMT and CRMP total production matches for the sector model, Synfield-1.



Figure 5-5: The CRMT and CRMP oil production matches for the sector model, Synfield-1.

For the CRMT, only one set of α and β are needed. In the CRMP there are two options for matching the field oil production rate. One can match the field oil production

by matching all individual well oil rates by the EPLFFM, which gives each producer a set of α and β , and then sum all the individual oil rates to calculate field oil production. Or the total production from the CRMP can replace the total production of the CRMT in Eq. 4-19 and only one α and β are evaluated for the field.

Individual producers' total and oil production matches are obtained by the CRMP. Since the producers BHP are kept constant, for 16 injectors and 32 producers, 576 unknowns shall be evaluated for the CRMP: 512 connectivities, f_{ij} ; 32 time constants, τ_j ; and 32 initial production rates, $q_j(t_0)$. Figure 5-6 shows the steady-state connectivity map for the sector model. The time constants range from 2 to 4 days. Connectivities associated with corner injectors and producers are the largest. As expected, larger connectivities are encountered for close injector-producer well-pairs and connectivities for the middle injectors, I4 to I6 and I12-I14, are symmetric.



Figure 5-6: Interwell steady-state connectivities obtained from the CRMP evaluation. Well locations for the sector model of peripheral water injection, Synfield-1.

The flood efficiency can be evaluated easily based on the connectivities and time constants for a single injector/producer or a group of injectors/producers. In this case study, the CRMP connectivities show that 46% of the steady-state injection rate of injector I1 is directed toward producers P1, P2 and P3. Either producers or injectors can be grouped and the contribution of a single injector or a group of injectors can be evaluated on the production rate of a single producer or groups of producers. The relative contribution of injector I1 for producers P1-P8, P8-P16, and P17-P24 are 67%, 16% and 5%, respectively. If injectors I1-I8, I9-I16, as well as producers P1-P8, P9-P16, P17-P24 and P25-P32 are grouped as one injector or producer, as shown in Fig. 5-7, this case study can be treated as a field with two injectors: I_{West} and I_{East} and four producers: L_1 , L_2 , L_3 and L_4 . The CRMP evaluation for this simplified case study shows that 73% and 17% of the injection rate of the injectors in I_{West} are directed toward L_1 and L_2 , respectively.



Figure 5-7: Group contribution of injectors and producers connectivities, Synfield-1.

Figure 5-8 shows the CRMP total production match for producers P1, P2, P9 and P10. The impacts of different injector's UISPs are indicated on these graphs. Injectors I1 to I8 mainly impact the production rates of producers P1 through P8 and then P9 through P18. Because other producers filter the UISPs of injectors I1 through I8, these pulses have minor impact on the production rate of producers P19 through P27 and do not affect the production rate of producers P25 through P32 at all. For the same reason, injectors I9 to I18 UISPs do not impact the production rate of the producer P1 through P8. Small contributions of injectors I9 through I18 can be seen on producers P9 through P18.

The EPLFFM(α_j, β_j) are used to match the oil production of each individual producer by the CRMP. Figure 5-9 shows the oil production match for Producers P1, P2, P9 and P10. The relative error encountered in the CRMP oil rate match is about 5%.



Figure 5-8: Total production match for producers P1, P2, P9 and P10 by CRMP demonstrating the impact of injector UISPs on individual producer's production rate, Synfield-1.



Figure 5-9: Oil production match for producers P1, P2, P9 and P10 by CRMP based on the EPLFFM, Synfield-1.

5.2.1.3- Connectivities as a Function of Well-pair Distances

Connectivities for new producers can be initialized based on the injectorproducers distance-connectivities correlation. In homogenous reservoirs, as the distance between injector-producer pair increases connectivities decrease. Some insights about the new producer's connectivities, f_{ij} 's, and the well-pair distances, d_{ij} can be obtained based on established historical connectivities.

It is possible to have a reasonable estimation of a new producer's connectivities based on adjacent producers' connectivities. As an example, in Fig. 5-10 the connectivities between injector I1 and I4 and producers in L_1 and L_2 groups are plotted as a function of well-pair distances. Strong correlation between connectivities and well-pair distances is apparent for injector I1 and I4 with producers in L1 and L2. Therefore, new producer connectivities can be estimated by indicating the group of producers to which it belongs and using the connectivity-distance correlations associated with the group of producers.

Producers between an injector-producer pair act as barrier to injection pulses and weaken or completely filter the UISPs. For example, producers P1 through P8 filter the UISPs of I1 and I4 for producers P9 through P16, which causes the connectivities between these two injectors and producers P9 through P16 to be smaller than any of the connectivities of producers P1 through P8, even for the closer producers. Major discontinuities occur in connectivity-distance relationship when other producers exist between an injector-producer pair.

AS strong correlation exists between the CRMP connectivities, f_{ij} 's, and initial connectivities, f_{ij} *'s, evaluated by Eq. 5-1. Despite the strong correlation, the slope of linear correlation is not unity and varies for different injector as shown on Fig. 5-11.



Figure 5-10: Connectivities as a function of distance for injector I1 and I4 with producers P1 through P8, L₁, and P9 through P16, L₂, Synfield-1.


Figure 5-11: Strong linear correlation between the CRMP connectivities and initial connectivities evaluated based on well-pair distances for injectors I1 and I4 with producers P1 to P8, L₁, Synfield-1.

This case study demonstrated: 1) the application of the UISPs in evaluating the CRM parameters, 2) the capabilities of the calibrated CRMT and CRMP in matching historical total and oil production rate by combining it with the EPLFFMs, 3) flood efficiency evaluation for a single/group of injector(s)/producer(s) by grouping them, and 4) the correlation between interwell connectivities and the well-pair distances.

5.2.2- Synfield-2: Seven-Spot Pattern

This case study tests the CRM capabilities in history-matching of total and oil production rates in a heterogeneous reservoir that has errors in the rate measurements. In this case study, the UISPs are used to satisfactorily calibrate the CRM while random rate measurement errors from 5 to 20% are added to the oil and total production simulated rates. The impact of oil rate measurement errors on the EPLFFMs is also examined.

5.2.2.1- General Information

This example is an 18 acre seven-spot pattern of a heterogeneous reservoir model with 17 wells, 10 injectors and 7 producers, as shown in Fig. 5-12. Table 5-2 provides the average reservoir and fluid properties and Table 5-3 shows the statistics for porosity, and permeabilities. The reservoir model is adapted from a field case study in Minas, Indonesia.

All of the wells are identical and completed in the entire 37 ft thickness of the reservoir in all the 17 layers. Except for the 10-day intervals when the UISPs are introduced in the injectors, injection rates are kept constant at 300 RB/D for injectors I1, I3, I8 and I10, and at 600 RB/D for the other injectors. All of the producers are operating at constant bottomhole pressure of 300 psi. Figure 5-13 shows the field injection signal and the production response of different producers to the UISPs. The magnitude of the impacts of the UISPs on the production responses of different producers are proportional to the connectivities between injector-producer well-pairs.



Figure 5-12: Well location map for 10 injectors, I1 to I10 and 7 producers, P1 to P7 in the seven-spot pattern, Synfield-2.

Parameter	Value				
Number of gridblocks	$35 \times 57 \times 5$				
Gridblock size, ft	$\Delta x = 75$, $\Delta y = 75$, variable Δz				
Oil viscosity, cp	8.0				
Water viscosity, cp	1				

Table 5-2: Reservoir and fluid properties for Synfield-2

Table 5-3: Porosity and permeabilities statistics for seven-spot pattern.

	Mean, \overline{X}	Standard Deviation, <i>S</i>	$\overline{X} - S$	$\overline{X} + S$	Coefficient of Variation, Cv
Porosity, fraction	0.285	0.029	0.256	0.541	0.101
$Log(k_x)$	1.823	0.526	1.297	2.349	0.289
k_x , md	66	NA	20	223	0.702
$Log(k_y)$	1.825	0.527	1.298	2.351	0.289
k_{v} , md	67	NA	19.876	225	0.703
$Log(k_z)$	1.524	0.578	0.946	2.103	0.379
k_z , md	33	NA	9	127	0.736



Figure 5-13: Total injection rate of 10 injectors, I1 to I10 and total and oil production rate of 7 producers, P1 to P7 in the seven-spot pattern, Synfield-2. Total rates are in RB/D and oil rate is reported in STB/D.

5.2.2.2- History-matching

During the course of total production history-matching by the CRMP 70 connectivities, 7 time constants and 7 initial production rates are evaluated simultaneously. Since there is no loss of injection out of the pattern and there is no other source of support beside the indicated injectors, we constrain the sum of the connectivities for each injector to unity. History-matching of the individual producers' oil production rates are obtained by using EPLFFMs.

Figs. 5-14 and 5-15 show producers P1, P2, P4 and P7 total and oil production match obtained by the CRMP. The relative errors for oil and total production historymatch for individual producers are less than 3% between the time interval of 180 to 720 days. Table 5-4 provides the CRMP parameters and the α_j and β_j associated with each producers for oil production estimation.



Figure 5-14: Total production rate match by the CRMP for producers P1, P2, P5 and P7, Synfield-2.



Figure 5-15: Oil production rate match by the CRMP for producers P1, P2, P4 and P7 using EPLFFM, Synfield-2.

The connectivity map is in Fig. 5-16. Considering the permeability distribution in the reservoir, small connectivities are obtained in the regions with very low permeability around producers 7, causing the production rate of this producer to be less than any other producer in the field. On the contrary, high permeability around producer P1 causes strong connectivities between this producer and neighboring injectors which makes this producer the biggest producer of the field.

Table 5-4: CRMP parameters evaluated by history-matching using UISPs, Synfield-2.

	$ au_{j,}$ days	f _{1j}	f _{2j}	f _{3j}	f _{4j}	f _{5j}	f _{6j}	f _{7j}	f _{8j}	f _{9j}	f _{10j}	<i>q_j(</i> 0), RB/D	α_j	β_{j}
P1 (j=1)	5.8	0.31	0.19	0.06	0.26	0.11	0.14	0.10	0.13	0.11	0.11	267	6.6E-06	1.015
P2 (j=2)	5.6	0.13	0.24	0.33	0.14	0.28	0.11	0.17	0.10	0.13	0.17	340	3.1E-06	1.058
P3 (<i>j</i> =3)	6.6	0.14	0.07	0.03	0.11	0.04	0.11	0.04	0.15	0.06	0.03	163	6.8E-07	1.214
P4 (<i>j</i> =4)	6.2	0.19	0.25	0.18	0.25	0.21	0.22	0.21	0.17	0.21	0.16	382	2.0E-06	1.065
P5(j=5)	5.3	0.13	0.14	0.29	0.10	0.23	0.10	0.25	0.07	0.14	0.29	282	4.7E-06	1.041
P6 (<i>j</i> =6)	5.6	0.09	0.08	0.06	0.10	0.07	0.26	0.12	0.32	0.24	0.10	236	3.2E-06	1.062
P7 (<i>j</i> =7)	6.0	0.02	0.04	0.04	0.04	0.05	0.07	0.11	0.06	0.11	0.15	156	5.3E-08	1.413
Sum $f_{ij} =$		1	1	1	1	1	1	1	1	1	1			



Figure 5-16: Steady-state connectivity map for seven-spot pattern shows good connectivities in high permeability regions of the field. Low permeability regions are indicated on the map, Synfield-2.

The quality of the oil production matches is controlled by both the total production match and the reliability of the EPLFFMs. Equation 4-21 has a linear form in the logarithmic scale. Therefore, a log-log plot of the water oil ratio (WOR), against cumulative water injected (CWI) for each producer can determine the applicability of the EPLFFMs. Figure 5-17 shows strong linearity on these plots for all the producers in the seven-spot pattern. Therefore, the EPLFFM can provide a good estimation of all the producers oil rate.



Figure 5-17: Log-log plot of water-oil ratio (WOR) against cumulative water injected (CWI) for each producer, P_j , used to determine α_j and β_j in EPLFFM, Synfield-2 (X-axis values are shifted to avoid overlapping of the curves).

The total and oil production rates for the seven-spot pattern are obtained by encountering relative errors of only 1%, as presented in Fig. 5-18 based on the CRMP. As in the first case study, few UISPs have allowed us to obtain a quality match by the CRM for this case study.



Figure 5-18: Total and oil production rate match by the CRMP and EPLFFM for the seven-spot pattern, Synfield-2.

5.2.2.3- Effect of Rate Measurement Error

Rate measurement errors are an inseparable part of all field practices which hinder history-matching and consequently reduce the reliability of future performance estimations. In this case study, random rate measurement errors are added to the total and oil production rate obtained from an Eclipse simulation run to check the reliability of the CRM under rate measurement errors. Increments of 5% relative errors are introduced to the total and oil production rates of each producer. The CRMP along with EPLFFM are used to match the total and oil production history accounting for 5, 10, 15, and 20 percent rate measurement errors, as shown in Fig. 5-19. As long as the UISP's impact on the total production rates are larger than the random error encountered, the CRM can reliably match the production history and establish connectivities between well-pairs.



Figure 5-19: Total and oil production rate match by the CRMP and EPLFFM for the seven-spot pattern considering 5, 10, 15 and 20 percent oil and total rate measurement errors, Synfield-2.

Figure 5-20 shows that despite a 20% total and oil rate measurement errors, a straight line can still be constructed for large CWI for each producer, which enables reliable application of the EPLFFM. Connectivities and time constant obtained in different cases with relative rate measurement errors have strong correlation that indicates the CRM stability for all the cases with rate measurement errors in this example.

Correlation ratio between CRMP connectivities between base case, without random error, and cases with 5, 10, 15, and 20 percent error are 0.99, 0.95, 0.93, and 0.89, respectively. Figure 5-21 shows a cross plot of the connectivities obtained in the CRMP without error compared with case studies with relative errors. The time constants are slightly affected with the random errors and stay very close to the original values of the time constant evaluated by the CRMP base case, Table 5-5.



Figure 5-20: Log-log plot of water-oil ratio (WOR) against cumulative water injected (CWI) for each producer, P_j , shows applicability of the EPLFFM for oil rate match under 20% random error, Synfield-2.



Figure 5-21: Cross plot of connectivities obtained in the CRMP with/without random errors, Synfield-2.

Table 5-5: The CRMP time constants for cases with different percentage of total production relative errors.

Time costants	Base	5% Error	10% Error	15% Error	20% Error
$ au_1$	5.82	6.17	6.30	5.50	5.82
$ au_2$	5.56	5.50	4.75	5.97	6.74
$ au_{3}$	6.62	6.44	6.61	6.69	7.11
$ au_4$	6.22	5.89	5.93	6.28	5.99
${ au}_{5}$	5.31	5.10	5.90	5.42	4.83
$ au_{6}$	5.56	5.52	5.16	5.54	6.66
$ au_7$	6.17	6.42	6.11	5.60	6.96

As shown in this case study, the CRM can consistently perform history-matching under rate measurement uncertainty. As long as the impacts of the injectors' signals on the production rate of producers are larger than the range of the rate measurement error, CRMs can reliably capture reservoir connectivities. This makes the CRM suitable for field case applications where rate measurement errors are inevitable.

5.2.3- Synfield-3: Five-Spot Pattern

This case study is set up by using CMG outputs to demonstrate the CRM capability in history-matching of all oil recovery phases: primary, secondary and tertiary recoveries. A five-spot pattern is selected in which waterflooding, as secondary recovery process, is followed with carbon dioxide (CO_2) flooding, as tertiary recovery process. Two slugs of CO_2 , each lasting for a year are injected during years 7 and 9 when the oil cut during secondary recovery has fallen below 1%. The CRMP is used to match the total production rates and oil matches that are obtained for water flood by EPLFFM and for the CO_2 flood by the logistic equation based fractional-flow model (LEBFFM) presented in Chapter 4.

5.2.3.1- General Information

This case study is a 40-acre inverted five-spot pattern consisting of 10 layers with different porosity and permeabilities with four producers at the corners and one central injector, Fig. 5-22. Table 5-6 provides the average reservoir and fluid properties and Table 5-7 shows the statistics for porosity and permeabilities in this case study.



Figure 5-22: Five-spot pattern well location map, Synfield-3.

Table 5-6: Reservoir and fluid properties for Synfield-3

Parameter	Value
Number of gridblocks	$31 \times 31 \times 10$
Gridblock size, ft	$\Delta x = 42.6, \Delta y = 42.6, \Delta z = 10$
Oil viscosity, cp	8
Water viscosity, cp	0.35

Table 5-7: Layer porosities and permeabilities for Synfield-3

Layer	Porosity	Permeabilities, md
		k_x, k_y, k_z
1	0.12	12.6, 8.5, 4.0
2	0.23	13.3, 13.5, 2.5
3	0.15	15.9, 15.5, 3.74
4	0.28	15.5, 15.7, 3.2
5	0.19	19.7, 13.1, 1.9
6	0.29	14.9, 14.8, 3.5
7	0.16	12.9, 21.3, 1.8
8	0.21	16.3, 16.8, 2.4
9	0.19	13.5, 18.0, 2.7
10	0.24	14.6, 13.3, 3.8

All of the wells are vertical and completed in all layers. Except for a 300 day interval in the 6th year when an injection pulse is introduced by reducing the field injection rate to 2500 RB/D, the injection rates are kept constant at 5000 RB/D. Reduction of the field total injection rate from 5,000 to 2,500 RB/D is to generate a pulse in the reservoir that facilitated the CRM calibration. Producers were operated at a constant bottomhole pressure of 2800 psia to obtain miscibility for the CO₂ flood. Figure 5-23 shows the field total injection signal and production response, as well as the two CO₂ slugs and the oil production of the field. The oil production encounters two increases because of the two slugs of CO₂ injection during years 9 and 12.

The total production shows a reduction around year 3 because of the breakthrough of the minimum mobility saturation.



Figure 5-23: Field total and carbon dioxide, CO₂, injection rates and field total and oil production rates, five-spot pattern, Synfield-3.

5.2.3.2- History-matching

The field and producer one, P1, the total and oil production history-matches based on the CRMP and using the EPLFFM for waterflood and LEBFFM for CO₂ flood, are presented in Figs. 5-24 and 5-25. The field production is simply the sum of all the producers' production. The CRMP results show the symmetry of the pattern in which the connectivities, f_{ij} , for all the producers are equal to 0.25 and the time constants, τ_j , are equal to 122 days. The empirical power law fractional-flow model, EPLFFM (2.63E-07, 6.05), perfectly matches the oil-cut during waterflooding and the two logistic equation curves, LE (1, 1, 1), based on Eq. 4-25 provide the oil rates during the CO₂ flooding intervals.

For each CO_2 slug injection, the Hubbert curves fitting parameters must be determined. The Hubbert model for oil production rate can be defined by

$$q_{o}(t) = \frac{dN_{P}}{dt} = \frac{rN\exp[r(t_{peak} - t)]}{\left(1 + \exp[r(t_{peak} - t)]\right)^{2}}$$
(4-25)

where *r*, the growth/decline rate, *N* the ultimate recoverable oil and t_{peak} are unknowns of the logistic equation and they are determined by history matching for each CO₂ event slug injection. Besides the symmetric Hubbert curves fitting parameters two additional values, t_{delay} , which account for the delays of oil production response to the CO₂ injections, must be evaluated during history matching. These values cause the Hubbert curves to shift along the time axis.

The Hubbert equation parameters are in Table 5-8. The t_{delay} 's are measured from the initiation of the CO₂ slugs injections and the t_{peak} 's are measured from the t_{delay} 's. N is the amount of oil ultimately recoverable for each CO₂ slug. CO₂ remaining in the reservoir from the first slug causes the delay time of the second slug to be shorter and the N to be slightly larger than the N of the first slug.

	Oil response delay <i>t_{delay}</i> , years	Growth or decline rate, <i>r</i> , 1/years	Ultimate recoverable oil, N , $365 \times RB$	Oil peak time, <i>t_{peak}</i> , years
Slug 1	1.05	2.34	1798	1.61
Slug 2	0.86	1.60	1997	1.45

Table 5-8: Hubbert logistic equation parameters for individual producers during CO₂ flooding.



Figure 5-24: History-match of producer P1 total and oil production rates, identical to other producers, by CRMP, EPLFFM for water flood and LEBFFM for CO_2 flood, Synfield-3.



Figure 5-25: History-match of field oil production rates by EPLFFM for water flood and LEBFFM for CO_2 flood, five-spot pattern, Synfield-3.

If this approach were to be applied to continuous CO_2 injection flooding, the cumulative oil production, the area under the Hubbert curve, will represent the residual oil after waterflooding. This example showed the flexibility of the CRM and EPLFFM and LEBFFM in history-matching of the oil production during secondary and tertiary recoveries.

5.2.4- Synfield-4: Streak Case

This case study presents applications of the CRMs: CRMT, CRMP and CRMIP in history-matching of oil and total production, and optimization for a very heterogeneous case study. The streak case was chosen in a continuation of previous studies of Albertoni et al. (2003), Yousef et al. (2006), and Liang et al. (2007). Different aspects of the CRM's capabilities to capture the variation of the injection rate and bottomhole pressure (BHP) of producers are demonstrated. Furthermore, a cumulative oil production rate optimization is attained with the CRMP by reallocating injected water. Thereafter, the optimized injection rates were fed into an Eclipse numerical flow-simulation model to quantify their impact on cumulative field oil production compared to a base case.

5.2.4.1- General Information

The streak case is a synthetic field consisting of five vertical injectors and four vertical producers. Figure 5-26 displays the well locations and the two high-permeability streaks. The permeability is 5 md everywhere except, where the two streaks occur, and a constant porosity of 0.18 is assigned globally. The total mobility ($\lambda_o + \lambda_w$) is 0.45 and independent of saturation. Oil, water, and rock compressibility are 5×10⁻⁶, 1×10⁻⁶ and 1×10⁻⁶ psi⁻¹, respectively.



Figure 5-26: Streak case well location map of five injectors, I1 to I5 and four producers, P1 to P4, and the permeability field consist of two high-permeability streaks of 500 and 1,000 md. (same example as in Yousef, 2006 and Liang, 2007)

Figure 5-27 shows the plots of monthly injection rates for all the injectors for 100 months. The average injection rate for all the injectors is about 1,000 RB/D. Nonetheless, average total production rates for P1 and P4 are dominating the total production, as presented in Fig. 5-28. The BHPs at the producers are kept constant at 250 psia.



Figure 5-27: Individual well injection rate for streak case.



Figure 5-28: Individual well total production rates for streak case.

5.2.4.2- History-matching and Optimization

Instead of using UISPs in this case study, rich injection signals generated based on injection rates of an Argentinean field case study (as presented in Yousef, 2005) are used to calibrate the CRMs. The CRMs are used to match total production based on 100 months of production history. There are just two model parameters for the CRMT, while the number of parameters increases to 28 for CRMP and 60 for CRMIP. In the CRMT, the field time constant, τ_F , is 14 days. Tables 5-9 and 5-10 show fitted parameters for CRMP and CRMIP models based on 100 months of production history, respectively.

Values of f_{ij} 's are powerful quantifiers of connectivity between wells; the time constants represent the delay of production response of a producer to the associated injection rate(s). As expected, because of the heterogeneous permeability field in this reservoir, the highest value of f_{ij} 's for producer P1 and P4 are associated with the two high permeability streaks. In contrast, small time constants associated with producers P1 and P4 for CRMP and associated with the high-perm streaks in the CRMIP represent the quick response of production rates of these two producers to injectors I1 and I3. The

water injected in injector I1 (or I3) almost totally flows along the streak and instantly increases the production rate of producer P1 (or P4).

	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)
f_{1j} (i=1)	0.96	0.01	0.00	0.03
f_{2j} (i=2)	0.47	0.02	0.19	0.32
f_{3j} (i=3)	0.10	0.02	0.02	0.86
$f_{4j} (i=4)$	0.18	0.15	0.00	0.67
$f_{5j} (i=5)$	0.16	0.02	0.19	0.63
$ au_{j,}$ Days	1	34	48	1
q _j (t ₀), RB/D	5996	159	151	6666

Table 5-9: Streak case CRMP parameters, based on injection rate variations.

Table 5-10: Streak case CRMIP parameters, based on injection rate variations.

	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)
f_{1j} (i=1)	0.94	0.01	0.01	0.04
f_{2j} (i=2)	0.51	0.01	0.19	0.30
f_{3j} (i=3)	0.09	0.02	0.03	0.86
f_{4j} (i=4)	0.20	0.15	0.00	0.65
f_{5j} (i=5)	0.13	0.02	0.18	0.67
${ au}_{1j}$, Days	1	19	10	10
${ au}_{2i}$, Days	34	4	41	49
$ au_{\scriptscriptstyle 3j}$, Days	9	23	16	1
$ au_{4\mathrm{j}}$,Days	81	37	3	38
$ au_{\mathit{5j}}$, Days	67	68	46	40
$\boldsymbol{q}_{1j}(\boldsymbol{0}),$ RB/D	5073	107	4	730
$oldsymbol{q}_{2j}(0),$ RB/D	0	68	2	0
q _{3j} (0), RB/D	1295	0	3	3118
$\boldsymbol{q}_{4j}(0)$, RB/D	146	31	102	453
$\boldsymbol{q}_{5j}(0)$, RB/D	0	0	104	0

Figure 5-29 shows the total production match for each of the CRMs with the simulated results of the numerical model. Using CRMP and CRMIP, one can match the total production for any of the producers. For example, Fig. 5-30 shows the total production match for producer P4 from both CRMP and CRMIP.



Figure 5-29: Streak case CRMs match of total field production.



Figure 5-30: CRMP and CRMIP match of total production for Producer P4, streak case.

After obtaining the weights and time constants for the CRMs, EPLFFMs are used to model oil-production. The parameters α_j and β_j for each producer for CRMP are in Table 5-11. These values are associated with the linear section of WOR against cumulative-water injection on a log-log graph for each of the producers. As an example, Fig. 5-31 shows the log-log graph of WOR and cumulative-water injected toward producer P4. The linear section of Fig. 5-31, after 50 percent watercut, resembles the same criterion of applicability of waterflood frontal advance, as discussed by Ershaghi and Abdassah (1984).

Table 5-11: Streak case oil fractional-flow parameters for CRMP.

_	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)
α_{j}	1.0E-05	3.7E-19	8.0E-15	1.1E-14
β_{j}	0.9112	3.3859	2.4541	2.2252



Figure 5-31: WOR and cumulative-water injected for producer P4.

Figures 5-32 and 5-33 show the oil-rate match for the entire field and producer P4, respectively. The mismatch at early time for oil production matches is caused as a result of nonlinearity of the log-log plot of CWI and WOR.



Figure 5-32: Streak case CRMP match of field oil production rate.



Figure 5-33: Streak case CRMP match of P4 oil production rate.

Based on the calibrated CRM and EPLFFMs, as shown in Fig. 5-1, one can match not only the production history, but also optimize the future field performance for a desired injection schedule. In this example, the first 100 months of the production history are used for model calibrations. Then the cumulative oil production rate for a 16-month period by reallocating water injection is maximized. Optimization results suggested that the maximum oil production would occur if injectors I2, I3, and I5 are shut-in and injector I1 and I4 remain active by injecting 3,862 and 4,000 RB/D, respectively. We provided the optimized injection rates into Eclipse and compared the associated oil production of the field with a base case. The base case is the simulated oil production rate for 16 months, if we kept the same injection rate as of the last month of history. Counterintuitive activation of injector I1, associated with the high-permeability streak, is explained by the existence of the majority of the remaining oil around I1 after 100 months of production, as shown in Fig. 5-34. Figure 5-35 shows a 35 percent increase of oil production during the optimized period compared to the base case scenario.



Figure 5-34: Oil saturation map after 100 months of production, streak case.



Figure 5-35: 35% oil production increase by optimized injection rate for CRMP, streak case.

5.2.4.3- BHP Variation and Data Frequency Effect

Considering the producers' BHP variation in the CRM makes it possible to attain productivity indices and capture high frequency fluctuation of the production rates. Productivity index and time constant multiplication provides a direct measure of the pore volume affected with the injection associated with the corresponding time constant: effective control volume of a producer for CRMP or an injector-producer pair in CRMIP.

This example shows simultaneous variation of injection rates and BHP of the producers on the total production for the streak case. In this scenario, the same injection rates as shown in Fig. 5-27 are applied while all the producers BHPs are randomly fixed to a new value with an average of 250 psia every 30 days. Figure 5-36 presents the total production rate match of the CRMP model with simulated data. Spikes in total production correspond to the variation of producers' bottom hole pressure, they are captured by CRMP. For clarity, Fig. 5-36 displays only a 300-day window. Fitting parameters for the CRMP model are presented in Table 5-12.

As before, f_{ij} 's are representative of reservoir heterogeneity. The smaller time constant and larger productivity indices are intuitively associated with wells connected to high-permeability streaks. Since the BHP data are available, in addition to the connectivities and time constant, a measure of each producers' productivity indices can be calculated in the CRM. Back calculated productivity indices associated with producer P1 and P4 are also a reflection of larger effective permeability associated with these producers. Productivity index of the producer P1 which is associated with streak with 1,000 md permeability is almost twice of that of producers P4 which is connected to the streak with 500 md permeability. On the other hand, multiplication of the time constants and productivity indices is a direct measure of the control volume for each of the producers. In this case study producer P1 and P4 have the minimum $\tau_j J_j$, meaning that either they have the smallest affected control volumes, or the compressibility of the fluid within their control volumes is negligible. On contrary, producer P2 and P3 show an estimate of control volumes of at least four times of producers P1 and P4. Based on this simple calculations for a constant compressibility we can conclude: $V_{P1} < V_{P4} < V_{P2} < V_{P3}$. This estimation of the control volume is possible because of including the producers BHP variation in the CRM and availability of the BHP measurements.

The quality of the production matches can be affected considerably by the frequency of the producers rate and BHP measurement, especially for cases that the producers' BHP are changing. To demonstrate this point, Figs. 5-37 and 5-38 show the CRM history-match by daily and monthly production rate measurements, respectively.



Figure 5-36: Field total production and variation of BHP.

	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)
f_{1j} (i=1)	0.83	0.05	0.00	0.12
f_{2j} (i=2)	0.24	0.02	0.37	0.37
f_{3j} (i=3)	0.08	0.05	0.00	0.86
$f_{4j} (i=4)$	0.58	0.01	0.00	0.41
f_{5j} (i=5)	0.42	0.00	0.00	0.58
${ au}_{j}$, Days	0.01	24.88	83.80	0.10
q _j (t ₀), RB/D	0	0	0	0
$oldsymbol{J}_{oldsymbol{j}}$ (RB/D)/psi	20.98	0.18	0.05	9.18

Table 5-12: Streak case CRMP parameters, based on injection rates and producer' BHP variations.

In this example, production rate variations are caused only by producers BHP variations. Figure 5-37 shows that if rate and BHP measurements at the time of the producers BHP changes are recorded, the real fluctuation of the production rates can be captured in history-matching model satisfactorily. On the contrary, Fig. 5-38 shows that if rate and BHP variations measurements are not available, especially at the point when the BHP is changed, the history-matching models can poorly mimic the shape of the field

production variations. The shape of the CRM match is mainly affected by the availability of the BHP data point. Therefore, high frequency rate and BHP measurements are the key to a reliable history-match by CRM or any other model, especially when the production rate is mainly affected with the BHP variations.



Figure 5-37: Total production variation caused by the producer's BHP variations and the CRM match by using daily measurements of rate and BHP.



Figure 5-38: Total production variation caused by the producer's BHP variations and the CRM match by using monthly measurements of rates and BHP.

The streak case example showed the capability of the CRMs to history-match and optimize the reservoir performance in an extremely heterogeneous reservoir. The impacts of the producer's BHP variation as well as the importance of the frequency of the rate and BHP measurements were demonstrated.

5.2.5- MESL Case Study

This example shows application of the CRMs in history-matching and optimization of the performance of a reservoir in Angola based on its numerical model. Optimization is performed by reallocating the field water injection to maximize field cumulative oil production in a year.

5.2.5.1- General Information

This field is located 50 miles offshore at a water depth of 1,300 ft. The oil API gravity is 24 with a viscosity of about 3 cp at the bubblepoint. The average horizontal permeability is approximately 1,400 md, vertical permeability of 166 md, and average porosity of 15%. The model has 159,654 (= $41 \times 59 \times 66$) cells with 40,432 of them being active. Each gridblock has a dimension of 100 m × 100 m. The oil relative permeability of 0.6 occurs at 8% water saturation and the water relative permeability of 0.19 corresponds to a water saturation of 76%.

Peripheral water injection commences at the start of production. The actual number of wells needed is to be guided by the performance of initial wells; however, four injectors and six producers are envisioned at the start, as shown in Fig. 5-39. Two of the producers, P2 and P6, are horizontal. This initial development scenario became the focal

point of our modeling, wherein variable-injection rate and constant-BHP at the producers are assigned.

Imposing a rich injection signal for a period of six years paved the way for estimating model parameters for CRMs and the oil-fractional-flow model. By rich signal we mean high-rate injection with variable rates. For instance, injectors I1, I2, and I3 are injecting at an average of 15,000 RB/D, while 4,000 RB/D is the average injection rate for injector I4. Among the producers, the average rate at P3 is 14,700 RB/D and that at P6 is 2,500 RB/D. These rates translate into 30% and 5% of the total field production at P3 and P6, respectively. Figure 5-40 selectively shows the total production rate for some of the producers and Fig. 5-41 presents the total field water-injection rate and the corresponding total production rate.



Figure 5-39: MESL field reservoir boundary and well locations.



Figure 5-40: Production rate of some producers in the MESL field .



Figure 5-41: Total injection and production rate of the MESL field. The fluctuation in injection rate provides a rich signal in the reservoir.

5.2.5.2- History-matching and Optimization

In this example, there are two unknowns for the CRMT, 36 unknowns for the CRMP, and 72 unknowns for the CRMIP. Based on the CRMT, the field time constant, τ_F , of 140 days was estimated by minimizing the average-absolute error. Figures 5-42 and

5-43 show total production match of the field and producers P5, respectively. Tables 5-13 and 5-14 present the corresponding parameters for CRMP and CRMIP.



Figure 5-42: CRMs total production match for MESL field.



Figure 5-43: CRMs total production match for producer P5.

	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)	P5 (<i>j</i> =5)	P6 (<i>j</i> =6)
f_{1j} (i=1)	0.322	0.492	0.073	0.000	0.114	0.000
f_{2j} (i=2)	0.018	0.055	0.421	0.208	0.291	0.008
f_{3j} (i=3)	0.011	0.091	0.346	0.274	0.179	0.099
f_{4j} $(i=4)$	0.015	0.000	0.339	0.427	0.000	0.219
$ au_{j,}$ Days	53	79	278	283	81	320
$\boldsymbol{q}_j(t_0)$, RB/D	4464	5977	7596	4285	5688	3872

Table 5-13: CRMP parameters, based on injection rate variations.

Table 5-14: CRMIP parameters, based on injection rate variations.

	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)	P5 (<i>j</i> =5)	P6 (<i>j</i> =6)
f_{Ij} (i=1)	0.307	0.407	0.180	0.000	0.106	0.000
f_{2j} (i=2)	0.028	0.105	0.339	0.156	0.302	0.069
f_{3j} (i=3)	0.016	0.123	0.346	0.267	0.192	0.057
f_{4j} (i=4)	0.003	0.000	0.336	0.649	0.011	0.000
${oldsymbol au}_{Ij}$, Days	51	58	412	142	398	70
${oldsymbol au}_{2j}$, Days	68	125	61	125	54	175
${oldsymbol au}_{\scriptscriptstyle 3j}$ Days	164	218	87	145	103	124
${ au}_{_{4\!j},}$ Days	28	30	500	496	18	500
$q_{1j}(0), RB/D$	0	167	0	4792	0	0
$\boldsymbol{q}_{2j}(\boldsymbol{0}),\;$ RB/D	0	0	5179	290	0	0
$q_{3j}(0)$, RB/D	0	0	23	0	0	0
$\boldsymbol{q}_{4j}(0),$ RB/D	12691	14728	0	0	23421	9980

After obtaining the weights and time constants, the EPLFFMs are used to match the oil production rates. The EPLFFM's parameters, α_j and β_j values, are estimated for each producer by minimizing the field wide oil production estimation error, as presented in Table 5-15 for CRMP. Figures 5-44 and 5-45 show oil production rate and its match for entire field and producer P5, respectively.

	P1 (<i>j</i> =1)	P2 (<i>j</i> =2)	P3 (<i>j</i> =3)	P4 (<i>j</i> =4)	P5 (<i>j</i> =5)	P6 (<i>j</i> =6)
α_{j}	6.5E-14	3.2E-15	2.0E-12	5.6E-13	8.9E-13	2.3E-14
β_{j}	1.9276	2.0288	1.6650	1.7742	1.7630	2.0275

Table 5-15: MESL field oil fractional-flow parameters for the CRMP.



Figure 5-44: CRMP oil production match for the MESL field.



Figure 5-45: CRMP oil production match for producer P5.

As presented for the streak model, we used the CRMP to maximize field wide oil production by reallocating injected water while keeping the total field-injection rate constant at the end of year eight. Optimization results suggested shutting in injector I1 while injectors I2 and I3 get their maximum injection rate of 24,000 RB/D and injector I3 stays active with 10,900 RB/D. These optimized injection rates were used in an Eclipse model to compare their impact on field oil production with a base case. Results showed a 6% increase in field oil production compared to the base case in which we kept the total injection rates constant from year 8 to 9, as shown in Figs. 5-46 and 5-47.



Figure 5-46: Relative changes in individual production rates after imposing optimized injection rate for the MESL field.



Figure 5-47: A 6% annual cummulative oil production increase by imposing an optimized injection rate for MESL field.
The MESL case study showed the capability of all the CRMs in evaluating field performance with complex geologic models. History-matching and optimization were performed in a short period of time by using a spreadsheet application of the CRM and only based on injection and production rate data.

5.3- SUMMARY

In this chapter, the CRMs' reliability was tested by using numerical simulation. Five case studies of simulated data to mimic field conditions for different CRMs were presented. History-matching of total and oil productions, flood efficiency evaluation and optimization of reservoir performance were performed by the CRMs and EPLFFM in a spreadsheet application. Generating rich signal to allow system characterization was the main driver behind creating the UISPs, or large perturbations in g rates and pressures. Predictably, field injection and production data contain measurement errors. Therefore, random error was introduced in the simulated rates to demonstrate its impact on estimation of CRM parameters. Optimization of the reservoir performance was demonstrated for two case studies by reallocation of the field injection rates. The CRM optimized results were validated with the numerical simulation models.

As in grid-based history-matching, CRM solutions may be nonunique. That is, multiple solutions may exist for the match of similar quality. Several approaches may be taken to obviate this issue. We found domain analysis, in terms of both time and space, to be very useful. For instance, for a reservoir of long production/injection history coupled with complex well-drilling sequence, segmenting the problem into multiple time domains become very useful. Similarly, segmenting the reservoir in spatial domain, commensurate with drilling history, makes the problem tractable.

Chapter 6: FIELD APPLICATIONS

To predict or optimize a reservoir performance, one must build a reservoir model that can reliably match the production history. Constructing a reservoir model and matching the production history are time-consuming processes that can take months. During reservoir simulation, the general field performance, despite its importance, might be overlooked if we do not start assessments with a simplified field model such as general material balance and/or if we only focus on numerical simulation model and their detail tunings. The capacitance resistive model (CRM) has the capability to quickly evaluate general reservoir performance, based on injection and production rates history, while history-matching based on numerical simulation models might take long time to complete.

In this chapter, first steps to apply the CRM in field applications are presented and then four field case studies are selected and CRMs capabilities in history-matching and flood efficiency evaluation as well as connectivity maps are presented. Information on the field examples are only given to the limit that has been requested from the data providers. Presented case studies are selected in a way to demonstrate CRMs capabilities in different settings: a tank representation of a field, its ability to determine connectivity between the producers and injectors, and understanding flood efficiencies for the entire or a portion of a field. This chapter demonstrates the application of the CRMs to multiple field examples of waterfloods and a pilot of CO_2 flood, each having a wide range of production/injection history and complex well-drilling schedule.

6.1- FIELD CASE REALITY AND LIMITATIONS

History-matching reservoir performance is a difficult inverse problem. Ordinarily, history-matching entails minimizing the difference between the observed and computed response in terms of gas/oil ratio, water/oil ratio, and reservoir drainage-area pressures. Systematic approaches have emerged to simplify history-matching because manual matching by adjusting global and/or local geological and flow properties is tedious and time-consuming.

Previous studies (Albertoni and Lake 2003; Yousef et al. 2006; Sayarpour et al. 2007) have shown the usefulness of the CRM in establishing connectivity between the producers and injectors, *en route* to matching historical performance. In most cases, the proof of concept was demonstrated with synthetic examples. Rooted in signal analysis and material-balance, CRM can rapidly attain a performance match without having to build an independent geologic model. Unlike conventional methods, one can also perform history-matching over any time segment of a field's producing life. Moreover, any arbitrary control volume consisting of any number of wells may be assigned because neither saturation nor pressure match is sought.

Field application of the CRMs might encounter any of the following which can complicate their application.

- i. Poor injection signal or flat fixed injection rate which prevents reliable CRM parameters evaluation.
- ii. Less injection than production or under injection.
- iii. Long shut-in period for a producer causes change of flood patterns.
- iv. Conversion of producers to injectors
- v. Data limitations and lack of high quality rate and bottomhole pressure (BHP) measurements.

6.2- SUGGESTED STEPS TO APPLY CRMS IN FIELD STUDIES

The following are recommendations to be considered in a field application of CRMs:

- Construct a cross plot of total field or group of wells of interest total production rates against total injection rates. This plot shows how well correlated injection and production rates are.
- 2- Initiate field evaluation with tank representation of the field, the CRMT, and then follow with producer control volume base CRM, CRMP and then injectorproducer control volume base CRM, CRMIP.
- 3- Represent the field or group of wells as a single injector and single producer using tank representation of the CRM. Grouping injectors and/or producers as one pseudo injector and/or one pseudo producer, facilitates the evaluation of the general field performance by reducing the number of unknowns in the CRM, and provides a general understanding of how well maintained is the field injection, or if other sources of pressure support exist.
- 4- Apply correction of the CRMT parameters based on the number of active producers in the field. Most of the field production rates during the early time field development are due to new producers, rather than field injection.
- 5- Initialize the time constants in CRMP or CRMIP based on the CRMT or grouped well assessment.
- 6- Initialize the connectivities, f_{ij} , for the CRMP based on the normalized inverse distance weighting for each injector.
- 7- Use large fluctuations of rates, such as injector's shut-in intervals to calibrate the CRMs for major field injectors.

6.3- FIELD CASE STUDIES

In this section we discuss four case studies, each depicting various facets of the CRM in complex reservoirs. For instance, in the Reinecke field we used the tank model (CRMT) but with different fractional-flow models to examine the relative fractions of oil and water production at the field level. In the South Wasson Clear Fork (SWCF) field, we explored both the total fluid match and individual well matches with CRMP, involving one time constant for each producer. The benefits of grouping producers and injectors in an analysis are demonstrated in the UP-Ford field study. Further analysis of pattern floods, such as in a CO₂ pilot, is examined in the McElroy field study. Mature waterfloods are the common thread in all cases.

6.3.1- Reinecke Field

In this case study we demonstrate the capability of the CRM tank application, CRMT, in matching the field total liquid and oil production rate for the Reinecke field. Then, we use five different fractional-flow models to match the oil production rate of the field.

6.3.1.1- General Information

The Reinecke field is an upper Pennsylvanian to lowest Permian carbonate buildup in the southern part of the Horseshoe Atoll in the Midland Basin of West Texas. The field was discovered in 1950 at depths of approximately 6,700 ft. The original oil was 42 °API with a gas-oil ratio of 1,266 scf/STB. Between 1950 and 1970, the reservoir pressure declined from 3,162 to 1,984 psia, just below the oil bubble point pressure of 2,000 psia. Water injection into the underlying aquifer began in the late 1960's as part of a pressure-maintenance program. Approximately 60% of the 83 MMSTB of the Reinecke field oil production is produced from the south dome, which covers one square mile. It is mainly limestone (70%) with an average porosity of 11%, a horizontal permeability of 166 md and a vertical permeability of 11 md. More than 50% of the original oil-in-place in south dome is recovered by bottom-water drive and crestal CO_2 injection (Saller et al. 2004). Figure 6-1 displays the field structure map based on a 3D seismic travel time and the location of the south dome.



Figure 6-1: Reinecke field structure map based on a 3D seismic time and the location of the south dome (after Saller et al., 2004).

6.3.1.2- Total Production Rate Match

Figure 6-2 shows both the production and water-injection history from 1972 to 1993 for the Reinecke field. Water breakthrough occurred in 1977 and the field's oil production declined even after drilling 20-acre infill wells in the mid to late 1980s. For this case study, we apply the tank representation for the field, CRMT, in which the field

liquid production rate, $q_F(t)$, is evaluated as a function of time and the k^{th} time interval field injection rate, $I_F^{(k)}$, after *n* time interval of Δt :

$$q_F(t_n) = q_F(t_0) \left(e^{-(\frac{t_n - t_0}{\tau_F})} \right) + \sum_{k=1}^n \left[(e_w + f_F I_F^{(k)}) e^{-(\frac{t_n - t_k}{\tau_F})} \left(1 - e^{-\frac{\Delta t_k}{\tau_F}} \right) \right]$$
(6-1)

where $\tau_{\rm F}$ is field-time constant given by:

$$\tau_F = \left(\frac{c_t V_P}{J}\right)_F \tag{6-2}$$

Equation 6-1 has four unknowns: 1) the total liquid production rate at the beginning of the production history, $q_F(t_0)$, 2) the fraction of the total field injection rate maintained in the reservoir control volume, f_F , 3) the field-time constant, τ_F , and 4) an unknown source of pressure support, e_w . Note that e_w has the same units as $I_F^{(k)}$ and is constant.

Minimizing the relative error between the CRMT estimate and the field measurements of the total liquid production rate from 1972 to 1993 yields the desired model parameters. Figure 6-3 presents the CRMT total production match, with a relative error of 8%, and the CRMT fitting parameters for the Reinecke field.



Figure 6-2: Injection-production history of the Reinecke reservoir.



Figure 6-3: CRMT match of the total liquid production rate, Reinecke reservoir.

Because of the strong correlation between the injection and production rates in the Reinecke field, the correlation ratio R being 0.95, CRMT yields a very good fit of the total liquid production rate, as exemplified by a low relative error of 8.8%.

6.3.1.3- Oil Production Rate Match

The Reinecke field the oil fractional-flow is predicted as a function of time based on several models. In these fractional-flow models, time is implicitly embedded in the cumulative water injected, W_i ; and normalized-water saturation, S, respectively. First, consider an empirical power-law fractional-flow model (EPLFFM), introduced by Gentil (2005) and used by Liang et al. (2007), presented in Chapter 4 and given by

$$EPLFFM(\alpha,\beta): \quad f_o(t) = \frac{1}{1 + \alpha W_i^{\beta}}$$
(4-12)

where α and β are the fitting parameters for the empirical power-law fractional-flow model, which is represented as EPLFFM(α , β). Here α has units of bbl^{- β}. A second model is based on Leverett's (1941) fractional-flow model by neglecting capillary pressure for horizontal reservoirs, presented in Chapter 4 and given by

BLBFFM(
$$M_o, m, n$$
): $f_o(\overline{S}) = 1 - \left[1 + \frac{\left(1 - \overline{S}\right)^m}{M_o \overline{S}^n}\right]^{-1}$ (4-4)

We consider two major forms of the BLBFFM in the Reinecke field by considering linear (m = n = 1) and nonlinear relative permeability curves ($m \neq 1, n \neq 1$). Figure 6-4 shows the match and fitted parameters for the oil cut, based on EPLFFM and BLBFFMs.

Fig. 6-5 exhibits the total and oil production matches based on the CRMT and different fractional-flow models applied to the Reinecke field. Perhaps the improved match of BLBFFMs owe to their superior modeling capability and more fitting parameters than the EPLFFM. The relative errors encountered for oil production rate estimation by BLBFFMs are 12 and 10% for linear and nonlinear forms of relative permeability, respectively. The EPLFFM shows a 24% relative error in matching the historical oil production.



Figure 6-4: Fractional-flow simulation of oil rate with different models, Reinecke reservoir.



Figure 6-5: Overall performance match with different fractional-flow models, Reinecke reservoir.

6.3.2- South Wasson Clear Fork (SWCF) Reservoir

In this example, the CRM's capability is tested in history-matching and evaluating the flood efficiency for an entire field compared to an arbitrary section of the reservoir.

6.3.2.1- General Information

The SWCF reservoir contains about 1,500 ft of dolomitized shallow-water carbonates at a burial depth of 6,900 ft in west Texas. Heterogeneity manifests by layer stratification with high-permeability layers causing early water breakthrough, bypassing the oil in low-permeability layers (Jennings et al., 2002). The average porosity is approximately 6% and the corresponding average permeability is about 3 md. Oil recovery is reported to be about 30% of OOIP after 55 years development with various waterflood patterns. Waterflood was initiated in 1980 with a nine-spot pattern, which was later converted into a five-spot pattern in 1987. Figure 6-6 shows a partial well location map containing 63 producers and 32 injectors as well as the location of an arbitrarily

selected section of the reservoir having six injectors and three producers. These are selected for evaluating the flood efficiency.



Figure 6-6: Well location map and selected reservoir section for flood efficiency evaluation of the SWCF reservoir.

Figure 6-7 shows the SWCF reservoir's monthly water injection, total production, and oil production from 1988 to mid 1999. Considerable fluctuation of the injection rate in this reservoir provides the desired signal quality, which makes this reservoir an ideal candidate for CRM treatment. As Fig. 6-7 shows, the first 44 months of the injection did not directly affect the total production rate, signifying the reservoir fill-up period. Consequently, we used data after the first four years of production.



Figure 6-7: Variable injection rates enrich signal quality, SWCF reservoir.

6.3.2.2- History-matching

Figure 6-8 exhibits the CRMT total production match and fitting parameters after the 44th month of production. The CRMT results reveal the existence of other source of support besides the injectors in the SWCF reservoir. Despite a very small correlation ratio of 0.07 between the injection and production rates, the CRMT matched the production rates with a relative error of only 9%. The correlation ratio between observed field total production rate and the CRMT estimation is 0.87.

The flood efficiency of any arbitrary section of the reservoir can be evaluated with CRMT by estimating the f_F . The sum of the injection rates of the six injectors and the production rates of the three producers for 14 years are treated as the injection and production rates of the pattern marked in Fig. 6-6. Figure 6-9 presents the production rates and the CRMT and CRMP matches for this pattern with estimation relative errors of 24% and 22%, respectively. These errors may appear a bit high at first glance, but they may be reduced easily by enlarging the control volume, thereby including more injectors.

An issue with an arbitrary control volume is that it may not account for the entire injection signal. In contrast, CRMT evaluation shows that 53% of injection, emanating from the six injectors, moves toward the three producers in the pattern, with a production-response-time constant τ_F of 12 months.



Figure 6-8: CRMT total production rate match for the SWCF case study.



Figure 6-9: Total liquid production match over a small region (6 injectors/3 producers).

The CRMP provides an estimation of the total production rate of producer *j* based on the injection rates, $\bar{t_i}$, of N_{inj} injectors. In the SWCF, the CRMP parameters obtained for the arbitrary pattern show that the middle producer has the smallest time constant of 7.1 months, while producers P1 and P3 time constants are 13.0 and 11.9 months, respectively, Table 6-1. Injector 3, or I3, provides the least support for the pattern in that only P3 receives 17% of its total injection rate. In contrast, I1 and I6 provides complete support, with P2 gaining more than 60% of their injection rates. These allocation factors can be obtained for any pattern or section of the reservoir and can be used to guide numerical-simulation modeling. The CRMP also allows evaluation of either a few individual producers or an entire field's production. Figure 6-10 shows the total and oil production match for producers P1 and P3 in the selected pattern. These oil production matches are obtained with the EPLFFMs.

Table 6-1: Selected pattern CRMP parameters in the SWCF.

	τ	f _{1j}	f _{2j}	f ₃ j	f _{4j}	f _{5j}	f _{6j}
P1 (j=1)	13.0	0.67	0.00	0.00	0.28	0.22	0.34
P2 (j=2)	7.1	0.33	0.29	0.00	0.08	0.41	0.66
P3 (j=3)	11.9	0.00	0.09	0.18	0.13	0.13	0.00



Figure 6-10: The CRMP total and oil production match for producers P1 and P3 in the selected reservoir section, SWCF reservoir

6.3.3- UP-Ford Zone, East Wilmington Field Case Study

In this example, we demonstrate the capability of the CRMP to match the field's and individual producers' total liquid and oil production rates by grouping injectors and producers and predicting reservoir performance based on a given historical production domain.

6.3.3.1- General Information

This horizon is a low-permeability turbidite that has undergone waterflooding since 1965 (Jenkins et al., 2004). The UP-Ford zone is about 900 ft thick comprising six subunits. Porosity averages about 20% and permeability ranges from 2 to 50 md. Peripheral water injection commenced in the 1970's, but crestal producers received minimal pressure support, leading to the drilling of pattern injectors in the 1990's. The change in flood pattern, hydraulic-fracture stimulation of the producers, and sand control all contributed to improved reservoir performance. For this reservoir unit, Al-Sharif and Rael (2003) used a combined decline-curve analysis and fractional-flow relationship to obtain reliable predictions for oil and water rates, respectively.

6.3.3.2- History-matching by Well-Grouping

We used the CRMT in history-matching and predicting reservoir performance by grouping an uneven number of active producers. This well-grouping scheme ensured the reliability of history-matching by reducing the number of unknown parameters. Corrections for the number of active producers in the CRMT enabled matching the production rate when the field injection rate was less than the total field production rate. In the following we explain this correction scheme. In CRMT, the field time constant, τ_F , must be modified as a function of time based on the number of active producers in a reservoir. Changes in the number of active producers cause an increase or decrease in the reservoir productivity index in the CRMT. As a result, if new producers are added to the same reservoir, the field production rate increases; however, the field-time constant decreases by the ratio of currently active producers to previously active producers in the field, η , which is defined as

$$\eta = \frac{\text{number of currently active producers}}{\text{number of previously active producers}}$$
(6-3)

The production rate only at the beginning of the second interval is multiplied by η to account for an increase or decrease of the production rate.

History-matching in UP-Ford unit entailed handling 191 producers and 71 injectors over 475 months. To manage these wells effectively we represented half of the field's injection and production by a single injector and a single producer. To do so, we ranked the producers and injectors based on their cumulative oil production and water injection, respectively. Then, we assigned the first 23 producers and 10 injectors to account for half of the field's oil production and water injection, while grouping the remaining of the producers as the 24th producer and the rest of the injectors as the 11th injector. Because of this well grouping scheme, the total number of unknowns to be evaluated were reduced from $N_{inj} \times (N_{pro} + 3) = 13,493$ to 312 for the CRMP total production match. Figure 6-11 presents the UP-Ford total production and injection rates and the number of active wells for 475 months of production.



Figure 6-11: Field production and injection rate history along with number of active wells, UP-Ford unit, Wilmington field.

Applying the CRMT to the UP-Ford reservoir, we found that 93% of the field injection conforms to the reservoir's control volume. The field time constant is approximately 18 months and the initial production rate is 69,577 RB/month. The relative errors for the total and oil production history-match with CRMT are 11 and 19%, respectively. Correcting the CRMT model based on the number of active producers enabled us to match the total production rate at the early stages of the field development, where the injection rate was less than the field production rate, as shown in Fig. 6-12. The quality of the CRMP match is quite evident from both the total liquid rate and oil production stand points. The normalized rates are used for clarity so that the large values for the monthly production and injection rates do not clutter the axes. The individual producers' total and oil production matches are presented in Appendix F.



Figure 6-12: Field total liquid and oil production match, UP-Ford unit.

To demonstrate the CRMT's ability to predict future performance, we used only the production history from 25 to 35 years to match the total production. Figure 6-13 indicates both the calibration or history-matched interval and the prediction interval. The fact that the CRMT could predict the historical production after the 35th year with a relative error of only 4% is a testament to its reliability. Figure 6-14 shows the cross plot of the CRMP estimation against the field measurements obtained by summing the match of all individual producers. This plot simply confirms the quality of the fit with the CRMP, which allows treatment of individual wells, unlike the CRMT.



Figure 6-13: Field total liquid production match and prediction capability of CRMT, UP-Ford unit.



Figure 6-14: Strong correlation between actual field production and model, UP-Ford unit.

6.3.4- McElroy Field CO₂ Pilot Study

This case study is selected to demonstrate the CRM compatibility with water alternating gas (WAG) injection case studies.

6.3.4.1- General Information

The McElroy field produces from the Permian Grayburg-San Andres dolomites at an average depth of 3,000 ft (Goolsby and Anderson, 1964). The gross oil column varies considerably across the field, from 100 ft on the west to a maximum of 550 ft on the east. The original-oil-in place is about 2.2 BSTB. The average porosity and permeability are approximately 6.5% and 2 md, respectively. Pilot waterflooding, initiated with an inverted seven-spot pattern in 1947 and 20-acre five-spot in 1953, showed considerable promise. Pattern realignment occurred in one section of the field in 1988 after 26 years of flooding (Lemen et al. 1990). Thakur (1998) summarized some of the modeling results of the waterflood performance.

A CO₂ pilot injection lasting for 12 years was initiated in late 1988. Figure 6-15 depicts the pilot area, where the water injectors shown in blue triangles were designed to contain the CO₂ injection within the flood area. The purple diamonds designate the wateralternating gas (WAG) injectors. Overall, 39 producers and 33 injectors make up this pilot. Our objective was to evaluate the success of this pilot. Figure 6-16 presents the pilot's production and injection history.

6.3.4.2- History-matching and Flood Efficiency Evaluation

Perhaps a cursory analysis of the effectiveness of CO_2 injection is in order before embarking upon further analysis. Figure 6-17 shows that the same volume of CO_2 injection yielded larger oil response when compare to the same volume of water injection. The tank model CRMT yields a good match of both the total fluid and oil production data, as testified by Figs. 6-18 and 6-19, respectively.



Figure 6-15: Well location map of McElroy CO₂ pilot area and Pattern-4 location.



Figure 6-16: Field performance in the McElroy CO₂ pilot area. 148



Figure 6-17: Oil production response to CO₂ injection demonstrated.

We performed the CRMT calibration only for six years after initiating the CO₂ injection to demonstrate the CRM's ability to evaluate future performance. Relative errors for both the total and oil production rates for this arbitrary training interval were roughly 13%. The calibrated model could predict the total fluid and oil production rates for the entire history with 20 and 22% relative errors, respectively. The empirical power law fractional-flow model (EPLMFF) was used to predict the oil production trend. The CRMT results, Fig. 6-18, indicate that 63% of the injection fluids are effective with a time constant of 92 days.



Figure 6-18: Performance match of total fluids with CRMT model, McElroy CO₂ pilot area.

To obtain the oil match, we invoked the EPLFFM to split the oil and water; the resultant coefficients of the fractional-flow model yield $\alpha = 3.40 \times 10^{-4}$ and $\beta = 0.616$. Figure 6-20 showing the CRMP modeling suggests a good quality match.



Figure 6-19: Performance match of oil production with the CRMT model, McElroy CO_2 pilot area.



Figure 6-20: Performance match of total fluids with the CRMP model, McElroy CO₂ pilot area.

The overall history-match of the entire pilot appears quite good with both the CRMT and CRMP approaches. As indicated previously, the CRM allows selection of any arbitrary control volume over any span of history. We show one such example from Pattern-4 containing 11 injectors and 9 producers, as shown in Fig. 6-15. The CRMT evaluation for this pattern suggests that 64% of the injection fluids are effective with an average time constant of 189 days. Relative errors of 23 and 24% result for the total and oil rate estimation, respectively. On the other hand, the CRMP revealed that more than 80% of the injection in the Pattern-4 injector, injector II1, is directed toward producers P9, P2, and P5, while injectors I4 and I9 contribute minimally, Table 6-2. Such insights into injectors' contributions for any pattern or segment of the field can have direct usage in grid-based simulations. Figure 6-21 displays the Pattern-4 match of both the total fluid and oil rates, obtained with the CRMP, individual well producers match are presented in the Appendix F.

	τ, Days	f _{1j}	f ₂	f _{3j}	f _{4j}	f _{5j}	f _{6j}	f _{7j}	f _{8j}	f _{9j}	f _{10j}	f _{11j}
P1 (j=1)	78	0.21	0.58	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P2 (j=2)	51	0.00	0.00	0.11	0.04	0.04	0.00	0.00	0.00	0.12	0.00	0.29
P3 (j=3)	50	0.30	0.04	0.35	0.14	0.03	0.15	0.00	0.00	0.00	0.02	0.04
P4 (j=4)	200	0.00	0.01	0.00	0.00	0.05	0.00	0.57	0.00	0.10	0.10	0.00
P5 (<i>j</i> =5)	60	0.17	0.01	0.09	0.00	0.00	0.21	0.12	0.00	0.00	0.24	0.17
P6 (<i>j</i> =6)	60	0.00	0.04	0.00	0.00	0.59	0.15	0.00	0.00	0.08	0.17	0.07
P7 (<i>j</i> =7)	50	0.17	0.00	0.05	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00
P8 (j=8)	91	0.00	0.05	0.00	0.00	0.17	0.00	0.00	0.00	0.05	0.00	0.06
P9 (j=9)	91	0.16	0.00	0.00	0.26	0.04	0.15	0.10	1.00	0.00	0.03	0.38
	sum $f_{ij} =$	1.00	0.73	0.65	0.44	0.91	0.67	0.80	1.00	0.36	0.56	1.00

Table 6-2: The CRMP parameters in the selected boundary in pattern-4, McElroy pilot area.

One of the key objectives of CRM analysis is to understand well connectivity so that appropriate measures, such as pattern realignment can follow for optimal-flood performance. Figure 6-22 presents steady-state well connectivity map, f_{ij} 's, of the entire 151

pilot area with arrows. The length of an arrow signifies the degree of communication intensity between an injector and a producer. Stronger connectivity between CO_2 injectors and the producers located in the southeast section of the pilot region explains the improvement in oil recovery, as indicated by Fig. 6-17.



Figure 6-21: Matching total liquid and oil production in Pattern 4, McElroy CO₂ pilot area.



Figure 6-22: Steady state interwell connectivity map, McElroy CO₂ pilot area.

6.4- SUMMARY

This chapter demonstrates applications of simple, yet powerful analytic tools for four field examples. We have successfully applied this tool to several field cases such as: 1) Reinecke, 2) Malongo 3) South Wasson Clear Fork, (SWCF), 4) UP-Ford 5) Rosneft 6) Lobitto, 7) Seminole 8) McElroy. Examples shown in this chapter are a subset of that effort. Two case studies involving daily rates and variation of bottomhole pressure (BHP) were also studied, but because of restriction on releasing the field information only two of the wells production matches are presented in the Appendix E to show the BHP effect on the CRM estimation. Individual producer matches based on the CRMP for selected wells of UP-Ford and McElroy are presented in the Appendix F.

These reservoirs are complex in that they contain extreme heterogeneity in both lateral and vertical directions. The success of a simple CRM tool may appear a bit perplexing at first. However, when we examine the model's underlying principle of connectivity based on signal and material-balance analyses, its ability to perform credible history-matching becomes transparent. In this context, the tortuosity of the flow path is unimportant from a signal transmission point of view.

Chapter 7: UNCERTAINTY QUANTIFICATION

The key to a good reservoir performance forecast lies in the availability of a reliable history match. Several equally probable sets of reservoir parameters can normally match a production history, but time and resource limitations allow evaluation of only a few of these solutions, thereby reducing the reliability of reservoir models' predictions.

Numerous sets of equiprobable history-matched solutions (EPHMSs), if obtained, can estimate the uncertainty in hydrocarbon recovery predictions. This chapter demonstrates the use of capacitance-resistive models (CRM) in evaluating reservoir uncertainty based on a given production history. Unconstrained-continuous range of uncertain reservoir properties are narrowed down to constrained-discontinues range by EPHMSs and proper combinations of uncertain reservoir parameters that satisfy the production history are achieved before performing comprehensive numerical simulations.

Because history-matching with a single geologic model does not assure attaining the 'correct' model, uncertainty in forecasting remains. Tavassoli et al. (2004) made this point very eloquently. The lack of forecasting certainty has prompted some to pursue history-matching and forecasting with an ensemble of models that carry geologic uncertainty. For instance, Landa et al. (2005), using clustered computing, showed how uncertainty in static modeling can be handled in both history-matching and forecasting. Similarly, Liu and Oliver (2005) explored applications of an ensemble Kalman filter in history-matching where continuous model updating with time is sought for an ensemble of initial reservoir models. In yet another approach, Sahni and Horne (2006) have used wavelets for generating multiple history-matched models using both geologic and production data uncertainty. Guevara (1996) and Ballin (1993) tried to minimize the load of uncertainty in predicting reservoir performance by combining the result of sensitivity analysis of both simple and complex reservoir model. Although Guevara (1996) and Ballin (1993) used simple models, neither considered the use of the simple models in screening and in providing some of the data required in their complex reservoir models.

In this chapter, first the impacts of uncertainty in a few reservoir properties, such as porosity and residual saturations, on the total and oil production response of a synfield to a fixed injection signal are evaluated. The Decision Management System (DMS) software is used to capture the sensitivity of the production responses. Then for a synthetic case study, numerous sets of EPHMSs are obtained from the CRM and Buckley-Leverett based fractional-flow model (BLBFFM). Independent sets of EPHMS provide probability distribution functions for major uncertain reservoir properties, such as the original oil and water in place, and residual oil and water saturations, before constructing comprehensive simulation models. Internally linked sets of reservoir parameters are selected based on mutual dependence. This biased sampling strategy provides systematic guidelines in selecting the proper combination of uncertain reservoir properties for numerical simulations considering uncertainty.

One can use the combination of the CRM and the BLBFFM, as independent predictive models, to evaluate probability distribution functions (PDF) for uncertain reservoir properties from the EPHMs. Internally dependent reservoir parameters, the EPHMS sets, provide a good restrictive sampling domain and reasonable guidelines for selecting appropriate input data for simulating large models with uncertainty. Significant engineering and computing time can be saved by limiting numerical simulation input data to the EPHMS sets of CRM/BLBFFM, which provide proper combination of uncertain parameters.

7.1- PRODUCTION RATE SENSITIVITY

A simple synthetic case study, involving a quarter of a five-spot pattern with an injector and a producer is modeled with Eclipse simulator for evaluation of production sensitivity. Reservoir and fluid properties of the streak case, the example provided in Chapter 5, are used in this example and the permeability field is modified to be a homogenous horizontal permeability of 40 md and a vertical permeability of 4 md. The injector and producer are located at the corner of the pattern, as shown in Fig. 7-1.



Figure 7-1: Simple quarter of five-spot synthetic case study for evaluating production rate sensitivity.

Three major uncertain reservoir properties are considered in this example: the porosity which has a normal distribution with an average of 0.18 and varies from 0.06 to 0.30, the residual oil saturation which has a uniform distribution with an average of 0.2 and ranges from zero to 0.4, and the residual water saturation which has a uniform distribution with an average of 0.35 and varies from 0.15 to 0.5. Collectively, these uncertain variables cause an uncertainty in the recoverable oil in a waterflood exercise.

Figure 7-2 shows 250 random realizations of these three uncertain variables generated by Latin Hypercube (LHC) sampling. The base case is shown in green in this figure. Using these realizations, Eclipse and Decision Management System (DMS) softwares were used to generate and visualize the impacts of the uncertain variables on the total and oil production responses to a fixed injection signal over 100 months. The injection signal, the injector I1 rate in the streak case, and 250 responses of the total and oil production rates are shown in Fig. 7-3. The base case is highlighted in pink, and those production responses that are within an arbitrary acceptable range of rate measurement error are highlighted in blue. The total production responses. The total production responses are significantly affected by the amount of original oil-in-place. Figure 7-4 shows the cross plots of the average field oil saturation after 100 months of production for the 250 realizations against porosity, residual-oil and water saturations.



Figure 7-2: Distribution of 250 realizations of three uncertain input reservoir parameters: the porosity, and the residual water and oil saturations. Samples are generated by LHC sampling. The base case, indicated with a cross, has a porosity of 0.18, a residual water saturation of 0.35 and a residual oil saturation of 0.2.



Figure 7-3: Total and oil production responses to 250 realizations of three uncertain reservoir parameters obtained by DMS. The oil production shows more variation than the total production responses.



Figure 7-4: Cross plots of the average field oil saturation and three uncertain reservoir parameters after 100 months of production for 250 realizations. The base case is indicated with a cross.

If the base case production is considered as the measured field production, within an accepted range of errors in the production rate measurements, shaded in blue in Fig. 7-3, different combination of equally likely uncertain parameters can be selected that result in these total and oil production responses. In Fig. 7-5, the highlighted in blue dots show the groups of uncertain parameters that have generated similar production responses, highlighted in blue curves. These scenarios are the equiprobable historymatched solutions (EPHMSs).



Figure 7-5: Highlighted curves and dots in blue are uncertain variables that generate similar total and oil production rates responses within a given range of rate measurement error of the base case. All the uncertain parameters can vary over their entire range. The blue dots are equiprobable history-matched solutions.

Failure to capture the possible scenarios that create the same production response can lead to an unrealistic reservoir performance prediction. A reliable history match shall provide different combinations of uncertain reservoir variables that satisfactorily match the production responses.

Reservoir uncertainty can be quantified and confined by finding numerous sets of equiprobable history-matched solutions (EPHMSs). The EPHMSs are normally found by minimizing the differences between observed and estimated model values. Often many solutions have an acceptable range of error that results in different combinations of the reservoir properties that satisfy the production history. These combinations generate a discrete domain of internally linked reservoir properties that can reasonably satisfy the production history and quantify the range of uncertain variables. As an example, Fig. 7-6 shows a schematic of iso-error contours between estimation and observation as a result of variation of parameters X_1 and X_2 . Six local minimums, each having different values for X_1 and X_2 are the best candidate, EPHMS#1 to #6, for error minimization. Evaluation of these interrelated sets is the key for a reliable history match and provides a sampling domain that constrains the range of uncertain variables and restricts their combinations to those that satisfactorily match the production history.



Figure 7-6: A schematic representation of six equiprobable history-matched solutions (EPHMSs) that confine the continuous range of uncertain parameters X_1 and X_2 to a discrete range. Estimation error of 10% is considered as threshold.

7.2- RESERVOIR UNCERTAINTY QUANTIFICATION BY THE EPHMS

In this section, numerous sets of EPHMSs that satisfactorily match the base case production rates are evaluated and, based on these solutions, the range of major reservoir properties, such as residual oil and water saturations and recoverable oil in place, are evaluated by a combination of CRM and BLBFFM. The Buckley-Leverett based fractional-flow model (BLBFFM) has the following form:

BLBFFM(
$$M_o, m, n$$
): $f_o(\overline{S}) = 1 - \left[1 + \frac{\left(1 - \overline{S}\right)^m}{M_o \overline{S}^n}\right]^{-1}$ (4-4)

The end-point mobility ratio, M_o , and relative-permeability curve exponents, m and n, are the unknowns in the Buckley-Leverett-based fractional-flow model, designated by BLBFFM(M_o , m, n). The normalized average water saturation \overline{S} is evaluated as a function of time based on material balance as presented in Chapters 4 and 6.

Simulator generated production responses for the base case are used as production history and matched with the CRM. Figure 7-7 shows the injection signal and the total production and the match obtained by the CRM. The BLBFFM is used to match the oil production rates for 10 years of production history.

In the fractional-flow model, the average residual oil and water saturations, \overline{S}_{or} and \overline{S}_{wr} , and the endpoint mobility ratio, M_o , explicitly, and the original-in-place oil and water, implicitly, are unknown parameters. These unknown parameters are evaluated by history matching the oil production rate. Numerous sets of the EPHMSs are evaluated by the CRM/BLBFFM. Figure 7-8 shows the cumulative distribution functions (CDF) of each parameter obtained from EPHMSs where 1st, 5th, 10th, 15th and 20th best solutions are marked. Fig. 7-9 shows the cross plots and Fig. 7-10 presents the CDF of the relative errors of the oil production history-match for 780 sets of EPHMSs. Best 20 EPHMSs are

provided in Table 7-1 and shown on Fig. 7-9. Each of the EPHMS provides a unique combination of uncertain reservoir variables, and can be selected as an input set for the numerical simulation model. The EPHMSs confine the continuous range of uncertain reservoir parameters and form a restricted sampling domain for numerical simulation.



Figure 7-7: Injection, total production and CRM total production match for the quarter of five-spot pattern shown in Fig. 7-1.

EPHMS	Residual water saturation, S_{wr}	Residual oil saturation, S _{or}	Mobiliy ratio, <i>M</i> _o	Water in place, W (STB)	Oil in place, N (STB)	Pore volume, V _P (RB)
1	0.380	0.313	0.367	4.88E+05	6.90E+06	7.87E+06
2	0.380	0.315	0.375	4.64E+05	6.91E+06	7.87E+06
3	0.376	0.302	0.345	4.20E+05	6.74E+06	7.66E+06
4	0.382	0.309	0.366	4.86E+05	6.86E+06	7.82E+06
5	0.380	0.313	0.372	4.77E+05	6.90E+06	7.86E+06
6	0.389	0.290	0.363	4.82E+05	6.66E+06	7.60E+06
7	0.375	0.306	0.348	4.86E+05	6.82E+06	7.78E+06
8	0.389	0.300	0.378	4.54E+05	6.76E+06	7.70E+06
9	0.385	0.287	0.346	4.62E+05	6.62E+06	7.55E+06
10	0.390	0.297	0.369	4.63E+05	6.71E+06	7.66E+06
11	0.395	0.278	0.357	4.64E+05	6.54E+06	7.46E+06
12	0.382	0.307	0.371	4.93E+05	6.86E+06	7.83E+06
13	0.376	0.306	0.361	4.72E+05	6.83E+06	7.78E+06
14	0.378	0.313	0.355	4.79E+05	6.89E+06	7.85E+06
15	0.383	0.301	0.377	4.54E+05	6.78E+06	7.72E+06
16	0.369	0.322	0.354	5.00E+05	7.00E+06	7.98E+06
17	0.391	0.294	0.373	4.40E+05	6.70E+06	7.63E+06
18	0.381	0.320	0.372	5.07E+05	6.99E+06	7.98E+06
19	0.392	0.283	0.376	4.32E+05	6.60E+06	7.51E+06
20	0.376	0.320	0.378	5.05E+05	7.01E+06	8.00E+06

Table 7-1: Best 20 equiprobable history-matched solutions (EPHSs).


Figure 7-8: The CDF of the 780 sets of EPHMs for the quarter of five-spot obtained by the CRM and application of BLBFFM(M_o ,1,1). The 1st, 5th, 10th, 15th and 20th best history match solutions are marked on the CDFs.



Figure 7-9: Cross plot of the sets of EPHMSs for a quarter of five-spot, Fig. 7-1, obtained by the CRM and application of BLBFFM(M_o ,1,1). The best 20 solutions are highlighted within the EPHMSs by purple. These internally related solutions can be used as input data for numerical simulations.

The distributions and the range of the EPHMSs for this family of solutions overestimate the residual oil and water saturations of the base case but in general agree with the original input data in the numerical simulation input file. Obtained EPHMSs rarely have a relative error more than 8.5%, as presented in Fig. 7-11. Figure 7-12 shows

the quality of the oil production match for two values of the relative errors of 8.6 and 9.7%, both larger than most of the EPHMSs. These errors indirectly ensure the quality of the oil-production match by the EPHMSs which have less the 8.5% of relative errors, especially for the best 20 solutions.



Figure 7-10: CDF of the oil production match relative error for EPHMSs.



Figure 7-11: Oil production relative errors against residual oil saturations for the EPHMSs. Note that the best 20 solutions (purple dots) cover almost the entire range of residual oil saturation of the EPHMSs.



Figure 7-12: Oil Production and the CRM/BLBFFM match for a quarter of fivespot pattern of Fig. 7-1. Two of the largest oil production history-match errors are shown to illustrate the quality of the EPHMSs that their relative errors are rarely above 8.5%.

7.3- SUMMARY

In this chapter, first the impacts of the uncertainty of some major reservoir properties, such as porosity, residual oil and water saturation, on the total and oil production responses were demonstrated by using DMS. Then, the equiprobable historymatched solutions (EPHMSs) were obtained both by the CRM and DMS applications. The CRM was used to match a simple case study and numerous sets of EPHMSs were obtained. These EPHMSs were used to quantify the CDF of average reservoir parameters, such as original oil and water in place, residual oil and water saturations, and the endpoint mobility ratio. The obtained range and combination satisfactorily cover the numerical simulation model that was used to generate the production responses. Equally probable combinations of uncertain reservoir parameters, the EPHMS sets, created an internally-related combination of reservoir parameters that satisfy the production history.

Chapter 8: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The main objectives of this work were evaluating reservoir performance accurately in a short period of time and also quantifying the reservoir uncertainty. Therefore, new analytical solutions for capacitance-resistive models (CRMs) as fast predictive techniques were developed and their application in history-matching, optimization, and evaluating reservoir uncertainty for water/CO₂ floods were demonstrated.

This work developed and tested the CRMs in synthetic and field case studies, and introduced an algorithm to narrow down the wide range of uncertainty of major reservoir properties based on numerous sets of the CRM history-matched solutions. Since the CRM only requires injection-production rate and if available bottomhole pressure data, it is used widely in many field applications. Insights gained from performing the CRM were used to evaluate reservoir operating conditions and flood efficiency during production history. Then the CRMs were used to predict and optimize future reservoir performance. Furthermore, the CRMs' results were used to narrow down the range of some of the parameters that are involved in predictive model from a wide-continuous to a narrow-discrete range.

8.1- TECHNICAL CONTRIBUTIONS

The analytical solutions for the continuity equation based on superposition in time and space were developed for three different reservoir control volumes: 1) CRMT, the entire field volume, 2) CRMP, the drainage volume of a producer and 3) CRMIP, the control volume between injector-producer pairs. The analytical solutions enable rapid estimation of the CRM unknown parameters: the interwell connectivity and production response time constant. These solutions are obtained based on stepwise or linear variation of injection rate and linear variation of the producers' bottomhole pressures projections. Furthermore, by considering a series of tanks between each injector/producer pair, CRM solutions were modified and CRM-Block analytical solutions based on superposition in time and space were developed.

Analytical solutions facilitated the CRMs' application for rapid assessment at different levels of a field study, from a single well, to a group of wells, and to an entire field. The CRM's analytical solutions, in conjunction with the physical meaning of its parameters, its capability to discern reservoir connectivities, its flexibility in taking variable timesteps, simplicity, and speed are major advantages over those developed previously.

The CRM provides an estimation of the total liquid production; therefore we evaluated and incorporated different forms of the fractional-flow model for miscible and immiscible floods to be able to estimate oil production. We mostly used the empirical power-law fractional-flow model (EPLFFM) as the preferred fractional-flow model in the course of history-matching and optimization of waterfloods. The Buckley-Leverett based fractional-flow models (BLBFFM) were suitable for uncertainty evaluation. The application of the logistic equation based fractional-flow models (LEBFFM) was demonstrated the flexibility to be applied to tertiary flooding processes. We used this model for miscible CO_2 floods.

The validation of the CRM was performed by testing its results against numerical simulation results. We presented several synthetic case studies in this work, including a peripheral water injection, a seven-spot pattern, a five-spot pattern, the streak case and the MESL case study. Each of these case studies were selected to test different aspect of

the CRM capabilities. Among these the optimization of reservoir performance was conducted for the streak and MESL cases by reallocating field water injection. The seven-spot case was used to analyze the impact of rate measurement errors on the CRM performance and the five-spot pattern was a CO_2 flood case study. In the peripheral water injection case study, we introduced the concept of unique injector shut in pulses (UISPs) to enhance the CRM calibration. The advantage provided by the use of UISPs was further confirmed when applied to three of the synthetic case studies.

The CRM ability to perform as a simple and reliable analytical tool was further verified using four field examples. We have successfully applied this tool to several cases such as: 1) Reinecke, 2) Malongo 3) South Wasson Clear Fork, (SWCF), 4) UP-Ford 5) Rosneft 6) Lobito, 7) Seminole 8) McElroy. We presented a subset of these evaluations in this work.

The application of the CRM yields to history-matched solutions. As presented in this work, numerous sets of equiprobable history-matched solutions (EPHMS) were obtained by applying the CRM. Then, EPHMSs were used to quantify and confine the range of uncertain reservoir parameters and provide cumulative distribution functions (CDF) of uncertain reservoir parameters. This approach provides a restricted sampling domain in which sets of internally linked reservoir parameters are indicated and can be use to efficiently design a more comprehensive reservoir simulation study.

8.2- CONCLUDING REMARKS

 Application of the CRM to several synthetic and field case studies showed its reliability and capability as a tool in history-matching and optimization of reservoir performance. Several case studies showed that the calibrated CRMs are capable of generating solutions that are comparable to those obtained from 3D numerical flowsimulation models and provide good quality history-match to the field case studies.

- 2. The CRMs are very fast and inexpensive to use. Thus, with a minimum available reservoir data, only injection and production data, and a small investment in computing and engineering time, it will be possible to have a preliminary evaluation, prediction of reservoir characteristics and future production.
- 3. The CRM should be a precursor to any history-matching exercise. The CRM's predictive capability allows first-order investigation of future flood performance without large time investment. CRMs' rapid history-matching capability in complex field environments serves as an excellent precursor to any grid-based modeling study.
- 4. Significant insights about the flood performance over time can be gained for any arbitrary pattern by estimating fractions of injected water being directed from an injector to various producers, and the time taken for an injection signal to reach a producer. Injector-to-producer connectivity is inferred directly during the course of error minimization. CRMs provide significant insights about a waterflood's overall performance and pattern-allocation factors.
- Field engineers can quickly manage and optimize real-time reservoir performance. Reallocation of injected water among the existing injectors can be investigated while maximizing cumulative oil production by the CRM.
- 6. Because the CRM circumvents geologic modeling and saturation-matching issues, it lends itself to frequent usage without intervention of expert modelers, this enable evaluation of numerous sets of the EPHMS and consequently quantification of reservoir uncertainty.

- 7. The EPHMS sets provide good-restricted sampling domains and reasonable guidelines for selecting appropriate input data for full-field numerical modeling by evaluating the range of uncertain reservoir parameters. Therefore, we can minimize the number of full-scale numerical runs by restricting selection of reservoir properties for comprehensive reservoir simulation to those that satisfy the production history in the CRMs. Tremendous engineering and computing time may be saved by limiting numerical simulation input data to those EPHMS sets obtained from the CRMs.
- 8. For case studies with flat injection signals, the UISPs showed to be a powerful way to calibrate the CRM. In most of the field production history the USIPs exist and can be used to evaluate the CRM parameters: the interwell connectivity and production response time constant.
- 9. Errors in rate measurements can also be entertained during history-matching. As long as the impacts of the injectors' signals on the production rate of producers are larger than the range of the rate measurement error, CRMs can capture reservoir connectivities reliably. This makes the CRM suitable for field case applications where rate measurement errors are inevitable.
- 10. The analogy between a reservoir oil production and the population size of bacteria enables application of the rich literature on logistic equations (LEs) in predicting the oil production for EOR processes.

8.3- RECOMMENDATIONS FOR FUTURE WORK

1. Interwell connectivities, f_{ij} , and time constants, τ_j , are considered to be independent of injection rates. Modification of f_{ij} and τ_j in the CRM over different time intervals shall be performed if major changes happen in the well schedules that causes major

changes of the streamlines. In this matter, a producer shut in for a long period of time is a good example.

- 2. Although, well-pair connectivities compare relative contribution of an injector to different producers, but when the focus is on a producer, multiplication of average injection rate and associated f_{ij} should be considered as a measure to recognize importance of different injectors on the production rate of a producer. Constructing a map that shows either the absolute support of different injectors, $f_{ij}\overline{i}$, or the fraction of the production of each producer provided by different injectors, $f_{ji} = f_{ij}\overline{i}/\overline{q}_j$, is recommended.
- 3. The correlation ratio between injection and production rates is a good measure to identify the injectors in the vicinity of a producer that should be included in the CRM application. However, the user shall consider the time lag that can exist between the production responses and the injection signals, which could disguise the connectivity between an injection-producer pair.
- 4. The application of the CRMs was demonstrated in this work. However, real-time field optimization has not been tested neither with unique injector shut-in pulses (UISPs) nor with the CRM optimization algorithm. We recommend the application of these two concepts in a real field application and optimization.
- 5. The CRM yields its best performance when obvious production and injection fluctuations exist throughout the production history of a field. No systematic study has been conducted to date to understand and quantify when a field historical production has enough character (rich production-injection signals) to successfully screen good reservoir candidates for CRM application. A study in this area is recommended.

- 6. Most of the case studies used in this work considered vertical wells. From the standpoint of the CRM development, this tool should be able to handle other types of well configurations. However, this should be verified in a systematic way. Testing the application of the CRM for horizontal or inclined well configuration is recommended.
- 7. The CRM is very similar to a simplified streamline simulation concept. Compared to the streamline simulation, connectivities can be represented by the ratio of the number of streamlines connecting an injector to a producer to the total number of streamlines originated from the injector. The time constant is comparable to the time of flight concept. A comprehensive comparison of both approaches is recommended.
- 8. Field production and injection data are often reported as cumulative monthly allocated rates. Reporting rate measurement as monthly cumulative and using allocation factor for assigning share of production of producers filter the injection signals and corresponding production responses. This can lead to an unrealistic evaluation of the field performance by the CRM. We recommend that the user be aware of this fact and spend the time mining and analyzing the field data before his/her evaluation.
- 9. Pattern allocation, specially for pattern boundary injectors can be directly calculated in the CRM for each injector from the interwell connectivities estimates. We recommend the use of these allocation factors from the CRM in any comprehensive numerical simulation designed for an element of symmetry, a pattern, or a sector model to reasonably estimate the contribution of injectors.
- 10. During the course of history-matching in this work, estimation error of, first, the total and then the oil production were minimized. Applying multi-objective optimization can facilitate the application of the CRM and oil fractional-flow model in historymatching and optimization. It is worth mentioning that multi-objective optimization

can be used to simultaneously maximize the correlation ratio and minimize the relative errors between observed and estimated total and/or oil rates which provide a better objective function.

11. The CRM as a simple and fast tool can provide many sets of history-matching solutions that can quantify the range of uncertain reservoir parameters. We recommend the application of CRM as a precursor of any comprehensive numerical simulation. The workflow in Fig. 8-1 shows an algorithm to be applied in this regard.



Figure 8-1: Workflow to apply EPHMSs obtained by the CRM in comprehensive numerical simulation modeling.

12. Application of unique injection shut in pulses (UISPs) enable reliable CRM calibration. This is demonstrated in evaluating the CRM parameters for history-matching of total and oil production in numerically simulated case studies presented in this work. It is strongly recommended to apply this in field case. In field operations there are times when injectors are shut, and their shut-in incidents can be used to investigate interwell connectivities and used for calibrating the CRMs.

13. CRM-Block can be applied to couple wellbores with reservoirs by considering three tanks in series: injector, reservoir and producer. Intuitively, two tanks with smaller time constants than the reservoir, which represent the wellbores, can be considered to model injector and producer wellbore effects, Fig. 8-2.



Figure 8-2: Schematic representation of wellbore and reservoirs with three tanks in series with different time constant to capture the wellbore effect in the CRM.

- 14. CRM can be combined with different fractional-flow model to provide oil production history match and estimation for different recovery processes. Development of compatible fractional-flow models for other enhanced oil recovery processes such as chemical flooding is recommended.
- 15. Development and application of the CRM have been mainly for slightly compressible systems, mostly waterfloods. It is recommended to check for development and applicability of the CRM for compressible system.
- 16. Out of pattern injections or supports such as aquifer influx are modeled as constant supports during production history. The CRM estimation can be improved if pseudo steady state aquifer models are used and combined with the CRMs.

Appendices

APPENDIX A: DERIVATION OF FUNDAMENTAL EQUATION OF THE CAPACITANCE-RESISTIVE MODEL (CRM)

A macroscopic material balanced equation (MBE) over an arbitrary control volume (CV) for phase j can be written as

$$\begin{cases} \text{Total Mass of } j \text{ in} \\ \text{the CV at } t + \Delta t \end{cases} - \begin{cases} \text{Total Mass of } j \\ \text{in the CV at } t \end{cases}$$

$$= \begin{cases} \text{Mass of } j \text{ entering} \\ \text{the CV during } \Delta t \end{cases} - \begin{cases} \text{Mass of } j \text{ leaving} \\ \text{the CV during } \Delta t \end{cases}$$
(A-1)

or

$$\frac{\mathrm{d}M_{j}}{\mathrm{d}t} = \rho_{j,in}q_{j,in} - \rho_{j,out}q_{j,out} \tag{A-2}$$

where M_j is the mass of component *j* in the CV and ρ_j is its density. Assuming an average constant density for component *j* in CV we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\overline{S_j \rho_j V_P}\right) \approx \frac{\mathrm{d}}{\mathrm{d}t} \left(\overline{S_j \rho_j V_P}\right) = \rho_{j,in} q_{j,in} - \rho_{j,out} q_{j,out}$$
(A-3)

where, $\overline{\rho}_j$ is the average density, \overline{S}_j is the average saturation of *j*, and \overline{V}_p represents the pore volume in the CV. Applying the chain rule we can differentiate Eq. A-3:

$$\overline{\rho}_{j}\overline{V}_{P}\frac{\mathrm{d}\overline{S}_{j}}{\mathrm{d}t} + \overline{S}_{j}\overline{V}_{P}\frac{\mathrm{d}\overline{\rho}_{j}}{\mathrm{d}t} + \overline{S}_{j}\overline{\rho}_{j}\frac{\mathrm{d}\overline{V}_{P}}{\mathrm{d}t} = \rho_{j,in}q_{j,in} - \rho_{j,out}q_{j,out}$$
(A-4)

Dividing by $\overline{S}_{j}\overline{\rho}_{j}\overline{V}_{P}$:

$$\frac{1}{\overline{S}_{j}}\frac{\mathrm{d}\overline{S}_{j}}{\mathrm{d}t} + \frac{1}{\overline{\rho}_{j}}\frac{\mathrm{d}\overline{\rho}_{j}}{\mathrm{d}t} + \frac{1}{\overline{V}_{p}}\frac{\mathrm{d}\overline{V}_{p}}{\mathrm{d}t} = \frac{1}{\overline{S}_{j}\overline{\rho}_{j}\overline{V}_{p}}(\rho_{j,in}q_{j,in} - \rho_{j,out}q_{j,out})$$
(A-5)

$$\frac{1}{\overline{S}_{j}}\frac{d\overline{S}_{j}}{dt} + \frac{1}{\overline{\rho}_{j}}\frac{d\overline{\rho}_{j}}{d\overline{p}_{j}}\frac{d\overline{\rho}_{j}}{dt} + \frac{1}{\overline{V}_{p}}\frac{d\overline{V}_{p}}{d\overline{p}_{j}}\frac{d\overline{\rho}_{j}}{dt} = \frac{1}{\overline{S}_{j}\overline{\rho}_{j}\overline{V}_{p}}(\rho_{j,in}q_{j,in} - \rho_{j,out}q_{j,out})$$
(A-6)

By using the definitions of total compressibility, we get:

$$\frac{1}{\overline{S}_{j}}\frac{\mathrm{d}\overline{S}_{j}}{\mathrm{d}t} + c_{j}\frac{\mathrm{d}\overline{p}_{j}}{\mathrm{d}t} + c_{f}\frac{\mathrm{d}\overline{p}_{j}}{\mathrm{d}t} = \frac{1}{\overline{S}_{j}\overline{\rho}_{j}\overline{V}_{P}}(\rho_{j,in}q_{j,in} - \rho_{j,out}q_{j,out})$$
(A-7)

$$\frac{1}{\overline{S}_{j}}\frac{\mathrm{d}S_{j}}{\mathrm{d}t} + (c_{j} + c_{f})\frac{\mathrm{d}\overline{p}_{j}}{\mathrm{d}t} = \frac{1}{\overline{S}_{j}\overline{\rho}_{j}\overline{V}_{P}}(\rho_{j,in}q_{j,in} - \rho_{j,out}q_{j,out})$$
(A-8)

Eq. A-8 can be written for any phase. For example, for an immiscible two phase of water and oil we can write:

$$\begin{cases} \frac{1}{\overline{S}_{w}} \frac{d\overline{S}_{w}}{dt} + (c_{w} + c_{f}) \frac{d\overline{p}_{w}}{dt} = \frac{1}{\overline{S}_{w} \overline{\rho}_{w} \overline{V}_{p}} (\rho_{w,in} \dot{i}_{w} - \rho_{w,out} q_{w}) \\ \frac{1}{\overline{S}_{o}} \frac{d\overline{S}_{o}}{dt} + (c_{o} + c_{f}) \frac{d\overline{p}_{o}}{dt} = \frac{1}{\overline{S}_{o} \overline{\rho}_{o} \overline{V}_{p}} (0 - \rho_{o,out} q_{o,out}) \end{cases}$$
(A-9)

In Eq. A-9, the oil injection rate, i_o , is assumed to be zero. Neglecting the capillary pressure effect and assuming the same densities for injected, produced and reservoir water as well as produced and reservoir oil, Eq. A-9 simplifies to

$$\begin{cases} \frac{1}{\overline{S}_{w}} \frac{d\overline{S}_{w}}{dt} + (c_{w} + c_{f}) \frac{d\overline{p}}{dt} = \frac{1}{\overline{S}_{w} \overline{V}_{p}} (i_{w} - q_{w}) \\ \frac{1}{\overline{S}_{o}} \frac{d\overline{S}_{o}}{dt} + (c_{o} + c_{f}) \frac{d\overline{p}}{dt} = \frac{-1}{\overline{S}_{o} \overline{V}_{p}} q_{o} \end{cases}$$
(A-10)

Summing the equations given in A-10, and eliminating the saturation derivative terms in by using $d\overline{S}_o = -d\overline{S}_w$ we obtain the continuity equation for CRM:

$$\left(\frac{\mathrm{d}\overline{S}_{w}}{\mathrm{d}t} + \frac{\mathrm{d}\overline{S}_{o}}{\mathrm{d}t}\right) + \left(c_{w}\overline{S}_{w} + c_{o}\overline{S}_{o} + c_{f}(\overline{S}_{o} + \overline{S}_{w})\right)\frac{\mathrm{d}\overline{p}}{\mathrm{d}t} = \frac{1}{\overline{V}_{P}}(i_{w} - q_{w} - q_{o})$$
(A-11)

$$c_t V_p \frac{\mathrm{d}\overline{p}}{\mathrm{d}t} = i_w - q_w - q_o \tag{A-12}$$

or

$$c_t V_p \frac{\mathrm{d}\overline{p}}{\mathrm{d}t} = i(t) - q(t) \tag{A-13}$$

where i(t) and q(t) represent the injection rate and total production rates, respectively. Based on the definition of productivity index *J*, the total production rate in reservoir volumes, q(t), is

$$q(t) = J(\overline{p} - p_{wf}) \tag{A-14}$$

Elimination of the average reservoir pressure from A-13 leads to the fundamental firstorder ordinary differential equation for the CRM.

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} + \frac{1}{\tau}q(t) = \frac{1}{\tau}i(t) - J\frac{\mathrm{d}p_{wf}}{\mathrm{d}t}$$
(A-15)

where J is assumed to be constant and the time constant, τ , is defined as

$$\tau = \frac{c_t V_p}{J} \tag{A-16}$$

and has units of time.

Equation (A-15) is developed for a system based on the following assumptions:

- Constant temperature
- Instantaneous equilibrium in the control volume, tank assumption
- Two immiscible phases coexist
- Two components
- Capillary pressure effect is neglected
- Small fluid compressibility which causes equal density for injected, reservoir and produced fluids.
- Darcy's law applies
- Productivity index is constant

APPENDIX B: PRIMARY TERMS IN THE CAPACITANCE-RESISTIVE MODEL

For case studies in which field injection is only implemented in a section of the reservoir, the CRM match and predictions will be more accurate if two different control volumes are used, one for primary recovery and one for secondary recovery. In-situ continuity equation during primary and secondary recoveries can be written in terms of the total production for a producer and a series of injectors as

$$\begin{cases} \left(c_{t}V_{p}\right)_{jp}\frac{\mathrm{d}\overline{p}_{j}}{\mathrm{d}t} = -q_{jp}(t) & t < t_{s} \\ \left(c_{t}V_{p}\right)_{js}\frac{\mathrm{d}\overline{p}_{j}}{\mathrm{d}t} = \sum_{i=1}^{N_{inj}}f_{ij}i_{i}(t) - q_{js}(t) & t \ge t_{s} \end{cases}$$
(B-1)

where the subscript *jp and js* indicate the properties of producer *j* during primary and secondary recoveries, and t_s is the starting time for secondary recovery. Eliminating the average pressure from Eq. B-1 by applying the productivity index definition leads to the following ordinary differential equations during primary and secondary recoveries:

$$\begin{cases} \frac{\mathrm{d}q_{jp}(t)}{\mathrm{d}t} + \frac{1}{\tau_{jp}}q_{jp}(t) = -J_{jp}\frac{\mathrm{d}p_{_{vf,j}}}{\mathrm{d}t} , q_{jp}(t_0) = q_{jp}(t_0) ; \tau_{jp} = \left(\frac{c_t V_p}{J}\right)_{jp} ; t < t_s \\ \frac{\mathrm{d}q_{js}(t)}{\mathrm{d}t} + \frac{1}{\tau_{js}}q_{js}(t) = \left[\frac{1}{\tau_{js}}\sum_{i=1}^{N_t} f_{ij}i_i(t) - J_{js}\frac{\mathrm{d}p_{_{vf,j}}}{\mathrm{d}t}\right] , q_{js}(t_s) = 0 ; \tau_{js} = \left(\frac{c_t V_p}{J}\right)_{js} ; t_s \le t \end{cases}$$
(B-2)

The solutions of these differential equations are:

$$\begin{cases} q_{jp}(t) = q_{jp}(t_0)e^{-(\frac{t-t_0}{\tau_{jp}})} - e^{\frac{-t}{\tau_{jp}}}\int_{t_0}^{t}J_{jp}e^{\frac{t}{\tau_{jp}}}\frac{dp_{wf}}{dt}dt \\ q_{js}(t) = \int_{t_0}^{t}\frac{1}{\tau_{js}}e^{\frac{t}{\tau_{js}}}\sum_{i=1}^{N_i}f_{ij}i_i(t)dt - e^{\frac{-t}{\tau_{js}}}\int_{t_0}^{t}J_{js}e^{\frac{t}{\tau_{js}}}\frac{dp_{wf}}{dt}dt \quad t_s \le t \end{cases}$$
(B-3)

For the special case of constant bottomhole pressure this solution simplifies to:

$$\begin{cases} q_{jp}(t) = q_{jp}(t_0) e^{-(\frac{t-t_0}{\tau_{jp}})} \\ q_{js}(t) = \begin{cases} 0 & t_s \ge t \\ \left(1 - e^{-(\frac{t-t_s}{\tau_{js}})}\right) \left[\sum_{i=1}^{N_i} \left[f_{ij}I_i(t)\right]\right] & t \ge t_s \end{cases}$$
(B-4)

Here the overall production is the contribution of primary and secondary production. In addition, the secondary production at the beginning of the secondary recovery, t_s , is equal to zero; therefore, overall production can be written as the sum of primary and secondary recoveries:

$$q_{j}(t) = q_{jp}(t) + q_{js}(t) = \begin{cases} q_{jp}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{jp}})} & t_{s} \ge t \\ q_{jp}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{jp}})} + \left(1 - e^{-(\frac{t-t_{s}}{\tau_{js}})}\right) \left[\sum_{i=1}^{N_{i}} \left[f_{ij}I_{i}(t)\right]\right] & t \ge t_{s} \end{cases}$$
(B-5)

Fig. B-1 presents a simple example in which the impacts of primary and secondary productions on the total production rate are shown.



Figure B-1: Primary and secondary production effect on overall production.

APPENDIX C: SUPERPOSITION IN TIME SOLUTION FOR CAPACITANCE-RESISTIVE MODEL

Figure C-1 shows series of linear stepwise injection rates. The capacitanceresistive model (CRM) total production rate for a fixed injection rate and constant bottomhole pressure is:

$$q(t) = q(t_0)e^{-\left(\frac{t-t_0}{\tau}\right)} + I\left[1 - e^{-\left(\frac{t-t_0}{\tau}\right)}\right]$$
(C-1)

Figure C-1: Stepwise change of injection rate schedule from time t_0 to t_n .

Based on the superposition in time one can write the impact of each injection pulse on the total production at time t_n as the following:

The impact of
$$I_1$$
 at $t_n = I_1 \left(1 - e^{-(\frac{t_n - t_0}{\tau})} \right) - I_1 \left(1 - e^{-(\frac{t_n - t_1}{\tau})} \right) = I_1 \left(e^{-(\frac{t_n - t_1}{\tau})} - e^{-(\frac{t_n - t_1}{\tau})} \right)$
$$= I_1 \left(e^{-(\frac{t_n - t_1}{\tau})} - e^{-(\frac{t_n - t_1}{\tau})} \times e^{-(\frac{t_1 - t_0}{\tau})} \right) = I_1 \left(1 - e^{-(\frac{\Delta t_1}{\tau})} \right) e^{-(\frac{t_n - t_1}{\tau})}$$
(C-2)

The impact of
$$I_2$$
 at $t_n = I_2 \left(1 - e^{-(\frac{t_n - t_1}{\tau})} \right) - I_1 \left(1 - e^{-(\frac{t_n - t_2}{\tau})} \right) = I_1 \left(e^{-(\frac{t_n - t_2}{\tau})} - e^{-(\frac{t_n - t_1}{\tau})} \right)$
$$= I_2 \left(e^{-(\frac{t_n - t_2}{\tau})} - e^{-(\frac{t_n - t_2}{\tau})} \times e^{-(\frac{t_2 - t_1}{\tau})} \right) = I_2 \left(1 - e^{-(\frac{\Delta t_2}{\tau})} \right) e^{-(\frac{t_n - t_1}{\tau})}$$
(C-3)

The impact of
$$I_k$$
 at $t_n = I_k \left(1 - e^{-(\frac{t_n - t_{k-1}}{\tau})} \right) - I_k \left(1 - e^{-(\frac{t_n - t_k}{\tau})} \right) = I_k \left(e^{-(\frac{t_n - t_k}{\tau})} - e^{-(\frac{t_n - t_{k-1}}{\tau})} \right)$
$$= I_k \left(e^{-(\frac{t_n - t_k}{\tau})} - e^{-(\frac{t_n - t_k}{\tau})} \times e^{-(\frac{t_n - t_{k-1}}{\tau})} \right) = I_k \left(1 - e^{-(\frac{\Delta t_k}{\tau})} \right) e^{-(\frac{t_n - t_k}{\tau})}$$
(C-4)

Therefore, at time t_n the impact of series of stepwise injection rate variations is equal to the secondary production rate:

$$q_s(t_n) = \sum_{k=1}^n I_k \left(1 - e^{-\left(\frac{\Delta t_k}{\tau}\right)} \right) e^{-\left(\frac{t_n - t_k}{\tau}\right)}$$
(C-5)

where subscripts *s* denotes the secondary production. Including the primary production effect provide the following solution for the CRM using superposition in time.

$$q(t_n) = q_p(t_n) + q_s(t_n) = q(t_0)e^{-(\frac{t_n - t_0}{\tau})} + \sum_{k=1}^n I_k \left(1 - e^{-(\frac{\Delta t_k}{\tau})}\right)e^{-(\frac{t_n - t_k}{\tau})}$$
(C-6)

APPENDIX D: CAPACITANCE-RESISTIVE MODEL TOOL IN MICROSOFT EXCEL

Each of the following botton creates a CAPACITANCE-RESISTIVE MODEL:

Injection rate and production rate (and if availabel producers bottomhole pressure) are the minimum data requiered to use this tool.

CRMT: considers one tank to represent the field (only estimates the entire field production: Oil Production is added) CRMP: considers the control volume of one producer (estimates field and individual producers production) CRMP1: Use if BHP data are available for producers. CRMP2: Use if there is NOT any data for producers'BHP CRMP3: Use if BHP data are available, and long and numerous producers shut in intervals exist. Production rates after each shut in are assumed to be known and equal to q(0)s. q(0)s won't be among model unknowns. CRMP4: Use if there is NOT any data for producers'BHP. Production rates after each shut in are assumed to be known and equal to q(0)s. q(0)s won't be among model unknowns CRMP5: Use if BHP data are available for producers. Oil production based on Power-Law Model is added. Production rates after each shut in are assumed to be known and equal to q(0)s. q(0)s won't be among model unknowns CRMP6: Use if there is NOT any data for producers'BHP. Oil production based on Power-Law Model is added. Production rates after each shut in are assumed to be known and equal to g(0)s. g(0)s won't be among model unknowns CRMP7: Use if BHP data are available for producers. Oil production based on Power-Law Model is added Production rates after each shut in are assumed to be known and equal to q(0)s. q(0)s won't be among model unknowns CRMP8: Use if there is NOT any data for producers'BHP. Oil production based on Power-Law Model is added. Production rates after each shut in are assumed to be known and equal to q(0)s. q(0)s won't be among model unknowns CRMIP: considers the control volume between each injector-producer pair (estimates field and producers production) CRMP 7, No q(0) CRMP 5 CRMP 3 CRMP 1 CRMG: With BHP With BHP No a(0) CRMIP v.1 With BHP With BHP With Oil Model With Oil Model CRMT (Linear Rate CRMIP v.2 CRMP 4 CRMP 6 CRMP 8 , No q(0) Change) CRMP 2 Without BHP Without BHF NO q(0) With BHP WO BHP WO BHP With Oil Model With Oil Model $q(t) = q(t_0)e^{-(\frac{\tau-\tau_0}{\tau})} + \left| i(t) - e^{-(\frac{\tau-\tau_0}{\tau})}i(t_0) \right| - \tau \left(1 - e^{-(\frac{\tau-\tau_0}{\tau})}\right) \left[\frac{\Delta i(t)}{\Delta t}\right]$ CRM-TANK : $q_{j}(t) = q_{j}(t_{0})e^{-(\frac{t-t_{0}}{\tau_{j}})} + \left(1 - e^{-(\frac{t-t_{0}}{\tau_{j}})}\right) \left[\sum_{i=1}^{N_{i}} \left[f_{ij}I_{i}\right] - J_{j}\tau_{j}\frac{\Delta p_{wf,j}}{\Delta t}\right]$ CRMP: Producer CV: $q_{j}(t) = \frac{1}{N_{i}}q_{j}(t_{0})\sum_{i=1}^{N_{i}}e^{-\frac{(t-t_{0})}{\tau_{j}}} + \frac{1}{N_{i}}\sum_{i=1}^{N_{i}}\left|\lambda_{ij}I_{i}\left(1 - e^{-\frac{(t-t_{0})}{\tau_{ij}}}\right)\right| - \frac{1}{N_{i}}\frac{\Delta p_{vof,j}}{\Delta t}\sum_{i=1}^{N_{i}}\left|\beta_{ij}\tau_{ij}\left(1 - e^{-\frac{(t-t_{0})}{\tau_{ij}}}\right)\right|$ CRMG-Injector Producer CV: Yousef's Model: $\lambda_{ij} = \frac{f_{ij}}{f'_{ij}}, \beta_{ij} = \frac{J_{ij}}{f'_{ij}}$ $q_{j}(t_{n}) = \sum_{i=1}^{N_{i}} q_{ij}(t_{n}) = \sum_{i=1}^{N_{i}} q_{ij}(t_{0}) e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} + \sum_{i=1}^{N_{i}} f_{ij} \left| i_{i}(t_{n}) - e^{-(\frac{t_{n}-t_{0}}{\tau_{ij}})} i_{i}(t_{0}) \right|$ CRMIP_Injector Producer CV $-\sum_{i=1}^{N_i} \left\{ \tau_{ij} \sum_{k=1}^{n} \left\{ e^{-\left(\frac{t_n - t_k}{\tau_{ij}}\right)} \left(1 - e^{\frac{-\Delta_k}{\tau_{ij}}} \right) \left[\frac{f_{ij} \Delta i_i^{\ k}}{\Delta t_k} + J \frac{\Delta p_{\forall j,j}^k}{\Delta t_k} \right] \right\} \right\}$ By Morteza Sayarpour sayarpour@mail.utexas.edu Last update 06/29/2008

Figure D-1: Interface of the CRM tool in Excel.

APPENDIX E: FIELD EXAMPLE OF THE IMPACT OF THE BOTTOMHOLE PRESSURE DATA ON THE CAPACITANCE-RESISTIVE MODEL PRODUCTION RESPONSE

In this section, the total production history-match by the CRMIP with and without producers' bottomhole pressure (BHP) are compared in a field case. In this example, there are three injectors and four producers where injection rates and producers BHP are measured daily. As shown in Figs. E-1 and E-2 high frequency production fluctuation are not captured if the BHP data are unavailable or discarded. Cross plots of the observed-estimated rates indicate the improvement of the estimation by including BHP data.



Figure E-1: The impact of the BHP data availability in improving the quality of the CRM prediction for producer two, without and with the BHP data, in a field study with daily rate and BHP measurements.



Figure E-2: The impact of the BHP data availability in improving the quality of the CRM prediction for producer four, without and with the BHP data, in a field study with daily rate and BHP measurements.

APPENDIX F: TOTAL AND OIL PRODUCTION MATCHES FOR INDIVIDUAL WELLS IN THE MCELROY AND UP-FORD CASE STUDIES





Figure F-1: The total and oil production match and cross plots for producer 1 and 2 in the McElroy CO2 pilot.



Figure F-2: The total and oil production match and cross plots for producer 3 and 4 in the McElroy CO₂ pilot area.



Figure F-3: The total and oil production match and cross plots for producer 5 and 6 in the McElroy CO_2 pilot area.



Figure. F-4: The total and oil production match and cross plots for producer 7 and 8 in the McElroy CO₂ pilot area.



Figure. F-5: The total and oil production match and cross plots for producer 9 in the McElroy CO_2 pilot area.



UP-Ford Field Individual Producers Total and Oil Rate Matches

Figure F-6: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 1 and 2 in the in the UP-Ford field.



Figure F-7: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 3 and 4 in the in the UP-Ford field.



Figure F-8: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 5 and 6 in the in the UP-Ford field.



Figure F-9: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 7 and 8 in the in the UP-Ford field.



Figure F-10: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 9 and 10 in the in the UP-Ford field.



Figure F-11: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 11 and 12 in the in the UP-Ford field.


Figure F-12: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 13 and 14 in the in the UP-Ford field.



Figure F-13: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 15 and 16 in the in the UP-Ford field.



Figure F-14: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 17 and 18 in the in the UP-Ford field.



Figure F-15: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 19 and 20 in the in the UP-Ford field.



Figure F-16: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 21 and 22 in the in the UP-Ford field.



Figure F-17: The total and oil production matches and cross plots of the CRMP estimation vs. field measurements for producer 23 and 24 in the in the UP-Ford field. Producer 24 accounts for the other half of the field production.

Nomenclature

- $c_o =$ oil compressibility (L²/F)
- $c_t =$ total reservoir compressibility (L²/F)
- $c_w =$ water compressibility (L²/F)
- CWI = cumulative water injected (L³)
- $e_w = \inf \left[u \left(\frac{L^3}{t} \right) \right]$
- f_{ij} = interwell connectivity between injector *i* and producer *j*, dimensionless
- $f_o =$ oil-fractional-flow, dimensionless
- H = heterogeneity factor
- i_i = variable injection rate (L³/t)
- I = fixed injection rate (L³/t)
- J = productivity index (L⁵/Ft)
- k = permeability (md)
- K = Koval factor
- m = relative permeability exponent, dimensionless
- M_{ij} = number of block between injector *i* and producer *j*
- M_o = endpoint mobility ratio, dimensionless
- n = time-like variable, relative permeability exponent, dimensionless
- N = oil in place, (L3)
- N_{inj} = total number of injection wells, dimensionless
- N_P = cumulative oil production
- N_{pro} = total number of production wells, dimensionless

OOIP = original oil in place (L3)

OWIP = original water in place (L3)

- p_{wf} = bottom-hole flowing pressure (F/L2)
- \overline{p} = average reservoir pressure (F/L2)
- q = fluid production rate (L3/t)
- $q_o =$ oil production rate (L3/t)
- $q_w =$ water production rate (L3/t)
- r = growth or decline rate in logistic equation (1/t)
- R = correlation coefficient, dimensionless

ROIP = recoverable oil in place (L3)

- S = saturation, dimensionless
- S_{or} = residual oil saturation, dimensionless
- $S_o =$ oil saturation, dimensionless
- S_{wr} = residual water saturation, dimensionless
- t = time(t)
- $V_P =$ pore volume (L3)
- W_i = cumulative water injected (L3)
- *WOR* = water oil ratio

Greek alphabets

- $\alpha =$ fractional-flow model or logistic equation coefficient
- β = fractional-flow model or logistic equation coefficient
- $\gamma =$ logistic equation coefficient

 ξ = integrating variable (t)

- $\eta =$ ratio of currently active producers to previously active producers
- κ = asymmetric Hubbert curve standard deviation coefficient
- σ = standard deviation

- $\lambda_o =$ oil mobility ratio
- $\lambda_w =$ water mobility ratio
- μ = viscosity (cp)
- σ = standard deviation
- τ = time constant (t)
- τ^* = equivalent blocks time constants (t)

Subscripts and superscripts

- b = block identifier
- D = dimensionless time
- F = field value indicator
- i = injector index
- *ij* = injector-producer pair index
- j = producer index
- k = timestep index
- o = oil index
- s = solvent index
- p = pattern value indicator
- r = residual indicator
- w = water index
- x = X direction indicator
- y = Y direction indicator
- z = Z direction indicator

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