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Essays on Housing and Labor Markets

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Essays on Housing and Labor Markets

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DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2009

Dedicated to

my wife Asli,

my son Tarik and

my parents Mustafa and Zulfiye.

Acknowledgments

I would like to thank my thesis supervisors Fatih Guvenen and P. Dean Corbae for their generous time and encouragement. I specifically owe a lot to Fatih Guvenen for his continuing support starting from the first day of my Ph.D. studies. With his leadership, research skills, hard work and scholarship, he has become a great role model for me. I am also very grateful for having an exceptional doctoral committee. I want to thank to my committee members Gianluca Violante, Burhanettin Kuruscu and Russell Cooper for their support and fruitful conversations.

My special thanks go to my wife Asli for her enormous support, understanding and patience, and of course to my son Tarik for being the ultimate joy of my of life. I would also like to thank to my parents for their continuous support through my entire life.

Last but not least, I would like to thank Yavuz Arslan, Pablo Derasmo, Borghan Narajabad and Juan Sanchez for their valuable comments about my thesis.

Essays on Housing and Labor Markets

Publication No. ____________________

Bulent Guler, Ph.D.
The University of Texas at Austin, 2009

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In the first chapter, I study the effects of innovations in information technology on the housing market. Specifically, I focus on the improved ability of lenders to assess the credit risk of home buyers, which has become possible with the emergence of automated underwriting systems in the United States in the mid-1990s. I develop a standard life-cycle model with incomplete markets and idiosyncratic income uncertainty. I explicitly model the housing tenure choice of the households: rent/purchase decision for renters and stay/sell/default decision for homeowners. Risk-free lenders offer mortgage contracts to prospective home buyers and the terms of these contracts depend on the observable characteristics of households. Households are born as either good credit risk types—having a high time discount factor—or bad types—having a low time discount factor. The type of the household is the only source of asymmetric information between households and lenders. I find that as lenders have better information about the type of households, the average downpayment fraction decreases together with an increase in the average mortgage premium, the foreclosure rate, and the dispersions of mortgage interest rates and downpayment

fractions, which are consistent with the trends in the housing market in the last 15 years. From a welfare perspective, I find that better information, on average, makes households better off.

In the second chapter, I focus on the labor market behavior of couples. Search theory routinely assumes that decisions about the acceptance/rejection of job offers (and, hence, about labor market movements between jobs or across employment states) are made by individuals acting in isolation. In reality, the vast majority of workers are somewhat tied to their partners—in couples and families—and decisions are made jointly. This chapter studies, from a theoretical viewpoint, the joint job-search and location problem of a household formed by a couple (e.g., husband and wife) who perfectly pool income. The objective of the exercise, very much in the spirit of standard search theory, is to characterize the reservation wage behavior of the couple and compare it to the single-agent search model in order to understand the ramifications of partnerships for individual labor market outcomes and wage dynamics. We focus on two main cases. First, when couples are risk averse and pool income, joint-search yields new opportunities—similar to on-the-job search—relative to the single-agent search. Second, when couples face offers from multiple locations and a cost of living apart, joint-search features new frictions and can lead to significantly worse outcomes than single-agent search.

Finally, in the third chapter, I focus on the relation between house prices and interest rates. Although interest rates and housing prices seem mostly to have a negative relation in the data, the relation does not seem to be stable. For example, the recent run up in the global housing prices is generally explained by globally low interest rates. On the other hand, there have been periods where housing prices and interest rates moved together. Motivated by these observations, I formulate a two period OLG model to find out the form of the relationship between interest rates and housing prices. It appears that the distribution of homeownership is also important for housing price dynamics. I show that housing prices in the equilibrium do not always have a negative relation with interest rates.

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Chapter 1

Innovations in Information Technology and the Mortgage Market

1.1 Introduction

The US housing market has witnessed important changes since the mid-1990s. Arguably, the most prominent technological change during this time was the emergence of automated underwriting systems (hereafter AUS), which allowed a better assessment of the credit risks of home buyers. In particular, advances in information technology (e.g., the rapid decline in the cost of storing and transmitting credit information) have enabled access to more comprehensive data on households, which in turn increased the predictive power of credit scores, thereby allowing lenders to assess the credit risk of home buyers more precisely. Accompanying these improvements in information technology, the housing market has experienced changes along some key dimensions. As reported in Table 1.1, a comparison of the 1991-1995 period and 2002-2006 period reveals that (i) the foreclosure rate has increased, (ii) average mortgage premium has gone up, (iii) average downpayment fraction has decreased, and (iv) the dispersions of mortgage interest rates and downpayment fractions have risen up.

Table 1.1: Summary Statistics*

Statistic	1991-1995	2002-2006
Foreclosure rate	0.33%	0.44%
Mortgage premium	0.32%	0.52%
Average downpayment fraction	14.5%	8.9%
Coef of variation-int rate	0.159	0.203
Coef of variation-downpayment	1.28	3.55

* Downpayment and mortgage interest rate data are from American Housing Survey and calculated for 30-year fixed rate mortgages. Mortgage premium, measured as the difference between 30-year fixed rate mortgage and AAA corporate bond yield, is from Federal Reserve Board. Foreclosure data is from Mortgage Bankers Association.

In this paper, I explore the effect of innovations in information technology—specifically, the increased ability of lenders to assess the credit risk of home buyers—on the housing market. I develop a standard life-cycle model with incomplete markets and idiosyncratic labor income uncertainty. I also model the housing tenure choice. There are two types of households: those with a high time discount factor (ie., the "good" type) and those with a low time discount factor (ie., the "bad" type). Households are born as renters with ex-ante heterogeneity in income and wealth. Every period, renters decide whether or not to purchase a house. There is a continuum of risk-neutral lenders who offer mortgage contracts to prospective home buyers. A mortgage contract consists of a mortgage interest rate, loan amount, mortgage repayment schedule and maturity. Mortgages are fully amortizing, that is, homeowners have to pay the mortgage back in full until the end of the mortgage contract, as specified by the maturity. However, homeowners also have the option to sell their houses or default on the mortgage and return to the rental market. Selling a house is different from defaulting, because a seller has to pay back the outstanding mortgage balance to the lender whereas a defaulter has no obligation. Therefore, default occurs in equilibrium as long as the selling price is lower than the outstanding mortgage debt. Upon default, the household becomes a renter again and is excluded from the mortgage market for a certain number of years as punishment.

There is free entry into the credit market, so in equilibrium lenders make zero profit on each contract. Since mortgages are long-term contracts, it is essential for the lenders to infer the default probability of each household at every date and state, which depends on the income risk as well as on the type of each household. Clearly, given an income realization, a household with a low time discount factor (bad type) has a higher probability of default, since she values the benefit of homeownership less compared to the good types. I explore two information structures. In one economy, lenders cannot observe the types although they can observe all the other characteristics of the household. This creates asymmetric information between the lenders and households, and I call this the asymmetric information economy (AI). In the other economy, lenders can observe all the characteristics of the household and therefore information is symmetric (hence, called symmetric information economy, SI).[1]

I interpret the AI economy as representing the US economy before the emergence of automated underwriting systems (before mid-1990s), whereas the SI economy represents the more recent period with AUS (mid-2000s). Because these two

[1] Chatterjee, Corbae and Rios-Rull (2007, 2008) build a model of reputation in credit markets and show how credit scores are informative about the type of the households in equilibrium. Good types are patient and value reputation more compared to the bad types. As a result, the credit score, which tracks the history of the ability and willingness of the household to make debt payments, becomes very informative in differentiating the households. However, due to the curse of dimensionality of such a model, I do not explicitly model the credit scores. Instead, I assume that the emergence of credit scores has enabled lenders to fully observe the type of the households.

time periods also differ in average interest rates and housing prices, each economy is calibrated to match these two empirical targets in their respective time period. The results indicate that the transition from the AI economy to the SI economy decreases the downpayment fraction and increases the mortgage premium, foreclosure rate, and homeownership rate, which are consistent with the current changes in the mortgage market. Moreover, consistent with the data, the transition brings an increase in the dispersion of the mortgage interest rates and downpayment fractions. However, the levels of the dispersion of mortgage interest rates that the model generates are much lower than their data counterparts. This is mainly due to omission of several important risk factors (i.e., risk-free interest rate risk, house price risk) in the model.

Because the AI and SI economies also differ in the average interest rate and housing price, I conduct the following (counterfactual) experiment to isolate the role of information technology. Basically, I simulate the AI economy with the same set of parameters in the SI economy. The results show that information structure is the main driving force behind the increase in the dispersion of mortgage interest rates and downpayment fractions as well as having an important role in the increase of the mortgage premium, the foreclosure rate, and the decrease in the downpayment fraction. Risk-free interest rate and house price are more important in explaining the increase in the homeownership rate and the decrease in the downpayment fraction. A higher foreclosure rate does not mean that lenders and households are worse off in the SI economy. When I measure the welfare gain of being born into the SI economy as opposed to the AI economy, the gain, in consumption equivalent terms, is between 0.25% and 0.29% depending on whether I use pooling contracts or separating contracts as equilibrium contracts in the AI economy. Furthermore, the zero-profit restriction on the contracts ensures that, ex-ante, lenders are indifferent between both economies. I, finally, check the robustness of the results to the selection of equilibrium in the AI economy. The counterfactual shows the equilibrium with pooling contracts. To explore the effect of an alternative equilibrium, I solve the AI economy with separating contracts. The new equilibrium with separating contracts shows very similar effects as the equilibrium with pooling contracts.

In the AI economy, lenders cannot observe the types and they face an adverse selection problem. This puts additional constraints on the contracts offered in the AI economy. The transition from the AI economy to the SI economy, both in the extensive margin and intensive margin, makes the credit terms more relaxed. In the extensive margin, those low income households who are rationed out in the AI economy become eligible for mortgages. Since these households are income constrained, they demand for lower downpayment fraction which requires higher mortgage premium. In the intensive margin, since bad types are, now, perfectly observed they get contracts that have lower downpayment fraction at the cost of higher mortgage premium[2]. Moreover, since good types face lower mortgage premium, they also demand for lower downpayment fraction loans. As a result downpayment fraction decreases whereas mortgage premium and homeownership rate increase. Since the

[2]Note that households face a trade-off between the downpayment fraction - short-term debt - and mortgage payment - long-term debt - during the house purchase. Bad types - impatient households - favor a decrease in the downpayment fraction more than an decrease in the mortgage payment compared to the good types - patient households.

new home purchasers are low income households and average downpayment fraction decreases together with an increase in the mortgage premium, the default risk in the market increases. Thus, the foreclosure rate increases.

There is a growing empirical literature suggesting that innovations in the mortgage market are the main reasons for the recent changes in the housing market. Testing the forecasting relationship between housing spending and future income, Gerardi, Rosen and Willen (2006) find that recent developments in the mortgage market have ensured that households have been more able to buy houses whose values are in line with their long-term income prospects. Mian and Sufi (2008) document that high unfulfilled demand zip codes experienced relative declines in mortgage application denial rates and mortgage interest rates and relative increases in mortgage credit and house prices despite the negative relative income and employment growth in these zip codes. They also find that the growth of securitization was significantly higher in high unfulfilled demand zip codes, suggesting a possible role of supply side changes in the mortgage market. Finally, using the American Housing Survey data, Doms and Krainer (2007) find that housing expenditures of the households facing the greatest financial constraints have increased substantially using, particularly, the newly designed mortgages.

Although, on the empirical side, financial innovations in the mortgage industry and its impact on the market and households seem to be well documented, the literature thus far has paid little attention to modeling this link. In a stylized model, Ortalo-Magne and Rady (2006) show the effect of credit constraints, especially the effect of downpayment requirement, on the extensive and intensive margin of homeownership. Chambers, Garriga and Schlagenhauf (2008), in a quantitative framework, analyze the effect of financial innovations on the homeownership rate. They find that the key to understanding the increase in the homeownership rate, especially for young households, is the expansion of the set of mortgage contracts. Nevertheless, their way of modeling the terms of mortgage credit is in a reduced form and cannot explain the reason for the expansion of the mortgage credit. Moreover, they do not model the default option.

The equilibrium model of mortgage credit and default used in this paper is related to the equilibrium models of unsecured borrowing and bankruptcy[3]. Closely related to my paper are three papers influenced by Narajabad (2007): Sanchez (2008), Athreya, Tam and Young (2008), and Livhits, MacGee and Tertilt (2008) who explore the effects of innovations in the unsecured credit market. These papers show that a transition from a partial information economy to a full information economy results in an increase in consumer bankruptcies and debt which is consistent with the trend in the data. These papers analyze the unsecured credit market and borrowing is only for one period. Different from these models, I analyze the mortgage market characterized by secured borrowing and long-term maturity contracts which requires us to model the lender's problem recursively.

This paper is organized as follows: Section 1.2 documents some recent innovations in the mortgage market, especially focusing on the emergence of credit scoring technology and its impacts on the market. Section 1.3 describes the envi-

[3] Chatterjee, Corbae, Nakajima (2007) and Livhits, MacGee and Tertilt (2007) are some prominent examples of such models.

ronment and sets up the model. Section 1.4 presents the main results of the model together with a counterfactual experiment separating the impact of the change in information structure. It also presents the results for an alternative equilibrium definition. Finally, Section 1.5 concludes with directions for future work. The Appendix presents a simpler model to analyze the potential existence problem.

1.2 Innovations in Information Technology

There are three basic components of single-family mortgage underwriting: the value of the collateral, the ability of the borrower to make monthly mortgage payments and the willingness of the borrower to pay back outstanding mortgage debt. They are summarized as the traditional "three C's": Collateral, Capacity and Credit. The loan-to-value ratio is the measure of the collateral, which is basically measured by the downpayment fraction and the real value of the house. Capacity is useful to understand the ability of the borrower to make the monthly mortgage payments and is measured through several economic variables regarding the home buyer such as debt-to-income ratio, debt-service ratio,employment status, and savings. Lastly, credit shows both the ability and willingness of the borrower to pay back the debt and is assessed through a credit report summarizing the historical performance of the home buyer in the credit market.

Until the mid 1990's, credit was the missing piece of the three C's. Insufficient available credit data for individuals was the main reason for the absence of credit reports in mortgage underwriting. Unlike unsecured credit, mortgages are long-term contracts and larger amount of loans are at risk or fraudulent. Knowing the ability of the home buyer to make the periodic mortgage payments is the most important information for the lenders. The home buyer's loyalty to the payments strongly depends on his credit history, which captures the historical performance of the individual in the credit market. Borrowers with poor credit records go into mortgage default at much higher rates than borrowers with good credit records. Since insufficient credit report data may be misleading, lenders hesitated to use the credit reports for a long time. Straka (2000) shows the relationship between credit scores and default rates using a 1995 assessment of a large sample of Freddie Mac loans which were originated between 1990 and 1991. The result shows that in a weak regional housing market, a mortgage holder with a credit score, measured as a FICO score, smaller than 620 is 17 times more likely to default than a mortgage holder with a credit score higher than 760. He also shows that even in a strong regional housing market, credit scores have a great predictability of mortgage default[4].

As the IT revolution has made computers part of our daily life, enabled data storage to become more efficient and less expensive and allowed computer networking through local area networks and the internet, there has been an explosion in the growth of credit report in the late 1980s and early 1990s[5]. As a result we see a shift in the mortgage landscape. Long-time dominant manual and decentralized underwriting and origination systems requiring labor and paper intensive loan processing and risk assessment and lasting for weeks and even months have

[4]See also Pennington-Cross (2003), Cutts and Green (2004) and Barakova, Bostic, Calem, and Wachter (2003) for further evidence of predictive power of credit scores in mortgage repayment and default.

[5]See Hunt (2005) on the evolution of consumer credit reports and Lacour-Little (2000) and Pafenberg (2004) for the adoption of credit reporting in the mortgage industry.

been rapidly replaced by automated and centralized underwriting systems based on credit scores, statistical model loan processing and risk evaluations which result in in-minute decisions and same-day closings. Before 1995, negligible amount of mortgage lenders had been using automated underwriting systems. In 1995, FreddieMac and FannieMae published industry letters that endorsed the use of credit scores to assess credit quality. In subsequent years, the mortgage industry has experienced a growing adoption of automated underwriting systems which rely on credit scores and statistical models[6].

This transition has brought two innovations to the mortgage industry: usage of credit scoring and automation of the underwriting process. These innovations have increased the ability of the lenders to assess the credit risk of the home buyers. Straka (2000) documents the result of an experiment which compares the performance of manual -without credit scores- and automated -with credit scores- underwriting. A pool of 1000 mortgages that originated between 1993 and 1994, were evaluated both by manual and automated underwriting systems[7]. Although both underwriting systems chose half of the loans as investment-quality loans, the overlapping was quite few. After three years, the performance of the loans was compared in four categories (share of the 30 days, 60 days, 90 days delinquent loans and foreclosed loans) and the results were striking. While investment quality loans determined by automated underwriting system performed quite better than

[6] According to Pafenberg (2004), among the loans Freddie Mac and Fannie Mae purchased from enterprises, the percentage of mortgages evaluated using automated underwriting systems by the enterprises prior to the purchase increased from 10% to 60% between 1997 and 2002.

[7] Straka (2000) notes that the assessment of all mortgages through manual underwriting lasted six months while through automated underwriting it lasted only a couple of hours.

the non-investment quality loans, there was essentially no difference between the investment and non-investment quality loans determined by manual underwriting in terms of delinquency rates. The results were quite striking, especially in terms of foreclosure rates . Non-investment quality loans ended up in foreclosure eight times more than investment quality loans according to the automated underwriting system selection. However, according to manual underwriting selection, investment quality loans ended up in foreclosure seventeen times more than the non-investment quality loans[8].

1.3 Model

I begin by describing the environment agents face in the economy. I then specify the decision problems of households and lenders. I finally define the equilibrium.

1.3.1 Environment

The economy is populated by overlapping generations of J period lived households and a continuum of lenders. Each generation has a continuum of households. Time is discrete and households live for a finite horizon. There is no aggregate uncertainty. Households face idiosyncratic uncertainty in labor income and markets are incomplete. There is mandatory retirement at the age J_r. Retirement income is constant and depends on the income of the household at age J_r and the average

[8] Gates, Perry and Zorn (2002) also provide a comparison of manual and automated underwriting systems. They also show how automated underwriting outperforms manual underwriting in terms of predicting delinquency and foreclosure.

income in the economy. They can save at an exogenously given interest rate r but they're not allowed to make unsecured borrowing. Ex-ante, households differ in three dimensions: initial asset, income and discount factor. Initial income is assumed to be the stationary distribution and the initial asset-income ratio is assumed to be log-normally distributed. There are two types of households: good types having high time discount factor and bad types having low time discount factor.

Households live in houses, which they can either rent or own. At the beginning of each period, a household is in one of the three housing statuses: *inactive renter*, *active renter*, or *homeowner*. Active renters are always allowed to purchase a house, while inactive renters are only allowed with a certain probability δ. Both rental price and purchase price for the houses are exogenous and constant[9]. The size of the house is fixed, i.e. there is no upgrading or downgrading of the house size. However, since houses are big and expensive, their purchase is only through mortgages, which is also the only source of borrowing in the economy. A mortgage contract is a combination of interest rate and loan amount, specified by the downpayment fraction and house value. Maturity of the mortgages is assumed to be the remaining life time of the household until retirement[10]. Lenders only of-

[9]I implicitly assume that the supply of rental and owner-occupied units is perfectly elastic. There is a fixed unit of housing and all units can be converted into a rental or owner-occupied unit without any cost. These assumptions ensure that the price stays constant and all the response to a demand increase occurs in the extensive margin as an increase in the homeownership rate.

[10]Maturity of the mortgage, in reality, is a choice variable. However, in the current context to save from an extra state variable, I avoid this choice for now. Moreover, I assume that all homeowners are forced to sell their houses by retirement and spend their remaining life as renters. Since after retirement there is no uncertainty, housing tenure choice becomes uninteresting. So, to simplify the problem of the retirees, I ignore their housing tenure choice and force them to live as renters. This formulation will greatly simplify the computation of the value function at the time of retirement.

fer fixed-payment mortgages, so the payment is constant throughout the life of the mortgage[11]. There is no mortgage refinancing or home-equity line of credit. Homeowners have the option to default at any time period. The details of the model are explained below.

1.3.1.1 Households

Households derive utility from consumption and housing services. Preferences are represented by

$$E_0 \left[\sum_{j=1}^{J_r} \beta_i^{j-1} u_k \left(c_j \right) + \beta_i^{J_r+1} W \left(w_{J_r}, y_{J_r} \right) \right]$$

where $\beta_i < 1$ is the discount factor for type $i \in \{g, b\}$ agent, c is the consumption and k is the housing status: renter or homeowner. W represents the value function of the household at retirement given wealth w_{J_r} and income y_{J_r}[12]. There are two types of households: good types and bad types. Good types have a higher time discount factor than the bad types: $\beta_g > \beta_b$. Types are fixed and the measure of the good types in the economy is μ. The house size is fixed and the utility from housing services is summarized as two different utility functions: one for the renter, u_r and one for the homeowner, u_h. A homeowner receives a higher utility than a renter from the same consumption: $u_h(c) > u_r(c)$.

[11] Since I assume constant interest rate, traditional fixed rate mortgages and adjustable rate mortgages would have fixed payments throughout the life of the mortgage and they both fall into this category. These mortgages are not necessarily optimal contracts. A more convenient formulation should also include the mortgage payment as part of the contract and be determined in equilibrium. However, for simplicity I abstract from that and focus on the fixed payment mortgage contracts which are the dominant mortgages in the U.S. history.

[12] Since there is no housing tenure choice and uncertainty after retirement, household's problem is trivial and can be calculated analytically.

The log of the income before retirement is a combination of a deterministic and a stochastic component whereas after retirement it is the λ fraction of the income at age J_r plus η fraction of the average income in the economy, $\bar{y}$:

$$y_j\left(j, z_j\right) = \left\{\begin{array}{ll} \exp\left(f\left(j\right)+z_j\right) & \text{if } j \leq J_r \\ \lambda y_{J_r}\left(J_r, z_{J_r}\right)+\eta\bar{y} & \text{if } j > J_r \end{array}\right\}$$

$$z_j = \rho z_{j-1}+e_j$$

where y_j is the income at age j, $f\left(j\right)$ is the age-dependent deterministic component of the log income, and finally z_j is the stochastic component of the log income. The stochastic component is modeled as an $AR(1)$ process with ρ as the persistency level. The innovation to the stochastic component, e_t, is assumed to be i.i.d and normally distributed: $N\left(0, \sigma_e^2\right)$. Households can save to smooth their consumption at the constant risk-free interest rate r, but there is no unsecured borrowing.

Households start the economy as active renters, and can purchase a house and become an owner at any period in time. However, an inactive renter is only allowed to purchase a house with probability δ. With $(1-\delta)$ probability, she is forced to live as a renter. Since houses are expensive items, their purchases can only be done through securitized borrowing: mortgages. A purchaser chooses among a menu of feasible mortgage contracts, each specified with a loan amount and interest rate[13]. Since the mortgages are fixed-payment mortgages, the contract together with the maturity, remaining time to retirement, determine the periodic mortgage payments.

[13]Not every combination of mortgage interest rate and loan amount is feasible for the household. Lenders' inference about the type of the household and competition among lenders restrict the contracts offered to the household.

As long as the household stays in the house, she has to make these payments. The homeowner has also the option to sell the house at any time period. However, selling the house is costly. There are some costs (transaction costs and maintenance costs) associated with selling the house. So, a seller incurs a proportional cost, φ, of the house price. Moreover, a seller has to pay the outstanding mortgage debt back to the lender.

There is another option for the household to quit the house. She can default on the mortgage. A defaulter has no obligation to the lender. Upon default, the lender seizes the house, sells it and pays back, if any, to the defaulter the amount net of outstanding mortgage debt and costs associated to selling the house. The lender's cost of selling the house is φ fraction of the house price. What makes default appealing for the household is the fact that a defaulter has no obligation to the lender whereas a seller has to pay back the debt in full. The same fact puts a risk of loss on the lender. The lender incurs a loss if the net value of the house is smaller than the outstanding debt upon default.

Default is not without any cost to the household. A defaulter becomes an inactive renter and can only enter to the housing market with probability δ. Lastly, at the end of the life cycle, homeowner sells the house and enjoys the utility from consuming the selling price. Again, the seller loses φ fraction of the house price during the transaction.

1.3.1.2 Lenders

There is a continuum of lenders and financial markets are perfectly competitive. Lenders are risk-neutral[14]. The economy is assumed to be an open economy and the risk-free interest rate, r, is set exogenously. Mortgage contracts are long-term contracts and the maturity of the contract is directly determined by the time to retirement, which is assumed to be certain and observable. Lenders have full commitment to the contract and renegotiation is not allowed.

Each contract is characterized by a loan amount, d, and interest rate, r_m. Since the households can default on the mortgage at any time period, and transaction and further costs make the loan not fully securitized, lenders face a risk of loss on mortgage loans. Moreover, there is an additional per period servicing cost for mortgage loans, τ, which is assumed to be proportional to the loan amount.

1.3.1.3 Timing

The timing of the events is the following: Households are born as active renters. For any other period, the household starts the period either as a homeowner, an active renter or an inactive renter. At the beginning of each period, households realize their income shock and decide about their housing statuses for the current period.

An active renter has two choices: continue to rent or purchase a house. If she decides to continue to rent, she pays the rental price, makes her consumption

[14] Securitization of mortgages helped lenders to diversify the risk they face and liquidate their asset holding. However, risk-neutrality assumption eliminates such benefits of the securitization.

and saving choices, and reaches to the next period as an active renter. If she decides to buy a house, she goes to a lender. The lender offers a menu of mortgage contracts depending on the observable of the household[15]. The household chooses the mortgage contract that maximizes her utility. Lastly, she pays the downpayment and periodic mortgage payment implied by the mortgage contract, makes her consumption and saving choices, and reaches to the next period as a homeowner.

A homeowner has three choices. If she decides to stay in the current house, she pays the fixed mortgage payment, makes her consumption and saving choices, and starts the next period again as a homeowner. If she decides to sell the house, she receives the selling price, pays the outstanding mortgage debt back to the lender, makes her consumption and saving choices and begins the next period as an active renter. If she decides to default, she receives any positive remaining balance - the selling price of the house to the lender minus the outstanding mortgage debt - from the lender, makes her consumption and saving choices, and starts the next period as an active renter with δ probability and inactive renter with $(1-\delta)$ probability.

An inactive renter has no housing tenure choice. She is forced to live as a renter. So, she pays the rental price, and only makes her consumption and saving choices and starts the next period as an active renter with δ probability and inactive renter with $(1-\delta)$ probability.

[15] Note that in SI economy and AI economy with pooling contracts, the lender only offers one contract depending on the observable of the household. In the AI economy with separating contracts, the lender offers two contracts to separate the good type and the bad type.

1.3.1.4 Information Structure

As I mentioned above, the menu of mortgage contracts offered by the lender depends on the observable of the household. I model the information structure in two different ways. In the first economy, which I call as the *"Asymmetric Information" (AI)* economy, the lender can observe the current characteristics of the household except the type - discount factor. I also assume the history of the household is not observable. The lender only knows the initial distribution of the households and can infer the type of the household given the current period observable. This informational asymmetry between households and lenders creates the problem of *adverse selection*. Since the lender cannot observe the type, any contract designed for the good type is also available for the bad type with the same observable.

In the second economy, which I call as the *"Symmetric Information" (SI)* economy, the lender observes all the characteristics of the household. This feature of the economy enables the lenders to separate all the households, evaluate the default risk of each household and set mortgage prices at the household level. So, in the SI economy, mortgage pricing is fully individualized, whereas in the AI economy, lenders face a pool of households with the same characteristics but different types.

1.3.2 Decision Problems

I now turn to the recursive formulation of the household's and lender's problem. Note that since the mortgages are long-term contracts, the lender's problem also has dynamic structure. The lender has to calculate the default risk of the household through the life of the mortgage. Here, I first start with the recursive formulation of the household's problem, then I set up the lender's dynamic programming problem which is also closely related to the household's problem.

1.3.2.1 Household's Problem

I only focus on household's problem before retirement. The value function at the time of retirement can be calculated analytically given the utility specification. At the beginning of each period, the household is in one of the three housing positions: inactive renter, active renter and homeowner. After the realization of the income shock, the active renter and the homeowner make their housing tenure choices for the current period and start the next period with their new housing statuses. Let's denote V_i^r as the value function for a type i active renter after the realization of the income shock and just before the housing choice. Similarly, let V_i^h be the value function for a type i homeowner and let V_i^e be the value function for a type i inactive renter. Note that in the current period inactive renter has no housing tenure choice.

Inactive Renter. I start with the problem of an inactive renter. An inactive renter's problem is simple. She does not have any housing tenure choice, she is forced to be a renter in the current period. The only decisions she has to make are the consumption and saving allocations. She starts the next period as an active renter with probability δ and an inactive renter with probability $(1-\delta)$. Denoting the value function of a type i inactive renter with age j, period beginning saving a

and income z as $V_i^e(a,z,j)$, the inactive renter's problem is given by:

$$V_i^e(a,z,j) = \max_{c,a'\geq 0}\left\{u_r(c) + \beta_i E\left[\delta V_i^r\left(a',z',j+1\right) + (1-\delta)V_i^e\left(a',z',j+1\right)\right]\right\} \tag{1.1}$$

subject to

$$c + a' + p_r = y(j,z) + a(1+r)$$

where c is the consumption, a' is the next period saving, and p_r is the exogenous rental price . Note that the inactive renter derives utility from consumption and being a renter.

Active Renter. Different from an inactive renter, an active renter has to make a housing tenure choice. After the realization of the income shock, an active renter has to decide whether to continue to stay as a renter or purchase a house in the current period. This means I need to define two additional value functions for the active renter. Define V_i^{rr} as the value function for a type i active renter who decides to stay as a renter and name such a household as *renter*. Her problem is very similar to the inactive renter's problem apart from the fact that she starts the next period as an active renter for sure. Given all these facts, I can write the problem of the renter as:

$$V_i^{rr}(a,z,j) = \max_{c,a'\geq 0}\left\{u_r(c) + \beta_i E V_i^r\left(a',z',j+1\right)\right\} \tag{1.2}$$

subject to

$$c + a' + p_r = y(j,z) + a(1+r)$$

The second possible choice of an active renter is to purchase a house.Define the value function for a type i active renter who decides to purchase a house as V_i^{rh} and name such a household as *purchaser*. Housing purchase is done through a mortgage contract. The purchaser, additional to the usual consumption and saving choices, has to choose a mortgage contract. Lenders design the mortgage contracts depending on the observable of the household. Due to the perfect competition in the financial market, lenders make zero-profit on these mortgage contracts. So, only the contracts which make zero-profit are feasible and offered to the household. I denote the set of feasible contracts for a household with observable θ as $\Upsilon(\theta)$. In the SI economy, $\theta \equiv (a, z, j, i)$ and in the AI economy $\theta \equiv (a, z, j)$. A mortgage contract is specified with a loan amount d and interest rate, r_m. So, a typical element of the feasible contract set is $(d, r_m) \equiv \ell \in \Upsilon(\theta)$. I leave the construction of $\Upsilon(\theta)$ to the section I define the lender's problem. Since mortgages are due by retirement, which is deterministic, household's age captures the maturity of the mortgage contract. Moreover, since I only focus on fixed payment mortgages, the choice of the loan amount and interest rate, together with the age of the household, determine the amount of mortgage payments, m. The calculation of these payments is shown in the lender's problem. Out of the total financial wealth, net of the mortgage payment and downpayment fraction, the household makes her consumption and saving choices and starts the next period as a homeowner. So, I can formulate the problem of the purchaser in the following way:

$$V_i^{rh}(a, z, j) = \max_{\substack{c, a' \geq 0 \\ (d, r_m) \in \Upsilon(\theta)}} \left\{ u_h(c) + \beta_i E V_i^h \left(a', z', j+1; d', r_m \right) \right\} \tag{1.3}$$

subject to

$$\begin{aligned} c + a' + m(d, r_m, j) + p_h - d &= y(j, z) + a(1 + r) \\ d' &= (d - m(d, r_m, j))(1 + r_m) \end{aligned} \tag{1.4}$$

where p_h is the exogenous fixed house price. The household makes the downpayment immediately upon the purchase of the house, or mortgage payments are due by the beginning of each period. Outstanding mortgage debt decumulates according to equation (1.4). It says that next period outstanding mortgage debt, d', is the current period outstanding mortgage debt reduced by the mortgage payment, net of interest payment. Note that since the purchaser becomes a homeowner in the current period, she derives utility from both consumption and being a homeowner.

The value function for the renter together with the value function for the purchaser characterize the value function for the active renter:

$$V_i^r = \max\left\{V_i^{rr}, V_i^{rh}\right\} \tag{1.5}$$

Homeowner. A homeowner has three housing choices: stay in the current house, sell the house, or default on the mortgage. This requires us to define three additional value functions. Let V_i^{hh} be the value of a type i homeowner who decides to stay in the current house and name such a household as *stayer*. Apart from the usual state variables (a, z, j), a stayer is also defined by her outstanding mortgage debt, d, and interest rate on the mortgage, r_m[16]. A stayer has to make her

[16]There are other possible combinations of state variables for the stayer. Since, the mortgage payments are fixed, one can formulate the stayer's problem by using the mortgage payment instead

consumption and saving allocations out of her wealth net of the periodic mortgage payment. The outstanding mortgage debt decumulates according to the same equation I defined in the purchaser's problem. In recursive formulation, the problem of the stayer becomes the following:

$$V_i^{hh}(a,z,j;d,r_m) = \max_{c,a'\geq 0}\left\{u_h(c) + \beta_i E V_i^h\left(a',z',j+1;d',r_m\right)\right\} \tag{1.6}$$

subject to

$$\begin{aligned} c + a' + m(d,j,r_m) &= y(j,z) + a(1+r) \\ d' &= (d - m(d,r_m,j))(1+r_m) \end{aligned}$$

The second possible choice for a homeowner is to sell the house and become a renter, and name such a household as *seller*. The selling price of the house is exogenously set to $(1-\varphi_h)$ fraction of the purchase price p_h. This feature tries to capture the possible transaction costs, maintenance costs etc. Moreover, a seller has to pay the outstanding mortgage debt, d, in full to the lender. Denoting V_i^{hr} as the value function for a type i seller, the recursive formulation of her problem is the following:

$$V_i^{hr}(a,z,j;d,r_m) = \max_{c,a'\geq 0}\left\{u_r(c) + \beta_i E V_{j+1}^r\left(a',z',j+1\right)\right\} \tag{1.7}$$

subject to

$$c + a' + p_r = y(j,z) + a(1+r) + p_h(1-\varphi) - d$$

of the outstanding debt. However, it'll be clear in the seller's problem that I also need to know the age of the individual at the time of the origination. To economize from the state variables, I find this formulation more convenient.

Again, since the seller becomes renter in the current period, she pays the rental price and enjoys the utility of a renter.

The third and the last possible choice for a homeowner is to default on the mortgage. Name such a household as *defaulter*. A defaulter has no obligation to the lender. The lender seizes the house, sells it in the market and pays any positive amount net of the outstanding mortgage debt and selling costs back to the defaulter. For the lender, selling price of the house is assumed to be $(1-\varphi_s)\,p_h$. So, the defaulter receives $\max\{(1-\varphi_s)\,p_h, 0\}$ from the lender. Defaulter starts the next period as an active renter with probability δ. With $(1-\delta)$ probability she becomes an inactive renter. Denoting V_i^d as the value function for a type i defaulter, her problem becomes the following:

$$V_i^d(a,z,j) = \max_{c,a'\geq 0}\left\{u_r(c) + \beta_i E\left[\delta V_i^r\left(a',z',j+1\right) + (1-\delta)\,V_i^e\left(a',z',j+1\right)\right]\right\} \tag{1.8}$$

subject to

$$c + a' + p_r = y(j,z) + a(1+r) + \max\{(1-\varphi)\,p_h - d, 0\}$$

Since the defaulter is a renter in the current period, she pays the rental price and enjoys the utility of a renter.

Lastly, I close the decision problem of a homeowner by characterizing her value function, which is the maximum of the above three value functions:

$$V_i^h = \max\left\{V_i^{hh}, V_i^{hr}, V_i^d\right\} \tag{1.9}$$

1.3.2.2 Lender's Problem

Since the mortgages are long-term contracts, the lender's problem is also a dynamic problem. The lender has to design a menu of contracts, $\Upsilon(\theta)$, depending on the observable, θ of the purchaser. As I mentioned above, a mortgage contract is a combination of a loan amount and an interest rate: $(d, r_m) \in \Upsilon(\theta)$. Note that I do not include mortgage payment, m and maturity as parts of the mortgage contract, because maturity is directly determined through the age of the household, which is observable, and mortgage payment is assumed to be fixed and becomes a function of the loan amount, interest rate and household's age.

Present Value Condition. I first show how the mortgage payments are computed. Since the mortgages are fixed-payment mortgages, the payments are constant through the life of the mortgage. They are directly computed from the *present value condition* for the contract. This condition says that given the loan amount and the mortgage interest rate, the present discounted value of the mortgage payments should be equal to the loan amount. Since the lender has full commitment on the contract, he calculates the payments as if the contract ends by the maturity. Assuming the interest rate on the mortgage is r_m and current age of the household is j, this gives me the following formulation for the per-period payments of a mortgage loan with outstanding debt d:

$$d = m + \frac{m}{1+r_m} + \frac{m}{(1+r_m)^2} + ... + \frac{m}{(1+r_m)^{J_r-j}}$$

$$m(d, r_m, j) = \frac{1-\alpha}{1-\alpha^{J_r-j+1}} d, \quad \text{where } \alpha = \frac{1}{1+r_m} \tag{1.10}$$

No-Arbitrage Condition. Next, given the mortgage payments and loan amount, the lender has to determine the mortgage interest rate. This rate is pinned down by the *no-arbitrage condition*. It says that given the expected mortgage payments, the lender should be indifferent between investing in the risk-free market and creating the mortgage loan. Note that the expected payments are not necessarily the above calculated mortgage payments. If the household defaults when the outstanding mortgage debt is d, the lender receives $\min\{(1-\varphi)\,p_h, d\}$[17].

Before formulating the no-arbitrage condition, let me denote the value of a mortgage contract with outstanding debt d and interest rate r_m, offered to a type i household with current period characteristics (a, z, j) as $V_i^{\ell}(a, z, j; d, r^m)$. Note that this function does not only represent the value of the contract at the origination, but also represents the continuation value of the contract at any time period through the mortgage life. Depending on the homeowner's tenure choices, the realized payments may change. If the household stays in the current house, the lender receives the calculated mortgage payment and the continuation value from the contract with the updated characteristics of the household and the loan amount. If the household defaults, then the lender receives $\min\{(1-\varphi)\,p_h, d\}$. If the household sells the house, the lender receives the outstanding loan amount, d.

Given that the opportunity cost of the contract is the risk-free interest rate, r, plus the per period transaction cost, τ, and the lender is risk-neutral, the value

[17]Since default is costly, as long as $p_h\,(1-\varphi) \geq d$, the household sells the house rather than defaulting. This means, in equilibrium, when the household defaults, the lender receives $p_h\,(1-\varphi) < d$ and incurs some loss.

function for the lender becomes the following:

$$V_i^\ell(a,z,j;d,r_m)=\left\{\begin{array}{lr}m(d,r_m,j)+\frac{1}{1+r+\tau}EV_i^\ell(a',z',j+1;d',r_m) & \text{if hh stays}\\ \\ \min\{p_h(1-\varphi),d\} & \text{if hh defaults}\\ \\ d & \text{if hh sells}\end{array}\right\} \tag{1.11}$$

where $d'=(d-m(d,r_m,j))(1+r_m)$, a' is the policy function to problems (1.3) and (1.6) and finally m is defined by equation (1.10).

Now, I am ready to formulate the no-arbitrage condition. At the time of the origination of the contract, the lender may not be able to observe all the characteristics of the household. So, I need to state the no-arbitrage condition conditional on the information structure. It is different for the SI economy and the AI economy.

Symmetric Information: In the SI economy, the lender observes all the characteristics of the household. This actually means mortgage contracts are individualized and independent from all the other households in the economy. The lender can solve the household's problem and obtain the necessary policy functions (saving choice and housing choice) to evaluate the value of the contract at the origination. So, the no-arbitrage condition for a mortgage contract offered to a type i household with characteristics (a,z,j) becomes:

$$V_i^\ell(a,z,j;d,r_m)=d \tag{1.12}$$

Note that initial loan amount d is determined by the downpayment fraction: $d=(1-\phi)\,p_h$.

Asymmetric Information: In the AI economy, the lender cannot observe the type of the household, but can observe the other characteristics: (a,z,j). Now, the lender faces a pool of households with the same saving level, income and age, but possibly different types. So, a contract offered to a type is available for the other type in the pool. This creates *adverse selection* problem. In the appendix, with a simple example, I show that contracts offered in the SI economy may yield negative profits if offered in the AI economy. Specifically, the contract offered to a good type household in the SI economy, is now attractive for a bad type household. The lender cannot differentiate the bad type and good type households, and contract offered to a good type household attracts both types. Since bad type individuals have higher risk of default, this results a loss in the contract designed for the good type household. So, the lender has to either pool different types into a pooling contract or screen different types by offering separating contracts. However, both types of contracts may suffer the problem of not being deviation-free. So, I may not have a Nash-equilibrium. Pooling contracts are always breakable by cream-skimming the good types and separating contracts can also be broken by offering a pooling contract or another separating contract which relies on cross-subsidization if the measure of the good types is sufficiently high. Fortunately, with certain modification in the equilibrium concept or the game structure, it is possible to support the pooling contract as an equilibrium. I leave the discussion of potential problems of existence and other related issues to the appendix, and for now assume the pooling contract is supportable as an equilibrium.

Since a pooling contract attracts both types in the pool, I need to revise the no-arbitrage condition. It should account for the possibility that both types of households have access to this contract. As a result, no-arbitrage condition for a pooling contract becomes the following:

$$\frac{\sum_i V_i^{\ell}\left(\theta; \ell\left(\theta; d, r_m\right)\right) \Gamma_i^r\left(\theta\right)}{\sum_i \Gamma_i^r\left(\theta\right)} = d \tag{1.13}$$

where $\frac{\Gamma_i^r(\theta)}{\sum_i \Gamma_i^r(\theta)}$ is the relative measure of each type in the pool of households with observable $\theta \equiv (a, z, j)$. This condition says that at the origination, the expected value of the contract to the lender should be the originated loan amount.

1.3.3 Equilibrium

I begin by defining the equilibrium for the SI economy, and then define the equilibrium for the AI economy. The definition for the SI economy is relatively simple, because in the SI economy markets are fully individualized, and the problem of the lender is trivial.

Define the set of state variables for the household as Ω with a typical element (a, z, j, i) [18],and let $\theta \in \Theta \subseteq \Omega$ be the observable characteristics of the household by the lender.

Definition 1. Symmetric Equilibrium: A symmetric equilibrium to the SI economy is a set of policy functions $\{c_s^*, a_s^*, \ell_s^*, i_s^*\}$ and a contract set Υ_s such that

(i) given the feasible contract set Υ_s, $c_s^* : \Omega \times \Upsilon_s \rightarrow \Re$, $a_s^* : \Omega \times \Upsilon_s \rightarrow \Re$, and $\ell_s^* : \Omega \times \Upsilon_s \rightarrow \Re^2$ solve equations (1.1) – (1.3) and (1.6) – (1.8), i_s^* is a policy indicator function which solves equations (1.5) and (1.9) ,

[18] The only relevant household for the lender is the purchaser, since contracts are only offered to them. And the state variable for a purchaser is, as mentioned earlier, (a, z, j, i)

(ii) given the policy functions each contract $\ell \in \Omega \times \Upsilon_s$ solves equation (1.12) and

(iii) no lender finds it profitable to offer another contract, which is not in the contract set, $\Omega \times \Upsilon_s$, i.e. $\nexists (d, r_m)$ such that $V^{\ell}(\theta; d, r_m) > d$ for $\forall \theta \in \Theta$, with V^{ℓ} defined as in equation (1.11).

However, in the AI economy, the lender's problem is more complicated. The nature of the equilibrium heavily depends on the type of environment, the definition of equilibrium and the type of equilibrium. I particularly focus on the pooling equilibrium and support the existence of the equilibrium by modifying the equilibrium concept as described in the Appendix. I leave the discussion of all the issues about the existence of equilibrium to the Appendix, and define the equilibrium for the AI economy in the following way:

Definition 2. Asymmetric Equilibrium - Pooling: An equilibrium to the AI economy is a set of policy functions $\{c_a^*, a_a^*, \ell_a^*, i_a^*\}$ and contract set Υ_a such that

(i) given the feasible contract set Υ_a, $c_a^* : \Omega \times \Upsilon_a \to \Re$, $a_a^* : \Omega \times \Upsilon_a \to \Re$, and $\ell_a^* : \Omega \times \Upsilon_a \to \Re^2$ solve equations (1.1) – (1.3) and (1.6) – (1.8), i_s^* is a policy indicator function which solves equations (1.5) and (1.9),

(ii) given the policy functions each contract $\ell \in \Omega \times \Upsilon_a$ solves equation (1.13),

(iii) no lender finds it profitable to offer another contract with the anticipation that the other competitors can withdraw their contracts and

(iv) Γ_i^r is consistent with the policy functions.

There are two main differences of the Asymmetric Equilibrium from the Symmetric Equilibrium. The first one is the zero-profit condition. In the AI economy, the equilibrium is pooling whereas it is separating in the SI economy. That is, while the market for each household is individualized in the SI economy, the segregation is much less in the AI economy. In the AI economy, since types are not observable, they are pooled and both types receive the same contract. As a result lender has to take the measure of each household into account in the calculation of zero-profit condition.

The second difference is about the equilibrium concept. In the SI economy, I use the well-known and commonly used Nash equilibrium as my equilibrium concept. However, as mentioned in the Appendix, in the AI economy, my environment suffers the problem of existence of equilibrium. So I modify the equilibrium concept following Wilson (1977). This new equilibrium concept is known as Anticipatory Equilibrium and it does notallow deviations of lenders which will be unprofitable upon the other lenders withdraw the initial contracts. Although it is an unusual equilibrium, it has the feature of supporting the pooling contract as an equilibrium. I provide further discussion of this issue in the Appendix. In the next section, I also explore another equilibrium concept, Reactive equilibrium which supports the least-cost separating contract as an equilibrium, and analyze the differences.

Note that the no-arbitrage condition for AI economy, equation (1.13), specifies a set of mortgage contracts. For each $d \in [0, p_h]$ there is a corresponding r_m such that this condition is satisfied. Actually, this set is the pooling iso-profit curve. However, perfect competition requires that the equilibrium should be deviation-free.

Although, the new equilibrium concept restricts the set of deviations, in the Appendix I show that the equilibrium with a pooling contract is a unique point. It is the point where the good type household receives the highest utility, i.e. good type household's indifference curve should be tangent to the pooling iso-profit curve. Formally, the equilibrium with pooling contract is characterized by the following equation:

$$\ell^* (\theta; d, r_m) = \arg\max V_g^r (\theta; \ell (\theta; d, r_m)) \tag{1.14}$$

subject to

$$d = \frac{\sum_i V_i^\ell (\theta; \ell (\theta; d, r_m)) \Gamma_i^r (\theta)}{\sum_i \Gamma_i^r (\theta)}$$

where $\theta \equiv (a, z, j)$ is the observable of the household by the lender.

1.4 Findings

I first present the calibration of the model. Then, I present the results. Lastly, I analyze a counterfactual experiment, and check the robustness of the results to an alternative equilibrium concept.

1.4.1 Calibration

A set of the parameters is directly taken from the literature. For the rest of the parameters, I calibrate the SI economy to match some relevant data moments for the 2002-2006 period. In particular, I calibrate the utility advantage of homeownership, γ_h, the mortgage servicing cost τ, and the ratio of discount factors of good type and bad type, $\frac{\beta_g}{\beta_b}$, to match the homeownership rate, mortgage premium and foreclosure rate in the 2002-2006 period. I first solve the SI economy with these parameters. As I mentioned earlier, the AI economy represents the period before the introduction of automated underwriting systems. Since these systems started to be used by the mid-1990s, I chose the 1991-1995 period representing the AI economy. This period was different from the 2002-2006 period not only in the information structure but also in the house price and risk-free interest rate. So, for the AI economy, I calibrate the rent-price ratio, $\frac{p_r^{AI}}{p_h^{AI}}$ to match the homeownership rate in the 1991-1995 period using the interest rate and house price in that period. Table 1.2 presents the results of the calibration.

Table 1.2: Calibration

Parameter	Explanation	Value	Source
σ	risk aversion	2	
ρ	persistence of income	0.84	literature
σ_ε	std of innovation to AR(1)	0.34	literature
φ	selling cost	10%	
r^{SI}	risk-free interest rate	3.2%	data
r^{AI}	risk-free interest rate	4.73%	data
$p_h^{SI}/\overline{y}$	price/income ratio	4.1	data
$p_h^{AI}/\overline{y}$	price/income ratio	3.1	data
p_r^{SI}/p_h^{SI}	rent-to-price ratio	3.1%	$r^{SI}/(1+r^{SI})$
μ_w	mean of initial wealth/income	-2.794	GP (2002)
σ_w	std of initial wealth/income	1.784	GP (2002)
β_g	discount factor - good	0.92	matches wealth-income ratio
β_b	discount factor - bad	0.84	matches foreclosure in 2002-2006
γ_h/γ_r	utility advantage of ownership	1.0818	matches ownership in 2002-2006
τ	transaction cost of mortgage	0.46%	matches premium in 2002-2006
p_r^{AI}/p_h^{AI}	rent-to-price ratio	3.49	matches ownership in 1991-1995
δ	prob. of being an active renter	0.17	matches 5-7 years exclusion

Households. A model period is 1 year and households live for 65 periods. The mandatory retirement age is 45. Utility function for the households is the standard CRRA utility function with a slight modification to account for the benefit of homeownership: $u_k(c) = \frac{(\gamma_k c)^{1-\sigma}}{1-\sigma}$, $k \in \{r, h\}$ and γ_k is the utility advantage of being a renter $(k = r)$ or homeowner $(k = h)$[19]. I normalize $\gamma_r = 1$, and calibrate γ_h to match the homeownership rate in the 2002-2006 period. This implies $\gamma_h = 1.0818$, which means being a homeowner gives 8.18% more consumption than being a renter. I set the risk-aversion parameter, σ, to 2. I assume the measure of the good types, μ, is 80%. The discount factor for the good type, β_g, is fixed to 0.92 and for the bad type, it is calibrated to match the foreclosure rate in the 2002-2006 period. This gives me $\beta_b = 0.84$.

For the income process before retirement, I take the parameters to be consistent with the findings of Hubbard, Skinner and Zeldes (1994), Carroll and Samwick (1997) and Storesletten, Telmer and Yaron (2004). Using their income process, I simulate an economy for a sufficiently long time and estimate the resulting income profile as an AR(1) process[20]. This gives us the income persistency, ρ, as 0.84 and standard deviation of the innovation to the AR(1) process, σ_ε, as 0.34. I ap-

[19] Given this utility specification, since there is no housing tenure choice and uncertainty after retirement, I can solve the value function at the time of retirement analytically: $W(w_r, y_r) = u_r(\bar{c}) \frac{1-\varkappa^{J-J_r+1}}{1-\varkappa}$, where w_r is the total wealth, including real estate, at the of retirement and y_r is the retirement income level, $\bar{c} = \frac{\alpha_1 y_r}{\alpha_2} + \frac{w_r}{\alpha_2}$, $\alpha_1 = \frac{1-\omega_1^{J-J_r+1}}{1-\omega_1}$, $\alpha_2 = \frac{1-\omega_2^{J-J_r+1}}{1-\omega_2}$, $\omega_1 = \frac{(\beta(1+r))^{1/\sigma}}{1+r}$, $\omega_2 = \frac{1}{1+r}$, and $\varkappa = \beta(\beta(1+r))^{\frac{1-\sigma}{\sigma}}$.

[20] More specifically, I assume the stochastic component of the log income as a combination of an AR(1) component and transitory component. Within the range of these papers, I assume the persistency of the AR(1) process as 0.96, the standard deviation of the innovation to the AR(1) process as 0.16, and the standard deviation of the transitory shock as 0.22.

proximate this income process with a 15-states first-order Markov process using the discretization method outlined in Adda and Cooper (2003)[21]. For after retirement income, I assume $\lambda = 0.35$ and $\eta = 0.2$, meaning the retiree receives 35% of the income at the time of retirement plus 20% of the mean income in the economy. The probability of becoming an active renter, while the household is an inactive renter, is set to 0.17, to capture the fact that the bad credit flag stays approximately 5-7 years in the credit history of the household. The loss in the selling price of the house is set to $\varphi = 10\%$[22]. The initial distribution of the income is assumed to be the stationary distribution. Following Gourinchas and Parker (2002), the initial distribution of the wealth to income ratio is assumed to be lognormal with mean $\mu_{w/y} = -2.794$ and standard deviation $\sigma_{w/y} = 1.784$.

Lenders. The annual risk-free interest rate is set to $r^{SI} = 3.2\%$ for the SI economy, which is the average real return on AAA corporate bond in the 2002-2006 period. The same rate is 4.73% in the 1991-1995 period. So, I set the risk-free interest rate in the AI economy to $r^{AI} = 4.73\%$. The annual transaction cost of mortgages to the lender is calibrated to match the mortgage premium in the 2002-2006 period. This gives me $\tau = 0.46\%$ of the loan amount.

[21] This approximation gives biased results as the persistency of the income process increases. To avoid this bias, I checked the accuracy of the approximation with 15-states Markov process and found that during computation setting $\rho = 0.85$ and $\sigma_\varepsilon = 0.33$ results the desired persistency and standard deviation.

[22] Gruber and Martin (2003) estimates this cost for the homeowner as 7% using CEX data. Note that I abstract from various other sources of selling the house like house price change, unemployment shock, medical expense shock and I also exclude the depreciation on the houses. So, I think 10% is a reasonable estimate of the transaction cost for selling the house.

Prices. For house prices, I use the metropolitan affordability index from Joint Center for Housing Studies. This index shows the median house price to median household income ratio. The ratio is 4.1 for the 2002-2006 period and 3.1 for the 1991-1995 period. So, I set the ratio of house price to mean income to $\frac{p_r^{SI}}{\bar{y}} = 4.1$ in the SI economy and $\frac{p_h^{AI}}{\bar{y}} = 4.1$ in the AI economy. Finally, rent-to-price ratio, $\frac{p_r^{SI}}{p_h^{SI}}$ is set to $\frac{r^{SI}}{1+r^{SI}} = 3.1\%$ in the SI economy[23]. For the AI economy, I calibrate this ratio to match the homeownership rate in the 1991-1995 period. This gives me $\frac{p_r^{AI}}{p_h^{AI}} = 3.49\%$

1.4.2 Results

I want to see whether the improvements in information technology - specifically the emergence of automated underwriting systems - can explain the recent changes in the mortgage market, particularly the decrease in the downpayment fraction and the increase in the mortgage premium, foreclosure rate, homeownership rate, loan-to-value ratio, debt-service ratio, and dispersion of mortgage interest rates and downpayment fractions. To pursue this goal, given the above set of parameters, I first solve the SI economy, which is my benchmark economy, and then compare the results to the AI economy. The AI economy represents the period before the introduction of automated underwriting systems, and the SI economy represents the period after the introduction of automated underwriting systems. These two periods not only differ from each other in terms of information structure but also in

[23]In the literature the imputed rent is calculated as the sum of cost of foregone interest, cost of property tax, maintenance cost, tax deductability of mortgage interest and expected capital gain. Since I abstract from all other dimensions, the imputed rent in my model corresponds to the cost of foregone interest, which is $\frac{r}{1+r}$.

terms of risk-free interest rate and house price. So, I solve the AI economy with a different set of parameters than the SI economy. Since the transition from the AI economy to the SI economy involves changes in the information structure, risk-free interest rate and house price, it is hard to measure the direct contribution of the information structure on the statistics of interest. To quantify the contribution of the information technology on the mortgage market, I run a counterfactual experiment. In the counterfactual, I focus on the effect of the information structure. I assume that during the transition from the AI economy to the SI economy, the risk-free interest rate and house price have changed to their SI economy counterparts, but the information structure has not changed. That is, I solve the AI economy with the same set of parameters of the SI economy. I define the difference between the results of the SI economy and this counterfactual as the contribution of the change in the information structure[24]. Lastly, I analyze the results for an alternative equilibrium concept, named as Reactive equilibrium. This equilibrium concept can support the least-cost separating contract as an equilibrium. Using Reactive equilibrium, I analyze the first counterfactual and check the robustness of my results to a change in the equilibrium concept.

I first start with the comparison of the AI economy and the SI economy. Table 1.3 shows the comparison of the two economies as well as how the model matches the data. Overall, the results show that the transition from the AI economy to the SI economy captures the recent trends in the mortgage market. During the

[24]Since I treat house price as exogenous, it is not exactly the right definition. In a world with endogenous house price, this counterfactual should result lower house price than the house price in the SI economy. So, the actual contribution should be lower than what I define here.

calibration, I target the homeownership rate in both economies and the mortgage premium and foreclosure rate in the SI economy. In that perspective, the model matches the data quite well. In the SI economy, the foreclosure rate is 0.44% and the mortgage premium is 0.52%. The homeownership rate increases from 64.2% to 68.6%. They are all consistent with their data counterparts.

Table 1.3: Benchmark Results - Symmetric Information vs Asymmetric Information

	Model		**Data**	
Economy	**AI**	**SI**	**1991-1995**	**2002-2006**
Statistic				
Homeownership rate [b]	**64.2%** [a]	**68.6%**[a]	**64.2%**	**68.6%**
Mortgage premium	0.49%	**0.52%**[a]	0.32%	**0.52%**
Foreclosure rate	0.32%	**0.44%**[a]	0.33%	**0.44%**
Average downpayment fraction	27.2%	2.9%	14.5%	8.9%
Coef of variation-downpayment	0.9	4	1.28	3.55
Coef of variation-int rate	0.014	0.015	0.159	0.203
Debt-service ratio [b]	14.3%	20.5%	14%	15%
Combined loan-to-value ratio [b]	52.2%	66.4%	58.3%	67.1%

[a] These are the variables matched to the data
[b] Homeownership data is from Census data, debt-service ratio is from Federal Reserve Board and combined loan-to-value ratio is from Flow of Funds Account.

In the other dimensions, the model does a fairly good job in capturing the trends and levels corresponding to the data. As we switch from the AI economy to the SI economy, the foreclosure rate increases from 0.32% to 0.44% while it increases from 0.33% to 0.44% in the data. The average downpayment fraction in the model decreases from 27.2% to 2.9% while it decreases from 14.5% to 8.9% in the data. Coefficient of variation for downpayment fractions increases from 0.9 to 4 and in the data it increases from 1.28 to 3.55. The average combined loan-to-value ratio increases from 52.2% to 66.4% whereas in the data the increase is from 58.3% to 67.1%. The debt-service ratio increases from 14% to 15% in the data and the model predicts an increase from 14.3% to 20.5%. The downpayment fraction in the SI economy is below its data counterpart. I think the main reason for this fact is the absence of repeat buyers in my model. Since I abstract from moving shocks and divorce shocks which are the main reasons to buy a house for repeat buyers, and repeat buyers on average put a higher downpayment on the house, my model produces a lower downpayment fraction compared to the data[25].

One of the main weakness of the model is its deficiency in creating enough mortgage premium and dispersion of mortgage interest rates. Although the model seems to capture the mortgage premium in the 2002-2006 period, it is basically through the high mortgage servicing cost I calibrated to match this moment. The mortgage premium net of the mortgage servicing cost, which is the real premium due to the credit risk of the household, in the SI economy is 0.06%. Similarly, although the coefficient of variation for mortgage interest rate increases - consistent with the data - the levels of the dispersion are far below the values observed in the data. I conjecture that the main reason for this big difference is the absence of several major risk factors that affect households' credit risk. In the model, for a certain type, the credit risk only comes from the income uncertainty. In reality, households face

[25]A national survey conducted by National Association of Realtors in 2007 reveals that the median downpayment of first-time buyers is just 2%. The same study indicates that the biggest downpayment resource for repeat buyers is the profit they made from their prior house sales. Moreover, I believe that the huge volume of refinancing after 2001 is another important factor to observe high downpayment fraction in the data.

more uncertainty like house price risk, risk-free interest rate risk, medical expense shock, etc. All these factors increase the mortgage premium and also increase the heterogeneity among households which in turn increase the dispersion of interest rates.

Note that the AI economy is different from the SI economy in three dimensions which all have impacts during the transition. Intuitively, a decrease in the risk-free interest rate makes houses more affordable and increases the homeownership rate. Moreover households can afford higher loans meaning the downpayment fraction decreases. However, the effect of interest rate on the loan-to-value ratio and the foreclosure rate is not clear. A decrease in the risk-free interest rate certainly decreases the mortgage interest rate, and as mortgage interest rate decreases the rate outstanding mortgage debt decumulates increases which means combined loan-to-value ratio decreases for the same loan. On the other hand, a decrease in the downpayment fraction increases the combined loan-to-value ratio. So, overall effect is not clear. For the foreclosure rate there are also two opposing effects. As the homeownership rate increases, the mean income of the homeowners decreases since the new home buyers are the lower income households. Further, as the downpayment fraction increases, the credit risk of the household increases and this pushes the foreclosure rate up. However decreasing mortgage interest rate decreases the likelihood of foreclosure. Thus, the net effect depends on the magnitude of these two effects. Similarly, while a decrease in the mortgage payments decreases the debt-service ratio, as low income households purchase houses the mean income of the homeowners decreases and the debt-service ratio increases. An increase in the house price, on the other hand, makes houses less affordable, decreases the downpayment fraction, increases the loan-to-value ratio and the foreclosure rate.

The effect of a change in the information structure is not trivial, more interesting and the main focus of this paper. I analyze the effect of the information structure in the next section.

1.4.3 Counterfactual: The Effect of the Information Structure

This counterfactual is designed to separate the effect of the information structure. For this purpose, I assume that during the transition from the AI economy to the SI economy, everything changed but the information structure. More specifically, I first solve the SI economy and compare its results to the economy I solve with exactly the same set of parameters but different information structure. I call this economy as the AI-2 economy. The only difference between these two economies is the fact that in the AI-2 economy lenders have partial information about the households whereas in the SI economy they have full information. Column 3 of Table 1.4 presents the results of this counterfactual. I argue that the change from Column 3 to Column 2 captures the effect of the information structure.

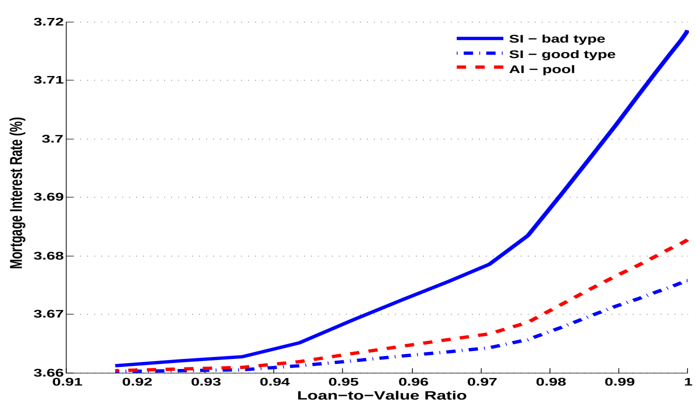
Table 1.4: Counterfactual-The Effect of the Information Structure

Economy	**AI** r^{AI}, p^{AI} pooling	**SI** r^{SI}, p^{SI}	**AI-2** r^{SI}, p^{SI} pooling	**AI-3** r^{SI}, p^{SI} separating
Statistic				
Homeownership rate	64.2%	68.6%	68.2%	67.3%
Mortgage premium	0.49%	0.52%	0.50%	0.49%
Foreclosure rate	0.32%	0.44%	0.36%	0.34%
Average downpayment	27.2%	2.9%	4.3%	5.4%
CV-downpayment	0.9	4	2.9	2.5
CV-int rate	0.014	0.015	0.01	0.014
Debt-service ratio	14.3%	20.5%	20%	20%
Combined loan-to-value ratio	52.2%	66.4%	65.5%	64.3%
Welfare Gain			−0.25%	−0.29%

All of the results presented in the benchmark calibration qualitatively hold. The homeownership rate, loan-to-value ratio, foreclosure rate, average downpayment fraction, debt-service ratio, and dispersion of interest rate and downpayment have all the same patterns as in the benchmark calibration but in smaller measures. Better information results an increase in the homeownership rate from 68.2% to 68.6%, the mortgage premium from 0.50% to 0.52%, the foreclosure rate from 0.36% to 0.44%, the coefficient of variation for mortgage interest rates from 0.01 to 0.015 and for downpayment fractions from 2.9 to 4, the loan-to-value ratio from 65.5% to 66.4%, the debt-service ratio from 20% to 20.5%. Moreover, the downpayment fraction decreases from 4.3% to 2.9%.

In the SI economy, lenders observe all the characteristics of the household and design contracts for each individual. Figure 1.1 shows mortgage interest rate as a function of loan-to-value ratio for both types. First of all, it shows a positive relation between the loan-to-value ratio and the mortgage interest rate. As the loan-to-value ratio increases the credit risk of the loan increases and the mortgage premium increases. Secondly, it shows a comparison of the mortgage interest rates for both types. For the same loan amount, good types qualify for a lower mortgage interest rate. Good types have a higher discount factor which makes them care more about the future benefits of the homeownership. As a result they have a lower default probability. This fact decreases the credit risk of the loan and requires a lower mortgage premium.

Figure 1.1: Mortgage Interest Rate as a Function of Loan Amount



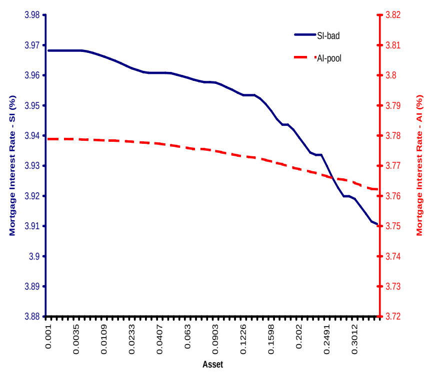
In the AI-2 economy, lenders cannot observe the types and they face an adverse selection problem. More specifically, if contracts designed in the SI economy are offered, bad types also demand for the contracts designed for the good types and make these contracts carry higher risk of default and yield negative profits. As a result lenders design pooling contracts which give higher utility for the bad type and lower the utility for the good type. Compared to the separating contracts offered in the SI economy, pooling contracts offer higher loan amount and lower mortgage interest rate for the bad types and lower loan amount and higher mortgage interest rate for the good types. Figure 1.2(a) shows the comparison of mortgage interest rate as a function of asset level given the same level of loan amount and income for the bad type. Figure 1.2(b) shows the same figure for the good type. As we switch from AI-2 economy to SI economy, for the same loan amount mortgage premium increases for the bad type while it decreases for the good type. The figures also show that in the SI economy, the dispersion of the mortgage interest rates along the asset dimension is much higher for the bad type, while it is slightly lower for the good type. Figure 1.2(c) and Figure 1.2(d) show the same comparison of both economies along the income dimension. Again, I have similar results in the income dimension. Mean and dispersion of mortgage interest rates increase for the bad type while they decrease for the good type. Overall, the transition from the AI-2 economy to the SI economy makes good types better off and bad types worse off. Moreover, higher premium for the bad type and lower premium for the good type make the loan amount offered to good types to increase while the loan amount offered to the bad types to decrease. Lower utility and higher downpayment fraction crowd some of the bad types who were homeowners in the AI-2 economy out. However, higher utility and higher loan amounts increase the homeownership rate of good types and we see a decrease in the downpayment fraction and an increase in the homeownership rate[26].

Lower downpayment fraction requires higher mortgage premium, and this pushes the mortgage premium up. Lower downpayment fraction together with a higher mortgage premium increase the combined loan-to-value ratio. Note that households who are rationed out in the AI-2 economy but qualify for home purchase in the SI economy are the lower income households. As they enter into the housing market, they lower the average income of the homeowners. This fact, together with an increase in the mortgage payments driven by an increase in the loan-to-value ratio and mortgage premium, increases debt-service ratio. Higher debt-service ratio increases the aggregate risk in the market and we observe an increase in foreclosure rate in the SI economy. Lastly, I calculate the welfare gain due to better information[27]. Although, mortgage premium is higher and foreclosure rate is higher in the SI economy, in consumption equivalent terms, the welfare of the household born into the SI economy is 0.25% higher than the welfare of the household born into the AI-2 economy. Although bad types' welfare is reduced in the SI economy, the loss is bounded below by quitting to the rental market. However, the welfare of the high types in the SI economy increases without any bound. That's why, overall welfare increases in the economy as information gets better.

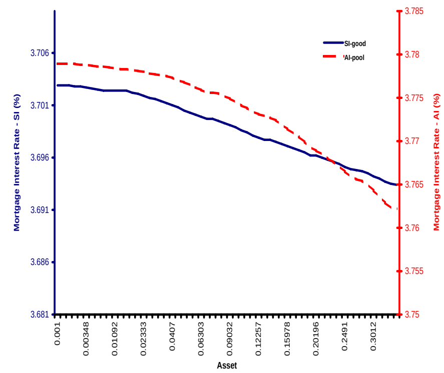
[26]Although bad types are crowded out in the SI economy, the increase in the homeownership rate of the good types dominates and overall homeownership rate increases. The downpayment fraction decreases for the good types, and increases for the bad types. However, some bad types decide not to buy a house upon an increase in the downpayment fraction. Thus, the average downpayment decreases.

[27]Welfare calculation is in line of Lucas (1987). It is the consumption equivalent gain for the household who is born to the SI economy as opposed to the AI economy.

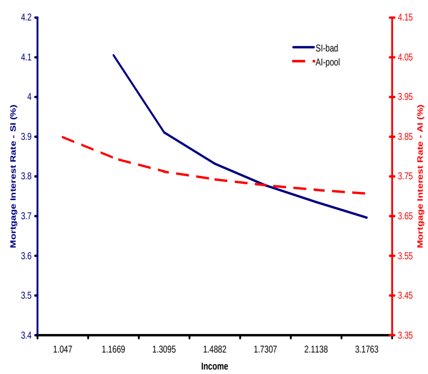
Figure 1.2: Mortgage Interest Rate in Both Economies



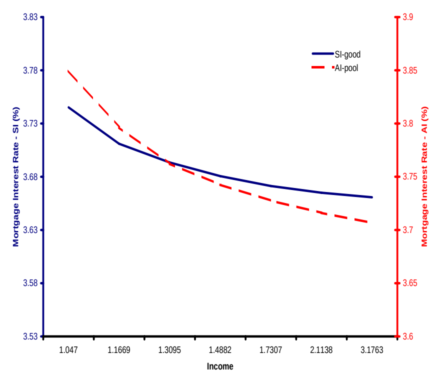
(a) Comparison for Bad Type - Asset Dimension



(b) Comparison for Good Type - Asset Dimension



(c) Comparison for Bad Type - Income Dimension



(d) Comparison for Good Type - Income Dimension

Figure 1.3: Homeownership Rate over the Life Cycle in SI vs AI

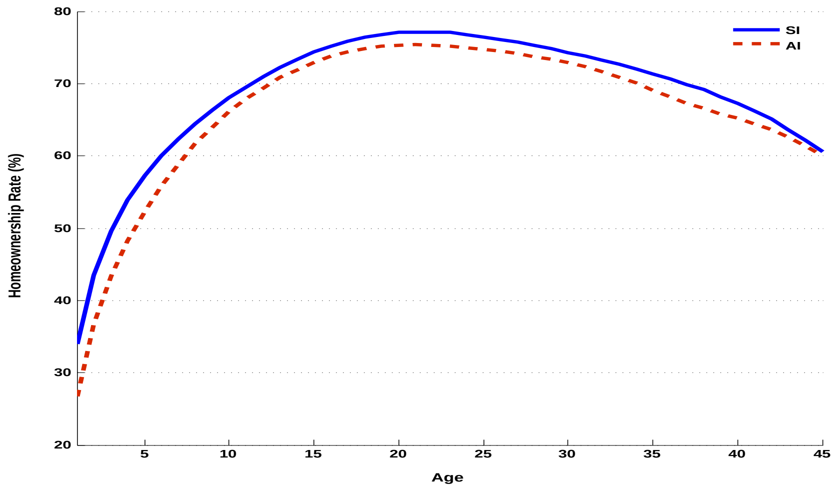
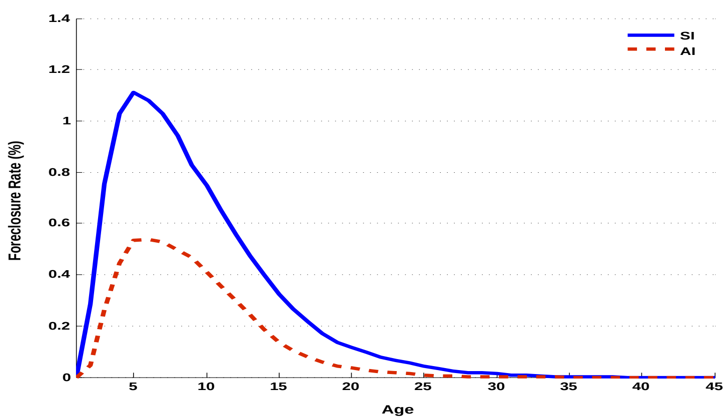


Figure 1.4: Foreclosure Rate over the Life Cycle in SI vs AI



I also look at the differences of these two economies over the life-cycle. Figure 1.3 and Figure 1.4 show how homeownership rate and foreclosure rate change over the life-cycle in both economies, respectively. Regarding the comparison of the two economies, supporting the aggregate results, homeownership rate and foreclosure rate are both lower in the AI economy compared to the SI economy. Moreover, both economies exhibit hump shape homeownership and foreclosure rates. They are both increasing in the early life-cycle, peak up through the middle age and decrease at the end of the life-cycle. Being a homeowner is more valuable early in the life, because the maturity of the mortgage is longer and the mortgage premium is smaller. However, not all the households can afford to buy a house due to income constraints. Since the income profile is initially increasing and households can save, some households can afford to purchase a house as they age. So, initially we see an increasing pattern in the homeownership rate. But later in the life, income decreases and purchasing a house needs higher amount of payments and larger premiums which decreases the demand for houses. Moreover, through the end of the life-cycle, lower income homeowners sell their houses to smooth consumption and I see a decrease in the homeownership rate.

Foreclosure rate has a similar pattern to the homeownership rate. Note that those who purchase a house in the early periods are the high income households who carry lower risk. As households age, lower income households, who carry higher risk of default, become eligible to purchase a house through saving. So, as households age, the credit risk of the homeowners increases. Moreover, home purchase in later periods requires higher downpayment fraction and mortgage premium, so they carry higher risk of default. Thus, in earlier periods, the credit risk of the average homeowner increases which increases the foreclosure rate. However, as households age, homeowners' home equity increases and lower income households exit the market by selling their houses. Both factors decrease the probability of default for the homeowners. Hence, late in the life-cycle, foreclosure rate decreases.

Figure 1.5 shows mean level of income for homeowners and Figure 1.6 shows mean level of income for renters. Both renters and owners have higher average income in the AI-2 economy. This actually shows that the transition from the AI-2 economy to the SI economy makes houses affordable for lower income households. These are the households that are borrowing-constrained and rationed out in the AI-2 economy. Since, in the AI-2 economy, they are in the upper tail of the income distribution for renters and have lower income than the homeowners, as they become homeowners they decrease the mean income of both renters and homeowners . Secondly, if we compare the income of renters and homeowners, we see that homeowners are richer than the renters.

The reason why we have a decreasing initial trend for homeowner's income over the life-cycle is related to the fact that households purchase a house when they can afford it. Initially only high income households can afford to buy a house, but later, lower income households can afford to buy a house by accumulating assets. So, over the life cycle, the pool of the homeowners start to include lower income households and the average income of the pool decreases despite the increase in the mean income due to the hump-shape profile of the income over the life-cycle. Although the hump-shape profile suggest the income to decrease through the end

Figure 1.5: Mean Income of Homeowner over the Life Cycle in SI vs AI

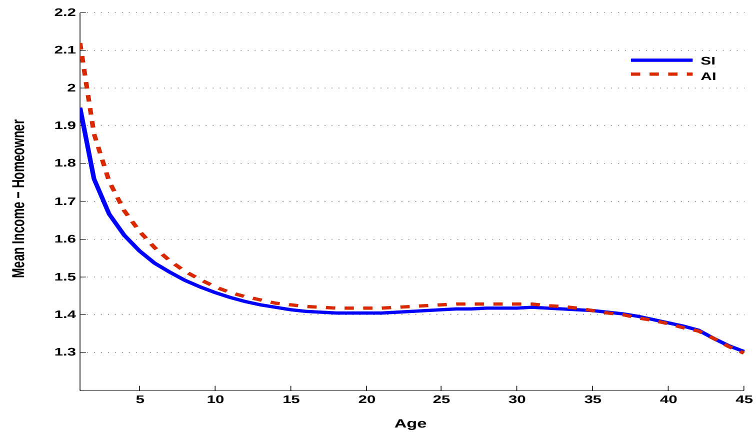
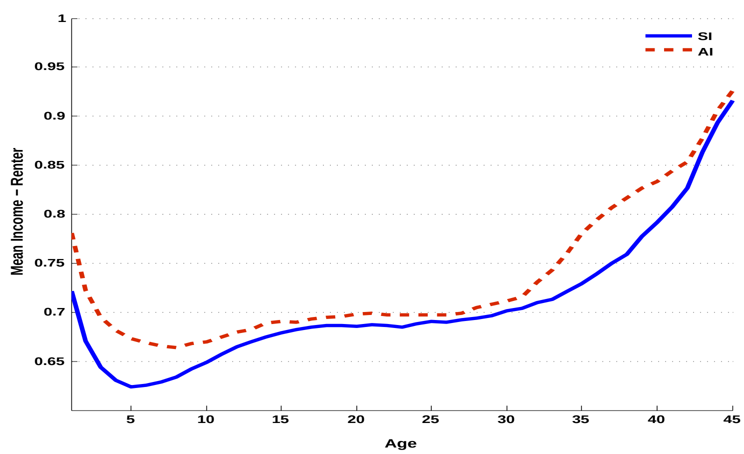


Figure 1.6: Mean Income of Renter over the Life Cycle in SI vs AI



of the life-cycle, we observe an increase in the homeowner's income close to the end of the life-cycle. This is related to the usual consumption smoothing argument. Households enjoy utility at the last period by selling the house. Risk averse households, especially the ones with lower income, sell their houses at earlier periods to smooth their consumption. As a result, through the end of the life-cycle we see the lower income households selling their houses and increasing the average income of the homeowners.

For the renters, we see a similar profile of the mean income over the life-cycle. Initially the mean income decreases as the households at the upper tail of income distribution for renters purchase houses and become homeowners. Later, the increase due to the hump shape of income profile dominates and mean income starts to increase. The rapid increase through the end of life cycle is again due to the households who sell their houses to smooth their consumption.

Figure 1.7 shows the debt-to-income ratio over the life cycle in both economies. The ratio is strictly smaller in the AI-2 economy compared to the SI economy over the life cycle. In both economies the ratio has an increase in the early periods for a short time followed by a monotonic decrease till the end of the life-cycle. In the earlier years, households with higher income prospects purchase a house. But, as households age we see those households with lower income prospects purchase houses. So, we observe an increase in the debt-to-income ratio initially. However, as households age, the mortgage debt decumulates, and lower income homeowners quit housing either by selling or defaulting. Thus, later in the life-cycle, we observe a decrease in the debt-to-income ratio.

Figure 1.7: Debt-to-Income Ratio over the Life Cycle in SI vs AI

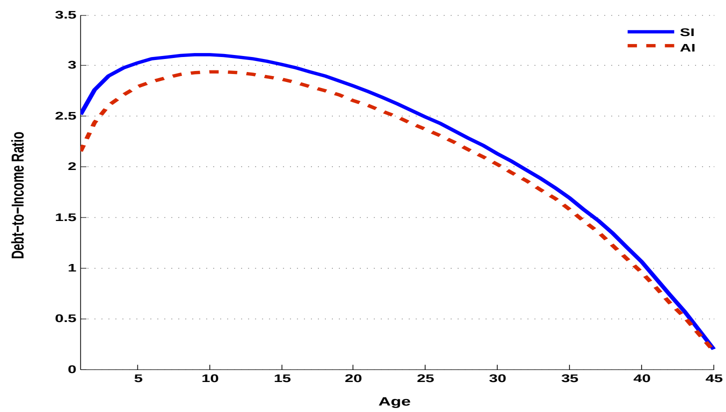


Figure 1.8: Debt-Service Ratio over the Life Cycle in SI vs AI

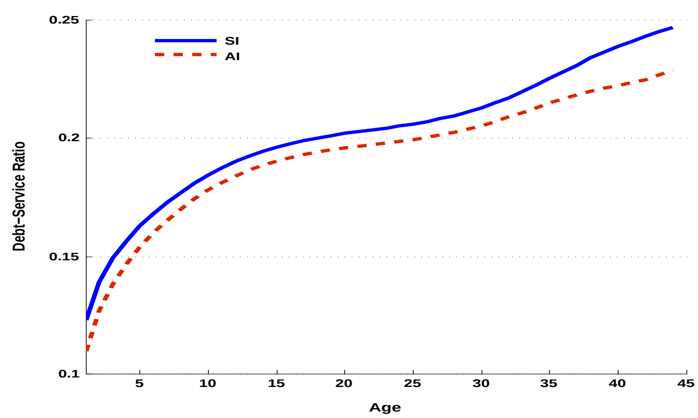


Figure 1.8 shows the debt-service ratio over the life cycle. The ratio increases over the life-cycle. In the initial periods, lower income households enter to homeownership and as Figure 1.5 shows we observe a decrease in mean income of homeowners despite the increasing income profile due to life-cycle hump-shape. The later entrants of homeonwership are the lower income households and they face higher mortgage payments, because they face higher premium and the maturity of the mortgages is shorter. Thus, since average mortgage payment increases and mean income decreases, we observe an increase in the debt-service ratio. Through the end of the life-cycle, the hump-shape profile of income forces the debt-service ratio to increase. If we compare the ratio across the two economies we observe a higher ratio in the SI economy compared to the AI economy. This is because of two facts. Compared to the AI economy, in the SI economy the mean income of the homeowners is lower and secondly households face higher mortgage premium and lower downpayment fraction, which both increase the mortgage payment. Lower mean income and higher mortgage payments increase the debt-service ratio and we observe a higher debt-service ratio in the SI economy.

Lastly, Figure 1.9 presents the consumption path for the homeowner and Figure 1.10 shows the consumption path for the renter over the life-cycle in both economies[28]. Both the homeowners and the renters have increasing consumption path over the life cycle. The initial increase in the consumption is due to the increasing pattern of the income process in the early life-cycle. However, contrary to what we observe in the data, later in the life-cycle consumption continues to

[28]The consumption levels are in terms of the mean income.

increase. This is due to the huge consumption jump through the end of the life-cycle driven by selling the house. When we compare both economies in terms of consumption paths, we see a very similar pattern in the SI and the AI-2 economies. Both homeowners and renters have higher consumption in the AI-2 economy. We know that in the SI economy, good type households with lower income prospects who are rationed out in the AI-2 economy face better mortgage contract terms. These households are the marginal households and they have better income prospects with respect to the ones in the pool of the renters. As we switch from the AI-2 economy to the SI economy, they become homeowners and we observe a decrease in the income level of the renters. This is the main reason for the lower consumption in the SI economy for the renters. For the homeowners, the logic is similar. In the SI economy, they have lower consumption because of two reasons. On the one hand lower income households join to the pool of the homeowners. This transition decreases the average income of the homeowners, which in turn decreases the average consumption of the homeowners. On the other hand, in the SI economy the average debt-service ratio is higher mainly due to the higher loans and higher mortgage premiums. This clearly increases the financial burden on the households and decreases their consumption.

1.4.4 Alternative Equilibrium Concept

As I mentioned earlier, the existence of Nash equilibrium is hard to justify in the current environment. That's why, I changed the equilibrium concept slightly and support the pooling contract as an equilibrium. In the literature, there is also another equilibrium concept proposed to overcome the problem of existence in these

Figure 1.9: Consumption of Homeowner over the Life Cycle in SI vs AI

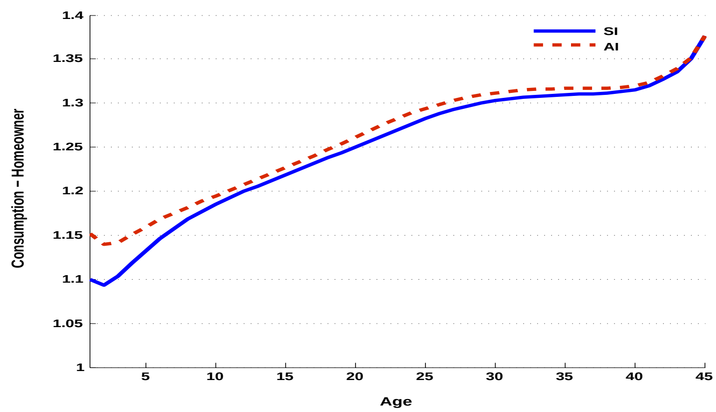
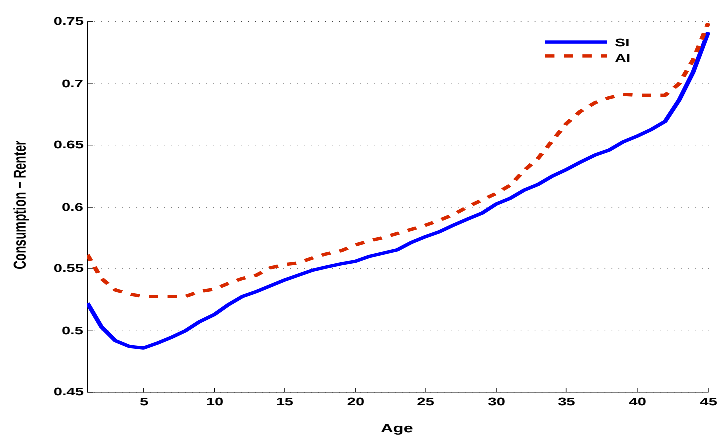


Figure 1.10: Consumption of Renter over the Life Cycle in SI vs AI



types of screening models. It is introduced by Riley (1979) and known as *Reactive Equilibrium*. Different than the Nash equilibrium and Anticipatory equilibrium it assumes that the lenders can react to a deviation by adding new contracts. So, deviations which would be unprofitable after the other lenders add new contracts are not allowed. This equilibrium concept has the feature of supporting the least-cost separating contract as an equilibrium.

A separating contract should satisfy two properties. First, it should yield zero-profit to the lender given that the targeted type takes the contract. Second it should be incentive compatible for the targeted household. This last property says that the contract designed for the other type in the same pool should not give a higher utility to the targeted household. Formally I can write the no arbitrage condition in the following way:

$$V_i^{\ell}\left(\theta;\ell_i\left(\theta;d,r_m\right)\right)=d \tag{1.15}$$

subject to

$$V_i^{r}\left(\theta;\ell_i\left(\theta;d,r_m\right)\right)\geq V_i^{r}\left(\theta;\ell_{i'}\left(\theta;d',r_m'\right)\right),\ \forall i'\text{ such that }\Gamma_{i'}^{r}\left(\theta\right)>0$$

where $\ell_i\left(\theta;d,r_m\right)\equiv\left(d,r_m\right)$ is the contract designed for type i household with observable $\theta\equiv\left(a,z,j\right)$ and $\Gamma_i^r\left(\theta\right)$ is the distribution of type i renters with observable θ and lastly $d=\left(1-\phi\right)p_h$.

Definition 3. Asymmetric Equilibrium - Separating: An asymmetric separating equilibrium to the AI economy is a set of policy functions $\{c_a^*, a_a^*, \ell_a^*, i_a^*\}$ and contract set Υ_a such that

(i) given the feasible contract set Υ_a, $c_a^* : \Omega \times \Upsilon_a \to \Re$, $a_a^* : \Omega \times \Upsilon_a \to \Re$, and $\ell_a^* : \Omega \times \Upsilon_a \to \Re^2$ solve equations (1.1) – (1.3) and (1.6) – (1.8), i_s^* is a policy indicator function which solves equations (1.5) and (1.9) ,

(ii) given the policy functions each contract $\ell \in \Omega \times \Upsilon_a$ solves equation (1.15),

(iii) no lender finds it profitable to offer another contract even after the other competitors can react by adding new contracts and

(iv) Γ_i^r is consistent with the policy functions.

There are two main differences of the Asymmetric Separating Equilibrium than the Symmetric Equilibrium. The first one is the zero-profit condition. In the AI economy, the equilibrium is separating as it is in the SI economy. That is, the market for each household is individualized. However, in the AI economy, the lender has to take into account all the incentive compatibility constraints which are summarized by equation (1.15). It says that each type picks the contract designed for her and the contract designed for the other type with the same observable yields no better utility. This constraint puts extra restriction on the equilibrium contracts compared to the ones offered in the SI economy. In general, good types qualify for "better" terms compared to the bad types since bad types carry higher risk of default. So, bad types have always incentive to take the contracts designed for the good types if only the equilibrium contracts in the SI economy are offered. This forces the good types differentiate themselves from the bad types. In my model they differentiate themselves using the downpayment fraction. Good types demand for lower loans which are not attractive for bad types. However, these loans give lower utility for the good types compared to the SI equilibrium contracts, which means it is costly for good types to differentiate themselves.

The equilibrium contract in the AI economy has the following feature. The bad type in a pool receives the contract offered to her in the SI economy and good type receives the contract such that the bad type is indifferent between the contract offered to her and this contract. This equilibrium contract is called the least-cost separating contract, because it is the contract in which good types differentiate themselves with the minimum cost.

The second difference is about the equilibrium concept. In the SI economy, I use the well-known and commonly used Nash equilibrium as my equilibrium concept. However, as mentioned in the Appendix, in the AI economy, my environment suffers the problem of existence of equilibrium. So, I modify the equilibrium concept as in the lines of Riley (1979), which is called Reactive Equilibrium. This equilibrium concept does not allow deviations of lenders which will be unprofitable upon the other lenders react by adding new contracts. Although it is an unusual equilibrium, it has the feature of supporting the least-cost separating contract as an equilibrium. I provide further discussion of this issue in the Appendix.

Using the Reactive equilibrium concept, I solve the model with the benchmark calibration. Column 4 of Table 1.4 presents the results for the new equilibrium. The results show that the changes are similar. Homeownership rate, loan-to-value ratio, foreclosure rate, debt-service ratio, mortgage premium and dispersion of mortgage interest rate are all very similar compared to the economy with the pooling contract as an equilibrium. As a welfare comparison, the results show that better information increases the welfare of the households by 0.29% in consumption equivalent terms.

1.5 Conclusion

In this paper I have explored the effect of technological improvements on the mortgage market. I show that, thanks to the automated underwriting systems, as lenders can better assess the credit risk of the households the market experiences a decrease in the downpayment fraction and consequently an increase in the homeownership rate, loan-to-value ratio, foreclosure rate, and dispersion of mortgage interest rates, which are all consistent with the recent trends in the data. I have also shown that the removal of informational asymmetry between lenders and households makes the households better off.

My quantitative work sheds some light on how the mortgage market responds to a change in the supply of credit and it has the potential to answer the implications of different policies directed to the mortgage market. However taking the house prices in the model exogenous masks the real effects of these policies. Understanding how house prices respond to the changes in the market seems to be important to fully capture the real effects of different policies. The extension of the current framework with endogenous house prices is an ambitious but necessary step forward.

Moreover, recent financial crisis stemmed from the subprime mortgage market has brought a lot of attention to how the mortgage market operates. Although my framework is useful to understand the interaction between lenders and households, it is mute in the interaction of lenders and investors which I think is the real cause of the current crisis. So, as a next step it is important to model the interaction between lenders and investors, in which there is significant informational asymmetry.

Chapter 2

Joint-Search Theory: New Opportunities and New Frictions [1]

2.1 Introduction

In year 2000, over 60% of the US population was married, the labor force participation rate of married women stood at 61%, and in one-third of married couples wives provided more than 40 percent of household income (US Census (2000); Raley, Mattingly and Bianchi (2006)). For these households, who make up a substantial fraction of the population, job search is very much a joint decision-making process.

Surprisingly, since its inception in the early 1970's, search theory has almost entirely focused on the single-agent search problem. The recent survey by Rogerson, Shimer and Wright (2005), for example, does not contain any discussion on optimal job search strategies of two-person households acting as single decision units. This state of affairs is rather surprising given that Burdett and Mortensen (1977), in their seminal piece on "Labor Supply Under Uncertainty," lay out a two-person search model and sketch a characterization of its solution, explicitly encouraging further work on the topic. This pioneering effort, which remained virtually unfollowed, rep-

[1] This chapter borrows extensively from one of my working papers joint with Fatih Guvenen and Gianluca Violante.

resents the starting point of our theoretical analysis. Only very recently, a renewed interest seems to have arisen in the investigation of household interactions in the context of frictional labor market models. Dey and Flinn (2007) study quantitatively the relationship between health insurance coverage and labor market outcomes at the household level. Gemici (2007) estimates a structural model of migration and labor market decisions of couples.

Our theoretical analysis focuses on the search problem of a couple who faces exactly the same economic environment as in the standard single-agent search problem of McCall (1970), and Mortensen (1970), without on the job search, and Burdett (1978) with on the job search. A couple is an economic unit composed of two individuals linked to each other by the assumption of perfect income pooling. There is an active and growing literature that attempts to understand the household decision making process, and emphasizes deviations from the unitary model we adopt here, e.g., Chiappori (1992). While we agree with the importance of many of those features, incorporating them into the present framework will make it harder to compare the outcomes of single-search and joint-search problems. The simple unitary model of a household adopted here is a convenient starting point, which helps to examine more transparently the role of the labor market frictions and insurance opportunities introduced by joint-search.

From a theoretical perspective, there are numerous reasons why couples would make a joint decision leading to choices different from those of a single agent. We start from the most obvious and natural ones. First, the couple has concave preferences over pooled income. Second, the couple can receive job offers from multiple locations but faces a utility cost of living apart. In this latter case deviations from the single-agent search problem occur even for linear preferences. One appealing feature of our theoretical analysis is that it leads to two-dimensional diagrams in the space of the two spouses' wages (w_1, w_2), where the reservation wage policies can be easily analyzed and interpreted.

As summarized by the title of our paper, joint search introduces new opportunities and new frictions relative to single-agent search. In the first environment we study, couples have risk-averse preferences and have access to a risk-free asset for saving but are not allowed to borrow. In this framework, joint-search works similarly to on-the-job search by allowing the couple to climb the wage ladder. In particular, a couple will quickly accept a job offer received when both members are unemployed (in fact, more easily than a single unemployed agent), but will be more choosy in accepting the second job offer (that is, when one spouse is already employed). This is because the employed spouse's wage acts as a consumption smoothing device and allows the couple to be effectively more patient in the job search process for the second spouse.[2] Furthermore, if the second spouse receives and accepts a very good job offer, this may trigger a quit by the employed spouse to search for a better job, resulting in a switch between the breadwinner and the searcher within the household. As is well-known, this endogenous quit behavior never happens in the standard single-agent version of the search model. We call this process—of quit-search-work that allows a couple to climb the wage ladder—the

[2] The ability to save does not help smooth consumption because in this model individuals and couples face non-decreasing wage earnings paths over their lifetime. Consequently, smoothing depends on the ability to borrow which is ruled out.

"breadwinner cycle." Overall, couples spend more time searching for better jobs, which results in (typically) longer unemployment durations but also leads to higher lifetime wages and welfare for couples compared to singles.

Second, the model with multiple locations and a cost of living apart shows some new frictions introduced by joint-search. Even with risk-neutral preferences (and no financial market frictions or constraints), the search behavior of couples differs from that of single agents in important ways. For example, the model generates what Mincer (1978) called tied stayers—i.e., workers who turn down a job offer in a different location that they would accept as single—and tied movers—i.e., workers who accept a job offer in the location of the partner that they would turn down as single. Therefore, the desire to live together effectively narrows down the job offers that are viable for couples, who end up choosing among a more limited set of job options. As a result, in this environment, couples are always worse off than singles as measured by their lifetime income. The set of Propositions proved in the paper formalizes the new opportunities and the new frictions in terms of comparison between reservation wage functions of the couple and reservation wage of the single agent. We also provide some illustrative simulations to show that the deviations of joint-search behavior from its single-agent counterpart can be quantitatively substantial.

The rest of the paper is organized as follows. Section 2.2 describes the single-agent problem which provides the benchmark of comparison throughout the paper. Section 2.3 develops and fully characterizes the baseline joint-search problem. Section 2.4 extends this baseline model in a number of directions: on-the-job search, exogenous separations, and access to borrowing (saving is always allowed). Section 2.5 studies an economy with multiple locations, and a cost of living apart for the couple. Section 2.6 concludes the paper.

2.2 The Single-Agent Search Problem

To warm up, we first present the sequential job search problem of a single agent—the well-known McCall-Mortensen (McCall, 1970; Mortensen, 1970) model. This model provides a useful benchmark against which we compare the joint-search model, which we introduce in the next section. For clarity of exposition, we begin with a very stylized version of the search problem, and then consider several extensions in Section 2.4.

Economic Environment. Consider an economy populated with individuals who all participate in the labor force: agents are either employed or unemployed. Time is continuous and there is no aggregate uncertainty. Workers maximize the expected lifetime utility from consumption

$$E_0 \int_0^\infty e^{-rt} u\left(c\left(t\right)\right) dt$$

where r is the subjective rate of time preference, $c\left(t\right)$ is the instantaneous consumption flow at time t, and $u\left(\cdot\right)$ is the instantaneous utility function.

An unemployed worker is entitled to an instantaneous benefit, b, and receives wage offers, w, at rate α from an exogenous wage offer distribution, $F\left(w\right)$ with support $[0, \infty)$. The worker observes the wage offer, w, and decides whether to accept or reject it. If he accepts the offer, he becomes employed at wage w forever.

If he rejects the offer, he continues to be unemployed and to receive job offers. All individuals are identical in terms of their labor market prospects, i.e., they face the same wage offer distribution and the same arrival rate of offers, α. There are no exogenous separations and no on-the-job search. Finally, we assume that individuals have access to risk-free saving but are not allowed to borrow. As will become clear below, in the present framework individuals face a wage earnings profile that is non-decreasing over the lifecycle (without exogenous separation risk) and, therefore, consumption smoothing only requires the ability to borrow, but does not benefit from the ability to save. As a result, individuals will optimally set consumption equal to their wage earnings every period even though they are allowed to save.[3]

Value functions. Denote by V and W the value functions of an unemployed and employed agent, respectively. Then, using the continuous time Bellman equations, the problem of a single worker can be written in the following flow value representation:[4]

$$rV = u\left(b\right) + \alpha \int \max\left\{W\left(w\right) - V, 0\right\} dF\left(w\right), \tag{2.1}$$

$$rW\left(w\right) = u\left(w\right). \tag{2.2}$$

This well-known problem yields a unique reservation wage, w^*, for the unemployed such that for any wage offer above w^*, she accepts the offer and below w^*, she rejects the offer. Furthermore, this reservation wage can be obtained as the

[3]Borrowing in financial markets, on-the-job search and exogenous job separation are introduced in Section 2.4.

[4]Below, when the limits of integration are not explicitly specified they are understood to be the lower and upper bound of the support of w.

solution to the following equation:

$$u(w^*) = u(b) + \frac{\alpha}{r}\int_{w^*}(u(w) - u(w^*))\,dF(w) \tag{2.3}$$
$$= u(b) + \frac{\alpha}{r}\int_{w^*} u'(w)(1 - F(w))\,dw,$$

which equates the instantaneous utility of accepting a job offer paying the reservation wage (left hand side, LHS) to the option flow value of continuing to search in the hope of obtaining a better offer in the future (right hand side, RHS). Since the LHS is increasing in w^* whereas the RHS is a decreasing function of w^*, equation (2.3) uniquely determines the reservation wage, w^*.

2.3 The Joint-Search Problem

We now study the search problem of a couple facing the same economic environment described above. For the purposes of this paper, a "couple" is defined as an economic unit composed of two individuals who are linked to each other by the assumption that they perfectly pool income. As before, because households are not able to borrow, they simply consume their total income in each period which is the sum of the wage or benefit income of each spouse. Couples make their job acceptance/rejection/quit decisions jointly, because each spouse's search behavior affects the couple's joint welfare.

A couple can be in one of three labor market states. First, if both spouses are unemployed and searching, they are referred to as a "dual-searcher couple." Second, if both spouses are employed (an absorbing state) we refer to them as a "dual-worker couple." Finally, if one spouse is employed and the other one is unemployed, we refer to them as a "worker-searcher couple." As can perhaps be anticipated, the most interesting state is the last one.

Value Functions. Let U denote the value function of a dual-searcher couple, $\Omega(w_1)$ the value function of a worker-searcher couple when the worker's wage is w_1, and $T(w_1, w_2)$ the value function of a dual-worker couple earning wages w_1 and w_2. The flow value in the three states becomes

$$rT(w_1, w_2) = u(w_1 + w_2), \tag{2.4}$$

$$rU = u(2b) + 2\alpha \int \max\{\Omega(w) - U, 0\}\, dF(w), \tag{2.5}$$

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2). \tag{2.6}$$

The equations determining the first two value functions (2.4) and (2.5) are straightforward analogs of their counterparts in the single-search problem. In the first case, both spouses stay employed forever, and the flow value is simply equal to the total instantaneous wage earnings of the household. In the second case, the flow value is equal to the instantaneous utility of consumption (which equals the total unemployment benefit) *plus* the expected gain in case a wage offer is received. Because both agents receive wage offers at rate α, the total offer arrival rate of a dual-searcher couple is 2α.[5] Once a wage offer is received by either spouse, it will be accepted if it results in a gain in lifetime utility (i.e., $\Omega(w) - U > 0$), otherwise it will be rejected.

[5] Because time is continuous, the probability of both spouses receiving offers simultaneously is negligible and is hence ignored.

The value function of a worker-searcher couple is somewhat more involved. As can be seen in equation (2.6), if a couple receives a wage offer (which now arrives at rate α since only one spouse is unemployed) there are three choices facing the couple. First, the unemployed spouse can reject the offer, in which case there is no change in the value. Second, the unemployed spouse can accept the job offer and both spouses become employed, which increases the value by $T(w_1, w_2) - \Omega(w_1)$. Third, and finally, the unemployed spouse can accept the job offer *and* the employed spouse simultaneously quits his job and starts searching for a better one.

As we shall see below, this third case is the first important difference between the joint-search problem and the single-agent search problem. In the single-search problem, once an agent accepts a job offer, she will never choose to quit her job. This is because an agent strictly prefers being employed to searching at any wage offer higher than the reservation wage. Because the environment is stationary, the agent will face the same wage offer distribution upon quitting and will have the same reservation wage. As a result, a single employed agent will never quit, even if he is given the opportunity. In contrast, in the joint-search problem, the reservation wage of each spouse depends on the income of the partner. When this income grows, for example because of a transition from unemployment to employment, the reservation wage of the previously employed spouse may also increase, which could lead to exercising the quit option. We return to this point below and discuss it in more detail.

2.3.1 Characterizing the couple's decisions

To better understand the optimal choices of the couple, it is instructive to treat the accept/reject decision of the unemployed spouse and the stay/quit decision of the employed spouse as two separate choices (albeit the couple makes them simultaneously). Before we begin characterizing the solution to the problem, we state the following useful lemma. We refer to Appendix B.1 for all the proofs and derivations.

Lemma 2.3.1. Ω *is a strictly increasing function, i.e.,* $\Omega'(w) > 0$ *for all* $w \in [0, \infty)$.

We are now ready to characterize the couple's search behavior. First, for a dual-searcher couple, the reservation wage—which is the same for both spouses by symmetry—is denoted by w^{**}, and is determined by the equation:

$$\Omega\left(w^{**}\right) = U. \tag{2.7}$$

Because U is a constant and Ω is a strictly increasing function (Lemma 2.3.1), w^{**} is a singleton.[6]

A worker-searcher couple has two decisions to make. The first decision is whether accepting the job offer to the unemployed spouse (say spouse 2) or not. The second decision, *conditional* on accepting, is whether the employed spouse (spouse 1) should quit his job or not. Let the current wage of the employed spouse be w_1 and denote the wage offer to the unemployed spouse by w_2.

[6] Note that no wage below w^{**} will ever be accepted by the couple, and therefore, observed in this model, which means that we can focus attention on the behavior of value functions and reservation functions for wages above w^{**}. Therefore, the statements we make below about the properties of certain function should be interpreted to apply to those functions only for $w > w^{**}$, and may or may not apply below that level.

Accept/Reject Decision. Let us begin by supposing that it is not optimal to exercise the quit option upon acceptance. In this case a job offer with wage w_2 will be accepted when $T(w_1, w_2) \geq \Omega(w_1)$. Formally, the associated reservation wage function $\phi(w_1)$ solves

$$T(w_1, \phi(w_1)) = \Omega(w_1). \tag{2.8}$$

Suppose now instead that it is optimal to exercise the quit option upon acceptance. Then, the job offer will be accepted when $\Omega(w_2) \geq \Omega(w_1)$, which implies the reservation rule

$$\Omega(\phi(w_1)) = \Omega(w_1). \tag{2.9}$$

Given the strict monotonicity of Ω, the reservation wage rule is very simple: accept the new offer (and the other spouse will quit the existing job) whenever $w_2 \geq w_1$. The worker-searcher reservation wage function $\phi(\cdot)$ is therefore piecewise, being composed of (2.8) and (2.9) in different ranges of the domain for w_1. The kink of this piecewise function, which always lies on the 45 degree line of the (w_1, w_2) space, plays a special role in characterizing the behavior of the couple. We denote this point by $(\hat{w}, \hat{w})$, and formally it satisfies: $T(\hat{w}, \phi(\hat{w})) = \Omega(\hat{w}) = \Omega(\phi(\hat{w}))$.[7] Since $rT(\hat{w}, \hat{w}) = u(2\hat{w})$, $\hat{w}$ solves

$$u(2\hat{w}) = \Omega(\hat{w}). \tag{2.10}$$

[7] At this stage we have not proved that $\hat{w}$ is unique, but it will turn out that it is.

Stay/Quit Decision. It remains to characterize the quitting decision. If $T(w_1, w_2) \leq \Omega(w_2)$ it is optimal for the employed spouse to quit his job when the unemployed spouse accepts her job offer (that is, this choice yields higher utility than him staying at his job and the couple becoming a dual-worker couple). This inequality implies the indifference condition:

$$T(w_1, \varphi(w_1)) = \Omega(\varphi(w_1)). \tag{2.11}$$

Two important properties of φ should be noted. First, φ is not necessarily a function; it may be a correspondence. Second, φ is the inverse of that piece of the ϕ function defined by (2.8). This is easily seen. By symmetry of T, from (2.8) we have that $T(\phi(w_1), w_1) = \Omega(w_1)$, or $T\left(w_2, \phi^{-1}(w_2)\right) = \Omega\left(\phi^{-1}(w_2)\right)$ which compared to (2.11) yields the desired result.

Since $\varphi = \phi^{-1}$ then φ will also cross the function ϕ on the 45 degree line at the point $\hat{w}$. Therefore, $\hat{w}$ is the highest wage level at which the unemployed spouse is indifferent between accepting and rejecting her offer *and* the employed partner is indifferent between keeping and quitting his job. To emphasize this feature, we refer to $\hat{w}$ as the "double indifference point."

In what follows, we characterize the optimal strategy of the couple in the (w_1, w_2) space. This means establishing the ranking between w^{**} and $\hat{w}$, especially in relation to the single-agent reservation wage w^*, and studying the function ϕ. Once we have characterized the shape of ϕ, that of ϕ^{-1} follows immediately. Overall, these different reservation rules will divide the (w_1, w_2) into four regions: one where both spouses work, one where both spouses search and the remaining two regions where spouse one (two) searches and spouse two (one) works.

2.3.2 Risk-neutrality

As will become clear below, risk aversion is central to our analysis. To provide a benchmark, we begin by presenting the risk-neutral case, then turn to the results with risk averse agents.

Proposition 1. [Risk neutrality] With risk-neutral preferences, i.e., $u'' = 0$, the joint-search problem reduces to two independent single-search problems. Specifically, the value functions are:

$$T(w_1, w_2) = W(w_1) + W(w_2),$$

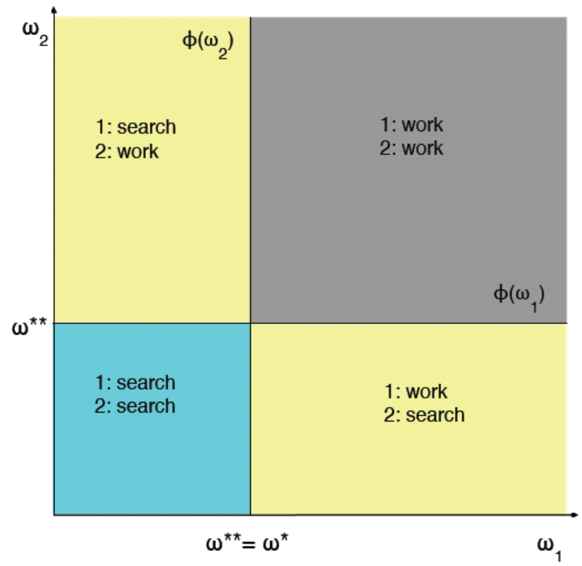
$$U = 2V,$$

$$\Omega(w_1) = V + W(w_1).$$

The reservation wage function $\phi(\cdot)$ of the worker-searcher couple is constant and is equal to the reservation wage value of a dual-searcher couple (regardless of the wage of the employed spouse) which, in turn, equals the reservation value in the single-search problem, i.e., $\phi(w_1) = w^{**} = w^*$.

Figure 2.3.2 shows the relevant reservation wage functions in the (w_1, w_2) space where w_1 and w_2 are the wages of the spouses 1 and 2, respectively. In this paper, when we talk about worker-searcher couples, we will think of spouse 1 as the employed spouse and display w_1 on the horizontal axis, and think of spouse 2 as the unemployed spouse and display the wage offer received by her (w_2) on the vertical axis.

Figure 2.1: Reservation Wage Functions of a Risk-Neutral Couple: Search behavior is identical to the single-search economy.



As stated in the proposition, the reservation wage function of a worker-searcher couple, $\phi(w_1)$ is simply the horizontal line at w^{**}. Similarly, the reservation wage for the quit decision is the inverse (mirror image with respect to the 45 degree line) of $\phi(w_1)$ and is shown by the vertical line at $w_1 = w^{**}$. The intersection of these two lines gives rise to four regions, in which the couple display distinct behaviors.

No wage below w^{**} is ever accepted in this model. Therefore, a worker-searcher couple will never be observed with a wage below w^{**}. As a result, the only wage values relevant for the employed spouse are above the $\phi(w_1)$ function. If the unemployed spouse receives a wage offer $w_2 < w^{**}$, she rejects the offer and continues to search. If she receives an offer higher than w^{**} she accepts the offer. At this point the employed partner retains his job, and the couple becomes a dual-worker couple.

For things to get interesting, risk aversion must be brought to the fore. In Section 2.5, we will also see that when the job-search process takes place in multiple locations and there is a cost of living separately for the couple, then even in the risk neutral case there is an important deviation from the single-agent search problem.

2.3.3 Risk-aversion

To introduce risk aversion into the present framework we employ preferences in the HARA (Hyperbolic Absolute Risk Aversion) class. This class encompasses several well-known utility functions as special cases. Formally, HARA preferences are defined as the family of utility functions that have linear risk tolerance: $-u'(c)/u''(c) = a + \tau c$, where a and τ are parameters.[8]

This class can be further divided into three sub-classes depending on the sign of τ. First, when $\tau \equiv 0$, then risk tolerance (and hence absolute risk aversion) is independent of consumption level. This case corresponds to constant absolute risk aversion (CARA) preferences also known as exponential utility $u(c) = -e^{-ac}/a$. Second, if $\tau > 0$ then absolute risk tolerance is increasing—and therefore risk aversion is decreasing—with consumption, which is the decreasing absolute risk aversion (DARA) case. A well-known special case of this class is the constant relative risk aversion (CRRA) utility: $u(c) = c^{1-\rho}/(1-\rho)$, which obtains when $a \equiv 0$ and $\tau = 1/\rho > 0$. Finally, if $\tau < 0$ risk aversion increases with consumption, and this class is referred to as increasing absolute risk aversion (IARA). A special case of this

[8] Risk tolerance is defined as the reciprocal of Pratt's measure of "absolute risk aversion." Thus, if risk tolerance is linear, risk aversion is hyperbolic.

class is quadratic utility: $u(c) = -(a-c)^2$, which obtains when $\tau = -1$.

2.3.3.1 CARA utility

We first characterize the search behavior of a couple under CARA preferences and show that it serves as the watershed for the description of search behavior under HARA preferences. The following proposition summarizes the optimal search strategy of the couple.

Proposition 2. [CARA utility] With CARA preferences, the search behavior of a couple can be completely characterized as follows:

(i) The reservation wage value of a dual-searcher couple is strictly smaller than the reservation wage of a single agent: $w^{**} < w^* = \hat{w}$.

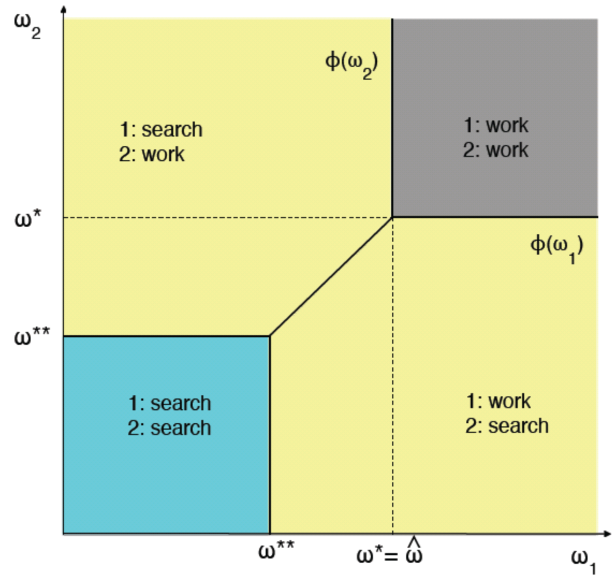
(ii) The reservation wage function of a worker-searcher couple is piecewise linear in the employed spouse's wage

$$\phi(w_1) = \begin{cases} w_1 & \text{if } w_1 \in [w^{**}, w^*) \\ w^* & \text{if } w_1 \geq w^*. \end{cases}$$

Figure 2.2 provides a visual summary of the contents of this proposition in the wage space. Three important remarks are in order.

First, the dual searcher couple is less choosy than the single agent $(w^{**} < w^*)$. With risk aversion, the optimal search strategy involves a trade-off between lifetime income maximization and the desire for consumption smoothing. The former force pushes up the reservation wage, the second pulls it down as risk-averse agents particularly dislike the low income state (unemployment). The dual-searcher couple

Figure 2.2: Reservation Wage Functions with CARA Preferences.



can use income pooling to its advantage: it initially accepts a lower wage offer (to smooth consumption across states) while, at the same time, not giving up completely the search option (to increase lifetime income) which remains available to the other spouse. In contrast, when the single agent accepts his job he gives up the search option for good which induces him to be more picky at the start. Notice that joint-search plays a role similar to on-the-job search in the absence of it. We return to this point later below.

Second, for a worker-searcher couple earning a wage greater than w^*, the reservation wage function is constant and equal to w^*, the reservation wage value of the single unemployed agent. This is because with CARA utility agents' attitude towards risk does not change with the consumption (and hence wage) level. As the wage of the employed spouse increases, the couple's absolute risk aversion remains unaffected, implying a constant reservation wage for the unemployed partner.

While the appendix contains a formal proof of this result, it is instructive to sketch the argument behind the proof here. To this end, begin by conjecturing that there is a wage level (to be determined below) above which it is never optimal to exercise the quit option. In this wage range, equation (2.6) simplifies to

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int_{\phi(w_1)} \{T(w_1, w_2) - \Omega(w_1)\} \, dF(w_2).$$

Substituting out Ω and T (using equations (2.4) and (2.8)) shows that the reservation wage function for the unemployed spouse must satisfy:

$$u(w_1 + \phi(w_1)) = u(w_1 + b) + \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] \, dF(w_2). \tag{2.12}$$

Finally, with exponential utility we have: $u(w_1 + w_2) = -u(w_1)\,u(w_2)$, which simplifies the previous condition by eliminating the dependence on w_1 :

$$u(\phi(w_1)) = u(b) + \frac{\alpha}{r} \int_{\phi(w_1)} (u(w_2) - u(\phi(w_1))) \, dF(w_2).$$

Notice that, since the dependence on the employed partner wage w_1 ceases, this condition becomes exactly the same as the one in the single-search problem (equation 2.3) and is thus satisfied by the constant reservation function: $\phi(w_1) = w^*$. Moreover, when ϕ is a constant function, its inverse is $\phi^{-1}(w_1) = \infty$. Thus, there is no wage offer w_2 that can exceed $\phi^{-1}(w_1)$ to trigger a quit, which in turn verifies our conjecture that the employed spouse does not quit in the wage range $w_1 > w^*$.

Breadwinner cycle. A third remark, and a key implication of the proposition, is that the reservation wage value of a dual-searcher couple w^{**} being strictly smaller than w^{*} activates the region where $\phi(w_1)$ is strictly increasing, and in turn gives rise to the "breadwinner cycle." Suppose that $w_1 \in (w^{**}, w^{*})$ and the unemployed spouse receives a wage offer $w_2 > w_1 = \phi(w_1)$, where the equality only holds in the specified region (w^{**}, w^{*}). Because the offer is higher than the worker-searcher couple's reservation wage, the unemployed spouse accepts the offer and becomes employed. However, accepting this wage offer also implies $w_2 > \phi^{-1}(w_1) = w_1$ which, in turn, implies $w_1 < \phi(w_2)$. This means that the threshold for the first spouse to keep his job now exceeds his current wage, and he will quit.

As a result, spouses simultaneously switch roles and transit from a worker-searcher couple into another worker-searcher couple with a higher wage level. This process repeats itself over and over again as long as the employed spouse's wage stays in the range (w^{**}, w^{*}), although of course the identity of the employed spouse (i.e., the breadwinner) alternates. Once both spouses have in hands job offers beyond w^{*}, the breadwinner cycle stops and so does the search process.

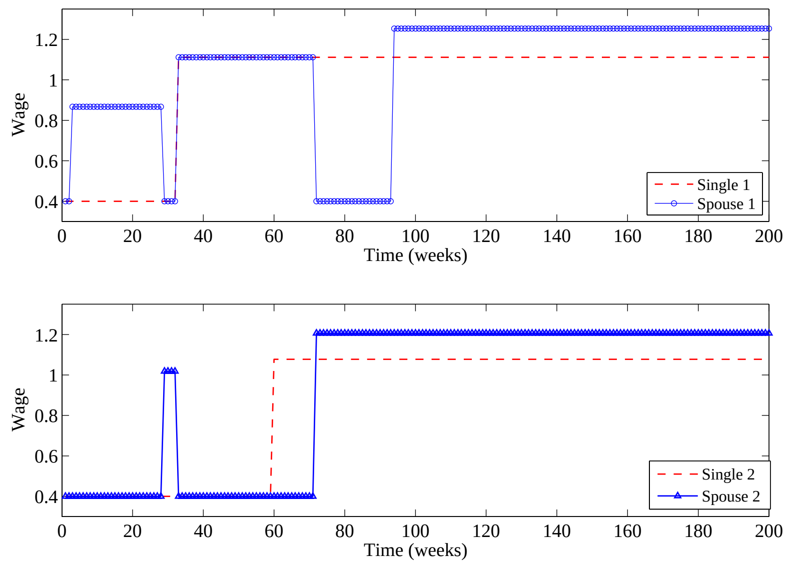
To provide a better sense of how the breadwinner cycle works, figure 2.3 plots the simulated wage paths of a couple when spouses behave optimally under joint-search (lines with markers) and for the same individuals when they act as two unrelated singles (dashed lines). To make the comparison meaningful, the paths are generated using the same simulated sequence of job offers for each individual when he/she is single and when they act as a couple. First, the breadwinner cycle is seen clearly here as spouses alternate between who works and who searches depending on the offers received by each spouse. Instead, when faced with the same job offer sequence the same individuals simply accept a job (agent 1 in period 33 and agent 2 in period 60) and then never quit. Second, in period 29, agent 2 accepts a wage offer of 1.02 when she is part of a couple, but rejects the same offer when acting as single, reflecting the fact that dual-searcher couples have a lower reservation wage than single agents. The opposite happens in period 60 when agent 2 accepts a job offer of 1.08 as single, but turns it down when married, reflecting the fact that worker-searcher couples are more picky in accepting job offers than single agents. It is also easy to see that in the long-run the wages of both agents are higher under joint-search—thanks to the breadwinner cycle, even though it may require a longer search process. Below we provide some illustrative simulations to show that on average joint-search always yields a higher lifetime income (i.e., even when later wages are discounted).

2.3.3.2 DARA utility

As noted earlier, DARA utility is of special interest, since it encompasses the well-known and commonly used CRRA utility specification $u(c) = c^{1-\rho}/(1-\rho)$. More generally, the coefficient of absolute risk aversion with DARA preferences is $-u''(c)/u'(c) = \rho/(c+\rho a)$, which decreases with the consumption (and hence the wage) level. The following proposition characterizes the optimal search strategy for couples with DARA preferences.

Proposition 3. [DARA utility] With DARA preferences, the search behavior of a couple can be characterized as follows:

Figure 2.3: Simulated Wage Paths for a Couple (Joint-Search) and for Same Individuals When they are Single.



(i) The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$ (with $w^{*} < \hat{w}$), which implies that the breadwinner cycle exists.

(ii) The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 \geq \hat{w}$, $\phi(w_1)$ is strictly increasing with $\phi' < 1$.

Figure 2.4 provides a graphical representation of the reservation wage functions associated to the DARA case. Unlike the CARA case, the reservation function of the worker-searcher couple is now increasing with the wage of the employed spouse at all wage levels.[9] This is because with decreasing absolute risk aversion a cou-

[9]This result is related to Danforth (1979) who showed that in the presence of saving and

ple becomes less concerned about smoothing consumption as household resources increase and, consequently, becomes more picky in its job search.

Again, it is useful to sketch the main idea behind the proof, which proceeds by assuming a non-increasing reservation wage function and showing that this leads to a contradiction. Specifically, begin by supposing that $\phi'(\cdot) \leq 0$ beyond a certain wage threshold. In this case, the quit option will not be exercised, so we have:

$$u(w_1 + \phi(w_1)) - u(w_1 + b) = \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] \, dF(w_2),$$

which is identical to the CARA case, except that we have rearranged the terms here for convenience. Divide both sides by the left hand side:

$$1 = \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2). \tag{2.13}$$

Now consider a wage level $\widetilde{w}_1 > w_1$ and replace $\phi(w_1)$ on the right hand side with $\phi(\widetilde{w}_1)$ (which is smaller, by our hypothesis that $\phi'(\cdot) \leq 0$). Then we have:

$$1 \leq \frac{\alpha}{r} \int_{\phi(\tilde{w}_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(\tilde{w}_1))}{u(w_1 + \phi(\tilde{w}_1)) - u(w_1 + b)} \right] dF(w_2). \tag{2.14}$$

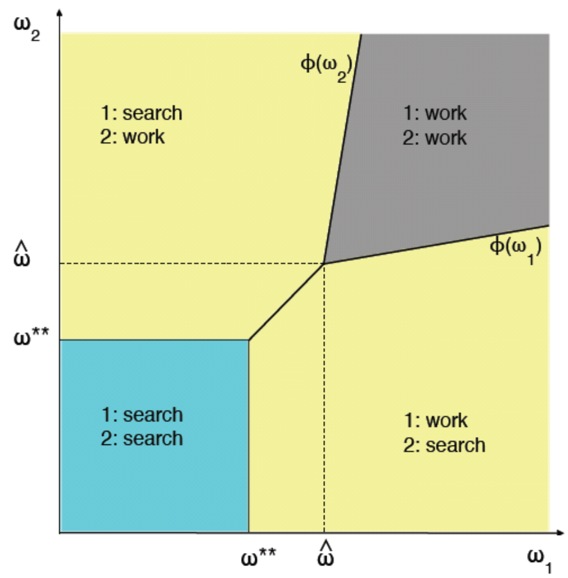
Next, applying a well-known result on DARA preferences established by Pratt (1964, Theorem 1), it can easily be shown that the following inequality holds for any $p > m > q$ and $\tilde{w}_1 > w_1$:

$$\frac{u(w_1 + p) - u(w_1 + m)}{u(w_1 + m) - u(w_1 + q)} < \frac{u(\tilde{w}_1 + p) - u(\tilde{w}_1 + m)}{u(\tilde{w}_1 + m) - u(\tilde{w}_1 + q)}. \tag{2.15}$$

---

no exogenous job separation, whether the reservation wage is increasing or decreasing in wealth depends on the degree of absolute risk aversion of the utility function.

Figure 2.4: Reservation Wage Functions with DARA Preferences (CRRA is a Special Case).



Setting $p \equiv w_2, m \equiv \phi(\tilde{w}_1)$, and $q \equiv b$; integrating both sides over w_2, and then combining with equation (2.14) yields:

$$1 < \frac{\alpha}{r} \int_{\phi(\tilde{w}_1)} \left[\frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)} \right] dF(w_2).$$

But notice that the right hand side of this last expression and of equation (2.13) are identical (when $\tilde{w}_1$ is replaced with w_1), whereas the left hand side of each expression is different. Therefore, we have reached a contradiction, establishing that $\phi'(w_1) > 0$ as stated in the proposition.

The proposition also shows that the breadwinner cycle continues to exist. In contrast to the CARA case, now the breadwinner cycle is observed over a wider range of wage values of the employed spouse. This is because, as can be seen in

Figure 2.4, ϕ is strictly increasing in w_1, so its inverse is not a vertical line anymore but is itself an increasing function. As a result, even when $w_1 > \hat{w}$, a sufficiently high wage offer—one that exceeds $\phi^{-1}(w_1)$—will not only be accepted by the unemployed spouse but it will also trigger the employed spouse to quit. One way to understand this result is by noting that the employed spouse will quit if his reservation wage upon quitting is higher than his current wage. If $w_2 > \phi^{-1}(w_1)$, this implies that upon quitting the job, the reservation wage for the currently employed spouse becomes $\phi(w_2) > \phi\left(\phi^{-1}(w_1)\right) = w_1$. Since this reservation wage is higher than his current wage, it is optimal for the employed spouse to quit the job. Finally, note that only if the wage offer is $w_2 \in \left(\phi(w_1), \phi^{-1}(w_1)\right)$, the job offer is accepted without triggering a quit.

2.3.3.3 IARA utility

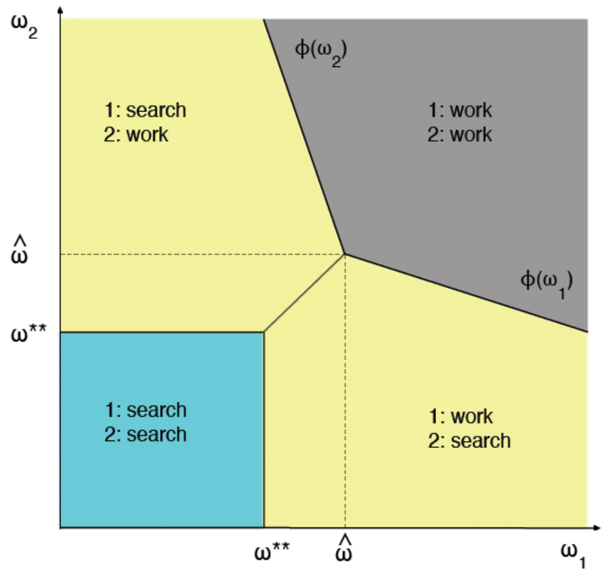
We now turn to IARA preferences, which display increasing absolute risk aversion as consumption increases. One well-known example of IARA preferences is quadratic utility: $-(a-c)^2$ where $c \leq a$.

Proposition 4. [IARA utility] With IARA preferences, the search behavior of a couple can be completely characterized as follows:

(i) The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$, which implies that the breadwinner cycle exists.

(ii) The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 \geq \hat{w}$, $\phi(w_1)$ is strictly decreasing.

Figure 2.5: Reservation Wage Functions with IARA Preferences (Quadratic Utility is a Special Case).



The proof of the proposition is very similar to the DARA case, and is therefore omitted for brevity.[10] Figure 2.5 graphically shows the IARA case.

The reservation wage function ϕ of a worker-searcher couple deviates from the CARA benchmark in the opposite direction of the DARA case. In particular, beyond wage level $\hat{w}$, the reservation function $\phi(w_1)$ is decreasing in w_1, whereas it was increasing in the DARA case. As a result, if the unemployed spouse receives a wage offer higher than $\phi^{-1}(w_1)$, she accepts the offer, the employed stays in the job and both stay employed forever. If the wage offer instead is between $\phi(w_1)$ and

[10]The logic of the proof is as follows. Conjecture that beyond some wage level w_1 the employed worker never quits, and verify the guess by using the property of IARA (also shown by Pratt (1964)) corresponding to (2.15), but with the inequality reversed. The rest of the proof is exactly as for the DARA case.

$\phi^{-1}(w_1)$, then the job offer is accepted followed by a quit by the employed spouse. This behavior is the opposite of the DARA case where high wage offers resulted in quit and intermediate wages did not. Moreover, now the breadwinner cycle never happens at wage levels $w_1 > \hat{w}$. This is a direct consequence of increasing absolute risk aversion which induces a couple to become less choosy when searching as its wage level rises.

Before concluding this section, it is interesting to ask why it is the *absolute* risk aversion that determines the properties of joint-search behavior (as shown in the propositions so far), as opposed to, for example, *relative* risk aversion. The reason has to do with the fact that individuals are drawing wage offers from a fixed probability distribution, regardless of the current wage earnings of the couple. As a result, the uncertainty they face is fixed and is determined by the dispersion in the wage offer distribution, making the attitudes of a couple towards a fixed amount of risk—and therefore, the absolute risk aversion—the relevant measure.[11]

2.3.4 An Isomorphic Model: Single-Search with Multiple Job Holdings

As the reader may have already noticed, the joint-search framework analyzed so far is isomorphic to a search model with a single agent who can hold multiple jobs at the same time. To see this, suppose that the time endowment of a worker can be divided into two sub-periods (e.g., day-shift and night-shift). The single agent can be (i) unemployed and searching for his first job while enjoying $2b$ units of home

[11]If, for example, individuals were to draw wage offers from a distribution that depended on the current wage of a couple, this would likely make the relative risk aversion relevant. This is not the case in the present setup.

production, (ii) working one job at wage w_1 while searching for a second one, or (iii) holding two jobs with wages w_1 and w_2. It is easy to see that the problem faced by this individual is exactly given by the same equations (2.4, 2.5, 2.6) for the joint-search model and therefore has the same solution.[12] Consequently, for example, when the agent works in one job and gets a second job offer with a sufficiently high wage offer, he will accept the offer and simultaneously quit the first job to search for a better one. In this case, it is not the breadwinner that alternates, but the jobs that the worker juggles over time. Although we do not pursue this alternative interpretation in this paper, it would be interesting to explore the implications of such a model including asymmetries between the first and second jobs (such as the wage offer distributions).

2.4 Extensions

The basic joint-search framework in the previous section was intended to provide the simplest possible deviation from the well known single-search problem. Despite being highly stylized, this simple model illustrated some new and potentially important mechanisms that are not operational in the single-agent search problem.

In this section, we enrich this basic model in four empirically relevant directions. First, we allow for nonparticipation. Second, we add on-the-job search. Third, we allow for exogenous job separations. Fourth, we allow households to borrow in financial markets. We are able to establish analytical results in some special

[12] There is a further assumption here that the arrival rate of job offers is proportional to the non-working time of the agent (that is, 2α when unemployed and α when working one job) which does not seem unreasonable.

cases. We also simulate a calibrated version of our model to analyze the differences between a single-agent search economy and the joint-search economy in more general cases.

2.4.1 Nonparticipation

We now extend the two-state model of the labor market we adopted so far to a three-state model where either spouse can choose nonparticipation. Nonparticipation means that the individual does not search for a job opportunity. Consistently with the rest of the paper, where we interpret b as income, we model the benefit associated to nonparticipation as $z > b$ consumption units (e.g., through home production).

We need to redefine some of the value functions for the couple. First, consider the two configurations where (i) both spouses are outside the labor force, and (ii) one spouse does not participate and the other is employed at wage w. Because of the absence of randomness, both these states are absorbing, like being a dual worker couple. Therefore, we can denote the flow value for a couple in the first state as $rT(z, z) = u(2z)$ and the flow value for a couple in the second state as $rT(z, w_2) = u(z + w_2)$. This formulation shows that nonparticipation is equivalent to a job opportunity which pays z (and entails foregoing search) that is always available to the worker.

The flow value for the state where one spouse does not participate and the other is unemployed is

$$r\Omega(z) = u(z + b) + \alpha \int \max\{T(z, w_2) - \Omega(z), \Omega(w_2) - \Omega(z), 0\} dF(w_2), \quad (2.16)$$

where the equation shows that upon spouse 2 accepting a job offer, spouse 1 can either remain out of the labor force, or start searching.

The value of a dual-searcher couple becomes

$$rU = u(2b) + 2\alpha \int \max\{T(z,w) - U, \Omega(w) - U, 0\}\, dF(w), \tag{2.17}$$

which shows that upon either spouse finding a job, the other one has the choice of either keep searching or dropping out of the labor force.

Finally, the value of a worker-searcher couple where spouse 1 is employed is:

$$r\Omega(w_1) = u(w_1 + b) + \alpha \int \max\{T(w_1,w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2). \tag{2.18}$$

The choices available to the couple when spouse 2 finds an acceptable job offer are either spouse 1 remains employed at w_1, or spouse 1 quits into unemployment. This state will arise only for $w_1 > z$, since z is always available.[13] As clear from this equation, once the couple reaches this state, nonparticipation will never occur thereafter. This observation is important, since it means that our definitions of w^{**}, $\hat{w}$ and $\phi(w)$ remain unchanged and these functions are independent of z.

Proposition 5. [Joint search with non participation] With either CARA or DARA preferences, the search behavior of a couple can be summarized as follows:

(i) if $z \leq w^{**}$, the search strategy of the couple is unaffected by nonparticipation, since the latter option is never optimal.

[13]More precisely, there is a third option in the max operator which is, theoretically, available to spouse 1: quitting into non-participation and accepting z forever with a gain $T(z,w_2) - \Omega(w_1)$ for the couple. However, the wage gain associated to spouse 1 keeping his/her current job, $T(w_1,w_2) - \Omega(w_1)$, must be larger since previously spouse 1 has accepted w_1 when z was available.

(ii) if $w^{**} < z < \hat{w}$, dual search is never optimal, and whenever a spouse is unemployed, the other is either employed or nonparticipant. The reservation wage of a nonparticipant-searcher couple is z, and the reservation function of a worker-searcher couple is the same function $\phi(w)$ as in absence of nonparticipation.

(iii) if $z \geq \hat{w}$, nonparticipation is an absorbing state for both spouses, and search is never optimal.

Since nonparticipation is like a job offer at wage z that is always available, if $z < w^{**}$ such offer is never accepted by a dual searcher couple, and nonparticipation is never optimal. When $w^{**} < z < \hat{w}$, then consumption smoothing motives induce the jobless couple to move one of its members into nonparticipation, say spouse 1, while spouse 2 is searching with reservation wage $\phi(z) = z$. As soon as a wage offer w_2 larger than z arrives, the unemployed spouse accepts the job and spouse 1 switches into unemployment, since search is equivalent to being employed at $\phi(w_2) \geq \hat{w} > z$. The inequality follows from the CARA or DARA assumption under which ϕ is a non decreasing function. It is immediate that if $z \geq \hat{w}$, then both spouses exit the labor force right away and no search occurs. As soon as one chooses not to search, the other spouse reservation wage becomes $\phi(z)$ which is always smaller than z in this region. As a result, nonparticipation is attractive for the other spouse as well.[14]

[14]In order to save space, we do not represent graphically this version of the model. It is immediate to see that one can generate the graph with nonparticipation corresponding to case (ii) by overlapping a squared area with coordinates $(x, y) = (z, z)$ to Figures 2 and 4. This area would substitute the dual-searcher couple with the nonparticipant-searcher couple.

The dual search problem is, once again, different from the single-agent search problem. For example, in the CARA case where $\hat{w} = w^*$, we can establish that under configuration (ii), a single agent would be always searching and non employment would never arise, whereas a jobless couple would choose to move one spouse out of the labor force for consumption smoothing purposes.

Finally, note that under this representation of non participation as income, we obtain a stark result: the couple will never be in a state where one spouse works and the other is a non-participant. If preferences are CARA or DARA, this state can only occur when wealth effects on labor supply are active (as in Burdett and Mortensen, 1977), or in presence of asymmetries between spouses. However, the next Lemma shows that under IARA, the worker-nonparticipant configuration may be optimal for the couple. Intuitively, since ϕ is decreasing in w (recall Figure 5), a wage offer $\tilde{w}$ could arrive, say to a dual searcher couple, that is so high to induce the couple to accept the offer and set the new reservation wage for the unemployed member to $\phi(\tilde{w}) < z$ –thus, the unemployed immediately exits the labor force.

Lemma 1. [Non participation with IARA preferences] With IARA preferences, both dual searcher couples and non-participant searcher couples can become a non-participant worker couple.

2.4.2 On-the-job search

Suppose that agents can search both off and on the job. During unemployment they draw a new wage from $F(w)$ at rate α_u whereas during employment they sample new job offers from the same distribution F at rate α_e. What we develop below is, essentially, a version of the Burdett (1978) wage ladder model with couples. The flow value functions in this case are:

$$rU = u(2b) + 2\alpha_u \int \max\{\Omega(w) - U, 0\}\, dF(w) \tag{2.19}$$

$$\begin{aligned} r\Omega(w_1) = {} & u(w_1 + b) + \alpha_u \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2) \\ & + \alpha_e \int \max\left\{\Omega\left(w_1'\right) - \Omega(w_1), 0\right\} dF\left(w_1'\right), \end{aligned} \tag{2.20}$$

$$\begin{aligned} rT(w_1, w_2) = {} & u(w_1 + w_2) + \alpha_e \int \max\left\{T\left(w_1', w_2\right) - T(w_1, w_2), 0\right\} dF\left(w_1'\right) \\ & + \alpha_e \int \max\left\{T\left(w_1, w_2'\right) - T(w_1, w_2), 0\right\} dF\left(w_2'\right). \end{aligned} \tag{2.21}$$

We continue to denote the reservation wage of the dual searcher couple as w^{**}, and the reservation wage of the unemployed spouse in the worker-searcher couple as $\phi(w_1)$. We now have a new reservation function, that of the employed spouse (in the dual-worker couple and in the worker-searcher couple) which we denote by $\eta(w_i)$.

It is intuitive (and can be proved easily) that under risk neutrality the joint-search problem coincides with the problem of the single agent regardless of offer arrival rates. Below, we prove another equivalence result that holds for any risk-averse utility function but for the special case of symmetric offer arrival rates $\alpha_u = \alpha_e$, i.e., when search is equally effective on and off the job.

Proposition 6. [On-the-job search with symmetric arrival rates] If $\alpha_u = \alpha_e$, the joint-search problem yields the same solution as the single-agent search problem, even with concave preferences. Specifically, $w^{**} = w^* = b$, $\phi(w_1) = w^{**}$ and $\eta(w_i) = w_i$ for $i = 1, 2$.

To understand this equivalence result, notice that one way to think about joint-search is that it provides a way to climb the wage ladder for the couple even without on-the-job search: when a dual-searcher couple accepts the first job offer, it continues to receive offers, albeit at a reduced arrival rate. Therefore, one can view joint-search as "costly" version of on-the-job search. The cost comes from the fact that, absent on the job search, in order to keep the search option active, the pair must remain a worker-searcher couple, and must not enjoy the full wage earnings of a dual-worker couple as it would be capable of doing with on the job search. As a result, when on-the-job search is explicitly introduced and the offer arrival rate is equal across employment states, it completely neutralizes the benefits of joint-search and makes the problem equivalent to that of a single-agent. The solution is then simply that the unemployed partner should accept any offer above b and the spouse employed at w_1 any wage above its current one.

The preceding proposition that characterizes joint search behavior when $\alpha_u = \alpha_e$ provides an alternative benchmark to the baseline model, which had $\alpha_u > \alpha_e \equiv 0$. The empirically relevant case is probably in between these two benchmarks, in which case joint-search behavior continues to be qualitatively different from single search (for example, the breadwinner cycle will be active), but the difference becomes smaller quantitatively as the effectiveness of on-the-job search increases. We provide some simulations in Section 2.4.5 below to illustrate these intermediate cases.

2.4.3 Exogenous separations

As discussed above, in the absence of exogenous separations agents optimally choose not to accumulate assets, so a simple borrowing constraint ensures that agents live as hand-to-mouth consumers. This is no longer true when exogenous separation risk is introduced, because in this case accumulated assets can be used to smooth consumption when agents lose their jobs. This saving motive, however, significantly complicates the analysis. Thus, to establish some theoretical results we disallow access to financial markets in this section.

Once again, under risk neutrality it is easy to establish that the joint-search problem collapses to the single agent problem. With risk aversion, however, this is not the case anymore. We first state the following proposition that characterizes joint-search behavior with exogenous separations and then discuss the intuition.

Proposition 7. [CARA/DARA utility with exogenous separations] With CARA or DARA preferences, no access to financial markets, and exogenous job separation, the search behavior of a couple can be completely characterized as follows:

(i) The reservation wage value of a dual-searcher couple satisfies: $w^{**} < \hat{w}$ (with $w^{*} < \hat{w}$), which implies that the breadwinner cycle exists.

(ii) The reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi(w_1) = w_1$, and for $w_1 \geq \hat{w}$, $\phi(w_1)$ is strictly increasing with $\phi' < 1$.

Two remarks are in order. First, for DARA preferences, the existence of exogenous separations has *qualitatively* no effect on joint-search behavior, as can be seen by comparing propositions 3 and 7.[15] Second, and perhaps more interestingly, for CARA preferences, $\phi(w_1)$ is no longer independent of the employed spouse's wage, but is now increasing with it. To understand this result, consider the problem of the worker-searcher couple with current wage w_1 contemplating a new job offer with wage w_2. At rate δ, the currently employed spouse is going to lose his job and if the couple turns down the offer at hand, its earnings will fall from $w_1 + b$ to $2b$ for a net change of $b - w_1 < 0$. Clearly, this income loss (and, therefore, the fall in consumption) is larger, the higher is the current wage of the employed spouse. If instead the couple accepts the job offer, the change upon the job loss of the employed spouse will be from $w_1 + b$ to $b + w_2$, for a net change of $w_2 - w_1$. On the one hand, setting the reservation wage to $\phi(w_1) = w_1$ would completely insure the downside risk of the employed spouse losing his job (because then $w_2 - w_1 \geq 0$). At the same time, letting the reservation wage rise this quickly with w_1 reduces the probability of an acceptable offer and increases the probability that the searcher will still be unemployed when the employed spouse loses his job. As a result, the optimal reservation wage policy balances these two considerations to provide the best self-insurance and, consequently, have $\phi(w_1)$ rise with w_1, but less than one for one: $\phi' < 1$.

[15]The only difference is that here we explicitly rule out saving, whereas previous propositions did not require this assumption as explained before. However, apart from the stronger assumption made here, the search behavior with DARA utility is the same in the two propositions.

Another intuitive way to understand this result begins with recalling that consumption smoothing provided within the household substitutes for the borrowing and saving in financial markets that is ruled out by the proposition. With this in mind, note that in the absence of exogenous separations, accepting a job offer entails a permanent rise in a couple's resources, which implies that the couple never has an incentive to save even when they could. In contrast, with exogenous separation risk, each employment spell is temporary, and when it ends the wage earnings reverts back to the constant amount b. The couple therefore would like to spread these temporary earnings over time, which requires them to save. Since that option is not available, the couple instead adjusts the job rejection/acceptance margin, by acting more patiently in job search—i.e., setting a higher reservation wage—when the current wage is high. Loosely speaking, this lowers the probability of finding a job too quickly—which would push the already high earnings and hence consumption further up—and provides a higher wage once a job accepted, which mitigates the fall in household resources when the currently employed spouse loses his job.

2.4.4 Borrowing in Financial Markets

With few exceptions, search models with risk-averse agents and a borrowing-saving decision are typically not amenable to theoretical analysis.[16] One such exception is when preferences are of CARA type and agents have access to a risk-free asset, an environment that has been used in some previous work to obtain analyt-

[16]It is therefore not surprising that most studies of search models with risk-aversion and savings restrict attention to quantitative analyses. For examples where the decision maker is a household, see Costain (1999), Browning, Crossley and Smith (2003), Lentz (2005), Lentz and Tranaes (2005), Rendon (2006) and Lise (2006).

ical results (Danforth (1979), Acemoglu and Shimer (1999), Shimer and Werning (2006)). Following this tradition, we consider the CARA framework studied in Section 2.3.3.1 extended to allow borrowing. Before analyzing the joint search problem, it is useful to recall here the solution to the single-agent problem.

Single-agent search problem. Let a denote the asset position of the individual. Assets evolve according to the law of motion

$$\frac{da}{dt} = ra + y - c, \tag{2.22}$$

where r is the risk-free interest rate, y is income (equal to w during employment and b during unemployment), and c is consumption. The value functions for the employed and unemployed single agent are, respectively:

$$rW(w,a) = \max_c \{u(c) + W_a(w,a)(ra + w - c)\}, \tag{2.23}$$

$$rV(a) = \max_c \{u(c) + V_a(a)(ra + b - c)\} + \alpha \int \max\{W(w,a) - V(a), 0\}\, dF(w), \tag{2.24}$$

where the subscript a denotes the partial derivative with respect to wealth. These equations reflect the non-stationarity due to the change in assets over time. For example, the second term in (2.23) is $(dW/dt) = (dW/da) \cdot (da/dt)$. And similarly for the second term in (2.24).

We begin by conjecturing that $rW(w,a) = u(ra + w)$. If this is the case, then the FOC determining optimal consumption for the agent gives $u'(c) = u(ra + w)$ which confirms the conjecture and establishes that the employed individual consumes his current wage plus the interest income on the risk free asset. Let us now guess that $rV(a) = u(ra + w^*)$. Once gain, it is easy to verify this guess through the FOC of the unemployed agent. Substituting this solution back into equation (2.24) and using the CARA assumption yields

$$w^* = b + \frac{\alpha}{\rho r} \int_{w^*} \left[u\left(w - w^*\right) - 1 \right] dF\left(w\right) \tag{2.25}$$

which shows that w^* is the reservation wage, which is independent of wealth. Therefore, the unemployed worker consumes the reservation wage plus the interest income on his wealth. This result highlights an important point: the asset position of an unemployed worker deteriorates and, in presence of a debt constraint, she may hit it. As the rest of the papers cited above which use this set up, we abstract from this possibility. The implicit assumption is that borrowing constraints are "loose" and by this we mean they do not bind along the solution for the unemployed agent.

Joint-search problem. When the couple search jointly for jobs, the asset position of the couple still evolves based on (2.22), but now $y = 2b$ for the dual searcher couple, $b + w_1$ for the worker-searcher couple, and $w_1 + w_2$ for the employed couple. The value functions become:

$$rT\left(w_1, w_2, a\right) = \max_c \left\{ u\left(c\right) + T_a\left(w_1, w_2, a\right)\left(ra + w_1 + w_2 - c\right) \right\}, \tag{2.26}$$

$$rU\left(a\right) = \max_c \left\{ u\left(c\right) + U_a\left(a\right)\left(ra + 2b - c\right) \right\} + \alpha \int \max\left\{ \Omega\left(w, a\right) - U\left(a\right), 0 \right\} dF\left(w\right), \tag{2.27}$$

$$\begin{aligned} r\Omega\left(w_1, a\right) = {} & \max_c \left\{ u\left(c\right) + \Omega_a\left(w_1, a\right)\left(ra + w_1 + b - c\right) \right\} \\ & + \alpha \int \max\left\{ T\left(w_1, w_2, a\right) - \Omega\left(w_1, a\right), \Omega\left(w_2, a\right) - \Omega\left(w_1, a\right), 0 \right\} dF\left(w_2\right). \end{aligned} \tag{2.28}$$

Solving this problem requires characterizing the optimal consumption policy for the dual-searcher couple $c_u\left(a\right)$, for the worker-searcher couple $c_\Omega\left(w_1, a\right)$, and for the dual-worker couple $c_e(w_1, w_2, a)$, as well as the reservation wage functions, now potentially a function of wealth too, which must satisfy, as usual: $\Omega(w^{**}(a), a) = U(a)$, $T(w_1, \phi(w_1, a), a) = \Omega(w_1, a)$ and $\Omega(\phi(w_1), a) = \Omega(w_1, a)$. The following proposition characterizes the solution to this problem.

Proposition 8. [CARA utility and access to financial markets] With CARA preferences, access to risk-free borrowing and lending, and "loose" debt constraints, the search behavior of a couple can be characterized as follows:

(i) The optimal consumption policies are: $c_u(a) = ra + 2w^{**}$, $c_\Omega(w_1, a) = ra + w^{**} + w_1$ and $c_e(w_1, w_2, a) = ra + w_1 + w_2$.

(ii) The reservation function ϕ of the worker-searcher couple is independent of (w_1, a) and equals w^{**}, so there is no breadwinner cycle.

(iii) The reservation wage w^{**} of the dual-searcher couple equals w^*, the reservation wage of the single-agent problem.

The main message of this proposition could perhaps be anticipated by the fact that borrowing and saving effectively substitutes for the consumption smoothing provided within the household, making the latter redundant. Consequently, each spouse in the couple can implement labor market search strategies that are independent from the other spouse's actions and, as a result, each acts as in the single-agent model. Of course, to the extent that borrowing constraints bind or preferences deviate from CARA, the equivalence result no longer applies.

The contrasting results obtained from these two benchmarks—the baseline model with no borrowing and the model in this section where borrowing is allowed subject to very loose constraints—provide a useful guide for future empirical work. In particular, they suggest that deviations from single-search behavior in the data (such as the breadwinner cycle) are more likely to be evident and detectable among young and poor households and may be less significant among older and wealthier households.

2.4.5 Some illustrative simulations

In this section our goal is to gain some sense about the quantitative differences in labor market outcomes between the single-search and the joint-search economy. We start from the case of CRRA utility and exogenous separations. Later we add on-the-job search. Thus the economy is characterized by the following set of parameters: b, r, ρ, δ, F, α_u and α_e. When on-the-job search is not allowed, we simply set $\alpha_e = 0$, and $\alpha \equiv \alpha_u$.

We first simulate labor market histories for a large number of individuals acting as singles, compute their optimal choices and some key statistics: the reservation wage w^*, the mean wage, unemployment rate and unemployment duration. Second, we pair individuals together and we treat them as couples solving the joint-search problem in exactly the same economy (i.e., same set of parameters $\{b, r, \rho, \delta, F, \alpha_u, \alpha_e\}$). We use the same sequence of wage offers and separation shocks in both economies. The interest of the exercise lies in comparing the key labor market statistics across economies. For example, it is not obvious whether the joint-search model would have a higher or lower unemployment rate: for the dual-searcher couples, $w^{**} < w^*$, but for the worker-searcher couple $\phi(w)$ is above

w^* at least for large enough wages of the employed spouse.

Calibration. We calibrate the model to replicate the salient features of the US economy. The time period in the model is set to one week of calendar time. The short duration of each period is meant to approximate the continuous time structure in the theoretical models (which, among other things, implies that the probability of both spouses receiving simultaneous offers is negligible). The coefficient of relative risk aversion ρ will vary from zero (risk-neutrality) up to eight in simulations. The weekly net interest rate, r, is set equal to 0.001, corresponding to an annual interest rate of 5.3%. Wage offers are drawn from a lognormal distribution with standard deviation $\sigma = 0.1$ and mean $\mu = -\sigma^2/2$ so that the average wage is always equal to one. We set $\delta = 0.0054$, which corresponds to a monthly employment-unemployment (exogenous) separation rate of 0.02. For each risk aversion value, the offer arrival rate, α_u, is recalibrated to generate an unemployment rate of roughly 0.055.[17] For the model with on the job search we set the offer arrival rate on the job, α_e, to match a monthly employment-employment transition rate of 0.02. Finally, the value of leisure b is set to 0.40 , i.e., 40% of the mean of the wage offer distribution.

Table 2.1 reports the results of our simulation. The first two columns confirm the statement in Proposition 1 that under risk neutrality the joint-search problem reduces to the single-search problem. Let us now consider the case with $\rho = 2$. The reservation wage of the dual-searcher couple is almost 25% lower than in the single-

[17] As risk aversion goes up, w^{**} falls and unemployment duration decreases. So, to continue matching an unemployment rate of 5.5% we need to decrease the value of α_u. For example, for $\rho = 0$, $\alpha_u = 0.4$ and for $\rho = 8$, $\alpha_u = 0.12$.

Table 2.1: A Comparison of Single- versus Joint-Search with CRRA Preferences

	$\rho = 0$		$\rho = 2$		$\rho = 4$		$\rho = 8$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
Res. wage w^*/w^{**}	1.02	1.02	0.98	0.75	0.81	0.58	0.60	0.48
Res. wage $\phi(1)$	–	n/a	–	1.03	–	0.941	–	0.84
Double ind. $\hat{w}$	–	1.02	–	1.02	–	0.94	–	0.82
Mean wage	1.06	1.06	1.07	1.10	1.01	1.05	1.001	1.01
Mm ratio	1.04	1.04	1.09	1.47	1.23	1.81	1.67	2.10
Unemp. rate	5.5%	5.5%	5.4%	7.6%	5.4%	7.7%	5.3%	5.6%
Unemp. duration	9.9	9.9	9.7	12.6	9.8	13.3	9.6	10
Dual-searcher	–	6	–	4.7	–	7.7	–	7.1
Worker-searcher	–	9.8	–	14.2	–	13.6	–	9.6
Job quit rate	–	0%	–	11.1%	–	5.55%	–	0.74%
EQVAR- cons.	–	0%	–	4.5%	–	14%	–	26%
EQVAR- income	–	0%	–	1.1%	–	2.8%	–	0.7%

search economy. And this is reflected in the much shorter unemployment durations of dual-searcher couples. At the same time, though, the reservation wage of worker-searcher couples is always higher than w^*. In the second row of the table we report the reservation wage of the worker-searcher couple at the mean wage offer. Indeed, for these couples, unemployment duration is higher. Overall, this second effect dominates and the joint-search economy displays a longer average unemployment duration–12.6 weeks instead of 9.7–and a considerably higher unemployment rate, 7.6% instead of 5.4%.

Comparing the mean wage tells a similar story. The job-search choosiness of worker-searcher couples dominates the insurance motive of dual-searcher couples and the average wage is higher in the joint-search model. The ability of the couple to climb higher up the wage ladder is reflected in the endogenous quit rate (leading to the breadwinner cycle) which is sizeable, 11.1%. Indeed, the region where the breadwinner cycle is active is rather big, as measured by the gap between w^{**} and $\hat{w}$ which is equal to 2.7 times the standard deviation of the wage offer.

The next six columns display how these statistics change as we increase the coefficient of relative risk aversion. As is clear from the first row, in the case when $\rho = 0$ the difference between w^* and w^{**} is zero. As ρ goes up, both reservation wages fall. Clearly, higher risk aversion implies a stronger demand for consumption smoothing which makes the agent accept a job offer more quickly. However, the gap between w^* and w^{**} first grows but then it shrinks. Indeed, as $\rho \to \infty$, it must be true that $w^* = w^{**} = b$, so the two economies converge again. As for $\phi(1)$, it falls as risk aversion increases, which means that for higher values of ρ the worker-searcher couple accepts job offers more quickly, reducing unemployment. Indeed, at $\rho = 8$ the unemployment rate and the mean wage are almost the same in the two economies.

We also report a measure of frictional wage dispersion, the mean-min ratio (Mm) defined as the ratio between the mean wage and the lowest wage, i.e. the reservation wage. Hornstein, Krusell, and Violante (2006) demonstrate that a large class of search models, in particular those without on the job search, when plausibly calibrated generate very little wage dispersion. The fifth row of Table 2.1 confirms this result. It also confirms the finding in Hornstein et al. that the Mm ratio increases with risk aversion. What is novel here is that the joint-search model generates more frictional inequality: the reservation wage for the dual searcher couple is lower, but the couple can climb the wage distribution faster which translates into a higher average wage. This result is consistent with the finding in Hornstein et al. that single-agent search models with on the job search fare better in terms of residual wage dispersion.

Next, we discuss two separate measures of welfare effects of joint-search in the simulated economy. Recall that the jointly searching couple has two advantages: first, it can smooth consumption better, second it can get higher earnings. The first measure of welfare gain is the standard consumption-equivalent variation and embeds both advantages. The second is the change in lifetime income from being married which isolates the second aspect–the novel one.[18] The consumption-based measure of welfare gain is very large, not surprisingly. What is remarkable is that also the gains in terms of lifetime income can be very large, for example around 2.8% for the case $\rho = 4$. As risk aversion goes up, the welfare gains from family insurance keep increasing, but as explained above, the ones stemming from better search opportunities fade away.

Table 2.2 presents the results when on-the-job search is introduced into this environment. The first four columns simply confirm the theoretical results established in previous sections. For example, when agents are risk-neutral, on-the-job search has no additional effect, and both the single- and joint-search problems yield the same solution regardless of parameter values. Similarly, as shown in proposition 6 when on-the-job search is as effective as search during unemployment ($\alpha_e = \alpha_u$) then, again, single- and joint-search coincide.

Overall, comparing these results to those in Table 2.1 shows that the effects

[18] To make the comparison between singles and couples meaningful, we assume that each spouse consumes half of the household's income (as opposed to "all income" assumed in the theoretical analysis). Notice that, with CRRA preferences, this alternative assumption would not affect any of the theoretical results proved before.

Table 2.2: Single- versus Joint-Search: CRRA Preferences and On-the-Job Search

	$\rho = 0$ $\alpha_u = 0.2, \alpha_e = 0.03$		$\rho = 2$ $\alpha_u = 0.1, \alpha_e = 0.1$		$\rho = 2$ $\alpha_u = 0.11, \alpha_e = 0.02$		$\rho = 4$ $\alpha_u = 0.11, \alpha_e = 0.02$	
	Single	Joint	Single	Joint	Single	Joint	Single	Joint
Res. wage w^*/w^{**}	0.98	0.98	0.4	0.4	0.78	0.67	0.62	0.54
Res. wage $\phi(1)$	–	0.98	–	0.4	–	0.85	–	0.74
Double ind. $\hat{w}$	–	0.98	–	0.4	–	0.87	–	0.8
Mean wage	1.13	1.13	1.16	1.16	1.08	1.09	1.08	1.09
Mm ratio	1.15	1.15	2.90	2.90	1.38	1.63	1.74	2.02
Unemp. rate	5.4%	5.4%	5.4%	5.4%	5.3%	5.8%	5.3%	5.4%
Unemp. duration	9.8	9.8	10.5	10.5	9.7	10.6	9.6	9.8
Dual-searcher	–	7	–	7.7	–	7.1	–	7
Worker-searcher	–	9.4	–	9.9	–	10.2	–	9.3
EU quit rate	–	0%	–	0%	–	0.93%	–	0.19%
EE transition	0.45%	0.45%	1.03%	1.03%	0.49%	0.47%	0.51%	0.49%
EQVAR-cons.	–	0%	–	4.6%	–	4.1%	–	15%
EQVAR-income	–	0%	–	0%	–	0.2%	–	0.05%

of joint-search on labor market outcomes are qualitatively the same as before, but they become much smaller quantitatively. This is perhaps not surprising in light of the discussion in Section 2.4, where we argued that joint-search is a partial substitute for on-the-job search (or a costly version of it). Therefore, once on-the-job search is available, having a search partner is not so useful any longer to obtain higher earnings. Although, it obviously remains a great way to smooth consumption, as evident from the last two lines of the table.

2.5 Joint-search with Multiple Locations

The importance of the geographical dimension of job search is undeniable. For the single-agent search problem, accepting a job in a different market could require a relocation cost that may be high enough to induce the agent to turn down the offer. In the joint-search problem, the spatial dimension introduces a new and interesting search friction. In addition to migration costs that also apply to a single agent, a couple is likely to suffer from the disutility of living apart if spouses accept jobs in different locations. This cost can easily rival or exceed the physical costs of relocation since it is a flow cost as opposed to the latter, which are arguably better thought of as one-time costs.

To analyze the joint-search problem with multiple locations, we extend the framework proposed in Section 2.2 by introducing a fixed flow cost of living separately for a couple. As we shall see below, the introduction of location choice leads to several important changes in the search behavior of couples compared to a single agent, *even* with risk-neutrality. Furthermore, many of these changes are not favorable to couples, which serves to show that joint-search can itself create new frictions. This is in contrast to the analysis performed so far, which only showed new opportunities of joint search.[19]

To keep the analysis tractable, we first consider agents that search for jobs in two symmetric locations, and provide a theoretical characterization of the solution. In the next subsection, we examine the more general case with $L(> 2)$ locations that is more suitable for a meaningful calibration and provide some results based on numerical simulations.

[19]This friction raises the issue of whether the couple should split. While the interaction between labor market frictions and changes in marital status is a fascinating question, it is beyond the scope of this paper. Here we assume that the couple has committed to stay together or, equivalently, that there is enough idiosyncratic non-monetary value in the match to justify continuing the relationship.

2.5.1 Two locations

Environment. As before, we define a couple as an economic unit composed of two individuals $(1, 2)$ who are linked to each other by the assumption that income is pooled and household consumption is a public good. Households can save but not borrow and there are no exogenous separations.

The economy has two locations. Couples incur a flow resource cost, denoted by κ, if they live apart. Denote by i the "inside" location and by o the "outside" location. Offers arrive at rate α_i from the current location and at rate α_o from the outside location. The two locations have the same wage offer distribution F. We assume away moving costs: the point of the analysis is the comparison with the single-agent problem and such costs would also be borne by the single agent.

A couple can be in one of four labor market states. First, if both spouses are unemployed and searching, they are referred to as a "dual-searcher couple." Second, if both spouses are employed in the same location (in which case they will stay in their jobs forever) we refer to them as a "dual-worker couple" but if they are employed in different locations we refer to them as "separate dual-worker couple" (another absorbing state). Finally, if one spouse is employed and the other one is unemployed, we refer to them as a "worker-searcher couple." As explained, individuals in a dual-searcher couple have no advantage from living separately, so they will choose to live in the same location. Let $U, T\left(w_1, w_2\right), S\left(w_1, w_2\right)$ and $\Omega\left(w_1\right)$

be the value of these four states, respectively. Then, we have

$$rT(w_1, w_2) = u(w_1 + w_2) \tag{2.29}$$

$$rS(w_1, w_2) = u(w_1 + w_2 - \kappa) \tag{2.30}$$

$$rU = u(2b) + 2(\alpha_i + \alpha_o) \int \max\{\Omega(w) - U, 0\}\, dF(w) \tag{2.31}$$

$$\begin{aligned} r\Omega(w_1) = u(w_1 + b) &+ \alpha_i \int \max\{T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2) \\ &+ \alpha_o \int \max\{S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0\}\, dF(w_2) \end{aligned} \tag{2.32}$$

The first three value functions are easily understood and do not require explanation. The value function for a worker-searcher couple now has to account separately for inside and outside offers. If an inside offer arrives, the choice is the same as in the one-location case since no cost of living separately is incurred. If, however, an outside offer is received, the unemployed spouse may turn down the offer or may accept the job, in which case the couple has two options: either it chooses to live separately incurring cost κ, or the employed spouse quits and follows the newly employed spouse to the new location to avoid the cost.

The decision of the dual-searcher couple is entirely characterized by the reservation wage w^{**}. For the worker-searcher couple, let $\phi_i(w_1)$ and $\phi_o(w_1)$ be the reservation functions corresponding to inside and outside offers. Once again, these functions are piecewise with one piece corresponding to the 45 degree line. By inspecting equation (2.32) it is immediate that, as in the one-location case, $\phi_i^{-1}(w_1)$ and $\phi_o^{-1}(w_1)$ characterize the quitting decision.

Single-agent search. Before proceeding further, it is straightforward to see that the single-search problem with two locations is the same as the one-location case, with the appropriate modification to the reservation wage to account for separate arrival rates from two locations. In the risk neutral case, we have:

$$w^* = b + \frac{\alpha_i + \alpha_o}{r} \int_{w^*} [1 - F(w)]\, dw. \tag{2.33}$$

Recall that in the one-location case, risk neutrality resulted in an equivalence between the single-search and joint-search problems. As the next proposition shows, this result does not hold in the two-location case anymore, as long as there is a positive cost κ of living apart:

Proposition 9. [Two locations] With risk neutrality, two locations and $\kappa > 0$, the search behavior of a couple can be completely characterized as follows. There is a wage value

$$\hat{w}_S = b + \kappa + \frac{\alpha_i}{r} \int_{\hat{w}_S - \kappa} [1 - F(w)]\, dw + \frac{\alpha_o}{r} \int_{\hat{w}_S} [1 - F(w)]\, dw$$

and a corresponding value $\hat{w}_T = \hat{w}_S - \kappa$ such that:

(i) $w^{**} \in (\hat{w}_T, \hat{w})$ whereas $w^* \in (\hat{w}, \hat{w}_S)$. Therefore, $w^{**} < w^*$ which implies that the breadwinner cycle exists.

(ii) For outside offers, the reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}_S$, $\phi_o(w_1) = w_1$, and for $w_1 \geq \hat{w}_S$, $\phi_o(w_1) = \hat{w}_S$.

(iii) For inside offers, the reservation wage function of a worker-searcher couple has the following properties: for $w_1 < \hat{w}$, $\phi_i(w_1) = w_1$, for $w_1 \in [\hat{w}, \hat{w}_S)$, $\phi_i(w_1)$ is strictly decreasing and for $w_1 \geq \hat{w}_S$, $\phi_i(w_1) = \hat{w}_T$.

Figure 2.6: Reservation Wage Functions for *Outside Offers* with Risk-Neutral Preferences and Two Locations

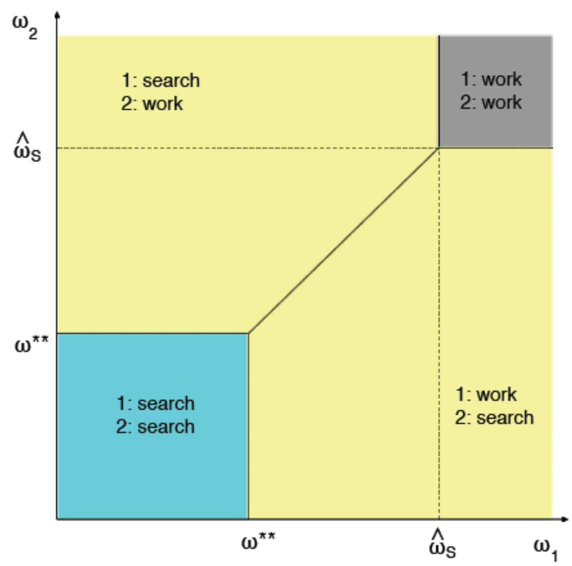
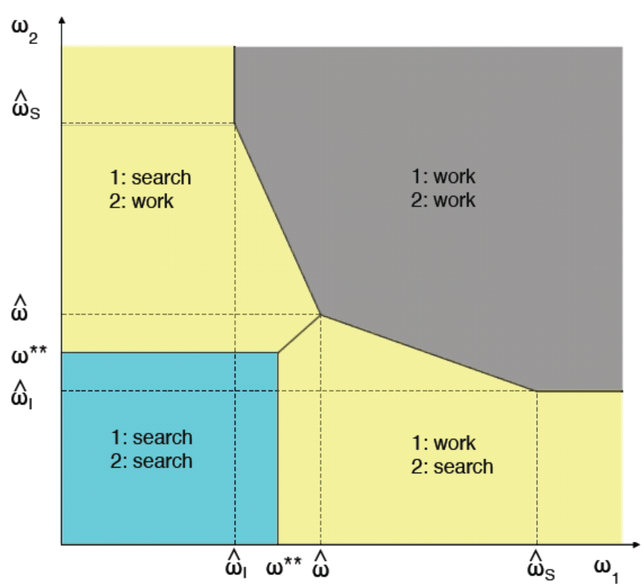


Figure 2.7: Reservation Wage Functions for *Inside Offers* with Risk-Neutral Preferences and Two Locations



Figures 2.6 and 2.7 graphically show the reservation wage functions for outside offers and inside offers, respectively. As seen in these figures, the reservation wage functions for both inside and outside offers are quite different from the corresponding ones of the model with one-location (Figure 2.3.2). In particular, the reservation wage functions for both inside offers and outside offers now depend on the wage of the employed spouse at least when $w_1 \in (w^{**}, \hat{w}_S)$. This has several implications.

Consider first outside offers for a worker-searcher couple where one spouse is employed at $w_1 < \hat{w}_S$ (Figure 2.6). The couple will reject wage offers below w_1, but when faced with a wage offer above w_1, the employed worker will quit his job and follow the other spouse to the outside location. The cost κ is too large to justify living apart while being employed at such wages. In contrast, when $w_1 > \hat{w}_S$ if the couple receives a wage offer $w_2 > \hat{w}_S$, it will bear the cost of living separately in order to receive such high wages.

Comparing Figure 2.7 for inside offers to Figure 2.6, it is immediate that the range of wages for which inside offers are accepted by a worker-searcher couple is larger, since no cost κ has to be paid. Interestingly, the reservation function $\phi_i(w_1)$ now has three distinct pieces. For w_1 large enough, it is constant, as in the single-agent case. In the intermediate range $(\hat{w}, \hat{w}_S)$ the function is decreasing. This phenomenon is linked to the reservation function for outside offers ϕ_o which is increasing in this range: as w_1 rises the gains from search coming from outside offers are lower (it takes a very high outside wage offer w_2 to induce the employed spouse to quit), hence the reservation wage for inside offers falls.

For w_1 small enough, the reservation function $\phi_i(w_1)$ is increasing and equal to the wage of the employed spouse. In this region, the breadwinner cycle is again active, so whenever the wage offer is higher than the employed spouse's wage but smaller than $\varphi_i(w_1)$, the couple goes through the breadwinner cycle. However, if the wage offer is high enough, the potential negative impact of the outside wage offers induces the couple to become a dual-worker couple. Using the same reasoning we applied to the range $(\hat{w}, \hat{w}_S)$, the reservation wage for being a dual-worker couple decreases as w_1 increases.

Tied-movers and tied-stayers. In a seminal paper, Mincer (1978) has studied empirically the job-related migration decisions of couples in the United States (during the 1960's and 70's). Following the terminology introduced by Mincer, we refer to a spouse who rejects an outside offer that she would accept when single as a "tied-stayer." Similarly, we refer to a spouse who follows her spouse to the new destination even though her individual calculus dictates otherwise as a "tied-mover." Using data from the 1962 BLS survey of unemployed persons, Mincer estimated that "22 percent or two-thirds of the wives of moving families would be tied-movers, while 23 percent out of 70 percent of wives in families of stayers declared themselves to be tied-stayers (page 758)."[20]

[20] More precisely, Mincer (1978) defines an individual to be a tied-stayer (a tied-mover) if the individual cites his/her spouses' job as the main reason for turning down (accepting) a job from a different location: Mincer wrote (page 758): "The unemployed were asked whether they would accept a job in another area comparable with the one they lost. A positive answer was given by 30 percent of the married men, 21 percent of the single women, and only 8 percent of the married women. Most people who said no cited family, home, and relatives as reasons for the reluctance to move. However, one quarter of the women singled out their husbands' job in the present area as the major deterrent factor".

Figure 2.8: Tied-Stayers and Tied-Movers in the Joint-Search Model

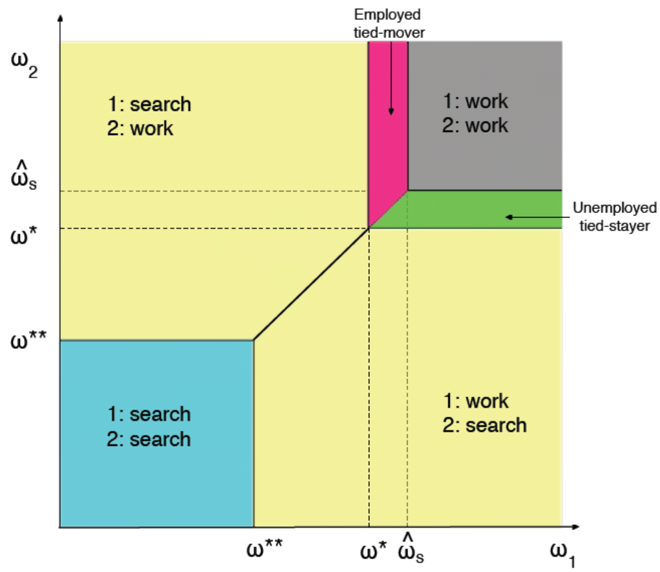


Figure 2.8 re-draws the reservation wage functions for outside offers and indicates the regions that give rise to tied-stayers and tied-movers in our model. First, if the wage of the employed spouse, w_1, is higher than w^*, then the unemployed spouse rejects outside offers and stays in the current location for all wage offers less than $\phi_i(w_1)$. In contrast, a single agent would accept all offers w_2 above w^*, which is less than $\phi_i(w_1)$ by Proposition 9. Therefore, an unemployed spouse who rejects an outside wage offer $w_2 \in (w^*, \phi_i(w_1))$ is formally a tied-stayer (as shown in figure 2.8). There is also another region where the *employed* spouse is a tied-mover. Suppose the wage of the employed, w_1, is smaller than $\hat{w}_S$, and the unemployed receives an outside wage offer higher than w_1, then the unemployed accepts the offer, the employed spouse quits the job and both move to the other location. Note that the employed spouse would not move to the other location if she were single, so the

employed spouse is also as a tied-mover (see figure 2.8).

Both set of choices involve potentially large concessions by each spouse compared to the situation where he/she were single, but they are optimal from a joint decision perspective. This opens the possibility of welfare costs of being in a couple versus being single with respect to job search, an aspect of the model which we analyze quantitatively, through simulation, in the next section.

2.5.2 Some illustrative simulations with multiple locations

Although the two location case serves as a convenient benchmark that illustrates all the key mechanisms, it is not a natural environment for a calibrated exercise, especially when the offer arrival rates from the two locations are not the same ($\alpha_i \neq \alpha_o$). But this is the more reasonable assumption since the outside location is more appropriately interpreted as the "rest of the world" and in many cases could offer more job opportunities than any one home location. This asymmetry between the two locations cannot be captured satisfactorily with two locations, for example by setting $\alpha_o \gg \alpha_i$, because this would imply that if one of the spouses moves to the rest of the world, the other spouse will have a very high probability of moving to the same location, where the couple will reunite.

For the simulation exercise, we therefore extend the framework described above to multiple locations and allow exogenous separations. Specifically, consider an economy with L geographically separate symmetric labor markets. Firms in each location generate offers at flow rate ψ for employed agents and at rate ψ_u for unemployed agents. A fraction θ of both types of offers are distributed equally to

Table 2.3: Single- versus Joint-Search: 9 Locations and Risk Neutral Preferences

	$\kappa = 0$		$\kappa = 0.1$	$\kappa = 0.3$
	Single	Joint	Joint	Joint
w^*/w^{**} (Reservation wage)	1.02	1.02	0.97	0.94
$\hat{w}_T$	–	1.02	0.95	0.88
$\hat{w}$ (Double indiff. point)	–	1.02	0.99	0.97
$\hat{w}_S$	–	1.02	1.04	1.13
$\phi_i(1)$ (Reservation wage)	–	n/a	0.984	0.95
Mean wage	1.058	1.058	1.06	1.045
Mm ratio	1.04	1.04	1.09	1.11
Unemployment rate	5.5%	5.5%	6.9%	13.7%
Unemployment duration	9.9	9.9	9.8	13.0
Dual-searcher	–	6.5	3.3	3.0
Worker-searcher	–	9.3	12.9	28.0
Movers (% of population)	0.52%	0.52%	0.74%	1.26%
Stayers (% of population)	1.12%	1.12%	1.53%	3.4%
Tied-movers/Movers	–	0%	29%	56%
Tied-stayer/Stayers	–	0%	11%	23%
Job quit rate	–	0%	23%	50%
EQVAR-cons	–	0%	–0.8%	–6.5%

the $L-1$ outside locations and the remaining $(1-\theta)$ is made to the local market.[21] The value functions corresponding to this economy are provided in the Appendix and are a straightforward extensions of value functions in (2.29)–(2.32).

The number of locations, L, is set to 9 representing the number of U.S. census divisions and θ is set to $1 - 1/L$, implying that firms make offers to all locations with equal probability. The remaining parameters are calibrated as before, i.e., to match certain labor market statistics in the single-agent version of the model. Table

[21] The assumption that there are a very large number of individuals in each location, combined with the fact that the environment is stationary (i.e., no location specific shocks) implies that we can take the number of workers in each location as constant, despite the fact that workers are free to move across locations and across employment states depending on the offers they receive.

2.3 presents the simulation results. A comparison of the first two columns confirms that the single- and joint-search problems are equivalent when there is no disutility from living apart ($\kappa = 0$).[22] The third and fourth columns show the results when $\kappa = 0.1$ and 0.3, respectively—representing a flow cost equal to 10% and 30% of the mean wage. First, the reservation wages are in line with our theoretical results in Proposition 9: $\hat{w}_T < w^{**} < w^* < \hat{w}_S$. Second, the presence of the cost κ makes outside offers less appealing, making the couple reject some offers that a single would accept. As a result, the average wage is lower and the unemployment rate is higher in the joint-search economy. In fact, when $\kappa = 0.3$ the unemployment rate is substantially higher—13.7% compared to 5.5% in the single-agent model. However, the average duration of unemployment is not necessarily longer under joint-search: when $\kappa = 0.1$ the average duration falls to 9.8 weeks from 9.9 weeks in the single agent case, but rises to 13 weeks when κ is further raised to 0.3. The next two rows decomposes the average unemployment duration figure into the component experienced by dual-searcher couples and by worker-searcher couples. The duration of the former group is shorter than that of single agents (since $w^{**} < w^*$) and gets even shorter as κ increases (falls from 6.5 weeks to 3 weeks in column 4). However, because worker-searcher couples face a smaller number of feasible job offers from outside locations, they have a much longer unemployment spells: 12.9 weeks when $\kappa = 0.1$ and 28 weeks when $\kappa = 0.3$, compared to 9.3 weeks when $\kappa = 0$. Overall, there are more people who are unemployed at any point in time, and some of these

[22]Since κ does not have any effect on the single-search problem, we present them only for the case with $\kappa = 0$.

unemployed workers—those in worker-searcher families—stay unemployed for much longer than they would have had they been single, while trying to resolve their joint-location problem.

We next turn the impact of joint-search on the migration decision of couples. In our context, we define a couple to be a "mover" if at least one spouse moves for *job-related* reasons. This includes dual-searcher couples who move to another location because one of the spouses accepts an outside job offer and worker-searcher couples if at least one spouse moves to another location because the unemployed spouse accepts an offer at another location.[23] Similarly, we define a couple to be a "stayer" if either member of the couple turns down an outside job offer.

Using this definition, the fraction of movers in the population is 0.52% per week when $\kappa = 0$; it rises to 0.74% when $\kappa = 0.1$ and to 1.26% when $\kappa = 0.3$. Part of the rise in moving rate is mechanically related to the rise in the unemployment rate with κ: because there is no on the job search, individuals only get job offers when they are unemployed, which in turn increases the number of individuals who accept offers and move. Notice also that while the fraction of movers appears high in all three cases, this is not surprising given that we are completely abstracting from physical costs of moving. Perhaps more striking is the fact that almost 56% of all movers are tied-movers when $\kappa = 0.3$, using the definition in Mincer (1978) described above. The fraction of tied-stayers is also sizeable: 21% in the high-friction

[23]However, consider a dual-worker couple where spouses live in separate locations. If one of the spouses receives a separation shock and becomes unemployed, she will move to her spouse's location. In this case the household is not considered to be a mover since the move did not occur in order to accept a job.

case. The voluntary quit rate—which is defined as the fraction of employment-to-unemployment transitions that are due to voluntary quits—is as high as 50% when $\kappa = 0.3$.

Finally, a comparison of lifetime wage incomes shows that the friction introduced by joint-location search can substantial: it reduces the lifetime income of a couple by about 0.8% (per-person) compared to a single agent when $\kappa = 0.1$ and by 6.5% when $\kappa = 0.3$. Overall, these results show that with multiple locations, joint-search behavior can deviate substantially from the standard single-agent search.

2.6 Conclusions

Our work extends naturally in two directions. First, from a theoretical viewpoint, one should explore other channels leading to joint-search decisions in the labor market. For example, complementarity/substitutability in leisure (Burdett and Mortensen, 1978), or more realistic consumption-sharing rules that deviate from full income pooling as in the collective model (e.g. Chiappori, 1992). One can also generalize the symmetry assumption we made on individuals and locations. One limit of the present framework, especially in the multiple location case where the couple may be worse-off than the single agent, is that we ignore the option to split up (see Aiyagari, Greenwood and Guner, 2000, for a quantitative model of marriage and divorce with frictional marriage market). A search-based analysis of labor and marriage market dynamics is an ambitious but necessary step forward.

Second, the qualitative features of the joint-search problem established here provide guidance on conducting quantitative work more effectively. For example, the qualitatively different behaviors implied by different sub-classes of HARA preferences (and specifically, the opposite behavior of the reservation wage functions under DARA and IARA preferences) cautions against using linear-quadratic approximations to CRRA utility functions often made in computational analysis. The model in Section 2.5 generalized to allow asymmetry in skills across spouses, asymmetry in locations in size, and, perhaps, also to allow for borrowing/saving can be brought to the data and estimated structurally. The challenge is to access micro data with household level information on the detailed labor market histories of both members of the couple and on their geographical movements.

Chapter 3

House Prices and Interest Rates [1]

3.1 Introduction

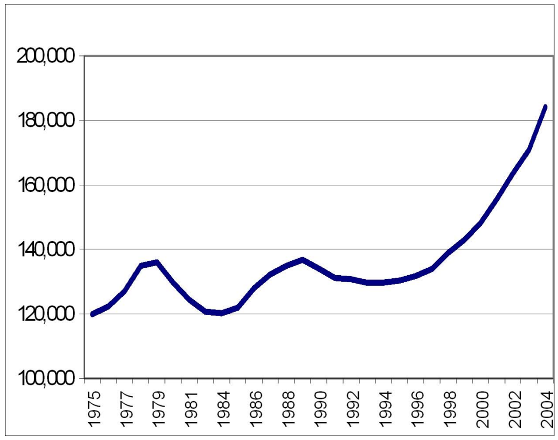
In this paper we study the relationship between interest rates and housing prices in a two period overlapping generations model. The existence of aggregate interest rate uncertainty, explicit long-term mortgage contracts and the illiquidity of the housing good distinguish our model from the existing literature.

In figure 1 we plot the evolution of the housing prices from 1975 to 2005. Figure 2 plots the real 30-year fixed rate mortgage rates in the U.S. for the same period. Comparing the mortgage rate movements in this graph to the changes in real median housing prices shown in figure 1 reveals a surprisingly strong (negative) relationship: housing prices were falling from 1979 to 1985 when real mortgage rates were high; and housing prices were "booming" in late 1990's when mortgage rates were low (and falling). Therefore, figures 1 and 2 are qualitatively consistent with the hypothesis that the housing prices and interest rates have a negative relationship.

The strong negative relationship between interest rates and housing prices has been used in the literature to explain the housing price movements observed in the data. Himmelberg, Mayer and Sinai (2005) and Martin (2005) emphasize

[1] This chapter borrows extensively from one of my working papers joint with Yavuz Arslan.

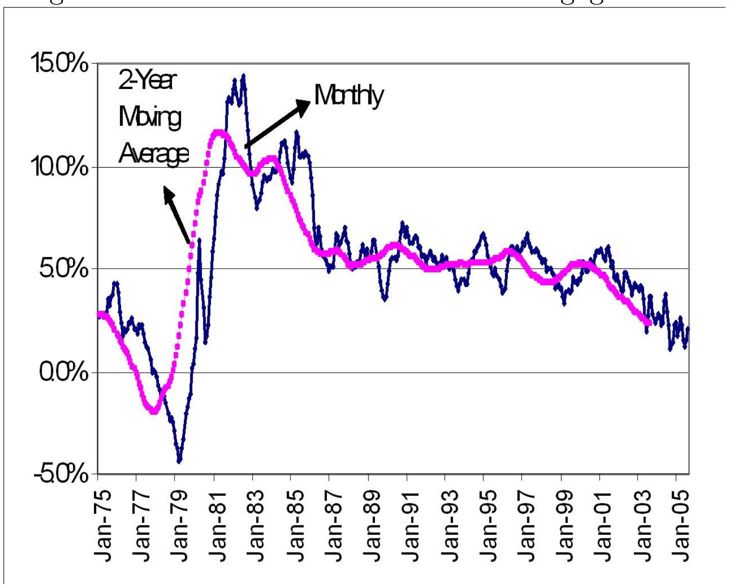
Figure 3.1: Median House Prices



the importance of interest rates in explaining the recent increase in housing prices. Himmelberg, Mayer and Sinai (2005) develop a formula to account for the changes in the annual cost of owning a house. In their formula, interest rates affect the homeowner's forgone earnings. Martin (2005) studies the baby boom's impact on the US housing prices by using a simple Lucas asset pricing model. He argues that the baby boom affected the demand for housing as well as the interest rates. Due to lower interest rates, his model can predict the housing price increase in the last decade as opposed to Mankiw and Weil (1988).

Although the negative relationship between interest rates and housing prices is both intuitive and well documented it has not been very stable. Figure 3 shows the relationship between the percentage change in real housing prices and percentage change in real mortgage interest rates. Between 75 and 79, they are both positive, meaning housing prices and mortgage interest rates both increased in this period. After 79, for a period of 10 years, housing prices and mortgage interest rates have

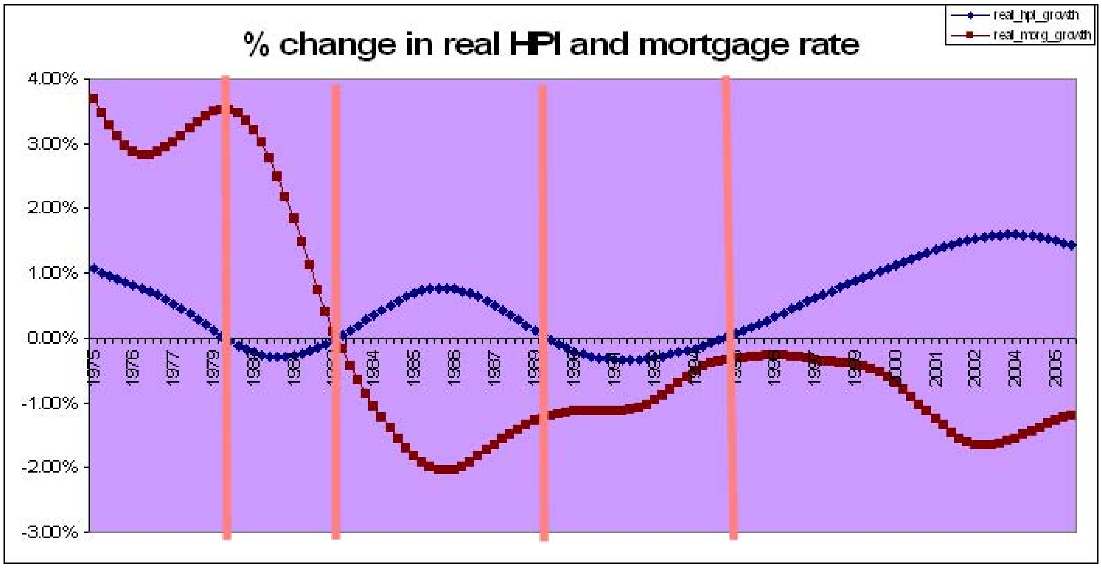
Figure 3.2: 30 Year Real Fixed-Rate Mortgage Rates.



negative correlation. At the beginning of 80s mortgage interest rates started to decline and housing prices started to increase, however through mid 80s this trend is just reversed. Starting by early 90s, housing prices and mortgage interest rates had a positive correlation. Both variables were decreasing in the early 90s. However, after mid 90s, this positive correlation disappeared and they started to move opposite to each other. Mortgage interest rates have been decreasing while housing prices have been increasing, and this relationship still exists although it's much weaker.

In this paper we analyze a two period overlapping generations model to understand the relationship between interest rates and housing prices. At the beginning of each period, the economy is hit by an exogenous interest rate shock. A young agent enters into the economy with a limited amount of resources which makes him borrow in the mortgage market to finance his housing purchase. After

Figure 3.3: Change in Housing Prices vs Change in Mortgage Rates



he buys his house he receives his remaining income and consumes the non-housing consumption good. At the end of the period he makes the first mortgage payment. Old agents receive an exogenous movement shock. If an old agent moves from his house, he gets the return from selling the house and pays the remaining mortgage debt coming from the previous house. Then he borrows in the mortgage market to finance his new house purchase. If he does not move from his house, he makes the second mortgage payment and uses the remaining income to purchase non-housing consumption. There are risk-neutral profit-maximizing banks which offer mortgage contracts. At the equilibrium, banks are indifferent between lending in the bond market and the mortgage market. For simplicity we assume that aggregate house supply is fixed and there is no rental market.

We show that, even though interest rates is the only exogenous process in the model, housing prices is a function of both interest rates and *"effective supply"*[2]. The joint dynamics of interest rates and the effective supply determines the interest rate-housing price relationship. Simulations of a plausibly calibrated version of model shows that interest rates and housing prices are 95% of the time are negatively related, but for 5% of the time they are not.

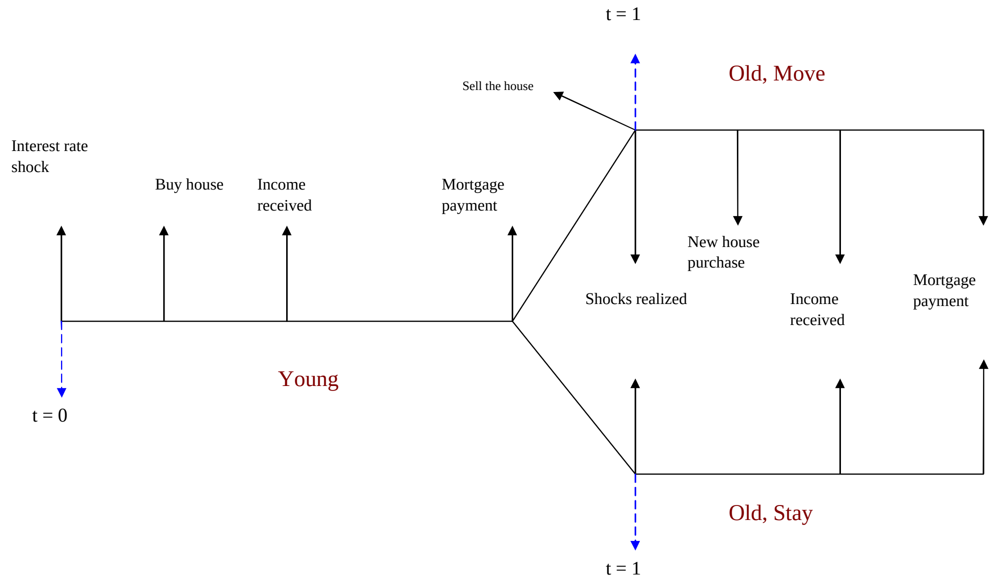
The paper is organized as follows. Section 2 describes the model. In Section 3 we solve the model. We do the calibration in Section 4. The results of the paper are reported in Section 5. In Section 6 we conclude.

3.2 The Model

The basic framework studied in this paper is a two-period overlapping-generations model. At the beginning of each period, the economy is hit by an exogenous interest rate shock. A young agent enters into the economy with a limited amount of resources which makes him borrow in the mortgage market to finance his housing purchase. After he buys his house he receives his remaining income and consumes the non-housing consumption good. At the end of the period he makes the first mortgage payment. Old agents receive an exogenous movement shock. If an old agent moves from his house, he gets the return from selling the house and pays the remaining mortgage debt coming from the previous house. Then he borrows in the mortgage market to finance his new house purchase. If he does not move from his house, he makes the second mortgage payment and uses the remaining income to

[2] What we call as "effective supply" is the housing amount in the economy available for sale. Since, some of the agents will not move, this effective supply will change in time.

Figure 3.4: Timing of Events



purchase non-housing consumption. There are risk-neutral profit-maximizing banks which offer mortgage contracts. At the equilibrium, banks are indifferent between lending in the bond market and the mortgage market. For simplicity we assume that aggregate house supply is fixed and there is no rental market.

Mortgage Market

Risk-neutral profit-maximizing banks offer fixed-rate 30 year mortgage contracts. We have two conditions to find the mortgage rates and the corresponding payments. The first condition is the *present value condition*, which means that the present value of the payments should be equal to the loan amount,

$$1 = \frac{D_t}{1+d_t} + \frac{D_t}{(1+d_t)^2}.$$

An agent should pay D_t if he borrows 1 dollar in the mortgage market when the mortgage rate is d_t. The next condition is the *no-arbitrage condition.* At equilibrium, banks should be indifferent between lending in the bond market and the mortgage market,

$$1 = \frac{D_t}{1+r_t} + (1-\pi)\frac{D_t}{(1+r_t)E[(1+r_{t+1})]} + \pi\frac{1-D_t+d_t}{1+r_t}.$$

The lender will receive the first mortgage payment D_t at the end of the first period, and he will discount this with current interest rate, $1+r_t$.[3] With probability $1-\pi$ the agent will not move and he will do his second mortgage payment, which is $\frac{D_t}{(1+r_t)E[(1+r_{t+1})]}$. With π probability the agent will move and prepay his loan by the beginning of the second period. The discounted value of this payment is $\frac{1-D_t+d_t}{1+r_t}$. In the solution of the model, we should also find D_t, d_t as functions of state variables.

The old agents who buy a house get a 15 year mortgage contract, and the interest on the mortgage is the same as the interest rate on the bond.

Old Agent's Problem

If an old agent receives a movement shock he chooses how big a house to purchase depending on the state of the market. If he does not move he stays in his old house. The value function of an old agent can be formulated as

$$V_{old}(H_{t-1}, r_t) = \left\{ \begin{array}{l} V_{move}, \text{ if the agent receives a movement shock} \\ V_{stay}, \text{ if the agent does not receive a movement shock} \end{array} \right\}.$$

[3]As the agents in the model can transact only after 15 years, the corresponding discount factor will be "15 to the power of *yearly interest rate*".

The aggregate state of the economy is summarized by H_{t-1} (the aggregate amount of housing they bought in the previous period) and r_t (interest rate). The value of movement is

$$V_{move}(H_{t-1}, r_t) = \underset{h_{t,o,m}}{Max} \left\{ U(c_{t,o,m}, h_{t,o,m}) \right\}. \tag{3.1}$$

In equation 3.1, $c_{t,o,m}$ is the consumption, $h_{t,o,m}$ is the housing choice of an old moving agent. The utility function is linear in consumption and quadratic in housing. The parameters of the utility function are α and θ. By the linear-quadratic utility function we are able to find all of the decision rules and the value functions analytically which simplifies the computation and the analysis significantly.

$$U(c_{t,o,m}, h_{t,o,m}) = c_{t,o,m} - \alpha/2(\theta - h_{t,o,m})^2$$

$$c_{t,o,m} + [P_t(h_{t,o,m} - h_{t-1,y}) + (1 - D_{t-1} + d_{t-1})P_{t-1}h_{t-1,y}] = w$$

If an old agent moves, first he will sell his old house, then pays off the remaining mortgage debt. The seller will receive $P_t h_{t-1,y}$ as the price. The remaining debt from the old mortgage is $P_{t-1}h_{t-1}(1 - D_{t-1} + d_{t-1})$.[4] Once he is done with the old house, he purchases a new house by borrowing in the mortgage market. His mortgage payment at the end of the period is $P_t h_{t,o,m}(1 + r_t)$. We discount this payment with $1 + r_t$ to measure the real cost of mortgage payment to the agent.

The optimal housing decision of an old agent is

$$h_{t,o,m} = \theta - \frac{P_t}{\alpha}.$$

[4]The total payment he made is $P_{t-1}h_{t-1}D_{t-1}$, but $P_{t-1}h_{t-1}d_{t-1}$ of this payment is gone for the interest

The quantity of housing demanded by moving old agent increases with the taste parameter θ, decreases with the price, P_t, and decreases with the other taste parameter α. The agents who do not receive movement shock will stay in their old houses. The value of staying in their old houses is,

$$V_{stay}(H_{t-1}, r_t) = U(c_{t,o,s}, h_{t,o,s}). \tag{3.2}$$

In equation 3.2, $c_{t,o,s}$ and $h_{t,o,s}$ are consumption and housing of an old agent if he stays in his old house. The consumption staying old agents can be written as

$$c_{t,o,s} = w - \frac{P_{t-1} h_{t-1,y} D_{t-1}}{1 + r_t}.$$

If the agents stay they will pay the remaining mortgage of their house, which is $P_{t-1} h_{t-1,y} D_{t-1}$ and consume the rest.

$$h_{t,o,s} = h_{t-1,y}$$

Their housing will be the same housing they lived in during the previous 15 periods, $h_{t-1,y}$.

Young agent's problem

The problem of young agents is a little more complicated. Young agents have to consider about the next period when they're making the house purchase. The maximization problem of a young agents is

$$V_{young}(H_{t-1}, r_t) = \max_{h_{t,y}} \left\{ U(c_{t,y}, h_{t,y}) + \beta \pi E[V_{move}] + \beta(1-\pi) V_{stay} \right\},$$

where

$$c_{t,y} = w - \frac{P_t h_{t,y} D_t}{1+r_t}.$$

In the utility function of the young agents $c_{t,y}$ is the consumption and $h_{t,y}$ is the housing at time t. They have a linear-quadratic utility function, same as the old's utility function. F.O.C of the young agent's problem yields:

$$\underbrace{P_t[\frac{D_t}{1+r_t} + \beta(1-\pi)E[\frac{D_t}{1+r_{t+1}}] + \beta\pi(d_t + 1 - D_t)]}_{\text{marginal cost}} =$$
$$\underbrace{\alpha\theta\,(1+\beta(1-\pi)) - \alpha(1+\beta(1-\pi))h_{t,y} + \beta\pi E[P_{t+1}]}_{\text{marginal benefit}}.$$

Left hand side of this equation is the marginal cost of an additional unit of housing for the young agent. The present value of the first mortgage payment is $P_t\frac{R_t^f}{1+r_t}$. With $1-\pi$ probability the agent will not move and will make the second mortgage payment which has the present value of $\beta(1-\pi)E[\frac{D_t}{1+r_{t+1}}]$. With probability π the agent will move and prepay the mortgage loan. An additional unit of housing has two marginal benefits for the young household. One is the enjoy of living in the house. This affect is expressed by $\alpha\theta\,(1+\beta(1-\pi)) - \alpha(1+\beta(1-\pi))h_{t,y}$. The second benefit is the expected monetary returns, $\beta\pi E[P_{t+1}]$, in case of movement.

We can write the previous equation in the following way to get an expression for housing demand of young agents:

$$h_{t,y} = \theta + \frac{\beta\pi E[P_{t+1}] - P_t[\frac{D_t}{1+r_t} + \beta(1-\pi)E[\frac{D_t}{1+r_{t+1}}] + \beta\pi(d_t + 1 - D_t)]}{\alpha(1+\beta(1-\pi))}.$$

As it's clear from this equation, young agents' house demand depends on both current period variables and next period expectations about price and interest rates. This equation states that young agents have to know about the evolution of housing prices to optimally choose their housing level.

Market Clearing Condition

There is only one market to be cleared in this economy, which is the housing market. As mentioned before, housing supply is fixed through time, so the market clearing condition will imply the following:

$$H_{t,y} + \pi H_{t,o,m} = H - (1-\pi)H_{t-1,y}.$$

where the capital letters indicate for aggregate variables, and H is the aggregate fixed total supply of housing. The only staying agents are the $(1-\pi)$ measure of young households of the last period. This means that effective supply in period t will be $H - (1-\pi)H_{t-1,y}$. The total demand of period t comes from the moving old agents which have π measure and new young agents who just entered to the economy. So, total demand of period t will be $H_{t,y} + \pi H_{t,o,m}$.

3.3 Solution of the Model

3.3.1 Case 1: $\pi = 0$: Agents don't move

The F.O.C. simplifies to the following expression:

$$h_{t,y} = \theta + \frac{-P_t[\frac{D_t}{1+r_t} + \beta E[\frac{D_t}{1+r_{t+1}}]}{\alpha(1+\beta)}.$$

and $1 = \frac{D_t}{1+r_t} + \frac{D_t}{(1+r_t)E(1+r_{t+1})} \Rightarrow D_t = \frac{(1+r_t)E(1+r_{t+1})}{1+E(1+r_{t+1})}$. Note that since agents don't move, they don't consider the next period price. Together with the market clearing condition, the Euler equation becomes:

$$H - H_{t-1,y} = \theta + \frac{-P_t[\frac{D_t}{1+r_t} + \beta E[\frac{D_t}{1+r_{t+1}}]}{\alpha(1+\beta)}.$$

Solving for the P_t gives

$$P_t = \frac{\alpha\left(1+\beta\right)\left(\theta - H + H_{t-1,y}\right)}{[\frac{D_t}{1+r_t} + \beta E[\frac{D_t}{1+r_{t+1}}]}.$$

If the current interest rates increase current housing prices will decrease since the denominator in price function increases. But if we look at the dynamics of the prices from period t to $t+1$, housing prices in period $t+1$ will not necessarily be lower than the housing prices in period t even if r_{t+1} is larger than the r_t. In this special case, where agents do not move, initial distribution of the hosing stock will be very effective in the housing price dynamics. The dynamics of housing distribution is straight forward and can be find by the following expressions,

$$H_{t,y} = H - H_{t-1,y},$$

$$H_{t+1,y} = H_{t-1,y}.$$

The dynamics of housing distribution is deterministic (at odd periods it will take one value and at even periods it will take another) and only depends on the initial distribution (it does not depend on interest rates). Suppose that the current period is a low interest rate period with low $H_{t-1,y}$. Then in the next period, $H_{t,y}$ will be large, which means a small amount of effective housing supply. Even if interest rates in the next period happens to be high, due to lower supply of housing prices may be higher.

3.3.2 Case 2: $\pi = 1$: All agents move

The F.O.C. simplifies to the following expression:

$$h_{t,y} = \theta + \frac{\beta E[P_{t+1}] - P_t[\frac{D_t}{1+r_t} + \beta(d_t + 1 - D_t)]}{\alpha}.$$

and $1 = \frac{D_t}{1+r_t} + \frac{1-D_t+d_t}{1+r_t} \Rightarrow d_t = r_t$ and $D_t = \frac{(1+r_t)^2}{2+r_t}$. Together with the market clearing condition, the Euler equation becomes:

$$H = 2\theta + \frac{\beta E[P_{t+1}] - P_t[\frac{1+r_t}{2+r_t} + \beta(r_t + 1 - \frac{(1+r_t)^2}{2+r_t})]}{\alpha}.$$

To solve for the prices in closed form we assume that interest rates follow a 2-state Markov process. We guess the following functional form for the housing prices:

$$P(r_i, H) = \mu_i + \gamma_i H \text{ where } i \epsilon \{1, 2\}.$$

Then the Euler equation, if the economy is in state 1 is

$$H\alpha = 2\theta\alpha + \beta[\delta P(r_1, H) + (1-\delta)P(r_2, H)] - P(r_1, H)(1+\beta)\frac{r_1+1}{r_1+2}]. \tag{3.3}$$

In equation 3.3, δ is the probability of staying in the existing state. Substituting the price guess gives

$$H\alpha = 2\theta\alpha + \beta\delta\mu_1 + \beta(1-\delta)\mu_2 - \mu_1\phi(r_1) + (\beta\delta\gamma_1 + \beta(1-\delta)\gamma_2 - \gamma_1\phi(r_1))H. \tag{3.4}$$

Where $\phi(r_1) = (1+\beta)\frac{1+r_1}{2+r_1}$. We can write a similar equation if the economy is in the state 2.

$$H\alpha = 2\theta\alpha + \beta\delta\mu_2 + \beta(1-\delta)\mu_1 - \mu_2\phi(r_2) + (\beta\delta\gamma_2 + \beta(1-\delta)\gamma_1 - \gamma_2\phi(r_2))H.$$

Then we can solve for $\mu_1, \gamma_{1,}\mu_2, \gamma_2$ by equating the constants and the coefficients of H in equations 3.3 and 3.4. The equations are,

$$\begin{aligned}
0 &= 2\theta\alpha + (\beta\delta - \phi(r_1))\mu_1 + \beta(1-\delta)\mu_2, \\
0 &= 2\theta\alpha + (\beta\delta - \phi(r_2))\mu_2 + \beta(1-\delta)\mu_1, \\
0 &= (\beta\delta - \phi(r_2))\gamma_2 + \beta(1-\delta)\gamma_1 - \alpha, \\
0 &= (\beta\delta - \phi(r_1))\gamma_1 + \beta(1-\delta)\gamma_2 - \alpha.
\end{aligned}$$

Solving the equation system gives,

$$\begin{aligned}
\mu_1 &= \frac{2\alpha\theta(\beta - 2\beta\delta + \phi(r_2))}{\beta^2(2\delta-1) + \phi(r_1)\phi(r_2) - \beta\delta(\phi(r_1) + \phi(r_2))}, \\
\mu_2 &= \frac{2\alpha\theta(\beta - 2\beta\delta + \phi(r_1))}{\beta^2(2\delta-1) + \phi(r_1)\phi(r_2) - \beta\delta(\phi(r_1) + \phi(r_2))}, \\
\gamma_1 &= \frac{\alpha(\beta(2\delta-1) - \phi(r_1))}{\beta^2(2\delta-1) + \phi(r_1)\phi(r_2) - \beta\delta(\phi(r_1) + \phi(r_2))}, \\
\gamma_2 &= \frac{\alpha(\beta(2\delta-1) - \phi(r_2))}{\beta^2(2\delta-1) + \phi(r_1)\phi(r_2) - \beta\delta(\phi(r_1) + \phi(r_2))}.
\end{aligned}$$

Since now we have the housing prices as a function of interest rates we can analyze how prices react to interest rate changes. Now suppose that the economy is in state 1. The question is, how will the housing prices change if r_1 goes up? To be able to answer the question we should analyze how μ_1 and γ_1 change with r_1.

$$P(r_1, H) = \mu_1 + \gamma_1 H$$

$$\begin{aligned}
P(r_1, H) &= \frac{2\alpha\theta(\beta - 2\beta\delta + \phi(r_2))}{\beta^2(2\delta-1) + \phi(r_1)\phi(r_2) - \beta\delta(\phi(r_1) + \phi(r_2))} + \\
&\quad \frac{\alpha(\beta(2\delta-1) - \phi(r_1))}{\beta^2(2\delta-1) + \phi(r_1)\phi(r_2) - \beta\delta(\phi(r_1) + \phi(r_2))}
\end{aligned}$$

If r_1 increases $\phi(r_1)$ will increase. The denominators of μ_1 and γ_1 will increase if $\phi(r_2)$ is larger than $\beta\delta$. In this case μ_1 will decrease as interest rates increase. Similarly, γ_1 will decrease as interest rates increase, since numerator decreases and the denominator increases. But, if $\phi(r_2)$ is smaller than $\beta\delta$, μ_1 will increase and γ_1 will decrease. In this case, the response of housing prices to an interest rate increase is ambiguous.

3.3.3 Case 3 $\pi \in (0,1)$: Some agents move

Note that all agents of same generation are identical, so the aggregate law of motion for households will be the same as the individual one, i.e., we can rewrite the market clearing condition in the following way:

$$P_t f(r_t) = B + (1-\pi)H_{t-1,y} + A\beta\pi E[P_{t+1}].$$

where $A = \frac{1}{\alpha(1+\beta(1-\pi))}$, $f(r_t) = \frac{\pi}{\alpha} + A\left[D_t(\frac{1}{1=r_t} + \frac{\beta(1-\pi)}{E(1+r_{t+1})}) + \beta\pi(d_t + 1 - D_t)\right]$, $B = (1+\pi)\,\theta - H$ and the first order condition for the young agents will be:

$$H_{t,y} = \theta + A\beta\pi E[P_{t+1}] - P_t g(r_t). \tag{3.5}$$

where $g(r_t) = f(r_t) - \frac{\pi}{\alpha_o}$

Young agents' problem depends on the next period price, and equation (3.3.3) shows that price depends on the last period effective supply. So, young agents have to know the evolution of the aggregate effective supply to guess the next period price level. Using equations (3.3.3) and (3.5), we can have the following equation for the evolution of the young agents' housing level, which will explicitly give the expectation of the evolution of effective supply (Note that effective supply at period t is $H-(1-\pi)H_{t-1,y}$:

$$E(H_{t+1,y}) = k_0(r_t) + k_1(r_t)H_{t,y} + k_2(r_t)H_{t-1,y}, \tag{3.6}$$

where

$$k_0(r_t) = Bq(r_t) - \left((1+\frac{1}{\beta})\theta - H\right),$$

$$k_1(r_t) = q(r_t) + \frac{1}{\beta},$$

$$k_2(r_t) = (1-\pi)q(r_t),$$

where $q(r_t) = \frac{g(r_t)(\alpha_y+\alpha_o\beta(1-\pi))}{\beta\pi} = \frac{D_t(\frac{1}{r_t}+\frac{\beta(1-\pi)}{E[r_{t+1}]})+\beta\pi(d_t+1-D_t)}{\beta\pi}$.A simple guess for $H_{t,y}$ is:

$$H_{t,y} = a(r_t) + b(r_t)H_{t-1,y}.$$

If we substitute this in (3.6) then we'll get:

$$E[a(r_{t+1})] + E[b(r_{t+1})]H_{t,y} = k_0(r_t) + k_1(r_t)H_{t,y} + k_2(r_t)H_{t-1,y},$$

which can be simplified as:

$$\begin{aligned} H_{t,y} &= \frac{k_0(r_t) - E[a(r_{t+1})] + k_2(r_t)H_{t-1,y}}{E[b(r_{t+1})] - k_1(r_t)} \\ &= a(r_t) + b(r_t)H_{t-1,y}. \end{aligned}$$

So, we have to solve the following functional equational system:

$$b(r_t) = \frac{k_2(r_t)}{E[b(r_{t+1})] - k_1(r_t)},$$

$$a(r_t) = \frac{k_0(r_t) - E[a(r_{t+1})]}{E[b(r_{t+1})] - k_1(r_t)},$$

and price can be found from the following equation:

$$P_t = \frac{\alpha}{\pi}\left[\theta\pi - H + (1-\pi)H_{t-1,y} + a(r_t) + b(r_t)H_{t-1,y}\right].$$

Since we have N states for the interest rates in this economy, then we'll have N corresponding equations for each function a and b. Let b_s represent the corresponding b function for interest rate $r_s \in \mathbb{R}$. And a_s represents the corresponding a function for interest rate $r_s \in \mathbb{R}$. Then, we can rewrite our functional equation system in the following way:

$$b_s = \frac{k_{2s}}{E(b|s) - k_{1s}} \quad \forall s \in \{1, ..., N\},$$

where k_{is} is the k_i ($i \in \{0, 1, 2\}$) corresponding to the state where $r_t = r_s$, $s \in \{1, ..., N\}$ and $E(b|s)$ is the expected value of next period b given that current state is i. Shortly:

$$b \times (P * b) - bk_1 = k_2, \tag{3.7}$$

where P is the transition matrix for the N-state Markov process and $*$ is the matrix multiplication whereas $\times$ is the elementwise multiplication. Given that, we have the solution for the above N equations N unknowns nonlinear system for b, we can apply the same to find the function a:

$$a_i = \frac{k_{0s} - E(a|s)}{E(b|s) - k_{1s}} \quad \forall s \in \{1, ..., N\},$$

or more shortly, in matrix form:

$$a \times (P * b - k_1) = k_0 - P * a. \tag{3.8}$$

and the corresponding law of motion for price and house demand will be:

$$H_{t,y}(s, H_{t-1,y}) = a_s + b_s H_{t-1,y},$$

$$P_t(s, H_{t-1,y}) = c_s + d_s H_{t-1,y}, \tag{3.9}$$

for $\forall s \in \{1, ..., N\}$ denoting for the state of the interest rate, and

$$c_s = \frac{\alpha(\theta\pi - H + a_s)}{\pi},$$

$$d_s = \alpha \frac{b_s + 1 - \pi}{\pi}.$$

Knowing the previous period young agents' housing level, equation (3.9) will determine equilibrium housing price level as a function of current period interest rate. In case of two states for interest rates, equation (3.7) becomes two simultaneous quadratic equations in two unknowns. The solutions, which could be obtained by using Mathematica, are complex and long so that we don't want to print them here. Moreover, complexity of solutions don't allow us to make comparative statistics.

3.4 Calibration

To simulate the model, we need to choose values for parameters in the model. Since the model is a two-period OLG model, total life time is summarized in two periods. Assuming consumers enter to the housing market at the age of 30 and lives till the age of 60, each time period in the model will correspond to 15 years.

Since the model period is 15 years (which means interest rates will stay constant for 15 years) it is hard to calibrate the interest rates directly. As the main objective of the paper is to emphasize a mechanism in the housing market, the numerical results have a secondary importance. However, we perform robustness tests, by solving the model for different combinations of parameters, to check how our results are affected by our specific parameters.

We assume the stochastic process in the model is the real interest rate on a 15 year fixed-rate mortgage. This fits into our model, because when old agents borrow in the market we assumed they could get loans with the period interest rate. To find the real rate on a 15 year fixed-rate mortgage, we simply subtract the inflation in that year from the nominal rate.[5] The average of real 15 year fixed-rate mortgage during 1975-2005 period is 5 percent. Then we estimate an AR(1) process of these real rates which gives the yearly autocorrelation coefficient (ρ) as 0.88. The coefficient of variation (σ) of real 15 year fixed-rate mortgage is around 0.5. Finally, we use Hussey and Tauchen (1991) method to approximate the AR(1) process with a 100 state Markov chain. We iterate this Markov process 15 times to obtain 15 year transition probabilities.

The second important parameter for our model is the moving probability of old households. PSID data shows that 5.44 percent of the respondents moved during the last year (Cocco (2000)). The corresponding number for 15 years would be 56 percent. We calibrate θ to 1 to have risk aversion of 2 for the housing consumption.

[5]This is equivalent to assuming current period's inflation is the best predictor of the future inflation.

Table 3.1: Calibrated Parameters

Description	Parameter	Value
Length of a period		15 years
Number of states in the Markov Process	N	100
Average of the real interest rates (1975-2006)	μ	5%
Annual autocorrelation of the interest rates (1975-2006)	ρ	0.88
Coefficient of variation of interest rates (1975-2006)	σ	0.5
Movement probability	π	0.56
Housing supply	H	1
Discount factor	β	$(1/1.05)^{15}$
A parameter in the utility function	α	1
A parameter in the utility function	θ	1

Note. –Coefficient of variation is calculated as standard deviation divided by mean

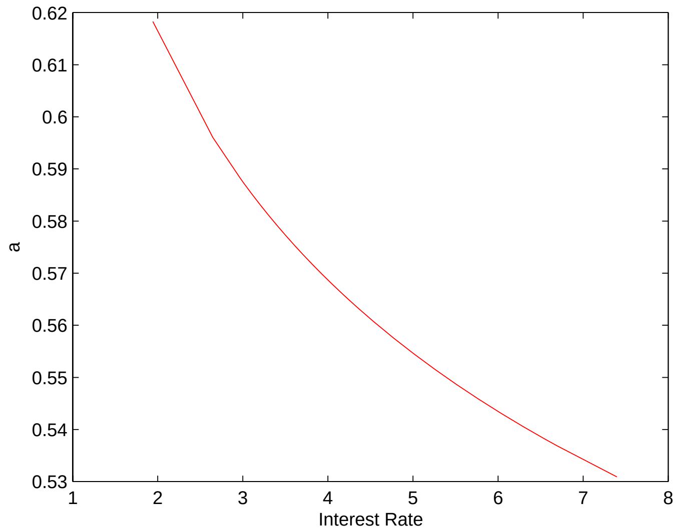
Aggregate housing supply is normalized to1. We calibrate β, the discount factor to $1/(1.05)^{15}$ (1.05 is the average yearly gross interest rate).

Since we do not have any prior information about the value of α and the value of α does not affect our results much we assume α is 1. Table 1 summarizes the parameter values for our benchmark model.

3.5 Results

We simulate the model 1,000 times assuming the economy lasts for 100 periods. The results show that for most of the times (around 95%), interest rates and housing prices are negatively related. However, about 1% of the fluctuations in housing prices and interest rates had a positive relation. For approximately 4% of the times, when the interest rates did not move, house prices still moved which shows the importance of the effective supply. The most effective parameter on the relationship between interest rates and housing prices is the moving probability,

Figure 3.5: The Constant Term in the Housing Demand Function



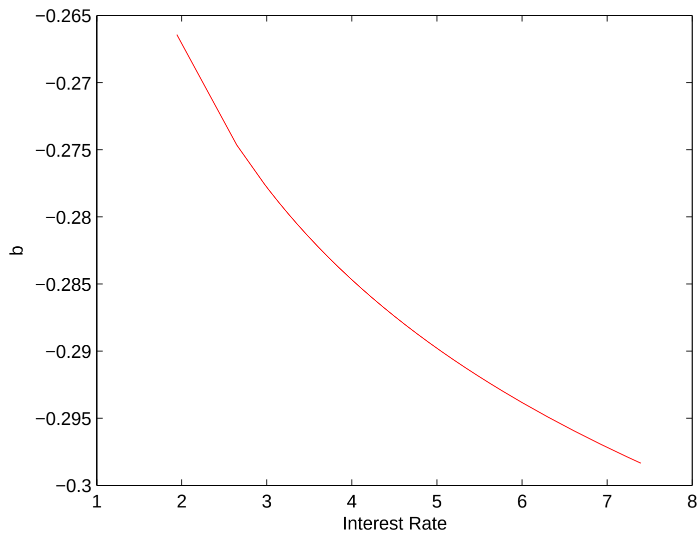
π. As π decreases, the influence of housing distribution becomes larger and the percentage of times when interest rates and housing prices comove increases in our simulations.

To assess the change in housing prices more closely, one has to pay attention to the evolution of young household's housing stock, $H_{t,y}$. Figures 3.5 and 3.6 show the evolution of young agents' house demand as a function of interest rates. Actually, young households' house demand function, $H_{t,y}$ consists of two functions of interest rates, $a(r_t)$ and $b(r_t)$. In the function, $a(r_t)$ is the constant term depending on interest rates and $b(r_t)$ is the coefficient of last period's house demand of young agents

$$H_{t,y} - H_{t-1,y} = a(r_t) + (b(r_t) - 1)\, H_{t-1,y}$$

As it is clear from the figure, $a(.)$ is positive and $b(.) - 1$ is negative. De-

Figure 3.6: The Coefficient of $H_{t-1,y}$ in the House Demand Function



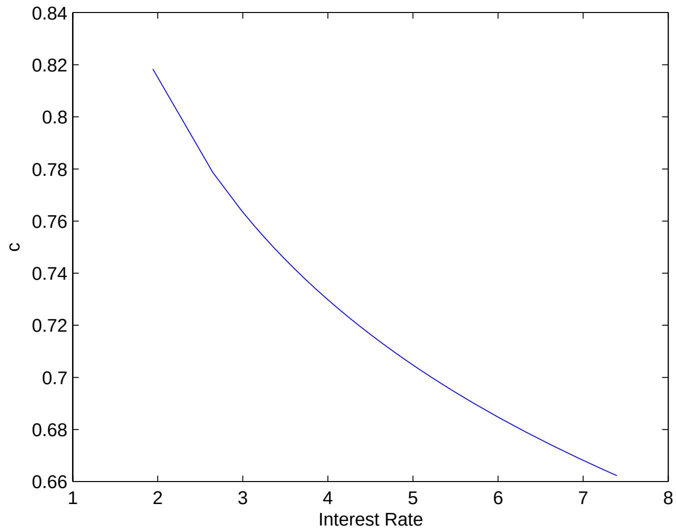
pending on the magnitude of the last period demand, young agents may increase or decrease their demand as a response to an increase in interest rate. Thus, it's not possible to definitely know whether young households demand less or more as interest rates increase without knowing the effective supply. This, in turn, effects the effective supply for the next period. Ambiguity in the demand of young agents as a function of interest rates results an ambiguity in effective supply for the next period. Note that effective supply for period $t+1$ is

$$H^e_{t+1} = H - (1-\pi)H_{t,y} = H - (1-\pi)(a(r_t) + b(r_t)H_{t-1,y}).$$

and effective supply for period t is

$$H^e_t = H - (1-\pi)H_{t-1,y}.$$

Figure 3.7: The Constant in the Housing Price Function



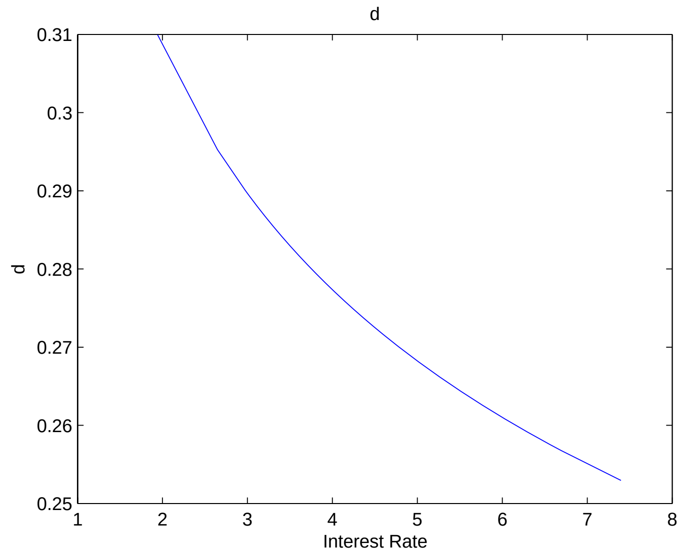
So the change in effective supply from period t to period $t+1$ is

$$dH^e_{t+1} = H^e_{t+1} - H^e_t = -(1-\pi)a(r_t) - (1-\pi)(b(r_t)-1)H_{t-1,y} = f(r_t) + g(r_t)H_{t-1,y}.$$

where $f(r_t) = -(1-\pi)a(r_t)$ and $g(r_t) = -(1-\pi)(b(r_t)-1)$. While $f(.)$ is negative, $g(.)$ is positive. This causes the ambiguity on the change of effective supply. So, depending on the magnitude of young agents' last period house demand, effective supply may increase or decrease as interest rates increase. For high values of $H_{t-1,y}$, effective supply increases while for low values of $H_{t-1,y}$, effective supply decreases.

Figures 3.7, and 3.8 show the equilibrium housing prices as a function of interest rates. We guessed the price function as $P_t = c(r_t) + d(r_t)H_{t-1,y}$. Both functions, $c(.)$, and $d(.)$ are decreasing functions of interest rates. This shows that as interest rates increase demand decreases, which means demand shifts to the left. We also know that effective supply moves ambiguously as interest rates increase.

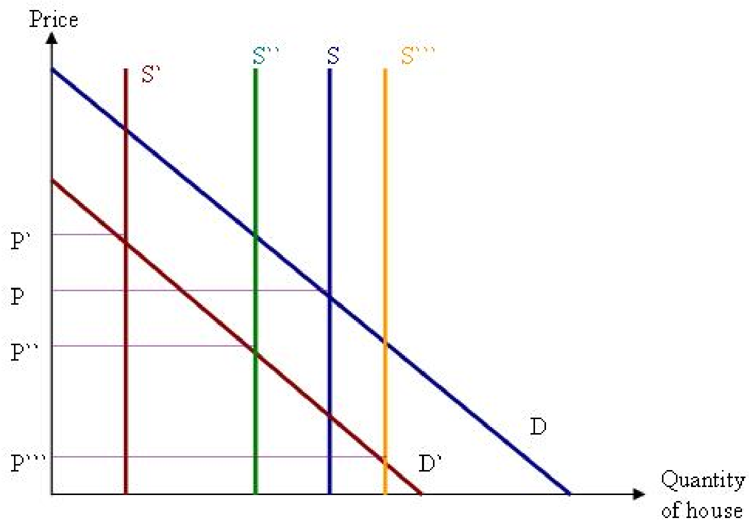
Figure 3.8: The Coefficient of $H_{t-1,y}$ in the Housing Price Function



Hence, price moves ambiguously also depending on the magnitude of the shifts of demand and effective supply.

As interest rates increase, price function tells that demand will decrease. However, direction and magnitude of the shift in the effective supply is not clear. It may go in both directions, and depending in the direction and magnitude of the shift, equilibrium prices may increase or decrease. If the previous period's house demand of young agents is low enough, then effective supply will shift to the left sufficiently and equilibrium price will increase. However, if it's not low enough, then effective supply may shift to the left slightly or to the right, which, in turn, results a decrease in equilibrium price level. Thus, the effect of an interest rate on housing prices is ambiguous and depends on the distribution of house stock between agents.

Figure 3.9: Equilibrium House Price: The Effects of Supply and Demand



3.6 Conclusion

In this paper we have analyzed the behavior of housing prices in response to fluctuations in interest rates by using a simple overlapping generations framework. We show that housing prices depend on both interest rates and effective housing supply. Effective housing supply in the market highly depends on the distribution of housing among agents. If young households have the biggest portion of the housing stock, this means, next period's effective supply will be low (due to exogenous movement shock), which in turn results an ambiguous change in housing prices in response to an increase in interest rates. Similarly, if young agents hold a smaller fraction of total housing, next period's effective supply will be more and housing prices will move ambiguously if interest rates decrease.

Appendices

Appendix A

Chapter 1 Appendix

A.1 Existence of Equilibrium - A Simplified Model

To better address the issues involved with the existence of the equilibrium, I now modify the model to a simpler version. Assume that there are only two periods, there is no saving and households are risk-neutral. Households are hand-to-mouth agents. I abstract from the first period choice problem, because the issues involved with the existence of equilibrium are relevant for the households who are offered contracts. So, in the first period, I assume that households are all purchasers. Income follows random walk: $\theta_t = \theta_{t-1} + \varepsilon$, where ε is mean-zero normal random variable with variance σ_ε^2. In the second period, after realizing the income shock, homeowner can either stay in the current house, then she has to make her mortgage payments and she enjoys the utility from being a homeowner, and in the final period she receives income from selling the house. If she defaults, she becomes a renter and enjoys the utility of being a renter: there is no other cost of default and no rental price. Suppose there is no selling option. From the same consumption, homeowners get $\gamma > 1$ times higher utility than the renter. I skip the problem of a renter in the second period, because I assume that she has no housing option in the second period. She basically has to stay as a renter. Moreover, set the price of a house to $p_h = 1$.

Using equation (1.10), I get mortgage payment to be $m = \frac{1+r_m}{2+r_m}$ when the mortgage interest rate is r_m. The last period utility of a type i stayer becomes $u_2^{hh}(\theta, i; d, r_m) = \gamma(\theta - md) + \beta_i$. Similarly, the utility of a defaulter becomes $u_2^d(\theta, i) = \theta$. Since the second-period beginning utility of a homeowner is $u_2^h(\theta, i; d, r_m) = \max\left\{u_2^{hh}(\theta, i; d, r_m), u_2^d(\theta, i)\right\}$, I get

$$u_2^h(\theta, i; d, r_m) = \left\{\begin{matrix} \gamma(\theta - md) + \beta_i & \text{if } \theta \geq \theta_i^* \\ \theta & \text{if } \theta < \theta_i^* \end{matrix}\right\}$$

where $\theta_i^* = \frac{\gamma md - \beta_i}{\gamma - 1}$.

Then first period utility of a purchaser becomes the following:

$$\begin{aligned} u_1^{rh}(\theta, i) &= \max_{(d, r_m) \in \Upsilon(\theta)} \{u(\theta, i; d, r_m)\} \qquad \text{where} \\ u(\theta, i; d, r_m) &= \gamma(\theta - md - (1-d)) + \beta_i \int^{\theta_i^* - \theta} (\theta + \varepsilon)\, dF(\varepsilon) + \beta_i \int_{\theta_i^* - \theta} [\gamma(\theta + \varepsilon - md) + \beta_i]\, dF(\varepsilon) \\ &= \gamma d (1 - m(1 + \beta_i(1 - F(\theta_i^* - \theta)))) + \kappa_i(\theta, \theta^*) \qquad \text{where} \qquad (\text{A.1}) \\ \kappa_i(\theta, \theta_i^*) &= \gamma(\theta - 1) + \beta_i \int^{\theta_i^* - \theta} (\theta + \varepsilon)\, dF(\varepsilon) + \beta_i \int_{\theta_i^* - \theta} (\gamma(\theta + \varepsilon) + \beta_i)\, dF(\varepsilon) \end{aligned}$$

Similarly, the value of a mortgage contract (d, r_m) offered to type θ household becomes:

$$\begin{aligned} v(d, r_m; \theta, i) &= md + \frac{1}{1+r} \int_{\theta_i^* - \theta} md dF(\varepsilon) \\ &= md\left(1 + \frac{1}{1+r}(1 - F(\theta_i^* - \theta))\right) \end{aligned}$$

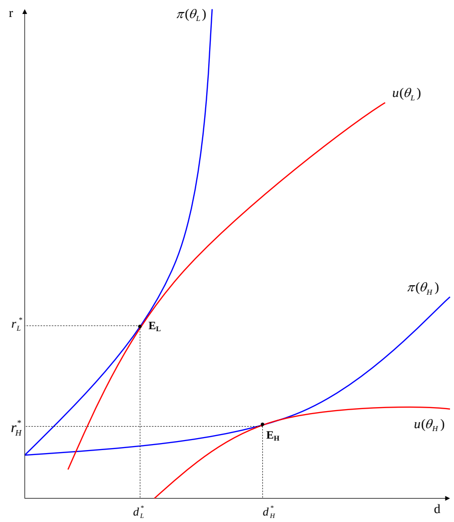
Then no-arbitrage condition simply implies that the profit to the lender is equal to zero:

$$\begin{aligned} \pi(d, r_m; \theta) &= md\left(1 + \frac{1}{1+r}(1 - F(\theta_i^* - \theta))\right) - d \qquad (\text{A.2}) \\ m &= \frac{1}{1 + \frac{1}{1+r}(1 - F(\theta_i^* - \theta))} \in \left[\frac{1+r}{2+r}, 1\right) \end{aligned}$$

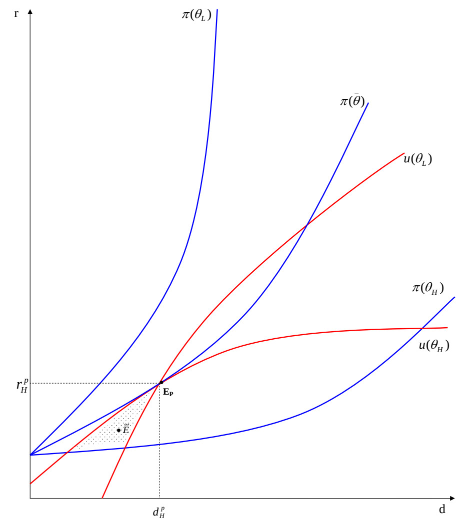
I now illustrate the equilibrium to the above economy in a phase-diagram of contract space: (d, r_m). Using equations $(A.1)$ and $(A.2)$, I can construct the indifference curve and the iso-profit curve for the household. Figure A.1(a) shows typical indifference curves. For a good type, it is denoted by $u(\theta_H)$, and for a bad type, it is denoted by $u(\theta_L)$. The iso-profit curve, for a good type, is denoted by $\pi(\theta_H)$, and for a bad type it is denoted by $\pi(\theta_L)$. The indifference curves yield higher utility as they shift to the right, so the equilibrium to the symmetric information economy is the point where the indifference curve is tangent to the iso-profit curve for each type. So, (d_L^*, r_L^*) and (d_H^*, r_H^*) are the equilibrium to the SI economy.

Problem of Existence of Nash equilibrium . In the asymmetric information economy, the types are not observable. So, both contracts are available for the households. Clearly, the contract designed for the good type gives a higher utility for the bad type. If both contracts are offered, both types choose the contract (d_H^*, r_H^*) and the lender makes negative profit. So, the equilibrium in the SI economy is not sustainable in the AI economy. In the literature two types of contracts are suggested as a potential equilibrium to the AI economy facing the adverse selection problem: *pooling contracts* and *separating contracts*. A pooling contract pools both types into a single contract, while a separating contract is able to separate the types. As it is analyzed in Rothschild and Stiglitz (1976), a pooling contract cannot be a Nash equilibrium to the AI economy. The intuition is simple. As it is seen in Figure A.1(b), point E_p is a candidate pooling equilibrium. It is on the iso-profit curve for

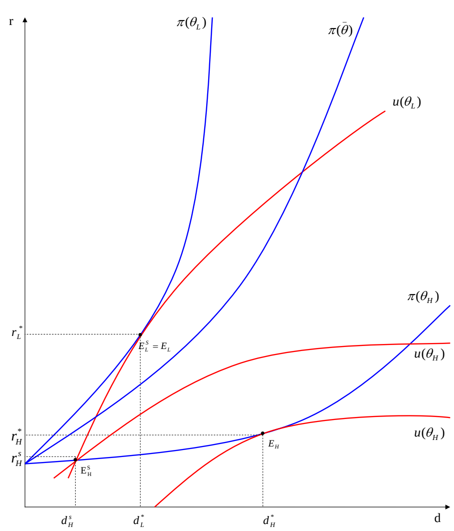
Figure A.1: Illustration of Equilibrium



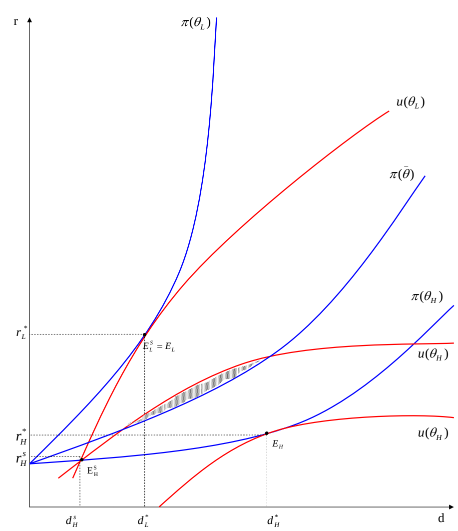
(a) Symmetric Equilibrium



(b) No Pooling Equilibrium



(c) Separating Equilibrium



(d) No Separating Equilibrium

the pool. However, a point in the dotted region, like point $\tilde{E}$ cream-skims only the good types and since it is on the left of the iso-profit curve for the good type, it yields a positive profit to the lender. So, such a deviation is profitable for the lender which results a pooling contract not to be an equilibrium contract. Although it is not guaranteed, a separating equilibrium may exist. Figure A.1(c) shows a candidate separating equilibrium. Such a separating contract is called *least-cost separating* contract. Contracts E_H^S and E_L^S separate both types. Either type finds it not optimal to choose the contract designed for the other type and both contracts make zero-profit. Since the iso-profit curve for the pool is to the left of the indifference curve for the good type, good types never prefer a pooling contract. So, pooling contracts cannot break the separating equilibrium. However, if the proportion of the good types is sufficiently high, then it is possible to break the candidate separating equilibrium by either a pooling contract or another separating contract which relies on cross-subsidization. Figure A.1(d) shows how a pooling contract breaks the separating equilibrium. Any point in the shaded region is a profitable deviation for the lender. It attracts both types and yields a positive profit. In such an environment no Nash equilibrium exists. Note that the least-cost separating contract can only be broken by contracts which rely on cross-subsidization. However, such contracts can always be broken with a separating contract which cream-skims only the good types.

Anticipatory Equilibrium. It is possible to support a pooling contract by modifying the equilibrium concept as in Wilson (1977). He proposes the anticipatory equilibrium concept, where any deviation which would become unprofitable if the initial contracts are withdrawn are not allowed. Note that a pooling contract is not a Nash equilibrium, because it can broken by cream-skimming the good types as it is seen in Figure A.1(b). However, such a deviation will be unprofitable if the others can withdraw their contracts. In the current example, it means that a deviation like point $\tilde{E}$ in Figure A.1(b) becomes unprofitable if the initial contract E_p is withdrawn. Because it'll attract both types and results a negative profit. So, such deviations do not threat the equilibrium and the pooling contract survives as an equilibrium.

A pooling contract attracts both types, so the no-arbitrage condition should account for this fact. Any point on $\pi\left(\bar{\theta}\right)$ in Figure A.1(b) is a candidate for equilibrium. However, the equilibrium should be deviation-free, where the set of deviations is restricted by the new equilibrium concept. The condition for the equilibrium to be deviation-free further restricts the equilibrium to a unique point at which the good type receives the maximum utility, i.e. it is the point where the indifference curve for the good type is tangent to the pooling iso-profit curve. Other pooling contracts where the good type receives lower utility are not equilibrium, because such a contract can be easily broken by offering a contract which gives slightly higher utility for the good type and lower utility for the low type but still above the pooling iso-profit curve.

At this point, it is also worthwhile to mention that this equilibrium can also be supported as a perfect Bayesian equilibrium of a modified game. Hellwig (1987) modifies the game in these types of screening games and shows that the above defined pooling contract is the unique stable perfect Bayesian equilibrium of the modified game. The modification is the follows. Suppose in the first stage, as in the original game, lenders offer contracts. However before households choose from these contracts, different than the original game, Hellwig introduces a second stage where the lenders can see the other lenders' contracts and are allowed to withdraw their contracts. Finally in the third stage, households choose from the remaining contracts and contracts are executed. This game is clearly in the spirit of the Wilson's Anticipatory equilibrium concept and supports the pooling contract as equilibrium contract.

Reactive Equilibrium. Riley (1979) offers another equilibrium concept, *Reactive Equilibrium*, so that the least-cost separating contract survives as an equilibrium. The only difference of the Reactive equilibrium from the Nash equilibrium is that it does not allow any deviations that would become unprofitable if they led the other lenders to react by adding new contracts. Note that the least-cost separating contract can only be broken by a contract which depends on cross-subsidization. Any contract with cross-subsidization can be broken with another separating contract by cream-skimming the good types and yields the lender offering such a contract only the low types and consequently negative profit.

Here, it is useful to mention that it is also possible to model the above economy as a signalling game rather than a screening game. In signalling games, the uninformed player moves first. In my economy, it corresponds to the following game. In stage one, households move and choose a loan amount. After observing the loan amount and other characteristics of the household, in the next and last stage, lenders compete by offering mortgage interest rate. The last stage of the game, due to perfect competition, is simple. Basically, lenders set the zero-profit mortgage interest rate corresponding to the observable and loan amount. In the signalling games, I have to deal with the beliefs. In my environment, the households can signal their types by choosing the loan amount. Then the lenders have to form their beliefs on the type of the households based on this signal. The common equilibrium concept used in the literature is the perfect Bayesian equilibrium. Wherever possible, the beliefs are formed using the household's strategies in a bayesian fashion. However, the lenders also have to assign beliefs for off-the-equilibrium strategies. This feature of the model gives potential multiplicity of the equilibria. It is possible to have a continuum of pooling and separating equilibria. Nevertheless, using the equilibrium refinements, specifically intuitive criterion, introduced in Cho and Kreps (1987), and universal divinity, introduced in Banks and Sobel (1987), the unique outcome of the game becomes the least-cost separating equilibrium.

Appendix B

Chapter 2 Appendix

B.1 Proofs

Proof. [Lemma 2.3.1] Rewrite equation (2.6) using equation (2.4):

$$r\Omega(w) = u\left(w + b\right) + \alpha g(w) \tag{B.1}$$

where

$$g(w) \equiv \int \max\left\{\frac{u\left(w + w_2\right)}{r}, \Omega(w_2) - \Omega(w), 0\right\} dF(w_2).$$

We construct the proof by contradiction. Let us assume $\Omega'(w) \leq 0$. From equation (B.1), $r\Omega'(w) - u'\left(w + b\right) = \alpha g'(w)$. Then, $g'(w) \leq \frac{-u'(w+b)}{\alpha} < 0$. If Ω is a decreasing function, then $\Omega(w_2) - \Omega(w)$ is increasing functions of w. This means that all the terms inside the max operator of the g function are increasing, which implies that g is an increasing function, i.e., $g'(w) \geq 0$, for each w, which is a contradiction. Thus $\Omega'(w) > 0$. □

Proof. [Proposition 1] From the definition of the worker-searcher reservation wage when the quit option is not exercised, the couple has to be indifferent between both partners being employed and only one being employed. This means that ϕ has to satisfy: $\Omega\left(w_1\right) = T\left(w_1, \phi\left(w_1\right)\right)$. We conjecture that the quitting option is never exercised. This allows us to disregard the second term inside the max operator in

(2.6) Using this last equality, equations (2.6) and (2.4) and the fact that workers are risk-neutral, the equation characterizing $\phi(w_1)$ becomes

$$\phi(w_1) = b + \frac{\alpha}{r} \int_{\phi(w_1)} [w_2 - \phi(w_1)] \, dF(w_2) \, .$$

It is clear that $\phi(w_1)$ does not depend on w_1, and the above equation is exactly equation (2.3) of the single-search problem. So, $\phi(w_1) = w^* = \hat{w}$. As a result, $\phi^{-1}(w_1) = \infty$, confirming the guess that the employed spouse never quits, since quits occur only if the wage offer w_2 exceeds $\phi^{-1}(w_1)$.

Now we will establish that $w^{**} = w^*$. Equation (2.7) implies that

$$r\Omega(w^{**}) = rU = 2b + \frac{2\alpha}{r} \int_{w^{**}} r\Omega'(w) [1 - F(w)] \, dw. \tag{B.2}$$

At $w_1 = w^*$, we can rewrite equation (2.6) in the following way

$$r\Omega(w^*) = w^* + b + \frac{\alpha}{r} \int_{w^*} r\Omega'(w) [1 - F(w)] \, dw. \tag{B.3}$$

Subtracting (B.3) from (B.3) multiplied by 2 and using the fact that $r\Omega(w^*) = 2w^*$ yields

$$r [\Omega(w^*) - \Omega(w^{**})] = \frac{2\alpha}{r} \int_{w^*}^{w^{**}} r\Omega'(w) [1 - F(w)] \, dw$$

Since Ω is strictly increasing, $w^* \geq w^{**}$ implies $\Omega(w^*) \geq \Omega(w^{**})$, but then the above equation in turn implies that $w^{**} = w^*$. Thus, the quit option will never be exercised. □

Proof. [Proposition 2] It is useful to begin by first proving part (ii) of the proposition. At the reservation wage for the worker-searcher couple we have $T(w_1, \phi(w_1)) =$

$\Omega(w_1)$. Let us begin by conjecturing that there is a value w_1 above which the employed worker never quits his job. Therefore in this range we do not have to worry about the second argument of the max operator in (2.6). Using equations (2.6) and (2.4), we get

$$\begin{aligned}
u\left(w_1+\phi\left(w_1\right)\right)-u\left(w_1+b\right) &= \frac{\alpha}{r}\int_{\phi(w_1)}\left[T\left(w_1,w_2\right)-\Omega\left(w_1\right)\right]dF\left(w_2\right)\\
&= \frac{\alpha}{r}\int_{\phi(w_1)}\left[u\left(w_1+w_2\right)-u\left(w_1+\phi\left(w_1\right)\right)\right]dF\left(w_2\right)\\
-\rho u\left(w_1\right)\left(u\left(\phi\left(w_1\right)\right)-u\left(b\right)\right) &= -\rho u\left(w_1\right)\frac{\alpha}{r}\int_{\phi(w_1)}\left[u\left(w_2\right)-u\left(\phi\left(w_1\right)\right)\right]dF\left(w_2\right)\\
u\left(\phi\left(w_1\right)\right)-u\left(b\right) &= \frac{\alpha}{r}\int_{\phi(w_1)}\left[u\left(w_2\right)-u\left(\phi\left(w_1\right)\right)\right]dF\left(w_2\right)
\end{aligned}$$

where the second line uses the definition of ϕ and the third line uses the CARA assumption $u\left(c_1+c_2\right)=-\rho u\left(c_1\right)u\left(c_2\right)$. Note that this is exactly the same equation characterizing the reservation wage of the single unemployed (equation 2.3). So, we can conclude that in this region $\phi\left(w_1\right)=w^*$. Moreover, $\hat{w}$ is a singleton since ϕ crosses the 45 degree line only once, so $\hat{w}=w^*$. If $w_1\geq w^*$, the employed spouse does not quit the job, since $\phi^{-1}\left(w_1\right)=\infty$ and quits take place if $w_2>\phi^{-1}\left(w_1\right)$, which confirms the initial guess.

Now that we have characterized the part of the ϕ function for $w_1\geq w^*$, we now turn to the part below w^*. Here we have $\phi\left(w_1\right)=w_1$ and quits are possible as long as $w^{**}<w^*$ as stated in part (i). This is what we prove next. When the wage of the employed agent is equal the double indifference point $\hat{w}$, we have $r\Omega\left(\hat{w}\right)=u\left(2\hat{w}\right)$ from (2.10). Subtracting (2.5) from this equation, we get

$$r\left[\Omega\left(\hat{w}\right)-\Omega(w^{**})\right]=u(2\hat{w})-u(2b)-2\alpha\int_{w^{**}}\left[\Omega\left(w\right)-\Omega(w^{**})\right]dF\left(w\right)$$

Evaluate equation (2.6) at $\hat{w}$, and note that $T(\hat{w}, w) = \Omega(w)$ to arrive at

$$u(2\hat{w}) = u(\hat{w} + b) + \alpha \int_{\hat{w}} [\Omega(w) - \Omega(\hat{w})] \, dF(w) .$$

Combining these two equations yields:

$$\begin{aligned} r[\Omega(\hat{w}) - \Omega(w^{**})] &= 2u(\hat{w} + b) - u(2\hat{w}) - u(2b) - 2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w)[1 - F(w)] \, dw \\ &= \rho[-2u(\hat{w}) u(b) + u(\hat{w}) u(\hat{w}) + u(b) u(b)] - 2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w)[1 - F(w)] \, dw \\ &= \rho[u(\hat{w}) - u(b)]^2 - 2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w)[1 - F(w)] \, dw, \end{aligned}$$

where the second line again uses the CARA assumption. Suppose now, ad absurdum, that $w^{**} \geq \hat{w}$, then clearly, $LHS \leq 0$. But since obviously $\hat{w} > b$, and $2\alpha \int_{w^{**}}^{\hat{w}} \Omega'(w)(1 - F(w)) dw \leq 0$, we have that $RHS > 0$, a contradiction. Thus, $w^{**} < \hat{w} = w^{*}$. □

Proof. [Proposition 3] We begin with part (ii). The proof proceeds by conjecturing that there is a value w_1 above which the employed spouse never quits his job and showing that this leads to a contradiction. For quit not to occur beyond a wage threshold, we need to have $\phi' \leq 0$ in that region since ϕ^{-1} would also be decreasing in this case. Indeed, suppose that the couple draws a wage $w_2 > \phi(w_1)$. The reservation wage of the employed spouse upon quitting would be $\phi^{-1}(w_2) < w_1$, where w_1 is the current wage, which would not justify quitting. Then, the equation characterizing $\phi(w_1)$ becomes, as usual,

$$u(w_1 + \phi(w_1)) - u(w_1 + b) = \frac{\alpha}{r} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))] \, dF(w_2) .$$

Consider a wage level $\tilde{w}_1 > w_1$. Then, rearranging, we get

$$\begin{aligned}
1 &= \frac{\alpha}{r} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\
&\leq \frac{\alpha}{r} \int_{\phi(\tilde{w}_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(\tilde{w}_1))}{u(w_1 + \phi(\tilde{w}_1)) - u(w_1 + b)} \right] dF(w_2) \\
&< \frac{\alpha}{r} \int_{\phi(\tilde{w}_1)} \left[\frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)} \right] dF(w_2) \\
&= 1,
\end{aligned}$$

which is a contradiction. The first weak inequality comes from the fact that $\phi' \leq 0$. The second strict inequality holds because of the DARA utility assumption (Pratt, 1964, Theorem 1): if u is in the DARA class, for any $k > 0$ and m, n, p, q such that $p < q \leq m < n$, we have

$$\frac{u(n) - u(m)}{u(q) - u(p)} < \frac{u(n+k) - u(m+k)}{u(q+k) - u(p+k)}. \tag{B.4}$$

Here $p = w_1 + b$, $q = m = w_1 + \phi(\tilde{w}_1), n = w_1 + w_2$ and $k = \tilde{w}_1 - w_1$.

The contradiction shows that the conjecture $\phi' \leq 0$ is not correct. Therefore $\phi(w_1)$ must be strictly increasing in w_1 over this range. In this case, the employed spouse may find it optimal to quit the job if the unemployed receives a sufficiently high wage offer, i.e., whenever $w_2 > \phi^{-1}(w_1)$. This leads us to another conjecture: for any $w_1 < \hat{w}$, $\phi(w_1) = w_1$ and for $w_1 \geq \hat{w}$, $0 < \phi' < 1$. Then, the equation characterizing $\phi(w_1)$ becomes

$$\begin{aligned}
1 = &\frac{\alpha}{r} \int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2) \\
&+ \frac{\alpha}{r} \int_{\phi^{-1}(w_1)} \left[\frac{r\Omega(w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \right] dF(w_2)
\end{aligned}$$

This conjecture implies that for any w_1, $\Omega(w_2) > T(w_1, w_2) = u(w_1 + w_2)$ for all $w_2 > \phi^{-1}(w_1)$. So, for any w_1, we can find an $\varepsilon > 0$, sufficiently small, such that $\int_{\phi^{-1}(w_1)} r\Omega(w_2)\, dF(w_2) \geq \int_{\phi^{-1}(w_1)} u(w_1 + w_2 + \varepsilon)\, dF(w_2)$. Then, for such an $\varepsilon > 0$, using the DARA property in equation $(B.4)$, we get

$$\begin{aligned}
1 &= \frac{\alpha}{r}\int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}\right] dF(w_2) \\
&\quad + \frac{\alpha}{r}\int_{\phi^{-1}(w_1)} \left[\frac{r\Omega(w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}\right] dF(w_2) \\
&< \frac{\alpha}{r}\int_{\phi(w_1)}^{\phi^{-1}(w_1)} \left[\frac{u(w_1+w_2+\varepsilon) - u(w_1+\varepsilon+\phi(w_1))}{u(w_1+\phi(w_1)+\varepsilon) - u(w_1+b+\varepsilon)}\right] dF(w_2) \\
&\quad + \frac{\alpha}{r}\int_{\phi^{-1}(w_1)} \left[\frac{r\Omega(w_2) - u(w_1+\phi(w_1)+\varepsilon)}{u(w_1+\phi(w_1)+\varepsilon) - u(w_1+b+\varepsilon)}\right] dF(w_2)
\end{aligned}$$

Moreover, since

$$\begin{aligned}
1 = {} & \frac{\alpha}{r}\int_{\phi(w_1+\varepsilon)}^{\phi^{-1}(w_1+\varepsilon)} \left[\frac{u(w_1+w_2+\varepsilon) - u(w_1+\phi(w_1+\varepsilon)+\varepsilon)}{u(w_1+\varepsilon+\phi(w_1+\varepsilon)) - u(w_1+b+\varepsilon)}\right] dF(w_2) \\
& + \frac{\alpha}{r}\int_{\phi^{-1}(w_1+\varepsilon)} \left[\frac{r\Omega(w_2) - u(w_1+\phi(w_1+\varepsilon)+\varepsilon)}{u(w_1+\phi(w_1+\varepsilon)+\varepsilon) - u(w_1+b+\varepsilon)}\right] dF(w_2)
\end{aligned}$$

, then $\phi(w_1) < \phi(w_1 + \varepsilon)$ for $\varepsilon > 0$ sufficiently small, implying $\phi' > 0$.

We now prove part (i) of the proposition. We first show that $w^{**} < \hat{w}$. Subtracting equation (2.5) from equation (2.10) we obtain

$$r\ \left[\Omega(\hat{w}) - \Omega(w^{**})\right] = u(2\hat{w}) - u(2b) - 2\alpha \int_{w^{**}} \left[\Omega(w) - \ \Omega(w^{**})\right] dF(w)\,. \quad \text{(B.5)}$$

At $w_1 = \hat{w}$, we can write equation (2.6) as

$$r\Omega(\hat{w}) = u(\hat{w} + b) + \alpha \int_{\hat{w}} \left[\Omega(w) - \Omega(\hat{w})\right] dF(w)$$

because for any wage offer $w_2 > \hat{w}$, the unemployed accepts the offer and the employed quits the job, meaning $\Omega(w_2) > T(\hat{w}, w_2)$. Multiplying the above equation by 2 and using equation (2.10) ,we arrive at

$$u\left(2\hat{w}\right)=2u\left(\hat{w}+b\right)-u\left(2\hat{w}\right)+2\alpha\int_{\hat{w}}\left[\Omega\left(w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w_2\right).$$

Substituting this expression for $u\left(2\hat{w}\right)$ into the RHS of the equation $(B.5)$ delivers

$$\begin{aligned}r\ \left[\Omega(\hat{w})-\Omega(w^{**})\right]&=2u\left(\hat{w}+b\right)-u\left(2\hat{w}\right)-u\left(2b\right)\\&\quad+2\alpha\left[\int_{\hat{w}}\left[\Omega\left(w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right)-\int_{w^{**}}\left[\Omega\left(w\right)-\ \Omega(w^{**})\right]dF\left(w\right)\right]\\&=2u\left(\hat{w}+b\right)-u\left(2\hat{w}\right)-u\left(2b\right)+2\alpha\int_{\hat{w}}^{w^{**}}\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.\end{aligned}$$

where the second line uses integration by parts. Now, by concavity of u, $2u\left(\hat{w}+b\right)-u\left(2\hat{w}\right)-u\left(2b\right)>0$. Suppose, ad absurdum, $w^{**}\geq\hat{w}$. Then, the RHS of the above equation is strictly positive, but the LHS is either negative or zero, which is a contradiction. Therefore, $w^{**}<\hat{w}$.

We now prove, by contradiction, that $\hat{w}>w^{*}$. Suppose that $w^{*}\geq\hat{w}$. Recall that equation (2.6) evaluated at $\hat{w}$ can be written as

$$r\Omega\left(\hat{w}\right)=u\left(\hat{w}+b\right)+\alpha\int_{\hat{w}}\left[\Omega\left(w\right)-\Omega\left(\hat{w}\right)\right]dF\left(w\right).$$

Since $r\Omega\left(\hat{w}\right)=u\left(2\hat{w}\right)$, we can rewrite the above relationship as

$$\begin{aligned}u\left(2\hat{w}\right)-u\left(\hat{w}+b\right)&=\frac{\alpha}{r}\int_{\hat{w}}\left[r\Omega\left(w\right)-u\left(2\hat{w}\right)\right]dF\left(w\right)\\&>\frac{\alpha}{r}\int_{\hat{w}}\left[rT\left(\hat{w},w\right)-u\left(2\hat{w}\right)\right]dF\left(w\right)\\&=\int_{\hat{w}}\left[u\left(\hat{w}+w\right)-u\left(\hat{w}+\hat{w}\right)\right]dF\left(w\right)\end{aligned}$$

Rearrange the above equation and, once again, use the property of DARA utility to get

$$\begin{aligned}
1 &> \frac{\alpha}{r} \int_{\hat{w}} \left[\frac{u(\hat{w}+w) - u(\hat{w}+\hat{w})}{u(\hat{w}+\hat{w}) - u(\hat{w}+b)} \right] dF(w) \\
&> \frac{\alpha}{r} \int_{\hat{w}} \left[\frac{u(w) - u(\hat{w})}{u(\hat{w}) - u(b)} \right] dF(w) \\
&\geq \frac{\alpha}{r} \int_{w^*} \left[\frac{u(w) - u(w^*)}{u(w^*) - u(b)} \right] dF(w) \\
&= 1
\end{aligned}$$

The second inequality is due to the property of DARA utility, the third weak inequality derives from the assumption $w^* \geq \hat{w}$ and from u being an increasing function. The last equality comes from the definition of reservation wage for the single agent. Since we reached a contradiction, it must be that $\hat{w} > w^*$.

Finally, we need to prove that $\phi' < 1$. Let us assume $\phi' > 1$. This means that for $w_1 > \hat{w}$, $\phi(w_1) > \phi^{-1}(w_1) = \varphi(w_1)$. For any $w_1 > \hat{w}$, if the wage offer $w_2 > \phi(w_1)$, the unemployed accepts the offer, meaning $T(w_1, w_2) > \Omega(w_1)$. But since $w_2 > \phi(w_1) > \phi^{-1}(w_1)$, the employed quits the job at the same time, which means $\Omega(w_2) > T(w_1, w_2) > \Omega(w_1)$. With the same logic, one can see that if $w_2 \in (w_1, \phi(w_1))$, we get $\Omega(w_2) > \Omega(w_1) > T(w_1, w_2)$. If $w_2 \in (\varphi(w_1), w_1)$, we have $\Omega(w_1) > \Omega(w_2) > T(w_1, w_2)$ and if $w_2 < \varphi(w_1)$, we have $\Omega(w_1) > T(w_1, w_2) > \Omega(w_2)$. Hence, if $w_2 > w_1$, then the unemployed accepts the job and the employed quits the job, forcing the reservation wage to be w_1. Hence $\phi(w_1) = w_1$, resulting in $\phi' = 1$, a contradiction. $\square$

Proof. [Proposition 5] There are three cases to consider.

(i) Consider a dual-searcher couple. Recall that by definition of w^{**}, $U = \Omega(w^{**}) > T(2w^{**}) > T(z + w^{*}) > T(z + w)$ for all $w < w^{**}$. Hence, no wage offer below w^{**} is accepted by the searching couple since dual search always dominates. For wage offers above w^{**}, $T(z, w) < T(w^{**}, w) < \Omega(w)$ since under CARA or DARA ϕ is a non decreasing function. Therefore a dual searcher couple which samples an offer above w^{**} becomes a worker-searcher couple. Simple inspection of equation (2.18) shows that the worker-searcher couple will never transit through nonparticipation. It remains to be proved that being a dual non participant couple is also dominated. This is straightforward, since $U = \Omega(w^{**}) > T(2w^{**}) > T(2z)$. Dual search dominates dual nonparticipation. Hence, nonparticipation never occurs.

(ii) Since $U = \Omega(w^{**}) < \Omega(z)$, search-nonparticipation is always preferred to dual search. Since we are in the range $z < \hat{w}$, where quitting is optimal, we know that $\phi(z) = z$. As soon as the searcher receives a job offer higher w than z, she becomes employed and the couple becomes a worker-searcher couple. From that point onward, the dynamics are as in the baseline model.

(iii) Under this configuration, $U = \Omega(w^{**}) < \Omega(\hat{w}) < \Omega(z)$ which proves that search-nonparticipation is always preferred to dual search. However, we can write $\Omega(z) = T(z, \phi(z)) < T(2z)$ since above $\hat{w}$ we have $\phi(z) \leq z$. Thus, both members enter to the non participation pool, which is an absorbing state. □

Proof. [Proposition 6] Let us conjecture that $\phi(w_1) = w^{**}$ for any value of w_1, i.e., $T(w^{**}, w_2) = \Omega(w_2)$. This implies that the quit option is never exercised since any observed w_1 will be greater than or equal to w^{**}. So, one can disregard the second argument in the max operator in (2.20). Evaluating (2.20) at w^{**} yields

$$r\Omega\left(w^{**}\right) = u\left(w^{**} + b\right) + 2\alpha_u \int \max\left\{\Omega\left(w\right) - \Omega\left(w^{**}\right), 0\right\} dF\left(w\right) dF\left(w_2\right)$$

where we have used the fact that $\alpha_e = \alpha_u$ and the conjecture. Since $\Omega\left(w^{**}\right) = U$, comparing the above equation to (2.19) yields that $w^{**} = b$. We now verify our conjecture. From (2.21) evaluated at $w_2 = w^{**}$:

$$\begin{aligned} rT\left(w_1, w^{**}\right) &= u\left(w_1 + b\right) + \alpha_e \int \max\left\{T\left(w_1', w^{**}\right) - T\left(w_1, w^{**}\right), 0\right\} dF\left(w_1'\right) \\ &+ \alpha_u \int \max\left\{T\left(w_1, w_2'\right) - T\left(w_1, w^{**}\right), 0\right\} dF\left(w_2'\right) \\ &= u\left(w_1 + b\right) + \alpha_e \int \max\left\{\Omega\left(w_1'\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_1'\right) \\ &+ \alpha_u \int \max\left\{T\left(w_1, w_2'\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2'\right) \\ &= \Omega\left(w_1\right), \end{aligned}$$

which confirms our conjecture, since $T\left(w^{**}, w_2\right) = \Omega\left(w_2\right)$ implies that $\phi\left(w_2\right) = w^{**}$. Finally, from equation (2.21), it is immediate that $\eta\left(w_i\right) = w_i$ which completes the proof. □

Proof. [Proposition 7] We begin with part (ii). The value functions (2.4) and (2.6) modified to allow for exogenous separations are:

$$(r + 2\delta) T\left(w_1, w_2\right) = u\left(w_1 + w_2\right) - \delta\left[\Omega\left(w_1\right) + \Omega\left(w_2\right)\right] \tag{B.6}$$

$$\begin{aligned} r\Omega\left(w_1\right) &= u\left(w_1 + b\right) - \delta\left[\Omega\left(w_1\right) - U\right] \qquad\qquad \text{(B.7)} \\ &+ \alpha \int \max\left\{T\left(w_1, w_2\right) - \Omega\left(w_1\right), \Omega\left(w_2\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2\right) \end{aligned}$$

From the definition of reservation function ϕ for the worker-searcher couple, $T(w_1 + \phi(w_1)) = \Omega(w_1)$, we have:

$$u(w_1 + \phi(w_1)) - \delta[\Omega(w_1) - \Omega(\phi(w_1))] = r\Omega(w_1).$$

Let us assume that there is a wage value w_1 beyond which the employed worker never quits. Then, in this range $\phi(w_1)$ is a non-increasing function. Using this property in $(B.7)$ and substituting into the above equation, we get:

$$\begin{aligned} u(w_1 + \phi(w_1)) &= u(w_1 + b) + \alpha \int_{\phi(w_1)} [T(w_1, w_2) - T(w_1, \phi(w_1))]\, dF(w_2) - \delta[\Omega(\phi(w_1)) - U] \\ &= u(w_1 + b) + h(\phi(w_1)) \qquad \text{(B.8)} \\ &+ \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} [u(w_1 + w_2) - u(w_1 + \phi(w_1))]\, dF(w_2) \end{aligned}$$

where

$$h(x) = \frac{\alpha\delta}{r + 2\delta} \int_x [\Omega(w_2) - \Omega(x)]\, dF(w_2) - \delta[\Omega(x) - U]$$

with h decreasing in x. Rearrange equation $(B.8)$ as:

$$1 = \frac{\alpha}{r + 2\delta} \int_{\phi(w_1)} \left[\frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)}\right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)}. \qquad \text{(B.9)}$$

Since $\phi(w_1)$ is a decreasing function of w_1 then, for any $\tilde{w}_1 > w_1$, we have:

$$0 \leq \frac{u(w_1 + w_2) - u(w_1 + \phi(w_1))}{u(w_1 + \phi(w_1)) - u(w_1 + b)} \leq \frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(w_1))}{u(\tilde{w}_1 + \phi(w_1)) - u(\tilde{w}_1 + b)} \leq \frac{u(\tilde{w}_1 + w_2) - u(\tilde{w}_1 + \phi(\tilde{w}_1))}{u(\tilde{w}_1 + \phi(\tilde{w}_1)) - u(\tilde{w}_1 + b)}$$

where the first weak inequality stems from the fact that u is CARA or DARA, and the second from the fact that ϕ is weakly decreasing. Overall, the above condition implies the first term in equation $(B.9)$ is an increasing function of w_1.

Since h is decreasing in x, and $\phi(\tilde{w}_1) \leq \phi(w_1)$ for $\tilde{w}_1 > w_1$, we have

$$\frac{h(\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} < \frac{h(\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)},$$

because the right hand side has a weakly greater numerator, and a strictly smaller denominator, than the left hand side. And we reach the following contradiction:

$$\begin{aligned}
1 &= \frac{\alpha}{r+2\delta}\int_{\phi(w_1)}\left[\frac{u(w_1+w_2) - u(w_1+\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)}\right] dF(w_2) + \frac{h(\phi(w_1))}{u(w_1+\phi(w_1)) - u(w_1+b)} \\
&< \frac{\alpha}{r+2\delta}\int_{\phi(\tilde{w}_1)}\left[\frac{u(\tilde{w}_1+w_2) - u(\tilde{w}_1+\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)}\right] dF(w_2) + \frac{h(\phi(\tilde{w}_1))}{u(\tilde{w}_1+\phi(\tilde{w}_1)) - u(\tilde{w}_1+b)} \\
&= 1
\end{aligned}$$

We conclude that $\phi(w_1)$ is strictly increasing in w_1. Once we have established this result, similar arguments used in the proof of Proposition 3 apply here for part (i) to complete the proof. □

Proof. [Proposition 8] We conjecture that $rT(w_1, w_2, a) = u(ra + w_1 + w_2)$. Then the RHS of equation (2.26) becomes

$$\max_c \left\{u(c) + u'(ra + w_1 + w_2)(ra + w_1 + w_2 - c)\right\}.$$

The FOC implies $u'(c) = u'(ra + w_1 + w_2)$, so $c_e(a, w_1, w_2) = ra + w_1 + w_2$. If we plug this optimal consumption function back into equation (2.26), we arrive at $rT(w_1, w_2, a) = (ra + w_1 + w_2)$, which confirms the conjecture.

Similarly, let us guess that $r\Omega(w_1, a) = u(ra + w_1 + \phi(w_1))$. Again, plugging this guess into RHS of equation (2.28) the FOC implies $c_\Omega(w_1, a) = ra + w_1 +$

$\phi(w_1, a)$. Substituting this function back into (2.28) gives

$$r\Omega(w_1, a) = u(ra + w_1 + \phi(w_1, a)) + u'(ra + w_1 + \phi(w_1, a))(b - \phi(w_1, a)) + \frac{\alpha}{r}\int \max\left\{\begin{array}{c} u(ra + w_1 + w_2) - u(ra + w_1 + \phi(w_1, a)), \\ u(ra + w_2 + \phi(w_1, a)) - u(ra + w_1 + \phi(w_1, a)), 0 \end{array}\right\} dF(w_2)$$

Using the CARA property of u, we can simplify the RHS and rewrite the above equation as:

$$r\Omega(w_1, a) = u(ra + w_1 + \phi(w_1, a))\left[\begin{array}{c} 1 - \rho(b - \phi(w_1, a)) - \\ \frac{\alpha}{r}\int \max\{u(w_2 - \phi(w_1, a)) - 1, u(w_2 - w_1) - 1, 0\} dF(w_2) \end{array}\right].$$

Now, using the definition of ϕ, and the expression for $rT(w_1, \phi(w_1, a), a)$ in the above equation, we have:

$$\phi(w_1, a) = b + \frac{\alpha}{\rho r}\int [u(\max\{w_2 - \phi(w_1, a), w_2 - w_1, 0\}) - 1] dF(w_2).$$

As in the CARA case without saving, conjecture that there is a value w_1 such that beyond that value the quitting option is never exercised. Then, in this range we can ignore from the second argument in the max operator and rewrite

$$\phi(w_1, a) = b + \frac{\alpha}{\rho r}\int_{\phi(w_1, a)} [u(w_2 - \phi(w_1, a)) - 1] dF(w_2) \tag{B.10}$$

which implies that ϕ is a constant function, independent of (w_1, a). Moreover, comparing $(B.10)$ to the equivalent equation for the single agent problem (2.25) yields that $\phi(w_1, a) = w^*$.

Finally, let us turn to U and conjecture that $rU(a) = u(ra + 2w^{**})$. Substituting this guess into equation (2.27) and taking the FOC leads to the optimal policy function $c_u(a) = ra + 2w^{**}$ which confirms the guess. Then, using the CARA assumption, equation (2.27) becomes

$$\begin{aligned} rU(a) &= u(ra + 2w^{**}) - \rho u(ra + 2w^{**})(2b - 2w^{**}) - \frac{2\alpha}{r} u(ra + 2w^{**}) \int_{w^{**}} [u(w - w^{**}) - 1] dF(w) \\ &= u(ra + 2w^{**}) \left[1 - \rho(2b - 2w^{**}) - \frac{2a}{r} \int_{w^{**}} [u(w - w^{**}) - 1] dF(w)\right]. \end{aligned}$$

and using $rU(a) = u(ra + 2w^{**})$ we arrive at:

$$w^{**} = b + \frac{a}{\rho r} \int_{w^{**}} [u(w - w^{**}) - 1] dF(w)$$

which, once again, compared to (2.25) implies that $w^{**} = w^*$. This concludes the proof. □

Proof. [Proposition 9] We first prove parts (ii) and (iii), which establish the behavior of the reservation wage functions. The reservation function for outside offer satisfies $S(w_1, \phi_o(w_1)) = \Omega(w_1)$. As before, we begin by conjecturing that quit option is never exercised beyond a certain wage threshold. In this range, from the definition of $\phi_o(w_1)$:

$$\begin{aligned} \phi_o(w_1) &= b + \kappa + \alpha_i \int_{\phi_i(w_1)} [T(w_1, w_2) - \Omega(w_1)] dF(w_2) + \alpha_o \int_{\phi_o(w_1)} [S(w_1, w_2) - \Omega(w_1)] dF(w_2) \\ &= b + \kappa + \alpha_i \int_{\phi_i(w_1)} T_2(w_1, w_2)(1 - F(w_2)) dw_2 + \alpha_o \int_{\phi_o(w_1)} S_2(w_1, w_2)(1 - F(w_2)) dw_2 \\ &= b + \kappa + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F(w_2)] dw_2 + \frac{\alpha_o}{r} \int_{\phi_o(w_1)} [1 - F(w_2)] dw_2 \quad \text{(B.11)} \end{aligned}$$

where the second line is obtained through integration by parts and the third line uses the risk neutrality assumption which assures $T_2(w_1, w_2) = S_2(w_1, w_2) = \frac{1}{r}$.

We now turn to inside offers. The reservation function for inside offer satisfies $T(w_1, \phi_i(w_1)) = \Omega(w_1)$. We keep analyzing the region of w_1 above $\hat{w}_S$ where we know the employed worker does not quit upon receiving outside offers. From the definition of $\phi_i (w_1)$:

$$\begin{aligned}\phi_i (w_1) &= b + \alpha_i \int_{\phi_i(w_1)} [T (w_1, w_2) - \Omega (w_1)] \, dF (w_2) + \alpha_o \int_{\phi_o(w_1)} [S (w_1, w_2) - \Omega (w_1)] \, dF (w_2) \\ &= b + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F (w_2)] \, dw_2 + \frac{\alpha_o}{r} \int_{\phi_o(w_1)} [1 - F (w_2)] \, dw_2 \qquad \text{(B.12)}\end{aligned}$$

where the second line is derived exactly as for the outside offer case.

Combining equations $(B.11)$ and $(B.12)$ we can verify that $\phi_o (w_1)$ and $\phi_i (w_1)$ are independent of w_1, and $\phi_i (w_1) = \phi_o (w_1) - \kappa$ for $w_1 \geq \hat{w}_S$. This confirms the conjecture, and results $\hat{w}_T = \hat{w}_S - \kappa$.

Let us extend our analysis of inside offers to the region where w_1 is lower than $\hat{w}_S$. Here, the reservation function ϕ_i satisfies

$$\phi_i (w_1) = b + \frac{\alpha_i}{r} \int_{\phi_i(w_1)} [1 - F (w)] \, dw + \frac{\alpha_o}{r} \int_{w_1}^{\hat{w}_S} \Omega' (w_2) [1 - F (w_2)] \, dw_2$$

since the employed worker will quit upon receiving outside offers. Clearly, $\phi_i (w_1)$ is decreasing in w_1 over this region. We conclude that for $w_1 \geq \hat{w}_S$, we have $\phi_i (w_1) = \hat{w}_T$ and in the range $[\hat{w}, \hat{w}_S)$ the function ϕ_i is decreasing, with $\hat{w}$ denoting the double indifference point, i.e. the intersection with the 45 degree line. As usual, below $\hat{w}$, $\phi_i (w_1) = w_1$. This completes the proof of parts (ii) and (iii).

We next prove part (i) of the proposition: $w^{**} \in (\hat{w}_T, \hat{w})$ and $w^{*} \in (\hat{w}, \hat{w}_S)$, so $w^{**} < w^{*}$. It is also useful to recall that $\hat{w}_T < \hat{w} < \hat{w}_S$.

Step 1: We first show $w^{**} \in (\hat{w}_T, \hat{w})$. Equation (2.32) evaluated at he point $w_1 = \hat{w}_T$ becomes

$$r\Omega (\hat{w}_T) = \hat{w}_T + b + (\alpha_i + \alpha_o) \int_{\hat{w}_T} \Omega' (w) [1 - F (w)] \, dw. \qquad \text{(B.13)}$$

The reservation wage of the dual-searcher couple w^{**} is characterized by the equation

$$r\Omega\left(w^{**}\right)=2b+2\left(\alpha_i+\alpha_o\right)\int_{w^{**}}\Omega'\left(w\right)\left(1-F\left(w\right)\right)dw. \tag{B.14}$$

Now subtract equation $(B.13)$ multiplied by 2 from equation $(B.14)$, and get

$$r\left[\Omega\left(w^{**}\right)-\Omega\left(\hat{w}_T\right)\right]=r\Omega\left(\hat{w}_T\right)-2\hat{w}_T+2\left(\alpha_i+\alpha_o\right)\int_{w^{**}}^{\hat{w}_T}\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.$$

Suppose $w^{**}\leq\hat{w}_T$, then LHS of the above equation is negative or zero. The second term of the RHS is positive. The term $r\Omega\left(\hat{w}_T\right)-2\hat{w}_T$ is also positive because for $w_1=\hat{w}_T$ the employed worker would prefer to quit his job than remaining employed (more precisely, he strictly prefers it for an outside offer, he's indifferent for an inside offer). Therefore the RHS is positive which is a contradiction. So $w^{**}>\hat{w}_T$.

Step 2: Similarly, consider equation (2.32) evaluated at $w_1=\hat{w}$. Note that at $w_1=\hat{w}$, for inside offers the employed spouse never exercises the quit option, while for outside offers, she does so. So, equation (2.32) evaluated at $w_1=\hat{w}$ becomes

$$r\Omega\left(\hat{w}\right)=\hat{w}+b+\frac{\alpha_i}{r}\int_{\hat{w}}\left[1-F\left(w\right)\right]dw+\frac{\alpha_o}{r}\int_{\hat{w}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw.$$

Also note that since $\hat{w}$ is the double indifference point for inside offers, $r\Omega\left(\hat{w}\right)=2\hat{w}$. Again, subtract this last equation multiplied by 2 from equation $(B.14)$ to get

$$r\left[\Omega\left(w^{**}\right)-\Omega\left(\hat{w}\right)\right]=\frac{2\alpha_i}{r}\left[\int_{w^{**}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]dw-\int_{\hat{w}}\left[1-F\left(w\right)\right]dw\right]+2\frac{\alpha_o}{r}\int_{w^{**}}^{\hat{w}}r\Omega'\left(w\right)\left[1-F\left(w\right)\right]$$

Now, suppose $w^{**}\geq\hat{w}$. Then the LHS becomes nonnegative. The last term in the RHS is negative. From the definition of $\phi_i\left(w_1\right)$, $r\Omega\left(w_1\right)=rT\left(w_1,\phi_i\left(w_1\right)\right)=w_1+\phi_i\left(w_1\right)$. Thus, $\phi_i'\left(w_1\right)=r\Omega'\left(w_1\right)-1$. But since we have proved that $\phi_i'\left(w_1\right)\leq 0$

above $\hat{w}$, we have that $r\Omega'(w_1) \leq 1$. Therefore, the first term in the RHS must also be negative, which delivers a contradiction and leads to $w^{**} < \hat{w}$. Steps 1 and 2 establish that $w^{**} \in (\hat{w}_T, \hat{w})$.

Step 3: We next prove $w^* \in (\hat{w}, \hat{w}_S)$. Combining the equation (2.32) evaluated at $\hat{w}$ with the fact that $r\Omega(\hat{w}) = 2\hat{w}$, we have

$$\hat{w} = b + \frac{\alpha_i}{r}\int_{\hat{w}} [1 - F(w)]\, dw + \frac{\alpha_o}{r}\int_{\hat{w}} r\Omega'(w)[1 - F(w)]\, dw$$

Subtracting this equation from equation (2.33) we get

$$w^* - \hat{w} = \frac{\alpha_i}{r}\int_{w^*}^{\hat{w}} [1 - F(w)]\, dw + \frac{\alpha_o}{r}\left[\int_{w^*} [1 - F(w)]\, dw - \int_{\hat{w}} r\Omega'(w)[1 - F(w)]\, dw\right]$$

Suppose, $w^* \leq \hat{w}$, then the LHS becomes non-positive, but the RHS is strictly positive since $r\Omega'(w) \leq 1$, a contradiction. Thus, $w^* > \hat{w}$.

Step 4: Finally we show that $w^* < \hat{w}_S$. Rewrite the equation for $\hat{w}_S$ as

$$\hat{w}_S = b + \kappa + \frac{\alpha_1}{r}\int_{\hat{w}_S - \kappa} (1 - F(w))\, dw + \frac{\alpha_2}{r}\int_{\hat{w}_S} (1 - F(w))\, dw$$

Subtracting equation (2.33) from the equation defining $\hat{w}_S$, we get

$$\hat{w}_S - w^* = \kappa + \frac{\alpha_i}{r}\int_{\hat{w}_S - \kappa}^{w^*} [1 - F(w)]\, dw + \frac{\alpha_o}{r}\int_{\hat{w}_S}^{w^*} [1 - F(w)]\, dw$$

Suppose $w^* \geq \hat{w}_S$, then the LHS is non-positive. However, since $\kappa > 0$, RHS is strictly positive. Thus, $w^* < \hat{w}_S$. Therefore, $w^* \in (\hat{w}, \hat{w}_S)$, and the proof is complete. $\square$

B.2 Additional value functions

Equations for the economy with multiple locations, exogenous separations, risk-neutral agents and on the job search.

First, consider the problem of a couple that is currently together. The arrival rate of wage offers for each spouse from the current location (in which case they can accept the job and still stay together) is $(1-\theta)\,\psi$. The total arrival rate of all outside offers for each spouse is $\theta\psi$ which is obtained by multiplying the number of offers (at rate $\theta\psi/(L-1)$ from each outside location) by the number of such locations $(L-1)$. The equation is:

$$\begin{aligned}
rT(w_1,w_2) = {} & w_1 + w_2 + (1-\theta)\,\psi \int \max\left\{T\left(w_1',w_2\right) - T(w_1,w_2),\Omega\left(w_1'\right) - T(w_1,w_2),0\right\} dF\left(w_1'\right) \\
& + (1-\theta)\,\psi \int \max\left\{T\left(w_1,w_2'\right) - T(w_1,w_2),\Omega\left(w_2'\right) - T(w_1,w_2),0\right\} dF\left(w_2'\right) \\
& + \psi\theta \int \max\left\{S\left(w_1',w_2\right) - T(w_1,w_2),\Omega\left(w_1'\right) - T(w_1,w_2),0\right\} dF\left(w_1'\right) \\
& + \psi\theta \int \max\left\{S\left(w_1,w_2'\right) - T(w_1,w_2),\Omega\left(w_2'\right) - T(w_1,w_2),0\right\} dF\left(w_2'\right) \\
& + \delta\left[\Omega(w_1) - T(w_1,w_2)\right] + \delta\left[\Omega(w_2) - T(w_1,w_2)\right]
\end{aligned}$$

Notice that in all cases, an offer to one spouse can trigger a quit for the other spouse, which is taken into account in this equation. Turning to a couple whose members currently live in different locations, call A and B, the problem is somewhat different. The couple could reunite if either spouse receives an offer from the location of the other spouse. The arrival rate of job offers at location A from B (and B from A) is $\theta\psi/(L-1)$. The arrival rate of offers that keep the couple separate is simply the total offer arrival rate minus the rate just calculated, which is $\psi\left(1-\theta/\left(L-1\right)\right)$ for each spouse:

$$
\begin{aligned}
rS\left(w_1,w_2\right) = w_1+w_2-\kappa &+ \frac{\theta\psi}{\left(L-1\right)}\int \max\left\{T\left(w_1',w_2\right)-S\left(w_1,w_2\right),\Omega\left(w_1'\right)-S\left(w_1,w_2\right),0\right\}dF\left(w_1'\right)\\
&+\frac{\theta\psi}{\left(L-1\right)}\int \max\left\{T\left(w_1,w_2'\right)-S\left(w_1,w_2\right),\Omega\left(w_2'\right)-S\left(w_1,w_2\right),0\right\}dF\left(w_2'\right)\\
&+\psi\left(1-\frac{\theta}{L-1}\right)\int \max\left\{S\left(w_1',w_2\right)-S\left(w_1,w_2\right),\Omega\left(w_1'\right)-S\left(w_1,w_2\right),0\right\}dF\left(w_1'\right)\\
&+\psi\left(1-\frac{\theta}{L-1}\right)\int \max\left\{S\left(w_1,w_2'\right)-S\left(w_1,w_2\right),\Omega\left(w_2'\right)-S\left(w_1,w_2\right),0\right\}dF\left(w_2'\right)\\
&+\delta\left[\Omega\left(w_1\right)-S\left(w_1,w_2\right)\right]+\delta\left[\Omega\left(w_2\right)-S\left(w_1,w_2\right)\right]
\end{aligned}
$$

Turning to a worker-searcher couple, their problem needs to account separately for offers received by the employed spouse and the unemployed spouse who receive offers at different rates. The unemployed spouse receives offers at rate $\left(1-\theta\right)\psi_u$ from the current location in which case the couple faces the same options as in the one-location problem. Second, the same spouse receives outside offers at rate $\theta\psi_u$ in which case (i) the unemployed spouse can choose to accept the offer and the couple could live separately, (ii) the offer could be accepted followed by quitting by the employed spouse, or (iii) the offer could be rejected. Finally, the total offer arrival rate of the employed spouse from all locations is ψ in which case the offer can either be accepted resulting in a transition to another worker-searcher couple with a higher wage, or could be rejected. Notice that in this last case, the location of the offer does not matter since the unemployed spouse will simply follow the employed one in case the offer is accepted.

$$
\begin{aligned}
r\Omega\left(w_1\right) = w_1 + b + (1-\theta)\,\psi_u \int \max\left\{T\left(w_1, w_2\right) - \Omega\left(w_1\right), \Omega\left(w_2\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2\right) \\
+ \theta\psi_u \int \max\left\{S\left(w_1, w_2\right) - \Omega\left(w_1\right), \Omega\left(w_2\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_2\right) \\
+ \psi \int \max\left\{\Omega\left(w_1'\right) - \Omega\left(w_1\right), 0\right\} dF\left(w_1'\right) + \delta\left[U - \Omega\left(w_1\right)\right] \\
rU = 2b + 2\psi_u \int \max\left\{\Omega\left(w\right) - U, 0\right\} dF\left(w\right).
\end{aligned}
$$

It is easy to see that when $L = 2$ all these equations reduce to those for the two-location problem with on-the-job search. To get the value functions used in the simulated exercise, set $\psi = 0$ to eliminate on the job search.

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This dissertation was typeset with LaTeX[†] by the author.

[†] LaTeX is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's TeX Program.