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IDENTIFICATION OF VULNERABLE TRANSPORTATION INFRASTRUCTURE AND HOUSEHOLD DECISION MAKING UNDER EMERGENCY EVACUATION CONDITIONS

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by

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Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

December, 2003

UMI Number: 3122770

ProQuest Information and Learning Company
300 North Zeeb Road
PO Box 1346
Ann Arbor, MI 48106-1346

Dedication

This dissertation is dedicated to the memory of all lives lost during the September 11, 2001 terrorist attacks and their families.

Acknowledgements

My tenure as a graduate student has spanned the most enjoyable four years of my life. I would like to take this opportunity to thank all of the Transportation Engineering professors at the University of Texas at Austin for their support, guidance, and instruction. Dr. Mahmassani has made the development of my research interests possible and served as a mentor, encouraging me to grow both personally and academically. Much appreciation is owed to Dr. Bhat and Dr. Machemehl who offered advice on personal and professional matters. The comments and constructive criticism offered by the committee members has been instrumental to the completion and focus of this dissertation.

Furthermore, I acknowledge the blessings of family bestowed upon me. I thank my husband Kenneth for his support, encouragement, and love. My parents, Sandra and Thomas Murray, and brother, Jeffrey Murray, have been unwavering in their support of my work and career.

This dissertation work was funded by the Southwest Region University Transportation Center.

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Publication No.______________

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The University of Texas at Austin, 2003

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This dissertation combines two primary problems under general disaster considerations. First, a methodology is presented to identify vulnerable transportation infrastructure, which is defined as the set of network links, the damage of which results in the maximum disruption of the network's origin-destination connectivity. The disrupting agent is permitted a limited number of resources with which to damage the network. The measure of disruption, resulting from the damage, is based on a given set of traffic conditions, the availability of alternate paths, and roadway design characteristics. A bi-level mathematical programming model represents the interaction of the traffic assignment and the disruption measure. This bi-level model allows the problem to be viewed as a game between an evil entity, who seeks to disrupt the network,

and a traffic management agency that routes vehicles so as to avoid vulnerable links to the greatest degree possible while meeting origin-destination demands.

The second problem is to mathematically describe household decision making behavior in an emergency evacuation. Traditional transportation network evacuation models have omitted a commonly observed sociological phenomenon – that families gather together before evacuating an area. This omission can lead to overly optimistic evacuation times, and the evacuation models fail to capture underlying traffic patterns that only arise during times of crises. Two linear integer programs are developed to model the decision making behavior; the first describes a meeting location selection process and the second assigns trip chains for drivers to pick up family members who may not have access to a vehicle. The mathematical programs are combined with a traffic assignment-simulation package for evacuation analysis.

Interactions between the two problems are also explored. Evacuation conditions are examined when the traffic management agency routes traffic around vulnerable links. The impact of the unusual traffic patterns, that arise using the household decision making behavior evacuation model, is evaluated in terms of shifts in the relative vulnerability of the transportation links. Finally, the routing strategies are evaluated for extensions in network evacuation times.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

Threats of terrorism, war, and natural disasters have created an environment in which the evacuation of a city or region may be necessary. The transportation network of the affected area plays a crucial role in the success of moving the area residents to safety. Transportation engineers and planners continually seek to improve the mobility of residents through the network, particularly during emergency situations. In preparation for these times of unusual and extreme traffic conditions, the ability to identify vulnerable transportation infrastructure, an understanding of evacuation behavior at the household level, and the associated simulation tools are of critical importance.

This chapter introduces the motivation for this work, the problem and related objectives, the contributions of this work to the fields of transportation engineering, evacuation planning, and critical infrastructure protection, and outlines the remainder of this dissertation.

1.1 MOTIVATION

Disaster management and related emergency evacuation are not new fields of study. Analyses of literature trends indicate that prior to the Cold War, much of the research was focused on evacuation procedures and the determination of factors that were more likely to cause people to leave their homes in response to natural threats, such as floods and hurricanes. Fear of nuclear attacks and the construction of nuclear power plants motivated a great deal of evacuation planning activity in the 1970's in response to a new type of threat. In the late twentieth and early twenty-first centuries, hurricanes caused the timing of evacuation orders to be evaluated more carefully. Now, amid the rising fear of terrorism in the United States, there has been another shift in focus from natural disasters to those caused by mankind.

Three time periods can be associated with a disaster. The first is the pre-disaster phase. During this time, especially for natural events, authorities may initiate the evacuation process. Warning technology, such as weather tracking devices, can be extremely valuable at this time. The second phase is the actual disaster strike and the limited time period that immediately follows. This period may encompass an earthquake or bombing and the immediate aftermath. During this time, victims may suddenly flee while rescue workers respond to the site. In the third phase, evacuees have abandoned the disaster area and only the rescue workers remain. At this time recovery begins. Due to the wide array of different evacuation scenarios, the focus of this work is on the first two phases, which are associated with the actual evacuation process.

Simulation methods are commonly used for transportation strategy evaluation. The transportation network is modeled and traffic movements are simulated through a series of behavior rules. However, previous models have not adequately captured the interaction among the existing transportation infrastructure, the provision and exchange of information enabled by modern information and communications technologies, and the behavior of evacuees. Traditional evacuation models assume that residents immediately leave the threatened area; however, this is not always the case. Parents may actually head toward danger to gather family members prior to evacuating the area. In this work, some of the apparently disorganized traffic caused by this behavior is explained by a series of mathematical programs, which emulate household decision-making behavior. To determine the importance of specific roads to the connectivity of origins and destinations, a measure called the vulnerability index was developed. For each link, the vulnerability indices are then aggregated across all origin-destination pairs into a disruption index, which allows for the identification of roadways that should be protected or where redundancy is needed in the transportation network.

1.2 Problem Statement and Objectives

The overall problem investigated in this dissertation is to develop a decision-aiding methodology for emergency evacuation planning for a city that considers transportation infrastructure vulnerability, realistic evacuee behavior, and the potential of information and communication technology. There are two primary problems addressed within the overall problem. The first involves the identification of vulnerable transportation infrastructure elements. The second pertains to accurately emulating network evacuation flow patterns resulting from the depiction of individual behavior at the household level. Each of these is explained further in the following sections.

1.2.1 Identification of Vulnerable Transportation Infrastructure Elements

The identification of vulnerable transportation infrastructure elements poses numerous challenges to the planning, engineering, and infrastructure protection communities. The definition of vulnerable, or critical, infrastructure elements may vary depending on the specific problem context. For instance, a bridge may be vulnerable to flooding. Another bridge may be vulnerable to a terrorist attack because of its history or landmark status. The definition of vulnerable, or critical, infrastructure used in this dissertation applies to a link, or set of links, the damage of which causes the most disruption to the origin-destination connectivity of the network. The problem of interest is to characterize the vulnerability of transportation infrastructure elements and identify the most vulnerable elements in a network for particular threat scenarios. More formally, the problem is to identify a set of transportation network links, the damage of which will maximally disrupt the origin-destination connectivity of the network.

The objectives pertaining to this problem are

1. To develop a mathematical measure of origin-destination connectivity vulnerability for a link, or set of links;
2. To extend the origin-destination vulnerability measure to the network level; and
3. To examine the impact of routing strategies and information on the vulnerability of transportation network links.

These objectives are addressed in detail in chapter 3. In chapter 5, the measures developed for objectives (1) and (2) are applied to a larger network. Objective (3) is explored in both chapters 3 and 5.

1.2.2 Model of Household Decision Making in an Emergency Evacuation

Accurately modeling emergency evacuation conditions is extremely difficult due to the lack of empirical evidence. Each emergency presents a different set of conditions. The differences may be due to the type of emergency, the experience of the community with similar events, the amount of warning that precedes the incident, the predicted severity and scope of the disaster, and conditions external to the community. Transportation evacuation model verification is highly impractical to conduct prior to an evacuation because there are ethical and practical constraints to conducting a "test" evacuation of a city.

Numerous evacuation studies have been conducted after the event has occurred. The majority of these have been conducted by agencies whose primary responsibility is not related to transportation engineering. A key finding from these studies that has been omitted from the majority of the transportation evacuation models is that families tend to gather together and then evacuate as a single unit. This omission leads to inaccuracy in many aspects of the evacuation model. Underlying traffic patterns, such as those that arise when parents go to schools to collect their children, are not captured in traditional models. Congestion is not properly predicted. As a result, the evacuation time prediction may be biased to the low side.

The problem addressed in chapter 4 of this dissertation is to develop a mathematical model of intra-household logistics during an emergency evacuation, including the processes by which family members gather and meet to evacuate jointly. The model would then be incorporated into a traffic assignment-simulation methodology to represent the dynamics of the resulting network flow patterns during the evacuation. Intra-household logistics modeling entails two primary decision dimensions: meeting location selection and the sequencing of pick-up assignments, resulting in trip chains to be completed by the household members using the transportation network, parts of which may be damaged or operationally modified (due to traffic management or vulnerability protection actions). The objectives associated with this problem consist of the following:

1. Formulate an optimization-based model of intra-household logistics decision-making behavior; the formulation captures trade-offs among key factors considered by the household in the decision process.
2. Examine the sensitivity of the decision behavior outcomes with respect to the relative weights associated in the above

trade-offs, and identify switchover points at which changes in behavior or pick-up assignments might result.

3. Represent and characterize the traffic conditions that arise when the resulting emergency trip chaining behavior of multiple households interact through the transportation network, using a state-of-the-art dynamic network traffic simulation-assignment methodology.

These three objectives are addressed in chapter 4. The model developed in objective (1), the results of objective (2), and the combined household decision making behavior – traffic simulation package of objective (3) are further explored in chapter 5. The difference in the approach to objective (3) in chapters 4 and 5 is the influence of non-driving entities, such as a traffic management agency. In chapter 4, the traffic management agency allows the use of any network link. In chapter 5, some links are assigned a very high cost which influences the route selection between origin-destination pairs.

1.2.3 Combining the Problems: Vulnerability of Networks under Evacuation Flow Patterns

The two primary problems discussed in sections 1.2.1 and 1.2.2 are considered jointly in chapter 5. This gives rise to two types of problem situations: (1) determining the vulnerability of network infrastructure elements under evacuation flow patterns; and (2) designing or inducing evacuation patterns that are less susceptible to disruption and are hence more likely to successfully and safely complete the evacuation process in the event of disruptive action. Traffic management agency routing strategies can then be devised and evaluated in terms of the effect on evacuation time and associated impact on transportation infrastructure vulnerability rankings.

1.3 RESEARCH SIGNIFICANCE AND CONTRIBUTIONS

This research contributes to the field of transportation engineering in two main arenas. The first area is network reliability and vulnerability. In this dissertation, an index is developed to characterize the relative importance of a given link, or set of links, to the network's origin-destination connectivity, for a given set of network flow conditions. The second area of significance is in evacuation modeling. Traditional engineering models have omitted an important factor at the family, or household, level. In this work, a series of linear integer programs is presented to describe the meeting location selection and the trip chaining assignment decisions for gathering family members, prior to evacuation. Without this component, evacuation models fail to capture an essential portion of the travel made within the city. The interaction of the drivers seeking to pick up family members and the drivers leaving the city has not been adequately studied. Integration of the mathematical programs for intra-household logistics decision with a network traffic simulation-assignment methodology leads to a more realistic representation of evacuation scenarios and the associate vehicular traffic flow patterns in the network.

1.4 STRUCTURE AND OVERVIEW OF THE DISSERTATION

This dissertation is organized in six chapters. Following the problem definition, motivation, and objectives discussed in the present chapter, the next chapter presents a general overview of the literature related to network reliability, evacuation behavior, and vehicle routing problems. Chapter 3 presents the modeling framework for the identification of vulnerable transportation infrastructure elements and an example of the methodology applied to a small transportation network. The underlying problem in chapter 3 is considered from a game theoretic perspective, in which an evil entity seeks to disrupt the network flow and a traffic management agency employs advanced traveler information systems and other means to route vehicles around vulnerable links. In the fourth chapter, the household evacuation behavior models are developed, including an application to a sample network. The behavior models assume that family members gather together prior to evacuating the city. Chapter 5 presents a hypothetical case study in which the models from Chapters 3 and 4 are applied to a moderately sized network. Finally, in chapter 6, the summary, conclusions, and directions for future work are presented.

Chapter 2

General Background

This chapter presents general background literature for this dissertation and is divided into three sections. In the first section, a general overview of previous studies pertaining to network reliability and vulnerability, which is the subject of chapter 3, is presented. The second part of the chapter discusses observed evacuation behavior. The third section identifies mathematical models that are related to the household evacuation model presented in Chapter 4.

2.1 NETWORK RELIABILITY

Network reliability has been a growing area of interest to the transportation community. Other fields, such as telecommunications and water resources, have addressed network reliability over the years (see for example Lee, 1980; Aggarwal, 1985; Yang et al, 1996). The definition of network reliability that is of interest here pertains to connectivity. Specifically, the network reliability is the probability that the origin and destination are connected due to the probabilities of link existence. Difficulties exist in directly applying the definitions and methodologies of the fields of telecommunications and water resources to the transportation arena.

2.1.1 Other fields

There are several characteristics of telecommunications systems that create difficulties in directly applying methodologies to the transportation networks. For example, the radio or optical signal, or whatever is flowing on the network, may degrade over the distance of the link (Caccetta, 1984). Another example is the treatment of the flow that is on the network. If a communications link is damaged, the calls using that link are dropped with little impact to the remainder of the network. In the transportation network, the vehicles may become damaged and cause queueing in the network. Finally, simplified versions of telecommunication networks can be represented as having equal probabilities of operating (Nel and Colbourn, 1990). Rarely, if ever, is this the case in a transportation network. Instead of information, there are vehicles flowing through the network and these vehicles are dispersed across nearly all competitive paths from an origin to a destination; only in extreme cases are roads completely closed, for any reason.

The use of nearly all paths, with similar travel times, is due to the fact that the vehicles are driven by people who have the ability to choose routes based on their perception of the state of the transportation network and not necessarily the actual state of the system. Furthermore, each driver may place different weights on factors that affect his or her route choice. For instance, one driver may decide that travel time is the most important criterion, regardless of the number of turns, the type of road, the safety of the road, and the number of traffic calming measures. Other drivers may prefer a simple route, with very few road changes and turns, even if the travel time is slightly longer.

2.1.2 Transportation Engineering

Unlike information or water, drivers have the ability to act as individual particles. Since there are differences in the commodity flowing on the network, methods from telecommunications and water resources need to be carefully adapted to the particular characteristics of a transportation system.

From the transportation engineering perspective, the focus of recent works related to network reliability has been on the probability of a pathway being completely operational and with damage to none of the links. Numerous methodologies, such as game theory (see Bell, 2000; Cassir and Bell, 2000; Bell, 1999), Monte Carlo simulation (see Chen et al, 1999), stochastic user equilibrium, and minimum cut sets have been employed, albeit for different problem formulations.

Iida and Wakabayashi (1989) proposed two approximation methods for determining the connectivity reliability between a pair of nodes in a transportation network. These methods were based on reliability graph analysis using minimal path sets and cut sets. In this work, Iida and Wakabayashi noted that to find an exact value for the reliability, complete enumeration of the minimal path sets and/or minimum cut sets was necessary. Due to the cumbersome nature of finding the exact solution, the authors presented a method that would approximate the reliability by using only partial sets. One of the assumptions that is critical to the use of this work is that the reliability of individual links is known *a priori*.

Iida (1999) presented basic equations for connectivity reliability when a system is in a series or in parallel. For Iida's (1999) work to apply to a network, one must be able to predict the probability that a link would be damaged and to what extent. In the case of terrorism a great deal of uncertainty exists in identifying specific transportation links that may be impacted. Additionally, even if the link is correctly identified as a target, slight miscalculations on the part of the actor may lead to that link being missed and an adjacent one being hit.

Asakura (1999) incorporated a stochastic user equilibrium model into a performance reliability model. In this work, he examined the role of information on user's route choice. Like Iida's (1999) work, the probability of the links existing was assumed known.

Many of the previous works presented above are more applicable to vehicle accidents or natural disasters, particularly flooding, where the probability of a roadway being affected is more easily quantifiable due to either historical data or the surrounding environment. The development of a mathematical measure of importance for links under any conditions and in any network, with or without a history of flooding or earthquake damage, will greatly aid both public and private sectors. By defining the value to be within given limits, one may determine the importance of a link in connecting an origin and a destination.

In the field of operations research, some work has addressed determining vital arcs in a network. Corley and Sha (1982), Malik, Mittal, and Gupta (1989) and Ball, Golden, and Vohra (1989) defined the most vital arcs problem as determining the subset of arcs whose removal from the network would result in the greatest increase in the shortest path between a given pair of nodes. This problem concept is similar to that found in the definition of edge persistence in the telecommunications industry (Caccetta, 1984). However, edge persistence pertains to the number of links that must be removed from the network and not the identification of the importance of particular links. Malik, Mittal, and Gupta (1989) proposed an exact algorithm for determining the ***k*** most vital arcs. Ball, Golden and Vohra (1989) showed that the most vital arcs problem is closely related to the most relevant arcs problem, the solution of which provides a lower bound on the optimal solution of the most vital arcs problem. The authors described an algorithm to solve the most relevant arcs problem. The most vital arcs problem is NP hard but the most relevant arcs problem admits a polynomial time solution algorithm.

Studies conducted in the operations research field are more directly applicable to the work presented here. The problems examined by the authors mentioned above are related to the links in the shortest path. In this work, all links are evaluated, not just the most critical ones. In chapter 3, a methodology for determining the relative importance of links in the network is presented. By examining the values of the disruption indices (see chapter 3), one can rank the links in terms of importance; the arcs with the highest rank correspond to the links that would be identified using the approaches of Corley and Sha (1982), Malik, Mittal, and Gupta (1989), and Ball, Golden and Vohra (1989).

2.1.3 Aggregation

In Chapter 3, a methodology for the determination of the vulnerability of a link, or set of links, is developed. An index is presented that represents the importance of the set of links to the connectivity of an origin-destination pair. This index is then aggregated over all origin-destination pairs to obtain a network level measure.

The issues of aggregating data with different units and different perspectives have been studied in detail. There are numerous approaches to grouping data and group decision making. Regarding the decision making, common methods include game theory and utility theory. As recognized by Keeney and Raiffa (1976), the decision maker can be a group of individuals, each of whom have a stake in the outcome, or a single individual, who must consider the groups but develop his or her own utility measure. The research presented here is related to the second type of decision maker, that of the single individual.

The vulnerability index that was developed for the links in a path connecting a single origin to a single destination can be likened to the concept of utility. Both measures are functions of variables in the immediate environment of the individual. If the strict utility (the utility measure is directly proportional to the choice probability) model holds, then both the criticality index and utility value are between 0 and 1 (see Luce, 1959; Ben-Akiva and Lerman, 1985). Due to these similarities, further discussion of previous works will focus on those related to utility aggregation.

One of the most common uses of utility aggregation can be found in the social welfare arena. Harsanyi (1955) supported the formulation of the social welfare function as a sum of the weighted utilities. This particular formulation has been prevalent even before the 1900's (Sen, 1973). Sen recognized the possibility of bypassing individual utilities and defining the welfare function directly on the distribution of incomes. This functional form is frequently used in public policy. Furthermore, Sen (1973) recognized that when utilities are employed, the function with the utilities is simply a special case of the more general form.

Keeney and Raiffa (1976) employed the formulation discussed above and reduced the problem of group preference aggregation to one of determining the relative weights that should be given to each party. The weights are assigned to the individual's utility function in the overall decision maker's utility function; thus, the decision maker's utility function is a function of the weighted individual parties' utility functions.

Kantor and Nelson (1979) introduced the concept of conditional utilities to the method employed by Keeney and Raiffa. The conditional utilities depended on the present state of the system and the possible actions by the decision maker, rather than the possible outcome of the actions by the decision maker. Conditional utilities allow for a more flexible model as states change over time.

Brock (1980) presented a theory of preference aggregation that characterizes an equitable distribution of utility gains. Brock's contribution to this area of weighted utilities is the distinction between hypothetical and operational interpersonal comparisons. In the hypothetical realm, utility distributions are identified for any possible situation which may arise. However, when the plan is put into practice, there may not be a need for interpersonal comparisons of utility.

Rawls (1971) acknowledged that there is no single answer to the problem of assigning weights when there are competing principles of justice. Intuition plays a role at this juncture. Based on this observation, the work presented here will provide the opportunity for the decision maker to employ his/her intuition for the particular environment in which he/she works.

The research presented in this dissertation draws from a variety of fields including operations research, water resources, telecommunications, and decision making. The initial formulation of the vulnerability index is related to the concepts of network reliability and vital arcs. Adjustment factors to this index are based on the idea of weighting, which comes from multiple decision maker problems. (Weights and multi-objective decision making are also employed in chapter 4). The vulnerability index is a new measure, and the adjustment factor is a response to initial difficulties identified with the interpretation of the index. The literature presented above shows the relationship of this work to that of previous researchers.

2.2 EVACUATION BEHAVIOR

A large number of studies have been conducted after all types of disasters. The majority of these works are more than twenty years old. More recent publications focus less on the evacuation itself and more on the technology employed during recovery and reconstruction efforts. This section presents an overview of evacuation literature.

2.2.1. Early Studies

Many of the early publications on evacuations were the result of observations of human behavior while others outlined plans for community preparedness. Advanced modeling of evacuation procedures, however, did not occur until computers were easily accessible.

Fritz and Mathewson (1957) observed a "convergence behavior" that occurs once a disaster has struck. People, information, and supplies have been noted to head toward the disaster area. This observation is related to the search and recovery aspects of a modern disaster.

Gillespie and Perry (1976) also focused on collective behavior during mass emergencies. They observed that when typical societal conditions no longer exist, a new "norm" is, at least temporarily, established. The establishment of a new "norm" is particularly observable during riots and other violent outbursts, but can also be found in panic situations, such as those that may be present in unexpected evacuation scenarios.

Herr (1984) reported that the work of Hans and Sell found that in 70 events, a state of panic, manifested in excessive driving speed, did not exist. Zelinsky and Kosinkski (1991) also rejected the idea that panic evacuations do not occur. Sattayhatewa and Ran (2000), however, stated that people do panic and disregard others while seeking to evacuate.

Regardless of whether panic occurs while people are driving, the time to evacuate an area using vehicles needs to be estimated so that officials can know when to give warnings and orders to evacuate an area. There have been numerous studies pertaining to community preparedness and estimations for evacuation times in the case of nuclear events (see, for example: Moore, et al, 1963; McLuckie, 1975; Brand, 1984; Gillespie, et al, 1993; Lindell and Perry, 1992). Other works, such as Palm and Hodgson (1993) and Perry and Mushkatel (1984) used surveys to identify characteristics of individuals who are more likely to evacuate in the event of natural disasters. Zelinsky and Kosinski (1991) also studied the importance of a number of variables to the propensity for an individual to evacuate.

As noted in Dow and Cutter (2002), one of the most important observations obtained from the early research is that household members being together is important to the decision to evacuate. This issue was researched by Perry, Lindell and Greene (1981), Johnson (1988), Sime (1993), and Zelinsky and Kosinski (1991), among others.

The use of survey information and advances in technology can improve the understanding of evacuees' behavior. Many of the survey studies were mentioned above. Some of the advances in technology and their application to evacuations is discussed in the next section.

2.2.2 Technological Advances

The most important advances in technology for evacuation have been in the area of information transfer. Satellites have become available for evacuation efforts. Walter (1990) explained the use of satellites for advanced warning and search-and-rescue efforts. Cellular phones are another example of how information may be relayed. Comfort (2000) observed the use of two-way radios, satellite telephones, cellular telephones, aerial photography, geographic information systems, satellite imagery, and computer modeling in the first three days following an earthquake in Turkey on August 17, 1999.

While rescue workers use the satellites and other technological advances for detailed information, more general information can be passed on to the public through other mass communication media, such as television, radio, and the world-wide-web. Rattien (1990) discusses the role of the media in disaster management. The influence of information on drivers' behavior leads to new modeling challenges. A brief overview of some of the evacuation models is presented in the next section.

2.2.3 Modeling

The need to model transportation related evacuation issues has been identified by numerous researchers. Ardekani and Hobeika (1988) sited the need for a "real-time microcomputer-based transportation decision tool" (p.123) in their aftermath study of the Mexico City Earthquake in 1985. Plowman (2001) sited a modeling tool for hurricane evacuations; however, the considerable advanced warning associated with hurricane scenarios leads to difficulty in directly applying tools for modeling hurricane evacuations to disasters, such as terrorist incidents, which occur with little advanced warning.

There have been numerous models developed to simulate evacuations of both structures and cities. Helbing has modeled pedestrian evacuation of a room using the principles of physics. For the transportation aspects, dynamic traffic assignment has become a common methodology; see for instance: Sattayhatewa and Ran (2000); and Sheffi, et al (1981). Two examples of urban or regional evacuation models are NETVAC (Sheffi, et al, 1981), a macroscopic traffic simulation model, and REMS (Tufekci and Kisko, 1991), a model with both macro- and microscopic features. Karbowicz and Smith (1983) employed a heuristic to determine the shortest (in terms of both time and distance) evacuation route in a stochastic network; the type of network they examined was that of a building. The concept of the heuristic is easily transferable to a transportation network, although the number of decision points increases dramatically from that of a building.

Use of simulation models can aid decision makers in determining where the important links are in a network. By examining queue lengths, one can easily identify problem areas; however, the relative importance of a particular link to the connectivity of specific origins and destinations is not always easily determined. This issue is examined more thoroughly in this dissertation.

2.3 VEHICLE ROUTING

This section of the literature review focuses on vehicle routing, which is instrumental to modeling the decisions of a household during an evacuation scenario. Vehicle routing and many of its variants have been studied extensively. The problem and several examples of prior research are presented below.

2.3.1. The Basic Vehicle Routing Problem

In the basic vehicle routing problem (VRP), there is a set of customers with a given demand. A fleet of vehicles is originally stationed at a central depot. The vehicles are sent to the customers to meet their demands. The problem is to minimize the travel cost for the fleet. Capacity constraints for the vehicles must be considered. A common simplifying assumption made by researchers is that the capacities of all of the vehicles are identical.

The VRP, adapted from the formulation of the vehicle routing problem with time windows by Desrochers et al (1988) is as follows:

$$\min \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{2.1}$$

subject to

$$\sum_{j\in N} x_{ij} = 1 \qquad \text{for } i \in N \tag{2.2}$$

$$\sum_{j\in N} x_{ij} - \sum_{j\in N} x_{ji} = 0 \qquad \text{for } i \in N \tag{2.3}$$

$$D_i + t_{ij} \le D_j \qquad \text{for } (i,j) \in I \tag{2.4}$$

$$y_i \le y_j + q_i \qquad \text{for } (i,j) \in I \tag{2.5}$$

$$0 \le y_i \le Q \qquad \text{for } i \in N \tag{2.6}$$

$$x_{ij} \in \{0,1\} \qquad \text{for } (i,j) \in A \tag{2.7}$$

where c_{ij} is the cost of using arc (i,j),

x_{ij} is an integer variable, taking the value 1 if arc (i,j) is used and 0 otherwise,

N is the set of nodes in the graph,

A is the set of arcs in the graph,

I is the set of customers requiring service,

D_i is the departure time from node i,

y_i is the load in the vehicle arriving at node i,

q_i is the demand at customer i, and

Q is the capacity of the vehicle.

The constraints are interpreted as follows. Constraint 2.2 requires every family member to be picked up only once. Equation (2.3) is the constraint that requires the number of vehicles entering an intermediate node is the same as the number of vehicles leaving that intermediate node. Constraint 2.4 ensures that the departure time from j must be greater than the departure time from i and the travel time from i to j. If a link between 2 nodes is used, then the load of the vehicle arriving at the first node is at most the load of the vehicle arriving at the second node plus the demand that was picked up from the first node. Equation 2.6 ensures that the load of the vehicle arriving at node i is less than capacity.

The vehicle routing problem has often been likened to the traveling salesman problem (TSP), in which a salesman starts at the home city and must visit each of the cities in the network once and only once and finally return home (see for example Lin and Kernighan, 1972). Due to the similarities, TSP heuristics can be employed in the solution of VRP's.

In the traveling salesman problem realm, one of the variants is the existence of multiple salesmen, who together must meet all of customer visitations. Simchi-Levi and Berman (1990) investigated the optimal locations and districting for the case where there are two salesmen. In this dissertation, the starting locations of the vehicles is fixed, but among the household's drivers, districting may be performed.

Clarke and Wright (1963) considered a case in which the capacities of the vehicles in the fleet varied. In their work, they noted that if the capacity of the largest vehicle was greater than the sum of all of the customer demands, the problem became a TSP. Some of the assumptions made by Clarke and Wright may not be applicable when the commodity being picked up is people and the household has a limited number of vehicles. The first assumption that may not be applicable is that the demand at the pick-up locations is such that each customer may be serviced by its own vehicle. The second assumption allows for the splitting of loads among vehicles. Initially, this assumption seems ridiculous when the commodity is people; however, this may be allowable when there are multiple children at one school and one of the vehicles has insufficient space for all of the children. Clarke and Wright provide a methodology for solving the problem by hand. The savings associated with connecting two pick-up locations is calculated and locations are linked so as to maximize the savings.

Nag et al (1988) also examined the vehicle routing problem with a heterogeneous fleet and the inability of certain types of vehicles to service some customers. This problem particularly relates to the evacuation problem where a household has more than one vehicle, such as a sports car and a sports utility vehicle (SUV), and different numbers of children at different schools. For instance, the SUV would be needed to pick up three children at elementary school because the sports car only has one additional seat. The sports car could be used to pick up the one child at middle school, or the SUV could be used to collect all of the children. Nag et al propose four heuristic methods to solve this more complicated version of the vehicle routing problem. In the simplest heuristic, the authors create an artificial capacity for all of the vehicles of the same type. This artificial capacity is not applicable to the evacuation scenario since uneven load concerns are ignored, rather, the goal is to collect everyone as rapidly as possible. Like other methodologies, clusters are formed and the nodes within the cluster are sequenced using traveling salesman techniques; again, this needs to be carefully adapted to the evacuation scenario since the vehicles are not necessarily returning to their points of origin.

Another variation of the VRP, that is relevant to the evacuation problem, has been investigated by Laporte, et al (1984). The variation was to constrain the maximum distance traveled by any vehicle. This distinction is particularly relevant to the case when family members are attempting to reach a meeting place at approximately the same time (see Chapter 4). Laporte, et al, treats the upper bounds on the maximum distance as constraints; whereas the formulation for this dissertation incorporates the desire for similar arrival times as part of the objective function.

In earlier work, Russell (1977) bounded the maximum travel distance for the M-tour TSP. Russell's (1977) description of the M-tour traveling salesman problem is nearly identical to that of the vehicle routing problem with differences being found in the constraints. Another of the constraints was related to timing. Some cities, or customers, were only available for visitation during certain time windows. This work appears to be an early generalization of the vehicle routing problem with time windows, which is discussed in the following section.

2.3.2. Vehicle Routing with Time Windows

The vehicle routing problem with time windows (VRPTW) is similar to the vehicle routing problem with additional constraints that require the vehicles to arrive at the customer location within a given time frame. Any early arrivals incur waiting time. Golden and Assad (1986) present a general description of the problem.

The VRPTW is known to be NP-hard, meaning that solution procedure is known to exist that is of polynomial computational complexity (Baker and Schaffer, 1986). Many of the previous works in this area present heuristic methods for solving this problem.

Solomon (1987) presented heuristics to solve the VRPTW that were extensions of previously developed VRP heuristics. The added complexity was in the incorporation of time. The assumption of a homogeneous fleet simplifies the problem by eliminating the need to associate different capacity constraints with individual vehicles. Among the heuristics extended were savings, time-oriented nearest neighbor, and insertion. The insertion technique was recommended based on the problems considered.

Kolen, et al (1987) used branch-and-bound techniques to solve the VRPTW. The underlying assumptions of Kolen, et al's research match those of Solomon (1987) in that there is a single depot for a fleet of homogeneous vehicles.

Baker and Schaffer (1986) modified the branch exchange techniques commonly used to solve the VRP to account for the additional constraints of time windows. The use of branch exchange techniques to improve existing heuristics was an extension of a then working paper by Solomon. For the branch exchange procedure, there may be a reordering of the nodes within a given vehicle's route or there may be a switching of two arcs between two vehicles' routes. By employing the branch exchange techniques, Baker and Schaffer (1986) were able to find solutions that were closer to optimality than the original tours generated using the original nearest neighbor and insertion heuristics.

Solomon, Baker, and Schaffer (1988) focused on the extension of the branch exchange solution improvement procedures to the time window constrained vehicle routing and scheduling problem and implementation methods for these procedures. In order to reduce computation time, the authors eliminated unnecessary feasibility checks that were due to the nature of the problem. The complexity of the algorithms is actually increased while the running time was decreased.

There are several differences between previous works and the research presented here. First, the vehicles are not located at a single depot. In this problem, the vehicles are assumed to be located wherever their drivers are at the time the evacuation begins. For instance, the starting location of vehicles may include work places, shopping or recreation areas, home, and high schools. Second, the fleet of vehicles available to a household is not assumed to be homogeneous. In the case where a household owns more than one vehicle, one may be a sports utility vehicle, a family sedan, or a sports car. The capacities of these vehicles vary.

The time windows as defined for the typical VRPTW are not directly applicable to the case at hand. In the initial formulation considered here, no time windows are considered; however, time windows could be included in certain scenarios, such as flooding or hazardous materials incidents. The nature of some emergencies requires that people close to the incident be evacuated first. In these situations, there may not be a specific time window for the affected citizens to be picked up; rather, if they are not picked up before a certain time, another agency will move them to a safer location.

This work shares several assumptions with the previous studies discussed in this section. Like Solomon (1987), all vehicles are initially assumed to leave their starting locations at the earliest possible time. In the evacuation problem, the driver sees no benefit to waiting at the origin. This assumption may be modified in the event that the incident is localized and initially contained, but allowed to spread after an initial evacuation has begun. Each vehicle is assumed to have a pre-specified capacity, though not all of the vehicles are assigned the same capacity. Additional similarities between the vehicle routing problem and the research presented here will be shown in chapter 4.

2.4 SUMMARY

This chapter has presented an overview of the literature related to this dissertation. The first section was related to network reliability and vulnerability, which pertains to chapter 3. In the second portion of this chapter, observed evacuation behavior was discussed. In the third section, the vehicle routing problem and some of its variants was presented. Both the second and third parts of this chapter relate to chapter 4. Since chapter 5 incorporates the methodologies from chapters 3 and 4, all of the literature presented in this chapter pertains to chapter 5.

Chapter 3

Identification Of Vulnerable Transportation Infrastructure

Critical transportation infrastructure consists of links that are particularly important to the connectivity of origins and destinations. Intuitively, these links are bridges, tunnels, or other roadways that connect multiple origins and destinations and carry heavy volumes of traffic. Proving that intuition is indeed correct can be difficult. As noted in chapter 2, the most vital links are defined as those whose removal from the network results in the greatest increase in shortest path travel time (Corley and Sha, 1982; Malik, Mittal, and Gupta, 1989; and Ball, Golden, and Vohra, 1989). Identifying the optimal solution to the most vital arcs problem is extremely difficult because it requires complete enumeration of all of the options. Furthermore, the most vital arc problem primarily applies to single origin-destination pairs and not the network as a whole. The classic minimum cut problem also identifies a set of links whose removal from the network will completely sever the destination from the origin. Neither of these two approaches allows for the determination of relative importance of links other than these vital arcs. The relative importance of all links can be used by traffic management agencies under emergency conditions due to natural disasters as well as anthropic disasters. Furthermore, the solution to the minimum cut problem may not be unique. Neither the most vital arcs problem nor the minimum cut problem accounts for the resources that may be required to remove the links from the network.

With the increase in global awareness of terrorism, the issue of physically disabling roadways, bridges, and tunnels has become of greater concern. Terrorists, or evil entities, have a limited amount of resources with which to cause damage to a transportation network. With the resources available, the evil entity seeks to inflict the maximum disruption to the network in terms of both connectivity and the amount of vehicles that are impacted.

In this chapter, a vulnerability index is developed that identifies the relative importance of a link, or set of links, to the connectivity of a given origin-destination pair. An aggregation of the vulnerability indices over the network's origin-destination pairs yields the disruption index. This disruption index is used to address the problem: given a limited amount of resources for causing damage to a transportation network, the transportation network itself, and a traffic assignment determine the set of links whose damage causes the maximum disruption to the network. The problem is formulated as a bi-level mathematical programming model. At the lower level is the system optimal traffic assignment problem. At the upper level is a linear integer program that has the objective of maximizing the damage to the network in terms of the disruption index.

This bi-level mathematical program can be viewed as a game between a traffic management agency and an evil entity. The evil entity's objective is represented by the upper level problem while the traffic management agency (TMA) is represented at the lower level. The evil entity selects a set of roads to target from a list of scenarios based on the available resources. The selected scenario has the greatest disruption index. The TMA's strategy depends on the information available to it. Four games of varying information are examined in this chapter. In the first, the traffic management agency has no knowledge of the threat from the evil entity. In the second game, the traffic management agency knows that the evil entity is planning to disrupt the network, but the evil entity is unaware that the traffic management agency has this information. In the third game, the traffic management agency knows that the evil entity is planning an attack and reroutes vehicles to avoid these links while ensuring that origin-destination demands are met. For this scenario, all of the resources are consumed simultaneously. Finally, the fourth game is similar to the third except that links are damaged sequentially, rather than simultaneously. In this chapter, the games are conducted on a simple network for ease of explanation. In chapter 5, a larger network is examined.

The remainder of this chapter is divided into the following sections. First, the vulnerability and disruption indices are developed. Second, the bi-level mathematical formulation of the problem is presented. Third, a small sample network is evaluated in terms of the four games. Finally, a summary of the chapter is presented.

3.1 DEVELOPMENT OF THE DISRUPTION INDEX

In this section, a methodology for the determination of two indices is presented. First, a vulnerability index is developed. This index is a measure of importance of a link, or set of links, to the connectivity of an origin-destination pair based on current traffic and infrastructure states. Second, a disruption index is developed. The disruption index is the aggregation of the vulnerability indices across all origin-destination pairs, thus providing a state-based measure of network vulnerability. The disruption index is the measure by which the "evil-entity" is envisioned to select links to damage.

The vulnerability index explicitly accounts for flow, the availability of alternate paths, travel time, marginal costs, and capacity of links. Traffic conditions may be generated by different methods. User equilibrium minimizes travel time from the individual driver's perspective. System optimal traffic assignment, used in this chapter, minimizes travel time at the network level. This assignment yields the best possible traffic conditions from the network, not the individual driver's perspective. When the traffic management agency has control over the traffic, the vehicles are routed to optimize conditions at the network level. In the games discussed in this chapter, the TMA seeks to route vehicles so as to avoid threatened links. Without guidance from the TMA, more drivers than necessary may choose paths containing vulnerable links and unnecessarily put themselves in danger.

Within a scenario, a set of links is examined. The amount of resources available to an evil entity determines the number of links in the scenario. The number of possible scenarios increases combinatorially with an increase in the amount of available resources. To determine the vulnerability index for a given origin-destination (O-D) pair, the flow on the scenario's links is examined. The formulation of the index seeks to find other paths, with excess capacity, for the flow on the link(s) of interest. Scenarios that consist of more than one link present a challenge. The flows are not necessarily additive. For instance, two links may lie on the same path and the flow exits one of the links and enters the other. This is the same flow and, therefore, cannot be added to itself. Therefore, the relationship between link and path flows must be known. In this work, the relationships between the links and paths are known.

When the flow on the scenario's link(s) belongs to more than one O-D pair (multi-commodity flow), the allocation of excess capacity becomes an issue. For this dissertation, no prioritization among the O-D pairs is permitted. If there is insufficient excess capacity to accommodate the flow on the links of interest for a given origin-destination pair, the links are critical to that O-D pair and the vulnerability index takes its maximum value of 1.0.

Examination of alternate paths for the accommodation of the flow on the link(s) of interest implements a utility of the alternate path. This utility incorporates the amount of excess capacity available for a given O-D pair, the maximum flow service rate, the free flow travel time, and the marginal path cost (travel time). Provided there is sufficient excess capacity to accommodate the flow on the scenario's link(s), greater utilities of the alternate paths indicate less vulnerable links. The methodology for the determination of the vulnerability index, and subsequently, the disruption index is presented below. The notation that is used for the development of the index and subsequent sections of this chapter is given in table 3.1.

The transportation network is represented as a directed graph ***G(N,A)*** consisting of a set of nodes ***N*** and a set of arcs ***A*** connecting those nodes. Some of these nodes are origins (***R***), some are destinations (***S***), and some are intermediate nodes with no vehicles entering or leaving the network at those points. The total demand from a given origin to a given destination $q^{r,s}$ is given as a parameter. Traffic is assigned to paths connecting the origin-destination pairs; the path flows are known. The non-negative cost t_l of using each link $l \in A$ is known, as well as the current link demand x_l.

Table 3.1 Notation for the Development of the Disruption Index and the Bi-Level Formulation

Notation	Interpretation
	Sets and Indices
h_j	Bottleneck link of path j
I	Set of possible scenarios for link damage
i	Scenario index
j	Path index
l, a	Link indices
L_j	Set of links in path j
L^i	Set of links in scenario i
R	Set of origin nodes
r	Origin index
S	Set of destination nodes
s	Destination index
	Parameters
$K^{r,s}$	Total number of paths connecting r and s – may be limited by user
ρ_l	Maximum service flow rate of link l (vph)
$\rho_j^{r,s}$	Maximum service flow rate of path j from origin r to destination s (vph)
$q^{r,s}$	Total origin-destination demand (vph) from r to s
$T_0^{r,s}$	Path travel time threshold for origin-destination pair (r,s)
T^0_j	Free flow path travel time for path j (min)
	Variables
c_l	Excess capacity of link l (vph) [0, ρ_l]
$C_j^{r,s}$	Excess capacity on path j available to r,s (vph)
D_i	Value of disruption for scenario i [0, |R|x|S|]
$f_j^{r,s}$	Flow on path j from r to s (vph)
$g_j^{r,s}$	Utility of alternate path j [0, 1]
$k_i^{r,s}$	Number of alternate paths needed to accommodate $x_i^{r,s}$
$M_i^{r,s}$	Adjusted vulnerability index for link l evaluated for r,s
$t_l(x_l)$	Flow dependent link travel time (min) on link l
τ_j	Marginal travel time (min) of path j
U	Value to be maximized in the upper level problem
$V_i^{r,s}$	Vulnerability index for scenario i evaluated for origin-destination pair (r,s)
x_l	Total link flow (vph)
$x_l^{r,s}$	Flow on link l from r to s (vph)
$X_{i,j}$	Amount of flow on link in L^i to be accommodated by alternate path j
X^i	Total flow on the links in L^i
$X_i^{r,s}$	Total flow on the links in L^i corresponding to origin r and destination s
y_i	Integer decision variable; = 1 if scenario i is selected and 0 otherwise
Φ	Arc-path incidence matrix
Φ_i	Arc-path incidence matrix for links in scenario i

For a given scenario *i*, the O-D flow that is affected is the sum of the O-D flows on the paths containing the links ($\boldsymbol{L^i}$) in scenario *i*:

$$X_i^{r,s} = \sum_j \Phi_i f_j^{r,s} \qquad (3.1)$$

The total flow is the sum of the flows on paths containing the links of interest, or the sum of the origin-destination specific flows:

$$X_i = \sum_{r,s} X_i^{r,s} \qquad (3.2)$$

If the set of links ($\boldsymbol{L}^{i}$) in the scenario were damaged, X_i is the amount of flow that would have to be accommodated by excess capacity on alternate paths. These alternate paths cannot contain any of the links in $\boldsymbol{L^i}$.

Let $\boldsymbol{L_j}$ be the set of links in path $\boldsymbol{j}$. Let $\boldsymbol{h_j}$ be the bottleneck link of path $\boldsymbol{j}$, where the bottleneck is defined as the link with the minimum excess capacity $\boldsymbol{c_l}$. Excess capacity is calculated as the difference in the link maximum service flow rate $\boldsymbol{\rho_l}$ and the current flow $\boldsymbol{x_l}$ on link $\boldsymbol{l}$.

$$c_l = \rho_l - x_l \qquad (3.3)$$

The path service rate $\rho_j^{r,s}$ is the minimum service rate of the links in the path:

$$\rho_j^{r,s} = \min_{l \in L_j} \rho_l \,. \qquad (3.4)$$

As a cursory first step to determining whether $X_i^{r,s}$ can be accommodated by the remainder of the network, the classical maximum flow problem is employed. Flow from and an origin is maximized, subject to flow conservation constraints, capacity constraints, and non-negativity constraints (Bertsimas and Tsitsiklis, 1997). This step is repeated for every O-D pair with flow on links in $\boldsymbol{L^i}$. In this cursory step, all of the excess capacity is allocated to the O-D pair under consideration. If the maximum flow through the network, without $\boldsymbol{L^i}$**,** using c_l as the capacity of link l is less than $X_i^{r,s}$, there is no need to continue, the origin-destination vulnerability $V_i^{r,s}$ is the maximum (1.0).

Once it has been determined that $X_i^{r,s}$ can be accommodated by the remaining network, the alternate paths that are considered are restricted by several factors. First, as previously mentioned, the path may not contain any of the links in the set $\boldsymbol{L^i}$. Second, the travel time $T_j^{r,s}$ on the alternate path j must be less than some threshold value $T_0^{r,s}$. This threshold is determined by the analyst and serves two purposes: (1) to eliminate paths with endless cycles and (2) to reduce the number of paths considered. Alternate paths are considered in order of marginal path travel times with the lowest marginal travel time path being considered first. The number of alternate paths ($k_i^{r,s}$) considered depends on the excess path capacity available to the O-D pair ($C_j^{r,s}$), which is defined as the minimum of adjusted link excess capacities. These link excess capacities are adjusted by the ratio of the scenario O-D flow to the total scenario flow that could use the link (see equation 3.5). In other words, a portion of the link's excess capacity is allocated for all of the affected origin-destination pairs that could use the link.

$$C_j^{r,s} = \min_{l \in L_j} c_l \left(\frac{X_i^{r,s}}{\sum\limits_{r',s'} X_i^{r',s'}} \right) \tag{3.5}$$

where (r',s') is an O-D pair with flow on the scenario's links.

Alternate paths are considered until $X_i^{r,s}$ has been accommodated, a predetermined maximum number of alternate paths $K^{r,s}$ have been considered, or no additional alternate paths exist. When $X_i^{r,s}$ has been accommodated on alternate paths, the number of paths used to accommodate this flow is $k^{r,s}$. If $X_i^{r,s}$ has not been accommodated by alternate paths before $K^{r,s}$ is reached or no additional alternate paths are available, the set of links L^i forms a "flow dependent cut set." For the purposes of this work, a flow dependent cut set is a set of links whose removal from the network will result in the inability of the network to transmit the origin-destination demand. The vulnerability index ($V_i^{r,s}$) takes its maximum value (1.0) for a scenario in which a flow dependent cut set is formed.

If the set of links $\boldsymbol{L^i}$ forms neither a cut-set (from the cursory step) nor a flow dependent cut set, the flow $X_i^{r,s}$ can be accommodated by alternate paths.

The utility (g_j) of alternate path j is then determined for the given $\boldsymbol{r}$ and $\boldsymbol{s}$. The utility of the alternate path is a measure of the relative usefulness of the alternate path. In this dissertation, "utility" is the combination of the relative capacity and the ratio of the free flow path travel time and the marginal path travel time for alternate path $\boldsymbol{j}$ shown in equation (3.6).

$$g_j^{r,s} = \frac{C_j^{r,s}}{\rho_{h_j}} \frac{T_j^0}{\tau_j} \tag{3.6}$$

In the denominator of the first term of the right hand side, the saturation flow is a characteristic of the link type, such as freeway or arterial, signalized or unsignalized. For the period of analysis, the saturation flow is treated as a constant. The excess capacity of the alternate path, in the numerator of the first term, may vary among evaluation periods (such as peak and off-peak). The excess capacity cannot exceed the saturation flow, resulting in a maximum value of 1.0 for that ratio. The first term indicates that as the excess capacity of the bottleneck link increases, the feasibility of that alternate path increases. If the excess capacity of path j ($C_j^{r,s}$) is zero, the bottleneck link is at capacity and that path is not a viable alternative.

As in the first term, the second ratio of the right hand side of equation (3.6) contains a term describing characteristics of the baseline path and a term describing the current state of the path/network. The ratio of the free flow travel time on path $\boldsymbol{j}$ and the marginal path travel time is bounded by 0.0 and 1.0. The upper bound may be reached but the lower bound is never achieved, only approached. Since each of the ratios in equation (3.6) are bounded by 0.0 and 1.0, the utility $g_j^{r,s}$ of alternate path j is bounded by 0.0 and 1.0, inclusive.

In the formulation of the vulnerability index (equation 3.7), the utility of the alternate paths required to accommodate the flow on the damaged link are multiplied by the proportion of that flow that would be diverted to that path. Use of the proportion bounds the vulnerability index when the adjusted utilities are aggregated to form the disruption index. The vulnerability index for scenario *i* with respect to origin *r* and destination *s* ($V_i^{r,s}$) ranges from 0.0 to 1.0 with 1.0 indicating that links in $\boldsymbol{L^i}$ are extremely important to the connectivity of *r* and *s* given the current state of the network.

$$V_a^{r,s} = \begin{cases} 1.0 & if\ k^{r,s} > K^{r,s} \\ 1.0 - \sum\limits_{j=1}^{k^{r,s}} g_j^{r,s} \dfrac{X_{a,j}}{x_a^{r,s}} & otherwise \end{cases} \tag{3.7}$$

When $k_i^{r,s}$ exceeds $K^{r,s}$, an insufficient number of alternate paths available are to accommodate the O-D flow from the link of interest due to network constraints or the method in which excess capacity is allocated. In order for scenario *i* to have an index of 0.0, there must be at least one alternate path that currently has no flow and can accommodate $X_i^{r,s}$. This unlikely set of conditions suggests that each link will have a positive vulnerability index.

To complete the interpretation of the vulnerability index at the origin-destination level, an adjustment factor is applied to $V_i^{r,s}$ based on the proportion of the origin-destination flow carried on the link(s) of interest. Let $\chi_i^{r,s}$ denote the coefficient of $V_i^{r,s}$.

$$\chi_i^{r,s} = \left(\frac{X_i^{r,s}}{q^{r,s}} \right) \tag{3.8}$$

The general form of the adjusted vulnerability index $M_i^{r,s}$ is given in equation (3.9).

$$M_i^{r,s} = \chi_i^{r,s} V_i^{r,s} \tag{3.9}$$

Finally, the disruption index (D_i) for scenario *i* is defined in equation (3.10) as the sum, over all origin-destination pairs, of the adjusted vulnerability indices of scenario *i*.

$$D_i = \sum_{r,s} M_i^{r,s} \tag{3.10}$$

The disruption index is bounded by 0.0 and the number of origin-destination combinations ($|\boldsymbol{R}|$x$|\boldsymbol{S}|$). A value of 0.0 indicates that, for the given traffic conditions, damaging the set of links $\boldsymbol{L^i}$, would have no impact on the given traffic. A value at the upper bound indicates that the set of links $\boldsymbol{L^i}$ affects every origin-destination pair in the network, these links carry all of the flow, and there is insufficient capacity to accommodate this flow on alternate paths.

3.2 BI-LEVEL FORMULATION

In this section, a bi-level formulation is presented to identify vulnerable links and sets of links in the transportation network. Game theory can be used to solve the formulation. In the game context, one player represents an "evil entity" seeking to disrupt the network to the greatest degree given the resources available. The disruption index, developed in section 3.1, is the criterion by which this player makes decisions. The other player in the game is a traffic management agency who seeks to route traffic away from the vulnerable links.

The transportation network is represented by a directed graph $\boldsymbol{G(N,A)}$ consisting of a set of nodes $\boldsymbol{N}$ and arcs $\boldsymbol{A}$. A flow dependent cost is associated with traversing each arc $\boldsymbol{l}$, and is in terms of travel time $\boldsymbol{t_l(x_l)}$.

3.2.1 Formulation

The upper level problem (P1) is a decision making problem for the "evil entity." This decision maker first examines his resources to determine how many links ($\boldsymbol{n}$) he can damage. Based on the resources, he creates scenarios, which are sets of ***n*** links to be damaged simultaneously. The scenario selected maximizes the disruption across all origin-destination pairs in the network.

(P1) $$\max U = \sum_{i=1}^{I} D_i\ y_i \qquad (3.11)$$

subject to

$$\sum_{i=1}^{I} y_i = 1 \qquad (3.12)$$

$$y_i \in \{0,1\} \qquad (3.13)$$

The decision variables of P1 are binary integers which take the value 1 if scenario ***i*** is selected and 0 otherwise. Equation (3.12) ensures that only one scenario is chosen.

The lower level problem (P2) represents the minimization of travel time for all users in the network. This system optimal traffic assignment (Sheffi, 1985) represents the case where traffic managers direct vehicles to different paths. By using this formulation, the network is at its best possible state from the collective view of the users.

(P2) $$\min z(x) = \sum_{l \in A} x_l \big(t_l(x_l)\big) \qquad (3.14)$$

s.t. $$\sum_j f_j^{r,s} = q^{r,s} \quad \forall\, r,s \qquad (3.15)$$

$$f_j^{r,s} \geq 0 \qquad \forall\ j,r,s \qquad (3.16)$$

$$x^A = \Phi^{|A| \times |J|} f^J \qquad (3.17)$$

where $\boldsymbol{x^A}$ is the vector of arc flows $x_l \in x^A$ and

$\boldsymbol{f^J}$ be the vector of path flows.

The objective function of P2 minimizes the flow dependent travel time for all network users. Equation (3.15) ensures that the sum of the flows on the paths connecting ***r*** and ***s*** meet the demand for the origin-destination pair. The second constraint is the non-negativity constraint. The final equation relates the link and path flows through an arc-path incidence matrix $\Phi^{|A|x|J|}$ (Jahn, et al, 2002); the values of the entries of this matrix are 0 if link *l* does not lie on path *j* and 1 if link *l* does lie on path *j*.

Since the lower level problem is to be solved before the upper level problem, P2 can be incorporated into the constraint set of P1 (see Shimizu, et al., 1997). Let $\boldsymbol{X}^*$ be an optimal solution vector of flows from the system optimal formulation. The coefficient D_i is a function of $\boldsymbol{X^*}$ and can be represented as $D_i(\boldsymbol{X^*})$. The two problem formulations P1 and P2 are combined into a bi-level formulation in P3.

(P3) $$\max U = \sum_{i=1}^{I} D_i\left(X^*\right) y_i \tag{3.18}$$

s.t. $$\sum_{i=1}^{I} y_i = 1 \tag{3.19}$$

$$y_i \in \{0,1\} \tag{3.20}$$

$$X^* \Leftarrow \min \sum_{l \in A} \sum_{r,s} x_l^{r,s} t_l \left(\sum_{r,s} x_l^{r,s} \right) \tag{3.21}$$

s.t. $$\sum_j f_j^{r,s} = q^{r,s} \quad \forall r,s \tag{3.22}$$

$$f_j^{r,s} \geq 0 \qquad \forall\ j,r,s \tag{3.23}$$

$$x_l^{r,s} \geq 0 \tag{3.24}$$

$$x^A = \Phi^{|A| \times |J|} f^J \tag{3.25}$$

The lower level problem is a nonlinear programming formulation in terms of the decision variables $\boldsymbol{x_l^{r,s}}$. The coefficients (D_i) of the decision variables of P1 are functions of the decision variables of P2’. Because the lower level problem is solved before the upper level problem, the upper level problem is linear in terms of $\boldsymbol{y_i}$. The upper level problem is solved for each alternative optimum found in the lower level problem.

3.2.2 Solution Framework

The bi-level problem is solved sequentially. The lower level problem is continuously differentiable over the allowable range of $\boldsymbol{x}$. As noted by Lee and Nie (2001) and Mouskos and Mahmassani (1989), the system optimal traffic can be solved by modified versions of the Frank-Wolfe algorithm (Bertsekas, 1999, p.215-218). The resulting vector of link flows is used in the calculation of the upper level decision variable coefficients. The upper level problem is not continuously differentiable in $\boldsymbol{y}$, due to the discrete nature of the variable. The linear nature of the upper level problem allows for solution methodologies such as the Simplex Algorithm (see Bertsimas and Tsitsiklis, 1997, Ch.3, pp. 81-137).

3.2.3 Sample Network

The simple network shown in figure 3.1 will be used for this discussion. The network consists of six nodes and eight links. Two of the nodes (1 and 5) are origins and two are destinations (2 and 6), resulting in four origin-destination pairs.

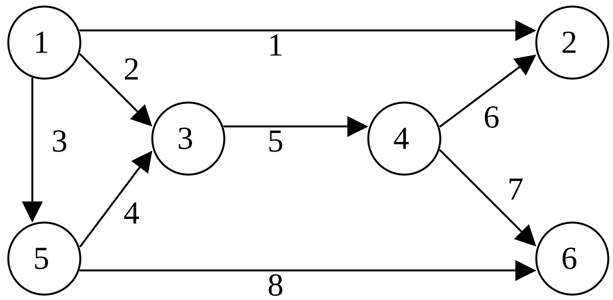

Figure 3.1 Sample Network

The link characteristics for the network shown in figure 3.1 are presented in table 3.2.

Table 3.2 Link Characteristics for the Sample Network

Link	1	2	3	4	5	6	7	8
Max Service Rate (vphpl)	2000	1700	1600	1700	1800	1800	1800	2000
Number of Lanes	2	1	1	1	2	1	1	2
Free Flow Travel Time	10	2	2	1.5	6	2	3	11

For the example presented in this paper, the flow dependent travel times are determined from the BRP formula (equation 3.26). The variable $\boldsymbol{t_f}$ represents the free flow travel time on the link.

$$t_l = t_f + 0.15\left(\frac{x_l}{\rho_l}\right)^4 \tag{3.26}$$

For illustration purposes, three different demand levels are examined. The first set of total origin-destination demands are $q^{1,2}$ = 3000 vph, $q^{1,6}$ = 1000 vph,

$q^{5,2}$ = 1100 vph, and $q^{5,6}$ = 2500 vph. The second set of demands are three quarters of the first set: $q^{1,2}$ = 2250 vph, $q^{1,6}$ = 750 vph, $q^{5,2}$ = 825 vph, and $q^{5,6}$ = 1875 vph. The third set of demands is half of the first set: $q^{1,2}$ = 1500 vph, $q^{1,6}$ = 500 vph, $q^{5,2}$ = 550 vph, and $q^{5,6}$ = 1250 vph. The resulting flow distributions and disruption indices are presented in table 3.3.

3.2.4 Results and Discussion for Single Links

The results for the sample network shown in figure 3.1 and the link characteristics in table 3.2 are displayed in tables 3.3 and 3.4. The first table presents one of the optimal flow distributions resulting from the system optimal traffic assignment and the resulting values of the vulnerability indices. The second table presents the values of the disruption indices for each of the links.

Table 3.3 Flow Distribution and Vulnerability Indices Results

	Demand Level	Arc 1	Arc 2	Arc 4	Arc 5	Arc 6	Arc 7	Arc 8
(x_l)	Original	2300	1700	1700	3400	1800	1600	1900
	¾	2250	750	1306.67	2056.67	825	1231.67	1393.33
	½	1500	500	1314.98	1814.98	550	1264.98	485.02
$x_l^{1,2}$	$q^{1,2}$ = 3000	2300	700	0	700	700	0	0
	$q^{1,2}$ = 2250	2250	0	0	0	0	0	0
	$q^{1,2}$ = 1500	1500	0	0	0	0	0	0
$x_l^{1,6}$	$q^{1,6}$ = 1000	0	1000	0	1000	0	1000	0
	$q^{1,6}$ = 750	0	750	0	750	0	750	0
	$q^{1,6}$ = 500	0	500	0	500	0	500	0
$x_l^{5,2}$	$q^{5,2}$ = 1100	0	0	1100	1100	1100	0	0
	$q^{5,2}$ = 825	0	0	825	825	825	0	0
	$q^{5,2}$ = 550	0	0	550	550	550	0	0
$x_l^{5,6}$	$q^{5,6}$ = 2500	0	0	600	600	0	600	1900
	$q^{5,6}$ = 1875	0	0	481.67	481.67	0	481.67	1393.33
	$q^{5,6}$ = 1250	0	0	764.98	764.98	0	764.98	485.02
$M_l^{1,2}$	$q^{1,2}$ = 3000	0.7667	0.1350	0*	0.1350	0.1350	0	0
	$q^{1,2}$ = 2250	1	0*	0*	0*	0*	0	0
	$q^{1,2}$ = 1500	1	0*	0*	0*	0*	0	0
$M_l^{1,6}$	$q^{1,6}$ = 1000	0	0.0029	0*	0.0029	0	0.0029	0*
	$q^{1,6}$ = 750	0	0.0411	0*	0.0008	0	0.0008	0*
	$q^{1,6}$ = 500	0	0.0036	0*	0.00001	0	0.00001	0*
$M_l^{5,2}$	$q^{5,2}$ = 1100	0	0	1	1	1	0	0
	$q^{5,2}$ = 825	0	0	1	1	1	0	0
	$q^{5,2}$ = 550	0	0	1	1	1	0	0
$M_l^{5,6}$	$q^{5,6}$ = 2500	0	0	0.1144	0.1144	0	0.1144	0.7600
	$q^{5,6}$ = 1875	0	0	0.0899	0.0899	0	0.0899	0.7431
	$q^{5,6}$ = 1250	0	0	0.0749	0.0749	0	0.0749	0.3880

* indicates that the value is zero based solely on the flow assignment; a path including that link does exist for the origin-destination pair. Arc 3 was omitted from the table because no flow was assigned to it and the resulting values of the vulnerability indices were 0 for all cases.

The data in the table above show some general trends and a couple of points that fall outside of generalities. The flow for origin-destination pair (1,2), is assigned to link 1 the majority of the time. For the highest demand case, not all of the demand is assigned to this path, unlike the lower demand cases. The reason for this result is that the alternate path (links 2, 5, and 6) becomes competitive, in terms of flow dependent travel time. For ODs (1,6) and (5,2) all of the flow is assigned to the same path for each of the demand levels examined. Only one path connects node 5 to node 2, but for (1,6), the traffic assignment is a result of travel time. Origin-destination pair (5,6) offers the opportunity to examine capacity constraints. The path consisting of links 4, 5, and 7 has a smaller travel time than the alternate path (link 8). The existence of only one path connecting nodes 5 and 2 leads to a priority assignment of this OD flow to the links in the single path, which shares links with the shortest time path for (5,6). At the highest demand level, the capacity constraint on link 4 prohibits any additional flow (above 600vph) for (5,6) from being assigned to path 4, 5, 7. The traffic assignments at the lower demand levels reflect the dependency of travel time on the link flow levels.

Although intuition may lead to the expectation that increased demand will lead to higher vulnerability indices, this is not always true. The index formulation captures more of the intricacies of traffic assignment and network design than intuition. The value of the indices for the links connecting node 5 and node 2 do not change, as expected, since there is only one path, and therefore, each link is critical. At the highest demand level in this example, link 1 achieves its lowest vulnerability index. This result captures the fact that only at this level is the OD flow distributed between two possible paths. The most counter-intuitive result occurs for OD pair (1,6). The lower two demand levels follow the expected trend of more traffic leads to a higher vulnerability index. This generalization is followed by links 5 and 7 for all demand levels. However, link 2 has a higher vulnerability index than links 5 and 7 for the lower two demand levels. The design of the network allows for three alternate paths to exist between node 1 and node 6. Only one of these paths contains link 2, while two of the paths contain links 5 and 7. This difference allows the value of the index to vary between link 2 and links 5 and 7. The reason for the index being greater for link 2 is that the alternate path with the lower marginal travel time also has less excess capacity so the overall utility for the vehicles that would be reassigned to that path is lower, resulting in a higher vulnerability index. The network does better as a whole using the marginal path cost as the order for reassignment rather than ordering by path excess capacity. At the highest demand level, the index is the same for all of the links in the path; this is due to one of the alternate paths having no excess capacity to accommodate the flow on link 2. Origin-destination pair (5,6) does follow the intuitive trend and the results show an increase in vulnerability indices for both paths as demand increases (see also figure 3.2).

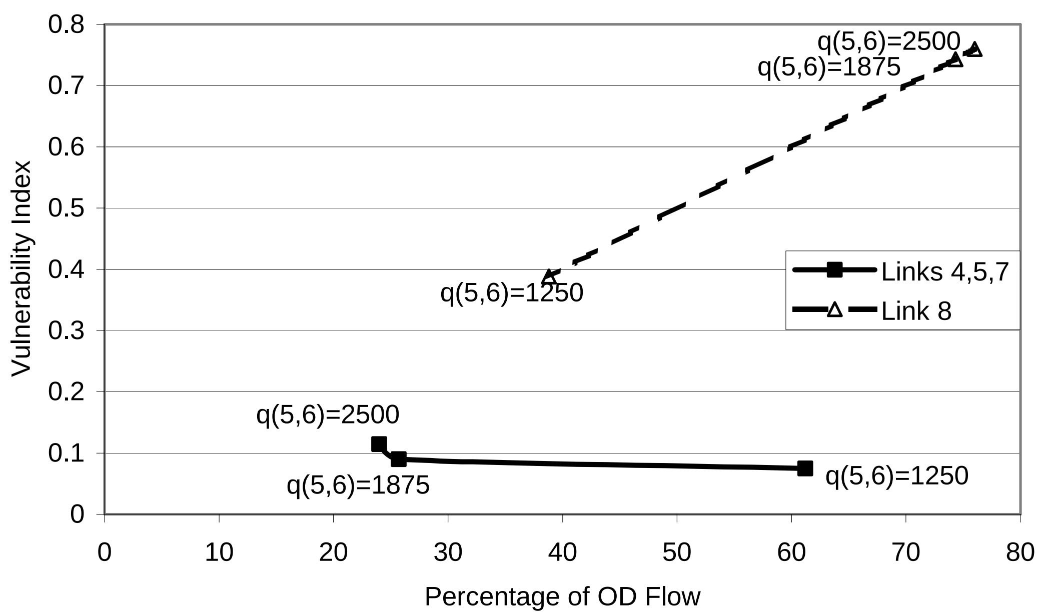

Figure 3.2 Comparison of Vulnerability Indices for Alternate Paths for OD (5,6)

Figure 3.2 shows some of the complexities in predicting the relative importance of a specific link to the connectivity of an origin-destination pair. The curve connecting the data points for link 8 is linear and has a positive slope; this reflects two attributes of the network state. First, insufficient capacity exists on alternate paths to accommodate the flow on link 8 from node 5 to node 6.

Second, the proportion of the OD flow assigned to link 8 maps directly to the vulnerability index. Links 4, 5, and 7 have the same vulnerability index for the given origin-destination pair. This result reflects the fact that these three links lie on the same path and no alternate path for that OD pair shares one of these links. The non-linear nature of the curve connecting the data points for these links indicates that an alternate path can accommodate the flow on the path consisting of links 4, 5, and 7, for at least some demand levels. Furthermore, the relatively low value of the vulnerability index emphasizes the existence of a viable alternative path with sufficient excess capacity.

The vulnerability index is an appropriate measure for determining the importance of a link to origin-destination connectivity, but the disruption index needs to be calculated for the network level. The disruption indices are calculated as the sum of the vulnerability indices across all origin-destination pairs. The disruption indices for the single arcs and different demand levels are given in table 3.4. This measure allows for an ordering of links in terms of vulnerability from the network perspective.

Figure 3.3 provides a demonstration of how the disruption index can be vastly different from the vulnerability index shown in figure 3.2. In the figure below, the disruption indices are not identical for links 4, 5, and 7. This is due to the assignment of various other OD flows to these links. The disruption index for link 8 is the same as the vulnerability index because the only traffic assigned to that link is for the origin-destination pair (5,6).

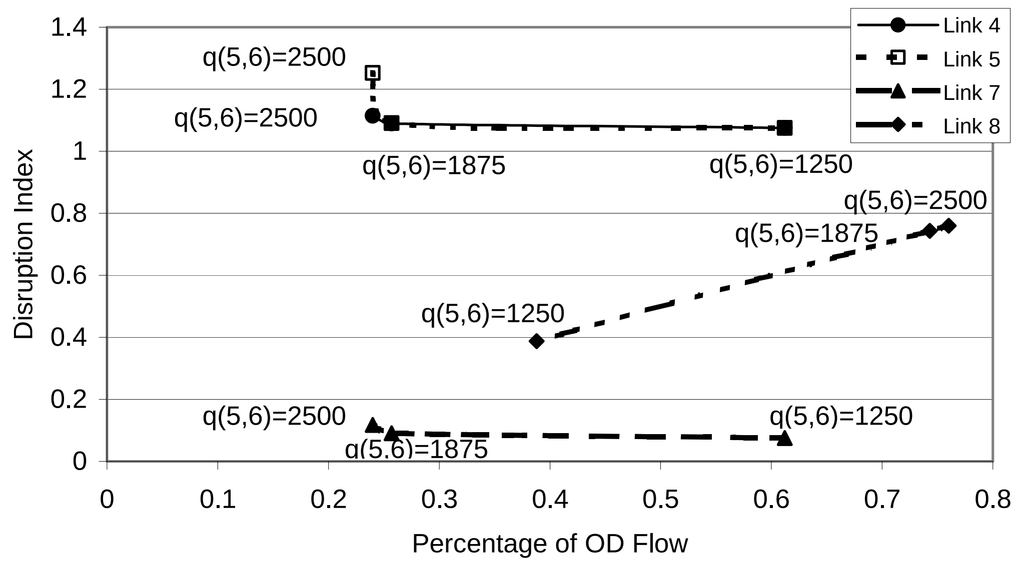

Figure 3.3 Comparison of Disruption Indices and Percentage of OD (5,6) Flow Assigned to Link

Table 3.4 Disruption Indices for the Sample Network and Various Demand Levels

Demand Level	Arc 1	Arc 2	Arc 3	Arc 4	Arc 5	Arc 6	Arc 7	Arc 8
Original	0.7667	0.1379	0	1.1144	1.2523	1.1350	0.1173	0.7600
¾	1.0000	0.0411	0	1.0899	1.0907	1.0000	0.0907	0.7431
½	1.0000	0.0036	0	1.0749	1.07491	1.0000	0.07491	0.3880

From the results shown in table 3.4, for all three demand levels, the evil entity would select link 5 as the target. One of the obvious reasons for this selection is that link 5 lies on paths connecting each of the origin-destination pairs. For any single origin-destination pair, link 5 does not dominate all of the other links in the network, but when the network is considered as a whole, link 5 becomes the most vulnerable link.

This section has presented results of single link vulnerability and disruption analysis. The next section allows for links to be considered jointly.

3.2.5 Results and Discussion for Joint Link Consideration

Due to the small nature of the sample network and for ease of discussion, the scenarios considered for multiple arc damage are first limited to two links. The results for the joint consideration of the links are presented below. The demand levels from the previous section are used here. The results are separated by origin-destination pair. Table 3.5 is for (1,2), table 3.6 is for (1,6), table 3.7 is for (5,2), and table 3.8 is for (5,6).

Table 3.5 Joint Vulnerability Indices for Origin-Destination (1,2)

	Demand	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Link 1	Original	1.0000	0.7667	0.7667	1.0000	1.0000	0.7667	0.7667
	¾ & ½	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Link 2	Original		0.1350	0.1350	0.1350	0.1350	0.1350	0.1350
	¾ & ½		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Link 3	Original			0.0000	0.1350	0.1350	0.0000	0.0000
	¾ & ½			0.0000	0.0000	0.0000	0.0000	0.0000
Link 4	Original				0.1350	0.1350	0.0000	0.0000
	¾ & ½				0.0000	0.0000	0.0000	0.0000
Link 5	Original					0.1350	0.1350	0.1350
	¾ & ½					0.0000	0.0000	0.0000
Link 6	Original						0.1350	0.1350
	¾ & ½						0.0000	0.0000
Link 7	Original							0.0000
	¾ & ½							0.0000

For the origin-destination pair (1,2), at the highest demand level, only links 1, 2, 5, and 6 had a vulnerability index greater than 0.0 due to the traffic assignment. These links represent two of the three possible paths connecting that O-D pair. Let path 1 contain link 1 and path 2 consist of links 2, 5, and 6. The third path (arcs 3, 4, 5, and 6) had no excess capacity available. Any combination of link 1 and a link from path 2 resulted in a joint vulnerability index of 1.0 since no alternate path was available. When one link from path 2 was considered with another arc from the same path, the joint index was the maximum of the individual link vulnerability indices, which in this case were identical. Any combination of links 3, 4, 7, and 8 resulted in a joint vulnerability index of 0.0 because none of the flow from node 1 to node 2 was assigned to these arcs. Finally, consideration of links 3, 4, 7, or 8 and one of the links from paths 1 or 2 yielded a joint index equivalent to the individual index for the arc from path 1 or 2.

At the lower demand levels, all of the flow for (1,2) was assigned to path 1. When link 1 was combined with any other link, the join vulnerability index was 1.0. If the combination of arcs did not include link 1, the joint index was 0.0.

Table 3.6 Joint Vulnerability Indices for Origin-Destination (1,6)

	Demand	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Link 1	Original	0.0029	0.0000	0.0000	0.0029	0.0000	0.0029	0.0000
	¾	0.0411	0.0000	0.0000	0.0008	0.0000	0.0008	0.0000
	½	0.0036	0.0000	0.0000	0.00001	0.0000	0.00001	0.0000
Link 2	Original		1.0000	0.0029	0.0029	0.0029	0.0029	1.0000
	¾		1.0000	0.0411	0.0411	0.0411	0.0411	1.0000
	½		1.0000	0.0036	0.0036	0.0036	0.0036	0.0036
Link 3	Original			0.0000	1.0000	0.0000	1.0000	0.0000
	¾			0.0000	1.0000	0.0000	1.0000	0.0000
	½			0.0000	1.0000	0.0000	1.0000	0.0000
Link 4	Original				0.0029	0.0000	0.0029	0.0000
	¾				0.0008	0.0000	0.0008	0.0000
	½				0.00001	0.0000	0.00001	0.0000
Link 5	Original					0.0029	0.0029	1.0000
	¾					0.0008	0.0008	1.0000
	½					0.00001	0.00001	1.0000
Link 6	Original						0.0029	0.0000
	¾						0.0008	0.0000
	½						0.00001	0.0000
Link 7	Original							1.0000
	¾							1.0000
	½							1.0000

Table 3.6 presents the joint vulnerability indices for origin-destination pair (1,6). There are three possible paths connecting this pair of nodes. Let path 4 consist of links 2, 5, and 7; path 5 consist of arcs 3, 4, 5, and 7; and path 6 consist of links 3 and 8.

Examination of the joint index of links 2 and 8 at different demands reveals that traffic levels play a crucial role in the calculation of vulnerability indices. At the two higher demands, link 4 does not have sufficient excess capacity to accommodate the flow on link 2, which means that path 5 is insufficient as the only alternate path from node 1 to node 6. Since arcs 2 and 8 are members of paths 4 and 6, respectively, their joint vulnerability index is 1.0. At the lowest demand level, path 5 can accommodate the flow on path 4. As a result, the joint vulnerability index of links 2 and 8 is the same as the individual index of link 2 (since arc 8 carries no flow from node 1 to node 6).

Arcs 2 and 3 form an obvious cut set for this OD pair; consequently, their joint vulnerability index is 1.0. Link combinations 3 and 5, 3 and 7, 5 and 8, and 7 and 8 also form cut sets and have joint indices of 1.0 for all demand levels.

Links 1 and 6, by the network configuration, cannot carry any flow between OD (1,6) so either of these links in combination with any other arc has a joint vulnerability index equivalent to that other arc's individual index.

Table 3.7 Joint Vulnerability Indices for Origin-Destination (5,2) and All Demand Levels

	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Link 1	0.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
Link 2		0.0000	1.0000	1.0000	1.0000	0.0000	0.0000
Link 3			1.0000	1.0000	1.0000	0.0000	0.0000
Link 4				1.0000	1.0000	1.0000	1.0000
Link 5					1.0000	1.0000	1.0000
Link 6						1.0000	1.0000
Link 7							0.0000

Table 3.7 demonstrates the simplicity of the case when only one path exists between an origin-destination pair, such as (5,2) in this example. The path (path 7) consists of links 4, 5, and 6. Each arc is a cut set for this OD pair. As

such, any combination of links with arcs 4, 5, or 6 yields a joint vulnerability index of 1.0. If none of the links of path 7 are in the combination under consideration, the index is 0.0.

Table 3. 8 Joint Vulnerability Indices for Origin-Destination (5,6)

	Demand	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Link 1	Original	0.0000	0.0000	0.1144	0.1144	0.0000	0.1144	0.7600
	¾	0.0000	0.0000	0.0899	0.0899	0.0000	0.0899	0.7431
	½	0.0000	0.0000	0.0749	0.0749	0.0000	0.0749	0.3880
Link 2	Original		0.0000	0.1144	0.1144	0.0000	0.1144	0.7600
	¾		0.0000	0.0899	0.0899	0.0000	0.0899	0.7431
	½		0.0000	0.0749	0.0749	0.0000	0.0749	0.3880
Link 3	Original			0.1144	0.1144	0.0000	0.1144	0.7600
	¾			0.0899	0.0899	0.0000	0.0899	0.7431
	½			0.0749	0.0749	0.0000	0.0749	0.3880
Link 4	Original				0.1144	0.1144	0.1144	1.0000
	¾				0.0899	0.0899	0.0899	1.0000
	½				0.0749	0.0749	0.0749	1.0000
Link 5	Original					0.1144	0.1144	1.0000
	¾					0.0899	0.0899	1.0000
	½					0.0749	0.0749	1.0000
Link 6	Original						0.1144	0.7600
	¾						0.0899	0.7431
	½						0.0749	0.3880
Link 7	Original							1.0000
	¾							1.0000
	½							1.0000

Origin-destination pair (5,6) is slightly more complex than (5,2) but less so than (1,2) and (1,6). There are only two paths connecting nodes 5 and 6. Let path 8 consist of link 8 and path 9 consist of arcs 4, 5, and 7. The joint vulnerability index of one of the remaining four links in the network and one of the arcs in path 8 or 9 is equivalent to the individual index of the link in path 8 or 9. Since only one alternate path to path 9 exists in the sample network, each link in path 9 has the same index value and this holds true when any combination of those arcs is considered. Finally, when link 8 is examined jointly with any of the links in path 9, a cut set is formed and the joint vulnerability index is 1.0.

The joint vulnerability indices reflect two interesting points. First, although links that lie on the same path have identical joint vulnerability indices for a given OD pair for the highest demand level, there are exceptions. One of these exceptions would occur when (1,6) is considered; either links 2 and 5 or 2 and 7 would have a higher index than links 5 and 7. When links lie on the same path, the maximum index for those arcs is taken as the joint vulnerability index. The second point is that a joint index of 1.0000 indicates either (a) only one path connects the OD pair, as in (5,2) or (b) those links taken together may indicate a cut set for that OD pair. An exception to (b) is: an alternate path may exist but has insufficient excess capacity to accommodate the necessary flow.

Like in the single link example, the disruption index is simply the sum of the vulnerabilities indices across all OD pairs. The following table provides the disruption indices for the pair-wise joint consideration of arcs in the sample network.

Table 3.9 Joint Disruption Indices for Sample Network

	Demand	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Link 1	Original	1.0029	0.7667	1.8811	2.1173	2.0000	0.8840	1.5267
	¾	1.0411	1.0000	2.0899	2.0907	2.0000	1.0907	1.7431
	½	1.0036	1.0000	2.0749	2.07491	2.0000	1.07491	1.3880
Link 2	Original		1.1350	1.2523	1.2523	1.1379	0.2523	1.8949
	¾		1.0000	1.1310	1.1310	1.0411	0.1310	1.7431
	½		1.0000	1.0785	1.0785	1.0036	0.0785	0.3916
Link 3	Original			1.1144	2.2494	1.1350	1.1144	0.7600
	¾			1.0899	2.0899	1.0000	1.0899	0.7431
	½			1.0749	2.0749	1.0000	1.0749	0.3880
Link 4	Original				1.2523	1.2494	1.1173	2.0000
	¾				1.0907	1.0899	1.0907	2.0000
	½				1.07491	1.0749	1.07491	2.0000
Link 5	Original					1.2523	1.2523	3.1350
	¾					1.0907	1.0907	3.0000
	½					1.07491	1.07491	3.0000
Link 6	Original						1.2523	1.8950
	¾						1.0907	1.7431
	½						1.07491	1.3880
Link 7	Original							2.0000
	¾							2.0000
	½							2.0000

For all demand levels, the maximum value of the disruption index occurs when links 5 and 8 are damaged simultaneously. Damage to these links will provide a cut set for origin-destination pairs (1,6), (5,2), and (5,6).

For the OD pair (1,2) the results for three or more links being damaged simultaneously can frequently be summarized in terms of the results for the n=2 case. The most interesting result occurs when links 1,2, and 3 or 1,2, and 4 are considered. Either of these two groups forms a traditional cut set for the origin-destination pair and the joint vulnerability index is 1.0. Any additional links examined with one of these triplets cannot contribute any vulnerability and the index remains at 1.0. Since link pairs 1 and 5 and 1 and 6 already form a cut set, any additional links that are considered with them will have a joint vulnerability index of 1.0. Links 7 and 8 cannot carry flow for OD (1,2) so disabling either or both of these two links will yield the same index as in the n-1 case (or n-2, if both are damaged). Links 3 and 4 lie on the same alternate path for OD pair (1,2) and only one alternate path contains each of these links; therefore, the vulnerability index for both of these links will be the same. Damaging both links 3 and 4 would be redundant for (1,2). Combinations of these two arcs and other arcs will yield an index equivalent to the n-1 case. A similar case arises with links 5 and 6 which lie on the same two alternate paths for OD (1,2) and the indices are treated in a similar manner to the case of links 3 and 4. Links 2, 3, and 4 are upstream of link 5 so damaging any of the previous links is redundant to damaging link 5. The value of the joint index is taken as the maximum value of the (n-1) index without consideration of link 5 and the (n-1) index without links 2, 3, or 4, respectively. A similar situation arises when links 2, 3, or 4 are examined in conjunction with link 6 since this link is downstream of the others.

Joint analysis of three or more links for OD (1,6) can also be summarized in terms of the n=2 scenario. Links 1 and 6 cannot contribute to the vulnerability of another grouping of arcs, so consideration of n-1 other links and one of these links yields a joint index equivalent to the joint index of the n-1 links. If both links 1 and 6 are considered with n-2 additional links, the joint index for the n links is the same as that for the n-2 additional arcs. Links 2 and 3, 5 and 8, and 7 and 8 form cut sets for this OD pair. Damage to any other link in conjunction with one of these pairs would be redundant and the value of the index remains at 1.0 as in the n=2 case. The triplet {2,4,8} also forms a cut set and in cases where n > 3, damage to any link in combination with this triplet would be redundant and the joint index value remains at 1.0. Link 7 is downstream of link 5 and is the only arc to which OD (1,6) traffic can flow after leaving link 5; therefore, from the single OD perspective, damaging both links 5 and 7 is redundant. The joint vulnerability index for a set of links containing both arcs 5 and 7 is the maximum of the joint vulnerability index of the set of links without link 5 and the joint index of the set of links without link 7. Using the set {2,5,7} as an example, the joint vulnerability index would be the maximum of $M^{1,6}_{\{2,5\}}$ and $M^{1,6}_{\{2,7\}}$, which are equivalent and the values for different demand levels are given in table 3.6. Links 2 and 4 are upstream of link 5 and no other path for this OD pair uses link 5 so damage to the pair {2,4} is redundant to disabling link 5. The joint vulnerability index for n links including {2,4,5} is the maximum of the index for the n-1 case where link 5 is not in the set and the n-2 set where links 2 and 4 are not in the set. For the triplet {2,4,5} and the ¾ demand case, $M^{1,6}_{\{2,4,5\}}$ is the maximum of $M^{1,6}_{\{2,4\}}$ (=0.0411) and $M^{1,6}_{\{5\}}$ (=0.0008). A comparable scenario arises when links 2,4, and 7 are among the links of interest. (Recall that link 7 is the only downstream arc of link 5 for this OD pair). As previously mentioned, link 5 is downstream of link 4 and damage to link 4 is redundant to disabling link 5; however, the converse is not true since link 5 is on two of the possible paths. When links 4 and 5 are in the set of links of interest, the joint vulnerability index is the maximum of the n-1 case without link 4 and the n-1 case without link 5. This analysis further extends to the situation in which links 4 and 7 are in the set of arcs of interest. Finally, traffic from link 3 must enter either link 4 or link 8. Damaging link 3 and the pair {4,8} is redundant. The joint vulnerability index for a set of n links containing {3,4,8} can be calculated as the maximum of the n-2 index without {4,8} and the n-1 index without link 3.

Joint vulnerability indices for OD (5,2) for any number of links can be easily determined since one path connects this pair of nodes. If the arc combination contains links 4, 5, and/or 6, the index is 1.0. If none of the three links are present, the value is 0.

Recall that for OD (5,6) there are only two possible paths - {4,5,7} and {8}. As in the n=2 case, any combination of link 8 and one of {4,5,7} results in a cut set and the joint vulnerability index is 1.0. Any additional link considered with the pair will have no impact on the value of the index; $M^{5,6}_{\{4,8,n-2\}} = M^{5,6}_{\{5,8,n-2\}} = M^{5,6}_{\{7,8,n-2\}} = 1.0$. If the triplet {4,5,7} is considered by itself or with links from the set {1,2,3,6} the joint index is the maximum value of the individual index for links 4, 5, and 7, such as in the n=2 case. If any combination of links 1, 2, 3, and 6 are considered jointly with link 8, the joint index is simply the value of $M^{5,6}_{\{8\}}$ or the n=1 case for link 8. When the links of interest solely consist of members of {1,2,3,6}, the joint index for this OD pair is 0.0.

3.3 APPLICATION OF GAME THEORY

A two player non-zero sum game is envisioned where one player is an evil entity, such as a terrorist cell, intent on destroying a transportation infrastructure link, or set of links, and the other player is the traffic management agency who tries to keep as many drivers safe as possible. The secondary objective of the traffic management agency is to allow each driver to reach their destination, if possible.

There are four cases of information in this game. In the first, the traffic management agency is not aware of an impending threat to the transportation system. In the second situation, the traffic management agency is suspects that the terrorists will take action, but the cell is not aware of that information has reached its opponent. Third, the agency perceives a general threat and the cell suspects that information has been leaked to the other team. Finally, the two players alternate moves with perfect information; the terrorists damage one link, the traffic management agency re-routes traffic, then the terrorists damage another link, and so on until either all of the resources have been used or there are no origin-destination pairs that remain connected.

Let Player M be the traffic management agency and Player T be the terrorist cell. Player M routes traffic to minimize the network travel time (system optimal). Let the payoff for player M be the percentage of vehicles that arrive safely at their destinations. Player T will seek to cause as much disruption to the flow of traffic as possible. Let the payoff for player T be the disruption index discussed in section 3.1; the objective function and constraints are given in P1 in section 3.2.1.

3.3.1 Game 1: No-Information for the Traffic Management Agency

When Player M is not aware that it should be involved in a game, the game becomes the bi-level mathematical program discussed in the previous section. For the sake of consistency, Player M routes traffic according to the system-optimal traffic assignment, but, in this game, does not try to avoid any links. Since this game reduces to the formulation in section 3.2, the results are the same.

3.3.2 Game 2: Some Information for the TMA

If Player M suspects that Player T will make a move, Player M will route traffic so as to minimize travel time around the suspected targeted set of links. Player M moves first in this game. Player M is aware of the objective function for player T; therefore, the game is simple to solve. Little guess work is required on the part of player M and none on the part of player T. In the event that there are multiple optima for player T, player M assigns probabilities to the strategies.

The mathematical program (P4) used by player M is a modified version (P2).

(P4)
$$\min z(x) = \sum_{l \in A, l \neq l^*} \sum_{r,s} x_l^{r,s} t_l \left(\sum_{r,s} x_l^{r,s} \right) + \sum_{l^*} \sum_{r,s} x_{l^*}^{r,s} H_{l^*} \tag{3.27}$$

s.t.
$$\sum_j \left(\min_{l \in L_j} x_l^{r,s} \right) = q^{r,s} \quad \forall r,s \tag{3.28}$$

$$x_l^{r,s} \geq 0 \tag{3.29}$$

$$\sum_{r,s} x_l^{r,s} \leq \rho_l \qquad \forall l \in A \tag{3.30}$$

where l^* is a link selected for damage by Player T and $\boldsymbol{H_{l^*}}$ is a high cost for using link $\boldsymbol{l^*}$. In this game, $\boldsymbol{l^*}$ is known by Player M. Equation (3.30) represents capacity constraints for every link in the network.

From the results in table 3.4 for n=1, Player M will seek to avoid link 5 for all of the demand levels. Player M knows that Player T will target link 5 based on the system optimal traffic assignment when Player M has no information about Player T. Examination of table 3.9 for the n=2 case reveals that Player M avoids routing traffic on links 5 and 8 for all of the demand cases. When n is 3 or higher, Player M cannot route traffic away from links 1, 5, and 8 simultaneously. These three links have a joint disruption index of 4.0 (for all demand levels), indicating that all four of the network's origin-destination pairs would be severed. Any higher value of n cannot disrupt the OD connectivity of this sample network further than the n=3 case.

Since Player T is unaware of the information that Player M has received, Player M's strategy is to maximize its own payoff, which in this case also minimizes the payoff to Player T (see table 3.11 in the next section). The next game allows for knowledge on the part of both players.

3.3.3 Game 3: One Move for Each Player, Full Information

In this game, each player optimizes their own position while trying to predict the opponent's strategy. Assume that each player has full knowledge of the other's payoffs. Player M investigates various strategies through the equations in (P4) and Player T employs (P1) for each of Player M's possible strategies. Let m denote the strategy of Player M and e denote the strategy of Player T. Let $p_{m,e}$ be the payoff to Player M and $D_{m,e}$ be the payoff to Player T, where $D_{m,e}$ is the value of the disruption index for strategy e when traffic is routed by strategy m. The payoff to Player M is calculated in the following manner:

$$p_{m,e} = \frac{\sum_{r,s} q^{r,s} - \sum_{r,s} \sum_{l^* \in e} x_{l^*}^{r,s}}{\sum_{r,s} q^{r,s}} \times 100 \qquad (3.31)$$

Equation (3.31) represents the percentage of network demand that is not routed on targeted links l^* dictated by Player T's strategy e.

Assume that each player has full knowledge of the other's payoffs. The general payoff matrix for both players is given in table 3.10.

Table 3.10 General Payoff Matrix

	Link Set 1	Link Set 2	…	Link Set y
Routing 1	$p_{1,1}$, $D_{1,1}$	$p_{1,2}$, $D_{1,2}$	…	$p_{1,y}$, $D_{1,y}$
Routing 2	$p_{2,1}$, $D_{2,1}$	$p_{2,2}$, $D_{2,2}$	…	$p_{2,y}$, $D_{2,y}$
…				
Routing w	$p_{w,1}$, $D_{w,1}$	$p_{w,2}$, $D_{w,2}$	…	$p_{w,y}$, $D_{w,y}$

Player M moves first. Player M may approach the same from several perspectives such as:

1. Minimize the maximum payoff to Player T:

$$z_1 = \min_m \left(\max_e D_{m,e} \right) \tag{3.32}$$

2. Maximize the minimum payoff to Player M:

$$z_2 = \max_m \left(\min_e p_{m,e} \right) \tag{3.33}$$

3. A combination of (1) and (2):

$$z_3 = f(z_1, z_2) \tag{3.34}$$

There may be an equilibrium point that is not an optimal solution for either player. According to Nash equilibrium, deviating from this point will cause at least one of the players to do “no better” than the equilibrium point.

For the original demand level, the resulting payoff matrix, evaluated for the potential damage of only one link, is given below. Table 3.11 is interesting in that one can observe the exact consequences of an error in Player M’s prediction of player T’s move. The flow distributions on which table 3.11 can be found in Appendix B.

Table 3.11 Payoff Matrix for n=1, Original Demand Level

	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Do nothing	60.52, 1.000	86.84, 0.110	100.00, 0	82.49, 1.053	69.33, 1.059	85.53, 1.000	83.80, 0.059	70.14, 0.908
Avoid 1	69.74, 0.767	77.63, 0.138	100.00, 0	77.63, 1.114	55.26, 1.252	76.32, 1.135	78.95, 0.117	75.00, 0.760
Avoid 2	60.52, 1.000	100.00, 0	86.84, 0.336	77.63, 1.202	77.63, 1.380	85.52, 1.000	92.11, 0.380	61.84, 1.134
Avoid 3	60.52, 1.000	86.84, 0.110	100.00, 0	82.49, 1.053	69.33, 1.059	85.53, 1.000	83.80, 0.059	70.14, 0.908
Avoid 4	60.52, 1.000	86.84, 0.698	100.00, 0	85.53, 1.000	72.34, 1.628	85.53, 1.000	86.84, 0.628	67.11, 1.000
Avoid 5	60.52, 1.000	100.00, 0	86.84, 0.001	85.53, 1.000	85.53, 1.000	85.53, 1.000	100.00, 0	53.95, 1.000
Avoid 6	60.52, 1.000	86.84, 0.132	100.00, 0	82.82, 1.048	69.66, 1.054	85.53, 1.000	84.13, 0.054	69.82, 0.918
Avoid 7	62.60, 0.947	97.92, 0.038	86.84, 0.089	85.53, 1.000	83.45, 1.038	83.45, 1.038	100.00, 0	53.95, 1.089
Avoid 8	60.52, 1.000	86.84, 0.003	100.00, 0	77.63, 1.114	64.47, 1.117	85.53, 1.000	78.95, 0.117	75.00, 0.760

The strategy "avoid 3" is redundant to the "do-nothing" alternative, so this row may be eliminated as a viable strategy for Player M. For 7 of the 8 (non-redundant) cases, Player T receives the greatest payoff by damaging link 5. The one exception is when Player M uses the strategy "avoid link 7" and Player T then damages link 8. From Player T's perspective, column "Link 5" dominates "Link 1," "Link 2," "Link 3," "Link 4," "Link 6," and "Link 7." From the resulting matrix, consisting of columns "Link 5" and "Link 8," one can see that row "Avoid 5" dominates "Avoid 7." Once this row is eliminated, column "Link 5" dominates "Link 8." Therefore, Player T's best strategy is to damage arc 5. To both maximize his own payoff and minimize the payoff to Player T, Player M will choose the strategy "Avoid Link 5."

By playing the strategy "Avoid Link 5" and "Link 5" is selected by Player T, the payoff for Player M is 85.53%, which means that 85.53% of the vehicles were able to safely reach their destinations. The "Do Nothing" alternative for

Player M only allows 69.33% of the vehicles to safely reach their destinations. By correctly predicting Player T's strategy, Player M is able to theoretically save 1231.2 vehicles/hour from damage or from becoming trapped in the network due to the directional nature of the links.

Table 3.12 is the payoff matrix for the two player game when the demand is at the ¾ level and Player T has limited resources and can damage only one link. The corresponding flow distributions for each of Player M's strategies are found in Appendix B.

Table 3.12 Payoff Matrix for n=1, 3/4 Demand Level

	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Do nothing	60.53, 1.000	86.84, 0.041	100.00, 0	77.08, 1.090	63.92, 1.091	85.53, 1.000	78.39, 0.091	75.56, 0.743
Avoid 1	77.63, 0.567	70.18, 0.194	99.56, 0.004	80.10, 1.064	50.28, 1.215	68.42, 1.138	81.86, 0.077	72.09, 0.848
Avoid 2 (opt 1)	60.53, 1.000	100.00, 0	86.84, 0.396	73.10, 1.182	73.10, 1.182	85.52, 1.000	87.57, 0.182	66.38, 1.018
Avoid 2 (opt 2)	60.53, 1.000	100.00, 0	86.84, 0.396	73.10, 1.372	73.10, 1.443	85.52, 1.000	87.57, 0.443	66.38, 1.022
Avoid 3	60.53, 1.000	86.84, 0.041	100.00, 0	77.08, 1.090	63.92, 1.091	85.53, 1.000	78.39, 0.091	75.56, 0.743
Avoid 4	60.53, 1.000	86.84, 0.513	100.00, 0	85.53, 1.000	72.37, 1.003	85.53, 1.000	86.84, 0.003	67.11, 1.000
Avoid 5	60.53, 1.000	100.00, 0	86.84, 2×10^{-4}	85.53, 1.000	85.53, 1.000	85.53, 1.000	100.00, 0	53.95, 1.000
Avoid 6	60.53, 1.000	86.84, 0.038	100.00, 0	77.02, 1.090	63.86, 1.091	85.53, 1.000	78.34, 0.091	75.61, 0.741
Avoid 7	62.61, 0.947	97.92, 0.028	86.84, 0.066	85.53, 1.000	83.44, 1.028	83.44, 1.028	100.00, 0	53.95, 1.066
Avoid 8	60.53, 1.000	86.84, 2×10^{-4}	100.00, 0	70.18, 1.117	57.02, 1.117	85.53, 1.000	71.49, 0.117	82.46, 0.533

*Options 1 and 2 for strategy Avoid 2 yielded equivalent objective value functions to four decimal places.

The results in table 3.12 are similar to those in table 3.11. Like the higher demand case, Player T's strategy Target Link 5 dominates all of the others except Target Link 8; the matrix can thus be reduced to the columns titled "Link 5" and "Link 8." For Player M, strategy Avoid Link 3 is equivalent to the "Do Nothing" alternative. “Avoid 1” is an inferior strategy to the “Do Nothing” alternative and can be eliminated from consideration. “Avoid 5” dominates “Avoid 7.” From the remaining matrix, Player T would target link 5. The cell “Avoid 5”, “Link 5” is a Nash equilibrium point; deviation from this point results in Player M doing worse.

Table 3.13 represents the payoff matrix for the case where Player T has the resources to damage only one link and the demand is half of the original.

Table 3.13 Payoff Matrix for n=1, 1/2 Demand Level

	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Do nothing/ Avoid 3	60.53, 1.000	86.84, 0.004	100.00, 0	77.08, 1.075	63.92, 1.075	85.53, 1.000	78.39, 0.075	75.56, 0.388
Avoid 1	93.42, 0.167	55.26, 0.087	98.68, 0.095	70.54, 1.083	25.81, 1.164	52.63, 1.052	73.18, 0.112	80.77, 0.639
Avoid 2 (opt 1)	60.53, 1.000	100.00, 0	86.84, 0.518	73.10, 1.153	73.10, 1.153	85.52, 1.000	87.57, 0.153	66.38, 0.777
Avoid 2 (opt 2)	60.53, 1.000	100.00, 0	86.84, 0.518	73.10, 1.589	73.10, 1.384	85.52, 1.000	87.57, 0.383	66.38, 0.659
Avoid 4	60.53, 1.000	86.84, 0.662	100.00, 0	85.53, 1.000	72.37, 1.001	85.53, 1.000	86.84, 0.001	67.11, 1.000
Avoid 5	60.53, 1.000	100.00, 0	86.84, 4×10^{-5}	85.53, 1.000	85.53, 1.000	85.53, 1.000	100.00, 0	53.95, 1.000
Avoid 6	60.53, 1.000	86.84, 0.005	100.00, 0	65.40, 1.075	52.24, 1.075	85.53, 1.000	66.72, 0.075	87.23, 0.388
Avoid 7	60.53, 1.000	100.00, 0	86.84, 4×10^{-5}	85.53, 1.000	85.53, 1.000	85.53, 1.000	100.00, 0	53.95, 1.000
Avoid 8	60.53, 1.000	86.84, 0.000	100.00, 0	70.18, 1.023	57.02, 1.023	85.53, 1.000	71.49, 0.023	82.46, 0.078

For Player T, the strategy “Target Link 5” dominates “Target Link 1,” “Target Link 2,” “Target Link 3,” “Target Link 6,” “Target Link 7,” and “Target Link 8.” From the payoff matrix consisting of only the columns “Link 4” and “Link 5,” Player M would select either “Avoid Link 5” or “Avoid Link 7.” Referring to table B.3, one can see that these two strategies yield equivalent flow distributions.

When Player T has the resources to damage two links simultaneously, there are numerous alternative optima for the routing strategies of Player M. A sample of the resulting flow distributions are included in Appendix B for the three demand levels. The payoff matrices, based on those flow distributions, are presented in tables 3.14, 3.16, and 3.18.

Table 3.14 Payoff Matrix for n=2, Original Demand Level

	Links 1,2	Links 1,3	Links 1,4	Links 1,5	Links 1,6	Links 1,7	Links 1,8
Avoid 1,2 (opt 1)	68.42 0.800	47.37 2.000	46.05 2.000	46.05 2.000	46.05 2.000	68.42 0.800	22.37 0.800
Avoid 1,2 (opt 2)	68.42 0.800	48.69 1.967	47.37 1.967	46.05 2.000	46.05 2.000	69.74 0.767	23.68 2.767
Avoid 1,3	47.37 2.000	69.74 0.767	47.37 1.944	25.00 3.240	46.05 2.000	48.68 2.007	44.74 1.527
Avoid 1,4	47.37 2.000	69.74 0.767	55.26 1.767	32.90 3.000	46.05 2.000	56.58 1.767	36.84 1.767
Avoid 1,5 (opt 1); 1,6 (opt 1); 1,7	60.53 1.000	56.58 1.767	55.26 1.767	46.05 2.000	46.05 2.000	69.74 0.767	23.69 2.767
Avoid 1,5 (opt 2); 1,6 (opt 2)	60.53 1.000	47.37 2.000	46.05 2.000	46.05 2.000	46.05 2.000	60.53 1.000	14.47 3.000
Avoid 1,5 (opt 3); 1,6 (opt 3)	60.53 1.000	51.97 1.883	50.66 1.883	46.05 2.000	46.05 2.000	65.13 0.883	19.08 2.883
Avoid 1,8 (opt 1)	60.53 1.000	56.58 1.767	47.37 2.007	38.16 2.240	46.05 2.000	61.84 1.007	31.58 2.527
Avoid 1,8 (opt 2)	60.53 1.000	56.58 1.767	47.37 2.295	38.16 3.000	46.05 2.000	61.84 1.367	31.58 2.167
Avoid 1,8 (opt 3)	60.53 1.000	56.58 1.767	47.37 2.167	38.16 2.380	46.05 2.000	61.84 1.187	31.58 2.347
Avoid 2,3 (opt 1)	47.37 2.000	60.53 1.000	43.01 2.073	29.86 3.092	46.05 2.000	44.33 2.092	30.67 1.908
Avoid 2,3 (opt 2); 2,8 (opt 3); 7,8 (opt 3)	60.53 1.000	47.37 2.000	38.16 2.357	38.16 2.351	46.05 2.000	52.63 1.372	22.37 2.627
Avoid 2,3 (opt 3)	53.84 1.508	54.05 1.492	40.43 2.241	33.75 3.112	46.05 2.000	48.22 1.767	26.78 2.233
Avoid 2,4; 2,5; 2,7; 3,5 (opt 2); 4,5; 5,6; 5,7; 6,7	60.53 1.000	47.37 2.000	46.05 2.000	46.05 2.000	46.05 2.000	60.53 1.000	14.47 3.000
Avoid 2,6 (opt 1); 2,8 (opt 2); 5,8 (opt 2); 7,8 (opt 2)	60.53 1.000	47.37 2.000	38.16 2.240	38.16 2.240	46.05 2.000	52.63 1.240	22.37 2.760
Avoid 2,6 (opt 2)	60.53 1.000	47.37 2.000	38.16 2.500	38.16 3.000	46.05 2.000	52.63 1.600	22.37 2.400
Avoid 2,6 (opt 3)	60.53 1.000	47.37 2.000	38.16 2.393	38.16 2.380	46.05 2.000	52.63 1.420	22.37 2.580

Avoid 2,8 (opt 1); 3,6; 3,7 (opt 2); 3,8; 4,8; 5,8 (opt 1); 6,8; 7,8 (opt 1)	47.37 2.000	60.53 1.000	38.16 2.177	25.00 3.240	46.05 2.000	39.47 2.240	35.53 1.760
Avoid 3,4; 3,5 (opt 1); 3,7 (opt 1); 4,6	47.37 2.000	60.53 1.000	46.05 2.000	32.90 3.000	46.05 2.000	47.37 2.000	27.63 2.000
Avoid 3,5 (opt 3); 3,7 (opt 3)	53.84 1.508	54.05 1.492	46.05 2.000	39.37 3.000	46.05 2.000	53.84 1.508	21.16 2.492
Avoid 4,7	60.53 1.000	49.45 1.947	48.13 1.947	46.05 2.000	46.05 2.000	62.60 0.947	16.55 2.947
Avoid 5,8 (opt 3)	49.42 1.844	58.48 1.156	46.05 2.000	34.94 3.000	46.05 2.000	49.42 1.844	25.58 2.156

	Links 2,3	Links 2,4	Links 2,5	Links 2,6	Links 2,7	Links 2,8	Links 7,8
Avoid 1,2 (opt 1)	78.95 1.121	77.63 1.121	77.63 1.121	77.63 1.121	100.00 0.000	53.95 2.000	53.95 2.000
Avoid 1,2 (opt 2)	77.63 1.135	76.32 1.135	76.32 1.135	76.32 1.135	98.68 0.019	52.63 2.019	53.95 2.000
Avoid 1,3	77.63 1.135	55.26 2.375	55.26 2.375	63.16 2.135	69.74 1.375	52.63 1.895	53.95 2.000
Avoid 1,4	77.63 1.135	63.16 2.135	63.16 2.135	63.16 1.676	77.63 1.135	44.74 2.135	53.95 2.000
Avoid 1,5 (opt 1); 1,6 (opt 1); 1,7	77.63 1.135	76.32 1.135	76.32 1.135	76.32 1.135	90.79 0.135	44.74 2.135	53.95 2.000
Avoid 1,5 (opt 2); 1,6 (opt 2)	88.84 1.000	85.53 1.000	85.53 1.000	85.53 1.000	100.00 0.000	53.95 2.000	53.95 2.000
Avoid 1,5 (opt 3); 1,6 (opt 3)	82.24 1.078	80.92 1.078	80.92 1.078	80.92 1.078	95.39 0.078	49.34 2.078	53.95 2.000
Avoid 1,8 (opt 1)	77.63 1.135	68.42 1.375	68.42 1.375	76.32 1.135	82.89 0.375	52.63 1.895	53.95 2.000
Avoid 1,8 (opt 2)	77.63 1.135	68.42 2.135	68.42 2.135	76.32 1.135	82.89 0.735	52.63 1.535	53.95 2.000
Avoid 1,8 (opt 3)	77.63 1.135	68.42 1.515	68.42 1.515	76.32 1.135	82.89 0.555	52.63 1.715	53.95 2.000
Avoid 2,3 (opt 1)	86.84 1.000	69.33 2.092	69.33 2.092	72.37 1.537	83.80 1.092	56.99 1.908	53.95 2.000
Avoid 2,3 (opt 2); 2,8 (opt 3); 7,8 (opt 3)	86.84 1.000	77.63 1.351	77.63 1.351	85.53 1.000	92.11 0.351	61.84 1.628	53.95 2.000
Avoid 2,3 (opt 3)	86.84 1.000	73.22 2.112	73.22 2.112	78.84 1.351	87.70 1.112	59.57 1.741	53.95 2.000
Avoid 2,4; 2,5; 2,7; 3,5 (opt 2); 4,5; 5,6; 5,7; 6,7	86.84 1.000	85.53 1.000	85.53 1.000	85.53 1.000	100.00 0.000	53.95 2.000	53.95 2.000

Avoid 2,6 (opt 1); 2,8 (opt 2); 5,8 (opt 2); 7,8 (opt 2)	86.84 1.000	77.63 1.240	77.63 1.240	85.53 1.000	92.11 0.240	61.84 1.760	53.95 2.000
Avoid 2,6 (opt 2)	86.84 1.000	77.63 2.000	77.63 2.000	85.53 1.000	92.11 1.000	61.84 1.400	53.95 2.000
Avoid 2,6 (opt 3)	86.84 1.000	77.63 1.380	77.63 1.380	85.53 1.000	92.11 0.380	61.84 1.580	53.95 2.000
Avoid 2,8 (opt 1); 3,6; 3,7 (opt 2); 3,8; 4,8; 5,8 (opt 1); 6,8; 7,8 (opt 1)	86.84 1.000	64.47 2.240	64.47 2.240	72.37 2.000	78.95 1.240	61.84 1.760	53.95 2.000
Avoid 3,4; 3,5 (opt 1); 3,7 (opt 1); 4,6	86.84 1.000	72.37 2.000	72.37 2.000	72.37 1.557	86.84 1.000	53.95 2.000	53.95 2.000
Avoid 3,5 (opt 3); 3,7 (opt 3)	86.84 1.000	78.84 2.000	78.84 2.000	78.84 1.335	93.32 1.000	53.95 2.000	53.95 2.000
Avoid 4,7	84.76 1.038	83.45 1.038	83.45 1.038	83.45 1.038	97.92 0.038	51.87 2.038	53.95 2.000
Avoid 5,8 (opt 3)	86.84 1.000	74.42 2.000	74.42 1.508	74.42 1.508	88.89 1.000	53.95 2.000	53.95 2.000

	Links 3,4	Links 3,5	Links 3,6	Links 3,7	Links 3,8	Links 6,7	Links 6,8
Avoid 1,2 (opt 1)	64.47 1.498	64.47 2.121	64.47 1.124	78.95 1.121	46.05 2.121	77.63 1.121	31.58 3.121
Avoid 1,2 (opt 2)	64.47 1.530	63.16 2.135	63.16 1.198	78.95 1.116	46.05 2.116	76.32 1.135	30.26 3.135
Avoid 1,3	77.63 1.114	55.26 2.249	76.32 1.135	78.95 1.114	75.00 0.760	55.26 2.375	51.32 1.895
Avoid 1,4	85.53 1.000	63.16 2.135	76.32 1.135	86.84 1.000	67.11 1.000	63.16 2.135	43.42 2.135
Avoid 1,5 (opt 1); 1,6 (opt 1); 1,7	72.37 1.415	63.16 2.135	63.16 1.550	86.84 1.000	53.95 2.000	76.32 1.135	30.26 3.135
Avoid 1,5 (opt 2); 1,6 (opt 2)	72.37 1.001	72.37 2.000	72.37 1.001	86.84 1.000	53.95 2.000	85.53 1.000	39.47 3.000
Avoid 1,5 (opt 3); 1,6 (opt 3)	72.37 1.207	67.76 2.078	67.76 1.285	86.84 1.000	53.95 2.000	80.92 1.078	34.87 3.078
Avoid 1,8 (opt 1)	64.47 2.421	55.26 3.135	63.16 1.556	78.95 2.000	61.84 1.760	68.42 1.375	38.16 2.895
Avoid 1,8 (opt 2)	72.37 1.421	63.16 2.135	63.16 1.556	86.84 1.000	53.95 2.000	68.42 2.135	38.16 2.535
Avoid 1,8 (opt 3)	68.42 2.421	59.21 2.223	63.16 1.556	82.89 1.088	57.89 1.880	68.42 1.515	38.16 2.715
Avoid 2,3 (opt 1)	82.49 1.053	69.33 2.053	85.53 1.000	83.80 1.053	70.14 0.908	69.33 2.092	55.67 1.908

Avoid 2,3 (opt 2); 2,8 (opt 3); 7,8 (opt 3)	67.38 2.336	67.38 2.111	72.37 1.336	81.85 1.111	58.94 1.848	77.63 1.351	47.37 2.628
Avoid 2,3 (opt 3)	75.361 1.331	68.68 2.072	79.05 1.259	83.15 1.072	64.32 1.380	73.22 2.112	51.78 2.233
Avoid 2,4; 2,5; 2,7; 3,5 (opt 2); 4,5; 5,6; 5,7; 6,7	72.37 1.001	72.37 2.000	72.37 1.001	86.84 1.000	53.95 2.000	85.53 1.000	39.47 3.000
Avoid 2,6 (opt 1); 2,8 (opt 2); 5,8 (opt 2); 7,8 (opt 2)	64.47 2.336	64.47 3.000	72.37 1.336	78.95 2.000	61.84 1.760	77.63 1.240	47.37 2.760
Avoid 2,6 (opt 2)	72.37 1.337	72.37 2.000	72.37 1.336	86.84 1.000	53.95 2.000	77.63 2.000	47.37 2.400
Avoid 2,6 (opt 3)	68.42 2.336	68.42 2.088	72.37 1.336	82.89 1.088	57.89 1.880	77.63 1.380	47.37 2.580
Avoid 2,8 (opt 1); 3,6; 3,7 (opt 2); 3,8; 4,8; 5,8 (opt 1); 6,8; 7,8 (opt 1)	77.63 1.114	64.47 2.114	85.53 1.000	78.95 1.114	75.00 0.760	64.47 2.240	60.53 1.760
Avoid 3,4; 3,5 (opt 1); 3,7 (opt 1); 4,6	85.53 1.000	72.37 2.000	85.53 1.000	86.84 1.000	67.11 1.000	72.37 2.000	52.63 2.000
Avoid 3,5 (opt 3); 3,7 (opt 3)	79.05 1.148	72.27 2.000	79.05 1.148	86.84 1.000	60.63 1.492	78.84 2.000	46.16 2.492
Avoid 4,7	72.37 1.094	70.29 2.038	70.29 1.132	86.84 1.000	53.95 2.000	83.45 1.038	37.40 3.038
Avoid 5,8 (opt 3)	83.48 1.079	72.37 2.000	83.48 1.079	86.84 1.000	65.06 1.156	74.42 2.000	50.58 2.156

	Links 4,5	Links 4,6	Links 4,7	Links 4,8	Links 5,6	Links 5,7	Links 5,8
Avoid 1,2 (opt 1)	77.63 1.121	77.63 1.121	77.63 1.121	31.58 2.498	77.63 1.121	77.63 1.121	31.58 3.121
Avoid 1,2 (opt 2)	76.32 1.135	76.32 1.135	77.63 1.116	31.58 2.530	76.32 1.135	76.32 1.135	30.26 3.135
Avoid 1,3	55.26 2.375	68.42 1.312	64.47 2.240	52.63 2.000	55.26 2.375	55.26 2.375	30.26 3.135
Avoid 1,4	63.16 2.135	76.32 1.135	72.37 2.000	52.63 2.000	63.16 2.135	63.16 2.135	30.26 3.135
Avoid 1,5 (opt 1); 1,6 (opt 1); 1,7	76.32 1.135	76.32 1.135	85.53 1.000	39.47 2.415	76.32 1.135	76.32 1.135	30.26 3.135
Avoid 1,5 (opt 2); 1,6 (opt 2)	85.53 1.000	85.53 1.000	85.53 1.000	39.47 2.001	85.53 1.000	85.53 1.000	39.47 3.000
Avoid 1,5 (opt 3); 1,6 (opt 3)	80.92 1.078	80.92 1.078	85.53 1.000	39.47 2.207	80.92 1.078	80.92 1.078	34.87 3.078

Avoid 1,8 (opt 1)	68.42 1.375	68.42 1.375	77.63 1.240	39.47 2.421	68.42 1.375	68.42 1.375	30.26 3.135
Avoid 1,8 (opt 2)	68.42 2.135	68.42 1.388	77.63 2.000	39.47 2.421	68.42 2.135	68.42 2.135	30.26 3.135
Avoid 1,8 (opt 3)	68.42 1.515	68.42 1.381	77.63 1.380	39.47 2.421	68.42 1.515	68.42 1.515	30.26 3.135
Avoid 2,3 (opt 1)	69.33 2.092	82.49 1.073	69.33 2.092	52.63 2.000	69.33 2.092	69.33 2.092	39.47 3.000
Avoid 2,3 (opt 2); 2,8 (opt 3); 7,8 (opt 3)	77.63 1.351	77.63 1.226	77.63 1.351	39.47 2.336	77.63 1.351	77.63 1.351	39.47 3.000
Avoid 2,3 (opt 3)	73.22 2.112	79.91 1.176	73.22 2.112	46.16 2.259	73.22 2.112	73.22 2.112	39.47 3.000
Avoid 2,4; 2,5; 2,7; 3,5 (opt 2); 4,5; 5,6; 5,7; 6,7	85.53 1.000	85.53 1.000	85.53 1.000	39.47 2.001	85.53 1.000	85.53 1.000	39.47 3.000
Avoid 2,6 (opt 1); 2,8 (opt 2); 5,8 (opt 2); 7,8 (opt 2)	77.63 1.240	77.63 1.240	77.63 1.240	39.47 2.336	77.63 1.240	77.63 1.240	39.47 3.000
Avoid 2,6 (opt 2)	77.63 2.000	77.63 1.202	77.63 2.000	39.47 2.336	77.63 2.000	77.63 2.000	39.47 3.000
Avoid 2,6 (opt 3)	77.63 1.380	77.63 1.221	77.63 1.380	39.47 2.336	77.63 1.380	77.63 1.380	39.47 3.000
Avoid 2,8 (opt 1); 3,6; 3,7 (opt 2); 3,8; 4,8; 5,8 (opt 1); 6,8; 7,8 (opt 1)	64.47 2.240	77.63 1.177	64.47 2.240	52.63 2.000	64.47 2.240	64.47 2.240	39.47 3.000
Avoid 3,4; 3,5 (opt 1); 3,7 (opt 1); 4,6	73.37 2.000	85.53 1.000	72.37 2.000	52.63 2.000	72.37 2.000	72.37 2.000	39.47 3.000
Avoid 3,5 (opt 3); 3,7 (opt 3)	78.84 2.000	85.53 1.000	78.84 2.000	46.16 2.148	78.84 2.000	78.84 2.000	39.47 3.000
Avoid 4,7	83.45 1.038	83.45 1.038	85.53 1.000	39.47 2.094	83.45 1.038	83.45 1.038	37.40 3.038
Avoid 5,8 (opt 3)	74.42 2.000	85.53 1.000	74.42 2.000	50.58 2.079	74.42 2.000	74.42 2.000	39.47 3.000

Reduction of the matrix shown in table 3.14 reveals a single equilibrium point. For Player T, "Target Links 5,8" dominates all other strategies except "Target Links 1,5." Using only these two columns, Player M's strategy "Avoid 1,5 (option 2); 1,6 (option 2)" dominates all other strategies except the strategy beginning with "Avoid 2,4; 2,5." The two remaining strategies for Player M are equivalent and referring to table B.4, they yield identical flow distributions. Although Player M's dominant strategy did not include a reference to "avoid 5,8," the flow distribution for the dominant strategy is also an alternative optimal flow pattern for "avoid 5,8." The original tables did not reflect this fact because the sample of alternative optimal solutions was limited to three for each set of links.

As a side note, if Player M had been misinformed about the amount of resources available to the "evil entity," the routing strategy selected would not have yielded as high a payoff as when Player M had the correct information. If Player T really only had the resources to damage one link but Player M thought there were enough resources to damage two links, the payoff matrix in table 3.15 would have resulted. This table is based on Player M's dominant routing strategy, determined from table 3.14.

Table 3.15 Payoff Matrix for Misinformation about Player T's Resources, Original Demand Scenario

Player	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
M	60.52	100.00	86.84	85.53	85.53	85.53	100.00	53.95
T	1.000	0	0.001	1.000	1.000	1.000	0	2.000

Based on the payoffs in table 3.15, Player T would select link 8 as the target. The payoff to Player T would be 2.000. Player M would receive a payoff of 53.95, which is greater than the predicted 39.47 from table 3.14. However, if Player M had known that Player T only had the resources to damage one link, and that link had still been link 8 (although this is not the optimal strategy for Player T), Player M would have routed traffic so as to receive a payoff of 75.00 (see table 3.11). Player T's payoff would have been only 0.760. The misinformation about the amount of resources would have been an advantage to Player T.

Table 3.16 Payoff Matrix for n=2, 3/4 Demand Level

	Links 1,2	Links 1,3	Links 1,4	Links 1,5	Links 1,6	Links 1,7	Links 1,8
Avoid 1,2 (opt 1)	75.44 0.622	47.37 2.000	45.61 2.011	45.61 2.011	46.61 2.000	75.00 0.633	29.83 2.609
Avoid 1,2 (opt 2)	75.44 0.622	49.56 1.944	48.25 1.944	46.05 2.000	46.05 2.000	77.63 0.567	31.58 2.567
Avoid 1,3 (opt 1)	47.37 2.000	77.19 0.578	57.61 1.687	27.79 3.001	46.05 2.000	58.93 1.720	49.41 1.422
Avoid 1,3(opt 2); 1,6(opt 2)	47.81 1.967	77.19 0.600	57.97 1.677	28.15 3.017	46.05 2.000	59.73 1.677	49.48 1.442
Avoid 1,3(opt 3)	47.81 1.967	77.19 0.600	57.97 1.703	28.15 3.017	46.05 2.000	59.73 1.700	49.48 1.422
Avoid 1,4	47.81 1.967	77.19 0.600	63.16 1.567	33.33 2.018	46.05 2.000	64.91 1.533	44.30 1.600
Avoid 1,5(opt 1); 2,4; 2,5; 2,7; 3,5(opt 2); 3,7(opt 2); 4,5; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 1)	60.53 1.000	47.37 2.000	46.05 2.000	46.05 2.000	46.05 2.000	60.53 1.000	14.47 3.000
Avoid 1,5(opt 2); 1,7	60.53 1.000	64.47 1.567	63.16 1.567	46.05 2.000	46.05 2.000	77.63 0.567	31.58 2.567
Avoid 1,5(opt 3)	60.53 1.000	55.73 1.788	54.41 1.789	46.05 2.000	46.05 2.000	68.88 0.788	22.83 2.788
Avoid 1,6(opt 1); 2,3(opt 1); 3,6; 5,8(opt 1)	47.37 2.000	60.53 1.000	37.55 2.174	24.38 3.604	46.05 2.000	38.86 2.226	36.14 1.741
Avoid 1,6 (opt 3)	47.37 2.000	68.88 0.788	47.09 1.940	25.58 3.001	46.05 2.000	48.41 1.986	43.31 1.566
Avoid 1,8 (opt 1)	47.37 2.000	77.19 0.578	47.37 1.869	17.54 3.121	46.05 2.000	48.68 2.044	59.65 1.111
Avoid 1,8 (opt 2)	47.81 1.967	77.19 0.600	47.81 1.860	17.98 3.108	46.05 2.000	49.56 2.000	59.65 1.133
Avoid 1,8 (opt 3)	47.81 1.967	77.19 0.600	47.81 1.889	17.98 3.105	46.05 2.000	49.56 2.020	59.65 1.113

Avoid 2,3(opt 2); 2,6(opt 3); 2,8(opt 1)	60.53 1.000	47.37 2.000	30.70 2.653	30.70 3.236	46.06 2.000	45.18 1.724	29.83 2.233
Avoid 2,3(opt 3); 7,8(opt 3)	54.20 1.481	53.69 1.519	35.00 2.295	28.68 3.416	46.05 2.000	43.15 1.807	31.85 2.141
Avoid 2,6 (opt 1)	60.53 1.000	47.37 2.000	33.62 2.280	33.62 2.280	46.05 2.000	48.10 1.280	26.90 2.622
Avoid 2,6 (opt 2)	60.53 1.000	47.37 2.000	33.62 2.720	30.70 2.346	46.05 2.000	45.18 1.346	29.83 2.533
Avoid 2,8 (opt 2)	60.53 1.000	47.37 2.000	30.70 2.346	30.70 2.346	46.05 2.000	45.18 1.346	29.82 2.533
Avoid 2,8 (opt 3)	60.53 1.000	47.37 2.000	30.70 3.752	30.70 3.471	46.05 2.000	45.18 2.064	29.82 1.933
Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	47.37 2.000	60.53 1.000	46.05 2.000	32.89 2.003	46.05 2.000	47.37 2.000	27.63 2.000
Avoid 3,5(opt 3); 3,7(opt 3)	54.82 1.500	54.82 1.500	46.05 2.000	40.35 2.104	46.05 2.000	54.82 1.500	21.93 2.500
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 2)	47.37 2.000	60.53 1.000	30.70 2.292	17.54 3.121	46.05 2.000	32.02 2.467	42.98 1.533
Avoid 4,7	60.53 1.000	49.46 1.947	48.15 1.947	46.05 2.000	46.05 2.000	62.62 0.947	16.57 2.947
Avoid 4,8 (opt 3)	47.37 2.000	60.53 1.000	37.28 2.18	24.12 3.607	46.05 2.000	38.60 2.232	36.40 1.733

	Links 2,3	Links 2,4	Links 2,5	Links 2,6	Links 2,7	Links 2,8	Links 7,8
Avoid 1,2 (opt 1)	71.93 1.132	70.18 1.144	70.18 1.144	70.61 1.133	99.56 0.011	54.39 1.987	53.95 2.000
Avoid 1,2 (opt 2)	69.74 1.138	68.42 1.138	68.42 1.138	68.42 1.138	97.81 0.018	51.75 2.018	53.95 2.000
Avoid 1,3 (opt 1)	70.18 1.138	50.59 2.139	50.59 2.139	55.70 1.651	65.07 1.280	42.39 1.982	53.95 2.000
Avoid 1,3(opt 2); 1,6(opt 2)	69.74 1.138	50.52 2.155	50.52 2.155	55.70 1.643	64.99 1.249	42.03 1.981	53.95 2.000
Avoid 1,3(opt 3)	69.74 1.138	50.52 2.155	50.52 2.155	55.70 1.643	64.99 1.271	42.03 1.961	53.95 2.000
Avoid 1,4	69.74 1.138	55.70 1.156	55.70 1.156	55.70 1.634	70.18 1.105	36.84 2.138	53.95 2.000
Avoid 1,5(opt 1); 2,4; 2,5; 2,7; 3,5(opt 2); 3,7(opt 2); 4,5; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 1)	86.84 1.000	85.53 1.000	85.53 1.000	85.53 1.000	100.00 0.000	53.95 2.000	53.95 2.000

Avoid 1,5(opt 2); 1,7	69.74 1.138	68.42 1.138	68.42 1.138	68.42 1.138	82.89 0.138	36.84 2.138	53.95 2.000
Avoid 1,5(opt 3)	78.48 1.094	77.17 1.094	77.17 1.094	77.17 1.094	91.64 0.094	45.59 2.094	53.95 2.000
Avoid 1,6(opt 1); 2,3(opt 1); 3,6; 5,8(opt 1)	86.84 1.000	63.86 2.604	63.86 2.604	72.37 1.512	78.34 1.604	62.45 1.741	53.95 2.000
Avoid 1,6 (opt 3)	79.48 1.094	56.70 2.095	56.70 2.095	64.01 1.607	71.17 1.483	52.91 1.872	53.95 2.000
Avoid 1,8 (opt 1)	70.18 1.138	40.35 2.258	40.35 2.258	55.70 1.638	54.82 1.604	52.63 1.671	53.95 2.000
Avoid 1,8 (opt 2)	69.74 1.138	40.35 2.246	40.35 2.246	55.70 1.629	54.82 1.571	52.19 1.672	53.95 2.000
Avoid 1,8 (opt 3)	69.74 1.138	40.35 2.243	40.35 2.243	55.70 1.629	54.82 1.592	52.19 1.652	53.95 2.000
Avoid 2,3(opt 2); 2,6(opt 3); 2,8(opt 1)	86.84 1.000	70.18 2.236	70.18 2.236	85.53 1.000	84.65 1.236	69.30 1.233	53.95 2.000
Avoid 2,3(opt 3); 7,8(opt 3)	86.84 1.000	68.15 2.416	68.15 2.416	79.20 1.305	82.63 1.416	65.00 1.621	53.95 2.000
Avoid 2,6 (opt 1)	86.84 1.000	73.10 1.280	73.10 1.280	85.53 1.000	87.57 0.280	66.38 1.622	53.95 2.000
Avoid 2,6 (opt 2)	86.84 1.000	73.10 1.446	73.10 1.446	85.53 1.000	87.57 0.446	66.38 1.055	53.95 2.000
Avoid 2,8 (opt 2)	86.84 1.000	70.18 1.346	70.18 1.346	85.53 1.000	84.65 0.346	69.30 1.533	53.95 2.000
Avoid 2,8 (opt 3)	86.84 1.000	70.18 2.471	70.18 2.471	85.53 1.000	84.65 1.471	69.30 0.933	53.95 2.000
Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	86.84 1.000	72.37 1.003	72.37 1.003	72.37 1.503	86.84 0.003	54.95 2.000	53.95 2.000
Avoid 3,5(opt 3); 3,7(opt 3)	88.60 1.000	79.82 1.104	79.82 1.104	79.82 1.302	94.30 0.104	55.70 2.000	53.95 2.000
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 2)	86.84 1.000	57.02	57.02 2.121	72.37 1.500	71.49 1.121	69.30 1.533	53.95 2.000
Avoid 4,7	84.75 1.028	83.43 1.028	83.43 1.028	83.43 1.028	97.91 0.028	51.85 2.028	53.95 2.000
Avoid 4,8 (opt 3)	86.84 1.000	63.60 2.607	63.60 2.607	72.37 1.507	78.07 1.607	62.72 1.733	53.95 2.000

	Links 3,4	Links 3,5	Links 3,6	Links 3,7	Links 3,8	Links 6,7	Links 6,8
Avoid 1,2 (opt 1)	57.02 1.674	57.02 2.141	57.46 1.136	71.49 1.141	39.47 2.119	70.18 1.144	25.00 3.119
Avoid 1,2 (opt 2)	57.46 2.121	55.26 2.138	33.26 1.216	71.93 1.121	39.04 2.121	68.42 1.138	22.37 3.138

Avoid 1,3 (opt 1)	80.42 1.062	50.59 2.199	68.86 1.137	81.73 1.06	72.21 0.845	50.59 2.139	41.07 1.982
Avoid 1,3(opt 2); 1,6(opt 2)	79.90 1.097	50.08 2.202	67.98 1.172	81.66 1.064	71.85 0.876	50.52 2.155	40.27 2.014
Avoid 1,3(opt 3)	80.34 1.092	50.52 2.197	67.98 1.172	82.10 1.058	71.41 0.889	50.52 2.155	40.27 1.994
Avoid 1,4	85.09 1.033	55.26 2.138	67.98 1.172	86.84 1.000	66.67 1.033	55.70 1.156	35.09 2.172
Avoid 1,5(opt 1); 2,4; 2,5; 2,7; 3,5(opt 2); 3,7(opt 2); 4,5; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 1)	72.37 1.000	72.37 2.000	72.37 1.000	86.84 1.000	53.95 2.000	85.53 1.000	39.47 3.000
Avoid 1,5(opt 2); 1,7	72.37 2.000	55.26 2.138	55.26 2.138	86.84 1.000	53.95 2.000	68.42 1.138	22.37 3.138
Avoid 1,5(opt 3)	72.37 1.281	64.01 2.094	64.01 1.376	86.84 1.000	53.95 2.000	77.17 1.094	31.12 3.094
Avoid 1,6(opt 1); 2,3(opt 1); 3,6; 5,8(opt 1)	77.02 1.090	63.86 2.090	85.53 1.000	78.34 1.090	75.61 0.741	63.86 2.604	61.14 1.741
Avoid 1,6 (opt 3)	78.21 1.081	56.70 2.176	77.17 1.094	79.53 1.081	74.42 0.778	56.70 2.095	51.59 1.872
Avoid 1,8 (opt 1)	70.18 1.117	40.35 2.254	68.86 1.137	71.49 1.117	82.46 0.533	40.35 2.258	51.32 1.671
Avoid 1,8 (opt 2)	69.74 1.15	39.91 2.258	67.98 1.172	71.49 1.120	82.02 0.567	40.35 2.246	50.44 1.705
Avoid 1,8 (opt 3)	70.18 1.150	40.35 2.255	67.98 1.172	71.93 1.116	81.58 0.580	40.35 2.243	50.44 1.685
Avoid 2,3(opt 2); 2,6(opt 3); 2,8(opt 1)	63.60 1.607	63.60 2.117	72.37 1.490	78.07 1.117	62.72 1.733	70.18 2.236	54.82 2.233
Avoid 2,3(opt 3); 7,8(opt 3)	68.58 1.415	62.259 2.126	78.69 1.289	76.73 1.126	70.38 1.212	68.15 2.416	56.85 2.141
Avoid 2,6 (opt 1)	59.94 2.396	59.94 2.182	72.37 1.396	74.41 1.182	66.38 1.622	73.10 1.280	51.90 2.622
Avoid 2,6 (opt 2)	72.37 1.396	72.37 2.000	72.37 1.396	86.84 1.000	53.95 2.000	73.10 1.446	51.90 2.055
Avoid 2,8 (opt 2)	57.02 2.396	57.02 2.224	73.37 1.396	71.49 1.224	69.30 1.533	70.18 1.346	54.82 2.533
Avoid 2,8 (opt 3)	70.18 1.519	70.18 2.029	72.37 1.490	84.65 1.029	56.14 1.933	70.18 2.471	54.82 1.933
Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	85.53 1.000	72.37 2.000	85.53 1.000	86.84 1.000	67.11 1.000	72.37 1.003	52.63 2.000
Avoid 3,5(opt 3); 3,7(opt 3)	79.82 1.096	74.12 2.000	79.82 1.096	88.60 1.000	61.40 1.500	79.82 1.104	46.93 2.500

Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 2)	70.18 1.117	57.02 2.117	85.53 1.000	71.49 1.117	82.47 0.533	57.02 2.121	67.98 1.533
Avoid 4,7	72.37 1.070	70.28 2.028	70.28 1.099	86.84 1.000	53.95 2.000	83.43 1.028	37.38 3.028
Avoid 4,8 (opt 3)	76.75 1.092	63.60 2.092	85.53 1.000	78.07 1.092	75.88 0.733	63.60 2.607	61.40 1.733

	Links 4,5	Links 4,6	Links 4,7	Links 4,8	Links 5,6	Links 5,7	Links 5,8
Avoid 1,2 (opt 1)	70.18 1.144	70.18 1.144	70.18 1.144	24.56 2.665	70.18 1.144	70.18 1.144	24.56 3.132
Avoid 1,2 (opt 2)	68.42 1.138	68.42 1.138	70.61 1.121	24.56 3.121	68.42 1.138	68.42 1.138	22.37 3.138
Avoid 1,3 (opt 1)	50.59 2.139	63.75 1.246	67.26 2.001	52.63 2.000	50.59 2.139	50.59 2.139	22.81 3.137
Avoid 1,3(opt 2); 1,6(opt 2)	50.52 2.155	63.23 1.249	67.62 2.017	52.19 2.033	50.52 2.155	50.52 2.155	22.37 3.138
Avoid 1,3(opt 3)	50.52 2.155	63.23 1.275	67.62 2.107	52.19 2.033	50.52 2.156	50.52 2.155	22.37 3.138
Avoid 1,4	55.70 1.156	68.42 1.138	72.81 1.018	52.19 2.033	55.70 1.156	55.70 1.156	22.37 3.138
Avoid 1,5(opt 1); 2,4; 2,5; 2,7; 3,5(opt 2); 3,7(opt 2); 4,5; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 1)	85.53 1.000	85.53 1.000	85.53 1.000	39.47 2.000	85.53 1.000	85.53 1.000	39.47 3.000
Avoid 1,5(opt 2); 1,7	68.42 1.138	68.42 1.138	85.53 1.000	39.47 3.000	68.42 1.138	22.37 3.138	22.37 3.138
Avoid 1,5(opt 3)	77.17 1.094	77.17 1.094	85.53 1.000	39.47 2.281	77.17 1.094	77.17 1.094	31.12 3.094
Avoid 1,6(opt 1); 2,3(opt 1); 3,6; 5,8(opt 1)	63.86 2.604	77.02 1.174	63.86 2.604	52.63 2.000	63.86 2.604	63.86 2.604	39.47 3.000
Avoid 1,6 (opt 3)	56.70 2.095	69.85 1.246	65.05 2.001	52.63 2.000	56.70 2.095	56.70 2.095	31.12 3.094
Avoid 1,8 (opt 1)	40.35 2.258	53.51 1.429	57.02 2.121	52.63 2.000	40.35 2.258	40.35 2.258	22.81 3.137
Avoid 1,8 (opt 2)	40.35 2.246	53.07 1.432	57.46 2.108	52.19 2.033	40.35 2.246	40.35 2.246	22.37 3.138
Avoid 1,8 (opt 3)	40.35 2.243	53.07 1.461	57.46 2.105	52.19 2.033	40.35 2.243	40.35 2.243	22.37 3.138
Avoid 2,3(opt 2); 2,6(opt 3); 2,8(opt 1)	70.18 2.236	70.18 1.469	70.18 2.236	39.47 2.490	70.18 2.236	70.18 2.236	39.47 3.000

Avoid 2,3(opt 3); 7,8(opt 3)	68.15 2.416	74.48 1.264	68.15 2.416	45.80 2.289	68.15 2.416	68.15 2.416	39.47 3.000
Avoid 2,6 (opt 1)	73.10 1.280	73.10 1.280	73.10 1.280	39.47 2.396	73.10 1.280	73.10 1.280	39.47 3.000
Avoid 2,6 (opt 2)	73.10 1.446	73.10 1.374	73.10 1.446	39.47 2.396	73.10 1.446	73.10 1.446	39.47 3.000
Avoid 2,8 (opt 2)	70.18 1.346	70.18 1.346	70.18 1.346	39.47 2.396	70.18 1.346	70.18 1.346	39.47 3.000
Avoid 2,8 (opt 3)	70.18 2.471	70.18 2.490	70.18 2.471	39.47 2.490	70.18 2.471	70.18 2.471	39.47 3.000
Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	72.39 1.003	85.53 1.000	72.37 1.003	52.63 2.000	72.37 1.003	72.37 1.003	39.47 3.000
Avoid 3,5(opt 3); 3,7(opt 3)	79.82 1.104	85.53 1.000	79.82 1.104	46.93 2.096	79.82 1.104	79.82 1.104	41.23 3.000
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 2)	57.02 2.121	70.18 1.292	57.02 2.121	52.63 2.000	57.02 2.121	57.02 2.121	39.47 3.000
Avoid 4,7	83.43 1.028	83.43 1.028	85.53 1.000	39.47 2.070	83.43 1.028	83.43 1.028	37.38 3.028
Avoid 4,8 (opt 3)	63.60 2.607	76.75 1.179	63.60 2.607	52.63 2.000	63.60 2.607	63.60 2.607	39.47 3.000

The payoff matrix shown in table 3.16 can be reduced to a six by three matrix. Player T's strategy "Target Links 5,8" dominates all other strategies except "Target Links 1,4" and "Target Links 1,5." The resulting three column matrix is further reduced by noting that Player M's strategy "Avoid 1,5 (option 2); 1,7" dominates "Avoid 1,2 (option 2)," "Avoid 1,3 (option 2); 1,6 (option 2)," "Avoid 1,3 (option 3)," "Avoid 1,4," "Avoid 1,8 (option 2)," and "Avoid 1,8 (option 3)." The strategy that begins "Avoid 1,5 (option 1); 2,4…" dominates "Avoid 1,2 (option 1)," "Avoid 1,6 (option 1);…," "Avoid 2,3 (option 2); …," "Avoid 2,3 (option 3); …," "Avoid 2,6 (option 1)," "Avoid 2,6 (option 2)," "Avoid 2,8 (option 2)," "Avoid 2,8 (option 3)," "Avoid 3,4; …," "Avoid 3,8; …," and "Avoid 4,8 (option 3)." Strategy "Avoid 4,7" dominates "Avoid 1,6 (option 3)" and "Avoid 1,8 (option 1)."

The 6 row, 3 column matrix can be further reduced. Player T's strategy "Target Links 5,8" dominates the other two columns. Player M chooses the routing strategy “Avoid 3,5 (option 3); 3,7 (option 3)” to receive the payoff of 41.22. Player T’s resulting payoff is 3.00. Although the strategy label does not include “Avoid 5,8,” the strategy is actually a fourth option for “Avoid 5,8.”

If Player M had over-estimated the amount of resources available to Player T, would have received an advantage and Player M a disadvantage. The payoff matrix associated with the routing strategy selected by Player M as a result of table 3.16 is presented in table 3.17.

Table 3.17 Payoff Matrix for Misinformation about Player T’s Resources, 3/4 Demand Scenario

Player	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
M	60.53	94.30	94.30	85.53	79.82	85.53	94.30	61.40
T	1.000	0.302	0.096	1.000	1.104	1.000	0.104	1.500

As in the previous demand case, Player T would select link 8 as the target. Player M would receive a higher payoff than in the case where Player T actually had the resources to damage two links, but had Player M known that link 8 was the only target, traffic could have been routed appropriately. As in table 3.12, Player M would have received a payoff of 82.46 if the correct information about Player T’s resources had been obtained. By providing misinformation, Player T would have obtained a payoff of 1.500 instead of 0.533.

Table 3.18 Payoff Matrix for n=2, 1/2 Demand Level

	Links 1,2	Links 1,3	Links 1,4	Links 1,5	Links 1,6	Links 1,7	Links 1,8
Avoid 1,2 (opt 1)	89.47, 0.267	51.32, 1.900	48.68, 1.929	44.74, 2.029	46.05, 2.000	92.11, 0.195	48.68, 2.127
Avoid 1,2 (opt 2)	89.47, 0.333	51.32, 1.833	48.68, 1.933	44.74, 2.100	46.05, 2.000	92.11, 0.267	48.68, 2.067
Avoid 1,3	47.37, 2.000	92.11, 0.200	63.08, 1.460	18.34, 2.954	46.05, 2.000	64.39, 0.922	73.76, 0.758
Avoid 1,4	48.68 1.761	92.11 0.267	78.95 1.167	34.21 2.029	46.05, 2.000	81.58 0.507	59.21 1.267
Avoid 1,5(opt 1); 1,7	60.53 1.000	80.26 1.167	78.95 1.167	46.05, 2.000	46.05, 2.000	93.42 0.167	47.37 1.995
Avoid 1,5(opt 2); 2,4; 2,5; 3,5(opt 2); 3,7(opt 2); 4,5; 4,7; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 2)	60.53 1.000	47.37 2.000	46.05 2.000	46.05 2.000	46.05 2.000	60.53 1.000	14.47 3.000
Avoid 1,6 (opt 1)	47.37 2.000	60.53 1.000	25.93 2.343	12.77 3.102	46.05 2.000	27.24 2.450	47.76 1.388
Avoid 1,6 (opt 2)	48.68 1.900	92.11 0.267	64.50 1.428	19.77 2.912	46.05 2.000	67.13 0.848	73.66 0.828
Avoid 1,8 (opt 1)	48.68 1.900	92.11 0.267	48.68 1.644	3.95 3.258	46.05 2.000	51.32 1.108	89.47 0.347
Avoid 1,8 (opt 2)	48.68 1.900	93.42 0.267	48.68 1.737	3.95 3.258	46.05 2.000	51.32 1.521	89.47 0.287
Avoid 2,3 (opt 1)	47.37 2.000	60.53 1.000	26.16 2.340	13.01 3.096	46.05 2.000	27.48 2.445	47.52 1.395
Avoid 2,3 (opt 2)	60.53 1.000	49.57 2.000	21.75 2.589	21.75 2.585	46.05 2.000	36.23 1.597	38.77 2.161
Avoid 2,3 (opt 3)	56.42 1.312	55.10 1.688	22.63 2.647	18.53 2.876	46.05 2.000	33.00 2.011	42.00 1.811
Avoid 2,6 (opt 1)	60.53 1.000	47.37 2.000	21.67 2.447	21.67 2.447	46.05 2.000	36.14 1.447	38.86 2.259
Avoid 2,6 (opt 2)	60.53 1.000	60.53 2.000	21.67 3.031	21.67 2.592	46.05 2.000	36.14 2.279	38.86 1.659
Avoid 2,6 (opt 3)	60.53 1.000	47.37 2.000	21.67 2.814	21.67 2.778	46.05 2.000	36.14 1.885	38.86 1.959
Avoid 2,8 (opt 1)	60.53 1.000	47.37 2.000	15.79 2.529	15.79 2.529	46.05 2.000	30.26 1.529	44.74 2.080
Avoid 2,8 (opt 2)	60.53 1.000	47.37 2.000	15.79 3.149	15.79 2.708	46.05 2.000	30.26 2.395	44.74 1.480
Avoid 2,8 (opt 3)	60.53 1.000	47.37 2.000	15.79 2.910	15.79 2.888	46.05 2.000	30.26 1.981	44.74 1.780

Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	47.37 1.831	60.53 1.000	46.05 2.000	32.90 2.001	46.05 2.000	47.37 2.000	27.63 2.000
Avoid 3,5(opt 3); 3,7(opt 3)	53.95 1.452	53.95 1.500	46.05 2.000	39.47 2.079	46.05 2.000	53.95 1.500	21.05 2.500
Avoid 3,6	47.37 2.000	60.53 1.000	25.93 2.343	12.77 3.102	46.05 2.000	27.24 2.450	47.76 1.388
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 1)	47.37 2.000	60.53 1.000	15.79 2.472	2.63 3.705	46.05 2.000	17.11 2.607	57.90 1.080
Avoid 4,8 (opt 3)	47.37 2.000	60.53 1.000	29.61 2.289	16.45 3.008	46.05 2.000	30.92 2.383	44.08 1.500
Avoid 5,8 (opt 1)	47.37 2.000	60.53 1.000	25.93 2.343	12.77 3.102	46.05 2.000	27.24 2.450	47.76 1.388
Avoid 7,8 (opt 3)	53.95 1.500	53.95 1.500	22.37 2.414	15.79 2.888	46.05 2.000	30.26 1.981	44.74 1.780

	Links 2,3	Links 2,4	Links 2,5	Links 2,6	Links 2,7	Links 2,8	Links 7,8
Avoid 1,2 (opt 1)	53.95, 1.052	51.32, 1.081	51.32, 1.081	52.63, 1.052	94.74, 0.035	51.32, 1.966	53.95, 2.000
Avoid 1,2 (opt 2)	53.95, 1.052	51.32, 1.152	51.32, 1.152	51.63, 1.152	96.05, 0.104	51.32, 2.004	53.95, 2.000
Avoid 1,3	55.26, 1.060	26.24, 2.014	26.24, 2.014	40.79, 1.563	40.71, 1.407	36.92, 1.618	53.95, 2.000
Avoid 1,4	53.95, 1.052	40.79, 1.081	40.79, 1.081	40.79, 1.521	55.26, 0.952	21.05, 2.052	53.95, 2.000
Avoid 1,5(opt 1); 1,7	53.95, 1.052	52.63, 1.052	52.63, 1.052	52.63, 1.052	67.11, 0.052	21.05, 2.052	53.95, 2.000
Avoid 1,5(opt 2); 2,4; 2,5; 3,5(opt 2); 3,7(opt 2); 4,5; 4,7; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 2)	86.84, 1.000	85.53, 1.000	85.53, 1.000	85.53, 1.000	100.00, 0.000	53.95, 2.000	53.95, 2.000
Avoid 1,6 (opt 1)	86.84, 1.000	52.24, 2.102	52.24, 2.102	72.37, 1.502	66.72, 1.102	74.07, 1.388	53.95, 2.000
Avoid 1,6 (opt 2)	53.95, 1.052	26.35, 1.964	26.35, 1.964	40.79, 1.523	40.82, 1.293	35.50, 1.613	53.95, 2.000
Avoid 1,8(opt 1)	53.95, 1.052	10.53, 2.310	10.53, 2.310	40.79, 1.516	25.00, 1.554	51.32, 1.132	53.95, 2.000
Avoid 1,8(opt 2)	53.95, 1.052	10.53, 2.751	10.53, 2.751	40.79, 1.516	25.00, 1.641	51.32, 1.072	53.95, 2.000
Avoid 2,3(opt 1)	86.84, 1.000	52.48, 2.096	52.48, 2.096	72.37, 1.507	66.95, 1.096	73.84, 1.395	53.95, 2.000
Avoid 2,3(opt 2)	86.84, 1.000	61.23, 1.585	61.23, 1.585	85.53, 1.000	75.70, 0.585	78.25, 1.161	53.95, 2.000
Avoid 2,3 (opt 3)	86.84, 1.000	58.00, 1.876	58.00, 1.876	81.42, 1.191	72.47, 0.876	77.37, 1.123	53.95, 2.000
Avoid 2,6(opt 1)	86.84, 1.000	61.14, 1.447	61.14, 1.447	85.53, 1.000	75.61, 0.447	78.34, 1.259	53.95, 2.000
Avoid 2,6(opt 2)	86.84, 1.000	61.14, 1.592	61.14, 1.592	85.53, 1.000	75.61, 0.592	78.34, 0.659	53.95, 2.000
Avoid 2,6(opt 3)	86.84, 1.000	61.14, 1.778	61.14, 1.778	85.53, 1.000	75.61, 0.778	78.34, 0.959	53.95, 2.000
Avoid 2,8(opt 1)	86.84, 1.000	55.26, 1.529	55.26, 1.529	85.53, 1.000	69.74, 0.529	84.21, 1.080	53.95, 2.000
Avoid 2,8(opt 2)	86.84, 1.000	55.26, 1.708	55.26, 1.708	85.53, 1.000	69.74, 0.708	84.21, 0.480	53.95, 2.000
Avoid 2,8(opt 3)	86.84, 1.000	55.26, 1.888	55.26, 1.888	85.53, 1.000	69.74, 0.888	84.21, 0.780	53.95, 2.000

Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	86.84, 1.000	72.37, 1.001	72.37, 1.001	72.37, 1.501	86.84, 0.001	53.95, 2.000	53.95, 2.000
Avoid 3,5(opt 3); 3,7(opt 3)	86.84, 1.000	78.95, 1.079	78.95, 1.079	78.95, 1.289	93.42, 0.079	53.95, 2.000	53.95, 2.000
Avoid 3,6	86.84, 1.000	52.24, 2.102	52.24, 2.102	72.37, 1.502	66.72, 1.102	74.07, 1.388	53.95, 2.000
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 1)	86.84, 1.000	42.11, 2.705	42.11, 2.705	72.37, 1.500	56.58, 1.705	84.21, 1.080	53.95, 2.000
Avoid 4,8(opt 3)	86.84, 1.000	55.92, 2.008	55.92, 2.008	72.37, 1.557	70.40, 1.008	70.40, 1.500	53.95, 2.000
Avoid 5,8(opt 1)	86.84, 1.000	52.24, 2.102	52.24, 2.102	72.37, 1.502	66.72, 1.102	74.07, 1.388	53.95, 2.000
Avoid 7,8(opt 3)	86.84, 1.000	55.26, 1.888	55.26, 1.888	78.95, 1.289	69.74, 0.888	77.63, 1.280	53.95, 2.000

	Links 3,4	Links 3,5	Links 3,6	Links 3,7	Links 3,8	Links 6,7	Links 6,8
Avoid 1,2 (opt 1)	42.11, 2.069	38.16, 2.069	39.47, 1.145	56.58, 1.063	26.32, 2.006	51.32, 1.081	7.89, 3.012
Avoid 1,2 (opt 2)	43.42, 1.948	39.47, 2.052	40.79, 1.135	56.58, 1.048	25.00, 1.948	51.32, 1.152	7.89, 2.952
Avoid 1,3	70.97, 1.077	26.24, 2.137	53.95, 1.060	72.29, 1.077	81.66, 0.558	26.24, 2.014	35.61, 1.618
Avoid 1,4	84.21, 1.100	39.47, 2.052	51.32, 1.152	86.84, 1.000	65.79, 1.100	40.79, 1.081	18.42, 2.152
Avoid 1,5(opt 1); 1,7	72.37, 2.000	39.47, 2.052	39.47, 2.052	86.84, 1.000	53.95, 2.000	52.63, 1.052	6.58, 2.935
Avoid 1,5(opt 2); 2,4; 2,5; 3,5(opt 2); 3,7(opt 2); 4,5; 4,7; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 2)	72.37, 1.000	72.37, 2.000	72.37, 1.000	86.84, 1.000	53.95, 1.714	85.53, 1.000	39.47, 2.778
Avoid 1,6 (opt 1)	65.40, 1.075	52.24, 2.075	85.53, 1.000	66.72, 1.075	87.23, 0.388	52.24, 2.102	72.76, 1.388
Avoid 1,6 (opt 2)	69.77, 1.183	25.03, 2.135	51.32, 1.152	72.40, 1.083	80.23, 0.661	26.35, 1.964	32.87, 1.713
Avoid 1,8(opt 1)	53.95, 1.135	9.21, 2.087	51.32, 1.152	56.58, 1.035	96.05, 0.180	10.53, 2.310	48.68, 1.232
Avoid 1,8(opt 2)	55.26, 1.133	10.53, 2.085	51.32, 1.152	57.90, 1.033	94.74, 0.220	10.53, 2.751	48.68, 1.172
Avoid 2,3(opt 1)	65.64, 1.075	52.48, 2.075	85.53, 1.000	66.95, 1.075	86.99, 0.395	52.48, 2.096	72.52, 1.395

Avoid 2,3(opt 2)	50.27, 1.655	50.27, 2.139	72.37, 1.516	64.75, 1.139	76.04, 1.328	61.23, 1.585	63.77, 2.161
Avoid 2,3 (opt 3)	56.68, 1.509	52.57, 2.106	76.47, 1.403	67.05, 1.106	77.85, 1.086	58.00, 1.876	67.00, 1.772
Avoid 2,6(opt 1)	47.98, 1.671	47.98, 2.153	72.37, 1.518	62.45, 1.153	78.34, 0.966	61.14, 1.447	63.86, 2.259
Avoid 2,6(opt 2)	61.14, 1.589	61.14, 2.071	72.37, 1.518	75.61, 1.071	65.18, 1.659	61.14, 1.592	63.86, 1.659
Avoid 2,6(opt 3)	54.56, 1.630	54.56, 2.112	72.37, 1.518	69.03, 1.112	71.76, 1.459	61.14, 1.778	63.86, 1.903
Avoid 2,8(opt 1)	42.11, 1.782	42.11, 2.138	72.37, 1.644	56.58, 1.138	84.21, 0.783	55.26, 1.529	69.74, 2.080
Avoid 2,8(opt 2)	55.26, 1.722	55.26, 2.078	72.37, 1.644	69.74, 1.078	71.05, 1.480	55.26, 1.708	69.74, 1.480
Avoid 2,8(opt 3)	48.68, 1.752	48.68, 2.108	72.37, 1.644	63.16, 1.108	77.63, 1.280	55.26, 1.888	69.74, 1.780
Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	85.53, 1.000	72.37, 2.000	85.53, 1.000	86.84, 1.000	67.11, 1.000	72.37, 1.001	52.63, 2.000
Avoid 3,5(opt 3); 3,7(opt 3)	78.95, 1.074	72.37, 2.000	78.95, 1.074	86.84, 1.000	60.53, 1.428	78.95, 1.079	46.05, 2.439
Avoid 3,6	65.40, 1.075	52.24, 2.075	85.53, 1.000	66.72, 1.075	87.23, 0.388	52.24, 2.102	72.76, 1.388
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 1)	55.26, 1.023	42.11, 2.023	85.53, 1.000	56.58, 1.023	97.37, 0.080	42.11, 2.705	82.90, 1.080
Avoid 4,8(opt 3)	69.08, 1.078	55.92, 2.078	85.53, 1.000	70.40, 1.078	83.55, 0.500	55.92, 2.008	69.08, 1.500
Avoid 5,8(opt 1)	65.40, 1.075	52.24, 2.075	85.53, 1.000	66.72, 1.075	87.23, 0.388	52.24, 2.102	72.76, 1.388
Avoid 7,8(opt 3)	55.26, 1.430	48.68, 2.108	78.95, 1.322	63.16, 1.108	84.21, 0.706	55.26, 1.888	69.74, 1.780

	Links 4,5	Links 4,6	Links 4,7	Links 4,8	Links 5,6	Links 5,7	Links 5,8
Avoid 1,2 (opt 1)	51.32, 1.081	51.32, 1.081	55.26, 1.074	10.53, 3.046	51.32, 1.081	51.32, 1.081	6.58, 3.052
Avoid 1,2 (opt 2)	51.32, 1.152	52.63, 1.052	53.95, 1.148	10.53, 2.948	51.32, 1.152	51.32, 1.152	6.58, 3.052
Avoid 1,3	26.24, 2.014	39.39, 1.320	57.82, 1.954	52.63, 2.000	26.24, 2.014	26.24, 2.014	7.89, 3.060
Avoid 1,4	40.79, 1.081	52.63, 1.052	73.68, 1.029	51.32, 2.100	40.79, 1.081	40.79, 1.081	6.58, 3.052
Avoid 1,5(opt 1); 1,7	52.63, 1.052	52.63, 1.052	85.53, 1.000	39.47, 3.000	52.63, 1.052	52.63, 1.052	6.58, 3.052
Avoid 1,5(opt 2); 2,4; 2,5; 3,5(opt 2); 3,7(opt 2); 4,5; 4,7; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 2)	85.53, 1.000	85.53, 1.000	85.53, 1.000	39.47, 2.000	85.53, 1.000	85.53, 1.000	39.47, 3.000
Avoid 1,6 (opt 1)	52.24, 2.102	65.40, 1.343	52.24, 2.102	52.63, 2.000	52.24, 2.102	52.24, 2.102	39.47, 3.000
Avoid 1,6 (opt 2)	26.35, 1.964	38.19, 1.313	59.24, 1.912	51.32, 2.100	26.35, 1.964	26.35, 1.964	6.58, 3.052
Avoid 1,8(opt 1)	10.53, 2.310	22.37, 1.529	43.42, 2.258	51.32, 2.100	10.53, 2.310	10.53, 2.310	6.58, 3.052
Avoid 1,8(opt 2)	10.53, 2.751	22.37, 1.623	43.42, 2.699	51.32, 2.100	10.53, 2.751	10.53, 2.751	6.58, 3.052
Avoid 2,3(opt 1)	52.48, 2.096	65.64, 1.340	52.48, 2.096	52.63, 2.000	52.48, 2.096	52.48, 2.096	39.47, 3.000
Avoid 2,3(opt 2)	61.23, 1.585	61.23, 1.517	61.23, 1.585	39.47, 2.516	61.23, 1.585	61.23, 1.585	39.47, 3.000
Avoid 2,3 (opt 3)	58.00, 1.876	62.10, 1.554	58.00, 1.876	43.58, 2.403	58.00, 1.876	58.00, 1.876	39.47, 3.000
Avoid 2,6(opt 1)	61.14, 1.447	61.14, 1.447	61.14, 1.447	39.47, 2.518	61.14, 1.447	61.14, 1.447	39.47, 3.000
Avoid 2,6(opt 2)	61.14, 1.592	61.14, 1.797	61.14, 1.592	39.47, 2.518	61.14, 1.592	61.14, 1.592	39.47, 3.000
Avoid 2,6(opt 3)	61.14, 1.778	61.14, 1.644	61.14, 1.778	39.47, 2.518	61.14, 1.778	61.14, 1.778	39.47, 3.000
Avoid 2,8(opt 1)	55.26, 1.529	55.26, 1.529	55.26, 1.529	39.47, 2.644	55.26, 1.529	55.26, 1.529	39.47, 3.000
Avoid 2,8(opt 2)	55.26, 1.708	55.26, 2.039	55.26, 1.708	39.47, 2.644	55.26, 1.708	55.26, 1.708	39.47, 3.000
Avoid 2,8(opt 3)	55.26, 1.888	55.26, 1.803	55.26, 1.888	39.47, 2.644	55.26, 1.888	55.26, 1.888	39.47, 3.000

Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	72.37, 1.001	85.53, 1.000	72.37, 1.000	52.63, 2.000	72.37, 1.001	72.37, 1.001	36.84, 3.000
Avoid 3,5(opt 3); 3,7(opt 3)	78.95, 1.079	85.53, 1.000	78.95, 1.079	46.05, 2.074	78.95, 1.079	78.95, 1.079	39.47, 3.000
Avoid 3,6	52.24, 2.102	65.40, 1.343	52.24, 2.102	52.63, 2.000	52.24, 2.102	52.24, 2.102	39.47, 3.000
Avoid 3,8; 4,8(opt 1); 5,8(opt 3); 6,8; 7,8(opt 1)	42.11, 2.705	55.26, 1.472	42.11, 2.705	52.63, 2.000	42.11, 2.705	42.11, 2.705	39.47, 3.000
Avoid 4,8(opt 3)	55.92, 2.008	69.08, 1.289	55.92, 2.008	52.63, 2.000	55.92, 2.008	55.92, 2.008	39.47, 3.000
Avoid 5,8(opt 1)	52.24, 2.102	65.40, 1.343	52.24, 2.102	52.63, 2.000	52.24, 2.102	52.24, 2.102	39.47, 3.000
Avoid 7,8(opt 3)	55.26, 1.888	61.84, 1.414	55.26, 1.888	46.05, 2.322	55.26, 1.888	55.26, 1.888	39.47, 3.000

An equilibrium solution was determined for the payoff matrix shown in table 3.18. Examination of the columns revealed that Player T's option to target 1,4 dominated strategy target 1,3. Targeting links 2 and 4 yielded a greater payoff to Player T than disrupting links 2 and 3. Option (2,5) had equivalent payoffs to option (2,4) for Player T. The strategy to target 1,5 dominated options (1,2), (1,6), (1,7), (2,5), (3,7), (3,8), (4,5), (4,6), (4,7), (5,6), and (5,7). Damaging links 5 and 8 provided Player T with a greater payoff than disrupting (1,8), (2,6), (2,7), (2,8), (3,4), (3,5), (3,6), (4,8), (6,7), (6,8), and (7,8). Player M's payoffs were then examined for the remaining three columns [(1,4), (1,5), and (5,8)] of the matrix. Strategy "avoid (1,5) option 1" dominated alternatives "avoid (1,2) options 1 and 2," "avoid (1,4)," "avoid (1,6) option 2," and "avoid (1,8) options 1 and 2." Strategy "avoid (1,5) option 2" dominated alternatives "avoid (1,6) option 1," "avoid (2,3) options 1, 2, and 3," "avoid (2,6) options 1, 2, and 3," "avoid (2,8) options 1, 2, and 3," "avoid (3,4)," "avoid (3,5) option 3," "avoid (3,6)," "avoid (3,8)," "avoid (4,8) option 3," "avoid (5,8) option 1," and "avoid (7,8) option 3." Only three strategies remained for Player M: "avoid (1,3)," "avoid (1,5) option 1," and "avoid (1,5) option 2." Examination of the remaining matrix revealed “target links (5,8)” as the dominant strategy for Player T. Player M’s best strategy was “avoid (1,5) option 2,” which was equivalent to “avoid (5,8) option 2.” At this equilibrium point, Player M received a payoff of 39.47% of drivers safely reaching their destinations and Player T’s payoff was 3.000.

If Player M had overestimated Player T’s resources and Player T really only had enough supplies to damage one of the links, the following payoff matrix, shown in table 3.19 would have resulted from the routing strategy “avoid (1,5) option 2 / avoid (5,8) option 2”.

Table 3.19 Payoff Matrix for Misinformation about Player T’s Resources, ½ Demand Scenario

Player	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
M	60.53	100.00	86.84	85.53	85.53	85.53	100.00	53.95
T	1.000	0.000	3.7×10^{-5}	1.000	1.000	1.000	0.000	1.778

Player T would select link 8 as the target to obtain the largest payoff. Player M’s payoff would then be 53.95% of the drivers safely reaching their destinations. This payoff was greater than the case where Player T actually did have the resources to damage both links 5 and 8. However, recall from table 3.13, that if Player M knew that Player T had selected link 8, Player M could have implemented a routing strategy that would have allowed 82.46% of travelers to safely reach their destinations. By employing misinformation about the amount of resources, Player T was able to obtain a payoff of 1.778 (table 3.19), which is more desirable than 0.533 (table 3.13).

The payoff matrices are not presented here for the n=3 case because Player T has a single dominant strategy. To achieve the largest payoff, Player T would target links 1, 5, and 8. These three links form a traditional cut set for the network and Player M cannot route traffic to avoid links 1, 5, and 8 simultaneously. The payoff for Player T is 4.0 and the payoff to Player M is 0.

3.3.4 Game 4: Perfect Information for Both Players, Multiple Moves

In Game 4, Player M and Player T alternate moves. First Player M assigns traffic to avoid a predicted set of links. Player T then damages a set of links. Player M then reassigns traffic to avoid the set that was damaged. Player T selects the next set of links to be damaged, and so on until Player T's resources are gone or Player M cannot route traffic to connect any origin-destination pair.

Due to the small nature of the sample network, assume that each successive set of damaged links only consists of one arc. For this game, the original demand levels are used.

From the results in the previous games, Player M first chooses the strategy "Avoid Link 5" and Player T selects "Link 5." The origin-destination pair (5,2) has now been severed – no route exists between the two nodes. The demand for this O-D pair is removed from consideration of the remaining links. The modified network is shown below in figure 3.4.

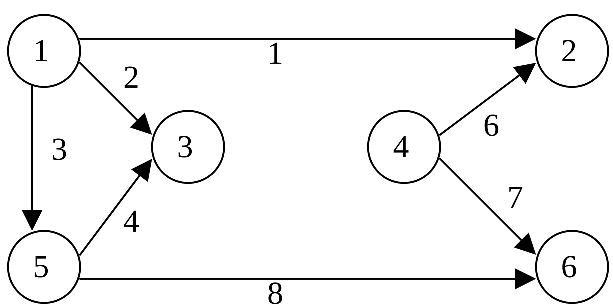

Figure 3.4 Sample Network after Player T's First Move

Clearly, links 2, 4, 6, and 7 are of no use to the connectivity of the remaining origin-destination pairs and the only links of interest are 1, 3, and 8. Recall that the O-D demands are $q^{1,2} = 3000$, $q^{1,6} = 1000$, and $q^{5,6} = 2500$. Since only one path connects each origin-destination pair and sufficient capacity is available on that path, all of the vehicles are assigned to link 1 for (1,2) and to links 3 and 8 for (1,6) and to link 8 for (5,6). The resulting vulnerability indices are $M_1^{1,2} = 1.0$, $M_3^{1,6} = 1.0$, $M_8^{1,6}= 1.0$, and $M_8^{5,6} = 1.0$. Summing over the O-D pairs, link 8 has the highest disruption index and will be targeted by Player T. Destroying this link will sever origins 1 and 5 from destination 6. After this move by T, only one O-D pair (1,2) is still connected. In the final move, Player T will damage link 1. Thus, after 3 moves by Player T, a cut set of the network has been determined.

3.4 SUMMARY

In this chapter, a bi-level mathematical programming formulation has been presented to identify vulnerable links in a road transportation network. Game theory may be used to solve this formulation, or interesting variants thereof. For illustrative purposes, a two player, non-zero sum game was presented, with four different cases of information. The payoff for the “evil entity” was the disruption index. The development of this measure was also presented in this chapter. Various attributes of the network were incorporated into the index, including current traffic flow, travel time, availability of alternate paths, and relative excess capacity on the alternate paths.

A small sample network was provided to illustrate the application of all of the concepts developed in this chapter. For each of the games envisioned, the most vulnerable link in the network was the one connecting the most origin-destination pairs and currently carrying flow to each of the destinations. The interplay of traffic assignment, flow dependent travel time, and network design allow the disruption index to capture effects of damage to one link, or set of links, on origin-destination connectivity and the network as a whole.

Chapter 4

Model Of Household Decision Making In An Emergency Evacuation

This chapter presents the mathematical modeling of household behavior in an emergency evacuation and integrates this behavior with a traffic simulation-assignment methodology to estimate network evacuation time. As noted in Chapter 2, families tend to unite prior to the evacuation of a building or town. Conventional evacuation models disregard this observed behavior. Stern and Sinuany-Stern (1989) and Sinuany-Stern and Stern (1993) did include some human factors into their evacuation model, but these aspects are related to the diffusion of evacuation instructions and individual's preparation time. These studies still overlook family gathering behavior. Omission of this phenomenon in simulation models has two major implications. First, the models fail to capture some of the complex travel patterns and the resulting network traffic patterns that are exhibited only during emergency situations. Second, the estimated evacuation times may be overly optimistic.

In this chapter, each household is modeled as a single entity that makes two primary decisions sequentially. The first decision is the location where the family will meet; this site may have been selected well in advance of the evacuation. The second decision is the assignment of drivers to pick up other family members who may not have access to vehicles. These decisions result in forming trip chains, thus addressing the first implication mentioned in the previous paragraph – that complex travel patterns have not been adequately captured by traditional engineering models.

The second implication, that predicted evacuation times are overly optimistic, is examined through traffic simulation. The impacts of trip chains and the factors involved in the activity chain generation process on evacuation time are examined in this chapter. Furthermore, network clearance comparisons are made with cases in which no trip chains are considered.

The remainder of this chapter is organized as follows. The first section presents the modeling framework and formulation, corresponding to the first objective identified in section 1.2.2 of chapter 1. The second portion of this chapter explains the experimental design. In the third section, the results are presented and discussed in terms of objectives 2 and 3 of section 1.2.2. Finally, a summary is provided.

4.1 MODELING FRAMEWORK AND PROBLEM FORMULATION

The notation used in this formulation is presented in table 4.1. The general problem can be stated as follows: given a set of households, their decision making rules, and a transportation network ***G(N,A)*** consisting of a set of nodes ***N*** and arcs ***A***, determine the evacuation time for the network. The characteristics of the network's nodes and links are known, and the associated time-varying OD demand pattern for regular (non emergency) peak period conditions is given.

The evacuation time (*E*) is defined as the time elapsed between the instant the evacuation is ordered and the instant the last vehicle exits the network.

$$E = T_{final} - T_{order} \tag{4.1}$$

where
$$T_{final} = \max_{v}\left(\theta^{v} + \sum_{i \in \xi} p_i^{v} + \sum_{j \in \psi} t_j^{v}\right). \tag{4.2}$$

ξ is the subset of intermediate nodes visited by vehicle v $\xi \in I^{h}$ and

ψ is the set of paths used by vehicle v.

Equation (4.2) determines the maximum time a vehicle takes to evacuate a particular area. The vehicle time is the elapsed time (θ^v) between the instant the evacuation order is given and the instant the vehicle enters the network or an entry queue to the network plus the dwell time ($\boldsymbol{p_i^v}$) at each intermediate location i visited by vehicle v plus the travel time (t_j^v) of each path j used by vehicle v. In the final term of equation (4.2), the path travel time is flow dependent and calculated as the sum of the travel times ($t_l(x_l)$) on the links in path j. To identify a lower bound on the evacuation time, the flow dependent link travel time is determined from the system optimal traffic assignment (Sheffi, 1985):

$$\min z(x) = \sum_{l \in A} \sum_{r,s} x_l^{r,s} t_l \left(\sum_{r,s} x_l^{r,s} \right) \tag{4.3}$$

subject to

$$\sum_j f_j^{r,s} = q^{r,s} \quad \forall r,s \tag{4.4}$$

$$x_l^{r,s} \geq 0 \qquad \forall l,r,s \tag{4.5}$$

$$f_j^{r,s} \geq 0 \qquad \forall\ j,r,s \tag{4.6}$$

$$x^A = \Phi^{|A| \times |J|} f^J \tag{4.7}$$

where $\boldsymbol{x^A}$ is the vector of arc flows $x_l \in x^A$,

$$x_l = \sum_{r,s} x_l^{r,s} \tag{4.8}$$

$\boldsymbol{f^J}$ is the vector of path flows.

In the system optimal traffic assignment, the objective function (4.3) minimizes the total network travel time. The first constraint ensures that origin-destination demands are met and the second and third constraints are for non-negativity. Equation (4.7) relates the vector of arc flows to the path flows through an arc-path incidence matrix Φ (Jahn, et al, 2002), the entries of which are 1 if link *l* lies on path *j* and 0 otherwise.

Table 4.1 Summary of Notation

Notation	**Interpretation**
E	Network evacuation time
T_{final}	Time at which the final vehicle exits the network
T_{order}	Time at which the evacuation order is given
v, w	Vehicle index
θ^v	Initial delay/waiting time of a vehicle entering the network
ξ^h	Set of intermediate destinations for vehicle v of household h
i	Index of the set I_v^h
I^h	Set of intermediate destinations for household h, not including the meeting location
p_i	Time spent at intermediate destination i
r,s	Origin-destination pair
l	Link index
L_j	Set of links in path j
t_l	Link travel time
$x_l^{r,s}$	Flow on link l pertaining to r,s
$q^{r,s}$	Origin-destination demand
G^h	Graph representing the household's evoked network
N^h	Set of nodes in G^h
A^h	Set of arcs in G^h
V^h	Set of vehicles belonging to household h
R^h	Set of origin nodes where household h has a vehicle
U^h	Set of possible meeting places for household h
u	Member of the set U^h
$\tau^{r,s}$	Perceived travel time from location r to s
S	Final destination/place of safety, such as a shelter
$d_{u,S}$	Distance between the meeting place and S
b_S	**Maximum tolerable distance between u and S**
F	Location of destruction, fire, storm, or other danger
$d_{u,F}$	**Distance between u and F**
b_F	Minimum tolerable distance between u and F
m_u	Maximum distance between u and the nodes in sets I_v^h and R^h
y_u	The decision variables are y_u which take the value 1 if location u is selected and 0 otherwise.

The set of intermediate destinations (ξ) visited by a vehicle is determined by a sequence of decisions at the household level. These intermediate destinations are determined through two linear integer programs. The first addresses the selection of a meeting location where the family gathers prior to evacuating the town, or threatened area. The second mathematical program assigns drivers to pick up sites where other family members, who may not have access to vehicles, are located. The second program also determines the sequence of these pick ups.

When making decisions, the household is assumed to ignore the actual nodes of the network and to consider the intermediate destinations as the "nodes" of their overall path. Thus, for decision making purposes, the family uses a perceived aggregated network that extracts relevant information from the actual underlying network. In other words, they create an evoked network that is specific to that household $\boldsymbol{G^h(N^h,A^h)}$, and solve their logistical problems without further consideration of the actual transportation network. In this household-specific network, the only nodes included in the set $\boldsymbol{N^h}$ are the origins of the household vehicles $\boldsymbol{R^h}$ and the intermediate household destinations $\boldsymbol{I^h}$, including possible meeting locations $\boldsymbol{U^h}$, $\boldsymbol{U^h} \subseteq \boldsymbol{I^h}$ (where the superscript h denotes the household).

$$\boldsymbol{N^h} = \boldsymbol{I^h} \cup \boldsymbol{R^h} \tag{4.6}$$

The formulation for deciding the household meeting location minimizes the maximum travel time from any of the locations at which there are family members $\left(\boldsymbol{I^h} \cup \boldsymbol{R^h}\right)$. The set of possible meeting locations is determined by the household's decision makers; this set may consist of home, schools, shopping areas, parks, or any subset of these. If the danger prevents two or more family members from meeting safely, they are considered separate entities. In order for a location to be considered, the site must be a minimum distance from the danger and within a given distance from the final shelter location. This last constraint ensures that the meeting location that is selected is not on the side of town furthest from the final evacuation path and safety. Once all family members have reached the meeting location, the family continues on to the final shelter as a single unit. The linear integer programming formulation is presented next, with the applicable notation defined in table 4.1.

The mathematical programming formulation is as follows:

$$\min_{U^h} Z^h = m_u \tag{4.7}$$

subject to

$$\left(y_u \tau^{j,u} \leq m_u \quad \forall\ j \in I^h \cup R^h\right) \qquad \forall u \in U^h \tag{4.8}$$

$$\sum_{u \in U^h} y_u = 1 \tag{4.9}$$

$$d_{u,S} \leq b_S \tag{4.10}$$

$$d_{u,F} \geq b_F \tag{4.11}$$

$$y_u \in \{0,1\} \qquad \forall\ u \in U^h \tag{4.12}$$

The objective function (4.7) minimizes the maximum perceived travel time of all family members' initial locations from the meeting place. The first constraint (4.8) determines the maximum travel time for every possible meeting place *u*. The second constraint (4.9) ensures that only one meeting location is selected. The third (4.10) and fourth (4.11) constraints are distance requirements. Finally, the fifth constraint (4.12) is the requirement that the decision variables y_u be zero or one (Desrochers et al, 1988).

The intermediate destination nodes are determined by the assignment of vehicles to pick up non-driving household members, based on the meeting location. The definition of the variables for the trip chain assignments is found in table 4.1. Additionally, $\boldsymbol{V^h}$ is the set of vehicles available to household *h*. The decision variables are x_v^{ij} which take the value 1.0 if vehicle v uses the hyperlink connecting nodes i and j. Indices for the vehicles are denoted by v and w; C_v is the capacity of vehicle v.

Two, possibly conflicting, objectives are involved in the pick-up, and resulting trip chain, assignments. Household decision makers may wish to (1) minimize the total travel time for household's fleet of vehicles and (2) minimize the waiting time at the meeting location. Risk is associated with traveling on the network because there could be an incident on the roadway; more household drivers on the network increases the risk to the household as whole. However, having one driver wait at the meeting location while another driver picks up all of the children could cause frustration and concern on the part of the driver who is waiting and one of the children may be waiting for the parent for a considerable amount of time. Thus, household decision makers must make trade offs between the dangers of multiple household vehicles traveling and waiting at the meeting location for an extended period of time. To minimize the time that family members in one vehicle are waiting for family members in another vehicle, their arrival times at the meeting location should be close together. This part of the objective function is given by:

$$\min \sum_{v,w \in V^h} \left| \sum_{i,j \in N^h} \tau^{ij} x_v^{ij} - \sum_{i,j \in N^h} \tau^{ij} x_w^{ij} \right| \tag{4.13}$$

The trade off between waiting time and multiple vehicles traveling can be mathematically expressed as a linear combination of equation (4.13) and the objective function of the classic VRP (see chapter 2). This second part of the overall objective function is as follows:

$$\min \sum_{i,j \in N^h} \tau^{ij} x^{ij}$$

Let λ be the weight associated with the total fleet travel time and ***(1-λ)*** be the weight assigned to the waiting time; $\lambda \in [0.0, 1.0]$. The specific weight assigned to each of the objectives may vary from household to household.

Recall that the links in the household's evoked network correspond to the paths (in the original network) between the intermediate destinations (nodes). For capacity considerations, each non-driving family member is considered an individual customer, regardless of whether there is another non-driving family member at the same location. If there is more than one family member at the same physical location, the evoked, aggregated network is modified to reflect virtual nodes connected by zero cost virtual links. In this manner, the driver with sufficient capacity does not incur any further cost for picking up more than one passenger at a given location. The capacity is adjusted appropriately.

Equation (4.13) is not a linear programming formulation, but may be converted to one (Bertsimas and Tsitsiklis, 1997) by observing that

$$\left| \sum_{i,j \in N^h} \tau^{ij} x_v^{ij} - \sum_{i,j \in N^h} \tau^{ij} x_w^{ij} \right| \tag{4.14}$$

is the smallest number $\boldsymbol{n_{v,w}}$ that satisfies

$$\left(\sum_{i,j \in N^h} \tau^{ij} x_v^{ij} - \sum_{i,j \in N^h} \tau^{ij} x_w^{ij} \right) \le n_{v.w} \quad \text{and}$$

$$-\left(\sum_{i,j \in N^h} \tau^{ij} x_v^{ij} - \sum_{i,j \in N^h} \tau^{ij} x_w^{ij} \right) \le n_{v.w} \tag{4.15}$$

The complete objective function, incorporating trade offs, is:

$$\min z = \lambda\left(\sum_{v \in V^h} \tau^{ij} x_v^{ij}\right) + (1-\lambda) \sum_{v,w \in V^h} n_{v,w} \tag{4.16}$$

The objective function is subject to the following constraints:

$$\sum_{v \in V^h} \sum_{j \in I^h} x_v^{ij} = 1 \qquad i \in (R^h \cup I^h) \tag{4.17}$$

$$\sum_{j \in I^h} x_v^{ij} - \sum_{j \in I^h} x_v^{ji} = 0 \qquad i \in I^h, v \in V^h \tag{4.18}$$

$$\sum_{j \in I^h} x_v^{ij} - \sum_{j \in I^h} x_v^{ju} = 0 \qquad i \in I^h, v \in V^h \tag{4.19}$$

$$\sum_{(i,u) \in A^h} x_v^{iu} = 1 \qquad \forall\ v \in V^h \tag{4.20}$$

$$\sum_{i \in (R^h \cup I^h),\ j \in I^h} x_v^{ij} \le C_v \qquad \forall\ v \in V^h \tag{4.21}$$

$$x_v^{ij} \in \{0,1\} \qquad \forall\ v \in V^h \tag{4.22}$$

$$\left(\sum_{i,j \in N^h} \tau^{ij} x_v^{ij} - \sum_{i,j \in N^h} \tau^{ij} x_w^{ij}\right),\ -\left(\sum_{i,j \in N^h} \tau^{ij} x_v^{ij} - \sum_{i,j \in N^h} \tau^{ij} x_w^{ij}\right) \le n_{v,w} \tag{4.23}$$

The first constraint (equation 4.17) ensures that each customer is picked up and delivered. The second constraint (4.18) is for the conservation of flow through the pick-up nodes. The third constraint (4.19) ensures that each family member that is picked up by a specific vehicle is delivered to the meeting location by the same vehicle. The fourth (4.20) indicates that each vehicle arrives at the meeting place only once. The fifth constraint (4.21) is the capacity constraint for the number of seats available in each vehicle. The sixth constraint (4.22) specifies the set of values that the binary decision variables may take. Equation (4.23) is required for the transformation of the absolute value in equation (4.14) into a linear programming formulation.

The two linear integer programs result in the selection of a meeting location and the trip chains assigned to each driving member of the household. In the following section, the experimental design for the application of this model is presented.

4.2 EXPERIMENTAL DESIGN

Three main steps are involved in the incorporation of the household behavior model for emergency evacuations within a network traffic modeling and simulation framework. First, a simulation is run to generate "typical" travel times to given locations. Then, these expected travel times are used in the linear integer programs that describe the decision-making process of the household meeting place selection and activity chain assignment of the family's vehicles. These linear integer programs are solved for each household in the network. Finally, the trip chains are employed in time-dependent traffic assignment-simulation software.

Step 1.

In the first step, an initial time-varying assignment is performed to generate the travel time characteristics on the network links for "everyday" traffic conditions, employing a k-shortest paths algorithm and the user equilibrium traffic assignment. Alternatively, actual measured travel times might be used, if available. These times serve as the perceived travel times ($\tau^{r,s}$) for the households' decision makers in the evoked network $\boldsymbol{G^h(N^h,A^h)}$. The time to reach family members and the meeting location greatly affect both the selection of the meeting location and the assignment of drivers to pick-up sites.

Step 2.

In the second step of the approach, the two linear integer programs described in the previous section, are solved sequentially. These formulations are used to determine the household meeting location and then, simultaneously, the assignment of drivers to pick-up locations and the sequence of those pick-ups.

The results from the linear integer programs are a set of trip chains for each participating vehicle in the household. This information is then used in step 3.

Step 3.

In the third step, the traffic is simulated using the results from step 2. The sequence of intermediate nodes in the trip chain is followed by the simulator. The availability and use of information may cause the driver to follow alternate paths in the underlying transportation network (which may not be part of the household's evoked network when the decisions were made). These decisions are handled according to the logic of the simulation-assignment methodology with trip chaining described in Abdelghany, Mahmassani, and Chiu (2001). The simulator keeps track of congestion levels and operating speeds, allowing for information on real-time travel conditions, which may be vastly different from those anticipated at the time household members made their decisions. Depending on information supply strategies, and the particular problem formulation and operational scenario, the drivers may make changes to their original plans.

The traffic simulation-assignment tool providing the network modeling capability for steps 1 and 3 is DYNASMART (DYnamic Network Assignment Simulation Methodology for Advanced Road Telematics), developed at the University of Maryland and the University of Texas at Austin. The key features of this software that are employed in this work included activity chains, zones, virtual centroids, user equilibrium traffic assignment, and system optimal traffic assignment (Mahmassani and Sbayti, 2003).

A simplified network model of the south-central portion of Fort Worth, Texas is used as a test bed for this work (see figure 4.1). The network consists of 184 nodes (only 180 are shown in the figure). In this model, two elementary schools (nodes 123 and 165), two middle schools (nodes 122 and 169), and one high school (node 170) are located throughout the network (Fort Worth Independent School District, 2003). Each school is modeled as a distinct zone because in DYNASMART, every zone has a virtual centroid to which demand is attracted or from which it is produced. The centroid may be connected to multiple links, but in this work, it is particularly important that only links entering or leaving the schools' nodes carry traffic related to picking up the children. Overall, the network shown in figure 4.1 is divided into 14 zones. Three of these are designated as business zones (7-9) ; five are school zones (10-14); and the remaining six are residential areas (see figure 4.2).

Four additional nodes (300, 301, 302, and 303) and seven links, not shown in figure 1, were added to the network to represent final shelter destinations. These nodes are considered to be outside of the evacuation area.

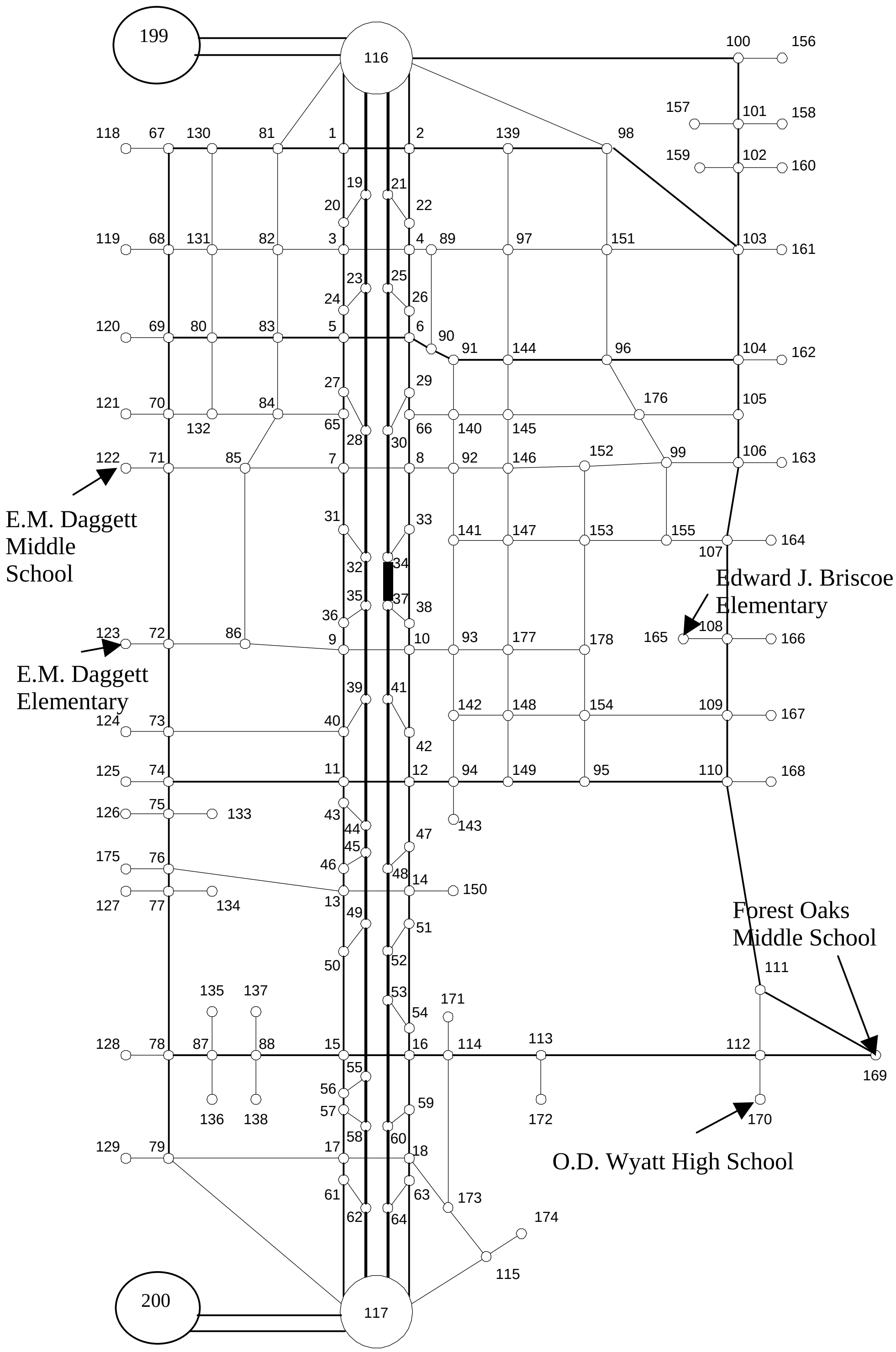

Figure 4.1 Sample Network

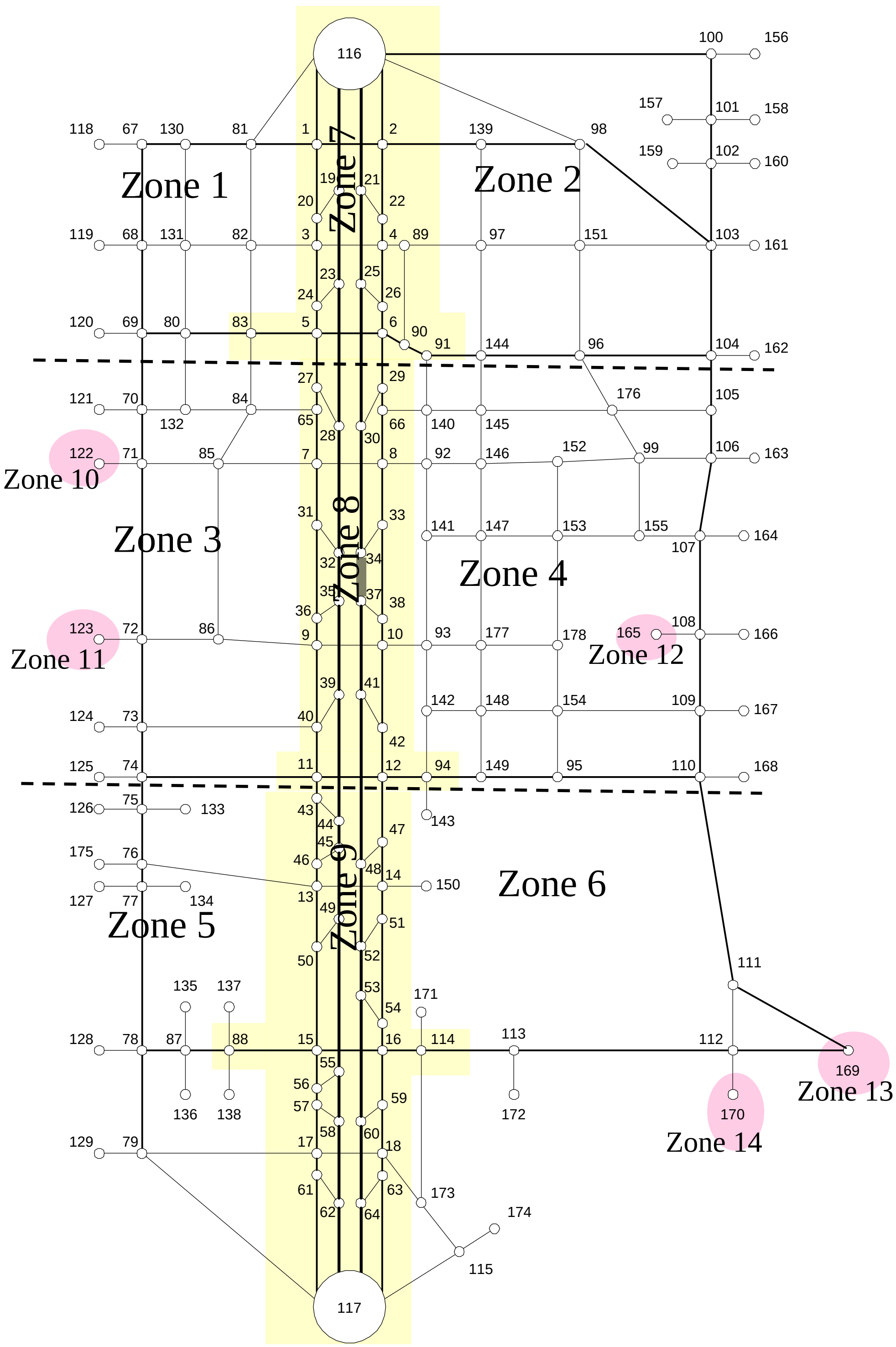

Figure 4.2 Sample Network with Zones

Census 2000 data for the Fort Worth, TX area indicated that of the 195,058 households, 65.4% are families and 34.6% are nonfamily households. The following data is relevant to the families, but is given in terms of percentage of the total households: 34.7% have children under 18 years old, 45.8% are married couple families, and 14.7% are single mother households. The average family size is slightly over 3 persons (Census 2000).

The percentages reported in the census tables are used to generate household types and the relative numbers of each type. Thirty different basic households are generated using the computer code found in Appendix C:

1. Single individual
2. Single parent with one elementary school child
3. Single parent with one middle school child
4. Single parent with one high school child
5. Single parent with two elementary school children
6. Single parent with two middle school children
7. Single parent with two high school children
8. Single parent with one elementary school child and one middle school child
9. Single parent with one elementary school child and one high school child
10. Single parent with one middle school aged child and one high school aged child
11. Couple (no children)
12. Two parents with one elementary school aged child
13. Two parents with one middle school aged child
14. Two parents with one high school aged child
15. Two parents with two elementary school aged children
16. Two parents with two middle school aged children

17. Two parents with two high school aged children
18. Two parents with one elementary school aged child and one middle school aged child
19. Two parents with one middle school aged child and one high school aged child
20. Two parents with one elementary school child and one high school child
21. Two parents with three elementary school children
22. Two parents with two elementary school children and one middle school child
23. Two parents with one elementary school child and two middle school children
24. Two parents with one elementary school, one middle school, and one high school child
25. Two parents with three middle school children
26. Two parents with two middle school children and one high school child
27. Two parents with one middle school and two high school children
28. Two parents with three high school children
29. Two parents with two high school children and one elementary school child
30. Two parents with one high school child and two elementary school children

These thirty are then repeated in the proportions previously mentioned until a total of 20,000 households are generated. The household's business and residential zones are assigned by a random number generator. The choice set for meeting locations of a household with two parents is limited to schools where the family had children in attendance and the home. In the case of single parents, only the school locations of the children are in the set. If the household does not have any children, the couple meets at the home. When the household is a single individual, there is no meeting location and the individual is assigned an immediate shelter destination.

Further assumptions that are used in the model include:

1. Households do not contain more than three children.
2. All household vehicles are capable of carrying the entire family.
3. Children are only transported by their parents' vehicles.
4. High school children would not drive themselves.
5. Traffic management agencies, combined with law enforcement personnel, would provide route guidance in an emergency situation so as to assign traffic according to the system optimum.
6. The order to evacuate would be given during school hours.

Once the households are generated, the framework developed in section 4.1 is followed. In accordance with step 1, a slight increase to typical peak period traffic is simulated using the user equilibrium traffic assignment. The travel times from business zones to residential zones, business zones to school zones, school zones to other school zones, and school zones to residential zones are used as the perceived trip costs for household decision making. The meeting locations and pick-up assignments are then generated for each household (step 2). Finally, the activity chains determined by the pick-up assignments are used in the system optimal traffic assignment (step 3).

4.3 EXPERIMENTAL RESULTS

Using the simulation tools, 20,000 households and 30,141 vehicles were generated. Approximately 49% of the households had only one adult and 51% had two. No children were present in 49% of the households, including both single and dual parent homes. Elementary school aged children (at least one) were found in 23% of the households; middle school aged children were in 23% of the households, and high school aged children were in 24% of the households. Approximately 13% of the households had multiple children in different schools. Based on these household characteristics, between 33 and 57% of the total generated vehicles would stop at schools to pick-up the children.

Typical travel times for the households were generated based on 34,021 vehicles. This increased number of vehicles allowed for a small amount of additional travel time, which may be part of the perceived travel time in an emergency situation. Based on 120 second cycle times for the lights, a delay of 1 minute was added for each traffic light. Delays at stop signs were assumed to be 30 seconds. The perceived zonal travel times with intersection delays are given in minutes in tables 4.2 and 4.3.

Table 4.2 Perceived Zonal Travel Times to Residential Zones

		To					
	Zone	1	2	3	4	5	6
	1	-	-	-	-	-	-
	2	-	-	-	-	-	-
	3	-	-	-	-	-	-
	4	-	-	-	-	-	-
	5	-	-	-	-	-	-
	6	-	-	-	-	-	-
From	7	1.44	4.71	9.11	9.13	7.95	8.78
	8	3.15	6.25	2.21	2.83	10.22	16.81
	9	13.01	9.60	9.32	7.38	3.68	5.49
	10	3.85	9.31	1.56	-	-	-
	11	5.08	-	1.54	-	7.79	-
	12	-	5.31	-	1.08	-	6.91
	13	-	-	-	6.04	12.03	1.08
	14	21.09	13.35	12.96	5.02	9.41	2.59

The blanks in tables 4.2 and 4.3 indicate that travel for that origin-destination (OD) zone pair is forbidden except when the vehicle is "passing through" one or both of those zones.

Table 4.3 Perceived Travel Times to School Zones

Zone	10	11	12	13	14
1	8.11	10.62	-	-	23.52
2	15.53	-	14.27	-	18.42
3	5.53	8.38	-	-	22.56
4	-	-	6.13	4.74	9.98
5	-	4.71	-	16.02	15.16
6	-	-	10.58	8.71	5.85
7	10.89	13.14	16.66	23.85	15.02
8	7.15	3.48	9.13	21.38	20.34
9	9.72	7.50	17.44	10.39	9.36
10	-	3.33	13.42	-	18.79
11	3.31	-	-	17.58	16.54
12	14.35	-	-	8.27	7.78
13	-	16.49	8.59	-	3.72
14	19.34	17.02	9.11	2.13	-

Using the perceived travel times given in tables 4.2 and 4.3 and equations (4.7-4.12), meeting locations were assigned to the households. Table 4.4 presents the relative frequency of meeting location selection based on household type. Due to the assumptions of household size and vehicles being able to accommodate all household members, several of the household types were grouped. For instance, household types 12, 15, and 21 differ only by the number of elementary school children; since all of these children were at the same location and vehicle capacity constraints were not an issue, household types 15 and 21 can be modeled as type 12. Household types 1-7 were limited to choice sets consisting of only one option, as described in the previous section. However, the particular node in the set was allowed to vary within household types; for instance, there were two possible elementary schools that were assigned based on the location of the home. When the home zone was available for consideration as a meeting location, families select this option the majority of the time. This result was due to the assignment of schools based on the zone in which the home was sited; such an assignment rule frequently placed the home zone in a central location relative to the schools.

Table 4.4 Meeting Location Selection

Household Type	Meet at Elementary School	Meet at Middle School	Meet at High School	Meet at Home	Evacuate immediately
1	NA	NA	NA	NA	100%
2 or 5	100%	NA	NA	NA	NA
3 or 6	NA	100%	NA	NA	NA
4 or 7	NA	NA	100%	NA	NA
8	50.43%	49.57%	NA	NA	NA
9	1.54%	NA	98.46%	NA	NA
10	NA	1.31%	98.69%	NA	NA
11	NA	NA	NA	100%	NA
12, 15, or 21	14.29%	NA	NA	85.71%	NA
13, 16, or 25	NA	18.16%	NA	81.84%	NA
14, 17, or 28	NA	NA	29.21%	70.79%	NA
18, 22, or 23	9.87%	6.89%	NA	83.24%	NA
19, 26, or 27	NA	0	18.39%	81.61%	NA
20, 29, or 30	0.63%	NA	18.56%	80.81%	NA
24	17.21%	0	0	82.79%	NA

After the meeting location was selected, the activity chains were determined using equations (4.16-4.23). Initially, a minimum dwell time, or delay, of five minutes was assumed for each of the intermediate destination nodes; smaller values of this waiting time were later considered (see below). This delay was intended to allow for parents to find their children in the mass of students that was likely to be awaiting transportation and secure them in the car. If the household had two vehicles, one of them was left at the meeting location and all of the family members evacuated in a single vehicle.

The value of the weight λ, associated with the total fleet travel time was varied from 0 to 1.0. The effect of varying this weight on the pick up assignments for one household of each type is shown in table 4.5 for the case when the waiting time at each intermediate node is 2.10 minutes. The vehicle and fleet times are the perceived travel times, including waiting times at the school nodes.

Table 4.5 Sample Trip Chains for Various Household Types

House -hold Type (hh #)	Weights (Total Fleet Time, Waiting Time at Meeting Node) (λ, 1–λ)	Car (start zone)	Stop 1 (zone)	Stop 2 (zone)	Stop 3 (zone)	Vehicle Time (min)	Fleet Time (min)
1	Any	1 (9)	-	-	-	-	-
2	Any	1 (7)	Elementary (11)	-	-	13.14	13.14
3	Any	1 (7)	Middle (10)	-	-	10.89	10.89
4	Any	1 (7)	High (14)			15.02	15.02
5	Any	1 (7)	Elementary (12)	-	-	16.66	16.66
6	Any	1 (7)	Middle (13)	-	-	23.85	23.85
7	Any	1 (9)	High (14)	-	-	9.36	9.36
8	Any	1 (9)	Middle (10)	Elementary (11)	-	15.15	15.15
9	Any	1 (9)	Elementary (12)	High (14)	-	27.32	27.32
10	Any	1 (7)	Middle (10)	High (14)	-	31.78	31.78
11	Any	1 (9)	Home (4)	-	-	7.38	16.51
		2 (7)	Home (4)	-	-	9.13	
12	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6) (0.5,0.5)	1 (9)	Home (2)	-	-	9.60	33.67
		2 (7)	Elementary (12)	Home (2)	-	24.07	
	(0.6,0.4) (0.7,0.3) (0.8,0.2) (0.9,0.1) (1.0,0.0)	1	Elementary (12)	Home (2)	-	24.85	29.56
		2	Home (2)	-	-	4.71	
13	Any	1 (7)	Home (5)	-	-	7.95	32.47
		2 (9)	Middle (13)	Home (5)	-	24.52	

14	Any	1 (7)	Home (6)	-	-	8.78	22.83
		2 (9)	High (14)	Home (6)		14.05	
15	Any	1 (7)	Home (4)	-	-	9.13	28.97
		2 (7)	Elementary (12)	Home (4)	-	19.84	
16	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6) (0.5,0.5) (0.6,0.4) (0.7,0.3) (0.8,0.2) (0.9,0.1)	1 (7)	Home (5)	-	-	7.95	37.10
		2 (5)	Middle (13)	Home (5)	-	29.15	
	(1.0,0.0)	1	Middle (13)	Home (5)	-	37.98	37.98
		2	Home (5)	-	-	0	
17	Any	1 (9)	High (14)	-	-	9.36	24.38
		2 (7)	High (14)	-	-	15.02	
18	Any	1 (9)	Middle (13)	Home (5)	-	24.52	41.91
		2 (9)	Elementary (11)	Home (5)	-	17.39	
19	Any	1 (9)	High (14)	Home (3)	-	24.42	38.97
		2 (7)	Middle (10)	Home (3)	-	14.55	
20	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6)	1 (7)	High (14)	Home (6)	-	19.71	46.16
		2 (9)	Elementary (12)	Home (6)	-	26.45	
	(0.5,0.5) (0.6,0.4) (0.7,0.3) (0.8,0.2)	1	Elementary (12)	Home (6)	Home (6)	25.67	39.72
		2	High (14)	Home (6)	-	14.05	
	(0.9,0.1) (1.0,0.0)	1	Elementary (12)	High (14)	Home (6)	31.23	36.72
		2	Home (6)	-	-	5.49	
21	Any	1 (9)	Elementary (11)	-	-	7.50	15.00
		2 (9)	Elementary (11)	-	-	7.50	

22	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6) (0.5,0.5)	1 (7)	Middle (13)	Home (6)	-	27.03	52.70
		2 (7)	Elementary (12)	Home (6)	-	25.67	
	(0.6,0.4) (0.7,0.3) (0.8,0.2) (0.9,0.1) (1.0,0.0)	1	Home (6)	-	-	8.78	38.99
		2	Elementary (12)	Middle (13)	Home (6)	30.21	
23	(0.0,1.0)	1 (9)	Middle (10)	Home (3)	-	13.38	30.16
		2 (7)	Elementary (11)	Home (3)	-	16.78	
	(0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6) (0.5,0.5) (0.6,0.4) (0.7,0.3) (0.8,0.2) (0.9,0.1)	1	Elementary (11)	Home (3)	-	11.14	25.69
		2	Middle (10)	Home (3)	-	14.55	
	(1.0,0.0)	1	Elementary (11)	Middle (10)	Home (3)	16.57	25.68
		2	Home (3)	-	-	9.11	
24	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6) (0.5,0.5)	1 (7)	Elementary (11)	Middle (10)	Home (3)	22.21	46.63
		2 (9)	High (14)	Home (3)	-	24.42	
	(0.5,0.5) (0.6,0.4) (0.7,0.3) (0.8,0.2) (0.9,0.1) (1.0,0.0)	1 (7)	Middle (10)	Elementary (11)	Home (3)	19.96	44.38
		2 (9)	High (14)	Home (3)	-	24.42	
25	Any	1 (9)	Middle (13)	Home (4)	-	18.53	27.66
		2 (7)	Home (4)	-	-	9.13	

26	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7)	1 (7)	Middle (13)	Home (4)	-	31.99	54.13
		2 (7)	High (14)	Home (4)	-	22.14	
	(0.4,0.6) (0.5,0.5) (0.6,0.4) (0.7,0.3) (0.8,0.2) (0.9,0.1) (1.0,0.0)	1 (7)	Home (4)	-	-	9.13	36.52
		2 (7)	High (14)	Middle (13)	Home (4)	27.39	
27	Any	1 (9)	Middle (10)	Home (3)	-	13.38	37.80
		2 (9)	High (14)	Home (3)	-	24.42	
28	Any	1 (7)	Home (4)	-	-	9.13	25.61
		2 (9)	High (14)	Home (4)	-	16.48	
29	Any	1 (9)	High (14)	-	-	9.36	36.68
		2 (9)	Elementary (12)	High (14)	-	27.32	
30	(0.0,1.0) (0.1,0.9) (0.2,0.8) (0.3,0.7) (0.4,0.6)	1 (9)	Elementary (12)	Home (6)	-	26.45	46.16
		2 (7)	High (14)	Home (6)	-	19.71	
	(0.5,0.5) (0.6,0.4) (0.7,0.3) (0.8,0.2)	1	High (14)	Home (6)	-	14.05	39.72
		2	Elementary (12)	Home (6)	-	25.67	
	(0.9,0.1) (1.0,0.0)	1	Home (6)	-	-	5.49	36.72
		2	Elementary (12)	High (14)	Home (6)	31.23	

Some of the household types did not show any change in the pick-up assignments when the weights on the total fleet time and the dwell time at the intermediate destinations varied. According to the assumptions made previously, type 1 households evacuated directly from their starting nodes so there was no opportunity to change pick-up assignments. As shown in table 4.5, household types 2-10 did not vary their intermediate nodes. These families consisted of one or two children and only one adult. Due to the restriction of the meeting location to one of the school nodes where the household's children were in attendance, once the meeting location was selected, there was only one (maximum) intermediate node on the evoked network before the meeting node. Household type 11 had no children, so there were no pick-ups to be made. In table 4.5, types 13, 14, 25, and 28 did not change which driver would collect the child. The starting location of the two household vehicles and the location of the home (and consequently the school) placed the child in much closer proximity to one of the adults. The other driver had a long trip to reach both the school and the meeting location (home) so minimization of the total fleet time and minimization of the waiting time at the meeting location yielded the same assignment. In the case of the household selected to represent type 15, the two household drivers started from the same zone making the perceived travel times identical for both adults and reassignment unnecessary. Household types 17 and 21 did not show any change in pick ups because the meeting locations were the schools where the household's children were located. For the household selected for type 18, the business zones for the two drivers were the same. Since there were two schools at which to stop, each driver went to one. In this case, the schools were relatively far apart and the assignment minimizes both the total fleet time and the waiting time at the meeting location. In the household for type 19, each driver picked up the child that was closest to him/her; thus minimizing both total fleet time and waiting time. The assignment shown in table 4.5 for the household of type 27 is optimal in terms of both total fleet time and waiting time so the weights in equation (4.16) have no effect. The representative of type 29 had two drivers starting at the same zone, the meeting location was one of the school zones, and there was only one additional pick-up to be made. These conditions led to there being no need to alter the assignments. For each of household types 13, 14, 15, 17, 18, 19, 21, 25, 27, 28, and 29, this lack of variation in assignment would not hold for every household of these types because of different residential, school, and work zones.

Changing the weights did affect the pick-up assignments for the representatives of household types 12, 16, 20, 22, 23, 24, 26, and 30. The first set of assignments represented the case where the waiting time at the meeting location was minimized. The last set of assignments resulted from the minimization of total fleet travel time. In the representative households with types 12, 22, and 24, when less than half of the weight was placed on the waiting time, the decision makers selected the second set of assignments. The case of 24 offered one additional interesting observation; the equilibrium point between the two solutions was captured at weights (0.5,0.5). The other cases that show multiple optimal solutions also had equilibrium points, but these were not captured by the weights selected for exploration. In the case of household type 16, the solution that minimized the total fleet time was only selected when that term was the only one considered (1.0, 0.0). The total fleet time for this second set of assignments was less than one minute below that of the first set while the waiting time was nearly 38 minutes, or approximately 16 minutes greater than the first set of trip chains. The representative of household type 20 had three optimal solutions. The first set minimized the waiting time at the meeting location and the third set minimized the total fleet travel time. The middle set had values of the total fleet travel time and waiting time between those of the first and third sets of assignments. Correspondingly, the mid-ranged weights made the second set of assignments optimal. This second set was also the most intuitive solution where each adult collected the child/children closest to him/her and the schools were considered on the way to the meeting location. A similar result occurred for the households representing types 23 and 30 and, again, the second set of assignments making the most intuitive sense of the three. Finally, the household for type 26 used the solution that minimized the total fleet time for the majority of the weights considered. The school locations corresponding to this household were relatively close together and could be considered on the way to both each other and the meeting location. Sending two drivers to the same relative area was not beneficial in the majority of the cases considered.

One of the key components to the pick-up assignments was the time each driver anticipated spending at the intermediate destination nodes collecting the children. The effect of varying these delay times at the school nodes and the weights in equation (4.16) was examined for household types 12-30. Three delays were examined – 1.00, 3.00, and 5.00 minutes. The household types that showed a change in the pick-up assignment due to the interaction of the weights and delays are shown in table 4.6. Only four representative households showed variation; the remainder yielded the same results as in table 4.5.

Table 4.6. Comparison of Pick-Up Assignments Due to Variation in Dwell Time and Weight on Perceived Total Fleet Travel Time

House-hold Type	Delay (p) at Pick-up (min)	Weight on Total Fleet Time (λ)	Vehicle	Trip Chain (zones)	Trip Chain Time (min)	Fleet Time (min)	Wait Time (min)
22	1.00	0.0, 0.1, 0.2, 0.3, 0.4, 0.5	1	7–13-6	25.93	50.5	1.36
			2	7–12-6	24.57		
		0.6, 0.7, 0.8, 0.9, 1.0	1	7-6	8.78	36.79	19.23
			2	7–12–13-6	28.01		
	3.00	0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6	1	7–13-6	27.93	54.5	1.36
			2	7–12-6	26.57		
		0.7, 0.8, 0.9, 1.0	1	7-6	8.78	40.79	23.23
			2	7–12–13- 6	32.01		
	5.00	0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6	1	7–13-6	29.93	58.5	1.36
			2	7–12-6	28.57		
		0.7, 0.8, 0.9, 1.0	1	7-6	8.78	44.79	27.23
			2	7–12–13-6	36.01		

24	1.00	0.0, 0.1, 0.2, 0.3, 0.4, 0.5	1	7-11-10-3	20.01	43.33	3.31
			2	9-14-3	23.32		
		0.5, 0.6, 0.7, 0.8, 0.9, 1.0	1	7-10-11-3	17.76	41.08	5.56
			2	9-14-3	23.32		
	3.00	0.0, 0.1, 0.2, 0.3, 0.4, 0.5	1	7-11-10-3	24.01	49.33	1.31
			2	9-14-3	25.32		
		0.5, 0.6, 0.7, 0.8, 0.9, 1.0	1	7-10-11-3	21.76	47.08	3.56
			2	9-14-3	25.32		
	5.00	0.0, 0.1, 0.2	1	7-11-10-3	28.01	55.33	0.69
			2	9-14-3	27.32		
		0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0	1	7-10-11-3	25.76	53.08	1.56
			2	9-14-3	27.32		
26	1.00	0.0, 0.1, 0.2	1	7-13-4	30.89	51.93	9.85
			2	7-14-4	21.04		
		0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0	1	7-4	9.13	34.32	16.06
			2	7-14-13-4	25.19		
	3.00	0.0, 0.1, 0.2, 0.3	1	7-13-4	32.89	55.93	9.85
			2	7-14-4	23.04		
		0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0	1	7-4	9.13	38.32	20.06
			2	7-14-13-4	29.19		
	5.00	0.0, 0.1, 0.2, 0.3, 0.4	1	7-13-4	34.89	59.93	9.85
			2	7-14-4	25.04		
		0.5, 0.6, 0.7, 0.8, 0.9, 1.0	1	7-4	9.13	42.32	24.06
			2	7-14-13-4	33.19		
30	1.00	0.0, 0.1, 0.2, 0.3, 0.4	1	9-12-6	25.35	43.96	6.74
			2	7-14-6	18.61		
		0.5, 0.6, 0.7	1	9-14-6	12.95	37.52	11.62
			2	7-12-6	24.57		
		0.8, 0.9, 1.0	1	9-6	5.49	34.52	23.54
			2	7-12-14-6	29.03		
	3.00	0.0, 0.1, 0.2, 0.3, 0.4	1	9-12-6	27.35	47.96	6.74
			2	7-14-6	20.61		
		0.5, 0.6, 0.7, 0.8	1	9-14-6	14.95	41.52	11.62
			2	7-12-6	26.57		
		0.9, 1.0	1	9-6	5.49	38.52	27.54
			2	7-12-14-6	33.03		
	5.00	0.0, 0.1, 0.2, 0.3, 0.4	1	9-12-6	29.35	51.96	6.74
			2	7-14-6	22.61		
		0.5, 0.6, 0.7, 0.8	1	9-14-6	16.95	45.52	11.62
			2	7-12-6	28.57		
		0.9, 1.0	1	9-6	5.49	42.52	31.54
			2	7-12-14-6	37.03		

In table 4.6, it was assumed that the household decision makers would know the dwell time at each pick up location, or that their perceived value of the delay was as stated. This delay time was simply added to the perceived travel time in the network.

Changing the value of the delay and varying the weight on the total fleet time did not cause any additional optimal solutions to be generated. As can be seen in table 4.6, the weights of the total fleet time associated with each assignment shifted. For the representatives of household types 22 and 26, there were two optimal routings on the household's evoked network. The first one presented in table 4.6 minimized the waiting time at the meeting location and assigned one pick-up to each driver. Since each driver saw the same increase in the dwell time at the school, the waiting time at the meeting location did not change for this assignment. The second set of routings minimized the total fleet time. All of the pick-ups were assigned to one of the vehicles; thus the increase in delay was experienced twice by the same vehicle. As the dwell time increased, the waiting time at the meeting location became greater for the second assignment than for the first (which remained constant); thus, greater weight on the total fleet time was required to shift the solution from the trip chain assignment that minimized the waiting time to the one that minimized the total fleet time. In the case of the household representing type 24, a delay of 5.00 minutes caused the individual vehicle time of driver 1 to become greater than that of driver 2 in the first assignment and less than that of driver 2 for the second assignment. The opposite situation held for dwell times of 1.00 and 3.00 minutes. The difference between the total fleet times and waiting times of the two assignments for the 1.00 and 3.00 minute delay cases both were 2.25 minutes. While the difference in the total fleet time remained the same for the 5.00 minute case, the difference in the waiting time was smaller thus making the second sequencing more appealing at lower weight on the total fleet time. Finally, the representative of household type 30 had three optimal solutions. The first set of trip chains shown in table 4.6 minimized the waiting time, the third minimized the total fleet time, and the second fell in between the first and the third. Switching from the second to the third solution required greater weight on the total fleet time for the greater delay cases because the waiting time increased by more than 15 minutes while the total fleet time decreased by only 3 minutes.

The activity chains for all of the households, not just the representatives, served as input to the traffic simulation-assignment package DYNASMART. The vehicles generated for the activity chains were distributed over slightly more than 3 minutes allowing for some delay in the receipt of the evacuation order. Referring to equation (4.2), the delay $\boldsymbol{w_v}$ of the vehicles ranged from 0.00 to 3.01 minutes. Several different waiting times ($\boldsymbol{p_i}$) at each intermediate node were considered. At school locations, the waiting times varied from 1.0 to 5.0 minutes. The final waiting time at the meeting location was also allowed to vary. The meeting location waiting time was taken to be the expected waiting time until the other vehicle arrived plus a constant. The average travel times (including entry queue waiting times) corresponding to the third term of the right hand side of equation (4.2) and evacuation times (left hand side of equation (4.2)) for these different cases are also displayed in table 4.7. The time of the evacuation order $\boldsymbol{T_{order}}$ was taken as 0.0; thus equation (4.1) reduced to equation (4.2). The resulting evacuation time of the network is presented in table 4.7 for the case where the weight on the total fleet time was 0.5; this weight was selected based on the results of tables 4.5 and 4.6. Two additional weights were examined and the results are provided in table 4.8.

Table 4.7 also presents times required to clear the network when various percentages of the peak period traffic (30,141 vehicles) were considered. Two methods for generating the traffic were employed. First, all of the vehicles were given the same start time. Second, the start times were distributed over 45 minutes.

Table 4.7 Network Clearance Times for Various Vehicle Loading Methods

Generation Method	**Number of Vehicles**	**Percentage of Activity Chain Vehicles**	**Average Network Travel Time (min)**	**Average travel time + entry queue time (min)**	**Network Clearance Time (min)**
Activity Chain (minimum wait 5.0 min at school and meeting location)	30141	100%	45.5	123.2	459.6
Activity Chain (minimum wait 4.0 min at school and meeting location)	30141	100%	43.3	118.8	440.5
Activity Chain (minimum wait 3.0 min at school and meeting location)	30141	100%	45.9	122.3	450.2
Activity Chain (minimum wait 2.0 min at school and meeting location)	30141	100%	42.3	117.7	436.4
Activity Chain (minimum wait 1.0 min at school and meeting location)	30141	100%	43.7	119.0	432.0
No activity chain, all at once	30141	100%	33.3	78.4	229.1
No activity chain, all at once	37675	125%	45.9	115.9	335.5
No activity chain, all at once	45206	150%	40.9	125.0	343.5
No activity chain, all at once	48221	160%	42.9	145.7	361.6
No activity chain, all at once	49732	165%	42.0	142.7	371.1
No activity chain, all at once	51240	170%	42.5	147.2	378.0
No activity chain, all at once	52742	175%	50.7	164.5	455.2
No activity chain, distribute over 45 minutes	30098	99.9%	17.2	18.9	157.7
No activity chain, distribute over 45 minutes	55259	183%	40.7	57.5	275.1
No activity chain, distribute over 45 minutes	60306	200%	41.5	60.7	334.9
No activity chain, distribute over 45 minutes	63336	210%	45.3	67.9	379.7

Examination of table 4.7 revealed that although decreasing the waiting times at intermediate nodes for activity chains resulted in shorter evacuation times, these clearance times are still between seven and eight hours. To achieve similar results without trip chains, approximately 175% of the original demand must be loaded simultaneously or between 180 and 207% of the original demand when vehicles were loaded over 45 minutes.

The measure of effectiveness of an evacuation may not be 100% of the vehicles clearing the network. Figures 4.3-4.9 and table 4.8 illustrate the interaction of the dwell times at the home and schools and the weight on the total fleet time.

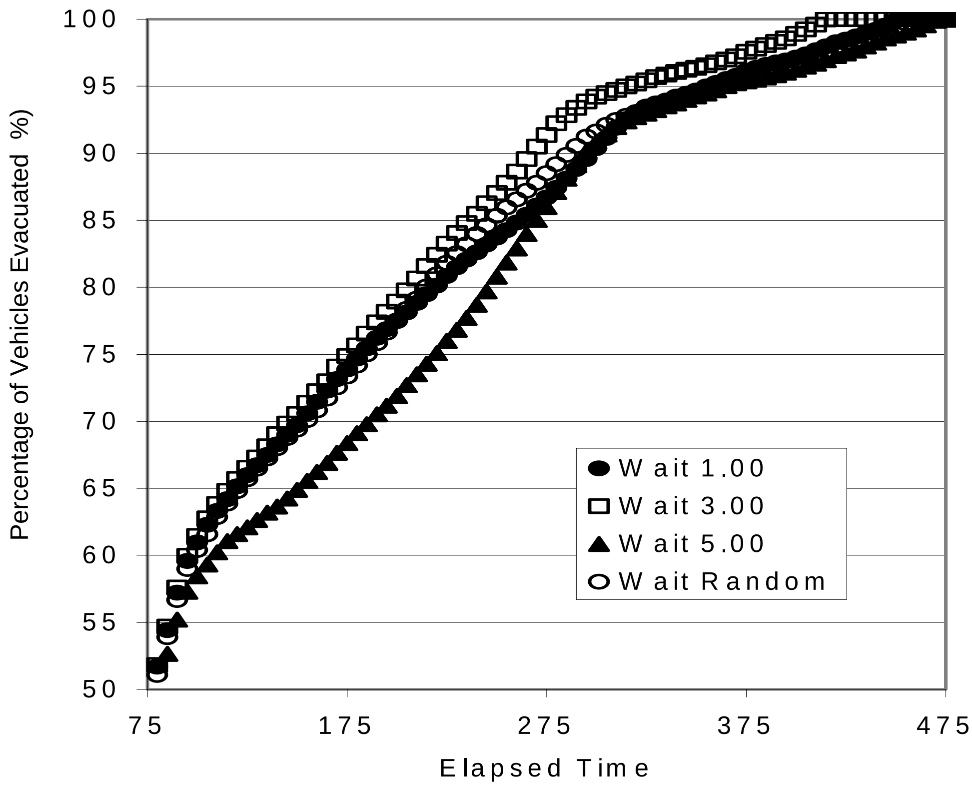

Figure 4.3 Network Clearance for All Weight (λ = 1.0) on Total Fleet Time

When all of the weight was placed on the total fleet time, the network clearance profile found in figure 4.3 was generated. All of the dwell times considered yielded similar time requirements to evacuate 50% of the population. At the 60% mark, the scenario of a 5.0 minute dwell time exhibited an increase in time requirements compared to the other three waiting time cases; this deviation persisted until approximately the 85% mark where the elapsed time for the 5.0

minute scenario was similar to the 1.0 minute case. This result was anticipated since the larger waiting times delay vehicles from reaching their final destinations. When the measure of effectiveness was 75% (or greater) of the population evacuated, a dwell time of 3.0 minutes yielded the quickest evacuation. This dwell time resulted in lower time requirements than the other cases because holding the vehicles at their intermediate destinations prevented some of the congestion that arose in the 1.0 minute case. The fact that the dwell time of 3.0 minutes was an improvement to the 5.0 minute case suggests that 5.0 minutes, although reducing network congestion, impeded the vehicles making stops. Finally, the randomly generated dwell time scenario produced evacuation times between the best case (3.0 min) and the worst case (5.0 min).

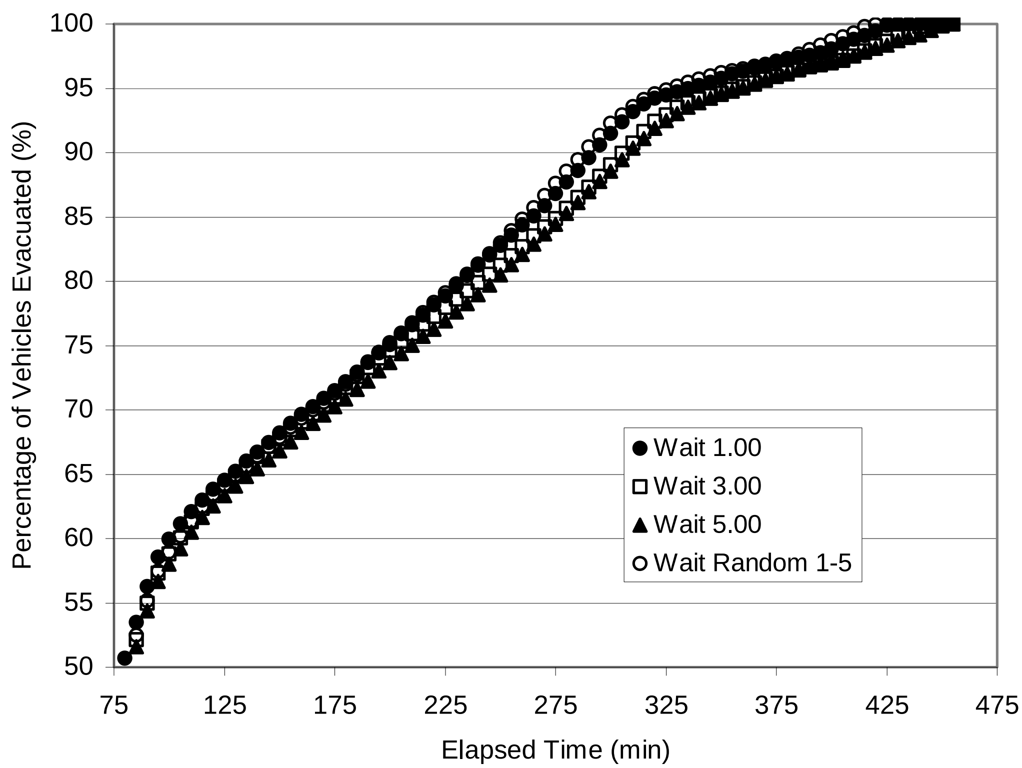

Figure 4.4 Network Clearance for Half Weight (λ = 0.5) on Total Fleet Time

There was less disparity in the evacuation times for the various dwell times when half of the weight was placed on minimizing the total fleet time and the other half on minimizing the waiting time at the meeting location than in the case where all of the weight was placed on the total fleet time. In figure 4.4, as in figure 4.3, the dwell time of 5.0 minutes yields longer evacuation times than the other three cases. However, in this weighting scenario, the 1.0 minute delay time yielded a faster evacuation than the 3.0 minute case. Recall from table 4.5 that the household pick-up assignments sometimes changed when the weight on the total fleet time was varied from 1.0 to 0.5. This trip chain alteration caused the difference in evacuation time requirements. Finally, when the measure of effectiveness was 85% or greater of the population evacuated, the randomly, uniformly generated dwell time produced the least network clearance time; this suggested that the interaction of various pick-up assignments and dwell times could improve the evacuation speed, compared to the uniform assignment of delays to all households.

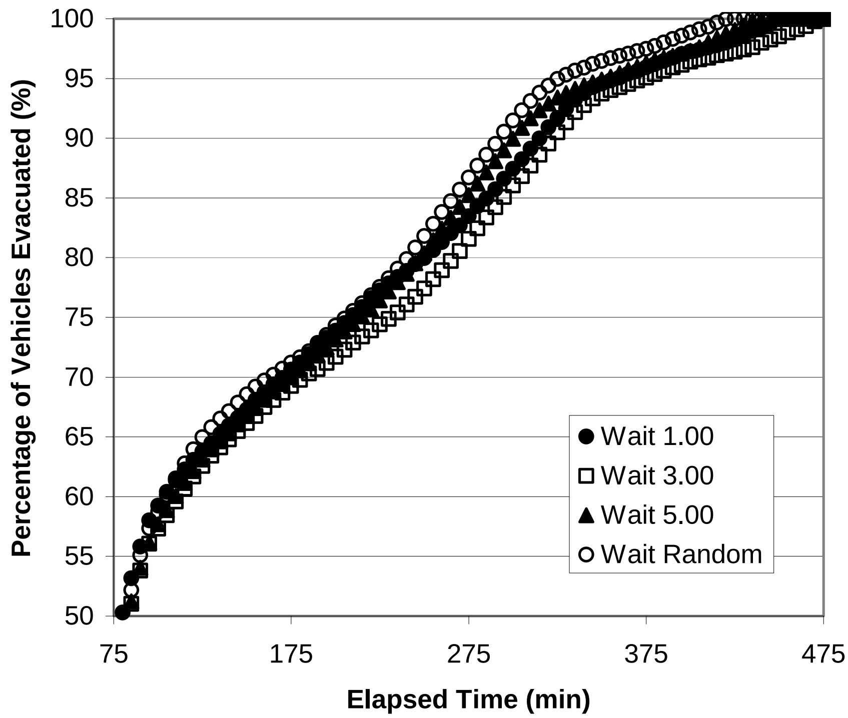

Figure 4.5 Network Clearance for No Weight on (λ = 0.0) Total Fleet Time

As noted in table 4.5, changing the weight among 1.0, 0.5, and 0.0 on the minimization of the total fleet time in equation 4.16 may result in up to three different household pick-up assignments. These variations in assignments account for the differences found in figures 4.3-4.5. As in figure 4.4, figure 4.5 showed that the randomly generated dwell time yielded the lowest evacuation time when the measure of effectiveness was 85%, or greater, of the population evacuated. At 70% and greater, the 3.0 minute dwell time required the most time to evacuate the network; this was opposite of the case where the weight of the total fleet time was 1.0. Figure 4.5 shows that a delay time of 5.0 minutes performed better than a dwell time of 1.0 minutes where the measure of effectiveness between 80 and 95% network clearance. The interaction of the pick-up assignments, dwell time of 1.0 minute, and congestion delayed vehicles more than the conditions associated with the case where the dwell time was 5.0 minutes.

Some of the particular points of interest from figures 4.3-4.5 are displayed in table 4.8, which presents the amount of time required to evacuate different percentages of the population for various delays when the weight on the total fleet time (see equation 4.16) was 1.0, 0.5, 0.0, and randomly, uniformly generated.

Table 4.8 Network Clearance Profiles for Various Dwell Times and Weights

Weight on Total Fleet Time	Min. Dwell Time at School & Home	Time for 50% to Evacuate (min)	Time for 60% to Evacuate (min)	Time for 70% to Evacuate (min)	Time for 80% to Evacuate (min)	Time for 90% to Evacuate (min)	Time for 100% to Evacuate (min)
1.0	1.00	77.03	96.52	151.69	219.01	297.75	456.4
	2.00	76.81	95.73	149.01	208.45	280.36	443.4
	3.00	76.56	95.14	146.14	206.35	267.75	422.8
	4.00	77.70	96.19	147.71	208.37	284.56	456.5
	5.00	80.03	108.81	186.41	246.41	294.23	486.3
	Random	78.06	98.61	154.37	214.65	285.96	478.0
0.5	1.00	78.72	100.23	162.88	232.11	292.04	432.0
	2.00	79.63	102.70	162.67	232.12	286.21	436.4
	3.00	81.20	104.75	167.27	240.8	305.39	450.2
	4.00	81.66	105.06	164.55	232.61	291.97	440.5
	5.00	82.11	108.00	173.14	246.89	308.07	459.6
	Random	80.64	104.16	165.08	231.27	287.81	425.8
0.0	1.00	79.46	103.29	170.40	250.32	315.09	453.2
	2.00	80.41	105.58	169.96	244.44	303.74	450.1
	3.00	83.04	111.94	182.05	266.66	322.38	478.0
	4.00	81.77	107.98	170.26	248.07	306.90	444.3
	5.00	82.70	109.87	175.13	247.76	300.32	449.7
	Random	81.09	104.50	162.93	240.66	292.33	425.8
Random	1.00	79.01	98.78	152.00	216.49	274.94	413.0
	2.00	79.50	100.25	155.30	223.45	276.13	398.4
	3.00	81.90	108.76	173.58	264.84	324.85	465.7
	4.00	80.98	104.12	164.20	232.86	282.94	413.9
	5.00	82.40	109.59	176.78	252.33	310.87	476.3
	Random	80.09	102.94	161.41	230.84	290.24	432.3

Examination of table 4.8 revealed several interesting points. Within each weighting scenario, the time to evacuate 50% of the population varied by less than 3.5 minutes, regardless of the dwell time. Over all of the weighting and dwell time cases, the evacuation time for 50% of the population differed by less than 6.5 minutes. The higher levels of evacuation showed greater disparity within a given weighting scenario and overall. At the 60% level, the difference in times varied between approximately 8 and 14 minutes within a given weighting case and just over 14 minutes for the entire sample. For the 70% level, the disparity increased and ranged between 19 and 40 minutes for the weighting cases and 40 minutes overall. The 80% level showed an even further spread of evacuation times with 60 minutes between the $\lambda = 1$, 3.0 minute dwell time and the $\lambda = 0$, 3.0 minute dwell time. Within a given weighting value, the difference in time requirements varied between 16 and 48 minutes. The 90% evacuation level presented less overall disparity (57 minutes) than the 80% clearance level. The range within a weight was 22 to 50 minutes. Finally, to completely clear the network, the time required varied between 34 and 78 minutes within a given weight and 88 minutes overall. Except for the 100% evacuation level, the $\lambda = 1$, 3.0 minute dwell time combination yielded the smallest time requirements; for the 100% level, the shortest evacuation time was obtained when the weight was randomly generated and the dwell time was 2.0 minutes. The combination of factors that generated the highest clearance times changed with the evacuation level. A dwell time of 3.0 minutes and $\lambda = 0$ yielded the highest evacuation time for the 50% and 80% clearance levels. For the 60% level, the combination yielding the highest network clearance time was randomly generated weights and 5.0 minute dwell times. At the 70% and 100% levels, the factors with the greatest evacuation time requirements were $\lambda = 1$ and 5.0 minute dwell time. Finally, for the 90% clearance level, randomly generated weights and 3 minute delay times yielded the highest evacuation times.

The data from table 4.8 for the measure of effectiveness 80% network clearance was used to create figure 4.6.

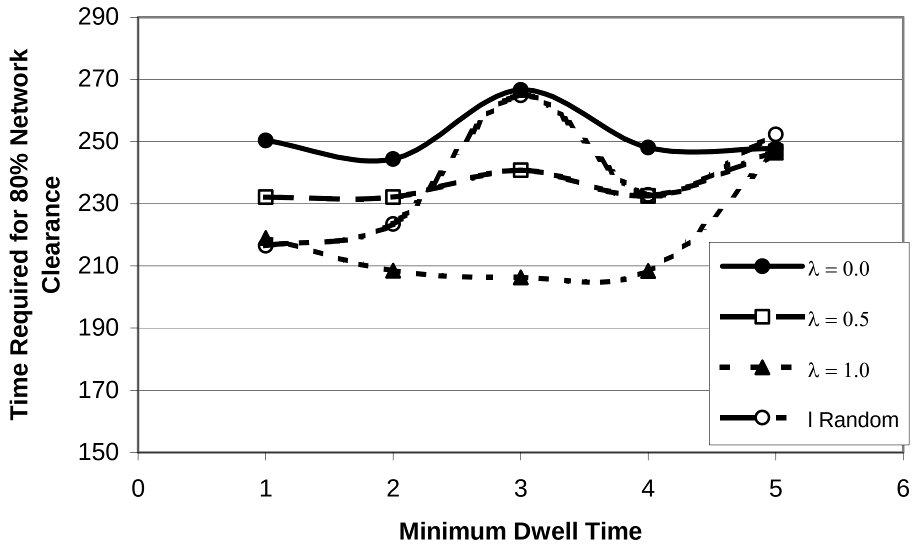

Figure 4.6 Comparison of 80% Network Clearance Times and Minimum Dwell Times for Various Total Fleet Time Weights

The complex interactions of dwell times, the weights on the two minimization objectives, and the related pick-up assignments were further demonstrated in figure 4.6. The nonlinear nature of the graphs demonstrated these complexities. The randomly generated weights led to the greatest distance between the peaks and valleys of the curve. The curve for $\lambda = 0$ was the most symmetrical about dwell time of 3.0 minutes. When none of the weight was placed on the minimization of total fleet travel time, the lowest point of the curve occurred at a dwell time of 3.0 minutes; this result was opposite to that of the case where all of the weight was placed on the minimization of total fleet travel time and the highest point of the curve was at a dwell time of 3.0 minutes. A weight of

1.0 on the total fleet time led to a fairly linear curve for dwell times of 1.0 to 4.0 minutes. This trend was broken for dwell times between 4.0 and 5.0 minutes.

There are several more observations of note pertaining to figure 4.6. The times for 80% of the population to evacuate for the 5.0 minute dwell time were fairly close for all of the fleet time weights examined. Excluding the random generation case, the weight of 0.0 on the minimization of the total fleet travel time (consequently all of the importance was placed on the minimization of the waiting time at the meeting location) yielded the highest evacuation time for all dwell times. Again, excluding the random scenario, the 80% evacuation time was the lowest when all of the weight was placed on the minimization of the total fleet time. Finally, equal weights on the minimization of total fleet time and the minimization of the difference in arrival times at the meeting location yielded 80% evacuation times between those associated with the extreme weights, for all dwell times considered.

Figures 4.7-4.9 allow the interaction of dwell time and weight on the total fleet time to be examined from the reverse perspective of figures 4.3-4.5. Figures 4.7, 4.8, and 4.9 present the time required to evacuate various percentages of the population for minimum dwell times of 1.0, 3.0, and 5.0 minutes, respectively.

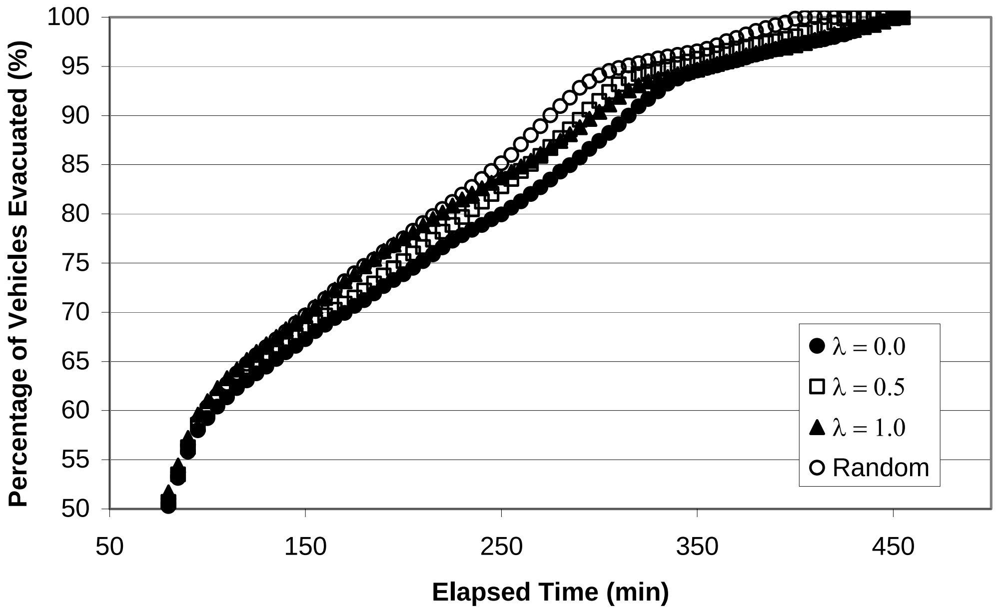

Figure 4.7 Comparison of Network Clearance for Total Fleet Time Weights with Minimum Dwell Time 1 Minute at School and Meeting Locations

The four curves shown in figure 4.7 had approximately the same shape although the slope varied. At 60% network clearance, the curve for the case where none of the weight was placed on the minimization of the total fleet travel time indicated that more time was needed than for the other weights. This trend persisted until the 93% network clearance level. The randomly generated weight scenario showed an improvement over the deterministic cases for 80% or higher levels of population evacuation. The case where equal weight was placed on the minimization of total fleet travel time and the minimization of waiting time at the meeting location yielded evacuation times that were between the two extreme weighting cases for 50-85% network clearance. After the 85% mark, the equal weight scenario produced lower evacuation times than either of the two extremes. This last observation, combined with the results for the randomly generated weights suggested that variable assignments improved the evacuation speed over the strict minimization of one of the two criteria in equation 4.16.

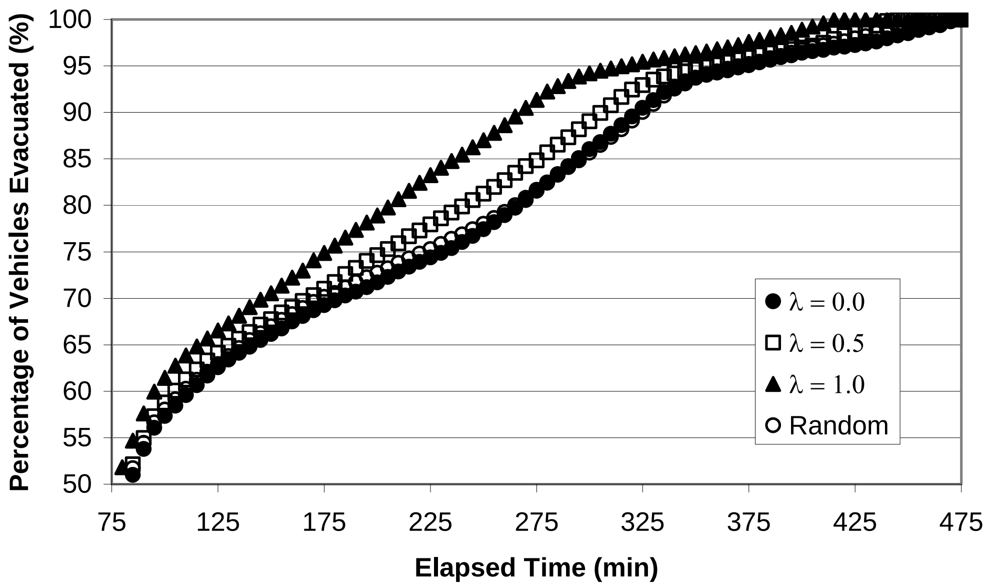

Figure 4.8 Comparison of Network Clearance for Total Fleet Time Weights with Minimum Dwell Time 3 Minutes at School and Meeting Locations

The results for a dwell time of 3.0 minutes were very different from those of a dwell time of 1.0 minute. At no point in figure 4.8 did the randomly generated weight scenario perform better than all of the deterministic waiting cases. Placing all of the weight on minimizing the total fleet travel time yielded the least evacuation times. As in the 1.0 minute dwell time case, the minimization of the waiting time at the meeting location required the most time to evacuate a given percentage of the population. Splitting the weight equally between the two criteria led to evacuation times between the two extremes.

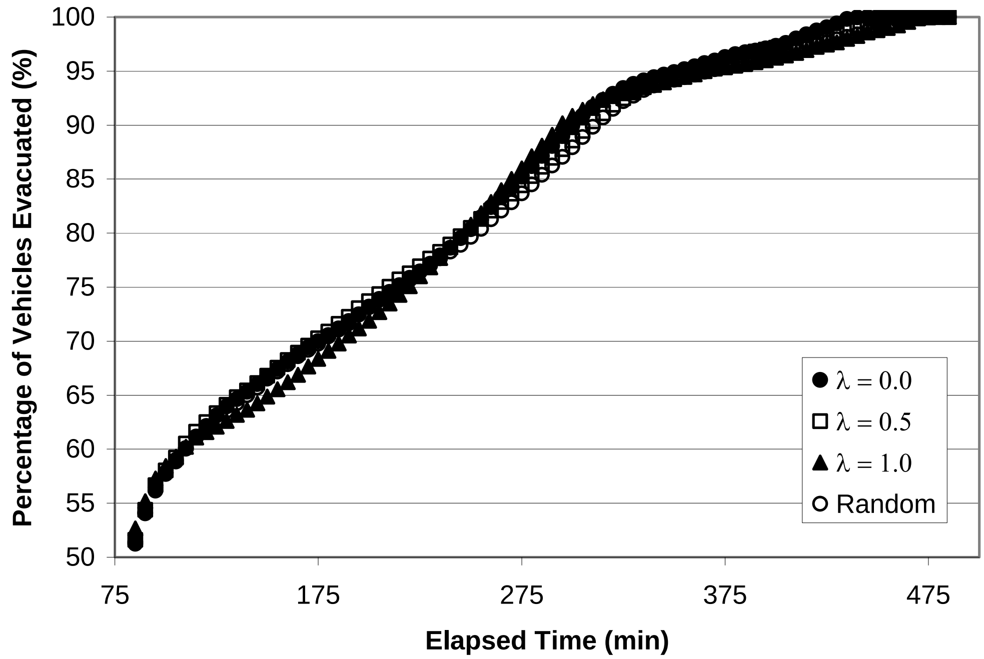

Figure 4.9 Comparison of Network Clearance for Total Fleet Time Weights with Minimum Dwell Time 5 Minutes at School and Meeting Locations

The most noticeable difference between figure 4.9 and figures 4.7 and 4.8 was the closeness of the four curves; the evacuation times varied little for the four weighting cases. For 60-75% network clearance levels, minimizing the total fleet travel time produced greater time requirements than the other three weighting scenarios. At the 78-92% evacuation level, the randomly generated weighting case yielded slightly higher time requirements than the other scenarios.

Part of the reason for the extensive evacuation time was the structure of the network. For the original results presented above, the links connecting four of the five schools to the rest of the network were extremely short. Although these lengths simulate an appropriate length of a driveway to the school, they do not capture the effects of parents parking in other areas during these usual conditions. To examine the scenario where additional space may be used, various lengths of the links connecting the schools at nodes 122, 123, 165, and 170 were examined for the case where waiting time at the school was 5.0 minutes and the minimum waiting time at the meeting location was 5.0 minutes. Table 5 shows the results for 1.25, 1.5, 1.75, and 2.0 times the original lengths of the links of interest. From these results, one can observe the complexities of the network simulation. Moderate increases (1.25 and 1.50) in the length of the link actually increases the total evacuation time and the corresponding average travel time, with and without entry queue delays. Higher multiples (1.75 and 2.0) of the original length decrease the total evacuation time from the original case, decrease the average network travel time, but slightly increase the average entry queue waiting time. Clearly, doubling the length of the school links provides the most significant improvement. The evacuation time decreases by slightly more than thirty minutes and the average travel times with and without entry queue delays decrease. For all of the cases examined, the average distance traveled did not vary by more than 0.9% of the original.

Table 4.9 Comparison of Average Times, Distances, and Evacuation Times for Different Lengths of School Links for Activity Chains with Minimum Waiting Times of 5.0 Minutes at Intermediate Nodes

Length of Link Leading To and From School Nodes (ft)	Average Network Travel Time (minutes)	Average Travel Time + Entry Queue Time (minutes)	Average Distance Traveled (mi)	Network Clearance Time (minutes)
528 (original)	45.8	115.9	6.9020	383.4
660 (1.25x orig)	46.9	127.6	6.9045	398.5
792 (1.50 x orig)	48.7	127.6	6.9150	399.5
924 (1.75x orig)	40.9	119.5	6.9456	381.4
1056 (2.00x orig)	43.5	114.2	6.9613	351.7

Comparing the evacuation times for the extended link length scenarios (table 4.9) with the various vehicle loading strategies displayed in table 4.7, indicates that between 150 and 175% of the original demand would be required when all of the vehicles are loaded at the same time. For the case when the length is doubled, this range can be narrowed to 150-160%. When the vehicles are spread over 45 minutes of generation time, more than 200% of the original demand would be required to achieve a similar network clearance time.

4.4 SUMMARY

The framework presented in this chapter allows for representation of more realistic evacuation scenarios, by incorporating critical aspects of household travel behavior that have been omitted from traditional evacuation models. The omission results in overly optimistic evacuation times. This framework was incorporated into a traffic assignment-simulation tool. The methodology used here allows for an evacuation simulation to more accurately capture the traffic flows that arise when parents pick up their children at the schools before evacuating the city. Such an emergency situation may arise in the case of sudden disasters or threats like earthquakes or terrorist incidents.

The results presented in this work are specific to the geometry of the network and the relative locations of schools, residential areas, and work areas, but some results can be generalized. First, the household's decision of where to meet plays a crucial role in the assignment of pick-ups to vehicles. Second, allowing the household's decision makers to assign weights to the total fleet travel time and the waiting time leads to a small solution set (in this work one, two, or three options) to be considered for vehicle routing. Two of the solutions were the one that minimized total fleet travel time and the one that minimized waiting time.

Third, attempting to coordinate arrival times is better applied to larger households than to smaller ones.

Comparison of network clearance times suggests that at least 175% of the original demand should be used if trip chains are ignored and vehicles proceed to their homes. Additional link length at school locations allows this factor to be reduced to some degree. The exact multiplicative factor depends on the loading scenario employed by the planning agency and any additional special emergency considerations that may be taken into account. The complex interactions of all of these factors require careful consideration by evacuation planners and transportation engineers.

Chapter 5

Case Study

This chapter presents a case study that illustrates how the concepts developed in chapters 3 and 4 interrelate. Following a threat to the transportation infrastructure, the public is advised to evacuate; the traffic management agency lacks the ability to control the heavy demand that will be placed on the network. Residents in a threatened area will logically attempt to evacuate, regardless of whether this sudden rush of traffic causes or exacerbates any transportation problems. Evacuations due to imminent disasters have somewhat different characteristics than those associated with advanced warning evacuations, such as those due to hurricanes. The immediate nature of the disaster causes the traffic to be concentrated over a much shorter period than for a planned evacuation. This chapter focuses on evacuations due to an immediate threat, whether natural or man-made.

As noted in chapters 2 and 4, a common observation about evacuating households is that the family members tend to gather prior to fleeing the area. The household level decision making model developed in chapter 4 is applied in this chapter. The household selects a meeting location and then assigns drivers to pick up family members without access to vehicles. All of the drivers then gather at the meeting location and evacuate as a single unit.

The evacuating traffic may create vulnerabilities in the network that do not exist under typical, everyday conditions. In this chapter, the network links are evaluated from the perspective of the evil entity for the unusual traffic patterns that arise in an emergency evacuation.

The specific objectives covered in this chapter are (1) to examine the impact of emergency trip-chaining behavior on the vulnerability of transportation network links and (2) to examine the impact of strategies, designed to route vehicles around vulnerable transportation infrastructure, on city evacuation times.

Several tasks are required to meet these objectives. First, baseline traffic conditions are simulated. Second, as in chapter 3, the vulnerabilities of the transportation network links are determined for this traffic pattern. Third, an evacuation is simulated using the trip-chaining assignment model developed in chapter 4. Fourth, the vulnerability of the links is determined under the evacuation conditions. Finally a comparison is made between the baseline case and the evacuation scenario.

The remainder of this chapter is directed toward accomplishing the tasks listed above and is organized in the following manner. First, the simulation test bed is described. Second, the experimental design is presented. Third, the experimental procedure is outlined. Fourth, the results are discussed. Finally, a summary of the chapter is provided.

5.1 SIMULATION TEST BED

The simulation test bed used for this case study was adapted from the network representing the south-central I-35 portion of Fort Worth, Texas. As in chapter 4, the original network was modified to include estimated school locations. The network is repeated in figure 5.1 for ease of discussion.

The network consists of 184 nodes (only 180 are shown in the figure). In this model, two elementary schools (nodes 123 and 165), two middle schools (nodes 122 and 169), and one high school (node 170) are located on the network. Each school is modeled as its own zone, and the remainder of the nodes are divided into three business zones and six residential zones.

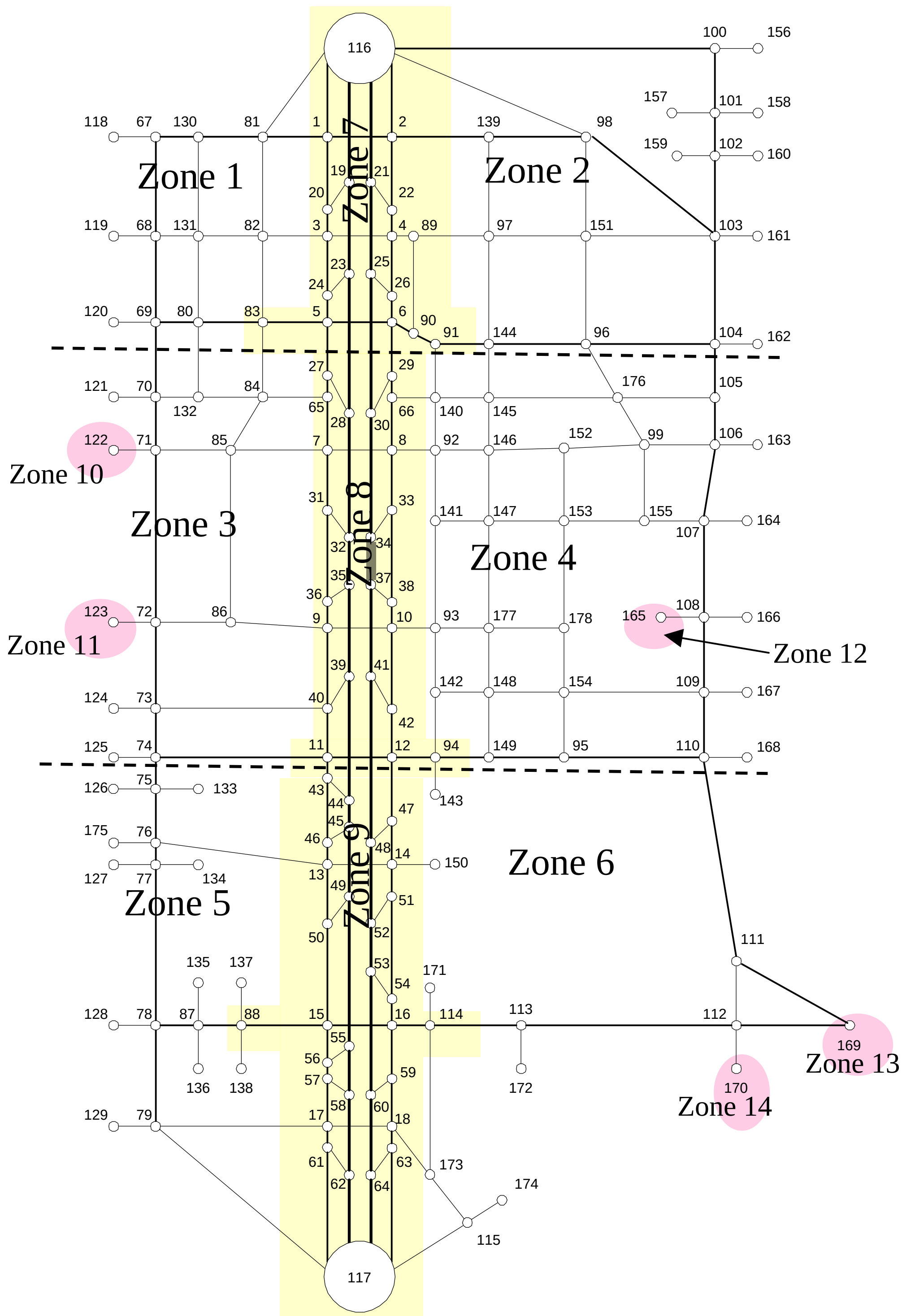

Figure 5.1 Simplified Version of South Central Fort Worth, TX

The link information pertaining to figure 5.1 is presented in Appendix D. The appendix contains the link number, the upstream and downstream nodes, the length of the link, the maximum service capacity, and the free flow speeds and travel times for each link.

5.2 EXPERIMENTAL DESIGN

This chapter takes the lessons learned from the variable exploration in chapters 3 and 4 and applies them to the network shown in figure 5.1. The following sections describe the values of the variables that are used in this case study. These variables include household characteristics, household decision making objective function weights, household dwell times at intermediate nodes in the trip chain, evil entity resources and targets, traffic management agency strategies, traffic assignment, number of alternate paths, and evaluation times. The final portion of section 5.2 outlines the combinations of these variables that are explored in this study.

5.2.1 Household Characteristics

In the simulations 20,000 households produced 30,141 vehicles. The household composition (number of adults and children of various ages) is identical to that found in chapter 4. Approximately 49% of the households have only one adult, and 51% have two. No children are present in 49% of the households, including homes with one or two adults. At least one elementary school aged child is found in 23% of the households, at least one middle school child in 23%, and at least one high school child in 24% of the households. Approximately 13% of the households have multiple children in different schools.

Due to these household characteristics, between 33 and 57% of the total generated vehicles would stop at schools to pick-up their children.

5.2.2 Household Decision Making Objective Function Weights

As in chapter 4, the network impacts of several weights associated with the components of the household trip chaining objective function – the minimization of total household fleet travel time and the minimization of waiting time spent at the meeting location - are examined. However, the ranges of values of these factors are more limited in this chapter. Based on the results in chapter 4, three deterministic weights on the minimization of the total fleet time are considered (λ = 0.0, 0.5, and 1.0). Case 1 reflects a weight of 0.0 on the minimization of total household fleet travel time; case 2 indicates a weight of 0.5 on the minimization of total fleet travel time; and case 3 reflects a weight of 1.0. These cases are the ones known to generate different household trip chains.

5.2.3 Household Dwell Times

Dwell times at intermediate nodes reflect the ease with which parents are able to locate their children at the schools and secure them in the vehicles. Recognizing that these dwell times will not be deterministic, households are assigned dwell times from a random uniform distribution that ranges from 1 to 5 minutes.

5.2.4 Evil Entity Resources and Targets and Traffic Management Agency Strategies

In this case study, the evil entity has the resources to damage only one link. For the baseline conditions and each of the evacuation weighting cases, each link is examined in a scenario where the evil entity targets that particular link. By limiting the number of links in the set of possible targets to one, different types of links can be more precisely examined.

Based on baseline vulnerability and trip chaining information, a sample of different types of links from figure 5.1 is presented for discussion. Both the traffic management agency and the evil entity consider this sample of links potential targets. Traffic management agency strategies avoid only one of the links in the sample. Similarly the evil entity's targeting scenario focuses on damaging only one of the sample links. The total sample includes arterial (link 306), overpass (link 43), freeway (link 123), frontage road (link 83), and residential (link 191) arcs. Also included are links between two schools (link 148) and between a school and a residential area (link 146). These links are circled denoted in figure 5.2. The link characteristics can be found in Appendix D.

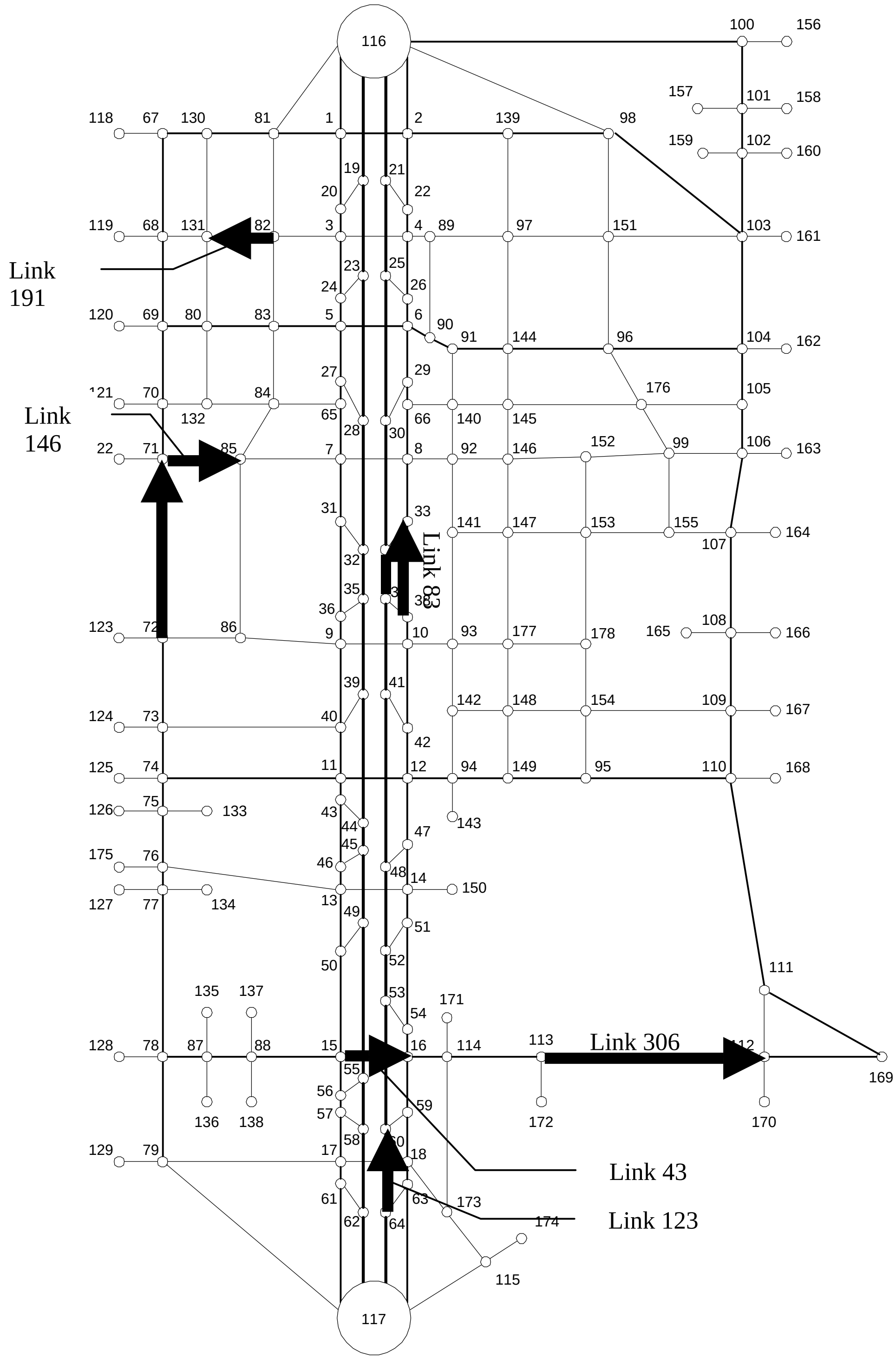

Figure 5.2 Selected Links

5.2.5 Traffic Assignment

The vehicle routing is roughly based on the system optimal traffic assignment. Only one additional iteration of the assignment procedure is used; thus generating an imperfect system optimal traffic assignment that is more likely to be seen in a true evacuation scenario. A traffic management agency would seek to influence vehicles but in the stress caused by danger, the response to guidance is variable.

Another advantage to the traffic assignment approach is the ability to add costs to threatened links, while maintaining connectivity. To incorporate this additional cost in DYNASMART-P, the links are converted to tolled facilities. By charging a toll, the threatened link can still be used for connectivity purposes, but the cost prohibits vehicles from using the link when an alternative path is available.

5.2.6 Alternate Paths

The disruption index developed in chapter 3 allows for the network analyst to limit the number of alternate paths considered for an origin-destination pair. In this case study, up to five paths are generated for each pair.

5.2.7 Evaluation Times

In a dynamic network, flows are not at steady state; thus, time instances must be selected for evaluation purposes. The vulnerability and disruption indices are calculated at 30 minutes. This time point is selected for two reasons. First, the simulation tool has enough time to load the network. Second, approximately 50% of the vehicles are still in the network for all of the simulations conducted. Two additional time points (60 and 120 minutes) are examined for a selected traffic management agency strategy. These additional time instances reveal changes in the traffic patterns as the simulation progresses.

Using a specific time instance in a dynamic network raises the question of how to measure the flow. In this work, instantaneous flow is used. Due to the nature of instantaneous flow, it may exceed link capacity; to adjust for this temporary condition, the maximum flow value is set to the maximum flow service rate of the link.

5.2.8 Combinations of Factors Examined

Table 5.1 provides a summary of the combinations of factors examined in the simulations. This table indicates that the weighting cases identified in section 5.2.2, the sample of links from section 5.2.4, and the time instances from section 5.2.7 are explored in different combinations. The remainder of the variables identified in above sections are treated as constants.

Table 5.1 Combinations of Experimental Factors

Time Instant (min)	Traffic Management Agency Strategy	Evil Entity Target Scenario	Baseline / Evacuation Case*
30	Do nothing	Each of links 1-452, individually	Baseline (30,149 vehicles)
			Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
30	Avoid Link 123	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
30	Avoid Link 146	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
30	Avoid Link 148	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
30	Avoid Link 191	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
30	Avoid Link 43	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)

30	Avoid Link 83	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
30	Avoid Link 306	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
60	Avoid Link 306	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)
120	Avoid Link 306	Each of links 123, 146, 148, 191, 306, 43, and 83, individually	Evacuation Case 1 (λ = 0.0)
			Evacuation Case 2 (λ = 0.5)
			Evacuation Case 3 (λ = 1.0)

* All of the evacuation cases use 30,141 vehicles.

Table 5.1 presents the combinations of variables that will be investigated in this chapter. The experimental procedure that outlines how these variables will be allowed to interact is presented in the following section.

5.3 EXPERIMENTAL PROCEDURE

The five steps of the experimental procedure outlined at the beginning of this chapter are explored in further detail in the following sections. The first portion describes tasks 1 (establishing baseline traffic conditions) and 2 (determining the link vulnerabilities associated with the baseline conditions). The second section corresponds to the third step (simulating evacuation conditions). The fourth (determining link vulnerabilities for evacuation conditions) and fifth tasks (comparing network conditions and link vulnerabilities from tasks 1-4) are described in section 5.3.3 and 5.3.4, respectively.

5.3.1 Establish Baseline Conditions

The first step to making a comparison is to establish a baseline. There are actually two reference points to be established. The first baseline reveals typical traffic patterns and network clearance time. The second is the set of disruption indices associated with the everyday traffic patterns at a given time. These points of reference correspond to the first two tasks previously described.

In the first task, peak period traffic conditions are simulated. The test bed is shown in section 5.1 (and in chapter 4). As in chapter 4, approximately 30,100 vehicles, generated over 45 minutes, are simulated to establish peak period conditions. The traffic simulation-assignment software DYNASMART-P (DYnamic Network Analysis Simulation Methodology for Advanced Road Telematics) is used for this purpose.

The network is then evaluated using the gaming approach developed in chapter 3 to establish the baseline vulnerabilities (task 2). To summarize the gaming approach, an evil-entity seeks to maximize disruption to the network by damaging a set of links. This damage disrupts origin-destination flows and possibly disconnects some destinations from certain origins. The opponent to the evil entity is a traffic management agency who, upon receiving information about a threat, seeks to route vehicles around the vulnerable links. In establishing the baseline, the traffic management agency is assumed to have no information of a threat to the infrastructure (this corresponds to game 1 developed in chapter 3).

5.3.2 Simulate Evacuation Conditions with Trip Chains

As in chapter 4, the number of households remains constant at 20,000. The number of vehicles that are associated with these households is also constant at 30,141; these vehicles are generated over 3.1 minutes. For each of the household decision making weights described in section 5.2.2, trip chains are generated using the code found in Appendix C. The activity chains generated in each case are used by DYNASMART-P to simulate corresponding traffic conditions.

5.3.3 Determine Infrastructure Vulnerabilities under Evacuation Conditions and Traffic Management Agency Strategies

For each weight on the total fleet travel time mentioned in section 5.2.2, the link vulnerabilities are evaluated at a given point in time using two gaming approaches. Link vulnerabilities are determined using the procedure developed in chapter 3. The disruption index is the measure of vulnerability and is calculated based on the state of the network. Recall that the disruption index directly accounts for the availability of alternate paths, traffic flow, excess path capacity, free flow travel time, and marginal path cost.

In the first game, the traffic management agency (Player M) is assumed to have no information about the evil entity's (Player T's) target. Recall that this lack of information corresponds to both game 1 in chapter 3 and the baseline established in task 2. In the second game, Player M has general information about the type of link to be targeted (e.g. freeway, arterial in a residential zone, arterial near a school zone) while Player T has perfect information about the other's moves and payoffs. Player M can route vehicles to avoid links but cannot prohibit vehicles from reaching their destinations.

5.3.4 Comparison of Peak Period and Evacuation Conditions

The network is examined from two perspectives – network clearance and link vulnerabilities. Each evacuation case is compared to the other evacuation cases and the baseline peak period conditions in terms of those two aspects. The traffic management strategies of avoiding types of targeted links are compared to the evacuation cases where none of the links are avoided and the peak period baseline. The comparison step of the evaluation framework is covered in the results section, presented next.

5.4 RESULTS

Using the experimental procedure described in section 5.3 and the factors outlined in section 5.2, results are generated. These results focus of link vulnerabilities and network clearance times for the baseline peak period conditions and the combinations of evacuation cases, traffic management agency strategies, and evil entity scenarios. The payoff matrix for Game 1 with the baseline peak period conditions is presented in Appendix E for time 30. Also displayed are the payoffs for the evacuation conditions where no links are intentionally avoided. The payoff to the evil entity is the value of the disruption index. As in chapter 3, the payoff to the traffic management agency (Player M) is the percent of vehicles safely reaching their destinations. Recall that in Game 1, Player M has no information about a threat to the network and Player T has the resources to damage only one link.

At such a large network scale and typical traffic patterns, the damage of a single link would have little effect on the network. Only eighteen links resulted in a payoff to Player M of less than 80%, of which six yielded payoffs of less than 70% (see Appendix E). In the peak-period baseline case, links 14 and 306 connected forty-one origin-destination pairs, which was the maximum of any of the 452 links.

The ten most vulnerable roadways under typical peak period conditions are discussed below and shown in figure 5.3. The highest disruption index value is 16.681, associated with link 306 (labeled 1 on figure 5.3). This link leads to the high school zone, a middle school zone, a residential zone, and an evacuation shelter. The second greatest disruption index (16.374) occurs for link 310 (numbered 2), which connects to link 306. Link 323 (labeled 3) has the third highest index; this link is a freeway section which leads from one of the major business zones. The fourth most vulnerable link is 107 (numbered 4), which is a freeway link downstream from link 323. The fifth greatest disruption index (13.712) is associated with link 123 (labeled 5), which is the freeway link between links 323 and 107. Sixth (11.692) is link 43 (numbered 6), which crosses the freeway. The seventh highest disruption index (11.488) is associated with link 117 (labeled 7), which is a freeway link downstream of link 306. The difference in the disruption index values for the freeway links are due to the presence of on and off ramps. Link 48 (numbered 8) had the eighth highest disruption index value (11.363); this link leads to link 310 and ias one of the downstream links from link 43. Ninth, link 105 (labeled 9) has an index of 11.005; this roadway segment is also on the freeway, downstream from link 117. Finally, the tenth greatest disruption index (9.000) is associated with link 305 (numbered 10), which is the approach to the high school.

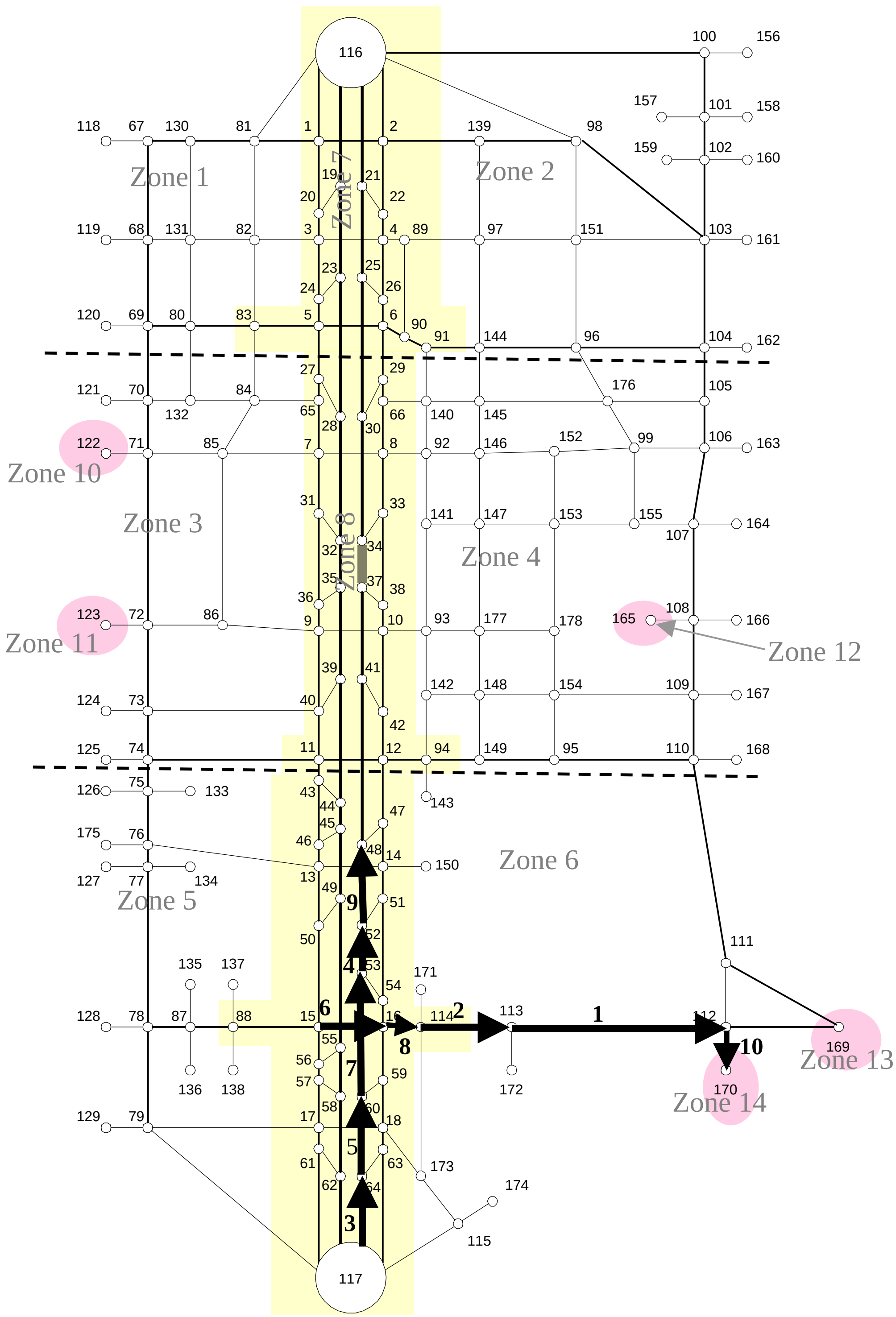

Figure 5.3 Ten Most Vulnerable Links for Peak Period Conditions

The peak period baseline conditions yielded the greatest disruption index of the four traffic patterns in Appendix E. It is important to note that the disruption index (payoff to Player T) did not capture vehicles that were not on the network at the instant under consideration. Any vehicle that was stopped at an intermediate node was not part of the index calculations.

Among the evacuation cases in Appendix E, the maximum disruption value increased with greater values of the weight associated with the total household fleet travel time. For each of the weights, the greatest disruption index was associated with link 305, which led to the high school. Since only one high school was available for all of the households in this particular network, this link could affect every other zone. Under typical traffic conditions, link 305 was the tenth most vulnerable; thus changes in vulnerability occur when parents pick up their children at school in an emergency compared to typical daily child transport. The ten most vulnerable links for each weighting case described in section 5.2.4 are shown in figures 5.4 (case 1), 5.5 (case 2), and 5.6 (case 3).

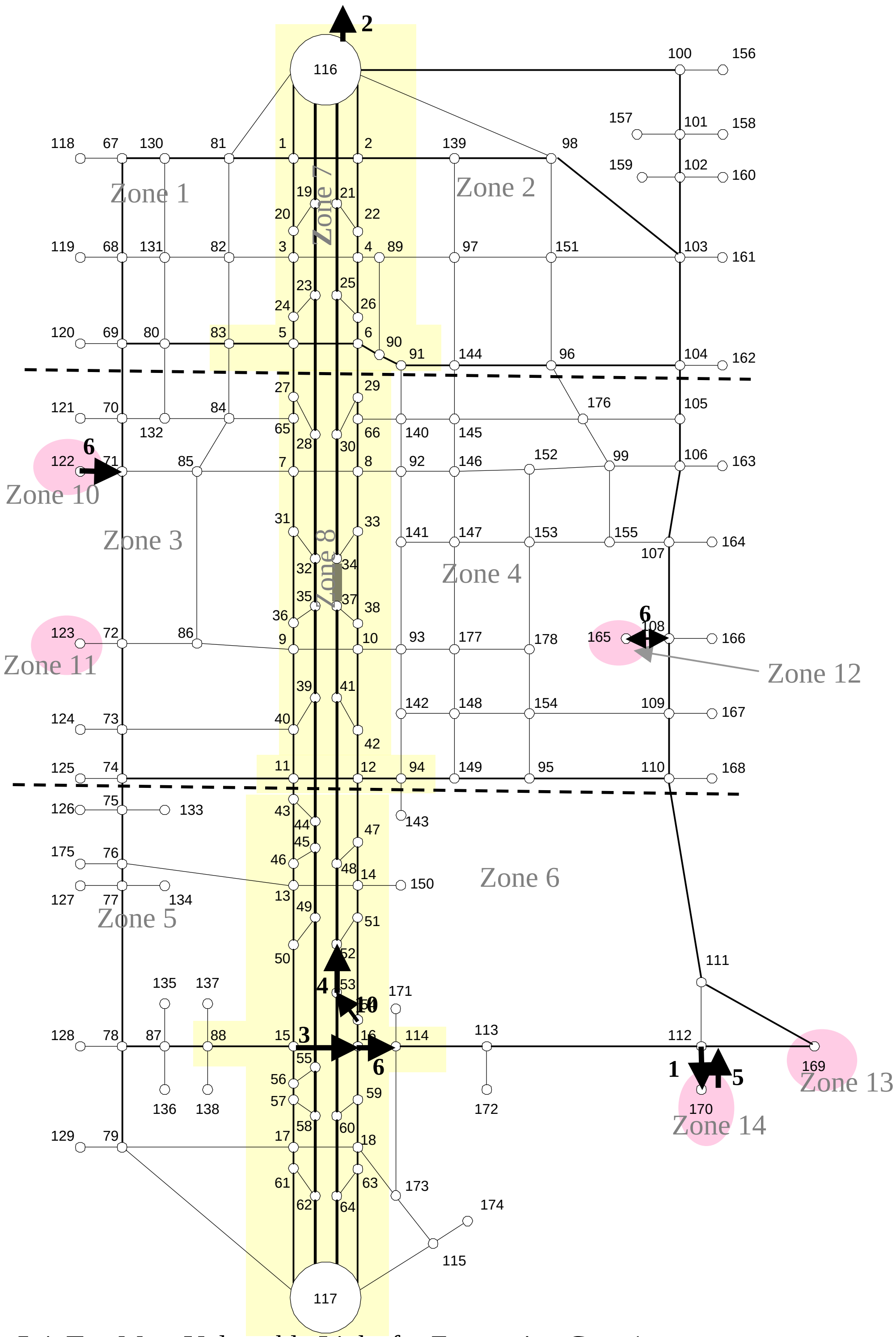

Figure 5.4 Ten Most Vulnerable Links for Evacuation Case 1

Figure 5.4 shows the ten most vulnerable links for evacuation case 1 when the traffic management agency strategy is to “do nothing,” i.e. no links are intentionally avoided. The most vulnerable link is the one leading to the high school. This particular link connects 13 different origin-destination pairs. The high school represents a high demand node; it is also a unique node because no alternative high school was available to households in the network. The second most vulnerable link is an evacuation freeway link leading out of the network to a shelter. This link is one of four possible links leaving the endangered area and may be used to connect 14 origin-destination pairs. The third most vulnerable link is an overpass; under peak period conditions, this link is the sixth most vulnerable. In this evacuation case, the fourth most vulnerable arc is the freeway link that corresponds to the fourth most vulnerable link under peak period conditions. The link leading from the high school to the remainder of the network is the fifth most vulnerable link in evacuation case 1. As with the most vulnerable link, this result is heavily dependent on the network design – there is only one link leading to/from a heavy demand node. Four links are tied as the sixth most vulnerable. Three of these links are associated with travel to and from the middle and elementary schools. Comparing this result to the peak period conditions emphasizes the impact of trip chaining on the traffic patterns. The fourth of the tied links leads to both the high school and a middle school. This particular link is the eighth most vulnerable under peak period conditions. Finally, the tenth most vulnerable link for evacuation case 1 is a freeway on ramp that connects to the fourth most vulnerable link.

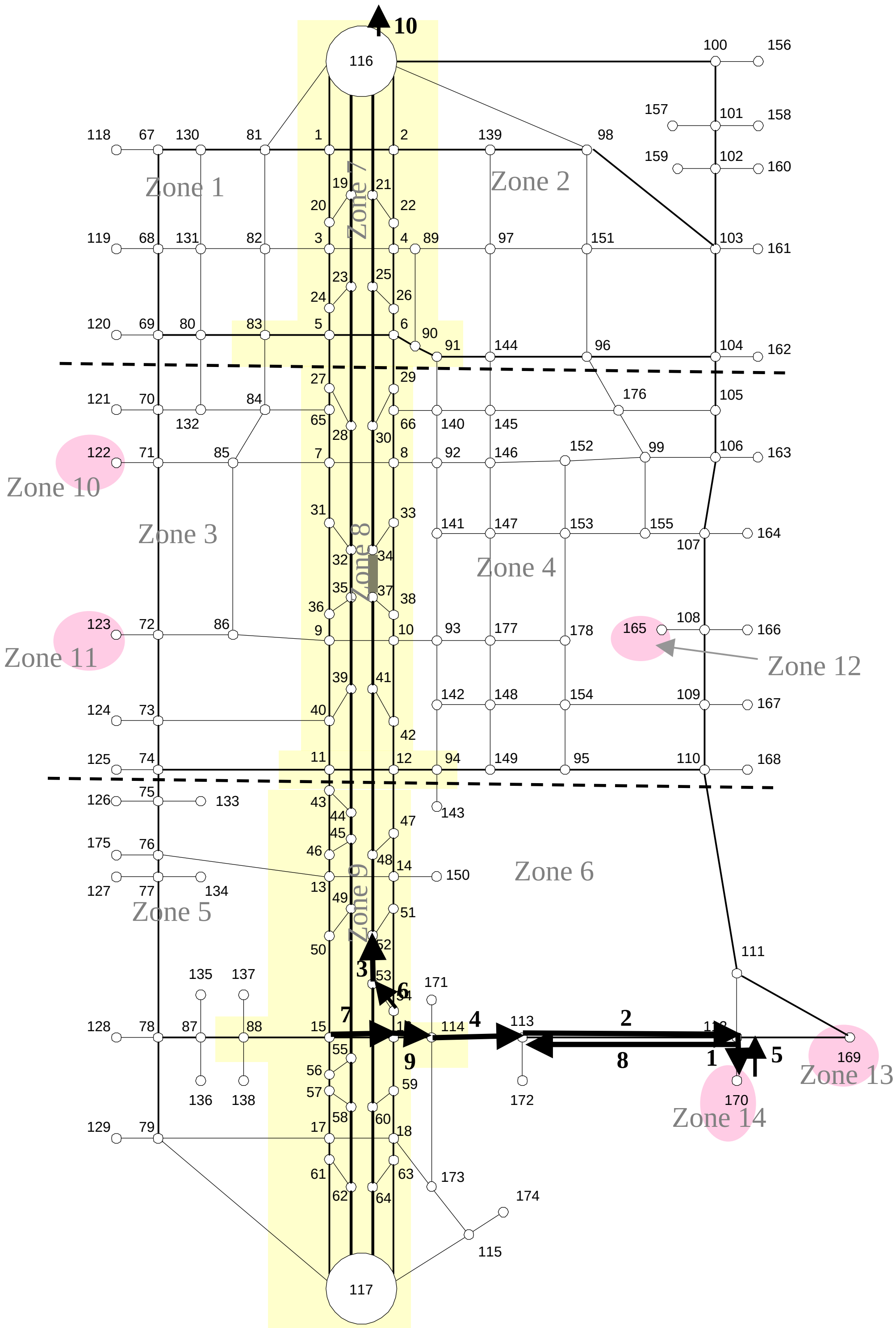

Figure 5.5 Ten Most Vulnerable Links for Evacuation Case 2

Figure 5.5 presents the ten most vulnerable links for evacuation case 2. As in the previous case, shown in figure 5.4, the first and fifth most vulnerable links are associated with the unique high demand node (the high school, in this study). The second most vulnerable link (arc 306) in case 2 is the most vulnerable link under baseline peak period conditions. This link leads to two different schools and (not shown on the map) one of the shelters outside of the endangered network. The third most vulnerable link in case 2 is the freeway link that is the fourth most vulnerable for baseline conditions and evacuation case 1. The fourth most vulnerable link (310) in case 2 is the upstream link of the second most vulnerable link (306). Link 310 is the second most vulnerable under peak period conditions. Case 2's sixth most vulnerable link is a freeway on ramp corresponding to the tenth most vulnerable link in case 1. The seventh most vulnerable link is the overpass that is the sixth most vulnerable for baseline conditions and the third most vulnerable for case 1. The eighth most vulnerable link leads from the high school and nearby middle school toward the freeway. Case 2 is the only case where this link is in the top ten, indicating that the three different sets of trip chains have different impacts on the link vulnerabilities. The ninth most vulnerable link is upstream of link 310 and corresponds to the sixth most vulnerable link in case 1 and the eighth most vulnerable link for baseline conditions. Finally, the tenth most vulnerable link for case 2 leads out of the endangered area and is the same link that is the second most vulnerable in case 1.

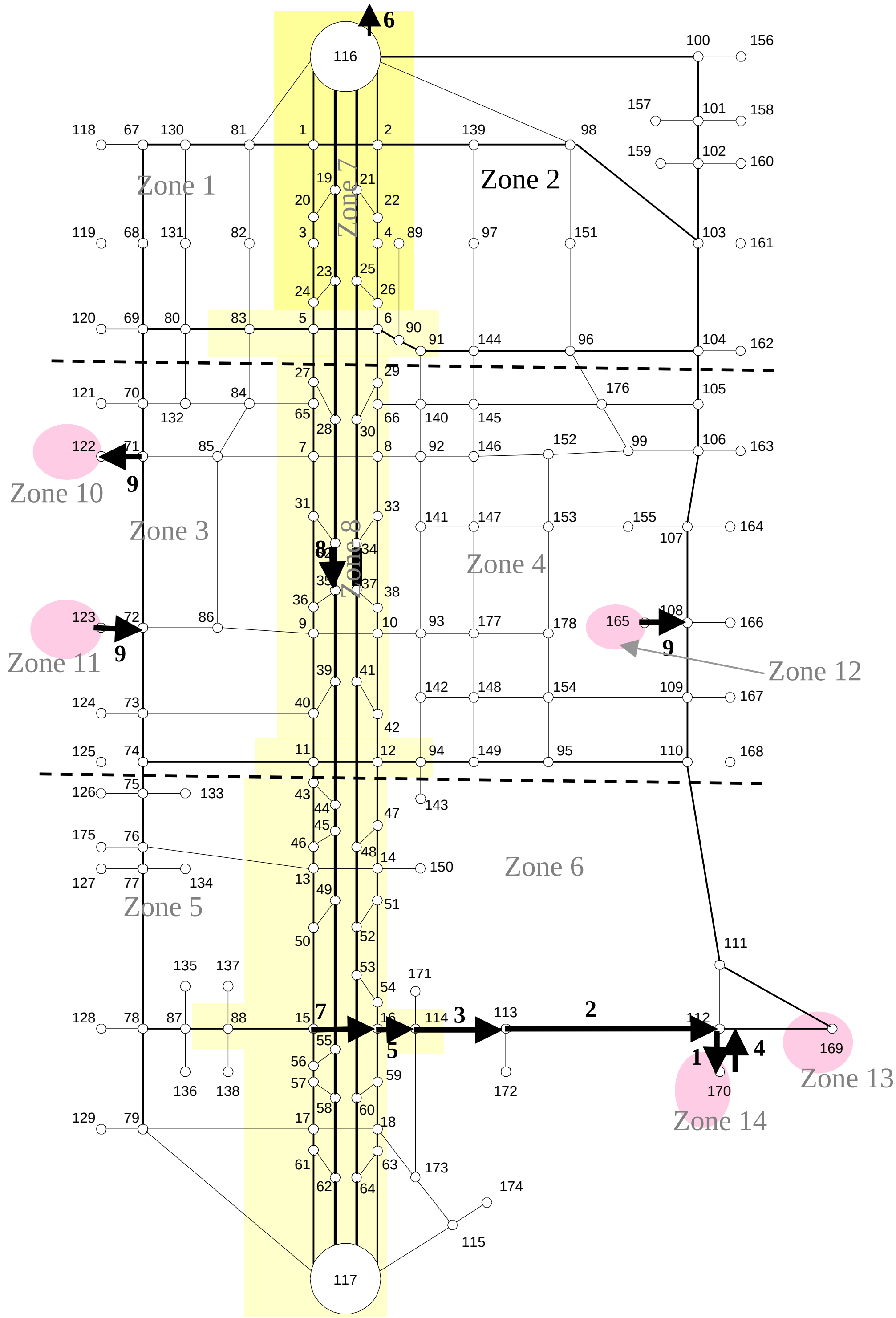

Figure 5.6 Eleven Most Vulnerable Links for Evacuation Case 3

Figure 5.6 shows the eleven most vulnerable links for evacuation case 3. One interesting observation is the freeway links that are in the top ten most vulnerable for the other evacuation cases and the baseline conditions are not in the top ten in this case. The traffic patterns generated by the trip chains vary across the cases examined. However, several links are found in the top ten for every case. The most vulnerable link is the one leading to the single high school in the network. The overpass (seventh most vulnerable in this case), one of the links immediately downstream of the overpass (fifth most vulnerable in this case), one of the links leading from the endangered network to shelter (sixth most vulnerable), and the link leading from the high school (fourth most vulnerable here) are also in the top ten of cases 1 and 2. Recall that the impact of the high school location is related to the network design which limits access to and egress from this unique node. As in case 2, the second most vulnerable link in case 3 is link 306, which also corresponds to the most vulnerable link for the baseline conditions. The third most vulnerable link is link 310, which is the fourth most vulnerable in case 2 and the second most vulnerable for peak period conditions. The one link unique to case 3 is the freeway link ranking eight in the most vulnerable links. As in case 1, links associated with middle and elementary schools (tied for ninth here) are among the most vulnerable.

Greater weights on the minimization of total household fleet travel time do not uniformly increase the vulnerabilities of school related links. For instance, link 147 has the same value of the disruption index for weights 0.5 and 1.0, but link 151 has higher values of the disruption index for weights 0 and 1.0 than for 0.5. These results reflect the fact that the weights generate different trip chains.

The effects of the weights are further investigated in the game where Player M (the traffic management agency) has general information about a threat to a type of link. As mentioned in section 5.2.4 and shown in figure 5.2, the types of links considered are arterial (link 306), freeway (link 123), arterial between two schools (148), arterial connecting a school and residential zone (146), frontage road (83), residential (191), and an overpass (link 43). Each one of these links is avoided, individually, by the traffic management agency. Table 5.2 provides the payoff matrices when these links are avoided and targeted, in turn, at time 30.

Table 5.2 Payoff Matrices for the General Information Game

Player M Strategy	Player T Strategy: Target	λ = 0.0 Player M Payoff	λ = 0.0 Player T Payoff	λ = 0.5 Player M Payoff	λ = 0.5 Player T Payoff	λ = 1.0 Player M Payoff	λ = 1.0 Player T Payoff
Avoid 123	123	99.86	0.183	99.95	0.852	99.82	0.328
	146	94.48	2.991	94.21	2.023	97.13	1.186
	148	97.67	2.623	96.68	1.02	98.43	3.088
	191	100	0	100	0	100	0
	306	85.62	8.619	79.33	10.06	79.69	9.505
	43	88.56	6.618	89.63	7.54	86.34	6.218
	83	100	0	100	0	100	0
Avoid 146	123	99.64	1.378	99.46	1.821	99.73	0.951
	146	98.03	0.864	100	0	100	0
	148	99.47	1.409	99.09	1.831	92.9	4.236
	191	100	0	100	0	100	0
	306	86.61	10.19	81.96	10.066	84.77	12.4
	43	88.66	8.764	84.36	8.95	89.17	9.206
	83	99.86	0.143	99.85	0.25	99.84	0.143
Avoid 148	123	98.95	1.924	98.25	2.19	98.99	1.999
	146	93.99	3.332	99.41	2.904	95.93	2.305
	148	94.72	2.473	99.71	0.834	97.74	1.248
	191	100	0	100	0	100	0
	306	90.49	8.161	87.6	9.94	89.73	9.119
	43	90.3	6.969	87.11	8.229	90.66	6.213
	83	100	0	100	0	100	0
Avoid 191	123	99.7	0.84	99.32	1.05	99.85	0.377
	146	96.3	1.884	96.67	2.668	96.96	1.369
	148	98.26	3.523	97.4	2.48	93.06	3.556
	191	100	0	100	0	100	0
	306	85.7	9.568	86.42	9.192	83.55	10.84
	43	87.67	7.529	86.97	8.961	88.81	7.12
	83	100	0	99.85	0.2	99.84	0.2
Avoid 306	123	99.72	1.438	99.57	2.576	99.56	0.716
	146	95.82	3.361	97.42	4.423	96.38	0.917
	148	98.56	3.218	95	2.506	94.69	3.203
	191	100	0	100	0	100	0
	306	99.1	0.49	100	0	100	0
	43	96.54	6.046	96.08	6.079	97.13	6.285
	83	100	0	100	0	100	0

Player M Strategy	Player T Strategy: Target	λ = 0.0 Player M Payoff	λ = 0.0 Player T Payoff	λ = 0.5 Player M Payoff	λ = 0.5 Player T Payoff	λ = 1.0 Player M Payoff	λ = 1.0 Player T Payoff
Avoid 43	123	97.47	1.597	96.79	1.68	96.75	1.59
	146	97.95	2.675	94.15	3.275	96.89	3.078
	148	95.47	3.842	96.48	3.583	97.2	2.599
	191	100	0	100	0	99.9	0.176
	306	88.8	7.323	90.72	7.191	85.12	6.886
	43	99.6	0.738	99.36	1.111	96	1.016
	83	100	0	100	0	100	0
Avoid 83	123	99.63	0.838	99.66	1.124	99.75	0.455
	146	96.71	2.727	96.62	2.692	94.44	3.603
	148	85.01	5.138	98.36	2.234	97.19	2.43
	191	100	0	100	0	100	0
	306	82.86	8.288	85.05	12.567	89.42	8.136
	43	85.22	8.006	90.51	7.297	91.21	6.635
	83	100	0	100	0	100	0

The weight associated with the minimization of total fleet travel time (and consequently the weight associated with the minimization of waiting time at the meeting location) cause different trip chains to be generated for the households. Avoiding a link affects the actual routing of the vehicles on the network but, in the simulations examined here, is not permitted to impact the sequencing of stops at intermediate nodes. Since the weights affect the trip chains and not the actual routing, no particular weight of the total fleet travel time consistently yields a higher payoff to Player M or Player T.

A comparison of the results from Appendix E and 5.2 indicates that charging an additional cost for using the link successfully reduces lane usage from the baseline peak period conditions and the evacuation cases where no links are intentionally avoided, regardless of the weight associated with the total household fleet travel time. The payoffs for Player M increase when a toll is charged for using link 123 (freeway) and that link is targeted by Player T. The payoff to Player T decreases for this strategy-targeting scenario combination. The same trend is observed for the arterial link (306), frontage road link (83), overpass (43), and the arterial road leading from a school zone to a residential zone (146).

However, this trend did not initially hold for link 148, which is an arterial roadway between two closely located schools. The weight of 0.5 on the minimization of total household fleet travel time follows the previously described pattern. The 0 and 1.0 weight cases result in a lower payoff for Player M than the baseline peak period conditions, but these payoffs are also lower than those for the evacuation conditions when no links were avoided. This result is explained by alternate paths becoming congested and drivers being willing to pay the additional cost of using the link. Subsequently examined higher tolls prohibited the use of link 148, resulting in a payoff of 100% to Player M and a payoff of 0 to Player T. Link 191 (residential) carries no flow in any of the evacuation case – traffic management agency strategy combinations so Player M received a payoff of 100%, which is higher than the payoff for the baseline peak period conditions.

For the majority of Player M's strategies shown in table 5.2 and the "do-nothing" evacuation strategies in Appendix E, targeting link 306 yields the highest payoff for Player T. The one exception occurs when Player M correctly anticipates Player T's move and avoids link 306. These observations indicate that link 306 lies on the shortest paths for many origin destination pairs, but there are alternate paths available. When link 306 is avoided, less flow is found on the link, but the times required for various percent network clearances are greater than for the other traffic management agency strategies.

The impact of the passage of time on the link vulnerabilities is further examined for Player M's strategy "avoid link 306." Table 5.3 displays the payoff values for times 30, 60, and 120 minutes for the different evacuation cases.

Table 5.3 Payoff Values for Player M's Strategy "Avoid Link 306" at Different Time Points

Player T Target	Time	$\lambda = 0$ Player M	Player T	$\lambda = 0.5$ Player M	Player T	$\lambda = 1.0$ Player M	Player T
123	30	99.72	1.438	99.57	2.576	99.56	0.716
	60	99.48	1.457	99.21	2.032	99.80	0.392
	120	100.00	0.000	100.00	0.000	100.00	0.000
146	30	95.82	3.361	97.42	4.423	96.38	0.917
	60	97.42	2.459	95.73	3.139	97.09	2.871
	120	98.93	1.235	98.97	0.736	95.57	2.645
148	30	98.56	3.218	95.00	2.506	94.69	3.203
	60	99.87	0.066	98.84	2.188	97.87	2.867
	120	98.78	0.232	97.90	0.489	99.39	0.693
191	30	100.00	0.000	100.00	0.000	100.00	0.000
	60	100.00	0.000	100.00	0.000	100.00	0.000
	120	100.00	0.000	100.00	0.000	100.00	0.000
306	30	99.10	0.490	100.00	0.000	100.00	0.000
	60	100.00	0.000	100.00	0.000	99.86	0.720
	120	100.00	0.000	100.00	0.000	100.00	0.000
43	30	96.54	6.046	96.08	6.079	97.13	6.285
	60	95.90	4.445	96.76	4.840	97.96	4.897
	120	100.00	0.000	99.18	2.000	100.00	0.000
83	30	100.00	0.000	100.00	0.000	100.00	0.000
	60	100.00	0.000	100.00	0.000	100.00	0.000
	120	100.00	0.000	100.00	0.000	100.00	0.000

The general trend observed from table 5.3 is that as the simulation progresses, the payoffs to Player M increase and the payoffs to Player T decrease. The results pertaining to Player T are consistent with intuition because as vehicles reached their destinations, the network became less congested, leading to additional excess capacity on alternate paths. As can be seen when link 123 is targeted, the payoff to Player M may not increase when the payoff to Player T decreases; this is due to the nature of Player M's payoff calculation as a percentage of the vehicles safely reaching their destinations based on the amount of vehicles in the network at the given time point. As the simulation progresses, links 146 and 148 showed an increase in the payoff to Player T, thus supporting the concept of traffic patterns evolving over time.

Player M's strategy affects the network clearance times during the evacuation. Figures 5.7-5.9 present the time required to clear different percentages of the network for the various Player M strategies. Figure 5.7 corresponds to the evacuation case 1 (no weight on the minimization of total fleet travel time); figure 5.8 is for evacuation case 2 (0.5 weight); and figure 5.9 is for the evacuation case 3 (1.0 weight).

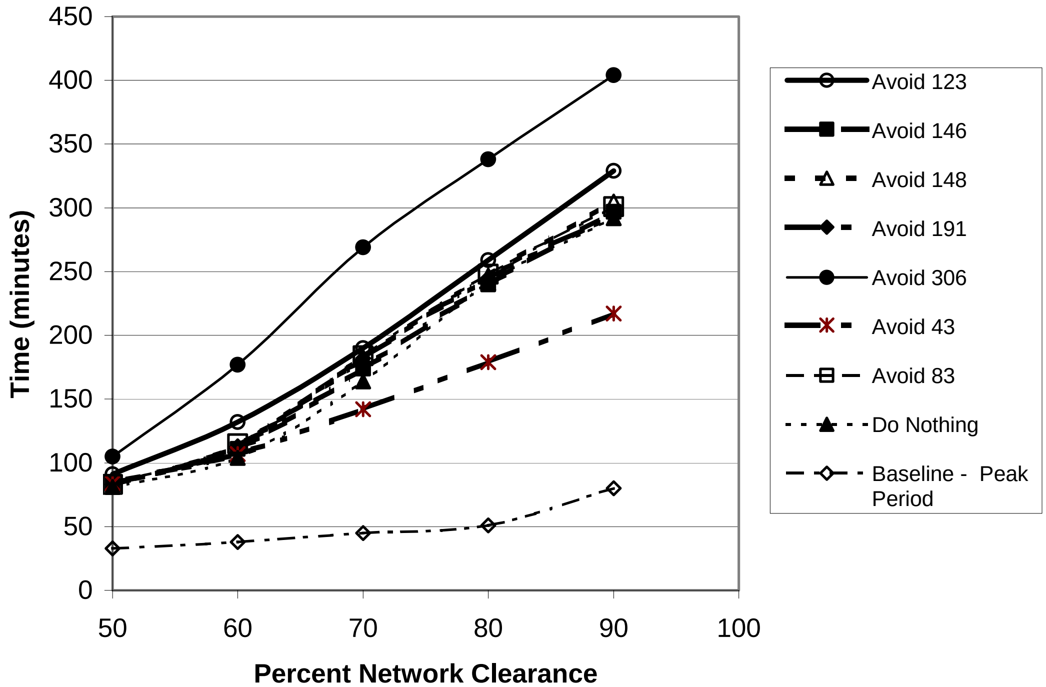

Figure 5.7 Network Clearance for Evacuation Case 1

Compared to the baseline peak period conditions, evacuation case 1, combined with traffic management agency strategies of avoiding threatened links, shows an increase in the time required to clear a given percentage of the network. Avoiding link 306 increases the network clearance time to the greatest degree. Referring to figure 5.1, avoiding this link would intuitively result in longer paths to reach two of the schools. Avoiding the freeway link (123) leads to the next largest increase in network clearance time. This link allows for the highest speeds and not using the link would force the vehicles to use slower roads and thus increase the network clearance times. Avoiding any of the remaining links, except 43, yields network clearance times similar to those of the "do nothing" strategy for the traffic management agency, suggesting that these links have little individual impact on the network. The clearance times for Player M's strategy to avoid link 43 are actually lower than those for the do nothing strategy. This result is due to the traffic simulation approach described in section 5.2.5. .

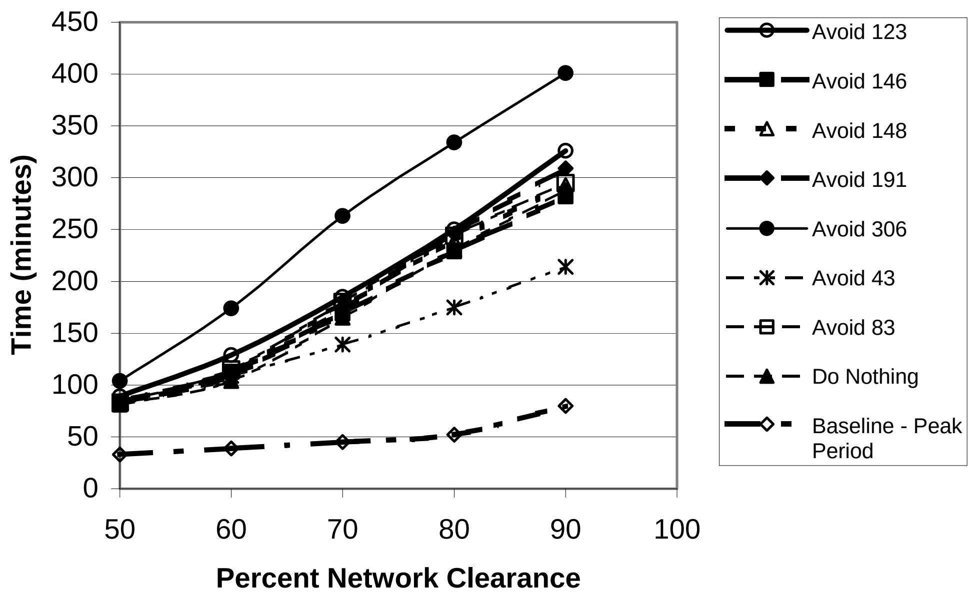

Figure 5.8 Network Clearance for Evacuation Case 2

Figure 5.8 reveals similar pattern to figure 5.7. The network clearance times for the case where the households place half of the weight on the minimization of total household fleet travel time and half on the minimization of waiting time at the meeting locations are lower than for the case shown in figure 5.7.

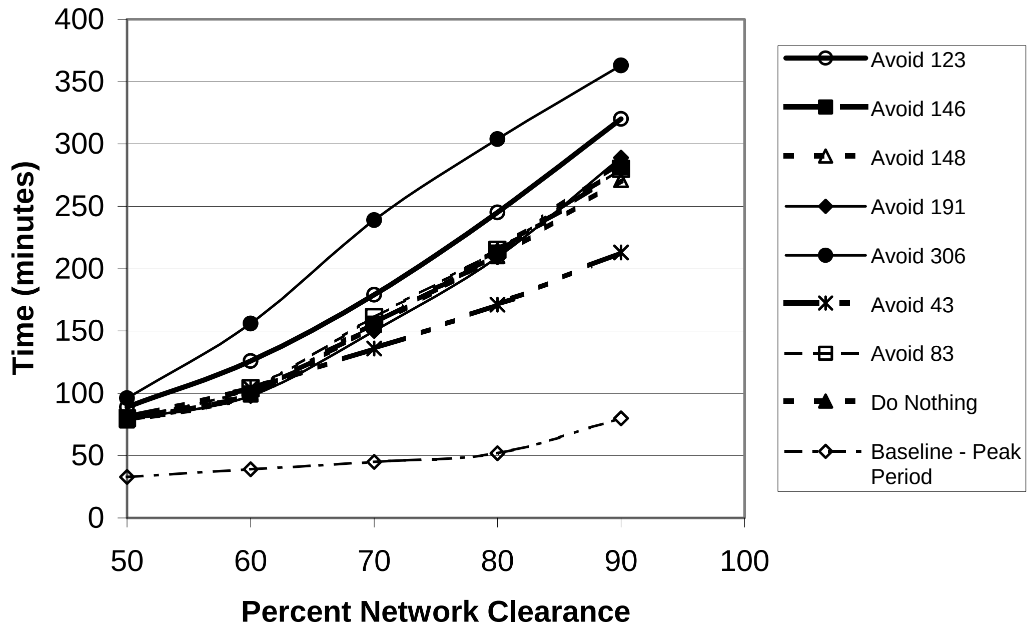

Figure 5.9 Network Clearance for Evacuation Case 3

Figure 5.9 shows that network clearance times for evacuation case 3 follow the same trends as in figures 5.7 and 5.8. In figure 5.9, the network clearance times for Player M's strategy "avoid 123" are consistently greater than the majority of the other traffic management agency strategies. In evacuation cases 1 and 2, the times for 70% clearance for strategy "avoid 123" are close to those for strategies "avoid 146," "avoid 148," "avoid 191," and "avoid 83." Overall, the network clearance times are smaller for case 3 compared to cases 1 and 2.

5.5 SUMMARY

In this chapter, a simplified model of a portion of Fort Worth, Texas was used to illustrate the logical and methodological interrelation between the approaches developed in chapters 3 and 4. Baseline peak period traffic conditions and the infrastructure vulnerabilities associated with those conditions were determined for the cases where the evil entity had the resources to damage one link.

In the baseline peak period case, the most vulnerable links were freeway links near a major business zone and the links leading to a node common to the most origin-destination pairs. These links would have intuitively been the most vulnerable, and this intuition was supported by the mathematical model.

The link vulnerabilities were also determined for evacuation conditions. The different traffic patterns that result from evacuations produced different disruption index values. Link vulnerability rankings changed as a consequence of the unusual traffic patterns. In both the evacuation and baseline scenarios, the most vulnerable link connected the greatest number of origin-destination pairs.

The type of link selected for damage, or avoidance, plays a critical role in the determination of payoffs and the resulting network clearance times. Seeking to avoid the vulnerable links generally yielded longer evacuation times; however, had those links been targeted, a high percentage of the vehicles would still safely reach their destinations. Thus, correctly predicting the target and avoiding the threatened link lengthens evacuation times (relative to the no-damage scenario) but ensures that a greater percentage of the population would successfully complete the evacuation in the event the link is indeed damaged.

Chapter 6

Summary And Conclusions

This chapter presents a summary and the conclusions of this dissertation. Both the conclusions specific to the examples presented in chapters 3, 4, and 5 and generalized conclusions are found in the following pages. This chapter is organized as follows. First, a summary of the work is given. Second, the conclusions specific to the networks shown in this dissertation and general conclusions and recommendations for the methodologies developed for this work are presented. Finally, directions for future work are suggested.

6.1 SUMMARY

In this dissertation two primary contributions are made to the transportation engineering field of knowledge. The first is in the area of network vulnerability. The second is in evacuation modeling. These two contributions have implications for fields outside of transportation engineering, such as evacuation planning, community and urban planning, military strategic strike planning, national defense, and antiterrorism defense.

The contribution to network vulnerability is primarily found in chapter 3 with a larger application provided in chapter 5. In chapter 3, two mathematical indices are presented. The vulnerability index is a measure of the importance of a specific link, or set of links, to the connectivity of an origin-destination pair. This index ias based on existing flow patterns, traffic conditions, and network design. The second measure, the disruption index, is an aggregation of the vulnerability indices across all origin-destination pairs in the network. The disruption index is a measure of the impact of damaging a specific link, or set of links, on the network as a whole. The disruption index - and the vulnerability index at the OD level – allows for the links of the network to be ranked in order of importance. A bi-level formulation, that uses the disruption index as one of its factors, was developed for the identification of the most vulnerable link, or set of links, in the transportation network. Several games were envisioned based on the bi-level formulation. The games consisted of two players: an evil entity, who seeks to maximize disruption of the network, and a traffic management agency, who routes vehicles in order to maximize the number of drivers who safely reach their destinations. Various rules and information for the games were considered, the conclusions for which are presented in section 6.2.

The second contribution to the field is related to evacuation modeling. In chapter 4, a series of mathematical expressions for household decision making under emergency evacuation conditions was presented. First, a family's decision makers select a meeting location. In this work, the objective is to minimize the maximum cost (time or distance) of reaching the meeting location from all of the sites at which household members are located at the time the evacuation order is given. Once the meeting location is chosen, the drivers are assigned to pick up other household members (such as school children) who are not able to drive themselves. The mathematical expression presented in chapter 4 allowes for trade offs between two criteria: (1) the minimization of total fleet travel time and (2) the minimization of waiting time at the meeting location. The advantage of the first criterion is the most efficient trip chains were generated. The disadvantage of (1) is that a single driver (in a two driver household) may be assigned to pick up all of the children allowing for the compilation of unforeseen delays. Criterion (2) also allows for the possibility of one driver picking up all of the children, but this result only occurs when the other driver has a longer perceived time to reach the meeting location. The disadvantage of the second criterion is more of the family members could be in the network (instead of waiting and worrying) and counter-intuitive assignments may be generated; for instance, a driver may be given a sequence of pick ups that requires more time than if the intermediate nodes are in a different order. The factors that influenced the trip chain assignments were thoroughly explored in chapters 4 and 5.

Chapter 5 presented a case study in which the methodologies of chapters 3 and 4 were applied to the same network. The impact of vulnerable link avoidance strategies on evacuation time was examined. The vulnerabilities of the links were determined at different times during an evacuation to identify weaknesses in the network design that are not evident under typical traffic conditions.

6.2 CONCLUSIONS AND RECOMMENDATIONS

The methodologies presented in the previous chapters are applicable to any transportation network. From the network designs and results obtained in this work, several conclusions and recommendations can be made.

For small networks, such as that in chapter 3, and a limited number of resources available to an evil entity, the disruption index can lead to the identification of cut sets. Larger, well connected networks, such as that found in chapter 5, require a substantially greater number of resources to completely sever the origins from the destinations. The vulnerability and disruption indices can, however, identify the links that are most vulnerable to damage given a specified number of resources.

Having accurate information about the amount of resources available to an evil entity is integral to the strategy selection of a traffic management agency. Underestimation leads to fewer vehicles safely reaching their destinations. Overestimation also yields an advantage to the evil entity. Although a greater number of vehicles will safely reach their destinations than originally anticipated, the routing strategy selected may cause a different set of links to be targeted. These links may not require as many resources to damage and the evil entity will receive a higher payoff than if the traffic management agency had perfect information.

Routing vehicles to avoid vulnerable transportation infrastructure yields longer evacuation times. Emergency evacuation scenarios utilize virtually all available capacity.

Using the decision making model presented in chapter 4 yields a more accurate evacuation model. Incorporating household interactions captures traffic patterns that do not exist under daily network demands. These traffic patterns lead to changes in link vulnerabilities from the typical conditions.

When the weights associated with the minimization of total fleet time and the minimization of the waiting time at the meeting locations are unknown, a deterministic dwell time of 5.0 minutes at the intermediate destinations should be used for the combination of network design and household characteristics employed in chapter 4. This dwell time yields the least disparity among the evacuation times for the different weights. However, using a random dwell time is more realistic. Varying the weight associated with the total fleet time across households should also yield a more realistic evacuation scenario since decision makers have different values.

Although no more than three solutions to the trip chain assignments were generated in this work, that is not the limit on the number of possible *pareto optimal* solutions to the problem, depending on the weights of the objective function criteria. The beginning and meeting locations of the household members may create a case where only one solution exists to the objective function regardless of the weight applied to the two criteria – provided that the sum of the weights is 1.0. The actual weights applied to the two criteria depend on the values of the household decision makers; for a generalized modeling scenario, an intermediate value for each weight should be selected.

The other factor that plays a critical role in the pick up assignment and sequencing of intermediate destinations is the dwell time at those intermediate destinations. Certain values of the dwell time may produce vastly different evacuation times, depending on the weights assigned to the two criteria. The dwell time not only affects the trip chain assignment but also allows for the possibility of overall network congestion alleviation. Removing some vehicles from the roads for a period of time (the delay) permits the drivers that are still in the network to proceed at a higher speed, thus resulting in a shorter evacuation. Simulation is required to find both the dwell time that offers the least disparity in evacuation times, regardless of the household's weighting selections, and the dwell time that minimizes evacuation time. Evacuation planners using this model should also keep in mind that they have little control over the individual household's delays at the intermediate nodes.

As should be obvious to planners, antiterrorists, and the military, the mostly likely targeted roads in an non-prioritized network are the links whose damage that can have the most impact on the transportation network as a whole. These links tend to be heavily traveled and used by drivers from more than one origin-destination pair. The joint disruption index for these links is higher than the joint index for other sets of links. In a prioritized network, where a particular origin-destination pair is valued more than other origin-destination pairs, the most likely targeted road is the most heavily traveled link connecting the OD pair of interest. If only the one origin-destination pair is considered, the vulnerability index aides in the identification of the most vulnerable link. Building additional roads to divert traffic away from these highly vulnerable links can save lives when limited threats (i.e. the disaster causing agent cannot destroy every roadway) are realized. Furthermore, this redundancy in the network can allow drivers to reach their desired destinations in a timely manner even if their primary route is unavailable.

6.3 FUTURE WORK

The work performed for this dissertation has lead to three future directions. First, the vulnerability and disruption indices may be used in location analysis. These indices may be used in the site selection for schools, government centers, and other buildings of interest. In terms of combining location analysis and evacuations, if a terrorist threat is perceived, officials may want to move the school children to another location, close to the original. The relocation may serve several functions including, but not limited to, the minimization of the disruption indices of the roads within a given radius of the school and the provision of greater access for the parents to reach their children and complete the evacuation in a more timely manner. Second, additional information levels for further gaming applications may be considered. Finally, the role and impacts of information supply strategies for travelers may be evaluated for evacuation purposes and routing around vulnerable transportation infrastructure.

APPENDIX A

Joint Vulnerability Index Computer Code

A.1 MODULE COMMON_VAR

```
parameter (imaxlink=8)
parameter (imaxnode=6)
parameter (imaxO=2)
parameter (imaxD=2)
parameter (imaxpath=3)
parameter (itotp=9)
integer iorigin
integer idest
integer ican
integer indlink ! number of damaged links
integer inumpath(imaxnode,imaxnode)
integer ipODid(imaxnode,imaxnode,itotp) ! assigns path number to OD
integer iplink(itotp,imaxlink) !1 if link on path
integer idlink(imaxlink) ! damaged link(s)
integer jO(itotp) ! origin node of path
integer jD(itotp)  ! destination node of path
integer iODaf(imaxnode,imaxnode) ! 1 if OD affected, 0 o.w.
integer inODaf ! number of ODs affected
integer ishare(itotp,itotp,imaxlink) ! 1 if paths share link
integer imshared(imaxlink) !1 if link is shared
integer ishrsmOD(imaxlink) !1 if link is shared by paths with same OD
integer ishrdfOD(imaxlink) !1 if link is shared by paths with dif OD
integer icantpath(itotp) ! path can't be used for reassignment
integer ibneklink(itotp) ! bottleneck link of path j
integer ibestp(imaxnode,imaxnode) !current best path from O to D
integer iflag(itotp) !flags path when full due to reassignment
integer iadflow(imaxnode,imaxnode,itotp) !flow reassigned to OD path
integer ictOsh(imaxlink) !counts the number of origins shared by a link
integer ictDsh(imaxlink) !counts the number of destinations shared
integer iOsh(imaxlink,imaxlink) !origins shared by link
integer iDsh(imaxlink,imaxlink) !destinations shared by link
```

```
integer ict_alt(imaxnode,imaxnode) !counts number of alt paths for OD

real totafl
real flow(imaxlink) ! flow on link
real extern(imaxlink) ! externality imposed by additional user to 1 other
real cap(imaxlink)  ! capacity of link
real excapOD(imaxnode,imaxnode) !excess capacity on all undamaged pathsOD
real exper(imaxlink) ! time experienced by traveler n+1
real marginal(imaxlink) ! link marginal time
real tfree(imaxlink) ! free flow travel time
real ptfree(itotp) ! free flow path travel time
real pathflow(itotp)
real pathmarg(itotp)
real plinkflow(itotp,imaxlink) !flow on link due to existing on path
real ODlinkfl(imaxnode,imaxnode,imaxlink) !flow on link from O to D
real ODflow(imaxnode,imaxnode) ! total OD flow
real proplfOD(imaxnode,imaxnode,imaxlink) !proportion of link flow for OD
real flowaOD(imaxnode,imaxnode) !flow on damaged link from O to D
real bnkexcap(itotp) ! minimum excess cap on path
real util(itotp) ! utility of alternate path
real utlp(imaxnode,imaxnode) !sum of adjusted path utilities
real cindex(imaxnode,imaxnode) !critical index
real mdftr(itotp) !modification factor for allocating excess capacity
real modfctr(imaxnode,imaxnode,imaxlink)

end
```

A.2 SUBROUTINE BOTTLENECK(IPATH)

```
! this subroutine finds the bottleneck of each path
use common_var
ibneklink(ipath)=0
bnkexcap(ipath)=0.0
temp_val=99999.9
do j=1,imaxlink
   if(iplink(ipath,j).eq.1)then
      if(ictOsh(j).gt.1)then
         call modify_xcap(j)
         mdftr(j)=modfctr(jO(ipath),jD(ipath),j)
         if(mdftr(j)*(cap(j)-flow(j)).lt.temp_val)then
            ibneklink(ipath)=j
            bnkexcap(ipath)=mdftr(j)*(cap(j)-flow(j))
            temp_val=bnkexcap(ipath)
         endif
      else
         if(cap(j)-flow(j).lt.temp_val)then
            ibneklink(ipath)=j
            bnkexcap(ipath)=cap(j)-flow(j)
            temp_val=bnkexcap(ipath)
         endif
      endif
   endif
enddo
return
end
```

A.3 SUBROUTINE FIND_BPATH(IORIG,JDEST,ICANPATH)

```
! this subroutine finds the best path for the OD pair that does not contain the
! damaged links
use common_var
integer icanpath
icanpath=0
temppm=999999.9
do iii=1,itotp
   do jjj=1,inumpath(iorig,jdest)
      if(ipODid(iorig,jdest,jjj).eq.iii)then
         call bottleneck(iii)
```

```
         if(bnkexcap(iii).lt.1.0) iflag(ipODid(iorig,jdest,jjj))=1
         if(icantpath(ipODid(iorig,jdest,jjj)).ne.1.and.
   +     iflag(ipODid(iorig,jdest,jjj)).ne.1)then
            icanpath=1
            call find_pathmarg(iii)
            if(pathmarg(iii).lt.temppm)then
               temppm=pathmarg(iii)
               ibestp(iorig,jdest)=ipODid(iorig,jdest,jjj)
            endif
         endif
      endif
   enddo
enddo
return
end
```

A.4 SUBROUTINE FIND_PATHMARG(IPATH)

```
use common_var

pathmarg(ipath)=0
ptfree(ipath)=0
do il=1,imaxlink
   if(iplink(ipath,il).eq.1)then
     call totmarg(il)
     pathmarg(ipath)=pathmarg(ipath)+iplink(ipath,il)*marginal(il)
     ptfree(ipath)=ptfree(ipath)+tfree(il)*iplink(ipath,il)
   endif
enddo
return
end
```

A.5 PROGRAM MAINPROG

```
use common_var
REAL DISRUPT
real playerM !payoff to player M

CALL read_input
Do i=1,imaxnode
   Do j=1,imaxnode
      inumpath(i,j)=0
   enddo
enddo

Do k=1, itotp
   inumpath(jO(k),jD(k))=inumpath(jO(k),jD(k))+1
   ipODid(jO(k),jD(k),inumpath(jO(k),jD(k)))=k
   call bottleneck(k)
   call utility(k)
enddo
Call ODaffected
Call SharedLinks
! DETERMINE IF A PATH HAS ANY EXCESS CAPACITY
do ii=1,itotp
   do jj=1,imaxlink
       if(iplink(ii,jj).eq.1.and.cap(jj)-flow(jj).lt.0.0) icantpath(ii)=1
   enddo
enddo
! DETERMINE AMOUNT OF OD FLOW THAT MUST BE
! ACCOMMODATED ON ALTERNATE PATHS
do i=1,imaxnode
   do j=1,imaxnode
      if(iODaf(i,j).eq.1)then
         do k=1,itotp
             jointpath=0
             if(jO(k).eq.i.and.jD(k).eq.j)then
               do idk=1,indlink
                  if(iplink(k,idlink(idk)).eq.1.and.jointpath.ne.1)then
                     flowaOD(i,j)=flowaOD(i,j)+plinkflow(k,idlink(idk))
                     totafl=totafl+plinkflow(k,idlink(idk)) ! total damaged flow
                     jointpath=1
```

```
                endif
              enddo
            endif
          enddo
        endif
    enddo
enddo

! DETERMINE IF THERE ARE ANY ALTERNATE PATHS
DO i=1,imaxnode
    do j=1,imaxnode
        if(iODaf(i,j).eq.1)then
          do k=1,itotp
              if(jO(k).eq.i.and.jD(k).eq.j)then
                  ict_alt(i,j)=ict_alt(i,j)+1
              endif
          enddo
        endif
    enddo
enddo

do i=1,imaxnode
    do j=1,imaxnode
        if(iODaf(i,j).eq.1)then
          do k=1,itotp
            if(jO(k).eq.i.and.jD(k).eq.j)then
                if(icantpath(k).ne.1)then
                    excapOD(i,j)=excapOD(i,j)+bnkexcap(k)
                    call utility(k)
                endif
            endif
          enddo
        endif
    enddo
enddo

open(unit=75, file='utility_n.dat', status='unknown')
do k=1,itotp
    write(75,700) k,util(k)
enddo
close(75)
! index needs to be adjusted by flow
```

```
do i=1,imaxnode
   do j=1,imaxnode
      if(iODaf(i,j).eq.1.and.flowaOD(i,j).gt.0.0)then
         ! if OD pair affected then need reassignment
         if(inumpath(i,j).eq.1)then
           ! no reassignment is possible
           cindex(i,j)=1.0
         else
            if(excapOD(i,j).lt.flowaOD(i,j))then
              cindex(i,j)=flowaOD(i,j)/ODflow(i,j)
            else
               Call Reassignment(i,j)
               if(ican.eq.0) then
                  cindex(i,j)=1.0
               else
                  do k=1,itotp
                     if(jO(k).eq.i.and.jD(k).eq.j)then
                       if(icantpath(k).ne.1)then
                         utlp(i,j)=utlp(i,j)+iadflow(i,j,k)/flowaOD(i,j)*util(k)
                         write(*,*)k,iadflow(i,j,k),flowaOD(i,j),util(k)
                       endif
                     endif
                  enddo
                  cindex(i,j)=(1-utlp(i,j))*flowaOD(i,j)/ODflow(i,j)
               endif
            endif
         endif
      else
         cindex(i,j)=0
      endif
   enddo
enddo

DISRUPT=0.0
open(unit=71, file='index.dat', status='unknown')
do i=1,imaxnode
   do j=1,imaxnode
      if(iODaf(i,j).eq.1) write(71,711) i,j,cindex(i,j)
      DISRUPT=DISRUPT+cindex(i,j)
   enddo
enddo
write(71,712) DISRUPT
```

```
playerM=(1-totafl/3800)*100     ! for ½ demand scenario
write(71,*) 'Percent safely reaching destinations: '
write(71,712) playerM
close(71)

700  format(I6, f15.9)
711  format(2I6,f15.9)
712  format(f15.9)

end program mainprog
```

A.6 SUBROUTINE MODIFY_XCAP(IILINK)

```
! this subroutine modifys the excess capacity of the bottleneck link
! by xa(O''D'')/sum(xaO'D') where O''D'' is the OD pair currently being
! examined and O'D' is the set of OD pairs affected by the damaged link
! and the bottleneck link of the path
! for the joint case, the denominator is the sum over all a and O'D'

use common_var
integer iODlflwck(imaxnode,imaxnode,imaxlink)
real modflow(imaxnode,imaxnode,imaxlink)
real modjflow(imaxlink)

do imo=1,imaxnode
   do imd=1,imaxnode
      iODlflwck(imo,imd,imaxlink)=0
   enddo
enddo

if(ictOsh(iilink).gt.1)then
   do imi=1,ictOsh(iilink)
      if(iODaf(iOsh(iilink,imi),iDsh(iilink,imi)).eq.1.and.
+      iODlflwck(iOsh(iilink,imi),iDsh(iilink,imi),iilink).ne.1.and.
+      ict_alt(iOsh(iilink,imi),iDsh(iilink,imi)).gt.1)then
         DO K=1,itotp
            IF(jO(K).eq.iOsh(iilink,imi).and.jD(K).eq.
    +       iDsh(iilink,imi))then
              if(icantpath(K).ne.1)then
```

```
                  if(iplink(K,iilink).eq.1)then
                     modjflow(iilink)=modjflow(iilink)+
     +               flowaOD(iOsh(iilink,imi),iDsh(iilink,imi))
                     iODlflwck(iOsh(iilink,imi),iDsh(iilink,imi),iilink)=1
                  endif
                endif
              endif
            enddo
          endif
        enddo
        do imi=1,ictOsh(iilink)
          if(iODaf(iOsh(iilink,imi),iDsh(iilink,imi)).eq.1)then
            modfctr(iOsh(iilink,imi),iDsh(iilink,imi),iilink)=
     +       flowaOD(iOsh(iilink,imi),iDsh(iilink,imi))/modjflow(iilink)
          endif
          if(iODaf(iOsh(iilink,imi),iDsh(iilink,imi)).eq.1.and.
     +   modjflow(iilink).le.0.000001)then
            modfctr(iOsh(iilink,imi),iDsh(iilink,imi),iilink)=1.0
          endif
        enddo
      endif
      return
      end
```

A.7 SUBROUTINE ODAFFECTED

```
! this subroutine identifies the OD pairs affected by the damaged link
! due to path ODs and the presence of OD flow on that link
use common_var
inODaf=0
do i=1, itotp
   do j=1, imaxlink
      do k=1, indlink
         if(iplink(i,j).eq.1.and.j.eq.idlink(k)) then
            iODaf(jO(i),jD(i))=1
            inODaf=inODaf+1
            icantpath(i)=1
         endif
      enddo
   enddo
enddo

return
end
```

A.8 SUBROUTINE READ_INPUT

```
use common_var
integer ilink, jpath

open(unit=1, file='linkstate.dat', status='old')
do i=1,imaxlink
   read(1,100) ilink, tfree(ilink), cap(ilink), flow(ilink)
enddo
close(1)
open(unit=2, file='paths.dat', status='old')
do i=1,itotp
   read(2,200) jpath,jO(jpath),jD(jpath),pathflow(jpath)
   do k=1, imaxlink
      read(2,201) iplink(i,k)
      plinkflow(i,k)=pathflow(jpath)*iplink(i,k)
   enddo
enddo
close(2)
do i=1,imaxnode
   do j=1,imaxnode
      do k=1,itotp
         if(jO(k).eq.i.and.jD(k).eq.j)then
            ODflow(i,j)=ODflow(i,j)+pathflow(k)
         endif
      enddo
   enddo
enddo
open(unit=3, file='damage_link.dat', status='old')
read(3,201) indlink
do i=1,indlink
   read(3,201) idlink(i)
   write(*,*) idlink(i)
enddo
close(3)
100   format(I6, 3f10.2)
200   format(3I6, f10.2)
201   format(I3)
return
end
```

A.9 SUBROUTINE REASSIGNMENT(IOR,JDE)

```
! this subroutine reassigns the traffic from the damaged link
! to the other paths connecting the O-D pair
use common_var
real freqrd(imaxnode,imaxnode)
freqrd(iOr,jDe)=0
do k=1,inumpath(iOr,jDe)
   if(icantpath(ipODid(iOr,jDe,k)).eq.1)then
      freqrd(iOr,jDe)=freqrd(iOr,jDe)+pathflow(ipODid(iOr,jDe,k))
   endif
enddo
CALL find_bpath(iOr,jDe,ican)
! now reassign one at a time
if(ican.ne.0)then
   do iflow=1, int(freqrd(iOr,jDe))
      pathflow(ibestp(iOr,jDe))=pathflow(ibestp(iOr,jDe))+1
      do il=1,imaxlink
         if(iplink(ibestp(iOr,jDe),il).eq.1)then
            flow(il)=flow(il)+1
         endif
      enddo
      iadflow(iOr,jDe,ibestp(iOr,jDe))=iadflow(iOr,jDe,ibestp(iOr,jDe))+1
      if(iflow.lt.int(freqrd(iOr,jDe)))then
         call find_bpath(iOr,jDe,ican)
      endif
      if(ican.eq.0) goto 666
   enddo
endif
666  return
return
end
```

A.10 SUBROUTINE SHAREDLINKS

```
! this subroutine identifies which paths share links
use common_var
integer ickODPL(itotp,imaxlink)
```

```
integer idfODpl(itotp,imaxlink)
do i=1,imaxlink
   ictOsh(i)=0
   ictDsh(i)=0    !THESE 2 VARIABLES HAVE IDENTICAL VALUES
enddo
do i=1, itotp-1
   do j=i+1, itotp
      do k=1, imaxlink
         if(iplink(i,k).eq.1.and.iplink(i,k).eq.iplink(j,k))then
            ishare(i,j,k)=1
            imshared(k)=1
            if(jO(i).eq.jO(j).and.jD(i).eq.jD(j))then
               ishrsmOD(k)=1
               if(ickODPL(i,k).ne.1)then
                  ODlinkfl(jO(i),jD(i),k)=ODlinkfl(jO(i),jD(i),k)+
    +             plinkflow(i,k)+plinkflow(j,k)
                  ickODPL(i,k)=1
                  ickODPL(j,k)=1
               endif
            else
               ishrdfOD(k)=1
               if(ickODPL(i,k).ne.1)then
                 ODlinkfl(jO(i),jD(i),k)=ODlinkfl(jO(i),jD(i),k)+plinkflow(i,k)
                 ickODPL(i,k)=1
               endif
               if(iODaf(jO(i),jD(i)).eq.1.and.iODaf(jO(j),jD(j)).eq.1.
    +          and.idfODpl(i,k).ne.1)then
                  ictOsh(k)=ictOsh(k)+1
                  ictDsh(k)=ictDsh(k)+1
                  iOsh(k,ictOsh(k))=jO(i)
                  iDsh(k,ictDsh(k))=jD(i)
                  idfODpl(i,k)=1
               endif
               if(ickODPL(j,k).ne.1)then
                 ODlinkfl(jO(j),jD(j),k)=ODlinkfl(jO(j),jD(j),k)+plinkflow(j,k)
                 ickODPL(j,k)=1
               endif
               if(iODaf(jO(i),jD(i)).eq.1.and.iODaf(jO(j),jD(j)).eq.1.
    +          and.idfODpl(j,k).ne.1)then
                  ictOsh(k)=ictOsh(k)+1
                  ictDsh(k)=ictDsh(k)+1
                  iOsh(k,ictOsh(k))=jO(j)
```

```
                iDsh(k,ictDsh(k))=jD(j)
                idfODpl(j,k)=1
               endif
             endif
           else
             ishare(i,j,k)=0
           endif
        enddo
     enddo
  enddo
! this section checks for the OD being affected
! if not, then the OD flow is not counted in the proportion
! calculated in the next section
  do i=1,imaxnode
     do j=1,imaxnode
        if(iODaf(i,j).ne.1)then
           do ip=1,itotp
              do k=1,imaxlink
                 ODlinkfl(i,j,k)=ODlinkfl(i,j,k)-plinkflow(ip,k)
                 if(ODlinkfl(i,j,k).lt.0.00001)ODlinkfl(i,j,k)=0.0
              enddo
           enddo
        endif
     enddo
  enddo
  do i=1,imaxnode
     do j=1,imaxnode
        if(iODaf(i,j).eq.1)then
           do ip=1,itotp
              if(jO(ip).eq.i.and.jD(ip).eq.j)then
                 do k=1,imaxlink
                    if(flow(k).le.0.00001)then
                       proplfOD(i,j,k)=0.0
                    else
                       if(imshared(k).ne.1) proplfOD(i,j,k)=1.0
                       if(iplink(ip,k).eq.1.and.imshared(k).eq.1)then
                         if(ishrdfOD(k).eq.1.and.plinkflow(ip,k).gt.0.00001)
     +                   then
                            proplfOD(i,j,k)=ODlinkfl(i,j,k)/flow(k)
                         else
                           proplfOD(i,j,k)=0.0
                         endif
```

```
                if(ishrsmOD(k).eq.1.and.ishrdfOD(k).ne.1)then
                  proplfOD(i,j,k)=1.0
                endif
                if(ishrsmOD(k).eq.1.and.ishrdfOD(k).eq.1)then
                  proplfOD(i,j,k)=ODlinkfl(i,j,k)/flow(k)
                endif
              endif
            endif
          enddo
        endif
      enddo
     endif
  enddo
enddo
return
end
```

A.11 SUBROUTINE TOTMARG(L)

```
! This subroutine calculates the total marginal cost of adding one user to the link
use common_var
if(flow(l).lt.cap(l))then
  extern(l)=0.6*((flow(l))**3)/((cap(l))**4)
  exper(l)=tfree(l)+0.15*((flow(l)/cap(l))**4)
  marginal(l)=exper(l)+flow(l)*extern(l)
endif
return
end
```

A.12 SUBROUTINE UTILITY(KPATH)

```
! this subroutine calculates the utility of an alternate path
use common_var
call find_pathmarg(kpath)
util(kpath)=(bnkexcap(kpath)/cap(ibneklink(kpath)))*
+   ptfree(kpath)/pathmarg(kpath)
return
end
```

Appendix B

Flow Distributions

Table B.1 Flow Distributions For Table 3.11, Original Demand Level

Player M Strategy	Flow	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8
Do nothing / Avoid 3	x_l	2300	1700	0	1700	3400	1800	1600	1900
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900
Avoid 1	x_l	2300	1700	0	1700	3400	1800	1600	1900
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900
Avoid 2	x_l	3000	0	1000	1700	1700	1100	600	2900
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900
Avoid 4	x_l	3000	1000	0	1100	2100	1100	1000	2500
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 5	x_l	3000	0	1000	1100	1100	1100	0	3500
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 6	x_l	3000	1000	0	1306	2306	1100	1206	2294
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	206	206	0	206	2294
Avoid 7	x_l	2842	158	1000	1100	1258	1258	0	3500
	$x_l^{1,2}$	2842	158	0	0	158	158	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500

Avoid 8	x_l	3000	1000	0	1700	2700	1100	1600	1900
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,2}$	0	0	0	1100	1100	1100	0	0
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900

Table B.2 Flow Distributions for Table 3.12 (n=1, 3/4 Demand)

Player M Strategy	Flow	Link 1	Link 2	Link 3	Link 4*	Link 5*	Link 6*	Link 7	Link 8
Do nothing / Avoid 3	x_l*	2250	750	0	1306.67	2056.67	825	1231.67	1393.33
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	481.67	481.67	0	481.67	1393.33
Avoid 1	x_l	1275	1700	25	1134.13	2834.13	1800	1034.13	1590.87
	$x_l^{1,2}$	1275	950	25	25	975	975	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	284.13	284.13	0	284.13	1590.87
Avoid 2 (opt 1)	x_l	2250	0	750	1533.49	1533.49	825	708.49	1916.51
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	708.49	708.49	0	708.49	1166.51
Avoid 2 (opt 2)	x_l	2250	0	750	1533.50	1533.50	825	708.50	1916.50
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	700	700	0	700	50
	$x_l^{5,6}$	0	0	0	8.50	8.50	0	8.50	1866.50
Avoid 4	x_l	2250	750	0	825	1575	825	750	1875
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 5	x_l	2250	0	750	825	1575	825	0	2625
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	0	750	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 6	x_l	2250	750	0	1309.79	2059.79	825	1234.79	1390.21
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	484.79	484.79	0	484.79	1390.21
Avoid 7	x_l	2131.22	118.78	750	825	943.78	943.78	0	2625
	$x_l^{1,2}$	2131.22	118.78	0	0	118.78	118.78	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 8	x_l	2250	750	0	1700	2450	825	1625	1000
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	875	875	0	875	1000

* Links 4, 5, and 6 always carry all of the OD (5,2) flow which is 825 vph in this case.

Table B.3 Flow Distributions for Table 3.13(n=1, 1/2 Demand)

Player M Strategy	Flow	Link 1	Link 2	Link 3	Link 4*	Link 5*	Link 6*	Link 7	Link 8
Do nothing / Avoid 3	x_l*	1500	500	0	1314.98	1814.98	550	1264.98	485.02
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	764.98	764.98	0	764.98	485.02
Avoid 1+ (opt 1)	x_l	250	1700	50	1119.35	2819.35	1800	1019.35	730.65
	$x_l^{1,2}$	250	1250	0	0	1250	1250	0	0
	$x_l^{1,6}$	0	450	50	0	450	0	450	50
	$x_l^{5,6}$	0	0	0	569.35	569.35	0	569.35	680.65
Avoid 2 (opt 1)	x_l	1500	0	500	1476.75	1476.75	550	926.75	823.25
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	926.75	926.75	0	926.75	323.25
Avoid 2 (opt 2)	x_l	1500	0	500	1476.84	1476.84	550	926.84	823.16
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	500	500	0	500	0
	$x_l^{5,6}$	0	0	0	426.84	426.84	0	426.84	823.16
Avoid 4	x_l	1500	500	0	550	1050	550	500	1250
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 5	x_l	1500	0	500	550	550	550	0	1750
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 6	x_l	1500	500	0	1314.81	1814.81	550	1264.81	485.19
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	764.81	764.81	0	764.81	485.19
Avoid 7	x_l	1500	0	500	550	550	550	0	1750
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 8	x_l	1500	500	0	1700	2200	550	1650	100
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	1150	1150	0	1150	100

* Links 4, 5, and 6 always carry all of the OD (5,2) flow which is 550 vph in this case.
\+ Options 1 and 2 for Avoid 2 have the equivalent values of the objective function to four decimal places.

Table B.4 Flow Distribution Corresponding to Table 3.14, n=2, Original Demand

Player M Strategy	Flow	Link 1	Link 2	Link 3	Link 4*	Link 5*	Link 6*	Link 7	Link 8
Avoid 1,2 (opt 1)	x_l*	2400	0	1600	1700	1700	1700	0	3500
	$x_l^{1,2}$	2400	0	600	600	600	600	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 1,2 (opt 2)	x_l*	2300	100	1600	1700	1800	1800	0	3500
	$x_l^{1,2}$	2300	100	600	600	700	700	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 1,3	x_l*	2300	1700	0	1700	3400	1800	1600	1900
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900
Avoid 1,4	x_l*	2300	1700	0	1100	2800	1800	1000	2500
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 1,5 (opt 1); 1,6(opt 1); 1,7	x_l*	2300	700	1000	1100	1800	1800	0	3500
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 1,5 (opt 2); 1,6(opt 2)	x_l*	3000	0	1000	1100	1100	1100	0	3500
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 1,5 (opt 3); 1,6(opt 3)	x_l*	2650	350	1000	1100	1450	1450	0	3500
	$x_l^{1,2}$	2650	350	0	0	350	350	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 1,8(opt 1)	x_l*	2300	700	1000	1700	2400	1800	600	2900
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900
Avoid 1,8(opt 2)	x_l*	2300	700	1000	1700	2400	1800	600	2900
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	0	1000	300	300	0	600	400
	$x_l^{5,6}$	0	0	0	300	300	0	0	2500
Avoid 1,8(opt 3)	x_l*	2300	700	1000	1700	2400	1800	600	2900
	$x_l^{1,2}$	2300	700	0	0	700	700	0	0
	$x_l^{1,6}$	0	0	1000	300	300	0	300	700
	$x_l^{5,6}$	0	0	0	300	300	0	300	2200
Avoid 2,3(opt 1)	x_l*	3000	1000	0	1330.96	2330.96	1100	1230.96	2269.04
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,6}$	0	0	0	230.96	230.96	0	230.96	2269.04

Avoid 2,3(opt 2); 2,8(opt 3); 7,8(opt 3)	x_l*	3000	0	1000	1700	1700	1100	600	2565.03
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	220.66	220.66	0	220.66	345.53
	$x_l^{5,6}$	0	0	0	379.34	379.34	0	280.50	2219.50
Avoid 2,3(opt 3)	x_l*	3000	507.95	492.05	1527.02	2034.97	1100	934.97	2565.03
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	507.95	492.05	146.51	654.45	0	654.47	345.53
	$x_l^{5,6}$	0	0	0	280.50	280.50	0	280.50	2219.50
Avoid 2,4; 2,5; 2,7; 3,5(opt 2); 4,5; 5,6; 5,7; 6,7	x_l*	3000	0	1000	1100	1100	1100	0	3500
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 2,6(opt 1); 2,8(opt 2); 5,8(opt 2); 7,8(opt 2)	x_l*	3000	0	1000	1700	1100	1100	600	2900
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	600	1900
Avoid 2,6(opt 2)	x_l*	3000	0	1000	1700	1700	1100	600	2900
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	0	600	600	0	600	400
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 2,6(opt 3)	x_l*	3000	0	1000	1700	1700	1100	600	2900
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	1000	300	300	0	300	700
	$x_l^{5,6}$	0	0	0	300	300	0	300	2200
Avoid 2,8(opt 1); 3,6; 3,7(opt 2); 3,8; 4,8; 5,8(opt 1); 6,8; 7,8(opt 1)	x_l*	3000	1000	0	1700	2700	1100	1600	1900
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,6}$	0	0	0	600	600	0	600	1900
Avoid 3,4; 3,5(opt 1); 3,7(opt 1); 4,6	x_l*	3000	1000	0	1700	2700	1100	1000	2500
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	1000	0	0	1000	0	1000	0
	$x_l^{5,6}$	0	0	0	600	600	0	0	2500
Avoid 3,5(opt 3);	x_l*	3000	507.95	492.05	1100	1607.95	1100	507.95	2992.05
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	507.95	492.05	0	507.95	0	507.95	492.05

3,7(opt 3)	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 4,7	x_l^*	2842.07	157.93	1000	1100	1257.93	1257.93	0	3500
	$x_l^{1,2}$	2842.07	157.93	0	0	157.93	157.93	0	0
	$x_l^{1,6}$	0	0	1000	0	0	0	0	1000
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500
Avoid 5,8(opt 3)	x_l^*	3000	844.35	155.65	1100	1944.35	1100	844.35	2655.65
	$x_l^{1,2}$	3000	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	844.35	155.65	0	844.35	0	844.35	155.65
	$x_l^{5,6}$	0	0	0	0	0	0	0	2500

Table B.5 Flow Distribution Corresponding to Table 3.16, n=2, 3/4 Demand

M's Strat.	Flow	Link 1	Link 2	Link 3	Link 4*	Link 5*	Link 6*	Link 7	Link 8
Avoid 1,2 (opt 1)	x_l^*	1400	0	1600	1700	1700	1675	25	2600
	$x_l^{1,2}$	1400	0	850	850	850	850	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	25	750
	$x_l^{5,6}$	0	0	0	25	25	0	0	1850
Avoid 1,2 (opt 2)	x_l^*	1275	125	1600	1675	1800	1800	0	2625
	$x_l^{1,2}$	1275	0	850	850	975	975	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 1,3 (opt 1)	x_l^*	1300	1700	0	1116.16	2816.16	1775	1041.16	1583.84
	$x_l^{1,2}$	1300	950	0	0	950	950	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	291.16	291.16	0	291.16	1583.84
Avoid 1,3 (opt 2); 1,6 (opt 2)	x_l^*	1275	1700	25	1120.61	2820.61	1800	1020.61	1604.39
	$x_l^{1,2}$	1275	975	0	0	975	975	0	0
	$x_l^{1,6}$	0	725	25	0	725	0	725	25
	$x_l^{5,6}$	0	0	0	295.61	295.61	0	295.61	1579.39
Avoid 1,3 (opt 3)	x_l^*	1275	1700	25	1120.61	2820.61	1800	1020.61	1604.39
	$x_l^{1,2}$	1275	975	0	0	975	975	0	0
	$x_l^{1,6}$	0	725	25	25	750	0	750	0
	$x_l^{5,6}$	0	0	0	270.61	270.61	0	270.61	1604.39
Avoid 1,4	x_l^*	1275	1700	25	825	2525	1800	725	1900
	$x_l^{1,2}$	1275	975	0	0	975	975	0	0
	$x_l^{1,6}$	0	725	25	0	725	0	725	25
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 1,5 (opt1); 2,4; 2,5; 2,7; 3,5 (opt2); 3,7 (opt2); 4,5; 5,6; 5,7; 5,8 (opt2); 6,7; 7,8 (opt 1)	x_l^*	2250	0	750	825	825	825	0	2625
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 1,5 (opt2); 1,7	x_l^*	1275	975	750	825	1800	1800	0	2625
	$x_l^{1,2}$	1275	975	0	0	975	975	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 1,5	x_l^*	1773.64	476.36	750	825	1301.36	1301.36	0	2625
	$x_l^{1,2}$	1773.64	476.36	0	0	476.36	476.36	0	0

(opt 3)	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 1,6 (opt1); 2,3 (opt1); 3,6; 5,8 (opt 1)	x_l^*	2250	750	0	1309.79	2059.79	825	1234.79	1390.21
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	484.79	484.79	0	484.79	1390.21
Avoid 1,6 (opt 3)	x_l^*	1773.64	1226.36	0	1242.00	2468.36	1301.36	1167.00	1458
	$x_l^{1,2}$	1773.64	476.36	0	0	476.36	476.36	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	417	417	0	417	1458
Avoid 1,8 (opt 1)	x_l^*	1300	1700	0	1700	3400	1775	1625	1000
	$x_l^{1,2}$	1300	950	0	0	950	950	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	875	875	0	875	1000
Avoid 1,8 (opt 2)	x_l^*	1275	1700	25	1700	3400	1800	1600	1025
	$x_l^{1,2}$	1275	975	0	0	975	975	0	0
	$x_l^{1,6}$	0	725	25	0	725	0	725	25
	$x_l^{5,6}$	0	0	0	875	875	0	875	1000
Avoid 1,8 (opt 3)	x_l^*	1275	1700	25	1700	3400	1800	1600	1025
	$x_l^{1,2}$	1275	975	0	0	975	975	0	0
	$x_l^{1,6}$	0	725	25	25	750	0	750	0
	$x_l^{5,6}$	0	0	0	850	850	0	850	1025
Avoid 2,3 (opt2); 2,6 (opt3); 2,8 (opt 1)	x_l^*	2250	0	750	1700	1700	825	875	1750
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	375	375	0	375	375
	$x_l^{5,6}$	0	0	0	500	500	0	500	1375
Avoid 2,3 (opt3); 7,8 (opt 3)	x_l^*	2250	360.48	389.52	1454.89	1815.37	825	990.37	1634.63
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	360.48	389.52	53.64	414.12	0	414.12	335.88
	$x_l^{5,6}$	0	0	0	576.25	576.25	0	576.25	1298.75
Avoid 2,6 (opt 1)	x_l^*	2250	0	750	1533.48	1533.48	825	708.48	1916.52
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0		708.48	708.48	0	708.48	1916.52
Avoid 2,6 (opt 2)	x_l^*	2250	0	750	1533.48	1533.48	825	708.48	1916.52
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	708.48	708.48	0	708.48	41.52
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 2,8 (opt 2)	x_l^*	2250	0	750	1700	1700	825	875	1750
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	875	875	0	875	1000
Avoid 2,8 (opt 3)	x_l^*	2250	0	750	1700	1700	825	875	1750
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	750	750	750	0	750	0
	$x_l^{5,6}$	0	0	0	125	125	0	125	1750

Avoid 3,4; 3,5 (opt1); 3,7 (opt1); 4,6; 4,8 (opt 2)	x_l*	2250	750	0	825	1575	825	750	1875
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	825	0	0	0	1875
Avoid 3,5 (opt3); 3,7 (opt 3)	x_l*	2250	325	325	825	1150	825	325	2200
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	325	325	0	325	0	325	325
	$x_l^{5,6}$	0			0	0	0	0	1875
Avoid 3,8; 4,8 (opt1); 5,8 (opt3); 6,8; 7,8 (opt 2)	x_l*	2250	750	0	1700	2450	825	1625	1000
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	875	875	0	875	1000
Avoid 4,7	x_l*	2130.69	119.31	750	825	944.31	944.31	0	2625
	$x_l^{1,2}$	2130.69	119.31	0	0	119.31	119.31	0	0
	$x_l^{1,6}$	0	0	750	0	0	0	0	750
	$x_l^{5,6}$	0	0	0	0	0	0	0	1875
Avoid 4,8 (opt 3)	x_l*	2250	750	0	1325	2075	825	1250	1375
	$x_l^{1,2}$	2250	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	750	0	0	750	0	750	0
	$x_l^{5,6}$	0	0	0	500	500	0	500	1375

* Links 4, 5, and 6 always carry all of the OD (5,2) flow which is 875 vph in this case.

Table B.6 Flow Distribution for Table 3.18, n=2, ½ Demand Level

Player M Strategy	Flow	Link 1	Link 2	Link 3	Link 4*	Link 5*	Link 6*	Link 7	Link 8
Do nothing	x_l^*	1500	500	0	1314.98	1814.98	550	1264.98	485.02
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	764.98	764.98	0	764.98	485.02
Avoid 1,2 (opt 1)	x_l	250	150	1600	1700	1850	1800	50	1700
	$x_l^{1,2}$	250	150	1100	1100	1250	1250	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	50	50	0	50	1200
Avoid 1,2 (opt 2)	x_l	250	150	1600	1700	1850	1800	50	1700
	$x_l^{1,2}$	250	100	1150	1150	1250	1250	50	0
	$x_l^{1,6}$	0	50	450	0	50	0	0	450
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 1,3	x_l	300	1700	0	1103	2803	1750	1053	697
	$x_l^{1,2}$	300	1200	0	0	1200	1200	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	553	553	0	553	697
Avoid 1,4	x_l	250	1700	50	550	2250	1800	450	1300
	$x_l^{1,2}$	250	1250	0	0	1250	1250	0	0
	$x_l^{1,6}$	0	450	50	0	450	0	450	50
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 1,5 (opt 1); 1,7	x_l	250	1250	500	550	1800	1800	0	1750
	$x_l^{1,2}$	250	1250	0	0	1250	1250	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 1,5(opt 2); 2,4; 2,5; 2,7; 3,5(opt 2); 3,7 (opt 2); 4,5; 4,7; 5,6; 5,7; 5,8(opt 2); 6,7; 7,8(opt 2)	x_l	1500	0	500	550	550	550	0	1750
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 1,6 (opt 1)	x_l	1500	500	0	1314.81	1814.81	550	1264.81	485.19
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	764.81	764.81	0	764.81	485.19

Avoid 1,6 (opt 2)	x_l	250	1700	50	1098.90	2798.90	1800	998.90	751.10
	$x_l^{1,2}$	250	1250	0	0	1250	1250	0	0
	$x_l^{1,6}$	0	450	50	0	450	0	450	50
	$x_l^{5,6}$	0	0	0	548.90	548.90	0	548.90	701.10
Avoid 1,8 (opt 1)	x_l	250	1700	50	1700	3400	1800	1600	150
	$x_l^{1,2}$	250	1250	0	0	1250	1250	0	0
	$x_l^{1,6}$	0	450	50	0	450	0	450	50
	$x_l^{5,6}$	0	0	0	1150	1150	0	1150	100
Avoid 1,8 (opt 2)	x_l	250	1700	50	1700	3400	1800	1600	150
	$x_l^{1,2}$	250	1250	0	0	1250	1250	0	0
	$x_l^{1,6}$	0	450	50	50	500	0	500	0
	$x_l^{5,6}$	0	0	0	1100	1100	0	1100	150
Avoid 2,3 (opt 1)	x_l	1500	500	0	1305.76	1805.76	550	1255.76	494.24
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	755.76	755.76	0	755.76	494.24
Avoid 2,3 (opt 2)	x_l	1500	0	500	1473.40	1473.40	550	923.40	826.60
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	83.82	83.82	0	83.82	419.18
	$x_l^{5,6}$	0	0	0	839.58	839.58	0	839.58	410.42
Avoid 2,3 (opt 3)	x_l	1500	155.95	344.05	1440.06	1596.01	550	1046.01	703.99
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	155.95	344.05	137.83	293.78	0	293.78	206.22
	$x_l^{5,6}$	0	0	0	752.23	752.23	0	752.23	497.77
Avoid 2,6 (opt 1)	x_l	1500	0	500	1476.75	1476.75	550	926.75	823.25
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	926.75	926.75	0	926.75	323.25
Avoid 2,6 (opt 2)	x_l	1500	0	500	1476.75	1476.75	550	926.75	823.25
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	500	500	0	500	0
	$x_l^{5,6}$	0	0	0	426.75	426.75	0	426.75	823.25
Avoid 2,6 (opt 3)	x_l	1500	0	500	1476.75	1476.75	550	926.75	823.25
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	250	250	0	250	250
	$x_l^{5,6}$	0	0	0	676.75	676.75	0	676.75	573.25

Avoid 2,8 (opt 1)	x_l	1500	0	500	1700	1700	550	1150	600
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	0	0	0	0	500
	$x_l^{5,6}$	0	0	0	1150	1150	0	1150	100
Avoid 2,8 (opt 2)	x_l	1500	0	500	1700	1700	550	1150	600
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	500	500	0	500	0
	$x_l^{5,6}$	0	0	0	650	650	0	650	600
Avoid 2,8 (opt 3)	x_l	1500	0	500	1700	1700	550	1150	600
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	0	500	250	250	0	250	250
	$x_l^{5,6}$	0	0	0	900	900	0	900	350
Avoid 3,4; 3,5 (opt 1); 3,7(opt 1); 4,6; 4,8(opt 2)	x_l	1500	500	0	550	1050	550	500	1250
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 3,5 (opt 3); 3,7 (opt 3)	x_l	1500	250	250	550	800	550	250	1500
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	250	250	0	250	0	250	250
	$x_l^{5,6}$	0	0	0	0	0	0	0	1250
Avoid 3,6	x_l	1500	500	0	1314.81	1814.81	550	1264.81	485.19
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	764.81	764.81	0	764.81	485.19
Avoid 3,8; 4,8 (opt 1); 5,8(opt 3); 6,8; 7,8(opt 1)	x_l	1500	500	0	1700	2200	550	1650	100
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	1150	1150	0	1150	100
Avoid 4,8 (opt 3)	x_l	1500	500	0	1175	1675	550	1125	625
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	625	625	0	625	625
Avoid 5,8 (opt 1)	x_l	1500	500	0	1314.82	1814.82	550	1264.82	485.18
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	500	0	0	500	0	500	0
	$x_l^{5,6}$	0	0	0	764.82	764.82	0	764.82	485.18

Avoid 7,8 (opt 3)	x_l	1500	250	250	1700	1700	550	1150	600
	$x_l^{1,2}$	1500	0	0	0	0	0	0	0
	$x_l^{1,6}$	0	250	250	250	250	0	250	250
	$x_l^{5,6}$	0	0	0	900	900	0	900	600

* Links 4, 5, and 6 always carry all of the OD (5,2) flow which is 550 vph in this case.

Appendix C

Household Generation Code

C.1 MODULE COMMON_VAR

```
parameter(ihhs=20000)
parameter(imaxkid=3)
parameter(imaxveh=35000)

integer iagek(ihhs,imaxkid)
integer ihomez(ihhs)
integer ibusz(ihhs,2)
integer ieschlz(ihhs)
integer imschlz(ihhs)
integer ihschlz(ihhs)
integer itype(ihhs)
integer inumkids(ihhs)
integer ivehnum(ihhs,2)
integer ictnofam
integer ictfam
integer ictmcf
integer ictsmf
integer ictkids
integer ictveh
integer iupn(imaxveh)
integer idwn(imaxveh)
integer iuserc(imaxveh)
integer ivehtype(imaxveh)
integer ivehocc(imaxveh)
integer inumndp(imaxveh) !number of nodes in path
integer inumdest(imaxveh)
integer iinfo(imaxveh)
integer ivtohh(imaxveh) !based on veh number tells hh number

real nofam
real fam
real mcf
real smf
```

```
real kids
real startt(imaxveh)
real sband(imaxveh)
real response(imaxveh) ! percent response for BR users, else 0
real acttime(6) ! activity duration

end
```

C.2 PROGRAM FAMILY_GEN

```
c       *********** Ft. Worth Census 2000 data ******************
c       Total number of households = 195,058 for entire city
c       Since we are only modeling a portion of the city, generate 20,000 hhs
c       Non families = 34.6%  (type 1)
c       Families = 65.4%
c        34.7% (of the total hhs) have children
c        45.8% (of the total hhs) are married couple families
c        14.7% (of the total hhs) are single mother families
c       **********************************************************
        use portlib
        use common_var
c       ****************** Family types ******************
c        1. Single individual
c        2. Single parent with one elementary school child
c        3. Single parent with one middle school child
c        4. Single parent with one high school child
c        5. Single parent with two elementary school children
c        6. Single parent with two middle school children
c        7. Single parent with two high school children
c        8. Single parent with one elementary school child and one middle school
c              child
c        9. Single parent with one elementary school child and one high school
c              child
c        10. Single parent with one middle school aged child and one high school
c              aged child
c        11. Couple (no children)
c        12. Two parents with one elementary school aged child
c        13. Two parents with one middle school aged child
c        14. Two parents with one high school aged child
c        15. Two parents with two elementary school aged children
c        16. Two parents with two middle school aged children
c        17. Two parents with two high school aged children
c        18. Two parents with one elementary school aged child and one middle
c              school aged child
c        19. Two parents with one middle school aged child and one high school
c              aged child
c        20. Two parents with one elementary school child and one high school
c              child
c        21. Two parents with three elementary school children
```

```
c          22. Two parents with two elementary school children and one middle
                school child
c          23. Two parents with one elementary school child and two middle school
                children
c          24. Two parents with one elementary school, one middle school, and one
                high school child
c          25. Two parents with three middle school children
c          26. Two parents with two middle school children and one high school
                child
c          27. Two parents with one middle school and two high school children
c          28. Two parents with three high school children
c          29. Two parents with two high school children and one elementary school
                child
c          30. Two parents with one high school child and two elementary school
                children
c************************************************************

         nofam=0.346*ihhs
         fam=0.654*ihhs
         mcf=0.458*ihhs
         smf=0.147*ihhs
         kids=0.347*ihhs

         ictveh=0

         do ii=1, ihhs
           r1=random(0)
           if(r1.lt.0.50.and.ictnofam.lt.int(nofam))then
                itype(ii)=1
                ictveh=ictveh+1
                ivehnum(ii,1)=ictveh
                ivehnum(ii,2)=0
                ivtohh(ictveh)=ii
                ictnofam=ictnofam+1
                inumkids(ii)=0
                ieschlz(ii)=0
                imschlz(ii)=0
                ihschlz(ii)=0
                r2=random(0)
                if(r2.lt.0.17)ihomez(ii)=1
                if(r2.ge.0.17.and.r2.lt.0.34)ihomez(ii)=2
                if(r2.ge.0.34.and.r2.lt.0.51)ihomez(ii)=3
```

```
            if(r2.ge.0.51.and.r2.lt.0.68)ihomez(ii)=4
            if(r2.ge.0.68.and.r2.lt.0.85)ihomez(ii)=5
            if(r2.ge.0.85)ihomez(ii)=6
            r3=random(0)
            if(r3.le.0.50)then
                  ibusz(ii,1)=7
                  call zone7(ictveh)
            endif
            if(r3.gt.0.50.and.r3.le.0.51)then
                  ibusz(ii,1)=8
                  call zone8(ictveh)
            endif
            if(r3.gt.0.51)then
                  ibusz(ii,1)=9
                  call zone9(ictveh)
            endif
            ibusz(ii,2)=0
      else !family
            ictfam=ictfam+1
            r4=random(0)
            if(r4.lt.0.30.and.ictsmf.lt.int(smf))then
             ictsmf=ictsmf+1
             ictveh=ictveh+1
             ivehnum(ii,1)=ictveh
             ivehnum(ii,2)=0
             ivtohh(ictveh)=ii
             ictkids=ictkids+1
             r2=random(0)
             if(r2.lt.0.17)ihomez(ii)=1
             if(r2.ge.0.17.and.r2.lt.0.34)ihomez(ii)=2
             if(r2.ge.0.34.and.r2.lt.0.51)ihomez(ii)=3
             if(r2.ge.0.51.and.r2.lt.0.68)ihomez(ii)=4
             if(r2.ge.0.68.and.r2.lt.0.85)ihomez(ii)=5
             if(r2.ge.0.85)ihomez(ii)=6
             r3=random(0)
             if(r3.le.0.49)then
                  ibusz(ii,1)=7
                  call zone7(ictveh)
             endif
             if(r3.gt.0.49.and.r3.le.0.51)then
                  ibusz(ii,1)=8
                  call zone8(ictveh)
```

```
endif
if(r3.gt.0.51)then
        ibusz(ii,1)=9
        call zone9(ictveh)
endif
ibusz(ii,2)=0
r5=random(0) ! number of kids
        if(r5.lt.0.6)then          ! 1 kid
          inumkids(ii)=1
          r6=random(0) ! age of kid
          if(r6.le.0.33) then
            iagek(ii,inumkids(ii))=1
                itype(ii)=2
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                     eq.5)then
                        ieschlz(ii)=11
                else
                        ieschlz(ii)=12
                endif
                imschlz(ii)=0
                ihschlz(ii)=0
          endif
          if(r6.gt.0.33.and.r6.le.0.66) then
            iagek(ii,inumkids(ii))=2
                itype(ii)=3
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                     eq.3)then
                        imschlz(ii)=10
                else
                        imschlz(ii)=13
                endif
                ieschlz(ii)=0
                ihschlz(ii)=0
          endif
          if(r6.gt.0.66) then
                iagek(ii,inumkids(ii))=3
                itype(ii)=4
                ieschlz(ii)=0
                imschlz(ii)=0
                ihschlz(ii)=14
          endif
        else     ! 2 kids
```

```
do k=1,2
 r6=random(0) ! age of kid
 if(r6.le.0.33) iagek(ii,k)=1
 if(r6.gt.0.33.and.r6.le.0.66) iagek(ii,k)=2
 if(r6.gt.0.66) iagek(ii,k)=3
enddo
if(iagek(ii,1).eq.1.and.iagek(ii,2).eq.1)then
        itype(ii)=5
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
            eq.5)then
                ieschlz(ii)=11
        else
                ieschlz(ii)=12
        endif
        imschlz(ii)=0
        ihschlz(ii)=0
 endif
if(iagek(ii,1).eq.2.and.iagek(ii,2).eq.2)then
        itype(ii)=6
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
            eq.3)then
                imschlz(ii)=10
        else
                imschlz(ii)=13
        endif
        ieschlz(ii)=0
        ihschlz(ii)=0
 endif
if(iagek(ii,1).eq.3.and.iagek(ii,2).eq.3)then
        itype(ii)=7
        ieschlz(ii)=0
        imschlz(ii)=0
        ihschlz(ii)=14
endif
if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.2).or.
    (iagek(ii,2).eq.1.and.iagek(ii,1).eq.2))then
    itype(ii)=8
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
            eq.5)then
                ieschlz(ii)=11
        else
                ieschlz(ii)=12
```

```
                    endif
                    if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                          eq.3)then
                                imschlz(ii)=10
                    else
                                imschlz(ii)=13
                    endif
                    ihschlz(ii)=0
          endif
         if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.3).or.
              (iagek(ii,2).eq.1.and.iagek(ii,1).eq.3))then
              itype(ii)=9
                    if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                          eq.5)then
                                ieschlz(ii)=11
                    else
                                ieschlz(ii)=12
                    endif
                    imschlz(ii)=0
                    ihschlz(ii)=14
         endif
         if((iagek(ii,1).eq.2.and.iagek(ii,2).eq.3).or.
              (iagek(ii,2).eq.2.and.iagek(ii,1).eq.3))then
              itype(ii)=10
                    if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                          eq.3)then
                                imschlz(ii)=10
                    else
                                imschlz(ii)=13
                    endif
                    ieschlz(ii)=0
                    ihschlz(ii)=14
         endif
        endif

else !married couple fam
  ictmcf=ictmcf+1
  r2=random(0)
  if(r2.lt.0.17)ihomez(ii)=1
  if(r2.ge.0.17.and.r2.lt.0.34)ihomez(ii)=2
  if(r2.ge.0.34.and.r2.lt.0.51)ihomez(ii)=3
  if(r2.ge.0.51.and.r2.lt.0.68)ihomez(ii)=4
```

```
if(r2.ge.0.68.and.r2.lt.0.85)ihomez(ii)=5
if(r2.ge.0.85)ihomez(ii)=6
ictveh=ictveh+1
ivehnum(ii,1)=ictveh
ivtohh(ictveh)=ii
r3=random(0)
if(r3.le.0.49)then
        ibusz(ii,1)=7
        call zone7(ictveh)
endif
if(r3.gt.0.49.and.r3.le.0.50)then
        ibusz(ii,1)=8
        call zone8(ictveh)
endif
if(r3.gt.0.50)then
        ibusz(ii,1)=9
        call zone9(ictveh)
endif
ictveh=ictveh+1
ivehnum(ii,2)=ictveh
ivtohh(ictveh)=ii
r8=random(0)
if(r8.le.0.48)then
        ibusz(ii,2)=7
        call zone7(ictveh)
endif
if(r8.gt.0.48.and.r8.le.0.49)then
        ibusz(ii,2)=8
        call zone8(ictveh)
endif
if(r8.gt.0.49.and.r8.le.0.99)then
        ibusz(ii,2)=9
        call zone9(ictveh)
endif
if(r8.gt.0.99)then
        ibusz(ii,2)=ihomez(ii)
        if(ihomez(ii).eq.1)call zone1(ictveh)
        if(ihomez(ii).eq.2)call zone2(ictveh)
        if(ihomez(ii).eq.3)call zone3(ictveh)
        if(ihomez(ii).eq.4)call zone4(ictveh)
        if(ihomez(ii).eq.5)call zone5(ictveh)
        if(ihomez(ii).eq.6)call zone6(ictveh)
```

```
endif
r7=random(0)
if(r7.le.0.30) then
        inumkids(ii)=0
        itype(ii)=11
endif
if(r7.gt.0.30.and.r7.le.0.60)then
        inumkids(ii)=1
        r6=random(0) ! age of kid
        if(r6.le.0.33) then
          iagek(ii,inumkids(ii))=1
          itype(ii)=12
                 if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                      eq.5)then
                           ieschlz(ii)=11
                 else
                           ieschlz(ii)=12
                 endif
                 imschlz(ii)=0
                 ihschlz(ii)=0
        endif
        if(r6.gt.0.33.and.r6.le.0.66) then
          iagek(ii,inumkids(ii))=2
          itype(ii)=13
                 if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                      eq.3)then
                           imschlz(ii)=10
                 else
                           imschlz(ii)=13
                 endif
                 ieschlz(ii)=0
                 ihschlz(ii)=0
        endif
        if(r6.gt.0.66) then
          iagek(ii,inumkids(ii))=3
          itype(ii)=14
                 ieschlz(ii)=0
                 imschlz(ii)=0
                 ihschlz(ii)=14
        endif
endif
if(r7.gt.0.60.and.r7.le.0.88)then
```

```
        inumkids(ii)=2
do k=1,2
          r6=random(0) ! age of kid
          if(r6.le.0.33) iagek(ii,k)=1
          if(r6.gt.0.33.and.r6.le.0.66) iagek(ii,k)=2
          if(r6.gt.0.66) iagek(ii,k)=3
enddo
        if(iagek(ii,1).eq.1.and.iagek(ii,2).eq.1)then
                itype(ii)=15
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                      eq.5)then
                         ieschlz(ii)=11
                else
                         ieschlz(ii)=12
                endif
                imschlz(ii)=0
                ihschlz(ii)=0
        endif
        if(iagek(ii,1).eq.2.and.iagek(ii,2).eq.2)then
                itype(ii)=16
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                      eq.3)then
                         imschlz(ii)=10
                else
                         imschlz(ii)=13
                endif
                ieschlz(ii)=0
                ihschlz(ii)=0
        endif
        if(iagek(ii,1).eq.3.and.iagek(ii,2).eq.3)then
                itype(ii)=17
                ieschlz(ii)=0
                imschlz(ii)=0
                ihschlz(ii)=14
        endif
        if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.2).or.
              (iagek(ii,1).eq.2.and.iagek(ii,2).eq.1))then
              itype(ii)=18
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                      eq.5)then
                         ieschlz(ii)=11
                else
```

```
                    ieschlz(ii)=12
                endif
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                    eq.3)then
                    imschlz(ii)=10
                else
                    imschlz(ii)=13
                endif
                ihschlz(ii)=0
        endif
        if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.3).or.
            (iagek(ii,1).eq.3.and.iagek(ii,2).eq.1))then
            itype(ii)=20
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                    eq.5)then
                    ieschlz(ii)=11
                else
                    ieschlz(ii)=12
                endif
                imschlz(ii)=0
                ihschlz(ii)=14
        endif
        if((iagek(ii,1).eq.2.and.iagek(ii,2).eq.3).or.
            (iagek(ii,1).eq.3.and.iagek(ii,2).eq.2))then
            itype(ii)=19
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                    eq.3)then
                    imschlz(ii)=10
                else
                    imschlz(ii)=13
                endif
                ihschlz(ii)=14
                ieschlz(ii)=0
        endif
endif
if(r7.gt.0.88)then
        inumkids(ii)=3
  do k=1,3
          r6=random(0) ! age of kid
          if(r6.le.0.33) iagek(ii,k)=1
          if(r6.gt.0.33.and.r6.le.0.66) iagek(ii,k)=2
          if(r6.gt.0.66) iagek(ii,k)=3
```

```
enddo
      if(iagek(ii,1).eq.1.and.iagek(ii,2).eq.1.and.
         iagek(ii,3).eq.1)then
         itype(ii)=21
            if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
               eq.5)then
                  ieschlz(ii)=11
            else
                  ieschlz(ii)=12
            endif
            imschlz(ii)=0
            ihschlz(ii)=0
      endif
      if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.1.and.
         iagek(ii,3).eq.2).or.(iagek(ii,1).eq.1.and.iagek(ii,2)
         .eq.2.and.iagek(ii,3).eq.1).or.(iagek(ii,1).eq.2.and.
         iagek(ii,2).eq.1.and.iagek(ii,3).eq.1))then
            itype(ii)=22
            if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
               eq.5)then
                  ieschlz(ii)=11
            else
                  ieschlz(ii)=12
            endif
            if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
               eq.3)then
                  imschlz(ii)=10
            else
                  imschlz(ii)=13
            endif
            ihschlz(ii)=0
      endif
      if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.2.and.
         iagek(ii,3).eq.2).or.(iagek(ii,1).eq.2.and.iagek(ii,2)
         .eq.2.and.iagek(ii,3).eq.1).or.(iagek(ii,1).eq.2.and.
         iagek(ii,2).eq.1.and.iagek(ii,3).eq.2))then
            itype(ii)=23
            if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
               eq.5)then
                  ieschlz(ii)=11
            else
                  ieschlz(ii)=12
```

```
        endif
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
           eq.3)then
              imschlz(ii)=10
        else
              imschlz(ii)=13
        endif
        ihschlz(ii)=0
endif
if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.2.and.
    iagek(ii,3).eq.3).or.(iagek(ii,1).eq.1.and.iagek(ii,2)
    .eq.3.and.iagek(ii,3).eq.2).or.(iagek(ii,1).eq.2.and.
    iagek(ii,2).eq.1.and.iagek(ii,3).eq.3).or.(iagek(ii,1)
    .eq.2.and.iagek(ii,2).eq.3.and.iagek(ii,3).eq.1).or.
    (iagek(ii,1).eq.3.and.iagek(ii,2).eq.2.and.iagek(ii,3)
    .eq.1).or.(iagek(ii,1).eq.3.and.iagek(ii,2).eq.1.and.
    iagek(ii,3).eq.2))then
        itype(ii)=24
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
              eq.5)then
              ieschlz(ii)=11
        else
              ieschlz(ii)=12
        endif
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
              eq.3)then
              imschlz(ii)=10
        else
              imschlz(ii)=13
        endif
        ihschlz(ii)=14
endif
if(iagek(ii,1).eq.2.and.iagek(ii,2).eq.2.and.
    iagek(ii,3).eq.2)then
        itype(ii)=25
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
              eq.3)then
              imschlz(ii)=10
        else
              imschlz(ii)=13
        endif
        ieschlz(ii)=0
```

```
        ihschlz(ii)=0
endif
if((iagek(ii,1).eq.2.and.iagek(ii,2).eq.2.and.
    iagek(ii,3).eq.3).or.(iagek(ii,1).eq.2.and.iagek(ii,2)
    .eq.3.and.iagek(ii,3).eq.2).or.(iagek(ii,1).eq.3.and.
    iagek(ii,2).eq.2.and.iagek(ii,3).eq.2))then
        itype(ii)=26
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
                eq.3)then
                imschlz(ii)=10
        else
                imschlz(ii)=13
        endif
        ieschlz(ii)=0
        ihschlz(ii)=14
endif
if((iagek(ii,1).eq.3.and.iagek(ii,2).eq.3.and.
    iagek(ii,3).eq.2).or.(iagek(ii,1).eq.3.and.iagek(ii,2)
    .eq.2.and.iagek(ii,3).eq.3).or.(iagek(ii,1).eq.2.and.
    iagek(ii,2).eq.3.and.iagek(ii,3).eq.3))then
        itype(ii)=27
        if(ihomez(ii).eq.1.or.ihomez(ii).eq.2.or.ihomez(ii).
            eq.3)then
                imschlz(ii)=10
        else
                imschlz(ii)=13
        endif
        ieschlz(ii)=0
        ihschlz(ii)=14
endif
if(iagek(ii,1).eq.3.and.iagek(ii,2).eq.3.and.
    iagek(ii,3).eq.3)then
        itype(ii)=28
        ieschlz(ii)=0
        imschlz(ii)=0
        ihschlz(ii)=14
endif
if((iagek(ii,1).eq.3.and.iagek(ii,2).eq.3.and.
iagek(ii,3).eq.1).or.(iagek(ii,1).eq.3.and.iagek(ii,2)
    .eq.1.and.iagek(ii,3).eq.3).or.(iagek(ii,1).eq.1.and.
    iagek(ii,2).eq.3.and.iagek(ii,3).eq.3))then
        itype(ii)=29
```

```
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                   eq.5)then
                      ieschlz(ii)=11
                else
                      ieschlz(ii)=12
                endif
                imschlz(ii)=0
                ihschlz(ii)=14
          endif
          if((iagek(ii,1).eq.1.and.iagek(ii,2).eq.1.and.
             iagek(ii,3).eq.3).or.(iagek(ii,1).eq.1.and.iagek(ii,2)
             .eq.3.and.iagek(ii,3).eq.1).or.(iagek(ii,1).eq.3.and.
             iagek(ii,2).eq.1.and.iagek(ii,3).eq.1))then
                itype(ii)=30
                if(ihomez(ii).eq.1.or.ihomez(ii).eq.3.or.ihomez(ii).
                   eq.5)then
                      ieschlz(ii)=11
                else
                      ieschlz(ii)=12
                endif
                imschlz(ii)=0
                ihschlz(ii)=14
          endif

      endif
     endif
  endif
enddo

call output_gen

stop
end
```

C.3 subroutine output_gen

```
use common_var

            ! this subroutine writes the output files
            crt_time=0.00
            do i=1, ictveh
              iuserc(i)=2
              ivehtype(i)=1
              ivehocc(i)=1
              inumndp(i)=1
              inumdest(i)=1
              iinfo(i)=0
              sband(i)=0.0
              response(i)=0.0
              if((mod(i,100)).eq.0)then
                        crt_time=crt_time+0.01
              endif
              startt(i)=crt_time
c             startt(i)=0.0
            enddo

            ! normal rush hour file
            open(unit=1, file='rushhour.dat', status='unknown')
            write(1,*) ictveh, 1, ' household data'
            write(1,*) '      #',' upstrm','  dnsm ',' start  ',' class',
                 ' type ',' occp ','#node ','#dest ',' info ',' band ',
                 '  respns'
            do i=1, ictveh
c            if(i.gt.60281)then
              write(1,100) i,iupn(i),idwn(i),startt(i),iuserc(i),ivehtype(i),
                  ivehocc(i),inumndp(i),inumdest(i),iinfo(i),sband(i),
                  response(i)
              do j=1,inumdest(i)
                        write(1,101) ihomez(ivtohh(i)),0
              enddo
c            endif
            enddo
            close(1)

            open(unit=2, file='members.dat', status='unknown')
            write(2,*) '   ihh# ',' type ',' e zone ',' m zone ',' h zone ',
```

```
      ' home  ',' numv '
      do i=1,ihhs
        if(ivehnum(i,2).eq.0)inumhhv=1
        if(ivehnum(i,2).ne.0)inumhhv=2
        write(2,200) i,itype(i),ieschlz(i),imschlz(i),ihschlz(i),
           ihomez(i),inumhhv
                write(2,201) ivehnum(i,1), ibusz(i,1)
                if(ivehnum(i,2).ne.0)write(2,201) ivehnum(i,2), ibusz(i,2)
      enddo
      close(2)


100   format(3I7,F8.2,6I6,2F8.4)
101   format(I12,F7.2)
200   format(7I8)
201   format(2I10)
      return
      end
```

C.4 SUBROUTINE ZONE1(IVEH)

```
! this subroutine assigns up and downstream nodes of origin links in zone 1 to veh

        use portlib
        use common_var

        r101=random(0)
        r102=random(0)

        if(r101.lt.0.50)then
                if(r102.lt.0.10)then
                        iupn(iveh)=81
                        idwn(iveh)=82
                endif
                if(r102.ge.0.10.and.r102.lt.0.20)then
                        iupn(iveh)=82
                        idwn(iveh)=83
                endif
                if(r102.ge.0.20.and.r102.lt.0.30)then
                        iupn(iveh)=130
                        idwn(iveh)=131
                endif
                if(r102.ge.0.30.and.r102.lt.0.40)then
                        iupn(iveh)=131
                        idwn(iveh)=80
                endif
                if(r102.ge.0.40.and.r102.lt.0.50)then
                        iupn(iveh)=68
                        idwn(iveh)=69
                endif
                if(r102.ge.0.50.and.r102.lt.0.60)then
                        iupn(iveh)=69
                        idwn(iveh)=68
                endif
                if(r102.ge.0.60.and.r102.lt.0.70)then
                        iupn(iveh)=80
                        idwn(iveh)=131
                endif
                if(r102.ge.0.70.and.r102.lt.0.80)then
                        iupn(iveh)=131
```

```
            idwn(iveh)=130
        endif
        if(r102.ge.0.80.and.r102.lt.0.90)then
            iupn(iveh)=80
            idwn(iveh)=132
        endif
        if(r102.ge.0.90)then
            iupn(iveh)=82
            idwn(iveh)=81
        endif
    else
        if(r102.lt.0.10)then
            iupn(iveh)=81
            idwn(iveh)=130
        endif
        if(r102.ge.0.10.and.r102.lt.0.20)then
            iupn(iveh)=82
            idwn(iveh)=131
        endif
        if(r102.ge.0.20.and.r102.lt.0.30)then
            iupn(iveh)=130
            idwn(iveh)=81
        endif
        if(r102.ge.0.30.and.r102.lt.0.40)then
            iupn(iveh)=131
            idwn(iveh)=82
        endif
        if(r102.ge.0.40.and.r102.lt.0.50)then
            iupn(iveh)=68
            idwn(iveh)=131
        endif
        if(r102.ge.0.50.and.r102.lt.0.60)then
            iupn(iveh)=69
            idwn(iveh)=80
        endif
        if(r102.ge.0.60.and.r102.lt.0.70)then
            iupn(iveh)=80
            idwn(iveh)=83
        endif
        if(r102.ge.0.70.and.r102.lt.0.80)then
            iupn(iveh)=131
            idwn(iveh)=68
```

```
        endif
        if(r102.ge.0.80.and.r102.lt.0.90)then
                iupn(iveh)=80
                idwn(iveh)=69
        endif
        if(r102.ge.0.90)then
                iupn(iveh)=82
                idwn(iveh)=3
        endif
endif

return
end
```

C.5 SUBROUTINE ZONE2(IVEH)

```
use portlib
        use common_var

        r201=random(0)
        r202=random(0)

        if(r201.lt.0.50)then
                if(r202.lt.0.10)then
                        iupn(iveh)=139
                        idwn(iveh)=97
                endif
                if(r202.ge.0.10.and.r202.lt.0.20)then
                        iupn(iveh)=97
                        idwn(iveh)=144
                endif
                if(r202.ge.0.20.and.r202.lt.0.30)then
                        iupn(iveh)=98
                        idwn(iveh)=151
                endif
                if(r202.ge.0.30.and.r202.lt.0.40)then
                        iupn(iveh)=151
                        idwn(iveh)=96
                endif
                if(r202.ge.0.40.and.r202.lt.0.48)then
                        iupn(iveh)=102
                        idwn(iveh)=103
                endif
                if(r202.ge.0.48.and.r202.lt.0.58)then
                        iupn(iveh)=103
                        idwn(iveh)=104
                endif
                if(r202.ge.0.58.and.r202.lt.0.68)then
                        iupn(iveh)=97
                        idwn(iveh)=139
                endif
                if(r202.ge.0.68.and.r202.lt.0.78)then
                        iupn(iveh)=144
                        idwn(iveh)=97
                endif
```

```
        if(r202.ge.0.78.and.r202.lt.0.88)then
                iupn(iveh)=151
                idwn(iveh)=98
        endif
        if(r202.ge.0.88.and.r202.lt.0.95)then
                iupn(iveh)=96
                idwn(iveh)=151
        endif
        if(r202.ge.0.95)then
                iupn(iveh)=104
                idwn(iveh)=103
        endif
else
        if(r202.lt.0.10)then
                iupn(iveh)=139
                idwn(iveh)=98
        endif
        if(r202.ge.0.10.and.r202.lt.0.20)then
                iupn(iveh)=98
                idwn(iveh)=139
        endif
        if(r202.ge.0.20.and.r202.lt.0.30)then
                iupn(iveh)=97
                idwn(iveh)=151
        endif
        if(r202.ge.0.30.and.r202.lt.0.40)then
                iupn(iveh)=151
                idwn(iveh)=97
        endif
        if(r202.ge.0.40.and.r202.lt.0.50)then
                iupn(iveh)=103
                idwn(iveh)=151
        endif
        if(r202.ge.0.50.and.r202.lt.0.60)then
                iupn(iveh)=144
                idwn(iveh)=96
        endif
        if(r202.ge.0.60.and.r202.lt.0.70)then
                iupn(iveh)=96
                idwn(iveh)=144
        endif
        if(r202.ge.0.70.and.r202.lt.0.80)then
```

```
            iupn(iveh)=96
            idwn(iveh)=104
        endif
        if(r202.ge.0.80.and.r202.lt.0.90)then
            iupn(iveh)=104
            idwn(iveh)=96
        endif
        if(r202.ge.0.90)then
            iupn(iveh)=151
            idwn(iveh)=103
        endif

    endif

    return
    end
```

C.6 SUBROUTINE ZONE3(IVEH)

```
use portlib
      use common_var

      r301=random(0)
      r302=random(0)

      if(r301.lt.0.50)then
        if(r302.lt.0.10)then
             iupn(iveh)=84
             idwn(iveh)=132
        endif
        if(r302.ge.0.10.and.r302.lt.0.20)then
             iupn(iveh)=132
             idwn(iveh)=70
        endif
        if(r302.ge.0.20.and.r302.lt.0.30)then
             iupn(iveh)=84
             idwn(iveh)=85
        endif
        if(r302.ge.0.30.and.r302.lt.0.50)then
             iupn(iveh)=85
             idwn(iveh)=86
        endif
        if(r302.ge.0.50.and.r302.lt.0.60)then
             iupn(iveh)=73
             idwn(iveh)=74
        endif
        if(r302.ge.0.60.and.r302.lt.0.70)then
             iupn(iveh)=74
             idwn(iveh)=75
        endif
        if(r302.ge.0.70.and.r302.lt.0.85)then
             iupn(iveh)=73
             idwn(iveh)=40
        endif
        if(r302.ge.0.85)then
             iupn(iveh)=74
             idwn(iveh)=11
        endif
```

```
else
  if(r302.lt.0.10)then
          iupn(iveh)=74
          idwn(iveh)=73
  endif
  if(r302.ge.0.10.and.r302.lt.0.20)then
          iupn(iveh)=86
          idwn(iveh)=9
  endif
  if(r302.ge.0.20.and.r302.lt.0.30)then
          iupn(iveh)=86
          idwn(iveh)=85
  endif
  if(r302.ge.0.30.and.r302.lt.0.40)then
          iupn(iveh)=85
          idwn(iveh)=7
  endif
  if(r302.ge.0.40.and.r302.lt.0.50)then
          iupn(iveh)=84
          idwn(iveh)=65
  endif
  if(r302.ge.0.50.and.r302.lt.0.60)then
          iupn(iveh)=84
          idwn(iveh)=83
  endif
  if(r302.ge.0.60.and.r302.lt.0.70)then
          iupn(iveh)=132
          idwn(iveh)=84
  endif
  if(r302.ge.0.70.and.r302.lt.0.80)then
          iupn(iveh)=132
          idwn(iveh)=80
  endif
  if(r302.ge.0.80.and.r302.lt.0.90)then
          iupn(iveh)=70
          idwn(iveh)=132
  endif
  if(r302.ge.0.90)then
          iupn(iveh)=70
          idwn(iveh)=69
  endif
endif
```

```
return
end
```

C.7 SUBROUTINE ZONE4(IVEH)

```
use portlib
        use common_var

        r401=random(0)
        r402=random(0)

        if(r401.le.0.50)then
                if(r402.lt.0.10)then
                        iupn(iveh)=145
                        idwn(iveh)=146
                endif
                if(r402.ge.0.10.and.r402.lt.0.20)then
                        iupn(iveh)=99
                        idwn(iveh)=176
                endif
                if(r402.ge.0.20.and.r402.lt.0.30)then
                        iupn(iveh)=105
                        idwn(iveh)=106
                endif
                if(r402.ge.0.30.and.r402.lt.0.40)then
                        iupn(iveh)=141
                        idwn(iveh)=93
                endif
                if(r402.ge.0.40.and.r402.lt.0.50)then
                        iupn(iveh)=177
                        idwn(iveh)=147
                endif
                if(r402.ge.0.50.and.r402.lt.0.60)then
                        iupn(iveh)=153
                        idwn(iveh)=198
                endif
                if(r402.ge.0.60.and.r402.lt.0.70)then
                        iupn(iveh)=148
                        idwn(iveh)=177
                endif
                if(r402.ge.0.70.and.r402.lt.0.80)then
                        iupn(iveh)=154
                        idwn(iveh)=95
                endif
```

```
        if(r402.ge.0.80.and.r402.lt.0.90)then
                iupn(iveh)=110
                idwn(iveh)=109
        endif
        if(r402.ge.0.90)then
                iupn(iveh)=92
                idwn(iveh)=140
        endif
else
        if(r402.lt.0.10)then
                iupn(iveh)=145
                idwn(iveh)=176
        endif
        if(r402.ge.0.10.and.r402.lt.0.20)then
                iupn(iveh)=176
                idwn(iveh)=105
        endif
        if(r402.ge.0.20.and.r402.lt.0.30)then
                iupn(iveh)=106
                idwn(iveh)=99
        endif
        if(r402.ge.0.30.and.r402.lt.0.40)then
                iupn(iveh)=99
                idwn(iveh)=152
        endif
        if(r402.ge.0.40.and.r402.lt.0.50)then
                iupn(iveh)=152
                idwn(iveh)=146
        endif
        if(r402.ge.0.50.and.r402.lt.0.60)then
                iupn(iveh)=141
                idwn(iveh)=147
        endif
        if(r402.ge.0.60.and.r402.lt.0.70)then
                iupn(iveh)=153
                idwn(iveh)=155
        endif
        if(r402.ge.0.70.and.r402.lt.0.80)then
                iupn(iveh)=198
                idwn(iveh)=177
        endif
        if(r402.ge.0.80.and.r402.lt.0.90)then
```

```
            iupn(iveh)=154
            idwn(iveh)=109
      endif
      if(r402.ge.0.90)then
            iupn(iveh)=110
            idwn(iveh)=95
      endif
endif

return
end
```

C.8 SUBROUTINE ZONE5(IVEH)

```
use portlib
        use common_var

        r501=random(0)
        r502=random(0)

        if(r501.lt.0.50)then
                if(r502.lt.10)then
                        iupn(iveh)=133
                        idwn(iveh)=75
                endif
                if(r502.ge.0.10.and.502.lt.0.20)then
                        iupn(iveh)=75
                        idwn(iveh)=76
                endif
                if(r502.ge.0.20.and.r502.lt.0.25)then
                        iupn(iveh)=76
                        idwn(iveh)=77
                endif
                if(r502.ge.0.25.and.r502.lt.0.50)then
                        iupn(iveh)=77
                        idwn(iveh)=78
                endif
                if(r502.ge.0.50.and.r502.lt.0.62)then
                        iupn(iveh)=135
                        idwn(iveh)=87
                endif
                if(r502.ge.0.62.and.r502.lt.0.78)then
                        iupn(iveh)=137
                        idwn(iveh)=88
                endif
                if(r502.ge.0.78)then
                        iupn(iveh)=134
                        idwn(iveh)=77
                endif
        else
                if(r502.lt.0.10)then
                        iupn(iveh)=75
                        idwn(iveh)=74
```

```
          endif
          if(r502.ge.0.10.and.r502.lt.0.25)then
                    iupn(iveh)=76
                    idwn(iveh)=75
          endif
          if(r502.ge.0.25.and.r502.lt.0.35)then
                    iupn(iveh)=77
                    idwn(iveh)=76
          endif
          if(r502.ge.0.35.and.r502.lt.0.50)then
                    iupn(iveh)=76
                    idwn(iveh)=13
          endif
          if(r502.ge.0.50.and.r502.lt.0.62)then
                    iupn(iveh)=87
                    idwn(iveh)=88
          endif
          if(r502.ge.0.62.and.r502.lt.0.75)then
                    iupn(iveh)=79
                    idwn(iveh)=78
          endif
          if(r502.ge.0.75)then
                    iupn(iveh)=79
                    idwn(iveh)=17
          endif
endif

return
end
```

C.9 SUBROUTINE ZONE6(IVEH)

```
use portlib
      use common_var
      r601=random(0)
      if(r601.lt.0.30)then
            iupn(iveh)=111
            idwn(iveh)=110
      endif
      if(r601.ge.0.30.and.r601.lt.0.60)then
            iupn(iveh)=110
            idwn(iveh)=111
      endif
      if(r601.ge.0.60.and.r601.lt.0.70)then
            iupn(iveh)=113
            idwn(iveh)=112
      endif
      if(r601.ge.0.70.and.r601.lt.0.80)then
            iupn(iveh)=112
            idwn(iveh)=113
      endif
      if(r601.ge.0.80.and.r601.lt.0.85)then
            iupn(iveh)=115
            idwn(iveh)=173
      endif
      if(r601.ge.0.85.and.r601.lt.0.90)then
            iupn(iveh)=173
            idwn(iveh)=115
      endif
      if(r601.ge.0.90.and.r601.lt.0.95)then
            iupn(iveh)=111
            idwn(iveh)=112
      endif
      if(r601.ge.0.95)then
            iupn(iveh)=112
            idwn(iveh)=111
      endif

      return
      end
```

C.10 SUBROUTINE ZONE7(IVEH)

! assigns up and downstream nodes of origin links to veh

```
      use portlib
      use common_var

      r701=random(0)
      r702=random(0)
      r703=random(0)

      if(r701.lt.0.97)then !southbound
       if(r703.lt.0.97)then !frontage road
            if(r702.lt.0.96)then
                  iupn(iveh)=199
                  idwn(iveh)=116
            endif
            if(r702.ge.0.96.and.r702.lt.0.97)then
                  iupn(iveh)=116
                  idwn(iveh)=1
            endif
            if(r702.ge.0.97.and.r702.lt.0.98)then
                  iupn(iveh)=1
                  idwn(iveh)=20
            endif
            if(r702.ge.0.98.and.r702.lt.0.99)then
                  iupn(iveh)=3
                  idwn(iveh)=24
            endif
            if(r702.ge.0.99)then
                  iupn(iveh)=5
                  idwn(iveh)=27
            endif
       else
            if(r702.lt.0.15)then
                  iupn(iveh)=116
                  idwn(iveh)=81
            endif
            if(r702.ge.0.15.and.r702.lt.0.27)then
                  iupn(iveh)=1
                  idwn(iveh)=81
```

```
            endif
            if(r702.ge.0.27.and.r702.lt.0.40)then
                        iupn(iveh)=3
                        idwn(iveh)=82
            endif
            if(r702.ge.0.40.and.r702.lt.0.52)then
                        iupn(iveh)=5
                        idwn(iveh)=83
            endif
            if(r702.ge.0.52.and.r702.lt.0.65)then
                        iupn(iveh)=83
                        idwn(iveh)=80
            endif
            if(r702.ge.0.65.and.r702.lt.0.77)then
                        iupn(iveh)=83
                        idwn(iveh)=82
            endif
            if(r702.ge.0.77.and.r702.lt.0.90)then
                        iupn(iveh)=83
                        idwn(iveh)=84
            endif
            if(r702.ge.0.90)then
                        iupn(iveh)=83
                        idwn(iveh)=5
            endif
  endif
else !northbound
  if(r703.lt.0.20)then !frontage road
            if(r702.lt.0.10)then
                        iupn(iveh)=2
                        idwn(iveh)=116
            endif
            if(r702.ge.0.10.and.r702.lt.0.50)then
                        iupn(iveh)=4
                        idwn(iveh)=22
            endif
            if(r702.ge.0.50)then
                        iupn(iveh)=6
                        idwn(iveh)=26
            endif
  else
            if(r702.lt.10)then
```

```
        iupn(iveh)=2
        idwn(iveh)=139
endif
if(r702.ge.0.10.and.r702.lt.0.13)then
        iupn(iveh)=4
        idwn(iveh)=89
endif
if(r702.ge.0.13.and.r702.lt.0.25)then
        iupn(iveh)=89
        idwn(iveh)=97
endif
if(r702.ge.0.25.and.r702.lt.0.28)then
        iupn(iveh)=89
        idwn(iveh)=4
endif
if(r702.ge.0.28.and.r702.lt.0.40)then
        iupn(iveh)=89
        idwn(iveh)=90
endif
if(r702.ge.0.40.and.r702.lt.0.45)then
        iupn(iveh)=6
        idwn(iveh)=90
endif
if(r702.ge.0.45.and.r702.lt.0.50)then
        iupn(iveh)=90
        idwn(iveh)=6
endif
if(r702.ge.0.50.and.r702.lt.0.65)then
        iupn(iveh)=90
        idwn(iveh)=89
endif
if(r702.ge.0.65.and.r702.lt.0.72)then
        iupn(iveh)=90
        idwn(iveh)=91
endif
if(r702.ge.0.72.and.r702.lt.0.78)then
        iupn(iveh)=91
        idwn(iveh)=90
endif
if(r702.ge.0.78.and.r702.lt.0.89)then
        iupn(iveh)=91
        idwn(iveh)=140
```

```
          endif
          if(r702.ge.0.89)then
                    iupn(iveh)=91
                    idwn(iveh)=144
          endif
  endif
endif

return
end
```

C.11 SUBROUTINE ZONE8(IVEH)

```
use portlib
        use common_var

        r801=random(0)
        r802=random(0)
        r803=random(0)

        if(r801.lt.0.50)then ! southbound
          if(r802.lt.0.45)then !frontage road
                if(r803.lt.0.20)then
                        iupn(iveh)=65
                        idwn(iveh)=7
                endif
                if(r803.ge.0.20.and.r803.lt.0.40)then
                        iupn(iveh)=7
                        idwn(iveh)=31
                endif
                if(r803.ge.0.40.and.r803.lt.0.65)then
                        iupn(iveh)=9
                        idwn(iveh)=40
                endif
                if(r803.ge.0.65.and.r803.lt.0.93)then
                        iupn(iveh)=40
                        idwn(iveh)=11
                endif
                if(r803.ge.0.93)then
                        iupn(iveh)=11
                        idwn(iveh)=43
                endif
          else
                if(r803.lt.0.20)then
                        iupn(iveh)=65
                        idwn(iveh)=84
                endif
                if(r803.ge.0.20.and.r803.lt.0.40)then
                        iupn(iveh)=7
                        idwn(iveh)=85
                endif
                if(r803.ge.0.40.and.r803.lt.0.60)then
```

```
			iupn(iveh)=9
			idwn(iveh)=86
		endif
		if(r803.ge.0.60.and.r803.lt.0.80)then
			iupn(iveh)=40
			idwn(iveh)=73
		endif
		if(r803.ge.0.80)then
			iupn(iveh)=11
			idwn(iveh)=74
		endif
  endif
else !northbound
  if(r802.lt.0.45)then !frontage road
		if(r803.lt.0.25)then
			iupn(iveh)=12
			idwn(iveh)=42
		endif
		if(r803.ge.0.25.and.r803.lt.0.50)then
			iupn(iveh)=10
			idwn(iveh)=38
		endif
		if(r803.ge.0.50.and.r803.lt.0.75)then
			iupn(iveh)=8
			idwn(iveh)=66
		endif
		if(r803.ge.0.75)then
			iupn(iveh)=66
			idwn(iveh)=29
		endif
  else
		if(r803.lt.0.15)then
			iupn(iveh)=12
			idwn(iveh)=94
		endif
		if(r803.ge.0.15.and.r803.lt.30)then
			iupn(iveh)=94
			idwn(iveh)=12
		endif
		if(r803.ge.0.30.and.r803.lt.50)then
			iupn(iveh)=94
			idwn(iveh)=142
```

```
            endif
            if(r803.ge.0.50.and.r803.lt.0.70)then
                        iupn(iveh)=94
                        idwn(iveh)=149
            endif
            if(r803.ge.0.70.and.r803.lt.0.80)then
                        iupn(iveh)=10
                        idwn(iveh)=93
            endif
            if(r803.ge.0.80.and.r803.lt.0.90)then
                        iupn(iveh)=8
                        idwn(iveh)=92
            endif
            if(r803.ge.0.90)then
                        iupn(iveh)=66
                        idwn(iveh)=140
            endif
  endif
endif

return
end
```

C.12 SUBROUTINE ZONE9(IVEH)

```
use portlib
        use common_var

        r901=random(0)
        r902=random(0)
        r903=random(0)

        if(r901.lt.0.03)then !southbound
          if(r902.lt.0.50)then !frontage road
                if(r903.lt.0.50)then
                        iupn(iveh)=13
                        idwn(iveh)=50
                endif
                if(r903.ge.0.50.and.r903.lt.0.75)then
                        iupn(iveh)=15
                        idwn(iveh)=56
                endif
                if(r903.ge.0.75)then
                        iupn(iveh)=17
                        idwn(iveh)=61
                endif
          else
                if(r903.lt.0.20)then
                        iupn(iveh)=13
                        idwn(iveh)=76
                endif
                if(r903.ge.0.20.and.r903.lt.0.40)then
                        iupn(iveh)=15
                        idwn(iveh)=88
                endif
                if(r903.ge.0.40.and.r903.lt.0.60)then
                        iupn(iveh)=17
                        idwn(iveh)=79
                endif
                if(r903.ge.0.60.and.r903.lt.0.80)then
                        iupn(iveh)=88
                        idwn(iveh)=15
                endif
                if(r903.ge.0.80)then
```

```
                  iupn(iveh)=88
                  idwn(iveh)=87
            endif
       endif
     else !northbound
       if(r902.lt.0.98)then !frontage road
            if(r903.lt.0.96)then
                  iupn(iveh)=200
                  idwn(iveh)=117
            endif
            if(r903.ge.0.96.and.r903.lt.0.97)then
                  iupn(iveh)=117
                  idwn(iveh)=63
            endif
            if(r903.ge.0.97.and.r903.lt.0.98)then
                  iupn(iveh)=18
                  idwn(iveh)=59
            endif
            if(r903.ge.0.98.and.r903.lt.0.99)then
                  iupn(iveh)=16
                  idwn(iveh)=54
            endif
            if(r903.ge.0.99)then
                  iupn(iveh)=14
                  idwn(iveh)=47
            endif
       else
            if(r903.lt.0.30)then
                  iupn(iveh)=117
                  idwn(iveh)=79
            endif
            if(r903.ge.0.30.and.r903.lt.0.60)then
                  iupn(iveh)=117
                  idwn(iveh)=115
            endif
            if(r903.ge.0.60.and.r903.lt.0.65)then
                  iupn(iveh)=114
                  idwn(iveh)=16
            endif
            if(r903.ge.0.65.and.r903.lt.0.80)then
                  iupn(iveh)=114
                  idwn(iveh)=113
```

```
        endif
        if(r903.ge.0.80.and.r903.lt.0.95)then
              iupn(iveh)=114
              idwn(iveh)=173
        endif
        if(r903.ge.0.95)then
              iupn(iveh)=16
              idwn(iveh)=114
        endif

  endif
endif

return
end
```

Appendix D

Link Characteristics For Figure 5.1

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
1	1	2	420	40	1800
2	1	20	1190	40	1800
3	1	81	1640	40	1800
4	2	1	420	40	1800
5	2	116	1410	40	1800
6	2	139	1200	40	1800
7	3	4	450	40	1800
8	3	24	1350	40	1800
9	3	82	1280	40	1800
10	4	3	450	40	1800
11	4	22	200	40	1800
12	4	89	390	40	1800
13	5	6	320	40	1800
14	5	27	700	40	1800
15	5	83	1450	40	1800
16	6	5	320	40	1800
17	6	26	400	40	1800
18	6	90	380	40	1800
19	7	8	320	40	1800
20	7	31	400	40	1800
21	7	85	1610	40	1800
22	8	7	320	40	1800
23	8	66	1500	40	1800
24	8	92	580	40	1800
25	9	10	320	40	1800
26	9	40	1730	40	1800
27	9	86	1830	40	1800
28	10	9	320	40	1800
29	10	38	670	40	1800
30	10	93	550	40	1800
31	11	12	320	40	1800
32	11	43	200	40	1800
33	11	74	3270	40	1800
34	12	11	320	40	1800
35	12	42	950	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
36	12	94	480	40	1800
37	13	14	320	40	1800
38	13	50	2720	40	1800
39	13	76	3790	40	1800
40	14	13	320	40	1800
41	14	47	2400	40	1800
42	14	150	100	30	1800
43	15	16	580	40	1800
44	15	56	1100	40	1800
45	15	88	1350	40	1800
46	16	15	580	40	1800
47	16	54	480	40	1800
48	16	114	250	40	1800
49	17	18	320	40	1800
50	17	61	650	40	1800
51	17	79	3370	40	1800
52	18	17	320	40	1800
53	18	59	1790	40	1800
54	18	173	330	40	1800
55	19	20	600	40	1800
56	19	23	1000	65	2200
57	20	3	400	40	1800
58	21	116	2200	65	2200
59	22	2	1390	40	1800
60	22	21	600	40	1800
61	23	24	1300	40	1800
62	23	28	3820	65	2200
63	24	5	450	40	1800
64	25	21	1200	65	2200
65	26	4	1400	40	1800
66	26	25	1180	40	1800
67	27	28	1370	40	1800
68	27	65	250	40	1800
69	28	32	2180	65	2200
70	29	6	150	40	1800
71	30	25	3520	65	2200
72	30	29	1220	40	1800
73	31	32	1400	40	1800
74	31	36	3740	40	1800
75	32	35	1160	65	2200

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
76	33	8	700	40	1800
77	34	30	2180	65	2200
78	34	33	1200	40	1800
79	35	36	1180	40	1800
80	35	39	1890	65	2200
81	36	9	630	40	1800
82	37	34	950	65	2200
83	38	33	3400	40	1800
84	38	37	1250	40	1800
85	39	40	1650	40	1800
86	39	44	3690	65	2200
87	40	11	850	40	1800
88	40	73	3270	40	1800
89	41	37	2450	65	2200
90	42	10	1630	40	1800
91	42	41	1100	40	1800
92	43	44	990	40	1800
93	43	46	2770	40	1800
94	44	45	510	65	2200
95	45	46	1270	40	1800
96	45	49	2630	65	2200
97	46	13	280	40	1800
98	47	12	850	40	1800
99	48	41	4520	65	2200
100	48	47	1620	40	1800
101	49	50	1640	40	1800
102	49	55	2600	65	2200
103	50	15	1880	40	1800
104	51	14	450	40	1800
105	52	48	2730	65	2200
106	52	51	1500	40	1800
107	53	52	850	65	2200
108	54	51	3670	40	1800
109	54	53	1320	40	1800
110	55	56	2020	40	1800
111	55	58	3500	65	2200
112	56	57	200	40	1800
113	57	17	1350	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
114	57	58	1280	40	1800
115	58	62	1670	65	2200
116	59	16	860	40	1800
117	60	53	4000	65	2200
118	60	59	1340	40	1800
119	61	62	950	40	1800
120	61	117	3050	40	1800
121	62	117	2500	65	2200
122	63	18	650	40	1800
123	64	60	1650	65	2200
124	64	63	550	40	1800
125	65	7	1500	40	1800
126	65	84	1410	40	1800
127	66	29	800	40	1800
128	66	140	580	40	1800
129	67	68	1860	40	1800
130	67	118	100	30	1800
131	67	130	500	40	1800
132	68	67	1860	40	1800
133	68	69	1930	40	1800
134	68	119	100	30	1800
135	68	131	500	40	1800
136	69	68	1930	40	1800
137	69	70	900	40	1800
138	69	80	580	40	1800
139	69	120	100	30	1800
140	70	69	900	40	1800
141	70	71	1730	40	1800
142	70	121	100	30	1800
143	70	132	500	40	1800
144	71	70	1730	40	1800
145	71	72	4240	40	1800
146	71	85	1610	40	1800
147	71	122	528	30	1800
148	72	71	4240	40	1800
149	72	73	1730	40	1800
150	72	86	1540	40	1800
151	72	123	528	30	1800
152	73	40	3270	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
153	73	72	1730	40	1800
154	73	74	1350	40	1800
155	73	124	100	30	1800
156	74	11	3270	40	1800
157	74	73	1350	40	1800
158	74	75	1280	40	1800
159	74	125	100	30	1800
160	75	74	1280	40	1800
161	75	76	1350	40	1800
162	75	126	100	30	1800
163	75	133	100	30	1800
164	76	13	3790	40	1800
165	76	75	1350	40	1800
166	76	77	580	40	1800
167	76	175	100	30	1800
168	77	76	580	40	1800
169	77	78	4410	40	1800
170	77	127	100	30	1800
171	77	134	100	30	1800
172	78	77	4410	40	1800
173	78	79	2510	40	1800
174	78	87	960	40	1800
175	78	128	100	30	1800
176	79	17	3370	40	1800
177	79	78	2510	40	1800
178	79	117	8040	40	1800
179	79	129	100	30	1800
180	80	69	580	40	1800
181	80	83	1190	40	1800
182	80	131	1930	40	1800
183	80	132	900	40	1800
184	81	1	1640	40	1800
185	81	82	1990	40	1800
186	81	116	2890	40	1800
187	81	130	1320	40	1800
188	82	3	1280	40	1800
189	82	81	1990	40	1800
190	82	83	1800	40	1800
191	82	131	1500	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
192	83	5	1450	40	1800
193	83	80	1190	40	1800
194	83	82	1800	40	1800
195	83	84	900	40	1800
196	84	65	1410	40	1800
197	84	83	900	40	1800
198	84	85	1730	40	1800
199	84	132	1730	40	1800
200	85	7	1610	40	1800
201	85	71	1610	40	1800
202	85	84	1730	40	1800
203	85	86	4240	40	1800
204	86	9	1830	40	1800
205	86	72	1540	40	1800
206	86	85	4240	40	1800
207	87	78	960	40	1800
208	87	88	800	40	1800
209	87	135	100	30	1800
210	87	136	100	30	1800
211	88	15	1350	40	1800
212	88	87	800	40	1800
213	88	137	100	30	1800
214	88	138	100	30	1800
215	89	4	390	40	1800
216	89	90	1700	40	1800
217	89	97	960	40	1800
218	90	6	380	40	1800
219	90	89	1700	40	1800
220	90	91	320	40	1800
221	91	90	320	40	1800
222	91	140	800	40	1800
223	91	144	800	40	1800
224	92	8	580	40	1800
225	92	140	1600	40	1800
226	92	141	900	40	1800
227	92	146	900	40	1800
228	93	10	550	40	1800
229	93	141	3830	40	1800
230	93	142	960	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
231	93	177	800	40	1800
232	94	12	480	40	1800
233	94	142	1650	40	1800
234	94	143	100	30	1800
235	94	149	800	40	1800
236	95	110	2510	40	1800
237	95	149	1000	40	1800
238	95	154	1650	40	1800
239	96	104	2580	40	1800
240	96	144	1300	40	1800
241	96	151	2100	40	1800
242	96	176	1300	40	1800
243	97	89	960	40	1800
244	97	139	2280	40	1800
245	97	144	2100	40	1800
246	97	151	1100	40	1800
247	98	103	3400	40	1800
248	98	116	2310	40	1800
249	98	139	1200	40	1800
250	98	151	1450	40	1800
251	99	106	1280	40	1800
252	99	152	1570	40	1800
253	99	155	1320	40	1800
254	99	176	1040	40	1800
255	100	101	1280	40	1800
256	100	116	5000	40	1800
257	100	156	100	30	1800
258	100	181	16000	40	1800
259	101	100	1280	40	1800
260	101	102	510	40	1800
261	101	157	100	30	1800
262	101	158	100	30	1800
263	102	101	510	40	1800
264	102	103	1410	40	1800
265	102	159	100	30	1800
266	102	160	100	30	1800
267	103	98	3400	40	1800
268	103	102	1410	40	1800
269	103	104	2090	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
270	103	151	2300	40	1800
271	103	161	100	30	1800
272	104	96	2580	40	1800
273	104	103	2090	40	1800
274	104	105	650	40	1800
275	104	162	100	30	1800
276	105	104	650	40	1800
277	105	106	1060	40	1800
278	105	176	1650	40	1800
279	106	99	1280	40	1800
280	106	105	1060	40	1800
281	106	107	1230	40	1800
282	106	163	100	30	1800
283	107	106	1230	40	1800
284	107	108	3210	40	1800
285	107	155	1000	40	1800
286	107	164	100	30	1800
287	108	107	3210	40	1800
288	108	109	1610	40	1800
289	108	165	528	30	1800
290	108	166	100	30	1800
291	109	108	1610	40	1800
292	109	110	1730	40	1800
293	109	154	2640	40	1800
294	109	167	100	30	1800
295	110	95	2510	40	1800
296	110	109	1730	40	1800
297	110	111	7330	40	1800
298	110	168	100	30	1800
299	111	110	7330	40	1800
300	111	112	1930	40	1800
301	111	169	3800	40	1800
302	112	111	1930	40	1800
303	112	113	5210	40	1800
304	112	169	3800	40	1800
305	112	170	528	30	1800
306	113	112	5210	40	1800
307	113	114	1990	40	1800
308	113	172	100	30	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
309	114	16	250	40	1800
310	114	113	1990	40	1800
311	114	171	100	30	1800
312	114	173	2800	40	1800
313	115	117	2730	40	1800
314	115	173	3530	40	1800
315	115	174	100	30	1800
316	116	1	1410	40	1800
317	116	19	2000	65	2200
318	116	81	2890	40	1800
319	116	98	2310	40	1800
320	116	100	8000	40	1800
321	116	179	8000	65	2200
322	117	63	3050	40	1800
323	117	64	2500	65	2200
324	117	79	8040	40	1800
325	117	115	2730	40	1800
326	117	180	5000	65	2200
327	118	67	100	30	1800
328	119	68	100	30	1800
329	120	69	100	30	1800
330	120	183	16000	30	1800
331	121	70	100	30	1800
332	122	71	528	30	1800
333	123	72	528	30	1800
334	124	73	100	30	1800
335	125	74	100	30	1800
336	126	75	100	30	1800
337	127	77	100	30	1800
338	128	78	100	30	1800
339	128	183	16000	30	1800
340	129	79	100	30	1800
341	130	67	500	40	1800
342	130	81	1320	40	1800
343	130	131	1860	40	1800
344	131	68	500	40	1800
345	131	80	1930	40	1800
346	131	82	1500	40	1800
347	131	130	1860	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
348	132	70	500	40	1800
349	132	80	900	40	1800
350	132	84	1730	40	1800
351	133	75	100	30	1800
352	134	77	100	30	1800
353	135	87	100	30	1800
354	136	87	100	30	1800
355	137	88	100	30	1800
356	138	88	100	30	1800
357	139	2	1200	40	1800
358	139	97	2280	40	1800
359	139	98	1200	40	1800
360	140	66	580	40	1800
361	140	91	800	40	1800
362	140	92	1600	40	1800
363	140	145	800	40	1800
364	141	92	900	40	1800
365	141	93	3830	40	1800
366	141	147	800	40	1800
367	142	93	960	40	1800
368	142	94	1650	40	1800
369	142	148	800	40	1800
370	143	94	100	30	1800
371	144	91	800	40	1800
372	144	96	1300	40	1800
373	144	97	2100	40	1800
374	144	145	920	40	1800
375	145	140	800	40	1800
376	145	144	920	40	1800
377	145	146	1580	40	1800
378	145	176	2300	40	1800
379	146	92	900	40	1800
380	146	145	1580	40	1800
381	146	147	900	40	1800
382	146	152	1000	40	1800
383	147	141	800	40	1800
384	147	146	900	40	1800
385	147	153	1000	40	1800
386	147	177	3830	40	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
387	148	142	800	40	1800
388	148	149	1650	40	1800
389	148	154	1000	40	1800
390	148	177	960	40	1800
391	149	94	800	40	1800
392	149	95	1000	40	1800
393	149	148	1650	40	1800
394	150	14	1600	40	1800
395	151	96	2100	40	1800
396	151	97	1100	40	1800
397	151	98	1450	40	1800
398	151	103	2300	40	1800
399	152	99	1570	40	1800
400	152	146	1000	40	1800
401	152	153	1000	40	1800
402	153	147	1000	40	1800
403	153	152	1000	40	1800
404	153	155	1500	40	1800
405	153	178	3830	40	1800
406	154	95	1650	40	1800
407	154	109	2640	40	1800
408	154	148	1000	40	1800
409	154	178	960	40	1800
410	155	99	1320	40	1800
411	155	107	1000	40	1800
412	155	153	1500	40	1800
413	156	100	100	30	1800
414	157	101	100	30	1800
415	158	101	100	30	1800
416	159	102	100	30	1800
417	160	102	100	30	1800
418	161	103	100	30	1800
419	162	104	100	30	1800
420	163	106	100	30	1800
421	164	107	100	30	1800
422	165	108	528	30	1800
423	166	108	100	30	1800
424	167	109	100	30	1800
425	168	110	100	30	1800

Link	Upstream Node	Downstream Node	Length (ft)	Speed (mph)	Max Service Rate (vphpl)
426	168	184	16000	30	1800
427	169	111	3800	40	1800
428	169	112	3800	40	1800
429	169	184	16000	40	1800
430	170	112	528	30	1800
431	171	114	100	30	1800
432	172	113	100	30	1800
433	173	18	330	40	1800
434	173	114	2805	40	1800
435	173	115	3530	40	1800
436	174	115	100	30	1800
437	175	76	100	30	1800
438	176	96	1300	40	1800
439	176	99	1040	40	1800
440	176	105	1650	40	1800
441	176	145	2300	40	1800
442	177	93	800	40	1800
443	177	147	3830	40	1800
444	177	148	960	40	1800
445	177	178	960	40	1800
446	178	153	3830	40	1800
447	178	154	960	40	1800
448	178	177	960	40	1800
449	179	116	8000	65	2200
450	179	181	8000	65	2200
451	180	117	8000	65	2200
452	180	182	11000	65	2200

Appendix E

Payoff Matrix for Baseline Conditions

	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
Link	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
1	100.00	0.000	98.68	1.163	99.15	0.876	99.53	0.946
2	99.82	0.072	99.1	0.402	98.88	0.262	98.97	0.333
3	95.42	2.506	100	0	100	0	100	0
4	99.15	0.379	99.27	0.518	99.94	0.074	100	0
5	100.00	0.000	99.07	0.663	98.9	0.949	99.41	1.052
6	100.00	0.000	99.41	0.645	99.28	0.736	100	0
7	99.51	0.539	97.24	0.879	96.13	1.089	94.41	0.963
8	98.92	0.603	98.21	0.566	96	1.355	98.23	0.54
9	79.17	3.359	100	0	100	0	100	0
10	78.44	2.625	99.89	0.027	99.96	0.008	99.96	0.009
11	99.33	0.307	93.29	2.534	93.26	2.042	89.68	2.725
12	99.51	0.539	100	0	97.03	1.233	100	0
13	91.02	3.844	96.23	2.901	96.95	2.28	97.84	1.233
14	93.86	2.423	97.23	1.321	94.19	2.805	94.87	2.78
15	92.86	2.050	93.21	2	95.88	1.191	97.72	1.65
16	91.51	2.327	98.74	1.677	98.91	1.197	98.24	1.887
17	86.42	1.734	96.53	2.211	92.78	2.65	91.26	2.645
18	86.32	5.099	97.83	1.775	97.14	2.082	99.82	0.265
19	100.00	0.000	96.89	2.571	97.26	1.508	94.08	2.463
20	91.71	3.593	98.64	1.8	98.32	2.065	97.18	2.301
21	93.40	3.041	99.27	3	98.99	3	99.21	3
22	93.62	3.079	98.95	3.4	98.84	3.167	99.21	3
23	96.89	1.434	99.57	0.123	99.45	0.3	94.72	1.892
24	97.40	1.585	97.43	2.247	97.46	1.252	95.34	2.596
25	99.57	1.000	92.82	2.758	90.57	2.222	91.79	2.851
26	95.73	2.268	100	0	99.64	0.083	100	0
27	94.97	3.045	93.98	2	91.91	2	95.2	2
28	91.86	3.854	95.35	1.764	93.45	1.802	96.69	1.683
29	98.17	0.948	94.37	0.97	98.8	1.104	94.84	0.831
30	98.18	2.261	97.36	1.495	97.01	0.753	94.31	1.852
31	94.64	2.871	95.43	1.488	94.16	2.28	94.2	2
32	92.40	3.629	97.59	0.67	98.15	1.456	96.91	2.065
33	99.52	0.147	98.61	2	98.68	2.285	99.57	1.268
34	99.52	0.147	98.61	2	96.94	3.525	98.38	1.666
35	92.56	4.518	92.81	3.052	98.48	2.125	94.69	2.319
36	91.34	4.817	95.53	3	92.78	4.98	94.2	2
37	97.63	1.022	99.68	0.286	99.68	0.131	99.72	0.167

	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
Link	**Player M**	**Player T**	**Player M**	**Player T**	**Player M**	**Player T**	**Player M**	**Player T**
38	87.91	6.156	99.3	1.178	99.22	1.001	98.89	1.289
39	85.98	4.459	100	0	98.95	0.875	99.11	0.974
40	82.36	6.700	99.3	1.178	98.4	1.72	98.46	1.986
41	98.13	1.841	100	0	99.81	0.819	99.26	0.184
42	96.68	1.048	100	0	100	0	100	0
43	75.25	11.692	89.52	7.688	88.46	8.696	89.75	7.683
44	84.71	2.595	95.04	1.29	95.78	2.985	95.55	1.348
45	86.96	5.246	98.28	2	99.53	1.268	98.23	2.108
46	90.98	0.345	94.81	3.118	96.8	3.8	93.44	3.987
47	87.55	5.346	97.64	4.411	95	6.617	97.62	3.884
48	70.23	11.363	90.05	7	88.48	8.638	89.34	9.051
49	88.19	0.860	98.53	2.079	96.44	2.444	95.63	3.315
50	95.80	0.902	95.99	1.114	96.27	2.013	99.43	0.634
51	98.16	0.466	100	0	100	0	100	0
52	99.37	0.096	99.04	1.347	97.2	2.401	99.33	1.262
53	97.71	0.862	98.96	1.88	98.74	2.543	99.52	2.216
54	83.77	3.185	99.51	0.213	96.75	1.352	95.86	2.139
55	99.51	0.539	100	0	100	0	100	0
56	97.28	0.994	80.96	3.679	81.67	4.142	85.24	4.386
57	99.34	0.612	98.26	0.558	96.19	0.847	98.27	0.53
58	99.49	0.025	79.84	5.091	89.66	3.937	78.42	4.654
59	99.33	0.307	100	0	100	0	99.88	0.017
60	100.00	0.000	93.29	3.325	93.26	2.042	89.8	2.618
61	100.00	0.000	93.37	1	96.03	1	97.86	1
62	97.28	0.994	87.58	2.679	85.64	3.142	87.38	3.386
63	98.92	0.339	91.39	1.604	91.66	2.414	96.02	1.566
64	99.49	0.025	86.55	3.07	96.4	1.884	88.62	2.036
65	86.53	1.734	97.85	1.581	95.12	1.898	95.52	1.452
66	99.49	0.025	98.64	0.638	97.61	0.761	95.7	1.203
67	94.86	1.229	97.23	1.321	94.82	1.996	96.06	1.78
68	99.00	0.203	99.68	0.4	98.77	1.475	98.12	1.833
69	92.14	2.221	84.82	4	80.46	5.138	83.44	5.167
70	75.39	5.879	99.54	0.445	96.23	2.354	91.96	3.204
71	100.00	0.000	87.91	2.433	98.78	1.229	92.92	0.834
72	95.30	0.707	99.54	0.445	97.56	1.079	97.44	1.133
73	92.87	1.666	98.48	2	98.32	2.065	97.18	2.301
74	98.84	0.584	100	0	100	0	100	0
75	85.01	3.888	83.3	6	78.79	7.203	80.62	7.467
76	88.43	7.397	98.95	3.4	98.84	3.167	99.02	3.522

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
77	95.30	0.707	87.44	2.878	96.35	2.202	90.36	1.966
78	90.26	6.433	99.11	3.2	98.99	3	99.02	3.522
79	100.00	0.000	94.2	1	92.22	1	95.3	1
80	85.01	3.888	89.1	5	86.57	6.203	85.32	6.467
81	98.84	0.584	94.2	1	92.22	1	95.3	1
82	85.56	6.989	86.55	6.078	95.33	5.202	89.37	5.489
83	98.17	0.948	99.84	0.2	99.85	0.167	100	0
84	100.00	0.000	94.21	1.771	98.8	1.104	94.84	0.831
85	98.29	0.325	97.4	1	94.77	2	94.2	2
86	86.72	3.707	91.7	4	91.8	4.203	91.12	4.467
87	94.84	2.123	97.4	1	93.49	2.297	92.48	3.198
88	99.01	0.050	100	0	100	0	100	0
89	85.56	6.989	92.33	5.306	96.53	4.162	94.54	4.658
90	93.52	4.703	99.58	1.052	98.83	2	99.16	1.184
91	98.89	0.449	93.23	2	99.64	0.189	95.52	1.135
92	99.15	0.770	97.59	0.658	98	1.518	96.91	2.054
93	92.58	2.844	100	0	100	0	100	0
94	85.86	4.477	89.29	4.658	89.8	5.722	88.03	6.521
95	97.02	0.588	100	0	100	0	99.96	0.026
96	88.84	3.889	89.29	4.658	89.8	5.722	88.07	6.495
97	89.60	3.432	100	0	100	0	99.96	0.026
98	89.26	6.952	97.92	3	97.86	3.943	99.12	1.219
99	86.67	6.690	99.11	3.306	96.89	3.973	99.02	3.522
100	92.50	4.749	97.92	3	98.34	3	99.86	1.035
101	91.14	3.340	91.7	4	91.65	4.11	92.74	4
102	97.70	0.540	97.59	0.658	98.15	1.611	95.33	2.495
103	73.62	9.767	90.51	5.869	89.97	5.821	90.92	6.043
104	82.01	6.823	99.3	1.178	98.22	2.539	97.72	2.17
105	79.17	11.005	97.02	6.306	95.23	6.973	98.87	4.557
106	90.99	4.289	99.3	1.178	98.22	2.539	98.68	1.624
107	70.16	15.215	96.32	7.485	93.45	9.513	97.56	6.181
108	91.02	2.635	100	0	100	0	99.04	0.547
109	96.28	3.705	96.48	6.635	93.6	8.782	97.74	5.098
110	97.92	0.511	99.56	0.199	98.64	0.513	96.66	1.669
111	99.79	0.038	98.03	0.459	99.52	1.16	98.67	0.827
112	78.06	5.392	93.81	2.689	93.67	4.268	91.97	4.205
113	87.08	0.942	95.98	2.181	96.54	1.686	96.42	2.852
114	90.97	3.070	97.84	0.508	97.13	2.589	95.55	1.338

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
115	90.77	3.108	95.86	0.953	96.65	3.666	94.22	2.161
116	80.36	3.743	98.44	2.417	97.81	3.232	98.47	2.954
117	73.88	11.488	99.84	0.85	99.85	0.731	99.81	1.083
118	91.67	1.933	99.95	0.108	99.82	0.379	99.93	0.154
119	95.86	0.785	95.89	1.56	96.04	2.607	99.31	0.874
120	99.38	0.157	99.95	0.012	99.95	0.017	100	0
121	86.63	3.891	91.75	2.483	92.68	5.217	93.53	3.023
122	93.74	2.731	100	0	100	0	100	0
123	65.55	13.712	99.79	0.958	99.67	1.11	99.75	1.237
124	98.58	1.038	100	0	100	0	100	0
125	94.48	1.149	98.43	1.902	98.49	1.598	98.61	1.649
126	99.81	0.032	100	0	99.37	0.809	98.82	1
127	80.09	4.363	100	0	98.67	1.275	94.52	2.071
128	98.96	1.052	99.57	0.123	100	0	100	0
129	98.21	0.781	100	0	100	0	100	0
130	100.00	0.000	100	0	100	0	100	0
131	99.82	0.024	99.52	1	99.83	0.114	99.94	0.04
132	99.82	0.024	99.52	1	99.83	0.114	99.94	0.04
133	95.43	1.252	100	0	100	0	100	0
134	98.96	0.459	100	0	100	0	100	0
135	97.97	0.814	100	0	96.74	0.896	100	0
136	100.00	0.000	99.52	1	96.56	1.01	99.94	0.04
137	93.97	3.483	93.79	0.937	96.26	0.943	98.01	0.929
138	98.92	0.147	97.3	1.91	97.78	1.648	95.38	1.487
139	98.07	1.040	100	0	99.81	0.198	100	0
140	97.07	0.405	93.91	3.91	94.24	3.244	95.31	1.777
141	94.37	2.800	92.65	2.538	94.42	3.891	95.77	3.947
142	99.88	0.020	97.25	1.135	98.26	1.27	94.4	1.554
143	94.75	1.066	95.32	2.7	98.24	1.708	96.76	1.681
144	92.34	1.213	92.94	5.598	92.15	5.838	92.34	3.204
145	98.97	0.211	98.39	2.131	99.56	1.087	98.06	1.193
146	94.48	1.991	98	1.645	97.61	2.599	92.62	3.957
147	92.95	4.000	91.43	6	93.61	7	95.29	7
148	91.32	2.123	95.17	2.836	95.87	3.633	98.58	2.069
149	100.00	0.000	98.7	1.556	97.07	2.25	96.31	3.375
150	99.42	1.027	93.93	2.806	94.9	1.204	94.05	2.302
151	94.16	3.000	93.43	4	91.6	3	93.85	4
152	99.82	0.040	100	0	99.08	0.213	98.28	1.198

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
153	90.50	3.507	99.74	1.067	99.85	1	99.59	1.553
154	100.00	0.000	98.92	1.199	98.65	1.037	98.77	1.138
155	99.82	0.040	99.63	0.43	100	0	99.7	0.229
156	94.32	4.112	95.62	1.659	99.12	0.438	98.63	0.856
157	90.50	3.507	99.65	1.132	99.85	1	99.85	1.35
158	99.51	0.112	99.35	1	97.78	2.16	98.97	1
159	99.51	0.084	97.13	1.21	98.5	1.431	99.54	0.607
160	83.25	8.129	98.03	0.308	98.51	0.906	97.99	1.523
161	98.09	1.131	99.06	0.152	98.81	0.287	99.6	0.201
162	100.00	0.000	100	0	100	0	99.06	0.422
163	99.43	0.350	99.78	0.333	99.32	0.775	99.78	0.169
164	97.88	1.384	100	0	99.77	0.148	99.58	0.25
165	83.25	8.129	100	0	98.95	0.875	98.97	1.082
166	99.49	0.372	99.06	0.152	98.9	0.286	99.32	0.367
167	99.31	0.365	100	0	99.96	0.011	100	0
168	97.43	2.990	100	0	100	0	99.86	0.108
169	99.34	0.468	99.06	0.152	98.9	0.286	99.32	0.367
170	100.00	0.000	100	0	100	0	100	0
171	100.00	0.000	100	0	100	0	100	0
172	97.43	2.990	100	0	100	0	99.86	0.108
173	99.51	0.411	100	0	100	0	100	0
174	97.37	1.919	98.37	0.791	98.6	0.953	99.32	0.367
175	99.88	0.093	100	0	100	0	100	0
176	97.08	0.898	99.24	0.267	99.35	0.501	99.16	0.5
177	96.55	4.156	99.31	0.639	99.7	0.667	100	0
178	94.70	3.195	99.95	0.009	99.96	0.011	99.96	0.02
179	97.87	0.717	100	0	99.95	0.001	100	0
180	97.63	1.512	93.65	0.967	96.26	0.943	98.01	0.929
181	93.57	1.018	96.6	2.726	97.55	1.429	93.13	1.93
182	100.00	0.000	97.41	1.637	99.29	0.261	100	0
183	99.65	0.048	99.43	1.063	99.62	0.248	99.71	0.721
184	100.00	0.000	98.39	1.433	99.18	0.835	99.49	0.969
185	99.15	0.424	99.55	0.416	99.05	0.737	98.85	0.671
186	100.00	0.000	97.91	1.247	97.93	1.447	99.88	0.05
187	96.09	2.200	100	0	100	0	100	0
188	99.82	1.000	97.24	0.879	95.98	1.589	94.41	0.963
189	100.00	0.000	98.95	0.287	99.23	0.477	99.65	0.206
190	96.09	2.200	99.16	0.916	98.9	1.237	98.85	0.671

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
191	96.09	1.162	100	0	100	0	100	0
192	87.31	4.484	96.22	3.477	96.45	2.665	96.17	2.21
193	92.34	2.308	93.21	2	95.71	1.721	97.72	1.65
194	99.76	0.033	98.22	0.529	98.21	1.337	94.06	1.168
195	94.30	0.995	100	0	100	0	100	0
196	95.30	0.979	98.75	1.502	99.09	0.931	99.31	0.816
197	91.95	2.419	98.67	0.365	98.04	1.866	98.25	0.777
198	98.98	0.300	100	0	100	0	99.13	0.287
199	96.71	0.752	97.28	1.11	99.37	0.809	98.62	1.492
200	97.01	1.097	97.57	1.768	97.49	1.489	92.79	2.866
201	96.20	2.256	99.27	3	98.86	3.25	98.58	4.297
202	91.18	2.704	97	1.188	98.59	0.978	98.47	0.993
203	98.76	0.425	100	0	100	0	99.71	0.084
204	99.57	1.000	97.36	1.495	96.66	0.837	95.05	1.668
205	95.44	2.789	93.98	2	91.91	2	95.2	2
206	97.01	1.096	96.57	1.312	98.24	0.367	98.72	0.718
207	98.75	0.576	100	0	100	0	99.86	0.108
208	97.37	1.919	98.18	0.971	98.07	1.292	99.32	0.367
209	100.00	0.000	100	0	100	0	100	0
210	100.00	0.000	100	0	100	0	100	0
211	82.33	8.256	97.42	2.071	96.9	2.442	98.89	1.517
212	98.75	0.576	100	0	100	0	98.97	1.218
213	94.99	1.682	98.48	1.056	99.61	0.343	99.45	0.419
214	99.79	0.075	99.92	0.158	99.62	0.84	99.44	0.654
215	91.24	1.333	98.16	0.872	99.15	0.275	99.7	0.319
216	99.16	0.611	100	0	100	0	100	0
217	100.00	0.000	100	0	97.75	0.994	100	0
218	97.69	1.241	97.86	1.743	95.65	1.296	99.52	0.36
219	93.95	0.720	100	0	100	0	99.82	0.126
220	85.84	4.951	98.23	1.487	96.95	2.138	99.82	0.265
221	91.99	1.907	98.84	1.352	95.8	1.265	99.64	0.416
222	94.85	3.085	99.65	0.345	100	0	100	0
223	94.72	1.625	98.18	1.615	96.99	2.111	99.82	0.265
224	99.27	0.151	99.95	0.1	99.41	0.364	96.04	1.75
225	94.97	1.755	99.6	0.474	99.01	0.207	99.29	1.257
226	96.98	4.000	97.58	1.878	99.04	0.681	97.2	0.89
227	99.72	0.107	99.9	0.269	100	0	98.9	0.366
228	95.13	1.453	94.57	1.52	99.66	0.144	94.84	0.831

	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
Link	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
229	95.53	1.560	99.6	0.474	100	0	100	0
230	100.00	0.000	99.66	0.275	99.05	0.39	99.3	0.517
231	98.84	2.000	98.15	0.887	97.46	0.726	97.24	1.025
232	99.52	0.089	93.61	3.052	97.61	3.285	95.14	2.368
233	98.26	2.989	95.55	1.509	99.42	0.54	97.62	0.256
234	100.00	0.000	94.04	2	97.16	2.018	93.39	2.338
235	93.56	4.488	95.94	2.491	93.11	4.48	97.08	1.023
236	99.10	3.119	94.66	2.808	93.85	1.732	94.48	2.215
237	94.74	2.014	97.15	2.877	98.68	2.285	99.42	1.896
238	85.54	3.814	99.03	0.967	99.55	0.667	100	0
239	100.00	0.000	100	0	100	0	100	0
240	98.33	1.137	99.69	1.114	99.61	0.266	99.88	0.029
241	98.81	0.038	99.86	0.103	99.34	0.298	99.73	0.066
242	99.84	0.006	93.99	2.962	92.27	2.029	98.24	1.25
243	96.94	0.685	98.16	0.872	99.87	0.036	99.88	0.193
244	100.00	0.000	100	0	100	0	100	0
245	100.00	0.000	100	0	99.85	0.021	100	0
246	100.00	0.000	99.61	1	100	0	100	0
247	100.00	0.000	100	0	99.85	0.333	99.86	0.25
248	99.33	0.137	98.27	1.671	95.98	1.692	97.07	1.388
249	100.00	0.000	100	0	99.74	0.073	100	0
250	99.20	0.076	95.35	1.346	93.27	0.955	98.38	1
251	100.00	0.000	99.03	0.865	99.77	0.04	99.92	0.05
252	100.00	0.000	100	0	100	0	100	0
253	100.00	0.000	94.77	2.135	92.21	2.655	98.04	1.7
254	96.39	1.253	99.41	1.257	96.12	2.11	99.61	0.095
255	99.60	0.405	100	0	100	0	100	0
256	97.17	1.863	100	0	100	0	100	0
257	98.36	0.390	100	0	100	0	100	0
258	100.00	0.000	100	0	100	0	100	0
259	98.63	1.152	100	0	100	0	100	0
260	99.60	0.405	100	0	100	0	100	0
261	100.00	0.000	100	0	100	0	100	0
262	100.00	0.000	100	0	100	0	100	0
263	99.20	1.036	100	0	100	0	100	0
264	99.60	0.405	100	0	100	0	100	0
265	100.00	0.000	100	0	100	0	100	0
266	100.00	0.000	100	0	100	0	100	0

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
267	100.00	0.000	99.04	0.724	97.27	0.856	97.58	0.797
268	99.82	0.036	100	0	100	0	100	0
269	100.00	0.000	100	0	99.85	0.333	99.86	0.25
270	98.76	0.576	100	0	100	0	100	0
271	99.82	0.036	100	0	97.63	0.737	99.8	0.323
272	100.00	0.000	100	0	100	0	100	0
273	100.00	0.000	99.04	0.724	97.27	0.856	97.58	0.797
274	100.00	0.000	100	0	99.85	0.333	99.86	0.25
275	100.00	0.000	97.2	1.174	99.29	0.248	97.24	0.957
276	100.00	0.000	95.75	2.724	96.91	0.991	94.53	1.951
277	100.00	0.000	100	0	99.85	0.333	99.86	0.25
278	100.00	0.000	100	0	99.76	0.114	100	0
279	100.00	0.000	99.77	1.052	99.7	0.145	100	0
280	100.00	0.000	95.75	2.724	96.67	1.104	94.53	1.951
281	100.00	0.000	99.03	0.865	99.62	0.373	99.78	0.3
282	100.00	0.000	99.9	0.174	100	0	100	0
283	100.00	0.000	95.52	3.775	96.84	1.022	94.53	1.951
284	97.73	3.000	93.51	3.731	91.67	3.133	93.39	3.917
285	95.89	1.420	98.87	1.036	95.82	2.295	95.18	2.24
286	100.00	0.000	99.92	0.053	97.63	0.824	99.42	0.384
287	95.89	1.420	94.39	4.811	92.51	3.421	89.05	4.925
288	100.00	0.000	95.13	3.117	96.41	2.8	95.56	4.351
289	89.15	5.000	91.42	7	90.91	5	90.4	4
290	93.11	5.000	95.84	3.748	95.84	3.523	92.96	2.953
291	89.23	5.493	97.31	4.269	98.43	3.027	95.61	2.294
292	100.00	0.000	96.24	2.957	95.01	3.036	95.36	3.708
293	90.60	1.468	100	0	98.57	0.773	99.38	0.536
294	98.04	3.000	96.77	1.687	97.73	0.66	97.06	1.317
295	81.72	5.533	97.15	2.877	98.68	2.285	98.72	2.283
296	87.78	3.969	98.76	2.753	98.88	2.181	95.6	2.297
297	99.34	3.000	92.06	4.801	89.27	4.624	91.26	5.361
298	85.59	3.327	98.61	2.301	97.43	1.581	98.06	2.284
299	68.14	8.738	97.04	5.694	97.57	4.493	94.76	6.177
300	99.66	2.000	91.76	4.269	92.96	4.517	91.6	6.262
301	86.28	2.912	94.22	2.833	88.87	4.329	95.45	2.754
302	69.84	8.617	93.29	5.641	94.13	5.486	92.49	6.497
303	99.93	0.028	93.61	3.844	89.05	8.655	92.36	5.117
304	85.66	5.853	97.55	1.721	97.76	2.603	94.81	4.545

Link	No Evacuation Player M	Player T	$\lambda = 0$ Player M	Player T	$\lambda = 0.5$ Player M	Player T	$\lambda = 1$ Player M	Player T
305	88.70	9.000	81.89	9.491	77.17	12.8	78.02	13.36
306	51.37	16.681	89.54	6.504	83.07	11.77	84.8	10.96
307	95.54	1.412	93.7	3.511	91.96	6.211	89.9	5.09
308	98.57	0.155	98.74	1.035	97.64	1.078	97.01	1.305
309	93.14	1.687	93.55	5.737	93.9	7.469	91.77	6.626
310	52.05	16.374	89.72	6.199	85.98	9.376	85.15	10.639
311	99.22	0.157	98.88	0.739	97.46	2.469	97.72	1.41
312	100.00	0.000	98.94	0.287	97.43	1.587	97.35	0.311
313	92.08	5.994	99.53	0.589	99.95	0.5	97.64	1.232
314	97.92	1.366	100	0	99.09	2	99.52	2
315	95.29	1.869	100	0	100	0	100	0
316	99.38	1.000	100	0	100	0	100	0
317	96.97	1.461	80.96	3.679	81.67	4.142	85.24	4.386
318	99.88	0.004	100	0	100	0	100	0
319	99.45	0.026	95.22	1.637	93.06	1.377	98.2	1.429
320	100.00	0.000	100	0	100	0	100	0
321	100.00	0.000	74.95	8.184	82.08	8.29	74.53	7.717
322	96.69	1.463	100	0	100	0	100	0
323	66.30	15.388	99.79	0.958	99.67	1.11	99.75	1.237
324	99.07	0.322	100	0	100	0	100	0
325	98.66	0.361	100	0	100	0	99.81	1
326	100.00	0.000	91.29	3.205	92.74	5.704	91.3	3.555
327	100.00	0.000	100	0	100	0	100	0
328	98.96	0.459	100	0	100	0	100	0
329	97.90	1.064	100	0	99.81	0.198	100	0
330	100.00	0.000	100	0	100	0	100	0
331	99.88	0.020	97.25	1.135	98.26	1.27	94.5	1.554
332	100.00	0.000	93.58	7	93.66	6	85.19	6
333	100.00	0.000	89.23	6	88.12	6	89.94	7
334	99.82	0.040	99.77	0.14	100	0	99.7	0.229
335	99.51	0.084	97.13	1.21	98.5	1.431	99.54	0.607
336	100.00	0.000	100	0	99.48	0.033	99.06	0.422
337	100.00	0.000	100	0	100	0	100	0
338	99.88	0.093	100	0	100	0	100	0
339	100.00	0.000	100	0	100	0	100	0
340	97.87	0.717	100	0	99.95	0.001	100	0
341	98.56	0.733	100	0	100	0	100	0
342	100.00	0.000	98.96	1	98.7	0.513	99.94	0.04

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
343	97.36	1.490	100	0	100	0	100	0
344	95.61	1.228	100	0	100	0	100	0
345	99.47	0.072	100	0	100	0	100	0
346	96.09	2.199	97.97	0.637	97.15	0.758	100	0
347	100.00	0.000	99.44	1	98.87	0.399	100	0
348	99.08	0.162	95.56	3.395	98.66	1.948	98.27	2.382
349	97.98	0.497	97.38	0.831	99.03	0.497	98.21	0.638
350	94.05	1.208	98.61	0.749	99.55	0.32	98.81	0.381
351	99.43	0.350	100	0	99.43	0.033	99.74	0.189
352	100.00	0.000	100	0	100	0	100	0
353	100.00	0.000	100	0	100	0	100	0
354	100.00	0.000	100	0	100	0	100	0
355	94.99	1.682	99.67	0.056	99.25	0.282	99.58	0.25
356	99.79	0.075	99.34	0.108	99.58	0.031	99.67	0.149
357	100.00	0.000	99.8	0.145	99.02	0.809	100	0
358	100.00	0.000	100	0	100	0	100	0
359	100.00	0.000	99.5	0.553	99.59	0.406	99.8	0.347
360	82.40	2.846	100	0	99.37	0.809	99.94	0.013
361	95.47	1.881	99.02	0.637	97.59	0.605	99.88	0.193
362	97.97	2.000	100	0	100	0	100	0
363	98.17	1.443	99.81	0.038	100	0	99.29	1.257
364	94.95	1.832	99.6	0.474	100	0	97.27	0.668
365	98.26	0.856	99.58	0.233	99.49	0.362	99.67	0.094
366	96.32	5.110	98	1.645	99.55	0.32	97.52	0.796
367	97.18	1.257	95.38	1.152	99.87	0.04	97.62	0.256
368	100.00	0.000	99.66	0.275	99.1	0.495	98.93	1
369	98.82	2.000	99.6	0.509	98.6	0.89	99.47	0.358
370	100.00	0.000	91.15	1	98.73	1.018	93.32	0.781
371	98.33	1.137	99.42	1.189	98.21	0.66	99.76	0.223
372	100.00	0.000	99.03	0.615	98.8	1.132	99.86	0.25
373	100.00	0.000	98.16	0.872	99.87	0.036	99.88	0.193
374	100.00	0.000	100	0	98.28	1.065	99.6	0.693
375	82.84	1.613	99.85	0.04	97.95	1.207	99.88	0.193
376	96.51	0.754	100	0	100	0	99.29	1.257
377	99.03	1.224	100	0	100	0	100	0
378	100.00	0.000	99.81	0.038	99.7	0.667	99.72	0.5
379	100.00	0.000	100	0	100	0	98.82	1
380	80.21	3.618	100	0	100	0	100	0

Link	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
381	99.20	1.122	99.9	0.269	100	0	99.17	0.274
382	99.83	0.073	100	0	100	0	99.72	0.092
383	97.58	1.163	100	0	100	0	97.27	0.668
384	81.62	1.718	100	0	100	0	98.82	1
385	96.89	5.000	99.45	1.074	100	0	96.52	1.092
386	99.12	0.577	98.45	0.84	100	0	99.44	0.162
387	97.74	0.268	99.42	0.152	99.1	0.495	99.1	0.841
388	100.00	0.000	99.88	0.056	99.08	0.505	99.61	0.224
389	94.94	3.356	97.24	1.864	95.4	2.598	98.94	0.614
390	94.64	0.763	100	0	100	0	99.94	0.013
391	99.15	1.415	97.53	2.777	96.7	3.79	98.9	1.982
392	98.14	1.251	96.11	2.204	94.81	1.827	96.91	1.224
393	91.87	2.670	99.12	0.495	99.55	0.833	100	0
394	97.81	0.651	100	0	100	0	100	0
395	99.84	0.006	94.96	2.346	93.47	0.897	98.38	1
396	96.94	0.685	100	0	100	0	100	0
397	99.82	0.036	99.86	0.103	99.13	0.356	99.73	0.066
398	100.00	0.000	100	0	100	0	100	0
399	99.25	0.078	100	0	100	0	100	0
400	98.87	0.311	100	0	100	0	100	0
401	99.83	0.073	100	0	100	0	99.72	0.092
402	94.94	1.249	100	0	99.7	0.165	96.08	1.668
403	98.12	0.388	100	0	100	0	100	0
404	97.73	3.000	99.72	0.731	100	0	96.24	1.184
405	97.80	2.073	98.97	1.174	99.7	0.165	99.49	0.477
406	99.52	2.000	97.58	1.572	98.59	0.571	98.27	0.604
407	92.06	5.000	96.78	2.818	96.12	2.864	99	0.533
408	96.94	0.375	100	0	98.78	0.67	99.38	0.536
409	79.94	3.928	100	0	99.79	0.104	100	0
410	97.14	1.175	99.63	0.206	96.42	1.965	99.61	0.095
411	97.73	3.000	94.48	2.865	92.21	2.655	94.28	2.884
412	98.74	0.245	99.24	0.83	99.4	0.33	95.57	2.145
413	98.36	0.390	100	0	100	0	100	0
414	100.00	0.000	100	0	100	0	100	0
415	100.00	0.000	100	0	100	0	100	0
416	100.00	0.000	100	0	100	0	100	0
417	100.00	0.000	100	0	100	0	100	0
418	99.82	0.036	100	0	97.63	0.737	99.8	0.323

	No Evacuation		$\lambda = 0$		$\lambda = 0.5$		$\lambda = 1$	
Link	Player M	Player T	Player M	Player T	Player M	Player T	Player M	Player T
419	100.00	0.000	97.47	0.941	99.79	0.072	97.78	0.721
420	100.00	0.000	99.9	0.174	100	0	100	0
421	100.00	0.000	99.92	0.053	97.63	0.824	99.42	0.384
422	100.00	0.000	89.95	7	89.56	6	87.81	7
423	93.11	5.000	95.84	3.748	95.92	3.054	91.01	3.359
424	98.04	3.000	96.92	1.395	97.73	0.66	97.06	1.317
425	85.59	3.328	98.7	1.442	97.93	0.487	98.56	1.202
426	100.00	0.000	99.81	1.059	99.48	1.102	99.22	1.164
427	84.90	2.614	98.68	3.465	96.39	2.459	97.69	4.574
428	94.17	1.665	94.04	3.185	93.63	4.846	97.06	2.449
429	100.00	0.000	96.32	0.861	97.91	2.296	96.66	1.067
430	100.00	0.000	90.8	7.206	88.77	8.796	85.32	9.559
431	99.78	0.070	98.88	0.739	97.46	2.469	97.72	1.41
432	98.57	0.155	98.74	1.035	97.7	1.026	94.8	1.537
433	98.93	0.587	99.26	0.711	96.43	3.517	99.22	1.63
434	79.06	5.175	99.56	0.199	97.35	1.738	95.72	2.588
435	98.48	0.292	99.58	0.089	100	0	97.78	0.232
436	95.29	1.869	100	0	100	0	100	0
437	99.31	0.365	100	0	99.96	0.011	100	0
438	96.39	1.253	99.55	1.218	96.51	1.415	99.61	0.095
439	100.00	0.000	93.8	3	91.97	2.695	97.96	1.75
440	100.00	0.000	100	0	100	0	100	0
441	100.00	0.000	99.85	0.04	99.37	0.809	100	0
442	95.40	0.644	99.66	0.275	99.79	0.104	99.77	0.172
443	84.74	1.024	100	0	100	0	99.26	0.184
444	99.69	0.467	97.82	1.068	96.65	1.205	98.75	0.798
445	98.69	2.000	98.78	0.659	99.76	0.171	98.33	0.523
446	93.13	2.392	100	0	100	0	100	0
447	97.36	3.134	98.09	1.558	99.76	0.171	98.33	0.523
448	85.93	0.855	99.66	0.275	99.49	0.269	99.49	0.477
449	100.00	0.000	100	0	100	0	100	0
450	100.00	0.000	74.95	8.184	82.08	8.29	74.53	7.717
451	100.00	0.000	100	0	100	0	100	0
452	100.00	0.000	91.29	3.205	92.74	5.704	91.3	3.555

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