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Increasing Multiplication and Division Fluency: Embedding Self-Regulation Strategies within Systematic, Strategic Instruction

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Increasing Multiplication and Division Fluency: Embedding Self-Regulation Strategies within Systematic, Strategic Instruction

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Dedication

This dissertation is dedicated to the elementary school teachers and students who opened their doors and allowed me to work with their students.

This dissertation is also dedicated to my wonderful husband who has supported me through the endeavor of graduate school. I love you!

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After four years it is hard to believe that the adventure of graduate school is complete. I am not sure where to begin to thank the family and friends that have been there to support me, but I know without their support this huge undertaking would not have been reached! As I am closing the door on graduate school, I reflect on the path that brought me to education, and the teachers that were so influential in building the love for knowledge and teaching. I only hope that I too have made such an impact on my former students, and those new teachers I want to develop.

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Increasing Multiplication and Division Fluency: Embedding Self-Regulation Strategies within Systematic, Explicit Instruction

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Students need to develop computational proficiency with basic facts (i.e., addition, subtraction, multiplication and division) to be successful in more advanced mathematics such as instruction in fractions, decimals, ratios, and rates (Gersten et al., 2009; NCTM, 2010; NMAP, 2008). Specifically, the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006) stresses the importance of automaticity in basic facts and the application of these skills to solving word problems. For older elementary students, it is vital that they are proficient in multiplication and related division facts in preparation for working with fractions and other algebra readiness skills. Thus, the purpose of this study was to teach multiplication and division facts using systematic, strategic instruction with and without self-regulation strategies. A single-subject, time-series design was employed to measure items correct on daily probes with nine, fourth grade students. The daily probes were designed with 15 review facts and 25 new facts to measure the ability to solve easy, review facts with automaticity and hard facts specifically taught during instruction. All instruction occurred in small groups (4 – 5 students), after school, with a trained instructor. The students received strategic, systematic instruction in hard multiplication and division facts (9s, 4/6/8s and

7s) with and without additional self-regulation components (self-correction, graphing and goal setting). Multiplication and division were taught together as a fact family, rather than apart, to increase conceptual understanding of the relation between multiplication and division. The findings showed that the students made positive growth in both operations in terms of items correct and fluency; with an increase in accuracy and decrease in time to reach phase change criteria when the intervention was embedded with self-regulation components. Findings from social validity measures from participants support the use of self-regulation as a means to increase motivation.

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Chapter 1: Introduction

RATIONALE

Basic numeracy skills and automaticity in mathematics are critical to student success in mathematics classes, post-secondary settings, and possible employment (Poncy, Skinner, & Jaspers, 2006). Historically, student skills in mathematics have been low as compared to other subject areas, and low skill levels have hindered student success in more advanced mathematics classes (Bryant, 2005; National Council of Teachers of Mathematics [NCTM], 2002; National Mathematics Advisory Panel [NMAP], 2008). Since 1960, the first National Report Card published findings based on a national assessment given to students in grade 3 through grade 12 (National Assessment of Educational Progress [NAEP], 2009). The NAEP measures number properties and operations, measurement, geometry, data analysis and algebra. In 2007 and 2009, 18% of fourth grade students were below basic level, 82% were at or above basic, 39% were above proficient, and 6% were advanced (NAEP, 2009). At the basic level, fourth-grade students are able to "estimate, use basic facts to perform simple computations with whole numbers; show some understanding of fractions; and solve simple real-world problems" (p. 20). Students performing at the proficient level "constantly apply integrated procedural knowledge and conceptual understanding to problem solving" in all areas tested (p. 20). At the advanced level students are able to "apply integrated procedural knowledge and conceptual understanding to complex and non-routine real-world problems" (NAEP, 2009, p. 20). Lower scores in mathematics for students in grades 4 and 8 have also been reported in the findings from another key mathematics measure, the Trends in International Math and Science Study [TIMSS] (TIMSS, 2007). The results of the TIMSS indicate that students in the U.S. are scoring lower than Asian and European students. Furthermore, on the fourth grade assessment, the U.S. students outperformed students in only eight developed countries out of 36, and the eighth grade students outperformed students in only seven out of 48 developed countries. Although the overall scores have increased since the early 1990's, a disheartening fact is that only 10% of fourth graders and 6% of eighth graders scored above the advanced benchmark (NAEP, 2009). In order to begin to address the mathematics performance issues of U.S. students, the National Mathematics Advisory Panel [NMAP] (2008), was appointed by President Bush in 2006 to recommend the ways to "foster greater knowledge of and improved performance in mathematics among American students." (p. xiii).

The report from the NMAP (2008) found that mathematical instruction in the U.S. lacks focus and breadth across grade levels and inadequately prepares students for the rigors of algebra, which is often viewed as the "gateway" for postsecondary education. The NMAP (2008) recommended that mathematics curriculum focus on pre-algebra skills, specifically, mastery of whole number computations, fractions, geometry, and measurement. Additionally, mathematics curriculum must build procedural knowledge, as well as conceptual understanding, computational fluency and problem solving. It is essential that mathematics curricula build students proficiency and fluency of multiplication and division facts through practice to develop automaticity, essential for fractions, "major obstacle to further progress in mathematics, including algebra" (NMAP, 2008, p.xix). Students who lack mastery of some of the fundamental prerequisite skills and concepts, such as fluency with basic facts, the ability to know when and how to use the associative and communicative property for solving facts, and efficiency with word problem solving, may experience challenges with the demanding rigor of algebra (NMAP, 2008). Unfortunately, the majority of elementary level mathematics textbooks does not include sufficient instruction and practice in arithmetic combinations (basic facts); rather, they only provide minimal exposure to facts (NMAP, 2008). This lack of focus on arithmetic combinations and mastery learning is discouraging because of the vital role both contribute to mathematics performance (Gersten, Jordan, & Flojo, 2005). Moreover, professional organizations have elevated the importance of computational fluency by including an emphasis in their standards on fluency with arithmetic combinations (National Core Standards, 2010; NCTM 2000, 2006).

Clearly national assessment data illustrate the need for practice and mastery to increase fluency and proficiency of basic facts in elementary grades. In addition, combining this instruction with word problems and application problems assists in the procedural and conceptual understanding of mathematical operations, leads to abstract reasoning, and allows students to discuss and model mathematics (National Core Standards, 2010).

Upper elementary students who continue to struggle with proficiency of basic facts and the application of the properties of mathematics find these issues can hinder their performance in fractions and geometry, and lead to motivational problems (Montague & Dietz, 2009; Montague, 2007; NMAP, 2008). Moreover, motivational problems can affect a student's ability to self-regulate (e.g., to set performance goals, complete all steps needed to solve, manage different strategies) and manage academic behaviors (e.g., time on-task, work completion, work accuracy) (Montague, 2007; Ryan, Reid, & Epstein, 2004). Self-regulation skills needed for mathematics include the ability to identify a specific strategy to solve a given problem and complete all steps while monitoring progress and/or behavior in learning (Kunsch, Jitendra, & Sood, 2007). Improved computational proficiency in basic mathematical skills, as well as the application of self-regulation skills, such as goal setting, graphing and self-monitoring, can lead to an increase of motivation and success in advanced mathematical classes

(Gersten, Chard, Jayanthis, Baker, Murphy, & Flojo, 2009). Understanding the motivational problems and the mathematical challenges faced by students with mathematical weaknesses needs to be considered when developing mathematical interventions.

CHARACTERISTICS OF MATHEMATICS DIFFICULTIES

As a result of Individuals with Disabilities Education Act of 2004 (IDEA) it is recommended that states begin to use a Response to Intervention (RtI) tiered model. In RtI, a universal screener is given to identify students below a benchmark in need of additional instruction. These students are often identified as having a mathematical and/or reading difficulty and should be provided small group, research-based instruction (Fuchs & Fuchs, 2005; Johnson, Jenkins, & Petscher, 2010). Within the last five years, RtI research in the area of mathematics has begun to investigate interventions and characteristics of Tier II students (Bryant, 2005; Fuchs, Fuchs, & Stecker, 2010). It is important to understand how working memory may affect strategy understanding and use, number sense and whole number computation can hinder students with mathematical difficulties. These characteristics should be considered in the development of effective intervention components and in the identification of Tier II students with mathematical difficulties.

It has been found that working memory can affect the ability to solve problems, retrieve facts, use strategies, and represent mathematical information, such as number sentence or schematic models, numeracy skills and estimation (Geary, 2004; Sood & Jitendra, 2007; Raghubar, Barnes, & Hecht, 2009; Swanson & Jerman, 2006). Deficits in working memory can be seen in accuracy and fluency weaknesses that “impairs performance” (Raghubar et al., 2009, p. 11) in basic arithmetic operations leading to ineffective strategy use and personal “rules” rather than a specific algorithm to solve problems, thus increasing the time needed to solve problems (Codding, Chan-Iannetta, Palmer & Lukito, 2009; Stein Kinder, & Silbert, 2006; Woodward, 2006). This ineffective strategy use can decrease rate and accuracy and increase error in multi-step problems (Burns, Codding, Boice, & Lukito, 2010; Bryant, 2005; Fuchs & Fuchs, 2005; Scheuermann, Deschler, & Schumaker, 2009). Explicit instruction of strategies, practice and application combined with a self-regulation strategy can help to remediate weaknesses in the area of strategy use, schematic models, numeracy skills and estimation (Gersten & Chard, 1999; Gersten et al., 2009b).

Number sense, whole number computation and the application of these skills in a word problem is often most cited as predictive characteristics of students with mathematical difficulties (Gersten & Chard, 1999; Jordan, Kaplan, Locuniak, & Ramineni, 2007). In the higher grades, a good foundation of number sense is applied to understanding and decomposing parts of numbers, as well building mathematical models in multiplication and division (National Core Standards, 2010). This flexibility of number, relates directly to arithmetic facts and the relationship between numbers (Bryant, 2005; Gersten, Jordan, & Flojo, 2005; Wagner & Davis, 2010; Woodward, 2006). NMAP (2008) reports that when students do not have a firm understanding of numbers and whole number computation, weaknesses become apparent when students are presented with more advanced, multi-step problems and algebra. Word problem solving is an important application of arithmetic facts, because students need a mastery of the facts in order to identify important information, write the correct algorithms and find the solutions. When facts are mastered, the students are more successful in understanding and solving word problems (Codding et al., 2009; Geary, 2004; Gersten et al., 2005; Powell & Fuchs, 2010). Through strong interventions within number sense and whole number computations, students are more successful in multi-step word problems. Mathematical interventions need to be strong in both procedures, as well as conceptual understanding to develop the number sense needed to master facts for use in word problems, fractions, and algebra (NMAP, 2008).

SELF-DETERMINATION, SELF-REGULATION AND MATHEMATICS

Self-Determination

The concept of self-determination originated with students and adults with developmental disabilities in an attempt to link school efforts and curriculum to post-secondary employment, as well adult living skills (Wehmeyer, Field, Doren, Jones & Mason, 2004). There are various definitions of self-determination within education and psychology, but the one most cited in research was developed by Field, Martin, Miller, Ward and Wehmeyer in 1998 as cited in Konrad, Fowler, Walker, Test and Wood (2007):

> Self-determination is a combination of skills, knowledge, and beliefs that enable a person to engage in a goal-directed, self-regulated, autonomous behavior. An understanding of one's strengths and limitations together with a belief in one self as capable and effective are essential. (p. 2)

Education is not the only arena with a focus on self-determination. Psychologists have examined the theory of self-determination within studies on motivation. Filak and Sheldon (2008) describe a self-determined person as one that experiences "quality motivation," meaning that the person "feels a sense of owning and causing his/her own behavior, even if he or she does not enjoy it" (p. 712, 714).

Self-determination is a theory with three distinct components, (1) autonomy, (2) competence and (3) relatedness (Filak & Sheldon, 2008). Autonomy is one component of the self-determination, that "involves feeling that one is the owner or author of one's current behavior" (Filak & Sheldon, 2008, p. 714), and acting in a self-realizing way

(Wehmeyer, Agran, & Hughes, 2000). Having a student monitor time on-task or setting a goal would be an example of autonomy. The next component of the self-determination theory presented by Filak and Sheldon (2008) is relatedness. Relatedness "involves feeling a meaningful connection between oneself and important others, rather than feeling like an object whose experiences are ignored and discounted" (Filak & Sheldon, 2008, p. 714). In the field of special education, educators are attempting to include students and parents in the educational planning process. Competence is the third component within the theory of self-determination. In an academic setting "students demonstrate competence by achieving command of increasingly difficult material" (Filak & Sheldon, 2008, p. 714). In studying behaviors, Fowler et al. (2007) found, transferring management, regulation, and social skills to the "self" or the student, interventions were more successful. The overall focus of self-determination was narrowed and self-regulation strategies were introduced in more intervention research.

Self-Regulation and Mathematics

Teaching academic skills embedded with self-regulation strategies may contribute to academic achievement for students with mathematical difficulties (Burns et al., 2010; Codding et al., 2009; Montague, 2007; Rock, 2005). The three components, autonomy, competence and relatedness, are deeply embedded and evident in self-regulation strategies. For example, Montague (2007) stated that self-regulation, or autonomy is often a weakness in students with learning difficulties, and teaching self-regulation may include, "self-instruction, self-questioning, and self-checking" (p. 75). Montague also noted that by infusing components of autonomy through self-regulation, students were given the tools, such as cue cards, to master word problem solving independently and attitudes towards mathematics material became more positive. Montague, Applegate and

Marquard (1993) and Montague (2007) learned that students with learning difficulties were more successful in applying mathematical strategies when a cue card was used by the student to self-regulate the steps of the mathematical strategy. Lee, Wehmeyer, Palmer, Soukup and Little (2008) also described the positive correlation between cue cards or simple visual prompts as self-regulation accommodations in the general education setting to increase success for students with learning difficulties.

In other areas of mathematics instruction, autonomy was demonstrated by offering choice in worksheets, writing goals related to mathematics achievement, or graphing progress alone or in conjunction with goals (Axtell, McCallum, Mee, Bell & Poncy, 2009; Burns, 2005; Flores, 2009; Flores, Houchins & Shippen, 2006; Jitendra, DiPi & Perron-Jones, 2002; Powell, Fuchs, Fuchs, Cirino, & Fletcher, 2009). Greater success for struggling students can be obtained when students become active participants by assisting in goal writing and progress monitoring (Konrad et al., 2007; Konrad & Test, 2007; Wehmeyer et al., 2004). For example, Rock (2005) used the self-regulation strategies of self-monitoring of behavior, time-on task and accuracy in the areas of mathematics and reading. By embedding self-regulation, it actively involved students in the process of data collection, and increased overall accuracy, productivity, and student engagement. In another study, Lee et al. (2008) conveyed that self-regulation strategy use was found to have a positive relationship in time on-task, work completion, and progress on educational goals. The integration of autonomy, relatedness and competence is the foundation for effective self-regulation strategies.

Incorporating self-regulation strategies may be an important component to promote successful academic interventions (Burns et al., 2010; Gersten et al., 2009a; Montague, 2007). For example, recent mathematics intervention studies have shown better outcomes when self-regulation strategies such as graphing and goal setting are combined with explicit, strategic instruction (Burns et al., 2010; Gersten et al., 2009b). Thus, studies have shown the benefits of including self-regulation strategies that promote autonomy, relatedness and competence can lead to an increase in motivation, as well as academic skills. Preliminary findings from the field of motivational research and longitudinal intervention studies report that increases in both student motivation and computation instruction are needed for students to be successful in post-secondary settings (Burns, 2005; Byman & Kansanen, 2008; Gersten et al., 2009a and 2009b; Flores, Houchins, & Shippen, 2006; Flores, 2009). Incorporating self-regulation strategies within mathematical interventions may be an important component leading to greater student success.

STATEMENT OF THE PROBLEM

Research in mathematics and self-regulation has focused on interventions to be completed independent of one another, but few researchers have examined multi-component strategies incorporating these two important skills. The corpus of research identifies that one of the characteristics of a student with a mathematical difficulty is lack of automaticity in basic facts. Students lacking computational fluency in solving the basic facts need to be identified early in their academic career and interventions need to be implemented for success in more advanced mathematical courses (Bryant, 2005; Gersten et al., 2005, Hecht et al., 2003; Sherin & Fuson, 2005; Woodward, 2006). Students with mathematical difficulties also struggle with self-regulation strategies, which can lead to academic failure (Auerbach, Gross-Tsur, Manor, & Shalev, 2008; Calhoon & Fuchs, 2003; Montague, 2007; Powell & Fuchs, 2010). By embedding self-regulation strategies within a mathematics intervention, students' mastery rates, accuracy, and productivity in arithmetic facts may increase.

PURPOSE OF THE RESEARCH

Students with mathematics difficulties may need evidence-based mathematics interventions combined with self-regulation strategies due to the high academic demands and rigor of the general education curriculum (Kunsch et al., 2007). It is vital that students be proficient in arithmetic combinations, and be able to apply computational skills and strategies to word problems. In addition, students need the flexible mathematical thinking required for success in advanced mathematical courses, future employment, as well as daily life tasks (Gray, Pinto, Pitta, & Tall, 1999). Thus, the purpose of this study was to teach whole number computational skills in multiplication and division facts, application of facts in word problems, and a component analysis of explicit instruction with and without self-regulation strategies. Mastery and accuracy data was collected to determine the self-regulation component affects in multiplication and division facts. This study sought to determine the effects of embedded self-regulation strategies (self-correction, goal setting and graphing) within a mathematics intervention on the performance of students with mathematics difficulties.

RESEARCH QUESTION

This research answered the following question for grade 4 students with mathematical difficulties:

To what extent will fluency and accuracy scores on multiplication and division fact probes differ as a result of a systematic, strategic intervention plus goal setting and graphing as compared to a systematic, strategic intervention without goal setting and graphing?

Chapter 2: Review of Related Literature

BACKGROUND AND RATIONALE

In 2008, the U.S. Department of Education, National Mathematics Advisory Panel (NMAP), released a report describing the mathematical weaknesses of today's students. NMAP highlighted the high percentage of students lacking proficiency in such prerequisite skills as fractions or algebra by 12th grade, leading to an increased need for remedial mathematics classes in postsecondary settings. NMAP asserted that motivated students perform better in mathematics, and that increases in both motivation and computation skills are needed for students to be successful in postsecondary settings. The panel also recommended that mathematics instruction be aligned with algebraic concepts and foundational skills, such as fractions, percentages, ratios, and computation. Kunsch et al. (2007) reported that students with mathematical difficulties lack the basic foundational number skills needed for understanding fractions, percentages, ratios, and multistep computations. Furthermore, there is a need for strong mathematical interventions to meet the high academic demands and rigor of the general education mathematics curriculum. Basic foundational number skills and facts that need to be reinforced and mastered for students to be successful in mathematics, as well as in future employment and postsecondary environments.

One means of reinforcing mastery of foundational number skills and facts is using self-regulation strategies within academic interventions. The combination of academics and self-regulation strategies may contribute to increased academic achievement and decrease the time needed to master specific skills (Rock, 2005) because it "enhances learning by helping students take control of their actions and move toward independence as they learn" (Montague, 2007, p. 76). For example, Rock used self-regulation strategies including students' self-monitoring behavior, time on task, and accuracy and productivity in mathematics and reading to increase academic engagement time, or time on task. When students were involved in the process of data collection within self-regulation, they decreased their disengaged time and increased time on task as well as productivity across "new versus previously learned material" (Rock, 2005, p. 13). Konrad and Test (2007) found similar results by including students as active participants in educational goal-setting and planning. Konrad and Test found that self-regulation strategies play a significant role in improving academic outcomes. However, limited evidence is found in the research of multicomponent strategies incorporating mathematics and self-regulation components (Chamber et al, 2007). Thus, there is a need for intervention studies that focus on components of self-regulation, such as self-monitoring, self-correction, graphing, and goal-setting within mathematics interventions to address the needs of students and increase success in the school environment. This chapter reviews the literature in the major domains of this study's research question: (a) the characteristics of students with mathematical difficulties, (b) instructional theories, (c) mathematics interventions and self-regulation reviews, (d) instructional routine features, and (e) assessment and progress monitoring.

CHARACTERISTICS OF STUDENTS WITH MATHEMATICAL DIFFICULTIES

The reauthorization of the Individuals with Disabilities Act of 2004 (IDEA) recommended that states alter their methods for diagnosing students with specific learning disabilities (SLD). Rather than using the traditional aptitude/achievement discrepancy model, the reauthorization of IDEA permitted the use of responsiveness to intervention as a substitute for, or supplement to, the discrepancy model (IDEA; P.L. 108-446). The goal of the response to intervention (RTI) model is for educators to intervene earlier in a child's academic career and reduce the number of inaccurate or false-positive labels of SLD (Fuchs & Fuchs, 2005). In an RTI model, a benchmark assessment is given to all students to identify those needing additional, supplemental instruction based on scores below a set benchmark (Fuchs & Fuchs, 2005; IDEA Partnership, 2011). Within RTI, a research-based intervention should be implemented for students scoring below the benchmark, and data are collected on an ongoing basis for making instructional decisions and/or special education placements (IDEA Partnership, 2011). RTI supports the notion that by providing "effective instruction early and intensively, [teachers can help students] make large gains in general academic achievement" (Lyon & Fletcher, 2001, p.24). As a result of the RtI movement, many intervention studies in mathematics include students with a SLD in mathematics and students with mathematical difficulties (Axtell et al., 2009; Bryant, et al, 2011; Flores, 2009; Powell et al., 2009). Based on the research on students having mathematics difficulties, five areas are the most difficult: (a) working memory, (b) number sense, (c) whole number computation, (d) word problem solving, and (e) self-regulation. Addressing these five areas within mathematics interventions can help remediate the deficits hindering success in mathematics.

Working memory

Working memory is defined as "a limited capacity central executive system that interacts with a set of two passive store systems used for temporary storage of different classes of information" (Braddeley & Logie, 1999, p. 343). A growing body of research has linked mathematical difficulties and working memory deficits in students at various ages (Geary, 2004, 2005; Raghubar et al., 2010; Swanson, Kehler, & Jerman, 2010; Swanson & Jerman, 2006; Swanson, Jerman, & Zheng, 2008), and over time this deficit "could affect the procedural competencies" (Geary, 2005, p. 306) needed in the “execution of mathematical procedures” (Geary, 2004, p. 9). In mathematics, weaknesses in working memory can affect a child’s ability to apply strategies to effectively solve whole-number computations and word problems (Geary, 1993, 2005; Raghubar et al., 2010; Swanson & Jerman, 2006; Swanson et al., 2008).

Deficits in working memory are evident when students struggle to complete whole-number computations. For example, when a typically developing student is solving 7 + 2, he or she “stores” the number 7 in his or her mind and then counts-on 2 more to solve the problem (Geary, 2004; Siegler, 1988). This computation task requires a student to identify the greater number, start counting from 7, and then count-on 2 more. Students with weaknesses in mathematics are not as efficient at solving whole-number computations as typically developing peers; rather students with mathematical difficulties count-all (i.e., start counting at 1, keep counting to 7, and then count 2 more) instead of using the more efficient count-on strategy (Geary, 2004, 2005) to solve the problem. These students struggle to apply strategies to solve whole-number computations because they cannot fully “see” how the algorithm is applied and represented (Hecht et al., 2003; Swanson & Jerman, 2006; Swanson et al., 2008). Swanson and Jerman (2006) and Miller and Hudson (2007) identified computation deficits as a type of mathematical difficulty procedural in nature due to ineffective strategy application. The consequences of procedural mathematical difficulties are seen in accuracy and fluency weaknesses and impaired performance in basic arithmetic operations (Raghubar et al., 2010). Mathematics interventions need to teach strategies to decrease cognitive load (Woodward, 2004) and increase the automaticity of solving facts. Increasing the automaticity is a “transition to memory-based processes [that] results in the quick solution of individual problems and reduction of the working memory demands” needed to solve facts (Geary, 2004, p. 7). Decreasing cognitive loads on working memory through efficient strategy instruction using research-based instructional approaches is necessary for students with mathematical difficulties.

Number sense

In addition to working memory weaknesses, another prevalent deficit for students with mathematical difficulties is number sense (Bryant, 2005; D. Bryant, Bryant, & Hammill, 2000). Gersten and Chard (1999) defined number sense as "the sense of what numbers mean and an ability to perform mental mathematics and to look at the world and make comparisons" (p. 19). A weakness in number sense is a significant predictor of mathematics difficulties (Bryant, 2005; Gersten et al., 2005; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Glutting, & Ramineni, 2010). Number sense often involves counting, and having a sense of flexibility with number discrimination, number transformation, number patterns, and estimating (Bryant, 2005; Jordan et al., 2007). In the higher grades, a good foundation of number sense is needed for the application of more advanced "break-apart" strategies in multiplication and in understanding the concept of parts and wholes in fractions (Woodward, 2006). Number sense development should not be limited to mathematics curriculum in the early elementary grades; rather, number sense should be taught and reinforced every year.

Jordan et al. (2010) examined performance on number sense assessments in kindergarten students and the same students in third grade. The kindergarten students with a more developed number sense were far more successful in third grade than those with low scores on the kindergarten number sense assessment. A more developed number sense leads to flexibility of number decomposition (Siegler, 1988) and an understanding of counting principles, such as cardinality, order, and abstraction (Geary, 2004). The mastery of number sense and flexibility in counting and breaking apart numbers directly relate to arithmetic facts because children need to comprehend the relationships between numbers (Bryant, 2005; Geary, 2004; Gersten et al., 2005; Siegler, 1988; Wagner & Davis, 2010; Woodward, 2004). Thus, developing number sense is vital for success in accuracy and fluency of facts.

Whole number computational fluency

Both working memory deficits and weaknesses in number sense affect whole-number computational fluency. Whole-number computation is defined by NCTM (2000) as "having efficient and accurate methods for computing" (p. 152). A deficit in number sense can dramatically affect a student's ability to solve facts, resulting in weaknesses in whole-number computational fluency, or the "ability to automatically retrieve…arithmetic facts" (Gersten et al., 2005, p. 294) and the automaticity is an integral component in mathematics curriculum (Schoenfeld, 2004). The inability to retrieve basic facts quickly has been labeled a processing delay, as well as a procedural delay (Bryant, 2005; Codding et al., 2009; Gersten et al., 2005; Hecht et al., 2003; Miller & Hudson, 2007; Pellegrino & Goldman, 1987; Woodward, 2006) and is often considered to be a hallmark of learning disabilities in mathematics (Geary, 2005; Gersten & Chard, 1999).

Hecht et al. (2003) explained that the underlying theory in procedural delays in mathematics is an immature development of both conceptual knowledge of facts, that is, an understanding "about the underlying principles that govern a domain" (p. 278). and "memorized facts involving arithmetical relations among numbers" (p. 278). Conceptual knowledge is an understanding of similar characteristics in numbers and the ability to generalize number patterns to whole-number computations and conversions (Miller & Hudson, 2007). Whole-number computation involves a marriage between understanding a strategy and generalizability of when to apply the strategy (Pellegrino & Goldman, 1987). It is the ability to generalize computations to different processes and representations, and an ability to communicate how to solve whole-number computations (NCTM, 2000).

NMAP (2008), NCTM (2006), and the National Core Standards (2010) recommend that third-grade students demonstrate mastery, defined as 80% or better in accuracy, on addition and subtraction facts and be introduced to multiplication and division by the end of third grade. By the end of fifth grade, students should be proficient in all basic fact operations—addition, subtraction, multiplication and division—and be able to complete multistep problems involving different operations. In a study examining whole-number computation, a large decrease in fluency and ability often occurred between third and fourth grade. Furthermore, the findings suggested, "learning or retention of fourth grade level computation skill items" is pivotal to long-term mathematical success (Calhoon, Emerson, Flores, & Houchins, 2007, p. 298). Automaticity in these areas is "important to estimation, mental calculation, and approximation skills" (Woodward, 2006, p. 287) and is a predictor of long-term mathematical success in more complex problem solving (NMAP, 2008).

Word problems

Word problems are a mix of numbers and words in which students apply mathematics instruction in the context of problem solving (Wyndhamm & Saljo, 1997). Word problems have gained national attention because they are the tools used to measure mathematical concepts locally, nationally, and internationally (NCTM, 2010; NAEP, 2009; TIMMS, 2007). Word problems measure students' ability to generalize different mathematical structures conceptually in a format that may not be familiar (Cogan et al., 2001; Gersten et al., 2009a and 2009b; Schoenfeld, 2004; Wyndhamm & Saljo, 1997). Word problems connect the application of mathematics to the process of mathematics. NCTM (2002) defined five process standards to be embedded within each mathematical strand: (a) problem solving, (b) reasoning and proof, (c) communication, (d) connections, and (e) representations. The process standards were developed to help students attain a deeper understanding of and mathematical flexibility with numbers, concepts, and mathematics language (Gray et al., 2005). Word problems connect the classroom to real-world applications (NCTM, 2002).

Difficulties in solving word problems can be a "major impediment for [students'] future success in any math-related discipline" (Gersten et al., 2009b, p. 26). Students with mathematical difficulties often struggle with word problems because they lack understanding of the language and vocabulary within the problems (Bryant, 2005; Fuchs et al., 2008; Gersten et al., 2005), are unable to apply multiple steps within word problems (Parmer, Cauley & Frazita, 1996), and experience difficulty in selecting and using the correct algorithms to solve word problems (Hecht et al., 2003). Students with mathematical difficulties have an inability to generalize strategies across different types of word problems (Gersten et al., 2009) and "infrequently abandon and replace ineffective strategies, rarely adapt previously learned strategies" (Montague & Dietz, 2009, p. 285). Students with mathematical difficulties require explicit instruction in the structure of word problems, identifying problem types, and applying a strategy to solve the problems (Gersten et al., 2009a; Jitendra et al., 2002; Jitendra et al., 2009; Montague & Dietz, 2009). To foster an understanding of word problems, cognitive strategy instruction is essential (Gersten et al., 2009; Jitendra et al., 2002; Jitendra et al., 2009; Montague & Dietz, 2009; Van Garderen, 2007), as well as the mastery and accuracy of whole-number computation fluency to increase abstract understanding in word problems (Jitendra et al, 2002; Van Garderen, 2007)

Self-regulation

Self-regulation in an academic setting involves using metacognitive strategies (Montague, 2007) for “planning, monitoring, and modifying” (Pintrich & DeGroot, 1990, p. 33), and effort in work completion (Konrad & Test, 2007; Pintrich & DeGroot, 1990; Rock, 2005). Mathematical challenges are becoming more evident in the upper grades because of the increased rigor of the mathematics curriculum (ESEA, 2001; NMAP, 2008). Many students often lack the self-regulation skills to manage both academics and life skills (Montague, 2007; Ryan et al., 2004). NMAP (2008) outlined the need to close the gap between struggling students and typically performing peers through interventions in number sense and computational fluency, while increasing motivation. Components of self-regulation can be used to increase student motivation in the demanding mathematics curriculum (Auerbach et al., 2008; Calhoon & Fuchs, 2003).

In educational settings, self-regulation involves teaching students specific strategies for learning, thereby increasing management of one’s actions. The self-regulation skills essential for mathematics success include the ability to identify a specific strategy to solve a problem and the ability to complete all steps while monitoring progress. Students with mathematical difficulties struggle to self-regulate the steps necessary to complete tasks (Miller & Hudson, 2007), and in paired peer interactions they tend to take a more passive role (Bottge, Heinrichs, Mehta, & Hung, 2002). Gersten et al. (2009) found large effect sizes in explicit interventions with students taking an active role in data collection through graphing and goal setting.

Summary

Due to the added demand of self-regulating learning, it is vital that students be proficient in facts and apply computational skills and strategies to word problems to gain the knowledge needed for success in advanced mathematical courses, employment, and everyday life tasks. Student involvement in the learning process can increase the rate at which a specific strategy is mastered, as well as accuracy in application problems (Lee et al., 2008; Rock, 2005). Building both procedural and conceptual knowledge of numbers and flexible mathematical thinking may not be enough; embedding self-regulation components in mathematical interventions is necessary for greater success for struggling students.

INSTRUCTIONAL APPROACHES

The methods of teaching mathematics vary between schools, classrooms, and teachers and are grounded in two instructional approaches, procedural and conceptual. Both instructional approaches will be explored to understand the differences and similarities and the role they play in mathematics education. Developing an understanding of the two approaches is important in recognizing the impact that specific components within each approach can bring to mathematics, specifically for students with mathematical difficulties.

Procedural

The procedural approach is often rule based and explicit. The procedural approach was an integral component of mathematical curricula and classrooms until the late 1980s. In the traditional classroom, the educator is the leader and teaches the specific strategies for solving problems. The role of the student is to be an active listener, to practice and to apply strategies to solve specific types of problems (Blanton, Westbrook and Carter, 2005; Schmittau, 2004; Van Oers, 2002). The procedural approach is often favored in special education literature and classrooms because explicit instruction has been an effective approach for students with academic weaknesses and SLD (Gersten & Chard, 1999; Gersten et al., 2005; Scheuermann et al., 2009; Stein et al., 2006).

The procedural approach has often been viewed as prescriptive (Stein et al., 2006; Swanson & Jerman, 2006) and as one that builds declarative knowledge (Miller & Hudson, 2007), because students practice facts and algorithms until they commit the answers to memory. The ability to retrieve information from memory, thus freeing the capacity of working memory, lends itself to support the procedural approach for remediating students with mathematical difficulties (Geary, 2005, 2004; Swanson & Jerman, 2006). The automaticity in fact retrieval is important because the student has more capacity to solve complex, multistep problems (Gray et al., 1999; NMAP, 2008). Proponents of the procedural approach believe that students need explicit instruction and time to practice the steps in order to develop a firm foundation of how to solve problems (Stein et al., 2006). NMAP (2008) reported that when students do not have a firm understanding of the basic foundational skills, weaknesses become more apparent when they are presented with advanced, multistep problems. When students are not taught foundational skills and are able to develop their own strategy and “rules” rather than a specific algorithm, more time is spent in retrieval of basic facts, thus increasing the time needed to solve problems (Miller & Hudson, 2007; Stein et al., 2006; Swanson & Jerman, 2006). This is echoed in the work completed by Codding et al. (2009) and Woodward (2006), which found that when students do not fully understand a specific strategy or its use, generalization to other skills, such as fractions and algebraic reasoning, is hindered. Ineffective strategy use can decrease rate and accuracy and increase errors in multistep problems. Students with mathematical difficulties struggle to form individual strategies that can be generalized to a variety of mathematical problem types (Burns et al., 2010; Bryant et al., 2000; Bryant, 2005; Fuchs et al., 2005; Scheuermann et al., 2009). Students need to master basic facts to be successful in multistep, higher order problems (National Core Standards, 2010; NMAP, 2008).

Conceptual

The conceptual approach centers on the framework that children develop their own way of learning and thinking. The conceptual approach was influenced by the work of Vygotsky and Davydov (Schmittau, 2004). These theorists completed extensive work in the area of applied learning. With a conceptual approach, learning is student centered, with a focus on real-world problems in an interactive environment (NCTM, 2010). The role of the educator is to provide minimal support, allowing students to develop their own procedures and strategies for solving problems (Miller & Hudson, 2007; NCTM, 2000; Woodward, 2004). When students display deficits or an inability to solve problems, an educator may provide a scaffold, encourage peer tutoring, or provide additional mathematical tools and representations (Stein et al., 2006). Wood, Bruner, and Ross (1976) defined scaffolding as a type of assistance a child needs to make academic gains. Scaffolds can be provided by both teachers and peers. Scaffolds are flexible and are slowly faded when a student begins to master the skill.

In a study completed by Blanton et al. (2005), secondary educators were observed so that the researchers could ascertain the number of times the educators provided assistance, or scaffolds, for students to arrive at a correct answer. The scaffolds were provided through peer tutors, educators, or a previously used picture representation. The results of the case studies found that following scaffolds, students were more successful in finding solutions to new problem types. Each student in the case study was able to solve application problems by utilizing a strategy that was developed individually, rather than using a specific strategy provided by the educator (Blanton et al., 2005). Finding an individual strategy or seeking a scaffold, such as a pictorial representation or a peer tutor, is basic to the student-centered framework of the conceptual approach.

Summary

NMAP (2008) and NCTM (2002) united procedural and conceptual approaches, by stressing the importance of basic-fact skills and application of facts in a student-centered approach. Proficiency in facts is pivotal to long-term mathematical success in more complex problem solving (NMAP, 2008). Using explicit instruction, cognitive strategies (Swanson & Sachs-Lee, 2000), and multiple representations (Gersten et al., 2009b; Miller & Hudson, 2007) to build both conceptual understanding and procedural knowledge of arithmetic facts results in students' ability to integrate these skills into problem solving and mathematical communications (Core Standards, 2010).

MATHEMATICAL INTERVENTION AND SELF-REGULATION REVIEWS

Mathematical intervention reviews

Three recent mathematics syntheses were identified that examined the effects of interventions in improving mathematical performance. In the Kunsch et al. (2007) synthesis of peer-mediated mathematics interventions, 17 articles were located. Participants included students with disabilities and those labeled as at risk in both general and special education settings. The authors examined the role of peer instruction in mathematical achievement. The overall effect size was .47, illustrating a moderate effect on students' mathematics performance. In examining the mathematical content measured, one finds that the strongest effect size (.63) was for peer-mediated mathematics interventions measuring computation facts and procedures. A small effect size (.34) was reported for interventions measuring computation and concepts with application. Overall, the results identified a need to perform additional research using group designs in various settings because of the small to minimal effect sizes. Although effect sizes were not large, students in all studies demonstrated growth in mathematical skills with the peer-mediated interventions (Kunsch et al., 2007).

Maccini, Mulcahy, and Wilson (2007) completed another synthesis of mathematics interventions that focused on secondary students in 6th through 12th grade with specific learning disabilities (SLD). A total of 23 articles were included. Maccini et al. reported an increase of 35% to 57% of intervention studies specifically measuring algebra, geometry, or word-problem solving, which is promising due to the additional pressure of high-stakes testing mandated by Elementary and Secondary Education Act (ESEA) IDEA (2004) legislation. The results mirror recommendations from NCTM, which expanded the scope of mathematics objectives for each grade level in 2006 to increase the rigor of algebra and geometry instruction. A total of five mathematical domains were represented among the 23 included articles: (a) fact instruction, (b) fractions, (c) geometry, (d) algebra, and (e) problem solving (Maccini et al., 2007). In both reviews, the authors noted the need for more rigorous research in the area of mathematics, utilizing state testing measures or standardized measures of mathematical domains to align interventions with educational legislation.

In addition to the two syntheses, a meta-analysis by Gersten et al. (2009b) was completed to identify effective mathematical interventions conducted with students with SLD. This meta-analysis included only quasi-experimental and randomized controlled studies with at least one treatment group and one comparison group. The authors reported results of 42 intervention studies. The instructional components and instructional design features were further analyzed, and statistical significance was reported. In the area of instructional components, explicit instruction, student verbalizations, visuals for both teacher and students, and student feedback were all found to be effective strategies with moderate to large effect sizes. Although the goal of the meta-analysis was not to identify self-regulation strategies, components of the interventions were examined and encompassed graphing, goal setting, and student feedback. Surprisingly, student goal setting alone was not significant, but when it was combined with giving students feedback both verbally and visually, as in a graph, students were more successful. A "key finding from this set of studies is that providing feedback to students enhances achievement" (Gersten et al., 2009b, p. 1221). Additionally, an examination of mathematical intervention instructional design features shows that careful planning of the sequence of skills, from easier to more difficult, combined with explicit instruction and utilization of multiple representations resulted in higher achievement levels. Overall, interventions are more effective and have higher statistical significance when explicit, systematic sequences of skills, multiple representations, and student feedback are combined than when these components are completed independently.

Self-regulation reviews

The reviews described in the previous section focused primarily on interventions in the mathematical domains (Kunsch et al., 2007; Maccini et al., 2007) and provided a deep analysis of the intervention components within a mathematical intervention (Gersten et al., 2009b). The next three syntheses measured aspects of self-regulation, both as it contributed to overall quality of life and impact within academic interventions. Components of self-regulation include graphing, goal setting, student feedback through self-correction, and the use of cue cards (Billington, Skinner, & Cruchon, 2004; Fowler Konrad, Walker, Test and Wood, 2007; Konrad & Test, 2007; Rock, 2005).

Self-regulation strategies are identified and categorized into three types; relatedness strategies, autonomy strategies, and competence strategies. Relatedness is the involvement of students in educational planning and progress monitoring (Filak & Sheldon, 2008). An example of relatedness is students participating in educational planning, such as an Individualized Education Program meeting. Autonomy involves the students' taking ownership of their learning and becoming active participants in data collection and goal setting. This is an area of weakness for struggling students and students with SLD. Montague (2007) and Jitendra et al. (2002) found that students were more successful in applying a strategy when a cue card was used and they were responsible for monitoring their own steps. The last area of self-regulation is competence. This component of self-regulation is when students master more difficult material, often mixed with easy material. Combining components of competence, such as reinforcement of accuracy and/or rewards for finishing more difficult work combined with individual goals (autonomy), is seen in intervention studies but often not labeled as a self-regulation strategy (Filak & Sheldon, 2008; Fowler, 2007).

Chambers et al. (2007) reviewed interventions that adhered to the self-regulation theory. Of the 31 studies in the Chambers et al. synthesis, 14 included studies that merely described self-regulation strategies without an intervention. Five of the 31 studies reported that adults with disabilities showed higher levels of self-regulation and had an increased chance of “membership in a high quality of life group” (p. 6) in employment settings and higher or more-developed independent living skills. In the area of self-regulation perceptions, people with disabilities rated self-regulation skills as more important than educators and family members. Based on the findings, Chambers et al.

recommended that educators receive more training and begin teaching components of self-regulation, such as using self-regulation strategies, to younger students, before late middle school or high school, for success to be reached in later life. In addition, the authors suggested that self-regulation needs to be generalized to all settings, including academic settings, the work place, and home, in order for people with disabilities to reach a higher level of self-regulation and enjoy a better quality of life.

Konrad et al. (2007) reviewed intervention studies conducted between 2000 and 2006 that included self-regulation strategies while measuring performance on academic tasks for students with SLD or attention-deficit disorder. Self-regulation was described as "self-monitoring, choice making, problem solving, goal setting, self-regulation, self-advocacy and self-awareness" (Konrad et al., 2007, p. 91). The dependent variables in the interventions measured academic outcomes in reading, mathematics, or written expression. A total of 34 articles met inclusion requirements and resulted in 312 total participants, the majority of whom (64.7%) were at the elementary level. In academics, quality and rate of academic assignments completed by students were measured in mathematics (*n* = 20), writing (*n* = 7), reading (*n* = 9), and spelling (*n* = 3), along with other academic tasks (*n* = 6) and homework (*n* = 1). Most of the studies included self-regulation components (*n* = 30). Konrad et al. concluded that when interventions involved a combination of self-regulation strategies (monitoring on-task time with goal-setting) students were more successful on measures of productivity. In the area of academic quality, the strongest effect was found for goal setting during a mathematics intervention. A limitation in this synthesis is the need for academic measures as the primary dependent measure variable, especially because the focus on high-stakes testing and accountability often decreases time devoted to teaching self-regulation strategies.

In the final synthesis, Fowler et al. (2007) reviewed academic intervention studies that incorporated self-regulation strategies for students with disabilities from 2000 to 2005. The authors defined self-regulation strategies as “choice making, decision making, problem solving, goal setting and attainment, self-advocacy, self-awareness, self-management or self-efficacy” (p. 272). The self-regulation strategy had to be part of a reading, writing, mathematics, or spelling intervention that measured quality of work, productivity, assessment results, or academic behaviors (time on-task, percentage of questions asked, and/or organization). In this review, 11 studies met inclusion requirements, for a total of 156 participants, the majority (61.8%) of participants were diagnosed with developmental disabilities. The quality of classwork completed by students was measured as a dependent variable in the area of language arts (*n* = 4 studies) and mathematics (*n* = 2 studies). Productivity of classwork was measured only in the area of language arts (*n* = 1). Six studies measured academic performance but did not use “direct measures of accuracy or productivity” (p. 276). Fowler et al.’s findings were similar to those of Konrad et al. (2007) in that combining “strategies of self-management, goal-setting, and problem solving were most effective and used most frequently” (Fowler et al., 2007, p. 282). Evidence linking a self-determined person with a better quality of life was found in all three reviews, but additional research is needed in school settings involving students diagnosed with various disabilities, and at all grade levels, before widespread use of interventions combining self-regulation and mathematics can be recommended (Chambers et al., 2007; Fowler et al., 2007; Konrad et al., 2007; 2007).

INTERVENTIONS COMBINING MATHEMATICS AND SELF-REGULATION

As more schools apply RtI to the identification of students with specific learning disabilities, scientific, research-based interventions are in demand for struggling students, especially in the area of mathematics. Specially, interventions from the standards and focal points developed by NCTM are needed as more states begin to adapt these national standards and/or develop curricula that include components of these standards. The standards and focal points were created for prekindergarten through eighth grade and include concepts, skills, and procedures for teaching mathematical skills within (a) number and operations, (b) algebra, (c) geometry, (d) measurement, (e) data analysis, and (f) probability (NCTM, 2000). In addition to interventions completed within a specific mathematical domain, a number of studies have been completed with self-regulation components embedded within the intervention. Some examples of self-regulation strategies include (a) self-monitoring through self-correction, (b) choice making, (c) self-advocacy, and (d) goal setting. Mathematical interventions including self-regulation strategies will be reviewed based on self-regulation components within specific mathematical domains.

Mathematical domains

From a review of intervention literature in mathematics with participants that were identified as struggling and/or SLD within the 9 years between 2001 and 2010, I found that 11 studies embedded self-regulation within the intervention. In 8 of 11 interventions, the mathematical domain was number and operations (Axtell et al., 2009; Burns, 2005; Calderhead, Filter, & Albin, 2006; Flores, 2009; Flores et al., 2006; Jitendra et al., 2009; Joseph & Hunter, 2001; Lee, Stansberry, Kubina, & Wannarka, 2005; Powell et al., 2009). Number and operations is a broad domain that includes computation in all operations (addition, subtraction, multiplication, division) as well as fractions and ratios, rates, and proportions. NCTM (2001) provides three basic principles of number and operations for all grades, kindergarten through twelfth: (a) understanding numbers and how they relate to one another, as well as number systems; (b) meanings of the different operations, such as understanding that an answer in subtraction and division is less than the beginning part; and (c) fluency in solving, estimation skills, and understanding when an answer is reasonable.

Four studies that pertained to self-regulation strategy focused on multiplication facts with a goal to increase fluency of the facts in third grade (Burns, 2005), middle school, sixth grade through eighth grade (Calderhead et al., 2006; Flores et al., 2006; Lee et. al., 2005). Four other studies taught strategies for increasing fluency in mathematical facts in division (Axtell et al., 2009), fractions, addition and subtraction with common and unlike denominators (Joseph & Hunter, 2001), subtraction with regrouping (Flores, 2009), and addition and subtraction without regrouping (Powell et al., 2009). Even when one examines mathematical interventions without self-regulation strategies, the majority of those interventions are found to be in the domain of numbers and operations (Hopkins & Egeberg, 2009; Rhymer, Skinner, Henington, D'Reaux, & Sims, 1998; Woodward, 2006). Practice of the facts was the approach to increase fluency and automaticity of basic mathematical facts in the cited interventions.

Three studies implemented a word problem solving intervention (Jitendra et al., 2002; Jitendra et al., 2009; Van Garderen, 2007). NCTM does not define *word problem solving* as a specific domain but, rather, defines it as an integrated goal within all domains as an application of the skills taught. Two of the word-problem studies (Jitendra et al., 2002; Van Garderen, 2007) taught schema-based diagrams to solve multistep computational word problems. In both of these studies the participants were in Grade 8. The last study (Jitendra et al., 2009) described students in Grade 7 using schema-based instruction to teach ratio and proportions. These three studies focused on students' selecting the correct operation to solve the word problem, versus using a specific strategy to solve a whole-number computation. This may be a reason for why the studies were completed in middle school rather than elementary school—due to the expectations and recommendations that students have mastery of basic facts by Grade 5 (NCTM, 2001; NMAP, 2008).

Self-regulation components

In addition to mathematical intervention in the studies reviewed, the studies included a self-regulation component. Research has shown that motivated students tend to perform higher in the areas of fluency and accuracy in a variety of academic skills (Burns et al., 2010; Codding et al., 2009; Montague, 2007; Morgan & Sideridis, 2006; Rock, 2005). Utilizing self-regulation strategies may be one way to develop a more effective academic intervention (Lee et al., 2008). In a meta-analysis contrasting fluency interventions in reading, the authors specifically examined the role of the self-regulation strategy based on slope analysis and found that goal-setting with feedback and reinforcement was most "associated with significant growth" (Morgan & Sideridis, 2006, p. 196). In addition, following a chi-square difference test, (a) goal setting and feedback and (b) goal setting with feedback and reinforcement and reinforcement alone had larger effect sizes than reading intervention components only, such as listening, repeated readings, and tutoring. The authors concluded that "goal setting interventions work, and they seem to work better than other types of fluency interventions" (Morgan & Sideridis, 2006, p. 203). The self-regulation components within each intervention were categorized into the three self-regulation components of relatedness, autonomy, and competence by the myself, rather than the study's authors.

Relatedness

Finding academic interventions that included the relatedness component of self-regulation was difficult. Relatedness is part of intervention studies but is primarily involved in interventions focused on educational planning, dropout prevention, or adults with disabilities (Destefano, Heck, Hasazi, & Furney, 1999; Garriott, 2007; Wood, Karvonen, Test, Browder, & Algozzine, 2004). Of the 11 intervention studies in the area of mathematics, only 2 had a relatedness component. Flores (2009) and Flores et al. (2006), prior to the start of the intervention, the students and teachers signed a contract stating that they would work together to increase mathematical skills. This is considered a strategy of self-regulation within relatedness because the student is making a choice and self-regulating his or her behavior.

Autonomy

The self-regulation component of autonomy was used most often within mathematical intervention studies. In nine studies (Axtell et al., 2009; Burns, 2005; Flores, 2009; Flores et al., 2006; Jitendra et al., 2002; Joseph & Hunter, 2001; Lee et al., 2005; Powell et al., 2009; Van Garderen, 2007), the autonomy component was found in a self-regulation strategy. In 63% of the studies (Axtell et al., 2009; Burns, 2005; Flores, 2009; Flores et al., 2006; Jitendra et al., 2002; Joseph & Hunter, 2001; Powell et al., 2009), self-graphing was the self-regulation strategy used as part of the mathematical intervention. In the Axtell et al. (2009) study, students were given many opportunities to practice missed facts on individual mastery levels. In addition, the students were "competing" only against themselves by setting a goal and then graphing the results, all within the same day of the intervention. In the above studies, the researchers reported that students self-graphed and examined the results, but no specific data were collected on how this may have assisted, or been an additional variable in, student success.

Although these data were not reported, they do provide evidence of the importance of student engagement and monitoring of mathematical outcomes. The findings suggest that self-graphing may enhance mathematical skills and are easily implemented within an intervention.

Self-regulation or self-monitoring through the use of cue cards was another common self-regulation strategy. Three studies combined graphing and cue cards (Jitendra et al., 2002; Joseph & Hunter, 2001; Van Garderen, 2007). In Jitendra et al. (2002) and Joseph and Hunter (2001), cue cards were used to self-regulate and help learn to apply the strategy independently, thus increasing self-advocacy. Finally, Van Garderen (2007) utilized self-regulation and self-reflection during the intervention and measurement phases of the design. The students were taught the strategy to solve word problems, and cue cards were given to check steps completed. In addition, the students had to self-reflect to identify how the problem was solved and check work to verify correct answers. In examining behavior or academic trends, interventions that actively included the participant were more effective than interventions and rewards managed strictly via adults, educators, or researchers (Fowler et al., 2007). Students with SLD struggle with self-regulation, but they can be explicitly taught these skills (Montague, 2007). For example, when taught how to use a cue card and regulate time engaged, or provided choices in work completion, overall motivation for the academic task increased (Montague, 2007; Rock, 2005).

Competence

The use of easy problems mixed with unknown facts is a strategy within the competence component of the self-determination theory. It can be considered self-regulation, as a mastery rate is applied in selection of the problem types. Two articles utilized self-regulation component as a direct part of the intervention and did define the use and purpose (Calderhead et al., 2006; Lee et al., 2005). As part of the design, the ratio of familiar problems to unfamiliar problems varied. The purpose of these studies was to increase mastery of problems while building student confidence. The authors applied the basic principles of competence to increase confidence, time on task, accuracy, and productivity.

INSTRUCTIONAL ROUTINE FEATURES

While the majority of mathematical intervention studies fall within the number and operation domain, specifically in the area of fact instruction, instructional routines vary slightly for each study. Some similarities among interventions designed for struggling students are the use of explicit instruction and multiple representations. In designing intervention components, researchers need to include procedural strategies as well as problem-based learning, and increase mathematical flexibility in understanding numbers and relationships among numbers (Bottge, Rueda, Serlin, Hung, & Kwon, 2007; Gray et al., 1999; Pellegrino & Goldman, 1987). Instructional routine features will be reviewed, including (a) multiple representations, (b) information processing, and (c) strategic, systematic instruction.

Multiple representations

Students struggling in mathematics often apply ineffective strategies and are weak in three areas; retrieval fluency; procedure; and higher level, or conceptual, understanding (Gray et al., 1999; Impecoven-Lind & Foegen, 2010; Jayanthi, Gersten, Baker, 2008). Interventions can remediate these areas through strategy instruction that incorporates multiple representations (Gersten et al., 2009; Jayanthi et al., 2008; NMAP, 2008; Stein et al., 2006; Woodward, 2006). Multiple representations involve representing mathematical concepts in three ways: (a) concrete, (b) visual/pictorial, and (c) abstract. NMAP (2008), NCTM Core Standards (2010), and In Focus Grade 4 (2009) provide examples of the multiple representations. Concrete representations involve the use of objects, manipulatives to represent numbers and build models of concepts in operations, and fractions, decimals, and algebraic reasoning. Visual or pictorial representations can include pictures of the manipulatives, number lines, hundred charts, and drawings of models, such as area models of multiplication. Abstract representations refer to the numbers alone, without the use of the concrete or pictorial scaffolds. Representations do not need to be isolated, and students can utilize all representations within one lesson to understand a particular mathematical concept.

As Witzel, Mercer, and Miller (2003) pointed out, students need to solve problems accurately at each stage prior to moving to fewer scaffolds or more abstract representations. This gradual transition leads to mastery of the content, not simply the process (Strickland & Maccini, 2010; Witzel, 2005). Gray et al. (1999) explained that students with a “flexible notion of process … have a cognitive advantage; they derive considerable mathematical flexibility” (p. 120). To strengthen mathematical flexibility and the idea of choice in how to derive an answer, students need to master the process to understand the concept. The Center on Instruction’s (2008) mathematical recommendations for teachers emphasizes the importance of both multiple representations and examples, for scaffolding (Jayanthi et al., 2008). A combination of examples and nonexamples presented through a carefully designed intervention using multiple representations can lead to accurate generalizations (Stein et al., 2006).

Flores et al. (2009) and Flores (2006) utilized a combination of strategy instruction and multiple representations. These authors used multiple representations (concrete, pictorial, abstract) for subtraction with regrouping, as well as strategy instruction for multiplication. The strategy phase began with concrete materials to introduce facts, and then transitioned to pictorial representations while using the DRAW strategy (Discover the sign, Read the problem, Answer the problem with pictures, and Write the answer). The DRAW strategy was also used to teach subtracting with regrouping in the Flores (2009) intervention. Similar to the strategy procedures in multiplication, the students first used base-ten materials, moved to drawing pictures and learning the DRAW strategy, and then to a worksheet with just numbers (abstract). Percentage of non-overlapping data points (PND) was calculated for the Flores (2009) study by the author but was not calculated for the 2006 study because visual representations were not provided in the article. In Flores (2009), no overlapping data points were evident for any of the six participants. All six participants increased in number of correct digits once instruction began. For all participants, one data point was reported in the maintenance stage, and this data point did not overlap to baseline, consistent with progress reported during the instructional phase.

Information processing

Merely teaching a student to solve a mathematical problem is not enough; understanding how an answer is obtained is a key to success across a variety of mathematical curricula (Gray et al., 1999). Mathematics "goes beyond arithmetic" (Saracho & Spodek, 2009, p. 297) and encompasses processes at different levels (Pellegrino & Goldman, 1987). Information processing shapes how students learn and the impact of ability levels, or the zone of proximal development, on learning concepts and procedures in mathematics (Gray et al., 1999; Pellegrino & Goldman, 1987; Saracho & Spodek; 2009; Schmittau, 2004; Van Oers, 2002). The idea that mathematical activities happen simultaneously is defined as *information processing*.

Using procedural-only interventions or application interventions in isolation has been criticized as a limited approach for students with SLD or those with mathematical difficulties. Information processing is a bridge between the conceptual underpinning and the procedures for computation-type work. Aligning the procedural nature of special education research with more constructivist practices increases mathematical flexibility and generalization to other types of problems (Bottge et al., 2007; Grey et al., 1999; Jitendra et al., 2009). Gray et al. explained the notion of schemas to switch between a "process, a concept output by that process, and a symbol that can evoke either process or concept" (p. 113). Students identified as low achievers tend to store nonessential information, resulting in poor and inefficient strategy use. The ability to move from counting to manipulating numbers is seen in average to high-average learners (Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009; Gray et al., 1999; Morgan, Farkas, & Wu, 2009; Woodward, 2006). One way to remediate a weakness in this area is through strategy instruction, specifically in schema-based instruction.

Linking schema-based and strategy instruction may be a missing component in procedural aspects of special education interventions. Connecting more efficient methods for solving problems may increase the visual ability and flexibility in understanding concepts and processes that are required in more advanced mathematical courses (Bottge et al., 2007; Gray et al., 1999). The use of schema-based instruction specifically in word problem solving was utilized in two articles with participants with SLD (Jitendra et al., 2002; Van Garderen, 2007) and two articles with typical middle school students in intact classrooms (Bottge et al., 2007; Jitendra et al., 2009).

In the two studies by Jitendra et al. (2002, 2009), the goal was to teach students word problems through schematic instruction. During the instructional phase, problems were presented in story format with a diagram. Following instruction and mastery (100%

for two consecutive sessions), the schema-based strategy instruction was taught for varying problems, multiplicative comparison problems (2002), and ratio and proportion (2009). Once instruction was over, each participant completed a generalization assessment and a maintenance measure or posttest. Parker, Vannest, and Brown (2009) classified PND greater than 90% as a large effect, 70.1% to 90 % as a moderate effect, 50.1% to 70 % as a low effect size, and any percentage lower than 50 as not effective. In Jitendra et al. (2002), only one of the four participants had an overlapping data point, resulting in 90% PND. In 2009, Jitendra and colleagues extended the previous work to classrooms assigned an intervention (70 students) and control classrooms (78 students). The authors reported a significant difference in condition "favoring the schema-based instruction" (p. 261) across all ability types. Van Garderen (2007) also implemented the intervention and used diagrams similar to the schema-based intervention of Jitendra et al. (2002, 2009), as well as a strategy to apply to multistep word problem solving. This intervention was divided into three phases and utilized modeling, questioning, guided and independent practice, rehearsal, and feedback. The first phase introduced the students to diagrams and how to use a diagram to solve problems. In Phase 2, the students were taught the Visualize strategy, which connects drawing a diagram and arranging the diagrams to solve one-step word problems. The third phase of instruction was using the Visualize strategy to solve two-step problems. Throughout the intervention, the students had a self-regulation/cue card sheet to guide them through solving one- and two-step problems. The Van Garderen (2007) intervention also used a single-subject design across three students. The baseline trends were not as stable as other single-subject designs and resulted in an overlap of points for two students, with PNDs of 80% and 83% considered a moderate effect size.

The last intervention connecting the idea of information processing and real-world application to word problems was a study completed by Bottge et al. (2007). In this intervention, the typical mathematics curriculum was anchored to real-world word problems. The goal of the intervention was to increase students' ability to estimate and interpret schematic plans and "apply what they learn in related hands-on problems" (Bottge et al., 2007, p. 32). Their results showed that all students, regardless of disability or mathematical ability, benefited from the anchored, schema-based instruction. In addition, students with SLD in inclusive classes showed greater improvement between pre- and posttests than peers in the control classrooms. Developing more efficient strategy use, especially in word problem solving (Bottge et al., 2007; Jitendra et al., 2002; Jitendra et al., 2009; Pellegrino & Goldman, 1987), can increase students' ability to create mental representations of mathematical problems, leading to better choices in how to solve problems (Gray et al., 1999). All the reviewed studies focused on teaching students background knowledge about the different types of problems and how to identify each problem type, allowing for greater generalizability to other types of problems. This finding further strengthens the need to explicitly teach how to solve word problems in conjunction with the schema of the problem types.

Systematic, strategic instruction

Much of effective instruction for students with mathematical difficulties or who have a diagnosis of SLD is explicit and systematic. Explicit instruction is adopted from Direct Instruction, developed by Engelmann and Carnine in 1991. It includes teacher modeling, student practice with support, and then independent practice through clear, slightly scripted examples and nonexamples of strategies in both reading and mathematics (Bryant, 2005; D. Bryant et al., 2000; Gersten et al., 2009; Impecoven-Lind

& Foegen, 2010; Jayanthi et al., 2008; Scheuermann et al., 2009; Stein et al., 2006; Woodward, 2006). The use of explicit, systematic instruction results in positive effects and is more effective than traditional or mediated instruction (Gersten et al., 2009; Kroesbergen & Van Luit, 2003; Maccini et al., 2007; Montague et al., 1993). In a study completed by Kroesbergen, Van Luit, and Maas (2004) directly comparing constructivist-only instruction and explicit, systematic instruction, students receiving the explicit instruction proved to be more effective at solving multiplication problems. It is interesting to note that both treatments increased student automaticity and rate of improvement, but for students with SLD, the explicit instruction was again more effective. While it is important to include components of information processing within interventions, students with mathematical difficulties need the structure of schema-based and explicit instruction for success and improvement in solving both word problems and whole-number computation problems (Gersten et al., 2009; Jayanthi et al., 2008; Jitendra et al., 2002; Jitendra et al., 2009; Kroesbergen et al., 2004; Kroesbergen & Van Luit, 2003; NMAP, 2008; Stein et al., 2006).

In recent mathematical interventions, explicit, systematic instruction was widely employed to teach specific mathematics strategies. Flores (2009) and Flores et al. (2006) utilized the strategic instructional model (SIM) and constant time delay (CTD) to teach multiplication facts (Flores et al., 2006). The SIM strategy builds “independent strategy use and self-regulation through explicit instruction” (p. 46). Flores (2009) and Flores et al. (2006) applied the explicit SIM strategy for one phase and then CTD for another phase. During both phases, the participants were taught facts through the presentation of flash cards. During the CTD phase, the students responded orally with the answer and then completed a worksheet containing the same facts. During the SIM phase, students applied the SIM strategy, DRAW, to both subtraction and multiplication and increased digits correct per minute in both interventions. Flores et al. (2006) also reported that all but one participant enjoyed the strategic instruction and reported that the skill taught in the intervention could be applied to other mathematical tasks. Stronger confidence levels were seen by the special educator who participated in this study.

In an experimental study, Powell et al., (2009) randomly assigned participants to four different conditions—fact practice, conceptual fact instruction with practice, procedural computation, and no tutoring (control)—and compared and reported effect sizes for the two types of participants (mathematics difficulty only, mathematics and reading difficulty) within the three tutoring conditions and control. Students assigned to fact practice began the intervention session on a computer, followed by flash card practice, math fact review, and then a final independent-practice worksheet. The conceptual fact instruction group had a similar tutoring session starting with flash cards, computer instruction, number line flash card practice, fact family review, and then the same final independent-practice worksheet. The third group, procedural computation, also worked on the computer, practiced flash cards, and then completed a computation and place value review ending with the same timed independent-practice worksheet. Overall, students having only mathematical difficulties outperformed students with mathematics and reading difficulties. The tutoring groups that received fact instruction showed significant gains as compared to the control group. The authors reported an interaction effect between tutoring groups and mathematics difficulty type, and a Tukey post hoc was completed. The fact retrieval of students with mathematics difficulty only was better for groups receiving no fact instruction, as well as for the control group (Powell et al., 2009).

In a study by Joseph and Hunter (2001), the mathematical domain of fractions was taught through explicit, individual instruction with the assistance of a cue card. The students received modeling in how to solve addition and subtraction fractions with common and unlike denominators using a cue card. Each cue card provided directions on how to solve the problems and was used during the 10-item probes. Following each probe, the students graphed their results, and the special educator encouraged them to utilize the cue cards to raise the accuracy of correct digits. For all three participants, the baseline was stable and an increase was seen in number of correct digits once the intervention began. For two of the three participants, there were few overlapping data points when comparing baseline to intervention, resulting in a high percentage of PND: 94% and 93%, respectively.

Another piece of explicit instruction is the use of examples within systematic instruction (Jayanthi et al., 2008; NMAP, 2008). Three studies (Axtell et al., 2009; Burns, 2005; Calderhead et al., 2006) incorporated the traditional modeling, guided practice, independent practice found in Direct Instruction, and infused explicit systematic strategy instruction with immediate feedback to increase fluency and accuracy. Axtell et al. (2009) randomly assigned students to a math strategy intervention (Detect, Practice and Repair [DPR]), or a reading intervention (control). The basis of DPR is that students are given a set time to complete each problem and then must begin the next problem when a metronome clicks, regardless of whether the current problem was completed. The students were given a worksheet with 48 problems and 1 min 20 s to complete the page, with the clicks occurring 40 beats a minute, which equates to about 1.5 s for answering each division problem. Following this timed worksheet, the students self-corrected and circled the first 5 problems that were not completed or were completed incorrectly. The students re-copied the missed or incorrect problems with the answer on a Copy, Cover, and Compare (CCC) worksheet. After writing the problem and the answer, the students had to verbally repeat it to themselves five times. The next step was to cover the problem and rewrite it in the next block on the CCC worksheet and then compare to the first box. After completing this portion of the intervention, the students were given 1 min to complete a fluency worksheet containing division problems. The students then graphed the results of this worksheet. This intervention lasted a total of 45 min for 18 school days.

Axtell et al. (2009) completed an ANCOVA to adjust means and account for any differences between intervention and comparison groups in pretest scores. The control group received only a reading intervention, with no additional support in mathematics. The intervention group received a mathematics intervention with a self-regulation component and produced a higher adjusted mean posttest score ($M = 47.52$, $SD = 31.56$) as compared to the comparison group ($M = 33.31$, $SD = 13.44$). The authors reported that the intervention, "not only [identified] a statistical significant difference between the adjusted means of the two groups, but also a large effect size" (p. 534). In addition, a post hoc Mann-Whitney *U* test was completed ($U = 49$, $p < .01$), further adding to the argument that the intervention group outperformed the control group on the posttest assessment. The effect size was recalculated for this synthesis using the adjusted means and standard deviations reported, and this produced an effect size of .28, Cohen's $d = .59$. This effect size accounts for 28% of the variability of mathematical performance as a result of the intervention. Although the authors reported a large effect size, a Cohen's *d* of .59 is considered medium by Gravetter and Wallnau (2007) in *Statistics for the Behavioral Sciences*.

A component of explicit instruction is the selection of items to be included in modeling, guided practice, and independent practice. The selection of items should include examples and nonexamples, as well as items to increase generalization. In both Calderhead et al. (2006) and Burns (2005), the instructional content of the intervention was taught using a procedure called *incremental rehearsal*. Incremental rehearsal is defined in both articles as a "gradually increasing ratio of known to unknown items reaching, at the final stage of implementation, 90% to 10%" (Burns, 2005, p. 238). For both interventions, the time devoted to the independent variable was 5 to 10 min per day. Calderhead et al. created worksheets that were mixed with mastered (addition and subtraction) and challenging (multiplication) problems. Each day the participants would complete three different worksheets containing 18 problems per worksheet. The first worksheet contained 6 challenging problems and 12 easy problems. The next worksheet contained 12 challenging problems and 6 easy problems. The last worksheet contained 18 challenging problems. While the author did report that students answered more problems correct on the mixed worksheets (challenging and easy) compared to worksheets with all challenging problems, PND could not be calculated for this review because graphs were not included in the study. Practically, mixing challenging and easy problems can increase the amount of problems completed, as well as accuracy (Calderhead et al., 2006). Burns' (2005) study maintained a 10%:90% ratio of new items to known items. The intervention was presented to the participants in a one-on-one setting using flash cards containing multiplication facts. The first unknown fact was taught to and orally rehearsed by the student. The unknown fact was then presented alongside known facts, increasing the amount of known facts between each display of an unknown fact. Because of differences among students, the intervention lasted 5 to 10 min, ending once the child had rehearsed all 10 fact cards or answered incorrectly three times for the unknown fact. Burns had three participants and no overlapping data, 100% PND. From baseline to intervention, all participants' scores on number of digits correct increased over time during the intervention phase.

Similar to incremental rehearsal is the use of high-preference tasks (Lee et al., 2005). In this intervention, the student was explicitly taught multiplication facts through a model, prompt, and check format. The teacher stated the fact and product (i.e. 4 x 3 = 12), which the student then repeated. Each fact was presented twice during modeling. In the prompt stage, the teacher stated the fact and the answer and then asked the student for the answer. During the check stage, the student stated the correct answer when presented with a flash card two times. Following the second round of checking with the flash card, the student completed a worksheet. The intervention was counter-balanced and alternated each day with either a high-p day (explicit instruction embedded with easy problems) or explicit instruction alone. On high-p-condition days, an easy problem (addition) was presented prior to a multiplication problem with the flash cards and on the worksheet. Similar to Calderhead et al.'s (2006) study, no effect sizes were reported or could be calculated for this review. The authors did report that explicit instruction with a high-p day resulted in students' solving more problems correct with greater fluency. The key to effective explicit instruction is a well-considered plan for the specific skills and strategies to teach. Including components from information processing using multiple representations in a structured, balanced intervention can mean greater access to grade-level mathematics and long-term success in advanced mathematical courses (Bottge et al., 2007; Gersten et al., 2009; Gray et al., 1999; Kroesbergen & Van Luit, 2003; Maccini et al., 2007; Montague et al., 1993; NMAP; 2008).

ASSESSMENT AND PROGRESS MONITORING

Important components in diagnosing SLD through an RtI model involve placing students in intervention groups and making data-based decisions using frequent assessments and ongoing monitoring of growth. Both NMAP (2008) and the IES recommendations for mathematical instruction (Jayanthi et al., 2008) highlight the need for formative assessments by both the teacher and the student. Ongoing data collection assists educators in making changes to whole class curricula and small-group interventions, and provides guidance on differentiation for diverse learners (Fuchs et al., 2010; NMAP, 2008; Shapiro, 2010; Shapiro, Edwards, & Zigmond, 2005). In the RtI process, growth and effectiveness of interventions can be assessed at all levels of the tiered system. Curriculum-based measures (CBM) are often used because progress monitoring measures are easy to create and administer (Shapiro et al., 2005). Variables measured via CBM include correct digits, correct items, mastery levels, and comparisons to both national and local norms (Foegen, 2008; Fuchs et al., 2010; Shapiro, 2010). An analytical tool used in conjunction with CBM is slope and regression (Fuchs, Fuchs, & Zumeta, 2008; Graney, Missall, Martinez, & Bergstrom, 2009; Morgan & Sideridis, 2006; VanDerHeyden & Burns, 2008). Slope and regression allows an educator to identify the rate of growth and determine whether the student is responding to the intervention at a rate similar to that of peers without weaknesses in mathematics.

CBM was designed to be used by educators as an easy assessment to track small changes in reading, writing, and mathematics in order to make decisions about instruction (Deno, 1985). CBM began in the special education field but has transcended into general education as a tool to measure progress as part of an RtI process, because it is sensitive to growth and reliable (Deno, 2003; Foegen, Jiban, & Deno, 2007; Graney et al., 2009; Shapiro et al., 2005; VanDerHeyden & Burns, 2008). CBM involves administering short, timed probes that provide information about students' fluency and accuracy. In the area of mathematics, CBM-M probes can be mixed-skill or single computation skills and take 2 to 5 min to administer. In the area of reading, extensive CBM research exists with passages normed and deemed technically adequate in the areas of fluency and comprehension. One challenge in the area of mathematics is the types of math problems to be included, as well as the decision to test mixed or single skills (Fuchs et al., 2008; Thurber, Shinn, & Smolkowski, 2002; VanDerHeyden & Burns, 2008). Fuchs et al. described two types of mathematical CBMs traditionally utilized in schools, CBM computation and CBM application.

CBM-M computation

CBM-M computation assessment traditionally includes basic, single-digit mathematical operations in addition, subtraction, multiplication, and division (Fuchs et al., 2008; Thurber et al., 2002). Probes can include mixed or isolated operations. Shapiro et al. (2005) created probes with mixed operations in their study with students in first grade through sixth grade. First- and second-grade probes had mixed addition and subtraction problems, with and without regrouping. Third-grade probes contained single-digit multiplication and division problems; fourth-grade probes included multidigit multiplication, multidigit division, and fractions. Decimals and division with remainders made up the fifth-grade and sixth-grade probes. In a similar study by Graney et al. (2009), all probes were mixed, third-grade performance was assessed on addition and subtraction problems, and fourth- and fifth-grade performance was assessed on addition, subtraction, multiplication, and division problems. The students in third grade had 2 min to work, and fourth and fifth graders were given 4 min. In another study (Thurber et al., 2002) examining CBM computation, researchers administered probes for fourth-grade students that contained mixed addition, subtraction, multiplication, and division problems, all simple with no regrouping or remainders with a 2-min time limit. All studies used a scoring procedure that counted digits correct per minute to assess student achievement.

When scoring a CBM-M probe for digits correct, all digits are counted to derive the score. For example 9 × 4 = 36 would result in a score of two digits correct, whereas 3 × 1 = 3 would be one digit correct. Digits correct are also counted for solutions that may be wrong; for example, for 9 × 4 = 32, the student would score 1 point because the 3 is in the tens place. For computation problems containing multiple steps, all digits above the equal line, except the problem, can be counted (i.e., 12 × 32 = 24 + 460 = 484; 8 digits correct). This allows an educator to examine for possible errors and gives students credit for partial completion. Although digits correct per minute can be calculated for all probes, traditionally probes in subtraction and division problems score total items correct to identify an overall percentage correct. This approach is used rather than digits correct per minute because most answers contain only one digit, if they are of the basic fact problem type (Shapiro, 2010; E. Shapiro, personal communication, February 2011). In the majority of mathematical intervention studies reviewed, digits correct was used for participant selection criteria as well as progress monitoring during the study (Axtell et al., 2009; Burns, 2005; Calderhead et al., 2006; Flores, 2009; Flores et al., 2006; Powell et al., 2009).

Shapiro (2010) identified classifications for labeling students in terms of their achievement levels. The classifications include mastery, instructional, and frustrational levels using a median for at least three scores using digits correct per minute. In first grade through third grade, a student at the frustrational level scores 0 to 9 digits correct, a student at the instructional level scores 10 to 19, and a student at the mastery level scores 20 and above. For fourth grade, students at the frustrational level scores 0 to 19 digits correct, students at the instructional level score 20 to 39, and students at the mastery level score 40 or above. Shapiro (2010) and NCRtI recommend using probes from the previous grade level and comparing those to the 25th percentile to identify students in need of supplemental intervention. Using spring normative data at the 25th percentile allows educators to identify students' computational skills as high or moderate. In fourth grade a student computational score is high at 17 digits correct and a student's computational score is moderate at 13 digits. In general, these guidelines can be applied, but commercial products, like AIMSweb ©, contain normative data. It is suggested that local norms be used as much as possible, because the class as a whole "should be monitored at its appropriate grade level" (Fuchs et al., 2008, p. 228). The use of local norms is echoed in the work of Graney et al. (2009) because an assumption exists that "one sample of students' progress on the CBM tools would provide general guidelines for other schools and districts in setting goals for their own students" (p. 123). However, not all students perform at the same rate. A few studies (Fuchs et al., 2008; Graney et al., 2009; Shapiro et al., 2005) have examined the rate of growth, that is, slope, of a particular group of students so that educators could make decisions and write educational goals that were equivalent to peers (not necessarily national norms). Even in these three studies, the rate of improvement per week is different, depending on the sample of students. Shapiro et al. (2005) reported a computation range of 0.123 to 1.77, with a mean of .38 digits correct per week. In Graney et al. (2009), weekly growth was calculated by grade level (Grade 3 through Grade 5) and totaled for 2 years. Contrary to other published findings, the authors found greater gains between winter and spring versus fall to winter, and the rate of growth was different from that reported by Shapiro et al. (2005). Fuchs et al. (2008) reported a rate of slope to be a range of .4 (Grade 2) to .7 (Grades 4–6) digits correct. These authors also supported the use of local norms or setting goals based on a student's ability by examining past performance to set future performance goals, called an "intraindividual framework" (p. 228).

CBM-M application

CBM-M application involves administering a probe containing mixed application problems that one would expect a student to be exposed to in his or her enrolled grade level (Fuchs et al., 25008; Shapiro et al., 2005). CBM-M application probes contain multistep problems with basic operations, as well as word problems, graphs, fractions, and other mathematical concepts. The use of this type of probe is more relevant to individual schools and districts because of the differences among state standards for mathematics (VanDerHeyden & Burns, 2008). In designing these probes, an educator also has to consider previous grade level application problems to identify where a weakness exists and/or identify misconceptions a child has developed in the area of mathematics (Shapiro, 2010). CBM-M application probes are still timed, but other variables, such as reading and slower processing of the higher level skills, may deflate a student's score (Shapiro 2010; Thurber et al., 2002; VanDerHeyden & Burns, 2008). This type of probe should be carefully analyzed for error patterns and examined alongside a computation probe to set appropriate long- and short-terms goals for students. Administering one or both types of CBM-M, computation or application, can provide invaluable data to educators concerning students in need of additional instructional support, such as a mathematics intervention. The data from a CBM-M can be used to design intervention components, as well as measure ongoing progress. In addition, the probes are simple, and students can be involved in the measurement process through self-regulation, such as graphing and goal-setting, and become active participants in educational planning.

SUMMARY OF FINDINGS

The purpose of this literature review was to better understand five overall domains of mathematics research: (a) characteristics of students with mathematical difficulties, (b) mathematical theories, (c) mathematical interventions, (d) instructional routine features, and (e) progress monitoring. According to the research, students with mathematical difficulties are characterized as having weaknesses in working memory, number sense, and computational fluency. Interventions aimed at improving these weaknesses need to be explicit, yet challenging for students. Students need a combination of both a constructivist approach and a deeper conceptual understanding to extend generalization and choice in how to answer multistep problems (Gray et al., 1999). Applying information processing and using examples, nonexamples, and strategy instruction can lead to long-term understanding and success. Without interventions in these areas, long-term success in algebra is hindered, thus increasing the need for remediation in high school and postsecondary settings (NMAP, 2008). A combination of the procedural components and conceptual understanding, combined with self-regulation of feedback, goal setting, and graphing, can lead to higher effect sizes and increased motivation for the task. The use of progress monitoring by both the educator and the student provides ongoing data to modify the intervention. Future research needs to be conducted to measure the effectiveness of the mathematical components and the self-regulation components when working with students with mathematical difficulties.

Chapter 3: Method

Students need to develop computational proficiency with facts (i.e., addition, subtraction, multiplication, and division) to be successful in more advanced mathematics, such as fractions, decimals, ratios, and rates (Gersten et al., 2009a; NCTM, 2010; NMAP, 2008). Specifically, the Curriculum Focal Points (NCTM, 2006) emphasize the importance of automaticity in computational skills, as well as students' ability to apply computational skills to solving word problems. The development and proficiency of facts needs to occur in elementary school. At the elementary level, addition and subtraction should be taught and mastered by the end of third grade, and multiplication and division should be introduced in third grade and mastered by the end of fifth grade (NCTM, 2006; NMAP, 2008; Shapiro, 2010; VanDerHeyden & Burns, 2009). However, instruction in basic facts is limited, and students often require remediation of facts beyond the elementary level for success in the upper grades (NMAP, 2008). Thus, educators need research-based, efficient, and effective interventions to assist students struggling with automaticity and accuracy in basic facts.

In addition to instruction in basic facts, the use of self-regulation strategies such as student feedback through self-correction, graphing, and goal setting has yielded positive gains and an increase in mastery levels in words read per minute and digits correct (Rock, 2005). Incorporation of self-regulation strategies for feedback has included graphing (accuracy and fluency) and goal-setting, and has shown increases in student engagement in mathematics (Konrad & Test, 2007; Rock, 2005). In one study, Billington, Skinner, and Cruchon (2004) found that by incorporating self-regulation strategies (graphing of accuracy), mathematics work completed in class increased and the time needed to complete the assignments decreased for students with a weakness in fact automaticity.

The number of studies published each year in the area of mathematics has increased recently but is limited in the area of mathematics interventions. When searching for studies that report the added value of including and measuring the effects of self-regulation strategies, such as goal-setting and graphing, the number of studies in mathematics drops significantly. Thus, the purpose of this study was to determine the effects of basic facts and self-regulation strategies, taught as part of an explicit and systematic mathematics intervention, on the mastery and accuracy of multiplication and division skills of fourth-grade students with mathematical difficulties. Fourth grade was selected on the recommendations of NCTM (2009) that students in Grade 4 need to understand and compute facts quickly to "help them become more efficient in multiplying multi-digit numbers. Students' success in mathematical content they encounter in later grades is enhanced by their ability to quickly recall basic multiplication and division facts" (NCTM , 2009, p. 11). Additionally, the National Core Standards (2010) hold that one of the three critical areas within mathematics curricula is fluency in multiplication and understanding of division.

Considering the importance of how basic facts relate to fractions, geometry, and algebra (Core Standards, 2010; Gersten et al., 2009a; NMAP, 2008), this study seeks to answer the following question for Grade 4 students with mathematical difficulties: To what extent will scores on multiplication and division fact computation probes differ as a result of a systematic, strategic intervention, plus goal setting and graphing, as compared to a systematic, strategic intervention without goal setting and graphing? This chapter describes the methodology for this study, including (a) setting and participants, (b) research design, (c) measures, (d) procedure, (e) instructional routine, (f) data analysis, and (g) social validity.

SETTING AND PARTICIPANTS

School

The elementary school was located in a district in central Texas. The school served students from prekindergarten through Grade 5. The demographics of the district were similar to state demographics, obtained from the Texas Education Agency. In the school district, a total of 22.6% of students were African American (14.2% state), 37.5% were Hispanic (47.9% state), 30.7% were Euro-American (34% state), .3% were Native American (.4% state), and 9% were Asian/Pacific Islander (3.6% state). In the district, 44.4% of students are categorized as of low socioeconomic status (56.7% state), 16% as limited English proficiency (LEP) (16.9% state), and 47.4% of students as at-risk, as defined by the state (48.3).

The elementary school was a Title I school, according to the number of students who received free/reduced-price lunch—58%, which was higher than both the district average (44.4%) and the state average (56.7%). This elementary school had a higher percentage of LEP students (25.4%) compared to both the district (16%) and the state (16.9%), as well as students identified as at-risk for academic failure (56.6%) (district, 47.4%, and state, 48.3%). The ethnic makeup of the school was similar to the district's: 17.4% were African American, 44.1% were Hispanic, 25.4% were Euro-American, and 12.3% were Asian/Pacific Islander. This school was chosen because of the high percentage of at-risk and low-socioeconomic status students. Student mobility tended to be higher, about 18% of total student population.

Intervention setting

All intervention sessions occurred within the students' elementary school. Tutoring was held in a small room near the student's classroom after school. The school was the recipient of a grant from the Texas Education Agency (called "21st Century") that provided after school tutoring and enrichment to students at risk of academic failure and/or drop-out risk. In addition, the classroom teachers provided tutoring twice a week after school for students who teachers feared might fail the state test in reading and mathematics. Nine students participated in this mathematics intervention. Of the nine participating students, eight were receiving after-school tutoring for the state test and three had been participating in the 21st Century project since the beginning of the school year.

The researcher worked closely with the classroom teachers and campus staff. The campus administrator approved the after-school tutoring time and secured the location for tutoring. The researcher and a certified teacher with a master's degree in special education completed all sessions of the pull-out tutoring. The master teacher attended a tutoring session and participated in training prior to tutoring. This teacher had been working with the researcher as a tutor on another project and was very familiar with the instructional components, pacing, and administration of instructional probes.

PARTICIPANT SELECTION

Consent was obtained from 77 (93%) students. All students with a returned consent were eligible for the study. Participants were not excluded if they were identified with a specific learning disability and/or as an English Language Learner, as long as their core mathematics instruction was the same as that for their peers. Several steps were taken to identify the participants for this study. In Step 1, the 77 students were given the General Operation Assessments in addition, subtraction, multiplication, and division on two separate days. In Step 2, following the completion of the assessments, local norms were developed for the results of digits correct per minute and items correct per minute; see Table 3.1 for local norms in multiplication and division. The scores were divided into three categories: (a) frustrational, (b) instructional, and (c) mastery. Shapiro (2004) classified 0 to 19 digits correct per minute as frustrational, 20 to 39 digits correct as instructional, and 40 or more digits correct per minute as mastery. The National Center on Response to Intervention (NCRTI) and Institute of Education Sciences (IES), What Works Clearinghouse, recommends that teachers administer a benchmark assessment to the class and school population and calculate both items correct and digits correct. Using the local norms allows a school to serve those students most in need (Fuchs et al., 2008). Thus, local norms were used to select the participants most in need of instruction in multiplication and division facts as compared to peers. Refer to Table 3.1 for local norms. Means, standard deviations, and percentiles are reported for both selection assessments given to the fourth graders. In multiplication, norms are reported for both items correct and digits correct per minute. In division, only items-correct norms are reported.

	Multiplication (*N* = 76)					
	Assessment 1: (IC)	**Assessment 1: (DC)**	**Assessment 2: (IC)**	**Assessment 2: (DC)**	**Assessment 1 & 2 (IC) Means**	**Assessment 1 & 2, (DC) Means**
Mean	21.32	34.71	23.78	38.13	22.55	36.42
Standard Deviation	8.446	14.423	8.391	14.403	8.4185	28.826
Percentiles:						
5th	9.8	14.7	11.8	15	10.8	14.85
10th	11.9	18.8	13	20.8	12.4	19.8
25th	16	26	18	29	17	27.5
50th	21	34	22	35	21.5	34.5
75th	25	41.5	30	49	27.5	45.25
90th	34.2	54.6	34.2	58.2	34.2	56.4

Note. IC = Items correct; DC = Digits correct.

	Division (*N* = 76)		
	Assessment 1: (IC)	**Assessment 2: (IC)**	**Assessment 1 & 2 (IC) Means**
Mean	10.89	13.7	12.30
Standard Deviation	7.84	9.22	8.53
Percentiles:			
5th	.00	.9	.45
10th	1.0	2.0	1.5
25th	4.25	7.0	5.63
50th	10	13	11.5
75th	16	19	17.5
90th	20.3	27	23.65

Note. IC = Items correct; DC = Digits correct.

Table 3.1: Beginning of Year Local Norms

In Step 3, students were selected for participation in the study based on the total number of items correct on the multiplication assessments. On the basis of the local norms, students scoring at or below the 25th percentile for total items correct on the multiplication general operations assessment were considered for this study. The 25th percentile was chosen based on the results from the sample of the lowest scoring students within Grade 4, often identified as Tier II and in need of intervention (Fuchs et al., 2008). Burns et al. (2010) applied a similar criterion in a review of mathematics interventions and found that interventions that focused on students at the frustrational level (<19) made greater gains, but success was still evident in students identified as instructional (>19). The goal of class wide assessments in basic operations is to determine who is below, and in need of support within, the sample population. Using local norms is the key in an academic area in which national norms are still emerging, such as mathematics (Shapiro, 2010). Addition is often cited as a prerequisite for multiplication (NCTM, In Focus Grade 4, 2009), but no literature could be located describing how addition proficiency was used as a criterion for inclusion for a multiplication intervention. It was originally proposed that students needed to score 40 or more digits correct (mastery level) on the addition assessments as part of the selection criteria, but according to the local norms this was too stringent and resulted in only one student who scored proficient in addition and frustrational in multiplication. Therefore, data were collected on performance on addition, subtraction, and division, but items correct on multiplication were the only scores used for the selection requirement.

In Step 4, following the selection of students, the researcher discussed student selection with the classroom teacher to confirm the data. The classroom teacher was asked how the students performed on classroom material and/or assessments and whether he or she believed that an intervention was warranted. All teachers reported that the selected students were low in fluency of basic facts. The teacher input confirmed participation of students for the intervention.

PARTICIPANTS

Following the General Operations Assessments, a total of 12 students met criteria for inclusion in the study. Of those 12, 2 were removed because they received mathematics instruction within a special education resource setting. Another student was removed because he was unable to stay after school for tutoring. Of the remaining 9 students included in the study, 8 were selected to participate in the school's after-school tutorials due to low ability in reading and/or mathematics and their risk for failing the state test. In addition to the educator's tutorials, 3 students were already part of the after-school enrichment grant. The 21st Century project is a state-sponsored grant targeting students at risk for academic failure. The grant provides enrichment opportunities in academics, technology, and physical education and provides students with transportation.

Demographic data for the participants are reported below. All participants' names have been changed; students were allowed to select their pseudo names. All students were in Grade 4 at the same elementary school. Seven of the nine participants received intervention from late January to the end of April. Two students (Cinderella and Rony) were removed from the 21st Century program due to behaviors on the afternoon bus in early March and did not have transportation to continue the study after school. The students were divided into two groups based on their mean digits correct per minute in multiplication. Group 1 mean scores were at or below 19 digits correct per minute—within the frustrational range. Group 2 mean scores were between 20 and 22 digits correct per minute and fell within the instructional range. Compared to local norms, the mean scores of digits correct per minute for each group were below the 10th percentile. The students are presented by group.

Group 1

Ambious

Ambious is a Euro-American male. At the start of the intervention he was 10 years 4 months old. He has been enrolled in the school since kindergarten. He had received mathematics intervention in Grades 1 and 2 and after-school tutorials in Grade 3. He passed the state test in Grade 3 in reading and mathematics. His teacher reported that he was a hard-working student but worked very slowly (though with minimal errors). On the selection assessments in multiplication, Ambious scored 8 items correct over 2 days, with 12 and 13 digits correct, respectively. He had no errors on the multiplication assessments. He answered only one hard fact (i.e., 3 × 4) and seven easy facts (0, 1, 2, 5, 10) each day. In addition, Ambious's mean score for items correct was 11, with a mean of 16 digits correct per minute. Again, his accuracy was high: 100%. Ambious reported that he "kind of liked math."

Aleva

Aleva is a Euro-American male. At the start of the intervention Aleva was 9 years 6 months old and qualified for free/reduced-price lunch. He has been enrolled in the school since Grade 2. He did not receive mathematical interventions in Grade 1 or 2 and was part of after-school tutorials in Grade 3 and Grade 4. His teacher reported that he struggled in mathematics but did pass the state test in reading and mathematics in Grade 3. On the selection assessments in multiplication, Aleva's mean scores in items correct and digits correct per minute were 14 and 19, respectively. He had less than 2 errors per assessment and answered primarily easy facts, with a mean of 12.5. He answered only one hard fact on each assessment. In division, his items-correct mean was 7 and only easy problems were completed. In addition, the mean scores for total items were 11 and 13, with an accuracy of 100%. Aleva reported that he “does not really like mathematics.”

Brittney

Brittney is a Hispanic female. At the start of the intervention Brittney was 10 years 4 months old and qualified for free/reduced-price lunch. She has been enrolled in the school since Grade 3 and received intervention in mathematics during after-school tutorials in Grade 3 and Grade 4. She passed the state test in Grade 3 in reading and mathematics. Brittney’s multiplication mean scores on the selection assessment in items correct and digits correct per minute were 9 and 15, respectively. Her accuracy was 98%. On the multiplication assessments, Brittney answered 8 and 7 easy facts over 2 days and 3 and 0 hard facts over 2 days. In addition, her mean scores for total items and digits correct per minute were 23 and 36. Her addition scores were some of the highest of the students selected for the intervention, showing 100% accuracy. Her mean score for items correct in division was 3. Brittney does not “really like math,” and her teacher reported that she often struggles in all subjects. Her teacher reported that Brittney was able to do well in class when she applied herself but lacked motivation when the work seemed difficult.

Avery

Avery is an African American female. At the start of the intervention, Avery was 9 years 11 months old. She enrolled in the school this year and did receive after-school tutorials. Her mean score on the selection assessment in multiplication was 11 items correct and 16 digits correct. Similar to the other students in Group 1, Avery only answered 1 and 2 hard facts over the 2 days and 10 and 8 easy facts over the 2 days, respectively. Her accuracy was high—98 % for all assessments. In division the mean score for items correct was 10. Avery reported that she does "not really like math, but does enjoy school." Her teacher reported that she was a very kind student who worked very hard in all subject areas.

Group 2

DJ

DJ is a Hispanic male. At the start of the intervention, DJ was 9 years 7 months old and qualified for free/reduced-price lunch. He had just enrolled in this school in Grade 4 but self-reported that he had attended after-school tutorials in Grade 3 as well. On the multiplication selection assessments, DJ scored 13 items correct and 21 digits correct over 2 days. He only answered 2 hard facts each day and 11 easy facts with 100% accuracy. On the division selection assessment DJ's mean score for items correct was 12. In addition, his mean score was 24 items correct and 37 digits correct. DJ reported that he "kind of likes math, but it is not my favorite subject." He did pass the state mathematics assessment in Grade 3. His teacher reported that his fluency hindered his ability in both reading and mathematics but that he did apply himself in class.

Taylor

Taylor is an African American female. At the start of the intervention Arianna was 10 years old. She has been enrolled at this school since kindergarten and received after-school tutorials in Grade 3 and Grade 4. She did not receive intervention in Grades 1 and 2 but did pass the state assessment in reading and mathematics. Her scores on the multiplication selection assessment were 11 and 13 items correct and 19 and 21 digits correct. She answered two hard facts each day and nine easy facts with 100% accuracy.

On the division selection assessment Taylor's mean score for items correct was 4. In addition Taylor scored 22 items correct over 2 days and 34 and 35 digits correct. Taylor reported that she "kind of likes math and enjoys school." Her teacher reported that she had to work really hard in all subjects to pass but was willing to stay after school for additional support.

Haley

Haley is an African American female. At the start of the intervention, Haley was 9 years 8 months old and was eligible for free/reduced-price lunch. She has been enrolled in this school since Grade 3 and received after-school tutorials in Grade 3 and Grade 4. Her mean scores in multiplication on the initial selection assessment were 12 items correct and 21 digits correct. In addition she scored 33 correct items both days and 53 and 52 digits correct. In multiplication she solved only 3 hard facts each day and 9 and 10 easy facts over the 2 days, with accuracy at 86%. She reported that she "kind of enjoys math." In division the mean score for items correct was 3. Her teacher reported that she worked hard but often did not turn in assignments on time. She did not do well on class tests and had to stay after school to make corrections numerous times.

Cinderella

Cinderella is a Hispanic female. At the start of the intervention Cinderella was 10 years old and was eligible for free/reduced-price lunch. She had enrolled in this school this year and reported that she has been to a total of five schools between kindergarten and Grade 4. She had been part of some kind of after-school tutorials/intervention from Grade 1 to Grade 4. On the multiplication selection assessment, Cinderella scored 14 and 13 items correct and 22 and 21 digits correct. She only answered 3 hard facts each day and 11 and 10 easy facts. Her accuracy was still relatively high—90% for both assessment days. In addition she scored 21 and 20 items correct and 32 and 30 digits correct. On the division selection assessment her mean score for items correct was She reported that she "totally likes math and school." Her teacher reported that Cinderella was a hard worker but her off-task behavior hindered her learning. She did get in trouble a few times prior to the start of the intervention, both in school and after school. She was removed from the intervention in March, when she was expelled from the 21st Century after-school grant for inappropriate behavior on the bus.

Rony

Rony is a Euro-American male. At the start of the intervention, Rony was 9 years 5 months old. He had been enrolled only this year at this school and was part of the after-school tutorials. His mean scores on the multiplication assessment were 17 items correct and 20 digits correct over 2 days. He only answered 1 hard fact each day and 16 and 17 easy facts, respectively. His accuracy was 100%, and he skipped around the assessment to answer all 1s and 10s facts. In addition his scores were very different on the two assessments: 26 and 16 items correct and 36 and 25 digits correct, respectively. His teacher reported that he lacked confidence in his ability and, depending on the day, scored very differently on classwork and tests. He had been in trouble in class for not completing work, being off task, and causing disruptive behavior, and did much better in small groups. On the division selection assessment the mean score for items correct was 4, and he scored very differently across the two days (1 and 7 items correct). He reported that he "kind of likes math and likes school some days." He was removed from the intervention in March, when he was expelled from the 21st Century after-school grant for inappropriate behavior on the bus.

RESEARCH DESIGN

A single-subject, reversal (A-B-C-B-C-B-C-D or A-C-B-C-B-C-B-D) treatment design was proposed to allow for three replications of the two intervention phases. Due to the lack of replication, a single-subject, time-series design was utilized for this study, resulting in an A-C-B-D (Group 1) and A-B-C-B-D (Group 2). The time –series design is appropriate for this study because it allows a comparison of slope across phases, as well as test for change of level. As stated by Crosbie (1993), this study allows for a "small number of observation," and "is ideal for clinicians and clinical researchers" (p. 969). Campbell and Stanley (1966) described a similar time – series method to determine the effectiveness of an intervention within a single-subject design. This studies design was used to test the differences between explicit, systematic intervention with and without self-regulation components, and time needed to meet the phase change criteria. (1) Fact intervention alone (Phase B) focused on explicit strategies for solving hard facts in multiplication and division and (2) fact intervention plus (Phase C) focused on explicit strategies for solving hard facts in multiplication and division plus self-regulation components (i.e., self-correction, goal setting, and graphing). Phase D was the maintenance phase. In Phase D the students were administered the General Operations Assessment 2 weeks and 3 weeks following the extinction of the intervention. This design was chosen because it allows a component analysis to be completed, which can then be compared to baseline (Phase A) and the two interventions, Phase B and Phase C, within groups and between groups. The results are considered preliminary or a pilot study due to the change of design.

Second, this design was chosen because the intervention can begin for all students immediately following baseline. Kennedy (2005) stated that the phase changes permit "strong statements about possible functional relations between the B – C - B components of the study" (p. 132). Horner et al. (2005) recommended that single-subject designs have at least three distinct phases. The design for this study had four phases: (a) baseline, (b) fact intervention alone, (c) fact intervention plus, and (d) maintenance, but did not demonstrate three replications of the intervention phases.

The intervention began in January, and the tutoring sessions ended the final week of April. That end date was selected because of state testing and the need to allow for 2 weeks between end of intervention and the first maintenance data point. The intervention design for Group 1 (Ambious, Aleva, Brittney, Avery) followed the A – C – B – D sequence. The intervention design for Group 2 (DJ, Taylor, Haley, Cinderella, Rony) followed the A – B – C – B –D sequence. The second phase B (Intervention Alone) includes only two data points that were completed prior to the extinction of the intervention date. The phase changed as a new set of facts was introduced after the group mean met the phase change criteria. The two interventions, *Intervention Alone* (Phase B) and *Intervention Plus* (Phase C) were counter-balanced so that both interventions began at the same time across each group of students. Also the intervention was counter-balanced, to control for possible familiarity with one of the other components, and reduce claims that the familiarity led to the changes. Both groups of students did complete the two maintenance data points.

DATA COLLECTION AND ANALYSIS

Both individual and group data were collected on items correct, digits correct (multiplication), accuracy, and problem type (easy and hard problems). Group means were used to determine when a group changed phases. Local norms were used to set the phase change criteria for both multiplication and division. The mean score for items correct was used rather than digits correct because it could be used for both multiplication and division. Digits correct was not used as progress monitoring data in division because most quotients have only one digit, thus resulting in students having to answer almost 40 division problems versus about 20 multiplication problems in a minute (Shapiro, 2010). For mastery, students needed to score above the 20th percentile in both multiplication and division in two out of three intervention sessions. For multiplication, the group mean for items correct had to be at or above 22 items correct in 1 min; for division, the group mean had to be 12 items correct in 1 min.

Data on accuracy and time needed to reach mastery (Burns, VanDerHeyden, & Jiban, 2006) were collected and analyzed. The group slope for each phase was computed and analyzed to determine statistical significance between phases of each group and within groups. This design adds to the research by examining the amount of instructional time needed to reach mastery under two intervention conditions, and to compare the strength of the self-regulation component plus explicit instruction using slope of performance (Bramlett, Cates, Savina, & Lauinger, 2010). In addition, dependent variables were operationally defined and measured repeatedly across controlled conditions (Horner, Carr, Halle, McGee, Odom & Wolery, 2005). The dependent variables in this study were items correct on the daily probes in multiplication and division. The probes were given twice a week following intervention. The instructional probes differed from the assessment completed at baseline to target and measure the specific fact strand taught. The use of a different measure was used in a class wide intervention to measure fluency in mathematics facts (Codding et al., 2009), as well as in a single-subject design measuring a specific fact type within two different interventions (Poncy et al., 2007). Data on accuracy and type of fact completed (easy or hard) were collected for each probe. Data were reported as means for each group.

MEASURES

General operation assessment

The General Operation Assessment (GOA) tested basic fact knowledge in addition, subtraction, multiplication, and division. Each assessment contained 100 problems, 10 rows and 10 columns. The number of items was selected to adequately test the majority of facts and related facts, as well as to limit a possible ceiling effect. The addition assessment contained single- and double-digit numbers, 0 to 10, without regrouping (e.g., 2 + 9, 9 + 2). The subtraction assessment contained 0 to 10, without regrouping (e.g. 9 – 2, 9 – 7). The multiplication assessment contained single- and double-digit factors timed 0 to 10 (e.g., 9 × 8, 8 × 9). The division assessment contained single- and double-digit dividend divided by 0 to 10 (e.g., 72 ÷ 9, 72 ÷ 8). All possible facts were written and then assigned a location on the assessment. The first rows began with easier problems, such as plus or minus 0, 1 or 2, and multiply and divide by 1, 2, 5, or 10, so that the students would possibly feel successful. The remaining problems were randomly placed on the assessment sheet so that all facts were tested at least once. Not all facts were tested because of the limit of 100 problems. Some communicative property facts for easier problems were removed so that more difficult problems were tested (e.g., 0 + 1 was tested but not 1 + 0). The assessments were created by the author based on the recommendations of the NCRTI for creating curriculum-based measurement for facts. See Appendix C for the GOA.

The GOA was given to the 77 students with a returned consent as part of the selection criteria. The students had 1 min to answer as many items as possible for each separate assessment. A test-retest using Pearson's product-moment coefficient was computed to test for construct validity of the assessment for all four general operations for items correct. The results were intercorrelated, and the resulting coefficients ranged from medium to high: addition, *r* = .82, subtraction, *r* = .83, multiplication, *r* =.79, and division, *r* = .81 (Gravetter & Wallnau, 2007). Correlations for digits correct were computed only for addition and multiplication, as it is not feasible to use results for digits correct for subtraction or division (Shapiro, 2010). Again, the correlations were high (*r* = .84) for addition and multiplication digits correct.

Instructional probes

Two probes containing 40 problems each were given following instruction twice per week. One probe was multiplication and the second probe was division. Each probe contained 15 review facts and 25 new facts from the specific phase strand (i.e., 9 facts). A mathematics worksheet generator (mathworksheets.com) was used to select the 15 review facts for each probe. The website allowed the user to select the facts and the amount desired for each worksheet. The author selected the review factors and excluded factors not yet instructed. The current fact strand problems were written using the communicative property fact (9 × 4, 4 × 9) and were placed on the worksheet. A tally system was used to verify that all current facts from that strand were tested an equal number of times across the four probes.

The probes were designed so that two current fact strand problems are presented before one review fact (i. e., 9 × 4, 6 × 9, 1 × 0). This pattern was continued for the remaining items on the probe. This ratio of review to current problem types was selected based on the work by Banda, Matuszny, and Therrien (2009) that found students' motivation to complete work increased when the material presented was a mix of new and unknown problems. A total of four probes (A, B, C, D) were created for each type of fact strand. For example, when students are taught multiplication and division facts for 9s (9 × 3, 9 × 4, 9 × 6, 9 × 7, 9 × 8, 9 × 9, and communicative property facts), the probes for the instructional week contained review items of factor times 0, 1, 2, 5, and 10. The 9s were the first hard fact instructed, so the review items only included the easy facts, or those considered mastered for this study. All remaining hard facts (4, 6, 8, 7, and 3) were not taught, so students were not tested over these facts. Once mastery of the 9s was obtained, the next set of probes for the 4/6/8s included 9 facts as part of the review items. For all probes, students were given 1 min to solve as many items as possible.

Students' progress was measured on items correct per minute. Graphs were completed for each individual student and by mean of items correct per group. The mean score of items correct by group was used to determine if the group met the phase change criteria, but individual data were still collected as another method to describe trends in the data. In addition to items correct, data were reported on the type of problems missed, hard (new fact) or easy (review problems) for each type of probe, multiplication, and division. Data were also collected on digits correct in multiplication, as well as accuracy on each probe.

Maintenance

Two weeks after the intervention, the participants were given the GOA (addition, subtraction, multiplication, division), as at the beginning of the study. A week after these data were collected, the general operation assessments were given to all Grade 4 students with a returned consent. These data were used as another maintenance point for the participants, as well as to calculate the local norms for end of year. The local norms were used to compare the participants' growth to that of their peers. The final data point was taken 3 weeks after the conclusion of the intervention.

PROCEDURE

The researcher contacted a local principal about participation in this study. This principal has worked with the researcher for 3 years in an early numeracy Response to Intervention grant. The previous study focused on the early elementary grades, so this study was the first contact with fourth-grade educators. Following the request to complete the study within this elementary school, district approval was obtained. A proposal was sent to the district in September and approved in October. No contact with the Grade 4 educators was made until district approval was obtained. Following district approval, the University of Texas at Austin IRB was completed and approved in late December. See Appendix A for copies of the parental consent and child assent forms. See Appendix B for copies of the teacher consent.

Baseline

Baseline data were collected three times in one week prior to the start of the intervention. During baseline, students completed the GOA in multiplication and division. A parallel version of the multiplication and division was created by reorganizing the complete rows of facts to control for repeated test effects. As in the initial assessment for inclusion criteria, the students had 1 min to complete as many items as possible. During baseline, prior to the start of the instruction in facts, participants were taught the definition of a goal and practiced writing short- and long-term goals for academic or social areas outside of mathematics. Goal-setting is a self-regulation component within *Intervention Plus*, Phase C. By teaching goal-setting to participants during baseline, less time was spent in the intervention teaching goals and how to write a goal.

The Q-TIP metacognitive strategy for solving word problems was also taught during baseline conditions. Q-TIP is an acronym to facilitate recall of the critical steps for solving a word problem. Q- TIP stands for “underline the Question, Think about the problem, Identify and circle important information, and then solve the Problem.” First, students were taught the steps of the Q-TIP strategy. Second, the students applied the strategy to Grade 2 and 3 word problems to practice using the steps of the strategy to mastery. Finally, the students completed a short quiz requiring writing of the parts of the Q-TIP strategy prior to completing grade-level word problems in intervention conditions (Deshler & Schumaker, 1988). As part of the intervention, every third day students were assigned application problems in multiplication and division using just the specific fact strand that was instructed during that week.

Intervention

Duration

The intervention sessions occurred 3 days per week from mid-January through the end of April. Intervention occurred Tuesday and Thursday after school and Friday during school. On Tuesdays and Thursdays, the intervention was completed in small groups, Group 1 followed by Group 2. On Fridays, all participants were tutored as a large group, due to scheduling conflicts. Explicit, systematic fact instruction occurred on Tuesday and Thursday, and application practice occurred on Fridays. A total of 26 sessions were conducted in fact instruction on Tuesdays and Thursdays, and a total of 10 application practices took place on Fridays. The majority of sessions (85%) were conducted by the researcher, and the remaining sessions were conducted by a trained tutor. The trained tutor held a master’s degree in special education and had experience in tutoring using explicit instruction. The intervention concluded prior to the last week of April due to state testing, as well as to allow maintenance to be completed prior to the end of the school year.

Instructional routine

The instructional routine on Tuesdays and Thursdays consisted of warm-up, modeling, guided practice, and progress monitoring using a multiplication and division probe. The strategic fact intervention was systematic, and explicit, with multiplication and division presented together as fact families. Only hard facts—9s, 4/6/8s, and 7s—were instructed within each phase (Silbert et al., 2006; Woodward, 2006). Students received error correction throughout the intervention session as part of the instructional routine. The pace of the instruction was efficient to allow for multiple opportunities for student engagement. Each intervention session lasted 25 to 30 min and began with a 3-min warm-up followed by 15 min of instruction that developed procedural and conceptual understanding of a strategy to solve the specific fact strand. The next 3 to 5 min were used to complete a guided practice sheet. The last 2 min of the intervention were the completion of the multiplication and division probes. Examples of the explicit, systematic lessons can be found in Appendix D and multiplication and division daily probes are in Appendix E.

On Fridays, the students applied the Q-TIP strategy to solve application problems adapted from released state tests, NAEP Grade 4 assessment, and core curriculum materials. All facts embedded in the application problems were facts from a specific strand taught on Tuesdays and Thursdays. A fact strand was the group of facts in which one factor was the same, (e.g., for 9s fact strand, all facts would contain a 9). The facts within each word problem reflected the fact strand presented during the 2 days of explicit, systematic instruction. No additional data were collected, but the researcher did maintain the student work samples. Examples of the application problems completed can be found in Appendix 2.

Warm-up

The warm-up focused on number recognition, counting, skip counting, and/or previous fact review (including ×0, ×1, ×2, ×5, and ×10). The purpose of the warm-up was for students to practice the prerequisite mathematical skills needed to solve or apply the specific strategy in instruction for the specific multiplication and division fact strand.

Modeling

During modeling, the researcher explained the strategy and modeled how to solve the facts by using think-aloud procedures to make the procedural steps transparent for the students. The think-aloud procedure also included an explanation of why the specific steps were chosen (Gray et al., 1999). Think-aloud procedures are used to "actively engage the learner in manipulating, linking and evaluating the information" (Rittle-Johnson, 2001, p. 2) a means of teaching self-exploration versus discovery learning. A direct instruction approach of modeling provided active engagement of the students, development of the procedural, strategic knowledge that can improve the conceptual understanding of the strategies (Rittle-Johnson, Siegler, & Alibali, 2001).

The strategy and the fact were then practiced in a fact family (e.g., 3 × 9 = 27, 9 × 3 = 27, 27 ÷ 9 = 3, 27 ÷ 3 = 9), thus linking multiplication and division throughout the intervention. A fact family is three numbers presented as factors, product or divisor, and dividend or quotient. The state objectives, National Core Standards, and NCTM (2009) recommend that multiplication be taught prior to division. This study taught the two operations together, so students could build an understanding of how the equal groups of multiplication are linked to division. Based on the students' success with addition and subtraction, it seemed probable that multiplication and division should also be taught concurrently.

All facts were taught through visual representations, using a concrete, pictorial, and abstract sequence (Jayanthi et al., 2008; Miller & Hudson, 2007). Students built the fact as an array using mathematical manipulatives (concrete) and then proceeded to draw the fact (pictorial) to solve the problem. The purpose of building the models was to show "how" and "why" the strategy worked to solve that specific fact strand. Rather than focusing on building just conceptual knowledge first, conceptual and procedural information was taught together, because "conceptual and procedural knowledge may develop in a hand-over-hand process, rather than one type strictly preceding the other" (Rittle-Johnson et al., 2001, p. 347). The students also used the arrays to identify the factors and products of the multiplication sentence and how the product is the dividend and the factors are the divisor and quotient in a division number sentence. Students discussed the procedural knowledge through showing the strategy and in self-explanations of why the strategy was effective for that particular fact strand (Rittle-Johnson, 2006).

Guided practice

Following instruction, the students solved problems together on a guided practice activity sheet using the strategy they had been taught. Both the instruction and the guided practice activity sheets included concrete representations during modeling, and pictorial scaffolds on the guided practice sheet. The purpose of the guided practice sheet was to allow the participant time to generalize the concrete examples to a paper-and-pencil task. As students practiced particular fact strands, the scaffolds (pictorial representations) on the guided practice were gradually removed.

Neighbor-share was also incorporated into the instruction. Part of the guided practice time was used for students to discuss and explain or teach the strategy to one another (Calhoon & Fuchs, 2003). Scheuermann et al. (2009) incorporated neighbor-share into word problems and found improved mathematical performance for the students. The same neighbor-share procedure was applied to allow students a chance to teach the strategy in their own words. The researcher acted as a facilitator and used error correction as needed. Another purpose of the neighbor-share was for the partner to check his or her own understanding of the strategy and correct the peer when it was his or her turn, if needed. See Appendix C for guided practice examples.

Lesson sequence

Each fact strand proceeded through a similar instructional routine until mastery was met, as measured by the group mean. The first two sessions of a new fact strand introduced the strategy using concrete manipulatives and scaffolds on the guided practice sheet. Examples of scaffolds included number lines, skip counting lines, and space provided to break apart the problem to solve. The next two sessions did not use manipulatives; rather, the students drew the problem to prove the answer. Throughout the phase, students first solved the multiplication fact, identified the three numbers in the fact family, wrote the communicative property fact, and then wrote the two division number sentences. By the third and fourth sessions, the students were skilled at writing four number sentences for each fact family using a fact family mat. The fact family mat was designed to assist students through pictorial scaffolds. The product was written on the roof in a triangle; the triangle then appeared after the equals sign. Throughout the intervention, hundreds charts and number lines were used for error correction, as well as scaffolds. Refer to Table 3.2 for the fact strand, warm-up examples, specific strategy, example, and representation within each lesson.

A priori decision criteria were identified to determine the scaffolds needed in instruction following the completion of four intervention sessions. If a group's mean in multiplication was less than 20 digits correct, the group received instruction from the beginning using concrete manipulatives to reintroduce the strategy. If a group's mean was 21 or greater digits correct, the second introductory lesson was taught followed by less structured lessons. If the group's mean was greater than 27 correct digits, the students completed instructional routines using less structured lessons without manipulatives.

Following these decisions, the students moved through the four lessons and then practiced specific fact strands using a copy, cover, and compare (CCC) constant time-delay instructional routine (Miller & Hudson, 2007). This strategy has been used with students from multiple grade levels with success and is used to increase fluency and/or accuracy following student mastery of a specific strategy (Codding et al., 2009; Poncy et al., 2006). During the CCC instructional routine, students reviewed the specific strategy, practiced "teaching" the strategy through neighbor-share, and then completed a CCC guided practice sheet. The guided practice sheet contained only multiplication and division facts from that phase strand. The students were given 3 s to complete a fact. After 3 s, a timer beeped and the student moved to the next fact, regardless of whether she or he had completed the previous fact (Miller & Hudson, 2007). Once the guided practice sheet was completed, the students wrote the correct answer to the problem in another column. The final step of the CCC instructional routine involved the students' comparing the two answers. If a problem was answered incorrectly or was skipped, the student practiced the problem by writing the number sentence three times.

Fact Strand	Prerequisite/ Warm-Up	Strategy	Example	Representation	Guided Practice Scaffolds
9s	• Count by 10's • Multiplying and dividing by 10 • Subtraction • Review 2s Fact Strand	Make Ten Minus the Factor	9x4 10 x 4 = 40 – 4 = 36	L. 1: Concrete ✓ Array ✓ Fact Family L. 2: Pictorial ✓ Hundreds Chart ✓ Fact Family L. 3 & 4: Abstract	L.1: Horizontal, scaffolds are present L. 2: Horizontal, scaffold structure only L. 3: Vertical, no scaffolds present L. 4: Vertical, no scaffolds, non-examples present
4/6/8s	• Count by 2's • Review 2s Fact Strand • Double facts (addition) • Review 0s and 1s Fact Strand	Double & Double Again & Double Three Times	4 x 3 2 x 3+2 x 3 6 + 6 = 12 6 x 6 2 x 6 + 2 x 6 + 2 x 6 12 + 12 + 12 6 x 8 2 x 8 + 2 x 8 + 2 x 8 16 + 16 + 16 32 + 16 = 48	L. 1: Concrete ✓ Break-apart numbers ✓ Fact Family L. 2: Pictorial ✓ Number Line ✓ Fact Family L. 3 & 4: Abstract	L.1: Horizontal, scaffolds are present L. 2: Horizontal, scaffold structure only L. 3: Vertical, no scaffolds present L. 4: Vertical, no scaffolds, non-examples present
7s	• Break apart numbers 0-10 • Addition facts • Review 2s Fact Strand • Review 5s Fact Strand	Break apart 7 and add	7 x 3 2 x 3 + 5 x3 6 + 15 21	L. 1: Concrete ✓ Break apart numbers ✓ Fact Family L. 2: Pictorial ✓ Number Line ✓ Fact Family L. 3 & 4: Abstract	L.1: Horizontal, scaffolds are present L. 2: Horizontal, scaffold structure only L. 3: Vertical, no scaffolds present L. 4: Vertical, no scaffolds, non-examples present

Note: L = Lesson

Table 3.2: Scope and Sequence of Intervention

Hard facts

Only hard facts were taught, due to mastery of easy facts (0, 1, 2, 5, and 10, based upon the results of the GOA. The first hard fact taught during intervention to both groups was the 9s. The strategy for 9s is Make Ten Minus the Factor. For this strategy, the students were taught to think of 9 as 10 and then multiply 10 with the other factor. Once this product was found, the students then subtracted the factor from the product (i.e., 9 × 4 = 10 × 4 – 4). A few students had already been taught the "finger trick" for the nines but were told they could not apply this trick unless they could explain why it worked. Following two lessons, two students were able to describe that using your 10 fingers was the same as the Make Ten Minus the Factor strategy. Only one student, Cinderella, used her fingers, but she was encouraged to solve the facts without this scaffold. All other students found the Make Ten Minus the Factor strategy to be more efficient than using their fingers or writing down the numerals 0 to 9.

The next fact strand taught was 4/6/8s. These hard facts were paired because the Double It strategy is used for all three. The Double It strategy means that the students break apart 4, 6, or 8 into groups of 2 and then multiply the other factor to 2 and add the products, so Double It, Double Again, or Double Three Times (e.g., 4 × 6 = 2 × 6 + 2 × 6). To introduce this strategy, the students were given cubes and told to make equal groups. Then they added additional cubes to show the other factor. In this hands-on activity with manipulatives, the 6s and 8s were divided into larger groups (e.g., two groups of three cubes to show 6), but the students realized that multiplying by 3 or 4 (e.g., solving 6 x 8 as 3 x 4 + 3 x 4 + 3 x 4 + 3 x 4) was difficult. Additional probing of mastered facts allowed the students to identify making multiple groups of two to multiply the other factor (e.g., 6 x 8 as 2 x 8 + 2 x 8 + 2 x 8). To increase fluency of these facts, time was spent in warm-up and during the lesson adding the double facts together.

The third fact strand of the intervention was 7s. The strategy for 7s is Break Apart. In Break Apart, the factor 7 is broken into 2 and 5; then the other factor is multiplied and the products added (i.e., 7 x 3 = 2 x 3 + 5 x 3). Only the students in Group

2 were taught this strategy for 2 days, but in that limited time the students were able to break apart a number based on the Double It strategy. The warm-ups and the daily probes contained review items of the easy facts, so the students did not struggle in multiplying the factor by 2 and 5. Warm-ups during this fact strand included addition review of the multiplication facts in times two and times five.

Intervention alone phase B

In Phase B the students received explicit, systematic, and strategic instruction in the fact strategy. The students completed all parts of the modeling and guided practice, as well as the multiplication and division probes. The students were timed for 1 min on each probe. When the students completed the probe, the researcher collected the probes. The students did not check the probes, nor did they receive a grade on the probes.

Intervention plus, phase C

In Phase C, the students received explicit, systematic, and strategic instruction in the fact strategy. The session was completed in much the same way as *Intervention Alone*, Phase B: 3-min warm-up, 15-min instruction, 3-min guided practice, and 2-min multiplication and division probes. The difference in this phase was the addition of the self-regulation components: self-correction, graphing, and goal setting. Following the 2-min probes in multiplication and division, the students self-corrected each probe. The total score and total number of errors were recorded on a “My Daily Data” worksheet. Students then wrote a goal for the next instruction session in both multiplication and division. Students graphed total scores on the multiplication graph and total scores on the division graphs. See Appendix F for a copy of the “My Daily Data” worksheets. As the students became proficient in grading, graphing, and goal setting, the additional time required to complete these components was about 5 min. Table 3.3 presents the phases completed for each group and the fact strand taught within each phase.

Group	9s	4/6/8s	7s
Group 1	Intervention Plus, Phase C	Intervention Alone, Phase C	Did not begin
Group 2	Intervention Alone, Phase B	Intervention Plus, Phase C	Intervention Alone, Phase B

Table 3.3: Hard Facts Completed

Fidelity of Implementation

Fidelity of implementation was assessed for 20% of the intervention sessions for each group. The fidelity form consisted of 15 items that addressed materials, lesson objectives, components of the lesson, and behavior. The questions were designed so that the observer could make a yes or no response if the action was observed. For the group receiving *Intervention Plus*, an additional 5 items were answered to determine if the self-regulation components (self-correction, goal-setting and graphing) were completed. The Fidelity of Implementation form can be found in Appendix G. All observations were conducted by a doctoral student in special education or the master tutor. Both observers were trained on how to use the form and in the lesson format. The researcher and the tutor were both observed, to ensure fidelity across tutors. All observed sessions received fidelity ratings at or above 95%.

DATA ANALYSIS

The students were placed in groups based on skill level on the selection assessments. Group 1 mean scores were below 19 digits correct (frustrational), and Group 2 mean scores were between 20 and 30 digits correct (instructional; Shapiro, 2010). Data are reported both individually and as a group using means for each data point. Individual student graphs were visually analyzed to identify any trends in the data and to assist, if needed, in the visual analysis of group graphs. To add to the visual analysis, improvement rate using slope was completed to compare the treatment phases, due to the close similarity between the two types of interventions (Parker, Hagan-Burke, & Vannest, 2007; Parker et al., 2009). A regression analysis was completed to determine whether the slope for each phase was significant, as well as to determine if the regression lines between groups and phases were significantly different (Armitage, 1980). Two procedures were used to analyze the data of the groups, percentage of non-overlapping data points (PND) and percentage of all non-overlapping data points (PAND). Both were used because controversy exists between which method is the best for reporting effect sizes in single-subject designs (Burns et al., 2010). However, effect sizes are generally reported in large, randomized control studies and have been proved to be statically sound. PAND is suggested as a means to add to the visual analysis to attach a "standardized expression of the amount of behavior change between phases" (Parker et al., 2009, p. 136).

Comparison of regression lines

In larger, experimental studies, slope is reported to test for significance of an intervention or a component of the intervention. Slope has been used to measure growth patterns in curriculum-based measurement to determine the amount of growth expected for students (Graney et al., 2009). Slope, or a regression analysis, is an important tool for measuring the amount of gains and determining whether the additional gains are a result of the intervention or the students would have made the same gains without the additional intervention time (Fuchs et al., 2008; Morgan & Sideridis, 2006). Testing the significance of an intervention can result in more sound judgments tied to a particular program. For this study, the regression lines were used to identify patterns in the intervention, positive or negative trends, and changes between phases. Each group was exposed to an *Intervention Alone* phase and an *Intervention Plus* phase. The goal of this analysis was to determine whether the slopes or regression lines in each phase are statistically different. In addition, the analysis added to the discussion of whether additional time is warranted for self-regulation components.

Prior to the availability of covariance analysis, statisticians applied a formula to compare regression lines (Armitage, 1980). The program to compare two regression lines is available online (http://www.stattools.net/Comp2Regs_Pgm.php) and was used to compare the regression lines between Group 1 and Group 2, during the 9s fact strand, and to compare the 4/6/8s fact strands. In addition the regression lines between phases for each group were tested: Group 1, Phases C and B; Group 2, Phases B and C were compared. Group 2 had only 2 data points in the third phase, so the second phase, B, was not included in the analysis. For the regression analysis, the independent variable was sessions and the independent variable was group mean scores. Although this is a relatively old analysis, it is new for the area of single-subject research. Only one article used a similar analysis in completing a meta-analysis of single-subject fluency interventions. In that meta-analysis, Morgan and Sideridis (2006), used a multilevel random coefficient modeling technique, which allows for "growth trajectories … and provides robust tests of statistical significance of both intercepts and slopes." (p. 192). This model allowed the researchers to compare different studies with uneven session times, as well as additional variables, such as school or teacher, which is not the case for the present study. In the present study, all students received the sessions at the same time, so a simpler analysis was used to demonstrate differences among slope.

Non-overlapping data points

Percentage of non-overlapping data points (PND) was calculated by identifying the number of data points that do not overlap between the baseline and intervention phases and then dividing by the total number of data points and, lastly, multiplying by 100 to obtain a percentage (Kennedy, 2005). Scruggs and Mastropieri (1994) defined "effect sizes" of PND greater than 70% as effective effects, between 50% and 69% PND as questionable effects, and less than 50% PND as no effect. These guidelines were used when determining the effectiveness of intervention with graphing and goal setting in comparison to intervention alone.

Percentage of all non-overlapping data

Percentage of all non-overlapping data (PAND) was calculated and converted to a phi coefficient. This method was described by Parker et al. (2007) as a superior means of comparing two treatments because it uses all data points, versus simply relying on "one unreliable data point" (p. 196). In addition, PAND can be converted to a phi coefficient, based on Pearson's phi, which is often reported as effect size within randomly controlled trials (Cohen, 1988). Parker and colleagues (2007) define the procedures for computing PAND and state that at least 20 data points are needed to compute the phi coefficient. This study is appropriate for PAND because the intervention has more than 20 data points.

PAND was calculated by counting the number of individual baseline data points and intervention data points and then dividing by the number of total intervention points that overlapped the highest baseline data point by the total number of data points (Burns et al., 2010; Parker et al., 2007). To calculate phi coefficient, a 2 x 2 table was created by starting with the percentage of data points in the baseline and intervention phases. Then the proportion of overlapping data is split, the "too high" scores in the baseline cell and the “too low” scores in the intervention phase (Parker et al., p. 197). Finally, the difference between the two cell ratios was computed for the effect size. A phi coefficient of .70 is considered a large effect; .50 to .69, a medium effect; .30 to .49, a small effect; and less than .29, no effect (Cohen, 1988).

SOCIAL VALIDITY

Student

Following the completion of the intervention, students filled out a seven-item Likert questionnaire based on the Children’s Intervention Rating Profile (CIRP; Witt & Elliott, 1985). This type of questionnaire was used in a study completed by Codding et al. (2009) and showed an internal consistency reliability of .75 to .89, which is considered adequate. The students completed the questionnaire in a group and all items were read out loud by the researcher. The answer choices were Totally Agree, Kind of Agree, Kind of Disagree, or Totally Disagree. In the CIRP a similar Likert answer choice is offered; smiling faces were to be used with younger students. The participants also answered open-ended questions regarding specifics about the intervention. See Appendix H for the student questionnaire. Five short-answer questions were asked:

(1) What did you like best about coming to learn about strategies to solve multiplication and division facts?

(2) What was hard for you when learning the facts?

(3) What would you tell your friends about our math tutoring time?

(4) Would you recommend it for your friends?

(5) What, if anything, would you change about the math tutoring time?

Teacher

The teachers completed a short questionnaire containing both open-ended questions and statements which require Likert type responses in the areas of (a) student selection, (b) fact instruction in the classroom, (c) classroom curriculum, and (d) fact intervention. The information obtained from the questionnaire was used to measure the social validity of the intervention, as well as to describe any possible history effects. History effects are events that can influence the progress of the students within the study, such as additional fact instruction outside of the intervention (Kennedy, 2005). The questionnaire took each teacher about 10 to 15 min to complete. The purpose of the questionnaire was to gain perspective on both fact instructions within the core curriculum and classroom and the changes in confidence and ability of the students selected for this study. See Appendix I for the teacher questionnaire.

SUMMARY

Using a time-series design, two groups of fourth graders received instruction in multiplication and division facts. Two intervention phases were instituted to measure the effect of self-regulation components within an explicit, systematic intervention; Phase B, Intervention Alone, and Phase C, Intervention Plus. Both groups were instructed in hard facts, specifically the 9s and 4/6/8s fact strands. Explicit, systematic instruction in facts occurred twice a week. Once a week, students practiced the facts from a specific fact strand within an application word problem. Three different analyses were completed: (a) percentage of non-overlapping data points, (b) percentage of all non-overlapping data points, and (c) a regression analysis to test for effects of the self-regulation components on items correct on a multiplication and division probes. Social validity of the intervention was measured through a questionnaire completed by the participants. In addition, classroom teachers completed a social validity measure to identify intervention effects in mathematics instruction.

Chapter 4: Results

The purpose of this study was to investigate the effects of a systematic, strategic intervention on the multiplication and related division fact performance of fourth grade students who were identified as having mathematical difficulties by scoring below the 10th percentile on the general operations multiplication assessment. In addition to the intervention, the participants corrected multiplication and division probes, graphed items correct, and set goals as a self-regulation component within the intervention. This research was guided by one research question:

> To what extent will fluency and accuracy scores on multiplication and division fact probes differ as a result of a systematic, strategic intervention, plus goal setting and graphing, as compared to a systematic, strategic intervention without goal setting and graphing?

To answer this question, both group data and individual data were analyzed. Multiple analyses were computed to determine the effectiveness of the intervention.

A time-series design for the study was utilized and resulted in an A-C-B-D (Group 1) or A-B-C-B-D (Group 2), in which the intervention phases were counterbalanced across groups. Baseline (Phase A) was followed by *Intervention Alone* (Phase B) or *Intervention Plus* (Phase C). A specific fact strand was taught in each intervention phase. A fact strand was the group of facts in which one factor was the same, (e.g., 9s fact strand, all facts would contain a 9- 9 × 3). In each phase the student groups were instructed with the same hard fact strand, but one group received the self-regulation components of graphing and goal setting with the systematic, strategic intervention while the other group received only systematic, strategic instruction with no graphing or goal setting. Hard fact strands included the 9s, 4/6/8s and 7s. A mastery criterion using group means of 22 items correct in multiplication and 12 items correct in division on 2 out of 3 days was utilized to define when a group was ready to move to the next phase of hard facts with or without the self-regulation components.

Local norms for items correct and digits correct for multiplication and items correct for division, were developed following the first and second selection assessment. The local norms were used to establish the phase change criteria, rather than national norms to more accurately match the school population. Based upon the normative data from the students in the school, a mastery score for multiplication and division was selected. The mastery score for items correct was the score at the 50th percentile of the local norms for both multiplication and division (Shapiro, 2010). This percentile was selected because it is similar to the recommendations for reading mastery using an RtI model (NCRTI; Fuchs and Fuchs, 2005).

Mastery for moving to a new phase with a new multiplication fact strand was defined as the group mean performance at 22 items correct or higher per minute for 2 out of 3 days. In division, the group mean performance must be 12 items correct or higher per minute for 2 out of 3 days. Only the group mean scores for items correct were used for phase change rationale. Additional data, digits correct per minute in multiplication, accuracy and the type of problems completed was also collected for each group and individual students. This data was collected to better describe the group data, using the individual data to explain inconsistent data points (high and/or low means), as well as understand the types of problems completed to better describe overall scores. The digits correct in multiplication were collected to triangulate the problem types completed. Hard fact answers often have two digits in the answer, possibly resulting in fewer items completed but greater digits correct. Group means were used rather than individual scores because a small- group intervention instead of individual tutoring time for Tier 2 students is more authentic to a typical school setting. This chapter reviews the analysis for this study, including (a) visual analysis, (b) effect size, and (c) social validity.

VISUAL ANALYSIS

Both individual data and group graphs were visually analyzed for trend and stability level in both multiplication and division. Individual data was analyzed as a mean to explain drastic changes in the group data. *Trend* is the "steepness of the data path" (Gast, 2010, p. 205). Direction (positive, negative or flat) and change in trend between intervention phases were examined. *Stability* refers to the level or the "amount of variability or range in data points" (Gast, 2010, p. 202). Each phase, (baseline, intervention, and maintenance) were analyzed separately. Refer to Figures 4.1 to 4.11 for group and individual student graphs, which show items correct in multiplication and division and digits correct in multiplication only. Refer to Figures 4.12 to 4.16 for percentages of easy and hard facts completed within each phase.

Baseline

Baseline data are used to establish a pattern of behavior (Kennedy, 2005), or in this study, ability level. The baseline data were used to identify students with a weakness in fluency (scoring lower than the 10th percentile) and in the inability to solve hard facts (i.e., 7 × 6, but not 7 × 2) in multiplication and division problems.

To identify a student's or a group's pattern of poor fluency and an inability to solve hard facts, a repeated-measure baseline was completed for multiplication facts and division facts, containing easy and hard facts. Data were collected on accuracy, type of problems completed (easy facts and hard facts) as defined by Silbert et al. (2006) and Woodward (2006), and fluency in solving multiplication and division facts. A total of five data points were collected during the baseline phase for each group, allowing for a pattern of ability to be established. Kennedy (2005) described the need for at least three data points but also recommended that the baseline be "as long as necessary, but no longer" (p. 38). Thus, baselines may be shorter or longer in duration, but a pattern should be established without undue harm to participants or, in the case of this study, test effects due to the repeated measures (Kennedy, 2005).

The first two data points were obtained during the selection assessments given to all students within their general education classrooms by the author. The last three data points were given with the assigned intervention groups (Group 1 and Group 2) and administered during the tutoring time after school. The General Operations Assessment in multiplication was used during baseline including a total of 100 problems, with 37 hard facts and 63 easy facts. The General Operations Assessment in division contains 100 problems (36 hard facts and 64 easy facts). The students were instructed to work across each row and to skip any problem they could not answer or found to be difficult. Baseline data were collected during the first 2 weeks in January following the students' winter break. Baseline data will be reviewed first for students groups and then for individual students.

Group 1

In multiplication, both items correct and digits correct per minute were examined. For Group 1, items correct in 1 min had a mean of 11.4 and fell within a range of 10 and 14. The stability level was stable because 80% of the data fell within 20% of the median value (Gast, 2010). The trend of the data was slightly positive, but relatively flat with a slope of 0.2.

For this group, the mean of items correct fell between the 5th and 10th percentile of the local normative data. Digits correct were similar, with a mean of 17.8 falling between the 5th and 10th percentiles of the local normative data and the overall range of 15 to 22. The third data point was the highest at 14 (items correct) and 22 (digits correct), but followed by a decreased mean of 10 and 12 (items correct) and 16 and 19 (digits correct) on the fourth and fifth data points. For the group, the accuracy during baseline phase was high, 96 %, with no more than two errors as a group per data point. In reviewing the student work, it was found that all students selected for the intervention skipped around the entire assessment, seeking easy facts. The group mean for easy problems across all five data points was 16 items correct with a range of 11 to 25. For hard facts, the mean was 4 items correct with a range of 0 to 11. Thus, this group demonstrated, on average, high accuracy on easy facts, but low fluency abilities with multiplication facts.

Overall, the group scores in division were much lower than multiplication as measured in both accuracy and fluency. Items correct were the only score reported for division because a majority of answers are single-digit, regardless of fact difficulty. Group 1 had a mean of 6.6 items correct with a range of 5 to 9 items correct. This is approximately the 25th percentile of the local norms. The trend of the group was positive, with a slope of 1.13 and a slight increase each day over 5 days, yet relatively stable. Similar to the multiplication assessments, the students skipped around seeking easier facts. Accuracy was low in division as compared to multiplication, with 33 errors across the five data points. The mean of easy problems completed was 5.95 items correct with a range of 0 to 13 items correct. The mean of hard facts completed was 0.47 items correct with a range of 0 to 2. Individual data from Group 1 participants (Ambious, Avery, Brittney, Aleva) are discussed below.

Ambious

Ambious was present for all five data collection assessment days. In multiplication, the mean of items correct was 9.2 with a range of 8 to 13. The level was stable due to the range of scores within 20% of the medium score. The trend was flat but positive with a slope of 0.2. The mean of digits correct was 15.2 with a range of 13 to 21. His accuracy was high, at 98% with only two total errors across the five data points. His baseline was relatively flat, with the exception of the third data point, which was higher than the other four data points. In division, the mean of items correct was 4.4 with a range of 0 to 11. Division stability was not as stable as multiplication due to an increase in scores on the fourth and fifth data points. Ambious's trend was positive, with a slope of 3, but accuracy was low at 69%. The first two data points resulted in a score of 0 due to solving all four problems incorrectly. The last two data points were higher, 8 and 11, with no errors, but he did skip around on the page to locate easy facts. During baseline, Ambious solved more easy facts correctly in both multiplication and division. In multiplication, the mean was 1.2 items correct for hard facts and 8 items correct for easy facts. In division, the mean was .2 items correct for hard facts and 4.2 items correct for easy facts.

Avery

Avery was also present for all five data collection assessment days. In multiplication, the mean of items correct was 11.4 with a range was 8 to 16. The trend was flat and stable. The mean of digits correct was 18.6 with a range of 13 to 27, with 27 on the third day of baseline collection. Her accuracy was 96%, with only two errors on the second assessment. In division Avery's mean of items correct was 11.4 with a range of 10 to 14, and an accuracy level of 100%. The baseline trend for division was positive and less stable due to an increase in the last two data points (14, 13). Although, her accuracy was high, Avery did skip around the page looking for easy facts to complete. In multiplication, her mean of hard facts completed was 1.6 items correct and mean of easy facts completed was 9.8 items correct. The mean for hard facts completed was 1.4 items correct and mean of easy facts completed was 10 items correct.

Brittney

Brittney was present for all five data collection assessment days. In multiplication, the mean of items correct was 12.8 with a range was 7 to 18. The mean for digits correct was 20 and the range was 11 to 28. Brittney was the only student who did not demonstrate an increase on the third data point, as compared to peers in Group 1. The overall trend of her baseline data was positive with a steady increase each day. The steady increase resulted in a less stable baseline. Her accuracy was also high, 96% with only three errors across all five assessments. In division, Brittney's mean of items correct was 4.2 and the range was 2 to 6, with an accuracy level of 64%. As in multiplication, the trend of her data was positive, but she showed less of an increase on the last 2 days. Brittney solved more easy facts than hard facts in both multiplication and division. In multiplication, the mean for hard facts completed was 2 items correct and the mean for easy facts completed was 10.8 items correct. In division, the mean for hard facts completed was 0.2 items correct and the mean of easy facts completed was 4 items correct.

Aleva

Aleva was present for only four data collection assessment data and was absent on the third day of baseline. This may explain the dramatic increase in the group baseline. In multiplication, the mean of items correct was 11.75 with a range of 9 to 15. The mean of digits correct was 16.5 and the range was 14 to 19. His data were stable due to the low variance in range. His accuracy was 98% with only one error on the initial selection assessment. His data trends were negative, with a slope of –.5, with the first two scores higher than the last two data points. Aleva’s data for division mirrored his multiplication, with a negative trend line with a slope of –.5. In division, the mean of items correct was 5.5 and the range was 4 to 7. His first two data points were higher (6, 7) than the last two data points (4, 5). His accuracy was also lower in division, 61%. Most of the errors occurred when he attempted to solve hard division problem types. He also skipped around the page and solved more easy facts than hard facts. In multiplication, the mean of hard facts completed was 0.75 items correct the mean of easy facts completed was 11 items correct. In division, the mean of hard facts completed was 0 items correct and the mean of easy facts completed was 5.5 items correct.

Group 2

Using the mean data for Group 2 in multiplication, the baseline mean was 17.27 items correct, with a range of 13 to 17. The trend of the data was positive, with a slope at 0.56. Overall, the trend was stable with a higher increase at the fourth data point followed by a decrease at the fifth data point. As a group, the mean of items correct was below the 10th percentile of the local normative data. The mean for digits correct was 22.6, and the range was 20 to 26 and stable. The trend of digits correct was positive, but decreased at the fifth data point. The accuracy for Group 2 was lower than Group 1: 75% with errors primarily made by three students (Taylor, Haley and Rony). Similar to Group 1, Group 2 students skipped around the assessment, locating the easy facts and skipping the hard facts. The mean for hard facts completed was 2.18 items correct, with a range of 0 to 5. The mean for easy facts completed was 12.5 items correct, with a range of 7 to 19.

Both fluency and accuracy were lower on the division assessment. In division, the mean for items correct was 6.4, which was slightly higher than the 25th percentile, and the range was 5 to 8. This group's trend was positive, with a peak of 8 items correct at the second data point, followed by a decrease for the third data point, slope of 1.13. Similar to the multiplication assessments, the students skipped around, seeking easy facts. The accuracy was low in division, 34 errors across the five data points. The mean for hard facts completed was 1.33 items correct, with a range of 0 to 4. The mean for easy facts completed was 6.05 items correct, with a range of 2 to 12. Individual data from Group 1 participants (DJ, Taylor, Haley, Cinderella, and Rony) are discussed below.

DJ

DJ was present for all five data collection assessment days. In multiplication, the mean for items correct was 14.6 and the range was 12 to 18 and stable. His trend was flat, with two peaks on Days 3 and 4 (17, 18) followed by a decrease on the last day. Digits correct had the same pattern, with high peaks and a mean of 23 with a range of 19 to 27. His accuracy was high, 96% with only three total errors. In division, the mean for items correct was 10.8 and the range was 10 to 14 and very stable. In four of the five data points, DJ scored 10 items correct, and in one data point (Day 2), 14 items correct. His accuracy was high, 96%, with errors only on the last day of data collection. DJ answered more easy facts than hard facts for both operations. In multiplication the mean of hard facts completed was 2 items correct and the mean of easy facts completed was 12.6 items correct. In division, the mean of hard facts completed was 2 items correct and the mean of easy facts completed was 8.8 items correct.

Taylor

Taylor was present for all five data collection assessment days. In multiplication, the mean of items correct was 12.8 and the range was 11 to 13. Her data trend was relatively flat and stable, with little variation among the data points. For digits correct, the mean was 20.2 and the range was 19 to 22, again a relatively flat and stable data trend. Her multiplication accuracy was still high, 91%, with six total errors. In division, the mean of items correct was 5.4 and the range was 2 to 10, with an accuracy level of 67%. The variance in her scores resulted in a less stable and positive trend line. Taylor had errors on every assessment across the data in baseline. She skipped around on the assessment and answered more easy facts than hard facts. In multiplication the mean for hard facts was 1.6 items correct and the mean for easy facts was 10.8 items correct. In division the mean of hard facts was 0.2 items correct and the mean for easy facts was 5.2 items correct.

Haley

Haley was also present for all five data collection assessment days. In multiplication, the mean for items correct was 17.8 and the range was 12 to 24. Haley's scores were positive and did increase greatly from the initial assessment (12) to the final assessment (22), resulting in a positive trend line and moderate stability. Accuracy was high at 91%. The items she answered were consistent, meaning she only attempted the same facts on each assessment. The mean of hard facts was 4 items correct, and the mean of easy facts was 13.8 items correct. The digits-correct trend was less sharply skewed, but still positive. The mean for digits correct was 28.4 and the range was 19 to 37. In division, the mean of items correct was 2.6 and the range was 2 to 4. The trend for division was flat and more stable than for multiplication. Haley's accuracy was negative, meaning she had more errors than items correct, resulting in 16 errors across the five data points. Haley did not answer any hard division facts correctly, and the mean for easy facts was 2.6 items correct.

Cinderella

Cinderella was only present for three data collection assessment days, the initial 2 days of assessment and Day 5. In multiplication, the mean for items correct was 12 and the range was 9 to 14. Digits correct produced a mean of 19.33 and the range was 15 to 22. Her overall pattern was negatively skewed due to the higher scores on the initial assessments. In division, the mean for items correct was 11.33 and the range was 8 to 13. Again, the first two data points were higher (13, 13) than the last data point of 8. Her accuracy was 100% for multiplication and 88% for division, which was still medium to high compared to other participants in the study. Cinderella's accuracy was high, but she still solved more easy facts than hard facts in both operations. In multiplication the mean for hard facts completed was 2.67 items correct and the mean for easy facts completed was 9.33 items correct. In division the mean for hard facts was 1.3 items correct and the mean for easy facts was 10 items correct.

Rony

Rony was present for four data collection assessment days. In multiplication, the mean of items correct was 15.75 and the range was 12 to 18. Digits correct resulted in a mean of 19.75 and the range was 18 to 21. Overall, his trend line was relatively stable and flat across the 4 days tested. Rony's accuracy in multiplication was lower than that of his peers: 87% with a total of 8 errors. In division, his accuracy was higher, 94%, but he attempted fewer problems. The range of division items correct was 1 to 7, with a mean of 4.5. More than any other participant, Rony solved easy facts and attempted only two hard facts. The mean for hard facts completed in multiplication was 0.5 items correct, and for division was 0 items correct. The mean of easy facts completed in multiplication was 15.25 items correct, and the mean for easy facts completed in division was 5 items correct.

Intervention

Both groups began with the first hard fact strand of 9s. This included 9 × 3, 9 × 4, 9 × 6, 9 × 7, 9 × 8, and 9 × 9. Following the baseline phase, Group 1 received *Intervention Plus* (Phase C) with the self-regulation components of graphing and goal setting. Following the baseline phase, Group 2 received *Intervention Alone* (Phase B). After instruction, the students completed probes with multiplication and division facts. The daily probes contained a total of 40 problems: 15 easy facts (factor times 0, 1, 2, 5, or 10) and 25 hard facts (only facts from that specific fact strand). Each group remained in the assigned intervention phase until mastery was obtained based on the mean of the group in both multiplication and division. Mastery was 22 items correct in multiplication and 12 items correct in division on 2 out of 3 days. Each phase change resulted in a fact strand change and change of phase, *Intervention Alone* (B) or *Intervention Plus* (C). Following the 9s, the next fact strands taught were 4/6/8s (4 × 3, 4 × 4, 4 × 6, 4 × 7, 4 × 8, 6 × 6, 8 × 8, 6 × 8) followed by 7s (7 × 3, 7 × 6, 7 × 7, 7 × 8).

Group 1

Group 1 completed one phase of intervention and moved to a second phase prior to the end of the intervention. The first phase was *Intervention Plus* (Phase C) with the 9s. A total of 14 data points were collected. In multiplication the mean of the group was 15.14 items correct and the range was 8 to 23, with a standard deviation of 4.18. It took all 14 data points for mastery to be obtained in multiplication. The overall trend was positive, with a slope of 1.08. In division, the group reached mastery after 5 data points but continued to practice and greatly improve in fluency until multiplication was mastered. In Phase C, the mean for division was 7.5 items correct and the range was 5 to 25 and standard deviation of 4.18. The slope was slightly higher and positive at 1.5. For both operations in this phase, accuracy was high. Multiplication accuracy was 97%, and in division, accuracy was 91%. The group means for hard facts in multiplication and division were 7.87 items correct and 9.51 items correct, respectively. The group means for completion of easy facts in multiplication and division were 7.48 items correct and 6.74 items correct.

The next phase, Phase B for Group 1, was *Intervention Alone* for the 4/6/8 fact strand. A total of 12 data points were collected in Phase B, but mastery was never reached in multiplication because the intervention ended. The trend line for multiplication and division was less steep than in the previous phase with slopes at 0.64 and 0.67 respectively. In multiplication the mean was 15.42 items correct and the range was 10 to 20, with a standard deviation of 3.15. In division the mean was 11.08 and the range was 6 to 16, with a standard deviation of 3.5. In both operations, the trend line was relatively flat. The accuracy scores were also lower in this phase: 88% in multiplication and 76% in division. The mean for hard facts completed in multiplication was 8.29 items correct, and the mean of hard facts completed in division was 6.24 items correct. The mean of easy facts completed in multiplication was 6.94 items correct and of easy facts completed in division was 5.68 items correct. Individual data for Group 1 intervention phases follow.

Ambious

Ambious was present for most of the intervention sessions (14 days in Phase C and 12 days in Phase B). He missed 1 day in Phase C and 2 days in Phase B. His scores increased but remained relatively flat in Phase C (fact strand 9s), with a mean of 10.54 items correct and the range was 4 to 15. In division, the mean for items correct was 12.54 and the range was 5 to 17. During Phase B, (fact strand 4/6/8s) in multiplication, his mean of items correct was 8 and the range was 3 to 13. In division, the mean of items correct was 10.6 and the range was 4 to 20. His accuracy was high across phases, with a score of 100% correct in multiplication and 99% correct in division in Phase C. In Phase B, accuracy remained high: 99% correct in multiplication and 97% correct in division. When completing the probes, Ambious went across the rows, versus skipping around to find only easy facts. For Phase C, his mean for hard facts in multiplication was 7.23 items correct and his mean for hard facts in division was 8.46 items correct. The mean for easy facts in multiplication was 3.31 items correct, and the mean for easy facts in division was 4.39 items correct. In Phase B the mean for easy facts in multiplication was 2.56 items correct and the mean for easy facts in division was 3.44 items correct. The mean of hard facts in multiplication was 5.67 items correct, and the mean of hard facts in division was 7.56 items correct.

Avery

Avery was present for most of the lessons. She missed 1 day during Phase C, (9s) and 1 day in Phase B, (4/6/8s). In Phase C, Avery's trend line was very steep in both multiplication and division. In multiplication, the mean of items correct was 23 and the range was 8 to 40. In division, the mean of items correct was 25 and the range was 7 to 39. In multiplication, Avery reached the phase change criteria on Days 4 (23 items correct), 5 (11 items correct) and 6 (22 items correct). Avery reached the phase change criteria in division after 2 days of instruction, scoring 12 items correct on Day 3 and 20 items correct on Day 4. In Phase B, Avery made progress, but the trend line was more sporadic and not stable, versus the nearly straight, positive trend line in Phase C. In Phase B, multiplication, the mean for items correct was 26 and the range was 14 to 36. In Phase B, Avery's score started high, at 19 items correct, and she scored 22 items correct, or mastery on Days 5 and 6. In division, the mean for items correct was 19 and the range was 11 to 25. Avery began this phase at mastery with 12 items correct, and then scored 17 and 18 on Days 2 and 3. Avery did not skip around the page, and this resulted in higher mean scores for hard facts in both multiplication and division. The mean scores for hard facts in multiplication for Phase B and Phase C were 14.92 items correct and 16.1 items correct respectively. Mean scores for division hard facts, Phase B and Phase C, were 15 items correct and 13.3 items correct. Means of easy facts in multiplication, Phase B and Phase C were 9.2 items correct and 9.9 items correct, respectively. The mean of easy facts in division was 10 items correct for Phase B and 7.2 items correct for Phase C. Her accuracy in both operations, across both phases, was high. Avery's accuracy in multiplication and division in Phase C was 98%. In Phase B, her accuracy in multiplication was 96% and in division was 99%.

Brittney

Brittney's attendance rate was similar to that of her peers in Group 1. She was present for all of Phase C (9s) and missed 2 days in Phase B (4/6/8s). Brittney's trend in Phase C was more positive but not as stable across the phase in multiplication. In multiplication the mean for items correct was 15.34 and the range was 8 to 22. She did reach the mastery score of 22 items correct on the last day of the phase, but did not score 22 items correct across 2 days. This was the highest score for Brittney in Phase C. In division her scores increased slowly, and then jumped from 7 items correct to 19 items correct on Days 7 and 8. Following Day 8, the scores remained well above the mastery score resulting in a very steep trend line. In division, the mean for items correct was 25.8 and the range was 4 to 31. In phase B, the scores in both operations were lower with a flat trend line. In multiplication, the mean for items correct was 23.1 and the range was 7 to 19. In division, the mean of items correct was 7.5 and the range was 2 to 10. Brittney tended to skip around the page, versus solving facts across a row. The mean score for multiplication hard facts in Phase C was 3.21 items correct and the mean of hard facts in Phase B was 8 items correct. In division the mean score for hard facts in Phase C was 8.36 items correct and in Phase B was 1.89 items correct. The means of easy facts completed were higher. In Phase C the mean for easy facts was 12.07 items correct and in Phase B the mean for easy facts was 5.44 items correct. In division the mean for easy facts in Phase C was 6.64 items correct and the mean for easy facts in Phase B was 5 items correct. Brittney's accuracy was high. In multiplication, Phase C, the accuracy was 97% and in division the accuracy was 93%. In Phase B, her accuracy in multiplication was 94% and in division her accuracy decreased to 66%.

Aleva

Aleva only missed only 1 day during the intervention phases. In Phase C (9s), Aleva had a positive trend line in both operations, with stable scores. In multiplication, the mean of items correct was 11.71 and the range was 5 to 19. In division, the mean of items correct was 12.64 and the range was 5 to 19. In phase B (4/6/8s), Aleva's trend was more flat without a consistent increase in scores. In multiplication, the mean of items correct was 12.25 and the range was 4 to 12. In division, the mean of items correct was 8.55 and the range was 4 to 12. Aleva varied in his attempts to solve the probes. At times he would go across a row, and at other times he would skip around, thus affecting the differences in scores within a phase. In Phase C the mean score for hard facts in multiplication was 6.57 items correct, and in Phase B the mean score for hard facts was 3.1 items correct. The mean score in division for hard facts in Phase C was 6.57 items correct and in division for hard facts in Phase B was 1.9 items correct. The means for easy facts across Phase C and Phase B were very similar. The mean for easy facts in multiplication in Phase C was 5.14 items correct, and the mean of easy facts in Phase B was 6.95 items correct. In division the mean for easy facts in Phase C was 6 items correct and the mean for easy facts in Phase B was 5.68 items correct. While Aleva's scores were not as constant, his accuracy was high across operations, at 99% in Phase C, and 98% in Phase B.

Group 2

Group 2 completed two phases and completed only 2 days of a third phase prior to the end of the intervention. In the first phase, *Intervention Alone* (9s, Phase B), a total of 16 data points was collected. In multiplication, the mean of items correct for the group was 17.25 and the range was 12 to 24; the standard deviation was 4.16. The overall trend was positive with a slope of 0.788. In division, the group reached mastery on Days 7 and 8 but remained in this phase until mastery was obtained in multiplication. In division, during Phase B the mean for items correct was 13.69, the range was 3 to 23 and the standard deviation was 5.58. The slope increased at a faster rate than in multiplication, at 1.13. In *Intervention Alone*, Phase B accuracy was moderate. In multiplication, the accuracy was 87% correct, and in division it was 80%. The means of hard problems in multiplication and division were 10.15 items correct and 9 items correct, respectively. Means of easy problems in multiplication and division were 7.09 items correct and 4.41 items correct, respectively.

Phase C, or *Intervention Plus* (4/6/8s), lasted for a total of 8 days. In multiplication, the trend was positive and steep, with a slope of 2.14, mean of 16.74 items correct, range of 10 to 25, and standard deviation of 5.39. In division the trend was positive but not as steep, with a slope of 0.988, a mean of 11.12 items correct, a range of 5 to 17 and a standard deviation of 3.6. Group 2 mastered both operations in about the same amount of sessions, but one additional session was needed for mastery in division. Accuracy was also slightly higher in Phase C than Phase B, at 91% in multiplication and 81% division. In multiplication, the mean of hard facts was 10.04 items correct and the mean of easy facts was 6.6 items correct. In division, the mean of hard facts was 7.32 items correct and the mean of easy facts was 4.48 items correct. The last phase, Phase B *Intervention Alone* (7s), consisted of only 2 data points for three students, DJ, Taylor and Avery. The group started high, with 17 items correct in multiplication and 13 items correct for division. Individual data for Group 2 intervention phases follow.

DJ

DJ missed a total of 5 days of instruction, 2 days in Phase B and 3 days in Phase C. For both phases his scores increased at a stable rate, resulting in positive trends for both operations. In multiplication, Phase B, the mean of items correct was 17.64 and the range was 6 to 28. His scores in multiplication met mastery on Days 10 and 11. In division, Phase B, the mean of items correct was 19.64 and the range was 3 to 30. In division, he reached mastery at Days 5 and 7, but was absent on Day 6. His accuracy was high in Phase B: 95% multiplication and 97% division. The mean for hard facts in multiplication was 11.64 items correct, and the mean for hard facts in division was 13.57 items correct. The mean for easy facts in multiplication was 6.71 items correct and for division was 6.07 items correct. In multiplication, Phase C, DJ's trend was steep, with a mean of 20.2 items correct and the range was 7 to 28. In division, the mean of items correct was 16 and the range was 9 to 21. His accuracy was again high, at 97% in multiplication and 95% in division. In the final phase, he scored 16 and 20 items correct in multiplication and 15 items correct for both days in division.

Taylor

Taylor missed only a total of 3 days; 2 days in Phase B and 1 day in Phase C. In Phase B, *Intervention Alone,* the trend line was positive but relatively flat. In multiplication her scores increased but then decreased, resulting in unstable data patterns. Division was slightly more consistent with a steeper trend line. In multiplication, the mean of items correct was 15 and the range was 2 to 20. Her scores did not meet the phase change criteria for this phase. In division, the mean for items correct was 14 and the range was 1 to 15. Taylor's mean score for hard facts in Phase B, in multiplication, was 7.21 items correct, and her mean score of hard facts in division was 5.79 items correct. Her mean score for easy facts in Phase B, in multiplication, was 5.79 items correct and her mean score for easy facts in division was 4.79 items correct. Her accuracy was high, at 92% in multiplication and 95% in division. In Phase C, *Intervention Plus*, the trend was positive but still showed highs and lows with no consistent pattern. In multiplication, the mean of items correct was 12.57 and the range was 9 to 17. In division, the mean of items correct was 10.42 and the range was 9 to 18. In Phase C, Taylor's scores in multiplication did not reach mastery, but mastery was met in division. In multiplication, the mean for hard facts in Phase C was 7.29 items correct and the mean for hard facts in division was 6.29 items correct. In Phase C, the mean score for easy problems in multiplication was 5.43 items correct and the mean of easy problems in division was 5.29 items correct. Her accuracy was again high, 94% in both multiplication and division.

Haley

Haley was present for a most of the intervention sessions, missing only 2 days in Phase B (9s). In Phase B, the trend in multiplication was positive and her initial score was at mastery, 24 items correct. She continued to increase her score across the 16 sessions. In multiplication, Phase B, the mean for items correct was 47.28 and the range was 14 to 33. In division, the trend was very steep and positive, with a mean of 14.5 items correct and the range was 1 to 26. She met the phase change criteria in both operations on Days 7 and 9. Haley tended to complete the problems moving across the row, resulting in a higher percentage of hard facts completed. The mean of hard facts in Phase B, in multiplication, was 17.07 items correct, and the mean of hard facts in division was 10 items correct. The mean for easy facts in Phase B, in multiplication, was 8.64 items correct, and the mean of easy facts in division was 4.5 items correct. Her accuracy was high across both operations: 99% in multiplication and 90% in division. In Phase C, Haley's trend was again positive, but steeper in multiplication than division. In multiplication, the mean of items correct was 17.88 and the range was 9 to 32. In division, the mean of items correct was 9.63 and the range was 2 to 16. In both operations, she did meet mastery in the same amount of session as the overall group. The mean for hard problems in Phase C, in multiplication, was 12.75 items correct, and the mean for hard facts in division was 7.13 items correct. The mean of easy facts in Phase C, in multiplication, was 6.75 items correct, and the mean of easy facts completed in division was 3.5 items correct. In Phase C, her accuracy remained at 99% in multiplication and increased to 92% for division. The last phase began with Haley almost at mastery after only 2 days, scoring 21 and 25 items correct in multiplication and 10 and 18 items correct in division.

Cinderella

Cinderella missed more days than her peers, 4 days in Phase B (9s) and 1 day in Phase C (4/6/8s) before removal from the after-school tutoring as a result of inappropriate behavior on the bus. In Phase C, she was only present for only 3 days, so only the completed Phase B results, *Intervention Alone,* were analyzed. In Phase B, the trend was positive but slightly flat in both operations. In multiplication, the mean for items correct was 14.33 and the range was 9 to 21. In division, the mean of items correct was 12.67 and the range was 6 to 20. Cinderella's mean for hard facts in multiplication was 9.67 items correct, and her mean of hard facts in division was 8.56 items correct. The mean for easy facts in multiplication was 4.67 items correct, and the mean of easy facts in division was 4.08 items correct. Her accuracy was high: 100% in multiplication and 99% in division.

Rony

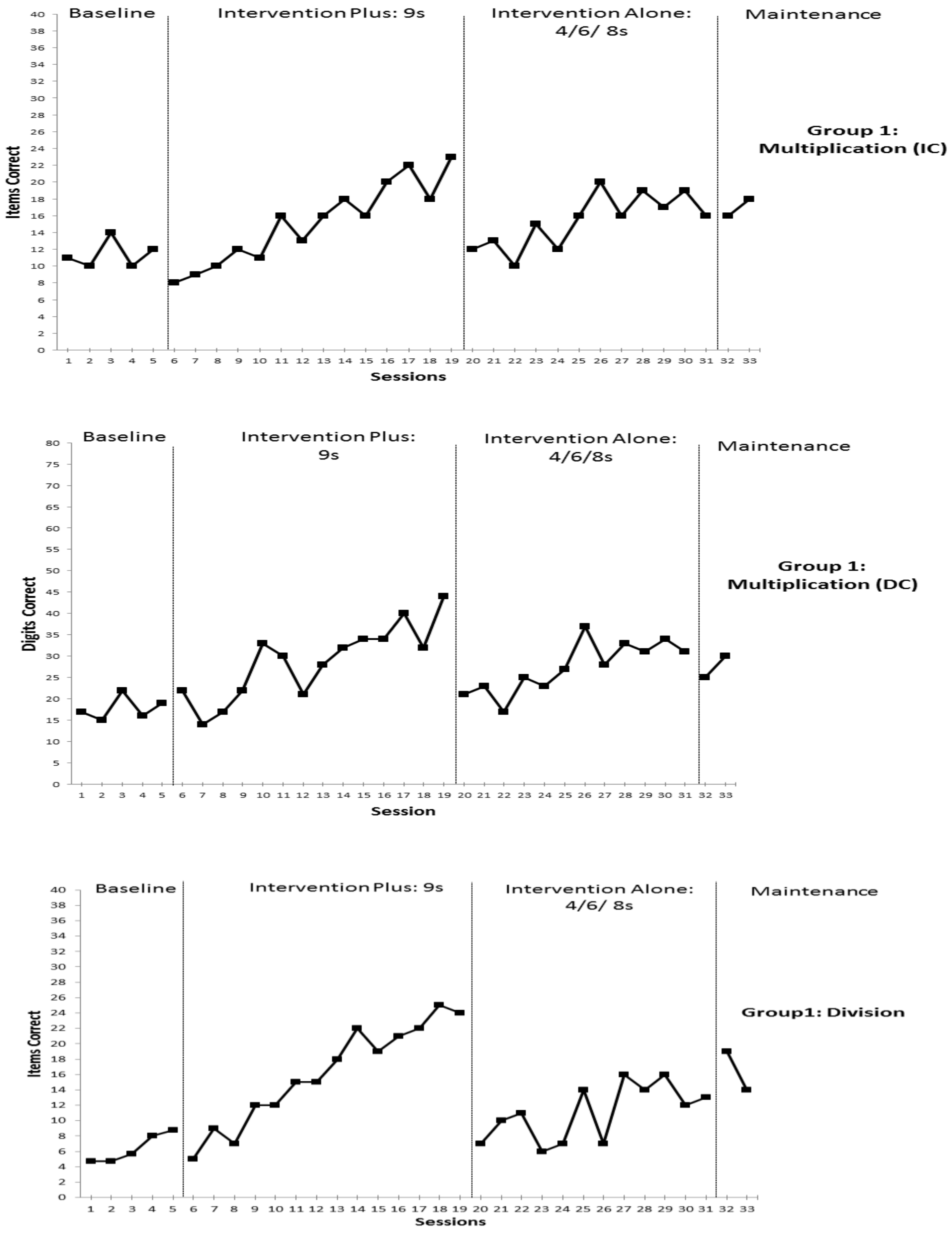
Similar to Cinderella's attendance, Rony missed 4 days in Phase B (9s), and 2 days in Phase C (4/6/8s). He only received 2 days of instruction in Phase C before removal from the after-school tutoring program as a result of inappropriate behavior on the bus. Only his completed Phase B will be analyzed. In Phase B, the trends were positive, with a steeper slope in division. In multiplication, the mean of items correct was 14 and the range was 11 to 20. His scores never met the phase change criteria in multiplication. In division, the mean of items correct was 9.33 and the range was 2 to 16. Rony did reach mastery in the same number of sessions as the overall group. He tended to skip around the multiplication probe, answering more easy facts than hard facts. The mean for hard facts in multiplication was 4.25 items correct, and the mean of hard facts in division was 6.67 items correct. The mean of easy facts in multiplication was 5.79 items correct, and the mean of easy facts in division was 2.25 items correct. Rony's accuracy was high in multiplication, at 95% and moderate in division, at 89%.

Maintenance

A total of 2 data points were collected once the intervention ended. The first maintenance data point was taken 2 weeks following the last day of intervention by the author and the second data point was completed 3 weeks after intervention, also administered by the author. The first maintenance data point was completed in a small group with all nine participants. The second data point was taken in the classroom with the participants and their peers. The second data point was used to determine end of year local norms. Group 1 mean scores for Day 1 and Day 2 in multiplication were 16, 18 items correct, and 25, 30 digits correct. The mean scores in division were 19, 14 items correct. Group 2 mean scores in multiplication were 24, 21 items correct and 38, 36 digits correct, and in division, 17, 14 items correct. Refer to Table 4.1 for the end of year local normative data for items correct and digits correct in multiplication and items correct in division. Group 1 means in multiplication were at or above the 16th percentile, an increase from the start of the intervention (below the 10th percentile, with a mean of 11.4 items correct in baseline). Group 2 means were at or above the 25th percentile, again an increase from the start of the intervention (below the 10th percentile, with a mean of 15 items correct in baseline). In division, both groups increased from the beginning, below the 10th percentile to at end of year at or above the 50th percentile.

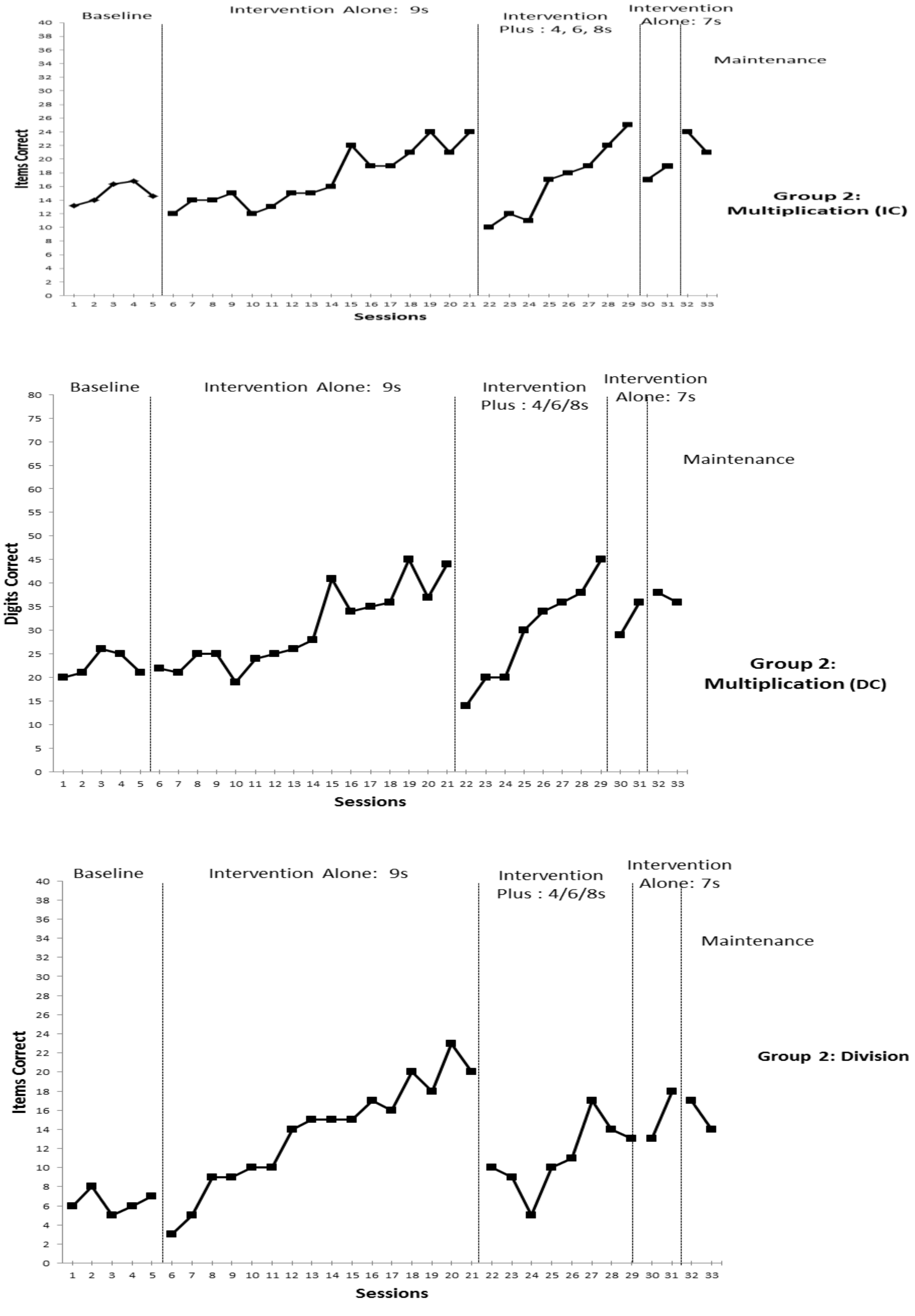
	Multiplication		**Division**
	Total Items (*N* = 68)	**Correct Digits**	**Total Items (*N* = 68)**
Mean	28.24	47.32	18.4
Standard Deviation	9.49	16.44	9.242
Percentiles:			
5th	14	23	4
10th	16.9	27.9	7.8
25th	21	34	12.25
50th	27	44.5	17
75th	34.5	57.5	24
90th	41	72.3	32

Table 4.1: End of Year Local Normative Data



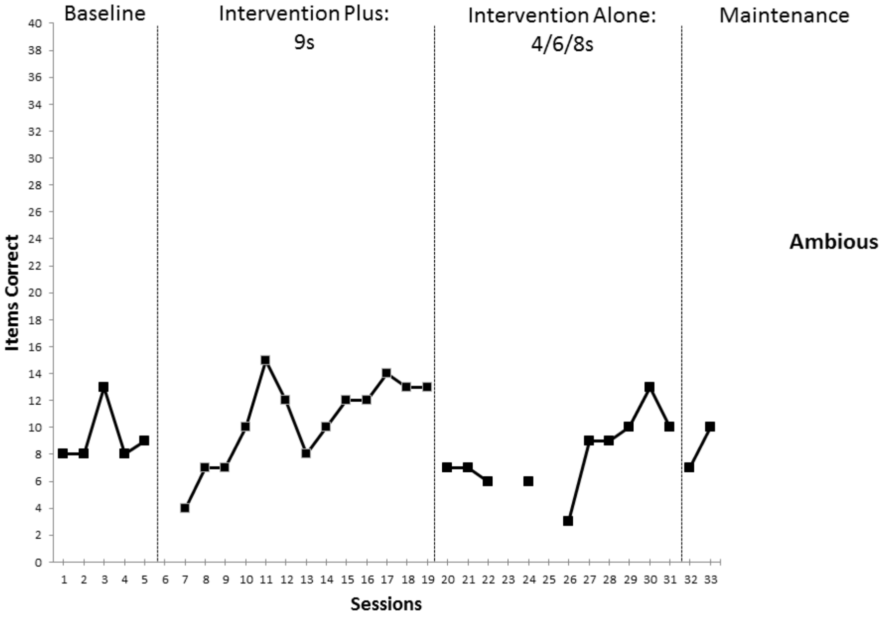
Note: IC = Items Correct, DC = Digits Correct

Figure 4.1: Group 1 Means – Multiplication and Division

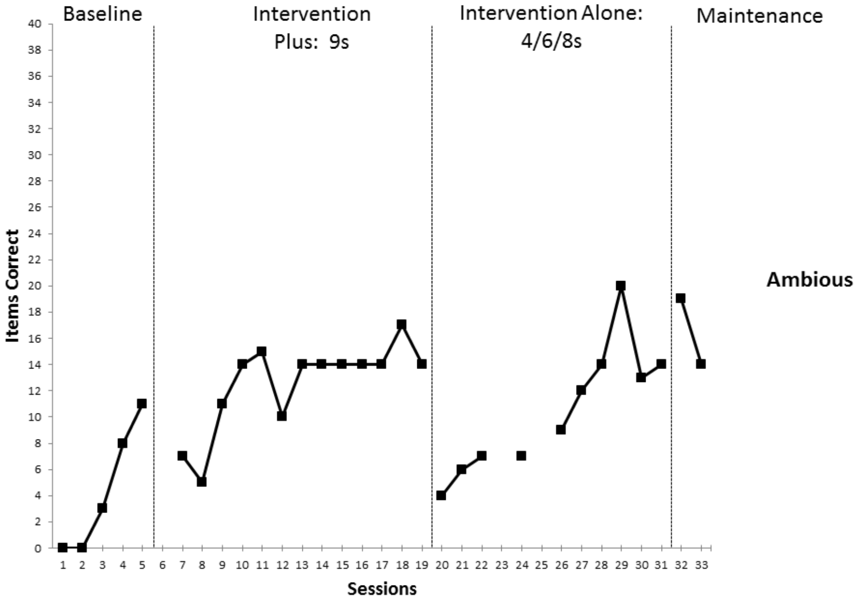


Note: IC = Items Correct, DC = Digits Correct

Figure 4.2: Group 2 Means – Multiplication and Division

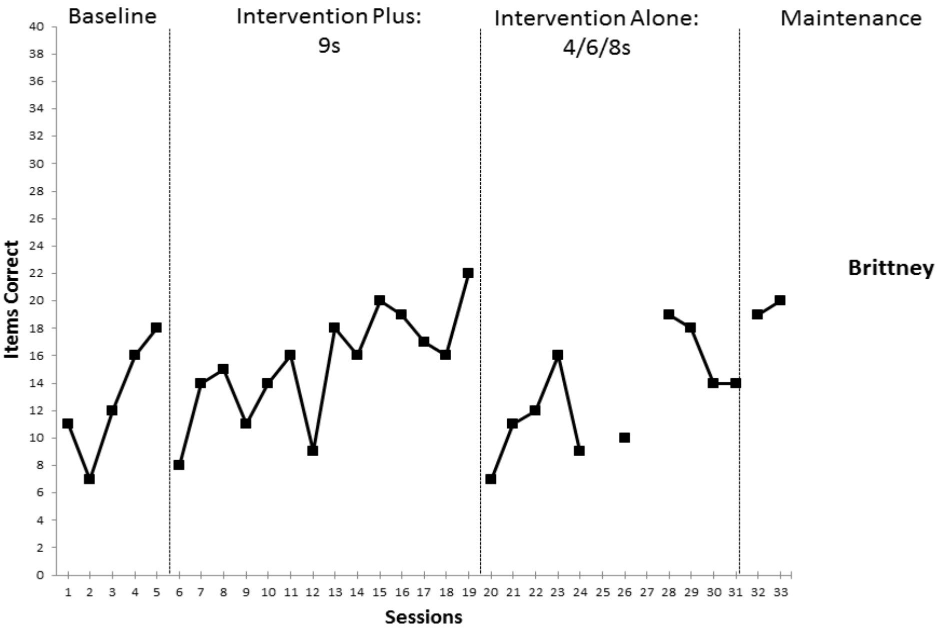


Note: Multiplication

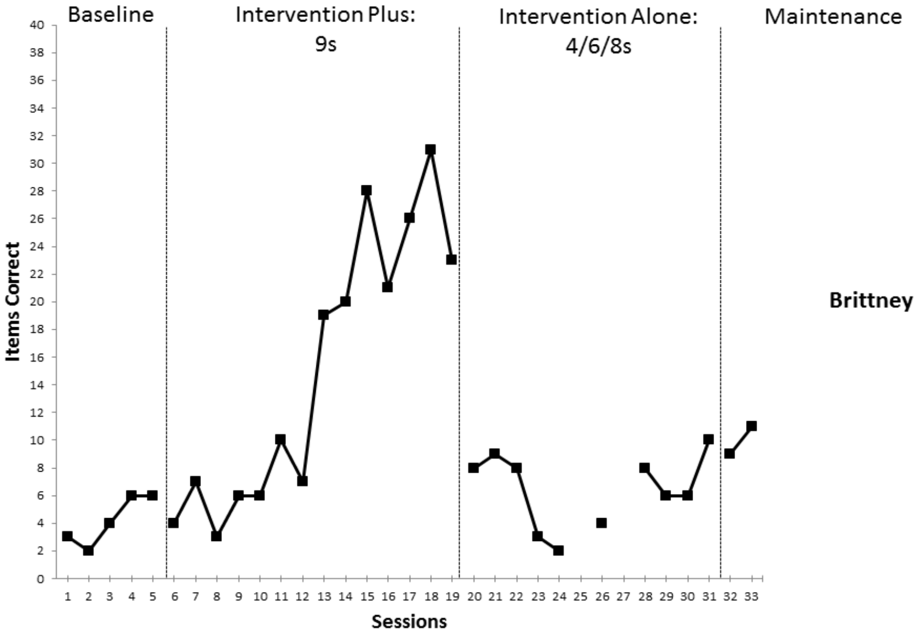


Note: Division

Figure 4.3: Ambious - Items Correct

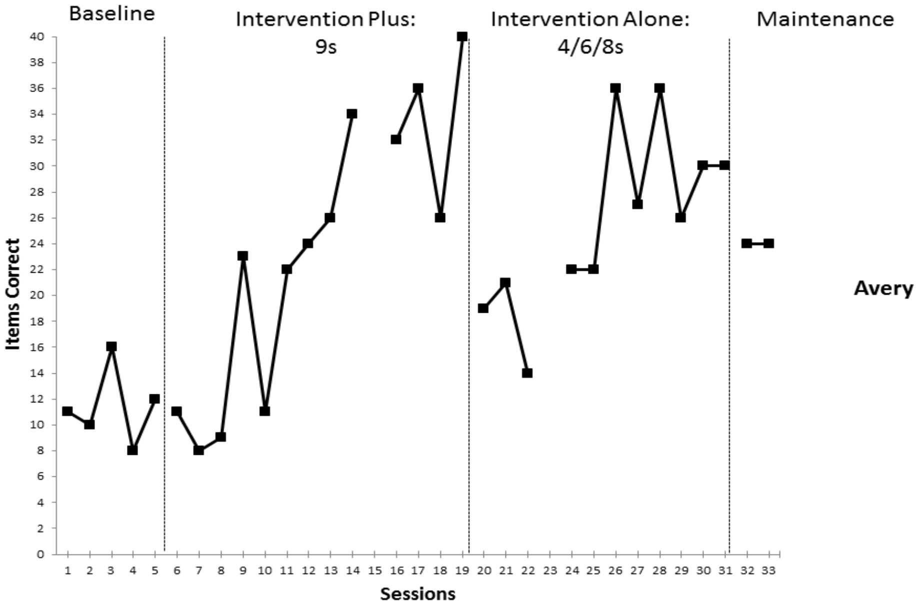


Note: Multiplication

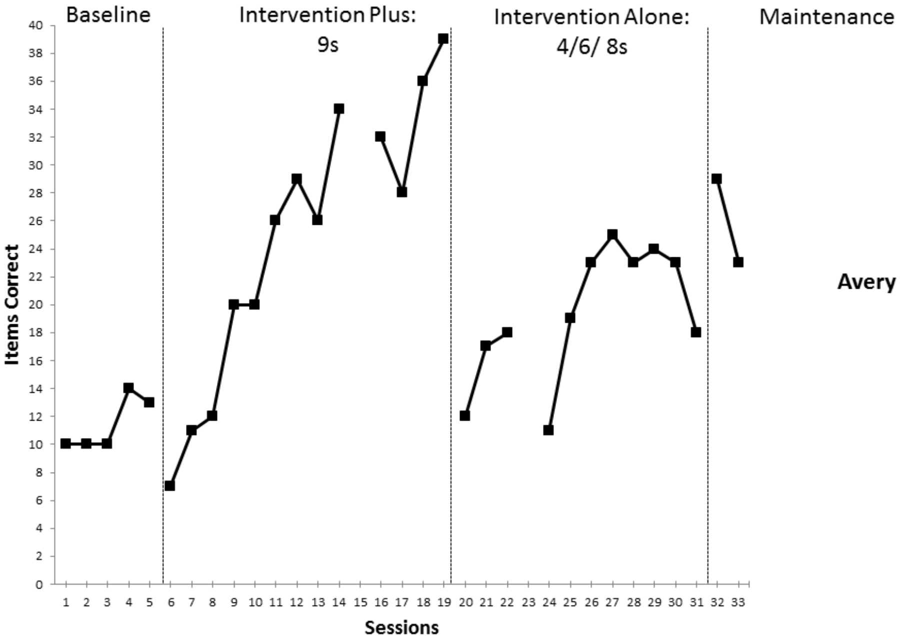


Note: Division

Figure 4.4: Brittney - Items Correct

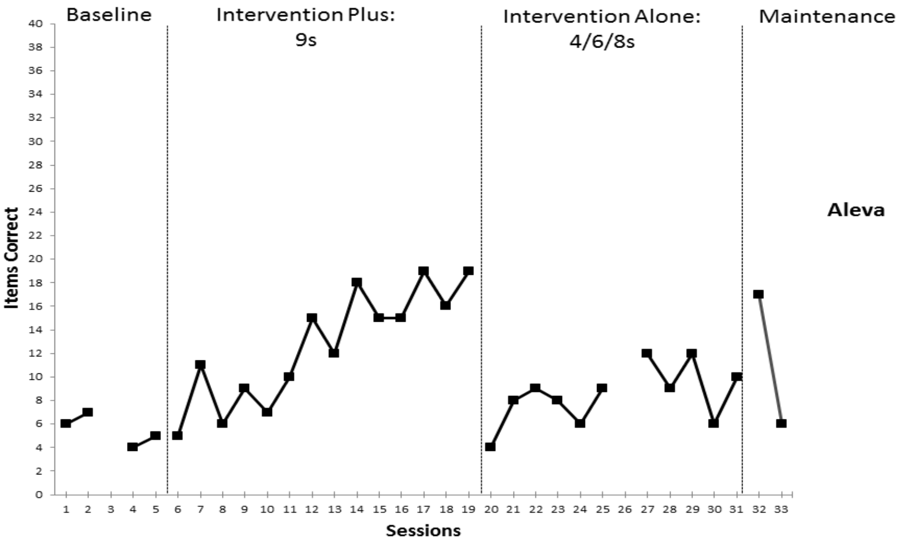


Note: Multiplication

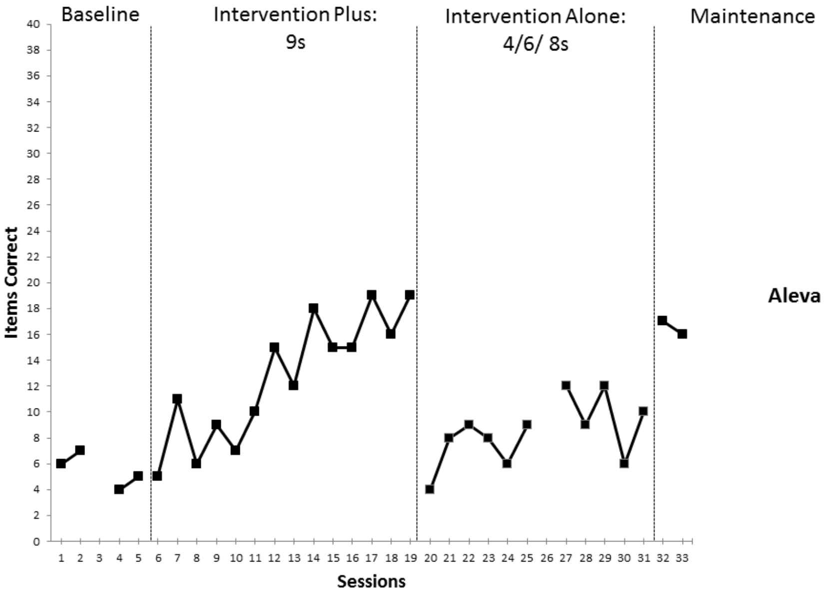


Note: Division

Figure 4.5: Avery - Items Correct

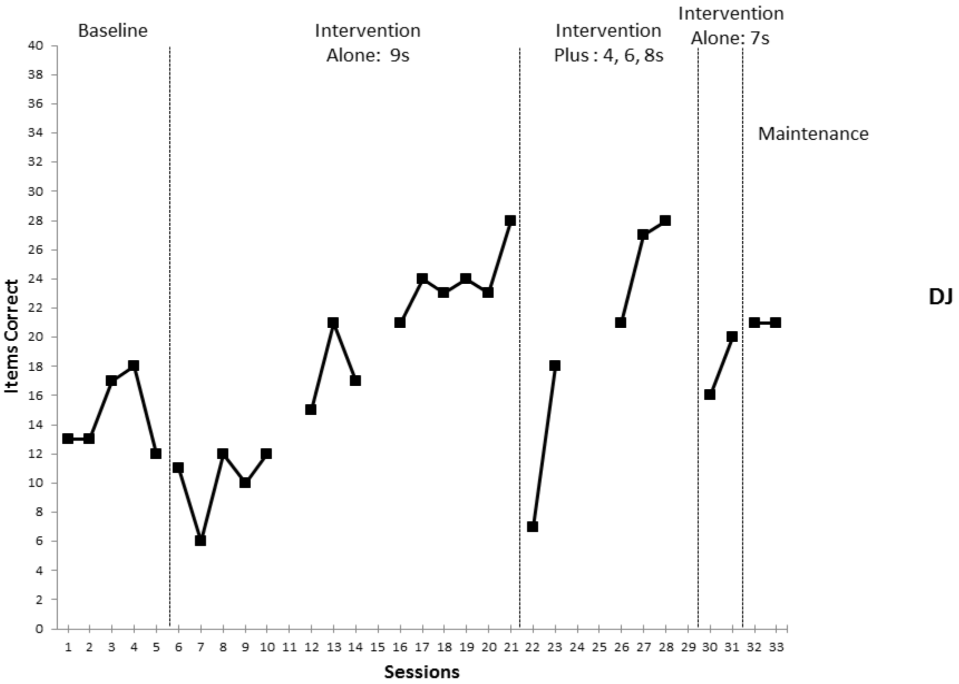


Note: Multiplication

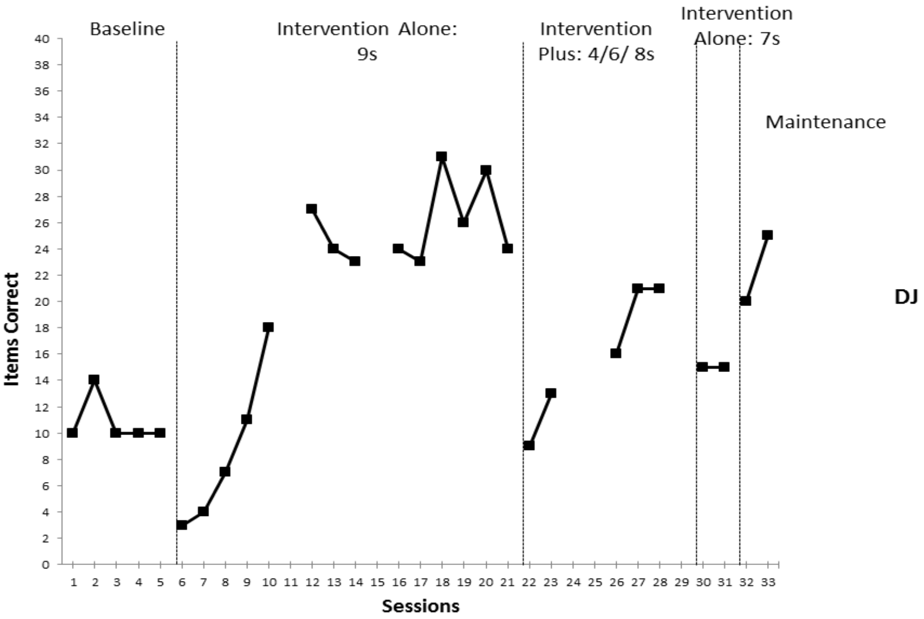


Note: Division

Figure 4.6: Aleva - Items Correct

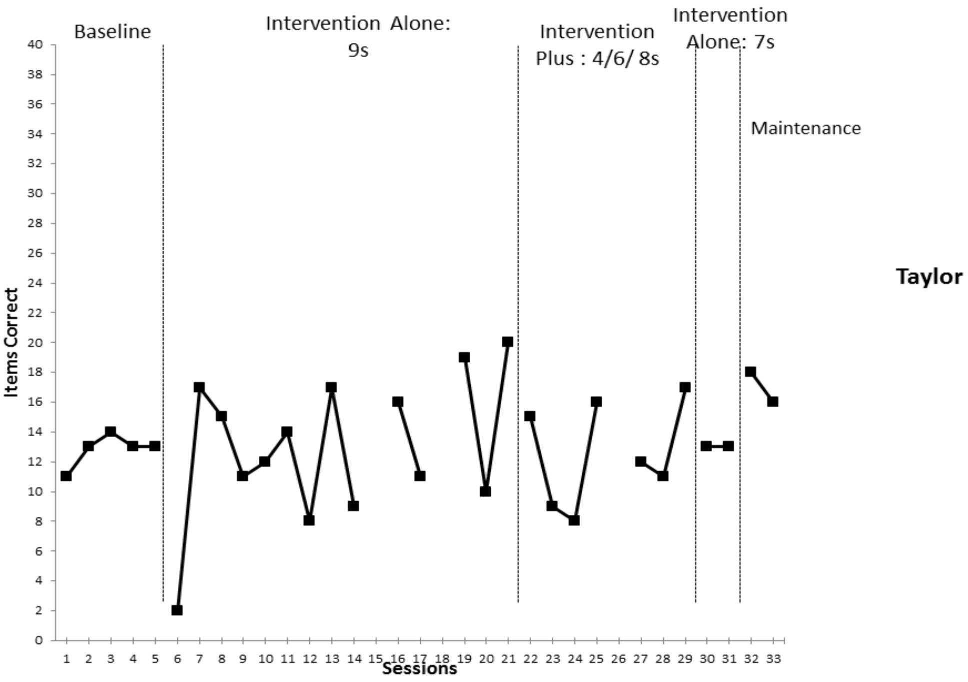


Note: Multiplication

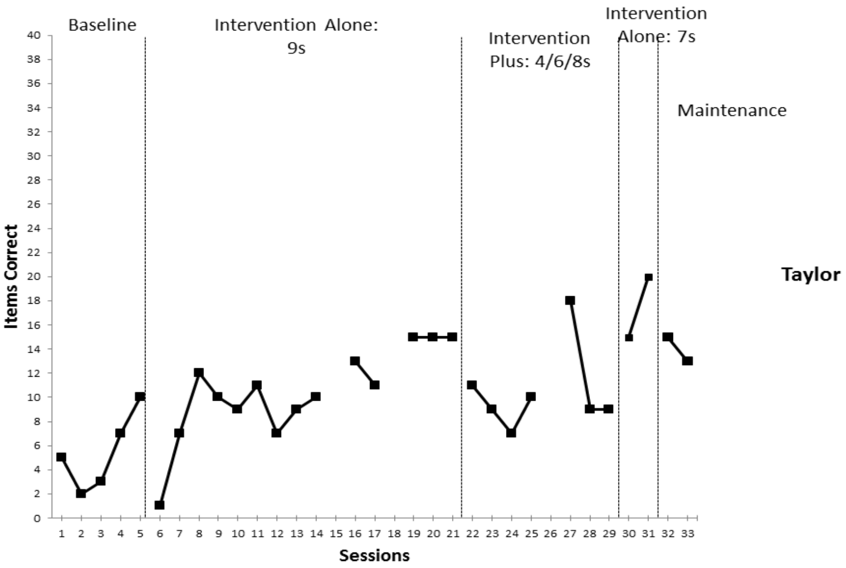


Note: Division

Figure 4.7: DJ - Items Correct

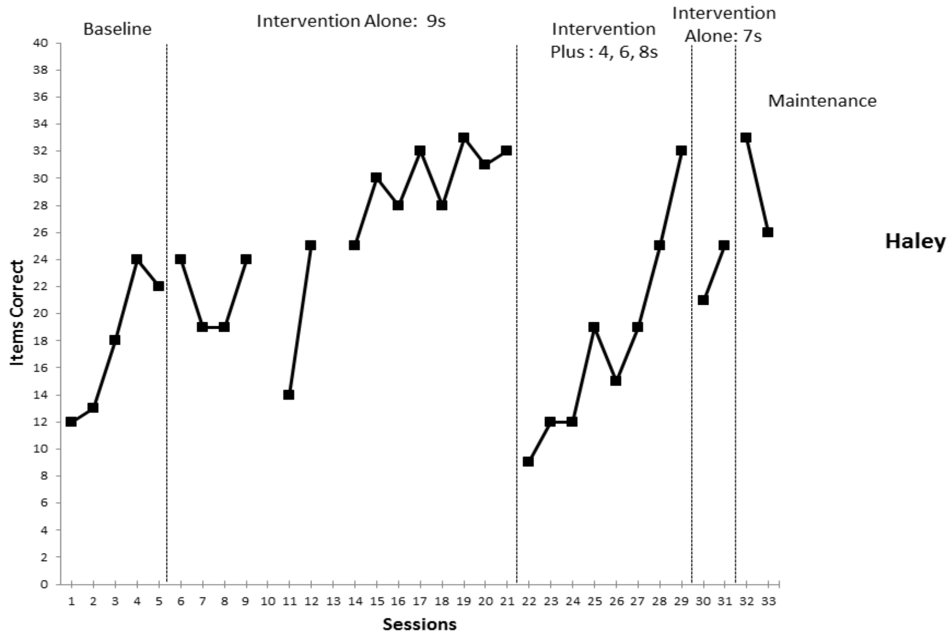


Note: Multiplication

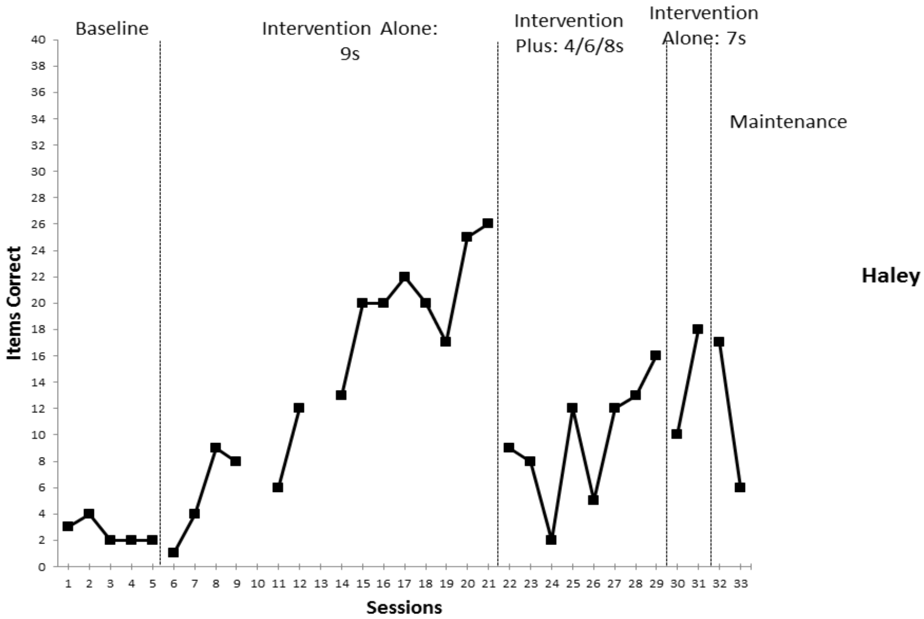


Note: Division

Figure 4.8: Taylor - Items Correct

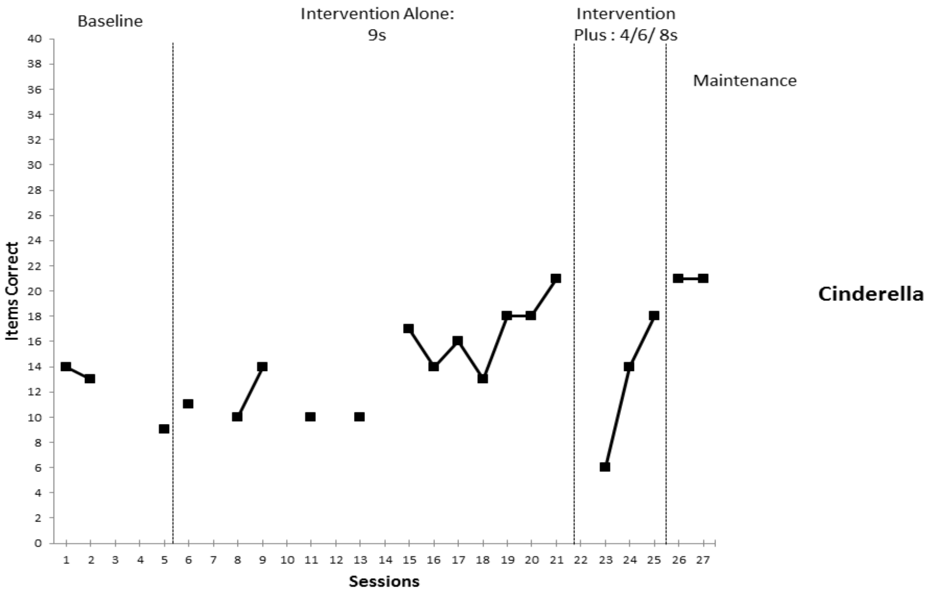


Note: Multiplication

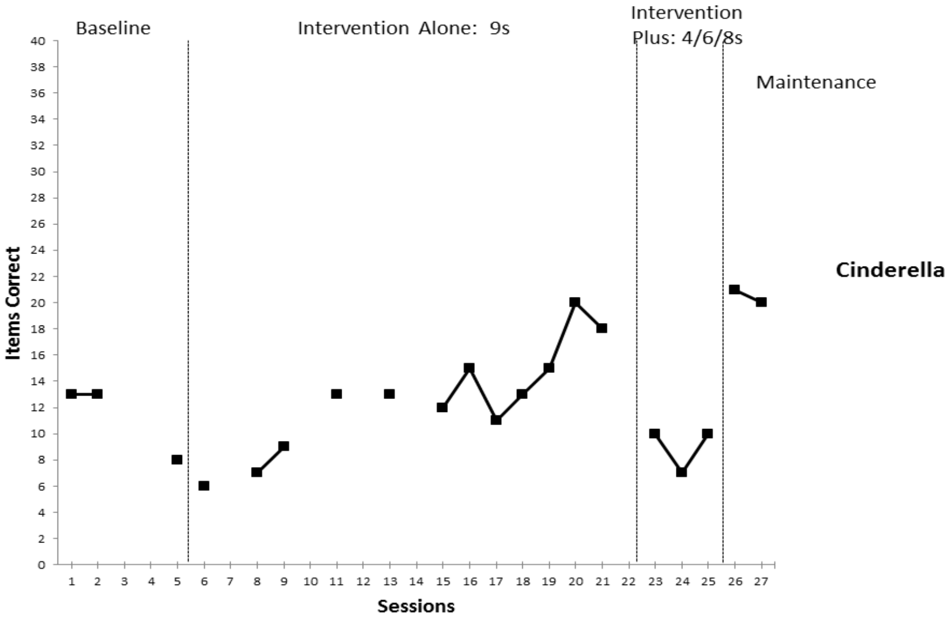


Note: Division

Figure 4.9: Haley - Items Correct

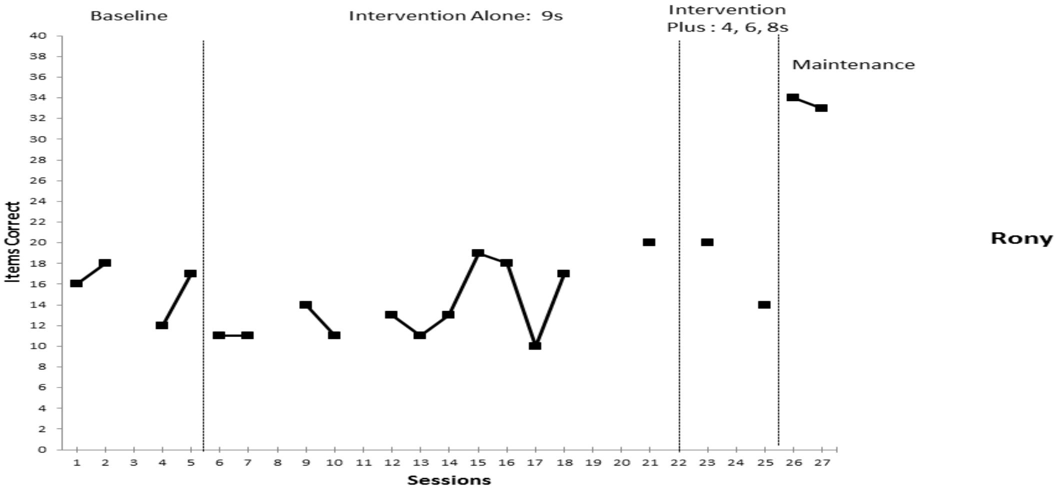


Note: Multiplication

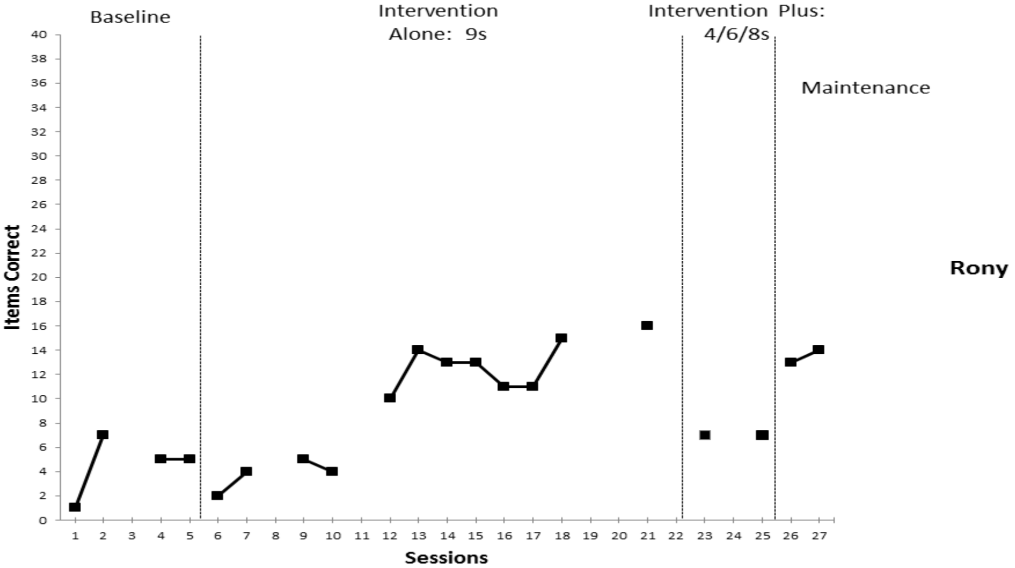


Note: Division

Figure 4.10: Cinderella - Items Correct



Note: Multiplication



Note: Division

Figure 4.11: Rony - Items Correct

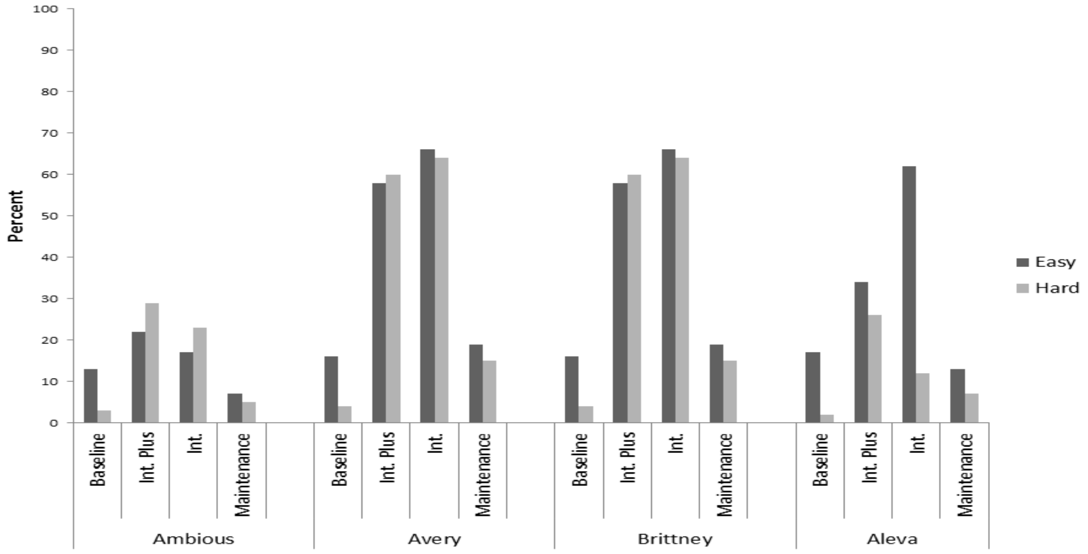


Figure 4.12: Group 1 - Multiplication Easy and Hard Problems Completed

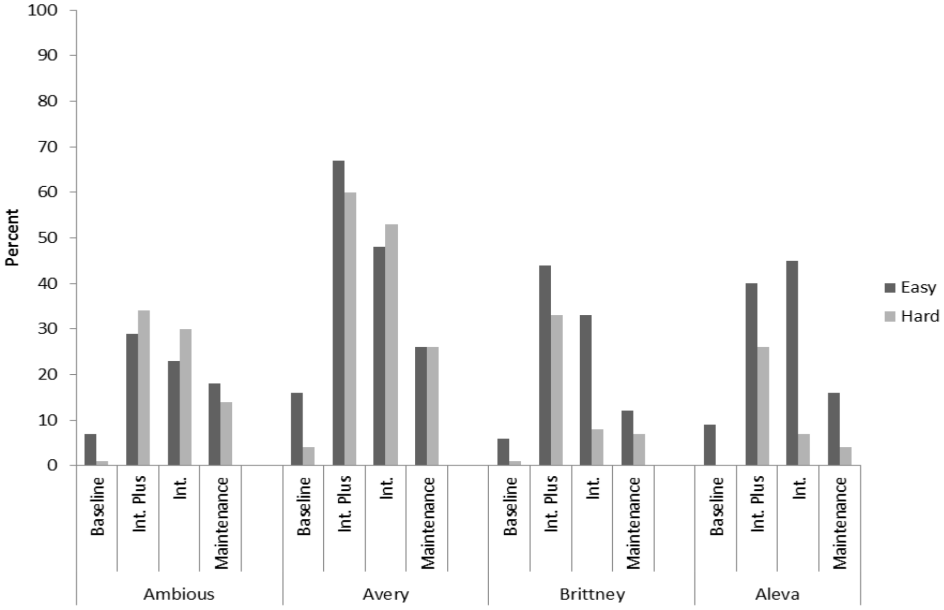


Figure 4.13: Group 1 - Multiplication Easy and Hard Problems Completed

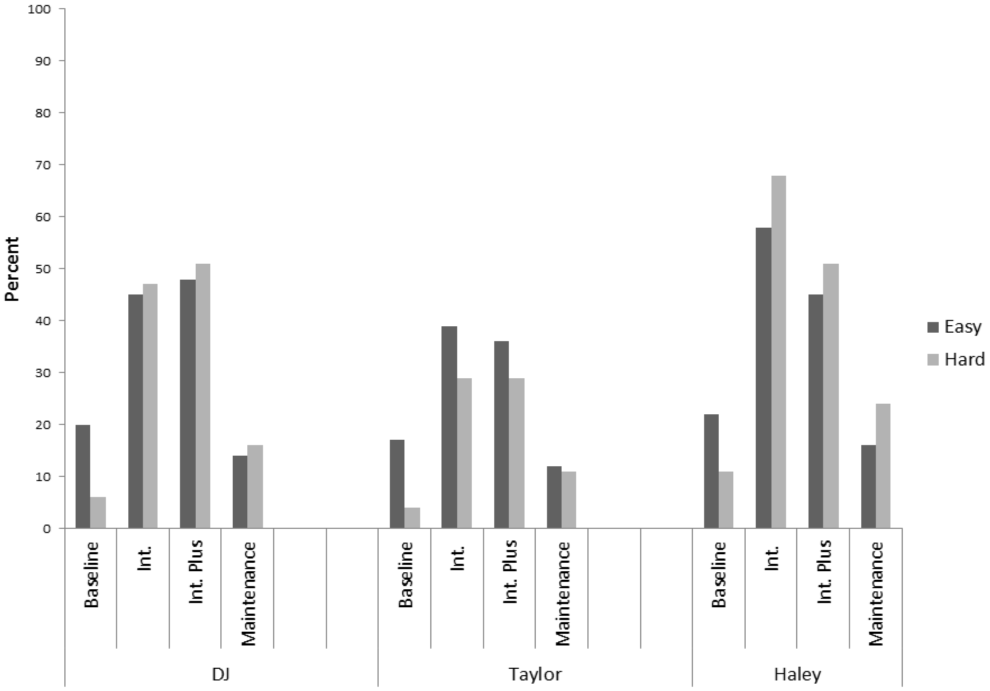


Figure 4.14: Group 2 - Multiplication Easy and Hard Problems Completed

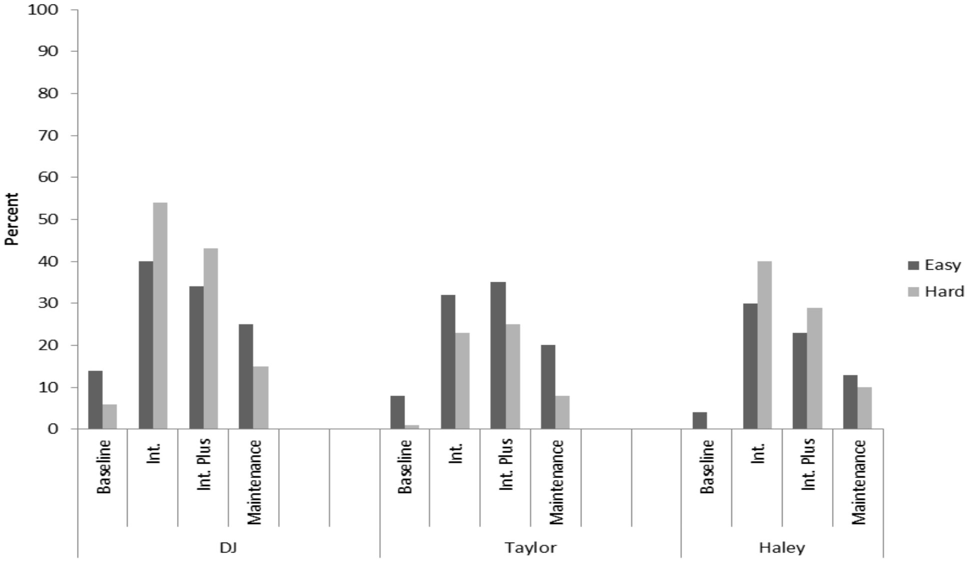


Figure 4.15: Group 2 - Multiplication Easy and Hard Problems Completed

EFFECT SIZES

To test for overall effects of the intervention, three types of analyses were completed: (a) percentage of non-overlapping data points (PND), (b) percentage of all non-overlapping data points (PAND), and (c) regression analysis. A comparison of regression lines was conducted to compare the slope and test for significant differences between and within groups.

Percentage of non-overlapping data points

PND was calculated by comparing baseline for each phase, as well as baseline to all phases, using group data. In multiplication, the measurement of items correct was used rather than digits correct because this was the measurement applied to change phases. PND was calculated to compare baseline to each intervention phase, Phase B and Phase C, to determine whether a difference in effect was evident as a result of the additional self-regulation components in *Intervention Plus*, Phase C. PND was also calculated to measure the effects of the intervention, because the intervention remained constant regardless of phase changes.

Group 1

PND was calculated between baseline and Phase C, *Intervention Plus* and baseline and Phase B, *Intervention Alone*. There were a total of 19 data points, 5 in baseline and 14 in Phase C. In multiplication, baseline to Phase C, a total of 6 data points overlapped, resulting in a PND of 68%, categorized as a questionable effect. Phase B had a total of 12 data points. In multiplication, baseline to Phase B, 4 data points overlapped, resulting in a PND of 76% categorized as effective instruction. In multiplication, baseline to intervention (Phase C plus Phase B) resulted in a total of 31 data points, 10 data points overlapped, and PND was 68%, categorized as questionable effects. The two maintenance data points did not overlap the baseline data points, resulting in a PND of 100%. In division, baseline to Phase C, 3 data points overlapped, resulting in a PND of 84%, categorized as effective instruction. In division, baseline to Phase B, 4 data points overlapped, resulting in a PND of 76%, categorized as effective instruction. In division, baseline to intervention (Phase C plus Phase B), 7 data points overlapped, resulting in a PND of 77%, categorized as effective instruction. The two maintenance data points did not overlap the baseline data points, resulting in a PND of 100%.

Group 2

PND was calculated between baseline and the first intervention phase, Phase B, *Intervention Alone* and Phase C, *Intervention Plus*. The second Phase B, was not calculated because only 2 data points were collected. There are a total of 21 data points between baseline (5 data points) and Phase B (16 data points). In multiplication, baseline to Phase B, 9 data points overlapped, resulting in a PND of 57%, categorized as questionable effects. Phase C had 8 total data points. In multiplication, baseline to Phase C, 4 data points overlapped, resulting in a PND of 69%, categorized as questionable effects. In multiplication, baseline to intervention (Phase B plus Phase C), a total of 24 data points existed, 13 data points overlapped, resulting in a PND of 55% categorized as questionable effects. The 2 maintenance data points did not overlap the baseline data points, resulting in a PND of 100%. In division, baseline to Phase B, 2 data points overlapped, resulting in a PND of 90%, categorized as effective instruction. In division, baseline to Phase C, 1 data point overlapped, resulting in a PND of 92% categorized as effective instruction. In division, baseline to intervention (Phase B plus Phase C), 3 data points overlapped, resulting in a PND of 92% categorized as effective instruction. The 2 maintenance data points did not overlap the baseline data points, resulting in a PND of 100%.

Percentage of all non-overlapping data points

PAND is a means of calculating a phi coefficient by splitting the high data points from baseline and intervention into a 2 x 2 table; and then the difference between two cells is computed for effect size (Parker et al., 2007). A phi coefficient of .70 is considered a large effect, .50 to .69 a medium effect, .30 to .49 a small effect, and less than .29 no effect (Cohen, 1988). It is recommended to use PAND when a data set has more than 20 data points (Parker et al., 2007), which is not the case for the present study using group means. Group 1, baseline and Phase C, totals only 19 data points. Group 1, baseline and Phase B, totals only 17 data points, and again not enough for PAND calculation. Group 2, baseline and Phase B, had a total of 21 points, but baseline and Phase C totals only 13 data points. Rather than calculate PAND for Group 2 baseline to Phase B only, PAND was calculated using the individual student scores for each phase as compared to baseline, within the assigned groups.

Group 1

There was a total of 73 individual student data points between baseline and Phase C in Group 1. See table 4.2 for individual statistics. In multiplication, 36 data points overlapped baseline in Phase C. This resulted in an overlap percentage of 49.3%. The overlap percentage was then subtracted from 100% to calculate PAND and was at 50.7%. The overlap, 49.3 %, was divided between the intervention and the baseline to split the high overlap. To calculate phi coefficient, the difference between the high and low data points was calculated using the formula from the assigned cells in the 2 x 2 table: [a/(a + c) – b/(b + d)]. See table 4.2 for the 2 x 2 table. Using items correct the phi coefficient was -0.27, with negative effects. PAND was also calculated using digits correct in multiplication to test for different results. The phi coefficient using digits correct resulted in no effect, phi = 0.08.

The same procedure was used to determine the phi coefficient between baseline and Phase B, *Intervention Alone*. In multiplication, a total of 61 data points, with 30 data points that overlapped resulted in an overlap of 49.1%. PAND was then calculated at 50.9 %. The overlap percentage was then split between the baseline and intervention in the 2 x 2 table to calculate the phi coefficient. Again, negative effects were seen following the computation of the phi coefficient at -0.15 for items correct, but a positive coefficient of 0.16 when calculated with digits correct. See table 4.2 for the statistics and the 2 x 2 tables for multiplication.

Statistics	**Aleva**	**Ambious**	**Brittney**	**Avery**
Phase scores	A: 12, 15, 11, 9 C: 6, 10, 9, 7, 8, 9, 6, 11, 12, 15, 16, 21, 17, 17	A: 8, 8, 13, 8, 9 C: 4, 7, 7, 10, 15, 12, 8, 10, 12, 12, 14, 13, 13	A: 11, 7, 12, 16, 18 C: 8, 14, 15, 11, 14, 16, 9, 18, 16, 20, 19, 17, 16, 22	A: 11, 10, 16, 8, 12 C: 11, 8, 9, 23, 11, 22, 24, 26, 34, 32, 36, 26, 40
Number of scores	18	18	19	18
Phase means	A: 11.75 C: 11.71	A: 9.2 C: 10.54	A: 12.8 C: 15.36	A: 11.4 C: 23.23
Overlap points	10	11	11	4

Note: A = Baseline, C = Intervention Plus

Overlap	**Intervention**	**Baseline**	**Total**
Higher	49.36 *cell a*	24.64 *cell b*	74 %
Lower	24.64 *cell c*	1.36 cell d	26 %
Total:	74 %	26 %	100 %

Statistics	**Aleva**	**Ambious**	**Brittney**	**Avery**
Phase scores	A: 12, 15, 11, 9 B: 13, 13, 9, 13, 12, 9, 12, 10, 15, 17, 13	A: 8, 8, 13, 8, 9 B: 7, 7, 6, 6, 3, 9, 9, 10, 13, 10	A: 11, 7, 12, 16, 18 B: 7, 11, 12, 16, 9, 10, 19, 18, 14, 14	A: 11, 10, 16, 8, 12 B: 19, 21, 14, 22, 22, 36, 27, 36, 26, 30, 30
Number of scores	15	15	15	16
Phase means	A: 11.75 C: 12.36	A: 9.2 C: 8	A: 12.8 C: 13	A: 11.4 C: 25.72
Overlap points	10	10	9	1

Note: A = Baseline, B = Intervention Alone

Overlap	**Intervention**	**Baseline**	**Total**
Higher	44.35 *cell a*	24.55 *cell b*	68.9 %
Lower	24.55 *cell c*	6.55 *cell d*	31.1 %
Total:	68.9 %	31.1 %	100 %

Table 4.2: Group 1 Items Correct – Multiplication

In division the effects were more promising with a total of 73 individual student data points in baseline and Phase C. Of those 73 data points, 14 overlapped between baseline and Phase C, resulting in an overlap percentage of 19.2%. PAND was calculated at 80.8%. The overlap percentage, 19.2% is split in the 2 x 2 table. The phi coefficient is calculated at 0.5, or medium effects. Baseline to Phase B resulted in 61 data points: 14 data points overlapped, resulting in an overlap percentage of 23%. PAND was then calculated at 77%. The overlap percentage, 23% was split between the baseline and intervention resulting in a phi coefficient of 0.46, or a small effect. See table 4.3 for the statistics and the 2 x 2 tables for division.

Statistics	**Aleva**	**Ambious**	**Brittney**	**Avery**
Phase scores	A: 6, 7, 4, 5 C: 5, 11, 6, 9, 7, 10, 15, 12, 18, 15, 15, 19, 16, 19	A; 0, 0, 3, 8, 11 C: 7, 5, 11, 14, 15, 10, 14, 14, 14, 14, 14, 17, 14	A: 3, 2, 4, 6, 6 C: 4, 7, 3, 6, 6, 10, 7, 19, 20, 28, 21, 26, 31, 23	A: 10, 10, 10, 14, 13 C: 7, 11, 12, 20, 20, 26, 29, 26, 34, 32, 28, 36, 39
Number of scores	18	18	19	18
Phase means	A: 5.5 C: 12.64	A: 4.4 C: 12.54	A: 4.2 C: 25.8	A: 11.4 C: 24.62
Overlap points	3	4	4	3

Note: A = Baseline, C = Intervention Plus

Overlap	**Intervention**	**Baseline**	**Total**
Higher	64.4 *cell a*	9.6 *cell b*	74 %
Lower	9.6 *cell c*	16.4 cell d	26 %
Total:	74 %	26 %	100 %

Statistics	**Aleva**	**Ambious**	**Brittney**	**Avery**
Phase scores	A: 6, 7, 4, 5 B: 4, 8, 9, 8, 6, 9, 12, 9, 12, 6, 10	A; 0, 0, 3, 8, 11 B: 4, 6, 7, 7, 9, 12, 14, 20, 13, 14	A: 3, 2, 4, 6, 6 B: 8, 9, 8, 3, 2, 4, 8, 6, 6, 10	A: 10, 10, 10, 14, 13 B: 12, 17, 18, 11, 19, 23, 25, 23, 24, 23, 18
Number of scores	15	15	15	16
Phase means	A: 5.5 B: 8.45	A: 4.4 B: 10.6	A: 4.2 B: 7.5	A: 11.4 B: 19.36
Overlap points	3	4	4	3

A = Baseline, B = Intervention Alone

Overlap	Intervention	Baseline	Total
Higher	57.4 *cell a*	11.5 *cell b*	68.9 %
Lower	11.5 *cell c*	19.6 cell d	31.1 %
Total:	68.9 %	31.1 %	100 %

Table 4.3: Group 1 Items Correct – Division

Group 2

To calculate PAND and the phi coefficient from baseline to Phase B, *Intervention Alone,* I used all five participants' individual scores. This resulted in a total of 88 data points. In multiplication, of those 88 data points, 36 overlapped between baseline and phase B, resulting in an overlap percentage of 40.9%. PAND was calculated at 59.1%. This percentage was then split between the high and the low in baseline and intervention in the 2 x 2 table. The phi coefficient was then calculated with a result of -0.22, negative effects for items correct. The same procedure was completed with digits correct to identify any difference, and this resulted in no effect at phi = 0.19. PAND was calculated for baseline to Phase C, but only three participants scores were used (DJ, Haley, Taylor) due to the removal of two participants at the beginning of the phase (Rony, Cinderella). This left a total of 35 data points; 12 data points overlapped between baseline and Phase C. The overlap percentage is 34.3%. PAND was calculated at 65.7%. The phi coefficient was found to be positive, at 0.3, with a small effect for items correct. Using digits correct for these three participants for baseline to Phase C, overlap percentage was 31.4% and PAND at 68.6%. The phi coefficient was calculated at 0.36, still a small effect. See table 4.4 for statistics and the 2 x 2 table for multiplication.

Statistics	Taylor	DJ	Avery	Rony	Cinderella
Phase scores	A: 11,13, 14, 13, 13 B: 2, 17, 15, 11, 12, 14, 8, 17, 9, 16, 11, 19, 10, 20	A: 13, 13, 17, 18, 12 B: 11, 6, 12, 10, 12, 15, 21, 17, 21, 24, 23, 24, 23, 28	A: 12, 13, 18, 24, 22 B: 24, 19, 19, 24, 14, 25, 25, 30, 28, 32, 28, 33, 31, 32	A: 16, 18, 12, 17 B: 11, 11, 14, 11, 13, 11, 13, 19, 18, 10, 17, 20	A: 14, 13, 9 B: 11, 10, 14, 10, 10, 17, 14, 16, 13, 18, 18, 21
Number of scores	19	19	19	16	15
Phase means	A: 12.8 B: 15	A: 14.6 B: 17.64	A: 17.8 B: 26	A: 15.75 B: 14	A: 12 B: 14.33
Overlap points	8	7	5	9	7

Note: A = Baseline, B = Intervention Alone

Overlap	Intervention	Baseline	Total
Higher	44.55 *cell a*	20.45 *cell b*	75 %
Lower	20.45 *cell c*	4.55 *cell d*	25 %
Total:	75 %	25 %	100 %

Statistics	Taylor	DJ	Avery
Phase scores	A: 11,13, 14, 13, 13 C: 15, 9, 8, 16, 12, 11, 17	A: 13, 13, 17, 18, 12 C: 7, 18, 21, 27, 28	A: 12, 13, 18, 24, 22 C: 9, 12, 12, 19, 15, 19, 25, 32
Number of scores	12	10	13
Phase means	A: 12.8 B: 12.57	A: 14.6 C: 20.2	A: 17.8 B: 17.88
Overlap points	4	2	6

Note: A = Baseline, C = Intervention Plus

Overlap	Intervention	Baseline	Total
Higher	39.9 *cell a*	17.2 *cell b*	57.1 %
Lower	17.2 *cell c*	25.3 *cell d*	42.5 %
Total:	57.1 %	42.9 %	100 %

Table 4.4: Group 2 Items Correct – Multiplication

In division, baseline to Phase B, there were a total of 88 data points between the five participants. Of those 88 data points, 25 overlapped, resulting in an overlap percentage of 28.4%. PAND was calculated to be 71.6%. The overlap percentage, 28.4%, was split, and the phi coefficient was 0.24, showing no effect. Baseline to Phase C was calculated with the three participants' individual data, resulting in 35 data points; 8 data points overlapped. The percentage of overlap was 22.9%, PAND at 77.1%. The phi coefficient was calculated at 1.04, or a large effect. See table 4.5 for the statistics and 2 x 2 tables for division.

Statistics	Taylor	DJ	Avery	Rony	Cinderella
Phase scores	A: 5, 2, 3, 7, 10 B: 1, 7, 12, 10, 9, 11, 7, 9, 10, 13, 11, 15, 15, 15	A: 10, 14, 10, 10, 10 B: 3, 4, 7, 11, 18, 27, 24, 23, 24, 23, 31, 26, 30, 24	A: 3, 4, 2, 2, 2 B: 1, 4, 9, 8, 6, 12, 13, 20, 20, 22, 20, 17, 25, 26	A: 1, 7, 5, 5 B: 2, 4, 5, 4, 10, 14, 13, 13, 11, 11, 15, 16	A: 13, 13, 8 B: 6, 7, 9, 13, 13, 12, 15, 11, 13, 15, 20, 18
Number of scores	19	19	19	16	15
Phase means	A: 5.4 B: 14	A: 10.8 B: 19.64	A: 2.6 B: 14.5	A: 4.5 B: 9.83	A: 11.33 B: 12.67
Overlap points	7	4	2	4	8

Note: A = Baseline, B = Intervention Alone

Overlap	Intervention	Baseline	Total
Higher	60.8 *cell a*	14.2 *cell b*	75
Lower	14.2 *cell c*	10.8 *cell d*	25
Total:	75%	25%	100

Statistics	Taylor	DJ	Avery
Phase scores	A: 5, 2, 3, 7, 10 C: 11, 9, 7, 10, 18, 9, 9	A: 10, 14, 10, 10, 10 C: 9, 13, 16, 21, 21	A: 3, 4, 2, 2, 2 C: 9, 8, 2, 12, 5, 12, 13, 16
Number of scores	12	10	13
Phase means	A: 5.4 C: 10.43	A: 10.8 C: 16	A: 2.6 C: 9.63
Overlap points	5	2	1

Note: A = Baseline, C = Intervention Plus

Overlap	Intervention	Baseline	Total
Higher	45.65 *cell a*	11.45 *cell b*	57.1
Lower	11.45 *cell c*	31.45 *cell d*	42.9
Total:	57.1%	42.9%	100

Table 4.5: Group 2 Items Correct – Division

Comparison of Regression

Comparison of regression was computed using an online calculator, http://www.stattools.net/Comp2Regs_Pgm.php. This program works by first computing the slope for each individual group and then estimating common slope testing for significance at *p* = .05. A standard error was computed to test if the slopes were significantly different. Significance of regression lines was tested between Group 1 and Group 2 for different fact strands (9s and 4/6/8s). In addition, within-group phases were tested to control for participants and to identify if differences existed in response to the additional components of *Intervention Plus*.

Between groups

The first fact strand taught across both groups was the 9s. Group 1 received *Intervention Plus*, Phase C, and Group 2 received *Intervention Alone*, Phase B. Group 1 had a total of 14 data points; Group 2 had 16 data points. The slope for Group 1 multiplication, Phase C, was 1.081, and the slope for Group 2, Phase B, was 0.788. The regression analysis for the individual groups was significant for Group 1, *p* = .0001, Group 2, *p* = .001. In comparing the two slopes, the difference between slopes was 0.2931; the standard error difference was 0.15. The slopes were approaching significance at *df* = 26, *p* = .06. In division, the same comparison of phases for the 9s produced significance for individual slopes: Group 1 slope, 1.49, *p* = .0001, Group 2 slope, 1.13, *p* = .0001. When the slopes were compared, the difference was 0.37, and the standard error difference was 0.14. The difference in slopes between the two groups was statically significant at, *df* = 26, *p* = .01.

The second comparison was between groups during the 4/6/8 fact strand. During this portion of the intervention, Group 1 received *Intervention Alone*, Phase B and Group 2 received *Intervention Plus*, Phase C. Group 1 had a total of 12 data points; Group 2 had

8 data points. In multiplication of 4/6/8s, the slope was significant for Group 1, slope 0.64, *p* = .0067, but not Group 2, slope .989, *p* = .07. When the slopes were compared, the difference was 0.3622, and the standard error difference was 0.52. The difference of slope was not significantly different, *df* = 16, *p* = .49. In division, the individual slope was significant for Group 1, slope 0.67, *p* = .0034, but not significant for Group 2, slope 0.99, *p* = .07. The difference between the two slopes was 0.31, with a standard error difference of 0.47, *df* = 21, *p* = .51 and was not statistically different.

Within groups

All students received the intervention, but to further test for effects of the self-regulation components, slopes were compared to test for statistical differences between phases. Group 1 received Phase C, *Intervention Plus,* and fact strand 9s with a total of 14 data points. The next phase received was Phase B, *Intervention Alone,* fact strand 4/6/8 with a total of 12 data points. In multiplication the slope was greater in Phase C, slope = 1.08, than Phase B, slope = 0.64, and was significant in both phases: Phase C *p* = <.0001, Phase B *p* = .0067. The difference between the two slopes was 0.441, with a standard error difference of 0.21. The slopes were significantly different, *df* = 22, *p* = .043. The slope in division was greater in Phase C, slope = 1.49 and significant at *p* = <.0001. Phase B had a slope = 0.626 and significant at *p* = .03. The difference between the slopes within Group 1 was 0.87, with a standard error difference of 0.25. The slopes were significantly different at *df* = 22, *p* = .0023.

Group 2 regression data was computed for Phase B, *Intervention Alone* and Phase C, *Intervention Plus*. Phase B had a total of 16 data points and Phase C had a total of 8 data points. In multiplication the slope of Phase B was significant, slope = 0.788, *p* = <0001. The slope of phase C was slightly steeper and significant, slope = 2.14, *p* =

<.0001. The difference between the two slopes was 1.25, with a standard error of 0.28. The slopes were significantly different, *df* = 20, *p* = <.0001. In division, the slope of Phase B is significant, slope = 1.13, *p* = <.0001. In Phase C the slope is not significant, slope = 0.988, *p* = .06. The difference between the two slopes was 0.1398, with a standard error difference of 0.337. There was not a significant difference between the two slopes in division, *df* = 20, *p* = .68.

HISTORICAL DATA

The nine participants were selected from five, Grade 4 classrooms. These classroom teachers filled out a short questionnaire to better describe the mathematical instruction, specifically fact instruction and any small-group instruction. These data were used to describe the general education classroom and explain any data points that could have been influenced by classroom instruction. The teachers were also asked to describe student selection, fact instruction in the classroom, and self-regulation components. It should be noted that the intervention was provided in addition to core mathematics instruction. The intervention was designed not to replace fact instruction but, rather, to strengthen and provide more practice for struggling students.

Student selection

This portion of the questionnaire examined the students selected for the intervention and inquired as to whether any additional instruction was occurring in conjunction with the intervention. All five teachers strongly agreed that the students selected were accurately selected and had been previously identified as requiring extra assistance and tutorials in mathematics. All the students except one (Ambious) received teacher tutorials on Tuesday and Thursday for 30 min, in addition to 30 min of flexible, small-group intervention during the school day. Three teachers reported that they agreed with the statement that the students receiving the intervention made better progress in facts than the students' peers and two teachers strongly agreed that greater improvement was seen in the intervention students than in the peers.

Fact instruction in the classroom

Starting this school year the teachers were responsible for testing fluency in addition, subtraction, multiplication and division. The teachers in Grade 4 reported that they conducted assessments for fluency in multiplication once a week (n = 2) or once a month (n = 3) and division weekly (n = 1), monthly (n =3), or not at all (n = 1). Three teachers worked specifically in multiplication and division facts with struggling students as part of whole-class instruction and in small groups. One teacher worked with students in multiplication and division facts as a whole class and in small groups and an interventionist worked with a few students in a pull-out program. Only one teacher worked with struggling students in facts individually.

The amount of time devoted to fact instruction varied among the five teachers. Two teachers spent 2 days per week on fact instruction, for about 10 to 12 min. Another teacher spent about 2 days per week, but 45 min per session. Two other teachers devoted 3 days per week to fact instruction, but the time varied between 10 and 20 min.

Although the time spent working with struggling students in multiplication and division facts differed, more consistency was evident in the time devoted to facts within the core curriculum. Fact instruction was listed as part of districts' scope and sequence for mathematics, and multiplication and division were listed as goals for Grade 4. All five educators agreed (n = 1) or strongly agreed (n = 4) that math facts are important in the curriculum, and once a week, multiplication and division facts were part of whole-class instruction. Four of the five teachers used strategies and manipulatives to teach multiplication and division facts. Some of the strategies and tools listed included arrays, multiplication chart, break-apart, and count-by. While the teachers did report that fact instruction is important, most time in after- school tutorials and during core instruction was devoted to preparing for the state test.

Self-regulation components

The teachers were directed by the district to test for fluency and accuracy in multiplication and division starting in the fall prior to the intervention. As reported previously, all five teachers collected data on multiplication probes, and four teachers collected data on division probes. All five teachers had the students grade the probes, but collective data on the probes were not shared with the students. None of the teachers had the students use the data to set individual goals or graph the results. The teachers added that the data they collected from the multiplication and division probes involved the total items correct, and they did not report completing any error analysis.

SOCIAL VALIDITY

Social validity was measured using a questionnaire for both the classroom teachers and the participants. The student questionnaire was a 4-point Likert scale scored as (4) *totally agree*, (3) *kind of agree*, (2) *kind of disagree* and (1) *totally disagree*. The average scores on each question measured ranged from 1 to 4 (M = 3.56). Table 4.6 reports the results of the participant's social validity measures. The students also answered five open-ended question:

(1) What did you like best about coming to learn about strategies to solve multiplication and division?

(2) What was hard for you when learning the facts?

(3) What would you tell your friends about our math tutoring time?

(4) Would you recommend it to your friends?

(5) What, if anything would you change about the math tutoring time?

The teachers' questionnaire consisted of 4-point Likert scale scored as (4) *strongly agree*, (3) *agree*, (2) *disagree* and (1) *strongly disagree*. The average score on each question measured ranged from 3 to 4 (M = 3.3). The teachers were also asked to answer four additional questions about duration and frequency of the strategy use, such as regularly, rarely, never or never, sometimes and frequently. Table 4.7 denotes the results of the teachers' social validity measures. In addition, the teachers were asked to answer three open-ended questions:

(1) Overall, did you feel that the intervention was successful? Why or why not?

(2) Was the time too much, not enough, or just right?

(3) Please provide any additional comments related to the intervention.

Participants

Overall, the students enjoyed the intervention and rated learning multiplication and division and strategies as the best part of the intervention (*n* = 4). Avery reported that the best part of the intervention was "because it helps me in class with my work in class." She added, " I like coming because she made it seem easy." Another participant, Cinderella, added that best part of the intervention was that "the teacher made it more easy to understand." Ambious reported simply that he liked it, and Brittney wrote very largely that she did not like it. When asked individually why she did not like the intervention, she simply shrugged her shoulders and stated, "I don't know." Students varied in their answers for what was hard, listing division (*n* = 3), division and multiplication (*n* = 1), the break-apart strategy (*n* = 2), mastering the nines (*n* = 1), and nothing (*n* = 2). Three students were not sure about what they would tell friends or if they would recommend the intervention to friends. Taylor simply wrote, “I would tell my class and my friends.” Six of the students would recommend the intervention to friends because, “it’s cool, yes it helps” (Rony). Avery added, “It is fun, easy and helpful for my brain. And it will help you memorize the steps and your skills.” Haley agreed, “I would do it every day to help people learn more math facts so they could get better at it.” Eight students stated that the intervention time needed to be changed. The students wanted the intervention to be during school, rather than after school, especially Rony and Cinderella. Two students, Haley and Avery would increase the time, “have more time like 40 minutes to practice more and learn more” (Haley). “Having it every day during snack time every day because I can get healthy and smart at the same time. It is helpful for me” (Avery). Only one student added that the instructor should give out prizes if the students paid attention and “don’t bother you” (Aleva). Overall, the students seemed to really enjoy the intervention time together after school and stated that they would miss the math time.

Statement:	Ambious	Aleva	Avery	Brittney	DJ	Taylor	Haley	Rony	Cinderella	**TOTAL**	**AVERAGE**
The strategies used to learn multiplication facts were easy.	4	4	4	3	3	3	4	4	3	**32**	**3.56**
The strategies used to solve multiplication facts would be good for my class mates to use.	4	1	4	3	4	4	4	3	4	**31**	**3.44**
I liked the strategies I learned to solve multiplication and division facts.	3	4	4	4	4	4	4	4	4	**35**	**3.89**
I think the strategies helped me to do better in math class.	4	3	4	4	4	4	4	3	4	**34**	**3.78**
I liked grading the 1-minute fact sheets.	4	1	4	4	3	4	4	4	4	**32**	**3.56**
I liked graphing my results.	4	1	4	4	3	4	3	4	4	**31**	**3.44**
I liked setting a goal for myself for the next fact sheet.	3	1	4	4	2	4	3	4	4	**29**	**3.22**

Table 4.6: Social Validity of Participants

Educators

The five classroom educators felt that the intervention was helpful for the selected students. Two teachers reported the students gained confidence and ability. Teacher 4 explained, “I do feel that the intervention was successful. I noticed the students were able to do more facts and able to use it in the long division and 2-digit by 2-digit multiplication.” Teacher 5 added, “The student gained confidence in her abilities to work out math facts and I saw an increase in fluency when times." Teacher 2 agreed, "The students gained confidence in their ability to multiply and divide." All five teachers felt that the time for the intervention was just right, "Students did not "burn out" but were consistent with practice" (Teacher 5). While the teachers did recognize the participating students were still behind peers, they felt the additional practice and/or starting the intervention earlier in the school year could make a difference. All of the educators were extremely pleased with the intervention and looked forward to using the strategies with their students during the coming school year.

Statement:	Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5	**TOTAL**	**AVERAGE**
The student's ability in solving multiplication and division facts increased as compared to peers.	3	4	3	3	3	**16**	**3.2**
The intervention helped my student's gain confidence in mathematics.	3	4	3	3	3	**16**	**3.2**
I would like to use the strategies with other students in the future.	4	4	4	4	3	**19**	**3.8**

Statement:	Teacher 1	Teacher 2	Teacher 3	Teacher 4	Teacher 5
Intervention students used the taught strategies in my classroom.	Regularly	Regularly	Regularly	Regularly	Regularly
The students shared the taught multiplication/division fact strategies with peers.	Sometimes	Sometimes	Sometimes	Never	Frequently
The students were able to use the strategies to solve multi-step/complex multiplication and division problems.	Sometimes	Frequently	Sometimes	Sometimes	Frequently
The student's shared their goals and progress on fact sheets with me.	Sometimes	Never	Sometimes	Never	Sometimes

Table 4.7: Social Validity of Educators

Chapter 5: Discussion

The purpose of this study was to evaluate the effectiveness of self-regulation components within an explicit and systematic multiplication and division intervention for students in Grade 4 with identified mathematical weaknesses. The intervention taught specific strategies to solve multiplication and division facts while developing conceptual understanding and procedural knowledge, along with vocabulary for multiplication and division (Core Standards, 2010; Gersten et al., 2009; NMAP, 2008).

The goal of the intervention was to increase accuracy and fluency of multiplication and division facts, which are considered to be critical foundation skills for algebra (NMAP, 2008). Another purpose of this study was to examine the effects of self-regulation components on mastery of the facts. The self-regulation components within the intervention include students' feedback through self-correction, graphing of scores, and goal setting.

The intervention content was influenced by the research base from the Curriculum Focal Points (NCTM, 2006); the Common Core State Standards (2010); recommendations from NMAP (2008); findings on strategy instruction (Woodward, 2006); Mathematics Instruction for Students with Learning Disabilities or Difficulty Learning Mathematics, from the Center on Instruction (Jayanthi et al., 2008); and Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools (Gersten et al., 2009). The instructional design of the intervention included the converging evidence for teaching mathematics and features of explicit, systematic instruction, including modeling, guided practice, and independent practice (Gersten et al., 2009; Jayanthis et al., 2008; Stein et al., 2006). The features of the instructional design also ensured that the mathematic fact strategies taught built procedural knowledge, while developing conceptual understanding. Vocabulary pertinent to multiplication and related division (Rittle-Johnson & Star, 2007) was taught, and students were encouraged to use mathematical vocabulary when describing processes associated with solving the facts (e.g., factor, product, quotient, divisor, dividend), and to connect multiplication and division via fact families to increase mathematical flexibility (Gray, Pinto, Pitta, & Tall, 1999; Jayanthi et al., 2008). Also, multiple representations were used to build conceptual understanding of a specific strategy (Gray et al., 1999; Rittle-Johnson, 2001; Rittle-Johnson & Star, 2007).

This research was guided by a single research question: To what extent will fluency and accuracy scores on multiplication and division fact probes differ as a result of a systematic, strategic intervention plus goal setting and graphing as compared to a systematic, strategic intervention without goal setting and graphing?

In this chapter the results in relation to the research question will be discussed. Additionally, a description of the study's limitations will be described, followed by suggestions for future research and implications for practice.

RESEARCH QUESTION AND DESIGN SUMMARY

The research question will be discussed in the following ways. First will be a description of the intervention alone, and then a discussion of the intervention with the self-regulation components will follow. Finally, there will be a discussion of the overall effect of the treatment, including the differences between the individual components.

The design of the study was a single-subject, time-series, A – B – C – D or A – C – B –D. Phase B and Phase C were intervention phases. In Phase B, *Intervention Alone,* the students received systematic, strategic instruction in multiplication and division facts. The students completed daily probes, and the instructor graded all probes. In Phase C, *Intervention Plus*, the students received the systematic, strategic instruction, but additional self-regulation components were added to the end of instruction. The self-regulation components included student self-correction of the probes, graphing, and goal-setting using the data from the multiplication and division probes. In Phase D, maintenance, the participants completed two additional probes in multiplication and division 2 weeks after the end of the intervention. A student group remained in an intervention phase until the phase change criteria was obtained, based on the group mean of items correct on the multiplication and division probe. The phase change criteria in multiplication were 22 items correct on 2 out of 3 days of instruction. In division, the phase change criteria were 12 items correct on 2 out of 3 days of instruction.

In each intervention phase a hard fact strand was taught. A fact strand is a specific fact in which all facts contain a common fact. The hard fact strands were as follows: (a) 9s, (b) 4/6/8s, and (c) 7s. The nine participants were divided into two groups. Group 1 had four participants and Group 2 had five participants (but then two Group 2 participants were removed, leaving three participants). Following the baseline phase, Group 1 received Phase C. The design for Group 1 was A – C – B –D. Following the baseline phase, Group 2 received Phase B. The design for Group 2 was A – B – C – B – D. The second Phase, B, had only just begin; 2 data points were collected prior to the conclusion of the intervention.

Intervention alone

Both groups received Phase B, or *Intervention Alone*. Group 1 received Phase B during the 4/6/8s fact strand, and Group 2 received Phase B during the 9s fact strand. In Phase B the students received the explicit instruction with multiplication and division taught and practiced together in a fact family. Systematic and strategic instruction in facts has been used successfully in multiplication studies (Burns, 2005; Calderhead, Filter, & Albin, 2006; Flores, Houchins, & Shippen, 2006). Yet, very few interventions explicitly teach division facts (Axtell, McCallum, Bell, & Poncy, 2009), and no studies could be located that teach multiplication and division facts together. By using systematic and strategic instruction to teach multiplication and division, this treatment will add to the research base in the area of mathematics. This intervention design taught the two operations as multiplication and related division facts from the start; results show that the participants made progress, as measured by items correct and accuracy in both multiplication and division probes.

To meet the phase change criteria, both groups had to complete more intervention sessions in Phase B than in Phase C. Group 1 received 12 sessions of Phase B but did not reach the phase change criteria in multiplication. Group 2 received 16 sessions of Phase B prior to phase change criteria in multiplication. Although the criterion was not reached, the trend of the data was positive, recognizing that students did need systematic, strategic fact instruction. In the Powell et al. (2009) study, students were categorized as having a mathematical difficult only or mathematics and reading difficulty, assigned to three tutoring conditions (fact-retrieval practice, conceptual instruction with fact retrieval practice, procedural computation/estimation instruction) and one control group (no tutoring). Overall, the students with only mathematical difficulties outperformed the students with mathematics and reading difficulties. The tutoring groups that received fact instruction showed significant gains as compared to the control group. For students with mathematics difficulty only, the fact-retrieval groups outperformed groups receiving no fact instruction, as well as the control group.

For Group 1 and Group 2, multiplication phase change criteria was more difficult to obtain than division. During baseline, division facts were difficult for the participants, as well as for typical peers. In January, 12 items correct was the 50th percentile using the local normative data. Following only a few days of intervention, the participants were scoring greater than the 50th percentile in items correct. For Group 1, the phase change criterion in division was reached following eight sessions. For Group 2, the phase change criterion in division was reached following seven sessions. In a study (Axtell et al., 2009) teaching division facts, intervention students were given many opportunities to practice missed facts on individual mastery levels. In addition, the students were "competing" only against themselves by setting a goal and then graphing the results all within the same day of the intervention. The students receiving the intervention in division outperformed the control group. This study reported similar results: Items correct for division were lower in baseline, but following instruction, practice, and a task involving identifying the related division fact from multiplication, the scores increased at a quick rate. Both the slopes of division were steeper than those of multiplication during Phase B. In Group 1, Phase B (4/6/8s), the slope of multiplication was 0.64 and the slope for division was 0.67. In Group 2, Phase B (9s), the slope in multiplication was 0.788 and the slope for division was 1.13. In both groups the slope was positive, showing that instruction alone and practice in facts, without the additional self-regulation components, can increase fluency of the multiplication and division facts for students with identified weakness in mathematics.

Intervention plus

Both groups received Phase C, or Intervention Plus. Group 1 began the intervention phases with Phase C during the 9s fact strand. Group 2 received Phase C following Phase B, during the 4/6/8s fact strand. In Phase C, the students continued to receive the strategic, systematic instruction, but additional self-regulation components were added. The components included self-correction of the multiplication and division probes, graphing results, and goal-setting. The self-regulation components occurred following the completion of the assessments. The self-regulation components added only an additional 5 min to the overall intervention time.

The students were grouped according to mean scores on items correct on the multiplication selection assessment. Group 1 scores were lower in baseline than were those of Group 2. Although Group 1 started lower, the students reached phase change criteria in Phase C (9s) two sessions prior to Group 2. Group 1 accuracy was also higher than Group 2's during the 9s fact strand. Accuracy for Group 1, Phase C, was 97% in multiplication and 91% in division, as compared to a decrease in Phase B, 88% in multiplication and 76% in division. A similar trend in accuracy was evident when Group 2 changed to Phase C. For Group 2, Phase B, accuracy in multiplication was 87% and in division was 80%. The accuracy increased to 91% in multiplication and 81% in division. The increase in accuracy may be the result of the addition of the self-correction components. For Group 2, who began the intervention without the additional components, simply grading and receiving immediate feedback seemed to be a motivational factor for these students. Group 2 students discovered that the other group was grading prior to the change in phase. The Group 2 students then started asking about their individual scores and errors and attempted to quickly count how many problems were completed prior to the collection of the probes. Two students in particular, Taylor and Haley, were very excited to correct and graph the results in Phase C.

An unexpected finding was that the rate to reach the phase change criteria also increased for both groups in Phase C, as measured by the slope. For Group 1, the slope for multiplication in Phase C (9s) was 1.08 and the slope for division was 1.5. For Group 2, Phase C (4/6/8s), the slope for multiplication was 2.14 and the slope for division was

0.99. The students in Group 2 started Phase C (4/6/8s) 2 days after Group 1 but reached phase change criteria in both multiplication and division after eight sessions. Compared to Phase B (9s), the session time was divided in half, allowing Group 2 to begin the 7s fact strand. The self-regulation components added an additional 5 min to instruction but decreased the number of session days needed to reach phase change criteria for both groups. Other studies (Axtell et al., 2009; Burns, 2005; Flores et al., 2006) have utilized graphing of digits correct within a multiplication (Burns, 2005; Flores et al., 2006) or division (Axtell et al., 2009) intervention with graphing but did not directly measure or compare the intervention without graphing, but their results were similar to those of this study. In the Burns (2005) study, three participants had no overlapping data, resulting in 100% PND. From baseline to intervention, all participants' scores on number of digits correct increased over time during the intervention phase, which included graphing of digits correct. Axtell et al.'s (2009) study included an intervention group that graphed digits correct and a control group that did not receive any intervention. An ANCOVA to adjust means and account for any differences between the intervention and comparison groups showed results supporting an intervention that "not only [identified] a statistical significant difference between the adjusted means of the two groups, but also a large effect size" (p. 534). Flores et al. (2006) utilized a brief amount of instructional time, 10 to 15 min, to teach one specific skill (multiplication) and saw an increase in correct digits in only 5 days.

COMPARISON OF INTERVENTION ALONE AND INTERVENTION PLUS

The students in both groups displayed positive trend lines across the intervention phases. Differences existed in the session days needed to meet the phase change criteria and in slope when comparing Phase B and Phase C. The effect sizes for each phase by group differed in each analysis—PND, PAND, and regression analysis—and do not provide clear results about the impact of the additional self-regulation components. A discussion of each type of effect size follows.

Percentage of non-overlapping data points

PND for this study was lower than expected. The skill was academic in nature, and it can be hypothesized that students do need more than one session to increase skill level. The students in both groups did not show immediate significant improvement when the intervention began, regardless of participation in Intervention Alone or Intervention Plus. This could be a result of the probe difference between the baseline and intervention conditions. In baseline the students skipped around the page in an attempt to locate easy facts. In the daily probes, the amount of easy facts was controlled—only 15 easy facts per probe. The probes were also designed in a manner that would force students to answer an increased amount of hard facts from the specific fact strand. The probes were designed based on the results of studies showing that students' accuracy and mastery increased when activity sheets were mixed with known and unknown problem types (Calderhead et al., 2006; Lee et al., 2005). The probes contained 25 hard facts and were designed systematically: two hard facts and then an easy fact, for the entire probe. As a result of the specifications in the daily probe, an improvement of scores on hard facts was evidenced by all students, as compared to baseline conditions. This is similar to the findings from Lee et al. (2005) that identified when problem types are mixed (50%/50% split of known to unknown, or 40%/60% split of known to unknown facts) students are more successful in solving.

Group 1

The overall results for PND are categorized as effective when one examines the gains in items correct as a result of students' receiving supplemental instruction in multiplication and division facts. The calculation of PND does not provide a clear argument that the intervention time was more effective as a result of the addition of the self-regulation components. In multiplication, baseline to Intervention Plus, PND (68%) was categorized as questionable. From baseline to Intervention Alone, PND (76%) was categorized as effective. In division, regardless of the intervention phase, PND percentages were higher (baseline to Phase C = 84%; baseline to Phase B = 76%), with less overlap between baseline and intervention phase data. For Group 1, it can be stated, using the PND data, that the self-regulation components added to *Intervention Plus* in division were more effective than *Intervention Alone*. The only other conclusion that can be justified was that the students made steady progress and increased fluency of multiplication and division facts.

To more closely match the instructional approach to supplemental instruction in schools, the students were placed in groups. The group mean was calculated each day following the multiplication and division probes. Individual data trends can skew a mean, as well increase PND as a result of insufficient progress. It is important to note that two participants' data (Ambious's and Brittney's) remained relatively flat, and phase change criteria was not obtained. Another student's data in Group 1 (Avery) often inflated the mean as a result of the high scores maintained across intervention phases. Her high scores on items correct allowed the group to meet the phase change criteria in fewer sessions than other students Avery's score on the probes increased the mean and resulted in phase change criteria met prior to the other participants in that group. In division, again, PND results were categorized as not very different across the two phases.

In both phases, PND results were categorized as effective instruction. In baseline to Phase C, PND was slightly higher, at 84%.

Group 2

In Group 2, PND effects did not differ in multiplication as a result of *Intervention Alone* or *Intervention Plus*. In multiplication, in both intervention phases, the PND resulted in questionable effects, slightly higher at 69% during Phase C. In division, PND resulted in large effects: 90% during Phase B and 92% in Phase C. Individually, the attendance of two participants (Cinderella and Rony) affected the group mean and may have been the reason for additional sessions in Phase B (9s). Rony tended to score low and, when present, acted to decrease the group mean. When Cinderella was present, the group mean was greater. Both Rony and Cinderella had sporadic attendance, and that could explain some of the variance in Phase B. In Phase C (4/6/8), the mean scores for items correct were more consistent, as a result of Rony and Cinderella's removal from intervention. In conclusion, the intervention was effective, but greater effects were not evident as a result of the self-regulation components.

Percentage of all non-overlapping data points

PAND was calculated as a means to show effect sizes using a phi coefficient. A limitation with this analysis is the amount of data points needed to calculate. Ideally, 20 or more data points were needed, and in this study there were not enough data points to use the group means. Using the individual scores within each phase resulted in negative effects, potentially due to the fact that group means were used to determine phase changes. The overlap is not as great when examining group data versus individual data. Unfortunately, PAND showed either no effect or small effects (Phase C, Group 2) for multiplication for both groups. PAND did show medium effects in division for Group 1 in both intervention phases, again supporting the conclusion that the intervention was effective regardless of the self-regulation components. For Group 2, no effect was found in Phase B but large effects were evident in Phase C. No literature could be located to compare results using PAND in a mathematical intervention. Parker et al. (2007) calculated PAND from published single-subject studies and concluded that effect sizes were lower using the PAND statistic than PND. These findings support the need for future research to include a self-regulation component within a fact intervention, specifically for teaching division.

Comparison of regression

Completing quantitative analysis within a single-subject design is a relatively new procedure but is suggested in the What Works Clearing House technical document (Kratochwill, Hitchcock, Horner, Levin, Odom, Rindskoph, & Shadish, 2010). In the technical manual, it describes the application of two group comparisons by stating that "the mean difference would typically be standardized by dividing by the control group variance or pooled within-group variance" (Kratochwill et al., 2010, p. 23). For this study, the two groups were able to be compared because, within fact strands, the groups received *Intervention Plus* or *Intervention Alone*. Although a relatively older statistical model was located to compare slopes, the results should be interpreted cautiously due to comparability concerns across groups (Kratochwill et al., 2010).

Overall, there were not statistical differences in multiplication when comparing the slopes between groups across intervention phases within the same fact strand. In division, the slopes were statistically different, favoring Group 1, with a steeper slope (1.49) during the 9s fact strand, with Group 1 receiving *Intervention Plus* and Group 2 receiving *Intervention Alone.* Across groups, the results are similar to the other analysis completed: The intervention resulted in growth/improvement in items correct, in participants, but only minimal to no effects with the addition of the self-regulation components.

As mentioned previously, slope regression is an analysis typically completed between two groups of students. The group means were reported to measure the dependent variable, and the intervention phases changed as a new fact strand was introduced. As an investigation analysis, the regression was compared across groups to identify whether differences in slope were statistically significant. Again, these results should be considered an investigational measure, rather than a strong statistical analysis. For Group 1, slopes in Phase B and Phase C were significant in multiplication, meaning the intervention produced effects on items correct when controlling for number of sessions. In comparing the slopes between groups, a significant difference was evident between Phase C and Phase B in both multiplication and division. An examination of the data points reveals that Group 1 began baseline at a frustration level and in Phase C mastered the 9s with a steep slope (slope = 1.08) and greater mean (mean = 7.5), and in fewer sessions, than Group 2 (slope = 0.79) with consistent increases each session. In Phase B the slope was more flat (slope = 0.63), with a lower mean (mean = 6.5), meaning that the individual students were not making progress each day, often earning the same score or a decreasing score.

In Group 2, baseline scores were at a higher level than Group 1, yet 2 additional data points were needed to reach phase change criteria in Phase B, the 9s fact strand. In this phase, accuracy was lower and students continued to make the same errors, as a possible result of no self-correction or feedback on the probes. In Phase B the slope was more flat (slope = .79) than in Phase C (slope = 2.14). In Phase C, the students' attitudes improved and the students were excited to self-correct the probes and set goals. In multiplication, the difference in slopes was significant, favoring Phase C (slope = 2.14), but not in division, with more similar slopes reported (Phase B slope = 1.13 and Phase C slope = 0.99). In division, the scores increased in both phases exponentially following explicit instruction in how to solve. The solid increase in scores in both phases resulted in the significance for the slope in Phase B but not in Phase C and no difference in comparison of phases.

SOCIAL VALIDITY

The results of the social validity questionnaires indicate that educators and participants found value in the intervention. Similar social validity findings were reported in Jitendra et al. (2005), in which both students and educators saw generalization of skills and increases in mathematical confidence. One educator anecdotally reported that two participants said that the state test was not as difficult, because they could apply the skill that was taught in the intervention. A similar finding was reported in Flores et al. (2006), where all but one participant enjoyed the strategic instruction and reported that the skills taught in the intervention could be applied to other mathematical tasks. The majority of students reported they liked the intervention and that it was beneficial to their classroom performance. Throughout the intervention the students seemed to enjoy working in small groups and saw the connection between the content in the intervention to their classroom. During *Intervention Plus*, the students were excited to grade the probes and set goals. Even when a goal was not obtained the following day, the students were still motivated in writing a new goal and focusing on a specific fact if an error had occurred. After a few sessions with Group 1 during the 9s, one student suggested the idea of writing her goal score on top of each probe. Soon the remaining group members began writing their goal scores, too. The students were taught the self-regulation components, but they made it meaningful to them.

The educators did not teach the intervention but observed changes in the performance of students who participated in the treatment within the classroom. Overall, the teachers reported that the participants were more confident in class and their academic engagement increased during mathematics. In addition, following an application problem session, three students asked if they could take the problems to work on with their peers. The students were excited to have learned a complex problem with many steps and operations and wanted to share this procedure with the class. During the course of the intervention, the principal requested a professional development for the school on fact instruction. One teacher reported that she wrote the Make Ten Minus the Factor strategy on the board and her student was excited, telling her that she had already learned that strategy and used it to solve her 9 facts. Overall, both participants and educators seemed to be positive about the intervention.

LIMITATIONS

Several limitations to this study must be considered when interpreting results. To begin, the use of a single-subject design limits the generalization of results to a larger population of students. The students selected for the study were identified as struggling in mathematical facts. This was triangulated with the educators, because eight of the nine students were already identified as at risk for failing the state mathematics test. The population was carefully selected using local normative data; thus, the use of the study's results should be considered as a pilot study to test for initial effectiveness, rather than making a casual claim. In this study, the use of local norms rather than standardized norms created selection criteria specific to this population.

A second limitation of the study design was the threat to internal validity, due to a lack of replication. Horner et al. (2005) stated that at least three demonstrations of experimental control should be evident in a single-subject study. It was proposed that students would be taught all hard facts, replicating each phase, resulting in an A-B-C-B-C-B-C-D or A-C-B-C-B-C-B-D design. Unfortunately, the students reached phase change criteria at a slower rate than expected. The first fact strand, 9s, took Group 1 about 7 weeks to complete and Group 2 about 8 weeks to complete. Due to slower-than-expected student progress and the end of the school year looming, there was not sufficient time to allow for replication across each group. Additionally, this study design did not allow for three demonstrations because each intervention phase was testing a component, not a different intervention, so it was expected that similar data trends would be reported in Phase B and Phase C. The dependent measure, items correct, was an academic skill that cannot be unlearned; therefore returning to baseline probably would have resulted in an increase in items correct versus the initial baseline.

A third limitation of this study involved comparison across groups. All students were identified with weaknesses in multiplication and division facts and were grouped with students of similar ability. Group 1's baseline scores were lower than Group 2's. All students identified in Group 1 were at the frustrational stage, whereas students in Group 2 were at the instructional phase. The group differences at the outset may have affected the rate at which the phase change criteria was obtained. Group 2 students began at a higher mean of items correct and took two additional sessions to reach the phase change criteria, but they started the next two phases, 4/6/8s and 7s, much higher and reached phase change criteria in about half the amount of sessions. This may be the result of the self-regulation components, or simply the students' ability level, which would mirror the results in a meta-analysis showing that students in the instructional group made more significant gains over the course of a year (Burns et al., 2010). Additional questions remain about the selection of students that may have been "too low," for only two days of strategic, systematic instruction and the overall results of the intervention.

Fourth, the reliability and validity of the instructional probes were not adequately assessed. Specificity rules were developed in the creation of the probes; 15 easy facts were randomly selected, hard facts were systematically assigned, and digits correct were almost equal across probes. Determining technical adequacy across probes within each fact strand would have strengthened the measures used in the study.

The last limitation pertains to control for history effects. The classroom teacher was asked to describe instruction and the amount of instruction devoted to facts, but the students were still receiving core mathematics instruction during the day. The effects of the intervention may have been influenced by timed probes administered in class, as well as additional practice provided as part of daily class assignments. While this is a limitation, the students were selected in January after receiving a semester of fact practice and were still not making the same progress as peers, which demonstrate that the participants were indeed students who have difficulty in mathematics.

PRACTICAL IMPLICATIONS

This study has several practical implications. First, the results show that the use of effective instructional components—modeling, guided practice, independent practice, and self-regulation—is promising for teaching multiplication and division facts to students with math difficulties. The instructional components have large effect sizes (Gersten et al., 2009; Jayanthi et al., 2008; NMAP, 2008), but creating the lessons and progress monitoring can be time consuming for classroom teachers. This intervention was created with the inclusion of effective instructional components, designed to increase student engagement, procedural knowledge, and conceptual understanding (Rittle-Johnson, Siegler, & Alibali, 2006) in a scripted lesson plan that would not require preparation time. The self-regulation components took an additional 5 min and appeared to decrease the number of sessions needed to reach the phase change criteria. The use of self-correction, graphing, and goal-setting is promising for students with academic weaknesses to increase motivation and self-worth and, therefore, would be beneficial to students and practitioners (Gersten et al., 2009; Montague, 2007; NMAP, 2008; Rock, 2005). By accessing an embedded self-regulation strategy within explicit, small-group instruction, students could increase their opportunities in general education and find success in school (Lee et al., 2007).

Another practical implication is the time required to implement the intervention. Only 2 days per week were devoted to strategy instruction. The third day was devoted to application problems and was completed with a larger group (all 9 students). The duration of the intervention was only 30 min and could be accomplished in groups of 3 to 5 students. Burns (2005) found similar results; the intervention took 10 to 15 min and required brief 2-hour training with the research assistants before implementation. The students were placed into small groups, they were exposed to a brief instructional period, and deficits were remediated, while incorporating self-regulation strategies. The teachers reported that the time needed for the intervention was appropriate and was similar to the time devoted to after-school tutoring (60 min, two times per week). Time is often listed as a challenge when implementing interventions within the RtI model (Fuchs, Fuchs, & Stecker, 2010), but given the limited amount of time needed to implement this treatment, it could be practically important to educators.

FUTURE RESEARCH

Although this study is promising, several proposals for future research have emerged from the analysis. First, the need to replicate the study to show experimental control at least three times should be completed. This can be accomplished by starting the intervention earlier in the school year using the same research design. This would allow more instructional sessions, thus teaching all of the hard fact strands. Replication could also be attained through a multiple-baseline or multiple-probe design, across groups or schools, leading to generalization of the study's effects. There is also a need to test the frequency of the intervention. The students did not "burn out," as stated by one teacher, and two students added that they would have liked additional intervention time. Perhaps another day of explicit instruction would lead to more rapid gains in meeting the phase change criteria of the hard facts, or if using group means a lower phase change criteria may be warranted. An analysis of frequency and duration is justified.

The additional time spent in the intervention phase using the self-regulation components is promising. More research needs to be conducted to determine the overall effectiveness of the intervention with the self-regulation components. *Intervention Alone* was promising, and the students did benefit from systematic and strategic instruction, but the self-regulation components increased slope, leading to greater improvement over time. A randomized control trial using *Intervention Plus* is warranted in the near future, and/or multiple baseline design with reversal to test for possible effects on fluency and accuracy due to the self-regulation components. Expanding to include different grade levels, such as fifth grade, may also be beneficial, as the participants were still behind their peers at the end of the year.

More development is also needed in the application element of the instructional routine. Most of the assessments given to students (e.g., state tests, NAEP) are presented as application problems. The practice in application needs to be expanded and data collected to measure if generalization from fact instruction is affecting students' ability to solve multi-step problems. Directly measuring multiplication and division objectives from state or national tests would add to the statistical power of a study and expand the ability to generalize this intervention.

SUMMARY

In sum, the results of this study indicate that systematic, strategic instruction may increase fluency of multiplication and division facts. Specifically, the purpose of this study was to determine if additional self-regulation components added to a systematic, strategic fact intervention would improve accuracy and fluency, as measured on multiplication and division fact probes. A visual analysis was completed, along with three different effect-size analyses. In the meta-analysis completed by Gersten and colleagues (2009), graphing led to large effect sizes when incorporated within an academic intervention. The present study incorporated both graphing and goal-setting but did not result in large effect sizes. It is interesting to note that the rate of to meet the phase change criteria decreased during Phase C in both groups, meaning total session time to reach the phase change criteria was less than in *Intervention Alone*. Overall, the results indicate positive trends in using systematic and strategic instruction; there is still no causal evidence indicating that the self-regulation components increased items correct.

Instruction in facts is pivotal for elementary students for long-term success in mathematics (NMAP, 2008). VanDerHeyden and Burns (2009) found that identification of students with low fluency scores in facts can facilitate early identification of students most in need of supplemental mathematics instruction. In reviewing mathematics interventions with self-regulation components (Axtell et al., 2009; Burns, 2005;

Calderhead et al., 2006; Flores, 2009; Flores et al., 2006; Jitendra et al., 2002; Joseph & Hunter, 2001; Lee et al., 2005; Powell et al., 2009; Van Garderen, 2007), the overall trends indicated that students performed well with interventions targeting mathematics combined with self-regulation strategies. When students with SLD and low mathematics performance are explicitly taught a specific skill, a defined mastery level can be achieved. In two studies, multiplication facts (Axtell et al., 2009) and addition and subtraction facts (Powell et al., 2009) were mastered by small groups of students when they were provided with small-group instruction.

This study incorporated multiplication and related division facts together. The growth in both operations supports teaching multiplication and related division facts at the same time for the duration of fact instruction within core mathematics curricula. In multiplication, the trends of both groups were positive and all students increased scores from baseline. Both maintenance data points were above baseline, and the participants did not skip around seeking only easy problems, as they did in baseline. According to end-of-year-local normative data, the students were no longer below the 10th percentile and the slope from January to May was comparable to that of their peers. Using the mean of items correct from the two multiplication assessments in January, the slope was 1.02. The slope of the intervention students was 1.02, confirming that the progress made was similar to that of typically developing peers.

In the area of division, the students in the study made incredible progress. At baseline, the majority of the participants were scoring less than 13 items correct. Following instruction, the students began scoring well above the 50th percentile of the local normative data and met the phase change criteria in division prior to multiplication. The trends for division were more steep and positive across both groups and participants. Using the end-of-year local normative data, slope was calculated for division from

January to May. Using the mean of the two division assessments given in January the slope of growth for typical peers was .97. The slope of the participants was 0.70—slightly lower but still positive. The positive trends in both multiplication and division, reinforce the need for students with mathematical difficulties, systematic and strategic instruction in multiplication and division facts are needed.

Another unique aspect of this study was the addition of self-regulation components. The additional components required only 5 min during instructional tutoring time and were easily implemented. The students were excited to be grading and reported that they enjoyed graphing and goal setting. As Fowler et al. (2007) found, when examining behavior or academic trends, interventions that actively included the participant were more effective than interventions and rewards managed strictly through adults (educators or researchers). As a recommendation to educators, incorporating self-regulation components may increase motivation and confidence in students, thus increasing their ability. These components can easily be integrated into supplemental instruction and/or core mathematics. Students need instruction in facts, especially students performing well below their peers, such as the participants selected for this study. Connecting multiplication and division helps to increase automaticity for facts and build on the foundational understanding of how the two operations are connected. Adding self-regulation components may not increase the slope significantly or increase accuracy, but the time needed to reach the phase change criteria is shorter.

Appendix A

Parental Permission for Child Participation in Research and Child Assent

The University of Texas at Austin

Your child is being asked to participate in a study of elementary mathematics. My name is Kathleen Hughes Pfannenstiel, and I am a student at the University of Texas at Austin, Department of Special Education. This form provides you with information about the study. I will provide you with a copy of this form to keep for your reference, and will also describe this study to you and answer all of your questions. Please read the information below and ask questions about anything you don't understand before deciding whether or not to take part. Your child's participation is entirely voluntary and you can refuse to have your child participate or withdraw your child from participation without penalty or loss of benefits to which you are otherwise entitled.

Title of Research Study:
Multiplication and Division Facts: Intervention for Success
IRB PROTOCOL # 2010-11-0113

Principal Investigator(s) (include faculty sponsor), UT affiliation, and Telephone Number(s):
Kathleen Hughes Pfannenstiel, M.Ed., Principal Investigator, College of Education, The University of Texas at Austin, 470-0651.

Diane Pedrotty Bryant, Ph.D. Faculty Sponsor, Department of Special Education, College of Education, The University of Texas at Austin, 784-7346.

What is the purpose of this study?
The purpose of this study is to teach strategies to solve multiplication and division facts and to see if it improves understanding of how to solve problems. The instruction will begin with easier division and multiplication problems and then move on to harder ones. To practice solving the multiplication and division problems, students will also solve the problems within a word problem.

What will be done if you take part in this research study?
If your child takes part in the study, he or she will be tested on addition, subtraction, multiplication and division facts for 10-12 minutes. Each fact assessment takes one-minute and will be completed in the child's classroom two times.

If your child scores lower on the multiplication assessment they may be invited to participate in additional math instruction time. The instruction will take place in small groups by me. Each small group will have 3-5 students. The tutoring will begin in January and may last until the end of the school year. The intervention will occur three days per week for 20-25 minutes. Your child will be completing worksheets, using different math tools and working closely with the tutor (Kathleen Hughes Pfannenstiel). Your child will complete daily assessments to monitor progress in multiplication and division. The assessments will include review problems, as well as problem types that have been taught and practiced.

If your child is selected to be part of the additional math instruction I will need access to your student's school permanent folder to gather the following information:

- Age
- Birth date
- Socioeconomic status
- Cultural/ethnic background
- Dominant language
- State test scores from grade 3 (TAKS)
- Scores from additional district assessments
    - Math
    - Reading (will be used to see how your child is performing as compared to math)

Following the intervention students will be asked three short questions about their time spent learning and practicing solving multiplication and division problems. (1)What did you like about coming to learn about facts? (2) What was hard for you when learning the facts? (3) What would you tell your friends about our math tutoring time? Would you recommend it for friends? All interviews will be short, 3-5 minutes and students will be asked the questions individually by the tutor.

The Project Duration is:

The project will begin in early January until May. Students selected for the intervention will begin pre-assessment in early January and intervention will begin later that month. The intervention may run until the end of the school year depending on accuracy and mastery of the multiplication and division facts.

What are the possible discomforts and risks?

The possible risks and discomforts your child may experience during participation in this study are expected to be minimal. None of the information I collect will affect your child's grades or be included in any school records. Any students not selected for intervention are still eligible for district interventions prescribed for students struggling in math facts. Students selected for the intervention will be pulled out of class and there is a possibility that this may make your child feel like an outsider. I will work closely with the classroom teacher to make the experience positive, as well as try to group the student with peers from their own classroom. At any time a student and/or parent may remove the student from the intervention.

What are the possible benefits to you or to others?

Your child may benefit from this project because I will

1. Provide your child (if he or she qualifies and is selected) extra math instruction in a small group of students to improve math fact ability at a time arranged by the classroom teacher. For students not invited to participate, classroom teachers will provide extra math instruction to students scoring lower on class wide math assessments.
2. Success in math fact ability has been connected with greater confidence, participation in class and greater enrollment in more advanced math courses.

If you choose to take part in this study, will it cost you anything?

There is no cost to you or your child to participate.

Will you receive compensation for your participation in this study?
There is no compensation for you or your child's participation in this study.

What if you are injured because of the study?
I cannot see any reason why your child would be injured during this project. Testing and extra math instruction is part of the school day and setting. If your child becomes sick during extra math instruction time, I will take your child back to the classroom and tell your child's teacher immediately.

If you do not want to take part in this study, what other options are available to you?
Your participation in this study is entirely voluntary. You are free to refuse to be in the study, and your refusal will not influence current or future relationships with The University of Texas at Austin. Your child will still be administered the district math assessment and the teacher will provide any additional instructional time for math.

How can you withdraw from this research study and who should you call if you have questions?

If you wish to stop your participation in this research study for any reason, you should contact the principal investigator: *Kathleen Hughes Pfannenstiel* at *(512) 470-0651* for any questions, concerns, or complaints about the research. You are free to withdraw your consent and stop participation in this research study at any time without penalty or loss of benefits for which you may be entitled. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision to remain in the study.

If you would like to obtain information about the research study, have questions, concerns, complaints or wish to discuss problems about a research study with someone unaffiliated with the study, please contact Jody Jensen, Ph.D., Chair, The University of Texas at Austin Institutional Review Board for the Protection of Human Subjects at (512) 232-2685. Anonymity, if desired, will be protected to the extent possible. As an alternative method of contact, and email may be sent to orsc@uts.cc.utexas.edu or a letter sent to IRB Administrator, P.O. Box 7426, Mail Code A3200, Austin, TX 78713.

How will your privacy and the confidentiality of your research records be protected?
Privacy: Your child's privacy will be protected in any writing or publication from this project because his or her name will not be used and the data will be combined from data from other students. If the results of this research are published or presented at scientific meetings, your child's identify will not be disclosed.

Confidentiality: (a) All data collected from students records will be given a number to protect student's privacy and confidentiality of information, (b) the staff involved in this project will take the ethics training for investigators using human subjects required by the University of Texas at Austin and will abide by rules of the University's Institutional Review Board, and (c) all data will be stored in a locked storage unit and on a secure computer that only the research staff will have access to.

If in the unlikely event it becomes necessary for the Institutional Review Board to review your research records, then The University of Texas at Austin will protect the confidentiality of those records to the extent permitted by law. Your research records will not be released without your consent unless required by law or a court order. The data resulting from your participation may be made available to other researchers in the future for research purposes not detailed within this consent form. In these cases, the data will contain no identifying information that could associate you with it, or with your participation in any study.

Will the researchers benefit from your participation in this study?
This study potentially benefits the Principal Investigator because I am confirming strategy instruction in multiplication and division facts that be used for further support for students who are identified as having math difficulties. The intervention is intended to improve math ability. There is little research on this topic to inform the educational community at present.

Signatures:

As a representative of this study, I have explained the purpose, the procedures, the benefits, and the risks that are involved in this research study:

Signature and printed name of person obtaining consent **Date**

Please review below and return to your child's teacher.

You have been informed about this study's purpose, procedures, possible benefits and risks, and you have received a copy of this form. You have been given the opportunity to ask questions before you sign, and you have been told that you can ask other questions at any time. You voluntarily agree to participate in this study. By signing this form, you are not waiving any of your legal rights.

Printed Legal Name of Your Child **Date**

Signature of Parent of Legal Guardian **Date**

CHILD ASSENT FORM

***Parents: After reading the permission form, please remember to read the form to your child or discuss the information with him/her.**

I agree to be in a study about math facts. This study was explained to my Parents/Legal Guardian and he/she/they said that I could be part of the intervention. The only people who will know about what I say and do in the study will be the people in charge of the study, my teacher and parents.

In this study I will be asked to work with different kinds of math tools and answer questions about how to solve problems. The people in charge of the study will also look at my school records to learn more things about me, like how old I am and my last TAKS test scores.

Writing my name on this page means that the page was read (by me/to me) and that I agree to be in the study. I know what will happen to me. If I decide to quit the study, all I have to do is tell the person in charge.

Signature of Child **Date**

Signature of Principal Investigator **Date**

Appendix B

Title Multiplication and Division Facts: Intervention for Success

IRB PROTOCOL # 2010-11-0113

Conducted By:

Kathleen Hughes Pfannenstiel, M.Ed., Principal Investigator, College of Education, The University of Texas at Austin, 470-0651.

Diane Pedrotty Bryant, Ph.D. Faculty Sponsor, Department of Special Education, College of Education, The University of Texas at Austin, 784-7346.

You are being asked to participate in a research study. This form provides you with information about the study. The person in charge of this research will also describe this study to you and answer all of your questions. Please read the information below and ask any questions you might have before deciding whether or not to take part. Your participation is entirely voluntary. You can refuse to participate or stop participating at any time without penalty or loss of benefits to which you are otherwise entitled. You can stop your participation at any time and your refusal will not impact current or future relationships with UT Austin or Pflugerville Independent School District. To do so simply tell the researcher you wish to stop participation. The researcher will provide you with a copy of this consent for your records.

The purpose of this study is to teach strategies to solve multiplication and division facts to 4th graders and to see if it improves understanding of how to solve problems. The instruction will begin with easier division and multiplication problems and then move on to harder ones. To practice solving the multiplication and division problems, students will also solve the problems within a word problem.

If you agree to be in this study, we will ask you to do the following things:

- Meet with the Principal Investigator to answer three, brief questions concerning the overall mastery and confidence level of students selected for the study

Total estimated time to participate in study is 20-30 minutes

The **risk** associated with this study is no greater than everyday life. You will be asked to share about the progress of your student(s) receiving the intervention. The interviews will be brief and will be used to validate the practical implications of the intervention.

Benefits of being in the study include extra instructional time for struggling students by the Principal Investigator. The PI will share all data collected to assist in your classroom teaching and in meeting district and school educational goals.

Compensation:

- There is no compensation for you or your student’s participation in this study.

Confidentiality and Privacy Protections:

- The data resulting from your participation may be made available to other researchers in the future for research purposes not detailed within this consent form. In these cases, the data will contain no identifying information that could associate you with it, or with your participation in any study.
- All data collected for this study will be given a number to protect privacy and confidentiality. The data will be stored in a locked storage unit and on a secure computer that only the research staff will have access to.

The records of this study will be stored securely and kept confidential. Authorized persons from The University of Texas at Austin, and members of the Institutional Review Board have the legal right to review your research records and will protect the confidentiality of those records to the extent permitted by law. All publications will exclude any information that will make it possible to identify you as a subject. Throughout the study, the researchers will notify you of new information that may become available and that might affect your decision to remain in the study.

Contacts and Questions:

If you have any questions about the study please ask now. If you have questions later, want additional information, or wish to withdraw your participation call the researchers conducting the study. Their names, phone numbers, and e-mail addresses are at the top of this page.

If you would like to obtain information about the research study, have questions, concerns, complaints or wish to discuss problems about a research study with someone unaffiliated with the study, please contact the IRB Office at (512) 471-8871 or Jody Jensen, Ph.D., Chair, The University of Texas at Austin Institutional Review Board for the Protection of Human Subjects at (512) 232-2685. Anonymity, if desired, will be protected to the extent possible. As an alternative method of contact, an email may be sent to orsc@uts.cc.utexas.edu or a letter sent to IRB Administrator, P.O. Box 7426, Mail Code A 3200, Austin, TX 78713.

You will be given a copy of this information to keep for your records.

Statement of Consent:

I have read the above information and have sufficient information to make a decision about participating in this study. I consent to participate in the study.

Signature:____________________________________ Date: __________________

Signature of Investigator:___________________________ Date: __________________

Appendix C

Name ______________________________

0 + 1	2 + 4	1 + 8	3 + 9	6 + 6	4 +10	7 + 2	6 + 1	7 + 8	0 + 5
2 + 3	1 + 1	4 + 7	5 + 7	8 + 2	9 + 5	10 + 8	6 + 0	2 + 5	9 + 1
4 + 6	8 + 3	2 + 9	6 + 9	7 + 5	8 + 4	0 + 3	8 + 8	9 + 3	1 + 5
6 + 5	4 + 0	5 + 5	4 + 8	7 +10	9 + 6	4 + 1	3 + 5	0 + 7	5 + 8
9 + 8	7 + 4	2 +10	4 + 5	10 + 0	8 + 5	3 + 3	6 + 2	1 +10	2 + 7
10 + 9	2 + 1	5 + 6	8 + 7	6 + 3	7 + 1	3 + 4	9 + 4	2 + 8	4 + 2
7 + 3	4 + 4	6 + 7	5 +10	3 + 8	0 + 9	5 + 4	8 + 6	3 + 2	6 +10
2 + 6	7 + 7	8 +10	5 + 3	2 + 0	10 +10	10 + 7	3 + 6	2 + 2	10 + 4
1 + 9	10 + 3	6 + 4	1 + 3	5 + 9	3 + 7	4 + 3	9 +10	4 + 9	8 + 9
3 +10	9 + 2	7 + 6	6 + 8	9 + 9	10 + 6	9 + 7	8 + 0	7 + 9	10 + 5

Name ______________________________

1 − 0	4 − 1	5 − 3	5 − 5	9 − 7	6 − 1	7 − 2	9 − 6	10 − 10	7 − 4
10 − 2	3 − 3	10 − 8	2 − 0	4 − 2	7 − 6	10 − 5	9 − 4	8 − 0	1 − 1
7 − 7	6 − 5	9 − 2	7 − 1	3 − 0	7 − 3	10 − 1	9 − 9	8 − 6	4 − 4
5 − 1	6 − 2	10 − 7	8 − 8	0 − 0	9 − 3	5 − 4	8 − 7	9 − 8	9 − 0
10 − 3	7 − 5	5 − 0	2 − 1	6 − 6	10 − 9	8 − 5	4 − 3	2 − 2	7 − 0
6 − 0	8 − 4	9 − 5	4 − 0	8 − 2	10 − 0	8 − 3	5 − 2	3 − 1	6 − 4
10 − 4	8 − 1	3 − 2	6 − 3	7 − 5	8 − 7	9 − 9	10 − 2	5 − 4	5 − 3
9 − 1	8 − 2	7 − 3	10 − 4	10 − 6	9 − 8	6 − 3	9 − 4	10 − 7	8 − 3
9 − 3	10 − 6	9 − 7	7 − 4	9 − 2	10 − 3	8 − 4	10 − 9	10 − 10	9 − 6
7 − 2	4 − 3	6 − 4	7 − 6	10 − 8	8 − 5	9 − 5	8 − 6	10 − 5	6 − 5

Name ______________________________

1	0	2	5	10	1	2	6	4	9
x 1	x 6	x 1	x 3	x 4	x 10	x 5	x 6	x 3	x 1
3	2	5	7	10	6	5	3	8	9
x 3	x 6	x 9	x 9	x 8	x 7	x 5	x 7	x 1	x 4
5	4	8	10	4	6	8	4	7	1
x 0	x 7	x 9	x 10	x 1	x 3	x 5	x 9	x 3	x 3
2	10	5	4	1	4	7	3	0	2
x 9	x 5	x 2	x 4	x 6	x 8	x 7	x 9	x 10	x 2
1	9	7	2	4	2	7	9	3	1
x 8	x 9	x 6	x 3	x 0	x 7	x 8	x 10	x 8	x 7
10	0	6	5	1	3	8	6	8	6
x 6	x 7	x 8	x 6	x 9	x 6	x 8	x 1	x 6	x 4
6	8	3	8	9	2	0	6	8	5
x 9	x 7	x 4	x 0	x 8	x 4	x 2	x 5	x 4	x 1
5	9	0	7	9	10	4	8	7	9
x 8	x 10	x 3	x 2	x 6	x 1	x 2	x 2	x 4	x 4
10	6	1	4	2	9	10	9	8	4
x 3	x 2	x 0	x 6	x 8	x 5	x 7	x 2	x 3	x 5
5	1	3	5	9	7	9	10	3	7
x 4	x 5	x 2	x 7	x 7	x 1	x 3	x 2	x 5	x 5

Name ______________________________

$1\overline{)0}$ $3\overline{)9}$ $2\overline{)4}$ $1\overline{)8}$ $10\overline{)10}$ $9\overline{)0}$ $8\overline{)8}$ $3\overline{)12}$ $4\overline{)8}$ $5\overline{)40}$

$6\overline{)24}$ $4\overline{)16}$ $4\overline{)4}$ $2\overline{)6}$ $4\overline{)12}$ $1\overline{)4}$ $3\overline{)0}$ $8\overline{)80}$ $5\overline{)45}$ $6\overline{)48}$

$2\overline{)20}$ $9\overline{)9}$ $6\overline{)12}$ $1\overline{)1}$ $8\overline{)48}$ $2\overline{)12}$ $3\overline{)15}$ $5\overline{)5}$ $4\overline{)20}$ $1\overline{)9}$

$3\overline{)3}$ $2\overline{)18}$ $4\overline{)32}$ $2\overline{)10}$ $7\overline{)28}$ $3\overline{)30}$ $1\overline{)5}$ $8\overline{)24}$ $6\overline{)30}$ $4\overline{)0}$

$4\overline{)24}$ $5\overline{)30}$ $7\overline{)7}$ $8\overline{)32}$ $10\overline{)0}$ $4\overline{)36}$ $2\overline{)16}$ $7\overline{)14}$ $3\overline{)6}$ $1\overline{)2}$

$8\overline{)16}$ $10\overline{)40}$ $7\overline{)21}$ $9\overline{)27}$ $8\overline{)56}$ $5\overline{)20}$ $4\overline{)28}$ $2\overline{)8}$ $9\overline{)45}$ $3\overline{)18}$

$7\overline{)49}$ $9\overline{)63}$ $8\overline{)40}$ $5\overline{)10}$ $7\overline{)35}$ $9\overline{)36}$ $6\overline{)54}$ $3\overline{)24}$ $2\overline{)2}$ $9\overline{)54}$

$6\overline{)0}$ $7\overline{)63}$ $5\overline{)50}$ $6\overline{)6}$ $10\overline{)70}$ $6\overline{)18}$ $10\overline{)30}$ $7\overline{)42}$ $9\overline{)72}$ $10\overline{)50}$

$3\overline{)21}$ $8\overline{)0}$ $10\overline{)90}$ $4\overline{)40}$ $8\overline{)64}$ $5\overline{)25}$ $9\overline{)18}$ $7\overline{)56}$ $6\overline{)60}$ $9\overline{)90}$

$6\overline{)42}$ $1\overline{)3}$ $6\overline{)36}$ $3\overline{)27}$ $5\overline{)35}$ $1\overline{)7}$ $8\overline{)72}$ $2\overline{)14}$ $9\overline{)81}$ $7\overline{)0}$

Appendix D

Lesson 1 Fact 9: Make Ten to Solve Nine's

Materials: Connecting cubes, wipe boards, fact family mats, multiplication and division fact cards (10's), vocabulary worksheet.

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 9.
- Write number sentences for multiplication and division facts of 9.
- State the "Make ten and subtract 9" strategy to solve multiplication and division facts.
- Identify when to use the strategy, "Make ten and subtract 9."

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will make arrays and learn a strategy to help us solve multiplication and division facts of 9.

Vocabulary:

We will be using some vocabulary today, rows, columns, factor, product, divisor and dividend.

Take your wipe board. A row goes across. Make small circles in a row on the wipe board.

A column is up and down. Make small circles in a column on the wipe board.

Point to the row and now point to the column.

We will learn the other vocabulary words in the lesson.

Review:

The strategy is to make ten and then subtract the factor. To help us make ten let's review multiplying and dividing by 10. (Hold up fact cards, x10 and divide by 10 and factors of 10)

Error correction: If students do not state the answers to x10 facts review the rule. When we multiply a number to ten the answer always has a zero in the ones place. Look at a hundreds chart (point to 10). Ten times 1 is 10. I can move down the column, 10 times 2 equals 20. What is 10 times 3? (30) Repeat. Show the division rule. Think about 10 times 2. The numbers in that fact family are 10, 2 and 20. So if we divide 20 by 2 what is the answer? (10) What is 20 divided by 10? (2)

Modeling:

1. **Give each student a wipe board.**

    A strategy is a plan to solve a problem. We will learn and use a strategy to multiply 9 times 3, 4, 6, 7, 8 and 9.

    A factor is a number in a multiplication problem. A factor is multiplied by another factor. What is a factor? *(A number in a multiplication problem)*

    Write the factors on your wipe board in a row, 3, 4, 6, 7, 8 and 9.

    Display fact 5x9. Is this a problem that you would use the Make Ten Minus 9 strategy? *(No)* To answer this fact it is easier to count-by 5's. Today's strategy is useful for the factors written on your board. What factors? *(3, 4, 6, 7, 8, 9)*

A multiplication problem has two factors. The other factor is 9. Write a 9 below each factor in a row.

We multiply two factors to find a product. A product is the answer of a multiplication problem. What is a factor? *(An answer to a multiplication problem)*

2. Give each student a vocabulary sheet. Have students write their names on top.

To help us remember the vocabulary words we will illustrate it. Write Factor x Factor=Product in a row. We can also write it in a column:

Factor
X Factor
Product

3. Have students place wipe board on top. Lay 100's chart on table.

xxxx
xxxx
xxxx
xxxx
xxxx
xxxx
xxxx
xxxx
xxxx

Put your wipe board on top of the vocabulary sheet. Write the problem 9x4 across the top, in a row. To solve this problem we can use the Make Ten and subtract factor strategy.

The first step is to think of 9 as 10. Write 10 below 9 and bring down the 4. Think, 10 times 4. What is 10 x 4? *(40)*

Write 40 after the equal sign. Now subtract the other factor. What is the factor we are multiplying to 9? *(4)* Count-back 4. *(Use a hundreds chart for error correction)*

What is 40 minus 9? *(36)* Write 36 after the equal sign in both problems.

9x4=36
10x4=40-4=36

4. Give each student 36 connecting cubes.

We can show this problem making an array. An array is rows and columns arranged in an order.

Make 4 groups of 9 cubes.

Lay the 4 groups in rows. Count the cubes. What is 4x9? *(36)* If you switched and made rows of 9 cubes in 4 columns would there still be 36? *(Yes)* Prove it. *(Students should switch the rows and columns to prove)*

This is a turnaround fact for multiplication. This only works when we add or multiply. What is it called when we switch the order of the factors? *(Turnaround fact)*

5. Give each student a Fact Family Mat and vocabulary sheet.

Just like in addition and subtraction we can write fact family sentences with the two factors and the product. What is one factor? *(9 or 4)* What is the other factor? *(4 or 9)* What is the product? *(36)*

In division, another name for the factors is divisor. What is the name of a factor in division? *(Divisor)* The product is the dividend. The answer to a division problem is the other factor or the quotient. What is the answer to a division problem? *(Quotient)*

You can write a division problem two ways. Write the vocabulary words on your vocabulary sheet.

Dividend ÷ Divisor=Quotient

Quotient
Divisor Dividend

Now we can write the four fact family sentences. The greatest number, the product or the dividend is written at the top of the roof. What is the product or dividend of 9x4? *(36)* Write 36.
What is one factor? *(9)* Write 9 in one corner. What is the other factor? *(4)* Write it.

What is one multiplication sentence? *(9x4=36)* Multiplication is like addition, we can turnaround the two factors and still equal the same product. What is the second multiplication sentence? *(4x9=36)*

Using the same 3 numbers we can write two division problems. What is the dividend? *(36)*
Write 36. The greatest number will always be split into equal parts. Think of the array we made. The first time we split 36 cubes into groups of how many? *(4)* Four is one divisor.

Write 4 on the next line. What is the quotient? How many groups of 4 did we make? *(9)* Write 9 after the equal sign.

We also split 36 into groups of 9. Nine is another divisor. Now think 9 times what equals 36? *(4)* Four is the divisor or the quotient for this number sentence? *(Quotient)*

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Have students write 10x[N]-N. Give each student a hundreds chart if they are struggling in counting back the other factor. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 9 facts on Fact Family Mat.

2 minutes: Independent Practice Probe

1. Collect all materials.
2. Give students the multiplication probe. Have students write their name and date. When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.
6. Assist students in writing a goal and graphing.

Lesson 2 Fact 9: Make Ten to Solve Nine's

Materials: Connecting cubes, wipe boards, fact family mats, multiplication and division fact cards (2's), vocabulary worksheet.

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 9.
- Write number sentences for multiplication and division facts of 9.
- State the "Make ten and subtract 9" strategy to solve multiplication and division facts.
- Identify when to use the strategy, "Make ten and subtract 9."

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will make arrays and practice the Make Ten Minus Factor strategy to help us solve multiplication and division facts of 9.

Vocabulary:

We will be using some vocabulary today, rows, columns, factor, product, divisor and dividend. Take out your vocabulary sheet. Write the word row. Does row go across the page or up and down? *(Across)*

How would you write column? *(Up and down)* Write column.

(Write 2x3 on wipe board) What is the product? *(6)* What is one factor?

(Write 6 ÷?) What number is missing? *(2 or 3)* This number is the divisor or dividend? *(Divisor)* What is 6 called? *(Dividend)*

Review:

Today we are going to review times 10 and divide by ten facts. We are also going to review facts of 2. (Hold up fact cards in multiplication and division)

Error Correction: If students do not know the 2 facts well remind them to double the factor, such as 4x2 think 4+4. Follow the multiplication problem with the division problem from the same fact family.

Modeling:

1. **Give each student a wipe board. Write 9x1 on teacher wipe board.**

    A strategy is a plan to solve a problem. We are going to practice using the Make Ten Subtract a Factor strategy. The Make Ten Subtract a Factor Strategy is used when we multiply 9 to 3, 4, 6, 7, 8 and 9. Write 3, 4, 6, 7, 8, and 9 in a row.

    Look at this fact, read it. *(9x1)* Do we use the Make Ten subtract 9 strategy to solve this fact? *(No)* Why? *(1 is not a factor we use it on)* What is 9x1? *(9)*

    Error Correction: When we multiply by one the answer is always that number.

6. **Lay a 100's chart on table.**

Write the problem 6x9 across the top on your wipe board. To solve this problem we can use the Make Ten and Minus Factor strategy. What strategy to solve? *(Make Ten Subtract a Factor)*

To use the strategy we think 10 instead of 9. Which factor changes to 10? *(9)* Write 10 below 9 and bring down the 6. Think, 6 times 10. What is 6x10? *(60)*

Write 60 after the equal sign. Nine is one factor. What is the other factor? *(6)* Now subtract 6. Count-back 6. *(Use a hundreds chart for error correction)*

What is 60 minus 6? *(54)* Write 54 after the equal sign in both problems.

6x9=54
6x10-6=54

7. **Place 54 cubes on the table.**

We can show this problem making an array. An array is rows and columns arranged in an order.

Help me make 6 groups of 9 cubes. How many groups? *(6)* How many in each group? *(9)*

Count the cubes. How many in all? *(54)*

Take the cubes apart. Make 9 groups of 6 cubes. How many groups? (9) How many in each group? *(6)*

How many cubes in all? *(54)*

8. **Give each student a Fact Family Mat.**

Write fact family sentences with the two factors and the product. What is one factor? *(9 or 6)* What is the other factor? *(6 or 9)* What is the product? *(54)*

Write the four fact family sentences. The greatest number, the product or the dividend is written at the top of the roof. What is the product or dividend of 6x9? *(54)* Write 54.
What is one factor? *(9)* Write 9 in one corner. What is the other factor? *(6)* Write it.

What is one multiplication sentence? *(6x9=54)* Multiplication is like addition, we can turnaround the two factors and still equal the same product. What is the second multiplication sentence? *(9x6=54)*

Using the same 3 numbers we can write two division problems. What is the dividend? *(54)*
Write 54. The greatest number will always be split into equal parts. Think of the array we made. We made groups. How many groups of 6 cubes? *(9)*

Write 6 as the divisor and 9 as the quotient.

We then made groups of 9 cubes. How many groups of 9 cubes? (6)

Write 9 as the divisor and 6 as the quotient.

9. **Repeat steps 3 and 4 with other 9 fact problems.**

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Have students write 10x[N]-N. Give each student a hundreds chart if they are struggling in counting back the other factor. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 9 facts on Fact Family Mat.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give the student's the multiplication probe. Have students write their name and date.

   When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson 3 Fact 9: Make Ten to Solve Nine's

Materials: Wipe boards, fact family mats, times ten facts product -9 facts, 100's chart (error correction) and vocabulary worksheet (error correction)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Write number sentences for multiplication and division facts of 9.
- State the "Make ten and subtract 9" strategy to solve multiplication and division facts.
- Identify when to use the strategy, "Make ten and subtract 9."

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will practice 9 facts in multiplication and division problems using the Make Ten Minus Factor strategy.

Review:

To warm-up our minds we are going to review 1 facts in multiplication and division.

To help us increase how fast we can solve 9 facts we will review times 10 facts and then subtract factor. (Review times ten and subtract factor)

Modeling:

1. **Give each student a wipe board. Write 5x9 on teacher wipe board.**

   We have been working on the Make Ten Minus Factor strategy. We use this when we multiply 9 to 3, 4, 6, 7, 8 and 9. When do we use this strategy? *(To multiply 9 to 3, 4, 6, 7, 8 or 9)*

   Give me an example of a problem that would use the Make Ten Minus Factor strategy. *(Allow students to state an example)*

   Look at my board. Read the problem. *(5x9)*

   Would I use the Make 10 Subtract Factor strategy? *(No)*

   To solve this problem I would count-by 5's since it faster. Count by 5s, 9 times. Ready, count. *(5, 10…45)* What is 5x9? *(45)*

2. **Write 7x9 on the wipe board vertically. If students do not have vocabulary memorized have them refer to vocabulary sheet.**

   Make your board match mine.

   What strategy do we use to solve? *(Make 10 Subtract Factor)*

   Both 7 and 9 are factors. Which factor will become 10? *(9)*

   Write 10 next to 9. Think, 10x7. What is 10 times 7? *(70)*

   What is the next step of the strategy? *(Subtract 7)*

   What is 70 minus 7? *(Have students count-back 7 or use 100's chart, 63)*

What is 7x9? *(63)*

Write the turnaround fact. What is the turnaround multiplication fact? *(9x7)*
What is 9 times 7? *(63)*

What are the two factors? *(7 and 9)*

What is the product? *(63)*

What is one divisor? *(7 or 9)* Write it and make the division bar.

What is the dividend? *(63)* Write 63 inside the division bar.

What is the quotient? *(9 or 7, depending on the divisor)*

What is another divisor? *(9 or 7)* Write it and make the division bar.

What is the dividend? *(63)* Write 63 inside the division bar.

What is the quotient? *(9 or 7, depending on the divisor)*

Erase your board.

3. Write 9 division bar, 72.

Make your board match mine. What part of the division problem is missing, divisor, dividend or quotient? *(Quotient)*

Think multiplication, 9 times what factor equals 72? *(8)*

***If students do not know answer, use following language:*
Use the Make Ten Minus Factor strategy. Think backwards. Round up, after what is the next ten when counting by tens, 70 and then? (80) Now count back from 80 to 72. (79, 78…72) How many numbers did we count? (8) What is 72 divided by 9? (8) What is 8x9? (72)

What three numbers are in this fact family? *(9, 8, 72)*

What is one multiplication number sentence? Write it.

What is the second multiplication number sentence? Write it.

What is one division number sentence? Write it.

What is the last division number sentence? Write it.

4. **Give each student a Fact Family Mat.**

   Practice other 9 facts, only giving two parts (such as one factor and a product)

   EX: 54 and 6.

   What part is missing, product, or factor? *(Factor)*

   What factor is missing? *(9)*

   Write the four number sentences on your fact family mat.

5. **Partner students for peer-sharing and teaching of Make 10 Subtract Factor Strategy.**

   Write another problem that uses the Make 10 Subtract Factor strategy. Work in partners and take turns being the teacher. Explain the steps to your neighbor. Once the first "teacher" is done another problem should be completed, but this time the student is now the teacher.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Have students write 10x[N]-N. Give each student a hundreds chart if they are struggling in counting back the factor. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 9 facts on Fact Family Mat.

2 minutes: Independent Practice Probe

2. Collect all materials.
3. Give the student's the multiplication probe. Have students write their name and date. When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

4. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

5. Correct each probe. Have students write the total number completed on top.
6. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

7. Assist students in writing a goal and graphing.

Lesson 4 Fact 9: Make Ten to Solve Nine's

Materials: Wipe boards, fact family mats, 100's chart (error correction) and vocabulary worksheet (error correction)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 9.
- Write number sentences for multiplication and division facts of 9.
- State the "Make ten and subtract 9" strategy to solve multiplication and division facts.
- Identify when to use the strategy, "Make ten and subtract 9."

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will review multiplication and division vocabulary and solve problems with factors and divisors of 9.

Review:

To help us subtract the factor let's review minus 10 facts. (Display 10-3, 10-4, 10-6, 10-7, 10-8, 10-9) We can also think addition Make Ten facts. (Display 3+__=10, 4+, 6+, 7+, 8+, and 9+=10)

We are going to review a variety of problems, multiplication and division facts 2, 10, 5's, and 1's. This will help us solve the facts faster on the Independent Practice. (Hold up a variety of problems)

Modeling:

1. **Give each student a wipe board. Write 9x9.**

    What strategy do we use when multiplying a factor with 9? (Make Ten Subtract Factor)

    Name a multiplication problem that you would use the Make 10 Minus 9 strategy. *(Accept answers in which 9 is multiplied by 3, 4, 6, 7, 8 or 9)*

    Read the problem. *(9x9)*

    Write the problem on your board.

    What is the first step of Make 10 Subtract 9? *(Make 9 into 10)*

    Write 10 next to one 9. What is the second step? *(Subtract 9)* Why do we subtract 9? *(The other factor)*

    What is 90 minus 9? *(81, use 100's chart to assist with counting back)*

    What is 9x9? *(81)*

    What are the factors in this problem? *(9 and 9)*

    What is another word for answer of a multiplication problem? *(Product)*

    What is the product of 9x9? *(81)*

What are the parts of a division problem? *(Have students refer to vocabulary sheet if struggling. Divisor, dividend and quotient)*

Eighty-one divided by 9 equals what? *(9)*

What is the answer for a division problem called? *(Quotient)*

Write the two division number sentences for this fact family.

2. **Give each student a Fact Family Mat. If students are proficient at writing number sentences, continue to use the wipe boards.**

Practice other 9 facts, only giving two parts (such as one factor and a product)

EX: 72 and 9.

What part is missing, product, or factor? *(Factor)*

What factor is missing? *(8)*

Write the four number sentences on your fact family mat.

3. **Partner students for peer-sharing and teaching of Make 10 Subtract Factor Strategy.**

Write another problem that uses the Make 10 Subtract Factor strategy. Work in partners and take turns being the teacher. Explain the steps to your neighbor. Once the first "teacher" is done another problem should be completed, but this time the student is now the teacher.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 1, 2, 5, 10 and 9 facts in multiplication and division.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give the student's the multiplication probe. Have students write their name and date.

When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   > Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson CCC (5) Fact 9: Make Ten to Solve Nine's

Materials: Wipe boards, C-C-P sheet

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Write number sentences for multiplication and division facts of 9.
- State the "Make ten and subtract 9" strategy to solve multiplication and division facts.
- Identify when to use the strategy, "Make ten and subtract 9."

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will practice multiplication and division problems with factors and divisors of 9.

Review: Allow more time to review (5 minutes)

To help us subtract the factor let's review minus 10 facts. (Display 10-3, 10-4, 10-6, 10-7, 10-8, 10-9) We can also think addition Make Ten facts. (Display 3+__=10, 4+, 6+, 7+, 8+, and 9+=10)

We are going to review a variety of problems, multiplication and division facts 2, 10, 5's, and 1's. This will help us solve the facts faster on the Independent Practice. (Hold up a variety of problems)

Modeling:

1. **Give each student a wipe board. Write 9x7 and 10x9.**

What strategy do we use when multiplying a factor with 9? (Make Ten Subtract Factor)

Read the problems. *(9x7 and 10x9)*

Write the problems on your board. Which problem would you use the Make Ten Subtract the Factor strategy? (9x7)

What is 10x9? (90)

What is the first step of Make 10 Subtract 9? *(Make 9 into 10)*

What is 10x7? (70)

What is the second step? *(Subtract 7)* Why do we subtract 7? *(The other factor)*

What is 70 minus 7? *(63)*

What is another word for answer of a multiplication problem? *(Product)*

What are the parts of a division problem? *(Have students refer to vocabulary sheet if struggling. Divisor, dividend and quotient)*

Write the other 3 number sentences for this family.

Sixty-three divided by 9 equals what? *(7)*

What is the answer for a division problem called? *(Quotient)*

2. **Give each student a Fact Family Mat. If students are proficient at writing number sentences, continue to use the wipe boards.**

Practice other 9 facts, only giving two parts (such as one factor and a product)

EX: 72 and 9.

What part is missing, product, or factor? *(Factor)*

What factor is missing? *(8)*

Write the four number sentences on your fact family mat.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet that should be tri-folded. The guided practice sheet is a Copy, Cover, Compare fluency practice. The students look at the problem in the "Compute" column and solve in 3 seconds. Once 3 seconds has passed tap on table and tell students to move to the next problem.

When I say begin start working on the first problem in the column. When I tap the table, go to the next problem even if you are not done with the current problem. You will only have 3 seconds to answer each problem.

Once the students have finished the compute column students turn to the compare column. As a group answer and write answers. The students then compare the columns (compute and compare). For every problem answered incorrectly or not solved the students write that problem 4 times in the practice column.

If extra time is available have students practice solving 1, 2, 5, 10 and 9 facts in multiplication and division. Hold up flashcards for 3 seconds and have students write the answers on the wipe boards.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give the student's the multiplication probe. Have students write their name and date.

When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

<u>5 minutes: Intervention Plus!</u>

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

    Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson: Double, Double Again, Double 3 times

Materials: Connecting cubes, wipe boards, fact family mats, multiplication and division fact cards (0, 1, 2, 5, 10, 9)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 4, 6 and 8.
- Write number sentences for multiplication and division facts of 4, 6, 8.
- Identify how to break apart 4, 6 and 8 into equal parts
    - 4 = 2 + 2
    - 6 = 3 + 3 or 2 + 2 + 2
    - 8 = 4 + 4 or 2 + 2 + 2 + 2
- State the "Double It" strategy to solve multiplication and division facts.
- Identify when to use the "Double It" strategy.

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will learn a new strategy to use when multiplying numbers to 4, 6 or 8.

Vocabulary:

First let's review our multiplication and division vocabulary. *(Students can use the vocabulary sheet for assistance).*

In multiplication the two parts or the numbers multiplied together are called factors. What are the two numbers called? What is the answer to a multiplication problem? *(Product)*

When we write a division problem the product is the dividend. What is the name of the product in a division problem? *(Dividend)* One factor is the divisor and the other factor is the answer or the quotient. What is the answer to a division problem? *(Quotient)*

Review:

How many numbers are in a fact family? *(3)*

First let's review the facts we have mastered, 0, 1, 2, 5, 9, and 10s. *(2-3 minutes)*

We also need to review doubles facts because this will help us with the strategy we are learning today. *(Start with easy facts, 4 + 4 through 10 + 10, 12 + 12, 14 + 14, 16 + 16, 12 + 12 + 12 and 32 + 32)*

Modeling:

1. **Give each student a wipe board and lay 36 cubes on the table.**

    A strategy is a plan to solve a problem. We will learn and use a strategy to multiply 4 times 3, 4, 6, 7, 8 and 6 x6, 6 x 8 and 8 x 8.

    Write the factors on your wipe board in a row, 3, 4, 6, 7, and 8.

    First we will practice the Double It strategy with 4 times these factors and then use it to answer the 6 and 8s.

    Display fact 9 x 4. Is this a problem that we are going to practice with the new Double it strategy? *(No)* What strategy do we use to solve this fact? *(Make Ten Minus the Factor)* Why*? (We already mastered it when learning 9s, faster)*

2. Write the fact 4 x 8 on the wipe board.

Make your board match mine
Read the fact. *(4 x 8)*

To solve a fact times 4 we can use the Double It strategy. What strategy? *(Double it)* We are going to break-apart the number 4, 6 and 8 into the double facts and then add.

What is a doubles fact? *(Same number added together)* Give me an example of a doubles fact. *(Allow a variety of answers)*

(Take out 4 cubes) How many cubes? *(4)*

What is half of 4? How do I break 4 into 2 equal parts? *(2 and 2, break 4 into 2 groups of 2)*

Here is one group of 2 and here is another group of 2.

The Double It strategy means you break apart the 4, 6 or 8 into equal groups of 2. How many 2s in 4? *(2)* How many 2s in 6? *(3)* How many 2s in 8? *(4)*

The first step of the Double It strategy is to break apart the 4, 6 or 8. What is the first step of the Double It strategy? *(Break apart 4, 6 or 8)*

After we break apart the 4, 6 or 8 we multiply 2 with the other factor. What is the next step? *(Multiply 2 with the other factor)*

The last step is to add. What is the last step? *(Add)*

Let's try it. Below the problem 4 x 8, write 2 x 8 and 2 x 8. We are doubling 8.

(Have a hundreds chart on table for assistance)
What is 2 x 8? *(16)* Write 16 + 16 below the 2 x 8 problems. Now we add. What is 16 + 16? *(32)* What is 4 x 8? *(32)*

What strategy did we use to solve 4 x 8? *(Double It)*

Let's prove the strategy by making an array.

First we make an array to show 4 x 8.

Look at half of the array, what fact? *(2 x 8)*

Or we can break it another way and think of it as 4 by 4. How many cubes are in each group of 4 by 4? *(16)* What was 8 x 2? *(16)*

3. **Give each student a Fact Family Mat .**

Write 4 x 6 on the fact Family mat. Where do we write the factors? (*In the lower corners of roof)*

What strategy do we use to solve 4 x 6? *(Double It.)*

How do we break apart 4 into equal parts? *(2 and 2)*

Now we multiply 2 to 4 or 2 to 6? *(2 to 6)* Why? *(It's the other factor)*

What is 2 x 6? *(12)*

What do we do with the two 12s? *(Add)*

What is 12 + 12? *(24)*

What is 4 x 6? *(24)*

Write the four fact family sentences.

Erase your mat. Now write 6 x 8.

What strategy? *(Double It.)*

2 times what equals 6? Or how many times do I count by 2s to reach 6? *(3)*

When multiplying 6 and 8 we first break apart 6 into 3 equal parts, basically we double it like we did with 4 x 6 and then double again.

Write 2 x 8 + 2 x 8 + 2 x 8.

What is 2 x 8? *(16)*

Now add 16 + 16 + 16. What is 16 + 16? *(32)* What is 32 + 16? *(48)*

What 3 numbers are in this fact family? *(6, 8 and 48)* Write the four fact family sentences.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Give each student a hundreds chart if they are struggling in adding when doubling. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 4, 6, and 8 facts on Fact Family Mat.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give students the multiplication probe. Have students write their name and date.

   When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   > Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson: Double, Double Again, Double 3 times

Materials: Connecting cubes, wipe boards, fact family mats, multiplication and division fact cards (0, 1, 2, 5, 10, 9)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 4, 6 and 8.
- Write number sentences for multiplication and division facts of 4, 6, 8.
- Identify how to break apart 4, 6 and 8 into equal parts
    - 4 = 2 + 2
    - 6 = 3 + 3 or 2 + 2 + 2
    - 8 = 4 + 4 or 2 + 2 + 2 + 2
- State the "Double It" strategy to solve multiplication and division facts.
- Identify when to use the "Double It" strategy.

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will learn and practice the Double It strategy to use when multiplying numbers to 4, 6 or 8.

Vocabulary:

First let's review the strategy Make Ten Minus the Factor. 9 times what factor would you use the Make Ten Minus the factor strategy? *(Allow a variety of answers 9 times 3, 4, 6, 7, 8, 9)*

What is the first step? *(Think of 9 as 10)* What is the next step? *(Minus the other factor)*

How many factors is a multiplication problem? *(2)*

How many numbers in a fact family? *(3)*

Review:

First let's review the 9s. *(Go through all 9 facts)*

To help us with the Double It strategy we need to have the 2s fast! Ready? *(Review all 2 facts)*

We also need to review doubles facts because this will help us with the strategy we are learning today. *(Start with easy facts, 4 + 4 through 10 + 10, 12 + 12, 14 + 14, 16 + 16, 12 + 12 + 12 and 32 + 32)*

Modeling:

1. **Give each student 28 cubes.**

Make an array that shows 7 x 4.
How many groups? *(4)*
How many in each group? *(7)*
What is 7 x 4? *(28)*

To make this problem easier and faster to solve we can use the Double It strategy.

The Double It strategy is used when we multiply a factor to 4. We double 2 times when multiplying a factor to 6, and double 3 times when the factor is 8.

What strategy do we use to solve 7 x 4? *(Double It)*

The first step of Double It is to see if a 4, 6 or 8 is in the problem. Is there a 4, 6 or 8? *(Yes, 4)*

Think of a doubles fact that equals 4. What number plus the same number equals 4? *(2 + 2)*

Show 2 x 7 with the cubes.

Now show 2 x 7 with the remaining cubes. What is 2 x 7? *(14)* What is 2 x 7? *(14)*

We have two arrays that we can add. What is 14 + 14? *(28)*

What is 7 x 4? *(28)*

2. **Give each student a Fact Family Mat.**

What strategy are we practicing? *(Double It)*

When do we use the Double It strategy, what factor has to be in the problem? *(4, 6 or 8)*

Write the factors 4 and 8 in the roof.

Break apart 4 into a 2 and a 2. What is 2 + 2? *(4)*

Now multiply 2 x 8. What is 2 x 8? *(16)*

We have to multiply 2 x 8 again. What is 2 x 8? *(16)*

Now add. What is 16 + 16? *(32)*

Write 32 in the roof. Is 32 a factor or the product? *(Product)*

What strategy? *(Double It)*

Write the four fact family sentences.

Erase your mat. Write 6 and 8 in the lower part of the roof. These are the factors.

What strategy? *(Double It)*

Is there a 4? *(No)*

There is not a 4, but there is a 6. When we multiply with a 6 we double the other factor and then double again. What is the other factor? *(8)*

How many times do we double 8? *(3 times)*

(Teacher should write these number sentences on a wipe board) Think, 8 x 2 + 8 x 2 + 8 x 2. What is 8 x 2? (16)

Now add 16 + 16? *(32)*

What is 32 + 16? *(48)*

What is the product of 6 x 8? *(48)*

Write the four fact family sentences.

(Continue with more 4 facts if time permits)

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Give each student a hundreds chart if they are struggling in adding when doubling. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 4, 6, and 8 facts on Fact Family Mat.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give students the multiplication probe. Have students write their name and date.

   When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a group score of 22 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson: Double It, Double Again, Double 3 Times

Materials: Wipe boards, fact family mats, times 4, 6, 8 flashcards, 100's chart (error correction) and vocabulary worksheet (error correction)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Write number sentences for multiplication and division facts of 4, 6, 8.
- Identify how to break apart 4, 6 and 8 into equal parts
    - 4 = 2 + 2
    - 6 = 3 + 3 or 2 + 2 + 2
    - 8 = 4 + 4 or 2 + 2 + 2 + 2
- State the "Double It" strategy to solve multiplication and division facts.
- Identify when to use the "Double It" strategy.

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will practice facts of factors times 4, 6 and 8 using the Double It strategy.

Review: Give each student a wipe board.

To warm-up let's review the 9s and the 2s, both mastered and easy facts.

To help us with the Double It strategy we need to memorize double facts. A double fact is the same number added together, like 2 + 2. Write the double fact and the answer. Ready? *(6 + 6, 8 + 8, 12 + 12, 14 + 14, 16 + 16, 32 + 32)* Also, try the triple doubles, 12 +12 +12 and 16 + 16 + 16. *(Have students keep these double facts displayed)*

Modeling:

1. **Give each student a wipe board to write the facts for Double It strategy.**

We have been working on the Double It strategy. We use this when we multiply 4 to 3, 4, 6, 7, 8. What strategy? (Double It) What is the common factor in all the problems for the Double It strategy? (4)

Write the facts.

We can also Double It again when multiplying 6 x 6 and 6 x 8. What strategy? (Double It again) When do we use this strategy? *(To multiply 6 x 6 and 6 x 8)* Write the facts.

We can solve 8 x 8 using the Double It 3 Times strategy. What strategy? *(Double It 3 times)* Write 8 x 8.

The Double It strategy takes the times 4, 6 or 8 and makes it a 2s fact. Think 2 times what equals 4? *(2)* 2 times what equals 6? *(3)* And 2 times what equals 8? *(4)*

First we break apart the 4, 6 or 8. 4 is broken into 2 and 2. How do we break up 4? *(2 and 2)* 6 is broken into 2, 2 and 2 or double and double again. How do we break apart 6? *(2, 2 and 2)* 8 is broken into 4, 2s or Double three times. How do we break apart 8? *(2, 2, 2, 2)*

After we break apart the 4, 6, or 8 we multiply 2 to the other factor. Then we add. We create a doubles fact. What do we do after we multiply the factor by 2? *(Add)*

2. **Write 4 x 9 and 9 x 8 on the wipe board.**

   Look at my board. Can I use the Double It strategy? *(No)* Why? *(Not a problem type)* What strategy do we use to solve 4 x 9 and 9 x 8? *(Make 10 Minus the factor)* What is 4 x 9 and 9 x 8? *(36, 72)*

3. **Give each student a fact family mat. Write the factors 8 x 4 on the Fact Family Mat.**

   Make your mat match mine.

   What strategy do we use to solve? *(Double It)*

   Both 8 and 4 are factors. What is the double fact that equals 4? *(2 + 2)*

   Think 2 x 8 plus 2 x 8. What is 2 x 8? *(16)* What is 16 + 16? *(32)*

   What is 4 x 8? *(32)*

   What 3 numbers are in this fact family? *(4, 8 and 32)*

   Write the sentences for this family.

   Erase your mat.

4. **Write 6 x 6 on the mat.**

   Make your mat match mine.

   What strategy? *(Double It or Double It Again)*

   2 times what equals 6? *(3)*

   We think 2 x 6 plus 2 x 6 plus 2 x 6. What is 2 x 6? *(12)*

   Now add. What is 12 + 12? *(24)* What is 24 + 12? *(36)*

   What is 6 x 6? *(36)*

   What 3 numbers are in this fact family? *(6, 6, 36)*

   Write the sentences for this family.

5. **Partner students for peer-sharing and teaching of Double It Strategy.**

   Write another problem that uses the Double It strategy. Work in partners and take turns being the teacher. Explain the steps to your neighbor. Once the first “teacher” is done another problem should be completed, but this time the student is now the teacher.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Have students write 10x[N]-N. Give each student a hundreds chart if they are struggling in doubling the product. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 4, 6, 8 facts on Fact Family Mat.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give the student’s the multiplication probe. Have students write their name and date.

   When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   > Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a group score of 22 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson: Double It, Double Again, Double 3 Times

Materials: Wipe boards, fact family mats, times 4, 6, 8 flashcards, 100's chart (error correction) and vocabulary worksheet (error correction)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Write number sentences for multiplication and division facts of 4, 6, 8.
- Identify how to break apart 4, 6 and 8 into equal parts
    - 4 = 2 + 2
    - 6 = 3 + 3 or 2 + 2 + 2
    - 8 = 4 + 4 or 2 + 2 + 2 + 2
- State the "Double It" strategy to solve multiplication and division facts.
- Identify when to use the "Double It" strategy.

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will practice facts of factors times 4, 6 and 8 using the Double It strategy.

Review: Give each student a wipe board.

To warm-up let's review the 5s both multiplication and division. (Add additional review items missed from previous IP)

To help us with the Double It strategy we need to memorize double facts. A double fact is the same number added together, like 2 + 2. Write the double fact and the answer. Ready? *(6 + 6, 8 + 8, 12 + 12, 14 + 14, 16 + 16, 32 + 32)* Also, try the triple doubles, 12 +12 +12 and 16 + 16 + 16. *(Have students keep these double facts displayed)*

Modeling:

1. **Give each student a wipe board to write the facts for Double It strategy.**

We have been working on the Double It strategy. We use this when we multiply 4 to 3, 4, 6, 7, 8. What strategy? (Double It) What is the common factor in all the problems for the Double It strategy? (4)

Write the facts.

We can also Double It again when multiplying 6 x 6 and 6 x 8. What strategy? (Double It again) When do we use this strategy? *(To multiply 6 x 6 and 6 x 8)* Write the facts.

We can solve 8 x 8 using the Double It 3 Times strategy. What strategy? *(Double It 3 times)* Write 8 x 8.

The Double It strategy takes the times 4, 6 or 8 and makes it a 2s fact.

First we break apart the 4, 6 or 8. What is the first step? (Break apart 4, 6 or 8)

How do we break apart 4? (2 and 2) How do we break apart 6? (2, 2 2) How do we break apart 8? (2, 2, 2, 2)

If students are struggling in how to break apart 4, 6 and 8 use the language from lesson 3: 4 is broken into 2 and 2. How do we break up 4? (2 and 2) 6 is broken into 2, 2 and 2 or double and double again. How do we break apart 6? (2, 2 and 2) 8 is broken into 4, 2s or Double three times. How do we break apart 8? (2, 2, 2, 2)

After we break apart the 4, 6, or 8 we multiply 2 to the other factor. Then we add. We create a doubles fact. What do we do after we multiply the factor by 2? *(Add)*

2. **Write 5 x 6 and 54 divided by 6 on the wipe board.**

   Look at my board. Can I use the Double It strategy? *(No)* Why? *(Not a problem type)* What strategy do we use to solve 5 x 6? *(Count by 5s)* What is 5 x 6? *(30)*

   What strategy do we use to solve 54 divided by 6? *(Make 10 Minus the factor)* What is 54 divided by 6? *(9)*

3. **Give each student a fact family mat. Write the factors 7 x 4 on the Fact Family Mat.**

   Make your mat match mine.

   What strategy do we use to solve? *(Double It)*

   Both 7 and 4 are factors. What is the double fact that equals 4? *(2 + 2)*

   Think 2 x 7 plus 2 x 7. What is 2 x 7? *(14)* What is 14 + 14? *(28)*

   What is 7 x 4? *(28)*

   What 3 numbers are in this fact family? *(4, 7 and 28)*

   Write the sentences for this family.

   Erase your mat.

4. **Write 12 (product) and 3 (factor) on the mat.**

   Make your mat match mine.

   Are we missing the factor or the product? *(Factor)*

   Think doubles fact. What number plus the same number equals 12? *(6 + 6)*

   Would 6 be the other factor? *(No)* The doubles fact equals 6, so that means we would have multiplied a factor with 2 to equal 6.

   2 times what equals 6? *(3)* Yes, but 3 times 3 is 9. If we can work backwards using the Double It strategy, what is the missing factor? *(4)*

   Write 4 on the mat. What is 12 divided by 3? *(4)*

What 3 numbers are in this fact family? *(3, 4, 12)*

Write the sentences for this family.

5. **Partner students for peer-sharing and teaching of Double It Strategy.**

   Write another problem that uses the Double It strategy. Work in partners and take turns being the teacher. Explain the steps to your neighbor. Once the first "teacher" is done another problem should be completed, but this time the student is now the teacher.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Have students write 10x[N]-N. Give each student a hundreds chart if they are struggling in doubling the product. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving 4, 6, 8 facts on Fact Family Mat.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give the student's the multiplication probe. Have students write their name and date.

   When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a group score of 22 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.
6. Assist students in writing a goal and graphing.

Lesson: Double It, Double Again, Double 3 Times

Materials: Wipe boards, CCP sheet

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Write number sentences for multiplication and division facts of 4, 6, 8.
- State the "Double It" strategy to solve multiplication and division facts.
- Identify when to use the strategy, "Double It."

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will practice multiplication and division problems with factors and divisors of 4, 6, 8. The strategy is the Double It.

Review: Allow more time to review (5 minutes)

To help us solve 4, 6, and 8s we need to practice the doubles. What is a double face? Solve. (6 + 6, 12+ 12, 14 + 14, 16 + 16)

We already mastered the 9s. What strategy can you use to solve 9s? (Make Ten Minus the Factor)

We are going to review a variety of problems, multiplication and division facts 2s, 10s, 5s, and 1s and 9s. This will help us solve the facts faster on the Independent Practice. (Hold up a variety of problems)

Modeling:

1. **Give each student a wipe board. Write 4 x 7 and 10 x 6.**

What strategy do we use when multiplying a factor to 4, 6 or 8? (Double It)

Read the problems.

Write the problems on your board. Which problem would you use the Double It strategy? (4 x 7)

What is 10 x 6? (60) How do we solve this problem? (Count by 10s)

What is the first step of Double It? *(Double the 7)*

What is 7 + 7 or 7 x 2? (14)

What is the second step? *(Add 14 + 14)*

What is 14 + 14? *(28)*

What is 4 x 7? (28)

What is another word for answer of a multiplication problem? *(Product)*

What are the parts of a division problem? *(Have students refer to vocabulary sheet if struggling. Divisor, dividend and quotient)*

Write the other 3 number sentences for this family.

What is the answer for a division problem called? *(Quotient)*

2. **Give each student a Fact Family Mat. If students are proficient at writing number sentences, continue to use the wipe boards.**

Practice other 4, 6, 8 facts, only giving two parts (such as one factor and a product)

EX: 36 and 6.

What part is missing, product, or factor? *(Factor)*

What factor is missing? *(6)*

Write the four number sentences on your fact family mat.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet that should be tri-folded. The guided practice sheet is a Copy, Cover, Compare fluency practice. The students look at the problem in the "Compute" column and solve in 3 seconds. Once 3 seconds has passed tap on table and tell students to move to the next problem.

When I say begin start working on the first problem in the column. When I tap the table, go to the next problem even if you are not done with the current problem. You will only have 3 seconds to answer each problem.

Once the students have finished the compute column students turn to the compare column. As a group answer and write answers. The students then compare the columns (compute and compare). For every problem answered incorrectly or not solved the students write that problem 4 times in the practice column.

If extra time is available have students practice solving 1, 2, 5, 10 and 9 facts in multiplication and division. Hold up flashcards for 3 seconds and have students write the answers on the wipe boards.

minutes: Independent Practice Probe

1. Collect all materials.
2. Give the student's the multiplication probe. Have students write their name and date.

When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.
6. Assist students in writing a goal and graphing.

Lesson: Break Apart 7 to Solve

Materials: Connecting cubes, wipe boards, fact family mats, multiplication and division fact cards (0, 1, 2, 5, 10, 9, 4)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 7.
- Write number sentences for multiplication and division facts of 7.
- Identify how to break apart 7 into two parts
    - 7 = 2 + 5
- State the "Break Apart" strategy to solve multiplication and division facts.
- Identify when to use the "Break Apart" strategy.

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will learn a new strategy to use when multiplying numbers to 7.

Vocabulary:

First let's review our multiplication and division vocabulary. *(Students can use the vocabulary sheet for assistance).*

In multiplication the two parts or the numbers multiplied together are called factors. What are the two numbers called? What is the answer to a multiplication problem? *(Product)*

When we write a division problem the product is the dividend. What is the name of the product in a division problem? *(Dividend)* One factor is the divisor and the other factor is the answer or the quotient. What is the answer to a division problem? *(Quotient)*

Review:

How many numbers are in a fact family? *(3)*

First let's review the facts we have mastered, 0, 1, 2, 5, 9, 4s and 10s. *(2-3 minutes)*

Modeling:

1. **Give each student a wipe board and 7 cubes.**

   A whole number can be broken apart into smaller parts. In the Double It strategy we broke apart 4, 6 and 8 into groups of 2 to help us solve facts quicker. For this strategy we are going to break apart the whole number 7.

   How many cubes do you have? *(7)*

   What facts do we have mastered? *(1, 2, 4, 5, 6, 8, 9, 10)* Write these factors on your board.

   How can we break apart 7 so that it we are using a fact that we have mastered? Can we break apart into 10 and another part? *(No)* Why? *(Greater than 7)* What other factors can we get rid of? *(8, 9)* Erase 8, 9 and 10.

   Break off 2 cubes. What part is left? *(5)* We can break apart 7 into 2 and 5 and then multiply. These are facts that we have mastered and can answer very fast!

This strategy is called Break Apart and can help us to solve times 7 facts. What is name of the strategy? *(Break Apart)* When do we use it? *(Multiplying to 7)*

2. **Give each student 14 more cubes.**

Put 2 cubes in a column and 5 cubes in a column. We are going to build 7 x 3 to show how the strategy works.

Write 7 x 3 on your board. First we break apart 7 into 2 and 5.

Now we will build an array to show 2 x 3 and 5 x 3. Build it.

XXX	XXX
XXX	XXX
	XXX
	XXX
	XXX

What is 2 x 3? *(6)* Write 6 below the array.

What is 5 x 3? *(15)* Write 15 below the array.

Now we add 6 + 15. What is 6 + 15? *(21, have 100s chart out for error correction)*

What strategy? *(Break Apart)*

When do we use Break Apart? *(When multiplying factors to 7)*

3. **Write 7 x 4, 7 x 9 and 7 x 8.**

Write these facts on your board.

How do we solve 7 x 4? *(Double it)* What is 7 x 4? *(28)*

How do we solve 7 x 9? *(Make 10 Minus the Factor)* What is 7 x 9? *(63)*

What strategy do we use to solve 7 x 8? *(Break Apart)*

What is the first step? How do we break apart 7? *(Into 2 and 5)*

Draw the array to show 2 x 8 and 5 x 8.

What is 2 x 8? (16) What is 5 x 8? *(40)*

What is the 2nd step of Break Apart? *(Add)* What is 16 + 40? *(56)*

4. **Give each student a Fact Family Mat .**

What 3 numbers are in this family? *(7, 8, 56)*

Write the factors and product in the roof.

Write the four number sentences.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Give each student a hundreds chart if they are struggling in adding after multiplying by 2 and 5. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving other times 7 facts on Fact Family Mat.

2 minutes: Independent Practice Probe

1. Collect all materials.
2. Give students the multiplication probe. Have students write their name and date.

   When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

3. Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

   Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Lesson: Break Apart 7 to Solve

Materials: Connecting cubes, wipe boards, fact family mats, multiplication and division fact cards (0, 1, 2, 5, 10, 9, 4)

Vocabulary:

- Multiplication: To add a number repeated times
- Factor: A number multiplied by another number
- Product: The answer to a multiplication problem
- Division: Grouping numbers into equal parts
- Divisor: The number that shows how many parts in each group
- Dividend: The total number to be divided into groups

Lesson Objective: The student's will

- Build an array to show multiplication and division facts of 7.
- Write number sentences for multiplication and division facts of 7.
- Identify how to break apart 7 into two parts
    - 7 = 2 + 5
- State the "Break Apart" strategy to solve multiplication and division facts.
- Identify when to use the "Break Apart" strategy.

12 minutes: Agenda, Vocabulary, Review, Modeling

Agenda:

Today we will learn a new strategy to use when multiplying numbers to 7.

Vocabulary:

First let’s review our multiplication and division vocabulary. *(Students can use the vocabulary sheet for assistance).*

What are the two parts in a multiplication problem called? *(Factors)*

What is the answer to a division problem? *(Quotient)*

Review:

How many numbers are in a fact family? *(3)*

First let’s review the facts we have mastered, 0, 1, 2, 5, 9, 4s and 10s. *(2-3 minutes)*

Modeling:

1. **Give each student a wipe board. Write 7 x 10, 7 x 2 and 7 x 6**

Read these 3 facts.

What is 7 x 10? *(70)* How do you solve it? *(Count by 10s, 7 times)*

What is 7 x 2? *(14)* How do you solve it? *(Count by 2s)*

We learned the strategy to solve 7 x 6. The strategy is called Break Apart. What is the strategy called? *(Break Apart)*

How do we break apart 7 to make multiplication easier? *(2 and 5)*

First write 2 x and 5 x below the 7 x 6 problem.

We broke apart 7, so which factor is left? *(6)* We have to still multiply 6 to 7, so we need to multiply 6 to 2 and 5.

Why do we need to multiply 6 to both 2 and 5? *(You have to multiply 6 to 7)*

Write 6 in both problems. What is 2 x 6? *(12)* What is 5 x 6? *(30)*

First we break apart 7. What is the first step of Break Apart? *(Break apart 7)*
How do we break apart 7? *(Into 2 and 5)*

Next we multiply the other factor to 2 and 5. What is the second step? *(Multiply the other factor with 2 and 5)*

Last we add the products. What is the last step? *(Add the products)* What is 12 + 30? *(42)*

What is 7 x 6? *(42)*

What strategy? *(Break Apart)*

Draw the array to show the Break Apart strategy used to solve 7 x 6.

7. **Give each student a Fact Family Mat. Write 7 x 7 in the roof.**

What two factors? *(7 and 7)*

What strategy? *(Break Apart)*

How do we break apart 7? *(2 and 5)*

What is 2 x 7? *(14)*

What is 5 x 7? *(35)*

What is the last step of the Break Apart Strategy? *(Add)*

What is 14 + 35? *(49)*

What 3 numbers are in this family? *(7, 7, 49)*

Write the four number sentences.

8 minutes: Guided Practice Sheet

Have students complete the guided practice sheet. Give each student a hundreds chart if they are struggling in adding after multiplying by 2 and 5. Once the student solves the multiplication problem write the division problem.

If extra time is available have students practice solving other times 7 facts on Fact Family Mat.

2 minutes: Independent Practice Probe

1. Collect all materials.
2. Give students the multiplication probe. Have students write their name and date.

3. When I say begin you will have 1-minute to solve as many items as possible. If you do not know an answer skip it and continue to the next problem. Do your best work. Ready, begin.

Repeat step 2 with the division probe.

5 minutes: Intervention Plus!

4. Correct each probe. Have students write the total number completed on top.
5. Give each student a daily data sheet. Graph the total amount completed. Have students write a daily goal.

> Now you are going to write a goal. A goal is a measurable aim to an end point. For us the aim is to complete each Independent Practice sheet, multiplication and division, with a score of 36 out of 40. An example of a goal is to have less than 3 errors on a multiplication sheet.

6. Assist students in writing a goal and graphing.

Name ______________________

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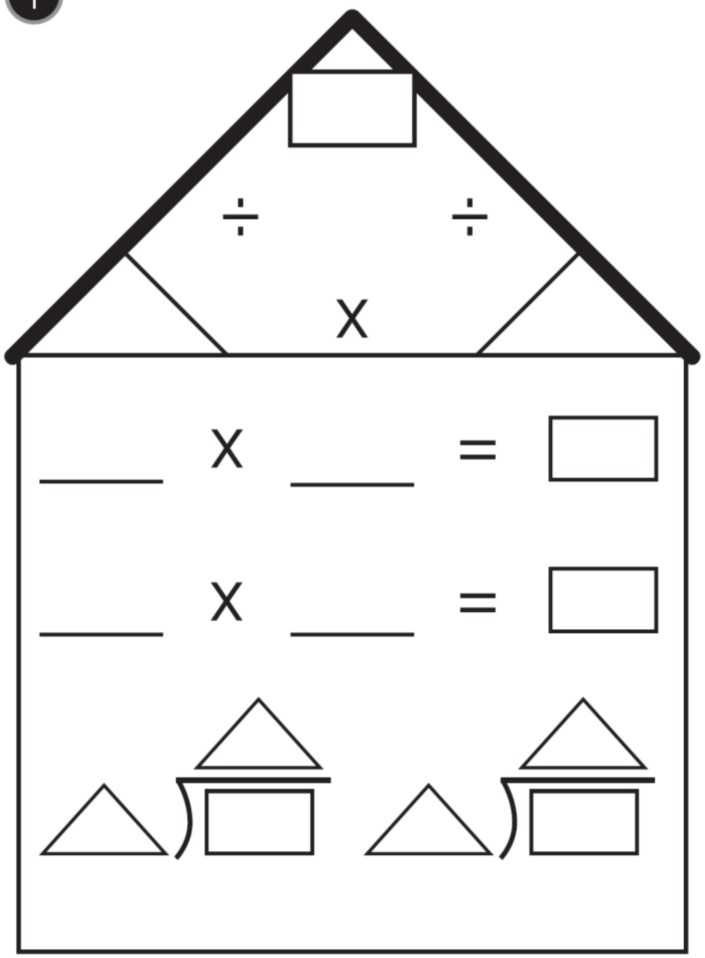

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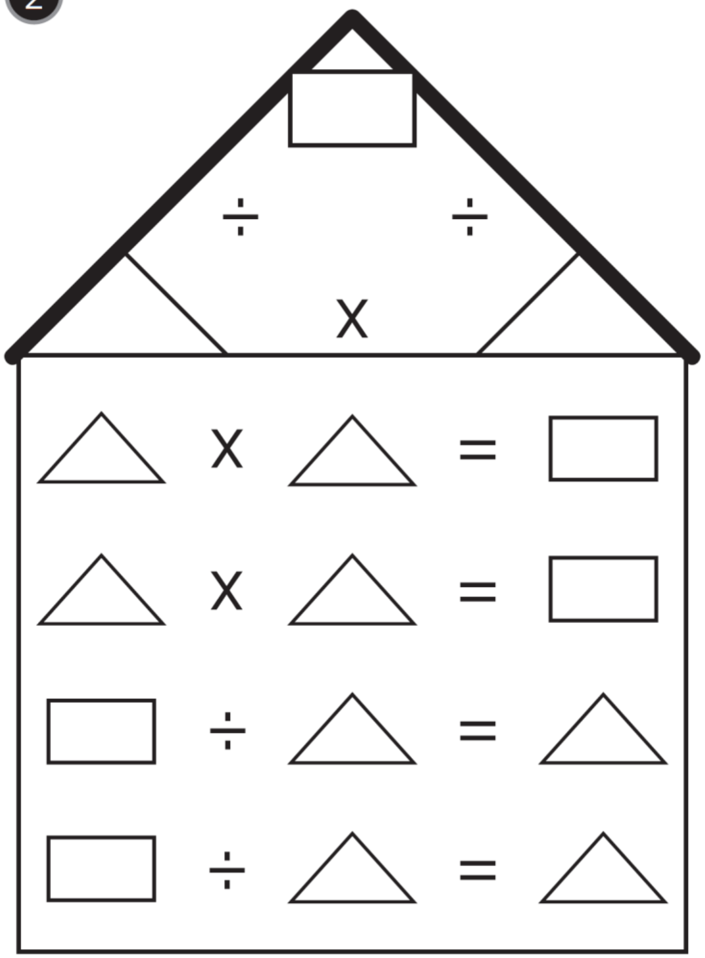

Name ______________________________ Date ________________

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- 9	- 9	- 9	- 9	- 9	- 9	- 9	- 9

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10 x 8 = ____

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❹ 9 x 7 = ____

10 x 7 = ____

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❺ 6 x 9 = ____

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❻ 8 x 9 = ____

8 x 10 = ____

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Name ______________________ Date ____________

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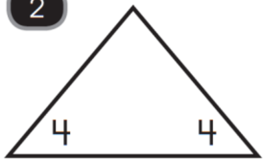

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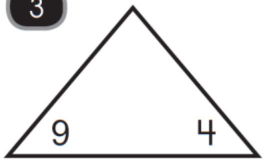

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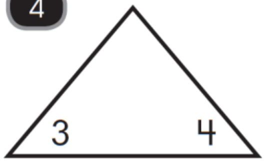

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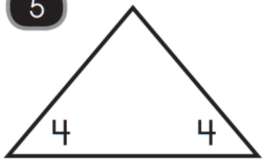

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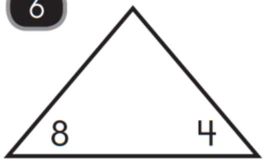

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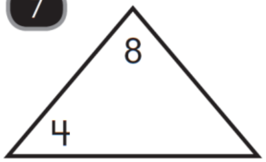

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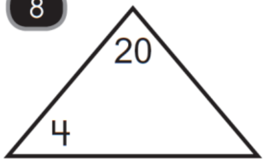

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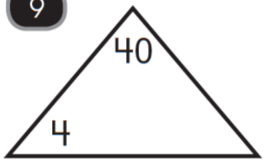

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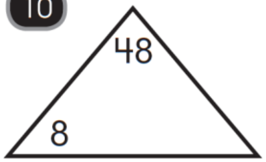
____ ÷ ____

10



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____ ÷ ____

Appendix E

Name ______________________________ **9a**

Date __________________

9 x 3 = 9 x 6 = 5 x 2 = 9 x 4 = 9 x 7 =

2 x 5 = 8 x 9 = 4 x 9 = 10 x 0 = 9 x 9 =

3 x 9 = 9 x 10 = 6 x 9 = 9 x 9 = 5 x 1 =

7 x 9 = 8 x 9 = 8 x 0 = 9 x 4 = 9 x 3 =

10 x 2 = 9 x 6 = 9 x 8 = 6 x 10 = 9 x 9 =

9 x 7 = 10 x 10 = 3 x 9 = 9 x 4 = 10 x 9 =

9 x 6 = 9 x 8 = 10 x 8 = 7 x 9 = 9 x 9 =

Name ______________________________

Date __________________

9c

9 x 8	3 x 9	3 x 0	9 x 9	9 x 6	5 x 9	7 x 9	4 x 9
10 x 9	8 x 9	9 x 3	1 x 4	6 x 9	9 x 9	5 x 6	9 x 7
9 x 4	5 x 2	9 x 3	9 x 8	8 x 1	6 x 9	9 x 9	0 x 10
4 x 9	9 x 7	9 x 0	9 x 8	3 x 9	2 x 5	9 x 6	9 x 4
8 x 10	7 x 9	9 x 9	2 x 1	9 x 7	5 x 5	10 x 4	10 x 1

Name ______________________________ **4d**

Date __________________

3	8	8	4	4	9	6	4
x 4	x 4	x 5	x 7	x 4	x 3	x 4	x 8
9	4	8	3	4	3	9	4
x 9	x 3	x 8	x 2	x 4	x 4	x 0	x 6
8	0	8	6	6	4	4	2
x 4	x 6	x 6	x 4	x 1	x 7	x 4	x 6
4	4	6	6	7	9	6	4
x 3	x 4	x 0	x 4	x 4	x 8	x 8	x 7
3	4	9	6	6	4	4	5
x 10	x 8	x 9	x 10	x 6	x 6	x 5	x 10

Name ______________________________

7a

Date __________________

$7\overline{)21}$	$7\overline{)49}$	$2\overline{)4}$	$8\overline{)56}$	$3\overline{)21}$
$4\overline{)40}$	$7\overline{)42}$	$7\overline{)21}$	$4\overline{)36}$	$6\overline{)42}$
$7\overline{)56}$	$7\overline{)28}$	$6\overline{)42}$	$8\overline{)56}$	$5\overline{)10}$
$7\overline{)21}$	$7\overline{)42}$	$2\overline{)10}$	$3\overline{)21}$	$7\overline{)49}$
$8\overline{)48}$	$7\overline{)49}$	$7\overline{)56}$	$2\overline{)14}$	$7\overline{)21}$
$6\overline{)42}$	$4\overline{)0}$	$3\overline{)21}$	$7\overline{)42}$	$4\overline{)28}$
$7\overline{)49}$	$3\overline{)21}$	$1\overline{)8}$	$7\overline{)56}$	$8\overline{)56}$
$4\overline{)4}$	$8\overline{)56}$	$10\overline{)50}$	$8\overline{)64}$	$9\overline{)63}$

Name ______________________________

Date __________________

9d

$9\overline{)54}$	$8\overline{)72}$	$2\overline{)6}$	$3\overline{)27}$	$9\overline{)36}$
$2\overline{)8}$	$9\overline{)27}$	$7\overline{)63}$	$1\overline{)4}$	$9\overline{)81}$
$9\overline{)63}$	$10\overline{)20}$	$4\overline{)36}$	$9\overline{)81}$	$5\overline{)20}$
$6\overline{)54}$	$9\overline{)27}$	$5\overline{)50}$	$9\overline{)72}$	$4\overline{)36}$
$5\overline{)10}$	$9\overline{)72}$	$7\overline{)63}$	$2\overline{)20}$	$6\overline{)54}$
$3\overline{)27}$	$2\overline{)12}$	$9\overline{)63}$	$8\overline{)72}$	$10\overline{)90}$
$9\overline{)36}$	$9\overline{)54}$	$10\overline{)30}$	$4\overline{)36}$	$7\overline{)63}$
$5\overline{)30}$	$9\overline{)54}$	$1\overline{)2}$	$2\overline{)16}$	$5\overline{)25}$

Appendix F

Name ______________________

✓My Daily Sheet

Date: ________
★ **Multiplication**
____ Errors
____ Correct
★ **Division**
____ Errors
____ Correct
GOAL: I will ______________________
____
____

• • • • • • • • • • • • • • • • • • • •

Date: ________
★ **Multiplication**
____ Errors
____ Correct
★ **Division**
____ Errors
____ Correct
GOAL: I will ______________________
____
____

• • • • • • • • • • • • • • • • • • • •

Date: ________
★ **Multiplication**
____ Errors
____ Correct
★ **Division**
____ Errors
____ Correct
GOAL: I will ______________________
____
____

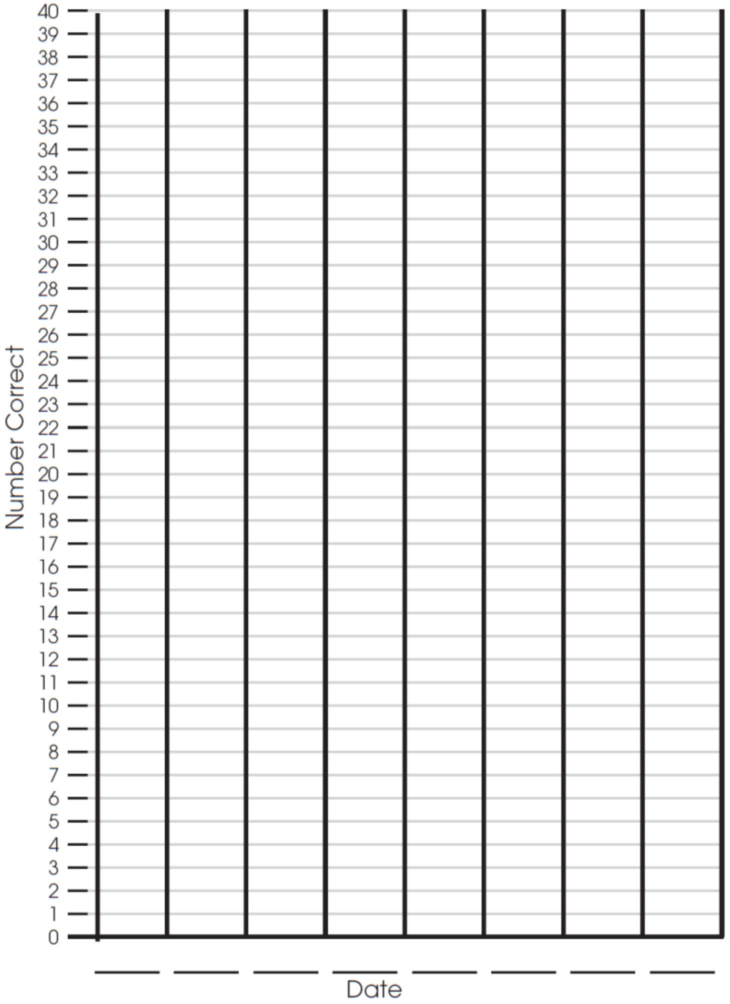
• • • • • • • • • • • • • • • • • • • •

Date: ________
★ **Multiplication**
____ Errors
____ Correct
★ **Division**
____ Errors
____ Correct
GOAL: I will ______________________
____
____

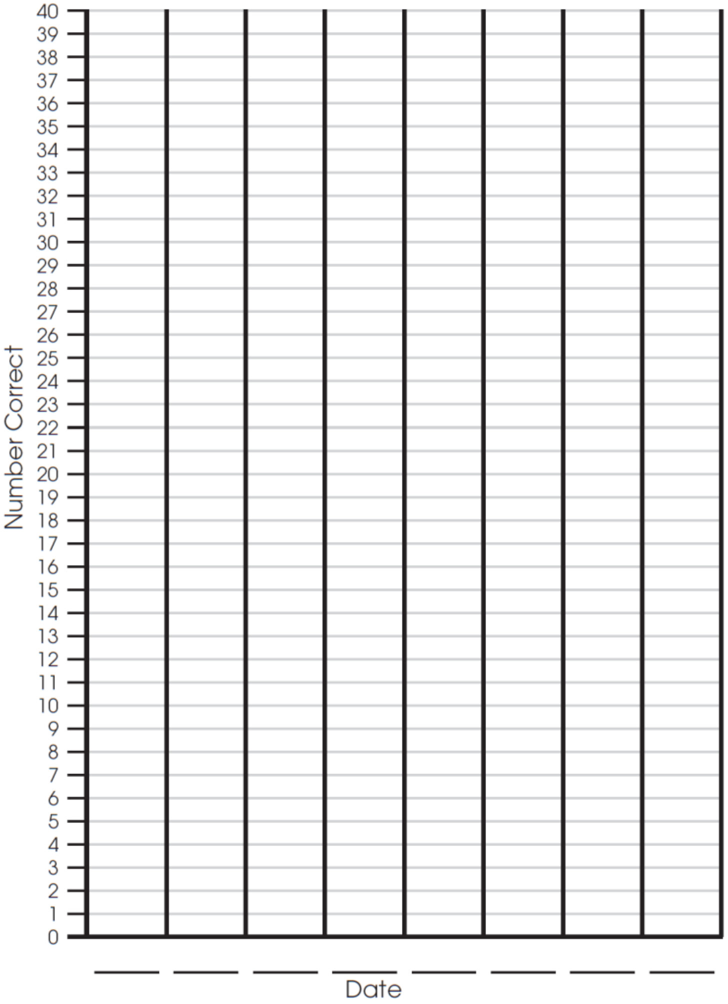
✓My Daily Chart

Name ______________________

Multiplication



Division



Appendix G

Fidelity Form

Materials	YES	NO	NOTES
Did the teacher provide a complete set of materials for student use as appropriate throughout the duration of the lesson?			
Did the teacher and students transition to different material in a short amount of time (less than 1 minute)?			

Objective's)	YES	NO	NOTES
Did the lesson address all objectives listed?			

Lesson	YES	NO	NOTES
Did the teacher complete the review?			
Was each step/activity completed within modeling?			
Did teacher maintain a brisk pace using the script?			
Did teacher provide feedback? (error correction and encouraging)			
After asking a question did teacher give wait time (3-5 seconds) and allow group or individual to answer?			
Did the teacher alternate between choral and individual response?			

Was the Guided Practice sheet completed by the whole group? (teacher may allow students to work at different pace, but teacher should be checking and assisting students)			
Was the Independent Practice sheet completed by all students?			
Did teacher use timer and adhere to time limits in lesson?			

Behavior	**YES**	**NO**	**NOTES**
Were students on-task a majority of the time? (Greater than 80%)			
Did teacher stop/redirect inappropriate behavior before it hindered others learning?			
Were students engaged? (Answering questions, following along/completing activities, completing Guided Practice, working during Independent practice)?			

Intervention Plus	**YES**	**NO**	**NOTES**
Did students correct their own probes? (Multiplication and division)			
Did students record amount correct?			
Did students graph correct items?			
Did students set goals?			
Did teacher assist students in graphing and goal setting?			

Additional Comments:

Appendix H

Name:

1. How long have you been at Northwest:
    - ☐ Since kindergarten
    - ☐ Since 1st grade
    - ☐ Since 2nd grade
    - ☐ Since 3rd grade

2. Have you ever been in math tutorials before? **Yes** **No**
    - When: **Kindergarten** **1st** **2nd** **3rd**
    - Setting: **In School** **After school**

3. Do you like math?

    Yes, totally! **Yes, kind of** **Not really** **No way!**

Think about our time in after school tutorials.

1. The strategies used to learn multiplication facts were easy.

    Totally Agree **Kind of agree** **Kind of disagree** **Totally disagree**

2. The strategies used to solve multiplication facts would be good for my class mates to use.

    Totally Agree **Kind of agree** **Kind of disagree** **Totally disagree**

3. I liked the strategies I learned to solve multiplication and division facts.

    Totally Agree **Kind of agree** **Kind of disagree** **Totally disagree**

4. I think the strategies helped me to do better in math class.

    Totally Agree **Kind of agree** **Kind of disagree** **Totally disagree**

5. I liked grading the 1-minute fact sheets.

    Totally Agree **Kind of agree** **Kind of disagree** **Totally disagree**

6. I liked graphing my results.
   Totally Agree Kind of agree Kind of disagree Totally disagree

7. I liked setting a goal for myself for the next fact sheet.
   Totally Agree Kind of agree Kind of disagree Totally disagree

What did you like best about coming to learn about strategies to solve multiplication and division facts?

____
____
____
____
____
_

What was hard for you when learning the facts?

____
____
____
____
____

What would you tell your friends about our math tutoring time?
Would you recommend it for your friends?

____
____
____
____
____

What, if anything, would you change about the math tutoring time?

____
____
____
____
____

Appendix I

Name: _______________________

Student Selection

1. The students were selected accurately for the fact intervention.

Strongly Disagree   Disagree   Agree   Strongly Agree

2. The students selected for the fact intervention were previously identified by me or the school as needing additional support in mathematics.

Strongly Disagree   Disagree   Agree   Strongly Agree

3. The students selected for fact intervention were already receiving additional instruction during the school day.

   YES   NO

   *If yes, what did the additional instruction look like?

   - Time (duration, frequency, days per week): ____________________
   - Groupings (number of students): _____________________
   - Setting (in class, pull-out). Circle:

   In-class   Pull-out   After school

4. Students selected for the intervention in January have made better progress in mathematical facts than other students with similar math fact ability.

Strongly Disagree   Disagree   Agree   Strongly Agree

5. If participating students are still behind their peers, what factors may be slowing their progress? (e.g.; more time needed, more practice needed, motivation, low fluency, lack accuracy, lack pre-requisite skills [addition/subtraction facts])

____

____

____

____

Fact Instruction in the Classroom

Based on your daily schedule and/or curriculum and select the statement(s) that best describes your teaching:

- ☐ I work with struggling students *specifically* in multiplication facts as part of my whole-class instruction.
- ☐ I work with struggling students *specifically* in multiplication facts in small group(s).
- ☐ I work with struggling students *specifically* in multiplication facts individually.
- ☐ A campus interventionist works with my struggling students *specifically* in multiplication facts.

- ☐ I work with struggling students *specifically* in division facts as part of my whole-class instruction.
- ☐ I work with struggling students *specifically* in division facts in small group(s).
- ☐ I work with struggling students *specifically* in division facts individually.
- ☐ A campus interventionist works with my struggling students *specifically* in division facts.

1. I teach struggling students in multiplication and division facts.

   Yes **No**

   a. **How many days per week? _____________**
   b. **How many minutes per day? ____________**

Curriculum

1. Math facts are an important part of my curriculum.

   Strongly Disagree **Disagree** **Agree** **Strongly Agree**

2. I include multiplication fact instruction into the mathematics curriculum.

   Never **Once a Month** **Once a Week** **Daily**

3. I include division fact instruction into my mathematics time.

   Never **Once a Month** **Once a Week** **Daily**

4. I teach specific strategies to solve multiplication facts.

   Yes **No**

5. I teach specific strategies to solve division facts.

   Yes **No**

*If yes for either or both #4 and #5, please list specific strategies taught to solve multiplication and/or division facts. (e.g.; Count-by, Double It, Make Ten Minus the factor, Break Apart, Multiplication chart)

____

____

____

____

6. Fact instruction is part of my district's curriculum scope and sequence.

Yes **No**

7. A majority of my mathematics instruction is focused around TAKS objectives rather than fact instruction.

Strongly Disagree **Disagree** **Agree** **Strongly Agree**

8. I collect data on my student's ability to solve multiplication facts.

Never **Once a Month** **Once a Week** **Daily**

9. I collect data on my student's ability to solve division facts.

Never **Once a Month** **Once a Week** **Daily**

10. I share data on fact fluency with my students'.

Never **Once a Month** **Once a Week** **Daily**

Additional Tutoring

Please provide a brief explanation of the after school tutoring.

____

____

____

____

Fact Intervention

Think about the students who received the fact intervention.

1. Intervention students used the taught strategies in my classroom.

Never **Rarely** **Regularly**

2. The student's ability in solving multiplication and division facts increased as compared to peers.

Strongly Disagree **Disagree** **Agree** **Strongly Agree**

3. The intervention helped my student's gain confidence in mathematics.

Strongly Disagree Disagree Agree Strongly Agree

4. The students shared the taught multiplication/division fact strategies with peers/me.

Never Sometimes Frequently

5. The students were able to use the strategies to solve multi-step/complex multiplication and division problems.

Never Sometimes Frequently

6. I would like to use the strategies with other students in the future.

Strongly Disagree Disagree Agree Strongly Agree

7. The students shared their goals and progress on fact sheets with me.

Never Sometimes Frequently

Overall, did you feel that the intervention was successful? Why or why not?

____
____
____
____
____

Was the time (2 days/week, 30 minutes) too much, not enough, just right?

____
____
____
____
____

Please provide additional comments related to the intervention that is not covered by the survey:

____
____
____
____
____

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Vita

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