

Modeling of Flexible Rotor Machines Supported With Hydrostatic Bearings

R.N. Headifen

The Center for Electromechanics
The University of Texas At Austin
Austin, Texas 78712

W.F. Weldon

The Center for Electromechanics
The University of Texas At Austin
Austin, Texas 78712

ABSTRACT

In this paper a method is described that takes the nonlinear dynamic stiffness and damping coefficients for multiple hydrostatic bearings and incorporates them into a rotordynamic FEM model for a rotating machine. A Newton-Raphson iteration scheme is presented that uses updated bearing coefficients at every iteration to the solution. A non-linear computer program was written using the method described which models transient and synchronous response and calculates damped eigenvalues.

SYMBOLS USED

C the damping matrix which also can contain gyroscopic terms
 K the stiffness matrix
 M the mass matrix
 Q is a coordinate transformation matrix
 F is a force vector
 c bearing clearance
 u displacement
 F_r radial force
 F_t tangential force
 m_e the imbalance mass multiplied by the geometric center offset from the rotor center of mass
 N the number of degrees of freedom (DOF)
 ϵ is the journal eccentricity in the bearing
 ϕ is the angle between the line of centers
 θ is the tangential coordinate
 μ is the bearing fluid viscosity
 ω is the journal rotation speed

INTRODUCTION

For high speed rotating machines running on hydrostatic bearings, the lack of analysis tools to accurately model the motion of the rotor in the bearing could lead to design errors resulting in the inability to operate the machine at design speed. In response to these requirements, a new computer analysis program was written to fully model the situation of a flexible rotor machine supported on

hydrostatic bearings. These could then be mounted into a flexible outer stator casing which could be hard or soft mounted to a skid mass or to ground. Current trends for high speed lightweight rotors is leading to more flexible structures, making this kind of analysis more valuable.

The literature revealed numerous articles on modeling flexible rotors in flexible stators (Rouch 1977). There is also a large amount of information on modeling of hydrostatic bearings (Rowe 1980, Tolk 1980). However information on a completely comprehensive method that included a flexible rotor, coupled to a stator structure or to ground through nonlinear hydrostatic bearings was not found in the literature search. There was information on work that had been performed on this problem using other fluid bearing types, mainly journal bearings.

Feria Kaiser (1987) discussed a program he had written for a flexible shaft mounted in hydrodynamic journal bearings. This program used energy methods. The program was not readily amenable to model more than 2 or 3 bearings in a machine due to the complexity involved in changing the equations. Adams (1980) described a computer program he developed for non-linear dynamics of flexible multi-bearing rotors. He used the finite element technique to model the distributed mass and elasticity of the rotor and stator structure. The bearings he modeled were also journal bearings. He treated gyroscopic stiffening of the shaft as an externally applied moment on the rotor. Several other papers present models of multi-mass flexible rotors supported in nonlinear journal bearings (McLean 1983, Greenhill 1982, Rabinowitz 1977). Vaughn (1989) wrote a program that modeled a rotor supported by a hydrostatic bearing at each end and a hydrostatic thrust bearing at one end. The program was a coupled five degree of freedom model that used a lumped mass approach.

DEVELOPMENT OF THE MODEL

The program developed was to model the schematic shown in figure 1. The side view in this figure shows the axisymmetric flexible rotor and stator structures and lumped foundation masses.

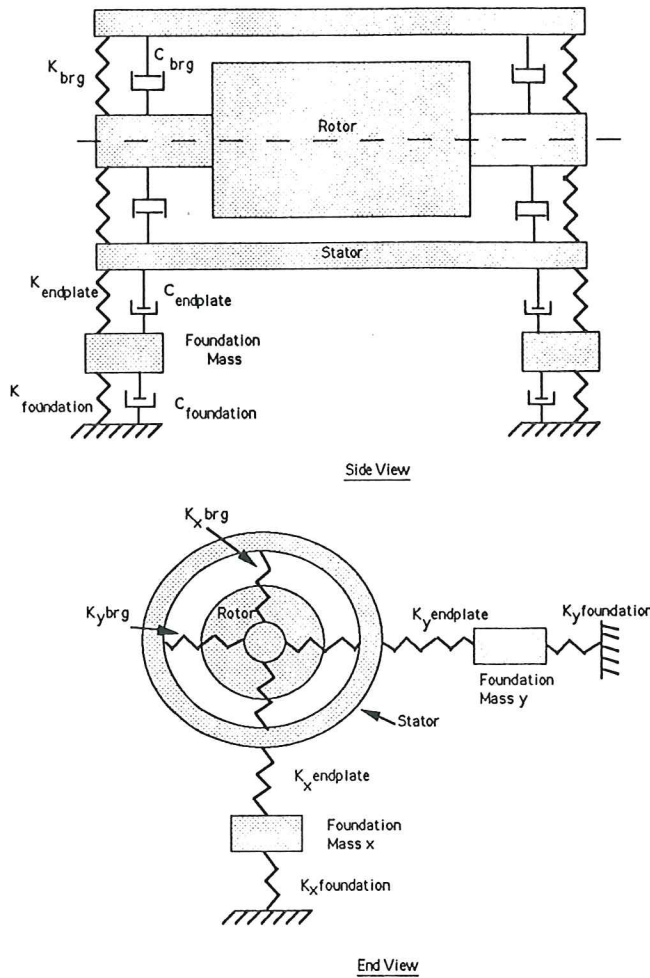


Figure 1. A schematic of the system to be modeled

The end view shows the two planes of motion and two of the eight stiffness and damping coefficients that the bearing adds to the system.

The final computer program was developed in three stages (Headifen 1991). The first was to write a flexible rotor dynamics finite element method (FEM) program that could accommodate linear springs and dampers as the boundary conditions. This model also included the stator structure and any additional nodal masses, such as a support skid, that were required.

The second stage was to write an analytical program that could model the nonlinear behavior of a hydrostatic bearing. These bearings have nonlinear load capacities and stiffness and damping properties over their eccentricity range (Ghosh 1979). Hence this program had to account for the nonlinearity of the bearing characteristics. The program had to be analytical because in the final program these bearing subroutines would be called many times to

calculate the bearing terms and pass them to the FEM portion of the program. A numerical approach would have been uneconomical timewise. The bearing program was a stand alone program that could model a lumped mass spinning and or whirling in a bearing. The program could perform transient response, synchronous response and could calculate the eight radial and tangential dynamic bearing coefficients at any position and speed in the bearing.

Once the bearing program was operating and had been verified, the two programs were merged in to one final program. The bearing program subroutines that were involved with calculating reaction forces and bearing coefficients were incorporated into the rotor program. The rotor program subroutines were then modified to account for the nonlinear coefficients.

THE LINEAR ROTOR PROGRAM

The method used to model the rotor and stator structure was to use the FEM. To keep the number of degrees of freedom (DOF) small, one dimensional elements were used that had four DOF per node. These were X, Y translation and α , β rotations (Bathe 1982). This formulation accounted for shear and bending deflection. Distributed stiffness (K) and mass (M) matrices were set up and gyroscopic terms were included in a C matrix. Even though the finite element method can require large matrices to be generated, it was chosen over the Transfer Matrix method because once the M, C and K matrices have been established, they are easy to manipulate in many different algorithms.

The equations for the FEM method are derived from the total kinetic and potential energy associated with lateral motion of the rotating shaft or stator. The energy terms come from a bending term, a shear term and a spin term. These energy equations are shown in (Manifold 1989). The final assembly form of the matrices is given by (Headifen 1991). The final dynamic equation can be written in the familiar form of

$$\mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{F} \quad (1)$$

where

\mathbf{F} = vector containing the model imbalances and gravity loads for horizontal machines

\mathbf{u} = vector of nodal displacements

In (1), the linear boundary conditions are included in the matrices on the left-hand side (LHS).

THE BEARING PROGRAM

This was a nonlinear program that at every time step or speed increment solved the equation of motion for a lumped mass in a hydrostatic bearing.

$$\mathbf{M}_B \ddot{\mathbf{U}} = \mathbf{F}(\dot{\mathbf{e}}, \mathbf{M}_B, \text{geom}, \mu, \dot{\phi}, \omega, \mathbf{U}) \quad (2)$$

where

\mathbf{F} = vector summation of the applied forces (imbalance and gravity) and the bearing reaction forces that depend on the bearing geometry and fluid, whirl speed, shaft speed and position. (Tolk 1980)

\mathbf{M}_B = 2x2 mass matrix for the vertical and horizontal planes.

\mathbf{U} = vector containing the horizontal and vertical displacements

A previously unseen method to account for bearing fluid compressibility was developed (Headifen 1991) and included in the program.

Bearing stiffness and damping coefficients could be calculated at any position and were evaluated by the small displacement method. This is the same techniques as used by Ghosh (1979) but with different definitions for the tangential coefficients. The definition of the coefficients used was

$$K_{rr} = \frac{\Delta F_r}{c\Delta\epsilon} \quad K_{tr} = \frac{\Delta F_t}{c\Delta\epsilon}$$

for a radial step, and

$$K_{tt} = \frac{(-F_{r,2} \sin \Delta\theta + F_{t,2} \cos \Delta\theta) - F_{t,1}}{c\epsilon\Delta\theta} \quad (3)$$

$$K_{tt} = \frac{(-F_{r,2} \sin \Delta\theta + F_{t,2} \cos \Delta\theta) - F_{t,1}}{c\epsilon\Delta\theta}$$

for a tangential step. When a tangential step is taken the direction that the forces are calculated in is changed by the rotation angle $\Delta\theta$, hence the components in the original direction must be determined.

For the damping coefficients, their definition was

$$C_{rr} = \frac{\Delta F_r}{c\dot{\epsilon}} \quad C_{tr} = \frac{\Delta F_t}{c\dot{\epsilon}}$$

for a radial velocity, and

$$C_{tt} = \frac{\Delta F_t}{c\epsilon\Delta\dot{\phi}} \quad C_{tr} = \frac{\Delta F_r}{c\epsilon\Delta\dot{\phi}} \quad (4)$$

for the tangential coefficients.

FINAL FEM PROGRAM WITH NON-LINEAR BEARING COEFFICIENTS

This program was a combination of the previous two programs. To incorporate the relative bearing subroutines into the rotor program, required writing linking subroutines that passed the position and speed to the bearing subroutines which then returned the reaction forces and coefficients back to the rotor part of the program.

At small eccentricities in the bearings, linear analysis could be used as the coefficients are not changing to any great extent. Over a wider range of operation, nonlinear analysis is required as follows. The general equation to solve is written as

$$M\ddot{U}(t) + C\dot{U}(t) + KU(t) = F(\dot{\epsilon}, M_B, \text{geom}, \mu, \dot{\phi}, \omega, U) \quad (5)$$

where

F = described in equation (2)

In equation (5), the matrices on the LHS only contain entries from the FEM assembly and any other linear stiffness and damping elements. These matrices are time invariant within each time step. The C matrix has a speed term in it so it is updated at the beginning of every step. Equation (5) has to be solved with a nonlinear solver. One method to achieve this is to use a Newton-Raphson technique and solve the problem as if the rotor was a free body acted upon by imbalance forces and bearing reaction forces only.

TIME RESPONSE

One method that can be used for time response is a non-linear version of the Newmark-Beta algorithm (Headifen 1991). This method uses finite difference approximations in the time domain to rearrange equation (5) to a form given by

$$\hat{K}^{t+\Delta t} U = {}^{t+\Delta t} \hat{R} \quad (6)$$

where

$$\hat{K} = K + a_0 M + a_1 C \quad (7)$$

$${}^{t+\Delta t} \hat{R} = {}^{t+\Delta t} R + M(a_0 {}^t U + a_2 {}^t \dot{U} + a_3 {}^t \ddot{U}) + C(a_1 {}^t U + a_4 {}^t \dot{U} + a_5 {}^t \ddot{U}) \quad (8)$$

a_1 to a_5 = constants defined in the Newmark-Beta method (Bathe 1982).

R = vector containing the applied forces such as imbalance and gravity, and the bearing reaction forces.

Equation (6) can be rearranged for a function G where

$$G_i(U) = \hat{K} U - \hat{R} = 0 \quad i = 1, 2, \dots, N \quad (9)$$

and the time superscripts have been dropped.

Using a Newton-Raphson iteration scheme (Press 1986), an equation which solves for the increments in the solution can be derived (Headifen 1991).

$$\left(\sum_{j=1}^N \frac{\partial G_i}{\partial U_j} \right) \delta U = -G_i(U) \quad (10)$$

The corrections are then added to the solution vector,

$$U_i^{\text{new}} = U_i^{\text{old}} + \delta U_i \quad i = 1, 2, \dots, N \quad (11)$$

The summation term on the left hand side of (10) is called the tangent stiffness matrix K^T . It is an $N \times N$ matrix where N is the number of DOF in model. Expanding the summation term in equation (10) gives

$$\sum_{j=1}^N \frac{\partial G_i}{\partial U_j} = \sum_{j=1}^N \frac{\partial (\hat{K} U - \hat{R})}{\partial U_j} = \hat{K} + \sum_{j=1}^N \frac{\partial \hat{R}_i}{\partial U_j} \quad (12)$$

Hence K^T is set equal to the \hat{K} matrix, then any terms in the \hat{R} vector that have a U dependence are differentiated with respect to U and added in. Recall that the time subscripts have been dropped and that U is really ${}^{t+\Delta t}U$, i.e. the displacement at the end of the time step. The only terms in the definition of \hat{R} that are dependent on ${}^{t+\Delta t}U$ are the ${}^{t+\Delta t}R$ terms. This vector is made up of the imbalance force and the bearing reaction forces. Only the reaction forces are dependent on the U terms. Therefore at a node that has a bearing on it, then

$$\frac{\partial \hat{R}_x}{\partial U_x} = k_{xx}, \frac{\partial \hat{R}_x}{\partial U_y} = k_{xy}, \frac{\partial \hat{R}_y}{\partial U_y} = k_{yy}, \frac{\partial \hat{R}_y}{\partial U_x} = k_{yx} \quad (13)$$

For all other DOF

$$\frac{\partial \hat{R}_i}{\partial U_j} = 0 \quad i, j = x, y \quad (14)$$

These are the bearing stiffness coefficients (relative to a reference frame that rotates with the rotor).

Similarly, though not quite so obvious due to the finite difference terms in the time domain, the terms in the definition of $\hat{\mathbf{r}}$ that are dependent on ${}^{t+\Delta t}\dot{\mathbf{U}}$ are also included. These are the reaction forces in the ${}^{t+\Delta t}\mathbf{R}$ term. Therefore at a node that has a bearing on it, then

$$\frac{\partial \hat{\mathbf{R}}_x}{\partial \dot{\mathbf{U}}_x} = C_{xx}, \frac{\partial \hat{\mathbf{R}}_x}{\partial \dot{\mathbf{U}}_y} = C_{xy}, \frac{\partial \hat{\mathbf{R}}_y}{\partial \dot{\mathbf{U}}_y} = C_{yy}, \frac{\partial \hat{\mathbf{R}}_y}{\partial \dot{\mathbf{U}}_x} = C_{yx} \quad (15)$$

For all other DOF

$$\frac{\partial \hat{\mathbf{R}}_i}{\partial \dot{\mathbf{U}}_j} = 0 \quad i, j = x, y \quad (16)$$

Thus for every node in the model that has a bearing on it, the stiffness and damping coefficients are added to their respective degrees of freedom in the tangent stiffness matrix. Equation (10) is then solved for the increments in the solution for the displacement. Then the tangent stiffness matrix and the RHS of (10) are reformed with the updated coefficients and equation (10) is resolved. This process is continued until either the increments in the displacements is less than a specified convergence or the RHS of (10) is less than a specified convergence.

REFERENCE FRAME TRANSFORMATION

Care must be taken to ensure that the bearing coefficients and the model have the same reference frame. Normally the bearing coefficients are expressed in a rotating reference frame that rotates with the shaft.

If the model uses a stationary global reference frame (for example X positive down and Y positive to the right) then the transformation from the rotating frame to the stationary frame is given by

$$\mathbf{Q}\mathbf{U}_G = \mathbf{U}_R \quad (17)$$

where

$$\mathbf{Q} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \quad (18)$$

ϕ = angle between the two frames.

The relationship between the coefficients in the stationary frame and the rotating frame is given by (Headifen 1991)

$$\mathbf{K}_G = \mathbf{Q}^T \mathbf{K}_R \mathbf{Q} \quad (19)$$

This transformation has to be performed prior to forming the tangent stiffness matrix.

FREQUENCY RESPONSE

Using a substitution for the synchronous displacement time variation of

$$\mathbf{U} = \mathbf{U}_0 e^{j\omega t} \quad (20)$$

in equation (5) allows the equation to be written as (Rouch 1980)

$$[-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}] \mathbf{U}_0 = \mathbf{F}(\omega) \quad (21)$$

or

$$[\mathbf{H}(\omega)] \mathbf{U}_0 = \mathbf{F}(\omega) \quad (22)$$

For the same nonlinear reasons as above, incorporation of the bearings had to be as reaction forces on the RHS and not by using the coefficients in the matrices on the LHS. Equation (22) is a non-linear complex equation. To solve this, a Newton-Raphson iteration scheme was employed in the same manner as above. Let

$$\mathbf{G}_i(\mathbf{U}) = \mathbf{H}\mathbf{U}_0 - \mathbf{F} = 0 \quad i = 1, 2, \dots, N \quad (23)$$

then following through equations (10,12) gives

$$\mathbf{H}^T = \sum_{j=1}^N \frac{\partial \mathbf{G}_i}{\partial \mathbf{U}_j} = \sum_{j=1}^N \frac{\partial (\mathbf{H}\mathbf{U} - \mathbf{F})}{\partial \mathbf{U}_j} \quad (24)$$

or

$$= \mathbf{H} + \sum_{j=1}^N \frac{\partial \mathbf{F}_i}{\partial \mathbf{U}_j} + j\omega \sum_{j=1}^N \frac{\partial \mathbf{F}_i}{\partial \dot{\mathbf{U}}_j} = \mathbf{H} + \mathbf{K}_{nm} + j\omega \mathbf{C}_{nm} \quad n, m = x, y$$

Hence the tangent stiffness matrix \mathbf{H}^T is set equal to the \mathbf{H} matrix and the $\delta \mathbf{F} / \delta \mathbf{U}$ terms are added to it at the respective DOF. Since equation (25) is complex, with the terms in the \mathbf{C} matrix being the imaginary components, then the damping coefficients are added to the imaginary components of the bearing DOF. They are first multiplied by the speed ω according to equation (21). The stiffness coefficients are added in to the real components of the same DOF. The complex solver then solves

$$\mathbf{H}^T \delta \mathbf{U}_0 = \mathbf{G}(\mathbf{U}_0) \quad (26)$$

similar to the time response. The Newton-Raphson iteration is repeated until a user supplied tolerance is reached then the next speed increment is made.

DAMPED EIGENVALUES AND LINEAR STABILITY

Eigenvalue analysis solves the homogeneous equation

$$\mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{K}\mathbf{U}(t) = 0 \quad (27)$$

Since there is no right hand side to add in the bearing reaction forces, the bearing coefficients can only be accounted for in the matrices on the left hand side. This poses a limitation on the validity of the analysis. The eigenvalue analysis will only be truly accurate over a region where the coefficients are relatively constant. This would be at small eccentricities. Since this is where most higher speed bearings operate, then it is a valid justification in most cases (Lund 1967). Once the coefficients have been added into equation (27), the equation can be converted to standard eigenvalue format by using a state vector notation as shown by Rouch (1980). Then a linear eigensolver can then be used to determine the eigenvalues and hence

linear stability at that position. As pointed out by Vance (1988), this type of analysis will only be able to predict the conditions for instability and the onset frequency of the whirling, it will not predict the actual bounded whirling amplitude as this requires equilibrium solutions to the nonlinear equations of motion. Vance points out that actual (measured) bounded whirling frequencies usually occur very close to the predicted frequencies for instability of the linearized system. A more detailed discussion of the consequences of linearized stability approach is presented by Vance. Nonlinear stability is not with the scope of this paper. McLean (1983) and Hahn (1979) present analyses on nonlinear stability for dampers.

The analyst should be aware that the eigenvalue solver will solve an eigenvalue for a mode in the tangential direction based on the tangential stiffness and damping coefficients. If the forcing function is rotor imbalance then this mode will never be excited. Only modes along the local k_{xx} direction will be excited, because the k_{xx} direction rotates with the shaft. An angle ϕ can be user defined for this option. This is similar to the angle ϕ described in the coefficient direction rotation. This allows the user to position the local direction relative to the global direction for calculation of the eigenvalues

EXAMPLES

To test the final coupled bearing-FEM program, comparisons were made against a solution of a short, thick section shaft. Testing of this type of shaft is a test of the coupling technique used to couple the nonlinear bearings to the rotor only. The linear part of the FEM program that models the flexibility of the rotor and stator had previously been verified against known analytical and numerical solutions. The only way to test the nonlinear part of the program was to use the short beam so that all the motion occurred in the bearing. Then direct comparisons could be made to the stand alone lumped mass bearing program with an equivalent loading. Orbit plots were made from the lumped mass bearing program, the nonlinear FEM program and a linear FEM program that used the bearing coefficients from the concentric position only. For the FEM programs a two bearing rotor was modeled. The model for the stand alone bearing program used the same bearing but half the rotor mass and half the imbalance. Figure 2 is a plot of the three orbits together. Each axis is the eccentricity in one of the two planes. Each orbit shows phase markers. The solid orbit line is the orbit in the bearing, the short dashed line is the orbit from the coupled FEM program and the long dashed line is from the linear FEM program. All orbits start off at the concentric position and spiral out to a steady state limit orbit. The bearing and nonlinear FEM orbits show almost exactly the same response. The linear FEM orbit shows a significantly different transient, before settling down to a steady state orbit which is similar to the other two orbits. At this small an eccentricity, this would be expected. Figure 3 is the same systems but now with twice the imbalance applied. The orbits show similar response as in figure 2 except with a larger eccentricity. The linear FEM orbit is a small amount larger than the other two because that type of analysis does not account for the increasing stiffness as the eccentricity increases. This type of diverging deviation shows the requirement for the nonlinear analysis to accurately model the response.

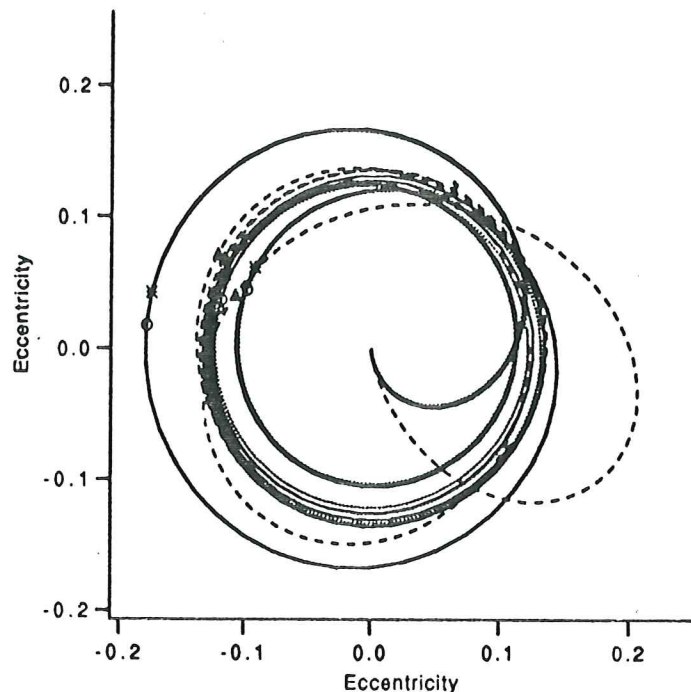


Figure 2. Whirl orbits of lumped mass program, linear FEM program and nonlinear FEM program

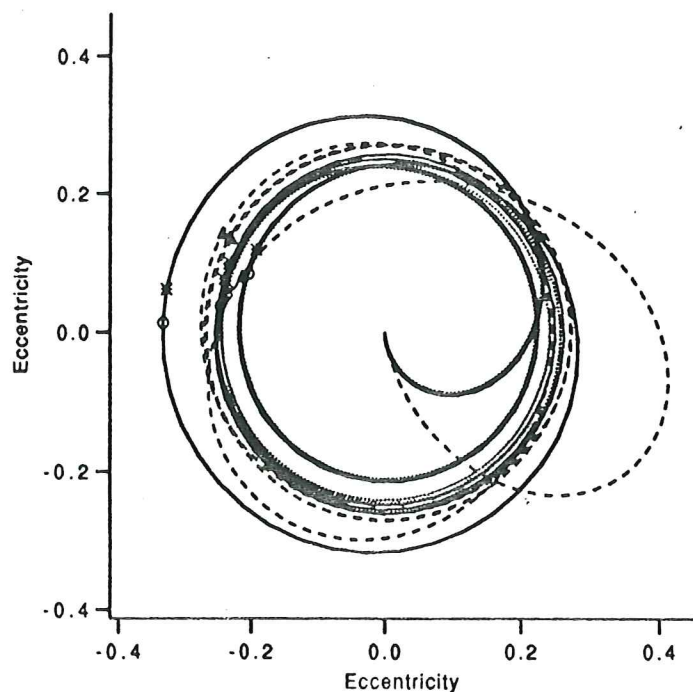


Figure 3. Whirl orbits of lumped mass program, linear FEM program and nonlinear FEM program with twice the imbalance used in figure 2

CONCLUSIONS

A method for incorporating nonlinear hydrostatic bearings into a full model rotordynamic analysis has been shown. The method was employed into a computer program that coupled the nonlinear hydrostatic bearings to a finite element representation of the machine structure. The method shown is completely general. It has the full simplicity of the FEM technique without any restrictions that other solution methods have imposed such as the Transfer Matrix approach. By treating the bearing nonlinearities as forces on the RHS, it avoids reformulating the general M, C and K matrices at every iteration. The program makes no assumptions about the mass assigned to a bearing such as a lumped mass approach requires. It is no more difficult to add in several bearings than it is to include only one. Any number of bearings and bearing geometry's can be used. The bearings can be in series or in parallel. The method shown can be used to model any other kind of bearing as well, provided that a method is available to calculate the stiffness and damping coefficients. The method shown, covered transient response, synchronous response and damped eigenvalue calculation.

REFERENCES

- Adams M. L., 1980, "Non-Linear Dynamics of Flexible Multi-Bearing Rotors," *Journal of Sound and Vibration*, 71(1), 129-144.
- Bathe K., 1982, *Finite Element Procedures in Engineering Analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- Feria Kaiser V., "Dynamic Modeling of Flexible Rotor Shafts Mounted on Hydrodynamic Bearings," Ph.D Dissertation, The University of Wisconsin, Milwaukee, 1987.
- Ghosh M., Majumdar B., "Steady-state and Dynamic Behavior of Multi-Recessed Hybrid Oil Journal Bearings," *Mechanical Engineering Science*, vol 21 No. 5 1979, pp 345-351.
- Greenhill L., Nelson H., "Iterative Determination of Squeeze Film Damper "Eccentricity For Flexible Rotor Systems," *J. Mechanical Design*, vol 104, No. 2 pp 334-338, 1982.
- Hahn E., "Stability and Unbalance Response of Centrally Preloaded Rotors Mounted in Journal and Squeeze Film Bearings," *J. Lub. Technology*, April 1979, vol 103, pp 120-128.
- Headifen R.N., 1991, "Rotordynamic Modeling of Flexible Rotor Machines Supported with Hydrostatic Bearings," Ph.D Dissertation, The University of Texas at Austin, Austin, TX.
- Lund J., Orcutt F., "Calculations and Experiments on the Unbalance Response of a Flexible Rotor," *J. Eng for Power*, Nov 1967 pp 785-796.
- Manifold S., "TXROTOR Users Guide," CEM-UT Internal Report, 1989.
- McLean L. J., 1983, "Unbalance Behavior of Squeeze Film Damped Multi-Mass Flexible Rotor Bearing Systems," *Journal of Lubrication Technology*, vol. 105, pp 22-28.
- Press W, Flannery B, 1986, *Numerical Recipes, The Art of Scientific Computing*, Cambridge University Press, Cambridge.
- Rabinowitz M. D., 1977, "Steady State Performance of Squeeze Film Damper Supported Flexible Rotors," *Journal of Engineering for Power*, October, pp 552-558.
- Rouch K. E., 1980, "Dynamic Reduction in Rotor Dynamics by the Finite Element Method," *Journal of Mechanical Design*, April, vol 102, pp 360-368.
- Rowe W. "Dynamic and Static Properties of Recessed Hydrostatic Journal Bearings by Small Displacement Analysis," *J. Lub. Tech.*, Jan 1980, Vol. 102 pp 71-79.
- Tolk K. "A Modeling Technique for Design and Simulation of Hydrostatic Journal Bearings," Ph. D Dissertation, The University of Texas at Austin, Austin TX May 1980.
- Vance J., *Rotordynamics of Turbomachinery* Texas A&M University, 1988 John Wiley and Sons, New York, New York.
- Vaughn, M., "A Clearance Sensing Hydrostatic Bearing Restrictor for the Homopolar Generator Systems Tester," Ph. D Dissertation, The University of Texas at Austin, Austin TX, August 1989.