

As table 3-21 shows, the greatest effect of monetary sector shocks occurs under model A-3, a model in which investment is insensitive to interest rates. Given a fixed money supply and a level of income determined by the real sector alone, the interest rate is the only clearing variable for the monetary sector; hence, it will have to change sufficiently to offset the monetary shocks. Since there is no change in income, this will not carry part of the responsibility to help the money market to clear, as would be the case when the real sector is sensitive to interest rates, as occurs in the other model specifications. Among those other specifications, for models A-1, A-2 case B and A-4 case B, the effect of monetary shocks are the same, but smaller than in model A-3; the reason for the smaller effect under these three situations lies precisely in the link from the monetary sector to the real sector through interest rates and from the real to the monetary sector through income, which, after the shock, plays a clearing-variable role together with interest rates.

We must now drive our attention to Model A-2, where we have specified two cases: case A where the "money supply" instrument is not the money supply itself but a monetary aggregate that is closer to the "sources" of reserves, insensitive to interest rates and hence easier to be controlled than the money supply. Case B is the usual, where money supply is the instrument. The first implication under case A is the possibility of money supply shocks playing a role in the economic system, in particular, of affecting the endogenous variables such as

income and interest rates. Second, the effect of money demand shocks is smaller under this case than under the alternative case B, due to the interest sensitivity of the money supply, which implies that the money supply now plays a role in helping the money market to clear after any shock.

Model A-5 is also interesting and special because the effect of monetary shocks on interest rates is diminished due to the wealth effect on consumption, which makes the link between monetary and real sectors stronger, causing income to play a greater role in the money market clearing process and therefore taking away the pressure from interest rate movements.

Finally, model A-4 case A is another special case in which the dynamic effects of monetary shocks will offset the initial impact effects. Interest rates may rise, fall or remain the same in the new equilibrium: a positive shock to the demand for money will cause higher interest rates but a lower income level, the latter effect leading to a reduction in price expectations and hence in investment. This in turn leads to reduced interest rates and further income contraction, the process continuing until a new equilibrium locus is reached.

On the other hand, the effect of real sector shocks is given in Table 3-22. The effect under models A-1, A-2 case B and A-4 case B are the same. The effect under A-2 case A, and A-5 is smaller than in the ones above, due to the interest sensitivity of the money supply in the

former and to the interest sensitivity of consumption in the latter. But the relative magnitude of the effect under these two models will depend upon the relative size of $|-c_3(1-b_1)|$ and $|b_0c'_1|$. (See Table 3-22). If the former is greater than the latter then the effect is greater under model A-5 than under A-2 case A. We then have A-4 case A where the dynamic multiplier is larger than the impact multiplier and hence the total effect on interest rates caused by the shock will be greater than in models A-1, A-2 and A-4 case B.

Under model A-3 the magnitude of the effect of the real sector shock is greater than in any other model. We now rank the models according to the magnitude of the effect of each type of shock on interest rates (in ascending order):

Monetary Sector Shocks

A-5 or A-2 case A, A-4, A-2 case B and A-1, A-3

Real Sector Shocks

A-2 case A or A-5, A-1, A-2 case B and A-4, A-3

If we now compare Tables 3-21 and 3-22 we notice that under any model structure the effect of real sector shocks on interest rates will be smaller than that of monetary sector shocks if b_0 is lower than $(1-b_1)$, and viceversa. However, under A-4 case A, the effect of monetary sector shocks is smaller than that of real sector shocks due

to the dynamic effects that are generated through price expectations.

As a final comment we must note that under model A-3, the effect of monetary shocks depends entirely upon the interest sensitivity of the demand for money (c_0), while the effect of real sector shocks also depends upon the marginal propensity to consume and the transactions demand for money coefficient (b_1 and b_0 respectively).

Conclusion: under a money supply policy, the effect of any of the different types of shocks on interest rates depends more on the characteristics of the economy than on the type of shock. Such effect will be lower the more sensitive are spending and the money supply to interest rates.

C. Effect of each type of shock under same model but different procedure.

1. Model A-1

As can be seen from Table 3-23, under model A-1 the effect of a real sector disturbance on income is greater under an interest rate policy than under a money supply policy, while the effect of monetary disturbances is greater under a money supply policy. On the other hand, we cannot establish a general conclusion about the relative effects of any shock on the different monetary variables that become endogenous on each policy procedure. This will depend upon the value of the

coefficients that determine the IS and LM slopes respectively. As long as $|b_0\beta + c_0| < 1$, the interest rate change caused by any type of shock under a money supply policy will be greater than the money supply change caused by the same shock under an interest policy.

2. Model A-2

Under model A-2, the same conclusions apply, but we must note that for case A the money supply is affected by any type of disturbance, under any type of policy, although this effect is lower under a money supply policy. Interest rates, on the other hand, are only affected under a money supply policy, as in model A-1 above. The conclusion about the relative variability of interest rates and money supply under the different policies is somehow changed as compared to case A: Here, if $|b_0\beta - c_3 + c_0| < 1$, then the effect of any shock on interest rates under a money supply policy is greater than the effect on the money supply caused under an interest rate policy; while under the money supply policy money supply volatility is not completely eliminated in this case A, it is smaller than under an interest rate policy.

Table 3-23. Models A-1 and A-2: Effects of Disturbances under Different Monetary Policy Procedures.

Model	Effect of Real Sector Disturbances		Effect of Monetary Sector Disturbances	
	Interest rate policy	Money supply policy	Interest rate policy	Money supply policy
A-1 Effect on Y Effect on Monetary variable	$\frac{\partial Y}{\partial U_e} = \frac{\partial Y}{\partial U_i} = \frac{1}{1-b_1} > 0$	$\frac{\partial Y}{\partial U_c} = \frac{c_0}{(b_0\beta+c_0)} \cdot \frac{1}{(1-b_1)}$	$\frac{\partial Y}{\partial U_{md}} = 0$	$\frac{\partial Y}{\partial U_{md}} = \frac{-\beta}{b_0\beta+c_0} < 0$
	$\frac{\partial M}{\partial U_c} = \frac{\partial M}{\partial U_i} = \frac{b_0}{1-b_1} > 0$	$\frac{\partial M}{\partial U_c} = \frac{-b_0}{(b_0\beta+c_0)} \cdot \frac{1}{(1-b_1)}$ where $\beta = c_1/(1-b_1)$	$\frac{\partial M}{\partial U_{md}} = 1$	$\frac{\partial r}{\partial U_{md}} = \frac{-1}{b_0\beta+c_0} > 0$
A-2	$\frac{\partial Y}{\partial U_c} = \frac{1}{1-b_1} > 0$	Case A (A ₃ *) $\frac{\partial Y}{\partial U_c} = \frac{-(c_3-c_0)}{(1-b_1)(b_0\beta-c_3+c_0)} > 0$ $\frac{\partial MS}{\partial U_c} = \frac{-c_3b_0}{(b_0\beta-c_3+c_0)} \cdot \frac{1}{(1-b_1)}$ $\frac{\partial r}{\partial U_c} = \frac{-b_0}{(b_0\beta-c_3+c_0)} \cdot \frac{1}{1-b_1}$ Case B (MS*) $\frac{\partial Y}{\partial U_c} = \frac{c_0}{(b_0\beta+c_0)(1-b_1)}$ $\frac{\partial r}{\partial U_c} = \frac{-b_0}{(b_0\beta+c_0)(1-b_1)}$	$\frac{\partial Y}{\partial U_{md}} = 0$	Case A (A ₃ *) $\frac{\partial Y}{\partial U_{md}} = \frac{-\beta}{b_0\beta-c_3+c_0} < 0$ $\frac{\partial MS}{\partial U_{md}} = \frac{-c_3}{b_0\beta-c_3+c_0} > 0$ $\frac{\partial r}{\partial U_{md}} = \frac{-1}{b_0\beta-c_3+c_0} > 0$ $\frac{\partial Y}{\partial U_{MS}} = \frac{\beta}{b_0\beta-c_3+c_0} > 0$ $\frac{\partial MS}{\partial U_{MS}} = 1$ $\frac{\partial r}{\partial U_{MS}} = \frac{1}{b_0\beta-c_3+c_0} < 0$ Case B (MS*) $\frac{\partial Y}{\partial U_{md}} = \frac{-\beta}{b_0\beta+c_0} < 0$ $\frac{\partial r}{\partial U_{md}} = \frac{-1}{b_0\beta+c_0} > 0$
	$\frac{\partial MS}{\partial U_c} = \frac{b_0}{1-b_1} > 0$		$\frac{\partial MS}{\partial U_{md}} = 1$	

3. Model A-3

Table 3-24 shows the multipliers under model A-3, where we see that the effect of a real sector disturbance on income is the same under both policy procedures. However, this type of disturbance would cause an interest rate variation of a greater magnitude under a money supply policy, than the variation there would be in the money supply under an interest rate policy.

On the other hand, although the effect of monetary disturbances on income is zero under either procedure, the variability they cause in interest rate when a money supply policy is followed is greater than the variability caused in the money supply when an interest rate policy is used.

4. Model A-4

The multipliers for model A-4 are also shown in Table 3-24, and Table 3-25 uses such multipliers to obtain comparative results of the effects of alternative policies. They show that if there is revision, real sector disturbances will have a greater impact on income under an interest rate policy than under a money supply policy, while monetary disturbances will affect income only if a money supply policy is followed. We can also conclude that as long as $|b_0\beta_2 + c_0| < 1$, any type of shock will cause an interest rate variation under a money supply policy of a greater magnitude

Table 3-24. Effects of Disturbances under Different Monetary Policy Procedures.

	Interest rate policy	Money supply policy	Interest rate policy	Money supply policy
A-3	$\frac{\partial Y}{\partial U_1} = \frac{1}{1-b_1}$ $\frac{\partial MS}{\partial U_C} = \frac{b_0}{1-b_1}$	$\frac{\partial Y}{\partial U_C} = \frac{1}{1-b_1}$ $\frac{\partial r}{\partial U_C} = \frac{-b_0}{c_0(1-b_1)}$	$\frac{\partial Y}{\partial U_{md}} = 0$ $\frac{\partial MS}{\partial U_{md}} = 1$	$\frac{\partial Y}{\partial U_{md}} = 0$ $\frac{\partial r}{\partial U_{md}} = -\frac{1}{c_0}$
A-4 revision	$\frac{\partial Y}{\partial U_C} = \frac{1}{1-b_1}$ $\frac{\partial MS}{\partial U_C} = \frac{b_0}{1-b_1}$	$\frac{\partial Y}{\partial U_C} = \frac{c_0}{(b_0\beta_2+c_0)(1-b_1)} > 0$ $\frac{\partial r}{\partial U_C} = \frac{-b_0}{(b_0\beta_2+c_0)(1-b_1)} > 0$	$\frac{\partial Y}{\partial U_{md}} = 0$ $\frac{\partial MS}{\partial U_{md}} = 1$	$\frac{\partial Y}{\partial U_{md}} = \frac{-\beta_2}{b_0\beta_2+c_0} < 0$ $\frac{\partial r}{\partial U_{md}} = \frac{-1}{b_0\beta_2+c_0} > 0 > 1$

Table 3-24. (continued)

	Interest rate policy	Money supply policy	Interest rate policy	Money supply policy
	Case A	Case A	Case A	Case A
no	$\frac{\partial Y_i}{\partial c_{ij}} = \frac{1}{1-b_1} \left[1 - \frac{\beta_2 A_{ij}^+}{Y_{i,j-1}} + \right.$	$\frac{\partial Y_i}{\partial c_{ij}} = \frac{1}{1-b_1} \left[\frac{c_0}{(b_0 \beta_2 + c_0)} - \frac{c_0^2 \beta_2 A_{ij}}{(b_0 \beta_2 + c_0)^2 Y_{i,j-1}} + \dots \right.$	$\frac{\partial Y_i}{\partial c_{ij}} = 0$	$\frac{\partial Y_i}{\partial c_{ij}} = \frac{-\beta_2}{b_0 \beta_2 + c_0} + \frac{c_0 \beta_2^2 A_{ij}}{(b_0 \beta_2 + c_0)^2}$
Revision	$+ \dots + (-1)^{k_{\beta_2} k_{A_{ij}}} \frac{A_{ij} A_{ij+1} \dots A_{ij+k+1}}{Y_{i,j-1} Y_{ij} \dots Y_{ij+k-2}} \Big] + \dots +$	$\frac{(-1)^{k_{c_0} k_{\beta_2} k_{A_{ij}}} \beta_2 A_{ij} A_{ij+1} \dots A_{ij+k+1}}{(b_0 \beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{ij+k-2}} \Big] + \dots +$	$\frac{\partial MS}{\partial c_{ij}} = 1$	$+ \dots + \frac{(-1)^{k_{c_0} k_{\beta_2} k_{A_{ij}}} c_0 \beta_2 A_{ij} A_{ij+1} \dots A_{ij+k-1}}{(b_0 \beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{ij+k-2}} < 0$
	$\frac{\partial MS}{\partial c_{ij}} = b_0 \left[\frac{\partial Y_i}{\partial c_{ij}} \right] > b_0$	$\frac{\partial r}{\partial c_{ij}} = \frac{1}{1-b_1} \left[\frac{-b_0}{b_0 \beta_2 + c_0} + \frac{b_0 c_0 \beta_2 A_{ij}}{(b_0 \beta_2 + c_0)^2 Y_{i,j-1}} \right.$		$\frac{\Delta r_i}{\Delta U} = \frac{-1}{(b_0 \beta_2 + c_0)} - \frac{b_0 \beta_2^2}{(b_0 \beta_2 + c_0)^2} + \dots +$
		$\frac{(-1)^{k_{c_0} k_{\beta_2} k_{A_{ij}}} c_0 \beta_2 A_{ij} A_{ij+1} \dots A_{ij+k+1}}{(b_0 \beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{ij+k-2}} \Big]$		$\frac{(-1)^{k_{b_0} k_{c_0} k_{\beta_2} k_{A_{ij}}} A_{ij} A_{ij+1} \dots A_{ij+k-1}}{(b_0 \beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{ij+k-2}} > 0$

Table 3-24. (continued)

Interest rate policy		Money supply policy		Interest rate policy		Money supply policy	
Case B		Case B		Case B		Case B	
$\frac{\partial Y_i}{\partial u_{ij}} = \frac{1}{1-b_1} \left[1 - \frac{\beta_2 W_{ij} b_0}{M_{i,j-1}} \right]$ $+ \dots + \frac{(-1)^k \beta_2 b_0^k W_{ij}^{k-1} \dots W_{i,j+k-1}}{(M_{i,j-1} \dots M_{i,j+k-1})}$ $\frac{\partial MS_i}{\partial u_{ij}} = b_0 \left(\frac{\partial Y_i}{\partial u_{ij}} \right) > b_0$ $= \frac{b_0}{1-b_1} \left[1 - \frac{\beta_2 W_{ij} b_0}{M_{i,j-1}} + \dots \right]$		$\frac{\partial Y_i}{\partial u_{ij}} = \frac{c_0}{(b_0 \beta_2 + c_0)(1-b_1)} > 0$ $\frac{\partial r_i}{\partial u_{ij}} = \frac{-b_0}{(b_0 \beta_2 + c_0)(1-b_1)} > 0$		$\frac{\partial Y_i}{\partial u_{ij}} = 0 \frac{\beta_2 W_{ij}}{M_{i,j-1}} + \frac{\beta_2^2 b_0 W_{ij}^{k-1} \dots W_{i,j+k-1}}{M_{i,j-1} \dots M_{i,j+k-1}}$ $+ \dots + \frac{(-1)^k \beta_2 b_0^k W_{ij}^{k-1} \dots W_{i,j+k-1}}{M_{i,j-1} \dots M_{i,j+k-1}} > 0$ $\frac{\partial MS_i}{\partial u_{ij}} = 1 + b_0 \left[\frac{\partial Y_i}{\partial u_{ij}} \right] > 1$		$\frac{\partial Y_i}{\partial u_{ij}} = \frac{-\beta_2}{b_0 \beta_2 + c_0} < 0$ $\frac{\partial r_i}{\partial u_{ij}} = \frac{-1}{b_0 \beta_2 + c_0} > 0$ (and > 1 if $b_0 \beta_2 + c_0 < 1$)	

than the money supply variation caused under an interest rate policy.

We now turn to the "no revision" cases. The first conclusion we draw is that the total effect (including dynamic effects) of a real sector disturbance is greater under an interest rate policy than under a money supply policy in both cases A and B. With respect to the effect of this disturbance upon monetary variables, the dynamic effects do not show a clear-cut conclusion. The short-term (one period) effect is the same as in the revision case: namely, that the interest rate variation under a money supply policy is of a greater magnitude than the money supply variation under an interest rate policy. However, when the dynamic effects are added, the conclusions may change. If price expectations are determined by past income growth rates (case A), the variation in interest rates under a money supply policy can be smaller, equal or greater than the variation of money supply under an interest rate policy.⁸⁶

If, on the other hand, price expectations depend upon past money growth rates, the short term effect of real sector shocks on interest rates under a money supply policy is greater than such effect on money supply under an interest rate policy. However, in the latter case dynamic effects are present while in the former they are not, and these might

Table 3-25. Model: A-4.

Effects of Real Sector Shocks under an Interest Rate (IR) Policy Relative to a Money Supply (MS) Policy		Effects of Monetary Shocks under an Interest Rate (IR) Policy Relative to a Money Supply (MS) Policy.	
On Income (Y)		On Income (Y)	
On Monetary variable (r & M)		On Monetary variables	
Revision	$\frac{\Delta Y}{IR_{pol}} > \frac{\Delta Y}{MS_{pol}}$	$\frac{\Delta r}{MS_{pol}} > \frac{\Delta M}{IR_{pol}}$	$\frac{\Delta r}{MS_{pol}} > \frac{\Delta M}{IR_{pol}}$
No revision			
case A	$\frac{\Delta Y}{IR_{pol}} > \frac{\Delta Y}{MS_{pol}}$	$\frac{\Delta Y}{IR_{pol}} < \frac{\Delta Y}{MS_{pol}}$	$\frac{\Delta Y}{IR_{pol}} < \frac{\Delta Y}{MS_{pol}}$
$P_e = f(LY)$	(dynamic included)	$\frac{\Delta r}{MS_{pol}} > \frac{\Delta M}{IR_{pol}}$	$\frac{\Delta r}{MS_{pol}} > \frac{\Delta M}{IR_{pol}}$
case B	$\frac{\Delta Y}{IR_{pol}} > \frac{\Delta Y}{MS_{pol}}$	$\frac{\Delta Y}{IR_{pol}} < \frac{\Delta Y}{MS_{pol}}$	$\frac{\Delta Y}{IR_{pol}} < \frac{\Delta Y}{MS_{pol}}$
$P_e = f(LM)$	(including dynamic)	$\frac{\Delta r}{MS_{pol}} > \frac{\Delta M}{IR_{pol}}$	$\frac{\Delta r}{MS_{pol}} > \frac{\Delta M}{IR_{pol}}$
(L-lag operator)			

NOTE: * = dynamic effects present
IR_{pol} = Interest rate policy
MS_{pol} = Money supply policy
Δr_{pol} = change in r under a money supply policy

cause the total effect on the money supply under the interest rate policy to be equal, or even greater than the effect on interest rates under a money supply policy.

Now, with respect to monetary disturbances, under the "no revision" assumption, the same conclusions we had for the "revision case" will hold with respect to the effect on income if we also assume that expectations depend upon past income growth rates.

But, if price expectations are a function of past money growth, under an interest rate policy there are only dynamic effects on income but no impact effects will occur.

If, on the other hand, a money supply policy is followed, only

growth, the effect of the monetary shock on interest rates

under a money supply policy (impact effect) will initially be

86. The key determinant of this is the value of $\frac{c_o^k}{(b_o\beta/[2]+c_o)^{k+1}}$ which appears on each of the terms that account for the dynamic effects. For the short term effect, since $=0$ it becomes $\frac{1}{b_o\beta_2+c_o}$ which is greater than 1 as long as $(b_o\beta_2+c_o)<1$. On the other hand, $\frac{c_o}{b_o\beta_2+c_o}<1$, and if we square both sides and divide both sides by c_o we get that

$\frac{c_o}{(b_o\beta_2+c_o)^2} < \frac{1}{c_o}$ where $\frac{1}{c_o}$ is greater than one. Hence, the term

$\frac{c_o}{(b_o\beta_2+c_o)^{k+1}}$ can be smaller,

equal or greater than 1; and this is what will determine if the dynamic effect of the shock on interest rates under a money supply policy is different from the dynamic effects on the money supply under an interest rate policy.

impact effects will occur, and these will be in the opposite direction of the dynamic effects under the interest rate policy (e.g., reducing income if U_{md} is positive, in the former, versus rising income if U_{md} is positive, in the latter). In addition, assuming that price expectations depend upon past income growth, the effect on interest rates if a money supply policy is followed can now be greater, equal or smaller than the effect on money supply when an interest rate is followed, because the dynamic effects in the former move in the opposite direction of the impact effects and may even cancel these out.

Lastly, if price expectations depend upon past money growth, the effect of the monetary shock on interest rates under a money supply policy (impact effect) will initially be greater than on the money supply under an interest rate policy. However, in the latter there are dynamic effects that could cause the total money supply variation to be the same or even greater than the variation caused to interest rates under the money supply policy.

5. Model A-5

We now proceed to draw the general conclusions for Model A-5. Tables 3-26 and 3-27 show the respective results. In this case, real sector shocks will have a greater impact on

Table 3-26. Model A-5: Effects of Disturbances under Different Monetary Policy Procedures.

Model	Effect of Real Sector Disturbances		Effect of Monetary Sector Disturbances	
	Interest rate policy	Money supply policy	Interest rate policy	Money supply policy
A-5	Case 1		Case 1	
	Effect on Y	$\frac{\partial Y}{\partial U_C} = \frac{1}{1-b_1} > 0$	$\frac{\partial Y}{\partial U_C} = \frac{c_0}{b_0(c'_1+c_1)+c_0(1-b_1)} > 0$	$\frac{\partial Y}{\partial U_{md}} = \frac{c'_1+c_1}{b_0(c'_1+c_1)+c_0(1-b_1)} < 0$
	Effect on C	$\frac{\partial MS}{\partial U_C} = \frac{b_0}{1-b_1} > 0$	$\frac{\partial r}{\partial U_C} = \frac{-b_0}{b_0(c'_1+c_1)+c_0(1-b_1)} > 0$	$\frac{\partial r}{\partial U_{md}} = \frac{-(1-b_1)}{b_0(c'_1+c_1)+c_0(1-b_1)} > 0$
Monetary variable		$\frac{\Delta GD}{\Delta U_C} = \frac{-b_0}{1-b_1}$	GD exogenous	GD exogenous
	Case 2		Case 2	
	Effect on Y	$\frac{\partial Y}{\partial U_C} = \frac{1}{1-b_1-b_0l} > 0$	$\frac{\partial Y}{\partial U_{md}} = \frac{1}{1-b_1-b_0l}$	$\frac{\partial Y}{\partial U_{md}} = \frac{-(c'_1+c_1)}{(c'_1+c_1)b_0+c_0(1-b_1)}$
Effect on C		$\frac{\partial MS}{\partial U_C} = \frac{b_0}{1-b_1-b_0l} > 0$	$\frac{\partial MS}{\partial U_{md}} = \frac{(1-b_1)}{1-b_1-b_0l}$	$\frac{\partial r}{\partial U_{md}} = \frac{-(1-b_1)}{b_0(c'_1+c_1)+c_0(1-b_1)}$
		$\frac{\partial GD}{\partial U_C} = \frac{-b_0}{1-b_1-b_0l} < 0$	$\frac{\partial GD}{\partial U_{md}} = \frac{-(1-b_1)}{1-b_1-b_0l}$	GD exogenous

income under an interest rate policy than under a money supply policy, assuming that net worth is defined as money supply plus government debt plus the value of capital stock (Case 1). If, on the other hand, net worth only includes money supply and capital stock, there is not a definite conclusion in this respect. The same inconclusive answer results for the effect of real sector shocks on the monetary variables under each policy, and this arises due to the different structural coefficients that are included in the respective multipliers, (see Table 3-27) and to the multiplier formula itself.

However, we must notice that under an interest rate policy, government debt becomes an endogenous variable, also affected by shocks.

With respect to monetary shocks, as long as net worth is defined as $MS+GD+K$, these do not affect income under an interest rate policy while they do affect income under a money supply policy. Also, the effect on interest rates under a money supply policy might be greater, equal or smaller than the effect on the money supply under an interest rate policy.

For the alternative case, in which net worth equals $MS+K$ only, monetary shocks do in fact affect income under either policy, but we cannot say a priori under which policy such impact will be greater. This same conclusion stands for the effects on the monetary variables under the alternative

Table 3-27. Model A-5: Comparison of the Effects of Shocks under different Policies.

	Effects of Real Sector Shocks under an Interest Rate (IR) Policy Relative to a Money Supply (M) Policy		Effects of Monetary Shocks under an Interest Rate (IR) Policy Relative to a Money Supply (M) Policy.	
	On income (Y)	On monetary variable (r & M)	On income (Y)	On monetary variables
Case 1 NW=MS+GD+K	$\Delta Y_{pol} > \Delta Y_{MS_{pol}}$	$\Delta r_{MS_{pol}} > \Delta r_{IR_{pol}}$	$\Delta Y_{IR_{pol}} < \Delta Y_{MS_{pol}}$	$\Delta r_{MS_{pol}} > \Delta r_{IR_{pol}}$
Case 2 NW=MS+K	$\Delta Y_{IR_{pol}} > \Delta Y_{MS_{pol}}$	$\Delta r_{MS_{pol}} > \Delta r_{IR_{pol}}$	$\Delta Y_{IR_{pol}} > \Delta Y_{MS_{pol}}$	$\Delta r_{MS_{pol}} > \Delta r_{IR_{pol}}$

Source: Table 3-26.

policies. Again, the relative impact of monetary shocks will depend upon the magnitude of the structural coefficients that make up the multipliers.

We therefore arrive to the following conclusions for model A-5:

1. Under an interest rate policy, income cannot be affected by monetary shocks if net worth equals $MS+GD+K$, but it can if net worth is defined only as $MS+K$.
2. Under an interest rate policy real sector shocks will affect income regardless of how we define net worth, although the effect will be greater if GD is not included in the definition $MS+K$ than if it is.
3. Under an interest rate policy, the impact of monetary shocks on income is lower than the impact of real sector shocks, but if net worth is defined as $MS+K$, this difference is smaller the more sensitive consumption is to changes in net worth.
4. Under a money supply policy, the alternative definitions of net worth make no difference with respect to the effects of any type of shocks on income and on the monetary variables. However, under an interest rate policy the effects of any shock are greater under the assumption that net worth is only $MS+K$.

Finally, model A-5 also allowed us to analyze the effects of the government budgetary changes under alternative monetary policy procedures, with the following conclusions:

1. When society views government debt as part of their wealth, past deficits will have an expansive effect on current income and cause interest rates to rise if a money supply is followed, or current money supply to expand if an interest rate policy is followed.
2. Independently of the society's concept of wealth, current budget changes will affect income and this effect will be stronger under an interest rate policy than under a money supply policy. However, under an interest rate policy the effect on income will be stronger if society does not view government debt as part of their wealth, as long as $\Delta G = \Delta T$, while if $\Delta G > \Delta T$ we cannot state with certainty that this will be the case.
3. If a money supply policy is followed, the effect on income will be the same independently of the society's concept of wealth if $\Delta G = \Delta T$; but if there is an unbalanced change in the budget, the effect on income is greater if society views government debt as part of their wealth.

CHAPTER 4

THEORETICAL ANALYSIS OF THE DISAGGREGATED MODELS

INTRODUCTION

The purpose of this chapter is to describe the theoretical analysis for the disaggregated models as described in Chapter 2. As we have said before, the special characteristic of these models is that the monetary sector has been highly decomposed in order to explicitly consider sources and uses of reserves.

Table 4-1 describes the model versions that will be considered subjects for the theoretical analysis of this chapter.

Table 4-1. Proposed Disaggregated Models D-1 through D-5 as Compared to A-1.

Model (identification)	Characteristics			
	Monetary Sector		Real Sector	
A-1	Money supply Exogenous	Money demand Function of income and interest rates	Investment Function of interest rates	Consumption Function of income
D-1	Disaggregated, exogenous	Demand for currency equals demand for deposits, functions of income and interest rates.	same as A-1	same as A-1
D-2	Disaggregated, exogenous	Demand for currency and for deposits interest-inelastic	same as A-1	same as A-1
D-3	Disaggregated. Borrowings and excess reserves, interest-elastic	same as D-1	same as A-1	same as A-1
D-4	same as D-1	Demand for currency and demand for deposits as functions of income and interest rates. Inclusion of interest bearing demand deposits.	same as A-1	same as A-1
D-5	same as D-3	same as D-4	same as A-1	same as A-1

Source: Chapter 2.

I. IMPLICATIONS OF EACH POLICY PROCEDURE.

A. MODEL D-1

This model introduces a disaggregated money supply in order to explicitly consider its sources. The purpose of this detailed specification is to be able to consider some of the sources of the monetary base as possible instruments of monetary policy, since these are more closely controllable. Furthermore, it will also be possible to specify some of those money sources as behavioral relationships if that is thought to be the case.

Tables 4-2 and 4-3 specify the model in mathematical form. As Table 4-2 shows, the model distinguishes two types of demand for money functions: Currency (CUR) and Demand Deposits (D). This is needed in order to properly specify the determinants of Required Reserves (RR), which are part of the demand for bank reserves, since $RR = rrD$ (where rr = required reserves ratio). Furthermore, the presence of disturbances to the demand for money may differently affect the monetary sector equilibrium depending upon the source of such disturbance, that is, whether it is a disturbance to the demand for deposits or to the demand for currency.

Table 4-2. Model D-1: Specification of Structural Equations

Real Sector:

$$Y = C + I + G$$

$$C = A_1 + b_1 Y + U_c$$

$$I = A_2 + c_1 r + U_i$$

Monetary Sector:

$$MS = CUR + D$$

$$CUR = c_2 r + b_2 Y + U_{CUR}$$

$$D = c_3 r + b_3 Y + U_{dep}$$

$$RT = RU + BOR$$

$$RT = RR + ER$$

$$RR = rr D$$

$$MB = RT + CUR$$

$$RU = Sma + Flt + Tco + CUR - Tch - Deg$$

where:

Y = real income

C = real consumption

I = real investment

G = real government expenditures

MS = money supply

CUR = currency

D = demand deposits

RT = total reserves

RU = unborrowed reserves

BOR = borrowings

ER = excess reserves

MB = monetary base

Sma = Fed's security portfolio

Flt = float

rr = required reserves ratio

Tco = treasury currency outstanding

Tch = treasury cash holdings

Deg = government deposits at Fed or monetary authority

Assumptions

$$b_1 > 0$$

$$b_2 > 0$$

$$b_3 > 0$$

$$A_1, A_2 \geq 0$$

$$c_1 < 0$$

$$c_2 < 0$$

$$c_3 < 0$$

$$0 < rr < 1$$

On the other hand, Table 4-3 shows the IS and LM equations. The latter is different from the one obtained in the aggregated model of chapter 3 in that it now shows the monetary base and excess reserves explicitly as part of this equilibrium relationship.

1. Interest Rate Policy

Determination of the Instrument Value

Given the target income level $Y=Y^*$, the instrument value r^* is determined by simultaneously solving the IS and LM equations. However, as has been the case before, in this situation r^* is solely determined by the IS curve, as :

$$r^* = \frac{Y^* - \alpha}{\beta}$$

where disturbances have been assumed to be zero.

With this r^* in mind, open market operations will be performed so as to keep interest rates as close as possible to such value. In the present model specification it is the variable S_{ma} that will move in order to keep r at r^* ; that is, S_{ma} becomes the endogenous variable under this policy procedure, since it directly reflects open market operations activity.

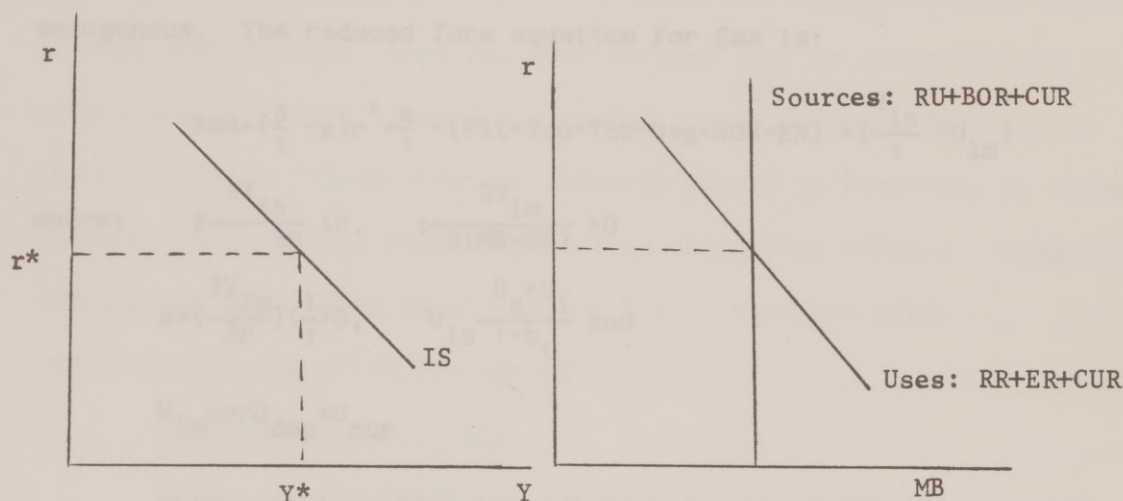
Figure 4-1 gives a graphical representation of the nature of this policy under the present model. Part A shows that the instrument value for interest rates, given Y^* , is determined by the IS; part B shows the equilibrium between sources and uses of the monetary base.

Table 4-3. Model D-1: IS and LM Relationships.

IS	$Y = \alpha + \beta r + U_{is}$ where: $\alpha = \left(\frac{A_1 + A_2 + G}{1 - b_1} \right) > 0$ $\beta = \left(\frac{c_1}{1 - b_1} \right) < 0$ $U_{is} = \left(\frac{U_c + U_i}{1 - b_1} \right)$
LM	$Y = \tau(MB-ER) + \tau_p r - \tau U_{LM}$ where: $\tau = \left(\frac{1}{rrb_3 + b_2} \right) > 0$ $\rho = -(rrc_3 + c_2) > 0$ $U_{LM} = rr U_{dep} + U_{cur}$ MB-ER=Sma+Flt+Tco-Tch-Deg-ER+BOR

Figure 4-1

Model D-1: Determination of the Interest Rate Value r^*
and Achievement of such Value through Open Market Operations



A. IS and determination of r^*

B. Sources and Uses of the
Monetary Base

NOTE: The vertical line representing "sources" in diagram B will move right or left such that it meets the "uses" function at r^* ; this is what goes on under the present policy.

The "sources", represented by the vertical line, must move such that it equals the "uses" relationship at r^* .

Implications of this Procedure

As was said before, under the present model this procedure implies that open market operations will become a function of the deviation of r from r^* . As a consequence S_{ma} , the Fed's Security Portfolio, becomes endogenously determined and hence affected by the model's parameters, autonomous variables and disturbances.

Furthermore, since it is part of the monetary base, this also becomes endogenous. The reduced form equation for S_{ma} is:

$$S_{ma} = \left(\frac{\beta}{\tau} - \rho \right) r^* + \frac{\alpha}{\tau} - (Flt + Tco - Tch - Deg + BOR - ER) + \left(\frac{U_{is}}{\tau} + U_{lm} \right)$$

where: $\beta = \frac{\partial Y_{is}}{\partial r} < 0, \quad \tau = \frac{\partial Y_{lm}}{\partial (MB - ER)} > 0$

$$\rho = \left(\frac{\partial Y_{lm}}{\partial r} \right) \left(\frac{1}{\tau} \right) > 0, \quad U_{is} = \frac{U_c + U_i}{1 - b_1} \text{ and}$$

$$U_{lm} = rrU_{dep} + U_{cur}$$

This equation shows that disturbances on either the real or the monetary sector affect the extent of open market operations and hence the level of S_{ma} . Similarly, S_{ma} is also affected by the IS and LM parameters and by the other sources and uses of the monetary base, as the equation above shows. Therefore, variations of S_{ma} will be caused by changes in any of those sources. For example, a sudden increase in Float (Flt) will lead to offsetting open market operations (hence

lowering Sma) so as to keep interest rates from deviating away from

According to our specifications, under the present case, authorities will observe changes in r and try to offset them, independently of the source of such change (which may be any of the elements in the Sma equation above). Then, if a particular change in r is due to a temporary change of an element affecting r (since authorities only see it as an interest rate change which must be offset), policy will be directed to change Sma in the needed direction, although such change might need to be reversed soon. If these temporal changes are repeated, they may lead to some over or undershooting of interest rate target, when the temporal change is going back to its permanent level (since when the temporal change is reversed, it works together with policy, thus accentuating the policy effect). Temporal changes may also cause volatility of Sma if they are quick (short-lived) but often (recurrent).

Reduced Form of Income

Since the instrument of monetary policy is the interest rate, it follows that under our model's specifications, it continues to hold (as in the aggregated mod) that the IS equation becomes the income reduced form equation:

$$Y = \alpha + \beta r^* + U_{is}$$

As before, it attaches any deviations of income from target to disturbances occurring in the real sector (U_{is}) and to parameter changes also in the real sector (α or β).

2. Security Portfolio Policy

Determination of the Instrument value

Given $Y=Y^*$, the instrument value Sma^* is established by estimating r from the IS assuming $U_{is}=0$ and then substituting such r into the LM. The resultant value for Sma is the following:⁸⁷

$$Sma^* = Y^* \left(\frac{1}{\tau} - \frac{\rho}{\beta} \right) - Flt + Tco - Tch - Deg - ER + \frac{\tau \rho \alpha}{\beta}$$

Implications of this Policy Procedure

Since within this procedure authorities will try to keep Sma at Sma^* , it follows that equilibrium between the real and monetary sectors will be attained by interest rates fluctuating as disturbances and/or structural changes occur. Therefore, interest rates become endogenous with the following reduced form equation:

$$r = \left(\frac{1}{\beta - \tau \rho} \right) (\tau Sma^* + \tau (Flt + Tco - Tch - Deg) + \tau (BOR - ER) - \alpha U_{is} - \tau U_{lm})$$

87. For derivation see Appendix. For the meaning of the coefficients see Table 4-3.

This equation shows that interest rates are affected by disturbances on both sectors, given the present policy procedure. 88

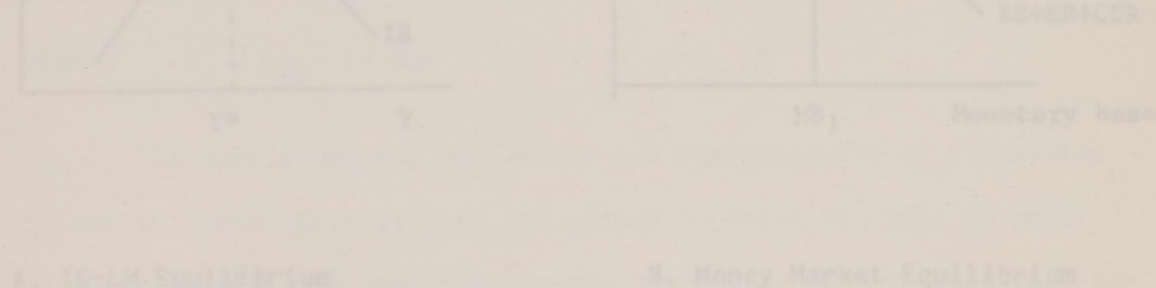
Reduced Form of Income

The reduced form equation is:

$$Y = \left(\frac{\tau}{\beta - \tau\rho} \right) (\beta Sma^* + \beta (Flt + Tco - Tch - Deg) + \beta (BOR - ER) - \rho\alpha - \rho U_{is} + \beta U_{lm})$$

Which shows that income is affected by the same elements that interest rates are. However, the direction of the effect of the monetary elements appearing in this equation is opposite to the direction of their effect on interest rates.

Figure 4-2 shows the implication of the present policy. In Part A, given Y^* , r is estimated and as $\bar{r}$ and Y^* are substituted into the LM, Sma^* is obtained. Part B shows the monetary base sources and uses equilibrium, which will be attained by movements in r , since Sma is fixed.



NOTE: Sma^* is determined so as to be consistent with Y^* . Then, interest rates would move to clear the markets if any changes occur (for example, 88. For derivations see Appendix. For meaning of particular coefficients see Tables 4-2 and 4-3.

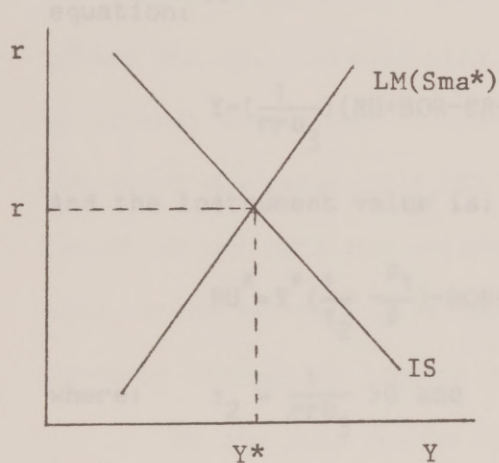
3. Borrowed Reserves Policy

Determination of the Instrument Value

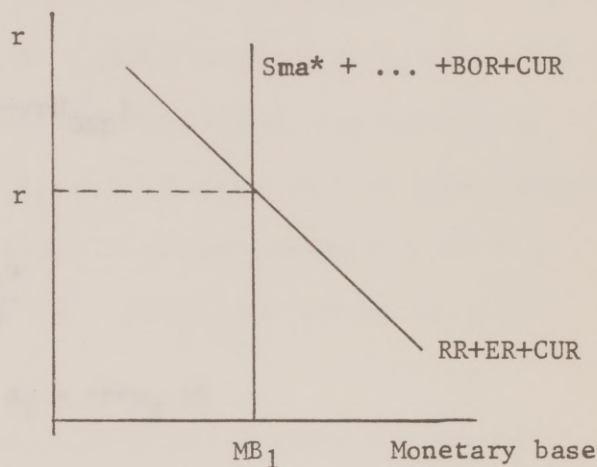
Figure 4-2

Model D-1, Sma Policy.

Determination of the Sma Value and Policy Implications.



A. IS-LM Equilibrium



B. Money Market Equilibrium

NOTE: Sma^* is determined so as to be consistent with Y^* . Then, interest rates would serve to clear the markets if any changes occur (for example, disturbances, etc.).

See derivation in Appendix.

3. Unborrowed Reserves Policy

Determination of the Instrument Value

Given $Y=Y^*$, the instrument value RU^* is established, as in the Sma^* case, estimating r from the IS, substituting it into the LM and then solving for the RU that will yield Y^* , given the model's structure.

However, the LM will now be different than under the previous policies, since the sources of the monetary base will be disclosed in a different way, and hence the relationship between monetary variables and income will be different. The LM is now given by the following equation:⁸⁹

$$Y = \left(\frac{1}{rrb_3} \right) (RU + BOR - ER - rrc_3 r - rrU_{dep})$$

And the instrument value is:

$$RU^* = Y^* \left(\frac{1}{\tau_2} - \frac{\rho_1}{\beta} \right) - BOR + ER + \frac{\rho_1 \alpha}{\beta}$$

where: $\tau_2 = \frac{1}{rrb_3} > 0$ and $\rho_1 = -rrc_3 > 0$

In such equation, BOR and ER are the estimated or forecasted values of these variables for the period in which RU^* will be kept; also, as before, the coefficients are assumed to be estimated in order

89. See derivation In Appendix.

to be able to determine RU^* .

Implications of this Procedure

The first implication of this procedure refers to the LM curve, which differs from that of the two policies considered previously.

Table 4-4 shows the LM equations under those policies and the present one. The main difference between them has to do with the currency equation coefficients and the error term (i.e., c_2 , b_2 and U_{cur}), since they do not appear in the LM under the RU^* policy. The reason for this is that under this policy any movements in the currency equation will be offset by movements in unborrowed reserves because currency changes affect unborrowed reserves. At the beginning of the policy period, currency will start at a given level, a level that will be already considered in determining RU^* ; from there, any changes in CUR, which may occur if r , Y , U_{cur} , c_2 or b_2 change, will be immediately cancelled out by a corresponding change in another element of RU (i.e., Sma), so as to keep RU at the level RU^* . Hence, the effect of such currency changes on income is zero.

We can also notice that the relationship between the exogenous monetary variables and income in the LM, or "partial income multiplier"⁹⁰ of those variables, are greater in case unborrowed reserves are used as the instrument (i.e., $\tau_2 > \tau_1$). The explanation is found in the mechanics of the present policy procedure: an expansive autonomous change in reserves, such as an increase in borrowings, will lead to higher income which itself will cause an increase in the demand

Table 4-4. Model D-1: LM Equations under the r^* , Sma and RU^* Policies.

Instrument used:	LM Equations
r	$Y = \tau(MB-ER) + \tau \rho r^* - \tau[\tau rrU_{dep}^* + U_{cur}]$
Sma	$Y = \tau Sma^* + \tau(Flt + Tco + BOR - Tch - Deg - ER) + \tau \rho r - \tau(rrU_{dep}^* + U_{cur})$
RU	$Y = \tau_2 RU^* + \tau_2(BOR-ER) + \tau_2 \rho_1 r - \tau_2(rrU_{dep})$ where: $\tau = \left(\frac{1}{rrb_3 + b_2} \right) > 0$ $\tau_2 = (1/rrb_3) > 0$ $\rho = -(rrc_3 + c_2) > 0$ $\rho_1 = -rrc_3 > 0$ $\tau_2 > \tau \text{ and } \rho > \rho_1$

Source: Appendix and Table 4-1.

for currency. However, as currency increases unborrowed reserves fall⁹¹ and so policy is directed to increase RU until RU^* is restored, offsetting the depressive effects of the increased currency demand. Hence any autonomous change in reserves will lead to a further (policy-caused) change in reserves which implies, ceteris paribus, that the income level necessary to keep the money market in equilibrium is even larger (larger LM shift). Therefore the partial income multiplier of reserves is greater than under previous policies. Figure 4-3 represents the LM shifts due to an increase in borrowings, under the Sma^* and RU^* policies.

Under the RU policy, if any of the elements that affect RU changes, open market operations will be performed to bring RU back to RU^* . This causes Sma to be endogenously determined by such elements and their determinants. We know that:

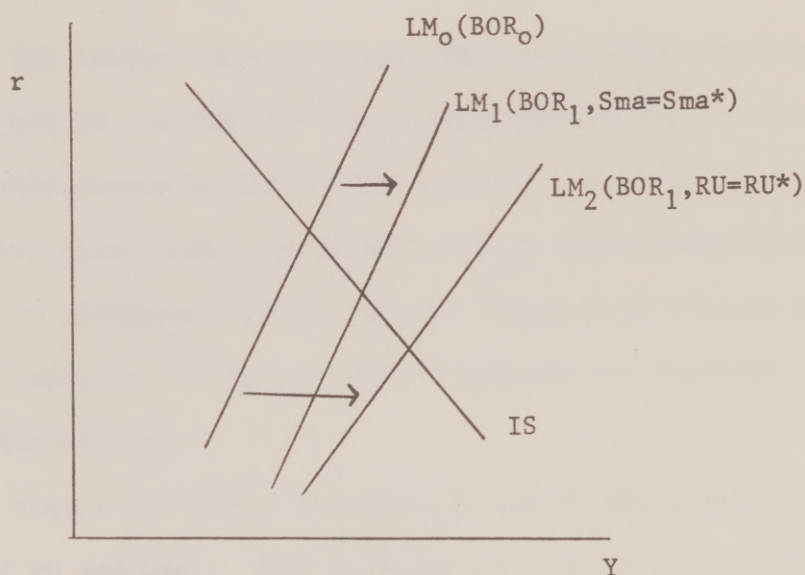
$$RU = Sma + Flt + Tco - CUR - Tch - Deg$$

$$\text{Hence: } \Delta RU = \Delta Sma + \Delta Flt + \Delta Tco - \Delta CUR - \Delta Tch - \Delta Deg$$

90. I have called it "partial income multiplier" because the "total income multiplier" should be the relationship between the monetary variables and equilibrium (IS-LM) income, and not only between the monetary variables and the LM income, which is the one I am now analyzing.

91. Recall that: $RU = Sma + Flt + Tco - CUR + Tch - Deg$

Figure 4-3
 Model D-1: Effects* of an Increase in Borrowings under
 the Sma and RU^* Policies.



LM_1 : LM after increase BOR , given Sma^* policy.

LM_2 : LM after increase in BOR , given RU^* policy.

But if $RU=RU^*$, and since authorities use open market operations as their way to control RU , Sma will change as there are any autonomous or behavioral changes in any of the elements that compose unborrowed reserves. Therefore :

$$\Delta Sma = -\Delta Flt - \Delta Tco + \Delta CUR + \Delta Tch + \Delta Deg$$

Because for the present policy period $\Delta RU = 0$. Then introducing the behavioral relationship for CUR we obtain:

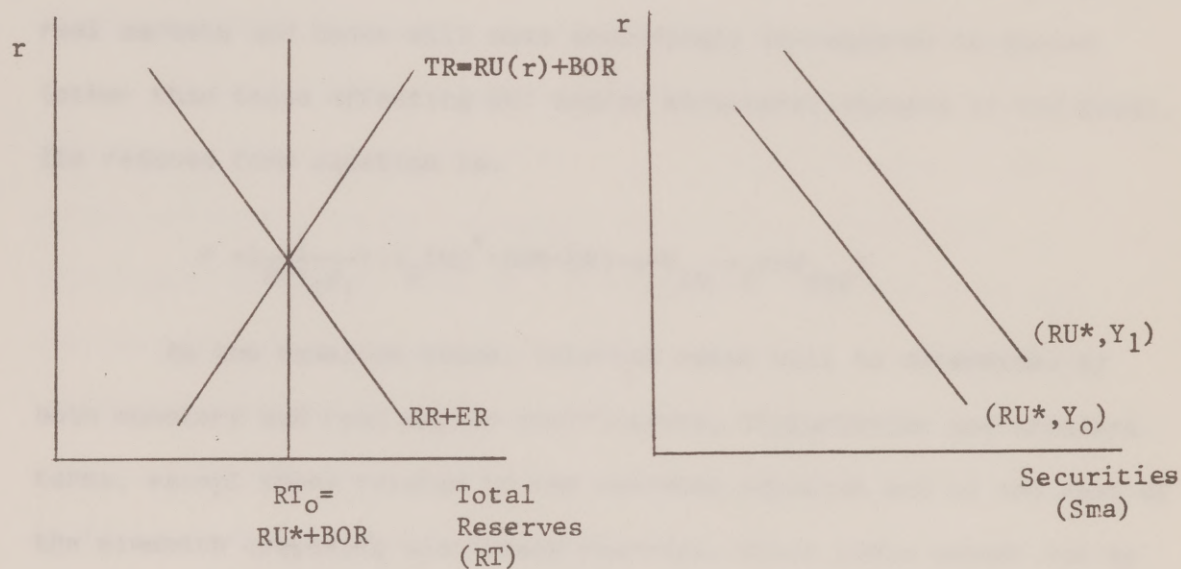
$$\Delta Sma = -\Delta Flt - \Delta Tco + \Delta(c_2 r + b_2 Y + U_{cur}) + \Delta Tch + \Delta Deg$$

The latter identity shows that Sma becomes interest and income elastic due to the CUR behavioral relationship which is part of RU , and to the role played by Sma within a policy where the instrument is RU . Therefore, since under the present policy unborrowed reserves are subject to endogenous forces but are supposed to stay at the RU^* level, monetary policy will be more active than in the previous case where the instrument was Sma .

Figure 4-4 shows the implications of the present procedure, in terms of RU and Sma . Part A shows the fact that as interest rates rise unborrowed reserves increase since currency demand falls. This renders the positive-sloped total reserves (TR) function in part A of the figure. But under the RU^* policy, Sma must fall to keep RU at RU^* with the resulting inverse relationship between Sma and r as depicted in Part B. On the other hand, a higher income level will cause CUR to

Figure 4-4

Model D-1: Unborrowed Reserves* (RU) and Securities (Sma) within the RU* Policy.



A. Market for reserves

B. Sma- r relationship
($Y_1 > Y_0$)

NOTES: 1. RU is interest-elastic due to CUR.

2. BOR has not been assumed interest-elastic so far.

rise, unborrowed reserves to fall and the resulting policy will make Sma rise to compensate such effect and bring RU back to RU^* . This implies a shift in the Sma-r relationship of part B of figure 4-4, upwards if income increases and leftwards if income falls.

With respect to the LM slope, we cannot say if it is greater or smaller under the RU^* policy than under the previous policies considered, since although τ_2 is greater than τ , ρ_1 is smaller than ρ . It will depend upon the empirical estimates of the coefficients.

On the other hand, this policy procedure implies an endogenous interest rate; that is, an interest rate that will clear the money and real markets and hence will move accordingly in response to shocks (other than those affecting RU) and/or structural changes in the model. Its reduced form equation is:

$$r = \left(\frac{1}{\beta - \tau_2 \rho_1} \right) (\tau_2 (RU^* + BOR - ER) - \alpha - U_{is} - \tau_2 rrU_{dep})$$

As the equation shows, interest rates will be determined by both monetary and real sector coefficients, disturbances and constant terms, except those related to the currency equation and to the rest of the elements composing unborrowed reserves, since these cancel out by the policy under effect.

Reduced Form of Income

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Its equation is the following:

$$Y = \left(\frac{\tau_2 \beta}{\beta - \rho_1 \tau_2} \right) (RU^* + BOR - ER - \frac{\rho_1 \alpha}{\beta} - \frac{\rho_1 U_{is}}{\beta} - rrU_{dep})$$

The equation shows that disturbances in both sectors can affect income and hence deviations of income from the target level.

Similarly, structural changes and changes in monetary variables which are not within the controllable band of the present policy, such as BOR and ER, can cause income to change.

Figure 4-5 shows the resulting relationship between Sma and interest rates under the three policy procedures discussed so far. The extreme cases, represented by the first two policies, imply a complete reliance in a price-clearing variable (Sma^* policy) or in a quantity-clearing variable (r^* policy). However, under the last policy considered (RU^*) both types of clearing variables play a role.

We must also note that under the RU^* policy, an autonomous increase in currency demand (i.e., $U_{cur} > 0$) will not affect interest rates, while an increase in deposit demand will. Figure 4-6 illustrates this, step by step. In diagram I, the effects on the sources and uses of reserves are shown. When U_{cur} is positive, the sources of reserves are affected, while when U_{dep} is positive it is the uses that is affected. Then, the resulting policy action when $U_{cur} > 0$ is to increase Sma , while it decreases Sma when $U_{dep} > 0$, with the result that in the former case interest rates go back to their initial level while in the latter they will be higher than initially. Diagram II

92. See Appendix for derivations

Figure 4-5
Model D-1: Securities-Interest Rate Relationship
under Different Policies

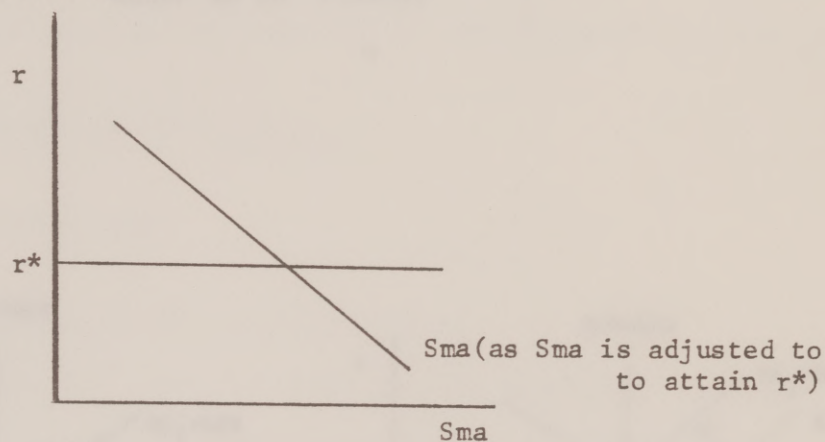
A. r^* Policy.

$$Sma = f(r)$$

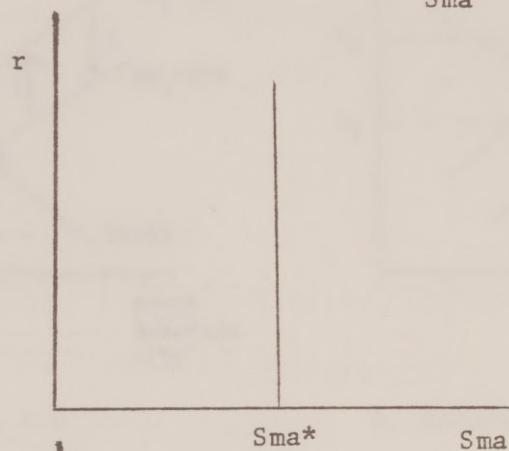
where $f'(r) < 0$

while r is in

process to be attained.



B. Sma^* Policy
 $Sma = Sma^*$



C. RU^* Policy.
 $Sma = f(r)$
where $f'(r) < 0$

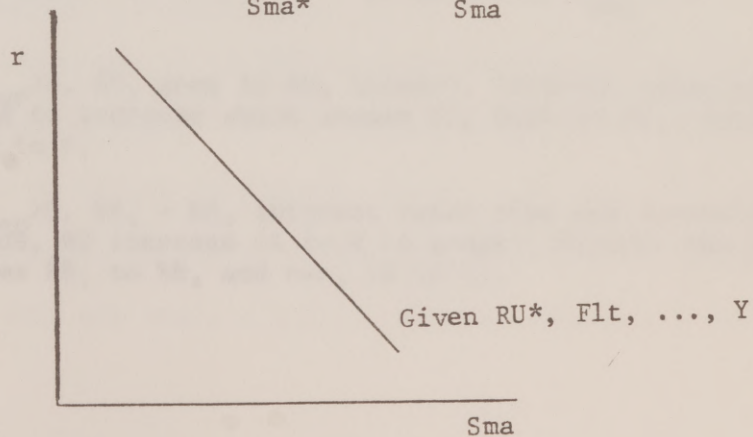
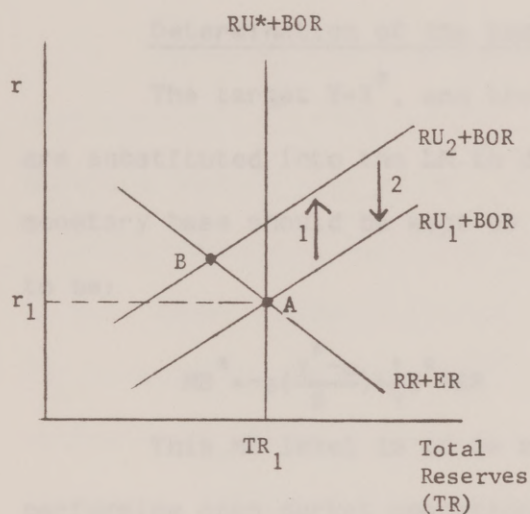
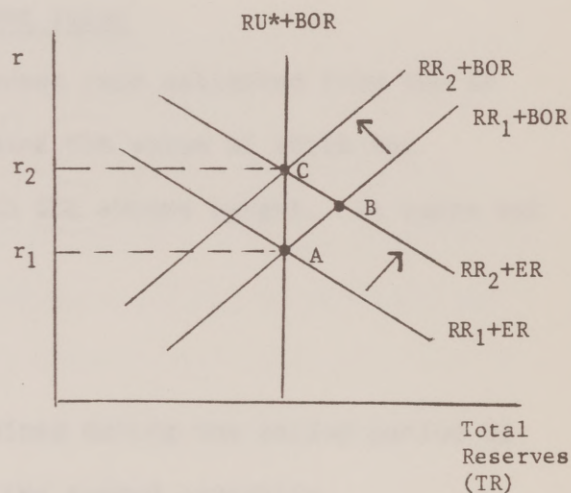


Figure 4-6

Model D-1: Effects* of U_{cur} and U_{dep}
under an RU Policy.



A. Effect of $U_{cur} > 0$



B. Effect of $U_{dep} > 0$

NOTES: 1. When $U_{cur} > 0$, RU_1 goes to RU_2 (lower), interest rates rise. Policy: Sma to increase which causes RU_2 back to RU_1 , interest rates back to r .

2. When $U_{dep} > 0$, $RR_1 \rightarrow RR_2$ interest rates rise and currency falls; hence, RU increase (A to B in graph). Policy: Sma to fall which causes RR_1 to RR_2 and $r=r_2$ (B to C).

shows the effects on income, which again are none if $U_{cur} > 0$ while income falls if $U_{dep} > 0$.

4. Monetary Base Policy

When using the monetary base as the policy instrument, the LM equation is again the same as under the r^* and Sma^* policies, that is:

$$Y = \tau(MB - ER) - \tau p r - \tau r r U_{dep} - \tau U_{cur}$$

where: MB = monetary base

Determination of the Instrument Value

The target $Y = Y^*$, and the interest rate estimated from the IS are substituted into the LM to determine the value at which the monetary base should be kept to attain the income target. It turns out to be:

$$MB^* = -\rho \left(\frac{Y^* - \alpha}{\beta} \right) + \frac{1}{\tau} Y^* + ER$$

This MB level is to be maintained during the policy period by performing open market operations in the needed direction.

Implications of this Procedure

Since in this case it is the monetary base which is used as instrument, a change in any of its components causing a deviation from the MB^* value will be offset by monetary policy. On the other hand, a change in any autonomous variable, structural parameter or a positive disturbance that does not cause a policy reaction (because it does not

affect MB), will be met by an interest rate change that will bring the monetary and real sectors back to equilibrium. The reduced form for the interest rate is:

$$r = \left(\frac{1}{\beta + \tau\rho} \right) (\tau MB^* - \tau ER - \alpha - \tau rr U_{dep} - \tau U_{cur} - U_{is})$$

Reduced Form of Income

Its equation is the following:

$$Y = \left(\frac{\beta\tau}{\beta - \tau\rho} \right) (MB^* - \frac{\rho\alpha}{\beta} - ER - U_{cur} - rr U_{dep} + \frac{\rho U_{is}}{\beta})$$

As we can see from the equation, Income will be affected by disturbances from both sectors. On the other hand, as long as the monetary base is kept at MB^* , only excess reserves (ER) among the monetary autonomous variables will affect income; that is, a change in ER will cause income to change. Furthermore, if ER was estimated as ER_0 in order to determine MB^* , and the true ER were different from the estimated ER_0 , then income will deviate from the target level Y^* . Similarly, if any of the structural coefficients change, or have been mis-estimated, income will be affected and will deviate from its target.

B. MODEL D-2 (Interest Inelastic Money Demand)

This model includes the disaggregated monetary sector, but assumes an interest-inelastic demand for money. Its complete specification is on Table 4-5.

Table 4-6, on the other hand, specifies the resulting IS and LM equations for such model. The main effect of the zero interest elasticity of the demand for money (for both CUR and D) is the zero interest elasticity of the LM equation, which therefore becomes a vertical line at some level of income. (See Figure 4-7).

Table 4-6 also shows two different LM equations, because, as in model D-1, under an unborrowed reserves policy the LM equation changes.

1. Interest Rate Policy

Determination of the Instrument value

Given the income target Y^* , the instrument value is determined from the IS as:

$$r^* = \frac{Y^* - \alpha}{\beta}$$

Implications of this Procedure

The implication of such a policy is to make S_{ma} (and hence RU) endogenous, since it is this variable that will keep the interest rate at r^* . Its reduced form is the following:

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93. See Appendix for derivations.

Table 4-5. Model D-2: Specification of Structural Equations

Real Sector:

$$Y = C + I + G$$

$$C = A_1 + b_1 Y + U_c$$

$$I = A_2 + c_1 r + U_i$$

Monetary Sector:

$$MS = CUR + D$$

$$CUR = c_2 r + b_2 Y + U_{CUR}$$

$$D = c_3 r + b_3 Y + U_{dep}$$

$$RT = RU + BOR$$

$$RT = RR + ER$$

$$RR = rr D$$

$$MB = RT + CUR$$

$$RU = Sma + Flt + Tco + CUR - Tch - Deg$$

where:

Assumptions

Y = real income	$b_1 > 0$
C = real consumption	$b_2 > 0$
I = real investment	$b_3 > 0$
G = real government expenditures	$A_1, A_2 \geq 0$
MS = money supply	$c_1 < 0$
CUR = currency	$c_2 < 0$
D = demand deposits	$c_3 < 0$
RT = total reserves	$0 < rr < 1$
RU = unborrowed reserves	
BOR = borrowings	
ER = excess reserves	
MB = monetary base	
Sma = Fed's security portfolio	
Flt = float	
rr = required reserves ratio	
Tco = treasury currency outstanding	
Tch = treasury cash holdings	
Deg = government deposits at Fed or monetary authority	

Table 4-6. Model D-2: IS and LM Equations.

IS	$Y = \alpha + \beta r + U_{is}$ where: $\alpha = \left(\frac{A_1 + A_2 + G}{1 - b_1} \right) > 0$ $\beta = \left(\frac{c_1}{1 - b_1} \right) < 0$ $U_{is} = \left(\frac{U_c + U_i}{1 - b_1} \right)$
LM ¹	$Y = \tau (MB - ER - U_{LM})$ where: $\tau = \left(\frac{1}{rrb_3 + b_2} \right) > 0$ $U_{LM} = rrU_{dep} + U_{cur}$
LM ²	$Y = \tau_2 (RU + BOR - ER - rrU_{dep})$ where: $\tau_2 = \frac{1}{rrb_3}$

1. This LM equation applies when either the monetary base, interest rates or security portfolio are used as instruments.

2. This LM equation applies when unborrowed reserves are used as instruments.

$$Sma = \frac{1}{\tau} (\alpha + \beta r^* + U_{is}) - BOR - Flt - Tco + Tch + CUR + Deg + ER + U_{lm}$$

This equation shows the resulting role that Sma plays if an interest rate policy is followed. Disturbance terms will affect it such as, for instance, a positive U_{lm} which will press interest rates up and policy will react to provide additional reserves to take the pressure from interest rates. These additional reserves are provided through open market operations, by buying securities which means an increase in Sma. The same type of mechanics applies for the rest of the elements in such Sma reduced form equation.

Reduced Form of Income

Under the present policy the reduced form equation for income coincides with the IS, indicating that only real sector disturbances or structural changes can cause income to deviate from target.

2. Security Portfolio Policy

Determination of the Instrument Value

Given $Y = Y^*$, Sma is obtained by substituting Y^* in the LM equation, leading to:

$$Sma = (rrb_3 + b_2) Y^* - Flt + Deg - BOR + ER - Tco + Tch$$

As usual, it is assumed that the autonomous variables and coefficients have been estimated for the policy period and used in determining Sma^* . Notice that, in contrast to model D-1, here the

determination of Sma^* is independent of the real sector because of the lack of the link through interest rates between monetary and real sectors when the demand for money is interest-inelastic.

Implications of this Procedure

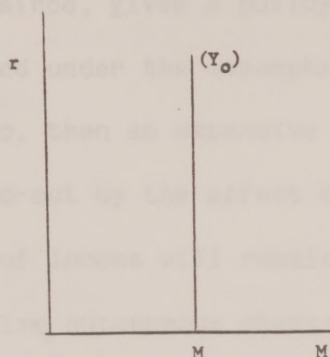
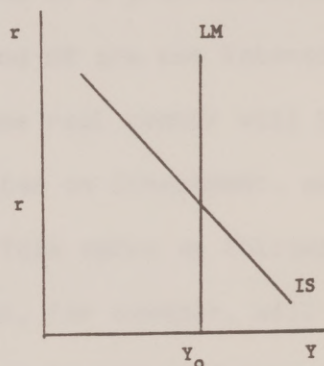
The interest rate becomes the clearing variable in the attainment of the equilibrium between the two sectors. The mechanics of this clearing process are the following: if at a given interest rate, there is a change in the system that causes a disequilibrium between monetary and real sectors, the interest rate will fluctuate until both sectors generate the same level of income. Figure 4-8 shows the determination of this equilibrium within the present policy. The initial, vertical LM is located at the target level of income Y^* , and it assumes the instrument value that is set at the beginning of the policy period (Sma^*). However, as the figure shows, as long as Sma is fixed at such value, a shift in either the LM or the IS will be met by a change in interest rates. This is corroborated by the interest rate reduced form equation, which shows the different autonomous variables, structural parameters and disturbances that, by shifting the LM or the IS, will induce interest rate to change:

$$r = \frac{\tau}{\beta} (Sma^* + BOR + Flt + Tco - Tch - Deg - ER - \frac{\alpha}{\tau} \frac{U_{is}}{\tau} - U_{lm})$$

where: $\frac{\tau}{\beta} < 0$ and $U_{lm} = U_{cur} + rrU_{dep}$

Figure 4-7.

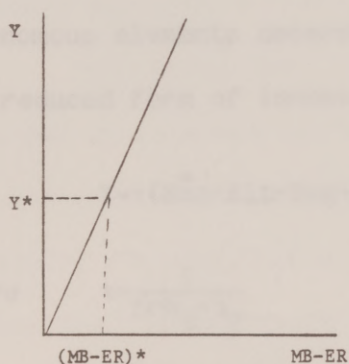
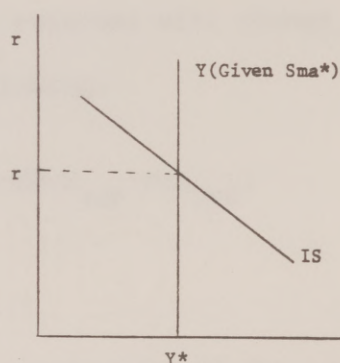
Model D-2: IS and Lm under Interest-Inelastic Md.

A. Demand for money,
given a level of income Y_0 .

B. IS and LM

Figure 4-8.

Model D-2: IS-LM Equilibrium under an Sma Policy.

A. Determination of
instrument value Sma^* ,
implied in $(MB-ER)^*$.B. IS-LM equilibrium given Sma^* .

1. $MB-ER = Sma + Flt + tc_0 - Tch - Deg + BOR - ER - ER$, so $(MB-ER)^* = Sma^* + Flt + tc_0 - Tch - Deg + BOR - ER$ where all elements except Sma^* are forecasted by authorities in order to determine Sma^* .

Reduced Form of Income

Within this model, income will be determined by the LM equation alone since, given a policy of keeping Sma at a predetermined level Sma^* , and under the assumption that BOR and ER are not interest-elastic, then an expansive movement in the real sector will be crowded-out by the effect of interest rates on investment, and the level of income will remain unchanged. This works as follows: an expansive autonomous change in investment, for example, will bring an initial increase in income, that will lead to higher demand for money, which will result in expanded demand for reserves. However, since the supply of reserves is fixed, interest rates start to rise causing investment to fall and hence driving income back to a lower level. This will continue until income is back to the level at which the demand for money equals supply and hence there is no more pressure for interest rates to change. This, of course, assuming that none of the autonomous elements determining total reserves will change. Therefore, the reduced form of income is the following:

$$Y = \tau (Sma^* + Flt - Deg + BOR + Tco - Tch - ER - U_{cur} - rr U_{dep})$$

where
$$\tau = \frac{1}{rrb_3 + b_2}$$

Hence, a change in any of the components of the monetary base will affect income, as well as the existence of monetary sector disturbances.

3. Unborrowed Reserves Policy

Determination of the Instrument Value

Given $Y=Y^*$, the instrument value for unborrowed reserves becomes:

$$RU^* = rrb_3 Y^* - BOR + ER$$

which is the level of unborrowed reserves that will be maintained during the policy period.

Implications of this Procedure

This policy procedure implies that, during the policy period, an autonomous change in any of the components of unborrowed reserves will be offset by a policy change. However, any other disturbance or structural change in the system will not induce a policy action and therefore interest rates will have to move in order to clear the monetary and real sectors.

This means that, given $RU=RU^*$, the IS-LM equilibrium will be attained by interest rates and income movements. However, under certain circumstances interest rates will be solely responsible for restoring the equilibrium while under others, both interest rates and income will change to attain equilibrium. This will become evident as we present the reduced form equations for both interest rates and income. The former is the following:

$$r = \frac{1}{\beta} (\tau_2^{RU*} + \tau_2^{BOR} - \tau_2^{ER} - \tau_2^{rrU_{dep}} - \alpha - U_{is})$$

where: $\tau_2 = \frac{1}{rrb_3} > 0$ and $\beta < 0$

As we can see from this equation, changes in borrowings, excess reserves, liquidity preference (as reflected in U_{dep}), autonomous consumption or investment, will be reflected in interest rate changes.

Reduced Form of Income

The income equation is the following:

$$Y = \tau_2 (RU^{*} + BOR - rrU_{dep} - ER)$$

where: $\tau_2 = \frac{1}{rrb_2}$

This is the same as the LM equation under the present policy.

It is important to note that both income and interest rates are affected by disturbances occurring in the demand for deposits but not in the demand for currency under the unborrowed reserves policy, while under the security portfolio policy both types of disturbances affected them. On the other hand, the components of unborrowed reserves e.g., float, etc. do not affect income and interest rates under this policy, while they do under the Sma^{*} policy. See the respective reduced forms in Tables 4-7 and 4-8.

Finally, disturbances or autonomous changes occurring in the real sector appear in the interest rate reduced form but not in the income reduced form equation under the present policy. This just confirms our earlier statement about the crowding out that occurs given

Table 4-7. Model D-2: Interest Rate Reduced Form Equations under Alternative Policies.

Policy	Equation
Sma^*	$r = \frac{\tau}{\beta} (Sma^* + BOR + Flt + Tco + Tch - Deg - ER - \frac{\alpha}{\tau} - \frac{U_{is} - U_{LM}}{\tau})$ where: $U_{LM} = U_{cur} + rrU_{dep}, \quad \tau = 1/(rrb_3 + b_2)$ $U_{is} = (Uc + Ui)/(1 - b_1)$
RU^*	$r = \frac{\tau_2}{\beta} (RU^* + BOR - ER - rrU_{dep} - \frac{\alpha}{\tau_2} - \frac{U_{is}}{\tau_2})$ where: $\tau_2 = \frac{1}{rrb_3}$
MB^*	$r = \frac{\tau}{\beta} (MB^* - ER - U_{LM} - \frac{U_{is}}{\tau} - \frac{\alpha}{\tau})$ where: $U_{LM} = U_{cur} + rrU_{dep}$ and $\tau = 1/(rrb_3 + b_2)$

the specification of the demand for money in this model.

4. Monetary Base Policy

Determination of the Instrument Value

Given $Y=Y^*$, the LM gives the value at which the monetary base will be kept during the policy period:

$$MB^* = (rrb_3 + b_2)Y^* + ER$$

where ER is the estimated excess reserves for the policy period.

Implications of this Procedure

The main difference of the present procedure, as compared to the previous one, is that here autonomous changes in borrowings (BOR) are automatically offset by the policy under effect. As before, real sector disturbances or structural changes will only induce interest rate changes, while monetary sector changes will lead to both interest rates and income changes. However, the possibility of a response to excess demand for money by increasing borrowings and so offsetting the crowding out is eliminated under the present policy, whereas it was possible under the RU^* policy. This is so because the monetary base is being used as instrument and BOR is part of it; therefore, a change in BOR will cause the monetary base to change and this will lead to a policy reaction to bring MB back to MB^* , which will offset the BOR change. The reduced form for interest rates is the following:

Table 4-8. Model D-2: Reduced Form Equations for Income under Alternative Policies.

Policy	Equations
Sma^*	$Y = \tau(Sma^* + Flt-Deg+BOR+Tco-Tch-ER-U_{LM})$ where: $U_{LM} = rrU_{dep} + U_{cur}$ $\tau = 1/(rrb_3 + b_2)$
RU^*	$Y = \tau_2 (RU^* + BOR-ER-rrU_{dep})$ where: $\tau_2 = 1/rrb_3$
MB^*	$Y = \tau(MB^* - ER - U_{LM})$ where: $U_{LM} = U_{cur} + rrU_{dep}$

$$r = \frac{1}{\beta} (\tau MB^* - \tau ER - \alpha - \tau U_{lm} - U_{is})$$

where: $U_{lm} = U_{cur} + rr U_{dep}$

In this equation we notice that BOR does not appear any more, since it is part of MB^* ; however, disturbances from both sectors can affect interest rates, including U_{cur} which is part of U_{lm} .

Hence, the interest rate will bring the markets into equilibrium when there has been a disruption that has not affected MB^* . However, when MB^* has changed it is Sma that will move to restore the monetary base to its instrument value. This means that since:

$$MB^* = Sma + Flt + Tco - Tch - Deg + BOR$$

Then: $\Delta Sma = -\Delta Flt - \Delta Tco + \Delta Deg - \Delta BOR$

which describes the elements that may induce policy actions.

Reduced Form of Income

Under this procedure and the present economic model, the income reduced form equation is:

$$Y = \frac{1}{rrb_3 + b_2} (MB^* - ER - U_{lm}),$$

where: $\frac{1}{rrb_3 + b_2} = \tau.$

which is just the LM equation.

From the above equations we can highlight several results:

- i) Changes in excess reserves (ER), autonomous consumption or investment (α) or a disturbance term in either sector, will induce interest rate changes. However, an expansive change in the monetary sector will have a greater impact on interest rates than an expansionary change of the same magnitude in the real sector. (Assuming $\tau > 1$, which is most likely).
- ii) Only monetary sector changes affect income, these being changes in excess reserves, liquidity preference (U_{dep}), or structural changes in the coefficients (b_3, rr, b_2).
- iii) Changes in the monetary sector that cause the monetary base to change, will induce open market operations, and hence a change in Sma , but income and interest rates will remain unchanged.

C. MODEL D-3 (Interest Elastic Borrowings and Excess Reserves).

Model D-3 introduces the assumption of an interest-elastic money supply by specifying Borrowings (BOR) and Excess Reserves (ER) as functions of interest rates. The complete specification of this model is shown in Table 4-9.

Table 4-9. Model D-3: Specification of Structural Equations.

Real Sector:

$$Y = C + I + G$$

$$C = A_1 + b_1 Y + U_c$$

$$I = A_2 + c_1 r + U_i$$

Monetary Sector:

$$MS = CUR + D$$

$$CUR = c_2 r + b_2 Y + U_{CUR}$$

$$D = c_3 r + b_3 Y + U_{dep}$$

$$RT = RU + BOR$$

$$RT = RR + ER$$

$$RR = rr D$$

$$MB = RT + CUR$$

$$RU = Sma + Flt + Tco + CUR - Tch - Deg$$

$$MB = RT + CUR$$

$$BOR = c_4 (r - r_d) + U_{bor}$$

$$ER = c_5 r + U_{er}$$

where: $A_1, A_2, G > 0, c_1, c_2, c_3, c_5 < 0, c_4 > 0$

$$0 < b_1, b_2, b_3 < 1 \quad 0 < rr < 1$$

and:

Y = real income, C = real consumption, I = real investment,
 G = real government expenditure, M = money supply, CUR = currency demand,
 D = demand deposits, RT = total reserves, RR = required reserves,
 ER = excess reserves,
 RU = unborrowed reserves, BOR = borrowings, FLT = float,
 Tco = treasury currency outstanding, Tch = treasury cash holdings,
 MB = monetary base, rd = discount rate, r = market interest rate,
 $U_c, U_i, U_{cur}, U_{dep}, U_{bor}, U_{er}$ = disturbances.

Before going ahead to describe the implications of different policy procedures under the present model, we must notice several aspects that are new within the present structure. The interest rate coefficients in the BOR and ER behavioral equations are part of the LM interest rate coefficient ρ_2 . See the LM equation in Table 4-10.

Comparing ρ_2 with the previous ρ and ρ_1 , we can see that:

$$\rho_2 > \rho \quad \text{and} \quad \rho_2 > \rho_1.$$

Figure 4-9 shows the LM relationship for models D-1 and D-3, the latter being the flatter one. This means that under model D-3, any interest rate change will require a greater change in income for the monetary sector to attain equilibrium.

1. Interest Rate Policy

Determination of the Instrument Value

The value of the instrument, in this case the interest rate, is determined as before, assuming $Y = Y^*$ and zero disturbances, turning out to be:

$$r^* = \frac{U^* - \alpha}{\beta}$$

Implications of this Procedure

If interest rates deviate from the instrument value r^* , open market operations will be performed in order to restore it, and therefore S_{ma} becomes the monetary endogenous variable subject to

Table 4-10. Model D-3: IS and LM Equations.

IS

$$Y = \alpha + \beta r + U_{is}$$

where:

$$\alpha = \frac{A_1 + A_2 + G}{1 - b_1}, \quad \beta = \frac{c_1}{1 - b_1}$$

$$U_{is} = \frac{U_c + U_i}{1 - b_1}$$

LM*

$$Y = \tau (Sma + Flt - Deg + Tco - Tch + \rho_2 r - c_4 rd - U_{LM}^1)$$

where:

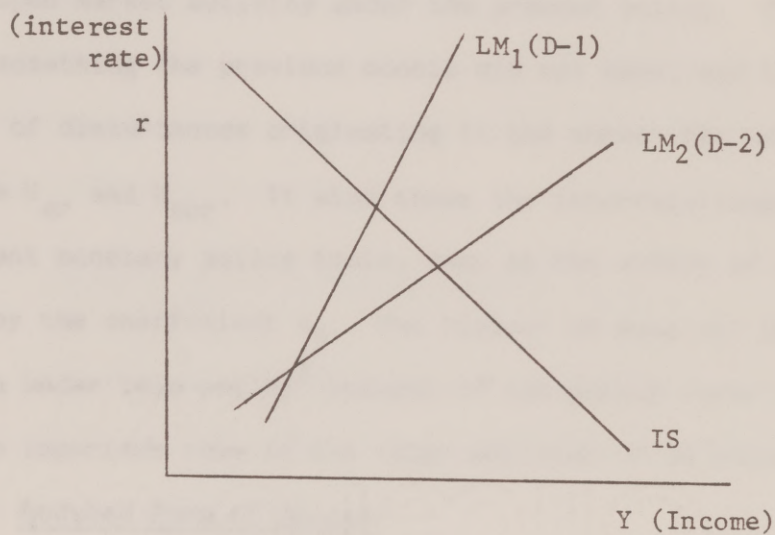
$$\tau = \frac{1}{rrb_3 + b_2} > 0,$$

$$\rho_2 = - (rrc_3 + c_5 + c_2 - c_4) > 0$$

$$U_{LM}^1 = U_{cur} + rrU_{dep} + U_{cur} - U_{bor}$$

* This LM equation applies for cases in which instrument is Sma or r, only.

Figure 4-9.
IS-LM Equilibrium under Models D-1 and D-3.



where:

$$LM_1: Y = \tau(MB-ER) + \tau\rho r - \tau U_{LM}$$

$$LM_2: Y = \tau = \rho(Sma) + Flt - Deg + Tc_0 - Tch)$$

$$- \tau\rho_2 r - \tau C_4 rd - \tau U_{LM}$$

or

$$Y = \tau(MB-BOR) - \tau\rho_2 r - \tau C_4 rd - \tau U_{LM}$$

8. Notice that ρ_2 does not appear in the variables or disturbance terms in the LM equation.

different types of disturbances, changes in autonomous variables and in structural parameters as its reduced form equation shows:

$$Sma = \left(\frac{\beta}{\tau} - \rho_2\right) r^* - Flt + Deg - Tco + Tch + c_4 r_d + \frac{\alpha}{\tau} \frac{U_{is}}{\tau} + U_{lm}^1$$

where: $U_{lm}^1 = rr U_{dep} + U_{er} + U_{cur} - U_{bor}$

and $\rho_2 = -(rr c_3 + c_5 + c_2 - c_4)$

This equation shows that disturbances from both sectors will cause open market activity under the present policy. Furthermore, it shows something the previous models did not show, and that is the effect of disturbances originating in the market for monetary reserves such as U_{er} and U_{bor} . It also shows the interrelationships between different monetary policy tools, such as the effect of r_d on Sma as given by the coefficient c_4 . The flatter LM does not have implications for Sma under this policy⁸ because of the policy itself, but it will play an important role in the other policies to be considered.

Reduced Form of Income

Under the present assumptions and policy, the reduced form coincides with the IS equation, and therefore only real sector disturbances will cause income changes. Monetary sector disturbances are accommodated by a subsequent policy action which restores income to

8. Notice that ρ_2 does not appear with the autonomous variables or disturbance terms in the Sma reduced form equation.

its initial level, as Figure 4-10 indicates. If there is an autonomous increase in money demand, for example, interest rates rise and S_{ma} will be increased (securities bought in open market) in order to restore r^* . Hence, the increased money demand has been met by an increased supply, leaving income and interest rates unchanged at their new equilibrium. The disturbance and policy effects are distinguished in diagrams A and B of figure 4-10.

2. Security Portfolio Policy

Determination of the Instrument Value

This is obtained by estimating the interest rate from the IS and substituting it, as well as the target income Y^* , into the LM equation. The resulting S_{ma} is:

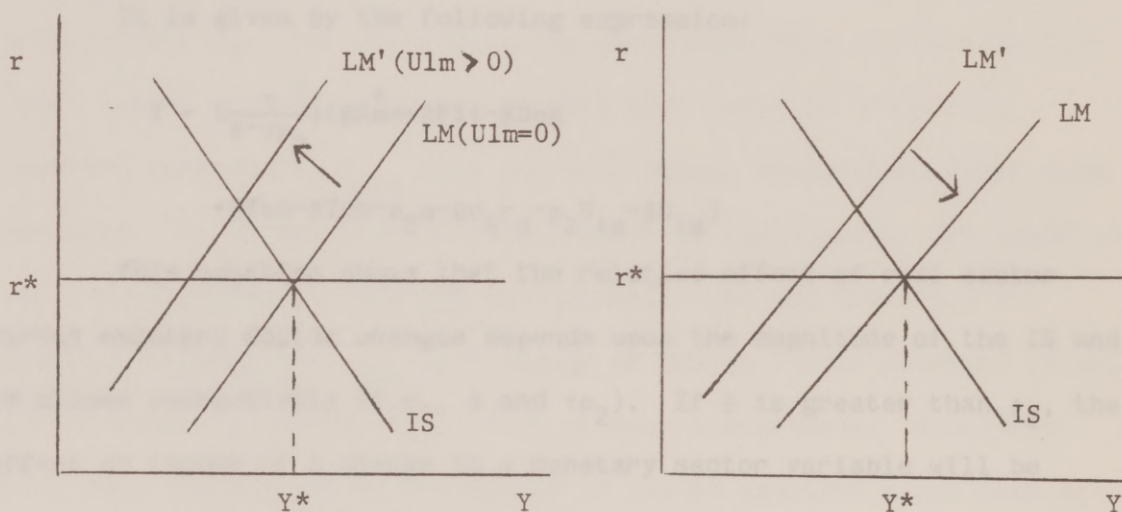
$$S_{ma}^* = Y^* \left(\frac{1}{\tau} - \frac{\rho_2}{\beta} \right) - Flt + Deg - Tco + Tch + \frac{\rho_2 \alpha}{\beta} - c_4 r_d$$

This type of policy is the less active of all, since S_{ma}^* is established at the beginning of the period and is under the direct control of the authorities. Therefore, there is hardly a chance that S_{ma} will deviate from S_{ma}^* , unless it is caused by policy.

Implications of this Procedure

Under this policy interest rates will fluctuate to keep markets in equilibrium, yielding the following reduced form equation:

Figure 4-10
 Model D-3: Effect of a Monetary Sector Shock on the
 IS-LM Equilibrium under an Interest Rate Policy.



A. Effect of the
 shock ($U_{LM} > 0$).

B. Policy effect.

$$r = \left(\frac{1}{\beta - \tau \rho_2} \right) (\tau Sma^* + \tau Flt - \tau Deg + \tau Tco - \tau Tch - \tau c_4 r_d^{-\alpha} U_{is} - \tau U_{lm})$$

This equation shows that the present type of policy does not provide any offsetting force to disturbances, parameter changes, or changes in autonomous variables since all of them appear in such reduced form. It again shows the relationship between the discount rate and the market interest rate, determined by the value of the following behavioral coefficients:

$$\frac{\Delta r}{\Delta r_d} = \frac{-\tau c_4}{\beta - \tau \rho_2} > 0$$

Reduced Form of Income

It is given by the following expression:

$$Y = \left(\frac{\tau}{\beta - \tau \rho_2} \right) (\beta Sma^* + \beta Flt - \beta Deg + \beta Tco - \beta Tch - \rho_2 \alpha - \beta c_4 r_d^{-\rho_2} U_{is} - \beta U_{lm})$$

This equation shows that the relative effect of real sector versus monetary sector changes depends upon the magnitude of the IS and LM slopes respectively (i.e., β and $\tau \rho_2$). If β is greater than ρ_2 , the effect on income of a change in a monetary sector variable will be greater than the effect of a same magnitude change in a real sector's variable. Hence, the flatter the IS (i.e., larger β), the greater the effect of the monetary shocks and the flatter the LM (i.e., larger ρ_2), the greater the effect of a real sector shock.

3. Unborrowed Reserves Policy

We must first analyze the LM equation under this policy since, as under previous mod, it changes when unborrowed reserves are used as instrument. The appropriate LM is the following:

$$Y = \tau_2 (RU^* + \rho_3 r - c_4 r_d - rr U_{dep} - U_{er} + U_{bor})$$

where: $\tau_2 = \frac{1}{rrb_3}$ and $\rho_3 = -(rrc_3 + c_5 - c_4)$

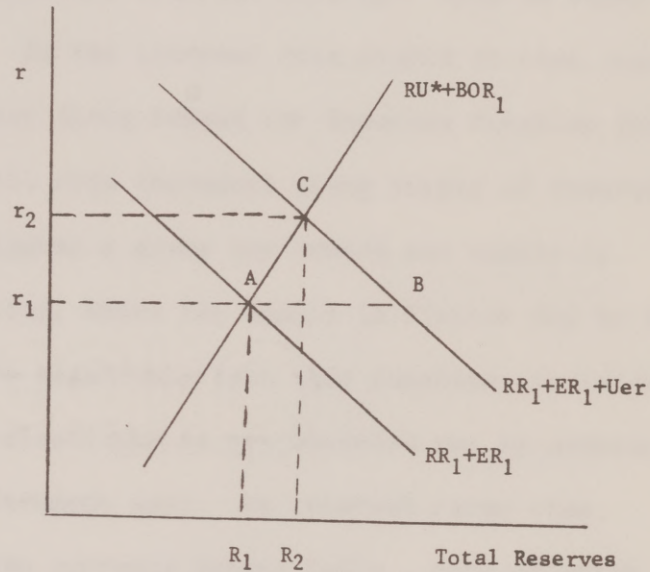
The LM slope ($\partial Y / \partial r$) is different than in previously considered policies: it is now $\tau_2 \rho_3$, while it was $\tau \rho_2$ before. However, we cannot say a priori which is higher since τ_2 is greater than τ , but ρ_3 is smaller than ρ_2 . But continuing the comparison with the LM equations under the r^* and Sma policies, we note that under an unborrowed reserves instrument, U_{cur} (the currency demand disturbance term) does not enter the LM equation, a result which is explained by the nature of the present policy, since CUR is included in RU and therefore a positive or negative U_{cur} will change RU and so induce monetary policy action that will offset the effect of such disturbance. Similarly, Flt, Deg, Tco and Tch do not appear in the LM equation since they are part of RU. Finally, the partial multiplier within the monetary sector, that is τ_2 , is greater under the present RU^* policy than the one (τ) under the previously considered policies.⁹⁵ The rationale for

95 . Recall that $\tau_2 = \frac{1}{rrb_3}$ while $\tau = \frac{1}{rrb_3 + b_2}$

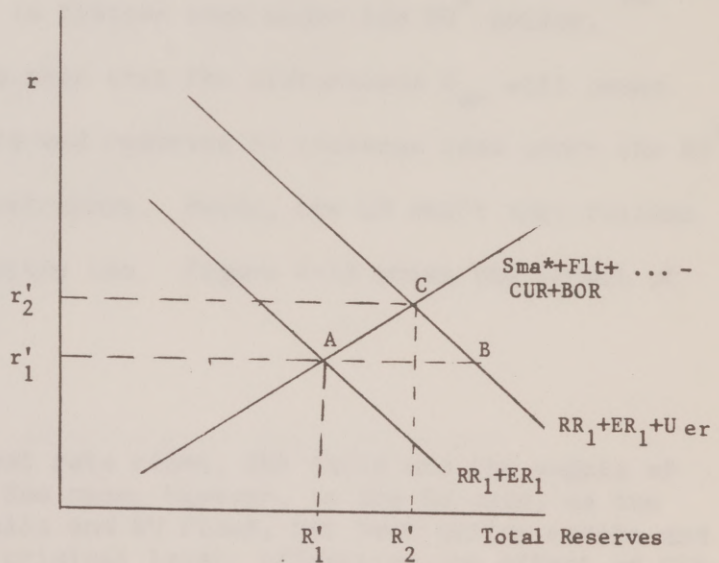
Figure 4-11.

Model D-3: Effect of a Positive U_{er} under the RU^* and Sma Policies.

A. Demand and supply of reserves.
Case RU^*



B. Demand and supply of reserves.
Case Sma .



NOTE: $(r_2 - r_1) > (r'_2 - r'_1)$.

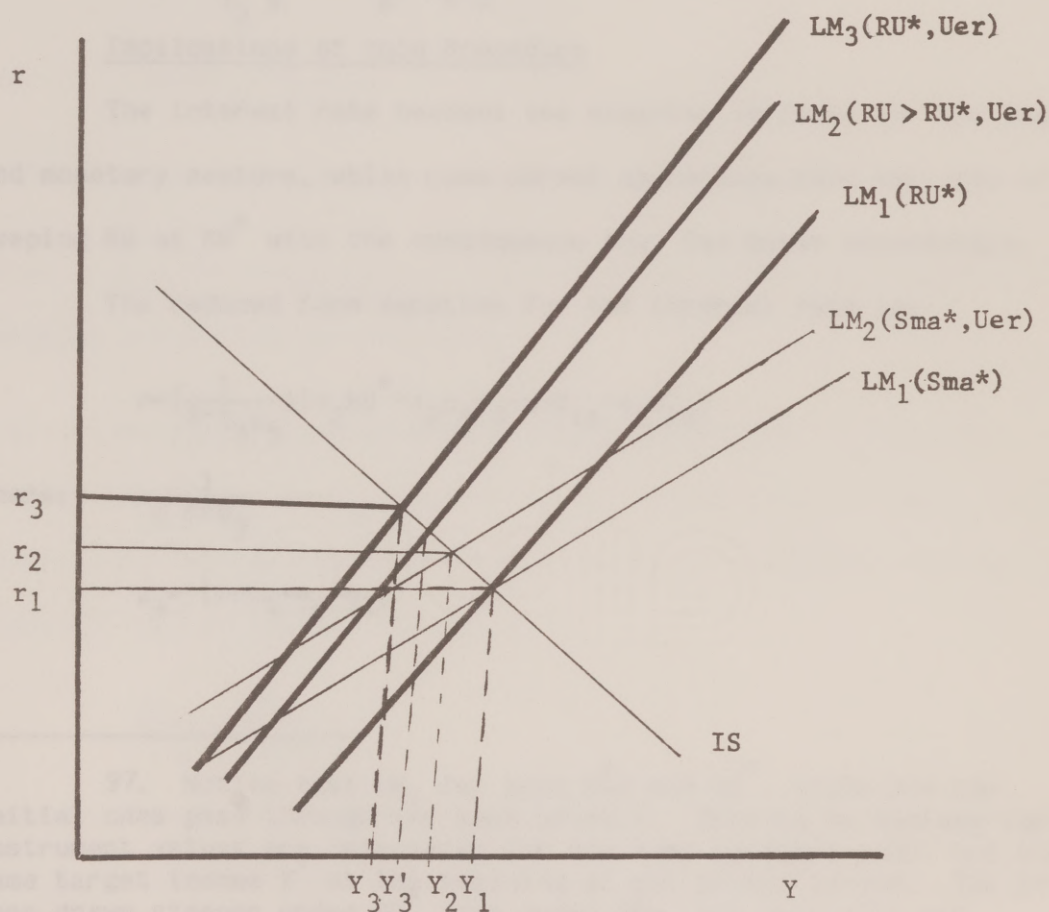
this can be better illustrated by an example. Suppose there is a sudden change in excess reserves (i.e., $U_{er} > 0$). The effect on the monetary sector is to increase the demand for reserves and hence at the initial interest rate, there will be an excess of demand for reserves over supply, which will press the interest rate up. This is shown in diagram A of figure 4-11. As the interest rate starts to rise demand deposits will fall (movement along Demand for Reserves function from B to C), while borrowings will rise (movement along supply of reserves function from A to C). Diagram B gives the demand and supply of reserves under the Sma^* policy, where the supply is flatter due to the fact that CUR, which enters negatively into this function, is interest elastic and this interest-elasticity is not canceled out by monetary policy when Sma is the instrument used. As interest rates rise, borrowings increase but also currency demand falls, both of which contribute to an increase in the supply of reserves; hence, to a supply of reserves function that is flatter than under the RU^* policy.⁹⁶

The implication is then that the disturbance U_{er} will cause interest rates to rise more and reserves to increase less under the RU^* case than if Sma is the instrument. Hence, the LM shift that follows is greater under the RU policy too. Figure 4-12 shows the effect of

96. As the interest rate rises, CUR falls and the supply of reserves increases in the Sma case; however, in the RU case, as the interest rate rises CUR falls and RU rises, but here policy reacts and so RU is decreased to its original level, offsetting the effect of CUR.

Figure 4-12

Model D-3: Effect of U_{er} (excess reserves shock) on the IS-LM Equilibrium under the RU^* and Sma Policies.



U_{er} under the alternative policies. First, the LM shifts from LM_1 to LM_2 under each policy ; then, under the RU^* policy there is an additional shift due to the policy action generated by the fact that U_{er} caused RU to be above RU^* , that is from LM_2 to LM_3 .⁹⁷

Determination of the instrument value

It is again determined by estimating r from IS and substituting it together with Y^* into the LM, giving:

$$RU^* = \left(\frac{1}{\tau_2} - \frac{\rho_3}{\beta} \right) Y^* + \frac{\rho_3 \alpha}{\beta} - c_4 r_d$$

Implications of this Procedure

The interest rate becomes the clearing variable for the real and monetary sectors, while open market operations have the role of keeping RU at RU^* with the consequence that Sma moves accordingly.

The reduced form equation for the interest rate is:

$$r = \left(\frac{1}{\beta - \tau_2 \rho_3} \right) (\tau_2 RU^* - \tau_2 c_4 r_d - \alpha - U_{is} - \tau_2 U_{lm}^2)$$

where: $\tau_2 = \frac{1}{r r b_3}$

$$\rho_3 = -(r r c_3 + c_5 - c_4)$$

97. Notice that LM_1 for both Sma^* and RU^* , which are the initial ones pass through the same point A. This is so because the instrument values are calculated for the same economic model and the same target income Y^* at the beginning of the policy period. The LM has been drawn steeper under RU^* than under Sma^* , but this will not necessarily be the case, as was already stated in the text above.

$$U_{lm}^2 = rrU_{dep} + U_{er} - U_{bor}$$

and
$$U_{is} = (U_c + U_i) / (1 - b_1)$$

This equation shows that disturbances in both sectors will cause interest rates to move. However, their effect is not the same, since it depends upon some different parameters:

$$\partial r / \partial U_{is} = -\frac{1}{\beta - \tau_2 \rho_3}$$

or
$$\partial r / \partial U_c = \partial r / \partial U_i = \frac{1}{1 - b_1} \left(-\frac{1}{\beta - \tau_2 \rho_3} \right)$$

and
$$\partial r / \partial U_{lm} = -\frac{\tau_2}{\beta - \tau_2 \rho_3}$$

where τ_2 is greater than 1 but also $1/(1 - b_1)$ is greater than 1.

Then:
$$\frac{\partial r / \partial U_c}{\partial r / \partial U_{lm}} = \frac{1}{(1 - b_1)(\tau_2)} = \frac{rrb_3}{1 - b_1}$$
 which says that the greater the marginal propensity to consume b_1

and/or the transactions demand for money coefficient b_3 are, the greater the effect of U_c relative to U_{lm} on the interest rate. On the other hand, the relationship between the market interest rate and the discount rate is now the following:

$$\frac{\Delta r}{\Delta r_d} = \frac{-\tau_2 \rho_4}{\beta - \tau_2 \rho_3}$$

And we know that τ_2 is greater than τ but ρ_3 is smaller than ρ_2 . Then, if $\tau_2 \rho_3$ is less or equal than $\tau \rho_2$, we can say that the effect of a change in the discount rate on the interest rate is larger under the

unborrowed reserves policy than under the security portfolio one.

Then, the reduced form for S_{ma} , which also becomes endogenous under the present policy, is obtained from:

$$S_{ma} = RU^* - Flt - Tco + Tch + Deg + c_2 r + b_2 Y + U_{cur}$$

where we would have to substitute the reduced forms of interest rates and income respectively, to get to the reduced form. However, because this would be a huge expression we have not included it, but what it implies is that all the factors affecting r and Y will affect S_{ma} , plus those factors appearing in the above expression for S_{ma} that do not appear in the former, such as U_{cur} and the components of unborrowed reserves e.g., float, etc.

Reduced Form of Income

Its reduced form equation under the unborrowed reserves policy is:

$$Y = \left(\frac{\tau_2}{\beta - \rho_3 \tau_2} \right) (\beta RU^* - \beta C_4 r_d - \beta U_{lm}^2 - \rho_3 \alpha - \rho_3 U_{is})$$

where: $U_{lm}^2 = rr U_{dep} + U_{er} - U_{bor}$

$$U_{is} = (U_c + U_i) / (1 - b_1)$$

$$\rho_3 = c_4 - r r c_3 - c_5 > 0.$$

and $\tau_2 = 1 / r r b_3$

For calculations see Appendix.

A shock to either the goods or monetary sectors would cause income to change; no policy reaction arises when one of these shocks occurs since unborrowed reserves are not affected by them. Therefore, there is no reason for the authorities to react. It also follows here, like under the Sma^* policy, that the flatter the LM (or larger its slope $\rho_3\tau_2$) the larger the effect of real sector changes on income; while a larger IS slope would mean a larger impact of monetary shocks on income. (See the reduced form equation above).

We also see here the link between income and the discount rate, which as we said above, if $\tau_2\rho_3$ is less or equal than $\tau\rho_2$, the effect of a discount rate change is stronger under the unborrowed reserves policy than under the security portfolio one.

4. Monetary Base Policy

Here, as we did when unborrowed reserves were the instrument, we must re-calculate the LM equation because the instrument used has changed and the LM may thus change. It turns out that the LM is different from the ones we obtained under Sma^* and RU^* policies
 98
 above.

98. For calculations see Appendix.

The resulting LM is:

$$Y = \tau (MB^* + \rho_4 r - rr U_{\text{dep}}^{-U} \text{cur}^{-U} \text{er})$$

where: $\tau = \frac{1}{rrb_3 + b_2}$

$$\rho_4 = -(rrc_3 + c_5 + c_2) > 0$$

and: $\tau \rho_4 = \text{LM slope}$

We must notice that the LM slope here may be greater, smaller or equal to the slope under the RU^* policy. This is so because we are comparing $\tau \rho_4$ in the present policy with $\tau_2 \rho_3$ in the RU^* policy, and although $\tau_2 > \tau$, ρ_3 can be larger, smaller or equal to ρ_4 . The relative magnitude of the LM slopes under these alternative cases will depend upon the absolute values of c_4 and c_2 , the interest sensitivity of Borrowings and Currency respectively. If c_4 is greater than c_2 , the LM slope will be larger (yielding a flatter LM) under an unborrowed reserves policy than under a monetary base policy. If c_2 is greater than c_4 , the slope differential would be smaller or it could even result in an LM which is flatter under the policy in which the monetary base is used.

Determination of the Instrument Value

Given Y^* , the interest rate is obtained from the IS and substituted into the LM to get:

$$MB^* = Y^* (rrb_3 + b_2 + \frac{\rho_4}{\beta}) + \frac{\rho_4 \alpha}{\beta}$$

which as usual assumes disturbances are zero.

Implications of this Procedure

Under this policy, interest rates as well as Sma and RU become endogenous variables. In order to keep the monetary base at the instrument value MB^* , any disturbance that may endogenously cause MB to change will induce a policy reaction and this will lead to changes in interest rates and in Sma and RU, which are affected by the open market operations carried out to bring MB back to MB^* . The reduced form for the interest rate is:

$$r = \left(\frac{1}{\beta - \tau \rho_4} \right) (\tau MB^* - \tau rr U_{dep} - \tau U_{cur} - \tau U_{er} - \alpha - U_{is})$$

In this equation the disturbance terms that appear are only those related to the demand for money and to the real sector. A disturbance related to the money supply, such as Borrowings, does not affect interest rates because it would cause the monetary base to deviate from MB^* and hence the resulting policy reaction would offset its effects on the interest rate.

On the other hand, as long as the monetary base is used as instrument, unborrowed reserves become endogenous since on the supply side we have:

$$MB^* = RU + BOR + CUR \quad \text{hence} \quad RU = MB^* - BOR - CUR$$

Since we know that:

$$RU = Sma + Flt - CUR - Deg + Tco - Tch$$

Then Sma, which is the item directly affected by open market operations, will depend upon:

$$Sma = MB^* - BOR - Flt + Deg - Tco + Tch$$

Hence, an unexpected change in BOR, Flt, etc. would cause Sma to change. Introducing the behavioral relationship for BOR, we obtain a more explicit description of the factors affecting Sma:

$$Sma = MB^* - c_4 r + c_4 r_d - U_{bor} - Flt + Deg - Tco + Tch$$

Therefore, Sma becomes a function of all factors affecting interest rates, of decisions taken with respect to the discount rate r_d , plus the other components of the monetary sector supply side as shown in this equation.

Reduced Form of Income

Its reduced form is the following:

$$Y = \left(\frac{\tau}{\beta - \rho_4 \tau} \right) (\beta MB^* - \rho_4 \alpha - \rho_4 U_{is} - \beta U_{lm}^3)$$

where: $U_{lm}^3 = r r U_{dep} + U_{cur} + U_{er}$

Indicating that the same type of disturbances affecting interest rates will affect income under this policy. The only additional comment here is to stress the fact that Borrowings do not appear in the reduced form

of either income or the interest rate. With this we refer to any of the elements in the BOR relationship: its interest sensitivity, the discount rate, and the disturbance. The reason of course is found in the mechanics of a monetary policy that sets the monetary base as instrument; since Borrowings are part of the monetary base, if they change, a policy action will be induced since MB must have changed too, and hence the effect of the change in BOR on interest rates or income is eliminated.

Finally Table 4-11 shows the reduced form equations for income and for the respective monetary variables under each of the alternative policy procedures considered in model D-3. The conclusions are the following:

1. Under an interest rate policy, monetary sector shocks do not affect income, but they do affect reserves through open market operations.
2. Under an Sma^* policy, monetary sector shocks affect both income and interest rates. The monetary sector shocks included are those that affect the demand for deposits, borrowings and excess reserves; these are the same that affect reserves and hence the money supply under the interest rate policy above.
3. Under an RU^* policy, monetary sector shocks affect both income and interest rates; however, shocks on the demand for currency do not appear among the type of shocks that can affect Y and r under this policy, while they did in the above cases.

Table 4-11. Model D-3: Reduced Form Equations under Alternative Monetary Policy Procedures.

Policy	Reduced Form Equations
r^*	$Sma = \left(\frac{\beta}{\tau} - \rho_2 \right) r^* - Flt + Degc_4rd$ $+ \frac{\alpha}{\tau} + \frac{U_{is}}{\tau} + U_{LM}^1$ $Y = \alpha + \beta R^* + U_{is}$
Sma^*	$r = \left(\frac{1}{\beta - \tau \rho_2} \right) \left(\tau Sma^* + \tau Flt - \tau Deg - \tau c_4rd \right.$ $\left. - \alpha - U_{is} - \tau U_{LM}^1 \right)$ $Y = \frac{\tau}{\beta - \tau \rho_2} \left(\beta Sma^* + \beta Flt - \beta Deg - \rho_2 - \beta c_4rd \right.$ $\left. - \rho_2 U_{is} - \beta U_{LM}^1 \right)$
RU^*	$r = \left(\frac{1}{\beta - \tau_2 \rho_3} \right) \left(\tau_2 RU^* - \tau_2 c_4rd - \alpha - U_{is} - \tau_2 U_{LM}^2 \right)$ $y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_3} \right) \left(\beta RU^* - \beta c_4rd - \beta U_{LM}^2 - \rho_3 \alpha - \rho_3 U_{is} \right)$
MB^*	$r = \left(\frac{1}{\beta - \tau \rho_4} \right) \left(\tau MB^* - \tau U_{LM}^3 - \alpha - U_{is} \right)$ $Y = \left(\frac{\tau}{\beta - \tau \rho_4} \right) \left(\beta MB^* - \rho_4 \alpha - \rho_4 U_{is} - \beta U_{LM}^3 \right)$

where:

$$\tau = 1/(rrb_3 + b_2)$$

$$U_{LM}^1 = rrU_{dep} + U_{er} + U_{cur} - U_{bor}$$

$$U_{LM}^2 = rrU_{dep} + U_{er} - U_{bor}$$

$$U_{LM}^3 = rrU_{dep} + U_{cur} + U_{er}$$

$$\rho_2 = -(rrc_3 + c_5 + c_2 - c_4)$$

$$\rho_3 = -(rrc_3 + c_5 - c_4)$$

$$\rho_4 = -(rrc_3 + c_5 + c_2)$$

4. Under an MB^* policy, monetary shocks also affect both income and interest rates, but again we have to point out that shocks on the Borrowings function do not affect them when the monetary base is the instrument.
5. We must recall that an increase in the discount rate will affect income and interest rates under the unborrowed reserves and the security portfolio policies. Also, under such policies the discount rate enters the formula for determining the instrument values Sma^* and RU^* , meaning that the discount rate can be used together with unborrowed reserves or the security portfolio, whichever is the policy being used, in the process of attaining the income target. This may be specially useful in a society where different monetary variables have different psychological effects in the financial markets, since then the authorities can alternate their use to create or avoid certain expectations or a different environment.

Although we cannot make a definitive comparison of the multipliers attached to the disturbances under each policy (since this will depend upon the magnitude of each of the parameters that compose such multipliers), the above conclusions are useful because they stress the importance of identifying the types of relationships that are less stable in the economy, since this will be necessary to evaluate alternative policies.

D. MODEL D-2 (Interest-bearing Demand Deposits)

Model D-2 introduces the possibility of interest-bearing demand deposits. Its complete specification is in Table 4-12. The introduction of the interest rate r_i with its paid on demand deposits (like r_d), with as the subject of our analysis in this model. In

Table 4-12. Model D-4: Specification of Model Equations.

Real Sector:

$$Y = C + I + G$$

$$C = A_1 + b_1 Y + U_c$$

$$I = A_2 + c_1 r + U_i$$

Monetary Sector:

$$MS = CUR + D$$

$$CUR = c_2 r + b_2 Y + U_{CUR}$$

$$NDD = c_3 a r + b_3 a Y + U_{NDD}$$

$$IDD = c_3 b (r - r_i) + b_3 b Y + U_{IDD}$$

$$D = NDD + IDD$$

$$RT = RU + BOR$$

$$RT = RR + ER$$

$$RR = rr D$$

$$RU = Sma + Flt + Tco + CUR - Tch - Deg$$

$$MB = RT + CUR$$

$$BOR = c_4 (r - r_d) + U_{BOR}$$

$$ER = c_5 r + U_{ER}$$

where:

NDD = non-interest bearing demand deposits.

IDD = interest bearing demand deposits.

r_i = rate paid on demand deposits.

For meaning of other variables, see model D-1, Table 4-2.

D. MODEL D-4 (Interest-bearing demand deposits).

Model D-4 introduces the assumption of interest-bearing demand deposits. Its complete specification is in Table 4-12. The introduction of the interest rate r_i , which is paid on some demand deposits (IDD), will be the subject of our analysis in this model. In this context, however, we will distinguish two cases: one which assumes there are ceilings on such r_i 's (i.e., $r_i = \bar{r}$), and the alternative case of no ceilings.

As was discussed in chapter 2, the effect of this new assumption is initially seen in the structural coefficients that characterize the demand form money; that is, c_{3a} , c_{3b} and c_2 . (See Table 4-12). The coefficients c_{3a} and c_{3b} will together determine the coefficient c_3 that would result if we add up the equations for NDD and IDD into one equation for demand deposits, such as:

$$D = c_{3a}r + c_{3b}(r - r_i) + b_{3a}Y + b_{3b}Y + U_{ndd} + U_{idd}$$

or
$$D = (c_{3a} + c_{3b})r - c_{3b}r_i + b_3Y + U_{dep}$$

In chapter 2 we reached the following conclusions about this Demand for Deposits equation:

1. Case 1. Ceilings on r_i ($r_i = \bar{r}$). The coefficient $c_{3a} + c_{3b}$ is lower than the c_3 obtained when we did not assume that deposits bear interest, and c_2 (currency demand coefficient) is larger here than in that case. This results in a steeper LM.

Table 4-13. Model D-4: IS and LM Equations.

IS	$Y = \alpha + \beta r + U_{is}$
LM^*	$Y = \tau(Sma + Flt + Tco$ $- Tch - Deg + rrc_{3a}r_i + \rho_s^2$ $- ER + BOR - U_{LM})$ where: $\rho_s = - [rr(c_{3a} + c_{3b}) + c_2]$ assuming: $b_3 = b_{3a} + b_{3b}$, $\tau = \frac{1}{rrb_3 + b_2}$ $U_{LM} = rrU_{dep} + U_{cur}$

* Applicable only to cases where instrument is r or Sma.

2. Case 2. No Ceilings on r_i . The coefficient $c_{3a}+c_{3b}$ is lower and c_2 larger than in Case 1. The implication is an even steeper LM than in case 1.

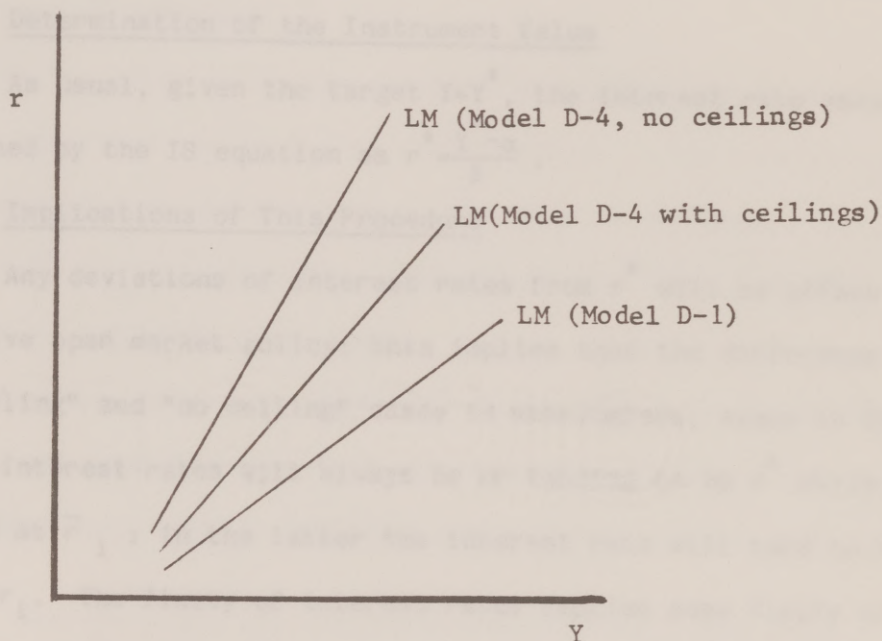
The LM equation (see Table 4-13), differs from the one in model D-1 in two main aspects:

1. The element $rrc_{3a}r_i$, which implies a leftward shift of the LM.
2. The slope given by:

$$\partial Y / \partial r = \frac{-rr(c_{3a}+c_{3b})-c_2}{rrb_3+b_2}$$

whose formula has not really changed since the previous c_3 has now become $c_{3a}+c_{3b}$. However, with the present assumptions we expect $c_{3a}+c_{3b}$ to be different from the c_3 obtained in previous mod, as we also expect the new c_2 to be different from the one in model D-1. Although c_2 would tend to be larger and the absolute value of $c_{3a}+c_{3b}$ would tend to be smaller, the larger c_2 will be directly reflected in lowering $c_{3a}+c_{3b}$, aside from the behavioral effect that causes c_{3a} and c_{3b} to be lower than before. Therefore, ρ_5 , which equals $-(rrc_{3a}+rrc_{3b}+c_2)$, will be lower than in previous model specifications, implying a steeper LM. This greater steepness is even more evident when we assume no ceilings. (See Figure 4-13).

Figure 4-13
The LM Relationship under D-1 and D-4.



Finally, we must note that we have implicitly assumed that the interest rate paid in demand deposits, r_i , is exogenous, even when there are not ceilings, since we have not specified a behavioral equation to explain it. In the future, a behavioral equation could be specified, for example:

$$r_{it} = f(r_{it-1}, r_{it-2}, \dots, r_{it-j})$$

and this may generate a new set of conclusions.

1. Interest Rate Policy

Determination of the Instrument Value

As usual, given the target $Y=Y^*$, the interest rate value is determined by the IS equation as $r^* = \frac{Y^* - \alpha}{\beta}$.

Implications of This Procedure

Any deviations of interest rates from r^* will be offset by the respective open market policy; this implies that the difference between the "ceiling" and "no ceiling" cases is meaningless, since in the former, interest rates will always be or tending to be r^* while r_i will be fixed at $\bar{r}_i$; in the latter the interest rate will tend to r^* and so will r_i . The fixity of interest rates implies some fixity of interest paid on deposits with the result that the speculative part of the demand for money becomes a fixed portion of such demand, in either of the cases under analysis.

Different values for the coefficients c_2 , c_{3a} and c_{3b} , under the ceiling and no ceiling cases, will not render different outcomes for equilibrium income and interest rates, but it might for the money supply that is necessary to keep interest rates at r^* . How the money supply is affected will really depend upon the slope and position of the demand for money under the ceiling and no ceilings cases. (See Figure 4-14 where the demand for money relationships, (diagram A) having different slopes and positions, generate different lev of money supply that will keep the target interest rate). This is necessary such that the LM under both cases passes through point A (Diagram B), where IS and r^* intersect.

On the other hand, the outcome for interest rates and income, from the time they change due to a disturbance to the time policy has been implemented to bring r back to r^* , will be different under the ceilings and no ceilings cases. This is evident from the different LM slopes under the alternative cases.

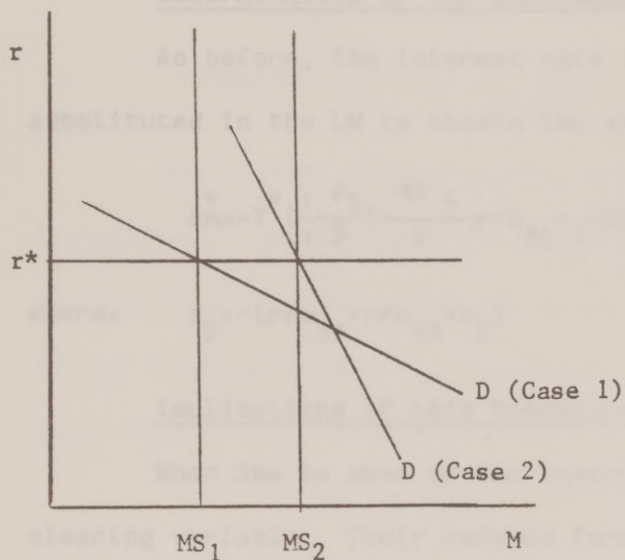
Finally, since the interest rate is kept at r^* by performing the necessary open market operations, the reduced form of Sma shows the factors that will lead to implement these operations so as to keep r^* :

$$Sma = \frac{\alpha}{\tau} + \frac{\beta - \rho_5}{\tau} r^* + \frac{U_{is}}{\tau} + (ER - BOR - Flt - Tco + Tch + Deg + rrU_{dep} + U_{cur})$$

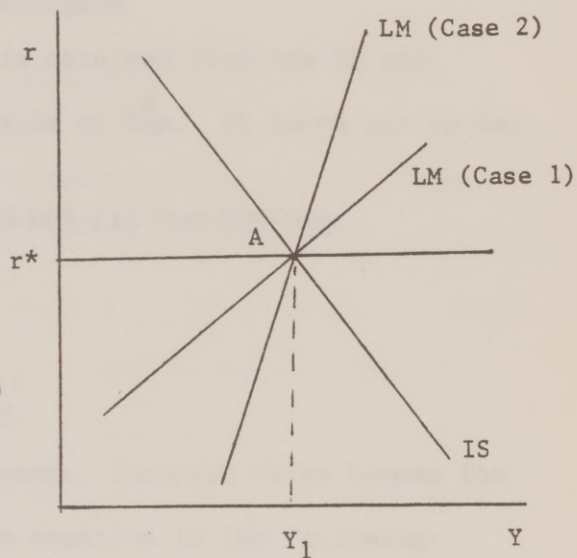
This equation shows that disturbances from both sectors will cause Sma to change; it also shows that a change in the sources and uses of the monetary base will generate an offsetting Sma change such that

Figure 4-14

Model D-4: Interest Rate Policy under Cases 1
(ceilings in r_i) and 2 (no ceilings).



A. Monetary sector:
demand and supply
of money



B. IS-LM equilibrium.

any interest rate movements is eliminated.

Reduced Form of Income

It is given by the IS equation:

$$Y = \alpha + \beta r^* + U_{is}$$

which implies that only real sector disturbances can cause income to deviate from target.

2. Security Portfolio Policy

Determination of the Instrument Value

As before, the interest rate is obtained from the IS and substituted in the LM to obtain the value of Sma^* . It turns out to be:

$$Sma^* = Y^* \left(\frac{1}{\tau} - \frac{\rho_5}{\beta} \right) + \frac{\alpha \rho_5}{\beta} - rrc_{3b} r_i + ER - BOR - Flt - Tco + Tch + Deg$$

where: $\rho_5 = -(rrc_{3a} + rrc_{3b} + c_2)$

Implications of this Procedure

When Sma is used as the instrument, interest rates become the clearing variable. Their reduced form equation is the following:

$$r = \frac{1}{\beta - \tau \rho_5} (\tau Sma^* + \tau Flt + \tau Tco - \tau Tch - \tau Deg + \tau rrc_{3a} r_i - \tau ER + \tau BOR - \tau U_{lm} - \alpha U_{is})$$

where: $U_{lm} = rrU_{idd} + rrU_{ndd} + U_{cur}$.

Now, we must recall that the parameter ρ_5 is smaller under model D-4 than the original ρ that we had under the original model D-1.⁹⁹ It is even smaller when there are no ceilings than when there are; therefore, as the equation shows, the effect on interest rates of a disturbance or a change in an autonomous variable will be greater when interest is paid on demand deposits, and even greater if there are no ceilings on such interest rates.

Reduced Form of Income

It is given by:

$$Y = \left(\frac{\tau}{\beta - \tau \rho_5} \right) (\beta Sma + \beta FIt + \beta Tco - \beta Tch - \beta Deg + \beta rrc_{3a} r_i - \rho_5 \alpha - \rho_5 is - \beta ER + \beta BOR - \beta U_{lm})$$

The effect of real sector and monetary sector disturbances, respectively, depend upon the magnitude of the parameters ρ_5 and β ; and we know that ρ_5 affects the LM slope while β is equal to the IS slope. If $|\beta|$ is greater than ρ_5 , the effect on income of monetary sector disturbances will be greater than that of real sector disturbances.

Then, comparing case 1 (ceilings) with case 2 (no ceilings), since in the former we have a larger ρ_5 (i.e., flatter LM) than in the latter, we conclude that when there are ceilings on r_i , the effect of

99. Recall that $\rho_5 = -(rrc_{3a} + rrc_{3b} + c_2)$ while $\rho = -(rrc_3 + c_2)$ and we have said that $c_{3a} + c_{3b} + c_2 < c_3 + c_2$.

real sector disturbances on income is greater than if no ceilings have been imposed.¹⁰⁰ On the other hand, given the presence of ρ_5 on the coefficient that multiplies all elements in the reduced form of income (i.e., $\frac{\tau}{\beta - \tau\rho_5}$), the larger ρ is, the lower the absolute value of that coefficient is and therefore the effect of monetary sector disturbances is lower, which really means that these have a larger impact on Y when there are no ceilings on r_i . A summary of these results is given on Table 4-14.

3. Unborrowed Reserves Policy

Under this policy, the first consequence is a different LM equation than under r^* and Sma policies. See Table 4-15. The LM equation shows a different slope than previously: $\tau_2\rho_2$ in the present policy, against $\tau\rho_5$ in the previous ones. However, we cannot tell which is greater since $\tau_2 > \tau$, but $\rho_5 > \rho_2$.

On the other hand, the error term is different since under RU^* policy, it does not include the disturbance of the currency equation, while it did under the policies analyzed before. These results are the same we obtained in the original model D-1 when we compared the RU^* policy with the other policies.

100. See Appendix for proof that as ρ_5 increases the effect of U_{is} on income, as given by $\frac{-\tau\rho_5}{\beta - \tau\rho_5}$, increases.

Table 4-14. Model D-4: Sma Policy.

Comparison of Cases 1 and 2.

	Case 1 (ceilings)	versus	Case 2 (no ceilings)
	ρ_s case 1	>	ρ_s case 2
Effect of U_{is} on Y	$\frac{-\tau\rho_s}{\beta-\tau\rho_s}$	>	$\frac{-\tau\rho_s}{\beta-\tau\rho_s}$
Effect of U_{LM} on Y	$\frac{-\beta\tau}{\beta-\tau\rho_s}$	<	$\frac{-\beta\tau}{\beta-\tau\rho_s}$
Effect of U_{is} on r	$\frac{-1}{\beta-\tau\rho_s}$	<	$\frac{-1}{\beta-\tau\rho_s}$
Effect of U_{LM} on r	$\frac{-\tau}{\beta-\tau\rho_s}$	<	$\frac{-\tau}{\beta-\tau\rho_s}$

where: $\rho_s = -(rrc_{3a} + rrc_{3b} + c_2)$

Table 4-15. Model D-4: LM Equations under Alternative Policy Procedures.

Policy	LM Equations
r^*	$Y = \tau(Sma + Flt + Tco - Tch - Deg + rrc_{3a} r_i + \rho_s r^* - ER + BOR - U_{LM})$
Sma^*	$Y = \tau(Sma^* + Flt + Tco - Tch - Deg + rrc_{3a} r_i + \rho_s r - ER + BOR - U_{LM})$
RU^*	$Y = \tau_2(RU^* + BOR - ER + \rho_6 r - rrc_{3a} r_i - rrU_{dep})$
MB^*	$Y = \tau(MB^* - ER + \rho_s r + rrc_{3a} r_i - rrU_{dep} - U_{cur})$
where: $\tau = \frac{1}{rrb_3 + b_2} \quad \tau_2 = \frac{1}{rrb_3}$ $\rho_5 = -rr(c_{3a} + c_{3b} - c_2) \quad \rho_6 = (-rrc_{3a} + c_{3b})$ $U_{LM} = rrU_{dep} + U_{cur}$ $\tau_2 > \tau \quad \quad \rho_5 \quad > \quad \quad \rho_6 \quad $	

NOTE: ρ_6 would equal ρ_2 if $c_{3a} + c_{3b} = c_3$.

Source: Appendix.

Determination of the Instrument Value

The value at which RU will be set with the objective of attaining the target income Y^* is:

$$RU^* = Y^* \left(\frac{1}{\tau_2} - \frac{\rho_6}{\beta} \right) - BOR + ER + \frac{\rho_6 \alpha}{\beta} + r r c_{3a} r_i$$

where: $\rho_6 = -rr(c_{3a} + c_{3b})$

Implications of this Procedure

Under this policy, both interest rates and Sma become endogenous variables and their respective reduced form equations are the following:

$$r = \left(\frac{1}{\beta - \tau_2 \rho_6} \right) (\tau_2 RU^* + \tau_2 BOR - \tau_2 ER - \tau_2 r r c_{3a} r_i - \tau_2 r r U_{dep} - \alpha - U_{is})$$

and: $Sma = RU^* - Flt - Tco + c_2 r + b_2 Y + U_{cur} + Tch + Deg$

where: $\tau_2 = 1/rrb_3$ and $\rho_6 = -rr(c_{3a} + c_{3b})$

and r and Y in Sma equation must be replaced by their form equations respectively.

Notice that interest rates are affected by all real sector disturbances but only the deposit demand disturbance from the monetary sector; however Sma, whose role is to offset changes in RU that cause it to deviate from RU^* , is affected by anything that changes r , Y and

also by the currency demand disturbance (U_{cur}). Since U_{cur} affects RU directly, it is prevented from affecting Y and r by the accommodative policy action it generates under the present policy of maintaining RU^* .

Reduced Form of Income

Its reduced form equation is:

$$Y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_6} \right) (\beta RU^* + \beta BOR - \beta ER - \rho_6 \alpha - \rho_6 U_{is} - \beta r r c_{3a} r_i - \beta r r U_{dep})$$

Where we see the same types of disturbances that appear in the interest rate equation. It also shows the importance of LM and IS slopes in determining the relative effects of real sector and monetary sector disturbances respectively. The larger is ρ_6 relative to β , the larger the effect of real sector disturbances as compared to the effect of monetary disturbances.¹⁰¹ On the other hand, the effect of a change in the interest paid on demand deposits will be larger the lower $|\beta|$ is.¹⁰²

Then comparing the cases 1 and 2, since we know they differ in the value of ρ_6 , which is larger in case 1 than in case 2, we must find out what happens to $\partial Y / \partial U_{is}$ and $\partial Y / \partial U_{dep}$ as ρ_6 changes. The conclusion is that:¹⁰³

101. Recall that IS slope is β , and LM slope is $\tau_2 \rho_6$.

102. See proof in Appendix.

103. See proof in appendix.

1. In case 1, the effect of U_{is} on Y is larger than in case 2.
2. In case 1, the absolute value of the effect of U_{dep} on Y is lower than in case 2.
3. In case 1, $r_i = \bar{r}_i$, hence $\Delta r_i = 0$; but in case 2, Δr_i is not necessarily zero and therefore income will be subject to an additional force that may cause it to change.

4. Monetary Base Policy

The first implication of using this procedure is the change in the LM equation, evident from table 4-15 above. The only difference with the LM equation obtained under the unborrowed reserves policy is that now the Borrowings (BOR) are eliminated from the LM relationship. This, of course, is due to the fact that BOR is part of the monetary base and hence it is included in the MB^* term which is the controlled variable under the present policy.

Determination of the instrument value

It turns out to be:

$$MB^* = Y^* \left(\frac{1}{\tau} - \frac{\rho_5}{\beta} \right) + ER - rrc_{3a} r_i + \frac{\rho_5 \alpha}{\beta}$$

Implications of this procedure

Both the interest rate and RU become endogenous variables under this policy. The reduced form equations are:

$$r = \frac{1}{\beta - \tau \rho_5} (\tau MB^* - \tau ER + \tau rrc_{3a} r_i - \tau r r U_{dep} - \tau U_{cur} - \alpha - U_{is})$$

and

$$RU = MB^* - BOR - c_2 r - b_2 Y - U_{cur}$$

where the reduced form equations for r and Y must be substituted into the RU equation.

On the other hand Sma , which is part of RU, will be determined by: 104

$$Sma = MB^* - Flt - Tco + Tch + Deg - Bor$$

Hence, as long as the right hand side elements of this equation are exogenous, Sma will be too. But if any of them, such as BOR in model D-3, is endogenous, then Sma would be endogenous too. This last possibility is considered in model D-5 below.

Reduced Form of Income

Its reduced form is:

$$Y = \frac{\tau}{\beta - \rho_5 \tau} (\beta MB^* - \beta ER - \rho_5 \alpha - \rho_5 U_{is} + \beta rrc_{3a} r_i - \beta r r U_{dep} - \beta U_{cur})$$

The shocks from the demand for money side as well as from the real sector would affect income; however, shocks on the supply side of the monetary sector will not affect it e.g., RU, BOR, etc.

With respect to the comparison between cases 1 and 2, the implications of this reduced form are the same as those we obtained under the RU^* policy, since the role played by τ and β is the same .

MODEL D-5 (Integration of Models D-3 and D-4).

This model incorporates the assumptions of models D-3 (money supply interest-elastic) and D-4 (interest-bearing demand deposits) together, as is shown in Table 4-16. The assumption of an interest-elastic money supply delivered an LM relationship that is flatter than the one in the original model D-1. On the other hand, the interest bearing demand deposit assumption caused a steeper LM than the one obtained under the D-1 model. Now, when we combine both assumptions, the latter tends to offset the effect of the former and therefore the resulting LM slope may be greater, lower or equal than the one under the original model specification.

However, we can surely say that under this D-5 model, the LM will be flatter than under D-4, since the interest elastic money supply offsets the lower interest-elasticity of the demand for money that is due to the fact that demand deposits bear interest. Similarly, the LM in model D-5 will be steeper than in model D-3 due to such demand for

Table 4-16. Model D-5: Specification of Structural Equations.

Real Sector:

$$Y = C + I + G$$

$$C = A_1 + b_1 Y + U_c$$

$$I = A_2 + c_1 r + U_i$$

—

Monetary Sector:

$$MS = CUR + D$$

$$CUR = c_2 r + b_2 Y + U_{CUR}$$

$$D = c_3 r + b_3 Y + U_{dep}$$

$$RT = RU + BOR$$

$$RT = RR + ER$$

$$RR = rr D$$

$$MB = RT + CUR$$

$$RU = Sma + Flt + Tco + CUR - Tch - Deg$$

$$MB = RT + CUR$$

$$BOR = c_4 (r - r_d) + U_{bor}$$

$$ER = c_5 r + U_{er}$$

where: $A_1, A_2, > 0$ $b_1 > 0$ $c_1 < 0$

$$c_2, c_{3a}, c_{3b} < 0$$

$$b_2, b_{2a}, b_{2b} > 0 \text{ but } < 1$$

$U_c, U_i, U_{cur}, U_{ndd}, U_{idd}, U_{bor}, U_{er}$ are disturbance terms.

money assumption. Other implications of the present model specification will be seen in the reduced form equation for income and for the endogenous monetary variables, under each alternative policy procedure.

Because this model is a summary of the previous two, we will present the analysis in a more synthetic way. Table 4-17 shows the resulting LM equations under each policy. The first difference between them has to do with the LM slope which is the same ($\tau\rho_7$) under the first two policies but then, under the RU^* policy, it becomes $\tau_2\rho_8$; and finally, under the MB^* policy it is $\tau\rho_9$. (See Table 4-18 too). From both tables 4-17 and 4-18 we can see that a steeper LM arises when the monetary variable used is the monetary base (MB^*), as opposed to r^* or Sma . However, comparing RU^* and MB^* , the outcome will depend upon the magnitude of the coefficients c_2 and c_4 :

$$\tau_2\rho_8 > \tau\rho_9 \quad \text{if} \quad |c_2| < c_4$$

But if $|c_2|$ is greater than c_4 , we cannot say with certainty which slope is greater. The key coefficients in this latter comparison, c_2 and c_4 , are crucial within the present model since c_2 is the interest-elasticity of the demand for currency and c_4 is the interest elasticity of borrowings. Under RU^* , c_4 offsets the greater slope of the LM caused by the less elastic demand deposit relationship; and under MB^* , the interest-elasticity of currency plays such role, since the role of BOR is eliminated given that it is included in the

Table 4-17. Model D-5: LM Equations under Alternative Monetary Policies.

Policy	LM Equations
r^*	$Y = \tau(Sma + Flt + Tco - Tch - Deg + \rho_7 r^* - c_4 rd + rrc_{3b} r^i + U_{bor} - rru_{ndd} - rru_{idd} - U_{er} - U_{cur})$
Sma^*	$Y = \tau(Sma + Flt + Tco - Tch - Deg + \rho_7 r^* - c_4 rd + rrc_{3b} r^i + U_{bor} - rru_{ndd} - rru_{idd} - U_{er} - U_{cor})$
RU^*	$Y = \tau_2(RU^* + \rho_8 r^* - c_4 rd + rrc_{3b} r^i + U_{bor} - rru_{ndd} - rru_{idd} - U_{er})$
MB^*	$Y = \tau(MB^* + \rho_9 + rrc_{3b} r^i - rru_{ndd} - rru_{idd} - U_{er} - U_{cur})$

where:

$$\tau = \frac{1}{b_2 + rrb_3} \text{ assuming } b_3 = b_{3a} + b_{3b}$$

$$\tau_2 = \frac{1}{rrb_3} \text{ assuming } b_3 = b_{3a} + b_{3b}$$

$$\rho_7 = -(rrc_{3a} + rrc_{3b} - c_4 + c_5 + c_2) > 0$$

$$\rho_8 = -(rrc_{3a} + rrc_{3b} - c_4 + c_5) > 0$$

$$\rho_9 = -(rrc_{3a} + rrc_{3b} + c_5 + c_2) > 0$$

and

$$\rho_8 < \rho_9$$

$$\rho_9 < \rho_7$$

$$\rho_8 > \rho_9 \text{ if } |c_2| < c_4$$

Table 4-18. LM Slopes ($\partial Y/\partial r$) for Models D-1, D-2, D-3, D-4 and D-5 under Alternative Policies.

Policy	Model D-1	Model D-2	Model D-3	Model D-4	Model D-5
r^*	$\tau\rho$	0	$\tau\rho_2$	$\tau\rho_5$	$\tau\rho_7$
Sma^*	$\tau\rho$	0	$\tau\rho_2$	$\tau\rho_5$	$\tau\rho_7$
RU^*	$\tau_2\rho_1$	0	$\tau_2\rho_3$	$\tau_2\rho_6$	$\tau_2\rho_8$
MB^*	$\tau\rho$	0	$\tau\rho_4$	$\tau\rho_5$	$\tau\rho_9$

where:

$$\tau = 1/(rrb_3 + b_2)$$

$$\tau_2 = 1/rrb_3$$

$$\rho = -(rrc_3 + c_2)$$

$$\rho_1 = -rrc_3$$

$$\rho_2 = -(rrc_3 + c_5 + c_2 - c_4)$$

$$\rho_3 = -(rrc_3 + c_5 - c_4)$$

$$\rho_4 = -(rrc_3 + c_5 + c_2)$$

$$\rho_5 = -(rr(c_{3a} + c_{3b}) + c_2) \quad c_{3a} + c_{3b} < c_3$$

$$\rho_6 = -(rr(c_{3a} + c_{3b}))$$

$$\rho_7 = -(rrc_{3a} + rrc_{3b} - c_4 + c_5 + c_2)$$

$$\rho_8 = -(rrc_{3a} + rrc_{3b} - c_4 + c_5)$$

$$\rho_9 = -(rrc_{3a} + rrc_{3b} + c_5 + c_2)$$

$$\rho_2 > \rho, \rho_2 > \rho_1$$

$$\rho_2 > \rho_3, \rho_2 > \rho_4$$

$$\rho_2 > \rho_5, \rho_5 > \rho_6$$

$$\rho_9 > \rho_5, \rho_8 < \rho_3$$

$$\rho_8 > \rho_6, \rho_8 < \rho_7$$

$$\rho_9 > \rho_6, \rho_9 < \rho_7$$

$$\rho_8 > \rho_9 \text{ if } c_4 > c_2 \mid \text{ and vice versa.}$$

instrumental variable MB.

Another difference in the LM equations has to do with the error terms that appear in each of them. Under the unborrowed reserves policy, U_{cur} does not enter the LM, while under the monetary base policy, it is U_{bor} which is canceled. Therefore, as we move from one policy to another, certain disturbances are not part of the picture any more; this may have important implications when a choice has to be made, since a careful analysis of the monetary relationships working in the economy would be necessary in order to choose the policy that cancels out the effect of the shocks that are more likely to affect the economy. This means that the stability of the relationships should be researched to establish which of them are the least stable (i.e., more subject to shocks or unpredicted changes).

Finally, table 4-19 shows the reduced form equations for the endogenous variables under each policy procedure. The results can be summarized as follows:

1. Under an interest rate policy the effect of monetary shocks on income is eliminated. However, these shocks do affect the endogenous monetary variable S_{ma} and hence RU and MB.
2. Under the Security Portfolio policy, the effect of real sector shocks on income is not as large as it was under the interest rate policy; but now, monetary sector shocks also affect income and of course, interest rates. Unborrowed reserves and the monetary base are also endogenous, affected by all factors

Table 4-19. Model D-5: Reduced Form Equations under Alternative Monetary Policies.

Policy	Reduced Form Equations
r^*	$Sma = \frac{\alpha}{1} + \left(\frac{\beta}{1} - \rho_7 \right) - Flt - Tco + Tch + Degc_{rd}$ $- rrc_{3b1} - \frac{U_{1s}}{1} - U_{bor} + rru_{ndd} + rru_{idd} + U_{er} + U_{cur}$ $Y = \alpha + \beta r^* + U_{1s}$
Sma^*	$r = \left(\frac{1}{\beta - \tau_1 \rho_7} \right) \left(\tau_1 Sma + \tau_1 Flt - \tau_1 Deg - \tau_1 Tco + \tau_1 Tch - \tau_1 c_{rd} - \alpha + \tau_1 rrc_{3b1} \right.$ $\left. - U_{1s} + \tau_1 U_{bor} - \tau_1 rru_{ndd} - \tau_1 rru_{idd} - \tau_1 U_{er} - \tau_1 U_{cur} \right)$ $Y = \left(\frac{\tau}{\beta - \tau_1 \rho_7} \right) \left(\beta Sma + \beta Flt - \beta Tco + \beta Tch - \beta Deg - \rho_7 \alpha - \beta c_{rd} \right.$ $\left. + \beta rrc_{3b1} - \rho_7 U_{1s} + \beta U_{bor} - \beta rru_{ndd} - \beta rru_{idd} - \beta U_{er} - \beta U_{cur} \right)$
RU^*	$r = \left(\frac{1}{\beta - \tau_2 \rho_8} \right) \left(\tau_2 RU - \tau_2 c_{rd} - \alpha + \tau_2 rrc_{3b1} - U_{1s} + \tau_2 U_{bor} \right.$ $\left. - \tau_2 rru_{ndd} - \tau_2 rru_{idd} - \tau_2 U_{er} \right)$ $Y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_8} \right) \left(\beta RU - \rho_8 \alpha - \rho_8 U_{1s} - \beta c_{rd} + \beta rrc_{3b1} + \beta U_{bor} \right.$ $\left. - \beta rru_{ndd} - \beta rru_{idd} - \beta U_{er} \right)$
MB^*	$r = \left(\frac{1}{\beta - \tau_3 \rho_9} \right) \left(\tau_3 MB - \tau_3 rru_{ndd} - \tau_3 rru_{idd} - \tau_3 U_{er} - \tau_3 U_{cur} - \alpha \right.$ $\left. - U_{1s} + \tau_3 rrc_{3b1} \right)$ $Y = \left(\frac{\tau}{\beta - \tau_3 \rho_9} \right) \left(\beta MB - \rho_9 \alpha - \beta rrc_{3b1} - \rho_9 U_{1s} - \beta U_{er} - \beta U_{cur} \right.$ $\left. - \beta rru_{idd} - \beta rru_{ndd} \right)$

affecting income and interest rates.

3. Under the unborrowed reserves and monetary base policies, the larger the LM slope is (τ_2^0 and τ_0 respectively), the greater is the effect of real sector shocks on income and the smaller is their effect on interest rates.¹⁰⁵
4. Under RU^* and MB^* , the effect of monetary shocks on income is greater the greater the absolute value of β is, that is, the flatter the IS is.¹⁰⁶
5. We must notice the presence of the discount rate in the reduced form equations for income and interest rates under the unborrowed reserves and the security portfolio policies. As we said with respect to model D-3, this may give the authorities the possibility of using a combined type of policy such as an Sma/r_d or an RU/r_d policy in order to attain the target Y^* .
6. Besides the discount rate, the rate paid on demand deposits can also be used as part of the policy procedure, under a model structure such as D-4 or D-5.

105 Recall that $\partial Y / \partial U_{is} = \frac{\tau_2^0}{\beta - \tau_2^0}$ and $\partial r / \partial U_{is} = \frac{-1}{\beta - \tau_2^0}$

106 See proof in Appendix.

Finally, it would be interesting to take a few lines to analyze and compare cases 1 and 2 under the assumption that in case 2 (no regulation), interest paid on deposits are a proportion of the market interest rates, such as:

$$r_{it} = pr_t \text{ where } 0 < p < 1, \text{ for case 2.}$$

and $r_{it} = \bar{r}_{it}$, for case 1.

If this is the case, the reduced form equations for income are those in table 4-20, which lead to the following conclusions:

1. If an unborrowed reserves policy is followed, the effect of U_{dep} shocks on income is larger under case 2 than under case 1 if it is true that $r_i = pr$. This is so because the multiplier has been enlarged with de-regulation. This result is consistent with those we obtained earlier when we assumed that ρ_2 is larger under case 1 than under case 2. Therefore, both analyses strengthen the conclusion that U_{dep} shocks have stronger effects on income under a de-regulated system (i.e., no ceilings).
2. Under the RU^* policy and the present assumption, the effect of real sector shocks has also been enlarged.¹⁰⁷ However, we

107. This is not so obvious, hence the mathematical proof is given in the Appendix.

Table 4-20. Model D-5: Reduced Form Equations for Income under r^* and RU^* Assuming that in the De-regulation Case (case 2), Interest Paid on Deposits (r_i) is a Proportion (P) of Market Interest Rates (r).

Policy	Reduced Form Equations	
	Case 1	Case 2
r^*	$Y = \alpha + \beta r^* + U_{is}$	$Y = \alpha + \beta r^* + U_{is}$
RU^*	$Y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_2} \right) (\beta RU^* + \beta BOR - \beta rr U_{dep} + \rho_2 U_{is} + \rho_2 \alpha)$	$Y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_2 - \tau_2 rrc_{3a} P} \right) (\beta RU^* + \beta BOR - \beta ER - \beta rr U_{dep} + (\rho_2 + rrc_{3a} P) U_{is} + (\rho_2 + rrc_{3a} P) \alpha)$

Source: Appendix.

previously concluded that in case 1, the effects of U_{is} on income are larger than in case 2 due to ρ_2 which is larger in the former. So, the two assumptions (de-regulation plus $r_i=pr$) together imply that the lower ρ_2 caused by the first assumption is offset by the new assumption $r_i=pr$, with the result that effects of U_{is} under case 2 have been enlarged.

However, the relative importance of the new effect will determine if despite it the effect of U_{is} on income is still greater under case 1 than under 2 or if the effect under case 2 has become greater (See the coefficient for U_{is} under both cases in Table 4-20).

3. If an interest rate policy is followed, it does not make any difference that $r_i=pr$, since the effect of monetary sector shocks on income is still zero, and that of real sector shocks remains as before.

When using the interest rate as the instrument, the IS equation represents also the reduced form equation for income, under any of the models considered. Under this case, the security portfolio becomes the endogenous monetary variable, as well as reserves and the monetary base.

When using any one of the monetary aggregates as instruments, S_{ms} , M_1 or M_2 , the interest rate becomes endogenous and hence its reduced form equation is shown. The other monetary aggregates, not

II. ANALYTICAL SIMULATION OF SHOCKS

A. SUMMARY OF RESULTS: MODELS D-1 TO D-5.

Table 4-21 presents the reduced form equations for income and for the resulting endogenous monetary variable, under each of the alternative policies. As we know by now, the models we have considered in this chapter allow us to analyze, on the money supply side, policies that we could not consider with the aggregated models of chapter 3. Here we have the interest rate policy (r^*), the security portfolio policy (Sma^*), the unborrowed reserves policy (RU^*) and the monetary base policy (MB^*).

When using the interest rate as the instrument, the IS equation represents also the reduced form equation for income, under any of the models considered. Under this case, the security portfolio becomes the endogenous monetary variable, as well as reserves and the monetary base.

When using any one of the monetary aggregates as instruments, Sma , RU or MB , the interest rate becomes endogenous and hence its reduced form equation is shown. The other monetary aggregates, not

Table 4-21. Summary of Results.

Model	Policy	Reduced Form Equations
D-1 (original)	r^*	$Y = \alpha + \beta r^* + U_{is}$ $Sma = \left(\frac{\beta}{\tau} \right) r^* + \frac{\alpha}{\tau} - (Flt + Tco - Tch + BOR - Deg - ER) + \frac{U_{is}}{\tau} + U_{LM}$ where: $U_{is} = \frac{Uc + Ui}{1 - b_1} U_{LM} = rrU_{dep} + U_{cur}$ $\rho = - (rrc_3 + c_2) \quad \tau = 1 / (rrb_3 + b_2)$
	Sma^*	$Y = \left(\frac{\tau}{\beta - \tau \rho} \right) \left(\beta Sma + \beta (Flt + Tco - Tch - Deg + BOR - ER) - \rho \alpha - \rho U_{is} - \beta U_{LM} \right)$ $r = \left(\frac{1}{\beta - \tau \rho} \right) \left(\tau Sma + \tau (Flt + Tco - Tch - Deg) - \alpha - U_{is} - \tau U_{LM} + \tau BOR - \tau ER \right)$
	RU^*	$Y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_1} \right) \left(\beta RU + \beta BOR - \beta ER - \rho_1 \alpha - \rho_1 \alpha - \rho_2 U_{is} - \beta rrU_{dep} \right)$ $r = \left(\frac{1}{\beta - \tau_2 \rho_1} \right) \left(\tau_2 RU + \tau_2 (BOR - ER) - \alpha - U_{is} - \tau_2 rrU_{dep} \right)$ where $\rho_1 = -rrc_3 \quad \tau_2 = 1 / (rrb_3 \quad \tau_2 > \tau \quad \rho > \rho_1)$
	MB^*	$Y = \left(\frac{\tau}{\beta - \tau \rho} \right) \left(\beta MB + \beta ER - \beta U_{LM} - \rho \alpha - \rho U_{is} \right)$ $r = \left(\frac{1}{\beta - \tau \rho} \right) \left(\tau MB + \tau ER - \alpha - \tau U_{LM} - U_{is} \right)$ where $U_{LM} = rrU_{dep} + U_{cur}$

Table 4-21. Summary of Results. (continued)

Model	Policy	Reduced Form Equations
D-2 (interest- inelastic demand for money).	r^*	$Y = \alpha + \beta r^* + U_{is}$ $Sma = \frac{\beta}{\tau} r^* + \frac{\alpha}{\tau} + \frac{U_{is}}{\tau} - BOR - Flt - Tco + Tch + Cur + Deg + ER + U_{LM}$
	Sma^*	$Y = \tau (Sma^* + Flt - Deg + BOR + Tco - Tch - ER - U_{LM})$ $r = \frac{\tau}{\beta} Sma^* + \frac{\tau}{\beta} \left(BOR + Flt - Tco - Tch - Deg - ER - \frac{\alpha}{\tau} - \frac{U_{is}}{\tau} - U_{LM} \right)$ $\text{where } U_{LM} = U_{cur} + rrU_{dep} \quad \tau = \frac{1}{rrb_3 + b_2}$
	RU^*	$r = \frac{\tau_2 RU^*}{\beta} + \frac{\tau_2 (BOR - ER - RU_{dep})}{\beta} - \frac{\alpha}{\beta} - \frac{U_{is}}{\beta}$ $\text{where } \tau_2 = 1/rrb_3 > 0$ $Y = \tau_2 (RU^* + \tau_2 BOR - \tau_2 rrU_{dep} - \tau_2 ER)$
	MB^*	$r = \frac{\tau_{MB}^*}{\beta} - \frac{\tau_{ER}}{\beta} - \frac{\alpha}{\beta} - \frac{\tau}{\beta} U_{LM} - \frac{U_{is}}{\beta}$ $\text{where } U_{LM} = U_{cur} + rrU_{dep}$ $Y = \tau_{MB}^* - \tau_{ER} - \tau U_{LM}$

Table 4-21. Summary of Results. (continued)

Model	Policy	Reduced Form Equations
D-3	r^*	$Y = \alpha + \beta r^* + U_{IS}$ $Sma = \left(\frac{\beta}{1} - \rho_2 \right) r^* - Fit + Deg + c_4 rd + \frac{\alpha}{1} + \frac{U_{IS}}{1} + U_{LM}^1$ where: $\rho_2 = -(rrc_3 + c_5 + c_2 - c_4)$ $U_{LM}^1 = rrU_{dep} + U_{er} + U_{bor} - U_{cur}$ $U_{IS} = (Uc + U_i) / (1 - b_1)$
	Sma^*	$Y = \left(\frac{1}{\beta \tau \rho_2} \right) \left(\beta Sma + \beta Fit - \beta Deg - \rho_2 \alpha - \beta c_4 rd - \rho_2 U_{IS} - \beta U_{LM}^1 \right)$ $r = \left(\frac{1}{\beta - \tau_2 \rho_3} \right) \left(\tau Sma + \tau Fit - \tau Deg - \tau c_4 rd - \alpha U_{IS} - \tau U_{LM}^1 \right)$
	RU^*	$Y = \left(\frac{1}{\beta - \tau_2 \rho_3} \right) \left(\beta RU - \beta c_4 rd - \rho_3 \alpha - \rho_3 U_{IS} - \beta U_{LM}^2 \right)$ $r = \left(\frac{1}{\beta - \tau_2 \rho_3} \right) \left(\tau RU - \tau c_4 rd - \alpha U_{IS} - \tau U_{LM}^2 \right)$ where $U_{LM}^2 = rrU_{dep} + U_{er} - U_{bor}$ $\rho_3 = -(rrc_3 + c_5 - c_4)$ $\tau_2 = 1 / rr b_3$
	MB^*	$Y = \left(\frac{1}{\beta - \tau \rho_4} \right) \left(\beta MB - \rho_4 \alpha - \rho_4 U_{IS} - \beta U_{LM}^3 \right)$ $r = \left(\frac{1}{\beta - \tau \rho_4} \right) \left(\tau MB - \tau rrU_{dep} - \tau U_{cur} - \tau U_{er} - \alpha U_{IS} \right)$ where: $\rho_4 = -(rrc_3 + c_5 + c_2)$ $U_{LM}^3 = rrU_{dep} + U_{cur} + U_{er}$

Table 4-21. Summary of Results. (continued)

Model	Policy	Reduced Form Equations
D-4 Interest-bearing demand deposits (ceilings and no ceilings) ¹	r^*	$Y = \alpha + \beta r^* + U_{1S}$ $Sma = \left(\frac{\beta - \rho_s}{\tau} \right) r^* + \frac{\alpha}{\tau} + \frac{U_{1S}}{\tau} + (ER - BOR - Flt - Tco + Tch + Deg) + rrU_{dep}^{cur} + U_{LM}$
	Sma^*	$Y = \left(\frac{1}{\beta - \tau \rho_s} \right) (\beta Sma + \beta Flt + \beta Tco - \beta Tch - \beta Deg + \beta rrc_{3a} r_i - \rho_s \alpha - \rho_s U_{1S} - \beta ER + \beta BOR - \beta U_{LM})$ $r = \left(\frac{1}{\beta - \tau \rho_s} \right) (\tau Sma + \tau Flt + \tau Tco - \tau Tch - \tau Deg + \tau rrc_{3a} r_i - \tau ER + \tau BOR - \tau U_{LM} - \alpha - U_{1S})$ where: $\rho_s = - (rrc_{3a} + c_{3b} + c_2)$ $U_{LM} = rrU_{dep} + U_{cur} + rrU_{idd} + rrU_{ndd}$
	RU^*	$Y = \left(\frac{1}{\beta} \right) (\beta RU + \beta BOR - \beta ER - \rho_s \alpha - \rho_s U_{1S} - \beta rrc_{3a} r_i - \beta rrU_{dep})$ $r = \left(\frac{1}{\beta - \tau \rho_s} \right) (\tau_2 RU + \tau_2 BOR - \tau_2 ER - \tau_2 rrc_{3a} r_i - \tau_2 rrU_{dep} - \alpha - U_{1S})$ where: $\tau_2 = 1/rrb_3$ $\rho_s = - (rrc_{3a} + c_{3b})$
	MB^*	$Y = \left(\frac{1}{\beta - \tau \rho_s} \right) (\beta MB + \beta ER - \rho_s \alpha - \rho_s U_{1S} + \beta rrc_{3a} r_i - \beta U_{LM})$ $r = \left(\frac{1}{\beta - \tau \rho_s} \right) (\tau MB + \tau ER + \tau rrc_{3a} r_i - \tau U_{LM} - \alpha - U_{1S})$ where: $U_{LM} = rrU_{dep} + U_{cur}$

1. The difference between the ceilings and no ceilings cases are not shown in this table. The real form equations are the same; it is the value of some coefficients that would change, mainly ρ .

Table 4-21. Summary of Results. (continued)

Model	Policy	Reduced Form Equations
D-5	r^*	$Y = \alpha + \beta r^* + U_{1s}$ $Sma = \frac{\alpha}{\tau} + \left(\frac{\beta}{\tau} - \rho \right) r^* - Flt - Tco + Tch + Deg + c_r d$ $- rrc_{3b} r_i + \frac{U_{1s}}{\tau} - U_{LM}^*$ where: $U_{LM}^* = U_{bor} - rrU_{ndd} - rrU_{idd} - U_{er} - U_{cur}$
	Sma^*	$Y = \left(\frac{\tau}{\beta - \tau \rho} \right) (\beta Sma + \beta (Flt - Tco + Tch - Deg) - \rho - \beta c_r d + \beta rrc_{3b} r_i - \rho) U_{1s} + \beta U_{LM}^*$ $r = \left(\frac{1}{\beta - \tau \rho} \right) (\tau Sma + \tau (Flt - Tco + Tch - Deg) - \tau c_r d - \alpha + rrc_{3b} r_i - U_{1s} + \tau U_{LM}^*)$
	RU^*	$Y = \left(\frac{\tau_2}{\beta - \tau_2 \rho_0} \right) (\beta RU - \rho_0 \alpha - \rho_0 U_{1s} - \beta c_r d + \beta rrc_{3b} r_i + \beta U_{LM}^*)$ $r = \left(\frac{1}{\beta - \tau_2 \rho_0} \right) (\tau_2 RU - \tau_2 c_r d - \alpha + \tau_2 rrc_{3b} r_i - U_{1s} + \tau_2 U_{LM}^*)$ where: $U_{LM}^* = (U_{bor} - rrU_{ndd} - rrU_{idd} - U_{er})$
	MB^*	$Y = \left(\frac{\tau}{\beta - \tau \rho_9} \right) (\beta MB - \rho_9 \alpha - \beta rrc_{3b} r_i - \rho_9 U_{1s} + \beta U_{LM}^*)$ $r = \left(\frac{1}{\beta - \tau \rho_9} \right) (\tau MB + \tau U_{LM}^* - \alpha - U_{1s} + rrc_{3b} r_i)$ where: $U_{LM}^* = -U_{cur} - rrU_{idd} - rrU_{ndd} - U_{er}$

used as instruments under a specific policy, may or may not be endogenous, depending on the policy under effect.

Model D-1 is our starting point where we explicitly consider the sources and uses of reserves. Model D-2 introduces the assumption of an interest inelastic money demand function, with important implications for the multipliers of autonomous variables and disturbances as is shown in the respective equations. Then model D-3 introduces behavioral equations for Borrowings and Excess Reserves, with the resulting new LM slope and the inclusion of new types of disturbances in the reduced form equations. Then model D-4 introduces the possibility of demand deposits that yield interest, whose main effect is to change the LM slope, making it steeper. Finally, model D-5 combines the assumptions considered in D-3 and D-4 with the result of a different LM slope, steeper than under D-3 but flatter than under D-4. As we will prove below, the LM slope $(\tau_i \rho_j)$ plays a very important role as it becomes part of the income multiplier as well as of the multiplier of the endogenous monetary variable.

B. EFFECT OF EACH TYPE OF SHOCK, SAME PROCEDURE BUT DIFFERENT MODELS

1. Interest Rate Policy

Interest Rate Policy: Effect of Shocks on Income

Under this policy, only real sector shocks affect income and such effect is of the same magnitude for any of the models considered. See the "Interest Rate Policy" column in tables 4-22, 4-23, 4-24, 4-25 and 4-26.

Interest Rate Policy: Effect of shocks on the Security Portfolio.

Since the interest rate is used as the instrument, open market operations will become endogenous, to be determined by the economy's parameters and autonomous variables. We then proceed to analyze the reduced form equation for the authority's security portfolio (S_{ma}), under the different models. Table 4-22 shows that the effect of real sector shocks on S_{ma} are the same in all the models. Then tables 4-23 and 4-24 show the effects of U_{cur} and U_{dep} , the two types of shocks directly related to the public's demand for money, whose effects on S_{ma} are 1 and rr , respectively, in all models. This means that a shock on the demand for currency has a greater impact than a shock on the demand for deposits under an interest rate policy (assuming, of course, that the reserve requirement ratio is less than one). However, this also means that a larger reserve requirement ratio would cause demand

Table 4-22. Chapter 4 Summary. Effect of Real Sector Disturbances under Different Monetary Policy Procedures.

Model	Interest Rate Policy	Security Portfolio Policy	Unborrowed Reserves Policy	Monetary Base Policy
D-1 original model	$\frac{\Delta Y}{\Delta U_C} = \frac{1}{1-b_1}$	$\frac{\Delta Y}{\Delta U_C} = \left(\frac{\tau}{\beta - \tau\rho} \right) \left(-\frac{\rho}{1-b_1} \right) > 0$	$\frac{\Delta Y}{\Delta U_C} = \left(\frac{\tau_2}{\beta - \tau_2\rho_1} \right) \left(-\frac{\rho_2}{1-b_1} \right)$	$\frac{\Delta Y}{\Delta U_C} = \left(\frac{\tau}{\beta - \tau\rho} \right) \left(-\frac{\rho}{1-b_1} \right)$
	$\frac{\Delta S_{ma}}{\Delta U_C} = \frac{1}{(1-b_1)\tau}$ where $\tau = 1/(rrb_3 + b_2)$	$\frac{\Delta r}{\Delta U_C} = -\frac{1}{(\beta - \tau\rho)(1-b_1)} > 0$ $\rho = -(rrc_3 + c_2)$	$\frac{\Delta r}{\Delta U_C} = -\frac{1}{(\beta - \tau_2\rho_1)(1-b_1)}$ $\rho_1 = -rrc_3$ $\tau_2 = 1/rrb_3$	$\frac{\Delta r}{\Delta U_C} = -\frac{1}{(\beta - \tau\rho)(1-b_1)}$
D-2	$\frac{\Delta Y}{\Delta U_C} = \frac{1}{1-b_1}$ $\frac{\Delta S_{ma}}{\Delta U_C} = \frac{1}{(1-b_1)\tau}$	$\frac{\Delta Y}{\Delta U_C} = 0$ $\frac{\Delta r}{\Delta U_C} = -\frac{1}{\beta(1-b_1)}$	$\frac{\Delta Y}{\Delta U_C} = 0$ $\frac{\Delta r}{\Delta U_C} = -\frac{1}{\beta(1-b_1)}$	$\frac{\Delta Y}{\Delta U_C} = 0$ $\frac{\Delta r}{\Delta U_C} = -\frac{1}{\beta(1-b_1)}$

Table 4-22. (continued)

Model	Interest Rate Policy	Security Portfolio Policy	Unborrowed Reserves Policy	Monetary Base Policy
D-3 (Money supply interest-elastic)	$\frac{\Delta Y}{\Delta U_c} = \frac{1}{1-b_1}$ $\frac{\Delta S_{ma}}{\Delta U_c} = \frac{1}{\tau(1-b_1)}$	$\frac{\Delta Y}{\Delta U_c} = \left(\frac{\tau}{\beta - \tau \rho_2} \right) \left(\frac{-\rho_2}{1-b_1} \right) > 0$ $\frac{\Delta r}{\Delta U_c} = \left(\frac{1}{\beta - \tau \rho_2} \right) \left(\frac{-1}{1-b_1} \right) > 0$ $\rho_2 = - (rrc_3 + c_5 c_2 - c_4)$	$\frac{\Delta Y}{\Delta U_c} + \left(\frac{\tau_2}{\beta - \tau_2 \rho_3} \right) \left(\frac{-\rho_3}{1-b_1} \right)$ $\frac{\Delta r}{\Delta U_c} = \left(\frac{1}{\beta - \tau_2 \rho_3} \right) \left(\frac{-1}{1-b_1} \right)$ $\rho_3 = 0 \quad (rrc_3 + c_5 - c_4)$ $\tau_2 = 1/rrb_3$	$\frac{\Delta Y}{\Delta U_c} - \left(\frac{\tau}{\beta - \tau \rho_4} \right) \left(\frac{-\rho_4}{1-b_1} \right)$ $\frac{\Delta r}{\Delta U_c} = \left(\frac{1}{\beta - \tau \rho_4} \right) \left(\frac{-1}{1-b_1} \right)$ $\rho_4 = - (rrc_3 + c_5 + c_2)$
D-4 (cases 1 and 2) Interest-bearing demand deposits.	$\frac{\Delta Y}{\Delta U_c} = \frac{1}{1-b_1}$ $\frac{\Delta S_{ma}}{\Delta U_c} = \frac{1}{\tau(1-b_1)}$	$\frac{\Delta Y}{\Delta U_c} = \left(\frac{\tau}{\beta - \tau \rho_5} \right) \left(\frac{-\rho_5}{1-b_1} \right) > 0$ $\frac{\Delta r}{\Delta U_c} = \frac{-1}{(\beta - \tau \rho_5)(1-b_1)}$ $\rho_5 = - (rrc_3 + c_5 + c_2)$	$\frac{\Delta Y}{\Delta U_c} = \left(\frac{\tau_2}{\beta - \tau_2 \rho_6} \right) \left(\frac{-\rho_6}{1-b_1} \right)$ $\frac{\Delta r}{\Delta U_c} = \frac{-1}{(\beta - \tau_2 \rho_6)(1-b_1)}$ $\rho_6 = - (rrc_3 + c_5 + c_2)$	$\frac{\Delta Y}{\Delta U_c} - \left(\frac{\tau}{\beta - \tau \rho_5} \right) \left(\frac{-\rho_5}{1-b_1} \right)$ $\frac{\Delta r}{\Delta U_c} = \frac{-1}{(\beta - \tau \rho_5)(1-b_1)}$ $\rho_5 = - (rrc_3 + c_5 + c_2)$

Table 4-23. Chapter 4 Summary: Effect of Disturbances to the Model for Currency (D) under Different Monetary Policy Frameworks

Model	Interest rate policy	Security portfolio policy	Unborrowed reserves policy	Monetary Base Policy
D-1	$\Delta Y / \Delta U_C = 0$	$- \tau / (\beta - \tau \rho_7) < 0$	0	$- \beta \tau / (\beta - \tau \rho_7) < 0$
D-2	$\Delta Y / \Delta U_C = 0$	$- \tau / (\beta - \tau \rho_7) > 0$	0	$- \tau / (\beta - \tau \rho_7) > 0$
D-3	$\Delta Y / \Delta U_C = 0$	0	0	0
D-4	$\Delta Y / \Delta U_C = 0$	0	0	0
D-5	$\Delta Y / \Delta U_C = \frac{1}{1-b_1}$ $\Delta Sma / \Delta U_C = \frac{1}{(1-b_1)\tau}$	$\Delta Y / \Delta U_C = \left(\frac{\tau}{\beta - \tau \rho_7} \right) \left(\frac{-\rho_7}{1-b_1} \right)$ $\Delta r / \Delta = \left(\frac{-1}{\beta - \tau \rho_7} \right) \left(\frac{1}{1-b_1} \right)$ $\rho_7 = - (rrc_{3a} + rrc_{3b} - c_4 + c_5 + c_2)$	$\Delta Y / \Delta U_C = \left(\frac{\tau_2}{\beta - \tau_2 \rho_8} \right) \left(\frac{-\rho_8}{1-b_1} \right)$ $\Delta r / \Delta U_C = \frac{-1}{(\beta - \tau_2 \rho_8)(1-b_1)}$ $\rho_8 = - (rrc_{3a} + rrc_{3b} - c_4 + c_5)$	$\Delta Y / \Delta U_C = \left(\frac{\tau}{\beta - \tau \rho_9} \right) \left(\frac{-\rho_9}{1-b_1} \right)$ $\Delta r / \Delta U_C = \frac{-1}{(\beta - \tau \rho_9)(1-b_1)}$ $\rho_9 = - (rrc_{3a} + rrc_{3b})$

Table 4-22. (continued)

Table 4-23. Chapter 4 Summary. Effect of Disturbances to the Demand for Currency (U_{cur}) under Different Monetary Policy Procedures.

Model:	Interest rate policy	Security portfolio policy	Unborrowed reserves policy	Monetary Base Policy
D-1	ΔY 0	$-\tau\beta/(\beta-\tau\rho) < 0$	0	$-\beta\tau/(\beta-\tau\rho) < 0$
	Δr 0	$-\tau/(\beta-\tau\rho) > 0$	0	$-\tau/(\beta-\tau\rho) > 0$
	ΔSma 1	0	1	0
D-2	ΔY 0	$-\tau < 0$	0	$-\tau < 0$
	Δr 0	$-\tau/(\beta) > 0$	0	$-\tau/\beta > 0$
	ΔSma 1	0	1	0
D-3	ΔY 0	$-\beta\tau/(\beta-\tau\rho_s) < 0$	0	$-\beta\tau/(\beta-\tau\rho_s) < 0$
	Δr 0	$-\tau/(\beta-\tau\rho_s) > 0$	0	$-\tau/(\beta-\tau\rho_s) > 0$
	ΔSma 1	0	1	0
D-4	ΔY 0	$-\beta\tau/(\beta-\tau\rho_s) < 0$	0	$-\beta\tau/(\beta-\tau\rho_s) < 0$
	Δr 0	$-\tau/(\beta-\tau\rho_s) > 0$	0	$-\tau/(\beta-\tau\rho_s) > 0$
	ΔSma 1	0	1	0
D-5	ΔY 0	$-\beta\tau/(\beta-\tau\rho_s) < 0$	0	$-\beta\tau/(\beta-\tau\rho_s) < 0$
	Δr 0	$-\tau/(\beta-\tau\rho_s) > 0$	0	$-\tau/(\beta-\tau\rho_s) > 0$
	ΔSma 1	0	1	0

deposit disturbances to have a greater impact on Sma .

Table 4-26 shows that the effect of U_{er} , the disturbance related to the institutions' demand for money, is also unity, while table 4-25 shows that the effect of U_{bor} , that represents the money supply side, is -1. Finally, comparing the effects of real sector shocks with those of monetary shocks on Sma , we see that the former may be greater, equal or smaller than the latter, depending upon the coefficients involved. If $\frac{rrb_3 + b_2}{1 - b_1}$ is greater than 1,¹⁰⁸ then the effect of U_c or U_i is greater than that of monetary shocks. If $\frac{rrb_3 + b_2}{1 - b_1}$ is less than 1, the effect of U_c or U_i is smaller than that of U_{cur} and U_{er} , but may be smaller, equal or larger than that of U_{dep} .

From there, the larger the transactions demand for money coefficients (b_3 and b_2), relative to the marginal propensity to save, the greater the effect of real sector shocks relative to the effect of monetary shocks. Notice however that for this effect the transactions demand for currency has a more important role than the transactions demand for demand deposits since the latter is multiplied by the reserve requirement ratio and so its effect on the numerator is diminished.

108. Recall that b_1 is the marginal propensity to consume, b_3 is the transactions demand for money coefficient for deposits and b_2 is the one for currency.

As a conclusion to our analysis of the interest rate policy case we must say that the effect of shocks on income and on Sma does not vary across the different models. However, we have not considered the possibility that some of the assumptions introduced, for example in D-2, D-3 and D-4, may cause a change in some of the coefficients that determine the effect of the shocks. For example, when we assume that demand deposits bear interest, we could think of the possibility that the transactions demand for currency (b_2) be affected. Although we have not assumed so, the present analysis would allow us to find out what the implications of such situation would be.

2. Security Portfolio Policy

Security Portfolio Policy: Effects of Shocks on Income

With the exception of model D-2, the reduced form equation for income, when Sma is used as instrument, shows that both real sector and monetary sector disturbances affect income. (See table 4-21).

The multipliers of real sector shocks are shown in Table 4-22, where we observe that the general form of such multiplier for all models, except D-2, is:

$$\left(\frac{\tau}{\beta - \tau \rho_i}\right) \left(-\frac{\rho_i}{1 - b_1}\right)$$

where $\rho_i = \rho, \rho_2, \rho_5$ and ρ_7 in models D-1, D-3, D-4 and D-5 respectively, while it is zero in model D-2.

Table 4-24. Chapter 4 Summary: Effect of a Disturbance to the Demand Deposit Equation (U_{dep}) under Different Monetary Policies.

Model:	Interest rate policy	Security portfolio policy	Unborrowed reserves policy	Monetary base policy
D-1	$\frac{\Delta Y}{\Delta U_{dep}}$	0	$-\tau r(\tau\beta)/(\beta-\tau\rho_1) < 0$	$-\tau r\beta\tau/(\beta-\tau\rho) < 0$
	$\frac{\Delta r}{\Delta U_{dep}}$	0	$-\tau_2\tau r/(\beta-\tau_2\rho_1) > 0$	$-\tau r\tau/(\beta-\tau\rho) > 0$
	$\frac{\Delta Sma}{\Delta U_{dep}}$	0	$-\frac{\tau r\tau_2(c_2+b_2\beta)}{(\beta-\tau_2\rho_1)} < 0$	0
D-2	$\frac{\Delta Y}{\Delta U_{dep}}$	0	$-\tau r\tau_2 < 0$	$-\tau r\tau$
	$\frac{\Delta r}{\Delta U_{dep}}$	0	$-\tau r\tau_2/\beta$	$-\tau r\tau/\beta$
	$\frac{\Delta Sma}{\Delta U_{dep}}$	0	$-b_2\tau r\tau_2\beta$	0
D-3	$\frac{\Delta Y}{\Delta U_{dep}}$	0	$-\tau r\tau_2\beta/(\beta-\tau_2\rho_3)$	$-\tau r\tau\beta\rho/(\beta-\tau\rho_4)$
	$\frac{\Delta r}{\Delta U_{dep}}$	0	$-\tau_2\tau r/(\beta-\tau_2\rho_3)$	$-\tau r\tau/(\beta-\tau\rho_4)$
	$\frac{\Delta Sma}{\Delta U_{dep}}$	0	$-\tau_2\tau r(c_2+b_2\beta)/(\beta-\tau_2\rho_3)$	0
D-4	$\frac{\Delta Y}{\Delta U_{dep}}$	0	$-\tau r\tau_2/(\beta-\tau_2\rho_5) < 0$	$-\tau r\tau\beta\tau/(\beta-\tau\rho_5)$
	$\frac{\Delta r}{\Delta U_{dep}}$	0	$-\tau r\tau/(\beta-\tau_2\rho_5) > 0$	$-\tau r\tau/(\beta-\tau\rho_5)$
	$\frac{\Delta Sma}{\Delta U_{dep}}$	0	$-\tau_2\tau r(c_2+b_2\beta)/(\beta-\tau_2\rho_5)$	0
D-5	$\frac{\Delta Y}{\Delta U_{dep}}$	0	$-\tau r\tau_2/(\beta-\tau_2\rho_7)$	$-\beta\tau r\tau/(\beta-\tau\rho_9)$
	$\frac{\Delta r}{\Delta U_{dep}}$	0	$-\tau r\tau/(\beta-\tau_2\rho_7)$	$-\tau r\tau/(\beta-\tau\rho_9)$
	$\frac{\Delta Sma}{\Delta U_{dep}}$	0	$-\tau_2\tau r(c_2+b_2\beta)/(\beta-\tau_2\rho_7)$	0

Table 4-25. Chapter 4 Summary. Effect of a Disturbance under Different Monetary Policy Procedures.

Model:	Interest rate policy	Security portfolio policy	Unborrowed reserves policy	Monetary base policy
D-1	ΔY	$\tau\beta/(\beta-\tau\rho) > 0$	$\beta\tau_2/(\beta-\tau_2\rho_1) > 0$	0
(ΔBOR)	Δr	$\tau/(\beta-\tau\rho) < 0$	$\tau_2/(\beta-\tau_2\rho_1) < 0$	0
	ΔSma	0	$\tau_2(c_2+\beta b_2)/(\beta-\tau_2\rho_1)$	-1
D-2	ΔY	$\tau > 0$	$\tau_2 > 0$	0
(ΔBOR)	Δr	$\tau/\beta < 0$	$\tau_2/\beta < 0$	0
	ΔSma	0	$b_2\tau_2 > 0$	-1
D-3	ΔY	$\beta\tau/(\beta-\tau\rho_2) > 0$	$\beta\tau_2/(\beta-\tau_2\rho_3) > 0$	0
(ΔBOR)	Δr	$\tau/(\beta-\tau\rho_2) < 0$	$\tau_2/(\beta-\tau_2\rho_3) < 0$	0
	ΔSma	0	$\tau_2(c_2+\beta b_2)/(\beta-\tau_2\rho_3)$	-1
D-4	ΔY	$\beta\tau/(\beta-\tau\rho_5) > 0$	$\beta\tau_2/(\beta-\tau_2\rho_6) > 0$	0
(ΔBOR)	Δr	$\tau/(\beta-\tau\rho_5) < 0$	$\tau_2/(\beta-\tau_2\rho_6) < 0$	0
	ΔSma	0	$\tau_2(c_2+\beta b_2)/(\beta-\tau_2\rho_6)$	-1
D-5	ΔY	$\beta\tau/(\beta-\tau\rho_7) > 0$	$\beta\tau_2/(\beta-\tau_2\rho_8) > 0$	0
(ΔBOR)	Δr	$\tau/(\beta-\tau\rho_7) < 0$	$\tau_2/(\beta-\tau_2\rho_8) < 0$	0
	-1	$\tau_2(c_2+\beta b_2)/(\beta-\tau_2\rho_8)$	-1	-1

On the other hand, we know from table 4-18 that $\rho_5 < \rho_7 < \rho_2$ and $\rho_5 < \rho < \rho_2$, and we proved that as the value of ρ increases, the value of such multiplier increases, therefore, the value of the multipliers $(\Delta Y / \Delta U_{is})$ can be ranked as follows:

Model D-3 > Model D-1 > Model D-4

Model D-3 > Model D-5 > Model D-4

Our conclusions are then that under the security portfolio policy:

1. Real sector shocks do not affect income when neither the demand nor the supply of money are interest elastic (Model D-2).
2. The effect of real sector shocks on income is greater when both the demand and supply of money are interest elastic than when only the demand is. (Model D-3 versus D-1).
3. If the money supply is insensitive to interest rates but the demand is not, the effect of real sector shocks on income is lower if interest is paid on demand deposits (model D-1 versus D-4), and even lower when there are not ceilings on such paid interest (case 2 of model D-4).
4. If both money supply and demand are interest elastic, the effect of real sector shocks on income is reduced when interest is paid on demand deposits (model D-3 vs. D-5), and even lower if no ceilings are imposed.

5. From above statements we conclude that the effect of real sector shocks on income is expected to diminish when demand deposits pay interest.
6. When demand deposits pay interest, the effect of real sector shocks on income is greater if the money supply is interest elastic than if it is not (model D-5 vs. D-4).

Now we must analyze the effect of monetary shocks on income. These shocks are those related to excess reserves, demand deposits, currency and borrowings. Whenever we talk about shocks, we will be referring to the disturbance in the respective equation or to an autonomous change in the variable if it has been specified exogenous. The general formula for the income multipliers ($\Delta Y / \Delta U$) in all models except D-2 is:

$$\text{Excess Reserves (} U_{er} \text{ or } \Delta ER) \quad \frac{-\beta\tau}{\beta-\tau\rho_i} < 0$$

$$\text{Borrowings, Flt, etc (} U_{bor}, \Delta BOR, \text{ etc) } \quad \frac{\beta\tau}{\beta-\tau\rho_i} > 0$$

$$\text{Demand Deposits (} U_{dep}, U_{ndd}, U_{idd} \text{) } \quad \frac{-rr\beta\tau}{\beta-\tau\rho_i} < 0$$

$$\text{Currency (} U_{cur} \text{) } \quad \frac{-\beta\tau}{\beta-\tau\rho_i} < 0$$

where again $\rho_i = \rho$ for model D-1, and $\rho_i = \rho_2$, ρ_5 and ρ_7 in models D-3, D-4 and D-5 respectively. For model D-2, $\tau\rho_i = 0$.

From the above formulas we can see that the effect of excess reserves or currency changes are the same, while those of demand deposits are smaller since these are multiplied by rr which is less than one. Lastly, the effect of borrowings is the same as that of excess reserves and currency but with the opposite sign of course, since borrowings are on the supply side of the money market. Then, comparing these multipliers for the different models, we know the only difference has to do with the value of ρ_1 : the larger ρ_1 is, the lower will be the absolute value of the multipliers. Since $\rho_5 < \rho < \rho_2$ and $\rho_5 < \rho_7 < \rho_2$, then under a security portfolio policy, the multiplier $\Delta Y / \Delta U_{lm}$ can be ranked as follows:

$$\text{Model D-3} < \text{Model D-1} < \text{Model D-4} < \text{Model D-2}$$

$$\text{Model D-3} < \text{Model D-5} < \text{Model D-4} < \text{Model D-2}$$

Therefore, we conclude that when the security portfolio is used as the instrument:

1. The types of monetary shocks that may affect income are those on: currency demand, demand deposits, excess reserves, borrowings, float, treasury currency or cash holdings and government deposits at the central bank.
2. When the money supply is interest elastic, the effect of monetary shocks is smaller than when it is not. (model D-3 versus D-1).

Table 4-26. Chapter 4 Summary. Effect of a Disturbance under Different Monetary Policy Procedures.

Model:	Interest rate policy	Security portfolio policy	Unborrowed reserves policy	Monetary base policy
D-1 (U _{er})	ΔY	0	$-\beta\tau/(\beta-\tau\rho) < 0$	$-\beta\tau/(\beta-\tau\rho)$
	Δr	0	$-\tau/(\beta-\tau\rho) > 0$	$-\tau/(\beta-\tau\rho)$
	ΔSma	+1	0	$-b_2\beta\tau_2/(\beta-\tau_2\rho_1) < 0$
D-2 (U _{er})	ΔY	0	$-\tau < 0$	$-\tau$
	Δr	0	$-\tau/\beta > 0$	$-\tau/\beta$
	ΔSma	+1	0	$-b_2\tau_2 < 0$
D-3 (U _{er})	ΔY	0	$-\beta\tau/(\beta-\tau\rho_2) < 0$	$-\beta\tau/(\beta-\tau\rho_4)$
	Δr	0	$-\tau/(\beta-\tau\rho_2) > 0$	$-\tau/(\beta-\tau\rho_4)$
	ΔSma	1	0	$-b_2(c_2+b_2\beta)(\beta-\tau_2\rho_3)$
D-4 (U _{er})	ΔY	0	$-\beta\tau/(\beta-\tau\rho_5) < 0$	$-\beta\tau/(\beta-\tau\rho_5)$
	Δr	0	$-\tau/(\beta-\tau\rho_5) > 0$	$-\tau/(\beta-\tau\rho_5)$
	ΔSma	1	0	$-\tau_2(c_2+b_2\beta)/(\beta-\tau_2\rho_6)$
D-5 (U _{er})	ΔY	0	$-\beta\tau/(\beta-\tau\rho_7) < 0$	$-\beta\tau/(\beta-\tau\rho_9)$
	Δr	0	$-\tau/(\beta-\tau\rho_7) > 0$	$-\tau/(\beta-\tau\rho_9)$
	ΔSma	1	0	$-\tau_2(c_2+b_2\beta)/(\beta-\tau_2\rho_8)$

3. When the demand for money is interest elastic, the effect of monetary shocks is greater when interest is paid on demand deposits (model D-1 vs. D-4).
4. When both the money supply and demand are sensitive to interest rates, the effect of monetary shocks is greater when interest is paid on demand deposits.
5. When interest is paid on demand deposits, the effect of monetary shocks on income is greater if the money supply is not sensitive to interest rates (model D-4 vs. D-5).
6. The less sensitive the demand for money and the money supply, the greater the effect of monetary shocks on income. The extreme case is model D-2 where none is sensitive to interest rates and the effect of monetary shocks on income is the largest when compared with the rest of the models studied.

Security Portfolio Policy: Effect of shocks on Interest Rates.

The effect of real sector shocks for all models is captured in the following general equation:

$$\Delta r / \Delta U_c = - \frac{1}{(\beta - \tau \rho_i)(1 - b_1)} > 0$$

where $\rho_i = \rho$, ρ_2 , ρ_5 , ρ_7 for models D-1, D-3, D-4 and D-5 respectively, and $\tau \rho_i = 0$ for model D-2.

As the value of ρ_i increases, the value of such multiplier diminishes. Therefore, under this policy the effect of real sector shocks on interest rates is ranked as follows:¹⁰⁹

Model D-2 > All Models

Model D-3 < Model D-1 < Model D-4

Model D-3 < Model D-5 < Model D-4

This leads to see that under the security portfolio policy:

1. The effect of real sector shocks on interest rates is largest when the demand for money is insensitive to interest rates.
2. The effect is greater when interest is paid on demand deposits than when it is not.
3. The effect is smaller when the money supply is sensitive to interest rates.

The effect of monetary shocks on interest rates is given by the following general formula:

$$\text{For Excess Reserves or Currency} \quad - \frac{\tau}{\beta - \tau \rho_i} > 0$$

$$\text{For Demand deposits} \quad - \frac{\tau r r}{\beta - \tau \rho_i} > 0$$

$$\text{For Borrowings} \quad - \frac{\tau}{\beta - \tau \rho_i} < 0$$

109. Based on the comparison of the ρ 's.

where $\rho_1 = \rho, \rho_2, \rho_5$ and ρ_7 for models D-1, D-3, D-4 and D-5, and $\tau\rho_1 = 0$ for model D-2.

Comparing between the different models we can rank them according to the magnitude of the effect of monetary shocks on income:

Model D-2 > Model D-4 > Model D-1 > Model D-3

Model D-2 > Model D-4 > Model D-5 > Model D-3

Therefore, when the security portfolio is used, we have the following conclusions with respect to the effect of monetary shocks on interest rates:

1. The effect of a demand deposit disturbance is smaller than that of currency or excess reserves.
2. When both the demand and supply of money are interest elastic, the effect on interest rates is smaller than when only the demand is (model D-3 versus D-1).
3. The effect on interest rates gets larger when interest is paid on demand deposits whether both the demand and the supply are interest elastic (model D-5 vs. D-3) or only the demand is (model D-4 vs. D-1).

110. Given that $\rho_5 < \rho < \rho_2$ and $\rho_5 < \rho_7 < \rho_2$.

4. When interest is paid on deposits, the effect of monetary shocks on interest rates is smaller if the money supply is sensitive to interest rates than if it is not.

3. Unborrowed Reserves Policy

Unborrowed Reserves Policy: Effect of shocks on Income

Under this policy, model D-2 shows that real sector shocks do not affect income. For the other models we again find a general formula:¹¹¹

$$\Delta Y / \Delta U_c = - \frac{\tau_2 \rho_i}{(\beta - \tau_2 \rho_i)(1 - b_1)}$$

where $\rho_i = \rho_1$, ρ_3 , ρ_6 and ρ_8 for models D-1, D-3, D-4 and D-5 respectively. On the other hand, since $\rho_6 < \rho_8 < \rho_3$ and $\rho_6 < \rho_1 < \rho_3$, and given the above formula, we know that as the value of ρ_i increases, the multiplier gets larger. Therefore, we can rank the value of this multiplier for the different models as follows:

Model D-3 > Model D-5 > Model D-4

Model D-3 > Model D-1 > Model D-4

111. See Table 4-22

We then conclude that under the unborrowed reserves policy:

1. The effect of real sector shocks on income is greater when both the demand for money and the money supply are interest elastic than when only the demand is (model D-3 versus D-1).
2. The effect of real sector shocks on income is diminished whenever interest is paid on demand deposits (model D-4 versus D-1 and D-5 versus D-3).
3. When interest is paid on deposits the effect of real sector shocks on income is greater if the money supply is interest elastic than if it is not (model D-5 versus D-4).

Now we turn to analyze the effect of monetary shocks. Table 4-23 shows that under the present policy, disturbances occurring in the demand for currency equation have no effect on income. For the other types of disturbances, the multiplier formulas have again a common pattern, namely that the only element changing among the models is the coefficient ρ_i . From tables 4-24, 4-25 and 4-26 we obtain these income multipliers:

$$\text{For Demand Deposits.} \quad -\frac{rr\tau_2\beta}{\beta-\tau_2\rho_i} < 0$$

$$\text{For Borrowings.} \quad -\frac{\tau_2\beta}{\beta-\tau_2\rho_i} > 0$$

$$\text{For Excess Reserves.} \quad -\frac{\tau_2\beta}{\beta-\tau_2\rho_i} < 0$$

where $\rho_i = \rho_1, \rho_3, \rho_6, \rho_8$ for models D-1, D-3, D-4 and D-5 respectively,

and $\tau_2 \rho_i = 0$ for model D-2.

From these equations, the absolute value of the multiplier of demand deposits is smaller than that of excess reserves, while the one for borrowings is equal to that of excess reserves but with opposite sign. Then, since $\rho_6 < \rho_1 < \rho_3$ and $\rho_6 < \rho_8 < \rho_3$ and since the multiplier formulas above show that the larger ρ_i is the lower the absolute value of the multipliers, we can rank the effects of monetary shocks on income among the models as follows:

Model D-2 > Model D-4 > Model D-1 > Model D-3

Model D-2 > Model D-4 > Model D-5 > Model D-3

We therefore conclude that if unborrowed reserves are used as instrument:

1. Disturbances to the demand for currency equation do not affect income. Similarly, autonomous changes in the float, treasury currency or cash holdings and government deposits at the central bank have no effect on income under this policy.
2. Among the rest of the monetary disturbances, those to the demand for deposits have the lowest impact on income under any of the models.
3. The effect of monetary shocks on income are greater when only the demand for money is sensitive to interest rates than when both the demand and the supply are (model D-1 versus D-3). The effect is largest when neither one is sensitive to

- interest rates (model D-2).
4. The effect of these shocks on income is enlarged if interest is paid on demand deposits (model D-4 vs. D-1 and D-5 vs. D-3); and it is even greater if no ceilings are imposed on such interest.

Unborrowed Reserves Policy: Effect of Shocks on Interest Rates.

The multipliers showing the effect of real sector shocks on interest rates are found in Table 4-22. The general formula is:

$$\Delta r / \Delta U_c = - \frac{1}{(\beta - \tau_2 \rho_i)(1 - b_1)}$$

where $\rho_i = \rho_1, \rho_3, \rho_6$ and ρ_8 for models D-1, D-3, D-4 and D-5 respectively, and $\tau_2 \rho_i = 0$ for model D-2. Hence, the larger ρ_i is, the smaller the multiplier and hence interest rates are less sensitive to real sector shocks. We can again rank the models according to the magnitude of such multiplier, as follows:

Model D-2 > Model D-4 > Model D-1 > Model D-3

Model D-2 > Model D-4 > Model D-5 > Model D-3

We therefore conclude that under the unborrowed reserves policy:

1. Real sector shocks have the greatest effect on interest rates when neither the demand or the supply of money are responsive to interest rates (Model D-2).

2. The effect of real sector shocks on interest rates is larger when only the demand for money is interest elastic than when both demand and supply are (model D-1 versus D-3).
3. When interest is paid on demand deposits, the effects of real sector shocks on interest rates are enlarged (model D-4 versus D-1 and D-5 versus D-3).
4. If interest rates are paid on deposits, the effect of real sector shocks on interest rates is greater if the money supply is not sensitive to interest rates than if it is (models D-4 versus D-5).

Now the effect of monetary shocks on interest rates will be discussed. First, as Table 4-23 indicates, disturbances to the currency equation have no effect. For the other types of disturbances or autonomous changes we have:

Demand Deposits $\Delta r / \Delta U_{\text{dep}} = - \frac{\tau_2^{rr}}{\beta - \tau_2 \rho_i} > 0$

Excess Reserves $\Delta r / \Delta U_{\text{er}} = - \frac{\tau_2}{\beta - \tau_2 \rho_i} > 0$

Borrowings $\Delta r / \Delta_{\text{bor}} = \frac{\tau_2}{\beta - \tau_2 \rho_i} < 0$

where $\rho_i = \rho_1, \rho_3, \rho_6, \rho_8$ for models D-1, D-3, D-4 and D-5 respectively, but $\tau_2 \rho_i = 0$ for D-2. The above multipliers give similar results to those obtained for the real sector shocks, in the sense that as ρ_i increases so does the multiplier. We can therefore conclude that under

the unborrowed reserves policy:

1. Among the monetary disturbances, those causing the demand for currency to change have no effect on interest rates.
2. An autonomous change in float, treasury currency or cash and the government deposits at the central bank have no effect on interest rates.
3. The effect of a disturbance to the demand deposit equation is smaller than one to the excess reserves equation.
4. The effect of the monetary shocks that can still affect interest rates under this policy will be greater if the money supply is not sensitive to interest rates than if it is. It will also be greater if interest is paid on demand deposits than if no interests are paid.

4. Monetary Base Policy

Monetary Base Policy: Effects of Shocks on Income

The effect of real sector shocks on income are shown in the last column of Table 4-22. Again, the effect of these shocks is zero in model D-2. For the other models, the general formula for their multiplier is:

$$\Delta Y / \Delta U_c = - \frac{\tau p_1}{(\beta - \tau p_1)(1 - b_1)}$$

where $\rho_1 = \rho$, ρ_4 , ρ_5 and ρ_9 for models D-1, D-3, D-4 and D-5 respectively. We also know that $\rho_5 < \rho_9 < \rho_4$ and $\rho_5 < \rho < \rho_4$ and in the formula above, if the value of ρ_1 increases, so will the value of the multiplier. Therefore, we can rank the models according to the magnitude of the effect of real sector shocks on income:

Model D-3 > Model D-5 > Model D-4

Model D-3 > Model D-1 > Model D-4

We conclude that under the monetary base policy:

1. The effect of real sector shocks on income is greater when the money supply is interest elastic than when it is not (model D-3 vs. D-1 and D-5 vs. D-4). It is zero when neither one is interest elastic.
2. The effect is lower when interest is paid on demand deposits than when it is not (model D-4 vs. D-1 and D-5 vs. D-3).
3. The more interest elastic the money supply and the demand are, the greater the effect of these shocks on income.

With respect to the effects of monetary shocks on income, first of all we must notice that borrowings will not affect income under a monetary base policy. (See Table 4-25). For the other types of shocks we can obtain a general formula for their income multipliers:

$$\text{For Currency or Excess Reserves } \Delta Y / \Delta U = - \frac{\beta \tau}{\beta - \tau \rho_1} < 0$$

For Demand Deposits

$$\Delta Y / \Delta U = - \frac{rr\beta\tau}{\beta - \tau\rho_i} < 0$$

where $\rho_1, \rho_4, \rho_5, \rho_9$ for models D-1, D-3, D-4 and D-5 respectively, but $\tau\rho_i=0$ in model D-2. These formulas show again that disturbances to the demand deposit function have a lower effect than the currency or excess reserves shocks. On the other hand, as we change from one model to another it is only the coefficient ρ_i that will make a difference; the larger ρ_i is the smaller the multiplier will be. Then since $\rho_5 < \rho_9 < \rho_4$ and $\rho_5 < \rho < \rho_4$, we can rank the models according to the effect of monetary shocks on income as:

Model D-2 > Model D-4 > Model D-5 > Model D-3

Model D-2 > Model D-4 > Model D-1 > Model D-3

Based on this ranking, we can say that when the monetary base is used as instrument:

1. An autonomous change in borrowings or a disturbance to the borrowings function will not affect income. Similarly, changes in Float, Treasury currency, etc., will not affect income.
2. The only monetary variables that may change exogenously and affect income are: currency, demand deposits and excess reserves.
3. The effects of the monetary shocks that are able to affect income will be greatest if neither the demand nor the supply of money are interest elastic (model D-2) and will be lowest

when both are responsive to interest rates (model D-3).

4. The effect of monetary shocks on income is greater if the demand is the only interest elastic relationship than if both demand and supply are (model D-1 versus D-3 and D-4 vs. D-5).
5. The effect of monetary shocks on income is enlarged when interest is paid on demand deposits (model D-4 vs. D-1 and D-5 vs. D-3).
6. From the above comments, it follows that the greater the total responsiveness to interest rates of both money demand and supply together, the lower the effect of this monetary shocks on income.

Monetary Base Policy: Effect of Shocks on Interest Rates

The effect of real sector shocks on interest rates is given by the following formula:

$$\Delta r / \Delta U_c = - \frac{1}{(\beta - \tau \rho_i)(1 - b_1)} > 0$$

while the effects of monetary shocks is given by:

$$\text{Demand Deposits} \quad \Delta r / \Delta U_{\text{dep}} = - \frac{\tau r r}{\beta - \tau \rho_i} > 0$$

$$\text{Currency and Excess Reserves} \quad \Delta r / \Delta U = - \frac{\tau}{\beta - \tau \rho_i} > 0$$

where $\rho_i = \rho$, ρ_4 , ρ_5 , ρ_9 , for models D-1, D-3, D-4 and D-5 respectively, while $\tau \rho_i = 0$ for D-2.

They show that as the value of ρ_i increases, the value of such multipliers will fall. Hence, the models' ranking according to the magnitude of the effect of real and monetary shocks on interest rates is just as the ranking we obtained above for the effect of monetary shocks on income:

Model D-2 > Model D-4 > Model D-5 > Model D-3

Model D-2 > Model D-4 > Model D-1 > Model D-3

And the conclusions are obviously the same:

1. Changes in borrowings have no effect on interest rates.
2. The effect of demand deposits shocks are smaller than currency or excess reserves shocks.
3. The greater the response of money demand and supply to interest rates, the lower the effect of shocks on interest rates.
4. The effect of shocks on interest rates is enlarged when interest is paid on deposits.

Regarding the disturbances on deposits and reserves, they will affect income and interest rates under the SM, BU and MB policies, although the effect of the deposit shock is smaller than that of excess reserves due to the reserve requirement ratio. Furthermore, the effects on income and interest rates under BU may be different than under SM and MB, again due to the LM slope given by ρ_i . (See

C. EFFECT OF EACH TYPE OF SHOCK UNDER THE SAME MODEL BUT DIFFERENT POLICY

1. MODEL D-1

Table 4-22 showed the effect of real sector shocks for model D-1 under each alternative policy. The effect of these shocks on income is largest under the interest rate policy. Among the other three policies we can say that the effect under Sma and MB is the same, but it is different under RU although we cannot say if greater or smaller. This will depend upon the value of $\tau_i \rho_i$ which is $\tau \rho$ in the former cases and $\tau_2 \rho_1$ in the latter. The same result holds for the effect on interest rates under this policies.

With respect to monetary shocks, these do not affect income nor interest rates under an interest rates policy. We also see that the disturbance on the currency equation does not affect income nor interest rates under an RU policy, and that the disturbance to borrowings does not affect income nor interest rates under the monetary base policy. (See Tables 4-23 and 4-25).

Regarding the disturbances on deposits and excess reserves, they will affect income and interest rates under the Sma, RU and MB policies, although the effect of the deposit shock is smaller than that of excess reserves due to the reserve requirement ratio. Furthermore, the effects on income and interest rates under RU may be different than under Sma and MB, again due to the LM slopes given by $\rho_i \tau_i$. (See

Tables 4-26 and 4-24).

It is also interesting to note that these two types of disturbances generate open market operations¹¹² under the RU policy but not under the MB or Sma policies. This is also true for the disturbances of the currency function. The reason is that unborrowed reserves are affected by these shocks; hence, under the RU policy action will follow to bring RU back to its instrument value RU^* . However, Sma and MB are not affected by these shocks, so no action is undertaken.

Finally, Table 4-25 showed the effects of a change in borrowings. In this case open market operations are undertaken under all policies except Sma. This, of course, is due to the fact that borrowings do not affect Sma, affect interest rates and hence RU through the change in currency demand, and affect MB directly. Therefore, actions are undertaken when either one of these latter is used as instrument.

2. MODEL D-2

Under this model, the IS turns out to be the reduced form for income under the interest rate policy, while the LM is the reduced form for income under the Sma, RU and MB policies. From there that real

¹¹². Reflected by ΔSma in the Tables.

sector shocks will affect income only under an interest rate policy, but not under the others. However interest rates are affected under any policy (excluding the interest rate policy), the effect being the same under any of them (Sma, RU or MB). With respect to the monetary shocks the same comments that were said with respect to model D-1 apply here.

3. MODELS D-3, D-4 AND D-5

For each of these models, the comparison of the effects of shocks under different policies leads to the same conclusions that we had for model D-1 (therefore, we will not repeat them here). This can be corroborated by looking at Tables 4-22 through 4-26 where each row, referring to a particular model, shows the same pattern as we move from one policy to another. This is true regarding the effect of real shocks for models D-1, D-3, D-4 and D-5, and for all models regarding monetary shocks.

However, there is an important feature that the specification of models D-3, D-4 and D-5 brings up and that may have important implications for monetary policy. This is the interrelationship between different tools of monetary policy, specifically of the discount rate with Sma when Sma is used as instrument (models D-3 and D-5); again, the discount rate with RU when RU is used as instrument (models D-3 and D-5), and then we have the link between the interest rate paid on deposits (r_1) and Sma or RU when either one is used as

instrument (models D-4 and D-5).

The relevance of these links arises as we look at the equations that determine the instrument value under each of the mentioned policies. Such equations include the target income (Y^*) and they also contain the discount rate, the rate paid on deposits or both, depending on the case. For example, in model D-3 when S_{ma} is used as instrument (see page 55), we can change the discount rate r_d and so adjust S_{ma}^* , still for the same target income.

An implication of this is that these other tools (r_d, r_i) also appear in the reduced form equation for income and interest rates, with their respective multipliers. These multipliers reveal that under certain circumstances such multipliers are larger under RU^* than under S_{ma}^* .

Finally, when the ceilings versus no ceilings cases are analyzed under models D-4 and D-5 we conclude that deregulation implies that the effect of demand deposits disturbances on income under policies other than r^* , are enlarged. On the other hand, the effect of real sector shocks on income is greater when there is regulation. However, if deregulation is allowed and the interest rate paid on deposits r_i is a function of the market interest rates ($r_i = f(r)$), the effect of the real sector shocks is enlarged.

D. CONCLUSIONS

To conclude on the comparative effect of shocks under different economic structures we have Table 4-27 showing the sensitivity of the effect of each type of shock under each policy, with respect to the structural characteristics that we have emphasized. The letter A in the Table is given to cases where the effect of the shock is greatest as compared to the effect under the alternative structural characteristics. Hence the comparison in the Table is done vertically, not horizontally.

From there we observe that the use of a particular policy under different economic structures may lead to different results depending on those structural characteristics and on the sources of the shocks. The one exception to this statement is the interest rate policy, since when it is followed, our different structures do not make a difference, the effect of shocks is invariant to the structural change. Under this policy only the source and the magnitude of the shocks may make a difference. This means that in two economies with different economic structures, both exposed to real sector shocks of the same magnitude, income will be affected equally. But if the magnitude of the shocks is different, or if one economy is exposed to real sector shocks while the other economy to monetary shocks, the effect on income will not be the same in the two economies.

Table 4-27. Comparative Effects of Shocks under Different Economic Structures¹.

Structural	Characteristics	Interest Rate			Policy			Security Portfolio			Policy	
		Effect of U_{1s} Shocks			Effect of Monetary Shocks ²			Effect of U_{1s} Shocks			Effect of Monetary Shocks ² *	
		on Income	on Sma		on Income	on Sma		on Income	on Sma		on Income	on Sma
Money Demand and Supply	Interest Paid on Deposits											
Md=f(r) MS=f(r)	Yes/ceilings	A	A		0	A		E	C		C	C
	Yes/no ceilings No	A	A	A	0	A	A	F	B		B	B
		A			0	A	A	D	D		D	D
Md=f(r) MS=f(r)	Yes/ceilings	A	A		0	A		0	A		A	A
	Yes/no ceilings No	A	A	A	0	A	A	0	A	A	A	A
		A			0	A	A	0	A	A	A	A
Md=f(r) MS=f(r)	Yes/ceilings	A	A		0	A		B	F		F	F
	Yes/no ceilings No	A	A	A	0	A	A	C	E		E	E
		A			0	A	A	A	G		G	G

Table 4-27 (continued)

Structural	Characteristics	Unborrowed				Reserves Policy				Monetary				Base Policy			
		Effect of U_{is} Shocks				Effect of Monetary Shocks				Effect of U_{is} Shocks				Effect of Monetary Shocks			
Money Demand and Supply	Interest Paid on Deposits	Effect of U_{is} Shocks				Effect of Monetary Shocks				Effect of U_{is} Shocks				Effect of Monetary Shocks			
		On income	On interest	On income	On r	$U_{cur}, U_{flt}, etc.$	U_{dep}, U_{er}, U_{bor}	On income	On r	On income	On interest	On income	On r	$U_{cur}, U_{dep}, U_{er}, U_{bor}, U_{flt}, etc$	On income	On r	On income
Md=f(r) MS=f(r)	Yes/ceilings	E	C	0	0	0	C	0	C	E	C	C	C	0	0	0	0
	Yes/no ceilings No	F D	B D	0 0	0 0	0 0	B D	0 0	B D	F D	B D	B D	B D	0 0 0	0 0 0	0 0 0	0 0 0
Md=f(r) MS=f(r)	Yes/ceilings	0	A	0	0	0	A	0	A	0	A	A	A	0	0	0	0
	Yes/no ceilings No	0 0	A A	0 0	0 0	0 0	A A	0 0	A A	0 0	A A	A A	A A	0 0 0	0 0 0	0 0 0	0 0 0
Md=f(r) MS=f(r)	Yes/ceilings	B	F	0	0	0	F	0	F	B	F	F	F	0	0	0	0
	Yes/no ceilings No	C A	E G	0 0	0 0	0 0	E G	0 0	E G	C A	E G	E G	E G	0 0 0	0 0 0	0 0 0	0 0 0

Notes: 1. This table compares the effect of shocks among different models. Letter "A" means the effect is largest under that economic structure, letter "F" or "G" represents the structure under which such effect is minimum, among the different structures analyzed in the table.

2. Monetary Shocks includes all types: $U_{cur}, U_{bor}, U_{er}, \Delta ER, \Delta BOR, \Delta FLT, etc.$

However, under the other three policies (Sma, RU and MB) the effect of shocks under different structures changes, although in the same direction in all three policies (See Table 4-27). For example, under any one of them real sector shocks have a greater effect on income when both demand and supply are interest elastic than when none is (see the letter A in the bottom row for any one of these policies).

This leads to conclude that if any one of these policies is used in a particular economy (with its own structural features), as some of its structural characteristics change, the effects of shocks on income and on interest rates will be different, even if the types of shocks are the same, or in other words, even if the type of instability has not changed. (for example, see that the order of the letters within the column for U_{is} under Sma, RU and MB are the same).

However there are some exceptions to the above conclusion, having to do with specific shocks. The first one is for the disturbance in the currency equation (U_{cur}), or for autonomous changes in the float, Tco , etc., when unborrowed reserves are used as the instrument; also, for the disturbance in the Borrowings equation, or for autonomous changes in float, etc., when the monetary base is used as instrument. For those shocks, the effect is the same (zero) independently of the structure, and the reason for this is that the effects of such shocks on income or interest rates are solely related to the policy and not to anything else.

Now, we want to compare the effect of real sector versus monetary sector shocks on income and interest rates under the policies Sma, RU and MB. We find that this comparison depends upon the following ratio:

$$\left| \frac{\Delta Y / \Delta U_c}{\Delta Y / \Delta U_{lm}} \right| = \frac{\rho_i}{\beta} = \frac{\rho_i}{c_1}$$

where ρ_i represents the interest sensitivity of the money demand and money supply together, while c_1 is the interest sensitivity of investment. And we have seen that ρ_i changes when the structure (models) changes as well as when certain policies are used. (See the Table 4-21).

On the other hand, the relative effect on interest rate is given by:

$$\left| \frac{\Delta r / \Delta U_c}{\Delta r / \Delta U_{lm}} \right| = \frac{1}{\tau_i (1 - b_1)}$$

where τ_i is $\frac{1}{rrb_3 + b_2}$ when Sma and MB are used as instruments, while it is $\frac{1}{rrb_3}$ when RU is used.

This ratio shows that the relative effect of real sector versus monetary sector shocks on interest rates does not depend upon the structural characteristics of the economy but upon the type of policy used.

CONCLUSIONS

The main objective of this dissertation is to contribute to the analysis of short run monetary policy, under the assumption that it is used to attain some long run macroeconomic target, specifically, some income target level.

In the context of short run monetary policy the key elements are what we have called instrument-indicators, defined as variables that serve both, as instruments used to attain a long run target, and as indicators of how close or how far we are from it. Hence, monetary authorities must look for a variable that will most closely fulfill this role. We described the properties that such variable should have according to what is needed from the policymaker's point of view. The different properties that were discussed are embodied in the idea that the economic structure must be such that there is a clear link between the chosen instrument and our target variable (i.e., income). This link, however, may be disturbed by sudden economic changes, thus preventing the attainment of the ultimate target. Hence the importance of analyzing the forces that may cause such distortion when a specific variable is used or proposed to be used as the monetary policy instrument. This analysis, of course, is done assuming a specific

economic structure, that is, a framework under which the proposed policy will be implemented.

This is precisely what we have tried to accomplish in this study. There have been a variety of "instruments" that have been used or proposed by monetary authorities or their economic advisors, and the choice has not been easy or successful all of the time. It has been argued- and diverse studies have concluded- that deviations of income from target can be minimized if the interest rate is used as the instrument in economies where the main source of instability arises in the monetary sector, while the money supply should be used when the real sector originates most of the economy's instability. Other policies, which use monetary aggregates not as broad as the money supply as instruments, have been suggested in the context of controlling the money stock.

In this study, we have further analyzed not only interest rates and the money supply as possible instruments to attain the ultimate target, income, but we have also studied the possibility of using the less aggregated but easier to control monetary variables, with the same objective of attaining the ultimate target. To this effect we specified two sets of models, the aggregated and the disaggregated ones. Each model within a set has some specific assumption that makes it different from the rest; furthermore, the second group of models discloses the monetary sector showing the initial sources of the money stock, but again each model within this group has some unique

assumption that distinguishes it from others.

The specification of the two sets of models allowed us to analyze the effects of using different instruments, under different economic structures. We defined a policy or policy procedure as that in which a specific instrument was used. For example, the interest rate policy is the one in which the interest rate is the variable to be used as the instrument of monetary policy. We then studied the implications of using each of the proposed policies, under each of the specified model structures.

Our analysis concentrated on the effect of different types of instabilities or shocks to the economic structure and hence to our ultimate target, under each policy procedure. The purpose was to identify the types of shocks that would prevent our attainment of ultimate targets, under each alternative policy. The specification of different structures under which the analysis was performed allowed us to capture the sensitivity of the results to a change in the economy's characteristics. Our conclusions are the following:

1. We cannot unequivocally say that when the monetary sector is the only (or the one with more instability), using an interest rate policy will assure that the income target will be achieved. We have shown that given certain structural characteristics, such instability will still cause income to deviate from target under an interest rate policy.

2. The structural characteristics under which income will still be affected by monetary sector instability even if an interest rate policy is used are: i) when investment is affected by the real expected interest rate, given that price expectations are a function of past money growth rate and short run revisions are not done; and ii) when consumption is affected by human and nonhuman wealth, the latter being perceived by society as the sum of the money and the capital stock only (government debt has been left out from such definition).
3. We cannot conclusively say that when most or all instability comes from the real sector, a policy of focusing on the money supply should be used. This will again depend upon some structural characteristics. If the interest sensitivity of spending (consumption and investment) is larger than the interest sensitivity of the demand for money, the effect of monetary sector instability on income will be greater than the effect of real sector instability, if the money supply policy is being used. Therefore, although the effect of real sector instability on income is smaller under a money supply policy than under an interest rate policy, as long as both types of instabilities exist, it would still be necessary to compare the total effect (of both) under each type of policy, thus selecting the one under which such effect is minimized.

4. When we introduce price expectations and a money supply policy is followed, the effect of monetary instability on income is smaller than in alternative models. This is more obvious when price expectations are a function of the money supply rather than of income. However, the effect on interest rates will be very large, since they will bear a greater responsibility in clearing the monetary and real sectors .
5. When a money supply policy is used, there should be concern about the behavior of interest rates, which may prove to be more volatile under this policy than under the alternative of using the interest rate as instrument. However we found that when the money supply policy is used, the greatest impact of any kind of shock on interest rates occurs when spending is not sensitive to interest rates (model A-3). The smallest impact on interest rates is found when consumption is affected by wealth, as well as when investment and consumption are interest-elastic.
6. Regardless of the model specification , we found that when the money supply is used as instrument, the larger the transactions demand for money coefficient relative to the marginal propensity to save, the greater the effect of real sector instability on interest rates relative to that of monetary instability. However, the difference between the effects of the different instability sources will be

accentuated in the case in which price expectations, defined as a function of past income growth, affect investment, assuming no revision of policy in the short run. This is due to the dynamic effects on interest rates, since real sector shocks cause interest rates to rise further, while monetary sector shocks will reverse the initial rise in interest rates.

7. It is usually true that if the only type of instability that exists comes from the real sector, a money supply policy will be superior since the effects on income are minimized. This is true with the exception of an economic structure where consumption is a function of wealth, and non human wealth is defined as money plus capital stock only. In this case, the effect of the real sector instability on income may be less, equal or greater under a money supply policy than under an interest rate policy.
8. If the interest rate policy is used in an economy where consumption is a function of wealth, but the definition of wealth (society's perception) changes, then the effect of real sector instability on income will also change. Such a slight change in a behavioral characteristic of the economy will have important effects on the outcome of such policy. The effect of real sector shocks on income will increase when government debt is incorporated in society's perception of nonhuman wealth. However, when a money supply policy is used, such

concept of wealth makes no difference with respect to the effect of any shock on income.

9. The way in which government deficits are financed will be affected by the monetary policy under effect, and by the way society views nonhuman wealth. Furthermore, under an interest rate policy, the distribution of the deficit financing between money supply and government debt will be affected by the relative values of important structural coefficients: the transactions demand for money, the consumption sensitivity to wealth and the marginal propensity to save.
10. Deficits per-se have a stronger effect on income when the interest rate rather than the money supply is used as the instrument of monetary policy.
11. Increasing budget deficits affect income under any policy, but again their effect is greater under the interest rate policy. If a money supply policy is followed, the effect of increasing deficits on income increases when society views the government debt as part of their net worth.
12. When spending is insensitive to interest rates we found that,
 - a) the effect of real sector instability on income is the same under any policy, b) the effect of monetary sector instability on income is zero under any policy, and c) the effect of real or monetary sector instability on the money supply under an

interest rate policy is not as strong as the effect on interest rates under a money supply policy. Therefore, we venture to say that under this type of economic structure, an interest rate policy would be preferable than a money supply policy. In this context, however, since monetary policy cannot be used to attain the income target because income is determined solely by the real sector, what the superiority of the interest rate policy really means is that the monetary policy should aim to stabilize money market conditions as reflected in interest rate movements. Of course that, if other assumptions are included into this structure, such as the wealth effect in consumption for example, the above conclusion may change. The existence of the wealth effect, given that money enters the concept of wealth, would imply that the proposed interest rate policy will cause monetary disturbances to result not only in money supply variation but also in income deviating from target. Therefore, although income is solely determined by the real sector, the monetary policy under effect (interest rate) activates the link between the monetary and real sectors in such a way that the the real sector is no longer the only one determining income. In a sense, monetary policy causes the real sector to become interest-responsive, with the result that the effect of monetary shocks on income is not zero any more.

13. The analysis undertaken when we introduced the effect of price expectations highlighted several interesting points. The revision versus the no revision cases represent the "discretion versus rule" issue, which has been one source of controversy between keynesians and monetarists. Our analysis shows that, in general, when income is used as target and given our model specification, economic shocks will cause greater instability on income under the no-revision cases, with one exception: When the money supply is used as the instrument, and price expectations are a function of past money growth rates, the revision and no revision cases deliver the same results, that is, the no revision case does not lead to greater income instability. The interaction of monetary policy and the price expectations mechanism are crucial in determining such outcome.

However, when price expectations are a function of income growth, an economic shock that leads to a change in current income, for example, will cause a change in price expectations, and if there is no revision of the instrument value, income will be further affected (dynamic effects). On the other hand, if there is short run revision, the key question is if the authorities have accurately estimated price expectations and incorporated them into the calculation of the instrument value. As long as they know the price expectations

mechanism, any misestimation will cause random deviations of income from target.

We concluded that when there is no revision and an interest rate policy is used, only past and current real sector instability affect current income, assuming price expectations are affected by past income growth. However, past monetary sector instability will also affect income if price expectations are determined by past money growth. Therefore, the effect of different types of instabilities (real or monetary sector) on income is not invariant to the specification of price expectations, as long as policy is not revised in the short run, that is, as long as "discretion" is ruled out.

If price expectations were "rational", then the no revision case would lead to the same conclusions of the revision case since price expectations would always be equal to the price variable as estimated by the authorities in their economic models, which is the one they would use when they determine the value of the monetary policy instrument they are using. Assuming, of course that they know that public expectations' are in fact rational. As long as true price expectations differ from the estimated price expectations, a policy rule will cause income to deviate from target. Then, if there is revision in the short run, such deviations may be

at least diminished as current economic outcomes are being incorporated into current policy decisions. But if there is not revision, the likelihood of successive income movements will be increased, depending on how different the true versus the expected price expectations mechanisms are.

We must mention at this point that we have not considered the case in which the demand for money is also affected by price expectations, a possibility that has been incorporated into recent theoretical and empirical economic models. This would be an interesting new case under which our analysis could be performed and may render new conclusions.

Finally, we must notice that price expectations have not been studied in the context of the labor market, since our analysis has only dealt with the demand side of the economy. To perform such analysis we would need to introduce the aggregate supply side of the economy. However, the results we have so far will still hold in terms of aggregate demand; the only difference that the analysis of price expectations in the labor market would make is that we would be able to analyze price versus real income behavior as a result of different shocks. Furthermore, we would be able to analyze supply-side shocks as well.

14. The disaggregation of the monetary sector, showing explicitly the sources and uses of reserves, but still keeping income as the target of monetary policy (as opposed to money control) has been helpful and illuminating mainly with respect to the following points:
 1. It shows the effects that institutional factors, working through monetary variables, might have on ultimate targets under each alternative monetary policy. For example, the float is a monetary item that reflects institutional settings in the U.S. monetary system, and it was found to play a role (i.e., affecting income) under an interest rate and a security portfolio policy, while its effect on income is nil under an unborrowed reserves or a monetary base policy.
 2. If an interest rate policy is followed, the structural characteristics we have studied in the different models do not make a difference with respect to the effect of real sector or monetary sector instability on income.
 3. The possibility of complementing a specific policy with another tool such as the discount rate, is found under the security portfolio, unborrowed reserves and monetary base policies. In the model in which interest is paid on demand deposits, such interest rate can also be used as part of the policy package, assuming of course that

regulation is allowed within the institutional framework.

The possibility of this "package" policy would be

advantageous under some circumstances. For example, at

some times it may be difficult to keep the "main

instrument" at the instrument value, or to immediately

bring it back to such value when it has changed. These

delays may cause temporary deviations of income from

target, but if another tool is available, it can be used

during such adjustment periods.

15. The sensitivity analysis we have undertaken will hold with respect to the specific assumption that each model incorporates. However, we could join two or more of the assumptions we have studied independently in this study, and this may lead to new and interesting results with respect to the adequacy of alternative policies. Although this has not been done here, our methodology can easily be adapted to such task. In fact, doing this will bring new cases, which may enclose the characteristics of a broader spectrum of real case studies and therefore yield the possibility of evaluating different policies using a framework that more accurately reflects the real world and that therefore can lead to policy guidelines that will be more adequate and exact.

From the above conclusions we can say that when the monetary authority needs to choose the best policy procedure to follow in order to attain the ultimate target, they must:

1. Define the structural characteristics of the economy as accurate as possible.
2. Investigate the more likely sources of economic instability.
3. Determine under which policy, given those characteristics and instability sources, will the deviation of the target (.e.g., income, etc.) from the target value be minimized.
4. Investigate the possibility that, using certain policy, the appearance of certain shocks may lead to behavioral changes in the economy, which may imply the need to revise the convenience of such policy.
5. Investigate the possibility that different types of shocks may be related, which may pose more difficult answers to the policy choice problem. This would be a new case to which we could apply our above analysis, with the new assumption that for example $U_{\text{cons}} = f(U_{\text{dep}})$. The alternative policies can be evaluated and compared and a new conclusion may be reached. Finding these relationships may not be as hard as it may seem when econometric models have been built and used for a reasonable amount of time, since from the residual of the model's equations, correlations and other types of analyses can be obtained to find links as the one just suggested.

The task of monetary policy is not an easy one. A procedure that may be best at one time may suddenly become a second best because behavioral changes in economic agents have occurred. In this context policymakers must continuously (short run) revise not only the value at which they have set the policy instrument they are using, but also the assumed economic structure, which if changed, may require a policy change.

CHAPTER 3

MATHEMATICAL DERIVATIONS AND PROOFS

1. MODEL 3-1

1. Interest Rate Instrument

a) Derivation of the Instrument Value V

From Eq 12: $V = \sum_{t=1}^T \frac{C_t}{(1+r)^t} + \frac{F}{(1+r)^T}$

Since we assume $C_t = C$

Reduced form of Eq 12:

APPENDIX

Eq 12 is $V = \sum_{t=1}^T \frac{C}{(1+r)^t} + \frac{F}{(1+r)^T}$

Using the reduced form of Eq 12:

$$V = \frac{C}{r} \left(1 - \frac{1}{(1+r)^T} \right) + \frac{F}{(1+r)^T}$$

2. Money Supply Instrument

a) Derivation of Instrument Value V

Assuming $V = 1$, from Eq 13 we get:

$$1 = \frac{1}{1+r}$$

Then into Eq:

$$1 = \frac{1}{1+r} \left(\frac{M}{P} \right)$$

CHAPTER 3

MATHEMATICAL DERIVATIONS AND PROOFS

I. MODEL A-11. Interest Rate Instrument

a) Derivation of the instrument value r^* .

From the IS: $Y^* = \alpha + \beta r + U_{is}$ Hence $r^* = \frac{Y^* - \alpha}{\beta}$

Since we assume $U_{is} = 0$.

b) Reduced form of Income.

The IS is $Y = \alpha + \beta r + U_{is}$

Hence the reduced form of income is:

$$Y = \alpha + \beta r^* + U_{is}$$

2. Money Supply Instrument

a) Derivation of Instrument Value M^*

Assuming $Y = Y^*$, from the IS we get :

$$r^* = \frac{Y^* - \alpha}{\beta}$$

Then into LM:

$$Y = \frac{1}{b_o} (MS - c_o r - U_{md})$$

and assuming $U_{md} = 0$,

$$Y^* = \frac{1}{b_o} (MS - c_o (\frac{Y^* - \alpha}{\beta}))$$

Hence $MS^* = \frac{b_o \beta + c_o}{\beta} Y^* - \frac{c_o \alpha}{\beta}$

b) Derivation of Reduced form for the interest rate

We substitute M^* into the LM and then the Y_{lm} into IS:

The LM is $Y = \frac{1}{b_o} (M^* - c_o r - U_{md})$

The r from the IS is: $r = \frac{Y - \alpha - U_{is}}{\beta}$

Hence we obtain: $r = \frac{1}{b_o \beta} (M^* - c_o r - U_{md} - b_o \alpha - b_o U_{is})$

From there the reduced form for interest rates is:

$$r = \frac{1}{b_o \beta + c_o} (M^* - b_o \alpha - U_{md} - U_{is} b_o)$$

c) Derivation of Reduced form for income

From the IS:

1) $r = \frac{Y - \alpha - U_{is}}{\beta}$

The LM is:

2) $Y = \frac{1}{b_o} (MS^* - c_o r - U_{md})$

Substituting 1) into 2):

$$Y = \frac{1}{b_o} (MS^* - c_o (\frac{Y - \alpha - U_{is}}{\beta}) - U_{md})$$

From where:

$$Y(b_o\beta + c_o) = \beta MS^* + c_o\alpha + c_o U_{is} - \beta U_{md}$$

Solving for Y we obtain its reduced form equation as:

$$Y = \frac{1}{b_o\beta + c_o} (\beta MS^* + c_o\alpha + c_o U_{is} - \beta U_{md})$$

d) Changes in interest rate and income due exclusively to IS and LM shocks.

From reduced form of interest rates:

$$\begin{aligned} dr &= \frac{\partial r}{\partial U_{md}} \partial U_{md} + \frac{\partial r}{\partial U_{is}} \partial U_{is} \\ dr &= -\frac{1}{b_o\beta + c_o} \partial U_{md} + \frac{b_o}{b_o\beta + c_o} \partial U_{is} \end{aligned}$$

where: $\frac{1}{b_o\beta + c_o} < 0$.

From the Reduced form of Income we get:

$$\begin{aligned} dY &= \frac{\partial Y}{\partial U_{md}} \partial U_{md} + \frac{\partial Y}{\partial U_{is}} \partial U_{is} \\ dY &= -\frac{\beta}{b_o\beta + c_o} \partial U_{md} + \frac{c_o}{b_o\beta + c_o} \partial U_{is} \end{aligned}$$

II) MODEL A-2

1. Interest Rate Instrument

a) Reduced Form of money supply

Given $r = r^*$, from the monetary sector equilibrium:

$$MS = a_3 + c_3 r^* + U_{ms} = Md = c_o r^* + b_o Y + U_{md}$$

So:

$$a_3 = (c_o - c_3) r^* + b_o Y + U_{md} - U_{ms}$$

And substituting income from the IS we obtain:

$$a_3 = (c_o - c_3) r^* + b_o \alpha + b_o \beta r^* + b_o U_{is} + U_{md} - U_{ms}$$

Then we substitute this into the MS equation to obtain:

$$MS = c_o r^* + b_o \beta r^* + b_o \alpha + b_o U_{is} + U_{md}$$

2. Money Supply Instrument

a) Determination of Instrument value a_3^* for case A and MS^* for case B.

$$\underline{\text{CASE A}} \quad (a_3 = a_3^*)$$

Equating money supply and demand:

$$MS = a_3 + c_3 r + U_{ms} = b_o Y + c_o r + U_{md}$$

Assuming $U_{ms} = U_{md} = 0$ and $Y = Y^*$:

$$a_3 + c_3 r = b_o Y^* + c_o r$$

$$a_3 = b_o Y^* + (c_o - c_3) r$$

Then substituting the value of r from the IS and assuming $U_{is} = 0$:

$$a_3 = b_o Y^* + (c_o - c_3) \frac{Y^* - \alpha}{\beta}$$

Which gives us the value for:

$$a_3^* = (b_o + \frac{c_o}{\beta} - \frac{c_3}{\beta}) Y^* - \frac{\alpha}{\beta} (c_o - c_3)$$

$$\underline{\text{CASE B}} \quad (MS = MS^*)$$

From the monetary sector equilibrium $MS = Md$, we obtain:

$$MS = b_o Y + c_o r + U_{md} \quad (\text{Supply} = \text{Demand})$$

And assuming $U_{ms} = U_{md} = 0$ and $Y = Y^*$, and substituting the r from the IS we obtain:

$$MS^* = b_o Y^* + c_o \frac{Y^*}{\beta} - c_o \frac{\alpha}{\beta}$$

$$\text{Hence} \quad MS^* = (b_o + \frac{c_o}{\beta}) Y^* - \frac{c_o \alpha}{\beta}$$

b) Derivation of Reduced Form of Interest rates, Case A.

The LM is:

$$Y = \frac{1}{b_o} ((c_3 - c_o) r + a_3 + U_{ms} - U_{md})$$

We substitute Y from LM into the IS and get:

$$r(b_o \beta - c_3 + c_o) = a_3^* + U_{ms} - U_{md} - \alpha b_o - b_o U_{is}$$

So the reduced form for the interest rate is:

$$r = \frac{1}{b_o \beta - c_3 + c_o} (a_3^* + U_{ms} - U_{md} - \alpha b_o - b_o U_{is})$$

c) Derivation of Reduced form of Interest Rates, Case B.

The LM is:

$$Y = \frac{1}{b_o} (-c_o r - U_{md} + MS^*)$$

Substituting the Y from the IS we obtain:

$$b_o(\alpha + \beta r + U_{is}) = -c_o r - U_{md} + MS^*$$

$$r = \frac{MS^*}{c_o} - \frac{U_{md}}{c_o} - \frac{b_o \alpha}{c_o} - \frac{b_o \beta r}{c_o} - \frac{b_o U_{is}}{c_o}$$

$$c_o r + b_o \beta r = MS^* - U_{md} - b_o \alpha - b_o U_{is}$$

So the reduced form for the interest rate is:

$$r = \frac{1}{c_o + b_o \beta} (MS^* - b_o \alpha - b_o U_{is} - U_{md})$$

d) Derivation of reduced form for a_3 , Case B.

Given:

$$MS^* = a_3 + c_3 r + U_{ms}$$

$$a_3 = MS^* - c_3 r - U_{ms}$$

And substituting r from the IS equation:

$$a_3 = MS^* - c_3 \frac{Y - \alpha - U_{is}}{\beta} - U_{ms}$$

Then substituting the reduced form for income in Y we get:

$$a_3 = MS^* - \frac{c_3}{\beta(\beta b_o + c_o)} MS^* - \frac{c_3}{\beta(\beta b_o + c_o)} c_o \alpha - \frac{c_3}{\beta(\beta b_o + c_o)} c_o U_{is}$$

$$+\frac{c_3}{(\beta b_o + c_o)}U_{md} + \frac{c_3}{\beta}\alpha + \frac{c_3}{\beta}U_{is} - U_{ms}$$

Hence

$$a_3 = MS^* \left(1 - \frac{c_3}{b_o \beta + c_o}\right) + \frac{\alpha}{\beta} \left(c_3 - \frac{c_3}{b_o \beta + c_o} c_o\right) - U_{is} \left(\frac{c_o}{\beta} \frac{c_3}{(b_o \beta + c_o)} - \frac{c_3}{\beta}\right) + U_{md} \frac{c_3}{(b_o \beta + c_o)} + U_{ms}$$

e) Derivation of Reduced Form for Income, Case A

The LM is:

$$Y = \frac{1}{b_o} (a_3^* + (c_3 - c_o)r) + U_{ms} - U_{md}$$

Inserting r from the IS we obtain:

$$Y = \frac{a_3^*}{b_o} + \left(\frac{c_3 - c_o}{b_o}\right) \left(\frac{Y - \alpha - U_{is}}{\beta}\right) + \frac{U_{ms}}{b_o} - \frac{U_{md}}{b_o}$$

Rearranging and solving for Y we obtain:

$$Y = \frac{1}{b_o \beta - c_3 + c_o} (\beta a_3^* - (c_3 - c_o)\alpha - (c_3 - c_o)U_{is} + \beta U_{ms} - \beta U_{md})$$

where: $\frac{1}{b_o \beta - c_3 + c_o} < 0$

f) Derivation of Reduced Form of Income, case B.

The LM becomes:

$$Y = \frac{1}{b_o} (MS^* - c_o r - U_{md})$$

Substituting r from IS we obtain:

$$Y = \frac{1}{b_o} (MS^* - c_o \left(\frac{Y - \alpha - U_{is}}{\beta}\right) - U_{md})$$

Then solving for income we obtain:

$$Y = \frac{1}{b_o \beta + c_o} (\beta MS^* + c_o \alpha + c_o U_{is} - \beta U_{md})$$

where: $\frac{1}{b_o \beta + c_o} < 0$

since: $b_o > 0, \beta < 0$ and $c_o < 0$.

g) Comparison of Multipliers, Case A versus Case B.

i) Case A Multiplier:

$$\xi_a = \frac{1}{b_o \beta + c_o - c_3} < 0$$

Case B Multiplier:

$$\xi_b = \frac{1}{b_o \beta + c_o} < 0$$

Conclusion: $|\xi_a| < |\xi_b|$

ii) Multiplier of U_{is} , Case A:

$$\frac{\partial Y}{\partial U_{is}} = \frac{-c_3 + c_o}{b_o \beta + c_o - c_3} > 0$$

Multiplier of U_{is} , Case B:

$$\frac{\partial Y}{\partial U_{is}} = \frac{c_o}{b_o \beta + c_o} > 0$$

Conclusion: $(\frac{\partial Y}{\partial U_{is}})_a - (\frac{\partial Y}{\partial U_{is}})_b =$

$$\frac{-\cancel{\beta b_o c_3} + \cancel{c_o \beta b_o} - \cancel{c_o c_3} + \cancel{c_o^2} - \cancel{c_o \beta b_o} + \cancel{c_o c_3} - \cancel{c_o^2}}{(b_o \beta - c_3 + c_o)(\beta b_o + c_o)} =$$

$$= -\frac{\beta b_o c_3}{(b_o \beta - c_3 + c_o)(\beta b_o + c_o)} > 0$$

since: $\beta < 0$, $c_3 > 0$ and $c_o < 0$

Hence $\left(\frac{\partial Y}{\partial U_{is}^a} \right) > \left(\frac{\partial Y}{\partial U_{is}^b} \right)$

iii) Multiplier of U_{md} , Cases A and B.

$$\frac{\partial Y}{\partial U_{md}^a} = -\xi_a \beta < 0$$

$$\frac{\partial Y}{\partial U_{md}^b} - \xi_b \beta < 0$$

Conclusion: $|\xi_a \beta| - |\xi_b \beta| = \beta (|\xi_a| - |\xi_b|) < 0$

Since $|\xi_a| < |\xi_b|$

Hence $\left(\frac{\partial Y}{\partial U_{md}^a} \right) < \left(\frac{\partial Y}{\partial U_{md}^b} \right)$

III) MODEL A-4

1. Interest Rate Policy

a) Reduced Form of Income, Case A: Price Expectations of Income growth,
Policy Revised in the short run.

The IS is:

$$Y = \alpha + \beta_2 r^* - \beta_2 \dot{p} + U_{is}$$

Then:

$$r^* = \frac{Y^* - \alpha - \beta_2 \tilde{p}_e}{\beta_2}$$

So :

$$Y = \alpha + \beta_2 \left(\frac{Y^* - \alpha + \beta_2 \tilde{p}_e}{\beta_2} \right) - \beta_2 \dot{p}_e + U_{is}$$

Hence:

$$Y = Y^* + \beta_2 \tilde{p}_e - \beta_2 \dot{p}_e + U_{is}$$

Then: If $\tilde{p}_e = \dot{p}_e$, $Y = Y^* + U_{is}$

But if $\tilde{p}_e \neq \dot{p}_e$, $Y = Y^* + \beta_2 \tilde{p}_e - \beta_2 \dot{p}_e + U_{is}$

b) Derivation of the Reduced Form of Income, Case A, Policy is not Revised.

The estimated price expectations to be used during all short periods (j's) within the long term period i are:

$$\begin{aligned} \tilde{p}_{e_i} = & \sum \sum a_{i-k,j} \left(\frac{Y_{i-k,j} - Y_{i-k,j-1}}{Y_{i-k,j-1}} \right) \\ & + \sum a_{i-k,1} \left(\frac{Y_{i-k,1} - Y_{i-k-1,4}}{Y_{i-k-1,4}} \right) \end{aligned}$$

Note that these estimated price expectations are based in past income growth (happenings of past long run periods i's).

The Actual price expectations for each short term period within the long run period i are:

$$\begin{aligned} \dot{P}_{e_{ij}} &= \sum_{k=1}^n \sum_{l=2}^4 a_{i-k,l} \left(\frac{Y_{i-k,l} - Y_{i-k,l-1}}{Y_{i-k,l-1}} \right) \\ &+ \sum_{k=1}^n a_{i-k,1} \left(\frac{Y_{i-k,1} - Y_{i-k-1,4}}{Y_{i-k-1,4}} \right) \\ &+ \sum_{l=2}^{j-1} a_{i,l} \left(\frac{Y_{i,l} - Y_{i,l-1}}{Y_{i,l-1}} \right) + a_{i,1} \left(\frac{Y_{i,1} - Y_{i-1,4}}{Y_{i-1,4}} \right) \quad \text{For } j > 1. \end{aligned}$$

Because if $j = 1$, $\dot{P}_{e_{ij}} = \tilde{P}_{e_{ij}} = \dot{P}_{e_i}$, since $\tilde{P}_{e_i}$ is based in all information prior to $j = 1$, which equals $\dot{P}_{e_{ij}}$ (true price expectations).

Then, the reduced form of income for $j > 1$ is:

$$\begin{aligned} Y_{ij} &= Y_{ij}^* + U_{is_{ij}} + \beta_2 (\tilde{P}_e - \dot{P}_e) \\ Y_{ij} &= Y_{ij}^* + U_{is_{ij}} + \beta_2 \left(- \sum_{l=2}^{j-1} a_{i,l} \left(\frac{Y_{i,l} - Y_{i,l-1}}{Y_{i,l-1}} \right) - a_{i,1} \left(\frac{Y_{i,1} - Y_{i-1,4}}{Y_{i-1,4}} \right) \right) \end{aligned}$$

c) Effect of a Real Sector Shock occurring in the long term period i and short term period j ($U_{is_{ij}}$), on income and money supply of the present and future short term periods within the present long term period i . Assuming an interest rate policy, case A and no revision.

$$\begin{aligned} Y_{ij} &= Y_{ij}^* + \beta_2 \left(-a_{i,1} \frac{Y_{i,1} - Y_{i-1,4}}{Y_{i-1,4}} - \right. \\ &\left. -a_{i,2} \frac{Y_{i,2} - Y_{i,1}}{Y_{i,1}} \dots a_{i,j-1} \frac{Y_{i,j-1} - Y_{i,j-2}}{Y_{i,j-2}} \right) \end{aligned}$$

$$+U_{is_{ij}}$$

where: $U_{is} = \frac{U_c}{1-b_1}$ or $\frac{U_i}{1-b_1}$

Suppose ΔU_c is positive, hence:

$$\Delta Y_{ij} = \frac{1}{1-b_1} \Delta U_{c_{ij}}$$

and $\Delta MS_{ij} = \frac{b_o}{1-b_1} \Delta U_{c_{ij}}$

Then $Y_{i,j+1} = Y_i^* + \beta_2 (-a_{i,1} \frac{Y_{i,1} - Y_{i-1,4}}{Y_{i-1,4}} - \dots - a_{i,j} \frac{Y_{i,j} - Y_{i,j-1}}{Y_{i,j-1}}) + \frac{U_{c_{i,j+1}}}{1-b_1}$

Hence $Y_{i,j+1} - Y_{i,j} = (U_{c_{i,j+1}} - U_{c_{ij}}) - \beta_2 a_{i,j} \frac{Y_{i,j} - Y_{i,j-1}}{Y_{i,j-1}}$

Assuming $U_{is_{i,j+1}} - U_{is_{i,j}} = 0$, which means no new shock, since we are investigating the effects of $U_{is_{ij}}$ only:

$$\Delta Y_{i,j+1} = -\beta_2 a_{i,j} \left(\frac{\Delta Y_{i,j}}{Y_{i,j-1}} \right)$$

Hence $\Delta Y_{i,j+1} = -\beta_2 a_{i,j} \left(\frac{\Delta U_{c_{i,j}}}{(1-b_1) Y_{i,j-1}} \right)$

And $\Delta MS_{i,j+1} = b_o \Delta Y_{i,j+1}$

$$\Delta MS_{i,j+1} = -b_o \beta_2 a_{i,j} \left(\frac{\Delta U_{c_{i,j}}}{(1-b_1) Y_{i,j-1}} \right)$$

And doing so for further future periods we obtain for the $j+k$ period:

$$\Delta Y_{i,j+k} = \frac{\beta_2^k a_{i,j} a_{i,j+1} \dots a_{i,j+k-1}}{Y_{i,j-1} Y_{i,j} \dots Y_{i,j+k-2}} (1-b_1) \Delta U_{c_{i,j}}$$

$$\text{Therefore: } \frac{\Delta Y_i}{\Delta U_{c_{i,j}}} = \left(1 - \frac{\beta_2 a_{i,j}}{Y_{i,j-1}} + \frac{\beta_2^2 a_{i,j} a_{i,j+1}}{Y_{i,j-1} Y_{i,j}}\right) \left(\frac{1}{1-b_1}\right) \\ + \dots + (-1)^k \beta_2^k \frac{a_{i,j} a_{i,j+1} \dots a_{i,j+k-1}}{2^{(1-b_1)} Y_{i,j-1} Y_{i,j} Y_{i,j+1} \dots Y_{i,j+k-2}} > 1$$

$$\text{And } \frac{\Delta MS_i}{\Delta U_{c_{i,j}}} = \frac{1}{1-b_1} \left(b_0 - \frac{\beta_2 a_{i,j}}{Y_{i,j-1}} + \frac{b_0 \beta_2^2 a_{i,j} a_{i,j+1}}{Y_{i,j-1} Y_{i,j}} \right. \\ \left. + \dots + b_0 (-1)^k \beta_2^k \frac{a_{i,j} a_{i,j+1} \dots a_{i,j+k-1}}{2^k Y_{i,j-1} Y_{i,j} Y_{i,j+1} \dots Y_{i,j+k-2}} \right) > b_0.$$

d) Effect of a real sector shock occurring in long term period i and short term period j ($U_{c_{i,j}}$) on income and money supply of present and future sort term periods within the present long term period i . Under assumption of an interest rate policy, Case B, and no short run revisions.

$$Y_{ij} = Y_i^* + \beta_2 \left[-w_{i,1} \frac{M_{i,1} - M_{i-1,4}}{M_{i-1,4}} - w_{i,2} \frac{M_{i,2} - M_{i,1}}{M_{i,1}} \right. \\ \left. - \dots - w_{i,j-1} \frac{M_{i,j-1} - M_{i,j-2}}{M_{i,j-2}} \right] + \frac{U_{c_{i,j}}}{1-b_1}$$

So if $\Delta U_{c_{i,j}}$ is not zero, then:

$$\Delta Y_{i,j} = \frac{1}{1-b_1} \Delta U_{c_{i,j}}$$

$$\text{And } \Delta M_{i,j} = \frac{b_0}{1-b_1} \Delta Y_{i,j} = \frac{b_0}{1-b_1} \Delta U_{c_{i,j}}$$

$$\text{So } \Delta Y_{i,j+1} = \Delta U_{c_{i,j}} - \beta_2 w_{i,j} \frac{\Delta M_{i,j}}{M_{i,j-1}}$$

And then assuming $\Delta U_{c_{i,j+1}} = 0$

$$\Delta Y_{i,j+1} = -\beta_2 w_{i,j} \frac{b_o \Delta U_{c,i,j}}{(1-b_1) M_{i,j-1}}$$

$$\Delta M_{i,j+1} = -\beta_2 b_o^2 w_{i,j} \frac{\Delta U_{c,i,j}}{(1-b_1)} M_{i,j-1}$$

Doing this for periods $j+2$, $j+3$, etc. and adding all $\Delta Y_{i,j}$ we obtain:

$$\frac{\Delta Y_i}{\Delta U_{c,i,j}} = \frac{1}{1-b_1} \left[1 - \frac{\beta_2 b_o w_{ij}}{M_{i,j-1}} + \frac{\beta_2^2 w_{ij} w_{i,j+1}}{M_{i,j-1} M_{i,j}} + \dots + (-1)^k \frac{\beta_2^k b_o^k w_{ij} w_{i,j+1} \dots w_{i,j+k-1}}{M_{i,j-1} M_{ij} M_{i,j+1} \dots M_{i,j+k-2}} \right] > 1$$

where: $\Delta Y_i = \Sigma Y_{ij}$

e) Effect of a Monetary Shock occurring in long term period i and short term period j ($U_{md_{ij}}$) on Income and Money Supply of the present and future short term periods within the present long term period. Assuming interest rate policy, case B and no revision.

$$\Delta MS_{ij} = 1 \Delta U_{md_{ij}}$$

$$\Delta Y_{ij} = 0$$

But
$$\Delta Y_{i,j+1} = -\beta_2 w_{ij} \frac{\Delta M_{ij}}{M_{i,j-1}}$$

$$\Delta Y_{i,j+1} = -\beta_2 w_{ij} \frac{U_{md_{ij}}}{M_{i,j-1}}$$

Then
$$\Delta M_{i,j+1} = b_o \Delta Y_{i,j+1} = -b_o \beta_2 w_{ij} \frac{\Delta U_{md_{ij}}}{M_{i,j-1}}$$

Doing so for future periods we get for period $j+k$:

$$\Delta Y_{i,j+k} = \beta_2^k b_o^{k-1} \frac{w_{ij} w_{i,j+1} \dots w_{i,j+k-1}}{M_{ij} M_{i,j-1} \dots M_{i,j+k-2}} \Delta U_{md_{i,j}}$$

$$\text{Hence } \frac{\Delta Y_i}{\Delta U_{md_{ij}}} = 0 - \beta_2 \frac{w_{ij}}{M_{i,j-1}} + \beta_2^2 b_o \frac{w_{ij} w_{i,j+1}}{M_{i,j-1} M_{ij}}$$

$$+ \dots + (-1)^k \beta_2^k b_o^{k-1} \frac{w_{ij} w_{i,j+1} \dots w_{i,j+k-1}}{M_{i,j-1} M_{ij} M_{i,j+1} \dots M_{i,j+k-2}} > 0$$

$$\text{And } \frac{\Delta M_i}{\Delta U_{md_{ij}}} = 1 + b_o \frac{\Delta Y_i}{\Delta U_{md_{ij}}} > 1$$

f) Effect of a Real Sector Shock ($U_{c_{ij}}$ or $U_{i_{ij}}$) on Income and interest rates, assuming a money supply policy (M^*), under case A and no revision.

$$Y_{ij} = Y_{i,A}^* + \frac{1}{b_o \beta_2 + c_o} + \frac{c_o \beta_2}{b_o \beta_2 + c_o} \left[-a_{i,1} \frac{Y_{i,1} - Y_{i-1,4}}{Y_{i-1,4}} - a_{i,2} \frac{Y_{i,2} - Y_{i,1}}{Y_{i,1}} - \dots - a_{i,j-1} \frac{Y_{i,j-1} - Y_{i,j-2}}{Y_{i,j-2}} \right]$$

$$\text{where: } A_{ij} = c_o U_{c_{ij}} - \beta_2 U_{md_{ij}}$$

If $U_{c_{ij}}$ is positive:

$$\Delta Y_{ij} = \frac{c_o}{b_o \beta_2 + c_o} \Delta U_{c_{ij}} = \frac{c_o}{b_o \beta_2 + c_o} U_{c_{ij}}$$

assuming that U_c was zero before period i,j .

$$\text{And } \Delta r_{ij} = \frac{b_o}{b_o \beta_2 + c_o} \Delta U_{c_{ij}}$$

$$\text{Then } \Delta Y_{i,j+1} = \frac{c_o}{b_o \beta_2 + c_o} \Delta U_{i,j+1} - \frac{c_o \beta_2}{b_o \beta_2 + c_o} a_{ij} \left[\frac{\Delta Y_{ij}}{Y_{i,j-1}} \right]$$

And assuming that $\Delta U_{is_{i,j+1}}$ is zero:

$$\Delta Y_{i,j+1} = \frac{c_o \beta_2}{b_o \beta_2 + c_o} a_{ij} \left(\frac{c_o U_{c_{ij}}}{b_o \beta_2 + c_o Y_{i,j-1}} \right)$$

which shows the effect of $U_{c_{ij}}$ in next period's income. It turns out to be:

$$\Delta Y_{i,j+1} = \frac{c_o^2 \beta_2^2 a_{ij} \Delta U_{c_{ij}}}{(b_o \beta_2 + c_o)^2 Y_{i,j-1}}$$

Doing this for periods $j+2$, $j+3$, etc, we get the general formula:

$$\Delta Y_{i,j+k} = (-1)^k c_o^{k+1} \beta_2^k \frac{a_{ij} a_{i,j+1} \dots a_{i,j+k-1}}{(b_o \beta_2 + c_o)^{k+1} Y_{i,j+k-2} \dots Y_{i,j-1}} \Delta U_{is_{ij}}$$

Then adding up the total effects of $U_{is_{ij}}$ on income we obtain:

$$\frac{\Delta Y_i}{\Delta U_{is_{ij}}} = \frac{c_o}{b_o \beta_2 + c_o} - c_o^2 \beta_2^2 \frac{a_{ij}}{(b_o \beta_2 + c_o)^2 Y_{i,j-1}} + c_o^3 \beta_2^2 \frac{a_{ij} a_{i,j+1}}{(b_o \beta_2 + c_o)^3 Y_{i,j-1} Y_i}$$

$$+ \dots + (-1)^k c_o^{k+1} \beta_2^k \frac{a_{ij} \dots a_{i,j+k-1}}{(b_o \beta_2 + c_o)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{i,j+k-2}} > 1$$

where: $U_{is_{ij}} = \frac{U_c}{1-b_1}$

The same procedure for the interest rates give:

$$\frac{\Delta r_i}{\Delta U_{is_{ij}}} = - \frac{b_o}{b_o \beta_2 + c_o} + \frac{b_o c_o \beta_2 a_{ij}}{(b_o \beta_2 + c_o)^2 Y_{i,j-1}} - \frac{c_o^2 b_o \beta_2^2 a_{ij} a_{i,j+1}}{(b_o \beta_2 + c_o)^3 Y_{i,j-1} Y_{ij}} + \dots + (-1)^{k-1} \frac{c_o^k b_o \beta_2^k a_{ij} \dots a_{i,j+k-1}}{(b_o \beta_2 + c_o)^{k+1} Y_{i,j-1} \dots Y_{i,j+k-2}}$$

g) Effect of a monetary sector shock ($U_{md_{ij}}$) on Income and Interest

rates, assuming a money supply policy, Case A, no revision.

If $U_{md_{ij}}$ is not zero, we have:

$$\Delta Y_{ij} = \frac{-\beta_2}{b_0\beta_2 + c_0} \Delta U_{md_{ij}} = \frac{-\beta_2}{b_0\beta_2 + c_0} U_{md_{ij}}$$

assuming $U_{md_{ij}}$ was zero before period i, j .

$$\text{Also } \Delta r_{ij} = \frac{-1}{b_0\beta_2 + c_0} \Delta U_{md_{ij}}$$

$$\text{Then } \Delta Y_{i,j+1} = \frac{\beta_2}{b_0\beta_2 + c_0} \Delta U_{md_{i,j+1}} - \frac{c_0\beta_2}{b_0\beta_2 + c_0} a_{ij} Y_{i,j-1} \frac{\Delta Y_{ij}}{Y_{i,j-1}}$$

but we assume $\Delta U_{md_{i,j+1}}$ is zero, therefore:

$$\Delta Y_{i,j+1} = \frac{c_0\beta_2}{b_0\beta_2 + c_0} a_{ij} \frac{-\beta_2 U_{md_{ij}}}{b_0\beta_2 + c_0 Y_{i,j-1}}$$

which equals:

$$\Delta Y_{i,j+1} = \frac{c_0\beta_2^2 a_{ij}}{(b_0\beta_2 + c_0)^2 Y_{i,j-1}} U_{md_{ij}}$$

Hence for period $J+k$ we have:

$$\Delta Y_{i,j+k} = (-1)^{k+1} \frac{c_0^k \beta_2^{k+1} a_{ij} a_{i,j+1} \dots a_{i,j+k+1}}{(b_0\beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{i,j+k-2}} U_{md_{ij}}$$

Doing the same for the interest rate we obtain:

$$\Delta r_{i,j+k} = (-1)^k \frac{b_0^k c_0^{k-1} \beta_2^{k+1} a_{ij} a_{i,j+1} \dots a_{i,j+k-1}}{(b_0\beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{i,j+k-2}} U_{md_{ij}}$$

Adding up all effects on Income we have:

$$\frac{\Delta Y_i}{\Delta U_{md_{ij}}} = -\frac{\beta_2}{b_0\beta_2 + c_0} + \frac{c_0\beta_2^2 a_{ij}}{(b_0\beta_2 + c_0)^2} - \frac{c_0\beta_2^3 a_{ij} a_{i,j+1}}{(b_0\beta_2 + c_0)^3 Y_{i,j-1} Y_{ij}} + \dots + (-1)^{k+1} \frac{c_0^k \beta_2^{k+1} a_{ij} a_{i,j+1} \dots a_{i,j+k-1}}{(b_0\beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{i,j+k-2}} < 0$$

And for the interest rate:

$$\frac{\Delta r_i}{\Delta U_{md_{ij}}} = -\frac{1}{b_0\beta_2 + c_0} - \frac{b_0\beta_2^2}{(b_0\beta_2 + c_0)^2} + \frac{b_0 c_0 \beta_2^3 a_{ij} a_{i,j+1}}{(b_0\beta_2 + c_0)^3 Y_{i,j-1} Y_{ij}} + \dots + (-1)^k \frac{b_0 c_0^{k-1} \beta_2^{k+1} a_{ij} a_{i,j+1} \dots a_{i,j+k-1}}{(b_0\beta_2 + c_0)^{k+1} Y_{i,j-1} Y_{ij} \dots Y_{i,j+k-2}}$$

In this case the initial effect is offset by the dynamic effects.

Hence interest rates could finally, after adjustment, be higher, lower or the same as before.

I. MODEL A-5

CASE 1

1. Derivation of the IS Equation

We substitute real sector equations into equilibrium:

$$(1) \quad Y = a_1 + b_1(Y-T) + c_1'r + l(GD+MS+K) + U_c + a_2 + c_1'r + U_i + G$$

Then from the model we have:

$$G = T + \Delta MS + \Delta GD$$

$$G = T + MS_t - MS_{t-1} + GD_t - GD_{t-1}$$

$$\text{Hence: } GD_t + MS_t + K = G - T + MS_{t-1} + GD_{t-1} + K$$

So we introduce this into (1) above and obtain:

$$(3) \quad Y - b_1 Y = a'_1 + a_2 - T(b'_1 + 1) + r(c'_1 + c_1) + lGD_{t-1} + lMS_{t-1} + lG + lK$$

$$(4) \quad Y = \frac{1}{1-b_1} (a_1 + a_2 - T(b_1 + 1) + lGD_{t-1} + lMS_{t-1} + lK + (1+l)G + r(c'_1 + c_1) + U_i + U_c)$$

Then from model equations we get:

$$G = T + MS_t - MS_{t-1} + GD_t - GD_{t-1}$$

$$G_{t-1} = T_{t-1} + MS_{t-1} - MS_{t-2} + GD_{t-1} - GD_{t-2}$$

$$GD_{t-1} = G_{t-1} - T_{t-1} - MS_{t-1} + MS_{t-2} + GD_{t-2}$$

$$\text{So: } GD_{t-2} = G_{t-2} - T_{t-2} - MS_{t-2} + MS_{t-3} + GD_{t-3}$$

For period $t-n$ we get:

$$GD_{t-n} = G_{t-n} - T_{t-n} - MS_{t-n} + MS_{t-n-1} + GD_{t-n-1}$$

Then substituting GD_{t-2} , ..., GD_{t-n} into GD_{t-1} we get:

$$(5) \quad GD_{t-1} = GD_{t-n-1} + (MS_{t-n-1} - MS_{t-1}) + \Sigma(G_{t-i} - T_{t-i})$$

Where:

GD_{t-1-n} = Initial government debt. $MS_{t-n-1} - MS_{t-1}$ = Increase in the money supply since initial period and $\Sigma(G_{t-i} - T_{t-i})$ = accumulated deficits. So we replace this equation 5 into 4 above and obtain:

$$Y_t = \frac{1}{1-b_1} \left[(a_1 + a_2 + 1(G_t - T_t) + (G_t - b_1 T_t) + 1 \Sigma(G_{t-i} - T_{t-i}) + 1 GD_{t-n-1} + 1 MS_{t-n-1} + r(c'_1 + c_1) + U_i + U_c) \right]$$

2. Money Supply Policy

a) Relationship between the interest elasticity of the demand for money (c_0) and effect of real sector shocks on income ($\partial Y / \partial U_i$).

$$\frac{\partial(\frac{\partial Y}{\partial U_i})}{\partial c_0} = \frac{c'_1 + c_1}{(b_0(c'_1 + c_1) + c_0(1-b_1))} < 0$$

This says that a larger c_0 will render a smaller effect of real sector shocks on income; but since c_0 is negative, larger c_0 means a smaller value for the coefficient, that is a smaller absolute value for c_0 .

b) Relationship between Interest elasticity of investment and consumption and the effect of money demand shocks on income ($\partial Y / \partial U_{md}$).

$$\frac{\partial(\partial Y/\partial U_{md})}{\partial(c'_1+c_1)} = \frac{c_o(1-b_1)}{(c'_1+c_1+c_o(1-b_1))^2} > 0$$

Therefore the larger the interest sensitivity of investment and consumption, the greater the effect of money demand shocks on income.

c) Derivation of the IS equation, case 1, assuming $MS_{t-1} = MS^*$.

Under a money supply policy, $\Delta MS = 0$ so:

$$Y = a_1 + b_1 Y - b_1 T + c'_1 r + 1(GD_{t-1} + G - T) \\ + 1MS^* + 1K + U_c + c_1 r + a_2 + U_i + G$$

$$\text{So: } Y = \frac{1}{1-b_1} (a_1 + a_2 - T(b_1+1) + 1GD_{t-1} + (1+1)G \\ + r(c'_1+c_1) + 1MS^* + 1K + U_i + U_c)$$

Notice that we are implicitly assuming that MS^* is not revised within the policy period.

d) Reduced Form of Interest Rates under money supply policy, case 1.

We substitute the LM into the above IS equation and get:

$$r = \frac{1}{(c'_1+c_1)b_o+c_o(1-b_1)} ((1-b_1-b_o)MS^* - b_o(a_1+a_2) \\ + b_o(1+b_1)T - 1b_oGD_{t-1} - b_o(1+1)G \\ - 1b_oK - b_o(U_c+U_i - (1-b_1)U_{md}))$$

e) Derivation of reduced form of income under a money supply policy,

case 1. possibilities are the following:

The interest rate is solved in the IS equation and substituted into the LM to obtain:

$$Y = \frac{1}{b_0(c'_1 + c_1) + c_0(1 - b_1)} \left[(c'_1 + c_1)MS^* + c_0(a_1 + a_2) - c_0(1 + b_1T) \right. \\ \left. c_0(1 + l)G + c_0(1)(MS^* + GD_{t-1}) + c_0(1)K.i10; + c_0U_i + c_0U_c - (c'_1 + c_1)U_{md} \right]$$

CASE 2

3. Derivation of IS

Substituting each real sector equation into the equilibrium condition we obtain:

$$Y = a_1 + b_1Y - b_1T + c'_1r + c_1r + lMS + lK + U_c + U_iG + a_2$$

Rearranging terms:

$$Y = \frac{1}{1 - b_1} (a_1 + a_2 + G - b_1T + lK + r(c'_1 + c_1) + lMS + U_i + U_c)$$

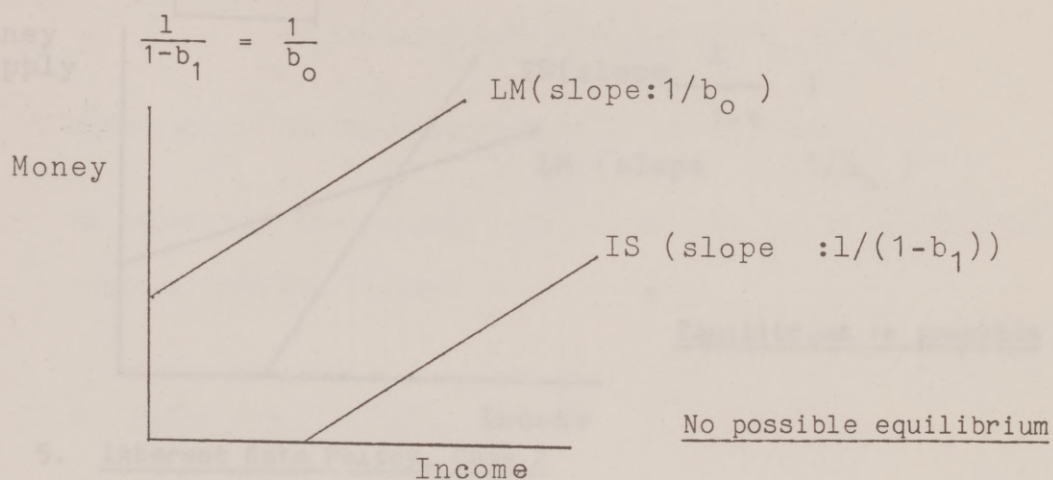
4. Stability of the System, case 2

To analyze the stability of the system we analyze the relationship between money supply and income on each sector, that is, on the IS and the LM. They are given by:

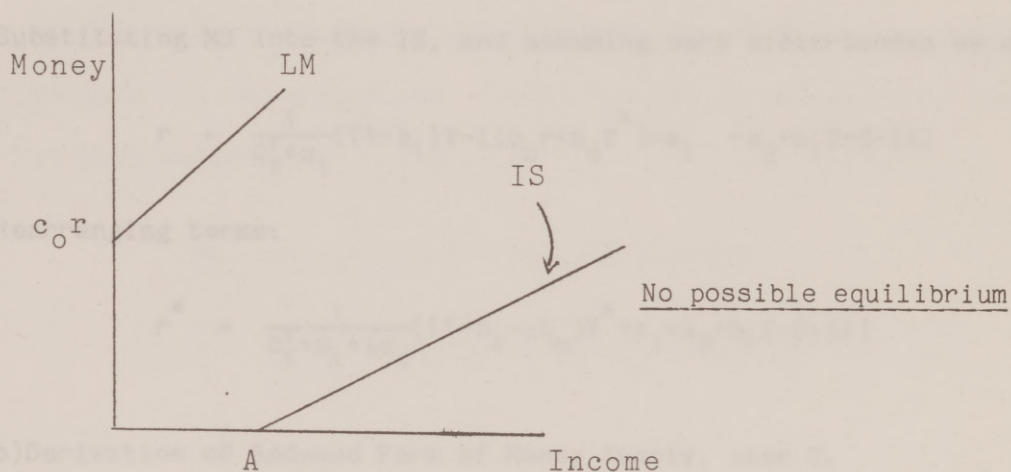
$$\frac{\partial Y}{\partial MS} = \frac{1}{1 - b_1} \quad \text{in Real Sector}$$

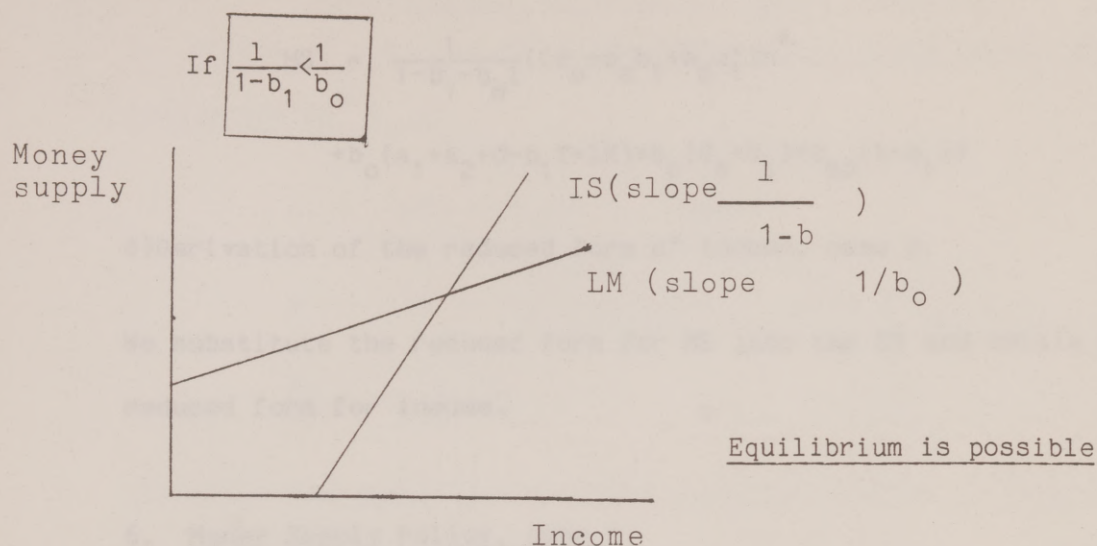
$$\frac{\partial Y}{\partial MS} = \frac{1}{b_0} \quad \text{in Monetary Sector}$$

Then the possibilities are the following:



If $\frac{1}{1-b_1} > \frac{1}{b_0}$





5. Interest Rate Policy, Case 2

a) Determination of Instrument Value

Substituting MS into the IS, and assuming zero disturbances we obtain:

$$r = \frac{1}{c'_1 + c_1} ((1-b_1)Y - 1(c_0 r + b_0 Y^*) - a_1 - a_2 + b_1 T - G - 1K)$$

Rearranging terms:

$$r^* = \frac{1}{c'_1 + c_1 + 1c_0} ((1-b_1 - 1b_0)Y^* - a_1 - a_2 + b_1 T - G - 1K)$$

b) Derivation of Reduced Form of Money Supply, case 2.

We substitute income (Y) from the IS into the money market equilibrium and obtain:

$$MS = c_0 r + \left(\frac{1}{1-b_1} (a_1 + a_2 + (G - b_1 T + 1K + r(c'_1 + c_1) + U_i + U_c) + U_{md} \right)$$

Rearranging terms we solve for MS:

$$MS = \frac{1}{1-b_1-b_o}((c_o-c_o b_1+b_o c_1^*)r^* + b_o(a_1+a_2+G-b_1T+lK)+b_o(U_c+U_i)+U_{md}(1-b_1))$$

c) Derivation of the reduced form of income, case 2.

We substitute the reduced form for MS into the IS and obtain the reduced form for income.

6. Money Supply Policy, case 2

a) Determination of Instrument value MS^*

We estimate the interest rate from the IS, which in this case will include MS, and substitute it into the LM, assuming zero disturbances in both markets:

$$MS^* = c_o r + b_o Y^* \quad \text{LM equilibrium}$$

$$\tilde{r} = \frac{1}{c_1^*+c_1}((1-b_1)Y^* - a_1 - a_2 + b_1T - G - lMS - lK)$$

So we obtain:

$$MS^* = \frac{1}{c_1^*+c_1+c_o}((c_o(1-b_1)+b_o)Y^* + c_o b_1T$$

$$-c_o G - c_o a_1 - c_o a_2 - c_o lK)$$

COMPARISONS OF CASE 1 VERSUS CASE 2

7. Effect of a ΔG and ΔT on the Money Supply assuming an Interest Rate

Policy and $\Delta G = \Delta T$

$$A = \Delta M = b_0 \Delta G \quad \text{Case 1}$$

$$B = \Delta M = \frac{b_0(1-b_1)}{(1-b_1-b_0)} \Delta G \quad \text{Case 2}$$

$$\frac{A}{B} = \frac{1-b_1-b_0}{1-b_1} < 1$$

Therefore the change in the money supply is greater in case 2 than in 1.

8. Analysis involving Government Debt

a) Derivation of Reduced Form of GD_t .

From our model equations we have:

$$1) \quad GD_t = G - T + GD_{t-1} - MS_t + MS_{t-1}$$

$$2) \quad GD_{t-1} = GD_{t-n-1} + MS_{t-n-1} - MS_{t-1} + \Sigma(G_{t-i} - T_{t-i})$$

Then substituting 2) into 1):

$$3) \quad GD_t = G - T + GD_{t-n-1} + MS_{t-n-1} + \Sigma(G_{t-i} - T_{t-i}) - MS_t$$

Then substituting into 3) the reduced form equation for MS_t we get:

$$4) \quad GD_t = G - T + GD_{t-n-1} + MS_{t-n-1} + \Sigma(G_{t-i} - T_{t-i})$$

$$-c_0 r^* - \frac{b_0}{1-b_1} [a_1 + a_2 + 1(G-T) + (G-b_1 T)]$$

$$+1\Sigma(G_{t-i}-T_{t-i})+1GD_{t-n-1}+1MS_{t-n-1}$$

$$+r^*(c_1'+c_1)+U_i+U_c) - U_{md}$$

Which after rearranging terms becomes:

$$\begin{aligned} 4') \quad GD_t &= (G-T) + \left(1 - \frac{1b_o}{1-b_1}\right) GD_{t-n-1} + \left(1 - \frac{1b_o}{1-b_1}\right) MS_{t-n-1} \\ &+ \left(1 - \frac{1b_o}{1-b_1}\right) (\Sigma(G_{t-i}-T_{t-i}) - c_o r^* - \frac{b_o}{1-b_1} (a_1+a_2)) \\ &- \frac{1b_o}{1-b_1} (G-T) - \frac{b_o}{1-b_1} (G-b_1 T) \\ &- \frac{b_o}{1-b_1} r^* (c_1'+c_1) - \frac{b_o}{1-b_1} (U_c+U_i) - U_{md} \end{aligned}$$

b) Derivation of the Effects of $\Delta G - \Delta T$ on the Money Supply and Income, assuming an interest rate policy and case 2.

Effects on the Money Supply:

$$\text{If } \Delta G = \Delta T \quad \Delta MS = \frac{b_o}{1-b_1-b_o} (1-b_1) \Delta G$$

Which is greater than in Case 1.

$$\text{If } \Delta G > \Delta T \quad \Delta MS = \frac{b_o}{1-b_1-b_o} (\Delta G - b_1 \Delta T)$$

Which is again greater than in Case 1.

Effects on Income:

$$\text{If } \Delta G = \Delta T \quad \Delta Y = \frac{1}{1-b_1-b_o} (1-b_1) \Delta G$$

$$\text{If } \Delta G > \Delta T \quad \Delta Y = \frac{1}{1-b_1-b_0} \frac{1}{1-b_1-b_0} (\Delta G - b_1 \Delta T)$$

c) Derivation of the effects of $\Delta G - \Delta T$ on Government Debt, assuming an interest rate policy, under case 2.

From the reduced form equation for GD (above) we obtain:

$$\Delta GD = \Delta G - \Delta T - \frac{b_0}{1-b_1-b_0} (\Delta G - b_1 \Delta T)$$

From there:

If $\Delta G = \Delta T$ ΔGD will be < 0 .

If $\Delta G > \Delta T$ ΔGD will be > 0 , its size depending on ΔMS .

Conclusion: The greater b_0 , the greater the increase in money supply and the lower the increase in government debt. The greater 1 and b_1 , the greater the ΔMS and the lower the ΔGD . The larger the marginal propensity to save, the lower the ΔMS and the greater the ΔGD .

d) Effect of the deficit per-se ($G_{t-1} - T_{t-1}$) on the money supply, under the interest rate policy, case 1

From the reduced form equation of the money supply:

$$\Delta MS_t = \frac{b_0 1}{1-b_1} \Delta (\Sigma (G_{t-i} - T_{t-i}))$$

$$\Delta MS_t = \frac{b_0 1}{1-b_1} \Delta \left[(G_{t-1} - T_{t-1}) + (G_{t-2} - T_{t-2}) + \dots + (G_{t-n} - T_{t-n}) \right]$$

$$\Delta MS_t = \frac{b_0 1}{1-b_1} (G_{t-1} - T_{t-1})$$

e) Effects of a deficit per-se on government debt.

$$\Delta GD_t = \left(1 - \frac{1b_0}{1-b_1}\right) \Delta(\Sigma(G_{t-i} - T_{t-i}))$$

which becomes:

$$\Delta GD_t = \left(1 - \frac{1b_0}{1-b_1}\right) (G_{t-1} - T_{t-1})$$

$$\Delta GD_{t+1} = \left(1 - \frac{1b_0}{1-b_1}\right) (G_t - T_t)$$

f) Comparison of the effect of ΔG and ΔT on income under a money supply and an interest rate policy, case 2.

$$\Delta Y = \frac{1}{1-b_1-b_0} (\Delta G - b_1 \Delta T) \quad \text{Under an Interest Rate Policy}$$

$$\Delta Y = \frac{c_0}{b_0(c'_1 + c_1) + c_0(1-b_1)} (\Delta G - b_1 \Delta T) \quad \text{Under a Money Supply Policy}$$

We know that $\frac{1}{1-b_1-b_0} > \frac{1}{1-b_1}$ and from case 1 we have:

$$\frac{1}{1-b_1} > \frac{c_0}{b_0(c'_1 + c_1) + c_0(1-b_1)}$$

Hence:

$$\frac{1}{1-b_1-b_0} > \frac{c_0}{b_0(c'_1 + c_1) + c_0(1-b_1)}$$

Hence, the effect is greater under an interest rate policy.

g) Comparison of the effect of ΔG and ΔT on income, under a money supply and an interest rate policy, case 1.

$$\Delta Y = \frac{1}{1-b_1} (1\Delta G - 1\Delta T + \Delta G - b_1 \Delta T) \quad \text{Under an Interest Rate Policy.}$$

$\Delta Y = \frac{c_0}{b_0(c_1' + c_1) + c_0(1 - b_1)} (1\Delta G - 1\Delta T + \Delta G - b_1\Delta T)$ Under a money supply policy.

The only difference is in the multipliers:

$$\frac{\text{Under } r^*}{\text{Under MS}^*} = \frac{1/(1 - b_1)}{(c_0 / (b_0(c_1' + c_1) + c_0(1 - b_1)))}$$

$$= \frac{b_0(c_1' + c_1) + c_0(1 - b_1)}{c_0(1 - b_1)} > 1$$

The multipliers of G and T are greater under an interest rate policy.

Hence if $\Delta G > \Delta T$ or $\Delta G = \Delta T$ in period T, the effect on income will be greater under an interest rate policy.

h) Effect of ΔG and ΔT on the monetary variables under different policies, cases 1 and 2.

CASE 1

If $\Delta G = \Delta T$:

$$r^* \quad \Delta MS = b_0 \Delta G = \frac{b_0}{1 - b_1} (\Delta G - b_1 \Delta T)$$

$$MS^* \quad \Delta r = \frac{-b_0}{b_0(c_1' + c_1) + c_0(1 - b_1)} (\Delta G - b_1 \Delta T)$$

If $\Delta G > \Delta T$:

$$r^* \quad \Delta MS = \frac{b_0}{1 - b_1} ((\Delta G - b_1 \Delta T) + 1(\Delta G - \Delta T))$$

$$MS^* \quad \Delta r = \frac{-b_0}{b_0(c_1' + c_1) + c_0(1 - b_1)} ((\Delta G - b_1 \Delta T) + 1(\Delta G - \Delta T))$$

CASE 2

$$r^* \quad \Delta M = \frac{b_0}{1-b_1-b_0} (\Delta G - b_1 \Delta T)$$

$$MS^* \quad \Delta r = \frac{-b_0}{b_0(c_1^* + c_1) + c_0(1-b_1)} (\Delta G - b_1 \Delta T)$$

1. MODEL 2-1

1. Unborrowed Resources Policy

a) Derivation of the effects of ΔG on ΔM .

From the reduced form equation for ΔM :

$$\Delta M = \frac{b_0}{1-b_1-b_0} (\Delta G - b_1 \Delta T)$$

$$\frac{\Delta M}{\Delta G} = \frac{b_0}{1-b_1-b_0}$$

$$\frac{\Delta M}{\Delta T} = \frac{-b_1 b_0}{1-b_1-b_0}$$

$$\frac{\Delta M}{\Delta T} = \frac{-b_1 b_0}{1-b_1-b_0}$$

since $c_1 > 0$, $b_1 > 0$ and $1-b_1 > 0$

11. MODEL 2-2

1. Unborrowed Resources Policy

a) Derivation of reduced form for ΔM .

CHAPTER 4

MATHEMATICAL DERIVATIONS AND PROOFS

I. MODEL D-11. Unborrowed Reserves Policy

a) Derivation of the effect of U_{dep} on Sma .

From the reduced form equation for Sma :

$$Sma = RU^* - Flt - Tco + Tch + c_2 r + b_2 Y + U_{cur} + Deg$$

$$\frac{\Delta Sma}{\Delta U_{dep}} = c_2 \frac{\Delta r}{\Delta U_{dep}} + b_2 \frac{\Delta Y}{\Delta U_{dep}}$$

$$\frac{\Delta Sma}{\Delta U_{dep}} = \frac{c_2 \tau_2 rr}{\beta - \tau_2 \rho_1} - \frac{b_2 rr \tau_2 \beta}{\beta - \tau_2 \rho_1} < 0$$

$$\frac{\Delta Sma}{\Delta U_{dep}} = \frac{rr \tau_2 (c_2 + b_2 \beta)}{\beta - \tau_2 \rho_1} < 0$$

since $c_2 < 0$, $b_2 \beta < 0$ and $\beta - \tau_2 \rho_1 < 0$

II. MODEL D-21. Unborrowed Reserves Policy

a) Derivation of reduced form for Sma .

$$RU^* = Sma + Flt + Tco - Tch - CUR - Deg$$

$$Sma = Sma + Flt + Tco - Tch - b_2 Y - U_{cur} - Deg$$

$$Sma = RU^* - Flt - Tco + Tch + b_2 Y + U_{cur} + Deg$$

b) Effect of U_{dep} and U_{cur} on Sma

$$\begin{aligned} \frac{\Delta Sma}{\Delta U_{dep}} &= \frac{\Delta Sma \Delta Y}{\Delta Y \Delta U_{dep}} \\ &= \frac{b_2 r r \tau_2 \beta}{\beta - \tau_2 \rho_1} < 0 \end{aligned}$$

2. Monetary Base Policy

a) Reduced form of Sma .

$$MB^* = RU + BOR + CUR$$

$$= Sma + Flt + Tco - Tch - CUR - Deg + BOR + CUR$$

$$Sma = MB^* - Flt - Tco + Tch + Deg - BOR$$

III. MODEL D-3

1. Unborrowed Reserves Policy

a) Derivation of reduced form for Sma .

This is obtained under the same procedure that we applied for model

D-2. It turns out to be:

$$Sma = RU^* - Flt - Tco + Tch + Deg + c_2 r + b_2 Y + CUR$$

b) Effect of U_{dep} on Sma .

$$\begin{aligned} \frac{\Delta Sma}{\Delta U_{dep}} &= c_2 \left(\frac{-\tau_2 rr}{\beta - \tau_2 \rho_3} \right) + b_1 \left(\frac{-rr \tau_2 \beta}{\beta - \tau_2 \rho_3} \right) \\ &= \frac{-\tau_2 rr (c_2 + b_2 \beta)}{\beta - \tau_2 \rho_3} < 0 \end{aligned}$$

IV. MODEL D-4

1. Interest Rate Policy

a) Derivation of reduced form of Sma

The IS and LM are equated:

$$\begin{aligned} \alpha + \beta r + U_{is} &= \tau (Sma + Flt + Tco - Tch - Deg + rrc_{3b} r_i \\ &+ \tau_5 r - ER + BOR - (rr U_{dep} + U_{cur})) \end{aligned}$$

From where we obtain:

$$Sma = \left(\frac{\beta}{\tau} - \rho_5 \right) r + \frac{\alpha}{\tau} - Flt - Tco + Tch$$

$$+ Deg - rrc_{3b} r_i + ER - BOR + rr U_{dep} + U_{cur}$$

2. Security Portfolio Policy

a) Derivation of instrument value Sma^* .

We substitute estimated r (r^*) from the IS into the LM and get:

$$(rrb_3 + b_2)Y^* = (-rrc_3 - c_2)\left(\frac{Y^* - \alpha}{\beta}\right) + rrc_{3a}r_i$$

$$-ER + BOR + Sma + Flt + Tco - Tch - Deg$$

From where we solve:

$$Sma^* = Y^* \left(\frac{1 - \rho}{\tau\beta} \right) + \frac{\alpha\tau}{\beta} - rrc_{3a}r_i + ER - BOR - Flt - Tco + Tch + Deg$$

b) Derivation of Reduced Form of Interest Rates

We equate IS and LM and solve for the interest rate:

$$\alpha + \beta r + U_{is} = \tau(Sma^* + Flt + Tco - Tch - Deg + rrc_{3a}r_i + \rho r - ER + BOR - U_{lm})$$

Solving for r we obtain:

$$r = \frac{1}{\beta - \tau\rho} \left(\tau Sma^* + \tau Flt + \tau Tco - \tau Tch - \tau Deg + \tau rrc_{3a}r_i \right.$$

$$\left. - \tau ER + \tau BOR - \tau U_{lm} - \alpha - U_{is} \right)$$

where: $U_{lm} = rrU_{dep} + U_{cur}$ and $\rho = -rrc_3 - c_2$

Note: $\rho_{\text{model D-1}} > \rho_{\text{model D-4 case 1}} > \rho_{\text{model D-4 case 2}}$

c) Derivation of Reduced form for income

We obtain r from the IS and substitute it into the LM :

$$Y = \tau Sma^* + \tau (Flt + Tco - Tch - Deg) + \tau rrc_{3a}r_i$$

$$+ \tau \rho \frac{Y - \alpha - U_{is}}{\beta} - \tau ER + \tau BOR - \tau U_{lm}$$

$$Y = \frac{\tau}{\beta - \tau\rho} (\beta Sma^* + \beta (Flt + Tco - Tch - Deg) + \beta rrc_{3a} r_i$$

$$- \rho\alpha - \rho U_{is} - \beta ER + \beta BOR - \beta U_{lm})$$

d) Effect of U_{is} on Income under cases 1 (ceilings) and 2 (no ceilings).

Define:

$$a = \frac{\partial Y}{\partial U_{is}} = \frac{\tau\rho}{\beta - \tau\rho}$$

$$\frac{\partial a}{\partial \rho} = \frac{-\beta\tau}{(\beta - \tau\rho)^2} > 0$$

Therefore the larger ρ is, the greater the effect of U_{is} on Income.

Since ρ when there are ceilings is larger than ρ when there are no ceilings. Hence the effect of U_{is} on income is greater when there are ceilings.

3. Unborrowed Reserves Policy

a) Derivation of reduced form for income, assuming $r_i = pr$

Substituting the interest rate from the IS into the LM we get:

$$Y - \frac{\tau_2 \rho_2}{\beta} Y = \tau_2 RU^* + \tau_2 BOR - \tau_2 ER - \frac{\tau_2 \rho_2}{\beta} \alpha$$

$$- \frac{\tau_2 \rho_2}{\beta} U_{is} + \tau_2 rrc_{3a} p \frac{Y - \alpha - U_{is}}{\beta} - \tau_2 \beta rr U_{dep}$$

Solving for Y after rearranging terms:

$$Y = \frac{1}{\beta - \tau_2 \rho_2 - \tau_2 rrc_{3a} p} (\beta \tau_2 RU^* + \beta \tau_2 BOR$$

$$-\beta\tau_2 ER + (U_{is} + \alpha)(\tau_2 rrc_{3a} p + \tau_2 \rho_2) - \tau_2 \beta r r U_{dep}$$

b) Proof that as β increases $\frac{\partial Y}{\partial r_i}$ decreases.

$$\text{Define } a = \frac{\partial Y}{\partial r_i} = \frac{-\tau_2 \beta r r c_{3a}}{\beta - \tau_2 \rho_2} > 0$$

$$\text{And } \frac{\partial a}{\partial \beta} = \frac{\tau_2 \rho_2 r r c_{3a}}{(\beta - \tau_2 \rho_2)^2} < 0$$

c) Sensitivity of $\frac{\partial Y}{\partial U_{is}}$ with respect to ρ_2

$$\text{Define: } b = \frac{\partial Y}{\partial U_{is}} = \frac{-\rho_2 \tau_2}{\beta - \rho_2 \tau_2} > 0$$

$$\frac{\partial b}{\partial \rho_2} = \frac{-\beta \tau_2}{\beta - \rho_2 \tau_2} > 0$$

So as ρ_2 increases, so does $\frac{\partial Y}{\partial U_{is}}$. But in case 1 such $\frac{\partial Y}{\partial U_{is}}$ is larger than in case 2.

d) Sensitivity of $\frac{\partial Y}{\partial U_{md}}$ to ρ_2 :

$$\text{Define: } c = \frac{\partial Y}{\partial U_{dep}} = \frac{-\beta \tau_2}{\beta - \rho_2 \tau_2} < 0$$

$$\frac{\partial c}{\partial \rho_2} = \frac{-\beta \tau_2^2}{(\beta - \tau_2 \rho_2)^2} > 0$$

Hence, as ρ_2 increases so does c , but since $c < 0$, this means that when ρ_2 increases, c becomes less negative, that is, its absolute value falls. In case 1, the absolute value of c is smaller, hence the absolute value of the effect of U_{dep} on income is smaller than in case 2.

e) Derivation of the Reduced Form for Income .

Substituting the interest rate from the IS into the LM we obtain:

$$Y = \tau_2^{RU*} + \tau_2^{BOR} - \tau_2^{ER} + \tau_2 \rho_2 \left(\frac{Y - \alpha - U_{is}}{\beta} \right)$$

In this last equation, we can substitute the interest rate from the IS into the LM equation of income and interest rates. The resulting equation will be:

Solving for Y we obtain:

$$Y = \frac{\tau_2}{\beta - \tau_2 \rho_2} (\beta^{RU*} + \beta^{BOR} - \beta^{ER} - \rho_2 \alpha - \rho_2 U_{is} + \beta^{rrc} \tau_2 r_i - \beta^{rrU} \tau_2 U_{dep})$$

f) Reduced Form for Interest Rates.

We equate LM Income and IS income:

$$\tau_2^{RU*} + \tau_2^{BOR} - \tau_2^{ER} + \tau_2 \rho_2 r - \tau_2^{rrc} \tau_2 r_i - \tau_2^{rrU} \tau_2 U_{dep} = \alpha + \beta r + U_{is}$$

Then we solve for r:

$$r = \frac{1}{\beta - \tau_2 \rho_2} (\tau_2^{RU*} + \tau_2^{BOR} - \tau_2^{ER} - \tau_2^{rrc} \tau_2 r_i - \tau_2^{rrU} \tau_2 U_{dep} - \alpha - U_{is})$$

g) Reduced Form for Sma.

By definition:

$$RU^* = Sma + Flt + Tco - Tch - Deg$$

$$RU^* = Sma + Flt + Tco - c_2 r - b_2 Y - U_{cur} - Tch - Deg$$

$$\text{Solving for } Sma = RU^* - Flt - Tco + Tch + c_2 r + b_2 Y + U_{cur} + Deg$$

In this last equation we would then have to substitute the reduced form equations of income and interest rates. The resulting equation will be the reduced form for Sma.

h) Derivation of LM equation

From equilibrium:

$$RR + ER = RU + BOR$$

where:

$$rr(c_3 r - c_{3a} r_i + b_3 Y + U_{dep}) + ER = RU^* + BOR$$

Solving for Y in terms of r we get:

$$Y = \tau_2 (RU^* + BOR - ER + \rho_2 r + RR c_{3a} r_i - rr U_{dep})$$

where:

$$\tau_2 = \frac{1}{rr b_3} \text{ and } \rho_2 = -rr c_3$$

i) Derivation of the Instrument Value for RU (RU^*)

Obtaining estimated r from the IS, assuming the target income $Y = Y^*$ and zero disturbances, the LM becomes:

$$Y^* = \tau_2^{RU^*} + \tau_2^{BOR} - \tau_2^{ER} + \tau_2 \rho_2 \left(\frac{Y^* - \alpha}{\beta} \right)$$

$$- \tau_2^{rrc} \rho_3 a r_i$$

Solving then for RU^* :

$$RU^* = Y^* \left(\frac{1}{\tau_2} - \frac{\rho_2}{\beta} \right) - BOR + ER + \frac{\rho_2 \alpha}{\beta} + rrc \rho_3 a r_i$$

4. Monetary Base Policy

a) Derivation of the Reduced Form of Interest Rates.

The LM is:

$$Y = \tau^{MB^*} + \tau \rho_4 r - \tau r r U_{dep} - \tau U_{er} - \tau U_{cur}$$

where: $\tau > 0$ and $\rho_4 > 0$

We substitute income from the LM into the IS :

$$r = \frac{\tau^{MB^*} + \tau \rho_4 r - \tau r r U_{dep} - \tau U_{cur} - \tau U_{er} - \frac{\alpha}{\beta} - \frac{U_{is}}{\beta}}{\beta}$$

Hence solving for r we obtain:

$$r = \frac{1}{\beta - \tau \rho_4} (\tau^{MB^*} - \tau r r U_{dep} - \tau U_{cur} - \alpha - \tau U_{er} - U_{is})$$

b) Derivation of the LM Equation

From the monetary sector equilibrium:

$$MB = RR + ER + CUR$$

$$MB^* = rr(c_3 r - c_3 a_i r + b_3 Y + U_{dep}) + ER + c_2 r + b_2 Y + U_{cur}$$

Solving for Y we obtain the LM:

$$Y = \tau (MB^* - ER + \rho r + rrc_3 a_i r - rrU_{dep})$$

where: $\tau = \frac{1}{rrb[3] + b_2}$ and $\rho = -rrc_3 - c_2$

b) Determination of Instrument Value MB^*

Substituting the interest rate from the IS into the LM;

$$Y = \tau \left(MB^* - ER + rrc_3 a_i r + \tau \left(\frac{Y - \alpha}{\beta} \right) \right)$$

Solving for MB^* we obtain:

$$MB^* = Y^* \left(\frac{1 - \rho}{\tau \beta} \right) + ER - rrc_3 a_i r + \frac{\rho \alpha}{\beta}$$

c) Derivation of the Reduced Form of Interest Rate.

Equating IS and LM :

$$\alpha + \beta r + U_{is} = \tau (MB^* - ER + \rho r + rrc_3 a_i r - rrU_{dep})$$

Then solving for r we get:

$$r = \frac{1}{\beta - \tau\rho} (\tau MB^* - \tau ER + \tau rrc_{3a} r_i - \tau rr U_{dep} - \tau U_{cur} - \alpha - U_{is})$$

d) Derivation of Reduced Form of Y.

We substitute r from the IS into the LM:

$$Y = \tau (MB^* - ER + \rho \left(\frac{Y - \alpha - U_{is}}{\beta} \right) + rrc_{3a} r_i - rr U_{dep} - U_{cur})$$

Then we solve for Y:

$$Y = \frac{\tau}{\beta - \tau\rho} (\beta MB^* - \beta ER - \rho\alpha - \rho U_{is} + \beta rrc_{3a} r_i - \beta rr U_{dep} - \beta U_{cur})$$

e) Derivation of the Reduced Form for RU and Sma.

From monetary sector definitions:

$$MB^* = RU + BOR + CUR$$

$$\text{so: } RU = MB^* - BOR - CUR = MB^* - BOR - c_2 r - b_2 Y - U_{cur}$$

Then substituting the reduced form for r and Y we obtain:

$$RU^* = MB^* - BOR - \frac{c_2}{\beta - \tau\rho} (\tau MB^* - \tau ER + \tau rrc_{3a} r_i - \tau rr U_{dep} - \tau U_{cur} - \alpha - U_{is})$$

$$-\frac{b_2 \tau}{\beta - \tau \rho} (\beta MB^* - \beta ER - \rho \alpha - \rho U_{is} + \beta rr c_{3a} r_i - \beta rr U_{dep} - \beta U_{cur}) - U_{cur}$$

Then for Sma:

$$MB^* = Sma + Flt + Tco - Tch - Deg - CUR + BOR + CUR$$

So the reduced form is:

$$Sma = MB^* - Flt - Tco + Tch + Deg - BOR$$

V. MODEL D-5

1. Interest Rate and Security Portfolio Policy

a) Derivation of LM Equation for r^* or Sma^* cases.

By definition

$$MB = RT = CUR$$

$$RU + BOR + CUR = RR + ER + CUR$$

$$Sma + Flt + Tco - Tch - Deg + BOR =$$

$$= rrD + c_5 r + U_{er} + c_2 r + b_2 Y + U_{cur} =$$

$$rr(c_{3a} r + b_{3a} Y + U_{ndd} + c_{3b} r - c_{3b} r_i + b_{3b} Y + U_{idd}) + c_5 r$$

$$+ U_{er} + c_2 r + b_2 Y + U_{cur}$$

We solve for Y and obtain:

$$Y = \frac{1}{b_2 + rrb_{3a} + rrb_{3b}} (Sma + Flt + Tco - Tch - Deg \\ + r(c_4 - c_5 - c_2 - rrc_{3a} - rrc_{3b}) - c_4 r_d + rrc_{3b} r_i \\ + U_{bor} - rrU_{ndd} - rrU_{idd} - U_{er} - U_{cur})$$

From where we define:

$$\rho_7 = -(-c_4 + c_5 + c_2 + rrc_{3a} + rrc_{3b})$$

And assuming $b_{3a} + b_{3b} = b_3$ we have:

$$\tau = \frac{1}{b_2 + rrb_3} = \frac{1}{b_2 + rrb_{3a} + rrb_{3b}}$$

b) Derivation of reduced form for Sma and Income under interest rate policy

Equating IS and LM:

$$\alpha + \beta r^* + U_{is} = \tau (Sma + \dots)$$

Solving for Sma we obtain:

$$Sma = \frac{\alpha + \beta r^*}{\tau} - \rho_7 r^* + \frac{U_{is}}{\tau} - Flt - Tco + Tch + Deg \\ + c_4 r_d - rrc_{3b} r_i - U_{bor} + rrU_{ndd} + rrU_{idd} + U_{er} + U_{cur}$$

The IS is just the IS equation, under this policy.

c) Derivation of reduced form for income under Sma policy.

This is done just as before (above models) by estimating r from IS equation and substituting it into the corresponding LM equation that we obtained in section a) above. For the interest rate, we equate IS and LM income and solve for the interest rate.

2. Unborrowed Reserves Policy

We start from the monetary sector equilibrium:

$$RU^* + BOR + CUR = RR + ER + CUR$$

We substitute the behavioral equations for ER, BOR and Demand deposits which is in $RR = rrD$:

$$RU^* + c_4 r - c_4 r_d + U_{bor} = rrc_{3a} r + rrb_{3a} Y + rrU_{ndd} + rrc_{3b} r - rrc_{3b} r_i + rrb_{3b} Y + rrU_{idd} + c_5 r + U_{er}$$

Rearranging and solving for Y we obtain:

$$Y = \frac{1}{rrb_{3a} + rrb_{3b}} (RU^* + (c_4 - rrc_{3a} - rrc_{3b} - c_5) - c_4 r_d + rrc_{3b} r_i + U_{bor} - rrU_{ndd} - rrU_{idd} - U_{er})$$

From where we define:

$$\rho_8 = -(rrc_{3a} + rrc_{3b} - c_4 + c_5)$$

And therefore $|\rho_8| < |\rho_7|$

3. Monetary Base Policy

a) Derivation of the LM equation.

From monetary sector equilibrium:

$$MB^* = RR + ER + CUR$$

$$\begin{aligned} & rrc_{3a}^r + rrb_{3a}^Y + rrU_{ndd} + rrc_{3b}^r - rrc_{3b}^r r_i + rrb_{3b}^Y \\ & + rrU_{idd} + c_5^r + U_{er} + c_2^r + b_2^Y + U_{cur} \end{aligned}$$

We solve for Y and obtain:

$$\begin{aligned} Y = & \left(\frac{1}{rrb_{3a} + rrb_{3b} + b_2} \right) (MB^* - r(rrc_{3a} - rrc_{3b} - c_5 - c_2) \\ & + rrc_{3b}^r r_i - rrU_{ndd} - rrU_{idd} - U_{er} - U_{cur}) \end{aligned}$$

From where we define:

$$\rho_9 = -(rrc_{3a} + rrc_{3b} + c_5 + c_2)$$

Note that: $|\rho_9| < |\rho_7|$

But: $|\rho_9| < |\rho_8|$ if $|c_2| < c_4$

And: $|\rho_9| > |\rho_8|$ if $|c_2| > c_4$

b) Reduced Form for Income and Interest Rates.

Again, the procedure is the same as that of above models, using the IS and the LM that corresponds to this policy case.

4. Model D-5: Case 1 versus Case 2: Assumptions about c_3 and c_2 coefficients

CASE:	c_3	c_2
1) Ceilings on interest r_i	Lower than when interest is not paid.	Greater than when interest is paid
2) No Ceilings (Deregulation)	Lower than in case 1	Greater than in case 1

Notes: c_3 = interest sensitivity of demand for deposits

c_2 = interest sensitivity of demand for currency.

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