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Adaptive Control for Double-Integrator Class Systems in the Absence of Velocity Feedback

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Adaptive Control for Double-Integrator Class Systems in the Absence of Velocity Feedback

by

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REPORT

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE IN ENGINEERING

THE UNIVERSITY OF TEXAS AT AUSTIN

December 2012

To my family and friends

Acknowledgments

First of all, I would like to express my gratitude to Dr. Maruthi R. Akella from the bottom of my heart. I believe that I was fortunate to start my graduate studies under his guidance. His intelligence and enthusiasm on researching and teaching he has shown to students are sincerely admirable. I have no doubt to say that he is a great teacher and researcher. With his valuable advice, this report was able to get started and completed.

My appreciation also extends to Dr. E. Glenn Lightsey for being a committee member in spite of his busy schedule this semester. Moreover, I would like to thank him as one of his students of hardware class. This report is a good opportunity to apply control theories to an actual plant. Without the experience of his class, the application of my research could not have happened.

I thank all friends in our research group for helping me to finish this report both directly or indirectly. Especially, I would like to acknowledge Sonia who helped to proofread this report.

Lastly, I always feel grateful to my wife and parents for their endless support and love.

Adaptive Control for Double-Integrator Class Systems in the Absence of Velocity Feedback

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This work considers formulation of new classes of adaptive controllers for double-integrator type systems where the underlying system parameters are uncertain and the complete state-vector is not available for feedback. Given the parameter uncertainty within the system model, a "separation principle" cannot generally be invoked towards an observer geared towards reconstruction of the full state vector using only measured variables.

In this report, controllers are designed for some important sub-classes of Euler-Lagrange type mechanical systems, where states are physically interpreted as position and velocity variables, and only the position part of the state vector is available as measured output. The typical approach to obtain velocity estimates using position interpolation (also known as dirty differentiation), is known to be strongly susceptible to measurement noise and therefore does not usually represent a robust option for feedback control implementation. The proposed control scheme achieves global asymptotic stability for

system dynamics subject to the condition that velocity states appear within the governing dynamics in a linear fashion. This arguably restrictive condition is loosened for the special case of scalar system with friction non-linearity as is typical within hardware implementations. The objective is to study prototypical mechanical systems with non-linearity appearing in the velocity rate equations with the eventual applications envisioned towards the attitude control problem accounting for the gyroscopic nonlinearity in the Euler rotational dynamics.

Based on classical certainty equivalence approaches for adaptive control, one cannot readily deal with cross terms associated with parameter estimates and unmeasured states in the Lyapunov function derivative in order to make the Lyapunov function negative definite or negative semi-definite. However, employing a new approach, this obstacle is shown in this report to be circumvented for scalar systems. In order to generalize the methodology for higher-order dynamics, a filtered state approach is used. Specifically, an auxiliary variable is introduced which plays an important role in determining restrictions on the control parameters and analyzing the stability. The proposed approach helps to overcome the uniform detectability obstacle. Additionally, this work can be applied to uncertain linear systems where independent control inputs are applied on each of the velocity state dynamics.

Lastly, the solution for the scalar is applied to the rotor speed control

system and is extended to the case where Coulomb friction is considered in addition to viscous friction. Since a sign function can be approximated as a hyperbolic tangent, the *tanh* model is used for the Coulomb friction. A controller is developed with the assumption that the coefficients of these frictions are unknown. The proposed control is then verified with Educational Control Product Model 750 Control Moment Gyroscope, and the simulation and actual test results are compared.

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Chapter 1

Introduction

Adaptive control is a certain type of non-linear feedback control for systems with uncertain parameters within the governing dynamics. When uncertain systems are considered, with a known mathematical structure, this control method not only provides a controller that guarantees asymptotic stability, but also estimates the unknown system parameters. In contrast to robust control, which uses a static control law and allows only certain range of parameter uncertainty, adaptive control employs a dynamic control law with estimates from the update laws and, thus, it is somewhat less restrictive regarding the magnitudes of parameter uncertainty.

The primary idea and motivation for adaptive control arose from design of autopilots for aircraft in the early 1950s [1]. Since then, it has been actively studied and developed, and now the framework for controller design is well established. Much of the earlier work is based on the full access of system states, a major limitation of adaptive control. Moreover, the construction of the control law is generally bounded by the certainty equivalence (CE) principle, which allows that the adaptive controller can retain the same structure of the corresponding deterministic case. The CE based adaptive approach gives a systematic controller design procedure but usually shows significantly poorer performance when compared with the deterministic case because the overall performance of the closed loop system is totally dependent upon the quality of parameter estimators and the underlying reference motion. Nowadays, as this topic has become deep and rich, much research has been done regarding relaxing the need for full-state feedback and/or the development of non-certainty equivalence (non-CE) based adaptive controllers to overcome these aforementioned performance drawbacks [2] [3] [4] [5].

In this report, motivated by need for adaptive output feedback control, second-order systems with unknown parameters are proposed as part of a new methodology for controller design. Since high-quality velocity measurements are usually expensive and harder to obtain in many applications of mechanical systems, it is assumed that only position signals are measured and used for feedback. Control strategies are developed based on a hybrid of CE and non-CE approaches using the results of the recent development in Immersion and Invariance (I&I) adaptive control [2]. The new strategies proposed throughout this report show the immense potential of the non-CE adaptive control techniques.

This report is organized as follows. In Chapter 2, a position feedback control law is developed for a scalar second order system. The unknown system parameters are estimated from the updated laws designed through the stability analysis. In Chapter 3, the design technique is extended to n-coupled linear second-order systems using linear low-pass filters. The proposed control method in Chapter 2 is extended to a scalar physical system with a dry friction which is non-linear in the velocity state and applied to the rotor speed control problem in Chapter 4. Then, conclusions are made in Chapter 5.

Chapter 2

Second-order Scalar Systems

As a starting point of developing the output feedback based adaptive control strategy, it is reasonable to consider the prototypical second-order system with its respective initial conditions:

$$\begin{aligned} \dot{x}_1 &= x_2; & x_1(0) &= x_{10} \\ \dot{x}_2 &= \theta_1^* f(x_1) + \theta_2^* x_2 + u; & x_2(0) &= x_{20} \\ y &= x_1 \end{aligned} \tag{2.1}$$

where θ_1^* and θ_2^* are constant but uncertain parameters, x_1 and x_2 are the position and velocity state respectively, y is the measured output, and u is the control input. $f(x_1)$ is assumed to be a Lipschitz continuous function and can be non-linear in x_1. The control objective is to find a control law for u that is independent of x_2 while estimating the uncertain system parameters, θ_1^* and θ_2^*. Of course, the parameter update laws must also be independent of x_2.

If some of the restrictions are loosened, this problem can be easily solved. First, let us assume that $f(x_1) = x_1$, and the parameters, θ_1^* and θ_2^* values are exactly known. Then, we can design a full-state or reduced-order observer and, according to the separation principle in linear control theory, utilize the estimated states in a full-state feedback control law. Specifically given the observability of the (A, C) pair, where

$$A = \begin{bmatrix} 0 & 1 \\ \theta_1^* & \theta_2^* \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

we have a full-state observer of the form

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + l_1(y - \hat{x}_1) \\ \dot{\hat{x}}_2 &= \theta_1^* \hat{x}_1 + \theta_2^* \hat{x}_2 + u + l_2(y - \hat{x}_1) \end{aligned} \tag{2.2}$$

where $\hat{x}_1$ and $\hat{x}_2$ are estimates for x_1 and x_2, and l_1 and l_2 are constant observer gains. Since the system is observable, the gains can always be chosen such that

$$\begin{bmatrix} -l_1 & 1 \\ \theta_1^* - l_2 & \theta_2^* \end{bmatrix}$$

is Hurwitz. If a reduced-order observer is considered, the following differential equation can be employed

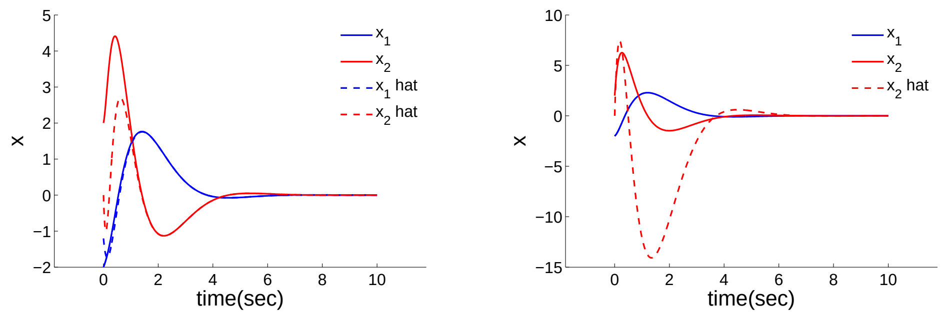
$$\dot{\hat{x}}_2 = -\lambda \hat{x}_2 - \lambda(\lambda + \theta_2^*)x_1 + \theta_1^* x_1 + u \tag{2.3}$$

where the convergence rate λ is a positive constant, and the estimate for x_2 is given by $\{\hat{x}_2 + (\lambda + \theta_2^*)x_1\}$. Using estimated states, the full-state observer based control law is obtained as

$$u = -(k_1 + \theta_1^*)\hat{x}_1 - (k_2 + \theta_2^*)\hat{x}_2 \tag{2.4}$$

or, with the reduced order observer, it is given by

$$u = -(k_1 + \theta_1^*)\hat{x}_1 - (k_2 + \theta_2^*)\{\hat{x}_2 + (\lambda + \theta_2^*)x_1\} \tag{2.5}$$



(a) Full-state observer (b) Reduced-order observer

Figure 2.1: Simulation results of observer based stabilization

where $k_1 > 0$ and $k_2 > 0$. Fig. (2.1) illustrates the two cases of the observer based control.

Secondly, when the full state is available for feedback but system parameters are uncertain, a conventional adaptive control scheme can be used. According to the certainty equivalence (CE) principle, the adaptive control law can have the same structure of the deterministic control law. Assuming the parameters are known, the control input given by

$$u = -(\theta_1^* + k_1)x_1 - (\theta_2^* + k_2)x_2 \tag{2.6}$$

with positive k_1 and k_2, stabilizes the system. By replacing θ_1^* and θ_2^* with its estimates, $\hat{\theta}_1$ and $\hat{\theta}_2$, the adaptive control law becomes

$$u = -(\hat{\theta}_1 + k_1)x_1 - (\hat{\theta}_2 + k_2)x_2 \tag{2.7}$$

Let us define parameter estimate errors as

$$
\begin{aligned}
\tilde{\theta}_1 &= \hat{\theta}_1 - \theta_1^* \\
\tilde{\theta}_2 &= \hat{\theta}_2 - \theta_2^*
\end{aligned}
\tag{2.8}
$$

and consider the following Lyapunov-like function

$$
V = \frac{1}{2}(x_1 + x_2)^2 + \frac{1}{\gamma_1}\tilde{\theta}_1^2 + \frac{1}{\gamma_2}\tilde{\theta}_2^2 \tag{2.9}
$$

where the learning rate parameters γ_1 and γ_2 are positive constants. Then, the time derivative of V is given by

$$
\begin{aligned}
\dot{V} &= (x_1 + x_2)(x_2 - k_1 x_1 - k_2 x_2 - \tilde{\theta}_1 x_1 - \tilde{\theta}_2 x_2) \\
&\quad + \frac{1}{2\gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{1}{2\gamma_2}\tilde{\theta}_2\dot{\hat{\theta}}_2 \\
&= -k_1(x_1 + x_2)\left(x_1 + \frac{k_2 - 1}{k_1}\right) \\
&\quad + \tilde{\theta}_1\left(\frac{\dot{\hat{\theta}}_1}{\gamma_1} - x_1(x_1 + x_2)\right) + \tilde{\theta}_2\left(\frac{\dot{\hat{\theta}}_2}{\gamma_2} - x_2(x_1 + x_2)\right)
\end{aligned}
\tag{2.10}
$$

If we choose the control parameters as

$$
\begin{aligned}
k_1 &> 0 \\
k_2 &= k_1 + 1
\end{aligned}
\tag{2.11}
$$

and the update laws as

$$
\begin{aligned}
\dot{\hat{\theta}}_1 &= \gamma_1 x_1(x_1 + x_2) \\
\dot{\hat{\theta}}_2 &= \gamma_2 x_2(x_1 + x_2)
\end{aligned}
\tag{2.12}
$$

we have

$$\dot{V} = -k_1(x_1 + x_2)^2 \leq 0 \tag{2.13}$$

which implies

$$\lim_{t\to\infty} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.14}$$

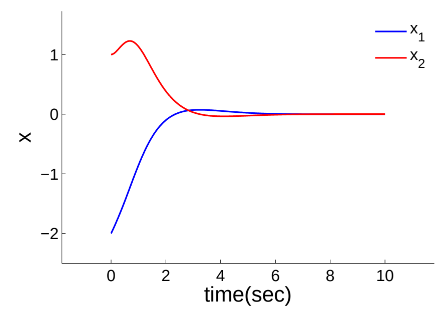
Although the parameter estimate errors $\tilde{\theta}_1(t)$ and $\tilde{\theta}_2(t)$ do not necessarily converge to zero, they are at least guaranteed to be bounded, but the states, x_1 and x_2 that we are interested in do tend to zero asymptotically for any initial condition. Simulation is performed with the following initial conditions

$$x_1(0) = -2,\ x_2(0) = 1,\ \hat{\theta}_1(0) = 0,\ \hat{\theta}_2(0) = 0$$

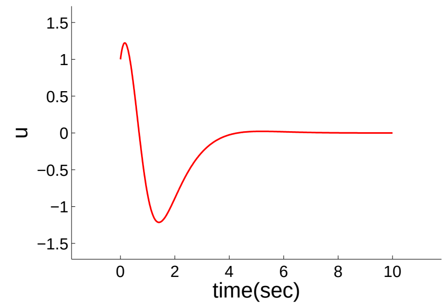
and parameter values

$$\theta_1 = 1, \theta_2 = 1,\ k_1 = 2,\ k_2 = 3,\ \gamma_1 = 1,\ \gamma_2 = 1$$

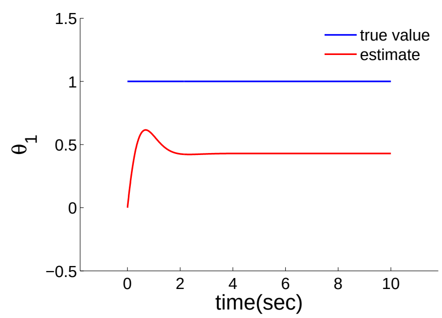
The results are shown in Fig. (2.2).



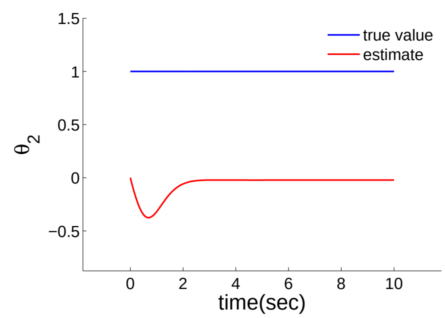
(a) Time histories of states, x_1 and x_2



(b) Time history of control, u



(c) Time history of estimate, $\hat{\theta}_1$



(d) Time history of estimate, $\hat{\theta}_2$

Figure 2.2: Simulation results of CE based stabilization

2.1 Controller Design

In our formulation, the two cases introduced in the previous section are more or less non-overlapping and, thus, neither of the above approaches are not directly applicable for more general situations. The linear observer approach requires the perfect knowledge of the parameters and linearity of the system. Therefore, even though the system is linear, without system identification, a linear observer cannot be designed. Moreover, for non-linear systems, one cannot expect the separation property in general. That is, the observer design cannot be separated from the design of the state feedback control and thus this approach is out of our options. One may use numerical differentiation for a velocity state and design a CE based adaptive controller. However, the computed velocity from the position measurements is, in general, corrupted by noise that can be amplified as sampling time decreases. No matter how good or noise-robust the numerical differentiation is, achieving the goal without using it performs better.

Since unknown parameters exist in Eq. (2.1) as it is originally stated, we still need to come up with some suitable adaptation technique. In the past, most of work on adaptive output feedback control focused on the systems where the cross terms of unmeasured states and unknown parameters do not appear. In [3], generalized version of this problem is introduced and solved with the assumption that the unknown parameters only appear in output-dependent terms, i.e., in our formulation, $\theta_2^* x_2$ is omitted in the velocity dynamics from

Eq. (2.1). Later, this limited formulation was extended to the dynamics where time-varying non-linear parametric uncertainty is allowed to occur coupled with unmeasured states in [4]. The solution proposed in [4] utilizes a high-gain non-linear observer and controller with a priori magnitude bounds on uncertain parameters associated with unmeasured states. Reference [4] suggests the existence of solutions for the general problem, but their synthesis is somewhat complex, and moreover high-gain control action may cause peaking phenomena in response.

In this report, relatively simple form of the controller will be introduced without designing an observer while estimating the unknown parameters. To tackle our problem, some concepts from a recently proposed non-CE based methodology, Immersion and Invariance (I&I) adaptive control, are employed [2]. The idea is that parameter estimate can be constructed as a sum of two signals, i.e., $(\hat{\theta} + \beta)$, where $\hat{\theta}$ comes from a update law and β is a function of system states. If we can find $\hat{\theta}$ and β independent of the unmeasured state x_2, the problem boils down to finding a control law which is also independent of x_2. Basically, this approach is in line with design of a reduced-order observer in which the estimates for unmeasured states are combinations of observer signals and measured state signals. The only difference is that the parameter update laws do not contain the control, u.

To further motivate this new approach, let us start with the assumption that θ_1^* and θ_2^* in Eq. (2.1) are fully known. The task is now to find a control input, u independent of x_2. The well known result from the passivity control leads us to the solution

$$u = -k_p x_1 - k_z \dot{z} - \theta_1^* f(x_1) - \theta_2^* x_2 \tag{2.15}$$

with a stable first order filter

$$\dot{z} = -\lambda z + \eta x_1; \quad z(0) = z_0 \tag{2.16}$$

where k_p, k_z, λ, and η are some constant positive real numbers [6]. The closed-loop system under action of the proposed control law in Eq. (2.15) is given by

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ z \end{bmatrix} = A_m \begin{bmatrix} x_1 \\ x_2 \\ z \end{bmatrix} \tag{2.17}$$

where

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ -(k_p + \eta k_z) & 0 & \lambda k_z \\ \eta & 0 & -\lambda \end{bmatrix}$$

Since the propagation matrix, A_m is Hurwitz by way of judicious selection of k_p, k_z, λ and η parameters, exponential stability for the closed loop system is guaranteed. This model is used as a target system when the I&I-like form of adaptive control scheme is applied. Thus, we can propose a controller of the form

$$u = -(\hat{\theta}_1 + \beta_1) f(x_1) - (\hat{\theta}_2 + \beta_2)\hat{x}_2 - k_p x_1 - k_z \dot{z} \tag{2.18}$$

where $(\hat{\theta}_1 + \beta_1)$ and $(\hat{\theta}_2 + \beta_2)$ are respectively the estimates for θ_1^* and θ_2^* and $\hat{x}_2$ is a replacement for x_2. Note that $\hat{x}_2$ is not an explicit estimate for x_2 and will be determined as a function of available signals, x_1 and z through stability analysis presented in the sequel.

2.2 Stability Analysis

When the system is closed with control input Eq. (2.18), we obtain

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\tilde{\theta}_1 f(x_1) - \tilde{\theta}_2 \hat{x}_2 + \theta_2^*(x_2 - \hat{x}_2) - k_p x_1 - k_z \dot{z} \\ \dot{z} &= -\lambda z + \eta x_1 \end{aligned} \tag{2.19}$$

where the parameter estimate errors have different definitions compared to Eq. (2.8) and are now taken as follows:

$$\begin{aligned}\tilde{\theta}_1 &= \hat{\theta}_1 + \beta_1 - \theta_1^* \\ \tilde{\theta}_2 &= \hat{\theta}_2 + \beta_2 - \theta_2^* \end{aligned} \tag{2.20}$$

Before proceeding further with the analysis, let us assume that the upper bound of θ_2^* is a priori known as $\bar{\theta}_2$ in order to deal with the cross term of the unknown parameter and the unmeasured state. Moreover, let us define an auxiliary variable

$$s = x_2 + \alpha x_1 - \dot{z} \tag{2.21}$$

where α is a positive constant number. This variable plays an important roll in the upcoming analysis and gives a simple structure of a Lyapunov-like function to help avoid the detecatability obstacle frequently confronted in adaptive control design [7].

Let us consider the following non-negative function

$$V = \frac{1}{2}s^2 + \frac{1}{2\gamma_1}\tilde{\theta}_1^2 + \frac{1}{2\gamma_2}\tilde{\theta}_2^2 \tag{2.22}$$

where the learning rates γ_1 and γ_2 are constant positive real numbers. When V is differentiated with respect to time, it follows that

$$\begin{aligned}
\dot{V} &= s\dot{s} + \frac{1}{\gamma_1}\tilde{\theta}_1(\dot{\hat{\theta}}_1 + \dot{\beta}_1) + \frac{1}{\gamma_2}\tilde{\theta}_2(\dot{\hat{\theta}}_2 + \dot{\beta}_2) \\
&= s(\dot{x}_2 + \alpha\dot{x}_1 - \ddot{z}) + \frac{1}{\gamma_1}\tilde{\theta}_1(\dot{\hat{\theta}}_1 + \dot{\beta}_1) + \frac{1}{\gamma_2}\tilde{\theta}_2(\dot{\hat{\theta}}_2 + \dot{\beta}_2) \\
&= s\{-\tilde{\theta}_1 f(x_1) - \tilde{\theta}_2\hat{x}_2 + \theta_2^*(x_2 - \hat{x}_2) - k_p x_1 - k_z\dot{z} + \alpha x_2 - \lambda\dot{z} + \eta x_2\} \\
&\quad + \frac{1}{\gamma_1}\tilde{\theta}_1(\dot{\hat{\theta}}_1 + \dot{\beta}_1) + \frac{1}{\gamma_2}\tilde{\theta}_2(\dot{\hat{\theta}}_2 + \dot{\beta}_2)
\end{aligned} \tag{2.23}$$

At this point, let us impose the following constraints on α, η, and λ in addition to the positiveness

$$\eta > \alpha > 0 \tag{2.24}$$

$$\lambda > \eta - \alpha > \max\{\bar{\theta}_2, 0\} \tag{2.25}$$

and choose k_p, k_z and $\hat{x}_2$ as

$$k_p = \alpha(\eta - \alpha) \tag{2.26}$$

$$k_z = \lambda - (\eta - \alpha) \tag{2.27}$$

$$\hat{x}_2 = -\alpha x_1 + \dot{z} \tag{2.28}$$

The time derivative of the Lyapunov-like function in Eq. (2.23) becomes

$$\begin{aligned}\dot{V} &= -(\eta - \alpha - \theta_2^*)s^2 \\ &\quad + \frac{1}{\gamma_1}\tilde{\theta}_1\{\dot{\hat{\theta}}_1 + \dot{\beta}_1 - \gamma_1(x_2 + \alpha x_1 - \dot{z})f(x_1)\} \\ &\quad + \frac{1}{\gamma_2}\tilde{\theta}_2\{\dot{\hat{\theta}}_2 + \dot{\beta}_2 - \gamma_2(x_2 + \alpha x_1 - \dot{z})(-\alpha x_1 + \dot{z})\} \end{aligned} \tag{2.29}$$

To eliminate the estimate error terms in $\dot{V}$, the time derivatives of the estimates can be chosen as

$$\begin{aligned}\dot{\hat{\theta}}_1 + \dot{\beta}_1 &= \gamma_1(x_2 + \alpha x_1 - \dot{z})f(x_1) \\ &= \gamma_1\{x_2 f(x_1) + (\alpha x_1 - \dot{z})f(x_1)\} \end{aligned} \tag{2.30}$$

$$\begin{aligned}\dot{\hat{\theta}}_2 + \dot{\beta}_2 &= \gamma_2(x_2 + \alpha x_1 - \dot{z})(-\alpha x_1 + \dot{z}) \\ &= \gamma_2\{x_2(-\alpha x_1 + \dot{z}) + (\alpha x_1 - \dot{z})(-\alpha x_1 + \dot{z})\} \end{aligned} \tag{2.31}$$

If we just use the classical CE approach, i.e., no β's are introduced, we cannot go further from this point. However, since

$$x_2 f(x_1) = \frac{d}{dt}\int_0^{x_1} f(\sigma)d\sigma \tag{2.32}$$

$$x_1 x_2 = \frac{d}{dt}\left(\frac{1}{2}x_1^2\right) \tag{2.33}$$

$$x_2\dot{z} = \frac{d}{dt}\left(-\frac{\eta}{2}x_1^2 + x_1\dot{z}\right) + \lambda x_1\dot{z} \tag{2.34}$$

the update laws are determined as

$$\dot{\hat{\theta}}_1 = \gamma_1(\alpha x_1 - \dot{z})f(x_1) \tag{2.35}$$

$$\dot{\hat{\theta}}_2 = \gamma_2\{\lambda x_1\dot{z} - (\alpha x_1 - \dot{z})^2\} \tag{2.36}$$

with

$$\beta_1 = \gamma_1 \int_0^{x_1} f(\sigma)d\sigma \tag{2.37}$$

$$\beta_2 = \gamma_2 x_1 \left(-\frac{\alpha+\eta}{2}x_1 + \dot{z}\right) \tag{2.38}$$

which are independent of the unmeasured signal x_2. Finally, by the constraint, Eq. (2.25), we have

$$\begin{aligned} \dot{V} &= -(\eta - \alpha - \theta_2^*)s^2 \\ &\leq 0 \end{aligned} \tag{2.39}$$

where $(\eta - \alpha - \theta_2^*)$ is a positive real number. Because $\dot{V}$ is negative semi-definite, we can conclude that signals, s, $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are bounded, i.e.

$$s,\, \tilde{\theta}_1,\, \tilde{\theta}_2 \in L_\infty \tag{2.40}$$

Furthermore, since

$$\begin{aligned} \lim_{t\to\infty} \int_0^t \dot{s}^2 dt &= \frac{V(0) - V(\infty)}{(\eta - \alpha - \theta_2^*)} \\ &< \infty \end{aligned} \tag{2.41}$$

we can also conclude that

$$s \in L_2 \tag{2.42}$$

To show that $\dot{s} \in L_\infty$, we need to examine the auxiliary variable, s. By Eq. (2.16), the states are expressed as functions of the filtered state, z.

$$x_1 = \frac{1}{\eta}(\dot{z} + \lambda z) \tag{2.43}$$

$$x_2 = \frac{1}{\eta}(\ddot{z} + \lambda \dot{z}) \tag{2.44}$$

Then, Eq. (2.21) becomes

$$\ddot{z} + \{\lambda - (\eta - \alpha)\}\dot{z} + \alpha\lambda z = \eta s \tag{2.45}$$

As the parameters are picked such that all coefficients of the above equation are positive, it is equivalent to the second order stable filter with input, s. Thus, $s \in L_\infty$ implies that

$$z,\ \dot{z},\ \ddot{z} \in L_\infty \tag{2.46}$$

which leads to

$$x_1,\ x_2 \in L_\infty \tag{2.47}$$

We can then show that

$$\dot{s} \in L_\infty \tag{2.48}$$

because

$$\dot{s} = -(\eta - \alpha - \theta_2^*)s - \tilde{\theta}_1 f(x_1) - \tilde{\theta}_2(-\alpha x_1 + \dot{z}) \tag{2.49}$$

By Barbalat's lemma, the fact that $s,\ \dot{s} \in L_\infty$ and $s \in L_2$ implies

$$\lim_{t\to\infty} s(t) = 0 \tag{2.50}$$

Finally, equations (2.43), (2.44), and (2.45) allow us to conclude that

$$\lim_{t\to\infty} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{2.51}$$

because the states of the stable filter converge to zero as its input vanishes.

Through the analysis, we assume that the value of the system parameter θ_2^* associated with the unmeasured state is unknown and arbitrary but its upper bound is a priori known. If the given plant is classified as a Euler-Lagrange type mechanical system, θ_2^* is usually physically associated with damping and thus non-positive, so that the system energy can be dissipated. In this case, the constraint (2.25) becomes simply

$$\lambda > \eta - \alpha \tag{2.52}$$

Otherwise, we must have

$$\lambda > \eta - \alpha > \bar{\theta}_2 \tag{2.53}$$

where $\bar{\theta}_2 > 0$ is the known upper bound of θ_2^*.

In summary, for second-order systems as stated in Eq. (2.1), the control law

$$u = -(\hat{\theta}_1 + \beta_1)f(x_1) - (\hat{\theta}_2 + \beta_2)(-\alpha x_1 + \dot{z}) - k_p x_1 - k_z \dot{z} \tag{2.54}$$

with

$$\dot{z} = \lambda z + \eta x_1 \tag{2.55}$$

and the update laws

$$\dot{\hat{\theta}}_1 = \gamma_1(\alpha x_1 - \dot{z})f(x_1) \tag{2.56}$$

$$\dot{\hat{\theta}}_2 = \gamma_2\{\lambda x_1 \dot{z} - (\alpha x_1 - \dot{z})^2\} \tag{2.57}$$

with

$$\beta_1 = \gamma_1 \int_0^{x_1} f(\sigma)d\sigma \tag{2.58}$$

$$\beta_2 = \gamma_2 x_1 \left(-\frac{\alpha + \eta}{2} x_1 + \dot{z}\right) \tag{2.59}$$

asymptotically stabilize the system if the control parameters are selected such that their constraints listed in Eq. (2.24) - (2.27) are satisfied.

2.3 Extension to Tracking Problem

Using the proposed design scheme, we can extend it to the tracking problem. Suppose we have predefined smooth and bounded reference trajectories, r_1 and r_2 for x_1 and x_2. By defining the tracking error states,

$$e_1 = x_1 - r_1 \tag{2.60}$$

$$e_2 = x_2 - r_2 \tag{2.61}$$

with the necessary matching condition

$$\dot{r}_1 = r_2 \tag{2.62}$$

the augmented error dynamics are obtained

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= \theta_1^* f(x_1) + \theta_2^* x_2 - \dot{r}_2 + u \\
\dot{z} &= -\lambda z + \eta e_1
\end{aligned} \tag{2.63}$$

Moreover, the auxiliary variable is defined in terms of the error states as

$$s = e_2 + \alpha e_1 - \dot{z} \tag{2.64}$$

However, the constraints on the parameters remain the same. With

$$u = -(\hat{\theta}_1 + \beta_1) f(x_1) - (\hat{\theta}_2 + \beta_2)(r_2 - \alpha e_1 + \dot{z}) + \dot{r}_2 - k_p e_1 - k_z \dot{z} \tag{2.65}$$

the closed loop system becomes

$$\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= -\tilde{\theta}_1 f(x_1) - \tilde{\theta}_2(r_2 - \alpha e_1 + \dot{z}) + \theta_2^* s - k_p e_1 - k_z \dot{z} \\
\dot{z} &= -\lambda z + \eta e_1
\end{aligned} \tag{2.66}$$

Once we choose

$$\dot{\hat{\theta}}_1 = \gamma_1(\alpha e_1 - \dot{z} - r_2) f(x_1) \tag{2.67}$$

$$\dot{\hat{\theta}}_2 = \gamma_2\{e_1(-\dot{r}_2 + \lambda \dot{z}) + (\alpha e_1 - \dot{z})(r_2 - \alpha e_1 + \dot{z})\} \tag{2.68}$$

and

$$\beta_1 = \gamma_1 \int_0^{x_1} f(\sigma) d\sigma \tag{2.69}$$

$$\beta_2 = \gamma_2 e_1 \left(r_2 - \frac{\alpha + \eta}{2} e_1 + \dot{z} \right) \tag{2.70}$$

the Lyapunov-like function

$$V = \frac{1}{2} s^2 + \frac{1}{2\gamma_1} \tilde{\theta}_1^2 + \frac{1}{2\gamma_2} \tilde{\theta}_2^2 \tag{2.71}$$

with its time derivative

$$\dot{V} = -(\eta - \alpha - \theta_2^*) s^2 \tag{2.72}$$

leads to the conclusion that the auxiliary variable, s converges to zero asymptotically, which implies the state errors also converge to zero.

2.4 Simulation

The two control cases developed in this chapter are simulated with the true values of $\theta_1^* = 2$ and $\theta_2^* = 2$. In addition, $f(x_1)$ is assumed to be x_1. The states are therefore propagated by the equations

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= 2x_1 + 2x_2 + u \end{aligned} \tag{2.73}$$

The system parameters are unknown, but the upper bound for θ_2^* is given by

$$\bar{\theta}_2 = 3$$

The design parameters are chosen so that they do not violate their constraints:

$$\alpha = 1,\ \eta = 5,\ \lambda = 20,\ \gamma_1 = 1,\ \gamma_2 = 1$$

The results are obtained using the following initial conditions:

$$x_1(0) = -1,\ x_2(0) = 1,\ z(0) = -0.25,\ \hat{\theta}_1 = 0,\ \hat{\theta}_2 = 0$$

For the tracking problem, the reference signals are set to

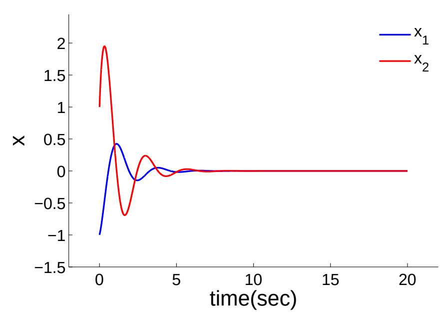
$$r_1 = \sin(t) \tag{2.74}$$

$$r_2 = \cos(t) \tag{2.75}$$

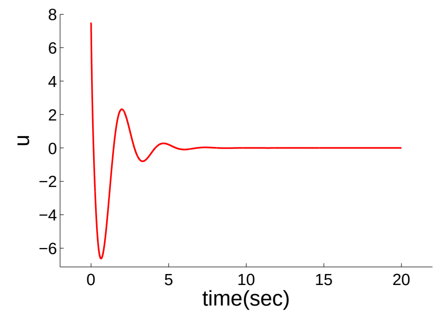
Since the reference is persistently exiting signals, it is expected that the parameter estimates tend to the true values [8].

2.4.1 Stabilization

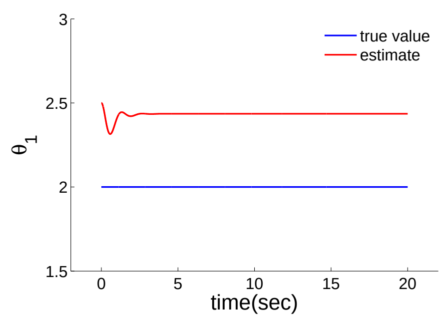
Figure 2.3 illustrates the simulation results for the stabilization case. As seen in the graphs, states are regulated to zero. The estimates do not go to the true values, but they are bounded and actually converge to some constants.



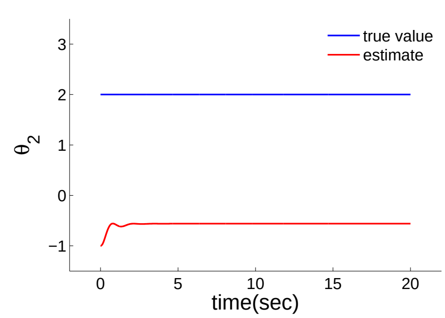
(a) Time histories of states, x_1 and x_2



(b) Time history of control, u



(c) Time history of estimate, $\hat{\theta}_1 + \beta_1$

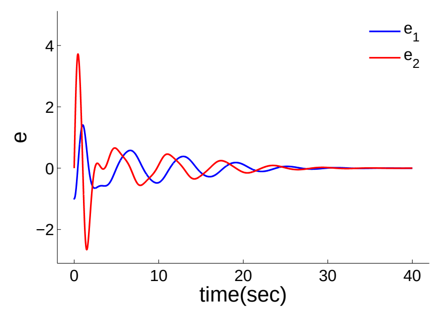


(d) Time history of estimate, $\hat{\theta}_2 + \beta_2$

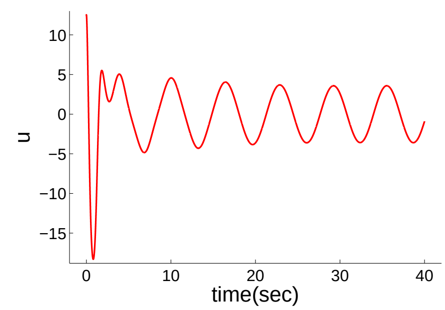
Figure 2.3: Simulation results of stabilization problem

2.4.2 Tracking

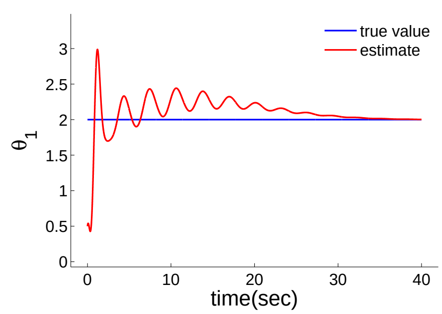
The convergence of the state and parameter errors are shown in the Figure 2.4. The difference from the stabilization problem is that the parameter estimates converge to their true values.



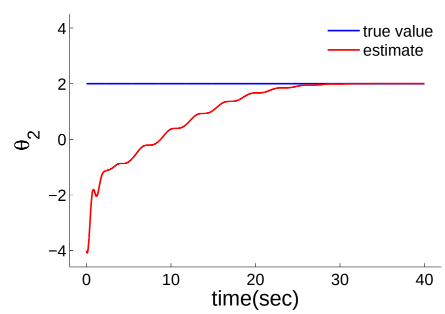
(a) Time histories of tracking errors, e_1 and e_2



(b) Time history of control, u



(c) Time history of estimate, $\hat{\theta}_1 + \beta_1$



(d) Time history of estimate, $\hat{\theta}_2 + \beta_2$

Figure 2.4: Simulation results of tracking problem

Chapter 3

Generalization to n-Coupled Systems

Using the previously developed technique, let us generalize the result. In stead of a single second order linear system, suppose we have coupled n number of systems of the form

$$\begin{aligned}\dot{\mathbf{x}}_1 &= \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= \Omega(\mathbf{x}_1,\, \mathbf{x}_2)\boldsymbol{\theta}^* + \mathbf{u}\end{aligned} \tag{3.1}$$

where $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{u}$ are n-dimensional vectors, $\boldsymbol{\theta}^*$ is an m-dimensional unknown constant vector, and $\Omega(\mathbf{x}_1,\, \mathbf{x}_2)$ is an $n \times m$ regressor matrix which depends on $\mathbf{x}_1$ and $\mathbf{x}_2$. Once we deal with vectors, introducing an additional variable β for estimates as presented in the previous chapter does not work due to the integrability obstacle associated with I&I control [2]. Therefore, a different approach is addressed. Instead of analyzing the position and velocity states, we will examine the so called filtered states which will be discussed in the sequel within the controller design section.

Similar to the scalar case, for this problem to be solvable, two things must be assumed. The first assumption is that $\Omega(\mathbf{x}_1,\, \mathbf{x}_2)$ can be separated into two regressors where we do not necessarily have linearity in $\mathbf{x}_1$, but assume linear dependence in $\mathbf{x}_2$. In other words, the regressor matrix can be expressed as

$$\Omega(\mathbf{x}_1, \mathbf{x}_2) = U(\mathbf{x}_1) + W(\mathbf{x}_2) \tag{3.2}$$

where $U(\cdot)$ is a Lipschitz continuous function and $W(\cdot)$ is a linear function that satisfies the following properties.

$$W(\mathbf{v} + \mathbf{w}) = W(\mathbf{v}) + W(\mathbf{w}) \tag{3.3}$$

$$W(a\mathbf{v}) = aW(\mathbf{v}) \tag{3.4}$$

$$\dot{W}(\mathbf{v}) = (\dot{\mathbf{v}}) \tag{3.5}$$

where $\mathbf{v}$ and $\mathbf{w}$ are n-dimensional vectors, and a is a scalar constant. Second, it is necessary to assume that the upper bounds of the absolute values of unknown parameters associated with $W(\mathbf{x}_2)$ are known. With these assumptions, the system we are dealing with is rewritten as

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= \sum_{i=1}^{p} \theta_i^* U_i(\mathbf{x}_1) + \sum_{j=1}^{q} \phi_j^* W_j(\mathbf{x}_2) + \mathbf{u} \end{aligned} \tag{3.6}$$

where $U_i(\mathbf{x}_1) \in \mathcal{R}^n$ and $W_j(\mathbf{x}_2) \in \mathcal{R}^n$ are the j^{th} column vectors of $U(\mathbf{x}_1)$ and $W(\mathbf{x}_2)$, and $\theta_i^* \in \mathcal{R}$ is the unknown parameter associated with U_i and $\phi_j^* \in \mathcal{R}$ is the partially known parameter associated with W_j with

$$|\phi_j^*| \leq \bar{\phi}_j, \quad j = 1, 2, \cdots, q \tag{3.7}$$

where $\bar{\phi}_j$'s are known positive values.

3.1 Controller Design

Let us consider the following filters

$$\dot{\mathbf{y}}_1 = -\nu \mathbf{y}_1 + \mathbf{x}_1 \tag{3.8}$$

$$\dot{\mathbf{y}}_2 = -\nu \mathbf{y}_2 + \mathbf{x}_2 \tag{3.9}$$

$$\dot{U}_{fi} = -\nu U_{fi} + U_i(\mathbf{x}_1); \quad i = 1, 2, \cdots, p \tag{3.10}$$

$$\dot{W}_{fj} = -\nu W_{fj} + W_j(\mathbf{x}_2); \quad j = 1, 2, \cdots, q \tag{3.11}$$

$$\dot{\mathbf{u}}_f = -\nu \mathbf{u}_f + \mathbf{u} \tag{3.12}$$

where ν is a positive real number [9]. From the filter definitions, we have

$$\ddot{\mathbf{y}}_1 = -\nu \dot{\mathbf{y}}_1 + (\dot{\mathbf{y}}_2 + \nu \mathbf{y}_2) \tag{3.13}$$

$$\begin{aligned} \ddot{\mathbf{y}}_2 = &-\nu \dot{\mathbf{y}}_2 + \sum_{i=1}^{p} \theta_i^* (\dot{U}_{fi} + \nu U_{fi}) + \sum_{j=1}^{q} \phi_j^* (\dot{W}_{fj} + \nu W_{fj}) \\ &+ (\dot{\mathbf{u}}_f + \nu \mathbf{u}_f) \end{aligned} \tag{3.14}$$

$$\dot{W}_{fj} = -\nu W_{fj} + \dot{W}_j(\mathbf{y}_2) + \nu W_j(\mathbf{y}_2) \tag{3.15}$$

The solutions are given by

$$\dot{\mathbf{y}}_1 = \mathbf{y}_2 + \boldsymbol{\epsilon}_1(t) \tag{3.16}$$

$$\dot{\mathbf{y}}_2 = \sum_{i=1}^{p} \theta_i^* U_{fi} + \sum_{j=1}^{q} \phi_j^* W_{fj} + \mathbf{u}_f + \boldsymbol{\epsilon}_2(t) \tag{3.17}$$

$$W_{fj} = W_j(\mathbf{y}_2) + \boldsymbol{\epsilon}_3(t) \tag{3.18}$$

where $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$ and $\boldsymbol{\epsilon}_3$ are exponentially decaying terms, and we can ignore them in the rest of the analysis without loss of generality [8].

We can now reformulate the problem. Given the dynamics

$$\dot{\mathbf{y}}_1 = \mathbf{y}_2 \tag{3.19}$$

$$\dot{\mathbf{y}}_2 = \sum_{i=1}^{p} \theta_i^* U_{fi} + \sum_{j=1}^{q} \phi_j^* W_{fj} + \mathbf{u}_f \tag{3.20}$$

$$\dot{U}_{fi} = -\nu U_{fi} + U_i(\mathbf{x}_1); \quad i = 1, 2, \cdots, p \tag{3.21}$$

with

$$W_{fj} = W_j(\mathbf{y}_2) \tag{3.22}$$

the goal is to find a control, $\mathbf{u}_f$, independent of $\mathbf{y}_2$ that drives $\mathbf{y}_1$ and $\mathbf{y}_2$ to zero. Let us follow a similar procedure of the scalar approach. First, with the additional filter

$$\dot{\mathbf{z}} = -\lambda \mathbf{z} + \eta \mathbf{y}_1 \tag{3.23}$$

where λ and η are positive constants, we can propose the control law

$$\mathbf{u}_f = -\sum_{i=1}^{p} \hat{\theta}_i U_{fi} - \sum_{j=1}^{q} \hat{\phi}_j W_{fj} - k_p \mathbf{y}_1 - k_z \dot{\mathbf{z}} \tag{3.24}$$

However, since W_{fj}'s are functions of $\mathbf{y}_2$, they cannot be directly used. Therefore, let us introduce estimates for W_{fj} and rewrite the proposed control law.

$$\mathbf{u}_f = -\sum_{i=1}^{p} \hat{\theta}_i U_{fi} - \sum_{j=1}^{q} \hat{\phi}_j \hat{W}_{fj} - k_p \mathbf{y}_1 - k_z \dot{\mathbf{z}} \tag{3.25}$$

where $\hat{W}_{fj}$'s are the estimates for W_{fj}'s. Obviously, $\hat{W}_{fj}$'s must be obtained from the available signals, i.e., they must be independent of $\mathbf{y}_2$. Once the system is closed with the proposed control, we have

$$\dot{\mathbf{y}}_1 = \mathbf{y}_2 \tag{3.26}$$

$$\dot{\mathbf{y}}_2 = -\sum_{i=1}^{p} \tilde{\theta}_i U_{fi} - \sum_{j=1}^{q} \tilde{\phi}_j \hat{W}_{fj} - \sum_{j=1}^{q} \phi_j^* \tilde{W}_{fj} \tag{3.27}$$

$$\dot{\mathbf{z}} = -\lambda \mathbf{z} + \eta \mathbf{y}_1 \tag{3.28}$$

$$\dot{U}_{fi} = -\nu U_{fi} + U_i(\mathbf{x}_1); \quad i = 1, 2, \cdots, p \tag{3.29}$$

$$W_{fj} = W_j(\mathbf{y}_2); \quad j = 1, 2, \cdots, q \tag{3.30}$$

where

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i^* \tag{3.31}$$

$$\tilde{\phi}_j = \hat{\phi}_j - \phi_j^* \tag{3.32}$$

$$\tilde{W}_{fj} = \hat{W}_{fj} - W_{fj} \tag{3.33}$$

with $i = 1, 2, \cdots, p$ and $j = 1, 2, \cdots, q$.

3.2 Stability Analysis

The analysis procedure follows closely to the scalar case, but it will be slightly more complex because more filters are involved and additional estimates are added. First of all, let us start by defining an auxiliary variable as

$$\mathbf{s} = \mathbf{y}_2 + \alpha \mathbf{y}_1 - \dot{\mathbf{z}} \tag{3.34}$$

Then, the time derivative of this vector is given by

$$\dot{\mathbf{s}} = -(\eta - \alpha)\mathbf{s} - \sum_{j=1}^{q} \phi_j^* \tilde{W}_{fj} - \sum_{i=1}^{p} \tilde{\theta}_i U_{fi} - \sum_{i=1}^{q} \tilde{\phi}_i \hat{W}_{fi} \tag{3.35}$$

if we choose

$$k_p = \alpha(\eta - \alpha) \tag{3.36}$$

$$k_z = \lambda - (\eta - \alpha) \tag{3.37}$$

where

$$\eta > \alpha \tag{3.38}$$

$$\lambda > \eta - \alpha \tag{3.39}$$

For the additional estimates, let us propose the following update law

$$\dot{\hat{W}}_{fj} = -k_j \tilde{W}_{fj} + \sum_{i=1}^{p} \hat{\theta}_i W_j(U_{fi}) + \sum_{i=1}^{q} \hat{\phi}_i W_j(W_i(\mathbf{y}_2)) + W_j(\mathbf{u}_f) \tag{3.40}$$

where k_j is a positive scalar constant, which leads to

$$\dot{\tilde{W}}_{fj} = -k_j \tilde{W}_{fj} + \sum_{i=1}^{p} \tilde{\theta}_i W_j(U_{fi}) + \sum_{i=1}^{q} \tilde{\phi}_i W_j(W_i(\mathbf{y}_2)) \tag{3.41}$$

Next, let us consider the Lyapunov-like function

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{s} + \frac{1}{2}\sum_{j=1}^{q} \tilde{W}_{fj}^T \tilde{W}_{fj} + \frac{1}{2}\sum_{i=1}^{p} \frac{1}{\gamma_i}\tilde{\theta}_i^2 + \frac{1}{2}\sum_{i=1}^{q} \frac{1}{\gamma_i'}\tilde{\phi}_i^2 \tag{3.42}$$

where the learning rates γ_i's and γ_i''s are positive constants. Differentiating V with respect to time, we have

$$\begin{aligned}\dot{V} = &-(\eta - \alpha)\mathbf{s}^T\mathbf{s} - \mathbf{s}^T \sum_{j=1}^{q} \phi_j^* \tilde{W}_{fj} - \mathbf{s}^T \sum_{i=1}^{p} \tilde{\theta}_i U_{fi} - \mathbf{s}^T \sum_{i=1}^{q} \tilde{\phi}_i \hat{W}_{fi} \\ &+ \sum_{j=1}^{q} \left[-k_j \tilde{W}_{fj}^T \tilde{W}_{fj} + \sum_{i=1}^{p} \tilde{\theta}_i \tilde{W}_{fj}^T W_j(U_{fi}) + \sum_{i=1}^{q} \tilde{\phi}_i \tilde{W}_{fj}^T W_j(W_i(\mathbf{y}_2)) \right] \\ &+ \sum_{i=1}^{p} \frac{1}{\gamma_i}\tilde{\theta}_i \dot{\hat{\theta}}_i + \sum_{i=1}^{q} \frac{1}{\gamma_i'}\tilde{\phi}_i \dot{\hat{\phi}}_i\end{aligned} \tag{3.43}$$

After some algebra, the equation is arranged as

$$\begin{aligned}\dot{V} = & -(\eta-\alpha)\mathbf{s}^T\mathbf{s} - \sum_{j=1}^{q}\phi_j^*\mathbf{s}^T\tilde{W}_{fj} - \sum_{j=1}^{q}k_j\tilde{W}_{fj}^T\tilde{W}_{fj} \\ & + \sum_{i=1}^{p}\tilde{\theta}_i\left[\frac{\dot{\hat{\theta}}_i}{\gamma_i} - \mathbf{s}^T U_{fi} + \sum_{j=1}^{q}\tilde{W}_{fj}^T W_j(U_{fi})\right] \\ & + \sum_{i=1}^{q}\tilde{\phi}_i\left[\frac{\dot{\hat{\phi}}_i}{\gamma_i'} - \mathbf{s}^T\hat{W}_{fi} + \sum_{j=1}^{q}\tilde{W}_{fj}^T W_j(W_i(\mathbf{y}_2))\right] \end{aligned} \tag{3.44}$$

Obviously, we can choose update laws such that the estimate error terms disappear.

$$\dot{\hat{\theta}}_i = \gamma_i\left[\mathbf{s}^T U_{fi} - \sum_{j=1}^{q}\tilde{W}_{fj}^T W_j(U_{fi})\right]; \quad i = 1, 2, \cdots, p \tag{3.45}$$

$$\dot{\hat{\phi}}_i = \gamma_i'\left[\mathbf{s}^T\hat{W}_{fi} - \sum_{j=1}^{q}\tilde{W}_{fj}^T W_j(W_i(\mathbf{y}_2))\right]; \quad i = 1, 2, \cdots, q \tag{3.46}$$

Then we have

$$\dot{V} = -(\eta-\alpha)\mathbf{s}^T\mathbf{s} - \sum_{j=1}^{q}\phi_j^*\mathbf{s}^T\tilde{W}_{fj} - \sum_{j=1}^{q}k_j\tilde{W}_{fj}^T\tilde{W}_{fj} \tag{3.47}$$

We can simplify $\dot{V}$ by defining a vector

$$\boldsymbol{\xi} = \begin{bmatrix} \tilde{W}_{f1}^T & \tilde{W}_{f2}^T & \cdots & \tilde{W}_{fq}^T & \mathbf{s}^T \end{bmatrix}^T \tag{3.48}$$

and using Kronecker product notation, i.e.,

$$\dot{V} = -\boldsymbol{\xi}^T(A \otimes I_{n\times n})\boldsymbol{\xi} \tag{3.49}$$

where

$$A = \begin{bmatrix} k_1 & 0 & \cdots & 0 & \frac{\phi_1^*}{2} \\ 0 & k_2 & \cdots & 0 & \frac{\phi_2^*}{2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & k_q & \frac{\phi_q^*}{2} \\ \frac{\phi_1^*}{2} & \frac{\phi_2^*}{2} & \cdots & \frac{\phi_q^*}{2} & (\eta-\alpha) \end{bmatrix} \in \mathcal{R}^{(q+1)\times(q+1)} \tag{3.50}$$

and $I_{n\times n}$ is an n-dimensional identity matrix. Since the Kronecker product of two positive definite matrices is positive definite, the only requirement for $A \otimes I_{n\times n}$ to be positive definite is that the determinant of A is positive. The determinant of A is easily obtained using LU factorization. In other words, A can be decomposed as

$$\begin{aligned} A &= LU \\ &= \begin{bmatrix} k_1 & 0 & \cdots & 0 & 0 \\ 0 & k_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & k_q & 0 \\ \frac{\phi_1^*}{2} & \frac{\phi_2^*}{2} & \cdots & \frac{\phi_q^*}{2} & \mu \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 & \frac{\phi_1^*}{2k_1} \\ 0 & 1 & \cdots & 0 & \frac{\phi_1^*}{2k_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \frac{\phi_1^*}{2k_q} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \end{aligned} \tag{3.51}$$

where

$$\mu = (\eta - \alpha) - \sum_{j=1}^{q} \frac{\phi_j^{*2}}{4k_j} \tag{3.52}$$

and we have

$$\begin{aligned} \det(A) &= \det(L)\det(U) \\ &= \mu \prod_{i=1}^{q} k_i \end{aligned} \tag{3.53}$$

Therefore, a positive constant μ ensures $\dot{V} \leq 0$. Since the upper bounds for ϕ_j^*'s are known, we can rewrite this constraint as

$$\eta - \alpha > \sum_{j=1}^{q} \frac{\bar{\phi}_j^2}{4k_j} \tag{3.54}$$

The signal chasing analysis analogous to the presentation in Chapter 2 gives us

$$\boldsymbol{\xi} \in L_\infty \cup L_2, \quad \dot{\boldsymbol{\xi}} \in L_\infty \tag{3.55}$$

and, by Barbalat's lemma, we can conclude that

$$\lim_{t \to \infty} \boldsymbol{\xi}(t) = \mathbf{0} \tag{3.56}$$

Finally, through additional signal chasing through the filter variable definitions, we have

$$\lim_{t \to \infty} \mathbf{x}_1(t) = \mathbf{0} \tag{3.57}$$

$$\lim_{t \to \infty} \mathbf{x}_2(t) = \mathbf{0} \tag{3.58}$$

$$\lim_{t \to \infty} \tilde{W}_{fj}(t) = \mathbf{0} \tag{3.59}$$

where $j = 1, 2, \cdots, q$.

3.3 Reconstruction of Signals

Since the analysis is not based on the actual signals but the filtered ones, and some of the filters require the unmeasured state, $\mathbf{x}_2$, as an input, we cannot directly apply the result of the preceding analysis for implementation. For example, the control law we proposed is not u, but u_f, and the filter (3.9) is not realizable because it requires $\mathbf{x}_2$. Therefore, the signals must be recovered in terms of known or measured ones with the realizable filters. Since $\dot{\mathbf{y}}_1$ converges to $\mathbf{y}_2$ exponentially fast, we can replace $\mathbf{y}_2$ with $\dot{\mathbf{y}}_1$. If we denote

$$\mathbf{y} = \mathbf{y}_1 \tag{3.60}$$

we have

$$\dot{\mathbf{y}} = -\nu \mathbf{y} + \mathbf{x}_1 \tag{3.61}$$

Then, the filter for $\mathbf{y}_1$ becomes

$$\dot{\mathbf{z}} = -\lambda\mathbf{z} + \eta\mathbf{y} \tag{3.62}$$

and consequently the auxiliary variable, $\mathbf{s}$, is of the form

$$\mathbf{s} = \dot{\mathbf{y}} + \alpha\mathbf{y} + \dot{\mathbf{z}} \tag{3.63}$$

When the same procedure is applied to the control law and the estimate update laws, we have

$$\mathbf{u}_f = -\sum_{i=1}^{p} \hat{\theta}_i U_{fi} - \sum_{j=1}^{q} \hat{\phi}_j \hat{W}_{fj} - k_p\mathbf{y} - k_z\dot{\mathbf{z}} \tag{3.64}$$

and

$$\dot{\hat{W}}_{fj} = -k_j\tilde{W}_{fj} + \sum_{i=1}^{p} \hat{\theta}_i W_j(U_{fi}) + \sum_{i=1}^{q} \hat{\phi}_i W_j(W_i(\dot{\mathbf{y}})) + W_j(\mathbf{u}_f) \tag{3.65}$$

$$\dot{\hat{\theta}}_i = \gamma_i \left[\mathbf{s}^T U_{fi} - \sum_{j=1}^{q} \tilde{W}_{fj}^T W_j(U_{fi}) \right]; \quad i = 1, 2, \cdots, p \tag{3.66}$$

$$\dot{\hat{\phi}}_i = \gamma_i' \left[\mathbf{s}^T \hat{W}_{fi} - \sum_{j=1}^{q} \tilde{W}_{fj}^T W_j(W_i(\dot{\mathbf{y}})) \right]; \quad i = 1, 2, \cdots, q \tag{3.67}$$

where

$$\tilde{W}_{fj} = \hat{W}_{fj} - W_j(\dot{\mathbf{y}}) \tag{3.68}$$

Lastly, for implementation, the actual control is reconstructed using the filter definition

$$\mathbf{u} = \dot{\mathbf{u}}_f + \nu\mathbf{u}_f \tag{3.69}$$

Here, we can say that $\mathbf{u}$ is independent of $\mathbf{x}_2$ by showing that $\dot{\mathbf{u}}_f$ is independent of $\mathbf{x}_2$ or $\ddot{\mathbf{y}}$.

$$\begin{aligned}
\dot{\mathbf{u}}_f &= -\sum_{i=1}^{p} \dot{\hat{\theta}}_i U_{fi} - \sum_{i=1}^{p} \hat{\theta}_i \dot{U}_{fi} - \sum_{j=1}^{q} \dot{\hat{\phi}}_j \hat{W}_{fj} - \sum_{j=1}^{q} \hat{\phi}_j \dot{\hat{W}}_{fj} - k_p \dot{\mathbf{y}} - k_z \ddot{\mathbf{z}} \\
&= -\sum_{i=1}^{p} \dot{\hat{\theta}}_i U_{fi} - \sum_{i=1}^{p} \hat{\theta}_i \dot{U}_{fi} - \sum_{j=1}^{q} \dot{\hat{\phi}}_j \hat{W}_{fj} - \sum_{j=1}^{q} \hat{\phi}_j \dot{\hat{W}}_{fj} - k_p \dot{\mathbf{y}} - k_z \left(-\lambda \dot{\mathbf{z}} + \eta \dot{\mathbf{y}}\right)
\end{aligned} \tag{3.70}$$

After some algebra, we have

$$\begin{aligned}
\mathbf{u} = &-\sum_{i=1}^{p} \left(\dot{\hat{\theta}}_i U_{fi} + \hat{\theta}_i \dot{U}_{fi} + \nu \hat{\theta}_i U_{fi} \right) \\
&- \sum_{j=1}^{q} \left(\dot{\hat{\phi}}_j \hat{W}_{fj} + \hat{\phi}_j \dot{\hat{W}}_{fj} + \nu \hat{\phi}_j \hat{W}_{fj} \right) \\
&- k_p \mathbf{x}_1 - k_z \left[\eta \dot{\mathbf{y}} + (\nu - \lambda) \dot{\mathbf{z}} \right]
\end{aligned} \tag{3.71}$$

3.4 Special case: $U(\mathbf{x}_1)$ is linear

If some columns of the regressor $U(\mathbf{x}_1)$ are linear, the order of the closed loop system can be reduced. Similar to $W(\mathbf{x}_2)$, the filtered regressors for the linear part of $U(\mathbf{x}_1)$ are replaced with

$$U_{fi} = U_i(\mathbf{y}) \tag{3.72}$$

$$\dot{U}_{fi} = U_i(\dot{\mathbf{y}}) \tag{3.73}$$

where U_{fi} are columns of $U(\mathbf{x}_1)$, which are linear.

3.5 Extension to Tracking Problem

For the tracking formulation, we can set up the error dynamics with predefined reference trajectories, $\mathbf{r}_1$ and $\mathbf{r}_2 = \dot{\mathbf{r}}_1$, which are bounded and smooth. In addition, if we define a new control

$$\mathbf{v} = \mathbf{u} - \dot{\mathbf{r}}_2 \tag{3.74}$$

we have

$$\begin{aligned}\dot{\mathbf{e}}_1 &= \dot{\mathbf{x}}_1 - \mathbf{r}_1 \\ &= \mathbf{e}_2 \end{aligned}\tag{3.75}$$

$$\begin{aligned}\dot{\mathbf{e}}_2 &= \dot{\mathbf{x}}_2 - \mathbf{r}_2 \\ &= \sum_{i=1}^{p} \theta_i^* U_i(\mathbf{x}_1) + \sum_{j=1}^{q} \phi_j^* W_j(\mathbf{x}_2) + \mathbf{v} \end{aligned}\tag{3.76}$$

In order to follow the analysis procedure discussed in the previous section, we need to construct the following filters

$$\dot{\mathbf{y}}_1 = -\nu \mathbf{y}_1 + \mathbf{e}_1 \tag{3.77}$$

$$\dot{\mathbf{y}}_2 = -\nu \mathbf{y}_2 + \mathbf{e}_2 \tag{3.78}$$

$$\dot{U}_{fi} = -\nu U_{fi} + U_i(\mathbf{x}_1); \quad i = 1, 2, \cdots, p \tag{3.79}$$

$$\dot{W}_{fj} = -\nu W_{fj} + W_j(\mathbf{x}_2); \quad j = 1, 2, \cdots, q \tag{3.80}$$

$$\dot{\mathbf{v}}_f = -\nu \mathbf{v}_f + \mathbf{v} \tag{3.81}$$

$$\dot{\mathbf{r}}_{f1} = -\nu \mathbf{r}_{f1} + \mathbf{r}_1 \tag{3.82}$$

$$\dot{\mathbf{r}}_{f2} = -\nu \mathbf{r}_{f2} + \mathbf{r}_2 \tag{3.83}$$

$$\dot{\mathbf{r}}_{f3} = -\nu \mathbf{r}_{f3} + \dot{\mathbf{r}}_2 \tag{3.84}$$

Note that the filters associated with the unmeasured state, (3.78) and (3.80) are only used for stability analysis. Moreover, with the matching conditions

$$\dot{\mathbf{r}}_{f1} = \mathbf{r}_{f2} \tag{3.85}$$

$$\dot{\mathbf{r}}_{f2} = \mathbf{r}_{f3} \tag{3.86}$$

(3.83) and (3.84) do not need to be implemented. From the filter definition, we have

$$\dot{\mathbf{y}}_1 = \mathbf{y}_2 + \boldsymbol{\epsilon}_1(t) \tag{3.87}$$

$$\dot{\mathbf{y}}_2 = \sum_{i=1}^{p} \theta_i^* U_{fi} + \sum_{j=1}^{q} \phi_j^* W_{fj} + \mathbf{v}_f + \boldsymbol{\epsilon}_2(t) \tag{3.88}$$

$$W_{fj} = W_j(\mathbf{y}_2) + W_j(\mathbf{r}_{f2}) + \boldsymbol{\epsilon}_3(t) \tag{3.89}$$

Now, the structure is the same as the stabilization case. Therefore, if one follows the proposed design procedure, the control and update laws which are independent of $\mathbf{x}_2$ are obtained.

3.6 Simulation

For the simulation, three dimensional states with two unknown parameters are considered. The regressors are selected arbitrarily such that $U(\mathbf{x}_1)$ is non-linear and $W(\mathbf{x}_2)$ is linear. The control objective in this simulation is to stabilize the plant with the closed system dynamics given by

$$
\begin{aligned}
\dot{\mathbf{x}}_1 &= \mathbf{x}_2 \\
\dot{\mathbf{x}}_2 &= \theta^* U(\mathbf{x}_1) + \phi^* W(\mathbf{x}_2) + \mathbf{u} \\
\dot{\mathbf{y}} &= -\nu \mathbf{y} + \mathbf{x}_1 \\
\dot{U}_f &= -\nu U_f + U(\mathbf{x}_1) \\
\dot{\hat{W}}_f &= -k\{\hat{W}_f - W(\dot{\mathbf{y}})\} + \hat{\theta} W(U_f) + \hat{\phi} W(W(\dot{\mathbf{y}})) + W(\mathbf{u}_f) \\
\dot{\hat{\theta}} &= \gamma \left[\mathbf{s}^T U_f - \{\hat{W}_f - W(\dot{\mathbf{y}})\}^T W(U_f) \right] \\
\dot{\hat{\phi}} &= \gamma' \left[\mathbf{s}^T \hat{W}_f - \{\hat{W}_f - W(\dot{\mathbf{y}})\}^T W(W(\dot{\mathbf{y}})) \right]
\end{aligned}
\tag{3.90}
$$

where the true values of unknown parameters are set to

$$\theta^* = 2, \quad \phi^* = 1$$

The regressors are chosen as

$$U(\mathbf{x}_1) = \begin{bmatrix} 2\sin(x_{13}) \\ -2x_{12}^2 \\ 4\sin(x_{11}) \end{bmatrix}, \quad W(\mathbf{x}_2) = \begin{bmatrix} x_{22} \\ -x_{21} \\ \frac{1}{2}x_{23} \end{bmatrix}$$

where

$$\mathbf{x}_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix}$$

The following values are used for the control parameters

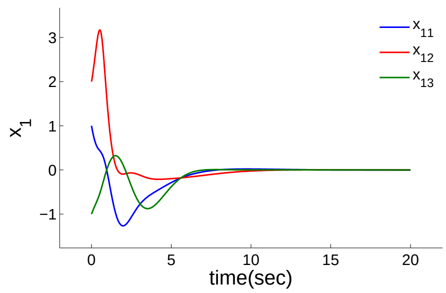
$$\alpha = 1,\ \eta = 2,\ \lambda = 5,\ \nu = 5,\ k = 3,\ \gamma = 1,\ \gamma' = 1$$

and for the initial conditions

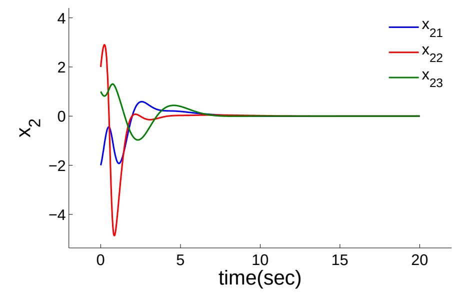
$$\mathbf{x}_{10} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{x}_{20} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{y}_0 = \begin{bmatrix} 0.2 \\ 0.4 \\ -0.2 \end{bmatrix}$$

$$\mathbf{z}_0 = \begin{bmatrix} 0.08 \\ 0.16 \\ -0.08 \end{bmatrix}, \quad \hat{W}_{f0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{\theta}_0 = 3, \quad \hat{\phi}_0 = 0$$

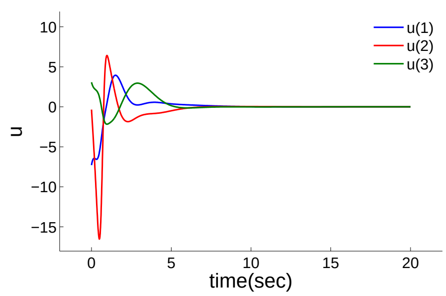
The simulation results are illustrated in Figure 3.1. In addition to the states, parameter estimates, and control, the difference between the filtered regressor and its estimate is also shown. As proven in the analysis, the states and regressor estimate error tend to zero asymptotically.



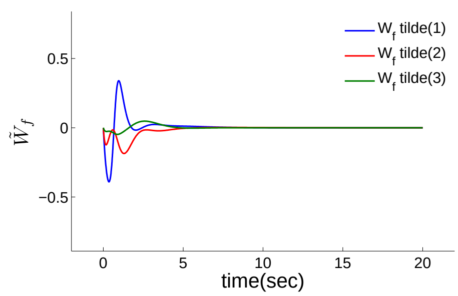
(a) Time history of position, $\mathbf{x}_1$



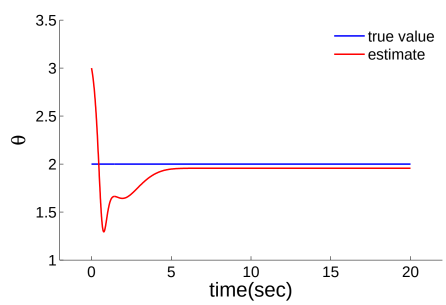
(b) Time history of velocity, $\mathbf{x}_2$



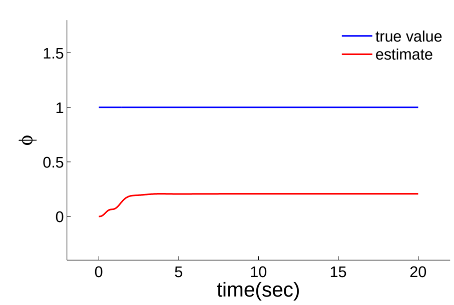
(c) Time history of control, $\mathbf{u}$



(d) Time history of $\tilde{W}_f$



(e) Time history of estimate, $\hat{\theta}$



(f) Time history of estimate, $\hat{\phi}$

Figure 3.1: Simulation results of stabilization problem

Chapter 4

Hardware Implementation

In this chapter, the previously proposed control scheme is applied to one dimensional rotating mass system actuated by a DC motor. Thanks to the passivity nature of the system, a control law can be designed even though there is a non-linear friction effect associated with a velocity state. The friction models used in the system dynamics are viscous friction which is linear in the angular speed and Coulomb friction which is approximated as a hyperbolic tangent function. As a physical device, Educational Control Product Model 750 Control Moment Gyroscope is used for the implementation [10]. The control goal is regulating the angular speed of the CMG rotor under the condition where uncertain parameters in the dynamics exist. Control parameters are chosen based on simulation so that control saturation does not occur and smooth control is guaranteed. Lastly, output states are compared with the simulation results.

4.1 Hardware and Software

4.1.1 Hardware

- Real-Time Computer with National Instruments 7831 R data acquisition card

- Host Computer
- Educational Control Products(ECP) Model 750 Control Moment Gyroscope with amplifier

4.1.1.1 Sensors

There are 4 optical encoders that measure angular positions of the rotor and the gimbals. The first encoder is used for the motor and has a resolution of 6667 counts/rev. The others with higher resolution are used for the gimbals.

4.1.1.2 Actuators

Two DC motors are installed on the first and second axes. One is used for a rotor speed control, and the other is used for a gimbal position control.

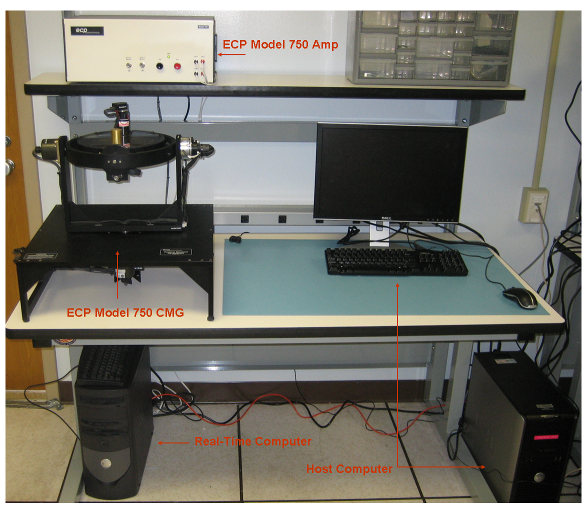


Figure 4.1: Hardware set-up

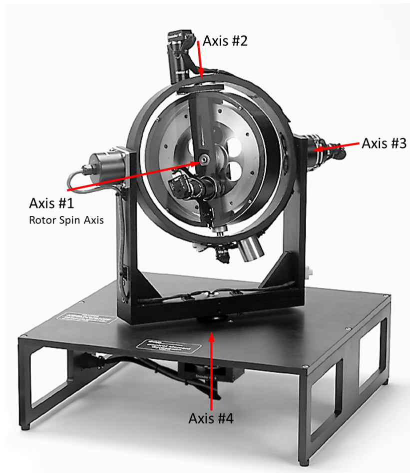


Figure 4.2: ECP Model 750 CMG

4.1.2 Software

Labview VI's designed by "The University of Texas at Austin Sensors and Actuators Laboratory" are used to control the CMG on the core level [11] [12]. These programs are developed for educational purposes. The Host VI is slightly adjusted for this application.

- FPGA VI - It handles the input/output and watch for system errors.
- Host VI - It is used to control inputs into the system.

4.2 Mathematical Description of the Physical System

4.2.1 Plant configuration and dynamics

Among several plant configurations of the Model 750 CMG, the simplest one is used where all brakes are applied on all gimbal axes to minimize the system order.

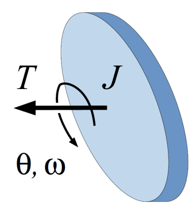


Figure 4.3: Plant configuration

This is a simple system of a rotating mass with the following equations of motion

$$\begin{aligned} \dot{\theta} &= \omega \\ J\dot{\omega} &= T \end{aligned} \tag{4.1}$$

where state variables θ and ω are angular position and velocity respectively, J is an inertia of the rotating mass, and T is an applied torque. If friction is considered, viscous plus Coulomb friction can be modeled. Viscous friction is the resisting force proportional to the angular velocity while Coulomb friction is the force with a constant magnitude in a direction opposite to the motion under the assumption of constant normal force. Neglecting static friction, the

Coulomb friction can be approximated using the hyperbolic tangent function rather than the sign function because the *tanh* model is more numerically stable and it represents the real system better. When the friction models are added to the system equations, we have

$$\dot{\theta} = \omega$$
$$J\dot{\omega} = -C\omega - F\tanh(A\omega) + T \tag{4.2}$$

where C and F and A are constant system parameters.

4.2.2 Unit conversion

It is convenient to use count based units because the encoder reading and motor control effort both are presented in count units. With the assumption that torque is proportional to the applied voltage, which corresponds to the control effort, we have

$$T = k_u u \tag{4.3}$$

where u is a control effort and k_u is a gain that converts the control effort to the applied torque. Also, as the encoder reading is in counts, we have

$$\theta = k_{c2r}\theta_c$$
$$\omega = k_{c2r}\omega_c \tag{4.4}$$

where θ_c is an angular position in counts, ω_c is an angular velocity in counts per second, and k_{c2r} is a gain that converts units from counts to radians. Once we define

$$
\begin{aligned}
x_1 &= \theta_c \\
x_2 &= \omega_c \\
c &= \frac{C}{J} \\
f &= \frac{F}{Jk_{c2r}} \\
a &= Ak_{c2r} \\
b &= \frac{k_u}{Jk_{c2r}}
\end{aligned}
\tag{4.5}
$$

the dynamics become

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -cx_2 - f\tanh(ax_2) + bu
\end{aligned}
\tag{4.6}
$$

It is assumed that a is known, and c, f, and b are partially known; signs of these parameters are known, but its magnitudes are not.

4.2.3 Hardware limitation

Since this is a physical plant, there exist a control saturation. The applied voltage cannot exceed $\pm$ 10 volts, which is equivalent to $-32768 < u < 32767$ counts. This saturation constraint will be enforced in the simulation while tuning for the various controller parameter values.

4.3 Controller Design

4.3.1 Control objective

The general objective is to design a control law that tracks the predefined reference trajectories. Since the design scheme is developed only for the systems where unknown parameters do not appear with non-linear functions of velocity states, the implementation seems to be inappropriate. However, if hyperbolic tangent friction model with viscous friction fits well and its mathematical structure is known, this non-linearity can be circumvented. Further discussion of Coulomb friction will be presented in the stability analysis section. Even if the slope of the hyperbolic tangent model at zero velocity does not match well, we can avoid the model mismatch effect by putting the reference velocity away from this region. Since the control goal is to track a constant speed, we can set the reference velocity as a constant value. However to obtain a smooth transient, a critically damped second order linear system with a constant input is introduced to govern the reference trajectories, such that the steady state will be the desired rotor velocity.

4.3.2 Controller design

Since the friction non-linearity is involved, the system we now deal with is different from Chapter 2. Thus, the control law needs to be re-formulated with the following dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -cx_2 - f\ \tanh(ax_2) + bu \end{aligned} \tag{4.7}$$

with the position, velocity, and acceleration reference trajectories

$$
\begin{aligned}
r_1 &= r \\
r_2 &= \dot{r} \\
r_3 &= \ddot{r}
\end{aligned}
\tag{4.8}
$$

Once the error states are defined as

$$
\begin{aligned}
e_1 &= x_1 - r_1 \\
e_2 &= x_2 - r_2
\end{aligned}
\tag{4.9}
$$

we have the following error dynamics

$$
\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= -cx_2 - f \ \tanh(ax_2) - r_3 + bu
\end{aligned}
\tag{4.10}
$$

As the target dynamics are given by

$$
\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= -k_p e_1 - k_z \dot{z}
\end{aligned}
\tag{4.11}
$$

with

$$
\dot{z} = -\lambda z + \eta e_1 \tag{4.12}
$$

where k_p, k_z, λ, and η are positive constants to be determined, we can rewrite the velocity error equation as

$$
\dot{e}_2 = -k_p e_1 - k_z \dot{z} + b[u - \theta_1^* x_2 - \theta_2^* \ \tanh(ax_2) + \theta_3^*(k_p e_1 + k_z \dot{z} - r_3)] \tag{4.13}
$$

where

$$\theta_1^* = \frac{c}{b} \tag{4.14}$$

$$\theta_2^* = \frac{f}{b} \tag{4.15}$$

$$\theta_3^* = \frac{1}{b} \tag{4.16}$$

are unknown constants. Before designing a control law, let us define two variables for convenience.

$$s = e_2 + \alpha e_1 - \dot{z} \tag{4.17}$$

$$\begin{aligned} \bar{s} &= s - x_2 \\ &= -r_2 + \alpha e_1 - \dot{z} \end{aligned} \tag{4.18}$$

where α is a positive number to be determined. s is a signal we want to drive to zero and $\bar{s}$ is a signal independent of x_2 so that we can implement it into our control law. Now, let us choose

$$u = -(\hat{\theta}_1 + \beta_1)\bar{s} - (\hat{\theta}_2 + \beta_2)\ \tanh(a\bar{s}) - (\hat{\theta}_3 + \beta_3)(k_p e_1 + k_z \dot{z} - r_3) \tag{4.19}$$

where $(\hat{\theta}_i + \beta_i)$'s are estimates for θ_i^*'s with $i = 1, 2, 3$. Here again, the saturation condition is not enforced explicitly. Therefore the control parameters will be chosen through the simulation so that the control effort do not exceed the limit values.

When this control law is applied, the closed-loop error dynamics become

$$
\begin{aligned}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= -k_p e_1 - k_z \dot{z} - cs - f[\,\tanh(a\bar{s}) + \tanh(ax_2)] \\
&\quad - b[\tilde{\theta}_1 \bar{s} + \tilde{\theta}_2 \tanh(a\bar{s}) + \tilde{\theta}_3 (k_p e_1 + k_z \dot{z} - r_3)]
\end{aligned} \tag{4.20}
$$

where

$$\tilde{\theta}_1 = (\hat{\theta}_1 + \beta_1) - \theta_1^* \tag{4.21}$$

$$\tilde{\theta}_2 = (\hat{\theta}_2 + \beta_2) - \theta_2^* \tag{4.22}$$

$$\tilde{\theta}_3 = (\hat{\theta}_3 + \beta_3) - \theta_3^* \tag{4.23}$$

The values of k_p and k_z will be determined as functions of the design parameters, α, λ, and η through a stability analysis. Some constraints will apply to these parameters in order to make the system stable.

4.3.3 Stability analysis

For the analysis, let us define a lower bounded function,

$$V = \frac{1}{2}\left(s^2 + |b| \sum_{i=1}^{3} \frac{\tilde{\theta}_i^2}{\gamma_i} \right) \tag{4.24}$$

where the learning rate γ_i's are positive constants. Then, the time derivative of V is given by

$$
\begin{aligned}
\dot{V} &= [-(\eta - \alpha)e_2 - k_p e_1 + (\lambda - k_z)\dot{z}]s - cs^2 - f[\tanh(a\bar{s}) + \tanh(ax_2)]s \\
&\quad - \operatorname{sign}(b)|b|[\tilde{\theta}_1 \bar{s} + \tilde{\theta}_2 \tanh(a\bar{s}) + \tilde{\theta}_3(k_p e_1 + k_z \dot{z} - r_3)]s \\
&\quad + |b| \left[\frac{1}{\gamma_1}\tilde{\theta}_1(\dot{\hat{\theta}}_1 + \dot{\beta}_1) + \frac{1}{\gamma_2}\tilde{\theta}_2(\dot{\hat{\theta}}_2 + \dot{\beta}_2) + \frac{1}{\gamma_3}\tilde{\theta}_3(\dot{\hat{\theta}}_3 + \dot{\beta}_3) \right]
\end{aligned} \tag{4.25}
$$

Once we choose

$$k_p = \alpha(\eta - \alpha) \tag{4.26}$$

$$k_z = \lambda - (\eta - \alpha) \tag{4.27}$$

with additional conditions on the design parameters

$$\eta > \alpha \tag{4.28}$$

$$\lambda > (\eta - \alpha) \tag{4.29}$$

we have

$$\begin{aligned}
\dot{V} &= -[(\eta - \alpha) + c]s^2 - f[\tanh(a\bar{s}) + \tanh(ax_2)]s \\
&\quad - \operatorname{sign}(b)|b|[\tilde{\theta}_1 \bar{s} + \tilde{\theta}_2 \tanh(a\bar{s}) + \tilde{\theta}_3(k_p e_1 + k_z \dot{z} - r_3)]s \\
&\quad + |b| \left[\frac{1}{\gamma_1}\tilde{\theta}_1(\dot{\hat{\theta}}_1 + \dot{\beta}_1) + \frac{1}{\gamma_2}\tilde{\theta}_2(\dot{\hat{\theta}}_2 + \dot{\beta}_2) + \frac{1}{\gamma_3}\tilde{\theta}_3(\dot{\hat{\theta}}_3 + \dot{\beta}_3) \right]
\end{aligned} \tag{4.30}$$

If update laws are determined such that the estimate error terms vanish, we will end up with

$$\begin{aligned}
\dot{V} &= -[(\eta - \alpha) + c]s^2 - f\left[\frac{\sinh(as)}{\cosh(a\bar{s})\cosh(ax_2)}\right]s \\
&\leq -(\eta - \alpha)s^2 \\
&\leq 0
\end{aligned} \tag{4.31}$$

Since s converges to zero asymptotically, the state errors also converge to zero, while the estimate errors remain bounded. The task remaining is to determine the update laws and the β's independent of x_2.

$$
\begin{aligned}
\frac{\dot{\hat{\theta}}_1 + \dot{\beta}_1}{\gamma_1 \operatorname{sign}(b)} &= s\bar{s} \\
&= e_1(r_3 - \lambda \dot{z}) + \bar{s}(\alpha e_1 - \dot{z}) + \frac{d}{dt}\left[e_1(-\frac{\eta - \alpha}{2}e_1 - r_2 + \lambda z)\right] && (4.32) \\
\frac{\dot{\hat{\theta}}_2 + \dot{\beta}_2}{\gamma_2 \operatorname{sign}(b)} &= s \tanh(a\bar{s}) \\
&= \tanh(a\bar{s})\left(\alpha e_1 - \dot{z} + \frac{\lambda \dot{z} - r_3}{\eta - \alpha}\right) + \frac{d}{dt}\left[-\frac{\ln\{\cosh(a\bar{s})\}}{a(\eta - \alpha)}\right] && (4.33) \\
\frac{\dot{\hat{\theta}}_3 + \dot{\beta}_3}{\gamma_3 \operatorname{sign}(b)} &= s(k_p e_1 + k_z \dot{z} - r_3) \\
&= e_1(\lambda k_z \dot{z} + \dot{r}_3) + (\alpha e_1 - \dot{z})(k_p e_1 + k_z \dot{z} - r_3) \\
&\quad + \frac{d}{dt}\left[e_1(\frac{k_p + \eta k_z}{2}e_1 - \lambda k_z z - r_3)\right] && (4.34)
\end{aligned}
$$

Therefore, the update laws and β's are

$$
\begin{aligned}
\dot{\hat{\theta}}_1 &= \gamma_1 \operatorname{sign}(b)[e_1(r_3 - \lambda \dot{z}) + \bar{s}(\alpha e_1 - \dot{z})] && (4.35) \\
\dot{\hat{\theta}}_2 &= \gamma_2 \operatorname{sign}(b)\left[\tanh(a\bar{s})(\alpha e_1 - \dot{z} + \frac{\lambda \dot{z} - r_3}{\eta - \alpha})\right] && (4.36) \\
\dot{\hat{\theta}}_3 &= \gamma_3 \operatorname{sign}(b)[e_1(\lambda k_z \dot{z} + \dot{r}_3) + (\alpha e_1 - \dot{z})(k_p e_1 + k_z \dot{z} - r_3)] && (4.37) \\
\beta_1 &= \gamma_1 \operatorname{sign}(b)\left[e_1(-\frac{\eta - \alpha}{2}e_1 - r_2 + \lambda z)\right] && (4.38) \\
\beta_2 &= \gamma_2 \operatorname{sign}(b)\left[-\frac{\ln\{\cosh(a\bar{s})\}}{a(\eta - \alpha)}\right] && (4.39) \\
\beta_3 &= \gamma_3 \operatorname{sign}(b)\left[e_1(\frac{k_p + \eta k_z}{2}e_1 - \lambda k_z z - r_3)\right] && (4.40)
\end{aligned}
$$

Note that we may have different forms of update laws depending on the choice of β's. By the analysis, we can conclude that the estimates $(\theta + \beta)$'s are bounded, but there are possibilities that individual θ's and β's go unbounded and it may cause a numerical instability. Therefore, choosing proper β's is important. In this case, β_1 and β_2 are chosen such that they are proportional to e_1, so that they converge to zero as the state errors go to zero. The function β_2 cannot be determined with a linear dependency on e_1, but it will be remained bounded because it is a function of $\bar{s}$ which converges to the constant reference velocity. In terms of the numerical stability, computing $\ln\{\cosh(a\bar{s})\}$ is also a large numerical overflow problem for a large $\bar{s}$. Therefore, a different form must be taken in order to avoid dealing with large numbers.

$$\ln\{\cosh(a\bar{s})\} = |a\bar{s}| + \ln\left\{\frac{1+e^{-2|a\bar{s}|}}{2}\right\} \tag{4.41}$$

4.4 Simulation

A reference trajectory needs to be chosen properly such that control saturation does not occur. If the velocity reference is a step response of the second order stable linear system with the target amplitude Ω, damping ratio ζ, and natural frequency ω_n, then we have a smooth trajectory. The reference is governed by the following differential equation

$$\frac{d^2}{dt^2}r_2 = -2\zeta\omega_n\frac{d}{dt}r_2 - \omega_n^2 r_2 + \omega_n^2\Omega \tag{4.42}$$

with zero initial conditions. Moreover, for the fastest convergence to Ω with a fixed natural frequency, a damping ratio should be equal to one. Thus, if we want a time constant, $\tau = 1$ which indicates approximately 4 seconds of settling time, then $\omega_n = 1$ will work.

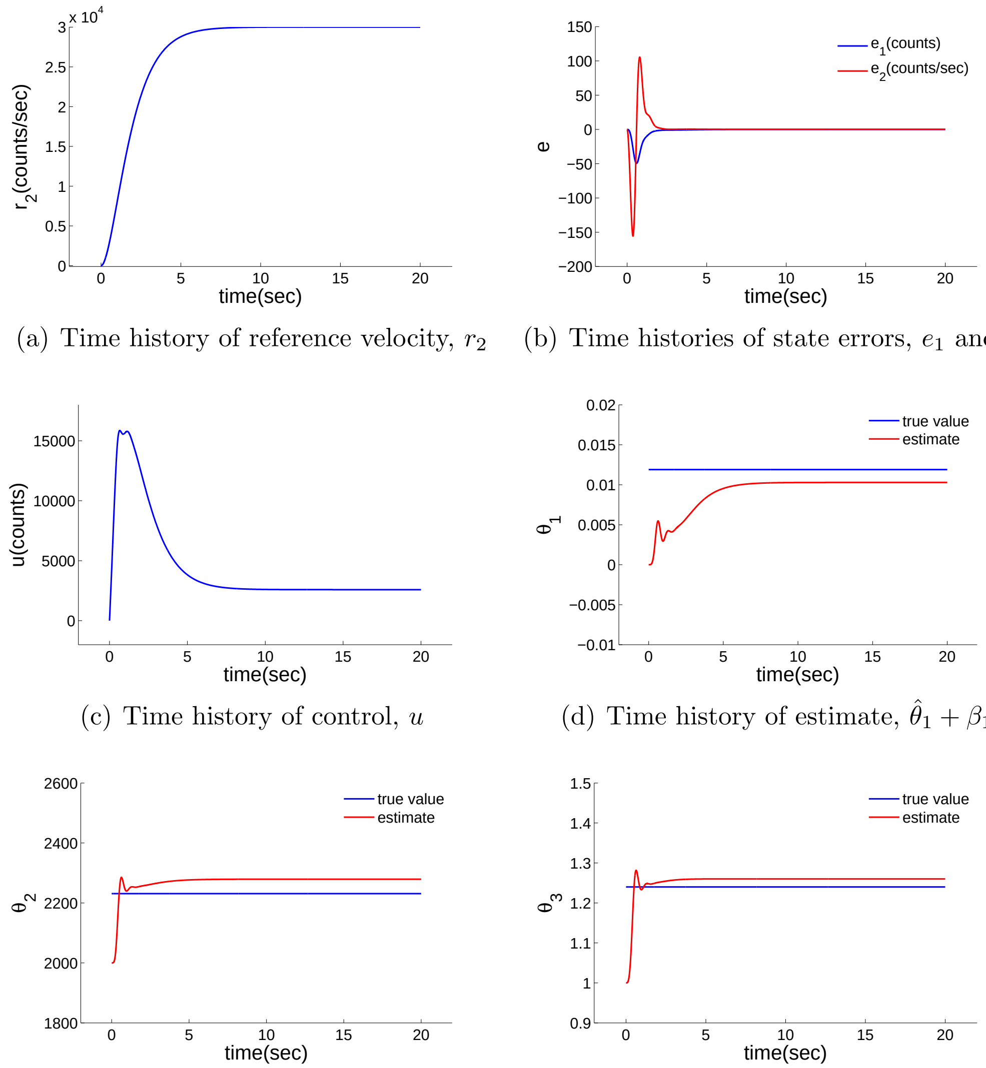
The simulation is performed with the following values:

$$a = 0.003, \quad b = 0.8065, \quad c = 0.0096, \quad f = 1799.3$$

The control parameters and initial conditions of update laws are chosen as

$$\alpha_1 = 1,\ \eta = 9,\ \lambda = 16$$
$$\gamma_1 = 10^{-7},\ \gamma_2 = 10,\ \gamma_3 = 10^{-6}$$
$$\hat{\theta}_1(0) = 0,\ \hat{\theta}_2(0) = 2000,\ \hat{\theta}_3(0) = 1$$

which will give us a non-oscillatory response. The results shown in Figure 4.4 are obtained when the reference rotor speed is set to $\Omega = 3 \times 10^4$ in counts, which is equivalent to 270 rpm.



(a) Time history of reference velocity, r_2

(b) Time histories of state errors, e_1 and e_2

(c) Time history of control, u

(d) Time history of estimate, $\hat{\theta}_1 + \beta_1$

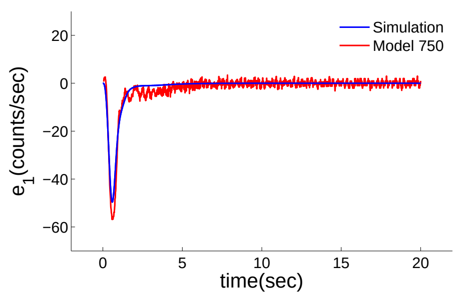
(e) Time history of estimate, $\hat{\theta}_2 + \beta_2$

(f) Time history of estimate, $\hat{\theta}_3 + \beta_3$

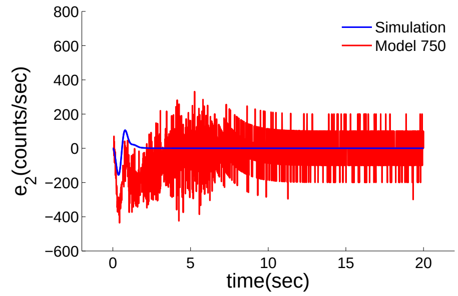
Figure 4.4: Simulation results of rotor speed control

4.5 Model 750 CMG Results

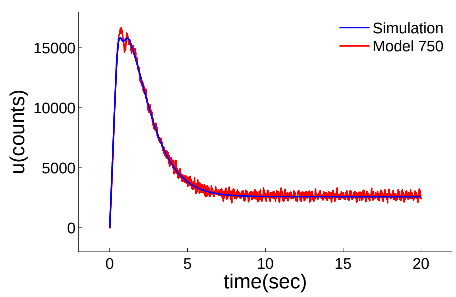
Using the same control parameters and initial conditions used in the simulation, the designed control law is applied to the Model 750. Figure 4.5 depicts the results of the hardware test. Since the velocity is not measured, numerical differentiation is used and compared with the simulation result. In the figure, it is observed that the velocity error is within noise level, which means that the velocity state tracks the reference. Moreover, as the overall performance is similar to the simulation, it can be said that the mathematical model used is reasonable to describe the real system.



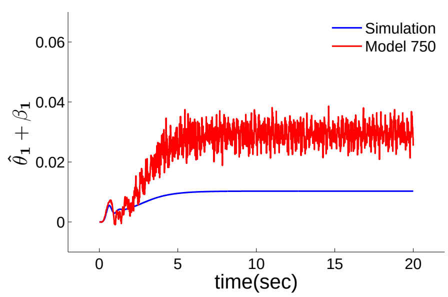
(a) Time history of position error, e_1



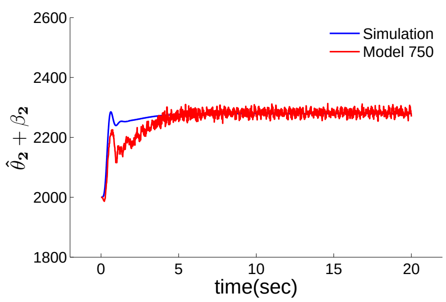
(b) Time history of velocity error, e_2



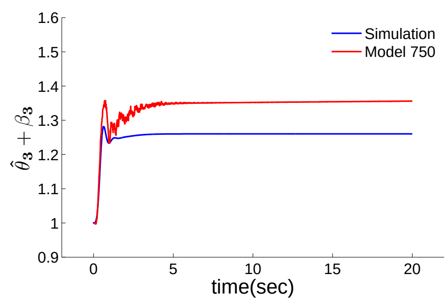
(c) Time history of control, u



(d) Time history of estimate, $\hat{\theta}_1 + \beta_1$



(e) Time history of estimate, $\hat{\theta}_2 + \beta_2$



(f) Time history of estimate, $\hat{\theta}_3 + \beta_3$

Figure 4.5: Comparison between simulation and hardware test

Chapter 5

Conclusions

In this report, a new adaptive control approach for second-order class linear systems is introduced. The objective of the research is to design a control law that stabilizes the system or tracks predefined reference trajectories under the situation where the velocity state is unavailable for feedback and the system parameters are unknown or partially known.

The control law is developed from a double integrator in the absence of unknown parameter effects. This is one of the results of passivity based control that uses a filtered signal out of the position state. That is, the feedback signal consists of the position and the filtered position states. From this feedback structure, we adopt the auxiliary variable, s which is key for the stability analysis.

Partially based on the non-CE adaptive control structure, the update laws for the uncertain parameters are established through a Lyapunov-like stability analysis. Two different approaches are suggested. The idea of the first approach is that the estimates are sums of two signals, one from the update laws and the other from the measured states. This method is well extended to the rotor speed control with the Coulomb friction in the dynamics. In spite of the main assumption that non-linearity associated with the velocity state does not appear in the dynamics, the control law independent of the velocity is successfully designed and implemented in the hardware.

However, the method derived in the scalar problems cannot be generalized to the more general case of n-coupled systems. Therefore, a filtered state approach is introduced for these vector cases. The main advantage of this approach is that we can handle the coupling effects and do not need to split the parameter estimation signals, but we are still bounded to the assumption that the unknown parameters appear in a linear fashion with the unmeasured velocity state.

This research is conducted as a first step towards the development of the angular-rate-free spacecraft attitude controller with an unknown inertia matrix. The application of the proposed methodology can be argued to be rather limited because of the restrictive nature of the assumptions we imposed: the linearity on the velocity state must be guaranteed and the bound of the absolute values of the uncertain parameters associated with the velocity must be known. However, as seen in the hardware implementation example, some special classes of non-linearity with respect to the unmeasured states such as friction can be successfully handled. Thus, we cannot ignore the possibility that the proposed technique can be extended to the desired application, the attitude control. Therefore, future work will deal with extending the control scheme for the more general non-linear cases.

Another restriction is observed regarding the dimension of the control input. Under the formulation, the control signal needs to be the same as the dimension of the position or velocity states. Of course, this is not always the case, but that suggests further explorations on under-actuated uncertain systems.

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