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**Formulation of Rigid Diaphragm Analysis Spreadsheet
by Stiffness Method**

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Report

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Abstract

Formulation of Rigid Diaphragm Analysis Spreadsheet by Stiffness Method

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Abstract: This report is the documentation for a stiffness formulation to perform rigid diaphragm analysis for wood structures subjected to wind loads. Traditionally, rigid diaphragm analysis has been performed using a vaguely-defined superposition approach; however, this report details a more rational stiffness approach to solving for forces placed on walls resulting from a rigid diaphragm, and its implementation is via a simple spreadsheet application. In addition to the formulation of the spreadsheet, the report contains a User's Guide and examples of the spreadsheet's use. The purpose of the spreadsheet is not as a replacement to more sophisticated and comprehensive finite element analysis software, but as a tool to aid designers who practice engineering and may not have access to such software. In general, the application is developed for wood diaphragms as will be noted by references to wood-related codes. However, much of the approach may be used for diaphragms constructed with other materials as well.

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1. INTRODUCTION

When analyzing the distribution of lateral forces placed on a building structure caused either by wind or seismic loads, there are two primary ways of idealizing the diaphragm. The two idealizations are either flexible or rigid, defined in the case of wood diaphragms in the Special Design Provisions For Wind And Seismic §2.2 (The American Wood Council, 2010). For brevity, this will be referred as SDPWS henceforth. In general, diaphragms made of wood fall in the category of flexible and as a result the lateral forces generated on the supporting walls are distributed in accordance with tributary areas. But, there are cases in which it may be required to idealize the wood diaphragm as rigid; for example, in an apartment building with many interior walls, especially if they are relatively closely spaced.

It is not uncommon in certain locations in the United States, such as California, with frequent seismic activity to analyze the distribution of the forces using both a rigid and flexible wood diaphragm analysis, and then assigning worst-case values to each wall. Over time, more engineers are becoming aware of the need for both analyses since the defining boundary between rigid and flexible diaphragm behavior—not the code-defined boundary—is vague at best. As most engineers know, the truth likely is somewhere between the two.

The purpose of this report is to provide the documentation of the formulation of an Excel[®] spreadsheet developed for rigid diaphragm analysis, a user's guide, and design examples. Although sophisticated finite element and 3D structural analysis software may be used, the proper use involves a large investment in time in order to accommodate all of the appropriate structural behavior, which may include connector slip and the relative deformation between sheathing panel edges, as well as others. Design engineers would

not be able to profit from such time consuming efforts. On the other hand, Excel[®] spreadsheet use is widespread and thus it is our choice for the rigid diaphragm analysis. By performing a rigid diaphragm analysis in addition to the conventional flexible analysis, there is a greater assurance that the supporting elements, specifically walls, are designed for the worst possible case.

2. FORMULATION

When analyzing a rigid diaphragm the typical procedure is to distribute the lateral load to each member proportional to the stiffness of that member resisting the lateral force. Then, the moment generated by the eccentricity between the center of rigidity and the resultant of the applied wind loads is then resolved into forces on the members. The final step is to use superposition and combine the forces of the two previous steps. The issue that arises is that when more than two walls not on the same plane are resisting lateral force in a particular direction the problem is indeterminate and cannot be solved using a simple analysis, so a different approach is being recommended, one that considers kinematic determinacy instead of static indeterminacy.

The Excel[®] Spreadsheet developed for analyzing rigid diaphragms was developed using a stiffness method approach that is analogous to having a rigid beam on spring supports. The spring supports represent the shear walls designed to carry the lateral load and the rigid beam represents the rigid diaphragm.

2.1 Assumptions

The only assumptions made in the analysis are:

1. The diaphragm undergoes small rotations and the consequence is that the geometry before and after loading are assumed to be the same.
2. The walls behave as linear elastic springs.
3. Walls do not provide stiffness perpendicular to their long direction; in other words they are uni-directional springs.

2.2 Spreadsheet Details and Design

2.2.1 WALL LOCATION AND STIFFNESS

The spreadsheet was designed in order quickly find forces caused by lateral wind loads. The spreadsheet initiates with the requirement that the start and end coordinates—in feet--of the walls be input. From this input the wall lengths, centroid, and direction are computed.

A relative stiffness among the walls is required to be input next. It has been provided by the spreadsheet a simple way of calculating an apparent stiffness of a wood shear wall. The stiffness is determined by first calculating the deflection under an assumed load using the shear wall deflection equation found in SDPWS §C4.3.2 and stated below.

$$\delta_{sw} = \frac{8vh^3}{EAb} + \frac{vh}{G_v t_v} + .75he_n + \frac{h}{b}\Delta_a \quad (1)$$

v : Induced unit shear, plf

h : Shear wall height, ft

b : Shear wall length, ft

E : Modulus of elasticity of end posts, psi

A : Area of end posts cross-section, in.²

$G_v t_v$: Shear stiffness, lb/in. of panel depth

e_n : nail slip in panel, in.

Δ_a : Total vertical elongation of wall anchorage system at wall ends by induced unit shear, in.

The definition for each variable can also be found in the SDPWS. With the deflection caused by the assumed load, the apparent stiffness per foot of wall, k_i , is found by dividing the assumed induced unit shear load, v , by the deflection of the shearwall, δ_{sw} .

$$k_i = \frac{v}{\delta_{sw}} \quad (2)$$

The stiffness computed for the wood shear walls is sensitive to the nail slip and the anchorage elongation term. Although it is not completely obvious from the equation, the nail slip term, e_n , given in Table C.4.2.2D of SDPWS is also a function of shear, except under a few circumstances. Table C.4.2.2D of SDPWS differentiates between the different sheathing materials. For wood structural panels or particleboard the nail slip is a function of shear. Fiberboard, gypsum board, and lumber used as sheathing contain constant slip regardless of shear. In the instances when the nail slip is a function of shear it is non-linear, and when nail slip is not a function of shear it is constant. Generally, for standard anchoring devices, manufacturers will provide deflections for their systems at allowable stress design levels. If no information is available on the anchorage system, engineering judgment should be used in accounting for all sources of elongation. The computed wall stiffness is then input into the relative stiffness cells. If the walls are not wood shear walls, then their stiffness maybe calculated by other means and then input into the relative stiffness cell. If the designer assumes that several wood walls are constructed the same--say, the same sheathing thickness and nailing schedule—then the unit shear load will be about equal, and the stiffness is proportional to the length, then a relative stiffness of 1 may be used for all walls. If a wall is sheathed on two sides identically, using the same fastener schedule and materials, then their unit shear capacity is twice that of a wall sheathed on one side. If one side of a wall is sheathed with gypsum wallboard and the other side using a structural panel then their shear capacities can be directly summed. SDPWS §C4.3.3 covers how to treat these walls and others.

The relative stiffness input is taken and normalized with respect to the highest relative stiffness. An equivalent length is calculated by multiplying the normalized stiffness by the length. The equivalent length together with the start and end coordinates are used to compute the location of the center of rigidity of the diaphragm. Multiplying

the x-coordinate, x_i , of all vertical walls by their equivalent length, L_e , and then dividing it by the sum of the equivalent lengths, calculates the x-coordinate of the center of rigidity, x_{cr} . The same is done for the y-coordinate with the horizontal walls; that is,

$$x_{cr} = \frac{\sum x_i L_e}{\sum L_e} \quad y_{cr} = \frac{\sum y_i L_e}{\sum L_e} \quad (3)$$

2.2.2 DISTRIBUTED WIND LOAD

The analysis accommodates only uniform loads on the diaphragm, but as many as five different values may be assigned along the loaded edge. In other words, the uniform load may be stepped. The following input required is the distributed wind load. The start and end coordinates of the uniformly distributed wind load, along with its magnitude are input. The resultant of the wind load is computed along with its location. It is also possible to input an accidental eccentricity that is taken as a percentage of the length of the applied wind load. Accidental eccentricity is usually applied as an addition and subtraction in seismic design. The purpose is to increase and decrease the moment generated by the applied load in order to obtain the worst loading conditions on the wall. The accidental eccentricity in wind design may be ignored, but is available to use if the designer chooses to use it.

2.2.3 STIFFNESS APPROACH

There are two main reasons behind using a stiffness approach to solve for the forces on each wall. The reasons are its ability to solve indeterminate structures, which most systems are, and its ease of programing. One way of viewing the stiffness method is via a superposition process as follows. Consider a simply supported beam with a uniform load $w(x)$, an applied moment M_0 , and unknown rotational degrees of freedom D_1 and D_2

shown on Figure 1. Superposition is used to decompose the original structure and loading into different cases whose total sum results in the original structure.

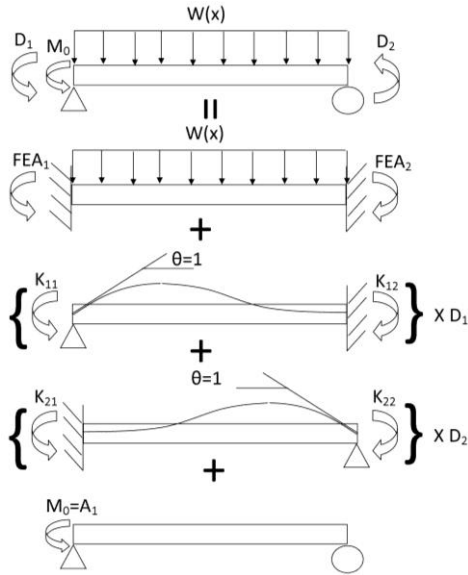


Figure 1: Simply Supported Beam with two Degrees of Freedom

The fixed-end actions, FEA, are the effects felt at the degrees of freedom as a result of loading between them, while holding those same degrees of freedom fixed. In this case the moment felt at degree of freedom 1, FEA₁, and 2, FEA₂. By releasing the first degree of freedom, D₁, and applying a moment that results in a unit rotation of one at that degree of freedom, the stiffness values for k_{11} and k_{12} are determined, where k_{11} and k_{12} are, respectively, the moment applied at degree of freedom 1 to cause a unit rotation and the reaction at degree of freedom 2--to hold the structure in equilibrium--due to the moment applied at degree of freedom 1. Degree of freedom 2 is released while degree of freedom 1 is held fixed. The process is repeated to find k_{21} and k_{22} . The applied moment, M_0 , being applied at degree of freedom 1 is an action corresponding with degree of freedom 1 and while there is no applied load corresponding with degree of freedom 2.

The degrees of freedom, \underline{D} , can be solved for in the general case by solving the equation $\mathbf{A} = \mathbf{FEA} + \mathbf{K D}$, where \mathbf{A} , \mathbf{FEA} , \mathbf{K} , and \mathbf{D} are matrices. In this example:

$$\mathbf{A} = \begin{bmatrix} M_0 \\ 0 \end{bmatrix} \quad \mathbf{FEA} = \begin{bmatrix} FEA_1 = \frac{w(x)L^2}{12} \\ FEA_2 = \frac{-w(x)L^2}{12} \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k_{11} = \frac{4EI}{L} & k_{12} = \frac{2EI}{L} \\ k_{21} = \frac{2EI}{L} & k_{22} = \frac{4EI}{L} \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

Another more relevant example is that of a rigid beam with two equally stiff springs supports, shown in Figure 2. The beam has two degrees of freedom at its center of rotation, the first, D_1 , a vertical translation and the second, D_2 , a rotation.

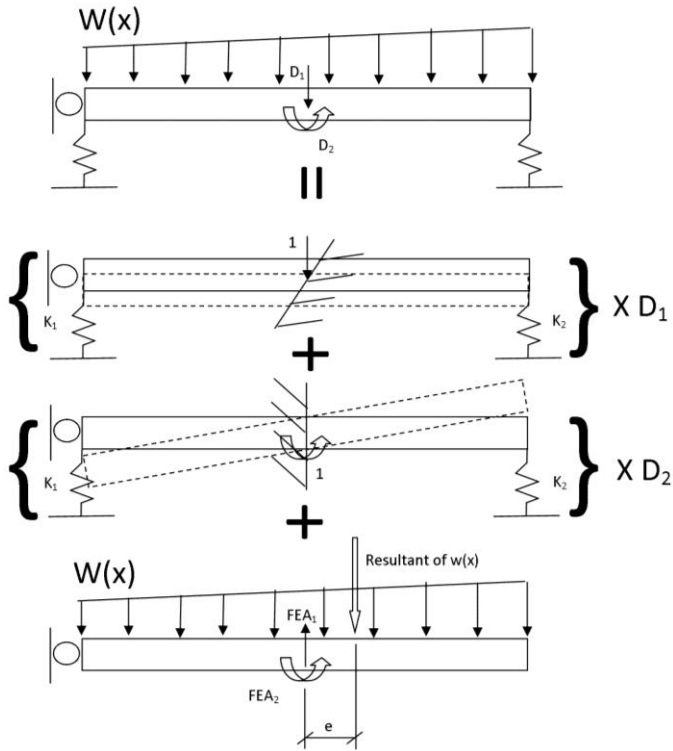


Figure 2: Rigid beam with two spring supports

In order to develop the equilibrium equations, the first degree of freedom is translated down one unit while restraining the rotational degree of freedom. The force generated in the direction of degree of freedom 1 by the springs under that unit displacement is taken as the k_{11} . The vertical translational degree of freedom is now restrained and then a unit rotation is applied. The summation of the force generated by each spring under that rotation multiplied by each ones distance to the center of rotation, location of degree of freedom, generates a moment. This moment is the k_{22} term. The fixed-end actions FEA_1 and FEA_2 are, respectively, the sum of the vertical forces and the moment generated by the eccentricity, e , between the resultant and the location of the degree of freedom. Again, this is solved by $\mathbf{A}=\mathbf{FEA}+\mathbf{K D}$ where in this example:

$$\mathbf{A} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \mathbf{FEA} = \begin{bmatrix} FEA_1 = Resultant \\ FEA_2 = (Resultant) * e \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k_{11} = k_1 + k_2 & k_{12} = 0 \\ k_{21} = 0 & k_{22} = k_1 \left(\frac{L}{2}\right)^2 + k_2 \left(\frac{L}{2}\right)^2 \end{bmatrix}$$

Notice that the off-diagonal terms are zero. k_{12} is zero because a vertical displacement causes no moment, since the springs are the same stiffness and spaced equally from the second degree of freedom. k_{21} is zero not only because the matrix must be symmetrical, but also because the force a unit rotation causes in both springs is equal and opposite.

Although this second example is simple, a rigid diaphragm can be modeled as a rigid beam. The only difference between the previous example and the diaphragm is a horizontal degree of freedom that is added to accommodate the horizontal springs representing walls. Figure 3 shows a diaphragm with springs representing walls; there are three independent degrees of freedom in this system: translations D_1 and D_2 and rotation D_3 , all assumed positive as shown. In this figure, each spring is parallel to the length

direction of the associated wall. The roller means that the wall has no stiffness perpendicular to its length, thereby not contributing any stiffness to the system. The stiffness associated with the horizontal degree of freedom is determined the same way as the vertical degree of freedom is, except it is now a horizontal unit displacement that is applied. The horizontal stiffness of the system, k_{11} , is the sum of the stiffness of all the horizontal springs. The vertical stiffness of the system, k_{22} , is the sum of the stiffness of all the vertical springs. The stiffness associated with the rotational degree of freedom is now calculated using the same procedure as before, but as a result of the two-dimensional nature of the problem there are other issues to consider.

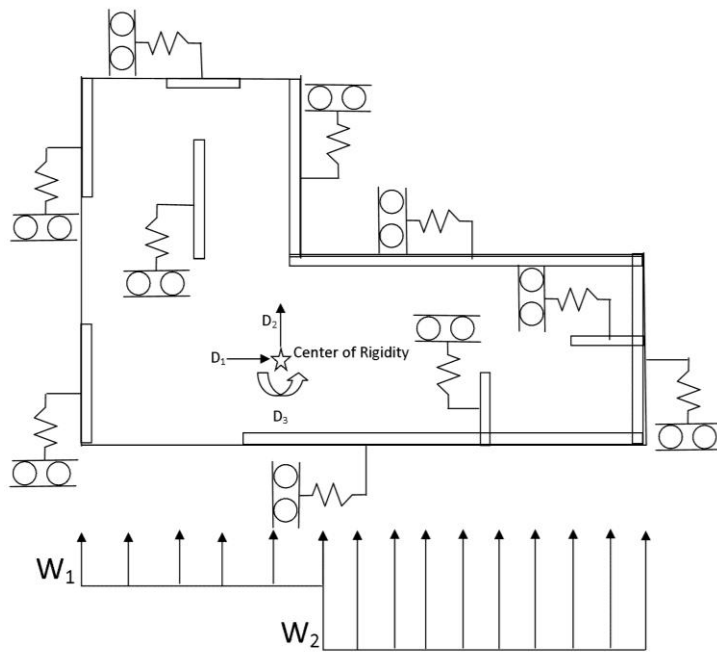


Figure 3: Diaphragm as rigid body with springs

The first issue for the 2-D problem is that walls that are further away from the point about which the diaphragm is rotating acquire larger displacements under rotations than walls that are closer to the point of rotation. What is done to find the displacement

and consequently the forces, and eventually moment, under such circumstances is to make use of the assumption of small rotations; that is, rotations are assumed to be sufficiently small as to cause wall displacements that are infinitesimal with respect to the dimensions of the wall. To explain this, Figure 4 shows the same spring before and after a rotation $\delta\theta$ with the radius R representing the distance to the center of rotation, CR. Because of the small rotation assumption, it is possible to assume that the line connecting the two spring locations is of length $R\delta\theta$ and that it is perpendicular to the radius R . From the geometry in Figure 4 it is evident that the vertical elongation of the spring is equal to $\Delta_y = |R\delta\theta \sin\phi|$. From the geometry it is also possible to determine that $\sin\phi = [x_1 - x_{CR}]/R$. Using the same procedure the horizontal displacement of horizontal springs can be found. For a horizontal spring the horizontal displacement is therefore $\Delta_x = |R\delta\theta \cos\phi|$, where $\cos\phi = [y_{CR} - y_1]/R$. The unit rotation that is applied is $\delta\theta$. The stiffness associated with the third degree of freedom is now determined after all the displacements have been found by taking the stiffness and displacement associated with each spring and multiplying it by the projected vertical distance, for horizontal springs, or the projected horizontal distance, for vertical springs, between the center of rotation and the original position. Figure 4 provides clarification on what the horizontal projected distance and vertical projected distance are.

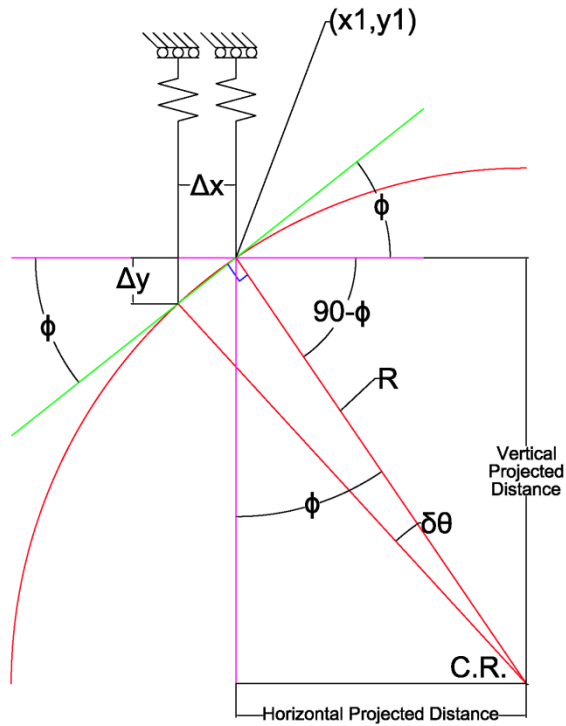


Figure 4: Displacement from unit rotation

The degrees of freedom are not placed at the centroid of the diaphragm. The reason for this being that since the diaphragm is modeled as rigid, the translation experienced by degree of freedom 1 or 2 and the rotation by degree of freedom 3 at any point in the diaphragm are the same. This is shown in Figure 5. As a result the degrees of freedom may be placed at any location. In the spreadsheet, the location where the degrees of freedom will be placed is the center of rigidity in order to be systematic.

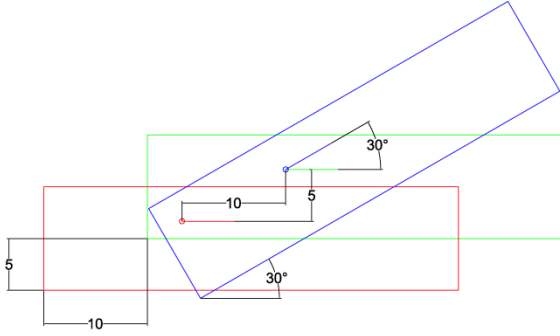


Figure 5: Equal translations and rotations in rigid body

The second issue for the 2-D problem is load application. The required fixed-end actions are found in a similar fashion as before. Consider the diaphragm shown in Figure 6, which has wind loads of two magnitudes, the higher one being, perhaps, a result of the diaphragm carrying wind loads from part of a first and second story. When the three degrees of freedom are restrained and the wind loads are applied, the sum of the horizontal forces is FEA_1 , the sum of the vertical forces is FEA_2 , and the moments caused by the two different wind loads are FEA_3 , all of which constitute the FEA vector. The degrees of freedom are found once more by solving $\mathbf{A} = \mathbf{FEA} + \mathbf{K} \mathbf{D}$ where in our example:

$$\mathbf{A} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{FEA} = \begin{bmatrix} FEA_1 = 0 \\ FEA_2 = -W_1 L_1 - W_2 L_2 \\ FEA_3 = W_1 L_1 e_1 - W_2 L_2 e_2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} k_{11} = \sum k_{ix} & 0 & 0 \\ 0 & k_{22} = \sum k_{iy} & 0 \\ 0 & 0 & k_{33} = \sum \Delta_{ix} k_{ix} d_{iH} + \sum \Delta_{iy} k_{iy} d_{iV} \end{bmatrix}$$

k_{ix} : Horizontal stiffness of individual spring

k_{iy} : Vertical stiffness of individual spring

d_{iH} : Horizontal distance to center of rigidity

d_{iV} : Vertical distance to center of rigidity

Δ_{ix} : Horizontal displacement caused by unit rotation

Δ_{iy} : Vertical displacement caused by unit rotation

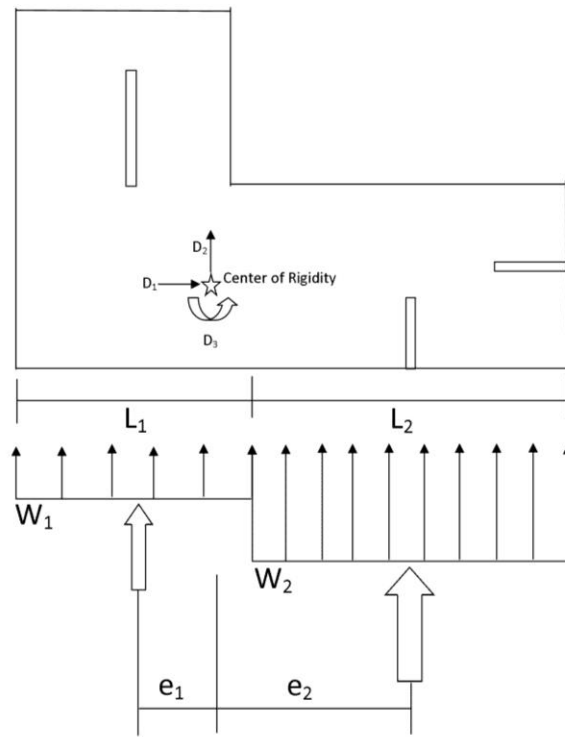


Figure 6: Fixed-end actions from wind load

Again, the off-diagonal terms are zero. This means that the degrees of freedom are uncoupled. Given the assumptions of this type of analysis, the off-diagonal terms always will be zero, which means that in the technical sense, a matrix formulation is not essential. However, given that wall displacements and forces are desired quantities, a matrix formulation allows for a very systematic approach.

2.2.4 ASSEMBLY OF STIFFNESS MATRIX AND LOAD VECTOR

Now that all the input is in place, the location of the springs representing the walls is placed at the centroid of each wall. The stiffness of the springs to be used in the assembly of the stiffness matrix are then computed by multiplying the equivalent length by some stiffness value, the resulting value will be used in the stiffness formulation and in calculating the final force in each spring. Theoretically it is not important what that stiffness value is since what matters in calculating their contribution to the stiffness matrix is their relative stiffnesses.

2.2.5 LOCATION OF SPRINGS AFTER LOADING

With the degrees of freedom solved for, the displacements of the springs must now be determined. First, what must be determined is the location of the springs after the rotational degree of freedom, D_3 , is applied to the rigid diaphragm. This is done by simply utilizing the same geometry, shown in Figure 4, which was used in the formulation of the stiffness corresponding to a unit rotation. Using a solver built into Excel[®], it is possible to accomplish the same task. What the solver is doing will be explained in the following section, but its purpose is to set up a framework for larger rotations. For small rotations, as assumed, both methods will yield the same result. The newly acquired spring locations are now translated vertically and horizontally by adding the vertical and horizontal degrees of freedom D_1 and D_2 respectively. The final displacement is then computed by subtracting the initial position, prior to any loading, from the final position. The total displacement of a spring is therefore:

$$\Delta_{xfinal} = RD_3 \cos\phi + D_1 = D_3[y_{CR} - y_1] + D_1 \quad (4)$$

$$\Delta_{yfinal} = RD_3 \sin\phi + D_2 = D_3[x_1 - x_{CR}] + D_2 \quad (5)$$

Final displacement is then multiplied by the stiffness of each spring determined earlier via the formulation of the stiffness matrix, in order to obtain the force on each wall. This

force can be divided by the actual length of the wall to determine the unit shear. By taking the force on each wall and dividing it by the stiffness of the wall, it is possible to check deflection serviceability requirements such as those in the 2012 International Residential Code Table R301.7 (International Code Council, 2011).

2.2.6 SOLVER

As mentioned earlier, the solver's purpose is to set up the frame work for larger rotations to be considered in future work. The solver considers four constraints that must be satisfied in order for an accurate solution to be found. Those constraints are:

1. No spring can be further away from the center of rigidity after the applied rotation then prior to the applied rotation
2. The straight line distance traveled by the spring under only rotation, found by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, must be equal to the distance h defined by:

$$h = \sqrt{(R - |R \cos \theta|)^2 + (R \sin \theta)^2} \quad (6)$$

where,

R= distance from original spring location to center of rigidity of the diaphragm

3. Equilibrium horizontally must be maintained
4. Equilibrium vertically must be maintained

The first two constraints are geometric constraints that are applicable to the movement associated with the rotational degree of freedom prior the implementation of the horizontal and vertical translational degrees of freedom. The first constraint is checked by simply calculating the straight line distance from the initial spring position to the center of rigidity and comparing it to the straight line distance from the location after

rotation to the center of rigidity. If it is assumed that the rotations will be small then constraint 2 can be simplified to $h=R\theta$. The simplification would assume that the straight-line distance traveled by the spring is equal to an arch segment, s , that can be defined as $s=R\theta$. The more complicated expression is the result of the direct application of geometry for any size angle.

The final two are associated with the final location of the springs after the rotation and translational degrees of freedom are applied, since it may be possible to obtain a solution that satisfies the geometric constraints but not satisfy equilibrium of the system. The locations of the springs are determined by the solver built into Excel[®]. The solver built into Excel[®] is a Generalized Reduced Gradient, GRG2, Algorithm for optimizing linear and nonlinear problems (Microsoft, 2011). The solver takes into consideration the four constraints stated above and solves for the point that best satisfies all the constraints. Because the optimization process, it is best to provide an initial guess that is close to the final answer to ensure a convergence on the solution.

The initial guess of the coordinates is provided by assuming that the rotation is small. The formulas are the following:

$$x_1 = D_3[y_{CR} - y_1] + x_0 \quad (7)$$

$$y_1 = D_3[x_1 - x_{CR}] + y_0 \quad (8)$$

Notice that if the rotations are small the solver will yield coordinates equal to the non-solver approach. The initial guess will result in values close enough to the solution to provide the solver with a good starting point.

2.3 Limitations

It is assumed that all the perimeter of the diaphragm consists of lines that are parallel or perpendicular to one another. All walls on the perimeter and on the interior of the diaphragm must be parallel to the outer sides. Diagonal wall are prohibited unless

they are modeled as a combined set of perpendicular walls that would result in the same stiffness. The spreadsheet has been limited to accommodate 15 walls.

Only uniform wind loads may be applied. The location of the resultant is calculated assuming the wind load is uniform. However, the spreadsheet allows for five different uniform wind loads, so at locations where wind loads change, like a second story, they can be accounted for.

As a result of the assumptions of small rotations and translations, when either get large the assumption of the direction of the resistance provided by the walls is no longer valid. Large rotations or displacements indicate that the walls are not providing sufficient stiffness for the given loads.

The “Final Displacement” column in the spreadsheet cannot be taken as the displacement of the wall. In order to obtain the final displacement of a particular wall take the load from the spreadsheet and divided it by that particular wall’s stiffness.

The last item is a warning. As mentioned in the **2.2.1 Wall Location and Stiffness** section, shear walls are sensitive to the nail slip and anchorage term. Because of this, it may be desirable to perform a second iteration in order to update the initially assumed induced unit shear force to the newly calculated one and obtain a new wall stiffness, but only after sound engineering judgment has been used in the first iteration. Additional iterations should be exercised with caution since it is possible that they may not converge as a result of the nail slip term, e_n , and anchorage term, Δ_a , especially if those terms are not updated for the new induced unit stiffness.

3. COMMENTS

Although the Excel[®] Spreadsheet was designed with wood shear walls in mind, it is possible to use the spreadsheet with walls of other materials, such as masonry, provided the relative stiffness among the walls is accurate. It is also possible to input a steel frame as some designers do, but as with a masonry wall, the total stiffness of the frame must be converted into a wall with an equivalent stiffness to the frame.

Future possible work on this spreadsheet would include the ability to include diagonal walls, allowance for more walls, the removal of the small rotation assumption, as well as the ability to find stiffness for walls of other materials.

Appendix A is a simple user's guide to the spreadsheet while Appendix B contains examples.

This Excel[®] Spreadsheet is not intended to replace sound engineering judgment or more sophisticated forms of analysis. It is only a tool to encourage and facilitate rigid diaphragm analysis as part of a more complete design process.

Appendix A

This section provides a user's guide to the Excel[®] Spreadsheet.

Step 1

Input start and end coordinates of each shear wall in feet.

Wall Location (ft.)				
No.	x1	y1	x2	y2
1	0	6.5	0	18.5
2	50	9.5	50	21.5
3	20	0	30	0
4	20	25	30	25
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

Optional Step

If using wood shear walls, it is possible to estimate their apparent stiffness in the spreadsheet. Input the values requested in the green area provided. The induced unit shear can be approximated as the allowable capacity of the wood framed shear walls, or the force obtained by a flexible analysis, or whatever unit shear the designer judges to be appropriate. The bottom value titled “wall stiffness” will be input into the next step.

Shear Wall Information (see Wind & Seismic Special Design Provisions for Wind and Seismic Section C4.3.2)

These values are for a given shear wall length	
v, plf, assumed induced unit shear	490
h, ft, Shear wall height	8
E, psi, modulus of Elasticity of end posts	1600000
A, in ² , Area of end posts cross-section	12.25
b, ft, Shear Wall Length	12
G _{vt} , lb/in., Shear Stiffness of panel depth	83500
Δa, in., total vertical elongation of wall anchorage system at the induced unit shear in the shear wall (including fastener slip, device elongation, etc.)	0.061
e _{nv} , in., nail slip	0.0218
bending deflection, in.	0.008533333
shear deflection, in.	0.046946108
nail slip, in.	0.1308
tie-down nail slip, in.	0.040666667
total deflection, in.	0.226946108
wall stiffness, lb/in. per foot of shear wall	2159.102902

Step 2

Input either relative or actual wall stiffness per foot of wall in the “Relative Stiffness” column. If the optional step performed above was done, this is the location where that stiffness should be placed.

Relative Stiffness
2159
1960
2140
2140
1
1
1
1
1
1
1
1
1
1
1
1

Step 3

Input uniformly distributed wind load. Positive is up or to the right

Distributed wind load						
	x1 (ft.)	y1	x2	y2	Load, lb/ft load applied up or to the right is positive	Accidental Eccentricity, %
1	0	0	50	0	150	0
2						
3						
4						
5						

Step 4

Copy “Values to be used as Initial guess”

Values to be used as Initial Guess	
0.003018764	-0.0057458
600.0030188	0.0063292
0.003018764	-0.0057458
-0.00301876	299.99425
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

Step 5

The values copied will be input into “Location after Rotation”. Perform “Paste Special” and select “Values” in order to only paste the values into the cells. Values could also be typed in directly.

	Location after rotation (Initial Guess)	
	x	y
1	0.003018764	-0.005745839
2	600.0030188	0.006329218
3	0.003018764	-0.005745839
4	-0.003018764	299.9942542
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	0	0
15	0	0

Step 6

From the data tab select “Solver” and press “Solve”. The initial guess should be close to the final answer. Depending on your version of Excel[®] the solver parameter window could appear different.

Solver Parameters

Set Objective:

To: ☐ Max ☐ Min ☒ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- \$D\$102:\$D\$116 <= \$E\$118
- \$D\$102:\$D\$116 >= -\$E\$118
- \$G\$84:\$G\$98 <= \$I\$82
- \$G\$84:\$G\$98 >= -\$I\$82
- \$P\$119 = 0

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

Step 7

Obtain wall forces. Positive is up or to the right.

Reaction Force by wall, lb (Positive is to the right or up)			force per unit length(plf)	
Wall no.	x	y	x	y
1	0	-3782.302317	0	-315.19
2	0	-3717.697683	0	-309.81
3	-64.605	0	-6.4605	0
4	64.6046	0	6.46046	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0

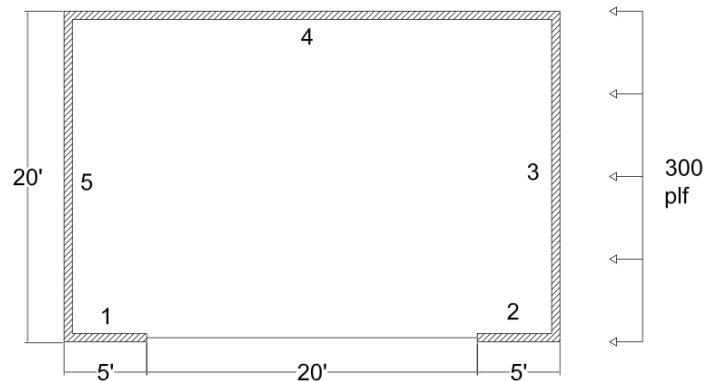
Appendix B

Three examples will be presented

EXAMPLE 1

This example's purpose is to introduce the overall procedure associated with the spreadsheet.

Problem: Solve for the forces on each wall. Assume each wall's stiffness is proportional to its length.



Solution:

Input wall locations and input relative stiffness of 1

Wall Location (ft.)					Wall Length Horz.Proj.	Wall Length Vert. Proj.	Total Length (ft.)	Relative Stiffness
No.	x1	y1	x2	y2				
1	0	0	5	0	5	0	5	1
2	25	0	30	0	5	0	5	1
3	30	0	30	20	0	20	20	1
4	0	20	30	20	30	0	30	1
5	0	0	0	20	0	20	20	1
6					0	0	0	1
7					0	0	0	1
8					0	0	0	1
9					0	0	0	1
10					0	0	0	1
11					0	0	0	1
12					0	0	0	1
13					0	0	0	1
14					0	0	0	1
15					0	0	0	1

Input wind load

Distributed wind load						
	x1 (ft.)	y1	x2	y2	Load, lb/ft load applied up or to the right is positive	Accidental Eccentricity, %
1	30	0	30	20	-300	0
2						
3						
4						
5						

Copy values for initial guess and past values into location after rotation

Wall no.	Values to be used as Initial Guess			Location after rotation (Initial Guess)	
1	29.9826429	0.0144643		x	y
2	329.982643	-0.014464		1	29.98264286
3	359.994214	119.98264		2	329.9826429
4	180.005786	240		3	359.9942143
5	-0.00578571	120.01736		4	180.0057857
6	0	0		5	-0.005785714
7	0	0		6	0
8	0	0		7	0
9	0	0		8	0
10	0	0		9	0
11	0	0		10	0
12	0	0		11	0
13	0	0		12	0
14	0	0		13	0
15	0	0		14	0
				15	0

Run solver

Solver Parameters

Set Objective:

To: ☐ Max ☐ Min ☒ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

-
-
-
-
-

☐ Make Unconstrained Variables Non-Negative

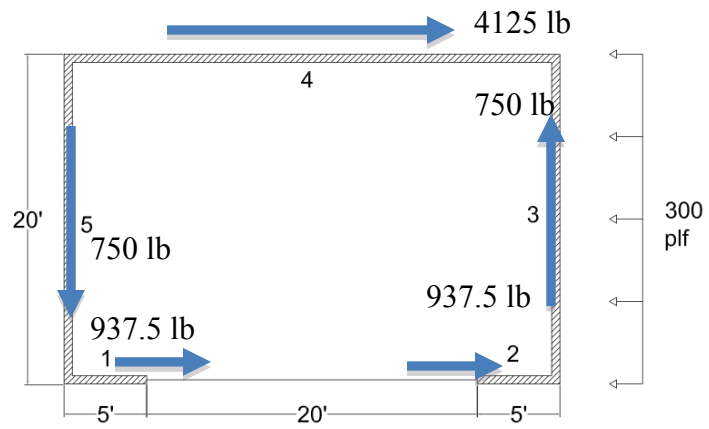
Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

Obtain values for force on each wall

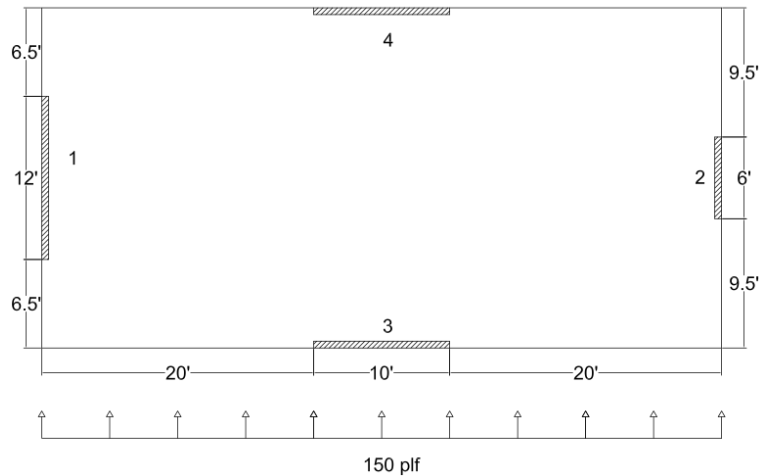
Reaction Force by wall, lb (Positive is to the right or up)			force per unit length(plf)	
Wall no.	x	y	x	y
1	937.5	0	187.5	0
2	937.5	0	187.5	0
3	0	750	0	37.5
4	4125	0	137.5	0
5	0	-750	0	-37.5
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0



EXAMPLE 2

The purpose of this example is to introduce the approximation of the wall stiffness

Problem: Solve for the forces on the shear walls. Shear wall vertical chords consist of one 8' tall 4X4 DF no.2. The structural sheathing is 7/16" OSB with 8d nails @ 4" o.c.



Solution:

From the NDS Table 1B (The American Wood Council, 2010) Area of chords is 12.25 in².

From NDS Table 4A (The American Wood Council, 2010) the modulus of elasticity is 1,600,000 psi.

From the wind and seismic provisions Table 4.3 (The American Wood Council, 2010) the allowable unit shear capacity in the panel, ASD, is 490 plf.

Shear Stiffness, $G_v t_v$, is 83,500 lb/in. from wind and seismic provisions Table C4.2.2A (The American Wood Council, 2010).

To calculate the wall stiffness, assume allowable unit shear is on wall.

Load per nail, $V_{\text{nail}} = 490 \text{ plf} / 3 \text{ nails/ft} = 163.333 \text{ lb/nail}$

Fastener slip, $e_n = 1.2[V_{\text{nail}}/616]^{3.018} = .0218$ in. from Wind and Seismic provisions Table C4.2.2D (The American Wood Council, 2010)

Vertical elongation of Anchorage system, Δ_a , shall be determined from the manufacturer's literature. In this example assume $\Delta_a = 0.03$ in. for every 6' of wall.

Locate Walls

Wall Location (ft.)				
No.	x1	y1	x2	y2
1	0	6.5	0	18.5
2	50	9.5	50	15.5
3	20	0	30	0
4	20	25	30	25
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

Obtain approximate wall stiffness

Shear Wall information (see Wind & Seismic Special Design Provisions for Wind and Seismic Section C4.3.2)

These values are for a given shear wall length	
v, plf, assumed induced unit shear	490
h, ft, Shear wall height	8
E, psi, modulus of Elasticity of end posts	1600000
A, in ² , Area of end posts cross-section	12.25
b, ft, Shear Wall Length	12
G _{v,t} , lb/in., Shear Stiffness of panel depth	83500
Δa, in., total vertical elongation of wall anchorage system at the induced unit shear in the shear wall(including fastener slip, device elongation, etc.)	0.06
e _n , in., nail slip	0.0218
bending deflection, in.	0.008533333
shear deflection, in.	0.046946108
nail slip, in.	0.1308
tie-down nail slip, in.	0.04
total deflection, in.	0.226279441
wall stiffness,lb/in. per foot of shear wall	2165.464072

Input wall stiffness

Relative Stiffness
2165.464072
2086.768921
2149.253763
2149.253763
1
1
1
1
1
1
1
1
1
1
1
1

Input distributed wind load

Distributed wind load						
	x1 (ft.)	y1	x2	y2	Load, lb/ft load applied up or to the right is positive	Accidental Eccentricity, %
1	0	0	50	0	150	0
2						
3						
4						
5						

Copy values for initial guess and “paste values” into “Location after rotation”

Wall no.	Values to be used as Initial Guess		Location after rotation (Initial Guess)	
	x	y	x	y
1	0	149.961711	0	149.9617112
2	600	150.079465	600	150.0794655
3	300.0294386	0.02058833	300.0294386	0.020588329
4	299.9705614	300.020588	299.9705614	300.0205883
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0

Run Solver

Solver Parameters

Set Objective:

To: ☐ Max ☐ Min ☒ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- <=
- >=
- <=
- >=
- = 0

☐ Make Unconstrained Variables Non-Negative

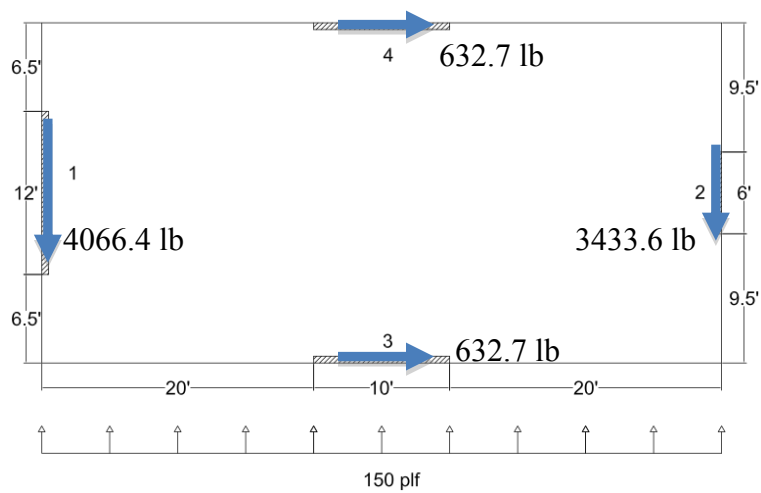
Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

Obtain wall forces

Reaction Force by wall, lb (Positive is to the right or up)			force per unit length(plf)	
Wall no.	x	y	x	y
1	0	-4066.354739	0	-338.863
2	0	-3433.645261	0	-572.274
3	-632.709	0	-63.2709	0
4	632.7095	0	63.27095	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0

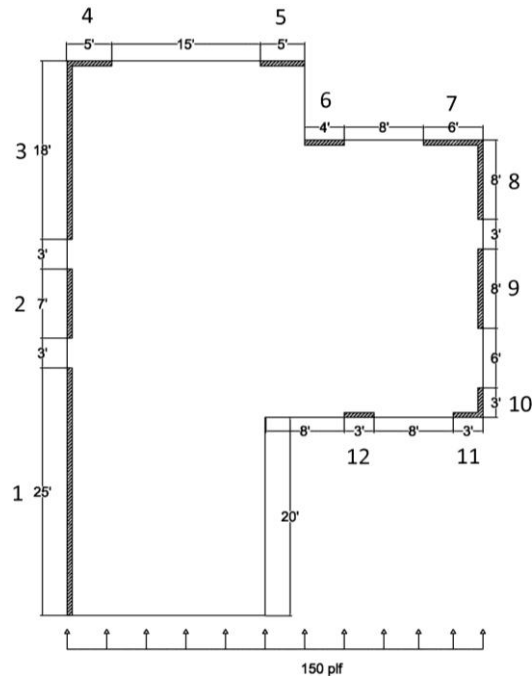


If desired, a second iteration can be done with updated wall stiffness approximations.

EXAMPLE 3

The purpose of this example is to illustrate its use on a typical house.

Problem: Solve for the forces on the shear walls. Shear wall vertical chords consist of one 8' tall 4X4 DF no.2. The structural sheathing is 7/16" OSB with 8d nails @ 4" o.c.



Solution:

From the NDS Table 1B (The American Wood Council, 2010) Area of chords is 12.25 in².

From NDS Table 4A (The American Wood Council, 2010) the modulus of elasticity is 1,600,000 psi.

From the wind and seismic provisions Table 4.3 (The American Wood Council, 2010) the allowable unit shear capacity in the panel, ASD, is 490 plf.

Shear Stiffness, $G_v t_v$, is 83,500 lb/in. from wind and seismic provisions Table C4.2.2A (The American Wood Council, 2010).

To calculate the wall stiffness, assume a unit shear of 150 plf is on wall.

Load per nail, $V_{\text{nail}} = 150 \text{ plf} / 3 \text{ nails/ft} = 50 \text{ lb/nail}$

Fastener slip, $e_n = 1.2[V_{\text{nail}}/616]^{3.018} = .00061 \text{ in.}$ from Wind and Seismic provisions Table C4.2.2D (The American Wood Council, 2010)

Vertical elongation of Anchorage system, Δ_a , shall be determined from the manufacturer's literature. In this example assume $\Delta_a = 0.047v_b/4500$

Locate Walls

<div>ONLY MODIFY WHAT IS IN GREEN</div> <div>Wall Location (ft.)</div>				
No.	x1	y1	x2	y2
1	0	0	0	25
2	0	28	0	35
3	0	38	0	56
4	0	56	5	56
5	20	56	25	56
6	25	48	29	48
7	37	48	43	48
8	43	48	43	40
9	43	37	43	29
10	43	23	43	20
11	43	20	40	20
12	32	20	29	20
13				
14				
15				

Obtain approximate wall stiffness

Shear Wall information (see Wind & Seismic Special Design Provisions
for Wind and Seismic Section C4.3.2)

**These values are for a given shear wall
length**

v, plf, assumed induced unit shear	150
h, ft, Shear wall height	8
E, psi, modulus of Elasticity of end posts	1600000
A, in ² , Area of end posts cross-section	12.25
b, ft, Shear Wall Length	3
G _{vt} , lb/in., Shear Stiffness of panel depth	83500
Δa, in., total vertical elongation of wall anchorage system at the induced unit shear in the shear wall(including fastener slip, device elongation, etc.)	0.0047
e _n , in., nail slip	0.00061
bending deflection, in.	0.01044898
shear deflection, in.	0.014371257
nail slip, in.	0.00366
tie-down nail slip, in.	0.012533333
total deflection, in.	0.04101357
wall stiffness,lb/in. per foot of shear wall	3657.326063

Input wall stiffness


Relative Stiffness

4714.243258
4280.48904
4643.087789
4072.326852
4072.326852
3906.115167
4191.222502
4349.974828
4349.974828
3657.636063
3657.636063
3657.636063
1
1
1

Input distributed wind load

Distributed wind load						
	x1 (ft.)	y1	x2	y2	Load, lb/ft load applied up or to the right is positive	Accidental Eccentricity, %
1	0	0	43	0	150	0
2						
3						
4						
5						

Copy values for initial guess and “paste values” into “Location after rotation”

Wall no.	Values to be used as Initial Guess			Location after rotation (Initial Guess)		
1	0.021826099	149.99259		x	y	
2	0.009147782	377.99259		1	0.021826099	149.9925894
3	-0.001195054	563.99259		2	0.009147782	377.9925894
4	29.99279943	671.99426		3	-0.001195054	563.9925894
5	269.9927994	672.0076		4	29.99279943	671.9942576
6	323.9981377	576.01061		5	269.9927994	672.0076032
7	479.9981377	576.01928		6	323.9981377	576.0106059
8	516.0008068	528.02128		7	479.9981377	576.0192806
9	516.0081469	396.02128		8	516.0008068	528.0212824
10	516.0158206	258.02128		9	516.0081469	396.0212824
11	498.0168215	240.02028		10	516.0158206	258.0212824
12	366.0168215	240.01294		11	498.0168215	240.0202815
13	0	0		12	366.0168215	240.0129414
14	0	0		13	0	0
15	0	0		14	0	0
	0	0	15	0	0	

Run Solver

Solver Parameters

Set Objective:

To: ☐ Max ☐ Min ☒ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- <=
- >=
- <=
- >=
- = 0

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

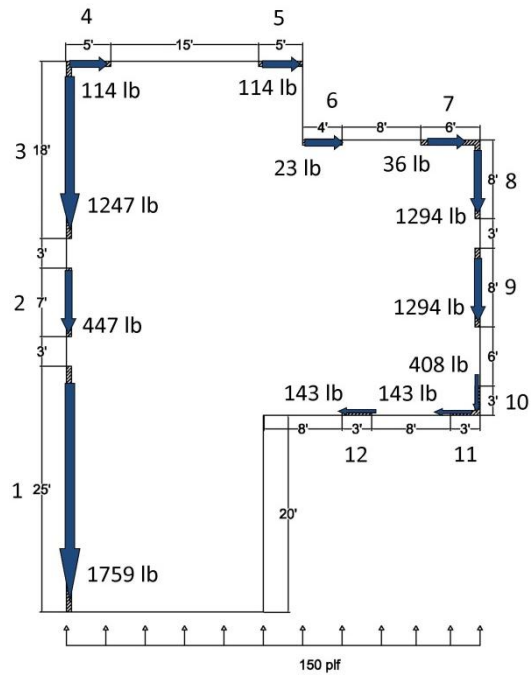
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options

Buttons: Help, Solve, Close

Obtain wall forces

Reaction Force by wall, lb (Positive is to the right or up)			force per unit length(plf)	
Wall no.	x	y	x	y
1	0	-1759.125743	0	-70.365
2	0	-447.2355485	0	-63.8908
3	0	-1247.453273	0	-69.303
4	113.745	0	22.7489	0
5	113.745	0	22.7489	0
6	22.5743	0	5.64358	0
7	36.333	0	6.0555	0
8	0	-1294.072329	0	-161.759
9	0	-1294.072329	0	-161.759
10	0	-408.0407767	0	-136.014
11	-143.198	0	-47.7328	0
12	-143.198	0	-47.7328	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0



If desired, a second iteration can be done with updated wall stiffness approximations.

References

- International Code Council. (2011, May). 2012 International Building Code. Country Club Hills, Illinois: International Code Council.
- Microsoft. (2011, September 19). *Microsoft*. Retrieved from Microsoft support: <http://support.microsoft.com/kb/82890>
- The American Wood Council. (2010). NDS. *National Design Specification for Wood Construction, 2005 Edition*. Washington, District of Columbia, United States of America: American Forest and Paper Association.
- The American Wood Council. (2010, February). Wind & Seismic. *Special Design Provisions For Wind And Seismic, 2005 Edition*. Washington, District of Columbia, United States of America: American Forest & Paper Association, Inc.