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Applications to Stellar Dynamics of a One-parameter Family of Triple Close Approaches

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Summary. This is the third and last of a series of papers dedicated to the effect of triple close approaches on the evolution of stellar systems. Previously obtained analytical and numerical results are applied to several astronomical models of triplets formed by Sun-like stars, white dwarfs, neutron stars, and galaxies.

Key words: three-body problem — triple close approach — escape — instability

Introduction

First the pertinent results of earlier papers (Szebehely, 1974a,b, referred to as Papers I and II) are shortly summarized. This is followed by the description of four astronomical models. Next, from the previously found one-parameter family, those members are selected which satisfy realistic astronomical conditions. Finally, the selected members are matched to the four models.

Previous studies using individual orbits or statistically established families exist in the literature (Agekian, 1967; Szebehely, 1967). Special dynamical models of considerable interest, and closely related to this problem, were investigated by Nahon (1973) and by Waldvogel (1977). The present study allows the continuous adjustment and matching of physical parameters because it is based on a family rather than on individual examples.

A preliminary short announcement of some of these results, without details, was offered by this writer in a review of the dynamics of triple systems (Szebehely, 1977).

Summary of Previous Results

In Paper I it is shown that (in the limit) the escape velocity (v_∞) of a member of a triple system is related to the semi-major axis of the binary (a_∞) left behind by $a_\infty v_\infty^2 = 2/3$, for a slightly perturbed asymmetric family of triple encounters. The initial conditions for all members of the family depend on a single parameter, the perturbing velocity v_0. All cases start from an equilateral triangle of unit sides. The three initial velocity vectors are parallel with one side of the triangle. The three masses are equal and in the original non-dimensional system $m_1 = m_2 = m_3 = 1$. The initial velocity vector of m_3 is parallel with side m_1m_2 and its magnitude is v_0. The initial velocity vectors ($v_0/2$) of m_1 and m_2 point in the opposite direction. These initial conditions are shown in Fig. 1 of Paper II and are described analytically in Paper I. The asymmetry and the closeness of the triple close approaches are controlled by v_0 which varies between 0.2 and 10^{-10}. The high side of the range corresponds to an ejection, the low side ($v_0 = 10^{-10}$) gives a high-velocity escape. For $0 < v_0 \leqq 10^{-2}$ the above-quoted limit-formula is applicable with satisfactory accuracy. To simplify the notation in the sequel we write $a = a_\infty$ and $v = v_\infty$ for the asymptotic values of the semi-major axis and of the hyperbolic escape velocity.

The motion is a contraction initially (see Paper II) and the first close approach occurs between m_1 and m_3. This distance, $\min r_{13} = r_{13}^*$, is the smallest distance occurring during the whole motion for $0 < v_0 < 10^{-1}$. Since in the range $0.1 \leqq v_0 \leqq 0.2$ both ejections and escapes occur, we use only results obtained in the lower velocity range ($10^{-2} \geqq v_0 \geqq 10^{-10}$) and consequently base the computations on the quantity r_{13}^* which for simplicity will be denoted by r.

In Paper II it was shown that $a \cong 12r$ for the members of the family, therefore $rv^2 \cong 1/18$. This last result means that if in the non-dimensional system used in Papers I and II, a minimum distance is selected, the velocity of escape may be computed. The member of the family of escape orbits corresponding to this preselected r is associated with a given value of the parameter v_0, however, this parameter is not used in the present study since it may be replaced by the physically more meaningful distance, r. The eccentricity of the binary formed is also given in Paper II as $e = 0.6$, independently of v_0 or of r.

The system reaches minimum size or rather minimum moment of inertia in approximately $\pi/\sqrt{24} = 0.6413$ non-dimensional time-units.

The Four Astronomical Models

In the non-dimensional system all computations are performed with $GM_mT_m^2/L_m^3 = 1$, consequently, the unit of time is $T_m = L_m^{3/2}(GM_m)^{-1/2}$ and the unit of velocity is $V_m = (GM_m/L_m)^{1/2}$, if the units of mass and length are M_m and L_m, with G as the constant of gravity. The subscript m refers to "model".

(i) The first model consists of three stars of solar mass, $M_m = M_\odot$ placed at the apices of an equilateral triangle at $L_m = 1$ pc apart. The unit of time is $T_m = 1.5\ 10^7$ yr and the unit of velocity is $V_m = 0.0655$ km s^{-1}.

(ii) Two models consisting of white dwarfs are studied. First (ii-a) three white dwarfs with solar masses are placed at 1 pc distances as in model (i). Note, however, that the radius of the participating bodies in case (i) is $R_\odot$ while in case (ii-a) $R_b = 0.0068\ R_\odot$, following Chandrasekhar's estimate. Consequently, in case (ii-a) much closer approaches are allowed without significant tidal effects than in case (i). The units are the same in cases (i) and (ii-a). If the model consists of three 40 Eridani-B-like stars (ii-b), then $M_m = 0.43\ M_\odot$ and $R_b = 0.016\ R_\odot$. The units are $T_m = 2.27\ 10^7$ yr and $V_m = 0.043$ km s^{-1}.

(iii) In the third model we place three neutron stars of solar masses at 1 pc distances, assuming for their radii $R_b = 10$ km. The units are the same as in case (i).

(iv) The fourth model consists of three galaxies of masses $M_m = 10^{10}\ M_\odot$ placed at $L_m = 100$ kpc distances. The units are $T_m = 4.71\ 10^9$ yr and $V_m = 20.7$ km s^{-1}. For the size of the galaxy $R_b = 5$ kpc is used.

Any other desired model might be established without difficulties. Note that in the following discussion lower case letters will denote the dimensionless quantities used in Papers I and II, while capitals will be reserved for dimensional quantities. The relations between these symbols are $R = rL_m$, $V = vV_m$, $T = tT_m$. In addition, it is expedient to introduce the symbols $\rho = R_b/R_\odot$, where R_b is the radius of the bodies participating in the motion, as well as $\mu = M_m/M_\odot$ and $\lambda = R/(2R_b)$, which will be used in the next section.

General Approach

The purpose of the paper is to find the principal characteristics of the models described in the previous section. Such characteristics are the velocity of escape (or ejection), the semi-major axis and eccentricity of the binary left behind and the time necessary to reach maximum contraction. The method of selection of the proper member of the family is based on the choice of a minimum distance from physical considerations. This minimum distance, R during a realistic, physically possible motion must be considerably larger than $2R_b$. Since $R = 2\lambda R_b$, the bodies touch when $\lambda = 1$. Clearly, such behavior violates our original dynamical model where tidal effects are excluded. If $\lambda = 100$, the tidal effects are of the same order of magnitude as between the Earth and the Moon. For the purposes of this paper, we will select the following three values for λ: $\lambda_1 = 10$ which is admittedly rather low; $\lambda = 100$ corresponding to 1 AU if $R_b = R_\odot$; and $\lambda_3 = 1000$.

At this point some simple results are offered.

i) the semi-major axis of the binary formed is $A = 12R = 24\lambda R_b$,

ii) the velocity of escape is

$$V_{esc} = \frac{1}{6}\left(\frac{\mu}{\lambda\rho}\right)^{1/2}\left(\frac{GM_\odot}{R_\odot}\right)^{1/2} = 72.74\left(\frac{\mu}{\lambda\rho}\right)^{1/2} (\text{km s}^{-1}),$$

iii) the highest velocity occurring at the first close approach is $V_{max} = 8V_{esc}$.

The first result (i) was mentioned before as $a = 12r$, i.e., the semi-major axis of the binary is 12 times the distance occurring at the first close binary approach. The second result (ii) is obtained from the relation, $v^2r = 1/18$ by substituting the definitions given at the end of the previous section. The third result (iii) concerning the maximum velocity was established by numerical integration, see Paper II.

Applications

In the first model $\mu = \rho = 1$, since this is the solar model with $M = M_\odot$ and $R_b = R_\odot$. Using $\lambda_1 = 10$, the closest distance is twenty solar radii ($R = 20\ R_\odot$), the semi-major axis of the binary is 240 $R_\odot$, the velocity of escape is 23 km s^{-1} and the highest velocity is 184 km s^{-1}.

Table 1 summarizes the results for all cases. Note that the highest escape and maximum velocities occur in case (iii), that is for the model of three neutron stars. The maximum velocity corresponds to approximately 16% of the velocity of light.

Table 1. Applications to models of triple systems

Model	Description	λ	R	A	V_{esc} km/sec	V_{max} km/sec
(i)	3 suns,	10	$20\ R_\odot = 0.1$ AU	$240\ R_\odot = 1.2$ AU	23	184
	$\mu = \rho = 1$,	100	$200\ R_\odot = 1$ AU	$2{,}400\ R_\odot = 12$ AU	7.3	58.2
	$T_c \cong 10^7$ yr	1,000	$2{,}000\ R_\odot = 10$ AU	$24{,}000\ R_\odot = 120$ AU	2.3	18.4
(ii-a)	3 white dwarfs	10	$0.136\ R_\odot = 9.45\ 10^4$ km	$1.13\ R_\odot = 8.2\ 10^{-3}$ AU	279	2,232
	with solar	100	$1.36\ R_\odot = 9.45\ 10^5$ km	$11.3\ R_\odot = 8.2\ 10^{-2}$ AU	88.2	706
	masses, $\mu = 1$,					
	$\rho = 0.0068$,	1,000	$13.6\ R_\odot = 9.45\ 10^6$ km	$113\ R_\odot = 0.82$ AU	27.9	223
	$T_c = 10^7$ yr					
(ii-b)	3 white dwarfs	10	$0.32\ R_\odot = 1.6\ 10^{-3}$ AU	$3.84\ R_\odot = 1.92\ 10^{-2}$ AU	119.25	954
	like 40 Eridani-B,	100	$3.2\ R_\odot = 1.6\ 10^{-2}$ AU	$38.4\ R_\odot = 0.19$ AU	37.7	302
	$\mu = 0.43$,	1,000	$32\ R_\odot = 0.16$ AU	$384\ R_\odot = 1.9$ AU	11.93	95.4
	$\rho = 0.016$,					
	$T_c = 1.45\ 10^7$ yr					
(iii)	3 neutron	10	200 km	$2.4\ 10^3$ km	6,062	48,493
	stars, $\mu = 1$,	100	$2\ 10^3$ km	$2.4\ 10^4$ km	1,917	15,336
	$\rho = 1.44\ 10^{-5}$,	1,000	$2\ 10^4$ km	$2.4\ 10^5$ km	606	4,849
	$T_c = 10^7$ yr					
(iv)	3 galaxies,	10	100 kpc	1,200 kpc	1,084	8,670
	$\mu = 10^{10}$,	100	1,000 kpc	$1.2\ 10^4$ kpc	343	2,742
	$\rho = 2.22\ 10^{11}$,	1,000	10^4 kpc	$1.2\ 10^5$ kpc	108	867
	$T_c = 3\ 10^6$ yr					

The time necessary from the beginning of the motion to the occurrence of the escape in all cases is approximately $t_c = 0.64$ time units (for many more significant figures, see Paper II). Therefore, the escape times are determined by the time-units applicable to the various cases. For instance, for case (i) we have $T_m = 1.5\ 10^7$ yr, therefore the escape time is $T_c = 0.96\ 10^7$ yr.

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