

NUMERICAL MODELLING OF WAX DEPOSITION USING SPH

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ABSTRACT

Phase change and deposition of solid particles in liquid flows are undesirable in some natural and industrial processes and can be hazardous in some cases. Considering the difficulties involved in petroleum exploitation in deep waters, precipitation and deposition of wax crystals in oil and gas pipelines are real concerns. This is due to the fact that during production, oil is exposed to more intense temperature gradients (the dominant mechanism in wax deposition) and as a result its heavier components migrate to the solid phase called deposit. According to various studies, wax layer deposited on a surface behaves like a gel with a complicated morphology. The gel behaves like a moving boundary porous media surrounding an internal flow. In order to model the presence of wax formation on the internal walls of the channel, a source term is added to the Darcy equation. It is assumed that other wax formation mechanisms such as shear dispersion and Brownian diffusion are neglected. Wax deposition simulation in channel, involves solution of the continuity, momentum, energy, species transport, and phase equilibrium equations. In the present study, the governing equations are solved using Smoothed Particle Hydrodynamics (SPH) method. Also, an algorithm for solid-liquid phase equilibrium calculations is proposed and compared with the experimental Wax Appearance Temperature (WAT). Both temperature and species distributions are compared with analytical solutions available for some simplified cases.

1. INTRODUCTION

Crystallization and deposition of solid phases in oil wells, pipelines and process facilities are undesired phenomena. Solid paraffin or wax is considered as one of these solid phases. The fundamental consequences of wax formation are pressure loss increase and reduction of oil production from petroleum reservoirs. Wax crystals increase viscosity and consequently, increase energy consumption for oil pumping and reduce pumping capacity. Wax precipitation also increases pipeline roughness which increases pressure loss. The other consequence is reduction in effective cross sectional area.

Therefore, modelling and analysis of wax particles' transport and their deposition in pipelines is a very important issue for petroleum engineers. The scientific society has made a major effort on innovation of different models to understand thermodynamics of wax formation such as solid-liquid phase

equilibrium and wax appearance temperature. The purpose of this work is to evaluate the feasibility of using SPH method for the simulation of transport deposition of wax. Studies have been accomplished on different aspects of wax deposition in pipes including molecular diffusion [1, 2], shear dispersion [3], Brownian diffusion [4, 5], and gravity settling [3-5].

Wax deposition is a moving boundary problem and it can be a great challenge to solve the equilibrium equations numerically. There are several techniques to solve a moving boundary problem. Banki et al. [6] implemented enthalpy-porosity method in simulation of flow with wax deposition and considered the gel layer as a pseudo-porous medium. Various studies have illustrated that wax crystallization in oil leads to gel layer formation with complex morphology. This gel behaves as a porous medium. In some works, the convective flow in gel layer was neglected and Fick's law was used to define molecular diffusion. Convective flow can be significant during initial gel layer formation. Studies on distillate fuels and oil fluids have shown that 2% of crystal deposition is enough to transform the fluid to gel [6]. We use a method based on thermodynamics of irreversible processes to define molecular and thermal diffusivities independently as proposed by Banki et al. [6].

Two major processes which influence wax deposition in pipelines are (1) heat transfer and (2) species transfer. Heat transfer is mainly due to conduction and convection. In some cases, viscous dissipation term can act considerably in heat balance equation. Species flux influence wax deposition by convection and diffusion.

This work's structure is as followings: At first, the problem is stated and a brief description of mathematical model for equilibrium equations, mass, energy and momentum equations is presented in SPH. Afterwards, the wax deposition results involving velocity and heavy component concentration profiles in channel cross sections are presented and this work is finished by a summary.

2. DOMAIN DEFINITION AND BASIC ASSUMPTIONS

Consider a horizontal channel with a length of L and width of H (Figure 1). A two-component fluid enters the channel from left side with volumetric flow rate Q and temperature T_{in} . The wall temperature is constant and equal to T_a which is lower than T_{in} . The outlet pressure is set to constant value P_o . Wax crystal

formation produces a gel layer (as shown in Figure1) which includes a liquid phase and a fixed solid phase. The following assumptions are considered:

- (1) Shear dispersion and Brownian diffusion are neglected,
- (2) Gravity is neglected,
- (3) Diffusion in solid phase is neglected,
- (4) Thermal capacities and conductivities, fluid viscosity, and mass diffusion coefficients are assumed independent of temperature,
- (5) Flow is laminar,
- (6) Multisolid phase model [7] is used in wax deposition.
- (7) Relative velocity of suspended solid phase respect to liquid phase is neglected.

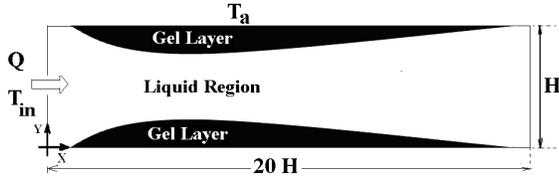


Figure 1. Computational domain and problem geometry in Cartesian system

3. MATHEMATICAL MODEL

The used mathematical model for wax deposition simulation in this work is similar to what was presented by Banki et al. [6]. The governing equations are mass, momentum and energy balances and species transfer and solid-liquid phase equilibrium which are described in the following.

3.1. Mass and momentum balance

The differential form of mass balance equation for a fluid is as follows:

$$\frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{v} = 0 \quad (1)$$

where ρ and \vec{v} are fluid density and velocity vector respectively, and D/Dt represents the material derivative. This equation is the compressible form of mass balance and is reduced to $\vec{\nabla} \cdot \vec{v} = 0$ for incompressible fluids.

Momentum equation for a fluid is the following

$$\frac{D\vec{v}}{Dt} = -\frac{1}{\rho} \vec{\nabla} P + \frac{1}{\rho} \vec{\nabla} \cdot \vec{\tau}' + \vec{g} \quad (2)$$

where P and $\vec{\tau}'$ are pressure and second-order stress tensor, respectively. Also, \vec{g} is the acceleration due to body forces such as gravity. For Newtonian fluids, the shear stress tensor is stated by a linear function of second-order strain tensor, ϵ_{ij} :

$$\tau'^{\alpha\beta} = 2\mu \epsilon^{\alpha\beta} \quad (3)$$

where μ is dynamic viscosity of fluid [8]. $\epsilon^{\alpha\beta}$ is defined according to the following relation:

$$\epsilon^{\alpha\beta} = \frac{1}{2} \left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha} \right) \quad (4)$$

Equation (2) is under the following boundary conditions:

$$\begin{aligned} \vec{v}(x, y=0, t) &= 0, & \vec{v}(x, y=H, t) &= 0, \\ \vec{v}(x=0, y, t) &= 6v_0 \frac{y}{H} \left(1 - \frac{y}{H} \right) \hat{i} \\ \frac{\partial \vec{v}}{\partial x}(x=L, y, t) &= 0, & P(x=L, y, t) &= P_0 \end{aligned} \quad (5)$$

In the above equations, v_0 is the mean velocity of internal flow and P_0 is the pressure outlet. In channel inlet, the flow is fully developed. The condition $\partial \vec{v} / \partial x = 0$ also denotes the fully developed flow at the outlet. Due to wax crystallization, we have a moving solid-liquid boundary. The main challenge is how the fluid flow deceleration in gel layer is described. In this work, we deal with the phase change as a pseudo-porous medium whose porosity ϵ changes from 1 to zero as the solid content alters from zero to 1. A source term based on Darcy equation is added to the right hand side of momentum equation (2) to describe velocity reduction in porous media:

$$\vec{S} = -\frac{C\mu}{\rho} \frac{(1-\epsilon)^2}{\epsilon^3} \vec{v} \quad (6)$$

where C is 10^6 m^{-2} [6].

For closing the above system of equations, an equation of state is needed to relate density change to pressure. A simple equation of state is:

$$P - P_0 = a^2 (\rho - \rho_0) \quad (7)$$

In the above equation, a is the speed of sound and in most cases, it is set to maximum fluid velocity multiplied by ten [9].

3.2. Energy balance

Energy balance equation in the form of enthalpy is written as below:

$$\frac{\partial}{\partial t} \left(\sum_{j=o,s} S_j c_j H_j \right) + \vec{\nabla} \cdot \left(\sum_{j=o,s} S_j c_j H_j \vec{v}_j \right) = \vec{\nabla} \cdot (k_{eff} \vec{\nabla} T) + \vec{\tau}' : \vec{\epsilon} \quad (8)$$

where the indices $j = o, s$ denote the oil and wax phases, respectively. H_j and S_j are molar enthalpy and volume fraction (saturation) of phase j , respectively. Also, k_{eff} is effective thermal conductivity of gel layer. $\vec{\tau}' : \vec{\epsilon}$ represents viscous dissipation.

The energy balance equation is solved under the following boundary conditions:

$$\begin{aligned} T(x, H, t) &= T_a, & T(x, 0, t) &= T_a, \\ T(x=0, y, t) &= T_{in}, & \frac{\partial T}{\partial x}(x=L, y, t) &= 0 \end{aligned} \quad (9)$$

In this work, solid and fluid thermal conductivities are assumed to be equal and hence, k_{eff} is equal to k_o .

3.3. Species transfer

Investigating gel structure formed by waxy oil flows reveals that at primary steps, there is 98% of fluid in gel structure [6]. Gel layer might grow as time goes by. If one considers the flow in two phase region and writes mass balance for component i , one obtains:

$$\frac{\partial}{\partial t} \sum_{j=o,s} (S_j c_{j,i}) + \bar{\nabla} \cdot \sum_{j=o,s} (S_j c_{j,i} \bar{v}_{j,i}) = 0, \quad i=1, \dots, n \quad (10)$$

where $c_{j,i}$ and $\bar{v}_{j,i}$ are molar density of component i in phase j and velocity vector of component i in phase j , respectively. The relation between overall velocity \bar{v}_j of phase j , diffusion flux $\bar{J}_{j,i}$ and velocity of component i in phase j , $\bar{v}_{j,i}$ follows:

$$S_j c_{j,i} \bar{v}_{j,i} = S_j c_{j,i} \bar{v}_j + S_j \bar{J}_{j,i}, \quad i=1, \dots, n \quad (11)$$

Equation (11) is solved under the following boundary conditions:

$$z_i \Big|_{x=0} = (z_i)_{in}, \quad \frac{\partial z_i}{\partial x} \Big|_{x=L} = 0, \quad (12)$$

$$(\bar{J}_{j,i}) \Big|_{y=0} = 0, \quad (\bar{J}_{j,i}) \Big|_{y=H} = 0$$

Molar diffusion flux J can be stated as a function of temperature and concentration gradients according to thermodynamics of irreversible processes [6].

3.4. Solid-liquid phase equilibrium

In general, there are two models for wax formation calculations. The first and the most general model is solid solution model [7]. In this model, the solid phase is considered as a homogeneous solution like liquid solutions. The second model known as multisolid phase model was proposed by Lira-Galeana in 1996 [10] which has found some industrial applications. In this model, the solid solution is not considered as a solution but is described as a mixture of pure components. The multisolid phase model is especially appropriate for the calculation of cloud point temperature. The highest temperature on which wax appears is known as wax appearance temperature which is theoretically equal to cloud point temperature. Both models are based on a balance between partial fugacity of each component in solid and liquid phases.

In this work, the solid-liquid phase equilibrium calculation is conducted using multisolid phase model. This model uses a two-step process:

- (1) Stability analysis
- (2) Phase split calculation

4. NUMERICAL MODELLING

The SPH method as a mesh free method has a fully Lagrangian nature. In this method, particles are used as the interpolating points to model and flow equations. The basis of SPH is kernel approximation based on an integral interpolation of functions. As soon as one selects an appropriate kernel function, one can write the partial differential equations in integral form. $W_{ij} = W(\bar{x}_i - \bar{x}_j, h)$ represents the kernel function value of particle i in the position of particle j . h is the smoothing length. The gradient of this function is given by

$$\nabla_i W_{ij} = \frac{\bar{x}_i - \bar{x}_j}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}}, \quad r_{ij} = |\bar{x}_i - \bar{x}_j|. \quad (13)$$

An SPH code was developed to solve mass and momentum balance equations. Then, the code was extended to solve energy balance equation considering the viscous dissipation term. In the next step, the capability to solve species transport equation was added to the code based on the same method as energy

balance equation. In the above two steps, the numerical and analytical solutions were compared for simplified cases. Then, the code was developed to solve local solid-liquid phase equilibrium equations and was evaluated. It was, then, added to the main simulation code. Finally, the source term presented by equation (6) was added to the momentum equation.

4.1. Solution of the mass and momentum balance equations

In this work, we have used the following discretized form of the mass balance equation using the SPH method:

$$\left\langle \frac{D\rho}{Dt} \right\rangle_i = \sum_{j=1}^N m_i \bar{v}_{ij} \cdot \nabla_i W_{ij} \quad (14)$$

where $\bar{v}_{ij} = \bar{v}_i - \bar{v}_j$. The discretized form of different terms in the momentum equation are:

$$\left\langle -\frac{\bar{\nabla}P}{\rho} \right\rangle_i = -\frac{1}{m_i} \sum_{j=1}^N \left(\frac{1}{\psi_i^2} + \frac{1}{\psi_j^2} \right) \frac{(\rho_j p_i + \rho_i p_j)}{(\rho_j + \rho_i)} \bar{\nabla}_i W_{ij} \quad (15)$$

and

$$\left\langle \bar{\nabla} \cdot \bar{\tau}' \right\rangle_i = \frac{1}{m_i} \sum_{j=1}^N \frac{4}{\psi_i \psi_j} \frac{\mu_i \mu_j}{\mu_i + \mu_j} \bar{v}_{ij} \frac{1}{r_{ij}} \frac{\partial W_{ij}}{\partial r} \quad (16)$$

where ψ_i is particle number density defined as ρ_i/m_i . We have used weakly compressible SPH method [9] to solve the system of mass and momentum equations. The mentioned method uses a compressible formulation to solve the checker-board problem and an intermediate velocity is defined in which all forces are involved except for the pressure gradient.

4.2. The solution of energy balance equation

For a multiphase flow containing wax, the energy balance equation in enthalpy form is like equation (8). Assuming zero velocity divergence and $H = \rho h$ and $h = \sum_{j=o,s} S_j c_j h_j$ one can write this equation as follows:

$$\rho \frac{Dh}{Dt} = \bar{\nabla} \cdot (k_{eff} \bar{\nabla} T) + \bar{\tau}' \cdot \bar{\epsilon}. \quad (17)$$

The discretized form of the energy balance equation in the SPH can be given by

$$\left\langle \frac{Dh}{Dt} \right\rangle_i = \frac{1}{m_i} \sum_{j=1}^N \left(\frac{4}{\psi_i \psi_j} \frac{k_i k_j}{k_i + k_j} T_{ij} \frac{1}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} \right) + \frac{2\mu_i}{\rho_i} \sum_{j=1}^N (\epsilon_{ij}^{\alpha\beta} \epsilon_{ij}^{\alpha\beta}) \quad (18)$$

where $T_{ij} = T_i - T_j$. The first term on the right hand side of equation (18) is the thermal conduction term [11]. The second term denotes the viscous dissipation part in the fluid enthalpy change [8]. $\epsilon_{ij}^{\alpha\beta}$ means the $\alpha\beta$ component of the second-order strain tensor between the SPH particles i and j . The summation indices in the expression $\epsilon_{ij}^{\alpha\beta} \epsilon_{ij}^{\alpha\beta}$ are α and β . $\epsilon^{\alpha\beta}$ is defined in equation (4) whose SPH form can be given by

$$\epsilon_{ij}^{\alpha\beta} = \frac{1}{2\psi_j} \left(\bar{v}_{ji}^\beta \frac{\partial W_{ij}}{\partial x_i^\alpha} + \bar{v}_{ji}^\alpha \frac{\partial W_{ij}}{\partial x_i^\beta} \right) \quad (19)$$

Here, it is assumed that velocity divergence is zero. The index j in equation (19) denotes the number of the particle in the neighbourhood of particle i and superscripts α and β for velocity and position vectors represent their α and β components.

4.3. The solution of species transport equation

One has to change species transport equation (10) to material derivative form in order to solve it by the SPH method. Assuming zero relative velocity for solid wax particles with respect to the liquid phase surrounding them and using the relation $cz_i = \sum_{j=0,s} S_j c_j x_{j,i}$ and total mass balance equation, the

species transport equation can be written as

$$\frac{Dz_i}{Dt} = -\frac{1}{c} \bar{\nabla} \cdot \sum_{j=0,s} (S_j \bar{J}_{j,i}), \quad i=1, \dots, n \quad (20)$$

where z_i and c are the total molar density and overall composition of component i , respectively. Because of negligible diffusion flux in solid phase with respect to the liquid phase, the species transport equation can be written as [6]

$$\frac{Dz_i}{Dt} = \frac{1}{c} (\bar{\nabla} \cdot (S_o c_o D^M \bar{\nabla} x_o) + \bar{\nabla} \cdot (S_o c_o D^T \bar{\nabla} T)), \quad i=1, \dots, n \quad (21)$$

Considering the similarities between equations (21) and (17), the species transport equation can be solved in the same way as the energy balance equation is solved. Thus, one can write the SPH form of the molecular diffusion term in equation (21) as

$$\left\langle \frac{1}{c} \bar{\nabla} \cdot (S_o c_o D^M \bar{\nabla} x_o) \right\rangle_i = \frac{1}{(m_i/MW_i)} \sum_{j=1}^N \left(\frac{4}{\psi_i \psi_j} \frac{d_i d_j}{d_i + d_j} x_{oj} \frac{1}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} \right) \quad (22)$$

where d_i and d_j are defined as

$$d_i = (S_o c_o D^M)_i, \quad d_j = (S_o c_o D^M)_j. \quad (23)$$

Also, the SPH form of thermal diffusion term can be given by

$$\left\langle \frac{1}{c} \bar{\nabla} \cdot (S_o c_o D^T \bar{\nabla} T) \right\rangle_i = \frac{1}{(m_i/MW_i)} \sum_{j=1}^N \left(\frac{4}{\psi_i \psi_j} \frac{d_i d_j}{d_i + d_j} T_{ij} \frac{1}{r_{ij}} \frac{\partial W_{ij}}{\partial r_{ij}} \right) \quad (24)$$

where d_i and d_j are defined as

$$d_i = (S_o c_o D^T)_i, \quad d_j = (S_o c_o D^T)_j. \quad (25)$$

In equations (22) and (24), MW_i denotes the molecular weight of particle i which depends on its composition. Wall boundary condition is set by neglecting the interaction between the fluid and wall particles.

4.4. The solution of phase equilibrium equations

Stability analysis can be conducted using the fugacity of pure component i in solid phase, $f_{s,i}^{pure}(p,T)$ as well as the fugacity of component i in liquid phase, $f_{o,i}(p,T,x_o)$ [7]. Phase split calculations can be done by the Newton-Raphson (NR) method. Since there are only two components in the mixture, the overall algorithm for phase equilibrium calculations can be summarized in a flowchart as shown in Figure 2. In this flowchart, the index 2 denotes the heavy species in the mixture. $x_{o,2}$ denotes the molar composition of heavy component in the liquid phase and is stated by x_o^{old} in this flowchart. x_o^{old} is initially equal to the composition of the heavy component in the channel entrance. ϵ_o is a minuscule value such as 10^{-10} . The

final value of x_o^{new} is the molar composition of heavy species in the liquid phase as a result of solid-liquid phase equilibrium.

5. NUMERICAL RESULTS

Heat transport and species diffusion are stated as the main reasons of wax deposition. Thus, a more precise solution of energy balance equation as well as species transport equation results into a better prediction of wax deposition inside pipelines.

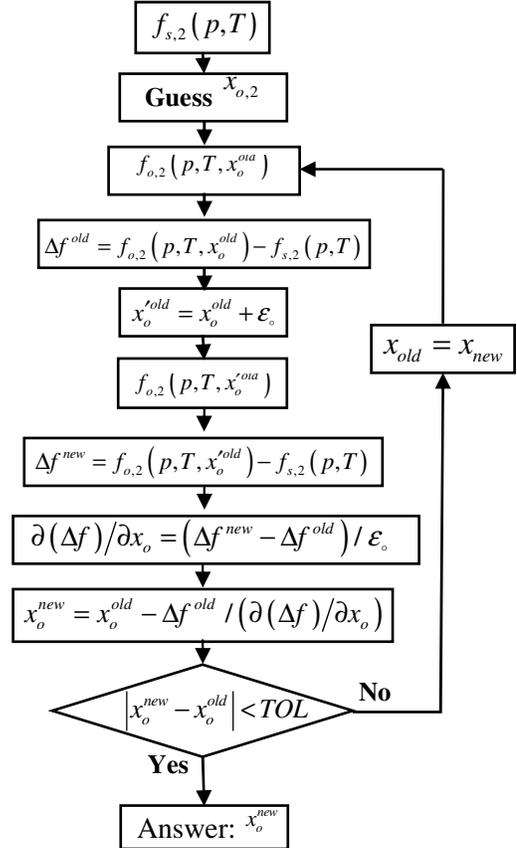


Figure 2. Flowchart of the phase equilibrium calculations

To evaluate the solution of energy balance equation in the presence of viscous dissipation term, one can use the analytical solution of this equation in its hydraulically and thermally developed form in a 2D channel of width H . Thus, before solving the main problem which involves wax deposition, the numerical and analytical solutions of energy balance equation have been compared for a simplified case. Hence, the proper precision of numerical solution for energy balance equation was proven.

Species diffusion in oil phase is known as the main reason for species transport. In this work, species diffusion has two mechanisms, temperature and concentration gradients. Temperature gradient can cause concentration diffusion and consequently concentration gradient. The wax precipitation from liquid phase and the formation of wax solid phase can cause concentration gradient independently. Here, the analytical solution for a simplified case of species transport equation has been used to validate the numerical solution of

this equation solely. For simplicity, the phase change process in fluid flow was neglected and concentration distribution was investigated just due to temperature gradient. Also, it was assumed that the diffusion of species in fluid does not change the fluid density. The simplified problem was solved numerically and the results match the analytical solution properly.

One parameter which has been used in various works to evaluate the solution of solid-liquid phase equilibrium equations is wax appearance temperature. This temperature is important since it specifies the initial point of wax deposition. Thus, for a specific composition, the WAT calculated by the prepared code has been compared with that of presented in [6]. The composition comprises of 90% n-Decane and 10% n-Dotriacontane. Pure n-Dotriacontane is solid in standard conditions. However, it is liquid in the pressure and temperature of the mixture which are respectively 20 bar and 333 Kelvin (the operating conditions) and can be considered as the wax. In the solid-liquid phase equilibrium calculations, WAT was obtained equal to 322.2 Kelvin. This parameter is reported equal to 322 Kelvin in [12]. So, one can conclude that the current phase equilibrium calculations have appropriate precision.

Wax deposition numerical solution

Here, temporal and spatial development of concentration and velocity is investigated in a specified cross section. Then, the importance of viscous dissipation and thermal diffusion are inspected in wax deposition.

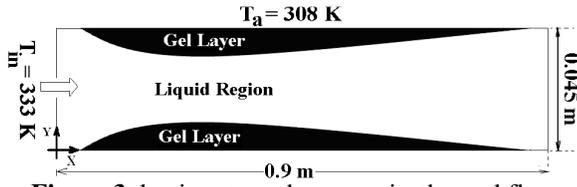


Figure 3. laminar two phase non-isothermal flow

The flow data and fluid properties are set to

$$\begin{aligned}
 u_{\max, in} &= 0.00989 \text{ m/s}, \quad (z_1)_{in} = 0.9, \\
 Q_{in} &= 0.15 \text{ m}^3/\text{hr}, \quad (u_{mean})_{in} = 0.0065495 \text{ m/s}, \\
 D_1^M &= 4.2 \times 10^{-6} \text{ m}^2/\text{s}, \quad D_1^T = -1.23 \times 10^{-9} \text{ m}^2/\text{s.K}, \\
 k_{mix}^L &= 0.144228 \text{ W/m.K}, \quad \mu_{mix}^L = 0.00112448 \text{ N.s/m}^2, \\
 \rho_{in} &= 738.37 \text{ kg/m}^3, \quad \text{Re} = 387.055, \\
 C_v &= 1725.396 \text{ J/kg.K}
 \end{aligned}$$

5.1. Development of cross sectional distribution of different variables

Figure 4 demonstrates the molar composition of light species (n-Decane) at different cross sections. The simulation time has been selected in order to have an approximately steady state solution. It is realized that after $X = 0.4$, the axial change in the profile is negligible.

The mentioned profile has a maximum point near wall and reduces as it becomes nearer to the wall. This trend can be

justified according to species transport equation. The simplified form of this equation is like $Dz_i/Dt = D^M \nabla^2 x_o + D^T \nabla^2 T$.

The term $D^M \nabla^2 x_o$ denotes the diffusion of one species due to concentration gradient. Thus, the species n-Decane in liquid phase diffuses from the place with high concentration (near wall) to a place with lower concentration (the center of the flow). The term $D^T \nabla^2 T$ causes the species n-Decane to diffuse due to temperature gradient. Because of the negative sign of D^T for the light component n-Decane, this species diffuses in the direction of temperature increase. The two diffusions mentioned above (Molecular and thermal) are in the same direction and cause the diffusion of n-Decane towards the centre of flow and produce the observed trend in Figure 4. It is obvious that far from the wall, the thermal diffusion is negligible due to minute temperature gradient and the tendency of n-Decane to diffuse towards the centre of the flow reduces. This phenomenon increases the concentration of heavy component and reduces the liquid phase saturation near wall. The second fact is due to filling the gel pores by more solid wax.

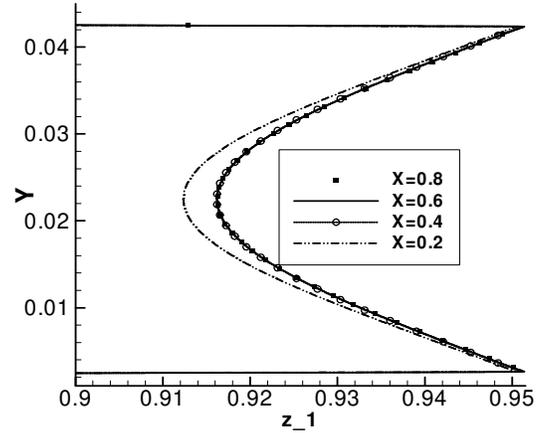


Figure 4. Total molar composition of n-Decane at various cross sections after $t = 32 \text{ sec}$.

Figure 5 demonstrates the difference between the velocity profiles at various cross sections. Investigating this figure, one can conclude that as one moves toward the flow direction, the maximum velocity increases at first and decreases afterward.

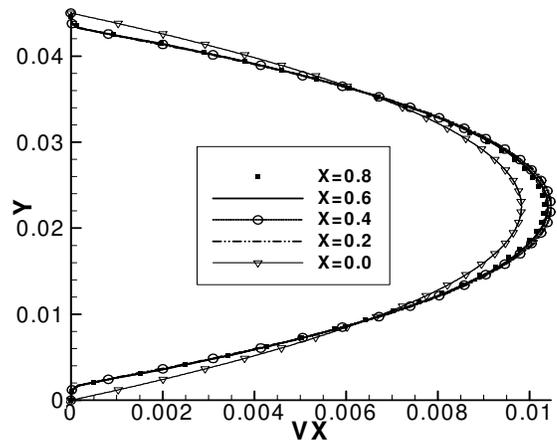


Figure 5. Horizontal component of velocity vector at various cross sections after $t = 32 \text{ sec}$.

This phenomenon can be a result of this fact that as one moves in the direction of fluid flow, solid wax thickness increases and then decreases. This circumstance can be justified by the fact that during the precipitation of heavy component, the concentration of the other component in liquid phase increases and as flow moves forward, fewer amount of the heavy component precipitates and thus, wax layer thickness reduces.

Figure 6 shows the velocity profile at a specified cross section and different simulation times. As time goes by, the velocity reduces in the vicinity of wall and increases near the centre line of the channel. The major reason for this behaviour is solid wax layer growth on wall and hence, greater decelerating force on the flow.

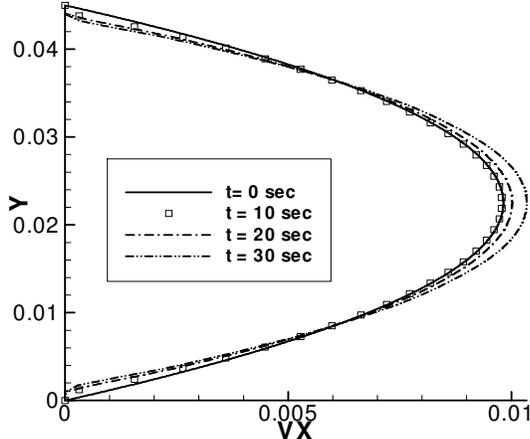


Figure 6. Horizontal component of velocity vector at various simulation times at $X = 0.8$.

5.2. The importance of viscous dissipation term and thermal diffusion

The problem was solved for three cases. In one of them, thermal diffusion term and in the other, viscous dissipation term were omitted from the species transport and the energy balance equations, respectively. The third case considered both terms omitted in other cases. Figure 7 demonstrates the effect of viscous dissipation and thermal diffusion terms upon velocity profile. This influence is negligible.

6. CONCLUSION

In this work, the feasibility of the SPH method to simulate the fluid flow containing a heavy component, wax, in a 2D channel was examined. A mathematical model based on the thermodynamics of irreversible processes was used for diffusion fluxes. It was presented that one can get the special features of a deposited wax layer such as converging-diverging geometry of the internal flow and its temporal growth by investigating cross-sectional velocity profiles. It was shown that neglecting thermal diffusion does not influence the species concentration profile in the channel cross section significantly. Also, neglecting viscous dissipation term does not influence the prediction of wax deposition significantly.

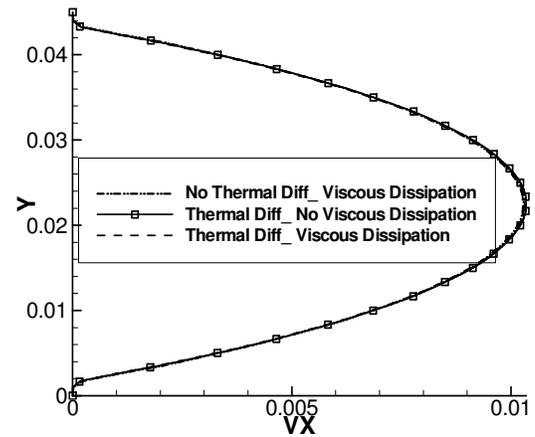


Figure 7. The comparison of horizontal velocity profile at $X = 0.8$ after $t = 32$ sec in the presence of various terms.

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