PHYSICAL SCALE MODELING TO PREDICT CURRENT DIFFUSION IN SOLID ARMATURES

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Abstract: Solid armatures are becoming increasingly important to the successful development of lower velocity (2 to 4 km/s) railguns. The ability to predict current distribution in an armature at a velocity is crucial to its thermal, electromechanical, and mechanical design. Numerical modeling techniques developed so far have not been verified by actual current distribution data. It is difficult to obtain data under actual operating conditions, so scaling laws have been developed to extrapolate data obtained in experiments under favorable conditions and lower velocity. An experiment has been designed to validate the scaling laws.

Introduction

Recent interest in rail accelerators for lower velocity (2 to 4 km/s) applications has led to renewed interest in the design of solid armatures. Benefits that would be realized from the use of solid armatures include greater electromechanical energy conversion efficiency and improved bore wear characteristics. However, in order to demonstrate their usefulness, solid armatures must perform consistently at higher velocity than they do at present.

Knowledge of thermal and mechanical loading is critical to the design of a solid armature. The designer is also concerned with optimizing the total armature mass and contact area. In order to have the necessary insight into these problems, a designer needs accurate information about the current distribution within the armature material.

Much research has been undertaken to predict the current distribution in a railgun armature and to develop methods to design working armatures based on those predictions [1,2,3]. A successful measurement of current distribution may provide insight necessary to the improvement of the distribution through clever mechanical design and choice of materials.

The electromagnetic phenomena associated with a solid armature may be investigated through mathematical means such as finite element analysis or through physical measurement of the parameters of interest. Numerical analyses to date have been based on assumptions that may significantly affect their accuracy. In order to evaluate the usefulness of numerical codes, physical data is needed. However, physical measurement is extremely difficult due to the environment that the armature must operate in and the velocity at which the material being measured is traveling.

A solution that exists is to devise an experiment that operates in a less harsh environment with an armature moving at a relatively low velocity. The data obtained from the low velocity experiment would be related to a high velocity condition through scaling laws based on equations that describe the behavior at any operating condition.

Model Theory

In order to make a physical scale model of an electromagnetic phenomena, a model system must be established in which the electromagnetic quantities of consequence are related by constants to corresponding quantities in the system for which the model is being made [4]. Electromechanical systems can be described to a high degree of accuracy by Maxwell's equations. In the case of a railgun, we are not interested in phenomena related to the propagation of electromagnetic waves since the time of interest (time of acceleration) is much longer than the propagation time for a wave. Furthermore, a railgun is an example of a magnetic field system since a large excitation results in a large current and a correspondingly large magnetic field. As a result, a railgun (as well as several other electromagnetic accelerators) can be considered a quasi-static magnetic field system. The assumption of a quasi-static magnetic field system allows the neglect of the displacement current term in the formulation of Ampere's law. Therefore, Maxwell's equations can be written:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (3)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (4)$$

where

$\mathbf{B}$ =magnetic field density
$\mathbf{E}$ = electric field intensity
$\mathbf{J}$ = current density
$\nabla = \mathbf{a}_x \frac{\partial}{\partial x} + \mathbf{a}_y \frac{\partial}{\partial y} + \mathbf{a}_z \frac{\partial}{\partial z}$
$\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ are unit direction vectors
x, y, z are displacements.

In addition, the constitutive law relating current density to electric field density is required to model the system under consideration:

$$\mathbf{J}' = \sigma \mathbf{E}' \quad (5)$$

where σ is the conductivity of the isotropic material to be studied. The prime notation indicates that the material under consideration is in motion. The electric field denoted by $\mathbf{E}'$ is that field which would be measured by an observer traveling with the material.

When the set of equations describing the system under study is written, the quantities are denoted with the subscript "s." When the set of equations represent the behavior of the model, the subscript "m" is used. If the model is to accurately represent phenomena occurring in the system for all conditions of interest, a quantity in the model must have a constant relationship to the corresponding quantity in the system [5]. For ease of analysis, the relationship between the model and system quantities are assumed to be linear in all cases. The relationships that establish this correspondence are

$$
\begin{array}{lll}
x_m = k_l\, x_s & y_m = k_l\, y_s & z_m = k_l\, z_s \\
t_m = k_t\, t_s & v_m = k_v\, v_s & \sigma_m = k_\sigma\, \sigma_s \\
\mathbf{J}_m = k_J\, \mathbf{J}_s & \mathbf{B}_m = k_B\, \mathbf{B}_s & \mathbf{E}_m = k_E\, \mathbf{E}_s
\end{array}
$$

Scaling relationships between the system under study and the model intended to represent the system are obtained by writing Maxwell's equations for the model and substituting the scaling equations into them.

For example, Amperes law (equation 1) for the model is written

$$\nabla'_m \times \mathbf{B'}_m = \mu_o \mathbf{J'}_m$$

Since

$$\frac{\partial}{\partial x_m} = \frac{\partial}{k_l\, \partial x_s}, \text{ etc.}$$

and by substitution of the correspondence relationships,

$$\frac{\nabla'_s}{k_l} \times k_B\, \mathbf{B'}_s = \mu_o k_J\, \mathbf{J'}_s$$

Collecting terms to the left yields

$$\frac{k_B}{k_l\, k_J}\, \nabla'_s \times \mathbf{B'}_s = \mu_o \mathbf{J'}_s$$

Ampere's law (equation 1) requires that

$$\frac{k_B}{k_l\, k_J} = 1 \qquad (6)$$

At this point, an examination of the linear scaling relationship is in order. The Biot - Savart law relates magnetic field intensity, current density and spatial quantities.

$$\mathbf{H} = \int_{vol} \frac{\mathbf{J} \times \mathbf{a}_R}{4\pi R^2}\, d(vol)$$

where R is the distance from the source point to the field point and $\mathbf{a}_R$ is the unit vector that gives the direction of R. By inspection of the Biot - Savart law, it can be seen that if current density and distance are scaled linearly, then magnetic field intensity will not also scale linearly. If the origin of the scaled source and the system source are chosen to be the same and the field point is chosen sufficiently distant from the source, the error introduced by the nonlinear correspondence will be small. However, in this case, the field point is contained within the volume of the source and the error cannot necessarily be assumed to be small. The problem can be circumvented by not scaling the spatial quantities, that is, by setting $k_l = 1$ so that the physical model is the same size as the actual system.

It has already been established that the current density (and therefore the resultant electric field) in the moving media is affected by the motion [3]. Therefore, an examination of the effects of motion on Faraday's law (equation 2) is in order before deriving the scaling relationship based on it.

By denoting the quantities measured in the moving time frame by a prime, Faraday's law can be rewritten

$$\nabla' \times \mathbf{E'} = -\frac{\partial \mathbf{B'}}{\partial t'} = 0 \qquad (7)$$

since an observer travelling with the armature "sees" no induced voltage.

It can be shown that the differential operator ∇' for coordinates x', y' and z' in the moving reference frame is the equivalent of ∇ in the fixed reference frame [6]. In addition, the right hand side of equation 7 can be expanded through application of the definition of the convective derivative

$$\frac{\partial \mathbf{B'}}{\partial t'} = \frac{\partial \mathbf{B'}}{\partial t} + \mathbf{v}\,(\nabla \cdot \mathbf{B'}) - \nabla \times \mathbf{v} \times \mathbf{B'}$$

By simplifying the above equation through the use of equation 3, rearranging terms and substituting into equation 8, Faraday's law for moving media is

$$\nabla \times \mathbf{v} \times \mathbf{B'} = \frac{\partial \mathbf{B'}}{\partial t} \qquad (8)$$

Using a procedure identical to the one used above with Ampere's law leads to the scaling relationship

$$\frac{k_l}{k_t\, k_v} = 1 \qquad (9)$$

The scaling law obtained from Ohm's law is found in a similar fashion

$$\frac{k_J}{k_\sigma k_E} = 1 \qquad (10)$$

The last scaling law relates the velocity of the armature to the current density in the armature. From figure 1 and by application of Kirchoff's voltage law, it can be seen that the potential field in the armature is related to the motional emf by

$$\frac{\mathbf{J}}{\sigma} = \mathbf{v} \times \mathbf{B}$$

and the scaling law derived from this expression is

$$\frac{k_J}{k_\sigma k_v k_B} = 1 \qquad (11)$$

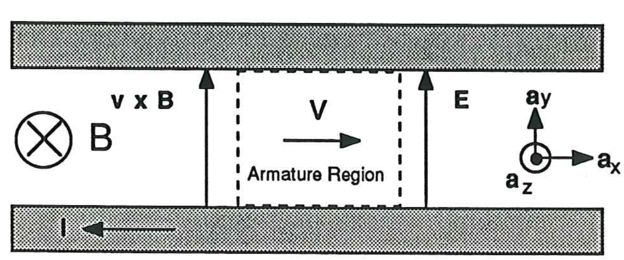

Figure 1. Schematic of a railgun near the armature

Equations 6, 9, 10, and 11 represent four scaling relationships containing seven quantities to be determined. Any three of the unknown scale factors can be set arbitrarily, leaving the remaining four to be determined from the scaling relationships. In reality, the scale factor for conductivity will be set by materials considerations and the velocity scale factor will be set by practical limitations of the power supply and accelerator. The spatial scale factor has already been set to one since linear scaling of distance is not possible.

Design of an Experiment to Verify Scaling Relationships

The current density in a moving medium is proportional to the medium's velocity and conductivity. Therefore, in order to represent phenomena taking place at a high velocity in a model at a lower, more manageable velocity, a model material with a higher conductivity must be chosen. If the system material is aluminum, a suitable material for the model would be copper. In the experiment cited above, the armature was constructed from 7075 Aluminum alloy having a conductivity of 39.2×10^6 $1/\Omega$-m. Copper has a conductivity of 63.3×10^6 $1/\Omega$-m , thus $k_\sigma = 1.62$.

In addition, the armatures will be fired from the same accelerator, thereby dictating the assignment of spatial scale factor, k_l, equal to 1 on practical as well as theoretical considerations. Examination of the scaling relationships obtained from Ampere's law (equation 6) and Ohm's law (equation 10) show that for $k_l = 1$, $k_B = k_J$ and $k_\sigma = 1/k_V$. Therefore, for the system and model chosen, $k_V = 0.617$.

At this point in the analysis, three unknowns remain to be determined, but only two independent equations remain to evaluate those unknowns. The solution to the problem is to determine the flux density scale factor, k_B, by simulating the performance of the accelerator numerically or through experiment and obtaining the field required to produce the specified operating conditions. However, in order to apply equations 10 and 11 at some operating point, information will be needed about the system's magnetic field density, electric field density, current density, and velocity. These parameters may be determined either through experiment or through numerical solution.

Both procedures contain advantages and disadvantages. Data gained from experiment is difficult to unravel and has low resolution due to size limitations on the sensors to be used. Additionally, modifications to the experimental system required to mount the sensors will undoubtedly affect the parameters to be measured. However, comparison of the experimental data for the "system" and "model" to be used for the validation of the scaling relationships will provide useful insight when scaled data is applied to armatures to be designed for high velocities.

On the other hand, numerical data has the advantage of clarity and near continuity. However, the assumptions made in the formulation of the numerical model are sure to introduce error into the data to used to develop scaling laws.

Figure 2 shows an armature designed to hold Rogowski coils to measure current distribution. The material to be used is 7075 aluminium. The armature is designed to be fired at up to 500 m/s. Figure 3 is the result of a two-dimensional electrothermal analysis of the armature at 364 m/s [7]. At this velocity and time, the current is flowing in the rear leaf only. In this region (the location of the first Rogowski coil in the actual armature), the average current density was calculated to be 1.5×10^9 A/m^2, the electric field (E_s) and magnetic field density (B_s) to be 48.5 V/m and 8.79 T, respectively.

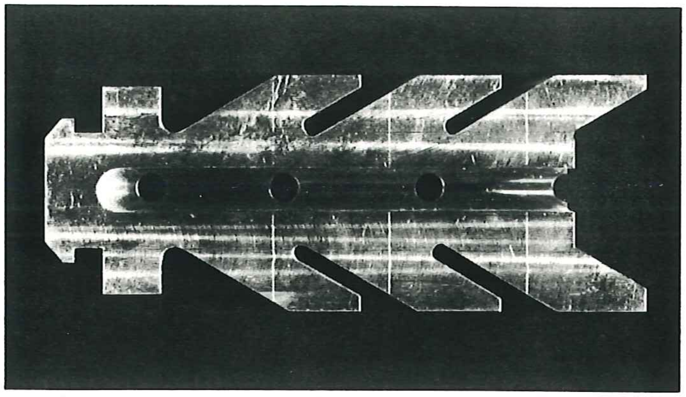

Figure 2. Aluminum armature for current distribution measurement (Holes are for Rogowski loops)

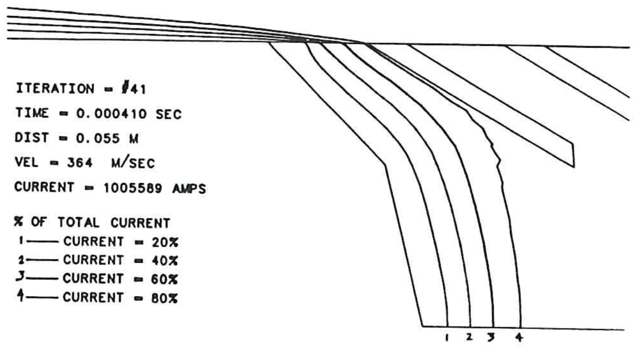

Figure 3. Current distribution obtained through two - dimensional electrothermal analysis

From the values obtained from the numerical analysis, the remaining scale factors, k_E, k_B, and k_J can be found. The scale factors will also be determined experimentally and compared to the numerical analysis, thus specifying the parameters for the model experiment to be used to verify the scaling relationships.

Conclusions

Scaling laws relating current diffusion in a high conductivity material at low velocity to current diffusion in a lower conductivity material have been written. An experiment has been designed to determine the validity of the scaling relationships. If the scaling is held to be valid, information for predicting thermal and mechanical loading in a solid armature at high velocity will be available by measuring current distribution at low velocity.

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