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Disintermediation and Co-opetition in Platform Ecosystems and Modern Value Chains

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Disintermediation and Co-opetition in Platform Ecosystems and Modern Value Chains

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DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

May 2015

Dedicated to my wife Jie Gao.

Acknowledgments

I am deeply grateful to my advisers, Dr. Ashish Agarwal and Dr. Stephen Gilbert, for their invaluable guidance. I am thankful for the time and effort they spent on nurturing me. I am also grateful to Dr. Guoming Lai and Dr. Jason Duan for their guidance in developing my methodological skills. I have also benefited a lot from interacting with Dr. Andrew Whinston, Dr. Anitesh Barua, and Dr. Sirkka Jarvenpaa. I am thankful for their advices.

I would like to express my gratitude to our graduate coordinators, John Lakavage, Caroline Walls, and Hillary Patterson, who have helped me a lot throughout the Ph.D. program. I am also grateful to my fellow Ph.D. students for their encouragement and spiritual support.

I am indebted to my wife, Jie Gao, for standing beside me throughout all these years. I would not be able to complete this dissertation without her love, patience, and sacrifice.

Disintermediation and Co-opetition in Platform Ecosystems and Modern Value Chains

Publication No. ________________

Zhuoxin Li, Ph.D.
The University of Texas at Austin, 2015

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This dissertation investigates partial disintermediation and co-opetition in platform-based ecosystems and modern supply chains. Disintermediation has been an intriguing puzzle for managers for the last several decades, but recent development in electronic commerce makes the management of this trade-off even more challenging. The first type of partial disintermediation I study, often referred to as "platform envelopment", is widely observed in platform-based businesses. Platform owners often rely on complementary innovations from third-party providers (i.e., third-party contents), while providing their own products/services to consumers (i.e., first-party contents). The second type of partial disintermediation I study is referred to as "supplier encroachment". Due to the fast development of electronic commerce, many manufacturers have established their direct-selling channels on the internet (e.g., online stores), instead of completely relying on third-party retailers to reach customers. The widespread observation of disintermediation and the resulting

co-opetition behaviors in various industries has motivated me to investigate two important questions: (1) what's the impact of partial disintermediation on consumer demand and firm profits? (2) what strategies can be used to manage the co-opetition relationship? I use both analytical modeling and empirical methods to study the impact of disintermediation on consumer behaviors, firm profits, and social welfare. The findings provide managerial insights into how to manage the co-opetition dilemma due to disintermediation.

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Chapter 1

Introduction

This dissertation investigates partial disintermediation and co-opetition in platform-based ecosystems and modern supply chains. Disintermediation has been an intriguing puzzle for managers for the last several decades, but recent development in electronic commerce makes the management of this trade-off even more challenging.

The first type of disintermediation I study is often referred to as supplier encroachment. Due to the fast development of eCommerce, many manufacturers have established their direct-selling channels on the internet (e.g., online stores), instead of purely relying on third-party retailers to reach customers. Examples of supplier encroachment abound: Amazon's Kindle is available through Amazon's own web-site as well as through major in-store and on-line channels of retailers such as Best Buy; Apple operates its own on-line and in-store direct channels, yet also sells through a variety of major retailers and telecommunications service providers; Many branded apparel manufacturers, e.g., Coach, Nike, and Tommy Hilfiger, etc., sell through traditional retailers as well as through their own "factory outle" channels; In the travel industry, hotels and airlines generate sales though their own channels and through brokerage services like Travelocity, Expedia, and Orbitz, etc.

The first essay Li et al. (2014) in the dissertation investigates how a supplier's direct channel impacts the material/information flow in the competing retailer channel. For a symmetric information setting, prior research has shown that supplier encroachment into a retailer's market can mitigate double marginalization and thus benefit the supplier and the supply chain. Our paper studies an asymmetric information setting where the retailer has private demand information of the local market. Supplier encroachment may alter the material/information flow in an intriguing way. We find that the launch of the supplier's direct channel can result in costly signaling behavior on the part of the retailer, in which he reduces his order quantity. Such a downward order distortion can amplify double marginalization. As a result, supplier encroachment can hurt both the retailer and the supplier. We further explore the implications of these findings for strategic information management. Complementing the conventional understanding, we show that with the ability to encroach, the supplier may prefer to sell to either a better informed or an uninformed retailer in different scenarios. On the other hand, as a result of a supplier developing encroachment capability, a retailer may either choose not to develop an advanced informational capability, or become more willing to find a means of credibly sharing his information with the supplier. Thus, competition between the supplier and the retailer may actually increase information transparency in the supply chain.

The second essay Li et al. (2015) in the dissertation investigates how a supplier's direct channel may interfere with her nonlinear pricing strategy. The literature have investigated the impact of supplier encroachment, assuming uniform wholesale price contracts and symmetric information between the supplier (she) and the retailer (he). However, in practices, retailers very often have private information about market conditions. In this paper, we investigate how the supplier's encroachment capability affects the retailer's information strategy, information rents, and the efficiency of nonlinear pricing. We find that the supplier's direct channel may either enhance or hinder her optimal pricing strategies, and fundamentally alter the structure of the optimal contracts. We observe upward distortion in equilibrium - the supplier intentionally sells an inefficiently large quantity through the retailer, in order to convince the retailer that she will not sell a large quantity through her direct channel. This upward distortion is in contrast to the well-known efficiency at the top property in screening contracts. Thus, supplier encroachment has two opposing effects. On one hand, the ability to shift sales to the direct channel allows the supplier to reduce information rents with less sacrifice of efficiency; but on the other hand, by introducing the possibility of her own opportunistic behavior (i.e., ex post encroachment), it can result in upward distortion of the quantities sold through the reselling channel, which is a new source of inefficiency. Depending upon the relative efficiency of the reselling channel and the demand distribution, either of these two effects may dominate and the supplier's ability to encroach may either benefit or hurt both the supplier and the retailer.

The second type of disintermediation I study, often referred to as platform envelopment, is widely observed in platform-based businesses. Platform owners often seek for complementary innovations from third-party providers (third-party contents), while at the same time provide their own applications to consumers (first-party contents). The third essay Li and Agarwal (2014) in the dissertation studies platform integration with first-party applications. Platform owners often choose to provide tighter integration with their own complementary applications (i.e., first-party applications) as compared to that with other complementary third-party applications. We study the impact of such integration on consumer demand for first-party applications and competing third-party applications by exploring Facebook's integration of Instagram in its photo-sharing application ecosystem. We find that consumers obtain additional value from Instagram after its integration with Facebook, leading to a large increase in the use of Instagram for Facebook photo-sharing. While consumer valuations of small third-party applications decrease, consumer valuations of big third-party applications slightly increase after the integration event. As a result, big third-party applications face much smaller reduction in demand as compared to small third-party applications. Interestingly, a large fraction of the new users Instagram attracted are new users who did not use any photo-sharing application, rather than incumbent users of third-party applications. As a consequence, the overall demand for the photo-sharing application ecosystem actually increases, which suggests that Facebook's integration strategy benefits the complementary market overall. Our results highlight the value of platform integration for first-party applications and the application ecosystem overall, and have implications for strategic management of first-party applications in the presence of third-party applications.

Chapter 2

Supplier Encroachment under Asymmetric Information

2.1 Introduction

Many upstream manufacturers have invested in direct channels such as online stores, catalog sales, and factory outlets (Nair and Pleasance 2005).[1] With these established direct channels, manufacturers may sell their products directly as well as indirectly through the reselling channels (e.g., distributors, wholesalers, retailers). Consequently, competition can arise between the resellers and their suppliers, a phenomenon often referred to as “supplier encroachment.”

While retail competition (competition among resellers) has been extensively studied and well understood, supplier encroachment has received much less attention and has distinct features. A study by Arya et al. (2007) shows that supplier encroachment endows the supplier with a mechanism to control the selling price in the retail market, and consequently motivates her to reduce her wholesale price. The combination of these two effects mitigates double marginalization and can benefit both the supplier and the reseller when the latter has a significant efficiency advantage in the retail process. While the existing literature has considered various elements that

[1] This chapter is based on Li et al. (2014). I appreciate my adviser and co-author Stephen Gilbert and Guoming Lai for their guidance and feedback when I was writing this paper.

may influence the effects of supplier encroachment, the information structure is often assumed to be symmetric in the supply chain. In practice, resellers often have better knowledge of the market potential than the upstream suppliers, due to their expertise and superior forecasting ability in the selling process as well as rich first-hand sales data. Although conventional wisdom holds that resellers necessarily benefit from having access to private market information, our results demonstrate that this is not always the case when the supplier has developed its own direct channel.

In this paper, we build upon the supplier encroachment framework based on Cournot competition from Arya et al. (2007) by incorporating an asymmetric information structure, where the reseller knows the realization of the market size but the encroaching supplier knows only the prior distribution of the market size. Credible information communication is not available. In the face of information disadvantage, the supplier wants to infer the true market size from the reseller's order quantity to more properly decide the direct selling quantity. Anticipating the supplier's strategy, the reseller may purposely distort his order quantity for his own benefit. The interaction between the supplier's and the reseller's incentives can result in an inefficient downstream signaling outcome. Regardless of which market size is observed by the reseller, he would like the supplier to believe it is small. Consequently, when the reseller observes a small market, he may need to distort his order quantity downward in order to send a credible signal that the market size is small. This downward distortion, if it occurs, amplifies double marginalization and may hurt both the supplier and the reseller. As a result, we find that, in the presence of asymmetric information, the supplier's development of encroachment capability can lead to “lose-win” or “lose-lose” outcomes for the supplier and the reseller in addition to the “win-win” and “win-lose” outcomes that are reported in Arya et al. (2007).

We then explore the implications of supplier encroachment for information management in supply chains. Prior literature has shown that with a reselling channel alone, the supplier is indifferent between a reseller who is better informed or equally informed. However, we find that when the supplier has the ability to encroach, she strictly prefers to sell to a better informed reseller when her efficiency disadvantage in the selling process is not large; otherwise, she prefers to sell to an equally informed reseller. On the other hand, prior literature shows that without the supplier's direct channel, the reseller always prefers to be better informed. Whereas, after the supplier launches her direct channel, the incumbent reseller can be discouraged from obtaining advanced information in a wide range of parameters. While this prevents downward distortion of the reseller's order quantity and benefits the supplier, it expands the range of parameters for which total supply chain profits are lower under encroachment. Our analysis also provides an interesting implication on information sharing. It is well known in the literature that, in standard bi-lateral monopoly settings in which production costs are linear, the supplier is restricted to setting a linear wholesale price, and the reseller determines the output quantity, the reseller benefits from having private information about demand. In contrast, we find that in the presence of the supplier's direct channel, the reseller may prefer that the supplier has access to the same information that he has over being privately informed.

2.2 Related Literature

The effects of supplier encroachment have been discussed in the literature. Empirical studies find that supplier encroachment can possibly lower a reseller's effort to sell a product (Fein and Anderson 1997) and it can also affect brand image (Frazier and Lassar 1996). However, several analytical studies have shown that supplier encroachment can mitigate double marginalization and thus benefit both the supplier and the reseller. For example, Chiang et al. (2003) demonstrate that a supplier's threat to sell through a direct channel causes the reseller to lower his selling price, which can benefit both parties. In another study, Tsay and Agrawal (2004) incorporate sales efforts that can be exerted by the supplier as well as the reseller, and show that in such a context, the launch of a supplier direct channel can still benefit both parties. The effect of mitigating double marginalization is also found by Cattani et al. (2006) based on a model with horizontal differentiation. They reveal that supplier encroachment can benefit both the supplier and the reseller if the supplier commits to the same selling price and the direct channel is not as convenient for consumers as is the existing reselling channel. Similarly, Arya et al. (2007) demonstrate based on a quantity competition model that the supplier's direct sale not only adds another source of revenue for her but also motivates her to offer a lower wholesale price to the reseller. Consequently, encroachment has the potential to benefit the supplier as well as the reseller, especially when the latter enjoys a significant cost advantage in the selling process. Our work complements this stream of research by allowing for the reseller to have private information about the market size, and this leads to results that are quite different from those found in the literature, including the fact that supplier encroachment can sometimes amplify double marginalization and hurt both the supplier and the reseller when the latter is privately informed about demand.

Our work is also related to the literature that investigates the incentives of information sharing in supply chains. Cachon and Lariviere (2001) explore contracts through which a downstream buyer can credibly share private demand information with a supplier. Li (2002) and Zhang (2002) investigate information sharing in a setting where a central supplier sells to multiple competing resellers and they show that without particular incentives, the resellers will withhold their private demand information instead of sharing it with the supplier in equilibrium. With a confidential agreement by which the supplier does not leak received information, competing resellers might be willing to share their private demand information with the supplier, which can drive down the wholesale price. Ha and Tong (2008) and Ha et al. (2011) study the incentives of information sharing within two competing supply chains, considering the effects of information accuracy, nonlinear production costs, as well as a nonlinear pricing schedule. Two recent papers are similar to ours in considering how a reseller's concern over leaking his private demand information may affect how he orders from a supplier. Anand and Goyal (2009) and Kong et al. (2013) investigate a one supplier-two reseller setting where the supplier may leak the market information learned from the incumbent reseller's order quantity to an entrant reseller. Anand and Goyal (2009) show under an exogenous wholesale price contract that the supplier always leaks information to stimulate downstream order quantity. Consequently, the incumbent reseller may purposely block information dissemination by ordering the same quantity for any market size. Kong et al. (2013) considers a similar setting but demonstrate that a revenue sharing scheme may prevent the supplier from leaking information, and consequently can result in pareto gains for all parties. Similar to this latter paper, we allow for an endogenous wholesale price, but we consider a setting in which the supplier's own direct channel, rather than a second reseller, is the potential beneficiary of information gained from the informed reseller. In addition, we consider the effect that a supplier's development of a direct channel can have upon her own as well as the reseller's preferences among different information structures, and find that encroachment may encourage the reseller to share his private information with the supplier. Consequently, our perspective of analyzing how supplier encroachment affects the flows of materials and information in a supply chain is quite different from either of these papers.

Finally, our work is related to the recent work of Jiang et al. (2011). In their study, an independent seller sells a product through a platform. The platform owner can also acquire the product and has monopolistic control over the access to the market. In particular, the platform owner can incur a fixed cost, to sell the independent seller's product and take away all of the demand from the latter if strong sales are revealed. They show a pooling outcome where the independent seller's incentive to hide the private demand information by exerting the same selling effort may hurt the platform owner but benefit himself. In contrast, we assume that both firms have the access to the market, but the supplier has full control over the access to the product (i.e., the reseller has no alternative source of supply). For this setting, we show that no pooling equilibrium can survive the intuitive criterion, and that, in the resulting separating equilibria, supplier encroachment can either benefit or hurt the two firms.

2.3 The Model

We consider a supplier (she) that sells a product through a reseller (he), but she also has her own direct channel and may sell the product directly to consumers. We normalize both the production cost of the supplier and the selling cost for the reseller to zero. To allow for the possibility that the supplier may be less efficient in retail operations than is the reseller, we assume that the supplier incurs a per-unit selling cost of c for each unit that she sells directly to consumers. Consumer demand follows a linear, downward sloping demand function, $P = \mathbf{a} - Q$, where Q is the total number of the product deployed for sale, P is the market clearing price, and $\mathbf{a}$ represents the market size. Note that it is without further loss of generality that we have normalized the slope of this demand function to be -1.

The above setup is nearly identical to that of Arya et al. (2007). However, to capture the notion that the reseller is closer to the market and may also have better expertise in forecasting the demand than the supplier, we assume that the market size $\mathbf{a}$ is, ex ante, random which can be either large ($\mathbf{a} = a_H$) with probability λ and small ($\mathbf{a} = a_L$) with probability $1 - \lambda$, where $a_H > a_L > 0$; the reseller can observe the true market size privately, before ordering from the supplier, while the supplier knows only the prior distribution of the market size.[2] Let $\mu = \lambda a_H + (1 - \lambda) a_L$,

[2]For simplicity, we assume here that the reseller learns the market size perfectly, while the supplier only has the prior knowledge. The insights we reveal, however, will continue to hold, even

representing the expected market size, and $\sigma^2 = \lambda(a_H - \mu)^2 + (1 - \lambda)(a_L - \mu)^2$ be the variance of the market size distribution. We restrict our attention to the cases in which $a_L > \frac{\mu}{2}$, so that, as we will show later, the reseller's equilibrium order quantity is strictly positive for both market sizes without supplier encroachment. Such an assumption will simplify the analysis and can also highlight the contrast between the cases with and without supplier encroachment. Finally, we assume that the supplier uses a linear wholesale price only contract. Linear pricing schemes are widely used in practice and also are commonly assumed in the literature that studies channel structure (e.g., McGuire and Staelin 1983; Lariviere and Porteus 2001; Cachon 2003; Arya et al. 2007). Similar results hold even if the supplier can implement nonlinear pricing through a menu of contracts.

Figure 2.1 details the timeline of the model. First, the supplier offers a wholesale contract to the reseller, which contains a unit wholesale price w. The reseller who has observed the true market size, $\mathbf{a} = a_H$ or $\mathbf{a} = a_L$, orders q_R units from the supplier. The supplier then decides the quantity q_S which she sells through her direct channel. The market clearing price P is realized according to $P = a_i - (q_R + q_S)$ for $i \in \{H, L\}$, and the two parties obtain their final profits.[3] The assumption that the reseller orders before the supplier determines her order quantity is justified by the fact that the supplier has no way to credibly commit to refrain from revising her own order quantity after receiving the reseller's order.

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if both of them receive noisy signals of the market size, as long as the reseller is more precisely informed about the demand than the supplier.

[3]This inverse demand function implicitly assumes that consumers perceive the two channels to be perfect substitutes. While allowing for partial substitutability would complicate the analysis, it

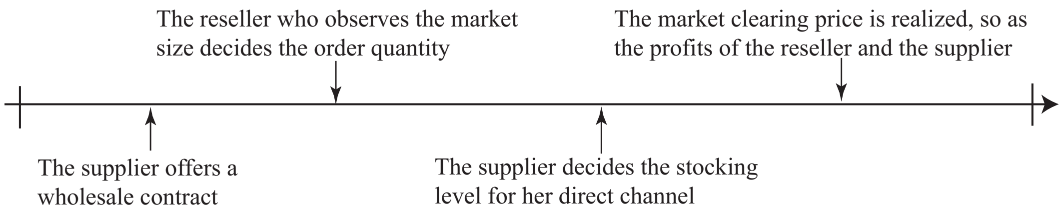


Figure 2.1: The timeline of the model.

2.4 Analysis

Before beginning our analysis of how the development of supplier encroachment capability (i.e., the launch of the direct channel) affects the interactions between a supplier and a reseller, we first present the benchmark in which the supplier lacks this capability and sells the product *only* through the reseller.

2.4.1 Benchmark without Encroachment

When the supplier lacks the infrastructure of a direct channel, she has effectively provided a credible commitment that she will not encroach. Then, given the wholesale price w and the market size $a_i, i \in \{H, L\}$, the reseller determines his order quantity as the solution to:

$$\max_{q_R}[a_i - q_R - w]q_R.$$

It is easy to obtain the optimal order quantity of the reseller for each market size a_i, $i \in \{H, L\}$:

$$q_R^N(w; a_i) = \frac{a_i - w}{2}.$$

would not provide additional insights.

Recall that the supplier does not observe the true market size, but anticipates the reseller's decision. Thus, the supplier chooses her wholesale price as the solution to:

$$\max_{w} \mathbf{E}\left[q_R^N(w;\mathbf{a})w\right].$$

In equilibrium, the supplier's optimal wholesale price is $w^N = \frac{\mu}{2}$, and the expected profits of the reseller and the supplier are:

$$\pi_R = \frac{\mu^2 + 4\sigma^2}{16} \text{ and } \pi_S = \frac{\mu^2}{8}, \tag{2.1}$$

where the lower case π indicates that there is no encroachment.

2.4.2 Encroachment Analysis

Let us now enhance the model to allow for supplier encroachment, by which we mean the supplier has her direct channel in place and can choose to sell directly. Since the supplier does not observe the market demand directly, she will use the information revealed from the reseller's order quantity to make her decision on the direct sale quantity. Because the reseller anticipates the supplier's reaction to his order, a signaling game may arise in which the reseller may purposely alter his order quantity. In particular, as is often the case, there are two mutually exclusive types of equilibria that might arise. In the first, the reseller orders a distinct quantity for each market size, and his order perfectly reveals the market size to the supplier. In the second, he orders the same quantity for both market sizes, and his order is uninformative. The former case represents a separating outcome while the latter corresponds to a pooling outcome. Typically, such signaling games can have multiple equilibria depending on the players' belief specification. However, this obstacle can be overcome by using the intuitive criterion, which is a classical equilibrium refinement developed by Cho and Kreps (1987). As we can show (see the appendix) that pooling equilibrium cannot survive the intuitive criterion in our model, we focus directly on separating equilibrium. That is, all of the formulation and analysis presented below are based on the fact that the supplier can perfectly infer the market size from the reseller's order quantity. Furthermore, we confine our formulation and analysis to those cases (with respect to c, λ, a_H and a_L) where the supplier optimally sells a positive quantity for each market size (the boundary condition is provided when we characterize the equilibrium). Displaying the full analysis for the cases where the supplier optimally chooses not to encroach would complicate the exposition without adding any interesting insights.

We first formulate the supplier's belief. We use $a_{j(q_R)}$ to indicate the market size that the supplier believes after receiving an order quantity q_R from the reseller. It is intuitive that the reseller will order more when the true market size is large than when the market size is small. Thus, we apply the following belief structure depending on a threshold order quantity $\hat{q}_R(w)$ for a given wholesale price w (other belief formulations exist that can lead to the same equilibrium result):

$$j(q_R) = \begin{cases} H & \text{if } q_R > \hat{q}_R(w), \\ L & o/w. \end{cases}$$

That is, the supplier believes that the market size is large if the reseller's order quantity $q_R > \hat{q}_R(w)$ and small otherwise. Then, after observing the reseller's order quantity q_R, the supplier determines her direct selling quantity by solving:

$$\max_{q_S}[a_{j(q_R)} - q_R - q_S - c]q_S,$$

which yields the optimal direct selling quantity:

$$q_S(q_R) = \frac{a_{j(q_R)} - q_R - c}{2}.$$

In anticipation of the supplier's belief and reaction, the reseller, who knows the true market size a_i, $i \in \{H, L\}$, solves:

$$\max_{q_R}[a_i - q_R - q_S(q_R) - w]q_R. \tag{2.2}$$

Let $q_R(w; a_i)$ denote the optimal solution of (2.2). Notice that for a given order quantity, the reseller would be better off if the supplier believed the market size were small than if she believed it were large. Thus, the reseller may purposely order a lower quantity to induce the supplier to believe the market size is small. The supplier will adjust $\hat{q}_R(w)$ taking the reseller's incentive into account. To solve this problem, we define the following equilibrium concept.

Definition 1. *Given any wholesale price w, a perfect Bayesian separating equilibrium is reached if $a_{j(q_R(w;a_i))} = a_i$ for each market size a_i, $i \in \{H, L\}$; that is, there exists a $\hat{q}_R(w)$ such that $q_R(w; a_H) > \hat{q}_R(w)$ while $q_R(w; a_L) \leq \hat{q}_R(w)$.*

To facilitate the characterization of the equilibrium, we define the following functions:

$$V_{ij}(q_R) = \left[a_i - q_R - \frac{a_j - q_R - c}{2} - w\right] q_R, \forall i, j \in \{H, L\}.$$

$V_{ij}(q_R)$ is the reseller's profit if the true market size is a_i while, given the reseller's order quantity q_R, the supplier believes that the market size is a_j.

Lemma 1. *$V_{ij}(q_R)$ is concave in q_R for any $i, j \in \{H, L\}$, and there is a unique maximizer of $V_{ii}(q_R)$, that is, $q_R = \left(\frac{a_i-2w+c}{2}\right)^+$, for each $i \in \{H, L\}$.*

Figure 2.2 illustrates the reseller's possible profit functions, $V_{ij}(q_R)$. From Lemma 1, we can clearly see that if the supplier could also observe the market size then the reseller would order $q_R = \left(\frac{a_i-2w+c}{2}\right)^+$, for each market size a_i. When the supplier does not have complete information, the reseller may have an incentive to place an order lower than $\left(\frac{a_i-2w+c}{2}\right)^+$. It is easy to see that the supplier would never benefit from setting a wholesale price, $w \geq \frac{a_H+c}{2}$, since this would prevent the reseller from ordering anything for each market size. Therefore, we implicitly assume $w < \frac{a_H+c}{2}$ for all the analysis below.

Lemma 2. *$V_{HL}(q_R) > V_{HH}(q_R)$ for any $q_R > 0$ and there exists*

$$\bar{q}_R(w) = \frac{2a_H - a_L - 2w + c - \sqrt{(a_H - a_L)(3a_H - a_L - 4w + 2c)}}{2} < \frac{a_H - 2w + c}{2}$$

such that $V_{HL}(\bar{q}_R(w)) = V_{HH}\left(\frac{a_H-2w+c}{2}\right)$, $V_{HL}(q_R) < V_{HH}\left(\frac{a_H-2w+c}{2}\right)$ when $q_R < \bar{q}_R(w)$, and $V_{HL}(q_R) > V_{HH}\left(\frac{a_H-2w+c}{2}\right)$ when $\bar{q}_R(w) < q_R < \frac{a_H-2w+c}{2}$. Furthermore, let $\overline{w} = \frac{3a_L-a_H+2c}{4}$. Then, $\bar{q}_R(w) \lesseqqgtr \left(\frac{a_L-2w+c}{2}\right)^+$ when $w \lesseqqgtr \overline{w}$.

Figure 2.2 provides an illustration of the results of Lemma 2 and the threshold $\bar{q}_R(w)$. Lemma 2 implies that if the threshold $\hat{q}_R(w)$ in the supplier's belief is above $\bar{q}_R(w)$, then if the reseller observes a large market he would order less than $\frac{a_H-2w+c}{2}$,

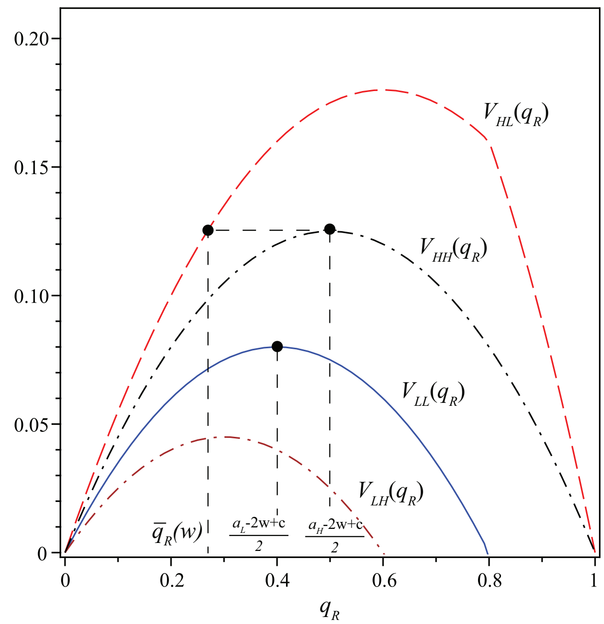


Figure 2.2: Demonstration of the reseller's possible profit functions and the threshold $\overline{q}_R(w)$. The parameters are: $\lambda = 0.3$, $a_H = 1.2$, $a_L = 1$, $w = 0.2$, and $c = 0.2$. In this example, $w < \overline{w} = 0.55$.

to induce the supplier to believe that the market size is small. Therefore, in order for a separating equilibrium to exist, $\hat{q}_R(w)$ cannot be greater than $\bar{q}_R(w)$. With this intuition, the following proposition characterizes a unique separating equilibrium that survives the intuitive criterion.

Proposition 3. *There exists a unique perfect Bayesian separating equilibrium that survives the intuitive criterion, in which the reseller's order quantity satisfies* $q_R(w; a_H) = \frac{a_H - 2w + c}{2}$ *and* $q_R(w; a_L) = \hat{q}_R(w) = \min\left\{\left(\frac{a_L - 2w + c}{2}\right)^+, \bar{q}_R(w)\right\}$*, and the supplier's direct selling quantity is* $q_S(q_R(w; a_i)) = \frac{a_i - q_R(w; a_i) - c}{2}$*,* $\forall i \in \{H, L\}$*.*

In this equilibrium, the reseller orders $\frac{a_H - 2w + c}{2}$ when the market size is large, which coincides with the optimal quantity he would order if the supplier also observes the market size. In other words, for the large market size, the reseller's order quantity will not be distorted as the information of the market size becomes private to the reseller. However, when the market size is small, distortion of the reseller's order quantity can arise due to information asymmetry. We can observe from Lemma 2 that only when $w \geq \overline{w}$, would $q_R(w; a_L)$ coincide with $\left(\frac{a_L - 2w + c}{2}\right)^+$, and when $w < \overline{w}$, $q_R(w; a_L) < \left(\frac{a_L - 2w + c}{2}\right)^+$. That is, if the wholesale price $w < \overline{w}$, then, for the small market size, the reseller will order less in the presence of asymmetric information than he would if the market size were observable to the supplier. In order to credibly signal that the market size is small, the reseller needs to downward distort the order quantity to such a level that he would have no incentive to mimic when observing the large market size even if that would allow him to deceive the supplier. Consequently, in equilibrium, the supplier can always learn the market size from the reseller's order quantity and determines her direct selling quantity accordingly.

Recall from Arya et al. (2007) that the potential for mutual benefit from encroachment arises because the supplier lowers the wholesale price at the same time that she stimulates the volume of sales through the reseller with the threat of her own direct sales. However, in the presence of asymmetric information, the reseller's propensity to downward distort his order quantity when the wholesale price is low can dampen the supplier's willingness to reduce her wholesale price. Consequently, informational asymmetry can reduce or eliminate the potential for mutual benefit from encroachment.

A deeper investigation of the reseller's order quantity for the small market size can draw the following conclusion.

Lemma 4. *The reseller's equilibrium order quantity for the small market size satisfies:* $\left|\frac{\mathrm{d}q_R(w;a_L)}{\mathrm{d}w}\right| = 1$ *when* $\overline{w} \leq w < \frac{a_L+c}{2}$ *and* $\left|\frac{\mathrm{d}q_R(w;a_L)}{\mathrm{d}w}\right| < 1$ *when* $w < \overline{w}$.

Lemma 4 asserts that the reseller's equilibrium order quantity under a small market size, $q_R(w;a_L)$, will be less responsive to the wholesale price (i.e., the order quantity increases at a slower rate as the wholesale price decreases) in the region with a distortion than without. As we will see later (at Proposition 5), the point where w drops below $\overline{w}$ corresponds to a discontinuous drop in the price elasticity of the reseller's order quantity and can drive up the supplier's wholesale price and thus amplify double marginalization.

The supplier's wholesale pricing decision, in anticipation of the subsequent subgames, can be expressed as:

$$\max_{w} \mathbf{E}\left[q_R(w;\mathbf{a})w + (\mathbf{a} - q_R(w;\mathbf{a}) - q_S(q_R(w;\mathbf{a})) - c)\, q_S(q_R(w;\mathbf{a}))\right]. \tag{2.3}$$

Proposition 5 provides the solution to (2.3) and the corresponding subgame equilibrium for the cases where the supplier encroaches for both market sizes.

Proposition 5. *Given* a_H *and* a_L*, for any* $\lambda \in (0,1)$*, there exists a threshold* $\overline{c}(\lambda)$ *such that in equilibrium,*[4]

(1) the supplier's optimal wholesale price and the reseller's order quantity follow:

[4] The threshold, $\overline{c}(\lambda)$, is the selling cost above which the supplier's direct selling quantity under at least one market size reaches zero in equilibrium.

i) if $c \in \left(0, \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right]$, *then* $w^* = \frac{3a_H - c}{6}$, $q_R(w^*; a_H) = \frac{a_H - 2w^* + c}{2}$ *and* $q_R(w^*; a_L) = 0$*;*

ii) if $c \in \left(\frac{3\sqrt{\lambda}(a_H - a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}\right]$, *then* $w^* = \frac{3\mu - c}{6}$, $q_R(w^*; a_H) = \frac{a_H - 2w^* + c}{2}$ *and* $q_R(w^*; a_L) = \frac{a_L - 2w^* + c}{2}$*;*

iii) if $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}, \overline{c}(\lambda)\right)$, *then* $w^* = \min\{\overline{w}, w_f\}$, *where* $w_f > \frac{3\mu - c}{6}$ *is the smallest solution of the first order condition of (2.3),*[5] $q_R(w^*; a_H) = \frac{a_H - 2w^* + c}{2}$ *and* $q_R(w^*; a_L) = \bar{q}_R(w^*)$*;*

(2) the supplier's direct selling quantity follows $q_S(q_R(w^*; a_i)) = \frac{a_i - q_R(w^*; a_i) - c}{2}$, $\forall i \in \{H, L\}$, *which is positive for* $c \in (0, \overline{c}(\lambda))$.

Proposition 5 shows that when the supplier's selling cost is relatively small, i.e., $0 < c \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, she chooses the wholesale price at a level such that the reseller orders a positive quantity only if the market size is large. With an intermediate selling cost, $\frac{3\sqrt{\lambda}(a_H - a_L)}{4} < c \leq \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}$, the supplier's optimal wholesale price induces the reseller to order a positive quantity for each market size. Further, the reseller who observes high demand will have no incentive to attempt to mimic even the undistorted quantity that he would order with a small market size. Consequently, when the supplier's selling cost is intermediate, we will see a natural separating equilibrium in which there is no distortion of the reseller's order quantity when the true market size is small. However, when the supplier's selling cost is relatively large, $\min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\} < c < \overline{c}(\lambda)$, the reseller who observes a

[5] The first order condition of (2.3) is $\lambda(3a_H - c - 6w) + (1 - \lambda)(a_H + 2a_L - c - 6w)\left(1 - \sqrt{\frac{a_H - a_L}{3a_H - a_L + 2c - 4w}}\right) = 0$.

large market size would have an incentive to mimic an undistorted small order quantity to deceive the supplier. As a result, the reseller will have to distort his order quantity downward when he truly observes a small market size, in order to credibly reveal the information to the supplier. In anticipation of this quantity distortion, the supplier's optimal wholesale price will exceed the $\frac{3\mu-c}{6}$ that she would otherwise offer in the absence of quantity distortion. Note that this is a direct outcome of the discontinuous reduction in the price elasticity of the reseller's order quantity as discussed below Lemma 4. Finally, note that we have restricted our analysis to the cases where $c < \bar{c}(\lambda)$ for which the supplier will sell a positive quantity through her direct channel for each market size. For $c \geq \bar{c}(\lambda)$, the analysis becomes quite complex because there may be cases in which the supplier sells nothing through her direct channel for either the small or the large market size. Since further analysis of these cases is unlikely to yield additional insights, we have restricted our attention to those cases for which the supplier always encroaches.

With Proposition 5, we can assess the impact of supplier encroachment on the supplier's as well as the reseller's profitability. Let us denote by uppercase Π_R and Π_S the equilibrium profits of the reseller and the supplier under encroachment when only the reseller knows the true realization of market size. We first focus on the cases with small and intermediate direct selling costs, i.e., $c \in \left(0, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}\right]$.

Proposition 6. *The supplier is always better off in expectation by encroachment (i.e., $\Pi_S > \pi_S$) when $c \in \left(0, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}\right)$.*

Proposition 6 shows that when the supplier's selling cost is relatively small or intermediate, encroachment always increases the supplier's profit. In particular, when the direct selling cost is small, $0 < c < \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, by encroachment, the supplier can set a wholesale price, $w^* = \frac{3a_H - c}{6}$, more appropriately targeting the large market size than what she would offer without encroachment; on the other hand, having the ability to sell the product directly with a small cost limits the potential loss if the reseller does not order when the market size is small. When the direct selling cost is intermediate, $\frac{3\sqrt{\lambda}(a_H - a_L)}{4} < c < \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}$, the situation faced by the supplier in our model is the most similar to what has been explored in Arya et al. (2007). In particular, the reseller is induced to order a positive but distinct quantity for each market size without any intentional distortion. Supplier encroachment not only introduces another stream of revenues to the supplier but also endows the supplier with a mechanism to control the selling price in the retail market, which mitigates double marginalization.

For the reseller, although supplier encroachment causes him to lose the monopoly power in his market, it may also lead to a lower wholesale price. As revealed in Arya et al. (2007) with symmetric and full demand information, supplier encroachment can benefit or hurt the reseller depending on the supplier's direct selling cost. We find a similar result with asymmetric information.

Proposition 7. *Given a_H and a_L, for any $\lambda \in (0, 1)$, there exists a threshold $\hat{c}_R(\lambda)$ such that the reseller is worse off in expectation by supplier encroachment (i.e., $\Pi_R < \pi_R$) when $c \in (0, \hat{c}_R(\lambda))$ and better off (i.e., $\Pi_R > \pi_R$) when $c \in \left(\hat{c}_R(\lambda), \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}\right]$.*

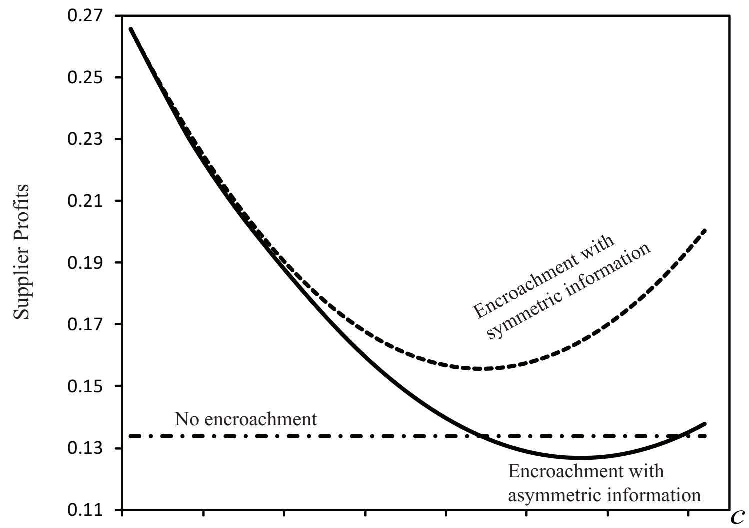
Proposition 7 shows the existence of a threshold, $\hat{c}_R(\lambda)$, with respect to the supplier's direct selling cost. When the supplier's direct selling cost, c, is lower than this threshold, the reseller is made worse off from losing the monopoly position in the downstream market as the supplier encroaches. In contrast, when the supplier's selling cost exceeds this threshold, the reseller enjoys a large efficiency advantage in the selling process, which helps him to gain more profit with a lower wholesale price offered by an encroaching supplier. The benefit from the reduction of the wholesale price outweighs the loss of demand due to the supplier's direct competition under encroachment. In particular, it can be shown that when $\lambda \to 0$, $\hat{c}_R(\lambda) \to \frac{3\sqrt{2}a_L}{8}$ and when $\lambda \to 1$, $\hat{c}_R(\lambda) \to \frac{3\sqrt{2}a_H}{8}$, which coincides with the threshold characterized in Arya et al. (2007) (with the market size a defined in their study being equal to either a_L or a_H). Therefore, supplier encroachment can still lead to a "win-win" outcome for the supplier and the reseller even under the setting with asymmetric market information in the channel.

The above analysis focuses on the cases where the supplier has a relatively small or intermediate selling cost. In such cases, the reseller has no incentive to purposely distort his order quantity under the supplier's optimal wholesale price. However, as we observe from Proposition 5, when the supplier's selling cost is relatively large, the supplier will set a wholesale price under which the reseller will downward distort his order quantity if the market size is small. Such a distortion can take a toll on both the supplier and the reseller.

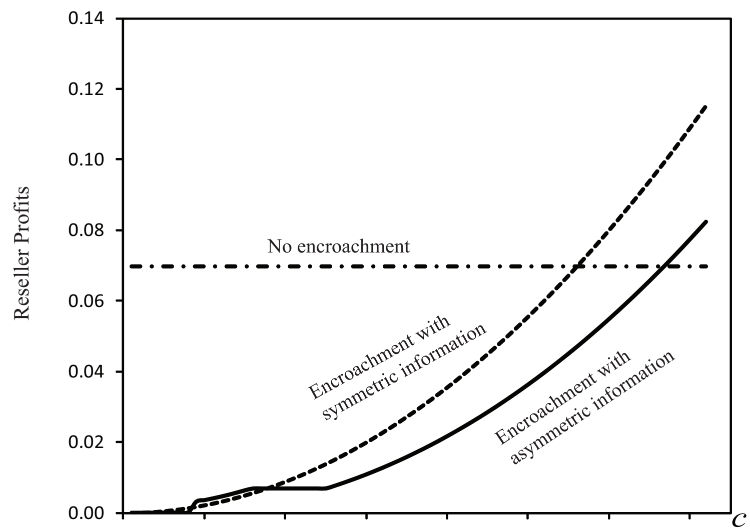
Remark 1. *There exist a_H, a_L, λ and $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}, \bar{c}(\lambda)\right)$ such that both the supplier and the reseller are worse off in expectation by supplier encroachment.*

While it is challenging to derive the sufficient and necessary condition under which having the ability to encroach hurts the supplier herself as well as the reseller, as a consequence of the fact that the supplier's optimal wholesale price takes a complex implicit form when $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}, \overline{c}(\lambda)\right)$, Remark 1 asserts the existence of such scenarios. With information asymmetry, the reseller observing a large market size has the incentive to order less to pretend to have observed a small market to induce the supplier to sell less in her direct channel. For a sufficiently small wholesale price, this incentive can cause the reseller to lower his order quantity to credibly signal his information when the market size is truly small. Recall from Lemma 4 that in the range of w for which distortion occurs, there is a reduction in the magnitude of the price elasticity of the reseller's order quantity. As a result, the anticipation of the reseller's potential distortion can cause the supplier to offer a higher wholesale price, which consequently amplifies double marginalization in the indirect channel. In contrast to the "win-win" outcome revealed in the earlier discussion, a "lose-lose" outcome can also arise if the supplier possesses the ability to encroach while she has an intermediate selling cost and the probability of a large market is low. This result does not occur in the analysis of Arya et al. (2007) because it is driven by the information asymmetry between the reseller and the supplier. It sounds an alarm over upstream encroachment when the downstream is better informed. Supplier encroachment can create downstream ordering distortion amid information dissemination, which harms channel efficiency.

To gain a deeper intuition, we further conduct a numerical analysis to reveal all possible outcomes. First, we find that the presence of information asymmetry



(a) $\frac{a_L}{a_H} < \frac{1}{2}$



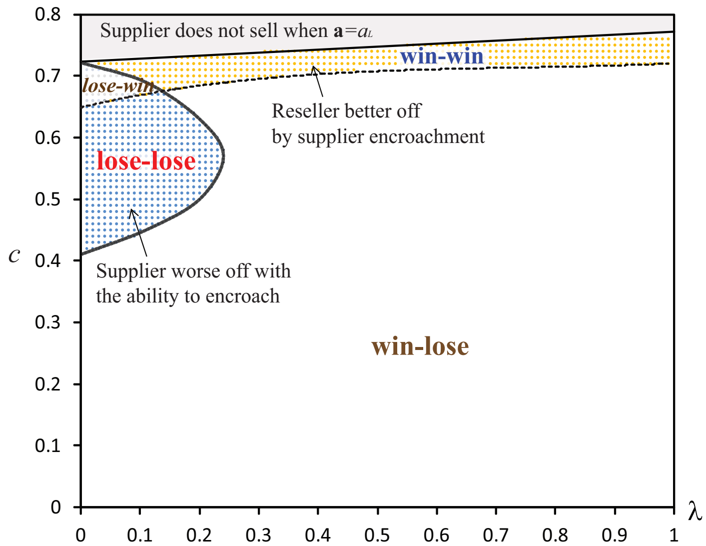
(b) $\frac{a_L}{a_H} > \frac{1}{2}$

Figure 2.3: Demonstration of the impacts of supplier encroachment on the supplier's and the reseller's profits with symmetric and asymmetric information settings. In this example, $a_H = 1.35$, $a_L = 1$, and $\lambda = 0.10$.

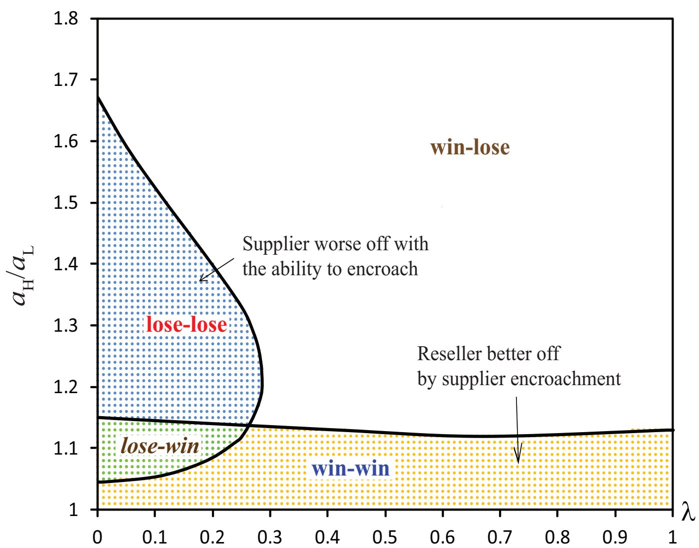
can have a significant impact on the benefits of encroachment for both the supplier and the reseller, and in general, this impact is stronger when the supplier's direct selling cost c becomes larger (see Figure 2.3). However, for some range of c, the presence of information asymmetry can also make supplier encroachment benefit the reseller more compared to the corresponding case with symmetric information. This is because an uninformed supplier cannot tailor her optimal wholesale price to the realized market size. Second and more interestingly, we can observe from Figure 2.4 that in the presence of information asymmetry, supplier encroachment can still lead to either a "win-win" or a "win-lose" outcome for the supplier and reseller respectively, but a "lose-win" or a "lose-lose" outcome is also possible. Recall from Arya et al. (2007) that neither of these latter two outcomes arises as a result of the development of encroachment capability under symmetric information. Specifically, even though the supplier benefits from having encroachment capability for a relatively wide range of parameters, she can still be worse off with a relatively small λ (the prior probability of the large market size) when her direct selling cost is intermediate (see the left subplot of Figure 2.4) or the ratio of the two market sizes is neither very large nor very small (see the right subplot of Figure 2.4). For the reseller, he will benefit from the supplier's ability to encroach, only if he enjoys a relatively large advantage in the selling process while the ratio of the two market sizes is small. Note that the equilibrium result in our study converges to that in Arya et al. (2007) as $\frac{a_H}{a_L}$ approaches one so that there is no information asymmetry and the signaling game does not occur. Although our results also converge perfectly to theirs as λ approaches one, we will not have similar convergence as λ becomes arbitrarily small, since the signaling game will always arise and there will be a discontinuity between the equilibrium result in our study and that in Arya et al. (2007). Such a discontinuity is common among signaling games and is not unique to our model.

2.5 Implications for Information Management

Throughout the previous analysis, we have assumed that the reseller is endowed with a better knowledge of the market scenario than that of the supplier. In this subsection, we explore how the supplier's development of encroachment capability alters the extent to which the supplier and the reseller benefit from the possession of information. We explore this issue from three different perspectives. First, we take the perspective of a supplier that may be able to choose among several resellers with which to do business, and we address the issue of whether the supplier should prefer to interact with a more or less informationally capable reseller (i.e., who can be better informed than the supplier or just equally informed). Second, we take the perspective of a supplier who is in a bilateral monopoly relationship with a single reseller, in which the reseller's decision to develop infrastructure to enhance his informational capability is an endogenous decision. Here, we address the question of how such an endogenized informational strategy affects the equilibrium profits of the supplier and the reseller. Finally, we consider the possibility that the reseller can share demand information with the supplier, and address the question of whether encroachment impedes or facilitates information sharing.



(a) $\frac{a_L}{a_H} < \frac{1}{2}$



(b) $\frac{a_L}{a_H} > \frac{1}{2}$

Figure 2.4: Demonstration of the impacts of supplier encroachment on the supplier's and the reseller's profitability. In this example, $a_L = 1.0$. In the left plot, $a_H = 1.35$; in the right plot, $c = 0.6$.

2.5.1 Preliminaries

In order to address the questions above, we must first perform some preliminary analysis to characterize the profits of the supplier and the reseller with and without encroachment capability under two additional information structures: one in which neither firm knows the realized market size, and another in which both firms know the realized market size. As before, we use lower case π_R and π_S (upper case Π_R and Π_S) to denote the profits of the reseller and the supplier without (with) supplier encroachment capability.

In the absence of encroachment it is straightforward to show that when neither the supplier nor the reseller knows the realization of the market size, their equilibrium expected profits can be characterized as:

$$\pi_R^{NI} = \frac{\mu^2}{16} \text{ and } \pi_S^{NI} = \frac{\mu^2}{8}, \tag{2.4}$$

where NI indicates no information. Similarly, if both the reseller and the supplier can observe the realization of the market size, presumably through some credible information sharing mechanism, then their expected profits can be expressed as:

$$\pi_R^{SI} = \frac{\mu^2 + \sigma^2}{16} \text{ and } \pi_S^{SI} = \frac{\mu^2 + \sigma^2}{8}, \tag{2.5}$$

where SI indicates shared information. Recall from section 2.4.1 that when the reseller has private information, the profits of the reseller and the supplier are $\pi_R = \frac{\mu^2+4\sigma^2}{16}$ and $\pi_S = \frac{\mu^2}{8}$, respectively. Therefore, in the absence of supplier encroachment, we have that $\pi_R \geq \pi_R^{SI} \geq \pi_R^{NI}$ while $\pi_S^{SI} \geq \pi_S = \pi_S^{NI}$, where the inequalities are strict if and only if $\sigma > 0$. That is, the reseller prefers to be privately informed to having shared information, and prefers shared information to no information. Although the supplier prefers having shared information to either no information or having a privately informed reseller, she is indifferent between the latter two. This finding has also been established in the literature (Li and Zhang 2002).

We now provide the profits of the firms for the case where the supplier has encroachment capability (the detailed derivation is provided in the appendix). In particular, if neither firm has information about the realization of demand, then the reseller's and the supplier's expected profits follow:

$$\Pi_R^{NI} = \frac{2c^2}{9} \text{ and } \Pi_S^{NI} = \frac{3\mu^2 - 6\mu c + 7c^2}{12}.$$

In contrast, if they both have information about the true market size, then their expected profits are:

$$\Pi_R^{SI} = \frac{2c^2}{9} \text{ and } \Pi_S^{SI} = \lambda\frac{3a_H^2 - 6a_H c + 7c^2}{12} + (1-\lambda)\,\frac{3a_L^2 - 6a_L c + 7c^2}{12}.$$

Note that, for both of the above cases of encroachment under symmetric information, the expected profit of the reseller is independent of the market size, and depends only upon the supplier's relative inefficiency, c.[6] The reason for this is that, with symmetric information, the supplier's equilibrium wholesale price induces the reseller to respond by ordering a quantity equal to $\frac{2c}{3}$, and the supplier subsequently sets her own quantity to ensure that the reseller's expected per-unit profit margin is $\frac{c}{3}$. As a result, the supplier's development of encroachment capability alters the reseller's

[6]This is also the case in Arya et al. (2007), though they do not discuss it.

preferences so that instead of preferring shared information to no information, he is indifferent between the two.

However, when the reseller has private information, an encroaching supplier can no longer ensure that the reseller's order quantity and profit margin are invariant with respect to the market size, and it is not obvious whether the reseller's having private information affects the expected profits of the supplier or the reseller. Recall that Π_S and Π_R are the expected profits for the supplier and reseller under encroachment when the reseller has private information that we analyzed in Section 2.4. In what follows, we compare these to the profit functions above and discuss the managerial implications.

2.5.2 Supplier Preference for an Informationally More or Less Capable Reseller

In settings in which a supplier can choose from among multiple potential resellers, it is of interest to understand the conditions under which she should prefer a reseller who is more or less capable of learning the market demand. Specifically, we consider the case where the supplier must choose between one reseller who is informationally more capable and knows the market size perfectly, and another who is less capable who has the same knowledge of the market size as the supplier. While such a choice is stylized, it reflects the reality that resellers are not homogenous in their abilities to collect and interpret data for the purpose of forecasting demand. Recall that, in the absence of encroachment, the supplier is indifferent regarding the reseller's informational capability. However, when the supplier has encroachment capability, we have the following result:

Proposition 8. *When* $c < \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}$*, the supplier prefers a better informed reseller.*

Once the supplier develops the ability to encroach upon the reseller's market, the reseller's knowledge of the true market size matters to the supplier. As long as her own direct selling operations are not too inefficient relative to those of the reseller, she prefers to interact with a reseller who has better knowledge of the market. Note that the condition (i.e., $c < \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}$) specified in Proposition 8 serves as a sufficient condition. From our numerical analysis (see Figure 2.5), however, we can make the following observation:

Observation 1. *When the supplier's direct selling cost exceeds a threshold level of inefficiency, she may prefer to interact with a reseller whose information is identical to her own in order to avoid the adverse effects of downward distortion that would arise with a better informed reseller.*

The above result is related to that of Taylor and Xiao (2010), who show that, for a supplier selling to a newsvendor, the supplier may or may not prefer to have a better informed reseller. However, our result is driven by entirely different forces that exist only in the presence of supplier encroachment.

2.5.3 Endogenous Encroachment and Information Strategies

We now consider how a supplier's decision to develop encroachment capability affects the decision of an existing reseller to develop/maintain the infrastructure that

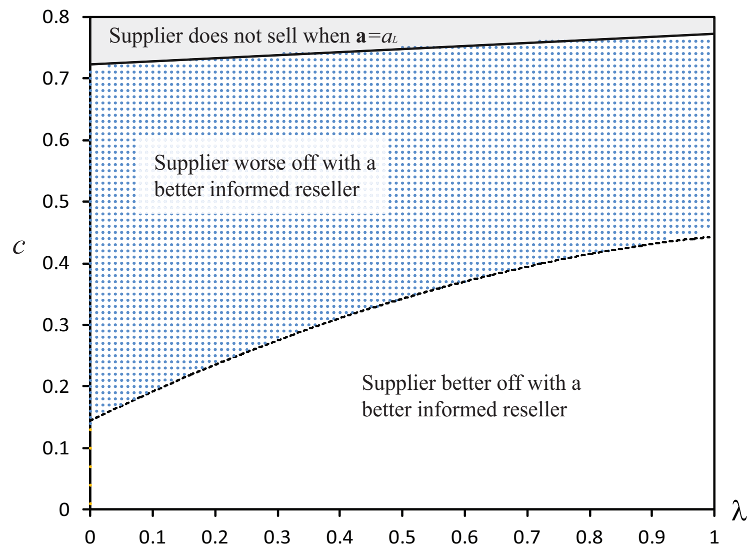


Figure 2.5: Demonstration of the impact of downstream information advantage on the supplier's profitability in the presence of supplier encroachment. In this example, $a_H = 1.35$ and $a_L = 1$.

provides him with better knowledge of the market demand. Specifically, we consider the following sequence of events: First, the supplier determines whether to develop encroachment capability by developing the infrastructure for a direct channel, and this decision is publicly observed. Second, the reseller decides whether to develop advanced informational capability. Third, the supplier observes the reseller's informational capability and sets the wholesale price. Fourth, the reseller responds with an order quantity. Finally, if the supplier has a direct channel, she determines her own volume of direct sales. Note that an alternative sequence of events would be to assume that the reseller decides whether to acquire specific market information after the wholesale price is announced. However, the sequence that we have proposed is more reasonable in environments where the reseller's information advantage is generated from his informational capability and expertise in predicting the demand, so that even if the supplier and the reseller receive the same data about the market demand, an informationally more capable reseller might be better able to interpret it and thus have a more accurate prediction of the true market size. To gain such informational capability would require a relatively long-term development of infrastructure, e.g., software, data structures, human resource capability; whereas, the wholesale price and ordering decisions can be made instantaneously. For simplicity, we normalize to zero the cost incurred by the reseller to develop informational capability (introducing a fixed cost would not change our results qualitatively). To understand how the reseller will choose his information strategy when the supplier develops encroachment capability, we need to compare his expected profits with private information, Π_R, from section 2.4.2 with his expected profits without information, Π_R^{NI}.

Proposition 9. *If* $c \in \left(\frac{3\sqrt{\lambda}(a_H - a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \bar{c}(\lambda)\right\}\right)$, *then the reseller will develop advanced informational capability relative to the supplier, which benefits both himself and the supplier (i.e.,* $\Pi_R > \Pi_R^{NI}$ *and* $\Pi_S > \Pi_S^{NI}$*). Otherwise, if* $c \in \left(0, \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right]$ *or* $c \in \left(\min\left\{\frac{3(1+3\lambda+4\sqrt{\lambda})(a_H - a_L)}{8}, \bar{c}(\lambda)\right\}, \bar{c}(\lambda)\right)$, *then* $\Pi_R < \Pi_R^{NI}$ *and the reseller will choose not to develop advanced informational capability.*

Proposition 21 confirms that even if the supplier has encroachment capability, the reseller can still benefit from having better knowledge of the demand, but only when his efficiency advantage in the selling process is intermediate so that he can avoid substantial distortion of his order quantity. In such a scenario, the dominant effect of private information is that it allows the reseller to tailor his order quantity to the realized market size. Furthermore, the supplier also strictly benefits from the reseller's endogenous decision to be better informed. Recall from the results of Li and Zhang (2002) that this will never occur in the absence of encroachment.

However, Proposition 21 also affirms that under supplier encroachment, the reseller is worse off by having better knowledge when either his advantage in the selling process is small or large. In particular, when $c \in \left(0, \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right]$, the supplier will set a high wholesale price for a privately informed reseller that induces him to order a positive quantity only if the market size is large. Because the supplier is only slightly less efficient than the reseller, she is willing to monopolize the market when demand is small in return for setting a wholesale price that is more precisely targeted at the large market size. This more precisely targeted (higher) wholesale price hurts the reseller. At the other extreme, when the reseller's selling cost advantage is large, i.e., $c \in \left(\min\left\{\frac{3(1+3\lambda+4\sqrt{\lambda})(a_H - a_L)}{8}, \bar{c}(\lambda)\right\}, \bar{c}(\lambda)\right)$, access to private demand information causes him to substantially distort his ordering quantity under low demand, which can result in his earning lower profit in expectation than if he did not have access to the information. Note that to compare the reseller's profits with and without advanced informational capability when $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \bar{c}(\lambda)\right\}, \min\left\{\frac{3(1+3\lambda+4\sqrt{\lambda})(a_H - a_L)}{8}, \bar{c}(\lambda)\right\}\right)$ is technically challenging, but we observe from our numerical analysis (see Figure 2.6) that, within this region, there exists a threshold on the cost, c, above (below) which the reseller is worse off (better off) with advanced informational capability.

In order to understand the impact of a reseller's endogenous information strategy upon the supplier's decision about whether to develop encroachment capability,

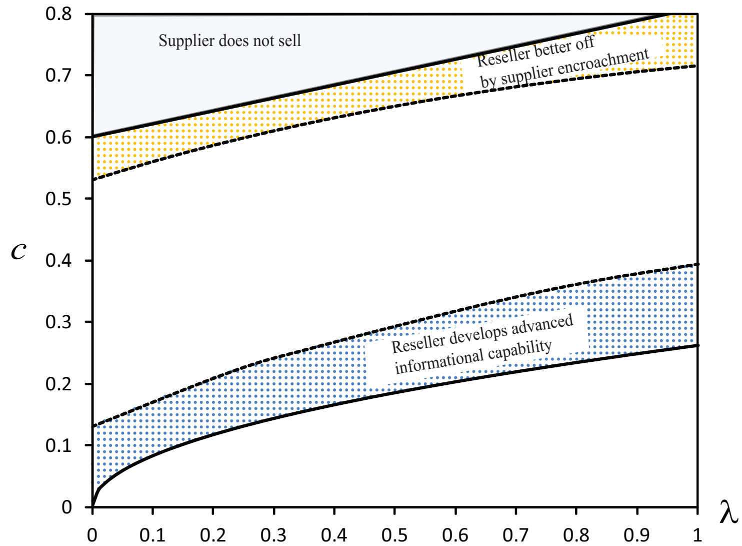


Figure 2.6: Demonstration of the impact of endogenous downstream information strategy on the reseller's profitability in the presence of supplier encroachment. In this example, $a_H = 1.35$ and $a_L = 1$.

we resort to a numerical investigation. For the example depicted in Figure 2.6, where $a_H = 1.35$ and $a_L = 1$, the reseller develops advanced informational capability for only a relatively small range of parameters. However, this is not enough to deter the supplier from encroaching. Regardless of the value of λ, the supplier benefits from encroachment over the entire range of c for which she would exercise her ability to encroach if she could.

To further explore how the development of encroachment capability affects the profits of the supplier and the reseller, we have plotted the ratio of profits with encroachment capability to profits without encroachment capability for the supplier, the reseller, and the supply chain in Figure 2.7. (A profit ratio larger than unity implies a benefit from encroachment capability.) Notice that in all three sub-figures, curves are not smooth. In particular, the plots of the profit ratios shift upward when there is positive variance and c is in the range for which the reseller develops informational capability. This occurs for $c \in (0.18, 0.3)$ for $\sigma^2 = 0.03$ and for $c \in (0.28, 0.48)$ for $\sigma^2 = 0.07$.

There are several things that are worthy of notice in these plots. First, observe that for the supplier, the profit ratio is greater than one over the entire range of c, i.e., she benefits from having encroachment capability even when it discourages the reseller from developing advanced informational capability. Second, notice that for the reseller, outside of the range of c for which he is better informed, greater demand variance reduces his profit ratio, i.e., he does not benefit as much from demand variance under encroachment as he does without it. As a consequence, we can see that as demand variance increases, there is a reduction in the range of c for which the reseller benefits from encroachment, i.e., where his profit ratio exceeds one. Also note that, in the range of c for which the reseller is better informed under $\sigma^2 = 0.03$ (and $\sigma^2 = 0.07$) his profit ratio exceeds that for $\sigma^2 = 0$. This is because when the reseller endogenously develops advanced informational capability, there is little or no ordering distortion, and demand variance reduces the extent to which he is harmed by encroachment, though not by enough to allow him to benefit.

Finally, we can observe that the range of c for which the entire supply chain is harmed by the supplier's development of encroachment capability is increasing in demand variance. This can be confirmed by the fact that, as σ^2 increases, a larger portion of the supply chain profit ratio curve lies below 1.0. This is a result of the fact that, encroachment discourages the development of advanced informational capability at the reseller, while in the absence of encroachment, the total supply chain profit increases when the reseller develops advanced informational capability.

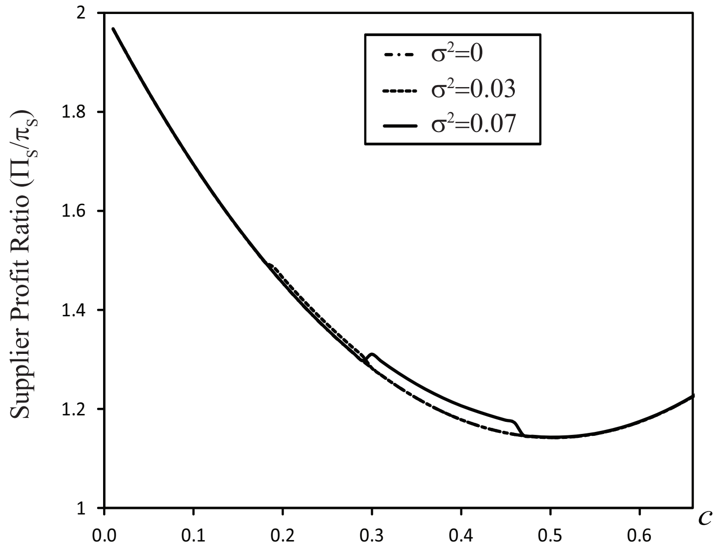
2.5.4 Credible Information Sharing

Thus far, we have assumed that there does not exist a credible mechanism for information sharing. Let us now relax this assumption and consider how the supplier's development of encroachment capability might affect the incentives for both firms to pursue a means for credibly sharing information.

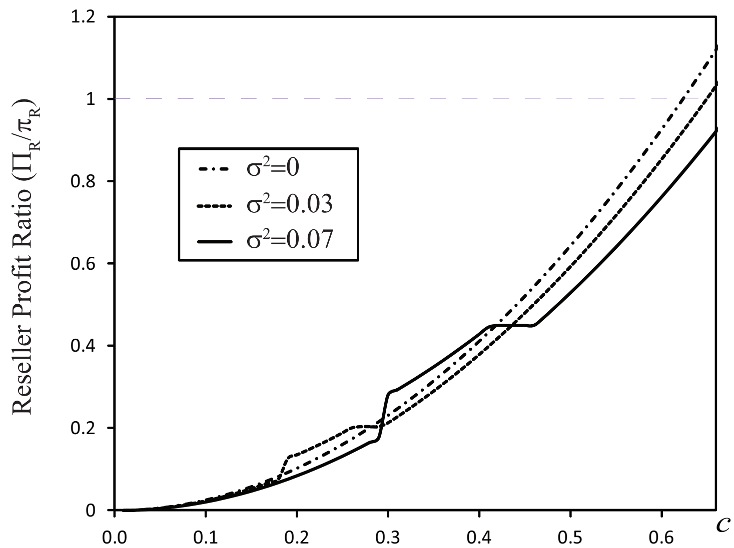
It is well established in the literature that, in the absence of supplier encroachment, $\pi_R \geq \pi_R^{SI}$, while $\pi_S^{SI} \geq \pi_S$, i.e., the reseller prefers being privately informed over having shared information with the supplier, while the supplier would prefer shared information over the reseller's having private information. (See, for example, Li and Zhang 2002.) However, once the supplier has encroachment capability, both the supplier's and the reseller's preferences between information structures can change as described in the following proposition.

Proposition 10. *i) When the supplier has encroachment capability, information sharing always benefits the supplier (i.e., $\Pi_S^{SI} \geq \Pi_S$). ii) However, the reseller prefers to share his information (i.e., $\Pi_R^{SI} > \Pi_R$) when $c \in \left(0, \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right]$ or $c \in \left(\min\left\{\frac{3(1+3\lambda+4\sqrt{\lambda})(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}, \overline{c}(\lambda)\right)$, and she prefers not to share his information (i.e., $\Pi_R > \Pi_R^{SI}$) when $c \in \left(\frac{3\sqrt{\lambda}(a_H-a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}\right)$.*

Knowing the exact market scenario always benefits the supplier because she can set a targeted wholesale price for each market size and avoid reseller order distortion. However, for the reseller, having shared information with the supplier is



(a) $\frac{a_L}{a_H} < \frac{1}{2}$



(b) $\frac{a_L}{a_H} > \frac{1}{2}$

Figure 2.7: The impact of demand uncertainty on the efficiency of the supply chain. In this example, $\lambda = 0.5$; $a_H = a_L = 1.175$ for the scenario with $\sigma^2 = 0$; $a_H = 1.35$ and $a_L = 1$ for the scenario with $\sigma^2 = 0.03$; and $a_H = 1.45$ and $a_L = 0.9$ for the scenario with $\sigma^2 = 0.07$.

a double-edged sword. On one hand, shared information allows the supplier to optimize the wholesale price for each market size. Of course, this hurts the reseller as a consequence of the fact that his profit function is concave in the wholesale price. On the other hand, when an encroaching supplier obtains access to the same market information as the reseller, it alters her strategy in setting the wholesale price so the reseller is induced to sell a positive quantity even when the market size is small and the supplier's cost disadvantage is small (i.e., $c \in \left(0, \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right)$); furthermore, information sharing will also cure the reseller's incentive to distort his order quantity when the supplier's direct selling cost is large (e.g., $c \in \left(\min\left\{\frac{3(1+3\lambda+4\sqrt{\lambda})(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}, \overline{c}(\lambda)\right)$), which will improve the efficiency. As a result, in contrast to the case without supplier encroachment, Proposition 10 asserts that under supplier encroachment, both the supplier and the reseller may benefit from the development of some means by which the reseller can credibly share its demand information. In other words, once the supplier has encroachment capability, a shift from an information structure in which the reseller is privately informed to one in which both firms have shared information is Pareto improving. This is not the case without encroachment, and suggests that the development of mechanisms for credibly sharing information might be more likely to occur in supply chains in which the supplier has its own direct channel.

2.6 Discussion and Conclusion

In this paper, we investigate supplier encroachment in the presence of information asymmetry where the reseller has private information about the market size.

We show that in such a setting, if the supplier uses a linear wholesale price, supplier encroachment can cause the reseller to practice costly signaling and distort his order quantity downwards. In addition, the supplier may increase her wholesale price and thus amplify double marginalization. Consequently, there are parameters for which supplier encroachment leads to "win-win", "win-lose", "lose-win" and "lose-lose" outcomes for the supplier and the reseller. These findings complement existing results that show how supplier encroachment mitigates double marginalization when both firms have the same information.

We also demonstrate that supplier encroachment can have significant implications for information management in supply chains. We find that when the supplier has the ability to encroach, she will strictly prefer to sell to a better informed reseller when her efficiency disadvantage in the selling process is not large; otherwise, she will prefer to sell to an equally informed reseller. This result complements prior literature that has shown that with a reselling channel alone, the supplier is indifferent toward the reseller's state of information. On the other hand, we find that when the supplier has encroachment capability, the reseller may prefer to remain uninformed about demand, which contrasts with existing results that have been obtained without encroachment. We further show that even though encroachment always benefits the supplier after the reseller's information strategy is endogenized, it can hurt the total supply chain performance as the reseller is discouraged from obtaining advanced information. Finally, our study reveals that both the supplier and the reseller may benefit from the development of a mechanism that will allow the reseller to credibly reveal his private demand information, which does not happen in the absence of encroachment.

Of course, our model also has some limitations. First, to avoid unnecessary complications, we have assumed that the reseller releases to the market all of the units that he orders, even if it might be ex-post sub-optimal to do so. If we were to allow the reseller a *free-disposal* option, this will tend to undermine the reseller's ability to commit to a sales quantity, but only when the wholesale price is relatively high. Consequently, a free disposal option plays a role only when both the ratio $\frac{a_H}{a_L}$ is large and the probability of a large market, λ, is sufficiently small. When it does play a role, it causes the reseller to order less for the large market size, and forces him to further distort his order quantity for the small market size. However, our main insights are robust to the free disposal option. For further discussion of this, we refer readers to the appendix.

A second limitation is the fact that we do not consider the possibility that once the supplier develops a direct channel, she may have access to a new source of information. In the appendix, we extend our model to allow for the supplier to receive a noisy signal about demand if she develops encroachment capability. This enables the supplier to tailor her wholesale price according to the signal that she receives. However, so long as the signal is imperfect, the signaling game between the supplier and reseller always arises, and all of our main results continue to hold qualitatively. Of course, it is also possible that both the supplier and the reseller may have imperfect signals about demand. Because it would introduce the possibility of signaling behavior for both the supplier and the reseller, this may alter the dynamics, which is a worthy subject for future research.

Finally, our model assumes that the supplier is limited to a linear wholesale price. Because linear wholesale prices are common in practice and are standard in the literature, it is useful to focus on them initially. However, it is also of interest to understand the implications of encroachment when a supplier can use a more sophisticated pricing mechanism. Under a non-linear pricing policy, the issues shift from signalling to screening. The analysis and insights are fundamentally different, as revealed in Li et al. (2015).

Chapter 3

Supplier Encroachment under Nonlinear Pricing

3.1 Introduction

In practice, firms may sell their products through both intermediaries (reselling channels) and their own direct channels.[1] For instance, electronic product makers may sell their products through third-party retail stores as well as their own stores or websites (e.g., Apple, Sony, Microsoft); apparel and fashion accessory makers may sell their products through independent retailers as well as their own factory outlets (e.g., Coach, Nike, Adidas); airlines and hotels sell tickets and rooms through both travel agencies and their websites (e.g., American Airlines, Hilton). This is not just limited to the large branded companies. More small and local firms also start to use direct sales besides the traditional distribution channels for various products (Reisinger 2012, Blank 2013).

There are a number of reasons why a firm might introduce its own direct channel in addition to relying on resellers (a practice that is often referred to as "supplier encroachment" in the literature), including: obtaining more direct feedbacks from consumers, increasing market size, and obtaining an additional source of

[1] This chapter is based on Li et al. (2015). I appreciate my adviser and co-author Stephen Gilbert and Guoming Lai for their guidance and feedback when I was writing this paper.

leverage with respect to resellers. Our focus is on the last one. In particular, a subtle benefit is revealed in the literature that supplier encroachment can mitigate double marginalization in the reselling channel and result in a "win-win" outcome even if the products sold through the two channels are perfect substitutes. In reaching such a conclusion, most studies assume that a wholesale price only contract (i.e., a linear contract) is adopted in the reselling channel and information is complete. However, this benefit will disappear if the supplier can use a nonlinear price scheme to coordinate the channel. Nonlinear pricing is not uncommon in practice. For instance, many firms offer quantity discounts to their customers. Nonlinear pricing has also been extensively studied in the economics, marketing and operations literatures (e.g., Spence 1977, Jeuland and Shugan 1983, Ha 2001).

In this study, we show that, although a supplier that can use nonlinear pricing gains nothing from the ability to encroach under symmetric demand information, this is not necessarily the case when the reseller has private demand information. Resellers often have access to better demand information than do their suppliers. Not only do they have access to data on a wider range of products, they often possess greater capability in interpreting whatever data is available as a result of their business focus on market mediation. Moreover, these informational advantages may persist even when suppliers operate their own direct channels. While the supplier encroachment literature has explored various factors, the interplay of nonlinear pricing and asymmetric information has not been investigated.

In our model, the supplier's ability to encroach endows her with the option to sell through her own direct channel as well as through a reseller (or both). To focus on how this ability affects her leverage with respect to the reseller, we assume that the products sold through the direct and indirect channels are perfect substitutes. To represent the fact that production must occur before the market clears, we assume that the two firms compete in quantities rather than prices, i.e., Cournot competition. In addition to having an informational advantage, we also assume that the reseller has an efficiency advantage that allows him to sell the product at a lower cost per unit than can the supplier. The sequence of events is as follows: The supplier acts as a Stackelberg leader by announcing a take-it-or-leave-it menu of quantity-price pairs (henceforth referred to as contracts) to the reseller. The reseller responds by choosing one of the price and quantity pairs from the menu. If the supplier has the infrastructure that allows her to have the option to encroach, then she determines her direct selling quantity. Finally, the market clears.

We first show that in the absence of information asymmetry, it is never beneficial for the supplier to possess the ability to encroach upon the reseller when she has the ability to use a nonlinear price scheme to contract with the reseller. With nonlinear pricing, the supplier can already capture the entire supply chain surplus for the first-best quantity while enjoying the efficiency of the reselling channel. Consequently, the only effect of her developing the ability to encroach is that it introduces the potential for her own opportunism, which interferes with her implementing the first-best solution. However, in the presence of information asymmetry, the supplier's ability to encroach is a double-edged sword for the supplier. Although the supplier's ability to encroach upon the reselling channel can reduce the efficiency loss that is required to reduce the information rents surrendered to the reseller, it also creates the possibility of her own opportunistic behavior, which will distort the reseller's order quantity. In contrast to the classical mechanism design problems where distortion occurs only in the less favorable states, in our context, the supplier's potential opportunism can result in distortion for even the most favorable state. Moreover, because the distortion of quantities has implications for both the magnitude of information rents as well as for the supply chain efficiency, the supplier and the reseller may either benefit from, or be hurt by, the supplier's ability to encroach, depending on the reseller's cost advantage in the selling process and the prior distribution of the market size. Specifically, we find that the supplier is always better off with the ability to encroach when the reseller's cost advantage in the selling process is small. When the reseller's cost advantage is intermediate, encroachment capability can be either beneficial or detrimental for the supplier, depending on the prior distribution of the market size. For the reseller, the supplier's ability to encroach always makes him worse off when his cost advantage is small and makes him better off when his cost advantage is intermediate. (When the reseller's cost advantage is sufficiently large, the supplier would never exercise her ability to encroach.) We reveal regions where the supplier's ability to encroach can lead to either "win-lose" or "lose-lose" outcomes for the two parties. These findings are robust to either a discretely or continuously distributed market size.

Hence, our study complements the existing literature on supplier encroachment. We demonstrate that in the presence of information asymmetry, supplier encroachment capability can be helpful even if the supplier can implement a nonlinear price scheme. Moreover, the effects of the supplier's encroachment capability on the two parties' profits are not monotone, and depend critically on the reseller's selling advantage and the information structure.

3.2 Related Literature

Manufacturers selling to multiple channels has been widely observed in practice (Nair and Pleasance 2005). The findings in the academic literature on supplier encroachment are however divided. There are studies that show supplier encroachment reduces the incentive of the resellers to promote the manufacturers' products and dilutes brand image (Fein and Anderson 1997, Frazier and Lassar 1996). Whereas, there is also a stream of research that shows supplier encroachment can improve the system efficiency by alleviating double marginalization. Specifically, Chiang et al. (2003) demonstrate that a supplier's threat to launch and sell through her direct channel can lower the reseller's selling price, while Cattani et al. (2006) and Arya et al. (2007) demonstrate, based on price and quantity competition models, that supplier encroachment can motivate the supplier to lower her wholesale price in the reselling channel. As shown by Tsay and Agrawal (2004), the result that the launch of a direct channel can mitigate double marginalization holds even in a context where the supplier and the reseller can exert sales efforts to promote the demand.

The above literature generally assumes that the supplier can use only a linear price scheme to contract with the reseller. As noted by Arya et al. (2007), if the supplier can alternatively use a nonlinear price scheme, then double marginalization would be completely resolved under the optimal supply contract, and encroachment could not provide strict gains for the manufacturer. While their observation is, of course, correct when both firms have symmetric market size information, we demonstrate that it may not hold when the reseller is endowed with private market size information. Specifically, under nonlinear pricing, encroachment generates a force that pushes the reseller's order quantities upward, and in general we no longer observe the usual "efficiency at the top" that one normally expects for nonlinear pricing problems.

Related to our work, there exist a few studies that explore the incentive of information sharing with different supply chain structures under asymmetric information. For instance, Li (2002) and Zhang (2002) investigate information sharing in a setting where a central supplier sells to multiple competing resellers that have better demand information, while Ha and Tong (2008) and Ha et al. (2011) focus on a setting with two competing supply chains and explore the incentive of each reseller to share information with his supplier. Different from the above studies where the resellers have the same information advantage, Anand and Goyal (2009) and Kong et al. (2013) investigate a one supplier-two competing reseller setting where the incumbent reseller has better information than the other parties. Anand and Goyal (2009) show that the supplier's incentive to leak the information learned from the incumbent reseller to an entrant reseller may block information sharing in the supply chain, while Kong et al. (2013) analyze a revenue sharing scheme to resolve information leakage. Finally, Guo and Iyer (2010) and Guo et al. (2011) investigate the effect of strategic ex post information sharing in a vertical supply chain where a party, either the supplier or the reseller, is able to acquire advanced information.

To our best knowledge, Li et al. (2014) is the only study to examine the impact of information asymmetry in a setting in which the supplier has encroachment capability. In that study, the supplier is assumed to use linear pricing, and it is shown that, as a consequence of the signaling game that the reseller initiates in response to a linear wholesale price, the supplier's ability to encroach can either mitigate or amplify double marginalization in the presence of information asymmetry with a wholesale price only contract. Of course, when the supplier can implement nonlinear pricing, double marginalization is no longer a concern, and that is why our focus is on investigating how encroachment capability affects the information rents and the efficiency of the pricing menu offered by the supplier. Under nonlinear pricing, the supplier's development of encroachment capability can have two opposing effects: By allowing the supplier to sell through the direct channel, it allows the supplier to reduce information rents with less sacrifice of sales volume. However, because the supplier's ability to encroach creates the potential for her own opportunism, it can also result in upward distortion of the quantities sold through the reselling channel for the best realization of market size. This upward distortion is in contrast to the usual *efficiency at the top* that we expect in screening contracts, and it is also distinct from the downward distortion that we observe under linear wholesale pricing.

3.3 The Model

We consider a supplier (she) that can sell her product either through a reseller (he), her direct channel, or both. To focus attention on the coordinating role of supplier encroachment, we assume that the products sold through the two channels are perfect substitutes, and thus adding a direct channel would not affect the total market size. This eliminates the possibility that a direct channel would allow the product to reach a broader set of consumers, which tends to favor the use of the direct channel. Specifically, we assume that the total consumer demand follows a linear, downward sloping function, $P = \mathbf{a} - Q$, where $\mathbf{a}$ represents the market size, Q is the total number of units of the product deployed for sale in the channels, and P is the market clearing price. To incorporate the notion of information asymmetry, we further assume that the market size $\mathbf{a}$ is, ex ante, random, which can be either large ($\mathbf{a} = a_H$) with probability $\lambda_H = \lambda$ or small ($\mathbf{a} = a_L$) with probability $\lambda_L = 1 - \lambda$, where $a_H > a_L > 0$. Denote by $\boldsymbol{\lambda} = [\lambda_H, \lambda_L]$ the vector of these two probabilities for high and low demands. This simple, two-point distribution of demand facilitates the demonstration of our main results. However, to confirm that our qualitative results do not depend upon the two-point distribution, we extend our analysis to a setting with a continuously distributed $\mathbf{a}$ in the appendix.

As in Arya et al. (2007), we assume that, because the supplier is less efficient in retail operations than the reseller, her per unit selling cost is c higher than that for the reseller. Such a premium can arise as a result of the supplier needing to pay higher transportation cost to ship items directly to consumers while the reseller can take advantage of bulk shipping to transport the items in bulk to a traditional retail location. To simplify the presentation of our results, we normalize the selling cost for the reseller to zero, and the selling cost of the supplier to c.

Finally, the supplier is the Stackelberg game leader who can provide a "take-it-or-leave-it" offer with a menu of contracts to the reseller. Without loss of generality, we assume the reseller's reservation profit is zero.

Figure 3.1 details the sequence of events in our model. First, the supplier designs a menu of contracts, $\{(w(a_i), q_R(a_i))\}_{i\in\{H,L\}}$, where $w(a_i)$ is the per unit wholesale price and $q_R(a_i)$ is the corresponding quantity in a contract. That is, if the reseller chooses one specific contract i, then he obtains $q_R(a_i)$ units and pays the supplier $w(a_i)$ per unit. Second, the reseller observes $\mathbf{a} = a_H$ or $\mathbf{a} = a_L$ and chooses one contract from the offer and the contract is executed immediately. Third, based on the contract chosen by the reseller, the supplier then stocks quantity, $q_S(a_i)$, of the product that she will sell through her direct channel. Lastly, the market clearing price P is realized according to $P = a_i - (q_R(a_i) + q_S(a_i))$ for $i \in \{H, L\}$, and the two parties obtain their final profits.

Note that the assumption that the two firms determine their stocking quantities sequentially, i.e., the reseller makes his ordering decision before the supplier determines her own stocking quantity, reflects the reality that the supplier typically has no means of making a credible commitment to not adjust her own stocking quantity in response to the order placed by the reseller. If the supplier does have such capability, then she can specify her own stocking quantity for each quantity-price pair in the menu of contracts that she offers to the reseller. Because this gives the supplier an additional degree of freedom in design of the menu of contracts without introducing the possibility of opportunism, it is intuitive that this would benefit the supplier and hurt the reseller.

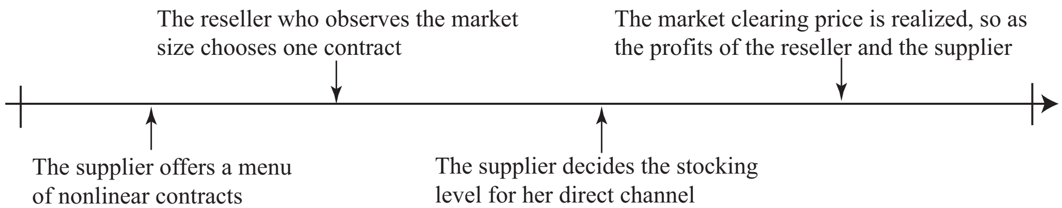


Figure 3.1: The timeline of the model.

3.4 Base Case with Perfect Information

In this section, we analyze a base case with perfect information. Specifically, the supplier and the reseller both know perfectly the realization of the market size, a_i, $i = H, L$. This analysis provides a useful benchmark with which to compare the results we will reveal later for asymmetric information.

We first derive the solution for the case where the supplier does not have the option to encroach upon the reselling channel. As the information is complete, the supplier will offer only one contract, $(w(a_i), q_R(a_i))$, corresponding to the realization of the market size, a_i. If the reseller takes this contract, his sales revenue will be $(a_i - q_R(a_i))q_R(a_i)$, which is maximized at $q_R(a_i) = \frac{a_i}{2}$. It is straightforward now that the optimal contract $(w(a_i), q_R(a_i))$ is equal to $(\frac{a_i}{2}, \frac{a_i}{2})$ for either realization of the market size. Under such a contract, the reseller obtains zero profit, while the supplier captures the entire surplus, $\frac{a_i^2}{4}$. Notice that this contract achieves the largest possible surplus of the system.

Now, we derive the solution for the case where the supplier has the ability to encroach upon the reseller. We apply backward induction. After the reseller takes the contract, $(w(a_i), q_R(a_i))$, the supplier determines her direct selling quantity by solving

$$\max_{q_S}(a_i - q_R(a_i) - q_S - c)q_S,$$

which yields the optimal direct selling quantity

$$q_S(a_i) = \left(\frac{a_i - q_R(a_i) - c}{2}\right)^+. \tag{3.1}$$

Notice that this quantity is a function of both the demand parameter, a_i, and the quantity sold by the reseller, $q_R(a_i)$. Of course, the reseller anticipates this direct selling quantity when he determines how to respond to the quantity-price pair offered by the supplier. Hence, when the supplier designs the contract, she must consider how her own subsequent incentive to encroach will affect the reseller's participation condition. Specifically, the supplier solves:

$$\begin{aligned} \max_{(w(a_i), q_R(a_i))} \quad & w(a_i)q_R(a_i) + (a_i - q_R(a_i) - q_S(a_i) - c)\, q_S(a_i) \\ s.t. \quad & (a_i - q_R(a_i) - q_S(a_i) - w(a_i))\, q_R(a_i) \geq 0. \end{aligned} \tag{3.2}$$

Let us denote by $q_R^{PI}(a_i)$ and $w^{PI}(a_i)$ the optimal solution to the supplier's optimization problem that is defined in (3.2), and let $q_S^{PI}(a_i)$ be the supplier's equilibrium direct selling quantity.

Proposition 11. *With perfect information of the market size a_i, $i = H, L$, and the option of encroachment, the supplier's optimal contract offer and her direct selling quantity are in Table 3.1.*

It is intuitive that when the supplier's selling cost is sufficiently large ($c > \frac{a_i}{2}$), the supplier will not use her direct channel, i.e., encroachment is not a practical

Scenarios	$w^{PI}(a_i)$	$q_R^{PI}(a_i)$	$q_S^{PI}(a_i)$
$c \in (0, \frac{a_i}{3}]$	$\frac{a_i - c}{2}$	$2c$	$\frac{a_i - 3c}{2}$
$c \in (\frac{a_i}{3}, \frac{a_i}{2}]$	c	$a_i - c$	0
$c \in (\frac{a_i}{2}, \infty)$	$\frac{a_i}{2}$	$\frac{a_i}{2}$	0

Table 3.1: Optimal contract offer and direct selling quantity under perfect information and supplier encroachment

option for the supplier. It is straightforward that the optimal contract under such a scenario follows: $(w^{PI}(a_i), q_R^{PI}(a_i)) = (\frac{a_i}{2}, \frac{a_i}{2})$, which achieves the maximum surplus of the system. When the supplier's selling cost is intermediate or small, the supplier's incentive to encroach plays a role. Recall that, when information is complete, the supplier always obtains the entire supply chain surplus under the optimal contract. However, in the presence of the ability to encroach, the supplier may fall victim to her own potential opportunism. That is, in anticipation of the supplier's ex post encroachment, the reseller may be unwilling to take the efficient contract, $(\frac{a_i}{2}, \frac{a_i}{2})$, to procure $\frac{a_i}{2}$ and pay $\frac{a_i^2}{4}$, since doing so would lead to a negative profit for himself. To mitigate this effect, the supplier must either reduce the per unit wholesale price or increase the quantity that is targeted at a reseller who observes a given market size. Note that these two actions have different effects. Reducing the per unit wholesale price would compensate the reseller for the lower retail price that he will receive as a consequence of the supplier's direct sales, whereas increasing the quantity would reduce the supplier's ex post incentive to sell through her direct channel. We can observe from Proposition 11 that the total output $(q_R^{PI}(a_i) + q_S^{PI}(a_i))$ is always greater than the first-best (efficient) quantity, $\frac{a_i}{2}$, for all $c \in (0, \frac{a_i}{2}]$. That is, the maximum supply chain surplus is not achieved in the presence of the supplier's ability to encroach. Proposition 12 formalizes this finding.

Proposition 12. *With perfect information of the market size a_i, $i = H, L$, the supplier (and also the supply chain) is never better off with the option of encroachment. In particular, the supplier is strictly worse off when $0 < c < \frac{a_i}{2}$. The reseller is indifferent as he always obtains zero profit with or without supplier encroachment capability.*

Because of the fact that, with complete information, the supplier can use nonlinear pricing to simultaneously achieve the first-best solution and to extract the full surplus from the reseller, encroachment does not add anything beneficial for the supplier or for the supply chain. In fact, because the ability to encroach creates the unavoidable possibility of supplier opportunism, it can only be detrimental. This result contrasts those revealed in the literature based on a linear price scheme where supplier encroachment can alleviate double marginalization and thus benefit the supplier and the supply chain.

3.5 Analysis with Asymmetric Information

In this section, we analyze our model with a binary distribution of the market size; i.e., $\mathbf{a} = a_H$ (a_L) with probability λ_H (λ_L), ex ante (where $\lambda_H = 1 - \lambda_L = \lambda$). In the absence of supplier encroachment capability, it is reasonable to assume that the reseller has access to better demand information than does the supplier. To represent this, we assume that the reseller observes the true realization of market size, while the supplier knows only the prior distribution at the time that she proposes the pricing policy. When the supplier develops the ability to encroach, this will change the strategic interactions that she has with the reseller. In addition, it may also give her access to her own demand information. To disentangle these two effects, for most of our analysis, we assume that encroachment capability does not alter the information that is available to the supplier before she announces her price policy. However, in the appendix, we confirm that our results are robust with respect to the possibility that, because encroachment capability puts the supplier in direct contact with end consumers, it also provides her with an independent signal about demand.

Let us begin by deriving the solution for the case where the supplier does not have the ability to encroach.

3.5.1 Without the Option of Encroachment

With asymmetric information, the supplier can implement nonlinear pricing through a menu of contracts, $\{(w(a_i), q_R(a_i))\}_{i\in\{H,L\}}$, one targeting the large market size and the other targeting the small market size. Without encroachment, the supplier's problem can be formulated as:

$$\begin{aligned} \Pi_S^N(\boldsymbol{\lambda}) \quad &= \max_{\{(w(a_i),q_R(a_i))\}_{i\in\{H,L\}}} \sum_{i\in\{H,L\}} \lambda_i w(a_i) q_R(a_i) \qquad (3.3) \\ s.t. \quad & \left(a_i - q_R(a_i) - w(a_i)\right) q_R(a_i) \geq 0, \forall i \in \{H, L\}, \\ & \left(a_i - q_R(a_i) - w(a_i)\right) q_R(a_i) \geq \left(a_i - q_R(a_j) - w(a_j)\right) q_R(a_j), \forall i, j \in \{H, L\}. \end{aligned}$$

The supplier designs the contracts to maximize her expected profit by satisfying the reseller's individual rationality (IR) and incentive compatibility (IC) constraints. From the revelation principle, in the optimal solution, the reseller will self-select the menu-option that corresponds to the true demand parameter, and we can represent the reseller's expected profit as:

$$\Pi_R^N(\boldsymbol{\lambda}) = \sum_{i\in\{H,L\}} \lambda_i \left(a_i - q_R^N(a_i) - w^N(a_i)\right) q_R^N(a_i) \tag{3.4}$$

where $q_R^N(a_i)$ and $w^N(a_i)$ for $i \in \{L, H\}$ denote the optimal solution to (3.3). We derive the following proposition from solving (3.3).

Proposition 13. *Without the option of encroachment, the optimal menu of contracts under asymmetric information is in Table 3.2.*

	Small Market Size		Large Market Size	
Scenarios	$w^N(a_L)$	$q_R^N(a_L)$	$w^N(a_H)$	$q_R^N(a_H)$
$\lambda \in (0, \frac{a_L}{a_H})$	$\frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$	$\frac{a_L}{2} - \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$	$\frac{a_H}{2} - \frac{2(a_H-a_L)q_R^N(a_L)}{a_H}$	$\frac{a_H}{2}$
$\lambda \in [\frac{a_L}{a_H}, 1)$	0	0	$\frac{a_H}{2}$	$\frac{a_H}{2}$

Table 3.2: Optimal menu of contracts under asymmetric information without supplier encroachment

Corollary 14. *The above menu of contracts induces the first-best (efficient) quantity $\frac{a_H}{2}$ to be sold when the realized market size is large, and it induces a less than the efficient quantity to be sold when the realized market size is small.*

The two properties in the above corollary are quite standard in mechanism design settings (see, e.g., Moorthy 1984) and are often referred to as "efficiency at the top" and "downward distortion", respectively. This can be observed from the fact that $q_R^N(a_H) = \frac{a_H}{2}$ and $q_R^N(a_L) < \frac{a_L}{2}$ for any λ.

Notice that in our problem, when $\lambda < \frac{a_L}{a_H}$ (or identically $a_L > \lambda a_H$), the possible profit that the supplier can achieve from the small market size is significant and thus it is beneficial for the supplier to offer positive quantities for both of the two market sizes. However, in order to induce the reseller observing the large market size to choose the optimal order quantity $\frac{a_H}{2}$, the supplier has to downward distort the order quantity targeting the small market size by $\frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ from the efficient level $\frac{a_L}{2}$. The supplier can capture the entire supply chain surplus when the market size is small, but she has to surrender some information rents to the reseller when the market size is large. Specifically, the information rents (or the expected profit the reseller obtains) are $\Pi_R^N = \lambda(a_H - a_L)q_R^N(a_L)$.

When $\lambda \geq \frac{a_L}{a_H}$ (or identically $a_L \leq \lambda a_H$), the information rents become sufficiently large so that the supplier prefers to avoid them by foregoing all sales when the market size is small. Consequently, the supplier's pricing policy induces a positive order quantity from only the reseller who observes the large market size. Note that, in this case, the information rents for the reseller are zero. Although the supplier extracts the full supply chain surplus conditional on the market size being large, neither firm earns anything when the market size is small.

3.5.2 With the Option of Encroachment

In many principal-agent settings with asymmetric information (such as the one analyzed in the above), it is possible to rely on the revelation principle in which there exists an optimal menu of contracts such that the principal learns the agent's true type from his choice of contract. However, in our setting with encroachment, because the supplier's ex-post output decision may depend upon the information that she obtains from the reseller's choice of contract, it is possible that the optimal contract will not induce each retailer to reveal his type. Consequently, we must consider two types of contract menus that the supplier can offer: a pooling menu, in which the reseller is offered a single quantity-price pair and accepts it regardless of the observed market size; and a separating menu, in which the reseller chooses a distinct quantity-price pair for each observed market size.

To formalize this, let $\Pi_S^{EP}(\boldsymbol{\lambda})$ be the maximum profit that the supplier can earn under encroachment conditional upon her using a pooling menu that causes the reseller to select a single quantity-price pair regardless of the observed market size, and let $\Pi_S^{ES}(\boldsymbol{\lambda})$ be the maximum profit that the supplier can earn under encroachment conditional upon her using a separating menu that causes the reseller to select a distinct quantity-price pair for each observed market size.

If the supplier offers a pooling menu, then she will not learn the market size from the reseller's response. Consequently, the supplier's output quantity will be:

$$q_S(q_R) = \left(\frac{\lambda a_H + (1-\lambda)a_L - q_R - c}{2}\right)^+ \tag{3.5}$$

and the conditionally optimal pooling contract can be identified as the solution to:

$$\begin{aligned} \Pi_S^{EP}(\boldsymbol{\lambda}) &= \max_{w,q_R} wq_R + (q_S(q_R))^2 \qquad (3.6) \\ s.t. \quad & (a_i - q_R - q_S(q_R) - w)\, q_R \geq 0, \forall i \in \{H, L\}. \end{aligned}$$

Alternatively, if the supplier offers a separating menu, then she will learn the true market size from the reseller's response, and her own optimal output quantity will be tailored to each market size, i.e.:

$$q_S(a_i) = \left(\frac{a_i - q_R(a_i) - c}{2}\right)^+ \tag{3.7}$$

and the conditionally optimal separating menu $\{(w(a_i), q_R(a_i))\}_{i\in\{H,L\}}$ can be identified as the solution to:

$$\begin{aligned} \Pi_S^{ES}(\boldsymbol{\lambda}) &= \max \sum_{i\in\{H,L\}} \lambda_i [w(a_i) q_R(a_i) + (a_i - q_R(a_i) - q_S(a_i) - c)\, q_S(a_i)] \qquad (3.8) \\ s.t. \quad & (a_i - q_R(a_i) - q_S(a_i) - w(a_i))\, q_R(a_i) \geq 0, \forall i \in \{H, L\}, \\ & (a_i - q_R(a_i) - q_S(a_i) - w(a_i))\, q_R(a_i) \geq \\ & (a_i - q_R(a_j) - q_S(a_j) - w(a_j))\, q_R(a_j), \forall i, j \in \{H, L\}. \end{aligned}$$

Note that, in both the pooling and separating menu design problems, the reseller's IR and IC constraints incorporate the supplier's direct selling quantities. However, because the reseller's type is revealed only in the separating menu, it is only there that the supplier can tailor his quantity to the realized market size. We derive the following proposition from solving (3.6) and (3.8).

Proposition 15. *With the option of encroachment, the optimal separating menu of contracts dominates the optimal pooling menu for the supplier, i.e.,* $\Pi_S^E(\boldsymbol{\lambda}) = \Pi_S^{ES}(\boldsymbol{\lambda})$. *The optimal separating menu of contracts is in Table 3.3, with* $w^E(a_L) = \mathbf{I}_{\{q_R^E(a_L)>0\}} \left(a_L - q_R^E(a_L) - q_S^E(a_L)\right)$ *and* $w^E(a_H) = a_H - q_R^E(a_H) - q_S^E(a_H) - \frac{(a_H - a_L) q_R^E(a_L)}{q_R^E(a_H)}$. *The supplier's direct selling quantity is* $q_S^E(a_i) = \left(\frac{a_i - q_R^E(a_i) - c}{2}\right)^+$, $i \in \{H, L\}$.

Under the above optimal separating menu of contracts, the reseller's expected profit is:

$$\Pi_R^E(\boldsymbol{\lambda}) = \sum_{i\in\{H,L\}} \lambda_i \left(a_i - q_R^E(a_i) - q_S^E(a_i) - w^E(a_i)\right) q_R^E(a_i). \qquad (3.9)$$

Corollary 16. *When the supplier has encroachment capability, the optimal nonlinear menu of contracts may no longer induce the reseller who observes the large market*

(a) Small Market Size

Scenarios		$q_R^E(a_L)$
$\lambda \in (0, \frac{a_L}{a_H})$	$c \in (0, \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}]$	$\left(2c - \frac{2\lambda(a_H - a_L)}{1-\lambda}\right)^+$
	$c \in (\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}, \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}]$	$a_L - c$
	$c \in (\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, +\infty)$	$\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$
$\lambda \in [\frac{a_L}{a_H}, 1)$	$c \in (0, \infty)$	0

(b) Large Market Size

Scenarios	$q_R^E(a_H)$
$c \in (0, \frac{a_H}{3}]$	$2c$
$c \in (\frac{a_H}{3}, \frac{a_H}{2}]$	$a_H - c$
$c \in (\frac{a_H}{2}, +\infty)$	$\frac{a_H}{2}$

Table 3.3: Optimal menu of contracts under asymmetric information and supplier encroachment

size to order the efficient quantity, $\frac{a_H}{2}$*, i.e., the optimal menu may lack the efficiency at the top property.*

The above Corollary highlights the fact that the reseller's willingness to pay for any given quantity is adversely affected by the supplier's own incentive to behave opportunistically after the reseller accepts the contract. Consequently, for the large market size, the supplier may no longer offer the efficient quantity $\frac{a_H}{2}$.

This result is driven by the fact that the supplier cannot pre-commit to her own output quantity, and her ex-post optimal quantity response is a function of the quantity that she sells to the reseller. In particular, it can be verified that $q_R^E(a_H)$ is the value of $q_R(a_H)$ that maximizes the total supply chain profit conditional upon the supplier selling $\left(\frac{a_H - q_R(a_H) - c}{2}\right)^+$ through her direct channel. Thus, although $q_R^E(a_H)$ is *conditionally* efficient, it is not *absolutely* efficient, i.e., it differs from the first-best solution. It can be confirmed that once we incorporate the functional form of the supplier's ex-post optimal direct selling quantity response into the reseller's utility function, which forms the basis for the IR and IC constraints, we continue to have the single crossing property in which a reseller's preference for a larger quantity is increasing in the size of market that he observes. In addition, the supplier's objective function is separable and concave in the quantities offered. Because of this structure, the solution to the mechanism design problem defined in (3.8) does have the efficiency at the top and downward distortion properies relative to this conditional optimization problem, but it may not have either of these properties relative to the first-best solution.

From Proposition 15, we can notice that when the supplier's selling cost is low ($c \leq \frac{a_H}{4}$), she sets $q_R^E(a_H)$ to less than the efficient quantity, and partially compensates by relying on her own direct channel.[2] But when her selling cost increases to the range, $c \in (\frac{a_H}{4}, \frac{a_H}{2})$, she sets $q_R^E(a_H)$ above the efficient quantity in order to credibly commit to limiting her own subsequent sales through her direct channel. For the small market size, the supplier's choice of $q_R^E(a_L)$ still involves the trade-off between information rents ($\Pi_R^E = \lambda(a_H - a_L)q_R^E(a_L)$) and wholesale revenue from the small market, but her ability to encroach gives her the ability to generate sales revenue from the small market without increasing information rents, especially when her selling cost is low. When $\lambda \geq \frac{a_L}{a_H}$, similar to the case without the ability of encroachment, the supplier does not induce the reseller to sell anything to the small market.

[2]Our solution requires that $c > 0$. Notice that in the limiting case of $c = 0$, the quantity for the reseller converges to $q_R^E(a_L) = q_R^E(a_H) = 0$, which would preclude the supplier from obtaining the market information from the reseller's order.

However, for $\lambda < \frac{a_L}{a_H}$, different from the case without the option of encroachment, the supplier will induce the reseller to order a positive quantity for the small market only if the reseller's cost advantage is sufficiently large, i.e., when $c > \frac{\lambda(a_H - a_L)}{(1-\lambda)}$. In such a scenario, the value that the supplier can obtain by selling through the reselling channel under the small market size outweighs the information rents that will be introduced. Otherwise, the supplier will prefer to forego any sales through the reselling channel and rely entirely upon her direct channel, when the market size is small. By doing so, the supplier saves the information rents that would otherwise be surrendered to the reseller under the contract targeting the large market size while still capturing some amount of sales through her direct channel. That is, in contrast to the complete information setting, supplier encroachment can be helpful from the perspective of reducing the information rents paid to the reseller under asymmetric information.

Hence, supplier encroachment can have two distinct effects on the supplier's profit under a nonlinear price scheme with asymmetric information. On the one hand, it can help reduce the information rents surrendered to the reseller; on the other hand, it can worsen the ordering distortion in the reselling channel. Below, we characterize when having the option to encroach benefits or hurts the supplier (i.e., comparing the supplier's profit under the decisions given by Propositions 13 and 15). We divide the comparison into two cases with $\lambda \geq \frac{a_L}{a_H}$ and $\lambda < \frac{a_L}{a_H}$ since the structures of the contracts differ significantly.

Proposition 17. *When $\lambda \geq \frac{a_L}{a_H}$, if $\frac{a_L}{a_H} \leq \frac{1}{2}$, then there exists one threshold c'_S such that encroachment capability makes the supplier strictly better off when $0 < c < c'_S$,* *weakly worse off when* $c'_S \leq c < \frac{a_H}{2}$*, and has no effect when* $c \geq \frac{a_H}{2}$*. If* $\frac{a_L}{a_H} > \frac{1}{2}$*, then there exist two thresholds* c'_S *and* c''_S *such that encroachment capability makes the supplier strictly better off when* $0 < c < c'_S$ *and* $c''_S < c < a_L$*, weakly worse off when* $c'_S \leq c \leq c''_S$*, and has no effect when* $c \geq a_L$*.*

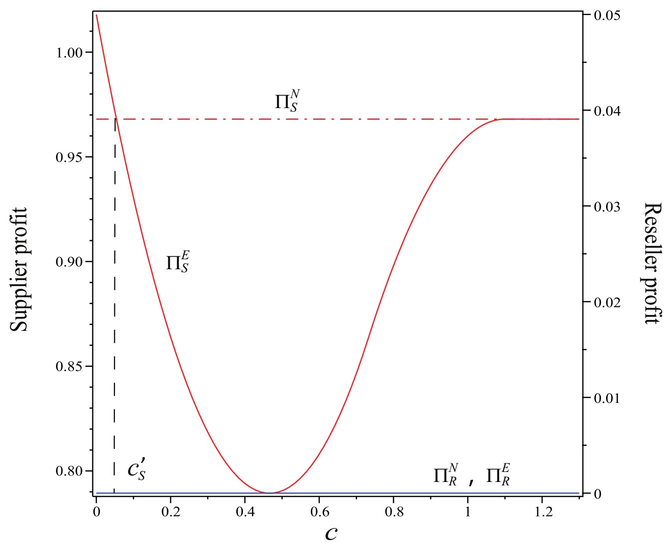
When $\lambda \geq \frac{a_L}{a_H}$, the supplier sells through the reseller only if the market size is large, regardless whether she has the option to encroach. It is apparent that, conditional upon the market size being small, the supplier can gain extra profit by having the option to sell directly. This gain however decreases as the supplier's direct selling cost (c) increases. On the other hand, conditional upon the market size being large, the supplier's ability to encroach can reduce her profit due to the ordering distortion in the reselling channel. Notice that this distortion loss is minimal when the supplier's direct selling cost is either small or large, while it can be significant when the supplier's selling cost is intermediate. As a result, it is intuitive that the supplier is better off by having the option to encroach when c is sufficiently small, and that there exists some intermediate range of c for which she is worse off. However, as c approaches $\frac{a_H}{2}$, she may be either better off or worse off. Specifically, when $\frac{a_L}{a_H} > \frac{1}{2}$, there exists a range, $c''_S < c < a_L$, for which the supplier is also better off by having the ability to sell directly. In this range, the extra profit the supplier obtains by selling directly for the small market size outweighs the cost of upward distortion for the large market size. In contrast, when $\frac{a_L}{a_H} \leq \frac{1}{2}$, the supplier's direct selling quantity for the small market size would be zero when c is close to $\frac{a_H}{2}$ and thus such a better-off region will not appear. Finally, for any $c \geq \max\{\frac{a_H}{2}, a_L\}$, the supplier will never sell directly as it is too costly. Consequently, she does not need to upward distort the reselling quantity for the large market size. Hence, the supplier is indifferent toward encroachment when $c \geq \max\{\frac{a_H}{2}, a_L\}$.

Proposition 17 in Figure 3.2. In this experiment, $\lambda = 0.80 > \frac{a_L}{a_H}$. Recall from Propositions 13 and 15 that this implies that, regardless of whether the supplier has encroachment capability, she sells nothing to the reseller when demand is low and she extracts the full surplus from the reseller, i.e., $\Pi_R^N = \Pi_R^E = 0$. In plot (a), we have $\frac{a_L}{a_H} < \frac{1}{2}$, while in plot (b), we have $\frac{a_L}{a_H} > \frac{1}{2}$. There is one threshold c in plot (a) that divides the regions where the supplier is better off and worse off by encroachment; and in plot (b), in addition to the pattern in plot (a), the supplier can also be better off when c is close to $\frac{a_H}{2}$.

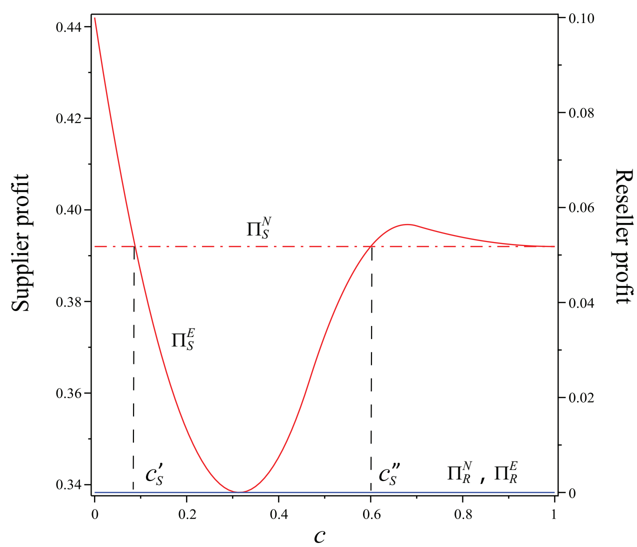
The comparison becomes relatively more involved when $\lambda < \frac{a_L}{a_H}$ under which the supplier sells through the reseller for both market sizes. We derive the following proposition.

Proposition 18. *When $\lambda < \frac{a_L}{a_H}$, the effect of encroachment capability upon the supplier can be characterized according to three thresholds: $c'_S \leq c''_S \leq c'''_S$, such that encroachment capability makes the supplier strictly better off when $0 < c < c'_S$ or $c''_S < c < c'''_S$, weakly worse off when $c'_S \leq c \leq c''_S$ or $c'''_S \leq c < \max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$, and has no effect when $c \geq \max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$.*

First, notice that supplier encroachment makes the supplier better off when c is small ($0 < c < c'_S$), worse off when c is relatively large (close to but smaller than $\max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$), and indifferent when c exceeds $\max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$. In particular, when c is small, the direct channel is nearly as efficient as the reselling



(a) $\frac{a_L}{a_H} < \frac{1}{2}$



(b) $\frac{a_L}{a_H} > \frac{1}{2}$

Figure 3.2: The effects of supplier encroachment and asymmetry information on the supply chain parties' profits when $\lambda \geq \frac{a_L}{a_H}$. The parameters are: $\lambda = 0.8$, $(a_L, a_H) = (1, 2.2)$ in the left plot, and $(a_L, a_H) = (1, 1.4)$ in the right plot.

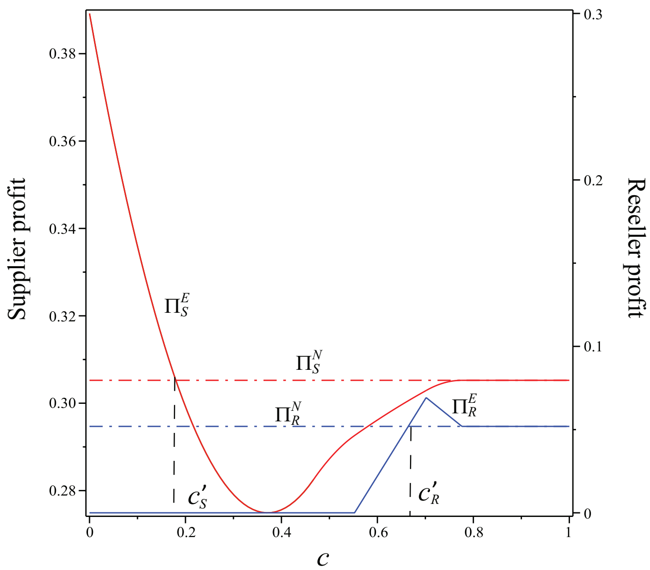
channel. In such a scenario, having the option to sell directly allows the supplier to reduce the quantity that she sells through the reseller for the small market size. Because the information rents are linearly increasing in $q_R(a_L)$, this reduces the information rents available to the reseller while losing little in the selling process, which benefits the supplier. However, as the direct channel becomes less efficient, the direct channel can lead to upward distortion of the quantities for both the large and small market sizes. This means that the direct channel not only introduces inefficient upward distortion for the large market size, it also increases the information rents. In particular, there exists a region close to $\max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$, where the supplier is worse off by having the option to encroach. In this region, the direct channel is relatively inefficient and is used only sparingly. The costs associated with the increment of information rents and the upward distortion exceed the benefit from direct sales. Obviously, as c continues to increase, the supplier becomes less and less efficient relative to the reseller, and the potential impact of the direct channel eventually vanishes. Indeed, when c is greater than $\max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$, the supplier does not use the direct channel and is indifferent toward the option to encroach.

Second, it is interesting to notice that in the middle range of c, supplier encroachment can be first detrimental for the supplier ($c'_S \leq c \leq c''_S$) and then become beneficial ($c''_S < c < c'''_S$), as c increases. Such a result can arise because both the benefit from reducing the information rents and gaining direct sales and the cost of upward distortion are not monotone in c. In fact, for the large market size, as c increases from zero, the cost of upward distortion caused by encroachment first increases from zero to some large value and then gradually decreases to zero. On the other hand, for the small market size, as c increases from zero, supplier encroachment first reduces the information rents and, at the same time, gains the supplier some extra profit from the direct sales. However, as c increases, these benefits decrease, and when c reaches a critical level, upward distortion in the reselling quantity may appear, and this increases the information rents. The above effects make it possible that in the middle range of c, supplier encroachment can first reduce the supplier's profit and then increase her profit relative to what it would be without encroachment. Note that two or three of these thresholds may coincide with each other depending on the prior distribution of the market sizes.

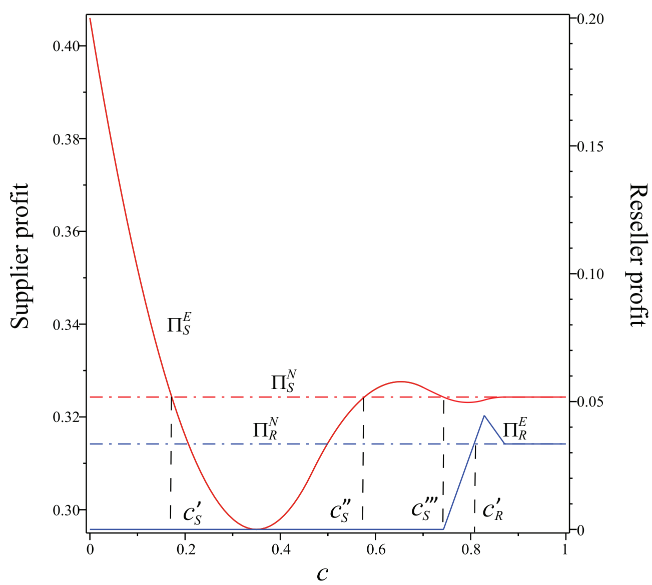
We demonstrate the results of Proposition 18 numerically in Figure 3.3. We can observe from the left plot that when $\lambda = 0.58$, the three thresholds are indistinguishable; as c increases, encroachment capability first makes the supplier better off, then worse off, and eventually has no effect. In contrast, in the right plot where $\lambda = 0.65$, there are three distinct thresholds below $\max\{\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2}\}$; as c increases, encroachment capability first makes the supplier better off, then worse off, then better off (again), then worse off (again), and eventually has no effect.

While the above discussion is from the supplier's perspective, the effects of supplier encroachment on the reseller's profit can be easily understood through the impact that is upon information rents and quantity distortion. The comparison between the gain and the loss for the reseller is, in fact, much simpler.

Proposition 19. *When $\lambda \geq \frac{a_L}{a_H}$, the reseller always obtains zero profit with or without supplier encroachment. When $\lambda < \frac{a_L}{a_H}$, there exists one threshold c'_R such*



(a) $\lambda = 0.58$



(b) $\lambda = 0.65$

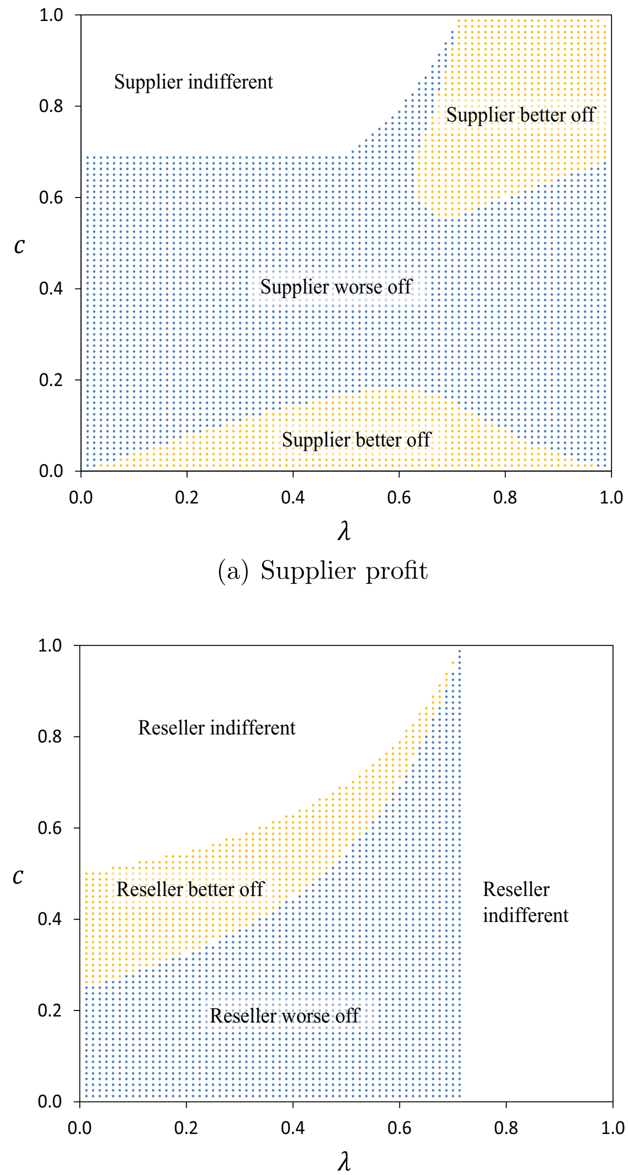
Figure 3.3: The effects of supplier encroachment and asymmetry information on the supply chain parties' profits when $\lambda < \frac{a_L}{a_H}$. The parameters are: $a_L = 1$ and $a_H = 1.4$.

that the reseller is weakly worse off when $c \leq c'_R$, strictly better off when $c'_R < c < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, and indifferent when $c \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, with supplier encroachment compared to without.

When $\lambda \geq \frac{a_L}{a_H}$, the supplier does not sell through the reseller for the small market size, with or without supplier encroachment, which yields zero information rent to the reseller. Hence the reseller's profit does not depend upon whether the supplier has the option to encroach. When $\lambda < \frac{a_L}{a_H}$, the supplier sells through the reseller when the market size is small, which yields positive information rents to the reseller. Note that the information rents increase linearly in the order quantity $q_R(a_L)$ targeting the small market size. We can thus find a threshold c'_R such that this ordering quantity with supplier encroachment is smaller than that without when $c \leq c'_R$, while it is the reverse when $c'_R < c < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ due to upward distortion. For any $c \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, the supplier does not use the direct channel and thus the reseller is indifferent whether the supplier has or has not the option to encroach. The results of Proposition 19 are depicted in Figures 3.2-3.3.

To have a full comparison of the parties' profits with and without supplier encroachment, we generate Figure 3.4 that shows the regions where the supplier and the reseller are better off, worse off, or indifferent by supplier encroachment with respect to c and λ.

It is of interest to compare these results to those of Arya et al. (2007), who study supplier encroachment under linear wholesale pricing and symmetric demand information. Recall that they find that the supplier is weakly better off with en-



(a) Supplier profit

(b) Reseller profit

Figure 3.4: Illustration of the regions where the supplier and the reseller are better off, worse off, and indifferent by supplier encroachment. The parameters are: $a_L = 1$ and $a_H = 1.4$.

croachment capability, and that there exists an intermediate range of cost advantage for the reseller for which he too is better off. However, they acknowledge that the benefits of encroachment disappear when the supplier can implement nonlinear pricing. In contrast, we have demonstrated that, when the information is asymmetric, the ability to encroach can either enhance or hinder a supplier's nonlinear pricing policy, depending upon the distribution of the demand parameter and her cost disadvantage.

Finally, it is natural to question how supplier encroachment affects the total supply chain surplus in our context. While it is challenging to assess the effects analytically, we can clearly observe from Figures 3.2-3.3 that the total supply chain surplus can be either increased or reduced by supplier encroachment for different parameters. The underlying intuition is similar to what we have explained for the results from the perspectives of the supplier and the reseller.

3.6 Extensions

We now discuss two extensions to our original model. In the first, to test the robustness of our model, we allow for the possibility that the reseller can freely dispose of units that he orders, so that he does not necessarily sell everything if it is ex-post suboptimal to do so. In the second extension, we consider how the supplier's encroachment capability will affect the reseller's willingness to acquire private demand information.

3.6.1 Free Disposal by the Reseller

It is easy to notice that without supplier encroachment, the reseller will not withhold any units if he orders optimally. In the following, we discuss the case in the presence of encroachment where the supplier offers a separating menu of contracts. The timeline of the game will be: First, the supplier offers a menu of contracts $\{(w(a_i), q_R(a_i))\}_{i\in\{H,L\}}$ to the reseller who then selects one to order. Second, based on the reseller's contract choice, the supplier prepares a direct selling quantity $q_S(a_i)$. Finally, the reseller and the supplier simultaneously determine the quantities to sell and dispose of any units they withhold.

A straightforward analysis of a simultaneous move Cournot competition can reveal that without any constraint, the optimal selling quantities of the reseller and the supplier should be $\frac{a_i+c}{3}$ and $\frac{a_i-2c}{3}$, respectively, for each market size. Certainly, in our context, the reseller cannot sell more than what he orders, i.e., his selling quantity will be $\min\{\frac{a_i+c}{3}, q_R(a_i)\}$. Therefore, comparing with the results in section 3.5.2, we can notice that the free disposal option will have an impact on the reseller's selling quantity when c is intermediate. Denote by $E = \max\{\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_L}{2}\}$, we have the following proposition.

Proposition 20. *With the reseller's option of free disposal, the optimal separating menu of contracts under asymmetric information and supplier encroachment is in Table 3.4, with* $w^E(a_L) = \mathbf{I}_{\{q_R^E(a_L)>0\}}\left(a_L - q_R^E(a_L) - q_S^E(a_L)\right)$ *and* $w^E(a_H) = a_H - q_R^E(a_H) - q_S^E(a_H) - \frac{(a_H-a_L)q_R^E(a_L)}{q_R^E(a_H)}$. *The supplier's direct selling quantity is* $q_S^E(a_i) = \left(\frac{a_i - q_R^E(a_i) - c}{2}\right)^+$, $i \in \{H, L\}$.

(a) Small Market Size

Scenarios		$q_R^E(a_L)$
$\lambda \in (0, \frac{a_L}{a_H})$	$c \in (0, \min\{\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_L}{5} + \frac{6\lambda(a_H-a_L)}{5(1-\lambda)}\}]$	$\left(2c - \frac{2\lambda(a_H-a_L)}{1-\lambda}\right)^+$
	$c \in (\min\{\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_L}{5} + \frac{6\lambda(a_H-a_L)}{5(1-\lambda)}\}, E]$	$\frac{a_L+c}{3}$
	$c \in (E, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}]$	$a_L - c$
	$c \in (\frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}, +\infty)$	$\frac{a_L}{2} - \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$
$\lambda \in [\frac{a_L}{a_H}, 1)$	$c \in (0, \infty)$	0

(b) Large Market Size

Scenarios	$q_R^E(a_H)$
$c \in (0, \frac{a_H}{5}]$	$2c$
$c \in (\frac{a_H}{5}, \frac{a_H}{2}]$	$\frac{a_H+c}{3}$
$c \in (\frac{a_H}{2}, +\infty)$	$\frac{a_H}{2}$

Table 3.4: Optimal separating menu of contracts under asymmetric information and supplier encroachment

Comparing with Proposition 15, we can verify that the supplier will be worse off when the reseller has the option of free disposal than without. The supplier now faces more constraints when optimizing the menu of contracts. Intuitively, free disposal limits the reseller's commitment power on his selling quantity, which enhances the supplier's ex-post encroaching incentive. From Figure A.1, we can observe that when $\frac{a_H}{5} < c < \frac{a_H}{2}$, the supplier's profit with free disposal is significantly lower than that without. Note that the reseller is also (weakly) worse off by the option of free disposal as his order quantity for the small market size becomes smaller.

It is worth noting that, because the reseller incurs no cost for obtaining the product other than the wholesale price, upward distortion will not arise in our current model if the reseller has the option of free disposal and cannot commit to selling everything that he orders. However, if the reseller incurs positive handling costs, then upward distortion can still be present. For example, suppose the reseller has a traditional, bricks and mortar, operation so that he incurs a positive per unit logistics handling cost c_R as soon as he orders and takes delivery, and the supplier sells through a direct online channel. Because the supplier's online channel requires her to ship units that she sells to individual consumers, her logistics costs are not only higher, i.e., $c_S > c_R$, they are also incurred at the point of sale rather than at the time that the units are produced. As long as we assume that the reseller can commit to selling everything that he orders, i.e., no free disposal, the reseller's (supplier's) cost can be normalized to zero ($c = c_S - c_R$), and this is exactly what we have done in the base model. However, when we allow for the reseller to have the free disposal option, his positive logistics cost will play an interesting role. Notice that with this logistics cost, the efficient quantity for the reseller to order and sell under complete information will become $\frac{a_i - c_R}{2}$. However, in the case with asymmetric information, at the second stage when the supplier and the reseller simultaneously choose their amounts to sell, this logistics cost is already sunk and will not play a role in the reseller's decision. As a result, if the reseller is not constrained by his order quantity, then he will sell $\frac{a_H + c_S}{3} = \frac{a_H + c_R + c}{3}$ when the market size is large. Clearly, supplier encroachment can still induce the reseller to sell more than his efficient quantity if $\frac{a_H - c_R}{2} < \frac{a_H + c_R + c}{3}$, or equivalently, $a_H < 5c_R + 2c$.

3.6.2 Downstream Information Acquisition

In the above analysis, we have assumed that the reseller knows the true market size. Here, we investigate how downstream information acquisition affects the two

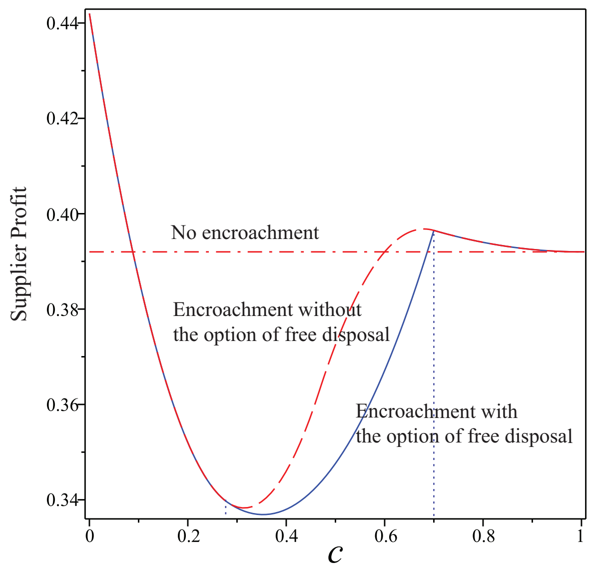


Figure 3.5: Demonstration of the impact of the reseller's free disposal option. In this example, $\lambda = 0.8$, $a_H = 1.4$ and $a_L = 1$. The dash-dotted curve: the supplier's profit without the option of encroachment; The dashed curve: the supplier's profit with the option of encroachment but without the reseller's option of free disposal; The solid curve: the supplier's profit with the option of encroachment and the reseller's option of free disposal.

parties' profits in the presence of supplier encroachment. To do so, we assume that without acquiring any information, the reseller has the same information set as does the supplier (i.e., both of them use the prior distribution of the market size, $\Pr(\mathbf{a} = a_H) = \lambda_H$ and $\Pr(\mathbf{a} = a_L) = \lambda_L$ with $\lambda_H = 1 - \lambda_L = \lambda$, to make their contracting and stocking decisions). Further, we assume that information acquisition is costless.

When neither firm observes the market size before the reseller orders, the supplier will offer one contract to the reseller that depends only on the expected market size. Specifically, let $(w(\mu), q_R(\mu))$ denote the contract where $\mu = \lambda a_H + (1 - \lambda)\, a_L$ is the expected market size. Because the reseller's order quantity conveys no information about demand, after the reseller accepts the contract, the supplier's direct selling quantity will be:

$$q_S(\mu) = \left(\frac{\mu - q_R(\mu) - c}{2} \right)^+ .$$

Consequently, the supplier's nonlinear pricing problem can be stated as:

$$\begin{aligned} \max_{(w(\mu), q_R(\mu))} \quad & w(\mu) q_R(\mu) + (\mu - q_R(\mu) - q_S(\mu) - c)\, q_S(\mu) \\ s.t. \quad & (\mu - q_R(\mu) - q_S(\mu) - w(\mu))\, q_R(\mu) \geq 0. \end{aligned}$$

The solution follows the same format as that in Proposition 11 with a_i replaced by μ.

Comparing the two parties' profits with and without downstream information acquisition, we obtain the following proposition.

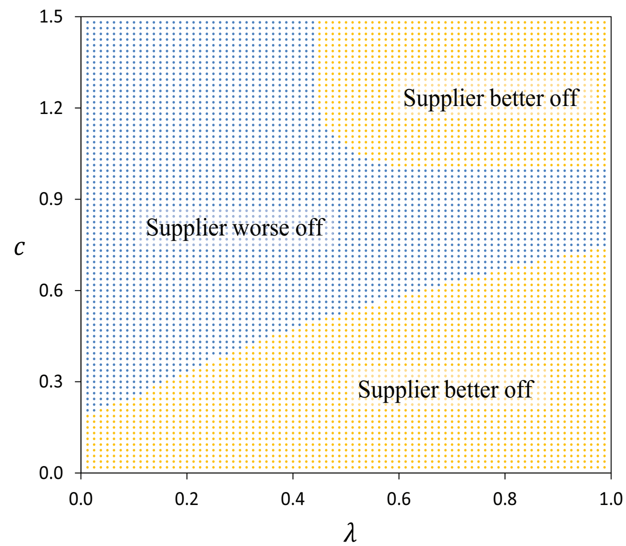
Proposition 21. *(i) For the reseller, when $\lambda \geq \frac{a_L}{a_H}$, he is indifferent toward acquiring information (since he always obtains zero expected profit); when $\lambda < \frac{a_L}{a_H}$, there exists a threshold c_R^I such that he is better off by acquiring information when $c > c_R^I$ and indifferent when $c \leq c_R^I$.*
(ii) For the supplier, there exist $\underline{c}_S^I$, $\overline{c}_S^I$ and $\overline{\lambda}$such that she is strictly better off by downstream information acquisition when $c < \underline{c}_S^I$, or $c > \overline{c}_S^I$ and $\lambda > \overline{\lambda}$.

For the reseller, without information acquisition, he will always obtain zero expected profit under the supplier's optimal nonlinear contract. In contrast, as revealed in Proposition 19, under some parameters, the reseller is able to obtain a positive expected profit when he has private information of the market size. Hence, the reseller is always (weakly) better off by acquiring the market information.

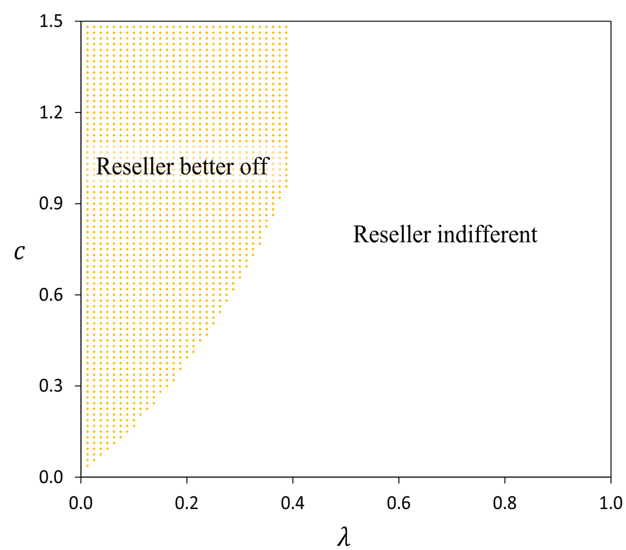
For the supplier, however, the reseller acquiring the market information can be a double-edged sword. On one hand, the supplier can screen out the market information and thus make more accurate contracting and direct selling decisions; on the other hand, the supplier will have to surrender some information rents to the reseller. Proposition 21(ii) provides two sufficient conditions for when downstream information acquisition benefits the supplier. In particular, when c is sufficiently small ($c < \underline{c}_S^I$), the direct channel will be very efficient, which limits the information rents paid to the reseller. In such a scenario, downstream information acquisition always benefits the supplier. When c is sufficiently large ($c > \overline{c}_S^I$) which prevents encroachment, downstream information acquisition benefits the supplier if the market size is very likely to be large ($\lambda > \overline{\lambda}$). In such a scenario, the gain from accurately contracting outweighs the information rents paid to the reseller. When these conditions do not hold, downstream information acquisition may make the supplier strictly worse off. We demonstrate the possible outcomes in Figure 3.6.

3.7 Conclusion and Discussion

The main contribution of our paper is to identify the complex trade-offs that are involved when a supplier develops encroachment capability in contexts where resellers have private demand information and nonlinear pricing can be implemented. Although encroachment capability provides the supplier with a more refined mechanism for managing information rents, it also introduces the possibility of her own opportunism, which can lead to inefficient distortions in the quantities sold through the reselling channel. As a consequence of these complex interactions, it is possible



(a) Supplier profit



(b) Reseller profit

Figure 3.6: Illustration of the regions where downstream information acquisition benefits or hurts the supplier and the reseller in the presence of supplier encroachment. The parameters are: $a_L = 1$ and $a_H = 2.5$.

for either/neither the supplier or/nor the reseller to benefit from encroachment.

In particular, we find that with some parameters, supplier encroachment can reduce the amount of efficiency that the supplier must sacrifice in order to reduce information rents received by the reseller. On the other hand, we also find that, because supplier encroachment capability allows the supplier to make an ex-post output decision, it may cause inefficient distortions in the quantities sold through the reselling channel. Specifically, the supplier's own potential to behave opportunistically in determining her own direct selling quantity can cause inefficient distortions in the quantities sold through the reselling channel.

From a practical perspective, our results clearly refute existing results that suggest that supplier encroachment would have no impact when a supplier can use nonlinear pricing. Indeed, we have shown that, if the supplier's direct selling channel is sufficiently efficient, then she can always benefit from developing encroachment capability, even if she is using nonlinear pricing. (For specific parameters, she may also benefit when her direct channel is at intermediate levels of efficiency.) Yet our results also highlight the dark-side of encroachment; there exists a moderate range of direct channel efficiency for which the supplier's ability to encroach renders both the supplier and the supply chain worse off.

In presenting our analysis, we have tried to simplify our model as much as possible to highlight the trade-offs that encroachment creates for the supplier between her enhanced ability to control information rents and the introduction of potential opportunism. However, in sections A.3 and 3.6.2, we consider two extensions of our base model. In the first, we confirm that our main results continue to hold, if the reseller can freely dispose of units that he acquires from the supplier when it is not optimal for him to release them all to the market. In the latter extension, we analyze how the supplier's encroachment capability affects the reseller's willingness to acquire information about the market size. Interestingly, although the reseller always at least weakly prefers to acquire information, the supplier may either benefit from or be hurt by downstream information acquisition.

Of course, when the supplier develops a direct channel to provide her with encroachment capability, her direct interactions with the market may provide her with a source of information that is independent from the reseller. In the appendix, we model this as a noisy signal of the true market size, and we show that the independent source of information has both a direct effect and an indirect effect. The direct effect is that the supplier can tailor her pricing menu according to the signal that she receives, and this helps to reduce information rents. In the extreme case where the signal is perfectly accurate, the information rents are eliminated. The indirect effect arises from the fact that, because the signal affects the price-quantity pairs that are offered to the reseller, it indirectly influences the supplier's direct selling quantity. Thus, the accuracy of the demand signal indirectly affects the supplier's direct selling quantity in spite of the fact that she is fully informed of the market size (via the reseller's order) at the time that she determines the quantity to sell directly. As a consequence of these two effects, the supplier may or may not benefit from the independent demand signal. More interesting is the observation that, for certain parameters (relatively high values of both c and λ), the reseller benefits from the supplier's development of encroachment capability only if it results in the supplier obtaining an independent source of demand information. Finally, the appendix also includes a demonstration that our qualitative results continue to hold when the market size follows a continuous (uniform) distribution.

Chapter 4

Platform Integration with First-Party Applications

4.1 Introduction

Platform-based technologies such as computer operating systems (e.g., Windows, Mac OS, and Linux), mobile operating systems (e.g., iOS and Android) and video game consoles (e.g., Xbox, Play Station, and Wii) have become an essential part of the information economy (Evans et al. 2006). As noted by Boudreau (2007), such platforms are defined as the set of core components whose functionalities can be extended by complementary applications. Platform owners often seek complementary innovations from third-party providers to meet the needs of heterogeneous users. This approach of complementary innovation has given rise to the model of a platform ecosystem which makes a platform more valuable (Gawer and Cusumano 2002, Tiwana et al. 2010). More recently, social networking services such as Facebook have also adopted this platform approach of complementary innovations. Facebook launched its platform in May 2007, providing a set of programming interfaces and tools for third-party software developers to create applications that interact with Facebook's core features (e.g., user profile and friendship network). As of February 2012, the Facebook platform supported more than 9 million applications in a variety of categories such as games, photo-sharing, music-sharing, news, entertainment, sports, travel, and lifestyle,[1] which in total attract more than 235 million users.[2]

While continuing to expand their ecosystem through third-party applications, platform owners may also provide their own applications to consumers (i.e., first-party applications), either by in-house development or by acquisition from third-party developers. These first-party applications tend to compete with third-party applications. A critical aspect of this vertical integration strategy is the platform owner's decision to make a tighter integration of first-party applications relative to the integration of other third-party applications. For example, Facebook acquired Instagram for $1 billion in April 2012. Instagram[3] and other Facebook photo-sharing applications offer social networking features for Facebook users to discover, like, comment on, or vote for photos from their friend network or even the entire Facebook network (the total number of active users of these applications was over 113 million in December 2012). After the acquisition, a partial integration was made by Facebook in June 2012 to facilitate photo-sharing between Instagram and Facebook. This integration provided Instagram users an easy-to-use interface to access Facebook data (e.g., user profiles and friendship network) and share photos on Facebook through Instagram automatically. However, users of third-party applications needed several extra steps to complete the same tasks.[4] Similarly, in May 2012, Facebook acquired

[1] http://www.insidefacebook.com/2012/04/27/facebook-platform-supports-more-than-42-million-pages-and-9-million-apps/.

[2] http://techcrunch.com/2012/08/14/facebook-says-it-now-has-235m-monthly-gamers-app-center-hits-150m-e monthly-users/.

[3] Instagram can be used independently of Facebook. Instagram users can use the application to interact with other social networking services, such as Twitter, Tumblr and Flickr. Our focus in this paper is Instagram's features that enable users to share photos on Facebook.

[4] http://androidcommunity.com/instagram-update-adds-deeper-facebook-integration-and-

a gift-giving application called Karma, which competes with other Facebook gift-giving applications (e.g., Wrapp).[5] Facebook also made tighter integration with Karma after the acquisition.[6]

Integration of first-party applications is an important platform design decision. As consumers value inter-product integration (Nambisan 2002), platform owners may benefit from providing tighter integration with their first-party applications. However, third-party developers may resist such moves and hesitate to contribute to the ecosystem as they fear the platform owner's ability to squeeze them ex post (Gawer and Henderson 2007). For example, Facebook's vertical integration strategy has raised concerns about the viability of the platform for third-party developers, as voiced in the following quote from the CEO of Wrapp: *"The $100 billion question now is whether Facebook will remain an open platform that partners and supports companies like Wrapp..."* The partial congruence of interests between the platform owner and third-party developers are evidenced in Facebook's response to its platform strategy: *"Our company is specifically looking for acquisitions that complement our core products. However, we've never been more invested in supporting and expanding the ecosystem of applications and developers that build with Facebook."*[7]

Managing the tension between first-party applications and third-party applications has been a critical part of major platforms' strategies (Gawer and Henderson

search-options-20120626/.

[5]http://www.businessweek.com/articles/2012-05-23/what-facebook-will-get-out-of-gift-giving-app-karma.

[6]http://blogs.wsj.com/venturecapital/2012/05/18/on-ipo-day-facebook-finds-time-to-buy-social-gifting-site-karma/.

[7]http://online.wsj.com/news/articles/SB10001424052702304441404577480611248317178.

2007, Huang et al. 2013). Platform owners have to carefully evaluate the impact of the integration of a first-party application on its application ecosystem. Analysis of the resulting consumer preferences and the substitution or complementary effects between first-party applications and third-party applications can help platform owners determine the overall impact of their strategy. While we focus on the Facebook platform, this study has implications for a number of other platforms that routinely provide first-party applications and have to devise their vertical integration strategy. For example, Apple has also introduced its own applications (e.g., Apple Maps, Facetime, and iMovie) for its iOS platform, while Google has launched a variety of first-party applications for its Android platform (e.g., Google Chat, Google Finance, and Google Maps).

The consequences of platform integration are multifold and not obvious. In the case of Facebook's integration of Instagram, one possibility is that the integration has little impact as it does not introduce new product features. Without the integration, users could still complete the same tasks using Instagram or any third-party application. However, past literature suggests that ease of use is positively associated with product adoption (Davis 1989, Cooper 2000, Dhebar 1995). Therefore, users may derive additional utility from the tighter integration of Instagram with Facebook as it enhances Instagram's ease of use as compared to that for other third-party applications. Integration by the platform owner may also signal high quality/credibility of the application. As a result, consumers may perceive the first-party application more viable than third-party applications. Due to these benefits, consumers may find the first-party application more appealing relative to third-party applications, resulting in higher demand for the first-party application and lower demand for third-party applications. However, the platform owner's integration strategy can also stimulate consumer demand for applications as users may perceive the platform owner's strategy as its commitment to grow the applications ecosystem. As a consequence, third-party applications may also benefit from this market expansion effect. Finally, the impact of the platform owner's integration strategy can vary across different third-party applications. Thus, the overall impact of platform integration on market demand in the ecosystem is not known.

Despite its importance, empirical research on consumer demand for first-party applications and third-party applications has been limited.[8] Furthermore, the impact of platform integration remains unclear. Gawer and Henderson (2007) use a qualitative approach to explore why Intel entered its complementary markets and how Intel balances its own strong incentives to enter against the risk of discouraging complementors' innovations. However, they do not empirically evaluate the effect of platform entry on consumer demand in complementary markets. Huang et al. (2013) focus on the role of intellectual property rights on third-party developers' incentives to join the SAP platform, but they do not study the impact of first-party application on consumer demand for third-party applications. Lee (2013) investigates the role of exclusive titles on platform competition in the U.S. videogame industry. However, he evaluates the impact of exclusive titles on the demand for competing videogame

[8]Platform owners conventionally do not release demand data about their tightly integrated first-party applications. For example, Facebook stops releasing application usage data once an application gets acquired by Facebook. The Instagram case we study is an exception - Facebook continued to provide publicly available data on consumers' use of Instagram for Facebook photo-sharing until December 2012.

platforms rather than competing titles. No empirical evidence on the impact of a platform's integration with its first-party applications has been documented. Furthermore, related theoretical papers have focused on how price competition influences market demand, leaving out consumer preferences for empirical studies. Thus, the effect of non-pricing strategy like platform integration in influencing consumer preferences and market demand is not known.

We fill the gap in the literature by studying the impact of Facebook's integration of Instagram on Facebook's photo-sharing applications market. The unique dataset from this integration event allows us to evaluate the changes in consumer preferences and market demand after Instagram became a tightly integrated first-party application. We aim to address the following questions:

(1) What is the impact of the platform owner's integration strategy on consumer demand for the first-party application and third-party applications?

(2) How does the integration strategy impact the overall demand in the complementary market?

We build a structural model of consumer choices and estimate demand for the first-party application, third-party applications and the overall photo-sharing application market before and after the integration event. Our model extends existing static structural demand estimation models (see, e.g., Berry et al. 1995) by incorporating network effects and switching costs arising from the social characteristics of photo-sharing applications. We estimate the model using a unique dataset that consists of daily usage of different applications on the Facebook platform.

The main findings are as follows. First, we find that consumers obtain additional utility from Instagram after its tighter integration with Facebook, leading to a dramatic increase in the demand for Instagram. This is possibly due to the real benefits from the integration as well as the perceived long-term viability of the first-party application. Second, while integration lowers consumer valuations of third-party applications with a small user base, it actually has a positive effect on consumer valuations of big third-party applications. This result suggests that the size of an application's user base influences consumer preferences for different types of third-party applications following the integration of the first-party application. As a consequence, the integration event has large negative impact on demand for competing third-party applications with a small user base, whereas it has much smaller negative impact on competing third-party applications with a large user base. Finally, we find that a large fraction of new users gained by Instagram are new users who did not use any photo-sharing application, rather than incumbent users of third-party applications. As a result, the overall demand in the market actually increases, which suggests that Facebook's integration strategy benefits the complementary market overall.

Our research makes several contributions. Our study contributes to the literature on platform strategies in complementary markets. Previous research has mostly relied on theoretical models to study strategic interactions between the platform owner and third-party developers (supply-side behavior), given various assumptions on consumer behavior (Farrell and Katz 2000, Hagiu and Spulber 2013). Additionally, they do not focus on platform's decision to have variable integration across applications and its implication for the consumer demand. Our paper is the first study that empirically evaluates consumer preferences for first-party applications vis-à-vis third-party applications (demand-side behavior) and its implication for the platform owner's integration strategy. Our research also contributes to the literature on network effects. Previous research has focused on the role of network effects on the adoption and diffusion of products (Brynjolfsson and Kemerer 1996, Kauffman et al. 2000, Fuentelsaz et al. 2012). Our research adds to this stream by evaluating the role of network effects on consumer preference for first-party applications and third-party applications and its implications in the context of platform ecosystems. Specifically, we show that third-party applications with a larger user base may actually benefit from a platform owner's integration of a first-party application.

To the best of our knowledge, our paper is the first one to empirically demonstrate the value of integration of an application by a platform owner. Previous research has assumed that cross-product integration is valuable and demonstrated how initial technology architecture/design enables future cross-product integration (Nambisan 2002, Baldwin and Clark 2000). In these studies, platform owners do not own first-party applications that compete with third-party applications. Our paper demonstrates the effect of platform integration on consumer valuations and demand for applications in the context of a platform ecosystem with first-party applications and competing third-party applications.

From the platform owner's perspective, our findings shed light on the efficacy of the platform's integration strategy. On one hand, such a strategy may be beneficial particularly in a market where network effects and switching costs are present.

In such a scenario, the platform owner may gain new users due to the appeal of the tightly integrated first-party application while not hurting third-party applications too much. On the other hand, our research informs small third-party applications, platform owners, and policy makers about the potential dark side of platform integration. As small third-party applications are more vulnerable to the negative shock from vertical integration, such strategies may cause small third-party developers to exit the market, which reduces the variety of products/services available in the complementary market. Platform owners and policy makers should evaluate the trade-off between the demand increase in the short-turn and the potential losses in product variety in the long-turn.

4.2 Related Literature

Our research is related to the literature on platform-based ecosystems with a focus on complementary markets and the literature on product adoption subject to network effects and switching costs. We discuss these two streams of research below.

Platform Ecosystems and Complementary Markets

Existing studies have mostly relied on analytical modeling to study the strategic interactions between the platform owner and third-party developers (i.e., supply-side behaviors). Eisenmann et al. (2011) study platform entry strategies when new entrants face entry barriers driven by strong network effects and high switching costs. Farrell and Katz (2000) evaluate how a platform owner's entry into its complementary market allows it to extract higher rents. Hagiu and Spulber (2013) investigate the strategic use of first-party applications and show that the level of investment in these applications is driven by the relationship between first-party applications and third-party applications and the market conditions. All these theoretical papers above focus on supply-side behavior and firm strategies, give various assumptions on consumer behavior. Our paper is the first study that empirically evaluates consumer preferences for first-party applications vis-à-vis third-party applications (i.e., demand-side behavior). Furthermore, theoretical papers have focused on how price competition influences market demand. Our study of the Facebook platform highlights the role of a non-pricing strategy like platform integration in influencing market demand.

Empirical research on platform-based ecosystems, with a focus on complementary markets, is limited. Chipty (2001) examines the consequences of vertical integration between programming and distribution in the cable television industry. She assesses the role of ownership structure in program offerings and finds that integrated operators tend to exclude rival program services from their distribution networks. In our study, the platform owner did not exclude rival applications and instead adopted an approach of tighter integration with its own application. Gawer and Henderson (2007) use a deductive, qualitative approach to explore why Intel entered its complementary markets and how Intel balanced its own strong incentives to enter against the risk of discouraging complementors' innovations. Our paper provides concrete empirical evidence on the effects of a platform owner's vertical integration strategy in shaping consumer demand in the complementary market.

Using firm-level financial data, Huang et al. (2013) highlights the role of intellectual property rights in third-party developers' incentives to join SAP's enterprise software platform. Third-party developers that hold patents and copyrights, which protect developers from being squeezed by the platform owner, are more likely to join the platform. The focus of their paper is on third-party developers' entry behavior, whereas our paper controls for entry behavior and focuses on understanding consumer choices of first-party and third-party applications before and after platform integration. Lee (2013) investigates the role of exclusive titles on platform competition in the U.S. videogame industry. However, he evaluates the impact of exclusive titles on consumer demand for competing videogame consoles rather than competing third-party applications. Furthermore, in our study of Facebook photo-sharing applications, consumers may prefer the tightly integrated first-party application for its ease of use, a characteristics that is absent in the videogame setting.

Product Adoption in the Presence of Network Effects

Several studies have focused on the role of network effects on product adoption. Brynjolfsson and Kemerer (1996) estimate the impact of installed base and compatibility on the price of packaged software. Their empirical analysis shows that the size of a product's installed base is positively associated with the price of the product. Using electronic banking as a context, Kauffman et al. (2000) provide empirical evidence on network externality as a determinant of product adoption and diffusion. They find that banks in markets that can generate a larger effective network size and a higher level of externalities tend to adopt electronic banking early. Xue et al. (2011) study consumer adoption of online banking services. They find that customers who reside in areas with a larger number of online banking adopters are faster to adopt online banking as well. Zhu et al. (2006) develop a conceptual model that captures network effects, expected benefits, and adoption costs as drivers in the adoption of internet-based interorganizational systems. They highlight that the extent to which a firm's trading partners is willing to support the same systems as a key driver of the focal firm's adoption decisions. Gallaugher and Wang (2002) show a positive effect of network size on the price of Web server software. They attribute network effects to three sources: exchange value, staying power, and extrinsic benefits. Fuentelsaz et al. (2012) analyze the role of switching costs and network effects in determining the level of competition in the European mobile communications industry. Consistent with theoretical predictions, their empirical results suggest that higher switching costs and stronger network effects lead to lower level of rivalry in the market.

Thus previous works have primarily focused on the role of network effects on product adoption and price competition. Our paper extends this stream of literature by evaluating the role of network effects in influencing consumer responses to a platform owner's integration of a first-party application.

4.3 Photo-Sharing Application Ecosystem on Facebook

In this paper we focus on photo-sharing applications that enable Facebook users to discover, edit and share photos on Facebook. Photo-sharing applications provide tools to create personalized photo collages, import pictures from an existing Facebook album, retouch, add filters or text, and share photos with friends. These applications also offer social networking features for Facebook users to discover, like, comment on, or vote for photos from their friendship network or even the entire Facebook network. These social features create a local social network within a focal photo-sharing application. Instagram has been one of the most popular photo-sharing applications on the Facebook platform.

On April 12, 2012, Facebook acquired Instagram for approximately $1 billion. After the acquisition, Facebook continued to run Instagram as an independent application, instead of fully integrating it into Facebook.com.[9] There were no significant changes to Instagram and Facebook.com after the acquisition deal, except that, on June 26, 2012, a partial integration was made to facilitate photo-sharing between Instagram and Facebook. After the tighter integration, if an Instagram user likes or comments a photo on Instagram, the photo along with the "like" or comment may automatically appear as the user's news feed on Facebook; if a Facebook user likes or comments the photo, the "like" or comment may appear in the original post on Instagram as well. The update also offers Instagram users enhanced capacity to find and connect to their Facebook friends and explore Facebook's network using this application. Users of third-party applications have to take several extra steps to complete these tasks.

We obtained a unique dataset from a business analytics company that tracks usage of applications on Facebook. While Facebook routinely stopped releasing application usage data after an application was acquired by Facebook, the Instagram case was an exception. Facebook continued to report data on consumers' use of Instagram for Facebook photo-sharing until December 2012. Our dataset consists

[9] https://newsroom.fb.com/News/321/Facebook-to-Acquire-Instagram.

of the number of daily active users for the top 20 photo-sharing applications on the Facebook platform from April 27, 2012 to December 15, 2012.[10] All the top 20 applications are free applications and cumulatively account for over 88% of the market share among all Facebook photo-sharing applications. Except Instagram, all other applications were owned by third-party developers. Besides the time-variant demand data, we also observe time-invariant product attributes such as release dates and distribution channels (Facebook canvas, iOS/Android applications). The dataset also consists of an app's average user star ratings on Facebook's application center. The star ratings remained constant during the panel period, suggesting there were no visible quality improvements to the photo-sharing applications during the panel period.

We divide the dataset into two subsamples, one for estimation and the other for model validation. The subsample for estimation covers the first 124 days (two months before and two months after the integration event), whereas the subsample for validation covers the remainder of 109 days.

Table 4.1 summarizes some descriptive statistics of the subsample used for estimation. We compute market shares by dividing the number of application users by the total number of Facebook users (see, e.g., Berry et al. 1995, Nevo 2001). The statistics show that a large fraction of Facebook users did not use any photo-sharing application regularly.

Figure 4.1 illustrates the demand changes for different applications before and

[10] After December 2012, Facebook stopped providing accurate application usage data (it only reports the range of application users, e.g., 10,000 – 50,000).

Variable	Mean	Std. dev.	Min	Max
Market Size (Daily Users of Facebook)	5.52×10^8	1.07×10^7	5.35×10^8	5.72×10^8
Natural Log of Market Size	20.12	16.19	20.10	20.16
Daily Users of Instagram	5.04×10^6	2.91×10^6	1.90×10^6	1.05×10^7
Natural Log of Daily Users of Instagram	15.43	14.88	14.46	16.17
Daily Users of Third-Party Apps	2.58×10^5	3.34×10^5	4.00×10^2	1.60×10^6
Natural Log of Daily Users of Third-Party Apps	12.46	12.72	5.99	14.29
Market Share of Instagram	0.90%	0.51%	0.35%	1.83%
Market Share of Third-Party Apps	0.047%	0.060%	7.07×10^{-5} %	0.29%

Table 4.1: Daily users of Instagram and other photo-sharing applications on the Facebook platform. The total number of applications in the sample is 20 and the length of the panel period is 124 days (two months before and two months after the integration event). Thus, the total number of observations in this balanced panel is 2,480.

after the integration event. Since its tighter integration with Facebook, Instagram has experienced significant growth in its user base. Moreover, the combined demand of the top third-party applications remains relatively stable, but the growth rate becomes noticeably lower after the integration event, suggesting certain degree of substitution between Instagram and third-party applications. The negative impact is particularly significant for third-party applications with a small network size. Finally, the total demand for Instagram and third-party applications is growing. By the end of August, 2012, the total demand in the photo-sharing category almost tripled. These results imply that a large fraction of users joining Instagram are new users, rather than incumbent users of third-party applications. In the remainder of the paper, we look into these results and their implications by building and estimating a random-coefficient discrete choice model that captures consumer choices.

4.4 The Model

Our objective is to estimate demand for different photo-sharing applications on the Facebook platform. We build a structural model of consumer choices based on the literature of structural demand estimation using aggregate data (Berry et al. 1995). This structural approach allows us to derive market share of each application as a function of product characteristics while accounting for unobserved consumer heterogeneity and demand shocks. Similar models have been used to study consumer choices in electronic markets and mobile applications markets (see, e.g., Ghose et al. 2012, Ghose and Han 2014, Danaher et al. 2014).

4.4.1 Model Setup

We observe period t, $t = 1, \ldots, T$, with M_t consumers. Each consumer chooses at most one application j, $j = 1, \ldots, J$, in each period. In our setup of Facebook photo-sharing applications, $J = 20$. All these photo-sharing applications are free and thus price is not relevant to consumer choices. We categorize the J applications into two groups: g ={first-party applications, third-party applications}. Denote $j = 0$ the option of outside good, the option of not using any of these J applications. Consumer i's utility of using application j in period t is specified as

$$U_{ij_gt} = y_{j(t-1)}\beta_i + \sum_g \gamma_g I_{j_gt} + \alpha_j + \tau_t + \varepsilon_{jt} + \epsilon_{ijt}, \tag{4.1}$$

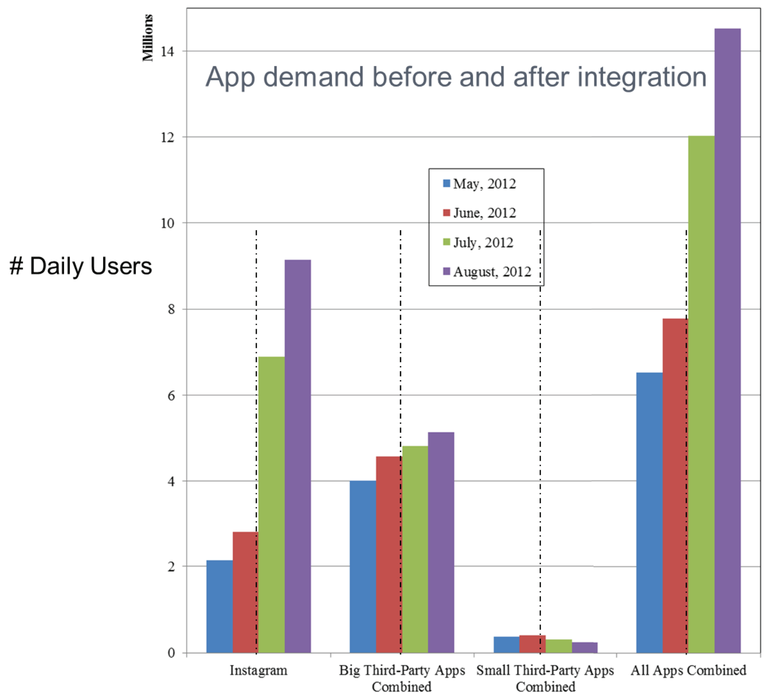


Figure 4.1: Market demand of Instagram, third-party applications, and the overall market before and after Facebook's tighter integration with Instagram. A tighter integration between Facebook and Instagram was made on June 26, 2012 (dashed line). We call the top 9 third-party applications (according to user base) big third-party applications and the remainder 10 applications small third-party applications. Instagram's user base (in millions) increased dramatically after the integration. However, the growth rate of big third-party applications became smaller and the user base of small third-party applications decreased. The total number of users for the photo-sharing category increased after integration.

where $y_{j(t-1)}$ is the lagged application user base that determines the direct network effect (see, e.g., Fuentelsaz et al. 2012). Network externalities are known to play a role in consumers' adoption of technology products (Katz and Shapiro 1986). Products and services with a larger user base may provide higher exchange value to users. Such installed base effect may also come from behavioral factors such as social preferences, observational learning, and word-of-mouth, which influence product diffusion (Bass 1969, Mahajan et al. 1990). Note that consumers may also derive utility from the overall user base of Facebook (i.e., indirect network effect). This common network effect, however, is not identified, as it enters the utility function for each application and will be cancelled out.

Consumer valuation of the first-party application may increase after its tighter integration with the platform due to better ease of use (Davis 1989, Cooper 2000, Dhebar 1995). For example, reciprocal sharing between Instagram and Facebook allow Instagram users to manage photos across Instagram and Facebook in a seamless fashion. This may have negative impact on third-party applications. These effects are capture by $I_{j_g t}$ which represents a vector of interaction terms $Integration \times AppGroup$ ($Integration \times Instagram$, $Integration \times ThirdPartyApp$), whose value is one if application j belongs to group g and $t \geq t_I$ (t_I is the integration time), and zero otherwise. These two interaction terms capture the impact of integration on consumer valuation of Instagram and third-party applications.

Our model controls for various unobserved shocks. In Equation (4.1), ε_{jt} is the app-specific shock that enters a consumer's utility in period t but are not observed by our econometrician, and ϵ_{ijt} is the idiosyncratic shock which is assumed to be drawn from the Type 1 extreme value distribution independently across consumers, applications, and time periods (Berry et al. 1995). The model also includes application dummies α_j and time dummies τ_t to control for time-invariant fixed effects and potential time trends that shift consumers' utility. Application dummies also account for observed and unobserved product characteristics that do not vary during the panel period. As noted by Nevo (2001), the rich specifications of fixed effects and time effects capture various components of unobservables such as unobserved promotional activities, unquantifiable product characteristics (e.g., brand equity), or systematic shocks to demand which are common across all photo-sharing applications. Such rich specifications provide a semi-parametric control that assuages potential misspecification concerns.

Following Berry et al. (1995), we model the distribution of consumers' taste parameters as multivariate normal, i.e.,

$$\beta_i = \bar{\beta} + \beta_v v_i, v_i \sim N(0, I_K), \tag{4.2}$$

where K is the dimension of product characteristics y_{jt}, $\bar{\beta}$ is a vector of the means of taste parameters, v_i is a vector of unobserved individual tastes, and β_v is a scaling diagonal matrix that represents the standard deviations of the taste distributions. In our setup, product attributes except application user bases do not change during the panel period and are already captured by the application dummy. Thus, we set $K = 1$.

We assume random coefficient for the key variable, i.e., the network effect $y_{j(t-1)}$, to account for the possibility that consumers may be heterogeneous in their valuation of the size of an application's user base. Identification of random coefficients for dummy regressors is difficult as these variables have very limited cross-sectional or temporal variations, which hinder the identification of β_v. As a result, we do not assume random coefficients for dummy regressors and interaction terms to avoid the explosion of parameters.[11] Finally, we normalize the mean utility from the outside option to zero, i.e., $U_{i0t} = \epsilon_{i0t}$.

Combining Equations (4.1) and (4.2), we have

$$U_{ij_gt} = \delta_{jt} + y_{j(t-1)}\beta_v v_i + \epsilon_{ijt}, \tag{4.3}$$

where

$$\delta_{jt} = y_{j(t-1)}) + \sum_g \gamma_g I_{j_gt} + \alpha_j + \tau_t + \varepsilon_{jt} \tag{4.4}$$

represents the mean utility and $y_{j(t-1)}\beta_v v_i+\epsilon_{ijt}$ corresponds to consumer i's individual-specific utility from using application j in time t.

A consumer's decision in current period may depend on her previous adoption and usage. For example, when facing high switching costs, consumers may continue to use the same product they have been using, even when more favorable alternatives are available. In the base model described above, we do not consider such dynamic behavior. In Section 4.6.3, we extend our base model to explicitly consider the role of switching costs in consumer choices.

Our model implicitly assumes that consumers are myopic and non-strategic. An individual consumer is "smal" relative to the size of the entire network such that

[11] Other studies using similar methodologies, such as Song (2015) and Gowrisankaran and Rysman (2012), assume only one random coefficient on the key explanatory variable.

she anticipates her individual adoption decision will not significantly influence the adoption behavior of other consumers. This is a common assumption in other papers using similar approaches (see, e.g., Ghose et al. 2012, Ghose and Han 2014, Danaher et al. 2014). In our context of Facebook photo-sharing applications, consumers have limited incentives to behave forward-looking because all applications are free and it is not impossible to switch to other applications. Therefore, we believe that our assumption of myopic and non-strategic consumers is appropriate.

Our structural model has several advantages over the traditional Difference-in-Differences (DD) technique used to measure the treatment effect. Platform integration impacts consumer choices over multiple periods, with both first-order effect (directly shifting consumer utility in current period) and second-order effect (indirectly shifting utility in future periods through lagged user base). Our model explicitly captures both these effects that are difficult to bed modeled using a DD approach. Further, the structural model allows us to run counterfactual simulations to estimate the market outcome in alternative scenarios, i.e., with and without integration. These counterfactual experiments are valuable in evaluating whether or not it is beneficial to conduct platform integration, instead of simply knowing the changes in market demand before and after integration.

4.4.2 Identification and Estimation

Our focus in this paper is consumer choices before and after platform integration. To reduce the interference from supply-side behaviors such as entry and exit following the integration, we restrict our analysis to a relative short horizon, two months before and two months after integration, such that application developers possibly have not yet responded to the integration event. This restriction allows us to estimate the demand equation in (4.1) without modeling the strategic interactions between the platform owner and third-party developers.

The panel structure of the dataset allows us to use the fixed-effects approach to control for potential unobserved/omitted time-invariant product characteristics and promotion efforts. The fixed-effects approach provides a semi-parametric control that assuages many misspecification concerns (Wooldridge 2010). However, fixed-effects estimators are inconsistent when the model includes predetermined explanatory variables such as lagged user base (Nickell 1981, Anderson and Hsiao 1982). The intuition for the inconsistency is that future adoptions are a function of current adoptions, implying that current unobservables are correlated with the size of application user base in all future periods. This violates the strict exogeneity condition required for the consistency of fixed-effects estimators. However, this inconsistency becomes insignificant when the number of time periods T is relatively large (Hahn and Kuersteiner 2002), as it is the case in our model. Hahn and Kuersteiner (2002) find that the magnitude of biases is close to $\frac{2}{T}$, which is about 0.01, a negligible number in our setting (parameter estimate of network effect is about 0.7 in our model.). As a robustness check, we also validate our results with various instruments for the lagged user base in Section 4.7.

It is possible that Facebook may coordinate the timing of integration based on some market trends for photo sharing. For example, an increased interest in photo sharing among consumers may influence the consumer response to integration.

We account for such time trends using the time dummies. It is also possible that Facebook may coordinate the timing of Integration with higher level of external promotions for the Instagram application. We conducted a comprehensive review of Instagram's internet activities during the same panel period. We went through historical news feeds and articles on major search engines (Google, Yahoo!, and Bing), mobile applications marketplaces (iTunes and Google Play), tech media websites (CNET and TechCrunch), and Instagram's company page on Facebook. We do not find any evidence that Instagram was executing unusual advertising or other promotional campaigns that may explain the demand patterns observed in Figure 4.1. However, it is possible that there are external unobserved market dynamics which influence consumer valuation and are correlated with the integration terms. As a robustness check, we validate our results with a suitable instrument for the integration variable in Section 4.7.

Details of the estimation algorithm are provided in the appendix. Here we present the intuition of the estimation procedure. The model is of individual behavior, yet only aggregate data is observed. Our goal is to estimate the mean and variance of the vector of model parameters while accounting for consumer heterogeneity. We apply iterative methods similar to the contraction mapping algorithm used by Berry et al. (1995) and Nevo 2001. With an initial value of β_v^0, we can predict individual utility and aggregate individual choices to obtain predicted market shares. We solve for the mean utility δ, such that the model-predicted market shares are equal to the observed market shares. We then form a minimal distance objective function based on the sum of squared errors (if instrument variables are used, we replace the minimal distance by a GMM objective function based on a set of moment conditions.) We then update the parameter value and use it as the starting point for the next iteration. This procedure is repeated until the algorithm finds the optimal value of β_v that minimizes the objective function. We tried different starting points and they routinely lead to the same estimates.

4.5 Empirical Analysis and Results

In this section, we explain the empirical results and provide evidence on the fit of the model. At the end of this section, we conduct counterfactual simulations and estimate market demand for a hypothetical scenario in which Facebook did not seek tighter integration with Instagram. By contrasting demand estimates from this counterfactual "without integration" scenario with those from the real "with integration" scenario, we are able to estimate the impact of platform integration on different types of applications.

4.5.1 Parameter Estimates

Estimation results are in Table 4.2. Estimates in the first column are from the model without consumer heterogeneity (fixing β_v to zero). The second column provides the results from the enhanced model with consumer heterogeneity on network effect. The last two columns present estimates from the same models, but with control of unobserved time trends. The sum of squared errors in Table 4.2 reveal that model fit increases as controls of consumer heterogeneity and time effects are included in the model (smaller squared errors mean better model fit).

Variable	Coefficients (Standard Errors) (1)	(2)	(3)	(4)
LaggedAppUserBase (log)	0.7430*** (0.0096)	0.7108*** (0.0096)	0.7471*** (0.0095)	0.7251*** (0.0095)
Integration×Instagram	0.3032*** (0.0407)	0.2976*** (0.0407)	0.3404*** (0.0407)	0.3323*** (0.0407)
Integration×ThirdPartyApp	-0.0424*** (0.0090)	-0.0367*** (0.0090)		
Time Dummy	No	No	Yes	Yes
Consumer Heterogeneity on AppUserBase (log)		0.0560*** (0.0006)		0.0439*** (0.0006)
Sum of squared errors	143.9190	143.8043	135.2909	135.2509

Table 4.2: Parameter estimates of the base models. Model (3)-(4) include time dummies and the interaction term $Integration \times ThirdPartyApp$ is omitted to avoid the dummy variable trap. Dummy variable trap occurs as there is perfect colinearity among the time dummies and the two interaction terms for all post-integration periods. Standard errors in parentheses and $^{*}p$ <0.10, $^{**}p$ <0.05, $^{***}p$ <0.01.

The coefficient of lagged application user base is positive and significant. This suggests that consumers derive a higher utility from using an application with a larger user base. The strong network effect may be attributed to the unique features of Facebook applications: social and sharing. Facebook users use these applications to share their photos with other users and comment/vote on photos posted by others. Many Facebook applications embed a local social network within the Facebook social network, a phenomenon we refer to as "social network within social network". Also note that the small but significant parameter of consumer heterogeneity indicates that users differ in their valuations of the network size. Some users value a large network size more than others. Ignoring this heterogeneity leads to overestimation of network effect.

The coefficient of $Integration \times Instagram$ is positive and significant, indicating that consumers derive additional value from Instagram after its tighter integration with Facebook. The additional value may come from better ease of use due to tighter integration. It may also come from consumers' perceived long-term viability of the first-party application after its tighter integration with the platform (Katz and Shapiro 1992, Gallaugher and Wang 2002). Tighter integration by the platform owner may also signal high quality/credibility of the application. Due to these benefits, users are more likely to choose the first-party application after the integration.

The integration event reduces consumer valuation of third-party applications, as evident from the negative coefficient of $Integration \times ThirdPartyApp$. If consumer valuation of integration is purely from the better ease of use of the first-party application being integrated, then the integration event should not have any effect on consumer valuations of third-party applications. However, the negative effect suggests that other factors such as stability or long-term viability of an application may also play a role in consumer valuation of the application. Consumers may perceive the integration event as lower future support of third-party applications from the platform owner, and as result, weaker perceived staying power of third-party applications.[12] To our best knowledge, these results provide the first empirical evidence on how consumer valuations of first-party applications and third-party applications

[12]The perception of weaker staying power could be driven by past consumer experiences in other contexts such as the desktop OS ecosystems where Microsoft's first-party applications eventually dominated in many complementary markets and drove away other competing third-party applications.

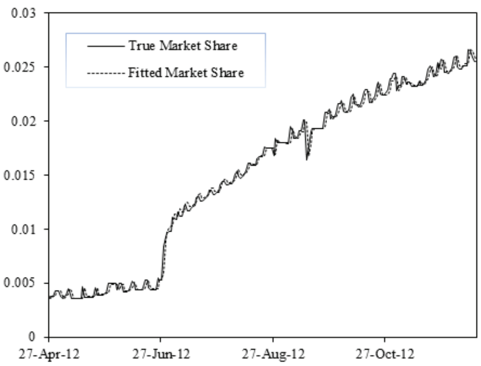
change following platform integration.

Evaluating the overall impact of platform integration in the presence of network effect is nontrivial. The parameter estimate of integration in Table 4.2 captures the one-period effect (first-order effect). The one-period effect impacts consumer choices in current period t, but the resulting application user base will give rise to network effect in the next period through the lagged application user base y_{jt}, which enters consumer's utility function in period $t+1$ (second-order effect). As a result, the overall effect of platform integration will be larger than the one-period effect. In other words, the second-order effect amplifies the one-period effect of platform integration by a multiplier that is strictly larger than one. In discrete choice models, it is impossible to derive a closed-form expression for the accumulated effect of platform integration. However, as demonstrated in Section 4.5.3, this accumulated effect can be easily computed by simulations using our structural model and parameter estimates.

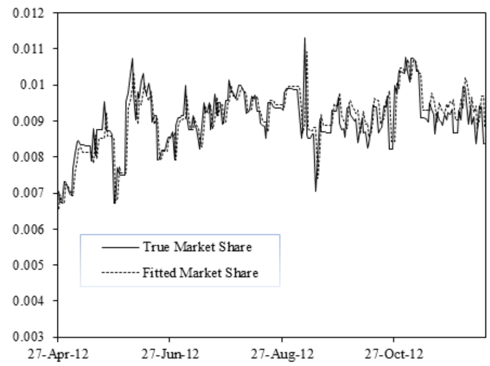
4.5.2 Model Fit and Validation

Before doing further analysis, we first evaluate the performance of our proposed model. The proposed model and parameter estimates enable us to predict each individual app's demand and market share. It allows us to evaluate in-sample fit and perform out-of-sample validation. To see the prediction power of our model, we simulate consumer choices and aggregate market shares. The mean absolute errors for the in-sample and out-of sample are 5.12×10^{-5} and 1.04×10^{-4}, respectively.

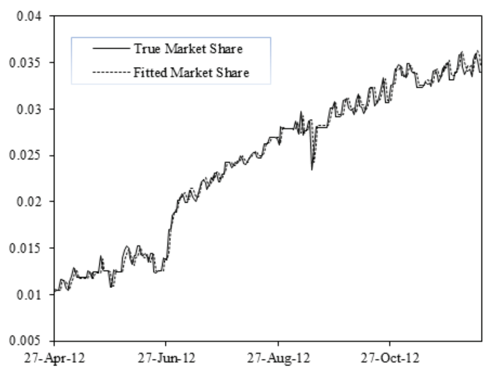
For expositional simplicity, we add up market shares from all third-party



(a) First-party application (Instagram)



(b) Third-party applications combined



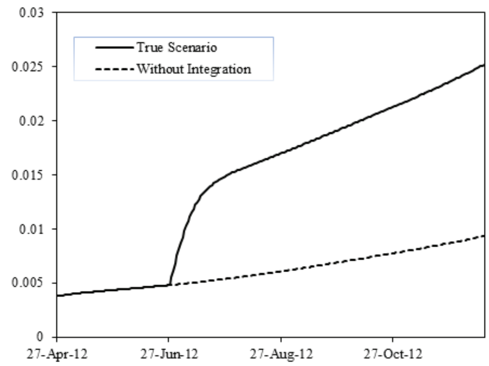
(c) All applications combined

Figure 4.2: Model predicted market share vs. true market share. Sample for estimation (April 27, 2012 - August 28, 2012); Hold-out sample for validation (August 29, 2012 - December 15, 2012)

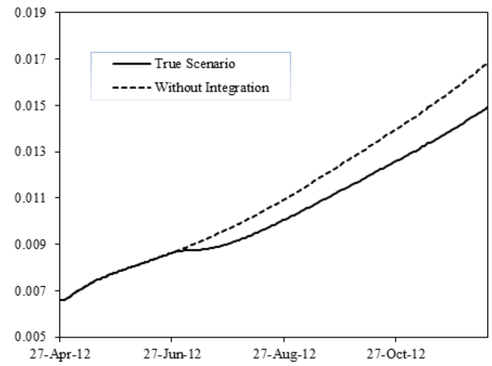
applications and present the combined predicted market shares for in-sample and hold-out sample. As we can see from Figure 4.2, the model predicted market shares are very close to the true market shares. The good out-of-sample prediction power confirms the validity of our model. It provides support for doing counterfactual simulations based on the model and parameter estimates.

4.5.3 The Impact of Integration on Market Demand

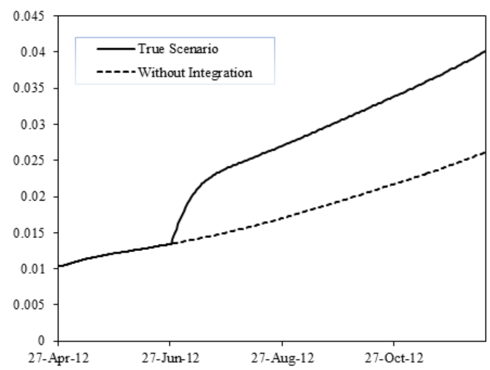
We first simulate consumer choices and compute market shares for each application under two alternative market scenarios: with integration (the true market scenario where integration occurred in late June 2012) and without integration (a counterfactual scenario where no integration was made). We then contrast the market shares under the counterfactual market scenario (s'_{jt}) to those under the true market scenario (s''_{jt}). Figure 4.3 shows the simulated market shares under the true market scenario (solid curve) and the counterfactual market scenario (dashed curve). The impact of platform integration can be identified by comparing the solid curve with the dashed curve. Figure 4.3(a) indicates that the first-party application experiences dramatic growth in market share due to its tighter integration with the platform. In addition, as shown in Figure 4.3(b), the integration event negatively impacts consumer demand for third-party applications. Compared to the "without integration" benchmark, the market shares for all third-party applications are lower in post-integration periods, indicating certain degree of substitution between the first-party application and third-party applications. However, as evidenced in Figure 4.3(c), the net impact of platform integration for the entire market is pos-



(a) First-party application (Instagram)



(b) Third-party applications combined



(c) All applications combined

Figure 4.3: Impacts of integration on market shares of the first-party application, third-party applications, and the overall market

itive. The result suggests that the majority of users gained by Instagram are new users who did not use any application, rather than the incumbent users of third-party applications. Therefore, the overall market demand increases after platform integration.

We compute the percentage change in market share due to platform integration. In period $t_I + \Delta t$ (i.e., Δt days after integration), this measure is calculated

Different Types of Apps	Percentage Changes in Market Share ($\Delta s_{(t\ +\Delta t\ )}$)		
	Δt 30	Δt 60	Δt 120
Instagram	177.74%	179.73%	174.61%
Third-Party Apps Combined	-6.70%	-8.16%	-9.89%
All Apps Combined	59.11%	58.88%	55.95%

Table 4.3: Changes in market share due to platform integration

as

$$\Delta s_{j(t_I+\Delta t)} = \frac{s''_{j(t_I+\Delta t)} - s'_{j(t_I+\Delta t)}}{s'_{j(t_I+\Delta t)}} \times 100\%. \tag{4.5}$$

Table 4.3 summarizes the changes in market shares for the first-party application, third-party applications, and the overall market. Compared to the "without integration" benchmark, tighter integration with Facebook increases the market share of the first-party application by about 177.74% within 30 days after the integration. In addition, the integration decreases the market share of third-party applications by 6.70%. However, the combined market share of all the 20 applications increases by 59.11%.

Our results suggest that tighter integration of Instagram has an overall positive effect on the ecosystem for photo sharing applications.

4.6 Additional Analysis

The impact of the platform owner's integration decision may vary across third-party applications. In this section, we investigate the effect of integration on big and small third-party applications. In addition, we extend the base model to capture the role of switching costs in consumer choices.

4.6.1 Variable Impact on Big and Small Third-Party Applications

In our main analysis we find that integration of the first-party application lowers consumer utility from using third-party applications (Table 4.2). The negative effect suggests that long-term viability of an application may play a role in consumer valuation. Consumers may lower their perception of the staying power of third-party applications after the platform exercised its integration strategy. However, for technology products that exhibit network effect, consumers associate a large user base with strong staying power (Katz and Shapiro 1992, Gallaugher and Wang 2002). As a result, the effect of integration may be different for third-party applications with different network sizes.

To capture this potential variable effect, we rank all the 19 third-party applications according to their user base in the first period (two months before the integration event). We create a dummy variable SmallThirdPartyApp where SmallThirdPartyApp equals to one if an app's user base ranks below 10th, and zero otherwise. Similarly, we create a dummy variable BigThirdPartyApp for the top 9 third-party applications. After the segmentation, there are now three groups of applications, i.e., g ={first-party applications, small third-party applications, big third-party applications}. We replace the interaction term $Integration \times ThirdPartyApp$ in Equation (4.1) by two interaction terms $Integration \times SmallThirdPartyApp$ and $Integration \times BigThirdPartyApp$ and re-estimate our model.

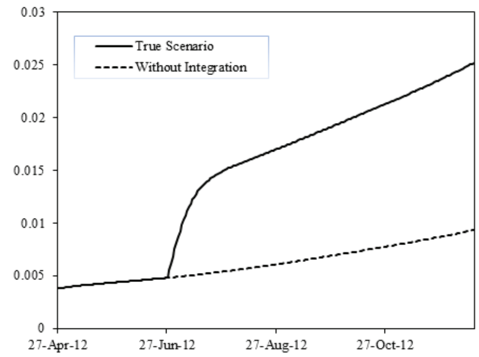
Results in Table 4.4 show that the platform's tighter integration with the first-party application creates different impact on big third-party applications and small third-party applications. Interestingly, consumer valuation of big third-party applications actually increases after platform integration, although the added value is not as large as that for the first-party application being integrated. A possible explanation is that consumers may not be concerned about the staying power of big third-party applications after the integration event. At the same time, integration of the first-party application may also signal high viability and credibility of the Facebook photo-sharing ecosystem. As a consequence, consumer valuation of big-third-party applications actually increases. In contrast to the increase in valuation of big third-party applications, consumer valuation of small third-party applications is reduced by a large amount. Small developers very often have limited budget and may lack the commitment to grow their user base in presence of unfavorable market conditions. As a result, small third-party applications are more vulnerable to platform integration. In this case, users of small applications are more likely to migrate to bigger applications when the perceived long-term viability of these small applications is weakened by the platform owner's integration behavior.

Note that the overall impact of integration on demand for each application depends on consumer utility of using the focal application relative other applications. Platform integration leads to a much larger increase in consumer utility of Instagram as compared to big third-party applications (Table 4.4). As a consequence, the overall market share for the big third-party applications grows at a slower rate after integration as compared to the scenario without integration (Figure 4.4).

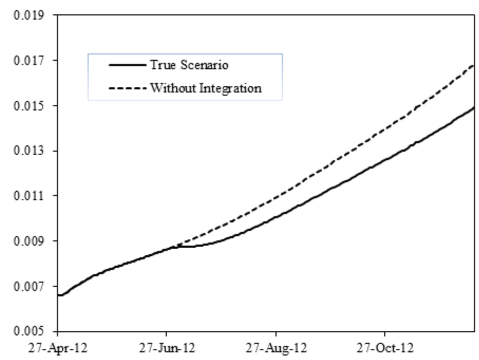
Our results suggest that Facebook's tighter integration with Instagram hurts

Variable	Coefficients (Standard Errors) (1)	(2)	(3)	(4)	(5)	(6)
LaggedAppUserBase (log)	0.7118*** (0.0102)	0.6358*** (0.0102)	0.7153*** (0.0101)	0.6182*** (0.0101)	0.7155*** (0.0101)	0.6278*** (0.0101)
Integration×Instagram	0.3419*** (0.0404)	0.3489*** (0.0403)	0.3090*** (0.0403)	0.2826*** (0.0402)	0.4639*** (0.0427)	0.4531*** (0.0426)
Integration×SmallThirdPartyApp	-0.1293*** (0.0136)	-0.1166*** (0.0135)	-0.1576*** (0.0184)	-0.1735*** (0.0183)		
Integration×BigThirdPartyApp	0.0307** (0.0124)	0.0618*** (0.0123)			0.1571*** (0.0184)	0.1737*** (0.0183)
Time Dummy	No	No	Yes	Yes	Yes	Yes
Consumer Heterogeneity on AppUserBase (log)		0.0847*** (0.0004)		0.1042*** (0.0004)		0.0936*** (0.0004)
Sum of squared errors	140.2847	139.7101	131.7789	131.3666	131.7984	131.3949

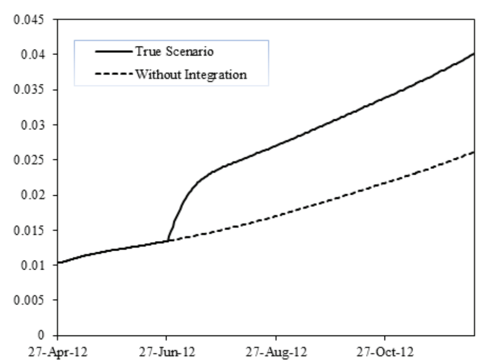
Table 4.4: Variable impact of integration on big and small third-party applications. Model (3)-(4) include time dummies and the interaction term $Integration \times BigThirdPartyApp$ is omitted to avoid the dummy variable trap. Dummy variable trap occurs because there is perfect colinearity among the time dummies and the three interaction terms for all post-integration periods. Similarly, Model (5)-(6) include time dummies and the interaction term $Integration \times SmallThirdPartyApp$ is omitted.



(a) First-party application (Instagram)



(b) Third-party applications combined



(c) All applications combined

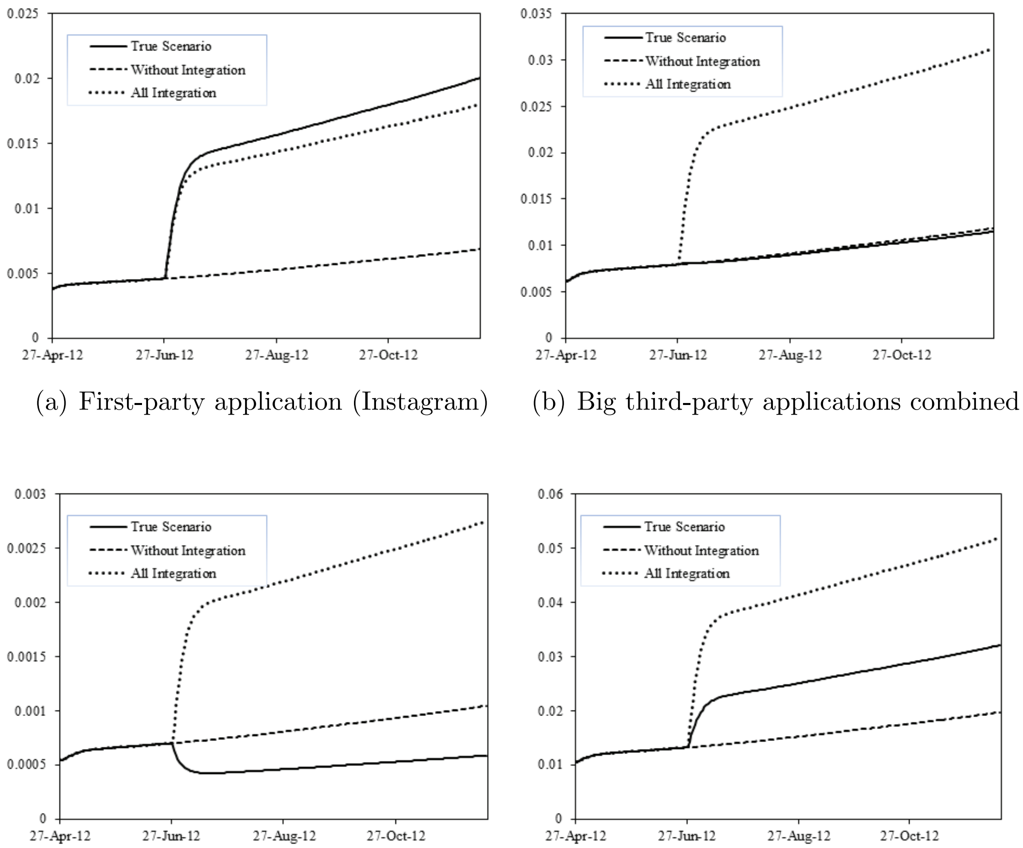
Figure 4.4: Impact of platform integration on market shares of the first-party application, big third-party applications, small third-party applications, and the overall market

small third-party applications more. This variable impact has important implications for the platform owner. In the short-run, the platform owner may benefit from demand increase following the integration of the first-party application. However, in the long run, the platform may suffer from the loss in product variety in the complementary market. The negative shock from platform owner's integration strategy may cause existing small third-party developers to exit the market. Further, potential new entrants may not see a fair chance to appropriate their innovations and choose not to participate in the platform ecosystem. This may hurt the platform in the long run.

4.6.2 Other Counterfactual Experiments

We conduct additional counterfactual experiments to highlight the impact of platform integration on the first-party application, third-party applications, and the entire marketplace.

Integration of Instagram vs. Integration of All Applications: We simulate a counterfactual scenario where all photo-sharing applications were integrated in late June 2012, assuming all integrated applications enjoy the increase in consumer utility as Instagram did after integration. This would represent the potential outcome of Facebook's plan to build a tightly integrated network with any application through its Open Graph interface. In Figure 4.5, the dotted curves correspond to the counterfactual scenario where all applications were integrated. Compared to the true market scenario where only Instagram was integrated, both big and small third-party applications are better off, but small third-party applications benefit more from this



(a) First-party application (Instagram) (b) Big third-party applications combined

(c) Small third-party applications combined (d) All applications combined

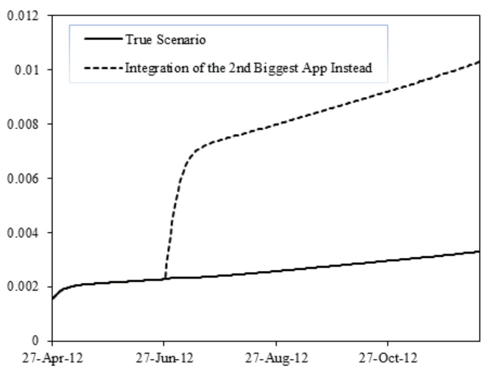
Figure 4.5: Integration of Instagram vs. integration of all applications

aggressive integration strategy. Additionally, a loss in demand for Instagram is compensated by an increase in demand for all third-party applications. As a result, the overall demand for the photo-sharing ecosystem is much higher. However, note that the platform owner has to evaluate the rent increase due to higher demand for third-party applications against the revenue loss due to lower demand for Instagram and the additional integration cost for integrating third-party applications.

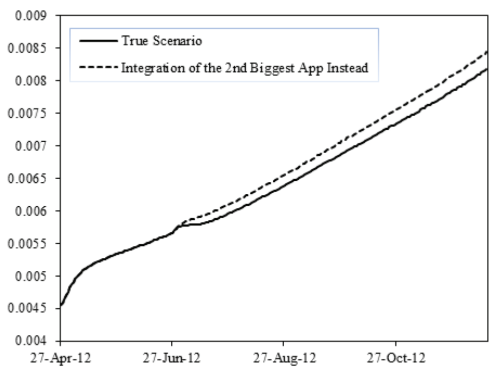
Integration of Instagram vs. Integration of the Second Biggest Application: Pixable was the second biggest photo-sharing application on the Facebook platform when Facebook made tighter integration with Instagram (Pixable's user base was about one-third of Instagram's). In this counterfactual experiment, we simulate an alternative scenario in which Facebook integrated Pixable instead of Instagram. The gain in demand for Pixable under the alternative is much lower as compared to the gain in demand for Instagram under the true scenario (Figure 4.6). Compared to integration of Instagram, integration of Pixable has smaller negative impact on other applications, but the total gain in market demand is also smaller. These results show that the impact of integration is proportional to the user base of the application being integrated. Clearly, the overall demand is lower as compared to the true scenario where Facebook integrated Instagram. If the cost of integration is comparable, our result would suggest that Facebook is better off by integrating Instagram rather than a third-party application.

4.6.3 Switching Costs

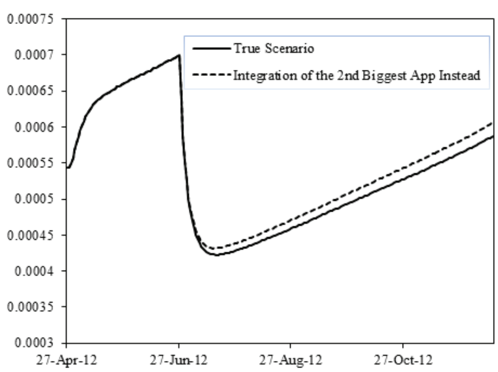
In our dataset we observe the daily usage of each photo-sharing application. As a consumer uses these applications repeatedly, her choice in the current period may depend on her previous choices, i.e., consumers may reveal state-dependent preferences. Such dynamic consumer behavior may be driven by switching cost which reduces a consumer's utility for other alternatives, or by variety-seeking behavior which reduces a consumer's utility from using the same application. For Facebook photo-sharing applications, state-dependent preferences may be attributed to the



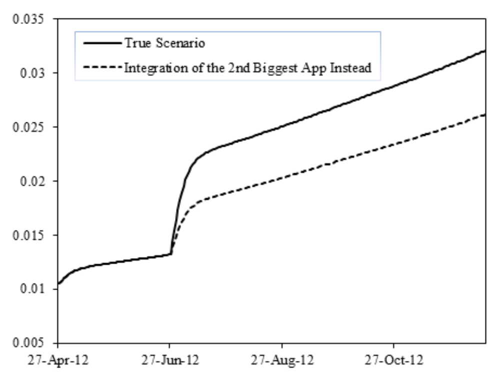
(a) First-party application (Instagram)



(b) Big third-party applications combined



(c) Small third-party applications combined



(d) All applications combined

Figure 4.6: Integration of Instagram vs. integration of the 2nd biggest application

social features (e.g., social network) in these applications. Switching to a new application means leaving the current network and joining a different community, which can be infeasible to some consumers with high switching costs but attractive to others with strong variety-seeking preferences.

The consumer utility function in Equation (4.1) can be modified to capture the effect of previous choices. Given that consumer i chose application $d_{i(t-1)}$ in period $t-1$, her utility from choosing application j in period t is

$$U_{ij_gt}(d_{i(t-1)}) = y_{j(t-1)}\beta_i + \sum_g \gamma_g I_{j_gt} + \alpha_j + \tau_t + \varepsilon_{jt} + \epsilon_{ijt} - c_i \mathbf{1}\left\{d_{i(t-1)} \notin \{0, j\}\right\}, \quad (4.6)$$

where c_i is consumer i' cost (or benefit if negative) from using another application and $\mathbf{1}\left\{d_{i(t-1)} \notin \{0, j\}\right\}$ is an indicator function defined as

$$\mathbf{1}\left\{d_{i(t-1)} \notin \{0, j\}\right\} = \begin{cases} 1, & \text{if } d_{i(t-1)} \neq 0 \text{ and } d_i(t-1) \neq j \\ 0, & \text{otherwise.} \end{cases}$$

Specifically, we assume that a consumer i incurs a cost for switching to a different application. However, switching to/from the outside option does not incur such cost.

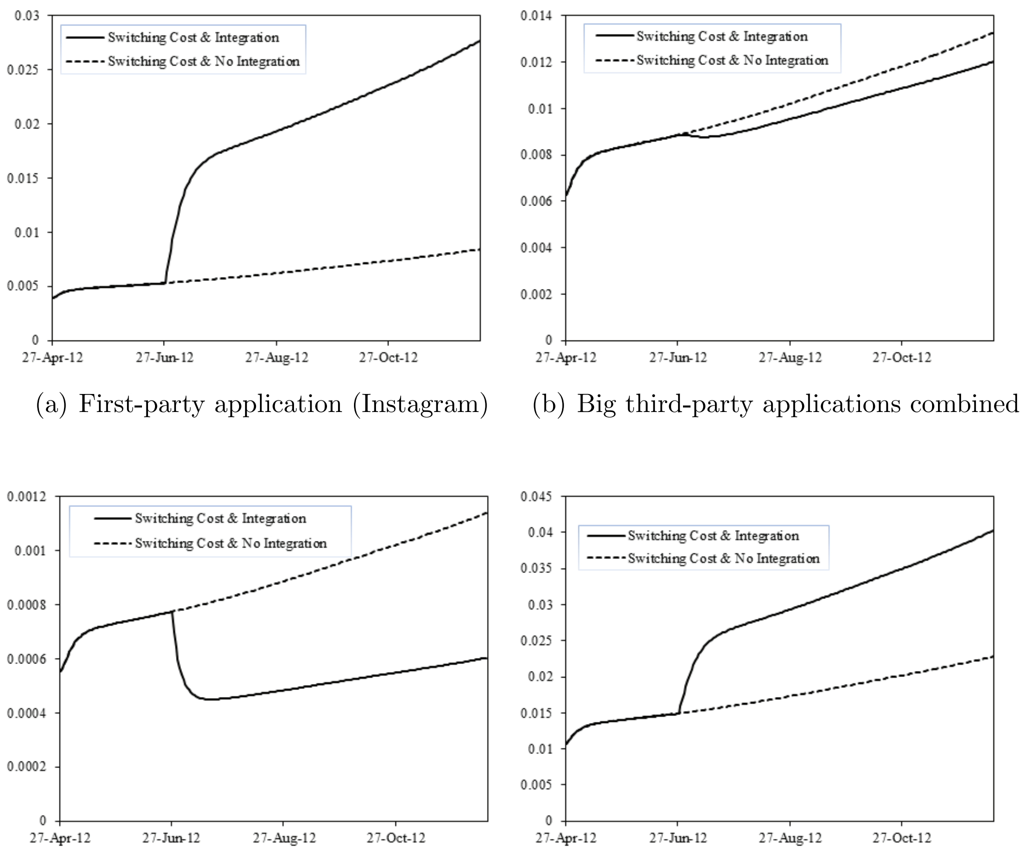
We assume c_i is drawn from a normal distribution $c_i \sim N(\bar{c}, \sigma_c^2)$, with mean $\bar{c}$ and variance σ_c^2. We fix $\sigma_c^2 = 1$ as it is difficult to identity both the mean and variance. Therefore, $c_i = \bar{c} + \varphi_i$, where φ_i follows the standard normal distribution. Again, we normalize the mean utility from the outside option to zero, i.e., $U_{i0t} = \epsilon_{i0t}$.

Identification of switching cost is methodologically challenging when only aggregate-level demand data are available. In the dataset, we do not observe an individual consumer's historical choices. Our identification strategy relies on the observed demand patterns. We observe the volume of incumbent application users as well as the number of new users joining the ecosystem. Incumbent application users face switching costs whereas new users do not. Given an initial set of parameter values, our iterative estimation procedure computes both existing and new users' probability of choosing an application based on product characteristics, users' previous choices, and switching costs. We equate the model-predicted market shares and the actual market shares in each period to solve for the mean utility and estimate the next set of parameter values. The algorithm iterates until the parameter estimates converges. The appendix provides the details of the estimation procedure. Estimates of the mean switching costs are shown in Table 4.5. Comparing the sum of squared errors with those in Table 4.2, we can see that the model fit improves after accounting for the switching costs.

Our results show that consumers on average incur high switching costs (Table 4.5). Figure 4.7 shows that demand predictions, after accounting for switching costs, are qualitatively similar to those from the base model. Switching costs may explain the observed demand patterns after the integration event. If there was no switching cost, the fast growth of Instagram's user base (and thus the increase in consumer utility from choosing Instagram) will attract many third-party applications users to Instagram. But we do not observe such big migration following the integration due to high switching costs which override the benefits of switching to Instagram. As outside option users and new users do not incur switching costs, they are far more likely to use Instagram after the tighter integration. Therefore, a large fraction of

Variable	Coefficients (Standard Errors)					
	(1)	(2)	(3)	(4)	(5)	(6)
LaggedAppUserBase (log)	0.6886*** (0.0102)	0.6087*** (0.0102)	0.6934*** (0.0102)	0.6496*** (0.0100)	0.6934*** (0.100)	0.6348*** (0.0100)
Integration×Instagram	0.0937*** (0.0403)	0.2500*** (0.0403)	0.0730* (0.0402)	0.1515*** (0.0402)	0.2321* (0.0402)	0.3454*** (0.0425)
Integration×SmallThirdPartyApp	-0.1376*** (0.0135)	-0.1199*** (0.0135)	-0.1623*** (0.0183)	-0.1685*** (0.0183)		
Integration×BigThirdPartyApp	0.0273** (0.0123)	0.0604*** (0.0123)			0.1621*** (0.0183)	0.1715*** (0.0183)
Time Dummy	No	No	Yes	Yes	Yes	Yes
Consumer Heterogeneity on AppUserBase (log)		0.1012*** (0.0106)		0.0793*** (0.0130)		0.0871*** (0.0077)
Switching Costs (Mean)	5.8706*** (0.0531)	3.3902*** (0.0360)	4.3118*** (0.0361)	3.8431*** (0.0069)	4.1115*** (0.0461)	3.5174*** (0.0268)
Sum of squared errors	139.5207	139.2849	131.0643	131.0100	131.0737	131.0220

Table 4.5: Parameter estimates of the models with switching costs. Model (3)-(4) include time dummies and the interaction term $Integration \times BigThirdPartyApp$ is omitted to avoid the dummy variable trap. Similarly, Model (5)-(6) include time dummies and the interaction term $Integration \times SmallThirdPartyApp$ is omitted.



(a) First-party application (Instagram) (b) Big third-party applications combined

(c) Small third-party applications combined (d) All applications combined

Figure 4.7: Impact of integration in the presence of switching costs

users Instagram gained are new users who did not use any application in previous period, rather than incumbent users of third-party applications.

4.7 Endogeneity Issues

We conduct several robustness checks to account for potential endogeneity issues. We discuss how to address these endogeneity concerns using appropriate instruments.

4.7.1 Endogeneity of Lagged User Base

As explained earlier, our model estimates might be biased due to the use of predetermined lagged user base which may be correlated with unobservables. In order to correct for this bias, we follow the approach of Arellano and Bover (1995) and use lagged differences of application user base as instruments for the mean utility function in Equation (4.4) and lagged application user base as instruments for the first-difference of this equation. The former equation is often referred to as "level equation", while the latter "first-differenced" equation. Blundell and Bond (1998) show that these instruments are correlated with explanatory variables and orthogonal to unobserved errors. These instruments have been successfully applied by researchers in a wide variety of fields within marketing and economics (see, e.g., Acemoglu and Robinson 2001, Durlauf et al. 2005, Clark et al. 2009, Yoganarasimhan 2012). To check the validity of these instruments in our context, we first perform weak identification tests on the instruments. The F statistic is greater than the recommended threshold of 10, suggesting the instruments are correlated with the suspected endogeneous variable (i.e., our instruments are not weak). We then perform the overidentification test (Hansen's J test) and cannot reject the null hypothesis of valid overidentifying restrictions. We apply these instruments using the generalized method of moments (GMM) method. Our estimation approach is explained in Appendix C.2.

GMM estimates of the models are in Table 4.6. Compared to the base model where instruments are not included, the two set of instruments both give qualitatively similar results. In models using lagged user bases as instruments, estimates of network effect are slightly larger and estimates of impact of integration are slightly smaller. In models using lagged differences of user bases as instruments, estimates of network effect are smaller and estimates of impact of integration are larger. Our estimates without using any instruments are located between the estimates using these two sets of instruments.

4.7.2 Endogeneity of Integration Timing

Facebook and Instagram might have chosen the integration timing such that the integration is more likely to lead to positive outcome. In other words, the integration event might be correlated with the unobserved shocks that enter a consumer's utility function but are unobservable to us. To control for this potential endogeneity, we use Facebook's stock price as an instrument for the integration timing.

Corporate investments are sensitive to stock prices (Baker et al. 2003, Chen et al. 2007). Additionally, firms are expected to increase their innovation activities and exploratory search after going public (Wu 2012). Facebook held its initial public

(a) Using lagged differenced user bases as instruments for the level equation

Variable	Coefficients (Standard Errors) (1)	(2)	(3)	(4)	(5)	(6)
LaggedAppUserBase (log)	0.6267*** (0.0800)	0.5575*** (0.0532)	0.5603*** (0.0942)	0.5437*** (0.0105)	0.5613*** (0.0940)	0.5537*** (0.0105)
Integration×Instagram	0.4473*** (0.1065)	0.4659*** (0.0411)	0.4658*** (0.1041)	0.4659*** (0.0421)	0.7213*** (0.1627)	0.7236*** (0.0445)
Integration×SmallThirdPartyApp	-0.1666*** (0.0374)	-0.1691*** (0.0126)	-0.2617*** (0.0659)	-0.2673*** (0.0192)		
Integration×BigThirdPartyApp	0.0506** (0.0224)	0.0678*** (0.0126)			0.2602*** (0.0656)	0.2626*** (0.0192)
Time Dummy	No	No	Yes	Yes	Yes	Yes
Consumer Heterogeneity on AppUserBase (log)		0.0655*** (0.0005)		0.0332*** (0.0010)		0.0236*** (0.0015)
Sum of squared errors	0.4460	0.4452	0.6284	0.6284	0.4393	0.4392

(b) Using lagged user bases as instruments for the first-differenced equation

Variable	Coefficients (Standard Errors) (1)	(2)	(3)	(4)	(5)	(6)
LaggedAppUserBase (log)	0.7354*** (0.0123)	0.7353*** (0.0102)	0.7439*** (0.0124)	0.7439*** (0.0101)	0.7441*** (0.0124)	0.7441*** (0.0101)
Integration×Instagram	0.3126*** (0.0415)	0.3126*** (0.0404)	0.2802*** (0.0421)	0.2802*** (0.0404)	0.4160*** (0.0454)	0.4160*** (0.0427)
Integration×SmallThirdPartyApp	-0.1189*** (0.0139)	-0.1189*** (0.0136)	-0.1384*** (0.0195)	-0.1384*** (0.0184)		
Integration×BigThirdPartyApp	0.0252** (0.0125)	0.0252** (0.0124)			0.1379*** (0.0195)	0.1379*** (0.0184)
Time Dummy	No	No	Yes	Yes	Yes	Yes
Consumer Heterogeneity on AppUserBase (log)		7.77E-05 (0.0037)		8.76E-05 (0.0061)		6.40E-05 (0.0057)
Sum of squared errors	0.5312	0.5312	0.7739	0.7739	0.7739	0.7739

Table 4.6: GMM Estimates with instrument variables. In both tables, Model (3)-(4) include time dummies and the interaction term $Integration \times BigThirdPartyApp$ is omitted to avoid the dummy variable trap. Similarly, Model (5)-(6) include time dummies and the interaction term $Integration \times SmallThirdPartyApp$ is omitted.

offering (IPO) in May 2012, but following that the share price dropped and the stock was considered disappointing. The company was under pressure from investors to generate more revenue (e.g., by monetizing Instagram) to improve its stock performance. Therefore, Facebook's subsequent investments were likely to be driven by its unsatisfactory stock price. Tighter integration of Instagram was one such innovation investment where Facebook explored seamless data exchange between Instagram and Facebook. Thus, the decision on the timing of integration was likely to be influenced by Facebook's stock price. The suspected correlation between the integration timing and Facebook's stock market performance is evident from the high correlation between the integration dummy and Facebook's stock price (correlation coefficient is -0.68).

Meanwhile, we expect that Facebook's stock price is not likely to influence consumers' relative preferences for various photo-sharing applications. The stock price may influence a user's decision to join Facebook. However, conditional on the fact that a consumer already joined Facebook, the stock price is not very likely to be directly correlated with the consumer's utility of using Instagram vis-à-vis any other photo-sharing application on the Facebook platform. Further, the Facebook platform was supporting over 9 million applications in different categories and only a small fraction (less than 1%) of Facebook users were using Instagram for photo-sharing on Facebook during our panel period (Table 4.1). As a result, although the stock price may be correlated with platform-specific unobservables, it is less likely to be correlated with unobservables specific to an individual application (e.g., promotions by Instagram).

We test the validity of using Facebook's stock price alone as an instrument for the integration timing. The F statistic is far greater than the recommended threshold of 10, suggesting the instruments are correlated with the integration timing. Note that we use stock performance data from SecondMarket for the month of April 2012 as Facebook's IPO took place in May 2012. We also test the validity of using Facebook's stock price together with lagged differences of application user base as instruments for both of the suspected endogenous variables (i.e., integration timing and lagged application user base). The F statistic is much larger than the recommended threshold of 10, suggesting the instruments are not weak. The Hansen's J test cannot reject the null hypothesis of valid overidentifying restrictions. These tests provide statistical evidence that the instruments are valid.

Estimates with this instrument, as reported in Table 4.7, remain qualitatively unchanged compared to estimates of the models without using any instruments. These additional analyses provide evidence that our results and main findings are robust.

4.8 Discussion and Conclusion

In this paper, we build a structural model of consumer choices and estimate application demand using aggregate data on application usage before and after platform integration. We find that consumers obtain additional value from Instagram after its tighter integration with Facebook, leading to dramatic growth in demand for Instagram. However, a large fraction of new users Instagram gained are new users who did not use any photo-sharing application, rather than incumbent users of third-

Variable	Coefficients (Standard Errors)					
	(1)	(2)	(3)	(4)	(5)	(6)
LaggedAppUserBase (log)	0.7075*** (0.0108)	0.7075*** (0.0102)	0.7138*** (0.0108)	0.7138*** (0.0101)	0.7040*** (0.0115)	0.7074*** (0.0101)
Integration×Instagram	0.3958*** (0.0613)	0.3958*** (0.0404)	0.3324*** (0.0616)	0.3324*** (0.0403)	0.5864*** (0.0688)	0.5864*** (0.0427)
Integration×SmallThirdPartyApp	-0.1312*** (0.0137)	-0.1312*** (0.0136)	-0.1566*** (0.0190)	-0.1566*** (0.0184)		
Integration×BigThirdPartyApp	0.0317** (0.0125)	0.0317** (0.0124)			0.1749*** (0.0205)	0.1749*** (0.0184)
Time Dummy	No	No	Yes	Yes	Yes	Yes
Consumer Heterogeneity on AppUserBase (log)		1.39E-04 (0.0071)		4.99E-05 (0.0081)		4.96E-05 (0.0319)
Sum of squared errors	1.5633	1.5633	1.5904	1.5904	1.3415	1.3415

Table 4.7: Parameter estimates with alternative instrument for integration timing. Model (3)-(4) include time dummies and the interaction term $Integration \times BigThirdPartyApp$ is omitted to avoid the dummy variable trap. Similarly, Model (5)-(6) include time dummies and the interaction term $Integration \times SmallThirdPartyApp$ is omitted.

party applications. As a result, the overall demand for Instagram and third-party applications actually increases, which suggest that Facebook's integration strategy benefits the complementary market. We find that the integration has different impact on big third-party applications and small third-party applications. Consumer valuations of small third-party applications are reduced by a larger amount, whereas valuations of big third-party applications are resistant to the integration shock. Such variable effects may be attributed to users' lower perceived staying power of small third-party applications after platform integration.

Our study makes several contributions. Managing the tension between first-party content and third-party content has been a critical part of major platforms' strategies. Previous research has mostly relied on theoretical models to study strategic interactions between the platform owner and third-party developers (i.e., supply-side behaviors). Our paper is the first study that empirically evaluates consumer preferences for first-party applications vis-à-vis third-party applications (i.e., demand-side behaviors). This paper is also the first to empirically demonstrate the impact of integration of an application by a platform on the application ecosystem. Our model and findings provide important implications for managing platform-based businesses. Analysis of the substitution and complementary effects between first-party applications and third-party applications may help platform owners determine the overall impacts of their platform strategies. Our structural demand analysis can also help platform owners evaluate whether it is beneficial to tightly integrate certain third-party applications with the platforms. Third-party developers may also benefit from better understanding of consumer preferences for first-party applications and third-party applications. Our models and results may help developers decide whether it is profitable to participate in a platform in the presence of first-party applications.

Our findings shed light on the effectiveness of the platform's strategy to provide tighter integration with the first-party application. On one hand, our results suggest that such a strategy may be beneficial particularly in a market where network effects and switching costs are present. In such a market scenario, the platform owner may gain new users due to the appeal of the tightly integrated first-party application while not hurting third-party applications too much. On the other hand, our research informs platform owners and policy makers about the potential dark side of platform integration. As small third-party applications are more vulnerable to the negative shock from vertical integration, such integration strategy may cause small third-party developers to exit the market, which may reduce the variety of products/services available in the complementary market. For platform owners and policy makers, our research informs the trade-off between the gains in accumulated demand in the short-run and losses in product variety in the long-run due to platform integration. As small third-party applications are more vulnerable to platform integration, platform owners may come up with certain subsidy schemes to incentivize small developers to stay in their ecosystems.

For third-party developers, our research has implications for their product design. Social applications like Facebook applications exhibit network effects and switching costs. Third-party developers may incorporate social features into their products/services to create large user base that mitigates the negative impact of platform integration. Building a large user base not only creates high exchange value for users, but also helps maintain users' perceived staying power of the products. For small third-party applications facing the threat of first-party applications, the priority of their business strategies may be given to continuously growing the user base, instead of rushing to monetize the existing customers.

Our study is not without limitations. The focus of this paper is short-run demand-side consumer behaviors, i.e., how consumers respond to platform integration and the resulting demand patterns for different types of applications in the complementary market. We do not model third-party developers' strategic decisions such as entry and exit, which require completely different models and assumptions. Future research may use a longer panel dataset to investigate these strategic responses and see how they impact the long-term viability of the ecosystem. Future research can also look into the role of product characteristics and product differentiation in influencing demand for first-party and third-party applications. Understanding the role of product differentiation may provide third-party developers important insights into optimal product design. It may also help platform owners decide what product attributes should be included in their first-party applications. Finally, out study is restricted to one platform ecosystem. Future studies may evaluate the robustness of the results in other platform-based ecosystems.

Appendices

Appendix A

Appendix for Chapter 2

A.1 Proofs of Main Results

Proof of Lemma 1. $V_{ij}(q_R)$ follows a piecewise quadratic function. The concavity property is relatively straightforward. Taking the first order condition of $V_{ii}(q_R)$ yields the unconstrained maximizer, $q_R = \frac{a_i - 2w + c}{2}$; when $w \geq \frac{a_i + c}{2}$, the unique maximizer would be $q_R = 0$.

Proof of Lemma 2 and Proposition 3. Based on Lemma 1, without the mimicking incentive, the reseller's optimal order quantity follows $q_R(w; a_i) = \frac{a_i - 2w + c}{2}$ (assuming it is positive). It is obvious that if $V_{HL}(\frac{a_L - 2w + c}{2}) \leq V_{HH}\left(\frac{a_H - 2w + c}{2}\right)$, then the reseller that observes a large market size has no incentive to mimic the ordering decision under a small market size (note that the reseller would never mimic the ordering decision under a large market size, when the true market size is small). This condition can be written as

$$\frac{(4a_H - 3a_L + c - 2w)(a_L + c - 2w)}{8} \leq \frac{(a_H + c - 2w)^2}{8},$$

or identically,

$$w \geq \overline{w} = \frac{3a_L - a_H + 2c}{4}.$$

In contrast, if $w < \overline{w}$, then the reseller that observes a large market size may attempt to mimic the ordering decision under a small market size. As a result, for a separating equilibrium to hold, the reseller, when observing a small market size, has to downward distort his order quantity to a level such that he would have no incentive to mimic when seeing a large market size; i.e., $V_{HL}(q_R(w;a_L)) \le V_{HH}\left(\frac{a_H-2w+c}{2}\right)$. Expanding this condition

$$[a_H - q_R(w;a_L) - \frac{a_L - q_R(w;a_L) - c}{2} - w]q_R(w;a_L) \le \frac{(a_H + c - 2w)^2}{8},$$

from which we obtain the threshold order quantity

$$\bar{q}_R(w) = \frac{2a_H - a_L - 2w + c - \sqrt{(a_H - a_L)(3a_H - a_L + 2c - 4w)}}{2}.$$

Then, given the specification of the supplier's belief system, one can verify that

$$q_R(w;a_i) = \begin{cases} \frac{a_H-2w+c}{2} & \text{if } i = H, \\ \hat{q}_R(w) & o/w \end{cases}$$

where $\hat{q}_R(w) = \min\left\{\left(\frac{a_L-2w+c}{2}\right)^+, \bar{q}_R(w)\right\}$ and

$$q_S(w;a_i) = \frac{a_i - q_R(w;a_i) - c}{2}$$

constitute a separating equilibrium. The result that this equilibrium uniquely survives the intuitive criterion is provided later).

Proof of Lemma 4. When $\overline{w} \le w < \frac{a_L+c}{2}$, $q_R(w;a_L) = \frac{a_L-2w+c}{2}$ and thus $\left|\frac{\mathrm{d}q_R(w;a_L)}{\mathrm{d}w}\right| = 1$. When $w < \overline{w}$, $q_R(w;a_L) = \bar{q}_R(w)$. Take the first derivative of $\bar{q}_R(w)$:

$$\frac{\mathrm{d}\bar{q}_R(w)}{\mathrm{d}w} = -1 + \frac{(a_H - a_L)}{\sqrt{(a_H - a_L)(3a_H - a_L + 2c - 4w)}} > -1.$$

Hence, $\left|\frac{\mathrm{d}q_R(w;a_L)}{\mathrm{d}w}\right| = 1$ when $\overline{w} \le w < \frac{a_L+c}{2}$ and $\left|\frac{\mathrm{d}q_R(w;a_L)}{\mathrm{d}w}\right| < 1$ when $w < \overline{w}$.

Proof of Proposition 5. Note that the supplier's expected profit follows

$$\begin{aligned}\Pi_S(w) = & \left[\lambda q_R(w;a_H) + (1-\lambda)\, q_R(w;a_L)\right] w \\ & + \lambda\left(\frac{a_H - q_R(w;a_H) - c}{2}\right)^2 + (1-\lambda)\left(\frac{a_L - q_R(w;a_L) - c}{2}\right)^2.\end{aligned}$$

We first show parts ii) and iii) where both parties' selling quantities are positive. When $w \ge \overline{w}$, the reseller does not distort his order quantity and thus $q_R(w;a_i) = \frac{a_i - 2w + c}{2}$. The supplier's expected profit in this natural separating equilibrium is (NS denotes natural separating, i.e., with no distortion):

$$\begin{aligned}\Pi_S^{NS}(w) &= \left[\lambda\left(\frac{a_H - 2w + c}{2}\right) + (1-\lambda)\left(\frac{a_L - 2w + c}{2}\right)\right] w \\ &\quad + \lambda\left(\frac{a_H + 2w - 3c}{4}\right)^2 + (1-\lambda)\left(\frac{a_L + 2w - 3c}{4}\right)^2 \\ &= \frac{-12w^2 + w(12\mu - 4c) + 9c^2 - 6\mu c + \lambda a_H^2 + (1-\lambda)a_L^2}{16}.\end{aligned} \tag{A.1}$$

When $w < \overline{w}$, the reseller distorts his order quantity when the market size is small and thus $q_R(w;a_L) = \bar{q}_R(w)$. The supplier's expected profit is (SD denotes separating with distortion):

$$\begin{aligned}\Pi_S^{SD}(w) = & \left[\lambda\left(\frac{a_H - 2w + c}{2}\right) + (1-\lambda)\,\bar{q}_R(w)\right] w \\ & + \lambda\left(\frac{a_H + 2w - 3c}{4}\right)^2 + (1-\lambda)\left(\frac{a_L - \bar{q}_R(w) - c}{2}\right)^2.\end{aligned}$$

The supplier selects the best wholesale price according to the above two types of separating equilibrium; that is, the supplier can solve two constrained optimization problems: $\max_w E[\Pi_S^{NS}(w)]$ s.t. $w \geq \overline{w}$, and $\max_w E[\Pi_S^{SD}(w)]$ s.t. $w < \overline{w}$, and choose the better outcome.

The first order condition of $\Pi_S^{NS}(w)$ yields the unconstrained optimal solution $w^{NS*} = \frac{3\mu - c}{6}$. The first order condition of $\Pi_S^{SD}(w)$ follows

$$\frac{\mathrm{d}\Pi_S^{SD}(w)}{\mathrm{d}w} = \frac{1}{4}\left[\lambda(3a_H - c - 6w) + (1-\lambda)(a_H + 2a_L - c - 6w)\left(1 - \sqrt{\frac{a_H - a_L}{3a_H - a_L + 2c - 4w}}\right)\right] = 0. \tag{A.2}$$

Notice that when $w < \overline{w} = \frac{3a_L - a_H + 2c}{4}$, $\sqrt{\frac{a_H - a_L}{3a_H - a_L + 2c - 4w}} < \frac{1}{2}$, which implies $0 < 1 - \sqrt{\frac{a_H - a_L}{3a_H - a_L + 2c - 4w}} < 1$. Thus, for $\frac{\mathrm{d}\Pi_S^{SD}(w_f)}{\mathrm{d}w} = 0$ to hold for $w_f < \overline{w}$, we must have $3a_H - c - 6w_f > 0$ and $a_H + 2a_L - c - 6w_f < 0$ (given $3a_H - c - 6w_f > a_H + 2a_L - c - 6w_f$). Then, we can derive

$$\begin{aligned} \frac{\mathrm{d}\Pi_S^{SD}(w_f)}{\mathrm{d}w} &> \frac{1}{4}\left[\lambda(3a_H - c - 6w_f) + (1-\lambda)(a_H + 2a_L - c - 6w_f)\right] \\ &> \frac{3\mu - c - 6w_f}{4}, \end{aligned}$$

which asserts that if $\frac{\mathrm{d}\Pi_S^{SD}(w)}{\mathrm{d}w} = 0$ has a solution $w_f \in (0, \overline{w})$, then w_f must be larger than the unconstrained maximizer, $\frac{3\mu - c}{6}$, of $\Pi_S^{NS}(w)$. Note that we can derive the second and third derivatives of $\Pi_S^{SD}(w)$. In particular, the third derivative $\frac{\mathrm{d}^3\Pi_S^{SD}(w)}{\mathrm{d}w^3}$ is always positive when $w \leq \overline{w}$ and thus the second derivative $\frac{\mathrm{d}^2\Pi_S^{SD}(w)}{\mathrm{d}w^2}$ is increasing when $w \leq \overline{w}$. We can also verify that $\frac{\mathrm{d}^2\Pi_S^{SD}(w)}{\mathrm{d}w^2}$ can be positive at $w = \overline{w}$ only if $c > \frac{(29+19\lambda)(a_H - a_L)}{8(1-\lambda)}$, and the first derivative $\frac{\mathrm{d}\Pi_S^{SD}(w)}{\mathrm{d}w}$ is positive at $w = \overline{w}$ only if

$c < \frac{(5+13\lambda)(a_H - a_L)}{8(1+\lambda)}$, which cannot hold simultaneously. Therefore, there at most exists one solution of $\frac{\mathrm{d}\Pi_S^{SD}(w)}{\mathrm{d}w} = 0$ in $(0, \overline{w})$.

Note that $\Pi_S^{NS}(w)$ and $\Pi_S^{SD}(w)$ coincide at $w = \overline{w}$ because the reseller's order quantity in the equilibrium without distortion coincides with that with distortion at $w = \overline{w}$. Therefore, if $\overline{w} \leq w^{NS*} = \frac{3\mu - c}{6}$, or identically, $c \leq \frac{3(1+2\lambda)(a_H - a_L)}{8}$, then w^{NS*} induces a natural separating equilibrium; moreover, $\Pi_S^{SD}(w)$ must be increasing at $w = \overline{w}$ given that $\Pi_S^{SD}(w)$ is increasing at $w = 0$ and any solution of the first order condition of $\Pi_S^{SD}(w)$ is larger than w^{NS*} or $\overline{w}$. Hence, if $c \leq \frac{3(1+2\lambda)(a_H - a_L)}{8}$, $w^* = w^{NS*} = \frac{3\mu - c}{6}$ is the supplier's optimal wholesale price. In contrast, if $\overline{w} > w^{NS*} = \frac{3\mu - c}{6}$, or identically, $c > \frac{3(1+2\lambda)(a_H - a_L)}{8}$, then w^{NS*} does not induce a natural separating equilibrium and the corner solution $\overline{w}$ would be the supplier's best choice achieving a natural separating equilibrium. Note that if $\frac{\mathrm{d}\Pi_S^{SD}(w)}{\mathrm{d}w} = 0$ has a solution $w_f \in (0, \overline{w})$, then w_f induces the distorted separating equilibrium which is the optimal solution. If $\frac{\mathrm{d}\Pi_S^{SD}(w)}{\mathrm{d}w} = 0$ does not have a solution in $(0, \overline{w})$, then $\Pi_S^{SD}(w)$ must be increasing in $(0, \overline{w})$and the corner solution $\overline{w}$ will be the supplier's optimal wholesale price. Given the optimal wholesale price, we can directly obtain the reseller's order quantity and then the supplier's direct selling quantity.

Notice that given the optimal wholesale price $w^* = \frac{3\mu - c}{6}$ in the separating equilibrium without distortion, the supplier's expected profit can be derived as:

$$\begin{aligned}
\Pi_S &= \left[\lambda\left(\frac{a_H - 2w^* + c}{2}\right) + (1-\lambda)\left(\frac{a_L - 2w^* + c}{2}\right)\right] w^* \\
&\quad +\lambda\left(\frac{a_H + 2w^* - 3c}{4}\right)^2 + (1-\lambda)\left(\frac{a_L + 2w^* - 3c}{4}\right)^2 \\
&= \frac{-12(\frac{3\mu - c}{6})^2 + (12\mu - 4c)\left(\frac{3\mu - c}{6}\right) + 9c^2 - 6\mu c + \lambda a_H^2 + (1-\lambda)a_L^2}{16} \\
&= \frac{4\mu^2 - 8\mu c + (9 + 1/3)c^2 + \sigma^2}{16}
\end{aligned} \tag{A.3}$$

where $\sigma^2 = \lambda\left(a_H - \mu\right)^2 + (1-\lambda)\left(a_L - \mu\right)^2$, the variance of $\mathbf{a}$. The supplier's expected profit in the separating equilibrium with distortion cannot be explicitly expressed.

The last step is to characterize the boundary conditions such that the supplier's and the reseller's selling quantities are strictly positive. If the supplier's selling cost is low, the reseller's selling quantity under a small market size will first go to zero. This always happens in a natural separating equilibrium, i.e., when $c \leq \frac{3(1+2\lambda)(a_H - a_L)}{8}$ (when the reseller's order quantity goes to zero under a small market size, the reseller would never mimic such an ordering decision when the market size is large; thus, it must be a separating equilibrium without distortion). It can be easily shown that when $c > \frac{3\lambda(a_H - a_L)}{4}$, the reseller's order quantity is positive. On the contrary, as the supplier's selling cost increases, the supplier's direct selling quantity may go to zero. Given a_H and a_L, we define the smallest threshold $\overline{c}(\lambda)$ that can be implicitly determined at which the supplier's direct selling quantity goes to zero under at least one market size.

Proof of Part i). The supplier can also choose a wholesale price such that the reseller does not order with a small market size. The supplier's profit function in this case is

$$\Pi_S(w) = \lambda \left[\left(\frac{a_H - 2w + c}{2} \right) w + \left(\frac{a_H + 2w - 3c}{4} \right)^2 \right] + (1-\lambda) \left(\frac{a_L - c}{2} \right)^2 .$$

There exists a unique maximizer $w^* = \frac{3a_H - c}{6}$ and the subgame equilibrium follows directly:$q_R(w^*; a_H) = \frac{2c}{3}$, $q_S(q_R(w^*; a_H)) = \frac{3a_H - 5c}{6}$, $q_R(w^*; a_L) = 0$, and $q_S(q_R(w^*; a_L)) = \frac{a_L - c}{2}$. The expected profit of the reseller is $\Pi_R = \lambda \frac{2c^2}{9}$ and the expected profit of the supplier is

$$\begin{aligned} \Pi_S &= \lambda \frac{3a_H^2 - 6a_H c + 7c^2}{12} + (1-\lambda) \frac{(a_L - c)^2}{4} \\ &= \frac{3\mu^2 - 6\mu c + 3c^2 + 3\sigma^2 + 4\lambda c^2}{12}. \end{aligned} \tag{A.4}$$

Comparing the supplier's expected profits in (A.3) and (A.4), we can find a threshold $\frac{3\sqrt{\lambda}(a_H - a_L)}{4}$ such that when $c \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, setting $w^* = \frac{3a_H - c}{6}$ is more beneficial for the supplier, under which the reseller does not sell when the market size is small, and when $c > \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, the supplier shall induce the reseller to order a positive quantity in both market scenarios.

Proof of Proposition 6. We first show that when $\frac{3\sqrt{\lambda}(a_H - a_L)}{4} < c \leq \min\left\{ \frac{3(1+2\lambda)(a_H - a_L)}{8}, \bar{c}(\lambda) \right\}$, the supplier benefits from encroachment. In this region, with encroachment, the natural separating equilibrium arises and the supplier's expected profit is as expressed in (A.3). From section 2.4.1, we know that without encroachment, the supplier's profit is $\pi_S = \frac{\mu^2}{8} = \frac{[\lambda a_H + (1-\lambda) a_L]^2}{8}$ with our presumption that $a_L > \frac{\mu}{2}$ (or identically $\lambda < \frac{a_L}{a_H - a_L}$; the reseller is induced to order a positive quantity in both market scenarios). Thus, we derive:

$$\begin{aligned}\Pi_S - \pi_S &= \frac{4\mu^2 - 8\mu c + (9 + 1/3)c^2 + \sigma^2}{16} - \frac{\mu^2}{8} \\ &= \frac{2(\mu - 2c)^2 + (1 + 1/3)c^2 + \sigma^2}{16} > 0.\end{aligned}$$

Second, we investigate the region where $c \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$. The supplier's expected profit with encroachment follows (A.4). Thus, we derive:

$$\Pi_S - \pi_S = \frac{\frac{3\mu^2}{2} - 6\mu c + 3c^2 + 3\sigma^2 + 4\lambda c^2}{12},$$

which decreases in c when $c < \frac{3\mu}{3+4\lambda}$. Given our presumption $\lambda < \frac{a_L}{a_H - a_L}$, we can verify that $\frac{3\mu}{3+4\lambda} = \frac{3(a_L + \lambda(a_H - a_L))}{3+4\lambda} > \frac{3(\lambda(a_H - a_L) + \lambda(a_H - a_L))}{3+4\lambda} > \frac{3\lambda(a_H - a_L)}{4}$.

Notice that when $\frac{3\lambda(a_H - a_L)}{4} \leq c \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, if the supplier followed the natural separating equilibrium, then the reseller would sell a positive quantity for each market size and thus the supplier's profit would be $\frac{4\mu^2 - 8\mu c + (9+1/3)c^2 + \sigma^2}{16}$ which is larger than $\frac{\mu^2}{8}$, as we have verified in the above. When the supplier optimizes her wholesale price, for $c \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, she chooses the strategy to sell only to the reseller when the market size is large, which implies that her profit under this strategy is larger than that under the natural separating equilibrium and thus larger than $\pi_S = \frac{\mu^2}{8}$. In other words, $\Pi_S - \pi_S > 0$ when $\frac{3\lambda(a_H - a_L)}{4} \leq c \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$. Given $\Pi_S - \pi_S$ is decreasing in c when $c < \frac{3\mu}{3+4\lambda}$, we assert that $\Pi_S - \pi_S > 0$ when $c < \frac{3\lambda(a_H - a_L)}{4}$.

Proof of Proposition 7. Notice that with supplier encroachment, the reseller's expected profit is

$$\Pi_R = \lambda \frac{2c^2}{9} \tag{A.5}$$

when $c \in \left(0, \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right]$ and

$$\begin{aligned} \Pi_R &= \lambda \frac{(a_H + c - 2w^*)^2}{8} + (1-\lambda)\frac{(a_L + c - 2w^*)^2}{8} \\ &= \lambda \frac{\left[(1-\lambda)(a_H - a_L) + \frac{4}{3}c\right]^2}{8} + (1-\lambda)\frac{\left[\lambda(a_H - a_L) - \frac{4}{3}c\right]^2}{8} \end{aligned} \tag{A.6}$$

when $c \in \left(\frac{3\sqrt{\lambda}(a_H - a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}\right]$. We can verify that Π_R increases in c and has an upward jump at $c = \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$.

On the other hand, the reseller's expected profit without supplier encroachment is

$$\pi_R = \lambda \frac{(2a_H - \mu)^2}{16} + (1-\lambda)\frac{(2a_L - \mu)^2}{16} \tag{A.7}$$

which is independent of c.

Comparing (A.5) with (A.7) yields a threshold $\hat{c}_R^l(\lambda) = \frac{3\sqrt{2(2a_H - \mu)^2 + 2(\frac{1}{\lambda} - 1)(2a_L - \mu)^2}}{8}$ where the two profits are equal; similarly, comparing (A.6) with (A.7) yields another threshold $\hat{c}_R^h(\lambda) = \frac{3\sqrt{(4\lambda^2 - 4\lambda)a_H a_L + (4\lambda - 2\lambda^2)a_H^2 + (2 - 2\lambda^2)a_L^2}}{8}$. Given the fact that Π_R increases in c while π_R is independent of c, it can be seen that if $\hat{c}_R^l(\lambda) \leq \frac{3\sqrt{\lambda}(a_H - a_L)}{4}$, then the reseller is worse off in expectation by supplier encroachment when $c \in \left(0, \hat{c}_R^l(\lambda)\right]$ and better off when

$$c \in \left(\hat{c}_R^l(\lambda), \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}\right];$$

otherwise, if $\hat{c}_R^h(\lambda) < \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}$, then the reseller is worse off when $c \in \left(0, \max\left\{\hat{c}_R^h(\lambda), \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right\}\right]$ and better off when

$$c \in \left(\max\left\{\hat{c}_R^h(\lambda), \frac{3\sqrt{\lambda}(a_H - a_L)}{4}\right\}, \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}\right];$$

otherwise, the reseller is always worse off by supplier encroachment when

$$c \in \left(0, \min\left\{\frac{3(1+2\lambda)(a_H - a_L)}{8}, \overline{c}(\lambda)\right\}\right].$$

Combining these scenarios, we can identify a threshold $\hat{c}_R(\lambda)$ that is either $\hat{c}^l_R(\lambda)$, $\max\left\{\hat{c}^h_R(\lambda), \frac{3\sqrt{\lambda}(a_H-a_L)}{4}\right\}$, or $\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}$. Notice that when $(1+\sqrt{2})a_L < a_H < \frac{13}{5}a_L$, there always exists a $\overline{\lambda}$ such that for $\lambda \in [0, \overline{\lambda})$, $\hat{c}_R(\lambda) < \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}$; also, $\hat{c}_R(\lambda) = \frac{3\sqrt{2}a_L}{8}$ when $\lambda \to 0$ and $\hat{c}_R(\lambda) = \frac{3\sqrt{2}a_H}{8}$ when $\lambda \to 1$. Therefore, there exist cases where the reseller is better off by supplier encroachment.

Proof of Remark 1. When $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}, \overline{c}(\lambda)\right)$, the supplier might be worse off in an equilibrium where the reseller downward distorts his order quantity. Since there is no closed-form solution for the supplier's optimal wholesale price when the market size is small, deriving the necessary and sufficient condition under which the supplier is worse off is technically challenging. Thus, we analyze the limiting case with $\lambda \to 0$.

Without supplier encroachment, as $\lambda \to 0$, the supplier's expected profit is $\pi_S = \frac{a_L^2}{8}$ and the reseller's expected profit is $\pi_R = \frac{{a_L}^2}{16}$. On the other hand, with supplier encroachment and when the separating with distortion arises, as $\lambda \to 0$, the supplier's optimal wholesale price is:

$$w^* = \begin{cases} \frac{a_H+2a_L-c}{6} & \text{if } \frac{5(a_H-a_L)}{8} \leq c < \overline{c}(\lambda), \\ \overline{w} & \text{if } \frac{3(a_H-a_L)}{8} < c < \frac{5(a_H-a_L)}{8}. \end{cases} \tag{A.8}$$

The supplier's expected profit is $\Pi_S = \bar{q}_R(w^*)w^* + \left(\frac{a_L-\bar{q}_R(w^*)-c}{2}\right)^2$ and the reseller's expected profit is $\Pi_R = \left(\frac{a_L-\bar{q}_R(w^*)+c-2w}{2}\right)\bar{q}_R(w^*)$. We can show that there exist a_H, a_L and $c > \frac{5(a_H-a_L)}{8}$ such that

$$\begin{aligned}
\Pi_S - \pi_S &= \frac{1}{72}\Big[33(a_H-a_L)^2 + 9(a_L-2c)^2 + 60c(a_H-a_L) + 6c^2 \\
&\quad -(7a_H - 7a_L + 8c)\sqrt{3(a_H-a_L)(7a_H-7a_L+8c)}\Big] < 0; \\
\Pi_R - \pi_R &= \frac{1}{72}\Big[-\frac{9a_L^2}{2} + (5a_H - 5a_L + 4c - \sqrt{3(a_H-a_L)(7a_H-7a_L+8c)}) \\
&\quad \times \left(-7a_H + 7a_L + 4c + \sqrt{3(a_H-a_L)(7a_H-7a_L+8c)}\right)\Big] < 0.
\end{aligned}$$

For example, when $a_H = 1.35a_L$, both $\Pi_S - \pi_S < 0$ and $\Pi_R - \pi_R < 0$ when $c \in [0.40a_L, 0.65a_L]$. Given the two parties' profit functions are continuous in λ, we can find such a_H, a_L, $\lambda > 0$ and $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}, \bar{c}(\lambda)\right)$, under which both parties are worse off by supplier encroachment.

Proof of Proposition 8. First, when $c \in \left(0, \frac{3\sqrt{\lambda}(a_H-a_L)}{4}\right]$, in the resulting equilibrium with information acquisition, the supplier's expected profit follows (A.4). Thus, the supplier gains from reseller information acquisition by:

$$\begin{aligned}
\Pi_S - \Pi_S^{NI} &= \frac{3\mu^2 - 6\mu c + 3c^2 + 3\sigma^2 + 4\lambda c^2}{12} - \frac{3\mu^2 - 6\mu c + 7c^2}{12} \\
&= \frac{3\sigma^2 - 4(1-\lambda)c^2}{12} \\
&= (1-\lambda)\frac{3\lambda(a_H-a_L)^2 - 4c^2}{12}
\end{aligned}$$

which is larger than zero for any $c \in \left(0, \frac{3\sqrt{\lambda}(a_H-a_L)}{4}\right]$.

Second, when $c \in \left(\frac{3\sqrt{\lambda}(a_H-a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}\right)$, with information acquisition, the supplier's expected profit follows (A.3). Thus, the supplier gains from reseller information acquisition by:

$$\Pi_S - \Pi_S^{NI} = \frac{4\mu^2 - 8\mu c + (9+1/3)c^2 + \sigma^2}{16} - \frac{3\mu^2 - 6\mu c + 7c^2}{12} = \frac{\sigma^2}{16} > 0.$$

Note that $c \in \left(0, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}\right)$ serves as a sufficient condition.

Proof of Proposition 21. First, when $c \in \left(\frac{3\sqrt{\lambda}(a_H-a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}\right)$, the resulting equilibrium with the reseller having private information is the natural separating equilibrium without distortion. The reseller's expected profit follows (A.6) and thus the reseller gains by

$$\begin{aligned}\Pi_R - \Pi_R^{NI} &= \lambda\frac{\left[(1-\lambda)(a_H-a_L)+\frac{4}{3}c\right]^2}{8} + (1-\lambda)\frac{\left[\lambda(a_H-a_L)-\frac{4}{3}c\right]^2}{8} - \frac{2c^2}{9} \\ &= \frac{\lambda(1-\lambda)(a_H-a_L)^2}{8} > 0.\end{aligned}$$

Second, when $c \in \left(0, \frac{3\sqrt{\lambda}(a_H-a_L)}{4}\right]$, with private information, the reseller will not sell when the market size is small and his expected profit is $\Pi_R = \frac{2\lambda c^2}{9}$, which is always smaller than $\Pi_R^{NI} = \frac{2c^2}{9}$.

Third, as shown in Proposition 3, the equilibrium wholesale price with the reseller having private information when $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \overline{c}(\lambda)\right\}, \overline{c}(\lambda)\right)$ satisfies $w^* > \frac{3\mu-c}{6}$. Moreover, it can be shown that the reseller's profit always decreases in w. Thus, by setting $w^* = \frac{3\mu-c}{6}$, we can obtain the following upper-bound on the reseller's profit for this range of c, which we denote by Π_R^{EB}:

$$\Pi_R^{EB} = \lambda\frac{(a_H-2w^*+c)^2}{8} + (1-\lambda)\left(\frac{a_L-\bar{q}_R(w^*)+c-2w^*}{2}\right)\bar{q}_R(w^*)$$

Recall that $\Pi_R^{NI} = \frac{2c^2}{9}$. With some algebra, we can show that $\Pi_R^{EB} <> \Pi_R^{NI}$ is equivalent to:

$$4\sqrt{3}\sqrt{(a_H-a_L)[(9-6\lambda)(a_H-a_L)+8c]} - 8c - (21-9\lambda)(a_H-a_L) <> 0.$$

It can further be shown that if $c > \frac{3(1+3\lambda+4\sqrt{\lambda})(a_H-a_L)}{8}$, then $\Pi_R^{EB} < \Pi_R^{NI}$ and thus $\Pi_R < \Pi_R^{NI}$.

Proof of Proposition 10. i) With information sharing, we can express the supplier's problem as:

$$\begin{aligned}\Pi_S^{SI} &= \max_{w_H,w_L}\left[\lambda\left(\frac{a_H-2w_H+c}{2}\right)w_H+(1-\lambda)\left(\frac{a_L-2w_L+c}{2}\right)w_L\right]\\ &\quad +\lambda\left(\frac{a_H+2w_H-3c}{4}\right)^2+(1-\lambda)\left(\frac{a_L+2w_L-3c}{4}\right)^2.\end{aligned}$$

Without information sharing, when $c \in \left(0, \frac{3\sqrt{\lambda}(a_H-a_L)}{4}\right]$, the supplier's profit follows:

$$\Pi_S = \max_w \lambda\left[\left(\frac{a_H-2w+c}{2}\right)w+\left(\frac{a_H+2w-3c}{4}\right)^2\right]+(1-\lambda)\left(\frac{a_L-c}{2}\right)^2,$$

which is clearly inferior to the one under information sharing.

When $c \in \left(\frac{3\sqrt{\lambda}(a_H-a_L)}{4}, \min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}\right)$, the supplier's profit follows:

$$\begin{aligned}\Pi_S &= \max_w\left[\lambda\left(\frac{a_H-2w+c}{2}\right)w+(1-\lambda)\left(\frac{a_L-2w+c}{2}\right)w\right]\\ &\quad +\lambda\left(\frac{a_H+2w-3c}{4}\right)^2+(1-\lambda)\left(\frac{a_L+2w-3c}{4}\right)^2,\end{aligned}$$

which is also inferior to the one under information sharing.

When $c \in \left(\min\left\{\frac{3(1+2\lambda)(a_H-a_L)}{8}, \bar{c}(\lambda)\right\}, \bar{c}(\lambda)\right)$, without information sharing, the distorted separating equilibrium will arise. We can rewrite the supplier's profit under information sharing as

$$\Pi_S^{SI} = \lambda\left[\left(\frac{a_H - 2w_H^* + c}{2}\right)w_H^* + \left(\frac{a_H + 2w_H^* - 3c}{4}\right)^2\right] + (1-\lambda)\left[q_R(w_L^*; a_L)w_L^* + \left(\frac{a_L - q_R(w_L^*; a_L) - c}{2}\right)^2\right]$$

where w_H^* and w_L^* are the optimal wholesale prices tailored to the two market sizes. We write the supplier's profit without information sharing as

$$\Pi_S = \lambda\left[\left(\frac{a_H - 2w^* + c}{2}\right)w^* + \left(\frac{a_H + 2w^* - 3c}{4}\right)^2\right] + (1-\lambda)\left[\bar{q}_R(w^*)w^* + \left(\frac{a_L - \bar{q}_R(w^*) - c}{2}\right)^2\right]$$

where w^* is the optimal wholesale price under the distorted separating equilibrium. Clearly,

$$\begin{aligned}
\Pi_S^{SI} - \Pi_S &> (1-\lambda)\left[q_R(w^*; a_L)w^* + \left(\frac{a_L - q_R(w^*; a_L) - c}{2}\right)^2 - \bar{q}_R(w^*)w^* - \left(\frac{a_L - \bar{q}_R(w^*) - c}{2}\right)^2\right] \\
&= \frac{(1-\lambda)\left[q_R(w^*; a_L) - \bar{q}_R(w^*)\right]\left[q_R(w^*; a_L) + \bar{q}_R(w^*) + 2c + 4w^* - 2a_L\right]}{4} \\
&= \frac{(1-\lambda)\left[q_R(w^*; a_L) - \bar{q}_R(w^*)\right]\left[\frac{a_L - 2w^* + c}{2} + \bar{q}_R(w^*) + 2c + 4w^* - 2a_L\right]}{4} \\
&= \frac{(1-\lambda)\left[q_R(w^*; a_L) - \bar{q}_R(w^*)\right]\left[\bar{q}_R(w^*) + \frac{5c - 3a_L}{2} + 3w^*\right]}{4} > 0
\end{aligned}$$

given $q_R(w^*; a_L) > \bar{q}_R(w^*)$ and $w^* > \frac{3\mu - c}{6} > \frac{3a_L - c}{6}$.

ii) Notice that $\Pi_R^{IS} = \Pi_R^{NI}$. Thus, the result with respect to the reseller's expected profit follows directly from Propositions 8 and 21.

A.2 Derivation of Profit Functions in Section 2.5

Recall that, in our analysis of encroachment in section 2.4.2, we assumed that the reseller knows the true market size while the supplier knows only the distribution of market size. Let us now extend that analysis to the case in which neither firm knows the true market size, i.e., no information. Subsequently, we will consider the case where they both know the true market size.

When neither firm knows the true market size, the supplier responds to the reseller's order quantity q_R, by choosing her own quantity as the solution to:

$$\max_{q_S}[\mu - q_R - q_S - c]q_S,$$

which yields the optimal direct selling quantity: $q_S(q_R) = \frac{\mu - q_R - c}{2}$. In anticipation of the supplier's reaction, the reseller solves:

$$\max_{q_R}[\mu - q_R - q_S(q_R) - w]q_R,$$

and his optimal order quantity is: $q_R^{NI}(w) = \frac{\mu - 2w + c}{2}$. The supplier's direct selling quantity is thus:

$$q_S^{NI}(q_R^{NI}(w)) = \frac{\mu + 2w - 3c}{4}.$$

We can express the supplier decision on the wholesale price as the solution to:

$$\begin{aligned} & \max_w q_R^{NI}(w)w + \left[\mu - q_R^{NI}(w) - q_S^{NI}(q_R^{NI}(w)) - c\right] q_S^{NI}(q_R^{NI}(w)) \\ = \quad & \max_w \frac{\mu - 2w + c}{2}w + \left(\frac{\mu + 2w - 3c}{4}\right)^2. \end{aligned}$$

The equilibrium wholesale price, the reseller's order quantity, and the supplier's direct selling quantity are:

$$w^{NI} = \frac{\mu}{2} - \frac{c}{6},\ q_R^{NI} = \frac{2c}{3},\ \text{and}\ q_S^{NI} = \frac{\mu}{2} - \frac{5c}{6},$$

and the reseller's and the supplier's expected profits without information acquisition are:

$$\Pi_R^{NI} = \frac{2c^2}{9} \text{ and } \Pi_S^{NI} = \frac{3\mu^2 - 6\mu c + 7c^2}{12}.$$

Let us now consider the case in which the two firms have shared information so that they both know the true market size. For each market size a_i, $i \in \{H, L\}$, the supplier responds to the reseller's order quantity q_R, by choosing her own quantity as the solution to:

$$\max_{q_S} [a_i - q_R - q_S - c] q_S,$$

which yields the optimal direct selling quantity: $q_S(q_R) = \frac{a_i - q_R - c}{2}$. In anticipation of the supplier's reaction, the reseller solves:

$$\max_{q_R} [a_i - q_R - q_S(q_R) - w] q_R,$$

and his optimal order quantity is: $q_R^{SI}(w; a_i) = \frac{a_i - 2w + c}{2}$. The supplier's direct selling quantity is:

$$q_S^{SI}(q_R^{SI}(w; a_i)) = \frac{a_i + 2w - 3c}{4},$$

and her decision on the wholesale price is the solution to:

$$\begin{aligned} & \max_w q_R^{SI}(w; a_i) w + \left[a_i - q_R^{SI}(w; a_i) - q_S^{SI}(q_R^{SI}(w; a_i)) - c\right] q_S^{SI}(q_R^{SI}(w; a_i)) \\ = \quad & \max_w \frac{a_i - 2w + c}{2} w + \left(\frac{a_i + 2w - 3c}{4}\right)^2. \end{aligned}$$

We can obtain the equilibrium wholesale price, the reseller's order quantity, and the supplier's direct selling quantity, corresponding to each market size:

$$w^{SI}(a_i) = \frac{a_i}{2} - \frac{c}{6},\ q_R^{SI}(a_i) = \frac{2c}{3}, \text{ and } q_S^{SI}(a_i) = \frac{a_i}{2} - \frac{5c}{6}.$$

Finally, the reseller's and the supplier's expected profits are:

$$\Pi_R^{SI} = \frac{2c^2}{9} \text{ and } \Pi_S^{SI} = \lambda \frac{3a_H^2 - 6a_H c + 7c^2}{12} + (1-\lambda)\,\frac{3a_L^2 - 6a_L c + 7c^2}{12}.$$

A.3 Extensions and Proofs

Extensions

In this section, we study several extensions to our base model. First, we allow for the possibility that the reseller cannot credibly commit that he will sell all of the units that he obtains from the supplier. Then, we allow for the possibility that the development of encroachment capability will provide the supplier with her own independent source of demand information. Finally, we consider the possibility of the supplier offering a two-part tariff.

Free Disposal by the Reseller

In the main text, we have implicitly assumed that the reseller will always sell all the units he orders from the supplier. We now relax this assumption by allowing the reseller to withhold some units for free disposal. The timeline of the game will be changed as follows: first, the supplier sets the wholesale price; second, the reseller observes the market size and places his order; third, the supplier produces the quantity ordered by the reseller and an additional quantity for her own use; finally the reseller and the supplier simultaneously determine the quantities that they will sell to the market. Of course, the quantities that are chosen in stage four cannot exceed the quantities produced in stage three.

Define a threshold by

$$\bar{\bar{q}}_R(w) = \frac{1}{6}\Big(6a_H - 3a_L - 6w + 3c - \sqrt{28a_H^2 - 36a_Ha_L + 20a_Hc - 48a_Hw + 9a_L^2 - 18a_Lc + c^2 - 12cw + 36a_Lw + 36w^2}\Big),$$

and the following proposition holds (which is parallel to Proposition 3).

Proposition 22. *Given any wholesale price offered by the supplier, there exists a unique perfect Bayesian separating equilibrium that survives the intuitive criterion, in which neither the supplier nor the reseller withholds any unit for free disposal. Furthermore, when* $w \geq \frac{a_H+c}{6}$*, the reseller's and the supplier's order quantities are the same as those in Proposition 3; when* $w < \frac{a_H+c}{6}$*, the reseller's order quantity satisfies* $q_R(w; a_H) = \frac{a_H+c}{3}$ *and* $q_R(w; a_L) = \min\left\{\left(\frac{a_L-2w+c}{2}\right)^+, \frac{a_L+c}{3}, \bar{\bar{q}}_R(w)\right\}$*, and the supplier's direct selling quantity is* $q_S(q_R(w; a_i)) = \frac{a_i - q_R(w;a_i) - c}{2}$*,* $\forall i \in \{H, L\}$*.*

Proposition 22 first asserts that the true market size will always be learned by the supplier, and that, in equilibrium, neither firm will withhold from the market any units that are ordered/prepared. However, although the free disposal option can alter the quantities that are ordered, the proposition also establishes that it plays a role only when the equilibrium wholesale price is below $\frac{a_H+c}{6}$. When this is the case, the free disposal option undermines the reseller's ability to commit to a quantity he wants to sell. As a result, the reseller will order less than what he would order without the option of free disposal even under a large market size.

Of course, when the supplier determines the wholesale price, she does so in anticipation of the above subgame equilibrium. To derive the optimal wholesale price analytically would be extremely tedious because it would involve comparing the supplier's profits in several different scenarios. To avoid this, we conduct a numerical analysis. Recall from Proposition 22 that, the free disposal option plays a role only when the equilibrium wholesale price exceeds $\frac{a_H+c}{6}$. As shown in Figure A.1, this occurs only when the prior probability of the large market size (λ) is small and the ratio of the two market sizes ($\frac{a_H}{a_L}$) is large. For these parameters, having the option of free disposal will clearly be a disadvantage for the reseller since he will lose the advantages of Stackelberg leadership for the large market size and may also need to distort more for the small market size. Hence, we can obtain largely similar managerial insights related to supplier encroachment as those without the free disposal option. Note that for the entire supply chain, the option of free disposal may have two opposing effects. On one hand, it restricts the reseller's and the supplier's selling quantities and thus avoids a very low market price. On the other hand, it may indirectly lower the reselling order quantity when the market size is small. The reseller who observes low demand may need to downward distort even further from $\frac{a_L-2w+c}{2}$.

Encroachment Provides Supplier with a Noisy Signal about Demand

In addition to the way in which the development of encroachment capability affects a supplier's strategic interactions with a reseller, which have been the focus of our analysis up until now, the development of her own direct channel may also provide a supplier with access to information about demand that is independent from what she learns from the reseller's order. Previously, we have ignored this possibility

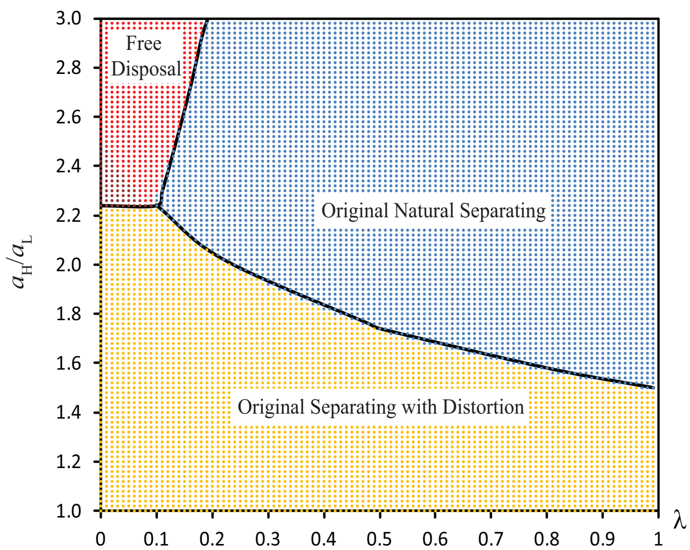


Figure A.1: Demonstration of the scenarios where free disposal by the reseller either has or does not have an impact. In this example, $a_H = 1.35$ and $a_L = 1$.

in order to focus exclusively on the strategic interactions with the reseller.

However, our model can easily be adapted to allow for the possibility that a supplier who develops encroachment capability will also receive an independent signal about market demand. To do this, we assume that the supplier receives a signal, denoted by $\mathbf{s} \in \{a_L, a_H\}$, after her decision to develop encroachment capability but before she announces her wholesale price to the reseller. In addition, we assume that the signal is accurate with probability $\phi \in [0.5, 1]$. Specifically, when the true market size is $\mathbf{a} = a_i$, $i = H, L$, the probability that the supplier receives a signal $\mathbf{s} = a_i$ is:

$$\text{Prob}(\mathbf{s} = a_i|\mathbf{a} = a_i) = \phi \text{ and } \text{Prob}(\mathbf{s} = a_j|\mathbf{a} = a_i) = 1-\phi, \ i = H, L \text{ and } j \in \{H, L\}\backslash i.$$

where the signal provides no information when $\phi = 0.5$, and is perfectly accurate when $\phi = 1$.

After the supplier receives the demand signal, she updates her prior on the probability that the demand parameter is a_H. Using Bayes' rule, it is easy to show that her updated prior that the market size is large will depend upon the signal that she receives in the following way:

$$\lambda_H'(\mathbf{s}) = \begin{cases} \frac{\phi\lambda}{\phi\lambda+(1-\phi)(1-\lambda)} & \text{if } \mathbf{s} = a_H \\ \frac{(1-\phi)\lambda}{(1-\phi)\lambda+\phi(1-\lambda)} & \text{if } \mathbf{s} = a_L \end{cases}$$

and $\lambda_L'(\mathbf{s}) = 1 - \lambda_H'(\mathbf{s})$ is her updated prior that the market size is small. Denote by $\boldsymbol{\lambda}'(\mathbf{s})$ the vector consisting of $\lambda_H'(\mathbf{s})$ and $\lambda_L'(\mathbf{s})$. With a slight abuse of notation, the expected optimal profits obtained by the supplier and by the reseller after when the supplier has encroachment capability can now be expressed as follows:

$$\begin{aligned} \mathbf{E}_{\mathbf{s}}[\Pi_S(\boldsymbol{\lambda}, \mathbf{s})] &= [\phi\lambda + (1-\phi)(1-\lambda)]\,\Pi_S(\boldsymbol{\lambda}'(a_H)) + [(1-\phi)\lambda + \phi(1-\lambda)]\,\Pi_S(\boldsymbol{\lambda}'(a_L)) \\ \mathbf{E}_{\mathbf{s}}[\Pi_R(\boldsymbol{\lambda}, \mathbf{s})] &= [\phi\lambda + (1-\phi)(1-\lambda)]\,\Pi_R(\boldsymbol{\lambda}'(a_H)) + [(1-\phi)\lambda + \phi(1-\lambda)]\,\Pi_R(\boldsymbol{\lambda}'(a_L)) \end{aligned}$$

where $\Pi_S(\boldsymbol{\lambda}'(a_H))$, $\Pi_S(\boldsymbol{\lambda}'(a_L))$, $\Pi_R(\boldsymbol{\lambda}'(a_H))$, and $\Pi_R(\boldsymbol{\lambda}'(a_L))$ each have the same structure that we have characterized with the supplier not receiving any signal.

The independent source of information allows the supplier to tailor her wholesale price according to the signal that she receives. However, it may have two opposing effects. When the supplier obtains a high demand signal (e.g., $\mathbf{s} = a_H$), the equilibrium wholesale price $w^*(\mathbf{s} = a_H)$ is higher than that in Proposition 5. Hence, as Lemma 2 and Proposition 3 suggest, the reseller is less likely to downward distort his order quantity. Whereas, when the supplier observes a low demand signal, the equilibrium wholesale price $w^*(\mathbf{s} = a_H)$ is weakly lower than that in Proposition 5 and the reseller is more likely to downward distort his order quantity. Note that the signaling game between the reseller and the supplier will always arise, unless the supplier's own signal is also perfect, i.e., when $\phi = 1$.

From our numerical analysis, we observe that the positive effect overall dominates, but the supplier benefits just slightly from obtaining the demand signal. In particular, Figure B.2 shows that the region in which the supplier is worse off shrinks, but very slightly, as the accuracy level of the supplier's signal, ϕ, increases. The region where the reseller benefits from supplier encroachment also expands slightly when ϕ increases.

Two-part Tariff

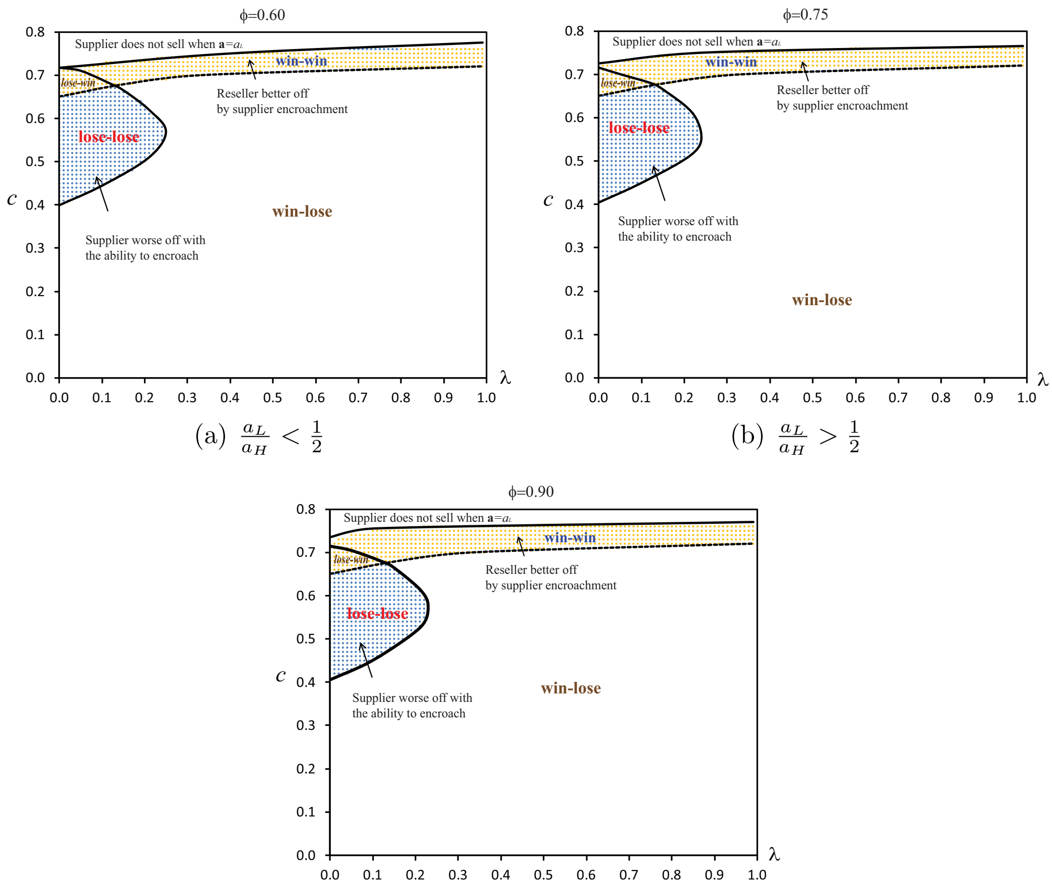
Previously, we have restricted our attention to contracts that involve only a per-unit wholesale price. We now extend our analysis to the case in which the supplier uses a single two-part tariff contract (T, w), where T is the fixed fee and w is the unit wholesale price.

Without Encroachment

When the supplier lacks the capability to encroach, she sets T and w to maximize the following:

$$\mathbf{E}\left[\left(T + wq_R^N(w;\mathbf{a})\right)\mathbb{I}\left((\mathbf{a} - w - q_R^N(w;\mathbf{a}))q_R^N(w;\mathbf{a}) - T\right)\right],$$

where $q_R^N(w;\mathbf{a}) = \frac{\mathbf{a}-w}{2}$ as in section 2.4.1 and $\mathbb{I}(x)$ is the indicator function such that $\mathbb{I}(x) = 1$ if $x \geq 0$ and $\mathbb{I}(x) = 0$ otherwise. Note that the indicator function captures the fact that the reseller orders a positive quantity and pays the fixed fee only if he does not make negative profit from doing so.



(a) $\frac{a_L}{a_H} < \frac{1}{2}$

(b) $\frac{a_L}{a_H} > \frac{1}{2}$

(c) $\frac{a_L}{a_H} > \frac{1}{2}$

Figure A.2: Demonstration of the impact of the supplier obtaining a noisy signal. The other parameters are: $a_H = 1.35$ and $a_L = 1.00$.

There are two possible solutions to this problem. The first is for the supplier to set T just low enough to induce the reseller who observes a small market size to order a positive quantity. If the supplier pursues this approach, then the conditionally optimal wholesale price and fixed fee are:

$$w^* = \mu - a_L \text{ and } T^* = \frac{(2a_L - \mu)^2}{4},$$

and the profits for the supplier and the reseller are:

$$\pi_S^{\mathcal{T}1} = \frac{a_L^2 + (\mu - a_L)^2}{4} \text{ and } \pi_R^{\mathcal{T}1} = \lambda \frac{a_H^2 - a_L^2}{4},$$

where we use the superscript $\mathcal{T}$ to indicate that the profit is achieved under a two-part tariff contract.

Alternatively, the supplier can offer a two-part tariff contract to exclude the reseller when he observes the small market size. In this case, the supplier sells nothing when the market size is small, but extracts the entire surplus from the reseller when the market size is large. Specifically, the conditionally optimal wholesale price and fixed fee are:

$$w^* = 0 \text{ and } T^* = \frac{a_H^2}{4},$$

and the profits for the supplier and the reseller are:

$$\pi_S^{\mathcal{T}2} = \lambda \frac{a_H^2}{4} \text{ and } \pi_R^{\mathcal{T}2} = 0.$$

By comparing $\pi_S^{\mathcal{T}1}$ with $\pi_S^{\mathcal{T}2}$, it is easy to confirm that the supplier will choose the first approach if and only if:

$$\lambda \leq \frac{a_H^2 - \sqrt{a_H^4 - 4a_H^2 a_L^2 + 8a_H a_L^3 - 4a_L^4}}{2(a_H - a_L)^2}.$$

With Encroachment

When the supplier has encroachment capability, she can similarly set T and w to either induce a positive order quantity when the market size is small or extract the entire surplus when the market size is large. In order to induce the reseller to order when the market size is small, the fixed fee cannot exceed:

$$T^{E1}(w) = \left(a_L - q_R(w; a_L) - q_S(q_R(w; a_L)) - w\right) q_R(w; a_L),$$

and the optimal wholesale price solves:

$$\Pi_S^{\mathcal{T}1} = \max_w T^{E1}(w) + \mathbf{E}\left[wq_R(w; \mathbf{a}) + \left(\mathbf{a} - q_R(w; \mathbf{a}) - q_S(q_R(w; \mathbf{a})) - c\right) q_S(q_R(w; \mathbf{a}))\right]$$

where $q_R(w; \mathbf{a})$ and $q_S(q_R(w; \mathbf{a}))$ follow Proposition 3. If the supplier sets T and w in this fashion and w^{E1} is the optimal wholesale price, then the reseller's profit is:

$$\Pi_R^{\mathcal{T}1} = \lambda\left[-T(w^{E1}) + \big(a_H - q_R(w^{E1}; a_H) - q_S(q_R(w^{E1}; a_H)) - w\big) q_R(w^{E1}; a_H)\right].$$

Alternatively, if the supplier does not induce a positive order quantity when the market size is small, then the fixed fee extracts the entire surplus from the reseller when the market size is large and is equal to:

$$T^{E2}(w) = \left(a_H - q_R(w; a_H) - q_S(q_R(w; a_H)) - w\right) q_R(w; a_H),$$

and the optimal wholesale price solves:

$$\Pi_S^{\mathcal{T}2} = \max_w \lambda\left(T^{E2}(w) + wq_R(w; a_H) + \left(a_H - q_R(w; a_H) - q_S(q_R(w; a_H)) - c\right) q_S(q_R(w; a_H))\right).$$

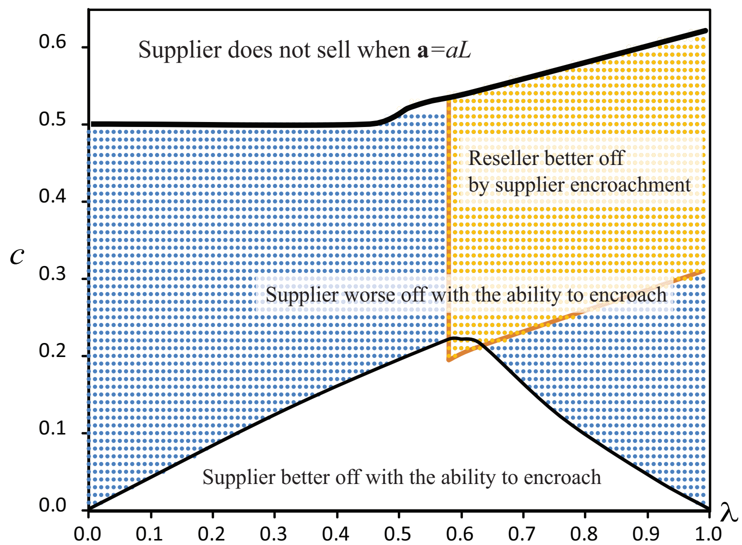
The supplier's optimal profit from using a two-part tariff when she has encroachment capability is the maximum of $\Pi_S^{\mathcal{T}1}$ and $\Pi_S^{\mathcal{T}2}$. Unfortunately, although the results of Proposition 3 continue to hold, obtaining an analytical characterization of the optimal two-part tariff is non-trivial. Therefore, we have performed a numerical study of how the supplier's ability to use a two-part tariff affects the extent to which the supplier and the reseller can benefit from the supplier's ability to develop encroachment capability. Figure A.3 shows the regions of parameters for which the supplier and the reseller benefit from the supplier's ability to encroach when the supplier uses a two-part tariff. By comparing it to Figure 2.4, we can see that the supplier's ability to use a two-part tariff dramatically alters the regions in which either the supplier or the reseller benefits. In particular, under a two-part tariff, there is a much smaller region of parameters for which the supplier benefits from the development of encroachment capability, and the region in which both the supplier and the reseller benefit becomes almost non-existent. To understand why the development of encroachment capability now only benefits the supplier when her direct selling cost, c, is sufficiently small, note that her ability to charge a fixed fee will generally cause her to set the per-unit price w to a lower value than if she relied entirely upon the per-unit price for income. When c is small, the supplier is willing to rely entirely upon her own direct channel for the small market size in return for setting a high fixed fee that captures the reseller's entire surplus when the market size is large, and thus, the reseller's ordering distortion is not a concern. However, when c is relatively large, as Proposition 3 suggests, the downward distortion effect is more serious when the wholesale price w is smaller. Such downward distortion adversely affects the supplier, both directly and indirectly. The direct negative effect is that downward distortion lowers the total supply chain surplus (and thus the supplier's profit) when the market size is small (notice that the supplier can extract the total supply chain surplus when the market size is small). The indirect negative effect is that the supplier needs to charge a lower fixed fee to induce the reseller who observes a small market size to sell. Therefore, the benefit of encroachment significantly shrinks.

With the above analysis by migrating from the system with a simple wholesale price only contract to a slightly more complex two-part tariff contract, a natural question surfaces: What if the supplier can use even more complex contracts, such as, a non-linear pricing scheme. In a separate manuscript, Li et al. (2015), we consider a general non-linear pricing policy and show how a supplier's development of encroachment capability is a double-edged sword that can either enhance or impede her ability to extract rents from a reseller. Note that the analysis there differs structurally from what we have done here because a screening problem arises under a non-linear pricing scheme, instead of a signaling problem as explored in this study.

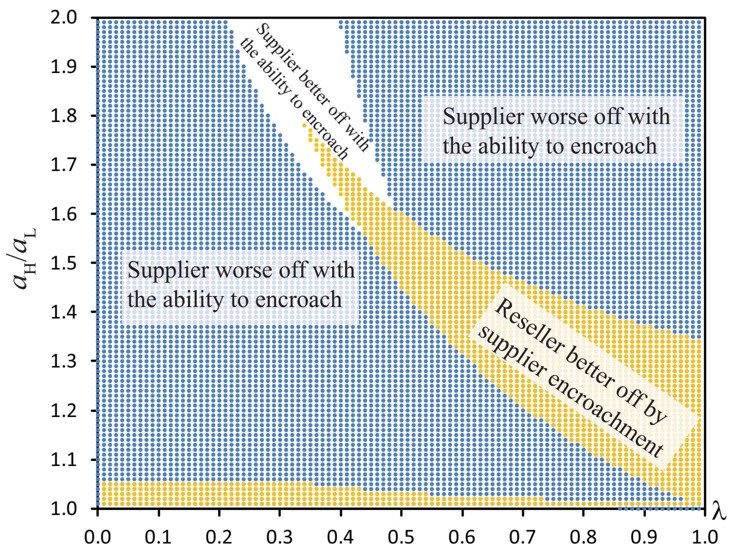
Proofs

Proof of Proposition 22. Let q_R^{stock} denote the quantity the reseller orders from the supplier and q_R^{sell} the quantity he sells to the market in the end. It is obvious that the supplier will not produce more than what she will sell. So we keep using the notation q_S to denote her selling quantity. With that, we can formulate the supplier's direct selling decision by:

$$\Pi_S = \max_{q_S} wq_R^{stock} + (a_i - q_R^{sell} - q_S - c)q_S,$$



(a) $\frac{a_L}{a_H} < \frac{1}{2}$



(b) $\frac{a_L}{a_H} > \frac{1}{2}$

Figure A.3: Demonstration of the impact of supplier encroachment on the supplier's and the reseller's profitability when the supplier can use a two-part tariff contract. In this example, $a_H = 1.35$ and $a_L = 1$.

which yields her optimal decision:

$$q_S(q_R^{sell}) = \frac{a_i - q_R^{sell} - c}{2}.$$

For the reseller, to determine his selling quantity, he solves:

$$\Pi_R = \max_{q_R^{sell}} -wq_R^{stock} + (a_i - q_R^{sell} - q_S)q_R^{sell},$$

which yields

$$q_R^{sell}(q_S) = \frac{a_i - q_S}{2}.$$

Substituting $q_R^{sell}(q_S)$ into $q_S(q_R^{sell})$, we obtain the equilibrium selling quantities:

$$q_R^{*sell} = \min\left\{\frac{a_i + c}{3}, q_R^{stock}\right\} \text{ and } q_S^* = q_S(q_R^{*sell}).$$

Clearly, the reseller will never order more than $\frac{a_i+c}{3}$. Hence, we first have:

$$q_R^{stock}(a_H) = \begin{cases} \frac{a_H+c}{3} & \text{if } w < \frac{a_H+c}{6}, \\ \frac{a_H-2w+c}{2} & \text{if } w \geq \frac{a_H+c}{6}. \end{cases}$$

Note that to have a separating equilibrium, the reseller may need to downward distort his ordering quantity when the market size is small, to deter himself from mimicking when he observes a large market size. Given $q_R^{stock}(a_H) = \frac{a_H-2w+c}{2}$ when $w \geq \frac{a_H+c}{6}$, we know the threshold ordering quantity under the small market size is $\bar{q}_R(w)$ according to Proposition 3. When $w < \frac{a_H+c}{6}$, $q_R^{stock}(a_H) = \frac{a_H+c}{3}$, and thus, we need to characterize another threshold order quantity in order for a separating equilibrium to hold. In particular, we have the following indifference condition:

$$\left(a_H - \frac{a_H + c}{3} - \frac{a_H - 2c}{3} - w\right)\frac{a_H + c}{3} = \left(a_H - q_R - \frac{a_L - q_R - c}{2} - w\right)q_R,$$

from which we can derive the threshold

$$\bar{\bar{q}}_R(w) = \frac{1}{6}\Big(6a_H - 3a_L - 6w + 3c \\ - \sqrt{28a_H^2 - 36a_H a_L + 20a_H c - 48a_H w + 9a_L^2 - 18a_L c + c^2 - 12cw + 36a_L w + 36w^2}\Big).$$

Hence, the reseller's order quantity when the market size is small follows:

$$q_R^{stock}(a_L) = \begin{cases} \min\left\{\bar{\bar{q}}_R(w), \frac{a_L+c}{3}, \left(\frac{a_L-2w+c}{2}\right)^+\right\} & \text{if } w < \frac{a_H+c}{6}, \\ \min\left\{\bar{q}_R(w), \left(\frac{a_L-2w+c}{2}\right)^+\right\} & \text{if } w \geq \frac{a_H+c}{6}. \end{cases}$$

A.4 Intuitive Criterion and Elimination of Pooling Equilibria

In this appendix, we prove that the separating equilibrium characterized in Proposition 3 uniquely survives the intuitive criterion refinement developed by Cho and Kreps (1987).

The Intuitive Criterion

The intuitive criterion uses two steps to examine an equilibrium of a signaling game between a signal sender and a signal receiver.

(i) **The first step** of the intuitive criterion derives a set Θ of the types of the sender, with which the highest utility that the sender can obtain by taking a specific off-equilibrium strategy is lower than that by keeping the equilibrium strategy. That is, under those types, the off-equilibrium strategy is dominated by the equilibrium strategy for the sender.

Specifically, in our model, suppose we have an equilibrium in which the reseller orders $q^e(w; a_H)$ when observing the large market size and orders $q^e(w; a_L)$ when observing the small market size. If $q^e(w; a_H) = q^e(w; a_L)$, then the equilibrium is pooling; otherwise, the equilibrium is separating. In the first step of the intuitive criterion refinement, for any off-equilibrium order quantity q, we derive a set of the market sizes:

$$\Theta(q) = \{a_{i \in \{H,L\}} : V(q^e(w; a_i); a_i) > \hat{V}(q; a_i)\}$$

where $V(q^e(w; a_i); a_i)$ denotes the reseller's equilibrium profit while $\hat{V}(q; a_i)$ denotes the highest profit that the reseller can obtain by ordering the off-equilibrium quantity q. Note that the highest profit for a given order quantity q is achieved if the supplier believes that the reseller has observed the small market size; that is,

$$\hat{V}(q; a_i) = \left(a_i - q - \frac{\max\{0, a_L - q - c\}}{2} - w\right) q.$$

Therefore, $\Theta(q)$ contains those market sizes under which the off-equilibrium strategy q is dominated by the equilibrium strategy $q^e(w; a_i)$ for the reseller.

If the set Θ^C, the complement of Θ, is an empty set, the second step becomes unnecessary since for both market sizes the off-equilibrium strategy is always dominated by the equilibrium strategy and the reseller will not deviate at all. In this case, the intuitive criterion imposes no constraint on the solution space. If Θ^C is nonempty (having one market size or both market sizes in the set in our model), then we need to carry out the second step.

(ii) **The second step** of the intuitive criterion checks if there exists a specific type in Θ^C such that the equilibrium utility of the sender with this type is lower than the lowest utility that the sender can obtain by taking a specific off-equilibrium strategy given that the receiver restricts his belief to Θ^C after observing such a deviation. If there does exist such a type, the equilibrium fails the intuitive criterion test; otherwise, the intuitive criterion imposes no constraint on the solution space.

Specifically, in our model, the second step of the intuitive criterion checks, for any order quantity q, if there exists a market size $a_i \in \Theta^C(q)$, the complement of $\Theta(q)$, such that with this market size the reseller's equilibrium profit $V(q^e(w; a_i); a_i)$ is lower than the lowest profit that the reseller can obtain by deviating to the order quantity q when the supplier's belief is restricted to $\Theta^C(q)$ for such a deviation. Let $\check{V}(q; a_i)$ denote this lowest profit, and it follows

$$\check{V}(q; a_i) = \begin{cases} \left(a_i - q - \frac{\max\{0, a_H - q - c\}}{2} - w\right) q & \text{if } a_H \in \Theta^C(q), \\ \left(a_i - q - \frac{\max\{0, a_L - q - c\}}{2} - w\right) q & o/w. \end{cases}$$

That is, if the large market size a_H is contained in $\Theta^C(q)$ (i.e., the strategy to deviate to q is not dominated by the equilibrium strategy for the reseller observing the large market size), then the lowest profit the reseller would obtain to deviate to q is achieved under the supplier belief that the reseller has observed the large market size for such a deviation. If the large market size a_H is not contained $\Theta^C(q)$, then $\Theta^C(q)$ contains only the small market size and thus the lowest profit the reseller would obtain to deviate to q is achieved under the supplier belief that the reseller has observed the small market size. If there exists such a market size $a_i \in \Theta^C(q)$ that $V(q^e(w; a_i); a_i) < \check{V}(q; a_i)$, then the equilibrium fails the intuitive criterion in our model; otherwise, the intuitive criterion imposes no constraint on the solution space.

Refinement over Other Equilibria

Now we use the above procedure of the intuitive criterion to refine the equilibria in our model. We first show that any pooling equilibrium cannot survive the intuitive criterion. Define the reseller's pooling profit

$$V_{iP}(q) = \left(a_i - q - \frac{\max\{0, \lambda a_H + (1-\lambda) a_L - q - c\}}{2} - w\right) q, \forall i \in \{H, L\}$$

where a_i is the true market size the reseller observes while the supplier obtains no information from the reseller's order quantity. Notice that all of those profit functions, $V_{ij}(q)$ and $V_{iP}(q)$, are concave. Moreover, $V'_{iL}(q) > V'_{iP}(q)$, $\forall i \in \{H, L\}$.

Now, suppose that given a wholesale price w, there is a pooling equilibrium in which the reseller orders q_P for each market size. Then, we can always find $q_F < q_P$ such that $V_{LL}(q_F) = V_{LP}(q_P)$. Notice that

$$\begin{aligned} V_{LL}(q_F) &= \left(a_L - q_F - \frac{\max\{0, a_L - q_F - c\}}{2} - w\right) q_F, \\ V_{LP}(q_P) &= \left(a_L - q_P - \frac{\max\{0, \lambda a_H + (1-\lambda) a_L - q_P - c\}}{2} - w\right) q_P. \end{aligned}$$

Substituting q_F into $V_{HL}(q)$, we obtain

$$V_{HL}(q_F) = \left(a_H - q_F - \frac{\max\{0, a_L - q_F - c\}}{2} - w\right) q_F.$$

Also,

$$V_{HP}(q_P) = \left(a_H - q_P - \frac{\max\{0, \lambda a_H + (1-\lambda) a_L - q_P - c\}}{2} - w\right) q_P.$$

Therefore,

$$\begin{aligned} V_{HL}(q_F) - V_{LL}(q_F) &= (a_H - a_L) q_F, \\ V_{HP}(q_P) - V_{LP}(q_P) &= (a_H - a_L) q_P. \end{aligned}$$

Given $q_F < q_P$ and $V_{LL}(q_F) = V_{LP}(q_P)$, it is obvious that

$$V_{HP}(q_P) - V_{LP}(q_P) > V_{HL}(q_F) - V_{LL}(q_F)$$

and thus,

$$V_{HP}(q_P) > V_{HL}(q_F).$$

As a result, we can find a $q_D = q_F + \epsilon$ such that the reseller when observing the small market size has an incentive to deviate from the pooling equilibrium q_P to q_D while he has no incentive to deviate from q_P to q_D when observing the large market size, assuming a deviation to q_D always leads the supplier to believe that the true market size is small; i.e., there is a q_D such that

$$\begin{aligned} V_{LP}(q_P) &< V_{LL}(q_D) \\ V_{HP}(q_P) &> V_{HL}(q_D). \end{aligned}$$

Hence, the pooling equilibrium fails the intuitive criterion (see Figure A.4 for a demonstration).

Besides pooling equilibria, there may exist other separating equilibria different from the one characterized Proposition 3. Notice that in order for a separating equilibrium to hold, the reseller's order quantity $q^e(w; a_L)$ when the market size is small must be smaller than $\hat{q}_R(w)(= \min\left\{\left(\frac{a_L-2w+c}{2}\right)^+, \bar{q}_R(w)\right\})$. It is obvious that given such a separating equilibrium, the reseller when observing the small market size would have an incentive to deviate from $q^e(w; a_L)$ to $\hat{q}_R(w)$ if the supplier holds the same belief that the market size is small for both quantities, while the reseller when observing the large market size would have no incentive to deviate from $\left(\frac{a_H-2w+c}{2}\right)^+$

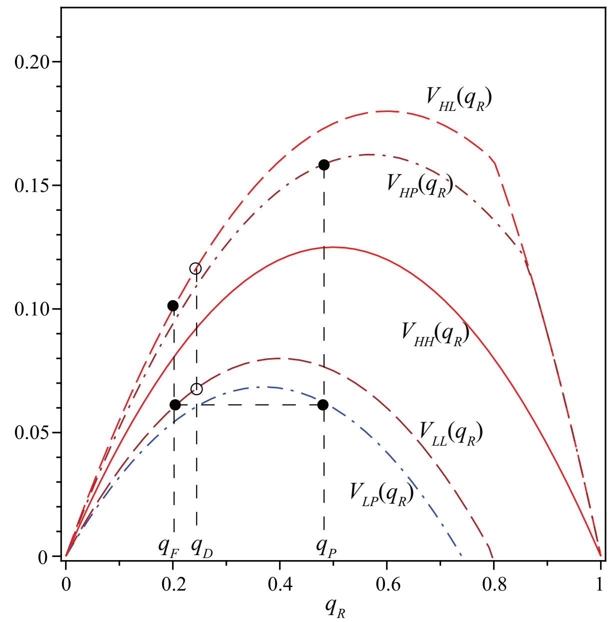


Figure A.4: Demonstration of the refinement over pooling equilibria by the intuitive criterion. The parameters are: $a_H = 1.2$, $a_L = 1$, $c = 0.2$, $\lambda = 0.3$, and $w = 0.2$.

to $\hat{q}_R(w)$ based on the definition of $\hat{q}_R(w)$. Hence, any separating equilibrium different from that in Proposition 3 will fail the intuitive criterion.

Appendix B

Appendix for Chapter 3

B.1 Proof of Main Results

Proof of Proposition 11. For each market size a_i, the reseller's participation constraint follows: $w(a_i) = a_i - q_R(a_i) - q_S(a_i)$. We first assume $q_S(a_i) = \frac{a_i - q_R(a_i) - c}{2} > 0$. Plugging $w(a_i)$ and $q_S(a_i) = \frac{a_i - q_R(a_i) - c}{2}$ into (3.2), we have the supplier's optimization problem as: $\max_{q_R(a_i)} q_R(a_i) \frac{a_i - q_R(a_i) + c}{2} + \left(\frac{a_i - q_R(a_i) - c}{2}\right)^2$. The first-order condition yields the optimal unbounded reselling quantity $q_R^{PI}(a_i) = 2c$. Notice that in order for $q_S(a_i) > 0$, we need $c < a_i - q_R^{PI}(a_i)$, i.e., the condition $c < \frac{a_i}{3}$. When this inequality does not hold, we have $q_S^{PI}(a_i) = 0$, and the supplier's optimization problem is simply $\max_{q_R(a_i)} (a_i - q_R(a_i))\, q_R(a_i)$, which yields $q_R^{PI}(a_i) = \frac{a_i}{2}$. Clearly, given $q_R^{PI}(a_i) = \frac{a_i}{2}$, in order for the supplier's direct selling quantity to be zero, we need $c \geq a_i - q_R^{PI}(a_i)$, i.e., the condition $c > \frac{a_i}{2}$. For the rest of the parameter space $\frac{a_i}{3} < c \leq \frac{a_i}{2}$, the optimal reselling quantity follows the corner solution: $q_R^{PI}(a_i) = a_i - c$. With $q_R^{PI}(a_i)$, the optimal wholesale price and equilibrium direct selling quantity can easily be obtained.

Proof of Proposition 12. This result follows directly from Proposition 11. One can easily verify that when $0 < c < \frac{a_i}{2}$, the total output $(q_R^{PI}(a_i) + q_S^{PI}(a_i))$ with supplier encroachment is larger than the efficient total output $\frac{a_i}{2}$. Hence, the total supply chain surplus with encroachment is lower than that without encroachment. The supplier's profit always equals the supply chain surplus as she can use nonlinear pricing to capture the entire supply chain surplus with perfect information.

Proof of Proposition 13. The classical mechanism design principle asserts that: there exists an optimal solution in which the two binding constraints are the reseller's individual rationality constraint for the small market size and his downward incentive comparability constraint. From these two binding constraints we can obtain:

$$\begin{aligned} w(a_L) &= a_L - q_R(a_L), \\ w(a_H) &= a_H - q_R(a_H) - (a_H - a_L)\frac{q_R(a_L)}{q_R(a_H)}. \end{aligned}$$

Substituting these expressions for $w(a_L)$ and $w(a_H)$ into the objective function of (3.3), we are left with an unconstrained objective function with only two variables, $q_R(a_L)$ and $q_R(a_H)$, and it is separable and concave. The result in Proposition 13 follows from applying the first-order conditions.

Proof of Proposition 15. In this proof, we solve the optimal separating menu of contracts. The comparison between the optimal separating menu of contracts and the optimal pooling contract is provided later.

To solve the optimal separating menu of contracts, notice that once we incorporate the functional form of the supplier's ex-post optimal direct selling quantity response into the reseller's utility function, which forms the basis for the IR and IC constraints, we continue to have the single crossing property in which a reseller's preference for a larger quantity is increasing in the size of market that he observes.

In addition, the supplier's objective function is separable and concave in the quantities offered. Therefore, this problem is a classical mechanism design problem. At optimum, the reseller's IR constraint for the small market size must be binding, i.e., $w(a_L) = a_L - q_R(a_L) - q_S(a_L)$. The reseller's IC constraint for the large market size must satisfy:

$$\begin{aligned}
&(a_H - q_R(a_H) - q_S(a_H) - w(a_H))q_R(a_H) \\
&\geq [a_H - (q_R(a_L) + q_S(a_L)) - (a_L - q_R(a_L) - q_S(a_L))]q_R(a_L) \\
&= (a_H - a_L)q_R(a_L).
\end{aligned}$$

Thus, the optimal wholesale price for the large market size satisfies:

$$w(a_H) = a_H - q_R(a_H) - q_S(a_H) - \frac{(a_H - a_L)q_R(a_L)}{q_R(a_H)}.$$

We can now substitute the above expressions into the supplier's objective of choosing $\{q_R(a_i)\}_{i\in\{H,L\}}$ to obtain:

$$\begin{aligned}
\max \quad & \lambda \left[\left(a_H - q_R(a_H) - \left(\frac{a_H - q_R(a_H) - c}{2} \right)^+ - \frac{(a_H - a_L)q_R(a_L)}{q_R(a_H)} \right) q_R(a_H) + \left(\left(\frac{a_H - q_R(a_H) - c}{2} \right)^+ \right)^2 \right] \\
& +(1-\lambda) \left[\left(a_L - q_R(a_L) - \left(\frac{a_L - q_R(a_L) - c}{2} \right)^+ \right) q_R(a_L) + \left(\left(\frac{a_L - q_R(a_L) - c}{2} \right)^+ \right)^2 \right] \\
& = \lambda \left[\left(a_H - q_R(a_H) - \left(\frac{a_H - q_R(a_H) - c}{2} \right)^+ \right) q_R(a_H) - (a_H - a_L)q_R(a_L) + \left(\left(\frac{a_H - q_R(a_H) - c}{2} \right)^+ \right)^2 \right] \\
& +(1-\lambda) \left[\left(a_L - q_R(a_L) - \left(\frac{a_L - q_R(a_L) - c}{2} \right)^+ \right) q_R(a_L) + \left(\left(\frac{a_L - q_R(a_L) - c}{2} \right)^+ \right)^2 \right].
\end{aligned}$$

Notice that the objective is separable and we can derive the optimal $q_R(a_L)$ and $q_R(a_H)$ separately.

i) We first optimize $q_R(a_H)$. Suppose $\left(\frac{a_H - q_R(a_H) - c}{2}\right)^+$ is positive (i.e., when

$q_R(a_H) < a_H - c$). We can maximize the term:

$$\begin{aligned}
& \lambda\left[\left(a_H - q_R(a_H) - \left(\frac{a_H - q_R(a_H) - c}{2}\right)\right) q_R(a_H) + \left(\frac{a_H - q_R(a_H) - c}{2}\right)^2\right] \\
= & \lambda\left[\left(\frac{a_H - q_R(a_H) + c}{2}\right) q_R(a_H) + \left(\frac{a_H - q_R(a_H) - c}{2}\right)^2\right],
\end{aligned}$$

which has the first-order condition as:

$$-\frac{q_R(a_H)}{2} + \left(\frac{a_H - q_R(a_H) + c}{2}\right) - \left(\frac{a_H - q_R(a_H) - c}{2}\right) = 0.$$

Therefore, the unbounded optimal quantity is:

$$q_R^{<1>}(a_H) = 2c.$$

Notice that when $c < \frac{a_H}{3}$, $\frac{a_H - q_R^{<1>}(a_H) - c}{2} > 0$.

Now, suppose $\left(\frac{a_H - q_R(a_H) - c}{2}\right)^+$ is zero (i.e., when $q_R(a_H) \geq a_H - c$). We can maximize the term:

$$\lambda\left[(a_H - q_R(a_H))\, q_R(a_H)\right].$$

The first-order condition yields the optimal quantity

$$q_R^{<2>}(a_H) = \frac{a_H}{2}.$$

It is clear that when $c \geq \frac{a_H}{2}$, $\frac{a_H - q_R^{<2>}(a_H) - c}{2} \leq 0$.

Therefore, when $c \geq \frac{a_H}{2}$, the optimal quantity is $q_R^E(a_H) = q_R^{<2>}(a_H) = \frac{a_H}{2}$; when $\frac{a_H}{3} \leq c < \frac{a_H}{2}$, $q_R^E(a_H) = a_H - c$; and when $c < \frac{a_H}{3}$, $q_R^E(a_H) = q_R^{<1>}(a_H) = 2c$.

ii) We use a similar procedure to optimize $q_R(a_L)$. Suppose $\left(\frac{a_L - q_R(a_L) - c}{2}\right)^+$ is positive (i.e., when $q_R(a_L) < a_L - c$). We can maximize the term:

$$
\begin{aligned}
&(1-\lambda)\left[\left(a_L - q_R(a_L) - \left(\frac{a_L - q_R(a_L) - c}{2}\right)\right) q_R(a_L) + \left(\frac{a_L - q_R(a_L) - c}{2}\right)^2\right] - \lambda(a_H - a_L)q_R(a_L) \\
= \quad &(1-\lambda)\left[\left(\frac{a_L - q_R(a_L) + c}{2}\right) q_R(a_L) + \left(\frac{a_L - q_R(a_L) - c}{2}\right)^2\right] - \lambda(a_H - a_L)q_R(a_L).
\end{aligned}
$$

The first-order condition is

$$(1-\lambda)\left[-\frac{q_R(a_L)}{2} + \left(\frac{a_L - q_R(a_L) + c}{2}\right) - \left(\frac{a_L - q_R(a_L) - c}{2}\right)\right] - \lambda(a_H - a_L) = 0,$$

which yields the optimal quantity

$$q_R^{<1>}(a_L) = \left(2c - \frac{2\lambda(a_H - a_L)}{1-\lambda}\right)^+.$$

Notice that when $c < \frac{a_L + \frac{2\lambda(a_H - a_L)}{1-\lambda}}{3}$, $\frac{a_L - q_R^{<1>}(a_L) - c}{2} > 0$.

Now, suppose $\left(\frac{a_L - q_R(a_L) - c}{2}\right)^+$ is zero (i.e., when $q_R(a_L) \geq a_L - c$). We can maximize the term:

$$(1-\lambda)\left(a_L - q_R(a_L)\right) q_R(a_L) - \lambda(a_H - a_L)q_R(a_L).$$

The first-order condition yields the optimal quantity

$$q_R^{<2>}(a_L) = \left(\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2\,(1-\lambda)}\right)^+.$$

It is clear that when $c \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, $\frac{a_L - q_R^{<2>}(a_L) - c}{2} \leq 0$.

Therefore, if $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} > \frac{a_L + \frac{2\lambda(a_H - a_L)}{1-\lambda}}{3}$ (i.e., $a_L > \lambda a_H$), then, when $c \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, the optimal quantity is $q_R^E(a_L) = q_R^{<2>}(a_L) = \frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$; when

$\frac{a_L+\frac{2\lambda(a_H-a_L)}{1-\lambda}}{3} \leq c < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$, $q_R^E(a_L) = a_L - c$; and when $c < \frac{a_L+\frac{2\lambda(a_H-a_L)}{1-\lambda}}{3}$, $q_R^E(a_L) = q_R^{<1>}(a_L) = \left(2c - \frac{2\lambda(a_H-a_L)}{1-\lambda}\right)^+$.

In contrast, if $\frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)} < \frac{a_L+\frac{2\lambda(a_H-a_L)}{1-\lambda}}{3}$ (i.e., $a_L \leq \lambda a_H$), then $q_R^{<2>}(a_L) = 0$. Therefore, when $c \geq a_L$, both the optimal directing selling and the optimal reselling quantities equal zero for the small market size. When $c < a_L$, we can notice that $q_R^{<1>}(a_L) = 0$. Therefore, the optimal reselling quantity is zero. The other results follow immediately.

Proof of Proposition 17. Without encroachment, the optimal contract follows Proposition 13, based on which we can derive the supplier's expected profit as:

$$
\begin{aligned}
\Pi_S^N &= \sum_{i\in\{H,L\}} \lambda_i w^N(a_i) q_R^N(a_i) \\
&= \lambda\left[\left(\frac{a_H}{2}\right)^2 - (a_H - a_L)q_R^N(a_L)\right] + (1-\lambda)\left(a_L - q_R^N(a_L)\right) q_R^N(a_L) \\
&= \lambda\left[\left(\frac{a_H}{2}\right)^2 - \mathbf{I}_{\{a_L>\lambda a_H\}}(a_H - a_L)\left(\frac{a_L}{2} - \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right)\right] \\
&\quad +(1-\lambda)\mathbf{I}_{\{a_L>\lambda a_H\}}\left[\left(\frac{a_L}{2}\right)^2 - \left(\frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right)^2\right].
\end{aligned}
$$

When $a_L \leq \lambda a_H$, the profit reduces to $\Pi_S^N = \lambda\left(\frac{a_H}{2}\right)^2$. With encroachment, the optimal solution of the supplier's problem follows Proposition 15 and the supplier's expected profit follows:

$$\begin{aligned}\Pi_S^E &= \sum_{i\in\{H,L\}} \lambda_i[w^E(a_i)q_R^E(a_i) + \left(a_i - q_R^E(a_i) - q_S^E(a_i) - c\right) q_S^E(a_i)] \qquad \text{(B.1)}\\ &= \lambda\left[\left(a_H - q_R^E(a_H) - q_S^E(a_H)\right) q_R^E(a_H) - (a_H - a_L)q_R^E(a_L) + \left(q_S^E(a_H)\right)^2\right]\\ &\quad +(1-\lambda)\left[\left(a_L - q_R^E(a_L) - q_S^E(a_L)\right) q_R^E(a_L) + \left(q_S^E(a_L)\right)^2\right].\end{aligned}$$

When $a_L \le \lambda a_H$, the above profit can be written as:

$$\Pi_S^E = \begin{cases} \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left(\left(\frac{a_L-c}{2}\right)^+\right)^2 & \text{if } c \ge \frac{a_H}{2},\\ \lambda\left[c\left(a_H - c\right)\right] + (1-\lambda)\left(\left(\frac{a_L-c}{2}\right)^+\right)^2 & \text{if } \frac{a_H}{3} \le c < \frac{a_H}{2},\\ \lambda\left[c\left(a_H-c\right) + \left(\frac{a_H-3c}{2}\right)^2\right] + (1-\lambda)\left(\left(\frac{a_L-c}{2}\right)^+\right)^2 & \text{if } c < \frac{a_H}{3}.\end{cases} \qquad \text{(B.2)}$$

When $a_L > c$, we have the first derivative as:

$$\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \begin{cases} -(1-\lambda)\frac{a_L-c}{2} & \text{if } c \ge \frac{a_H}{2},\\ \lambda a_H - (1-\lambda)\frac{a_L}{2} - \frac{5\lambda-1}{2}c & \text{if } \frac{a_H}{3} \le c < \frac{a_H}{2},\\ \frac{4\lambda+1}{2}c - \frac{\lambda a_H + (1-\lambda)a_L}{2} & \text{if } c < \frac{a_H}{3}.\end{cases} \qquad \text{(B.3)}$$

When $a_L \le c$, we have the first derivative as:

$$\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \begin{cases} 0 & \text{if } c \ge \frac{a_H}{2},\\ \lambda(a_H - 2c) & \text{if } \frac{a_H}{3} \le c < \frac{a_H}{2},\\ \lambda\left(\frac{5}{2}c - \frac{a_H}{2}\right) & \text{if } c < \frac{a_H}{3}.\end{cases} \qquad \text{(B.4)}$$

From the upper branch of (B.2), we can observe that when $c \ge \frac{a_H}{2}$, if $a_L \le c$, then $\Pi_S^E = \lambda\left(\frac{a_H}{2}\right)^2 = \Pi_S^N$, while if $a_L > c$, then $\Pi_S^E > \Pi_S^N$ and is decreasing in c. This observation implies that if $a_L \le \frac{a_H}{2}$, then we always have $\Pi_S^E = \Pi_S^N$ for any $c \ge \frac{a_H}{2}$. However, if $a_L > \frac{a_H}{2}$, then there exists an interval $c \in [\frac{a_H}{2}, a_L)$ in which Π_S^E is larger than Π_S^N but decreases to Π_S^N as c approaches a_L. Hence, we divide the analysis into two cases with (i) $\frac{a_L}{a_H} \le \frac{1}{2}$ and (ii) $\frac{a_L}{a_H} > \frac{1}{2}$, respectively.

(i) For the case in which $\frac{a_L}{a_H} \leq \frac{1}{2}$, we must show that $\Pi_S^E > \Pi_S^N = \lambda \left(\frac{a_H}{2}\right)^2$ when $c \to 0$, that Π_S^E is first decreasing, and then increasing in c, and that $\Pi_S^E = \Pi_S^N = \lambda \left(\frac{a_H}{2}\right)^2$ at $c = \frac{a_H}{2}$.

From the expression for Π_S^E shown in (B.2), we can see that, when $c \to 0$, we have $\Pi_S^E = \lambda \left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left(\frac{a_L}{2}\right)^2 > \Pi_S^N = \lambda \left(\frac{a_H}{2}\right)^2$. In addition, we can see from (B.4) that $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \lambda(a_H - 2c) > 0$ when $c \in [\max\{a_L, \frac{a_H}{3}\}, \frac{a_H}{2})$. Therefore, Π_S^E is increasing in c for $c \in [\max\{a_L, \frac{a_H}{3}\}, \frac{a_H}{2})$ and $\Pi_S^E = \Pi_S^N = \lambda \left(\frac{a_H}{2}\right)^2$ when $c = \frac{a_H}{2}$.

It remains to be shown that $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c}$ is decreasing and then increasing over the range $c \in [0, \max\{a_L, \frac{a_H}{3}\}]$. From (B.3), we can see that for c sufficiently small, $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \frac{4\lambda+1}{2}c - \frac{\lambda a_H + (1-\lambda)a_L}{2} < 0$. In addition, we can observe from both (B.4) and (B.3) that $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c}$ is increasing in c for all $c < \frac{a_H}{3}$.

For the range, $c \in [\frac{a_H}{3}, \frac{a_H}{2})$, we need to consider two possibilities: First, if $a_L < \frac{a_H}{3}$, then we have that $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \lambda(a_H - 2c) \geq 0$ and $\Pi_S^E < \Pi_S^N$ for all $c \in [\frac{a_H}{3}, \frac{a_H}{2})$. Alternatively, if $a_L \geq \frac{a_H}{3}$, then we have $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \lambda a_H - (1-\lambda)\frac{a_L}{2} - \frac{5\lambda-1}{2}c$ in the range of $c \in [\frac{a_H}{3}, a_L]$, while we have $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \lambda(a_H - 2c) > 0$ for all $c \in [a_L, \frac{a_H}{2})$.

If $5\lambda \leq 1$, then $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c}$ is non-decreasing in c for $c \in [\frac{a_H}{3}, a_L]$. By assumption, we have $a_L \leq \lambda a_H$, which implies that $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} > 0$ at the point $c = \frac{a_H}{3}$. It follows that $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} > 0$ for all $c \in [\frac{a_H}{3}, \frac{a_H}{2})$.

If $5\lambda > 1$, then $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \lambda a_H - (1-\lambda)\frac{a_L}{2} - \frac{5\lambda-1}{2}c$ is decreasing in c for $c \in [\frac{a_H}{3}, a_L]$. However, because $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c}$ is continuous at $c = a_L$ for $a_L \geq \frac{a_H}{3}$, and $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \lambda(a_H - 2c) > 0$ for $c \in [a_L, \frac{a_H}{2})$, it follows that we must have $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} > 0$ for $c \in [\frac{a_H}{3}, a_L]$ as well.

Combining the analysis for the above two situations, we can conclude that there exists one threshold c'_S such that $\Pi_S^E > \Pi_S^N$ when $0 < c < c'_S$, $\Pi_S^E \leq \Pi_S^N$ when $c'_S \leq c < \frac{a_H}{2}$, and $\Pi_S^E = \Pi_S^N$ when $c \geq \frac{a_H}{2}$.

(ii) The analysis for the case with $a_L > \frac{a_H}{2}$ is similar to the above, except that now we can have another threshold c''_S such that $\Pi_S^E > \Pi_S^N$ when $c''_S < c < a_L$.

Proof of Proposition 18. First, it is clear from Proposition 15 that when $c \geq \max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$, the supplier never sells directly even if she has the option to encroach, which suggests $\Pi_S^E = \Pi_S^N$.

Second, we can show that the supplier's expected profit with encroachment is increasing in c when $\max\left\{\min\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}, \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_H}{3}\right\} < c < \max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$. There can be two cases:

i) If $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$ (i.e., $\lambda > \frac{1}{2}$), then when $\max\left\{\frac{a_H}{2}, \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}\right\} < c < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$, the supplier's expected profit (according to Equation (B.1)) is:

$$\Pi_S^E = \lambda\left[\left(\frac{a_H}{2}\right)^2 - (a_H - a_L)(a_L - c)\right] + (1-\lambda)c(a_L - c).$$

Taking the first derivative: $\frac{d\Pi_S^E}{dc} = \lambda(a_H - a_L) + (1-\lambda)(a_L - 2c) > 0$ when $c < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$. Hence, Π_S^E increases in c when $\frac{a_H}{2} < c < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$.

ii) If $\frac{a_H}{2} > \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$ (i.e., $\lambda < \frac{1}{2}$), then when $\max\left\{\frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}, \frac{a_H}{3}\right\} < c < \frac{a_H}{2}$, the supplier's expected profit is:

$$\Pi_S^E = \lambda\left[c(a_H - c) - (a_H - a_L)\left(\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}\right)\right] + (1-\lambda)\left(\left(\frac{a_L}{2}\right)^2 - \left(\frac{\lambda(a_H - a_L)}{2(1-\lambda)}\right)^2\right),$$

which is obviously increasing in c when $c < \frac{a_H}{2}$.

Hence, $\Pi_S^E < \Pi_S^N$ when $\max\left\{\min\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}, \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_H}{3}\right\} < c < \max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$ and $\Pi_S^E \to \Pi_S^N$ as c increases to $\max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$.

Third, when $c < \min\left\{\frac{\lambda(a_H-a_L)}{1-\lambda}, \frac{\lambda a_H+(1-\lambda)a_L}{1+4\lambda}, \frac{a_H}{3}\right\}$, we can easily show that the supplier's expected profit Π_S^E under encroachment is decreasing in c. Notice from Proposition 15 that $q_R^E(a_L) = \left(2c - \frac{2\lambda(a_H-a_L)}{1-\lambda}\right)^+ = 0$ under encroachment when $c < \min\left\{\frac{\lambda(a_H-a_L)}{1-\lambda}, \frac{\lambda a_H+(1-\lambda)a_L}{1+4\lambda}, \frac{a_H}{3}\right\}$. Thus,

$$\Pi_S^E = \lambda\left[c\,(a_H - c) + \left(\frac{a_H - 3c}{2}\right)^2\right] + (1-\lambda)\left(\frac{a_L - c}{2}\right)^2,$$

when $c < \min\left\{\frac{\lambda(a_H-a_L)}{1-\lambda}, \frac{\lambda a_H+(1-\lambda)a_L}{1+4\lambda}, \frac{a_H}{3}\right\}$. By taking the first derivative of Π_S^E, it can be confirmed that that Π_S^E decreases in c when $c < \frac{\lambda a_H+(1-\lambda)a_L}{1+4\lambda}$. Further, notice that when c goes to zero, Π_S^E goes to $\lambda\frac{a_H^2}{4} + (1-\lambda)\frac{a_L^2}{4}$ which is larger than Π_S^N.

Combining the above results, we assert that there must exist a threshold c'_S such that when $c < c'_S$, $\Pi_S^E > \Pi_S^N$, and a threshold c'''_S such that when $c'''_S \le c < \max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$, $\Pi_S^E \le \Pi_S^N$. Furthermore, we can assert by comparing Π_S^E and Π_S^N according to the equilibrium solutions of Propositions 13 and 15 that there exists at most one more threshold $c''_S \in [c'_S, c'''_S]$ at which $\Pi_S^E = \Pi_S^N$, and these three thresholds may coincide with each other (the detailed comparison is long but mainly algebraic, which is thus omitted). Hence, the full comparison can be characterized by three thresholds: when $0 < c < c'_S$, $\Pi_S^E > \Pi_S^N$; when $c'_S \le c \le c''_S$, $\Pi_S^E \le \Pi_S^N$; when $c''_S < c < c'''_S$, $\Pi_S^E > \Pi_S^N$; when $c'''_S \le c < \max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$, $\Pi_S^E \le \Pi_S^N$; and when $c \ge \max\left\{\frac{a_H}{2}, \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}\right\}$, $\Pi_S^E = \Pi_S^N$.

Proof of Proposition 19. Without encroachment, the optimal contract follows

Proposition 13, based on which we can derive the reseller's expected profit as:

$$\Pi_R^N = \lambda(a_H - a_L)q_R^E(a_L) = \lambda \mathbf{I}_{\{a_L > \lambda a_H\}}(a_H - a_L)\left(\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}\right).$$

With encroachment, the optimal solution of the supplier's problem follows Proposition 15 and we can formulate the reseller's expected profit as:

$$\Pi_R^E = \sum_{i \in \{H,L\}} \lambda_i \left(a_i - q_R^E(a_i) - q_S^E(a_i) - w^E(a_i)\right) q_R^E(a_i) = \lambda(a_H - a_L)q_R^E(a_L).$$

Proposition 15 reveals that if $\frac{a_L}{a_H} \leq \lambda < 1$, $q_R^E(a_L) = q_R^N(a_L) = 0$ for any c. Thus, the reseller is always indifferent with or without encroachment.

If $\lambda < \frac{a_L}{a_H}$, $q_R^E(a_L) = q_R^N(a_L) = \frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ when $c \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, so the reseller is indifferent with or without encroachment in this region. When $\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)} < c < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, $q_R^E(a_L) > q_R^N(a_L) = \frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, which suggests that the reseller gains a larger profit under encroachment. When $c < \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$, $q_R^E(a_L)$ decreases and approaches zero as c decreases. Hence, there exists a threshold c'_R such that the resller is worse off when $c \leq c'_R$, better off when $c'_R < c < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, and indifferent when $c \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ with encroachment compared to without.

Proof of Proposition 20. An analysis of a simultaneous move Cournot competition can reveal that without any constraint, the optimal selling quantities of the reseller and the supplier should be $\frac{a_i + c}{3}$ and $\frac{a_i - 2c}{3}$, respectively, for each market size. Certainly, in our context, the reseller cannot sell more than what he orders, i.e., his selling quantity will be $\min\{\frac{a_i + c}{3}, q_R(a_i)\}$.

As in the proof of Proposition 15, the reseller's individual rationality constraint for the small market size must be binding at optimum and thus we have $w(a_L) = a_L - q_R(a_L) - q_S(a_L)$. The reseller's incentive compatibility constraint for the large market size must also bind at optimum and thus satisfy:

$$\begin{aligned}
&(a_H - q_R(a_H) - q_S(a_H) - w(a_H))q_R(a_H) \\
&\quad = [a_H - (q_R(a_L) + q_S(a_L)) - (a_L - q_R(a_L) - q_S(a_L))]q_R(a_L) \\
&\quad = (a_H - a_L)q_R(a_L).
\end{aligned}$$

Thus, the optimal wholesale price for the large market size satisfies:

$$w(a_H) = a_H - q_R(a_H) - q_S(a_H) - \frac{(a_H - a_L)q_R(a_L)}{q_R(a_H)}.$$

We can now substitute the above expressions into the supplier's objective of choosing $\{q_R(a_i)\}_{i\in\{H,L\}}$ to obtain:

$$\begin{aligned}
\max \quad & \lambda\left[\left(a_H - q_R(a_H) - \left(\frac{a_H - q_R(a_H) - c}{2}\right)^+ - \frac{(a_H - a_L)q_R(a_L)}{q_R(a_H)}\right)q_R(a_H) + \left(\left(\frac{a_H - q_R(a_H) - c}{2}\right)^+\right)^2\right] \\
& +(1-\lambda)\left[\left(a_L - q_R(a_L) - \left(\frac{a_L - q_R(a_L) - c}{2}\right)^+\right)q_R(a_L) + \left(\left(\frac{a_L - q_R(a_L) - c}{2}\right)^+\right)^2\right] \\
& = \lambda\left[\left(a_H - q_R(a_H) - \left(\frac{a_H - q_R(a_H) - c}{2}\right)^+\right)q_R(a_H) - (a_H - a_L)q_R(a_L) + \left(\left(\frac{a_H - q_R(a_H) - c}{2}\right)^+\right)^2\right] \\
& +(1-\lambda)\left[\left(a_L - q_R(a_L) - \left(\frac{a_L - q_R(a_L) - c}{2}\right)^+\right)q_R(a_L) + \left(\left(\frac{a_L - q_R(a_L) - c}{2}\right)^+\right)^2\right] \\
s.t. \quad & q_R(a_i) \le \frac{a_i + c}{3}, i = H, L
\end{aligned}$$

Notice that in the proof of Proposition 15, without the constraint $q_R(a_i) \le \frac{a_i+c}{3}$, the optimal quantity $q_R^E(a_i)$ is an interior solution to an optimization with a concave objective function. Therefore, with the constraint $q_R(a_i) \leq \frac{a_i+c}{3}$, the optimal quantity is $\min\{\frac{a_i+c}{3}, q_R^E(a_i)\}, i \in \{H, L\}$. In summary, the option of free disposal will lower the optimal quantity to $\frac{a_H+c}{3}$ for the large market size when $\frac{a_H}{5} < c < \frac{a_H}{2}$. For the small market size, the option of free disposal will lower the optimal reselling quantity to $\frac{a_L+c}{3}$ when $\min\{\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_L}{5} + \frac{6\lambda(a_H-a_L)}{5(1-\lambda)}\} < c < \max\{\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}, \frac{a_L}{2}\}$.

Proof of Proposition 21. (i) For the reseller, it is obvious that when $\lambda \in [\frac{a_L}{a_H}, 1)$, his profit is always zero either with or without information acquisition, and thus he is indifferent. When $\lambda \in (0, \frac{a_L}{a_H})$, without information acquisition, the reseller obtains zero profit; with information acquisition, he can obtain a positive profit when c is larger than $\frac{\lambda(a_H-a_L)}{(1-\lambda)}$ and zero otherwise. Therefore, there exists a threshold $c_R^I = \frac{\lambda(a_H-a_L)}{(1-\lambda)}$ such that the reseller is better off by information acquisition when $c > c_R^I$ and indifferent when $c < c_R^I$.

(ii) For the supplier, we first derive her profit in the case of no information acquisition. When $c \in (0, \frac{\mu}{3}]$, we have:

$$\begin{aligned}
\Pi_S^{ENI} &= \sum_{i\in\{H,L\}} \lambda_i \left[2c\frac{\mu - c}{2} + \frac{\mu - 3c}{2}(a_i - 2c - \frac{\mu - 3c}{2} - c)\right] \\
&= \frac{\mu^2 - 6c\mu + 9c^2 + 4c\mu - 4c^2}{4} \\
&= \frac{\mu^2 - 2c\mu + 5c^2}{4};
\end{aligned}$$

when $c \in (\frac{\mu}{3}, \frac{\mu}{2}]$, we have: $\Pi_S^{ENI} = c(\mu - c)$; and when $c \in (\frac{\mu}{2}, +\infty)$, we have: $\Pi_S^{ENI} = \frac{\mu^2}{4}$.

(ii-a) We show that there exists $\underline{c}_S^I$ such that $\Pi_S^E > \Pi_S^{ENI}$ when $c < \underline{c}_S^I$. Notice that when $c \leq \min\{\frac{\lambda(a_H-a_L)}{1-\lambda}, \frac{\mu}{3}\}$ (which implies that $c \leq \frac{a_H}{3}$ and $c < \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$), we can derive:

$$\begin{aligned}\Pi_S^E - \Pi_S^{ENI} &= \lambda\left[c\left(a_H - c\right) + \left(\frac{a_H - 3c}{2}\right)^2\right] + (1-\lambda)\left(\frac{a_L - c}{2}\right)^2 - \frac{\mu^2 - 2c\mu + 5c^2}{4} \\ &= \frac{\sigma^2}{4} - (1-\lambda)\, c^2.\end{aligned}$$

Clearly, $\Pi_S^E > \Pi_S^{ENI}$ if $c < \frac{\sigma}{2\sqrt{1-\lambda}}$. Hence, there exists a threshold $\underline{c}_S^I = \min\{\frac{\lambda(a_H - a_L)}{1-\lambda}, \frac{\mu}{3}, \frac{\sigma}{2\sqrt{1-\lambda}}\}$ such that $\Pi_S^E > \Pi_S^{ENI}$ when $c < \underline{c}_S^I$.

(ii-b) We show that there exist $\overline{c}_S^I$ and $\overline{\lambda}$ such that $\Pi_S^E > \Pi_S^{ENI}$ when $c > \overline{c}_S^I$ and $\lambda > \overline{\lambda}$. Notice that when $c > \max\{\frac{a_H}{2}, a_L\}$ (which implies $c > \frac{\mu}{2}$), the supplier never sells through her direct channel either with or without downstream information acquisition. we compare Π_S^E and Π_S^{ENI} for $\lambda \in [\frac{a_L}{a_H}, 1)$ and $\lambda \in (0, \frac{a_L}{a_H})$, respectively.

1) When $\lambda \in [\frac{a_L}{a_H}, 1)$, we have

$$\Pi_S^E - \Pi_S^{ENI} = \lambda\frac{a_H^2}{4} - \frac{\mu^2}{4},$$

which is positive when $\lambda > \frac{a_L^2}{(a_H - a_L)^2}$.

2) When $\lambda \in (0, \frac{a_L}{a_H})$, we have

$$\begin{aligned}\Pi_S^E - \Pi_S^{ENI} &= \lambda\left[\left(\frac{a_H}{2}\right)^2 - (a_H - a_L)\left(\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2\,(1-\lambda)}\right)\right] \\ &\quad +(1-\lambda)\left(\left(\frac{a_L}{2}\right)^2 - \left(\frac{\lambda(a_H - a_L)}{2\,(1-\lambda)}\right)^2\right) - \frac{\mu^2}{4} \\ &= \frac{\lambda[(1-\lambda+\lambda^2)a_H^2 - 2(2-2\lambda+\lambda^2)a_H a_L + (3-3\lambda+\lambda^2)a_L^2]}{4(1-\lambda)}.\end{aligned}$$

Let $U \equiv (1-\lambda+\lambda^2)a_H^2 - 2(2-2\lambda+\lambda^2)a_H a_L + (3-3\lambda+\lambda^2)a_L^2$. We have

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}\lambda} &= (-1+2\lambda)a_H^2 - 2(-2+2\lambda)a_H a_L + (-3+2\lambda)a_L^2;\\ \frac{\mathrm{d}^2U}{\mathrm{d}\lambda^2} &= 2a_H^2 - 4a_H a_L + 4a_L^2 = 2(a_H-a_L)^2 > 0.\end{aligned}$$

As a result, U is convex, and $U > 0$ when $\lambda > \frac{a_H - 3a_L + \sqrt{-3a_H^2 + 10a_H a_L - 3a_L^2}}{2(a_H - a_L)}$.

Combining cases 1) and 2), we confirm that there exist $\bar{c}_S^I$ and $\overline{\lambda}$ such that $\Pi_S^E > \Pi_S^{ENI}$ when $c > \bar{c}_S^I$ and $\lambda > \overline{\lambda}$. It is worth noting that the above conditions are all sufficient conditions. To derive the sufficient and necessary conditions is technically challenging.

B.2 Dominance of Separating Equilibrium

In this section, we show that the optimal separating solution always dominates the optimal pooling solution in our model.

B.2.1 Derivation of the Optimal Pooling Solution

We first derive the pooling solution. Notice that after the reseller accepts the contract (w, q_R), the supplier solves

$$\max_{q_S} \left[\lambda\left(a_H - q_R - q_S - c\right) + (1-\lambda)\left(a_L - q_R - q_S - c\right)\right] q_S$$

for her direct selling quantity. The optimal solution is $q_S(q_R) = \left(\frac{\lambda a_H + (1-\lambda)a_L - q_R - c}{2}\right)^+$. Given the reseller anticipates the supplier's direct sale decision, when designing the contract (w, q_R), the supplier solves

$$\begin{aligned}
\max_{w,q_R} \quad & wq_R + \left(\left(\frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2}\right)^+\right)^2 \\
s.t. \quad & \left(a_L - q_R - \left(\frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2}\right)^+\right) q_R - wq_R \geq 0; \\
& \left(a_H - q_R - \left(\frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2}\right)^+\right) q_R - wq_R \geq 0.
\end{aligned}$$

Clearly, the second constraint is redundant, and at optimum, the first constraint must bind. Therefore, $w^* = a_L - q_R - \left(\frac{\lambda a_H + (1-\lambda)a_L - q_R - c}{2}\right)^+$. We first solve the case where $q_R > 0$ and $\frac{\lambda a_H + (1-\lambda)a_L - q_R - c}{2} > 0$ at optimum. Then, the above program can be rewritten as:

$$\max_{q_R} \left(a_L - q_R - \left(\frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2}\right)\right) q_R + \left(\frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2}\right)^2.$$

From the first-order condition

$$\left(a_L - q_R - \left(\frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2}\right)\right) - \frac{q_R}{2} - \frac{\lambda a_H + (1-\lambda)\,a_L - q_R - c}{2} = 0,$$

we have $q_R^* = 2\,(a_L + c - (\lambda a_H + (1-\lambda)\,a_L))$, and thus, $q_S^* = \frac{3(\lambda a_H + (1-\lambda)a_L - c)}{2} - a_L$. Now, we discuss four subcases.

(1) Notice that $q_R^* > 0$ and $q_S^* > 0$ if $(\lambda a_H + (1-\lambda)\,a_L) - a_L < c < (\lambda a_H + (1-\lambda)\,a_L) - \frac{2}{3}a_L$. In this case, the supplier's profit is

$$\begin{aligned}
\Pi_S^{EP} = & ((\lambda a_H + (1-\lambda)\,a_L) - c)\,(a_L + c - (\lambda a_H + (1-\lambda)\,a_L)) \\
& + \left(\frac{3\,(\lambda a_H + (1-\lambda)\,a_L - c)}{2} - a_L\right)^2.
\end{aligned}$$

(2) When $c \leq (\lambda a_H + (1-\lambda) a_L) - a_L$, $q_R^* = 0$; that is, the supplier would sell the product only through her direct channel. The initial assumption is violated. Therefore, instead of the above optimization program, the supplier solves

$$\max_{q_S} \left[\lambda (a_H - q_S - c) + (1-\lambda)(a_L - q_S - c)\right] q_S.$$

In this case, the optimal quantity is $q_S^* = \frac{\lambda a_H + (1-\lambda) a_L - c}{2}$ and the supplier's profit is $\Pi_S^{EP} = \left(\frac{\lambda a_H + (1-\lambda) a_L - c}{2}\right)^2$.

(3) When $c \geq (\lambda a_H + (1-\lambda) a_L) - \frac{2}{3} a_L$, $q_S^* = 0$; that is, the supplier would sell the product only through the reseller. The initial assumption is violated. Therefore, instead of the above optimization program, the supplier solves

$$\begin{aligned}
\max_{w, q_R} \quad & w q_R \\
s.t. \quad & \frac{\lambda a_H + (1-\lambda) a_L - q_R - c}{2} \leq 0; \\
& (a_L - q_R) q_R - w q_R \geq 0; \\
& (a_H - q_R) q_R - w q_R \geq 0.
\end{aligned}$$

Clearly, the last constraint is redundant, and at optimum, the second constraint must bind. Therefore, $w = a_L - q_R$.

(3i) When the first constraint does not bind, we have $q_R^* = \frac{a_L}{2}$ and the supplier's profit $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$. This case arises if $c > (\lambda a_H + (1-\lambda) a_L) - \frac{a_L}{2}$.

(3ii) When the first constraint binds, we have $q_R^* = \lambda a_H + (1-\lambda) a_L - c$ and the supplier's profit

$$\Pi_S^{EP} = (a_L + c - (\lambda a_H + (1-\lambda) a_L)) (\lambda a_H + (1-\lambda) a_L - c).$$

This case arises if $(\lambda a_H + (1-\lambda)\, a_L) - \frac{2}{3} a_L \le c < (\lambda a_H + (1-\lambda)\, a_L) - \frac{a_L}{2}$.

We summarize the supplier's profit:

$$\Pi_S^{EP} = \begin{cases} \left(\frac{\lambda a_H + (1-\lambda) a_L - c}{2}\right)^2 & \text{if } c \le \lambda\,(a_H - a_L)\,, \\ ((\lambda a_H + (1-\lambda)\,a_L) - c)\,(a_L + c - (\lambda a_H + (1-\lambda)\,a_L)) \\ \quad + \left(\frac{3(\lambda a_H + (1-\lambda) a_L - c)}{2} - a_L\right)^2 & \text{if } \lambda\,(a_H - a_L) < c \\ & < \lambda\,(a_H - a_L) + \frac{1}{3} a_L, \\ (a_L + c - (\lambda a_H + (1-\lambda)\,a_L))\,(\lambda a_H + (1-\lambda)\,a_L - c) & \text{if } \lambda\,(a_H - a_L) + \frac{1}{3} a_L \le c \\ & < \lambda\,(a_H - a_L) + \frac{1}{2} a_L, \\ \left(\frac{a_L}{2}\right)^2 & \text{if } c > \lambda\,(a_H - a_L) + \frac{1}{2} a_L. \end{cases} \tag{B.5}$$

B.2.2 Profit Comparison

In the following, we compare the supplier's profits under the optimal separating and pooling solutions. Note that the separating solution (i.e., the menu $\{(q_R^E(a_L), w^E(a_L)), (q_R^E(a_H), w^E(a_H))\}$ and the direct selling quantities $(q_S^E(a_L), q_S^E(a_H)))$ is given in Proposition 4. We carry out the comparison for a list of cases depending on the supplier's direct selling cost c.

When $c \le \lambda\,(a_H - a_L)\,.$

In this case, the supplier does not sell through the reseller under the optimal pooling solution. On the other hand, notice that the pair of $q_R(a_L) = 0$ and $q_R^E(a_H)$ (as given in Proposition 4) is always a feasible solution for the supplier's problem of the separating case. Under this pair, the supplier's profit is $\left(\frac{a_L - c}{2}\right)^2$ after learning the market size is small and her profit is always larger than $\left(\frac{a_H - c}{2}\right)^2$ after learning the market size is large. Therefore, the supplier's profit under the optimal separating solution must be larger than $\lambda\left(\frac{a_H-c}{2}\right)^2+(1-\lambda)\left(\frac{a_L-c}{2}\right)^2$ which, by Jensen's inequality, is larger than the supplier's profit $\left(\frac{\lambda a_H+(1-\lambda)a_L-c}{2}\right)^2$ under the optimal pooling solution. Hence, the pooling solution is dominated the separating solution.

When $\lambda\left(a_H-a_L\right)<c<\lambda\left(a_H-a_L\right)+\frac{1}{3}a_L$.

In this case, the supplier sells through both the direct and the reselling channels under the optimal pooling solution, and her corresponding profit is

$$\begin{aligned}\Pi_S^{EP}=&\left(\left(\lambda a_H+\left(1-\lambda\right)a_L\right)-c\right)\left(a_L+c-\left(\lambda a_H+\left(1-\lambda\right)a_L\right)\right)\\&+\left(\frac{3\left(\lambda a_H+\left(1-\lambda\right)a_L-c\right)}{2}-a_L\right)^2.\end{aligned}$$

We discuss two subcases with $\lambda\left(a_H-a_L\right)<c\leq\frac{2}{3}\lambda\left(a_H-a_L\right)+\frac{a_L}{3}$ and $\frac{2}{3}\lambda\left(a_H-a_L\right)+\frac{a_L}{3}<c<\lambda\left(a_H-a_L\right)+\frac{a_L}{3}$ in sequence.

1) When $\lambda\left(a_H-a_L\right)<c\leq\frac{2}{3}\lambda\left(a_H-a_L\right)+\frac{a_L}{3}$, we have $q_R^*\leq a_L-c$ under

the pooling solution. Thus, we can derive:

$$
\begin{aligned}
\Pi_S^{ES} &= \lambda\left[\left(a_H - q_R^E(a_H) - \left(\frac{a_H - q_R^E(a_H) - c}{2}\right)^+\right) q_R^E(a_H) - (a_H - a_L) q_R^E(a_L) + \left(\left(\frac{a_H - q_R^E(a_H) - c}{2}\right)^+\right)^2\right] \\
&\quad + (1-\lambda)\left[\left(a_L - q_R^E(a_L) - \left(\frac{a_L - q_R^E(a_L) - c}{2}\right)^+\right) q_R^E(a_L) + \left(\left(\frac{a_L - q_R^E(a_L) - c}{2}\right)^+\right)^2\right] \\
&> \lambda\left[\left(a_H - q_R^* - \left(\frac{a_H - q_R^* - c}{2}\right)\right) q_R^* - (a_H - a_L) q_R^* + \left(\frac{a_H - q_R^* - c}{2}\right)^2\right] \\
&\quad + (1-\lambda)\left[\left(a_L - q_R^* - \left(\frac{a_L - q_R^* - c}{2}\right)\right) q_R^* + \left(\frac{a_L - q_R^* - c}{2}\right)^2\right] \\
&= \left(\lambda a_H + (1-\lambda) a_L - q_R^* - \lambda\left(\frac{a_H - q_R^* - c}{2}\right) - (1-\lambda)\left(\frac{a_L - q_R^* - c}{2}\right)\right) q_R^* \\
&\quad + \lambda\left(\frac{a_H - q_R^* - c}{2}\right)^2 + (1-\lambda)\left(\frac{a_L - q_R^* - c}{2}\right)^2 - \lambda (a_H - a_L) q_R^* \\
&\geq \left(\lambda a_H + (1-\lambda) a_L - q_R^* - \frac{\lambda a_H + (1-\lambda) a_L - q_R^* - c}{2}\right) q_R^* + \left(\frac{\lambda a_H + (1-\lambda) a_L - q_R^* - c}{2}\right)^2 - \lambda (a_H - a_L) q_R^* \\
&= \left(a_L - q_R^* - \frac{\lambda a_H + (1-\lambda) a_L - q_R^* - c}{2}\right) q_R^* + \left(\frac{\lambda a_H + (1-\lambda) a_L - q_R^* - c}{2}\right)^2 \\
&= \Pi_S^{EP}
\end{aligned}
$$

The last inequality holds by Jensen's inequality. Hence, the pooling solution is dominated the separating solution.

2) When $\frac{2}{3}\lambda (a_H - a_L) + \frac{a_L}{3} < c < \lambda (a_H - a_L) + \frac{a_L}{3}$, we construct a feasible solution of the supplier's problem of the separating case: $(q_R(a_L), q_R(a_H)) = (a_L - c, a_H - c)$ with the corresponding optimal wholesale prices determined from the IC and IR constraints (the direct selling quantity is always zero). Under this solution, the supplier's profit is: $\Pi_S' = \lambda [(a_H - c) c - (a_H - a_L)(a_L - c)] + (1-\lambda)(a_L - c) c$. We can obtain

$$\Pi_S' - \Pi_S^{EP} = \frac{(5\lambda a_H - 5\lambda a_L + a_L - 3c)(-\lambda a_H + \lambda a_L - a_L + 3c)}{4}$$

We can verify that $5\lambda a_H - 5\lambda a_L + a_L - 3c \geq 0$ and $-\lambda a_H + \lambda a_L - a_L + 3c \geq 0$ when $\frac{2}{3}\lambda (a_H - a_L) + \frac{a_L}{3} < c < \lambda (a_H - a_L) + \frac{a_L}{3}$. Therefore, $\Pi_S' \geq \Pi_S^{EP}$. The supplier's profit Π_S^{ES} under the optimal separating solution must be larger than Π'_S. Hence, the pooling solution is dominated the separating solution.

When $\lambda\,(a_H - a_L) + \frac{1}{3}a_L \le c < \lambda\,(a_H - a_L) + \frac{1}{2}a_L$.

In this case, the supplier does not sell through her direct channel under the optimal pooling solution. Her profit is

$$\Pi_S^{EP} = [a_L + c - (\lambda a_H + (1-\lambda)\,a_L)]\,[\lambda a_H + (1-\lambda)\,a_L - c]\,.$$

We apply the same feasible solution $(q_R(a_L), q_R(a_H)) = (a_L - c, a_H - c)$ as discussed in the above for the separating case, under which the supplier's profit is: $\Pi'_S = \lambda\,[(a_H - c)\,c - (a_H - a_L)\,(a_L - c)] + (1-\lambda)\,(a_L - c)\,c$. We can obtain

$$\begin{aligned}
\Pi'_S - \Pi_S^{EP} &= \lambda\,[(a_H - c)\,c - (a_H - a_L)\,(a_L - c)] + (1-\lambda)\,(a_L - c)\,c \\
&\quad - [c - \lambda(a_H - a_L)]\,[\lambda(a_H - a_L) + (a_L - c)] \\
&= \lambda\,(a_H - c)\,c - \lambda(a_H - a_L)\,(a_L - c) + (1-\lambda)\,(a_L - c)\,c \\
&\quad -\lambda(a_H - a_L)c + \lambda^2(a_H - a_L)^2 - (a_L - c)\,c + \lambda(a_H - a_L)\,(a_L - c) \\
&= \lambda\,(a_H - c)\,c - \lambda\,(a_L - c)\,c - \lambda(a_H - a_L)c + \lambda^2(a_H - a_L)^2 \\
&= \lambda^2(a_H - a_L)^2 > 0.
\end{aligned}$$

The supplier's profit Π_S^{ES} under the optimal separating solution must be larger than Π'_S. Hence, the pooling solution is dominated the separating solution.

When $c \ge \lambda\,(a_H - a_L) + \frac{1}{2}a_L$.

Now, we consider Case (3i) of the pooling solution under which the supplier's profit $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$. We present the comparison according to $\lambda \le \frac{a_L}{a_H}$ and $\lambda > \frac{a_L}{a_H}$ in sequence.

(Case A) $\lambda \leq \frac{a_L}{a_H}$

1) We first consider the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \frac{a_H}{2})$. We can obtain:

$$\begin{aligned}
\Pi_S^{ES} - \Pi_S^{EP} &= \lambda\left[\left(\frac{a_H}{2}\right)^2 - \frac{(a_H - a_L)\,a_L}{2} + \frac{\lambda\,(a_H - a_L)^2}{2(1-\lambda)}\right] \\
&\quad +(1-\lambda)\left(\left(\frac{a_L}{2}\right)^2 - \left(\frac{\lambda\,(a_H - a_L)}{2(1-\lambda)}\right)^2\right) - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 - \frac{\lambda\,(a_H - a_L)\,a_L}{2} + \frac{\lambda^2\,(a_H - a_L)^2}{2(1-\lambda)} \\
&\quad +(1-\lambda)\left(\frac{a_L}{2}\right)^2 - \frac{\lambda^2\,(a_H - a_L)^2}{4(1-\lambda)} - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 - \frac{\lambda\,(a_H - a_L)\,a_L}{2} + \frac{\lambda^2\,(a_H - a_L)^2}{4(1-\lambda)} - \lambda\left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 - \frac{\lambda a_H a_L}{2} + \frac{\lambda\,(a_L)^2}{2} + \frac{\lambda^2\,(a_H - a_L)^2}{4(1-\lambda)} - \lambda\left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 - \frac{\lambda a_H a_L}{2} + \lambda\left(\frac{a_L}{2}\right)^2 + \frac{\lambda^2\,(a_H - a_L)^2}{4(1-\lambda)} \\
&> 0.
\end{aligned}$$

2) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, a_H - c)$. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in \left(\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, \infty\right)$ and $c \in \left(\frac{a_H}{3}, \frac{a_H}{2}\right]$ which implies $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} < \frac{a_H}{2}$. We can obtain

$$\Pi_S^{ES} - \Pi_S^{EP} = \lambda\,(a_H - c)\,c - \frac{\lambda a_H a_L}{2} + \lambda\left(\frac{a_L}{2}\right)^2 + \frac{\lambda^2\,(a_H - a_L)^2}{4(1-\lambda)}. \qquad \text{(B.6)}$$

This difference is the smallest when c is at the lower support, i.e., either $c = \frac{a_H}{3}$

or $\frac{a_L}{2}+\frac{\lambda(a_H-a_L)}{2(1-\lambda)}$, whichever larger. We first assume $\frac{a_H}{3}>\frac{a_L}{2}+\frac{\lambda(a_H-a_L)}{2(1-\lambda)}$ (i.e., $a_L<\frac{2a_H}{3}-\frac{\lambda(a_H-a_L)}{(1-\lambda)}$). So the smallest difference is achieved at $c=\frac{a_H}{3}$ and we can obtain:

$$\begin{aligned}
\Pi_S^{ES}-\Pi_S^{EP} &= \frac{\lambda\left(a_H-a_L\right)^2}{4}-\lambda\frac{1}{36}\left(a_H\right)^2+\frac{\lambda^2\left(a_H-a_L\right)^2}{4(1-\lambda)} \\
&= \frac{\lambda\left(a_H-a_L\right)^2}{4(1-\lambda)}-\lambda\frac{1}{36}\left(a_H\right)^2 \\
&> \lambda\left[\frac{\left(\frac{a_H}{3}+\frac{\lambda(a_H-a_L)}{(1-\lambda)}\right)^2}{4(1-\lambda)}-\frac{1}{36}\left(a_H\right)^2\right]>0.
\end{aligned}$$

Now, we assume $\frac{a_H}{3}<\frac{a_L}{2}+\frac{\lambda(a_H-a_L)}{2(1-\lambda)}<\frac{a_H}{2}$, which implies $\frac{\lambda}{1-\lambda}<1$. So the smallest difference is achieved at $c=\frac{a_L}{2}+\frac{\lambda(a_H-a_L)}{2(1-\lambda)}$ and we can obtain:

$$\begin{aligned}
\Pi_S^{ES}-\Pi_S^{EP} &= \lambda\left(a_H-\left(\frac{a_L}{2}+\frac{\lambda\left(a_H-a_L\right)}{2(1-\lambda)}\right)\right)\left(\frac{a_L}{2}+\frac{\lambda\left(a_H-a_L\right)}{2(1-\lambda)}\right) \\
&\quad -\frac{\lambda a_H a_L}{2}+\lambda\left(\frac{a_L}{2}\right)^2+\frac{\lambda^2\left(a_H-a_L\right)^2}{4(1-\lambda)} \\
&= \lambda a_H\frac{a_L}{2}+\frac{\lambda^2 a_H\left(a_H-a_L\right)}{2(1-\lambda)}-\frac{\lambda a_H a_L}{2}+\lambda\left(\frac{a_L}{2}\right)^2+\frac{\lambda^2\left(a_H-a_L\right)^2}{4(1-\lambda)} \\
&\quad -\lambda\left(\left(\frac{a_L}{2}\right)^2+\left(\frac{\lambda\left(a_H-a_L\right)}{2(1-\lambda)}\right)^2+\frac{\lambda\left(a_H-a_L\right)a_L}{2(1-\lambda)}\right) \\
&= \frac{\lambda^2\left(a_H-a_L\right)^2}{2(1-\lambda)}+\frac{\lambda^2\left(a_H-a_L\right)^2}{4(1-\lambda)}\left(1-\frac{\lambda}{1-\lambda}\right)>0.
\end{aligned}$$

3) Now, consider the scenario: $c\in\left(\frac{a_L}{2}+\frac{\lambda(a_H-a_L)}{2(1-\lambda)},\infty\right)$ and $c\in\left(0,\frac{a_H}{3}\right]$ under which the optimal separating solution is $(q_R^E(a_L),q_R^E(a_H))=(\frac{a_L}{2}-\frac{\lambda(a_H-a_L)}{2(1-\lambda)},2c)$. Clearly, this scenario will arise only if $\frac{a_H}{3}>\frac{a_L}{2}+\frac{\lambda(a_H-a_L)}{2(1-\lambda)}$. To compare the supplier's profits under the optimal separating and pooling solutions, we rely on the feasible solution of the supplier's problem of the separating case: $(q_R(a_L),q_R(a_H))=$

$\left(\frac{a_L}{2} - \frac{\lambda(a_H - a_L)}{2(1-\lambda)}, a_H - c\right)$ with the optimal wholesale prices determined from the IC and IR constraints. We compare the supplier's profit $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ under the pooling solution with her profit Π_S^{ES} under this feasible solution of the separating case. Notice from scenario 2) that (B.6) is increasing in c. So the smallest difference of $\Pi_S^{ES} - \Pi_S^{EP}$ is achieved at the smallest c; i.e., at $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$. In scenario 2), we have shown that Π_S^{EP} is smaller than Π_S^{ES} when $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ under the condition $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} < \frac{a_H}{2}$. Clearly, this condition holds in scenario 3). Therefore, the optimal pooling solution is dominated by the optimal separating solution.

4) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (a_L - c, \frac{a_H}{2})$. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in (\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}, \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}]$ and $c \in (\frac{a_H}{2}, \infty)$. We can obtain

$$\begin{aligned}
\Pi_S^{ES} - \Pi_S^{EP} &= \lambda\left[\left(\frac{a_H}{2}\right)^2 - (a_H - a_L)(a_L - c)\right] \\
&\quad + (1-\lambda)(a_L - c)\,c - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)(a_L - c)\left[c - \frac{\lambda}{1-\lambda}(a_H - a_L)\right] - \left(\frac{a_L}{2}\right)^2 .
\end{aligned} \tag{B.7}$$

Taking the derivative of the difference with respect to c, we obtain $\frac{d\left(\Pi_S^{ES} - \Pi_S^{EP}\right)}{dc} = 2(1-\lambda)\left(\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} - c\right) > 0$ given the range of c which we focus on. Therefore, the smallest of the difference is achieved either at $c = \frac{a_H}{2}$ or $c = \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$ whichever larger. Suppose $\frac{a_H}{2} > \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$. We obtain the smallest difference by plugging $c = \frac{a_H}{2}$ into the above equation:

$$\begin{aligned}
\Pi_S^{ES} - \Pi_S^{EP} &= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left(a_L - \frac{a_H}{2}\right)\left[\frac{a_H}{2} - \frac{\lambda}{1-\lambda}(a_H - a_L)\right] - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left(a_L\frac{a_H}{2} - \left(\frac{a_H}{2}\right)^2 - \frac{\lambda}{1-\lambda}(a_H - a_L)a_L + \frac{a_H}{2}\frac{\lambda}{1-\lambda}(a_H - a_L)\right) - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)a_L\frac{a_H}{2} - (1-\lambda)\left(\frac{a_H}{2}\right)^2 - \lambda(a_H - a_L)a_L + \frac{a_H}{2}\lambda(a_H - a_L) - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)a_L\frac{a_H}{2} - (1-\lambda)\left(\frac{a_H}{2}\right)^2 - \lambda a_H a_L + \lambda a_L^2 + \lambda\frac{a_H^2}{2} - \lambda\frac{a_H a_L}{2} - \left(\frac{a_L}{2}\right)^2 \\
&= \frac{4\lambda - 1}{4}a_H^2 - \frac{4\lambda - 1}{2}a_H a_L + \frac{4\lambda - 1}{4}a_L^2 \\
&= \frac{4\lambda - 1}{4}(a_H - a_L)^2 .
\end{aligned}$$

Given $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, we know $\lambda > \frac{1}{2}$. Therefore, $\Pi_S^{ES} - \Pi_S^{EP} > 0$.

Suppose $\frac{a_H}{2} \leq \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$. We obtain the smallest difference by plugging $c = \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$ into the above equation:

$$\begin{aligned}
\Pi_S^{ES} - \Pi_S^{EP} &= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left[a_L - \left(\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}\right)\right]\left[\left(\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}\right) - \frac{\lambda}{1-\lambda}(a_H - a_L)\right] - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left[\frac{2a_L}{3} - \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}\right]\left[\frac{a_L}{3} - \frac{\lambda(a_H - a_L)}{3(1-\lambda)}\right] - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + (1-\lambda)\frac{2}{9}\left[a_L^2 + \frac{\lambda^2(a_H - a_L)^2}{(1-\lambda)^2} - 2\frac{\lambda(a_H - a_L)a_L}{(1-\lambda)}\right] - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\left(\frac{a_H}{2}\right)^2 + \frac{2}{9}(1-\lambda)a_L^2 + \frac{2}{9}\frac{\lambda^2(a_H - a_L)^2}{(1-\lambda)} - \frac{4}{9}\lambda(a_H - a_L)a_L - \left(\frac{a_L}{2}\right)^2 \\
&= \lambda\frac{a_H^2}{4} + \frac{2}{9}(1-\lambda)a_L^2 + \frac{2}{9}\frac{\lambda^2(a_H - a_L)^2}{(1-\lambda)} - \frac{4}{9}\lambda(a_H - a_L)a_L - \left(\frac{a_L}{2}\right)^2 \\
&= \frac{1}{(1-\lambda)}\left[\lambda(1-\lambda)\frac{a_H^2}{4} + \frac{2}{9}(1-\lambda)^2 a_L^2 + \frac{2}{9}\lambda^2(a_H - a_L)^2 - \frac{4}{9}\lambda(1-\lambda)(a_H - a_L)a_L - (1-\lambda)\left(\frac{a_L}{2}\right)^2\right].
\end{aligned}$$

Taking the derivative of the term in the bracket with respect to λ, we can obtain

$$\begin{aligned}
\frac{\mathrm{d}\left[(1-\lambda)\left(\Pi_S^{ES}-\Pi_S^{EP}\right)\right]}{\mathrm{d}\lambda} &= (1-2\lambda)\frac{a_H^2}{4}-\frac{4}{9}(1-\lambda)a_L^2+\frac{4}{9}\lambda\left(a_H-a_L\right)^2 \\
&\quad -\frac{4}{9}(1-2\lambda)\left(a_H-a_L\right)a_L+\left(\frac{a_L}{2}\right)^2; \\
\frac{\mathrm{d}^2\left[(1-\lambda)\left(\Pi_S^{ES}-\Pi_S^{EP}\right)\right]}{\mathrm{d}\lambda^2} &= -\frac{a_H^2}{2}+\frac{4}{9}a_L^2+\frac{4}{9}\left(a_H-a_L\right)^2+\frac{8}{9}\left(a_H-a_L\right)a_L \\
&= -\frac{a_H^2}{2}+\frac{4}{9}a_L^2+\frac{4}{9}\left(a_H^2+a_L^2-2a_Ha_L\right)+\frac{8}{9}\left(a_Ha_L-a_L^2\right) \\
&= -\frac{a_H^2}{36}<0.
\end{aligned}$$

Therefore, $\frac{\mathrm{d}\left[(1-\lambda)\left(\Pi_S^{ES}-\Pi_S^{EP}\right)\right]}{\mathrm{d}\lambda}$ is decreasing in λ. Notice that in this region we have $\lambda<\frac{a_L}{a_H}$. Plugging $\lambda=\frac{a_L}{a_H}$ into $\frac{\mathrm{d}\left[(1-\lambda)\left(\Pi_S^{ES}-\Pi_S^{EP}\right)\right]}{\mathrm{d}\lambda}$, we derive

$$\frac{\mathrm{d}\left[(1-\lambda)\left(\Pi_S^{ES}-\Pi_S^{EP}\right)\right]}{\mathrm{d}\lambda}=\left(\frac{a_H}{2}-\frac{a_L}{2}\right)^2>0.$$

We assess the sign of $\Pi_S^{ES}-\Pi_S^{EP}$ at the smallest possible λ. Given the condition $\frac{a_H}{2}\leq\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$, the smallest possible λ is the one under which $\frac{a_H}{2}=\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$. Note that when $\frac{a_H}{2}=\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$, $c=\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}=\frac{a_H}{2}$. In the above, we have already shown that when $c=\frac{a_H}{2}$, $\Pi_S^{ES}-\Pi_S^{EP}>0$. Thus, we know $\Pi_S^{ES}-\Pi_S^{EP}>0$ when $\frac{a_H}{2}<\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$.

5) Now, we consider the scenario $c\in(0,\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}]$ and $c\in(\frac{a_H}{2},\infty)$ under which the optimal separating solution is

$$\left(q_R^E(a_L),q_R^E(a_H)\right)=\left(\left(2c-\frac{2\lambda\left(a_H-a_L\right)}{1-\lambda}\right)^+,\frac{a_H}{2}\right).$$

Clearly, this scenario will arise only if $\frac{a_H}{2}<\frac{a_L}{3}+\frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$. To compare the supplier's profits under the optimal separating and pooling solutions, we rely on the feasible solution of the supplier's problem of the separating case: $(q_R(a_L), q_R(a_H)) = (a_L - c, \frac{a_H}{2})$ with the optimal wholesale prices determined from the IC and IR constraints. We compare the supplier's profit under this feasible solution of the separating case with her profit $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ under the optimal pooling solution (i.e., equation (B.7)). In scenario 4), we have shown that (B.7) is increasing in c. Thus, the smallest difference is achieved at $c = \frac{a_H}{2}$. Notice that given $\frac{a_H}{2} < \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$, we have $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$. In scenario 4), we have shown (B.7) is positive at $c = \frac{a_H}{2}$. Hence, the optimal pooling solution must be dominated by the optimal separating solution.

6) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under $(q_R^E(a_L), q_R^E(a_H)) = (a_L - c, a_H - c)$ under the separating equilibrium. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in (\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}, \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}]$ and $c \in (\frac{a_H}{3}, \frac{a_H}{2})$. We can obtain

$$\begin{aligned} \Pi_S^{ES} - \Pi_S^{EP} &= \lambda\left[(a_H - c)\, c - (a_H - a_L)\,(a_L - c)\right] + (1-\lambda)\,(a_L - c)\, c - \left(\frac{a_L}{2}\right)^2 \qquad \text{(B.8)} \\ &= \lambda\,(a_H - a_L)\,[2c - a_L] + (a_L - c)\, c - \left(\frac{a_L}{2}\right)^2. \end{aligned}$$

We can derive $\frac{\mathrm{d}\left(\Pi_S^{ES} - \Pi_S^{EP}\right)}{\mathrm{d}c} = 2\left(\frac{a_L}{2} + \lambda\,(a_H - a_L) - c\right) < 0$ given the condition of the optimal pooling solution $(c > \lambda\,(a_H - a_L) + \frac{a_L}{2})$. Therefore, the smallest of the difference is achieved either at $c = \frac{a_H}{2}$ or $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ whichever smaller.

Suppose $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$. We obtain the smallest difference by plugging $c = \frac{a_H}{2}$ into the above equation:

$$\begin{aligned}
\Pi_S^{ES} - \Pi_S^{EP} &= \lambda \left[\left(a_H - \frac{a_H}{2} \right) \frac{a_H}{2} - (a_H - a_L) \left(a_L - \frac{a_H}{2} \right) \right] + (1-\lambda) \left(a_L - \frac{a_H}{2} \right) \frac{a_H}{2} - \left(\frac{a_L}{2} \right)^2 \\
&= \lambda \left[\frac{a_H^2}{4} - a_H a_L + a_L^2 + \frac{a_H^2}{2} - \frac{a_H a_L}{2} \right] + (1-\lambda) \left(\frac{a_H a_L}{2} - \frac{a_H^2}{4} \right) - \left(\frac{a_L}{2} \right)^2 \\
&= \frac{4\lambda - 1}{4} (a_H - a_L)^2 .
\end{aligned}$$

Given $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, we know $\lambda > \frac{1}{2}$. Therefore, $\Pi_S^{ES} - \Pi_S^{EP} > 0$.

Suppose $\frac{a_H}{2} \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$. We obtain the smallest difference by plugging $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ into the above equation:

$$\begin{aligned}
\Pi_S^{ES} - \Pi_S^{EP} &= \lambda (a_H - a_L) \frac{\lambda (a_H - a_L)}{(1-\lambda)} \\
&\quad + \left(\frac{a_L}{2} - \frac{\lambda (a_H - a_L)}{2(1-\lambda)} \right) \left(\frac{a_L}{2} + \frac{\lambda (a_H - a_L)}{2(1-\lambda)} \right) - \left(\frac{a_L}{2} \right)^2 \\
&= \frac{\lambda^2 (a_H - a_L)^2}{(1-\lambda)} - \frac{\lambda^2 (a_H - a_L)^2}{4(1-\lambda)^2} \\
&= \frac{\lambda^2 (a_H - a_L)^2}{(1-\lambda)} \frac{3 - 4\lambda}{4(1-\lambda)} .
\end{aligned}$$

Given $\frac{a_H}{2} \geq \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, we know $\lambda \leq \frac{1}{2}$. Therefore, $\Pi_S^{ES} - \Pi_S^{EP} > 0$.

7) Now, consider the scenario $c \in (\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}, \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}]$ and $c \in (0, \frac{a_H}{3})$ under which the optimal separating solution $(q_R^E(a_L), q_R^E(a_H)) = (a_L - c, 2c)$. Clearly, this scenario will arise either if $\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)} < \frac{a_H}{3} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ or if $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} < \frac{a_H}{3}$. To compare the supplier's profits under the optimal separating and pooling solutions, we rely on the feasible solution of the supplier's problem of the separating case: $(q_R(a_L), q_R(a_H)) = (a_L - c, a_H - c)$ with the optimal wholesale prices

determined from the IC and IR constraints. We compare the supplier's profit under this feasible solution of the separating case with her profit $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ under the optimal pooling solution (i.e., equation (B.8)). In scenario 6), we have shown that (B.8) is decreasing in c. So the smallest difference for this scenario 7) is achieved at $c = \frac{a_H}{3}$ if $\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)} < \frac{a_H}{3} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ and at $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ if $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} < \frac{a_H}{3}$. We first consider $\frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)} < \frac{a_H}{3} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$. Notice that if $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, then we have shown in scenario 6) that (B.8) is positive at $c = \frac{a_H}{2}$, which implies (B.8) is positive at $c = \frac{a_H}{3}$ given (B.8) is decreasing in c and $\frac{a_H}{2} > \frac{a_H}{3}$. Similarly, if $\frac{a_H}{2} > \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, then we have shown in scenario 6) that (B.8) is positive at $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$, which implies (B.8) is positive at $c = \frac{a_H}{3}$ given (B.8) is decreasing in c and $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} > \frac{a_H}{3}$. Second, if $\frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)} < \frac{a_H}{3}$, then $\frac{a_H}{2} > \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$ and we have shown in scenario 6) that (B.8) is positive at $c = \frac{a_L}{2} + \frac{\lambda(a_H - a_L)}{2(1-\lambda)}$. Consequently, the optimal pooling solution must be dominated by the optimal separating solution.

8) Now, consider the scenario $c \in (0, \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}]$ and $c \in (\frac{a_H}{3}, \frac{a_H}{2})$ under which the optimal separating solution is

$$(q_R^E(a_L), q_R^E(a_H)) = \left(\left(2c - \frac{2\lambda\,(a_H - a_L)}{1-\lambda}\right)^+, a_H - c\right).$$

This scenario will arise either if $\frac{a_H}{3} < \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)} < \frac{a_H}{2}$ or if $\frac{a_H}{2} < \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$. Again, we rely on the feasible solution of the supplier's problem of the separating case: $(q_R(a_L), q_R(a_H)) = (a_L - c, a_H - c)$ with the optimal wholesale prices determined from the IC and IR constraints. We compare the supplier's profit under this feasible solution of the separating case with her profit $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ under the optimal pooling solution (i.e., equation (B.8)). In scenario 6), we have shown that (B.8) is decreasing in c. So the smallest difference is achieved at $c = \frac{a_L}{3} + \frac{2\lambda(a_H - a_L)}{3(1-\lambda)}$ if $\frac{a_H}{3} < \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)} < \frac{a_H}{2}$ and at $c = \frac{a_H}{2}$ if $\frac{a_H}{2} < \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$. We first consider $\frac{a_H}{3} < \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)} < \frac{a_H}{2}$. Notice that if $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$, then we have shown in scenario 6) that (B.8) is positive at $c = \frac{a_H}{2}$, which implies (B.8) is positive at $c = \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$ given (B.8) is decreasing in c. Similarly, if $\frac{a_H}{2} > \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$, then we have shown in scenario 6) that (B.8) is positive at $c = \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$, which implies (B.8) is positive at $c = \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$ given (B.8) is decreasing in c and $\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)} < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$. Second, if $\frac{a_H}{2} < \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}$, then $\frac{a_H}{2} < \frac{a_L}{2} + \frac{\lambda(a_H-a_L)}{2(1-\lambda)}$ and we have shown in scenario 6) that (B.8) is positive at $c = \frac{a_H}{2}$. Consequently, the optimal pooling solution must be dominated by the optimal separating solution.

9) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (\left(2c - \frac{2\lambda(a_H-a_L)}{1-\lambda}\right)^+, 2c)$. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in (0, \frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)}]$ and $c \in (0, \frac{a_H}{3})$. Moreover, the condition for the optimal pooling solution is $c > \lambda(a_H - a_L) + \frac{1}{2}a_L$. Therefore, we must have $\frac{a_H}{3} > \lambda(a_H - a_L) + \frac{1}{2}a_L$ and $\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)} > \lambda(a_H - a_L) + \frac{1}{2}a_L$.

From $\frac{a_H}{3} > \lambda(a_H - a_L) + \frac{1}{2}a_L$, we know $\lambda < \frac{2a_H - 3a_L}{6(a_H - a_L)}$. From $\frac{a_L}{3} + \frac{2\lambda(a_H-a_L)}{3(1-\lambda)} > \lambda(a_H - a_L) + \frac{1}{2}a_L$, we know $\lambda < \frac{2a_H - 3a_L - \sqrt{4a_H^2 + 12a_Ha_L - 15a_L^2}}{12(a_H-a_L)}$ or $\lambda > \frac{2a_H - 3a_L + \sqrt{4a_H^2 + 12a_Ha_L - 15a_L^2}}{12(a_H-a_L)}$. Notice that $\frac{2a_H - 3a_L - \sqrt{4a_H^2 + 12a_Ha_L - 15a_L^2}}{12(a_H-a_L)} < 0$ and $\frac{2a_H - 3a_L + \sqrt{4a_H^2 + 12a_Ha_L - 15a_L^2}}{12(a_H-a_L)} > \frac{2a_H - 3a_L}{6(a_H-a_L)}$. Hence, this case never arises.

(Case B) $\lambda > \frac{a_L}{a_H}$

We now consider the scenario with $\lambda > \frac{a_L}{a_H}$ under which $q_R^E(a_L)$ in the optimal separating solution is always zero.

1) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (0, \frac{a_H}{2})$. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in (\frac{a_H}{2}, \infty)$. It is straightforward to notice that

$$\begin{aligned}\Pi_S^{ES} &= \lambda \left(\frac{a_H}{2}\right)^2 + (1-\lambda)\left(\left(\frac{a_L - c}{2}\right)^+\right)^2 \\ &\geq \lambda \left(\frac{a_H}{2}\right)^2 \geq \frac{a_H a_L}{4} > \left(\frac{a_L}{2}\right)^2.\end{aligned}$$

2) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (0, a_H - c)$. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in (\frac{a_H}{3}, \frac{a_H}{2})$. Recall the condition for the optimal pooling equilibrium is $c > \lambda(a_H - a_L) + \frac{1}{2}a_L$. Thus, we must have $\frac{a_H}{2} > \lambda(a_H - a_L) + \frac{1}{2}a_L$; i.e., $\lambda < \frac{1}{2}$. Since $\lambda > \frac{a_L}{a_H}$, we have $a_H > 2a_L$.

The difference in profits is

$$\Pi_S^{ES} - \Pi_S^{EP} = \lambda(a_H - c)c + (1-\lambda)\left(\left(\frac{a_L - c}{2}\right)^+\right)^2 - \left(\frac{a_L}{2}\right)^2. \tag{B.9}$$

We first assume $\frac{a_L - c}{2} > 0$. Then, taking derivative of the profit difference with respect to c, we obtain

$$\frac{\mathrm{d}\left(\Pi_S^{ES} - \Pi_S^{EP}\right)}{\mathrm{d}c} = \lambda(a_H - 2c) - (1-\lambda)\left(\frac{a_L - c}{2}\right) = \frac{2\lambda a_H - (1-\lambda)a_L}{2} - \frac{5\lambda - 1}{2}c.$$

Plugging $c = \frac{a_H}{2}$, we have

$$\frac{\mathrm{d}\left(\Pi_S^{ES} - \Pi_S^{EP}\right)}{\mathrm{d}c} = \frac{(1-\lambda)(a_H - 2a_L)}{4} > 0.$$

Plugging $c = \frac{a_H}{3}$, we have

$$\frac{\mathrm{d}\left(\Pi_S^{ES} - \Pi_S^{EP}\right)}{\mathrm{d}c} = \frac{(1+\lambda)a_H + 3\lambda a_L - 3a_L}{6} > 0$$

given $(1+\lambda)a_H > a_H + a_L > 3a_L$. Therefore, $\frac{\mathrm{d}\left(\Pi_S^{ES} - \Pi_S^{EP}\right)}{\mathrm{d}c} > 0$ for any $c \in (\frac{a_H}{3}, \frac{a_H}{2})$. Plugging $c = \frac{a_H}{3}$ into $\Pi_S^{ES} - \Pi_S^{EP}$, we have

$$\Pi_S^{ES} - \Pi_S^{EP} = \frac{(1+7\lambda)a_H^2 - 6(1-\lambda)a_H a_L - 9\lambda a_L^2}{36}.$$

Notice that the above difference is increasing in λ. Thus, the smallest difference is achieved at $\lambda = \frac{a_L}{a_H}$. Plugging $\lambda = \frac{a_L}{a_H}$, we have $\Pi_S^{ES} - \Pi_S^{EP} = \frac{a_H^2 + a_H a_L + 6a_L^2}{36} - \frac{a_L^3}{4a_H} > 0$ given $a_H > 2a_L$. Therefore, $\Pi_S^{ES} > \Pi_S^{EP}$ for any $c \in (\frac{a_H}{3}, \frac{a_H}{2})$ if $\frac{a_L - c}{2} > 0$. Furthermore, we can notice that given c, (B.9) is decreasing in a_L. Hence, we must have $\Pi_S^{ES} > \Pi_S^{EP}$ for any $c \in (\frac{a_H}{3}, \frac{a_H}{2})$.

3) We compare $\Pi_S^{EP} = \left(\frac{a_L}{2}\right)^2$ with the supplier's profit Π_S^{ES} under the separating solution $(q_R^E(a_L), q_R^E(a_H)) = (0, 2c)$. Notice that this solution $(q_R^E(a_L), q_R^E(a_H))$ will arise only when $c \in (0, \frac{a_H}{3})$. Recall the condition for the optimal pooling equilibrium is $c > \lambda\left(a_H - a_L\right) + \frac{1}{2}a_L$. Thus, we must have $\frac{a_H}{3} > \lambda\left(a_H - a_L\right) + \frac{1}{2}a_L$; i.e., $\frac{a_L}{a_H} < \lambda < \frac{2a_H - 3a_L}{6(a_H - a_L)} = \frac{1}{3} - \frac{a_L}{6(a_H - a_L)} < \frac{1}{3}$. From $\frac{2a_H - 3a_L}{6(a_H - a_L)} > \frac{a_L}{a_H}$, we have $a_H > \frac{9+\sqrt{33}}{4}a_L$.

The difference in profits is

$$\Pi_S^{ES} - \Pi_S^{EP} = \lambda\left[\left(a_H - c\right)c + \left(\frac{a_H - 3c}{2}\right)^2\right] + (1-\lambda)\left(\left(\frac{a_L - c}{2}\right)^+\right)^2 - \left(\frac{a_L}{2}\right)^2. \tag{B.10}$$

We first assume $\frac{a_L - c}{2} > 0$, i.e., $c < a_L$. Then, taking derivative of the profit difference with respect to c, we obtain

$$\frac{\mathrm{d}\left(\Pi_S^{ES}-\Pi_S^{EP}\right)}{\mathrm{d}c} = \lambda\left(a_H-2c\right)-\lambda\frac{3(a_H-3c)}{2}-(1-\lambda)\left(\frac{a_L-c}{2}\right)$$
$$= -\frac{\lambda a_H+(1-\lambda)a_L}{2}+\frac{4\lambda+1}{2}c.$$

Solving $\frac{\mathrm{d}\left(\Pi_S^{ES}-\Pi_S^{EP}\right)}{\mathrm{d}c}=0$, we have $c=\frac{\lambda a_H+(1-\lambda)a_L}{4\lambda+1}$. The smallest $\Pi_S^{ES}-\Pi_S^{EP}$ is achieved at this c. Denote $U=(3\lambda^2+\lambda)a_H^2+(2\lambda^2-2\lambda)a_Ha_L-(5\lambda^2-\lambda+1)a_L^2$. We have

$$\Pi_S^{ES}-\Pi_S^{EP}\big|_{c=\frac{\lambda a_H+(1-\lambda)a_L}{4\lambda+1}}=\frac{U}{4(4\lambda+1)}$$

where $U>0$ if and only if $a_H>\frac{-\lambda^2+\lambda+\sqrt{16\lambda^4+3\lambda^2+\lambda}}{\lambda(3\lambda+1)}a_L$. This condition always holds because it is implied by $a_H>\frac{1}{\lambda}a_L$ (i.e., $\frac{1}{\lambda}>\frac{-\lambda^2+\lambda+\sqrt{16\lambda^4+3\lambda^2+\lambda}}{\lambda(3\lambda+1)}$ for any $\lambda<\frac{1}{3}$). Thus, $\Pi_S^{ES}>\Pi_S^{EP}$.

We now assume $\frac{a_L-c}{2}\leq 0$, i.e., $c\geq a_L$. Then, taking derivative of the profit difference with respect to c, we obtain

$$\frac{\mathrm{d}\left(\Pi_S^{ES}-\Pi_S^{EP}\right)}{\mathrm{d}c}=\lambda\left(a_H-2c\right)-\lambda\frac{3(a_H-3c)}{2}=-\frac{\lambda a_H}{2}+\frac{5\lambda}{2}c.$$

The smallest $\Pi_S^{ES}-\Pi_S^{EP}$ is achieved at $c=\frac{a_H}{5}$. Plugging this c into (B.10), we have

$$\begin{aligned}\Pi_S^{ES}-\Pi_S^{EP} &= \frac{\lambda a_H^2}{5}-\frac{a_L^2}{4}\geq\frac{4a_La_H-5a_L^2}{20}\\ &\geq \frac{(4+\sqrt{33})a_L^2}{20}>0\end{aligned}$$

where the first inequality holds because $\lambda a_H>a_L$ and the second one holds because $a_H>\frac{9+\sqrt{33}}{4}a_L$.

B.3 Extensions of the Base Model

In this section, we present two extensions of our base model. In B.3.1, we extend our model to a setting with a continuously distributed market size. We allow for the possibility that the development of encroachment capability allows the supplier to receive demand information through her direct channel in B.3.2.

B.3.1 Robustness with Uniform Distribution

In the base model, we have used a two-state distribution to model the market size. Here, we extend our model to a setting where the market size **a** follows a continuous, uniform distribution $\mathbf{U}[\underline{a}, \bar{a}]$ (it is technically cumbersome to compare the expected profits under more general continuous distributions; see B.3.3 for the detailed derivations). Under such a setting, the supplier designs a continuum of contracts, $(w(a), q_R(a))$, corresponding to each market size $a \in [\underline{a}, \bar{a}]$, to maximize her profit. In the following, we first characterize the optimal separating contracts for the two cases without and with supplier encroachment (we focus only on the separating cases in this analysis).

When the supplier does not have the option to encroach, the reseller that observes the true market size a will choose the contract by solving:

$$\max_{\hat{a} \in [\underline{a}, \bar{a}]} [a - q_R(\hat{a}) - w(\hat{a})] q_R(\hat{a}).$$

By the revelation principle, we can focus on the truth-telling solution; that is, at optimum, the reseller always chooses the contract that corresponds to the true market size he observes. Therefore, the supplier's problem can be formulated as

$$\begin{aligned}
\max_{\{(w(a),q_R(a))\}_{a\in[\underline{a},\bar{a}]}} \quad & \mathbf{E}\left[w(\mathbf{a})q_R(\mathbf{a})\right] \\
s.t. \quad & a = \arg\max_{\hat{a}\in[\underline{a},\bar{a}]}[a - q_R(\hat{a}) - w(\hat{a})]q_R(\hat{a}), \forall a \in [\underline{a},\bar{a}], \\
& \left(a - q_R(a) - w(a)\right) q_R(a) \geq 0, \forall a \in [\underline{a},\bar{a}].
\end{aligned}$$

Applying the classical mechanism design technique, we derive the following proposition.

Proposition 23. *Without the option of encroachment, the optimal order quantity and the corresponding wholesale price for the reselling channel follow:*

$$q_R^N(a) = \left(a - \frac{\bar{a}}{2}\right)^+ \quad \textit{and } w^N(a) = a - q_R^N(a) - \frac{\int_{\underline{a}}^{a} q_R^N(\tilde{a})\mathrm{d}\tilde{a}}{q_R^N(a)}.$$

In contrast, when there is a direct channel, the supplier may sell directly after she learns the market size from the reseller's contract choice. In particular, if the reseller takes a contract corresponding to $\hat{a}$, the supplier's optimal direct selling quantity follows: $q_S^*(\hat{a}) = \left(\frac{\hat{a}-q_R(\hat{a})-c}{2}\right)^+$. Anticipating the supplier's response, the reseller that observes the true market size a will choose the contract by solving:

$$\max_{\hat{a}\in[\underline{a},\bar{a}]}[a - q_R(\hat{a}) - q_S^*(\hat{a}) - w(\hat{a})]q_R(\hat{a}).$$

Thus, we can formulate the supplier's problem with the option of encroachment as:

$$\begin{aligned}
\max_{\{(w(a),q_R(a))\}_{a\in[\underline{a},\bar{a}]}} \quad & \mathbf{E}\left[w(\mathbf{a})q_R(\mathbf{a}) + (\mathbf{a} - q_R(\mathbf{a}) - q_S^*(\mathbf{a}) - c)q_S^*(\mathbf{a})\right] \\
s.t. \quad & a = \arg\max_{\hat{a}\in[\underline{a},\bar{a}]}[a - q_R(\hat{a}) - q_S^*(\hat{a}) - w(\hat{a})]q_R(\hat{a}), \forall a \in [\underline{a},\bar{a}], \\
& a - q_R(a) - q_S^*(a) - w(a) \geq 0, \forall a \in [\underline{a},\bar{a}],
\end{aligned}$$

which yields the following proposition.

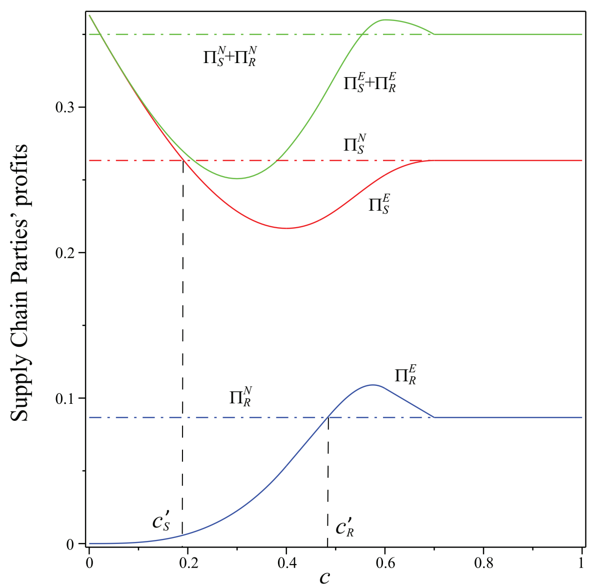


Figure B.1: The effects of supplier encroachment and asymmetry information on the supply chain parties' profits when the market size **a** is uniformly distributed in the support $[1.00, 1.40]$.

Proposition 24. *With the option of encroachment, the optimal order quantity in the reselling channel is in Table B.1, with the corresponding wholesale price satisfying:* $w^E(a) = a - q_R^E(a) - \left(\frac{a-q_R^E(a)-c}{2}\right)^+ - \frac{\int_{\underline{a}}^{a} q_R^E(\tilde{a})\mathrm{d}\tilde{a}}{q_R^E(a)}$*, and the supplier's equilibrium direct selling quantity is* $q_S^E(a) = \left(\frac{a-q_R^E(a)-c}{2}\right)^+$.

Scenarios	$c \in (0, \frac{\bar{a}}{3}]$	$c \in (\frac{\bar{a}}{3}, \frac{2\bar{a}-\underline{a}}{3}]$		$c \in (\frac{2\bar{a}-\underline{a}}{3}, \frac{\bar{a}}{2}]$	$c \in (\frac{\bar{a}}{2}, +\infty)$
	$a \in [\underline{a}, \bar{a}]$	$a \in [\underline{a}, 2\bar{a} - 3c)$	$a \in [2\bar{a} - 3c, \bar{a}]$	$a \in [\underline{a}, \bar{a}]$	$a \in [\underline{a}, \bar{a}]$
$q_R^E(a)$	$2(c + a - \bar{a})^+$	$2(c + a - \bar{a})^+$	$(a - c)^+$	$(a - c)^+$	$\left(a - \frac{\bar{a}}{2}\right)^+$

Table B.1: Optimal separating menu of contracts under uniform demand distribution

Comparing the supplier's and the reseller's profits with and without supplier encroachment, we derive the following proposition.

Proposition 25. *Under certain parameters of the market size distribution and the direct selling cost, the supplier as well as the reseller can be either better off or worse off by the development of the option of supplier encroachment.*

Hence, supplier encroachment can still either benefit or hurt the supplier and the reseller when the market size is continuously distributed, which is numerically illustrated in Figure B.1. Note that the results revealed under a two-state distribution may have a richer pattern than that under a uniform distribution because the market size is evenly distributed under the uniform distribution, while under a two-state distribution it can be skewed to either end. Nevertheless, the managerial insight remains largely the same (even for more general distributions) that supplier encroachment can help reduce the information rents but it can also induce the supplier to behave opportunistically and hurt herself.

B.3.2 Encroachment Provides Supplier with a Noisy Demand Signal

In addition to the way in which the development of encroachment capability affects a supplier's strategic interactions with a reseller, which have been the focus of our analysis up until now, the development of her own direct channel may also provide a supplier with access to information about demand that is independent from what she learns from the reseller's order. Previously, we have ignored this possibility in order to focus exclusively on the strategic interactions with the reseller.

However, our model can easily be adapted to allow for the possibility that a supplier who develops encroachment capability will also receive an independent signal about market demand. To do this, we assume that the supplier receives a signal, denoted by $\mathbf{s} \in \{a_L, a_H\}$, after her decision to develop encroachment capability but before she announces her menu of quantity-price pairs for the reseller. In addition, we assume that the signal is accurate with probability $\phi \in [0.5, 1]$. Specifically, when the true market size is $\mathbf{a} = a_i$, $i = H, L$ the probability that the supplier receives a signal $\mathbf{s} = a_i$ is

$$\text{Prob}(\mathbf{s} = a_i|\mathbf{a} = a_i) = \phi \text{ and } \text{Prob}(\mathbf{s} = a_j|\mathbf{a} = a_i) = 1-\phi,\ i = H, L \text{ and } j \in \{H, L\}\backslash i.$$

Note that when $\phi = 0.5$, the signal provides no information, and when ϕ =1, the signal is perfectly accurate.

After the supplier receives the demand signal, she will update her prior on the probability that the demand parameter is a_H. Using Bayes' rule, it is easy to show that her updated prior will depend upon the signal that she receives in the following way:

$$\lambda_H'(\mathbf{s}) = \begin{cases} \frac{\phi\lambda}{\phi\lambda+(1-\phi)(1-\lambda)} & \text{if } \mathbf{s} = a_H \\ \frac{(1-\phi)\lambda}{(1-\phi)\lambda+\phi(1-\lambda)} & \text{if } \mathbf{s} = a_L \end{cases}$$

and $\lambda_L'(\mathbf{s}) = 1 - \lambda_H'(\mathbf{s})$ is her updated prior for the probability that the demand parameter will be a_L. Denote by $\boldsymbol{\lambda}'(\mathbf{s})$ the vector consisting of $\lambda_H'(\mathbf{s})$ and $\lambda_L'(\mathbf{s})$. The expected optimal profits obtained by the supplier and by the reseller after when the supplier has encroachment capability can now be expressed as follows:

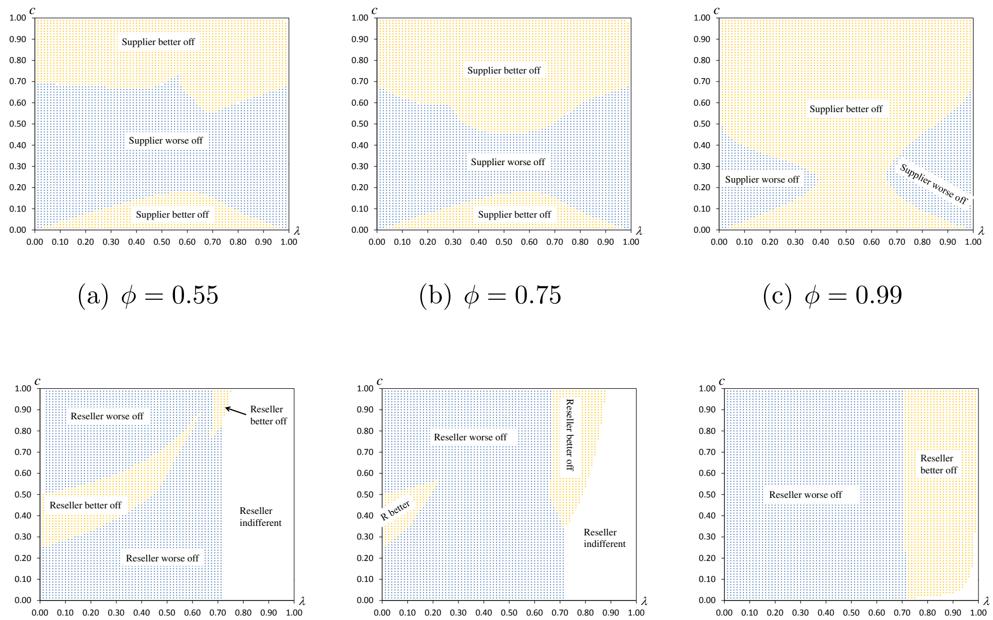
$$\begin{aligned} E_s\left[\Pi_S^E(\boldsymbol{\lambda}, \mathbf{s})\right] &= [\phi\lambda + (1-\phi)(1-\lambda)]\,\Pi_S^E(\boldsymbol{\lambda}'(a_H)) + [(1-\phi)\lambda + \phi(1-\lambda)]\,\Pi_S^E(\boldsymbol{\lambda}'(a_L)) \\ E_s\left[\Pi_R^E(\boldsymbol{\lambda}, \mathbf{s})\right] &= [\phi\lambda + (1-\phi)(1-\lambda)]\,\Pi_R^E(\boldsymbol{\lambda}'(a_H)) + [(1-\phi)\lambda + \phi(1-\lambda)]\,\Pi_R^E(\boldsymbol{\lambda}'(a_L)) \end{aligned}$$

where $\Pi_S^E(\boldsymbol{\lambda}'(a_H))$, $\Pi_S^E(\boldsymbol{\lambda}'(a_L))$, $\Pi_R^E(\boldsymbol{\lambda}'(a_H))$, and $\Pi_R^E(\boldsymbol{\lambda}'(a_L))$ each have the same structure that we have characterized previously.

The independent source of information has both a direct effect and an indirect effect. The direct effect is that the supplier can tailor her pricing menu according to the signal that she receives, and this helps to reduce information rents. In the extreme case where the signal is perfectly accurate, the information rents are eliminated. The indirect effect arises from the fact that, because the signal affects the price-quantity pairs that are offered to the reseller, it indirectly influences the supplier's direct selling quantity. Thus, the accuracy of the demand signal indirectly affects the supplier's direct selling quantity in spite of the fact that she is fully informed of the market size (via the reseller's order) at the time that she determines the quantity to sell directly. Figure B.2 illustrates the effects of the supplier's independent source of market information with different accuracy levels. Figure B.2(a)-B.2(c) show that the regions where the supplier is better off expand as ϕ increases for the reasons described above. By comparing these results to those in Figure 3.4(a), where $\phi = 0.5$ implicitly, even a relatively inaccurate signal, $\phi = 0.55$, dramatically expands the region in which the supplier benefits from encroachment.

In Figure B.2(d)-B.2(f) we show how the reseller is affected by encroachment when the supplier receives an independent demand signal of varying levels of accuracy. Note that the region depicted in Figure 3.4(b) corresponds to the case for $\phi = 0.5$, i.e., the demand signal that the supplier receives carries no information. We can see that, as the accuracy level of the supplier's signal increases, the original region in which the reseller is better off shrinks. However, a new region (around $\lambda = \frac{a_L}{a_H}$) where the reseller is better off emerges. Note that the reseller makes zero profit for $\lambda > \frac{a_L}{a_H}$ when the signal contains no information, i.e., when $\phi = 0.5$, because the optimal menu of quantity-price pairs does not induce the reseller to order when demand is low. However, when the supplier observes even a noisy signal of low demand before she announces the quantity-price pairs, she becomes more willing to induce the reseller to order a positive quantity when demand is low. Consequently, both the supplier and the reseller are better off under encroachment in this region.

This last observation is intriguing because it implies that for certain parameters (relatively high values of both c and λ), the reseller benefits from the supplier's development of encroachment capability only if it results in the supplier obtaining an independent source of demand information.



(a) $\phi = 0.55$ (b) $\phi = 0.75$ (c) $\phi = 0.99$

(d) $\phi = 0.55$ (e) $\phi = 0.75$ (f) $\phi = 0.99$

Figure B.2: Illustration of the regions where supplier obtaining a noisy signal benefits or hurts the supplier and the reseller in the presence of supplier encroachment. The supplier/reseller is better off in the orange regions, worse off in the blue regions, and indifferent in the blank regions. The other parameters are: $a_L = 1.00$ and $a_H = 1.40$.

Proofs

Proof of Proposition 23. Define the reseller's profit function:

$$\Pi_R(\hat{a}|a) = [a - q_R(\hat{a}) - w(\hat{a})]q_R(\hat{a})$$

where a is the observed market size while $\hat{a}$ is the index of the contract the reseller chooses. By the revelation principle, we focus on the truth-telling equilibrium where the reseller truthfully reports the observed market size and thus his profit follows:

$$\Pi_R(a|a) = [a - q_R(a) - w(a)]q_R(a). \tag{B.11}$$

To ensure the truth-telling equilibrium, we must have $\frac{\partial \Pi_R(\hat{a}|a)}{\partial \hat{a}}|_{\hat{a}=a} = 0$. The implicit function theorem implies that

$$\frac{\mathrm{d}\Pi_R(a|a)}{\mathrm{d}a} = \left[\frac{\partial \Pi_R(\hat{a}|a)}{\partial a} + \frac{\partial \Pi_R(\hat{a}|a)}{\partial \hat{a}}\frac{\partial \hat{a}}{\partial a}\right]|_{\hat{a}=a} = \frac{\partial \Pi_R(\hat{a}|a)}{\partial a}|_{\hat{a}=a} = q_R(a).$$

Integrating the above equation, we have the local IC constraint (where $\Pi_R(\underline{a}|\underline{a}) = 0$):

$$\Pi_R(a|a) = \Pi_R(\underline{a}|\underline{a}) + \int_{\underline{a}}^{a} \frac{\mathrm{d}\Pi_R(\tilde{a}|\tilde{a})}{\mathrm{d}\tilde{a}} d\tilde{a} = \int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}. \tag{B.12}$$

(B.12) suggests that for all $a \in [\underline{a}, \bar{a}]$, $\Pi_R(a|a) \geq 0$. Hence, we do not have to explicitly consider the IR constraint. Given the uniform distribution $\mathbf{U}[\underline{a}, \bar{a}]$, the reseller's expected profit is

$$\Pi_R = \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left[\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}\right] \mathrm{d}a = \int_{\underline{a}}^{\bar{a}} \frac{\bar{a}-a}{\bar{a}-\underline{a}} q_R(a)\mathrm{d}a.$$

The last equality holds because of integration by parts. By comparing (B.11) and (B.12), we can derive the supplier's optimal wholesale price:

$$w(a) = a - q_R(a) - \frac{\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}}{q_R(a)}. \tag{B.13}$$

Hence, the supplier's optimization problem can be formulated as:

$$\begin{aligned} \Pi_S = \max_{q_R(a)} \quad & \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} w(a) q_R(a)\mathrm{d}a \\ = \quad & \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left(a - q_R(a) - \frac{\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}}{q_R(a)}\right) q_R(a)\mathrm{d}a \\ = \quad & \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left[2a - \bar{a} - q_R(a)\right] q_R(a)\mathrm{d}a. \end{aligned}$$

The last equality holds because of integration by parts. Denote $H(a) = [2a - \bar{a} - q_R(a)]\, q_R(a)$. We have:

$$\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = 2a - \bar{a} - 2q_R(a).$$

Solving $\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = 0$, we have the optimal reselling quantity: $q_R^E(a) = \left(a - \frac{\bar{a}}{2}\right)^+$. Plugging $q_R^E(a)$ into (B.13), we have the optimal wholesale price: $w^E(a) = a - q_R^E(a) - \frac{\int_{\underline{a}}^a q_R^E(\tilde{a})\mathrm{d}\tilde{a}}{q_R^E(a)}$.

Proof of Proposition 24. With encroachment, after observing the reseller's order quantity $q_R(\hat{a})$, the supplier will set her direct selling quantity at $q_S^*(\hat{a}) = \left(\frac{\hat{a}-q_R(\hat{a})-c}{2}\right)^+$. Thus, the reseller's profit function can be written as:

$$\Pi_R(\hat{a}|a) = [a - q_R(\hat{a}) - \left(\frac{\hat{a} - q_R(\hat{a}) - c}{2}\right)^+ - w(\hat{a})]q_R(\hat{a})$$

where a is the observed market size while $\hat{a}$ is the index of the contract the reseller chooses. Under the truth-telling equilibrium, we have:

$$\Pi_R(a|a) = [a - q_R(a) - \left(\frac{a - q_R(a) - c}{2}\right)^+ - w(a)]q_R(a).$$

As a result, we can derive the supplier's optimal wholesale price:

$$w(a) = a - q_R(a) - \left(\frac{a - q_R(a) - c}{2}\right)^+ - \frac{\int_{\underline{a}}^a q_R(\tilde{a})\mathrm{d}\tilde{a}}{q_R(a)}. \tag{B.14}$$

Further, we can formulate the supplier's optimization problem under the uniform distribution as:

$$\begin{aligned}
\Pi_S^E = \max_{q_R(a)} \quad & \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ w(a)q_R(a) + [a - q_R(a) - \left(\frac{a-q_R(a)-c}{2}\right)^+ - c]\left(\frac{a-q_R(a)-c}{2}\right)^+ \right\} \\
= \quad & \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ \frac{(a-c)^2 - [q_R(a)]^2 + 4cq_R(a)}{4} - (\bar{a}-a)q_R(a) \right\} \mathrm{d}a.
\end{aligned}$$

Denote $H(a) = \frac{(a-c)^2 - [q_R(a)]^2 + 4cq_R(a)}{4} - (\bar{a} - a)q_R(a)$, and we have

$$\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = \frac{2c - q_R(a)}{2} - (\bar{a} - a).$$

Solving $\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = 0$, we derive the optimal reselling quantity:

$$q_R^E(a) = 2(c + a - \bar{a})^+,$$

under which the supplier's equilibrium direct selling quantity is

$$q_S^E(a) = \left(\frac{a - q_R(a) - c}{2}\right)^+ = \left(\frac{2\bar{a} - a - 3c}{2}\right)^+.$$

Note that the above derivation assumes the reselling quantity is positive. In case the reselling quantity is zero at a demand realization a, the supplier's equilibrium direct selling quantity will be $q_S^E(a) = \left(\frac{a-c}{2}\right)^+$. On the other hand, the direct selling quantity can also be zero in equilibrium.

The regions with respect to c and a that correspond to the above cases with specific pairs of $q_R^E(a)$ and $q_S^E(a)$ can be easily obtained as presented in the proposition. Finally, plugging $q_R^*(a)$ into (B.14), we have the optimal wholesale price: $w^E(a) = a - q_R^E(a) - \left(\frac{a - q_R^E(a) - c}{2}\right)^+ - \frac{\int_{\underline{a}}^{a} q_R^E(\tilde{a})\mathrm{d}\tilde{a}}{q_R^E(a)}$.

Proof of Proposition 25. Below, we examine the effect of supplier encroachment on the reseller and the supplier, respectively. We restrict the proof to the case where $\underline{a} \geq \frac{2\bar{a}}{3}$.

Reseller Profit

We can derive, based on Propositions 23 and 24, the reseller's expected gain from supplier encroachment:

$$\Pi_R^E - \Pi_R^N = \int_{\underline{a}}^{\bar{a}} \frac{\bar{a} - a}{\bar{a} - \underline{a}} \left[q_R^E(a) - q_R^N(a)\right] \mathrm{d}a.$$

In particular, we can have four cases:

i) When $c \geq \frac{\bar{a}}{2}$, $q_R^E(a) = q_R^N(a)$ for any $a \in [\underline{a}, \bar{a}]$. Hence, $\Pi_R^E = \Pi_R^N$.

ii) When $c \in (\frac{2\bar{a}-\underline{a}}{3}, \frac{\bar{a}}{2})$, $q_R^E(a) - q_R^N(a) = (a-c)^+ - (a-\frac{\bar{a}}{2})^+$. It is straightforward to see that $q_R^E(a) - q_R^N(a) \geq 0$ for any a, and $q_R^E(a) - q_R^N(a) > 0$ for any $a > c$. Hence, $\Pi_R^E > \Pi_R^N$. We also know that Π_R^E is decreasing in c when $c \in (\frac{2\bar{a}-\underline{a}}{3}, \frac{\bar{a}}{2})$ given $q_R^E(a)$ is decreasing in c.

iii) When $c \in (\frac{\bar{a}}{3}, \frac{2\bar{a}-\underline{a}}{3}]$, we have

$$\begin{aligned}
\Pi_R^E &= \int_{\underline{a}}^{\bar{a}} \frac{\bar{a}-a}{\bar{a}-\underline{a}} q_R^E(a) \mathrm{d}a \\
&= \int_{\underline{a}}^{2\bar{a}-3c} \frac{\bar{a}-a}{\bar{a}-\underline{a}} \left[2(c+a-\bar{a})\right] \mathrm{d}a + \int_{2\bar{a}-3c}^{\bar{a}} \frac{\bar{a}-a}{\bar{a}-\underline{a}} (a-c) \mathrm{d}a \\
&= \frac{-3\bar{a}^3 - 3c\bar{a}^2 + 27c^2\bar{a} - 27c^3 + 4\underline{a}^3 - 12\bar{a}\underline{a}^2 + 6c\underline{a}^2 - 12c\bar{a}\underline{a} + 12\bar{a}^2\underline{a}}{6(\bar{a}-\underline{a})}.
\end{aligned}$$

Taking the first derivative, we obtain

$$\frac{\mathrm{d}\Pi_R^E}{\mathrm{d}c} = -\frac{3\bar{a}^2 - 54\bar{a}c + 81c^2 - 6\underline{a}^2 + 12\bar{a}\underline{a}}{6(\bar{a}-\underline{a})}.$$

We can observe that Π_R^E is increasing when $\frac{3\bar{a}-\sqrt{6}(\bar{a}-\underline{a})}{9} < c < \frac{3\bar{a}+\sqrt{6}(\bar{a}-\underline{a})}{9}$ and decreasing otherwise. Notice that $\frac{3\bar{a}-\sqrt{6}(\bar{a}-\underline{a})}{9} < \frac{\bar{a}}{3}$ and $\frac{3\bar{a}+\sqrt{6}(\bar{a}-\underline{a})}{9} < \frac{2\bar{a}-\underline{a}}{3}$. Hence, Π_R^E is increasing in c when $c \in \left(\frac{\bar{a}}{3}, \frac{3\bar{a}+\sqrt{6}(\bar{a}-\underline{a})}{9}\right)$ and decreasing in c when $c \in \left(\frac{3\bar{a}+\sqrt{6}(\bar{a}-\underline{a})}{9}, \frac{2\bar{a}-\underline{a}}{3}\right)$.

iv) When $c \in (0, \frac{\bar{a}}{3})$, Π_R^E is increasing in c. Moreover, $\Pi_R^E \to 0$ when $c \to 0$.

Summarizing from the above analysis for cases i) to iv), we can conclude that as c increases, Π_R^E is first increasing (from 0 to a value greater than Π_R^N), then decreasing, and finally stabilizes at Π_R^N. Therefore, there exists a threshold c_R' such that the reseller is worse off with encroachment when $c < c_R'$, better off when $c_R' < c < \frac{\bar{a}}{2}$, and indifferent when $c \geq \frac{\bar{a}}{2}$.

Supplier Profit

We can derive, based on Propositions 23 and 24**,** the supplier's expected gain from supplier encroachment. In particular, we can have the following five cases:

i) When $c \geq \frac{\bar{a}}{2}$, $q_R^E(a) = q_R^N(a)$ and $q_S^E(a) = 0$ for any $a \in [\underline{a}, \bar{a}]$. Hence, $\Pi_S^E = \Pi_S^N$.

ii) When $c \in (\frac{2\bar{a}-\underline{a}}{3}, \frac{\bar{a}}{2})$, the supplier's expected profit with encroachment is

$$\begin{aligned}
\Pi_S^E &= \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ \frac{(a-c)^2 - \left[q_R^E(a)\right]^2 + 4cq_R^E(a)}{4} - (\bar{a}-a)q_R^E(a) \right\} \mathrm{d}a. \\
&= \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ c(a-c) - (\bar{a}-a)(a-c) \right\} \mathrm{d}a \\
&= \frac{-\bar{a}^2 + 6c\bar{a} - \underline{a}\bar{a} + 2\underline{a}^2 - 6c^2}{6}.
\end{aligned}$$

Taking the first derivative, we have $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \bar{a} - 2c$, which implies that Π_S^E is increasing in c when $c \in (\frac{2\bar{a}-\underline{a}}{3}, \frac{\bar{a}}{2})$. Hence, given Π_S^E approaches Π_S^N at $c = \frac{\bar{a}}{2}$, we know the supplier is worse off by encroachment when $c \in (\frac{2\bar{a}-\underline{a}}{3}, \frac{\bar{a}}{2})$.

iii) When $c \in (\frac{\bar{a}}{3}, \frac{2\bar{a}-\underline{a}}{3}]$, the supplier's profit with encroachment is

$$\begin{aligned}
\Pi_S^E &= \int_{\underline{a}}^{2\bar{a}-3c} \frac{1}{\bar{a}-\underline{a}} \left\{ \frac{(a-c)^2 - [2(c+a-\bar{a})]^2 + 8c(c+a-\bar{a})}{4} - 2(\bar{a}-a)(c+a-\bar{a}) \right\} \mathrm{d}a \\
&\quad + \int_{2\bar{a}-3c}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ c(a-c) - (\bar{a}-a)(a-c) \right\} \mathrm{d}a \\
&= \frac{24\bar{a}\underline{a}c - 24c\bar{a}^2 - 9c\underline{a}^2 + 12\bar{a}\underline{a}^2 + 42\bar{a}c^2 - 15\underline{a}c^2 - 12\underline{a}\bar{a}^2 + 6\bar{a}^3 - 5\underline{a}^3 - 27c^3}{12(\bar{a}-\underline{a})}
\end{aligned}$$

Taking the first derivative, we have

$$\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \frac{-24\bar{a}^2 + 84\bar{a}c - 9\underline{a}^2 - 30\underline{a}c - 81c^2 + 24\bar{a}\underline{a}}{12(\bar{a}-\underline{a})}.$$

It can be shown that Π_S^E is increasing when $\frac{14\bar{a}-5\underline{a}-2\sqrt{(\bar{a}-\underline{a})(14\underline{a}-5\bar{a})}}{27} < c < \frac{14\bar{a}-5\underline{a}+2\sqrt{(\bar{a}-\underline{a})(14\underline{a}-5\bar{a})}}{27}$ and decreasing otherwise. It is also easy to check that $\frac{14\bar{a}-5\underline{a}+2\sqrt{(\bar{a}-\underline{a})(14\underline{a}-5\bar{a})}}{27} > \frac{2\bar{a}-\underline{a}}{3}$ and $\frac{14\bar{a}-5\underline{a}+2\sqrt{(\bar{a}-\underline{a})(14\underline{a}-5\bar{a})}}{27} < \frac{\bar{a}}{3}$ for any $\underline{a} \geq \frac{2\bar{a}}{3}$. Therefore, Π_S^E is increasing in c and $\Pi_S^E < \Pi_S^N$ when $c \in (\frac{\bar{a}}{3}, \frac{2\bar{a}-\underline{a}}{3}]$.

iv) When $c \in (\bar{a}-\underline{a}, \frac{\bar{a}}{3}]$, the supplier's expected profit with encroachment is:

$$\begin{aligned}\Pi_S^E &= \int_{\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ \frac{(a-c)^2 - [2(c+a-\bar{a})]^2 + 8c(c+a-\bar{a})}{4} - 2(\bar{a}-a)(c+a-\bar{a}) \right\} \mathrm{d}a \\ &= \frac{5\bar{a}^2 + 5\underline{a}^2 + 9c\underline{a} - 7\bar{a}\underline{a} + 15c^2 - 15c\bar{a}}{12}.\end{aligned}$$

Taking the first derivative, we have $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \frac{5c}{2} - \frac{5\bar{a}-3\underline{a}}{4}$. Hence, Π_S^E can be first decreasing and then increasing in this region.

v) When $c \in (0, \bar{a}-\underline{a}]$, the supplier's profit with encroachment is:

$$\begin{aligned}\Pi_S^E &= \int_{\underline{a}}^{\bar{a}-\underline{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ \frac{(a-c)^2}{4} \right\} \mathrm{d}a \\ &\quad + \int_{\bar{a}-\underline{a}}^{\bar{a}} \frac{1}{\bar{a}-\underline{a}} \left\{ \frac{(a-c)^2 - [2(c+a-\bar{a})]^2 + 8c(c+a-\bar{a})}{4} - 2(\bar{a}-c)(c+a-\bar{a}) \right\} \mathrm{d}a \\ &= \frac{\bar{a}^3 - 3\bar{a}^2c + 3\bar{a}c^2 - 3(\underline{a}^3 - 3\underline{a}^2c + 3\underline{a}c^2)}{12(\bar{a}-\underline{a})}\end{aligned}$$

The first derivative is: $\frac{\mathrm{d}\Pi_S^E}{\mathrm{d}c} = \frac{-3\bar{a}^2+6\bar{a}c+(-3\underline{a}^2+6\underline{a}c)}{12(\bar{a}-\underline{a})}$. It can be shown that Π_S^E is decreasing when $c < \frac{\bar{a}^2+3\underline{a}^2}{2(\bar{a}+3\underline{a})}$. Notice that $\frac{\bar{a}^2+3\underline{a}^2}{2(\bar{a}+3\underline{a})} > \bar{a} - \underline{a}$ when $\underline{a} \geq \frac{2\bar{a}}{3}$. Thus Π_S^E is decreasing in the whole region $(0, \bar{a} - \underline{a}]$. Combining the analysis for cases iv) and v), we assert that the supplier's expected profit with encroachment is first decreasing and then increasing in c when $c \in (0, \frac{\bar{a}}{3}]$. The maximum is achieved when $c \to 0$, and Π_S^E goes to $\frac{\mu^2+\sigma^2}{4}$ which is larger than Π_S^N.

Summarizing the analysis for the cases i) to v), we conclude that as c increases in the interval $(0, \frac{\bar{a}}{2}]$, Π_S^E is first decreasing, then increasing, and finally stabilizes at Π_S^N. Therefore, there exists a threshold c'_S such that the supplier is better off with encroachment when $c < c'_S$, worse off when $c'_S < c < \frac{\bar{a}}{2}$, and indifferent when $c \geq \frac{\bar{a}}{2}$.

B.3.3 Analysis with General Continuous Demand Distribution

In this supplement, we extend the analysis for a special case with uniform distribution of demand realization to the general case with a general continuous distribution $G(a)$ and density $g(a)$. We briefly discuss why it is technically challenging to come up with analytical results for a general distribution, especially when the supplier holds the option of encroachment.

Without the Option of Encroachment

When the reseller observes a realization a but chooses the contract for realization $\hat{a}$, the reseller's profit is

$$\Pi_R(\hat{a}|a) = [a - q_R(\hat{a}) - w(\hat{a})]q_R(\hat{a}).$$

Due to the revelation principle, we, without loss of generality, focus on truth-telling equilibrium where the reseller truthfully reports her observed demand. If the reseller selects the truth-telling contract (the incentive compatibility constraints hold), we have

$$\Pi_R(a|a) = [a - q_R(a) - w(a)]q_R(a) \tag{B.15}$$

To ensure that the reseller chooses the truth-telling contract, we must have $\frac{\partial \Pi_R(\hat{a}|a)}{\partial \hat{a}}|_{\hat{a}=a} = 0$, i.e., truth-telling is the reseller's best strategy. The implicit function theorem implies that

$$\frac{\mathrm{d}\Pi_R(a|a)}{\mathrm{d}a} = \left[\frac{\partial \Pi_R(\hat{a}|a)}{\partial a} + \frac{\partial \Pi_R(\hat{a}|a)}{\partial \hat{a}}\frac{\partial \hat{a}}{\partial a}\right]|_{\hat{a}=a} = \frac{\partial \Pi_R(\hat{a}|a)}{\partial a}|_{\hat{a}=a} = q_R(a).$$

Integrating the above equation, we have the local incentive-compatability constraint

$$\Pi_R(a|a) = \Pi_R(\underline{a}|\underline{a}) + \int_{\underline{a}}^{a} \frac{\mathrm{d}\Pi_R(\tilde{a}|\tilde{a})}{\mathrm{d}\tilde{a}} d\tilde{a} = \int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}, \tag{B.16}$$

because $\Pi_R(\underline{a}|\underline{a}) = 0$. Equation (B.16) suggests that for all $a \in [\underline{a}, \bar{a}]$, $\Pi_R(a|a) \geq 0$. Hence, we do not have to explicitly consider the individual rationality constraint in this mechanism design problem. The reseller's expected profit is

$$\begin{aligned}\Pi_R &= \int_{\underline{a}}^{\bar{a}} g(a)\left[\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}\right]\mathrm{d}a \\ &= \int_{\underline{a}}^{\bar{a}} [1 - G(a)]\, q_R(a)\mathrm{d}a.\end{aligned}$$

Comparing Equations (B.15) and (B.16), we have supplier's the optimal wholesale price as

$$w(a) = a - q_R(a) - \frac{\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}}{q_R(a)}. \tag{B.17}$$

Denote the supplier's profit when demand realization is a by $\Pi_S(a|a) = w(a)q_R(a)$. Plugging Equation (B.17) into the supplier's profit function, we obtain the supplier's maximization problem as

$$\begin{aligned}
\Pi_S = \max_{q_R(a)} \quad & \int_{\underline{a}}^{\bar{a}} g(a)w(a)q_R(a)\mathrm{d}a. \\
= & \int_{\underline{a}}^{\bar{a}} g(a) \left(a - q_R(a) - \frac{\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}}{q_R(a)} \right) q_R(a)\mathrm{d}a \\
= & \int_{\underline{a}}^{\bar{a}} g(a) \left(a - q_R(a)\right) q_R(a)\mathrm{d}a - \int_{\underline{a}}^{\bar{a}} g(a) \left[\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a} \right] \mathrm{d}a \\
= & \int_{\underline{a}}^{\bar{a}} g(a) \left\{ \left[a - q_R(a)\right] q_R(a) - \frac{\left[1 - G(a)\right] q_R(a)}{g(a)} \right\} \mathrm{d}a
\end{aligned}$$

The last equality holds because by using integration by parts, we have

$$\begin{aligned}
\Pi_R = \int_{\underline{a}}^{\bar{a}} g(a) \left[\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a} \right] \mathrm{d}a \quad &= \quad \left[G(a) \int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a} \right] |_{\underline{a}}^{\bar{a}} - \int_{\underline{a}}^{\bar{a}} G(a)q_R(a)\mathrm{d}a \\
&= \quad \int_{\underline{a}}^{\bar{a}} q_R(a)\mathrm{d}a - \int_{\underline{a}}^{\bar{a}} G(a)q_R(a)\mathrm{d}a \\
&= \quad \int_{\underline{a}}^{\bar{a}} \left[1 - G(a)\right] q_R(a)\mathrm{d}a
\end{aligned}$$

Denote $H(a) = \left[a - q_R(a)\right] q_R(a) - \frac{[1-G(a)]q_R(a)}{g(a)}$, we have

$$\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = a - 2q_R(a) - \frac{1 - G(a)}{g(a)}.$$

We assume that the inverse hazard rate $\frac{1-G(a)}{g(a)}$ is monotone non-increasing in a (notice that it is a standard assumption in the mechanism design literature that the hazard rate $\frac{g(a)}{1-G(a)}$ is monotone non-decreasing). In this case, $\frac{\mathrm{d}H}{\mathrm{d}q_R}(a)$ is monotone. Solving $\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = 0$, we have the optimal wholesale quantity

$$q_R^N(a) = \frac{a}{2} - \frac{1 - G(a)}{2g(a)}.$$

Notice that $q_R^N(a)$ is increasing in a. Plugging this wholesale quantity into Equation (B.17), we have the optimal wholesale price as

$$w^N(a) = a - q_R^N(a) - \frac{\int_{\underline{a}}^{a} q_R^N(\tilde{a})\mathrm{d}\tilde{a}}{q_R^N(a)}.$$

With the Option of Encroachment

When the supplier holds the option of encroachment, the analysis of the contract design problem is similar. We only need to add the supplier's direct selling quantity into relevant equations. After observing the reseller's order quantity $q_R(\hat{a})$, the supplier's direct selling quantity follows Equation(3.1). The reseller's profit is

$$\Pi_R(\hat{a}|a) = [a - q_R(\hat{a}) - \left(\frac{\hat{a} - q_R(\hat{a}) - c}{2}\right)^+ - w(\hat{a})]q_R(\hat{a}).$$

If the reseller chooses the truth-telling contract (IC constraints hold), we have

$$\Pi_R(a|a) = [a - q_R(a) - \left(\frac{a - q_R(a) - c}{2}\right)^+ - w(a)]q_R(a).$$

Similar to the derivation of Equation (B.13), the supplier's optimal wholesale price follows

$$w(a) = a - q_R(a) - \left(\frac{a - q_R(a) - c}{2}\right)^+ - \frac{\int_{\underline{a}}^{a} q_R(\tilde{a})\mathrm{d}\tilde{a}}{q_R(a)}. \tag{B.18}$$

When demand realization is a, the supplier's profit function under truth-telling is

$$\Pi_S(a|a) \quad = \quad w(a)q_R(a) + [a - q_R(a) - \left(\frac{a - q_R(a) - c}{2}\right)^+ - c]\left(\frac{a - q_R(a) - c}{2}\right)^+.$$

Assume both firms sell positive quantities in any demand realization. The supplier's optimization problem is

$$\begin{aligned}
\Pi_S^E = \max_{q_R(a)} \quad & \int_{\underline{a}}^{\bar{a}} g(a) \left\{ w(a) q_R(a) + [a - q_R(a) - \left(\frac{a - q_R(a) - c}{2} \right)^+ - c] \left(\frac{a - q_R(a) - c}{2} \right)^+ \right\} \\
= & \int_{\underline{a}}^{\bar{a}} g(a) \left\{ \frac{a - q_R(a) - c}{2} \times \frac{a + q_R(a) - c}{2} + c q_R(a) - \frac{[1 - G(a)]\, q_R(a)}{g(a)} \right\} \mathrm{d}a \\
= & \int_{\underline{a}}^{\bar{a}} g(a) \left\{ \frac{(a-c)^2 - [q_R(a)]^2 + 4 c q_R(a)}{4} - \frac{[1 - G(a)]\, q_R(a)}{g(a)} \right\} \mathrm{d}a.
\end{aligned}$$

Denote $H(a) = \frac{(a-c)^2 - [q_R(a)]^2 + 4cq_R(a)}{4} - \frac{[1-G(a)]q_R(a)}{g(a)}$, and we have

$$\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = \frac{2c - q_R(a)}{2} - \frac{[1 - G(a)]}{g(a)}.$$

Solving $\frac{\mathrm{d}H}{\mathrm{d}q_R}(a) = 0$, we have the optimal wholesale quantity

$$q_R^E(a) = \left(2c - \frac{2\,[1 - G(a)]}{g(a)} \right)^+.$$

For $q_R^E(a)$ to be positive, we need $c > \frac{1-G(a)}{g(a)}$. The supplier's direct selling quantity is

$$q_S^E(a) = \left(\frac{a - q_R^E(a) - c}{2} \right)^+.$$

From Equation (B.18), we have $w^E(a) = a - q_R^E(a) - \left(\frac{a - q_R^E(a) - c}{2}\right)^+ - \frac{\int_{\underline{a}}^{a} q_R^E(\tilde{a}) \mathrm{d}\tilde{a}}{q_R^E(a)}$.

For $q_S^E(a) > 0$, i.e., $q_R^E(a) < a - c$, we need $c < \bar{c}(a) = \frac{a}{3} + \frac{2[1-G(a)]}{3g(a)}$. Notice that $\bar{c}(a)$ may not be monotone even we assume that the inverse hazard rate $\frac{1-G(a)}{g(a)}$ is monotone non-increasing in a. Hence, the supplier's direct selling quantity may go to zero either when the market size is small or when the market size is large. This non-monotonicity causes technical difficulties in characterizing the wholesale quantity and the supplier's direct selling quantity when either of them goes to zero. Moreover, it is technically challenging to analytically derive the supplier and the reseller's profits for a general distribution function. There do not exist simple expressions for the integrals under a general continuous distribution $G(a)$.

Appendix C

Appendix for Chapter 4

C.1 Estimation of the Random Coefficients Model

C.1.1 Calculation of Market Shares

Consumers are assumed to purchase one unit of the product that gives the highest utility. Since in this model an individual is defined as a vector of individual, time, and product-specific shocks, $(v_i, \varepsilon_{1t}, \ldots, \varepsilon_{Jt}, \epsilon_{i0t}, \ldots, \epsilon_{iJt})$, this implicitly defines the set of individual attributes that lead to the choice of product j. Formally, let A_{jt} defines the individuals who choose product j in period t, then

$$A_{jt}(\delta_{.t}; \beta_\nu) = \{(\nu_i, \varepsilon_{1t}, \ldots, \varepsilon_{Jt}, \epsilon_{i0t}, \ldots, \epsilon_{iJt}) | U_{ijt} \geq U_{ikt}, \forall k = 0, 1, \ldots, J\} \qquad \text{(C.1)}$$

where $\delta_{.t} = (\delta_{1t}, \ldots, \delta_{Jt})'$ are mean utilities of all products, respectively. Assuming ties occur with zero probability, the probability of the jth product is just an integral over the mass of consumers in the region A_{jt}. Hence, we can calculate the market share for product j as the probability of this product being chosen. Formally, it is given by

$$
\begin{aligned}
s_{jt}(\delta_{.t};\beta_v) &= \int_{A_{jt}} \mathrm{d}P(v,\epsilon) \\
&= \int_{A_{jt}} \mathrm{d}P(v)\mathrm{d}P(\epsilon) \\
&= \int \frac{\exp(\delta_{jt} + y_{j(t-1)}\beta_v v_i)}{1+\sum_{k=1}^{J}\exp(\delta_{kt} + y_{k(t-1)}\beta_v v_i)} \mathrm{d}P(v) \qquad \text{(C.2)}
\end{aligned}
$$

where $P(\cdot)$ is the distribution function. The second-last equality holds as we assume individual-specific coefficients v and time-individual-product specific idiosyncratic shocks ϵ are independent, while the last equality holds from the assumption that ϵ follow a Type I extreme value distribution. Market shares given in Equation (C.2) do not have a closed-form expression. We use a Monte Carlo simulation to approximate it. Recall that v_i follows a multivariate normal distribution $v_i \sim N(0, I_K)$, we can obtain an unbiased estimator of this integral by taking n random draws of v_i (each v_i corresponds to an individual consumer) and compute the average choice probability from these n consumers as

$$
s_{jt}(\delta_{.t};\beta_v) = \sum_{i=1}^{n} s_{ijt} = \sum_{i=1}^{n} \frac{\exp(\delta_{jt} + y_{j(t-1)}\beta_v v_i)}{1+\sum_{k=1}^{J}\exp(\delta_{kt} + y_{k(t-1)}\beta_v v_i)} \qquad \text{(C.3)}
$$

C.1.2 Estimation Procedure

The estimation involves two nested loops. In the outer loop, the parameters corresponding to the individual heterogeneity distribution are heuristically learned, whereas the inner loop involves computing the unknown parameters embedded in the mean utility. More specifically, we ran the estimation algorithm as follows.

1. Generate 1000 i.i.d. draws of v_i from the standard normal distribution.

2. Assign starting values to β_v^0. Also, initiate values for δ^0.

3. Compute market share for product j according to Equation (C.3) using Monte Carlo simulation.

4. The inner loop computation takes place based on a contraction-mapping procedure. Fixing the nonlinear parameters β_ν at their current values, iterate over the values of the mean utility δ to minimize the distance between the predicted market share and the observed market share.

5. Given the δ obtained from last step, extract the time-product specific unobserved characteristic ε from the linear equation as

$$\varepsilon = \delta - y\bar{\beta} + I\gamma + \alpha + \tau. \tag{C.4}$$

If instrument variables are not used, $\bar{\beta}$ in the equation above is the least squares estimator. If instruments are used, $\bar{\beta}$ is the GMM estimator.

6. Form an objective function. If instrument variables are not used, the objective function is the sum of squared errors, i.e., $f = \varepsilon'\varepsilon$. If instruments are used, we form a GMM-type objective function by interacting the unobserved characteristic ε with the set of instruments Z, i.e.,

$$f = \varepsilon' Z W^{-1} Z' \varepsilon, \tag{C.5}$$

where $W = E[Z'Z]$ is the GMM weighting matrix.

7. The inner loop computation takes place again. Use quasi-Newton algorithm to update the parameter values for β_v. Iterate from Step 3 until the algorithm finds the optimal combination of β_v and δ that minimizes the objective function.

C.2 Instrument Variables and GMM Estimation

We address the endogeneity issue related to integration timing by using Facebook's stock price as an instrument variable. Correlation of the endogeneity issue related to application user bases using valid instruments is detailed below. All instruments are applied in Step 5 of the estimation procedure in Appendix C.1.2. We turn to the GMM style estimators of dynamic panel data models that exploit the lags and lagged differences of explanatory variables as instruments. Although these GMM estimators are often used in linear models, we show that it is straightforward to apply them to our nonlinear model as well. We apply the instruments to the mean utility function, in Step 4 of the estimation procedure detailed in Appendix C.1.2. Essentially, the dependent variable is the mean utility δ and the error terms are time-product specific unobserved characteristic ε. Denoting the interaction terms by X_{jt}, we can rewrite Equation (C.4) as a standard panel model as follows

$$\delta_{jt} = y_{j(t-1)})\bar{\beta} + X_{jt}\gamma + \alpha_j + \varepsilon_{jt}. \tag{C.6}$$

The dynamic panel model literature has documented how to estimate panel model with fixed effects. We follow Nickell (1981) and Blundell and Bond (1998) and come up with two types of instruments below. As description in Appendix C.1, there are only minor changes to Step 5 and Step 6 of the estimation algorithm when instrument variables are used. Basically, we form a GMM objective function based on different moment conditions.

C.2.1 Moment Conditions for First-Differenced Equations

The first set of moment conditions apply instruments to first-differences of Equation (C.6). Denote the first-difference operator by Δ, the first-differenced equation is

$$\Delta\delta_{jt} = \Delta y_{j(t-1)}\bar{\beta} + \Delta X_{jt}\gamma + \Delta\varepsilon_{jt}, \tag{C.7}$$

where $\Delta\delta_{jt} = \Delta\delta_{jt} - \Delta\delta_{j(t-1)}$, $\Delta y_{j(t-1)} = y_{j(t-1)} - y_{j(t-1)}$, $\Delta X_{jt} = X_{jt} - X_{j(t-1)}$, and $\Delta\varepsilon_{jt} = \varepsilon_{jt} - \varepsilon_{j(t-1)}$. Notice that first differencing has eliminated fixed effect and thus the correlation between the explanatory variables and α_j is no longer an issue. However, by first differencing we have introduced another kind of bias. Now the error term $\Delta\varepsilon_{jt}$ is correlated with the explanatory variable $\Delta y_{j(t-1)}$. However, we can show that y_{jp}, $\forall p \leq t-2$ are valid instruments as they are not correlated with $\Delta\varepsilon_{jt}$, but correlated with $\Delta y_{j(t-1)}$. We therefore formulate the first set of moment conditions as follows

$$E[y_{jp}\Delta\varepsilon_{jt}] = 0,\ \forall p \leq t-2. \tag{C.8}$$

C.2.2 Moment Conditions for Level Equations

Consider the level equation (C.6), the fixed effect α_j is correlated with $y_{j(t-1)}$ because the fixed effect influences consumer utility and aggregate demand in each period. Notice that $E[y_{jt}\varepsilon_{js}] = 0$ if $s > t$ and $E[y_{jt}\varepsilon_{js}] \neq 0$ if $s \leq t$. Thus, we have that y_{jp} is uncorrelated with ε_{jt} for all $p \leq t-1$. By extension, this implies that Δy_{jp} is uncorrelated with ε_{jt} for all $p \leq t-1$. As y_{jp} is linearly correlated with α_j, Δy_{jp} is uncorrelated with α_j.Therefore, Δy_{jp} is uncorrelated with both α_j and ε_{jt}, for all $p \leq t-1$. Therefore, we have the second set of moment conditions as follows

$$E[\Delta y_{jp}\varepsilon_{jt}] = 0,\ \forall p \leq t-1.$$

C.3 Estimation with Switching Costs

C.3.1 Calculation of Market Shares

With switching costs, an individual's choices also depend on her choice in last period. An individual i is defined as a vector of individual, time, product-specific shocks, and her last period usage $d_{i(t-1)} = d$. A full vector of consumer characteristics consists of $(d_{i(t-1)}, \varphi_i, \nu_i, \varepsilon_{1t}, \ldots, \varepsilon_{Jt}, \epsilon_{i0t}, \ldots, \epsilon_{iJt})$, which implicitly defines the set of individual attributes that lead to the choice of product j. Formally, let A_jt defines the individuals who choose product j in period t, then

$$A_{jt}(d, \delta_{.t}; \beta_\nu) = \left\{(d_{i(t-1)}, \varphi_i, \nu_i, \varepsilon_{1t}, \ldots, \varepsilon_{Jt}, \epsilon_{i0t}, \ldots, \epsilon_{iJt}) | U_{ijt} \geq U_{ikt}, \forall k = 0, 1, \ldots, J\right\} \tag{C.9}$$

where $\delta_{.t} = (\delta_{1t}, \ldots, \delta_{Jt})'$ are mean utilities of all products. Assuming ties occur with zero probability, the average probability of the jth product being chosen by those consumers who chose d in last period is just an integral over the mass of consumers in the region A_{jt}. Hence, we can calculate this average probability as

$$\begin{aligned} s_{jt}(d, \delta_{.t}; \beta_v, \bar{c}) &= \int_{A_{jt}} \mathrm{d}P(\varphi, v, \epsilon) \\ &= \int_{A_{jt}} \mathrm{d}P(\varphi)\mathrm{d}P(v)\mathrm{d}P(\epsilon) \\ &= \iint \frac{\exp\left(\delta_{jt} + y_{j(t-1)}\beta_v v_i - (\bar{c} + \varphi_i)\mathbf{1}\left\{d_{i(t-1)} \notin \{0, j\}\right\}\right)}{1 + \sum_{k=1}^{J} \exp\left(\delta_{kt} + y_{k(t-1)}\beta_v v_i - (\bar{c} + \varphi_i)\mathbf{1}\left\{d_{i(t-1)} \notin \{0, k\}\right\}\right)} \mathrm{d}P(\varphi)\mathrm{d}P(v) \end{aligned} \tag{C.10}$$

where $P(\cdot)$ is the distribution function of consumer heterogeneity. The second-last equality holds as we assume individual-specific coefficients and time-individual-product specific idiosyncratic shocks are independent, while the last equality holds from the assumption that ϵ follow a Type I extreme value distribution. Market shares given in Equation (C.10) do not have a closed-form expression. We use a Monte Carlo simulation to approximate it. We can obtain an unbiased estimator of this integral by taking n independent draws of v_i and φ_i and compute the average choice probability from these n consumers as

$$s_{jt}(d,\delta_{.t};\beta_v,\bar{c}) = \sum_{i=1}^{n} \frac{\exp\left(\delta_{jt} + y_{j(t-1)}\beta_v v_i - (\bar{c}+\varphi_i)\mathbf{1}\left\{d_{i(t-1)} \notin \{0,j\}\right\}\right)}{1+\sum_{k=1}^{J}\exp\left(\delta_{kt} + y_{k(t-1)}\beta_v v_i - (\bar{c}+\varphi_i)\mathbf{1}\left\{d_{i(t-1)} \notin \{0,k\}\right\}\right)} \tag{C.11}$$

Let M_t denote the market size, i.e., the total number of consumers (including outside good users) in period t. Let N_t denote the number of new consumers arriving in the market in period t. The market size in period t is

$$M_t = M_{t-1} + N_t. \tag{C.12}$$

For expositional simplify, denote $s_{jt}(d) = s_{jt}(d,\delta_{.t};\beta_\nu,\bar{c})$. Denote by Q_{jt} the total number of consumers who chose j in period t, then

$$Q_{jt} = \sum_{k=0}^{J} Q_{k(t-1)} s_{jt}(d=k) + N_t s_{jt}(d=0), \tag{C.13}$$

where $Q_{k(t-1)}$ is the number of consumers who chose option k in the previous period $t-1$. The unconditional market share for product choose j in period t is simply

$$s_{jt} = \frac{Q_{jt}}{M_t} \tag{C.14}$$

Estimation Procedure

The estimation procedure is similar to the estimation of the base mode in Appendix C.1.2, with some very minor modifications. More specifically, we ran the estimation algorithm as follows.

1. Generate 1000 i.i.d. draws of v_i and 1000 i.i.d. draws of φ_i from a standard normal distribution.

2. Assign starting values to $(\beta_v^0, \bar{c}^0)$. Also, initiate values for δ^0.

3. Compute market share for product j according to Equation (C.14) using Monte Carlo simulation.

4. The inner loop computation takes place based on a contraction-mapping procedure. Fixing the nonlinear parameters $(\beta_v, \bar{c})$ at their current values, iterate over the values of the mean utility δ to minimize the distance between the predicted market share and the observed market share.

5. The same as in Appendix C.1

6. The same as in Appendix C.1

7. The inner loop computation takes place again. Use quasi-Newton algorithm to update the parameter values for $(\beta_v, \bar{c})$. Iterate from Step 3 until the algorithm finds the optimal combination of β_v, $\bar{c}$, and δ that minimizes the objective function.

Bibliography

Acemoglu, D., J. A. Robinson. 2001. A theory of political transitions. *American Economic Review* **91**(4) 938–963.

Anand, K. S., M. Goyal. 2009. Strategic information management under leakage in a supply chain. *Management Science* **55**(3) 438–452.

Anderson, T. W., C. Hsiao. 1982. Formulation and estimation of dynamic models using panel data. *Journal of Econometrics* **18**(1) 47–82.

Arellano, M., O. Bover. 1995. Another look at the instrumental variable estimation of error-components models. *Journal of Econometrics* **68**(1) 29–51.

Arya, A., B. Mittendorf, D. E. M. Sappington. 2007. The bright side of supplier encroachment. *Marketing Science* **26**(5) 651–659.

Baker, M., J. C. Stein, J. Wurgler. 2003. When does the market matter? stock prices and the investment of equity-dependent firms. *Quarterly Journal of Economics* **118**(3) 969–1005.

Baldwin, C. Y., K. B. Clark. 2000. *Design rules: The power of modularity*, vol. 1. MIT Press.

Bass, F. M. 1969. A new product growth for model consumer durables the bass model. *Management Science* **15**(5) 215–227.

Berry, S., J. Levinsohn, A. Pakes. 1995. Automobile prices in market equilibrium. *Econometrica* **63**(4) 841–90.

Blank, C. 2013. US seafood companies boost online sales. *SeafoodSource.* Accessed August 2013, http://www.seafoodsource.com/newsarticledetail.aspx?id=21474.

Blundell, R., S. Bond. 1998. Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics* **87**(1) 115–143.

Boudreau, K. J. 2007. Does opening a platform stimulate innovation? the effect on systemic and modular innovations. *MIT Sloan Research Paper* .

Brynjolfsson, E., C. F. Kemerer. 1996. Network externalities in microcomputer software: An econometric analysis of the spreadsheet market. *Management Science* **42**(12) 1627–1647.

Cachon, G. P. 2003. Supply chain coordination with contracts. eds. G. Graves, T. de Kok, ed., *Handbooks in Operations Research and Management Science: Supply Chain Management*. North Holland, Amsterdam, 229–340.

Cachon, G. P., M. A. Lariviere. 2001. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Science* **47**(5) 629–646.

Cattani, K., W. Gilland, H. S. Heese, J. Swaminathan. 2006. Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. *Production and Operations Management* **15**(1) 40–56.

Chen, Q., I. Goldstein, W. Jiang. 2007. Price informativeness and investment sensitivity to stock price. *Review of Financial Studies* **20**(3) 619–650.

Chiang, W. K., D. Chhajed, J. D. Hess. 2003. Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science* **49**(1) 1–20.

Chipty, T. 2001. Vertical integration, market foreclosure, and consumer welfare in the cable television industry. *American Economic Review* **91**(3) 428–453.

Cho, I. K., D. M. Kreps. 1987. Signaling games and stable equilibria. *Quarterly Journal of Economics* **102**(2) 179–221.

Clark, C., U. Doraszelski, M. Draganska. 2009. The effect of advertising on brand awareness and perceived quality: An empirical investigation using panel data. *Quantitative Marketing and Economics* **7**(2) 207–236.

Cooper, R. G. 2000. *Product leadership: creating and launching superior new products*. New York: Perseus Publishing.

Danaher, B., Y. Huang, M. D. Smith, R. Telang. 2014. An empirical analysis of digital music bundling strategies. *Management Science* **60**(6) 1413–1433.

Davis, F. D. 1989. Perceived usefulness, perceived ease of use, and user acceptance of information technology. *MIS Quarterly* **13**(3) 319–340.

Dhebar, A. 1995. Complementarity, compatibility, and product change: breaking with the past? *Journal of Product Innovation Management* **12**(2) 136–152.

Durlauf, S. N., P. A. Johnson, J. R. Temple. 2005. Growth econometrics. *Handbook of Economic Growth* **1** 555–677.

Eisenmann, T., G. Parker, M. Van Alstyne. 2011. Platform envelopment. *Strategic Management Journal* **32**(12) 1270–1285.

Evans, D. S., A. Hagiu, R. Schmalensee. 2006. *Invisible engines: how software platforms drive innovation and transform industries*. Cambridge,MA:MIT Press.

Farrell, J., M. L. Katz. 2000. Innovation, rent extraction, and integration in systems markets. *Journal of Industrial Economics* **48**(4) 413–432.

Fein, A. J., E. Anderson. 1997. Patterns of credible commitments: territory and brand selectivity in industrial distribution channels. *Journal of Marketing* **60** 19–34.

Frazier, G. L., W. M. Lassar. 1996. Determinants of distribution intensity. *Journal of Marketing* **61**(2) 39–51.

Fuentelsaz, L., J. P. Maicas, Y. Polo. 2012. Switching costs, network effects, and competition in the european mobile telecommunications industry. *Information Systems Research* **23**(1) 93–108.

Gallaugher, J. M., Y.-M. Wang. 2002. Understanding network effects in software markets: evidence from web server pricing. *MIS Quarterly* **26**(4) 303–327.

Gawer, A., M. A. Cusumano. 2002. *Platform leadership: How Intel, Microsoft, and Cisco drive industry innovation*. Boston,MA:Harvard Business School.

Gawer, A., R. Henderson. 2007. Platform owner entry and innovation in complementary markets: Evidence from intel. *Journal of Economics & Management Strategy* **16**(1) 1–34.

Ghose, A., S. P. Han. 2014. Estimating demand for mobile applications in the new economy. *Management Science* **60**(6) 1470–1488.

Ghose, A., P. G. Ipeirotis, B. Li. 2012. Designing ranking systems for hotels on travel search engines by mining user-generated and crowdsourced content. *Marketing Science* **31**(3) 493–520.

Gowrisankaran, G., M. Rysman. 2012. Dynamics of consumer demand for new durable goods. *Journal of Political Economy* **120**(6) 1173–1219.

Guo, L., G. Iyer. 2010. Information acquisition and sharing in a vertical relationship. *Marketing Science* **29**(3) 483–506.

Guo, L., T. Li, H. Zhang. 2011. Strategic information sharing in competing supply chains. *Working paper, Hong Kong University of Science and Technology, Hong Kong* .

Ha, A. Y. 2001. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Research Logistics* **48**(1) 41–64.

Ha, A. Y., S. Tong. 2008. Contracting and information sharing under supply chain competition. *Management science* **54**(4) 701–715.

Ha, A. Y., S. Tong, H. Zhang. 2011. Sharing demand information in competing supply chains with production diseconomies. *Management Science* **57**(3) 566–581.

Hagiu, A., D. Spulber. 2013. First-party content and coordination in two-sided markets. *Management Science* **59**(4) 933–949.

Hahn, J., G. Kuersteiner. 2002. Asymptotically unbiased inference for a dynamic panel model with fixed effects when both n and t are large. *Econometrica* **70**(4) 1639–1657.

Huang, P., M. Ceccagnoli, C. Forman, D. Wu. 2013. Appropriability mechanisms and the platform partnership decision: evidence from enterprise software. *Management Science* **59**(1) 102–121.

Jeuland, A. P., S. M. Shugan. 1983. Managing channel profits. *Marketing Science* **2**(3) 239–272.

Jiang, B., K. Jerath, K. Srinivasan. 2011. Firm strategies in the "mid tail" of platform-based retailing. *Marketing Science* **30**(5) 757–775.

Katz, M. L., C. Shapiro. 1986. Technology adoption in the presence of network externalities. *Journal of Political Economy* **94**(4) 822–41.

Katz, M. L., C. Shapiro. 1992. Product introduction with network externalities. *Journal of Industrial Economics* **40**(1) 55–83.

Kauffman, R. J., J. McAndrews, Y.-M. Wang. 2000. Opening the "black box" of network externalities in network adoption. *Information Systems Research* **11**(1) 61–82.

Kong, G., H. Zhang, S. Rajagopalan. 2013. Revenue sharing and information leakage in a supply chain. *Management Science* **59**(3) 556–572.

Lariviere, M. A., E. L. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management* **3**(4) 293–305.

Lee, R. S. 2013. Vertical integration and exclusivity in platform and two-sided markets. *American Economic Review* **103**(7) 2960–3000.

Li, L. 2002. Information sharing in a supply chain with horizontal competition. *Management Science* **48**(9) 1196–1212.

Li, L., H. Zhang. 2002. Supply chain information sharing in a competitive environment. eds. J. S. Song, D. D. Yao, ed., *Supply Chain Structures:Coordination, Information and Optimization*. Kluwer Academic Publishers, Norwell, MA, 161–201.

Li, Z., S. M. Gilbert, G. Lai. 2014. Supplier encroachment under asymmetric information. *Management Science* **60**(2) 449–462.

Li, Z., S. M. Gilbert, G. Lai. 2015. Supplier encroachment as an enhancement or a hindrence to nonlinear pricing. *Production and Operations Management* **24**(1) 89–109.

Li, Z., A. Agarwal. 2014. The impact of platform integration on consumer demand in complementary markets: Evidence from facebook's integration of instagram. *Working Paper, University of Texas at Austin* .

Mahajan, V., E. Müller, F. M. Bass. 1990. New product diffusion models in marketing: A review and directions for research. *Journal of Marketing* **54** 1–26.

McGuire, T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(2) 161–191.

Moorthy, K. S. 1984. Market segmentation, self-selection, and product line design. *Marketing Science* **3**(4) 288–307.

Nair, G., D. Pleasance. 2005. Mitigating channel conflict. *McKinsey Quarterly* **3** 16–17.

Nambisan, S. 2002. Complementary product integration by high-technology new ventures: The role of initial technology strategy. *Management Science* **48**(3) 382–398.

Nevo, A. 2001. Measuring market power in the ready-to-eat cereal industry. *Econometrica* **69**(2) 307–342.

Nickell, S. J. 1981. Biases in dynamic models with fixed effects. *Econometrica* **49**(6) 1417–26.

Reisinger, B. 2012. Middle Tennessee manufacturers boost online sales to bypass distributors, retailers. *Nashville Business Journal.* Accessed August 2013, http://www.bizjournals.com/nashville/print–edition/2012/12/07/middle–tennessee–manufacturers–boost.html.

Song, M. 2015. A hybrid discrete choice model of differentiated product demand with an application to personal computers. *International Economic Review* **56**(1) 265–301.

Spence, A. M. 1977. Nonlinear prices and welfare. *Journal of Public Economics* **8**(1) 1–18.

Taylor, T. A., W. Xiao. 2010. Does a manufacturer benefit from selling to a better-forecasting retailer? *Management Science* **56**(9) 1584–1598.

Tiwana, A., B. Konsynski, A. A. Bush. 2010. Research commentary-platform evolution: Coevolution of platform architecture, governance, and environmental dynamics. *Information Systems Research* **21**(4) 675–687.

Tsay, A. A., N. Agrawal. 2004. Channel conflict and coordination in the e-commerce age. *Production and Operations Management* **13**(1) 93–110.

Wooldridge, J. M. 2010. *Econometric analysis of cross section and panel data*. Boston,MA:MIT Press.

Wu, G. A. 2012. The effect of going public on innovative productivity and exploratory search. *Organization Science* **23**(4) 928–950.

Xue, M., L. M. Hitt, P.-y. Chen. 2011. Determinants and outcomes of internet banking adoption. *Management Science* **57**(2) 291–307.

Yoganarasimhan, H. 2012. Impact of social network structure on content propagation: A study using youtube data. *Quantitative Marketing and Economics* **10**(1) 111–150.

Zhang, H. 2002. Vertical information exchange in a supply chain with duopoly retailers. *Production and Operations Management* **11**(4) 531–546.

Zhu, K., K. L. Kraemer, V. Gurbaxani, S. X. Xu. 2006. Migration to open-standard interorganizational systems: network effects, switching costs, and path dependency. *MIS Quarterly* **30**(1) 515–539.