**The Thesis Committee for Shannah Kathryn Hoffman**
**Certifies that this is the approved version of the following thesis:**

# Instruction for Discovery Learning: Levels of Implementation Exhibited by a Sample of Algebra I Teachers

**APPROVED BY**
**SUPERVISING COMMITTEE:**

**Supervisor:** ______________________
Anthony J. Petrosino

**Co-Supervisor:** ______________________
Mark L. Daniels

# Instruction for Discovery Learning: Levels of Implementation Exhibited by a Sample of Algebra I Teachers

**by**

**Shannah Kathryn Hoffman, B.S.**

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# Abstract

## Instruction for Discovery Learning: Levels of Implementation Exhibited by a Sample of Algebra I Teachers



Shannah Kathryn Hoffman, M.A.

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Supervisors: Anthony J. Petrosino and Mark L. Daniels



One type of instruction that is of particular interest in STEM education is instruction that actively engages students in inquiry and discovery. The author develops an operational definition of instruction for discovery learning (IDL) that adopts some of the fundamental commonalities among many reform-oriented instructional frameworks such as inquiry-based and project-based instruction. Four teachers—who received their bachelor's degree in mathematics and teacher certification from the same undergraduate teacher-preparation program—and their Algebra I classes were observed with the focus on how particular features of IDL were being implemented in their classrooms. To gain further perspective on classroom practices and interactions, student surveys were administered to a total of 142 students and each teacher was interviewed. The student surveys focused on student orientations toward IDL, attitudes toward mathematics, and their perspective of IDL implementation in their class. Student survey data was analyzed through ANOVA, post hoc tests were used to identify significant pair-wise differences between teachers for which the ANOVA identified significance, and a factor analysis was used to evaluate the component loadings for the survey questions. The surveys revealed significant differences between perceived activities in the classes ($p<0.05$), but did not show very significant differences between student orientations toward IDL. All four teachers expressed familiarity with and commitment to reform-oriented frameworks such as inquiry-based and project-based instruction, and certainly experienced inquiry-based learning as students themselves in their undergraduate program. However, only *one* teacher—the one teaching in a New Tech high school that was structured on the framework of project-based instruction (PBI)—showed consistent differences in both student perspectives of IDL and observed implementation of IDL. The author discusses the levels at which these teachers implemented IDL, the differences among student perceptions across the classes, teacher orientations toward mathematics and learning, and the importance of a supportive school culture and administration in order to fully implement IDL and influence both student and teacher orientations toward reform-oriented pedagogy.

**Keywords**: discovery learning; inquiry-based learning; project-based instruction; teacher orientation; mathematics education; student surveys; classroom observation; school culture

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# Chapter 1: Introduction

Efforts to reform individual components of the American education system have come and gone over the past several decades, all of which stake a claim in how some feature of education can be enhanced. STEM fields have often been targeted by the more rigorous of these efforts (Kliebard, 2004). Mathematics, in particular, is often the focus of many overhauls and changes (Kliebard, 2004). As a result of all of these efforts to change particular aspects of mathematics education, some common themes emerge: (1) attention to rigor in terms of conceptual depth and procedural knowledge and (2) attention to student engagement by using more innovative instructional methods (Krajcik, McNeill, & Reiser, 2008). Many educators have interpreted the first of these themes as a need for standardization of curriculum and assessment, and have occasionally disregarded the second theme. This is likely due to the difficulty in implementing innovative methods that focus on genuine student engagement. This difficulty stems from a large range of factors including the school environment, the teacher's experience learning about innovative methods, support from professional developers and school administrators, teacher beliefs about the content and student learning, and student perspectives of innovative methods. In this paper, the author reports the various levels at which a sample of teachers incorporated more innovative methods into their classrooms.

In particular, the author developed an operational definition for instruction for discovery learning (IDL) from the ideas and research findings of several different mathematics education experts. The purpose of developing this definition was to provide flexibility across different classrooms in which teachers were enacting various formalizations of discovery learning (e.g. inquiry-based instruction and project-based instruction). Because the individual practices associated with these various pedagogical frameworks are quite different, this definition was intended to only capture the similarities at the core of these frameworks.

All four teachers involved in this study were certified through the same nationally-recognized university program, that emphasized innovative, reform-oriented teaching practices in mathematics (and science) classrooms. However, they naturally had very divergent characteristics and beliefs that greatly influenced their teaching practice. In order to develop a more thorough understanding of how these features of IDL were being implemented in these classrooms, the author utilized three different data sources: (1) classroom observations, (2) student surveys, and (3) post-observation interviews with each teacher. The results of the study were analyzed both qualitatively and quantitatively. Student survey responses were coded numerically (based on the Likert scales used for the different questions) and were analyzed through an analysis of variance (ANOVA), post hoc difference of means tests were conducted if deemed appropriate by the ANOVA, and a factor analysis was run on the data to further investigate patterns that arose in student responses.

Results from the qualitative analysis and statistical analysis of the quantitative data revealed significant distinctions among teacher practices and orientations and student perspectives. In particular, one teacher showed consistently significant differences from the other teachers in both the observed classroom practices and student perspectives in the surveys. Teachers attributed their struggle with IDL implementation to factors such as administrative support, the pressure of standardized tests, and experience in the classroom. A major conclusion from the study is that the environments in which these teachers taught had some contrasting critical features that profoundly impacted the teachers' pedagogical practices and even their orientations toward teaching mathematics.

The following chapters are organized as follows. Chapter 2 discusses the theoretical framework used to operationalize *discovery learning* for the purpose of this study, existing research on teacher and student orientations toward mathematics and reform-oriented pedagogy, and previous research conducted to measure implementation of such practices. Chapter 3 lays out the methods used in both carrying out the study and analyzing the qualitative and quantitative data. Chapters 4 and 5 provide a summation of the results and a discussion of the implications of these findings for students, practicing teachers, teacher educators, and educational administrators. Limitations and suggestions for further research conclude Chapter 5.

# Chapter 2: Theoretical Framework

The purpose of this study was to learn more about how teachers implement various features of instruction for discovery learning (IDL) and the effects that these pedagogical practices have on student orientations toward learning mathematics and their perspectives of classroom interactions and activities. The conceptual framework for this study depicts how different pedagogical frameworks approach student learning of mathematics and how various features of these individual formalizations were adapted to form an operational definition of *discovery learning*. To provide background information on the literature used to formulate the study and to analyze the results, this chapter is divided into the following sections: (1) discovery learning, (2) actual use of IDL practices, and (3) orientations toward IDL.

## DISCOVERY LEARNING

Many different titles have been given to the form of teaching that emphasizes teaching and learning practices in which students actively make sense of what they are learning and engage with concepts in a "dynamic space where power, authority, and control are necessarily shared between the teacher, students, and disciplinary practices…[involving] a reconceptualization of what it means to teach and learn and a need to create new and different opportunities for teaching in their own classrooms" (Rogers, Cross, Gresalfi, Trauth-Nare, & Buck, 2009). *Discovery learning* (DL) is a term that is most often used in to describe instruction methods in mathematics such as the Modified Moore Method or other inquiry-based frameworks that are typically used in undergraduate and graduate settings (Renz, 1999). Renz explains that in this context, the "essentials of discovery learning are: Motivation, Discovery, and Presentation" and that inquiry in mathematics is just students "doing research at their level" (1999). For this study, the definition of DL embodies the same frame of mind, but is adapted to fit more in line with the expectations of middle and high school students and teachers.

While DL highlights some fundamental commonalities of several reform-oriented approaches such as inquiry-based, problem-based, and project-based instruction, it is not intended to capture *all* distinguishing features of the individual frameworks—nor is it intended to equate the individual frameworks as identical. This term emphasizes the focus on students' ideas and contributions to their own learning and recognizes students as active collaborators in developing collective understanding with their teacher and peers—as opposed to a more passive student role as receivers of knowledge passed on to them from their teachers.

While the teachers involved in this study highlighted two approaches—inquiry-based instruction (IBI) and project-based instruction (PBI)—it is important to point out the commonalities not just between the two approaches, but among other approaches under this category of instruction for discovery learning (IDL). From literature on the various other approaches like PBI and constructivist learning (Barron et al., 1998; Boaler, 2000; Hmelo-Silver, et al., 2007; Krajcik, et al., 1998; Renza, 1999), the author identifies some consistent features of IDL that encompass all of these related teaching styles and frameworks. Figure 2 shows the operational definition of IDL by identifying these four essential features in contrast to how those features are exhibited in traditional instruction.

Table 1: Essential features of IDL in comparison to traditional instruction

| Features of IDL | Features of Traditional Instruction |
|---|---|
| 1. Students take an active role in learning | 1. Students passively gain knowledge by listening to the teacher |
| 2. Emphasizes higher-order thinking and critical reasoning | 2. Emphasizes procedural understanding, fact memorization, and single-correct answers |
| 3. Students engage in authentic collaboration with peers | 3. Students work individually with little or superficial collaboration with peers |
| 4. Incorporates technology to extend student learning | 4. Insufficiently or superficially incorporates technology |

Many studies and ideologies involving inquiry-based instruction (IBI) do not go into much more detail than these four features, and occasionally are even less structured than I have outlined above. For example, many descriptions of IBI do not include the incorporation of technology. Technology is included in this operational definition of IDL, however, because of the power technology has to engage students with the mathematics to extend their understanding and the importance of familiarizing students with the use of technology in many STEM-related careers (Blumenfeld, Fisherman, Krajcik, Marx, & Soloway, 2000; Milrad, 2002). Effective incorporation of tools and technology was also a point of emphasis throughout the teacher-preparation program in which all four teachers were certified (Barron et al., 1998; Marshall, Petrosino, & Martin, 2010).

The philosophy of IBI has its roots in practices of scientific inquiry, emphasizing the importance of student-centered activities that require students to put forward their own questions, collect and analyze data through their individual lens of scrutiny, and construct reasonable arguments based on evidence that they find themselves (Kuhn, Black, & Keselman, 2000). While this framework intentionally minimizes the teachers role in directly providing information and direction to students' learning processes, it is not intended to overshadow the influence a teacher should have on guiding students to build up their own sense of identity using critical reasoning and scientific/mathematical thinking. As some proponents of IBI and PBI have noted, this is an important role that the teacher must play by providing "extensive scaffolding and guidance to facilitate student learning" (Hmelo-Silver, Duncan, & Chinn, 2007). The origins of IBI appear in the works of John Dewey and have been more recently aligned to the constructivist view of learning (Li, Moorman, & Dyjur, 2010). As derived from Wilhelm and Walters' description of IBI from the scientific perspective (2006), mathematical inquiry involves students asking interesting questions, conducting their own investigations regarding those questions, analyzing arguments made by others, and using appropriate tools and logical reasoning to justify their solutions to themselves, their teachers, and their peers (as cited in Li et al., 2010).

Whereas IBI is loosely defined and can look quite different from context to context, the enactment of project-based instruction is much more structured. PBI is based on the idea that "learning of all kinds and in its all desirable ramifications best proceeds in proportion as wholeheartedness of purpose is present" (Kilpatrick, 1918). Marshall et al., identify several elements of PBI: "(a) driving question; (b) tangible product; (c) investigation; (d) use of cognitive tools; (e) collaborative nature of the activity; (f) the nature of the assessments for the task; and (g) the scaffolding provided for the task" (2010). In addition to these seven elements, they include the length in time of the activity as an important feature of PBI projects (Marshall, Petrosino, & Martin, 2010).

In response to some critics who have claimed that PBI activities engage students in "doing for the sake of doing," Barron and colleagues identify four design principles for effective project-based instruction that "can lead to doing with understanding rather than doing for the sake of doing" (1998). These four principles are "(1) Learning-appropriate goals, (2) Scaffolds that support student and teacher learning, (3) Frequent opportunities for formative self-assessment and revision, and (4) Social organizations that promote participation and results in a sense of agency" (1998). The intended outcomes of such instruction include student gains in both content knowledge and the awareness and ownership of their own learning (Barron, et al., 1998).

### ACTUAL USE OF IDL PRACTICES

Are teachers actually using IDL practices in their classroom? Are they developing lessons and activities based on these innovative features? There have been many studies that have investigated the influence of IDL practices—in particular with IBI and PBI—on student learning (Rogers et al., 2011). However, little research has focused on how successful teachers are at implementing various components of these ideologies in their classrooms after learning about the quality and efficacy of such practices.

In their research involving enactment of PBI in apprentice teaching, Marshall et al. measured the extent to which seven of the eight elements of PBI were being implemented by apprentice teachers (ATs) in their lessons that were planned and implemented after taking a course on PBI and near the end of the program (2010). They developed and used an implementation rubric to analyze the extent to which PBI practices were being implemented in the classrooms (see Table 2). These researchers found that most of the ATs involved in their study "tended to focus more on superficial aspects of PBI than did the experts" and few were described as fully implementing PBI in their classes, even if the ATs had high scores for PBI affinity (Marshall et al., 2010).

As part of this study, a similar rubric to examine the level at which teachers implement the features of IDL was used. Similar to the way in which Marshall and her colleagues developed the rubric based on the major components of PBI, a rubric was developed based on the four essential features of IDL shown in Figure 2 (See Appendix B).

Table 2: Implementation rubric (for PBI) from Marshall, Petrosino, & Martin (2010)

| Component | Minimal Implementation | Moderate Implementation | Full Implementation |
|---|---|---|---|
| Driving question | Question is supplied by instructor; predetermined answer | Students have some say in selecting or narrowing question. Requires multiple data sources. Answer not completely constrained | Question is meaningful to students, real-world problem with multiple, interdisciplinary data sources. Answer not previously known |
| Tangible product | Tangible product of some kind | Students use new concepts and apply information to create tangible product | Students use new concepts and apply information to create tangible product; includes multiple ways of representing information |
| Investigation | Hands-on, minds-on activity | Student-driven, complex task | Authentic, student driven, complex. Students learn concepts, apply info, represent knowledge in multiple ways |
| Cognitive tools | Access to learning tools of some kind | Access to multiple tools | Cognitively oriented collaboration and visualization tools |
| Collaborative activity | Students report results to others | Task includes collaboration between students to generate product | Task requires collaboration between students and others to generate product |
| Assessment | Some form of formative assessment | Authentic formative and summative assessments | Authentic assessment requiring multiple forms of knowledge representation |
| Scaffolding | Some form of scaffolding | Instructor provides scheduling milestones, inquiry is seeded with powerful ideas | Scheduling milestones, benchmark lessons, social structures to facilitate collaborative learning |

## ORIENTATIONS TOWARD IDL

### Teacher perspectives

Because many teachers are familiar with traditional teaching practices from their own K-12 education and from observing other practicing teachers, IDL usually requires a shift in a teacher's orientation toward instruction and the nature of mathematics. Mason describes this orientation change as "a revised didactic contract, seeking to balance developing competency with enculturation into mathematical thinking, rather than succumbing to student desire to minimize effort and simply be trained in requisite behavior" (2001). This aspect of learning as mathematical *enculturation* is often overlooked by many teachers because they—as most members of society do—view mathematics as a static discipline that is analogous to a set of rules for a complex game. Particularly in an education system that predominantly defines student success in terms of standardized test scores and defines learning as a process of checking boxes on a list of content standards, teachers often view the practices of mathematical reasoning and constructive knowledge building as secondary to these other priorities (Desimone, 2009).

Rogers et al. found that during a first-year implementation of PBI, teachers had varied notions of what it meant for their students to be successful, the teachers' goals were occasionally incompatible with those of PBI, and the teachers expressed views of science and mathematics as more rigid and prescriptive (2011). Because PBI and other IDL approaches "shift the focus of learning away from 'correct, indisputable' answers to the process of converging on solutions," creating an active environment where the notions of power and authority are balanced between students and teachers, this conflict with the more traditional teacher-as-authority or teacher-as-expert orientations may stand in the way of genuine IDL practices. Although teachers learn more innovative teaching strategies and general principles of reform-oriented instruction, many teachers revert to a method of teaching that is more indicative of their personal experiences as students than of their experiences preservice teachers in their preparation programs (Ball, 1988; Liljedahl, Rolka, & Rösken, 2007). This is no easy hurdle for teacher educators to overcome.

**Student perspectives**

In Jo Boaler's three-year study of two high schools (which she calls Amber Hill and Phoenix Park—the former a more traditional school and the latter using an IBL curriculum) Boaler reveals varied student perspectives on mathematics as a discipline and general learning (2002). Boaler found that the students at Phoenix Park "believed mathematics to be an active, inquiry-based discipline" (2002). However, she found that students at Amber Hill were much more likely to prioritize memory over thinking than those students at Phoenix Hill (2002). This is not surprising since a more traditional learning environment typically places much more value (in terms of grades and assessment) on students' memory of algorithms and theorems. This distinction between student perspectives is important because it is common for students who move from more traditional classrooms into a IDL environment may have hesitancy or difficulty with their new responsibilities for their learning. "Those working to promote educational reforms have generally overlooked the fact that students not only need to develop new ways of working in reform-oriented classrooms, but an understanding and commitment toward the changes in their roles" (Boaler, 2002). Of course, many students quickly notice the benefits of IDL in mathematics and appreciate the contrast between the new learning experiences with the traditional mathematics lectures (Bailey, Briggs, & Cooper, 2012; Boaler, 2002).

# Chapter 3: Methodology

While much of literature is devoted to parsing out the various aspects of reform-oriented teaching frameworks and analyzing the efficacy of such practices in terms of both teacher and student learning gains, little research has shown the levels and forms of implementation of such pedagogies by the large teacher workforce. To address the need for such research, this study was aimed to investigate how 4 Algebra I teachers and 142 students of their students use and perceive various aspects of IDL. To explain the details of the setting in which the study took place, the particular processes that occurred, and the way in which the data was analyzed, this chapter is divided into the following sections: (1) participants—case descriptions, (2) data collection, and (3) data analysis. Within the data collection section, the author provides more detail about the various components of the study—the classroom observations, student surveys, and teacher interviews.

## PARTICIPANTS—CASE DESCRIPTIONS

All four of the participating teachers obtained their teaching certification through the same teacher-certification program within a large research university, obtaining certification and a bachelor's degree in mathematics. The program included taking multiple courses within the mathematics department (many of the same courses that are required for pure mathematics degrees), several courses through the school of education (e.g. focusing classroom interactions and instruction methods), and several on-site requirements in which the preservice teacher planned, implemented, and received feedback on individual lessons. In particular, each teacher participated in at least one course that was taught using the *Modified Moore Method*, and some took several other mathematics courses under this type of instruction. The *Modified Moore Method* is a particular formalization of inquiry-based instruction in which the instructor(s) provide students with a well thought-out compilation of theorems in the subject area (e.g. Euclidean geometry or number theory), and the student is expected to work from a simple set of axioms and definitions to prove each theorem for the course. A major component of this is student presentations of these proofs. The students present their proofs to the course, which in turn critiques and provides feedback. Throughout this process the professor may ask students to restate parts of other students' proofs to check for clarity or offer counterexamples or alternative proofs. This similar experience, held by all four teachers, is a strong parallel because of the rigor and depth of inquiry expected in each of these courses is quite high.

At the time of this study, the four teachers taught at three different schools. The three schools are located in two districts. See Appendix E for information about how the districts differ in terms of student populations and achievement on standardized state tests. Ms. Anderson taught at Spring Middle School in a large suburban district.[1] Ms. Brown and Ms. Clark taught at Graham High School in a slightly smaller suburban district, and Ms. Day taught at Graham New Tech High School in that same district. Both districts are located near the university at which the teachers obtained their bachelors degrees and teacher certification. Spring Middle School and Graham New Tech High School were relatively young establishments—both less than 10 years old at the time of the study. Although the original Graham High School was established well before, there are had been recent growth spurts in student population within the past decade because of growth in the city's population and in the populations of neighboring areas.

[1] All of the teacher, student, and school names included in this paper are pseudonyms.

**Ms. Anderson**

After completing the university's certification program, Ms. Anderson started teaching at Spring Middle School. At the time of the study, she was in her ninth year teaching mathematics, and in addition to teaching two sections of Algebra I, she was currently teaching two sections of Geometry. In the past she had taught $7^{th}$ and $8^{th}$ Grade Math, Geometry, and Algebra—all at the same middle school. Only a handful of sections of Algebra I were taught per year at this middle school, some of which were taught by Ms. Anderson and other taught by another teacher who was also a graduate from the same university certification program.

Provided that the two teachers' schedules were usually at odds with each other and the occasional department meeting that was reserved for various other responsibilities during the few times that they would have to collaborate, the amount of collaborative planning between the two teachers is limited to infrequent and usually rushed meetings that occur once every other week at most. "It's kind of hit or miss…we don't get to plan together that often," states Anderson. All of the Algebra I classes take a common assessment (written by Ms. Anderson and the other Algebra I teacher) at the end of each six-week period. Ms. Anderson claimed that she did have the freedom to do what she wanted in terms of daily lesson planning and the sort of activities the students do in the class, but she mentioned other factors that may interfere with her ability to do what she truly wants to do. These included the large class sizes, the short 50-minute class periods, and the pressure to "still get through *all* of the algebra curriculum."

Two of this teacher's sections of Algebra I were involved in this study and included only $8^{th}$ grade students. The first section consisted of twenty-seven students and met during the second period of the day. The second section consisted of twenty-six students and met during the fourth period of the day, right before their lunch period. In between these two sections, the teacher had an off period that was typically her one planning period of the day, since her other off period at the end of the school day was almost always taken up by other responsibilities.

**Ms. Brown**

At the time of the study, Ms. Brown was a first year teacher teaching 3 sections of Algebra I and one section of Statistics at Graham High School. In addition to her experience in the certification program, Ms. Brown mentioned her experience as a student in the "rigorous AP and Pre-AP programs" as shaping her perspective on teaching. She explained that she did some planning with the entire math department, but more often as an "Algebra team." In this district, the Algebra I course is divided into trimesters A, B, and C. There were four other teachers at Graham High School teaching Algebra I, some of whom were teaching Trimester A, and most of whom were teaching Trimester B at the time of the study. In addition to the course being broken down into trimesters, content specialists in the district also broke down the content standards into "bundles" which were intended to cover a time span of three weeks in each classroom. Ms. Brown described the team planning process as follows:

> "Well, once a week we'll meet formally to either map out a 3-week bundle or to discuss, like, which activities we want to do. And then most of it is just designing tests. So sometimes we'll do the bundle mapping, designing a test or…analyzing data from previous tests. It's very data driven and this is actually the formula of what all of the content teams are supposed to be doing at our school. It's very focused on…backwards design, in that you make your test a few weeks before you give it. So you try to align your—what your teaching, to cover stuff that will be on the test."

She also mentioned that the group of teachers did meet informally throughout each week, such as passing by in the hallway to give feedback about how an activity went in their class. She described these interactions as more "scattered," though.

This teacher's two sections that were involved in this study consisted of only 9th grade students, all in Trimester B of Algebra I. The first section met during the first period of the day and was quite small—starting with seven students at the start of the trimester, and ending with just five students at the last observation. Ms. Brown's second section met right after their lunch break and consisted of twenty-two students. She explained that these two sections were tracked into her class based on their below-average standardized test scores from middle school. In between these two sections, the teacher had an off period and some additional time for lunch, which she explained was most often used for planning, but would occasionally be interrupted for meetings and other events.

**Ms. Clark**

As mentioned above, Ms. Clark also taught at Graham High School. She started her teaching career by taking on a classroom from a teacher that left halfway through the year—about two years prior to the study. Ms. Clark was the department head for the mathematics department of the school, a district league teacher ("a group of teachers who facilitate professional development on their campuses"), a mentor teacher, and an AVID teacher. She had taught various sections of on-level and Pre-AP Algebra I throughout those two years and was currently teaching one section of Trimester A Algebra I, one section of Trimester B Algebra I, and one section of Geometry.

Of these classes, only one was included in this study: the section of Trimester A Algebra I. It consisted of twenty-eight 9th and 10th grade students, many of whom were identified as English language learners (ELLs). During every class period, there was an ELL specialist in the classroom whose responsibility was to ensure effective communication between the teacher and the ELLs. According to Ms. Clark, there was very little collaboration between the ELL specialist and any of the teachers at the school. From an observer's perspective, the specialist's role was portrayed very simply as a translator between teacher and student, with very little (to no) influence on the lessons and activities.

### Ms. Day

Although Ms. Day worked in the same district as Ms. Brown and Ms. Clark, she worked in an entirely different environment. She taught Algebra I at Graham New Tech High School, which is based on the "New Tech Model" developed by Anthony Carnavale and Donna Desrochers (2003). At the time of the study Ms. Day had been teaching at this school for three years, but prior to that Ms. Day taught at Graham High School for three years. The year prior to the study, she and another teacher co-taught all of the sections of Algebra I. Ms. Day explained that since she was currently the only teacher teaching Algebra I, most of activities and projects that the students did were based on the planning she and the other teacher had done the year before, with some minor edits. The entire school is structured around project-based instruction and all of the teachers were trained and expected to carry out PBI in their content area, particularly the STEM teachers.

## Data Collection

In order to develop a more thorough understanding of how these features of IDL were being implemented in these classrooms, the author utilized three different data sources: (1) classroom observations, (2) student surveys, and (3) post-observation interviews with each teacher. Each data source offered a separate perspective into teaching practice. The study extended over a period of approximately three months, with multiple data collection methods occurring on the same day while. Only one site was visited in a day; however, for Graham High School, this meant that data from both Ms.

Brown's and Ms. Clark's classrooms were collected in each visit. To provide an overall picture of the progression of the study, the sequence of events are outlined in Figure 1.

| | | | | |
|---|---|---|---|---|
| Dec. 10 1st observation at Spring MS (Ms. Anderson) | 11 2nd observation at Spring MS | 12 | 13 | 14 1st observation at Graham HS (Ms. Brown & Ms. Clark) |
| Jan. 7 | 8 3rd observation and student surveys at Spring MS | 9 Interview at Spring MS | 10 | 11 2nd Observation Day at Graham HS |
| Jan. 21 | 22 | 23 | 24 3rd observation at Graham HS | 25 Student surveys and interviews at Graham HS |
| Feb. 20 | 21 | 22 | 23 1st observation at Graham New Tech HS (Ms. Day) | 24 |
| Mar. 1 | 2 | 3 | 4 2nd observation at Graham New Tech HS | 5 3rd observation, student surveys, & interview at Graham New Tech HS |

Figure 1: Sequence of Data Collection Events

### Classroom observations

Roughly a month before classroom observations began, the author was trained in using the UTeach Observation Protocol (UTOP), which was designed by math and science educators in the UTeach College of Natural Sciences program at the University of Texas at Austin and is used at universities across the nation to evaluate teachers' use of innovative teaching practices. It is much more than a simple checklist of various techniques, and has been shown to be reliable in effectively and accurately observing and analyzing teacher practice (MET Project, 2012). The protocol requires the observer to rate the quality of different components of a lesson on a scale of 0-5. Each component on the protocol includes various indicators that are used collectively to rate the overall quality of that component. For example, the subject matter component includes eight individual indicators—such as (4.2) whether or not the content was "consistent with deep knowledge and fluency with the mathematics concepts of the lesson" and (4.6) whether or not "it was made explicit to students why the content is important to learn."

Although the actual scores that the teachers received on this evaluation are not included in this paper, the use of the protocol greatly influenced the perspective through which all classroom activities and interactions were observed. The UTOP indicators guided the author's observation by highlighting particular aspects of classroom interactions on which to focus and take note. Classroom observations are a valuable source of abundant data; however, without an observation protocol, this amount of data can be quite overwhelming. The focus of the UTOP on particular aspects of the classroom environment, planning process, implementation, and the math or science content helped guide the observers attention to important features of classroom activities and interactions that are especially relevant to IDL.

Each section of Algebra I was observed from start to finish on three separate dates. The observer provided no form of instruction to the students, only interacting with students in situations where clarification of dialogue or explanations of activities were needed. For example, the observer occasionally asked students to repeat a statement made to a fellow classmate related to the activity they were working on. A few times, the observer asked students to explain how they were using a graphing calculator or how they were solving a problem on their paper. These observations were not video or audio recorded, but the observer typed field notes throughout each class period. Exact quotes from the students and teachers, as well as descriptions of classroom interactions and classroom environment, were captured in these notes.

These observations took place over a time span of approximately three months, and were scheduled at the teachers' convenience and preference. The only criteria provided to the teachers in selecting observation dates was that the class time could not be spent administering a summative assessment taken individually by the students and the class time needed to include some components that they themselves had some control in planning. These criteria likely skewed some of the findings in favor of more innovative classroom practices; therefore, some of my observations may not be reflective of the usual use of class time in each classroom. For this reason, the student surveys are useful in representing a perspective that likely reflects the typical experience in the class.

**Student Surveys**

Survey questions were written to target the various aspects of IDL. There were two sections of the survey. Section A (the *Student Orientation* section) consisted of questions for which the students rated, on a 5-point Likert scale, the extent to which they agreed or disagreed (-2=Strongly disagree and 2=Strongly agree). These questions were written to measure students' orientations toward inquiry-based learning and the extent to which they associate these principles with what they do in their classes. The *Independence & Autonomy* construct is focused on students' included questions such as "It is my responsibility to learn mathematics" and "I'm learning how to think for myself in this class." The *IDL Learning Style* construct identifies the students' reliance or conviction in IDL practices and how they influence their own learning of mathematics—e.g. do they see reading from the textbook, doing projects, or listening to step-by-step instructions from the teacher as valuable to their learning. The *IDL Perceived in Class* construct identifies students' perceptions of whether or not particular IDL features are experienced in their class—e.g. are they learning "mathematical ideas" or "tricks."

Section B consisted of questions, for which the students rated, on a 4-point Likert scale, the extent to which various activities occurred in their class. This section was partitioned into constructs twice: the *Participant* Partition distinguishes among *who* is involved in each type of activity, and the *Nature of Activity* Partition distinguishes among the *type* of activity. For example, the activities of working with a group on a project and working with a group on a worksheet both fall under the *Peer Interaction* construct within the *Participant* Partition, but the first falls under the Basic DL Practices construct and the second falls under the *Traditional/Procedural* construct under the *Nature of Activity* Partition. It is important to notice that not all items appear in both partitions, but each item is included in at least one of the partitions. Table 4 and Table 5 show how the questions in Section B of the student survey were broken into their constructs.

Table 3: Constructs for Section A in Student Surveys

| Construct | Survey Items |
|---|---|
| Independence & Autonomy | 3. It is my responsibility to learn mathematics.<br>6. I'm learning how to think for myself in this class.<br>14. It is my teacher's responsibility to make sure that I learn mathematics.[R] |
| Comfort | 10. I feel rushed in this class (I don't have enough time to think things through).[R]<br>12. I feel comfortable asking for help when I don't understand a problem.<br>16. I feel like there is too much work to do in this class.[R] |
| IDL Learning Style | 2. I learn a lot from reading the textbook.[R]<br>5. I learn a lot from doing practice problems (from a textbook or on worksheets).[R]<br>7. It is helpful when the teacher lets us figure problems out by ourselves.<br>9. It is helpful when the teacher walks us through the problems step-by-step.[R]<br>13. I learn a lot from doing projects and presentations.<br>15. I learn a lot from my classmates.<br>17. I learn a lot from listening to my teacher explain things in front of the whole class.[R]<br>20. I learn a lot from listening to my teacher explain things one-on-one.[R] |
| IDL Perceived in Class | 1. We have a classroom routine that we follow every day.[R]<br>4. I'm learning a lot about mathematical ideas.<br>11. I'm learning a lot of tricks on how to solve math problems.[R]<br>18. It is okay to disagree with my teacher about how to solve math problems.<br>19. I'm learning a lot of skills in this class that will help me in other classes. |

[R] *These items were inversely scored—i.e. agreement with these statements resulted in a lower score for this construct.*

Table 4: *Participant* Partition of Section B in the Student Surveys

| **Construct** | **Survey Items** |
|---|---|
| Individual Activities | 1. Individually work on worksheets |
| | 10. Individually work on projects |
| | 16. Take tests |
| | 21. Discover something new on my own |
| | 22. Take quizzes |
| Teacher Interaction | 11. Talk to the teacher about what I am learning |
| | 12. Talk to the teacher about things I like and dislike about the class |
| | 18. Learn something new from the teacher |
| Peer Interaction | 4. Work on a project with other classmates |
| | 5. Give a presentation (or speak in front of the class) |
| | 6. Talk about what I am learning with my classmates |
| | 7. Contact experts that work in fields related to what we are learning |
| | 8. Work on worksheets with other classmates |
| | 13. Talk about the importance of mathematics in the real world |
| | 14. Analyze other people's arguments |

Table 5: *Nature of Activity* Partition of Section B in the Student Surveys

| Construct | Survey Items |
|---|---|
| Traditional/Procedural | 1. Individually work on worksheets |
| | 8. Work on worksheets with other classmates |
| | 16. Take tests |
| | 18. Learn something new from the teacher |
| | 22. Take quizzes |
| Basic DL | 4. Work on a project with other classmates |
| | 5. Give a presentation (or speak in front of the class) |
| | 10. Individually work on a project |
| | 11. Talk to my teacher about what I am learning |
| | 19. Explain how I solved a problem |
| Innovative DL | 3. Reflect on what it means to learn and understand math |
| | 6. Talk about what I am learning with my classmates |
| | 7. Contact experts that work in fields related to what we are learning |
| | 12. Talk to the teacher about things you like and dislike about the class |
| | 13. Talk about the importance of mathematics in the real world |
| | 14. Analyze other people's arguments |
| | 15. Make concept maps (or other diagrams to model what I am learning) |
| | 17. Prove why a mathematical statement is true |
| | 20. Learn about what mathematicians do |
| | 21. Discover something new on my own |
| Technology-integration | 2. Use a computer to look up information |
| | 9. Use a computer to make models or representations |

## Teacher interviews

Because observations provide a limited perspective into the daily activities of a classroom, the teacher interviews served as another source of data that illuminated various aspects of planning, teaching environment, administrative support, and teacher beliefs about the subject matter and student learning. The interviews did not follow a strict protocol, but were guided a selection of key questions such as "How do you typically plan your lessons?" and "Do you feel as though you have the freedom to use lessons and activities that you choose or create?" In all four interviews, the teachers at least briefly spoke about their experience in their preparation program and its influence on their teaching practice. Further questions asked teachers to explain how they assess understanding in their classroom and how often they do so.

The interviews were conducted in the individual teachers' classrooms either during a conference period or after school. The interviews occurred after all observations were completed. As a result of the time available for each teacher and the time devoted to a subset of questions, no teacher addressed all of the questions included in Appendix A.

### DATA ANALYSIS

A total of 124 students across the 7 classes completed the student survey. Student survey responses were entered into a spreadsheet organized by class. Scores were calculated per student for each of the constructs by calculating the average of their responses to the questions within that construct. Based on these student scores, an average score for each construct was calculated per teacher. A one way analysis of variance (ANOVA) was run per construct to determine if significant differences arose among the teachers. Post hoc tests were also run to determine between which teachers these differences occurred ($t$ statistics). A factor analysis was used to further investigate how the survey responses clustered the questions.

Informal qualitative analysis began as soon as data collection began. Because the author was the only individual observing the classes, administering student surveys, and interviewing the teachers, analysis initially took the form of organizing and commenting on observation notes and open-coding interviews to identify recurring themes and concerns. As mentioned before, the observations were not recorded, therefore, the intent of the observations was to get an overall perspective into the daily classroom activities and interactions without keeping track of every single activity or interaction. The coding of the observation notes was for the purpose of identifying instances throughout the observations that connected with the four essential features of IDL either in effective portrayal of these features or in portrayal of the parallel, traditional feature. For example, instances in which students were directly instructed what to do and how to do it, this were coded under Feature1—Traditional, whereas instances in which students were able to explore their own ideas and manage the direction of the activity were coded under Feature1—IDL. While coding these observation notes, two other themes arose. These two emerging themes were (1) orientations toward IDL principles and (2) classroom management. The codes were created and revised based on coherence with the research questions. The interviews were coded separately, but followed the same structure: first they were coded according to the four features, but emerging themes included the two mentioned above as well as (3) teacher background, (4) collaboration with other teachers at the school, and (5) barriers to IDL implementation.

# Chapter 4: Results

After data was collected and organized, the teacher interviews and observation notes were coded according to the four features of IDL:

1. Student-centered, active learning
2. Higher-order reasoning and critical thinking
3. Authentic collaboration, and
4. Technology-integration.

In the first five sections of these results, the author discusses the evidence for each teacher's implementation for each IDL feature and reports the implementation level for each teacher using the IDL Implementation Rubric in Appendix B, with a summary of these implementations in the subsection, Overall implementation. In the section, Perceived Barriers to Full Implementation , the author discusses findings from the teacher interviews, as well as evidence from both classroom interactions and indirect comments throughout the interviews that uncover the elements of teaching that cause struggle for teachers to fully implement all of the IDL features effectively. Finally, the results from the ANOVA, post hoc tests, and the factor analysis are included in the section titled Student Survey Data.

### FEATURE 1: STUDENT-CENTERED, ACTIVE LEARNING

In a context in which IDL practices are being fully implemented, students play a key role in defining the questions to be studied as well as the direction that learning takes (Li et al., 2010). Some of the classroom activities did involve student choice in "the direction that learning takes," however there were few instances in which the students actually constructed their own questions, and even fewer in which these questions were explored during the class time. The teachers had varying beliefs about constructive learning, but all thought it was *important* for the students to have opportunities to think critically about the concepts and create their own understanding. This belief, however, was not as easily detected in their observed teaching practices. The authenticity of students as the primary knowledge constructors and as the decision-makers within the activities was quite limited among all four classrooms, however.

3. Silvia sells purses and watches. She sells purses for $15 each and watches for $20 each. She would like to make at least $350 by the end of the week from selling these purses and watches.
   a. Write an inequality that represents the number of purses, *p*, and the number of watches, *w*, that she needs to sell. Graph this inequality on graph paper.
   b. What are the *x*- and *y*-intercepts for this inequality? What do these intercepts mean to Silvia?
   c. What are two sample solutions to this inequality?
   d. What is an example of a solution for the inequality that does *not* make sense in this situation?

Figure 2: Sample problem from Ms. Anderson's activity

Ms. Anderson's group activity during the first two observation days involved students creating algebraic expressions (inequalities) and graphical representations appropriate for real-world scenarios. A sample problem is shown below in Figure 2. Desks were arranged in groups of 3 or 4, where students were faced toward one another. The teacher intentionally only provided a single copy of the worksheet for each group in order to encourage them to discuss the problems and ask each other questions.

Table 6: Evidence of Feature 1: Student-centered, constructive learning

| Teacher | Observed Evidence For or Against | Evidence of Teacher Beliefs from Interviews | Level of IDL Observed |
|---|---|---|---|
| Anderson | • Students *often* expected to determine how to solve problems, rather than being given a prescribed solution method<br>• Teacher provided opportunities for students to "struggle" with the concepts instead of immediately explaining a way of solving or thinking about a problem<br>• Students often asked to recall and apply prior knowledge in novel situations<br>• Students felt comfortable asking questions (student surveys)<br>• Students asked and explored very few questions of their own<br>• Discussions/activity rarely strayed away from the specified problems on worksheets | • Students should come to their own realizations and understandings of the patterns and relationships among the various mathematical concepts covered in the class<br>• Would like to do more student-directed activities (e.g. "independent research projects") | *Moderate* |
| Brown | • Rare instances in which teacher elaborated on student-generated questions<br>• Survey data revealed that students feel comfortable asking questions in class<br>• Predominantly direct instruction, while students took notes<br>• Students completed worksheets with practice problems after the teacher modeled "the steps"<br>• Instructions from teacher were very prescriptive and allowed little (or no) room for exploration<br>• If students asked for clarification, the teacher often reiterated previous statements without offering multiple representations or alternative explanations<br>• Student-generated questions and suggestions were often immediately redirected to the "correct" method for solving the problems | • Students sometimes have deficiencies that are difficult to get past<br>• Sometimes students struggle with the easy stuff, like memorizing the steps to solving a problem | *Minimal* |
| Clark | • Rare instances in which teacher elaborated on student-generated questions<br>• Predominantly direct instruction, while students took notes<br>• Students completed worksheets with practice problems after the teacher modeled "the steps"<br>• Instructions from teacher were very prescriptive and allowed little (or no) room for exploration<br>• If students asked for clarification, the teacher often reiterated previous statements without offering multiple representations or alternative explanations<br>• Student-generated questions and suggestions were often immediately redirected to the "correct" method for solving the problems<br>• Students *did not* feel comfortable asking questions in class (student surveys). | • Students benefit from the teacher modeling the strategies | *Minimal* |

Table 6 continued

| | | | |
|---|---|---|---|
| Day | • Students began new project by making lists of "Knows" and "Need-to-Knows" based on the introduction provided by the teacher about the project<br>• Students *often* expected to determine how to solve problems, rather than being given a prescribed solution method<br>• Students often asked to recall and apply prior knowledge in novel situations<br>• Teacher occasionally provided students with time to struggle with concepts before providing hints or suggestions<br>• Teacher rarely directly explained a procedure for solving a problem before providing ample time for students to explore on their own | • Experience in teacher preparation program has "opened my eyes" to see the student as the focus<br>• Tries to only use workshops if the students request instruction on a particular concept | *Full* |

Although these problems were predetermined and students were expected to arrive at a single-correct solution for each problem, the types of questions asked by the teacher and by the students demonstrated a significant amount of focus on student-driven ideas and solutions. For these reasons, the level at which Ms. Anderson implemented Feature 1 was determined to be *partial*. With a bit more student autonomy present in these activities this teacher would have been considered to have fully implemented this IDL feature. A sample of such a conversation is included below.

Student: I keep getting zero, but that's not right.

Anderson: It *could* be right. Why do you think it's wrong?

Student: Wait…what?

Anderson: How did you find the intercepts? How did you get zero?

Student: I plug in zero for the other one.

Anderson: And what did you get?

Student: Um…Well, that's how I got zero…and I get the same thing for $y$.

Anderson: Okay…what does that mean?

Student: Oh! No solution!

Anderson: Hold on. That's not what that means.

Student: Oh…there's only one solution?

Anderson: I want you think about it a little more. Graph the intercepts on the paper, and you guys made a table so use that. You guys have great ideas, just spend a little more time thinking about those.

Only in Ms. Day's classroom, were the students almost entirely in charge of determining the direction in which the activity would be carried out. Of course, the teacher provided a great deal of scaffolding on the worksheets to help guide students' thinking, but the students approached and completed the activity with the mindset that they were responsible for acknowledging what they already knew, needed to know, and how they would go about researching and building their collective knowledge to solve the problem. This focus on students as active participants was not only evident in the first observations—during which students presented their synopses of various topics learned in the weeks prior to the observation—but also in the other activities that took place in which students began work on "The Pool Problem." This project was one of the many projects developed by Ms. Day and the teacher that had also taught Algebra I at the same school the year before. The driving question of the project was, "What is the maximum area of a pool given a fixed perimeter?" Throughout the duration of the project the students were expected to provide evidence for their solutions with graphs, tables, equations, algebraic methods, and verbal descriptions. As expressed in the presentation the teacher used to initiate the project, the overall goal of the project was for students to "make connections between linear and quadratic intercepts and solutions."

The activities within the project were designed so that students took on the responsibility for determining how they could use their prior knowledge, interests, and group members to analyze the problem at hand. Because of the effort in planning and implementing a project focused on student engagement, it is easy to see that Ms. Day fully implemented the IDL feature of student-centered, active learning.

In Ms. Brown's and Ms. Clark's classrooms, however, the emphasis was not on student ideas and interests. The following interaction shows an example of how the emphasis of the activities in Ms. Brown's classroom was on testing strategies and how she actually disregards a student's suggestion to emphasize the strategy that she determined to be more efficient.

> *Teacher shows a slide in the PowerPoint that displays the question: "Which ordered pair is a solution to the inequality* $10x-4y<2$*?" with the ordered pairs, (5,3), (-3,8), (2,-2), and (-3,-4), listed as possible answers.*
>
> *Students work quietly, individually for about one minute.*
>
> Brown: I'm proud of those who are working hard and focusing. This is good practice for your test on Tuesday.
>
> … So does anybody have the answer?
>
> Student: I don't—not yet, but can't we just plug in the points and check them?
>
> Brown: Well, first you need to solve for *y*.
>
> Student: Oh. Okay.
>
> Brown: That's usually just something in general that you should do so that you can get it into slope-intercept form, and you can know the slope and stuff like that.
>
> *Another minute goes by as students continue to work on the problem.*
>
> Brown: You know what guys, what I said works just fine, but what you guys would probably prefer is to just start plugging in these points to see which one works. We did a few of these examples in your notes less than a week ago. So you could try that if you want to.

In this example, the student had actually suggested a more efficient method for finding the answer to this problem and showed an understanding the term *solution* in reference to an inequality. Instead of immediately exploring this student's suggestion, she reinforced the strategy that she had in mind initially—it wasn't until after work continued that she conceded to the idea proposed by the student, and even then the student was not acknowledged for contributing a valid solution method. This IDL feature of student-centered activities would be more evident in a classroom in which the teacher more promptly and willingly acknowledges and explores student suggestions and questions.

**FEATURE 2: HIGHER-ORDER THINKING AND CRITICAL REASONING**

In contrast to a didactic, prescribed approach to instruction, IDL practices involve more rigorous reasoning skills that potentially involve multiple perspectives and multiple solutions. This type of interaction with and perception of the content leads to increased student motivation, more retention of conceptual and procedural knowledge, and more *creative* [emphasis added] approaches to problem solving" (Li, Q., Moorman, L., & Dyjur, P., 2010). Mathematics is not normally considered by students to be a creative discipline, but many experts have emphasized the more aesthetic, creative perspective of mathematics and encourage this component of the subject to be underscored in classroom instruction (Craft, 2005; Hämäläinen & Vähäsantanen, 2011; Rogers et al., 2011). Craft has shown that creative construction of conceptual understanding in classrooms can be supported by activities such as critical questioning and challenging, exploring relationships among ideas, and engaging in metacognitive discussions (2005). These notions of higher-order knowledge construction were exhibited at largely varied degrees in all four classrooms.

As shown in Table 7, Ms. Brown and Ms. Clark displayed minimal implementation of Feature 2. There was limited evidence that students were engaged in higher-order thinking throughout any of the observations. In particular, the focus of the activities and discussions had quite the opposite intent. Students were explicitly instructed to use specific methods and solve problems according to the teachers' predetermined and supposedly more efficient steps. These two teachers started each class period with an activity they called “Do Now!” warm-ups. These were standard practice problems that are typical of standardized mathematics assessments—multiple choice questions that are aligned to one or two content standards. During none of the observed class periods in these teachers' classrooms, were students asked to explain their reasoning or explore more deeply the mathematical concepts underlying the algorithms they were using to solve problems. An example of a conversation from Ms. Brown's class exhibits the focus on memorization and procedural understanding of inequalities:

> Brown: [to a single student] Which symbol here? Which way does it face? [pointing to the graph of an inequality where the solution set was above the boundary line]
>
> Student: This one [pointing to the “<” symbol on the sheet]
>
> Brown: No. If it's shaded above you should have $y$ greater than or $y$ greater than or equal to [writing “$y >$ ___” and “$y \geq$ ___ on the student's sheet]
>
> [to the class] You guys should know that memorizing which way to shade is not hard, so this is not something that you should be missing problems on the [state standardized test] just because you can't remember where to shade…This is the easy part. You guys just need to take this worksheet home this weekend and really just memorize it.

This example shows the teacher's focus on very narrow strategies that would help students, if they were successful at memorizing, to perform better on standardized tests.

Ms. Anderson, on the other hand consistently implored students to provide a rationale for their answers, questioning them when they contributed both valid and invalid solutions. Similarly, Ms. Day regularly asked students to explain their thinking, make connections

Table 7: Evidence of Feature 2: Higher-order thinking and critical reasoning

| Teacher | Observed Implementation | Evidence of Teacher Beliefs from Interviews | Level of IDL Observed |
|---|---|---|---|
| Anderson | • Teacher *often* asked higher-order questions that required students to explain their solutions and conceptual ideas<br>• Students *often* evaluated the soundness of their solutions and interpreted the contextual meaning of their solutions/answers<br>• Many students focused on finding "the correct" answer<br>• Occasional missed opportunity to further discuss conceptual ideas behind the procedures/solutions | • Has "high expectations" for students<br>• Expects students to explain their reasoning and strive for conceptual understanding<br>• Getting students used to higher-order thinking requires preparation/ practice at the start of the year<br>• Without the proper "training" students would struggle a lot with writing proofs and explaining their reasoning in general | *Significant* |
| Brown | • Teacher did not ask students to explain *why* they gave a particular answer/solution—attention was dedicated to whether or not their answer was correct and what strategy they used to arrive at that answer<br>• Teacher presented several "tricks" for memorizing a strategy<br>• Teacher had negative responses when students used a solution method different from the one she expected them to use<br>• Teacher often mentioned standardized tests and test-taking strategies throughout class time | • Some students are learning a lot, but some students just "need to get their head in the game" so that they can complete the assignments and do well on the tests<br>• Students need to focus on organizing their work, keep note of the important features, and follow the algorithms<br>• It's disheartening when some students put in a lot of effort and just don't perform well on the tests | *Minimal* |
| Clark | • Students were rarely asked to explain their reasoning<br>• Teacher instructed students to copy the steps from the board as she wrote them so that they could memorize them and use the strategy on the practice problems | • Sometimes the students just don't want to put in the work, they think it's too hard | *Minimal* |
| Day | • Teacher *often* asked higher-order questions that required students to explain their solutions and conceptual ideas<br>• Teacher posed scenarios that required students to apply understanding in new contexts—even an instance that connected to a concept several students were currently learning in their Biology class<br>• Because of the design of the activity, students were rarely concerned about finding an *answer*<br>• A few instances in which the teacher provided teacher-directed "workshops" to teach students algorithms (e.g. using a grid to multiply binomials) | • This is difficult to do consistently with the pressures of standardized tests, but the students have become accustomed to the more conceptual and critical questions | *Significant* |

to other concepts that they had learned previously in the year, and even made a connection to Punnett squares which students were learning about in their Biology classes. Amid these frequent interactions involving higher-order thinking, the two teachers still spent a significant amount of time directly teaching students algorithms and tricks without providing much attention to the mathematical concepts. For example, Ms. Day spent roughly half an hour in each of her classes presenting a method for multiplying two binomial expressions in what she called a *workshop*. In PBI, a *workshop* usually refers to a student-requested session in which a teacher or fellow classmate directly teaches a concept that the student identifies as important to know. Although the students did occasionally ask clarifying questions during this workshop,

### FEATURE 3: AUTHENTIC COLLABORATION

Collaboration is a term that is used in a diverse array of contexts. Here, *authentic collaboration* entails individuals discussing and negotiating their own ideas to build a shared, coherent understanding of concepts and relationships (Hmelo-Silver & Barrows, 2008). Typical interactions among teachers and students follow the sequence of (1) someone *initiates* a question or problem—usually the teacher, (2) someone *responds*—usually a student, and (3) someone *evaluates* the validity of that response. This is referred to as the IRE (initate-respond-evaluate) structure that is commonly used throughout most classrooms (Cazden, 1986). Authentic collaboration, in contrast, does not follow this prescribed order of interactions and challenges the roles teachers and students play when they collaborate. For example, students may evaluate each others responses to questions, and they may even evaluate the meaning of the question before responding. Although designing an activity that intends for students to work in groups is a quality start, getting students to collaborate with genuine interest and motives takes some fostering on behalf

Table 8: Evidence of Feature 3: Authentic collaboration

| Teacher | Observed Implementation | Evidence of Teacher Beliefs from Interviews | Level of DL Observed |
|---|---|---|---|
| Anderson | • *Often* encouraged students to talk to group members<br>• Teacher created worksheets/activities that centered around group interactions<br>• Teacher asked several open-ended questions intended for students to discuss with group members<br>• Teacher occasionally collaborated with students, positioning herself as a learner<br>• Several students still worked individually on the worksheets<br>• Several students just checked their answers with group members without collaborating on the problem<br>• Students did not feel as though they learned a lot from their classmates (student surveys) | • Getting students familiar with group work and the expectations of collaboration takes a significant amount of preparation/practice at the start of the year<br>• It is important for the students to discuss work with each other before asking the teacher | *Moderate* |
| Brown | • Rare instances of students helping other students by explaining their solution<br>• Majority of students worked individually on all tasks<br>• Interactions between teacher and students were almost entirely directive (Teacher never took on a learner role)<br>• Students did not feel as though they learned a lot from their classmates (student surveys) | • Sometimes the teacher tries to engage the students in "meaningful discussions" | *Minimal* |
| Clark | • Students rarely discussed the activity except to ask questions about directions<br>• Most interaction happened between the teacher and individual students<br>• Students did not feel as though they learned a lot from their classmates (student surveys) | • Not mentioned | *Minimal* |
| Day | • Majority of class time was spent talking and working with classmates to either share ideas and solve problems collectively<br>• Teacher often collaborated with the students to foster a collegial work environment<br>• Students asked a significant number of meaning-seeking questions of their peers—very few were just interested in an answer<br>• Students felt as though they learn a lot from their classmates (student surveys) | • Even when students might seem off task, they may still be talking about the mathematics<br>• Students need to build meaningful relationships with their classmates in order to have a sense of trust and work toward a meaningful goal | *Full* |

of the teacher and students. The quality of the collaboration relies on whether a team of students is able to "build new and novel knowledge or improve shared conceptual understanding through such interactions" (Hämäläinen, R. & Vähäsantanen, K, 2008).

There were several instances in Ms. Anderson's and Ms. Day's classes in which students were strongly encouraged to work in dynamic collaboration with their peers. For example, Ms. Anderson would more often instruct students to talk about problems with their group members if they had a question that directly provide an answer, or even a hint. She would typically only provide a suggestion or hint *after* the students have discussed it to some extent with each other and were still struggling. Ms Day actually structured projects in her class so that students were *required* to depend on their group members to brainstorm ideas and critique each others reasoning. In fact, during the second observation, these students were starting a new project—"The Pool Problem"—and to form groups, Ms. Day had each student write down his or her strengths and weaknesses on a sticky note. After they had all finished, she grouped students with the attempt to fit strengths and weaknesses in groups so that they could use their self-reported strengths to help those who feel weak in those areas. Of course, some of the students' strengths and weaknesses didn't match up perfectly, but in most groups, there were at least a couple of areas in which one student felt weak that another student in that group reported as a strength.

These strategies employed by Ms. Anderson and Ms. Day to encourage authentic collaboration were quite different from the independent activities that were used in both Ms. Brown's and Ms. Clark's classes. In both classrooms, teachers used worksheets (that were to be completed individually) and notes as the primary activity in each lesson. Students took notes—usually copying as the teacher wrote on the same worksheet or notebook as the students—and then completed practice problems on worksheets. *Occassionally* students would check their answers with their classmates, but these interactions did not extend to discussions about the meaning of the problems, algorithms, or concepts.

### FEATURE 4: INCORPORATION OF TECHNOLOGY

IDL frameworks such as PBI emphasize the importance of technology and tools to extend students' conceptual understanding and apply this understanding in a variety of real-world contexts. Students can use technology to access real data on the internet to study patterns that are relevant to their local or even larger community (Krajcik & Blumenfeld, 2006). They can reach out to peers and even experts in the field through the use of networks, and use appropriate software to create models of complex systems (Krajcik & Blumenfeld, 2006). The distinction of this feature in IDL is that the incorporation of technology in these ways *extends* student abilities to explore the mathematical concepts as they relate to their world around them—in particular, in ways that would not be possible by other means.

As Table 9 shows, none of the teachers fully incorporated this feature in their classrooms. While Ms. Day was able to incorporate technology into the classroom to the greatest extent, the uses of the technology still heavily focused on representations or solutions that could have also been achieved through other means. For example, the first observation day included the students' presentations reviewing the concepts they had learned in the prior weeks. All of the students used some form of presentation software, but none of the students incorporated technological aspects that made the presentation significantly different from another means of presenting ideas such as displaying a poster. Ways in which the presentations *could* have incorporated technology more effectively would be to utilize examples of linear relationships they found online (appropriate to their individual topics) or to use software applications that allowed them to manipulate graphical representations of their linear systems to share with the class a different perspective on the relationship between various systems.

Table 9: Evidence of Feature 4: Incorporation of technology

| **Teacher** | **Observed Implementation** | **Evidence of Teacher Beliefs from Interviews** | **Level of DL Implementation Observed** |
|---|---|---|---|
| Anderson | • Only two instances in which students used a graphing calculator | • Not mentioned | *Minimal* |
| Brown | • Class set of graphing calculators were available, and most students used these to compute basic operations (most commonly, basic operations)<br>• Almost all students had a tablet computer either on their desk or underneath that they used for reasons not related to the activities | • Students can use calculators on their summative assessments, so they need to be familiar with how to do basic operations with the graphing calculators | *Moderate* |
| Clark | • Class set of graphing calculators were available, and most students used these to compute basic operations (most commonly, basic operations)<br>• Almost all students had a tablet computer either on their desk or underneath that they used for reasons not related to the activities | • Not mentioned | *Moderate* |
| Day | • Students used multiple technological resources to present their findings, share their ideas, and represent problems in multiple ways<br>• Occasional instances in which technology was used for entertainment purposes not related to lesson (e.g. instant messaging) | • Only mentioned how the computer stations were set up to establish group workspace in which they can collaborate easily | *Significant* |

In Ms. Anderson's classroom, there was very little use of technology—even the graphing calculators were rarely pulled from storage. In Ms. Brown's and Ms. Clark's classrooms, practically all of the students had a tablet computer that they either did not use at all or used for entertainment purposes not connected to the activities in the class—e.g. instant messaging and listening to music. These two teachers reminded students to keep these tablets tucked in their backpacks or under their desks during the activities, which created a perception of these tools as *distractions* during the classroom activities. Other than one instance in which a student used the device to compute an arithmetical calculation, these devices were not used to support the activities. Furthermore, none of the teachers mentioned the use of technology in their classrooms—except when Ms. Day mentioned that the computer stations were set up in a manner that distinguished the various groups so that the students were encouraged to collaborate with only their particular group members on their work.

### Overall implementation

The teachers showed compellingly different levels of implementation. Table 10 shows a summary of the levels at which each teacher implemented the four features of IDL.

Table 10: Overall Implementation of IDL by each Teacher

| **Teacher** | **1. Student-centered, constructive learning** | **2. Higher-order thinking and critical reasoning** | **3. Authentic collaboration** | **4. Technology-integration** |
|---|---|---|---|---|
| Anderson | *Moderate* | *Significant* | *Minimal* | *Minimal* |
| Brown | *Minimal* | *Minimal* | *Moderate* | *Moderate* |
| Clark | *Minimal* | *Minimal* | *Moderate* | *Moderate* |
| Day | *Full* | *Significant* | *Significant* | *Significant* |

## PERCEIVED BARRIERS TO FULL IMPLEMENTATION

According to Ms. Clark, the certification program portrays this perspective on teaching in a "perfect world," that is helpful for student teaching when teachers have the freedom to plan and implement a lesson that they believe in and that incorporates all of the components of more reform-based instruction. Clark claims that "actually being in the classroom now and teaching full-time, you realize how many other elements kind of become distractors as you're lesson-planning." She lists schedule changes, benchmark testing, end-of-course exams, conflicting schedules amongst the team of algebra teachers, and the structure that is enforced throughout the district. While she claimed that her students do engage in what she called “mini-projects,” she argued that semester-long projects just did not fit in with the way they are told to teach particular standards in the curriculum at specific times of the year.

Ms. Brown expressed several of the same concerns as Ms. Clark. Brown emphasized that she was expected to use all of the same assignments and worksheets as everyone else teaching Algebra I at the school. Despite this department organization, she explained how she still attempted to incorporate some elements of IDL or inquiry:

> I definitely have a bend towards inquiry-based, constructivism, and project-based instruction. It does influence my teaching, although I feel as though once you get into the classroom, especially maybe your first year, you’re somewhat conservative. It’s very hard, even for a good teacher to do a lot of that stuff. Sometimes…you may just not know a way to make it work. I guess it’s just not the way most teachers teach algebra. You have to be very committed to do it…I feel like I still find ways to sort of imbed bits of inquiry on the fly, like when I try to engage the class in a discussion about, ‘How do you think you would do this problem based on what you already know?’…but it’s not like all of the kids are going to engage in that conversation. They may just think, ‘This is not required so I don’t have to do it.’

Ms. Clark elaborates on the structure of the teamwork at their school:

> What we typically do is take ownership of a set of lessons and create those lessons and share them with the team, so if you were to go through each classroom…you would notice that the material would be exactly the same, maybe with a different style…but we do this so that you‘re not creating 187 days worth of lessons by yourself. So even if they look a little different from classroom to classroom, we try to make it as similar as possible…so that if a student from [Ms. Brown’s] class is moved here, into my classroom, they’re not completely thrown off.

Ms. Brown and Ms. Clark emphasized the structure of the lesson planning in their school as a barrier to implementing IDL in their classroom, yet both actually expressed many positive opinions about the structure. Not having to do everything individually, having more time for other responsibilities, and all being on the same page if issues arise were just some of the benefits of the structure that these teachers identified. The teachers expressed a certain amount of complacency and even appreciation of with the system. This reflects Ball's vision of teachers' past experiences in their own education that makes these more traditional teaching strategies more comfortable and easier. The teachers in this particular school, had just, in general, adopted a laissez faire attitude toward the way lesson planning and teaching is carried out in their classroom and throughout the school. Ms. Day, who was once a teacher at the high school, described her experience moving from this environment to her new position at the New Tech school:

> Whenever I used to plan, I would think more in terms of units, and I would have daily lessons, of course. And now here…I'm definitely really only thinking in terms of projects, but it's…just totally different. I'm thinking about how I can *rearrange* the [state standards] that I am told to teach in a way that's really going to give the students a deep conceptual understanding, but also give them that—the desire to actually learn all of that stuff that I think is interesting. That's a challenge, but I just don't feel like I was even able to do any of that when I was working at the other high school, in that traditional setting.

Ms. Day explained that she was still told by her administrators which standards she was expected to teach at various times of the year, but the expectation of the teachers in the school was just very different—they were *expected* to create projects and use more innovative teaching strategies. It was that expectation, in Ms. Day's opinion, that had the greatest influence on teachers' actual implementation of PBI and other IDL practices.

This sentiment is further iterated in Ms. Anderson's description of her school's structure for planning in quite the opposite setting. Anderson expressed how the *lack* of such expectations and support made it difficult for her to carry through with more IDL practices such as independent student research projects. She actually asserted a desire for these expectations from other teachers so that there would be a more powerful effort to incorporate more inquiry and active learning in the classrooms. Of course, this administrative support may come with new pressures and complications, but Ms. Day closed her interview with the following sentiment:

> I think the *best* thing—it's what keeps me sane—is knowing that if I need to just totally stop what I'm doing—like, say I realize that a project that I'm doing is just going badly, or the students just don't seem to be getting involved or interested—I can totally change it. I am not *bound* to someone else…telling me what to do. While my bosses do really care about me teaching the [state standards], there's just a certain level of respect and trust that I get from them, and that's what is just so different at this school.

### STUDENT SURVEY DATA

Because the survey was composed of two sections of items that reflected different aspects of student perceptions, this section is divided into sections to discuss (1) Section A of the student surveys, (2) Section B of the student surveys. In each section, the author first discusses the overall results and then talks about the individual constructs that make up the respective section of the survey. In the final section, the results from the factor analysis are analyzed in connection to the constructs that the author developed from the theoretical framework.

#### Section A of Student Surveys

Students in all classes exhibited similar views of mathematics and learning in their responses to Section A questions. Recall that the survey questions in this section were based on a 5-point Likert scale (-2=Strongly disagree, 0=Neutral, 2=Strongly agree). Figure 3 shows that most students did not express particularly strong agreement or disagreement with these questions. In particular, the responses across classrooms were much the same, with very few significant differences. One notable pattern that appears in all classrooms is the small variations (standard deviations) for both the *IDL Learning Style* and *IDL Perceived in Class*—meaning that very few students across all classrooms felt strongly about either one of these constructs. The *Comfort* and *Independence & Autonomy* constructs had slightly more agreement but also larger variance.

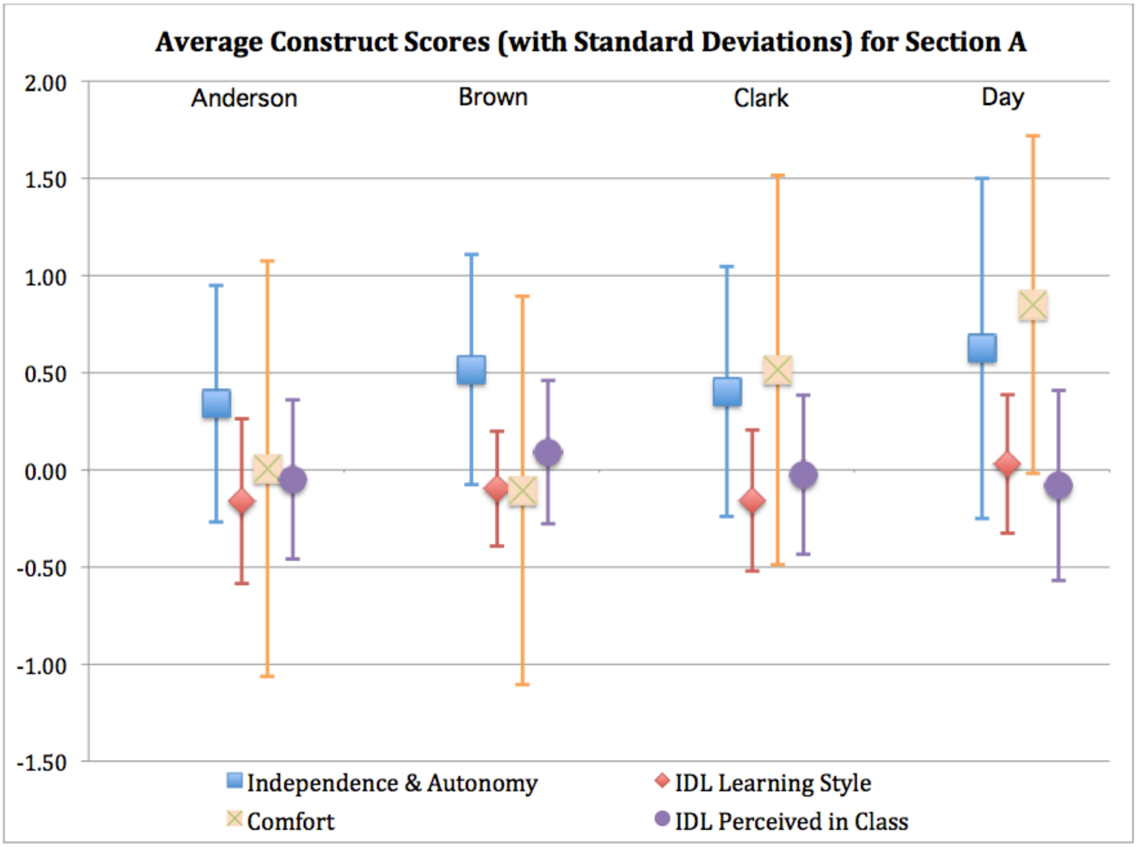



Figure 3: Average Construct Scores for Section A in Student Surveys.
*Note*. Student responses were based on a 5-point Likert scale (-2=Strongly disagree,0=Neutral, 2=Strongly agree).

The ANOVA on the constructs for Section A revealed that a statistically significant difference appeared only for the *Comfort* construct (See Table 11). The post hoc analysis determined that this significant difference was only between Ms. Day's score and each of the other three teachers. Table 12 shows the difference of means (in terms of *t* statistics) for the *Comfort* construct of Section A. The tables for the differences of means tests for the other constructs of Section A—which showed no significant differences—are included in Appendix C.

Table 11: ANOVA for Section A Constructs of Student Surveys

| | | **Sum of Squares** | **df** | **Mean Square** | **F** |
|---|---|---|---|---|---|
| **Independence & Autonomy** | Between Groups | 1.503 | 3 | 0.501 | 0.903 |
| | Within Groups | 65.443 | 118 | 0.555 | |
| | Total | 66.946 | 121 | | |
| **IDL Learning Style** | Between Groups | 0.643 | 3 | 0.214 | 1.476 |
| | Within Groups | 17.121 | 118 | 0.145 | |
| | Total | 17.763 | 121 | | |
| **Comfort** | Between Groups | 16.106 | 3 | 5.291 | 5.291** |
| | Within Groups | 119.741 | 118 | 1.015 | |
| | Total | 135.847 | 121 | | |
| **IDL Perceived in Class** | Between Groups | 0.388 | 3 | 0.129 | 0.735 |
| | Within Groups | 20.778 | 118 | 0.176 | |
| | Total | 21.166 | 121 | | |

* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level

Table 12: Difference of Means Between Teachers for *Comfort* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.1125<br>$t$=0.4420 | ----- | | |
| **Clark** | 0.5075*<br>$t$=2.1120 | 0.6200<br>$t$=2.1381 | ----- | |
| **Day** | 0.8408**<br>$t$=3.4172 | 0.9533**<br>$t$=3.2363 | 0.3333<br>$t$=1.1865 | ----- |

† Table includes mean differences and $t$ statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level

## Section B of Student Surveys

As mentioned previously, Section B of the student surveys was partitioned twice to reflect two different dimensions of the questions: (1) the *Participant* partition, which delineated who was involved in the activities and (2) the *Nature of Activity* partition, which delineated the *types* of the activities. In the analysis of these survey items, several significant differences appeared across the classes. As shown in Figure 4, Ms. Anderson's, Ms. Brown's, and Ms. Clark's students revealed similar patterns in their perspective of how often the activities they did in their classrooms involved individual work, interaction with the teacher, and interaction with their peers. These students' responses indicate that most often the activities were such that they worked independently, slightly fewer activities involved interaction with the teacher, and even fewer involved collaboration with their peers. In Ms. Day's classes, however, the pattern was almost reversed. These students still expressed that they sometimes (or often) worked individually, but they perceived slightly *more* interaction with their teacher and peers. The ANOVA results for this partition of Section B are shown in Table 13.

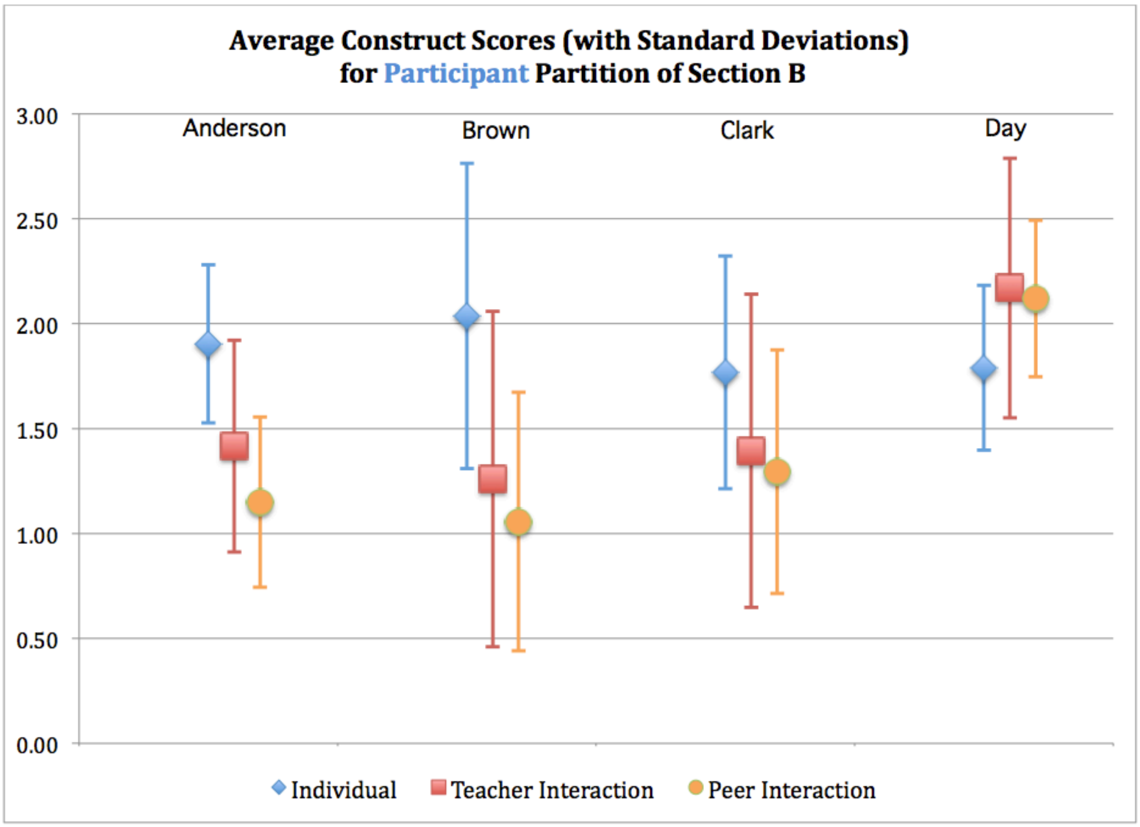



Figure 4: Average Construct Scores for *Participant* Partition of Section B in Student Surveys.
*Note*. The graph is cut off at zero because the survey responses ranged only from 0-3 (0=Never, 3=Very often).

Table 13: ANOVA for *Participant* Partition of Section B Constructs of Student Surveys

| | | Sum of Squares | df | Mean Square | F |
|---|---|---|---|---|---|
| **Individual** | Between Groups | 0.950 | 3 | 0.317 | 1.323 |
| | Within Groups | 26.346 | 110 | 0.240 | |
| | Total | 27.296 | 113 | | |
| **Teacher Interaction** | Between Groups | 11.826 | 3 | 3.942 | 9.752*** |
| | Within Groups | 42.444 | 105 | 0.404 | |
| | Total | 54.271 | 108 | | |
| **Peer Interaction** | Between Groups | 17.266 | 3 | 5.755 | 25.141*** |
| | Within Groups | 25.182 | 110 | 0.229 | |
| | Total | 42.448 | 113 | | |

* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

In contrast to the constructs in Section A of the surveys, the constructs in Section B consistently showed significant differences. In particular, the constructs of *Teacher Interaction* and *Peer Interaction* showed significant differences at the $p$=0.001 level. The following tables (Table 14 and Table 15) show the $t$ statistics for the means differences between each teacher. The mean differences for the *Individual Activities* construct is included in Appendix C, because it did not show significant differences in the ANOVA—i.e. students in all classes rated individual activities as happening at roughly the same frequency in their classes. Notice in both tables that the only post hoc tests that showed significant differences are those between Ms. Day and each of the other teachers. This supports the qualitative observations since Ms. Day's students were much more involved in collaborative learning with both the teacher and their peers.

Table 14: Difference of Means Between Teachers for *Teacher Interaction* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.1574<br>*t*=0.6188 | ----- | | |
| **Clark** | 0.0227<br>*t*=0.0946 | 0.1347<br>*t*=0.4647 | ----- | |
| **Day** | 0.7572**<br>*t*=3.0523 | 0.9147**<br>*t*=3.0840 | 0.7800**<br>*t*=2.7405 | ----- |

† Table includes mean differences (absolute values) and *t* statistics calculated for independent differences of means with unequal group sizes since the class sizes varied by teacher.
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

Table 15: Difference of Means Between Teachers for *Peer Interactions* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.0926<br>*t*=0.3639 | ----- | | |
| **Clark** | 0.1456<br>*t*=0.6060 | 0.2381<br>*t*=0.8218 | ----- | |
| **Day** | 0.9696***<br>*t*=3.9326 | 1.0622***<br>*t*=3.6008 | 0.8240**<br>*t*=2.9138 | ----- |

† Table includes mean differences and *t* statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

The other partition of Section B was the *Nature of Activity* partition. In this partition some *very* significant differences appeared across the classrooms, as shown in

Figure 5. Similar to the patterns that arose in the *Participant* partition, Ms. Day's scores exhibit a markedly different pattern from the other teachers. Students in all classes rated *Traditional/Procedural* items as happening very frequently. While all of the other teachers' scores for the remaining constructs were drastically lower, Ms. Day's scores for the *Basic DL* and *Technology-integration* were actually higher than the *Traditional/Procedural* construct score, and this teacher's *Innovative DL* score was not as substantially lower than the others.

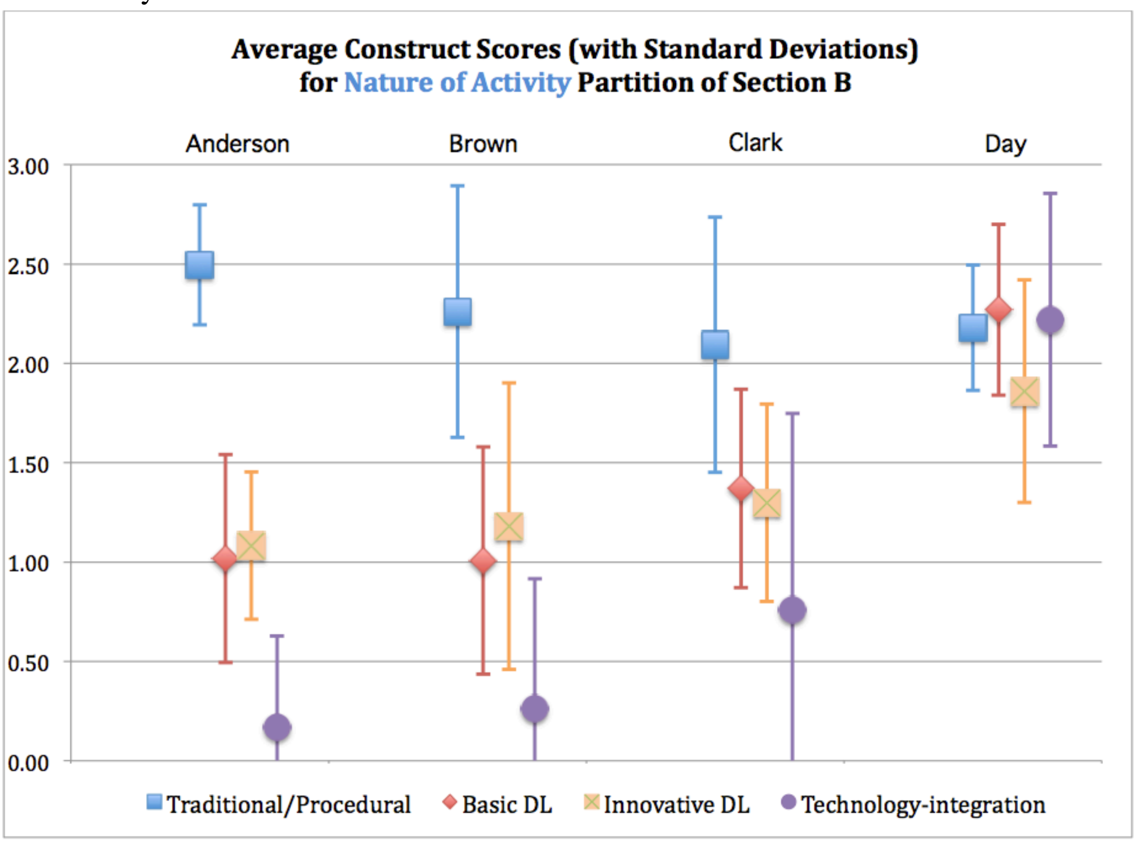



Figure 5: Average Construct Scores for *Nature of Activity* Partition of Section B in Student Surveys.
*Note*. The graph is cut off at zero because the survey responses ranged only from 0-3 (0=Never, 3=Very often).

Most notable is the contrast between Ms. Day's *Technology-integration* construct score, since the three other teachers' scores actually show that a significant amount of the students perceive technology-integration as inexistent (as interpreted from the fact that the standard deviations for these teachers actually extend below zero). This partition showed the greatest amount of significance in the ANOVA (see Table 16). All four constructs showed significant differences at the $p < 0.01$ level.

Table 16: ANOVA for *Participant* Partition of Section B Constructs of Student Surveys

| | | Sum of Squares | df | Mean Square | F |
|---|---|---|---|---|---|
| **Traditional/ Procedural** | Between Groups | 3.233 | 3 | 1.078 | 5.206** |
| | Within Groups | 22.977 | 111 | 0.207 | |
| | Total | 26.211 | 114 | | |
| **Basic DL** | Between Groups | 27.173 | 3 | 9.058 | 34.810*** |
| | Within Groups | 28.883 | 111 | 0.260 | |
| | Total | 56.055 | 114 | | |
| **Innovative DL** | Between Groups | 9.751 | 3 | 3.250 | 12.567*** |
| | Within Groups | 27.933 | 108 | 0.259 | |
| | Total | 37.685 | 111 | | |
| **Technology-integration** | Between Groups | 70.693 | 3 | 23.564 | 54.113*** |
| | Within Groups | 48.337 | 111 | 0.435 | |
| | Total | 119.030 | 114 | | |

* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

Table 17, Table 18, and Table 19 show the mean differences and *t* statistics for *Nature of Activity* partition for Section B. Again, the differences between Ms. Day's student construct scores and those of the other three teachers in this partition are exceptionally large.

Table 17: Difference of Means Between Teachers for *Basic DL* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.0101<br>*t*=0.0397 | ----- | | |
| **Clark** | 0.3516<br>*t*=1.4635 | 0.3617<br>*t*=1.2482 | ----- | |
| **Day** | 1.2516***<br>*t*=5.0730 | 1.2617***<br>*t*=4.2749 | 0.9000**<br>*t*=3.1804 | ----- |

† Table includes mean differences and *t* statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

Table 18: Difference of Means Between Teachers for *Innovative DL* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.0980<br>*t*=0.3853 | ----- | | |
| **Clark** | 0.2158<br>*t*=0.8985 | 0.2381<br>*t*=0.4066 | ----- | |
| **Day** | 0.7774**<br>*t*=3.1477 | 0.6794*<br>*t*=2.2989 | 0.5615*<br>*t*=1.9814 | ----- |

† Table includes mean differences and *t* statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

Table 19: Difference of Means Between Teachers for *Traditional/Procedural* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.2351<br>$t$=0.9244 | ----- | | |
| **Clark** | 0.4021*<br>$t$=1.6742 | 0.1670<br>$t$=0.5764 | ----- | |
| **Day** | 0.3152*<br>$t$=1.2791 | 0.0801<br>$t$=0.2728 | 0.0870<br>$t$=0.3056 | ----- |

† Table includes mean differences and $t$ statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

Table 20: Difference of Means Between Teachers for *Technology-integration* Construct†

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.0932<br>$t$=0.3662 | ----- | | |
| **Clark** | 0.5909*<br>$t$=2.4593 | 0.1670<br>$t$=1.7171 | ----- | |
| **Day** | 2.0474***<br>$t$=8.3060 | 1.9542***<br>$t$=6.6270 | 1.4565***<br>$t$=5.1518 | ----- |

† Table includes mean differences and $t$ statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).
* Significant at $p < 0.05$ level
** Significant at $p < 0.01$ level
*** Significant at $p < 0.001$ level

## Factor Analysis

The factor analysis was used to determine how the student survey questions loaded into constructs quantitatively. The following tables show the rotated component loadings (Varimax with Kaiser normalization), organized into the various components that emerged from Section A and Section B.

Table 21: Rotated Component Loadings from Factor Analysis on Section A of Student Surveys

| **Question** | Comp 1 | Comp 2 | Comp 3 | Comp 4 | Comp 5 | Comp 6 | Comp 7 |
|---|---|---|---|---|---|---|---|
| 4. I'm learning a lot about mathematical ideas. | .656 | | | | | | |
| 5. I learn a lot from doing practice problems (from the textbook or on worksheets). | -.724 | | | | | | |
| 11. I'm learning a lot of tricks on how to solve math problems. | -.741 | | | | | | |
| 19. I'm learning a lot of skills in this class that will help me in other classes. | .579 | | | | | | |
| 10. I feel rushed in this class (I don't have enough time to think things through). | | .717 | | | | | |
| 12. I feel comfortable asking for help when I don't understand a problem. | | .550 | | | | | |
| 16. I feel like there is too much work to do in this class. | | .783 | | | | | |
| 9. It is helpful when the teacher walks us through the problems step-by-step. | | | -.816 | | | | |
| 17. I learn a lot from listening to my teacher explain things in front of the whole class. | | | -.597 | | | | |
| 1. We have a classroom routine that we follow every day. | | | | -.657 | | | |
| 13. I learn a lot from doing projects or presentations. | | | | .583 | | | |
| 20. I learn a lot from listening to my teacher explain things one-on-one. | | | | -.639 | | | |
| 3. It is my responsibility to learn mathematics. | | | | | .704 | | |
| 6. I'm learning how to think for myself in this class. | | | | | .497 | | |
| 15. I learn a lot from my classmates. | | | | | .578 | | |
| 7. It is helpful when the teacher lets us figure problems out by ourselves. | | | | | | .502 | |
| 18.It is okay to disagree with my teacher about how to solve math problems. | | | | | | .800 | |
| 2. I learn a lot from reading explanations in a textbook. | | | | | | | .743 |
| 14. It is my teacher's responsibility to make sure that I learn mathematics. | | | | | | | .707 |
| Eigenvalue | 4.884 | 1.840 | 1.424 | 1.282 | 1.249 | 1.100 | 1.032 |
| % Variance | 13.192 | 10.955 | 10.389 | 8.910 | 8.555 | 6.131 | 5.925 |
| **Total Variance Explained** | | | | 64.058 | | | |

The component loadings for Section A match up fairly well with the delineation of the constructs in the Methodology chapter except for the constructs of *IDL Learning Style* and *IDL Perceived in Class*. Because these two constructs are so closely related, however, this mixture of the questions across the factor analysis components is not surprising. Figure 6 shows that the factor analysis did cluster all of the *Comfort* construct questions into a single component (Component 2), and only split up one question from the *Independence & Autonomy* construct between two components (Component 5 and Component 7). Therefore, the factor analysis does support the validity of these two constructs. This also provides further evidence that student responses varied quite widely for the constructs based on IDL practices.

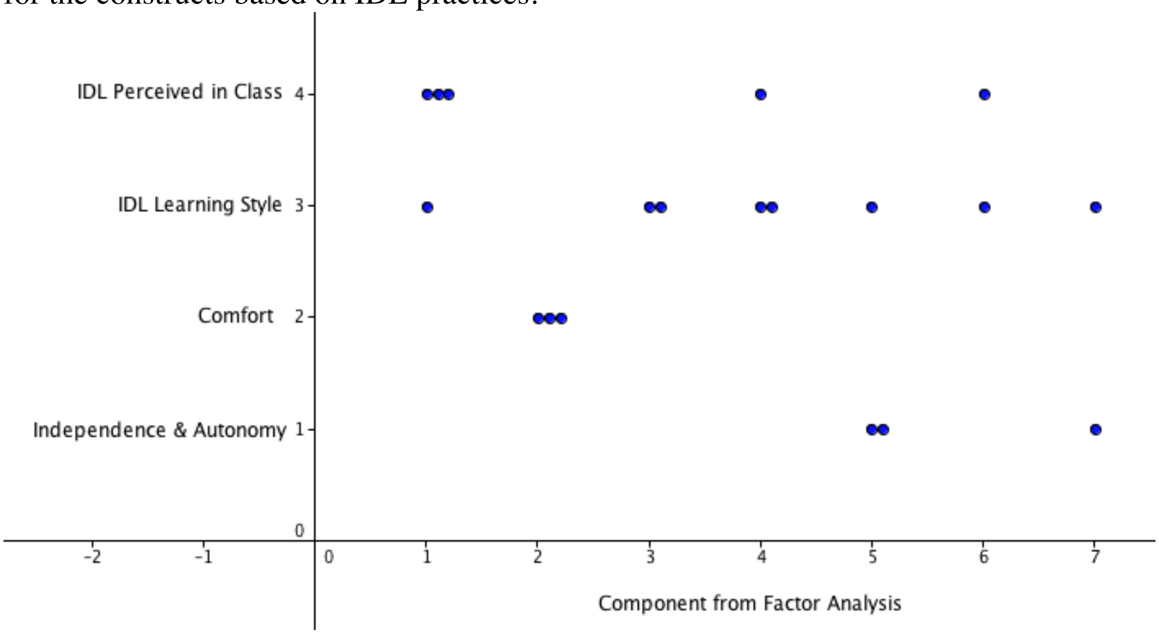



Figure 6: Comparison Between Component Loadings from Factor Analysis with Constructs
*Note*. Each point represents a question from Section A of the student surveys.

Table 22: Rotated Component Loadings from Factor Analysis on Section B of Student Surveys

| **Question** | Comp 1 | Comp 2 | Comp 3 | Comp 4 | Comp 5 | Comp 6 |
|---|---|---|---|---|---|---|
| 2. Use a computer to look up information | .747 | | | | | |
| 3. Reflect on what it means to learn and understand math | .565 | | | | | |
| 4. Work on a project with other classmates | .760 | | | | | |
| 5. Give a presentation (or speak in front of the class) | .691 | | | | | |
| 9. Use a computer to make models or representations | .688 | | | | | |
| 12. Talk to the teacher about things you like and dislike about the class | | .697 | | | | |
| 13. Talk about the importance of mathematics in the real world | | .732 | | | | |
| 14. Analyze other people's arguments | | .582 | | | | |
| 15. Make concept maps (or other diagrams to model what I am learning) | | .441 | | | | |
| 17. Prove why a mathematical statement is true | | .421 | | | | |
| 6. Talk about what I am learning with my classmates | | | .397 | | | |
| 11. Talk to my teacher about what I am learning | | | .571 | | | |
| 18. Learn something new from the teacher | | | .644 | | | |
| 19. Explain how I solved a problem | | | .708 | | | |
| 21. Discover something new on my own | | | .541 | | | |
| 7. Contact experts that work in fields related to what we are learning | | | | .668 | | |
| 10. Individually work on a project | | | | .768 | | |
| 20. Learn about what mathematicians do | | | | .502 | | |
| 8. Work on worksheets with other classmates | | | | | .734 | |
| 16. Take tests | | | | | .618 | |
| 22. Take quizzes | | | | | .758 | |
| 1. Individually work on worksheets | | | | | | .794 |
| Eigenvalue | 6.104 | 2.389 | 1.682 | 1.328 | 1.230 | 1.043 |
| % Variance | 15.521 | 12.273 | 10.258 | 9.261 | 9.148 | 6.157 |
| **Total %Variance Explained** | 62.617 | | | | | |

These component loadings reveal further evidence that students perceived particular questions as associated, even though they were intentionally not grouped together on the actual surveys. For example, students associated questions 8, 16, and 22, whereas the author associated these three questions (in addition to others) under the construct of *Traditional/Procedural* activities.

# Chapter 5: Discussion

The teachers involved in the study reflect a great variety of IDL implementation. Two important aspects that teachers referred to when discussing obstacles and struggles with implementing IDL practices are administrative and departmental support structures and pressure to teach to the test. Based on observations of classroom interactions and indirect comments made by teachers during the interviews, three more plausible conflicts that inhibit more complete implementation are the lack of access to quality *ongoing* professional development, the lack of access to appropriate technology-focused professional development, and teachers' traditional orientations toward mathematics and learning. In order to discuss each issue and its implications to mathematics education, this chapter is divided into sections to focus on the matters of (1) the pressure of standardized testing, (2) technology-focused professional development, (3) ongoing professional development, (4) teacher orientations toward mathematics and learning, (4) student perspectives and orientations, and (5) school culture. Finally, some limitations and suggestions for further research are discussed in the final section of this chapter.

## THE PRESSURE OF STANDARDIZED TESTING

Research has shown the importance of a supportive and knowledgeable school principal and other forms of administrative support in influencing successful implementation of IDL practices such as inquiry-based instruction (Towers, 2012). The teachers involved in this study all consistently pointed out various attributes of their administrative and departmental structures and how these factors inhibit or contribute to their successful execution of IDL practices. Much of the support (and lack of support) stems from pressures locally and nationally for students to perform on standardized tests (Hochberg & Desimone, 2010). Ms. Anderson and Ms. Day, especially, expressed as a great concern the balance between basing our evaluation of student gains on standardized test scores and assessing more rigorous and conceptual learning.

While Ms. Anderson and Ms. Day were heavily concerned with these tests and deep learning, Ms. Brown and Ms. Clark were overly concerned with “the coverage issue,” as discussed by Yoshinobu and Jones (2012). Mirroring some of the examples of teacher remarks about “the coverage issue” discussed by Yoshinobu and Jones, Ms. Brown describes her own struggle after being asked how she incorporates innovative features of learning in her classroom:

> Sometimes we don’t get time. Lately, we’ve been very crunched for time with what is prescribed by—everything that we need to cover by the end of the year. So we’re just…going way too fast to do very much of that.

This was a major concern for these teachers, and this concern is reflected in reports involving many other teachers (Hochberg & Desimone, 2010; Valli & Buese, 2007). These teachers have identified the pressure to generate swift improvement of student test scores and “staying on schedule with district instructional pacing guides” as the primary sources of this stress (Hochberg & Desimone, 2010).

> If curriculum changes are not made carefully with adequate planning and support, we risk a political backlash that favors back-to-basics and rote learning over authentic inquiry (Barron et al., 1998).

### TECHNOLOGY-FOCUSED PROFESSIONAL DEVELOPMENT

IDL and any other innovative teaching framework gains more potential strength if technological tools are incorporated in meaningful ways (Leiken & Grossman, 2013). In particular, technology can help teachers more strategically attack struggles with implementing PBI and other IDL frameworks such as generating more student interest in the subject of mathematics, providing opportunities for teachers and students to access a larger amount of potentially useful information, and aiding in the production of artifacts (Blumenfeld Soloway, Marx, Krajcik, Guzdial, & Palincsar, 1991).

The findings from this study reveal that teachers still struggle with fully implementing practices of IDL in their classrooms. Even in a school that has taken great effort to implement PBI in all of its classrooms, not all four features of IDL were fully brought to fruition. In particular, the feature of authentic incorporation of technology was particularly lacking in the observed classes. The rapid changes that occur in the field of technology from year to year likely intimidate several teachers, but this is an area in which professional development and teacher preparation experts should invest not only to improve mathematics instruction, but also students' comfort and expertise in a quickly changing society that relies greatly on technological advancement. Instead of just having tools available, teachers can learn how the tools could be used to broaden their own conceptualization of the mathematical concepts, as well as that of their students.

### ONGOING PROFESSIONAL DEVELOPMENT

"Although tools to help teachers prepare for problem- and project-based learning are important, support is also needed as teachers carry out problem- and project-based work" (Blumenfeld et al., 1991; Polly & Hannafin, 2011). Various formats for professional development that could be especially effective for IDL practices include professional learning communities, online discussion groups in which teachers can collaborate about experiences with the various types of IDL practices, and specialized lesson studies in which teachers collaboratively evaluate projects that they intend to use in their classroom (Blumenfeld et al., 1991; Lewis, Perry, & Murata, 2006; Loucks-Horsley et al., 2010).

A major challenge that has strained those advocating more innovative teaching practices is the difficulty of facilitating supportive environments for the teachers to develop and refine the necessary knowledge and skills for full implementation of IDL features (Blumenfeld et al., 1991). The practices associated with IDL can be quite overwhelming for many teachers to take on. An example of a common struggle that has been observed among teachers is that they will often not provide ample time for students to engage in authentic collaborate (Blumenfeld et al., 1991; Hmelo-Silver & Barrows, 2008). Perhaps one explanation for this struggle is that teachers are unaware of strategies that are appropriate and beneficial for facilitating collaborate knowledge building (Hmelo-Silver & Barrows, 2008). However, Kracjik & Blumenfeld offer another explanation:

> [It] might be that teachers don't see collaboration as essential to the meaning making process. This reason, unfortunately, is much harder to overcome, because it lies at teachers' belief about what fosters understanding (2006).

**TEACHER ORIENTATIONS TOWARD MATHEMATICS AND LEARNING**

As shown by the student survey responses and in the interviews with the teachers, students and teachers have deeply engrained orientations toward mathematics as a discipline and mathematics as a school subject. These deeply seeded attitudes toward mathematics greatly influence teacher practices in both positive and negative manners. As Ball asserted, preservice teachers "have developed a web of interconnected ideas about subject matter, about teaching and learning, and about schools" well before they enter any certification program (1988). Although all four teachers received their bachelor's degree in mathematics and teacher certification program through the same university—taking courses from many of the same professors and particularly engaging in rigorous mathematics courses that employed the *Modified Moore Method*—teachers tended to talk about mathematics and learning differently in their interviews.

Ms. Brown and Ms. Clark focused heavily on students' ability to *organize* their work in their notebooks and their ability memorize properties and algorithms that will help them perform on standardized tests. Ms. Anderson and Ms. Day, on the other hand, consistently emphasized their desire to have students understanding *deeply* the concepts such as *why* an answer to a problem makes sense and in what ways a graphical representation of an inequality is helpful.

Although at the start of the study, all four teachers expressed their familiarity and claimed that their teaching incorporated at least to some extent the practices of IBI, PBI, and other reform-oriented teaching frameworks that exemplify the features of IDL, three of the teachers (Ms. Anderson, Ms. Brown, and Ms. Clark) did not feel as confident about the extent to which they were implementing these pedagogical principles. Even in this lack of confidence, teachers pointed to particular practices such as "engaging students in discussion" or "asking students *why*" as ways in which they *do* practice IDL. Their remarks both in the interviews and in their teaching indicate that these teachers would like to more fully implement inquiry and discovery. The barriers that they perceived, however, kept them from fully carrying out this aspiration.

Ms. Brown particularly pointed out in her interview that years of experience can also greatly impact a teacher's zeal and ability to implement IDL practices. She argued that in the future she would be more likely to implement more reform-based practices. Although first-year teachers may have more pressure to "be conservative" (as Ms. Brown said) and less experience to incorporate the various IDL features, even veteran teachers still express attitudes about mathematics and learning that are just incompatible with the IDL mindset. In particular, the district in which Ms. Brown, Ms. Clark, and Ms. Day have high rates of poverty and the district has historically had low academic performance on state tests in mathematics (TEA, 2012). See Appendix E to see how the two districts compare.[2] The students in Ms. Brown's class, in fact, were tracked together based on their unsatisfactory scores on the middle school standardized state assessments. This was an issue that the teacher did not directly imply as a barrier to implementation, but it was reiterated several times throughout the interview. In contrast, Ms. Day—who taught in the same district—did not bring up student ability as an issue. These beliefs about student abilities and who can do mathematics—in particular, authentic, engaging mathematics—obviously differ between these teachers.

Certification and professional development programs have only large amounts to gain by investing in support systems that provide genuine opportunities for teachers to discuss and analyze these deeply held beliefs and convictions about the nature of teaching and mathematics as a field of expertise. Once teachers confront their own beliefs, they can better develop a more in-depth understanding of mathematical concepts and an inclination to more critical reasoning and pattern-seeking. These perspectives can then be shared with their students and authentic discovery encouraged.

### STUDENT PERSPECTIVES AND ORIENTATIONS

The data revealed large differences between students' perceptions of what types of activities were occurring in their classrooms, yet there were not large, statistically significant differences on the measures of students' orientations toward IDL practices. Students in all classes greatly valued their teacher's direction—many agreeing and even strongly agreeing with the statement, "It is helpful when the teacher walks us through the problems step-by-step." Students in the New Tech classroom engaged more in higher-

[2] Actual names of the districts are not provided.

order reasoning, yet they still spent some time focusing on finding *the* correct answer to many problems or questions. While this classroom is an exemplary case of how PBI or other IDL frameworks can work in a high school setting, this study shows that teachers and students still need some sort of push to focus more on conceptual understanding and authentic collaboration that is genuine and student-directed. As shown in the various descriptions of the classroom interactions many students lack familiarity with the more innovative features of discovery learning. For example, even in environments that encouraged authentic collaboration, many students still worked on their own. Working on an activity that lent itself to higher-order thinking and critical reasoning, students still allocated a significant amount of emphasis on finding *the* correct answer and less on conceptual understanding.

While the qualitative analysis of the interviews and observations revealed quite different levels of implementation between all four teachers, student perspectives of classroom practices did not always reflect these differences. For example, Ms. Anderson's implementation of Feature 2 (higher-order thinking and critical reasoning) was significant, yet this teacher's students reported lower levels of agreement with DL principles. These students typically did not feel that they often discovered new things on their own, analyzed arguments, or felt a strong personal responsibility to learn mathematics. School culture may be one possible reason for this contrast between observation and student survey results.

### SCHOOL CULTURE

One possible explanation for such drastic differences in student perceptions between Ms. Day's classroom and the other three is the strong reform-based mindset that encompasses the entire student body and faculty at Graham New Tech High School.

Students are there to learn and *actively* participate, whereas this frame of mind was lacking at both Graham High School and Spring Middle School. This suggests that in order to have the greatest impact on *student* attitude and orientation toward IDL, the school culture must be compatible with the IDL framework—not just individual classrooms (Blumenfeld et. al., 1991; KnowledgeWorks, 2011). Although many districts are adopting the New Tech Network Model ("New Tech High Model is Spreading to More Schools", 2009), it is important for *all* teachers and administrators to take away valuable lessons about the overall culture that is conveyed at their school. Changing various aspects about students' daily activities and expectations can help students to be more willing and prepared to engage more with the assortment of conceptual knowledge they encounter.

### LIMITATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Although, the results from the student surveys were not intended to be generalizable to larger student populations, it is important to point out that the constructs were not pilot tested before use in this study. Constructs were developed based on the author's intent of the questions, and students may have interpreted questions slightly differently. Because of this, the individual student scores, and hence the teacher averages, may not accurately reflect perceptions of the classroom dynamics.

The number of students (N=142) affords another limitation to the claims about student perceptions and orientations. More research is needed to further investigate the distinction between the way students perceive IDL features in their classroom—in particular, how it effects their view of mathematics as both a theoretical and applicable body of knowledge and as a socially-constructed structure. In addition to this, the another limitation is that this study did not take into account the difference in overall achievement statistics between the schools and districts included. It may be that "student ability" as defined by standardized assessment scores has a significant impact on the level at which a teacher feels comfortable implementing IDL features. This, of course, is reliant on teacher orientation toward learning and mathematics. Since the purpose of training teachers in the use of frameworks such as IBI and PBI is to make mathematics and science more approachable, interesting, and valuable to *all* students, this limitation should ideally not be a significant issue.

Since the number of observations was limited, the observed practices may not reflect typical practices that the teachers used on a daily basis. In particular, the level of technology incorporation throughout the classes may drastically change from day to day based on the aim of the teachers' lesson plans, so this may be one feature that could have been markedly different if observations took place on particular days of the year. However, as mentioned previously, the criteria provided to the teachers for selecting observation dates actually increased the likelihood of the author observing *more* IDL features than would be observed on average any other day of the year.

Further research needs to be conducted to more accurately portray the typical day-to-day activities and interactions that occur in teachers' classrooms. Of course, this small study involved only four teachers; therefore, generalizable claims are not of much merit in analyzing the IDL implementation levels of a larger population of teachers. However, because these teachers were selected for this study on the basis of recommendations from professors and administrators in this nationally-recognized preparation program, this study does serve as a valuable indication that other teachers *likely* struggle with similar conflicts involving their orientations toward mathematics and learning, their individual school environment and support, and areas within their content and pedagogical knowledge that could be strengthened. Ensuring that teachers not only confront these conflicts throughout their preservice training, but continue to tackle these issues throughout their careers with the support of professional developers and their own individual teacher colleagues.

## CONCLUSION

All four teachers expressed familiarity with and commitment to reform-oriented practices such as inquiry and constructivism that permeate IDL, and certainly experienced inquiry-based learning as students themselves in their undergraduate program. However, only *one* teacher—the one teaching in a New Tech high school that was structured on the framework of project-based instruction (PBI)—showed consistent differences in both student perspectives of IDL and observed implementation of IDL. The teachers attributed their struggle with full implementation to factors such as administrative support, the pressure of standardized tests, and experience in the classroom. The author adds to this list—based on observations of classroom interactions and indirect teacher comments during interviews—the teachers' and students' orientation toward mathematics and IDL, ongoing professional development that emphasizes comprehensive implementation, and technological training and support.

The importance of a supportive and engaging school culture is exemplified by this study because it shows how teachers with almost identical teacher preparation backgrounds showed quite astounding differences in their teaching practices. It could be argued that moving any one of the other three teachers into the New Tech Framework environment would not only reveal to them the contrast between their current teaching practice and more authentic student engagement and discovery, it could allow them the *opportunity* and *resources* to more fully implement these more innovative and engaging pedagogical practices.

# Appendix A: Teacher Interview Questions

## TEACHER INTERVIEW QUESTIONS

1. What are some of the most important factors that effect how well your lesson goes as you plan?
2. How can you tell when students are engaged with the mathematics or not?
3. How would you describe the homework you assign? Why do you assign this sort of work/activities?
4. Do you plan particular opportunities during the lesson for you to formatively assess student understanding?
5. What are some limitations that you face when assessing student understanding (formally and informally)?
6. How does a student's prior understanding effect what you do in your classroom?
7. Do you feel that it is important for students to talk in front of the class or in a group about their ideas in a mathematics class? Or is it more appropriate that they do this on their own?
8. Do you think your students are capable of writing mathematical proofs or justifications in this class?
9. Should more responsibility be put on you or your students for their learning of Algebra I topics?
10. Would you say that your Algebra I class is difficult for the average student?
11. If your students were to take away one thing from your class, what would you want them to understand?

# Appendix B: IDL Implementation Rubric

| IDL Feature | Minimal Implementation | Moderate Implementation | Significant Implementation | Full Implementation |
| --- | --- | --- | --- | --- |
| Student-centered, active learning | Students rarely explore their own questions, ideas, or interests; Majority of classroom interactions involve teacher-prescribed questions and strategies for solving those problems | Students occasionally encouraged to come up with their own strategies for problem situations, but the activities are still prescriptive heavily guided with teacher-selected goals and solution methods | Activities are deliberately designed for students to develop their own solution strategies and represent the problem in multiple ways; Teacher provides occasional direct instruction | Activities designed to provide students complete freedom to explore the concepts with enough scaffolding to guide thinking toward a common goal, teacher provides minimal direct instruction |
| Higher-order thinking and critical reasoning | Students learn and memorize tricks and algorithms that they can use to solve problems on standardized tests | Students learn algorithms for problem solving that can be used on exams, but students are occasionally asked to explain their reasoning and discuss how particular strategies make sense in different situations | Students develop critical reasoning skills that they can use to analyze the logic and purpose of various strategies; Students are occasionally asked to reflect on their own understanding; Students explicitly make connections among different concepts within mathematics | Students develop critical reasoning skills that they use to analyze the logic and purpose of various strategies and concepts; Students apply their understanding and skills in novel situations; Students explicitly make connections among different concepts within mathematics and other subject areas |
| Authentic collaboration | Insufficient amount of group work | Students occasionally work in groups, but much of the collaboration is focused on superficial interactions such as checking answers and passively sharing information | Students occasionally work in groups and share responsibility for building a coherent understanding of the concepts; Students ask each other meaning-seeking questions rather than just asking for answers | Students often work in groups and share responsibility for building a coherent understanding of the concepts; Students ask each other meaning-seeking questions rather than just asking for answers; Students build off of each others' strengths to expand their individual proficiency |
| Incorporation of technology | Insufficient use or minimal access to technology | Some access and use of technology, but only in ways that do little to extend student understanding | Teachers and students incorporate the use of technological tools that involve multiple ways of representing and sharing ideas | Teachers and students incorporate the use of technological tools that involve extending student understanding in ways that are not possible otherwise |

# Appendix C: Other Difference of Means Tables†

These constructs showed no statistical significance at $p \leq 0.05$.
† $t$ statistics for independent differences of means with unequal group sizes (since the class sizes varied by teacher).

Difference of Means Between Teachers for *Independence & Autonomy* Construct

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.6896 | ----- | | |
| **Clark** | 0.2624 | 0.3878 | ----- | |
| **Day** | 1.1556 | 0.3720 | 0.7847 | ----- |

Difference of Means Between Teachers for *IDL Learning Style* Construct

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.2535 | ----- | | |
| **Clark** | 0.0107 | 0.2136 | ----- | |
| **Day** | 0.7742 | 0.4288 | 0.6658 | ----- |

Difference of Means Between Teachers for *IDL Perceived in Class* Construct

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.5540 | ----- | | |
| **Clark** | 0.1041 | 0.4001 | ----- | |
| **Day** | 0.1216 | 0.5789 | 0.1943 | ----- |

Difference of Means Between Teachers for *Individual Activities* Construct

| | **Anderson** | **Brown** | **Clark** | **Day** |
|---|---|---|---|---|
| **Anderson** | ----- | | | |
| **Brown** | 0.5235 | ----- | | |
| **Clark** | 0.5640 | 0.9271 | ----- | |
| **Day** | 0.4606 | 0.8360 | 0.0771 | ----- |

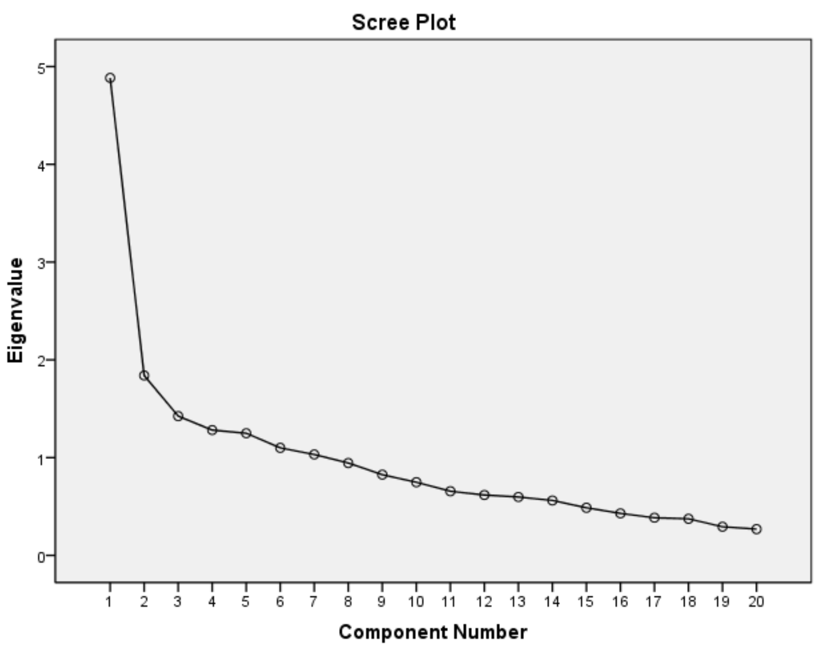
# Appendix D: Raw Results from Factor Analysis

*Section A:*

**Total Variance Explained**

| Component | Initial Eigenvalues | | | Rotation Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 4.884 | 24.420 | 24.420 | 2.638 | 13.192 | 13.192 |
| 2 | 1.840 | 9.200 | 33.620 | 2.191 | 10.955 | 24.147 |
| 3 | 1.424 | 7.122 | 40.743 | 2.078 | 10.389 | 34.536 |
| 4 | 1.282 | 6.408 | 47.151 | 1.782 | 8.910 | 43.446 |
| 5 | 1.249 | 6.246 | 53.397 | 1.711 | 8.555 | 52.001 |
| 6 | 1.100 | 5.499 | 58.897 | 1.226 | 6.131 | 58.132 |
| 7 | 1.032 | 5.161 | 64.058 | 1.185 | 5.925 | 64.058 |
| 8 | .945 | 4.723 | 68.781 | | | |
| 9 | .826 | 4.128 | 72.909 | | | |
| 10 | .748 | 3.741 | 76.650 | | | |
| 11 | .656 | 3.279 | 79.929 | | | |
| 12 | .618 | 3.091 | 83.020 | | | |
| 13 | .597 | 2.986 | 86.006 | | | |
| 14 | .562 | 2.809 | 88.815 | | | |
| 15 | .487 | 2.434 | 91.249 | | | |
| 16 | .429 | 2.147 | 93.396 | | | |
| 17 | .385 | 1.925 | 95.321 | | | |
| 18 | .374 | 1.872 | 97.193 | | | |
| 19 | .293 | 1.464 | 98.657 | | | |
| 20 | .269 | 1.343 | 100.000 | | | |

Extraction Method: Principal Component Analysis.



**Rotated Component Matrix[a]**

| | Component | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| A1 | -.188 | -.114 | -.104 | -.657 | -.040 | -.261 | .108 |
| A2 | -.285 | .020 | .064 | -.022 | -.232 | -.123 | .743 |
| A3 | .054 | .227 | -.024 | .210 | .704 | .127 | .038 |
| A4 | .656 | .242 | .161 | .357 | .140 | .090 | .083 |
| A5 | -.724 | .076 | .019 | .090 | -.212 | .053 | .075 |
| A6 | .423 | .120 | .185 | -.162 | .497 | .060 | -.143 |
| A7 | .154 | .349 | .470 | -.167 | .128 | .502 | -.069 |
| A8 | .258 | .270 | .550 | .193 | -.174 | .144 | -.053 |
| A9 | .049 | -.018 | -.816 | -.062 | -.159 | .057 | .000 |
| A10 | .239 | .717 | .163 | -.059 | -.050 | -.023 | .047 |
| A11 | -.741 | -.137 | -.196 | -.205 | .083 | .053 | .001 |
| A12 | .298 | .570 | .329 | .205 | .232 | -.105 | .037 |
| A13 | .112 | .051 | -.137 | .583 | .551 | .014 | -.188 |
| A14 | .274 | .019 | -.138 | -.115 | .178 | .108 | .707 |
| A15 | .118 | -.393 | .396 | .058 | .578 | -.142 | .094 |
| A16 | -.048 | .783 | .039 | .089 | .116 | -.055 | -.016 |
| A17 | -.392 | -.278 | -.597 | -.144 | .026 | -.003 | .068 |
| A18 | -.055 | -.182 | -.024 | .099 | .050 | .800 | .013 |
| A19 | .579 | .326 | .077 | .432 | .144 | .125 | .164 |
| A20 | -.039 | .092 | -.282 | -.639 | -.084 | .363 | -.003 |

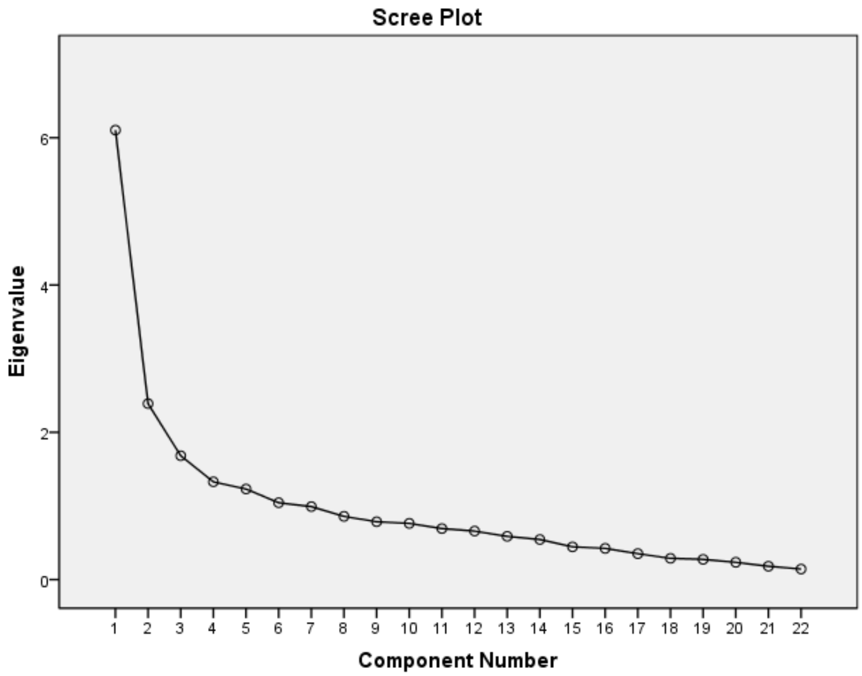
Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 15 iterations.

*Section B*:

**Total Variance Explained**

| | Initial Eigenvalues | | | Rotation Sums of Squared Loadings | | |
|---|---|---|---|---|---|---|
| Component | Total | % of Variance | Cumulative % | Total | % of Variance | Cumulative % |
| 1 | 6.104 | 27.745 | 27.745 | 3.415 | 15.521 | 15.521 |
| 2 | 2.389 | 10.859 | 38.605 | 2.700 | 12.273 | 27.794 |
| 3 | 1.682 | 7.644 | 46.249 | 2.257 | 10.258 | 38.051 |
| 4 | 1.328 | 6.035 | 52.284 | 2.037 | 9.261 | 47.312 |
| 5 | 1.230 | 5.590 | 57.874 | 2.013 | 9.148 | 56.460 |
| 6 | 1.043 | 4.743 | 62.617 | 1.354 | 6.157 | 62.617 |
| 7 | .991 | 4.506 | 67.123 | | | |
| 8 | .859 | 3.904 | 71.027 | | | |
| 9 | .785 | 3.569 | 74.596 | | | |
| 10 | .763 | 3.470 | 78.066 | | | |
| 11 | .693 | 3.148 | 81.215 | | | |
| 12 | .658 | 2.993 | 84.208 | | | |
| 13 | .587 | 2.667 | 86.875 | | | |
| 14 | .544 | 2.474 | 89.348 | | | |
| 15 | .445 | 2.022 | 91.370 | | | |
| 16 | .424 | 1.927 | 93.297 | | | |
| 17 | .352 | 1.599 | 94.896 | | | |
| 18 | .288 | 1.309 | 96.205 | | | |
| 19 | .275 | 1.250 | 97.455 | | | |
| 20 | .235 | 1.068 | 98.524 | | | |
| 21 | .181 | .824 | 99.348 | | | |
| 22 | .144 | .652 | 100.000 | | | |

Extraction Method: Principal Component Analysis.




**Rotated Component Matrix[a]**

| | Component | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| B1 | -.227 | .123 | -.213 | .077 | .113 | .794 |
| B2 | .747 | .202 | .111 | .209 | -.109 | -.100 |
| B3 | .565 | -.028 | .336 | .143 | .018 | .516 |
| B4 | .760 | .141 | .092 | .216 | .097 | -.074 |
| B5 | .691 | .345 | .097 | -.006 | -.271 | .007 |
| B6 | .135 | .355 | .397 | -.194 | .315 | -.306 |
| B7 | .226 | .315 | -.060 | .668 | -.128 | .022 |
| B8 | .347 | -.025 | -.045 | -.138 | .734 | .075 |
| B9 | .688 | .434 | .192 | .219 | -.122 | -.036 |
| B10 | .127 | .083 | -.020 | .768 | .010 | .064 |
| B11 | .309 | .331 | .571 | .146 | .116 | -.010 |
| B12 | .043 | .697 | .236 | .306 | -.111 | .095 |
| B13 | .213 | .732 | .027 | .080 | .102 | -.034 |
| B14 | .351 | .582 | .051 | .079 | .028 | .224 |
| B15 | .348 | .441 | .074 | .140 | -.169 | .403 |
| B16 | -.290 | .015 | .187 | -.062 | .618 | .083 |
| B17 | .251 | .421 | .276 | .236 | .296 | -.123 |
| B18 | .090 | -.044 | .644 | .062 | .052 | -.156 |
| B19 | -.027 | .379 | .708 | -.240 | -.055 | .066 |
| B20 | .203 | .300 | .393 | .502 | -.216 | .184 |
| B21 | .160 | -.063 | .541 | .500 | .270 | .149 |
| B22 | -.376 | .039 | .035 | .096 | .758 | -.084 |

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

# Appendix E: District Comparison Data

Texas Education Agency, 2012

| | Total number of students | Percent Economically Disadvantaged | Percent meeting 2010-11 Accountability Standards in Mathematics |
|---|---|---|---|
| **District A (Anderson)** | ~32,000 | 22.5% | 92.0% |
| **District B (Brown, Clark, & Day)** | ~7,000 | 79.4% | 77.0% |
| **State Average** | n/a | 59.2 | 84.0% |

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