# Transition probabilities of astrophysical interest in the niobium ions $\mathbf{N b}^{+}$and $\mathbf{N b}^{2+}$ 

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#### Abstract

Aims. We attempt to derive accurate transition probabilities for astrophysically interesting spectral lines of Nb II and Nb III and determine the niobium abundance in the Sun and metal-poor stars rich in neutron-capture elements. Methods. We used the time-resolved laser-induced fluorescence technique to measure radiative lifetimes in Nb II. Branching fractions were measured from spectra recorded using Fourier transform spectroscopy. The radiative lifetimes and the branching fractions were combined yielding transition probabilities. In addition, we calculated lifetimes and transition probablities in Nb II and Nb III using a relativistic Hartree-Fock method that includes core polarization. Abundances of the sun and five metal-poor stars were derived using synthetic spectra calculated with the MOOG code, including hyperfine broadening of the lines. Results. We present laboratory measurements of 17 radiative lifetimes in Nb II. By combining these lifetimes with branching fractions for lines depopulating the levels, we derive the transition probabilities of 107 Nb II lines from $4 \mathrm{~d}^{3} 5 \mathrm{p}$ configuration in the wavelength region $2240-4700 \AA$. For the first time, we present theoretical transition probabilities of 76 Nb III transitions with wavelengths in the range $1430-3140 \AA$. The derived solar photospheric niobium abundance $\log \epsilon_{\odot}=1.44 \pm 0.06$ is in agreement with the meteoritic value. The stellar $\mathrm{Nb} / \mathrm{Eu}$ abundance ratio determined for five metal-poor stars confirms that the $r$-process is a dominant production method for the $n$-capture elements in these stars.


Key words. atomic data - atomic processes - Sun: abundances - stars: abundances

## 1. Introduction

About half of the stable nuclei heavier than iron are believed to be synthesized by means of a slow neutron-capture process (the " $s$-process") in the late stages of the evolution of stars with masses in the range $0.8-8 M_{\text {Sun }}$. This process occurs when the star is in the "asymptotic giant branch" (AGB) phase of its life. During this phase, the star experiences a series of thermal pulses, which consist of recurrent thermal instabilities that lead to rich nucleosynthesis by means of the $s$-process. After each thermal pulse, the deep convective envelope of the star penetrates the region where $s$-process elements have been produced and brings them to the stellar surface, where they become observable (see e.g., Lugaro et al. 2003).

The $s$-process elements technetium, niobium, and ruthenium are particularly interesting because they give information about the timescales involved in addition providing insight into the $s$-process. Niobium is not produced directly by the $s$-process (unless the neutron density is extremely low), and the niobium abundance is probably dominated by ${ }^{93} \mathrm{Nb}$ from the decay of ${ }^{93} \mathrm{Zr}$ (generated by the $s$-process) with a lifetime of about $1.5 \times 10^{6}$ years (see e.g., Allen \& Porto de Mello 2007).

The abundance of technetium, niobium, and ruthenium are important for constraining evolutionary lifetimes during the thermally-pulsing AGB phase. However, only one study was
devoted to the time evolution of these elements in stars. Wallerstein \& Dominy (1988) investigated the technetium, niobium, and zirconium abundances in a sample of AGB M and MS stars and derived timescales of the shell flashes. The abundances were derived from equivalent width measurements (which are sensitive to line blends) using the $\log g f$ values of Duquette et al. (1986) for NbI and the $f$-values of Garstang (1981) for Tc I.

Technetium, niobium, and ruthenium studies are relatively uncommon simply because it is difficult to derive accurate abundances of these elements. Their available transitions are few in number and mostly weak and/or blended. Fortunately, sophisticated model atmospheres and synthetic spectrum techniques are now available for deriving reliable abundances from a small number of transitions. A few studies have been devoted to technetium abundance determinations (e.g., Van Eck \& Jorissen 1999; Lebzelter \& Hron 1999; Vanture et al. 2007). In this paper, we derive accurate niobium abundances for the Sun and several very metal-poor stars with enhanced neutron-capture abundances.

Lifetimes in Nb II were reported by Salih \& Lawler (1983), which were measured using time-resolved laser-induced fluorescence (TR-LIF) on a $\mathrm{Nb}^{+}$beam. Hannaford et al. (1985) measured 27 lifetimes for low-lying $5 d^{3} 5$ p energy levels using TR-LIF and a sputtered metal vapor. In addition, they
derived transition probabilities by using the branching fractions taken from Corliss \& Bozman (1962), and used them to determine the solar abundance of niobium. Transition probabilities for 145 lines in Nb II were reported by Nilsson \& Ivarsson (2008), who measured branching fractions (BFs) in the wavelength interval 2600-4600 Å. In addition, Nilsson \& Ivarsson (2008) combined the BFs with the lifetimes from Hannaford et al. (1985) to obtain transition probabilities for the 145 lines. Nilsson \& Ivarsson (2008) also reported hyperfine splitting constants for 28 even and 24 odd levels. The present paper is a continuation of the work reported by Nilsson \& Ivarsson (2008).

The only previous theoretical work on Nb II was that of Beck \& Datta (1995), who used a relativistic configuration-interaction (RCI) method to compute $f$-values for transitions connecting the $J=2,3,4(4 \mathrm{~d}+5 \mathrm{~s})^{4}$ states to the $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{3}^{\circ}$ and ${ }^{3} \mathrm{D}_{3}^{\circ}$ levels. To our knowledge, there are no transition probabilities or radiative lifetimes available for $\mathrm{Nb}^{2+}$ in the literature.

In the present work, we report $f$-values of 107 Nb II transitions in the spectral range $2242-4700 \AA$. In addition, a first set of 76 astrophysically interesting transition probabilities in Nb III are calculated using a theoretical model similar to that used in Nb II.

## 2. Measurements of lifetimes and branching fractions

Transition probabilities can be measured in several ways (see Huber \& Sandeman 1986, for a review). We used the emission technique, where radiative lifetimes measured with TR-LIF are combined with $B F$ s derived from spectra recorded using Fourier transform spectroscopy (FTS). With this technique, it is possible to obtain a large amount of accurate transition probabilities over a wide wavelength range.

The lifetime of an upper state $u$ can be written as
$\tau_{u}=1 / \sum_{k} A_{u k}$,
and the $B F$ of a line, from the upper state $u$ to a lower state $l$, is defined as
$B F_{u l}=A_{u l} / \sum_{k} A_{u k}$.
Combining these two equations provides us with the transition probabilities
$A_{u l}=B F_{u l} / \tau_{u}$.
In the following sections, the experimental measurements of the radiative lifetimes and $B F$ s are described.

### 2.1. Radiative lifetimes

We report the experimental lifetimes of 17 short-lived, oddparity levels in Nb II belonging to the $4 d^{3} 5$ p configuration. The measurements were performed using TR-LIF on ions in a laserproduced plasma. The TR-LIF technique previously provided accurate lifetimes in many different systems (see Fivet et al. 2006, 2008). A detailed description of the experimental setup is reported in Bergstöm et al. (1988) and Xu et al. $(2003,2004)$ and only a brief description is presented in this paper.

Niobium ions in different excited states were generated in a laser-produced plasma obtained by focusing a $5320 \AA$

Nd:YAG laser pulse (Continuum Surelite) onto a rotating niobium target. The different ionization stages have different velocities and can be separated by selecting an appropriate delay time between the ablation and the excitation pulses.

Excitation pulses, with a duration of $1-2 \mathrm{~ns}$, were obtained from a frequency-doubled Nd:YAG laser (Continuum NY-82) temporally compressed in a stimulated Brillouin scattering cell. To generate the required excitation wavelengths, the compressed pulses were used to pump a dye laser (Continuum Nd-60). Excitation wavelengths as low as $1910 \AA$ can be obtained using the DCM dye and non-linear processes, such as frequency doubling and tripling in KDP and BBO crystals and stimulated Raman scattering in a hydrogen gas cell.

The excitation beam interacted with the niobium ions about 1 cm above the target. The fluorescence emitted from the excited levels was focused with a fused-silica lens onto the entrance slit of a $1 / 8 \mathrm{~m}$ monochromator, and detected by a Hamamatsu R3809U micro-channel-plate photomultiplier tube with a risetime of 0.15 ns . The fluorescence signal was recorded by a transient digitizer with a time resolution of 0.5 ns . The temporal shape of the laser pulse was recorded simultaneously with a fast diode. Each decay curve was typically averaged over 1000 laser pulses.

The computer code DECFIT was developed to analyze the fluorescence signals. DECFIT operates on a Windows platform and provides the user with an efficient graphical environment. Lifetimes are extracted using a weighted least squares fit to a single exponential decay convolved with the shape of the laser pulse to the fluorescence signal. In addition, a polynomial background representation can be added in the fit.

The new experimental lifetimes for 17 levels in Nb II are reported in Table 1. The lifetimes are averages from at least ten recordings. The error estimates take into account the statistical uncertainties in the fitting as well as variations between the different recordings. The results reported by Salih \& Lawler (1983) and by Hannaford et al. (1985) are shown for comparison as well as the theoretical data obtained by Beck \& Datta (1995).

### 2.2. Branching fractions

The intensity of a spectral line is dependent on the population $\left(N_{u}\right)$, the transition probability $\left(A_{u k}\right)$, and the statistical weight $\left(g_{u}\right)$ of the upper level. The $B F$ can therefore be derived by measuring the intensities of all lines from a given upper level $u$
$B F_{u l}=A_{u l} / \sum_{k} A_{u k}=I_{u l} / \sum_{k} I_{u k}$,
where $I_{u l}$ is the intensity of a line from upper level $u$ to lower level $l$ corrected for the wavenumber dependent efficiency of the detection system. One advantage of this method is that neither the population of the upper level $u$ nor the population mechanisms have to be known.

The intensities were measured with FTS. Spectra between 20000 and $50000 \mathrm{~cm}^{-1}$ ( $5000-2000 \AA$ ) were recorded with the Lund UV Chelsea Instruments FT spectrometer. A custom-built hollow cathode discharge lamp (HCDL) was used to produce the $\mathrm{Nb}^{+}$ions. The HCDL was operated with a mixture of argon and neon as carrier gases at 1-2 Torr. Two different detectors were used. Between 20000 and $40000 \mathrm{~cm}^{-1}$, we used a Hamamatsu R955 PM tube, whereas the 30000 to $50000 \mathrm{~cm}^{-1}$ region was recorded with a solar blind Hamamatsu R166 PM tube. The first region was intensity-calibrated using known

Table 1. Calculated and experimental lifetimes for Nb II levels of the $4 \mathrm{~d}^{3} 5$ p configuration.

| Desig. ${ }^{\text {a }}$ | $\begin{gathered} E^{a} \\ \left(\mathrm{~cm}^{-1}\right) \end{gathered}$ | Theory |  |  | Experiment |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\tau(\mathrm{ns})$ | $\tau(\mathrm{ns})$ | $\tau(\mathrm{ns})$ | $\tau$ (ns) | $\tau(\mathrm{ns})$ |
|  |  | HFR(A) this work | HFR(B) this work | previous | this work | previous |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{2}$ | 33351.09 | 5.0 | 5.6 |  |  | 5.9(4) ${ }^{\text {e }}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{3}^{0}$ | 33919.24 | 4.9 | 5.5 | $5.16{ }^{\text {b }}, 6.53^{c}$ |  | 6.2(3) ${ }^{d}, 5.8(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{4}^{\circ}$ | 34632.03 | 4.7 | 5.3 |  |  | 5.8(3) ${ }^{d}, 5.5(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{1}^{0}$ | 34886.35 | 5.1 | 5.9 |  | 5.6(3) | 5.5(3) ${ }^{d}, 5.7(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{5}^{\circ}$ | 35474.20 | 4.6 | 5.2 |  |  | 5.3(3) ${ }^{d}, 5.3(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{2}^{\circ}$ | 35520.82 | 4.8 | 5.7 |  | 5.5(3) | 5.7(3) ${ }^{d}, 5.7(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{6}^{\circ}$ | 36455.46 | 4.4 | 5.0 |  |  | $5.0(3)^{d}, 5.1(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{3}^{0}$ | 36553.24 | 4.2 | 5.0 | $4.67^{b}, 5.60^{c}$ |  | $5.0(3)^{d}, 5.0(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{~F}_{1}^{\circ}$ | 36731.81 | 4.0 | 4.7 |  |  | $4.7(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{~F}_{2}^{\circ}$ | 36962.77 | 4.0 | 4.7 |  |  | $4.7(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{0}^{\circ}$ | 37298.24 | 2.3 | 2.7 |  | 2.6(2) | $3.1(3){ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{~F}_{3}^{\circ}$ | 37376.90 | 3.9 | 4.6 |  |  | $4.7(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{1}^{\circ}$ | 37480.08 | 2.4 | 2.7 |  |  | 3.1(3) ${ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{~F}_{4}^{\circ}$ | 37528.38 | 3.8 | 4.4 |  |  | $4.6(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{2}^{0}$ | 37797.32 | 2.6 | 2.8 |  | 2.7(2) | $3.2(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{~F}_{5}^{\circ}$ | 38024.34 | 3.9 | 4.6 |  |  | $4.6(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{3}^{\circ}$ | 38216.39 | 2.7 | 3.0 |  |  | 3.4(3) ${ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{4}^{\circ}$ | 38291.25 | 2.6 | 3.0 |  |  | 3.3(3) ${ }^{\text {e }}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{G}_{3}^{\circ}$ | 38684.96 | 4.5 | 5.1 |  |  | 5.0(3) ${ }^{\text {e }}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{~F}_{2}^{\circ}$ | 38984.39 | 6.8 | 7.3 |  |  | 7.3(3) ${ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{G}_{4}^{\circ}$ | 39335.27 | 4.5 | 5.1 |  |  | 5.1(3) ${ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{~F}_{3}^{\circ}$ | 39779.92 | 6.3 | 6.9 |  |  | 6.8(3) ${ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{G}_{5}^{\circ}$ | 40103.61 | 4.4 | 5.0 |  | 5.1(3) | $5.2(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{~F}_{4}^{\circ}$ | 40561.02 | 5.8 | 6.3 |  |  | $6.4(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{P}_{2}$ | 41710.14 | 7.9 | 8.4 |  | 8.5(7) |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{P}_{1}^{\circ}$ | 42132.65 | 6.5 | 7.2 |  | 7.2(5) |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{0}^{\circ}$ | 42596.55 | 6.3 | 7.2 |  | 7.5(7) |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{2}$ | 43290.34 | 6.9 | 7.3 |  | 7.5(4) |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{1} \mathrm{D}_{2}$ | 43618.39 | 6.4 | 6.7 |  | 6.7(3) |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{1}^{0}$ | 43649.19 | 8.4 | 8.1 |  | 7.2(4) | 12.8(3) ${ }^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{3}^{\circ}$ | 43887.08 | 5.9 | 6.3 |  | 5.7(5) |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{1}$ | 44066.67 | 5.8 | 6.7 |  | 6.4(5) | $5.3(3)^{e}$ |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{P}_{0}^{\circ}$ | 44285.94 | 6.6 | 7.5 |  | 7.0(5) |  |
| $\left.4 \mathrm{~d}^{3}{ }^{(2} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}^{\circ}$ | 44924.60 | 5.9 | 6.6 |  | $6.2(3)$ |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{4}^{+}$ | 44970.71 | 5.1 | 5.8 |  | 5.2(5) |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{3}^{\circ}$ | 45802.47 | 4.9 | 4.6 |  | 4.6 (3) |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{H}\right) 5 \mathrm{p}{ }^{3} \mathrm{H}_{6}^{\circ}$ | 48770.84 | 4.5 | 3.7 |  |  | $4.2(3)^{e}$ |

Notes. ${ }^{(a)}$ From Ryabtsev et al. (2000); ${ }^{(b)}$ from Beck \& Datta (1995): length form; ${ }^{\left({ }^{(c)}\right.}$ from Beck \& Datta (1995): velocity form; ${ }^{(d)}$ from Salih \& Lawler (1983); ${ }^{(e)}$ from Hannaford et al. (1985). $A(B)$ is written for $A \pm B$.
branching ratios in argon (Whaling et al. 1993), while the UV spectra were intensity-calibrated with a deuterium lamp, itself calibrated at the Physikalisch-Techniche Bundesanstalt (PTB), Berlin, Germany.

The lines were checked for possible effects of self-absorption by recording several spectra at different HCDL currents thus changing the populations of the different levels.

In Table 2, we list all BFs measured in this work as well as the theoretical values (see next section). The results are compared with theoretical $B F$ s calculated within the framework of the HFR + CPOL approach. The lines are sorted by upper level. The level designation and energies are from Ryabtsev et al. (2000). The uncertainties given in Col. 10 are calculated according to the method suggested by Sikström et al. (2002). In addition, we list the transition probabilities and the oscillator
strengths deduced from the experimental lifetimes as well as the measured branching fractions.

Not all transitions are strong enough to be seen in the spectra recorded by the HCDL, but the sum of all these unmeasurable weak transitions, the residuals, can be estimated with theoretical calculations. The differences between the experimental and the theoretical BFs are shown in Fig. 1. The signal-to-noise ratio of a line affected by hyperfine structure is lower because the intensity of the line is distributed between the different components. This is a problem for weak transitions. In addition, the theoretical $B F$ s of the weakest transitions are more difficult to calculate accurately than those of the strongest lines because they are sensitive to small configuration interaction effects and to possible cancellation effects in the line strengths. This explains the larger scatter in the left part of Fig. 1.

Table 2. Experimental branching fractions $(B F), A$-values, and $g f$-values in $\mathrm{Nb} \mathrm{II} . A(B)$ is written for $A \times 10^{B}$.

| Upper level | Lower level Design. | $\lambda_{\mathrm{air}}$ <br> ( $\AA$ ) | $\begin{gathered} \sigma \\ \left(\mathrm{cm}^{-1}\right) \end{gathered}$ | Calc. | $\begin{array}{r} B F \\ \text { Exp. } \end{array}$ | Exp/Calc | $\begin{gathered} A \\ \left(\mathrm{~s}^{-1}\right) \end{gathered}$ | $\log (g f)$ | Unc. $(\% \text { in } g f)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{~F}_{4}^{\circ}$ | $4 \mathrm{~d}^{4}{ }^{\text {D }}$ 3 | 2514.3531 | 39759.692 | 0.012 | 0.013 | 1.089 | 2.06(6) | -1.76 | 13 |
|  | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{4}$ | 2541.4244 | 39336.195 | 0.067 | 0.069 | 1.033 | 1.09(7) | -1.02 | 12 |
| $40561.02 \mathrm{~cm}^{-1}$ | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{5} \mathrm{~F}_{4}$ | 2700.5523 | 37018.457 | 0.026 | 0.035 | 1.336 | 5.47(6) | -1.27 | 12 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{5} \mathrm{~F}_{5}$ | 2745.3118 | 36414.981 | 0.079 | 0.086 | 1.087 | 1.35(7) | -0.86 | 12 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{4}$ | 3100.7738 | 32240.645 | 0.025 | 0.025 | 0.984 | 3.88(6) | -1.30 | 17 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{G}_{5}$ | 3372.5601 | 29642.544 | 0.055 | 0.065 | 1.197 | 1.02(7) | -0.80 | 8 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{3}$ | 3421.1646 | 29221.428 | 0.015 | 0.018 | 1.158 | 2.77(6) | -1.36 | 12 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{4}$ | 3717.0600 | 26895.332 | 0.348 | 0.368 | 1.055 | 5.75(7) | 0.03 | 8 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{3}$ | 3720.4532 | 26870.790 | 0.080 | 0.105 | 1.304 | 1.64(7) | -0.51 | 8 |
|  | $4 \mathrm{~d}^{41} \mathrm{G}_{4}$ | 3879.3421 | 25770.263 | 0.171 | 0.089 | 0.523 | 1.40(7) | -0.55 | 6 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{G}\right) 5 \mathrm{~s}^{3} \mathrm{G}_{4}$ | 4061.9711 | 24611.648 | 0.021 | 0.027 | 1.274 | 4.27(6) | -1.02 | 9 |
|  | Residual |  |  |  | 0.099 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{P}_{2}^{\circ}$ | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{3}$ | 2956.8884 | 33809.499 | 0.061 | 0.065 | 1.073 | 7.89(6) | -1.29 | 21 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{2}$ | 3237.9954 | 30874.306 | 0.248 | 0.199 | 0.802 | 2.42(7) | -0.72 | 11 |
| $41710.14 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{\text {d }} \mathrm{D}_{2}$ | 3394.9714 | 29446.922 | 0.151 | 0.153 | 1.008 | 1.85(7) | -0.80 | 11 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{2}$ | 3541.2454 | 28230.683 | 0.099 | 0.124 | 1.244 | 1.50(7) | -0.85 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{1}$ | 3691.1738 | 27083.944 | 0.071 | 0.073 | 1.040 | 5.60(6) | -1.24 | 14 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{2}$ | 3695.8960 | 27049.388 | 0.183 | 0.193 | 1.057 | 2.35(7) | -0.62 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2}\right.$ D 2 ) $5 \mathrm{~s}^{3} \mathrm{D}_{1}$ | 4367.3789 | 22890.623 | 0.035 | 0.037 | 1.041 | 4.48(6) | $-1.19$ | 16 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{1}$ | $4699.5812$ | 21272.640 | 0.041 | 0.046 | 1.110 | $5.53(6)$ | -1.04 | 19 |
|  | Residual |  |  |  | 0.110 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{P}_{1}^{\circ}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{0}$ | 2372.7316 | 42132.646 | 0.024 | 0.014 | 0.598 | 1.93(6) | -2.31 | 22 |
|  | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{1}$ | 2381.7200 | 41973.662 | 0.026 | 0.018 | 0.690 | 2.37(6) | -2.22 | 18 |
| $42132.65 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{P}_{2}$ | 2866.8425 | 34871.322 | 0.038 | 0.015 | 0.403 | 2.03(6) | -2.13 | 52 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{2}$ | 2887.0836 | 34626.881 | 0.042 | 0.060 | 1.426 | 8.00(6) | -1.52 | 21 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{1}$ | 3175.7747 | 31479.219 | 0.263 | 0.273 | 1.038 | 3.64(7) | -0.78 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{2}$ | 3194.2782 | 31296.809 | 0.259 | 0.281 | 1.083 | 3.75(7) | -0.77 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{2}$ | 3639.0501 | 27471.891 | 0.132 | 0.131 | 0.990 | 1.75(7) | -0.98 | 12 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{0}$ | 3641.3782 | 27454.328 | 0.037 | 0.034 | 0.936 | 4.59(6) | -1.56 | 20 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{0}$ | 4588.9970 | 21785.149 | 0.033 | $0.028$ | 0.836 | 3.67(6) | -1.46 | 23 |
|  | Residual |  |  |  | 0.146 |  |  |  |  |
| $\begin{aligned} & 4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}^{3} \mathrm{P}_{0}^{\circ} \\ & 44285.94 \mathrm{~cm}^{-1} \end{aligned}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{1}$ | 2265.4874 | 44126.956 | 0.025 | 0.04 | 1.612 | 5.66(5) | -2.36 | 28 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{P}_{1}$ | 2624.3286 | 38093.630 | 0.003 | 0.10 | 31.295 | 1.41(6) | -1.84 | 21 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}^{5} \mathrm{P}_{1}$ | 2972.4398 | 33632.513 | 0.246 | 0.14 | 0.558 | 1.96(6) | -1.59 | 47 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{1}$ | 3370.6053 | 29659.741 | 0.601 | 0.60 | 0.997 | 8.56(6) | -0.84 | 13 |
|  | Residual |  |  |  | 0.120 |  |  |  |  |
| $\begin{aligned} & 4 d^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}^{5} \mathrm{D}_{0}^{\circ} \\ & 42596.55 \mathrm{~cm}^{-1} \\ & 4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}^{5} \mathrm{D}_{2}^{\circ} \end{aligned}$ | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{1}$ | 3129.6522 | 31943.127 | 0.756 | 0.756 | 1.000 | 1.05(8) | -0.814 | 12 |
|  | Residual |  |  |  | 0.244 |  |  |  |  |
|  | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{2}$ | 2332.8990 | 42851.977 | 0.029 | 0.021 | 0.731 | 2.86(6) | -1.93 | 21 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{2}$ | 2793.6766 | 35784.573 | 0.029 | 0.047 | 1.649 | 6.30(6) | -1.43 | 19 |
| $43290.34 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{1}$ | 3063.1193 | 32636.911 | 0.120 | 0.132 | 1.103 | 1.76(7) | -0.91 | 14 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{2}$ | 3080.3475 | 32454.501 | 0.309 | 0.346 | 1.120 | 4.62(7) | -0.48 | 10 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{3}$ | 3128.9145 | 31950.748 | 0.054 | 0.047 | 0.855 | 6.21(6) | -1.34 | 22 |
|  | $4 d^{4}{ }^{1} D_{2}$ | $3222.0580$ | $31027.117$ | $0.226$ | $0.178$ | 0.789 | 2.38(7) | -0.73 | 12 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{D} 2\right) 5 \mathrm{~s}^{3} \mathrm{D}_{1}$ | $4085.3468$ | $24470.818$ | 0.030 | 0.031 | 1.034 | 4.16(6) | -1.28 | 18 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{1} \mathrm{P}_{1}$ | 4492.9563 | 22250.809 | 0.072 | 0.066 | 0.921 | 8.83(6) | -0.87 | 14 |
|  | Residual |  |  |  | 0.130 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{1} \mathrm{D}_{2}^{\circ}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{1}$ | 2300.2897 | 43459.407 | 0.019 | 0.013 | 0.685 | 1.93(6) | -2.12 | 15 |
|  | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{2}$ | 2315.1745 | 43180.030 | 0.062 | 0.040 | 0.639 | 5.81(6) | -1.63 | 11 |
| $43618.39 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{3}$ | 2334.8014 | 42817.065 | 0.037 | 0.030 | 0.820 | 4.40(6) | -1.75 | 14 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{P}_{2}$ | 2749.6848 | 36357.067 | 0.044 | 0.057 | 1.299 | 8.37(6) | -1.32 | 12 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{3}$ | 2798.9044 | 35717.747 | 0.063 | 0.108 | 1.704 | 1.58(7) | -1.03 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}^{5} \mathrm{P}_{1}$ | 3032.6351 | 32964.964 | 0.021 | 0.054 | 2.615 | 7.96(6) | -1.26 | 19 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}^{5} \mathrm{P}_{2}$ | 3049.5039 | 32782.554 | 0.038 | 0.066 | 1.767 | 1.30(7) | -1.04 | 13 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{3}$ | 3097.1160 | 32278.801 | 0.192 | 0.138 | 0.719 | 2.03(7) | -0.84 | 11 |
|  | $4 \mathrm{~d}^{4}{ }^{1} \mathrm{D}_{2}$ | 3188.3466 | 31355.170 | 0.034 | 0.023 | 0.685 | 3.40(6) | -1.59 | 30 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{D}_{2}$ | 3244.5078 | 30812.426 | 0.047 | 0.048 | 1.030 | 7.02(6) | -1.26 | 15 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{D}_{1}$ | 3277.7583 | 30499.863 | 0.020 | 0.019 | 0.918 | 2.72(6) | -1.66 | 32 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{3}$ | 3340.3740 | 29928.163 | 0.018 | 0.021 | 1.170 | 3.11(6) | -1.59 | 26 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{1}$ | 3448.2179 | 28992.192 | 0.083 | 0.067 | 0.814 | 9.86(6) | -1.06 | 14 |

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Table 2. continued.

| Upper level | Lower level Design. | $\begin{aligned} & \lambda_{\text {air }} \\ & (\AA) \end{aligned}$ | $\begin{gathered} \sigma \\ \left(\mathrm{cm}^{-1}\right) \end{gathered}$ | Calc. | $\begin{array}{r} B F \\ \text { Exp. } \end{array}$ | Exp/Calc | $\begin{gathered} A \\ \left(\mathrm{~s}^{-1}\right) \end{gathered}$ | $\log (g f)$ | $\begin{aligned} & \text { Unc. } \\ & (\% \text { in } g f) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{1}^{\circ}$ | $4 d^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{2}$ | 3452.3409 | 28957.636 | 0.162 | 0.153 | 0.946 | 2.24(7) | -0.70 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{2}$ | 4522.1931 | 22107.015 | 0.027 | 0.028 | 1.029 | 4.12(6) | -1.20 | 20 |
|  | Residual |  |  |  | 0.133 |  |  |  |  |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{P}_{2}$ | 2747.3562 | 36387.869 | 0.102 | 0.249 | 2.431 | 3.40(7) | -0.94 | 10 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{2}$ | 2765.9299 | 36143.428 | 0.186 | 0.076 | 0.409 | 1.04(7) | -1.45 | 12 |
| $43649.19 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{1}$ | 3029.8088 | 32995.766 | 0.028 | 0.138 | 4.942 | 1.88(7) | -1.11 | 11 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{2}$ | 3448.6696 | 28988.438 | 0.095 | 0.073 | 0.770 | 1.51(7) | -1.09 | 12 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{0}$ | 3450.7619 | 28970.875 | 0.301 | 0.178 | 0.590 | 2.36 (7) | -0.90 | 10 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2}\right.$ D2) 5s ${ }^{3} \mathrm{D}_{1}$ | 4026.3014 | 24829.673 | 0.046 | 0.049 | 1.063 | 4.09(6) | -1.53 | 23 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{D} 2\right) 5 \mathrm{~s}{ }^{3} \mathrm{D}_{2}$ | 4114.5320 | 24297.245 | 0.046 | 0.035 | 0.752 | 4.77(6) | -1.44 | 21 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{1} \mathrm{P}_{1}$ | 4421.6458 | 22609.664 | 0.034 | 0.041 | 1.224 | 5.62(6) | -1.31 | 29 |
|  | Residual |  |  |  | 0.162 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{3}^{\circ}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{2}$ | 2300.8551 | 43448.717 | 0.015 | 0.012 | 0.780 | 1.98(6) | -1.96 | 21 |
|  | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{3}$ | 2320.2400 | 43085.752 | 0.033 | 0.028 | 0.834 | 4.73(6) | -1.57 | 16 |
| $43887.08 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{4}$ | 2343.2745 | 42662.255 | 0.010 | 0.012 | 1.220 | 2.09(6) | -1.92 | 29 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{P}_{2}$ | 2729.5123 | 36625.754 | 0.014 | 0.013 | 0.925 | 2.20(6) | -1.77 | 24 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{4}$ | 2810.7918 | 35566.705 | 0.088 | 0.122 | 1.384 | 2.48(7) | -0.69 | 15 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{2}$ | 3024.7316 | 33051.241 | 0.477 | 0.401 | 0.841 | 6.86(7) | -0.18 | 15 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}^{5} \mathrm{P}_{3}$ | 3071.5463 | 32547.488 | 0.262 | 0.305 | 1.165 | 5.23(7) | -0.29 | 15 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{D}_{3}$ | 3242.4077 | 30832.382 | 0.011 | 0.018 | 1.656 | 3.08(6) | -1.47 | 49 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{2}$ | 4467.8849 | 22375.702 | 0.016 | 0.015 | 0.949 | 2.62(6) | -1.26 | 22 |
|  | Residual |  |  |  | 0.074 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}{ }^{5} \mathrm{D}_{1}^{\circ}$ | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{2}$ | 2291.3844 | 43628.304 | 0.023 | 0.014 | 0.609 | 2.18(6) | -2.29 | 26 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{2}$ | 2734.3521 | 36560.900 | 0.083 | 0.124 | 1.487 | 1.94(7) | -1.19 | 17 |
| $44066.67 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{1}$ | 2991.9493 | 33413.238 | 0.379 | 0.276 | 0.727 | 4.31(7) | -0.76 | 19 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{1}$ | 3395.7224 | 29440.466 | 0.131 | 0.065 | 0.498 | 1.02(7) | -1.28 | 22 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{2}$ | 3399.7112 | 29405.910 | 0.154 | $0.291$ | 1.894 | 4.55(7) | -0.63 | 17 |
|  | Residual |  |  |  | $0.230$ |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{2}^{\circ}$ | $4 d^{4}{ }^{5} D_{3}$ | 2265.6748 | 44123.277 | $0.059$ | $0.072$ | $1.217$ | $1.10(7)$ | $-1.37$ | $13$ |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{2}$ | $2671.6567$ | $37418.838$ | $0.028$ | $0.016$ | $0.582$ | $9.32(6)$ | $-1.30$ | 14 |
| $44924.60 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{3}$ | 2700.1527 | 37023.959 | 0.211 | 0.250 | 1.187 | 3.82(7) | -0.68 | 12 |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{~s}{ }^{5} \mathrm{P}_{1}$ | 2917.0516 | 34271.176 | 0.186 | 0.225 | 1.206 | 3.43(7) | -0.66 | 13 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{2}$ | 3179.2265 | 31445.143 | 0.021 | 0.025 | 1.200 | 3.86(6) | -1.53 | 65 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}{ }^{3} \mathrm{P}_{1}$ | 3299.5510 | 30298.404 | 0.098 | 0.055 | 0.563 | 8.47(6) | -1.16 | 17 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{3} \mathrm{P}_{2}$ | 3303.3216 | 30263.848 | 0.114 | 0.093 | 0.819 | 1.43(7) | -0.93 | 15 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{D} 2\right) 5 \mathrm{~s}{ }^{3} \mathrm{D}_{2}$ | 3909.3197 | 25572.655 | 0.022 | 0.008 | 0.361 | 1.22(6) | -1.85 | 32 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{~s}^{1} \mathrm{P}_{1}$ | 4185.5335 | 23885.074 | 0.048 | 0.042 | 0.890 | 6.46(6) | -1.07 | 32 |
|  | Residual |  |  |  | 0.212 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{P}\right) 5 \mathrm{p}^{5} \mathrm{D}_{4}^{\circ}$ | $4 d^{4}{ }^{5} D_{3}$ | $2263.3106$ | $44169.386$ | 0.014 | 0.010 | 0.709 | 1.85(6) | -1.89 | 20 |
|  | $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{4}$ | 2285.2235 | 43745.889 | 0.066 | 0.054 | 0.813 | 1.04(7) | -1.14 | 12 |
| $44970.71 \mathrm{~cm}^{-1}$ | $4 d^{3}\left({ }^{4} P\right) 5 s^{5} P_{3}$ | $2972.5710$ | $33631.122$ | $0.890$ | $0.907$ | $1.020$ | $1.75(8)$ | $0.32$ | $10$ |
|  | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{4}$ | 3193.4495 | 31305.026 | 0.008 | 0.006 | 0.857 | 1.24(6) | -1.77 | 52 |
|  | Residual |  |  |  | 0.023 |  |  |  |  |
| $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{P}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{3}^{\circ}$ | $4 \mathrm{~d}^{4} \mathrm{D}_{4}$ | 2242.5800 | 44577.650 | 0.017 | 0.055 | 3.162 | 1.20(7) | -1.20 | 10 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{4}$ | 2667.1467 | 37482.100 | 0.068 | 0.157 | 2.300 | 3.41(7) | -0.59 | 9 |
| $45802.47 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{H}_{4}$ | 2754.5496 | 36292.869 | 0.123 | 0.105 | 0.852 | 2.27(7) | -0.74 | 11 |
|  | $4 \mathrm{~d}^{4}{ }^{\text {d }} \mathrm{D}_{2}$ | 2980.7132 | 33539.252 | 0.217 | 0.251 | 1.158 | 5.46(7) | -0.29 | 11 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{D}_{2}$ | 3029.7413 | 32996.508 | 0.120 | 0.087 | 0.722 | 1.88(7) | -0.74 | 16 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{4}$ | 3110.7976 | 32136.787 | 0.029 | 0.021 | 0.730 | 4.62(6) | -1.33 | 28 |
|  | $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}^{3} \mathrm{~F}_{3}$ | 3113.1748 | 32112.245 | 0.012 | 0.029 | 2.370 | 6.32(6) | -1.19 | 21 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{G}\right) 5 \mathrm{~s}^{3} \mathrm{G}_{3}$ | 3304.7036 | 30251.221 | 0.161 | 0.067 | 0.415 | 1.46(7) | -0.78 | 14 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{G}\right) 5 \mathrm{~s}^{3} \mathrm{G}_{4}$ | 3348.7755 | 29853.103 | 0.074 | 0.050 | 0.675 | 1.08(7) | -0.90 | 14 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{H}\right) 5 \mathrm{~s}{ }^{3} \mathrm{H}_{4}$ | 3528.4778 | 28332.703 | 0.078 | $0.078$ | 1.002 | 1.70(7) | -0.66 | 17 |
|  |  |  |  |  | $0.101$ |  |  |  |  |
| $4 d^{3}\left({ }^{2} \mathrm{H}\right) 5 \mathrm{p}{ }^{3} \mathrm{H}_{6}$ | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{H}_{5}$ | 2566.0734 | 38958.386 | 0.021 | 0.013 | 0.621 | 3.06(6) | -1.41 | 14 |
|  | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{H}_{6}$ | 2590.9428 | 38584.448 | 0.358 | 0.478 | 1.333 | 1.14(8) | 0.17 | 11 |
| $48770.84 \mathrm{~cm}^{-1}$ | $4 \mathrm{~d}^{4}{ }^{3} \mathrm{G}_{5}$ | 2641.0561 | 37852.364 | 0.031 | 0.042 | 1.362 | 1.01(7) | -0.86 | 12 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{G}\right) 5 \mathrm{~s}^{3} \mathrm{G}_{5}$ | 3055.5159 | 32718.166 | 0.257 | 0.095 | 0.368 | 2.25(7) | -0.39 | 24 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{H}\right) 5 \mathrm{~s}{ }^{3} \mathrm{H}_{5}$ | 3175.8669 | 31478.359 | 0.246 | 0.252 | 1.024 | 5.99(7) | 0.07 | 17 |
|  | $4 \mathrm{~d}^{3}\left({ }^{2} \mathrm{H}\right) 5 \mathrm{~s}^{3} \mathrm{H}_{6}$ | 3189.2788 | 31345.949 | 0.086 | 0.120 | 1.393 | 2.86(7) | -0.25 | 20 |
|  | Residual |  |  |  | 0.001 |  |  |  |  |



Fig. 1. The ratio of our experimental to theoretical $B F$ s plotted against the experimental $B F \mathrm{~s}$.

For the convenience of the user, the Nb II wavelengths and oscillator strengths ( $\log g f$-values) are sorted by increasing wavelengths in Table 3. The wavenumbers and wavelengths are derived from energy levels reported by Ryabtsev et al. (2000). The starred wavelengths and the corresponding oscillator strengths are taken from Nilsson \& Ivarsson (2008).

## 3. Hyperfine structure

Niobium has a nuclear spin of $9 / 2$ and a magnetic moment $\mu / \mu_{N}=6.2$ (Sheriff \& Williams 1951). Therefore, many of the lines manifest hyperfine structure (hfs) effects as illustrated in Fig. 2 for the $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s} \mathrm{a}^{5} \mathrm{~F}_{5}-4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p} \mathrm{z}{ }^{3} \mathrm{~F}_{4}^{\circ}$ transition in Nb II at $2745.31 \AA$. The upper levels investigated in this paper are highly excited 5 p levels that interact only weakly with the nucleus and thus have a small hfs. The hfs in the lower levels is generally far more significant, especially for the 5 s levels that penetrate the nucleus to a higher extent. Magnetic dipole hfs constants ( $A_{\mathrm{hfs}}$ ) for many of the lower levels involved in the transitions reported here were published by Nilsson \& Ivarsson (2008). In Table 5, we indicate the hfs of the ${ }^{5} \mathrm{~F}_{1}-\mathrm{Z}^{5} \mathrm{G}_{2}^{\circ}, \mathrm{a}^{5} \mathrm{~F}_{2}-\mathrm{Z}^{5} \mathrm{G}_{3}^{\circ},{ }^{5} \mathrm{~F}_{3}-\mathrm{z}^{5} \mathrm{G}_{3}^{\circ},{ }^{5} \mathrm{~F}_{4}-\mathrm{z}^{5} \mathrm{G}_{4}^{\circ}, \mathrm{a}{ }^{3} \mathrm{P}_{1}-\mathrm{z}^{3} \mathrm{D}_{2}^{\circ}$, a ${ }^{3} \mathrm{~F}_{3}-\mathrm{z}{ }^{3} \mathrm{D}_{2}^{\circ}$, and ${ }^{3} \mathrm{~F}_{2}-\mathrm{z}{ }^{3} \mathrm{G}_{3}^{\circ}$ transitions, which are of astrophysical interest. The relative intensities for the components of the hypermultiplets have been calculated assuming LS coupling. This hfs information is required for calculations of stellar line profiles, which are necessary for accurate abundance determinations.

## 4. The theoretical model

The first analysis of Nb II was that of Humphreys \& Meggers (1945). They published a list of 1494 lines in the 2002-7026 $\AA$ spectral region, which were identified as transitions between 183 Nb II levels belonging to the $4 \mathrm{~d}^{4}, 4 \mathrm{~d}^{3} 5 \mathrm{~s}, 4 \mathrm{~d}^{3} 5 \mathrm{p}$ and $4 d^{2} 5 \mathrm{~s} 5 \mathrm{p}$ configurations. More recent measurements for the region 1579 to $2211 \AA$ were provided by Iglesias (1954), who identified 20 new levels, mostly belonging to the $4 d^{2} 5 \mathrm{~s} 5$ p configuration. The most recent analysis was that of Ryabtsev et al. (2000), who confirmed all but three of the previously known levels and found 153 additional Nb II levels, giving a total of 353 known levels. We used the energy levels of Ryabtsev et al. (2000).

Table 3. Experimental oscillator strengths $(\log g f)$ in Nb II sorted by air wavelengths.

| $\lambda_{\text {air }}$ <br> (A) | $\begin{aligned} & \sigma \\ & \left(\mathrm{cm}^{-1}\right) \end{aligned}$ | Lower level $\left(\mathrm{cm}^{-1}\right)$ | $\log (g f)$ |
| :---: | :---: | :---: | :---: |
| 2242.5800 | 44577.650 | 1224.823 | -1.20 |
| 2263.3106 | 44169.386 | 801.326 | -1.89 |
| 2265.6748 | 44123.277 | 801.326 | -1.37 |
| 2285.2235 | 43745.889 | 1224.823 | -1.14 |
| 2291.3844 | 43628.304 | 438.361 | -2.29 |
| 2300.2897 | 43459.407 | 158.984 | -2.12 |
| 2300.8551 | 43448.717 | 438.361 | -1.96 |
| 2315.1745 | 43180.030 | 438.361 | -1.63 |
| 2320.2400 | 43085.752 | 801.326 | -1.57 |
| 2332.8990 | 42851.977 | 438.361 | -1.93 |
| 2334.8014 | 42817.065 | 801.326 | -1.75 |
| 2343.2745 | 42662.255 | 1224.823 | -1.92 |
| 2372.7316 | 42132.646 | 0.000 | -2.31 |
| 2381.7200 | 41973.662 | 158.984 | -2.22 |
| 2514.3531 | 39759.692 | 801.326 | -1.76 |
| 2541.4244 | 39336.195 | 1224.823 | -1.02 |
| 2566.0734 | 38958.386 | 9812.452 | -1.41 |
| 2590.9428 | 38584.448 | 10186.390 | +0.17 |
| 2641.0561 | 37852.364 | 10918.474 | -0.86 |
| 2646.253* | 37778.026 | 438.361 | -0.55 |
| 2656.075* | 37638.332 | 158.984 | -0.61 |
| 2666.590* | 37489.926 | 801.326 | -0.79 |
| 2667.1467 | 37482.100 | 8320.373 | -0.59 |
| 2667.291* | 37480.076 | 0.000 | -0.77 |
| 2671.6567 | 37418.838 | 7505.765 | -1.30 |
| 2671.926* | 37415.061 | 801.326 | -0.33 |
| 2675.939* | 37358.955 | 438.361 | -0.73 |
| 2678.654* | 37321.092 | 158.984 | -1.56 |
| 2691.770* | 37139.258 | 158.984 | -0.79 |
| 2697.059* | 37066.429 | 1224.823 | +0.06 |
| 2698.858* | 37041.715 | 438.361 | -0.56 |
| 2700.1527 | 37023.959 | 7900.644 | -0.68 |
| 2700.5523 | 37018.457 | 3542.561 | -1.27 |
| 2702.194* | 36995.990 | 801.326 | -0.51 |
| 2702.517* | 36991.564 | 1224.823 | -0.75 |
| 2706.397* | 36938.540 | 438.361 | -1.25 |
| 2716.306* | 36803.790 | 158.984 | -1.18 |
| 2716.622* | 36799.513 | 1224.823 | -0.24 |
| 2721.630* | 36731.805 | 0.000 | -1.42 |
| 2721.982* | 36727.056 | 801.326 | -0.29 |
| 2729.5123 | 36625.754 | 7261.324 | -1.77 |
| 2733.256* | 36575.575 | 801.326 | -0.60 |
| 2733.462* | 36572.821 | 158.984 | -1.05 |
| 2734.3521 | 36560.900 | 7505.765 | -1.19 |
| 2737.085* | 36524.413 | 438.361 | -0.87 |
| 2745.3118 | 36414.981 | 4146.037 | -0.86 |
| 2747.3562 | 36387.869 | 7261.324 | -0.94 |
| 2749.6848 | 36357.067 | 7261.324 | -1.32 |
| 2754.5496 | 36292.869 | 9509.604 | -0.74 |
| 2765.9299 | 36143.428 | 7505.765 | -1.45 |
| 2768.124* | 36114.877 | 438.361 | -0.50 |
| 2780.235* | 35957.568 | 4146.037 | -0.40 |
| 2793.041* | 35792.709 | 3542.561 | -0.71 |
| 2793.6766 | 35784.573 | 7505.765 | -1.43 |
| 2798.9044 | 35717.747 | 7900.644 | -1.03 |
| 2810.7918 | 35566.705 | 8320.373 | -0.69 |
| 2827.075* | 35361.838 | 158.984 | -0.83 |
| 2829.750* | 35328.415 | 1224.823 | -1.55 |
| 2841.143* | 35186.758 | 3029.629 | -0.69 |
| 2842.643* | 35168.184 | 2629.132 | -0.59 |
| 2846.279* | 35123.260 | 2356.816 | -0.73 |
| 2861.092* | 34941.426 | 2356.816 | -0.68 |
| 2865.609* | 34886.354 | 0.000 | -1.18 |
| 2866.8425 | 34871.322 | 7261.324 | -2.13 |

Table 3. continued.

| $\begin{aligned} & \lambda_{\text {air }} \\ & (\AA \mathrm{A}) \\ & \hline \end{aligned}$ | $\left(\mathrm{cm}^{-1}\right)$ | Lower level $\left(\mathrm{cm}^{-1}\right)$ | $\log (g f)$ |
| :---: | :---: | :---: | :---: |
| 2887.0836 | 34626.881 | 7505.765 | -1.52 |
| 2917.0516 | 34271.176 | 10653.427 | -0.66 |
| 2956.8884 | 33809.499 | 7900.644 | -1.29 |
| 2868.521* | 34850.944 | 2629.132 | -0.37 |
| 2875.390* | 34767.687 | 3029.629 | -0.17 |
| 2877.038* | 34747.769 | 2629.132 | -0.39 |
| 2883.174* | 34673.826 | 3542.561 | -0.04 |
| 2888.829* | 34605.958 | 2356.816 | -0.59 |
| 2897.806* | 34498.753 | 3029.629 | -0.37 |
| 2899.233* | 34481.775 | 3542.561 | -0.34 |
| 2908.240* | 34374.989 | 2356.816 | -0.40 |
| 2910.587* | 34347.272 | 3029.629 | -0.16 |
| 2911.742* | 34333.642 | 2629.132 | -0.28 |
| 2927.811* | 34145.215 | 4146.037 | +0.16 |
| 2931.464* | 34102.673 | 2629.132 | -0.92 |
| 2941.543* | 33985.821 | 3542.561 | -0.05 |
| 2946.895* | 33924.106 | 2629.132 | -1.00 |
| 2950.880* | 33878.299 | 4146.037 | +0.24 |
| 2972.5710 | 33631.122 | 11339.590 | +0.32 |
| 2980.7132 | 33539.252 | 12263.221 | -0.29 |
| 2982.102* | 33523.609 | 3029.629 | -0.85 |
| 2991.9493 | 33413.238 | 10653.427 | -0.76 |
| 2994.722* | 33382.345 | 4146.037 | -0.29 |
| 3014.439* | 33164.006 | 2356.816 | -1.61 |
| 3024.7316 | 33051.241 | 10835.837 | -0.18 |
| 3028.441* | 33010.677 | 3542.561 | -0.20 |
| 3029.7413 | 32996.508 | 12805.965 | -0.74 |
| 3029.8088 | 32995.766 | 10653.427 | -1.11 |
| 3032.6351 | 32964.964 | 10653.427 | -1.26 |
| 3039.397* | 32891.690 | 2629.132 | -1.26 |
| 3049.5039 | 32782.554 | 10835.837 | -1.04 |
| 3055.5159 | 32718.166 | 16052.672 | -0.39 |
| 3063.1193 | 32636.911 | 10653.427 | -0.91 |
| 3071.5463 | 32547.488 | 11339.590 | -0.29 |
| 3073.236* | 32529.538 | 2356.816 | -1.19 |
| 3074.270* | 32518.595 | 7261.324 | -1.69 |
| 3076.863* | 32491.193 | 3029.629 | -0.55 |
| 3080.3475 | 32454.501 | 10835.837 | -0.48 |
| 3094.174* | 32309.420 | 4146.037 | +0.56 |
| 3097.1160 | 32278.801 | 11339.590 | -0.84 |
| 3099.181* | 32257.222 | 2629.132 | -0.93 |
| 3100.7738 | 32240.645 | 8320.373 | -1.30 |
| 3110.7976 | 32136.787 | 13665.686 | -1.33 |
| 3113.1748 | 32112.245 | 13690.228 | -1.19 |
| 3128.9145 | 31950.748 | 11339.590 | -1.34 |
| 3129.6522 | 31943.127 | 10653.427 | -0.81 |
| 3130.783* | 31931.636 | 3542.561 | +0.41 |
| 3135.925* | 31879.275 | 7900.644 | -1.32 |
| 3145.402* | 31783.232 | 8320.373 | +0.12 |
| 3163.140* | 31605.006 | 6192.310 | -1.38 |
| 3163.401* | 31602.404 | 3029.629 | +0.27 |
| 3175.7747 | 31479.219 | 10653.427 | -0.78 |
| 3175.8669 | 31478.359 | 17292.479 | +0.07 |
| 3177.766* | 31459.546 | 8320.374 | -1.80 |
| 3179.2265 | 31445.143 | 13479.460 | -1.53 |
| 3180.285* | 31434.626 | 7900.644 | +0.10 |
| 3181.398* | 31423.636 | 7261.324 | -1.12 |
| 3188.3466 | 31355.170 | 12263.221 | -1.59 |
| 3189.2788 | 31345.949 | 17424.889 | -0.25 |
| 3191.094* | 31328.160 | 4146.037 | -0.33 |
| 3193.4495 | 31305.026 | 13665.686 | -1.77 |
| 3194.2782 | 31296.809 | 10835.837 | -0.77 |
| 3194.974* | 31290.112 | 2629.132 | +0.12 |
| 3206.340* | 31179.195 | 7505.765 | +0.04 |

Table 3. continued.

| $\lambda_{\text {air }}$ <br> (Å) | $\begin{aligned} & \sigma \\ & \left(\mathrm{cm}^{-1}\right) \end{aligned}$ | Lower level $\left(\mathrm{cm}^{-1}\right)$ | $\log (g f)$ |
| :---: | :---: | :---: | :---: |
| 3207.331* | 31169.564 | 5562.241 | -1.37 |
| 3215.594* | 31089.472 | 8320.374 | -0.24 |
| 3216.187* | 31083.743 | 7900.644 | -1.52 |
| 3222.0580 | 31027.117 | 12263.221 | -0.73 |
| 3223.326* | 31014.897 | 8320.374 | -0.74 |
| 3225.471* | 30994.274 | 2356.816 | -0.01 |
| 3229.557* | 30955.063 | 7261.324 | -0.75 |
| 3236.400* | 30889.615 | 7900.644 | -0.30 |
| 3237.9954 | 30874.306 | 10835.837 | -0.72 |
| 3242.4077 | 30832.382 | 13054.696 | -1.47 |
| 3244.5078 | 30812.426 | 12805.965 | -1.26 |
| 3247.470* | 30784.316 | 7900.644 | -0.74 |
| 3248.932* | 30770.464 | 6192.310 | -1.11 |
| 3254.062* | 30721.958 | 2629.132 | -0.50 |
| 3267.673* | 30594.001 | 10604.229 | -1.38 |
| 3273.505* | 30539.495 | 6192.310 | -1.55 |
| 3273.880* | 30535.992 | 7261.324 | -1.68 |
| 3277.7583 | 30499.863 | 13118.528 | -1.66 |
| 3291.051* | 30376.683 | 3542.561 | -1.54 |
| 3297.045* | 30321.461 | 3029.629 | -1.63 |
| 3299.5510 | 30298.404 | 14626.199 | -1.16 |
| 3302.616* | 30270.315 | 9509.604 | -1.49 |
| 3303.3216 | 30263.848 | 14660.755 | -0.93 |
| 3304.7036 | 30251.221 | 15551.252 | -0.78 |
| 3319.585* | 30115.577 | 7261.324 | -0.91 |
| 3340.3740 | 29928.163 | 13690.228 | -1.59 |
| 3341.596* | 29917.215 | 10186.390 | -0.46 |
| 3343.892* | 29896.672 | 7900.644 | -1.31 |
| 3343.966* | 29896.014 | 8320.374 | -0.89 |
| 3346.751* | 29871.136 | 7505.765 | -1.23 |
| 3348.7755 | 29853.103 | 15949.370 | -0.90 |
| 3365.587* | 29703.963 | 8320.374 | -0.74 |
| 3365.872* | 29701.450 | 7261.324 | -1.54 |
| 3372.5601 | 29642.544 | 10918.474 | -0.80 |
| 3374.246* | 29627.738 | 7900.644 | -1.19 |
| 3386.238* | 29522.818 | 9812.452 | -0.46 |
| 3388.929* | 29499.376 | 10604.229 | -1.31 |
| 3391.587* | 29476.257 | 7900.644 | -1.85 |
| 3394.9714 | 29446.922 | 12263.221 | -0.80 |
| 3395.7224 | 29440.466 | 14626.199 | -1.28 |
| 3399.7112 | 29405.910 | 14660.755 | -0.63 |
| 3408.673* | 29328.512 | 6192.310 | -0.58 |
| 3409.185* | 29324.113 | 5562.241 | -0.87 |
| 3412.932* | 29291.914 | 7261.324 | -0.50 |
| 3420.625* | 29226.040 | 7505.765 | -1.12 |
| 3421.1646 | 29221.428 | 11339.590 | -1.36 |
| 3425.420* | 29185.131 | 10918.474 | -0.26 |
| 3426.528* | 29175.690 | 10604.229 | -0.78 |
| 3426.568* | 29175.356 | 9509.604 | -0.50 |
| 3436.821* | 29088.315 | 10246.955 | -1.30 |
| 3439.918* | 29062.130 | 7900.644 | -1.04 |
| 3440.581* | 29056.528 | 8320.373 | -0.69 |
| 3448.2179 | 28992.192 | 14626.199 | -1.06 |
| 3448.6696 | 28988.438 | 14660.755 | -1.09 |
| 3450.7619 | 28970.875 | 14678.318 | -0.90 |
| 3452.3409 | 28957.636 | 14660.755 | -0.70 |
| 3478.786* | 28737.432 | 10246.955 | -0.84 |
| 3479.560* | 28731.041 | 10604.229 | -0.46 |
| 3484.046* | 28694.044 | 6192.310 | -1.06 |
| 3489.087* | 28652.594 | 7900.644 | -1.08 |
| 3515.416* | 28438.005 | 10246.955 | -0.59 |
| 3528.4778 | 28332.703 | 17469.770 | -0.66 |
| 3537.622* | 28259.498 | 7261.324 | -0.96 |

Table 3. continued.

| $\lambda_{\text {air }}$ <br> ( $\AA$ ) | $\begin{aligned} & \sigma \\ & \left(\mathrm{cm}^{-1}\right) \\ & \hline \end{aligned}$ | Lower level ( $\mathrm{cm}^{-1}$ ) | $\log (g f)$ |
| :---: | :---: | :---: | :---: |
| 3540.959* | 28232.865 | 8320.373 | -0.43 |
| 3541.2454 | 28230.683 | 13479.460 | -0.85 |
| 3591.194* | 27837.946 | 10186.390 | -1.31 |
| 3619.509* | 27620.178 | 7900.644 | -0.42 |
| 3633.121* | 27516.698 | 12263.221 | -1.53 |
| 3639.0501 | 27471.891 | 14660.755 | -0.98 |
| 3641.3782 | 27454.328 | 14678.318 | -1.56 |
| 3651.182* | 27380.589 | 7505.765 | -0.48 |
| 3688.189* | 27105.862 | 10918.474 | -1.44 |
| 3691.1738 | 27083.944 | 14626.199 | -1.24 |
| 3695.8960 | 27049.388 | 14660.755 | -0.62 |
| 3717.0600 | 26895.332 | 13665.686 | +0.03 |
| 3720.4532 | 26870.790 | 13690.228 | -0.51 |
| 3740.720* | 26725.223 | 13054.696 | -0.31 |
| 3741.288* | 26721.166 | 12263.221 | -1.39 |
| 3781.372* | 26437.919 | 13665.686 | -0.47 |
| 3801.136* | 26300.459 | 13479.460 | -0.65 |
| 3804.012* | 26280.574 | 13054.696 | -0.98 |
| 3818.856* | 26178.422 | 12805.965 | -0.22 |
| 3828.243* | 26114.233 | 13665.686 | -0.86 |
| 3831.844* | 26089.691 | 13690.228 | -0.25 |
| 3855.489* | 25929.691 | 13054.696 | -1.17 |
| 3863.042* | 25878.995 | 12805.965 | -0.85 |
| 3865.004* | 25865.859 | 13118.528 | -0.83 |
| 3879.3421 | 25770.263 | 14790.755 | -0.55 |
| 3898.285* | 25645.042 | 13690.228 | -0.69 |
| 3909.3197 | 25572.655 | 19351.948 | -1.85 |
| 3919.701* | 25504.927 | 13479.460 | -0.93 |
| 3949.445* | 25312.850 | 14790.755 | -1.06 |
| 3952.363* | 25294.159 | 13690.228 | -0.93 |
| 4000.603* | 24989.164 | 14790.755 | -1.03 |
| 4026.3014 | 24829.673 | 18819.520 | -1.53 |
| 4059.674* | 24625.566 | 13665.686 | -1.18 |
| 4061.9711 | 24611.648 | 15949.370 | -1.02 |
| 4073.080* | 24544.515 | 14790.755 | -1.36 |
| 4085.3468 | 24470.818 | 18819.520 | -1.28 |
| 4104.160* | 24358.650 | 13665.686 | -1.24 |
| 4110.309* | 24322.205 | 13054.696 | -1.61 |
| 4114.5320 | 24297.245 | 19351.948 | -1.44 |
| 4126.178* | 24228.667 | 15551.252 | -1.49 |
| 4138.452* | 24156.809 | 12805.965 | -1.65 |
| 4156.671* | 24050.933 | 16052.672 | -1.12 |
| 4185.5335 | 23885.074 | 21039.529 | -1.07 |
| 4216.226* | 23711.215 | 13665.686 | -1.45 |
| 4254.385* | 23498.542 | 13054.696 | -1.23 |
| 4321.481* | 23133.708 | 15551.252 | -1.20 |
| 4367.3789 | 22890.623 | 18819.520 | -1.19 |
| 4367.960* | 22887.552 | 13665.686 | -1.03 |
| 4421.6458 | 22609.664 | 21039.529 | -1.31 |
| 4467.8849 | 22375.702 | 21511.376 | -1.26 |
| 4492.9563 | 22250.809 | 21039.529 | -0.87 |
| 4522.1931 | 22107.015 | 21511.376 | -1.20 |
| 4527.636* | 22080.389 | 12805.965 | -1.29 |
| 4579.444* | 21830.594 | 13690.228 | -1.02 |
| 4588.9970 | 21785.149 | 20347.497 | -1.46 |
| 4699.5812 | 21272.640 | 20437.503 | -1.04 |

Notes. For the starred values, see the text.

Our knowledge of the Nb III spectrum remains incomplete. Only 19 levels were reported by Moore (1958), which were deduced from the pioneering work of Gibbs \& White (1928) and Eliason (1933). Additional work performed by Iglesias (1955) established 58 new levels belonging to the $4 \mathrm{~d}^{3}, 4 \mathrm{~d}^{2} 5 \mathrm{~s}, 4 \mathrm{~d}^{2} 5 \mathrm{~d}$, $4 d^{2} 6 \mathrm{~s}$, and $4 \mathrm{~d}^{2} 5$ p configurations. The most recent analysis of this


Fig. 2. The $4 d^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s} \mathrm{a}^{5} \mathrm{~F}_{5}-4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p} \mathrm{z}{ }^{3} \mathrm{~F}_{4}^{\circ}$ transition in Nb II at $2745.31 \AA$. The large structure (more than $0.1 \AA$ ) is produced by the hfs splitting in the $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5$ s a ${ }^{5} \mathrm{~F}_{5}$ energy level (Nilsson \& Ivarsson 2008).
spectrum was completed by Gayazov et al. (1998), who identified 908 transitions.

Atomic structure calculations in Nb II- Nb III are realistic only if relativistic and correlation effects are considered simultaneously. As has been frequently discussed (e.g., Biémont \& Quinet 2003; Biémont 2005), the HFR approach developed by Cowan (1981), although based on the non-relativistic Schrödinger equation, is well suited to heavy atoms or ions because it incorporates the most significant relativistic effects (Blume-Watson spin-orbit interaction, mass-velocity, and onebody Darwin terms). This HFR approach was adopted here. Configuration interaction (CI) was included in the calculations extensively. In addition, a least squares fitting procedure was applied to the radial integrals using the experimental energy levels.

In Nb II, two different physical models were considered. In the first model (designated HFR(A)), a 4d ionic core surrounded by 3 valence electrons was chosen. Valence-valence type interactions were considered by including the following configurations in the CI expansions: $4 \mathrm{~d}^{4}, 4 \mathrm{~d}^{3} 5 \mathrm{~s}, 4 \mathrm{~d}^{3} 6 \mathrm{~s}, 4 \mathrm{~d}^{3} 5 \mathrm{~d}, 4 \mathrm{~d}^{2} 5 \mathrm{~s}^{2}$, $4 \mathrm{~d}^{2} 5 \mathrm{p}^{2}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 6 \mathrm{~s}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 5 \mathrm{~d}, 4 \mathrm{~d} 5 \mathrm{~s}^{2} 6 \mathrm{~s}$, and $4 \mathrm{~d} 5 \mathrm{~s} 5 \mathrm{p}^{2}$ (even parity), and $4 d^{3} 5 \mathrm{p}, 4 \mathrm{~d}^{3} 4 \mathrm{f}, 4 \mathrm{~d}^{3} 5 \mathrm{f}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 5 \mathrm{p}, 4 \mathrm{~d}^{2} 4 \mathrm{f} 5 \mathrm{~s}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 5 \mathrm{f}, 4 \mathrm{~d}^{2} 5 \mathrm{p} 6 \mathrm{~s}$, $4 d^{2} 5 \mathrm{p} 5 \mathrm{~d}, 4 \mathrm{~d}^{2} 4 \mathrm{f} 5 \mathrm{~d}$, and $4 \mathrm{~d} 5 \mathrm{~s}^{2} 5 \mathrm{p}$ (odd parity).

Core-valence interactions were taken into account using a polarization model potential and a correction to the dipole operator following a well-established procedure (see Biémont \& Quinet 2003; Quinet et al. 1999) giving rise to the HFR+CPOL method.

In the present context, the polarization model for Nb II was based on a $\mathrm{Nb}^{4+}$ ionic core surrounded by 3 valence electrons. For the dipole polarizability $\alpha_{\mathrm{d}}$, we used the value calculated by Fraga et al. (1976), i.e., $\alpha_{\mathrm{d}}=3.64 a_{0}^{3}$, corresponding to the ionic core NbV . The value of the cut-off radius $r_{\mathrm{c}}$ was the HFR mean value $\langle r\rangle$ of the outermost 4 d core orbital, i.e., $r_{\mathrm{c}}=1.85 a_{0}$.

The HFR+CPOL method was combined with a least squares optimization routine that minimized the discrepancies between calculated and experimental energy levels published by Ryabtsev et al. (2000). In the fitting process, we included all the experimentally known energy levels below $80000 \mathrm{~cm}^{-1}$. These levels belong to the $4 d^{4}, 4 d^{3} 5 s, 4 d^{3} 6 s, 4 d^{3} 5 d$, and $4 d^{2} 5 s^{2}$ configurations (even parity) and to the $4 d^{3} 5 p$ and $4 d^{2} 5 s 5 p$ configurations (odd parity). For these configurations, the fitted parameters were the center-of-gravity energies $\left(E_{a v}\right)$, both the single-configuration direct $\left(F^{k}\right)$ and exchange $\left(G^{k}\right)$ electrostatic

Table 4. Comparison between theoretical $f$-values obtained in this work for Nb II and those calculated previously.

| Lower level ${ }^{a}$ |  | Upper level ${ }^{a}$ |  |  | $f$-values |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Desig. | $E\left(\mathrm{~cm}^{-1}\right)$ | Desig. | $E\left(\mathrm{~cm}^{-1}\right)$ | this work | $\mathrm{RCI}^{\text {b }}$ |  |
|  |  |  |  |  | velocity | length |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}{ }^{5} \mathrm{~F}_{2}$ | 2629.13 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{5} \mathrm{G}_{3}^{\circ}$ | 33919.24 | 0.277 | 0.239 | 0.315 |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}{ }^{5} \mathrm{~F}_{3}$ | 3029.63 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}^{5} \mathrm{G}_{3}^{\circ}$ | 33919.24 | 0.072 | 0.065 | 0.074 |
| $4 \mathrm{~d}^{4}{ }^{5} \mathrm{D}_{2}$ | 438.36 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{3}^{\circ}$ | 36553.24 | 0.064 | 0.054 | 0.068 |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}{ }^{5} \mathrm{~F}_{2}$ | 2629.13 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}^{3} \mathrm{D}_{3}^{\circ}$ | 36553.24 | 0.022 | 0.020 | 0.027 |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}{ }^{5} \mathrm{~F}_{3}$ | 3029.63 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}^{3} \mathrm{D}_{3}^{\circ}$ | 36553.24 | 0.022 | 0.026 | 0.031 |
| $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{~s}{ }^{5} \mathrm{~F}_{4}$ | 3542.56 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{3}^{\circ}$ | 36553.24 | 0.065 | 0.052 | 0.068 |
| $4 d^{4}{ }^{3} \mathrm{P}_{2}$ | 7261.32 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}{ }^{3} \mathrm{D}_{3}^{\circ}$ | 36553.24 | 0.051 | 0.065 | 0.069 |
| $4 \mathrm{~d}^{4}{ }^{3} \mathrm{~F}_{4}$ | 8320.37 | $4 \mathrm{~d}^{3}\left({ }^{4} \mathrm{~F}\right) 5 \mathrm{p}^{3} \mathrm{D}_{3}^{\circ}$ | 36553.24 | 0.040 | 0.038 | 0.041 |

Notes. ${ }^{(a)}$ Ryabtsev et al. (2000); ${ }^{(b)}$ Beck \& Datta (1995).
interaction integrals and the spin-orbit parameters $\left(\zeta_{n l}\right)$. All the non-fitted $F^{k}, G^{k}$, and $R^{k}$ integrals were scaled by a factor of 0.85 as suggested by Cowan (1981), while the ab initio values of the spin-orbit parameters were used. The standard deviations of the fits were found to be equal to $296 \mathrm{~cm}^{-1}$ and $292 \mathrm{~cm}^{-1}$ for the even and odd parities, respectively.

In the second model (referred to as $\operatorname{HFR}(B)$ ), a $4 d^{2}$ ionic core surrounded by 2 valence electrons was considered. The valence-valence interactions were taken into account by including the following configurations on a vectorial basis: $4 \mathrm{~d}^{4}$, $4 d^{3} 5 \mathrm{~s}, 4 \mathrm{~d}^{3} 6 \mathrm{~s}, 4 \mathrm{~d}^{3} 5 \mathrm{~d}, 4 \mathrm{~d}^{2} 5 \mathrm{~s}^{2}, 4 \mathrm{~d}^{2} 5 \mathrm{p}^{2}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 6 \mathrm{~s}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 5 \mathrm{~d}, 4 \mathrm{~d}^{2} 4 \mathrm{f} 5 \mathrm{p}$, $4 d^{2} 5 \mathrm{p} 5 \mathrm{f}, 4 \mathrm{~d}^{2} 6 \mathrm{~s}^{2}, 4 \mathrm{~d}^{2} 5 \mathrm{~d}^{2}, 4 \mathrm{~d}^{2} 5 \mathrm{~d} 6 \mathrm{~s}$, and $4 \mathrm{~d}^{2} 5 \mathrm{p} 6 \mathrm{p}$ (even parity), and $4 d^{3} 5 \mathrm{p}, 4 \mathrm{~d}^{3} 6 \mathrm{p}, 4 \mathrm{~d}^{3} 4 \mathrm{f}, 4 \mathrm{~d}^{3} 5 \mathrm{f}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 5 \mathrm{p}, 4 \mathrm{~d}^{2} 5 \mathrm{~s} 6 \mathrm{p}, 4 \mathrm{~d}^{2} 4 \mathrm{f} 5 \mathrm{~s}$, $4 d^{2} 4 f 5 d, 4 d^{2} 5 s 5 f, 4 d^{2} 5 p 6 s, 4 d^{2} 5 p 5 d$, and $4 d^{2} 6 s 6 p$ (odd parity). In this model, the CPOL effects were included using the dipole polarizability corresponding to the ionic Nb IV core given in Fraga et al. (1976), i.e., $\alpha_{\mathrm{d}}=5.80 a_{0}^{3}$. As previously, the cut-off radius was chosen to be equal to $r_{\mathrm{c}}=1.85 a_{0}$.

For the odd parity, the fitting procedure was more complicated, partly because of the poor knowledge of the $4 \mathrm{~d} 5 \mathrm{~s}^{2} 5$ p configuration. Since all the experimental lifetimes obtained for Nb II correspond to $4 d^{3} 5 \mathrm{p}$ levels, we focused on the low-lying $4 d^{3} 5 p$ configuration including the odd experimental levels below $50000 \mathrm{~cm}^{-1}$. The parameters of the $4 \mathrm{~d}^{2} 5 \mathrm{~s} 5 \mathrm{p}$ configuration were not adjusted, except for the center-of-gravity energy $\left(E_{a v}\right)$. The standard deviations of the fits were found to be $280 \mathrm{~cm}^{-1}$ and $148 \mathrm{~cm}^{-1}$ for the even and odd parities, respectively ${ }^{1}$.

Table 1 shows a comparison between lifetimes calculated with the two theoretical models $\operatorname{HFR}(A)$ and $\operatorname{HFR}(B)$. We can see that the $\operatorname{HFR}(A)$ results appear systematically shorter than the experimental values (on average by $12 \%$ ). This is probably caused by our not introducing a sufficient number of $4 \mathrm{~d}^{2} n n^{\prime} l^{\prime}$ configurations to accurately account for the valencevalence interactions. Because of computer limitations, it was impossible to add more configurations of this type to the HFR(A) model. Consequently, we considered a second model with additional $4 \mathrm{~d}^{2} n l n^{\prime} l^{\prime}$ configurations and a $4 \mathrm{~d}^{2}$ ionic core surrounded by 2 valence electrons (HFR(B) model). From Table 1, we can see that there is good agreement between the $\operatorname{HFR}(\mathrm{B})$ results and the experimental lifetimes (within 4\%). However, for the $4 d^{3}\left({ }^{4} F\right) 5 p^{5} D^{\circ}$ levels, the values obtained by Hannaford et al. (1985) are about $10 \%$ longer than the lifetimes measured in the present work. The excellent agreement of the HFR+CPOL lifetimes with the new measurements presented here indicate that

[^0]the measurements of lifetimes by Hannaford et al. (1985) could be too long.

Table 4 compares the HFR oscillator strengths with those previously published by Beck \& Datta (1995) and shows that the agreement is very good.

Using the $\operatorname{HFR}(\mathrm{B})$ model, a particularly close agreement is found between theory and experiment for the lifetimes (see Table 1). Consequently, we decided to adopt a similar model for Nb III, an ion for which no experimental lifetimes at all are available. In this case, the following configurations were included: $4 \mathrm{~d}^{3}, 4 \mathrm{~d}^{2} 5 \mathrm{~s}, 4 \mathrm{~d}^{2} 6 \mathrm{~s}, 4 \mathrm{~d}^{2} 5 \mathrm{~d}, 4 \mathrm{~d} 5 \mathrm{~s}^{2}, 4 \mathrm{~d} 5 \mathrm{p}^{2}, 4 \mathrm{~d} 5 \mathrm{~s} 6 \mathrm{~s}, 4 \mathrm{~d} 5 \mathrm{~s} 5 \mathrm{~d}$, $4 d 5 p 4 f, 4 d 5 p 5 f, 4 d 6 s^{2}, 4 d 5 d^{2}, 4 d 5 d 6 s, 4 d 5 p 6 p$ (even parity), and $4 d^{2} 5 \mathrm{p}, 4 \mathrm{~d}^{2} 6 \mathrm{p}, 4 \mathrm{~d}^{2} 4 \mathrm{f}, 4 \mathrm{~d}^{2} 5 \mathrm{f}, 4 \mathrm{~d} 5 \mathrm{~s} 5 \mathrm{p}, 4 \mathrm{~d} 5 \mathrm{~s} 6 \mathrm{p}, 4 \mathrm{~d} 4 \mathrm{f} 5 \mathrm{~s}, 4 \mathrm{~d} 4 \mathrm{f} 5 \mathrm{~d}$, $4 d 5 s 5 f, 4 d 5 p 6 s, 4 d 5 p 5 d$, and $4 d 6 s 6 p$ (odd parity).

As far as polarization effects are concerned, a $\mathrm{Nb}^{4+}$ ionic core was considered to be surrounded by a valence electron. For the dipole polarizability $\alpha_{\mathrm{d}}$, we used the value calculated by Fraga et al. (1976), i.e., $\alpha_{\mathrm{d}}=3.64 a_{0}^{3}$. The value of the cut-off radius $r_{\mathrm{c}}$ was the HFR mean value $\langle r\rangle$ of the outermost 4 d core orbital, i.e., $r_{c}=1.85 a_{0}$.

In the fitting procedure, we used the experimentally established levels from Iglesias (1955). For the even parity, all the available experimental levels were included in the fitting procedure. For the odd parity, several levels were not established with certainty and were marked with a question mark (?) in Iglesias' analysis or were not adequately reproduced in our calculations. Consequently, we excluded the levels at 85 999.3, 74 907.7, $85585.1,83259.4$, and $81953.4 \mathrm{~cm}^{-1}$ from the fitting procedure.

The standard deviations of the fitting procedures were $197 \mathrm{~cm}^{-1}$ (even levels) and $248 \mathrm{~cm}^{-1}$ (odd levels). The lifetime values obtained in Nb III (with and without polarization effects) are reported in Table 6, only the values larger than 1 ns being quoted to limit the length of the table.

The oscillator strengths ( $\log g f$-values) and transition probabilities $(g A)$ of the most intense transitions in Nb III are reported in Table 7. We verified that none of these transitions were affected by cancellation effects in the line strengths (Cowan 1981).

## 5. Solar and stellar abundances

Using the new Nb II transition data, we rederived niobium abundances in the solar photosphere and in five $n$-capture-rich metalpoor giant stars. In general, we followed procedures that have been used in a number of papers on the rare-earth elements (Sneden et al. 2009, and references therein).

Table 5. Hyperfine splitting and oscillator strengths (log gf) for Nb II transitions.

| Transition ${ }^{a}$ | $F$ |  | $\begin{gathered} \sigma^{b} \\ \left(\mathrm{~cm}^{-1}\right) \end{gathered}$ | $\begin{gathered} \hline \lambda_{\text {air }}{ }^{b} \\ (\AA) \\ \hline \end{gathered}$ | $\log (g f)^{b}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | upper | lower |  |  |  |
| $\mathrm{a}^{5} \mathrm{~F}_{1}-\mathrm{z}{ }^{5} \mathrm{G}_{2}^{\circ}$ | 2.5 | 3.5 | 30993.613 | 3225.540 | -0.93 |
|  | 3.5 | 3.5 | 30993.756 | 3225.525 | -1.02 |
|  | 3.5 | 4.5 | 30993.928 | 3225.507 | -1.21 |
|  | 4.5 | 3.5 | 30993.941 | 3225.506 | -1.32 |
|  | 4.5 | 4.5 | 30994.113 | 3225.488 | -0.90 |
|  | 4.5 | 5.5 | 30994.323 | 3225.466 | -1.67 |
|  | 5.5 | 4.5 | 30994.338 | 3225.464 | -0.86 |
|  | 5.5 | 5.5 | 30994.548 | 3225.443 | -1.02 |
|  | 6.5 | 5.5 | 30994.815 | 3225.415 | -0.56 |
| $\mathrm{a}^{5} \mathrm{~F}_{2}-\mathrm{Z}^{5} \mathrm{G}_{3}^{\circ}$ | 1.5 | 2.5 | 31289.962 | 3194.990 | -1.12 |
|  | 2.5 | 2.5 | 31290.014 | 3194.984 | -1.23 |
|  | 2.5 | 3.5 | 31289.954 | 3194.990 | -1.27 |
|  | 3.5 | 2.5 | 31290.085 | 3194.977 | -1.63 |
|  | 3.5 | 3.5 | 31290.025 | 3194.983 | -1.04 |
|  | 3.5 | 4.5 | 31289.948 | 3194.991 | -1.45 |
|  | 4.5 | 3.5 | 31290.118 | 3194.974 | -1.19 |
|  | 4.5 | 4.5 | 31290.040 | 3194.982 | -0.98 |
|  | 4.5 | 5.5 | 31289.946 | 3194.991 | -1.72 |
|  | 5.5 | 4.5 | 31290.153 | 3194.970 | -0.90 |
|  | 5.5 | 5.5 | 31290.059 | 3194.980 | -1.03 |
|  | 5.5 | 6.5 | 31289.948 | 3194.991 | -2.16 |
|  | 6.5 | 5.5 | 31290.192 | 3194.966 | -0.69 |
|  | 6.5 | 6.5 | 31290.081 | 3194.977 | -1.22 |
|  | 7.5 | 6.5 | 31290.235 | 3194.962 | -0.52 |
| $\mathrm{a}{ }^{5} \mathrm{~F}_{3}-\mathrm{Z}{ }^{5} \mathrm{G}_{3}^{\circ}$ | 1.5 | 1.5 | 30889.762 | 3236.385 | -1.89 |
|  | 1.5 | 2.5 | 30889.689 | 3236.392 | -1.80 |
|  | 2.5 | 1.5 | 30889.813 | 3236.379 | -1.80 |
|  | 2.5 | 2.5 | 30889.740 | 3236.387 | -2.79 |
|  | 2.5 | 3.5 | 30889.637 | 3236.398 | -1.59 |
|  | 3.5 | 2.5 | 30889.812 | 3236.379 | -1.59 |
|  | 3.5 | 3.5 | 30889.709 | 3236.390 | -3.17 |
|  | 3.5 | 4.5 | 30889.576 | 3236.404 | -1.51 |
|  | 4.5 | 3.5 | 30889.801 | 3236.380 | -1.51 |
|  | 4.5 | 4.5 | 30889.669 | 3236.394 | -2.06 |
|  | 4.5 | 5.5 | 30889.507 | 3236.411 | -1.50 |
|  | 5.5 | 4.5 | 30889.781 | 3236.383 | -1.50 |
|  | 5.5 | 5.5 | 30889.620 | 3236.399 | -1.58 |
|  | 5.5 | 6.5 | 30889.428 | 3236.420 | -1.56 |
|  | 6.5 | 5.5 | 30889.753 | 3236.386 | -1.56 |
|  | 6.5 | 6.5 | 30889.562 | 3236.406 | -1.25 |
|  | 6.5 | 7.5 | 30889.341 | 3236.429 | -1.76 |
|  | 7.5 | 6.5 | 30889.715 | 3236.389 | -1.76 |
|  | 7.5 | 7.5 | 30889.495 | 3236.413 | -1.01 |
| $\mathrm{a}{ }^{5} \mathrm{~F}_{4}-\mathrm{Z}{ }^{5} \mathrm{G}_{4}^{\circ}$ | 0.5 | 0.5 | 31089.935 | 3215.546 | -2.46 |
|  | 0.5 | 1.5 | 31089.886 | 3215.551 | -2.02 |
|  | 1.5 | 0.5 | 31089.953 | 3215.544 | -2.02 |
|  | 1.5 | 1.5 | 31089.904 | 3215.550 | -4.06 |
|  | 1.5 | 2.5 | 31089.821 | 3215.558 | -1.79 |
|  | 2.5 | 1.5 | 31089.934 | 3215.546 | -1.79 |
|  | 2.5 | 2.5 | 31089.851 | 3215.555 | -3.05 |
|  | 2.5 | 3.5 | 31089.736 | 3215.567 | -1.67 |
|  | 3.5 | 2.5 | 31089.893 | 3215.551 | -1.67 |
|  | 3.5 | 3.5 | 31089.777 | 3215.563 | -2.30 |
|  | 3.5 | 4.5 | 31089.629 | 3215.578 | -1.60 |
|  | 4.5 | 3.5 | 31089.831 | 3215.557 | -1.60 |
|  | 4.5 | 4.5 | 31089.683 | 3215.572 | -1.88 |
|  | 4.5 | 5.5 | 31089.501 | 3215.591 | -1.58 |
|  | 5.5 | 4.5 | 31089.748 | 3215.566 | -1.58 |
|  | 5.5 | 5.5 | 31089.567 | 3215.584 | -1.58 |
|  | 5.5 | 6.5 | 31089.352 | 3215.607 | -1.60 |
|  | 6.5 | 5.5 | 31089.645 | 3215.576 | -1.60 |
|  | 6.5 | 6.5 | 31089.430 | 3215.599 | -1.35 |
|  | 6.5 | 7.5 | 31089.182 | 3215.624 | -1.69 |

Table 5. contined.


Notes. ${ }^{(a)}$ Level designations and center of gravity energies from Ryabtsev et al. (2000); ${ }^{(b)}$ derived from the data reported by Nilsson \& Ivarsson (2008).

### 5.1. Line selection

The principal difficulty for Nb II abundance studies is that there are no strong transitions in the visible spectral range. It is thus necessary to utilize the crowded UV region, where few if any transitions are unblended, and placement of the stellar continuum is often very uncertain. In Fig. 3, we plot the relative strengths of Nb II lines as a function of wavelength. Details of the line strength arguments can be found in Sneden et al. (2009) and references therein. Briefly, to first approximation the relative logarithmic absorption strength of a transition within a particular species is proportional to $\log g f-\theta \chi$, where $\chi$ is the excitation potential in eV and $\theta=5040 / \mathrm{T}$, the inverse temperature. Additionally, the first ionization potential for niobium is

Table 6. HFR lifetimes ( $\tau$ in ns) (without and with core-polarization effects included) of the low-lying levels of $\mathrm{Nb} \operatorname{III}$ ( $4 \mathrm{~d}^{2} 5 \mathrm{p}$ configuration). Only lifetime values larger than 1 ns are quoted.

| $E\left(\mathrm{~cm}^{-1}\right)^{a}$ | Term $^{a}$ | $J$ | $\tau(\mathrm{HFR})$ | $\tau(\mathrm{HFR}+\mathrm{CPOL})$ |
| :---: | :---: | :---: | :---: | :---: |
| 63686.7 | ${ }^{4} \mathrm{G}^{\circ}$ | $5 / 2$ | 2.25 | 2.58 |
| 65005.9 | ${ }^{4} \mathrm{G}^{\circ}$ | $7 / 2$ | 2.13 | 2.44 |
| 65904.2 | ${ }^{4} \mathrm{~F}^{\circ}$ | $3 / 2$ | 1.01 | 1.16 |
| 66456.2 | ${ }^{4} \mathrm{G}^{\circ}$ | $9 / 2$ | 2.02 | 2.32 |
| 66598.4 | ${ }^{2} \mathrm{~F}^{\circ}$ | $5 / 2$ | 1.21 | 1.39 |
| 67094.1 | ${ }^{4} \mathrm{~F}^{\circ}$ | $5 / 2$ | 1.20 | 1.37 |
| 67775.0 | ${ }^{2} \mathrm{D}^{\circ}$ | $3 / 2$ | 1.17 | 1.33 |
| 68061.7 | ${ }^{4} \mathrm{G}^{\circ}$ | $11 / 2$ | 1.96 | 2.25 |
| 68382.1 | ${ }^{2} \mathrm{~F}^{\circ}$ | $7 / 2$ | 1.56 | 1.78 |
| 69110.4 | ${ }^{2} \mathrm{D}^{\circ}$ | $5 / 2$ | 1.20 | 1.37 |
| 74726.7 | ${ }^{4} \mathrm{D}^{\circ}$ | $1 / 2$ | 1.40 | 1.62 |
| 75383.1 | ${ }^{4} \mathrm{D}^{\circ}$ | $3 / 2$ | 1.02 | 1.13 |
| $76387.8 ?$ | ${ }^{2} \mathrm{D}^{\circ}$ | $5 / 2$ | 1.00 | 1.16 |
| $76913.3 ?$ | ${ }^{2} \mathrm{D}^{\circ}$ | $3 / 2$ | 1.03 | 1.20 |
| 77247.5 | ${ }^{2} \mathrm{~F}^{\circ}$ | $5 / 2$ | 1.28 | 1.46 |
| 78085.9 | ${ }^{4} \mathrm{P}^{\circ}$ | $1 / 2$ | 1.16 | 1.34 |
| 78372.2 | ${ }^{4} \mathrm{P}^{\circ}$ | $3 / 2$ | 1.20 | 1.38 |
| 79429.7 | ${ }^{4} \mathrm{P}^{\circ}$ | $5 / 2$ | 1.18 | 1.35 |
| 82152.1 | ${ }^{2} \mathrm{H}^{\circ}$ | $11 / 2$ | 1.33 | 1.52 |
| 84523.0 | ${ }^{2} \mathrm{~F}^{\circ}$ | $7 / 2$ | 1.08 | 1.23 |

Notes. ${ }^{(a)}$ From Iglesias (1955).
relatively low $(6.6 \mathrm{eV})$. In the solar and stellar photospheres considered in this paper, virtually all niobium is in the form of $\mathrm{Nb}^{+}$; there are essentially no "Saha" corrections for other niobium ionization stages. Therefore, the relative strengths of Nb II lines can be compared to those of other ionized-species with similarly low ionization potentials (such as all of the rare-earth elements) by writing the relative strengths as $\log \epsilon g f-\theta \chi$, where $\epsilon$ is the elemental abundance. In Fig. 3, we show the adopted $\theta=1.0$, a compromise between the $\theta_{\text {eff }}$ values of the Sun and the metalpoor giant program stars. The strengths of the Nb II lines are quite sensitive to the exact choice of $\theta$. For this figure, we also assumed that $\log \epsilon_{\odot}=1.4$, which is close to the photospheric and meteoritic abundances.

In Fig. 3, we indicate the minimum strength at which Nb II lines can be detected in the solar spectrum, and the value of this quantity at which the lines become strong (so that line saturation might become an issue; see Sneden et al. 2009, and references therein). These two levels are very approximate, but for example lines with strengths $<-0.6$ are simply too weak to be reliably identified and used in an abundance study. They may be safely ignored in our search for useful Nb II lines. This limit (very roughly) holds also for the kinds of very metal-poor, $r$-process-rich giant stars considered in this paper.

It is clear from Fig. 3 that no detectable lines of Nb II occur above $4000 \AA$. This is confirmed by the Moore et al. (1966) solar line compendium. They identified 26 transitions as at least partly attributable to Nb II. Their two longest-wavelength identifications are 3818.99 and $4492.96 \AA$. But the latter transition (not considered in the present work or by Nilsson \& Ivarsson 2008) is blended with a Ce II line (Palmeri et al. 2000; Lawler et al. 2009) that accounts for a large fraction of the very weak solar absorption at this wavelength. Its attribution to Nb II is therefore doubtful, and thus all useful Nb II transitions are in the

Table 7. Transition probabilities ( gA , in $\mathrm{s}^{-1}$ ) and oscillator strengths $(\log g f)$ for the most intense transitions $(\log g f>-0.50)$ of Nb III.

| $\lambda(\AA)^{a}$ | Int. ${ }^{a}$ | Transition ${ }^{\text {a }}$ | $\log g f^{b}$ | $g A^{b}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1431.92 | 60 | $\mathrm{a}^{4} \mathrm{P}_{5 / 2}-\mathrm{Z}^{4} \mathrm{P}_{5 / 2}^{\circ}$ | -0.46 | 1.13(9) |
| 1445.43 | 80 | a ${ }^{4} \mathrm{~F}_{3 / 2}-\mathrm{z}^{4} \mathrm{D}_{1 / 2}^{\circ}$ | -0.37 | 1.36(9) |
| 1445.98 | 80 | a ${ }^{4} \mathrm{~F}_{5 / 2}-\mathrm{z}^{4} \mathrm{D}_{3 / 2}^{\circ}$ | -0.20 | 2.02(9) |
| 1447.09 | 80 | a ${ }^{4} \mathrm{~F}_{7 / 2}-\mathrm{z}^{4} \mathrm{D}_{5 / 2}^{\circ}$ | -0.08 | 2.65(9) |
| 1448.50 | 50B | $\mathrm{a}^{2} \mathrm{G}_{9 / 2}-\mathrm{y}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | -0.35 | 1.41(9) |
| 1451.63 | 60 | $\mathrm{a}^{2} \mathrm{H}_{11 / 2}-\mathrm{Z}^{2} \mathrm{H}_{11 / 2}$ | 0.03 | 3.40 (9) |
| 1456.68 | 100 | $\mathrm{a}^{4} \mathrm{~F}_{9 / 2}-\mathrm{Z}^{4} \mathrm{D}_{7 / 2}^{\circ}$ | 0.13 | 4.26(9) |
| 1495.94 | 100 | a ${ }^{4} \mathrm{~F}_{9 / 2}-\mathrm{z}^{4} \mathrm{~F}_{9 / 2}^{\circ}$ | 0.26 | 5.44(9) |
| 1499.45 | 80 | $\mathrm{a}^{4} \mathrm{~F}_{7 / 2}-\mathrm{z}^{4} \mathrm{~F}_{7 / 2}^{\circ}$ | 0.13 | 3.97(9) |
| 1501.99 | 100 | $\mathrm{a}^{4} \mathrm{~F}_{5 / 2}-\mathrm{z}^{4} \mathrm{~F}_{5 / 2}^{\circ}$ | -0.35 | 1.31(9) |
| 1513.81 | 80 | $\mathrm{a}^{2} \mathrm{H}_{9 / 2}-\mathrm{y}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | 0.39 | 7.08(9) |
| 1517.38 | 50 | $\mathrm{a}^{4} \mathrm{~F}_{3 / 2}-\mathrm{z}^{4} \mathrm{~F}_{3 / 2}^{\circ}$ | -0.23 | 1.70(9) |
| 1524.91 | 100 | $\mathrm{a}^{2} \mathrm{H}_{11 / 2}-\mathrm{y}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | 0.44 | 7.84(9) |
| 1537.50 | 60 | a ${ }^{4} \mathrm{P}_{3 / 2}-\mathrm{z}^{2} \mathrm{P}_{3 / 2}^{\circ}$ | -0.48 | 9.36 (8) |
| 1546.50 | 40 | $\mathrm{a}^{2} \mathrm{~F}_{7 / 2}-\mathrm{x}^{2} \mathrm{~F}_{7 / 2}^{\circ}$ | -0.10 | 2.23(9) |
| 1566.92 | 50B | $\mathrm{a}^{2} \mathrm{D}_{5 / 2}-\mathrm{y}^{2} \mathrm{D}_{3 / 2}^{\circ}$ | -0.35 | 1.21(9) |
| 1590.21 | 100 | $\mathrm{a}^{2} \mathrm{G}_{9 / 2}-\mathrm{z}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | 0.09 | 3.24(9) |
| 1598.86 | 80 | $\mathrm{a}^{2} \mathrm{~F}_{7 / 2}-\mathrm{x}^{2} \mathrm{D}_{5 / 2}^{\circ}$ | 0.08 | 3.14(9) |
| 1604.72 | 80 | $\mathrm{a}^{2} \mathrm{G}_{7 / 2}-\mathrm{z}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | 0.00 | 2.60(9) |
| 1639.51 | 80 | a ${ }^{4} \mathrm{P}_{5 / 2}-\mathrm{z}^{4} \mathrm{D}_{7 / 2}^{\circ}$ | -0.30 | 1.24(9) |
| 1682.77 | 100 | $\mathrm{a}^{2} \mathrm{H}_{11 / 2}-\mathrm{z}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | -0.07 | 2.03(9) |
| 1705.44 | 100 | $\mathrm{a}^{2} \mathrm{H}_{9 / 2}-\mathrm{z}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | -0.27 | 1.23(9) |
| 1808.70 | 50 | $\mathrm{a}^{2} \mathrm{D}_{5 / 2}-\mathrm{z}^{2} \mathrm{~F}_{7 / 2}^{\circ}$ | -0.48 | 6.77(8) |
| 1892.92 | 100 | $\mathrm{a}^{2} \mathrm{~F}_{7 / 2}-\mathrm{z}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | -0.49 | 6.07(8) |
| 1938.84 | 100 | $\mathrm{a}^{2} \mathrm{~F}_{5 / 2}-\mathrm{z}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | -0.49 | 5.72(8) |
| 2060.29 | 50 | $\mathrm{a}^{2} \mathrm{~F}_{7 / 2}-\mathrm{z}^{2} \mathrm{~F}_{7 / 2}^{\circ}$ | -0.35 | 7.02(8) |
| 2112.31 | 40 | $\mathrm{b}^{2} \mathrm{~F}_{7 / 2}-\mathrm{x}^{2} \mathrm{D}_{5 / 2}^{\circ}$ | -0.38 | 6.16(8) |
| 2240.31 | 60 | $\mathrm{b}^{4} \mathrm{P}_{3 / 2}-\mathrm{Z}^{4} \mathrm{P}_{5 / 2}^{\circ}$ | -0.31 | 6.54(8) |
| 2249.52 | 30 | $\mathrm{b}^{4} \mathrm{P}_{5 / 2}-\mathrm{z}^{4} \mathrm{P}_{5 / 2}^{\circ}$ | -0.28 | 6.97(8) |
| 2265.63 | 60 | $\mathrm{b}^{4} \mathrm{~F}_{7 / 2}-\mathrm{z}^{4} \mathrm{D}_{7 / 2}^{\circ}$ | -0.41 | 5.09(8) |
| 2273.92 | 80 | $\mathrm{b}^{4} \mathrm{~F}_{3 / 2}-\mathrm{z}^{4} \mathrm{D}_{1 / 2}^{\circ}$ | -0.23 | 7.65(8) |
| 2275.23 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{z}^{4} \mathrm{D}_{3 / 2}^{\circ}$ | -0.09 | 1.05(9) |
| 2279.36 | 80 | $\mathrm{b}^{4} \mathrm{P}_{1 / 2}-\mathrm{Z}^{4} \mathrm{P}_{3 / 2}^{\circ}$ | -0.22 | 7.81(8) |
| 2281.51 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{7 / 2}-\mathrm{z}^{4} \mathrm{D}_{5 / 2}^{\circ}$ | -0.00 | 1.28(9) |
| 2284.40 | 80 | $\mathrm{b}^{2} \mathrm{~F}_{7 / 2}-\mathrm{y}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | -0.20 | 8.02(8) |
| 2290.36 | 100 | $\mathrm{b}^{2} \mathrm{G}_{9 / 2}-\mathrm{x}^{2} \mathrm{~F}_{7 / 2}^{\circ}$ | 0.28 | 2.40 (9) |
| 2304.78 | 60 | $\mathrm{b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{z}^{2} \mathrm{D}_{5 / 2}^{\circ}$ | -0.38 | 5.26(8) |
| 2309.92 | 50 | $\mathrm{b}^{4} \mathrm{P}_{3 / 2}-\mathrm{z}^{4} \mathrm{P}_{1 / 2}^{\circ}$ | -0.38 | 5.15(8) |
| 2313.30 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{9 / 2}-\mathrm{z}^{4} \mathrm{D}_{7 / 2}^{\circ}$ | 0.23 | 2.12(9) |
| 2349.21 | 90 | $\mathrm{b}^{4} \mathrm{~F}_{3 / 2}-\mathrm{z}^{2} \mathrm{D}_{3 / 2}^{\circ}$ | -0.31 | 5.97(8) |
| 2355.54 | 80 | $\mathrm{b}^{4} \mathrm{P}_{3 / 2}-\mathrm{y}^{2} \mathrm{~F}_{5 / 2}^{\circ}$ | -0.25 | 6.84(8) |
| 2362.06 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{7 / 2}-\mathrm{z}^{4} \mathrm{~F}_{9 / 2}^{\circ}$ | -0.15 | 8.54(8) |
| 2362.50 | 80B | $\mathrm{b}^{2} \mathrm{D}_{5 / 2}-\mathrm{z}^{4} \mathrm{P}_{5 / 2}^{\circ}$ | -0.15 | 8.52(8) |
| 2372.73 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{z}^{4} \mathrm{~F}_{7 / 2}^{\circ}$ | -0.19 | 7.65(8) |
| 2387.41 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{3 / 2}-\mathrm{z}^{4} \mathrm{~F}_{5 / 2}^{\circ}$ | -0.18 | 7.72(8) |
| 2388.23 | 80 | $\mathrm{b}^{2} \mathrm{D}_{5 / 2}-\mathrm{y}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | -0.18 | 7.81(8) |
| 2413.94 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{9 / 2}-\mathrm{z}^{4} \mathrm{~F}_{9 / 2}^{\circ}$ | 0.41 | 2.90(9) |
| 2414.50 | 60 | $\mathrm{b}^{4} \mathrm{~F}_{7 / 2}-\mathrm{z}^{4} \mathrm{~F}_{7 / 2}^{\circ}$ | 0.20 | 1.80(9) |
| 2417.16 | 2 | $\mathrm{b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{z}^{4} \mathrm{~F}_{5 / 2}^{\circ}$ | -0.33 | 5.39(8) |
| 2421.91 | 100 | $\mathrm{b}^{2} \mathrm{G}_{9 / 2}-\mathrm{z}^{2} \mathrm{H}_{11 / 2}^{\circ}$ | 0.59 | 4.43(9) |
| 2446.10 | 40 | $\mathrm{b}^{4} \mathrm{P}_{1 / 2}-\mathrm{y}^{4} \mathrm{D}_{3 / 2}^{\circ}$ | -0.24 | 6.43(8) |
| 2456.99 | 100 | $\mathrm{b}^{4} \mathrm{~F}_{9 / 2}-\mathrm{Z}^{4} \mathrm{G}_{11 / 2}^{\circ}$ | 0.62 | 4.56(9) |
| 2457.24 | 20 | $\mathrm{b}^{4} \mathrm{~F}_{3 / 2}-\mathrm{z}^{4} \mathrm{~F}_{3 / 2}^{\circ}$ | -0.21 | 6.83(8) |
| 2468.72 | 80 | $\mathrm{b}^{4} \mathrm{~F}_{9 / 2}-\mathrm{z}^{4} \mathrm{~F}_{7 / 2}^{\circ}$ | -0.34 | 5.01(8) |
| 2475.87 | 80 | $\mathrm{b}^{2} \mathrm{D}_{3 / 2}-\mathrm{y}^{2} \mathrm{D}_{3 / 2}^{\circ}$ | -0.08 | 8.97(8) |
| 2486.02 | 50 | $\mathrm{b}^{4} \mathrm{P}_{1 / 2}-\mathrm{y}^{4} \mathrm{D}_{1 / 2}^{\circ}$ | -0.34 | 4.97(8) |

Table 7. continued.

| $\lambda(\AA)^{a}$ | Int $^{a}$ | Transition $^{a}$ |  | $\log g f^{b}$ |
| :---: | ---: | :---: | :---: | :---: |
| 2488.74 | 60 | $\mathrm{~b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{4} \mathrm{~F}_{3 / 2}^{\circ}$ | -0.41 | $4.14(8)$ |
| 2499.73 | 100 | $\mathrm{~b}^{4} \mathrm{~F}_{7 / 2}-\mathrm{Z}^{4} \mathrm{G}_{9 / 2}^{\circ}$ | 0.43 | $2.88(9)$ |
| 2545.64 | 100 | $\mathrm{~b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{4} \mathrm{G}_{7 / 2}^{\circ}$ | 0.27 | $1.93(9)$ |
| 2557.94 | 80 | $\mathrm{~b}^{4} \mathrm{~F}_{9 / 2}-\mathrm{Z}^{4} \mathrm{G}_{9 / 2}^{\circ}$ | -0.20 | $6.43(8)$ |
| 2567.44 | 50 | $\mathrm{~b}^{2} \mathrm{D}_{3 / 2}-\mathrm{y}^{4} \mathrm{D}_{5 / 2}^{\circ}$ | -0.14 | $7.69(8)$ |
| 2586.05 | 40 | $\mathrm{~b}^{4} \mathrm{P}_{5 / 2}-\mathrm{Z}^{2} \mathrm{P}_{3 / 2}^{\circ}$ | -0.12 | $7.47(8)$ |
| 2593.75 | 60 | $\mathrm{~b}^{4} \mathrm{~F}_{7 / 2}-\mathrm{Z}^{4} \mathrm{G}_{7 / 2}^{\circ}$ | -0.22 | $5.99(8)$ |
| 2598.86 | 80 | $\mathrm{~b}^{4} \mathrm{~F}_{3 / 2}-\mathrm{Z}^{4} \mathrm{G}_{5 / 2}^{\circ}$ | 0.12 | $1.30(9)$ |
| 2612.31 | 20 | $\mathrm{~b}^{2} \mathrm{D}_{5 / 2}-\mathrm{y}^{4} \mathrm{D}_{3 / 2}^{\circ}$ | -0.37 | $4.20(8)$ |
| 2628.67 | 50 | $\mathrm{~b}^{2} \mathrm{G}_{7 / 2}-\mathrm{Z}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | 0.21 | $1.57(9)$ |
| 2633.17 | 80 | $\mathrm{~b}^{2} \mathrm{G}_{9 / 2}-\mathrm{y}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | 0.41 | $2.44(9)$ |
| 2634.15 | 30 | $\mathrm{~b}^{4} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{4} \mathrm{G}_{5 / 2}^{\circ}$ | -0.46 | $3.38(8)$ |
| 2638.12 | 40 bl | $\mathrm{b}^{2} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{2} \mathrm{G}_{7 / 2}^{\circ}$ | 0.20 | $1.51(9)$ |
| 2657.99 | 80 | $\mathrm{~b}^{2} \mathrm{~F}_{7 / 2}-\mathrm{Z}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | 0.31 | $1.93(9)$ |
| 2930.26 | 40 | $\mathrm{~b}^{2} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{2} \mathrm{D}_{3 / 2}^{\circ}$ | -0.39 | $3.15(8)$ |
| 2937.71 | 50 | $\mathrm{~b}^{2} \mathrm{~F}_{7 / 2}-\mathrm{Z}^{2} \mathrm{D}_{5 / 2}^{\circ}$ | -0.19 | $4.98(8)$ |
| 2989.95 | 20 | $\mathrm{~b}^{2} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{4} \mathrm{~F}_{5 / 2}^{\circ}$ | -0.39 | $3.03(8)$ |
| 3001.84 | 80 | $\mathrm{~b}^{2} \mathrm{~F}_{7 / 2}-\mathrm{Z}^{2} \mathrm{~F}_{7 / 2}^{\circ}$ | 0.09 | $9.14(8)$ |
| 3034.87 | 10 | $\mathrm{~b}^{2} \mathrm{~F}_{5 / 2}-\mathrm{Z}^{2} \mathrm{~F}_{5 / 2}^{\circ}$ | -0.37 | $3.12(8)$ |
| 3142.26 | 80 | $\mathrm{~b}^{2} \mathrm{G}_{9 / 2}-\mathrm{Z}^{2} \mathrm{G}_{9 / 2}^{\circ}$ | -0.37 | $2.86(8)$ |

Notes. ${ }^{(a)}$ From Iglesias (1955); ${ }^{(b)}$ HFR + CPOL: this work. $A(B)$ is written for $A \times 10^{B}$; bl: blend.


Fig. 3. Relative strengths of Nb II lines (defined as $\log \epsilon_{\odot} g f-\theta \chi$, see text), plotted as a function of wavelength. The atmospheric transmission cutoff wavelength is shown as a vertical line. A dotted horizontal line indicates the approximate strength for barely detectable lines in the solar spectrum, and a dashed horizontal line shows the corresponding strength for strong lines, as discussed in the text.
crowded UV domain. A good discussion of the (lack of) easily accessible Nb II lines was given by Hannaford et al. (1985).

We therefore adopted the strategy of first identifying promising transitions in one of the " $n$-capture-rich" stars known to have enhanced niobium abundances. In these stars, the contrast between Nb II line strengths and those of contaminant transitions should be at its greatest. A similar technique was applied to the
analyses of Ho II (Lawler et al. 2004), Pt I (Den Hartog et al. 2005), and Tm II (Sneden et al. 2009).

For niobium, we chose CS 31082-001, the "uranium star" discovered by Cayrel et al. (2001) and discussed at length by Hill et al. (2002). Our spectrum for CS 31082-001 is described in Sneden et al. (2009). Considering all the laboratory line data in the present paper (Table 4) and in Nilsson \& Ivarsson (2008), we were able to identify only seven transitions in the CS 31082-001 spectrum as promising niobium abundance indicators, which are listed in Table 8. This small number of lines occurred in spite of the order-of-magnitude overabundances of $n$-capture elements with respect to the Fe-group in CS 31082-001 and other $r$-process-rich stars. The chosen lines include two that are in common with the Nb II lines used by Hannaford et al. (1985): $3215.59 \AA$ and $3740.72 \AA$.

In Fig. 4, we show four of the strongest Nb II lines in the spectra of the Sun and CS 31082-001. Inspection of this figure illustrates the difficulties mentioned above: all of the lines are weak and/or blended in both the Sun and the $n$-capture-rich giant. We discuss Fig. 4 further in the next subsection.

### 5.2. Niobium in the solar photosphere

In an attempt to derive a solar niobium abundance from the seven Nb II lines identified in the CS 31082-001 spectrum, we computed synthetic spectra within small wavelength intervals about each candidate transition. In assembling the required atomic and molecular lines, we began with the transitions in the Kurucz $(1998)^{2}$ database, and updated the transition probabilities, and the hyperfine and/or isotopic splits of $n$-capture lines described with literature references in Sneden et al. (2009). The Nb II transition probabilities and hyperfine structure data were taken entirely from this paper and Nilsson \& Ivarsson (2008); since naturally-occurring niobium exists only as ${ }^{93} \mathrm{Nb}$, isotopic wavelength shifts need not be considered. The line lists and the solar empirical model photosphere of Holweger \& Müller (1974) were used as inputs to the current version of the stellar line analysis code MOOG (Sneden 1973) ${ }^{3}$ to generate synthetic spectra. We adopted a microturbulent velocity of $0.8 \mathrm{~km} \mathrm{~s}^{-1}$. These spectra were then compared to the solar photospheric center-of-disk spectrum of Delbouille et al. (1973) ${ }^{4}$, after smoothing to account for solar macroturbulence and instrumental broadening (empirically determined as $1.5 \mathrm{~km} \mathrm{~s}^{-1}$ ).

We adjusted the transition probabilities for lines in which accurate experimental values are missing, to reproduce the overall line absorption in the solar and CS 31082-001 spectra. The niobium abundance for each feature was then determined by comparing the synthetic and observed spectra. The line-by-line abundances obtained are given in Table 8. Values in parentheses are rough estimates, and included as consistency checks only. They were not used in computing the mean photospheric niobium abundance. From the four strongest Nb II lines, we derived $\left\langle\log \epsilon_{\odot}\right\rangle=1.47(\sigma=0.02)$.

The close agreement among the abundances deduced from the four strongest photospheric Nb II lines infers a very small formal sample standard deviation. However, this underestimates the true uncertainty, because every line is at least partially blended (Fig. 4), and continuum placement in the dense UV solar spectrum is not easy to establish. From repeated trial synthesis/observation matches, we estimate a more realistic

[^1]H. Nilsson et al.: Transition probabilities in Nb II and Nb III

Table 8. Solar and stellar niobium abundances.

| $\lambda$ <br> $(\AA)$ | $\chi$ <br> $(\mathrm{eV})$ | $\log g f$ | Sun | $\mathrm{BD}+173248$ | $\mathrm{CS} 22892-052$ | $\mathrm{CS} 31082-001$ | HD 115444 | HD 221170 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\log \epsilon$ | $\log \epsilon$ | $\log \epsilon$ | $\log \epsilon$ | $\log \epsilon$ | $\log \epsilon$ |  |  |  |
| 3194.975 | 0.326 | 0.120 | 1.50 | -0.30 | $\ldots$ | -0.48 | $(-0.9)$ | $(-0.6)$ |
| 3206.339 | 0.930 | 0.038 | $(1.7)$ | -0.08 | $\ldots$ | -0.55 | $\ldots$ | $\ldots$ |
| 3215.593 | 0.439 | -0.235 | 1.53 | -0.23 | -0.85 | -0.66 | -0.98 | -0.82 |
| 3225.467 | 0.292 | -0.008 | $(1.5)$ | -0.35 | -0.72 | -0.53 | -0.95 | -0.79 |
| 3540.959 | 1.031 | -0.431 | $\ldots$ | $(-0.1)$ | $(-0.6)$ | -0.50 | $\ldots$ | $\ldots$ |
| 3717.060 | 1.693 | 0.030 | 1.48 | $(-0.2)$ | $\ldots$ | -0.61 | $\ldots$ | $(-0.6)$ |
| 3740.720 | 1.617 | -0.307 | 1.47 | $(-0.2)$ | $\ldots$ | -0.55 | $\ldots$ | $(-0.5)$ |
| mean |  |  | 1.49 | -0.24 | -0.78 | -0.55 | -0.96 | -0.80 |
| $\sigma$ |  |  | 0.02 | 0.12 | 0.09 | 0.06 | 0.02 | 0.02 |
| \#lines |  |  | 4 | 4 | 2 | 7 | 2 | 2 |



Fig. 4. Observed (points) and synthetic spectra (lines) of Nb II lines in the solar (left-hand panels) and CS 31082-001 (right-hand panels) spectra. The solar photospheric spectrum is from Delbouille et al. (1973), resampled at $0.024 \AA$ for display purposes. The CS 31082-001 spectrum is described in Sneden et al. (2009). In each panel, the heavy black line represents the best-fit synthesis for that feature, the dotted line shows the synthesis for an increase in the Nb abundance by 0.5 dex, the dashed line shows the synthesis for a decrease by 0.5 dex, and the thin solid line shows the synthesis with no Nb contribution.
internal uncertainty for each line to be $\sim \pm 0.08$, so that from four lines the standard deviation of the mean is 0.04. In Lawler et al. (2009) and references therein, we suggested that the external scale errors of solar abundances deduced from low-excitation ionized-state transitions for elements such as rare earths amount to $\sim \pm 0.03$. Therefore, by combining internal and external errors we recommend adopting $\left\langle\log \epsilon_{\odot}\right\rangle=1.47 \pm 0.06$.

Our new photospheric niobium abundance is in reasonable agreement with literature values. Hannaford et al. (1985) analyzed 11 mostly very weak Nb II lines, deriving $\left\langle\log \epsilon_{\odot}\right\rangle=$ $1.42 \pm 0.06$. Asplund et al. (2009) produced a new set of
solar photospheric abundances, which included calculated or estimated corrections for more physically realistic 3-dimensional hydrodynamic solar models and line formation computations that account for departures from LTE. For niobium, they recommend $\left\langle\log \epsilon_{\odot}\right\rangle=1.46 \pm 0.04$. Comparing their rare-earth $57 \leq Z \leq 72$ abundances to those published by the WisconsinTexas group (Sneden et al. 2009), we find $\langle\Delta \log \epsilon\rangle=-0.03$, in the sense Asplund et al. minus Sneden et al. This correction should apply approximately equally to niobium. If we were to adopt this small offset, then our final abundance would be $\left\langle\log \epsilon_{\odot}\right\rangle=1.44 \pm 0.06$. Both the raw and adjusted photospheric values closely agree with abundances of chondritic meteorites, $\left\langle\log \epsilon_{\text {met }}\right\rangle=1.43 \pm 0.04$ (Lodders et al. 2009). The solar-system niobium abundance appears to be well-determined.

### 5.3. Niobium abundances in r-process-rich metal-poor giants

We used the same type of synthetic/observed spectrum matches to derive niobium abundances in $5 r$-process-rich giants. A description of the observed spectra, model stellar atmospheres, and references to previous analyses of these stars is given by Sneden et al. (2009), and we summarize the model parameters in Table 9.

At near-UV wavelengths, the continuum opacities cease to be dominated by $\mathrm{H}^{-}$in the spectra of cool, metal-poor giants. A large amount of the opacity originates instead from Rayleigh scattering, as emphasized by, e.g., Cayrel et al. (2004). Including this scattering opacity alters the equation of radiative transfer, since the source function cannot be approximated simply by the Planck function. The MOOG analysis code was modified to account for this more complex radiative transfer environment in which the Planck function is linked to the continuum pure absorption opacity and the mean intensity is linked to the scattering (Sobeck et al. 2009). We applied the modified code to our analysis of the Nb II lines in the $r$-process-rich giants.

In Fig. 4, we compare synthetic and observed spectra for CS 31082-001. Comparison of these with the same lines in the solar spectrum shows that derivation of reliable niobium abundances are challenging in $r$-process-rich stars in spite of their order-of-magnitude agreement between $n$-captureelement overabundances. Of the five program metal-poor stars, CS 31082-001 is the most suitable candidate for a niobium abundance study. The other four stars are challenging in one or more ways: they have poorer quality spectra (in the case of CS 22898-052), smaller $n$-capture enhancement (HD 115444), higher Fe-peak metallicity (HD 221170), or a combination of these effects. The number of transitions used for each star, rather than the formal line-to-line scatter $\sigma$, should indicate to the reader the reliability of the mean abundance.

Table 9. Stellar model parameters.

|  | BD+17 3248 | CS 22892-052 | CS 31082-001 | HD 115444 | HD 221170 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $T_{\text {eff }}(\mathrm{K})$ | 5200 | 4800 | 4825 | 4800 | 4510 |
| $\log g$ | 1.80 | 1.50 | 1.50 | 1.50 | 1.00 |
| $[\mathrm{Fe} / \mathrm{H}]$ | -2.10 | -3.12 | -2.91 | -2.90 | -2.19 |
| $v_{t}(\mathrm{~km} \mathrm{~s}$ |  |  |  |  |  |
| Ref. | 1.90 | 1.95 | 1.90 | 2.00 | 1.80 |

References. 1: Cowan et al. (2002); 2: Sneden et al. (2003); 3: Hill et al. (2002); 4: Westin et al. (2000); 5: Ivans et al. (2006).

Previous analyses of these stars only used the strong $3215.59 \AA \mathrm{Nb}$ II line, and it is also the only line that we could reliably employ in all of these stars and the Sun. We therefore used this line to connect to past work in two ways. First, we repeated our synthetic spectrum calculations for this line with the assumption of a pure Planck source function. As expected, we found that the disagreements between the "scattering" and "Planck" abundances vary as a function of temperature. With $T_{\text {eff }}=5200 \mathrm{~K}$ (Table 9), BD+173248 is found to have essentially identical abundances with the two methods. The stars CS 22892052, CS 31082-001, and HD 115444 have $T_{\text {eff }} \simeq 4800 \mathrm{~K}$, and for these stars inclusion of scattering in the continuum source function results in abundances that are lower by $\simeq 0.06$ than those based on Planck-function calculations. However, in HD 221170, with $T_{\text {eff }}=4510 \mathrm{~K}$, the effect is more severe: the new calculations are lower by 0.45 dex. Caution is required when interpretating our niobium abundance for this star.

We also compared our results with previously published abundances. Our value for CS 31082-001 should be the most reliable since it is based on seven transitions that yield internally consistent results (Table 8). This abundance, $\log \epsilon(\mathrm{Nb})=-0.55$, is identical to that of Hill et al. (2002). The new abundance for BD +173248 is 0.06 less than that of Cowan et al. (2002), and for CS 22892-052 it is 0.02 more than that of Sneden et al. (2003). Westin et al. (2000) did not present a niobium abundance for HD 115444. The outlier to the general agreement is HD 221170, for which our new abundance is 0.35 dex lower than that of Ivans et al. (2006). This can be understood from the difference in analytical technique described in the preceding paragraph.

Comparisons of niobium europium abundances are useful for studying the relative $r$ - and $s$-process strengths. About $2 / 3$ of the solar-system niobium content is produced by the $s$-process, while nearly all of the solar-system europium has an $r$-process origin (e.g., Simmerer et al. 2004, their Table 10). The total solar-system abundance ratio is $\log \epsilon(\mathrm{Nb} / \mathrm{Eu}) \simeq 0.9$ (Lodders 2003; Asplund et al. 2009; Lodders et al. 2009). The $r$-process only component abundance ratio is much lower, after substracting the fraction of niobium produced by the $s$-process: $\log \epsilon(\mathrm{Nb} / \mathrm{Eu})_{r-\text { only }} \simeq 0.1$ (Simmerer et al. 2004). Adopting the Eu abundances for the $r$-process-rich stars studied by Sneden et al. (2009), for CS 22892-052, CS 31082-001, and HD 221170, we derive $\log \epsilon(\mathrm{Nb} / \mathrm{Eu}) \simeq 0.2$, in reasonable agreement with the solar-system $r$-only ratio. This re-emphasizes the assertions of previous authors that these stars' $n$-capture abundance distributions closely mimic the $r$-process solar-system abundances. The other two stars have significantly higher niobium abundances relative to their europium contents: $\log \epsilon(\mathrm{Nb} / \mathrm{Eu}) \simeq 0.4$ for BD +173248 , and $\simeq 0.7$ for HD 115444. It is clear that another $n$-capture mechanism must be invoked for these stars. Whether that extra amount is produced by a slow or rapid $n$-capture process is unclear from the $\mathrm{Nb} /$ Eu ratio alone.

## 6. Summary

We have reported radiative parameters in Nb II and Nb III determined from a combination of theoretical and experimental approaches. New transition probabilities $(g A)$ and oscillator strengths ( $\log g f$-values) have been obtained for 107 Nb II transitions of astrophysical interest. They have been inferred by combining experimental lifetimes and measured branching fractions. Most of the transition probabilities reported in Table 2 for Nb II, have uncertainties between 6 and $25 \%$. In addition, a first set of theoretical results is reported for 76 Nb III transitions. The accuracy of the results has been assessed by comparing HFR+CPOL lifetime values with experimental lifetimes obtained using the TR-LIF technique. The agreement between theory and experiment in Nb II is gratifying. We propose that our results supplant those previously published in the literature, but we provide data to allow users to draw their own conclusions.

We present hyperfine components with individual oscillator strengths for the strongest NbII lines, which are important for stellar atmosphere analyses.

Application of the Nb II data to the solar spectrum yields a photospheric abundance of $\log \epsilon_{\odot}=1.44 \pm 0.06$ in good agreement with the meteoritic value. The niobium abundances that we have derived in $r$-process-rich stars are much more reliable than those reported by previous analyses.

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[^0]:    ${ }^{1}$ Because of space limitations the values of the fitted parameters are not given here but are available upon request from the authors.

[^1]:    ${ }^{2}$ Available at http://cfaku5.cfa.harvard.edu/
    ${ }^{3}$ Available at: http://verdi.as.utexas.edu/moog.html
    ${ }^{4}$ Available at http://bass2000.obspm.fr/solar_spect.php

