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Essays on the Macroeconomic Implications of Financial Frictions

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Essays on the Macroeconomic Implications of Financial Frictions

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Dedicated to my daughter, Lily.

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Essays on the Macroeconomic Implications of Financial Frictions

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This dissertation explores the macroeconomic implications of financial frictions from three aspects. Chapter 1 develops an industry evolution model to explore the quantitative implications of endogenous financing constraints for job reallocation. In the model, firms finance entry costs and per period labor costs with long-term financial contracts signed with banks, which are subject to asymmetric information and limited commitment problems. Financing constraints arise as a feature of the optimal contract. The model generates endogenous firm exit and job reallocation in a stationary industry equilibrium. A quantitative analysis shows that endogenous financing constraints can account for a substantial amount of job reallocation observed in US manufacturing and the observed negative relationship between job reallocation rates and firm size as measured by employment.

Chapter 2 studies the quantitative impact of costly external finance on aggregate productivity. Empirical studies document that resource reallocation

across production units plays an important role in accounting for aggregate productivity growth in US manufacturing. Distortions in financial market could hinder the reallocation process and hence may adversely affect aggregate productivity growth. This chapter studies the quantitative impact of costly external finance on aggregate productivity through resource reallocation across firms with idiosyncratic productivity shocks. A partial equilibrium model calibrated to the US manufacturing data shows that costly external finance causes inefficient output reallocation from high productivity firms to low productivity firms and as a result leads to 1 percent loss in aggregate TFP. This is a significant loss considering that the aggregate TFP growth rate for the US manufacturing has averaged less than 2 percent per year during the post war period.

Chapter 3 illustrates how occasionally binding constraints in international borrowing can help explain business cycle asymmetries in small open economies. In the model, if the borrowing constraint binds, it binds when the economy transits from a recession to an expansion, but not vice versa. As a result, on average downward movements are sharper and quicker than upward movements. The model is calibrated to the Canadian economy. A quantitative exercise suggests that international borrowing constraints can account for 16 percent of steepness asymmetry in the Canadian real per capita GDP. The model also generates high degree of deepness asymmetry in investment, and steepness asymmetry in capital stock.

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Chapter 1

Employment Flows with Endogenous Financing Constraints

1.1 Introduction

This chapter explores the quantitative implications of financial frictions for simultaneous expansion and contraction across firms and the resultant reallocation of employment. The quantitative significance of such reallocation is reflected in the high turnover rates of jobs and firms. According to Davis, Haltiwanger and Schuh (1996), on average, 10.3 percent of manufacturing jobs were destroyed and 9.1 percent were created over a twelve-month interval during the 1973-88 periods. Based on Census of Manufactures, over forty percent of the manufacturing firms disappeared over five year periods and were replaced by new ones. Such events often impose considerable costs on individual workers and the society in general. Empirical studies, such as Evans (1987), Hall (1987), Dunne, Roberts and Samuelson (1989), and Davis, et al (1996), also find interesting regularities in the turnover. Notably, firm exit rates, job creation, destruction and reallocation rates are decreasing in firm size (measured by employment) and age.

Despite the economic and social significance, economic theories underlying the turnover have not been fully developed. Existing labor market theories, including labor demand models that incorporate labor adjustment costs and search theories of equilibrium job flows, have found difficulty in accounting for the negative size and age dependence of gross employment flows. Recent development in theories of firm dynamics and industry evolution has shed some light on re-solving this problem. The learning theory of Jovanovic (1982) has the potential to account for the negative age dependence of the turnover. Hopenhayn (1992) introduces persistent idiosyncratic technology shocks to drive the entry, exit and size dynamics of firms. This model can explain the negative dependence of firm exit rates on firm size. However, as pointed out by Cooley and Quadrini (2001), without some restrictions on the transition probabilities of the shock process, it is hard to derive a general pattern of job reallocation on firm size.

This chapter studies firm dynamics from another angle: frictions in firm financing and the resultant financing constraints. There is considerable empirical evidence suggesting that financing constraints might be important determinants of firm dynamics (See Fazzari, Hubbard, and Peterson (1988), Gilchrist and Himmelberg (1995), and Whited (1992)). Theoretically, if smaller firms are more subject to financing constraints, then any short-term fall in sales will tend to have a much larger impact on smaller firms because these firms are either unable to obtain or cannot afford sufficient loans to sustain their employment levels. However, no formal model has been posited to test this hypothesis and quantitatively evaluate its implications for employment dynamics. This chapter explicitly models the micro foundations of financing constraints and develops a model of industry evolution in which firms' growth and failure are fully driven by the endogenous financing constraints. Firm entry, exit and job reallocation emerge in the steady state of the industry. This provides the environment in which quantitative implications of endogenous financing constraints can be evaluated. Specifically, two questions are addressed. First, how much job reallocation can be accounted for by the financing constraints? And second, how much of the negative size dependence of job reallocation can be accounted for by the financing constraints?

The particular type of frictions in firm financing examined are asymmetric information and limited commitment problems. In the model, firms are endowed with identical risky projects that require a fixed initial investment that firms cannot afford with their initial wealth and per period labor inputs to produce a homogeneous product at each period. Entrepreneurs are assumed to have private information about the outcomes of their production, where the idiosyncratic production shocks are assumed to be i.i.d. across firms and over time. A firm can be scrapped at the beginning of a period and generate a positive scrap value. Upon entry, a firm signs long-term financial contract with a competitive bank which would finance its initial investment and per period wage bills and in exchange receive payments from the firm in every period. But banks cannot fully commit themselves; a bank would renege on a contract if the continuation value of the contract to herself falls below zero. A firm's life cycle (employment growth and failure) is completely regulated by the optimal contract in relation to its shock realizations.

The contracting problem is an extention of Clementi and Hopenhayn (Hereafter C-H) (2002), where they model a long-term borrowing/lending relationship with asymmetric information between an entrepreneur and a bank and show that borrowing constraints emerge as a feature of the optimal contract. The model predicts that when the firm is smaller, it is more financially constrained, grows faster, and has higher probability of being liquidated. These implications are consistent with empirical findings. However, due to the full commitment assumptions for both parties, the contract predicts that the firm repays all revenues to the bank, until its equity value grows to the unconstrained efficient level. Thereafter, the bank advances the unconstrained efficient amount of working capital to the firm in every period while the firm pays nothing back to the bank. This feature of the contract has several implications that are inconsistent with the data. First, it implies that a firm, if not liquidated at an early stage of its life, will grow in finite periods to a stage where it will operate at the unconstrained efficient level and will never be liquidated. While in the data, one observes even large firms going bankrupt and adjusting scales of operation. Second, once a firm grows to that stage, the bank it signs contract with will keep losing from the contract. In reality it's hard to imagine a bank would continue such an contract even though it is ex-ante optimal. Finally, in an industry equilibrium model, if firms are financed by the Clementi and Hopenhayn contracts, eventually all incumbent firms would grow to that stage, and there would be no long run firm entry and exit, and no job creation and destruction. As a consequence, the quantitative implications of financial frictions for firm dynamics couldn't be addressed.

This chapter modifies the Clementi and Hopenhayn contract by assuming limited commitment for banks, and then incorporates the contracting problem into an industry equilibrium model. A lemma shows that the limited commitment constraint can be reduced to an upper bound to the value entitlement that a bank can faithfully promise to an entrepreneur (the state variable in the recursive formulation of the dynamic contract), which ensures the tractability of the contracting problem. The optimal contract exhibits quite different features from those of C-H (2002). Notably, a firm is financing constrained throughout its life cycle with the financing constraints relax as it grows larger. A firm has positive probability of being liquidated from the perspective of any stage of its life. And it never stops the process of job creation and job destruction during its life cycle. It is these features that are crucial to generate endogenous firm entry, exit and job reallocation across firms in the stationary industry equilibrium. The optimal contract also predicts more realistic decision rules concerning a firm's dividend and repayment policy.

I define a stationary competitive industry equilibrium and prove that there exists a unique stationary equilibrium with entry and exit. In the equilibrium, aggregate employment is constant over time while individual firms continually adjust employment levels. In contrast, in the equilibrium without the financing frictions, all firms produce at the same unconstrained efficient level, employing the same efficient amount of labor, and there is no firm entry and exit. Therefore, the employment dynamics in the model are fully driven by the financial frictions. The baseline calibration picks key parameters to match the exit rate, employment share and relative size of exiting firms due to bankruptcy and liquidation for the US manufacturing. The model generates an annual job reallocation rate of 9.5%, which is nearly 50% of the job reallocation rate (19.4%) documented in the data. The quantitative analysis of simulated data also shows that firm exit rates, job creation, destruction, and reallocation rates are decreasing with firm size (as measured by employment) and firm age [1], as observed in the data. Notably, the correlation coefficient between average firm size and job reallocation rate is -0.51, slightly lower than its data counterpart (-0.59) in magnitude, which implies that 87% of the negative size dependence of job reallocation can be accounted for by the financial frictions in the model.

This chapter is along the same line of Cooley and Quadrini (2001), which introduces financial frictions into the basic framework of Hopenhayn (1992) and show that the integration of persistent shocks and financial market frictions allow the model to generate the simultaneous dependence of firm dynamics on size (measured by capital)[2] and age. Their financing constraints arise from two exogenous assumptions on financial market imperfections. Furthermore, firm exit is assumed to be exogenous. This chapter, by modeling the

[1]The negative age dependence is derived from the size dependence. Once we control for firm size, age dependence disappears.

[2]In Cooley and Quadrini (2001), firm size is measured by capital. Since labor and capital are assumed to be complements, the size dependence also holds if size is measured by employment.

micro foundations of financing constraints, can simultaneously account for firm exit and growth. The model can well capture the negative size dependence of firm exit and job reallocation. It also provides a quantitative evaluation of the significance of financial frictions for gross employment flows. But due to its simple structure, the model cannot account for the negative age dependence conditional on firm size. It once again highlights the role of financial market frictions in accounting for the dynamics of firms.

Finally, I want to point out that on the contrary to the creative nature of job reallocation driven by technology shocks, as in Hopenhayn and Rogerson (1993), the job reallocation in this model is not creative, or is inefficient. Firms in the model have identical technology. Without the financial frictions, they would produce at the same efficient level, and no firm would exit. The presence of financial frictions causes inefficient job reallocation across firms, and results in considerable losses in aggregate output and employment. The quantitative significance of these impacts might justify the intensified exercise of government watchdog accounting procedures at the Securities and Exchange Commission. A comparative static analysis is also executed to see how these impacts vary with the discount factor, project riskiness, and other primitives of the model. The results show that the model exhibits comparative static properties that are consistent with the data and existing literature.

The rest of the chapter is arranged as follows. Section 1.2 describes the model. Section 1.3 characterizes the equilibrium, in particular, the optimal financial contract. Section 1.4 describes the results of the quantitative analysis.

And Section 1.5 concludes.

1.2 The Environment

Time is discrete and the horizon is infinite. The industry is composed of a continuum of firms and banks. In each period, a continuum of infinitely lived entrepreneurs are born with net worth M and a risky project, which requires an initial fixed investment $I > M$ and per-period labor input to produce a homogeneous product. The labor cost must be paid before the production. Projects are subject to idiosyncratic production shocks θ in each period, where $\theta \in \{H, L\}$ with prob$\{\theta = H\} = \pi$. If $\theta_t = H$, a project produces output $f(l_t)$ in period t, where l_t is the number of workers employed in period t. The function f is continuous, strictly increasing and strictly concave. If $\theta_t = L$, output in period t is zero. Production shocks are assumed to be independent over time and across projects. A project can be scrapped at the beginning of a period, in which case it generates a positive scrap value S. I assume $S < I - M$.

As in C-H (2002), realizations of production shocks of a project are assumed to be private information of the entrepreneur who manages the project. Outsiders cannot observe or verify it. Since $I > M$, to undertake her project, an entrepreneur needs the financial services of banks. As discussed in C-H(2002), presence of asymmetric information gives rise to a long-term credit relationship between an entrepreneur and a bank, under which the bank provides funds for the entrepreneur to finance the initial investment and per-

period labor cost of her project, and in exchange, receives payments from the entrepreneur. In every period an entrepreneur is assumed to be liable for payments only to the extent of current revenues. This is the limited liability constraint for entrepreneurs. The remaining revenues are fully consumed by the entrepreneur (dividends of the firm). The assumed commitment problem that is crucial for the model states that a bank can renege on a contract at the beginning of a period without punishment if the expected discounted value of the contract to herself falls below zero. It is simply assumed here courts will not force a bank to stay with a contract she does not like. In that case, the bank may scrap a firm, grab all scrap value without fulfilling the promise to the entrepreneur she signed the contract with. The optimal contract subject to these constraints is carefully defined and characterized in Section 1.3.

Banks are competitive and infinitely-lived. They participate in the long-term credit market in which they provide funds for entrepreneurs in exchange for repayments. They also have access to an external one-period credit market which opens at the end of each period, where they can freely borrow or lend funds at interest rate r. Entrepreneurs are excluded from this market [3]. Throughout the discussion, banks are summarized into a single agent that contracts with all entrepreneurs.

In every period a new-born entrepreneur is offered lifetime contracts by the bank. If she accepts a contract, her project gets financed and a new firm

[3]Actually due to the information structure, one-period credit relationship with entrepreneurs is not feasible.

enters the industry. From then on, she simply follows the contract for labor, repayment and dividend decisions. If her firm gets liquidated, the entrepreneur exits the industry and never enters again. If she does not accept any contract, she stays out of the industry and simply consumes her initial wealth.

Incumbent firms behave competitively, taking prices in the output(p) and labor(w) markets as given. Aggregate demand for the product is given by the inverse demand function, $p = D(Q)$, where the function D is continuous, strictly decreasing, and satisfies $\lim_{Q\to\infty} D(Q) = 0$. Following Hopenhayn and Rogerson (1993), the wage rate is normalized to be 1, $w = 1$.

Both entrepreneurs and banks are risk neutral, and discount future cash flows at the same rate $\beta = 1/(1 + r)$.

The timing of events in one period is summarized as follows. At the beginning of a period, some incumbent firms are scrapped and exit the industry, and some new firms come in. Then firms hire labor in the competitive labor market and pay wage bills with loans from the bank. Production is undertaken and production shocks are realized for every firm. Entrepreneurs sell output in the product market, and make reports about production outcomes to the bank. Conditional on their reports, revenues are divided between the bank and entrepreneurs. Finally, the bank borrows or lends in the re-opened credit market.

1.3 Equilibrium

A stationary equilibrium is considered. Since there is no aggregate uncertainty, a constant output price of p is assumed. To facilitate an understanding of how the presence of asymmetric information affects the dynamics of a firm and the industry, I first consider the case of symmetric information, where the bank also observes the production shocks of firms.

1.3.1 Symmetric Information

Since all entrepreneurs are identical from the perspective of birth, the bank offers them the same optimal lifetime contract in equilibrium. Once an entrepreneur accepts the contract, the bank provides funds of $I - M$ to help finance the initial investment of her project. The contract also specifies how much funds the bank provides in each period for the firm to employ labor, how much the firm has to repay the bank in each period, and under what conditions the firm is scrapped. With symmetric information, the optimal contract achieves the first-best outcome, i.e., the bank provides an entrepreneur with funds to employ the unconstrained efficient amount of labor in each period, which is given by

$$l^\star(p) \equiv \text{argmax}_l \quad \pi p f(l) - l. \tag{1.1}$$

Then the total value of the contract, defined as the total expected discounted value of future net cash flows from the project, is given by

$$\tilde{W}(p) = \frac{\pi p f(l^\star(p)) - l^\star(p)}{1 - \beta}. \tag{1.2}$$

It is divided between the bank and the entrepreneur. Denote the value of the contract to the entrepreneur by V, then

$$V = \frac{\pi(pf(l^\star(p)) - \tau)}{1-\beta}, \tag{1.3}$$

where τ is the entrepreneur's repayment to the bank in a period if her production is successful in that period. Because of the limited liability constraint, the entrepreneur does not need to pay anything if her project fails in that period. Following C-H (2002), throughout the discussion the value of a contract to an entrepreneur is also called the equity value of her firm in the sense that it is the expected discounted value of the firm's future net cash flows or dividends implied by the contract. The value of the contract to the bank is then given by

$$B(V) = \tilde{W}(p) - V. \tag{1.4}$$

Competition among banks imply that

$$B(V) = I - M. \tag{1.5}$$

Recall that $I - M$ is what the bank has to pay to undertake the project [4]. Equation (1.5) says that there is no gain from participating in the contract for the bank. Since $I > M$, $B(V) > 0$. So the bank will not renege on the contract. Another condition comes from the free entry of firms.

$$V = M. \tag{1.6}$$

[4] If instead the bank invests this amount of funds in the one-period credit market, the present value of this investment to the bank would also be $I - M$, since $\beta = \frac{1}{1+r}$.

Combining equations (1.3) and (1.6) gives the firm's repayment in a period if production is successful, $\tau(p)$. Equations (1.4), (1.5) and (1.6) imply that

$$\tilde{W}(p) = I. \tag{1.7}$$

This pins down the equilibrium output price p. The total demand for the output is thus given by

$$Q = D^{-1}(p).$$

And the total mass of incumbent firms N is determined by

$$Q = N\pi f(l^{\star}(p)).$$

Since $S < I - M$, the bank never scraps an incumbent firm. So in equilibrium firms with total mass of N stay in the industry, hiring the efficient amount of labor and producing the efficient level of output in every period. There is no firm entry and exit. Incumbent firms never expand or contract. Hence there is no job creation and destruction.

1.3.2 The Optimal Financial Contract with Asymmetric Information

Again since entrepreneurs are ex-ante identical, in equilibrium the bank offers them the same optimal contract. Without loss of generality, consider the contracting problem between the bank and an entrepreneur born at period 0. Conditional on the history of reports of the entrepreneur, $h^t = (\hat{\theta}_0, \hat{\theta}_1, ..., \hat{\theta}_t)$, the contract specifies a contingent policy of liquidation probabilities $\alpha_t(h^{t-1})$, transfers from the bank to the entrepreneur in case of liquidation $X_t(h^{t-1})$, labor input $l_t(h^{t-1})$, and transfers from the entrepreneur to the bank in case of no liquidation $\tau_t(h^t)$ to maximize the value of the contract to the bank, subject to a set of conditions.

If both parties have unlimited commitment, the contracting problem would essentially be the same as C-H (2002). Similar repeated moral hazard problems have been studied in Green (1987), Atkeson and Lucas (1992), Spear and Srivastava (1987) and others, where they show such problems have a recursive formulation when the agent's expected discounted utility is used as a state variable. Likewise, the contracting problem with full commitment has a recursive formulation, taking the entrepreneur's value entitlement (the value of the contract to the entrepreneur) at the beginning of a period, V, as the state variable.

With limited commitment for the bank, a recursive formulation is not so straightforward. Lack of enforcement mechanism implies that the optimal contract has to be self-enforcing. In other words, the optimal contract must be such that the bank will not renege on it conditional on any history of reports of the entrepreneur, or equivalently, the value of the contract to the bank at the beginning of a period after any history of reports of the entrepreneur must be non-negative. A contract like C-H(2002) does not satisfy this constraint. For instance, it predicts that if the project is not scrapped at an early stage, V will eventually reach $\tilde{V}(p) \equiv \frac{\pi p f(l^{\star}(p))}{1-\beta}$. Thereafter, the firm will produce at the unconstrained efficient level in every period. Note that at $\tilde{V}(p)$, the value of the contract to the bank is $\tilde{W}(p) - \tilde{V}(p) < 0$. Intuitively, the limited commitment constraint is equivalent to putting an upper bound on the value of the contract to the entrepreneur at the beginning of any period, since the total value of the contract is bounded above by $\tilde{W}(p)$, the total value of the unconstrained efficient contract as defined in equation (1.2). Denote this upper bound as $\bar{V}$ and take it as given for the moment. Then the optimal contract has a recursive formulation with $V \leq \bar{V}$ as the state variable. This argument is consistent with Phelan (1995), where he considers an insurance contract between a firm and an agent with privately observed endowment, assuming that both parties can walk away from the contract at the beginning of a period (under some conditions or with some cost). He shows that the two limited commitment constraints can boil down to a restriction on the set of feasible continuation utilities for the agent such that the efficient contract is recursive.

Note that V is also bounded below by zero because the limited liability constraint ensures the entrepreneur a non-negative net cash flow in every period. For a given $V \in [0, \bar{V}]$, the bank's problem is to choose the choice variables to maximize $B(V)$, the value of the contract to herself, or equivalently, to maximize the total value, $W(V) \equiv V + B(V)$. The first choice to be made is whether to liquidate the project, obtaining the scrap value S, or keep it in operation. If the project is not scrapped, the problem for the continuation stage is to choose labor input, repayment to the bank, and etc. For a given output price p, a recursive formulation for the liquidation problem is given by

$$
\begin{aligned}
(P_1) \quad W(V;p) \quad &= \quad \max_{\alpha\in[0,1],X,V_c} \left\{\alpha S + (1-\alpha)\hat{W}(V_c;p)\right\} \\
\text{subject to} \quad & \\
& V = \alpha X + (1-\alpha)V_c \qquad (1.8) \\
& X \geq 0, V_c \geq 0.
\end{aligned}
$$

Here, α is the liquidation probability. As argued in C-H(2002), a stochastic liquidation would be optimal due to the non-convexity introduced by a constant scrap value. X is the transfer from the bank to the entrepreneur in case of liquidation. V_c is the value entitlement to the entrepreneur at the continuation stage if her firm is not liquidated. Equation (1.8) is a promise-keeping constraint. It states that the contract delivers an expected value equal to V to the entrepreneur such that the bank's promise to the entrepreneur is fulfilled.

A recursive formulation for the continuation problem is given by

$$(P_2) \quad \hat{W}(V_c;p) = \max_{l,\tau,V^H,V^L} \left\{ \pi p f(l) - l + \beta\{\pi W(V^H;p) + (1-\pi)W(V^L;p)\} \right\}$$

subject to

$$V_c = \pi(pf(l) - \tau) + \beta\{\pi V^H + (1-\pi)V^L\} \tag{1.9}$$

$$pf(l) - \tau + \beta V^H \geq pf(l) + \beta V^L, \text{ i.e.}$$

$$\tau \leq \beta(V^H - V^L), \tag{1.10}$$

$$\tau \leq pf(l), \tag{1.11}$$

$$V^H \leq \bar{V}, \tag{1.12}$$

$$V^L \leq \bar{V}, \tag{1.13}$$

$$l, \tau, V^H, V^L \geq 0.$$

Here the state variable is V_c, the value entitlement to the entrepreneur at the continuation stage of a period. l is the amount of labor the firm can hire with funds from the bank. τ is the repayment to the bank if a high production shock is reported. V^H and V^L are the continuation value entitlements to the entrepreneur at the beginning of next period if she reports a high or a low shock respectively. (1.9) is the promising-keeping constraint. (1.10) is the incentive compatibility constraint to ensure that the entrepreneur truthfully reports when a high shock is realized. Note that the entrepreneur cannot misreport when a low shock is realized. (1.11) is the limited liability constraint for the entrepreneur. Conditions (1.12) and (1.13) are imposed to ensure that the limited commitment constraint for the bank is satisfied.

Given p and $\bar{V}$, by standard argument of dynamic programming, one can show the existence and uniqueness of the value function $W(V;p)$ and $\hat{W}(V_c;p)$, and can also show that the policy functions $\lambda(V;p)$, $X(V;p)$, $V_c(V;p)$, $l(V_c;p)$, $\tau(V_c;p)$, $V^H(V_c;p)$ and $V^L(V_c;p)$ are single-valued and continuous.

The upper bound to feasible value entitlements, $\bar{V}$, which has been taken as given so far, must satisfy

$$B_{[0,\bar{V}]}(\bar{V};p) = W_{[0,\bar{V}]}(\bar{V};p) - \bar{V} = 0,$$

where the subscript $[0,\bar{V}]$ is imposed to highlight the state space associated with the value functions. If $B_{[0,\bar{V}]}(\bar{V};p) < 0$, the bank will renege on the contract so that $\bar{V}$ cannot be faithfully promised. If $B_{[0,\bar{V}]}(\bar{V};p) > 0$, then competition among banks would drive the bank to promise a higher value than $\bar{V}$, in which case $\bar{V}$ is not the highest possible value entitlement to the entrepreneur. Using this result, **Lemma 1.3.1** proves the existence and uniqueness of $\bar{V}$.

Lemma 1.3.1. *For a given $p > 0$, there exists a unique upper bound to feasible value entitlement to an entrepreneur, $\bar{V}(p)$.*

PROOF: See Appendix A.

The following Lemmas and Propositions characterize the features of the optimal contract. First consider problem (P_1). As in C-H(2002), there exists a stochastic liquidation region, $[0, V_r]$, where $V_r > 0$. For values $0 \leq V \leq V_r$, it is optimal to give the entrepreneur a lottery with values of $X = 0$ in case of liquidation and $V_c = V_r$ in case of continuation. The probability of liquidation,

$\alpha(V)$ is decreasing in the entrepreneur's value entitlement. The total value of the contract in this region is given by a linear combination of S and $\hat{W}(V_r;p)$, with weights $\alpha(V)$ and $1-\alpha(V)$ respectively. These results are summarized in **Proposition 1.3.1**.

Proposition 1.3.1. *There exists $0 < V_r < \bar{V}$, such that*

(i) $\alpha(V) = 1 - \frac{V}{V_r}$ *for* $V \in [0, V_r]$, *and* $\alpha(V) = 0$ *for* $V \in [V_r, \bar{V}]$;

(ii) $X(V) = 0$ *for* $V \in [0, \bar{V}]$;

(iii) $V_c(V) = V_r$ *for* $V \in [0, V_r]$, *and* $V_c(V) = V$ *for* $V \in [V_r, \bar{V}]$;

(iv) $W(V) = S + \frac{\hat{W}(V_r;p)-S}{V_r}V$, *for* $V \in [0, V_r]$, *and* $W(V;p) = \hat{W}(V;p)$ *for* $V \in [V_r, \bar{V}]$.

Now consider problem (P_2). By (iii) of **Proposition 1.3.1**, the state variable V_c lies in $[V_r, \bar{V}]$ in equilibrium. But I consider a larger space $[0, \bar{V}]$ for V_c to establish the results. Since f is strictly concave, it's not hard to establish the following result.

Proposition 1.3.2. $\hat{W}(V_c)$ *is strictly increasing and strictly concave for* $V_c \in [0, \bar{V}]$. *And* $W(V;p)$ *is linearly increasing for* $V \in [0, V_r]$ *and strictly increasing and strictly concave for* $V \in [V_r, \bar{V}]$.

For a given $V_c \in [0, \bar{V}]$, if the limited commitment constraints (10) and (11) are not binding, the continuation problem (P_2) would be the same as that of C-H(2002) and shares similar properties. The following Lemma defines such a region.

Lemma 1.3.2. *There exists* $0 < V_1 \leq \bar{V}$*, such that* $V^H(V_1) = \bar{V}$*. For any* $V_c < V_1$*,* $V^H(V_c) < \bar{V}$ *and* $V^L(V_c) < \bar{V}$*.*

PROOF: See Appendix A.

It can be shown that $V_r < V_1 < \bar{V}$. This relationship is established after I characterize the contract for $V_c \in [0, V_1]$. Basically, since the limited commitment constraints are not binding on this region, the contract shares same features as those of C-H(2002); the entrepreneur is borrowing constrained, transfers all revenues to the bank and consumes nothing (zero dividends).

Proposition 1.3.3. *For* $V_c \in [0, V_1]$*,*

(i) the limited liability constraint (1.11) is binding;

(ii) the incentive compatibility constraint (1.10) is binding;

(iii) $l(V_c) < l^\star(p)$*;*

(iv) $V^L(V_c) < V_c < V^H(V_c)$ *as long as* $V_c > 0$*,* $V^H(V_c)$ *is strictly increasing and* $V^L(V_c)$ *is non-decreasing;*

PROOF: See Appendix A.

The repayment policy stated in Part (i) implies a zero dividend policy for the firm. This allows the equity value of the firm to reach its upper bound

$\bar{V}$ in the shortest possible time. Part (iii) says that the firm is borrowing constrained in the sense that its employment is less than the unconstrained efficient level, which is what it would be if private information is not present. This results follows from a binding incentive compatibility constraint, as seen from the proof. Part (iv) implies that the bank promises the entrepreneur a higher beginning-of-next-period value entitlement if a high shock is reported today, and a lower value entitlement if a low shock is reported today. Such report-dependent future value entitlements are crucial for inducing truthful report of the entrepreneur. Since V^H is less than $\bar{V}$ on this region, it is strictly increasing. V^L is nondecreasing. And since V^L is bounded below by zero, there might exist a region of V_c where V^L is zero.

The following lemma states the relationship between V_r, V_1 and $\bar{V}$.

Lemma 1.3.3. $V_r < V_1 < \bar{V}$.

PROOF: See Appendix A.

For $V > V_1$, the limited commitment constraint in a good state (1.12) becomes binding. So the contracting problem exhibits different features, which are summarized in the following proposition.

Proposition 1.3.4. *For* $V_c \in [V_1, \bar{V}]$,

(i) $V^H(V_c) = \bar{V}$*, i.e., the limited commit constraint (1.12) is binding;*

(ii) the incentive compatibility constraint (1.10) is binding;

(iii) $l(V_c) < l^\star(p)$;

(iv) $V^L(V_c) < V_c$, *and* $V^L(V_c)$ *is strictly increasing in* V_c;

(v) $\tau(V_c) > 0$ *and strictly decreasing in* V_c;

(vi) there exists $\hat{V} \in [V_1, \beta\bar{V})$[5] *such that for* $V_c \in [V_1, \hat{V}]$, *the limited liability constraint (1.11) is binding; for* $V_c \in (\hat{V}, \bar{V}]$, *(1.11) is not binding, and* $l(V_c)$ *is strictly increasing.*

PROOF: See Appendix A.

Part (i) actually establishes that the limited commitment constraint (1.12) is binding for $V_c > V_1$. Part (ii) and (iii) show that the incentive compatibility constraint is still binding and as a result borrowing is constrained. Part (iv) is the most crucial result for generating steady state firm exit. Recall that in C-H(2002), when equity value reaches the threshold $\tilde{V}(p)$, $V^H(\tilde{V}(p)) \geq \tilde{V}(p)$ and $V^L(\tilde{V}(p)) = \tilde{V}(p)$. This implies that once a firm's equity value reaches $\tilde{V}(p)$, it will never fall down. As a result the firm ceases to be borrowing constrained and will never be liquidated ($\alpha(\tilde{V}(p)) = 0$). As stated in C-H(2002), the evolution process of equity values has two absorbing states, $V = 0$ and $V \geq \tilde{V}(p)$. "Eventually, either the first one is reached and the firm is liquidated, or the second one is reached and borrowing constraints cease forever."

[5]There is no simple analytical result for the value of $\hat{V}$. Computation shows that it is very close to V_1 under various parameterizations.

If firms are financed by this type of contracts, in a stationary industry equilibrium, all incumbent firms would reach the second state, employing the efficient amount of labor and producing at the efficient level in every period. There would be no job creation and destruction, no firm entry and exit. With the limited commitment assumption, however, part (iv) together with (iv) of Proposition 1.3.3 shows that $V^L(V_c) < V_c$ for all feasible values of V_c, including the highest value $\bar{V}$. So starting from any level, the equity value of a firm can fall down to the liquidation region following a sequence of bad shock realizations. In other words, a firm faces a positive liquidation probability from the perspective of any stage of its life. It is this feature that generates endogenous firm exit, firm heterogeneity and job reallocation in the stationary industry equilibrium to be described next.

Part (v) and (vi) imply different repayment and dividend policy from C-H(2002), where the firm transfers all the revenues to the bank and pays zero dividends until its equity value reaches $\tilde{V}(p)$. Thereafter the firm pays nothing back to the bank and all revenues are paid as dividends. Here, for $V_c > \hat{V}$, $d(V_c) \equiv pf(l(V_c)) - \tau(V_c) > 0$, and $d(V_c)$ is strictly increasing since $l(V_c)$ is strictly increasing and $\tau(V_c)$ is strictly decreasing. So as its equity value reaches $\hat{V}$, the firm ceases to transfer all revenues to the bank and begins to pay dividends in a good state and the amount of dividends is strictly increasing in the firm's equity value. This seems to be a more realistic dividend policy compared to C-H(2002).

The optimal contract is solved numerically (See Appendix A for the solution method) and Figure 1.1-1.6 of Appendix A plot the value functions and the policy functions for the baseline calibration to be described later. Among the features of the optimal contract, a notable one is the endogenous borrowing constraint: employment is strictly less than its unconstrained efficient level for all feasible equity values. Even though monotonicity of employment does not hold throughout the whole range of equity values, a firm with large equity values tends to employ more workers (See Figure 1.5). In other words, the endogenous financing constraints tend to relax as the firm's equity value grows. This feature combined with the evolution dynamics of equity values drives the job reallocation process. If a firm receives a high production shock this period, its equity value for next period will increase, which would dictate more employment for next period (except in the small decreasing regions), i.e., the firm will create jobs. On the contrary, if a firm receives a bad shock, its equity value for next period will decrease, which would dictate the firm to lay off workers. If the equity value falls to the liquidation region, the firm may be liquidated and exit the industry. In both cases, the firm destroys jobs. Such job creation and destruction is an ongoing process during a firm's life cycle, since the optimal contract dictates that a firm never reaches a stage where such process stops until the firm is liquidated.

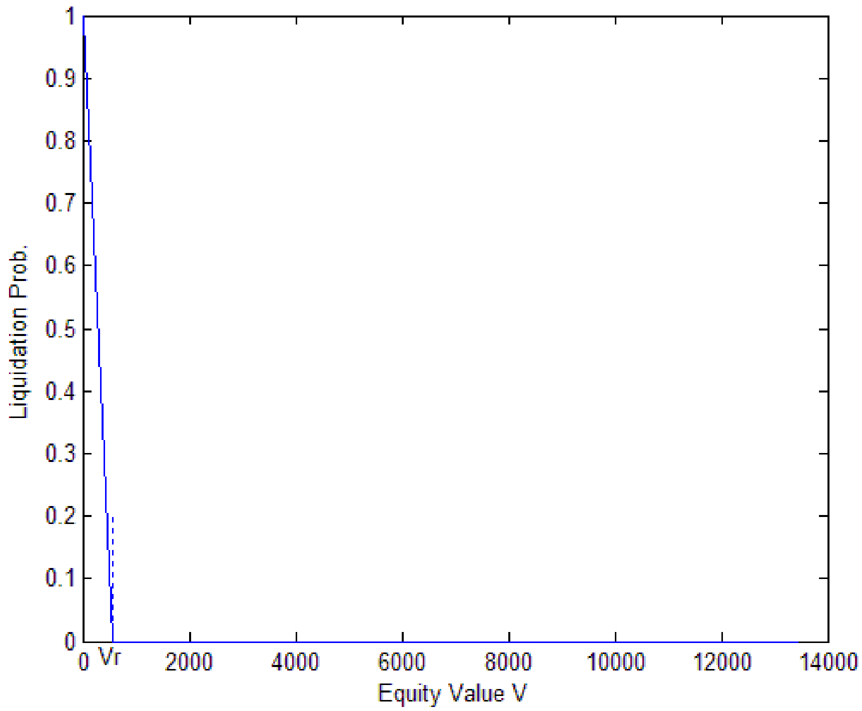

Figure 1.1: liquidation probability

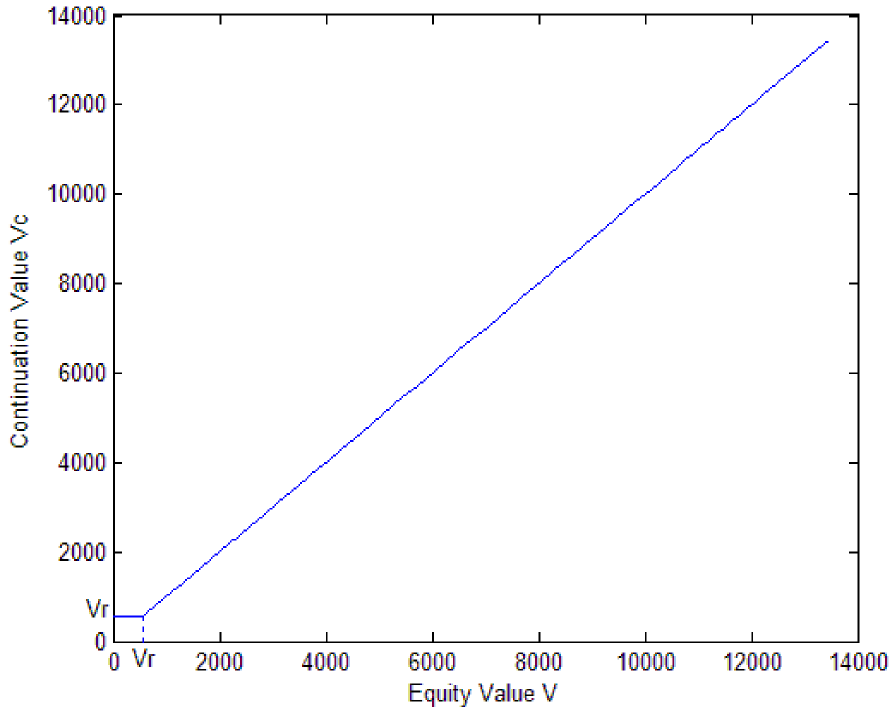

Figure 1.2: Continuation Value Entitlement (Vc)

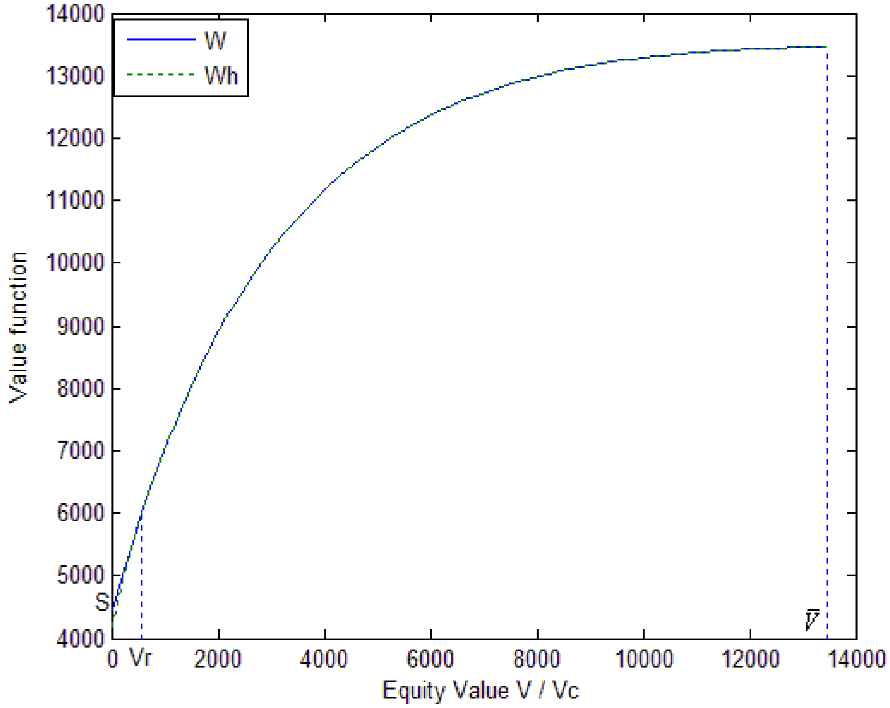

Figure 1.3: Value Functions

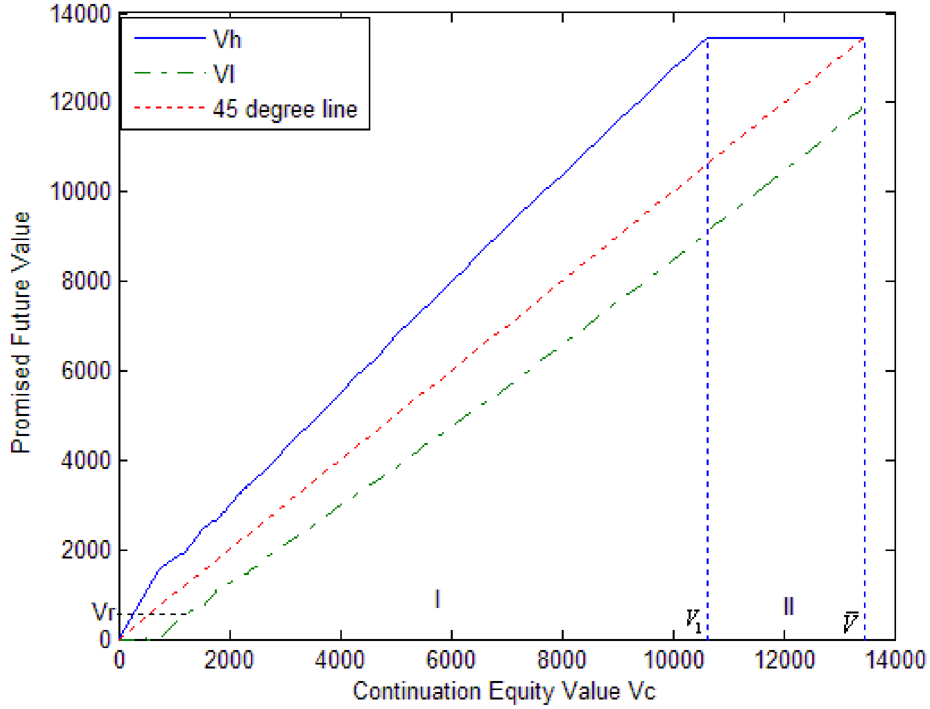

Figure 1.4: Equity Dynamics

Note: Region I denotes $[0, V_1]$, and II, $[V_1, \bar{V}]$.

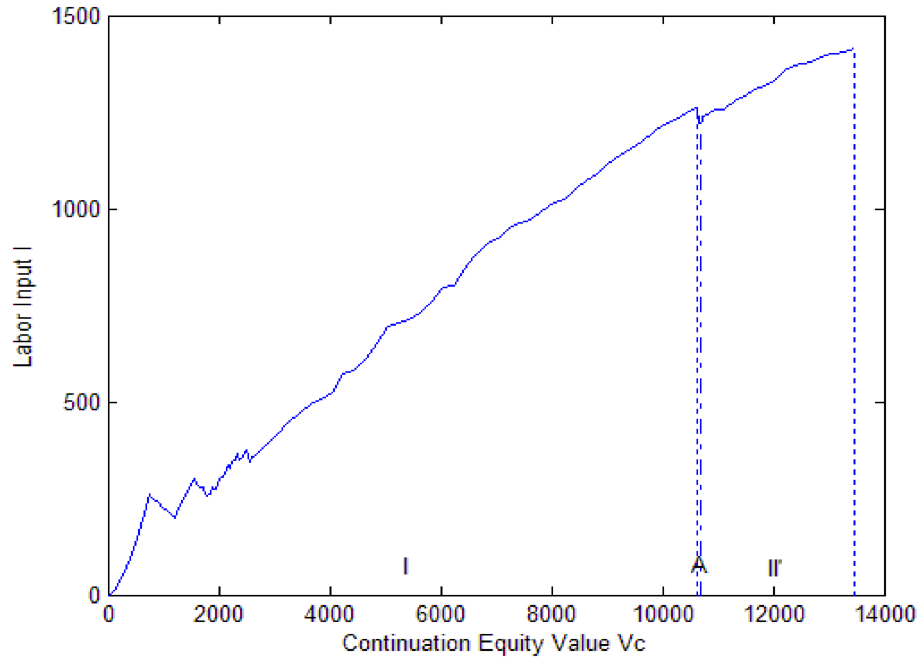

Figure 1.5: Decision Rule for Employment

Note: Region I denotes $[0, V_1]$, region A, $[V_1, \hat{V}]$, and region II', $[\hat{V}, \bar{V}]$. In the computation, the value functions W and $\hat{W}$ are computed by following the procedure described in Appendix A. Once W and $\hat{W}$ converge, the state space is discretized into even finer grids, especially for those regions where the labor input is not monotone. Then the labor input is re-computed for each grid point. This gives more accurate decision rule for labor input even though it looks choppier. The value functions and decision rules for V^H and V^L are not sensitive to this refinement.

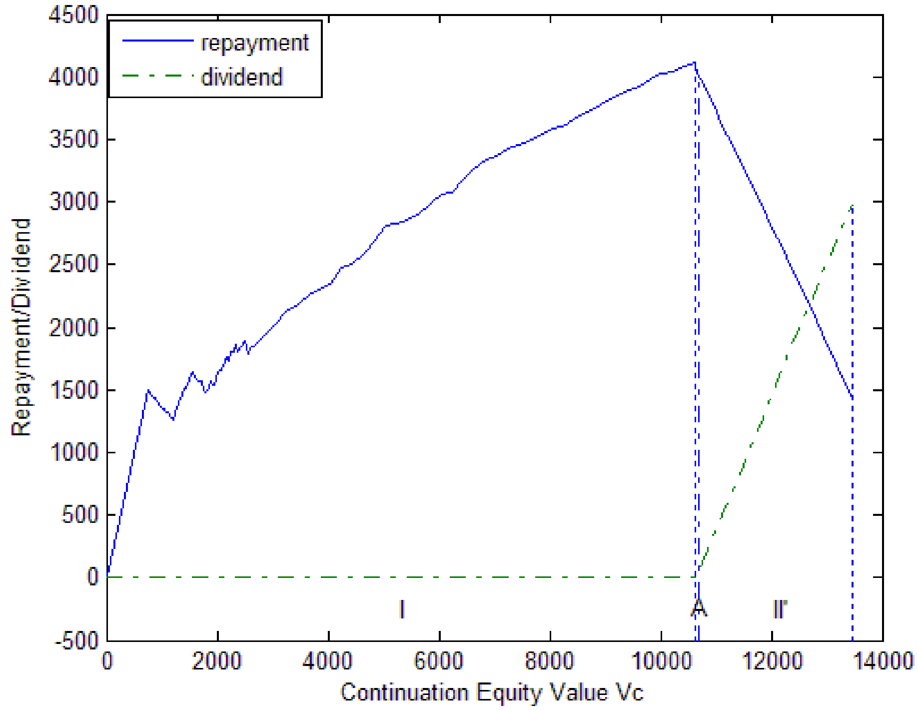

Figure 1.6: Repayment/Dividends in a Good State

Note: Region I, A, and region II' are defined as in Figure 1.5.

1.3.3 Entry of New Firms

In every period, new born entrepreneurs are offered lifetime contracts by banks and may accept whatever contracts give them the highest expected discounted value. In equilibrium they are offered the same contract ex-ante, which is the optimal contract characterized previously. If an entrepreneur accepts the contract, her project gets financed and a new firm enters the industry. Because of competition among banks, the value entitlement to an entrepreneur upon entry or the initial equity value of a new firm, V_0, is determined by

$$(P_3) \quad \max_{V_0 \in [0, \bar{V}(p)]} \quad V_0$$
$$\text{s.t.} \quad \hat{B}(V_0; p) \equiv \hat{W}(V_0; p) - V_0 \geq I - M, \tag{1.14}$$

where (1.14) is the participation constraint for the bank. For a given p, if a solution to (P_3) does not exist, the project is not financially feasible. If there exists a solution, since $\hat{W}$ is concave and $\hat{B}(0; p) = \beta S < I - M$, $\hat{B}(V; p)$ is hump-shaped, as shown in Figure 1.7 Notice that there exists a region of V such that the value of the contract to the bank increases with the value of the contract to the firm, so both parties would find it beneficial to renegotiate the contract once the firm's equity value evolves to this region. Therefore the contract is not renegotiation-proof. Renegotiation-proof contracts with repeated moral hazard are studied in Cheng Wang (2000) and Quadrini (2003), where renegotiation-proofness is obtained by imposing some lower bound to attainable expected utilities of the agent. It would be an interesting extension to derive renegotiation-proof contract in my context, and explore its implications for employment dynamics.

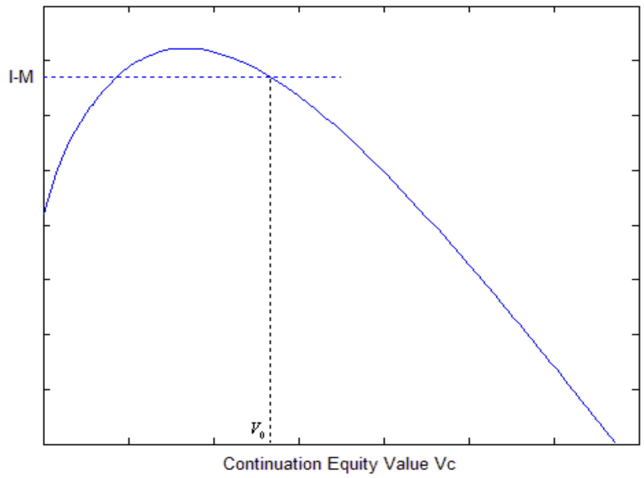

Figure 1.7: Value of the Contract to the Bank ($\hat{W}(V_c; p) - V_c$)

Denote the solution to (P_3) as $V_0(p)$. Then (1.14) is binding at $V_0(p)$, i.e., there is no gain to bank participation. Once the initial equity value is determined, from then on, the evolution of a new firm's equity value is completely regulated by the contract in relation to its production shock realizations. The firm simply follows the contract for its decisions on employment, repayments, dividends, and whether to exit or not.

The following lemma establishes the dependence of $V_0(p)$ on the output price p. It is to be used for establishing the existence and uniqueness of a stationary equilibrium with entry and exit.

Lemma 1.3.4. *$V_0(p)$ is continuous and strictly increasing in p.*

PROOF: See Appendix A.

1.3.4 Evolution of the Industry

The state of an incumbent firm is fully described by its equity value at the continuation stage of a period, V_c. So the state of the industry can be described by the distribution of all incumbent firms over V_c. The state of the industry can also be described by the distribution of firms over equity values at the beginning of a period. I choose to use the former for simplicity because it is the distribution directly related to aggregate production. Let $\mu_t(V_c; p)$ denote the distribution of incumbent firms over equity values at the continuation stage of period t. And denote the total mass of new entrant firms at the beginning of period t by E_t. Then μ satisfies the law of motion

$$\begin{aligned}\mu_{t+1}(A;p) = \int \Big\{ & \pi \left(1-\alpha(V^H(V))\right) \chi_A \left(V_c(V^H(V))\right) \\ & +(1-\pi)\left(1-\alpha(V^L(V))\right) \chi_A \left(V_c(V^L(V))\right) \Big\} \mu_t(dV;p) \\ & +E_{t+1}\, \chi_A(V_0(p)), \end{aligned} \tag{1.15}$$

for $\forall A \in \mathcal{V}(p)$, where $\mathcal{V}(p)$ is the σ-algebra generated by the state space $[0, \bar{V}(p)]$. $\chi_A(\cdot)$ is an indicator function, i.e., $\chi_A(V)$ equals 1 if $V \in A$ and equals 0 otherwise.

The transition from μ_t to μ_{t+1} can be written as $\mu_{t+1} = T^\star(\mu_t, E_{t+1}; p)$. It can be shown that $T^\star$ is linearly homogeneous in μ and E jointly. If the industry has twice as many firms of each type at the continuation stage of period t, and entry is doubled at the beginning of period $t+1$, then the industry will end up with twice as many firms of each type at the continuation stage of period $t+1$. This property turns to be useful in the computation of an invariant distribution (See Appendix A). Proposition 1.3.5 established this result.

Proposition 1.3.5. *$T^\star$ is linearly homogeneous in μ and E jointly.*

PROOF: See Appendix A.

With a measure of firms, μ, total labor demand L^D, output Y, repayment to the bank T, dividends Π, and total scrap value of liquidated firms R can be defined respectively.

$$
\begin{aligned}
L^D(\mu;p) &= \int l(V;p)\mu(dV;p), \\
Y(\mu;p) &= \pi \int f\left(l(V;p)\right)\mu(dV;p), \\
T(\mu;p) &= \pi \int \tau(V;p)\mu(dV;p), \\
\Pi(\mu;p) &= pY(\mu;p) - T(\mu;p), \\
R(\mu;p) &= \int \Big\{\pi\alpha(V^H(V)) + \\
&\qquad (1-\pi)\alpha(V^L(V))\Big\} S\ \mu(dV;p).
\end{aligned}
$$

Proposition 1.3.6 states that all aggregate quantities are linearly homogeneous μ. This result is similar to Proposition 1.3.5. Its proof is trivial and hence skipped.

Proposition 1.3.6. *The aggregate quantities defined above are linearly homogeneous in μ.*

1.3.5 Balance Sheet of the bank

In the model banks are assumed to be competitive. They participate in the long-term credit market in which they provide funds for entrepreneurs in exchange for payments. They also have access to an external short-term (one-period) credit market, where they can borrow or lend at interest rate r. Depending on their performance in the long-term credit market, some banks may borrow and others may lend funds in the short-term credit market to balance their budgets. It's interesting to know the total amount of short-term funds banks as a whole would hold in equilibrium. Summarizing the banks into a single agent, this quantity can be derived from her balance sheet.

Suppose at the beginning of period t, the bank holds short-term funds, B_t, which are raised in the short-term credit market at the end of last period. Some firms are liquidated, and the bank receives total scrap value R_t. These funds are used for financing the initial investment costs of new entry firms and the labor costs of all incumbent firms. After production, the bank receives total payment T_t from incumbent firms. Finally, the bank pays back B_t with interests, and borrows or lends new funds B_{t+1} in the re-opened short-term credit market. So her balance sheet for period t is given by

$$B_{t+1} + R_t + T_t = E_t(I - M) + L_t^D + (1 + r)B_t. \tag{1.16}$$

1.3.6 Stationary Competitive Equilibrium

A stationary competitive equilibrium for the industry consists of an output price $p^\star \geq 0$ and total output $Q^\star$; policy functions $\alpha(V; p^\star)$, $X(V; p)$,

$V_c(V;p^\star)$, $l(V_c;p^\star)$, $V^H(V_c;p^\star)$, $V^L(V_c;p^\star)$, $\tau(V_c;p^\star)$, as well as value functions $W(V;p^\star)$ and $\hat{W}(V_c;p^\star)$; a measure of incumbent firms $\mu^\star$ and a mass of entrants $E^\star \geq 0$; aggregate quantities $Y(\mu^\star;p^\star)$, $L^D(\mu^\star;p^\star)$, $T(\mu^\star;p^\star)$, $\Pi(\mu^\star;p^\star)$ and $R(\mu^\star;p^\star)$; and bank short-term credit $B^\star$ such that

(i) the value functions and policy functions solve (P_1) and (P_2);

(ii) $p^\star = D(Q^\star)$, and $Q^\star = Y(\mu^\star;p^\star)$;

(iii) $V_0(p^\star) \leq M$, with equality if $E^\star > 0$;

(iv) $\mu_t \equiv \mu^\star$ and $E_t \equiv E^\star$ solve equation (1.15);

(v) $R(\mu^\star;p^\star) + T(\mu^\star;p^\star) = E^\star(I - M) + L^D(\mu^\star;p^\star) + rB^\star$.

Condition (ii) states that demand must equal supply in the output market. Condition (iii) is the free entry condition of firms. Since there is unlimited supply of potential entrants, in equilibrium, $V_0(p^\star)$ cannot be strictly bigger than the initial wealth of an entrepreneur, M. If $E^\star$ is strictly positive, $V_0(p^\star)$ must equal M to ensure that firms are willing to enter. In this case, since (1.14) is binding at $V_0(p^\star)$, then

$$\hat{W}(M;p^\star) = I. \tag{1.17}$$

This condition says that the total expected discounted value from undertaking the project equals the initial setup cost of the project, or in other words the total expected value from entering equals the total entry cost. It is used to pin down $p^\star$ in a stationary equilibrium with positive entry. Condition (iv) states that $\mu^\star$ and $E^\star$ are such that the state of the industry is reproduced in every period through the optimal actions of firms and banks. Finally condition (v) states that the aggregate quantities satisfy the balance sheet of the bank.

It follows from the definition that the stationary equilibrium may take two different forms: with entry and exit or without. Since the data used to calibrate the model in a later section exhibits significant amount of entry and exit, an equilibrium with entry and exit is of greater interest. The industry evolution model of Hopenhayn (2002) has the property that if the entry cost is less than a critical value, there exists a equilibrium with entry and exit, and it is the unique stationary equilibrium. A similar property holds here. Theorem 1.3.1 establishes this result.

Theorem 1.3.1. *There exists $I^\star > 0$, $M^\star > 0$ such that for $I < I^\star$ and $M \geq M^\star$ a stationary equilibrium with entry and exit exists and it is the unique stationary equilibrium for the industry.*

PROOF: See Appendix A.

Intuitively, if the entry cost I is not too big, undertaking the project would be profitable. If a firm's initial wealth M is not too small, or equivalently, if the part of the entry cost undertaken by the bank is not too big, a contractual relationship between a firm and the bank would be feasible. The bank would like to offer contracts and firms would like to accept the contracts and enter the industry. As long as there are some firms in the industry, there is always firm entry and exit in the stationary industry equilibrium because the optimal contract predicts that a firm has a positive probability of being liquidated from the perspective of any stage of its life. At this point, this model improves over most previous models on firm dynamics, which either assume firm exit is exogenous, or depends on certain restrictions on parameters to generate endogenous firm exit. This property also ensures that the mixing condition required for the convergence of the firm distribution is satisfied.

1.4 Quantitative Analysis

In this section, the model laid out in the last two sections is solved numerically. An algorithm for finding the stationary equilibrium with entry and exit is provided in Appendix A. The stationary equilibrium has the property that aggregate variables are constant over time while individual firms continually adjusting over time. At any point in time there are some firms expanding, some firms contracting, some entering and others exiting. Entering and expanding firms hire workers and create jobs, while contracting and exiting firms fire workers and destroy jobs. On the contrary, the equilibrium with symmetric information has no such dynamics at all, as discussed in Section 1.3.1. This provides the setting in which the questions outlined in the Introduction can be addressed.

1.4.1 Baseline Calibration

To execute a quantitative analysis, we need to specify functional forms and assign parameter values. The production function is assumed to take the form

$$f(l) = Al^{\gamma}, 0 < \gamma < 1,$$

where γ is the labor share of income, and A is a scale factor. The inverse demand function takes the form

$$p = D(Q) = \frac{1}{aQ},$$

where a is a positive constant.

Assuming that a period is one year, I set the interest rate r to 6.5 percent, which is the average annual real interest rate over the last century. So $\beta = \frac{1}{1.065}$. Since the labor share of income has averaged about 0.64 over the postwar period, γ is set to 0.64. The assignment of remaining parameters requires a value of the stationary equilibrium output price $p^{\star}$. Following Hopenhayn and Rogerson (1993), $p^{\star}$ is normalized to unity and values of remaining parameters are chosen to be consistent with it. The scale factor A is chosen such that the unconstrained efficient level of employment is 1500 workers[6].

The probability of realizing a good production shock π, scrap value S and entrepreneur's initial wealth (a new firm's initial equity value) M are crucial for the entry and exit behavior of the model. They are chosen to match three moments of the US manufacturing data. The first one is the mean annual exit rate of manufacturing firms due to bankruptcy and liquidation. According

[6] I tried other values of efficient employment, such as 1000 and 2000 workers, and found that the results are not sensitive to this amount.

to Dunne, Roberts and Samuelson (1988), the exit rate of manufacturing firms during a 5-year period is 35.2 percent on average during the period of 1963 to 1982, implying a mean annual exit rate of 7 percent. Because firms may exit due to a lot of forces and the model formulated here only considers one driving force–financing problems, calibrating the model to match a 7 percent exit rate would over estimate the quantitative significance of the financial frictions for job reallocation. The lack of data and relevant empirical studies prevents me from obtaining an accurate measure of the exit rate due to bankruptcy and liquidation. Fortunately Compustat data records the year and reasons of deletion of a firm from Compustat, which allows me to compute the mean annual exit rate of Compustat manufacturing firms due to bankruptcy and liquidation. This number is 0.31 percent for the period of 1973 to 1988 [7]. Since Compustat firms are typically large and larger firms tend to have lower bankruptcy and liquidation rates, this measure provides a lower bound to the exit rate of all manufacturing firms due to bankruptcy and liquidation during the period of 1972 to 1988. The second moment is the mean annual employment share of exiting manufacturing firms. Again, using the Compustat data I obtain a lower bound of this moment for the period of 1972 to 1988, 0.05 percent. This measure is much lower than the mean annual employment share of exiting plants during 1972 and 1988, which is 2.34 percent according to Davis, et al. (1996). The last moment is the relative size of exiting firms due

[7] I want to compare the model predictions to those reported in Davis, et al. (1996), where the ASM for the period of 1972 to 1988 is used to document job reallocation statistics.

to bankruptcy and liquidation. Dunne, et al. (1988) defined the relative size of exiting firms as the ratio of average output of exiting firms to the average output of non exiting firms, and estimated it to be 0.34 on average during the period of 1967 to 1982. Since there is no evidence that shows systematic size difference between exiting firms due to bankruptcy and exiting firms due to other reasons, I take this number as the measure of the third moment. Computation shows that there is a unique choice of π, S and M that matches these three moments.

Once M is chosen, simply set $I = \hat{W}(M; p^\star)$. Recall that this is equation (1.17), a condition that has to be satisfied in an equilibrium with positive entry. Finally, the scale parameter a in the inverse demand function is set such that the total employment in the stationary equilibrium equals the mean annual employment of the manufacturing industry during 1972 and 1988, 18,135,000 employees according to Annual Survey of Manufactures [8].

Table 1.1 summarizes the baseline parameter values and matched quantities. Since the model assumes a simple structure of production technology. To see whether the model's predictions are sensitive to π, I also consider other values of π and find that the results are not sensitive to this parameter once other parameters are re-calibrated to match the three moments.

[8]This number is computed using the job creation and destruction data available on John Haltiwanger's web site.

Table 1.1: Baseline Calibration

	Parameter	Value
Discount factor	β	0.939
Interest rate of short-term funds	r	0.065
Probability of a good production shock	π	0.51
Labor share	γ	0.64
Scale factor in production function	A	42.62
Scrap value	S	4,460
Entrepreneur's initial wealth	M	3,254
Initial investment of a project	I	10,478
Scale factor in demand function	a	$3.25 \cdot 10^{-8}$
Matched quantities	Target	Model
Exit rate of firms	0.31%	0.31%
Employment share of exiting firms	0.05%	0.05%
Relative size of exiting firms	0.34	0.33
Total employment of industry	$1.8135 \cdot 10^{7}$	$1.8135 \cdot 10^{7}$

1.4.2 How Much Job Reallocation Can be Accounted for by the Endogenous Financing Constraints?

With parameter values determined, the equilibrium is numerically solved. The full information equilibrium is also solved by following the descriptions of Section 1.3.1, taking as given all the parameter values in Table 1.1. Table 1.2 presents the summary statistics for both equilibria. Note that the job creation and destruction rate in the stationary equilibrium with financial frictions is 4.75 percent per year, implying an annual job reallocation rate of 9.5 percent, while its frictionless counterpart is zero (See the third panel of Table 1.2). According to Davis, et al (1996), where the Annual Survey of Manufactures is exploited to document job creation and destruction statistics, the mean annual job creation rate for US manufacturing is 9.1 percent and job destruction rate

Table 1.2: Summary Statistics for Equilibria with and without Financial Frictions (Baseline Calibration)

	Symmetric Information	Asymmetric Information	Data (ASM)
Avg. firm size	1,137	1,157	
Std. of firm size	0	322	
Maximum firm size	1,137	1,413	
Minimum firm size	1,137	153	
Entry/exit rate	0	0.31%	
Size of entrants		447	
Avg. size of exiters		206	
Emp. share of entrants	0	0.12%	1.41%
Emp. share of exiters	0	0.05%	2.34%
Job creation rate	0	4.75%	9.1%
Job destruction rate	0	4.75%	10.3%
Job creation share of entrants	0	2.52%	15.5%
Job destruction share of exiters	0	1.16%	22.9%
Output price	0.905	1	
Total output($\cdot 10^7$)	3.3961	3.0736	
Total employment($\cdot 10^7$)	1.9671	1.8135	
Total # of firms($\cdot 10^6$)	1.7303	1.5669	
Total revenues($\cdot 10^7$)	3.0736	3.0736	
Total payment to bank($\cdot 10^7$)	2.7300	2.0811	
(fraction of total revenues)	(88.8%)	(67.7%)	
Total dividends of firms($\cdot 10^6$)	3.4365	9.9257	
(fraction of total revenues)	(11.2%)	(32.3%)	
Total scrap value($\cdot 10^5$)	0	2.1600	
bank short-term liability($\cdot 10^7$)	11.736	3.9103	

Notes: The statistics for ASM are from Davis, et al. (1996), which are based on plant-level data.

is 10.3 percent during the 1972-1988 periods. So conditional on the model 51.1 percent of job creation, 46.1 percent of job destruction and 49 percent of job reallocation observed in the data can be accounted for by the financial frictions in the model. To check the robustness of this result, I considered various parameterization. The model always generates a lot of job reallocation. For instance, for $\pi = 0.55$, the job creation or destruction rate is 9.65 percent. It suggests that if financial frictions are as severe as assumed in the model, they could potentially have significant impact on job reallocation. On the other hand, this result may over estimate the effect since the private information problem is probably more applicable to smaller firms while smaller firms have higher job creation and destruction rates.

According to ASM, a large fraction of job creation and destruction is accounted for by start-up (15.5%) and shut-down plants (22.9%) despite the employment shares of startups and shutdowns are relatively small. Since I calibrate the model to match a very low exit rate, the job creation and destruction shares of entrants and exiters are much smaller, 2.52 and 1.16 percent respectively. But they are relatively big compared to the employment shares of entrants and exiters, which are 0.12 and 0.05 percent respectively. This property is consistent with the data.

1.4.3 How Much of the Negative Size Dependence Can be Accounted for by the Endogenous Financing Constraints?

As described in the Introduction, the turnover of firms and jobs in the data exhibits a negative dependence on firm size and age. In the model, the turnover of firms and jobs is driven by the endogenous financing constraints due to the asymmetric information and limited commitment problems in firm financing. It's interesting to know whether the turnover exhibits same properties, and if so, quantitatively how close they are to those observed in the data. To explore these questions, I draw 100,000 firms from the stationary firm distribution over equity values (Figure 1.8 plots the distribution), and simulate them for 170 periods (10 simulations of 17 periods). For each simulation, firm exit rates and mean annual job flow rates are computed for each size and age category. The figures reported in Table 1.3-1.9 are averages across the 10 simulations.

1.4.3.1 Exit Rates by Size

Table 1.3 reports the 1-year, 2-year, 5-year, 10-year and 15-year exit rates for each firm size category. Firm size here refers to employment in the base year, i.e. the initial year of the time interval over which a particular exit rate is calculated. Classification I is a broad size classification and classification II is a more detailed one. Table 1.3 displays a strong negative relationship between firm exit rates and the size of firms. For example, 50 percent of firms with employment less than 300 workers exit in 10 years, while only 3 percent

Table 1.3: Exit Rates by Employer Size Category (%)

	1-year	2-year	5-year	10-year	15-year
All	0.3148	0.6271	1.5428	2.9224	4.2029
Classification I					
Small (< 500)	4.0206	8.0334	18.9037	29.7434	36.0803
Medium (500-999)	0	0	0.4580	3.5562	6.7242
Large (1000+)	0	0	0	0.0743	0.4442
Classification II					
< 300	11.8257	23.5589	38.7332	50.0709	56.3057
300-399	0	0.1029	12.8942	25.4604	31.4855
400-499	0	0	4.8047	13.8254	20.8168
500-599	0	0	2.6545	8.7813	13.6167
600-699	0	0	0.3244	5.6757	9.6386
700-799	0	0	0	3.0407	6.9052
800-899	0	0	0	1.8645	4.5861
900-999	0	0	0	1.1881	2.9100
1000-1099	0	0	0	0.4231	1.6608
1100-1199	0	0	0	0.1668	0.9718
1200-1299	0	0	0	0.0943	0.5382
1300+	0	0	0	0.0061	0.1883

Notes: This table gives the percentage of firms with base year employment in each category that exit in 1 year, 2 years, 5 years, 10 years and 15 years.

of firms with employment between 700 and 799 workers exit in 10 years. In particular, the 10-year and 15-year exit rates are strictly decreasing as the size of firms increases.

The driving force underlying the negative size dependence is a negative relationship between liquidation probabilities and firm equity values and an overall positive relationship between equity values and employment.

1.4.3.2 Employment Flows by Size

Several related but distinct concepts of employer size have been adopted by empirical studies in computing job flow rates and classifying firms or plants. A traditional measure (See Dunne, Roberts, and Samuelson (1989) and Evans (1987)) is the base year employment. Using this measure job creation rates for new firms are not well defined. Davis et al. (1996) argue that the base year size concept is subject to several other defects, and instead propose two new concepts for plant size: current plant size and average plant size. Despite different size concepts adopted, the empirical studies all find a negative size dependence of job flow rates. Following Davis et al. (1996), I define current firm size and average firm size, where current firm size equals the simple average of a firm's current employment and its employment 1 year ago and average firm size equals the weighted mean annual employment over the life cycle of the firm. My analysis considers the three concepts of firm size. Specifically, to explore the relationship between job flow rates and base year size, job flow

Table 1.4: Correlation of Job Flow Rates with Firm Size (average size)

	ρ(size, jcrate)	ρ(size, jdrate)	ρ(size, jrrate)
base-year size	-0.9325	-0.7591	-0.8408
current size	-0.6327	-0.6080	-0.6111
average size	-0.5073	-0.5273	-0.5105
data (ASM)	-0.6260	-0.5433	-0.5882
ratio	0.81	0.97	0.87

rates are defined using base year employment[9] and firms are classified by their base year employment. To explore the relationship between job flow rates and current firm size or average firm size, job flow rates are defined using current firm size [10], and firms are classified by their current size or average size. This is consistent with the practice in Davis et al. (1996), despite the production units considered here are firms rather than plants.

Table 1.4 reports the correlation coefficients between job flow rates and firm size for the three size measures. Note that all the figures are significantly less than zero, implying a strong negative correlation of job flow rates with firm size. In particular, the correlation coefficients of job creation rates, job destructions rates and job reallocation rates (sum of creation rates and destruction rates) with average firm size are -0.5073, -0.5273, -0.5105 respectively. These figures are very close to those in the ASM data, which are approximately -0.626, -0.5433 and -0.5882 respectively[11]. This result shows

[9]For example, job creation rate from t-1 to t for a firm is defined as the ratio of employment gains to the firm's employment in t-1.

[10]For example, job destruction rate from t-1 to t for a firm is defined as the ratio of employment losses to the firm's average employment of t-1 and t.

[11]These figures are computed using the third panel of Table 4.1 in Davis, et al (96). The

that the model can predict 81 percent of the negative size dependence for job creation, 97 percent for job destruction, and 87 percent for job reallocation. The negative size dependence is a robust finding as I vary parameter values. In particular, for $\pi = 0.55$ (other parameters are reset to match the moments) the correlation coefficients of job flow rates with average firm size are -0.49, -0.55 and -0.52 respectively.

To further explore the size dependence of gross employment flows, I also report the mean annual job flow rates shares by firm size category in Table 1.5-1.7, where Table 1.5 is based on base year size, Table 1.6 current firm size, and Table 1.7 average firm size. Except for very small firms, job creation rates are monotonically decreasing with firm size. This pattern is similar for the three measures of firm size. The weak positive relationship between job creation rates and firm size for small firms results from the non-monotonic labor policy function in low regions of equity values. Job destruction rates exhibit a more pronounced negative correlation with firm size in all three tables. In particular, job destruction rates are strictly decreasing with average firm size throughout the range of employment. Job reallocation rates also display a strong negative correlation with firm size. In terms of net growth rates of employment, Table

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job flow rates reported there for each size category are weighted average plant-level job flow rates across those plants whose parent firms' average size are within that category. So even though the reported job flow rates are not the firm-level job flow rates for that average firm size category, they provide an upper bound to the firm-level job flow rates. Replacing each size category with its mid-point, then we have a group of values for average firm size and corresponding job flow rates so that a set of correlation coefficients between average firm size and job flow rates can be computed. I take these measures as an approximation to the true correlation coefficients between firm-level job flow rates and average firm size in the ASM data.

Table 1.5: Net and Gross Job Flow Rates by Employer Size Category (base-year size)

	Job creation*	Job destruction	Job reallocation	Net growth	Job creation share	Job destruction share	Employment share
Classification I							
Small (< 500)	18.0930	12.8874	30.9803	5.2056	11.4814	6.3856	2.3515
Medium (500-999)	14.0389	9.8457	23.8846	4.1931	31.3000	21.9518	10.5813
Large (1000+)	3.1189	3.9062	7.0251	-0.7872	57.2185	71.6626	87.0672
Classification II							
< 300	15.9229	17.3073	33.2302	-1.3844	1.8396	1.9996	0.5483
300-399	18.0653	11.8733	29.9387	6.1920	2.8989	1.9053	0.7616
400-499	19.2498	11.3003	30.5501	7.9494	4.2103	2.4717	1.0380
500-599	18.4555	10.9812	29.4367	7.4742	6.6809	2.4778	1.0708
600-699	16.5949	11.7924	28.3872	4.8025	4.6034	3.2713	1.3165
700-799	15.5283	10.2380	25.7662	5.2903	7.8125	5.1511	2.3878
800-899	12.8517	8.8391	21.6908	4.0127	4.1094	2.8264	1.5176
900-999	11.7485	9.1039	20.8524	2.6447	10.6328	8.2395	4.2953
1000-1099	9.7433	6.1009	15.8441	3.6424	13.7877	8.6337	6.7161
1100-1199	8.2420	6.4817	14.7237	1.7603	7.1393	5.6147	4.1111
1200-1299	6.3879	5.2290	11.6169	1.1589	21.0220	17.2088	15.6188
1300+	1.1950	3.1473	4.3424	-1.9523	15.2633	40.2002	60.6180

*: Since the base-year size for new entry firms is zero so that the job creation rates are not well defined, I exclude new entry firms when computing the job creation rate.

Table 1.6: Net and Gross Job Flow Rates by Employer Size Category (current size)

	Job creation	Job destruction	Job reallocation	Net growth	Job creation share	Job destruction share	Employment share
Classification I							
Small (< 500)	16.9158	16.9849	33.9007	-0.0692	7.8028	7.8360	2.1892
Medium (500-999)	11.7427	11.5286	23.2712	0.2141	24.4259	23.9843	9.8720
Large (1000+)	3.6575	3.6789	7.3364	-0.0215	67.7712	68.1797	87.9388
Classification II							
< 300	31.8266	22.7705	54.5971	9.0560	3.2479	2.3243	0.4845
300-399	12.2513	15.9398	28.1911	-3.6885	1.6733	2.1775	0.6485
400-499	12.9908	15.0068	27.9976	-2.0160	2.9415	3.3987	1.0750
500-599	14.5801	13.9481	28.5282	0.6320	3.6395	3.4823	1.1851
600-699	13.9011	13.6902	27.5913	0.2109	4.1538	4.0912	1.4186
700-799	10.7059	12.7641	23.4700	-2.0582	3.6036	4.2972	1.5981
800-899	10.4312	12.9131	23.3443	-2.4820	6.4995	8.0472	2.9582
900-999	11.4042	7.1310	18.5352	4.2733	6.5327	4.0849	2.7194
1000-1099	8.6455	7.7884	16.4339	0.8572	9.6512	8.6963	5.3000
1100-1199	7.0879	7.6302	14.7180	-0.5423	15.4338	16.6186	10.3385
1200-1299	2.8509	4.9846	7.8355	-2.1337	7.9499	13.9034	13.2397
1300+	2.7886	2.3220	5.1105	0.4666	34.6733	28.8784	59.0344

Table 1.7: Net and Gross Job Flow Rates by Employer Size Category (average size)

	Job creation	Job destruction	Job reallocation	Net growth	Job creation share	Job destruction share	Employment share
Classification I							
Small (< 500)	15.7944	18.3129	34.1072	-2.5185	3.6013	4.3227	1.1082
Medium (500-999)	11.6887	11.5101	23.1988	0.1786	17.3340	17.0341	7.0671
Large (1000+)	4.0867	4.0640	8.1507	0.0228	79.0647	78.6431	91.8247
Classification II							
< 300	9.2761	28.5274	37.8035	-19.2513	0.1797	0.7035	0.1074
300-399	15.4513	19.0430	34.4942	-3.5917	1.3649	1.6888	0.4217
400-499	16.6048	15.8706	32.4754	0.7342	2.0942	1.9807	0.5938
500-599	15.0314	14.5872	29.6185	0.4442	2.3459	2.2628	0.7394
600-699	13.8551	13.4696	27.3247	0.3854	2.7431	2.6545	0.9409
700-799	12.5013	12.3327	24.8339	0.1686	3.2702	3.2199	1.2465
800-899	11.1362	11.1217	22.2579	0.0145	3.9465	3.9420	1.6934
900-999	9.8325	9.7207	19.5532	0.1118	5.1028	5.0387	2.4805
1000-1099	8.2317	8.1842	16.4159	0.0475	7.5246	7.4809	4.3583
1100-1199	6.5850	6.4982	13.0832	0.0868	12.9419	12.7748	9.3290
1200-1299	4.8357	4.8042	9.6399	0.0315	24.4192	24.2704	23.9916
1300+	2.9906	2.9824	5.9730	0.0082	34.0670	33.9830	54.0975

Table 1.8: Net and Gross Job Flow Rates by Employer Size Category (average firm size): Mean Annual Rates, 1973-1988, ASM data

	Job creation	Job destruction	Job reallocation	Net growth	Job creation share	Job destruction share	Employment share
Small (< 500)	11.94	12.86	24.8	-0.92	40.6	38.8	30.9
Medium (500-4999)	9.0	9.8	18.8	-0.8	19.9	19.2	19.5
Large (5000+)	7.0	8.45	15.45	-1.45	39.7	42.1	49.6

1.5-1.7 do not show systematic correlation between them and firm size. This finding is consistent with Davis et al. (1996).

Table 1.8 is obtained by combining Table 4.1 and 4.3 in Davis, et al (1996). It displays the job flow rates by a crude classification of average firm size for ASM. Even though the small, medium and large firms are classified differently, a comparison between Table 1.7 and Table 1.8 gives some information of how the model works. It shows that the magnitudes of job flow rates for each size category are compatible to their data counterparts. But the model predicts higher job creation, destruction and reallocation rates than the data for small and medium firms, while lower rates for large firms. This observation makes a lot sense considering that the only driving force for job reallocation in the model is financing constraints. I also find a dominant role of large firms in job creation and job destruction, despite the higher creation and destruction rates among smaller firms. This property is also observed in the data, but more pronounced in the model.

Table 1.9: Exit Rates by Employer Age Category (%)

Age categories	1-year	2-year	5-year	10-year
≤ 5 years old	1.0438	2.9549	8.9274	15.4221
6-10 years old	1.9889	3.4123	7.2847	12.8015
11-15 years old	1.4500	2.4761	5.8652	9.7864
15 years more	0.2251	0.4493	1.1034	2.1535

Table 1.10: Net and Gross Job Flow Rates by Employer Age Category

	Job creation	Job destruction	Job reallocation	Net growth	Job creation share	Job destruction share	Employment share
≤ 5 years old	0.4925	0.2318	0.7243	0.2608	0.0476	0.0145	0.0076
6-10 years old	0.1778	0.1985	0.3763	-0.0207	0.0199	0.0154	0.0090
11-15 years old	0.1352	0.1471	0.2824	-0.0119	0.0162	0.0131	0.0099
15 years more	0.0838	0.0996	0.1833	-0.0158	0.9163	0.9570	0.9753

1.4.4 Exit Rates and Employment Flows by Age

The model also generates an unconditional negative age dependence of firm exit and job reallocation. Table 1.9 reports firm exit rates and Table 1.10 reports job flow rates by the age of firms. Note that firm exit rates, job creation rates, destruction rates and reallocation rates are all decreasing with firm age. However, the negative age dependence results from the negative size dependence. Older firms are typically larger and larger firms have lower exit rates and job reallocation rates, therefore older firms have lower exit rates and job reallocation rates. Once firm size is controlled, the negative age dependence disappears.

1.4.5 A Comparison between the Equilibria with and without Financial Frictions

The model also has interesting implications of financial frictions in other aspects. Table 1.2 presents a comparison between the equilibria with and without the financial frictions, which gives us an understanding of the quantitative impacts of financial frictions on firm size distribution, aggregate output, employment, financial depth, and etc.

First, the financial frictions introduce variation to firm size distribution. In the frictionless world all firms have identical size, while with financial frictions there is a lot of heterogeneity, firm size ranging from 153 to 1,413 workers. Figure 1.9 depicts the stationary distribution of firms over employment, which is derived from the stationary firm distribution over equity values as plotted in Figure 1.8. Note that the distribution is very skewed to the right, i.e., a majority of firms are large firms with employment more than 1,000 workers. While in the data, small firms account for a large fraction of total number of firms. Firm size distribution is also more dispersed in the data. The model's prediction for firm size distribution can be improved by adding heterogeneity into the production technology. The model abstracts from this to focus on the role of financial frictions.

Second, with financial frictions the total number of firms is less than its frictionless counterpart, while the average firm size is larger. This observation seems to suggest that financial frictions also play a role in the determination of industry size and market structure.

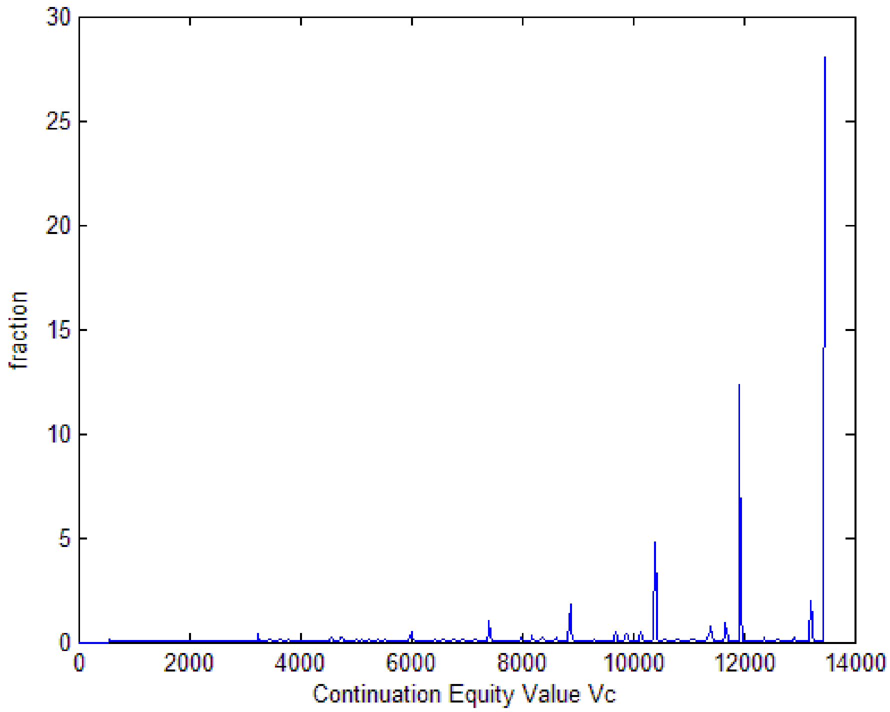

Figure 1.8: Firm Distribution (Equity Value)

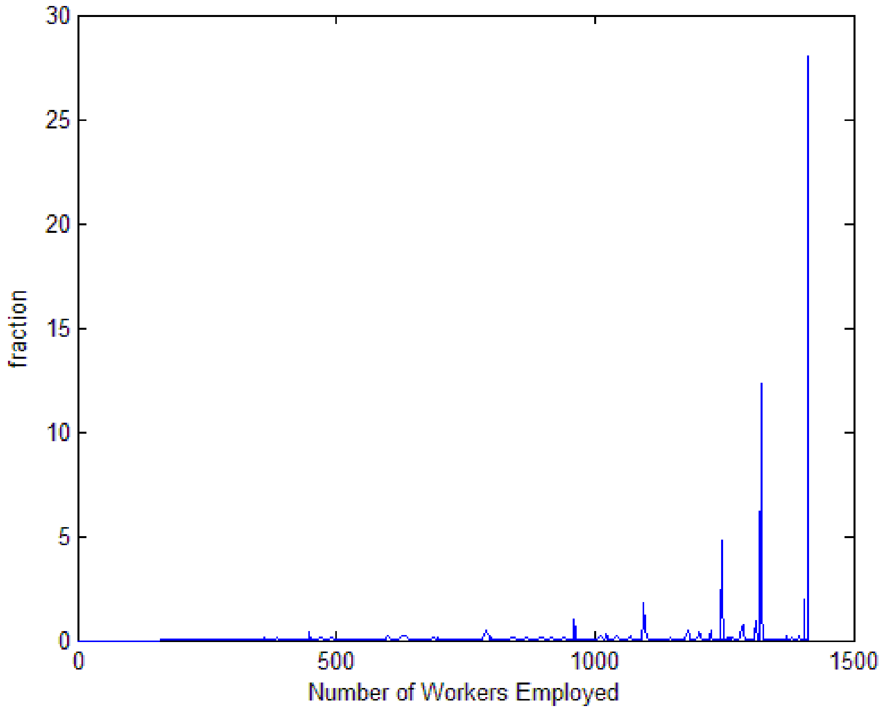

Figure 1.9: Firm Distribution (Employment)

Third, the presence of financial frictions can cause sizable losses in aggregate output and employment and considerable increases in output price. Compared to the frictionless world, the equilibrium output with frictions has decreased by 9.5 percent, employment decreased by 7.8 percent, and output price increased by 10.5 percent. This observation might justify the intensified exercise of government watchdog accounting procedures at the Securities and Exchange Commission, which aims to increase information transparency and eliminate the private information problem.

Fourth, the presence of financial frictions can greatly shift the division of revenues between banks and firms. The last panel of Table 1.2 reports the cash flow items in both equilibria. Note that with asymmetric information, the total payments to the bank is relatively lower (67.7 percent versus 88.8 percent of total revenues) and the total dividends of firms are relatively higher (32.3 percent versus 11.2 percent of total revenues). With informational asymmetries, the bank has to provide incentives for entrepreneurs to truthfully reveal their private information, which is achieved by giving relatively more to the entrepreneurs.

Finally, the total short-term liability held by the bank is 3 times less in the equilibrium with financial frictions. In a general equilibrium setting, this item would be the amount of funds that are intermediated through banks, which is an indicator of the financial depth. This result suggests that the presence of financial frictions may greatly hinder the development of financial market.

1.4.6 Comparative Static Properties

The discussions above show that presence of informational asymmetries and limited commitment problems in firm financing causes inefficient firm exit and job reallocation, output loss, employment loss and rise in equilibrium output price. In this section, I execute a comparative static analysis to see how these impacts vary with key primitives of the model. I consider the effects of changes in the interest rate or discount factor, riskiness of projects, entry cost, and entrepreneurs' initial wealth. The results are summarized in Table 1.11. For each set of new parameterization, the equilibrium with financial frictions is resolved to find the first four items in each panel of Table 1.11, where relative size of entrants and exiters are defined following the definition of Dunne, et al. (1989)[12]. The frictionless equilibrium with same parameterization is also solved such that a comparison is made to find the last three items in each panel.

The first panel shows that the higher the interest rate charged on banks' short-term liability, or the lower the discount factor, the higher the exit rate and job reallocation rate, and the larger the rise in output price and the losses in aggregate output and employment. This result suggests that increases in the interest rate worsen the adverse effects of the financial frictions. The second panel displays the effects of changes in project riskiness, which is measured by the probability of realizing a good shock. The lower π is, the riskier the

[12]Relative size of entrants= $\frac{\text{total output of entrants/number of entrants}}{\text{total output of incumbent firms/number of incumbent firms}}$

Table 1.11: Comparative Static Properties

Interest Rate(r)	6%	baseline(6.5%)	7%
Entry/exit rate(%)	0.24	0.31	0.39
Relative size of entrants	0.54	0.54	0.55
Relative size of exiters	0.33	0.33	0.33
Job Creation/destruction rate (%)	3.99	4.75	5.26
Increase in output price(%)	9.67	10.5	11.41
Output loss(%)	8.81	9.5	10.24
Employment loss (%)	6.79	7.81	8.82
Project Riskiness (π)	0.50	baseline(0.51)	0.55
Entry/exit rate(%)	0.33	0.31	0.23
Relative size of entrants	0.54	0.54	0.54
Relative size of exiters	0.33	0.33	0.34
Job Creation/destruction rate (%)	4.98	4.75	3.83
Increase in output price(%)	10.96	10.5	9.02
Output loss(%)	9.88	9.50	8.27
Employment loss (%)	8.28	7.81	5.98
Entry Cost (I)	10,000	baseline(10,478)	11,000
Entry/exit rate(%)	0.345	0.31	0.36
Relative size of entrants	0.63	0.54	0.48
Relative size of exiters	0.395	0.33	0.29
Job Creation/destruction rate (%)	4.51	4.75	5.37
Increase in output price(%)	8.26	10.5	16.76
Output loss(%)	7.63	9.50	14.35
Employment loss (%)	7.49	7.81	9.08
Firms' Initial Wealth (M)	2500	baseline(3,254)	4,000
Entry/exit rate(%)	0.33	0.31	0.29
Relative size of entrants	0.42	0.54	0.67
Relative size of exiters	0.285	0.33	0.367
Job Creation/destruction rate (%)	4.78	4.75	4.56
Increase in output price(%)	17.39	10.5	6.86
Output loss(%)	14.81	9.50	6.42
Employment loss (%)	8.23	7.81	7.55

projects are. Panel two shows that increases in project riskiness lead to more firm exits and more job creation and destruction, and enlarge the output and employment loss. The third panel considers entry cost. Higher entry cost causes more job reallocation, more output and employment loss, and larger increases in output price. But the effect of entry cost on firm entry or exit rate is not monotonic. It depends on the relative magnitude of the scrap value S, firm's initial wealth M and the entry cost I. Higher entry cost implies relatively smaller S and M, while smaller S decreases firm exits and smaller M increases firm exits. The last panel displays the effects of changes in firms' initial wealth M. Higher M reduces firm exits, job reallocation and losses in output and employment, i.e., more firm internal funds mitigates the adverse effects of financial frictions. This result is consistent with the literature on agency costs, such as Bernanke and Gertler (1989) and Gertler (1992), where a uniform finding is that higher net worth of firms moderates agency costs.

Another interesting finding is how the relative size of entrants and exiters are affected by these key factors. The size determination of entrants and exiters has been an interesting issue in the Industry Organization literature. Table 1.11 shows that changes in interest rate and project riskiness have no significant effects on the relative size of entrants and exiters, while changes in entry cost and firm initial wealth do. Higher firm initial wealth leads to a larger relative size of both entrants and exiters. Lower entry cost implies relatively higher firm initial wealth and thus has same effects.

1.5 Conclusion

Empirical evidence suggests that financing constraints might play an important role in the determination of firm dynamics. This chapter explores the quantitative implications of endogenous financing constraints for job reallocation, where the financing constraints are equilibrium outcomes of asymmetric information and limited commitment problems in firm external financing. A numerical analysis shows that endogenous financing constraints can account for a substantial amount of job reallocation and the negative size dependence of gross job flow rates. This exercise contributes to the literature in two aspects. First, it explictly models the micro foundations of financing constraints and show that such endogenous financing constraints can be the driving force underlying the growth and failure of a firm and the evolution of an industry. The model generates steady state firm entry, exit and job reallocation. In this sense, it provides a theoretical contribution to the literature on firm dynamics. Second, it quantitatively evaluates the significance of financing constraints for job reallocation, which has not been emphasized in the literature.

This exercise can be viewed as a first step toward the study of how financial market frictions affect gross job flows. For simplicity, the model abstracts from capital accumulation, technology progress, aggregate uncertainty and etc, which are all closely related to employment decisions of firms. In the model, financial frictions affect employment flows directly through imposing constraints in the labor finance of firms. It would be interesting to study how financial frictions impact job flows through other channels. The model also suggests that financial frictions play a role in accounting for firm size distribution, industry size, market structure, output growth and etc. Each of them deserves further investigation.

Chapter 2

Costly External Finance and Aggregate Productivity with Heterogenous Firms

2.1 Introduction

This chapter studies the quantitative impact of financial frictions on aggregate productivity in a setting with heterogenous firms. Recently there has been an increased interest in understanding the microeconomic dynamics of aggregate productivity growth. Corresponding to this literature is a surge of empirical work that exploits establishment-level data to explore the relationship between microeconomic productivity dynamics and aggregate productivity growth. Representative work includes Baily, Hulten, and Campbell (1992), Bartelsman and Dhrymes (1998) and Foster, Haltiwanger, and Krizan (2000). Even though their findings vary with the specific data sets and decomposition methodology used, a uniform finding in these studies is an important role of reallocation in accounting for aggregate productivity growth for the US manufacturing [1]. For instance, Bailey, Hulten and Campbell (1992) document

[1] Petrin and Levinsohn (2004) argue that the popular measurement of industry productivity growth adds a "reallocation" term to the growth accounting measure and fails to use the correct weights in the aggregation such that they call into question the literature's interpretation of "reallocation" as productivity growth. Instead, they propose a new method for separating real productivity growth from reallocation effects and find that such reallo-

that about half of overall productivity growth in U.S. manufacturing in the 80's can be attributed to factor reallocation from low productivity to high productivity plants.

Frictions in financial market can hinder the resource reallocation process among heterogenous production units by constraining a firm's ability to finance profitable investment opportunities, and therefore may potentially hamper the growth of aggregate productivity. This chapter formulates a simple partial equilibrium model to quantitatively assess this adverse effect. I abstract from modeling the microfoundations of financial frictions. Instead, financial market imperfections are summarized into a simple external finance cost function capturing the basic idea that external funds are more costly than internal funds if financial imperfections present. Then the costly external finance function is incorporated into a standard capital accumulation problem of a firm with idiosyncratic productivity shocks. The model is calibrated to Compustat US manufacturing data. The stationary properties of the industry are then compared with those if financial market is frictionless, i.e., external finance is costless. The results show that costly external finance leads to a reallocation of output shares from high-productivity firms to low productivity firms such that the output-weighted aggregate productivity is 1 percent less than it would be if external finance is costless. This is a significant loss considering that aggregate TFP growth for US manufacturing has averaged less

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cation effects are reasonably stable within industries and almost always positively impact aggregate productivity growth.

than 2 percent per year in the last 3 decades (See Baily, Hulten, and Campbell (1992), Foster, Haltiwanger, and Krizan (2000), Wheeler (2005) and Bartelsman and Dhrymes (1998)[2]). Considering firm entry and exit does not change this result significantly. A comparative static analysis shows that such adverse effect increases with external finance costs and the diversity of firms (variance of productivity shocks).

This chapter also gives interesting implications on the aggregate consequences of financial market frictions on output growth, which has been an important research issue. Most of the literature is based on the neoclassical growth model that abstracts from heterogeneity in production units. Not surprisingly, much of this literature has been concerned with understanding the role of aggregate accumulation and how aggregate accumulation is affected by financial market frictions. However, the empirical evidence shows that it is not only the level of factor accumulation that matters for aggregate output but how these factors are allocated across heterogenous production units. In our model, costly external finance decreases aggregate output through two channels. One is the traditional channel–capital accumulation. Costs associated with external finance increase the aggregate relative price of capital, and as a result decrease aggregate investment and lower aggregate capital accumulation. The other channel is through resource misallocation which results in lower aggregate productivity. Our results show that with heterogeneity in

[2]Even though the productivity growth rates documented in these studies differ from each other due to different data sets and aggregation methods used, they are no more than 2 percent per year in magnitude

firm-level productivity, both channels matter for aggregate output, but the effect on aggregate output through the second channel seems relatively small. This seems to suggest that for an economy as advanced as the US economy, the traditional neoclassical model may not be a bad framework for studying the long run consequences of financial frictions on aggregate output even though it ignores the effect through resource reallocation across heterogenous production units.

The rest of this chapter is organized as follows. Section 2.2 reviews a popular measurement of aggregate productivity and a decomposition methodology of aggregate productivity growth widely adopted by the empirical studies, which helps formulate our model and understand the results. Section 2.3 describes the model. Section 2.4 details the calibration and simulation methods. Section 2.5 describes the results. And Section 2.6 concludes.

2.2 Measurement of Aggregate Productivity and Decomposition of Aggregate Productivity Growth

A lot of empirical studies use the sum of output-weighted firm/plant level TFP to measure the aggregate productivity of an industry. According to Baily, Hulten and Campbell (1992), the definition of aggregate productivity is as follows. Suppose the production function for plant i in period t is

$$Q_{it} = F(K_{it}, L_{it}, M_{it}),$$

where K, L and M are capital, labor and intermediate inputs, respectively. The plant level TFP is defined as

$$lnTFP_{it} = lnQ_{it} - \alpha_K lnK_{it} - \alpha_L lnL_{it} - \alpha_M lnM_{it},$$

where α_K, α_L and α_M are return to scale factors for capital, labor and intermediate inputs respectively. Then the level of productivity for the industry in year t is represented by the following index:

$$TFP_t = \sum_i \theta_{it} TFP_{it},$$

where θ_{it} is the output share of the ith plant in industry output.

The industry productivity growth is typically decomposed into several parts characterizing the relative contributions of the stayers, the entrants and the exits. According to Baily, Hulten and Campbell (1992), the change in industry productivity between $t - \tau$ and t can be decomposed into 3 parts.

$$\begin{aligned} \Delta TFP_t = & \sum_{i \in C} \theta_{i,t-\tau} \Delta TFP_{it} + \sum_{i \in C} (\theta_{it} - \theta_{i,t-\tau}) TFP_{it} \\ & + \Big(\sum_{i \in N} \theta_{it} TFP_{it} - \sum_{i \in X} \theta_{i,t-\tau} TFP_{i,t-\tau} \Big). \end{aligned} \tag{2.1}$$

The first two terms reflect contribution of stayers to aggregate productivity growth, where the first term reflects the contribution of within plant productivity growth and the second term reflects the contribution of reallocation across plants which leads to changes in output shares. The last term reflects the contribution of net entry. Empirical studies find a significant role of the second term. That is, the shift of output shares from low productivity plants to more productive plants (within the stayers) is an important contributor to productivity growth in US manufacturing.

In this chapter, I formulate a version of the growth model in which capital accumulation and production is carried out by heterogenous firms with idiosyncratic productivity shocks. I compare the steady state output-weighted aggregate productivity in two cases: external finance is costly and costless. In other words, I consider the change in aggregate productivity from $t - \tau$ to t, imagining that in period $t - \tau$ the industry is in the steady state with costless external finance, while in period t the industry is in the steady state with costly external finance. It is shown that in the decomposition equation (2.1), the third term is zero since firm entry and exit is excluded in the model. The first term is also zero, since the two periods have exactly the same productivity distribution. Therefore the change in aggregate productivity is completely characterized by the second term–reallocation of output shares due to costly external finance.

2.3 The Model

The analysis is of partial equilibrium type, in that it focuses on a single firm's dynamic capital accumulation problem. When assessing the aggregate implications of costly external finance, a large number of such firms are considered.

The firm is infinitely lived. I exclude firm entry and exit in the analysis. One reason is for simplicity. Another reason is that the data set used to calibrate the model does not exhibit a lot of entry and exit. In a later section, I discuss how the result would change if considering firm entry and exit. In period t, the firm's operating cash flow is generated by a profit function given by

$$\pi(k_t, z_t) = e^{z_t} k_t^{\alpha},\ \alpha < 1.$$

Here, k_t is the firm's capital stock at the beginning of period t. Capital depreciates at rate δ and must be decided one period in advance. The relative price of capital good is p. z_t is the firm's idiosyncratic productivity shock. It is assumed to follow a $AR(1)$ process given by

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

where ε follows a truncated normal distribution with zero mean, standard deviation of σ and finite support $[-4\sigma, 4\sigma]$. Note that the firm's TFP in period t is e^{z_t}, according to the definition in Section 2.2.

As in Gomes (2001) and Whited (2004), financial market imperfections are summarized with a simple external finance cost function that takes the linear form given by

$$\lambda = \lambda_0 + \lambda_1 \times \text{amount of external funds}.$$

Equivalently, there is a fixed cost λ_0 and per unit cost λ_1 associated with external finance. This specification is intended to capture a variety of costs of going to financial market to raise capital, which would include the fixed and variable costs of public stock offerings, costs of monitoring the firm and the discounted present value of any premia associated with external debt and equity finance. Clearly the firm will only choose to use external finance when it exhausts internal funds and current investment opportunities justify the additional cost of external funds.

The firm's objective is to maximizes its expected discounted sum of future net cash flow, taking the price of capital good p as given. The firm's problem has the following recursive formulation.

$$V(k,z) = \max_{k' \geq 0} \quad \pi(k,z) - p\,i(k,k') - \lambda_0 I\{p\,i(k,k') > \pi(k,z)\}$$
$$-\lambda_1 \max\{p\,i(k,k') - \pi(k,z), 0\} + \beta E_{z'|z} V(k',z'), \quad (2.2)$$

where $i(k,k') = k' - (1-\delta)k$, and $I\{.\}$ is an indicator function. The right-hand side of (2.2) specifies the decisions the firm has to make. The first four terms reflect the current net cash flow: profits minus investment spending and financing costs. The last term is the expected continuation value.

Applying standard arguments of dynamic programming, one can show that a unique solution to this problem exists and establish some useful properties for the value function.

Proposition 2.3.1. *For a given p, there is a unique function $V(k,z)$ that satisfies (1); $V(k,z)$ is continuous and increasing in both k and z, and concave in k.*

Associated with this solution there is a decision rule concerning capital accumulation, denoted by $k'(k,z)$. If external finance is costless ($\lambda_0 = \lambda_1 =$

0), $k'(k,z)$ would be a function of current productivity shock z only, i.e., it is independent of current capital stock. Costly external finance introduces dependence of k' on k. The following proposition characterizes the decision rule $k'(k,z)$.

Proposition 2.3.2. *For a given z, there exists $0 < k_e(z) < k_f(z)$ and $0 < k_1'(z) < k_2'(z)$, such that*
(i) For $k < k_e(z)$, the firm resorts to external finance and $k'(k,z) \equiv k_1'(z)$;
(ii) For $k_e(z) \leq k \leq k_f(z)$, the firm's investment is constrained by its profits, i,e.,$k'(k,z) = (1-\delta)k + \frac{\pi(k,z)}{p}$;
(iii) For $k > k_f(z)$, the firm's investment achieves its unconstrained efficient level, i.e., $k'(k,z) \equiv k_2'(z)$.

PROOF: See Appendix B.

Proposition 2.3.2 states that for a given current productivity level, if the firm's current capital stock is relatively small, using external finance is profitable. But since the profit function exhibits decreasing return to scale, when the firm's capital stock passes some level ($k_e(z)$), current investment opportunities would not justify the additional cost of external finance and hence the firm's investment is constrained by its operating profit. If the firm's capital stock is big enough (greater than $k_f(z)$) such that it could generate enough cash flow to finance desired level of investment, the firm's investment is no longer financially constrained.

Figure 2.1 plots the policy function $k'(k,z)$ for a low level of current productivity z and a high level of z [3]. The figure is based on the baseline parameterization to be described in next section. In both plots, the solid line corresponds to the case of costly external finance, and the dashed line corresponds to costless external finance. Note that with costless external finance, k' is independent of current capital stock k. Costly external finance introduces dependence of k' on k, as described in Proposition 2.3.2. The figure also shows that with costly external finance $k'(k, z)$ may be discontinuous at $k_e(z)$ (For characterization of $k_e(z)$, see the proof of Proposition 2.3.2 in Appendix B). This is due to the nonlinearity introduced by a fixed external finance cost. Comparing the two plots suggests that due to costly external finance, high productivity firms tend to accumulate less capital than they would if external finance is costless, while this adverse effect is less severe for low productivity firms. Notice that for a low current productivity, the value of k' on the unconstrained region with costly external finance is bigger than the unconstrained efficient level corresponding to costless external finance, implying that low productivity firms have the incentive to accumulate more capital to generate more operating cash flow for next period when they are able to do so such that they could possibly avoid the use of external finance when higher productivity becomes available in next period. This property will help explain why the presence of costly external finance has an adverse effect on aggregate productivity, as will be clear in a later section.

[3] In the computation, I approximate the productivity shock process with a 10-state Markov chain. Here, the low z refers to the third state, and the high z refers to the 10^{th} state.

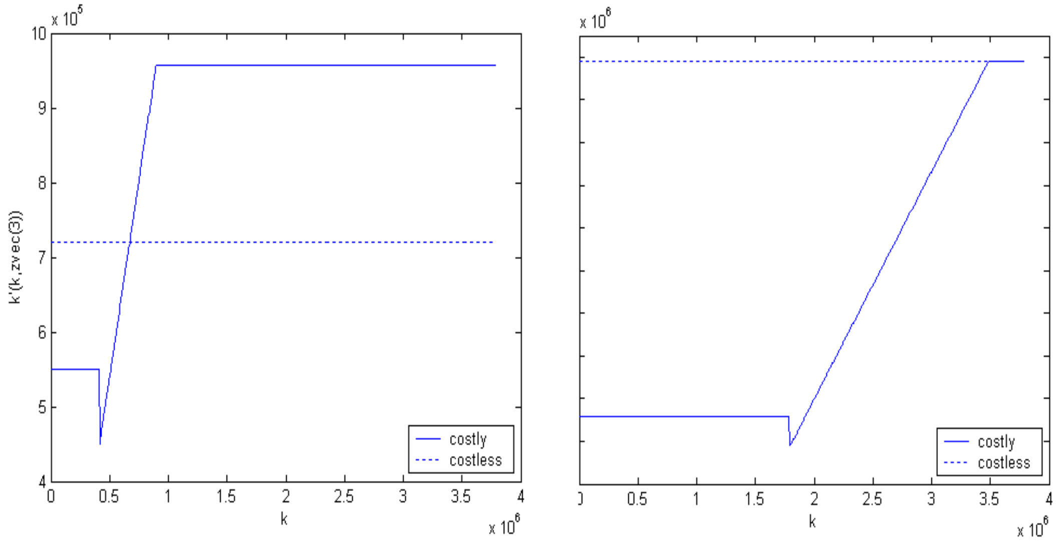

Figure 2.1: Decision rule for $k'(k, z)$

Proposition 2.3.2 implies that small firms (with smaller capital stock) resorts to external finance more often. This seems to contradict the commonly held belief that small firms are more financially constrained and rely on internal funds more often. Fazzari, Hubbard and Peterson (1988) document ratios of debt to total sources of funds by asset class for Value Line manufacturing firms in the period of 1970 to 1984, and find a weak positive relationship between the ratios and firm assets (See Table 2.1). Since equity finance is only a small fraction of external finance, this finding implies that large firms tend to use relatively more external finance. I compute external finance ratios by asset class for Compustat manufacturing firms in the period of 1989 and 2003. On the contrary, I find a strong negative relationship between external finance ratios and the total assets of firms (See Table 2.2). That is larger firms

Table 2.1: External Finance Ratio by Asset Class, Value Line Manufacturing Firms, 1970-84

	debt/sources of funds
All firms	0.289
< $10 million	0.241
$10-50 million	0.302
$50-100 million	0.213
$100-250 million	0.251
$250 million-$ 1 billion	0.237
> $ 1 billion	0.321

Note: This table is from Table 1 of Fazzari, Hubbard, and Peterson (1988)).

Table 2.2: External Finance Ratio by Asset Class, Compustat Manufacturing Firms, 1989-2003

	external funds/ sources of funds	external funds/ uses of funds
All firms	0.1077	0.1123
< $250 million	0.9337	0.9660
$250 million - $ 1 billion	0.2593	0.2974
$ 1-2 billion	0.1691	0.1844
> $ 2 billion	0.0784	0.0800

have lower external finance ratios. Since Compustat firms are mainly large mature firms, I cannot conclude whether this negative relationship holds for all manufacturing firms. However, this finding suggests that the commonly held belief may not hold in the data. Hence the decision rule predicted by the model may not be unrealistic.

2.4 Calibration and Simulation

To execute a quantitative analysis, I need to set values for parameters of the model, including the relative price of capital good, p, the discount factor, β, the depreciation rate of capital, δ, the return to scale of capital, α, the parameters describing the productivity shock, ρ and σ, and parameters in the external finance cost function, λ_0 and λ_1. The data I use to estimate or calibrate the parameters is taken from the Compustat North American industry annual file. I only consider firms in the manufacturing sector (with SIC codes between 2000 and 3999) during the period of 1989 to 2003. This time period is chosen since there are substantial changes in the reporting and accounting methods since 1988. Observations with missing data are deleted from the sample. Similar to Whited (1992) and Gilchrist and Himmelberg (1995), I exclude observations with large changes in the book value of capital stock, considering that they may indicate expansions or contractions of firms at margins other than capital expenditure (See Appendix B for details). Finally I end up with an unbalanced panel of firms from 1989 to 2003 with between 2210 and 3265 observations per year. Appendix B gives a detailed description of the variables in this data sample.

Following Cooper and Ejarque (2001), I set $p = 1$ and $\beta = 0.95$. The external finance cost function was estimated by Smith (1977) and Altinkilic and Hansen (2000), both using data on costs associated with new equity issuance. Their estimations yield λ_1=0.028 and 0.0241 respectively. Since in the data external finance mainly takes the form of debt finance rather than equity finance[4], I re-estimate this parameter by a panel regression of interest expenses of debt on debt issuance [5]. It gives a similar result, $\lambda_1 = 0.028$. Since λ_0 is sensitive to units of measure, it is estimated together with α, δ, ρ and σ to match five moments of the data. The first moment is the mean annual investment rate defined as the ratio of total investment to total capital stock, which is 0.17 for the data sample. The second moment is the cross-sectional average investment rate, which is 0.22. The third moment is the cross-sectional standard deviation of investment rate, which is 0.19. The fourth moment is the autocorrelation of investment rate, which is 0.21. In constructing investment rates for each firm at each year, the book values of the gross capital stock are converted into its replacement values following the perpetual inventory method described in Salinger and Summers (1983). Appendix B gives a detailed description of this procedure. The last moment is the fraction of total investment financed externally, i.e. the ratio of external finance used for investment to total investment. Compustat does not have enough information to directly calculate this moment. But it can be reasonably approximated by the ratio of total external finance to total uses of funds, which is 0.072, since in the data sample 86% of total uses of funds are for new capital purchase. These five moments are selected for their informativeness about the underlying structural parameters as well as their prominence in the literature.

For a given set of parameter values, the productivity shock is approxi-

[4]For my data sample, equity finance is about 10% of total external finance

[5]Data on total expenses of external finance is not available in Compustat. Otherwise, the cost function of external finance could be directly estimated.

Table 2.3: Baseline Calibration

	Parameter	Value
Price of capital	p	1
Discount factor	β	0.95
Returns to scale	α	0.9
Depreciation rate	δ	0.17
Persistence of shock	ρ	0.87
Variability of shock	σ	0.04
Fixed cost of external finance	λ_0	600
Unit cost of external finance	λ_1	0.028
Matched Moments	Data	Model
I/K	0.17	0.17
Avg. of i/k	0.22	0.19
Std. of i/k	0.19	0.18
Autocorr. of i/k	0.21	0.16
External finance $/I$	0.072	0.075

mated by a 10-state Markov process and the firm's problem is solved by value function iteration to obtain the decision rules $k'(k, z)$. Using the decision rules, an invariant distribution of firms over capital stock and productivity types, $\mu(k, z)$, can be computed. It is independent of the initial distribution of (k, z). Then I draw 20,000 firms from the invariant firm distribution and carry out the simulation for 15 periods[6], and construct an artificial panel data set. The five moments are computed for this artificial data set and compared with the corresponding data moments. This procedure is continued until the moments of the simulated data set are close enough to the data moments or cannot be improved. A more detailed estimation procedure is described in Ap-

[6] It is 15 years from 1989 to 2003.

pendix B. Table 2.3 summarizes the choice of parameter values and matched moments.

The high degree of nonlinearities in the solution makes it hard to match all moments exactly. Nevertheless the approximation appears reasonably close, as shown in Table 2.3. Note that the estimated value of α is 0.9, which is pretty close to 1, suggesting that the technology does not substantially depart from constant return to scale. This is consistent to most of previous studies (See Burnside (1996) and Gomes (2001)). Cooper and Haltiwanger (2005) estimate a much lower α of about 0.6 using the LRD plant level data. This estimate does not contradict mine since Compustat file is composed of bigger and more mature firms as compared to LRD. The estimated depreciation rate is 0.17, higher than those of most previous studies based on data before 1990s. Considering the rapid technological progress since 1990s, a higher depreciation rate of capital seems reasonable. The estimated degree of persistence and variability in productivity shocks is consistent with Gomes (2001). But the variability is much smaller than that of Cooper and Haltiwanger (2005).

2.5 Results

With the parameters determined, the question outlined in the Introduction can be addressed. This section summarizes the quantitative impacts of costly external finance on aggregate productivity, capital accumulation and output. A comparative static analysis is executed to see how these impacts are affected by the primitives of the model. Finally, I briefly discuss whether adding firm entry and exit would change the results.

2.5.1 Impact of Costly External Finance on Aggregate Productivity

To evaluate the quantitative impact of costly external finance on aggregate productivity, I compute the output-weighted aggregate productivity and compare it with its counterpart if external finance is costless [7]. As described in Section 2.2, to compute the output-weighted aggregate productivity, a distribution of output shares across different productivity types is needed. The invariant measure of firms over capital and productivity, $\mu(k, z)$, enables me to do so. As shown in Table 2.4, the output-weighted aggregate productivity with costly external finance is 1.0384, while its costless counterpart is 1.0485. This implies a 1% loss in aggregate productivity due to costly external finance. It is quantitatively significant considering that the total factor productivity growth for US manufacturing has averaged less than 2 percent per year during most of the post-war periods. Let us examine this result from several aspects.

First, as shown in Figure 2.2, the productivity distributions with costly or costless external finance are the same: firms with each of the 10 productivity types account for 10% of all firms. So in Table 2.4 the average productivity is 1 in both cases. Therefore the productivity change due to within firm productivity change is zero, i.e., the first item in the decomposition of aggregate

[7] The costless counterpart is computed by taking all parameter values as in Table 2.3 except that $\lambda_0 = 0$ and $\lambda_1 = 0$ and re-computing the firm's problem and the invariant firm distribution.

Table 2.4: Quantitative Impacts of Costly External Finance on Aggregate Productivity, Capital Accumulation and Output

	costly ext. finance	costless ext. finance	ratio (costly/costless)
Average productivity	1	1	1
Output-weighted productivity	1.0384	1.0485	0.9904
Aggregate capital stock:	$1.3597 \cdot 10^6$	$1.4674 \cdot 10^6$	0.927
Aggregate output:	$3.3784 \cdot 10^5$	$3.6298 \cdot 10^5$	0.931

Note: The aggregates are based on a unit measure of firms in both cases.

productivity growth (equation (2.1)) is zero. So the 1% loss in aggregate productivity due to costly external finance is completely through the second item–reallocation. This is shown clearly in Figure 2.3, which plots the distribution of output shares across productivity types for the two cases. Note that with costly external finance, the output shares of firms with high level productivities are smaller than their costless counterparts, while the output shares of firms with low productivities are larger than their costless counterparts. It follows that the presence of costly external finance leads to a shift of output shares from high productivity firms to low productivity firms and hence results in lower aggregate productivity. The driving force underlying this result is the distortion in firms' investment behavior due to costly external finance. As mentioned earlier, the adverse effect that costly external finance decreases capital accumulation is more severe for high productivity firms because low productivity firms have the incentive to accumulate more capital to avoid the use of external finance in the future. Higher capital accumulation leads to higher output in these low productivity firms, as a result leads to an output

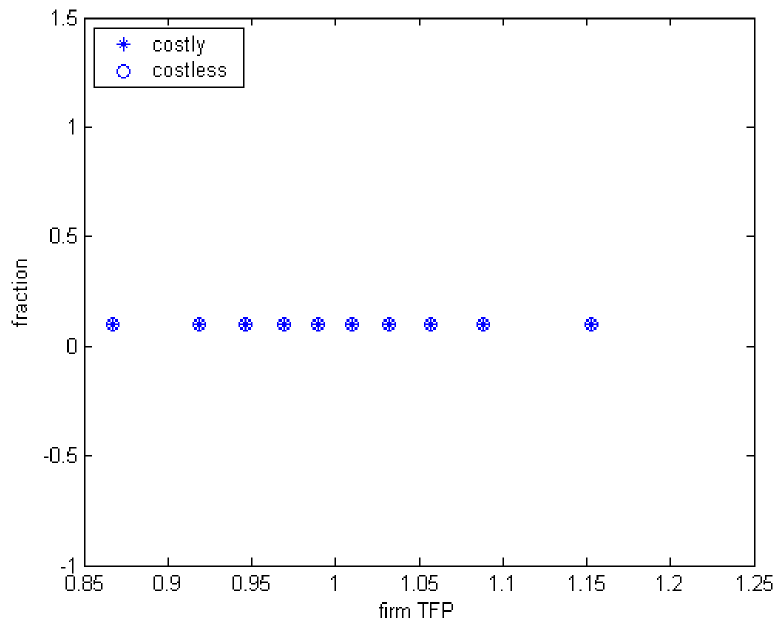

Figure 2.2: Firm Distribution over Productivity Types

reallocation from high productivity to low productivity firms.

Finally, Figure 2.4 plots the firm distribution over capital stock in the two cases. If external finance is costless, the firm distribution is a uniform distribution over the 10 efficient levels of capital stock corresponding to the 10 productivity types. While with costly external finance, the distribution is skewed to the left, with more firms having lower capital stock. This feature is consistent with the data.

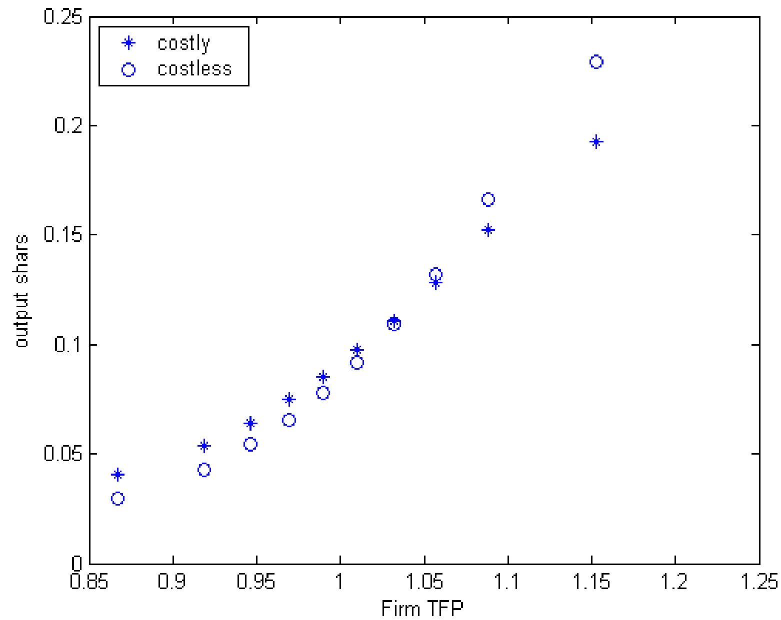

Figure 2.3: Distribution of Output Shares over Productivity Types

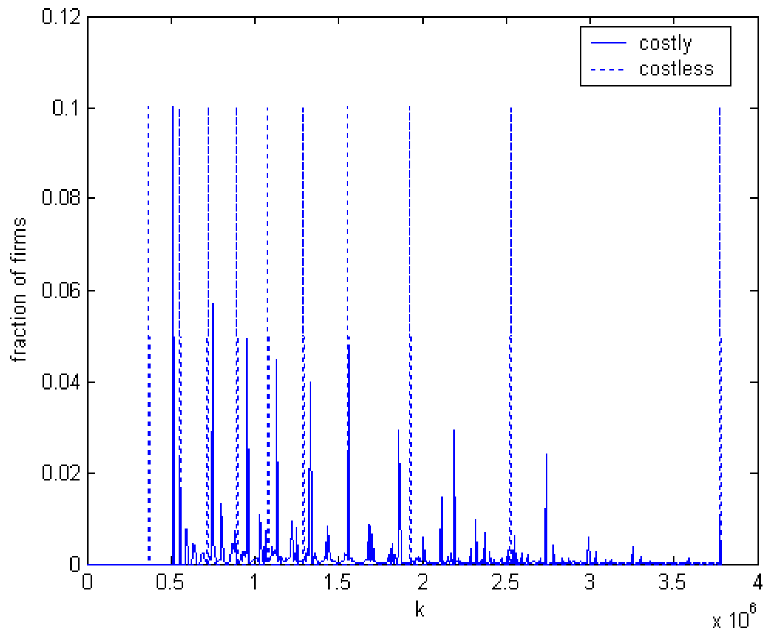

Figure 2.4: Firm Distribution over Capital Stock

Table 2.5: Quantitative Impacts of Costly External Finance on Aggregate Productivity and Output

	costly ext. finance	costless ext. finance	ratio (costly/costless)
Price of capital good:	1	1.0077	0.9924
Average productivity	1	1	1
Output-weighted productivity	1.0384	1.0485	0.9904
Aggregate output:	$3.3784 \cdot 10^5$	$3.389 \cdot 10^5$	0.9968

2.5.2 Impact of Costly External Finance on Capital Accumulation and Output

The quantitative impacts of costly external finance on capital accumulation and output are more pronounced in this partial equilibrium setting. As shown in Table 2.4, costly external finance results in a 7% loss in both aggregate capital and aggregate output. This result requires several remarks.

First, the result is based on a partial equilibrium analysis. When solving the costless problem, I keep the price of capital unchanged, $p = 1$. In a general equilibrium setting, the price of capital goods would increase to discourage investment as investment demand rises. As a result, the aggregate capital accumulation and aggregate output would not be so high as reported in Table 2.4. So the adverse effects of costly external finance on aggregate capital accumulation and output in a general equilibrium analysis would be smaller than suggested by Table 2.4.

Second, costly external finance can decrease aggregate output through two channels. One channel is decreasing aggregate productivity. Another is lowering aggregate capital accumulation since additional costs of external finance imply a higher aggregate cost of investment. To disentangle these two channels, I do another experiment. When solving the costless problem, I vary the price of capital good p, such that aggregate capital stock is the same as its counterpart with costly external finance. In this way, I keep the aggregate capital accumulation the same in both scenarios. Any change in aggregate output is completely through changes in aggregate productivity. The result is summarized in Table 2.5. Note that the effect on output-weighted aggregate productivity is not affected by the change of capital price [8]. A 1% decrease in aggregate productivity due to costly external finance leads to about 0.3% decrease in aggregate output. This seems a small contribution compared to the 7% total loss in aggregate output. In a general equilibrium setting, this contribution would probably be bigger considering that the total loss in aggregate capital accumulation due to costly external finance would be smaller.

A large literature that attempts to explore the relationship between financial market development and output growth adopts the framework of neoclassical growth model that abstracts from heterogeneity in production units. Not surprisingly, therefore much of this literature has been concerned with understanding the role of aggregate accumulation and how aggregate accumulation is affected by financial market development. The role of reallocation is completely neglected. With heterogeneous firms, the model can characterize both roles of aggregate accumulation and reallocation. where the role of

[8]This result suggests that the former result concerning the quantitative impact of costly external finance on aggregate productivity does not hinge on the partial equilibrium analysis adopted.

reallocation is characterized by the change in output-weighted aggregate productivity. The quantitative analysis above suggests that the impact of costly external finance on aggregate output is mainly through aggregate capital accumulation rather than reallocation. This result seems to suggest that such framework is not so bad in characterizing the long run relationship between financial market development and output growth for the US economy.

2.5.3 Comparative Statics

The previous results are based on the baseline calibration. In this section, I execute a comparative static analysis to see how the effects of external finance vary with key parameters of the model. I consider the effects of changes in the return to scale, in the persistence and variability of the productivity shocks, and in the external finance costs. For each new parameterization, the firm's problem is re-solved and the model is simulated to generate the four moments: cross-sectional mean, standard deviation and autocorrelation of investment rates, and fraction of total investment financed externally [9]. The corresponding problem with costless external finance is also re-solved to compute the ratios of aggregate productivity, aggregate capital stock and aggregate output to their costless counterparts. Smaller ratios imply more severe adverse effects of costly external finance. Table 2.6 summarizes the results. Note that the middle column of each panel refers to the baseline calibration.

[9] Aggregate investment rate is mainly determined by the depreciation rate of capital. It is about 0.17 in all these scenarios and hence is skipped in Table 2.6.

Table 2.6: Comparative Statics

	$\alpha = 0.85$	$\alpha = 0.9$	$\alpha = 0.95$
Average investment rate:	0.1785	0.19	0.2374
Std. of investment rate:	0.1212	0.18	0.4310
Autocorrelation of inv. rate:	0.1154	0.16	0.1227
External finance ratio:	0	0.075	0.2853
Aggregate capital stock *	0.97	0.927	0.734
Aggregate output	0.973	0.931	0.741
Aggregate productivity	0.9936	0.9904	0.9833
	$\sigma = 0.03$	$\sigma = 0.04$	$\sigma = 0.05$
Average investment rate:	0.179	0.19	0.2
Std. of investment rate:	0.129	0.18	0.251
Autocorrelation of inv. rate:	0.143	0.16	0.157
External finance ratio:	0.02	0.075	0.145
Aggregate capital stock	0.956	0.927	0.897
Aggregate output	0.958	0.931	0.903
Aggregate productivity	0.994	0.9904	0.987
	$\rho = 0.9$	$\rho = 0.87$	$\rho = 0.84$
Average investment rate:	0.191	0.187	0.184
Std. of investment rate:	0.206	0.181	0.159
Autocorrelation of inv. rate:	0.160	0.159	0.152
External finance ratio:	0.104	0.075	0.053
Aggregate capital stock	0.918	0.927	0.933
Aggregate output	0.923	0.931	0.937
Aggregate productivity	0.989	0.9904	0.991
	$\lambda_0 = 0$	$\lambda_0 = 600$	$\lambda_0 = 1000$
Average investment rate:	0.1874	0.1866	0.187
Std. of investment rate:	0.178	0.181	0.182
Autocorrelation of inv. rate:	0.176	0.159	0.149
External finance ratio:	0.08	0.075	0.073
Aggregate capital stock	0.9269	0.9266	0.925
Aggregate output	0.931	0.9307	0.93
Aggregate productivity	0.9906	0.9904	0.9903
	$\lambda_1 = 0.02$	$\lambda_1 = 0.028$	$\lambda_1 = 0.035$
Average investment rate:	0.191	0.1866	0.186
Std. of investment rate:	0.207	0.181	0.166
Autocorrelation of inv. rate:	0.151	0.159	0.153
External finance ratio:	0.122	0.075	0.054
Aggregate capital stock	0.934	0.927	0.919
Aggregate output	0.938	0.931	0.923
Aggregate productivity	0.9923	0.9904	0.989

*: Figures in the second part of each panel are ratios to their costless counterparts

The first panel of Table 2.6 shows that the adverse effects of costly external finance on aggregate productivity, aggregate capital accumulation and output increase with the return to scale parameter, α. Note that α indicates market power. The closer α is to 1, the lower the market power, or the more competition among firms. This result suggests that the adverse effects of costly external finance is more severe for an economy that is more competitive, *ceteris paribus*. The reason underlying this result can be seen from the reported external finance ratios. Note that external finance ratio increases with α. With lower market power, operating profits are relatively low so that firms have to resort to external finance more heavily. Therefore costly external finance imposes more adverse impacts the economy.

The second panel shows that the adverse effects of costly external finance increase with the variability in idiosyncratic productivity shocks. Higher σ implies greater heterogeneity among firms. This result suggests that the more diversified the productions units are, the greater the loss in aggregate productivity through resource misallocation due to costly external finance. Considering that some studies based on more comprehensive data set for US manufacturing give a much higher estimate for σ ($\sigma = .64$ in Cooper and Haltiwanger (2005)), my quantitative result may underestimate the actual impact of costly external finance on aggregate productivity. Finally, even though a rigorous assessment cannot be made in this partial equilibrium analysis concerning whether the relative importance of reallocation in transmitting the effect of costly external finance to aggregate output increases with the variability of productivity shocks, I believe that the performance of the standard neoclassical growth model would become worse if more diversity among production units are observed in the data.

The third panel shows how the impacts of costly external finance change with the persistence in productivity shocks. If the shock process is more persistent (higher ρ), the adverse impacts are more severe. At the first look, this may seem hard to understand. But note that the standard deviation of the productivity shock z is given by $\frac{\sigma}{\sqrt{1-\rho^2}}$. For a given σ, higher ρ implies higher variability in the productivity shocks. So the results are consistent with the previous discussion.

The last two panels consider how the impacts vary with the external finance costs. Not surprisingly, either higher fixed cost or higher unit cost of external finance leads to more severe adverse effects in aggregate productivity, aggregate capital accumulation and aggregate output.

2.5.4 Considering Firm Entry and Exit

The model laid out previously does not consider firm entry and exit. Even though Compustat data does not exhibit a lot of firm entry and exit, it is a common behavior of the US manufacturing industry. According to Dunne, Roberts and Samuelson (1988), by average approximately 4.5% of firms entered the US manufacturing industry every year during the period of 1963 to 1982 and similar percentage of firms exited every year. This section presents a brief discussion of how the quantitative impact of costly external finance on aggregate productivity would change if adding firm entry and exit to the model. Rather than doing a comprehensive analysis, I consider some simple cases of firm entry and exit.

I assume that the firm's exit is exogenous: every period, the firm has a probability of η to exit, where $\eta = 0.045$. Upon exit, the firm secures a zero exit value. Now the firm's problem is given by

$$\begin{aligned} V(k,z) &= \max_{k' \geq 0} \quad \pi(k,z) - p\, i(k,k') - \lambda_0 I\{p\, i(k,k') > \pi(k,z)\} \\ &\quad -\lambda_1 \max\{p\, i(k,k') - \pi(k,z), 0\} + \beta(1-\eta) E_{z'|z} V(k',z'). \end{aligned} \tag{2.3}$$

In the data, there are high-productivity entrants and low-productivity entrants. So I consider two cases of firm entry. First, new entry firms are of the highest productivity. This form of firm entry and exit is also adopted by Cooley and Quadrini (2001). Second, new entry firms are of the lowest productivity. The actual case probably lies in between. Upon entry, a new firm chooses its next period capital to maximize its expected continuation value. The entry problem is as follows.

$$V_0(z_0; p) \equiv \max_{k_0} \quad \int V(k_0, z') P(z_0, dz') - \lambda_0 - p(1+\lambda_1)k_0, \tag{2.4}$$

where $z_0 = \bar{z}$ for the first case, and $z_0 = \underline{z}$ for the second case. Free entry condition implies that

$$V_0(z_0; p) = c_e,$$

where c_e is a fixed entry cost.

Table 2.7: Aggregate Productivity and Moments with Firm Entry and Exit

	New firms are of highest productivity	New firms are of lowest productivity
Aggregate Productivity	1.0726	1.0169
Ratio to its frictionless counterpart	0.9917	0.9908
Aggregate investment rate	0.1702	0.1712
Average investment rate	0.1774	0.2317
Std. of investment rate	0.1963	0.1977
Autocorrelation of inv. rate	0.1831	0.20
External finance ratio	0.23	0.1132

To solve the problem, let p and all parameter values are as given in Table 2.3, and simply choose c_e such that the free entry condition is satisfied. For the costless problem, just let $\lambda_0 = \lambda_1 = 0$ in problem (2.3) and (2.4), and choose the price of capital good, p^c such that the free entry condition is satisfied [10]. Table 2.7 reports the results.

First, note that the ratio of the aggregate productivity measure to its frictionless counterpart is 0.9917 if new firms are of the highest productivity, and 0.9908 if new firms are of the lowest productivity. Both figures are a little bit bigger than the one obtained in the previous model, implying that the adverse impact of costly external finance on aggregate productivity mitigate a little bit when allowing for firm entry and exit. The five targeted moments are also computed for the new model. Except for the external finance ratio, all moments are quantitatively compatible to their data targets reported in Table

[10] Actually, I find that the aggregate productivity measure in the frictionless case is independent of the level of p^c.

2.3. Even though a more rigorous analysis would require a re-calibration of the new model, but since the model moments are not far from their data targets, it may be safely concluded that adding firm entry and exit does not change the magnitude of the impact of costly external finance on aggregate productivity significantly.

2.6 Conclusion

This chapter incorporates an external finance cost function into a firm's capital accumulation problem with idiosyncratic productivity shocks to study the quantitative impact of costly external finance on aggregate productivity. The results show that presence of costly external finance leads to a reallocation of output shares from high productivity firms to low productivity firms so that the output-weighted aggregate productivity is 1 percent smaller than it would be if external finance is not costly. Such loss is quantitatively significant considering the less than 2 percent average annual TFP growth rate of US manufacturing. Adding firm entry and exit to the model does not change this result significantly.

Costly external finance also has an adverse effect on aggregate output. The result shows that this effect is mainly through aggregate capital accumulation rather than reallocation across heterogenous firms. In this sense, the traditional neoclassical growth model that abstracts from firm heterogeneity may not be a bad framework in studying the long run consequences of financial market frictions on aggregate output growth. However, the result hinges on a homogeneous external finance cost function and a stationary analysis. There is empirical evidence suggesting that firms differ in external finance costs along a lot of dimensions, such as firm size, age, credit worthiness, and etc. It's not clear whether such heterogeneity matters a lot for the problem outlined here. In addition, the model abstracts from aggregate fluctuations. There are beliefs that aggregate consequences of financial frictions are more severe under some circumstances of the business cycle through the interaction among heterogeneous firms. These questions are open for future research.

Chapter 3

Business Cycle Asymmetries via Occasionally Binding International Borrowing Constraints

This chapter is a co-work with Dean Corbae and Scott Dressler.

3.1 Introduction

The behavior of macroeconomic variables over phases of business cycle has long been an object of interest to economists. A critical aspect of this is the symmetry or asymmetry of business cycles. Asymmetries of business cycles were noted by early economists. "There is, however, another characteristic of what we call the trade cycle which our explanation must cover; namely, the phenomenon of the crisis–the fact that the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when an upward is substituted for a downward tendency." So wrote Keynes in 1936. Such asymmetry in the transitions of business cycles was later defined as *steepness*. In other words, steepness is defined as the feature that the arrival of a recession is prompt while the recovery from a recession appears protracted. It is also known as growth-rate asymmetry or asymmetry in transitions or at turning points of business cycles. Economists also identified other types of asymmetry in business cycles. For instance, Sichel (1993) defines the feature that troughs are further below trend than peaks are above trend as *deepness*.

These asymmetries have been examined extensively since 1980s. Empirical research of Neftci (1984), Hamilton (1989), Falk (1986), Sichel (1993), Acemoglu and Scott (1997), and etc. has confirmed that a variety of U.S. macroeconomic variables such as GDP, GNP, industrial production, and unemployment rates are to some degree characterized by both types of asymmetry, although the reverse is found in Delong and Summers (1986). Razzak (2001), Belaire-Franch and Contreras (2003) provide international evidence of asymmetries in the cyclical fluctuations of real GDP's despite the degree of deepness and steepness differ across countries. Section 3.1 documents the deepness and steepness measures for real per capita GDP of a variety of countries.

Although asymmetry was identified as an important stylized fact of business cycles, theories of business cycle asymmetry are far from developed. Standard equilibrium business cycle models, such as those studied in Kydland and Prescott (1982), Hansen (1985) and others, are closely approximated by a linear dynamic system. So the business cycles in these models are symmetric fluctuations around trend, i.e. the response of the economy to a positive shock is the mirror image of the response to an equal sized negative shock. Existing literature provides few explanations for business cycle asymmetry. A previous explanation relies upon the existence of increasing returns, either directly in the production process, as in Acemoglu and Scott (1997), or in adjustment costs, as in Caballero and Engel (1991). Hansen and Prescott (2000) use occasionally binding capacity constraints to prevent booms from being as large a deviation from trend as recessions. Their model generates deepness asymmetry. Most explanations for steepness asymmetry rely on learning and information aggregation. Either agents response differently upon receiving good and bad news, as in Chalkley and Lee (1998), or information aggregation differs at the turning points of business cycles, as in Nieuwerburgh and Veldkamp (2004).

This exercise investigates whether credit constraints in international borrowing play a role in accounting for business cycle asymmetries in small open economies. Economists have noted the interactions between credit market and business cycles for a long time. Correspondingly, there is a large literature on the business cycle implications of credit market imperfections, such as Bernanke and Gertler (1989), Gertler (1992), Lamont (1995), Calstrom and Fuerst (1997), Kiyotaki and Moore (1997), Kocherlakota (2000), Cordoba and Ripoll (2002) and a lot of others. Most of this literature, however, has focused on how credit market imperfections help amplify and propagate aggregate shocks. In occasions where imperfections take the form of credit constraints, it was generally assumed that the credit constraints are always binding, at least around the steady state such that the business cycles of these models are symmetric fluctuations around trend. Kocherlakota (2000) is the first one that proposed the idea that credit constraints may play a role in generating business cycle asymmetries. But this hypothesis has not been tested and quantitatively evaluated.

This exercise is the first step toward this direction. We use a simple real business cycle model of small open economies to illustrate how business cycle asymmetries can be generated through constraints in international borrowing. Recently, there is an emerging literature that appeals to credit frictions in international capital market to explain financial or economic crises in developing economies. For instance, Atkeson and Rios-Rull (1996) develops a model in which a country faces a balance of payments crises if constraints on its international borrowing bind. See Arellano and Mendoza (2002) for a survey of this literature. In our model, international borrowing is constrained by a fraction of the capital stock of the country. Such borrowing constraint can be justified by sovereign risk, enforcement or information frictions that exist in international capital market. It binds occasionally. Depending on the state of the economy, the borrowing constraint may bind when the economy transits from a recession to an expansion. If this happens, upward movements in investment, capital stock and output are gradual and protracted. However, the borrowing constraint never binds during the transition from an expansion to a recession. So when bad shock occurs, capital stock can be quickly downsized by paying off previously accumulated debt or lending to the world. As a result, on average downward movements are sharper and quicker than upward movements. Degree of asymmetry depends on how often the international borrowing constraint is binding.

The model is calibrated to the Canadian economy. It generates a steepness of -0.083 for output, which is 16 percent of the steepness in the real per capita Canadian GDP. Compared to the data, the model generates lower asymmetry for output and consumption, much higher deepness asymmetry for investment, and high steepness asymmetry for capital stock. These results suggest that occasionally binding international borrowing constraints have the potential to account for business cycle asymmetries in small open economies. Adding labor decisions to the model may improve the model's prediction for asymmetry in output, since then the adjustments of output upon receiving a new shock is no longer constrained by the pre-determined capital. This is our next exercise. The results also suggest that international borrowing constraints play a role in accounting for the positive correlation between domestic investment and domestic saving, which is one of the most important stylized facts for small open economies.

The rest of the chapter is arranged as follows. Section 3.2 briefly reviews some empirical evidence of asymmetries in cyclical international GDP. Section 3.3 describes the model and section 3.4 describes the model's predictions. Section 3.5 concludes.

3.2 Business Cycle Asymmetry: International Evidence

According to Sichel (1993), if a time series exhibits deepness, then it should exhibit negative skewness relative to mean or trend; that is, it should have fewer observations below its mean or trend than above, but the average deviation of observations below the mean or trend should exceed the average

Table 3.1: Asymmetry in real per capital real GDP

	Deepness	Steepness
U.S.	-0.40	-0.45
Canada	-0.44	-0.52
Australia	-0.72	-0.51
Argentina	-0.28	-0.21
Chile	-0.13	-1.56
Mexico	-0.21	-0.44
Mylasia	-0.59	-0.91
S. Africa	-0.52	-0.17
Singapore	0.70	-1.27
Turkey	0.046	0.11
Taiwan	0.18	0.052
Brazil	0.01	-0.04

deviations above. If a time series exhibits steepness, then its first differences should exhibit negative skewness. That is, the sharp decreases in the series should be larger, but less frequent, than the more moderate increases in the series. So, deepness in a time series can be generally measured by the skewness of the detrended series, and steepness can be measured by the skewness of the first-differenced series, even though the empirical literature has developed more complicated parametric and nonparametric methods to test business cycle asymmetries. Significant negative skewness indicates asymmetry.

Table 3.1 above documents the measures for deepness and steepness in cyclical real per capita GDP for a variety of economies. Among them, asymmetry measures for U.S., Canada and Australia are based on quarterly per capita GDP data, while measures for other countries are based on annual per capita GDP due to the unavailability of quarterly data. Notice that for many countries, real per capita GDP exhibits both deepness and steepness asymmetry despite that degree of asymmetries differs a lot across countries, suggesting that business cycle asymmetry is an international phenomenon. But there are also countries whose real GDP does not exhibit asymmetry, such as Turkey and Brazil. This result may be due to the annual frequency of the data. Differences in the degree of business cycle asymmetry across economies require further investigation both empirically and theoretically.

3.3 Structure of the Model

We consider a small open economy version of a real business cycle model. The economy is populated by identical, infinitely-lived individuals who value consumption streams $C = \{C_t\}_{t=0}^{\infty}$ according to

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \tag{3.1}$$

where β denotes the discount factor.

Individuals have access to a production technology which uses capital to produce a composite commodity that can be consumed, invested or internationally traded. The production function is given by

$$Y_t = z_t f(K_t), \tag{3.2}$$

where Y_t denotes gross domestic product (GDP), f is continuous, strictly increasing and strictly concave, z_t is an exogenous shock whose law of motion is governed by the following equation

$$log(z_{t+1}) = \rho log(z_t) + \xi_{t+1}, \tag{3.3}$$

where ξ_t is normally distributed with zero mean and standard deviation σ. z_t measures changes in domestic productivity. Since GDP is a tradable commodity, z_t also incorporates the effects of disturbances in the terms of trade, which are viewed as important real shocks by several economists. As in Mendoza (1991), the model ignores the existence of nontraded goods.

Individuals are endowed with initial capital stock $K_0 > 0$, and capital depreciates at rate δ, where $0 \leq \delta \leq 1$. So the law of motion for domestic capital is given by

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{3.4}$$

The aggregate resource constraint of the economy dictates that the sum of consumption, investment and the balance of trade (TB_t) cannot exceed gross domestic product.

$$C_t + I_t + TB_t \leq z_t f(K_t), \tag{3.5}$$

Individuals can also borrow or lend one-period assets in the world capital market at interest rate r, where $r < \frac{1}{\beta} - 1$, to finance trade imbalances. The holdings of these assets evolve according to

$$B_{t+1} - B_t(1+r) + TB_t = 0, \tag{3.6}$$

where B_t is what the country borrows at period $t-1$ and has to pay off in period t, and B_{t+1} is the country's new credit at period t. The initial debt B_0

is assumed to be zero. We assume that international borrowing is constrained and the borrowing constraint takes the form

$$(1+r)B_{t+1} \leq \theta(1-\delta)K_{t+1},$$

or equivalently,

$$B_{t+1} \leq \theta \frac{1-\delta}{1+r} K_{t+1} \equiv \theta_1 K_{t+1}. \tag{3.7}$$

In other words, the country's international borrowing (with interests payment) cannot exceed a fraction of its capital stock. Here we assume that foreign lenders have limited sanction that they could impose on the country if it were to default. They could seize only a fraction of the country's available capital stock such that their lending to the country is constrained by this value. Note that the country's international debt is fully secured by its capital stock. In Kiyotaki and Moore (1997), Kocherlakota (2000), Campbell and Hercowitz (2004) and others, debt is assumed to be fully secured by collateralized asset, such as land or other durable goods. This is a common form of credit constraints. In general, international borrowing constraints can be justified by sovereign risk, information asymmetry, lack of enforcement, or other imperfections in international capital market.

Substituting TB_t in the budget constraint (3.5) with (3.6) gives

$$C_t + I_t + (1+r)B_t \leq z_t f(K_t) + B_{t+1}, \tag{3.8}$$

So the problem is to choose C_t, K_{t+1}, B_{t+1} to maximize (3.1), subject to (3.8), (3.7), (3.3), $K_0 > 0$ and $B_0 = 0$.

This problem has a recursive formulation, taking K, B and z as state variables. It doesn't possess an analytical solution and has to be solved numerically. Despite z can be approximated by a discrete Markov process, presence of two continuous state variables K and B entails difficulty in the computation. However, by re-defining variables, we can reduce it to a simpler problem. Let

$$W_{t+1} = \theta_1 K_{t+1} - B_{t+1}.$$

Then the credit constraint (3.7) is reduced to

$$W_{t+1} \geq 0.$$

The budget constraint (3.8) can be rewritten as

$$C_t + (1-\theta_1)K_{t+1} + W_{t+1} = z_t f(K_t) + (1-\delta)(1-\theta)K_t + (1+r)W_t.$$

Define

$$x_t = z_t f(K_t) + (1-\delta)(1-\theta)K_t + (1+R)W_t,$$

then a recursive formulation of the problem above is

$$\begin{aligned} V(x,z) \quad &= \max\nolimits_{K'\geq 0, W'\geq 0} \; u\big(x - (1-\theta_1)K' - W'\big) + \beta E_{z'|z} V(x', z') \\ \text{s.t.} \quad & \quad x' = z' f(K') + (1-\delta)(1-\theta)K' + (1+r)W' \end{aligned}$$

The Euler equations for the problem are given by

$$\begin{aligned} &-(1-\theta_1)u'(x-(1-\theta_1)K'-W') + \beta E_{z'|z}\Big\{[z' f'(K') + (1-\delta)(1-\theta)] \\ &\qquad\qquad\qquad\qquad u'(x'-(1-\theta_1)K''-W'')\Big\} = 0 \\ &-u'(x-(1-\theta_1)K'-W') + \beta(1+r)E_{z'|z}u'(x'-(1-\theta_1)K''-W'') + \mu = 0 \\ &\mu \geq 0, \; \mu W' = 0, \end{aligned}$$

where μ is the Lagrange multiplier on the constraint $W' \geq 0$. Recall that f is assumed to take a form that ensures an interior solution for K'.

Since $\beta(1+r) < 1$, it follows from the Euler equations that the international borrowing constraint is binding in a steady state of the non-stochastic problem ($z_t \equiv 1$). The assumption $r < \frac{1}{\beta} - 1$ also ensures the stability of the system. It prevents the economy from lending (not constrained) too much to accumulate assets in an explosive way. However, the credit constraint is not always binding, which will be clear in the discussions below. This introduces nonlinearity to the problem so that the standard linearization methodology is not applicable. For this simple model, we have a well defined value function, so standard dynamic programming approach still applies. In next section, we calibrate the model to the Canadian economy, and describe the business cycle properties implied by the model.

3.4 Findings

3.4.1 Calibration

To solve the model, values must be chosen for the parameters that describe preferences, technology and stochastic shocks. The model is parameterized so as to make it roughly consistent with some of the structure of the Canadian economy, which is typically viewed as a small open economy because of the high degree of integration of its financial markets with those of the U.S.. The data we used correspond to quarterly observations for the period 1971:Q1-2004:Q4, expressed in per capita terms by dividing by the population older than 14 years, transformed into logarithms, and detrended by the Hodrick-Prescott filter [1]. Some statistical moments are reported in the first panel of Table 3.3.

We set a period as a quarter. Let $f(K) = K^{\alpha}$. Let the world interest rate $r = 0.065/4$, which is the average real interest rate of the U.S. in the past century. Capital depreciation rate δ is set to 0.1/4. β is assumed to be smaller than $1/(1+r) = 0.984$. But if β is too small, it would introduce asymmetry to the model because individuals would be too willing to borrow and not willing to lend. To avoid this distortion we set $\beta = 0.9837$. The productivity shock z_t is approximated by a 2-state Markov chain following Tauchen (1986), $z \in \{z_g, z_b\}$, with transition probabilities $P_{z'|z}$. The return to scale factor α is determined such that the investment to output ratio is about 16.81%, as observed in the Canadian data. The variation and persistence parameters of the shock, σ and ρ are chosen to match the standard deviation and autocorrelation of the Canadian GDP, which are approximately 1.56% and 0.78 respectively. Finally, the credit constraint parameter θ is determined such that the ratio of trade balance to output is about 1.45%, as observed in the Canadian data. Values of the parameters and matched moments are listed in Table 3.2 below. The dynamic programming problem is numerically solved by value function iteration.

[1] Data source: CANSIM Data Retrieval, Statistics Canada.

Table 3.2: Parameter Values and Matched Moments

Parameter	Value	Moments	Model	Data
r	0.0163			
β	0.9837			
δ	0.025			
α	0.28	Investment to output ratio	16.8%	16.81%
σ	0.065	Standard deviation of output	1.80%	1.56%
ρ	0.98	Persistence of output	0.74	0.78
θ	0.18	Trade balance to output ratio	1.55%	1.45%

3.4.2 Business Cycle Moments

First, we document the key business cycle moments for the model economy, which are reported in the second panel of Table 3.3. These statistics are based on 250 simulations of 136 periods[2]. For each simulation, we compute the cyclical properties of the artificial data set by applying the Hodrick-Prescott filter.

The means of various summary statistics over the 250 simulations are reported. Compared to the data, the most striking moments of the model economy are the extremely high volatility of investment and the low autocorrelation of investment and contemporaneous correlation between investment and output. This result is due to the quick adjustments of investment, especially the downward adjustments, with a fixed interest rate. Other business cycle moments are not far from their data counterparts. In particular,

[2]We compute simulations of 136 periods since this is the number of quarters in the Canadian data sample.

Table 3.3: Statistical Moments: Canadian Data and the Model Economy

$(x =)$	σ_x	σ_x/σ_y	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
Canadian data (71:Q1-04:Q4)				
GDP	1.56	1	0.78	1
Investment	8.37	5.37	0.81	0.75
Saving	6.35	4.07	0.82	0.86
Consumption	0.91	0.58	0.57	0.76
$Corr$(Saving,Investment)		0.74		
Model economy				
GDP	1.80	1	0.74	1
Investment	45.72	25.4	-0.075	0.31
Saving	7.93	4.40	0.75	0.99
Consumption	0.54	0.3	0.74	0.91
Capital	1.42	0.79	0.69	0.68
$Corr$(Saving,Investment)		0.3 (0.072)		

the model generates high autocorrelation of output, saving, consumption and capital stock, and high contemporaneous correlation with output of saving, consumption and capital stock. As documented in Backus and Kehoe (1989), one significant stylized fact of modern open economies is that domestic savings and domestic investment are positively correlated. For the Canadian economy, this correlation is 0.74. The model also generates a positive correlation of domestic saving and investment, which is 0.3 with a standard error of 0.072. This result suggests that international borrowing constraints may play a role in accounting for this positive correlation.

3.4.3 Business Cycle Asymmetries

We then simulate the model to examine whether it provides an mechanism to generate business cycle asymmetry. If the borrowing constraint is never binding or always binding, the model would generate symmetric business cycles, as standard business cycle models do. However, the borrowing constraint in the model is occasionally binding. Whether it binds or not depends on the state of the economy, i.e., the state of capital stock, international debt, and the productivity shock. We do two experiments to illustrate why the credit constraint is occasionally binding and how occasionally binding constraint are capable of generating business cycle asymmetries.

We consider two states of the economy. Starting from each state, we simulate 20 bad shocks, then 20 good shocks to investigate how the economy respond differently upon receiving bad and good shocks. In experiment 1, the economy is originally in a good productivity state with a capital stock of 14.72 and a debt level of 2.25 [3]. In experiment 2, the economy starts from same capital stock and productivity, but the debt level is zero. Figure 3.1 and 3.2 plot the sample paths of output, capital stock, investment, consumption, borrowing, and credit line minus borrowing (W_t) for experiment 1 and 2 respectively.

Figure 3.1 shows that the downward movements in output, capital stock and investment during the transition from good states to bad states (expansion to recession) are sudden and big, while the upward movements during the transition from bad states to good states (recession to expansion) are gradual and moderate. This feature is less pronounced for consumption due to the consumption smoothing nature of the model. The last two plots of Figure 3.1 answered why the transitional dynamics exhibit such asymmetry. Note that the credit constraint is not binding ($W > 0$) during the transition from good states to bad states, while it binds for some periods during the transition from bad states to good states. When the economy receives bad shocks, individuals reduce investment sharply by paying off some debt accumulated earlier, such that there is a sharp decline in the debt level. When the economy receives good shocks, since borrowing is constrained, investment has to increase gradually. While in Figure 3.2, the downward movements and upward movements in output, investment and capital stock appear quite symmetric. This is due to a

[3] 14.72 is the highest level of capital stock that the economy can achieve. If the economy stays in a good state long enough, this capital level would be achieved. 2.25 is a pretty high debt level in the sense that it's close to the highest credit limit ($\theta_1 \cdot 14.72$).

non-binding borrowing constraint during both transitions, as shown in the last plot of Figure 3.2. The two experiments suggest that the credit constraint is occasionally binding. It may bind during the transition from a recession to an expansion, but not vice versa. As a result, on average the upward movements in output, investment and capital are sharper and quicker than downward movements. The degree of asymmetry depends on how often the economy behave like what is shown in Figure 3.1.

We have seen that the model has the mechanism to generate business cycle asymmetry. To quantify the asymmetry of the model economy, for each simulation, we compute the skewness of percentage deviations from Hodrick-Prescott trend, average deviations from below and above the trend, and skewness of first-differenced series. Means of these statistics across the 250 simulations are reported in Table 3.4. Measures of steepness asymmetry for output, capital, investment and consumption are -0.083, -2.35, -0.1 and 0.34 respectively. And measures of deepness asymmetry are -0.043, -0.015, -2.76 and -0.12 respectively. Besides, for all variables, average deviations below trend are bigger than average deviations above trend. These results indicate that the model generates both types of asymmetry for output, investment and capital stock and some deepness for consumption. Compared to the data, the model generates much higher deepness for investment, a high degree of steepness for capital stock, and lower asymmetry for output. In particular, The steepness measure for output is 16 percent of its data counterpart, suggesting that international borrowing constraints can account for 16 percent of observed

Table 3.4: Business Cycle Asymmetry: Canadian Data and the Model Economy

	GDP	K	I	C
Canadian data(71:1-04:4)				
Deepness	-0.44		-0.75	-0.07
Avg. % deviations above	1.18		5.94	0.66
Avg. % deviations below	-1.29		-5.94	-0.72
Steepness	-0.52		-0.63	-0.68
Model economy				
Deepness	-0.043	-0.015	-2.76	-0.12
Avg. % deviations above	1.42	1.02	12.89	0.401
Avg. % deviations below	-1.47	-1.13	-21.16	-0.403
Steepness	-0.083	-2.35	-0.1	0.34

steepness asymmetry in Canadian real GDP. The lower degree of asymmetry for output may be remedied by including labor choice in the model, since then the adjustments of output upon receiving a new shock is no longer constrained by the pre-determined capital.

3.5 Conclusion

The role of credit market imperfections in complicating with business cycles have been an intriguing and challenging research issue to economists. With a simple real business cycle model of a small open economy, this exercise shows that international borrowing constraints can help explain observed business cycle asymmetries in small open economies. The occasionally binding borrowing constraints produce non-linear dynamics during the transitions of business cycles. Quantitatively, the model seems to generate lower asymmetry in GDP than exhibited in most small open economies. Our next exercise is to include labor choice and non-tradables to see whether the quantitative implications can be improved. Occasionally binding credit constraints may also play a role in accounting for business cycle asymmetries in large economies, such as the U.S. economy. So a further extension is to extend the current model to a general equilibrium framework of a closed economy, which may require new computation approaches to deal with the nonlinearity introduced by occasionally binding constraints.

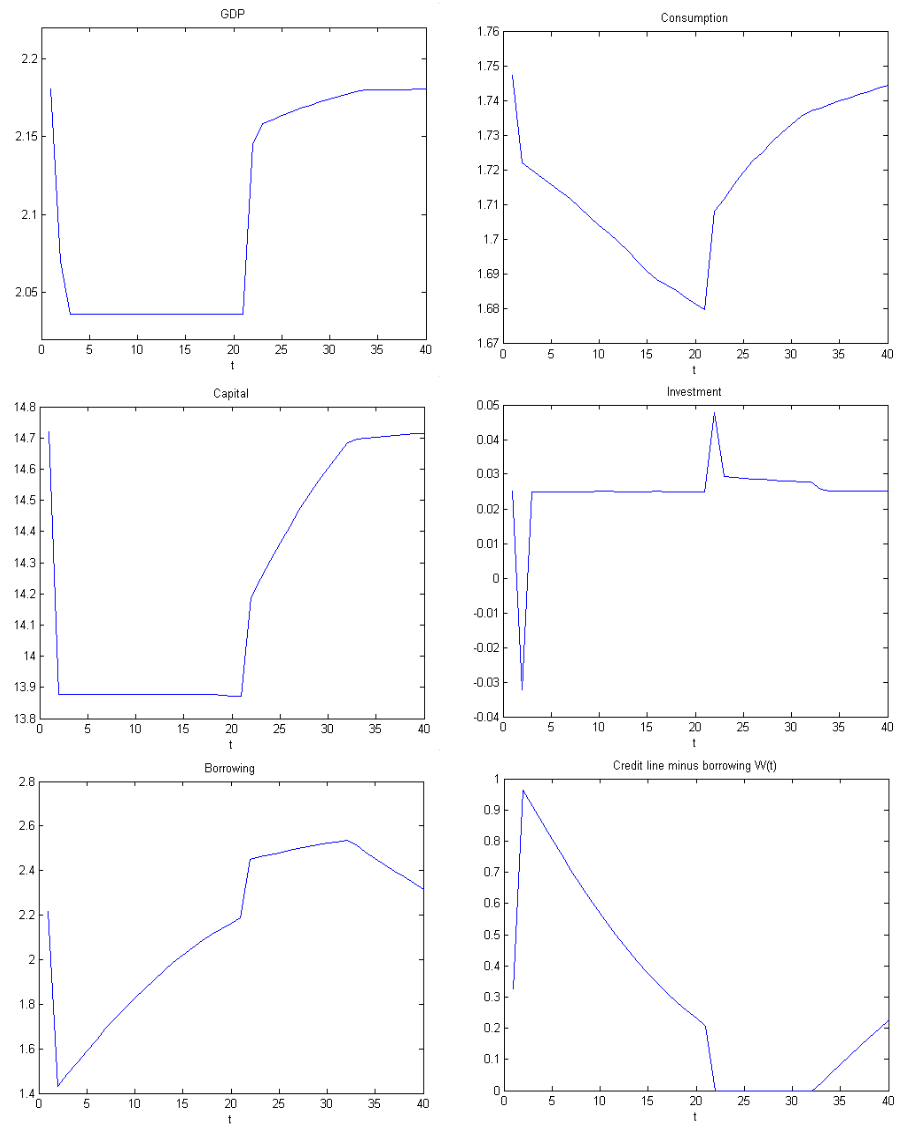

Figure 3.1: Transitional dynamics of the Model Economy (Experiment 1)

Notes: $K_0 = 14.72, z_0 = z_g, B_0 = 2.25$.

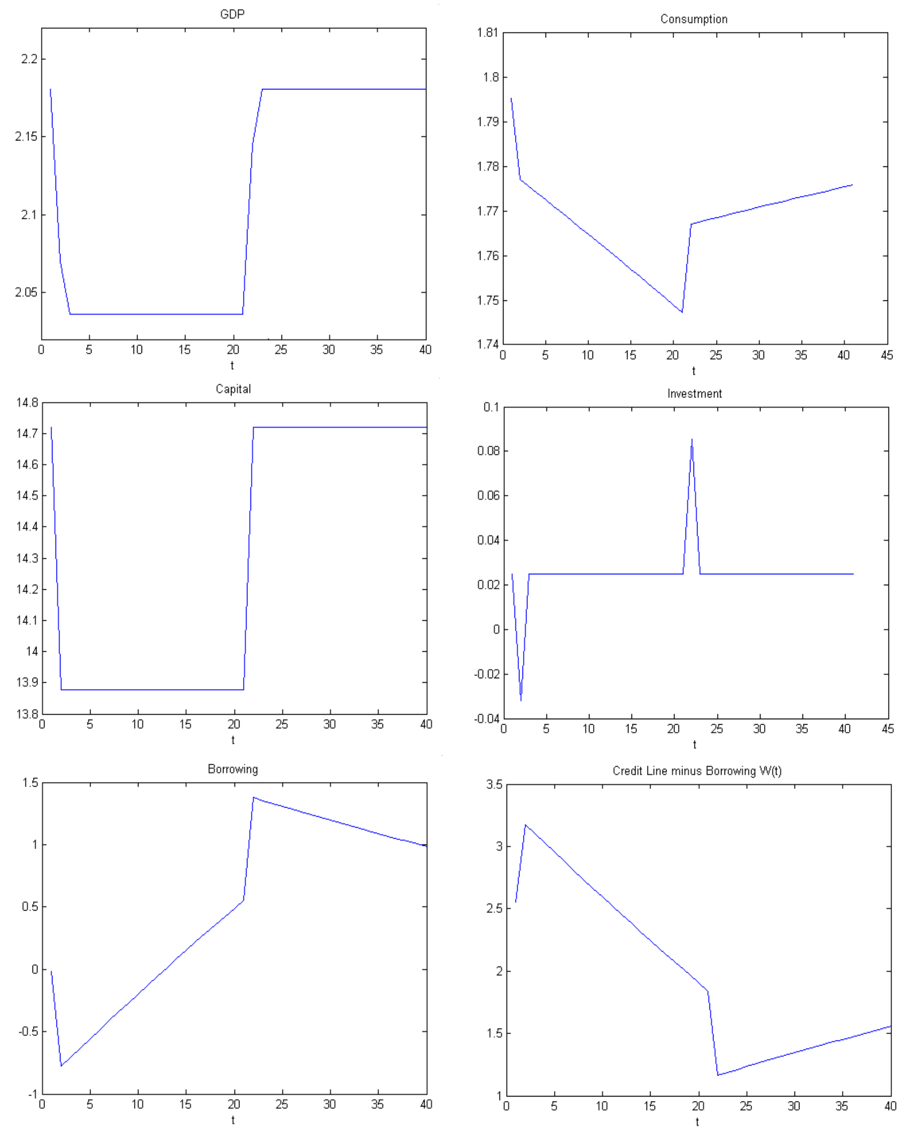

Figure 3.2: Transitional dynamics of the Model Economy (Experiment 2)

Notes: $K_0 = 14.72, z_0 = z_g, B_0 = 0$.

Appendices

Appendix A

Appendix to Chapter 1

A.1 Analytical Proofs

A.1.1 Proof of Lemma 1.3.1

Consider a sequence of candidates for $\bar{V}$, $\{\bar{V}_0, \bar{V}_1, \bar{V}_2 \ldots\}$. Let $\bar{V}_0 = \tilde{V}(p)$. Then $W_{[0,\bar{V}_0]}(\bar{V}_0; p) = \tilde{W}(p)$, so $B_{[0,\bar{V}_0]}(\bar{V}_0; p) < 0$. Since $B_{[0,\bar{V}_0]}(0; p) = W_{[0,\bar{V}_0]}(0; p) - 0 = S > 0$, by continuity of $B_{[0,\bar{V}_0]}$, there exists $\bar{V}_1 \in (0, \bar{V}_0)$, such that $B_{[0,\bar{V}_0]}(\bar{V}_1; p) = 0$. Since $\bar{V}_1 < \bar{V}_0$, $W_{[0,\bar{V}_1]}(\bar{V}_1; p) \leq W_{[0,\bar{V}_0]}(\bar{V}_1; p)$. So $B_{[0,\bar{V}_1]}(\bar{V}_1; p) \leq B_{[0,\bar{V}_0]}(\bar{V}_1; p) = 0$. Again continuity implies that $\exists\, \bar{V}_2 \in (0, \bar{V}_1]$, such that $B_{[0,\bar{V}_1]}(\bar{V}_2; p) = 0$. Continuing this process defines a non-increasing sequence $\{\bar{V}_0, \bar{V}_1, \bar{V}_2 \ldots\}$, with $0 < \bar{V}_i \leq \tilde{V}$, $B_{[0,\bar{V}_i]}(\bar{V}_{i+1}) = 0$. It converges to a unique limit, call it $\bar{V}$. Berge's Theorem of the Maximum guarantees that the value function W and hence B moves continuously with its constraint set parameter. So $B_{[0,\bar{V}]}(\bar{V}; p) = 0$. By construction, for any $V > \bar{V}$, $B_{[0,V]}(V; p) < 0$. So any $V > \bar{V}$ cannot be feasibly promised. Finally, since $W_{[0,\bar{V}]}(V; p)$ is concave and increasing in V, and $W_{[0,\bar{V}]}(0; p) = S > 0$, we have $W_{[0,\bar{V}]}(V; p) > V$ for all $V < \bar{V}$, i.e. $B_{[0,\bar{V}]}(V; p) > 0$ for $V < \bar{V}$. In other words, any $V < \bar{V}$ can be feasibly promised. So $\bar{V}$ is the upper bound to feasible value entitlements.

PROOF OF LEMMA 1.3.2: First note that for $V = 0$ [1], $V^H(V) = 0 < \bar{V}$. For $V = \bar{V}$, we can show that $V^H(V) = \bar{V}$ by contradiction. Consider (P_2) with $V = \bar{V}$. Suppose its solution is (l, τ, V^H, V^L) with $V^H < \bar{V}$. If $\tau = pf(l)$, then $\bar{V} = \beta(\pi V^H + (1-\pi)V^L) < \beta\bar{V}$. If $\tau < pf(l)$, increasing τ and V^H can make the objective strictly higher. So $V^H(\bar{V}) = \bar{V}$. By continuity of the policy function $V^H(V)$, there exists $0 < V_1 \leq \bar{V}$ such that $V^H(V_1) = \bar{V}$, $V^H(V) < \bar{V}$ for $V < V_1$. By (1.10), $V^L(V) \leq V^H(V)$ for all $V \in [0, \bar{V}]$. So $V^L(V) < \bar{V}$ for $V < V_1$.

A.1.2 Proof of Proposition 1.3.3

(i) If $V = 0$, this is obvious, since $l(0) = 0$, $\tau(0) = 0$. Consider an arbitrary $0 < V < V_1$, suppose the solution to (P_2) is (l, τ, V^H, V^L). From Lemma 1.3.2, we know $V^H < \bar{V}$. If $\tau < pf(l)$, since W is strictly increasing, the objective of (P_2) can be strictly increased by increasing τ and V^H in a way that keeps all constraints hold. So $\tau(V) = pf(l(V))$ for any $V < V_1$. By continuity of $\tau(V)$ and $l(V)$, the equality also holds for $V = V_1$. So (1.11) is binding for $V \in [0, V_1]$.

(ii) This is obviously true for $V = 0$. Suppose there exists $V_0 \in (0, V_1]$ such that $\tau(V_0) < \beta(V^H(V_0) - V^L(V_0))$. Since $\tau(V_0) = pf(l(V_0)) > 0$, $V^H(V_0) > V^L(V_0)$. Now consider two cases.

Case 1. $V^H(V_0) > V_r$. Since $\tau(V_0) < \beta[V^H(V_0) - V^L(V_0)]$, there exists

[1]For simplicity, throughout the proofs, I use V to denote either the value entitlement to an entrepreneur at the beginning of a period or the continuation value entitlement (V_c).

$\xi > 0$, such that $\tau(V_0) \le \beta\left[(V^H(V_0) - (1-\pi)\xi) - (V^L(V_0) + \pi\xi)\right]$. Consider a choice vector $(l(V_0), \tau(V_0), V^H(V_0) - (1-\pi)\xi, V^L(V_0) + \pi\xi)$. It's easy to see that it satisfies all the constraints of (P_2). Since $W(V)$ is strictly increasing, linear for $V \in [0, V_r]$ and strictly concave for $V > V_r$, by Jensen's inequality,

$$\pi W(V^H(V_0) - (1-\pi)\xi) + (1-\pi)W(V^L(V_0) + \pi\xi) > \pi W(V^H(V_0)) + (1-\pi)W(V^L(V_0)).$$

This contradicts that $(l(V_0), \tau(V_0), V^H(V_0), V^L(V_0))$ is the optimal solution.

Case 2. $V^H(V_0) \le V_r$. I first show that $l(V_0) < l^\star(p)$. By (i), $\tau(V_0) = pf(l(V_0))$, so $\pi V^H(V_0) + (1-\pi)V^L(V_0) = \frac{V_0}{\beta}$. Since $V^L(V_0) < V^H(V_0) \le V_r$, $\frac{V_0}{\beta} < V_r$ and hence $V_0 < V_r$. Since $W(V)$ is linear for $V \le V_r$, $\pi W(V^H(V_0)) + (1-\pi)W(V^L(V_0)) = W(\frac{V_0}{\beta})$. So $\hat{W}(V_0) = \pi pf(l(V_0)) - l(V_0) + \beta[\pi W(V^H(V_0)) + (1-\pi)W(V^L(V_0)] = \pi pf(l(V_0)) - l(V_0) + \beta W(\frac{V_0}{\beta})$. Since $V_0 < V_r$, $\hat{W}(V_0) < W(V_0)$. By Proposition 1.3.1, for $V \le V_r$, $W(V) = S + \frac{W(V_r) - S}{V_r}V$. So $\pi pf(l(V_0)) - l(V_0) + \beta\left[S + \frac{W(V_r)-S}{V_r}\frac{V_0}{\beta}\right] < S + \frac{W(V_r)-S}{V_r}V_0$, which implies $S > \frac{\pi pf(l(V_0)) - l(V_0)}{1-\beta}$. Since $S < I - M < I = \frac{\pi pf(l^\star(p)) - l^\star(p)}{1-\beta}$, $l(V_0) < l^\star(p)$. With this result and $\tau(V_0) < \beta[V^H(V_0) - V^L(V_0)]$, we can find $\xi, \xi' > 0$, such that $\tau + \xi \le \beta[V^H(V_0) - V^L(V_0)]$, $\tau + \xi = pf(l(V_0) + \xi')$, and $l(V_0) + \xi' \le l^\star(p)$. It's easy to see that the choice vector $(l(V_0) + \xi', \tau(V_0) + \xi, V^H(V_0), V^L(V_0))$ also satisfies (1.9). But it yields a higher value for the objective of (P_2), because $\pi pf(l) - l$ is strictly increasing for $l < l^\star(p)$. This is a contradiction.

In both cases, we get contradictions. So the incentive compatibility constraint is binding for $V \in [0, V_1]$.

(iii) First $l(0) = 0 < l^\star(p)$. For any $V \in (0, V_1]$, also consider the two cases: $V^H(V) > V_r$ and $V^H(V) \leq V_r$. For the second case, $l(V) < l^\star(p)$ is already proved in (ii). Now consider the first case. Since both (1.10) and (1.11) are binding, V^H and V^L can be solved in terms of V and l, $V^H = \frac{V+(1-\pi)pf(l)}{\beta}$, $V^L = \frac{V-\pi pf(l)}{\beta}$. Then (P_2) can be reduced to

$$(P_2')\begin{cases} \hat{W}(V) & = \max_{l\geq 0}\{\pi pf(l) - l + \beta\{\pi W(V^H) + (1-\pi)W(V^L)\}\} \\ s.t. & V^H = \frac{V+(1-\pi)pf(l)}{\beta} \\ & V^L = \frac{V-\pi pf(l)}{\beta} \geq 0 \end{cases}$$

Since $V > 0$, $l(V) > 0$ and it satisfies the first order condition

$$\pi pf'(l(V)) \geq \frac{1}{1-(1-\pi)\left[W'(V^L(V)) - W'(V^H(V))\right]}, \text{with equality if } V^L(V) > 0.$$

Note that by (i) and (ii), $\beta[V^H(V) - V^L(V)] = \tau(V) = pf(l(V)) > 0$, so $V^L(V) < V^H(V)$. Since W is linear for $V \leq V_r$ and strictly concave for $V > V_r$, $W'(V^L) > W'(V^H)$. So $\pi pf'(l(V)) > 1$, while $\pi pf'(l^\star(p)) = 1$. By strict concavity of f, $l(V) < l^\star(p)$.

(iv). First, since $l(V) > 0$ for $V > 0$, $V^H(V) = \frac{V+(1-\pi)pf(l(V))}{\beta} > V$. Now consider problem (P_2) with $V \in (0, V_1]$. Since both (1.10) and (1.11) are binding, (P_2) can be rewritten as

$$(P_2")\begin{cases} \hat{W}(V) & = \max_{V^H\geq 0, V^L\geq 0}\{\pi\beta(V^H - V^L) - f^{-1}(\frac{\beta(V^H-V^L)}{p}) + \\ & \qquad \beta\{\pi W(V^H) + (1-\pi)W(V^L)\}\} \\ s.t. & V = \beta\{\pi V^H + (1-\pi)V^L\} \end{cases}$$

Since $V^H(V) > 0$, $V^L(V) \geq 0$, $V^H(V)$ and $V^L(V)$ satisfies the first order conditions

$$W'(V^H) = \lambda - \left[1 - \frac{f^{-1'}(\frac{\beta(V^H-V^L)}{p})}{\pi p}\right],$$

$$W^{'}(V^L) \leq \lambda + \frac{\pi}{1-\pi}\left[1 - \frac{f^{-1^{'}}(\frac{\beta(V^H - V^L)}{p})}{\pi p}\right], \text{with equality if} \quad V^L > 0,$$

where $\lambda > 0$ is the Lagrangian multiplier on the constraint. If $V^L(V) = 0$, then $V^L(V) < V$. If $V^L(V) > 0$, then we have equality in the first order condition with respect to V^L. Since $l(V) \equiv f^{-1}(\frac{\beta(V^H - V^L)}{p}) < l^\star(p)$, and f^{-1} is convex, $f^{-1^{'}}(\frac{\beta(V^H - V^L)}{p}) < f^{-1^{'}}(l^\star(p)) = \pi p$. So $W^{'}(V^L(V)) > \lambda$. By the Envelope theorem, $\lambda = \hat{W}^{'}(V) \geq W^{'}(V)$. So $W^{'}(V^L(V)) > W^{'}(V)$, and by concavity of W, $V^L(V) < V$. This proves $V^L(V) < V < V^H(V)$ for $0 < V \leq V_1$.

Now suppose there exist $V, V' \in [0, V_1], V < V'$, such that $V^H(V) \geq V^H(V')$. Then $V^L(V') > V^L(V)$ by the constraint of (P_2"). So $\beta(V^H(V') - V^L(V')) < \beta(V^H(V) - V^L(V))$. Since f^{-1} is strictly convex,

$$f^{-1^{'}}\left[\frac{\beta(V^H(V') - V^L(V'))}{p}\right] < f^{-1^{'}}\left[\frac{\beta(V^H(V) - V^L(V))}{p}\right].$$

So

$$\begin{aligned}
\hat{W}^{'}(V') = \lambda(V') &= W^{'}(V^H(V')) + \left[1 - \frac{f^{-1^{'}}(\frac{\beta(V^H(V') - V^L(V'))}{p})}{\pi p}\right] \\
&\geq W^{'}(V^H(V)) + \left[1 - \frac{f^{-1^{'}}(\frac{\beta(V^H(V') - V^L(V'))}{p})}{\pi p}\right] \\
&> W^{'}(V^H(V)) + \left[1 - \frac{f^{-1^{'}}(\frac{\beta(V^H(V) - V^L(V))}{p})}{\pi p}\right] \\
&= \lambda(V) = \hat{W}^{'}(V),
\end{aligned}$$

i.e. $\hat{W}^{'}(V') > \hat{W}^{'}(V)$, which is a contradiction to the fact that $\hat{W}$ is concave. So for any $V, V' \in [0, V_1]$, $V < V'$, $V^H(V) < V^H(V')$, i.e. $V^H(V)$ is strictly increasing on $[0, V_1]$.

To prove that $V^L(V)$ is non-decreasing on $[0, V_1]$, consider $V, V' \in [0, V_1]$, $V < V'$. If $V^L(V) = 0$, then since $V^L(V') \geq 0$, it's obviously true that $V^L(V') \geq V^L(V)$. If $V^L(V) > 0$, $V^L(V') \geq V^L(V)$ can be proved by contradiction. If $V^L(V') < V^L(V)$, then $V^H(V') > V^H(V)$. So

$$\begin{aligned}
\hat{W}'(V') = \lambda(V') &\geq W'(V^L(V')) - \frac{\pi}{1-\pi}\left[1 - \frac{f^{-1'}(\frac{\beta(V^H(V') - V^L(V'))}{p})}{\pi p}\right] \\
&\geq W'(V^L(V)) - \frac{\pi}{1-\pi}\left[1 - \frac{f^{-1'}(\frac{\beta(V^H(V') - V^L(V'))}{p})}{\pi p}\right] \\
&> W'(V^L(V)) - \frac{\pi}{1-\pi}\left[1 - \frac{f^{-1'}(\frac{\beta(V^H(V) - V^L(V))}{p})}{\pi p}\right] \\
&= \lambda(V) = \hat{W}'(V),
\end{aligned}$$

Again, we get a contradiction to $\hat{W}$ being concave. Therefore $V^L(V)$ is non-decreasing on $[0, V_1]$.

A.1.3 Proof of Lemma 1.3.3

First, by (iv) of Proposition 1.3.3, $V^L(V_1) < V^H(V_1) = \bar{V}$. So

$$V_1 = \beta(\pi\bar{V} + (1-\pi)V^L(V_1)) < \beta\bar{V} < \bar{V}.$$

Suppose $V_1 \leq V_r$, consider problem (P_2') with $V = V_1$. By the Envelope theorem, $\hat{W}'(V_1) = \pi W'(V^H(V_1)) + (1-\pi)W'(V^L(V_1)) = \pi W'(\bar{V}) + (1-\pi)W'(V^L(V_1))$. Recall that W is linear on $[0, V_r]$, and strictly concave on $[V_1, \bar{V}]$. Since $V^L(V_1) < V_1 \leq V_r$, $W'(V^L(V_1)) = W'(V_1)$. Since $\bar{V} > V_1$ and $\bar{V} > V_r$, $W'(\bar{V}) < W'(V_1)$. So $\hat{W}'(V_1) < W'(V_1)$. However, $V_1 \leq V_r$ implies that $\hat{W}'(V_1) \geq W'(V_1)$. So $V_1 > V_r$.

A.1.4 Proof of Proposition 1.3.4

(i) By construction, $V^H(V_1) = \bar{V}$, and it has been proved that $V^H(\bar{V}) = \bar{V}$ in the proof of Lemma 1.3.2. Now suppose there exists $V_0 \in (V_1, \bar{V})$, such that $V^H(V_0) < \bar{V}$. If $\tau(V_0) < pf(l(V_0))$, the objective of (P_2) can be strictly increased by increasing $\tau(V_0)$ and $V^H(V_0)$ in a way that keeps all constraints hold. If $\tau(V_0) = pf(l(V_0))$ and $\tau(V_0) < \beta(V^H(V_0) - V^L(V_0))$, then $V_0 = \beta(\pi V^H(V_0) + (1-\pi)V^L(V_0))$, $V^H(V_0) > V^L(V_0)$. Since $V_0 > V_1 > V_r$, $V^H(V_0) > V_r$. Then as proved before the objective of (P_2) can be strictly increased by lowering $V^H(V_0)$ and increasing $V^L(V_0)$. If $\tau(V_0) = pf(l(V_0))$ and $\tau(V_0) = \beta(V^H(V_0) - V^L(V_0))$, by the argument of part (iv) of Proposition 1.3.3, $V^H(V_0) > V^H(V_1) = \bar{V}$. So $V^H(V) = \bar{V}$ for all $V \in [V_1, \bar{V}]$.

(ii) It holds for $V = V_1$. Suppose there exists $V_0 \in (V_1, \bar{V}]$, such that $\tau(V_0) < \beta(V^H(V_0) - V^L(V_0))$, where $V^H(V_0) = \bar{V} > V_r$. Again, by concavity of W, the objective of (P_2) can be made strictly higher by lowering $V^H(V_0)$ and increasing $V^L(V_0)$ in a way that makes all constraints hold. So (1.10) is binding on $[V_1, \bar{V}]$.

(iii) Consider any $V \in [V_1, \bar{V}]$, the problem of (P_1) and (P_2) can be reduced to

$$(P') \begin{cases} W(V) & = \max_{l \geq 0} \left\{ \pi pf(l) - l + \beta\{\pi W(\bar{V}) + (1-\pi)W(V^L)\} \right\} \\ s.t. & V^L = \frac{V - \pi pf(l)}{\beta} \geq 0 \\ & (1-\pi)pf(l) \geq \beta\bar{V} - V \end{cases}$$

where the second constraint is the limited liability constraint. Note that the first constraint implies $pf(l) \leq \frac{V}{\pi}$. And the second constraint implies $pf(l) \geq$

$\frac{\beta\bar{V}-V}{1-\pi}$. So a necessary condition for (P') to be meaningful is $\frac{V}{\pi} \geq \frac{\beta\bar{V}-V}{1-\pi}$ or equivalently, $V \geq \beta\pi\bar{V}$. Note that this is true for V_1, since $V_1 = \beta(\pi\bar{V} + (1-\pi)V^L(V_1)) \geq \beta\pi\bar{V}$. So this condition holds for any $V \in [V_1, \bar{V}]$.

Consider two possible cases. First, $(1-\pi)pf(l(V)) = \beta\bar{V} - V$. Since $\bar{V} = V^H(V_1) = \frac{V_1+(1-\pi)pf(l(V_1))}{\beta}$, then $(1-\pi)pf(l(V)) = \beta\frac{V_1+(1-\pi)pf(l(V_1))}{\beta} - V \leq V_1+(1-\pi)pf(l(V_1)) - V_1 = (1-\pi)pf(l(V_1)) < (1-\pi)pf(l^\star(p))$. So $l(V) < l^\star(p)$. Second, $(1-\pi)pf(l(V)) > \beta\bar{V} - V$. Then $l(V)$ satisfies the first order condition

$$\pi pf'(l(V)) \geq \frac{1}{1-(1-\pi)W'(V^L(V))}, \text{with equality if} \quad V^L(V) > 0.$$

Clearly $\pi pf'(l(V)) > 1$, so $l(V) < l^\star(p)$.

(iv) Consider the problem (P') for any $V \in [V_1, \bar{V}]$. Since (1.10) is binding, l can be solved in terms of V^L, $l = f^{-1}\left(\frac{V-\beta V^L}{\pi p}\right)$. And (1.11) can be rewritten as $\beta(1-\pi)V^L \leq V - \pi\beta\bar{V}$. So the problem can be rewritten as

$$\begin{aligned}(P\text{''}) \quad W(V) = \max_{V^L\in[0,\bar{V}]} \quad & \left\{V - \beta V^L - f^{-1}\left(\frac{V-\beta V^L}{\pi p}\right)\right. \\ & \left. + \{\pi W(\bar{V}) + (1-\pi)W(V^L)\}\right\} \\ s.t. \quad & \beta(1-\pi)V^L \leq V - \pi\beta\bar{V}\end{aligned}$$

The first order condition for V^L is

$$W^{'}(V^L) \geq \frac{1 - f^{-1'}\left(\frac{V-\beta V^L}{\pi p}\right)\frac{1}{\pi p}}{1-\pi} + \mu, \text{with equality if} \quad V^L < \bar{V},$$

where $\mu \geq 0$ is the Lagrangian multiplier on the constraint. By the Envelope theorem,

$$W^{'}(V) = 1 - f^{-1'}\left(\frac{V-\beta V^L}{\pi p}\right)\frac{1}{\pi p} + \mu.$$

Note that $V_1 = \beta\pi\bar{V} + (1-\pi)V^L(V_1) < \beta\bar{V}$. So V can fall in $[V_1, \beta\bar{V})$ or $[\beta\bar{V}, \bar{V}]$. First, consider $V \in [\beta\bar{V}, \bar{V}]$. In this case, the constraint is not binding, since $(1-\pi)pf(l(V)) > 0 \geq \beta\bar{V} - V$. So $\mu = 0$. And $W'(V^L(V)) \geq \frac{W'(V)}{1-\pi} > W'(V)$. This implies $V^L(V) < V$. So $V^L(V) < \bar{V}$, which gives us equality in the first order condition, $W'(V^L(V)) = \frac{W'(V)}{1-\pi}$. Since $\beta\bar{V} > V_1 > V_r$, $W(V)$ is strictly concave on $[\beta\bar{V}, \bar{V}]$. So $W'(V^L(V))$ is strictly decreasing in V, and hence $V^L(V)$ is strictly increasing in V. Now consider $V \in [V_1, \beta\bar{V})$. First note that $V^L(V) \leq \frac{V - \pi\beta\bar{V}}{\beta(1-\pi)} < \frac{\beta\bar{V} - \pi\beta\bar{V}}{\beta(1-\pi)} = \bar{V}$. So

$$\begin{aligned}
W'(V^L(V)) &= \frac{1 - f^{-1'}\left(\frac{V - \beta V^L(V)}{\pi p}\right)\frac{1}{\pi p}}{1-\pi} + \mu(V) \\
&= \frac{1 - f^{-1'}\left(\frac{V - \beta V^L(V)}{\pi p}\right)\frac{1}{\pi p}}{1-\pi} + W'(V) - \left\{1 - f^{-1'}\left(\frac{V - \beta V^L}{\pi p}\right)\frac{1}{\pi p}\right\} \\
&= \frac{\pi}{1-\pi}\left\{1 - f^{-1'}\left(\frac{V - \beta V^L(V)}{\pi p}\right)\frac{1}{\pi p}\right\} + W'(V).
\end{aligned}$$

By (iii), $l(V) = \frac{V - \beta V^L(V)}{\pi p} < l^\star(p)$, so $f^{-1'}\left(\frac{V - \beta V^L(V)}{\pi p}\right) < f^{-1'}(l^\star(p)) = \frac{1}{f'(l^\star(p))} = \pi p$. And hence $W'(V^L(V)) > W'(V)$, which implies $V^L(V) < V$. Also,

$$W'(V^L(V)) = \frac{W'(V) - \mu(V)}{1-\pi} + \mu(V) = \frac{W'(V)}{1-\pi} - \frac{\pi}{1-\pi}\mu(V).$$

Consider any $V, V' \in [V_1, \beta\bar{V})$, $V' > V$. Suppose $V^L(V') \leq V^L(V)$. Then

$$W'(V^L(V')) = \frac{W'(V')}{1-\pi} - \frac{\pi}{1-\pi}\mu(V') \geq W'(V^L(V)) = \frac{W'(V)}{1-\pi} - \frac{\pi}{1-\pi}\mu(V).$$

Since $W'(V') \leq W'(V)$, $\mu(V') \leq \mu(V)$. On the other hand, since $V' - \beta V^L(V') > V - \beta V^L(V)$, and f^{-1} is strictly convex, $f^{-1'}\left(\frac{V' - \beta V^L(V')}{1-\pi}\right) >$

$f^{-1'}\left(\frac{V-\beta V^L(V)}{1-\pi}\right)$. So $W'(V^L(V')) = \frac{1-f^{-1'}\left(\frac{V'-\beta V^L(V')}{\pi p}\right)\frac{1}{\pi p}}{1-\pi} + \mu(V') < \frac{1-f^{-1'}\left(\frac{V-\beta V^L(V)}{\pi p}\right)\frac{1}{\pi p}}{1-\pi} + \mu(V) = W'(V^L(V))$. This implies $V^L(V') > V^L(V)$, which contradicts our assumption that $V^L(V') \leq V^L(V)$. So $V^L(V') > V^L(V)$ for any $V', V \in [V_1, \bar{V}), V' > V$, i.e., $V^L(V)$ is strictly increasing on $[V_1, \bar{V})$. In summary, $V^L(V) < V$ and strictly increasing on both $[V_1, \beta\bar{V})$ and $[\beta\bar{V}, \bar{V}]$, so $V^L(V) < V$ on $[V_1, \bar{V}]$. And by continuity, $V^L(V)$ is strictly increasing on $[V_1, \bar{V}]$.

(v) In (P_2), since the incentive constraint (1.10) is binding, $\tau(V) = \beta\bar{V} - V^L(V)$. Since $V^L(V) < V \leq \bar{V}$, $\tau(V) > 0$. And since $V^L(V)$ is strictly increasing in V, $\tau(V)$ is strictly decreasing in V.

(vi) Consider the problem (P') formulated in part (iii) for some $V \in [V_1, \bar{V}]$. Note that the limited liability constraint is binding for $V = V_1$. Since $l(V) > 0$ for $V > 0$, the constraint is not binding for $V \geq \beta\bar{V}$. By continuity, there exists $\hat{V} \in [V_1, \beta\bar{V})$, such that the limited liability constraint is binding for $V \in [V_1, \hat{V}]$, and not binding for $V \in (\hat{V}, \bar{V}]$. For $V \in (\hat{V}, \bar{V}]$, $l(V)$ satisfies the first order condition

$$\pi p f'(l(V)) = \frac{1}{1-(1-\pi)W'(V^L(V))}.$$

Since $V^L(V)$ is strictly increasing and W is strictly concave on $(\hat{V}, \bar{V}]$, $f'(l(V))$ is strictly decreasing, i.e., $l(V)$ is strictly increasing in V.

A.1.5 Proof of Lemma 1.3.4

Since the periodic profit function $\pi p f(l) - l$ is continuous and strictly increasing in p, $\hat{W}(V; p)$ is continuous and strictly increasing in p. And hence

$\hat{W}(V;p) - V$ is continuous and strictly increasing in p. So $V_0(p)$ is continuous and strictly increasing in p as long as a solution to (P_3) exists at p.

A.1.6 Proof of Proposition 1.3.5

Suppose $\mu_t'(V;p) = \kappa\mu(V;p)$, $E_{t+1}' = \kappa E_{t+1}$, where $\kappa > 0$ is an arbitrary real number. Since for every $A \in \mathcal{V}\,(p)$, $\mu_t(A;p) = 0$ implies $\mu_t'(A;p) = 0$, μ_t' is absolutely continuous with respect to μ_t. By Radon-Nikodym Theorem, $d\mu_t' = \kappa d\mu$. So

$$
\begin{aligned}
\mu_{t+1}'(A;p) = &\int \Big\{ \pi \left(1 - \alpha(V^H(V))\right) \chi_A \left(V_c(V^H(V))\right) \\
&+ (1-\pi)\left(1 - \alpha(V^L(V))\right) \chi_A \left(V_c(V^L(V))\right) \Big\} \mu_t'(dV;p) + E_{t+1}' \, \chi_A(V_0(p)) \\
= &\int \Big\{ \pi \left(1 - \alpha(V^H(V))\right) \chi_A \left(V_c(V^H(V))\right) \\
&+ (1-\pi)\left(1 - \alpha(V^L(V))\right) \chi_A \left(V_c(V^L(V))\right) \Big\} \kappa\, \mu_t(dV;p) + \kappa E_{t+1} \, \chi_A(V_0(p)) \\
= \kappa &\Big\{ \int \Big\{ \pi \left(1 - \alpha(V^H(V))\right) \chi_A \left(V_c(V^H(V))\right) \\
&+ (1-\pi)\left(1 - \alpha(V^L(V))\right) \chi_A \left(V_c(V^L(V))\right) \Big\} \mu_t(dV;p) + E_{t+1} \, \chi_A(V_0(p)) \Big\} \\
= \kappa &\mu_{t+1}(A;p).
\end{aligned}
$$

A.1.7 Proof of Theorem 1.3.1 (Sketch)

For any given distribution of firms, μ, there exists a unique p that satisfies condition (ii) in the definition of equilibrium. Denote p^0 as the output price corresponding to $\mu = 0$. p^0 may be infinity. Define $I^\star = \hat{W}(M;p^0)$, then if $I < I^\star$, the problem (P_3) has a solution at p^0 and $V_0(p^0) > M$. Note that

(P_3) has no solution for sufficiently small p, since $\hat{W}(V; p = 0) = \beta S < I - M$, for $\forall V > 0$. Define $p^1 = \min\{p : \exists V \in [0, \bar{V}(p)], \text{ s.t.} \hat{W}(V; p) - V \geq I - M\}$, i.e., p^1 is the smallest p at which (P_3) has a solution. Since $\hat{W}(V; p) - V$ is continuous and strictly increasing in p, p^1 is well defined and strictly greater than zero. And p^1 is continuous and strictly decreasing in M. So by Lemma 1.3.4, $V_0(p^1(M))$ is continuous and strictly decreasing in M. Note that for M close to zero, $V_0(p^1(M)) > M$. And for M close to I, $V_0(p^1(M))$ is close to zero, which is less than M. So there exists $M^\star$, $0 < M^\star < I$, such that $V_0(p^1(M^\star)) = M^\star$. If $M \geq M^\star$, then $V_0(p^1(M)) \leq M$. By the continuity and monotonicity of $V_0(p)$, there exists a unique $p^\star$, $p^1(M) \leq p^\star < p^0$, such that $V_0(p^\star) = M$.

With $p^\star$, decision rules $l(V; p^\star)$, $V^H(V; p^\star)$, $V^L(V; p^\star)$, $\tau(V; p^\star)$, $\alpha(V; p^\star)$ and $V_c(V; p^\star)$ as well as value functions $W(V; p^\star)$ and $\hat{W}(V; p^\star)$ can be uniquely determined by solving the contracting problem (P_1) and (P_2).

The final step is to establish the existence and uniqueness of an invariant measure $\mu^\star$ and mass of entry $E^\star$ that satisfy the equilibrium conditions. First let $E^t \equiv 1$ in equation (1.15), then the operator $T^\star$ has a unique fixed point μ. It's not hard to show that the transition function defined by the decision rules is monotone, has the Feller property, and satisfies the mixing condition of Assumption 12.1 of Stokey and Lucas (1989). Since entry is positive and 0 is not in the ergodic set of μ (firms with equity values equal to zero are scrapped for sure and cannot survive to the continuation stage), μ puts all positive mass on firms with equity values greater than zero. Since

$l(V) > 0$ for $V > 0$, the aggregate output $Y(\mu; p^\star) > 0$. Let $E^\star$ be determined by $Q^\star \equiv D^{-1}(p^\star) = E^\star Y(\mu; p^\star)$. Since $p^\star < p^0$, $Q^\star > 0$. So $E^\star > 0$. By the linear homogeneity of $T^\star$, $\mu^\star = E^\star \mu$ is the unique fixed point of $T^\star$ when entry is $E^\star$.

A.2 A Brief Description of the Solution Method

The behavior of the industry cannot be characterized analytically. I construct a numerical approximation to the stationary competitive equilibrium with entry and exit defined in Section 3.6. For a given set of parameter values, the computation strategy involves the following steps.

1. Solving the dynamic contracting problems and computing the optimal decision rules, which involves an iteration on the following steps.

    - For a given W, find its fixed point $\bar{V}$;
    - Solve the problem (P_2) to obtain $\hat{W}$;
    - Solve the problem (P_1) to obtain a new W.

      (((P_1) and (P_2) are solved by piecewise linear approximation. The state space $[0, \bar{V}]$ is divided into 100 grids, with finer grids for smaller regions).

2. Solving the entry problem (P_3) to determine a firm's initial equity value V_0.

3. Iterating on (1.15) to compute the stationary measure μ with $E = 1$, and

4. Using an exogenously given level of total labor demand to determine the equilibrium level of entry $E^{\star}$ and the corresponding stationary measure $\mu^{\star}$.

Appendix B

Appendix to Chapter 2

B.1 Data Description

B.1.1 Rule for deleting major capital changes

I exclude observations for which

$$|Gk_{i,t} - Gk_{i,t-1} - i_{i,t} + Retr_{i,t}| > 0.15 \cdot Gk_{i,t-1},$$

where $Retr_{i,t}$ denotes retirements (DATA184). In the instances where the retirement number is missing, I assume it is zero unless the discrepancy was negative. In this case, a value of $0.1 \cdot Gk_{i,t-1}$ is substituted for $Retr_{i,t}$.

B.1.2 Variables

- Investment: Investment expenditures, $i_{i,t}$, is reported capital expenditure on property, plant and equipment (DATA30).
- Gross PPE: book value of gross plant, property and equipment (DATA7).
- Depreciation: reported value of depreciation and amortization (DATA14).
- External finance: sum of net debt issuance, net equity issuance and net changes in current debt (DATA313+DATA127).

- Sources of funds: sum of operating net cash flow and net cash flow from financing activities (DATA308+DATA313).
- Uses of funds: sum of capital expenditures, acquisitions and increases in financial assets (-DATA311).
- Debt: sum of long-term debt and debt in current liabilities (DATA9+DATA34).
- Interest expenses on total debt (DATA15).

B.2 Procedure for Constructing Investment Rates

A major work for constructing investment rates for each firm at each year involves converting the book value of capital stock into its replacement value. Denote $k_{i,t}$ as the replacement value of firm i's capital stock at the beginning of period t (or at the end of period $t-1$). It is constructed by the perpetual inventory method described in Salinger and Summers (1983).

- First, set the replacement value of the capital stock equal to the book value of gross plant, property and equipment for the first year the firm appears on Compustat file if it is later than 1979 or for year 1979 [1] otherwise, i.e., $k_{i,0} = Gk_{i,0}$, where $Gk_{i,t}$ is the reported value of gross PPE at the end of period t.

[1] Here, I use 1979 as the starting year. Using years earlier than 1979 does not change the moments of investment rates that are relevant to the calibration.

- Then estimate the useful life of capital goods in any year using the formula $L_{i,t} = \frac{Gk_{i,t-1}+i_{i,t}}{Depr_{i,t}}$, where $Depr_{i,t}$ is the reported value of depreciation and amortization. Take the time average of $L_{i,t}$, denoted by L_i.

- Define the replacement value of the capital stock using the double declining balance method of depreciation.

$$k_{i,t} = \left[k_{i,t-1} \frac{P_t^k}{P_{t-1}^k} + i_{i,t} \right] (1 - 2/L_i), t = 1, 2, \cdots ,$$

  where P_t^k is the deflator for non-residential investment, which is downloadable from the BEA website.

In calculating the cross sectional mean, standard deviation and autocorrelation of investment rates, observations with investment rates over 300% are excluded.

B.3 Proof of Proposition 2.3.2

To characterize $k'(k,z)$, I rewrite the problem (2.2) as

$$\begin{aligned} V(k,z) &= \max \Big\{ \max_{k' > (1-\delta)k + \frac{\pi(k,z)}{p}} \pi(k,z) - p\,(k' - (1-\delta)k) - \lambda_0 \\ &\quad - \lambda_1 [p(k' - (1-\delta)k) - \pi(k,z)] + \beta E_{z'|z} V(k', z'), \\ &\quad \max_{k' \leq (1-\delta)k + \frac{\pi(k,z)}{p}} \pi(k,z) - p\,(k' - (1-\delta)k) + \beta E_{z'|z} V(k', z') \Big\} \end{aligned} \tag{B.1}$$

The firm can choose to use external finance or not. The first inner maximization problem is the decision faced by the firm if external funds are needed to finance the investment, and the second inner maximization problem is the decision if investment can be fully financed by the firm's operating profits. The first order condition for the first inner maximization problem is given by

$$\beta E_{z'|z} V_1(k', z') = p(1 + \lambda_1), \quad \text{if} \quad k' > (1 - \delta)k + \frac{\pi(k, z)}{p}. \tag{B.2}$$

Note that for given z, (B.2) determines a unique k', denoted by $k'_1(z)$. Equating $k'_1(z) = (1-\delta)k + \frac{\pi(k,z)}{p}$ gives a unique k, denoted by $k_e(z)$. Then if $k < k_e(z)$, $k'(k, z) \equiv k'_1(z)$.

The first order condition for the second inner maximization problem is

$$\begin{aligned} \beta E_{z'|z} V_1(k', z') = p, \quad \text{if} \quad k' < (1 - \delta)k + \frac{\pi(k, z)}{p}, \\ \text{otherwise}, k' = (1 - \delta)k + \frac{\pi(k, z)}{p}. \end{aligned} \tag{B.3}$$

Note that there is a unique $k'_2(z)$ that satisfies $\beta E_{z'|z} V_1(k', z') = p$. Since $\lambda_1 > 0$ and $V(k, z)$ is concave in k, $k'_2 > k'_1$. Let $k = k_f(z)$ satisfy $k'_2(z) = (1 - \delta)k + \frac{\pi(k,z)}{p}$. Then $k_f(z) > k_e(z)$. For $k > k_f(z)$, $k'(k, z) \equiv k'_2(z)$. For $k_e \leq k \leq k_f$, $k'(k, z) = (1 - \delta)k + \frac{\pi(k,z)}{p}$.

Note that the policy function $k'(k, z)$ may be discontinuous at the cutoff point k_e, since that is no guarantee that $k'_1 = (1 - \delta)k_e + \frac{\pi(k_e,z)}{p}$.

B.4 Estimation Procedure

For a given set of values for $(\alpha, \delta, \rho, \sigma, \lambda_0)$,

1. solve the firm's problem by value function iteration

    - approximate the productivity shock process by a 10-state Markov chain, as described in Tauchen (1986);

    - let the state space for k be $[0.01, \bar{k}_0]$, where $\bar{k}_0$ is the steady state capital stock in a deterministic problem with productivity being the highest level, discretize $[0.01, \bar{k}_0]$ into 301 equally spaced points, and do value function iteration until convergence is obtained;

    - let the state space for k be $[0.01, \bar{k}]$, where $\bar{k} = \max_{(k,z)} k'(k, z)$, discretize $[0.01, \bar{k}]$ into 801 equally spaced points, take the value function obtained in the last step as the initial value function, and do value function iteration until convergence.

2. starting from a uniform distribution over (k, z) and using the decision rule $k'(k, z)$, do another function iteration to obtain the stationary firm distribution, $\mu(k, z)$;

3. generate 20,000 firms from the stationary firm distribution and carry out the simulation for 15 periods, compute the five moments using the simulated panel data set, and compare them with the data moments. If they are close enough, stop. Otherwise, choose another set of values, and repeat steps 1 to 3.

Bibliography

Acemoglu, D., and A. Scott (1997): "Asymmetric Business Cycles: Theory and Time-Series Evidence," *Journal of Monetary Economics*, 40, 501–533.

Alti, A. (2003): "How Sensitive Is Investment to Cash Flow When Financing is Frictionless," *Journal of Finance*, 58, 707–722.

Arellano, C., and E. Mendoza (2002): "Credit Frictions and 'Sudden Stops' in Small Open Economies: An Equilibrium Business Cycle Framework for Emerging Markets Crises," NBER Working Paper 8880.

Atkeson, A., and R. Lucas (1992): "On Efficient Distribution with Private Information," *Review of Economic Studies*, 59, 427–453.

Atkeson, A., and J.-V. Rios-Rull (1996): "The Balance of Payments and Borrowing Constraints: An Alternative View of the Mexican Crises," Federal Reserve Bank of Minneapolis, Research Department Staff Report 212.

Backus, D., and P. Kehoe (1989): "International Evidence on the Historial Properties of Business Cycles," Working Paper No. 402R, Federal Reserve Bank of Minneapolis.

Baily, M. N., C. Hulten, and D. Campbell (1992): "Productivity Dynamics in Manufacturing Plants," *Brookings Papers: Microeconomics*, pp. 187–267.

Bartelsman, E. J., and P. J. Dhrymes (1998): "Productivity Dynamics: U.S. Manufacturing Plants, 1972-1986," *Journal of Productivity Analysis*, 9, 5–34.

Belaire-Franch, J., and D. Contreras (2003): "An Assessment of International Business Cycle Asymmetries Using Clements and Krolzig's Parametric Approach," *Studies in Nonlinear Dynamics & Econometrics*, 6(4).

Bernanke, B., and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, 79, 14–31.

Campbell, J. R., and Z. Hercowitz (2005): "The Role of Households' Collateralized Debt in Macroeconomic Stabilization," NBER Working Paper 11330.

Carlstrom, C. T., and T. S. Fuerst (1997): "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *The American Economic Review*, 87, 893–910.

Chalkley, M., and I. H. Lee (1998): "Learning and Asymmetric Business Cycles," *Review of Economic Dynamics*, 1, 623–645.

CHRISTIANO, L. J., AND J. FISHER (2000): "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints," *Journal of Economic Dynamics and Control*, 24, 1179–1232.

CLEMENTI, G. L., AND H. HOPENHAYN (2002): "A Theory of Financing Constraints and Firm Dynamics," Manuscript, University of Rochester.

COOLEY, T., AND E. C. PRESCOTT (1995): *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, NJ.

COOLEY, T., AND V. QUADRINI (2001): "Financial Markets and Firm Dynamics," *The American Economic Review*, 91, 1286–1310.

COOPER, R., AND J. EJARQUE (2001): "Exhuming Q: Market Power vs. Capital Market Imperfections," NBER working paper No. 8182.

——— (2003): "Financial Frictions and Investment: A Requiem in Q," *Review of Economic Dynamics*, 6, 710–728.

COOPER, R., AND J. HALTIWANGER (2005): "On the Nature of Capital Adjustment Costs," .

CORDOBA, J. C., AND M. RIPOLL (2004): "Credit Cycles Redux," *International Economic Review*, 45, 1011–1046.

DAVIS, S., J. HALTIWANGER, AND S. SCHUH (1996): *Job Creation and Destruction*. The MIT Press, Cambridge.

DELONG, B. J., AND L. H. SUMMERS (1986): "Are Business Cycles Symmetrical?," in *The American Business Cycle: Continuity and Change*, ed. by R. J. Gordon, pp. 166–178. University of Chicago Press, Chicago.

DUNNE, T., M. ROBERTS, AND L. SAMUELSON (1988): "Patterns of Firm Entry and Exit in U.S. Manufacturing Industries," *Rand Journal of Economics*, 19, 495–515.

——— (1989): "The Growth and Failure of U.S. Manufacturing Plants," *Quarterly Journal of Economics*, 104, 671–698.

EVANS, D. S. (1987): "The Relationship Between Firm Growth, Size and Age: Estimates for 100 Manufacturing Industries," *The Journal of Industrial Economics*, 35, 567–581.

FALK, B. (1986): "Further Evidence on the Asymmetric Behavior of Economic Time Series over the Business Cycle," *Journal of Political Economy*, 94, 1096–1109.

FAZZARI, S., R. G. HUBBARD, AND B. C. PETERSON (1988): "Financing Constraints and Corporate Investment," *Brooking Papers on Economic Activity*, 1, 141–206.

FOSTER, L., J. HALTIWANGER, AND C. KRIZAN (2000): "Aggregate Productivity Growth: Lessons from Microeconomic Evidence," in *New Developments in Productivity Analysis, NBER*, ed. by C. R. Hulten, E. R. Dean, and M. J. Harper.

GERTLER, M. (1988): "Financial Structure and Aggregate Economic Activity: An Overview," *Journal of Money, Credit and Banking*, 20, 559–588.

——— (1992): "Financial Capacity and Output Fluctuations in an Economy with Multi-Period Financial Relationships," *Review of Economic Studies*, 59, 455–472.

GILCHRIST, S., AND C. P. HIMMELBERG (1995): "Evidence on the Role of Cash Flow for Investment," *Journal of Monetary Economics*, 36, 541–572.

GOMES, J. (2001): "Financing Investment," *The American Economic Review*, 91, 1263–1285.

GREEN, E. (1987): "Lending and the Smoothing of Uninsurable Income," in *Contractual Arrangements for Intertemporal Trade*. University of Minnesota Press, Minneapolis, MN.

GREENWALD, B., AND J. E. STIGLITZ (1993): "Financial Market Imperfections and Business Cycles," *Quarterly Journal of Economics*, 108, 77–114.

HALL, B. H. (1987): "The Relationship Between Firm Size and Firm Growth in the US Manufacturing," *The Journal of Industrial Economics*, 35, 583–606.

HAMILTON, J. D. (1989): "A New Approach to the Economic Analysis of Non stationary Time Series and the Business Cycle," *Econometrica*, 57, 357–384.

Hansen, G. D., and E. C. Prescott (2000): "Capacity Constraints, Asymmetries, and the Business Cycle," Manuscript.

Hopenhayn, H. (1992): "Entry, Exit and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 60, 1127–1150.

Hopenhayn, H., and R. Rogerson (1993): "Job Turnover and Policy Evaluation: a General Equilibrium Analysis," *Journal of Political Economy*, 101, 915–938.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50, 649–670.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

Kocherlakota, N. R. (2000): "Creating Business Cycles Through Credit Constraints," *Federal Reserve Bank of Minneapolis Quarterly Review*, 24, 2–10.

Lamont, O. (1995): "Corporate-Debt Overhang and Macroeconomic Expectations," *The American Economic Review*, 85, 1106–1117.

Mendoza, E. G. (1991): "Real Business Cycles in a Small Open Economy," *The American Economic Review*, 81, 797–803.

Neftci, S. N. (1984): "Are Economic Time Series Asymmetric over the Business Cycle?," *Journal of Political Economy*, 92, 307–328.

Nieuwerburgh, S. V., and L. Veldkamp (2004): "Learning Asymmetries in Real Business Cycles," Manuscript, New York University.

Petrin, A., and J. Levinsohn (2004): "Measuring and Mismeasuring Industry Productivity Growth Using Plant-level Data," Manuscript.

Phelan, C. (1995): "Repeated Moral Hazard and One-Sided Commitment," *Journal of Economic Theory*, 66, 488–506.

Quadrini, V. (2003): "Investment and Liquidation in Renegotiation-Proof Contracts with Moral Hazard," Manuscript, New York University.

R., C., and E. E. (1991): "Dynamic (S, s) economies," *Econometrica*, 59, 1659–1686.

Razzak, W. A. (2001): "Business Cycle Asymmetries: International Evidence," *Review of Economic Dynamics*, 4, 230–243.

Rogerson, R., and D. Restuccia (2004): "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," No. 69 in Society for Economic Dynamics 2004 Meeting Papers.

Salinger, M., and L. Summers (1983): "Tax Reform and Corporate Investment," in *Behavioral Simulation Methods in Tax Policy Analysis*, ed. by M. Feldstein, Chicago. University of Chicago Press.

Sichel, D. E. (1993): "Business Cycle Asymmetry: A Deeper Look," *Economic Inquiry*, 31, 224–236.

Spear, S. E., and S. Srivastava (1987): "On Repeated Moral Hazard With Discounting," *Review of Economic Studies*, 54, 599–617.

Stokey, N. L., R. E. Lucas, and E. C. Prescott (1989): *Recursive methods in economic dynamics*. Harvard University Press, Cambridge, MA.

Tauchen, G. (1986): "Finite State Markov Chain Approximations to Univariate and. Vector Autoregressions," *Economics Letters*, 20, 177–181.

Veldkamp, L. (2004): "Slow Boom, Sudden Crash," *Journal of Economic Theory*, forthcoming.

Wang, C. (2000): "Renegotiation-Proof Dynamic Contracts with Private Information," *Review of Economic Dynamics*, 3, 396–422.

Wheeler, C. H. (2005): "Job Flows and Productivity Dynamics: Evidence from U.S. Manufacturing," Federal Reserve Bank OF St. Louis, Working Paper 2005-017A.

Whited, T. (1992): "Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data," *Journal of Finance*, 47, 1425–1460.

Whited, T., and C. Hennessy (2005): "Debt Dynamics," *Journal of Finance*, 60, 1129–1165.

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