The Thesis Committee for Stephen Christopher Young
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Stability Issues in Kalb-Ramond/Dilaton Braneworld Scenarios

APPROVED BY
SUPERVISING COMMITTEE:

Supervisor: ______________________________
Willy Fischler

Reader: ______________________________
Sonia Paban

Stability Issues in Kalb-Ramond/Dilaton Braneworld Scenarios

by

Stephen C. Young, B.S.

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Stability Issues in Kalb-Ramond/Dilaton Braneworld Scenarios

Stephen C. Young, M.A.
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Supervisor: Willy Fischler

I summarize the Randall-Sundrum braneworld scenario, and its application to solving the hierarchy problem in the Standard Model of elementary particles. A generalized Randall-Sundrum scenario is presented, which includes the presence of string-inspired massless Kalb-Ramond and dilaton fields, and includes their backreaction on the metric. It is shown that in such a scenario, solutions exist which can achieve the desired warping on the Standard Model brane, and which stabilize the modulus corresponding to the radius of the extra dimension.

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Chapter 1

Introduction

The Standard Model of elementary particle physics has been extremely successful in accounting for experimental observations. It has, however, a number of confusing features that suggest the existence of new physics that are not incorporated into this model. One of these features is what is called the *Gauge Hierarchy Problem*. This refers to the difference between the energy scale of electroweak unification (246 GeV) and the Planck scale (1.22×10^{19} GeV). This difference in scale is unnatural within the minimal Standard Model since it requires a fine tuning order by order within perturbation theory. A number of ideas on how to extend the Standard Model have been proposed to solve this problem, such as technicolor and low energy supersymmetry.

An alternative solution to the hierarchy problem can come from the existence of large compactified extra dimensions [2, 1]. In these, the observed Planck mass M_{Pl} is related to the fundamental higher dimensional Planck mass M via $M_{Pl}^2 = M^{n+2}V_n$, where V_n is the volume of the compact dimensions. By making V_n large, we can obtain a fundamental M which is on the order of the electroweak scale. However, unless the number of extra dimensions is large, this introduces another hierarchy between the compactification scale $V_n^{-1/n}$ and the mass M.

Randall and Sundrum [15] proposed another method to resolve the hierarchy problem which does not require the use of large extra dimensions. In their scenario, the metric is not factorizable, but instead takes the form of a "warped" product

$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2\phi^2. \tag{1.1}$$

Here, k is a scale on the order of the Planck scale, x^μ are the familiar 4 dimensional coordinates, and $0 \leq \phi \leq \pi$ is the coordinate of an extra compact fifth dimension whose size is set by r_c. This metric is a solution to Einstein's equations in a setup with two 3-branes and an appropriate five-dimensional cosmological constant term. The extra dimension takes the form of an $S^1/\mathbb{Z}_2$ orbifold, with (x, ϕ) and $(x, -\phi)$ identified, and the two 3-branes lie at the fixed points at $\phi = 0$ and $\phi = \pi$. These two branes extend in the x^μ directions and form the boundaries of the five-dimensional space. In this space, four-dimensional mass scales are related to five-dimensional mass scales via the warp factor $e^{-2kr_c\phi}$. Since the warp factor is an exponential function of the size of the compactified dimension, large extra dimensions are not required in order to achieve a substantial hierarchy.

There is another important issue to be resolved in this model, however. The value of r_c is associated with the vacuum expectation value of a massless four-dimensional scalar field. This modulus field has zero potential, and so its value is not determined by the dynamics of the model. So some other mechanism has to be included which generates a potential for this field. Goldberger and Wise [9] showed that by including a bulk scalar in the setup, with interaction terms localized to the two branes, a potential could be generated for r_c which stabilized it at a value which generated the desired warp factor. Further studies [13] analyzed the effect of the backreaction of this scalar field on the metric, and found that stability could still be achieved. [16] and [3] give pedagogical introductions to braneworld scenarios and the stabilization issue.

Given that we have a scenario for resolving the hierarchy problem in the context of an higher-dimensional spacetime, one might ask whether this scenario could be realized in a theory which involves higher dimensions, such as string theory. Indeed, elements of warped geometry have been applied in explicit string theory models which also serve as models for inflation. See eg the work of KKLMMT [11].

With an eye towards the construction of string theory-based models, we consider the presence of additional background fields which may be present in a Randall-Sundrum type scenario. In addition to gravity, two other massless closed string modes can propagate in the bulk, namely the scalar dilaton and the Kalb-Ramond field. One might hope that the dilaton, given some mass in a particular string theory compactification, could serve as the scalar that Goldberger and Wise used to stabilize the size of the fifth dimension. Das et al. [7] studied this setup in the context of small fluctuations around the original Randall-Sundrum solution, while taking into account the full backreaction of the scalar dilaton and Kalb-Ramond field strengths on the metric. They concluded that solutions which achieved sufficient warping to resolve the hierarchy problem could only be attained by fine-tuning the Kalb-Ramond field to a very small value, and that in general, such solutions could not be stabilized.

We extend this analysis to the case of solutions which are not fluctuations around the Randall-Sundrum solution, and find that for a wide range of adjustable parameters within the theory, solutions can be found which achieve the desired warping and in addition stabilize the value of the extra dimension.

We begin with a review of the basic structure of the Randall-Sundrum model and the Goldberger-Wise stabilization mechanism, and then move on to a summary of the results of Das et al. We then present our analysis and discuss open questions.

Chapter 2

The Randall-Sundrum Braneworld

2.1 The original RS setup

The Randall-Sundrum model [15] exists on a five-dimensional *warped* spacetime with metric

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2. \tag{2.1}$$

We choose our metric to have signature $\eta_{\mu\nu} = (+,-,-,-)$. The fifth dimension is the orbifold $S^1/\mathbb{Z}_2$ so that (x^μ, ϕ) and $(x^\mu, -\phi)$ are identified. There are two 3-branes, located at the fixed points $\phi = \pi$ and $\phi = 0$, which are referred to as the visible and hidden branes, respectively. These serve as boundaries of the five-dimensional spacetime. Both branes couple to the purely four-dimensional components of the metric:

$$g_{\mu\nu}^{vis}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = \pi), \quad g_{\mu\nu}^{hid}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = 0) \tag{2.2}$$

where G_{MN}, $(M, N = \mu, \phi)$ is the five-dimensional metric.

The action describing this set-up is given by

$$
\begin{aligned}
S &= S_{gravity} + S_{vis} + S_{hid} \\
S_{gravity} &= \int d^4x \int_{-\pi}^{\pi} r_c \, d\phi \sqrt{G} \, [\Lambda + 2M^3 R] \\
S_{vis} &= \int d^4x \sqrt{-g_{vis}} \, [L_{vis} - V_{vis}] \\
S_{hid} &= \int d^4x \sqrt{-g_{hid}} \, [L_{hid} - V_{hid}]. \qquad (2.3)
\end{aligned}
$$

Here, M is the five-dimensional Planck mass, V_{vis} and V_{hid} are constant vacuum energies on the two branes which act as gravitational sources even in the absence of any matter content, and L_{vis} and L_{hid} are matter lagrangians on the branes, which will not affect the form of the classical 5-dimensional metric.

Varying the action with respect to the metric, we obtain the Einstein equations:

$$
\begin{aligned}
\sqrt{G}\left(R_{MN} - \frac{1}{2} G_{MN} R\right) &= -\frac{1}{4M^3}[\Lambda \sqrt{G} \; G_{MN} + V_{vis} \sqrt{-g_{vis}} \; g_{\mu\nu}^{vis} \; \delta_M^\mu \delta_N^\nu \; \delta(\phi - \pi) \\
&+ \; V_{hid} \sqrt{-g_{hid}} \; g_{\mu\nu}^{hid} \; \delta_M^\mu \delta_N^\nu \; \delta(\phi)]. \qquad (2.4)
\end{aligned}
$$

The key assumption of the Randall-Sundrum solution is that a solution exists which obeys *four*-dimensional Poincare invariance, motivating the form of the metric in Eq.(2.1). With this ansatz, the Einstein equations following from Eq. (2.4) are

$$\frac{6\sigma'^2}{r_c^2} = \frac{-\Lambda}{4M^3}, \qquad (2.5)$$

$$\frac{3\sigma''}{r_c^2} = \frac{V_{hid}}{4M^3 r_c}\delta(\phi) + \frac{V_{vis}}{4M^3 r_c}\delta(\phi - \pi). \qquad (2.6)$$

which have the following solution, consistent with the orbifold symmetry

$$\sigma = r_c|\phi|\sqrt{\frac{-\Lambda}{24M^3}}. \qquad (2.7)$$

This solution only makes sense if $\Lambda < 0$, so the five-dimensional space in between the two 3-branes will be a slice of an AdS_5 geometry.

As the metric is a periodic function in ϕ, Eq.(2.7) implies

$$\sigma'' = 2r_c\sqrt{\frac{-\Lambda}{24M^3}}[\delta(\phi) - \delta(\phi - \pi)]. \tag{2.8}$$

From this we see that we only obtain a solution to Eq.(2.6) if V_{vis}, V_{hid} and Λ are related in terms of a single scale k,

$$V_{hid} = -V_{vis} = 24M^3k, \quad \Lambda = -24M^3k^2. \tag{2.9}$$

This relation must hold between the boundary and bulk cosmological terms if we are to obtain a solution which respects four-dimensional Poincare invariance. Substituting these relations into the ansatz 2.1 gives us the form of the metric mentioned in the introduction,

$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2\phi^2. \tag{2.10}$$

Since we are considering a small r_c, the fifth dimension cannot be detected in present experiments, and it makes sense to consider a four-dimensional effective field theory description. This yields a value for the four-dimensionsal Planck mass of

$$M_{Pl}^2 = \frac{M^3}{k}[1 - e^{-2kr_c\pi}]. \tag{2.11}$$

Although this does not produce a significant reduction in the four-dimensional Planck mass relative to the five-dimensional one, in this setup, the mass parameters that we observe are those of fields confined to the visible 3-brane. The warp factor will change any fundamental mass parameter m_0 on the visible 3-brane in the five-dimensional theory to another *physical* mass m, which is what is measured by the metric in the effective four-dimensional theory.

$$m \equiv e^{-kr_c\pi} m_0 \tag{2.12}$$

Thus if $e^{kr_c\pi}$ is of order 10^{15}, or equivalently $kr_c \approx 12$, this mechanism produces TeV physical mass scales from fundamental mass parameters not far from the Planck scale, 10^{19} GeV.

2.2 Goldberg-Wise stabilization of r_c

In Randall and Sundrum's initial scenario, r_c is the vacuum expectation value of a massless four-dimensional field. This field has zero potential and r_c is therefore not determined by the dynamics of the model. Goldberger and Wise [9] found a mechanism by which to generate a potential to stabilize r_c at a particular value. To do this, they included an additional massive scalar in the bulk, by adding an additional term to the action 2.3.

$$S_{scal} = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} r_c \, d\phi \sqrt{G} \left(G^{AB} \partial_A \Phi \partial_B \Phi - m_{scal}^2 \Phi^2 \right), \tag{2.13}$$

where G_{AB} with $A, B = \mu, \phi$ is given by Eq. (2.1). They also included interaction terms on the hidden and visible branes given by

$$S_{int} = - \int d^4x \sqrt{-g_{hid}} \, \lambda_{hid} \left(\Phi^2 - v_h^2 \right)^2 - \int d^4x \sqrt{-g_{vis}} \, \lambda_{vis} \left(\Phi^2 - v_v^2 \right)^2 \tag{2.14}$$

The terms on the branes cause Φ to develop a ϕ-dependent vacuum expectation value $\Phi(\phi)$ which is determined classically by solving the equations of motion derived by varying the action $S_{int} + S_{scal}$. Plugging this solution back into the action and integrating over the compact dimension yields an effective four-dimensional potential for r_c.

Goldberger and Wise considered a simplified case in which the parameters λ_{hid} and λ_{vis} are infinitely large. They were able to find minima of the potential which yielded $kr_c \approx 12$ without having to extremely fine tune any other parameters in the model. In this case, $kr_c \approx 12$ corresponded to $v_{hid}/v_{vis} = 1.5$ and $m_{scal}^2/k^2 = 1/10$.

A more exact analysis of this scenario for λ_{hid} and λ_{vis} large but finite was carried out by Das et al. [8]. They found the existence of closely space minima and maxima of the potential, with the value of r_c associated with a minima slightly shifted from the value derived by Goldberger and Wise. It remained the case that no fine-tuning of parameters was necessary to achieve sufficient warping at the visible brane.

Chapter 3

Kalb-Ramond and dilaton background with Backreaction

3.1 The Kalb-Ramond-dilaton braneworld

In the context of string theory, the Neveu-Schwarz Neveu-Schwarz (NS-NS) sector contains massless closed-string modes corresponding to the graviton and scalar dilaton, as well the second-rank antisymmetric Kalb-Ramond field. Higher order effects in string theory can give the dilaton a mass, and so we might wonder if the dilaton could serve as the massive Goldberger-Wise scalar. All of these modes propagate in the bulk, so it is natural to consider a more general version of the Randall-Sundrum model with all three of these fields present. Das et al. [7] investigated the resolution of the hierarchy problem, as well as the stabilization of the modulus within the context of a scenario such as this. They included the backreaction of the scalar diliaton and Kalb-Ramond fields on the metric, determined an expression for the modified warp factor, and found solutions which had the form of small fluctuations around the original RS solution, Eq.(2.7). Within this perturbative regime, they found that solutions with sufficient warping caused the fine-tuning problem to appear in another guise: the value of the background Kalb-Ramond energy density had to be fine-tuned to a value $b \lesssim 10^{-64}$.

In what follows, we will substitute $y = r_c\,\phi$, $d\phi = dy/r_c$, and define our warp factor as $A(y)$, in contrast to the $2\sigma(\phi)$ that was used in Eq.(2.1). In this form, the metric is

$$ds^2 = e^{-A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2. \tag{3.1}$$

Now consider the action for the massless Kalb-Ramond field, conformally coupled to the scalar dilaton, in the Einstein frame:

$$S_{KR} = \int d^4x\, dy \sqrt{G}\ \exp(-\Phi/M^{3/2})\ [2H_{MNL}H^{MNL}]. \tag{3.2}$$

Note the sign of the H^2 term, which is taken as positive due to our choice of metric signature. We add this to equations (2.3), (2.13), and (2.14) to get the total action for the Kalb-Ramond-scalar dilaton-gravity system,

$$S = S_{gravity} + S_{vis} + S_{hid} + S_{KR} + S_{dilaton} + S_{int}, \tag{3.3}$$

where

$$\begin{aligned}
S_{gravity} &= \int d^4x\, dy \sqrt{G}\,[\Lambda + 2M^3R] && (3.4)\\
S_{vis} &= \int d^4x \sqrt{-g_{vis}}\,[L_{vis} - V_{vis}] && (3.5)\\
S_{hid} &= \int d^4x \sqrt{-g_{hid}}\,[L_{hid} - V_{hid}] && (3.6)\\
S_{KR} &= \int d^4x\, dy \sqrt{G}\ \exp(-\Phi/M^{3/2})\ [2H_{MNL}H^{MNL}] && (3.7)\\
S_{dilaton} &= \int d^4x\, dy \sqrt{G}\ \frac{1}{2}[G^{AB}\partial_A\Phi\partial_B\Phi - m_{dil}^2\Phi^2] && (3.8)
\end{aligned}$$

$$S_{int} = -\int d^4x \sqrt{-g_{hid}}\,\lambda_{hid}\left(\Phi^2 - v_h^2\right)^2 - \int d^4x \sqrt{-g_{vis}}\,\lambda_{vis}\left(\Phi^2 - v_v^2\right)^2 \tag{3.9}$$

Varying this action with respect to the metric, we obtain the five-dimensional Einstein equations (here we define $' \equiv d/dy$),

$$\frac{3}{2}A'^2 = -\frac{\Lambda}{4M^3} - \frac{1}{2M^3}[3G^{\nu\beta}G^{\lambda\gamma}H_{y\nu\lambda}H_{y\beta\gamma}\exp(-\frac{\Phi}{M^{3/2}}) + \frac{1}{4}(\Phi'^2 - m_{dil}^2\Phi^2)] \quad (3.10)$$

$$\begin{aligned}\frac{3}{2}(A'^2 - A'') &= -\frac{\Lambda}{4M^3} + \frac{1}{8M^3}[\Phi'^2 + m_{dil}^2\Phi^2] \\ &- \frac{1}{2M^3}\exp(-\frac{\Phi}{M^{3/2}})[-12\eta^{\lambda\gamma}H_{y0\lambda}H_{y0\gamma} + 3G^{\nu\beta}G^{\lambda\gamma}H_{y\nu\lambda}H_{y\beta\gamma}\eta_{00}]\end{aligned} \quad (3.11)$$

$$\begin{aligned}\frac{3}{2}(A'^2 - A'') &= -\frac{\Lambda}{4M^3} + \frac{1}{8M^3}[\Phi'^2 + m_{dil}^2\Phi^2] \\ &- \frac{1}{2M^3}\exp(-\frac{\Phi}{M^{3/2}})[-12\eta^{\lambda\gamma}H_{yi\lambda}H_{yi\gamma} + 3G^{\nu\beta}G^{\lambda\gamma}H_{y\nu\lambda}H_{y\beta\gamma}\eta_{ii}]\end{aligned} \quad (3.12)$$

In Eq.(3.12), the index i on the right hand side runs over the three spatial components x, y, z, and there is no sum over i. Also, $\eta^{ij} \equiv g^{im}g^{jn}\eta_{mn}$.

Adding Eq.(3.11) to the x, y, z components of Eq.(3.12), we get

$$\frac{3}{2}[A'^2 - A''] = -\frac{\Lambda}{4M^3} + \frac{1}{8M^3}[\Phi'^2 + m_{dil}^2\Phi^2]. \quad (3.13)$$

Subtracting Eq.(3.13) from Eq.(3.10) yields

$$\frac{3}{2}A'' = -\frac{1}{4M^3}\Phi'^2 - \frac{3}{2M^3}G^{\mu\nu}G^{\alpha\beta}H_{y\mu\alpha}H_{y\nu\beta}\exp(-\Phi/M^{3/2}) \quad (3.14)$$

To get the equation of motion for the Kalb-Ramond 2-form field, we vary the action (3.4-3.9) with respect to B_{AC}, which gives

$$\partial_A \left(\exp(-2A)\exp(-\Phi/M^{3/2})H^{ABC}\right) = 0, \tag{3.15}$$

where $H_{ABC} = \partial_{[A}B_{BC]}$ is the associated field strength.

We now make the assumption that Φ is a function only of the y direction, in which case this equation becomes

$$\begin{aligned}\left(-2A' - \frac{\Phi'}{M^{3/2}}\right) e^{-2A}e^{-\Phi/M^{3/2}}H^{y\mu\nu} \quad &+ \quad e^{-2A}e^{-\Phi/M^{3/2}}\partial_y H^{y\mu\nu} \\ &+ \quad e^{-2A}e^{-\Phi/M^{3/2}}\partial_\rho H^{\rho\mu\nu} = 0 \end{aligned}\tag{3.16}$$

We will also assume that $H^{\rho\mu\nu} = 0$, which allows us to drop the last term. A solution to this simplified equation is

$$\begin{aligned} H^{y\mu\nu} &= k^{\mu\nu}\exp(2A)\exp(\Phi/M^{3/2}) \\ H_{y\mu\nu} &= k_{\mu\nu}\exp(\Phi/M^{3/2}) \end{aligned}\tag{3.17}$$

where $k_{\mu\nu}$ is a constant antisymmetric tensor, independent of y.

We will express this solution in terms of a squared Kalb-Ramond field strength,

$$\begin{aligned} G^{\mu\alpha}G^{\nu\beta}H_{y\mu\nu}H_{y\alpha\beta} &= \exp(2A)\eta^{\mu\alpha}\eta^{\nu\beta}H_{y\mu\nu}H_{y\alpha\beta} \\ &= \exp(2A)\eta^{\mu\alpha}\eta^{\nu\beta}k_{\alpha\beta}k_{\mu\nu}\exp(2\Phi/M^{3/2}) \\ &= bM^5\exp(2A)\exp(2\Phi/M^{3/2}) \end{aligned}\tag{3.18}$$

where $bM^5 = \eta^{\mu\alpha}\eta^{\nu\beta}k_{\alpha\beta}k_{\mu\nu}$ and we have defined b as a dimensionless parameter measuring the energy density of the Kalb-Ramond field.

Finally, varying the action with respect to the dilaton gives its equation of motion,

$$\Phi'' - 2A'\Phi' - m_{dil}^2\Phi + \frac{6\exp(-\Phi)}{M^{\frac{3}{2}}}(G^{\mu\alpha}G^{\nu\beta}H_{y\mu\nu}H_{y\alpha\beta}) = 0 \tag{3.19}$$

We now substitute the expression (3.18) for the squared Kalb-Ramond field strength into the equations of motion (3.19) and (3.14) for the dilaton and the metric, and obtain the following two equations:

$$\Phi'' - 2A'\Phi' - m_{dil}^2\Phi + 6M^{\frac{7}{2}}b\exp(2A)\exp(\Phi/M^{3/2}) = 0 \tag{3.20}$$

$$A'' = -\frac{1}{6M^3}\Phi'^2 - bM^2\exp(2A)\exp(\Phi/M^{3/2}) \tag{3.21}$$

These two coupled, non-linear differential equations will be the starting point of the bulk of our analysis.

3.2 Perturbative solutions to Randall-Sundrum

We will first look for solutions of Φ which correspond to solutions for A which are close to the Randall-Sundrum solution, $A = 2kr_c\phi = 2ky$. We will assume that this corresponds to taking the value of b to be small, so that we are perturbing about the Randall-Sundrum solution with a small Kalb-Ramond field energy density. We look for a solution as a power series in b. To zeroth order, Eq.(3.20) becomes

$$\Phi'' - 4k\Phi' - m_{dil}^2\Phi = 0 \tag{3.22}$$

which has a solution $\Phi_0(y) = C_0\exp(2ky(1-\nu))$, where $\nu = \sqrt{4 + m^2/k^2}$.

To first order in b, we assume the solution is of the form $\Phi(y) = \Phi_0(y) +$

$b\Phi_1(y)$ and Taylor expand the $\exp(\Phi/M^{3/2})$ term in Eq.(3.20), yielding

$$\Phi'' - 4k\Phi' - m_{dil}^2\Phi + 6M^{\frac{7}{2}} b \exp(4ky) \sum_{n=0}^{\infty} \frac{\left(\Phi/M^{3/2}\right)^n}{n!} = 0 \tag{3.23}$$

from which we can obtain a series solution for the first order b term. Thus to leading order in b, the solution reads

$$\Phi(y) = C_0 \exp\left[2kr(1-\nu)\phi\right] + b \sum_{n=0}^{\infty} \frac{6M^{7/2}}{k^2} \frac{\left(\Phi_0/M^{3/2}\right)^n}{n!} \frac{\exp\left[kr\phi(2n(1-\nu)+4)\right]}{(\omega_n^2 + 4\omega_n - m_{dil}^2/k^2)} \tag{3.24}$$

Note that in order for this perturbation series to be valid over the entire bulk spacetime, we must require that $b \lesssim \exp(-4kr\pi) \approx 10^{-64}$. In other words, the existence of a perturbative solution around RS requires b to be severely fine-tuned. The leading order contribution in the above equation comes from the $n = 0$ term in the summation. Considering only this term, and using $\nu = \sqrt{4 + m_{dil}^2/k^2}$, the truncated solution for Φ is

$$\Phi(y) = \Phi_0 \exp[-m_{dil}^2 y/4k] - \frac{6bM^{7/2}}{m_{dil}^2} \exp[4ky] \tag{3.25}$$

Substituting this into Eq.(3.21), we obtain a solution for the warp factor $A(y)$ to leading order in b,

$$\begin{aligned} A(y) &= ky - \frac{\Phi_0}{M^{3/2}} \exp[-m_{dil}^2 y/2k] - 32b \frac{\Phi_0 M^{1/2} k^2}{(16k^2 - m_{dil}^2)^2} \exp(4k - m_{dil}^2/4k)y \\ &- bM^2 \int \left[\int \exp\left[2ky + \frac{\Phi_0}{M^{3/2}} \exp[-m^2 y/4k]\right] dy \right] dy \end{aligned} \tag{3.26}$$

In the last term on the RHS of the above equation, $\frac{\Phi_0}{M^{3/2}} \exp[-m_{dil}^2 y/4k] << 2ky$, so we can approximate the integrand and obtain

$$\begin{aligned} A(y) &= ky - \frac{\Phi_0}{M^{3/2}} \exp[-m_{dil}^2 y/2k] - 32b \frac{\Phi_0 M^{1/2} k^2}{(16k^2 - m_{dil}^2)^2} \exp(4k - m_{dil}^2/4k)y \\ &- \frac{bM^2 \exp(\frac{\Phi_0}{M^{3/2}})}{(2k - m_{dil}^2/4k)^2} \exp[(2k - m_{dil}^2/4k)y] \end{aligned} \tag{3.27}$$

Eq.(3.27) is the back-reacted expression for the warp factor $A(y)$, where the second and third/fourth terms on the RHS are the contributions from the dilaton and KR field respectively. In the absence of the dilaton field, this expression reduces to the one for the warp factor in a KR-gravity bulk spacetime [6].

We now consider the expression for the physical mass parameter on the visible brane, given by

$$m_{phys} = m_0 \exp[-A(y)]_{y=r\pi} \tag{3.28}$$

where m_0 is the fundamental mass parameter. We use Eq.(3.27) and take $kr_c \sim 12$, $k \sim M_{Pl}$, and $r_c \sim l_{Pl}$ in order to agree with the values that Randall and Sundrum used in their original model. We also take $\Phi_0 \sim M^{3/2}$. Using these values, we estimate the warp factor at the visible brane as

$$[A(y)]_{y=\pi} = 37 - 10^{-16} - b10^{62} - b10^{31} \tag{3.29}$$

Since the perturbative solution is only valid for $b \lesssim 10^{-64}$, the backreacted value of $A(y)$ on the visible brane is always positive, and very close to the RS value. Thus the above solution results in a small fluctuation to the RS value of $A(y)$ in a self-consistent manner. We conclude from this analysis that the hierarchy problem can be resolved even in the presence of the dilaton and Kalb-Ramond fields, albeit at the cost of the parameter b necessarily being very fine-tuned. It is important to emphasize that this conclusion applies only to the case of perturbative solutions around the original Randall-Sundrum model. The question remains of whether there exist non-perturbative solutions to the equations of motion (3.20) and (3.21) was not treated by Das et al., and it is to this question that we now turn.

Chapter 4

Non-perturbative solutions to the Kalb-Ramond-dilaton braneworld

4.1 Goals and the nondimensionalized equations of motion

There may be solutions to the equations of motion for this system that are not simply small fluctuations around the Randall-Sundrum solution. A goal would be to find solutions which satisfy three criteria:

- **Sufficient warping at the visible brane.** We need to achieve a warp factor at $y = \pi$ which achieves a reduction in the physical mass scale on the visible brane to $\sim$ the electroweak scale. Ie reduction by a factor of $10^{-16} - 10^{-17}$ from the Planck scale.

- **No fine-tuning of any parameters.** In the perturbative case, we achieved sufficient warping, but only at the cost of fine-tuning $b \lesssim 10^{-64}$. We would like to find solutions for values of the Kalb-Ramond field strength that are not so fine-tuned. *although we will be limited by observational constraints on just how big this background value can be.. see torsion.*

- **Stability of the r_c modulus.** Our solutions should generate a potential for the value of r_c which has a local minimum at a value which is $\sim l_{Pl}$.

We return to the equations of motion,

$$\Phi'' - 2A'\Phi' - m_{dil}^2\Phi + 6M^{\frac{7}{2}} b \exp(2A) \exp(\Phi/M^{3/2}) = 0 \tag{4.1}$$

$$A'' = -\frac{1}{6M^3}\Phi'^2 - bM^2 \exp(2A) \exp(\Phi/M^{3/2}) \tag{4.2}$$

Before looking for solutions to these equations, we first express them in terms of dimensionless parameters. We define $\Phi = PM^{3/2}$ and $m_{dil} = m'M$, where P and m' are dimensionless. We also go back to a dimensionless bulk interval, $0 < \phi < \pi$, by substituting $y = r_c\,\phi$. Plugging these into Eq.(4.1), we get

$$\begin{aligned} \frac{1}{M^{7/2}\,r_c^2} M^{3/2} P''(\phi) \quad &- \quad \frac{2}{M^{7/2}\,r_c^2} M^{3/2} A'(\phi) P'(\phi) \\ &- \quad \frac{m'^2}{M^{3/2}} M^{3/2} P(\phi) \\ &+ \quad 6b \exp(2A(\phi)) \exp P(\phi) = 0 \end{aligned} \tag{4.3}$$

where the prime now denotes differentiation with respect to ϕ.

Lastly, we define $r_c = r' l_p \sim r'/M$, where l_p is the Planck length, and obtain

$$P''(\phi) - 2A'(\phi)P'(\phi) - r'^2 m'^2 P(\phi) + 6\,b\,r'^2 \exp(2A(\phi)) \exp P(\phi) = 0 \tag{4.4}$$

Nondimensionalizing Eq.(4.2) in a similar fashion yields

$$A''(\phi) = -\frac{1}{6}P'(\phi)^2 - b\,r'^2 \exp(2A(\phi)) \exp P(\phi) \tag{4.5}$$

4.2 Boundary conditions and the computational setup

In order to solve this system of equations, we begin by fixing boundary conditions, and then employ a shooting method in an attempt to find solutions which gain a value of the warp factor at $\phi = \pi$ sufficiently large to reduce the physical mass by $10^{-16} - 10^{-17}$ orders of magnitude. This translates to a value of $A(\pi)$ between 36.84 and 39.14. We have two second-order ODEs, so we must choose four boundary conditions in order for the problem to be well posed. We choose to fix

1. The value of $A(0) = 0$.
2. The value of $A'(0)$. Note that in the original Randall-Sundrum scenario, the slope of A is discontinuous at the orbifold fixed points at $\phi = 0, \pi$. This is due to the localized energy density in the V_{hid} and V_{vis} terms on the branes.
3. The value of $P(0)$
4. The value of $P'(0)$

The shooting method will vary the value of $P'(0)$ for a given choice of values $(b, m', A'(0), P(0))$, and look for a solution with $36.84 \leq A(\pi) \leq 39.14$.

We now consider some properties of the individual terms in Eqs.(4.4) and (4.5), in order to identify characteristics that a general solution must have. The following arguments come from experience in looking at numerical solutions to the equations, and serve as a qualitative explanation of why the solutions we present will have the form and parameter values that they do.

Looking at Eq.(4.5), we see that both terms on the right hand side are always negative. Thus any solution for A will have a slope that is monotonically decreasing over the interval. If A' decreases too quickly, we will not achieve the desired warping of $A(\pi) = 36.84$. Therefore we must attempt to either minimize the magnitude of these two terms, pick a sufficiently large value of $A'(0)$, or some combination of the two. The exponential term in Eq.(4.5) presents a particular problem, since e.g. for $b \sim 1$, the factor of $\exp(2A)$ will quickly turn A' negative. The only way to minimize the influence of this term is to make the value of the dilaton *negative* over the interval. P must be negative enough to make this exponential term sufficiently negligible over the entire interval. This exponential term, which also occurs in Eq.(4.4), is the only place that b is present. It follows that we can take b as large as we like, at the cost of P being sufficiently negative to make the exponential term negligible [1].

The $P'(\phi)^2$ term in Eq.(4.5) also presents a problem. We would also like to keep this as close to zero as possible.. Naively, we might look for a solution with $P'(0) = 0$ and hope that it does not change much over the interval, but a quick inspection of Eq.(4.4) shows that this is impossible. In Eq.(4.4) we can minimize the exponential term in the same fashion as we did for Eq.(4.5), but minimizing $P'(\phi)$ then implies $P''(\phi) \approx r'^2 m'^2 P(\phi)$. This results in $|P'(\phi)|$ rapidly becoming large, A' becoming negative, and the occurrence of the kink in P. See Fig. 4.2.

The kinklike rapid change in P is correlated with A' becoming negative, and our goal in finding a solution which realizes $A(\pi) \geq 36.84$ will be trying to eliminate this kink. To accomplish this, we will attempt to minimize the $P''(\phi)$ term in eq. (4) over as much of the interval as possible. This is equivalent to minimizing the

[1]Note that the requirement of a large and negative value of P differs from the the assumption by Das et al. above Eq.(3.29) that $P \equiv \Phi_0 \sim M^{3/2}$. This choice was for the perturbative case, however.

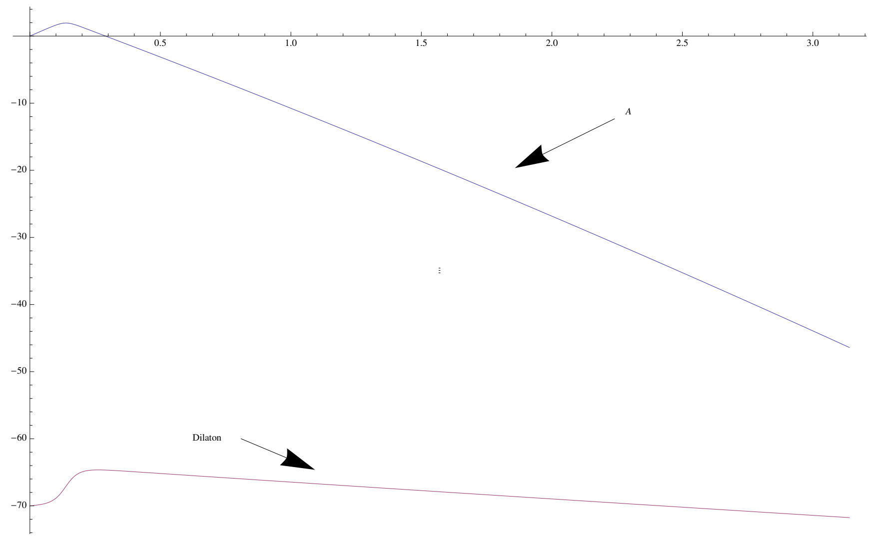

Figure 4.1: Solutions are sensitive to $P'(0)$.

exponential term and also requiring

$$A'(\phi)P'(\phi) \approx -r'^2 m'^2 P(\phi). \tag{4.6}$$

This latter condition, which we will refer to as the *slope-tuning condition*, turns out to be possible for a wide range of initial parameters r',m', and $A'(0)$. Since $P(\phi)$ is always negative, we can satisfy this condition for $A'(\phi)$ and $P'(\phi)$ both positive. We can envision a solution with $A'(\phi)$ positive over the entire interval, so our only constraint is that $P'(\phi)$ must be positive. A sufficient fine-tuning of $P'(\phi)$ at $\phi = 0$ can result in solutions in which the kink is pushed essentially all the way to $\phi = \pi$, as in Fig. 4.2.

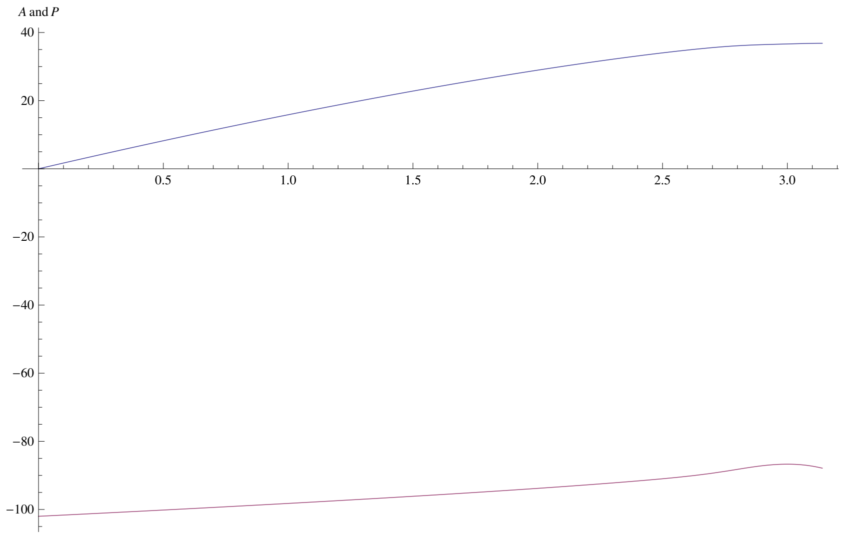

Figure 4.2: A good solution. $A(\pi) = 36.84$, $b = 10^{-3}$, $r' = 12$, $m' = 1/11$, $P(0) = -102$, and $A'(0) = 17$.

4.3 Value of the dilaton and higher order curvature terms

In the solutions that we have described so far we have set $r' = 12$ in accordance the original Randall-Sundrum solution. We still take $k \sim M_{Pl}$. Due to the monotonically decreasing nature of $A'(y)$ in our solutions, it is clear that we will require a value of $A'(0)$ which is greater than the Randall-Sundrum value in order to achieve the desired warping at the visible brane. For example, in Randall-Sundrum, we have a linearly increasing warp factor with a constant value of $A'(\phi) = \ln(10^{16}/\pi) \simeq 11.7$. In contrast, the solution in Fig. 4.2 has $A'(0) = 17$, which decreases over the bulk interval to $A'(\pi) = -0.37$. The curvature is clearly not constant, and if we compute the value of the Ricci scalar, we obtain

$$R = g_{\mu\nu} R^{\mu\nu} = \frac{5A'(\phi)^2 - 4A''(\phi)}{r'^2} \tag{4.7}$$

which for $r' = 12$ and $A'(\phi) = 11.7$ yields an R which has a maximum at the Planck brane of $\simeq 4.75$.

The original Randall-Sundrum model with the scalar dilaton included could potentially have terms which involve coupling of the dilaton to scalars formed out of higher order derivative terms of the metric. Terms such as $\Phi^{2/3}R^2$, $\Phi^{2/3}R_{\mu\nu}R^{\mu\nu}$, or $\Phi^{2/3}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ could potentially be large enough that our classical approximation would no longer be valid. In the case of Goldberger-Wise stabilization, however, the constraint that determines the value of the minimum is

$$kr_c = \left(\frac{4}{\pi}\right)\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right]. \tag{4.8}$$

Goldberger and Wise avoid any fine-tuning of the value of v_h with respect to v_v by taking $\ln(v_h/v_v) \sim 1$, but otherwise the magnitudes of v_h and v_s are not constrained. Thus v_h^2/M^3 and v_h^2/M^3 can be taken small enough to be to make $\Phi^{2/3}R^2$ or any other higher derivative term in the Lagrangian small enough that we do not have to worry about these terms affecting the validity of our approximation. Indeed, v_h^2/M^3 and v_h^2/M^3 being small was taken by G&W as a condition in order to be able to neglect the backreaction of the scalar field on the background geometry for the computation of $V(r_c)$.

Our non-perturbative solutions present a problem in the context, however, as a large negative value of the scalar dilaton is required to achieve sufficient warping at the visible brane. A term such as $\Phi^{2/3}R$ is clearly not negligible here, and other higher derivative couplings may also present problems. To resolve this problem, note that given a solution, we can rescale the parameters b, m', and r' to obtain an otherwise identical looking solution. These parameters must satisfy $m'r' = \text{const}$ and $br'^2 = \text{const}$. For example a solution with $m' = 1/11$, $r' = 12$, and $b = 10^{-3}$ looks the same as one with $m' = 1/110$, $r' = 120$, and $b = 10^{-5}$. Looking at at Eq.(4.7), the latter solution will have a scalar curvature which looks identical to the former, except 2 orders of magnitude smaller. This still does not introduce a significant fine tuning in either r', m', or b, but it reduces the magnitude of terms such as $\Phi^{2/3}R^2$ to manageable levels.

In fig. 4.3, we show R^2 plotted over the bulk interval for the solution shown in fig. 4.2, except with $b = 10^{-5}$, $r' = 120$, and $m' = 1/110$, as described above. R^2 for the original solution would look identical, but with a value on the Planck brane of ~ 100 rather than .01. Fig. 4.4 shows the associated value of $\Phi^{2/3}R^2$.

Other curvature invariants such as the Ricci tensor squared and Riemann tensor squared can appear in higher derivative couplings. In our warped geometry, these have the form

$$R_{\mu\nu}R^{\mu\nu} = \frac{5A'(\phi)^4 - 8A'(\phi)^2 A''(\phi) + 5A''(\phi)^2}{r'^4}, \tag{4.9}$$

and

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{3A'(\phi)^4 + 2\left(A'(\phi)^2 - 2A''(\phi)\right)^2}{2r'^4} \tag{4.10}$$

Figs. 4.5 and 4.6 show the behavior of these curvature invariants over the bulk interval, for the $r' = 120$ solution. As these are even smaller in magnitude than R^2, terms such as $\Phi^{2/3}R_{\mu\nu}R^{\mu\nu}$ or $\Phi^{2/3}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ will also be sufficiently small. Thus we see that we can adjust the parameters r', b, and m' to obtain solutions where higher derivative coupling terms do not ruin our classical approximation.

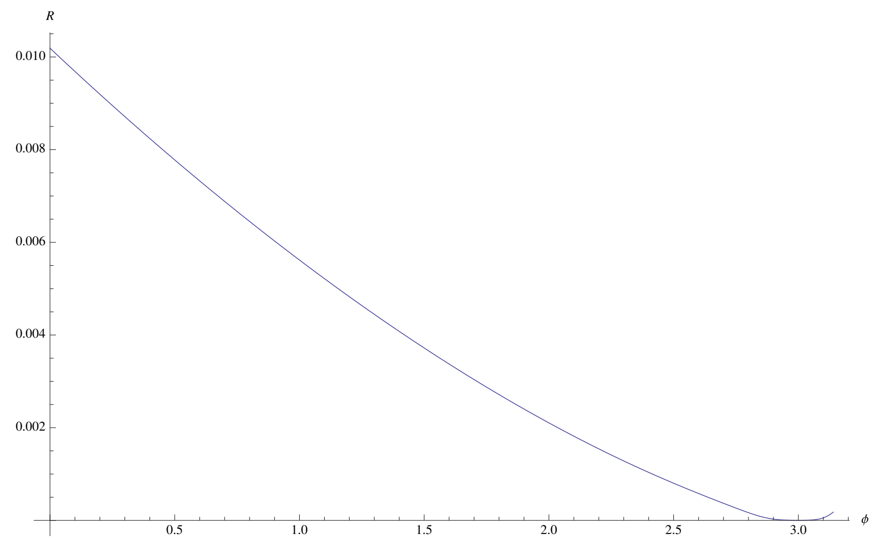

Figure 4.3: The scalar curvature squared. $A(\pi) = 36.84$, $b = 10^{-5}$, $r' = 120$, $m' = 1/110$, $P(0) = -102$, and $A'(0) = 17$.

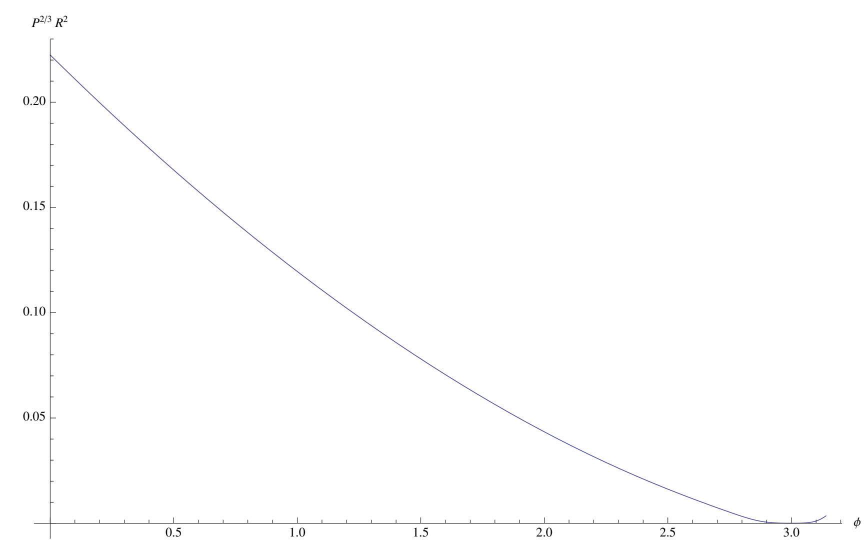

Figure 4.4: $|P|^{2/3}R^2$ for $A(\pi) = 36.84$, $b = 10^{-5}$, $r' = 120$, $m' = 1/110$, $P(0) = -102$, and $A'(0) = 17$.

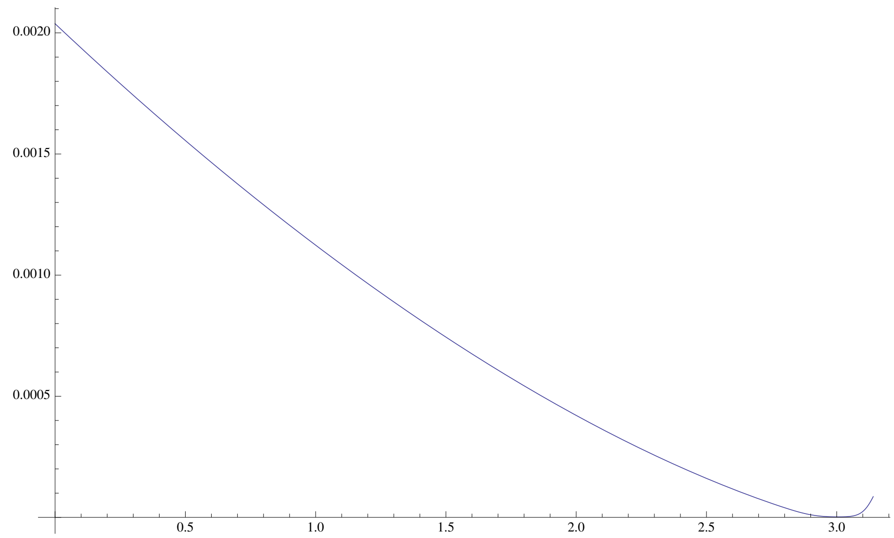

Figure 4.5: The Ricci tensor squared, $R_{\mu\nu}R^{\mu\nu}$. $A(\pi) = 36.84$, $b = 10^{-5}$, $r' = 120$, $m' = 1/110$, $P(0) = -102$, and $A'(0) = 17$.

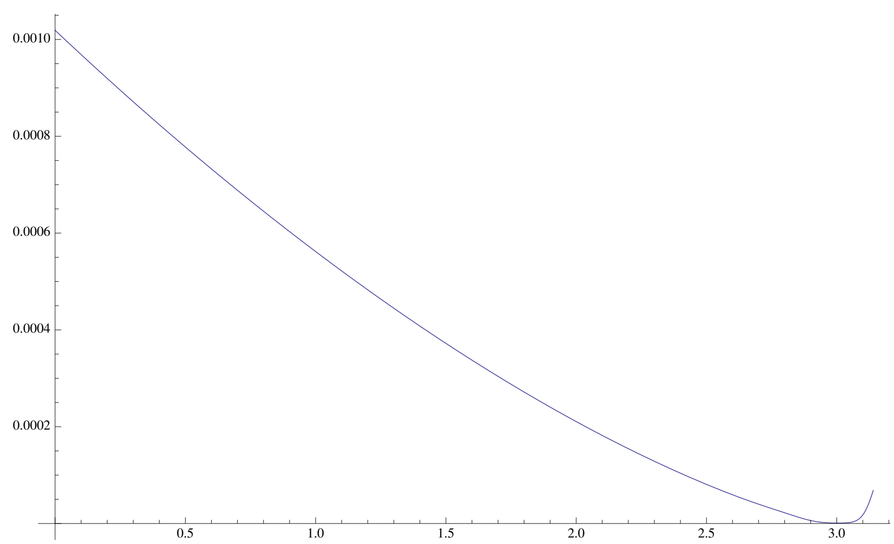

Figure 4.6: The Riemann tensor squared, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. $A(\pi) = 36.84$, $b = 10^{-5}$, $r' = 120$, $m' = 1/110$, $P(0) = -102$, and $A'(0) = 17$.

4.4 Parameter space of solutions

Having found that non-perturbative solutions do in fact exist, we would like to find the region of parameter space $(b, m', A'(0), P(0), P'(0))$ in which solutions exist. We do this here for a subset of this parameter space, namely for $b = 10^{-5}$and $m' = 1/110$. After fixing the boundary condition $A(0) = 0$, we look for solutions which gain a warp factor of at least $A = 36.84 = \ln(10^{16})$ on the visible brane. We find that there is minimum value of $A'(0) = A'_{crit}$ for which sufficient warping can be achieved. As we vary $A'(0)$, there exists a variable range of values of $P(0)$, always negative, for which we can find solutions. These solutions will not necessarily be stable, which is an issue will we treat in the next section in order to refine our space of solutions.

As previously mentioned, due to the $-(1/6)P'(\phi)^2$ term on the RHS of Eq. (4.5), $A'(\phi)$ will be montonically decreasing over the interval. Thus $A'(0)$ must be greater than the Randall-Sundrum value, $A'(\phi)_{RS} \simeq 11.7$. The value of $P(0)$ must also be sufficiently negative to make the exponential term negligible, but as can be seen from Eq.(4.5), it cannot be *too* negative, or else A' will decrease too fast to ever achieve the desired warping at the visible brane, see Fig. 4.7. We will thus have a range of values of $P(0)$, which will vary as a function of $A'(0)$, for which we can obtain solutions with sufficient warping.

In Fig. 4.8, we show a picture of the shape of this solution space. The vertical lines correpond to values of $A'(0)$ where we calculated the corresponding range of valid $P(0)$ values. Only a limited number of $A'(0)$ values were considered due to limited time and computational resources, but the general shape of the solution space is indicated. We see that A_{crit} is approximately 15.1, and that there is an increasing range of valid $P(0)$ values corresponding to increasing values of $A'(0)$. The solution space is unbounded as $A'(0)$ increases, but this correponds to the dilaton become increasing more large and negative. This may present problems for solutions with large values of $A'(0)$ due to higher order derivative couplings as mentioned earlier, but we leave discussion of this problem to future works.

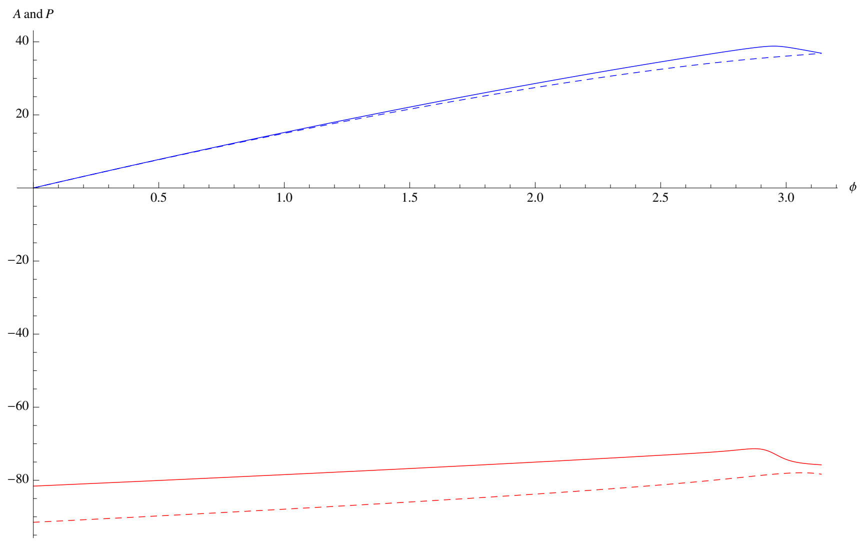

Figure 4.7: Solutions at boundaries of allowed $P(0)$ range for $b = 10^{-5}, m' = 1/110, A'(0) = 16$.

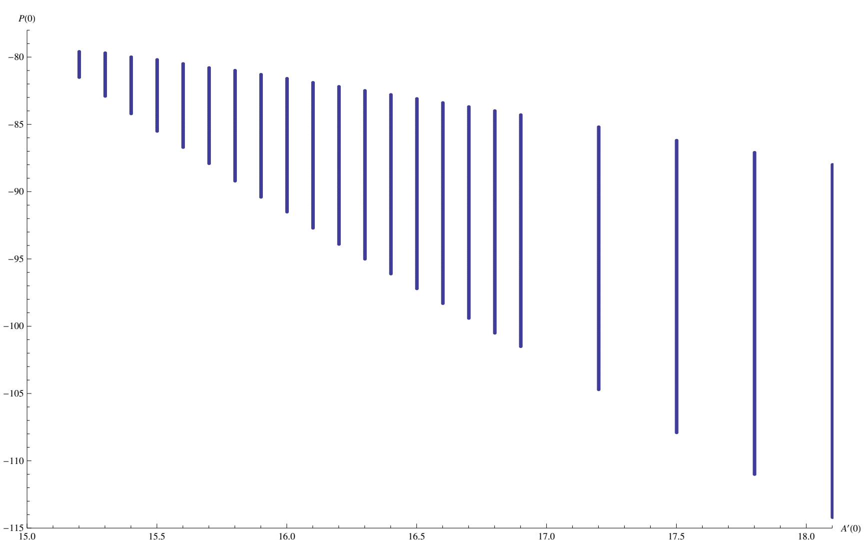

Figure 4.8: Partial map of solution space for $b = 10^{-5}, m' = 1/110$.

4.5 A space of stable solutions

In the previous section, we fixed b, m', and r_c and found a range of values of $A'(0)$ and $P(0)$ for which solutions existed which achieved sufficient warping on the visible brane. However we have not addressed the issue of stabilization of the value of r_c. To generate a potential for r_c, we take our solutions for the scalar dilaton, plug them back into the action, and integrate over ϕ (we are back to dimensionful parameters, ie ($' \equiv d/dy$)),

$$\begin{aligned} V_\Phi(y_\pi) &= \int_0^{y_\pi} dy \exp(-2A(y)) \exp(-\Phi(y)/M^{3/2})[-2H_{MNL}H^{MNL}] \\ &+ \int_0^{y_\pi} dy \exp(-2A(y)) \frac{1}{2}[\Phi'(y)^2 + m_{dil}^2 \Phi(y)^2] \\ &+ \exp(-2A(0))\lambda_p(\Phi^2(0) - v_p^2)^2 + \exp(-2A(y_\pi))\lambda_s(\Phi^2(y_\pi) - v_s^2)^2. \end{aligned} \tag{4.11}$$

Here, the factors of $\exp(-2A)$ in the bulk and interaction terms come from the determinants of the bulk and brane metrics. For the ground state configuration of Φ, we take

$$\Phi(0) = v_p \quad \Phi(\pi) = v_s \tag{4.12}$$

as we are considering the case in which λ_p and λ_p are infinite, and in this case it is energetically favorable to do so. We therefore ignore the brane interaction terms.

In the first line of Eq.(4.11), we use $H^{\mu\nu\rho} = 0$ to write

$$-2H_{MNL}H^{MNL} = -6H_{y\mu\nu}H^{y\mu\nu} = 6G^{\mu\alpha}G^{\nu\beta}H_{y\mu\nu}H_{y\alpha\beta}. \tag{4.13}$$

We then use Eq.(3.18) to get

$$V_\Phi(y_\pi) = \int_0^{y_\pi} dy \left(6bM^5 \exp\left(\Phi(y)/M^{3/2}\right) + \frac{1}{2}\exp\left(-2A(y)\right)[\Phi'(y)^2 + m_{dil}^2\Phi(y)^2]\right). \tag{4.14}$$

We again nondimensionalize this equation as in section 4.1, yielding

$$V_\Phi(r') = \int_0^{\pi} r'M^4 d\phi \left(6b\exp\left(P(\phi)\right) + \frac{1}{2}\exp\left(-2A(\phi)\right)\left[\frac{P'(\phi)^2}{r'^2} + m'^2P(\phi)^2\right]\right) \tag{4.15}$$

where we have used $dy = r_c d\phi \sim (r'd\phi/M)$, and we are back to $' \equiv d/d\phi$. Given this expression, we can compute the potential for the solutions that were found in the previous section.

These solutions were generated based solely on the condition of achieving a sufficient warp factor on the visible brane, and so far we have said nothing about whether they are near a minimum of the potential $V_\Phi(r')$, or whether a such a minimum even exists. However, by looking at the behavior of our solutions in the region where the kink in the dilaton is smoothed out, we can construct a general argument for why such stable solutions should be expected to exist:

As we have seen in the process of tuning the value of $P'(0)$ in order to achieve a sufficiently warped solution, a failure to precisely match the slope-tuning condition will result in a solution which has a kink in the dilaton field which either turns up or down, and an accompanying downturn in the slope of $A(\phi)$. As we get closer to satisfying the slope-tuning condition, these two solutions can converge to one in which the dilaton has no kink. Consider a solution with no kink in the dilaton, for which $r' = r_{crit}$. For this solution, $A(\phi)$ will be larger than for solutions which have a dilaton kink. Start by perturbing the value of r' slightly downward, which results in a solution with the kink up, and a smaller value of $A(\phi)$ over the rightmost part of the interval. Now increase r', passing through r_{crit}, and consider what happens to the terms in Eq.(4.15). As r' increases, $A(\phi)$ increases until we pass r_{crit}, after which it decreases again. Thus $\int \exp(-2A(\phi))$ achieves a local minimum. The $P'(\phi)^2$ term also decreases and then increases, due to the kink vanishing as we pass r_{crit}, although it is affected by a constantly increasing r'^2 in the denominator. Finally, the behavior of $P(\phi)^2$ is more complicated, but it is multipled by $\exp(-2A(\phi))$. We see that, neglecting the term proportional to b, we expect to possibly see a local minimum in the potential integral at some point around $r' = r_{crit}$.

To confirm this, we employ a simple potential difference method to look at the slope of the potential for our solutions, and find that if we vary the value of r', we always come across a closely related solution which does lie at a minimum, see Fig. 4.9. The shape of the potential $V_\Phi(r')$ that we generally find is far more sensitive to changes in r' for $r' > r'_{min}$ than for $r' < r'_{min}$.

Note that the main effect of perturbing r' on the shape of a solution is to disturb the slope-tuning condition, Eq.(4.6). We can achieve the same effect by perturbing the value of $P'(0)$ in the opposite direction. In addition, perturbing r' will slightly change the bounds on our solution space through its effect on the exponential term in Eq.(4.1). For this reason, we choose to vary $P'(0)$ instead of r'. This will allow to keep our previously generated solution space fixed, and look for stable solutions within this space which lie at a fixed value of $r' = 120$.

Our approach will be to take each of the solutions from the previous section

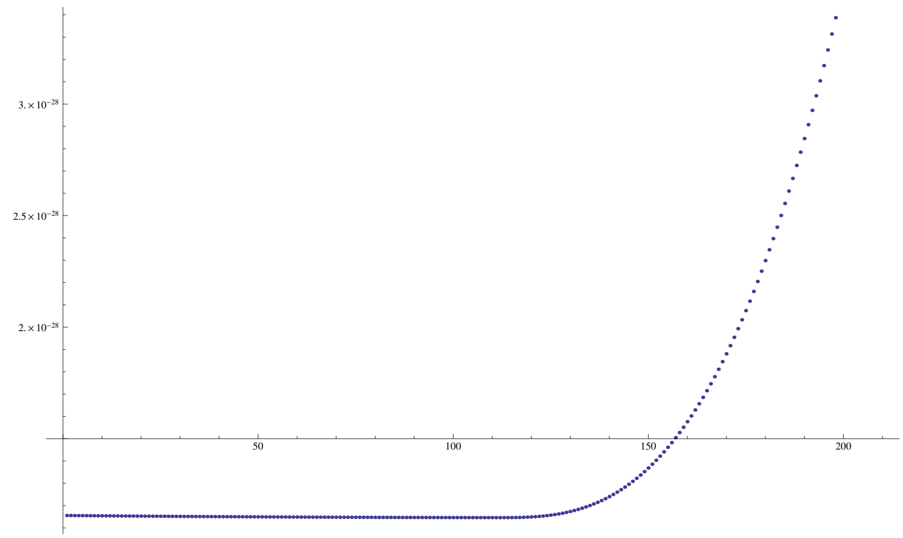

Figure 4.9: A typical potential $V_\Phi(r')$.

which achieve sufficient warping at the visible brane, and vary the value of $P'(0)$ (equivalently, v_s) until a first order difference approximation to the slope dV/dr' indicates we are sufficiently near a minimum. We then consider this new solution, and look at how the value of $A(\pi)$ has changed.

Fig. 4.10 shows our results for $b = 10^{-5}, m' = 1/110,$ and $r' = 110$. Within our solution space, there is a region in which we can find a value of $P'(0)$ which simultaneously achieves a minimum of $V_\Phi(r')$ and results in a sufficient warping on the visible brane. Notice that even though our general solution space is unbounded except for limits due to higher order corrections, our stable solution space exists for only a limited range of $15.2 \lesssim A'(0) \lesssim 18.2$.

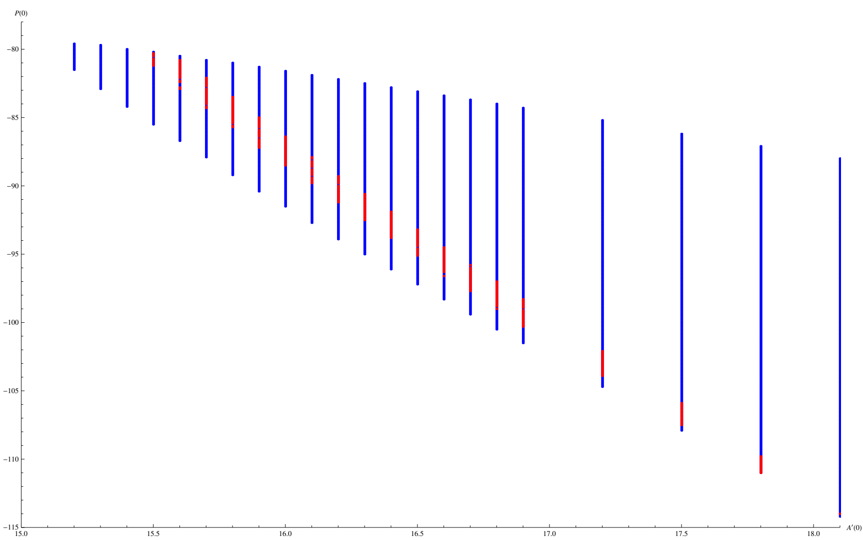

Figure 4.10: Stable solutions. $b = 10^{-5}, m' = 1/110, r' = 110$

Chapter 5

Discussion and Conclusions

We have analyzed the scalar dilaton-Kalb-Ramond-gravity system in a five-dimensional Randall-Sundrum setup, interpreting the dilaton as the Goldberger-Wise scalar which stabilizes the size of the extra dimension. Although solutions perturbative around the original Randall-Sundrum solution require a fine-tuning of the strength of the Kalb-Ramond field, we have found that non-perturbative solutions exist which require no such fine-tuning of any parameters, and which generate a potential for the radius of the extra dimension which in some cases has a minimum.

A number of issues remain open for further analysis. We have not yet calculated the steepness of the potential minima that we have found, corresponding to the mass of the radius modulus. There are various observational constraints lower bounding this value, see [10, 4, 5], and calculation of this value for our solutions, as well as other regions of the parameter space would be a natural next step in the analysis.

We also have not investigated the contact between our chosen value of $b = 10^{-5}$ and observational constraints. The Kalb-Ramond field is associated with torsion in our bulk spacetime [14], although this torsion is suppressed on the visible brane due to the warp factor. It would be interesting to explore the limits on the

b parameter in our model due to the observed limits on the torsion. We have chosen $b = 10^{-5}$ semi-arbitrarily: big enough to distance ourselves from the fine-tuned value of $b \simeq 10^{-64}$ necessary for perturbative solutions, but small enough to not require extremely large values of the scalar dilaton, and with a glance towards the observational constraints just mentioned.

Another interesting feature of our solutions is that some of them have negative value of $A'(\pi)$, suggesting that the visible brane has an positive tension, in contrast to the negative tension required in the original Randall-Sundrum scenario. The total cosmological constant on the visible brane is a sum of the negative bulk cosmological constant and the visible brane tension, so this suggests the possibility of a deSitter solution in the segment of the model that is supposed to represent our visible universe. See [12] for another example of a model which does not require negative tension branes.

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Vita

Stephen Christopher Young was born in Harbor City, California on March 26th, 1978, the son of Dr. Stephen J. Young and Veronica M. Young. He received a B.S. in Physics from the University of California, San Diego in 2001. He was accepted into the physics program at the University of Texas at Austin in fall 2002.

Permanent address: 6500 Nancy Road, Rancho Palos Verdes, CA 90275

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