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Interaction and Marginal Effects in Nonlinear Models: Case of Ordered Logit and Probit Models

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Abstract

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Interaction and marginal effects are often an important concern, especially when variables are allowed to interact in a nonlinear model. In a linear model, the interaction term, representing the interaction effect, is the impact of a variable on the marginal effect of another variable. In a nonlinear model, however, the marginal effect of the interaction term is different from the interaction effect. This report provides a general derivation of both effects in a nonlinear model and a linear model to clearly illustrate the difference. These differences are then demonstrated with empirical data. The empirical study shows that the corrected interaction effect in an ordered logit or probit model is substantially different from the incorrect interaction effect produced by the *margins* command in Stata. Based on the correct formulas, this report verifies that the interaction effect is not the same as the marginal effect of the interaction term. Moreover, we must be careful when interpreting the nonlinear models with interaction terms in Stata or any other statistical software package.

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Chapter 1: Introduction

Researchers are often interested in the estimation of interaction terms to infer how the effect of one independent variable on the dependent variable depends on the level of another independent variable. They would also like to measure the marginal effect to find a good approximation to the amount of change in a dependent variable for each one unit change in an independent variable.

In linear models, the estimation and interpretation of the coefficient associated with the interaction term between two variables are not complicated. As demonstrated by Ai and Norton (2003) with nonlinear models, however, the estimation and interpretation of the coefficient associated with the interaction term becomes more complicated. Norton, Wang, and Ai (2004) also pointed out that the marginal effect of a change solely in the interaction term is completely separate from that of a change in both the variables included in the interaction.

Moreover, we must be careful of the sign that may be different for different observations. Buis (2010) maintained the following:

> "The marginal effect is an approximation of how much the dependent variable is expected to increase or decrease for a unit change in an explanatory variable: that is, the effect is presented on an additive scale. The exponentiated coefficients give the ratio by which the dependent variable changes for a unit change in an explanatory variable: that is, the effect is presented on a multiplicative scale." (p. 305)

Thus, it is very important to understand that the marginal effect in a nonlinear model with any interaction term differs from the marginal effect in the model without an interaction term. When reviewing 13 economics journals between 1980 and 2000, Norton, Wang, and Ai (2004) found 72 articles that mentioned interaction terms in nonlinear models and the articles misinterpreted the coefficient associated with the interaction term. The complicated marginal effect for a logit or a probit model can be easily computed by using Norton's inteff command which is a user-written add-on module for Stata. However, the command is not applicable for ordered logit and probit models which will be discussed in this report. Hence, this report will provide the correct mathematical formula and will demonstrate the correct computation of the marginal effect for a change in the two variables included in the interaction term in ordered logit and probit models.

This report will first present the estimation of interaction effects for linear models and nonlinear models with formulas, followed by an explanation of ordered logit and probit models. The report will also employ Korean data drawn from the *Asian Barometer Survey* to correctly estimate interaction effects in ordered logit and probit models using Stata. The methods would be applicable to other software packages that estimate ordinal response models. The appendix contains the summary for the data used in Chapter 4.

Chapter 2: Estimation of Interaction Effects

To explain a general derivation of interaction effects in both linear and nonlinear models, this chapter closely follows Norton, Wang, and Ai (2004).

Linear Models

Consider that the dependent variable *y* depends on two independent variables, x_1 and x_2, their interaction term ($x_1 x_2$), and a vector of an additional independent variable *Z*, including the constant term. The expected value of the dependent variable *y*, conditional on the independent variable, is

$$E: = E[y|x_1, x_2, Z] \; = \; \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + Z\beta,$$

where the parameters βs are unknown and the vector *Z* excludes x_1 and x_2.

Suppose that two independent variables, x_1 and x_2, are continuous, and the marginal effect of x_1 on *E* depends on x_1 if β_{12} is non-zero:

$$\frac{\partial E}{\partial x_1} = \; \beta_1 + \beta_{12} x_2.$$

The interaction effect, which is the impact of a marginal change in x_2 on the marginal effect of x_1, comes out by taking the derivative of the above with respect to x_2:

$$\frac{\partial^2 E}{\partial x_1 \, \partial x_2} = \; \beta_{12}.$$

From the above result, in linear models, the interaction effect, $\frac{\partial^2 E}{\partial x_2 \, \partial x_1}$, is equivalent to the marginal effect, $\frac{\partial E}{\partial (x_1 x_2)}$, of the interaction term, $x_1 x_2$. For nonlinear models, however, this equality generally is not the same, as is demonstrated in the following section.

Nonlinear Models

The probit model, one type of nonlinear model, is now used in order to show the derivation of the interaction effect. This is similar to the previous example, but the dependent variable y is a dummy variable, not a continuous variable. The response is modeled as a transformation of the standard normal cumulative distribution function as follows:

$$\begin{aligned}\Pr[y = 1|x_1, x_2, Z] &= \Phi(\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + Z\beta) \\ &= \Phi(u),\end{aligned}$$

where $\Phi(u)$ is the standard normal cumulative distribution[1] and u represents $\beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + Z\beta$. If two independent variables, x_1 and x_2, are continuous, then the marginal effect of just the interaction term $x_1 x_2$ is

$$\frac{\partial\Phi(u)}{\partial(x_1 x_2)} = \beta_{12}\phi(u),$$

where $\phi(u)$ is $\Phi'(u)$. However, the full interaction effect is the cross partial derivative of $E[y|x_1, x_2, Z]$:

$$\frac{\partial^2\Phi(u)}{\partial x_1\,\partial x_2} = \beta_{12}\phi(u) + (\beta_1 + \beta_{12}x_2)(\beta_2 + \beta_{12}x_1)\,\phi'(u).$$

We can see that the full interaction effect is obviously different from the marginal effect of the interaction term, $x_1 x_2$, $\beta_{12}\phi(u)$.

Norton, Wang, and Ai (2004) pointed out that there are some crucial implications by drawing the above equation for nonlinear models. First of all, even if β_{12} is zero, the interaction effect could be nonzero. For example, for a probit model including β_{12} that

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[1] $\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} \exp\left(\frac{-t^2}{2}\right) dt$

is equal to zero, the interaction effect could be $\beta_1\beta_2\phi'(u)$ which is definitely nonzero. Also, the test for determining the statistical significance of the interaction effect is not simple. Instead of a conducting a *z* test for the statistical significance of the coefficient of just β_{12}, we can determine its statistical significance with a test associated with the entire cross derivative. Moreover, the interaction effect in nonlinear models is conditional on the independent variables. Finally, since there are two additive terms, each of which can be positive or negative, the interaction effect may have opposite signs for different observations. Therefore, the sign of β_{12} does not always reflect the sign of the interaction effect.

Consider *F*(*u*) as a nonlinear function of $u \coloneqq \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + Z\beta$. For example, *F* is the probability that *y* equals 1. Also, for logit and probit models, we clarify the interaction effect to be the change in the predicted probability that *y* equals 1 per unit change in bothx_1 and x_2. Now we can think about some of the general formulas for the interaction effects resulting from nonlinear models as in the following argument.

If x_1 and x_2 are both continuous variables, the interaction effect is the double derivative with respect to x_1 and x_2:

$$\frac{\partial^2 F(u)}{\partial x_1\, \partial x_2} = \frac{\partial\{F(\beta_1+\beta_{12}x_2)F^{'}(u)\}}{\partial x_2}$$

$$= \beta_{12}F'(u) + (\beta_1 + \beta_{12}x_2)(\beta_2 + \beta_{12}x_1)\, F''(u),$$

where *F'*(*u*) and *F''*(*u*) denote the first and second derivatives.

If x_1 and x_2 are both dummy variables, the interaction effect is the discrete double difference:

$$\frac{\Delta^2 F(u)}{\Delta x_1 \Delta x_2} = \frac{\Delta\{F(\beta_1+\beta_2x_2+\beta_{12}x_2+Z\beta)-F^{'}(\beta_2x_2+Z\beta)\}}{\Delta x_2}$$

$$= F(\beta_1 + \beta_2 + \beta_{12} + Z\beta) - F(\beta_1 + Z\beta) - F(\beta_2 + Z\beta) + F(Z\beta)$$[2].

If x_1 is a continuous variable and x_2 is a dummy variable, the mixed interaction effect is as follows:

$$\frac{\Delta}{\Delta x_2}\left(\frac{\partial F(u)}{\partial x_1}\right) = \frac{\Delta}{\Delta x_2}(F'(u)(\beta_1 + \beta_{12}x_2))$$
$$= F'(\beta_1 x_1 + \beta_2 + \beta_{12}x_1 + Z\beta)(\beta_1 + \beta_{12}) - F'(\beta_1 x_1 + Z\beta)\beta_1.$$

For the probit model, we can use the cumulative normal distribution, $\Phi(u)$, instead of $F(u)$ mentioned above. Then, $F'(u)$ can be substituted with the density function of the standard normal distribution, $\varphi(u)$, and $F''(u)$ can be substituted with $\varphi'(u) = -u\varphi(u)$. Likewise, we can also apply this to a logit model if $F(u)$ is substituted with $\Lambda(u)$[3]. $F'(u)$ is substituted with $\Lambda'(u) = \Lambda(u)(1 - \Lambda(u))$, and $F''(u)$ is replaced with $\Lambda''(u) = (\Lambda(u)(1 - \Lambda(u)))' = \Lambda(u)(1 - \Lambda(u))(1 - 2\Lambda(u))$ (Frondel and Vance, 2009).

This report focuses on the most common interaction effect between two variables. One may take three derivatives or three discrete differences to find the correct interpretation for a model with three interacting variables.

Odds Ratio

With an estimated regression coefficient in a logit regression model, the interpretation of the associated odds ratio renders a more meaningful understanding of effects in medical and epidemiological studies (Kutner, Nachtsheim, Neter, and Li,

2 $\frac{\Delta^2 F(u)}{\Delta x_1 \Delta x_2} := \frac{\Delta}{\Delta x_2}(\frac{\Delta F(u)}{\Delta x_1}) = \frac{\Delta}{\Delta x_2}(E[y|x_1 = 1, x_2, Z] - E[y|x_1 = 0, x_2, Z])$

3 $\Lambda(u) = \frac{1}{1 + e^{-u}}$

2005). Accordingly, many researchers prefer to fit a logit model rather than a probit model because odds ratio are not able to be computed in probit models. The odds is the ratio of the probability, π, to one minus the probability:

$$\pi = \frac{1}{1+\exp(-X\beta)},$$

$$\text{odds} = \frac{\pi}{1-\pi} = \frac{1}{\exp(-X\beta)} = \exp(X\beta).$$

Let us take an example. A researcher would like to look into the probability of eating breakfast every morning, which depends on whether the person is female, as well as on other explanatory variables (X). The odds ratio for gender is the odds for female (female = 1) divided by the odds of male (female = 0)[4]:

$$\text{odds for female} = \frac{\pi(\text{eating}|\text{female})}{1-\pi(\text{eating}|\text{female})} = \exp(\beta_{female} + X\beta),$$

$$\text{odds for male} = \frac{\pi(\text{eating}|\text{male})}{1-\pi(\text{eating}|\text{male})} = \exp(X\beta),$$

$$\text{odds ratio} = \frac{\text{odds for female}}{\text{odds for male}} = \exp(\beta_{female}).$$

Norton, Wang, and Ai (2004) mentioned that there are two main advantages in using odds ratios. First, the calculation is simple because the exponentiation of the estimated coefficient is only required. Second, with smaller π, the odds ratio approaches the risk ratio, which is easy to figure out conceptually. The risk ratio can be described as the ratio of two probabilities. For example, the risk ratio for the eating breakfast example is the probability of eating breakfast for female divided by the probability of eating breakfast for male:

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[4] It is important to assume that all other variables remain constant, since a researcher is able to focus on the relation between gender and eating breakfast every morning.

$$\text{risk ratio} = \frac{\pi(\text{eating}|\text{female})}{\pi(\text{eating}|\text{male})}.$$

If the risk ratio represents 1.5, for example, females are fifty percent more likely to eat breakfast every morning than males, holding all other variables constant.

Consider the odds ratio when there is an interaction between two dummy variables, x_1 and x_2. One may think that the odds ratio for the interaction term is the same as $\exp(\beta_{12})$. However, this is an incorrect assumption. The expression $\exp(\beta_{12})$ is not the odds ratio, but the ratio of odds ratios:

$$\text{odds ratio for } x_1 \mid_{x_2 = 1} = \frac{\frac{\pi(y=1|x_1=1;x_2=1)}{1-\pi(y=1|x_1=1;x_2=1)}}{\frac{\pi(y=1|x_1=0;x_2=1)}{1-\pi(y=1|x_1=0;x_2=1)}} = \frac{\exp(\beta_1+\beta_2+\beta_{12}+X\beta)}{\exp(\beta_2+X\beta)},$$

$$\text{odds ratio for } x_1 \mid_{x_2 = 0} = \frac{\frac{\pi(y=1|x_1=1;x_2=0)}{1-\pi(y=1|x_1=1;x_2=0)}}{\frac{\pi(y=1|x_1=0;x_2=0)}{1-\pi(y=1|x_1=0;x_2=0)}} = \frac{\exp(\beta_1+X\beta)}{\exp(X\beta)},$$

$$\text{ratio of the odds ratios for } x_1 \text{ and } x_2 = \exp(\beta_{12}).$$

From the above result, we can find that $\exp(\beta_{12})$ is neither a risk ratio nor an odds ratio. Since these concepts are confusing, we need to take a closer look at the interpretation of the interaction terms.

Chapter 3: Ordered Logit and Probit Models

With empirical data, ordered logit and probit models will be used to show the significance of the interaction effect in a nonlinear model in Chapter 4. Thus, understanding the concept of ordered logit and probit models is important. One may be unfamiliar with these models because of the term *ordered*. However, the methodology is very useful to researchers when handling ordinal responses from survey data. In statistics, a regression model for ordinal dependent variables is an extension of logit and probit models for dichotomous dependent variables but, has more than two (ordered) response variables.

Main Concept

Ordered logit and probit models are a useful analysis method in the social sciences with various response scales. They are more developed than traditional regression models, especially with respect to the handling of survey responses on a Likert scale[5], a popular scale in the social sciences. The responses on Likert scales are ordered with respect to agreement and/or lack of agreement.

Traditional regression models often used the mean of the response code as it is to estimate a regression equation. However, when a response is code (*1 = very satisfied, 2 = satisfied, 3 = normal*), the mean of *2.5* cannot tell whether the disposition of respondents is *satisfied* or *normal*. There is no analytical evidence that the mean of *2.5* is significant. Hence ordered logit and probit models take care of the response type with the probabilistic concepts similar to the way that binary responses are handled.

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[5] For example, it has the format like *strongly agree, agree, disagree, neither agree nor disagree,* and *strongly disagree.* A 5 or 7 point scale is usually used in a questionnaire.

Model Structure

The form of ordered logit and probit models that we apply now was proposed by McKelvey and Zavoina (1975). The structure of an ordered logit or probit model is:

$$y_i^* = X_i\beta + u_i \quad \text{for } i=1,\ldots,n,$$

where y_i^* is a latent variable; X_i is a vector of independent variables; β is a vector of parameters; and u_i is an unobserved error term.

Consider that y_i has K possible outcomes ($y_i = k$, with $k = 1,\ldots,K$). The model is appropriate when outcomes have a natural ordering that means that $k + 1$ is "better" than k. Assume that the observed ordinal variable y_i is related to the latent variable according to the following scheme:

$$y_i = k \text{ if } \mu_{\text{k-1}} \leq y_i^* \leq \mu_{\text{k}} \text{ for } k = 1,\ldots,K.$$

Again, it is as in the following:

$$\begin{aligned} y_i &= 1 \text{ if } \mu_0 \leq y_i^* \leq \mu_1 \\ &= 2 \text{ if } \mu_1 \leq y_i^* \leq \mu_2 \\ &= 3 \text{ if } \mu_2 \leq y_i^* \leq \mu_3 \\ &\;\vdots \\ &= K \text{ if } \mu_{\text{k-1}} \leq y_i^* \leq \mu_{\text{k}}, \end{aligned}$$

where $\mu_{0,} \mu_{1,\ldots,} \mu_{\text{k}}$ are thresholds with $\mu_0 = -\infty$ and $\mu_{\text{k}} = \infty$.

The conditional probability of observing $y_i = k$ is

$$\begin{aligned} \Pr(y_i = k \mid X_i) &= \Pr(\mu_{\text{k-1}} \leq y_i^* \leq \mu_{\text{k}}) \\ &= \Pr(\mu_{\text{k-1}} \leq X_i\beta + u_i \leq \mu_{\text{k}}) \end{aligned}$$

$$= \Pr(\mu_{k-1} - X_i\beta \leq u_i \leq \mu_k - X_i\beta)$$
$$= \Pr(u_i \leq \mu_k - X_i\beta) - \Pr(u_i \leq \mu_{k-1} - X_i\beta)$$

for $k = 1,\ldots,K$.

Ultimately, we can find the conditional probability of observing $y_i = k$. To obtain the conditional probability we make assumptions about the distribution of u_i. That is, if u_i is regarded as a logistic random variable, the conditional probabilities will correspond to an ordered logit model whereas, if u_i is regarded as standard normal random variable, they will correspond to an ordered probit model.

When u_i follows a logistic distribution, then

$$\Pr(y_i = 1 \mid X_i) = \Lambda(\mu_1 - X_i\beta) - \Lambda(- X_i\beta)$$
$$\vdots$$
$$\Pr(y_i = k \mid X_i) = \Lambda(\mu_k - X_i\beta) - \Lambda(\mu_{k-1} - X_i\beta)$$
$$\vdots$$
$$\Pr(y_i = K \mid X_i) = 1 - \Lambda(\mu_{K-1} - X_i\beta).$$

Likewise, when u_i follows a standard normal distribution, it is straightforward to find the conditional probabilities using $\Phi(u_i)$ instead of $\Lambda(u_i)$. This report skips them.

Chapter 4: Empirical Study

Based on the theoretical methodology as mentioned earlier, this chapter demonstrates the correct estimation of the interaction effect in ordered logit and probit models.

Dataset Description

The dataset used in this report comes from the *Asian Barometer Survey 2005-2008*[6] conducted by Academia Sinica and National Taiwan University. The unit of analysis is individuals in South Korea. In particular, this study demonstrated that the national election in South Korea is affected by mass media as a function of demographic factors, such as gender, age, and education. The mass media is closely connected with voters and plays an important role in the elections. As such, most voters get the information about the election and the candidates from the mass media, such as television, radio, newspapers, internet, and so on.

The dependent variable is voter evaluation of the 2007 South Korean presidential election[7], measured on a four-point ordinal Likert scale, with 1 indicating *Not free or fair* and 4 indicating *Completely free and fair*. The independent variables considered in the models are the frequency of internet use and the demographic factors (gender, age, and education). The frequency of internet use is measured by how often individuals use the internet, with 1 indicating *Never* and 6 indicating *Almost daily*. Gender is coded by 0 if

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[6] The survey project involves collaboration among thirteen East Asian countries. Under the data sharing agreement among East Asian collaborators, the dataset covers the issues of citizens' attitudes and values toward politics, power, reform, and democracy in countries. All the variables contained in core questionnaire freely accessible to scholars and experts worldwide upon application. (*Source: http://www.asianbarometer.org/*)

[7] The 17th South Korean presidential election was held on 19 December 2007. The election was won by Lee Myung-bak of the Grand National Party. He beat Chung Dong-young who was a United New Democratic Party candidate and Lee Hoi-chang who was an independent candidate. Voter turnout was 63.0% according to the National Election Commission.

male and 1 if female, with male chosen as the reference category. Age is divided into a four-point scale, ranging from under 30 years old to over 51 years old. Education is measured by a ten-point scale, ranging from *No formal education* to *Post-graduate degree*. These demographic factors are also defined as control variables.

We can look into the relationship between election (fairness and freeness) perceptions and internet use through the data analysis. The primary interests, however, are the interaction effects between internet use and gender, between internet use and age, and between internet use and education. The interaction between the independent variables may imply that the impact of frequency of internet use on individual evaluation of election differs depending on gender, age, or education. In other words, we could find how the effect of frequency of internet use changes for a unit change in each of demographic factors. For these reasons, we need to consider varied models which include interaction variables to correctly analyze data.

Model Estimation without Interaction Effects

Ordered logit or probit models are appropriate when analyzing these data since the dependent variable has more than two ordered response level. We can easily fit these models using Stata. Then the output of ordered logit or probit models shows the cut-points (a.k.a. thresholds) unlike binary logit or probit models. In an ordered logit model, the cut-points are interpreted as the adjusted log odds of being in category *k* or lower on the response variable (i.e. as conditional cumulative logits). In general, since interpretation of the ordered logit or probit model is not dependent on the points, this report focuses only on the effects of substantive predictors and their interactions.

Table 1 shows the estimates for the ordered logit and probit models without interaction effects included. We can first focus on the individual coefficients and interpret them. The estimated coefficients of Gender and Age in both models are statistically significant at the 5% or 10%-level. The remaining predictor variables (Internet Use and Education) are not statistically significant. For the significant Gender effect in the ordered logit model, we would say that for a one unit increase in Gender, we expect a 0.2374 decrease in the log odds of being in a higher level of Voter Evaluation holding other variables in the model constant. Female's odds of a higher evaluation of the election are $1 - e^{-.237}$ or about 20% lower than male's. On the other hand, for a one unit increase in Education, we expect a 0.0609 decrease in the log odds of being in a higher level of Voter Evaluation holding the other variables in the model constant. Again, the coefficients for Internet Use and Education are not statistically significant and thus we need to check whether they are involved in an interaction with other variables.

	Ordered Logit Model		Ordered Probit Model	
	Coefficient	Marginal Effect	Coefficient	Marginal Effect
Internet Use	0.0287	-0.0028	0.0173	-0.0032
	(0.0378)	(0.0037)	(0.0216)	(0.0040)
Gender	-0.2374*	0.0229*	-0.1450**	0.0269**
	(0.1235)	(0.0120)	(0.0708)	(0.0132)
Age	0.1872**	-0.0181**	0.1039**	-0.0193**
	(0.0715)	(0.0070)	(0.0408)	(0.0076)
Education	-0.0609	0.0059	-0.0395*	0.0073*
	(0.0391)	(0.0038)	(0.0226)	(0.0042)

Note: * denotes significance at the 10%-level and ** at the 5%-level
Standard errors in parentheses

Table 1: Ordered Logit and Probit Models for Voter Evaluation

The marginal effects in an ordered logit or probit model obtained from Stata's *margins* command may have opposite signs from their coefficients. The reason is that increasing an independent variable actually shifts the distribution to the right while the coefficient and threshold estimates are held constant (Greene 2008). The marginal effects in Table 1 are calculated at the mean values of the model covariates. The marginal effects of the independent variables are the change in the probability of observing Voter Evaluation, if the independent variables change by one unit, while all the other variables remain unchanged. For example, with a one unit increase in Internet Use from its mean[8], the probability of evaluating *Not free or fair*[9] is expected to decrease by 0.28 percent, holding all other variables constant in the ordered logit model. The probability of evaluating *Not free or fair* from voters is expected to increase by 0.59 percent for a one unit increase in Education from its mean.

	Odds Ratio	Std. Err.
Internet use	1.0292	0.0389
Gender	0.7887*	0.0974
Age	1.2059**	0.0863
Education	0.9409	0.0368

Note: * denotes significance at the 10%-level and ** at the 5%-level

Table 2: Odds Ratios for Ordered Logit Model

In Table 2, the results are displayed as proportional odds ratios obtained from the ordered logit model. The interpretation is pretty much the same as that of a binary logit

8 See Appendix.

9 Table 1 reports the marginal effect when Y = Pr(Voter Evaluation = 1). *Not free or fair* is coded as 1 in Stata.

model. For Internet Use, we would say that for a one unit increase in Internet Use, (i.e., going from *Almost daily* to *At least once a week*), the odds of the high evaluation of voters versus the lower categories are 1.0292 greater, controlling for the other variables. Likewise, for a one unit increase in Internet Use, the odds of the other categories versus low evaluation of voters are 1.0292 times greater, given that all of the other higher variables are unchanged. For a one unit increase in Age, the odds of being in the higher category of Voter Evaluation versus the lower categories are 1.2059 times greater, given that the other variables are held constant. By the proportional odds assumption[10], the same increase, 1.2059 times, can be found between low evaluation and the other categories combined. Actually the calculation is straightforward with the exponentiated logit coefficient. In other words, the odds ratio for Age, 1.2059, is obtained from $e^{0.1872}$.

Model Estimation with Interaction Effects

In the model with interaction terms, we should be careful of analyzing and interpreting the marginal effect. As previously stated, the marginal effect in nonlinear models is complicated, especially when it involves interactions. If we only consider the *margins* command as before, we will incorrectly estimate the marginal effect for the variables included in the interaction term. This chapter verifies it and correctly computes the interaction effect with the correct mathematical formula.

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[10] Ordered logit or ordered probit regression assumes that the relationship between each pair of outcome groups is the same. That is, the assumption means that the coefficients that explain the relationship between, for example, the highest versus all lower categories of the dependent variable are the same as those that explain the relationship between the next highest category and all lower categories. We call it the proportional odds assumption or the parallel regression assumption (Long and Freese, 2006).

First of all, to see whether or not there is a relationship among the independent variables in the previous empirical model, we can check the bivariate correlations between all possible pairs of variables with Stata.

	Voter Evaluation	Internet Use	Gender	Age	Education
Voter Evaluation	1.0000				
Internet Use	-0.0576	1.0000			
Gender	-0.0524	-0.1307	1.0000		
Age	0.1186	-0.6187	0.0228	1.0000	
Education	-0.0883	0.5571	-0.2307	-0.5259	1.0000

Table 3: Dataset Correlations

From looking at Table 3, we would examine the high correlations among these variables and guess the interaction effects from those correlations. If there is any interaction effect between variables, we can produce a better model specification and an improved interpretation of the relationship in the data than before.

	Ordered Logit Model				Ordered Probit Model			
	Model 1 Coeff.	Model 2 Coeff.	Model 3 Coeff.	Model 4 Coeff.	Model 1 Coeff.	Model 2 Coeff.	Model 3 Coeff.	Model 4 Coeff.
Internet Use	0.0287	0.1217	-0.1732	0.0333	0.0173	0.0706	-0.1041	0.0198
	(0.0378)	(0.1221)	(0.1111)	(0.0381)	(0.0216)	(0.0700)	(0.0647)	(0.0217)
Gender	-0.2374*	-0.2292*	-0.2336*	-0.2427**	-0.1450**	-0.1412**	-0.1435**	-0.1481**
	(0.1235)	(0.1239)	(0.1234)	(0.1236)	(0.0708)	(0.0710)	(0.0709)	(0.0709)
Age	0.1872**	0.3362*	0.1968**	0.4662	0.1039**	0.1895*	0.1084**	0.2681
	(0.0715)	(0.1996)	(0.0718)	(0.2957)	(0.0408)	(0.1140)	(0.0409)	(0.1726)
Education	-0.0609	-0.0510	-0.1474**	0.0665	-0.0395*	-0.0335	-0.0903**	0.0360
	(0.0391)	(0.0410)	(0.0594)	(0.1367)	(0.0226)	(0.0238)	(0.0341)	(0.0804)
Internet Use & Age		-0.0292				-0.0168		
		(0.0366)				(0.0209)		
Internet Use & Education			0.0301*				0.0180**	
			(0.0156)				(0.0091)	
Age & Education				-0.0370				-0.0218
				(0.0380)				(0.0222)

Note: * denotes significance at the 10%-level and ** at the 5%-level
Standard errors in parentheses

Table 4: Ordered Logit and Probit Models with Interaction Term

Based on the results in Table 3, we can have ordered logit and probit models with interaction terms included (see Table 4). Model 1 shows the estimated coefficients without any interaction as before. Model 2 includes the interaction term involving Internet Use and Age and Model 3 includes the interaction term associated with Internet Use and Education. Also, there is the interaction between Age and Education included in Model 4. As shown in Table 4, the interaction between Internet Use and Education is statistically significant in Model 3 and in both the ordered logit and probit model. In fact, Internet Use does not have a strong influence on Voter Evaluation in all models. In addition, Education is statistically significant only when interacted with Internet Use as seen in Model 3. From the results, we can conclude that there is an interaction effect between Internet Use and Education in this model, even though Internet Use is not statistically significant in Model 3.

Now let us take a look at the problem of the magnitude of the interaction effect, which is the primary interest in this report. As mentioned earlier, it is a little complicated to compute the marginal effect of an interaction term compared to that of the main effects in a nonlinear model. In Chapter 2, we saw that the interaction effect is equivalent to the marginal effect of the interaction term in a linear model. Using the *margins* command in Stata, we can simply find the interaction effect considered as the marginal effect of that. However, this idea does not work in nonlinear models. For example, the the interaction effect between x_1 and x_2 should be calculated as $\frac{\partial^2 Pr(Y=k)}{\partial x_2\,\partial x_1}$, not as $\frac{\partial Pr(Y=k)}{\partial(x_1 x_2)}$, which is the marginal effect for the interaction term, $x_1 x_2$. That is, the result will be incorrect, if we use simply the *margins* command in Stata to estimate the interaction effect in a nonlinear model.

	Ordered Logit Model		Ordered Probit Model	
	Incorrect	Correct	Incorrect	Correct
Internet Use	-0.0029*	0.0006**	-0.0033**	0.0004**
& Education	(0.0015)	(0.0013)	(0.0017)	(0.0010)

Note: * denotes significance at the 10%-level and ** at the 5%-level
Standard errors in parentheses

Table 5: Interaction Effects for Voter Evaluation

The correct and incorrect interaction effects are reported in Table 5 for both ordered logit and probit models. Correct values are obtained by computing $\frac{\partial^2 Pr(Y=1)}{\partial Education\, \partial Internet\ Use}$, and they come from using the *predictnl* command after invoking the *margins* command in Stata. Also, incorrect values come from merely the *margins* command in Stata by computing $\frac{\partial Pr(Y=1)}{\partial Education * Internet\ Use}$. We can see from the results that the interaction effect is clearly distinct between the correct and the incorrect formulations. The correct value, in both logit and probit models, has a smaller standard error and a positive sign. Therefore, for nonlinear models with interaction terms, we must carefully estimate the model with respect to the marginal and interaction effects.

Study Results

In this chapter, we have estimated interaction effects in two nonlinear models (i.e. ordered logit and probit models) using data from a survey of Voter Evaluation from Korea. The data contain an explanatory variable (Internet Use) and several demographic factors. Data analysis carried out using Stata shows that the interaction term between Internet Use and Education is statistically significant in both ordered logit and probit models. Simply looking at it, we can interpret this interaction effect to infer that the impact of Internet Use depends on the level of Education or that the impact of Education depends on the level of Internet Use. Estimation of the marginal effect of an interaction term, however, is not an issue to be taken lightly. This report discussed and demonstrated problems inherent with interaction effects in nonlinear models. Applying a general derivation of interaction effects in nonlinear models as outlined by Norton, Wang, and Ai (2004), we were able to correct misleading results obtained from Stata's *margins* command. Fortunately, invoking the *predictnl* command after the *margins* command in Stata provided the correct marginal effects. For these data, the correct interaction effect has a smaller standard error and opposite sign compared to the incorrectly estimated one. Therefore, this chapter has illustrated the importance of carefully interpreting the terms involved in an interaction in nonlinear models, as mentioned in previous chapters.

Chapter 5: Conclusion

This report introduced the general derivation of marginal and interaction effects in nonlinear models as well as in linear models in Chapter 2. The interaction term in linear models is sufficient to infer the interaction effect that is the influence of a variable on the marginal effect of another variable. However, the marginal effect of an interaction term in nonlinear models is not the same as the interaction effect in linear models. That is, for nonlinear models we must distinguish $\frac{\partial \Pr(Y=k)}{\partial(x_1 x_2)}$ from $\frac{\partial^2 \Pr(Y=k)}{\partial x_2\, \partial x_1}$ when variables interact with each other. In Chapter 4, it was demonstrated that the correct interaction effect in ordered logit and probit models is substantially different from the results obtained directly from Stata's *margins* command without the subsequent correction using the *predictnl* command. We saw the wrong magnitude and standard error of the interaction term (with opposite sign in the demonstration in this report). We have confirmed that the interaction effect is very different from the marginal effect of an interaction term in nonlinear models. When fitting the nonlinear models with interaction terms in Stata and other standard software, we have to be cautious of their interpretation. Also, extra effort is required to estimate the correct values. This is because the inference of interaction terms in linear models does not extend to that in nonlinear models. Special purpose routines such as *margins* and *predictnl* commands can be very helpful for researchers who use Stata. But similar types of utilities may be lacking in other popular software packages.

Appendix

	Obs	Mean	Std. Dev.	Min	Max
Voter Evaluation	1023	2.7263	0.8586	1	4
Internet Use	1196	4.1881	2.1377	1	6
Gender	1212	1.5091	0.5001	1	2
Age	1212	2.6469	1.1338	1	4
Education	1212	6.7995	1.9814	1	10

Table A1: Summary of Variables

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