# Using adaptive ruled layers for Rapid Prototyping: principles and first results 

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#### Abstract

Current 2.5D layered rapid prototyping has as disadvantage the staircase effect, requiring thin layers to be used to achieve a reasonable accuracy. Slices with inclined outer surfaces can be constructed using linear interpolation between adjacent contours, resulting in ruled slices. A methodology to approximate a given model geometry within a specified accuracy using ruled slices and an adaptive layer thickness is described. This involves matching successive contours and analysing the geometry for curvature and inclination to calculate allowed layerthicknesses. First results show a significant reduction in the number of layers when compared to adaptive slicing using 2.5D layers. A proof-of-concept software, the Delft University of Technology Improved Slicer (DUTIS) has been developed to perform the adaptive slicing using either 2.5D or ruled layers allowing a comparison between the two alternative methods.


Keywords zero order approximation, first order approximation, comparison, direct adaptive slicing, matching contours, ruled slices

## 1. INTRODUCTION

In the commonly used 2.5D Layered Manufacturing Techniques (LMT), which in effect use a zero-order approximation of the original geometry, a relatively small layer thickness $d$ must be used to constrain the errors that are introduced by this approximation (the stair-case effect). The layer thickness $d$ can be increased when a first-order approximation is used in which the side surfaces of the slices can be inclined. A possible solution to accomplish this is by using ruled slices, interpolating between two adjacent planar contours. Moreover, when also an adaptive layer thickness is used, the number of layers required for a user defined error $\delta$ can be further reduced. In this paper the zero and first order approximation algorithms, combined with direct adaptive slicing of BREP CAD model geometry will be compared.

An advantage of this so-called ruled slicing is that, because the edges of the adjacent slices meet, $\mathrm{C}^{0}$ continuity in the slicing direction (z-direction) is guaranteed, eliminating the staircase effect as present in the 2.5 D approach. An improved surface finish is the result. Another advantage is the reduction in the number of layers, resulting in faster manufacturing. A disadvantage is that manufacturing of the ruled slices requires a process with 4 degrees of freedom, which is generally more expensive than a process with 2 degrees of freedom.

The Delft University of Technology Improved Slicer (DUTIS) software has been developed using the ACIS® geometric modelling kernel (ACIS, 1996) and C++ on an SGI platform to
approximate a given BREP CAD model geometry using either a zero or a first order approximation within the specified $\delta$. The paper is divided into the following sections. Section 2 describes the related research. In section 3 the general procedure will be introduced. In section 4 the calculation of an adaptive layer thickness will be explained and the method of matching arbitrary contours from adjacent slices will be discussed. In section 5 the results for some examples will be shown. Finally in section 6 the results will be discussed and conclusions will be given.

## 2. RELATED RESEARCH

A lot of work is carried out by other researchers in the area of adaptive slicing and first order approximation of model geometry.

In our software the algorithms for the zero order approximation are based on the work of (Suh and Wozny, 1994) and (Kulkarni and Dutta, 1995 and 1996). A horizontal layer (in the xy-plane) with a vertical outer surface can be regarded as a zero order approximation in the vertical slicing direction $(z)$ of a part of the original model geometry. The usual procedure to reconstruct such a layer is to slice the model and to extrude the slice into the slicing direction to give the slice a certain thickness. Each contour is independently used for this reconstruction. The reconstruction error depends on the local curvature and the $z$ component of the surface normals of the model geometry. The basic idea behind the calculation of an adaptive layer thickness is that a surface can be locally approximated in a certain plane by a circle with radius R equal to the local curvature in the plane. The approximation error can be calculated and from this an expression for the allowed layer thickness for a convex surface can be derived as shown in (Suh and Wozny, 1994) and (Kulkarni and Dutta, 1995 and 1996)

$$
\begin{equation*}
d=-R \sin \theta+\sqrt{R^{2} \sin ^{2} \theta+2 R \delta+\delta^{2}} \tag{1}
\end{equation*}
$$

in which R radius of curvature in vertical direction, $\delta$ the user-defined error and $\theta$ in $[0 . . \pi]$ the angle between the surface normal and the horizontal plane at the bottom of the slice. Using the same approach it is also possible to derive expressions for convex surfaces and for $\theta$ in $[-\pi . .0]$.

Other researchers have been working on the subject of manufacturing layers with inclined side surfaces. The term twisted profile layers is used in (Barlier et al.1995) to describe ruled slices as a means to improve layered manufacturing. Layers with inclined side surfaces are also used by (Hope, 1996) for the manufacturing of large objects. Ruled slices are also used for the CAM-LEM process (Newman et al.1995) and the Shapemaker II process (Thomas et al.,1996). These processes are capable of producing ruled slices by using either a 5 -axis laser cutting system or a 4axis hot-wire cutter (for foam material). In general, because ruled slices are described by means of a set of non-intersecting straight line segments connecting the upper and lower contours of the slice, ruled slices can be produced from sheet material by processes that employ line visibility (Woo, 1994) and have at least 4 degrees of freedom (translation in the $\mathrm{x}, \mathrm{y}$ plane and additional rotations $\phi, \theta$ ). Among these processes are wire EDM, hot-wire cutting, laser cutting, water-jet cutting, CNC side milling and more.

## 3. GENERAL PROCEDURE IN DUTIS

DUTIS is an interactive program that allows the user to load a CAD model, and manipulate this CAD model by rotation, translation and scaling options if desired. Also boolean operations are implemented, allowing for a project to be assembled from multiple CAD models that must be sliced as a whole. When the CAD model has the desired orientation, such that the z -axis of the CAD model coincides with the required approximation direction, a number of steps have to be taken in sequential order to complete the process for a non-equidistant layered approximation of a given BREP CAD model within a user-defined $\delta$. These steps are

1. User input: The approximation method must be chosen, either zero- or first order
2. User input: The required accuracy must be specified
3. Automated procedure:

- The CAD model is analysed for important features that must be preserved, such as horizontal areas and sharp edges
- The $z$-heights of the found features are used to divide the CAD model into segments
- The CAD model segments are sliced with an adaptive layer thickness according to $\delta$ using the equations for either 2.5D or ruled slices
- Either 2.5D or ruled slices are reconstructed

The resulting layered approximation can be interactively viewed on screen and, if accepted, the data can be used for manufacturing.

The segmentation procedure samples the curves between adjacent surfaces as well as the surfaces to find segment borders, based on the extrema of the curves and horizontal areas of the surfaces.


Figure 1. Problems with reconstructing a surface between 2 contours: the branching and correspondence problems

The differences between the zero and first-order approximation approaches are the calculation of an allowed layer thickness and the reconstruction of the slice. The CAD model segmentation is a necessary step in this procedure for the reconstruction of ruled slices, because the reconstruction of a three-dimensional shape from 2 successive contours is a complex problem. A surface connecting the contours must be found. This is difficult because of two problems, (a) branching
and (b) correspondence as shown in figure 1. The branching problem indicates that the number of contours is different between two successive slices. The correspondence problem indicates that the number of contours per slice is the same but larger than one for two successive slices. In both cases it is unknown how the contours should be connected from one slice to the other. The branching problem is eliminated when the model is first segmented as explained above. We will discuss the correspondence problem further in section 4.4.

## 4. ADAPTIVE SLICING OF SEGMENTS

In this section it is assumed that the CAD model is divided into a set of segments. These segments are processed individually. The subdivision of the segments into a set of slices with an adaptive layer thickness must take account of the curvature, slope of the CAD model surface and the user-specified $\delta$.

### 4.1. First order approximation

The first order approximation is based on linear interpolation between successive contours using geometrical and topological information from the original geometry. In effect a ruled surface (Farin, 1990) between 2 adjacent contours is constructed. For ruled slicing an expression for the error and thus for the allowed layer thickness can be derived as

$$
\begin{equation*}
d=\cos \alpha \cos \theta \sqrt{2 R \delta-\delta^{2}} \tag{2}
\end{equation*}
$$

in which R radius of curvature in the plane of interest, $\delta$ the user-defined error, $\theta$ in $[-\pi . . \pi]$ the angle between the surface normal and the horizontal plane at the half height of the slice and $\alpha$ in $[-\pi / 2 . . \pi / 2]$ the angle between the vertical direction and the line connecting the points on the upper and lower contour of the ruled surface.

### 4.3. Implementation of the slicing algorithm

The algorithms that are implemented in DUTIS to slice a segment in the z-direction uses as input a CAD model segment and the user-specified $\delta$. DUTIS also takes explicitly into account the limitations of the manufacturing process. Depending on the type of process used to manufacture the ruled slices and the available sheet materials either a set of $n$ allowed layer thicknesses $t h_{l . . n}$ or the fact that an arbitrary layer thickness is possible can be indicated. The general procedure can be described as

1. Define the extent of the segment in z -direction: $\mathrm{z}_{\min }, \mathrm{z}_{\text {max }}$
2. Approximate the segment using a stepwise approach:

- Calculate the intersection at $\mathrm{z}=\mathrm{z}_{\mathrm{i}}$ (starting at $\mathrm{z}_{\min }$ ), resulting in a set of contours $\mathrm{C}_{\mathrm{i}}$
- Sample $C_{i}$ and calculate the maximum allowed layer thickness $d_{i}$ according to either eq. (1) or eq. (2) for zero- or first-order approximation respectively.
- If only a limited set of layer thicknesses is available, select the nearest smaller layer thickness $\mathrm{d}_{\mathrm{i}}:=\mathrm{d}_{\mathrm{i}}^{\prime} \leq \mathrm{d}_{\mathrm{i}}$ from the available set of thicknesses
- Check if new sample height is not exceeding the segment dimensions: $\left(z_{i}+d_{i} \leq z_{i}\right)$
- Check at suggested new sample height $z_{i}+1$ if the assumptions used in the calculation for the layer thickness are still correct (see below)
- if correct, continue
- if not correct, repeat with layer thickness $d_{i}=\alpha d_{i}$ with $0<\alpha<1$

In this algorithm a two-way approach is used to ensure that regions of higher curvature are not missed. First in the upward direction a new layer thickness is calculated. Secondly in the downward direction at the new proposed slice height a check-calculation is made. This checkcalculation is made to ensure that the proposed layer thickness does not result in exceeding the user-specified $\delta$ in the approximation. This check is necessary because of a possible changing curvature in the upward direction of the CAD model. Furthermore the third point in step 2 of the described algorithm selects the appropriate sheet thickness from the available set of thicknesses for the manufacturing process. This set is user-specified and can thus be adapted to local circumstances.

### 4.4 Constructing the slices and solving the correspondence problem

After all segments are processed, a list of contours must be processed to generate the slices. For a zero-order approximation, this is a simple procedure that exists of an extrusion in the positive z direction to give the slice the desired thickness. For a first-order approximation, two successive contours must be matched using a more difficult procedure. This matching can be accomplished using either matching in parameter space (see (de Jager, 1996A) and (de Jager et al., 1996B)) or matching in geometric space such that the correct starting points on the successive contours can be found.

A calculated intersection of the CAD model and a plane consists of a set of planar contours. A contour consists of a closed and ordered set of connected curves which are originating from slicing the BREP geometry. In the slicing procedure we keep track of the original surfaces in order to evaluate the geometry around the contours. We can use the topological information from the original CAD model to match corresponding curves. All contour curves from one contour have a corresponding curve in the adjacent contour, because as a result of the segmentation procedure they are originating from the same surfaces. Therefore we can always find a correspondence between adjacent contours.

## 5. Results

The results for a simple test object (a sphere with radius 25 mm ) show clearly that depending on the ratio between the curvature of a model and the allowed error, the ratio between the number of layers for a zero-order approximation and a first-order approximation varies between approx. 4 and 14 times as shown in table 1.

In figure 2 the result of the first order approximation using ruled slices for a non-rotational symmetric shape (a perfume bottle) is shown. You can see the original model and the approximation with 29 ruled slices. For the same accuracy 1342.5 D slices were needed, which is a factor 4.6. In table 2 the results for different $\delta$ 's are summarised.

| $\delta$ | number of 2.5D <br> slices | number of ruled <br> slices | ratio 2.5D/ruled |
| :---: | :---: | :---: | :---: |
| 1.0 | 26 | 6 | 4.3 |
| 0.5 | 54 | 8 | 6.8 |
| 0.1 | 253 | 18 | 14.1 |

Table 1 Layered approximation of a sphere with radius $\mathrm{R}=25.0 \mathrm{~mm}$

| $\delta$ | number of $2.5 D$ <br> slices | number of ruled <br> slices | ratio 2.5D/ruled |
| :---: | :---: | :---: | :---: |
| 30.0 | 69 | 10 | 6.9 |
| 10.0 | 134 | 29 | 4.6 |
| 1.0 | 288 | 60 | 4.8 |

Table 2 Layered approximation of the model 'bottle' with size $454 \times 104 \times 300 \mathrm{~mm}$

In figure 3 the results for a more complex geometry are shown. This example consists of a more difficult geometry with different features: straight faces, a hole, a cylinder and different radii. The dimensions of this model are $200 \times 100 \times 50 \mathrm{~mm}$ and the allowed $\delta$ is 1.0 mm . The zero order approximation requires 23 layers, while the first order approximation requires 11 layers, a reduction of 2 times the number of layers. It can be noted that the correspondence between the adjacent contours is solved correctly.


Figure 2. Object "Bottle" and the approximation with 43 ruled slices. The size of the object is $454 \times 104 \times 300 \mathrm{~mm}$, and the $\delta$ is 10 mm .


Figure 3. The approximation of a more difficult geometry with 11 ruled slices. The size of the object is $200 \times 100 \times 50 \mathrm{~mm}$, and the $\delta$ is 1 mm .

The figures in this section are screenshots of the geometry before and after the approximation procedure. The user of DUTIS can interactively manipulate the view of the geometry to evaluate the result before taking a decision whether or not to manufacture the model.

For demonstration purposes, we have experimented with a 6-axis robot milling workcell (Vergeest and Tangelder, 1996) using side milling, simulating a 4 -axis NC milling machine and have managed to use the robot set-up for 4 -axis machining. Some models have been manufactured using the robot to manufacture individual slices and by manually assembling these slices.

## 6. Discussion and conclusions

We have shown in this paper that it is possible to get a significant reduction in the required number of layers for a user-specified $\delta$ when adaptive ruled layers are used. The theoretical results in this paper are valid for a range of processes that employ line-visibility. Although this type of equipment to manufacture a ruled slice is more expensive than the equipment necessary to manufacture a 2.5 D slice, we are confident that for large smooth shapes (with not too many surface details) it is very profitable to use ruled slices. Depending on the model geometry and the userdefined $\delta$, the actual reduction in number of layers for the examples shown varies between 2 and 14 times, based on the use of a continuously varying layer thickness. However, DUTIS is also capable of taking into account user-defined discrete sets of thicknesses. The reduction in the number of layers improves the time and costs to manufacture and assemble the model. The interactive viewing of the result gives an opportunity to evaluate the approximation before the actual manufacturing.

We are aware that the overall improvement depends also on several other factors. For a more thorough analysis the time and costs to produce one layer, the costs of material and the time and
costs of assembly should also be taken into account. Therefore both the 2.5 D as well as the ruled slicing methods have been implemented in DUTIS to give the user the opportunity to evaluate the results for both methods and make a proper choice of an appropriate manufacturing method.

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