The Dissertation Committee for Erin Elizabeth Turner
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Critical Mathematical Agency:

Urban Middle School Students Engage in Mathematics to

Investigate, Critique, and Act Upon Their World

Committee:

____
Susan B. Empson, Supervisor

____
Angela Calabrese Barton

____
Uri Treisman

____
Angela Valenzuela

____
Lisa Goldstein

____
Taylor Martin

Critical Mathematical Agency:

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by

Erin Elizabeth Turner, B.A., M.A.

Dissertation

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Doctor of Philosophy

The University of Texas at Austin

December, 2003

Critical Mathematical Agency:

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Publication No. ________________

Erin Elizabeth Turner, Ph.D.
The University of Texas at Austin, 2003

Supervisor: Susan B. Empson

This dissertation presents the findings of a collaborative critical ethnographic study of the impact of 'teaching and learning mathematics for social justice' for urban youth, specifically, for students in a diverse middle school mathematics classroom in New York City. The study drew upon critical theory's stance towards schooling, mathematics, and society to craft a vision of teaching and learning mathematics for social justice, and investigated how this approach might support students in developing and enacting a sense of critical mathematical agency. *Critical Mathematical Agency* refers to students capacity to (a) view the world with a critical mind set and imagine how the world might become a more socially just, equitable place, and (b) identify themselves as powerful mathematical thinkers who construct rigorous mathematical understandings, and who participate in mathematics in personally and socially meaningful ways.

To examine how students enacted critical mathematical agency, and how their sense of agency interacted with their engagement with the discipline of mathematics, data from 12 case study students, including individual / group interviews, classroom observations, and student work samples, were collected over a seven month period. Analysis of this data revealed that students enacted agency in a variety of ways, including through acts of asserting intention, positioning and authoring, improvisation, critique, and transformational resistance. Analysis of students' participation also indicated that students' efforts to assert personal intentions, and therein, enact agency, can generate possibilities for potentially rigorous, and personally meaningful, mathematical investigations. The extent to which students' ideas for analysis materialized into mathematically significant and personally transformative investigations depended on the affordances and constraints of the figured worlds in which they participated, in particular, how their intended investigations were supported, mathematically, by the dominant activity and discourse of their classroom. Detailed case studies of four students are presented to illustrate these themes, and tensions that cut across students' stories are discussed.

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CHAPTER ONE: RATIONALE

It was an unseasonably warm morning in mid May, and between the humidity, the lack of ventilation, and the heat generated by thirty middle school students who had just walked up five flights of stairs, many in jackets, to get to their school, the temperature in Ms. Font's math classroom was stifling. Students welcomed the opportunity to leave the classroom and start collecting data for their small group investigations of their school space. I accompanied two groups of students, one that was studying the school's gymnasium, and a second, Tyrell, Vellez, and Carlos, that had chosen to analyze the boys' restroom space, which they were convinced was "way too small!" for the number of male students and staff at their school. And they wanted to "prove it!"

As we made our way across the gym to the restroom facility, I was struck by several aspects of the space: its incredibly small size, its remote location - a good five to ten minute walk from many of the classrooms – and its comprised condition – peeling paint, stalls without doors or doors that did not close, and missing and chipped tiles on the floor and walls. Tyrell, Vellez, and Carlos were clearly well-accustomed to the place, and after making a series of jokes about its distinctive smell and the 'prohibited' presence of an adult female in the 'boys' restroom, they launched into the task of measuring the dimensions of the odd-shaped room.

Their persistence and level of engagement in the activity was impressive. Using make-shift measuring sticks that consisted of wooden rods approximately one meter in length that had paper meter strips taped on with masking tape that covered up a good portion of the numbers, the boys worked together to measure the length of each wall of the restroom. They engaged in animated discussions about which dimension was the width, about how to record measurements like 2 meters and 18 centimeters using a single unit, and about how to account for the "extra little space" that was accumulated each time they laid down their measuring sticks, which were slightly longer than a meter, end to end along the floor. They attempted to sketch the outline of the room in their notebooks, to help them keep track of each dimension that they measured. Given the restroom's odd shape - it more resembled a 'Z' than a standard rectangle - this task was inherently complex. Moreover, 15 to 20 minutes of sliding the measuring sticks all over the floor had taken its tool on these fragile tools, and the boys were left with sections of paper measuring strips dispersed across the floor, which I helped them to piece back together.

On several occasions I had to restrain myself from suggesting that the boys approximate the restroom space with a simple rectangle, in an effort to make the task more accessible. I was worried about their ability to later analyze this complicated space. But the boys persisted, even as they had to redraw the outline of the space multiple times because *they* wanted "to get it right." They were engaged in measuring distances, figuring out how to use tools, converting between units, creating representations, justifying their thinking to one another, and solving the many 'real' problems that spontaneously arose as they worked through this task. In other words, they were actively participating in mathematics, but within the context of an issue that they cared about (the boys' bathroom space), that was consequential to them, and that impacted, on a daily basis, their experiences at school.

I begin with this brief narrative of interactions with students in Beatriz Font's sixth grade mathematics class for two reasons. First, I think that it begins to paint a picture of some of the challenges that face students and teachers in many low-income, urban schools: limited resources, lack of space, overcrowding, and buildings in need of repair, among others. Second, and more importantly, it presents a sense of possibility, an image of what doing and learning mathematics might look like, and might mean for students. It suggests that participating in school mathematics has the potential to be a meaningful, relevant, and even transformative experience. Although the curricular and pedagogical practices are not at all representative of how most students in such schools experience mathematics, the building conditions and the lack of resources in this story are typical of many low-income urban schools. The following section further documents the inequities and challenges that plague urban mathematics education.

Introduction

Disturbing discrepancies exist in how students experience mathematics in schools (Campbell & Silver, 1999; Oakes, 1990). Low-income and minority students are disproportionately represented in low-level, skills oriented classrooms, and are more likely to encounter deteriorated school buildings, decreased access to modern technology and quality resources, and less experienced and less qualified teachers (Campaign for Fiscal Equity, 1999; Campbell & Silver, 1999; Kozol, 1991; Oakes, 1990; Tate, 1995). Discrepancies persist on measures of achievement in mathematics, such as entrance into advanced courses and performance on standardized assessments (Campbell & Silver, 1999; National Assessment of Education Progress, 2003; Ortiz-Franco & Flores, 2001;

Secada, 1992). For instance, when the results of the recent Third International Mathematics and Science Study-Repeat [TIMSS-R] assessment were desegregated, versus averaged to "hide the true extremes," the analysis indicated striking disparities (Berliner, 2001, title). A critical consideration of issues of equity in mathematics education is urgent, because currently, instead of facilitating democratic ideals, "mathematics education generates selection, exclusion and segregation" along the lines of gender, race, language, and socioeconomic status (Skovsmose & Valero, 2001, p. 41).

The NCTM Reform Movement: Laudable, But Insufficient

To address these inequities, the National Council of Teachers of Mathematics (NCTM) launched a reform movement aimed at quality mathematics education for all students. Though the reform movement's intent is laudable, it insufficiently addresses the challenges that face many poor, urban schools. First, although the *Principles and Standards* document (NCTM, 2000) adamantly supports high expectations, highly qualified teachers and ample resources for *all* students in *all* classrooms, it fails to consider the specific actions and supports needed to bring about this transformation (Allexsaht-Snider & Hart, 2001; Gutstein, 2003; Hart, 2003). Without a focus on issues within and beyond the classroom that impact instruction, reform efforts become isolated interventions that ignore the larger structural problems that undermine the efforts of individual teachers and schools (Apple, 1995; Secada, 1995).

Second, reform documents emphasize national standards to improve mathematics education, yet fail to question who might benefit from increased standardization (Apple, 1995). Standardization may only exacerbate existing inequalities by institutionalizing as ‘official knowledge’ the perspectives and ways of knowing of the dominant culture, thereby enhancing educational opportunities for already advantaged students (Tate, 1994) and further marginalizing culturally, linguistically, and racially diverse students (Apple, 1995; Zevenbergen, 2000). Moreover, high-stakes standardized assessments aimed at holding schools accountable for students’ achievement of these rigorous standards are particularly devastating to students in large, inequitably funded urban districts, like New York City, where severe deficits in human and capital resources deprive students “of a fair opportunity” to pass the exams (Campaign for Fiscal Equity, 1999, p. 1).

Third, the pedagogical framework advocated by the reform movement, ‘Teaching for Understanding’ (e.g., Hiebert, Carpenter, Fennema et al., 1997), is based on a body of research that, though rigorous and substantial, is limited in scope, which makes problematic its applicability to the challenges faced by low income, urban schools. Certain groups of students, namely those historically marginalized and underserved by public education, have been excluded, or at best inadequately represented, in this research (Secada, 1991). Moreover, the majority of recent research in mathematics education – including those studies that have informed the ‘Teaching for Understanding’ framework - is narrowly focused on children’s mathematical thinking (e.g., see Hiebert & Carpenter, 1992, for a review of this research), and lacks sufficient attention to the socio-political contexts that surround teaching and learning endeavors (Skovsmose & Valero, 2002). Such analysis is imperative to understanding the complexity of students’ experience and performance in school mathematics (Martin, 2000; Valero, 2002). As Empson (2002) argued, “Although I acknowledge the power of this framework [teaching for understanding] to help teachers effect changes in practices and student outcomes, I suggest that as we consider the educational experiences of US children at the periphery of mainstream ways, it offers an incomplete view of possible solutions” (p. 590).

Fourth, the reform movement fails to critique the goals of learning mathematics (Apple, 1995). The focus remains on functional mathematical literacy and conceptual understanding, yet understanding for what purpose? Unfortunately, this understanding is often disconnected from students’ lives and intentions (Chazan, 2000; Noddings, 1993; Skovsmose, 1994), and fails to prepare students for critical, active participation in society (Tate, 1994). Even students who develop identities as powerful mathematical thinkers may fail to find personal reasons, beyond instrumental ones (Martin, 2000), why they should learn mathematics. And for many students, the instrumental importance of mathematics may not be sufficient (Noddings, 1993), as they may feel that there is a low probability that they or their communities will benefit from the mathematics that they learn in school (Martin, 2000; Valero, 2000). Students’ refusal to engage and perform may be their way of resisting an educational system that fails to take their purposes and intentions into account, and that deprives them of opportunities to make important decisions, to participate in negotiating the curriculum, and to assume more power (Chazan, 2000; Shor, 1992, 1996).

As Boaler and Greeno (2000) argue, students’ experience of math “runs counter to their developing identification as responsible, thinking agents” and against the kinds of people they want to be and become – creative, humane, connected citizens (p. 171).

Supporting students' sense of themselves as people who can and do make a difference, their sense of *agency*, is not only a critical component of schooling (Bruner, 1996; Jackson, 2003; Pruyn, 1999), but an important step towards achieving equity in mathematics education (Gutstein, 2003a; see also Ernest, 2002; Valero, 2002).

Therefore, while the NCTM documents have drawn increased attention to equity concerns -- the centrality of equity in the 2000 *Principles and Standards* marked a vast improvement over the previous version (NCTM, 1989), which argued for equity as an "economic necessity" (p. 4) – this consideration has been narrow and insufficient. There remains an urgent need for educators to go beyond the *Standards* as they strive for a just mathematics education for all students.

Beyond the Standards: A Focus on Student Agency

In considering *how* to work towards a just and equitable mathematics education, I agree with Gutstein (2003b) who has argued, "we need to broaden the conception of what it means to work for equity *in* mathematics education, and to consider how to work for equity *through* mathematics education as well. That is, mathematics education *itself* can be a tool for equity and justice" (p. 27, emphasis original). Just how might math education become a tool for justice, both within and beyond the classroom? This question captures the essence of this dissertation. Drawing on the critiques leveled above, a core component of achieving equity through mathematics education may be an increased focus on students' sense of *agency* – their sense of themselves as "agents whose actions count in, and account for, the world" (Holland, Lachiotte, & Skinner et al., 1998, p. 285).

An increasing number of researchers, both within mathematics education (Empson, 2002; Ernest, 2002; Gutstein, 2002, 2003a, 2003b; Hart, 2003; Martin, 2000; Mellin-Olsen, 1987; Valero, 2002) and beyond (Bruner, 1996; Noddings, 1993; Pruyn, 1999; Varenne & McDermott, 1998) have argued for increased attention to student identity and agency as a way to promote equity. These arguments have taken various forms, from Noddings' (1993) call to consider students' purposes in learning mathematics, and their sense of themselves in relation to the world, as they confront questions such as:

> Who am I? Who will I be? How hard should I work and toward what end? How am I doing and how can I tell? Can I make a difference in the world? Do I have any control over my own life? (p. 153)

to Valero's (2002) plea for educators to recognize the agency that students bring to the mathematics classroom as a way of positioning students with more power relative to school mathematics practices. Yet they converge on the imperative to conceptualize students as social, cultural, political beings whose intentions in learning surpass the narrow view of a mathematically active, cognitive subject (Valero, 2002).

Researchers also vary in how they link an emphasis on student agency with equity in/through mathematics education. Some scholars advocate attending to student agency as a way of motivating mathematical learning by fostering higher levels of engagement (e.g., Allexsaht-Snider & Hart, 2001; Jackson, 2003; Mellin-Olsen, 1987). While I do not disagree with this argument, my own thinking more closely parallels that of researchers who take an explicitly political stance, linking student agency to broader notions of empowerment and social justice (e.g., Ernest, 2001; Gutstein, 2003a; 2003b; Valero, 2002). From this perspective, fostering student agency can support equity not only through its impact on students' sense of themselves as doers and creators of mathematics, but also as it encourages students to see themselves as capable citizens who have the power to be "key participants in the struggles for equity and justice" (Gutstein, 2003b, p. 27). In other words, in mathematics classrooms that provide opportunities for students to develop and enact agency, students' participation can become a means of fostering equity *through* mathematics education.

Curriculum and Pedagogy that Support Agency:

Teaching Mathematics for Social Justice

In this study, I present one curricular/pedagogical approach, '*Teaching and Learning Mathematics for Social Justice*'[1], and argue that this approach provides fertile ground for students to both engage in significant mathematics and draw upon and enact a sense of agency as they investigate issues of importance in their lives. I use teaching and learning for social justice in a Freirian (1970/1993) sense to describe education grounded in developing ways of knowing that support students in critically reading and acting upon the world. In other words, from a teaching for social justice perspective, the aim of mathematics education becomes

> to help people become critical thinkers who can use mathematics as a tool for analyzing social and political issues and can reflect on that tool use, including its

[1] I further describe the components of Teaching and Learning Mathematics for Social Justice in Chapter 2.

limitations … both for the empowerment of the individual and for the betterment of the society. (Mukhopadhyay & Greer, 2001, p. 300)

As students participate in mathematics classrooms aimed at *Teaching and Learning for Social Justice*, they have opportunities to negotiate and shape their study of mathematics, to experience mathematics as a discipline that can help them make sense of the conditions of their lives, and to identify themselves not only as capable mathematical thinkers, but as active, critical citizens whose actions can and do make a difference. That is, teaching for social justice creates a particular kind of community of practice (Lave & Wenger, 1991), or *figured world* (Holland, Skinner, Lachiotte et al., 1998), "in which children and teachers take on certain roles and help define who they are" (Boaler & Greeno, 2000, p. 173). This figured world is one that engages students as (a) critically reflective citizens who address questions of personal and social importance and use what they learn to act upon their lives, and (b) capable students that assume power over their learning and construct deep understanding of significant mathematics, and of the potential transformative power of mathematics in their lives.

I argue that students' participation in the figured world of a social justice oriented mathematics classroom fosters a particular kind of agency that I refer to as *critical mathematical agency*[2]. This enhanced sense of agency encompasses, but goes beyond, "the realized capacity of people to act upon their world … purposively and reflectively" (Inden, 1990, as cited in Holland et al., 1998, p. 23). To begin, it is *critical*. That is,

[2] In Chapter 2, I draw on relevant literature to construct a conceptual framework for 'critical mathematical agency.' In subsequent chapters, I employ and refine this framework in my analysis of the data.

critical mathematical agency implies students' capacity to view the world with a critical mind set, to imagine how the world might be a more just and equitable place, and to engage in action aimed at personal and social transformation.

Moreover, as this enhanced sense of agency refers to the particular figured world of a mathematics classroom, it is m*athematical*. That is, it includes students' capacity to identify themselves as powerful mathematical thinkers, who construct and use mathematics in ways that are personally and socially meaningful. In summary, *critical mathematical agency* allows students to draw upon and construct mathematical understanding to investigate and critique situations in their lives and in the world around them, and to act transformatively upon those conditions. In this way, classroom practices that foster this enhanced sense of agency among students - such as teaching and learning mathematics for social justice - have the potential to engage students as participants in struggles for justice, thereby supporting equity *in* and *through* mathematics education.

Need for Research

Although research that documents teaching mathematics for social justice in k-12 schools is limited (e.g., Gutstein 2000, 2003a, 2003b; Skovsmose, 1994), a growing number of researchers argue that this pedagogy can support the ongoing struggle for equity in mathematics education (Skovsmose & Valero, 2001; see also Frankenstein, 1995; Gutstein, 2003a; Tate, 1995), through the cultivation of student agency (Gutstein, 2003b; Valero, 2002). As Ernest (2001) argued,

> Once mathematics becomes a 'thinking tool' for viewing the world critically, it contributes to the political and social empowerment of the learner, and ultimately to the promotion of social justice and a better life for all. (p. 287)

Unfortunately, most researchers have based their arguments on theoretical constructs that are not often examined in relation to actual classroom data (e.g., Cotton, 2001; Ernest, 2001; Skovsmose & Valero, 2001; Valero, 2002). Empirical studies that do exist are primarily teachers' anecdotes of their own practice (e.g., Bigelow & Peterson, 2002; Peterson, 1999), isolated classroom examples included to illustrate a point (e.g., Ladson-Billings, 1995; Skovsmose & Valero, 2002; Tate, 1995), or suggestions of projects that teachers might implement (e.g., Zavalsky, 1996). While such work offers important contributions -- it is not at all my intent to devalue the merits of theoretical work or classroom stories -- a more thorough, extended, and systematic analysis of teaching and learning mathematics for social justice is needed.

Recent work by Gutstein (2000, 2002, 2003a, 2003b) has begun to address this void in the literature. His description (2003b) of student agency among middle school mathematics students in his 'teaching for social justice classroom' is compelling, yet still lacking are analyses of *how* student agency develops, *what* this agency means in a mathematics classroom and *how* it interacts with the discipline of mathematics, and moreover, *how* student agency in mathematics can contribute to a broader struggle for equity and justice. A central goal of this study was to address this dearth in the literature.

Research Questions

This study investigated how teaching and learning mathematics for social justice might foster *critical mathematical agency* among students, the interaction and tensions between student agency and their engagement in the discipline of *mathematics*, and how critical mathematical agency might contribute to equity *in* and *through* mathematics education. In particular, the following questions guided the investigation.

1) How do students enact *critical mathematical agency* within mathematics classrooms aimed at teaching and learning for social justice?

2) How does students' engagement in the discipline of mathematics support and interact with their enactment of *critical mathematical agency*?

3) As student agency is shaped and constrained by the social, cultural, political and historical forces that characterize a particular context (Baez, 2000; Holland et al., 1998; Varenne & McDermott, 1999), what tensions and contradictions arise as students exert a sense of agency within, upon and against those structures, and how are those tensions negotiated by students and teachers?

Constraints on the Study's Claims

Given my focus on the enactment of student agency within a mathematics classroom aimed at teaching and learning for social justice, this study was not able to make claims about how students exhibited agency across different academic, social, or cultural contexts. Students were encouraged to talk about experiences in other contexts, but the classroom-based focus of the study did not support analysis of how factors beyond the school (e.g., family interactions, church experiences, etc.) or beyond their mathematics classroom (e.g., experiences in a humanities class) might have contributed to or supported the agency that students enacted. Moreover, the duration of the study prevented making claims about the development of study agency over time.

The claims of this study were related to the kind of data that were collected and how that data were analyzed. For instance, data collection methods (e.g., clinical interviews, classroom observations) enabled assessing students' mathematical understanding of certain concepts. However, since the mathematical content of the study was not predetermined, but emerged as the study progressed, I was not able to formally measure growth in individual students' understanding of concepts through traditional measures such as pre- and post-assessments. In addition, while the questions I asked centered on students' classroom participation, and while data analysis attended to how certain classroom practices facilitated students' "agentive" action (Pruyn, 1999), the aim of the study was not to identify teacher actions that functioned to support student agency. In fact, the nature of the questions I asked (i.e., student centered) led to methods of data collection and analysis that focused more on students' participation than on the teacher's actions.

Outline of the Dissertation

The remainder of the dissertation is organized as follows. In Chapter 2, I review the relevant literature, drawing upon theories of agency as shaped and enacted in figured worlds (Holland et al., 1998), and on theoretical and empirical work in critical mathematics education and teaching and learning for social justice, to develop a conceptual framework for *critical mathematical agency*. In Chapter 3, I describe the methodological details of this critical ethnographic study. I begin by justifying critical ethnography as an appropriate methodological orientation for this study, and then detail the methods of generating and analyzing data that I employed to make sense of students' critical mathematical agency.

In Chapter 4, I portray the research setting: Beatriz Font's[3] sixth grade mathematics classroom at Francis Middle School in New York City. I begin with an overview of the school that includes historical information, a description of the physical environment, and insights into the beliefs and values of Francis staff and students. I then present a detailed picture of the figured world of Beatriz's classroom, including the physical environment, her pedagogy, and the norms and expectations that guided classroom interactions.

In Chapter 5, I focus on the first two research questions that guided this study: a) How did students enact *critical mathematical agency* within a mathematics classroom aimed at teaching and learning for social justice?, and b) How did students' engagement in the discipline of mathematics support and interact with their sense of agency? The chapter draws upon the stories of four case study students, Angel, L.J., Joel, and Naisha, to examine these questions. In Chapter 6, I focus on the third research question: the tensions that arose as students exerted agency within, upon and against the constraints of the figured world of their mathematics classroom, and how those tensions were

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[3] Beatriz collaborated in the creation, analysis and in some cases, writing of this report, and as such, preferred for her actual name to be used. The names of all other teachers, staff and students, as well as the name of the school, have been changed to protect participants' confidentiality.

negotiated by students and their teacher. In the final chapter, Chapter 7, I discuss the implications of this study and questions for future research.

CHAPTER TWO: A CASE FOR CRITICAL MATHEMATICAL AGENCY

The focus of this study is the *critical mathematical agency* that students enacted as they participated in a classroom aimed at teaching mathematics for social justice, how this agency interacted with their engagement with the discipline of mathematics, and the tensions and contradictions that arose. But what do I mean by critical mathematical agency? How does it differ from agency in general? What does it look like in classrooms? What does it entail, and how and why does it matter? The aim of this chapter is to draw on both theoretical and empirical work to situate critical mathematical agency in the relevant literature, and in doing so, outline a conceptual framework that further elaborates its multiple components.

<u>In the first section</u> of this chapter, I construct a working definition of *human agency*, and review key concepts that are crucial to how agency is understood within the context of this study. I then explore the connection between agency and social justice, moving from the broad notion of *agency* to the more particular construct of *critical agency*. <u>In the second section</u>, I shift to a description of pedagogical practices that might support students in developing and enacting a sense of critical agency: Teaching and learning mathematics for social justice. I situate this pedagogical approach in the literature, outline its central tenets, and review the limited empirical studies in the field. <u>In the third section</u>, I present *critical mathematical agency* as a way of describing the particular kind of agency that may be fostered in mathematics classrooms driven by the pedagogical practices previously described. After detailing the components of critical

mathematical agency, I draw on theoretical work in critical mathematics education to examine the possible interactions between students' agency and their engagement with the discipline of mathematics.

Conceptualizing Agency

The construct of agency is increasingly prominent in educational research, particularly among scholars concerned with equity (e.g., Bruner, 1996; Martin, 2000; Varenne & McDermott, 1999), and those who ascribe to socio-cultural-historical theories of learning and activity (e.g., Holland, Skinner, Lachiotte & Cain, 1998; Pruyn, 1999). While researchers differ in how they operationalize agency in their work, there is general agreement that agency involves individuals' sense of themselves as "agents whose actions count in, and account for, the world" (Holland et al., p. 285). Inden (1990, cited in Holland et al.) defined human agency as:

> The realized capacity of people to act upon their world and not only to know about or give personal or intersubjective significance to it. That capacity is the power of people to act purposively and reflectively, in more or less complex interrelationships with one another, to reiterate and remake the world in which they live. (p. 42)

Along similar lines, Pruyn (1999) has defined agency as, "purposeful action taken by an individual, or group of individuals, in order to bring about change" (p. 20). Drawing on these definitions, I use the term agency to refer to: individuals or groups acting upon the world in purposeful ways, with the aim of creating, impacting, and/or transforming themselves and/or the conditions of their lives. The sections that follow present key concepts that are integral to how I have conceptualized agency within this study.

Agency in the Context of Figured Worlds

Identity and Agency

Identity and agency are developed and enacted in social practice. Identities are multiple and dynamic, constantly in the process of forming and (re)forming as individuals participate in the activities of social worlds, and often "thickening over time" (Holland, 2003, p. 6), sometimes into durable, agentive senses of self. Identities can become means through which individuals assert themselves, care about the conditions of their lives, and attempt to direct their own behavior, or in other words, means through which individuals enact agency (Holland, 2003). Drawing on Holland et al.'s (1998) words, identities – including the conceptions one has of oneself, and residue from past experiences (one's "History in Person") -- "are important bases from which people create new activities, new worlds, and new ways of being" (Holland et al., 1998, p. 5, p. 18).

Agency in Figured Worlds

However, human capacity to act agentively is not without constraint. In any given social interaction, one's "History in Person" meets with a particular combination of anticipated and unanticipated circumstances, constraints, and expectations, all of which may impact that individual's actions (Holland et al., 1998, p. 46). As such, it is imperative to examine human agency as situated within the practices of "historically contingent, socially enacted, culturally constructed [figured] worlds" (p. 7). Holland et al. defined *figured worlds* as "socially and culturally constructed realm[s] of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52)[1]. They proceeded to argue that agency, which emerges from the self-in-practice, is developed through participation in the day-to-day activities organized by figured worlds (p. 57).

For instance, individuals develop agency as they (a) consciously accept or reject the subject positions cast upon them by others, (b) as they "identify themselves as actors with more or less influence …. privilege … or power" (p. 60), (c) as they challenge or attempt to transform the norms and values that constrain their activity, (d) as they participate in acts that are consequential both personally and collectively, and (e) as they develop skills and understandings that enhance their capacity to direct their own behavior in these worlds. In the case of schools, classrooms become figured worlds, and the identities and agencies that develop and are enacted within classrooms are closely related to how students participate, the roles they assume, and the shared understandings that they construct (Boaler & Greeno, 2000; Lave & Wenger, 1991).

Agency Constrained by Figured Worlds

Figured worlds function not only to facilitate agency, but also to shape and limit its enactment. Both the form agency assumes – how it is enacted and for what purposes -- and the consequence agency has for participants are impacted by the discourses, ideologies, practices, cultures, and histories that characterize a particular figured world (Holland et al., 1998; see also Baez, 2000; Varenne & McDermott, 1998). For instance, certain actions may be valued over others, particular actors may be cast into positions of

[1] In Chapter 4, I present detailed descriptions of two figured worlds in which students interact in this study: their school, and their sixth grade mathematics classroom.

power and influence, while others have limited opportunities to make decisions or participate in consequential ways.

Agency is always limited (Varenne & McDermott, 1998). As Holland et al. (1998) argued, “the author [the agent] works within, or at least against, a set of constraints that are also a set of possibilities for utterance” (p. 171). For example, a mathematics teacher’s intention to diverge from a traditional textbook and invite student input to help her design a more engaging, problem solving lesson is constrained by numerous aspects of the figured world of the school, including: the length of a class period, available tools and resources, the teacher’s understanding of the mathematical concepts and of the students’ knowledge, administrative pressures not to diverge from the official text, the culture of the math department, pressure of standardized assessments, and student resistance, among others.

Resisting and Remaking Figured Worlds Through Improvisation

Yet, just as human agency is limited by figured worlds, so too are humans capable of using agency to resist and transform figured worlds. According to Holland et al. (1998), this often occurs through agentive acts of *improvisation*. For instance, when individuals find themselves in disjoint situations, or unlikely, ambiguous spaces (Behar, 1995), spaces that pull them in contradictory directions, or spaces that by nature of their ambiguity allow a range of possible actions, they must improvise a response to the situation. These improvisations are shaped both by what individuals bring to the figured world (their past experiences, understandings, subjectivities) and by the positions and resources afforded by the present situation (Holland et al., 1998).

These impromptu actions, which are not as likely to emerge when activity is more scripted and predictable, can then serve to interrupt, resist, contradict, and even remake aspects of the figured world. Even within figured worlds where certain participants have very limited power, improvisation is still possible, and has the potential to become "openings by which change comes" (Holland et al., 1998, p. 18). Holland et al. argued

> Even within grossly asymmetrical power relations, the powerful participants rarely control the weaker so completely that the latter's ability to improvise resistance becomes irrelevant. (p.277)

Continuing with the previous example, a mathematics teacher torn between administrative pressure to use textbook materials that focus on preparing students for standardized assessments, and her own beliefs of how to teach mathematics for understanding, may respond by improvising her own curriculum, drawing both on what she brings to the classroom, and on the resources and liberties her particular teaching context affords. In this way, she remakes aspects of her own practice, and moreover, challenges aspects of the figured world of the school, such as the status accorded to textbooks and standardized assessments, historically accepted models of directive instruction, and the power of the administration to dictate classroom practices. In other words, her agentive act of improvisation becomes a "mediator to change oneself and others, and perhaps even the figured worlds in which one acts" (Holland et al., 1998, p. 46).

Tension Between Human Agency and the Constraints of Figured Worlds

Discussion of this ever-present tension between human agency on one hand, and the power of context on the other, abounds in the literature (e.g., Baez, 2000; Foucault, 1977; Holland et al., 1998; Vithal, 1999). Rather than isolate one construct, such as agency, at the expense of the other, the context of a figured world, it is imperative to consider both. As Varenne & McDermott (1998) argued, quoting Bentley (1908), "if we throw emphasis on either one of the two to the exclusion of the other, and deny the complement, we are constructing a world out of stuff that has definition only in terms of the very opposition we attempt to deny" (p. 171). Moreover, rather that placing human agency and contextual dimensions on opposite ends of a dichotomy, it may be more helpful for research to examine the interplay between these two forces (Baez, 2000; Vithal, 1999). The present study addresses this interplay by considering questions such as: How might students' agency be shaped, limited, and/or facilitated by the figured world of a mathematics classroom aimed at teaching for social justice? How do students' acts of agency impact the multiple figured worlds -- at the classroom level and beyond -- that students inhabit?

Thus far, I have discussed agency in broad terms, as purposeful action situated within figured worlds. Yet what do these purposeful acts look like? How might one recognize agency among students within the figured world of a middle school mathematics classroom? The section that follows draws on the literature to make this somewhat elusive notion of agency more tangible and concrete.

Agency Enacted Within Figured Worlds

"Human agency may be frail, especially among those with little power, but it happens daily and mundanely, and it deserves our attention" (Holland et al., 1998, p. 5). I begin with this quote because it emphasizes the daily nature of agency. Agency, like related concepts of *empowerment* and *liberation*, is easily romanticized, evoking images of large-scale protest and resistance, or idealized examples of success in the face of unfavorable conditions. While such images may reflect a powerful sense of agency, it is my intent to draw attention to how people, in particular middle school students, enact a sense of agency in more subtle and daily – but no less powerful – ways. As Varenne & McDermott (1998) argued, "our task is to give examples of the things that kids can do that nonetheless disappear in the normal telling of their lives" (p. 65).

Agency as Positioning and Authoring

One such daily enactment of agency are the ways individuals position and author themselves with respect to their lives, to others, and to their surroundings. Holland et al. (1998) referred to positioning as *taking a stance*, and argued that this stance taking occurs as part of participating in figured worlds (see also Calabrese Barton, Drake, Pérez, et al., 2003). Situations and interactions cast individuals into certain positions (e.g., that of passive student, or all-knowing teacher) that can strongly affect how they respond (Holland et al.; Pruyn, 1999). As individuals respond to the positions cast upon them, as they answer the world, they simultaneously make meaning of the world and themselves, a process which Holland (2003) has referred to as "self-authoring" (p. 5; Holland et al., p. 173). In this way, acts of authoring, to the extent that they are aimed at transforming individuals themselves and/or the conditions of their lives, are expressions of agency. As Holland (2003) wrote, “self-authoring ... can become a basis for agency rendering a person or a collective, a sustained, generative source of meaning and action even in the face of powerful forces of positioning” (p. 5-6).

What might agency as authoring and positioning look like within the figured world of a middle school mathematics classroom? To begin, students continually position themselves and are positioned by others as part of ongoing classroom interactions. Through taking a stance as leaders of small group activities, tutors who assist struggling peers, or as students who share novel strategies, pose challenging questions, or critique hegemonic classroom practices, students communicate something to themselves and others about their mathematical understanding, and moreover, about their role and authority within, and perhaps beyond, the mathematics classroom. By asserting their needs, concerns, interests and goals, and by posing questions of personal and collective importance, students author spaces for themselves in the classroom, spaces that have the potential to transform their experience in the figured world. Documenting how students position and author themselves, and how they respond to the positions cast upon them, may provide further insight into their sense of agency.

Agency as Transformational Resistance and Critique

A second notion of agency prevalent in the literature is *agency as acts of resistance and critique* (Baez, 2000; Gellert, Jablonka & Keitel, 2001; Gutstein, 2003b; McLaren, 1989; Pruyn, 1999; Varenne & McDermott, 1998). Even when confronted with intense external pressure to conform and significant constraints upon possible courses of action, people can and do resist, and this resistance occurs as they participate in, and sometimes against, figured worlds. At times, this resistance is subtle – choosing not to participate in a class discussion as a way of resisting a curriculum that the student experiences as irrelevant – and on other occasions, more forceful – openly critiquing a school policy or classroom practice as unjust. Students may resist the roles cast upon them, challenge the norms and expectations that guide the activity of their classroom, or even critique the subject matter they study as limited, biased, or exclusionary.

Researchers have been careful to distinguish between reactionary and self-destructive resistant behavior (e.g., dropping out of school, acting out in class), and resistant acts aimed at critiquing and transforming unjust conditions, referred to by Solorzano & Delgado Bernal (2001) as *transformational resistance* (see also Gutstein, 2003b; Pruyn, 1999). The former mode of resistance may actually be counter-productive, reinforcing and recreating those ideologies, practices, and structures that it intends to resist (McLaren, 1989; Solorzano & Delgado Bernal, 2001; Willis, 1977). However, such acts still function to challenge hegemonic authority, and while they alone are not enough to create change, they deserve notice (Pruyn, 1999).

In contrast, *transformational resistance,* which is motivated and supported by a *critique* of oppressive conditions, ideologies, and norms, and a sense that individual and social change is possible, is agentive (Solorzano & Delgado Bernal, 2001). This resistance often begins as individuals interrogate and critique their own reality, and begin to see larger social, economic, historical, and political forces reflected in their experiences within particular figured worlds (Gellert, Jablonka, & Keitel, 2001; Peterson, 1999). Students might start with critical reflection upon local circumstances, such as overcrowding at their school or unjust standardized assessment practices, which can help them to realize that "the injustices of our society are not isolated exceptions; they are logical consequences of the institutional structures of our society, and it does not have to be that way – there are other choices we can make to reorganize our institutions" (Frankenstein, 1995, p. 180). As students question their daily lives and recognize the impact of the culture of power – how it affects their opportunities to learn, their access to information and services, and their school environment – they enact a sense of critique that can in turn support their acts of resistance.

Summary

Authoring and positioning, and resistance informed by critique are useful constructs for making sense of agency, constructs that at their core include elements of *improvisation*. For instance, improvised actions can *resist* scripted or anticipated ways of participating in figured worlds. Improvisations allow individuals to challenge subject positions cast upon them, and to work against and around contextual constraints. In a similar manner, acts of *authoring* and *positioning* involve improvisation. As individuals author themselves, or to use Holland et al.'s (1998) words, as they "answer the world" they do so in ways that are not scripted or predetermined (p. 272). Instead, they draw upon their experiences, understandings, and histories, and upon the resources and positions afforded by the figured world, to craft an "in the moment" response.

In so far as these improvisations support individuals (and groups) in purposeful action upon their lives and/or their world, and "to the extent that these productions [improvisations] are used again and again, they can become tools of agency" (Holland et al., 1998, p. 40). Examining the role of improvisation in students' acts of agency, and in particular, how students make use of improvisation as a tool for future action, facilitates a deeper understanding of the richness and complexity of their agentive actions.

It is also possible that students may exhibit agency in ways that do not easily map onto the constructs presented in this section; this is an issue that will be addressed in subsequent chapters.

Linking Agency and Social Justice: Critical Student Agency

When agency goes beyond students' sense of themselves as people whose actions can and do make a difference, to include actions informed by an awareness of oppressive and unjust conditions and aimed at social transformation, it becomes what is referred to in the literature as *critical student agency* (Pruyn, 1999). Pruyn defined this enhanced sense of agency as, "purposeful action taken by a student, or a group of students, to facilitate the creation of counter hegemonic *pedagogical* (i.e. educational) practices" (p. 4, emphasis original). In this study, I use critical student agency more broadly, not only to refer to actions aimed at transforming classroom curriculum and pedagogy, but also actions that attempt to transform practices, policies, and conditions outside the classroom – actions motivated by a concern for social justice. For middle school students, such actions could include: critique of gender inequities in the workplace, speaking out against injustice in the community, investigation and resistance of district tracking or assessment policies, critical analysis of data followed by some kind of responsive action, or challenging knowledge presented in official school textbooks, among others.

According to Pruyn (1999), this process of enacting critical agency and working towards social justice is cyclical. He argued

> It is through taking [critical] agentive stances and actions that individuals begin to see themselves … as potential socio-political actors. … Through participation in agentive acts, individuals begin to recognize, and struggle against, attempts by hegemonic cultural institutions to position them as passive followers and conformists. (p. 20)

In other words, by enacting critical agency, students increase their awareness of injustice in society and strengthen their sense of themselves as political actors, which in turn motivates them to continue to engage as critical agents, thereby furthering the struggle for justice.

Challenges and Tensions of Critical Student Agency

The limited empirical studies that have examined critical student agency (e.g., Gutstein, 2000, 2003b; Pruyn, 1999) discuss various challenges inherent to their work. To begin, agency is not a stable, immutable construct, but rather a fluid aspect of one's identity that develops and "thickens" over time (Holland, 2003). The same individual can demonstrate a strong sense of critical agency in some contexts, and assume a passive, powerless stance in others. Gutstein (2003b) described this contradictory nature of agency among his students: "In my classes, I have seen the same students act disruptively one day, give up the next day, and even show aspects of transformational resistance on other days" (p. 13).

Other challenges to fostering critical student agency discussed in the literature include: (a) schools' role in socializing students as passive recipients of information, thereby minimizing their creative, critical power (Freire, 1970/1993); (b) students' acquiescence to these hegemonic instructional practices and their self-positioning as passive subjects, because "that's just the way school is" (Pruyn, 1999; see also Martin, 2000; Shor, 1996); (c) oppressive high-stakes testing practices that limit teachers' use of liberatory pedagogies and therein deny students opportunities to develop the knowledge and dispositions needed to understand, critique, and transform the conditions of their lives (Gutstein, 2003b); and (d) students' sense of powerlessness about the injustices in the world, feelings of "oh well, what can I do about it?" (Gutstein, 2003a, p. 48).

These challenges are real, and deserve attention. And yet I argue that precisely because of these pressing challenges, because of the current educational and political situation that is particularly detrimental to students in poor, urban schools, there exists an even more urgent need to support curricula and pedagogies that foster a sense of critical agency among students. As Bruner (1996) argued

> Education is risky, for it fuels a sense of possibility. But a failure to equip minds with the skills for understanding and feeling and acting in the cultural world is not simply scoring a pedagogical zero. It risks creating alienation, defiance, and practical incompetence. … We must constantly reassess what school does to the young student's sense of his own powers (his sense of agency). (p. 39, 42)

The previous section reviewed concepts that are crucial to how agency is understood within the context of this study, and introduced critical agency as a way to connect agency and a struggle for social justice. In the next section, I describe a curricular/pedagogical approach that can support students in developing and enacting a sense of critical agency within the figured world of a mathematics classroom: teaching and learning mathematics for social justice.

In Support of Agency: Teaching and Learning Mathematics for Social Justice

Grounding Teaching Mathematics for Social Justice in the Literature

My vision of mathematics education for social justice draws heavily on two bodies of literature: (a) research in critical pedagogy, especially that of Paulo Freire (1970/1993), and others inspired by his work (e.g., McLaren, 1989; Nieto, 1999); and (b) work in critical mathematics education (e.g., Skovsmose, 1994; Frankenstein, 1983, 1987, 1990, 1995, 1997).

According to Freire, a Brazilian educator who spent his life developing critical pedagogy to support the empowerment of oppressed groups, education is an inherently political act that should serve to foster conscientization - critical reflection and action - among students (Freire, 1970/1993). For Freire, injustices such as poverty and illiteracy are inescapably linked to larger societal and educational structures that serve to advantage some individuals and oppress others. The central struggle of education then, is to help students develop ways of critically reading and acting upon the world in order to transform those structures, policies and assumptions that contribute to their oppression. At the heart of Freire's critical pedagogical approach is that students develop a socio-political consciousness that supports them in action aimed at self and social transformation, or in other words, that they develop and enact a sense of *critical agency* (Freire, 1970/1993; see also Horton & Freire, 1990; Pruyn, 1999).

Freire's theory about the relationship between education and social change has provided a powerful theoretical lens for educators across the disciplines. However, the vast majority of research in critical pedagogy, including that of Freire, has focused on language and literacy. Applying these ideas to mathematics education has been more challenging. Recently, a small but growing number of researchers have drawn upon critical pedagogical theory as a way to reframe their work in mathematics, thereby establishing the field of *critical mathematics education*[2] (e.g., Skovsmose, 1994; see also Ernest, 2002; Frankenstein, 1987, 1995; Gutstein, 2003a, Valero, 2002). Skovsmose (1994) described some of the questions and challenges that researchers in this field have confronted:

> Could 'mathemacy'[3] be substituted for literacy? … Could mathemacy also be used for the purpose of empowerment? … Mathemacy, as a radical construct, has to be rooted in the spirit of critique and the project of possibility that enables people to participate in the understanding and transformation of society. … What might the meaning of mathemacy be, if it is to fit this formulation? … Does mathematics have anything to do with the critical structure of society? (p. 26-27)

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[2] Also referred to in the literature as: (a) mathematics education for empowerment (Ernest, 2002), (b) Critical Mathematical Literacy (Frankenstein, 1983), and (c) Teaching Mathematics for Social Justice (Gutstein, 2002, 2003a, 2003b).

[3] Skovsmose (1994) defined mathemacy as the "ability to calculate and use mathematical and formal techniques" (p. 26).

While researchers differ in the particular ways they answer these questions, they seem to agree on two fundamental implications of Freire's work for mathematics education. First, they have embraced Freire's claim that knowledge is neither static nor neutral, as it does not exist apart from how and why it is used, and in whose interest (Freire, 1970/1993). Applied to mathematics education, this means challenging mathematical knowledge and practices, so that learners gain "an appreciation and awareness of the nature and value of mathematics and the uses to which it is put, as well as an understanding and critique of its social uses" (Ernest, 2002, p. 4; see also Frankenstein, 1987, 1997; Skovsmose, 1994; Vithal, 2000). Second, they have drawn upon Freire's theoretical lens to examine and redefine the goals and purposes of learning mathematics, so that the aim of critical mathematics education becomes enabling understanding, critique, and action upon the world (Ernest, 2001), in addition to supporting conceptual understanding of the discipline.

Empirical Research Investigating Teaching Mathematics for Social Justice

Research that has documented critical pedagogy in action in k-12 mathematics classrooms is extremely limited. In the following section, I review several classroom-based research programs that do exist, and discuss their relevance to the present study.

Marilyn Frankenstein

Over the past two decades, Marilyn Frankenstein (1987, 1994, 1995, 1997) has provided numerous examples of critical mathematics education through her work with adult students in basic mathematics courses. Although Frankenstein's research occurred in post-secondary versus k-12 settings, I have chosen to include it here, because her work represents the most persistent, structured attempt to link mathematics education with a struggle for social justice. Central in Frankenstein's approach was the use of statistical tools to critically analyze social conditions such as unemployment rates, income data, and military versus domestic spending, among others. The aim of this critical analysis of quantitative data was to challenge hegemonic ideologies and inequalities, and to foster in students the ability to question and critique the conditions in which they live (Frankenstein, 1987,1997).

In addition to providing examples and analysis of curricular activities, Frankenstein's (1987, 1997) work has suggested, through brief classroom stories and vignettes, that it is possible for students to develop a more critical awareness and understanding of society through their participation in mathematics. Yet, Frankenstein's research has lacked a systematic analysis of the impact of critical mathematics education on students. She has presented broad claims about her students' reactions, "students do become angry and committed to social change" (1987, p. 201), yet has not documented or analyzed interactions that evidenced this transformation. Absent is a detailed description of how students expressed critical awareness, how their mathematical understanding may have supported this awareness, and moreover, a discussion of the complexities and tensions inherent in this kind of work.

Moreover, Frankenstein (1987) has limited the mathematical content of her curriculum to numerical reasoning about statistical data, focusing her work on fostering "the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues and the ability to present data to change people's perceptions of those issues" (p. 12). While statistical analysis is a clear link between mathematics and issues of equity and justice, and definitely the one most reported in the literature (e.g., Bigelow & Peterson, 2002; Tate, 1995), also needed is research that examines how students' participation in other mathematical domains (e.g. geometry, algebra) can contribute to a struggle for social justice.

Ole Skovsmose

In Denmark, a group of researchers led by Ole Skovsmose (1994; Skovsmose & Valero, 2001, 2002) has advanced a critical approach to mathematics education in both elementary and secondary schools. While their work has made significant theoretical contributions, analysis of empirical data has been limited. An exception are Skovsmose's (1994) descriptions of mathematics "projects" designed to investigate local issues so that students learn and use mathematics for real purposes. For example, the "Economic Relationships in the World of a Child" project focused on analyzing children's personal use of money, critiquing social policies that award money to families based on the number of children, and designing a budget for a community youth organization (p. 63). A definite strength of Skovsmose's work were these rich, detailed descriptions of the project-based units, descriptions that both lent a concreteness to teaching mathematics for social justice, and provided a varied sense of what is possible. However, it is important to note that these descriptions were based on teachers' recollections of their own practice, and not on Skovsmose's or another researcher's observations. Skovsmose utilized each project as a case to illuminate a particular aspect of his theory of critical mathematics education. In this way, through grounding theoretical constructs in stories of classroom activity, Skovsmose began to link theory and practice, therein making a significant contribution to the field.

However, much like Frankenstein, Skovsmose (1994) focused on theorizing critical mathematics curricula and pedagogy from the perspective of the researcher and in some cases the teacher, not on documenting students' participation or analyzing their perceptions and experiences. In order for scholars in the field to continue to claim that critical mathematics education promotes equity and justice, research needs to pay careful attention to students and their experiences. Greatly needed is work that (a) documents students' participation in classrooms aimed at teaching mathematics for social justice, and (b) analyzes the mathematical learning and the sense of personal and collective agency evidenced in what students say and do.

Danny Martin

Though Danny Martin's (2000) work did not focus on teaching and learning mathematics for social justice – although the teachers at the Oakland high school that he studied drew upon the Algebra Project[4] (Moses, 2001) to inform their curriculum - I include his research here, because of his ambitious efforts to incorporate the constructs of identity and agency into analyses of student success (or failure) in school mathematics. Martin's study aimed to understand how African-American middle-school students' performance in mathematics was impacted not just by curricular programs or pedagogical

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[4] The Algebra Project, founded by civil rights activist and mathematics educator Robert Moses in the 1980's, is a national mathematics literacy effort aimed at helping low income students and students of color--particularly African-American and Latino/a students--successfully achieve mathematical skills that are prerequisites for college preparatory mathematics programs.

practices, but by a combination of forces across multiple contexts, including individual, school, community, and socio-historical. Martin found that mathematically successful students exhibited high levels of “achievement-oriented individual agency” and rejected “the negative influences that surrounded them,” such as low expectations for achievement (p. 123). In addition, these students recognized the instrumental importance of mathematics in their lives, considering mathematics as essential to future success in school, work, and life.

While Martin’s research made a significant contribution, directing our attention to the powerful impact of students’ identities and agencies on their engagement and success in school mathematics, his analysis of agency was limited. First, he narrowly focused on the *achievement* oriented agencies of *individual* students. Agency, as it is defined in the present study, is aimed at more than the ability to succeed in school, and the development of agency is a collective, rather than an individual effort. Moreover, missing in Martin’s research is an analysis of how student agency was situated within and interacted with their participation in particular social practices. As Empson (2002) argued in an essay review that discussed Martin’s study, “Not considered is how students are positioned by particular curriculum arrangements, instructional practices, or other educational processes, and the interaction of context and agency in practice” (p. 595). Greatly needed is research that examines these interactions.

Eric Gutstein

Perhaps the most relevant research to the present study is Eric Gutstein’s recent work with Latino/a middle school students in a large mid-western city (Gutstein, 2000, 2002, 2003a, 2003b). Gutstein spent two years teaching a 7th and 8th grade mathematics class, combining a reform oriented mathematics program with a series of real-world projects designed to build on students' lived experiences and their sense of justice. In addition to describing his approach to teaching mathematics for social justice in detail (2003a), Gutstein has analyzed students' participation in his classroom, including their mathematical power and changed orientations towards mathematics (2003a), and the sense of agency that they developed (2002, 2003b). Gutstein argued (2003b) that certain classroom conditions, including opportunities to learn more about personal histories, to see that positive change is possible, and to use mathematics to discover misconceptions about social reality and to take action, can foster agency among students. Moreover, he documented how students evidenced agency, albeit inconsistently, in the ways they spoke out against injustices, questioned and challenged situations in their own communities, and suggested possible actions.

Gutstein's work is unique both in how it positioned students' voices as a focal point of analysis, and because he was able to trace the development and enactment of agency among his students over several years. However, Gutstein (2003b) has separated his discussion of student agency from his analysis of their mathematical learning (2003a). That is, his work has not explicitly addressed how students' critical agency interacts with their engagement in mathematics. In order to better understand the nature of student agency in a *mathematics* classroom, versus in a language arts or social studies classroom, I argue that analysis of the interaction between students' agency and their participation in the discipline is essential. Moreover, Gutstein's pedagogy (2003a), though innovative and theoretically grounded, did not create spaces for students' intentions to influence the classroom curriculum. Given the potentially powerful relationship between asserting intentions and fostering agency, I argue that allowing students' intentions to inflitrate curriculum design is an essential component of any pedagogy aimed at teaching mathematics for social justice. The present study aims to build on Gutstein's research, and that of Frankenstein (1987, 1997), Skovsmose (1994), and Martin (2000), by addressing these voids in the literature.

Components of Teaching Mathematics for Social Justice and Student Agency

As previously stated, my vision of mathematics education for social justice draws on two bodies of literature: (a) critical pedagogy (e.g., Freire, 1970/1993), and (b) research in critical mathematics education. In the following section, I pull together work from each field to outline the central tenets of teaching and learning mathematics for social justice, and describe how these tenets might facilitate critical student agency. These tenets are:

(a) a focus on praxis: critical reflection and transformative action
(b) a sharing of power and authority among teachers and students
(c) a negotiated, organic, problem-posing curriculum that draws on students' interests, experiences, mathematical understanding and sense of justice
(d) a critique of the discipline of mathematics, and its uses in society.

A Focus on Praxis: Critical Reflection and Transformative Action

A central tenet of mathematics education for social justice is its emphasis on praxis – the process of critically reflecting and acting upon the world in order to transform it (Freire, 1970/1993, p. 6). Praxis is a collective endeavor that begins by encouraging students to generate, discuss and reflect upon issues that impact their lives. Students' experiences and intentions are the starting point, not the content matter (D'Ambrosio, 1990; Ernest, 2001; Frankenstein, 1987; McLaren, 1989; Tate, 1994; Valero, 2002), because immediate contexts, such as classrooms, schools, and neighborhoods, are students' favored places of praxis (Shor, 1992, 1996). Moreover, focusing on real situations that concern students' lives allows students to interrogate their own reality, and to see larger societal issues reflected in their experiences (Gellert, Jablonka, & Keitel, 2001; Peterson, 1999). Such critically reflective discussion can foster a critical mind set among students, thereby contributing to their sense of agency.

In addition to reflection, a focus on *praxis* contends that students need opportunities to engage in action aimed at self-empowerment and social transformation (Ernest, 2001, 2002; Gutstein, 2003a, 2003b). The knowledge that students construct, in particular the mathematical knowledge, should help them to participate in issues that affect their lives (D'Ambrosio, 1990; Tate, 1995). In this way, mathematical content and practices are not ends in themselves, but tools for participation in democratic life, tools to fight oppression (Gutstein, 2000, p. 2). In fact, students' desire to understand and address the inequities they experience in low income, urban schools and communities can motivate their use of mathematics (Tate, 1995). Moreover, their participation in action aimed at personal / social transformation, even if that action does not have a broad or lasting impact, draws on and may help to deepen their sense of critical agency.

A Sharing of Power and Authority Among Teachers and Students

A second tenet of this pedagogical approach is an authentic sharing of power and authority among teachers and students (Ernest, 2002; Freire, 1970/1993; Vithal, 1999). Typically, schools mirror authoritarian power structures in society that remove power from students, teaching them that they do not have the authority to disagree or to question, and that their role is to passively assimilate knowledge presented by the teacher. If instead, education becomes "something done *by* and *with* students rather than by the teacher *for* and *over* them" classroom interactions might challenge, rather that recreate unjust power relations (Shor, 1996, p. 148, emphasis original).

In their efforts to authentically share power with students, teachers can begin by challenging discursive practices traditionally accepted within mathematic classrooms and recognizing that such practices are mechanisms that can support or undermine students' sense of power and agency. They might reflect on questions such as: Who addresses who, and for what purposes? Is it safe for students to question and challenge? What are the rules for speaking? Who benefits, and whose ways of seeing, knowing, and acting upon the world are excluded? (Oakes & Lipton, 1999, p. 124). Critical reflection on classroom discourse can assist teachers in reconciling traditionally antagonistic and authoritarian student-teacher relationships (Freire, 1970/1993; Shor, 1996), which will then support students' power and agency in the classroom.

A Negotiated, Emergent, Problem-posing Curriculum

One of the primary outcomes of teachers and students sharing power and authority within classrooms, and the third tenet of teaching mathematics for social justice, is a *negotiated*, *emergent* curriculum (Shor, 1992, 1996; Boomer, 1992) that builds on students' experiences, mathematical understanding, and sense of justice. As Boomer (1992) explained, "negotiating the curriculum means deliberately planning to invite students to contribute to, and to modify, the educational program" (p. 14). It means considering students' intentions and experiences, and allowing students to jointly determine the processes, content and purposes of an activity or unit of study (Ernest, 2001, 2002; Skovsmose, 1994; Skovsmose & Valero, 2002; Valero, 2002)[5]. Freire refers to this negotiating process as *problem-posing* driven by *generative themes* that emerge from students' lives (Freire, 1970/1993). Generative themes are those ideas and experiences which give meaning to students' lives, and which "contain the possibility of unfolding into again as many themes, which in their turn call for new tasks to be fulfilled" (p. 83). The teacher's task in this negotiation is to consider what mathematics is implied by, or would help to clarify, students' generative themes (Frankenstein, 1987), and how those concepts and processes might build upon students' current mathematical understanding.

Within a 'Teaching Mathematics for Social Justice' perspective, negotiating the curriculum is important because providing students opportunities to express their intentions and make decisions democratizes authority and challenges undemocratic power structures in schools (Vithal, 1999). As Skovsmose and Valero (2002) argued, "the very process of planning, carefully and in detail, access to any kind of ideas, obstructs the possibility of making access democratic" (p. 399). Moreover, insofar as students have

[5] The curriculum negotiation (between teacher, students, and researcher) that occurred in this study is described in detail in Chapter 3.

opportunities to act transformatively upon the classroom curriculum, this process of negotiation may contribute to their sense of agency.

A Critique of the Discipline of Mathematics, and its Uses in Society

Just as schooling is inherently political, so is mathematics. Mathematics is not a neutral, objective discipline, as it cannot be detached from the interests, values, and intentions of the people who have created and used it within particular socio-cultural historical contexts (Anderson, 1990; Bishop, 1994; D'Ambrosio, 1990; Fasheh, 1997, Frankenstein, 1983; Frankenstein & Powell, 1994; Mellin-Olsen, 1987; Restivo, 1993; Skovsmose & Valero, 2001, 2002, among others). Thus a final tenet of 'Teaching Mathematics for Social Justice' is that is raises questions about how the discipline is defined -- what counts as mathematics, what is excluded, and who has the power to decide (D'Ambrosio, 1990; Powell & Frankenstein, 1997) --- and how the discipline is used, such as the power of mathematics in society to advance or block particular political agendas (Tate, 1994), or to present particular views of reality, that benefit some over others, in seemingly objective ways (Ernest, 2001, 2002; Frankenstein, 1987).

This call to challenge the discipline does not imply that students do not have access to the mathematics curriculum traditionally valued by the culture of power, because they clearly need this knowledge to participate and succeed in school (Delpit, 1988, 1995). However, students can see school mathematics as a socially constructed body of knowledge, not as a set of pre-existing, unquestionable truths (Anderson, 1990; Bishop, 1994). As Valenzuela (1999) explained

> I by no means suggest that students should not master the dominant knowledge that is customarily embodied in the standard curriculum, but rather that it should be openly recognized as dominant and exclusive so that an additive thought process may supervene to both challenge and counterbalance its undue influence. (p. 270)

Critiques of Teaching and Learning Mathematics for Social Justice

Not all researchers who strive to promote equity in mathematics education support a “Teaching for Social Justice” approach. In the following section, I present and respond to several critiques of this pedagogy. To begin, Knight and Pearl (2000) leveled a general critique of critical pedagogy, arguing that while critical pedagogy draws upon theory that “reveals the pervasive, deep-seated antidemocratic nature of schooling,” it fails to offer any realistic way of “reconstructing” schools (p. 197). Knight and Pearl agree that the aim of education should be to prepare informed, critical, responsible citizens, but contend that critical pedagogy does little to advance this agenda because (a) it lacks “readily understandable practical suggestions” (p. 206), (b) it has failed to demonstrate, through empirical research, that it results in academic success for low-income students and students of color; and (c) it has minimal standing in political contexts “where it counts,” such as in state legislatures (p. 222).

I agree with Knight and Pearl (2000) that critiquing hegemonic practices is not sufficient, and that offering specific pedagogical alternatives is essential. The series of project-based units developed as part of this study, and the descriptions of how each unit was negotiated and enacted (see Chapter 3) were aimed at responding to such a critique.

I also concur that there exists a pressing need for critical mathematics educators to examine proposed pedagogies in actual classroom settings, and to include in this examination an assessment of students' mathematical learning. The present study was aimed at addressing this dearth in the literature.

Other researchers deeply committed to equity have also critiqued critical pedagogical approaches for their failure to adequately consider the beliefs, values, and ways of knowing and understanding the world of the students and families that they serve. Scholars of color in particular (e.g., Delpit, 1988) have argued that historically this pedagogy, though aimed at supporting students in low-income, urban schools, has been developed and promoted by white, middle-class educators. For instance, Delpit (1988) stated that liberal educators' goal that children "become autonomous, [and] develop fully who they are in the classroom setting, without having arbitrary, outside standards forced upon them" may be an appropriate goal for children who are already members of the culture of power, but that parents of students outside the dominant culture may want something else for their children (p. 285). That is, parents of low-income students and students of color may " want to ensure that the school provides their children with discourse patterns, interactional styles, and spoken and written language codes that will allow them success in the larger society" (p. 285).

Applying Delpit's (1988) argument to urban mathematics education, one could argue that most important is not that students draw on mathematics to investigate local circumstances or engage in personally and socially transformative action, but that they develop the mathematical concepts and skills needed to perform well on standardized assessments, and to gain entrance in to advanced level coursework. Delpit's argument also implies that the teacher assume a more active role in sharing mathematical vocabulary, conventions and procedures (or mathematical "codes of power") with students, in order to maximize their opportunities to participate in the culture of power. I concur that all students, particularly those whom our educational system has historically underserved, need access to these codes. However, I do not believe that critical and liberal pedagogies are in conflict with students' capacity to learn – and challenge – the codes of the culture of power. In fact, as the stories in Chapters 5 and 6 demonstrate, students in this study were able to both assume a critical stance and challenge individuals in positions of power, and at the same time, develop understanding of traditionally valued mathematical skills and concepts (i.e., codes of power). Yet I find Delpit's call for critical pedagogues to pay closer attention to the ideas, values, and understandings of teachers and students of color quite powerful, and urgent. Continuing to advance critical pedagogy in mathematics, without making a conscious effort to include the perspectives of individuals in the community one serves, would be a definite mistake.

Within mathematics education, the work of Bob Moses (2001) might be seen as a critique of critical pedagogy in mathematics education. While Moses has adamantly supported the empowerment of minority youth in relation to mathematics, his vision of empowerment differs from that advocated within a teaching for social justice framework. For instance, Moses's Algebra Project was based upon providing "real-life" experiences (e.g., data collection experiments, physical events) that allowed students to investigate fundamental algebraic concepts and to "move towards their standard expression in school mathematics" (p. 118). In other words, Moses has developed very innovative and potentially empowering instructional programs, and his fundamental goal has been access to advanced level, college preparatory mathematics courses for all students. While I do not disagree with this goal, I believe that it is limited. Teaching and learning mathematics for social justice not only advocates rigorous mathematical understanding for all students, but also calls for a redefinition of the goals of purposes of mathematics education. That is, learning mathematics is not just about gaining entrance into higher level classes, but about opportunities to draw upon the discipline as a tool for describing, critiquing, and acting transformatively upon the world.

The previous section outlined the central tenets of teaching mathematics for social justice, reviewed research that investigated this pedagogy in classroom settings, and responded to critiques of this approach. In the next section, I describe the particular kind of agency fostered in classrooms aimed at teaching mathematics for social justice.

Critical Mathematical Agency

Teaching and learning mathematics for social justice creates a particular kind of *figured world* (Holland et al., 1998) within which students investigate questions of personal and social importance, reflect on issues of equity and justice, and use what they learn to act upon their lives. Together, these interactions may foster a sense of *critical agency* (Pruyn, 1999), by supporting students' capacity to (a) view the world with a critical mind set, (b) imagine how the world might become a more socially just and equitable place, and (c) engage in action aimed at personal and social transformation. But what might critical agency look like within a *mathematics* classroom? In this last section, I move from the general notion of *critical agency*, which might be fostered in any number of classrooms, to the particular kind of agency supported in classrooms aimed at teaching mathematics for social justice.

Critical Mathematical Agency Defined

What do I mean by *critical mathematical agency*? To begin, this enhanced form of agency is *critical*, and as such, encompasses the components of critical student agency described in Table 2.1. In addition, it is *mathematical*. That is, it includes students' capacity to (a) understand mathematics, (b) identify themselves as powerful mathematical thinkers, and (c) construct and use mathematics in personally and socially meaningful ways (e.g., Empson, 2002, p. 598).

Understanding is central to any notion of agency. As Bruner (1996) argued, "since agency implies not only the capacity for initiating, but also for completing our acts, it also implies skill or know-how," or in other words, it implies understanding (p. 36). By understanding mathematics I refer to students' ability to see how an idea or concept is related or connected to something already known (Hiebert, Carpenter, Fennema et al., 1997). The more connections the student establishes, and the more mathematically powerful those connections, the deeper the understanding (Hiebert et al., 1997; Hiebert & Carpenter, 1992). Within math classrooms, understanding may manifest itself through students' ability to make sense of, solve and pose mathematical problems, and to judge the validity of strategies and solutions (Ernest, 2001).

The second *mathematical* component of critical mathematical agency is students' identification of themselves as powerful mathematical thinkers, which refers to how students author and position themselves in relation to mathematical content and practices. To the extent that students author their own knowledge, take control of their learning and position themselves as capable doers and creators of mathematics (Carpenter & Lehrer, 1999), they may identify themselves as powerful mathematical thinkers. Understanding may play a pivotal role in this self-identification (Empson, 2002).

Table 2.1

Critical Mathematical Agency Defined

CRITICAL MATHEMATICAL AGENCY	
Is Critical	Involving students' capacity to: •View the world with a critical mind set •Imagine how the world might become a more socially just, equitable place •Engage in action aimed at personal and social transformation
Is Mathematical	Involving students' capacity to: • Understand mathematics • Identify themselves as powerful mathematical thinkers • Construct and use mathematics in personally and socially meaningful ways

The final *mathematical* component of critical mathematical agency is students' capacity to use mathematics in personally and socially transformative ways. That is, students can draw on mathematics to assist them in realizing individual and collective intentions, or to critically make sense of and act upon the world (e.g., Tate, 1995). They may employ the results of mathematical investigations to critique conditions in their lives, or to resist unjust policies. This dual nature of critical mathematical agency, the sense that it is *critical* and at the same time *mathematical*, is summarized in Table 2.1.

Interactions Among the Elements of Critical Mathematical Agency

Given the dynamic and multi-dimensional nature of critical mathematical agency, there will invariably be interaction between and among its various components. For instance, how might students' engagement in and understanding of mathematics support and/or enhance their sense of agency? How might students' general sense of themselves as competent people who can and do make a difference relate to their sense of mathematical agency? And what about students' capacity to view the world with a critical mind set, how might that mind set be supported by and/or challenge their participation in mathematics? While subsequent chapters further illuminate these interactions through analysis of the data, the next section draws on conceptual and empirical work to outline possible interactions among the elements of students' critical mathematical agency.

Disciplinary Concepts and Practices and Their Relation to Student Agency

Mathematical concepts and practices can be powerful to the extent that they facilitate the construction of new understandings and/or the deepening or challenging of existing knowledge (Skovsmose & Valero, 2002). Within this perspective, examples of potentially powerful practices include problem solving, which affords thinking about and operating upon concepts, in addition to proof, conjecture, and the construction of mathematical models. However, consistent with a situated view of learning (Lave & Wenger, 1991), I want to argue that mathematical ideas and practices are *only* vehicles for agency to the extent that they "support people's empowerment in relation to their life conditions" (Skovsmose & Valero, 2002, p. 394). In other words, mathematical ideas and practices become powerful, or agentive, when they (a) enhance opportunities for individuals to participate in the practices of a particular figured world, authoring spaces for themselves and positioning themselves as knowledgeable and capable in relation to their activity; or (b) provide opportunities for students to envision and create a desirable range of future possibilities, sometimes through critiquing and/or resisting those ideologies, norms, actions, and conditions that they deem unfavorable.

Pickering (1995) has described the power, or agency of disciplinary practices in more dialectic terms. He argued that the routine, highly structured practices of scientific disciplines, "carry human practices along. … lead[ing] us through a series of manipulations within an established conceptual system," (p. 115). According to Pickering, disciplinary practices result in an ongoing *dance of agency* between human actors, in this case, students engaging in mathematics, and rules for participating in the discipline, which impose structures, routines, and ways of thinking upon their practice. Pickering's *dance of agency* provides a powerful analytic tool, or metaphor, for thinking about the interaction between the discipline of mathematics and student agency, particularly how student agency (include here their critical mind set, their purposes and intentions) might function to challenge the discipline (Ernest, 2001).

The Power of Mathematical Understanding and its Relation to Student Agency

Understanding and skill are central to any notion of agency, insofar as that notion includes the capacity to meaningfully engage in purposeful action, for "without skill, we are powerless" (Bruner, 1996, p. 94). Peterson, Fennema & Carprenter (1991), cited in Empson (2002), described how a first grade student, Billy, experienced a personal transformation by virtue of his mathematical understanding. Though Billy was not able to count or recognize numbers at the beginning of the year, after repeated interactions aimed at building on his mathematical thinking and increasing his understanding, Billy came to identify himself as a mathematical thinker who took great pride in his love of math. Empson argued that as Billy's understanding increased, so did his sense of himself as a capable doer of mathematics, and therein, his sense of agency, suggesting that mathematical understanding can contribute to students' sense of agency.

However, in order for mathematical understanding to enhance students' *critical mathematical agency*, not just their agency as learners or as students of mathematics, that understanding must serve broader goals that are negotiated by the classroom community and that reflect personal and/or collective needs and purposes, often needs and purposes that extend beyond the figured world of the mathematics classroom. That is, students' intentions and the capacity to act upon those intentions can infiltrate the process of understanding mathematics. Valero (2002), drawing on Foucault's (1972) understanding of power and agency, echoed this claim.

> Empowerment does not emerge from the "possession" [*or understanding*] of mathematics, but from the position that students adopt to influence the social practices where mathematics are taught and learned. Empowerment, then, is not passed from the teacher to the student by means of the transference of a "powerful knowledge." … Power is not an intrinsic characteristic of a person or a thing, but the manifestation of a relation in which people position themselves in order to influence the outcomes of a situation. (pp. 11-12)

While Valero speaks of power and empowerment, her statement might just as easily be about agency. While both disciplinary practices and understanding of powerful mathematical ideas can to a certain extent support students' critical mathematical agency, in and of themselves, they are insufficient. Rather, what seems to matter most is how students make use of what they know, and how what they know facilitates the possibility for transformative action (Empson, 2002). I refer to this notion that mathematics can support students' capacity to make sense of and act upon, situations of personal and social importance, as the *transformative power of mathematical activity*[6].

The Transformative Power of Mathematical Activity and Student Agency

Describing this transformative power of mathematics, Ernest (2002) argued, "Social empowerment through mathematics concerns the ability to use mathematics to better one's life chances in study and work and to participate more fully in society through critical mathematical citizenship" (p. 1). Insofar as students' *participation* in mathematics supports their capacity for critical understanding, reflection, decision-

[6] Skovsmose and Valero (2002) referred to the use of mathematical ideas as resources for action in society as the "sociological power of mathematics" (p. 395).

making, and action, it acts in service of students' critical mathematical agency. But what does this mean in practice?

A descriptive tool. In concrete terms, students might engage mathematics as a descriptive tool for investigating, making sense of, and modeling reality. For instance, they can draw on the 'thinking tools' of mathematics (Mellin-Olsen, 1987) -- such as statistical tools for collecting, representing, and manipulating data, and numerical tools for performing and reasoning about calculations, comparing amounts, and identifying inequalities (Frankenstein, 1990) -- to assist them in gaining insights about particular situations, in ways that other disciplines may not allow. This may occur on a very personal level, as students use mathematics to describe situations in their daily lives, and then draw on those descriptions to justify their needs and concerns.

Or, students may employ mathematics as a descriptive tool to make sense of issues at the local, national, or global level. Peterson (1994, 1999) described how elementary school students might use mathematics to collect and analyze data about bias in the media, considering for example, the depiction of women, the presence of diverse perspectives, or the gender and racial breakdown of newspaper staff, television hosts, or radio announcers. Gutstein (2000, 2001) discussed how middle school students have used mathematics as a tool to analyze the racial composition and wealth distribution of various nations, using children to represent the respective population of each continent and chocolate chip cookies to represent their portion of the wealth. Frankenstein (1990) presented statistics as a powerful tool that can help students challenge the social and economic disparities that plague society and result in the oppression of women, the poor, and minorities.

Engaging mathematics as a descriptive tool to make sense of and model reality can support students' critical mathematical agency because an understanding of how mathematical models of social phenomenon are constructed and interpreted allows students to judiciously evaluate the mathematical underpinnings of social and political decision-making (Ernest, 2002). Which, in addition to enhancing agency, is "an essential component of education for responsible citizenship" (Mukhopadhyay & Greer, 2001, p. 303). Moreover, as students use mathematics to investigate situations of personal and social importance, they may come to see mathematics as a powerful and relevant force in their lives, and to see themselves as capable users and producers of mathematics – both views that can support their sense of agency.

A tool for reasoning and reflection. In terms of supporting students' critical mathematical agency, mathematics might also serve as a tool for reasoning and reflection. The mathematical models and representations that students construct, and the understanding that those models afford, become tools for further reasoning and reflection (Lehrer & Schauble, 2000). For example, as they work with data students can "reflect explicitly about the qualities and affordances of different data representations" (p. 131), asking questions about how the data are structured, and how the structure of the data impacts the kinds of questions one can ask, and the statements one can make. They can then reflect upon several possible models or representations, and determine which model bests describes a given situation (Greer, 1997), or best supports a particular argument

(Lehrer & Schauble, 2000), or political point of view (Tate, 1995). Students can also use models that they have constructed to critically reflect upon conditions in the world and in their lives, and to imagine ways that those conditions might be changed to be more socially just.

A tool for decision-making and transformative action. A final way that mathematics might support students' critical mathematical agency is by functioning as a prescriptive, decision-making tool for transformative action. Transformative (or socio-political) action refers to action aimed at elucidating and transforming unjust and inequitable conditions in society (Hodson, 1998). In the case of mathematics, students can use mathematics to make and communicate decisions, to envision and represent new possibilities (Skovsmose, 1999), to strategically argue their point of view (Tate, 1995), and to share their views with others, particularly those in positions of power (Skovsmose, 1994). For example, students in Gutstein's (2000, 2001) middle school classroom used mathematics to argue that the world map projection commonly used in schools, including their own, was inaccurate and distorted, enlarging some areas (Europe, Greenland) and shrinking others (Africa), and placing predominantly white countries "in the middle of the world" (2000, p. 18). The mathematics was critical to their argument. As one student commented, "All of this information was brought up just by using mathematics to get answers of social issues. Like I always say, we couldn't have done it without the mathematics" (2000, p. 25). Experiences which provide students opportunities to use mathematics as they act in transformative ways for social change, and to see that their actions can make a difference, support students in developing and deepening a sense of critical mathematical agency.

Student Agency and its Capacity to Challenge the Discipline of Mathematics

Thus far, discussion of the interactions between and among the components of critical mathematical agency has focused on the impact of students' engagement with the discipline on students' sense of agency. This last section considers these interactions from a different angle. Rather than students' participation in mathematics impacting their sense of agency, how might student's agency serve to challenge and push against the discipline of mathematics? Returning to Pickering's (1995) metaphor, how might the dance of agency result in a shaping of the discipline? Unfortunately, discussion of this interaction is virtually absent in the literature. In what follows, I review several studies that have begun to address this issue.

At the college mathematics level, Henderson (1996) examined the extent to which students in his geometry course showed him new mathematics in a classroom environment that afforded them opportunities to draw on their own reasoning and understandings to solve problems. Over a six-year period, he found that 31 percent of his students generated mathematics that was new to him. The majority of these students were women, and/or students of color. Henderson argued that groups historically underrepresented in mathematics, such as these students, are most likely to bring different meanings and understanding to the classroom, different ways of asking questions and validating ideas, and as a result, when such students are able to assert their beliefs, experiences, and ways of making sense of the world through their participation in mathematics, their actions can challenge traditionally accepted mathematical norms and practices. While Henderson's study did not explicitly address student agency, his findings suggest that when students have opportunities to author and position themselves as knowledgeable and capable, as people whose ideas can and do matter within the figured world of their mathematics classroom, their agentive actions can challenge mathematical practices.

At the elementary and middle school level, Marta Civil and her colleagues (2001; see also Gonzalez, Andrade, Civil & Moll, 2001; Kahn & Civil, 2001) have studied the forms and funds of mathematical knowledge that emerge from the activity of students and their families, in particular, their mothers. They documented the rich mathematics embedded in the practices of Señora María, an immigrant woman who worked out of her home as a seamstress (González et al., 2001). As they unearthed Señora Maria's mathematical activity, they challenged standard conceptions of the discipline by privileging mathematical practices and insights that are often marginalized or unacknowledged in an academic context. In fact, some of the mathematics this woman employed was so deeply embedded in her practice, that is was almost invisible and "difficult to understand" for researchers who viewed her practice "through eyes trained by formal education" (p. 124).

Civil and her colleagues have argued that students' lived, daily experiences, the activities they engage in outside of school and the situations they care deeply about, may also be rich funds of mathematical knowledge (Civil, 1994, 1998; Gonzalez, Civil, Andrade, et al., 1997). If students' "social worlds" are allowed to enter the mathematics classroom and to shape both the content and processes of classroom activities (which could occur as they author spaces for themselves and assert their intentions), students' agentive actions may push against the mathematical norms and practices that typically guide the activity of the figured world. One aim of this study is to further illuminate these interactions between student agency on one hand, and mathematical practices on the other.

Conclusion

In this chapter, I drew on both theoretical and empirical work to situate critical mathematical agency in the relevant literature, and in doing so, construct a conceptual framework that elaborated its multiple components. After reviewing general features of agency that are central to how agency is understood within the context of this study, I presented the notion of *critical mathematical agency* as a way to describe the particular kind of agency that may be fostered in mathematics classrooms aimed at teaching and learning for social justice. While numerous researchers have conjectured about how critical mathematics education might contribute to student learning, empowerment, and to larger struggles for justice and equity, systematic empirical research has been extremely limited. A primary aim of this study was to address this void in the literature. In the next chapter, Chapter 3, I present the methodological details of this study.

CHAPTER THREE: RESEARCH METHODOLOGY

This critical ethnographic study investigated (a) how middle school mathematics students participating in a classroom aimed at teaching and learning for social justice enacted a sense of critical mathematical agency, (b) the interactions between their sense of agency and their engagement with the discipline, and (c) the tensions that arose and how they were negotiated. The study took place in a diverse, urban middle school in New York City, and was a collaborative effort between myself, a sixth grade mathematics teacher (Beatriz[1]), and her students.

This chapter begins with a discussion of the research methodology that guided the study, critical ethnography. I present critical ethnography as an appropriate tool for research aimed at promoting social justice, and then describe its guiding principles and research practices and how they were applied in this study. Next, I present general methodological information about the study, including a brief description of the setting[2], and of the processes of gaining entrance, selecting participants, and collaboratively negotiating the curriculum with Beatriz and her students. The final section details methods of generating and analyzing data, and criteria for establishing validity and trustworthiness.

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[1] Beatriz's name used with permission. The names of students, staff members, and schools have been changed to protect confidentiality. Students choose their own pseudonyms.

[2] Chapter 4 provides a more detailed description of the research context, including the school, the students, Beatriz's classroom and her pedagogy.

Research Methodology: Critical Ethnography

As a mathematics education researcher hoping to promote social justice through my work, I must consider which research practices best support my efforts. As Lather (1986) argued

> We who do empirical research in the name of emancipatory politics must discover ways to connect our research methodology to our theoretical concerns and commitments. At its simplest, this is a call for critical enquirers to practice in their empirical endeavors what they preach in their theoretical formulations. (p. 258; see also Vithal, 2000)

The following section presents one research methodology that has particular potential for social justice oriented research, critical ethnography.

Guiding Principles of Critical Ethnography

Critical ethnography has grown out of the convergence of the work of educational ethnographers, who seek to understand the complexity and contextual nature of all that occurs in schools, and critical theorists, who raise questions about the role of schools and other societal institutions in creating and sustaining inequities (Anderson, 1989). Through this union, critical theory pushed ethnographers to be more political and more critically aware of relations of power, and ethnography challenged critical theorists to be less idealistic and more grounded in empirical research (Calabrese Barton, 2000). Critical ethnography is characterized by the following set of epistemological (nature of knowledge) and ontological (nature of reality) beliefs and assumptions. While this list is not intended to be exhaustive, it does capture some of the core ideas that underlie the methodology.

1. All knowledge is subjective, and socially constructed to reflect a particular place, time, and social / cultural / political context.
2. There is no single, objective reality that exists and can be known. Rather there are multiple socially, historically constructed realities that are produced and reproduced in the context of social interactions.
3. The understandings that emerge from critical ethnographic studies are co-constructions that reflect the positions, experiences, values and beliefs of both researchers and participants, and that can help to challenge dominant ways of knowing and seeing the world.

Like traditional ethnographers, critical ethnographers seek to gain insights, illuminate situations, and generate understanding (Anderson, 1989). What distinguishes critical ethnography, what makes it *critical*, is that its purpose and goal is not just understanding, but praxis, defined as "the political commitment to struggle for liberation and in defense of human rights" (Trueba, 1999, p. 593). As critical ethnographers strive to make sense of schools and classrooms, they recognize that "all education is intrinsically political", and they focus on the need to "reflect seriously on equity matters" to examine "culturally hegemonic practices and to document cultural conflict" (p. 593-594). Their daily research activities include efforts to raise awareness and impact change. Thus the central aim of critical ethnography is critique and transformation (Guba & Lincoln, 1994), and advocacy for the oppressed (Trueba, & McLaren, 2000).

To achieve this goal of "advocacy for oppressed," critical ethnographers have outlined a process that can guide researchers' work (Trueba, 1999). The process includes (a) documenting the nature of the oppression, (b) documenting the process of empowerment, (c) accelerating the conscientization of the oppressed and the oppressors, (d) sensitizing the research community to the implications of the research for the quality of life, thereby linking intellectual work to real-life conditions; and (e) reaching a higher level of understanding of the historical, political, sociological and economic factors supporting the abuse of power and oppression (Trueba, 1999, p. 593).

I argue that there is significant overlap between these guiding principles of critical ethnography and the core components of teaching and learning mathematics for social justice discussed in the previous chapter. In each case, recognition of political, social and economic factors that impact education and result in inequities is imperative; critical reflection and awareness about the nature of domination and oppression in one's own life is essential; and the need to go beyond critique to action is emphasized, suggesting that critical ethnography is a powerful methodology for research aimed at promoting social justice in mathematics education (Skovsmose & Borba, 2000; Vithal, 2000).

Critical Ethnographic Research Practices

In working towards the principles outlined by Trueba (1999), critical ethnographers have drawn upon a number of research practices. The following section describes these practices, both the challenges they pose and the potential they offer, and explains how each practice was manifested in this study.

Stories in Their Own Voices

First, it is essential that participants, who often represent marginalized, oppressed positions, have opportunities to tell and craft their own stories. As Pignatelli (1998) argued, "in telling their own stories in their own words, a critical dialogue about power and equity can be further clarified and invigorated" (p. 404; see also Pizarro, 1998). In presenting stories, it is important that multiple positions, ideas and experiences are included, so that ethnography stands "less as a seamless narrative and more as a juxtaposition of voices and competing perspectives" (Pignatelli, 1998, p. 420).

This study provided participants with multiple opportunities to share their stories, ideas, and experiences. Efforts were made to ensure a balance between stories elicited by the researcher, and therefore positioned and framed in certain kinds of ways, and stories which participants decided to share on their own, in the context of more informal interactions, including contexts outside the mathematics classroom. For instance, I accompanied students on a school field trip to the ice skating rink, and later recorded field notes about our conversations. The inclusion of participant-driven conversations is necessary because even interviews that are intended to be open-ended and that rely on emergent interviewing techniques (Erlandson, Harris, Skipper, et al., 1993) are characterized by asymmetrical relationships that place the researcher in a position of power and control (Patai, 1991). In this study, prolonged engagement with participants facilitated the sharing of stories both in contexts framed by the researcher and contexts where the participants had more decision making power (e.g., informal conversations at lunch or after school).

Critical Reflexivity

Sharing participants' stories is not without challenges. Critical ethnographers admit that all research is subjective and that their own assumptions and perspectives play a key role in all phases of the investigation, including how they understand and represent participants (Foley, 2002; Young, 2000). To address my subjectivities and privileged position within the research, I engaged in an on-going process of reflexivity, of "turning in on oneself in a critical manner" (Foley, 2002, p. 473). As Young (2000) described, "Reflexivity involves self-reflection on one's research process and findings, self-awareness of one's social positionality, values and perspectives, and self-critique of the effects of one's words and actions upon the individuals and groups being studied" (p. 642).

As part of this process of critical reflexivity, I prepared a researcher as instrument statement (Erlandson et al., 1993, see Appendix A). In this statement, I discuss my beliefs, experiences, and assumptions about (a) teaching mathematics for social justice, (b) the goals and purposes of education, including its relation to student agency; (c) the discipline of mathematics, and (d) teacher-researcher collaborations. To further document my thoughts and reactions throughout the study, I made regular entries in a reflexive journal (Erlandson et al., 1993). This journal provided a space for me to reflect on interactions with participants, to record changes in my own beliefs and assumptions, and to keep track of critical questions and insights that emerged.

Critical ethnographers not only need to be reflexive about their relation to the research, but about the methods and practices they employ. As Apple (1995) argued, "a double reflexivity is needed – one that is critical of dominant approaches and agendas, and another that is constantly self-critical of the alternatives we propose" (p. 341). This double reflexivity is especially important in research that seeks to promote social justice, as it can help researchers to avoid practices and representations that could potentially exploit, dominate, or further disempower the teachers, students and families with whom they work (Young, 2000). Throughout the study, I met on a semi-regular basis with the classroom teacher to critically reflect upon our teaching and research practices, questioning the merits of our approach, and making adjustments and changes as we deemed appropriate.

Collaboration

For critical ethnographic research to act in support of social justice, participants and researchers need opportunities to collaborate with one another. Engaging participants as co-researchers helps address the power imbalances inherent to all research relationships (Bernal, 1998; Patai, 1991; Pizarro, 1998; Young, 2000). For instance, researchers typically retain control over study design and implementation, which may decrease the usefulness of the study for participants, and/or make it more difficult for their perspectives to be heard. If instead, participants are invited to contribute to all stages of the research process, the investigation is more likely to reflect the values, beliefs and epistemologies of the participants, and to serve their individual and collective needs (Bernal, 1998; Pizarro, 1998; Young, 2000).

This study involved ongoing collaboration between myself, the classroom teacher (Beatriz), and her students[3]. This collaboration began several months prior to the beginning of data collection, through a series of discussions about the initial plan for the study (my dissertation proposal) and how it might take form in Beatriz's classroom. Though I drafted the initial list of research questions, I sought Beatriz's feedback and insight on multiple occasions, including during a visit to her school several months before the study began, and adjusted the questions accordingly. During that initial visit, Beatriz and I discussed her interests and preferences (e.g., she was interested in organizing the curriculum around a series of projects), her concerns (e.g., she wanted to begin with several short term activities, to help students "get used to the idea" of investigating personal, school, and community concerns in their mathematics class), along with relevant logistical information (e.g., she had a student teacher working with her 4th period math class, and requested that we limit the study to interactions with students from her 1st and 2nd periods). Moreover, I had the opportunity to speak with the school principal, who expressed her full support of the study.

Beatriz and I continued to collaborate on a regular basis throughout the study. While our collaboration was not the focus, but an integral component, of this research, I want to describe how that collaboration manifested itself in both structured and more emergent ways.

Initial unit planning meetings. At the very beginning of the study, and before each of the three project-based curricular units, Beatriz and I met for extended

[3] Collaboration with students is discussed in a subsequent section that describes the process of negotiating the curriculum.

brainstorming and planning sessions. These meetings generally lasted about 2 hours, were audio-taped, and occurred away from the school over lunch or dinner.

During these and other planning meetings, Beatriz and I each contributed our own experiences and areas of expertise. I brought my experiences as a novice researcher in urban schools, my knowledge of the theoretical literature that informed the study, and my understanding of children's mathematical thinking. Beatriz contributed her knowledge of the students, their families, the community, and school context, and her expertise with the middle school mathematics curriculum. She also brought her own experiences as a Latina, as a individual born outside the mainland United States (she was born and spent a portion of her childhood in Puerto Rico), and a native Spanish speaker, experiences she shared with many of her students.

Ongoing weekly planning meetings. Once we began a unit of study, Beatriz and I met each week to outline the details of daily lessons and activities, homework assignments, and informal assessments. These meetings generally lasted about an hour, either after school or during Beatriz's planning period, and were not audio-taped. Both Beatriz and I took notes during these meetings, which I then typed up in the form of rough lesson plans (see Appendix B for an example lesson plan). Though I intended for these weekly planning meetings to last longer so that they might include a stronger reflexive component, due to time constraints, and the ever present difficultly of finding a block of time when Beatriz and I could meet, weekly reflection was sometimes overshadowed by the more immediate need to outline upcoming lessons.

Daily conversations. Between weekly planning meetings, Beatriz and I discussed the details and outcomes of particular lessons. These conversations occurred on a continual basis, during and between the 1st and 2nd period classes, over lunch, walking to the subway station, and occasionally on the phone in the evening. In addition to the logistics of day-to-day lesson planning, these conversations explored a variety of other issues, including concerns about a particular student, results of staff meetings, and frustrations about upcoming standardized testing.

Shared responsibilities. My collaboration with Beatriz also manifested itself in more subtle and unexpected ways. These daily, and often spontaneous collaborations involved Beatriz and I sharing responsibilities typically reserved for the 'teacher,' particularly as the study progressed and the task of managing an organic, negotiated curriculum became more time intensive. For instance, I began to assist Beatriz with daily tasks such as: making copies, writing homework problems, reviewing student work, gathering needed supplies, assisting on field trips, and managing small groups of students. At times, this assistance occurred at Beatriz's request; other times, I initiated the collaboration.

While sharing responsibilities helped to establish a more equitable teacher-researcher relationship, Beatriz and I maintained somewhat unique roles that implicated different kinds of power and privilege (Goldstein, 2000, p. 524). For instance, while I assisted in lesson planning and interacted with students during classroom activities, Beatriz assumed the primary responsibility for all classroom instruction. Moreover, as the researcher, I clearly had forms of power and freedom that Beatriz did not enjoy. I was able to leave before the school day ended, miss a week of classes for a professional conference, and attending staff meetings was always an option, not an obligation. Moreover, I had the freedom to challenge school policies, such as those that governed standardized testing practices, without any fear of negative consequences. Beatriz and I spoke openly about these inherent power differentials, in hopes that addressing imbalances in power would help us to a craft a more mutually beneficial research relationship.

Continued collaboration after data collection. Beatriz and I continued to collaborate, albeit on an irregular basis, after the completion of data collection in July 2002. This collaboration included informal conversations about my analysis of the data, and more formal discussions / feedback sessions about drafts of dissertation chapters, the emerging conceptual framework, and reports of the research that I presented at professional conferences. In each case, I attempted to incorporate Beatriz's suggestions and feedback into the final documents and presentations.

Recently, Beatriz and I were able to co-present the results of this study at a national conference (Turner & Font, 2003), an experience which we found both rewarding and problematic. Despite genuine efforts to include Beatriz as a primary participant in the session, and explicit attempts to privilege her voice as the teacher, the power and status awarded to academic, theoretical knowledge, versus what Beatriz referred to as her experiential, classroom based knowledge, permeated the conference, and made Beatriz's experience of presenting more frustrating and demeaning, than empowering.

Transformative Action

Finally, for critical ethnography to be a useful tool for working towards social justice, the research must involve opportunities for action. Research cannot be separated from the active process of social transformation, because it is not “research on empowerment … but research as empowerment” (Pizarro, 1998, p. 61), or as Lather (1986) argued, research *as* praxis. In fact, the extent to which researchers and participants move beyond critique to action that supports social justice should be used to measure the strength and value of the research (Pizarro, 1998). Just as it is important for students within mathematics classrooms to make the transition from conscientization to praxis, it is imperative that critical ethnography, with its emphasis on action, facilitates this process. This action does not occur as an end result of a critical ethnographic study, but as an ongoing part of daily research activity. Valero (2000) referred to this as the “Responsiveness and Responsibility Principle” which states that social justice oriented research requires “a constant and open ‘giving back’ to the people of the situation involved in the research” (section entitled The Dilemma of Scientific Distance, ¶ 3).

To achieve this aim, the proposed study provided multiple opportunities for participants to engage in transformative action. Students participated in important school affairs, and used mathematics as a tool to present and argue their positions. Most importantly, the students had opportunities to make purposeful decisions about the kinds of action(s) they wanted to participate in, the kind of information they wanted to share, and with whom, and the form that information should assume.

Tensions Inherent to Collaborative Critical Ethnographic Research

Collaborative ethnographic research is not without challenges and limitations. The very way that schools and universities operate, and the expectations and structures they impose on teachers and researchers work against this kind of collaboration, and towards the reproduction of the status quo (Pizarro, 1998).

Collaboration versus exploitation. Teachers in poor, urban schools usually face overcrowded classrooms, heavy workloads, and extracurricular responsibilities, leaving them limited time to devote to extra projects such as collaborative research. When collaboration is possible, researchers need to be careful that they do not abuse their position of power and that their practices do not become exploitative and oppressive. As Patai (1991) argued, "It is in fact exceedingly difficult to strike a balance that neither exploits the researched nor imposes on them our own psychological demands" (p. 147). While incorporating participants' perspectives is essential, researchers benefit professionally from the time intensive process of conducting investigations, and in many cases, participants do not. It is important to always consider the possibility of exploitation otherwise this methodology could work against, rather than for, social justice.

In this study, Beatriz and I spoke openly about this delicate balance between collaboration and exploitation. While she communicated, and continues to communicate, a genuine interest in participating in this collaborative study, I made efforts to ensure that she felt comfortable stating her own needs, even if those needs involved some level of withdrawal from the collaboration. While I do not want to claim that Beatriz always felt empowered, rather than exploited by virtue of her participation in this research, the fact that she at times set limits to her involvement (e.g., I can't meet today, I can't look over that chapter right now, I didn't have time to review the article you gave me to read), and on other occasions sought to increase her participation (e.g., attempting to re-organize her schedule to present with me at a professional conference, volunteering to author portions of a paper, requesting copies of student interviews to read) affirms that we were somewhat successful in establishing a mutually beneficial collaborative relationship.

Collaboration and blurring of roles. When our collaboration led Beatriz and I to share responsibilities typically assumed by the classroom teacher, this began to complicate and blur my role as a researcher. For instance, as Beatriz and her students began to view me more as a fellow participant and less as an observer, I at times found myself pulled from a particular group of students, who I had intended to observe and interact with during that day's lesson, to another group of students who either needed assistance, or had a question, or wanted my opinion. Given that there were always 25 to 30 students present in Beatriz's classroom, students welcomed and even sought out the support of an additional adult. I experienced an ongoing tension between my needs and responsibilities as a researcher to diligently document interactions with case study students, and my role as a participant, which implicated – at least from the students' perspective – some sense of responsibility to the classroom community as a whole. Vithal (2000) described a similar tension in a collaborative study with pre-service teachers.

> When standing at the back of the classroom with the video camera, I was to all intents and purposes a non-participating observer. That immediately changed

> when the student teacher asked my opinion about something, or the pupils drew me in with a question. A significant difficultly was in trying to simultaneously reflect and understand what was going on as a researcher while at the same time acting as a participant in the process as a teacher or teacher educator. (section entitled reflexivity, ¶ 2)

I reflected repeatedly on this tension, both during and after the study, and although my actions at times compromised aspects of my effectiveness as a researcher – such as my ability to gather uninterrupted data from case study students -- I felt an ethical responsibility to be a responsive participant, even when that responsiveness pulled me from my primary research responsibilities.

General Methodological Concerns

Brief Overview of the Research Context[4]

This critical ethnographic study was conducted in a sixth-grade mathematics classroom at a low-income, urban school (Francis Middle School) in New York City. During the 2001-2002 school year (the year of the study) Francis had approximately 211 students in sixth, seventh, and eighth grade. 34 per cent of the students at Francis were African American, 38 % Latino/a (primarily of Puerto Rican or Dominican decent), 15% European American, and 12% Asian, Pacific Islanders, Alaskan Natives, and Native American[5]. This diverse student body was reflected in Beatriz's classroom. While the

[4] A full description of the research context is presented in Chapter 4.
[5] All data based on statistics from the New York Board of Education 2001-2002 Annual School Report.

majority of the school's Latino population spoke Spanish, many students grew up using both English and Spanish on a daily basis. Only 7% of the students were officially classified as English Language Learners. Seventy percent of students at Francis were eligible for the district's free or reduced lunch program.

The 6th grade mathematics teacher who participated in this study, Beatriz, was chosen for her dedication to teaching mathematics in low income, urban schools, her interest in issues related to equity and social justice, and her desire to collaboratively explore the meaning of teaching mathematics for social justice in her classroom.

The Process of Entering

Consistent with the ethnographic principle of prolonged engagement (Erlandson et al., 1993), data was generated over a six-month period, beginning in January of 2002 and continuing until the end of the school year in June, 2002. Data collection occurred on a daily basis. During the first several weeks of the study, I spent the entire day at the school, in order to familiarize myself with the setting and to establish an accepted role for myself as an active participant observer. In an effort to get to know students both as individuals and as students of mathematics, I interacted with students throughout the day, including during classroom activities, before and after school, and at lunch recess. I also had a series of conversations with the school's principal, and participated in several school-community events. Moreover, as I lived several blocks from Francis, I began an ongoing effort to familiarize myself with the neighborhood / community context. My proximity allowed me to interact with some students in non-school related contexts, such as at the corner store, walking down the street, or in church.

Selecting Case Study Students

Within each class period, Beatriz and I selected a smaller group of four or five case study students (see Table 3.1). (Her first period class had a heterogeneous mix of 29 students, and her second period class, 28 students.) As the school had a low mobility rate, approximately 5%, it was not necessary to begin with a larger sample of students, as researchers often do to account for students who may leave the school and withdraw from the study.

The purpose of involving students from two classes was to gain a more complex picture of the nature of student agency within a middle school mathematics classroom aimed at teaching mathematics for social justice, *not* to set up a comparison between one group of students and another. While each class investigated the same broad themes, the particulars of the curriculum were unique with each group of students, according to their interests, needs, and experiences. Different opportunities to use mathematics for individual and collective empowerment emerged, different conversations occurred, and students chose to pursue different kinds projects.

Beatriz and I selected case study students according to the following general principles. First, we chose students that represented a range of perspectives and beliefs about mathematics. That is, we chose students who openly expressed interest in mathematics (e.g., Vellez), others whose feelings towards the subject were ambivalent (e.g., Jhana), and others who claimed that math was boring, insignificant, and irrelevant to their lives (e.g., Angel).

This assessment of students' stance towards the subject matter was based on Beatriz's interactions with students throughout the year, and my informal conversations with students during the first two weeks of the study. These conversations included questions such as: When you think of math, what do you think of? How do you feel about math? Is math something that is important to you, if so, why?

Second, case study students represented a range of mathematical abilities, which was determined by considering Beatriz's judgment of students' abilities against the results of classroom assessments. Though this was not intentional, two case study students were eligible for, but did not receive, special education services in mathematics. Finally, we attempted to select an equal number of girls and boys, and students who reflected the diverse ethnicities present in Beatriz's classroom. All case study students were fluent English speakers, and while several of the students' parents were born in Puerto Rico or the Dominican Republic, the students were all born in New York City or surrounding communities.

I want to emphasize that our selection of a diverse group of case study students was ***not*** driven by a desire for generalization, but by the belief that diversity among participants would assist us in understanding the complex nature of student agency in this particular mathematics classroom.

Table 3.1

Case Study Students

Name	Age	Gender	Ethnicity
	First period class		
Jhana	11	female	Latina/Puerto Rican
Angel	12	female	African American
Vellez	12	male	Latino/Puerto Rican
Manny	13	male	African American
	Second period class		
Lianna	12	female	Latina / Puerto Rican
Naisha	11	female	African American
Andrés	12	male	Latino/ Dominican
L.J.	11	male	Latino / Puerto Rican
Joel	11	male	Latino / Puerto Rican

Negotiating an Emergent Curriculum

One of the most challenging aspects of this study was addressing all of the questions and issues that arose from the ongoing process of sharing power and negotiating an emergent curriculum. Over the course of the study, Beatriz, her students, and I negotiated three theme-based units where students used rigorous mathematics to investigate aspects of their lives and their world. Since prior to the study Beatriz did not characterize her pedagogy as "Teaching and Learning Mathematics for Social Justice," we were striving to create, and then study, what did not already exist, or what only existed in theory, or in practices in some other context. According to Skovsmose & Borba (2000), this process of investigating "what could be brought about," of imagining and then negotiating alternatives to the present situation, is not only a creative act, but a core characteristic of critical research in mathematics education (p. 7; see also Vithal, 2000).

This negotiation involved responding to tensions between (a) "a hypothetical situation inspired by a theoretical landscape" (e.g., the vision of "teaching mathematics for social justice" that I constructed, or imagined, based on a review of the literature), and (b) "the actual situation," or what actually existed in Beatriz's classroom at Francis Middle School (Vithal, 2000, introduction, ¶ 1). To address these tensions, we negotiated "an arranged situation" (which involved the enactment of the three theme-based units previously mentioned) that allowed us to investigate how the ideals of the theoretical landscape played out in actual classroom practice (section entitled The Arranged Situation, ¶ 1).

Since sharing power with students was a key component of the curriculum approach we were attempting to enact, developing the units prior to beginning the study would have been problematic. As Vithal (2000) argued, "A curriculum approach that seeks to value the intentions, participation and actions of learners in the arranged learning situation, needs to value and invite the teachers [and students] in similar ways into learning about such a curriculum approach" (section entitled Reflections on Actual, Hypothetical and Arranged Situations, ¶ 4). Moreover, while negotiating an emergent curriculum made designing carefully sequenced mathematical activities difficult, if not impossible, this ongoing negotiation helped to balance the unequal power relationships that existed between myself as the researcher, and Beatriz and her students as participants, because the process facilitated a continuous critique and questioning of ideas by participants.

Ultimately, Beatriz, her students and I negotiated a series of three "Mathematics for Social Justice" units. The first two units lasted approximately four to five weeks each, and the final unit lasted two weeks. Other instructional activities, such as preparing for and taking standardized assessments, were scattered throughout the three units. The following section briefly describes each of the units, and the various factors (such as negotiation with students, grade level objectives, and available materials) that influenced their development. While this was not a study of collaborative curriculum development, but of student agency, describing the curriculum is critical because it provided the context that framed, at least partially, students' participation in mathematics and their opportunities to enact agency.

When I entered Beatriz's classroom in January of 2002, her students were working on a unit from the mathematics curriculum adopted by the school, the *Connected Mathematics Program (CMP).* This unit, *Prime Time*, focused on relationships among whole numbers such as primes and prime factorization, square numbers, factors and multiples, and operations with odd and even numbers. Previously, students had studied other units in the CMP sequence, including one on data analysis. Upon completing *Prime Time* at the end of January, Beatriz felt that she needed to proceed with a unit that would allow her students to investigate rational numbers, a central component of the middle school mathematics curriculum.

The First Unit: Looking at the World Through Numbers

Around this time, Beatriz and I began to plan the first unit. In the beginning, our discussions were broad brainstorming sessions. I brought up ideas from the literature, including projects described in teacher journals (e.g. Rethinking Schools) and at educational conferences. I also voiced suggestions that Beatriz's students had shared with me during informal conversations and interviews. For example, students were intrigued by relationships among different countries in the world, particularly those between the United States and its adversaries. Given that this study began only several months after the events of September 11, 2001, students' concern for international affairs was not surprising. Beatriz contributed her own assessment of students' interests, along with her knowledge of their mathematical understanding. For instance, she felt very strongly (and I agreed) that students needed continued opportunities to investigate rational numbers and hoped that we could design a project with that mathematical content in mind.

We ultimately decided to develop a series of lessons, that later extended into a five week unit, that would allow students to use rational numbers (fractions, decimals and percents) to investigate living conditions in different parts of the world. Students began by exploring the worldwide distribution of population, wealth and energy use. They proceeded by generating data about local economic conditions, including how much people of different professions earn, how much basic necessities (e.g., milk, eggs, rent, shoes) cost, and how long people had to work in order to buy what they needed to survive. Students then had opportunities to exchange this data with students from other parts of the world, such as Spain, Zimbabwe, and Malaysia, through a collaborative internet project (Orillas, 2002). This exchange was motivated by students' questions about whether or not disparities in the global distribution of wealth mattered, because, as one student commented, "maybe stuff is just cheaper over there [in Asia, or Africa], so they don't need that much money." Since the exchange facilitated a comparison of living conditions in different countries, it also helped students reflect on their questions about why other nations might feel animosity towards the United States.

At a certain point, students became very intrigued by salary statistics, and the unit shifted to an exploration of income data from the U.S. Census website. Students compared incomes by profession, race and gender, expressed annual salaries in terms of weekly and hourly pay, and calculated the amount of time that different individuals would have to work to pay for certain basic items, such as food, clothing, and housing.

In the final week of the unit, students formed small groups to conduct what Beatriz referred to as "Educate!" projects. She emphasized, "You learned a lot of things in here about the world, or our community. We can't just keep that information here… cause then what good is it?" Students were instructed to choose an aspect of the data that they would like to further analyze, and then to make decisions about how and with whom they wanted to share the results of their analysis. Several groups chose to share their small group investigations at a school-wide meeting, others decided to distribute flyers in the community, and still others opted to survey community members about their reactions to the data. (See Table 3.2 for a list of the small group projects).

Table 3.2

Focus of Small Group Projects in the Looking at the World Through Numbers Unit

First Period	Interview Francis teachers about inequities in teacher salary
	Comparison of NBA and WNBA salaries
	Community feedback about inequities in doctors' salaries
	New York teacher pay within and outside of Manhattan
	Distribution of fliers that analyze the gender gap in salaries
Second Period	Commercial about price comparisons in U.S. vs. Zimbabwe[a]
	Skit about inequality in salaries earned by whites vs. blacks
	Rap about non-living wages in Zimbabwe[a]
	Analysis of police officer, fire fighter salaries
	Analysis of teacher salaries

[a] Students in the second period class exchanged data with students in Zimbabwe through a collaborative internet project.

Beatriz and I drew on a number of outside resources to assist us in planning specific investigations in this unit, including (a) descriptions of a 'World Wealth Simulation' lesson in Peterson (1995) and Gutstein (2001), (b) activities included in an on-line global networking project entitled, "Connecting Mathematics to our Lives" (Orillas, 2002), and (c) teaching resources that accompanied the U.S. Census database. Initially, I had intended for students to play a greater role in determining the focus of this first unit, but after talking repeatedly with Beatriz, we decided that *we* needed time to develop *our* ability to collaboratively develop curriculum before we invited students into the process. This is not to say that students did not have choices within this initial unit. They had many. They determined what aspects of the data were most interesting to them, the kinds of questions they wanted to ask about the data, and how they wanted to share the results of their small group investigations. I just want to be clear that initial decisions about the focus of the unit were made by Beatriz and myself, not by the students.

The Second Unit: Space at Francis Middle School[6]

While all three units aimed to address and connect with students' lives, this second project in particular was driven by generative themes that emerged from students' experiences (Freire, 1970/1993), namely their concerns about limited space and overcrowded hallways at the school. As we reflected on the first unit, Beatriz and I agreed that as beneficial as it was to take a global perspective, students also needed opportunities to explore more local, and as Beatriz said, "close to home" issues.

[6] While students' participation in each of the three units was included in the analysis, due to space constraints and redundancy, stories from the second unit, "Space at Francis Middle School," are the focus of the analysis presented in Chapters 5 and 6.

Moreover, I was very interested in students playing a more active role in negotiating the focus of the unit. With this in mind, we solicited students' ideas for our next 'math project,' asking them each to make a list of questions and issues about the school, neighborhood, and/or larger community context that concerned them. Shor (1996) referred to this process as front-loading student expression. We then grouped and tallied all the issues students mentioned, and met to discuss the results.

The issues most commonly mentioned by students included (a) concerns about overcrowding at the school, (b) relationships between the U.S. and other nations and the looming possibility of war, (c) violence in the community, (d) global health crisis, namely AIDS; and (e) racism and sexism in the media and the workplace. Initially, Beatriz and I considered presenting this list of five topics back to the students, so that they could further discuss each issue, and then agree on one particular issue for the class to investigate (Shor, 1992). Given time constraints, the unlikely possibility of reaching full consensus, and our need to ensure that the unit would provide fertile ground for exploring significant mathematics, we opted to make the decision ourselves.

Our ultimate decision to pursue the issue of overcrowding at the school was based on a number of factors, including (a) the mathematical content the unit would draw upon, (b) the opportunities it would provide for students to generate their own data, versus merely analyzing data retrieved from an external source; and (c) the salience this issue seemed to have for students (e.g., it was an issue that had also emerged during interviews and informal conversations). The fact that exploring the school space would draw upon a mathematical domain other than statistical analysis was particularly influential in our decision. As Beatriz commented during our meeting,

> I want to make [the unit] more personal, but also mathematically - not that I am worried about covering a certain topic, but I want to know, can this [teaching mathematics for social justice] only be done with data? Or can this be done with another [math] topic? I think we should look at that. … With the space issue, measuring, and obviously geometry would get in there.

Ultimately, this unit drew upon numerous mathematical concepts and skills, some we had anticipated (e.g., linear measurement, calculation of area, ratio) and others that we had not (e.g., multiplying mixed numbers and fractions). At times, the need for a particular mathematical concept emerged, as Stevens (2000) found, as students "pursued non-mathematical objectives." For instance, when a group of students decided they wanted to share their analysis of the school space with district administrators, they encountered the need to convert their measurements from metric to standard form, because according to the district, they "only spoke" the language of standard units.

The unit began with a series of five lessons that focused on developing the concept of area. Next, students spent two weeks measuring classrooms and hallways, and collecting relevant information about district building codes and space regulations. During these two weeks, Beatriz continued to present mini-lessons on an as needed basis. Her role became negotiating an intersection between the students' context, their needs, experiences and purposes within the project, and the academic context, the disciplinary knowledge and practices that would support their endeavor (Shor, 1992). Towards the end of the unit, students formed small groups to investigate a particular aspect of 'Space at Francis Middle School' in greater depth. During this phase, groups worked independently to formulate questions, generate necessary data, and construct arguments based on their analysis. For instance, while one group compared the bathroom space per student ratio at Francis to that of a neighboring middle school, another group focused on overcrowding in the hallways (see Table 3.3 for a summary of small group projects). Ultimately, some arguments were shared with individuals outside the classroom.

Table 3.3

Focus of Small Group Projects in the Space at Francis Middle School Unit

First Period	Comparison of total hallway area at Francis vs. Longmore[a]
	Analysis of hallway width against district building codes
	Ratio of students to hallway space at Francis vs. Longmore
	Bathroom space per student at Francis vs. Longmore
	Ratio of classroom space to students at Francis vs. Longmore
Second Period	Comparison of total hallway area at Francis vs. Longmore
	Ratio of gym space to students at Francis
	Analysis of fire hazards created by hallway width and area
	Analysis of gym use by students at Francis vs. Longmore
	Safety hazards created by poles in classrooms and the gym

[a]Longmore was another district middle school located directly below Francis, on the 4th floor of the same building.

The Third Unit: Global Sweatshops

The final theme-based unit, “Global Sweatshops,” was developed as a way to further explore questions related to the transnational production and sale of products that emerged during the “Looking at the World through Numbers” unit. For example, for their final project in that unit, one group of students created a commercial that critiqued the athletic shoe industry. They were outraged that brand name tennis shoes, manufactured in Asian countries at extremely low production costs, were then sold in the United States and in developing countries at ridiculously inflated prices. (Students generated the data for their commercial from the information they exchanged with other students in the “Connecting Math to our Lives” project, particularly students in India and Zimbabwe.) When other students viewed the commercial, they were intrigued, and questions about whether or not similar practices occurred in other industries (e.g., clothing, electronics, toys) surfaced.

It was with these questions in mind that Beatriz and I developed this third unit. We drew upon a number of resources, including a collection of articles, statistics and images designed for classroom use, entitled *Rethinking Globalization* (Bigelow & Peterson, 2002), and video and text materials from the National Labor Committee. As in the previous units, after a brief introduction to global sweatshops and a series of activities that helped students to think about how to analyze the production and sale of products mathematically, students formed small groups and selected particular aspects of “Global Sweatshops” to investigate in greater depth. For instance, one group chose to research the

J.Lo clothing line, while another focused on the production of baseball caps in the Dominican Republic. This third unit was much shorter than the first two, lasting only two weeks, and did not introduce any new mathematical content.

Methods of Generating Data

Over the course of the study, I generated data through a variety of ethnographic research methods. I conducted individual interviews and focus group discussions with each of the case study students, I observed and videotaped classroom interactions on a daily basis, and collected samples of student work and assessments. In addition, I recorded detailed field notes about informal interactions with students, made entries in a reflexive journal, and collected relevant documents distributed by the school and district. In the sections that follow, I describe each type of data generated, and its relation to the focus of this research: students' critical mathematical agency.

Interviews

Math Talk Interviews

I conducted two 'math talk' interviews with each case study student, one at the beginning and one at the end of the study (See Appendix C and Appendix D for sample interview protocols). All the initial interviews were individual, and lasted approximately one hour. In some instances, the interview was divided into two sections, conducted on subsequent days. Each section lasted approximately 30 minutes. Due to time constraints at the end of the school year, some of the final interviews were conducted with a small focus group of three to four case study students. The content of the final interview was very similar to the initial interview. However, students were also asked reflect upon their experiences over the course of the semester as they participated in a mathematics classroom aimed at teaching and learning for social justice (see Appendix D). All interviews, both individual and small group, were audio taped and later transcribed.

All interviews were semi-structured and invited students to share stories about past, present, and future experiences with school, the community, and mathematics. Interviews were structured around sharing stories because stories can offer insights into how individuals understand themselves, their lives and experiences (Drake, Spillane, & Hufferd-Ackles, 2001; Goodson, 1991), which include insights into their beliefs, interests, intentions, and values, and I would argue, into their sense of agency. While the 'math talk' interviews were intended to be open-ended enough so that students were able to tell those stories that they felt were important to tell, given the focus of proposed study, the interview attempted to elicit stories that related to students' sense of critical mathematical agency.

As defined in this study, critical mathematical agency has a *critical* component that refers to students' capacity to view the world with a critical mind-set, to imagine how the world might become a more socially just, equitable place, and to engage in action aimed at personal and social transformation. Moreover, critical mathematical agency has a *mathematical* component that includes students' capacity to understand mathematics, to identify themselves as powerful mathematical thinkers, and to construct and use mathematics in personally and socially meaningful ways.

To investigate these multiple aspects of agency, the interview included prompts that elicited students' stories about (a) their school and local community, particularly those aspects of their school and community that concerned them; (b) their role within the school and community, their participation in actions aimed at personal and social transformation, and the extent to which they felt their actions could make a difference; (c) their needs, interests, and priorities, and the extent to which school in general and mathematics in particular helped them to explore, or might help them to explore, those personal intentions; (d) mathematics as a discipline, what it means to do and know math, how mathematics is used in society, and how mathematics might be used; and (e) themselves as capable mathematical thinkers, and as users and producers of mathematics.

The first part of the interview focused on students' beliefs and experiences related to their school and community. For instance, questions three and four asked:

> 3) Can you tell me a story about two things about your community / neighborhood that you really like, or that are really important to you?
>
> 4) Now can you think something about your community / neighborhood that you wish was different, or that you might like to change?

While the intent of these questions was to gain insight into students' capacity to engage in critique and to envision their role in affecting change, I felt it was important to ask not only about the needs of the community, but also about what the community had to offer.

The remaining questions in the first part of the interview focused on students' experiences in school, in particular, their participation in projects and activities aimed at personal or collective transformation. Students were asked to share stories about projects that were especially important to them, to talk about the role (or possible role) of mathematics in these projects (questions 5 and 6, see Table 3.4), and then to envision projects that they would like to see happen (question 7, see Table 3.4). The purpose of these questions was to elicit students' ideas about how school in general and mathematics in particular might help them to explore personal intentions and engage in transformative action.

Table 3.4

Selected Math Talk Interview Questions

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5. I am wondering if you have ever been a part of any kind of project, in school or outside of school, where you have done something that helped your school or your community in some way. This could be a project where you did something to help, or to make something better, or to change something in the community.
   Probes: Did the project involve any math?
      If yes: Can you tell me about how it involved math? How did you use math?
      If no: Can you imagine doing a project like that one that did involve math?
      If yes: What would it be like?
      If no: Why not?

6. I am interested in other kinds of things you have done in school, or experiences that you have had in school, that have been really important to you. What is some project or activity or experience that was really important to you?
   Probes: same as above

7. Now I want you to think about what kinds of projects you wish you could do in school, or experiences you would like to have in school. What is a project / activity / experience that you would really like to do at school?
   Probes: What would it be like? What would you do?

Who would be involved?
Why would you like to participate in this?
Do you think this could happen? Why or why not?
Would this project involve any math?

The second part of the interview focused on students' beliefs and experiences related to mathematics and mathematics education. The first four questions, adapted from Drake et al., (2001) asked students to share various parts of their math life story. For example, question nine read:

> Can you tell me about a time in your life that was a real high point for you in math? This would be a time when you felt excited, or happy, or good about your experience with math.
>
> > Probes: Can you tell me more about what happened? Who was involved? What did you do? How did you feel? Why was this such a high point for you?

These questions were designed to elicit students' beliefs and ideas about mathematics as a discipline, and about themselves as capable mathematical thinkers. If students' ideas about the discipline did not emerge in their responses to the math life story questions, additional questions were used. For example, question 13 asked students to think about how mathematics is used, or could be used, in society.

> Who do you know that does math or uses math?
>
> How do they use math? What do they use it for?
>
> Do you ever use math outside of school? How do you use it?

Can you think of other ways that people use math?

Focus Group Interviews, also known as "Pizza Lunches"

Towards the middle of the study, as students were participating in the three 'teaching mathematics for social justice' units described above, I conducted two focus group interviews with small groups of case study students. Students affectionately referred to these interviews as "Pizza Lunches," because they involved staying in for lunch and eating pizza in Beatriz's classroom. These focus group interviews (Morgan, 1997) were less structured, as I wanted to promote a more natural, dialogical conversation with students, and to provide students with an opportunity to listen to and reflect upon the stories of their classmates.

The first pizza lunch occurred during the 'Looking at the World Through Numbers' unit, and lasted approximately 30 minutes. The conversation was motivated by the students' need to share the results of their data collection and analysis with other classrooms in the "Connecting Math to our Lives" project. I began the discussion by posing questions such as, "What would you like to share with the other classes? What have you found out that you think is really important, or that surprised you, or that you think other people need to know about?" Our discussion focused on students' reactions to the data they had been collecting and generating, and students expressed that they were particularly interested in sharing their opinions about inequities in income along the lines of race and gender.

The second pizza lunch took place during the 'Space at Francis Middle School' unit. This discussion centered on what students had learned over the course of the unit, and whether or not they felt the results of their investigations would have any kind of impact (e.g., on district policy). Students also reflected on how the 'Space at Francis Middle School' unit was different from other projects they had participated in. I guided the conversation by posing questions such as, "What have you learned in the project? What do you think helped you to learn that? Did math help you in any way?" and "Do you think people will listen [to your concerns]? What do you think you as kids can do [about the overcrowding at Francis]?" Both pizza lunch conversations were audio taped and later transcribed.

Clinical Problem-Solving Interviews

At the end of the second unit (Space at Francis Middle School), I conducted an individual clinical interview with each of the case study students (Ginsburg, Kossan, Schwartz, et al., 1983; Goldin, 2000). The purpose of this clinical interview was to assess students' understanding of the mathematical content that emerged during the unit, such as measuring the area of regular and irregular shapes, and multiplying fractions and mixed numbers (see Appendix E for clinical interview protocol). The interview included problem solving tasks similar to those students had worked on in class (e.g., Find the area of a classroom at Longmore that measures 7 and 3/4 meters by 12 1/2 meters), in addition to tasks that asked them to apply concepts they had explored to solve novel problems. For instance, the final problem in the interview presented students with information about two different school gyms, and the number of students at each school who planned to attend a dance in the gym, and then asked students to determine which school dance would be more crowded.

In addition to a list of specific questions to ask, the clinical interview protocol contained probes related to each question, and alternate questions to use when the standard questions were either too easy or too challenging for students (Goldin, 2000). I asked children to explain their thinking and justify their strategies throughout the interview, by posing questions such as "How did you figure that out?" and "How do you know?" I made notes about the strategies they used, and collected any written work that they produced as they solved the problems. Each interview lasted approximately 45 minutes, and took place in a quiet place at the school, such as an unoccupied classroom. All interviews were audio taped and later transcribed.

Assessing student understanding was important in this study, as understanding involves students taking control of their own learning, authoring their own knowledge, and developing personal investments and interests that guide their mathematical activity (Carpenter & Lehrer, 1999). In this way, as students' understanding increases, so might their sense of themselves as capable doers of mathematics, and therein, their sense of agency (Empson, 2002; see also Bruner, 1996).

However, as I was interested not only in students' mathematical understanding, but also in their thoughts about that understanding and its possible relationship to their personal and collective intentions (an aspect of their agency), the second part of the clinical interview included a series of questions that asked students to reflect on their experiences in the unit. For example, I asked students

> In the past month we have been working on the "Overcrowding at Francis" project. I am interested in what you think you have learned by working on that

project? Do you think you have learned a little or a lot? *Probe for mathematical learning if student does not mention.* Is there anything else you think you have learned? How do you think you learned that? Was what you learned useful to you in any way? How was it useful?

This second part of the interview was also audio taped, and later transcribed.

Classroom Observations

Daily interactions between students, their teacher, and other members of the school and local community were a significant source of data for this study. As the figured world in which students participate undoubtedly impacts the goals they pursue and the beliefs, values, and understandings that they develop, including those beliefs and understandings related to their sense of agency (Holland et al., 1998), it was essential to document the kinds of activity and interactions that occurred (Cobb, 2000).

For the first several weeks of the study, as I was "gaining entrance" into the school community, I recorded detailed field notes of my interactions and observations. I recorded these 'jottings' (Emerson, Fretz, & Shaw, 1995) throughout the school day, and later expanded them to more comprehensive field notes. After three to four weeks, once case study students were selected and the process of negotiating the curriculum began, I supplemented my field notes by video-taping classroom interactions. The purpose of video-taping was both to capture an accurate record of the dialogue among students and to document important visual / physical components of their interactions (e.g., what they recorded on paper, how they used tools and diagrams, their body language, etc). As I anticipated that students' sense of agency would be evidenced in their comments and conversations, as well as in their actions, it was important to have accurate records of interactions, records which would not have been possible relying solely on field notes.

As I had anticipated, the study generated about three and a half months, or 75 hours worth, of video taped classroom interactions. Not all tapes were fully transcribed. Instead, I first viewed each tape and wrote a detailed summary, but not a word for word transcript, of all dialogue. I then went back and fully transcribed the portions of each tape that contained critical interactions with case study students.

Student Work Samples

At several points during the study, students were asked to reflect in writing about their participation in classroom activities. These reflections were collected and analyzed as additional sources of data. Other student work samples that were created over the course of the project and that seemed to evidence students' sense of agency, their perceptions about themselves, mathematics, and/or about education for social justice, were also collected and analyzed. For instance, each group of students kept a folder where they organized all of their questions, calculations, ideas, and arguments related to the "Space at Francis Middle School" unit. The contents of these folders were drawn upon to craft "stories" of particular case study students.

Reflexive Journal Entries

Throughout the data collection and analysis phases of this study, I made semi-regular entries in a reflexive journal. These entries allowed me reflect to on methodological aspects of the study, such as my own role in the research process, and how I noticed my subjectivities and biases influencing the generation and interpretation of data. I focused on my interactions and relationships with the students and their teacher, how I saw issues of power and position evidenced in those interactions, and how I might be using my position of power in ways that could undermine the collaborative nature of the study. Journal entries also allowed me to make notes about interpretations and possible themes that I saw emerging from the data. In this way, these entries, like 'analytic memos' (Emerson, Fretz, & Shaw, 1995) helped to direct my attention to critical insights that emerged and helped to focus future observation and analysis.

Relevant Documents

In addition to the forms of data previously mentioned, I collected various supporting documents that provided relevant information about the school and district context. These documents included signs students posted around the school as part of their protest of the school's lunchtime detention policy, and letters sent home to families regarding upcoming standardized assessments. The primary purpose of these documents was to enhance my understanding of the uniqueness of the school context.

Methods of Data Analysis

Ongoing Analysis

Data analysis was an ongoing process in this study, one that began with the first day of interacting with the students and continued throughout the preparation of the final report. In qualitative research methodologies, such as critical ethnography, data generation and analysis are concurrent, inseparable processes that inform one another. As Erlandson et al. (1993) described, "the human instrument responds to the first available data and immediately forms very tentative working hypotheses that cause adjustments in interview questions, observational strategies, and other data collection procedures" (p. 114).

Initial Structured Review of the Data

After transcribing and summarizing the taped interviews and classroom interactions, and expanding upon field notes and reflexive journal entries, I conducted a general read-through of the data (Emerson, Fretz & Shaw, 1995), paying attention to the data as a whole, and making analytic memos about the insights, patterns, possible themes and questions that came to mind. I drew upon Wolcott's (1990) notion of "guiding questions," to focus this initial read through of the data (p. 32; see also Emerson, Fretz & Shaw, 1995). For example, I read all the data from the initial and final math talk Interviews with the following questions in mind, "What are students' conceptions of mathematics?" and "How do students relate mathematics to themselves and to their lives?" and "What changes do I notice, if any, between the two interviews?" among others. Reading through all the interviews provided me with an initial, albeit tentative sense of possible themes and categories that were emerging across case study students.

I then selected a particular case study student and reviewed all data from that student, including interviews, classroom interactions, work samples, and focus group discussions. I repeated this process with six of the nine case study students. Again, these reviews were focused around guiding questions (Wolcott, 1990), such as "What does agency seem to look like for Angel?" and "How does participation in mathematics fit into Angel's story of agency?" and "What does Angel's participation look like across time, in different settings, in different projects?" among others. The purpose of reviewing the data student by student was to gain a sense of that student's story over the course of the study, and in particular, a sense of how that student enacted and developed a sense of critical mathematical agency. The general impressions that I gained through these initial structured reviews of the data informed future phases of analysis.

Organizing and Coding the Data

Next, I began the process of "chunking and labeling" the data, a process of dividing the data into "pieces of data that may stand alone as independent thoughts" and then assigning a word or phrase that represented the meaning of each unit (Erlandson et al., 1993, p. 117). My intent was not to chunk and label every single utterance, classroom behavior, and statement that a student made during a focus group discussion or interview. The shear quantity of data made such analysis unproductive. Instead I identified interactions (from classroom observations) and stories and comments (from field notes, student interviews and focus groups) that seemed to (a) illuminate something important about a particular case study student, (b) reveal some aspect of students' critical mathematical agency, including a lack of a particular aspect, such as a lack of mathematical understanding; and/or (c) evidence some kind of tension (e.g., among students, between students and the curriculum, between students' interests and the mathematics, between students' activity and the school context etc..). These significant interactions then became "chunks", or what Ely et al. (1991) referred to as "thinking units" of data (p. 143). In some instances, a single "chunk" evidenced multiple aspects of agency.

As I began to organize the data, I was very careful not to divide the data into units that were too small, which would remove the students' actions, comments, or questions from the larger context of the interaction. This was particularly important when coding students' participation in Beatriz's classroom, because any given student action was shaped by a number of contextual factors, including interactions with the teacher, conversations with other students, the nature of the task, and the larger meanings surrounding the activity.

As I proceeded to code these "chunks" of data, I used a combination of pre-established codes (drawn from the literature) and emergent/open codes (drawn from my initial structured review of the data). For example, to examine the data for instances of student agency, I began with an initial coding scheme based on the various aspects of agency outlined in my conceptual framework (e.g., agency as authoring and positioning, agency as critique, agency as resistance, agency as improvisation, etc.). However, upon reviewing the data, I noticed that many students seemed to enact agency by asserting both personal and collective intentions. Thus I revised my initial coding scheme to include this category - agency as asserting intentions – a category which was later expanded to encompass a number of different codes.

As I analyzed the data to better understand the tensions that arose as students enacted critical mathematical agency, I relied almost exclusively on open coding (Emerson, Fretz, & Shaw, 1995), a process of generating as many codes as possible to describe the data, without relying on pre-established codes or categories. I say "almost exclusively" because I entered the analysis with a general sense (from past experience, from reviewing the literature) that tensions would emerge between a classroom curriculum centered on students and their intentions, and a curriculum that at the same time needed to address particular mathematical content and skills. While my understanding of this very broad tension did guide me in coding the data, I still relied on open coding to greatly expand and refine my understanding of this tension, and the others that emerged.

Emerging Tensions and Themes

Once I identified a set of core themes about students' critical mathematical agency, and about the tensions that arose as they enacted that agency, I reviewed all classroom observation, interview, focus group and field note data with these themes in mind (Emerson, Fretz, & Shaw, 1995). This process involved "physically grouping segments of the data on a theme in order to more easily explore their meanings" (p. 159). This process of sorting according to themes (tensions) assisted me in understanding the complexity of each theme, and in examining how a given theme played out in different way with different students, and in different contexts. The analysis of these themes, stated as tensions, is presented in Chapter 6.

Developing Case Studies

Concurrent to my analysis of the themes described above, I constructed a set of three case studies, which I presented in the form of stories that detail the participation of four case study students. (One story involved two students who regularly worked together on classroom projects). I chose to tell the stories of these four students not because their participation was in any way more or less agentive than that of their peers. Rather I chose these students because together, their stories provide a rich, and varied sense of *how* students enacted critical mathematical agency through their participation in a teaching mathematics for social justice oriented classroom. These case studies are presented in Chapter 5.

Quality Criteria for Establishing Validity

Throughout the ongoing process of data generation and analysis, I made a number of efforts to establish what Lincoln and Guba (1985) referred to as "trustworthiness"-- the truth-value, or quality, of the findings (see also, Anderson, 1989; Ely, Anzul, Friedman, et al., 1991; Glesne, 1999).

Credibility

The first aspect of trustworthiness, credibility, refers to how well the findings of the study match the perceptions of the participants. To ensure that my findings accurately reflected students' and teachers' perspectives, I used a series of member checks (Ely et al., 1991; Erlandson et al., 1993; Lincoln & Guba, 1985) during interviews, classroom interactions, and when possible, at the completion of the study. For example, during interviews I asked students questions such as, "Okay, I think I hear you saying ….. Is that right?" or "During our last conversation, you told me about …. , let me know if there is anything you want to add, or if that does not sound right." During classroom interactions I asked students questions such as, "Can you tell me about what you are doing?", "Why did you decide to do that?", or "What did you think about what we talked about today?"

I chose to focus on member checks *during* interactions with students for two reasons. First, after an interaction occurred students may have had difficulty remembering what they said or felt or did, and second, member checking after interactions would have been a time intensive, tedious process, which could potentially function to burden, or even exploit participants (Patai, 1991). During member checking, whenever my understanding of the participants' beliefs or experiences did not match their perceptions, I adjusted my notes to more accurately represent the participants' views.

Both during and after the study, I also discussed my interpretations of student interviews and observations with Beatriz. These discussions contributed to the credibility of the study by helping me reconsider and clarify my interpretations of the data. For instance, as a tension between the mathematics students needed to know to move forward with their intentions in the projects and the mathematics those same students were able to conceptually understand emerged from the data, I discussed the tension with Beatriz. She confirmed that she had consciously experienced this tension as she was interacting with students, and shared insights with me about why she responded to the tension in the ways that she did. (This tension is discussed at length in Chapter 6).

Triangulation of Data Sources

This study drew upon multiple forms of data to investigate students' critical mathematical agency, including: (a) individual student interviews (both 'math talk' interviews and clinical problem-solving interviews); (b) focus group interviews, (c) classroom interactions, (d) student work samples, and (e) field notes and reflexive journal entries. To triangulate the data, I reviewed each emergent theme (tension) across multiple sources of data (various case student students, the teacher, and the researcher) and across multiple types of data (interviews, classroom interactions, field notes, and written documents). This process of triangulating data of various forms from various points of view adds credibility to the study and helps to establish trustworthiness (Ely et al., 1991; Erlandson et al., 1993).

Transferability

A second aspect of trustworthiness, transferability, refers to "the extent to which [the study's] findings can be applied in other contexts or with other respondents" (Erlandson et al., 1993, p. 31). This is distinct from the notion of generalizability, which allows researchers to make generalizations across populations and settings. With transferability, it is the reader, not the researcher, who must decide whether or not aspects of the study's findings might transfer to other contexts. "Transferability across contexts may occur because of shared characteristics," characteristics which are made visible through the use of thick description (Erlandson et al., 1993, p. 32). In an effort to provide the reader with sufficient detail to make judgments about transferability, I presented my findings in the form of rich, descriptive case studies, and included the participants' actual voice as much as possible. In addition, I selected case studies that represented a diversity of perspectives in an effort to "maximize the range of specific information that can be obtained from and about the context," further enhancing the reader's ability to identify shared characteristics between the context of the study and their own context (p. 33).

Catalytic Validity

While standard practices of establishing trustworthiness are necessary and important, they are not sufficient in critical ethnography. Critical ethnographic research, research that is explicitly ideological (Lather , 1986) and that is framed by an agenda of social critique (Anderson, 1989), raises questions about validity that go beyond mainstream qualitative research. To address these tensions around validity, Lather (1986) proposed the notion of catalytic validity, which "represents the degree to which the research process reorients, focuses, and energizes participants towards knowing reality in order to transform it" (p. 272). According to Lather, catalytic validity is achieved if participants gain self-understanding, and self-determination through their participation in the study. As I analyzed the data, I identified evidence of this notion of validity. These moments of realization, transformation, and subtle change are discussed in the case studies presented in Chapter 5, and in the tensions outlined in Chapter 6.

Conclusion

In this chapter, I presented critical ethnography as an appropriate tool for research aimed at promoting social justice. I reviewed the principles and research practices that characterize critical ethnography, and offered examples of how they were applied in this study. Next, I discussed general methodological information about the study, including the process of collaboratively negotiating the curriculum with Beatriz and her students. Finally, I detailed the methods of generating and analyzing data that this study employed.

In Chapter 4, I offer a detailed description of the research context, including the school, Beatriz's classroom, her students, and her pedagogy.

CHAPTER FOUR: THE RESEARCH CONTEXT

In this chapter, I present an overview of the research context: Beatriz Font's 6th grade mathematics classroom at Francis Middle School. I begin the chapter with a discussion of the school setting, including: a brief history of school, a description of its physical environment, and a summary of the school's philosophy and vision. Next, I situate Beatriz and her practice within the larger school context. I describe her decision to teach at Francis, a decision based in part on the congruence between her own teaching philosophy and that of the school, her classroom environment, and her pedagogical practices. The chapter closes with an introduction to Beatriz's students, whose stories will be presented in detail in Chapter 5.

The Setting: Francis Middle School

Francis Middle School opened in 1990, following a district wide decision to support the creation of more 'small schools' for middle grade students. At the time, the school's current and founding director, Carol Williams, was a teacher in a neighboring district. She submitted a proposal to start a school whose mission would be "to create a multi-racial and multi-ethnic community of students, staff and parents who take seriously the politics of urban life, who study vigorously and inquire intensively, who co-construct knowledge, curriculum, and assessment" (Fine, 1996, p. 13). Carol's proposal was accepted and she embarked on the near impossible task of finding available space within the district for a new school. At that time, the elementary school that currently shares a building with Francis had vacated the top floor because of continual problems with a leaking roof, holes in the floor, and pigeons that nested in the gym and nearby classrooms. Carol claimed the space for Francis, and after covering large holes with pieces of plywood and placing buckets beneath persistent leaks, the school opened. Soon thereafter, Francis was adopted by a neighborhood church, and repairs began.

Initially, Francis was a very small school of 60 students, 3 teachers, 1 instructional aide, and Carol. All decisions were made consensually, including hiring staff, admitting students, and distributing funds. Since that first year, the school has continued to grow in size. Yet the principles upon which it was founded, such as the importance of democratic leadership, the belief that "adults must know children well to teach them effectively," and the central role of students' needs and interests in the curriculum, have remained a strong focus (Fine, 1996, p. 37).

The School Space

During the 2001-2002 school year (the year of the study), Francis had approximately 211 students in sixth, seventh, and eighth grade. This represented a 24% increase over the number of students the previous year, and a 52% increase over the school enrollment two years prior. Enrollment was projected to increase another 15 to 20 per cent during the 2002-2003 academic year. This increase in enrollment – but not space – was a matter of great concern for students and staff members. As Beatriz reflected, "As students and teachers climbed the five flights of stairs to reach the once pigeon infested fifth floor which now housed their school, they worried about how the narrow halls and lack of classroom space might accommodate the already cramped community in upper Manhattan."

This tension around the issue of space, along with the school's desire for democratic leadership, was evident in the school's physical layout. For instance, Francis did not have an official 'office' space. Instead, one medium sized classroom housed the principal's office, the teachers' lounge and lunchroom, the secretary's office, and the copy room. The room was not subdivided with temporary walls. Instead, the work of administrators, staff, faculty, and sometimes students, coexisted within one open and very dynamic space. This 'multi-purpose' room existed because of logistical concerns – there were no other empty rooms at Francis --, but also because of the school's desire to reflect, in its physical structure, their goal of democratic leadership. Carol did not have her own private office because the existence of a separate 'principal's' office would have created unnecessary division between school staff and administration.

During the first several weeks of the study, as I entered the "office space" one morning, I was struck by how democratically the space was used. A group of 8th grade students were just leaving the office, after distributing flyers in all of the teachers' mailboxes. The flyer was not an "official" school communication, but rather a student-made memorandum protesting the school's lunchtime detention policy, otherwise known as "P.B.L." or "Peanut Butter Lunch." (Students received PBL for a variety of infractions, including repetitive tardiness, failing to turn in homework assignments, and disrespectful behavior in class.) Teachers and administrators observed as students passed out the flyers, asking them about its contents, and respecting their right to communicate their opinion. Students were never told that they could not place materials in teachers' mailboxes, nor were they critiqued for their blatant protest of a long-standing school policy. The flyer, which students later posted on walls around the school, read

> As students of the United States we have the power to end P.B.L. All it takes to stop this school prison from continuing is your help. It's time for us to take a stand and do what we think is right. Because of the fact that I have P.B.L. everyday it's only a joke to me. P.B.L. takes place during lunchtime when we can be outside having more fun with out friends and getting some air. 5% of the students that go to Francis skip P.B.L., sometimes they skip because they don't know that they have it, and sometimes they skip because a friend asks them to. It's time to make a big change and it takes all of us to do it.

While the students' plea to end the lunchtime detention program had minimal impact on school policy, their actions, and the way those actions were supported (at least partially) by the school, alluded to the emphasis on democratic participation that permeated the school space. Students were encouraged to speak their minds; voicing protests and dissenting opinions in public spaces at the school was not atypical. (See Appendix F for an example of another flyer posted by two 8th grade students protesting the force of sexism in their lives). Poetry and artwork honoring the lives of civil rights activists and other "movers and shakers" of society adorned the school's walls. Large pieces of butcher paper were often hung in the gym to gather students' opinions about various school issues, such as resolving conflicts and testing practices.

In other words, many students at Francis were comfortable, and encouraged by their teachers to raise “taboo” topics such as racism, discrimination, prejudice, and injustice. Such actions were supported not only by the school’s ideology, which was reflected in how students used and experienced the physical space of the school, but in the school’s history. Students at Francis were entering a figured world (Holland et al., 1998) that had a tradition of pushing boundaries and promoting social justice. For instance, in the semester before this study began, students, families and staff members joined forced to protest a district decision to eliminate para-professional (i.e. instructional aide) positions at the school. Their actions, along with some creative budget manipulations, allowed the school to retain a parent and community member who had supported Francis students and staff in multiple capacities for many years.

Thus when students in Beatriz’s 6th grade mathematics classroom began to draw upon mathematics to investigate issues of justice and fairness in their lives, they acted in ways that built upon, and nourished the story of the school. Drawing on Lynn Arthur Steen’s (1990) image, they were building “On the Shoulders of Giants,” the “giants” including: previous students, faculty, and community members who had participated in enacting the vision of Francis Middle School, individuals in their own families and neighborhoods - including themselves - whose stories were sources of inspiration, and the voices of other social justice activists like Martin Luther King, Rosa Parks, and Cesar Chavez, whose words and images permeated their classrooms and hallways. Even as they were building on the work of others, they were also making the story of Francis Middle School their own, as stories presented in Chapters 5 and 6 demonstrate.

The Philosophy of Francis Middle School

Francis has a small, but collaborative staff of 11 teachers and 2 administrators. What follows is Beatriz's description of the working environment at the school.

> The school is located in west Harlem, yet both students and teachers traveled to Francis each day from all five boroughs and other neighboring communities (e.g., Long Island, New Jersey). Several of the teachers living outside Manhattan commented that they chose to make up to two-hour daily commutes for one reason, the students. At Francis Middle School, teachers have the unique opportunity to work collaboratively on the development of the policies and procedures that run the school. The teachers meet as a team for more than an hour and a half each week to plan curriculum, discuss strategies for school improvement, study topics of interest to the school, and to modify and adjust school policies as needed. The mission at Francis is to honestly, respectfully, and genuinely*, leave no child behind*. Students are always at the center of each staff meeting and many times the only topic of conversation is how to help a particular child in need. Each teacher designs her own curriculum combining both the requirements of the district and the needs of her particular group of students. Resources used and topics discussed are at the discretion of each teacher.

One reason that Beatriz chose to work at Francis, and that I consequently chose to conduct this study at the school, was the school's unique commitment to serve students as whole people, recognizing and nurturing their individual strengths, and attending to their academic as well as emotional, physical, and social needs. The Francis staff understands their role as preparing students not only to succeed in school, but more importantly, to live meaningful, fulfilling lives. They strive to nurture students' sense of justice, and their interest in learning about and caring for the surrounding community. In describing her view of the school's philosophy, Carol stated:

> First and foremost, I want the kids to come away with skills, reading, writing, mathematics, and critical thinking skills. But even more than that, I want the children to come away empowered. Many children do not come from empowered families, but families that have been systemically disempowered. I want students coming away feeling that their education is their business, and that they have some control over what happens.

One way that the school provided students "some control over what happens" was through how the school curriculum was structured. All academic classes other than mathematics (i.e., language arts, science, social studies), integrated students from across 6th, 7th, and 8th grade. Students were not grouped by ability, but an effort was made to establish a heterogeneous mix of ages, grades and academic achievement levels in each classroom. This integration of students across grade levels allowed students to nurture friendships with older and younger peers, to explore topics at a variety of levels of complexity, and (particularly for the 7th and 8th graders) to assume genuine roles as leaders and mentors. In addition to academic course work, students were allowed to select two elective classes which met anywhere from one and a half to three hours a week. Electives were also multi-graded, and included such topics as swimming, community service, yoga, art museum visits, astronomy, choir and musical theater, hip-hop musical interpretation and analysis, and feminist literature. Electives were added and removed each semester according to both student and teacher interest.

Tensions at Francis Middle School

While Francis was clearly a unique place, and while its philosophy and vision were very consistent with the aims of this study, it was by no means a perfect school. Over time, subtle contradictions, or points of tension, between the school's espoused mission and its enacted practice became more apparent. For instance, although a significant percentage of students' families spoke Spanish as their primary language, Francis lacked a system for translating school-home correspondence. At times, letters and flyers were sent home only in English, and on other occasions, one of the two bilingual teachers at the school (Beatriz included) was asked to quickly translate correspondences during their lunch hour or planning period. Likewise, the school's progress reports, which were used in lieu of traditional report cards because they provided parents which much more descriptive information about their child's learning, were only available in English. While Beatriz made concerted efforts to explain the forms to Spanish speaking parents - over the phone, in person, or through a cover letter - mono-lingual teachers were left with no option but to send the forms home and rely on students to translate the material for their parents.

Since Francis was a small school with a small budget, it also lacked many of the resources and support structures common in larger city middle schools. For instance, there was no library, no nurse's office, and no budget to pay for substitute teachers. Whenever a teacher was absent, classes were combined, or another teacher would cover the absent teacher's classroom during his/her planning period. The budget for administrative staff was also minimal, and so teachers assumed many responsibilities typically performed by school counselors, secretaries, or other support personnel (e.g. organizing standardized test administration, planning the master schedule, assigning students elective classes, ordering equipment, etc.). Most teachers seemed very willing to perform these extra duties; there was definitely a strong spirit of collaboration at the school. But over time, administrative demands coupled with the already time intensive process of student-centered, organic teaching became taxing for teachers at Francis.

The School's Support for the Study

Throughout the study, Carol openly supported my collaboration with Beatriz and her students, along with the aims of the research. From her perspective, math was a critical area of the curriculum, one that "has the power to open doors for students, especially for girls, and even more so for girls of color." "That's why all of our math teachers are women," she commented. According to Carol, while issues of social justice had been infused into other curricular areas at Francis, such as humanities, social studies, and science, the mathematics curriculum had remained relatively unchanged and unchallenged. For this reason, among others, Carol embraced the notion that students in Beatriz's classroom might have opportunities to engage in mathematics in personally and socially meaningful ways.

The Participants: Beatriz and her 6th Grade Mathematics Students

Beatriz

Beatriz came to Francis Middle School in the fall of 2000, after spending one year working in central Texas on a research and professional development project aimed at systemic reform in mathematics education, a project that I also participated in as a graduate student. Prior to that year, Beatriz had received a Master's degree in mathematics education, and spent several years teaching fifth grade. During the year that we worked together, Beatriz and I regularly discussed our ideas about teaching and learning mathematics, about the purpose of education, and about the needs of urban schools. We shared many beliefs and values, and grew to respect one another not only as colleagues, but also as friends.

In the spring of 2000, when Beatriz made the decision to return to full-time classroom teaching, she choose to apply to schools in the New York City area. She shared with me that she had always wanted to teach in a truly "urban" school, and in particular, to work in one of New York City's Puerto Rican and/or Dominican communities. She also sought out a school that was implementing a reform-oriented curriculum, and that supported collaboration among teachers, students and their families. With these criteria in mind, Francis Middle School seemed like an ideal fit.

In the fall of 2002, the beginning of her second year at Francis, Beatriz and I discussed the possibility of conducting a collaborative research project in her classroom. (We had previously mentioned the idea, but neither she nor I thought the study would ever become a reality.) At the time, I was in the process of writing my dissertation proposal and trying to identify possible sites for my research. I invited Beatriz to participate because of her dedication to teaching mathematics in low income, urban schools, her interest in issues related to equity and social justice, and her desire to collaboratively explore the meaning of 'teaching mathematics for social justice' in her classroom.

Beatriz's Classroom

Like all classrooms at Francis, Beatriz's was a unique and very alive space, one that simultaneously reflected the history of the school, Beatriz's own story, and the ideas and experiences of her students. Along the back wall of the classroom, in addition to floor radiators, an upright piano and an old church pew (Beatriz also directed the school choir and taught a musical theater elective), and several outdated computers that were occasionally functional, stood a floor to ceiling metal shelf unit – the kind often sold in home improvement centers and used for storage in a garage -- that was affectionately known by Beatriz and her students as the "Big Ugly Shelf Thing." While this assortment of classroom furnishings may seem odd, each piece became an integral part of daily life in Beatriz's classroom. Math problems were written about the shelf unit, which stored materials from teachers who had previously used her classroom, and who intermittently sent students in the middle of a class period to retrieve particular items. The computer tables doubled as a storage area for coats and backpacks, and the church pew became a space for students to 'cool off' or to talk with Beatriz in 'private.'

Beatriz made concerted efforts to transform this 'unique' classroom into a more inviting and livable space. She covered the spaces between bookshelves with patterned fabric, and used the shelves to display copies of children's picture books such as *Math Curse*, and *Sir Cumference and the Round Table: A Math Adventure*. Windows were topped with brightly colored curtains, folding tables with pieces of cloth, and trinkets and *recuerdos* from Beatriz's childhood and adolescence were scattered throughout the room. Walls displayed striking images of Puerto Rico, Beatriz's place of birth, in addition to drawings, letters, and poems from past students. Beatriz hung seasonal paper cut outs from the ceiling, and jotted personal notes, such as "Today is my dad's birthday," across the front board.

However, the task of creating an aesthetically pleasing, and livable classroom space was not without its challenges. The classroom included two floor-to-ceiling poles that created large 'blind spots,' or spaces within the classroom where students could not sit if they had any intention of seeing the chalkboard. Students were constantly adjusting their desks to avoid these blind spots, which added to the constant hum of squeaky chairs and wobbly tables. Beatriz's ability to approach the structural limitations of the classroom with a sense of humor, and her students' ability to adapt to the 'uniqueness' of the space, was remarkable.

Beatriz's pedagogy

I want all my math teachers to be like Ms. Font, the way she *smoothed* things down on us. (Carlos, individual interview)

Interactions within Beatriz's classroom were equally lively and personal. Students usually entered the classroom in the midst of conversation, talking back and forth across the room about their plans for lunch, a recent basketball game, or the latest news about fellow Francis students. In between classes, observing students break out in song, or challenge each other with dance moves, was not uncommon. Once each day's lesson formally began, students continued to interact in a dynamic, but very focused manner. If their participation ever fell below Beatriz's standard, she would stop the lesson and engage the entire class in two to three minutes of 'energizing exercises,' after which their participation regained its usual energy. What follows are Beatriz's comments about her approach to classroom instruction.

> Rather than a set of rules to be memorized and formulas to be learned, my students and I view mathematics as something that can be questioned, discovered, and acted upon. It is through these mathematical interactions and investigations that students become true mathematicians. As a community of learners, my students are allowed and encouraged to disagree with and question each other. I begin each lesson with a short story. At first the stories were funny anecdotes about my childhood or my commute to school via three trains, but as the year progressed the stories set the stage for the problem of the day. Each story was carefully woven so that before they knew the story had ended the students had begun to think about the problem. "I can't do it" and "I don't get it" are two phrases that are left at the door when a student enters my classroom. It is too often that children of urban communities enter a math classroom with the attitude that they cannot do math or that math is inaccessible to a female, a Hispanic, an African American student. As a result, part of what I teach is Math confidence. The attitude that I can do math and I will do math is something that we work on

> for the better part of a semester. Two posters hung in the corner of my room: One the NACME poster "Math is Power: Demand it" which depicts the word MATH tattooed on the knuckles of a gang member's hand and the other Robert Frost's "The Road Less Traveled." Not all of my students will have the chance to go to college because of economic, social or cultural constraints, but my job is to give every one of them a fighting chance to take the road less traveled.

Clear in Beatriz's comments is her faith in students' competence as mathematical thinkers, and her belief that students can deepen their understanding by participating in classroom activities that push them to engage with the material, ask questions, and justify their thinking.

In the following section, I present some of the key features of Beatriz's approach to teaching mathematics and interacting with students. These "key features" initially emerged from my analysis of field note data. I then shared the analysis with Beatriz, to check my interpretations against her own understandings of her pedagogy. Describing Beatriz's pedagogy is important, because how she interacted with students and the kinds of participation and activity that she valued became prominent aspects of the figured world in which students participated. What follows is a brief overview of these "key features," how they related to a "Teaching Mathematics for Social Justice" approach, and how they were conducive to fostering student agency.

Positioning students in ways that helped them to claim mathematical authority. Beatriz frequently positioned students as capable mathematical thinkers who had the capacity to understand concepts, improvise viable problem solving strategies, pose questions, and invent mathematical terminology. For instance, whenever she introduced a new mathematical concept (e.g., prime number, prime factorization), she encouraged students to invent and improvise their own language for talking about the patterns and relationships they noticed, and their own ways of notating and keep track of their strategies. This resulted in students generating language such as “stretching out the numbers” to describe the process of prime factorization. Although Beatriz later informed students that the process had a mathematical name (i.e., prime factorization), students were allowed and encouraged to use their own terminology and to share their terminology in the public space of classroom discussion.

Beatriz commented that earlier in the school year, during a lesson introducing the Venn diagram as a way to represent relationships among data, she mentioned the term “Venn Diagram” in passing, and then introduced students to the “Font Diagram,” which was essentially the same, only drawn in the shape of two overlapping squares instead of circles. Constructing and naming her own diagram was a conscious, deliberate action. Beatriz felt that (a) too many mathematical tools and concepts had been named for men, and (b) students needed to see that anyone, including themselves, had the authority to construct unique models and strategies and then name them as they saw fit. This resulted in a regular flow of improvised strategies that were generated by individual or small groups of students, named accordingly, and then presented in the public space of the classroom (e.g., “I used Jhana’s way” and “You could really do it Roberto’s way.”)

Beatriz positioned students as capable mathematical thinkers not only by supporting them in naming strategies, processes, and concepts, but by making them responsible for evaluating the validity of their solutions. She made statements such as "I don't have the answer. You better not be waiting for me to tell you, cause it's not going to happen." In addition, she encouraged students to raise questions whenever they disagreed with something she or another student did, which they did, making comments such as "Ms. Font, you forgot to do that little space over there." When the class could not agree upon whether or not a particular strategy or solution was valid, she would assign that strategy as a homework problem for students to investigate further.

Promoting dialogue and valuing diversity in thinking. Discussion and debate were common activities in Beatriz's classroom. Time permitting, Beatriz invited multiple students to share their strategies for each problem that the class solved. Special attention was awarded to strategies that were novel, unexpected, or in some other way unique. For instance, when L.J. improvised a unique strategy for solving an equal share fraction problem, the class, Beatriz included, applauded his solution. They seemed to recognize both the validity and the creativity of his approach, criteria which were highly valued in their classroom.

When students improvised ways of approaching problems that seemed either invalid or very inefficient, Beatriz encouraged students to think through their strategies. She valued diverse ways of thinking about problems, and did not want to stifle what could develop into a unique, powerful strategy by prematurely redirecting a student's thinking. To support students in thinking through their own ideas, she posed questions such as, "Well what do you think would happen if you tried ________?" or "Tell me more about why you think _________ would work?"

Explicitly emphasizing issues of fairness. Another "key feature" of Beatriz's classroom was her explicit emphasis on equity and fairness, particularly as it related to students' participation in mathematics. For instance, after calling on several male students to share their strategies, she would "think aloud" for the students as she commented, "Okay, I just called on two boys, so now I am going to call on two girls." Beatriz felt that making her thinking explicit to students was important not only because it evidenced her "fairness" as a teacher, but because it communicated to students that she believed that *all* students were capable of contributing to class discussions, not just those who tended to participate more frequently (i.e. the boys) or those who identified themselves as "good at mathematics."

Beatriz's concern for fairness and equity was also evident in the way that she handled classroom activities and homework assignments. Early in the year, she promised students that she would never assign "busy work" and that she would never send homework problems that they were not prepared to solve. As a result, whenever the class did not get far enough in a given lesson, Beatriz refrained from sending the planned homework assignment. Moreover, she repeatedly emphasized to students that everything they did in her class would be meaningful, and that if they ever failed to see the purpose of a particular assignment, they should ask, which they did. Genuine, honest questions such as "Why are we doing this?" and "What's the point of this?" were not uncommon.

Making the class personal: Attending to students (and herself) as whole people, not just doers of mathematics. A final feature of Beatriz's pedagogy was the way that she considered students (and herself) as whole people, people who had needs, ideas, and wants, people whose lives extended beyond the figured world of her classroom. Her concern for students as individuals was evident in the kinds of discussions she fostered (e.g., conversations about friendships, about how to resolve conflicts, about how to present themselves and assert their needs, etc.), and in the ways she helped students to connect mathematical concepts with their own lives and experiences (e.g., eliciting stories of students' experiences with percents and fractions).

Beatriz also made the class personal by allowing her identity to become visible to students through the stories and anecdotes that she shared. In revealing personal information about herself (e.g., stories about family members, weekend adventures, childhood experiences, and morning train rides, etc.), Beatriz invited students to make their own lives and experiences more visible in the classroom. The role of story in facilitating this identity sharing process was critical. In my field notes I wrote:

> The students now seem to expect stories from Bea, they ask her to start each lesson in this way, almost like they are yearning for some kind of real life connection. They want to know about her, her life, her story. "I want a story" they demand. … When she begins the telling, there is absolute silence. The kids seem to crave every minute, even when the stories are fairly simple. There is utmost respect, a longing to hear what she has to say.

Beatriz's stories functioned to make the class more personal in a variety of ways, including (a) helping students connect mathematical concepts with their daily lines, (b) helping students connect with Beatriz not just as a teacher, but as an individual who cared deeply about them, and who could relate to aspects of their experience. For instance, one morning students entered Beatriz's classroom particularly fatigued and lacking in energy. Beatriz proceeded to relate an anecdote from her own experience as a middle school student, trying to pay attention in class when she was tired or not interested. The story began with an introduction to the teacher.

> When I was in school, there was this very interesting, that would be a stretch, she was actually a very boring teacher and her name was Ms. Newton. And she had the – the teacher you always see when you watch like Arnold. She had this huge like bee-hive hairdo. You could hide like 50 pencils in there and you would never see it. And she wore these long big skirts, that like maybe she had 6 people under there. And you could smell her from miles away cause she wore a lot of powder and perfume. Well, "Ms. Newton is coming down the hall," cause you could always small her. And also, her purse always matched her shoes, cause when she had little purple shoes she had a purple purse. It got pretty nauseating.

She then continued to share the strategies she developed, as a student, to "stay awake" when she felt tired in this teacher's classroom. For example, she challenged herself to take notes with her non-dominant hand. She commented

> So I decided I would have to have something to keep me awake, so I decided let me try to take notes with my left hand. Why not? So I started writing with my left hand and the effort that it took me to think about writing, how can I keep up with what she is saying, kept me awake in class, and now I write with both hands.

Stories like this one not only helped Beatriz's students to "wake up" and engage with the remainder of the lesson, but the stories nurtured a relationship between Beatriz and her students as people. Stories became a tool that Beatriz employed to address the teacher-student contradiction (Freire, 1970/1993). Telling personal stories constructed bridges between Beatriz's experiences and identity and those of her students, which resulted in blurring the lines of this false teacher-student dichotomy.

Beatriz's pedagogy, teaching mathematics for social justice, and agency. While Beatriz did not consider her instruction prior to the beginning of the study as an example of "Teaching and Learning Mathematics for Social Justice," I want to argue that the key features of her pedagogy were consistent with the aims of this approach. For example, her acts of positioning students as capable mathematical thinkers and encouraging them to claim mathematical authority helped to establish a genuine sharing of power among students and teachers. Moreover, by centering students' experiences, lives, needs, and concerns, and by providing students with a modicum of choice in all activities (the latitude of students' choices greatly expanded once the class began the three "Math for Social Justice" units described in Chapter 3), Beatriz involved students in negotiating an emergent, problem posing curriculum. Together, these actions helped to reconcile traditionally antagonistic teacher-student relationships, which then supported students' power and agency in her classroom[1].

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[1] Other elements of "Teaching Mathematics for Social Justice," such as (a) a focus on praxis: critical reflection and transformative action, became more apparent as Beatriz and her students negotiated and enacted the three theme-based "Math for Social Justice" units outlined in Chapter 3. The aim of the section above was to document the norms, values, and ways of interacting that characterized the figured world of her classroom both before, during, and I would suspect after the completion of the study.

The Students

Students from Beatriz's first and second period math classes participated in this study. Students came to Beatriz's sixth grade mathematics classroom from various elementary schools both within and outside of the district. The district's official elementary mathematics text was *Investigations,* a reform oriented program designed to address the NCTM standards. However, students' descriptions of their elementary math experiences indicated that teachers used a variety of curriculum materials, including traditional textbooks, *Investigations*, and what students referred to as 'worksheets.'

While presenting particular students' stories is the focus of Chapter 5, in this chapter I wanted to offer a general description of the student culture in Beatriz's classroom, a description that addresses questions such as, "How do the students see themselves? What kinds of things do they like and care about? What do they think about their school and their community? How do they feel about mathematics?"

During the first theme-based unit, "Looking at the World Through Numbers," a group of students (including seven of the nine case study students) gathered during lunch recess to compose an introduction of themselves and their school to share with other students participating in the on-line networking project. While the intended audience for this introduction was others students from around the world, and not readers of a dissertation that documented their work, I have chosen to draw on this e-mail message here because I believe that students' own words provide a vivid, dynamic, and honest picture of who they are as people, and how they view their school and community. What follows are excerpts of the students' introductory message.

Dear Connecting Math to our Lives Classes,

Have you ever been to New York City? We are students from Francis Middle School. We are 11 and 12 years old and we are in sixth grade. And we love our teachers. And Ms. Font is our teacher and she teaches math and we get a good education. And we have a wonderful chorus at out school, and guess who conducts it? Ms. Font!

We like to have fun, we like basketball, and we like chess. We also like dancing, the Harlem shake (it's shaking your shoulders), hanging out with friends. We also like music, like rap, reggae, rock and roll. We like to play games like board games and games that you play outside like man and woman hunt, out of respect. We get along with each other. We have a lot of opportunities in our school that other schools don't have. We have a community of harmony. This school helps us do what we want, not just what other people expect of us. At this school we get to go out for lunch, and we have half days every Friday. We have a lot of after school activities like chess, dance and basketball, talent show. We get to take classes with the 8th and 7th graders so we are all equal. We get books in our classes. Before Francis was a small school now it is bigger, we have 200 kids. It is bigger than what Francis is supposed to be. We get to have drama, yoga, art, boy talk, women's talk, we also have field biology, dance, and aerobics. And we don't have a lot of money at our school, we don't have a basketball team, but we have many teams in the school so we can support ourselves. Don't think just because we live in New York doesn't mean we have a lot of money or that we think we are better than everyone else, because we are all equal. In our school we have an advisory and an advisor is a teacher that guides you through the school.

…. The teachers let us talk and tell them how our life is and what we think and what we are feeling. And when we get something wrong they try to say it in a way that doesn't hurt it. The people in this school are not racist, because "we are family." Some teachers let us listen to music while we are working.
We love math. Our teachers take math slowly so that we will all get it. It is not too hard or too easy, it is perfect. She takes it step by step so we can all get it. She will take a few days with each math lesson. Ms. Font gets it through our brains so it will be fun, and if it gets boring, Ms. Font will tell us a story. And she will help us with the homework if we have questions. We do fractions in the class, and I know some people think that fractions are boring but Ms. Font makes it ghetto. She is with the new styles.

Recently we were talking about how some countries have more wealth than other countries and they use more energy and we have been talking about the population that all the countries have. We have learned that we have the most money, but we don't have the most people, which is kind of weird to us. Right now we are learning about how much energy other countries use, and which one uses more. And there are countries that have lots of people and less money. We are wondering what you thought about all these questions? We think a lot of people want to come here, because we have more freedom than some other countries. What is your idea of freedom?

…. New York has a lot of traffic and in the morning when you get ready to go to school, it takes a long a time to get from place to place. New York is a great place to live because you are free. New York attracts a lot of people because all the buildings and all the fun things to do. It has a lot of places that are educational, like things about computers. New York I have lived there my whole life and I think if something like ever happens to New York it would be very bad because New York is a very special place to live.

By Andrés, Jhana, Cristina, Naisha, Lianna, L.J., Joel. Vellez, Chantelle, Evan

Francis is a happy school.

Many races. Speak our mind. Teachers let us speak out.

Conclusion

In this chapter, I offered an overview of the research context: Beatriz Font's 6^{th} grade mathematics classroom at Francis Middle School. I began by describing the history, philosophy, and physical setting of the school, and the then situated Beatriz and her classroom practice within the larger school context. The chapter concluded with an introduction to Beatriz's students, including their ideas about themselves, their school, and their community. In Chapter 5, I present the in-depth stories of four of the nine case study students.

CHAPTER 5: STUDENTS' STORIES

In this chapter, I present the stories of four of the nine case study students from Beatriz's math classroom. The chapter opens with Angel's story, continues with the story of L.J. and his friend Joel, and concludes with the story of Naisha. I chose to tell the stories of these four students not because their participation was in any way more or less agentive than that of their peers. All the students demonstrated agency, at different times, and in different ways. Rather I chose these students because together, their stories provide a rich, and varied sense of *how* students enacted critical mathematical agency through their participation in a teaching mathematics for social justice oriented classroom. Moreover, each of the stories alludes to one or more of the *tensions* that arose. Chapter 6 is dedicated to analyzing these tensions, including how they were addressed by students and teachers.

Angel

> I want to become the president. And if I don't become the president, I want to become vice president. And if I don't become vice president, I want to be a model. … But at the same time, I am not going to forget my family. Because other presidents, they be acting like they don't have no family. And my mom takes care of me, and I am supposed to take care of my mom when I get older. My mom says, 'you gonna take care of me?' and I say 'yeah!' And so, I would build my mom her own house, but I don't want her to be alone. She will have somebody with her, that I trust and that she trusts.

Angel is a tall, 12 year old, African-American girl who lives in upper Manhattan with her mother, a younger brother and sister, and her mother's partner. As the opening quote illustrates, family relationships, in particular her relationship with her mother, were of utmost importance to Angel. Angel enjoyed spending time with her mother, who she referred to as “my friend, my *best* friend.” Though she emphasized the value of honest, open communication with her mother, Angel also recognized that other youth her age might not understand her unique mother-daughter relationship. She explained,

> Most kids don’t really think about certain things. Like they don’t know things that I know. Maybe at their house their mother don’t tell them stuff, because they don’t know most of the stuff that I know, because my mother sits down and talks to me about right and wrong, and she makes me learn from my mistakes. …. We do a lot of stuff together. We talk about each other’s thoughts. And I have this friend, I *used* to have this friend and he used to try to say, “Oh, you are always going to your mommy for everything.” And I was like, “Yeah?”

Angel drew upon close family relationships not only for advice and emotional support, but also as a source of motivation in school. She spoke of striving to “get good grades” and “get scholarships” not only for herself, but also “to show my mom that I can do it.” Angel valued honoring her mother, and viewed success in school as one way to communicate that honor. Yet just as Angel spoke of how she was responsible for achieving in school, she also recognized her mother’s role in supporting her efforts. At times, this support came in the form of small pieces of advice, such as “And like my mom always told me, take your time, so when you take your time and you finish your work it shows that you respect yourself and you do a good job.” On other occasions, support meant that parents should, according to Angel, respond to their children’s requests for educational resources. For instance, as she explained that she wanted her mother to buy her a new book that she was “dying to read”, Angel argued that although parents should not buy their children everything they want, they do have a responsibility to supply their children with those materials that are important to their learning, like books.

Enacting Critical Agency

The quote at the beginning of this chapter also alludes to Angel's strong sense of personal agency. The goals of "becoming president" or "become the first black president" were ones that she stated adamantly, and repeatedly, over the course of the study. Yet in Angel's case, this goal evidenced more than personal agency; it also demonstrated a sense of *critical agency* that involved (a) an awareness of unjust conditions, (b) the capacity to imagine that the world could be a more socially just and equitable place, and (c) the ability to imagine her own role in that transformation. That is, motivating Angel's desire to be president was an explicit critique of this country's present administration, led by white men, and its inability to effectively address important social issues, such as those that impact the poor, the sick, women, and children. Responding to a question about whether or not she thought she would be able to achieve to her goal, Angel argued:

> Angel: My mom said I should, because most people really think that it's not really right cause it's all those men, and then all of them are white too, and so - But then when they see that someone is trying to make a change, my mom said that most people will vote for you.
>
> Erin: Ahh. You're thinking people are ready for a change.

Angel: And women, men don't really pay attention to the problems that you need and stuff, and my aunt, she says that they need a women to make sure that everything is alright, and they are more organized. And --- that's why I want to become president too.

Erin: What would you want to do?

Angel: First of all, I would make like this big building, for people who don't have any money or stuff, and it would just be—like Medicaid and stuff, because a lot of people are sick and stuff. And I would make sure there are no terrorists– and George Bush, he's not even doing anything. I would make sure that people are getting along. And I would make more programs, for like girls, like [girls] here that gets pregnant and stuff, get them in a building, and make sure they're alright, and yeah. And you know people that has to be outside [on the street], they need help and all that George Bush and like the mayor is doing is telling them [the police] to arrest them, and they aren't really doing anything [wrong]. … And I would, instead of using our money to build all these weapons you could use that money for schools, and for programs, and for kids that are in the hospital.

As this conversation demonstrates, Angel went beyond critique to suggest possible changes and assert her potential role in those changes. With confidence, she communicated her belief that she could make things better.

Aspects of critical agency were also evident in the ways Angel talked about practices at Francis Middle School. She had very definite ideas about the qualities of a good teacher –caring, respectful, and fair - and critiqued both teachers and school policies that failed to demonstrate these criteria. For instance, Angel argued that Ms. Jackson[1], a language arts teacher at Francis, was “not always fair” because her pedagogy did not allow for an honest assessment of what students learned. By allotting students “only five or ten minutes” to work on an exam, and then deciding to “take up the tests,” she prevented students like Angel, who “was really paying attention” from demonstrating their knowledge. Angel commented, “And I started crying, because I really wanted to complete that test, and if I get a bad grade, then my mom’s not going to be happy with that.” Moreover, Angel critiqued Ms. Jackson’s lack of respect for students, stating that she “is rude”, because she frequently answered her cell phone during class activities, and spent class time talking with her own children, instead of her students. Angel emphasized, “And I am like, you are teaching *this* class, you *can’t* be on that cell phone.”

Angel leveled similar critiques against broader school policies, such as the school’s lunchtime detention program, otherwise know as “P.B.L.” (peanut butter lunch). Predictably, many students spoke unfavorably of PBL, which consisted of staying inside during the lunch hour and silently consuming a peanut butter sandwich supplied by the school. Yet, Angel’s critique was unique in that she based her comments on the principle that school is supposed to “teach you something” and thus PBL is not a “fair” practice because it has no educational value. Moreover, Angel challenged the “fairness” of the program by arguing that students could receive PBL for actions such as tardiness that were sometimes beyond their control. For instance, students came to Francis from all

[1] A pseudonym

five boroughs, and trains and buses could be late, and traffic unpredictable. Angel generated numerous ideas for how PBL might be more educational and thus more fair, such as "students could write a letter or something, saying how they should have improved their behavior."

Although Angel was able to challenge aspects of the school as a figured world that constrained, or negatively impacted her activity, and to imagine how the school might be different (ways of enacting critical agency), her ability to imagine her own role in that transformation was limited. Regarding PBL, she was fairly certain that nothing could be done to change the school's policy. With Ms. Jackson, even when presented with the opportunity to (possibly) make a change - her mother offered to speak with the school principal about removing her from the class– Angel responded, "I told my mom, it's cool. I'll stay in there." It seems that for Angel, the possible negative repercussions of being removed from a teacher's classroom outweighed the potential benefits.

Why is it, at least in these examples, that Angel did not bring her sense of the power of transformative action, and her confidence in her own ability to contribute to that transformation, to her interactions within the figured world of her school? Why was she able to talk adamantly and passionately about the changes she would make as president, and yet simply responded "it's not possible" when asked about changing the school's PBL detention program? I would like to argue that this difference is influenced, at least in part, by the fact that the constraints and limitations that are invariably part of public schooling are all too familiar to students. That is, even at Francis, where centering students' needs and experiences is an explicit part of the school mission, the probability that students would be able to change school structures, such as discipline policies, is low. Six years of experience in public schools has made this exceeding clear to students. The point here is *not* that schools should do away with discipline policies, or even that students should be invited to collaboratively determine how schools are run. Instead, I want to argue that (a) schools, as figured worlds, involve a number of constraints and special considerations (e.g. having a parent remove you from a teacher's classroom could have negative consequences in a small school environment); (b) that students, as participants in the figured world, are aware of those constraints, and (c) that their awareness and interpretation of those constraints impacts how they enact agency.

In other words, Angel understood the lunchtime detention program as a stable school structure, not one that was "vulnerable" and "open to redefinition" (Baez, 2000, p. 339). This understanding, which reflected both her present encounter with the constraint, and the sediment of years of prior experiences with similar institutional constraints, made it difficult for Angel to move from envisioning changes in the school to engaging in action aimed at that transformation. Interestingly, several weeks prior to this conversation with Angel, a group of 8th grade students distributed flyers in protest of the school's PBL detention policy[2]. Angel seemed to be unaware of their actions, actions which might have provided a powerful example of how other individuals, like herself, challenged the constraints of the figured worlds in which they participate. Moreover, if actors in positions of power in the figured world (e.g., teachers, administrators) had publicly recognized and genuinely considered these students' complaints, such actions

[2] See Chapter 4 for a more complete description of these students' actions.

might have helped Angel to understand that institutional constraints, though durable and undeniable, are human constructions (in this case constructions of teachers and administrators), that are open to redefinition.

Thus for Angel, transformative action within a hypothetical, or imagined context, like becoming president, felt much more plausible than the capacity to affect change in a setting whose structures and constraints she has encountered on a daily basis for the past six years. This tension between student agency, contextual constraints, and students' understanding of those constraints, is further developed in Chapter 6.

Angel's Participation in Mathematics

Angel had a quiet, and sometimes even aloof presence in Beatriz's first period mathematics class. She typically entered the room alone, and then joined a few of her close girl friends at a grouping of desks towards the back of the classroom. Some days, Angel would sit and talk quietly with her friends as she waited for class to begin. On other days, Beatriz's request that students take out their homework and begin the "Do Now!" warm-up problem would catch her with her head resting on the desk, as she struggled to work through early morning fatigue. But if Angel happened to be in the midst of a good novel she would enter the room with her nose in the book and devour as many pages as possible before the math lesson 'officially' began.

Angel spoke very candidly about her preference for other subjects, such as language arts and humanities, over mathematics. During conversations at the beginning of the study, she commented, "in this school, math is hard", and "math is alright, I guess, but I am not really good at it." Interestingly, Angel's most enthusiastic comments about mathematics occurred as she was describing her early experiences learning mathematics at home with her mother.

> I started doing math when I was home, before I got to school. Yeah, and my mother used to teach me, and she would say that I am a fast learner, so she didn't have any problems from me. … Yeah, she did addition and subtraction, and she taught me … it was with pictures and stuff, or she would write it down and she would work with me.

Once we began to discuss her school math experiences, Angel's answers became shorter, and her tone less animated. She spoke of math as "numbers" and "operations," and recalled topics she did not understand (e.g., "that division thing, when you put the sign"), and teachers who "don't know how to explain." While Angel was certain that she would need mathematics in the future, she was unable to describe how math would benefit her, beyond "in school" or "for certain jobs." Consistent with Angel's own comments, Beatriz considered her, based on her performance and participation in classroom activities, to be a fairly average, or even below average mathematics student.

Critical Mathematical Agency Through Asserting Intentions

Given Angel's somewhat negative, or at least disinterested attitude towards the subject matter, what happened to her strong sense of critical agency when she entered the figured world of her mathematics classroom? How was she able to integrate her critical stance, her ideas about justice, and her sense of the power of transformative action with her participation in mathematics? In other words, how did she enact critical *mathematical* agency? As the subsequent examples demonstrate, Angel's critical mathematical agency was most visible and consequential when she had opportunities to investigate questions of personal importance. That is, she enacted agency, and in ways that impacted her interactions with the subject matter, as she *asserted personal intentions*. By *intentions* I refer to desires, needs, plans, hopes, interests, and goals that are grounded both in what an individual brings to a particular situation (i.e. a person's socio-cultural history), and in how that individual, within a given social situation, perceives and defines a set of possibilities for the future (Skovsmose, 1994)[3]. Consider the following story of Angel's participation in the "Space at Francis Middle School Unit."

Angel was a girl who was not always comfortable with her appearance, in particular, her height and strong build, and entering the school's girls' restroom and finding it difficult to navigate among the 10 or 12 other people who might be using the space, caused her significant frustration. She was especially bothered that all females in the school, including 103 students and 15 teachers, administrators and interns, had to share one rather small facility. "Adults shouldn't be using children's bathrooms" she repeatedly commented. When Beatriz presented students with the opportunity to select an aspect of the school space to investigate in greater depth (all students completed a "What's your issue?" assignment, which asked them to define a concern about the school space and to think about what information they would need to investigate that concern mathematically), the choice for Angel was obvious: "We want to know, why are the girls' bathrooms *so small*?"

[3] The notion that students enacted agency as they asserted personal intentions is elaborated in Chapter 6. This theme arose across case study students, and could be distinguished from students' acts of agency as positioning and authoring or transformational resistance and critique.

Angel believed that mathematics would support her investigation of the school restrooms, referring to math as "one of those things that helps you solve your problems." In fact, I argue that the critical agency Angel exerted as she *asserted personal intentions* and *critiqued* the current school space pushed her to engage in and begin to develop understanding of significant mathematics. That is, as she brought her intentions to the center of this negotiated curriculum and began to ask questions that mattered to her, her desire to better understand and act upon the 'space crisis' at her school created a genuine, and seemingly natural need for mathematics. As she stated in a small group discussion with one of her classmates, Naisha,

> Angel: I think that every time you did [the space project], it makes you feel quizzitive.
>
> Naisha: What's quizzitive?
>
> Angel: Curious, It's like – curious about different things, like it makes you want to go deeper into the project, like learn more stuff. Like the thing with the world, the thing with the area and stuff, the thing with the school, you want to learn more. Like the legal width of the hallway, the different dimensions of each room, and other stuff. It makes you feel mad curious because you want to know different things. And when you learn different things, that will help you with different stuff. … Cause you know, you know how much space you got, and then you know how much space everyone is going to get.

It is important to note that Angel's capacity to ask questions that mattered to her, or "to go deeper into the project" in her words, was supported by her participation in the figured world of her mathematics class. Beatriz opened a space for students to pose questions of personal importance (e.g., What's your issue? What do you want to talk to the district about?), validated those concerns (e.g., "Yes, bathroom space is a really important issue.") and then supported students with the resources that they needed to carry out their intended investigations (e.g., in Angel's case, information about the number of female students and staff members at Francis, and class time to measure the girls' restroom, etc.). Moreover, as students progressed in their investigations, Beatriz encouraged them to think critically about the data they were generating, and how to draw upon that data to construct powerful, convincing arguments (e.g., "How can you prove that point? What information do you need? How can you show that concretely and specifically?" and "Can you make that into a comparison?"). In this way, Angel's math classroom became a place that valued, invited, and even fostered the critical world-view and strong sense of justice that she brought with her to school.

Continuing with the story, Angel's group began by constructing a floor plan of the restroom, measuring its dimensions, and calculating the area (which was a challenging task in itself due to the irregular shape and odd dimensions of the room). They ultimately decided to approximate the area with a rectangle that enclosed most of the space and that measured 5 1/4 meters by 2 3/4 meters. Angel, with the assistance of other members of her group, was able to figure the area by partitioning the 5 1/4 by 2 3/4 meters area into smaller rectangular spaces that allowed her to deal with whole and fractional square meters separately (see Figure 5.1). This strategy was invented by one of Angel's classmates, made public in a whole group discussion facilitated by Beatriz, and then appropriated (with varying degrees of success and understanding) by many students.

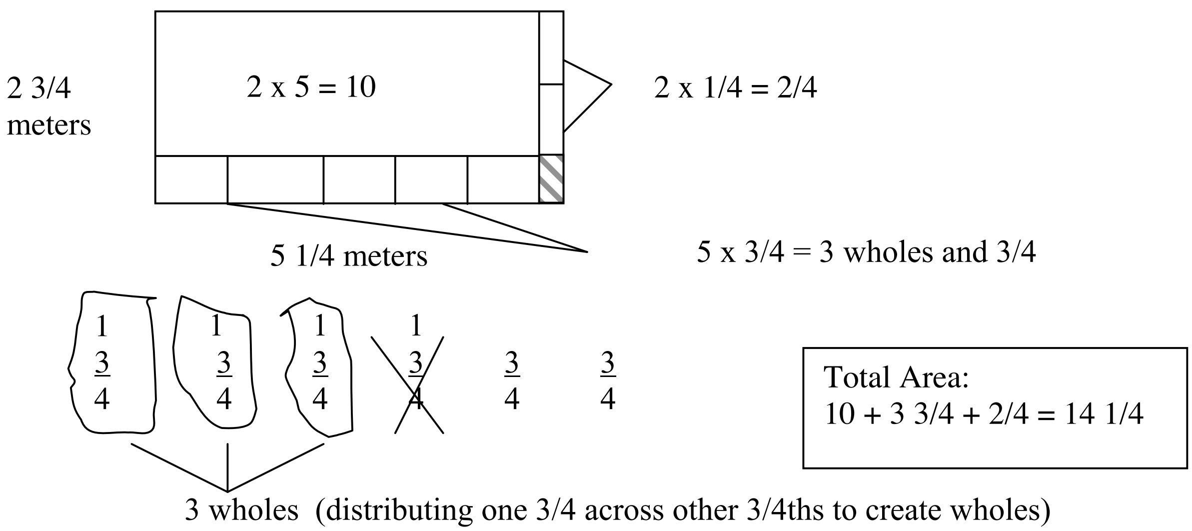

Figure 5.1. Figuring the Area of the Girls' Restroom. Angel's strategy for partitioning a drawing of the girls' restroom into smaller rectangular spaces to assist her in calculating the area. Angel combined most, but not all, of the partial areas to find the area of the entire space.

As Figure 5.1 demonstrates, Angel had a developing understanding of the meaning of area, and of ways to calculate the area of a space that encompassed both whole and partial square units. Also evident is her understanding of multiplying a whole number by a fraction, and of combining fractional parts to create wholes. As she solved this and similar problems, she was typically able to explain her thinking, including how her operations with fractions and whole numbers related to particular portions of area (e.g., adding five 3/4ths and then labeling five sections on the diagram that each measured 3/4 of a square meter).

The skills and understandings that Angel developed were supported by her participation in the figured world of her math classroom. For instance, Beatriz presented a series of mini-lessons aimed at helping students to mathematically investigate their concerns. These mini-lessons addressed topics such as accurate measurement, calculating area by partitioning a space into standard square units, and making sense of mixed number dimensions, concepts that Beatriz and I anticipated would be helpful to students. Though these mini-lessons were usually planned, at times they occurred spontaneously in response to a student's need to tackle a particular kind of problem. For instance, a student's question about "How do you multiply with 3/4?" prompted Beatriz to facilitate a discussion focused on multiplying common fractions by whole numbers. This mini-lesson generated the strategy of redistributing fourths to make wholes that Angel drew upon to solve the problem presented in Figure 5.1.

However, as this example illustrates, Angel's calculation of area was incomplete. Though she accounted for the majority of the space, she disregarded the small 1/4 by 3/4 meter rectangular area in the lower right hand corner of her drawing (shaded in Figure 5.1) [4]. More importantly, Angel did not recognize that her calculation of the total area was an estimate. In fact, when calculating the area of a rectangular space that had two mixed number dimensions, Angel consistently ignored small portions of the area (e.g., for the area of a classroom that measured 5 1/4 meters by 8 1/2 meters she would include the

---

[4] Angel's disregard for the 'small little corner space' was also an artifact of instruction, and reflected one of the ways that the tension between allowing students' intentions to drive the mathematical content while at the same time ensuring that students were able to conceptually understand that (sometimes quite challenging) content, was resolved. Chapter 6 includes a more detailed discussion of this tension.

40 whole square meters, the eight 1/4 square meters, and the five 1/2 square meters, but leave out the space that measured 1/4 meter by 1/2 meter).

The point here is not to emphasize gaps in Angel's understanding, but to point out that as she exerted critical agency by asserting her personal intentions and assuming a critical stance towards the school space, Angel *began* to understand significant mathematical concepts, such as area and operations with fractions and mixed numbers. Moreover, her understandings, though partial, enhanced her capacity to participate in personally meaningful, and even transformative ways, in the figured worlds of her classroom and school. In Angel's words, her developing understanding afforded her "more defense" in her efforts to engage in transformative action. She explained,

> With math .. it's like you have more defense. You know the length and the width, and you know – let's say you go and have an argument with somebody, and you say the hallway or the bathroom is small, and they say, "what do you mean it's small?" And you don't even know how big it is. And it's like, this room, you know the lengths, and the widths, and then it's like you have more defense right there, because you know more stuff that they didn't even know about.

In this way, her emergent mathematical understanding supported her critical mathematical agency.

However, Angel's story is not just about how the agency that students enact as they assert personal intentions can motivate and support their opportunities to learn mathematics. Her story also illuminates the power of students' intentions to challenge the mathematical norms and expectations of the figured world in which they participate.

Continuing with the previous example, once Angel and her group members had calculated the area of the girls' restroom, they were not sure what to do with the information. That is, they weren't convinced that a single measurement like "area" would assist them as they framed their argument about the bathroom space crisis. So they began to improvise other ways to analyze the girls' restroom, based on the number of stalls, the number of females in the school who may need to use the facility at any given time, the estimated wait time during peak use periods (e.g., between classes), and the space available for waiting. They discussed comparing Francis's restroom with the girls' restrooms on other floors, and talked about how "taking pictures" or "recording a video" might help them to convince others of the urgency of the problem.

The improvised, mathematically rich approaches to analyzing space that Angel and her peers generated to investigate their concerns about the girls' restroom, were encouraged, but not adequately supported mathematically, by their participation in the figured world. For example, during the final days of the project, Angel expressed frustration as her group struggled to pull together all of the variables they were considering to construct some kind of mathematical argument. Beatriz had presented a series of mini-lessons aimed at supporting students in this task, but the content of these mini-lessons (i.e. measuring, understanding and calculating area, figuring ratios) did not provide Angel with the mathematical support that she needed to investigate her unique concerns. While many other groups calculated a space per student ratio, Angel openly resisted this idea, even when it was (inappropriately) suggested by Beatriz and myself. For Angel, it did not make sense to think about how much of a square meter of bathroom space could be allotted to each female in the school. Her primary concern was the number of stalls available, and for how long and in what conditions girls and women at the school had to wait to use those stalls.

As Angel negotiated her participation in this project, she was driven by her personal experiences waiting to use the restroom, and at times entering and deciding that she did not even have time – or space - to wait based on the number of people already in line. She was *not* driven by a desire to apply a particular mathematical concept (e.g., area or ratio) that had been emphasized in classroom instruction. In this way, the personal intentions that Angel asserted as a form of critical mathematical agency challenged more simplistic ways of evaluating crowding, such as a space to person ratio, and brought a level of complexity to the analysis that was not as apparent in other groups who approached the project less as an opportunity to investigate questions of personal importance and more as a mathematical exercise. Angel's story alludes to the potential power of students negotiating their intentions into the curriculum to not only transform their experience with and relationship to the mathematics, but also to challenge the mathematical content that they investigate.

Critical Mathematical Agency Through Critique

Within the "Space at Francis Middle School" unit, the fact that Angel *asserted her intentions*, and thereby enacted a sense of agency is not surprising, as the project was explicitly designed to provide students with such opportunities. But it raises an important question. How might Angel's critical mathematical agency look different within the context of classroom activities that were not designed, from the outset, to draw upon students' experiences and intentions? For instance, in the first project, "Looking at the World through Numbers[5]," Angel's opportunities to make explicit choices about the content she studied or the activities she participated in, were limited. The focus of the unit, the mathematical topics addressed, and the day-to-day sequence of activities were all determined by Beatriz and myself. Granted, we continually adjusted the unit design in response to students' questions and concerns and their understanding of the mathematical concepts, but the unit did not open up a space for Angel to assert personal intentions in the same way as the unit previously discussed.

Yet Angel still managed to enact a strong sense of critical agency, and moreover, that agency supported her mathematical learning. The important difference is that in this unit, Angel's enactment of agency began not with opportunities to assert intentions, but to raise critiques. Consider the following story. Several weeks into the unit, the class began to analyze income data they had acquired from the U.S. Census website. This was part of the students' participation in a global networking project entitled, "Connecting Math to my World" (Orillas, 2002)[6]. Students had the task of collecting local data about the cost of living (e.g., the price of food, rent, bills, and other basic necessities) and wages earned by various professions, and then sharing that data with other classes from around the world. While most classes in the project submitted single figures for the income earned by each profession, since Beatriz's students generated their information from the Census database, they were able to access data that was disaggregated according to gender and race.

[5] See Chapter 4 for a more detailed description of this unit of study.
[6] See Chapter 4 for a discussion of this project.

For Angel, whose participation in the unit up to this point had been limited, the opportunity to reason about this data was important, in that it became an entry point for her sense of agency. For instance, when the class first accessed the Census website, Angel requested to view to data by gender. She was shocked at the income gap that she discovered, and raised questions such as, "They [men and women] don't get the same amount of money?? But you said they have the same job? I don't get it." She resisted accepting that women actually earn less than their male counterparts, and generated a number of scenarios, like working extra hours, that "justified" the gap. She argued

> Angel: Okay, so say like you have two parents, and they working the same job, and one of your parents might be making more money than the other. Well one of your parents, maybe your dad, might be working overtime and when you work overtime you get more money on your check.
>
> Manny: But still the man is going to make more. Cause if she works overtime and he works overtime, still he would get paid more.
>
> Angel: Yeah well my mother makes more than my father.

In a subsequent lesson, Beatriz provided students with tables that displayed much of the data that they had collected from the Census website. When Beatriz invited students to examine the tables and "see what [you] notice," Angel was adamant about exploring disparities along the lines of gender and race. The following conversations illustrate how Angel drew on her strong sense of justice to levee critiques and raise questions that were important to her. Those questions in turn prompted her to draw on mathematics, albeit in fairly simple ways, to further investigate the situation.

> Angel: [Reasoning about the data collected by the class from the census website]. Okay, well right here, for the female doctor. Why is it that the white female got more money than both of them [the black and Hispanic female doctors], if they do the same job? What's the difference? I don't get that. What are they doing that they get more money?
>
> Beatriz: That's a question. Why is that happening?
>
> Angel: And then the same for this white male? He makes more.
>
> Beatriz: Why is that?
>
> Angel: But doesn't it depend on how much you know as a doctor? Or how much you work?
>
> Beatriz: Yeah, it should. ….
>
> Erin: [Angel is now analyzing the data in her small group]
> What are you working on?
>
> Angel: Mostly the men, the men make more. Wait, let me tell you. All the men make more money than the females, but right here [points to the income figures they collected for doctors], it's the only one. This is the only one where the

woman makes more than the men [indicates that the white female doctor's average salary is higher than the black male doctor's salary].

Erin: Ok, so you noticed that the women make less –

Angel: -- We are going to, now we are going to compare how much more money [the men] make than the women. And how much more would it take for a woman to catch up. If you want to add, you could add this amount of money, like adding it on, or you could times it, until you get this amount of money that the man makes, or close to it.

Following this interaction, Angel proceeded to calculate differences between the average annual salary earned by male and female doctors of different races. She then tried to determine ratio relationships between the salaries earned by a white male doctor, and female doctors of different races. This involved rounding figures and using numerical reasoning to construct relationships such as, "the black female doctor earns about half as much as the white male doctor, and so she has to work for two years to make what he makes [in one year]." Other ratios were slightly more challenging for Angel, because the numbers did not fall into a straightforward 'twice as much' or 'three times as much' relationship (e.g., the average salary for a white male doctor was about one and a half times as much as that of the white female doctor).

The math here is fairly simple. I do not intend to argue that in analyzing this data Angel engaged in rigorous mathematics. Rather, what seems to be important is that she drew upon her sense of justice to raise critiques, and then to pose her own questions,

questions that mattered to her given her position as a young African-American female who would someday enter the workforce, and her mother's position as someone currently impacted by the inequities she noted. Also important is how features of the figured world of her mathematics classroom supported her in raising these critiques. Beatriz encouraged Angel to ask questions, to pose conjectures, to generate personal examples. She facilitated discussions among students that provided Angel access to differing opinions (e.g., Manny's comment above) that helped her to critically reflect on the data.

Moreover, Beatriz left the responsibility for investigating questions that arose to Angel (e.g., "I don't get that. What are they doing that they get more money?"), which positioned her as someone who was capable of critically reasoning about and analyzing data. Then, as Angel used mathematics in ways that made sense to her to investigate those questions, she positioned herself as a someone who was capable of doing and understanding mathematics, and more importantly, of participating in mathematics in ways that were personally meaningful and even transformative. And therein, she enacted a sense of *critical mathematical agency*.

Transformation: Angel's Reflections on Her Experiences

In contrast to her comments about school mathematics at the beginning of this story, Angel recognized that the kind of learning that occurred within the units that were part of this study was different. She commented,

> Look, it's like you are learning about other things. You are learning about things that you *be* in every day, and it's a part of your life. … Because you know something more, it's like adding more to your knowledge. Because you can

remember that. So when we did that [referring to both the “Space at Francis” unit and the unit that investigated income inequities], it was something you could keep with you, it is like information that could involve *you*.

While I do not want to claim that Angel experienced a complete transformation in her views about mathematics, her comments suggest that she was beginning to recognize that mathematics could be consequential in her life in realms beyond the classroom. She was developing understanding of what critical mathematics educators have referred to as the socially transformative power of the discipline (Skovsmose & Valero, 2002). In her own words, drawing on her own experiences, she was starting to recognize the power of participating in mathematics in ways that drew upon her strong sense of agency.

Summary: Angel’s Critical Mathematical Agency

In Table 5.2 I present a summary of the central components of Angel’s critical mathematical agency. The first row of the table highlights the critical world-view and strong sense of justice that Angel brought to Beatriz’s math classroom, and indicates that Angel drew upon these understandings and sensibilities in the various ways that she enacted agency. Next, the table summarizes the role of asserting intentions, of critique, and of improvisation in Angel’s acts of agency.

Table 5.2.

Summary of Angel's Critical Mathematical Agency

ANGEL'S CRITICAL WORLD VIEW AND STRONG SENSE OF JUSTICE ↓ ↓ ↓	
THE ROLE OF	
Asserting Intentions	• Entry point for enacting critical mathematical agency • Created a seemingly natural need to engage in, and begin to develop understanding of, significant mathematics • Served to challenge dominant ways of thinking about the mathematical content within the figured world (e.g., challenged the value of a people to space ratio to evaluate overcrowding)
Understanding	• Supported Angel's efforts to engage in transformative action, even when that understanding was partial
Critique	• Entry point for enacting critical mathematical agency • Prompted Angel to pose personally and socially relevant questions, and then draw upon mathematics to investigate those questions
Improvisation	• Allowed Angel to go beyond the norms of the figured world to generate alternate ways of evaluating the school space that were more useful in investigating her personal intentions

L.J. and Joel

"In the future, I want to get a good education, and I want to make it to the NBA, cause it's fun to play basketball, and they make a lot of money. … and I also want to go to college and get a good education, and then play basketball, cause then they won't call you stupid, cause you have a very good education. Sometimes they think jocks are dumb, but if you got a good education so you know how to do your own signing, and handle your money and stuff, then they won't make a fool of you. -- L.J.

This passion for basketball was something that united L.J. and Joel, two students in Beatriz's second period math class. Both boys played on the school's basketball team, and conversations about upcoming games, new moves, and plans to practice during lunch recess often dominated their interactions as friends. For L.J., a tall and muscular 12 year old who played on up to four recreational and competitive teams at the same time, basketball was a central part of life. L.J. lived with his mother in a neighborhood close to the school, a neighborhood that he "really liked" because of its proximity to several outdoor courts. Both his parents supported his interest and aptitude for basketball, particularly his father who had played competitively throughout the mainland United States and Puerto Rico, where L.J.'s grandparents were originally from. L.J. viewed playing basketball as a vehicle for gaining entrance and financial support to attend a "good high school" and a "good college," and a way to visit places beyond New York City. As a sixth grader, he already had scouts from several high schools tracking his performance in games, and had previously traveled to states like California and Arizona to participate in national tournaments.

L.J.'s good friend Joel also expressed a strong passion for basketball. Though he was shorter and much smaller framed, Joel was a quick and agile player who made a strong contribution to the school team. Joel also participated in a neighborhood basketball league in Queens, where he lived with his mother, her husband, and his two brothers. Joel's family had moved frequently during his school years, back and forth between Manhattan and the outer boroughs, moves which he found difficult because each move meant leaving behind a group of friends and basketball teammates. In spite of these frequent moves, Joel always attended Manhattan schools. His mother traveled into the city each morning, and rather than enroll Joel in a series of neighborhood schools that would change whenever they moved, she chose to bring him with her. Joel's family was originally from Puerto Rico, though he and his parents were born in New York City.

Views About Francis Middle School

Both L.J. and Joel spoke very positively about their school. L.J. relished in the opportunity to play basketball with his friends, and appreciated the non-competitive nature of the games. He felt that Francis was "a good school," with "nice teachers" who enforced reasonable discipline policies. He stated, "when they got to be mean, they be mean, but then you also have a good time with them and you learn a lot of stuff." L.J.'s critique of the school was the ongoing conflict between students at Francis and Raymond Park, a larger middle school down the street. Joel shared this concern, wishing that they "could just get along" and "have no more fights." Joel was especially worried about potential problems with students that were "much bigger" than him and felt that Francis students "should make them some kind of offer" in an effort to end the conflicts.

Other than problems with Raymond Park, and frustration with several teachers who always gave him "a hard time," Joel enjoyed his time at Francis. As he wrote in a personal narrative, "Now I am in a Jr. High School named Francis. I'm connecting with some of my teachers. This is one of the schools I actually like, and I don't want to leave." Academically, Joel had a history of success in school. He was very aware of his own capabilities, but felt that others often underestimated his academic skills, because he did not fit the image of "a smart kid." He commented, "People, they think I am like in the regular skill level, but then when they see how smart I am, they get like, confused, they think I got left back." Despite his academic capabilities, Joel had frequent conflicts with many of the teachers at Francis (other than Beatriz), and as a result, often spent class periods sitting in the office or visiting his advisor[7]. I suspect that it was these discipline issues that led many teachers and students to assume that Joel lacked academic skills.

Participation in Math Class . . . The Beginnings of Agency

Both L.J. and Joel were active participants who had a strong presence in Beatriz's second period class. L.J., who claimed that math was his favorite class (even though he "used to *hate* math"), typically entered the room in a relaxed, but confident strut, smiling and flashing peace signs to his friends, as if walking from the classroom door to his desk was a performance. His comfort level both with Beatriz and his peers was exceedingly clear in the way he moved throughout the classroom, the way he made jokes and playful comments, and in the way he would transform a break in the typical classroom routine into a moment of *improvisation*. For instance, during a class discussion about measuring and calculating the area of different objects in the classroom, when the opportunity arose, L.J. took it upon himself to stand in, or *position himself*, as the teacher.

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[7] Every teacher at Francis also served as an advisor for a group of 15 to 18 students. When students had problems with one of their teachers during the day, they were often sent to their advisor.

[Beatriz draws a picture of the shelf unit at the back of the classroom, otherwise known as the 'Big Ugly Shelf Thing,' on the board, and prepares to label its dimensions].

Beatriz: [to students] And what were the dimensions of the B – U – S – T? [writes B-U-S-T, for "Big Ugly Shelf Thing" on the board as she talks. She doesn't realize that it spells out 'BUST' until she has written it on the board. She begins to laugh, at first quietly, but as students catch on to the humor, she starts to crack up, and has to walk away from the board.]

L.J. [immediately stands up, walks quickly to the front of the room, and raises his arms, as he says] *I* am the new teacher now!

Beatriz: [starts to walk back towards the board]

L.J. No Ms. Math, I took your place. *I 'm* the teacher.

Beatriz: [smiling, asks L.J. to sit down]

Though L.J. repeatedly stated that he enjoyed mathematics, it was not a subject that came easily to him. He described frustration and confusion whenever he had to execute paper and pencil calculations such as long division, which he "never understood." He talked about math as numbers and operations that were "hard" because you had to "think it all in your head." Before coming to Francis, L.J. received special education services in both mathematics and reading. At Francis, consistent with the school's philosophy of emphasizing children's strengths, L.J. was to receive extra support in his regular classroom through a special education inclusion model. However, due to funding cuts at the beginning of the year, the school was forced to eliminate the positions of several paraprofessionals (i.e. instructional aides) who assisted with special education services. As a result, the only extra support that L.J. and other special education students in Beatriz's classroom received was from Beatriz, and at times, myself.

Like L.J., Joel had a visible, and well-respected position in Beatriz's classroom. He was well liked by his peers, and seen by many, the boys in particular, as a desirable math partner. Joel enjoyed participating in class discussions, particularly when he felt he could make an important mathematical contribution. For instance, during an introductory lesson on fractions, Joel was adamant about sharing his thinking with the class. As Beatriz was still discussing students' strategies for the first problem, Joel called out, "Can I do the back one?" meaning that he wanted to share his strategy for solving the more challenging homework problem on the back of the page. Less then a minute later, when Beatriz had not responded to his request, he pleaded, "Can I share *my* way?" followed by "Ughh!" when his request was not immediately acknowledged. During the next several minutes, he continued to assert his desire to participate through comments such as "I know! I know!" and "Please Ms. Font!" He ultimately decided that rather than waiting for an invitation to share his thinking, he would introduce his ideas into the public space of the group discussion himself, by calling out whatever he happened to know about the particular fractions that the class was discussing. For example, when one student stated that the answer to a problem was 3/4, Joel claimed "and 3/4 is 6/8!" which though true, had nothing to do with the student's solution to the problem.

In contrast to his active participation in Beatriz's classroom, like L.J., Joel spoke of how he used to "hate math." According to Joel, things changed "when I came to this school and people were saying that I was good at math, so I just started getting into it." Interestingly, the two teachers in the school that Joel had positive relationships with, Beatriz and his advisor, both taught mathematics, which Beatriz felt supported his positive feelings towards the subject.

I include this discussion of Joel's and L.J.'s ways of participating in Beatriz's classroom because I think that it lays a foundation for a discussion of their critical mathematical agency. In contrast to Angel, whose presence in the classroom was quiet, and more subtle, and whose expressions of critical mathematical agency began not with her active, assertive participation in class discussions, but with the sense of critique and fairness that she brought with her to the classroom and with her desire to assert personal intentions, for L.J. and Joel, what seemed to provide an entry point for their critical mathematical agency were their patterns of participating in classroom interactions. That is, Joel and L.J.'s strong and visible presence in the classroom, and the ways in which they *authored* and *positioned* themselves in relation to their peers, and to the subject matter, made a difference in terms of their agency. In authoring a place for himself in the group discussion, and then positioning himself as a capable mathematical thinker, Joel was enacting a sense of agency. Likewise, when L.J. positioned himself in a place of authority in the classroom (e.g., "*I* am the new teacher now!"), this improvised action, though short-lived, drew on his sense of agency as a person. In the section that follows, I examine how L.J. and Joel positioned themselves and authored roles for themselves as they enacted a sense of critical mathematical agency through their participation in Beatriz's classroom.

Critical Mathematical Agency Through Authoring and Positioning

During the first several weeks of the study, it became clear to me that Joel had authored a space for himself in the classroom as 'the explainer.' He relished in the opportunity to clarify an idea for one of his classmates, or to justify his thinking to a questioning teacher or peer. His explanations often transformed into performances that included animated facial expressions and voice inflections, and sometimes even movement. Moreover, as he was justifying an idea, he would often spontaneously generate a related example or hypothetical situation, just to make his point clearer. For instance, during the second week of the "Space at Francis Middle School" unit, Beatriz and her students discussed how to figure the area of a space that had mixed number dimensions, such as the school gym that measured 19 meters by 23 1/2 meters. While most students agreed that multiplying 19 times 23 was insufficient, because "you have to add the halves" (from the 23 1/2), they struggled to articulate exactly *what* to do with the 'halves', and moreover, *how* the half square meter spaces related to the total area, which Beatriz had represented with a rectangular figure on the board. At one point in the conversation, L.J. even posed a question challenging the *need* to consider the "halves." In response to these questions posed by both his peers and his teacher, Joel stood up and presented an improvised demonstration aimed at helping the class visualize the impact of failing to consider the 19 half square meters.

> Beatriz: now what do I need to do? [A student has just finished multiplying 19 times 23]

Carlos: You have the multiply the half, 19 times the half.

Beatriz: Why?

Alfonso: Cause if you don't multiply by the half, it doesn't make sense.

Beatriz: But why?

Alfonso: It's like tiles you have a lot of half tiles. So you have 19 halves of tiles.

Beatriz: Good, we have 19 halves of tiles, not just one.

Student: but why?

……..

L.J.: Yeah, what if you didn't multiply any of the halves?

Beatriz: He said what if you didn't multiply any of the halves?

Katrina: It wouldn't work cause you have to add the halves.

Beatriz: Why? [pushing the students' thinking] It's just a little half, it's just a little piece.

Student: no

Joel: No, cause look, it's like 1/4 of the room. [Motions with his arm, tracing an imaginary line in the air that cuts off a portion of the room. Let's say this is your room, without the halves it's like …. just this part. You don't have this part. You don't have all the 19 halves. [Stands up, moves towards the back wall of the classroom, and standing parallel to the wall, extends both arms out to his sides to show with his arms how disregarding the half square meter spaces would be equivalent to disregarding a section of the classroom space all along the back wall of the room.]

Beatriz: [asks Joel to stay there and hold up his arms, and asks Ms. Green to join him] So you would have figured out all that space up to where Joel and Ms. Green are, and you would have missed all that space back there.

In the previous interaction, Joel drew on a sense of mathematical agency as he positioned himself as a knowledgeable, capable doer of mathematics. Through his words and his actions, he communicated a sense of 'mathematical authority' to the class, an authority that was taken seriously by his teacher and incorporated into the subsequent class discussion. The way that Beatriz validated Joel's improvised explanation, and then involved Ms. Green in extending the physical model that he was attempting to create, is important to note. Joel's act of positioning himself as a capable mathematical thinker did not occur in isolation, but as part of his participation in a figured world that encouraged and supported students in explaining their reasoning and justifying their ideas. That is, Beatriz's ways of interacting with students positioned them as mathematically competent, and in this interaction, Joel claimed that position[8]. In addition, once Joel assumed the position of an "explainer" of mathematical ideas, Beatriz supported him by validating his model and re-voicing his explanation to the class.

[8] See Chapter 4 for a more complete discussion of this aspect of Beatriz's pedagogy.

Joel is the not the only student who enacted agency in this interaction. For instance, L.J. also drew upon a sense of agency as he positioned himself both as someone who had the right to conceptually understand the mathematical concepts, and as someone capable of posing challenging questions for rest of the class to consider. As he asked, “Yeah, but what if you didn’t multiply any of the halves?” L.J. simultaneously asserted his own right to understand the meaning of calculating the area of the 19 by 23 1/2 meter gym space, and at the same time, as he voiced this question to his peers during a whole group discussion, his right to push his classmates’ thinking further.

Later in this conversation, L.J. again positioned himself in this agentive role of ‘question poser.’ He recalled a classroom at the school that he had measured, and noted that its dimensions included 3/4 of a meter, rather than 1/2 a meter, and asked, “Yeah, but what about the other room? We did [measured] a room, and there was no halves, it was 3/4. It was like 8 and then 3/4. What about that?” Beatriz replied, “You are talking about what happens if it is 3/4 instead of 1/2? Let’s do an example. Like Mr. Bronson’s room, it’s 10 meters by 6 3/4. So what should I do?” Through this question, L.J. not only asserted his right, and need, to understand a related example, but he also positioned himself in the role of setting tasks for the class to investigate. In posing L.J.’s question as an interesting problem for all students to consider, Beatriz supported his agentive act of positioning. Her pedagogy invited students to pose their own problems, and whenever possible, incorporated students’ tasks into the flow of the lesson.

Several days later, during a class discussion of how to figure the area of a rectangular space with two mixed number dimensions (i.e., a classroom that measured 8

1/2 by 6 1/2 meters), L.J. again, through the questions he posed, asserted both his own right to understand the mathematics, along with his capacity to challenge the thinking of his classmates. He claimed that he understood how in this particular example, figuring the area of a space with ‘two fractions’ in the dimensions involved calculating the area of four separate spaces (see Figure 5.3), but at the same time wondered whether or not that would be true for all rectangular spaces with two mixed number dimensions? Would you always end up with four partial areas?

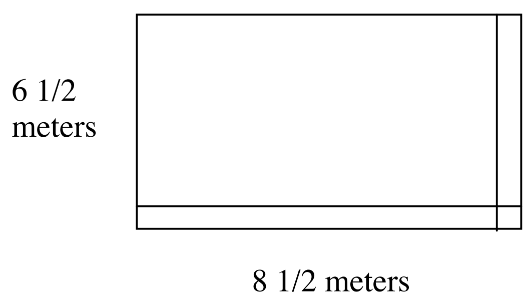

Figure 5.3. Figuring the area of a rectangular space with two mixed number dimensions

In each of these instances, L.J.’s questions expressed a genuine desire, or even ‘right,’ to understand the mathematical concepts, as well as a capacity to challenge his classmates’ thinking and set tasks for them to investigate. At a minimum, Joel’s and L.J.’s capacity to pose questions and grapple with challenging concepts demonstrates a sort of mathematical agency that one would hope to find in all mathematics classrooms. One could argue that their participation, though agentive, does not evidence critical mathematical agency, for there is no critique, no engagement in personally or socially transformative action. Taken in isolation, I would agree.

Yet consistent with the understanding that agency is always situated, it is essential to consider L.J.'s and Joel's actions within the larger context of this unit. For both students, accurately figuring the area of school spaces, in particular the gym, was consequential in realms beyond this particular lesson, and as subsequent examples will demonstrate, beyond the classroom. This was not just a review of a homework problem, but an investigation of a part of their school – the gym – that was extremely important to them, and that later became the focus of a small group project that critiqued the limitations of the space. Considering how L.J.'s and Joel's acts of positioning themselves as capable mathematical thinkers might be connected to their personal intentions alludes to the potentially animating role that these intentions might play.

Supporting Acts of Positioning and Authoring with Mathematical Understanding

While L.J. and Joel definitely positioned themselves as capable mathematical thinkers, in terms of agency, it is also important to consider how they drew upon mathematical understanding to support those positions. As Bruner (1996) argued, "since agency implies not only the capacity for initiating, but also for completing our acts, it also implies skill or know-how" (p. 36). To what extent were Joel's and L.J.'s agentive acts of positioning themselves as 'explainers' or 'question posers' supported by mathematical understanding?

To begin, both L.J. and Joel demonstrated increasingly rigorous mathematical understanding as the "Space at Francis Middle School" unit progressed. They were capable of measuring and then constructing and labeling a floor plan of irregular spaces such as the boys' restroom. They were quite proficient, L.J. in particular, with moving flexibly between exact measurements given in meters and centimeters to approximations of these measurements based on fractions of a meter (e.g., the area of a classroom object that measured 92 cm by 1 m and 84 cm was estimated by calculating the area of a 1 m by 1/34 m space). When the need the arose, because of a school district request, to convert all measurements from metric to standard units, both L.J. and Joel efficiently converted their group's measurements using a strategy generated by Joel. Given the fact that 1 inch is approximately equal to 2.5 centimeters, Joel successively doubled both quantities to find a series of equivalent ratios: 2 in ≈ 5 cm, 4 in ≈ 10 cm, and then 40 in ≈ 100 cm, or 1 m, and then operated on one of the ratios (40 in ≈ 1 m) to find that 1/2 m ≈ 20 in, 1/4 m ≈ 10 in, etc. L.J. and Joel then used these relationships to convert all of their hallway, classroom, and gym measurements into feet and inches.

Both students also demonstrated a strong understanding of the meaning of area, and of strategies for figuring the area of spaces with mixed number dimensions. For instance, the following example (see Figure 5.4) illustrates how L.J. responded to the task of calculating the area of a Francis classroom that measured 5 1/4 meters by 8 1/2 meters.

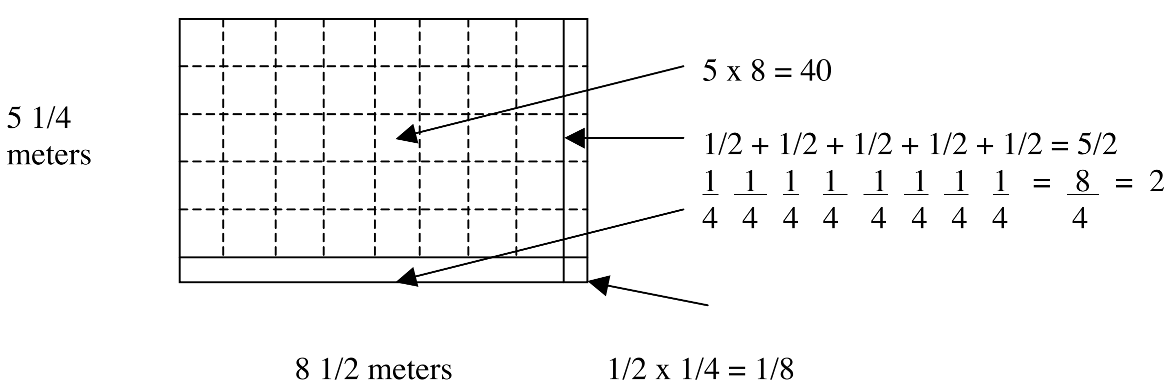

Figure 5.4. L.J.'s Strategy for Calculating the Area of a Classroom

L.J. explained his strategy stating,

> What I did was times 5 by 8 and I got 40, and then I put 1/4, 8 times, and I got 8/4. Then I knew 4 times 2 is 8, so that makes 2 wholes for my answer. And then I put 1/2 5 times and got 5/2. Then I said that makes 2 wholes, and another 1/2. And then for the little square, I knew it was 1/2 by 1/4, and I knew that 1/2 of 1/4 was 1/8. For my answer total I got 40 + 2 + 2 1/2 + 1/8, so I got 44 1/2 and 1/8.

Though L.J. was not able to add 1/2 and 1/8 when he represented his final answer (which he recognized), his strategy and accompanying explanation demonstrated a strong understanding of area and of operations with benchmark fractions. He recognized that he could measure area in square units (evidenced by the dashed lines he drew to represent whole square meters and 1/2 square meters), and was able to explain how each operation that he completed related to a specific portion of the total space. Given that at the beginning of the unit, most students, including L.J., had very limited previous experience with both area and fractions – as evidenced by their performance on initial tasks and Beatriz's pre-assessment of their understanding - this understanding is significant.

Even more significant is that L.J.'s response to this problem reflected a combination of strategies that he had improvised, or invented, at different points in the unit, and then continued to use as tools for future action. For instance, early in the unit L.J. began to use lines to partition the rectangular space into whole and partial square meters; later he developed a strategy for multiplying fractions by whole numbers that involved repeated addition of unit fractions. Beatriz fostered L.J.'s capacity to improvise strategies by (a) encouraging him to solve problems in ways that made sense to him, (b) questioning him in ways that supported his thinking (Jacobs & Ambrose, 2003), and (c) facilitating small and large group discussions that allowed L.J. to draw on other students' thinking to support him in constructing his own problem solving strategies. Each of L.J.'s improvisations became tools that he employed to enhance his capacity to participate in meaningful ways in the figured world of his classroom. He used these tools to tackle increasingly complicated problems, and eventually, to justify his arguments about the school space. That is, his improvised strategies not only supported his mathematical understanding, but to the extent that they were used "again and again, they can become tools of agency or self-control and change" (Holland et al., 1998, p. 40).

On a regular basis, Joel demonstrated equally impressive understanding. He often assumed the role of assisting his classmates in revising their work, particularly when it involved figuring out how to add or multiply a set of fractions. As previously demonstrated, Joel was always eager to share and defend his thinking with the class, even on the several occasions when that reasoning was not supported by a viable way of understanding the problem. For instance, early in the "Space at Francis Middle School Unit," Beatriz presented students with a series of problems to help them think mathematically about the issue of overcrowding at the school. In one problem, students were given the dimensions of two dance halls (gymnasiums) and the number of students that would attend a dance in each hall. Students then had the task of determining which of two dances would be *less* crowded.

Joel was adamant as he tried to convince his classmates that Dance A, which was attended by 240 students and took place in a gym that measured 20 meters by 20 meters, would be *less* crowded that Dance B, which was attended by 80 students and occurred in a smaller hall that measured 10 meters by 20 meters. Earlier in the discussion, L.J. had reasoned that since the number of students in Dance A was more than half of the total available space (240 students, 400 square meters of space) while the number of students at Dance B was less than half of the available space (80 students, 200 square meters of space), Dance A would be *more* crowded. Even as several other students supported L.J.'s line of reasoning, Joel remained convinced that according to his way of thinking about the situation, Dance A would be *less* crowded. He argued,

> I picked dance A [as the less crowded dance]. Because this is how I did it. First I shaded in 240 [square meters] into dance A and I counted what was left, and 160 was left, so I added that together, and it was equal to 400. And then I did that for dance B, and they have 120 space left, so I think dance A gets more room, cause like -- Here's an example. Like say there is like 2 people, and the maximum [total] space is 5. They don't get more room than like [some other group], and they are not even half yet [the amount of people is not even half the amount of space]. And they don't' get more room than like people that are 45, and it's 80 boxes [square meters]. Cause they [the 45 people] have like more room.

I include this example for two reasons, not among them to point out gaps in Joel's understanding. First, although Joel employed a subtractive strategy (e.g., comparing the number of square meters left in each dance hall after assigning each person one square meter of space) that was not sufficient to consider questions of crowding, at least in the way crowding is traditionally understood (a ratio of the space to the people), his strategy did demonstrate mathematical understanding, and given his interpretation of the problem, actually made sense. For instance, in response to Joel's reasoning, Beatriz asked students to think again about Jessica's strategy of passing out squares of space to each person, and then determining in which dance did a person receive more space, or about L.J.'s strategy of considering whether the students attending each dance would fill up more or less than half the total space. Her intent was to draw on other students' strategies (positioning them as mathematically capable) to support Joel's understanding of the problem. Yet Joel resisted her attempt to shift his thinking. He replied, "Ms Font. Wait a minute. Cause I was about to say something. Cause not everybody is going to dance at the same time."

According to Joel's way of conceptualizing the problem, if a large enough portion of the students in Dance A were crowded along the walls, not dancing, then those students who were dancing may actually have had more room to dance, given that their dance hall was 400 versus 200 square meters. In fact, Joel's line of reasoning adds a layer of complexity to the problem. Students might investigate whether different percentages of students dancing at any given time would impact, or not, the overall crowding. This leads to my second reason for including this example. Students' acts of positioning themselves in ways that claim mathematical authority, even those that do not seem to be supported by strong mathematical understanding, deserve notice, because students' arguments and justifications, particularly those that are as persistent as Joel's, are based on some kind of logic and understanding. Joel improvised an alternate conceptualization of the problem, one that he defended vehemently in an effort to assert his competence as a mathematical thinker. His understanding of the situation, though different from that which the problem intended to elicit, served to challenge the constraints of the original task, and how the task was interpreted by other actors in the figured world. In this way, Joel's act of presenting and validating an alternate way of analyzing the problem was an act of critical mathematical agency.

Yet L.J. and Joel drew upon more than mathematical understanding as they positioned themselves in agentive ways in relation to their peers, their teacher, and the subject matter. As the class investigations became more personal for L.J. and Joel, and as their actions became increasingly aimed at affecting change, engaging in critique or participating in transformative action, they began to draw on personal experiences and intentions to support their arguments. For instance, approximately three weeks into the "Space at Francis Middle School" unit, a group of students investigating the school's narrow hallways discovered, with the help of Ms. Green, that the district had a building code that included a minimum acceptable width for school hallways. As the building code listed all measurements in standard units, students' first task was to convert their own hallway width measurements from meters to feet and inches. When students discovered that only one of the school's eleven hallways was 'legal' according to the district regulations, they were understandably concerned. Around this time, the school principal, Ms. Williams, was preparing to attend a meeting with district administrators, and Ms. Green mentioned that students might provide her with information to share with the district. What follows is L.J.'s response to Ms. Green's suggestion.

Ms. Green: What could Ms. Williams say to the district about the hallways?

L.J. Well what ***I*** would say to the district, it that it's **too small**. The hallways is too small and they are not even 5 feet 6 inches [the 'legal' width according to district building codes]. And I can't even lay across it! And I may have to take a picture of me laying across it, so they see how I couldn't fit.

In responding to Ms. Green, L.J. almost literally positioned himself as an authority on the school's hallways as he emphatically stated what *he* would say to district administrators, rather than what he would tell *Ms. Williams* to convey. To establish this authority, he drew not only on numerical data ("They are not even 5 feet 6 inches"), but also on his personal relationship to the narrow hallways ("And I can't even lay across it!"). Later in the conversation, Joel also drew upon his own experiences trying to navigate the narrow hallways as he suggested a way to communicate the urgency of the problem to the district.

I was thinking that like the last project, we did a commercial, and I was thinking for like my group to do another commercial about the school. Like how our school is overcrowded, we could have kids changing classes, and have a whole bunch of kids coming through the hallways and we are all stuffed.

As L.J. and Joel thought of ways to convince district administrators about the problems caused by their school's narrow hallways, they drew to some extent on mathematical information, but more so on their personal experiences (kids changing classes and 'stuffing' the hallways) and relationships to the situation. In other words, for L.J. and Joel, the argument about the school's narrow hallways was as much personal, as it was mathematical. As L.J. and Joel drew upon personal experiences and intentions to enact a sense of critical mathematical agency, how did their experiences and intentions interact with their engagement in mathematics? That is, like Angel, did their personal intentions push them to explore the situation mathematically? And if so, how did the figured world of their mathematics classroom support them in asserting those intentions? Did their experiences and intentions challenge mathematical constructs typically used to evaluate issues such as crowding? Or did their intentions in some way conflict with their engagement with the subject matter? In the following section, I present a final story of L.J. and Joel's participation in this unit to investigate these questions.

Critical Mathematical Agency Through Asserting Intentions

For L.J. and Joel, the most pertinent aspect of the school space was not navigating small bathrooms, or feeling cramped inside classrooms that were clearly too small for 28 to 30 middle school students, but the presence of 'poles' in the gymnasium. They were both frustrated by the numerous large columns that jutted from floor to ceiling in the middle of their gym court. Although the columns were heavily padded to minimize injury inflicted upon the countless students (and teachers) who accidentally ran into them on a daily basis, they still posed significant challenges during basketball games, obstructing players' views, blocking passes, and creating frustrating obstacles as team members tried to make their way from one end of the court to the other.

While the class discourse about space tended to gravitate towards the legal status of hallways and the space per student ratio in different classrooms (topics that conveniently involved lots of rich mathematics), L.J. and Joel continually interjected their own interests into class discussions. For instance, after listening to several classmates report on results of speaking about the project with parents and district administrators, L.J. interrupted and asked, “Did you all talk about the gym to the board of education, like how we don‘t get to use it and stuff like that? Like how we can’t have [basketball] tournaments because of the poles in the gym?”

In response to L.J.’s question, the principal, Ms. Williams, who was visiting Ms. Font’s classroom at the time to talk with students about the outcome of the meeting, stated that they chose not to talk about the school’s inability to host basketball tournaments because “there is nothing [the district] can do about the poles in the gym.” She proceeded to discuss with students the historical context of the school. When the building was constructed over 100 years ago, steel beams were not available as support structures, and thus in order to support the building’s tile roof, large weight bearing columns were installed all over the 5th floor, which was intended to be ‘an attic.’ When the 5th floor was converted into a school, it was constructed around these poles, which now lie in middle of classrooms, in the office, and throughout the gym.

In spite of the principal’s decisive statement that the poles were not removable (thereby imposing an undeniable constraint that negated the possibility of any action that would result in change), Joel proceeded to ask, “Well how long do you think it would take for them to take down the poles and put in steel [beams] in the ceiling?” Persistent in his desire to envision a different school space, Joel disregarded the reality of the constraint. Once again, Ms. Williams confirmed that the poles were not removable. Several days later when students had the opportunity to choose a particular issue to investigate in greater depth, L.J. and Joel decided to focus their analysis on 'the poles in the gym.' What follows are several excerpts of a discussion with the students during their initial stages of planning.

> Erin: Do you have any ideas?
>
> Joel: Yeah, the poles in the gym, Joel Productions. [describing plan for the video] We would be like walking in the hallway to the gym, and we would be like, man these hallways are small. So then when we finally get into the gym, we are like, ah'ight let's play some basketball. And we are playing and we keep getting hit by the poles cause they are all over. And then we are like, we should move these poles and then Ms. Williams [the principal] comes and she is like, yes we should. And we are gonna **try**. And she is talking about how dangerous the poles are and people could really get hurt.

Striking in Joel's opening comment is how he drew upon the task of "making a video" (a task he chose) to imagine a world not bound by the constraints he encountered in his classroom and school. In planning the video, Joel imagined a school space that was different in the physical sense (i.e., a space where removing the poles in the gym was possible), but also different in terms of how individuals in power responded to students' personal and collective concerns. When he and L.J. previously expressed their concerns about the gym to Ms. Williams, she replied, "There is nothing [the district] can do about the poles in the gym." In the imagined world of his video skit, Ms. Williams supported the boys' desire to remove the poles, stating, "Yes we should. And we are going to **try**."

As Joel imagined himself in a world in which he asserted his intentions and participated in transformative action, and then proceeded to try to create that world, to act it out, to take it seriously and make it consequential (Holland et al., 1998, p. 280), he enacted a sense of agency. However, in the midst of this conversation, I didn't recognize Joel's agentive act of "figuring a world" as a way of circumventing the constraints that limited his ability to assert his personal intentions. Instead, I focused on making sure he and his fellow group members defined their project in a way that would allow for significant mathematical activity. With this in mind, the conversation continued.

> Erin: Okay, here's my question, you need to figure out how math might help you to think about this situation?
>
> Joel: You could be like man there's a lot of poles in there, let's count them.
>
> L.J.: You could ask how many there are in this whole building and we could estimate, even though we don't really know the answer, we could be like estimating and saying yeah, there's about that many.
>
> Roberto: We could ask each other, what is the area of the gym?

Joel: I know, we could start the video in the math class, first Ms. Font is talking about math, so it could be about math, right, and she is like alright, everybody be quiet so we can be dismissed. And then she is like, L.J.'s table can leave.

L.J.: Yeah she could be talking about the poles, about how many poles, and we be like, yeah let's go really see about the poles. Let's go see about the poles so we can see if we can play basketball. So we go in the gym and we see the poles, and then we count them, and then we play basketball, and we bump into the poles, and then at the end, one of us gets hurt, and we can say like yeah Ms. Font was right.

Erin: So your argument is that the poles are in the way so you can get hurt? Are you also going to deal with the poles in the classrooms, and how they block kids' view of the board?

Ms. Font: I already told them they should not be concentrating on the details of the video they should be concentrating on some facts.

Erin: You have to DO some math. It's a good idea to start in math class, but you also have to use some math. What about looking at how your paths get blocked? Or how if you are standing in a certain place, the pole blocks your line of sight? Or what about looking at the size of the gym, and how many students from three different schools use it?

L.J.: Didn't I just say we were going to talk about how many poles are in the school?

Roberto: We can say like what's the area of this – and we are going to be in the gym doing the area on the floor [figuring out the area on a piece of paper], and then I am going to be like, let's play some basketball.

> Erin: How does the area help you make your argument? Think about it, if I'm on the school board how does that convince me that the gym really has some problems?
>
> L.J.: (frustrated) We are gonna do Joel's idea.

This excerpt, as frustrating and humbling as it is, is important to include because it illuminates the complexity of this ongoing tension between students' agency in negotiating the curriculum, and the desire to keep mathematics at or near the center of students' activity, a challenge that is intensified when teachers and students negotiate the curriculum on an ongoing basis, when activities that provide fertile ground for mathematics are not predetermined, but rather emerge as students "pursue non-mathematical objectives" (Stevens, 2000, p. 116).

From the beginning of the conversation, L.J. and Joel were clear about their objective: to communicate to others, through video, problems posed by the poles in the gymnasium. Their description of these problems was quite general ("getting hit by the poles cause they are all over"), and the mathematics that emerged inconsequential (counting or estimating the total number of poles). Yet their objective was clear, and their enthusiasm about the project, until I entered the picture and began pushing them on the mathematics, contagious. (They had already outlined a seven-scene script for the video, and were beginning to write the dialogue.)

To the students' credit, they were active negotiators in this interaction, intent at all times on moving their own goals for the project forward, while at the same time trying to frame those intentions as ones that involved 'doing' and 'using' mathematics, in other words, ones that fit within the constraints imposed upon them. The context of the activity placed them in a potentially contradictory, or "disjoint situation," one that positioned them both as students of mathematics responsible to the demands of the discipline and as young citizens encouraged to investigate and act upon issues of personal importance (Holland et al., 1998, p. 16). The tension arose because their idea of 'doing mathematics' as part of the project (e.g., counting poles, starting the video in a math class, video taping themselves working on some area problems), that is, their concept of what mathematical agency might have meant in this context, differed from my own. I was intent [as was Beatriz] on ensuring that significant mathematics did emerge; we felt a genuine need to address certain mathematical concepts through this unit, and this was a constraint that we imposed upon students' activity. Thus, I focused my questions on helping them to 'pull out' the mathematical content while at the same time structuring that content into their experience, not apart from it. In this task, I was unsuccessful. Though I alluded to, but failed to make explicit, possible mathematical connections (e.g., poles obstructing view and motion, resulting in reduced options for passing angles, and diminished visibility, especially when compared to options available for players with no poles in their gym), L.J. actively resisted my suggestions, restating "Didn't I just say we were going to talk about how many poles are in the school?"

Did they not 'see' the mathematical possibilities? Or were they merely choosing to reassert their own intentions and subvert my attempts to transform *their* project into something that it was not? Perhaps, their actions were aimed at consciously resisting the constraints (i.e., this project must include significant mathematics) imposed upon their activity by the figured world. What might it have meant to structure mathematical content into Joel and L.J.'s experience with the gym space, and in particular, their interest in the danger of the poles? What kind of mathematical understanding would such an investigation have required? These are not trivial questions, nor do they have clear answers. Was it even reasonable to think that rich mathematics might have emerged from the boys' intention to investigate the poles? As I reflect on that question, I am reminded of Steven's (2000) argument that we must "allow projects to be real enough in their organization to allow mathematical problems to emerge or not" (p. 120). Maybe mathematics was not the best tool to understand or evaluate this situation? Perhaps a visual depiction of the space, such as the one L.J. and Joel were advocating through their video, would have presented a more convincing argument. Or is it just that my own beliefs and assumptions about what counts as 'valid' mathematics, and what it means to know and do mathematics, limited the opportunities for mathematics that I saw in the activity of these two students (Civil, 1998; González, Andrade, Civil & Moll, 2001; Kahn & Civil, 2001; Stevens, 2000)?

Ultimately, L.J., Joel and Roberto did script and produce a 3-minute video about the school gym that incorporated many of their initial ideas. In addition, to meet Beatriz's (and my own) expectation that they also engage in meaningful mathematics, they reluctantly agreed to figure out the area of the gym, and analyzed how that space might be shared among 12 players in a half-court game. (Due to the poles in the middle of the court, half-court games were more feasible.) In this way, they improvised a project that allowed them to address their own goals and intentions, as well as the expectations and constraints that guided their activity in the figured world. I suspect their interactions with Beatriz and myself supported them in generating this improvised solution that not only addressed, but also challenged, the constraints of the project. We both felt strongly that students should have opportunities to investigate issues of personal importance, and thus encouraged them to investigate the gym space, even though we suspected that the project would lack mathematical rigor.

In fact, the mathematics was minimally interesting, and more importantly, it did little to support their central argument about obstructions created by the poles. In fact, when L.J. and Joel presented their video investigation to their classmates at the end of the 5-week project, they quickly glossed over the measurements, areas, and space per player ratios they had calculated, indicating the minimal importance of these 'more mathematical arguments' for the central focus of their discussion: danger imposed by the poles. In a similar manner, when L.J. wrote a letter about their project to Paula Rundberg, the district superintendent, his primary emphasis was the safety issue presented by the lockers and poles. His letter read:

> My name is L.J. Mansfield. I am a 6th grade student from Francis Middle School. I am writing to you because I am concerned about the defect in our gym. Me and my group did a project about the gym. We discovered there is no space in the gym and that the poles and the lockers are dangerous because if we were playing a half court game with the whole school there, we would only get 2 square meters each.

So when we are playing, we can hit the poles and lockers going after the ball, since there is no space to play.

What I think the district should do is remove the poles and the lockers, and put something to hold up the gym roof so we would not need the poles, and we can just remove the lockers. I think if we did that, it would be much safer for the students. Thank you for listening to my letter, and I hope you consider my ideas.

Transformation: L.J. and Joel's Reflections on Their Experiences

During the final weeks of the "Space at Francis Middle School" unit, and several weeks later at the end of the semester, I spoke at length with L.J. and Joel about their experiences in this project. The mathematical understanding they developed was evident, and their capacity to position themselves as capable mathematical thinkers, clear. But how did they make sense of the experience? To begin, both L.J. and Joel commented that participating in projects in Beatriz's classroom was significantly different from their previous school mathematics experiences. Joel stated:

Joel: Yeah [I liked the "Space at Francis Middle School" unit]. Like I never done anything like this. Since I started doing it, I started getting used to it, and now it's easy.

Erin: What do you mean?

Joel: Like finding the area of a room, and then using it for useful information. I never did anything like that, in my other school.

Joel went on to say that he enjoyed learning "useful information", such as how to find the area of different spaces because, "It's important, and I am not just doing it for an assignment. We are learning about things that are important. And we are not just learning it to learn it. It's important, and it's about our school." L.J. shared similar comments, contrasting the "regular math" that he used to study with the "math to help our school" that the projects involved. He claimed

> Before, we were just doing regular math, now we are doing math about our school and other schools, and to help our school. ... And [referring to the first project, "Seeing the World through Numbers"] I think it's important that we are doing math to learn about other people's lives, and not always learn[ing] about the people-that-got-a-lot-of-money's lives.

Moreover, in contrast to their comments about the pertinence of mathematics to their lives at the beginning of the study (both students initially felt that math was important, but were unable to provide specific example of how mathematics might be useful, other than in helping them to manage their money and avoid "getting cheated"), as they reflected on their experiences in the "Space at Francis Middle School" unit, L.J. and Joel spoke about the power of mathematics to "prove a point." Joel argued that presenting an argument without some kind of data is futile, because "we wouldn't have no proof, no information to tell them what we have, and how little the school is." Similarly, L.J. stated that the mathematics was central to their argument about overcrowding at the school, because

> Without math you wouldn't [be able to] show how overcrowded it is, and how small it is, so they wouldn't agree with us if they didn't know. Like [math] shows that their smallest room is like our biggest room. And you have to measure and stuff to show that. You can't just guess. You can't just say we have small rooms, cause they [the district] don't know that. They didn't see our rooms yet.

While my intent is not to claim that L.J. and Joel transformed their ideas about mathematics through their participation in Beatriz's classroom, I do want to argue that their experiences pushed them to broaden their conception of mathematics, particularly its relationship to their lives. Like Angel, they were beginning to recognize that participation in mathematics could result in knowledge and understandings that might (a) be important to their lives, (b) be important to their school, or (c) be useful in supporting, or 'proving' their arguments. In other words, they were starting to see that mathematics could support them in investigating and acting transformatively upon their world.

Summary of L.J.'s and Joel's Critical Mathematical Agency

In Table 5.5 I present a summary of the central components of L.J.'s and Joel's critical mathematical agency. The first row of the table highlights the active, engaged, and public nature their participation in Beatriz's math classroom, and indicates that L.J. and Joel drew upon these preferred ways of interacting as they enacted critical mathematical agency. Next, the table summarizes the roles of positioning and authoring, asserting intentions, and improvisation in their agentive actions.

Table 5.5.

Summary of L.J.'s. and Joel's Critical Mathematical Agency

<table>
<tr><td colspan="2">PATTERNS OF ACTIVE PARTICIPATION IN CLASSROOM INTERACTIONS
↓ ↓ ↓</td></tr>
<tr><td colspan="2">The Role Of …</td></tr>
<tr><td>Positioning and Authoring</td><td>• Entry point for enacting critical mathematical agency
• Supported students in claiming and demonstrating mathematical understanding and authority
• Involved justifying ideas, explaining concepts, posing tasks, and challenging dominant ways of thinking about a problem
• Students drew upon mathematical understanding and personal experience to defend and justify their positions</td></tr>
<tr><td>Asserting Intentions</td><td>• Resulted in tension between students' goals and interests and the norms, expectations, and constraints of the figured world (e.g., the need to engage in significant mathematics)</td></tr>
<tr><td>Improvisation</td><td>• Supported the generation of problem solving strategies which facilitated understanding and served as tools for future action
• Allowed students to address the tension with their own intentions and the constraints imposed by particular tasks, and by the expectation of the figured world
• Supported students in challenging the constraints previously mentioned</td></tr>
</table>

Naisha

> My favorite subject is history. Cause I love to learn about *my* history. About *my* history. I am studying Madame CJ Walker now. She was the first black millionaire, cause she made hair products. So I am learning more about her and I am going to do a project about her. … And maybe [in school] we could spend more time, do more research, more projects on black history month. Like some people just do one black history month, but maybe you could do more than that, like at least spend two months on black history, so students could do [learn] more.

The last story that I present in this chapter is that of Naisha, a spirited, opinionated, and very self-assured eleven year-old African-American student in Beatriz's second period math class. As the opening quote illustrates, Naisha had a strong passion for learning about *her* history. She critiqued the limited attention her school awarded to African-American history, and to compensate, asked her mother to enroll her in an after school program run by a local church where she heard she would able to do "lots of black history projects." Her mother and grandmother, with whom she lived as an only child, supported and encouraged her interest. In fact, Naisha shared stories of family gatherings where female relatives would retell the story of her great aunt's courageous participation in the civil rights movement in the segregated south. Her conviction and willingness to protest, even when that protest resulted in negative consequences, such as time in jail, were sources of great pride for Naisha and her family.

Other than her critique of the school's social studies curriculum, and occasional complaints about teachers that "talked too much" and left no time for students to "actually work," Naisha spoke positively about her experiences at Francis. When asked about her teachers, she commented

> I think what's good about all my teachers, is that if you get in a lot of trouble they give you the right to speak your mind. And to express your feelings, and I think that is very important in teaching. And so I like my teachers for that.

The right to "speak her mind" was valued by Naisha, and it was a right that she asserted on a regular basis, and that became an entry point for her sense of agency. Comments such as, "Well what *I* think," or "What *I* think we should do," or "Well, *I* have something to say" were an integral part of her participation in classroom discussions and activities. Frequently, 'speaking her mind' also involved raising a critique and/or suggesting some kind of transformative action. For instance, when students were sharing their concerns about the community, Naisha argued

> We need to help keep the community clean, and like some people in my community -- I live in the projects -- and some people don't like to do that. But we try to keep it as clean as possible. No litter. My little cousin, if he throws something on the ground, I pick it up and throw it in the garbage. Cause I don't like to litter. But everybody just does their own thing. We don't really get together as a group, and like talk, [about] what should we do in the community. We just do our own thing. I think we should talk.

Evident in Naisha's comments are a number of significant ideas. First, she asserted an issue that was important to her, then critiqued her community's lack of response to that issue, and finally, advocated for the potential power of collective action. The sense that something *can* and *should* be done in response to unfair or undesirable conditions, and that that 'something' is more effective when it includes collective, versus individual action, is one that Naisha expressed repeatedly. She spoke of wanting to protest unfair standardized testing policies, the need to complain to the city about the lack of fire escapes in all government funded housing, and as the stories that follow demonstrate, the need to press the school district for more space. Implicit in each of her pleas for action was also a call for that action to be a collective effort. She argued

> Now *we* [referring to African Americans and other racial minorities] really have the ability to express ourselves. *We* won't just take anything that white people say. *We* would overpower them. Because we have more strength than we had back in the day.

What happened to Naisha's sensibility for critique and transformative action, in other words, to her sense of critical agency, when she entered Beatriz's mathematics classroom? To what extent was she able to draw upon this critical agency to enhance her participation in a classroom aimed at "Teaching and Learning Mathematics for Social Justice"? And how did her sense of agency interact with her engagement with the discipline? The stories that follow investigate these questions.

Naisha's Participation in Mathematics

Despite her small frame and young appearance, Naisha had a strong, visible presence in Beatriz's second period class. This presence was not so much due to regular participation or frequent demonstration of mathematical competence, for although Naisha had moments of active participation, she chose those moments carefully, and was not a student who dominated class discussions. Her strong presence was more a reflection of how she carried herself, how she entered the room with confidence, how every comment she made communicated a sense of self-assurance and authority. Naisha was articulate and passionate when she spoke, and her peers recognized and respected this about her.

When I spoke with Naisha about her views about mathematics at the beginning of the study, she responded, "what do I think of math -- you mean, numbers?" Numerical operations dominated Naisha's ideas about the discipline, which she described as something that previously, she "couldn't stand," but that more recently (in 4th or 5th grade), she "started to like." She said she felt good about mathematics whenever she "got all of [the] answers right." For instance, once she learned a procedure for solving division problems, math "started to be fun" for Naisha, because she knew she could solve the problems and arrive at the correct answers. Yet Naisha still felt some apprehension towards the subject. In particular, she expressed strong dislike for tedious paper and pencil calculations. She stated, "[the teacher] would give us like six numbers to multiply, and it would take at least 30 minutes to do one problem. I didn't want to do it."

Critical Mathematical Agency Through Critique and Transformation: The "Seeing the World Through Numbers" Unit

Naisha's sensibility towards critique and transformative action (including transformational resistance) became very apparent during the first unit of study: "Seeing our World Through Numbers." When students began to investigate disaggregated income data, Naisha's participation was especially agentive. In fact, examining this data became an entry point for her to enact a sense of critical agency. She raised critiques in small and large group discussions, and resisted other students' attempts to justify the income inequities that they discovered. For Naisha, speaking from her position as an

African-American young woman who in the future planned to enter the work force, critiquing income disparities was a personal matter. For instance, as her group reviewed income statistics they collected from the Census database, Naisha interjected

> Can I say something? I think that is unfair. … I think it is unfair as I am looking at this sheet right here, I am looking at doctors [income]. I am thinking about being a doctor and **I AM** going to be a doctor when I grow up and I see that a white male he makes a lot more money and a black male he makes a whole lot lesser than that. And then the Hispanic make, he makes a little bit, a little bit of what the white male [does]. And the white female … My point is that if I look in this chart I see that, I notice that the black male and the black female make a lot less than the white male and the white female.

Naisha spoke repeatedly of her desire to become a doctor, and realizing that both male and female African-American physicians earned less than their white counterparts was disturbing to her, in a very personal way. She voiced her critique, which later became the basis for transformative action aimed at resisting, or 'speaking out against' these injustices (story to follow).

Beatriz supported Naisha's capacity to raise critiques in multiple ways. For instance, the way that she (a) posed the task to students (e.g., "So here is a chart that shows some of the annual salaries that you wanted to know about. Pick a job to look at. Figure out what the chart shows, and record your comments. Maybe you want to compare some things, maybe you want to look at gender, or maybe race, or maybe you want to do some other kind of analysis.); (b) the kinds of questions that she asked as students were examining the data (e.g., "What kind of trends do you notice? … That's interesting, why do you think that is? …. What could you compare that to?); and (c) the way that she positioned students as capable "researchers" capable of analyzing data and reporting results (e.g., "You've done some research here, some analysis. So let's make a poster about what you have learned and we can put it in the hallway), served to facilitate Naisha's capacity to enact critical agency.

Several days later, during a lunchtime conversation about the project, students began to form conjectures about the income disparities they noticed. When several of the boys in the class attempted to justify the significantly higher average salary of male (NBA) as compared to female (WNBA) professional basketball players based on men's superior capability as players and/or the widespread popularity of the NBA league, Naisha challenged their thinking by openly naming the salary differences as 'sexist'.

Erin: So what did you notice about the basketball salaries?

Naisha: The races mean something.

Devon: Yeah, I think it depends on what they are capable of.

Vellez: I that maybe basketball players, men basketball players, they are a lot more popular, and too the NBA has been around for a very long time.

Naisha: I think it is a kind of 'a way of the sexes' and women and men. How men seem to always make more than women, like in the WNBA. And women make less. And it would take them at least a couple of years to reach what the men make.

Cristina: 54 years [a group of students previously figured this out]

Naisha: Also, I think the reason why they kind of make it that way is that they think that women can't reach the same goals as men. So that is why they did it that way. … I think that is kind of racist because --

Vellez: ---you mean sexist

Naisha: Yeah, sexist, because a woman can do the same things as a man. They can do the jumping in the air and putting the ball in the hoop, and stuff like that. Slam dunks and all that stuff like that. So it's sexist. That's it.

Naisha resisted her classmates' efforts to 'explain away' the disparities, by arguing instead that income equalities were the result of discriminatory beliefs about women ("they think women can't do the same things as men") which resulted in deliberate actions on the part of those in power ("so that is why they did it that way") that served to structure inequality into the way salaries were determined. In raising this critique, in openly naming income disparities as racist (doctors' salaries) and sexist (basketball players' salaries), she enacted a sense of critical agency. Later in the same conversation, Naisha proceeded to link these inequities with historical forces of discrimination and oppression, such as slavery, as the following excerpt illustrates.

Joel: Well it is not fair that the white people get paid more than any other, they probably think they should get paid more, because of their race.

Naisha: I think it has something to do with the slavery time, how they [whites] used to be higher.

Devon: Yeah, exactly.

Naisha: Yeah, so they think that now, they could still do that.

Devon: We are just a day away from slavery really. You never know what could happen.

…

Naisha: Well I think if slavery would start again, the black people wouldn't listen to the owners. … We have the power to speak out. We got the power to speak out.

With her comment, "I think it has something to do with the slavery time, how they [whites] used to be higher," Naisha shifted the direction of the conversation among her peers. She pushed them to consider disparities in the salaries earned by whites and blacks not as an isolated inequity, but as "the logical consequence of the institutional [and historical] structures of our society" (Frankenstein, 1995, p. 180). Naisha followed her critique with a call for resistance and action, stating, "we have the power to speak out."

In fact, 'speaking out' was a recurring theme for Naisha, and one that was valued and encouraged in the figured world of her mathematics classroom. Later in unit, Beatriz presented students with the opportunity to investigate a certain aspect of the "Seeing the World through Numbers" project in greater depth, and to share the results of their investigations with others. She presented the task as follows

> You learned a lot of things in here about the world, or our community. We can't just keep that information here … cause then when good is it? Some people have seen the posters you put up in the hall, but some people don't ever make their way

> down this hall. So we need to get our message our farther, to the school, or the community. Today you are going to get into groups and decide what you want to do, what you want to investigate and what you want to do about it.

Beatriz proceeded to share several examples of "what other people have done" to educate the broader community about issues of social importance. These examples included (a) a series of "Truth Campaign" commercials aimed at educating youth about the dangers of smoking, (b) a children's book, The House that Crack Built (Taylor, 1992), which illustrated the devastating impact of drug trafficking on poor communities; and (c) a collection of graphs published in the New York Times that presented an overview of those who lost their lives in the World Trade Center disaster.

As she shared each example, Beatriz emphasized, "That was *their* issue. They did some research, and they wanted to do something about it, and this is how *they* decided to communicate their message." She then supported students in brainstorming ways that *they* might "educate others" about *their* issues. Students generated a multitude of ideas, including: distributing flyers and posters, interviewing community members, planning a protest, designing artwork to display in public spaces (e.g., street murals, sidewalk art), and creating videos, commercials, skits, and newspaper articles, among others. In this way, by sharing examples of what others have done, and by helping students to think about what they could do, Beatriz supported students, Naisha included, in moving from critique to engaging in transformative action.

Naisha embraced this opportunity to "speak out." In particular, she wanted to tell others about the disturbing gender and race-based disparities in income that she and her classmates had discovered. She decided to script and direct a talk show, which she eventually videotaped and shared with other students during an all school meeting. In this way, producing the skit was an act of improvisation that supported her in resisting a particular set of injustices that characterized her world. Reflecting on her reasons for wanting to produce the talk show skit, she said,

> We thought that if a person was to go on a [talk] show protesting like the white versus black racism, it would be kind of a good idea. So we thought that we would take it into our own hands, and like go ahead of ourselves, and do it.

"Taking it into her own hands," was exactly was Naisha did. She organized a group of five of her classmates, and together, they created a 10-minute talk show where 'guests' shared information about and reactions to the "reality that whites make more than blacks." Naisha, the self-appointed host of the talk show, was clearly the leader of the group, telling her classmates when to enter, how they should answer questions, and what information they needed to collect or calculate. For instance, one 'guest' on the show argued, "it's not right. See a black women gets paid 16 dollars an hour if she a teacher, but a white woman, she gets paid 20 dollars an hour, and they doing the same job."

Naisha's ability to view the world with a critical mind set, her capacity to see larger social, historical, and political forces reflected in the data she investigated, and her commitment to take responsibility for transformative resistance and action "into her own hands" were remarkable examples of critical agency. Yet what about the mathematics? Comparing and contrasting income data was a fairly trivial use of the discipline. Naisha and other members of her small group did participate in more interesting problem solving activities when they calculated hourly earnings based on yearly salaries, or when they figured out how long a particular individual would have to work to purchase a given item (calculations they did in preparation for their skit), but Naisha's role in the problem solving was fairly passive. She often relied on teachers or other students to direct her mathematical activity, posing questions such as "Are we supposed to divide?" or "Do I just keep adding this number up until I get to the answer?" When she found herself working in a small group with a student who was very strong mathematically, like Joel, she would often position herself in the role of 'recording answers' or 'sharing strategies with the class,' but rarely in the role of 'problem solver.' That is, she would often sit back as another student worked through a problem mentally or on paper, and when that student reached a solution, would ask questions such as, "Okay, so the answer is $1.24?"

This is not to imply that Naisha assumed a passive role in her small group. She was a strong leader, who as the previous example indicated, orchestrated the activity of her peers. She realized what data needed to be collected, and had ideas about how to strategically use that data to argue her point of view. While she did not demonstrate rigorous problem solving capabilities, she definitely knew how to draw upon mathematical (i.e., statistical) information to level critiques, and to support her efforts towards transformative action, both components of a *critical mathematical agency*.

Critical Mathematical Agency Through Critique and Transformative Action: The "Space at Francis Middle School" Unit

Given Naisha's heightened critical awareness of power relationships and discrimination, and her astute understanding of the need to 'speak out' against dominant ideologies, the fact that Naisha enacted a sense of critical agency as she investigated disaggregated income data is not surprising. Yet how did her sense of critical agency manifest itself within a much different project, a project that did not intentionally or explicitly address issues of race, class, and gender, but instead investigated a local issue at the school? The following section addresses this question through several stories of Naisha's participation in the "Space at Francis Middle School" unit.

To begin, unlike some students who seemed to view overcrowding at the school as a local, isolated problem, Naisha spoke critically and explicitly about the connections between their particular situation at Francis and broader educational inequalities. Specifically, as she compared the space at her own school to that at Longmore, Naisha was invited to think about the basis for the discrepancies she noticed. During an after school conversation about the project with her friend Cristina, Naisha argued that Longmore had one of the most selective acceptance policies in the district, and as a result, the school was populated by "Whites! And a few Hispanics." Cristina added, "At Longmore, there are more white people than anything at that school, I mean a few, a few Hispanics. I am not saying that no black people can get in, but …." For Naisha and Cristina, the overrepresentation of white students in certain district middle schools was not inconsequential, but directly related to (a) special educational programs, such as gifted and talented; (b) better school facilities, such as wider hallways and larger classrooms; and (c) modern resources, such as the laptop computers given to students at Longmore. Naisha summarized the disparities between the schools as she stated, "It's an upper class thing, for the white kids. They get a better education. Better everything."

Significant here is the interplay and symbiotic relationship between Naisha's general critique of the inequity in the world and her specific critique of the unfairness of the overcrowding at her school. The critical mind set about the world that Naisha brought with her to this project, and the ease with which she talked about issues of race, gender, and class discrimination, helped her to critically interpret her own experience, and name it as an instance of institutionalized racism/classism. In addition, the mathematical investigation of space that Naisha conducted at the local level helped her to see how broader inequalities played out in local circumstances. In a sense, Naisha's participation allowed her to write her own particular story into a larger critique about injustice and inequality. Similar to Esperanza, the indigenous Mexican woman in Behar's work (1995), Naisha plotted herself into a narrative and a system that has been historically oppressive to her, in an effort to resist and transform that oppression. In doing so, she made a larger story of inequality and oppression her own, repositioning herself, rewriting her own script, or as Behar (1995) wrote, "cutting a new window for [herself] in these narratives" (p. 315).

Unfortunately, Naisha's critical interpretation of overcrowding at her school never made its way into the public space of classroom discussions. Instead, Naisha shared these insights with me, and her friend Cristina, in an after school conversation on one of the final days of the unit. On several occasions, students did raise questions aimed at understanding the discrepancies between their own school and Longmore middle school, such as: "*Why* do students at Longmore have computers when we don't?" or "*Why* are their hallways wider than ours?" But for various reasons, primarily the need to continue with the flow of the lesson (which when these questions were raised focused on sharing multiple strategies for calculating area), these questions were never taken up by Beatriz or other students in the classroom. Instead, the dominant discourse focused on investigating, describing, and proving the disparities between Francis and Longmore middle schools, not on critically reflecting on possible reasons for the differences.

As the unit progressed, Naisha strove to share her critique of the Francis school space with others. In doing so, she enacted a sense of critical mathematical agency. For instance, when the principal offered several students the opportunity to speak about the project at a school advisory council meeting (attended by parents, district administrators and community members), Naisha volunteered to prepare a speech. She drew on her critical mindset and her strong sense of self as she crafted a speech that incorporated mathematical arguments to illustrate the problem of overcrowding. What follows is Naisha's presentation of the speech to the class.

> Naisha: Good evening, my name is Naisha Watson. I am a sixth grader at Francis Middle School, and I am going to talk about overcrowding at our school. Our math class has been comparing our school to Longmore. We have noticed as a class, that we have no space for kids to sit, and that some kids that can sit, can't see the chalkboard. The reason why they can't see is because of all the poles. We have counted all the poles in Francis. And in total there are 25 poles in the whole school. As you can see on our graph, there are poles in many of the classrooms. (Refers to a large floor plan of the school that she and several classmates created for this presentation.) All the Ps in the circles stand for all the poles in the

classrooms. The board of education building code says that the hallways must be 5 feet and 8 inches. As you can see from our graph, there is only one hallway that is 5 feet and 8 inches. (Motions to the floor plan)

Student: It's that one right there, 5 feet and 10 inches.

Naisha: All the other hallways that are red on the graph do not meet the board of education building code. So all the ones that are red, these, all of these that are red, they do not meet it. And another thing that makes our hallways smaller is all the lockers and the radiators. The board of education building code has another building code that the classrooms have to be at least 750 sq feet for 30 children. As you can see on the graph, only 3 classrooms are big enough, the rest of the classrooms that are orange on the graph are smaller than 750 sq feet. In our school we have 213 kids, if there was to be a fire in our school there would be a hazard to get through our narrow halls. So as a school we think we should have less students or more space. By Naisha Watson. Thank you very much for listening to our talk.

Clear in Naisha's speech is her use of measurements, numerical data and diagrams to argue her point of view. For Naisha, mathematics became a tool to investigate an important issue in her life, and to make sense of that issue in ways that other disciplines might not have allowed. In this way, mathematics supported her agency. It is important to note that the speech Naisha read in the transcript above was not the original version. On the day of the meeting, Naisha met with Ms. Williams, the school principal, to review her presentation. In Naisha's initial version, she had repeated references to Longmore middle school, and drew upon comparable data from the two schools to 'convince' the advisory council members of the urgency and 'unfairness' of Francis's overcrowding problem. However, Ms. Williams instructed her to remove these comparisons, because the issue "wasn't really about Longmore," but about their own school, and said that the repeated references to another school might "be confusing" for the council members. When Naisha insisted that the issue for her class *was* about Longmore and the disparate conditions between the two schools, Ms. Williams conceded and allowed her to retain one mention of the comparison.

When I spoke with the principal later in the day, she reframed this concern slightly, explaining that repeated mention of a neighboring (and better resourced) school would only add tension to an already adversarial relationship. (Several years earlier, Francis lobbied the district for the same space – the 4th floor of the building – that was later awarded to Longmore.) Ms. Williams also worried that statements such as "Longmore Middle School has this much …. and we only have this much ……" might be understood by district administrators as unfounded complaints rather than valid, well supported arguments.

I include this contextual information for two reasons. First, Naisha's negotiation of the content of her speech offers a thought provoking example of the tension between students' acts of agency and the structures and pressures that function to constrain that agency (Baez, 2000; Holland et al., 1998; Varenne & McDermott, 1999). Even within a curriculum driven by students asserting their intentions and investigating real issues and conditions in their lives, those 'real conditions' (e.g., the language of the district building code and the adversarial relationship between Francis and Longmore) function to shape and constrain students' expressions of agency.

Yet this relationship between agency and context/structure is not a one-way interaction, but rather a dynamic interplay between the two forces. Just as human agency is limited and molded by contexts and structures, so too is agency capable of resisting, subverting, and transforming those structures (Baez, 2000; Holland et al., 1998; Pérez, 1999; Behar, 1995). What is problematic in this interaction is that those 'real conditions' were not open for critique. The space was not created, or rather, Beatriz and Ms. Williams and I did not create the space for students to pose questions such as: Why does the district rely on standard units of measure? Is this an arbitrary choice or one based on practical, political, financial, or other concerns? And in terms of the content of the code, is 750 square feet of classroom space per 30 students even a reasonable amount? How does this compare to class size regulations in other districts? Related to the tension between the two schools, why isn't it acceptable to base our arguments on a critical comparison of space allocation? Why are we not able to speak more openly about racism and classism in our educational system? In failing to create the space for such critical questions, and to acknowledge that structures and codes can be vulnerable and open to redefinition, we missed a potentially rich opportunity for students' agencies to push against and challenge the constraints of the figured worlds in which they participate.

Second, in reflecting on the evolution of Naisha's speech, I want to argue that her intended use of mathematics as a tool for comparative analysis might have been more powerful than using mathematics to describe a single set of conditions. That is, while evaluating the school space against a district building code provided a point of reference, it failed to illuminate how Francis School compared to other district middle schools. Was it possible that many schools housed in older buildings were in violation of district regulations? If so, then the space crisis at Francis was no more urgent than overcrowding at many other city schools. Or was it the case that Francis was not like the other schools in terms of space, as Ms. Williams herself repeatedly stated, in which case the students' concerns would merit (from the district's perspective) increased attention? The point here is that the structures and constraints that forced Naisha to alter her speech may have also functioned to make the speech less powerful.

Supporting Critical Mathematical Agency with Mathematical Understanding

Naisha clearly drew upon mathematical ideas and the results of rigorous problem solving activities (e.g., measuring and calculating the area of all classrooms in the school) as she crafted her speech, yet in many instances, the ideas and the data she utilized were products of the collective effort and understanding of a number of students. To what extent was her action supported by her own mathematical understanding? As in the previous project, Naisha demonstrated a developing, but tentative understanding of the mathematical concepts that were central to this unit of study (e.g., area, operations with fractions and mixed numbers, ratios).

For example, during the second week of the unit, Naisha embarked on the task of figuring the area of a hypothetical room that measured 3 1/2 meters by 6 1/4 meters (see Figure 5.6). She began by sketching the room, and drawing in lines that partitioned the larger rectangle into whole and partial square meter spaces. When I asked her about her strategy, she explained

> Naisha: What I did, is the ones [square meters] on the bottom I didn't count, and the ones [square meters] on the side I didn't count, I counted all the other ones. And it was equal to 18 [square meters]. And then the ones on the side right here [points to right side of figure] they're all 1/4ths. And so I added all the 1/4s and it equaled 4/4. (She counted the rectangular space in the corner that measured 1/2 meter by 1/4 meter as another 1/4 square meter space). And then I added all of these [points to 1/2 square meter spaces along the bottom of the diagram] and two of them equaled a whole. Each of them equaled a half, but I added two of them at the same time and it equaled a whole. So I added one whole on to the 18, and then another whole, and then another, and then I got 21. And then I added the 4/4 and I got 21 and 4/4 square meters.

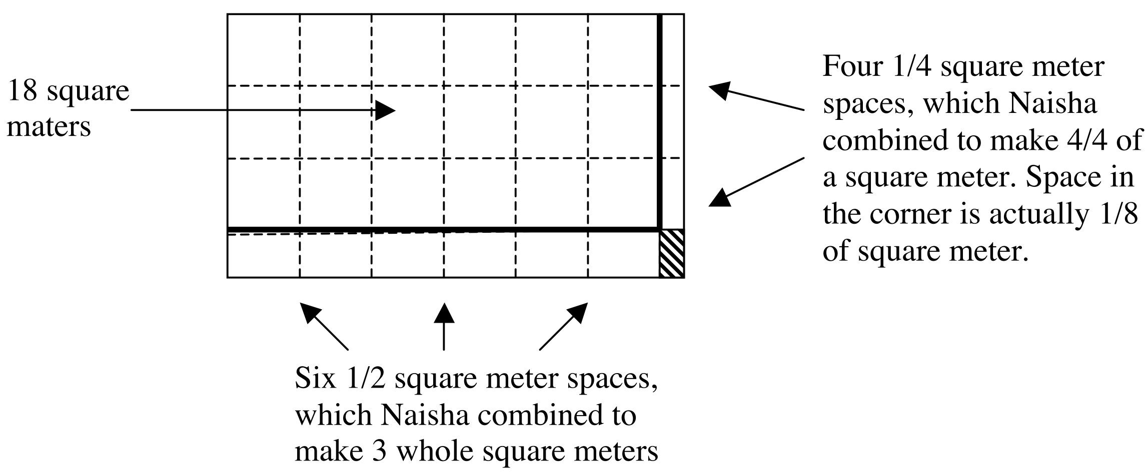

Figure 5.6. Naisha's strategy for calculating the area of a 3 1/2 meter by 6 1/4 meter rectangular space.

Naisha's approach to calculating the area of this space clearly demonstrated a developing understanding of area. Considering the students' lack of prior experience with area, coupled with the fact that Naisha was attempting to solve a rather difficult problem fairly early in the unit, this understanding was significant. However, when I questioned Naisha about the small rectangular area in the corner of the diagram, which she had incorrectly identified as a 1/4 square meter space, it became clear that her understanding was incomplete.

Erin: Now how many 1/4 square meters do you have?

Naisha: 4. 1, 2, 3, 4. (counting corner space as 1/4 square meter)

Erin: Hmm, This little one in the corner, it doesn't look the same size. I'm not sure that it is a 4^{th}. (pointing to corner space)

Naisha: No, it's a 1/4. Cause they're all 4ths. (points to four rectangular spaces along the right side of the diagram)

Erin: How do you know they are all 4ths?

Naisha: Cause that's how I do it.

Erin: What do you mean?

Naisha: Cause if it's 6 1/4 then I put in the 4ths and there are 4/4, so that makes 21 and 4/4, or really that makes 22.

Erin: So you are thinking if those are all 1/4, then the area is 22.

Naisha: Yeah

Erin: And you told me this side along the bottom is 6 1/4, but how long is the other side?

Naisha: 3 1/2.

Erin: So, if it's 3 1/2, how could there be four 1/4 square meters?

Naisha: That doesn't matter, I just put the 4ths and there are four 4ths.

Erin: Okay.

While there were clearly gaps in Naisha's understanding (e.g., she was not able to make sense of the small rectangular space in the corner of the diagram), I want to emphasize that at the beginning of the unit, she demonstrated a developing *conceptual* understanding of the meaning of area, and of ways to calculate the area of a space that encompassed both whole and partial square units. Interestingly, as the unit progressed, Naisha continued to misinterpret, or even disregard the "small corner space" that was created in rectangular rooms with two mixed number dimensions[9]. For instance, during the third week of the unit, Naisha drew the following sketch to accompany her calculation of the area of a Francis classroom that measured 9 1/2 meters by 4 1/2 meters (see Figure 5.7).

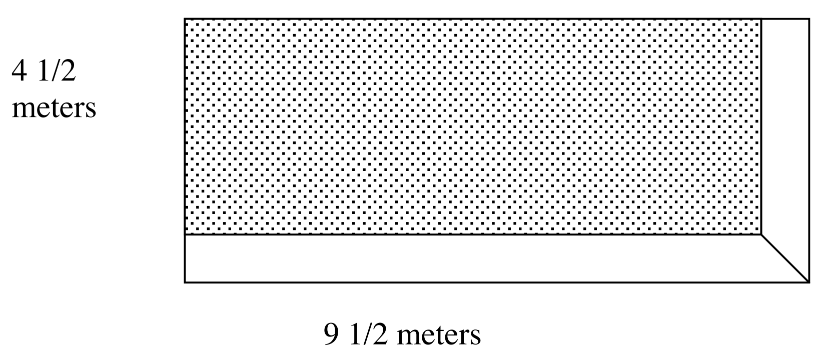

Figure 5.7. Naisha's sketch of the area of a Francis classroom

[9] Her disregard for the "small corner space" may have been an artifact of instruction, which I further discuss in Chapter 6.

Clear in the drawing, and her accompanying calculations, is her complete disregard for the 1/2 by 1/2 meter space formed in the lower right hand corner of the diagram. Naisha conceptualized the room as a large 4 by 9 meter space, and a collection of 1/2 square meter spaces along the bottom and right sides of the figure. Her drawing reflected that conceptualization. As the unit progressed, Naisha began to abandon these sketches and diagrams as she developed increasingly procedural ways of calculating area. For example, when I asked her to explain how a series of calculations she completed to figure the area of a 10 1/4 by 8 1/2 meter space related to a visual diagram of the space, she commented, "I don't do it like that. I don't know … I don't do it like that. I just multiply the numbers." When I persisted and asked, "You told me you multiplied 10 times 1/2, and that gave you 5 square meters. Can you show me where that space is on this picture of the room?" she again replied, "No, I just multiplied the half times ten, that's how you do it."

These inconsistencies in Naisha's developing understanding, and the way that her understanding seemed to become more procedural over time, as the conceptual difficultly of the unit increased, and more importantly, as the pressure to compile a significant amount of data to share at the district meeting intensified, alludes to a significant tension that arose between (a) supporting students with the mathematical content that they needed to move forward in the unit (e.g., to present at the district meeting); while at the same time (b) ensuring that students developed conceptual understanding of that content. This tension, and how it was addressed by Beatriz and her students, is further discussed in Chapter 6.

Transformation: Naisha's Reflections on the Projects

When I spoke with Naisha towards the end of the semester, and asked her to reflect on her experiences in Beatriz's math class, like L.J. and Joel, she felt that the class was different, and in important ways, from her previous school math experiences. She explained that in other classes, she studied material, but never had the opportunity to 'do anything' with what she learned. In contrast, in Beatriz's class, "we did the project, and then we did something about it- we did something with it." This opportunity to respond in some way to what she learned was important to Naisha, particularly given the nature of that learning. That is, discovering injustices that personally impact you, your family and your peers, without the opportunity to actively respond to or resist that injustice, even if those actions are just small steps like scripting and performing a talk show for the school, can leave students feeling helpless and further victimized, rather than empowered (Gutstein, 2003a). Regarding the issue of overcrowding at her school. Naisha felt that "speaking out" as a way of resisting the inequities she and her classmates discovered was not only necessary, but potentially effective. She argued

> I think it's good [that we talked to the district], because if you keep talking to them [the district] then they will probably listen, and you will get on their nerves and maybe then they will want to give us more space, or let us be in a different building with more space, [space] that is lawful. And we won't have to be in an attic, like we are. … I think people will listen to us, cause we do deserve a better school and more space.

Moreover, because she drew upon mathematics to support her transformative action, Naisha had the opportunity to challenge her ideas about the discipline. She stated,

> I learned that you can do a lot of things with math. Not just fractions, multiplication and division. Things in your community and stuff like that. … Because without the math, then, we wouldn't have the area of the school, and we wouldn't really know. We would have to say that we estimated it, that we think that it's that amount. And the meeting wouldn't have been as powerful as it was.

Like Angel, L.J. and Joel, she was beginning to recognize what Skovsmose and Valero (2002) referred to as the transformative power of mathematics, the sense that mathematical ideas and practices can be resources for action in her own life, and in society.

Summary of Naisha's Critical Mathematical Agency

In Table 5.8 I present a summary of the central components of Naisha's critical mathematical agency. The first row of the table highlights the strong sense of justice, and the desire to "speak her mind" that Naisha brought to her participation in Beatriz's math classroom, and indicates that Naisha drew upon this tendency to "speak out" as she enacted critical mathematical agency. Next, the table summarizes the roles of critique, transformative action and resistance, and improvisation in her agentive actions.

Table 5.8.

Summary of Naisha's Critical Mathematical Agency

STRONG SENSE OF JUSTICE and DESIRE TO "SPEAK HER MIND" ↓ ↓ ↓	
The Role Of …	
Critique	• Entry point for enacting critical mathematical agency • Supported Naisha in critically interpreting her own experience, and allowed her to link local instances of injustice (e.g., inequities in income and in school space) to broader, and sometimes historical forces of discrimination and oppression • Pushed Naisha to investigate statistical data
Transformative Action and/or Resistance	• Provided a way of responding to, challenging, or acting upon, the inequities that she noticed • Supported by the results of mathematical investigations (e.g., measurements, area calculations, hourly salaries), including mathematical comparisons of data
Improvisation	• Served as a mechanism that facilitated Naisha's ability to resist the injustices that characterized her world (e.g., improvising a skit that resisted disparities in income between whites and blacks)

Conclusion

In this chapter, I presented the stories of four case study students from Beatriz's math classroom. I chose these students because together, their stories provided a rich and varied sense of *how* students enacted critical mathematical agency through their participation in a teaching mathematics for social justice oriented classroom, and how their acts of agency interacted with their engagement with the discipline of mathematics.

Several tensions arose across these (and other) students' stories. In the next chapter, Chapter 6, I describe these tensions and discuss how they were addressed by Beatriz and her students.

CHAPTER 6: TENSIONS

As the stories in the previous chapter demonstrate, when students enact a sense of critical mathematical agency, even within a context aimed at fostering and supporting that agency, their actions are not without constraint. Students' efforts to assert their intentions push against the norms and expectations of the figured worlds (the classroom, the school) in which they participate. Their plans for transformative action must be negotiated in response to the history of the figured world, the resources it makes available, and its ways of positioning actors, such as students, teachers, and administrators, in relation to one another, and in relation to the established activity of the figured world. In other words, students' efforts to enact agency and teachers' efforts to support their actions necessarily create a multitude of difficult, disjoint spaces, spaces inherently full of tension, spaces which Holland et al. (1998) referred to as "contested space, a space of struggle" (p. 282).

I want to argue that *understanding* students' agency demands paying attention to these "contested spaces." To begin, these spaces offer insight into the *interplay* between human agency and contextual constraints. Focusing on this interplay helps avoid a potentially dichotomous, and I would argue counterproductive, analysis which either (a) overemphasizes social constraints, minimizing opportunities for human agency, or (b) "neglects the cultural and social contexts that inform the 'playing field' to which human action is directed and by which it is shaped" (Holland et al., 1998, p. 278; see also Baez, 2000; Varenne & McDermott, 1998). Moreover, focusing on these spaces of struggle helps to illuminate how students' participation in figured worlds functions to shape, limit and/or facilitate their agency, and at the same time, how participants' agentive acts impact, and even transform the figured worlds that they inhabit.

What kinds of "contested spaces," or spaces of tension, arose for Beatriz and her students in this study? How did they experience these 'spaces' and more importantly, how did they address the tensions that inevitably surfaced? In this chapter, I discuss two tensions that arose both across the stories of Angel, Joel, L.J. and Naisha (see Chapter 5), and across the stories of other case study students in Beatriz's classroom. The first "space of struggle" focuses on the ongoing tension between (a) students' efforts to assert their intentions, and (b) the values and expectations of the figured world -- a mathematics classroom -- that framed their activity. The second "contested space" centers on the tension that arose when students' acts of asserting intentions created a need for resources – in this case, particular mathematical skills and understandings – which were (a) not already part of the figured world, and (b) beyond the realm of what some students were ready, or able to understand[1]. Given the central role of *asserting intentions* in each of these tensions, before analyzing these "spaces of struggle", I want to explicitly discuss acts of asserting intentions as a powerful form of agency.

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[1] Chapter 7 discusses a final tension that cut across students' stories. This tension highlights the fact that as students engaged in critique and planned for transformative action, they were acting from positions of limited power within the various figured worlds that they inhabited -- that of a student, that of a child, that of a low income individual of color – and that they were aware of, and yet hopeful about, those limitations.

Asserting Intentions as a Form of Agency

A central theme that emerged from this study is that one of the primary ways that students enacted critical mathematical agency was through asserting personal and collective intentions. While previous theoretical and empirical work has drawn attention to acts of *authoring and positioning* (Holland et al., 1998; Holland, 2003; Martin, 1999), and *critique and transformational resistance* (Gutstein, 2002, 2003b; Pruyn, 1999) as enactments of agency, the stories of Angel, L.J. and Joel, and their classmates suggest that the notion of *asserting intentions* is another important lens through which one can view students' agentive actions.

Skovsmose (1994) has argued repeatedly for the role of students' intentions in learning mathematics, stating that:

> A condition for productive teaching-learning process is that a situation is established where students are given opportunities to investigate reasons and goals for suggested teaching-learning processes, and by doing so, to accentuate their own intentions and to incorporate some of them as part of the learning process. (p. 184)

Skovsmose claimed that intention is imperative to learning, because in order for learning to be a purposeful and potentially empowering act for the student, instead of one that is overly directed by the teacher, students' intentions must be integrated into the learning process. I argue that intention is also closely linked to agency. As students assert intentions in an effort to impact their experiences in a particular figured world (e.g., the classroom, the community), their expressions of intention become enactments of agency.

The point here is not only to highlight asserting intentions as a powerful form of agency, but also to examine how acts of asserting intention related to, and in some instances animated, the forms of agency previously mentioned. Asserting intentions, much like drawing on a sense of justice and fairness, seemed to be a point of entry for students, a way for them to begin enacting agency within and beyond the classroom. That is, many students began by expressing needs, wants, and concerns about their school space (e.g., Angel's concern about how long she had to wait to use the restroom, L.J. and Joel's desire to play basketball in a gym unobstructed by poles). In many cases, these acts of asserting intention, when supported and developed over time, led students to enact other forms of critical mathematical agency, such as critique or transformational resistance. For instance, for Angel, what began as expressing a personal concern resulted in a critical analysis of the girls' restroom aimed affecting some kind of change. Similarly, L.J. and Joel's initial desire to play basketball in a "normal" gym pushed them to produce a video aimed at convincing others to remove the poles because of the safety hazards they presented. In other words, what began as asserting intentions developed into transformative action upon the situation.

Asserting intentions was such a predominant feature of students' acts of agency, one that seemed to flow effortlessly for students, that it became a useful way of making sense of their activity. While previous research (Gustein, 2002, 2003b) has documented that students' strong of sense of justice can support their agentive actions, this study points to the power of *asserting intentions* as another entry point to agency, particularly in contexts where students have opportunities to negotiate and shape the curriculum, and to bring their personal stories, needs, and desires into the classroom. Unfortunately, these opportunities are rarely created in other classrooms, because of tightly structured curricular materials, the pressure of high-stakes standardized assessment (McNeil & Valenzuela, 2000), and district mandates that dictate a particular instructional scope and sequence, among other reasons. The stories of Beatriz's students argue for the need, or at least the potential benefit in terms of fostering agency, of creating opportunities for students to interject their needs, wants, and experiences into the curriculum, and therein, assert their intentions.

Yet, as previously mentioned, students' acts of asserting intention necessarily create spaces of tension. In the sections that follow, I analyze the tensions that arose and the ways that Beatriz and her students addressed those tensions.

Tension #1: Students' Intentions within he Figured World of the Math Classroom

As the stories in Chapter 5 demonstrate, when students attempted to interject their personal intentions into the mathematics curriculum, tensions sometimes emerged between (a) students' interests, purposes and goals, and (b) the norms and values of the figured world – Beatriz's mathematics classroom – in which they participated. I want to emphasize that while many of these norms and values were negotiated and renegotiated by Beatriz and her students as participants in, and creators of, the figured world, others were imposed upon their activity by larger institutional goals, structures, and policies.

For instance, over the course of the study, Beatriz and her students negotiated the expectation that participating in mathematics could be a personally and socially meaningful endeavor. Discussions around questions such as "What's your issue? What do you want to know about?" and "How can math help you to make that argument? How can you use math to find out more about the situation?" played pivotal roles in the negotiation of this expectation. Beatriz and her students also negotiated norms about communicating mathematical thinking, ways of expressing disagreement, and the importance of mathematical understanding.

In contrast, the expectation that all 6th grade students (a) investigate a particular set of mathematical concepts, and (b) demonstrate understanding of those concepts on city wide standardized assessments, was not something that Beatriz and her students could negotiate. I do not intend to argue that district goals and testing policies were invulnerable to change, because redefining institutional structures – over time, through political pressure, changes in leadership, or large-scale resistance – is certainly possible. But the reality that Beatriz, as a 6th grade mathematics teacher in New York City public schools was responsible for addressing certain mathematical objectives, and the reality that the school's ability to make curricular decisions and maintain some level of autonomy was dependent on students' performance of those objectives, was not about to change. These were undeniable, obdurate forces that imposed their own set of norms, values and expectations on the activity in Beatriz's classroom.

Beatriz was acutely aware of these expectations, and while she refused to allow district guidelines to dictate her daily practice, she felt a responsibility as her students' teacher to ensure that they engage in rich mathematical activity, and investigate certain mathematical domains (e.g., rational number, data and statistics, and geometry and measurement). Equally important to Beatriz was that students participated in mathematics in ways that were personally and socially meaningful, that they asked questions that they cared about, and sought ways to "share what they learned" with others. Thus, as students asserted intentions in Beatriz's classroom, they also negotiated those intentions in response to the norms, values, and expectations of the figured world, some of which they had helped to create, and others that were imposed from external structures. In some instances, as the stories in the following section demonstrate, this negotiation was a smooth, and fairly effortless process.

Students' Intentions Facilitating their Participation in Mathematics

At times, students' intentions supported, or even animated particular norms and values – doing significant mathematics in personally and (potentially) socially meaningful ways -- of the figured world in which they participated. The way that Angel's personal concerns about the girls' restroom motivated her mathematical investigation of the space is a clear example. And yet Angel was not the only student who evidenced this dynamic. Other students' efforts to better understand and act upon the 'space crisis' at their school created a genuine, and seemingly natural need for mathematics. These students sought out measurements and district building regulations, asked classmates and teachers to help them multiply the sometimes 'unfriendly' fractions and mixed numbers they needed to work with as they calculated areas, and looked for ways to express the relationships they noticed mathematically. For these students, "the subject positions foisted upon them," as a student of mathematics and a student charged with examining and acting upon matters of personal importance, were not replete with contradictions, but instead supported one another (Holland et al, 1998, p. 46).

Consider the story of another case study student, Lianna, whose group decided to compare the area of the hallway space at Francis with the area of the hallways at Longmore, a smaller middle school housed on the fourth floor of the building. Lianna, a very soft-spoken girl, who felt a special bond with Beatriz because of their common Puerto Rican heritage, was unusually animated during this small group portion of the project. Because Lianna spent a significant portion of her day in Beatriz's classroom (morning homeroom, 2nd period math class, afternoon advisory, and twice a week choir practice), she often faced the challenge of exiting the room and navigating through one of the school's narrowest, and most densely populated hallways. Not a student who was comfortable pushing her way through oncoming crowds of up to 80 children at a time, Lianna was often left standing just outside the door for five or six minutes while other students passed in and out of adjacent classrooms.

The measurements and area calculations that Lianna's group produced would qualify as rigorous mathematics in any sixth grade classroom. The students developed their own strategies for multiplying measurements such as 18 3/4 meters by 1 1/4 meters, and then faced the task of totaling the areas of 12 different hallways. But Lianna was not content with merely stating the total hallway area of each school, she wanted to make her argument stronger, or to use her words, to "use more specifics so people will listen."

When she overheard that a classmate, Thomas, utilized the total hallway area and the number of students to calculate a hallway space per student ratio, she was intrigued.

> Lianna: How did you do that? We already found out the [hallway] area of Longmore, and I want to see how much [space] they will each get. You found out how much each person will get in Francis, and I want to do the same thing in Longmore. But I don't know how to do it.
>
> Thomas: You got to know how many students there are.
>
> Lianna: 60
>
> Thomas: 60 students, and how much is the area?
>
> Lianna: 246 and 3/4 meters squared.
>
> Thomas: So I am going to divide 60 into 246, cause that way I can find out how much each person gets, cause it kind of divides it [the space] up.

Lianna very clearly stated her need, and with the help of Thomas, she figured out that if all the Longmore students were to enter the hallways, each student would have 4.1 square meters of space. She was shocked when she compared that figure to the less than 1 square meter of hallway space allotted to each student in her own school. She said

> I learned that Longmore's hallway space is bigger, cause that's what I studied. And they have less kids. And that our hallway space is a lot smaller than theirs and we have more students. And I learned that when you put all the kids in the hallways, in Francis, they don't even get a full meter[2]. The get like about 9/10ths.

---

[2] Lianna is referring to square meters as "meters."

> And in Longmore they get like four meters of space. …. And math helped us because it proved to the district that our school was smaller and that the hallways were small and that there is NOT enough space for the students.

Ratio proved to be a very useful mathematical tool for Lianna, and one that she continued to draw upon, and was encouraged by her teacher to draw upon, as she investigated subsequent problems involving space. I present Lianna's story here, as a complement to the story of Angel, because I think that taken together, their experiences help to illuminate the complexity of this tension. First, it is significant that Lianna not only came to understand something about ratios as she solved this problem, but that her personal intentions motivated that understanding, and shaped the meaning it held for her. I suspect that Lianna's understanding of ratio might have been different if she had only studied the concept through traditional textbook problems. Second, it is significant that Lianna's personal intentions nudged her to investigate mathematical concepts (e.g., measurement, area, ratio) that were anticipated and supported through her participation in the figured world of her classroom. Her 'need to know' about the concept of ratio was consistent with the kind of mathematical support that other actors in the figured world, principally Beatriz and myself, were prepared to offer. We expected that figuring space to student ratios would assist students in investigating overcrowding at their school, and were prepared to support students in this manner. This consistency is not inconsequential. I want to argue that it helps to explain why in contrast to Angel, who struggled to pull together a final mathematical investigation of the bathroom space, Lianna successfully presented a coherent, compelling, and mathematically rigorous analysis of the hallways.

In both Angel's and Lianna's stories, the girls' personal intentions motivated a potentially compelling, and personally transformative mathematical analysis of their school space. What seemed to distinguish their stories, in particular their ability to move from critiques, questions, and ideas for analysis to actually carrying out that analysis, was the extent to which their intentions were supported, mathematically, by the actors and expectations present in the figured world of their classroom. Lianna's (and many other students') more conventional approach of measuring classrooms, calculating areas and figuring ratios was supported by her peers, taken up in whole group discussions, and addressed in mini-lessons, homework problems, and classroom assessments. In contrast, Angel's personal intentions motivated a 'need to know' about analyses related to time (e.g., wait time during peak use periods) and quantities other than area (e.g., concern about the number of stalls, and not the area of the restroom), analytic approaches which were neither addressed nor supported in the public classroom space. While both Beatriz and I attempted to support Angel in her investigation, sometimes in relatively unproductive ways, I believe that we had not anticipated the kind of analysis that Angel wanted to embark upon, and that as a result, we were not prepared to adequately support her, mathematically, in investigating and asserting her intentions.

In Figure 6.1, I present a visual diagram aimed at illuminating the interactions between Angel's and Lianna's intentions, and the norms, expectations and mathematical activity of the figured world. Clear in the diagram is how both girls' intentions animated a potentially compelling mathematical analysis of the school space (indicated by arrows in the diagram), and how that 'potential' was supported by certain values of the figured world. The diagram also illustrates that while Lianna's intended analysis was supported by the mathematical activity that dominated the figured world, Angel's intended analyses were not, and as a result, the two students differed in their ability to move from critiques, questions, and ideas to actually executing a mathematical analysis that would support subsequent transformative action upon the situation.

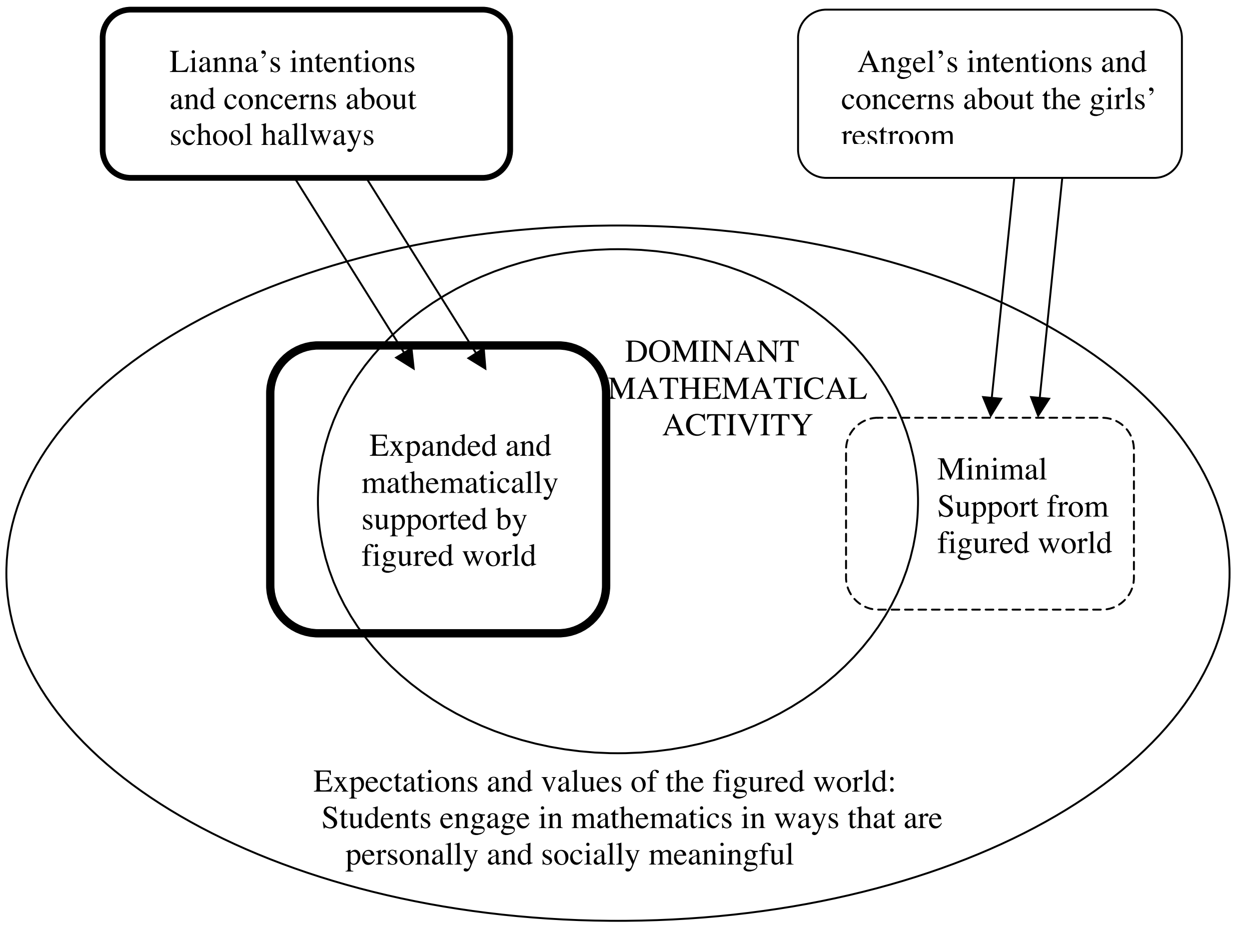

Figure 6.1. Interactions between Angel's and Lianna's intentions and the norms and expectations of the figured world

Students Intentions' Challenging Mathematical Content

The issue of mathematically supporting Angel's concerns raises another question about the inherent strain between students' intentions and the expectations of the mathematics classroom. I suspect that one reason Angel's participation in the figured world was not able to sufficiently support her in her intended analysis was that her intentions challenged both (a) how other actors in the figured world defined the mathematical content, and (b) more conventional ways of evaluating overcrowding.

Just as Angel was not the only student whose personal intentions motivated their use of mathematics, nor was she only student whose intentions challenged prevailing understandings of the content. Consider the story of Marlene and Jerica, two students in Beatriz's classroom whose primary concern was the fire hazard created by the narrow hallway outside their classroom. In the following discussion, they improvised a way of evaluating the hazard mathematically. Rather than relying on their classmates' methods for investigating hallway space (multiplying dimensions to calculate area), they decided to evaluate the hallway based on the array of people that could fit within the space.

Jerica: We are doing fire hazards. What we came up with was doing how many kids across, and how many kids could go up and down the hallway. And it came out to 96, but that's really really up against the wall, and we said that in a fire, no kids could be up against the wall, so we took away 3 [rows] and now it would be 87. That's how many kids. So now we are trying to find out how many kids is in each of these three classes for this hallway. …

Erin: I didn't quite get that, can you tell me again where you got the 87 from?

Marlene: We went in the hallways, and we measured the width, and then we measured the length, and when we measured the width it was three people.

Jerica: And we measured it by kids because that's how many people is going to be in the hallway [if there's a fire] so we measured it in kids. 3 kids can stand across, and then 32 people can stand this way (motions with hand to indicate the length). What we did was we lined up 1-2-3, and every time somebody moved, it would become – we would count one. [they stand up to demonstrated]. So it's 32 long by 3 wide.

Erin: So you found the area in people?

Jerica: Yes, but then we are subtracting because if we did 32, that's this whole hallway, that would be up up against the wall. You are not actually going to walk up against the wall, so we minused the three, which is the three rows up against the wall, and it became a 29, so then we times'd the 29 times 3. And now we are going to ask Ms. Font if we could see how many kids are in each of these three classrooms, so how many kids could fit in this hallway. Cause only 87 kids could fit in this hallway, if we are trying to get out of a fire. But maybe not even 87.

Marlene: Cause some kids are big.

Jerica: And some kids don't walk in a single file. … And we are going to compare it to Longmore, and if they have a fire hazard how many kids

> could fit in their hallway. We are going to put 'More kids could be killed in our school than in their school, because more kids can fit downstairs'.

In this interaction, Jerica and Marlene improvised a model for measuring space (i.e., an array of people that would fit within the space), and in doing so, challenged how most actors in the figured world were evaluating school spaces (measuring dimension and calculating areas). From their perspective, calculating area in the traditional sense - using square units – did not seem appropriate, and so they improvised, measuring the hallway space in terms of an array of people, and then adjusting that array based on actual conditions of the context – "No kids could be up against the wall, so we took away 3 [rows] and now it would be 87." Jerica and Marlene's improvised model for examining space then became a tool they used to investigate and compare other hallway spaces, and to argue that their school's hallways did in fact present a greater fire hazard: "more kids could be killed in our school than in their school, because more kids can fit downstairs."

In this way, their story is not only an example of how students' efforts to pursue their intentions (e.g., investigating fire hazards created by a large number of people moving through a narrow hallway) can result in the construction of mathematical models and interpretations that challenge dominant ways of thinking about the content, but also of the power of improvisation to facilitate the process. That is, asserting their intention to evaluate the hallway space using an improvised mathematical model, that though related to area, did not make use of area, within a figured world where the dominant discourse was about measuring dimensions and calculating area in square meters, was bound to create tension. Through their agentive act of improvisation, Jerica and Marlene were able to push against, move around, and challenge the prevailing mathematical norms of the figured world in order to investigate their intentions in a way that made sense to them.

Yet, it is also important to consider how Jerica and Marlene's improvised actions were supported by other elements of the figured world. For instance, Angel's ideas for examining the girls' restroom space were also improvisations aimed at asserting personal intentions. Why were Jerica and Marlene able to use their improvised model to analyze fire hazards at the school, and moreover, draw on the strategy as a tool to support future action, while Angel was not? I want to argue that Jerica and Marlene's ability to move from improvised ideas to action is related to how their ideas were taken up by the figured world. The improvised model that they generated challenged the dominant discourse about area in the classroom, but their ideas were ones that could be easily incorporated into classroom discussions about measuring space and evaluating crowding.

In contrast to Angel, who challenged the very notion of measuring space as a primary tool for investigating overcrowding at the school, Jerica and Marlene (through their model) didn't challenge the centrality of quantifying space, but how the space should be measured. As a result, their improvised attempts to assert their intentions were supported by classroom discussions about the meaning of area and the importance of using standard units, and by suggestions from peers to compare the areas of similar spaces in the two schools. They were able to draw on the affordances of the figured world, and then adjust those affordances to meet their needs and intentions (e.g., approximating a standard unit of measure through an array of people standing in lines).

Figure 6.2 presents a diagram that illuminates the interactions between Jerica's and Marlene's intentions, their improvised mathematical model, and the dominant mathematical activity of the figured world. Like Angel and Lianna, their intentions animated a potentially powerful mathematical analysis. Figure 6.2 illustrates how their improvised model simultaneously challenged and was supported by the prevailing mathematical activity of their classroom. The support afforded by the figured world allowed Jerica and Marlene to move from their improvised model to a compelling investigation of the fire hazards created by narrow hallways in the school.

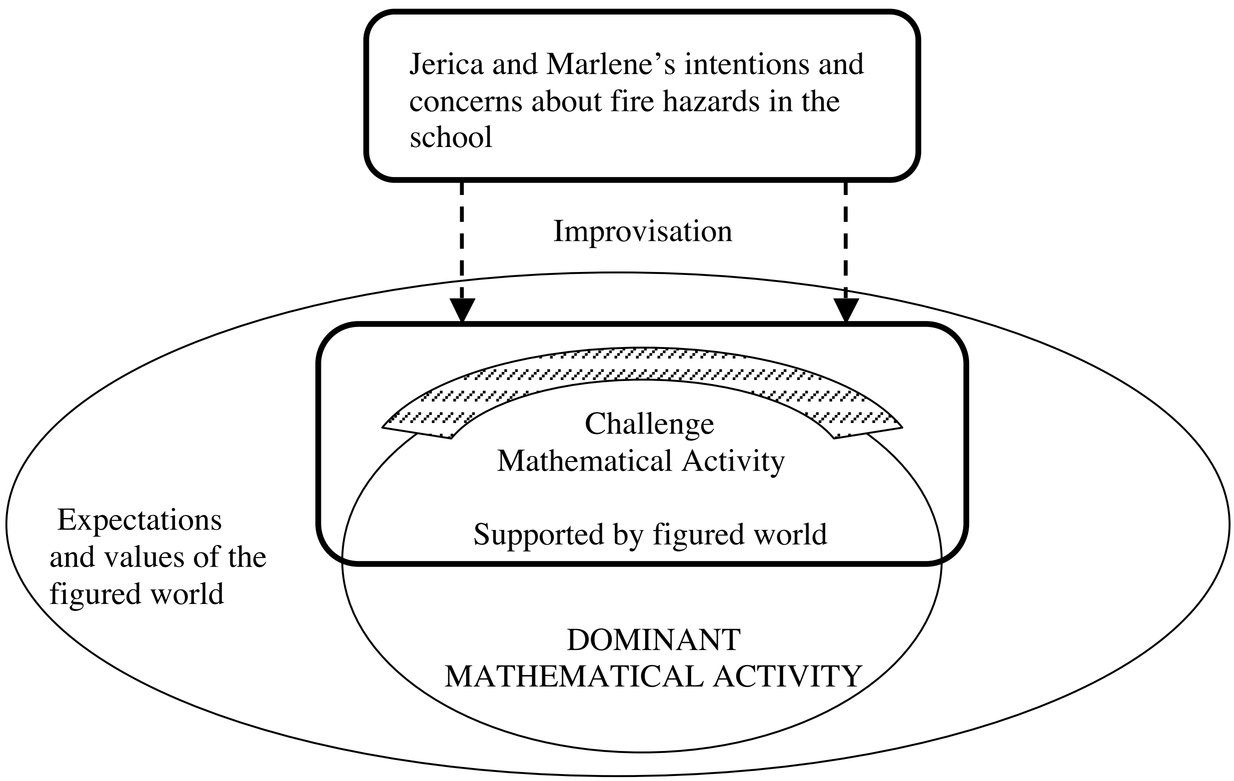

Figure 6.2. Interactions between Jerica and Marlene's intentions, their improvised mathematical model, and the dominant mathematical activity of the figured world.

Strain Between Students' Intentions and Expectations of the Mathematics Classroom

While the stories of Angel, Lianna, and Jennifer and Marlene all illustrated how students' intentions animated their opportunities to engage in the mathematical activity of the figured world, not all stories evidenced this dynamic. At times, students asked questions about the school space that were non-mathematical in nature (e.g., What can we do about all the poles in the gym?), and therefore asserting their intentions did not naturally lead them to rigorous mathematical investigations, as L.J. and Joel's story demonstrated. Given our expectation that (a) all students would investigate and develop understanding of mathematical concepts such as measurement and area through their participation in the unit, and (b) that mathematics could serve as a tool to support students' efforts, when students asserted intentions that evoked minimal mathematical content, tension arose. Both Beatriz and I felt a responsibility to ensure that students' participation in each of the projects would support and extend their mathematical understanding. Moreover, there were particular mathematical domains that Beatriz needed to address with her students (e.g., measurement, geometry, rational number concepts), and this need necessarily constrained the latitude that students had to purse any aspect of the school, mathematical or not, that happened to interest them.[3]

Within these spaces of tension, one of the primary roles of the teacher was to ensure that the mathematics did emerge, to pull out the content and yet also structure the content into students' experience, not apart from it. Beatriz (and on occasion, I) faced the

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[3] See Chapter 3 for a more complete discussion of how the need to address particular mathematical domains, among other factors, shaped the units that were developed.

challenge of negotiating some intersection between the students' context, their intentions and purposes in the project, and the academic context, the disciplinary knowledge and practices that would support their endeavor (Shor, 1992). Beatriz posed questions to small and large groups such as: "What information do you need?", "What measurements might help you?", "How would knowing the area help you?", "What might you compare this [area / measurement] to?", and "How could you make your argument more powerful?" Despite our efforts, we were not always successful in constructing a bridge between students' intentions and the mathematical expectations of the figured world.

Consider the story of another case study student, Andrés, whose group was intent on investigating fire hazards at Francis, in particular, the hazard created by limited access to stairwells. In the following conversation, I attempted to highlight possibilities for mathematical investigations within his intentions, possibilities which he resisted.

Erin: What are you interesting in looking at?

Andrés: We are going to do about the fire hazards, and how fast the building would burn up.

Erin: Okay, so tell me what you are concerned about with the fire hazard.

Andrés: Cause there's no stairs, like no stairs come up to the 5th floor, and then we couldn't get out and we would die. And you would have to walk a really long way, and you can't get out fast.

Erin: So you are thinking about how far you would have to walk to get to the stairs? How could you investigate that?

Andrés: [not really paying attention to me] -- and then the building would burn down so fast, like in 10 minutes, and we couldn't get out.

Erin: Why are thinking you couldn't get out?

Andrés: Because the building would already be burning down, and we're on the top floor, and we wouldn't get out.

Erin: Okay, but let's think about how you could use math to look at that. You said something about how far you would have to walk to get to the stairs, and then how long it would take you to get out. Do you think you could look at that? Maybe it would help you to know how far it is from the classroom to the staircase, or maybe you could even look at how far you have to walk compared to kids on other floors.

Andrés: No, we want to do about how the building would burn down and how we couldn't get out. Cause that's really dangerous and it's a problem, and it's not fair. Cause people will just be running to get out, and pushing each other, and getting hurt, and then we won't be able to get out!

Erin: Okay, but if you want to talk about how you couldn't get out, you need to think about how to show that, how math would help you show that. Maybe you could look at how small the hallways are.

Andrés: No, whatever, we don't want to measure the hallways. We're doing about the fire hazard. And how we need more stairs to get out.

This interaction was particularly frustrating because while I saw definite possibilities for mathematical investigations within Andrés's proposed project (e.g., time it would take students to escape as compared to how quickly the building burned), I was not able to help him see these connections. He resisted suggestions about measuring the size of hallways or the distance between his classroom and the stairwell, and instead repeatedly asserted his claim that Francis needed "more stairs." Ultimately, his group abandoned the 'fire hazards' project, frustrated by the pressure to incorporate mathematical activity into an issue that from their perspective was very straightforward.

In reflecting on what might have motivated Andrés's resistance, I think it is significant that the issue of limited access to stairwells is one that Andrés and other students had complained about on a regular basis, even before the project began. They were frustrated that Francis students were restricted to select staircases (due to building rules about staircase use, and to the limited number of staircases that reached the 5th floor), while students attending other schools in the building had the freedom to use the staircase of their choice. While this issue was partly about fairness, it was also about genuine safety concerns. A building across the street had recently been damaged in a fire, and just months after the World Trade Center disaster, students were all too aware of the dangers of being trapped in a burning building. This was a real and urgent problem for Andrés, one that had real consequences, "and then we won't be able to get out!" The fact that the issue of limited access to stairwells was already an issue for Andrés, and one that in his mind, already had a solution - "more stairs" - may have diminished his motivation to further investigate the situation mathematically.

In addition, Andrés's resistance may have been influenced by how he interpreted the task. Andrés was already convinced that limited access to stairwells created a safety problem in the school, and did not seem to be aware that one way to persuade others, who did not already share his urgent need to remedy the situation, would be to construct a well-formed mathematical argument to support his claim. The task of investigating the situation, collecting data, and building an argument seemed irrelevant to Andrés, who was already convinced that a problem existed. While other stories demonstrate how students' convictions can animate meaningful mathematical investigations, Andrés's story illuminates the potential for strain and tension that is part of the complex interplay between students' intentions, and the expectations of the mathematics classroom.

While I believe that mathematics would definitely have added a powerful dimension to Andrés's intended argument, the stories of *other* students, like L.J. and Joel, suggest that mathematics may not always be the best tool to understand or evaluate a situation (Stevens, 2000, p. 119; see also Skovsmose, 1994). For instance, perhaps L. J. and Joel's idea of creating a video, which would have drawn on visual images rather than quantitative data to convince the district of the space problems at the school, would have been more powerful. Perhaps photographs of students struggling to pass through hallways, or video of students running into the poles in the gym during a basketball game would have better communicated the urgency of the problem. L.J. and Joel's story challenged an implicit assumption of the figured world, and one expressed by numerous students, that arguments are more convincing when they involve "proof with numbers."

Moreover, the notion that mathematics may not be the most useful tool to investigate students' intentions raises challenging questions for teachers. What happens when students' intentions lead them to valid, engaging and potentially transformative questions that are best examined through disciplines other than mathematics (e.g., L.J. and Joel's investigation of poles in the gym)? What is the place of such investigations within the figured world of the mathematics classroom? And if such intentions are better addressed in other contexts, or through other disciplines, how do teachers communicate these boundaries to students? The task of creating genuine opportunities for students to assert intentions, while at the same time, sufficiently guiding those opportunities so that students also engage in significant mathematics, is complex and challenging, as the stories of students in Beatriz's class demonstrate.

The Value of Working Within This Difficult, Contested Space

The complexity of negotiating intersections between students' intentions and the norms and expectations of the mathematics classroom raises questions about the value of working within this difficult space. What might students gain from struggling to connect their intentions, with varying degrees of success, with the mathematical skills and understandings that are valued within the figured world of their classroom? I argue that students can and did benefit, in multiple ways, from navigating these tensions. As stories like Angel's, Lianna's, and others demonstrated, students found ways to interject their intentions into the curriculum, and in the process, develop mathematical understanding[4].

[4] Refer to Chapter 5 for a discussion of the mathematical understanding that students developed through their participation in these projects.

Moreover, even when students were not able to *fully* connect their personal intentions with significant mathematics, by (a) struggling to do so, and (b) participating in a figured world where other actors were striving to make those same connections, they were pushed rethink their relationship to the discipline, and its potentially transformative power in their lives. This shift in students' beliefs and ideas about mathematics was apparent in each of the stories discussed in the previous chapter. For instance, Naisha initially described mathematics as a subject she "couldn't stand" and one that centered on "numbers" and paper and pencil calculations. Reflecting on her participation in the projects in Beatriz's classroom, she argued, "you can do a lot of things with math. Not just fractions, multiplication and division. Things in your community and stuff like that." Naisha's comments were not unique. During final interviews, focus group discussions, and classroom interactions, *each* of the case study students spoke about mathematics in ways that evidenced the beginnings of shifts and transformations in their understanding of the discipline.

In particular, three central themes about the power and potentially transformative role of mathematics emerged across students' comments. Students spoke of how mathematics was a tool that could help them to (a) investigate situations of personal and social importance, (b) explore issues of equity and fairness, and to level critiques, and (c) prove and argue their point of view, which supported them in acting transformatively upon situations. The sections that follow explore each of these themes.

Math as a tool to investigate situations of personal and social importance. Students spoke directly and openly about the difference between their participation in

Beatriz's classroom, and their previous school math experiences. L.J. commented, "Before we were just doing regular math, now we are doing math about our school, and other schools, and to help our school." Lianna spoke of how math helped her "learn about our school and our environment" and that in Beatriz's class, "it's not just math, it's about the community, and real things that are happening around the world, and how much people get paid, and stuff like that." Angel was even more direct as she argued, "Look, it's like you are learning about other things. You are learning about things that you *be* in everyday, and it's a part of your life. … it [the math she learned through the "Space at Francis Middle School" unit] was something you could keep with you, it is like information that could involve *you*." Jhana added that

> You could do so many things with [math] to help you out. … by trying to figure out something, by trying to find out something with it. … and [math] actually tells like what is happen [in the situation] cause it gives more *detail* to it.

Repeatedly, students' comments alluded to how they employed mathematics as a descriptive tool (e.g., Mellin-Olen, 1987; Peterson, 1994, 1999) to help them make sense of situations of personal and social importance. Moreover, present across these comments is the notion that learning mathematics in contexts that strive to link mathematical content with students' experiences and intentions results in learning that is consequential in realms beyond the classroom, specifically, learning that is consequential in students' lives. Given that students often struggle to identify personal reasons, beyond instrumental ones, why they should learn mathematics (Martin, 2000; Noddings, 1993), this understanding is significant.

Mathematics as a tool to explore issues of equity and fairness. As students described how mathematical investigations helped them to learn about the world, they emphasized that these investigations allowed them to consider issues of fairness and equity, and to level critiques. For instance, Carlos argued, “I learned about how stuff like space is not shared evenly, and how we can prove that with math. … Now that we know, we can use that in our life, and do something about it. ” Lianna commented that analyzing the income data “was important, because we were talking about how it was unfair for women and men and for some races, and it’s good because it’s the truth, and we told people about it.” For Andrés, analyzing the data not only revealed issues of inequity, but evoked a strong emotional response, and a desire to act. He commented, “I was going and doing the research, and finding things out, and I am upset because I am finding out that these people are getting treated bad, and it’s not fair. And we have to do something.”

In some cases, reasoning about data or about the results of mathematical investigations not only served to explore particular instances of injustice, but to open the door for much broader critiques. For example, Naisha and Cristina’s conversation about how mathematics helped illuminate and confirm the inequity of their school space led them to a larger critique that linked disparities in resources and space to discrimination along the lines of race and class. Naisha asserted, “It’s an upper class thing, for the white kids. They get a better education. Better everything.” Similarly, students’ calculation of hourly wages and their subsequent discussion of the income disparities they noticed prompted them to consider the reasons for the inequities, and to connect them with broader, historical forces of oppression, such as slavery. In this way, mathematics served as a *tool for further reasoning and reflection* (Lehrer & Schauble, 2000), in particular, for critical reflection about how particular injustices are linked to much broader, societal level inequalities.

Mathematics as a tool to prove and argue a point of view, as a way of acting transformatively upon a situation. The most prevalent theme that emerged from students' comments was that mathematics supported them in proving arguments and defending their point of view. For students, the need to "prove a point" was closely connected to their desire to act transformatively upon situations of personal and social importance. For instance, Lianna spoke of how math provided her with "details and specific numbers. Because we would need specific numbers for our arguments about the school space." She explained, "we measured how big everything was, and so that helped us because it proved to the district that our school was smaller and the hallways were small and there is not enough space for the students." Angel added that mathematics offered her "defense" to help her "solve problems." She claimed,

> Math is just like one of those things that helps you solve your problems. It's like you have more defense. … Like let's say you go and have an argument with somebody, and you say the hallway is small, and they say, what do you mean it's small, and you don't even know how big it is. … Say it's this room, and you know the length and the width, and then it's like you have more defense right there, because you know more stuff that they didn't even know about.

Jhana argreed with Angel, arguing that the power of her argument was greatly enhanced when she drew upon mathematics to support her points. She stated,

> And [math] was something that you actually learned how to prove a point. And try to tell somebody, like give somebody proof and information about the thing that you are fighting against. …[Before] I wouldn't really use math. I would just say like LOOK how much space they have instead of what we have [in our school]. Cause it's like you can fit so many of our kids in their space. But I would really use math [now] and so I thought this was different. Math made my argument make more sense.

As the comments above demonstrate, insofar as mathematical activity helped students to defend their arguments, they saw mathematics as a tool that enhanced their ability to respond to and act upon "real problems." A mathematical analysis of the school space afforded them "proof" and "details" to "show" and "convince" district authorities of the urgency of their school's overcrowding problem. As students struggled to connect their mathematical activity with issues that they cared about, they began to see mathematics not only as a tool for understanding or critically reflection, but for acting transformatively on situations by strategically arguing their point of view and sharing that view with others, particularly those in positions of power (Skovsmose, 1999; Tate, 1995).

A related tension: Uncritically accepting the power of the discipline. Yet as students used mathematics in support of their empowerment, did they at the same time accept the power of the discipline unproblematically? What did the "Math is Power: Demand It" poster in their classroom come to mean for them? Was their understanding of the power of mathematics limited to how they might employ mathematics as a descriptive tool, or a resource for action in society? Or did students also have a critical sense of the sometimes oppressive ways that mathematics is used in society?

Mathematics has a formatting power; it provides a way of describing and interpreting reality (Vithal & Skovsmose, 1997) that not only "touches reality, but also squeezes and transforms it" (Skovsmose, 1994, p. 102; Gellert, Jablonka, & Keitel, 2001). When mathematical models are created, certain elements of reality are deemed important and included in the model, while others are left out. Certain relationships are considered, and others, disregarded. In this way, models reflect particular values, beliefs and perceptions of reality—those of the individuals who construct the model – and may serve some interests over others (Skovsmose, 1994; see also Frankenstein, 1983). Students should have opportunities to critique the use of mathematically-based models, rather than accept these models uncritically (Gellert, Jablonka, & Keitel, 2001). How might students have extended their sense of the power of mathematics (e.g., math as a tool to prove and argue a point of view) to also critique how mathematics is used in the world? What might such a critique of the formatting power of mathematics have looked like with this unit of study? Perhaps they could have reflected on what a mathematical representation of their school space revealed, and more importantly, what it kept silent. What mathematical models and codes might have been open for critique? What would it look like for students to construct their own models and codes, based on the needs, values, and beliefs of members of their community? For instance, students might have critiqued the model used by their school district to determine how many students were “slotted” to each school, and then proposed their own model based on a combination of factors that they deemed important.

The previous section discussed the ongoing tension between (a) students’ efforts to assert their intentions, and (b) the values and expectations of the figured world -- a mathematics classroom -- that framed their activity. In addition to analyzing how this tension played out in students’ activity, the section argued that students can and do benefit from navigating this contested space, particularly in terms of how they conceptualize their relationship to the discipline, and its potentially transformative power in their lives. In the next section, I examine the tension that arose when students’ acts of asserting intentions created a need for resources – in this case, particular mathematical skills and understandings – which were (a) not already part of the figured world, and (b) beyond the realm of what some students were ready, or able to understand.

Tension #2: The Mathematical Understandings Implicated by Students’ Intentions

Students’ Intentions Eliciting Challenging Mathematical Content

As students began to assert their intentions within the “Space at Francis Middle School” unit, their concerns and questions created the need for a variety of mathematical skills and understandings, some of which were beyond the conceptual reach of the students. This is, when teachers allowed students’ intentions about “real” situations to shape mathematical content, versus basing lessons on textbooks containing carefully sequenced, ‘sanitized’ activities designed to address specific concepts, the content quickly became difficult, and necessarily complex. Consider Lianna, who was very concerned about the school's lack of hallway space and wanted to argue that Francis students had significantly less space than students in other middle schools. To investigate this concern, she began with the task of measuring and finding the area of the hallway outside her mathematics classroom. The approximate shape of this hallway is illustrated in Figure 6.3.

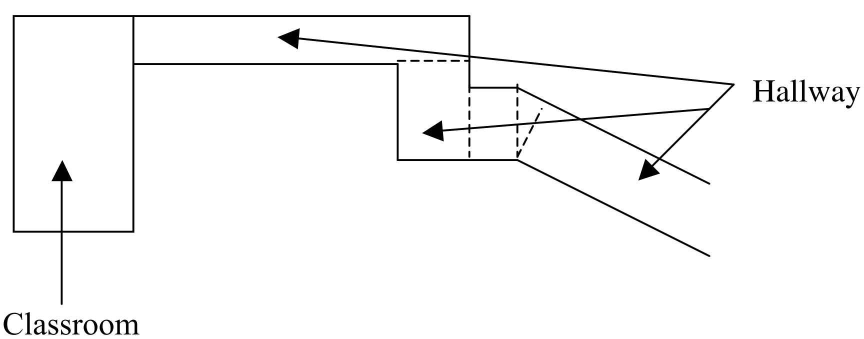

Figure 6.3. Hallway outside of students' math classroom
[a] Dashed lines indicate how students partitioned space into smaller rectangles to calculate the area.
[b] Small triangular space not accounted for in the total area

To approach this task, Lianna and her peers had to consider how to:

- Partition an irregular space into a collection of spaces that are easier to measure, such as rectangles (see dashed lines in Figure 6.3)
- Accurately measure the length and width of each rectangle
- Express measurements taken in multiple units (6 m and 20 cm) in a single unit, typically meters, which students accomplished by relating lengths in centimeters to the closest fractions of a meter (6 m and 20 cm became 6 1/4 m)

- Use the linear dimensions (e.g., 8 1/2 meters by 1 1/4 meters) to figure the area of the space, which could involve partitioning each rectangle into whole and partial square meter spaces, or directly multiplying the dimensions
- Multiply fractions, to find the area of small spaces, or portions of the total space (e.g., 1/2 x 1/4, or 1/4 X 1/4)
- Add partial areas to find to total area, which involved adding mixed numbers, such as 10 5/8 m^2 + 3 1/4 m^2 + 5 5/8 m^2 + 3/16 m^2

This task, like many others that students encountered as they investigated their school space, required an extensive list of mathematical skills and understandings. It was a task that would present significant challenges for most middle school students, and certainly for sixth graders. While this task was one of the more difficult ones, it fails to encompass the breadth of concepts students' intentions pushed them to explore. For instance, as students began measuring different sections of their school space, the need to construct a visual diagram of the space (i.e. floor plan) arose. Students investigated how to record measurements on a floor plan, how to interpret a floor plan provided by the district, and how to make sense of diagrams that failed to accurately represent the relative size of different spaces. Moreover, when the district informed students that they needed to present all measurements in standard, versus metric, units, students faced the challenge of converting all their measurements from meters to feet and inches.

In most cases, these were new, difficult, and at the same time, highly necessary concepts for the students. That is, students *had* to be able to operate with fractions and mixed numbers (adding, multiplying) in order to calculate the area of school spaces that concerned them. Likewise, they *needed* to be able to convert linear and area measurements from metric to standard units in order to communicate their arguments to district authorities. At times, these mathematical skills and understandings were beyond students' conceptual reach. It was in this contested space that tension arose. Beatriz faced the challenge of (a) supporting students with the mathematics that they needed to move forward with their intentions, and at the same time, (b) ensuring that they developed conceptual understanding of the content.

The sections that follow examine different ways that Beatriz and her students responded to this tension. The first section discusses instances, particularly during the early phases of the unit, when Beatriz strongly emphasized students' conceptual understanding, and demonstrates how students' drew on the understanding they developed to improvise strategies for increasingly difficult problems. The second section examines how as the mathematics became more complex, and the students' need to share their findings with others more urgent, instructional interactions became more focused on ensuring that students could "do" the mathematical operations they needed to support their emerging arguments. The third section presents a final response to this ever-present tension: the acceptance of approximate answers. When it became clear that some students could not accurately figure the area of a given space without executing procedures that they did not understand (e.g., multiplying a fraction by a fraction), Beatriz opted for students to approximate the total area.

Struggling to Support Students' Intentions <u>and</u> Their Mathematical Understanding

Response #1: A focus on conceptual understanding, which supported student improvisation. At the beginning of the "Space at Francis Middle School" unit, when it became clear that students were interested in investigating *how much* space they had at their school, Beatriz planned a series of mini-lessons to introduce students to the concept of area, and to strategies for finding the area of rectangular spaces. She started with activities that asked students to use square color tiles (manipulative materials) to construct rectangles of different sizes. (e.g., How many rectangles can you make that have an area of 36 tiles? 24 tiles? etc.) Students drew on knowledge of multiplication and factor pairs to successfully solve these tasks, and while the term *area* was new to many students (evidenced by a whole group pre-assessment of their understanding), they easily made a connection between the number of square tiles that could fill a space and the area of the space. Realizing that students would also need to think about spaces in terms of their dimensions, Beatriz began to talk about the length and the width of the rectangles, and asked students to label the dimensions of the shapes they constructed.

In a follow-up lesson, Beatriz presented students with outlines of rectangular spaces, and asked them to find the area of each space using the square tiles. While some students needed to entirely fill the shape with the tiles and then count the tiles one by one to find the area, others students had more efficient ways of calculating the total number of tiles, such as multiplying the number of rows by the number of columns or skip counting by the number of tiles in each row or column. Beatriz pushed students to find more efficient strategies, and after solving several similar problems involving rectangular arrays, the vast majority of students in the class recognized and could justify that by multiplying the dimensions of the shape they could find the total number of tiles, or total area. Beatriz then discussed with students that while square tiles were useful tools to measure the area of small spaces, a larger unit, such as square meters, would be necessary to measure the area of spaces like hallways and classrooms.

At this point in the unit, while students were beginning to talk about shapes in terms of their dimensions (e.g., this is a 7 by 10 rectangle, or this is 8 meter by 9 meter room), most students thought about area not only as the result of multiplying two numbers, but as the number of whole or partial squares needed to completely fill a given space. This was a *conceptual* understanding of area based on the notion that area is measured by "filling a space" with an array of square units, and an understanding that Beatriz actively supported. For instance, she frequently posed questions such as, "How many tiles would you need to fill up this space? So what is the area?" or "So you multiplied 9 times 8 (the dimensions of the space) to get the area. How do you know that will give you the area? How could you show that with the tiles?" The point here is that early in the unit, when the mathematical content was (a) less challenging, and (b) largely controlled by teacher designed mini lessons and carefully selected tasks, the impending conflict between supporting students' intentions (which at this stage was limited to preparing them to think about the concept of area, and to find the area of basic rectangular shapes), and fostering their mathematical understanding, was not apparent. This tension began to emerge as students' intensions pushed them to investigate more complex problems.

Towards the end of the first week of the unit, Beatriz presented students with the task of finding the area, in square meters, of a series of rectangular rooms. One of the problems involved a room that measured 4 meters by 2 1/2 meters (see Figure 6.4). Beatriz purposefully included a mixed number dimension (2 1/2 meters), realizing that as students began measuring classrooms and hallways in the school, they would encounter similar and even more difficult tasks. Although students had explored fractions earlier in the semester, this was their first encounter with fractional measurements in area problems. Thus the task presented students with a problem for which they had no set response or prescribed strategy, opening a space for improvisation. At the same time, the task was designed to be accessible to students. They could apply their understanding of the meaning of area (the number of square tiles that would fill the space) and of how to calculate area (multiplying the dimensions of a rectangular space) to support them in solving this more difficult task.

Consider how Vellez, one of the case study students, tackled this problem with great confidence. What follows are Vellez's comments (to himself) as he invented a strategy for making sensing of "what do to with the fractions".

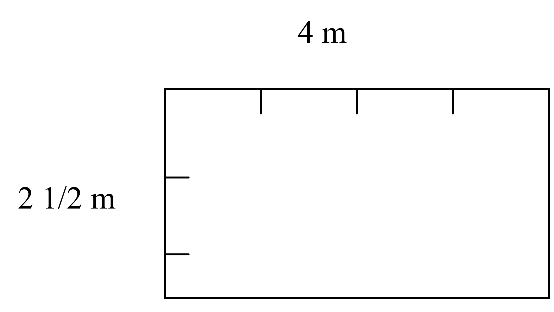

Figure 6.4. Task presented to students: Find the area of a 4 meter by 2 1/2 meter space

Vellez: (working on his own, looking at the diagram of a 4 by 2 1/2 meters rectangular space) Okay. Oh. Yeah. That's 8. (Draws lines to subdivide the entire rectangular space into individual square meters, and then counts the whole square meters, see Figure 6.5).

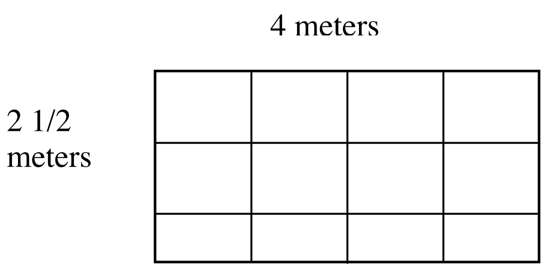

Figure 6.5. Vellez drew lines to subdivide space into square meter areas

Vellez: Oh. Oh. Wait. I see! (erases something) Okay, 2 times 4, equals 2 – oops. (pause) I know how to do it, yeah! I did it! Yeah. Look (holding his paper up to the video camera). I multiplied a half by four, because there are four of them. Because there's 4 halves (points to half square meter spaces), so I got 2. And then 2 times 4 is 8, plus 2 more is 10. Yeah. (smiling) And that's how many [square meters] is here.

Erin: And how did you know there were four halves there?

Vellez: Because it's 2 and 1/2, and that means four, two and a halves, so four halves. (Points to 4 spaces that are each a 1/2 square meters in the diagram)

I include this example because it illustrates how some students were able to draw upon the conceptual understanding they developed to improvise strategies for solving increasingly difficult problems. Vellez began by drawing lines to help him visualize the whole and partial tiles that would cover the rectangular space, a strategy he had previously used to find the area of shapes with whole number dimensions. That representation then helped him to make sense of multiplying the dimensions, and to verify that his proposed calculations (4 x 2 = 8, and 4 x 1/2 = 2) would find the entire area of the space. He explained, "I multiplied a half by four, because there are four of them. Because there's 4 halves (points to half square meter spaces), so I got 2. And then 2 times 4 is 8, plus 2 more is 10."

This strategy of separating the mixed number measurement (2 1/2) into its whole (2) and fractional (1/2) parts (essentially a re-invention of the distributive property of multiplication: 4 x 2 1/2 = (4 x 2) + (4 x 1/2)) was one that Vellez, and consequently many of his peers, continued to draw upon throughout the 5-week unit as a tool to support their analysis of the school space. Vellez consistently broke apart mixed number multiplication problems ("6 1/2 times 4, that's a 4 by 6 [rectangle] and then 4 halves [square meters]"), and then related these sub problems to corresponding portions of the total space.

Drawing on an understanding of area to solve problems involving mixed number dimensions created more tension for other students. For example, when Angel worked on the same problem (see Figure 6.4) she struggled to make sense of how to deal with the side that measured 2 1/2 meters. Responding to a disagreement between two classmates, one of whom felt the total area was 8 1/2 m^2, while the other was convinced that the area was 10 m^2, Angel commented

Angel: I disagree. You could still get 8 1/2, and you could still get 10. The way she did it, I think, is that she did 4 times 2 and she got 8, and then she just added that half and got 8 1/2. And also, you could – if you put in the little boxes, like to show the halves.

Beatriz: Like this? [adds lines to figure on the board to show the 1/2 square meter spaces]

Angel: Yeah, and if you got 4 times 2, if you put your hand on the bottom part (the half square meter spaces), you would count the top squares and it would be 8. And when you count them all together, with the halves, it would be 10. I don't really know, it's hard.

Beatriz: Okay, so now how are you getting from 8 to 10? Angel, tell us what you are adding to it [the 8 square meters]?

Angel: The halves. Half and half make one, and then half and half make another one.

Beatriz: This half and this half make one. This half and this half make another one. So 9, 10. So that's a great strategy.

Beatriz proceeded to ask other students in the class what they thought of Angel's strategy, and moreover, whether or not her explanation helped clarify whether the total area was 8 1/2 or 10 square meters. Students almost unanimously agreed that 10 square meters was the correct answer, and many began to rely on Angel's strategy of "covering up" part of the area to facilitate thinking about whole square meters and partial square meters separately. I include this example because it demonstrates how both Beatriz and her students responded to the tension that arose as students explored new mathematical content in ways that supported conceptual understanding. Angel reasoned through how to represent and account for the half square meter areas, and Beatriz supported that reasoning through questions, comments, and visual representations.

This strong focus on students' conceptual understanding was not limited to the first portion of the unit. Beatriz continued to emphasize understanding as students began to gather measurements from around the school, and as the need to solve increasingly complex problems arose. For instance, during the second week of the unit, students were attempting to calculate the area of a classroom at Francis that measured 10 meters by 6 3/4 meters. Though most students were comfortable dealing with 1/2 square meter spaces, this was the first problem that required thinking about other partial square meter areas. When Lianna raised a question about, "What to do with all the 3/4ths?" Beatriz stopped the discussion and posed the following problem for all students to think about, "So I have 10 3/4ths, what do I do with that? Let me write them out. (writes out 3/4 on the board, 10 times). What should I do? Let's think about that."

Students proceeded to make suggestions, such as "you add them," "you change the numerator, you add them up, like 3, 6, 9," and "you add them and you get 30/4, and then I think you can do how many 4s go into 30." Beatriz listened and asked questions about each suggestion students presented, such as "Okay, but why would I want to do how many 4s go into 30?" These questions provided students with opportunities to justify their thinking to one another, and to consider various strategies for approaching the problem. When a group of students struggled to make sense of 30/4, and how 30/4 related to a certain number of whole and partial square meter spaces, Beatriz introduced a real life example to help them think about the problem. She explained,

> I have 30/4, it's like I have, it's like I have a personal pizza, picture a personal pizza, it does not come sliced in 8 pieces, it usually comes in 4 pieces. Right? Then four slices of pizza make a whole pizza. So if I have 30 slices, 30/4, how many pizzas does that make?

The students immediately began making notes on their papers, counting by four on their fingers, and calling out, "it makes 7", or "it's 7, and then there's 2 pieces left." They quickly agreed that 30/4 was equivalent to 7 whole square meters, and 1/2 square meter, which they added to the area of the 6 by 10 meter rectangle to get a total area of 67 1/2 meters squared. At the end of this interaction, Naisha (one of the students who had struggled) commented, "I'm glad we did that one, cause I got 66 meters squared, cause I didn't really understand that 3/4 thing. But now I do."

The stories presented in this section demonstrate that the potential conflict between supporting student's intentions while at the same time supporting their mathematical understanding was sometimes addressed in ways that were productive both for students' understanding and for their ability to investigate aspects of their school space that mattered to them. In other words, the tension was addressed in ways that supported students' critical mathematical agency. These dually productive responses occurred when instruction maintained a strong focus on developing conceptual understanding, which in turn facilitated students' ability to improvise strategies for solving novel problems (e.g., Vellez) and to make sense of new and challenging concepts (e.g., Angel and Naisha). Moreover, these responses were more concentrated in the first portion of the unit, when (a) the mathematical content was more accessible, and more importantly, when (b) the urgent opportunity for students to share the results of their investigations with others (e.g., district authorities) had not yet arisen. When this opportunity emerged, and students had less than a week to pull together their arguments to present at a district advisory council meeting, there was a shift in how Beatriz and her students responded to the challenge of supporting both students' intentions and their mathematical understanding. The following section examines this shift.

Response #2: Instructional interactions focused on students' ability to "do" the mathematics that they needed to build and support their arguments. During the third week of the unit, the school principal informed Beatriz and her students that she wanted to invite students to share their analysis of overcrowding at the school at an upcoming district advisory council meeting. While students enthusiastically welcomed this invitation (from the first day of the project they had expressed interest in talking with the mayor, the superintendent, and the chancellor about their school's space crisis), the invitation arrived at a time when most students were just beginning to define the focus of their small group investigations, and when they were still in the midst of gathering data, figuring areas, measuring hallways, and building their arguments. This opportunity, as potentially transformative as it was, was accompanied by a time limitation that further strained an already tense balance between supporting students' intentions and ensuring that they develop mathematical understanding. While only several students would actually present at the district meeting, Naisha among them, all students were aware that their arguments and concerns had the potential to be heard in a context where they could be consequential, and this awareness created a sense of urgency that impacted classroom interactions.

At the same time, the problems students were investigating implicated increasingly complex mathematics. For instance, students analyzing the school's hallway space had the task of measuring and calculating the area of 12 different hallways, some which had "difficult" dimensions involving two mixed numbers, such as 18 3/4 meters by 1 1/4 meters. The following conversation illustrates how Beatriz and her students began to address this tension between the challenging mathematics that students needed to be able to *do* as they prepared their arguments, and the mathematics that they were ready to conceptually *understand*. Prior to this discussion, several students expressed concerns about how to find the area of hallways and classrooms with "two fractions" (in the dimensions). Beatriz decided to address these concerns by guiding the students through several examples.

> Beatriz: Okay, some of the rooms or hallways have two fractions. Let's talk about what we need to do if there are two fractions. Let's say this is my hallway. (Draws picture on overhead, see Figure 6.6) Some of you will have to work on some hallways or rooms in a minute that have two fractions. Say this was 2 1/2 meters by 10 1/2 meters.

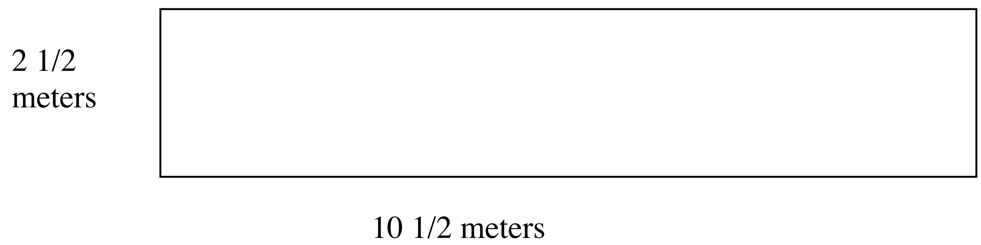

Figure 6.6. Finding the area of a hallway that measures 2 1/2 meters by 10 1/2 meters

L.J.: That's easy, that's real easy.

Beatriz: Okay, so 2 1/2 by 10 1/2. I now have two fractions here. Shantae used nice language here when she talked about 'hold the fractions'. That's a good way to think about it, so let's hold it in our head, and deal with it [the fractions] later. So let's do what we know. Let's hold the fractions in our head and we could do (covers up fractions)

L.J.: 10 times 2

Beatriz: Okay, 10 by 2, that space right here. (Draws lines to form 10 by 2 rectangle, writes 10 x 2 inside the rectangle). There are two fractions here, if it were only 10 by 2 1/2, this space right here (points to long rectangle along the bottom, that measures 10 by 1/2), would be 10 by ---?

Student: A half.

Beatriz: 10 by a half. (writes 10 x 1/2 inside the rectangle) There would be a 10 by 2, and a 10 by a 1/2 (See Figure 5). So, I would have to multiply 10 times 2, and 10 times 1/2. I know the half goes here, because the measurement of this line goes two and one half. Now what if it were 2 times 10 1/2? (covers up the 1/2 that is part of the 2 1/2) Remember how Shantae said we could hold one of the fractions. We have already done 10 times 2, what do we have to do with the 1/2?

Naisha: Times it by 2.

Beatriz: Times is by 2. It would be here. And there would be two halves. (Draws in lines to make small rectangle that measure 2 by 1/2, draws lines to show each of the half square meters) So we did that space that is 2 times 1/2 (writes 2 x 1/2, see Figure 6.7). Did you see that?

Beatriz then quickly reviewed this portion of the procedure with students, emphasizing covering one fraction, e.g. the 1/2 on the 10 1/2, and calculating 10 times 2 and 10 times 1/2, and then covering the other fraction, such as the 1/2 on the 2 1/2, and then multiplying 2 times 1/2, "since we already did the 2 times the 10."

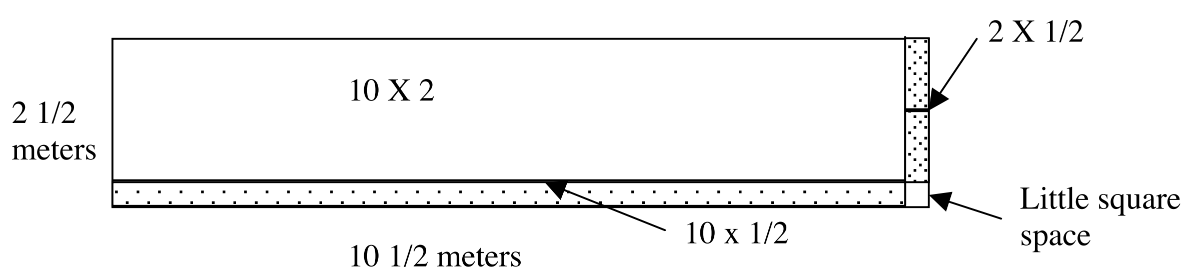

Figure 6.7. Shading part of the area of the 10 1/2 by 2 1/2 meter hallway

Next, Beatriz addressed the "small little square space" in the corner of the rectangle (see Figure 6.7). She stated, "We are going to be left with a little corner right here, if you can deal with that, that's fine. If you can't, that's okay, that's close enough.

We'll deal with that later." Several students called out that the area of the little square would be 1/4 of a square meter, because it was 1/2 of 1/2 of a square meter, an explanation which Beatriz acknowledged before continuing with her explanation.
The issue of how to account for this "little square" has arisen in previous lessons, and while a few students were able to reason about the space as a fraction of a fraction of a square meter to calculate its area (i.e. 1/2 of a 1/2), most students disregarded the space, or simply counted it as another 1/2 or 1/4 square meter. (Beatriz's decision to deemphasize the importance of this portion of the total area is discussed in greater detail in the next section).

I include this example because it illustrates a shift in the way that Beatriz and her students responded to the tension between students' need to engage in challenging mathematics and their ability to conceptually understand the content. In Beatriz's classroom, most discussions were characterized by asking students to share strategies, to justify their thinking, to pose questions, and to comment on their classmates' solutions. In this interaction, there is a shift in who controls the conversation – the teacher, and in the focus of the conversation – preparing students to execute procedures that will assist them in calculating the information they need to support their arguments. This is not to imply that from this point in the unit all classroom discussions were characterized by a more teacher directed, procedural tone. In fact, the very next day, and in numerous subsequent lessons, Beatriz pushed students who were analyzing the gym space (23 1/2 meters by 19 meters) to explain *why* they needed to multiply 19 times 1/2, and *how* the 19 half square meter spaces related to the total area.

What did occur is that as the opportunity for students to bring their arguments to the district board meeting approached, classroom discussions became more teacher directed, and more focused on presenting and explaining efficient strategies for calculating area. Beatriz wanted students to be able to construct arguments and if possible, present their findings at the meeting. She also recognized that even if students shared the responsibility of measuring and calculating among all members of the class, the amount of work students' intended analyses required, along with the mathematical difficultly of the problems they were attempting to solve, made it unlikely that students would improvise efficient methods for calculating the area of spaces with "difficult dimensions" on their own. For many students, understanding how to find the area of "easier" spaces, such as a classroom that measured 6 1/2 meters by 8 meters, presented enough of a challenge. Moreover, the limited amount of time that students had to prepare their arguments further impeded their ability to grapple with the concepts in ways that might help them construct their own, conceptually-based strategies to solve the problems. So in an effort to support students' ability to move forward with their intentions in the project, classroom discussions became more focused on making sure that students knew *how* to calculate the information they need to frame their arguments.

This shift in the way that Beatriz and her students responded to the tension between the mathematics students needed to know and the mathematics they were able to understand raises questions about students' opportunities to enact a sense of critical mathematical agency. One could argue that deemphasizing conceptual understanding would undermine agency, by diminishing students' capacity to justify their thinking, to understand rigorous mathematics, and to view themselves as powerful mathematical thinkers. In general, I strongly agree with that argument. Yet stories from Beatriz's classroom challenge such a straight-forward interpretation. For some students (e.g., Naisha), access to efficient procedures, regardless of whether those procedures were fully understood, facilitated the ability to quickly generate the data (measurements and areas) needed to support their arguments. In other words, these algorithmic procedures also acted in support of students' critical mathematical agency, particularly in their capacity to engage in action aimed at personal and social transformation.

There is definitely a trade-off here, one that other research has alluded to (Gutstein, 2003a), and one that raises significant questions. What happens when procedural understanding supports student agency, sometimes in the absence of a deep and profound understanding of the mathematical concepts? Is this truly an instance of critical mathematical agency? Students clearly used mathematics in personally and socially meaningful ways, and yet what did it mean to them to employ the results of mathematical investigations that they didn't fully understand? What did it mean for Naisha to speak at a district meeting and share measurements and calculations that she may not have been able to explain? Did it matter? In terms of the ideas she constructed about her relationship to the discipline, I suspect that it didn't. She stated,

> I learned you can do a lot of things with math. … Math helped me with space, because you could measure – you could measure the floor, and then you could multiply to get the area. ... Because without the math, then we wouldn't have the

> area of the school, and we wouldn't really know, we wouldn't really know, what's the area. And then meeting wouldn't have been as powerful as it was.

In terms of her fragile sense of herself as a capable mathematical thinker, I am not sure. I hesitate to argue that engaging in mathematics without understanding could ever foster students' sense of their own mathematical agency. Yet it seems plausible that the shift in Naisha's stance towards the discipline, coupled with her experience positioning herself in a place of authority relative to the results of the class's mathematical investigation could support her in developing mathematical agency in the future.

On the other hand, what if instruction had only permitted students to investigate those problems that they were able to conceptually understand? How might such an emphasis on conceptual understanding have truncated students' ability to investigate issues that mattered to them? For instance, how would students have dealt with hallways that measure 1 3/4 meters by 18 1/4 meters? They might have rounded all measurements to the nearest whole meter, but that not only makes for minimally interesting mathematics, but also raises questions about the viability of sharing inaccurate data with district authorities. They might have focused only on the width of the hallways, but how would that constraint have been experienced by students whose real concern was not just narrow hallways, but the overall lack of hallway space? Given that students' intentions led them to investigate "real" situations that implicated inherently complex mathematics, is it even realistic to think that students could conceptually understand all the concepts and procedures that they employed? And moreover, would it not have been equally, if not more detrimental to deny students the opportunity to communicate their arguments to the district, just because some of the mathematics that they needed was beyond their conceptual reach? I argue that it would have been.

In concluding this section, it is also important to note that a number of students *were* able to understand the procedures they employed. That is, while some students appropriated these procedures algorithmically, with minimal conceptual understanding (e.g., Aliyah asked, "Is it always the opposite?" meaning, "Do you always multiply each whole number by the fraction in the other, or 'opposite' dimension?), over time, others were able to justify and explain the steps they were taking. For example, though Lianna initially appeared to be executing procedures rather automatically, as she explained her strategy, it became clear that she understood how each operation corresponded to a portion of the total area.

> Lianna: (Explaining how she found the area of a room that measured 8 1/4 meters by 6 1/2 meters, see Figure 6.8) Ms. Font told us – see cause remember when Ms. Font told us that when you have two fractions you have to multiply it by the opposite. 1/2 times 8 is what I did. And 1/4 times 6. And so when I got that, my answers, I added all of them up. See first I did 6 times 8, cause that's the easiest. And then I did 1/2 times 8, cause there's 8 halves. And then I did 1/4 times 6, cause there's 6 one-fourths. And then all of my answers, I added them up, and I got 53 1/2. (disregarding the "small square space" in the corner)
>
> Erin: And how did you know you had to multiply all those different numbers?

Lianna: Cause Ms. Font told us that before, but also like, look, cause you know that there's 6 one-fourth meters, the little squares (traces outline of a rectangle that measures 6 meters by 1/4 meter, draws lines to show individual 1/4 square meter areas). If you draw it out, there is going to be 6 one-fourth meters, and if you draw it out there is going to be 8 one-half meters (traces outlines of rectangle that measures 8 meters by 1/2 meter, draws in lines to show individual 1/2 square meter areas).

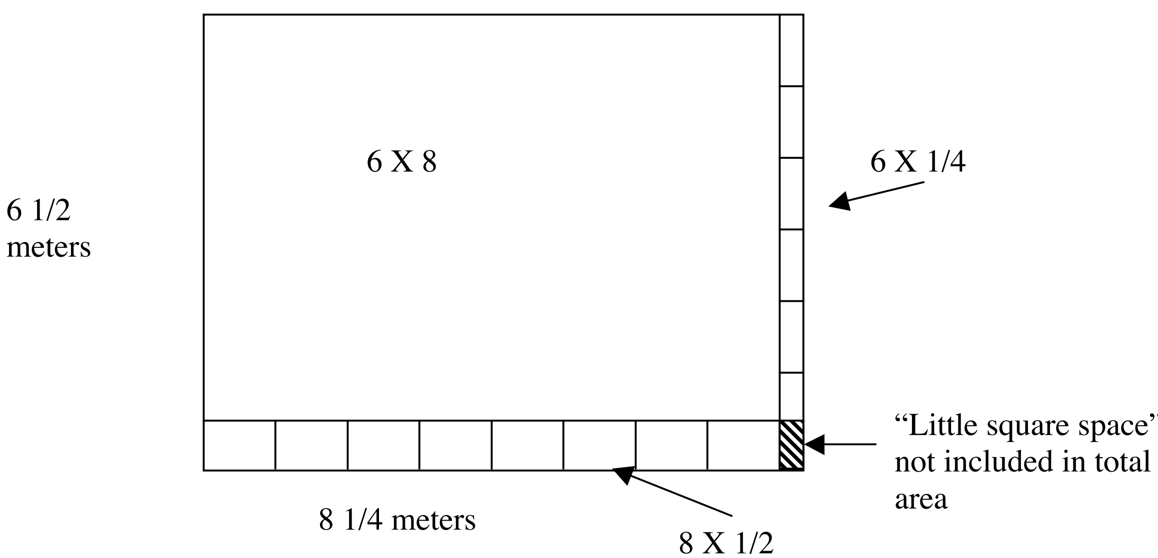

Figure 6.8. Lianna's explanation of the area of a 8 1/4 by 6 1/2 meter room

Lianna's strategy not only demonstrates her understanding of the procedure that "Ms Font told [her] about", but also alludes to another issue previously mentioned: students' disregard for a small portion of area, referred to by the class as "the little square space.[5]" When problems that evoked this "little square space" first arose, many students disregarded the space as on oversight, or automatically considered it to be another 1/2 or 1/4 square meter. Later, when Beatriz explicitly told students that they didn't have to consider the "little square", because "it's [the area is] close enough," disregarding the space was less of an oversight and more of an artifact of instruction. The following section describes the development of this "little square space" artifact.

Response #3: Accepting approximate answers and partial solutions. What prompted Beatriz to make the instructional decision described above? Interestingly, this decision was a third way that Beatriz and her students responded to this ongoing tension. When Beatriz determined that many students could not accurately figure the area of a given space without executing procedures that they did not at all understand (e.g., multiplying a fraction by a fraction), Beatriz opted for students to approximate the total area by "leaving out" this small corner space. Reflecting on this decision, Beatriz said she typically did not address multiplication of fractions as part of the 6th grade curriculum, and that if students were able to multiply whole numbers times fractions as they solved a variety of area problems, she would be thrilled.

Yet as the unit progressed, it became clear that while some students were aware that they were leaving out a small portion of the area each time, other students, particularly those who approached the problems algorithmically, did not seem to recognize this unaccounted for area. Beatriz and I both felt it was important that students

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[5] Students referred to the small rectangular space formed in corner of rectangular rooms with two mixed number dimensions as the "little square space" even when that space was actually not a square.

at least acknowledged that they were omitting part of the area, and that they considered whether or not these "small spaces" would make a significant difference in their emerging arguments about the school space. To address this concern, Beatriz focused students' attention on the "little corner space" during a whole group discussion. Prior to this interaction, students shared their strategies for finding the area of a Francis classroom that measured 6 1/2 meters by 8 1/4 meters (the same problem Lianna solved, see Figure 6.8). As expected, students failed to consider all of the space.

Beatriz: Ok, now I have a question for you. We figured out this space right here, we figured out this space, we figured out this space, all of this together. (shades in the 3 rectangular spaces that students have accounted for). Are we done?

Tyron: No, there's a little space.

Beatriz: Yes, there's a little spot right there. Have we figured out that little spot right there? Do we have to do it?

Student: No

Beatriz: Why not?

Tyron: Cause it's small.

Beatriz: If I want to cover this room with tiles, and I only buy 53 1/2 tiles (area students accounted for), will I have enough to cover this little spot too?

Students: NO!

Beatriz: Okay, this little spot. So what do I have to do to figure out this little space?

(silence)

Marcus: Well you could tell because going down (one dimension) it is 1/2, and so the side could be 1/2, and then the top could be 1/4. Multiply 1/4 times 1/2.

Beatriz: Is this the one we are missing? This space right here? 1/2 times 1/4? Have you been doing that with your classrooms? Have you been remembering to do that little space right here?

Students: NO!

Beatriz: No, right, nobody has been doing that little space. Let's see how much that comes out to be.

Beatriz proceeded to guide students in thinking about the size of the small corner piece. She commented, "1/2 times 1/4 Marcus says. Okay, How do I take 1/2 of 1/4?" After students called out a variety of answers, she posed a slightly different question, again aimed at helping students to make sense of 1/2 times 1/4. She asked, "If I have 1/4 and I am cutting it in 1/2, what would that be?" Several students responded that they space would be 1/8 of a square meter. Beatriz then drew outlined a 1/8 square meter rectangle on the board, so that students could visualize the size of the area they had been omitting. When she asked students whether or not they felt that a space this size would "make a difference" when they "compare the space of Francis, to Longmore," most students felt that the different was inconsequnetial. For instance, Geena commented, “It’s not going to change anything, because Longmore is still a lot bigger.” Other students justified the insignificance of the “little space” to their arguments about overcrowding at Francis using more mathematical reasoning. Vellez asserted

> Like, it would take, with 1/8 a square meter, it would take 8 of them to make 1 [whole square meter]. I don’t know how many classrooms there are, but if you multiply 12 [imagine 12 classrooms] times 1/8, you won’t get more that two [whole square meters], you won’t even get like 2.

As previously mentioned, Beatriz made the decision to have students approximate the area of the spaces they were investigating when she determined that calculating the area of the “small corner space” would require students to execute procedures (i.e. multiplying a fraction times a fraction) that they did not understand. This decision allowed students to move forward in the project, and therein, supported them in critiquing and acting transformatively upon the space crisis at their school. The decision became problematic however when students demonstrated that they did not understand their area calculations as “approximations” of the total area, but exact measurements. Beatriz had valid reasons for the decision that she made, namely minimizing students’ experience of executing procedures without understanding. But students’ lack of awareness about whether or not they were omitting a portion of the total area, and if so, why they were omitting that area, highlights the need for teachers to explicitly share their decision making with students.

Gutstein (2003a) has also argued for the need to openly talk with students about certain decisions that one makes as a teacher. In reflecting on how he veered from his typical emphasis on student understanding to a more directive teaching style when he implemented a series of "real-world" mathematics project with middle school students, he recognized that he was often forced to make choices between (a) providing students opportunities to develop rigorous mathematical understanding, and (b) meeting the other goals he had for the projects, such as critical reflection on community and global issues. Gutstein argued that these trade-off were real, and unavoidable (p. 65). The tensions that emerged in Beatriz's classroom clearly support and further elucidate his claim. Gustein argued that one way to minimize the negative impact of such trade-offs is "when [teachers] are conscious of [the trade-offs] and communicate them explicitly to students. (e.g., 'OK, I'm going to tell you how to do this, only because we're short on time, because you know I never do this normally')" (p. 65).

In Beatriz's classroom, this might have resulted in repeated comments such as, "Remember, there is a little rectangular space here, and it's part of the total area, but since finding the area of that space involves calculations that are very challenging, and since the area is really small, we are going to leave it out." Explicitly communicating these kinds of decisions to students is important, because it supports them in enacting agency from a position of awareness and understanding. However, I want to argue that even more important is inviting students to share in the decision making process.

Conclusion

In this chapter, I discussed two tensions, or "contested spaces" that arose across the stories presented in Chapter 5, and those of other case study students. The first section focused on the ongoing tension between (a) students efforts to assert their intentions, and (b) the norms, values, and expectations of the figured world in which they participate. The second section addressed the tension that arose when students' acts of asserting intentions created a need for mathematical skills and understandings which were (a) not already part of the figured world, and (b) beyond the realm of what some students were ready to conceptually understand. In each section, I described the tension, its impact on students' activity, and how it was addressed in classroom interactions. In the next chapter, Chapter 7, I explore the primary themes that emerged from this study, and discuss its implications both for research and for teaching.

CHAPTER SEVEN: CONCLUSION

In the opening chapter of this dissertation, I set forth a series of questions that guided this collaborative, critical ethnographic study. I sought to understand (a) how students enacted a sense of critical mathematical agency within a mathematics classroom aimed at teaching and learning for social justice, (b) how students' engagement with the discipline of mathematics interacted with their enactment of critical mathematical agency, and (c) what tensions and contradictions arose as students exerted a sense of agency within, upon, and against the contextual constraints that were part of the figured world of their classroom, and (d) how students and teachers negotiated those tensions. In this final chapter, I summarize the themes that emerged as I investigated these questions, highlighting the ever present tension between students' enactments of agency and the constraints and affordances of the figured worlds in which they participated. I then outline the implications of this study for future teaching and research aimed at promoting equity and justice in/through mathematics education. The chapter concludes with a brief discussion of what is next for students and teachers at Francis middle school.

Cross Cutting Themes

Entry Points for Enacting Critical Mathematical Agency

Students enacted critical mathematical agency in various ways as they participated in the figured world of Beatriz's classroom, through acts of asserting intention, positioning and authoring, critique, transformational resistance, and improvisation. However, looking across students' stories it became clear that certain actions that flowed effortlessly for students served as powerful entry points for further development and enactment of agency. The most significant, and at the same time, potentially conflict-ridden entry point, were students' opportunities to assert intentions. Stories such as Angel's, Jerica and Marlene's, and Lianna's (discussed at length in Chapters 5 and 6) clearly demonstrated that students' acts of asserting intentions can generate opportunities for them to participate in mathematics in ways that are personally meaningful and potentially transformative. Moreover, acts of asserting intention, when supported and developed over time, impelled students to enact other forms of critical mathematical agency, such as critique or transformational resistance.

A related entry point was students' opportunities to draw upon a sense of justice and fairness to level critiques[1]. Students brought to the classroom a collective sense of justice that extended beyond individual concerns about fairness (e.g., this is not fair for me) to a sense that issues of justice – and injustice – were also about members of one's school, family, community, ethnicity, and gender, and for the students in Beatriz's classroom, the members of other oppressed groups (e.g., Angel's and Naisha's concerns about income inequities discussed in Chapter 5, or other students' desire to film a commercial protesting unjust working conditions described by partner classes in India and Zimbabwe). Students drew upon this sense of justice to raise critiques (e.g., white doctors earn more than black and Latino/a doctors, and men earn more than women), critiques that generated justice oriented questions that were important to students (e.g., How much more? How long would a woman have to work to earn what a man earns?). Thus, much like students' propensity for asserting intentions, students' sensibility for

[1] See Gutstein (2003b) for a parallel discussion of how building on students' sense of justice can foster agency.

critique also served as an entry point for engaging in mathematics in personally and socially meaningful ways.

Complexity of Interactions Between Students' Intentions and Their Participation in Mathematics

While asserting intentions emerged as a powerful entry point for agency, students' stories also revealed that their efforts to interject their intentions into the curriculum encountered numerous constraints, and varying degrees of support. Interactions between students' intentions and their participation in the mathematical activity of the figured world were particularly complex. In some instances, students' experienced a productive intersection between (a) their personal intentions, (b) the potential for significant mathematical activity evoked by those intentions, (c) their capacity to understand the mathematical skills and concepts needed to address those intentions, and (d) how those intentions were supported, mathematically and otherwise, through their participation in the figured world. When these elements intersected (as they did for Lianna, Jerica and Marlene, and other students, see Chapter 6), this intersection supported students in moving from ideas for potentially compelling mathematical investigations to actually carrying those investigations; they were able to shift from general critiques of their school space (e.g., the hallways are too narrow), to constructing well-supported arguments aimed at affecting change.

Yet not all students experienced such powerful, productive intersections between their intentions, their mathematical understanding, and the supports and constraints of the figured world. At times, students asserted intentions that were best addressed by disciplines other than mathematics, as L.J. and Joel's story about "the poles in the gym" demonstrates (see Chapter 5). These students were forced to negotiate their intentions against the norms and expectations of the figured world, namely that all students participate in the projects in ways that involve significant mathematics. On other occasions, students' intentions evoked potentially powerful mathematical investigations, but the mathematics that they needed was not supported by the dominant discourse and activity of the figured world. Angel's desire to evaluate the girls' restroom based on wait time and number of stalls - instead of space - is a clear example of this dynamic (see Chapter 5). In other instances, students' intentions created the need for mathematical skills and understandings that were beyond their conceptual reach (see Chapter 6). Elucidating these complex interactions between students' intentions and the mathematics curriculum is a significant contribution of this research, and one that has been called for by other researchers in the field (e.g., Hart, 2003; Skovsmose & Valero, 2002).

Impact of Navigating These Contested Spaces

Even when students did not experience an intersection between each of the factors discussed in the previous section, their stories demonstrated that they benefited from navigating these difficult, contested spaces. By participating in a figured world where actors were striving to connect personal and collective intentions with significant mathematical activity, students experienced a shift, or the beginnings of a transformation, in their understanding of the discipline and its relation to their lives. Students came to see mathematics as a "life-long thing," and a vehicle for investigating situations that "you care about" and that are consequential in realms beyond the classroom. They identified mathematics as a tool to (a) investigate issues of personal and social importance, (b) explore issues of equity and fairness, and support critiques; and (c) to prove and argue their point of view. Moreover, during final interviews and focus group discussions students demonstrated that they could extend their developing understanding about the transformative power of participating in mathematics (Skovsmose & Valero, 2000) to other situations in their lives. They identified issues that concerned them (e.g., lack of support for the homeless, teenage pregnancy, bias in the media), and generated multiple, and specific ideas about how they could use mathematics as a tool for investigating, critiquing, and acting transformatively upon each situation.

Cross Cutting Tensions

Various tensions emerged across the stories of students' participation in a figured world aimed at "Teaching and Learning Mathematics for Social Justice." In chapter 6, I presented an analysis of these contested spaces. In this chapter, I want to discuss a broader, more general tension - between human agency and contextual constraints - that cut across the tensions previously described, and that emerged in each of the students' stories. In what follows, I outline how students experienced this tension through their participation in Beatriz's classroom, and how they made sense of the tension in ways that allowed them to retain a sense of hope that change and transformation were possible.

Students' Experience of the Tension

On a daily basis, students confronted the reality that human agency is not without constraint. As they attempted to interject their needs and purposes into the curriculum, they had to negotiate those intentions against a set of constraints (and affordances) that were very real. Students were aware of these constraints, which included things such as: available resources, limited time, the looming pressure to perform well on standardized tests, the expectation that participation in math class involved engaging in significant mathematics, among others. Other constraints were not as salient for students, but were quite present in Beatriz's mind, as the teacher. These included district instructional guidelines, the goals of the research project (which though designed to create spaces for empowerment, also acted as an imposition upon the 'regular' activity of Beatriz and her students[2]), students' conceptual understanding, and our developing understanding of the pedagogy we were attempting to enact.

This tension also emerged as students imagined ways to act transformatively on situations that concerned them (e.g., the overcrowding problem at their school), and even more so, as they began to carry out those actions. For instance, Naisha was forced to adjust the form and content of her speech to district administrators, in order to comply with "the language" of district building codes (e.g., standard versus metric units), and to remove potentially provocative references to other middle schools (see Chapter 5). As other students considered how to respond to the "space crisis" at Francis, they grappled with questions such as: What can we really do about the overcrowding? It is even possible to get a new school? Could we switch floors with Longmore? Is that even possible? And what about the poles in the gym, can they be removed? Could we build a new gym?

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[2] See Vithal (2000) for a discussion of how research interventions aimed at promoting equity and justice can also function as impositions, and even have a disempowering effect.

An increasingly critical analysis of their school space (e.g., students' outrage when they learned that only one hallway, and several classrooms were 'legal' according to district building codes), coupled with a growing awareness about the constraints that limited the possibility of transforming that space (e.g., structural, financial, historical, political considerations), could have been disempowering to students, raising their consciousness about unjust conditions, but leaving them feeling like they are powerless to affect change. For this reason, it was important that students had opportunities to engage in some kind of responsive action, even if that action was as simple as sharing their findings with other students at the school. As Gutstein (2003b) argued, "these may be small steps, but they provide openings for students to develop a sense, and imagine the potential, of their own actions in the world" (p. 23).

But how did students make sense of these "small steps"? Did they find them meaningful? Did they view actions such as writing letters to district superintendents as worthwhile activities, or as pointless efforts to transform a situation that had no possibility for change? In some cases, students concluded that change was unlikely, For instance, L.J., who produced a video and wrote a letter aimed at convincing the school district to remove the poles in the gym commented

> I learned that sometimes you can change stuff and sometimes you can't. Maybe we can change that this space is too small, but maybe we can't. Maybe this time, we just have to learn how to use it.

Yet many other students, despite their acute awareness of the constraints imposed by the figured worlds of their classroom and school, and by broader institutional structures largely beyond their control, retained a sense of hope that change was indeed possible, and that their actions could contribute to that change. What accounted for this powerful sense of hope?

Hope for Change and Transformation

During class discussions and individual and small group interviews, I asked students whether or not they felt that their actions (as part of the "Space at Francis Middle School" unit) would make a difference, and if not, what would need to happen for change to occur. What emerged from our conversations was that students' found a sense of hope and possibility in particular kinds of actions that involved (a) collective endeavors, (b) resistance, (c) building on the struggles and success of others, or (d) persuading individuals in positions of power. The following sections describe each of these "sources of hope."

Collective, resistant action. Students felt very strongly that their efforts to impact change would be more effective if they were collective endeavors that involved some kind of resistance. For instance, Carlos stated,

> maybe we should talk to other kids that are also at schools that don't have a lot of supplies or that are short on space and then we could all go together and stand outside and protest. I think then people would be sympathetic and want to help us out.

Jhana echoed Carlos's reasoning, explaining that while the district would never listen to an argument presented by "one or two of us," they would be more likely to respond to an argument presented "by the whole 6^{th} grade." She commented

> There are a lot of kids in our school and the whole 6th grade, well the 6th grade math classes that did this, there was a lot of us. And I think if there was only one or two of us they wouldn't really think about it, like noone really listens – but since there were so many of us I think that they will actually think about it and might do something about it. Not necessarily enlarge the space, which you can't really do, but like give us more floors or next year not put in as many students.

Jhana not only argued for the power of collective action in her comment, but also demonstrated a clear awareness of how structural constraints impacted the kind of change that was possible.

Action that builds on the struggles and successes of others. Students also experienced a sense of hope when they were able to link their own struggles with those of others like them, who had faced and surmounted similar challenges in the past. For example, in an effort to defend his position that transforming the school space to reduce overcrowding was indeed possible, Evan shared the following story. He explained

> In my old school, they had a very little space. I went back to visit and they weren't there and so I figured out that they went to another building. They were fighting for more space, and I found out that they won and they got to move to another building because they won the battle.

For Evan, the actions of students and teachers at his "old school" provided an example of how 'ordinary' people in his own community can struggle for justice and impact change. This example supported his *own* efforts to engage in transformative action, and his ability to find a sense of hope that his actions could make a difference.

Action that convinces individuals in positions of power. Finally, students were acutely aware that addressing the problem of overcrowding at their school required persuading individuals in positions of power, such as the superintendent, or school board members, that the school needed more space. Angel thought that school board members might respond for personal motives, to make themselves look better by demonstrating that they really did care about the students. She commented

> Because the amount of kids that we have in this school, it counts a lot. It's kind of like bad [in the school right now], and it's showing like how much we care for the students, and the board of education is hardly doing anything right now, so let's get some points on that [meaning the board of education might think they can earn 'points' by helping students and teachers at Francis].

Naisha doubted that any change would occur while she was still at the school, but believed it was nonetheless important to keep pressuring district administration, because "if you keep talking to them, then they will probably listen, and you will get on their nerves, and maybe then they will want to give us more space, or let us be in a different building with more space, [a building] that is lawful."

Yet aside from whether or not students believed that their efforts would result in any lasting change, they experienced a need, or in some cases a sense of responsibility to speak out and take action against unjust conditions, particularly because their unique experience of those conditions was something that they alone could contribute. As Lianna argued, "we *have* to say something because *we* are the students and *we* are the ones that have to live in the school everyday."

In the previous section, I summarized the primary themes that emerged across students' stories, emphasizing the ever present tension between student agency and contextual constraints, and describing how students made sense of that tension in ways that allowed them to retain a sense of hope for the possibility of change. In the next section, I outline the implications of this study for future teaching and research aimed at promoting equity and justice in/through mathematics education.

Implications for Teaching and Learning Mathematics for Social Justice

The complexities of "Teaching and Learning Mathematics for Social Justice" are many, as the stories of students in Beatriz's classroom illustrated. The constraints that challenged teachers' and students' enactments of agency were real, and the tensions that arose, varied. Given these challenges, I find myself asking the following questions. First of all, is this approach worth it? I feel strongly that it is, and trust that the stories of how students enacted critical mathematical agency, and the descriptions of the shifts in students' understanding of the discipline and its relation to their lives, present a compelling enough case (see Chapters 5 and 6). Secondly, if we acknowledge the potential power of this pedagogical / curricular approach, how might the themes and tensions that emerged from this study inform the work of other mathematics educators who aim to promote social justice? The following section is an attempt to address this second question.

1. Create spaces for students to assert personal and collective intentions. If we accept Gutstein's (2003b) argument that in mathematics classrooms that provide opportunities for students to develop and enact agency, students' participation can become a means of fostering equity *in* and *through* mathematics education, finding ways to create such opportunities becomes very important. The stories of students in Beatriz's classroom suggest that a powerful way to animate students' agentive action is to create spaces for students to assert personal and collective intentions. I want to argue that "spaces for students to assert intentions" should be created as part of the daily life of the classroom, not as an add-on activity that is situated within an otherwise teacher controlled environment. At times, these "spaces" might allow students to negotiate core aspects of the curriculum, such as the focus of a unit of study. On other occasions, students might have opportunities to interject their intentions in smaller, day-to-day activities, such as posing problems for the class to solve, or choosing between a set of activities. What is important, from the perspective of fostering agency, is that students' intentions are invited into the classroom space on a regular basis.

2. Recognize and reflect on the challenge of constructing intersections between students' intentions and the mathematics they need to study. Teaching and learning mathematics for social justice requires ongoing critical reflection not only on the part of the students, but also their teacher. Negotiating intersections between students' context, their intentions and purposes in the project, and the academic context, the disciplinary knowledge and practices that would support their endeavor, is challenging. To support teachers in this challenge, they need to critically reflect on issues such as, (a) how students' ideas and conceptions might be challenging the dominant ways of thinking about mathematics in the figured world, (b) how their own ideas about mathematics might limit their capacity to see the mathematical potential imbedded in students' intentions, and (c) when students' intentions would be best addressed by disciplines other than mathematics.

In order to leave room for mathematics to emerge (or not) from students' investigations of their intentions, teachers may opt to balance the curriculum between units of study driven by the goal of students' mathematical understanding (e.g., reform curricula), and other projects that hold the development of students' critical mathematical agency – which includes their mathematical understanding, among other elements - as central. Creating such a balance might afford teachers more freedom to allow students to genuinely explore personal and social intentions and issues of justice during this second set of projects, because the responsibility for supporting students in developing mathematical understanding is shared by other, more content driven units of study (see Gutstein, 2003a, for a discussion of this balance).

3. *Recognize the inherent trade-offs of simultaneously working towards multiple instructional goals (that cut across disciplines), and be explicit about the choices that are made*. When Freire (1970/1993) argued for a "problem-posing" pedagogy based on generative themes that emerged from students' lives as a vehicle for fostering the empowerment of oppressed groups and promoting social justice in education, he did so from a place not bound by the kind of powerful constraints that characterize our current system of public education. His teachers and students were not held "accountable" for their performance on standardized tests, and no district or state guidelines mandated the content that should be covered in his classrooms.

When teachers like Beatriz attempt to enact a problem-posing pedagogy with their students, their actions are necessarily constrained by the social/political/historical reality within which “school” takes place. As a result, tensions arise, and trade-offs are inevitable. What seems to important is that teachers (a) recognize these tensions, and more importantly (b) make conscious, explicit choices about how to address these tensions in their classrooms. That is, if teachers, like Beatriz, decide at a certain point to accept less than rigorous mathematical understanding, or to “present” students with skills and concepts that they might not fully understand, but that they need to move forward with their intentions in the project, teachers need to explicitly communicate these decisions, first to themselves, and when appropriate, to students.

Implications for Schools

In order for schools and districts to foster the pedagogical and curricular suggestions outlined in the previous section, there are numerous structures, policies, and other elements that would support their efforts. To begin, teachers need time to meet with one another, and to critically reflect upon the development and enactment of critical pedagogy in their classrooms. Teachers would benefit from opportunities to discuss common dilemmas, and to learn from how other teachers resolve the tensions that arise. Forming teacher study groups at the school level, that address particular issues of concern such as (a) responding to the pressure of standardized testing, (b) supporting students’ understanding of the mathematical content that emerges as units of study develop, (c) balancing a reform oriented curriculum (such as *Connected Mathematics*) with a project-based approach, and (d) inviting families and community members to collaborate in curriculum development, among others. It is important that whenever possible, this time is structured into teachers' standard school day, so that teachers feel respected and supported in their efforts to enact a negotiated, emergent curriculum in their classrooms.

In addition, the way that schools structure a typical day could have a significant impact on teachers' ability to implement teaching mathematics for social justice. For example, 45 to 50 minute class periods are not as conducive to this pedagogy as block scheduling (e.g., math periods that last 90 minutes every day, or 120 minutes every other day) might be. Creating opportunities for a single teacher to address multiple disciplines (e.g., mathematics and social studies, mathematics and science), would also be beneficial, as allowing students to investigate situations of personal and collective importance invariably leads to projects that integrate multiple disciplines. Allowing teachers to teach more than one subject matter at the middle school level can create challenges, because teachers are often not certified in multiple subjects, or do not feel comfortable teaching across disciplines. In such cases, schools might support teachers in forming grade level teams, where teachers could meet regularly to share the results of their problem-posing pedagogies with one another, and whenever possible, coordinate their efforts.

Teaching mathematics for social justice, like many student-centered, progressive pedagogies, would also be enhanced by reductions in class size. Supporting the mathematical understanding of 30 or more students is a significant challenge. Add the demands of supporting students with a diverse set of mathematical skills and concepts that become necessary as projects develop, and implementing a teaching mathematics for social justice pedagogy with a class of 30 or more students can become an overwhelming task. Efforts at the state, district, and school level to reduce class size to 20 or 25 students would greatly facilitate teachers' efforts.

Directions for Future Research

Given the paucity of empirical, classroom based studies that offer an extended, systematic analysis of teaching and learning mathematics for social justice, and its impact on student agency, there remains an urgent need for this kind of research. To begin, future research might examine how student agency develops over time, and across contexts, for as Holland et al., (1998) has argued, "Conceiving oneself as an agent whose acts count in, and account for, the world cannot happen overnight" (p. 285). Of particular interest would be how certain pedagogical / curricular practices (e.g., elements of teaching and learning mathematics for social justice) support the development and enactment of student agency over time. Such research could address questions such as: How do students' extended experiences in a social justice oriented mathematics classroom influence their capacity to act as agents of change, over time? And does their sense of themselves as people whose actions can and do make a difference extend (or not) into other contexts of their lives?

A second direction for future research would be to more closely examine teachers' decision-making processes as they negotiate the myriad of tensions and challenges that this kind of teaching evokes. Research could strive to understand how teachers address these tensions in ways that are productive (or not so productive) for certain valued outcomes such as student agency, and students' mathematical understanding. Understanding how different teachers, working in different conditions, and faced with different sets of affordances and constraints manage the tensions that invariably arise when teaching and learning mathematics is driven by a broad set of goals (e.g., student understanding, the development of critical agency, opportunities for students to participate in mathematics in personally and socially meaningful ways, etc.), could offer powerful insights for other teachers and researchers aimed at promoting social justice in their work.

A third direction for future research would be to integrate the perspectives of multiple actors, such as students, parents, community members, other teachers and administrators into a study of the *meaning* and *impact* of a teaching and learning mathematics for social justice approach. Research might seek to understand how students' families and community member make sense of students' school mathematics experiences, and how insights from families and community members might challenge, enhance, or in some way change the way that teachers and researchers conceptualize "mathematics education for social justice." For instance, studies might explore questions such as: What does teaching and learning mathematics for social justice mean for parents and family members? How might community members' ideas, perspectives, and values related to mathematics education be incorporated into students' school math experiences?

Final Thoughts:

The Impact of Students' and Teachers' Actions at Francis Middle School

At the end of the study, it was still unclear whether or not the district would increase the school's allocated space or make any adjustment to the number of incoming students that Francis had to accept. So unfortunately, Beatriz's students ended 6th grade not knowing whether or not their efforts would have any immediate, direct impact. However, at some point over the summer, as the district was making final determinations about space and student enrollment, they decided to reduce Francis' incoming 6th grade class, by approximately 25 students, so that the number of 6th graders entering would be equivalent to the number of 8th graders leaving the school, thereby allowing the school to retain its current size and level of overcrowding. While this result may seem inconsequential – the school is almost as overcrowded as it was last year, the district's action prevented the situation from deteriorating even more. For a school whose total enrollment had been officially set to increase from 213 students to over 240 students, a district decision to reverse that increase was welcomed as one small success.

Yet, what seems most important is not whether or not this particular "battle," to use Evan's words, was won or lost, but that students and teachers had the opportunity to conceptualize their struggle as one step in a larger process of conscientizacao (Freire, 1970/1993). The history of social movements tells us that whether or not one wins or loses a particular struggle (and there were definitely struggles that students lost, such as their campaign against the poles in the gym), one has to accumulate the lessons that were learned, recognize what was gained that cannot be lost (e.g., critical consciousness, increased understanding), and develop a more protracted view. Jhana demonstrated signs of this understanding in her reflections on the impact of the project. In response to the question, "Do you think what students did made a difference?" She commented

> Yes, because we could – well yes, because first of all we found out something for ourselves and we actually proved a point. We actually, like WE made the

> difference, like we can. Yeah, because if you want to prove a point then it doesn't matter [what the point] is, you need to have learned what we learned. And we learned that, and we told people. And maybe they are not going to do anything about it, but we still proved a point.

Teachers, researchers, and other adults can play pivotal roles in helping students to come to this understanding. One of tenets of this pedagogical approach, and of the broader struggle to promote equity *through* mathematics education (Gutstein, 2003b) is to help students "see themselves as key participants in the struggles for equity and justice" (p. 27). Coming to "see themselves" as active agents of change means that students not only have opportunities to engage in transformative action, but opportunities to develop an understanding of how their own actions and struggles for justice fit into a larger story of social struggle, struggles that develop over time, and that consist of many gains and losses, including less visible, but enduring gains, such as shifts in understanding and increased critical awareness. These enduring gains, or to use Jhana's words, what students "learned" and "found out" and "proved" for themselves, are important because they can strengthen students' sense of themselves as capable, political actors, which in turn motivates them to continue to engage as critical agents, thereby furthering the struggle for justice.

Appendix A

Researcher as Instrument Statement

As the collaborative critical ethnography I plan to engage in this semester begins to take form in my mind, I find myself reflecting on my own role and relationship to the study. I am, after all, the research instrument, and therefore I feel it is imperative for me to recognize and share and those experiences, values, beliefs, and expectations that will undoubtedly influence aspects of this investigation. The general topic I will explore in this study centers on the critical mathematical agency that students enact as part of their participation in a mathematics classroom aimed at teaching and learning for social justice. Given this focus, I think it is important to examine my experiences and beliefs about teaching, about math, and about the nature of math education for marginalized groups of students.

As many researchers have observed, we often choose to investigate that which we are passionate about, those topics that have relevance, meaning, and significance in our lives. In my case, this observation is definitely true. There are few issues in life that I feel more passionate about than teaching, particularly teaching in urban settings. Yet now that I have left the classroom, I still call myself a teacher, think of myself as a teacher, and feel a deep sense of connection and solidarity with those who work in schools. Before returning to graduate school, I spent five years teaching fourth and fifth grade in a bilingual school near downtown Phoenix. I carefully choose this school for its commitment to honoring the cultures and languages of all students, and for its reputation for challenging traditional notions of the purposes and practices of schools. It was consider by many in the local educational community as a radical place of learning, which attracted me even more. At the time, I don't think I fully realized why I was drawn to that particular school, nor was I aware of the emotional, mental, and physical journey that I was about to begin. But I couldn't imagine working anywhere else. And I sensed it had something to do with an abiding desire to be involved in setting that I considered to be transformative and empowering.

For as long as I can remember, I have been drawn to issues surrounding empowerment and justice. I have followed intently the struggles of the Zapatistas in Chiapas, Mexico, the Hopi and Navajo in northern Arizona, and the sweatshop workers in Asia, among others, all stories of communities uniting in their common struggle for justice, freedom and power. The writings of Paulo Freire and other critical theorists, and their call for praxis, the process of reflection and action to challenge patterns of domination and oppression, have impacted me tremendously. I attempted to incorporate some of these ideas into my practice as a classroom teacher. I was a strong advocate for bilingual education, and for the true involvement of parents and community members in all aspects of the educational process, from curriculum design to school finance to assessment practices. I tried to listen to the voices of students in my classroom, and to collaborate with them to construct our classroom curriculum and pedagogy.

Through interactions with students and their families, I knew students' lives were rich sources of knowledge, experiences, and ideas, including ideas about their lives, futures, and communities. Themes such as immigration, poverty, fair working conditions, and discrimination surfaced repeatedly in our conversations. While these issues were salient for students, they were issues that school in general, and mathematics in particular, failed to help them address. As a teacher, I reflected on how urban schools might provide mathematics education that builds on students' diverse strengths, interests, and experiences, and helps them recognize mathematics as a transformative force in their lives. I struggled to create these connections in my classroom.

Coming to graduate school has definitely helped me to articulate my ideas about empowerment, specifically about the role of schooling in creating spaces and opportunities for students, families, and community members to empower themselves. I believe that one of the central purposes and outcomes of schooling should be empowerment. This belief shapes my view of the purpose of learning academic content, particularly subjects like math and science, which are historically inaccessible for many minority groups. In my opinion, the purpose of learning math goes far beyond learning the content for the content's sake. Instead, I think that students can learn to use math as a tool to understand, challenge and better their own lives and communities. In addition, knowledge of math can allow students entry into fields, schools and jobs that traditionally have not had equal representation of minority groups (e.g. medicine, research scientists, engineering). I also think it is important for students to have opportunities to critique the discipline of math, to challenge assumptions such as who can do math, what counts as real math, and what "doing math" looks like.

I believe that the math that gets taught in schools should be relevant and meaningful to students' lives. In fact, to me what math is all about is exploring and making sense of *your* world, and then using that knowledge to better your world. As a teacher, I tried to make connections between the content I was teaching and the children's lived experiences whenever possible. In fact, not only should math education strive to make connections to students and their lives, but I believe that the process of learning should center on the students, and not on the subject matter. Not that mathematics is not an important, powerful discipline, but I believe its value is enhanced insofar as it relates to the lives of those who study. I realize this is a very difficult ideal for classroom teachers to put into practice, especially with designated curricula, standardized tests, and pressures from administrators. This was a continual struggle for me as a teacher, but one that I felt was worth the effort.

When I think about myself as a teacher, and the role that teachers might have in math classrooms, the words facilitator, advocate, and co-constructor come to mind. I see myself as working with the students and sharing power to make important decisions about what gets taught and how it gets taught. I feel a responsibility to prepare students with the knowledge they will need to be successful in our current educational system, but I also feel a strong sense of agency. I believe that I have the ability and the right to challenge current curricular documents and to adapt or recreate them to meet the needs of the students in my classroom. I also believe that teaching is inherently a political act, and as a teacher, particularly one who works with marginalized populations, I am a cultural and political worker for my students. As a teacher, I also see myself as a continual learner. I value learning from students, their families, from colleagues and experts, and I am very willing to admit that there are many answers I do not know, and many experiences I had not had.

I have continued to think critically about these issues throughout my graduate studies: as a graduate research assistant investigating systemic reform in urban schools, in teaching mathematics methods courses for pre-service bilingual teachers, in mentoring novice teachers as they craft mathematics lessons that draw on students' experiences, and in my study of the work of critical and feminist theorists such as Paulo Freire, Sonia Nieto, and Nell Noddings. These experiences have offered me more powerful ways of thinking about relationships between students, schools, and society, and about the potential role of math education to promote equity and social justice. Over the last three years these questions have evolved into the focus of my dissertation study.

As I enter this critical ethnographic study with Beatriz and her students, I expect to find a teacher who is deeply concerned about her students, and who wants to do everything possible to facilitate her students' learning and success. I also expect to encounter challenges in our research relationship, challenges due to time limitations, differences in goals and perspectives, and most importantly, challenged related to the fact that Beatriz and I are not only co-research, but friends. In terms of the students, I expect to find students that vary considerably in their views of mathematics, in their mathematical understanding, and in their propensity for actions such as critique, resistance and transformation. I expect some students to speak out on a regular basis, to resist what they perceive to be inequities, and I expect for others to acquiesce.

Honestly, after much thought and reflection, I do not believe that there is anything that I am not willing to discover, although there are some things that I *hope* I do not discover. For example, I would be disappointed, and even concerned I discover that the students have no interest in drawing on mathematics to investigate issues in their lives, or if I discover that they are very resistant to engaging in mathematics in ways that might differ considerably from their previous school experiences.

I hope that the results of my research will help others interested in promoting equity in and through mathematics education, particularly in urban settings. I hope to gain insights into students sense of agency, and in particular, how their engagement in the discipline of mathematics supports / works against / interacts with their sense of agency.

I also hope that the experience of participating in this investigation will be beneficial to the students. While I realize that this study is rather short in duration (8 months), I hope that some aspect of the project will be helpful to them. As Richardson writes, "A continuing puzzle for me is how to do sociological research and how to write it so that the people who teach me about their lives are honored and empowered" (as quoted in Erlandson, Harris, Skipper & Allen, 1993 p. 27). This is also a puzzle to me, though one that I think is important to think carefully about. Hopefully the opportunity to participate in projects that encourage students to relate mathematics to their lives with be helpful to students, in terms or their motivation to learn mathematics, their feelings about the discipline, and of course, their mathematical understanding.

Appendix B

Sample Lesson Plan (Expanded with Field Note Entries)

Income Data Lesson

Part I, Wednesday March 27th - Part II, Wednesday April 10th
(spring break and two days of test prep in between)

Context / Background

Students had been participating for about 4 weeks in a collaborative on-line project, entitled "Connection Math to our Lives", hosted by Orillas (www.orillas,org), and I-EARN. The aim of the project was 'joining others around the world in examining their own lives and communities and broader issues relating to social justice and equality from a mathematical perspective.' Students had previously collected local data about the prices of various shopping list items (on a shopping list that they came up with). They research prices and analyzed how and why prices might be different for different brands/stores. They then compared their lists and prices with the data submitted by other students in the project – form various parts of the world. One of the questions that arose was about how to compare the prices, and how to tell, relative to how much money people have to spend, how expensive a given item was. (e.g. what does it mean to spend $2.00 on milk for someone in New York, versus spending the equivalent of $0.50 on milk in India?… in other words, what are the 'real costs' to people making different amounts of money in different places?) The students hypothesized that people earned less in other countries because they didn't need as much money, because things (food and other items) were so much less expensive. To help us think about these 'real costs,' our students, and other students in the project, gathered information about the salaries earned by different workers in their area/country – which is where this lesson fits in.

<u>Lesson Break Down:</u>

PART ONE:

I. **Small group discussion:** The 'real costs' of our shopping list.

Students discuss the following questions in small groups:

So how much money would we need to buy all these items?

Does that amount of money mean the same / or feel the same to different people who work at different jobs? For example, does it mean the same for a teacher to spend this money? For a lawyer to spend this money?

II. **Large group discussion:** Finding income data on-line

Students share comments from their small group discussions, included is a discussion of where we might be able to find out how much different people earn? We introduce the concept of a census….. (though this source of information is problematic) and talk about how the census makes their information available on-line. Using a computer projection system, students view the website and decide which jobs there are interested in learning more about, and how they want to view those jobs.

(Field Notes about this discussion)

When we did this activity with the kids they were very interested in a number of jobs, including: actors, NBA players, doctors, lawyers, store workers, sanitation workers.

Students had a number of ideas about different ways that they wanted to look at the income data. Miranda and Shania wanted to look at comparisons between black males and black females. Carlos wanted information about new reporters and Rico about vets. Antoinette wanted to know how much plastic surgeons wanted. Julia wanted to look at teachers – but in particular to see how much females of different races make as teachers. Other students wanted information about actors and actresses – in particular comparing how much black female and black male actors and actresses make. Emilee talked about how much Asians make – and how come that group is not represented – or at least accounted for in the break down of the census data by race and gender. Andrés yelled out: It's not fair! Carlos asked: I want to know why that is? We left that question open for now.

[After this part of the lesson, we compiled the income data that the students were interested in looking at into a chart. Originally, we intended for students to use the website to access this information themselves, but due to time constraints, and internet access issues, we decided to find the information students requested ourselves, and then share it with them.]

PART TWO

Mathematical Goals:

- Interpret chart, identify patterns/trends in the data.
- Make mathematical statements (involving comparisons, ratios, differences) based on the information in the chart.

- Pose questions based on the information in the chart and then use the chart and mathematical operations and comparisons to investigate those questions.

I. **Whole Class Introduction: Discussion of male/female income chart**

Pass out male/female salary graph, from Field Economy book. Students record individual thoughts, discuss meaning as a large group. [This chart shows changes in male/female salary gap in recent years, through a bar graph. The chart shows that a gender gap exists for whites, blacks, Hispanics – only groups shown on the chart. Data is presented in terms of percents. Class discussion focuses on relating percent figures to amounts of money. E.g. If a white woman makes 87% of a white man's salary, how much does she earn for every dollar a white man earns? What about for every 5 dollars?]

II. Partner Work: Analyze Income Data Charts

Share INCOME data chart with students. [Income data chart is basically a table we made that shows the average annual salary – taken from the 2000 census data, for the 10 jobs most requested by students. Salary data is divided by gender and race].

- Students take time in their pairs to look over the chart and to reflect on the questions on the work sheet. What do they notice? How can they use math to think and talk about what they notice?
- Each pair of students needs to pose one question, based on the information in the chart, that mathematics would help them to investigate.

(Field Notes about partner work)

•Jhana and Manny focus on the difference between NBA and WNBA salary. They pose the problem of 'how many years would it take for a WNBA player to make what a NBA player makes in one year?" Jhana begins by successively adding 55,000 (WNBA average salary)…. Then decide it will take her forever to get to 3,000,000. They decide to divide.

•Vellez decides he wants to use fractions to talk about the relationships between the salaries. He starts with the WNBA to NBA ratio (55,000 / 3,000,000), and asks himself, is the NBA ten times as much, twenty times as much? He tries to get to a 'familiar fraction' …. Which is difficult because of the ratio he is working with. He finally comes up with a 1/50 relationship – which he says is a close estimate.

• Shayna and Nickole discuss general trends in data in terms of gender/race. They focus particularly on why white women(and men) make more, and then test this idea by looking at white vs. black, white vs. Latino data for each job. They start to think about how to figure out how much more.

• Carlos subtracts to find the salary difference between HM and BM lawyer, doubles the difference to find difference for 2 years, and continues on in this manner, to find the difference in three years, four years, etc..

• Andrés spends most of the time looking / talking about the trend in the data that white men always make more. He link higher salary of white men to ideas about power, white men feeling superior, in control.

• Joel use repeated addition to calculate how many years WNBA player needs to work to earn what NBA player makes

• Chantelle focuses on the differences in male/female salary, across race. She talks about females working two jobs so that they could make the same as men, questions why whites make more.

II. **Whole group sharing of findings.**

Call on groups to share the patterns / trends they identified, the questions the posed, how they investigated those questions, and what they found.

Appendix C

Math Talk Interview Protocol (Initial)

Part One: Stories, beliefs, and experiences about school and community

School and Community

I am really interested in your ideas about your school and your community.

1) What are two things about your school that you really like, or that are really important to you?

Probes: Why is it really important to you? What do you like about it? What does it mean to you?

2) Now what is something about your school that you wish was different, or that you might like to change?

Probes: Why would you like change it? How do you wish that it was? What do you think you could do to change it? Do you think that would make a difference? What do you think would need to happen for it to change? Is there anything else you would really like to change?

3) Can you tell two things about your community / neighborhood that you really like, or that are really important to you?

Probes: Why is it really important to you? What do you like about it? What does it mean for the community?

4) Now can you think something about your community / neighborhood that you wish was different, or that you might like to change?

Probes: Why would you like change it? How do you think it got to be that way? How do you wish that it was? What do you think you could do to change it? Do you think that would make a difference? What do you think would need to happen for it to change? Is there anything else you would really like to change?

Specific Educational Experiences

5) I am wondering if you have ever been a part of any kind of project, in school or outside of school, where you have done something that helped your school or your community in some way. This could be a project where you did something to help, or to make something better, or to change something in the community.

If no follow with probes:

Could you imagine doing something like that in school? Why or why not?

What do you think it would be like?

If yes, follow with probes:
What was that project like? What did you do? Why do you think you did it?
Did it make a difference? (Why or why not?)
What did you think of the project? How did you feel about the project?
What did it mean to you? What did it mean to the community / school?
Did the project involve any math?
If yes: Can you tell me about how it involved math? How did you use math?
If no: Can you imagine doing a project like that one that did involve math?
If yes: What would it be like?
If no: Why not?

6) I am interested in other kinds of things you have done in school, or experiences that you have had in school, that have been really important to you. What is some project or activity or experience that was really important to you?

Probes: What did you do? Who was involved? How did you feel?
Why do you think you did this project? Who decided on the project?
What did it mean to you? Why was it important to you?
Did the project involve any math?
If yes: Can you tell me about how it involved math? How did you use math?
If no: Can you imagine doing a project like that one that did involve math?
If yes: What would it be like?
If no: Why not?

7) Now I want you to think about what kinds of projects you wish you could do in school, or experiences you would like to have in school. What is a project / activity / experience that you would really like to do at school?

Probes: What would it be like? What would you do? Who would be involved?
Why would you like to participate in this?
Do you think this could happen? Why or why not?
Would this project involve any math?

Part Two: Stories, beliefs, and experiences with mathematics

Questions in this section are adapted from the 'Mathematics life-story interview for teachers' presented in Drake et al.,(2001, p. 21-22), and based on McAdams (1993).

Today I am going to ask you to talk about some of your experiences with mathematics. You can share stories about your experiences with math in school, or outside of school.

8) Earliest experience

Can you think back on when you were younger, and tell me about your first memory of math? This could be a memory about the first time you remember having an experience with math, at home, or at school.

Probes: Can you tell me more about what happened? Who was involved? What did you do? How did you feel?

9) High point experience

Can you tell me about a time in your life that was a real high point for you in math? This would be a time when you felt excited, or happy, or good about your experience with math.
Probes: Can you tell me more about what happened? Who was involved? What did you do? How did you feel? Why was this such a high point for you?

10) Low point experience

Can you tell me about a time in your life that was a low point for you in math? This would be a time when you did not feel very good about your experience with math, or when you didn't like what was happening, or you felt frustrated or upset.
Probes: Can you tell me more about what happened? Who was involved? What did you do? How did you feel? Why was this such a low point for you?

11) Visions for the future

Now that you have told me a little bit about your past, I would like you to think about your future. Can you tell me what you would your experiences with math to be like in the future?
Probes: What would your future be like? How would math be a part of your future? What kinds of things would you be doing? Can you imagine using math to change something, or make something better? What goals and dreams do you have for your self? Do you think this future will happen?

Other beliefs about mathematics

Questions in this section are based on previous "Math Talk" interviews conducted by the researcher with elementary school students. (Ask these questions only if these issues did not arise in students' responses to the previous questions)

12) Conceptions of math

What would you say to your little brother or sister (or to a new student to the school, if the child does not have a younger sibling) if he or she were wondering what math was? What would you tell them? How would you describe math to them? Are there other things that you think of? What would you say to a student at your school that said, "What's the point of learning math?" What would you say if the student said that they have learned enough math already?

13) Use of mathematics in the world

(For these questions, think about doing and learning math outside of your classroom)

Who do you know that does math or uses math?
How do they use math? What do they use it for?
Do you ever use math outside of school? How do you use it?
Can you think of other ways that people use math?
Can you think of other ways that you could use math?

Appendix D

Math Talk Interview Protocol (Final)

Part One: Stories, beliefs, and experiences with mathematics

Reflections on use of math in Bea's classroom

1) I want you to think about the **math you have done this year in Ms. Font's class**. If someone else had never been to your math class but wanted to know about it, what would you tell them?
    What is your math class like?
    What kinds of things do you do?
    What was math like this semester? What did you think about this semester? How was this semester different from the fall semester?
    How is math the same or different from your other classes?

Reflections on the three projects

2) **The three projects**. I want you to think now about the three projects that we did in Ms. Font's class. First – the project about the world distribution of wealth and how much things cost and how much people make. Second – the project about overcrowding at Francis. And Third – the project about the global sweatshops.

    What did you think (feel) about each of these projects? What was interesting to you? What did you like and not like?
    How were these projects the same or different from other things you have done in math class?
    How were these projects the same or different from other things you have done in other classes?
    Was one of these projects more interesting to you … and why?
    Was one of these projects less interesting to you …. And why?
    Which project do you think helped you to learn the most? Why?
    What did you learn by doing these projects? (general)
    What kind of math did you learn?
    How do you think these projects were alike? How were they different?
    If you could pick another topic to investigate what would it be?
    Would you like to do any projects like these again? Why or why not?

Part Two: Stories, beliefs, and experiences with mathematics

Conceptions of math

What would you say to your little brother or sister (or to a new student to the school, if the child does not have a younger sibling) if he or she were wondering what math was? What would you tell them? How would you describe math to them? Are there other things that you think of?

What would you say to a student at your school that said, "What's the point of learning math?" What would you say if the student said that they have learned enough math already?

Use of mathematics in the world

Who do you know that does math or uses math?

How do they use math? What do they use it for?

Do you ever use math outside of school? How do you use it?

Can you think of other ways that people use math?

Can you think of other ways that you could use math?

Scenarios (if time)

1) [Scenario taken directly from issues and comments that students generated in class discussions]

A group of middle school students was learning about the newspaper and they started to ask some questions about the articles they were reading. They noticed that a lot of the articles about crime and drugs were about black and latino men….and there were not a lot of article about good things that blacks and Latinos were doing. One student also said that a lot of the articles about businesses and artists and people in government were all about whites. This group of students decided they wanted to investigate this issue.

What do you think of the issue?

How could you investigate it?

What would you need / want to find out?

How could math help you to investigate the issue?

How could math help you to present your argument?

2) [Scenario taken from school wide discussion of standardized testing, and 8th graders decision to boycott the tests]

Here were have some data about how kids have done on tests they take in highschool. Take a few minutes to look at the data. What do you think? What do you notice? What else would you want to know? What questions does this raise for you? Could you do anything? How could math help you?

Appendix E

Clinical Interview Protocol

Section One: Area Problems

1) Find the area of this Dance Room in square meters
(show drawing#1)

> *Probes: How did you solve that? What did you do? How did you know to do that?*
> *How do you know that is the whole area? Is there another way you could have solved this problem? How do you know that would work?*
> *(also probe student to relate multiplication calculations to the diagram – e.g. where do we see 12 X 6 or 12 X 1/2 on the diagram?)*

1z) Easier version for students who are unsuccessful
(show drawing #2)

> *Probes: How did you solve that? What did you do? How did you know to do that?*
> *How do you know that is the whole area? Is there another way you could have solved this problem? How do you know that would work?*

2) This is a classroom downstairs at P.S. 165. Find the area in square meters.
(show drawing #3)

> *Probes: How did you solve that? What did you do? How did you know to do that?*
> *How do you know that is the whole area? Is there another way you could have solved this problem? I see that you multiplied _______ . How did you know to do that? What part of the area did that help you find? (in particular probe about the connection between procedures students do and how those procedures relate to finding the area of different parts of the diagram)*

2z) Easier version for students who do not solve successfully OR who I suspect may need an easier problem to start
(show drawing #4)

> *Probes: How did you solve that? What did you do? How did you know to do that?*
> *How do you know that is the whole area? Is there another way you could have solved this problem? I see that you multiplied _______ . How did you know to do that? What part of the area did that help you find?*

Section Two: Multiplication of Fractions Problems

3) Solve this problem. (multiplication of fractions)

Start with problem A and proceed to problem B if successful and problem C if not successful. (Except students who have struggled with previous multiplication of fractions…. Then start with problem C).

A. 1/2 X 3/4 =
B. 2/4 X 1/8 =
C. 1/2 X 1/4 = or (harder) 1/2 X 1/3 =

Section Three: Ratio and Issues of Crowding

4) Which gym is more crowded**?**
(show drawing #6) Drawing depicts area of two gyms and total number of people in each gym. Students need to decide which gym is more crowded.

Probes: How did you solve that? How do you know that gym ___ is more crowded? How much space would each person get in gym ___ ? How did you figure that out? Would everyone get that much space? Is there any way you could share that space that is let over? (if students say each person gets 1 square meter and then there is some space left over)

Section Four: Reflections on the Unit

5) Reflective Questions

In the past month we have been working on the "Overcrowding at Francis" project. I am interested in what you think you have learned by working on that project?
Do you think you have learned a little or a lot? *Probe for mathematical learning if student does not mention.*
Is there anything else you think you have learned?
How do you think you learned that?
What do you think is going to happen with the project? Do you think people are going to listen? What do you think you can do? How do you think math might heklp you?

Appendix F

Flyer Posted by Two 8th Grade Girls Protesting Sexism

Let's make a difference

Ok, was up girl. I guess you know what the word sexism means. I think that we should do something to stop it or at least try, because it isn't right. We experience this everyday life, and it isn't fair. We are humans and equal to men so why should we have to deal with this. So help me. SERIOUSLY.

Help me stop sexism and maybe our lives can be happier. And I know it seems hard for me to say, but even Jay-Z [popular male rap artist] is sexist and I don't like that about him. I have rights and I should be treated equally. Now are you gonna to help me or just sit there and let it happen? FIGHT FOR WOMYNS RIGHTS!!!!!!!!!!!!! (sic)
DON"T LET IT KEEP HAPPENING TO YOU!!!!!!!!!!
WOMYN POWER!!!!!!!!!!!!!!!!!!!

By: Chantelle Gutierrez and Monica Macías.

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VITA

Erin Elizabeth Turner was born in Cincinnati, Ohio on May 8, 1974, the daughter of Marcia Middleton Turner and Terry Charles Turner. After graduating from Saguaro high school in Scottsdale, Arizona, in 1991, she enrolled in Arizona State University in Tempe, Arizona. She received a Bachelor of Arts in Elementary Education, with an emphasis on Bilingual/ESL education, from Arizona State in December 1994. During the following years she was employed as a 4th and 5th grade bilingual teacher at W.T. Machan elementary school in Phoenix, Arizona. During her first year of teaching, she began a Masters of Arts degree from Arizona State University, which she completed in August 1999. In September 1999 she entered the Graduate School of The University of Texas at Austin.

Permanent Address: 1906 Pearl Street, Apt. 203, Austin, Texas 78705

This dissertation was typed by the author