The Report committee for Huan Wang

Certifies that this is the approved version of the following report:

The Therapist Scheduling Problem for Patients with Fixed Appointment Times

APPROVED BY

SUPERVISING COMMITTEE:

Supervisor: ______________________________

Jonathan F. Bard

____

Ahmad I. Jarrah

The Therapist Scheduling Problem for Patients with Fixed Appointment Times

by

Huan Wang, B.S.

Report

Presented to the Faculty of the Graduate School

of the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science in Engineering

The University of Texas at Austin

December 2011

Acknowledgement

I wish to express my sincere gratitude to my supervisor, Prof. Jonathan Bard, Department of Operations Research and Industrial Engineering, University of Texas at Austin for his support and guidance in carrying out this report. Also, I sincerely thank to my second reader, Prof. Ahmad I. Jarrah and I wish to express my gratitude to Yufen Shao, Jin Zhang and Yao Hu who rendered their help during the period of my report work. Last but not least I thank my beloved parents and all my friends for their manual support, strength, help and for everything.

The Therapist Scheduling Problem for Patients with

Fixed Appointment Times

by

Huan Wang, M.S.E
The University of Texas at Austin, 2011
SUPERVISOR: Jonathan F. Bard

This report presents a series of models that can be used to find weekly schedules for therapists who provide ongoing treatment to patients scattered around a geographical region. In all cases, the patients' appointment times and visit days are known prior to the beginning of the planning horizon. Variations in the model include single vs. multiple home bases, homogeneous vs. heterogeneous therapists, lunch break requirements, and a nonlinear cost structure for mileage reimbursement and overtime. The single home base and homogeneous therapist cases proved to be easy to solve and so were not investigated. This left two cases of interest: the first includes only lunch breaks while the second adds overtime and mileage reimbursement. In all, 40 randomly generated data sets were solved that consisted of either 15 or 20 therapists and between roughly 300 and 540 visits over five days. For each instance, we were able to obtain the minimum cost of providing home healthcare services for both models using CPLEX 12.2. The results showed that CPU time increases more rapidly than total cost as the total number of visits grows. In general, data sets with therapists who have different starting and ending locations are more difficult to solve than those whose therapists have the same home base.

Table of Contents

List of Tables

List of Figures

1. Introduction

Home healthcare is the business of providing professional healthcare services in the patient's place of residence. These healthcare services include nursing, physical therapy, speech therapy, occupational therapy, medical treatments, and general assistance to the infirm (Kergonsien 2009). As the population ages, the need for healthcare professionals grows. The Census Bureau projects that by 2030 there will be more than 70 million Americans aged 65 and older, more than twice their number in 1995. By 2040, one of out of every five Americans will be over 65. The number of "old old," aged 85 and older, is projected to triple or quadruple (Super 2002). There are now more than 10,000 home healthcare organizations in the U.S. alone. This number will continue increasing at an accelerated rate leading to greater competition between providers. To remain profitable, these organizations must find new ways to better manage their human resources. A critical aspect of this effort is the derivation of cost-efficient schedules for up to a week at a time for their therapists. This involves constructing travel routes and assigning patients to therapists while taking into account a number of operational and logics constraints. (Begur 1997). In fact, effective staff planning and scheduling must aim at cost minimization.

In this report, we present a wide range of models that can be used to solve the weekly therapist scheduling problem but only investigate the two most comprehensive. The objective of the models is to minimize the total cost of providing home healthcare services for up to a week at a time without violating availability constraints, the requirement for lunch breaks, and patient appointment times. The type of organization that we have in mind is an independent agency that contracts with therapists to work on an hourly basis. Costs include wages for treatment and drive time, mileage reimbursement, and a premium for overtime. Because of different training levels, different therapists get paid at different rates. Travel cost arises from the fact that therapists must often travel for several hours between various locations to see their patients on any day of the week. Indeed, it is shown in the report of The National Association for Home Care and Hospice that healthcare providers drive nearly 5 billion miles each year. It's important to minimize the cost of the travel time.

In the next section, we review the literature related to home healthcare scheduling. In Section 3, a description of operations is provided that includes scheduling issues and cost structure. The problem statement and models are giving in Section 4. In Section 5, we give an example and present our computational results obtained with CPLEX 12.2 for the 40 randomly generated data sets tested. Instances include either 15 or 20 therapists and upwards of 540 visits per week, which reflects the actual demand faced by a Midwestern rehabilitative company that provides physical, occupational, and speech therapy services. We close with a summary of the work and suggestions for future research.

2. Literature Review

The problem discussed in the report is posed in the context of a home healthcare delivery, but it is not unique to that industry. More generally, we are solving a vehicle scheduling problem with time window and multiple depots. There is a vast majority of literature on algorithms for multiple depot vehicle scheduling problem (MDVSP), which was introduced by Carpaneto et al. (1989). They present a branch and bound algorithm for the MDVSP which is base on the additive lower bound and the use of dominance criteria. Rieiro and Soumis (1994) improved Carpaneto's results by introducing a new formulation of the MDVSP as a set partitioning (SP) problem with side constraints whose continuous relaxation can be solved by column generation. They showed that the continuous relaxation of the SP formulation provides a much tighter lower bound than the additive bound procedure. Binaco and Mingozzi (1993) studied the same problem using a similar approach. However, in addition to column generation, they describe a procedure for computing a lower bound to the MDVSP that is based on heuristically solving the dual of the continuous relaxation of SP without using the SP matrix. The dual solution obtained is used to reduce the number of variables in the SP in such a way that the resulting SP problem can be solved by standard branch and bound techniques. Dell'Amico and Fischetti (1993) aim at both minimizing the number of vehicles used and the overall operational cost. They designed a new polynomial-time heuristic which always guarantees the use of the minimum number of vehicles. Extensive computational results on test problems involving up to 1,000 trips and 10 depots are reported.

The multiple depot crew scheduling problem (MDCSP) which is quite similar to our home healthcare problem is an extension of the MDVSP. In this case, each crew must return to their starting depot, and limits are imposed on both the elapsed time and the working time of any duty. Boschetti and Mingozzi (2004) developed a bounding procedure based on Lagrangian relaxation and column generation. Their procedure is an effective alternative to classical column generation as it is not affected by the typical degeneration drawbacks in solving the linear relaxation of the master problem with a standard LP code.

There is also some literature that focuses directly on the home healthcare routing and scheduling problem. Cheng and Rich (1998) considered healthcare staff planning with full-time and part-time workers. They developed two mixed-integer linear programming formulations, both of which include the lunch break constraints. A two-phase heuristic was implemented and numerical is presented for small data sets with up to 2 full-timers, 2 part-timers and 10 patients. Bertels and Fahle (2006) present the core optimization components of PARPAP which is an optimization and planning tool for highly constrained staff routing problems, again for home healthcare. A combination of linear programming, constraint programming and heuristics are used. Their model is flexible enough to accommodate many of the components of real-world problems. Different than others who aim to minimize the total cost, Steeg and Schröder (2008) choose the objective of minimizing the number of nurses that visit patients during the schedule. Their model combines the nurse rostering problem with a periodic vehicle routing problem. Their algorithm first applies constraint programming to compute an initial feasible solution. Then a metaheuristic called adaptive large neighborhood search is used in an effort to improve the results.

3. Description of Operations

Therapists that provide rehabilitation services can be divided into three categories: physical, occupational and speech. Physical therapists are healthcare professionals who diagnose and treat individuals of all ages. Their patients range from newborns to the very old who have medical problems or other health-related conditions, illnesses, or injuries that limit their ability to move and perform functional activities to the degree that they would like in their daily lives. Physical therapy is performed by either a physical therapist (PT) or sometimes by a physical therapist assistant (PTA) acting under their direction. Occupational therapists work with individuals who suffer from a mentally, physically, developmentally, or emotionally disabling condition. They provide treatments that develop, recover, or maintain a person's daily living functions. Speech and language pathologists assess, diagnose, treat, and help to prevent disorders related to speech, language, cognitive-communication, voice, swallowing, and fluency (J.F. Bard 2011). To simplify the problem, we only consider PTs and PTAs in this report.

Therapists who contract to work 40 hours per week are classified as "full-time"; otherwise, they are "part-time" and work on a fixed pre-defined schedule. For example, a specific part-timer may work Tuesday morning and all day Thursday. Full-timers are typically but not always scheduled five days a week between 8:00 am and 5:30 pm. Both PTs and PTAs could be either full-timers or part-timers, but the wages of the later are usually higher so if the aim is cost minimization, it's better to schedule part-timers first then full-timers.

In this report, we focus on patients who have fixed appointment times which means the number of treatments each week for a patient and the start and end time of the treatment is precisely defined. Patients can be treated at hospitals, nursing centers, medical lodges, clinics or their home. Most of the patients need one to two treatments weekly and very few patients need treatments every day. Each treatment lasts from 15 minutes to an hour. We assume there are no patients that must be treated exclusively by a PT.

The above factors determine both routing and scheduling must be considered in the problem. Given the information of therapists and time of the appointments, the objective is to develop a weekly schedule for each therapist that minimizes the total cost of providing home healthcare services while satisfying a collection of hard constraints that are now discussed.

3.1 Scheduling issues

Typically, therapists start their day at home, visit a series of patients at different sites and then return home after seeing their last patient. As in most service organizations, when a therapist is assigned 6 or more hours of work including driving time, a lunch break must be provided. Because most patients not residing in their homes receive lunch at a set time (e.g., 12:00 pm), this is often the best time for therapists to have a break. Here we assume, the lunch break is 30 minutes and should be assigned between 11:00 am and 1:00 pm. In addition to treating patients, a therapist's work also includes performing some administrative tasks. Such tasks do not need to be performed until the end of the day but in the case of treatment it is best to do them while memory is fresh. Therefore, administrative time is added to treatment time in the models.

3.2 Cost issues

Therapists get paid for time spent seeing patients and for driving between patient locations. Usually, the treatment rate and driving rate is therapist dependent. Therapists are also paid for the time required to perform administrative functions that typically include document treatment and notifying relatives. To determine administrative time, an important parameter— *productivity* is introduced. Productivity is defined as the ratio of treatment time to treatment plus administrative time. It is included in the information of therapists along with the administration rate.

For a given week, the total working time for a therapist is derived by adding all treatment, driving and administrative time together. When a therapist works more than

40 hours in a week, overtime begins to accumulate. He/she is paid one and a half times the regular wage rate for each overtime hour.

Another component of cost is the mileage reimbursement. On a given day, if a therapist drives more than 25 miles, he is additionally paid at a rate decided by the company for the extra miles.

3.3 Travel time estimation

In order to determine travel time, we have to figure out the distance between one location to another and also the speed of driving. In this report, we use Euclidean metric to calculate the distance between any two points. Given two coordinates (X_i, Y_i) and (X_j, Y_j) expressed in degrees of longitude and latitude, the distance (in mile) between i and j is calculated by the following equation.

$$D(i,j) = \sqrt{\left(53\left(X_i - X_j\right)\right)^2 + \left(69.1\left(Y_i - Y_j\right)\right)^2}$$

In determining the driving speed for therapists, we use the algorithm below for computing vehicle velocity (*MPH*).

If $(D(i,j) < 1)$ then $D(i,j) = 1$

If $(D(i,j) \leq 20$ and Metro = "Y") then

$MPH(i,j) = 18.285 + 0.45159 \times D(i,j)$

ElseIf $(D(i,j) \leq 20$ and Metro = "N") then

$MPH(i,j) = 5.447 \times log_e\left(D(i,j)\right) + 11.11$

ElseIf $(D(i,j) \geq 20)$ then

$MPH(i,j) = 17.326 + 14.4335 \times log_e\left(D(i,j)\right)$

End If

$MPH(i,j) = \min\{MPH(i,j), 50\}$

Note that the ''Metro'' flag is set to ''N'' for selected depots located in sparsely populated regions (Bard and Jarrah 2009).

4. Model Formulations

The problems described below are referenced by their generic names which come from the vehicle routing literature but will be defined in the context of therapist scheduling. In particular, a therapist is equivalent to a vehicle, a patient is equivalent to a customer, and treatment is equivalent to service. The full problem spans five days and has the objective of minimizing treatment, driving, administrative, mileage and overtime costs while observing flow balance, time, and resource constraints. The first model we present is the single depot vehicle scheduling problem (SDVSP). The second model we have is multiple depot vehicle scheduling problem (MDVSP) with lunch break while the last one is MDVSP with overtime, mileage and lunch breaks. The following basic notation will be used in the formulations. Additional notation will be introduced as needed.

Indices and sets

i, j	index for patients
k	index for therapists
d	index for days
$o(k)$	origin (and destination) of therapist k
K	set of all therapists
I	set of patients to be seen over the planning horizon
$IF(i,d)$	set of patients that can follow patient i in a schedule on day d including $o(k)$
$IP(i,d)$	set of patients that can precede patient i in a schedule on day d including $o(k)$
$IL(d)$	set of patients that can precede a lunch break on day d
$IL(i,d)$	set of patients that can follow patient $i \in IL(d)$ in a schedule on day d

Data and parameters

c_{ij}^{k}	hourly traveling rate from patient i to patient j by therapist k
c_{id}^{k}	hourly treatment rate by therapist k to see patient i on day d
τ_{ij}	travel time between patient i and j
s_{id}	time required to provide treatment to patient i on day d
a_{id}	appointment time for patient i on day d

Decision variables

x_{ijd}^{k} 1 if therapist k visits patients i and j in succession on day d, 0 otherwise

4.1 Single depot vehicle scheduling problem (SDVSP)

In this version of the problem, all therapists in the set K are identical, which means that they have the same working windows, treatment rates and travel rate, and are based at the same clinic where they start and end their day. The SDVSP decomposes by day and can be formulated as a min-cost flow problem (MCFP) (Desrosiers et al. 1995, Lenstra and Rinnooy Kan 1981). The homogeneity assumption allows us to drop the index k. This leads to the following model for day d.

$$\phi_d^{SDVSP} = \text{Minimize} \sum_{i \in I \cup \{o\}} \sum_{j \in IF(i,d)} \bar{c}_{ijd} x_{ijd} \tag{1a}$$

$$\text{subject to} \sum_{j \in IF(i,d)} x_{ijd} = 1, \ \forall\, i \in I \tag{1b}$$

$$\sum_{i \in IP(j,d)} x_{ijd} - \sum_{i \in IF(j,d)} x_{jid} = 0, \ \forall\, j \in I \cup \{o\} \tag{1c}$$

$$\sum_{j \in I} x_{ojd} \le |K| \tag{1d}$$

$$x_{ijd} \in \{0,1\}, \ \forall\, i \in I \cup \{o\}, j \in IF(i,d) \tag{1e}$$

The objective in (1a) is to minimize the cost of treating all patients on a specific day. The coefficients $\bar{c}_{ijd}$ include the travel cost between location i and j and the cost of providing treatment for patient i on day d. Constraints (1b) ensure that each patient is visited exactly once while identifying her immediate successor. Constraints (1c) guarantee flow balance at each location visited while (1d) limits the number of schedules each day to at most $|K|$--one for each therapist. Variable definition is given in (1e). This model is quite simple and is used as a foundation.

4.2 Multiple depot vehicle scheduling problem (MDVSP) with lunch break

In this problem, each therapist k has his unique location $o(k)$ which is referred to as a home base where he starts and ends the day. Therapists in the set K are no longer assumed to be identical, which means that they may have different working windows, treatment rates and mileage reimbursement rates. Therefore, it is necessary to reintroduce the index *k* in the notation.

In most organizations, when a worker's shift extends beyond a certain number of hours, an uncompensated lunch break must be provided. In our case, when a therapist is assigned 6 or more hours of work, including driving and administrative time, he is entitled to a ½-hour break between 11:00 am and 1:00 pm. To identify when a lunch break can be taken, we define pairs of patients (i,j) whose appointment times are far enough apart to allow the break to be inserted between their treatments. For therapist *k* on day *d*, let $IL(i,d)$ be the set of immediate successors of patient *i* that admit a lunch break. The conditions for $j \in IL(i,d)$ are as follows: (i) $a_{jd} - (a_{id} + s_{id} + \tau_{ij}) \geq 0.5$, that is, at least 30 minutes of idle time exist between the end of patient *i*'s treatment and the beginning of patient *j*'s treatment, and (ii) $|[a_{id} + s_{id}, a_{jd}] \cap [11,13]| \geq 0.5$, that is, the ½-hour break can fit between 11:00 am and 1:00 pm.

Let $IK(k,d)$ be the set of patients that therapist k can be assigned on day d. By updating model (1) to accommodate lunch breaks and to take into account the fact that therapists must be viewed individually, we get the following model for day d.

$$\phi_d^{MD-L} = \text{Minimize} \sum_{k \in K} \sum_{i \in I \cup \{o(k)\}} \sum_{j \in IF(i,d)} \bar{c}_{ijd}^k x_{ijd}^k \tag{2a}$$

$$\text{subject to} \sum_{k \in K} \sum_{j \in IF(i,d)} x_{ijd}^k = 1, \ \forall\, i \in I \tag{2b}$$

$$\sum_{i \in IP(j,d)} x_{ijd}^k - \sum_{i \in IF(j,d)} x_{jid}^k = 0, \ \forall\, k \in K, j \in IK(k,d) \cup \{o(k)\} \tag{2c}$$

$$\sum_{j \in I} x_{o(k),j,d}^k \leq 1, \ \forall\, k \in K \tag{2d}$$

$$\sum_{i \in IL(d)} \sum_{j \in IL(i,d)} x_{ijd}^k \geq 1 \quad , \ \forall\, k \in K \tag{2e}$$

$$x_{ijd}^k \in \{0,1\}, \ \forall\, k \in K, i \in I \cup \{o(k)\}, j \in IF(i,d) \tag{2f}$$

Model (2) is the same as model (1) except for the addition of (2e) and the reintroduction of the index *k*. These constraints enforce the lunch break requirement for all therapists by insisting that each schedule include a lunch break arc (i,j), where the set $IL(d)$ is limited to those patients whose appointment times are early enough in the day to satisfy the above two conditions on day d. Note that the depot $o(k)$ is an element in $IL(i,d)$ if only to ensure that (2) is feasible. If $x^k_{iod} = 1$ for some i, it means that the lunch break is taken after the last patient is treated. In practice, the earliest appointment time is at 8:00 am so it is not possible for this situation to occur, given that the break must conclude no later than 1:00 pm. Constraint (2e) does not permit the break to precede the first appointment.

In general, the binary requirements for the x-variables in (2f) cannot be relaxed. A special case exists, however, that preserves the MCFP characteristics of (2) when there is a single depot, that is, $o(k) = o$ for all k. In particular, when all circuits in the graph G starting and ending at o contain at most one lunch break arc, model (2) remains a MCFP and can be solved as an LP. The proof is given in Shao (2011). It is an open question whether the more general case is polynomial solvable.

4.3 MDVSP with overtime, mileage and lunch break

The above models decompose by day. When a therapist works more than 40 hours in a week he is paid time and a half for each overtime hour. This factor ties the days of the week together and prevents a daily decomposition. To incorporate overtime in the model, it is necessary to keep track of the billable, travel, treatment and administrative time that each therapist accumulates each day. To calculate the administration time, we use the following formula

$$adm_{ikd} = \left({}^{1}\!/_{p_k} - 1\right) \times c^k_{id}$$

where p_k is the productivity for therapist k.

To add the overtime constraints in the model, it is necessary to keep track of all the travel, treatment and administrative time that each therapist accumulates each day. A second factor not yet considered is the nonlinear nature of mileage reimbursement. For the first 25 miles, therapist k is not reimbursed but for all miles above that, he is paid at the rate of c_k^{miles}. In the formulation, we make use of these parameters and the following additional notation.

Indices and sets

$DU(i)$ set of days in the planning horizon that patient i is to be treated

$DK(k)$ set of days in the planning horizon that therapist k works

Data and parameters

c_k^{reg} hourly treatment rate for therapist k

c_k^{miles} hourly travel cost for every mile that therapist k drives in a day beyond 25 miles

adm_{ik} time required by therapist k to perform administrative functions after treating patient i on day d

$\tau_k^{\max\_over}$ upper limit on the amount of overtime permitted for therapist k

$dist_{ij}$ distance between locations i and j

Decision variables

T_{dk}^1 number of hours to go from the depot to the first patient that therapist k sees on day d

T_{dk}^2 number of hours it takes for therapist k to return to the depot after treating his last patient on day d

T_{dk} total time for which therapist k is paid on day d

T_k^{over} number of hours that therapist k works in a week beyond 40

D_{dk} number of miles that therapist k drives on day d

ΔD_{dk} number of miles above 25 that therapist k drives on day d

Based on model (2), we add constraints for overtime and mileage reimbursement. This leads to model (3).

$$\phi^{MD-L-O-M} = \text{Minimize} \sum_{k\in K}\sum_{d\in DK(k)}\sum_{i\in IK(k,d)\cup\{o(k)\}}\sum_{j\in IF(i,k,d)}\left(c_{ij}^k + c_{id}^k\right)x_{ijd}^k$$

$$+ \sum_{d\in D}\sum_{k\in K} c_k^{miles}\Delta D_{dk} + \sum_{k\in K} 1.5 c_k^{reg} T_k^{over} \quad (3a)$$

subject to
$$\sum_{k\in K(i,d)}\sum_{j\in IF(i,k,d)} x_{ijd}^k = 1, \ \forall\, i \in I, d \in DU(i) \quad (3b)$$

$$\sum_{j\in IK(k,d)} x_{o(k),j,d}^k \le 1, \ \forall\, k \in K, d \in DK(k) \quad (3c)$$

$$\sum_{i\in IP(j,k,d)} x_{ijd}^k - \sum_{i\in IF(j,k,d)} x_{jid}^k = 0, \ \forall\, k \in K, d \in DK(k), j \in IK(k,d) \cup \{o(k)\} \quad (3d)$$

$$\sum_{j\in IK(k,d)} \tau_{o(k),j} x_{o(k),j,d}^k = T_{dk}^1, \ \forall\, k \in K, \ d \in DK(k) \quad (3e)$$

$$\sum_{i\in IK(k,d)} \left(\tau_{i,o(k)} + s_{id} + adm_{ikd}\right) x_{i,o(k),d}^k = T_{dk}^2, \ \forall\, k \in K, d \in DK(k) \quad (3f)$$

$$T_{dk}^1 + T_{dk}^2 + \sum_{i\in IK(k,d)}\sum_{j\in IF(i,k,d)\setminus\{o(k)\}} \left(\tau_{ij} + s_{id} + adm_{ikd}\right) x_{ijd}^k = T_{dk},$$

$$\forall\, k \in K, d \in DK(k) \quad (3g)$$

$$\sum_{d\in DK(k)} T_{dk} - 40 \le T_k^{over}, \ \forall\, k \in K \quad (3h)$$

$$\sum_{i\in I\cup\{o\}}\sum_{j\in IF(i,d)} dist_{ij} x_{ijd}^k = D_{dk}, \ \forall\, d \in D, k \in K \quad (3i)$$

$$D_{dk} - 25 \le \Delta D_{dk}, \ \forall\, d \in D, k \in K \quad (3j)$$

$$\sum_{i\in IK(k,d)}\sum_{j\in IL(i,k,d)} x_{ijd}^k \ge 1, \ \forall\, k \in K, d \in DK(k) \quad (3k)$$

$$x_{ijd}^k \in \{0,1\}, \ T_{dk}^1 \ge 0, \ T_{dk}^2 \ge 0, \ T_{dk} \in \left[0, \tau_{dk}^{max}\right], \ T_k^{over} \in \left[0, \tau_k^{max\_over}\right], \ D_{dk} \ge 0,$$

$$\Delta D_{dk} \ge 0, \ \forall\, i \in I \cup \{o(k)\}, d \in DU(i), j \in IF(i,k,d), k \in K(i,d) \quad (3l)$$

The objective function (3a) has three terms. The first minimizes the cost of traveling between all pairs of patients plus the cost of providing treatment. For those patients to be seen on day d, the set $IK(k,d)$ must be defined accordingly. The second and third terms minimize the mileage reimbursement and overtime payments, respectively.

Constraints (3b) ensure that each patient has exactly one successor on a route on day d. The first summation is over the set of therapists who are qualified to provide treatment to patient i and the second is over all patients who are compatible successors of i and hence can be on the same route. An ordered pair of patients (i,j) is said to be *compatible* on day d for therapist k if it satisfies the following conditions: $a_{id} + s_{id} + \tau_{ij} \leq a_{jd}$, $e_{kd} + \tau_{o(k),i} \leq a_{id}$, and $a_{jd} + s_{jd} + adm_{jkd} + \tau_{j,o(k)} \leq f_{kd}$. As such, if $j \notin IF(i,k,d)$, then either i and j are separated by too great a distance, therapist k cannot reach i at her appointment time, patient j's appointment time does not allow sufficient time for therapist k to return to his home base prior to the end of his day, or therapist k is not suited to treat patient j. These situations are sorted out in a preprocessing step where all the sets are defined. The requirement that patient i be seen by a particular therapist k on day d, can be accommodated by appropriately defining the set $K(i,d)$. We treat this as a hard constraint but it is an easy matter to treat patient preferences as a soft constraint, penalizing non-preferred assignments.

Constraints (3c) limit each therapist $k \in K$ to at most one schedule per day. By implication, when $x^k_{o(k),j,d} = 0$ for all $j \in IK(k,d)$, therapist k is not given a schedule on day d. If a subset of therapists, say, n_k of them, have the same profile and can be viewed as interchangeable (that is, homogeneous), then the right-hand side of (3c) can be replaced with n_k and K redefined to be the set of therapist types rather than individuals. Constraints (3d) impose route continuity for each therapist k and each compatible patient $j \in IF(i,k,d)$ by requiring that tours (loops) are constructed rather than open paths. The start and end of a tour for therapist k is assumed to be the same location, $o(k)$. Given positive flow on network G^k, when (3d) is combined with (3b), we see that each patient j who is treated by k has a unique predecessor and a unique successor.

Constraints (3e) – (3f) keep track of the amount of paid time accrued daily by therapist k. Therapists may work for more than 8 hours a day but overtime does not apply until their billable plus administrative plus travel time exceeds 40 hours in a week. The summation on the left-hand side of (3e) determines the time to go from the home base of therapist k to his first patient. The left-hand side of (3f) determines the time required by therapist k to treat the last patient in his schedule on day d, perform the associated administrative functions, and then return to his home base.

The total number of working hours that therapist k accrues on day d is calculated in (3g). An upper bound is placed on this value in (3*l*). Constraint (3h) sums the working hours over the week and subtracts 40 to provide a lower bound on the number of overtime hours for therapist k. An upper bound is likewise imposed in (3*l*). Because we are trying to minimize overtime compensation in (3a), we are able to write the constraints in (3h) as inequalities rather than equalities; T_k^{over} will always assume its smallest value in any solution and hence will be binding in either (3h) or (3*l*). Constraints (3i) are similar to those in (3g) and are used to determine the total distance that therapist k drives each day. Whether mileage reimbursement is due is determined by (3j) in a manner similar to (3h).

Constraints (3k) are written for those therapists whose time window on day d spans 6 or more hours and ensures that they each traverse at least one arc (i,j) that admits sufficient time for the lunch break. Because the demand for therapists generally exceeds the supply, we have assumed for modeling purposes that all those whose time windows are at least 6 hours will be assigned at least 6 hours of work and hence will require a break. In any case, $j \in IL(i,k,d)$ if and only if (i) $a_{jd} - (a_{id} + s_{id} + \tau_{ij}) \geq 0.5$, that is, at least 30 minutes of idle time exist between the end of patient i's treatment and the beginning of patient j's treatment, and (ii) $|[a_{id} + s_{id}, a_{jd}] \cap [11,13]| \geq 0.5$, that is, the ½-hour break can fit between 11:00 am and 1:00 pm. The equations needed to determine whether or not these conditions hold and hence whether a therapist is indeed entitled to a break on day d are given by Shao (2011). Finally, variable definitions are stated in (3*l*).

5. Computational Results

5.1 Date description

Both models (2) and (3) were tested using the same 40 randomly generated instances for each. In the first case, all therapists have the same home base (single depot case), but otherwise have different working windows, treatment rates and reimbursement rates. In the second case, all therapists start at unique locations and have different weekly profiles and cost characteristics. Of the 40 instances, five contain 15 therapists and 150 patients, five contain 15 therapists and 195 patients, five contain 20 therapists and 200 patients and the remaining five contain 20 therapists and 280 patients. Most patients require multiple treatments during the week.

Before presenting the results, an example is given to illustrate the input data and the type of results obtained from the analysis. A 15-therapist, 150-patient instance is used for this purpose. Table 1 indicates the location of each facility. Table 2 highlights the input data for each therapist for Monday (the data for the remaining four days is similar and will be omitted). Treatment rate (TR), driving rate (DR), administration rate (AR) and productivity (Prod) are listed under the cost columns. As can be seen, the ID numbers begin with 0. Longitude and latitude are given for each therapist's home base as is the clinic he reports to. The latter information is not used in the models. Note that therapist 9 is not available on Monday.

Table 3 provides the basic information for each patient, including his or her ID, the facility code associated with the treatment location, and the appointment time on Monday. Patient ID begins with 1000.

Table 1. Facility data

ID	Facility name	Longitude	Latitude
KS109	Medical Lodge	-97.0054215	37.5155992
KS130	Medical Lodge	-97.4397647	37.7330952
KS245	Rest Haven	-97.246866	37.560118
KS249	Rest Haven	-97.3146551	37.6525878
KS532	Medical Lodge	-97.572676	37.73481
KS593	Medical Lodge	-97.645003	37.3863735
KS605	Medical Lodge	-97.234295	37.5519869
KS636	Hospital	-97.1839959	37.657345
KS637	Hospital	-97.200431	37.7034049
KS763	Medical Lodge	-97.2496716	37.6993938
KS803	Nursing Center	-97.366046	37.648453
KS863	Nursing Center	-97.447053	37.73173
KSH01	Home Health	-97.267299	37.68347
KSH02	Home Health	-97.2113605	37.7685329
KSH03	Home Health	-97.58307	37.661173
KSH04	Home Health	-97.466316	37.710647
KSH05	Home Health	-97.657722	37.869227

Table 2. Input data for therapists

ID	Type	Cost data ($)				Clinic ID	Longitude	Latitude	Availability (Monday)
		TR/hr	DR/hr	AR/hr	Prod/hr				
0	PT	45	45	45	0.65	KS863	-97.4456	37.7313	8:00am-5:30pm
1	PT	40	40	40	0.77	KS763	-97.2289	37.7035	7:00am-6:00pm
2	PT	50	50	50	0.75	KS763	-97.2289	37.7035	1:00pm-5:00pm
3	PT	42	37	37	0.8	KS249	-97.3123	37.6499	7:00am-6:00pm
4	PT	45	45	45	0.65	KS637	-97.2014	37.7025	1:00pm-5:00pm
5	PT	45	45	45	0.65	KS803	-97.366	37.6313	7:00am-6:00pm
6	PTA	27.5	27	27	0.9	KS763	-97.2289	37.7035	7:00am-6:00pm
7	PT	38.5	38.5	38.5	0.65	KS763	-97.2289	37.7035	7:00am-6:00pm
8	PT	30	30	30	0.65	KSH02	-97.2406	37.749	7:00am-6:00pm
10	PT	45	45	45	0.8	KS249	-97.3123	37.6499	8:00am-12:00pm
11	PT	40	40	40	0.77	KS130	-97.4432	37.7318	7:00am-6:00pm
12	PT	50	50	50	0.77	KSH04	-97.4665	37.7168	8:00am-5:30pm
13	PT	40	40	40	0.77	KS130	-97.4432	37.7318	7:00am-6:00pm
14	PT	50	50	50	0.77	KS863	-97.4456	37.7313	7:00am-6:00pm

Table 3. Input data for patients

ID	Facility code	Appointment (Monday)
1001	KSH01	10:30am-11:00am
1005	KSH04	8:00am-8:30am
1007	KS605	1:00pm-1:30pm
1008	KSH05	11:00am-12:00pm
1013	KS605	8:00am-8:30am
1014	KS803	10:30am-11:00am
1015	KSH01	2:00pm-2:45pm
1016	KSH02	11:00am-11:30am
1021	KS593	1:00pm-1:30pm
1022	KSH01	9:30am-10:00am
1025	KSH02	9:30am-10:00am
1027	KS593	11:00am-12:00pm
1028	KS245	2:00pm-2:30pm
1029	KS593	10:30am-11:00am
1030	KS245	1:00pm-1:30pm
1032	KS130	4:00pm-4:30pm
1033	KS636	10:30am-11:00am
1034	KS637	11:30am-12:00pm
1035	KS130	1:30pm-2:00pm
1036	KS636	10:00am-10:30am
1038	KSH05	2:00pm-2:45pm
1039	KSH03	3:30pm-4:15pm
1040	KS245	10:00am-10:30am
1041	KSH04	4:00pm-4:45pm
1042	KS605	10:30am-11:00am
1052	KS249	2:00pm-2:30pm
1053	KSH02	10:00am-10:30am
1054	KSH04	1:00pm-1:30pm
1055	KS593	3:30pm-4:15pm
1057	KS593	3:30pm-4:15pm
1058	KSH03	1:45pm-2:30pm
1059	KS593	4:00pm-4:30pm
1061	KSH02	10:00am-10:15am
1062	KS803	3:30pm-4:15pm
1063	KS803	2:00pm-2:30pm
1064	KS763	9:00am-9:30am

ID	Facility code	Appointment (Monday)
1065	KS605	3:30pm-4:00pm
1066	KS532	1:00pm-1:30pm
1068	KS593	11:15am-11:45am
1071	KS130	2:30pm-3:15pm
1073	KS763	4:00pm-4:45pm
1074	KSH02	4:00pm-4:45pm
1075	KSH01	1:30pm-2:00pm
1081	KS593	7:30am-8:00am
1087	KS803	2:45pm-3:30pm
1088	KSH05	11:00am-11:30am
1089	KS130	4:30pm-5:00pm
1090	KS249	2:30pm-3:00pm
1091	KS636	9:00am-9:30am
1094	KSH02	9:00am-9:30am
1095	KS863	4:00pm-4:30pm
1097	KS593	8:00am-8:30am
1098	KS130	8:00am-8:30am
1100	KS636	11:00am-11:30am
1101	KS763	10:00am-10:30am
1102	KSH02	3:00pm-3:30pm
1105	KS863	1:00pm-1:30pm
1106	KS605	3:00pm-3:30pm
1107	KS763	9:00am-9:30am
1110	KS249	11:00am-11:45am
1111	KS803	1:45pm-2:30pm
1112	KSH03	8:30am-9:00am
1115	KS130	3:30pm-4:15pm
1116	KS763	4:30pm-5:00pm
1117	KS637	1:00pm-1:30pm
1120	KS245	8:30am-9:00am
1124	KS803	10:00am-10:30am
1129	KS249	10:30am-11:00am
1136	KS637	1:30pm-2:00pm
1138	KS130	2:00pm-2:30pm
1139	KS636	8:30am-9:00am
1147	KS532	2:00pm-2:30pm

5.2 Sample results

Models (2) and (3) were solving using Java and CPLEX Concert Technology. In the computations, the travel cost for every mile that therapist k drives in a day beyond 25 miles, c_k^{miles}= $0.55, and the upper limit on the amount of overtime permitted for therapist k is 20 hours, that is, $\tau_k^{\max\_over}$ = 20.

Table 4. Monday schedule for therapists from model (2)

Therapist ID	Route (patient-appointment time)
0	1098 - 8:00 am, 1105 - 1:00 pm, 1095 - 4:00 pm
1	1022 - 9:30 am, 1001 - 10:30 am, 1034 - 11:30 am, 1117 - 1:00 pm, 1136 - 1:30 pm, 1102 - 3:00 pm, 1074 - 4:00 pm
2	1075 - 1:30 pm, 1015 - 2:00 pm, 1073 - 4:00 pm
3	1013 - 8:00 am, 1042 - 10:30 am, 1007 - 1:00 pm, 1052 - 2:00 pm, 1090 - 2:30 pm, 1062 - 3:30 pm
5	1120 - 8:30 am, 1040 - 10:00 am, 1068 - 11:15am, 1021 - 1:00 pm, 1059 - 4:00 pm
6	1139 - 8:30 am, 1091 - 9:00 am, 1036 - 10:00 am, 1033 - 10:30 am, 1100 - 11:00 am, 1030 - 1:00 pm, 1028 - 2:00 pm, 1106 - 3:00 pm, 1065 - 3:30 pm, 1116 - 4:30 pm
7	1064 - 9:00 am, 1101 - 10:00 am, 1088 - 11:00 am, 1038 - 2:00 pm, 1057 - 3:30 pm
8	1094 - 9:00 am, 1025 - 9:30 am, 1053 - 10:00 am, 1008 - 11:00 am, 1066 - 1:00 pm, 1147 - 2:00 pm, 1055 - 3:30 pm
10	1129 - 10:30 pm, 1110 - 11:00 am
11	1005 - 8:00 am, 1107 - 9:00 am, 1061 - 10:00 am, 1016 - 11:00 am, 1035 - 1:30 pm, 1138 - 2:00 pm, 1071 - 2:30 pm, 1115 - 3:30 pm, 1089 - 4:30 pm
12	1112 - 8:30 am, 1058 - 1:45pm, 1039 - 3:30 pm
13	1081 - 7:30 am, 1097 - 8:30 am, 1029 - 10:30 am, 1027 - 11:00 am, 1054 - 1:00 pm, 1063 - 2:00 pm, 1087 - 2:45pm, 1041 - 4:00 pm
14	1124 - 10:00 am, 1014 - 10:30 am, 1111 - 1:45pm, 1032 - 4:00 pm

The solution to model (2) gives the minimum total cost by day without considering overtime and mileage reimbursement. Table 4 identifies the schedules derived for the 14 therapists who are available on Monday for the example data set. The first column is the therapist's ID. The second column is his corresponding route order specified by patient ID and appointment time. For instance, on Monday morning, therapist 7 starts at his home base, visits patient 1064 from 9:00 am to 9:30 am (see Table 3 for treatment times), and then patient 1101 from 10:00 am to10:30 am. The third patient is 1088 whose appointment is from11:00 am to 11:30 am. After a lunch break, therapist 7 visits patient 1038 between 2:00 pm and 2:45 pm followed by 1057 from 3:30 pm to 4:15 pm. At that point his day is complete and he is free to return to his home base.

Table 5. Monday schedule for therapists from model (3)

Therapist ID	Route (patient-appointment time)
0	1112 - 8:00 am, 1066 - 1:00 pm, 1147 - 2:00 pm, 1095 - 4:00 pm
1	1022 - 9:30 am, 1001 - 10:30 am, 1034 - 11:30 am, 1117 - 1:00 pm, 1136 - 1:30 pm, 1102 - 3:00 pm, 1074 - 4:00 pm
2	1075 - 1:30 pm, 1015 - 2:00 pm
3	1013 - 8:00 am, 1042 - 10:30 am, 1007 - 1:00 pm, 1063 - 2:00 pm, 1087 - 2:45pm, 1062 - 3:30 pm
5	1120 - 8:30 am, 1040 - 10:00 am, 1068 - 11:15am, 1059 - 4:00 pm
6	1139 - 8:30 am, 1091 - 9:00 am, 1036 - 10:00 am, 1033 - 10:30 am, 1100 - 11:00 am, 1030 - 1:00 pm, 1028 - 2:00 pm, 1106 - 3:00 pm, 1065 - 3:30 pm, 1116 - 4:30 pm
7	1107 - 9:00 am, 1101 - 10:00 am, 1008 - 11:00 am, 1038 - 2:00 pm, 1055 - 3:30 pm
8	1094 - 9:00 am, 1025 - 9:30 am, 1053 - 10:00 am, 1088 - 11:00 am, 1105 - 1:00 pm, 1052 - 2:00 pm, 1090 - 2:30 pm, 1073 - 4:00 pm
10	1129 - 10:30 pm, 1110 - 11:00 am
11	1098 - 8:00 am, 1064 - 9:00 am, 1061 - 10:00 am, 1016 - 11:00 am, 1035 - 1:30 pm, 1138 - 2:00 pm, 1071 - 2:30 pm, 1115 - 3:30 pm, 1089 - 4:30 pm
12	1005 - 8:00 am, 1054 - 1:00 pm, 1041 - 4:00 pm
13	1081 - 7:30 am, 1097 - 8:30 am, 1029 - 10:30 am, 1027 - 11:00 am, 1021 - 1:00 pm, 1057 - 3:30 pm
14	1124 - 10:00 am, 1014 - 10:30 am, 1111 - 1:45pm, 1032 - 4:00 pm

The solution to model (3) gives the minimal total cost for the week considering overtime and mileage reimbursement. Table 5 reports each therapist's schedule who is available on Monday. Comparing Tables 4 and 5, we see that the schedules for therapists 0, 2, 3, 5, 7, 8, 11, 12 and 13 change while the others remain the same. The reason is due to the mileage reimbursement since none of the schedules include overtime, that is, $T_k^{over} = 0$, for all $k \in K$. The objective of model (3) is to minimize the total cost of providing treatment over the week. The second term in the objective function (3a) is the total mileage reimbursement, that is, $\sum_{d \in D} \sum_{k \in K} c_k^{miles} \Delta D_{dk}$ and directly affects the solution. Looking at Table 6 we see that the total number of miles above 25 that all therapists drive on Monday is 212. 37 when model (2) was solved. When model (3) was solved, this number dropped to 176.43 miles, as might have been expected. The inclusion of mileage costs push down the number of miles driven.

Table 6. Comparison of driving distance for all therapists on Monday

Therapist ID	Model (2) distance	Model (2) distance above 25 miles	Model (3) distance	Model (3) distance above 25 miles
0	3.00	0.00	21.53	0.00
1	15.39	0.00	15.39	0.00
2	5.04	0.00	4.92	0.00
3	21.67	0.00	20.43	0.00
4	0.00	0.00	40.86	15.86
5	54.76	29.76	54.76	29.76
6	23.83	0.00	23.83	0.00
7	90.20	65.20	90.20	65.20
8	94.39	69.39	58.46	33.46
9	0.00	0.00	0.00	0.00
10	2.00	0.00	2.00	0.00
11	31.96	6.96	29.88	4.88
12	14.55	0.00	2.00	0.00
13	66.06	41.06	52.31	27.31
14	15.14	0.00	15.14	0.00
total	438.00	212.37	431.72	176.47

The minimal total cost for model (2) for the week was $12092.49 while the minimal total cost for model (3) was $12565.20. Somewhat surprisingly, runtimes were dramatically different. It took 53.49 CPU sec to get the optimal solution for model (2) and 1705.30 CPU sec for model (3). The 30-fold increase was due directly to the vastly greater number of constraints in model (3).

5.3 Output summary

We tested 40 instances of models (2) and (3) on a workstation running Ubuntu linux with a dual core, hyperthreading 3.73GHz Xeon processors and 24GB of shared memory. Output summaries are presented in Tables 7 – 10. In the majority of instances, the stopping criteria were an optimality gap of 0.5% or 10,000 nodes, whichever occurred first. The gaps reported are those realized at the final node in the search tree. When model (3) was solved for problems 16 – 20, the node limit was raised to 20,000 as seen in Table 10.

Table 7 gives the results for model (2) for the 20 instances associated with the case of different therapists who begin and end their day at the same location, while Table 8 gives the results for the case of different therapists at different locations, also for 20 instances and model (2). Tables 9 and 10 give parallel results for model (3).

Each table has the same format. Column 1 gives the problem number. The next three columns indicate the number of patients, the total number of patient visits per week, and their average treatment time per visit in hours, respectively. Columns 5 – 10 are associated with the therapists. Column 5 gives the number of therapists, columns 6 and 7 are data input that indicate the average wage rate and productivity for each therapist, respectively. Date output are presented in column 8-9. Column 8 reports the average number of days per week that each is available, and column 9 reports the average number of working hours per therapist per week. Column 10 gives the average number of visits per therapist per day. The last four columns highlight the results. The total cost of providing treatment for a week is given in column 11. Column 12 indicates the CPU time in seconds for solving the model. Column 13 shows the optimality gap provided by

CPLEX while column 14 gives the number of final nodes in the search tree upon termination. The + in the last column denotes a node generated by the heuristic.

All runs terminated with a feasible solution. For instances with the same number of therapists, more patients means more total visits, a greater average number of working days, longer average working hours, and a greater average number of visits per therapist per week. The total cost and CPU time increase with number of therapists and patients in the data sets. In other words, when the total number of visits grows, the total cost grows linearly and the CPU time grows approximately exponentially. These relationships are illustrated in Figures 1 and 2 where the horizontal axes represent the total number of visits per week. In Figure 1, the vertical axis is the corresponding total cost in Table 7, and in Figure 2, the vertical axis is the corresponding CPU time. Comparing the two figures, we see that CPU time increases more rapidly than total cost.

For Tables 8, 9 and 10, we can draw the same conclusions as we did from Table 7. As the total number of visits increases, it takes more time to solve the problem and we get higher total cost. Figures 3 to 8 illustrate again that CPU time increases more rapidly than total cost. Tables 9 and 10 indicate when model (3) was tested with the same data sets, it took much more time to obtain a solution. Although the number of integer variables is the same in each model, the fact that the constraints span the full week in model (3) greatly adds to its complexity. On average, it took 576.103 CPU seconds to obtain the results for the 20 instances associated with different therapist at the same location using model (2), and an average of 12531.05 seconds to obtain the results for the same instances using model (3). This corresponds to a nearly 22-fold increase in runtime. For the 20 instances associated with different therapist at different location, it took 770.30 seconds on average for model (2) and 18181.82 seconds on average for model (3), a 23.6-fold increase. A final observation from these results is that it is slightly more difficult to solve either model when the therapists are at different locations than when they all start and end their day at the same location.

6. Conclusions

In this report, we used 40 data sets to evaluate two different models for scheduling therapist visits over the week. The first model is generically referred to as the MDVSP with lunch breaks, and the second as the MDVSP with overtime, mileage reimbursement and lunch breaks. For all instances, CPLEX was able to find feasible solutions well within 2.04% of optimality and often within 1%. For model (2) with 15 therapists, the gap was within 0.5%, averaging 0.17%; for the 20 therapist case, the gap averaged 0.21%. For model (3) with 15 therapists, the gap averaged 0.73% ; for the 20 therapist case, the gap averaged 1.18%.

For future research, it would be worth investigating the more general problem in which not all patients have fixed appointment times, but can be seen almost any time during the day. The literature on "dynamic patient requests" is limited in the home healthcare field but is more common in the context of vehicle routing. Nevertheless, it is an open question as to whether algorithms can be developed to optimally solve the dynamic patient problem when the additional constraints discussed in this report are included in the model, and when patients have multiple treatment requirements per day. These features introduce much greater level of complication than when the appointment times are fixed. Nevertheless as the demand of home healthcare increases, dynamic patient requests will more likely become part of the input and so will need to be included in the formulation of therapist routing and scheduling problems.

Table 7. Results from model (2) for different therapists at the same location

	Fixed patients			Therapists						Results			
				Input			Output						
Prob.no.	No.	No. Visits	Avg. treatment time/visit	No.	Avg. wage/hr	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/therapist per day	Cost ($)	CPU (sec)	Gap (%)	Nodes no.
1	150	314	0.61	15	39.03	0.78	4.20	21.79	4.19	12351.52	140.05	0.33%	0+
2	150	300	0.58	15	40.37	0.74	4.47	20.98	4.00	11628.82	67.99	0.16%	0+
3	150	322	0.50	15	39.47	0.75	4.13	22.45	4.29	12572.60	85.16	0.04%	0+
4	150	303	0.60	15	40.30	0.77	4.07	21.12	4.04	11801.22	87.55	0.02%	0+
5	150	303	0.60	15	37.79	0.77	4.27	21.19	4.04	11021.32	90.98	0.18%	0+
6	195	363	0.60	15	39.00	0.81	4.27	23.89	4.84	13079.29	112.05	0.04%	0+
7	195	374	0.59	15	37.69	0.78	4.40	25.02	4.99	12978.33	292.30	0.14%	0+
8	195	379	0.58	15	40.33	0.74	4.40	25.59	5.05	14492.26	287.93	0.23%	0+
9	195	361	0.59	15	37.97	0.76	4.27	24.86	4.81	12799.61	138.11	0.14%	0+
10	195	378	0.59	15	37.19	0.77	4.47	25.12	5.04	12768.01	241.56	0.26%	0+
11	200	416	0.59	20	35.90	0.78	3.90	19.75	4.16	12530.81	442.99	0.24%	0+
12	200	393	0.62	20	37.32	0.76	4.20	20.36	3.93	14468.96	352.61	0.17%	0+
13	200	457	0.55	20	39.72	0.75	4.50	21.51	4.57	15849.28	583.02	0.15%	0+
14	200	452	0.50	20	38.67	0.79	4.20	18.81	4.52	13177.75	687.18	0.50%	336
15	200	426	0.59	20	37.73	0.78	4.05	20.55	4.26	14363.94	286.14	0.01%	0+
16	280	554	0.59	20	38.57	0.75	4.70	26.64	5.54	18387.26	1692.36	0.03%	0+
17	280	567	0.59	20	36.24	0.76	4.70	27.55	5.67	17967.23	1808.24	0.40%	0+
18	280	540	0.59	20	38.95	0.76	4.45	26.64	5.40	18962.48	1441.52	0.40%	0+
19	280	539	0.60	20	37.07	0.80	4.60	25.51	5.39	17368.71	1622.66	0.03%	0+
20	280	532	0.60	20	39.85	0.77	4.65	26.15	5.32	19379.01	1061.66	0.50%	0+

Table 8. Results from model (2) for different therapists at different locations

Prob.no.	Fixed patients			Therapists						Results			
				Input			Output						
	No.	No. Visits	Avg. treatment time/visit	No.	Avg. wage/hr	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/therapist per day	Cost ($)	CPU (sec)	Gap (%)	Nodes no.
1	150	306	0.60	15	39.33	0.74	4.60	20.89	4.08	11670.11	72.67	0.16%	0+
2	150	325	0.59	15	37.33	0.75	4.40	21.17	4.33	10838.48	80.89	0.14%	0+
3	150	307	0.60	15	38.87	0.78	4.40	19.65	4.09	10147.60	85.76	0.36%	0+
4	150	312	0.61	15	41.20	0.75	4.40	20.98	4.16	12092.49	53.49	0.00%	0+
5	150	306	0.62	15	41.40	0.76	4.33	21.10	4.08	12606.49	44.05	0.23%	0+
6	195	386	0.59	15	39.73	0.74	4.60	26.41	5.28	14463.08	300.70	0.50%	0+
7	195	383	0.60	15	40.65	0.76	4.53	24.41	5.11	13935.74	144.99	0.10%	0+
8	195	387	0.60	15	37.39	0.75	4.53	25.69	5.16	13187.67	199.83	0.15%	0+
9	195	376	0.60	15	36.93	0.78	4.40	23.95	5.01	12562.94	262.86	0.08%	0+
10	195	351	0.60	15	35.79	0.76	4.40	23.06	4.68	11417.17	132.27	0.06%	0+
11	200	402	0.60	20	41.75	0.76	4.20	19.65	4.02	15349.36	188.97	0.08%	0+
12	200	419	0.60	20	39.47	0.76	4.45	20.53	4.19	14792.05	2105.04	0.28%	1702
13	200	421	0.59	20	38.84	0.77	4.25	19.84	4.21	13771.91	370.12	0.15%	0+
14	200	432	0.60	20	40.05	0.76	4.50	20.71	4.32	15161.04	545.55	0.24%	0+
15	200	416	0.61	20	43.55	0.77	4.45	19.94	4.16	16090.59	223.30	0.16%	0+
16	280	504	0.60	20	36.54	0.79	4.65	23.16	5.04	15251.15	4151.89	0.27%	1443
17	280	524	0.60	20	40.50	0.77	4.45	25.13	5.24	19070.66	797.38	0.07%	0+
18	280	540	0.60	20	35.58	0.76	4.50	25.87	5.40	17138.18	1654.47	0.24%	410
19	280	549	0.60	20	38.61	0.75	4.80	27.86	5.49	18651.15	2302.81	0.14%	210
20	280	534	0.61	20	40.55	0.74	4.65	26.59	5.34	20432.90	1688.89	0.21%	311

Table 9. Results from model (3) for different therapist at the same location

Prob.no.	Fixed patients			Therapists						Results			
				Input			Output						
	No.	No. Visits	Avg. treatment time/visit	No.	Avg. wage/hr	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/therapist per day	Cost ($)	CPU (sec)	Gap (%)	Nodes no.
1	150	314	0.61	15	39.03	0.78	4.20	21.66	4.19	12849.09	1211.93	0.54%	10000
2	150	300	0.58	15	40.37	0.74	4.47	20.75	4.00	12221.41	1422.89	1.13%	10000
3	150	322	0.50	15	39.47	0.75	4.13	22.40	4.29	13164.55	2175.74	0.54%	9481
4	150	303	0.60	15	40.30	0.77	4.13	21.02	4.04	12418.14	383.86	0.47%	603
5	150	303	0.60	15	37.79	0.77	4.33	21.00	4.04	11573.14	5176.30	0.63%	10000
6	195	363	0.60	15	39.00	0.81	4.33	23.81	4.84	13751.38	2115.18	0.73%	10000
7	195	374	0.59	15	37.69	0.78	4.40	24.88	4.99	13680.13	2120.95	0.81%	10000
8	195	379	0.58	15	40.33	0.74	4.53	25.53	5.05	15102.68	1848.04	0.50%	3985
9	195	361	0.59	15	37.97	0.76	4.27	24.66	4.81	13639.73	5328.43	0.68%	10000
10	195	378	0.59	15	37.19	0.77	4.47	24.95	5.04	13444.85	3242.68	0.57%	10000
11	200	416	0.59	20	35.90	0.78	3.95	19.65	4.16	13086.67	6946.45	0.77%	10000
12	200	393	0.62	20	37.32	0.76	4.20	20.30	3.93	15097.79	4775.64	1.38%	10000
13	200	457	0.55	20	39.72	0.75	4.50	21.52	4.57	16410.64	9830.18	1.07%	10000
14	200	452	0.50	20	38.67	0.79	4.30	18.66	4.52	13698.76	5975.20	1.31%	10000
15	200	426	0.59	20	37.73	0.78	4.10	20.55	4.26	14955.84	6876.39	0.79%	10000
16	280	554	0.59	20	38.57	0.75	4.80	26.48	5.54	19048.09	56382.74	1.25%	10000
17	280	567	0.59	20	36.24	0.76	4.75	27.32	5.67	18850.23	47451.51	1.09%	10000
18	280	540	0.59	20	38.95	0.76	4.55	26.47	5.40	19865.01	33404.91	1.26%	10000
19	280	539	0.60	20	37.07	0.80	4.60	25.39	5.39	18139.03	26397.81	1.15%	10000
20	280	532	0.60	20	39.85	0.77	4.70	26.03	5.32	20144.04	27554.21	1.11%	10000

Table 10. Results from model (3) for different therapists at different locations

Prob.no.	Fixed patients			Therapists						Results			
				Input			Output						
	No.	No. Visits	Avg. treatment time/visit	No.	Avg. wage/hr	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/therapist per day	Cost ($)	CPU (sec)	Gap (%)	Nodes no.
1	150	306	0.60	15	39.33	0.74	4.60	20.81	4.08	12177.52	1566.60	1.08%	10000
2	150	325	0.59	15	37.33	0.75	4.40	21.07	4.33	11270.85	2133.87	1.09%	10000
3	150	307	0.60	15	38.87	0.78	4.40	19.53	4.09	10442.12	1575.07	0.66%	10000
4	150	312	0.61	15	41.20	0.75	4.40	21.06	4.16	12565.20	1705.30	0.68%	10000
5	150	306	0.62	15	41.40	0.76	4.33	20.99	4.08	13076.97	1487.60	0.59%	10000
6	195	386	0.59	15	39.73	0.74	4.73	26.05	5.28	14991.18	7689.80	0.88%	10000
7	195	383	0.60	15	40.65	0.76	4.20	24.34	5.11	14446.38	2512.21	0.54%	10000
8	195	387	0.60	15	37.39	0.75	4.53	25.59	5.16	13740.74	5841.48	0.99%	10000
9	195	376	0.60	15	36.93	0.78	4.40	23.89	5.01	13016.69	3383.83	0.71%	10000
10	195	351	0.60	15	35.79	0.76	4.53	22.97	4.68	11952.35	1404.21	0.74%	10000
11	200	402	0.60	20	41.75	0.76	4.20	19.63	4.02	15914.56	5817.41	0.98%	10000
12	200	419	0.60	20	39.47	0.76	4.45	20.42	4.19	15363.32	6235.48	1.39%	10000
13	200	421	0.59	20	38.84	0.77	4.25	19.72	4.21	14333.70	7645.86	1.16%	10000
14	200	432	0.60	20	40.05	0.76	4.50	20.58	4.32	15653.36	8726.33	1.45%	10000
15	200	416	0.61	20	43.55	0.77	4.60	19.71	4.16	16490.05	8602.79	0.84%	10000
16	280	504	0.60	20	36.54	0.79	4.65	23.17	5.04	15920.99	37649.13	2.04%	20000
17	280	524	0.60	20	40.50	0.77	4.45	25.14	5.24	20003.24	39894.68	0.87%	20000
18	280	540	0.60	20	35.58	0.76	4.55	25.76	5.40	17695.48	46769.14	1.14%	20000
19	280	549	0.60	20	38.61	0.75	4.80	26.39	5.49	19391.73	109289.64	1.44%	20000
20	280	534	0.61	20	40.55	0.74	4.65	26.42	5.34	21148.66	63705.99	1.12%	20000

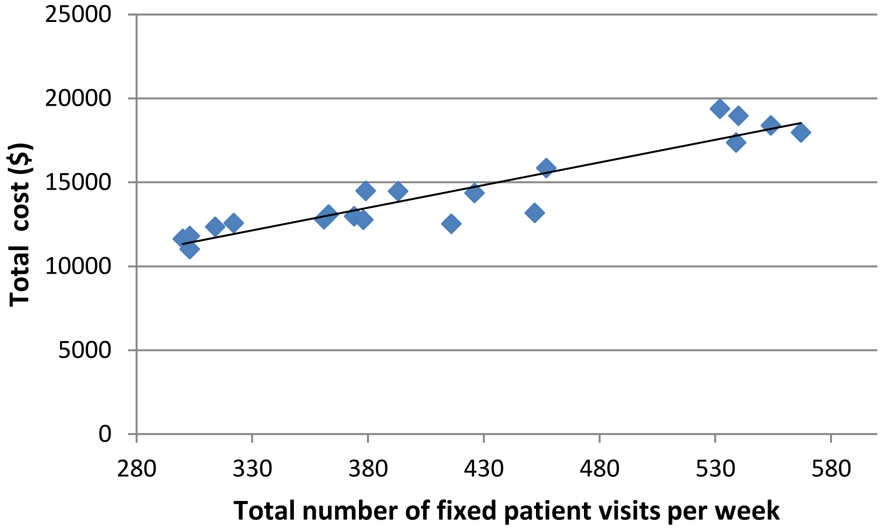

Figure 1. Total visits and total cost from model (2) for different therapists at the same location

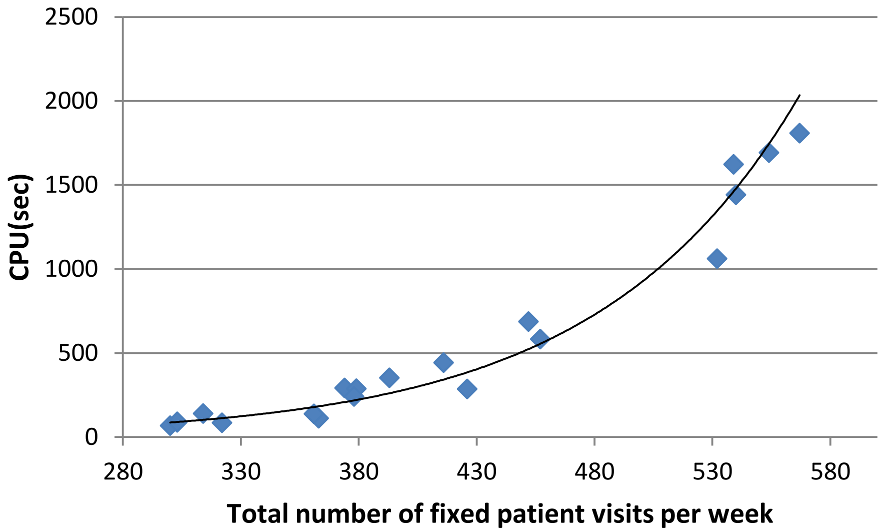

Figure 2. CPU time and total cost from model (2) for different therapists at the same location

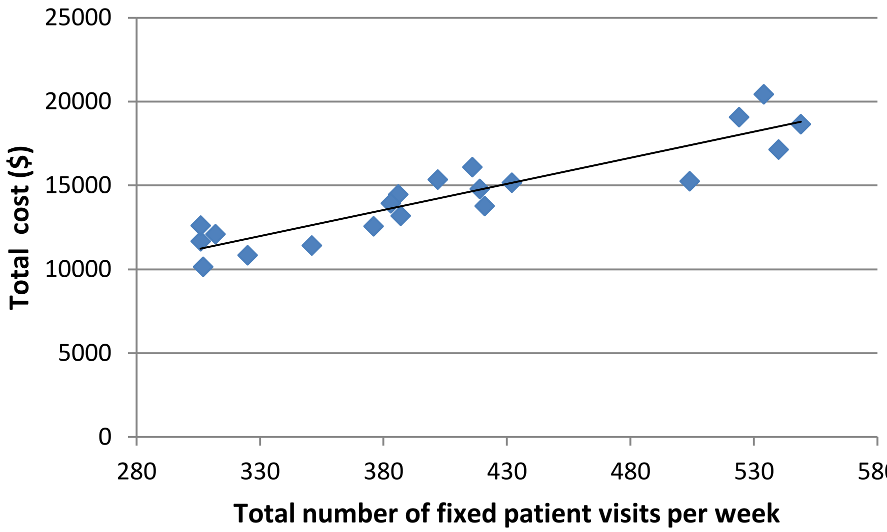

Figure 3. Total visits and total cost from model (2) for different therapists at different location

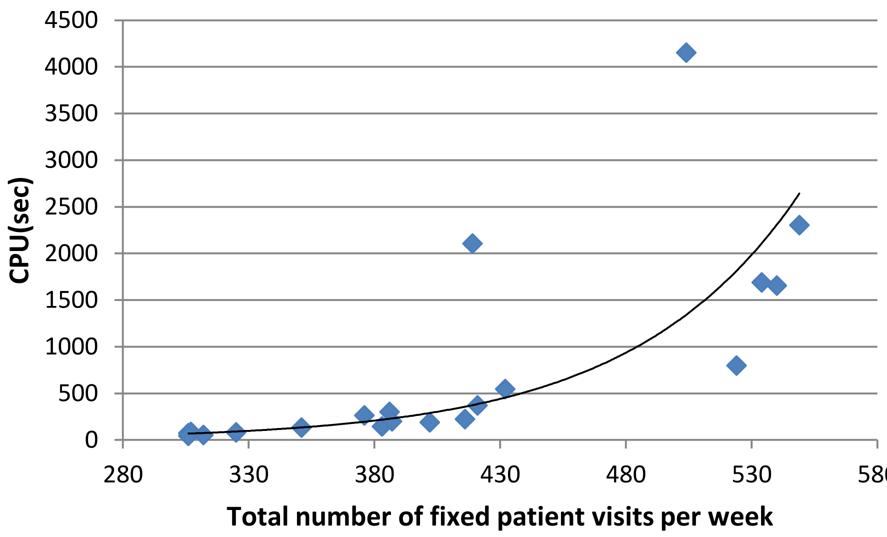

Figure 4. CPU time and total cost from model (2) for different therapists at different location

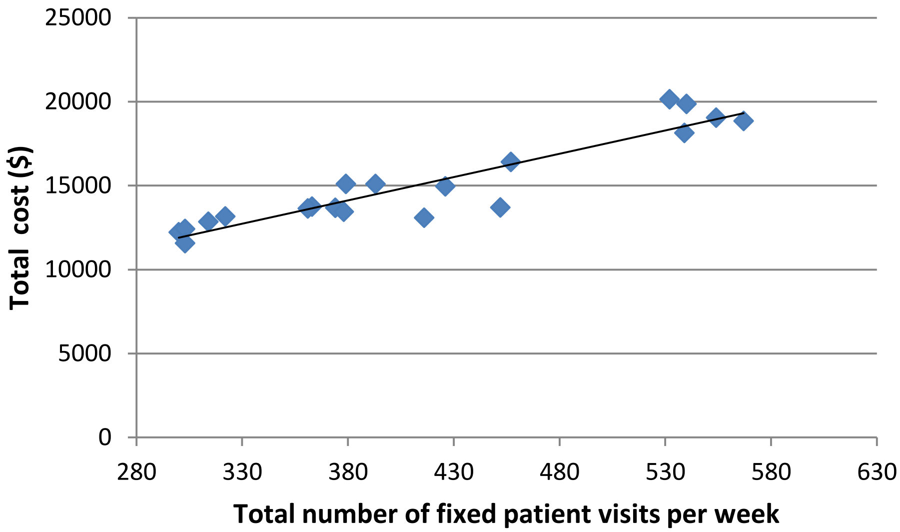

Figure 5. Total visits and total cost from model (3) for different therapists at the same location

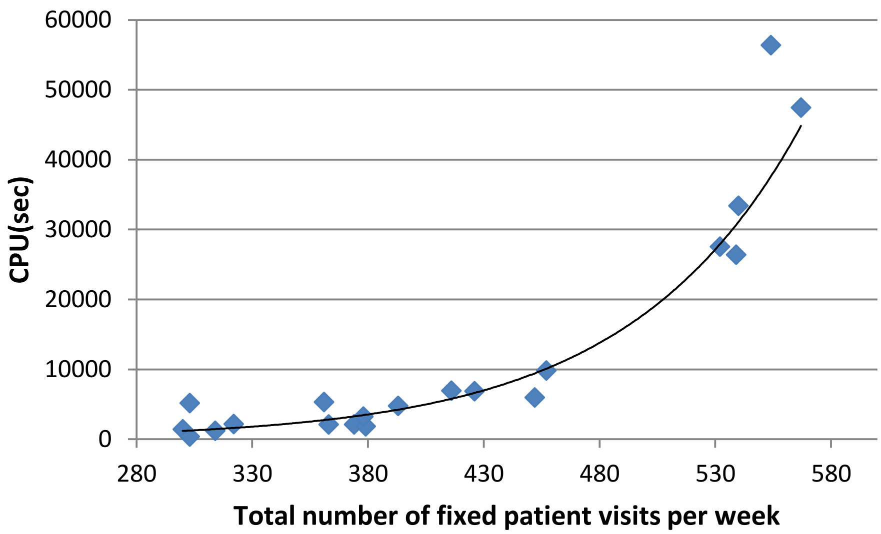

Figure 6. CPU time and total cost from model (3) for different therapists at the same location

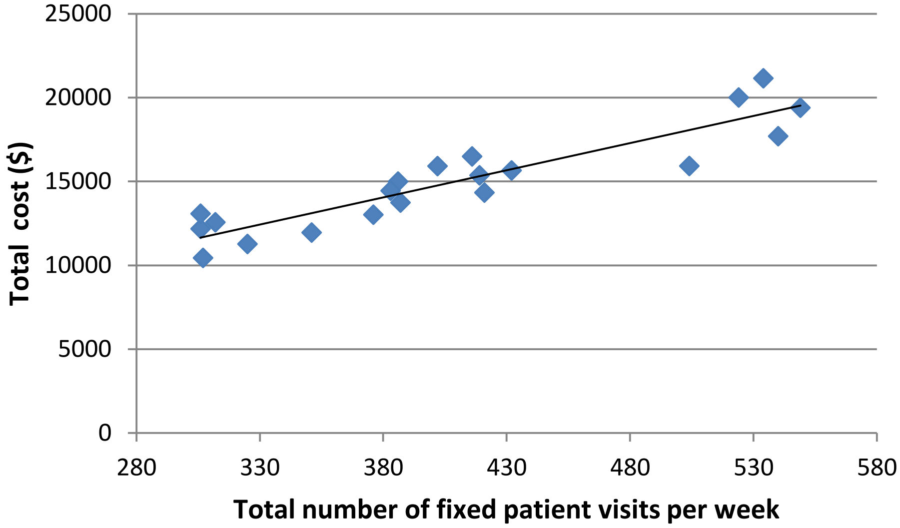

Figure 7. Total visits and total cost from model (3) for different therapists at different location

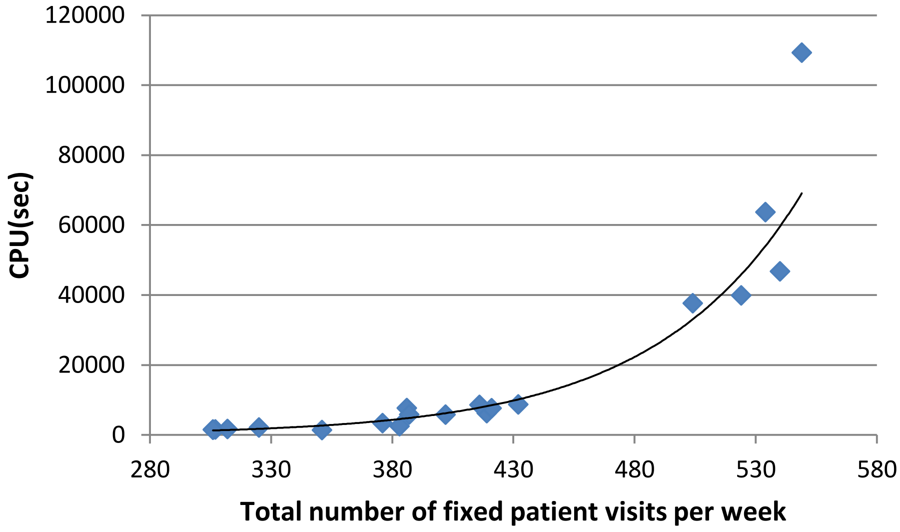

Figure 8. CPU time and total cost from model (3) for different therapists at different location

7. References

Bard, J.F. (2011). Therapist routing and scheduling problem. Working paper, Graduate Program in Operations Research & Industrial Engineering, The University of Texas, Austin, Texas.

Bard, J.F. and A.I. Jarrah (2009). A large-scale constrained clustering for rationalizing pickup and delivery operations. *Transportation Research*, 43(5), 542–561.

Begur, S. V., D. M. Miller and J. R. Weaver (1997). An integrated spatial DSS for scheduling and routing home-health-care nurses. *Interfaces*, 27(4), 35–48.

Bertels, S., T. Fahle (2006). A hybrid setup for a hybrid scenario: Combining heuristics for the home health care problem. *Computers & Operations Research*, 33(10), 2866–2890.

Boschetti. M.A, A. Mingozzi and S. Ricciardelli (2004). An exact algorithm for the simplified multiple depot crew scheduling problem, *Annals of Operations Research*, 127, 177-201.

Carpaneto. G and M. Dell'Amico (1989), A branch and bound algorithm for the multiple depot vehicle scheduling problem, *Networks*, 19(5), 531-548.

Cheng, E and J.L. Rich (1998). A home health care routing and scheduling problem. Technical report, Rice University, Houston, Texas, USA.

Dell'Amico.M, M. Fischetti and P. Toth (1993). Heuristic algorithms for the multiple depot vehicle scheduling problem, Technical report, University of Bologna, Bologna, Italy.

Desrosiers, J., Y. Dumas, M.M. Solomon and F. Soumis (1995). Time constrained routing and scheduling. In *Handbooks in Operations Research and Management Science*, Volume 8, *Network Routing*, T.L. Magnanti, C.L. Monma and G.L. Nemhauser (eds.), North-Holland, Amsterdam, 35-139.

Kergonsien, Y., C. Lente and J. Billaut (2009). Home health care problem, an extended multiple Traveling Salesman problem. *4th Multidisciplinary International Scheduling Conference: Theory and Appllications*, 85-92, Dublin, Ireland.

Lenstra, J.K., K. Rinnooy and A.H.G. (1981). Complexity of vehicle routing and scheduling problems. *Networks*, 11(2), 221-228.

Ribeiro. C and F. Soumis (1994). A column generation approach to the multiple-depot vehicle scheduling problem, *Central European Journal of Operations Research*, 42(1), 41-52.

Shao, Y. (2011), Home therapist network modeling. PhD dissertation, Graduate Program in Operations Research & Industrial Engineering, The University of Texas, Austin, Texas.

Steeg, J. and M. Schröder (2007). A hybrid approach to solve the periodic home health care problem, In *Operations Research Proceedings*, Volume 2007, Part XII, J.Kalcsics and S. Nickel (eds.), SpringerLink, German, 297-302.

Super, N. (2002). Who will be there to care? The growing gap between caregiver supply and demand. *National Health Policy Forum*, 1-19, George Washington University, Washington DC. http://www.nhpf.org/library/background-papers/BP_Caregivers_1-02.pdf