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# Effective Power Factor: Analysis and Implementation

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# Effective Power Factor: Analysis and Implementation

**by**

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# Dedication

To my parents, Keyu and Shiyong

To Jiao

I love you

# Acknowledgements

First, I would like to thank my supervisor, Professor Surya Santoso. Without his help, I would not have been able to complete this research, and would not have chosen to further my studies at the University of Texas at Austin.

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Finally, I would like to give thanks to my parents for their constant support.

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# Abstract

Effective Power Factor: Analysis and Implementation

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The study reviews and examines the definitions of reactive power, apparent power, and power factor. Among the different definitions of power factor in three-phase circuits under a non-sinusoidal condition, this study adopts the definition of the effective power factor, which is also advocated by IEEE Standard 1459-2010. The effective power factor is defined as the ratio of the real power consumed by the load over the effective apparent power. The effective apparent power is the maximum power transmitted to the load (or delivered by a source) while keeping the same line losses and the same load (or source) voltage and current. The effective power factor theory gives apparent power a definite physical significance and provides more insights than other definitions in unbalanced circuits. Another merit of the effective power factor definition is that it only involves measurements and computations in the time domain. This study implements the computation of the effective power factor in MATLAB for use in PSCAD/EMTDC. The latter simulates the power system and provides three-phase voltage and current

measurements. MATLAB performs the effective power factor computation and sends the results back to PSCAD.  A number of simulations are provided in this report to demonstrate the validity and the accuracy of this implementation.

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# Chapter 1 Introduction

## 1.1 POWER FACTOR DEFINITION

Power factor is an important steady-state quantity in power quality. It has been used to describe the effectiveness of electrical load in expending power delivered by the supply system. Power companies, such as transmission and distribution utilities, are particularly concerned with this issue. In low power factor circuits, , a large amount of power produced by the generators cannot be effectively expended by the load. The transmission and distribution utilities attempt to avoid this situation because first of all it wastes transmission and generation resources. If a specific amount of demands needs to be met, more generators and lines have to be built, which in turn increases operational costs. Secondly, although only a part of the delivered power is consumed by the load, a significant amount of power is actually travelling in the power system. It is carried around by large inductive currents that are out of phase with the voltage. A large current increases the power loss on transmission lines, which further lowers transmission efficiency. More serious consequences could include the overheating of equipment and other accidents (transmission line sagging into a tree), which in the worst cases could lead to a system breakdown.

Due to the above reasons, utilities encourage power system entities to maintain the power factor, and penalize operations that deteriorate the power factor. The discussion regarding definitions of power factor is ongoing.

In single-phase circuits under a sinusoidal condition, the power factor is the cosine of the phase angle difference between the voltage and current waveforms. The problem lies with the presence of harmonic distortion, causing a non-sinusoidal condition on single-phase circuits. Power factor can no longer be defined by the cosine of the phase angle difference because in addition to the phase angle of the fundamental frequency, there are also phase angles of harmonic frequencies. Chapter 2.3 shows that although the definition of real power is widely accepted in single-phase circuits under non-sinusoidal conditions, there exist a number of different definitions of reactive power, apparent power, and power factor.

## 1.2 Scope of This Study

This study advocates a specific definition for power factor, that is, the effective power factor. This definition was first proposed by F. Buchholz in 1922 and explained by W. M. Goodhue in 1933. And, it was recently adopted by IEEE's Standard 1459-2010 as the definition for the measurement of electric power quantities. This study constructs a measurement system consisting of a MATLAB function that calculates the effective power factor and a FORTRAN based interface program. This program passes the data between the MATLAB and the time-domain power system simulation software, PSCAD/EMTDC. The proposed system can be used by PSCAD as a meter block. It measures the effective power factor of any three-phase circuits during a given simulation run-time.

Chapter 2 reviews the prevalent power theories from single-phase circuits to three-phase circuits, and from sinusoidal to non-sinusoidal ones. Some of these theories are closely related to the development of the effective power factor theory, while others oppose it. These theories will then be compared with the effective power factor theory in this Chapter.

Chapter 3 explains the effective power factor theory. It starts by clarifying the physical significance of the effective power factor, and then introduces the definitions of effective voltage, effective current, effective apparent power, and most importantly, the effective power factor. Finally, using two examples, this chapter identifies the advantages of effective power factor over other power factor definitions in unbalanced circumstances.

Chapter 4 shows the structure and the feature of the proposed measurement system. It consists of a MATLAB function and a FORTRAN interface program. The chapter also describes how the measurement system works with PSCAD, the power system simulation platform.

Chapter 5 presents a few cases in which the effective power factors are measured using the proposed measurement mechanism. The results are compared with other power factor definitions and verified by IEEE Standard 1459-2010.

Chapter 6 concludes this research, and discusses the future scope of this work.

# Chapter 2 Review of Prevalent Power Theories

Effective power theory is one among a number of power theories. To provide a relevant background of effective power theory and to better understand its advantages over other power factor theories, Chapter 2 reviews a number of prevalent theories. These theories include those of single-phase circuits and three-phase circuits, under both sinusoidal and non-sinusoidal conditions. Supporting and opposing arguments of effective power theory are presented.

## 2.1 Power Theory in Single-Phase Circuits under a Sinusoidal Condition

### 2.1.1 Voltage and Current

When a sinusoidal voltage source is serving linear loads, such as resistors, inductors, and capacitors, the waveshape of the current waveform is identical to that of the voltage waveform as shown in Fig. 2.1. In other words, linear loads do not modify the waveshape and frequency of the load current.

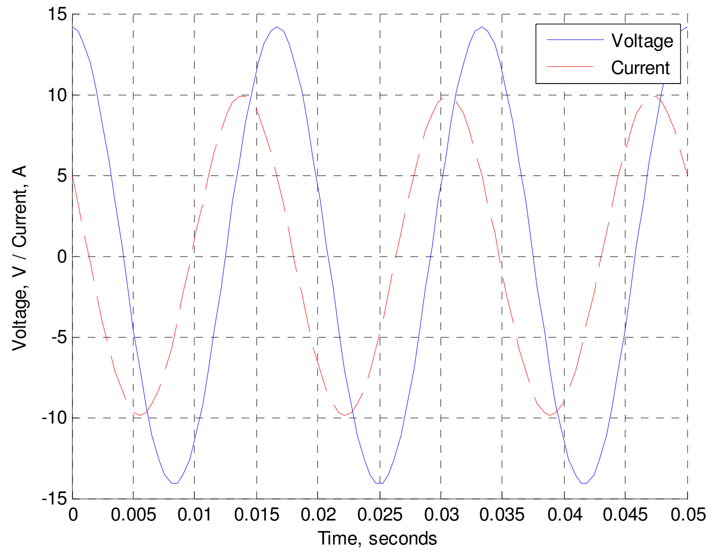


Fig. 2.1 Sinusoidal voltage and current waveforms in single-phase circuits

Voltage and current are usually defined with a cosine wave, as shown below [1] [2] [3]:

Voltage waveform:

$$v(t) = V_{\max} \cos(\omega t + \alpha) \tag{1}$$

Current waveform:

$$i(t) = I_{\max} \cos(\omega t + \beta) \tag{2}$$

where

$V_{\max}$ and $I_{\max}$ are the crest values of the voltage and current waveforms, respectively;

$\alpha$ and $\beta$ are the phase angles of the voltage and current waveforms; and,

$\omega$ is the angular frequency of the voltage and the current, for a 60-Hz system, $\omega = 120\pi$.

The root mean square (rms) value of any periodic signal is defined below [1]:

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} = \frac{V_{\max}}{\sqrt{2}} \tag{3}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \frac{I_{\max}}{\sqrt{2}} \tag{4}$$

Conventionally the subscript of rms can be omitted; thus the rms value of voltage and current can also be written as $V$ and $I$. The voltage and current waveforms can then be expressed as:

$$v(t) = \sqrt{2}V \cos(\omega t + \alpha) \tag{5}$$

$$i(t) = \sqrt{2}I \cos(\omega t + \beta) \tag{6}$$

### 2.1.2 Instantaneous Power

The instantaneous power *p(t)* varies with time. At any time, $t$, it is defined as the product of the voltage and current at the same moment.

$$p(t) = v(t) \cdot i(t) = V_{\max} I_{\max} \cos(\omega t + \alpha)\cos(\omega t + \beta) \tag{7}$$

Instantaneous power represents the rate of the flow of energy per unit time. It can be further decomposed into two components [3] [4] [5]:

$$p(t) = VI\cos(\alpha - \beta) + VI\cos(2\omega t + \alpha + \beta) \tag{8}$$

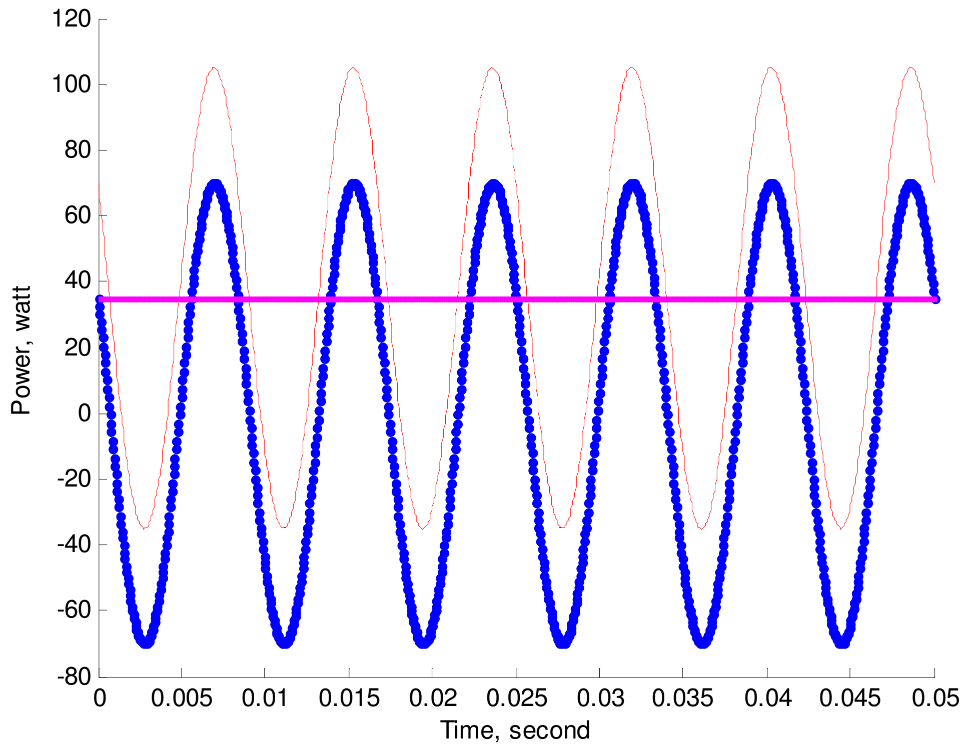


Fig. 2.2 Decomposition of instantaneous power

As shown in Fig. 2.2, the first component of instantaneous power is a constant term that is independent of time. The second component is a double frequency oscillating term, the average of which is zero. Therefore, the average of the instantaneous power over a period of time is equal to the first term given in (8).

Das, Kirtley, and Grainger propose another approach to decompose the instantaneous power [4] [5] [6]:

$$p(t) = VI\cos\varphi(1 + \cos 2\omega t) + VI\sin\varphi\sin 2\omega t \tag{9}$$

where, $\varphi = \alpha - \beta$ is the phase angle difference between the voltage and the current.

This decomposition is illustrated graphically in Fig. 2.3

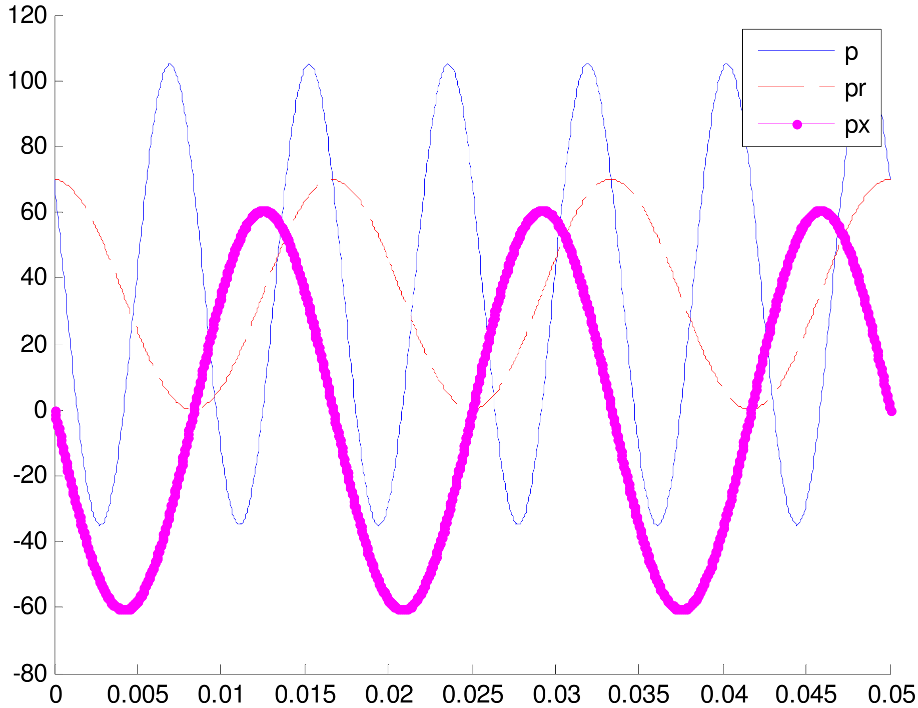


Fig. 2.3 Another method of instantaneous power decomposition

Grainger previously explained the physical interpretation behind this decomposition method in [6]. According to his findings, any sinusoidal current waveform can undergo decomposition into two components, the first of which is the current in phase with the voltage:

$$i_R = \sqrt{2}I\cos\varphi\cos\omega t \tag{10}$$

The other component is the current lagging voltage by $90°$

$$i_X = \sqrt{2}I\sin\varphi\cos\omega t \tag{11}$$

Since the instantaneous power produced by $i_R$ and $i_X$ are:

$$p_R = v\cdot i_R = 2V\cdot I\cdot\cos\varphi\cdot\cos^2\omega t = V\cdot I\cdot\cos\varphi\cdot(1+\cos 2\omega t) \tag{12}$$

$$p_X = v\cdot i_X = 2V\cdot I\cdot\sin\theta\cdot\sin\omega t\cdot\cos\omega t = V\cdot I\cdot\sin\varphi\cdot sin2\omega t \tag{13}$$

these values are respectively equal to the first and second term of the instantaneous power defined in (9). Therefore, the first term of the instantaneous power in (9) is the active power that is consumed by the circuit, while the second term in (9) is called the instantaneous reactive power [6], which represents the flow of energy sloshing back and forth between the source and the load.

### 2.1.3 Active Power

Active power is defined as the time average of the instantaneous power in a period of time $T$ [2] [3]:

$$P = \frac{1}{T}\int_0^T p(t)dt = VI\cos\varphi = S\cos\varphi \tag{14}$$

Active power represents the average power that the circuit consumes over a period. The first decomposition of instantaneous power expressed in (8) generates a DC term and an AC term that oscillates about zero. Therefore, the first DC term in (8) is exactly the same as the time average of instantaneous power; or, in other words, it is active power. Active power is also called real power [6].

### 2.1.4 Reactive Power

Similar to active power, reactive power is defined as the amplitude of the second term $p_X$ in (9) [2] [3] [6]:

$$Q = VI\sin\varphi \tag{15}$$

Conventionally, the physical meaning of reactive power denotes a power component, the average of which is zero. In a passive network, the reactive power represents the maximum energy stored in inductors or capacitors.

One of the physical interpretations of reactive power in single-phase circuits under a sinusoidal condition is that it equals the maximum instantaneous power stored in the magnetic or electric field. The energy stored in an inductor's magnetic field at any time is equal to:

$$E = \frac{1}{2} i^2(t) L \tag{16}$$

Therefore, assuming a constant inductance over time, the instantaneous power flowing into the inductor is equal to the derivative of the stored energy:

$$p_L(t) = \frac{\partial E}{\partial t} = i(t) \frac{\partial i(t)}{\partial t} L \tag{17}$$

If a sinusoidal voltage is connected to a series RL load, the current phasor is:

$$\bar{I} = \frac{\bar{V}}{R + j\omega L} \tag{18}$$

where $\bar{I}$ and $\bar{V}$ represent the current and voltage in phasor form. The following shows the conversion of the current from the phasor to time domain:

$$i(t) = \sqrt{2} \cdot \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \beta) \tag{19}$$

Thus, the time derivative of current is:

$$\frac{\partial i(t)}{\partial t} = \sqrt{2} I \omega \cos(\omega t + \beta) \tag{20}$$

where $I = V / \sqrt{R^2 + \omega^2 L^2}$ is the root mean square value of the current. Substitute (19) and (20) into (17):

$$\begin{aligned} p_L(t) &= 2I^2 \omega L \sin(\omega t + \beta) \cos(\omega t + \beta) \\ &= I^2 \omega L \sin(2\omega t + 2\beta) \end{aligned} \tag{21}$$

Therefore, the maximum instantaneous power flowing into the inductor is:

$$\begin{aligned} p_{L,\max} &= I^2 \omega L \\ &= VI \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \\ &= VI \sin \varphi \end{aligned} \tag{22}$$

As the result shows, the maximum instantaneous power that flows into an inductor in a circuit is equal to the reactive power. Above is an example of an inductor. The use of a capacitor yields similar results.

### 2.1.5 Complex Power

With active and reactive power defined, complex power in single-phase circuits under a sinusoidal condition is given as a complex number, or a vector whose real part is the same as active power and whose imaginary part is the same as reactive power [1] [2] [3] [4] [5] [6]:

$$\overline{S} = P + jQ \tag{23}$$

The over bar means this variable is in a phasor form.

The introduction of the phasor expression for both the voltage and the current to complex power yields the following:

$$\overline{V} = V \cdot e^{j(\omega t+\alpha)} = V\angle\alpha \tag{24}$$

$$\overline{I} = I \cdot e^{j(\omega t+\beta)} = I\angle\beta \tag{25}$$

The complex power can be expressed as

$$\begin{aligned}\overline{S} &= P + jQ \\ &= VI\cos\varphi + jVI\sin\varphi \\ &= VI \cdot e^{j(\alpha-\beta)} \\ &= \left(V \cdot e^{j(\omega t+\alpha)}\right)\left(I \cdot e^{-j(\omega t+\beta)}\right) \\ &= \overline{V} \cdot \overline{I}^{*}\end{aligned} \tag{26}$$

where the asterisk * represents the conjugate of a vector.

Apparent power in single-phase circuits under a sinusoidal condition is defined as the length of the complex power.

$$S = \left|\overline{S}\right| = \sqrt{P^2 + Q^2} = V \cdot I \tag{27}$$

The relationship between active power, reactive power and apparent power in single-phase circuits under a sinusoidal condition is often referred to as a power triangle [1] [6], as shown in Fig. 2.4

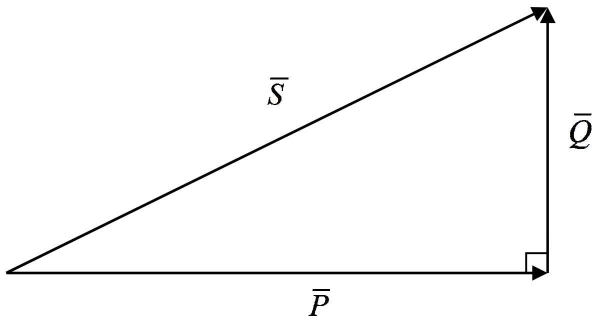


Fig. 2.4 The power triangle

**2.1.6 Power Factor**

Power factor is the ratio between the real power and the apparent power.

$$PF = \frac{P}{S} \tag{28}$$

Power factor is a very important index in power systems. It indicates the portion of transmitted apparent power that is actually exhausted. For a load of constant active power demand, a lower power factor means a higher apparent power. Since the voltage level of the power grid is fixed, the higher the apparent power, the higher the line current. This not only lowers the efficiency of transmission, but also can cause serious problems for regulation of the power system.

Strzelecki calls this value the displacement power factor [2] because in single-phase circuits under a sinusoidal condition, the power factor is equal to the cosine of the phase angle difference between the voltage and current vectors.

$$PF = \frac{P}{S} = \frac{VI\cos\varphi}{VI} = \cos\varphi \tag{29}$$

## 2.2 POWER THEORY IN THREE-PHASE CIRCUITS UNDER A SINUSOIDAL CONDITION

### 2.2.1 Balanced Three-Phase Circuits

#### *2.2.1.1 General Power Definitions*

In balanced three-phase circuits under a sinusoidal condition, magnitudes of line to neutral voltages and the line currents are identical in each phase:

$$|V_p| = |V_{an}| = |V_{bn}| = |V_{cn}| \tag{30}$$

$$|I_p| = |I_{an}| = |I_{bn}| = |I_{cn}| \tag{31}$$

The total three phase active power and reactive power are defined below [6]:

$$P = 3|V_P||I_P|\cos\varphi_P \tag{32}$$

$$Q = 3|V_P||I_P|\sin\varphi_P \tag{33}$$

The total three phase apparent power of the load is:

$$|S| = \sqrt{P^2 + Q^2} = 3|V_P||I_P| \tag{34}$$

#### *2.2.1.2 Instantaneous Power in Balanced Three-Phase Circuits*

One of the most significant features of three-phase circuits is that although the instantaneous power in each single phase is time variant, the total instantaneous power consumed by a three-phase load remains constant.

If the line to neutral voltage and line currents of the three phases are as follows:

$$\begin{aligned} v_{an}(t) &= V_{\max}\cos(\omega t + \delta) & i_a(t) &= I_{\max}\cos(\omega t + \beta) \\ v_{bn}(t) &= V_{\max}\cos(\omega t + \delta - 120°) & i_b(t) &= I_{\max}\cos(\omega t + \beta - 120°) \\ v_{cn}(t) &= V_{\max}\cos(\omega t + \delta + 120°) & i_c(t) &= I_{\max}\cos(\omega t + \beta + 120°) \end{aligned} \tag{35}$$

then the instantaneous powers in the three phases are respectively:

$$p_a(t) = v_{an}(t)i_a(t) = V_{rms}I_{rms}\cos(\delta - \beta)[1 + \cos 2(\omega t + \delta)] + V_{rms}I_{rms}\sin(\delta - \beta)\sin 2(\omega t + \delta) \tag{36}$$

$$p_b(t) = v_{bn}(t) i_b(t) = V_{rms} I_{rms} \cos(\delta - \beta)[1 + \cos 2(\omega t + \delta - 120°)] + V_{rms} I_{rms} \sin(\delta - \beta) \sin 2(\omega t + \delta - 120°)$$
(37)

$$p_c(t) = v_{cn}(t) i_c(t) = V_{rms} I_{rms} \cos(\delta - \beta)[1 + \cos 2(\omega t + \delta + 120°)] + V_{rms} I_{rms} \sin(\delta - \beta) \sin 2(\omega t + \delta + 120°)$$
(38)

The total real power of the three-phase circuit is the sum of the three single-phase real powers:

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t) \tag{39}$$

At any time $t$, the sum of the following terms is zero:

$$\cos 2(\omega t + \delta) + \cos 2(\omega t + \delta + 120°) + \cos 2(\omega t + \delta - 120°) = 0 \tag{40}$$

$$\sin 2(\omega t + \delta) + \sin 2(\omega t + \delta + 120°) + \sin 2(\omega t + \delta - 120°) = 0 \tag{41}$$

Therefore, all the terms that are varying with the time $t$ are cancelled out in the expression of three-phase instantaneous power. The simplified expression is:

$$p_{3\phi}(t) = 3 V_{rms} I_{rms} \cos(\delta - \beta) \tag{42}$$

Given these conditions, the instantaneous power of three-phase circuits under a sinusoidal condition can yield a constant figure with no pulsating components. This result is shown in Fig. 2.5. Solid curves exhibit single-phase instantaneous powers, while the dashed level line overhead represents the sum of the three sinusoidal waves, or the three-phase instantaneous power.

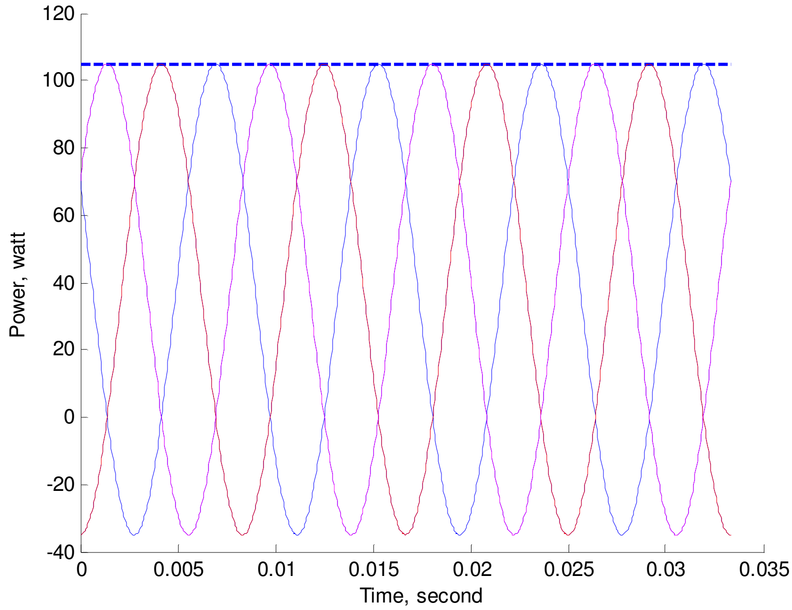


Fig. 2.5 Phase and total instantaneous power

### 2.2.2 Unbalanced Three-Phase Circuits

Unlike in single-phase circuits, the rms values and the relationship between voltage and current waveforms in different phases differ according to load characteristics as well as other factors. If this is the case, the general power theory defined for the balanced three-phase system no longer applies. All three phases must be treated individually.

#### *2.2.2.1 Active Power in Three-Phase Unbalanced Circuits*

Active power in three-phase circuits is defined as the sum of all three active powers:

$$P = P_a + P_b + P_c \tag{43}$$

This still represents the time average of total instantaneous power. Since active power must be consumed in one of the three phases, the average of the total instantaneous power must be equal to the sum of the average of instantaneous power in each phase.

#### *2.2.2.2 Apparent Power in Three-Phase Unbalanced Circuits*

The definition of apparent power in three-phase unbalanced circuits is not unanimous. If the voltage and current are sinusoidal and the circuit is well balanced, the results under different definitions remain the same. However, under certain conditions, different definitions can yield different results. Some of these definitions are listed below [2] [4]:

Vector apparent power

$$S_V = \left|\overline{S}_a + \overline{S}_b + \overline{S}_c\right| \tag{44}$$

Arithmetic apparent power

$$S_A = \left|\overline{S}_a\right| + \left|\overline{S}_b\right| + \left|\overline{S}_c\right| \tag{45}$$

where, $\overline{S}_k, k \in \{a,b,c\}$ is the $k$ th phase complex power. Since three-phase circuits under a non-sinusoidal condition also introduce these definitions, these issues will be discussed further in Chapter 2.4. At this point, however, one can conclude that the arithmetic apparent power is always greater than or equal to the vector apparent power. Moreover, if the circuit is balanced, then:

$$\overline{S}_a = \overline{S}_b = \overline{S}_c \tag{46}$$

Under this condition, the vector apparent power and the arithmetic apparent power are the same:

$$S_V = S_A = 3S_p \tag{47}$$

In [3], Hase adopts the vector apparent power theory and further develops the theory in the sequence domain. These adaptions enable engineers to treat any arbitrary unbalanced three-phase system as the sum of three balanced circuits, with one in positive sequence, one in negative sequence, and one in zero sequence.

The following represents the transformation of the voltage and the current from the phase domain to the sequence domain.

$$\overline{V}_{abc} = \begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix} = \mathbf{a}^{-1} \begin{bmatrix} \overline{V}_0 \\ \overline{V}_1 \\ \overline{V}_2 \end{bmatrix} = \mathbf{a}^{-1}\overline{V}_{012} \tag{48}$$

$$\overline{I}_{abc} = \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix} = \mathbf{a}^{-1} \begin{bmatrix} \overline{I}_0 \\ \overline{I}_1 \\ \overline{I}_2 \end{bmatrix} = \mathbf{a}^{-1}\overline{I}_{012} \tag{49}$$

where matrix $\mathbf{a}$ is the 0-1-2 transformation matrix defined as

$$\mathbf{a} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad , \quad a = e^{j120°} = 1\angle 120° \tag{50}$$

The three phase instantaneous apparent power is defined as

$$\begin{aligned} S_{3\phi} &= \overline{V}_{abc} \cdot \overline{I}_{abc}^{\;*} \\ &= \left(\mathbf{a}^{-1}\overline{V}_{012}\right)^{\mathrm{T}} \cdot \left(\mathbf{a}^{-1}\overline{I}_{012}\right)^{*} \\ &= \overline{V}_{012}^{\;\mathrm{T}} \cdot \left(\mathbf{a}^{-1}\right)^{\mathrm{T}} \cdot \overline{I}_{012}^{\;*} \cdot \left(\mathbf{a}^{-1}\right)^{*} \end{aligned}$$

$$\because \left(\mathbf{a}^{-1}\right)^{\mathrm{T}} = \mathbf{a}^{-1}, \mathbf{a}^{-1} = 3\mathbf{a}$$

$$\begin{aligned} S_{3\phi} &= 3\left\{\overline{V}_0 \cdot \overline{I}_0^{\;*} + \overline{V}_1 \cdot \overline{I}_1^{\;*} + \overline{V}_2 \cdot \overline{I}_2^{\;*}\right\} \\ &= 3\left\{S_0 + S_1 + S_2\right\} \end{aligned} \tag{51}$$

where the T represents a matrix transposition operation. According to (51), the summation of the complex power in the sequence domain is equal to the average of the complex power in the a-b-c domain.

This theory is based on a sequence domain transformation, which helps to solve unbalanced three-phase systems. However, when the circuit is non-sinusoidal, the theory needs further elaboration. As mentioned before, Hase's theory adopts the vector apparent power for the three-phase apparent power. But, the validity of the definition for vector apparent power is still under discussion.

### 2.3 Power Theory in Single-Phase Circuits under a Non-Sinusoidal Condition

The study of power theory in non-sinusoidal circuits has been ongoing for more than a hundred years. In 1892, Steinmetz was the first to notice [7] the impact of harmonic distortion on apparent power in electric circuits. He found that in a circuit with mercury rectifiers, the apparent power is higher than the active power. Since the voltage and the current were exactly in phase, this must have been caused by the distortion of voltage and current waveforms.

In 1927, Budeanu proposed [8] a set of power definitions based on the frequency domain. Four years later, Fryze developed [9] another set of power definitions based on the decomposition of current in the time domain. Budeanu's and Fryze's findings have become fundamental power theories and remained relevant for nearly a century. During this time, discussion regarding this topic never ceased. Additionally many scientists tried to add to or adjust the definitions, while others looked to find a physical interpretation for these occurrences. Several of these theories and interpretations are presented below.

#### 2.3.1 Real Power under Arbitrary Voltage and Current Waveforms

Although different power theories have different interpretations for apparent power and reactive power or current components, the definition of real power under non-sinusoidal condition is almost unanimous. Suppose that the voltage and current are still sinusoidal, but of different frequencies, $\omega_1$ and $\omega_2$:

Voltage:

$$v(t) = V_{\max}\cos(\omega_1 t + \alpha) = \sqrt{2}V\cos(\omega_1 t + \alpha) \tag{52}$$

Current:

$$i(t) = I_{\max}\cos(\omega_2 t + \beta) = \sqrt{2}I\cos(\omega_2 t + \beta) \tag{53}$$

Under these conditions instantaneous power is the product of the voltage and current

$$\begin{aligned} p(t) &= v(t)\cdot i(t) = V_{\max} I_{\max}\cos(\omega_1 t + \alpha)\cos(\omega_2 t + \beta)a \\ &= VI\cos\left[(\omega_1 - \omega_2)t + \alpha - \beta\right] + VI\cos\left[(\omega_1 + \omega_2)t + \alpha + \beta\right] \end{aligned} \tag{54}$$

The product turns out to be the sum of two oscillating components, both of which have an average of zero. Therefore, the time average of instantaneous power produced by voltage and current of different frequencies is zero.

According to Fourier's theory, any arbitrary periodic waveform can be decomposed into the sum of a series of sinusoidal waves of different frequencies,

$$v(t) = \sum_{n=0}^{\infty}\sqrt{2}V_n\sin(n\omega_1 t + \alpha_n) \tag{55}$$

$$i(t) = \sum_{n=0}^{\infty}\sqrt{2}I_n\sin(n\omega_1 t + \beta_n) \tag{56}$$

where $V_n$ and $I_n$ are the root mean square values of the $n$th harmonic component of voltage and current and $\alpha_n$ and $\beta_n$ are the phase angles respectively. Since the average of the products of different components are zero, real power is defined as the time average of instantaneous power and consists of only the production of the voltage and current harmonic components that are under the same frequency:

$$P = \sum_{n=0}^{\infty}V_n I_n\cos(\varphi_n) \tag{57}$$

where, $\varphi_n = \alpha_n - \beta_n$ is the phase angle difference between the $n$ th harmonic components of voltage and current.

### 2.3.2 Budeanu's Power Theory

Budeanu's definition of active and reactive power in a non-sinusoidal condition is a direct extension of his work on a sinusoidal condition [8]. He defined the active power as:

$$P = \sum_{n=0}^{\infty} V_n I_n \cos\varphi_n = \sum_{n=0}^{\infty} P_n \tag{58}$$

and the reactive power, similarly, as:

$$Q = \sum_{n=1}^{\infty} V_n I_n \sin\varphi_n = \sum_{n=1}^{\infty} Q_n \tag{59}$$

where $V_n$ and $I_n$ are the root mean square values of the $n$th harmonic components of voltage and current and $\varphi_n$ is the phase angle difference between the $n$th harmonic components of voltage and current. The apparent power is still defined as the product of the root mean square value of voltage and current:

$$S = V \cdot I \tag{60}$$

Under a non-sinusoidal condition, the geometric summation of active power and reactive power is not equal to the apparent power. Budeanu defined the difference between them as distortion power:

$$D = \sqrt{S^2 - P^2 - Q^2} \tag{61}$$

Despite its widespread acceptance, Budeanu's power theory has received quite a few criticisms. For example, Czarnecki published [10] [11] his critiques of Budeanu's theory, which included the following viewpoints. Budeanu's definition introduces the distortion power $D$ as a complementary part of apparent power. But, it fails to link this value with any load properties and consequently fails to embody this definition with substantial physical meaning. The reactive power, according to Budeanu's definition, also lacks physical meaning. It is not a measure of energy oscillation. For example, the quantity can be zero while one-directional power components may exhibit significant amplitudes and cause the system to oscillate. The reactive power and distortion power are of no significance for the power factor correction. Therefore, the reactive power cannot be minimized, and the power factor cannot be improved. In some situations, compensation of reactive power $Q$ may even negatively impact the power factor.

### 2.3.3 Fryze's Power Theory

Fryze proposed [9] to decompose the current into two components: active current $i_a$ and reactive current $i_b$:

$$i = i_a + i_b \tag{62}$$

The active current is equal to the current if the same voltage is connected to a pure resistive load that can generate the same active power:

$$i_a = \frac{P}{V^2} v = Gv \tag{63}$$

$P$ represents the active power consumed by the circuit, and $G$ is named as the equivalent conductance. The rest of the current is defined as the reactive current:

$$i_b = i - i_a \tag{64}$$

The scalar product of these two components is:

$$\begin{aligned} \langle i_a, i_b \rangle &= \frac{1}{T}\int_0^T i_a \cdot i_b dt = \frac{1}{T}\int_0^T i_a \cdot (i - i_a) dt \\ &= \frac{P}{V^2 T}\int_0^T v \cdot i dt - \frac{P^2}{V^4 T}\int_0^T v^2 dt \\ &= 0 \end{aligned} \tag{65}$$

which means that the two currents are orthogonal and their geometric summation is equal to the current:

$$|i|^2 = |i_a|^2 + |i_b|^2 \tag{66}$$

Multiplying $|v|^2$ on both sides of the expression yields:

$$S^2 = P^2 + {Q_F}^2 \tag{67}$$

where $Q_F = |v| \cdot |i_b|$ is defined as reactive power. It is also the component of the apparent power that does not contribute to the active power [12].

R. Strzelecki argues against this power theory [2]. The reactive power defined as $Q_F = |v| \cdot |i_b|$ does not satisfy the principle of energy conservation. Therefore, the direction of the reactive power flow cannot be determined by the sign of $Q_F$. Fryze fails to give a concrete physical explanation for the reactive component of current $|i_b|$ or the relationship between this current and the circuit or load parameters. One of the apparent drawbacks of this approach is that the active power factor defined by Fryze reaches its maximum if and only if the current is proportional to the voltage. However, under non-sinusoidal condition, the fact that current and voltage are proportional does not guarantee optimal power flow [2].

In their original forms, Budeanu's and Fryze's definitions only apply to single-phase circuits, but their definitions can be easily expanded for balanced three-phase circuits.

### 2.3.4 Shepherd and Zakikhani Power Theory

Shepherd and Zakikhani adopted Budeanu's approach to defining powers in a frequency domain [13]. They proposed to decompose the apparent power in a new manner and tried to lend distortion power some physical meaning.

Shepherd and Zakikhani proposed that apparent power consists of three orthogonal components: active apparent power $S_R$, reactive apparent power $S_X$, and distortion apparent power $S_D$:

$$S^2 = {S_R}^2 + {S_X}^2 + {S_D}^2 \tag{68}$$

The three power components are defined respectively:

$$ {S_R}^2 = \sum_1^n {V_n}^2 \sum_1^n {I_n}^2 \cos^2(\varphi_n) \tag{69}$$

$$S_X{}^2 = \sum_1^n V_n{}^2 \sum_1^n I_n{}^2 \sin^2(\varphi_n) \tag{70}$$

$$S_D{}^2 = \sum_1^n V_n{}^2 \sum_1^p I_p{}^2 + \sum_1^m V_m{}^2 (\sum_1^n I_n{}^2 + \sum_1^p I_p{}^2) \tag{71}$$

Shepherd and Zakikhani gave [13] the reactive power a distinct physical significance; Budeanu's theory did not. They proved that by connecting a capacitor in parallel, the reactive power $S_X$ can be minimized. Consequently, the power factor can be optimized. The capacitance of a capacitor that optimizes the power factor is [2] [13]:

$$C = \frac{\sum_{n=1}^{N} n V_n I_n \sin \varphi_n}{\omega_1 \sum_{n=1}^{N} n^2 V_n{}^2} \tag{72}$$

Although their theory succeeds in connecting the reactive apparent power with the parameter selection for power factor optimization, Shepherd and Zakikhani's power theory was not successful in giving complementary reactive power a distinct physical meaning [2]. Moreover, the active part of the apparent power is not equal to the average of the instantaneous power; this denies a physical interpretation of active power in their definition [12]. According to Strzelecki, Ryszard Benysek, and Grzegorz, Shepherd and Zakikhani's theory has never been extended to polyphase circuits [2].

### 2.3.5 Sharon's Power Theory

Sharon proposed [14] the following manner to decompose the apparent power:

$$S^2 = P^2 + S_Q{}^2 + S_C{}^2 \tag{73}$$

where $P$ is active power and $S_Q$ is a reactive power in quadrature. It is further defined below:

$$S_Q = V \sqrt{\sum_1^n I_n{}^2 \sin^2(\varphi_n)} \tag{74}$$

$S_C$ is a complementary reactive power that consists of the remaining part of apparent power:

$$S_C = \sqrt{S^2 - P^2 - S_Q{}^2} \tag{75}$$

The similarity between Sharon's and Shepherd and Zakikhani's power theories is that the maximization of a power factor can be achieved by minimizing the $S_Q$ in the model. What is more, Sharon introduced a more acceptable definition of active power $P$ . Unlike $S_R$ which was defined by Shepherd and Zakikhani, active power $P$ is the time average of instantaneous power. Yet, Sharon did not explain explicitly the physical significance of complementary apparent power $S_C$.

### 2.3.6 Emanuel's Power Theory

Since the major contribution to reactive power comes from the fundamental harmonic component, Emanuel proposed [15] the following manner to decompose apparent power:

$$P_C{}^2 = S^2 - P^2 - Q_1{}^2 \tag{76}$$

where, $S$ is the apparent power, $P$ is the active power, and $P_C$ is the complementary power that contributes to the apparent power. $Q_1$ is the fundamental harmonic component of reactive power defined as:

$$Q_1 = V_1 I_1 \sin\varphi_1 \tag{77}$$

Since this set of definitions is based on frequency domain, it also serves as an extension of Budeanu's power theory.

### 2.3.7 Kusters and Moore's Power Theory

Kusters and Moore's power theory adopts Fryze's method of decomposing the current in time domain [16]. They proposed to further decompose the current into three components:

- Active current: this component has the same waveform and is in phase with the voltage
- Capacitive reactive current: this has the same waveform and phase as that of the current in a capacitor with the same voltage across it.
- Residual reactive current: the remaining current

Apparent power is decomposed correspondingly into three parts: the active power $P$, the capacitive reactive power $Q_c$, and the residual reactive power $Q_{cr}$:

$$S = \sqrt{P^2 + Q_c{}^2 + Q_{cr}{}^2} \tag{78}$$

According to [16], if the capacitive reactive current is negative, then a shunt capacitor can compensate for the reactive power. To optimize the power factor, all the needed values can be measured in the time domain; this makes it easier to implement than Shepherd and Zakikhani's theory, especially with less advanced measurement apparatuses available in the past.

Regardless of these issues, the residual reactive current or power still lacks physical significance.

### 2.3.8 Czarnecki Power Theory

Using Fryze's concept of active current $i_p$ and Shepherd and Zakikhani's concept of reactive current $i_r$ and then introducing a new component $i_s$, or scattered current, Czarnecki proposed [17] the decomposition of current in the manner shown below:

$$i = i_p + i_r + i_s \tag{79}$$

The whole decomposition is not absolute but rather is based on the reference of voltage and defined as:

$$v = \sqrt{2}\,\mathrm{Re}\sum_{n=0}^{\infty} V_n \exp(jn\omega_1 t) \tag{80}$$

The active current definition is the same used in Fryze's theory:

$$i_a = \frac{P}{V^2} v = Gv \tag{81}$$

The scattered current is defined as:

$$i_s = \sqrt{2}\,\mathrm{Re}\sum_{n=0}^{\infty}(G_n - G_e)V_n \exp(jn\omega_1 t) \tag{82}$$

Finally, the reactive current is defined as:

$$i_r = \sqrt{2}\,\mathrm{Re}\sum_{n=1}^{\infty} jB_n V_n \exp(jn\omega_1 t) \tag{83}$$

where, $G_n + jB_n$ is the load admittance for the $n$-th harmonic and $G_e$ is the equivalent conductance defined according to Fryze's theory.

The decomposition of source current into three components in a linear circuit is extended to a non-linear circuit by the separation of the generated current $i_g$ . This current consists of harmonics generated in a load due to the load's non-linearity or the circuit time-variant parameters. Thus, in most cases, the current was decomposed into four components:

$$i = i_p + i_r + i_s + i_g \tag{84}$$

According to Czarnecki [17], each component of the source current can be connected with a specific kind of physical phenomena found in the electric circuit. Therefore, Czarnecki's power theory is often referred to as the theory of the current's physical components.

### 2.4 Power Theory in Three-Phase Circuits under a Non-Sinusoidal Condition

Apparent power in unbalanced three-phase circuits can be calculated using different definitions, several of which are listed below:

Vector Apparent power

$$S_V = \sqrt{\left(\sum_k P_k\right)^2 + \left(\sum_k Q_{bk}\right)^2 + \left(\sum_k D_k\right)^2} \tag{85}$$

Arithmetic Apparent power

$$S_A = \sum_k \sqrt{P_k^{\ 2} + Q_{bk}^{\ \ 2} + D_k^{\ 2}} \tag{86}$$

where, $P_k$ is the active power in the $k$th phase, $Q_{bk}$ is the reactive apparent power in the $k$th phase, and $D_k$ is the distortion power in the $k$th phase. In three phase sinusoidal circuits, the definitions are equivalent to:

$$S_V = \left|\overline{S}_a + \overline{S}_b + \overline{S}_c\right| \tag{87}$$

$$S_A = \left|\overline{S}_a\right| + \left|\overline{S}_b\right| + \left|\overline{S}_c\right| \tag{88}$$

where $\overline{S}_a$, $\overline{S}_b$, and $\overline{S}_c$ represent the complex power of each phase.

Both the vector and arithmetic apparent power definitions inherited Budeanu's definition of apparent power in the single phase. The vector apparent power is the norm of the vector summation of three single phase complex powers. The arithmetic apparent power is the summation of the three single phase apparent powers.

These two definitions are widely accepted by major international organizations. The American Institute of Electrical Engineers held a discussion in 1920 and tried to decide which of these should be adopted for apparent power. These definitions also appear in IEEE Standard 1459.

The following two definitions of apparent power are based on Fryze's definition of reactive power:

Apparent rms power

$$S_E = \sum_k \sqrt{(P_k^{\ 2} + Q_{fk}^{\ \ 2})} = \sum_k V_k I_k \tag{89}$$

System apparent power

$$S_S = \sqrt{(P^2 + Q_f^{\ 2})} = \sqrt{\sum_k V_k^{\ 2}}\sqrt{\sum_k I_k^{\ 2}} \tag{90}$$

In these definitions, $Q_f$ and $Q_{fk}$ are the total reactive power and the reactive power in each phase respectively.

According to [12], the vector apparent power (87) is generally the lowest among the four definitions shown in (87), (88), (89), and (90). The system apparent power given in (90) calculates the voltage and current in different phases respectively so it is generally the highest. Arrillaga states in [12] that the following would also apply:

$$S_V \le S_A \le S_E \le S_S \tag{91}$$

Czarnecki further extended [18] his power theory from single-phase circuits to three-phase circuits. In this theory he decomposed the load current into five orthogonal components, each of which has a distinct connection with physical phenomena and a corresponding compensation condition. Like his theory for single-phase circuits, Czarnecki's three-phase theory only requires a time domain measurement, which allows for easy implementation. However, this theory has received only limited acceptance.

P. J Rens and P. H. Swart claim that Czarnecki's three-phase power theory only conforms to a symmetric voltage source with zero impedance [19]. Violation of either of the above conditions will introduce inconsistent results.

Power theories in single-phase circuits under sinusoidal conditions have the simplest form, and are widely adopted. Power theories in single-phase circuits under non-sinusoidal conditions should have a similar form. However, under non-sinusoidal or unbalanced conditions, it becomes harder to find physical interpretations for reactive power and apparent power. This is the main reason why scholars would propose different power theories applying to single-phase circuits under non-sinusoidal conditions and unbalanced three-phase circuits.

This study adopts the effective power factor theory described in Chapter 3. It is developed from Budeanu's single-phase power theory discussed in Chapter 2.3. It will be shown that this new power theory gives more accurate results in unbalanced three-phase circuits than the other three-phase power theories discussed in Chapter 2.4.

# Chapter 3 Effective Power Factor Theory

Effective power factor theory was originally proposed by F. Buchholz [20] in 1922 and explained by W. M. Goodhue [21] in 1933. IEEE Standard 1459 adopted this theory and developed a set of definitions for power system properties. These definitions were developed because, according to Emanuel [24], "Buchholz-Goodhue's effective apparent power $S_e$ is the only definition true to the physical meaning of apparent power in polyphase circuits."

Effective power factor theory is another way to define physical properties in a power system. Since the major objective of this study is to measure the power factor, effective power factor theory has some advantages over its competitors. Power factor, as an important index of power quality, is defined as the ratio of real power to apparent power. Real power always has a clear and widely accepted definition. It is equal to the time average of instantaneous power. A detailed discussion of this issue can be found in Chapter 2. Therefore, the definition of apparent power remains the major argument and concern in the study of the power factor.

This chapter explains the effective power factor theory and shows its advantages over other power factor definitions. This study adopts the effective power factor theory because, first, unlike other theories, the effective power factor theory has a clarified physical interpretation for apparent power. Secondly, it gives more accurate results in unbalanced three-phase circuits, especially when the neutral current cannot be neglected. And finally, effective power factor theory only involves measurements and computations in the time domain.

### 3.1 PHYSICAL MEANING OF APPARENT POWER

Other theories always begin to define apparent power by attempting to understand reactive power. Once this problem is solved, apparent power is defined as the Euclidian summation of reactive power and real power. While this seems like a viable approach for the study of apparent power, it does have certain drawbacks. First, the definition and physical interpretation of reactive power remains open to discussion; and, consequently, the further summation or apparent power is even less unanimous. Additionally, since apparent power is defined as a purely mathematical function, it is difficult to pinpoint the physical significance of this term.

Effective power factor theory approaches this problem from the opposite direction. It defines all power system terms based on the interpretation of apparent power. Apparent power, not reactive power, remains the major concern in the study of the power factor. Furthermore, an effective power factor theory makes a clear statement about the physical significance of apparent power. This makes effective apparent power preferable over other theories in the context of this particular study.

W. V. Lyon [22] referred to apparent power as “the greatest possible power that would be absorbed by any load taking the same rms. line currents and the same rms. voltages”. A similar definition can also be found in Curtis and Silsbee’s work [23], where “the apparent power is the maximum possible active power for the given effective values of current and potential difference”. The most straightforward demonstration of this definition is a single-phase circuit under sinusoidal condition, as shown in Fig. 3.1.

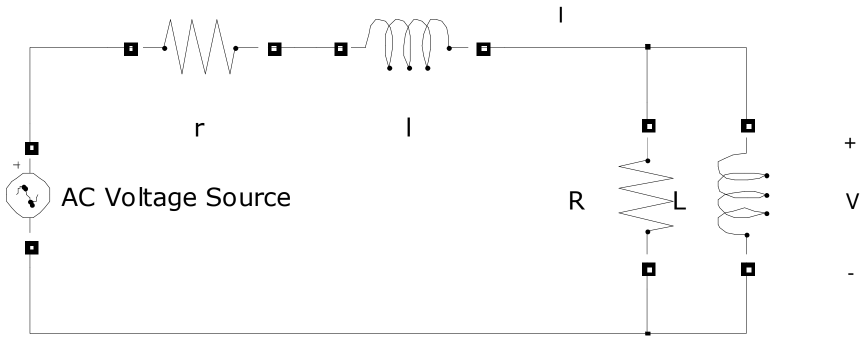


Fig. 3.1 Effective apparent power definition in single phase circuit

A load with a resistance of $R$ and an inductance of $L$ is connected to an AC power supply through a power line, the resistance and inductance of which is $r$ and $l$. Suppose the load voltage is $V$, the line current is $I$, and both values are constant. Then, the current flowing through the resistive load $R$ is:

$$I_R = I\frac{j\omega L}{R + j\omega L} \tag{92}$$

And, the real power consumed by the load is consequently equal to:

$$P_R = I_R^2 R = I^2 R\left|\frac{j\omega L}{R + j\omega L}\right|^2 \tag{93}$$

In this case, since the norm of the complex term is no larger than 1, it is easy to see that in order to optimize the real power transmission a compensative capacitor needs to be connected in parallel to the load. With this in place, the parallel impedance goes to infinity, and the norm term is equal to 1. Using this setup, the real power transferred reaches a maximum:

$$P_{\max} = VI \tag{94}$$

According to Lyon's definition, apparent power is the greatest possible real power absorbed by the load; the apparent power in this case is

$$S = P_{\max} = VI \tag{95}$$

With regards to physical meaning, this result indicates that a constant load voltage and line current allow the single-phase circuit to exhibit the best energy transfer condition when the load is purely resistive.

### 3.2 EQUIVALENT EFFECTIVE CIRCUIT

While this definition of apparent power can be applied to three-phase circuits, it is harder to find the optimum condition for energy transfer directly into three-phase circuits due to the presence of unbalance and harmonic distortion. Both effective apparent power theory and IEEE Standard 1459 adopt a slightly updated definition of apparent power to compensate for this issue. Emanuel concludes [24] that the enhanced definitions of apparent power and power factor are as follows:

i. Apparent power is the maximum power transmitted to the load (or delivered by a source) while keeping the same line losses and the same load (or source) voltage and current.

ii. Power factor is the ratio of the actual power to the maximum power that can be transmitted while keeping the line power loss and the load voltage constant. (Here, the word power can also be replaced with energy transmitted during a given time interval.)

Although at first glance this might seem similar to the original definition for apparent power, it is different. This new definition adds one more equality constraint to the optimization problem: the line loss has to be constant. One important implication of this enhanced definition is that as long as two circuits have the same line loss under the same load (or source) voltage and current, they should have the same apparent power. Thus, they are equivalent in the sense of calculating the apparent power. This serves as a theoretical foundation for the equivalent effective circuit method.

An arbitrary unbalanced three-phase circuit is shown in Fig. 3.2. Here, a three-phase balanced voltage source is connected to a three-phase resistive load. Since the load resistances $R_a$ , $R_b$ and $R_c$ are not equal, the load voltage and current remain unbalanced. The conductor resistances of the three transmission lines are $r_a$, $r_b$ and $r_c$. Neutral conductor resistance is noted as $r_n$.

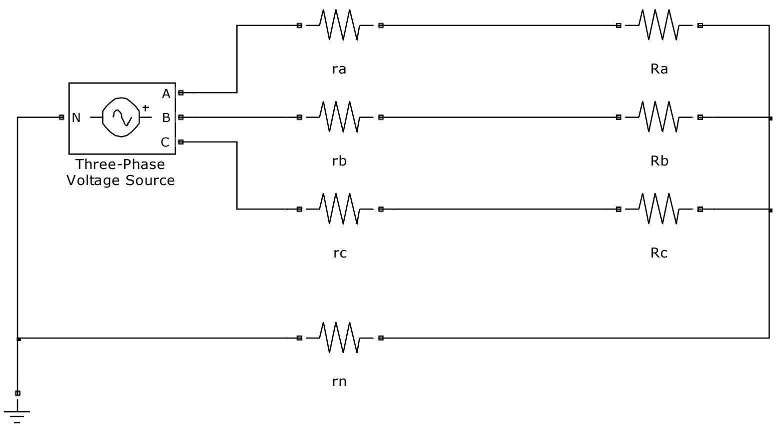


Fig. 3.2 Arbitrary three-phase unbalanced circuit

The total line loss of this arbitrary three-phase unbalanced circuit is:

$$\Delta P = I_a{}^2 r_a + I_b{}^2 r_b + I_c{}^2 r_c + I_n{}^2 r_n{}^2 \tag{96}$$

An equivalent effective circuit would balance a three-phase three-wire circuit, which consumes the same amount of line loss under the same voltage and current as the original arbitrary circuit. Since balanced circuits are much easier to analyze, not much effort is required to find the maximum amount of power that can be transferred in the equivalent effective circuit. According to the enhanced definition of apparent power and its implications, the apparent power of the equivalent effective circuit is equal to the original circuit. This process reveals the apparent power in a three-phase circuit for effective apparent power theory.

### 3.3 EFFECTIVE VOLTAGE AND CURRENT

While the construction of an equivalent circuit seems easy, there is an additional problem: the equivalent circuit exhibits a different voltage and current from the original circuit.

As previously described, the load voltages and the line currents in the circuit shown in Fig. 3.2 are different due to an unbalanced load. However, if the equivalent circuit is a balanced system, the load voltages and line currents of different phases remain identical. If they have different voltages and currents, it is not possible to compare the line loss between these two circuits. Moreover, the effective circuit is not equivalent to the original because it does not meet the prerequisite of the implication, which requires that they have the same voltage and current.

To make the effective circuit comparable with the original, it is necessary to define effective voltage and effective current. Shown in Fig. 3.3 is the effective version of the equivalent circuit shown in Fig. 3.2. Emanuel calls this an "optimized circuit".

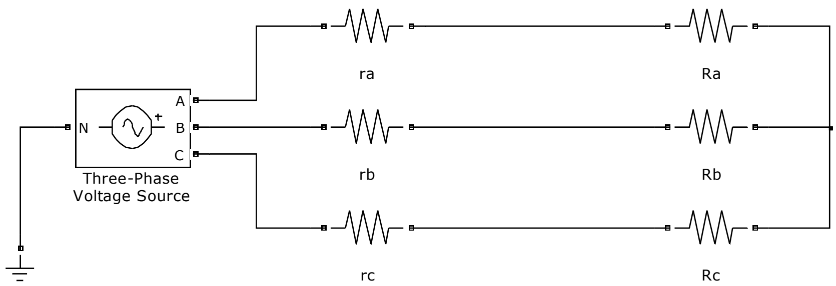


Fig. 3.3 Effective equivalent circuit for the arbitrary circuit

The effective equivalent circuit is a balanced circuit, so the three line resistances are equal, $r_a = r_b = r_c = r$, and the load resistance are equal as well, $R_a = R_b = R_c = R_e$. The line current of the effective circuit $I_e$ is defined as the equivalent current. The total line loss of the effective circuit is:

$$\Delta P = 3I_e^2 r \tag{97}$$

When comparing this expression with the line loss of the original circuit, the line loss of the two circuits should remain the same:

$$3I_e^2 r = (I_a^2 + I_b^2 + I_c^2)r + I_n^2 r_n \tag{98}$$

Solving the equation leads to the expression of effective current:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + I_n^2 \rho}{3}}, \quad \rho = \frac{r_n}{r} \tag{99}$$

One of the significant features of this expression is that it is independent of the load condition. Further, it is almost independent of the line condition. In a four-wire system, the only necessary value for the transmission line is the ratio of the line resistance $r$ to the neutral conductor resistance $r_n$. In a three wire system, the ratio is equal to zero since neutral conductor resistance is zero. The expression can be simplified as:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} \tag{100}$$

In practical systems, $\rho$ might be time dependent and difficult to measure. This complicated expression is a function of many factors including humidity and temperature. IEEE Standard 1459 suggests the ratio $\rho$ takes the value of 1.0.

The equivalent voltage is obtained by decomposing the active component of the load into three equivalent resistances $R_Y$ connected in wye, and three equivalent resistances $R_\Delta$ connected in $\Delta$. Fig. 3.4 shows this more developed equivalent circuit, where

$$R_a = R_b = R_c = R_Y \tag{101}$$

$$R_{ab} = R_{bc} = R_{ca} = R_{\Delta} \tag{102}$$

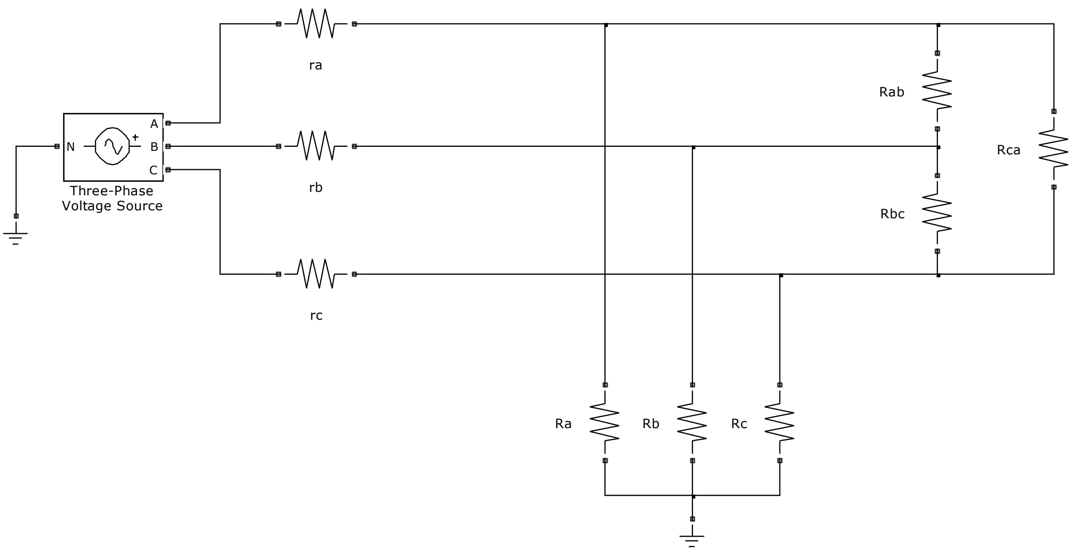


Fig. 3.4 A more developed effective equivalent circuit for the original circuit

The power equivalence between the original circuit and the effective equivalent circuit yields the following equation:

$$\frac{V_a^{\ 2} + V_b^{\ 2} + V_c^{\ 2}}{R_Y} + \frac{V_{ab}^{\ 2} + V_{bc}^{\ 2} + V_{ca}^{\ 2}}{R_{\Delta}} = 3\frac{V_e^{\ 2}}{R_Y} + 3\frac{V_e^{\ 2}}{R_{\Delta}/3} \tag{103}$$

where $V_e$ is the effective line-to-neutral voltage.

The real power consumed by the Y-connected resistances and $\Delta$-connected resistances are defined respectively as

$$P_Y = \frac{3V_e^{\ 2}}{R_Y} \tag{104}$$

$$P_{\Delta} = \frac{9V_e^{\ 2}}{R_{\Delta}} \tag{105}$$

Define the ratio of the two real power components $\xi$ as

$$\xi = \frac{P_{\Delta}}{P_Y} = \frac{3R_Y}{R_{\Delta}} \tag{106}$$

Substitute this ratio $\xi$ into the former power equivalence equation and solve for effective line-to-neutral voltage $V_e$:

$$V_e = \sqrt{\frac{3(V_a^{\;2} + V_b^{\;2} + V_c^{\;2}) + \xi(V_{ab}^{\;2} + V_{bc}^{\;2} + V_{ca}^{\;2})}{9(1+\xi)}} \tag{107}$$

If the value of ratio $\xi$ is unknown, IEEE Standard 1459 suggests it take the value of 1.0. This leads to the following expression:

$$V_e = \sqrt{\frac{3(V_a^{\;2} + V_b^{\;2} + V_c^{\;2}) + V_{ab}^{\;2} + V_{bc}^{\;2} + V_{ca}^{\;2}}{18}} \tag{108}$$

### 3.4 Effective Apparent Power and Power Factor

The construction of the equivalent effective circuit is accomplished only after calculating the effective voltage and current. The apparent power of this equivalent circuit remains equal to the apparent power of the original circuit.

By adopting Budeanu's theory, IEEE Standard 1459 defines the apparent power in single-phase circuits as:

$$S = VI \tag{109}$$

where $V$ and $I$ are the rms value of the voltage and current

$$V^2 = \frac{1}{kT}\int_{\tau}^{\tau+kT} v^2 dt = \sum_{h=0}^{\infty} V_h^{\;2} \tag{110}$$

$$I^2 = \frac{1}{kT}\int_{\tau}^{\tau+kT} i^2 dt = \sum_{h=0}^{\infty} I_h^{\;2} \tag{111}$$

Since the equivalent circuit is perfectly balanced, the total apparent power is three times as large as the apparent power of any single phase. Also, because $V_e$ and $I_e$ are defined as the rms value of the effective line-to-neutral voltage and the effective line current, the total effective apparent power can be calculated as:

$$S_e = 3V_e I_e = 3\sqrt{\frac{I_a^{\;2} + I_b^{\;2} + I_c^{\;2} + I_n^{\;2}}{3}} \cdot \sqrt{\frac{3(V_a^{\;2} + V_b^{\;2} + V_c^{\;2}) + (V_{ab}^{\;2} + V_{bc}^{\;2} + V_{ca}^{\;2})}{18}} \tag{112}$$

If the original circuit is a three-wire circuit, the expression for the apparent power can be simplified as:

$$S_e = 3\sqrt{\frac{I_a^{\,2}+I_b^{\,2}+I_c^{\,2}}{3}}\sqrt{\frac{V_{ab}^{\,2}+V_{bc}^{\,2}+V_{ca}^{\,2}}{9}} = \sqrt{\frac{(I_a^{\,2}+I_b^{\,2}+I_c^{\,2})(V_{ab}^{\,2}+V_{bc}^{\,2}+V_{ca}^{\,2})}{3}} \tag{113}$$

In accordance with the second part of the enhanced definition of apparent power proposed by Emanuel, the power factor is defined as the ratio of real power transmitted from the source to the load over the apparent power of the circuit:

$$PF_e = P / S_e \tag{114}$$

where, the definition of real power $P$ is the same as previously discussed:

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} (v_a \cdot i_a + v_b \cdot i_b + v_c \cdot i_c)dt \tag{115}$$

### 3.5 Effective Power Theory's Advantage

As presented in Chapter 2, there are many different definitions for apparent power in three-phase circuits. Arithmetic apparent power and vector apparent power are the two most popular definitions, and each has been adopted by different national and international organizations as the official definition of apparent power. Despite their popularity, these definitions actually need further clarification. Under certain circumstances, the two definitions do not agree with one another, and they can both lead to meaningless results with regards to a power factor study.

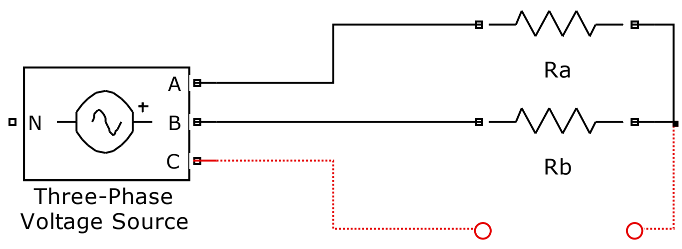


Fig. 3.5 Unbalanced three-phase resistive load

Fig. 3.5 shows a three-phase circuit with a balance three-phase voltage source connected to an unbalanced three-phase resistive load. Loads in phase A and phase B have identical resistance $R$, while phase C is open circuited. Suppose the line-to-neutral voltage on the source side is $V_{LN}$. Given these conditions, the real power consumed by the load is:

$$P = \frac{V_{ab}^{\ 2}}{2R} = \frac{3V_{LN}^{\ 2}}{2R} \tag{116}$$

The complex power in phase A is:

$$S_a = V_a \cdot I_a{}^* = V_a \cdot (\frac{V_{ab}}{2R})^* = \sqrt{3}\frac{V_{LN}^{\ 2}}{2R}\angle -30° \tag{117}$$

Similarly, the complex power in phase B is:

$$S_b = V_b \cdot I_b{}^* = V_b \cdot (-\frac{V_{ab}}{2R})^* = \sqrt{3}\frac{V_{LN}^{\ 2}}{2R}\angle 30° \tag{118}$$

Because there is no current in phase C, the apparent power in phase C is zero.

$$S_c = 0 \tag{119}$$

According to the definition of arithmetic apparent power (45), the apparent power of the circuit is:

$$S_A = |S_a| + |S_b| + |S_c| = \sqrt{3}\frac{V_{LN}^{\ 2}}{R} \tag{120}$$

Thus, the arithmetic power factor of this circuit is:

$$PF_A = \frac{P}{S_A} = \left(\frac{3V_{LN}^{\ 2}}{2R}\right) / \left(\sqrt{3}\frac{V_{LN}^{\ 2}}{R}\right) = \frac{\sqrt{3}}{2} \tag{121}$$

According to the definition of vector apparent power (44), the apparent power of the circuit is:

$$S_V = |S_a + S_b + S_c| = \left|\sqrt{3}\frac{V_{LN}^{\ 2}}{2R}\angle -30° + \sqrt{3}\frac{V_{LN}^{\ 2}}{2R}\angle 30° + 0\right| = \frac{3V_{LN}^{\ 2}}{2R} \tag{122}$$

So, the vector power factor of this circuit is:

$$PF_V = \frac{P}{S_V} = \left(\frac{3V_{LN}^{\;2}}{2R}\right) / \left(\frac{3V_{LN}^{\;2}}{2R}\right) = 1 \tag{123}$$

This example highlights two factors needing further clarification. First, in unbalanced three-phase circuits, the arithmetic power factor (apparent power) and vector power factor (apparent power) can be different from one another. Secondly, in this particular case, the conclusion of the vector power factor as equal to 1 is problematic. Power factor is an important index of power quality and should be capable of indicating the level of unbalance in the system. The circuit shown in Fig. 3.5 is obviously not balanced and is not in optimal condition. However, the vector power factor shows that this same circuit is in its best condition. It can be anticipated that if the vector power factor is introduced as the indicator of power quality in the power industry, it will fail to sense similar problematic scenarios.

According to the definitions of effective apparent power, the effective current is:

$$I_e = \sqrt{\frac{I_a^{\;2} + I_b^{\;2} + I_c^{\;2}}{3}} = \sqrt{\frac{2}{3}} I = \frac{V_{LN}}{\sqrt{2}R} \tag{124}$$

and the effective voltage is:

$$V_e = \sqrt{\frac{V_{ab}^{\;2} + V_{bc}^{\;2} + V_{ca}^{\;2}}{9}} = V_{LN} \tag{125}$$

Therefore, the effective apparent power is:

$$S_e = 3V_e I_e = \frac{3V_{LN}^{\;2}}{\sqrt{2}R} \tag{126}$$

and the effective power factor is:

$$PF_e = \frac{P}{S_e} = \left(\frac{3V_{LN}^{\;2}}{2R}\right) / \left(\frac{3V_{LN}^{\;2}}{\sqrt{2}R}\right) = \frac{\sqrt{2}}{2} \tag{127}$$

Table 3.1 Comparison of different power factors in the first example

| $PF_A$ | $PF_V$ | $PF_e$ |
|---|---|---|
| 0.866 | 1 | 0.707 |

Comparing the results from three different definitions, the effective power factor is the smallest in this particular case:

$$PF_e < PF_A < PF_V \tag{128}$$

Shown in Fig. 3.6 is another example. Here, a balanced three-phase four-wire sinusoidal voltage source is connected to an unbalanced three-phase RLC load. Phase A is a resistor R. Phase B is an inductor L. Phase C is a capacitor C.

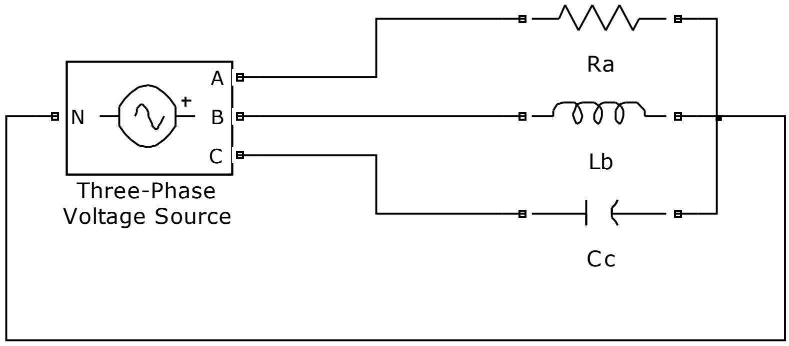


Fig. 3.6 Unbalanced three-phase RLC load

Suppose that the impedances of the three phases are the same:

$$R = \omega L = \frac{1}{\omega C} \tag{129}$$

The real power consumed by the three phases is:

$$P_a = \frac{{V_{LN}}^2}{R}, \qquad P_b = 0, \qquad P_c = 0 \tag{130}$$

The reactive power consumed by the three phases is:

$$Q_a = 0, \qquad Q_b = \frac{V_{LN}{}^2}{R}, \qquad Q_c = -\frac{V_{LN}{}^2}{R} \tag{131}$$

According to the definition of arithmetic apparent power, the apparent power of the circuit is:

$$S_A = |S_a| + |S_b| + |S_a| = \frac{3V_{LN}{}^2}{R} \tag{132}$$

Thus, the arithmetic power factor is:

$$PF_A = \frac{P_a + P_b + P_c}{S_A} = \frac{1}{3} \tag{133}$$

According to the definition of vector apparent power, the apparent power of the circuit is:

$$S_V = |S_a + S_b + S_c| = \left|(P_a + P_b + P_c) + j(Q_a + Q_b + Q_c)\right| = P_a \tag{134}$$

Therefore, the vector power factor of this circuit is:

$$PF_V = \frac{P}{S_V} = \frac{P_a}{P_a} = 1 \tag{135}$$

Again, this is one of the scenarios in which the arithmetic power factor and vector power factor are not the same. For this reason, it can lead to problematic results if the vector power factor is introduced to the power industry as an index for power quality.

According to the definitions of effective apparent power, the effective current of the circuit shown in Fig. 3.6 is:

$$I_e = \sqrt{\frac{I_a{}^2 + I_b{}^2 + I_c{}^2 + I_n{}^2}{3}} \tag{136}$$

Since the neutral current is:

$$I_n = |I_a + I_b + I_c| = \left|\frac{V_{LN}}{R}\angle 0° + \frac{V_{LN}}{R}\angle(-120-90)° + \frac{V_{LN}}{R}\angle(120+90)°\right| = (\sqrt{3}-1)\frac{V_{LN}}{R} \tag{137}$$

Substitute (137) into the expression for the equivalent current (136):

$$I_e = \sqrt{\frac{3+(\sqrt{3}-1)^2}{3}}\frac{V_{LN}}{R} = 1.1786\frac{V_{LN}}{R} \tag{138}$$

The effective voltage is:

$$V_e = \sqrt{\frac{V_{ab}{}^2 + V_{bc}{}^2 + V_{ca}{}^2}{9}} = V_{LN} \tag{139}$$

Therefore, the effective apparent power is:

$$S_e = 3V_e I_e = 3.5359\frac{V_{LN}{}^2}{R} \tag{140}$$

and the effective power factor is:

$$PF_e = \frac{P}{S_e} = \left(\frac{V_{LN}{}^2}{R}\right) / \left(\frac{3.5359V_{LN}{}^2}{R}\right) = \frac{1}{3.5359} = 0.2828 \tag{141}$$

Table 3.2 Comparison of different power factors in the second example

| $PF_A$ | $PF_V$ | $PF_e$ |
|---|---|---|
| 0.866 | 1 | 0.707 |

Comparing the result from three different definitions, the effective power factor is still smallest in this particular case:

$$PF_e < PF_A < PF_V \tag{142}$$

In conclusion, compared with the arithmetic power factor and the vector power factor, the effective power factor is a more reliable indicator of power quality. This is due to its clarified physical interpretation of apparent power; this is especially true in unbalanced three-phase circuits or four-wire circuits where the neutral conductor currents cannot be neglected. Because both the arithmetic and vector power factors are problematic, as shown in Chapter 3.5, the effective power factor is the only accurate definition of power factor.

# Chapter 4 the Implementation of IEEE Standard 1459 in Simulation Software

One of the objectives of this study is to implement IEEE Standard 1459 in simulation software. More specifically, a meter program has to be developed, which can be invoked by the simulation software PSCAD. This meter program must also measure the effective power factor as defined by IEEE Standard 1459 and account for any unbalanced or distorted three-phase circuits. Since PSCAD is capable of invoking MATLAB functions, the calculation of the effective apparent power and the effective power factor are implemented in a MATLAB function. PSCAD needs an interface code to call a MATLAB function. The interface described here is designed specifically for the signal flow of this project and is written in FORTRAN language.

## 4.1 SIMULATION AND MEASUREMENT SYSTEM

Fig. 4.1 shows the block diagram of the complete simulation and measurement system. The whole system consists of three major parts: the PSCAD software, the interface, and the MATLAB function.

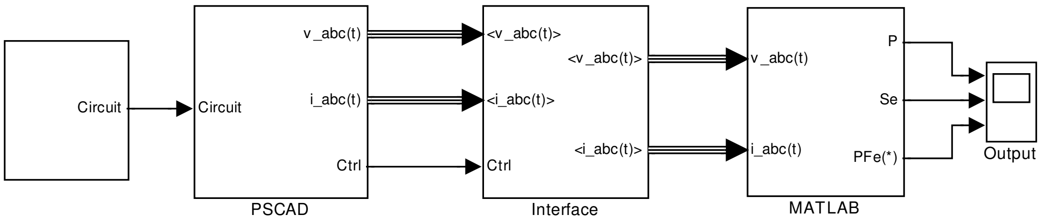


Fig. 4.1 Block diagram of the measurement system

PSCAD is a full-featured power system simulation and design platform. Given any circuit design, PSCAD can simulate the behavior of the system and track changes in state variables. To measure the effective power factor, PSCAD must measure the time varying line currents and line-to-neutral voltages at the point of interest. This can be accomplished using a current meter and a volt meter that are integrated with PSCAD.

The real time readings of currents and voltages are transmitted into the MATLAB function through the interface program. PSCAD also has to generate a control signal to enable the interface program periodically. The point is that if the interface program is called in every cycle of the simulation, it will consume a great amount of hardware resources. This is not necessary, especially when the PSCAD simulation time step is set at a very small value. The control signal is generated with an impulse generator integrated in PSCAD. It will generate a logical 1 signal in a specific frequency while maintaining logical zero in the other time. For this particular study, the frequency of the enable control signal is set at 500 Hz.

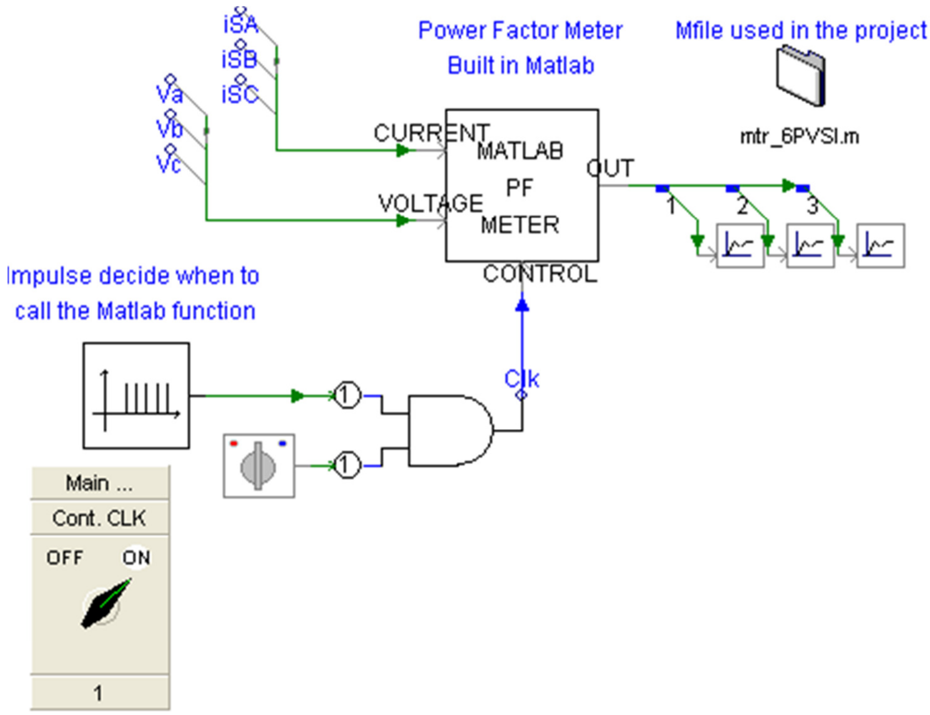


Fig. 4.2 Signal flow diagram of the system depicted in PSCAD

## 4.2 MATLAB FUNCTION

### 4.2.1 Basic Feature of the MATLAB Function

The basic objective of the MATLAB function is to calculate the three-phase real power $P$, the effective power $S_e$, and the effective power factor $PF_e$.

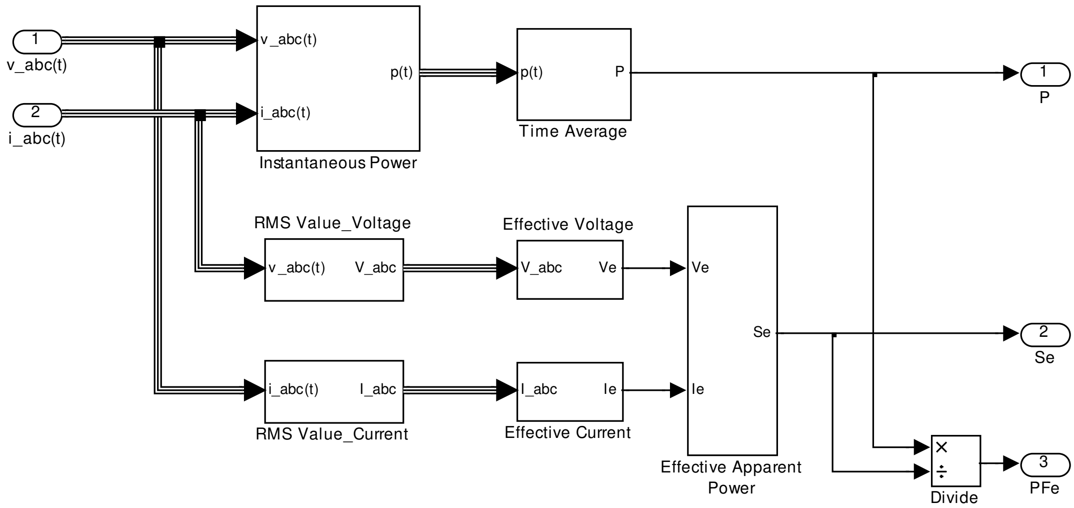


Fig. 4.3 Block diagram of the MATLAB function

Real power $P$ is defined as the time average of the instantaneous power, while instantaneous power is the product of the line-to-neutral voltage and current at a given time. Therefore, the real power $P$ can be calculated directly from the time variant voltage and current readings:

$$P = \frac{1}{T}\int_{t_0}^{t_0+T}\left[p_a(t)+p_b(t)+p_c(t)\right]dt = \frac{1}{T}\int_{t_0}^{t_0+T}\left[v_a \cdot i_a(t)+v_b \cdot i_b(t)+v_c \cdot i_c(t)\right]dt \tag{143}$$

Because the line current and line-to-neutral voltages are measured discretely, in order to calculate the real power in MATLAB, this equation needs a minor modification, i.e.,

$$P = P_a + P_b + P_c = \frac{1}{N}\left(\sum_{n=1}^{N} v_a(n) i_a(n) + \sum_{n=1}^{N} v_b(n) i_b(n) + \sum_{n=1}^{N} v_c(n) i_c(n)\right) \tag{144}$$

Effective apparent power $S_e$ is defined as three times the product of effective voltage and effective current:

$$S_e = 3V_e I_e \tag{145}$$

Therefore, before calculating the effective power, one needs to calculate the effective voltage and the effective current first.

To calculate the effective voltage, line-to-line voltages must be converted into root mean square values as well as line-to-neutral voltages. There are several ways to accomplish this task. Since this particular system measures and transmits all three time varying line-to-neutral voltages to the MATLAB function, they can be used to calculate the line-to-line voltage

$$\begin{aligned} v_{ab}(t) &= v_a(t) - v_b(t) \\ v_{bc}(t) &= v_b(t) - v_c(t) \\ v_{ca}(t) &= v_c(t) - v_a(t) \end{aligned} \tag{146}$$

All line-to-neutral voltages and line-to-line voltages can then be converted in to an rms value according to the definition of root mean square value:

$$\begin{aligned} V_a &= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v_a^{\,2} dt} \\ V_b &= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v_b^{\,2} dt} \\ V_c &= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v_c^{\,2} dt} \\ V_{ab} &= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v_{ab}^{\,2} dt} \\ V_{bc} &= \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v_{bc}^{\,2} dt} \end{aligned} \tag{147}$$

$$V_{ca} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} {v_{ca}}^2 dt}$$

Additionally, the time varying line-to-neutral voltages must be measured in a discrete manner. In the context of this study, the expressions of rms value for the voltages are modified as follows:

$$V_a = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_a^2(n)}$$
$$V_b = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_b^2(n)}$$
$$V_c = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_c^2(n)}$$
$$V_{ab} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_{ab}^2(n)} \tag{148}$$
$$V_{bc} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_{bc}^2(n)}$$
$$V_{ca} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} v_{ca}^2(n)}$$

By substituting these rms line-to-neutral and line-to-line voltages into the definition of effective voltage (108):

$$V_e = \sqrt{\frac{3({V_a}^2 + {V_b}^2 + {V_c}^2) + {V_{ab}}^2 + {V_{bc}}^2 + {V_{ca}}^2}{18}} \tag{149}$$

Similarly, to calculate the effective current one needs to calculate the neutral current $I_n$ first. Using Kirchhoff's current law, the current that flows through the neutral current is the sum of the three line currents:

$$i_n(t) = i_a(t) + i_b(t) + i_c(t) \tag{150}$$

After obtaining all needed time varying currents, transfer them into an rms value in a discrete manner:

$$I_a = \sqrt{\frac{1}{N}\sum_{n=1}^{N} i_a^2(n)}$$
$$I_b = \sqrt{\frac{1}{N}\sum_{n=1}^{N} i_b^2(n)}$$
$$I_c = \sqrt{\frac{1}{N}\sum_{n=1}^{N} i_c^2(n)} \tag{151}$$
$$I_n = \sqrt{\frac{1}{N}\sum_{n=1}^{N} i_n^2(n)}$$

By substituting these rms currents into the previously mentioned expression, the effective current can be calculated.

$$I_e = \sqrt{\frac{I_a^{\ 2} + I_b^{\ 2} + I_c^{\ 2} + I_n^{\ 2}}{3}} \tag{152}$$

At this level of calculation, another significant advantage of effective power factor theory adopted by IEEE Standard 1459 can be observed. Effective power factor theory is developed from Budeanu's power theory, because it inherits the same definition of apparent power in single-phase circuits under a non-sinusoidal condition. However, the computation of effective apparent power does not involve any frequency domain operations, like Budeanu's theory does. Every term involved in the calculation is a root mean square value which can be directly converted from time domain quantities. This not only guarantees the simplicity of the calculation procedure but also lowers the hardware requirements for the measurement of the effective power factor. Although current technology is capable of realizing onsite real time frequency domain analysis, simpler calculations such as this can greatly reduce the cost of equipment and earn greater acceptance within the power industry.

### 4.2.2 Customizations and Optional Features

To make the MATLAB function work with the interface program, several customizations, including introducing a group of global vectors, have to be made. Also, several visualizations allow for more straightforward and more easily understood measurements. While these visualizations consume part of the hardware resources, they are optional features that do not have to be realized in onsite measurement equipment.

Whenever the interface program is invoked by the PSCAD software, 500 Hz in this particular case, three line-to-neutral voltages and three line currents are transferred to the MATLAB function. However, as discussed, the calculation of the effective power factor is based on root mean square values and time averages. This means that the calculation cannot be performed solely using the six data points. In fact, the data acquired in at least one period of time needs to be stored in order to perform root mean square and time average operations. Moreover, the time window needs to be shifted, and the data needs to be updated continually so that the effective power factor calculated in the function reflects the condition of the circuit in real time.

Once the function is complete, it automatically releases the memory space and deletes all related data. Therefore, to store the previously acquired data, this project employs the global function in MATLAB in order to generate six vectors that can be used to store the data. After establishing the storage vectors, the bubble algorithm is introduced to move the time window and refresh the data. This allows for the earliest data points to be released. Additionally, the indices of all other data points increase by 1, and the newest data points are then stored as the first entries in the corresponding vectors.

To check if the data has successfully refreshed and to make the result more straightforward, the function then plots the waveforms of three-phase line-to-neutral voltages and line currents in the last two time periods. The waveforms move forward with the time; this shows that the data has been successfully updated and stored.

With at least one time period of data stored, it is not very hard to perform the Fourier decomposition for the voltages and currents. MATLAB has an integrated FFT function that is very convenient to use. Most prevalent micro-processors and DSPs can complete the FFT operation. In case the FFT becomes a burden for the onboard system of measurement equipments or creates an issue with cost control, the following paragraphs show that the FFT operation can be replaced by a series of equivalent algebraic operations with reasonable accuracy.

According to the definition, for a periodic function $f(x)$ that is integratable on $[-\pi,\pi]$, the Fourier coefficients of $f(x)$ are defined as follows:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx, \qquad n \geq 0 \tag{153}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx, \qquad n \geq 1 \tag{154}$$

In engineering problems, the function $f(x)$ is always presumed to converge at the partial sum:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(nx) + b_n \sin(nx)\right] \tag{155}$$

Therefore, the amplitude of each harmonic component is equal to:

$$f_n = \sqrt{a_n^{\ 2} + b_n^{\ 2}} \tag{156}$$

Although the calculation of $a_n$ and $b_n$ employs an integral operation, it can be replaced here by summation because the value of the function $f(x)$ is a series of discrete data rather than a continuous curve.

$$a_n = \frac{1}{T}\sum_{t=0}^{T} f(t)\cos(nt)dt \tag{157}$$

$$b_n = \frac{1}{T}\sum_{t=0}^{T} f(t)\sin(nt)dt \tag{158}$$

In this manner, Fourier transformation can be accomplished by a series of addition, multiplication and square root operations. The following is the MATLAB code written for the Fourier transformation operation. Unlike the FFT function that is integrated in MATLAB--the result of which is not equal to the harmonic amplitude and requires further interpretation--this function gives the amplitude of each harmonic component directly. And, although it uses summation in the place of integration, the error is less than 0.2%.

```
function [mag,ang] = fouriertrans(sig,f0,j)
T=1/f0;
dt=2*T/length(sig);
f=f0*j;
w=2*pi*f;
t=linspace(0,2*T,length(sig));
an=1/T*sum(sig.*cos(w*t)*dt);
bn=1/T*sum(sig.*sin(w*t)*dt);
mag=sqrt(an^2+bn^2);
ang=atan(bn/an);
```

Given these conditions, it is not necessary to utilize Fourier transformation if the objective is merely to measure the effective power factor. This remains a completely optional feature.

### 4.2.3 MATLAB Code

The MATLAB function consists of several paragraphs: Data Collection, Output of Waveforms, Output of the FFT Spectrum, Calculation of the Effective Power Factor, and the Return of the Result.

```
function y=store2(in_c,in_v)
%% Data Collection
global ia_t ib_t ic_t va_t vb_t vc_t
```

```
fs=500;                              % Sample rate
Tnum=2;                              % Number of period of storage
number=fs/60*Tnum;

ia_t(number)=0;
ib_t(number)=0;
ic_t(number)=0;
va_t(number)=0;
vb_t(number)=0;
vc_t(number)=0;

for i=number-1:-1:1
    ia_t(i+1)=ia_t(i);
    ib_t(i+1)=ib_t(i);
    ic_t(i+1)=ic_t(i);
    va_t(i+1)=va_t(i);
    vb_t(i+1)=vb_t(i);
    vc_t(i+1)=vc_t(i);
end
ia_t(1)=in_c(1);
ib_t(1)=in_c(2);
ic_t(1)=in_c(3);
va_t(1)=in_v(1);
vb_t(1)=in_v(2);
vc_t(1)=in_v(3);

%Define time axis and plot the waveform
t=0:1/fs:(number-1)/fs;
grid on,
subplot(3,1,1)
plot(t,va_t,t,vb_t,t,vc_t);    % Plot the three phase voltage waveform
ylabel('Voltage/V'), xlabel('Time/s')
subplot(3,1,2)
```

```
plot(t,ia_t,t,ib_t,t,ic_t);   % Plot the three phase current waveform
ylabel('Current/A'), xlabel('Time/s')

%% FFT the current waveform (Optional)
N=length(ia_t);                % Sample frequency = N/((N-1)*sstep)
ia_fft=fft(ia_t,N)/(length(ia_t)/2);       % Absolute value of the fft
freq0=60;                      % Base current
freq=[0:(N-1)]/N*N/((N-1))*fs;       % Harmonic frequencies
harmonic=freq/freq0;           % Harmonic = H frequency / Base frequency
subplot(3,1,3)
bar(harmonic(1:length(ia_fft)/2),abs(ia_fft(1:length(ia_fft)/2))),title
('FFT result for ia(t)'),xlabel('nth harmonic'), ylabel('Current / A')

%% Calculate the three phase real power
pa_t=va_t.*ia_t;
pb_t=vb_t.*ib_t;
pc_t=vc_t.*ic_t;
Pa=1/(length(t)-1)*sum(pa_t);
Pb=1/(length(t)-1)*sum(pb_t);
Pc=1/(length(t)-1)*sum(pc_t);
P=Pa+Pb+Pc;

%% Calculate the three phase effective apparent power
% Calculate effective voltage Ve
%  Calculate the line to line voltage
vab_t=va_t-vb_t;
vbc_t=vb_t-vc_t;
vca_t=vc_t-va_t;
Vab=sqrt(1/(length(t)-1)*sum(vab_t.^2));
Vbc=sqrt(1/(length(t)-1)*sum(vbc_t.^2));
Vca=sqrt(1/(length(t)-1)*sum(vca_t.^2));
%  Calculate the effective voltage
Va=sqrt(1/(length(t)-1)*sum(va_t.^2));
Vb=sqrt(1/(length(t)-1)*sum(vb_t.^2));
Vc=sqrt(1/(length(t)-1)*sum(vc_t.^2));
```

```
eta=1;
%Ve=sqrt((3*(Va^2+Vb^2+Vc^2)+eta*(Vab^2+Vbc^2+Vca^2))/(9*(1+eta)));
Ve=sqrt((Vab^2+Vbc^2+Vca^2)/9);     % 3-line simplification

% Calculate effective current Ie
Ia=sqrt(1/(length(t)-1)*sum(ia_t.^2));
Ib=sqrt(1/(length(t)-1)*sum(ib_t.^2));
Ic=sqrt(1/(length(t)-1)*sum(ic_t.^2));
% Calculate the neutral current
in_t=ia_t+ib_t+ic_t;
%plot(t,in_t),
T=t(length(t));
In=sqrt(sum(in_t.^2*T/(length(t)-1))/T);
ro=1.43;
%Ie=sqrt((Ia^2+Ib^2+Ic^2+ro*In^2)/3);
Ie=sqrt(sum(Ia^2+Ib^2+Ic^2)/3);     % 3 line simplification

% Calculate effective apparent power Se
Se=3*Ve*Ie;

%% Calculate the power factor
display(Se);
display(P);
PFe=P/Se;
y=[P Se PFe];
```

### 4.3 PSCAD – MATLAB INTERFACE

The purpose of this interface program is to collect and sort the voltage and current measurement results, to call the specific MATLAB function, to pass the measurement data to the function, and finally to receive the output from the MATLAB function. PSCAD is actually based on FORTRAN language. It translates everything in the circuit into a FORTRAN code. Because there is no customized interface block that can be employed directly, the interface code must be written in FORTRAN language

The construction of this interface in PSCAD starts with building a subsystem. The subsystem has three inputs: a $\boldsymbol{R}^3$ vector, INPUT_V, as the three line-to-neutral voltages; another $\boldsymbol{R}^3$ vector, INPUT_C, as the three line currents; and a scalar $\boldsymbol{R}$, INPUT1, as the enabling control signal. Here, the bolded $\boldsymbol{R}$ represents the real number set.

As discussed before, the MATLAB function does not need to run in every simulation cycle. Therefore, an impulse generator is employed in the system in order to generate an enabling control signal every 2ms. In each simulation cycle, the interface program will check if the control signal is 1. If not, the program bypasses the whole procedure and waits for the next cycle. If control signal does equal 1, the program stores the three voltages on the top of the stack and then stores the three currents on the top of the stack with the STORF(NSTORF) command, where NSTORF is the position of the pointer.

The directory and name of the MATLAB function is defined on the mask of the subsystem, and it can be adjusted directly on the front panel of PSCAD. These two parameters are stored as “Path” and “Name” and can be employed by the FORTRAN functions in the backstage of the subsystem. PSCAD can call a MATLAB function with a CALL MLAB_INT("%:Dir\$Path", "$Name", "In1 In2 …" , "Out" ) command. In1 and In2 are the type and length of the input parameters; Out is the type and length of the output argument. In this particular case, they all can be replaced by R (3). R stands for real number, and 3 is the length of the arguments. By calling a CALL MLAB_INT() command, the interface program automatically groups the first 3 real numbers from the pointer position together as a vector and then passes it to the MATLAB function as the first parameter. Then, it will pass the next 3 real numbers to the MATLAB function as the second parameter. It will store the output from the MATLAB function as the 7th to 9th real number starting from the pointer position.

Finally, the interface program defines the 7th to 9th real number starting from the pointer position as the output of the interface, and the program returns these values to the PSCAD front panel for visual output. Before starting the next cycle, the pointer has to be moved nine spaces forward.

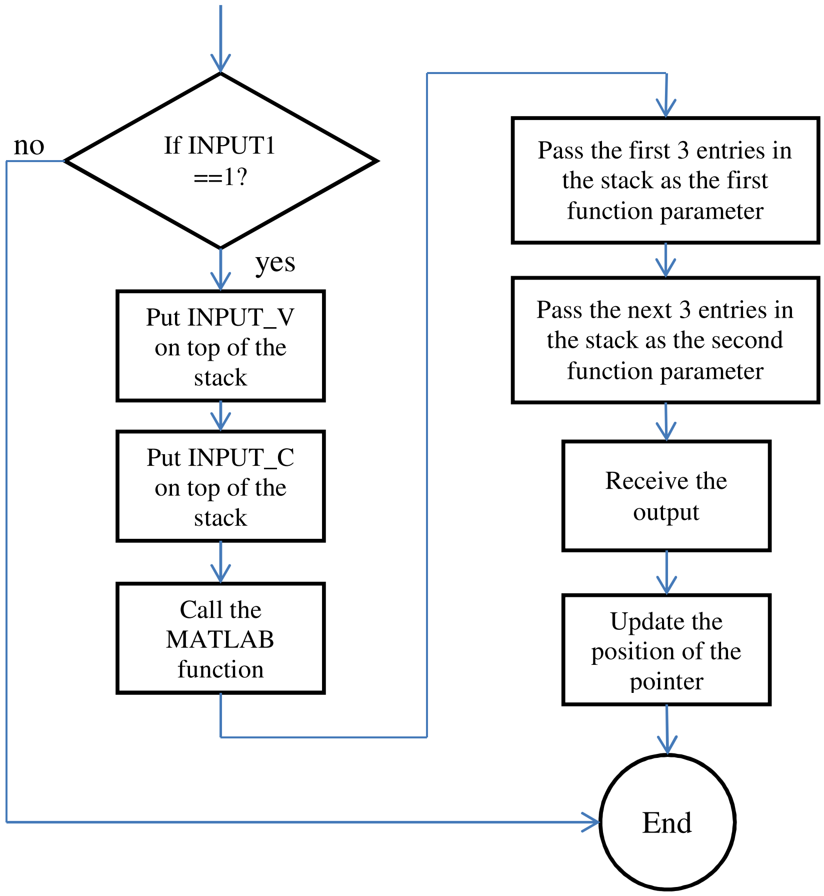


Fig. 4.4 Block diagram of the interface program

Fig. 4.4 shows the block diagram of the interface program. The following paragraphs present the interface code written in FORTRAN.

```
#STORAGE REAL:9 INTEGER:1
! --------------------------------------------------
! --------------------------------------------------
! PSCAD/EMTDC - MATLAB INTERFACE
! Module: $Name
#LOCAL INTEGER I_CNT
     IF ($INPUT1 == 1) THEN
! --------------------------------------------------
! Transfer EMTDC Input Variables to Matlab Interface
! --------------------------------------------------
!
! First Input Array (REAL(3))
      I_CNT = 1
      DO WHILE ( I_CNT .LE. 3 )
            STORF(NSTORF+I_CNT-1) = $INPUT_C(I_CNT)
            STORF(NSTORF+3+I_CNT-1)= $INPUT_V(I_CNT)
        I_CNT = I_CNT + 1
      END DO
!
! --------------------------------------------------
! Call PSCAD/EMTDC Matlab Interface:
! CALL MLAB_INT("MFILEPATH","MFILENAME","Input Format","Output Format")
! --------------------------------------------------
      CALL MLAB_INT("%:Dir\$Path", "$Name", "R(3) R(3)" , "R(3)" )
!
! --------------------------------------------------
! Transfer Matlab Output Variables from Matlab Interface
! --------------------------------------------------
!
! First Output Array (REAL())
      DO I_CNT = 1,3
      $OUTPUT(I_CNT) = STORF(NSTORF+5+I_CNT)
      ENDDO
     ENDIF
```

```
! Update STORx Pointers
     NSTORF = NSTORF + 9
! ---------------------------------------------------
! ---------------------------------------------------
```

# Chapter 5 Application and Simulation

The effective power factors of several circuits are measured in this chapter, using the simulation system presented in Chapter 4. The examples cover simple three-phase circuits as well as very complicated ones. This proves the capability of the measuring mechanism proposed in this research and also implies the possibility of this mechanism's acceptance in the power industry.

## 5.1 Balanced Resistive Load and Inductive Load

### 5.1.1 Resistive Load

Any new or more generalized theory has to give the same result as the old ones when applied to comparatively simple situations because the old theory likely explained the same situation very well. Therefore it is reasonable to expect that the effective power factor of a balanced resistive load is equal to 1.0, and the effective power factor of balanced inductive load is equal to $\cos\phi$, where $\phi$ is the phase angle difference between the voltage and current waveform.

In this 4-wire balanced resistive load system, $2\Omega$ resistors are connected to 480V (line to line) voltage sources, as shown in Fig. 5.1.

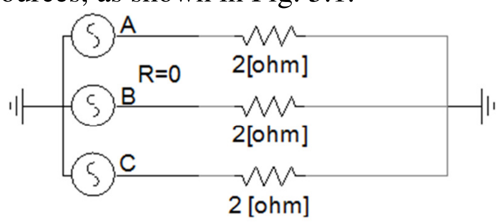


Fig. 5.1 4-wire balanced resistive load system

As shown in Fig. 5.2, the voltage and current waveform are sinusoidal and perfectly in phase. It can also be anticipated that the line current has only one component, or the base component.

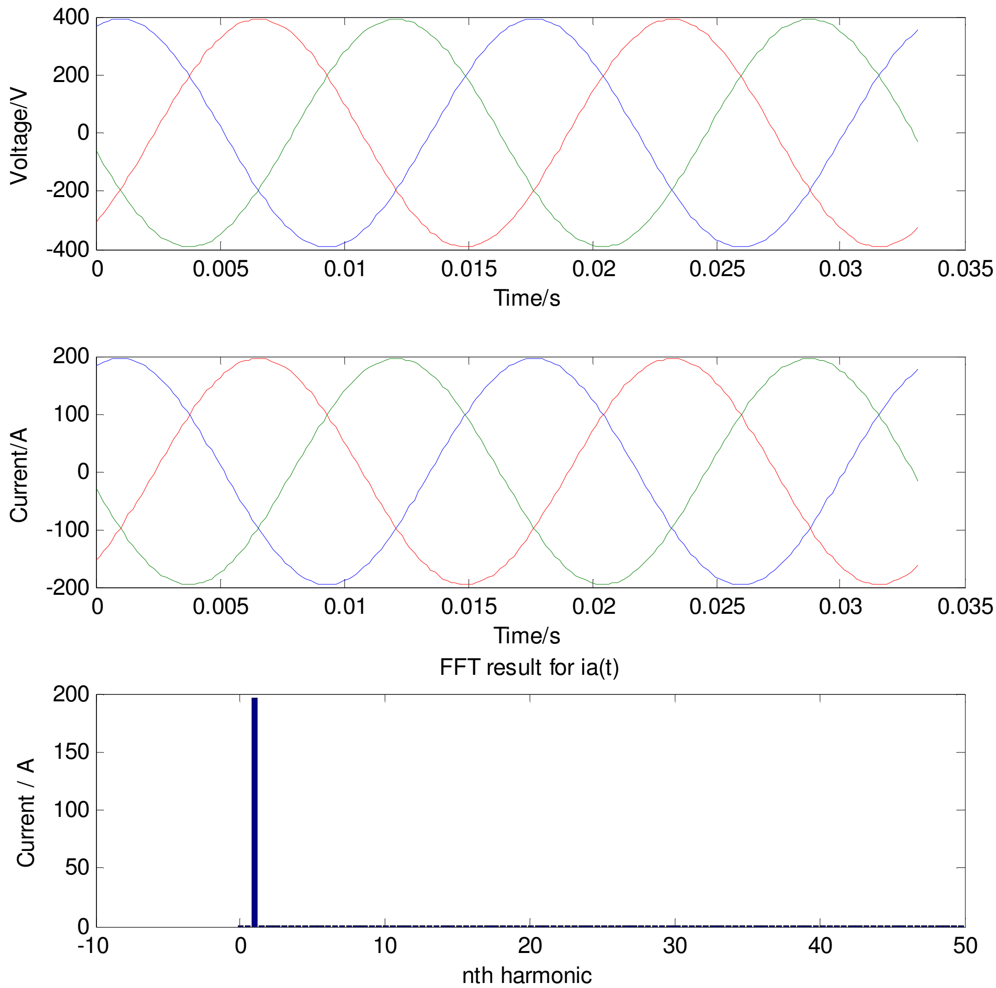


Fig. 5.2 Voltage and current waveform of balanced resistive load

In the end, calculations reveal that the real power and the effective apparent power are identical. Consequently, the effective power factor is equal to 1. This result perfectly agrees with the conventional definition of a power factor. The effective voltage $V_e$, effective current $I_e$, effective apparent power $S_e$, real power $P$, and effective power factor $PF_e$ are shown in Table 5.1.

Table 5.1 Result for balanced resistive load system

| $V_e$(V) | $I_e$ (A) | $S_e$ (kVA) | $P$ (kW) | $PF_e$ |
|---|---|---|---|---|
| 277.170 | 138.585 | 115.235 | 115.235 | 1.000 |

### 5.1.2 Inductive Load

In the following system, a $2\Omega$ resistor is connected in series with a 3.063mH inductor to a 480V (line to line) voltage source in every single phase. At a frequency of 60Hz, the phase impedance is $2 + j2/\sqrt{3}\Omega$. It can be expected that the power factor is equal to $\cos 30° = 0.866$. This circuit is shown in Fig. 5.3

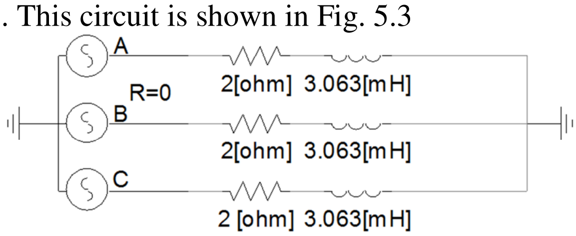


Fig. 5.3 4-wire balanced inductive load system

Fig. 5.4 shows that the voltage and current waveforms are sinusoidal with a phase difference of $30°$. And again, there is no harmonic source in this circuit, so only one base component can be observed in the frequency spectrum.

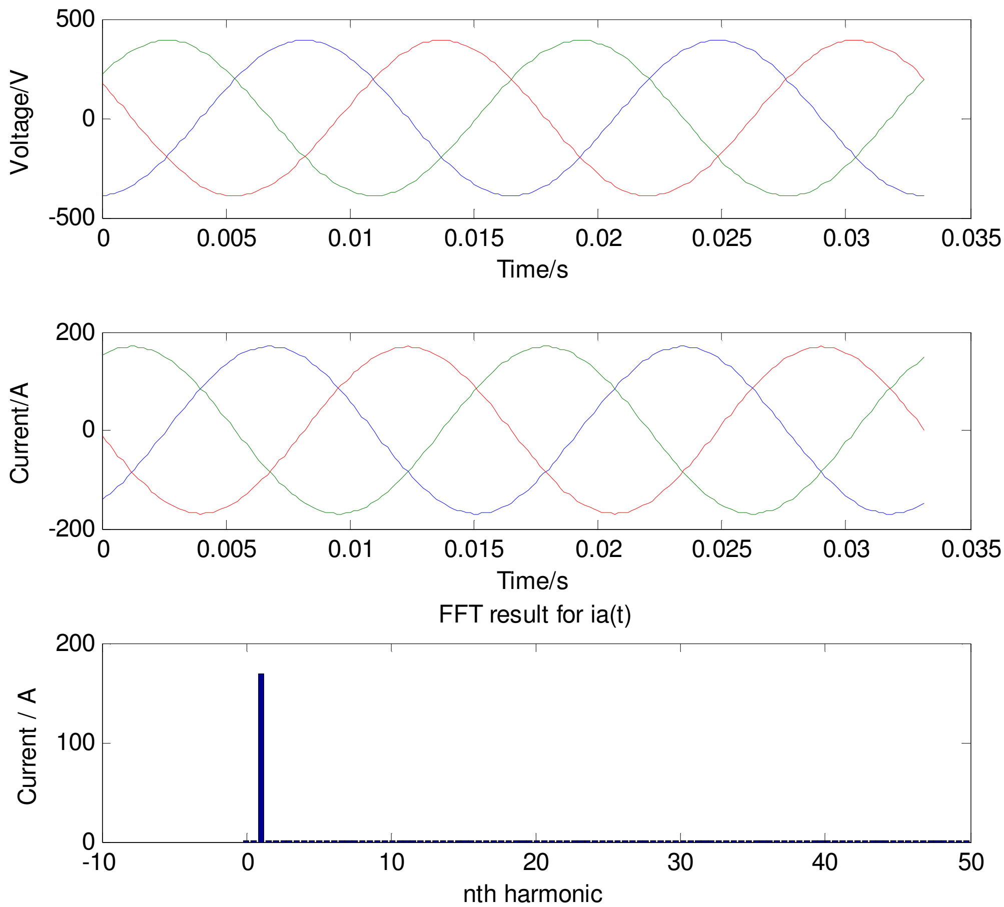


Fig. 5.4 Voltage and current waveform of balanced inductive load

The computation result reveals that the real power absorbed by the load is 86.425kW while the effective power transmitted is 99.796kVA. Therefore, the effective power factor is 0.866. This result also complies with the conventional definition of a power factor.

Table 5.2 Result for balanced inductive load system

| $V_e$ (V) | $I_e$ (A) | $S_e$ (kVA) | $P$ (kW) | $PF_e$ |
|---|---|---|---|---|
| 277.170 | 120.017 | 99.796 | 86.425 | 0.866 |

## 5.2 Power Electronic Devices

### 5.2.1 Six-Pulse Current Source Inverter

The proliferation of harmonic loads in power systems raises much discussion among different power theories, and this has led to a need for a unified definition of the power factor. The following examples will measure the effective power factor of different inverters, which are all typical sources of harmonic distortions. These examples will show that the power factor measurement mechanics that this paper proposes can work in harmonic dominated circumstances.

First, in a 6-pulse current source inverter, the current waveform is distorted; thus, it can be expected that the power factor would be lower than one. The circuit of a 6-pulse current source inverter is shown in Fig. 5.5. The inverter is connected to a 480V (line to line) voltage source. A dc current source is connected to the load to represent the constant load current.

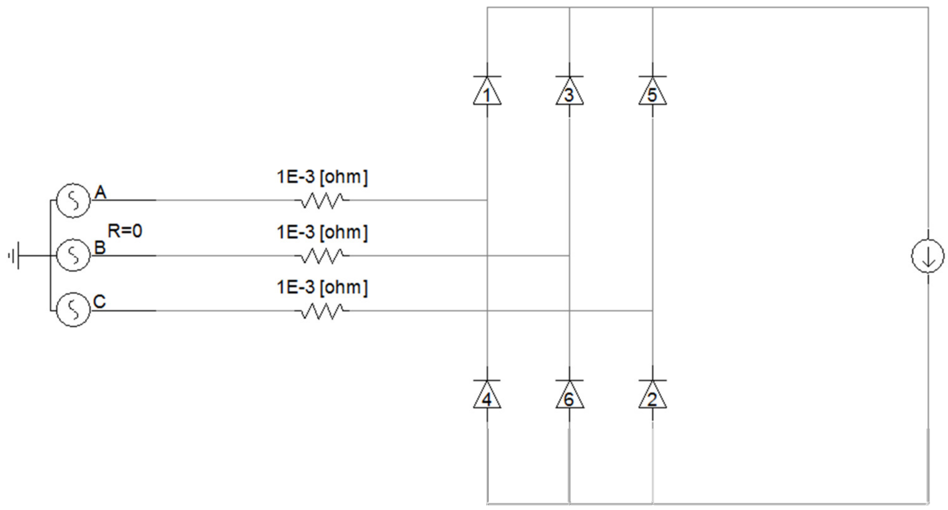

Fig. 5.5 6-pulse current source inverter circuit

The source side line-to-neutral voltages and currents waveforms are shown in Fig. 5.6. The current now has a square shape, so harmonic components can be observed on the FFT spectrum. But, the base component of the current is still in phase with the voltage.

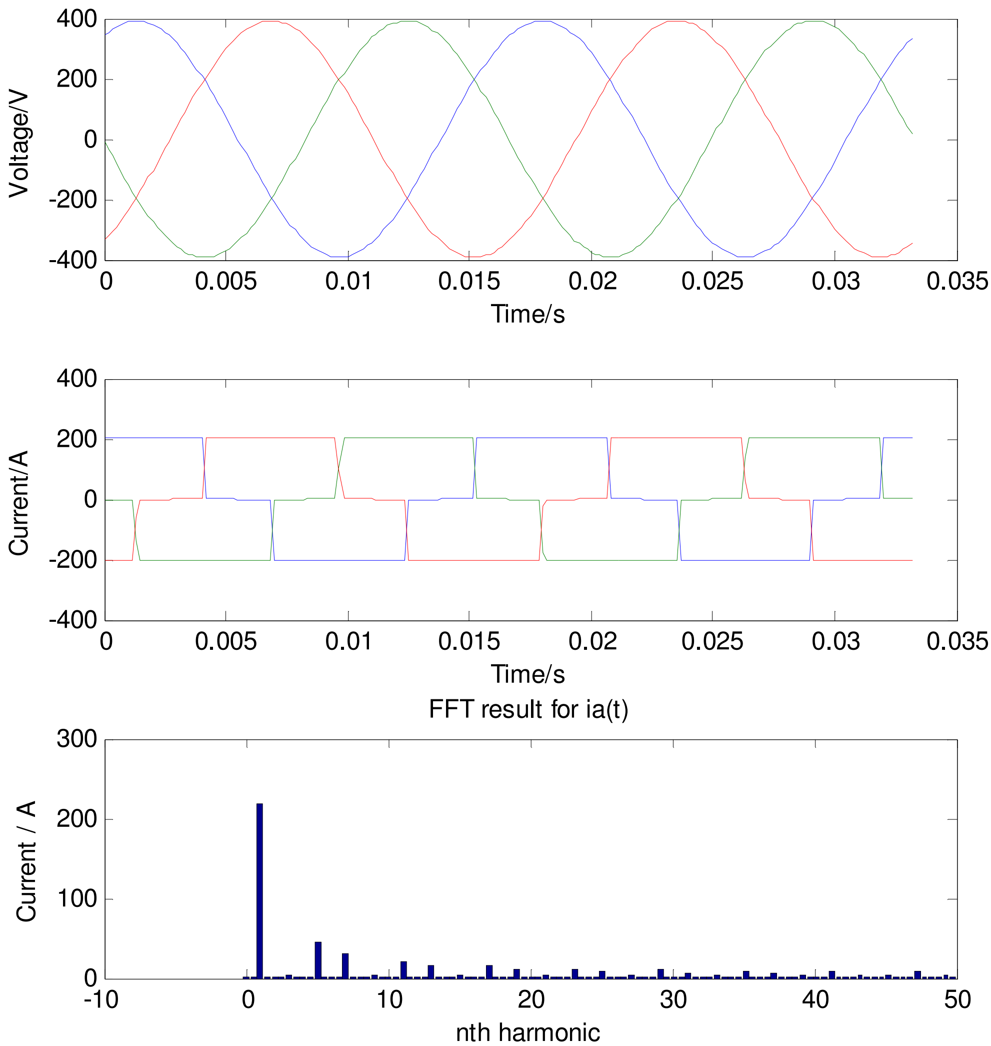


Fig. 5.6 Voltage and current waveform of 6-pulse current source inverter

Computation reveals that the effective power factor in this case is 0.955.

Table 5.3 Result for 6-pulse current source inverter system

| $V_e$ (V) | $I_e$ (A) | $S_e$ (kVA) | $P$ (kW) | $PF_e$ |
|---|---|---|---|---|
| 277.170 | 163.240 | 135.736 | 129.688 | 0.955 |

### 5.2.2 Six-Pulse Voltage Source Inverter

Next, a 6-pulse voltage source inverter circuit, which is shown in Fig. 5.7, is evaluated. The inverter is connected to a 480V (line to line) voltage source. A 0.1F capacitor is connected in parallel to maintain output voltage. A dc voltage source is connected as the dc load, the inner resistance of which is set to 1 $\Omega$.

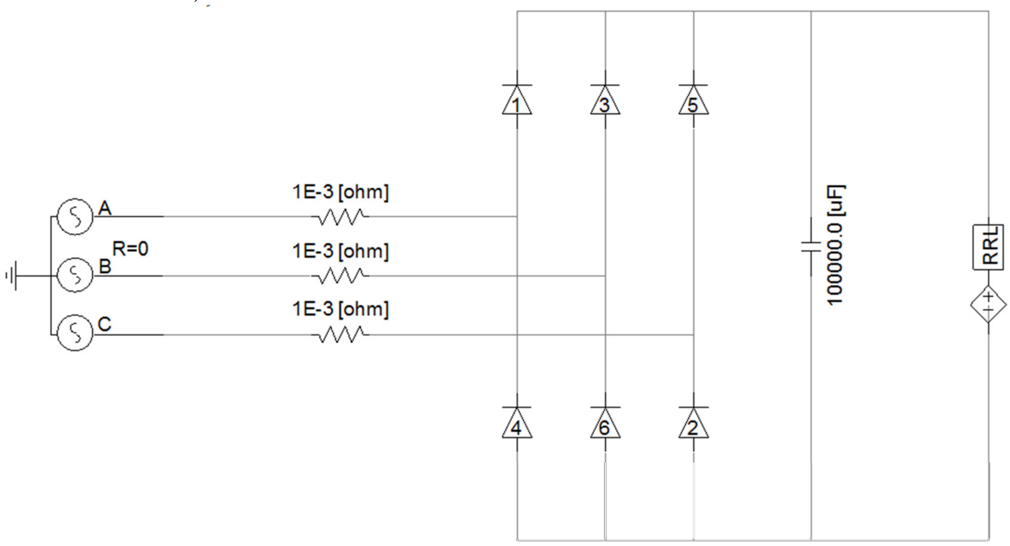


Fig. 5.7 6-pulse voltage source inverter circuit

The voltage and current waveforms are shown in Fig. 5.8. The current now has a double impulse pattern, which introduces more harmonic distortions than the 6-pulse current source inverter. The comparison between the line current spectrums is shown in Fig. 5.9. Computation reveals that the total harmonic distortion of the voltage source inverter is 109%, or larger than that of the current source inverter, which is 229%. This indicates that the voltage source inverter has a much lower effective power factor.

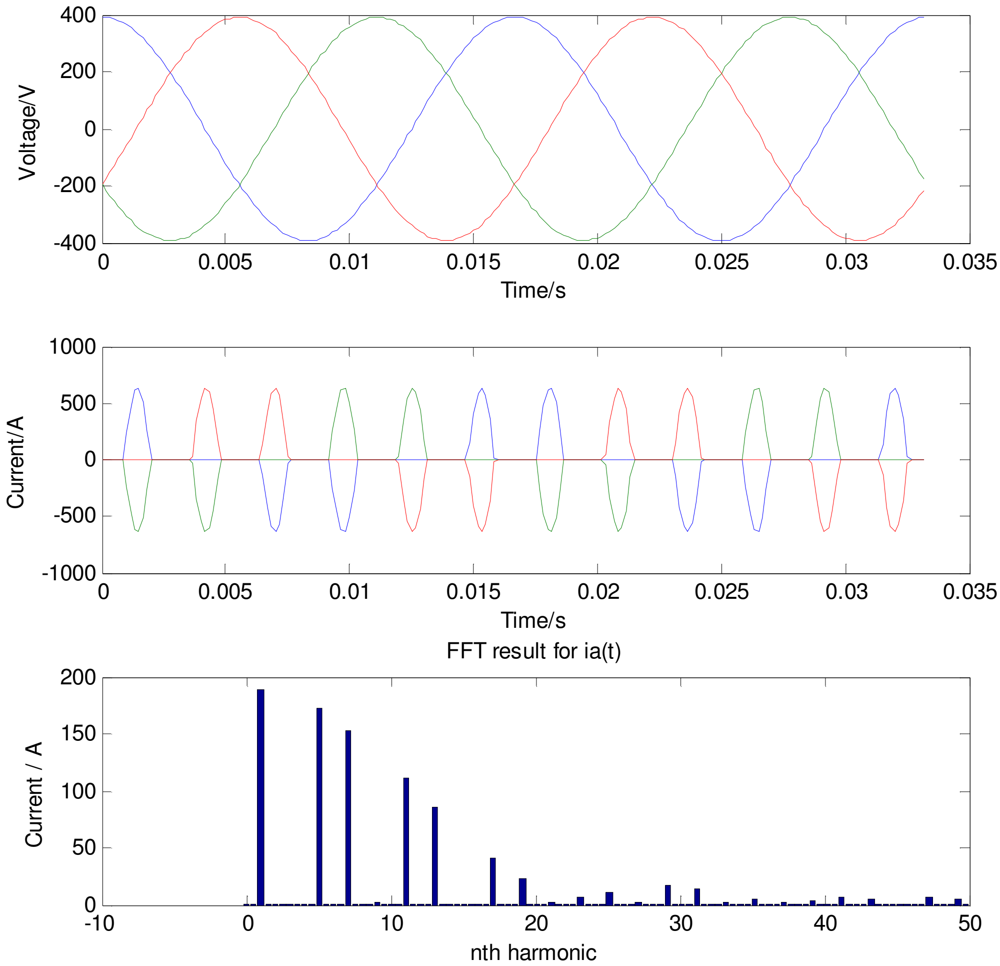


Fig. 5.8 Voltage and current waveform of 6-pulse voltage source inverter

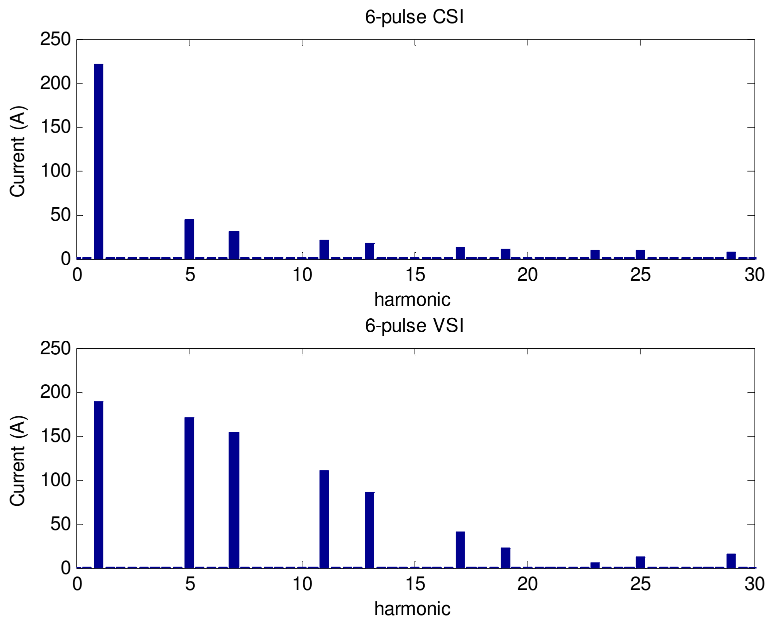


Fig. 5.9 Line current harmonic components in current and voltage source inverters

Further computation reveals that the effective power factor for this voltage source inverter system is 0.567. As expected, it is lower than that of the current source inverter.

Table 5.4 Result for 6-pulse voltage source inverter system

| $V_e$ (V) | $I_e$ (A) | $S_e$ (kVA) | $P$ (kW) | $PF_e$ |
|---|---|---|---|---|
| 277.170 | 236.568 | 196.709 | 111.557 | 0.567 |

### 5.2.3 Twelve-Pulse Voltage Source Inverter

The last inverter tested is the 12-pulse voltage source inverter. Two 6-pulse inverters are connected in parallel. The upper inverter is connected to the three-phase voltage source via a D-D connected transformer, while the lower inverter is connected to the same voltage source via a D-Y connected transformer. The difference in the configuration of transformers leads to $30°$ of phase angle difference between the two 6-

pulse inverters; this doubles the pulse number of the larger inverter. One of the significant advantages of 12-pulse inverters is that the DC output voltage ripples are smaller, and the AC input currents are smoother; both of these factors lower the harmonic components and enhance the power quality. Therefore, the effective power factor is expected to be significantly closer to 1.0 than the 6-pulse ones. Fig. 5.10 shows the circuit of the 12-pulse current source inverter.

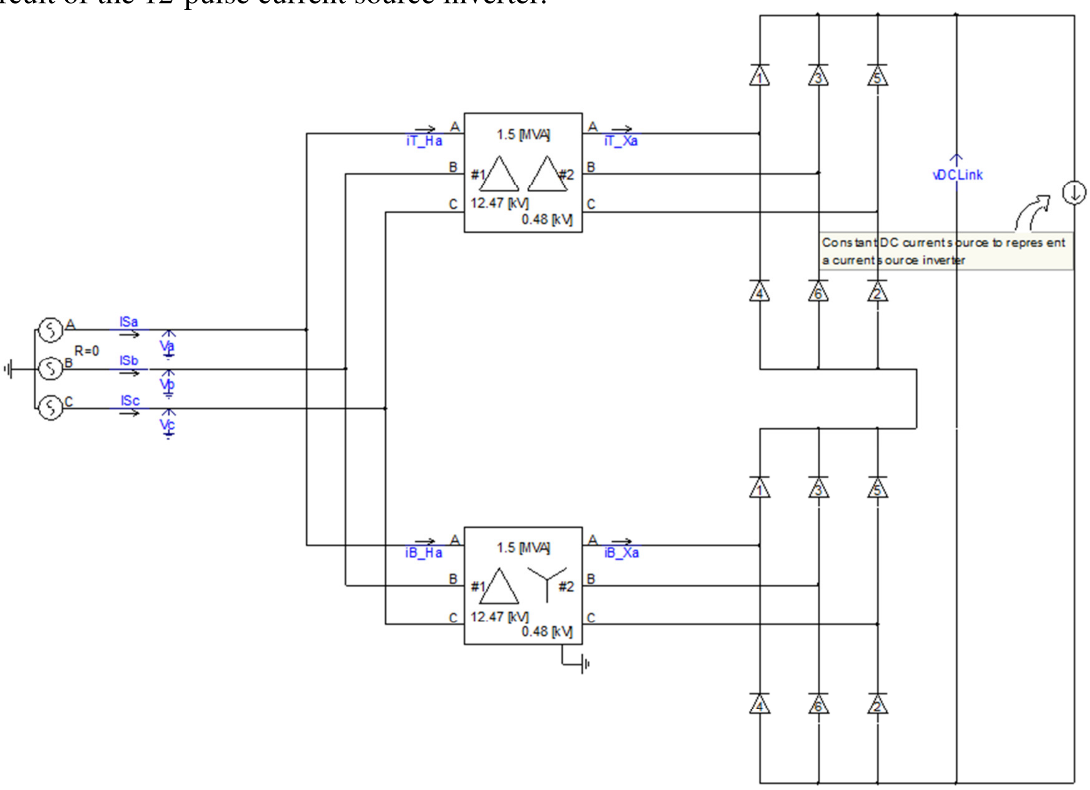


Fig. 5.10 12-pulse current source inverter

The line-to-neutral voltages and line currents observed on the source side are presented in Fig. 5.11. Compared with the current waveform of 6-pulse inverters, this current waveform is much closer to a sinusoidal wave. As expected, the harmonic components of the current waveform are insignificant.

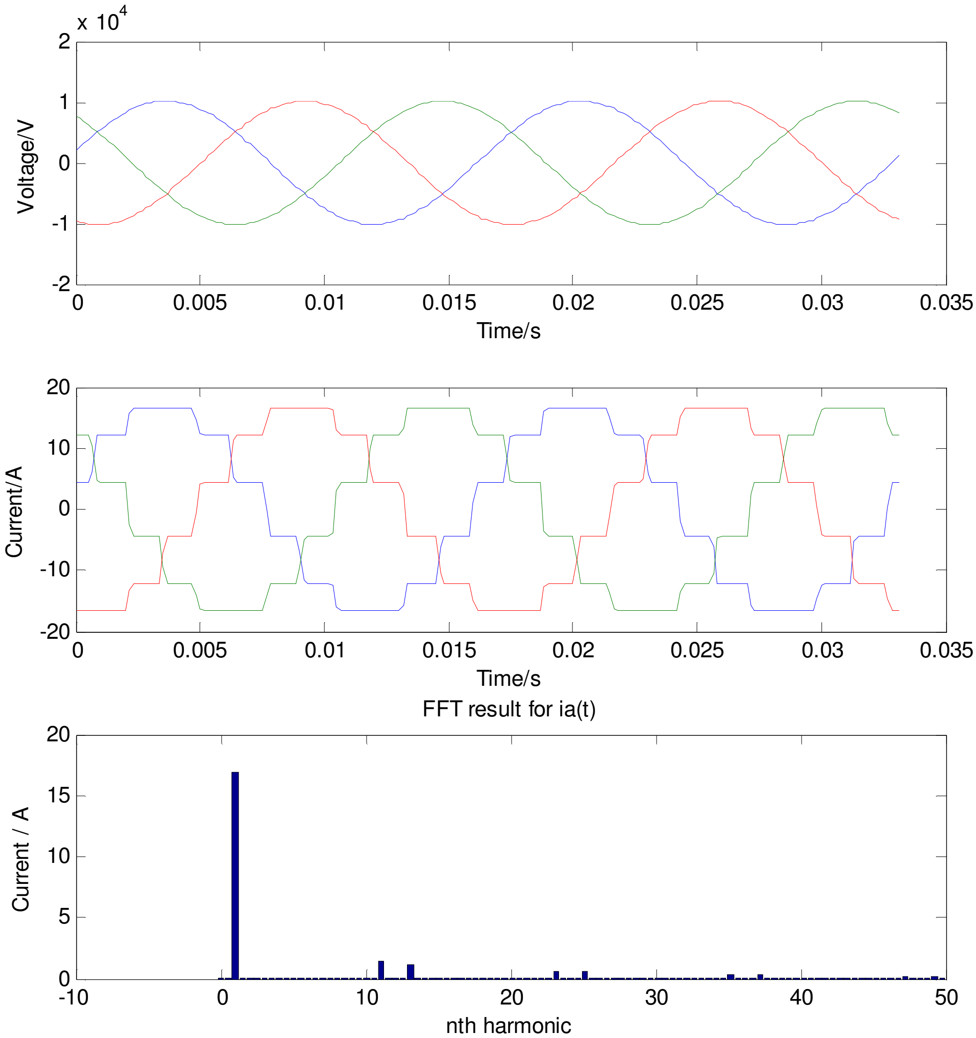


Fig. 5.11 Voltage and current waveforms of 12-pulse current source inverter

Calculation shows that the real power transmitted from the source is 260.062kW, while the effective apparent power of the system is 262.684kVA. Therefore, the effective power factor is 0.9900, which is closer to 1—especially when compared with 6-impulse inverts.

## 5.3 COMPREHENSIVE EXAMPLE

The last simulation example examines a circuit that is unbalanced and displays harmonic distortion. This example can test the performance of the effective power factor theory under the very worst conditions. This type of simulation was first tested by Emanuel [24]. If the results of this study are the same as Emanuel's, the level of compliance of this mechanism with the IEEE Standard 1459 can be verified.

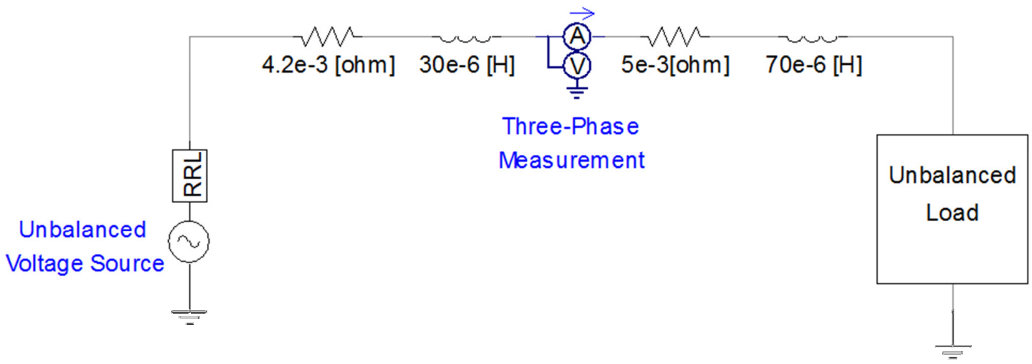


Fig. 5.12 One-line diagram of the comprehensive example circuit

The following frequency domain data was measured from a three-phase load containing 90-kVA single-phase nonlinear loads and 45-kVA linear loads consisting of small motors and electric heaters; this is shown in Fig. 5.12. The load is supplied by a slightly asymmetrical three-phase, four-wire system of 480 V and 60 Hz. The harmonic components of line-to-line voltages are exhibited in Table 5.5, and the components of line currents are exhibited in Table 5.6. The $h$ in the table represents the harmonic order. $V_{ah}$, $V_{bh}$, and $V_{ch}$ represent the root mean square values of the $h$th harmonic components of the corresponding line-to-neutral voltages; $I_{ah}$, $I_{bh}$, and $I_{ch}$ represent the root mean square values of the $h$th harmonic components of the corresponding line currents. THD represents the total harmonic distortion.

Table 5.5 Harmonic components of line-to-line voltages

Base Voltage: $V_a = 272.45$

| $h$ | $V_{ah}$ | $V_{bh}$ | $V_{ch}$ |
|---|---|---|---|
| 1 | $100\angle 0°$ | $103.65\angle -126°$ | $102.08\angle 123°$ |
| 3 | $5.77\angle 49°$ | $5.79\angle 50°$ | $5.37\angle 50°$ |
| 5 | $1.59\angle -144°$ | $1.14\angle -70°$ | $0.49\angle 170°$ |
| 7 | $0.49\angle -37°$ | $0.77\angle -129°$ | $0.54\angle 168°$ |
| 9 | $1.02\angle 96°$ | $1.04\angle 91°$ | $0.99\angle 109°$ |
| 11 | $0.75\angle 179°$ | $0.52\angle -132°$ | $0.49\angle 174°$ |
| 13 | $0.31\angle -124°$ | $0.47\angle 148°$ | $0.41\angle 154°$ |
| 15 | $0.56\angle 40°$ | $0.57\angle 33°$ | $0.47\angle 65°$ |
| 17 | $0.57\angle 133°$ | $0.41\angle 173°$ | $0.47\angle 140°$ |
| %THD | 6.20 | 5.92 | 5.48 |

Table 5.6 Harmonic components of line currents

Base Current: $I_{a1} = 133.92A$

| $h$ | $I_{ah}$ | $I_{bh}$ | $I_{ch}$ |
|---|---|---|---|
| 1 | $100\angle -23.67°$ | $123.12\angle -121.6°$ | $68.48\angle 98.0°$ |
| 3 | $57.60\angle 142°$ | $57.50\angle 145°$ | $31.52\angle 151°$ |
| 5 | $35.80\angle -65°$ | $33.89\angle 60°$ | $22.75\angle -169°$ |
| 7 | $14.52\angle 82°$ | $12.25\angle -38°$ | $12.46\angle -133°$ |
| 9 | $3.17\angle 161°$ | $4.33\angle 151°$ | $4.33\angle -111°$ |
| 11 | $5.12\angle -144°$ | $4.91\angle 21°$ | $2.16\angle -165°$ |
| 13 | $3.43\angle 21°$ | $2.37\angle -101°$ | $2.85\angle -159°$ |
| 15 | $1.21\angle 97°$ | $1.90\angle 90°$ | $1.85\angle -137°$ |
| 17 | $2.00\angle -173°$ | $1.79\angle -30°$ | $0.74\angle -160°$ |
| %THD | 69.74 | 55.45 | 60.03 |

The time varying waveform of line-to-neutral voltages and line currents can be constructed from these two data tables. Both waveforms are shown in Fig. 5.14. The voltage waveform is slightly distorted from a sinusoidal wave, while currents are severely distorted. What is more, the amplitudes of currents in three phases are different, which implies that the circuit is unbalanced.

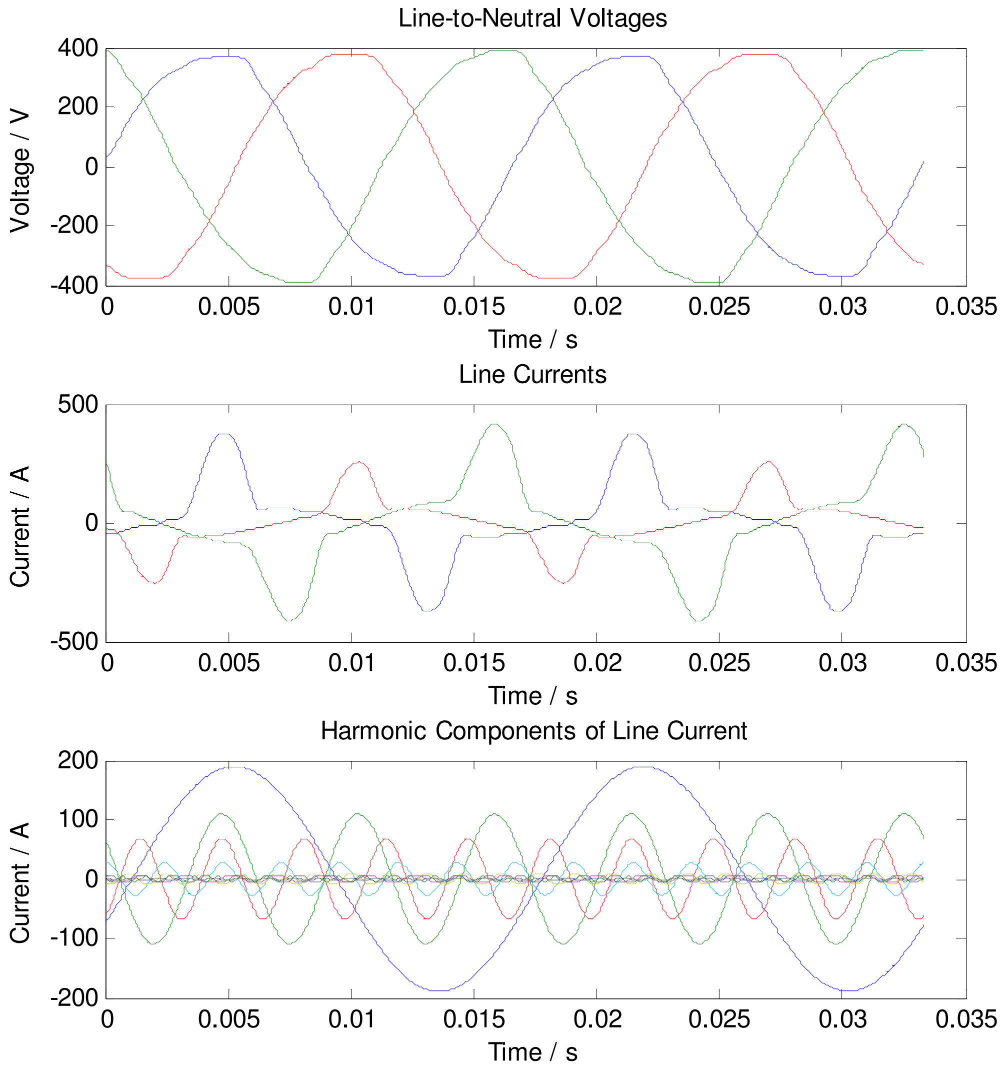


Fig. 5.13 Voltage and current waveforms of the comprehensive example

Calculation reveals that the real power consumed is:

$$P = 102.71kW \tag{159}$$

The effective voltage is:

$$V_e = 278.02V \tag{160}$$

The effective current is:

$$I_e = 210.33A \tag{161}$$

The effective apparent power is:

$$S_e = 3V_eI_e = 175.426MVA \tag{162}$$

And, the effective power factor is:

$$PF_e = \frac{P}{S_e} = 0.5855 \tag{163}$$

The arithmetic power factor is also analyzed for this example. The arithmetic apparent power of the circuit is:

$$S_A = V_aI_a + V_bI_b + V_cI_c = 127.913MVA \tag{164}$$

Therefore, the arithmetic power factor of the circuit is:

$$PF_A = \frac{P}{S_A} = 0.804 \tag{165}$$

Again, the arithmetic power factor is larger than the effective power factor.

The effective power factor that Emanuel records in [24] is 0.585. Since the two results are very close, the measurement system designed in this study gives reasonably accurate results that comply with IEEE Standard 1459, even under the most complicated situations.

# Chapter 6 Conclusion and Future Work

## 6.1 Conclusion

This study designed and constructed a measurement system that could be called by the power system simulation software, PSCAD, to analyze the voltage and current waveforms and to measure the effective power factor in real time. As a new theory that could be applied to circuits as complicated as unbalanced non-sinusoidal three-phase circuits, it proved capable of explaining the simple balanced sinusoidal three-phase circuits as well as the conventional definition of a power factor. Simulation on power electronic devices evaluated the performance of the measurement system under harmonic distorted situations. The system ran smoothly in all cases, and the results were consistent with general knowledge and available research. Finally, the system was evaluated under the most complicated circumstances that might be encountered in the real world, or more simply put using a three-phase circuit that was neither balanced nor sinusoidal. The proposed system worked smoothly during the test. The resulting measurement matched the IEEE Standard 1459 very well; thus, the accuracy of the system was verified. Other power factor definitions were also applied to the same circuit; and the arithmetic power factor was higher than the effective power factor. As explained in Chapter 3, the arithmetic and vector power factors are problematic in unbalanced systems, so it is highly recommended to replace those definitions with the effective power factor definition.

In this author's opinion, the effective power factor theory is the most suitable theory to address three-phase circuits under the non-sinusoidal and unbalanced condition.

First, the effective power factor theory has a clarified physical interpretation of the apparent power: the maximum power transmitted to the load (or delivered by a source) while keeping the same line losses and the same load (or source) voltage and current. Other theories attempt to interpret the reactive power, and then define the apparent power as a Euclidean summation of real and reactive power. This mathematical definition makes it difficult to pinpoint the physical significance of apparent power. However, as pointed out in Chapter 3, the study of power factor is more concerned with the apparent power than with the reactive power.

Secondly, effective power factor theory provides more accurate results in unbalanced three-phase circuits, especially when the neutral current cannot be neglected. As shown in Chapter 3.5, vector power factor and arithmetic power factor cannot represent precisely the level of unbalance in a power system. On the other hand, the effective power factor gives more reasonable results in unbalanced three-wire three-phase circuits. Moreover, since it takes into account the neutral current explicitly in the definition, the effective power factor theory would be more accurate than other theories in a four-wire system where the neutral current cannot be neglected.

Finally, effective power factor theory only involves measurements and computations in the time domain. Effective power factor theory is developed from Budeanu's power theory, because it inherits the same definition of apparent power in single-phase circuits under a non-sinusoidal condition. However, the computation of effective apparent power does not involve any frequency domain operations. Every term involved in the calculation is a root mean square value, which can be directly converted from time domain quantities. This not only guarantees the simplicity of the calculation procedure but also lowers the hardware requirements for the measurement of the effective power factor.

## 6.2 FUTURE WORK

Calculations of the root mean square values and time average values require knowledge of the signal frequency. At this stage, the frequencies of the voltage and current are presumed to remain constant at 60 Hz. However, the system frequency fluctuates in the real world due to the equilibrium of power supply and demand. Although the fluctuation is insignificant most of the time, it might change dramatically in rare cases, such as when a generator is tripped in a microgrid. Therefore, the inability to track system frequency will limit the use of this technology.

In this author's opinion, the ultimate goal of measuring the power factor is to help minimize power losses. The downstream application of this effective power factor measurement system should allow for the coordination of power factor correction operations and the location of harmonic sources. FACTS technology has been evolving for decades. In comparison, harmonic source locating is a comparatively new field and still requires additional work on the definition of harmonic contribution. However, very limited work in both fields uses the effective power factor as an indicator of power quality. The resolution of effective apparent power mentioned in IEEE Standard 1459-2010 shows that it has the potential to contribute to these fields.

Right now the measurement system is realized in MATLAB and FORTRAN. If the standard is widely accepted by the power industry, the realization of the system for onboard systems needs to be addressed.

## Appendix

## State of Ethics and Academic Integrity

I certify that I have completed the online ethics training modules, particularly the Academic Integrity Module[3], of the University of Texas at Austin - Graduate School. I fully understand, and I am familiar with the University policies and regulations relating to Academic Integrity, and the Academic Policies and Procedures[4]. I also attest that this thesis is the result of my own original work and efforts. Any ideas of other authors, whether or not they have been published or otherwise disclosed, are fully acknowledged and properly referenced. I also acknowledge the thoughts, direction, and supervision of my research advisor, Prof. S. Santoso.

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[3] The University of Texas at Austin – Graduate School online ethics training modules, http://www.utexas.edu/ogs/ethics/, and the ethics training on academic integrity, http://www.utexas.edu/ogs/ethics/transcripts/academic.html, accessed on Nov. 18, 2010.

[4] The University of Texas at Austin, General Information, 2006 – 2007, Chapter 11, Sec.11 101, http://www.utexas.edu/student/registrar/catalogs/gi06-07/app/appc11.html.

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## Vita

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