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Cross-Temporal Relations

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Cross-Temporal Relations

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Report

Presented to the Faculty of the Graduate School

of the University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

Master of Arts

The University of Texas at Austin

December 2010

Dedicated to:

Sharmane Johnson

You're my Pube Shirt

Acknowledgement

I would like to express my deepest appreciation to my committee supervisor, Dr. Josh Dever, who provided endless support and uncountably many helpful ideas without which this paper would not have been possible.

I would also like to thank Dr. Ray Buchanan and Dr. Paul Teller for there helpful comments and guidance as well.

Lastly I'd like to thank my wife for listening to me blab about space-time and instantaneous velocities.

Cross-Temporal Relations

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University of Texas at Austin 2010

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In this paper I argue that the presentist cannot deal adequately with cross-temporal relations. I look at several attempts to solve the cross-temporal relations objection and find only one that might work. Still I argue that even it can't deal with cross-temporal *spatial* relations such as continuity. I defend Sider here against two plausible responses. The first is that instantaneous velocities can be employed on the presentist's behalf to get them out of trouble. I argue that this response won't work. The second is a response by Dean Zimmerman in which the presentist accepts that past space-time points exist at present. I argue that his response does indeed provide us with a solution but that the cost of that solution is far too high.

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1 Introduction

There's a well known divide in the philosophy of time between presentists and eternalists. Presentism is the view that the only things that exist *simpliciter* are those existing presently. This is not the trivial thesis that the only things that exist *now* are those existing presently, or the false claim that the only things that exist *timelessly* are those that exist now. Rather, it's the view that the only things that exist, with quantifiers wide open, are the things presently existing. Eternalists, by contrast, think that when we open our quantifiers all the way we get all past, present, and future objects. Each view has its merits and demerits but overall I feel eternalism is superior. One reason for this assessment is presentism's inability to adequately deal with the problem of cross-temporal relations—at least that's what I'll argue in this paper.

Roughly stated the problem is that if past objects don't exist, then present objects can't stand in relations to them, but there are numerous relations holding between past and present objects. This problem is not unfamiliar to the presentist and many have proposed solutions to it. In this paper I'll consider several of those proposals and show why they either don't work or are unacceptable. I'll then identify a view of presentism put forth by Sider [11] that may have a shot at dealing with cross-temporal relations adequately. The rest of the paper argues that it too fails. I defend a later argument by Sider [12] to the effect that presentism, in any form, can't distinguish various cross-temporal *spatial* relations—like accelerated, inertial, continuous or discontinuous motion. In defending Sider's argument I consider two main responses. The first, involves the central claim that instantaneous velocity is an intrinsic property. I argue that it is not, and that even if it were, Sider's argument still goes through. The second is Dean Zimmerman's surrogate and empty-box view. I argue that the surrogate view doesn't represent a solution to the problem but that though the empty-box view does it comes at much too high a price. I conclude that cross-temporal relations represents a serious threat to the presentist's position and that we should prefer eternalism.

2 The Problem of Cross-Temporal Relations

What are cross-temporal relations? Roughly they are relations that have relata that exist at different times. Many relations that can hold between relata existing simultaneously can also hold between relata existing at distinct times. I can admire my wife, for instance, a person I'm contemporaneous with, but I can also admire Aristotle, a person I'm not. *Taller than*, *smarter than*, *uglier than* and other general comparative, *despising*, *loving*, *admiring* and other intensional relations, *father of*, *descendent of*, *philosophic parent of*, and other ancestor relations all appear, sometimes, cross-temporal. Some miscellaneous but no less important examples include, change, set membership, reference, causation, and the purely temporal before and after. Cross-temporal relations are trouble for the presentist. Here's an abstract version of the argument from cross-temporal relations against presentism:

(1) Presentism: The only things that exist *simpliciter* are those existing presently.

(2) Principle of Relations (PR): For any relation R, if R is an n-placed relation holding between $\langle x_1, x_2, x_3, ..., x_n \rangle$ then all of $x_1, x_2, x_3, ..., x_n$ must exist.

(3) Cross-Temporality (CT): Certain sentences concerning no longer presently existing entities that have a relation to presently existing entities (CT-sentences) are correctly evaluated as true.[1]

(4) So let R be an n-placed cross-time relation *holding* between $\langle x_1, x_2, x_3, ..., x_n \rangle$.

(5) Since R is a cross-temporal relation at least two of the relata, say x_i and x_j, are *not* contemporaneous.

(6) Let x_i exist i.e. be present.

(7) Then x_j doesn't exist (by 5, and presentism).

[1] From Torrengo [16]).

(8) But x_j does exist (by PR).

(9) So presentism is false.

In the next four sections we'll look at various proposals for dealing with this argument. The first three proposals I'll dispatch rather quickly (within the sections they're elaborated in) but the proposal Sider gives in section 5 will take the rest of the paper to deal with. We begin with frivolous presentism.

3 Frivolous Presentism

A quick way out of the problem of relations is to deny that the holding of a relation entails the existence of that relation's relata i.e. to deny PR. Meinong and others have found motivation for such a denial in the relation of intensionality since it can sometimes seem directed toward non-existent objects like God or Santa. Since we can stand to Santa in the love relation, and since Santa doesn't exist, it must be that loving Santa doesn't entail his existence. But if intensional relations can hold between existent and non-existent relata, why can't cross-temporal relations hold between existent and non-existent relata? The view that they can is called *frivolous presentism.* Thomas M. Crisp [6] defines frivolous presentism as follows,

Frivolous Presentism: It is always the case that: (i) for every x, x is present, but (ii) it is, was or will be the case that, for at least one x, x has a property or stands in a relation at a time t such that x does not exist at t.

Denying PR allows the frivolous presentists to easily avoid the problem of cross-temporal relations. For instance 'Caroline is the daughter of JFK' is about a genuine cross-temporal relation holding between Caroline and JFK, but only Caroline exists, so presentism is preserved at the expense of PR. Is this a price we should be willing to pay? Thomas Crisp thinks not, he writes,

> The main problem with this reply to the objection from cross-time relations is that it is so difficult to believe. The suggestion here is that Caroline bears a relation R to JFK and Jackie, but there is nothing to which she bears R. Bizarre. To be sure, some presentists of the more Meinongian bent will see no problem here. But for those suspicious of propertied non-existents, the frivolous presentist's reply simply is not a serious option.

I agree with Crisp. To my mind frivolous presentism (alone, i.e. not in conjunction with Meinongianism) is a dead hypothesis, to employ a notion of William James' [8]. It's just too implausible to take seriously.

But what about those Meinongians for which frivolous presentism would seem natural? What should we say about the Meinongian presentist? Fortunately there seems to be a very good reason we won't have to delve into the murky waters of Meinongianism. Here's why. Suppose a presentist is also a Meinongian. Such a combination, with respect to time, is indistinguishable from eternalism. Both employ different names for the same ontological categories. Past and future objects would not exist but would have being and could stand in cross-temporal relations. Such objects are simply those that exist-at a time for the eternalist. The objects that exist for the presentist Meinongian are those that exist-at present (where 'present' is an indexical) for the eternalist.[2] They may conceive of the categories differently but given the roles they each want the objects in those categories to play it's difficult to see any difference between the two views.

Of course there will be other stuff in the Meinongian presentists ontology. For instance they may accept that Santa Clause has being though he doesn't exist. Where is he on the eternalist picture? This is irrelevant. As far as time goes the Meinongian presentist and the eternalist are indistinguishable. What they say about fictitious characters (the eternalist is free to be a Meinongian) is none of my concern. So frivolous presentism, alone, doesn't really represent a serious option and frivolous presentism in conjunction with Meinongianism turns out to be indistinguishable from eternalism and therefore won't help the presentist avoid the problem of cross-temporal relations. We now turn to Crisp's response.

[2] I assume the eternalist is a detenser but nothing turns on this assumption.

4 Crisp's Response

In this section I'll explicate Crisp's [6] response and then say why I think it fails. Crisp's main line of thought is simple though the details get slightly complicated. Consider,

(1) Clinton admires JFK.

(2) Bush is of the same political party as Lincoln.

(3) Today's flood was caused by yesterday's downpour.

(4) Caroline is the daughter of JFK and Jackie.

and,

(1′) Clinton bears the *admires* relation to JFK.

(2′) Bush bears the *same political party* relation to Lincoln.

(3′) Today's flood bears the *is caused by* relation to yesterday's downpour.

(4′) Caroline bears the *daughter of* relation to JFK and Jackie.

Crisp claims that the argument from cross-temporal relations requires the following two thesis:

(MF): Sentences of ordinary language like (1)-(4) express Moorean facts.[3]

(NP): Sentences of ordinary language like (1)-(4) are predicative with respect to names of non-present entities.

His main argument runs as follows: either (1′) - (4′) are what's actually meant by (1)-(4) or they're not. If so then (1)-(4) are not Moorean facts. If not, then they are not predicative with respect to the names of non-present entities. In either case the conjunction of MF and NP is false.

[3]Moorean facts are the type of things we know better than any premise in a philosophical argument to the contrary. 'This is a hand' uttered while holding up a hand is the classic example.

But since the argument from cross-temporal relations requires the truth of this conjunction it must be unsound. That's the main line of argument. Let's see how he supports his premises.

Assume ($1'$) - ($4'$) are strict philosophic ways of saying (1)-(4), then the latter are not Moorean facts since the former aren't. Why aren't the former Moorean facts? To be a Moorean fact they would have to fall into one of three categories: deliverance of our sense, truths of reason, or such that they have Moorean evidence for them. They clearly don't fall into the first two categories and Crisp argues that they don't fall into the last category either as follows. He starts by giving the below account of Moorean evidence:

ME: q is Moorean evidence for a proposition p iff (i) q is in either the set of propositions that are deliverances of reason or the senses for which there is no knock down argument and which one would be crazy not to believe, or the set of propositions for which there are good arguments such that one would be crazy not to accept them once grasping those arguments, and (ii) the conditional epistemic probability of p on q is high. Where the conditional epistemic probability of p on q—$P(p/q)$—is the degree to which a human being of sound understanding could believe p if she fully believed q, had no other evidence for or against p, and considered the evidential bearing of q on p.

The question then is, can we find a proposition that would count as Moorean evidence for, say, ($4'$)? Crisp argues no. If ($4'$) is just the philosophically sophisticated way of saying (4) then any Moorean evidence for the one would be evidence for the other. So let E be the conjunction of all the Moorean evidence for (4). This will include legal documents, testimonials, medical records, newspaper reports, and generally the type of stuff a historian would produce as evidence for such a claim. But E won't be Moorean evidence for ($4'$). This is because ($4'$) entails (5)

(5) Quantifying unrestrictedly, something is identical with JFK.

so by the probability calculus $P(4'/E) \leq P(5/E)$. But E doesn't make (5) probable at all. E is just the wrong type of evidence required to support (5). Legal documents and old newspapers won't settle a debate concerning ontology. Since Crisp can't think of any other Moorean facts that could be conjoined with E to establish (5) he concludes that it's not a Moorean fact and therefore neither is (4′). But if (4′) isn't a Moorean fact then neither is (4) under the assumption that the former is just a philosophically sophisticated way of saying the latter. That's the first disjunct. On to the second.

Suppose (1′) - (4′) are *not* strict philosophic ways of saying (1) - (4), then the latter are not predicative with respect to names. Assume they are. Then, sticking with our example, (4) commits one to (5). If (4) is predicative of 'JFK' then there must be something that is identical with JFK. But now the reasoning above that showed that (4′) wasn't a Moorean fact since it commits one to (5) can also be used to show that (4) is no Moorean fact. Thus Crisp concludes that either (1) - (4) are not Moorean facts or they are not predicative with respect to the names of non-present entities. And since he thinks the cross-temporal relations objection requires both MF and NP that argument must be unsound.

But Crisp's response fails. First it's not at all obvious why the argument from cross-temporal relations requires the conjunction of MF and NP. But even if it does Crisp's argument isn't convincing. The fact that (4′) is not Moorean doesn't show that (4) is not Moorean, even supposing the former is the proper analysis of the latter. I say this for two reasons: First, if this line of thought were right it would prove too much, just take any putative Moorean fact and it's philosophic analysis. I wager that the analysis won't be a Moorean fact. But then it would follow that the putative Moorean fact isn't a Moorean fact either. And it would be special pleading to think that this line of reasoning works here but not in others cases. Second, and related, it just seems to me that the analysand, if informative, is likely to have a different epistemic status than the analysandum. So the fact that x is a Moorean fact and y is its analysand, doesn't entail that y is Moorean.

5 Overlap Approach

In this section we'll consider the overlap view.[4] We'll see that it's not very plausible in its skeletal form and offer a few modifications. I'll then point to some problems I see with the modifications considered and leave it at that. The overlap view is tempting and maybe there are modifications that will work but that's not a task I've seen completed and it's not a task I see easily completed.

The overlap approach denies neither PR nor CT but attempts to show how presently existing things can stand in relation to non-present and therefore non-existing things. On this view all that is needed for x to stand in relation R to y where x is present and y is not, is for there to exist an object[5] O such that y once stood to O in some relation R' and x now stands to O in some relation R''. For anyone that believes in necessarily existing object there always will be such an object. The abstract property *being blue* for instance might work. Crisp [6] gives the following general recipe:

> (i) take a sentence S such that, for some proper names α and β and some two-term predicate R: S's grammatical form is "$R(\alpha, \beta)$," α denotes some present object α^*, β does not denote anything but *was* or *will be* such that it denotes some object β^*...(ii) find some object x such that, to put it loosely x's existence "overlaps" α^* and β^* (in the sense that x coexists with β^*); then (iii) translate S as a claim to the effect that α^* bears some relation R' to x and it was or will be the case that x bears some relation R'' to β^*.

So for example, (4) comes out true because the overlap object *being blue*, as the case may be, bears some relation R' to Bush and once bore some relation R'' to Lincoln.

Let's now see why this initial proposal is a nonstarter. Matthew Davidson [7] writes,

> I can't see how this works as a direct answer to the problem of relations. First of all, if this picture is to work there must be some specificity of the relations involved. Surely it's

[4]The overlap view is sometimes attributed to Chisholm [5] but, for my part, I find it difficult to find in that paper.

[5]I'll use 'object' to follow the literature. Russell's use of 'term' might be more appropriate. It could be anything from an event, a property, a number, a concrete physical object, basically anything that exists.

not sufficient for me to stand in the *being of the same political party* relation to FDR that I stand in any relation or other with an object that did stand in any relation or other to FDR. Can it really be that the fact that some relation or other holds between me and an object and some relation or other held between that object and another suffices for the fact that a *particular* relation – *being of the same political party* – holds between me and the other object?

Davidson's objection is exactly right; the account over generates if the nature of the relations involved, and the overlap object, are not restricted. Hitler for instance once bore the coexisting relation with the abstract object *being blue* as Obama presently does, yet Hitler and Obama aren't of the same political party. But how do we modify the view?

5.1 Modified Overlap Approach

Both Davidson and Crisp modify the overlap account in plausible ways[6] but I'm still skeptical that the view is viable. Let's start with Crisp. He thinks the overlap theorist takes (2) and (4) to express,

($2''$) Bush belongs to a political party P such that WAS(Lincoln belongs to P).

and,

($4'$) Caroline is such that WAS(She is born to JFK and Jackie).

respectively. He claims, plausibly, that the overlap objects in question are the political party P and Caroline herself. There's no appeal here to any necessarily existing objects tenuously connected to the relevant subjects. Both the political party P and Caroline (whether abstract or not) seem highly relevant to the relation in question. But how do we determine what object is the actual overlap object when there are many plausible candidates? Consider the following claim,

(6) I admire Aristotle.

[6]Crisp doesn't explicitly modify the account but the examples he chooses illustrate certain restrictions.

The *admires* relation, the abstract information that is Aristotle's writings, and the numerous objects that link Aristotle to me (physical books and such) are all plausible overlap objects. But which is the correct one? An account is desperately needed.

Davidson offers on the overlapper's behalf that we "require that the relation that is supposed to hold between the present entity and the past entity be such that it also is the relation that holds between the two present entities." This suggestions while getting a lot right still misses much.[7] Caroline, for example, is the most plausible object of overlap in (4) yet she isn't the daughter of herself.

The considerations above hopefully suggest that the overlap response is not very promising. At minimum they should show there are details to be worked out for those who find the view viable. This is the last of the three responses I consider implausible. We're now going to turn to a response I think is more plausible.

[7]See Davidson [7] for his own rejection of his own suggestion.

6 Super and Quasi-supervenience Basis

In this section I'll do some preliminary discussion of a plausible and popular response before moving on to Sider's specific version of it.

Many presentists wrestle with the grounding problem. Roughly how can the presentist ground (or find truthmakers for) sentences about the past like, 'Lincoln was tall'? Let's suppose they can do it. These grounding facts would provide a supervenience basis that could potentially ground CT-sentences as well. Bourne [3] develops an *ersatzer* presentism that takes sets of propositions as times. The propositions contained in a time are, intuitively, the ones true of that time. This gives him a supervenience basis for which he thinks he can ground the truth of CT-sentences. Bigelow employs facts true at the present. Two physically indistinguishable present times could differ with respect to the facts true of those times. In one Hannibal won at Cannae in the other he didn't, for instance. These facts work, in effect, as his supervenience basis.

Let's look at a simple case they handle reasonably well.

(7) Lincoln was taller than Gary Coleman is.

This CT-sentence is true because the height of Lincoln is greater than the height of Gary Coleman. So all they need is the height of Lincoln and Coleman. But both are in their supervenience basis. For Bourne they're given by propositions in the relevant times, for Bigelow by facts true at present.

I don't care how they get their respective supervenience basis. My concern is the richness of such a basis. I'll argue, following Sider [12], that it won't be rich enough to deal with cross-temporal spatial relations.

6.1 Sider 1999)

Sider [11] adopts a version of the softening approach. He doesn't paraphrase CT-sentences but rather claims they are, strictly speaking, false. Since doing this conflicts with our ordinary common sense and since we often have good evidence for CT-sentences he tries to mitigate this offense. He offers underlying truths in the neighborhood of the CT-sentences to accomplish this goal. He claims that the CT-sentences are *quasi-true* and that this sufficiently discharges the presentist's obligation to explain our intuitions.[8] Sider gives the following technical definition to *quasi-truth*:

(QT) S is *quasi-true* iff there is a true proposition p such that, were X true, p would have been true and would have entailed the truth of S.

Where X ranges over theses of ontology like presentism, eternalism, or platonism etc. In the present debate eternalism is substituted for X. The set of underlying truths is a *quasi-supervenience base* for the *quasi-truths*. To illustrate consider Sider's example,

(8) Abraham Lincoln was tall.

For Sider's presentist (8) is strictly speaking false but *quasi-true*. The presentist has at his disposal a whole array of true tensed propositions concerning Abraham Lincoln. For example, WAS(Lincoln is tall), WAS(Lincoln is honest), WAS(Lincoln signing the Emancipation Proclamation) etc. More than this they have an array of tensed facts about Lincoln that just mention the subatomic goings on. These underlying truths are the *quasi-supervenience base* for the *quasi-truths* about Lincoln. Were eternalism true these underlying truths would also be true and would entail the truth of (8). Thus (8) is *quasi-true*.

We need to make a quick distinction before we can see what the central problem with this view is. The distinction is between internal and external relations. Internal relations hold solely in virtue of intrinsic properties had at times. Michael Jordan being taller than Spud Webb is an internal relation. External relations require something more. Change is an external relation. The change of an iron that goes from hot to cold doesn't depend solely on the intrinsic features of the iron at times. If it did it would be indistinguishable from a change in an iron that goes from cold to hot. The intrinsic properties matter, but so does the temporal order.

[8] For a critique of Sider's notion of quasi-truth see Crisp [6].

Can a *quasi-supervenience basis* be provided for both internal and external relations? Let T be the totality of all sentences of the form 'It WAS/Will BE the case n units of time ago/hence that: ϕ'. T determines a series of "snapshots" of the universe at successive moments, complete with the order and temporal relations between each snapshot. It tells us about all the intrinsic properties that any object of history has at any given time. Since internal relations only require such facts all sentences about cross-time *internal* relations will have a *quasi-supervenience basis* making them *quasi-true*. But what about cross-temporal *external* relations? And in particular, what about cross-temporal spatial relations? Sider [12] argues not. In the remaining paper I'll explicate and then defend his argument.

7 Sider's (2004) Argument from Cross-Time Spatial Relations

Consider an object, O, moving through space during some interval. Here are three possible states of motion for O:

a) O moves along a continuous unaccelerated path

b) O moves along a continuous accelerated path

c) O moves along a discontinuous path.

The problem for the presentist, Sider argues, is that it's not clear how they can distinguish between these three possibilities.

(1) T determines a series of "snapshots" of the universe at successive moments, complete with the order and temporal relations between each snapshot.

(2) However, T does not determine how these snapshots line up spatially.

(3) But then any one of the three possibilities (a, b or c) is compatible with T.

(4) Thus the theory requires a necessary "bridge principle" that would give us a rule for lining up the snapshots.

(5) No such bridge principle exists.

(6) So presentism cannot distinguish between a, b or c.

T tells us a great deal about the world at any given time. It tells us all the relative spatial relations between all objects in the world at a given time. It tells us the temporal ordering of each snapshot. But what T doesn't tell us is how these snapshots line up spatially. To put it another way, a, b, and c represent three distinct cross-temporal *external* relations that O may enter into, but presentism doesn't tell us which. In terms of *quasi*-truth: even if eternalism were true, the *quasi*-supervenience basis wouldn't tell us which of many possible block universes is correct.

So the problem for the presentist, if he wants to distinguish between a, b and c, amounts to finding a necessarily true bridge principle that would tell us how to line up the snapshots spatially. Sider writes, "The only course open to the presentist would be to provide some sort of necessarily true 'bridge principles' that say: if the series of snapshots takes a certain form, then the snapshots 'automatically' line up in such and such a way" [12]. Sider thinks no such bridge principle exists for the presentist to make use of.

To support this claim Sider rejects three plausible bridge principles, I'll call the 'Absolute Space principle', the 'Maximizing Continuity principle' and the 'Instantaneous Velocity principle (IVP)'. We're primarily interested in IVP but let's briefly look at the others to help us get a feel for the problem and what a solution might look like.

7.1 The Absolute Space Principle

Newton believed that space was a substance that objects moved about in. On his view each point of space endures the passage of time and is always spatially related to every other point of space in the same way. Due to this the notion of the sameness-of-position had a definite meaning for Newton: an object is at the same position if it occupies the same (enduring) point of space over time. If this were true (i) would be easily satisfied because we could tell exactly *where* an object was at any given time (and what properties it had at that time).

If space is absolute we have a simple bridge principle: line up all the enduring points of space. Every snapshot has information about the position of the enduring points of space and by lining them up we get an automatic (and correct) alignment of the spatial relations of objects across time. Roughly, the problem with this principle is that there is no good reason to think space is absolute. In neither Minkowski nor Neo-Newtonian geometry is the notion of sameness-of-position meaningful (though the notion of inertial motion is).

7.2 The Maximizing Continuity Principle

If we were to maximize the continuity between every snapshot and the unaccelerated motion of each snapshot, then we should be able to distinguish between possibilities a, b and c. Since each snapshot does tell us the spatial relations of each object at that time, we can evaluate how well a given alignment of the snapshots maximizes continuity and unaccelerated motion by measuring the distance between objects at one time and comparing them to the distance between those same objects at different times. Done for all objects this would vastly constrain the possible alignments and would distinguish the three possibilities.

But this only works if the world we are considering is sufficiently complex. If the world contains only one object, there would be no way to distinguish the three possible state of motion the object could be in. Thus the bridge principle, "Maximize continuity and unaccelerated motion," fails because it's not necessary.

7.3 The Instantaneous Velocity Principle

If each snapshot told us not only the position of a given object but also its instantaneous velocity (often 'i.v.'), then the presentist could use the bridge principle: align the snapshots so that, at each instant, the value of the derivative of the position function of each object (at that instant) equals the objects i.v. (at that instant). This may at first sound question begging since were not sure what the position function of any object is. However, once an alignment is given there will be a position function for each object. If we consider just the alignments that give continuous position functions for each object (this can be done as shown above) then we can just look at those alignments and find the only one such that the value of the derivative of the position function, at instant t, equals the value of the instantaneous velocity of each object, at instant t for all times t.

This one might work. Sider rejects this bridge principle because he doesn't think the instantaneous velocity of an object is an intrinsic property of an object at that instant. I think Sider's response is correct but rushed and I'm going to give a much fuller defense in the next two sections.

Intrinsicality is a tough topic and there's little agreement as to the correct analysis. There is more consent when it comes to instantaneous velocity though it's by no means universal. Most people accept the calculus definition of instantaneous velocity (see appendix 1) though there is wide disagreement about whether or not it's an intrinsic or extrinsic property. To make matters slightly worse there are some, notably, Tooley [14] that think the calculus definition fails entirely and he gives an account on which instantaneous velocity is certainly intrinsic. Since the only way the IVP will work is if instantaneous velocity is intrinsic I'll need to consider any account of instantaneous velocity on which it turns out to be intrinsic. Since this would require too much space I'll argue in the following manner. First I'll assume the calculus definition of instantaneous velocity and then I'll lay out the case against it being intrinsic. Second, I'll give up the assumption of the calculus definition and assume that instantaneous velocity is an intrinsic property. I'll then try to show that on any such account Sider's argument will still go through.

8 The Case Against IVP

In this section I'll argue that the calculus definition of instantaneous velocity is not intrinsic. There are two parts to this argument. First I'll argue that on none of the most plausible accounts of intrinsicality does the calculus instantaneous velocity turn out to be intrinsic. I'll take this as good evidence but as the claim is too important I won't stop there. So second, I'll arbitrate a debate between Sheldon Smith and Frank Arntzenius regarding this very question and side with Arntzenius by concluding that the calculus velocity is *not* intrinsic.[9]

8.1 Accounts of Intrinsicality

Defining 'intrinsic' would bog us down. Here I'll show that on three accounts of intrinsicality, instantaneous velocity comes out extrinsic. Perhaps every account I consider is wrong. But I assume they agree on the clear cut cases. And the fact that instantaneous velocity isn't intrinsic on any of them suggests that it's not even a boarderline case.

8.1.1 Relational/Non-realtional

One way of making the intrinsic/extrinsic distinction is to take extrinsic properties to by be relational properties and take intrinsic properties to be non-relational properties. A relational property is a property that, when instantiated, entails that it's bearer stand in some relation to another entity. If instantaneous velocity is a relational property, an object with one must be related to some other relata. But the instantaneous velocity of an object is always relative to a frame of reference. Standing on a boat I throw a rock into the sea. At time t_0 it reaches a velocity of 100 mph *relative to me*. Relative to a person standing on the shore the rock reaches a velocity of 120 mph, since the boat is moving at 20 mph relative to her. So the calculus velocity is a relational property where the relation holds between the object "in motion" and some other object fixing the frame of reference. Thus on this account the calculus velocity turns out relational and hence an extrinsic property.

[9]Actually my conclusion is a little more subtle but that's the important part for our purposes.

8.1.2 Kim's (1982) account

Here's Kim's [9] account:[10]

D1 (Chisholm) G is *rooted outside times at which it is had* = df

> Necessarily for any object x and any time t, x has the property G at t only if x exists at some time before or after t.

D2 (Kim) G is *rooted outside the objects that have it* =df

> Necessarily any object x has G only if some contingent object wholly distinct from x exists.

D3 (Kim) G is *internal* = df

> G is neither rooted outside times at which it is had nor outside the objects that have it.

Again the calculus definition turns out not to be an intrinsic (internal) property of objects. Suppose we are endurantists and suppose O has an instantaneous velocity of 100 mph at time t. In order for O to satisfy the calculus definition (i.e. to actually have a calculus velocity) O must have endured throughout some open interval of time. The calculus i.v. is rooted outside t.[11] But it is not rooted outside O. If we are purdurantists and O is a temporal slice that has an i.v. of 100 mph at time t we get a similar result. The calculus i.v. is both rooted outside t and O, since it implies the existence of temporal parts other than just O and times other than t. So again the calculus velocity is not intrinsic on this account.

8.1.3 Duplication

Lewis [10] writes, "The intrinsic properties of something depend only on that thing; whereas the extrinsic properties of something may depend, wholly or partly, on something else. If something has an intrinsic property, then so does any perfect duplicate of that thing; whereas duplicates situated in different surroundings will differ in their extrinsic properties" pp. 197. His tight knit circle of

[10]See Lewis [10] for a rejection of Kim's account.

[11]I think that even if our endurantist is a presentist the calculus i.v. is rooted outside t. The spirit of the definition would remain if we read 'x exists at some time before' as 'x existed at some time before'.

interdefinables. An exact duplicate of a is something that shares all the same intrinsic properties of a. a' is an exact duplicate of a iff for every intrinsic property Fx such that Fa we also have Fa'.

Consider an object O *at just* t and suppose it has an instantaneous velocity of v at time t. If instantaneous velocity is really an *intrinsic* property of O at just t then any exact physical duplicate O' of O will share this intrinsic property. But we can imbed O' in a history that would require a different value for the velocity. Imagine O' lonely in a universe with no history, it's the only object in a single object world. Since it's an exact physical duplicate of O, and since instantaneous velocity is supposed to be an intrinsic property, O' has a velocity of v despite the fact that there's no open interval around t at which the position development of O' is defined. But this contradicts the calculus definition. So again the calculus i.v. turns out to be not intrinsic since duplicates situated in different surroundings differ with respect to it.

These considerations suggest that the calculus velocity is not an intrinsic property. But let us not leave the matter unsettled. We're now going to look at a debate primarily between Frank Arntzenius and Sheldon Smith that will help assuage any doubt that may remain. If the reader is already convinced by the above considerations or wishes to avoid a technical discussion he or she can reasonably skip to the next section and not miss any of the main argument.

8.2 Arbitrating the Debate

David Albert [1] and Frank Arntzenius [2] don't think velocity is intrinsic. We'll start with their arguments and then consider Sheldon Smith's [13] response and further argumentation.

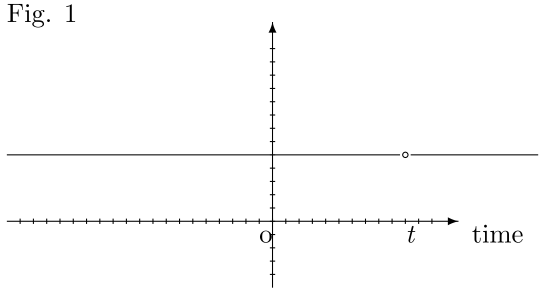
8.2.1 Albert and Arntzenius' Arguments

Albert's Implication Case

Let object O have a constant position function of 5 defined everywhere in some interval around t and let its position at t be undetermined (see figure 1). By applying only logic and definition to this information, we can come to know conditional information about O's instantaneous velocity at

t. Namely, if O's position is 5 at t, then its instantaneous velocity at t is zero and if O's position is anywhere other than 5 at t, then it's position function would be discontinuous and it would have an undefined velocity at t. So given just our knowledge about the position of O at non-t points we learn that O's instantaneous velocity at t is either zero or undefined. But if position and velocity are really part of the instantaneous state of objects at instants, then the position and velocity of O at t will not be logically and conceptually independent from its positions and velocities during the interval around t. So velocity is not part of a genuine instantaneous state (cf. footnote 2).

Fig. 1

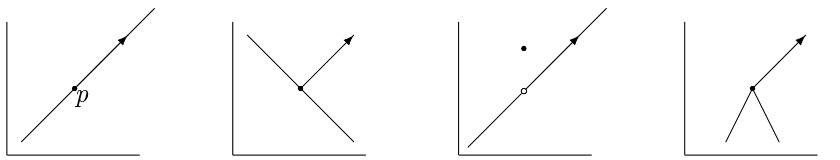


Albert concludes that velocity is not truly part of genuine instantaneous states of objects. Rather, he claims, they are properties of the "immediate vicinity of t" which is a larger temporal span than just t.

Arntzenius' Constraint Case

Let O's position at t be represented by a point p on a piece of paper and its velocity by a vector emanating from p. If velocity is really an intrinsic property of objects at instants, then this would impose (by the mere application of logic and definition to information about the velocity at t) a serious constraint on the possible position developments of O. Any position development of O around t, represented here as a line, would have to be tangential to the vector at t (as in figure 2-A) which rules out many possible histories. For example O could not have a position function non-tangent to the vector, nor could it have a gap at t, nor could it peak at t (see figure 2-B, C, and D respectively). But while we may expect the laws of nature to impose such a constraint on the physically possible histories of O, we do not expect mere application of logic and definition to information about the velocity at t to do so.

Fig. 2



A. Possible B. Impossible C. Impossible D. Impossible

Arntzenius [2] concludes that the calculus velocity isn't part of instantaneous states. Rather, he claims, the calculus velocity at t is a property of the position development of the object during any finite neighborhood of t on which the position development occurs.

What can be said about these two cases? Sheldon Smith thinks both arguments are flawed and we now move to his response.

8.2.2 Smith's (2003) Objection

Smith's [13] argument is directed at Albert and Arntzenius. It has two parts: First, he identifies a shared intuition that he believes drives both arguments. Second, he argues that this intuition cannot be motivated. Since this intuition can't be motivated Smith concludes that we need not accept their argument.

Independence Intuition

Smith notes that both Albert and Arntzenius feel that instantaneous states must have a certain independence.

Independence Intuition: The instantaneous state of an object at t should be logically and conceptually independent from its state at instants other than t (before the laws of motion are given).

Albert *seems to* endorse the independence intuition.[12] Arntzenius, although not as explicit as Albert, also *seems to* accept the independence intuition since it's a hidden premise getting him from the fact that the velocity at t constrains the position development within $(t-\delta, t+\delta)$ to the conclusion that velocity is not part of the instantaneous state at time t. I also employed this intuition in my argument that velocity is not intrinsic on Lewis' account and Tooley worlds presuppose it as well. So all three aspects of the case against IVP rely on this intuition–hence my concern. Unfortunately, neither Albert nor Arntzenius offers any support for the independence intuition. Smith claims not to have the intuition and claims that most of us who have become accustom to viewing both position and instantaneous velocity as comprising the instantaneous state of objects will not have this intuition either.

Smith asks why velocities' lack of independence suggests to Albert and Arntzenius that it's not instantaneous. Can the independence intuition be motivated? Smith attempts to motivate it on their behalf:

> If non-t points carry an implication for the velocity at t like the one described above, [Alberts implication case] then the intuition is that the velocity is as much a property of the non-t points that carry that implication as it is of t... Again, it would seem that the claim would be that if velocity at t constrains non-t behavior [Arntzenius constraint case] in some fashion, then the velocity has to be considered, in part, a property of the

[12]Albert writes:

> Let's start by thinking through what it means to give a complete description of the physical situation of the world at an instant.
>
> There would seem to be two things you want from a description like that:
>
> a. that it [a complete description of the physical situation of the world at an instant] be genuinely *instantaneous* (which is to say that descriptions of the world at different times have the appropriate sort of logical or conceptual or metaphysical *independence* of one another, that a perfectly explicit and intelligible *sense* can be attached to *any temporal sequence whatever* of the sorts of descriptions we have in mind here—whether the sequence happens to be in accord with the dynamical laws or not, that any such sequence whatever is *readable—against the background or within the context or relative to the framework* of the best or last or canonical metaphysical interpretation of whatever complete theory of the world is under discussion—as a *story of the physical world*); and
> b. that it be *complete* (which is to say, that *all* the physical facts about the world can be read off from the full temporal set of its descriptions).
>
> Good. Lets call whatever satisfies (a) and (b) an instantaneous physical state of the world.

non-t stuff that is constrained by the t velocity.

Smith's argues that no matter how one cashes out the expressions "non-t points that carry the implication" and "constrained non-t points" they will either have no denotation or denote a set of points that we shouldn't attribute the velocity to. This leaves the independence intuition lacking motivation.

Lack of Motivation

What is the denotation of "the non-t stuff" that is so constrained or carries implications? Smith considers two suggestions roughly given by Albert (A1) and Arntzenius (A2).

A1 The non-t stuff that carries implications for, or is constrained by the velocity at t is the immediate vicinity of t. The velocity should be considered as much a property of the immediate vicinity of t as it is of t.

A2 The velocity at t should be considered a property of any set of points that carry implications for the velocity at t or are constrained by the velocity at t. So while there is no unique non-t stuff that carries implications for, or is constrained by the velocity at t, any set of the form $(t-\delta, t+\delta)$ with t deleted (hence forth $(t-\delta, t+\delta)-t$) is non-t stuff that is constrained by (and carries implications for) the velocity at t. The velocity should be considered as much a property of these sets as it is of t.

Smith quickly dispatches A1. For any point thought to be close or immediate to t we can always find points closer and still more immediate to t in the standard continuum. So 'the immediate vicinity of t' lacks denotation in the standard continuum. But suggestion A2 is more difficult to refute and can be seen as two claims:

Constraint Claim: The velocity at t should be considered a property of the position development on $(t-\delta, t+\delta)-t$ *because* the position development of the object is constrained within $(t-\delta, t+\delta)-t$.

and

Implication Claim: The velocity at t should be considered a property of the position development on $(t-\delta, t+\delta) - t$ *because* that collection has implications for the velocity at t.

Smith's objection to A2, and the above claims, is, roughly, that a set of points having implications or being constrained (in Arntzenius' sense) is just not enough to merit attributing the velocity at t to them. A set of points can't just be constrained *within* by the velocity at t it must be constrained *throughout* by the velocity at t if those points are to merit attributing the velocity to them. Similarly, a set of points can't just have implications for the velocity at t, the velocity must *depend* on the set of points if they are to merit attributing the velocity to them. Let's turn to his argument. I'm going to first shamelessly quote his argument then attempt to make it precise.

The Argument

Concerning the implication claim Smith writes,

> We should *reject* the claim that the velocity is a property, in part, of *all* of $(t-\delta, t+\delta)$ with t deleted just because this set has implications for the velocity at t. Most of those points are not responsible for the implication. But, one might think that it must be the set of points we left out of $(t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$, the ones that are in $(t-\epsilon, t+\epsilon)$ with t deleted, that the velocity is a property of, for when they are there we *do* get the implication. Cannot we claim that the implication arises from *that* set of points and include them in the class of points upon which the velocity at t depends, and thus claim that a velocity value at t is a description *in part* of what is going on *throughout* that interval? No. All of the claims above are true no *matter what* ϵ *is*, as long as it is not zero. We cannot say what set of non-t points is responsible for that implication for the velocity at t, because we can always move in toward t in a way that it captures any point, and $(t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$ where $\epsilon > 0$ is less that $\delta > 0$ *never* carries the

> implication. While it is true that $(t-\epsilon, t+\epsilon)$ with t deleted *always* has the implication, this fact does not help us to answer what set of points besides t the velocity depends upon because we can run the same argument again to suggest that most of the points in that interval are too far away to matter.[13]

And concerning the constraint claim Smith writes,

> Rather than non-t behavior constraining velocity, Arntzenius points out that t velocity constrains non-t behavior in that once I stipulate the velocity at a point t, the position behavior within $(t-\delta, t+\delta)$ with t deleted has to be such that the velocity comes out to be the stipulated value, but the points in that set are all non-t points. A genuinely instantaneous property, so he claims, should not imply such a constraint on a non-t set… Thus, one would like to know *why* the fact that the velocity at t has such implications for a set of non-t points suggests that it is not instantaneous, for many of us do not feel that it does suggest that. Again, it would seem that the claim would be that if velocity at t constrains non-t behavior in some fashion, then the velocity has to be considered, in part, a property of the non-t stuff that is constrained by the t velocity. But, what exactly is the non-t stuff that is so constrained? It seems that one will either claim falsely that it is all of $(t-\delta, t+\delta)$ or claim that it is something like the "immediate vicinity of t" which has no denotation.[14]

Smith accepts both that the velocity at t constrains the position development $(t-\delta, t+\delta) - t$ and that the position development on $(t-\delta, t+\delta) - t$ has implications for the velocity at t. What he denies is that these are strong enough conditions to merit property attribution. He argues that we shouldn't attribute the velocity at t to *all* of the points in $(t-\delta, t+\delta) - t$; the points on the outskirts of $(t-\delta, t+\delta) - t$ are not constrained nor are they "responsible" for the implications it carries.

[13] Smith [13] pp. 276
[14] Ibid. pp. 277

Clarifying Smith's Argument

Here we'll clarify Smith's arguments against A2. They seem initially clear but actually the use of terms is not. What's meant by 'responsibility', 'dependence', or 'relevance'. Smith argues that the velocity depends on just the instant t. But is his account of dependence for *instants* the same as his account of dependence for *collections of instants*. Or is he working with different notions here? Similarly for 'responsibility' and 'relevance'.

If we look at Smith's first argument we begin to wonder why he doesn't follow it out to it's mathematical conclusion. Why doesn't he reason as follows: Let i be an instant in $(t-\delta, t+\delta) - t$. Then pick ϵ so that $i \in (t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$. Then i is not responsible, or is irrelevant to the implications $(t-\delta, t+\delta) - t$ carries. But since i is arbitrary we have that every single instant in $(t-\delta, t+\delta) - t$ is irrelevant. Surely Smith doesn't neglect this line of reasoning because he only wishes to establish the weaker conclusion that *most* of the points in $(t-\delta, t+\delta) - t$ are irrelevant. He plausibly neglects this line because it seems somewhat paradoxical. We are given a collection of instants and we are given that this collection has implications for the velocity at t. If we reason as above we conclude that not a single instant in that collection is responsible for the implications the collection has. We might be tempted from here to conclude that the collection of instants (taken as a whole) is not responsible for the implications that it carries. This would be too quick. A collection can have properties that no individual member of the collection has. So whether or not a collection of instants is responsible for the implications that it carries wouldn't necessarily have anything to do with whether or not its individual members are *individually* responsible for the implications that the collection carries. Smith might hold the position that every single instant is *individually* irrelevant but that the instants are *collectively* relevant or responsible. We need an account. Every instant is a member of a collection that has implications (so one might think that every instant is relevant) but it's also a member of some collection that doesn't have implications (so one might think that every instant is irrelevant). At any rate I think the arguments are interesting and in need of formalizing.

Definitions

To make Smith's argument precise we need some definitions[15]. Throughout we will be concerned with the spatial trajectory of an object, in one dimension, as a function of time. The object is assumed to have positions in the temporal interval, (a, b), and we are interested in the velocity at t_0, where $a < t_0 < b$. f, g vary over functions of t with domain (a, b). X, Y vary over subsets, proper or improper, of (a, b). Since we are interested in finding non-t points that are constrained within or have implications for the velocity at t we will assume that X does not contain t. $f|_X$ is the function with domain X, and $f|_X \subset f$ means that f is a function with domain (a, b) that extends $f|_X$. v ranges over all the possible values of the velocity at t_0 including the "undefined" value unless otherwise noted.

If $(\exists f|_X)(\forall f \supset f|_X)(f'(t_0) \neq v)$ for defined v, then we say that the velocity at t_0 constrains the position development *within* X and write $CW(v, f, X)$.[16]

If $(\exists f|_{(a,b)-X})(f|_{(a,b)-X})'(t_0) = v$ and $(\exists f|_X)(\exists g|_X)((f|_{(a,b)-X} \cup f|_X)'(t_0) \neq (f|_{(a,b)-X} \cup g|_X)'(t_0))$ then we say the position development on X has implications for the values of the velocity at t_0 and write $I(f|_X, v)$.

For a proof of $CW(v, f|_X) \equiv I(f|_X, v)$ see appendix 2.

$WR(X) =^{\mathrm{df}}$ If $I(X)$ or $CW(X)$, then X is *weakly responsible* for the value of the velocity at t. Or equivalently (for those who read the proof) if X is nice, then X is weakly responsible for the value of the velocity at t.

Smith accepts that $I((t - \delta, t + \delta) - t)$ and $CW((t - \delta, t + \delta) - t)$ but denies that this is enough to attribute the velocity to *all* of $(t - \delta, t + \delta) - t$ because it fails as follows:

Crucial Failing: There exists proper subsets Y, Y' of X such that $WR(Y)$ and $\sim WR(Y')$

[15] In what follows I'm greatly indebted to Paul Teller for suggesting helpful notation, criticizing and refining the definitions, and most importantly for recognizing the logical relations between dependence, constraint throughout, and the condition of failure Smith endorses.

[16] We require that v be defined since it makes little sense to think that an undefined velocity constrains anything.

Smith advocates a stronger condition than just A2. In the implication case it's not enough for a set to have implications for the velocity at t the velocity must depend on that set. And in the constraint case it's not enough for a set to be constrained within it must be constrained *throughout*.

$D(X) =^{\text{df}} WR(X) \wedge (\forall Y \subset X) \sim WR(Y)$, that is, X is weakly responsible[17], but none of its proper subsets are weakly responsible—here we say the velocity v *depends on the position of the object in set* X.

$CT(X) =^{\text{df}} WR(X) \wedge (\forall Y \subseteq X) WR(Y)$, that is X and all its proper subsets are weakly responsible [18] We say X *is constrained throughout* by the velocity.

Dependence and constraint throughout are the two different ways of avoiding that which Smith insists is a potential failing of $WR(X)$. Smith agrees that $WR(X)$ is a necessary condition for the course of values in X being somehow responsible for constraining the value of v. But he maintains that as a second necessary condition we must also have:

I. $\sim (\exists Y_1 \subset X)(\exists Y_2 \subset X)[WR(Y_1) \wedge \sim WR(Y_2)]$

This is just a formal denial of the crucial failing and it's easily equivalent to:

II. $(\forall Y_1 \subset X)(\forall Y_2 \subset X)[\sim WR(Y_1) \vee WR(Y_2)]$

which is, in turn, equivalent to

III. $(\forall Y_1 \subset X) \sim WR(Y_1) \vee (\forall Y_2 \subset X) WR(Y_2)$,

which when conjoined with the other necessary condition, $WR(X)$ gives

IV. $[WR(X) \wedge (\forall Y_1 \subset X) \sim WR(Y_1)] \vee [WR(X) \wedge (\forall Y_2 \subset X) WR(Y_2)]$

which is just,

[17] Think "has implications for".
[18] Think "constrained within".

V. $CT(X) \vee D(X)$

which we will call strongly responsible written $SR(X)$. Smith denies that A2 is a strong enough condition. There will be sets that satisfy it but that we should still not attribute the velocity to. We need something stronger. We need a set that is not only weakly responsible but also strongly responsible for the implications it carries. We might call this Smith's velocity attribution thesis (for short SVAT) and formulate it thus:

SVAT: The velocity at t should only be considered a property, in part, of the object on X, if $SR(X)$.

Formalized Argument

Since strong responsibility is a disjunctive condition Smith must argue both that the velocity at t does not depend on $(t-\delta, t+\delta)$, and that $(t-\delta, t+\delta)$ is not constrained throughout. This is what I take it Smith is doing in the quotes in 6.3. With these resources we formalize Smith's argument.

1. $(t-\delta, t+\delta) - t$ has implications for the velocity at t.
2. $(t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$ does not have implications for the velocity at t.
3. The velocity depends on X only if every subset of it has implications.
4. But $(t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$ is a subset of $(t-\delta, t+\delta) - t$ yet has no implications for the velocity at t.
5. So the velocity does not depend on $(t-\delta, t+\delta) - t$.
6. The position development is constrained within $(t-\delta, t+\delta) - t$.
7. The position development is not constrained within $(t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$.

8. The position development is constrained throughout Y only if every subset of it is constrained within Y.

9. But the position development on $(t-\delta, t-\epsilon) \cup (t+\epsilon, t+\delta)$ is a subset of $(t-\delta, t+\delta) - t$ yet is not constrained within.

10. So the position development is not constrained throughout $(t-\delta, t+\delta) - t$.

11. So by conclusion five and nine we have that $(t-\delta, t+\delta) - t$ is not strongly responsible for the implications it has for the velocity at t.

12. By SVAT we should not attribute the velocity to *all* of $(t-\delta, t+\delta-t)$.

Responding to Smith

First, I think both parties are making a mistake in thinking that intrinsic and extrinsic are contradictories rather than merely contraries. No property or relation can be both intrinsic and extrinsic (i.e. they are contraries). But not all properties are either intrinsic or extrinsic (i.e. they are not contradictories), there is a third and forth option. To see this let's introduce some terminology. Call a property or relation *accompanied* iff it coexists with some *determinable* wholly distinct contingent object. Both parties assume that all extrinsic properties imply accompaniment. But consider loneliness (the ontological status not the mental state). This property is extrinsic. You can not be alone merely in virtue of the way you are i.e. it's not intrinsic.[19] Yet loneliness does not imply accompaniment. Call extrinsic properties that don't entail accompaniment *negatively extrinsic*. Call extrinsic properties that do imply accompaniment *positively extrinsic*. Both parties assume that if velocity is extrinsic it must be positively extrinsic. Smith's argument shows that the calculus velocity is not positively extrinsic. Having a velocity does not entail the existence of a determinable unique open interval of time responsible or constrained by the i.v. then. But showing that the velocity is not positively extrinsic doesn't thereby show that it's intrinsic—it's possible

[19]See Lewis [10].

that it's negatively extrinsic (in fact it is). Albert and Arntzenius' make the same mistake. Their arguments show that the instantaneous velocity is not intrinsic. But then they try to pin it on some larger interval of time suggesting they think 'extrinsic' means positively extrinsic. Velocity is local. It requires an open interval of time around t but no particular one. It's extrinsic, but only negatively.[20]

Second, Smith is playing down the popularity of the independence intuition. Tooley [14] held that the intuition could be used as an argument against the calculus definition. Given that velocity is an intrinsic property, and given the independence intuition, the calculus definition can't be right since it would violate independence. Whether right or wrong, the point is that even those who argue that the velocity <u>is</u> intrinsic seem to hold the intuition.

Lastly and related, Smith fails to recognize a plausible motivation for the independence intuition. To my mind it's motivated by the notion of intrinsicality itself. The instantaneous state of an object at t is characterized by all the intrinsic properties had by that object at t.[21] And it's intrinsicality that demands the independence. Just recall what people want out of a notion of intrinsicality: Lewis says, "The intrinsic properties of something depend only on that thing; whereas the extrinsic properties of something may depend, wholly or partly, on something else." Kim's account involved not being rooted outside times at which it is had i.e. "Necessarily for any object x and any time t, x has the property G at t only if x exists at some time before or after t." It's clear that the concept of intrinsicality demands this type of independence and all our arguments, Albert and Arntzenius, Tooley worlds (below), and myself, assume velocity is intrinsic.

We now move on to the second part of the argument.

[20]See Butterfield [4] for a similar response.

[21]The instantaneous state regards just the physical state. If not the intuition would be clearly false. Truth value links must be preserved on any account of time. If E occurs at t then at t', with $t < t'$, it must be true that E occurred at t. That's not logical or conceptual independence.

9 Tooley worlds

In the previous section we argued that the calculus definition of instantaneous velocity is not intrinsic. We took up the calculus definition because it is the most widely accepted account. In this quick section I'll drop that assumption and assume rather that instantaneous velocity is an intrinsic property. I'll argue that even if instantaneous velocity is intrinsic, Sider's argument still holds.

Sider points to small worlds as problems for the maximizing continuity principle. I'm going to employ the same strategy here. Tooley [15] imagines a class of worlds that are problematic for the instantaneous velocity principle. In these worlds the position of an object at any given time is completely random. As it happens some lucky objects end up with continuous and differentiable trajectories. Furthermore instantaneous velocity is an intrinsic feature of objects at times in these worlds. Call such worlds *Tooley worlds.* Now we have a problem for IVP. The problem is not that it can't distinguish a, b, and c. But that it would get many Tooley worlds wrong. If we fix the discontinuously moving objects so their position developments match up with their instantaneous velocities we will have completely screwed up the world. In such a world the velocity of an object is independent of it's position development. So matching the two up will get the wrong result.[22]

Thus even if instantaneous velocity is intrinsic, a version of Sider's argument still looms. This concludes our discussion of the IVP principle. We have only one final matter to attend to.

[22]This type of example will generalize for acceleration. Suppose our Tooley worlds also had acceleration as an intrinsic feature of objects. If they are really intrinsic then they would be independent of velocity and position. So again a bridge principle like, make sure the acceleration match's up with the changes in velocity of objects, would fail to get it right as well.

10 Zimmerman's Response

In this section we're going to consider Dean Zimmerman's [17] response to Sider's argument. He's one of the few presentists whom directly addresses the issue. His response has three parts. First, he assumes substantivalism with respect to GR. Second, he argues for an A-theoretically privileged foliation of the GR space-time manifold. Lastly, he presents three different views—the ghostly box, the empty box, and the surrogate strategy—regarding the ontological status of currently unoccupied points within the manifold. Essentially these three strategies attempt to incorporate the backward looking portions of the Minkowski space-time manifold, complete with metrical structure, into the presentist picture. This is supposed to get him all the facts required to fully determine the trajectories of objects and hence how to line up the "snapshots" thus saving presentism.

10.1 An A-theoretically Privileged Foliation

In SR and GR it's often taken that there is no privileged now. Time is relative to ones inertial reference frame. But Zimmerman argues that there is in fact a privileged now. Or better that there is a slicing up of the GR manifold (a foliation) into sets of points of the manifold all of which were happening all at once. Here's his argument that there is one and what it's kinda like (1)-(8) and how to arrive at it (9)-(D2). I'll lay out the argument and formulation then pick at them.

(1) There is an objective, important difference between events that are really happening to me, and ones that merely *did* or *will* happen to me; and the events that are really happening to me are confined to a tiny region, r, on the world-line I will eventually have traced through the manifold.

(2) I am not metaphysically special, unique among all human beings with respect to some important, objective feature of the manifold; neither is the region r, nor is my world-line.

(3) If the only events in the universe that are really happening are the ones happening to *me* at r, then r and I would be very special.

(4) Events are really happening to me, at r, and to many other objects at points on their paths through the manifold. (From 1, 2, & 3).

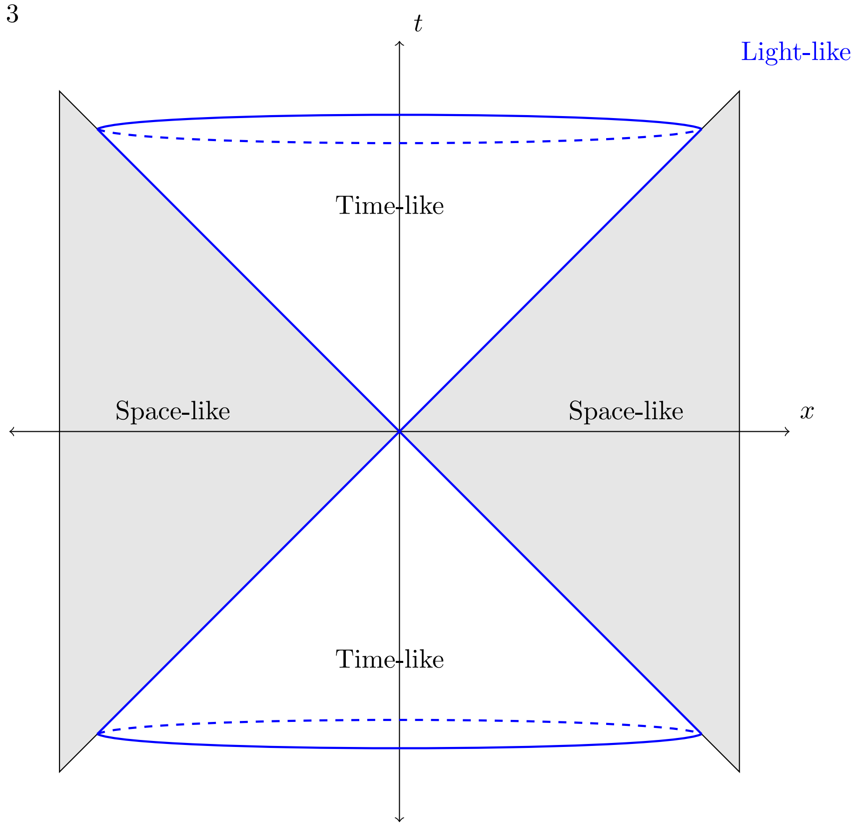
(5) According to SR, the only geometrically distinguished subsets of points that include r, along with many other locations in the manifold, are the following: (a) the points at space-like distance from r,...(b) the points in or on r's forward light-cone; (c) the points in or on r's backward light-cone; (d) the points on the various planes associated, by the Radar method, with continuous paths passing through r; (e) three "hyperboloids of revolution" about r; or (f) some set of points definable in terms of these distinctions. See figure 3.

(6) If the region in which events are happening were restricted to (a), (b), (c), (d), or (e), I or r or my world-line would be very special.

(7) If the region in which events are really happening coincided with a set of points including r that are geometrically distinguished, according to SR, then I or r or my world-line would be very special. (From 5, & 6).

(8) There is a region of the manifold in which events are really happening, it includes r and many other points, and it does not coincide with any region that is geometrically distinguished, according to SR. (From 1, 2, 4, & 7)

Fig. 3



I think this argument is seductive though it's invalid. Although (4) doesn't strictly follow from (1)-(3) we can see a sorites series of regions that eventually crosses over into the clearly *not*-special cases once they start containing "many" events. That is, all we can really conclude is that there is at least one event that's really happening in region $r' \neq r$, but $r \cup r'$ will still be special. The move from (6) to (7) also doesn't follow since option (f) was never ruled out, but this too can be overlooked. We should also not be too worried about the ambiguity in the assumed uniqueness of "the region in which events are really happening". I'm assuming that a region of the manifold is just a set of space-time points and that no connectedness is assumed (as that would be unwarranted). So I find no serious objects to Zimmerman's argument thus far; assuming presentism were true there would be a unique present not geometrically distinguished by SR. So far so good. Zimmerman then tries to further describe the geometric properties of the region in which events are really happening i.e. the present.

(9) For any events e^1 and e^2, e^2 is causally dependent upon e^1 only if, when e^2 was happening, e^1 had already happened.

(10) If a particle, photon, or wave occupies a path in the manifold, its occupancy of a point r on that path is causally dependent upon its having occupied the points on that path that stand in light-like or inertial accessibility relations to r.

(D1) S is the *immediate causal environment* of the current state s of x[23] $=_{\text{df}}$ S is the set of all pairs $\langle y, z\rangle$ such that y is a particle, wave, or other process and z is a current state of y that could, at some point, come to have an effect upon a state of x that is also partly causally dependent upon x's current state s.

(D2) R is the universe of x, relative to the current state s of x $=_{\text{df}}$ R is the smallest region satisfying the following recursive condition: for every pair $\langle y^1, z^1\rangle$ in the immediate causal environment of x's current state s, R includes the location at which z^1 is happening to y^1; for each such $\langle y^1, z^1\rangle$, and for every pair $\langle y^2, z^2\rangle$ in the immediate causal environment of y^1's current state z^1, R includes the location at which z^2 is happening to y^2; and so on.

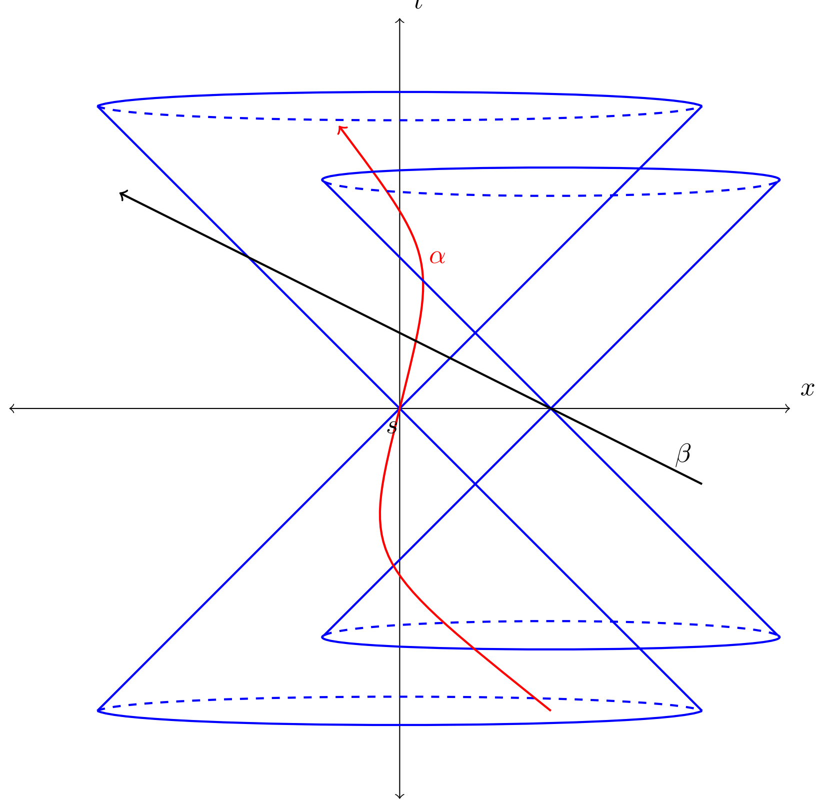
According to Zimmerman the present (the universe of x, relative to current state s of x) is a very thin slice through the space-like region of x's light-cone. The argument is quite simple. First, let r be the location of x relative to the current state s. Then R can't include a point p in or on the forward or backward light-cone of r. Suppose it did. Then there would be an inertial or light-like path from r to p or from p to r. Suppose the former and that a particle β followed that path from r to p. Then by (10) β's occupying p would be causally dependent upon it's having occupied the points on that path that stand in light-like or inertial accessibility relations to p. Specifically β's occupying p would be partially causally dependent upon it's occupying r. But then by (9) when β's occupying p was happening, β's occupying r had already happened. So p can't be in R since any state of an object at p wouldn't be a current state. The other case is handled similarly. So

[23] I'll write $ICE(x, s)$.

(9) and (10) establish that the present cuts across the space-like region. But how do we know it's exceedingly thin? Well suppose both $\langle y, z\rangle$ and $\langle y, z'\rangle$ were both in $ICE(x, s)$. Then both z and z' are supposed to be the current state of the same object y. But either z' is causally dependent upon z or vice versa. In either case when one was happening the other had already happened. This makes R skinny.

Now that we kinda get the picture let me note some problems. First (9) rules out the possibility of backwards causation. I see no reason to do so but I won't make a fuss. Second, (10) has an important counterexample. Consider an object β moving faster than light.

Fig. 4



In figure 4 β moves faster than light and relativity theory doesn't rule out the possibility of faster than light travel. It prohibits particles that are at one time not moving faster than light from accelerating faster than light (as this would require an infinite amount of energy) but it doesn't prohibit particles from always traveling above light speed. But even if physics did rule out the possibility of faster than light travel I don't think one's metaphysics of space-time should do so.

Notice also that the universe of x may not contain the *whole* universe. There could for instance be a particle separated by so much space-time from x and the things in it's ICE and the things in their ICE and so on, that it's causally unrelated to anything in the universe of x. Now intuitively this particle is still part of the universe and, maybe, present. Yet Zimmerman's account will rule it out.

Despite all my objections there's really no use in stalling on Zimmerman's account of a privileged foliation. If presentism is true then there really is one and it doesn't matter what it looks like. Still this doesn't mean the presentist automatically has a solution to Sider. Let's get to the part where Zimmerman offers a solution.

10.2 Two Views

Zimmerman develops three frameworks he thinks can resolve Sider's objection but I'll only consider the two most plausible the surrogate and the empty-box view. But before we get to them we must make a distinction. Presentism is the view that the only *objects* that exist are the ones that presently exist. This is compatible with two distinct views: 1) the only space-time points that exist are the ones that presently exist as well and 2) more than just the present space-time points exist! The first view Zimmerman calls the "one slice presentist". For them the only space-time points that exist are the ones constituting the privileged foliation. The second view can be cashed out in infinitely many ways but Zimmerman discusses only one plausible version, the "growing manifold presentist". The growing manifold presentist accepts the existence of not just present space-time points but also those that were formerly occupied. Thus as time flows the space-time manifold grows.

Zimmerman thinks Sider's objection relies on the assumption that the presentist must be a one slice presentist. A "snapshot" of the one slice presentist's world tells us only the relative distance relations between objects along with any intrinsic properties had by them. But since no other space-time points exist, a whole wealth of information is left untapped.

To remedy this Zimmerman conceives of a growing manifold presentist that takes the existence of past space-time points very seriously. This "empty-box" view makes two important assumptions about space-time points: first, that they are substantival in nature, and second, their metric relations are preserved over time. I'll call this second assumption *metric stability.* A "snapshot" of the empty-box presentist's world tells us not only the relative distances between objects along with their intrinsic properties but also it's exact, enduring, *past and presents* space-time locations. Since the metric relations between these space-time points are preserved the empty-box presentist can read the properties of an object's path directly off the space-time points it occupied. In this way the presentist can distinguish between inertial, accelerated, continuous and discontinuous motion thus resolving Sider's cross-temporal objection.

Though this approach seems to work Zimmerman thinks he's got a "still more excellent way". He writes,

> If one takes for granted the metric structure of Minkowskian space-time or a not-too-bizare manifold satisfying GR's constraints, surrogates for past points can easily be constructed out of the points in the present slice. For each past point, there is a region in the presently existing slice of the manifold that contains all and only the points on the slice that were inertially or light-like accessible from the past point; the region in question is the presently existing slice of the point's forward light-cone. In SR and foliable GR space-times, these regions could be used as descriptive names for each formerly-filled, now non-existent space-time point — each such point has exactly one point-surrogate in the presently existing slice. If the presentist is allowed to help herself to the facts about which collections of points constitute point-surrogates, the current geometry of the present slice will include enough information to recover all the facts about which past space-time points constituted inertial and light-like paths. [17]

Now though this is Zimmerman's preferred solution I don't think it's as excellent as the empty-box view. Consider the following problem the empty-box view can solve. Suppose a particle follows a path β past a large star. The star curves space-time and thus β is accelerated. But the past space-time points in the empty-box don't contain the star, they don't contain anything. So β is inertial rather than accelerated. The empty-box view has a simple fix. Since space-time is substantival the empty-box view has the fact that the large star was at a certain space-time location and since space-time is metrically stable they know, *now*, exactly which point that is. Given this and facts about the intrinsic nature of the star (it's mass and shape and such) the empty-box view can figure out whether β was inertial or accelerated. The substantival and metric stability assumptions generate a fabric of cross-temporal spatial relations between space-time points. All other cross-temporal spatial relations piggy back off them and the intrinsic properties of objects had at times. If you drop either of these assumptions the view fails so we can now see why the surrogate view fails.

The surrogate view almost looks like it can handle this problem. But in fact the surrogate view begs the question. Suppose we ask the proponents of the surrogate view if β is inertial or accelerated. All they need to do is look at the particle at each moment and its distance relation to the star, how massive the star is etc., and piggy back off the cross-temporal relations between the space-time points constituting β. In this way they can distinguish inertial from accelerated paths. But the reason they can solve this problem is because it already grants them cross-temporal relations they don't have access to. Let me explain.

Let t_1 be the present. We take every possible light sphere at t_1 to be a surrogate for some past space-time point. We take the present space-time points as surrogates for themselves. Then we add accessibility relations to these surrogates, namely, light-like, space-like, and time-like accessibility relations. Now we have our space-time points and we have our metric. We can now add all the objects that exist at t_1 to the present surrogates. But now what? Suppose O is at $s_1 = \langle\langle x_1, x_2, x_3\rangle, t_1\rangle$ and we want to figure out its past trajectory. Which surrogate represents its space-time point at say $t_0 < t_1$? That is where was it earlier in relation to where it is now? The surrogate view can't say. Although it has all the cross-temporal relations between space-time points worked out it doesn't have any for objects! So while it can tell us which space-time points do constitute an inertial or accelerated path it still can't tell us where objects are across time and thus can't tell us whether *their* path is accelerated or inertial i.e. because it can't tell us what it's path is in the first place.[24]

So Zimmerman's preferred surrogate view fails. But the empty-box view does solve Sider's cross-temporal spatial relations objection. So we do have a solution. But what is the cost of that solution? Zimmerman argues that the cost is minimal. He writes,

> The space-time manifolds of SR and GR resemble quarks and dark matter more than they resemble horses and wars, with respect to our reasons for believing in them. They are theoretically posited entities that earn their keep by the crucial roles they play in successful scientific theories. Suppose I come to believe in a four-dimensional manifold with a specified structure because interactions among objects alone are not enough to to explain why observable things behave as they do. Should this bother me, *as a presentists*? Not much, I think. A space-time manifold is a strange beast — at least, when it is construed substantivally, as a sort of four-dimensional, invisible, permeable cosmic jell-o. The manifold of Galilean or Minkowskian space-time, and the manifolds allowed by GR, and not the kinds of thing one should have posited, had they not seemed necessary to play a role in some well-confirmed scientific theory. An A-theorist, like everyone else, should look to science for information about the structure of such things, including their metrical properties and the number of dimensions they have. My convictions about the unreality of past and future objects and events, on the other hand, are convictions about horses and wars and people; they have little to do with questions about what sorts of theoretical entities should be allowed to figure in scientific theories.

What is the argument? The conclusion is that the cost of accepting the ongoing existence of formerly occupied parts of the manifold (the empty-box) is not very high. But the mentioned

[24] I'd like to thank Josh for pointing this problem out to me.

considerations fail to bare on that conclusion? For instance we can grant that space-time manifolds resemble quarks more than horses with respect to why we believe in them. We believe in horses because we can sense them. But we believe in space-time manifolds because they "earn their keep" in scientific theories. We can grant that manifolds are strange, theoretical entities. But does this mean that a metaphysical theory that posits them in its ontology is thereby not charged for them? No. The fact is that this "merely theoretical" entity is pulling a great deal of weight for the presentist. Without it they have no solution to Sider's objection. So the fact that it is theoretical, or strange, or not the kind of thing one would typically posit unless science deemed it necessary, is irrelevant. One might even argue that these considerations bear in the opposite direction but I won't push it.

So, who am I to judge? I'm an B-theorist after all and have the entire space-time manifold and objects in my metaphysics. So what cost are the presentists paying that I'm not? First, the empty-box view doesn't "look to science" in the way Zimmerman suggests. Minkowskian space-time is an E^4 Euclidian four dimensional space-time. But the empty-box view imposes extra structure on space-time making it $E^3 \times E^1$ thus making the notions of temporal separation, spatial separation, and simultaneity well defined. Maybe this is a minor cost. Secondly, the empty-box view has a very strange asymmetry with respect to substance. Consider some substance, say water. It exists now at t_1 and is located at the substance (the space-time point) $\langle\langle x_1, x_2, x_3\rangle, t_1\rangle$. But a moment later at t_2 one substance, the water at $\langle\langle x_1, x_2, x_3\rangle, t_1\rangle$ ceases to exist while the other substance $\langle\langle x_1, x_2, x_3\rangle, t_1\rangle$ itself continues to exist. How does one reconcile this tension? If space-time isn't substantival then there is no solution to Sider, if it is, then why does it get to stick around while other substances don't? I think this brings out the *ad hoc* nature of Zimmerman's solution. Thirdly, the empty-box view seems to have little justification for accepting past but not future space-time points. Scientists making predictions or plotting a trajectory to the moon are employing the future space-time points of the manifold. That is, cross-temporal spatial relations don't just hold for past objects. This is a minor and contentious point so I don't stress it. Lastly, the presentist that adopts this solution must be a substantivalist. But the substantivalism/relationalism debate is hardly settled and one wouldn't have thought they would be forced into substantivalist just to answer Sider's objection. The B-theorist has none of these problems.

So although Zimmerman does give us a response to Sider's cross-temporal relations problem, what's required to accept it seems, to my mind, and probably even more so to the average presentist's mind, (since they fancy themselves the champions of common sense), much too high a price to pay.

11 Conclusion

What's occurred in this paper? My main argument is this. Presentism can't deal with cross-temporal relations. We considered a few moves the presentist could make against the argument from cross-temporal relations. I found several unsatisfactory. But not all were doomed. I identified a strain of presentism with enough resources to provide a supervenience basis that could deal with potentially all CT-sentences. But there was a glitch. Any such view still couldn't account for cross-temporal spatial relations. In particular they could not distinguish continuous accelerated, continuous unaccelerated, and discontinuous motion. The supervenience basis simply wasn't rich enough. We considered two ways of enriching it. First the instantaneous velocity principle. I argued that this attempt fails because instantaneous velocity (as defined by calculus) is not intrinsic. Second Zimmerman's surrogate and empty-box view. I argued that while the surrogate view doesn't represent a solution the empty-box view does enrich the supervenience basis enough to answer Sider's objection. Unfortunately the cost of accepting the empty-box view seems too high. The conclusion then is that the cross-temporal relations represent a serious threat to the presentist's position. This is one of the main reasons I prefer the eternalist picture of space-time.

Appendix 1: The Calculus Definition of Instantaneous Velocity

In this appendix we'll discuss the calculus definition of instantaneous velocity. For this the concept of a limit is crucial. Heres the intuitive idea of a limit: if, as the inputs of a function get closer to some number a the outputs get closer to some number L, then L is the limit of that function as the inputs tend toward a. For example, lets see what $f(x) = 2x^2 + 3$ does as the inputs tend toward 2. When x is 3 $f(x) = 21$, and when x is $1 f(x) = 5$ so that the limit is somewhere between 5 and 21. If we take numbers closer to 2 say 1.9 and 2.1 we get 10.22 and 11.82 respectively. Still closer numbers, say 1.99 and 2.01, give us 10.9202 and 11.802 respectively. Still closer numbers, say 1.9999 and 2.0001, give us 10.99920002 and 11.00080002 respectively. Clearly as we take inputs closer to 2, from either the right or the left, the outputs get closer and closer to 11. So the limit as x tends toward 2 of $f(x)$ is 11. And thats expected since $f(2) = 11$. We write this as

$$\lim_{x \to 2} 2x^2 + 3 = 11$$

and read it,"$2x^2 + 3$ tends toward 11 when x tends toward " or "the limit, as x tends toward 2, of $2x^2 + 3$ is 11". Thats the intuitive notion, now here's the precise definition of

$$\lim_{x \to a} f(x) = L$$

Assume f is defined on some interval around a. If for any positive number ϵ there exists a positive number δ such that $0 <| x - a |< \delta$ implies that $| f(x) - L |< \epsilon$, then L is said to be the limit of f as x tends toward a and is written

$$\lim_{x \to a} f(x) = L$$

ϵ is viewed as a challenge and δ is the response. You give me a small number, say .01 and I try to find a small (usually smaller) number δ such that any inputs in the δ-neighborhood of a will be within a distance of .01 from L. For example, consider the function $f(x) = 3x$ and say we want to know if the limit as x approaches 2 is 6—as it looks. We can start by playing the game: you give me .1 and now I need to find a δ that satisfies the conditions. Since we want $\mid 3x-6 \mid < .1$ we have $3 \mid x-2 \mid < .1$ and $\mid x-2 \mid \frac{.1}{3} = .033333333$. So if we pick $\delta = .033333333$ then $\mid 3x-6 \mid$ will be less than .1 when $\mid x-2 \mid < \delta$. This does not prove that

$$\lim_{x \to 2} 3x = 6$$

since we have considered only one of an uncountably many ϵs needing consideration. Since we cant look at each number individually we look at an arbitrary ϵ and find a general δ in terms of it. Let ϵ be given. We want $\mid 3x-6 \mid < \epsilon$ when $\mid x-2 \mid < \delta$ for some δ. From the above it should be clear that when $\delta = \frac{\epsilon}{3}$ and $\mid x-2 \mid < \delta$ we have $\mid 3x-6 \mid < \epsilon$. So $\lim_{x \to 2} 3x = 6$ does indeed equal 6. The notion of an instantaneous velocity can now be precisely defined. Let f be the position function of an object O, defined on at least some open interval of time around t and Δt be a small change in time (the inputs of f are instants and the outputs are the positions O has at those instants). Then if

$$\lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

exists and is finite, it is called the instantaneous velocity of O at t.

Appendix 2: Proof of $CW(v, f|_X) \equiv I(f|_X, v)$

Here we show that $CW(v, f|_X) \equiv I(f|_X, v)$.

Proof:

Call a set X nice iff for any open set O with $t_0 \in O, O \cap X \neq \emptyset$. We will show that $CW(v, X) \equiv X$is nice $\equiv I(X, f|_X)$.

Part 1: $CW(v, X)$ iff X is nice.

$(\leftarrow)$

Suppose $\neg CW(v, X)$. Then $(\forall f|_X)(\exists f \supseteq f|_X)(f'(t_0) = v)$. Suppose for reductio that X is nice. Then for any open interval O there is an $x \neq t_0 \in O$. Another way to put this is that for any positive δ there exists an $x \in X$ such that $0 < |x - t_0| < \delta$. Being nice doesn't tell us a whole lot about X but we at least know that X has a countably infinite number of members that get progressively closer to t_0. Let a set of those members be X'. Index X' in the obvious way letting the furthest member from t_0 be indexed by 0 (if there is no furthest just pick an arbitrary element and index in from there).

Now let

$$f|_X = \{ f\ |_X(x) = c, \text{ if } x \in X' \text{ and } x \text{ is indexed by an even number} f|_X(x) = m \text{ otherwise}$$

So $f|_X$ maps the evenly indexed elements in X' to c and every other element, including the oddly indexed elements in X' to m. Let m and c be such that $m < c$ and $|c - m| = 1$. Let v be given and let f be the (or an) extension of $f|_X$ such that $f'(t_0) = v$. Since v is defined f is differentiable at t_0 and is therefore continuous at t_0. But f can't be continuous at t_0 given that it extends $f|_X$. To see this let $\epsilon = 1$. Since f is supposed to be continuous at t_0 there must exist a δ such that for any $t \in dom(f)$ such that $|t - t_0| < \delta$ implies that $|f(t) - f(t_0)| < \epsilon$. But for any positive δ there exists an evenly indexed x and an oddly indexed $y < x$ in X' such that both $0 < |x - t_0| < \delta$ and

$0 < |y - t_0| < \delta$. By continuity we should have both $|c - f(t_0)| < \epsilon$ and $|m - f(t_0)| < \epsilon$. There are five cases: if $f(t_0) = c$ or $f(t_0) = m$ then we have that $|1| < \epsilon = 1$ which is bad. If $f(t_0) < m$ then $|c - f(t_0)| \not< \epsilon$ and vice versa if $c < f(t_0)$. If $m < f(t_0) < c$ then we just pick an ϵ less than 1/2 and one or the other requirement will fail. So f is not continuous at t_0. So X is not nice.

$(\rightarrow)$

Again by contraposition. Assume X is not nice. Then there is an open interval O such that $X \cap O = \emptyset$. Let v and $f|_X$ be given. Now all we do is define f so that within O it behaves so that $f'(t_0) = v$. But then we have $(\forall f|_X)(\exists f \supseteq f|_X)(f'(t_0) = v)$ since $f|_X$ was arbitrary. So by contraposition we have that if $C(v, X)$, then X is nice.

Part 2: X is nice iff $I(X, v)$.

$(\rightarrow)$

Suppose X is nice and $\neg I(X, v)$. Then

$$\forall f|_{(a,b)-X}, (f|_{(a,b)-X})'(t_0) \neq v$$

or

$$\forall f|_X \forall g|_X [(f|_{(a,b)-X} \cup f|_X)'(t_0) = (f|_{(a,b)-X} \cup g|_X)'(t_0)]$$

Suppose $\forall f|_{(a,b)-X}, (f|_{(a,b)-X})'(t_0) \neq v$. Then there is an open interval O such that $(a, b) - X \cap O = \emptyset$. So X is really nice (it has an entire open interval as a subset) and so we have that $(\exists f|_X)(\exists g|_X)((f|_{(a,b)-X} \cup f|_X)'(t_0) \neq (f|_{(a,b)-X} \cup g|_X)'(t_0))$.

Either $(f|_{(a,b)-X})'(t_0) \neq v$ where $v = u$ or $(f|_{(a,b)-X})'(t_0) \neq v$ where v is real. It's always possible for $(f|_{(a,b)-X})'(t_0) = u$. So $v \neq u$. Either there are other real values m (at least one) such that there is an $(f|_{(a,b)-X})'(t_0) = m$ or this constraint holds for every real number. If it holds for every real number v then $(f|_{(a,b)-X})'(t_0)$ is neither real nor undefined which is bad. If there is an $f|_{(a,b)-X}$ such that $(f|_{(a,b)-X})'(t_0) = m$ for $m \neq v$ and $m \in \Re$, then the velocity is completely determined within $(a,b)-X$. So there must be an open interval O such that $O \subseteq (a,b)-X$ on which the velocity is defined. But then $O \cap X = \emptyset$. But this contradicts the niceness of X.

Now suppose $\forall f|_X \forall g|_X ((f|_{(a,b)-X} \cup f|_X)'(t_0) = (f|_{(a,b)-X} \cup g|_X)'(t_0))$. Then the velocity is completely determined by $f|_{(a,b)-X}$. So there is an open interval O such that $O \subseteq (a,b)-X$. But then $O \cap X = \emptyset$. But this contradicts the niceness of X. So our hypothesis leads to a contradiction in both cases. So X must have implications for the values of the velocity at t_0.

$(\leftarrow)$

Suppose $I(X,v)$ yet X is not nice. Since X is not nice there is an open interval O such that $O \cap X = \emptyset$. So $O \in (a,b)-X$ and the velocity is completely determined by $f|_O$. But then $\forall f|_X \forall g|_X ((f|_O \cup f|_X)'(t_0) = (f|_O \cup g|_X)'(t_0))$ and so $\forall f|_X \forall g|_X ((f|_{(a,b)-X} \cup f|_X)'(t_0) = (f|_{(a,b)-X} \cup g|_X)'(t_0))$. But then it's not the case that $I(X,v)$. A contradiction. So X is nice after all.

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