The Dissertation Committee for Hong Xu
certifies that this is the approved version of the following dissertation:

Essays on Certification Mechanism Design in Strategic Communications

Committee:

Andrew B. Whinston, Supervisor

Maxwell B. Stinchcombe, Supervisor

John Mote

Thomas Wiseman

Bin Gu

Essays on Certification Mechanism Design in Strategic Communications

by

Hong Xu, B. Management; M.S.

DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

August 2010

Essays on Certification Mechanism Design in Strategic Communications

Publication No. ____________________

Hong Xu, Ph.D.
The University of Texas at Austin, 2010

Supervisors: Andrew B. Whinston
Maxwell B. Stinchcombe

Certifiers have an crucial role in facilitating effective communication in the online and the traditional world. As a way of generating statistically meaningful information, certification has been adopted in financial statements evaluation and more recently in various online communities as well. This dissertation examines three related issues along this common theme: online reputation market, moderation in user-generated content, and strategic communications in the market for certifications, and is consisted of three essays. The first essay analyzes the impact of various dispute mechanisms on online identity trading. Online identities with a good reputation profile is a valuable and tradable asset. However, the fact that identity creation is free leaves room for low quality sellers to free-ride high quality sellers. When there is a lack of incentive for sellers to maintain a good reputation, identity trading becomes ineffective. This essay focuses on the role of an auditing system, such

as eBay dispute center, and shows that even a small amount of objective information from the auditors can reverse the negative result and sustain reliable reputation and identity trading. The second essay investigates the impact of moderation on the quality of information in an user-generated content(UGC) environment. In most UGC communities, content contributors have incentive to publish biased or false information. For example, companies hire people to write positive reviews about themselves. This essay establishes a framework for the mechanism design of moderation, and provides insight on how to optimally allocate moderation resource. The third essay examines a market for certification and certifiers' strategic reporting behaviors. The central question is how to induce certifiers to provide statistically meaningful information to investors when they are paid by their client firms. We provide insights on how certifier competition plays an role in firms' certifier choice, how certifiers degrade their accuracies to achieve maximum profit, and how the legal environment impacts the information quality.

Table of Contents

List of Figures

Chapter 1

An Introduction to Certification in Online and Traditional World

An online community is a virtual environment where users participate through an online information system to achieve a common goal. Online communities exist for various purposes. For example, a Bulletin Board system enables users to share content and have discussions; a electronic commerce system, such as eBay or Amazon marketplace, establishes a platform for buyers and sellers to carry out transactions; online communities with a social network focus, including Facebook, YouTube, and MySpace, become a channel for communication through voice, video or text. In many aspects, the different purposes of online communities complement each other instead of differentiating one from another. For instance, eBay has developed a feedback forum, which is a Bulletin Board system, for the buyers to share their purchase experience and discuss possible ways to detect and avoid dishonest sellers. On the other hand, online as well as brick-and-mortar retailers have gradually discovered the great potential of social network websites as a sales venue. More and more sellers publish video tutorials of their products on YouTube to promote sales. Indeed, various online communities have in common the need for reliable communication in order for success in business, learning, and expanding social network. Certification mechanisms has been adopted by many online communities for different purposes. For example, Slashdot has constructed a reputation system and has been recognized for its quality of content. The site uses "karma" points to measure commentators' reputations based on the quality of their past comments. Amazon has a profound database of user reviews on various products. To facilitate potential buyers in finding relevant and reliable reviews, they have a function for people to rate on the helpfulness of each piece of product review. Wikipedia, on the other hand, has a editorial board that review the content contributed by its users, and remove content that they deem to be incorrect or inappropriate.

Certifiers are also widely used in the traditional world to facilitate the decision-making process of investors, banks, and customers. The essential problem for certifications is how to guarantee that the net result, statements and their (non-)certification, convey the information relevant for investors. Operating firms, issuers of securitized assets, and sometimes government agencies have an interest that is not aligned with the investors. Realizing this, investors are not willing to believe the information provided unless verified by the external certifier, such as auditing firms and credit rating agencies. However, the collapse of Enron Corp., the demise of the Lehman Brothers, and the recent financial crisis have put the credibility of certifiers under controversy. An examiner, appointed by the US bankruptcy court, in the bankruptcy of the Lehman Brothers revealed evidence that the firm has prepared misleading financial statement using an accounting gimmick called "repo 105"(Baer and

Thomas, 2010), which was also used by Bank of America Corp. and Citigroup Inc. to hide billions of dollars of debt (Rapoport, 2010). Lehman's auditor, Ernst & Young, was also the target in a British investigation into whether the auditing firm properly audited Lehman Brother's accounts in the months leading up to the bank's collapse (Murphy and Masters, 2010). Meanwhile, the Senate subcommittee has launched an investigation on Moody's and Standard & Poor's, which confirmed their act of skewing their assessments to please their clients (Krugman, 2010). On the other hand, when the users of the certification are the buyers, they can directly reward or punish the certifier for accuracies or inaccuracies. Auditors hired by lenders as part of the loan approval process are in this position, especially in-house auditors. For the first three decades of its existence, Underwriters Laboratory was in a similar position.

Chapter 2

Auditing in a Market for Reputation

2.1 Introduction

Reputation, as an economic concept to deal with moral hazard and/or adverse selection problems, has been studied by many researchers (Fudenberg and Levine 1992; Mailath and Samuelson 2001). In these studies, the principal's willingness to pay depends only on the agent's past performance as measured by reputation, and transaction outcomes are not contractible. This is typically justified by the fact that if the outcome were contractible, the principal and agents could simply assign a contingent contract to resolve any problems. In many scenarios in reality, however, the outcome can be verified or audited (at some cost). One such example is grid computing (Hidvegi 2006), where a principal can check computational results by repeating the computation process. When verification is not performed all the time due to the cost or imperfect audit, there is still room for moral hazard and adverse selection problems. This paper addresses the problems of both moral hazard and adverse selection, with imperfect audit.

We introduce an unbiased auditor to perform verifications on a random basis. Each agent operates a firm that has a reputation, and the reputation is updated according to the auditing result. Agents can trade firms together with reputations in the reputation market, and the shift of ownership is unobservable to consumers. Our major result shows the existence of a separating equilibrium, in which high-type agents buy firms with good reputation, and low-type agents buy firms with low reputation. Our result contrasts with that of Tadelis (2002), who assumes unverifiable outcome and reputation is based on consumer reports. Tadelis (2002) concludes that no separating equilibrium exists. We attribute this difference to the contingent payments we introduced, under which low-type agents have less incentive to mimic high types, since they may fail the audit and get no benefit from mimicking.

2.2 The model

We consider three entities: producers, consumers, and an auditor. We normalize the population of producers to 1, and producers can be one of two types, denoted by $\theta \in \{H, L\}$. The proportion of H-type producers is λ_H, and that of L-type producers is $1-\lambda_H$. There are two markets: the product market and the reputation market. Each product is associated with a probability of being successful. For simplicity, we assume reputation is the number of continuous successes j before a firm fails the audit. Firms' built-in reputations are separated from agents and can be traded on the reputation market. Both markets open for infinite periods, and in each period, the timeline of events can be described as follows.

First, producers trade reputations in the reputation market. The price for a reputation j is denoted by V_j. Producers have three choices: staying with the current reputation, selling that reputation and buying another one, or creating a new reputation. Second, each producer chooses an effort level $w \in [0,1]$ at cost $c(w) = k_\theta w^2$ to produce one product, where $\theta \in \{H, L\}$ and $k_L > k_H$. For simplicity, we assume w is also the probability of the product being successful. Each producer produces a batch of r products in each period, and within a batch, whether one product is successful is independent of another. The quality of a certain product is unobservable to consumers, but is verifiable by the auditor. Next, the auditor randomly picks one product from each batch to check.[1] If it turns out to be successful, the producer's reputation is adjusted from j to $j+1$ and it is allowed to sell the batch of products; if it turns out to be a failure, the producer's reputation is changed to 0 and the batch of products is taken away by the auditor.[2] The random audit follows from the well-established audit sampling procedure (Rittenberg and Schwieger 2005). A successful product has value 1 to a consumer, and a failed product has value 0 to a consumer. We assume that consumers are risk neutral and have no bargaining power, so they pay their expected value of products. Thus, given $\tilde{w}$ is their belief about a product being successful, the price for this product is $\tilde{w}$.

[1]Notice that, since the unit chosen is independent from the rest in the batch, the audit is imperfect in the sense that the quality of the remaining products is not reflected in the audit result.

[2]The purpose of taking away failed batches is to punish producers for being caught by the audit. Since these batches contain successful products, the auditor can still sell them to consumers to avoid efficiency loss.

We assume that creating a new reputation is free (i.e., $V_0 = 0$), as a normalization. We also assume that shifts in reputation ownership are unobservable to consumers. This indicates the separation of producers from reputations. In reality, to observe shifts of owners is usually very costly, especially to individual consumers. Technically, we let $2k_\theta - 1 > 0$. This means the marginal cost of production is higher than the marginal revenue when the highest effort($w = 1$) is exerted. Otherwise, it is possible that exerting the highest effort is agents' best choice, and thus the model will become trivial.

2.3 Existence of a Separating Equilibrium

A separating equilibrium consists of a set of values $\{w_j, V_j\}$, $j = 0, 1, ...$ and a cutoff reputation h: any reputation $j < h$ is chosen by low-type producers only and any $j \geq h$ is chosen by high-type producers only. Denote $\tilde{w}_j$ as the market belief of the effort level by producers with reputation j, while w_j is the equilibrium effort level chosen by the producer. The expected payoff from producing r units is $[wr\tilde{w} - rk_\theta w^2 + (wV_{j+1} - V_j)]$. The first term is the product of price and the expected probability of passing the audit; the second term is simply the cost of effort; and the third term is the expected change in reputation value. We simply consider the expected payoff per unit product, and define π_j^θ as the maximum expected payoff per unit for a type θ producer under reputation j. We then write π_j^θ in the following form:

$$\pi_j^\theta = \max_w w\tilde{w}_j - k_\theta w^2 + \frac{wV_{j+1} - V_j}{r}. \tag{2.1}$$

In the separating equilibrium described above, all low-type producers must be indifferent to holding any reputation $j < h$. Otherwise, reputation prices would have to change to balance the market. Similarly, all high-type producers are indifferent to holding reputations $j \geq h$. Denote equilibrium profits for low and high types as π_L and π_H, respectively. For now, we use π_L and π_H as given, and we will show later how π_L and π_H can be calculated.

Low-type Producers' Effort and Payoff: In a separating equilibrium, low-type producers' optimal effort level, termed w_j, is determined by the first-order condition of the objective function (2.1):

$$\tilde{w}_j - 2k_L w_j + \frac{V_{j+1}}{r} = 0, \text{ for } j = 0, 1, ..., h-1. \tag{2.2}$$

Notice that in equilibrium, the market's belief about effort levels is consistent with producers' equilibrium effort levels, i.e., $\tilde{w}_j = w_j$. Based on this, we can solve for the equilibrium effort levels as:

$$w_j = \frac{V_{j+1}}{(2k_L - 1)r}. \tag{2.3}$$

Furthermore, we can derive the equilibrium profit π_L by substituting the first-order condition (2.2) into the payoff function (2.1):

$$\pi_L = k_L w_j^2 - \frac{V_j}{r}. \tag{2.4}$$

Notice that $V_0 = 0$. Provided the equilibrium profit is π_L, we can derive w_0 by (2.4), V_1 by (2.3), w_1 by (2.4), and so on, until we get all w_j, $j = 1, 2, ..., h-1$, and all V_j, $j = 1, 2, ..., h$.

Lemma 1. *In a separating equilibrium, both reputation prices* V_j *and effort levels* w_j *increase in reputation* j*.*

This lemma can be inferred from (2.3) and (2.4). First, we have $V_1 > V_0 = 0$. By (2.4), if $V_{j+1} > V_j$, then $w_{j+1} > w_j$, which in turn indicates $V_{j+2} > V_{j+1}$ by (2.3). Thus, $V_{j+1} > V_j$ and $w_{j+1} > w_j$ for $j = 0, 1, ..., h-1$. This result means that in the separating equilibrium, low-type producers are induced to work harder as the reputation goes higher in order to lower the probability of being caught and losing the value of reputation.[3]

High-type Producers' Effort and Payoff: To capture a plausible equilibrium, we assume that in a separating equilibrium all reputations $j \geq h$ have the same value V_h, and all high-type producers exert the same level of effort w_h. One justification is that producers' types can be inferred from reputations, and there is no need to further differentiate among high-type producers. Applying an analysis similar to that for low-type producers, we can derive high types' equilibrium effort level and payoff as:

$$w_h = \frac{V_h}{(2k_H - 1)r}, \tag{2.5}$$

$$\pi_H = k_H w_h^2 - \frac{V_h}{r}. \tag{2.6}$$

Notice that V_h can be obtained from (2.3): $V_h = r(2k_L - 1)w_{h-1}$.

[3]Notice that the monotonicity here is certainly related to the punishment scheme in our reputation system: once a product fails, an agent's reputation goes to zero. The monotonicity may be absent for general reputation systems.

Determining π_L: From the above analysis, all other variables (i.e., w_j, V_j, and π_H) can be determined in equilibrium as functions of π_L. In this subsection, we show that the composition (λ_H) of the producer population determines π_L. Denote λ_j as the proportion of reputation j. We study a steady state case where proportions of reputations $\lambda_j, j = 1, ..., h, ...$ remain the same across periods, despite the dynamics of the producer population.

$$1 - \lambda_H = \sum_{j=0}^{h-1} \lambda_j = \lambda_0 + \sum_{j=1}^{h-1} \lambda_0 \prod_{i=0}^{j-1} w_i, \tag{2.7}$$

$$\lambda_H = \sum_{j=h}^{+\infty} \lambda_j = \sum_{j=h}^{+\infty} \lambda_0 \prod_{i=0}^{j-1} w_i. \tag{2.8}$$

The first equation means that in a separating equilibrium the total proportion of reputations below h equals the proportion of low-type producers. The second equation is for high-type producers and has a similar interpretation. From the above two equations, we have[4]

$$\lambda_H = \frac{\prod_{i=0}^{h-1} w_i}{(1 - w_h) \sum_{j=0}^{+\infty} \prod_{i=0}^{j-1} w_i}. \tag{2.9}$$

Notice that w_i's are functions of π_L. We can show there exists a unique π_L satisfying (2.9), so we can express π_L as $\pi_L(\lambda_H, k_L, k_H)$. (The existence and uniqueness of π_L are presented in the appendix.)

Existence of a Separating Equilibrium: To show that the separating equilibrium does exist, we need to prove that both low-type and high-type producers have no incentive to deviate, i.e., we need to check incentive compatibility (IC) conditions for both types.

[4] We define $w_{-1} = 1$ in order to better arrange the equation.

(i) IC for low-type producers: To mimic high types, low-type producers maximize (2.1) under $\tilde{w}_h = w_h$. Based on the first order condition, we get $w_h^L = \frac{w_h + \frac{V_h}{r}}{2k_L} = \frac{k_H}{k_L} w_h$, and $\pi_h^L = k_L (w_h^L)^2 - \frac{V_h}{r}$. The incentive compatibility requires $\pi_h^L \leq \pi_L$.

(ii) IC for high-type producers: Similarly, we get $w_j^H = \frac{w_j + \frac{V_{j+1}}{r}}{2k_H} = \frac{k_L}{k_H} w_j$ and $\pi_j^H = k_H (w_j^H)^2 - \frac{V_j}{r}$. Notice that $\pi_j^H = k_H (\frac{k_L}{k_H} w_j)^2 - k_L w_j^2 + \pi_L$, which is increasing in j since w_j increases in j and $k_H (\frac{k_L}{k_H} w)^2 - k_L w^2 + \pi_L$ increases in w. This means π_{h-1}^H is the highest profit that a high type can get by mimicking a low type. Therefore, it is sufficient to show $\pi_{h-1}^H \leq \pi_H$.

Proposition 1 (Existence of a separating equilibrium)**.** *A separating equilibrium exists if* $0 < \pi_L(\lambda_H, k_L, k_H) < 1 - k_H - \frac{k_L(k_L - k_H)}{k_H} \left(\frac{2k_H - 1}{2k_L - 1} \right)^2$.

Intuitively, this condition means that as long as low-type producers' profit is low enough, high-type producers will not be interested in pooling with them. On the other hand, this profit should be no less than zero so that low-type producers are willing to participate and to stay with low reputations.

Proposition 2. *In a separating equilibrium, the equilibrium profit for low-type producers* π_L *increases in the proportion of high-type producers* λ_H*.*

This is because a higher λ_H means a higher demand but a lower supply of reputation h, which leads to a higher V_h. Thus, a low-type producer who luckily reaches h will be paid higher, which indicates a higher π_L.

2.4 Conclusion and Discussion

In this paper, we targeted the category of verifiable products, and showed the existence of a separating equilibrium in a reputation market. We introduced an auditor to check on outcomes and to implement contingent payments to agents based on the auditing results. By combining incentives from both reputation and audit, we are able to derive the separating equilibrium, which is lacking with non-verifiable products (Tadelis 2002). A separation has an important interpretation in the online environment, where people face a high risk of manipulation and fraud.

The separating result derived is robust to different reputation measures and auditing procedures.[5] For example, if we consider the proportion of success for the last T samples (1 in each period) as the reputation measure, it can be shown that a similar separating equilibrium exists.[6] In terms of auditing procedures, we can relax the current strict rule by, for example, randomly auditing two products from r, and requiring at least one success for payments and reputation to increase. We can also show that a similar separation exists.

As in many signaling games, however, multiple equilibria may arise. In that case, certain refinements can be applied to rule out some of the equilibria (e.g., intuitive criterion, proposed by Cho and Kreps (1987)). On the other hand, it is worth pointing out that, for a given h, our analysis generates a

[5] We thank the reviewer for comments on the robustness of the model.

[6] Notice that the equilibrium in the special case $T = 1$ is equivalent to the equilibrium of $h = 1$ in the current reputation measure.

unique separating equilibrium, if any, although there may exist multiple h's that lead to separation.

Chapter 3

Moderated Online Communities and User-generated Content

3.1 Introduction

The rise of social computing and online communities has ushered in a new era of content delivery, where information can be easily shared and accessed. A large number of applications have emerged that facilitate collective actions for content generation and knowledge sharing. Examples include blogs, online product reviews, wiki applications such as Wikipedia.com, and online forums such as slashdot.org. Due to the anonymity of Internet users, however, ensuring information quality or inducing quality content remains a challenge. To deal with that, this paper introduces a moderation system and examines its effect on the content quality of online communities.

Information sharing and user-generated content have become ubiquitous online phenomena. For example, Wikipedia, a free online encyclopedia, is dedicated to massive distributed collaboration by allowing visitors to add, remove, edit, and change content. In online product reviews, like the ones on Amazon.com, any user can post reviews on any item, even if he or she has not bought the item on Amazon. Online forums such as Slashdot are another example. Slashdot, a website that supports discussions on user-submitted news stories and articles related to technology, is one of the most frequently visited sites on the Internet. On Slashdot, all users can express their opinions simply by posting the comment under a selected topic.

As these applications have gained popularity and importance, the quality of content has become a concern. On Wikipedia, readers may be provided with content that is misleading or even incorrect. Product reviews on Amazon can be manipulated by sellers or book publishers to boost their products. On Slashdot, commentators may post biased or useless comments; For example advertisers from hardware companies may post biased comments to promote their products. Wikipedia is still experimenting with different approaches to ensure the quality of content. As one of its co-founders pointed out, Wikipedia, as an encyclopedia, lacks both the usual review process and the respect for expertise of most encyclopedias.[1] Amazon has introduced a voting system in which consumers can vote on whether a particular review is helpful, and the vote result affects the continued ranking of that review. Voting mitigates the information manipulation problem, but it has its pitfalls, since the voting process itself can be manipulated.

Unlike many other social networks, Slashdot has constructed a reputation system and has been recognized for its quality of content. The site uses "karma" points to measure commentators' reputations based on the quality of

[1] http://www.kuro5hin.org/story/2004/12/30/142458/25

their past comments. Karma points or reputations are clustered into a small set of labels (e.g., terrible, bad, positive, excellent). Each comment posted by a commentator receives a score ranging from -1 to 5. The score often signals the quality of the comment and affects its readership, as a comment with a higher score typically attracts more readers. Comments' default scores differ from each other according to the commentators' reputations: Commentators with a good reputation receive a high default score and high expected readership, which fulfills the goal of reputation systems in the sense that commentators with high reputations get rewards in some way.

One additional step taken by Slashdot beyond the regular reputation systems is its moderation system. Once a comment is posted, it may be checked or "moderated" by selected users who can change its score and assign a label, such as "informative" or "redundant." The moderation result affects the comment's score and thus its readership, and it also changes the commentator's reputation. In a sense, moderation plays the same role here as auditing in accounting. Slashdot selects moderators randomly from among eligible users and then limits the moderator status, both in number of posts to be moderated (five) and in time (three days). This restriction ensures no moderator can have an undue effect on the system.

Similar moderation processes have been adopted by other online communities, such as kuro5hin.org and plastic.com. In fact, the moderation process was introduced mainly to screen information. As stated by one of Slashdot's founders, "The purpose of moderation is to help people organize information"; it can help users "pick up hidden gems on the sandy beach of comments" Chromatic et al. (2002). However, it seems the actual impact of moderation is more extensive. In particular, the refined review process could have a significant effect on commentators' incentive to generate quality content.

Introducing moderation to online communities shows promise for ensuring content quality. However, little research in information systems has been done to study the effect of moderation or the design of a moderation system. As a starting point, this paper examines the effect of moderation on the performance of an online community. We consider a community consisting of both dedicated and opportunistic commentators. The former behave altruistically while the latter behave strategically. Commentators possess reputations in the community and post their comments of different quality. A moderation system moderates each comment. The moderation result affects both a comment's readership and the commentator's reputation.

We start with a simple case in which the moderation system monitors comments from commentators with different reputations with the same frequency. We find that moderation has a direct impact on the opportunistic commentators' incentive to exert effort. When adequate moderation is applied, opportunistic users always exert effort regardless of their reputations, whereas if moderation is very limited, they may exert no effort at all. When the level of moderation is in the middle, both types of commentators may adopt a strategy mixing exertion and no exertion. We also demonstrate that a reputation system that includes moderation is superior to a pure reputation system in terms of the expected performance of the community.

We also consider differentiated moderation probabilities for different reputations. We find that when the moderation system monitors low-reputation commentators more carefully, commentators may display reputation oscillation. In particular, they work hard to generate useful information for building up a high reputation in one period and then exploit it in the next. In this case, interestingly, the expected performance from high-reputation commentators can be inferior to that from low-reputation ones, which again illustrates the critical impact of moderation on commentators' incentives.

Finally, we discuss the optimal moderation resource allocation, which appears to be an important issue when the moderation resource is costly. We find that when dedicated commentators play a significant role in a community and opportunistic commentators are able to generate as high quality content as the former, optimal moderation involves either moderating all commentators equally or moderating low-reputation commentators only. In other words, it is never optimal to monitor high-reputation commentators more closely.

As information quality has been identified as an important factor in the success of information systems (DeLone and McLean, 1992), the quality of content is a natural concern for online communities. A large volume of the existing literature focuses on reputation systems. For example, Dellarocas (2005) has studied the reputation mechanism in eBay-like trading environments, with a focus on how mechanism parameters (e.g., a user's feedback profile) affect sellers' effort levels and market efficiency. Ba and Pavlou (2002) examine whether good reputations generate product price premiums on eBay-like trading platforms. The study on reputation in economics can be traced back several decades. In their seminal work, Kreps and Wilson (1982) and Milgrom and Roberts (1982) concluded that reputation effects arise with even a small amount of incomplete information on agents' types. Later, Cripps et al. (2004) showed that with imperfect monitoring, reputation cannot be sustained infinitely—if a long-run player stays in the game long enough, short-run players will eventually learn the long-run players' true type, and the game will inevitably revert to one of static Nash equilibrium. In contrast to well-understood reputation systems, the moderation system has attracted little notice. Lampe and Resnick (2004) document some observations of the moderation practice and point out that "important challenges remain for designers of such systems." Our paper tries to build up a game-theoretic model to analyze commentators' incentives and study the impact of moderation, and we aim to address why moderation works and how it can be improved.

Several authors have looked at user-generated content from different perspectives. For example, based on a unique data set from Amazon, Forman et al. (2008) study the relationship between reviews and sales and suggest that online community users rate reviews containing identity-descriptive information more positively and that disclosure of identity information is related with online product sales. Study of online communities also has been concerned with users' motivation for their voluntary participation in and contribution to communities. Based on their data, Wasko and Faraj (2005) find several factors related to users' motivation to contribute, including the perception that it enhances their professional reputation. Bateman et al. (2006) study this issue, drawing on organizational commitment theory. Benabou and Tirole (2006) develop a theory of prosocial behavior to systematically explain this motivation issue. They attribute the individuals' motivation to the intrinsic value, monetary benefit, and reputation effect derived from the participation. Our paper assumes that two different types of commentators, the dedicated type and the opportunistic type, participate in online communities for their own reasons. The former behaves like altruists who may be motivated by dominant intrinsic value and/or reputation effect, whereas the latter acts strategically.

Our work is also related to the studies on moral hazard problems under other settings, such as insurance contracts and owner-management contracts. For example, in an insurance setting, agents (insured) have incentive to misreport the loss from accidents, and principals (insurers) have the right to audit agents' reports. Optimal auditing has long been discussed in the literature (Mookherjee and Png, 1989). Our study stands out from the above literature in that we develop a repeated game with reputation to capture the unique feature of online communities, in which moderation/auditing affects not only agents' current period payoff but also their future payoff.

The rest of the paper is organized as follows. In section 2 we lay out our model. We analyze the equilibrium effort choice under the same moderation probabilities in section 3 and under differentiated moderation probabilities in section 4. In section 5, we investigate the optimal moderation resource allocation. Some extensions and discussion are offered in section 6. Section 7 concludes the paper.

3.2 Model

We consider an online community in an infinite-period horizon, in which a large number of commentators post comments and develop reputations in doing so. At the beginning of each period, the commentators post their comments, and the comments are moderated at some point within that period. At the end of the period, the commentators' reputations are updated based on the revealed quality of their comments as determined by the moderation. For simplicity, we assume that comments are available to readers for the current period only.

We categorize the commentators into two different types: dedicated and opportunistic. Dedicated commentators post their opinions and behave like altruists. This is because they derive a great deal of intrinsic value from the community and are thus dedicated to the community posting. Due to the heterogeneity in the commentators' knowledge, some of their comments are of high quality, whereas others may be of low quality. We assume that the proportion of high-quality comments or the probability of a comment being of high quality is s, $0 < s < 1$. In contrast, opportunistic commentators behave strategically. They can exert effort ($e = 1$) at cost c or exert no effort ($e = 0$) at zero cost to generate a comment. The comment with effort is of high quality with probability k, $0 < k < 1$, while the comment with no effort is of low quality with probability 1. One special case is $k = s$, which means the probability that an opportunistic commentator's comment with effort is of high quality is the same as that for a dedicated commentator's comment. Cost c can be interpreted as the time that commentators spend in properly organizing their opinions or investigating the topic under discussion. We assume that among all commentators the proportion of dedicated type is μ, and hence the proportion of the opportunistic type is $(1 - \mu)$.

The quality of comments is unobservable to readers ex ante but is revealed once readers go over the comments. To motivate opportunistic commentators to exert effort to generate high-quality comments and to guide readers toward those of higher quality, a moderation system is implemented, in which some moderators check the quality of comments (by going over the comments) and label them as either high quality or low quality. For example, on Slashdot, comments might be moderated to be "Insightful" or "Informative," or to be "Redundant" or "Offtopic." We here use "high quality" to refer to the former category and "low quality" to the latter category. With probability α ($0 \leq \alpha < 1$), a comment is moderated at the very beginning of the period; otherwise, the comment is moderated at the end of the period. The former we call early moderation or moderation, and the latter we call late moderation or feedback(as if consumers report quality feedback after consuming a product). The result of early moderation affects both the number of readers of the comment and the reputation of the corresponding commentator. Late moderation affects commentators' reputations, but it does not affect the number of readers of the comment, since the quality of the comment is revealed at the end of the period.

A commentator may have a high reputation or a low reputation. We consider the commentator's reputation as high if the last comment is judged to be high quality and as low if it is deemed low quality. Such an assumption is mainly for technical simplification and imposes little restriction since the primary purpose of our reputation system is to examine opportunistic commentators' incentive. The above simple reputation measure plays an effective sanctioning role (i.e.,the threat of future punishment).

We are interested in the impact of the moderation system on opportunistic commentators' behavior. We assume opportunistic commentators derive utility from others reading their comments. In particular, we assume the utility is linear in the number of readers. Notice that here readers may include commentators as well as *lurkers*(i.e., the users who read comments generated by commentators but do not generate comments themselves). The high-quality comments revealed by early moderation get the maximum readership, normalized to 1, and the low-quality comments revealed by early moderation get 0 readership. For comments with late moderation, the readership level is equal to the likelihood of their being high quality, which is also termed their expected quality. To rule out a trivial case, we assume $c < k$; otherwise, expected maximum readership cannot compensate for the effort cost, and no opportunistic commentators exert effort.

We use subscript i, $i \in \{0, 1\}$, to indicate a commentator's reputation (with 1 representing high reputation), and we denote v_i as the commentator's expected payoff. Thus, the payoffs of opportunistic commentators at period t can be formulated as follows:

$$v_1^t = \max_{e\in\{0,1\}} \alpha ek + (1-\alpha)\, r_1^t + \beta ekv_1^{t+1} + \beta\,(1-ek)\, v_0^{t+1} - ce, \quad (3.1)$$

$$v_0^t = \max_{e\in\{0,1\}} \alpha ek + (1-\alpha)\, r_0^t + \beta ekv_1^{t+1} + \beta\,(1-ek)\, v_0^{t+1} - ce. \quad (3.2)$$

where β is a discount factor and r_i^t is the expected quality of comments from commentators with reputation i.

We will be concerned with *steady states* in which r_i^t and v_i^t are independent of time. (They, of course, depend on the state variable—reputation i.) In other words, timing does not play a role in commentators' decisions. For this reason, we simply omit the period indicator t for our discussion and rewrite the above payoff functions as:

$$v_i = \max_{e\in\{0,1\}} \left[\alpha ek + (1-\alpha)\, r_i\right] + \beta\left[ekv_1 + (1-ek)\, v_0\right] - ce, \text{ for } i \in \{0,1\}. \quad (3.3)$$

The term in the first square bracket represents the expected payoff from the current-period readership, and the term in the second square bracket captures the future payoff.

Notice the nature of the dynamic programming in the above payoff function: The current effort choice affects not only the commentator's current stage payoff but also his or her future payoff through the realized reputation. Also, it is worth pointing out that we can treat e as a continuous variable, since e can also be interpreted as the probability of exerting effort in our game-theoretic framework.[2]

3.3 Equilibrium Performance

Moderation probabilities have a critical impact on opportunistic commentators' incentive to exert effort. In this section, we investigate three cases where, in equilibrium, opportunistic commentators exert effort definitely, exert no effort definitely, and exert effort with some probability, respectively.

Notice that the marginal benefit from exerting effort is the probabilistic increase in the current period payoff (αk) and the increase in discounted future payoff ($\beta k(v_1 - v_0)$). On the flip side, exerting effort incurs cost c. The balance between the marginal benefit and the marginal cost is captured by the first-order derivative of the payoff functions (3.3),

$$\alpha k + \beta k(v_1 - v_0) - c, \tag{3.4}$$

which determines the commentators' equilibrium choice. If the above is positive, meaning the marginal benefit outweighs the marginal cost, the commentator will exert effort. Otherwise, he or she prefers not to exert effort. It is worth noting that commentators have symmetric incentives in the sense that if it is optimal for them to exert effort when their reputation is high, they also find it optimal when their reputation is low.

[2]Provided a linear effort-cost structure, the analysis of the case with continuous effort level e, $e \in [0, 1]$, is equivalent to the analysis we conduct under the current setting.

Meanwhile, dedicated commentators do not behave strategically, and with probability s their comments are of high quality regardless of their current reputation. Therefore, a proportion s of dedicated commentators possess high reputation.

3.3.1 The Equilibrium with Effort

When the probability of early moderation (*moderation probability* afterwards) is high, opportunistic commentators have great incentive to exert effort because, otherwise, their comments would fail the early moderation and thus receive no readership. More precisely, the equilibrium with opportunistic commentators exerting effort requires high moderation probabilities, such that the marginal benefit outweighs the marginal cost(i.e., $\alpha k+\beta k(v_1-v_0)-c \geq 0$).

According to equation(3.3), the opportunistic commentators' expected payoffs in equilibrium are:

$$v_1 = \alpha k+(1-\alpha)\, r_1+\beta[kv_1+(1-k)v_0]-c, \text{ and } v_0 = \alpha k+(1-\alpha)\, r_0+\beta[kv_1+(1-k)v_0]-c. \tag{3.5}$$

The difference between the above expected payoffs, $v_1-v_0 = (1-\alpha)(r_1-r_0)$, plays a role in determining opportunistic commentators' incentives. Notice that the difference is a function of the moderation probability. If $\alpha = 1$, then $v_1 - v_0 = 0$, which means that the expected payoffs are the same under either reputation and this case is reduced to a trivial one. In fact, $\alpha = 1$ means each comment will be moderated and the quality will be revealed immediately, and hence the payoff is solely determined by the moderation result.

For this reason, under $\alpha = 1$, reputations do not matter to either readers or commentators. To exclude this trivial case, we assume $\alpha < 1$.

Recall the proportion of dedicated commentators with high reputations is s. Proportion k of opportunistic commentators have high reputations when they exert effort. So the size of the population in high reputations will be $\mu s + (1-\mu)\,k$, consisting of dedicated commentators (the first term) and opportunistic ones (the second term). Since the expected qualities of comments from dedicated commentators and from opportunistic ones are s and k, respectively, we can formulate the expected quality of comments from high reputation commentators as follows:

$$r_1 = \frac{\mu s s + (1-\mu)\,kk}{\mu s + (1-\mu)\,k}. \tag{3.6}$$

Similarly, we can formulate the expected quality of comments from low reputation commentators as:

$$r_0 = \frac{\mu(1-s)s + (1-\mu)\,(1-k)k}{\mu(1-s) + (1-\mu)\,(1-k)}.$$

Based on the expected payoff functions (3.5), we can rearrange the first-order incentive condition as:

$$\alpha k\left[1 - \beta\left(r_1 - r_0\right)\right] + \beta k\left(r_1 - r_0\right) - c \geq 0. \tag{3.7}$$

Clearly, the left hand side is increasing in α. In other words, the higher the moderation probability, the more likely the opportunistic commentators are to exert effort. Intuitively, increasing moderation probability means increasing the chance of receiving early moderation, which encourages opportunistic commentators to exert effort because they would get caught easily and their comments would be revealed as low quality. Therefore, a higher moderation probability is more likely than a lower one to induce opportunistic commentators to exert effort.

We define α_H as the value of α that binds the above inequality (3.7), which is:

$$\alpha_H = \frac{c/k - \beta\,(r_1 - r_0)}{1 - \beta\,(r_1 - r_0)}. \tag{3.8}$$

Thus, we obtain the following lemma.

Lemma 2. *Under any $\alpha \geq \alpha_H$, exerting effort can be sustained as an equilibrium.*

It is worth noting that when the effort cost c is high enough (e.g., $c > k$), no moderation scheme can induce opportunistic commentators to exert effort. Recall that the maximum readership/benefit that commentators can achieve is 1 at each period. Therefore, when the cost is beyond the expected maximum readership (k), no opportunistic commentators will exert effort in any cases. For this reason, we assume that $c < k$.

From the definition of α_H, $\alpha > c/k$ is a sufficient condition to induce opportunistic commentators to exert high effort. Intuitively, $\alpha > c/k$ means that the expected increase in the current period payoff (αk) outweighs the marginal cost c, which provides commentators with adequate incentive to exert effort.

3.3.2 The Equilibrium with No Effort

Because dedicated commentators can have high reputations and low reputations, readers have a certain quality expectation of the comments even from the lowreputation commentators. As a result, opportunistic commentators may catch a "free ride" on those dedicated ones by receiving some readership without exerting any effort, as long as they are not caught in early moderation. Thus, when the moderation probability is low enough, the "free-ride" strategy would be opportunistic commentators' best choice. More precisely, when the marginal benefit from exerting effort is not enough to compensate for the marginal cost(i.e., $\alpha k + \beta k(v_1 - v_0) - c < 0$), opportunistic commentators exert no effort in equilibrium. The equilibrium expected payoffs are $v_0 = (1-\alpha)\, r_0 + \beta v_0$ and $v_1 = (1-\alpha)\, r_1 + \beta v_0$. Their difference is $v_1 - v_0 = (1-\alpha)\,(r_1 - r_0)$.

In this case, opportunistic commentators maintain low reputations because they exert no effort. As a result, the high-reputation commentators are composed purely of dedicated commentators, and therefore the expected quality of comment from them is s, i.e., $r_1 = s$. Low-reputation commentators consist of both dedicated commentators and opportunistic ones. Notice that a proportion $1-s$ of dedicated commentators is in the low-reputation category with the opportunistic ones. We can then formulate the expected quality of comments from low-reputation commentators as:

$$r_0 = \frac{\mu(1-s)s}{\mu(1-s) + 1 - \mu}. \tag{3.9}$$

Substituting r_1 and r_0 in the first-order derivative and rearranging the terms, we have

$$\alpha k \left[1 - \beta \left(s - r_0\right)\right] + \beta k \left(s - r_0\right) - c \leq 0. \tag{3.10}$$

Clearly, the left hand side is increasing in α. In other words, the lower the moderation probability, the less likely opportunistic commentators are to exert effort. The intuition is similar to the earlier case: Decreasing the moderation probability also decreases the marginal benefit from exerting effort. We define α_L as the value of α binding in the above inequality, which is:

$$\alpha_L = \frac{c/k - \beta \left(s - r_0\right)}{1 - \beta \left(s - r_0\right)}. \tag{3.11}$$

Thus, we can derive the following lemma.

Lemma 3. *Under any $\alpha \leq \alpha_L$, exerting no effort can be sustained as an equilibrium.*

The intuition is as we mentioned at the beginning of this subsection. Opportunistic commentators can expect a certain level of readership even if they do not exert any effort, as long as they do not get caught by early moderation. In this case, the certain level of expectation in the performance is attributed to the dedicated commentators, since they always contribute. This provides opportunistic commentators with a chance to free-ride. When the moderation probability is low and hence there is only a low chance of getting caught and ending up with zero readership, opportunistic commentators have an incentive to free-ride off the dedicated ones. Thus, low moderation results in no effort.

However, when the cost of effort is low enough (such that $c/k \leq \beta(s - r_0)$ and then $\alpha_L \leq 0$), exerting no effort cannot be sustained as an equilibrium, no matter how low the moderation probability is. This is because when free-riding is expected, the expected readership is also adjusted to a lower level in equilibrium. Meanwhile, opportunistic commentators always have the option to exert effort, join the high-reputation group, and obtain high expected readership. When the effort cost is very low, the benefit from free-riding will be overcome by the net benefit from exerting effort. As a result, regardless of how low the moderation probability is, opportunistic commentators choose to exert effort.

3.3.3 Mixed Strategy Equilibrium

The above analysis characterizes the opportunistic commentators' equilibrium effort choice when the moderation probability is very high or very low. What will be their equilibrium choice if the moderation probability is between the two, say $\alpha_L < \alpha < \alpha_H$?[3] In such cases, we can speculate that in equilibrium some opportunistic commentators may exert effort, whereas others do not, or they sometimes exert effort but other times do not. This involves mixed strategy equilibria.

[3]Technically speaking, $\alpha_L > \alpha_H$ may occur. In that case, multi equilibria exist for a certain range of moderation probabilities.

For a mixed strategy (between exerting effort and not exerting effort) to arise in equilibrium, opportunistic commentators must be indifferent about exerting effort or not—otherwise they could always go with the more profitable option. So the marginal benefit balances the marginal cost in equilibrium, that is, $\alpha k + \beta k(v_1 - v_0) - c = 0$. We consider a symmetric case where opportunistic commentators exert effort with probability m in each reputation.[4] In such a case, the difference in expected payoffs associated with high and low reputations is again equal to the difference in the current period payoff; i.e., $v_1 - v_0 = (1 - \alpha)(r_1 - r_0)$ (refer to (3.3).) The proportion of opportunistic commentators with high reputations will be mk. Then, we can characterize the expected qualities of comments from high reputation and low reputation, respectively, as:

$$r_1 = \frac{\mu s s + (1-\mu)\, mkmk}{\mu s + (1-\mu)\, mk}, \tag{3.12}$$

$$r_0 = \frac{\mu\,(1-s)\, s + (1-\mu)\,(1-mk)\, mk}{\mu\,(1-s) + (1-\mu)\,(1-mk)}. \tag{3.13}$$

Based on the first-order condition, we derive the mapping between the moderation probability and the mixed strategy:

$$\alpha\,(m) = \frac{c/k - \beta\,(r_1 - r_0)}{1 - \beta\,(r_1 - r_0)}. \tag{3.14}$$

Lemma 4. *For any $\alpha \in [\alpha_L, \alpha_H]$, exerting effort with probability m can be sustained as an equilibrium, where m? is determined by (3.14).*

[4] In fact, under moderation probability in this range, asymmetric equilibria may exist. For example, it could be that high-reputation opportunistic commentators have a higher probability to exert effort in some equilibrium.

A mixed strategy may arise as an equilibrium because of the externality of the benefit from free-riding. Opportunistic commentators benefit from pooling with or free-riding on dedicated ones when they do not exert effort and do not get moderated. However, as the number of free-riders increases, the readers' expectation of the pool decreases. As a result, opportunistic commentators get less readership and less benefit from free-riding. If the benefit from free-riding is greater than the net benefit from exerting effort, the number of free-riders will increase and thus the benefit declines. Otherwise, the number of free-riders decreases and the benefit from free-riding increases. In equilibrium, the benefit from free-riding balances the net benefit from exerting effort, which also determines the number of free-riders (i.e., the probability that opportunistic commentators will exert effort).

In summary, we characterize the full equilibrium under different moderation probabilities in the following proposition.

Proposition 3. *[Equilibrium Effort] The following describes an equilibrium: For $\alpha > \alpha_H$, opportunistic commentators exerting effort; for $\alpha < \alpha_L$, opportunistic commentators exerting no effort; for $\alpha \in [\alpha_L, \alpha_H]$, opportunistic commentators exerting effort with probability $m(\alpha)$ (determined by (3.14)).*

Since different moderation arrangements provide different incentives for opportunistic commentators to exert effort, moderation plays a critical role in determining the equilibrium expected performance. When the moderation probabilities are the same for high and low reputations, as we have discussed so far, the equilibrium expected performances associated with each reputation appear in a uniform rank as summarized in the following proposition. This is in contrast to the case with differentiated moderation probabilities, shown in the next section.

Proposition 4. *In equilibria described above, the expected performance of high-reputation commentators is (weakly) higher than that of low-reputation commentators. Formally,* $r_1 \geq r_0$.

This result looks very natural, since a high reputation is normally perceived as an indicator of good performance. However, it is not trivial. In our case, the expected performance of a reputation is essentially determined by the population composition (dedicated or opportunistic) under that reputation and the opportunistic commentators' performance. (Recall that dedicated commentators perform at the same level under each reputation.) Notice that in each equilibrium described above, opportunistic commentators exert the same level of effort under each reputation because of the symmetric incentive (which is due to the same moderation probability). Therefore, the above proposition, in fact, says that the higher performance commentators dominate in the high-reputation group more than in the low-reputation group.

3.3.4 Reputation Without Moderation

Reputation systems are used ubiquitously in online marketplaces and communities to provide information on users' abilities and trustworthiness. In most cases, however, they are not combined with a moderation system. In this subsection, we compare the moderated reputation system with a pure reputation system. Setting $\alpha = 0$ reduces the above moderation system into a pure reputation system.

Without moderation, the marginal benefit of exerting effort is from the increase in discounted future payoff ($\beta k(v_1 - v_0)$) only, which is in contrast to the increase in both the current period payoff and discounted future payoff in the case with moderation. The marginal cost is c, as before, so compared to the case with moderation, the marginal benefit from exerting effort diminishes while the marginal cost stays the same. As a result, we have:

Corollary 1. *The overall performance under a moderation system ($\alpha > 0$) is (weakly) better than that under a pure reputation system ($\alpha = 0$).*

The proposition indicates that moderation is generally desirable for better performance in an online community, if the cost of moderation is zero or minimal. When the moderation incurs considerable cost, the extent of moderation needs to balance the cost and the benefit. Slashdot, for instance, employs a massively distributed moderation approach, in which all eligible readers have the potential to be invited as moderators, voluntarily checking or auditing for the Slashdot community. Such an arrangement provides a cost-effective way to implement a moderation system in online communities.

3.4 Differentiated Moderation Probabilities

So far, we have taken for granted that the same moderation probability is applied to commentators in both the high- and low-reputation categories. It is plausible that the community may arrange different moderation schemes for each reputation group since, after all, reputation to some degree implies commentators' types or effort. For example, the moderation system may watch low-reputation commentators more carefully, considering that they perform poorly.

In this section, we study a more general case in which the moderation system moderates comments from different reputations with different probabilities. We denote α_1 (α_0) as the moderation probability for high- (low-) reputation commentators. Replacing the moderation probability α with the differentiated ones α_i in the payoff function (3.3), we can get a similar payoff function.

The basic tradeoff in commentators' decisions remains the same, except that now we have differentiated moderation probabilities. Similar to (3.4), the incentive to exert effort is determined by $\alpha_i k + \beta k(v_1 - v_0) - c$, $i \in \{0, 1\}$. Because of the differentiated moderation probabilities, unlike the previous case, opportunistic commentators may choose asymmetric effort in equilibrium: They may choose to exert effort when they are in one reputation category and choose not to do so when they are in the other reputation category.

We first consider the case $\alpha_1 < \alpha_0$, meaning the system watches low-reputation commentators more closely. Similar to the case in which there is no discrimination in moderation, we still can derive the upper bound and lower bound of the moderation probability to identify when opportunistic commentators do and do not exert effort. Notice that in the current case, opportunistic commentators have asymmetric incentive to exert effort when possessing different reputations. In particular, low-reputation opportunistic commentators have more incentive to exert effort, since they are more likely to get early moderation.

We are more interested in the case where opportunistic commentators may adopt different strategies under different reputations. In general, more moderation gives these commentators more incentive to exert effort. Given the moderation probabilities $\alpha_1 < \alpha_0$, it may arise as an equilibrium that opportunistic commentators exert no effort when possessing high reputations, whereas (some) opportunistic commentators exert effort when possessing low reputations. We first consider a steady state equilibrium in which a relatively small proportion w ($w < \frac{k}{1+k}$) of opportunistic commentators has high reputations and a proportion $1-w$ has low reputations (and the number of opportunistic commentators with each reputation is invariant over time). Under such a scenario, it must be the case that low-reputation opportunistic commentators exert effort with probability $\frac{w}{(1-w)k}$ to make the number of opportunistic commentators with high reputation stable. The expected performance can be formulated as:

$$r_1(w) = \frac{\mu s s}{\mu s + (1-\mu) w}, \quad (3.15)$$

$$r_0(w) = \frac{\mu (1-s) s + (1-\mu) w}{\mu (1-s) + (1-\mu)(1-w)}. \quad (3.16)$$

When a relatively large proportion ($w > \frac{k}{1+k}$) of opportunistic commentators have high reputation in a steady state equilibrium, it must be the case that some high-reputation opportunistic commentators also exert effort and all low-reputation exert effort. If the probability of high-reputation opportunistic commentators exerting effort is x, from the steady state condition, we have $w = (1-w)k + wxk$ and thus $x = 1 - \frac{k-w}{wk}$. We can then similarly formulate:

$$r_1(w) = \frac{\mu s s + (1-\mu)[w - (1-w)k]}{\mu s + (1-\mu) w}, \quad (3.17)$$

$$r_0(w) = \frac{\mu (1-s) s + (1-\mu)(1-w) k}{\mu (1-s) + (1-\mu)(1-w)}. \quad (3.18)$$

We denote

$$H_1(\alpha_0|w) \equiv \frac{[1+\beta r_0(w)]\alpha_0 + \beta[r_1(w) - r_0(w)] - c/k}{\beta r_1(w)}, \text{ for } w \in [0, \frac{k}{1+k}],$$

and

$$H_2(\alpha_0|w) \equiv \frac{\beta[k - r_0(w)]\alpha_0 + \beta[r_0(w) - r_1(w)] + c/k}{1 + \beta k - \beta r_1(w)}, \text{ for } w \in [\frac{k}{1+k}, k].$$

As we shall see, $\alpha_1 = H(\alpha_0|w)$ defines a line on which w of opportunistic types having high reputation are sustained as an equilibrium. Such an equilibrium is characterized by the following proposition.

Proposition 5. *[Reputation Oscillation] For any* (α_0, α_1) *with* $\alpha_1 \leq \alpha_0$*, the following is an equilibrium (refer to Figure 3.1):*

(a) If $H_1(\alpha_0|\frac{k}{1+k}) < \alpha_1 \leq H_1(\alpha_0|0)$*, a proportion* w *(*$0 \leq w < \frac{k}{1+k}$*) of opportunistic commentators are in high reputation and exert no effort in each period, and the other opportunistic ones are in low reputation and exert effort with probability* $\frac{w}{(1-w)k}$*, where* w *is determined by* $H_1(\alpha_0|w) = \alpha_1$*.*

(b) If $H_2(\alpha_0|\frac{k}{1+k}) < \alpha_1 \leq H_2(\alpha_0|k)$*, a proportion* w *(*$\frac{k}{1+k} < w \leq k$*) of opportunistic commentators are in high reputation and exert effort with probability* $1 - \frac{k-w}{wk}$ *in each period, and the other opportunistic ones are in low reputation and exert effort, where* w *is determined by* $H_2(\alpha_0|w) = \alpha_1$*.*

(c) If $\alpha_1 \leq H_1(\alpha_0|\frac{k}{1+k})$ *and* $\alpha_1 \leq H_2(\alpha_0|\frac{k}{1+k})$*, a proportion* $\frac{k}{1+k}$ *of opportunistic commentators are in high reputation and exert no effort in each period,, and the other opportunistic ones are in low reputation and exert effort.*

The proposition predicts that the reputations of opportunistic commentators oscillate between high and low: They build up high reputations when they are in low-reputation states, and then exploit the reputation (with some probability in case (b)) when they are in high-reputation states.

As shown in Figure 3.1, $\alpha_1 = H(\alpha_0|w)$ or $\alpha_2 = H(\alpha_0|w)$ defines a line such that each pair of (α_0, α_1) on this line can support reputation oscillation with w of opportunistic commentators in high reputation in equilibrium.

The condition $\alpha_1 \leq H_1(\alpha_0|0)$ is to make sure that it is at least in some opportunistic commentators' interest to exert effort. In fact, when $\alpha_1 >$

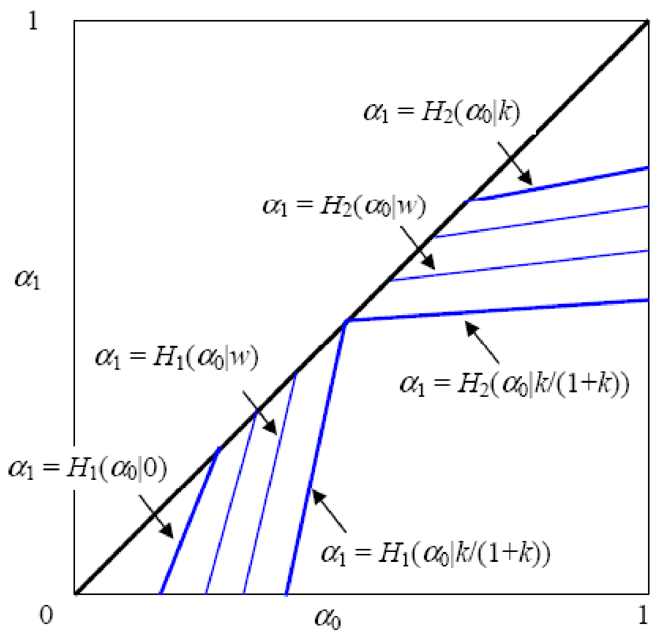

Figure 3.1: Equilibrium under $\alpha_1 < \alpha_0$

$H_1(\alpha_0|0)$, all opportunistic commentators stay at low reputation and exert no effort. (See the bottom left-hand corner in Figure 3.1.) So, similar to α_L in the case with uniform moderation probabilities, $\alpha_1 = H_1(\alpha_0|0)$ defines the boundary condition beyond which no opportunistic commentators exert effort. Similarly, $\alpha_1 = H_2(\alpha_0|k)$ defines the boundary condition beyond which all opportunistic commentators exert effort.

The above discussion shows the importance of moderation. In general, moderation plays a role in inducing opportunistic commentators' effort, and the frequency of moderation affects opportunistic commentators' incentives to exert effort. As shown above, when low-reputation commentators are moderated more frequently, opportunistic ones could optimally choose to exert more effort when they have low reputations than when they have high reputations. As a result, the overall performance of low-reputation commentators may be even better than that of high-reputation ones. The following result exemplifies the condition for such a circumstance.

Corollary 2. *[Reverse Reputation] When the equilibrium w ($w \leq \frac{k}{1+k}$), determined by $H_1(\alpha_0|w) = \alpha_1$ in Proposition 5, is greater than $\frac{\sqrt{\mu}}{1+\sqrt{\mu}}s$, the expected performance of high-reputation commentators is lower than that of low-reputation ones, that is, $r_0(w) > r_1(w)$.*

In these scenarios, high reputation, in fact, means something "bad" (and in equilibrium readers anticipate that). This is in sharp contrast to the standard reputation measure, where high reputation is believed to be an indicator of high quality (in adverse selection settings) or high effort (in moral hazard settings). Reputation under this moderation would be simply a symbol with no definite meaning, which again highlights the significant impact of moderation on online communities.

In a distributed moderation system as in Slashdot, moderators may have different preferences for checking high-reputation or low-reputation comments more frequently, as there is no direct control on their preference. As a result, it may occur that, overall, the moderators check the low-reputation commentators more often. In such instances, readers should be informed of such a fact or be guided to read comments from low-reputation commentators first, since reputation is a misleading indicator of comment quality.

Along a similar line, we can derive equilibria under moderation schemes with $\alpha_1 > \alpha_0$. In these cases, high-reputation commentators have more incentive to exert effort. In a steady state equilibrium with proportion w of the opportunistic commentators in high reputation, it must be that high-reputation commentators exert effort and low-reputation commentators exert effort with probability x such that $w = wk + (1-w)xk$ and thus $x = \frac{w(1-k)}{(1-w)k}$. Then:

$$r_1(w) = \frac{\mu ss + (1-\mu)wk}{\mu s + (1-\mu)w}, \tag{3.19}$$

$$r_0(w) = \frac{\mu(1-s)s + (1-\mu)w(1-k)}{\mu(1-s) + (1-\mu)(1-w)}. \tag{3.20}$$

The incentive conditions require that high-reputation opportunistic commentators are induced to exert effort while their low-reputation counterparts are indifferent; formally:

$$\beta k(v_1 - v_0) + \alpha_1 k - c > 0 \text{ and } \beta k(v_1 - v_0) + \alpha_0 k - c = 0. \tag{3.21}$$

Notice that $v_1 - v_0 = (\alpha_1 - \alpha_0)k + (1 - \alpha_1)r_1 - (1 - \alpha_0)r_0$. By substituting in $r_i(w)$, the incentive condition for low-reputation commentators can be reorganized as

$$\alpha_1 = M(\alpha_0|w) \equiv \frac{-(1 - \beta k + \beta r_0(w))\alpha_0 + c/k - \beta(r_1(w) - r_0(w))}{\beta(k - r_1(w))}. \tag{3.22}$$

Similarly, $\alpha_1 = M(\alpha_0|w)$ defines a line such that each pair of (α_0, α_1) on this line can support an equilibrium with w of opportunistic commentators in high reputations. (See Figure 3.2.) When the moderation probability for high-reputation commentators is below a lower bound ($M(\alpha_0|0)$), no opportunistic commentators exert effort; and when the moderation is above an upper bound ($M(\alpha_0|k)$), all opportunistic commentators exert effort.

3.5 Optimal Moderation Allocation

When the community has adequate resources for moderation, it is always desirable to moderate the comments as much as possible. For example, if the community has a total moderation resource greater than the minimum moderation requirement needed to induce the highest effort (α_H, defined by (3.8)), moderating comments with equal probability regardless of the commentators' reputations can induce opportunistic commentators to exert effort.

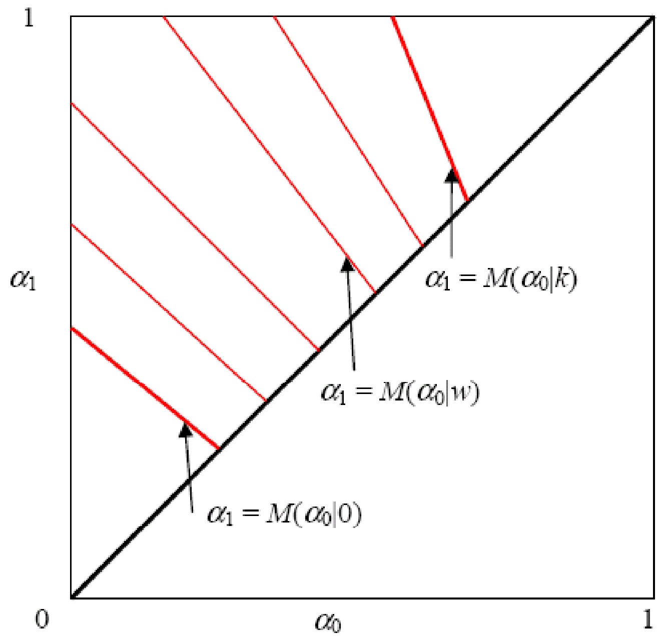

Figure 3.2: Equilibrium under $\alpha_1 > \alpha_0$

In reality, however, resources for moderation are often limited and scarce, and moderation is costly. So the community designer needs to balance the increase of overall "system performance" and the cost of doing so. In other words, the community designer faces a decision on optimal moderation. We define the overall system performance as the expected quality of all comments, or the average quality of comments from each reputation weighted by its respective population size, $n_1r_1 + n_0r_0$, where n_i is the number of commentators with reputation i. Such a definition does measure the overall system performance, since it reflects the total size of the readership of a community. Also, we assume the moderation cost is an increasing convex function of total moderation resources $(n_1\alpha_1 + n_0\alpha_0)$ and denote it as $C(n_1\alpha_1 + n_0\alpha_0)$. Then the community designer's objective function can be formulated as:

$$\max_{\alpha_1,\alpha_0}(n_1r_1 + n_0r_0) - C(n_1\alpha_1 + n_0\alpha_0). \tag{3.23}$$

Notice that the system performance $(n_1r_1 + n_0r_0)$ is determined by the expected number of opportunistic commentators who exert effort in equilibrium, as dedicated commentators' performance is not affected by the moderation system design. If proportion w of opportunistic commentators stay at high reputation over time, there must be w/k of opportunistic commentators who exert effort (considering the expected quality of a comment with effort k), and thus $n_1r_1 + n_0r_0 = \mu s + (1-\mu)w$. We next examine the minimum moderation resource required to achieve a proportion w, or

$$\min_{\alpha_1,\alpha_0}(n_1\alpha_1 + n_0\alpha_0) \tag{3.24}$$

$$\text{s.t.} \quad n_1 r_1 + n_0 r_0 = \mu s + (1-\mu)w. \tag{3.25}$$

We are interested in whether the moderation system should moderate high-reputation commentators more or low-reputation ones more. To avoid the technical complexity, we next consider a case in which the performance of opportunistic commentators with effort is at least as good as that of dedicated ones (i.e., $k \geq s$), and $\mu s \geq 1/2$, which means dedicated commentators contribute significantly.

Proposition 6. *Considering the steady-state equilibrium under* $k \geq s$ *and* $\mu s \geq 1/2$*,*

(a) For all (α_0, α_1) *with* $\alpha_1 \geq \alpha_0$*, equally moderating all commentators (i.e.,* $\alpha_0 = \alpha_1$*) is superior to any other schemes.*

(b) For all (α_0, α_1) *with* $\alpha_1 \leq \alpha_0$*, moderating low reputation only (i.e.,* $\alpha_1 = 0$*) is superior to any other schemes if* $w \leq \frac{k}{1+k}$*; otherwise, equally moderating all commentators (i.e.,* $\alpha_0 = \alpha_1$*) is superior to any other schemes.*

When moderators pay more attention to high-reputation commentators, high-reputation opportunistic commentators have higherincentive to exert effort than their low-reputation counterpart. In cases with some opportunistic commentators exerting effort, it must be that high-reputation opportunistic commentators exert effort and low-reputation opportunistic ones do not. In these cases, increasing moderation probability on low-reputation commentators (resulting in less free-riding) lowers the value of staying at low reputation. In contrast, to lower the high-reputation value, we need to reduce the moderation probability on high reputation, considering that high-reputation opportunistic commentators benefit from early moderation (by receiving maximum readership) if their performance with effort is better than that of dedicated commentators (i.e., $k \geq s$). Therefore, properly increasing moderation on low reputation and decreasing that on high reputation can keep opportunistic commentators' incentives unchanged and thus keep the system performance unchanged. (Recall the difference in reputation value influences commentators' incentives.) In some sense, moderation on high reputation and that on low reputation are substitutes. This also explains why the slope of $\alpha_1 = M(\alpha_0|w)$ is negative in Figure 3.2. The above proposition shows that moderation on low reputation is more effective when there are significantnumber of high-reputation dedicated commentators (i.e., $\mu s \geq 1/2$).

When moderators pay more attention to low-reputation commentators, high-reputation opportunistic commentators have less incentive to exert effort and may free-ride. When the moderation probabilities are not high enough (to induce high-reputation opportunistic commentators to exert effort), reducing the moderation on high-reputation commentators here results in higher value for them because of the increased chance of free-riding. Meanwhile, reducing the moderation on low-reputation commentators can increase the value of staying at low reputation. (Consider the value from exerting no effort since they are indifferent.) Therefore, properly reducing moderation on high reputations and reducing that on low reputations can keep opportunistic commentators' incentives unchanged and thus keep the system performance unchanged. Thus, the most cost-effective(efficient?) approach is to moderate low reputation at the minimum probability for a certain level of performance. The line $\alpha_1 = H_1(\alpha_0|w)$ in Figure 3.1 illustrates such intuition: As the slope of the line is positive, any point with smaller α_0 and α_1 can lead to the same system performance as other points while requiring less moderation resources; and thus the most cost-effective approach is to not moderate high reputations at all. When the moderation probabilities are high enough to induce some high-reputation opportunistic commentators to exert effort, decreasing the moderation probability on low-reputation commentators decreases the value for them because of the lower chance of receiving maximum readership if $k \geq s$. To decrease the value for high-reputation commentators, we need to reduce the moderation probability for them, using the same argument for the case with $\alpha_1 > \alpha_0$. This explains why the slope of $\alpha_1 = H_2(\alpha_0|w)$ in Figure 3.1 is positive and thus moderating high and low reputations equally is most cost-effective.

The above proposition predicts that it is optimal either to moderate high and low reputations with equal probability or to moderate low reputation only. With the condition specified in the proposition, it is not difficult to pinpoint the condition for either scheme to be optimal.

Corollary 3. *To sustain a proportion w of opportunistic commentators in high reputation in a steady-state equilibrium under $k \geq s$ and $\mu s \geq 1/2$, (a) if*

$w \leq \frac{k}{1+k}$, it is optimal to moderate low reputations only; (b) if $w > \frac{k}{1+k}$, it is optimal to moderate all commentators equally.

The reason for the optimality is as follows. When $w \leq \frac{k}{1+k}$, according to Proposition 6, for all (α_0, α_1) with $\alpha_1 \geq \alpha_0$, equally moderating commentators (i.e., $\alpha_0 = \alpha_1$) is the best choice. Meanwhile, $\alpha_0 = \alpha_1$ is a dominated moderation scheme among all (α_0, α_1) with $\alpha_1 \leq \alpha_0$—moderating low reputation only (i.e., $\alpha_1 = 0$), which is thus the optimal among all possible schemes. When $w > \frac{k}{1+k}$, equally moderating commentators is unambiguously optimal for all (α_0, α_1) with $\alpha_1 \geq \alpha_0$ and all (α_0, α_1) with $\alpha_1 \leq \alpha_0$.

Based on the above corollary and noting $n_1 r_1 + n_0 r_0 = \mu s + (1-\mu) w$, the optimization problem in (3.23) reduces to:

$$\max \left\{ \max_{0 \leq w \leq \frac{k}{1+k}} [\mu s + (1-\mu) w] - C\left(n_0(w)\alpha_0(w)\right), \max_{\frac{k}{1+k} < w \leq k} [\mu s + (1-\mu) w] - C\left(\alpha_0(w)\right) \right\}$$

where the first $\alpha_0(w)$ is determined by $H_1(\alpha_0|w) = \alpha_1 = 0$ and the second $\alpha_0(w)$ is determined by $H_2(\alpha_0|w) = \alpha_1 = \alpha_0$.

The following example illustrates that moderating low-reputation commentators only may indeed appear as the optimal solution.

Example 2 *Let $\mu = 3/4$, $s = 2/3$, $c = 1/2$, $\beta = 4/5$, $k = 1$, and specify the cost function as $2(n_1\alpha_1 + n_0\alpha_0)^2$. We can verify that simply moderating low-reputation commentators only with probability $1/2$ can yield net value 0.55, whereas the best result with an equal moderation probability is to moderate nobody, which yields net value $1/2$.*

In fact, the relative size of the dedicated commentators population to that of the opportunistic ones plays an important role in the choice of optimal moderation. When dedicated commentators are the majority and most of them are in high reputation, as in the above example, moderating high reputation becomes very costly without much benefit, since those dedicated commentators contribute anyway. In contrast, moderating low reputation is less costly due to the relatively small population there, and properly imposing some moderation may motivate some opportunistic commentators to exert effort.

Connecting the above observation to Proposition 5 and Corollary 2, we conclude that the interesting and seemingly abnormal results on reputation oscillation and reverse reputation can arise as an optimal solution.

In some special cases, the total moderation resource (say α_T) is exogenously given, or the moderation cost is extremely high beyond a certain levelin light of the cost function we discussed above. According to the above proposition, it is optimal eitherto moderate commentators with equal probability (i.e., *even moderation*) or to moderate the low-reputation group only. If the moderation probability is low (lower than α_L), even moderation is unable to induce opportunistic commentators to exert effort. In contrast, if the same moderation resource is applied to the low-reputation group, it may provide incentive to some commentators in that group to exert effort. So we have the following result.

Corollary 4. *When the total moderation α_T is exogeneously given and less than α_L, if $\mu s \geq 1/2$, moderating the low-reputation group only is optimal.*

In general, when the moderation resource is limited, equal allocation of the moderation resource dilutes the moderation frequency and thus dilutes opportunistic commentators' incentive to exert effort. As a result, opportunistic commentators may exert no effort in either reputation category. In contrast, by concentrating the resources on one reputation category, it may provide enough incentive for the opportunistic commentators with that reputation to exert effort because of the increased current-stage payoff.

In distributed moderation systems like that in Slashdot, moderation is performed not by a central moderator but by distributed ones, as mentioned earlier. In these instances, it is advised that system designers provide detailed moderation guidance for potential moderators. For example, designers should tell them to focus more on high reputation, or the reverse. Furthermore, the guidance should depend on components of the commentator population and should be adjusted accordingly as the population changes.

3.6 Conclusion

In this paper, we investigated the impact of a moderation system on the performance of online communities. First, we considered equal moderation probability for different reputations and found that moderation probabilities critically affect opportunistic commentators' behavior. In particular, there is a lower bound on the moderation probability to induce effort and an upper bound to induce no effort. If a reputation system without moderation is viewed as a benchmark, we showed that the reputation system with moderation always outperforms the benchmark system. Then, we studied a model with differentiated moderation probabilities for different reputations, where we discovered reverse reputation and reputation oscillation. It was shown that agents in the low-reputation category may exert more effort than those in the high-reputation category and then exploit their reputation when they reach the high levels. As a result, the expected performance from the low-reputation category is even better than that from the high-reputation one. Finally, we discussed the optimal moderation resource allocation. We found that when moderation is costly, optimal moderation involves moderating commentators with equal probability or moderating low-reputation commentators only under some mild conditions. We also illustrated that reputation oscillation and reverse reputation can arise in equilibrium, even under the optimal moderation allocation.

Our study provides insights for online community governance. For the purpose of inducing quality content, an online community should introduce a moderation system to monitor commentator-generated content. Promotional chats are commonly observed over the Internet (Mayzlin, 2006). Moderation not only effectively screens out this biased information but also regulates the advertisers or other commentators who otherwise would easily take advantage of the anonymity in the communities. Also, it is worth noting that the frequency of moderation is critical and should be properly chosen for better performance of the online community. Especially when moderation resources are limited (e.g., in terms of personnel, system capacity, and so on), resources should be directed toward users according to their reputation; it is generally effective to start by moderating one group of users with the same reputation.

The current study can be extended in several directions. First, it is sensible to introduce an adverse selection problem with opportunistic commentators. In general, opportunistic commentators can differ in their hidden abilities to generate high-quality information. Such hidden abilities are due to various factors, such as their knowledge level and the opportunity cost of their effort/time. As a result, a simple reputation measure that considers only the recent moderation outcome is insufficient because the reputation not only is about the threat of future punishment but also involves learning about agents' types. A model with an adverse selection problem can be expected to offer more significant results. In addition, once a richer reputation measure is introduced, the moderation scheme can then be further refined based on agents' reputation/history. How to tailor moderation for commentators with different reputations is another question for future research.

Second, it is interesting and important to consider competition among online communities for the next step. Competition is ubiquitous on the Internet. While our paper derives insight about the effect and design of moderation policy isolated from the competition, studying the moderation mechanism in a competitive environment can complement and enrich our results.

Finally, conducting further empirical or experimental tests on our results remains to be considered as interesting direction. The results derived from our analytical model provide many testable predictions about the effect of moderation policy, such as that high moderation probability generally leads to high quality of comment and that reverse reputation may occur when the moderation probability on low-reputation commentators is higher than on high-reputation ones. For example, the quality of comments on Slashdot can be readily measured, as the score that a comment receives reflects its quality. The moderation probability can be approximated using the percentage of comments being moderated out of the total comments in each category or the percentage of the number of moderation being conducted out of the number of total comments.

Chapter 4

Aspects of the Market for Certification

The truth is rarely pure and never simple. (Oscar Wilde)

4.1 Introduction

The essential problem in the market for certification is how to guarantee that the net result, statements and their (non-)certification, convey the information relevant for investors. Issuers of debt or securities, be they operating firms raising capital, financial institutions selling securitized debts, or governments issuing bonds, are interested in obtaining capital at a lower cost. Realizing this, investors are not willing to believe the information provided unless verified by an external certifier, such as an auditing firm or a credit rating agency. However, recent events have cast doubt on the credibility of certifiers.[1]

[1]For example, following the collapse of the Enron Corp., their auditor, Arthur Anderson LLP surrendered its CPA licenses and its right to practice in 2002. Ernst & Young, as the auditor of the Lehman Brothers, has faced inquiry by the U.K. government after the US bankruptcy court examiner revealed evidence that the Lehman Brothers has prepared misleading financial statement using an accounting gimmick called "repo 105" (Baer and Thomas, 2010), which was also used by Bank of America Corp. and Citigroup Inc. to hide billions of dollars of debt (Rapoport, 2010). Meanwhile, the Senate subcommittee has launched an investigation of Moody's and Standard & Poor's, which confirmed their act of skewing their assessments to please their clients (Krugman, 2010).

The previous literature on auditing and credit rating agencies mainly investigated how to motivate certifiers to exert appropriately high effort in gathering evidence and evaluating clients under the assumption that the certifiers always truthfully reveal their evaluation. We identify two questions as key to determinants of certifiers' reliability: Who buys the certification, the firms or the investors; and what is the balance of power between firms and certifiers? There is an extensive literature studying how to optimally motivate a single certifier hired by the investors to evaluate the actions of an operating firm. By contrast, we focus on the potential conflicts of interest that arise when the operating firm hires a certifier in a competitive market to evaluate its actions.[2]

We also examine the balance of power by analyzing the possibility that certifiers may strategically misreport their findings in the direction favored by the operating firms. This possibility is limited by the observation that investors ignore "fluff." More formally, if misreporting is too large, the information value of certification does not induce investment.

4.1.1 Historical background of certification

When the users of the certification are the buyers, they can directly reward or punish the certifier for accuracies or inaccuracies. Auditors hired by lenders as part of the loan approval process are in this position, especially in-house auditors. For the first three decades of its existence, Underwriters

[2]To the best of our knowledge, we are the first to analyze competition between certifiers.

Laboratory was in a similar position.

Supported by a consortium of insurance companies, Underwriters Laboratory (UL) developed expertise in rating the safety of electric appliances of all sorts. The insurance companies used these ratings and the safety standards behind them in the design and pricing of their policies, making a UL certification very valuable.

In 1935, UL broke from the insurance companies, reorganizing itself as a non-profit company and beginning to charge manufacturing companies to have goods and processes certified. Perhaps because of UL's near monopoly, or their corporate culture, or perhaps because of the small size of the individual certification purchases, the manufacturers, that is, the certification buyers, have never achieved significant power in this market. As evidence of this, UL sets prices for their services and sets the safety standards. Being a non-profit, the prices are small enough that the manufacturers have not systematically complained. Being a self-consciously expert organization, the standards are set in consultation with industry and government, and again, there have not been systematic complaints.

In a similar fashion, during the late 19^{th} and early 20^{th} century, the rating agencies, Moody's, Fitch's, and Poor's (later of Standard and Poor's) specialized in collecting, aggregating and interpreting huge amounts of data on the railroads. The buyers of these analyses were interested investors. Especially with the passage of the 1934 Securities Exchange Act, the investor pool buying the raters' analyses expanded to cover institutions constrained to hold only "investment grade" paper.

In the 1970's, perhaps as a result of the plunging cost of photocopies, the market structure shifted, and the asset issuers became the buyers of the certification. By itself, this change might not have been problematic, but in the 1990's, three developments shifted the balance of power toward the financial institutions issuing the assets. First, the assets being issued began to change from corporate bonds to structured financial assets. Second, in the interest of "transparency," the ratings agencies made public the criteria by which they assigned ratings. This included a description of the 'fudge factors,' the places where judgement calls by the ratings agency can adjust the criteria. Third, the issuers began to hire the ratings agencies as consultants, and began to hire individuals raters from the agencies at salaries much higher than the agencies were paying them.

The first and second change combined to open room between ratings assigned by different agencies to the same asset. The first change turned the asset issuer's problem into a linear programming problem, finding e.g. the cheapest mix of assets achieving a given rating will, if the prices of the assets are accurate reflections of their risk, systematically lowers the average quality of bonds receiving that rating.[3] The second change gave the issuers the ability to play the ratings agencies off against each other, to shop for ratings.[4] The

[3]This problem was exacerbated by the raters' decision to model the correlationsbetween assets in the same industry as being 0.3, and as being 0.0 across industries. See Partnoy (2009) for this.

[4]Senator Franken's recent proposed amendment to the Nelson and Schumer financial

third change gave the issuers yet more clout in shopping for ratings.

Our analysis will focus on the possibility that liability might ameliorate the problems of the sort that have arisen from raters' behavior. However, this has little relevance for this particular set of business, they have never lost a case trying to hold them liable for damages arising from their actions. The 1934 Securities Exchange Act specifically exempts the raters from liability, and it seems to be settled law that their ratings are "commercial speech," hence under stringent 1'st Amendment protection.

Auditors of firms' financial statements are in a position intermediate between UL and the ratings agencies. The buyers of the certification are the firms whose statements are being certified, and auditors routinely perform consulting services for the firms buying the certification. However, unlike the ratings agencies, the consulting activities are both somewhat limited by regulation, and, at least since 2001, consulting fees must be reported.[5] Further, auditing firms must be partnerships rather than limited liability corporations, they have been held liable for damages resulting from errors (however they arise). Further, reputational concerns play a large role for auditors, both as a marketing tool for their services and as a potent dissuader of potential lawsuits.

---

regulation bill is directly aimed at this problem.

[5]If one looks only at the effects on share price of high consulting fees, one cannot reject the hypothesis that there is no effect of high consulting fees. However, if one looks at cases in which there are quarterly earnings surprises, higher consulting fees are associated with a significant negative and long lasting effects on share prices (Fracis and Ke, *Review of Accounting Studies* 2006).

4.1.2 Theoretical Issues

Information is a public good, and public goods are typically underprovided by markets. This problem is compounded by the low cost of reproducing information. To counteract this, publicly traded firms must file quarterly financial statements.

Talk is cheap, and without some kind of verification requirements on the financial statements, so are financial statements. A standard response to the cheap talk dilemna is for those who wish to distinguish themselves to take an action that is so costly for others that it serves as a credible signal. We study cases in which other credible signals are too expensive, and auditing is accurate enough and cheap enough that firms who wish to distinguish themselves would gladly submit to audits of their statements.

4.1.3 Description of Model and Results

We consider a multi-stage game with two types of firms, two types of certifiers, and homogeneous investors. We start with a simple case where the certifiers are non-strategic. This represents the ideal certifier who is dedicated to supply reliable information without additional incentives. We focus on each firms' decision on which certifier to patronize. Since the better certifier conveys a better signal to investors, and leads to a higher chance of investment, it may be intuitive to conjecture that ethical firms would prefer the better certifier. However, when the unethical firms realize this signaling role, they will choose the better certifier as well because the high investment probability justifies the higher cost associated with the better certifier. We find the only stable equilibrium in which investors earn the market rate of return. This is equivalent to the auditing literature, where the two types of certifiers are equivalent to the moral hazard issues.

We then expand the analysis by considering strategic certifiers who are not subject to litigation. This presents the current situation of the rating agencies. On the one hand, certifiers now are economic agents who seek for maximal profit instead of quality of information; on the other hand, firms may not actually prefer reliable information as long as it helps them to attract investment. The two parties usually reach an agreement on what certification to provide through the payment contract between them, which is the focus of this section. We are able to derive the equilibrium contract that firms will offer when they discover the market for certifications. We then examine the impact of this payment contract on the quality of information conveyed through the certifiers, and how this change of quality takes place through a Markov scramble process.

In our third model, we introduce litigation over certifiers to the framework. Litigation serves as a disincentive for certifiers to participate in the market for certifications, and in other words, motivates certifiers to supply quality information with their expertise. This model resembles the auditing industry, which is one of the most influential and established industry in the certification business. We identify two questions in this setting: What is the impact of litigation on firms' choice of certifiers, investors' investment, and certifiers' incentive to distort information? and, What is the proper level of litigation that may increase the reliability of certification?

We have two main conclusions from our analysis: (1) contingent fee is necessary to induce quality reporting; even well-performing operating firms have to provide a contingent payment contract. (2) operating firms have a tendency to lower the accuracy of high quality auditing firms and to narrow the difference between auditing firms. (3) excessive liability may lead to overly cautious audit and have a negative impact on financial reporting.

The rest of the paper is organized as follows. Section 2 provides a review of literature in information communication and certification industries. Section 3 lays out a general model along with critical assumptions. Section 4 & 5 examines certifiers' strategic role with and without litigation. Section 6 concludes the paper.

4.2 Review of Literature

This paper is related to the literature in several areas. The fundamental question we are addressing is how to turn a cheap talk scenario into effective communication between firms and investors. Without certifiers such as auditors or rating agencies, low-type firms have little incentive to reveal their performance, and investors simply ignores firms' announcement (Crawford and Sobel, 1982). When the relative proportion of low-type firms' is large enough, investors' confidence in the market becomes low and do not invest. Even high-type firms have difficulty in getting financing and are driven out of the capital market, which leads to a scenario of market for "Lemons" (Akerlof, 1970). Certifiers, as an intermediary between firms and investors, provide their third-party opinion to facilitate investors making more efficient investment decisions. Below we discuss in detail literature on certification, credit rating agencies, and auditing.

4.2.1 Certification

Certification industries are usually concentrated with a few oligopolies (Furchtgott-Roth et al. (2006), Coskun (2008)). One major topic in the certification literature is the market for certification, or in other words, who induces the certification market, the buyer or the seller? This question is interesting because certification plays a significantly different role when induced by the buyer than when induced by the seller. Certification plays a role of costly inspection device when demanded by the buyer, while a role of signaling device when demanded by the seller. The certifiers put in their knowledge and effort to provide a third-party opinion on the situation so that buyers make better decisions. [put in costly-state verification] We view most certification scenarios as a mixture of both inspection and signaling. We take the industry structure of certification as given, and focus on certifiers' strategic decisions in reporting.

Fasten and Hofmann (2010) studies a two-sided market for certifications under the assumption of a non-strategic certifier with a perfect information revelation technology. The certifier determines a price for certification in order to maximize overall profit. We look at certifiers' strategic certification issuance instead. Stahl and Strausz (2009) looks at a similar problem in the auto production setting. They compare buyer-induced certification versus seller-induced certification to obtain the maximum profit for the certifier.

Lizzeri (1999) looks at an intermediary's strategic decisions in both pricing and degree of disclosure. Specifically, this paper looked at different disclosure rules including full disclosure, no disclosure or noisy disclosure, which could mean to disclose a partition. Somehow the paper showed that the profit maximizing disclosure rule is to only certify firms above a certain threshold. (He models non-contingent fees and ratings concerning the value of an object. In equilibrium a monopoly CRA will only reveal whether the object has positive or negative value to capture maximum surplus.)

4.2.2 Credit Rating Agencies

Credit rating agencies are in a very similar position to auditing firms. Although there are important differences between the ratings industry and the accounting industry, a parallel is often made with the rapid fall of the accounting firm Arthur Andersen following its implication in the Enron scandal. Rating agencies often argue that they have their reputation at stake for offering reliable ratings, the fundamental conflict of interest generated by their business model has drawn increasing attention.

One intuitive way to promote honest behavior by rating agencies is through financial institution regulation. Rating agencies are designated by the Securities and Exchange Commission(SEC) and have to obtain approval to become a Nationally Recognized Statistical Rating Organization(NRSRO). Stolper (2009) adopted a principal-agent framework to study rating agencies' moral hazard problem in a setting where a regulator approves credit rating agencies. When there are multiple rating agencies who can potentially collude with each other, it becomes difficult for the regulator, in this case the SEC, to evaluate the performance of each rating agency. This is especially so in the financial market because of the market risk. When a client default, it does not necessarily indicate rating inflation by a certain rating agency, but could also be a result of the market risk.

Mathis et al. (2009) studies the impact of reputation on rating agencies' truthful behavior. When rating agencies' income mainly comes from security issuers, they tend to manipulate their reputation: they report truthfully to build a good reputation, and then they become lax which cause their reputation to fall brutally.

Another force for truthful behavior that many people believes in is industry competition. However, research has shown that while competition intuitively provides incentive for quality reporting, it also causes unintended consequences. For example, Skreta and Veldkamp (2009) show that if issuers can choose one rating from among multiple ratings to disclose, then an increase in the complexity of securities produces rating inflation. Sangiorgi et al. (2009) discusses potential winner's curse where the most favorable rating is the only rating published when multiple rating agencies exist.

4.2.3 Auditing

The main difference of this paper from previous literature is that we focus on the impact of liability on auditors' reporting decisions instead of audit effort decisions. Throughout the audit engagement, auditors' involvement takes place in stages, i.e. client acceptance decision, audit quality decision, audit report decision. Auditor independence as well as the reliability of the certification are concerns in each of the four stages. The previous literature has examined the determinants to whether an auditor accepts an client (Laux and Newman, 2010) and to the level of effort chosen by the auditors (Dye (1993), Carcello and Palmrose (1994)), while our paper take a different perspective by examining auditors' decision to truthful report after they have made an effort to audit their clients.

Choi et al. (2008) provides both analytical and empirical analysis on audit fee premium in variant to the legal environment and the auditing firm's status, i.e. Big 4 and non-Big 4. In their theoretical framework, the model is similar to the settings in this paper in the following aspects: first, an auditor examines the type of a project, and their opinion influences the investment decision; second, audit is imperfect and the auditor makes legal payment with a certain probability in the case of an audit failure. However, this model is significantly different from our model in that: first, they assume that auditor will report exactly what they discover; second, the owners are also the investors, which eliminates the misreport problem and turns the model into a costly-state-verification scenario; also, the owners/investors do not update their belief with the information from the audit, instead, they behave naively and invest if and only if the auditors issue good opinion; and lastly, although they have considered two types of auditors, the model is fairly static and generates no insight on the competition between the two. These differences have led to a different focus in our paper. On the one hand, we are interested in the fundamental issue in auditing, that is the misreport by the firms and the conspiracy between auditors and firms. We examine auditors' fee premium, as well as the spread between Big 4 and non-Big 4 firms in this strategic setting. In this sense, our paper provides a closer-to-reality explanation of auditing firms' behavior and the impact of the different legal regimes. Secondly, and very importantly, we intend to capture a more complete picture of the audit industry where the Big 4 firms are in fact competing with the non-Big 4 firms, and client companies face a choice of two types of auditors.

Hillegeist (1999) compares different liability apportionment rules in a setting with strategic reporting by clients and strategic auditing by the auditors. One important difference is that this paper looks at the case when investors will invest even without additional information from the audit, while our model intend to address the ineffectiveness due to lack of information. Also, our paper allows for competition between different auditors. The main different is that our model looks at auditors as strategic agent too. They may end up reporting truthfully what they discover from their audit, but it will occur in our paper as an equilibrium outcome instead of an assumption.

Lu and Sapra (2009), in a specific setting of firm acquisition, investigates the link between auditor conservatism and litigation, where auditor conservatism is defined as auditors disapproves a report when in doubt. They adopted an audit technology that induces conclusive evidence, from which the auditor is certain about their clients' performance, or inconclusive evidence, from which the auditor in not sure about their clients' performance and make an judgment call. In terms of audit fee structure, they have separated audit and non-audit fees, where the former is non-contingent, while the latter is contingent on auditors' opinions. One advantage of our model is that we interpret auditors' strategic decisions without emphasizing the size of non-audit fees. As long as there is client pressure, we predict deterioration in audit quality. One of their major conclusion is that non-audit fees may improve audit quality, because it induces auditor approval when they have to make judgment call. We reach the same result but from a different angle, i.e., non-audit fees are necessary to compensate for the risk that auditors take when they attest approval. Also, our Markov scramble result demonstrates their conclusion that mandatory restriction of client pressure/non-audit fee, from Section 201 of the Sarbanes-Oxley Act of 2002, increases auditor conservatism.

Schwartz (1997) focus on damage measurements, or the liability level, in order to prevent wealth transfer from auditors to investors. When the liability level is determined by the actual investment level, over-investment occurs due to the insurance effect of litigation. Therefore, this paper concludes that the appropriate liability should be independent of the actual investment. In comparison, although our paper focus on auditors' strategic behaviors instead of investment levels, we provide implications on the impact of litigation on investors' over-investment decisions.

Laux and Newman (2010) studies the impact of litigation on client acceptance rate. This paper proposes a triangle effect of litigation. High litigation (damage payments) leads to high audit fees paid by the firms. However, Laux and Newman (2010) suggests that the increase in audit fee is justified by the increase in investment. Our study reveals a different impact of high audit fee, that is, it serves as a risk premium/insurance to auditors for issuing favorable report, but not necessarily increase the audit quality or the level of investment. One interesting aspect of this paper is that they separated the types of project from the types of the firm. Auditors only report on the types of the project, whereas the types of the firm determines whether they accept the client. However, it is tricky that the success rate of the project depends on both the firm type and the project type. Two doubts about the paper: 1) investors finance the whole project but receives only partial cash flow (see page 6.) 2) investors make naive decision without updating beliefs.

Auditor competition can happen in two aspects: audit quality and capacity to pay damages. Bar-Yosef and Sarath (2005) provided an analysis of the audit industry structure where Big-4 firms hold an extremely large market share.

Auditor independence has been a widely recognized issue by regulators and researchers. The collapse of Enron Corp has led to a debate over auditor independence and non-audit fees. Frankel et al. (2002) presented evidence that firms purchasing more non-audit services from their external auditor are more likely to just meet or beat analysts' forecasts and to report larger absolute discretionary accruals.

4.3 The Model

We analyze a game with three risk-neutral entities: firms, certifiers, and investors. We consider two types of firms, H and L, who are seeking to raise capital from outside investors. The proportion of H-firms is μ and the proportion of L-firms is $1-\mu$. The type of a firm is private information to the firm itself, while μ is public information. The return of investment depends on the type of the firm. H-firms who are in a good financial situation can generate more cash flow from the projects than L-firms, whose financial performance is relatively poor. Investors have a higher willingness to pay for H-firms than L-firms due to the different cash flows they generate. However, since firms' types are unobservable, the investors need to learn the types of the firms in order to determine their willingness to pay for a specific firm. In other words, if properly priced, both H and L-firms deliver the same return to the investors. L-firms have a tendency to disguise as a H-firm, and induce investors to over-pay for their securities and lower investors' return. For example, a few months before Lehman Brothers filed for bankruptcy in September 2008 with $613 billion in bank debt, it has shifted $49 billion debt using a process called Repo 105 to make its balance sheet look better to investors. A certification is frequently needed for provide a third-party opinion on the performance of the firm. We consider two types of certifiers, A and B. Certifier A is capable of providing more accurate opinions than certifier B. The different accuracy levels can be caused by the certifier's experience or knowledge. Certifiers' types are known to the certifiers and also are observable to both firms and investors. Certifiers express their opinion by issuing a certification, which can be of two types, h, which means a high assessment of the associated firms' expected return, and l, which means a low assessment of the expected return. Investors would like to know the true type of the firm so that the price can be properly determined. (They do not prefer high to low instead just have an interest in learning the knowledge.)

The timeline of events is shown in Figure 4.1. In the first stage of the game, both types of firms announce a type to investors. Then they choose between two certifiers for a certification about their type. Meanwhile, they make a take-it-or-leave-it offer to the certifier which specifies the payment scheme. In the third stage, the certifier investigate their patron and issue an certification. The investors observe the announcement and the certification, as well as certifiers' types, and decide whether to invest in the firms. The investors have an outside option that gives a market rate of return, r.

We present the payoff structure for both firms and investors in the following table.

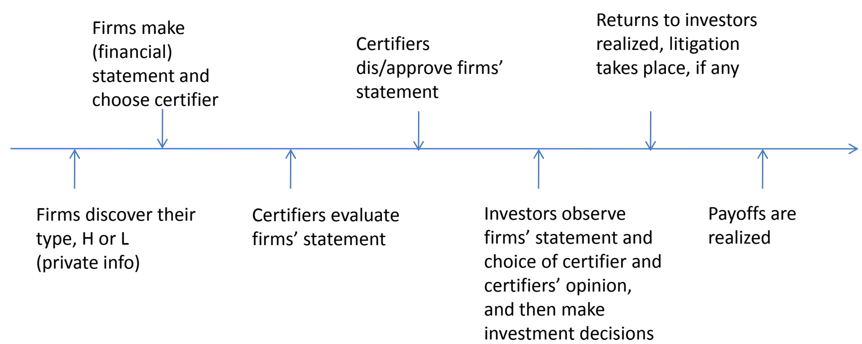

Figure 4.1: Sequence of play by operating firms, certifiers, and investors

Firms' payoff	H-Firms	L-Firms
Investment	1	a
No Investment	b	0

Investors' payoff	H-Firms	L-Firms
Investment	1	0
No Investment	r	r

H-firms always have a higher rate of return than L-firms and investors strictly prefer to invest in H-firms.

The technology for certification are defined in the following table. We follow the economics and accounting literature to describe the technology in terms of its false-positive and false-negative probabilities. For instance, when an A certifier faces a H-firm, the certifier believes it is a H-firm with confidence level s_A. We let $s_A > s_B$ and $t_A > t_B$. In other words, certifier A is less likely to make a misjudgement on firms' types. The certifiers also incur a cost of investigation, denoted by c_A and c_B. We normalize the cost for certifier B to $c_B = 0$ and assume that $c_A > 0$.

Certifier A	H-Firms	L-Firms	Certifier B	H-Firms	L-Firms
H	s_A	$1 - t_A$	H	s_B	$1 - t_B$
L	$1 - s_A$	t_A	L	$1 - s_B$	t_B

We focus on cases where information is necessary for investment to occur. If there is no certifier, investors would act based on their initial belief, μ. When the high type firms predominate, i.e. $\mu > r$, all investors invest since the expected payoff is higher than the outside option, the market rate of return, which yields r. The outcome is a cheap talk scenario, because both types of firms will announce H, and knowing this, investors ignores their announcement. However, cheap talk does not lead to market breakdown as in the Lemon's market because there are sufficient H-firms that are preferred by investors. When the sufficiency is not longer true, there is a role for additional statistically meaningful information. To investigate this case, the following assumption is in force throughout.

Assumption 1. *Information is needed for investment to occur,* $\mu < r$.

We also take into account potential regulation and litigation on certifiers. When investment failure occurs, investors can sue the certifier. If the court judges the certifier to be liable for misleading investors, the certifier will be penalized to compensate for investors' loss. Let ξ denote the expected level of liability and ψ denote the proportion of penalty allocated to investors. We will consider three different contexts where the litigation environment differs.

The following table summarizes a list of notations that will be used in the rest of the paper.

H	Firms that generate a higher cash flow from the project
L	Firms that generate a lower cash flow from the project
A	Certifiers with higher accuracy
B	Certifiers with lower accuracy
a	L-firms' payoff when they receive investment
b	H-firms' payoff when they do not receive investment
r	Return from investors' outside investment opportunity
α	Probability of L firms choosing certifier B in equilibrium
β	Probability of H firms choosing certifier B in equilibrium
p	Probability of investment upon certification h from certifier A in equilibrium
q	Probability of investment upon certification h from certifier B in equilibrium
ξ	The expected level of liability when certifiers issue a false positive certification
ψ	Proportion of liability relocated to investors

We summarize the differences between the three models in below.

Model 1: Non-strategic Certifiers We first analyze firms' certifier choice and reporting strategy when the certifiers are non-strategic. Certifiers are created to provide their independent and unbiased third-party opinions. They are often required to disclose any conflict-of-interest with the firms or the investors. For instance, rating agencies and analysts, after offering their stock or bond recommendations, have to state whether they own the stock that they recommend. The auditing industry has also put in force several standards to reduce the potential conflicts of interest, including required rotation of engaged partners and limiting the non-audit service that auditors can perform for the same client firm. We study non-strategic certifiers as a benchmark model.

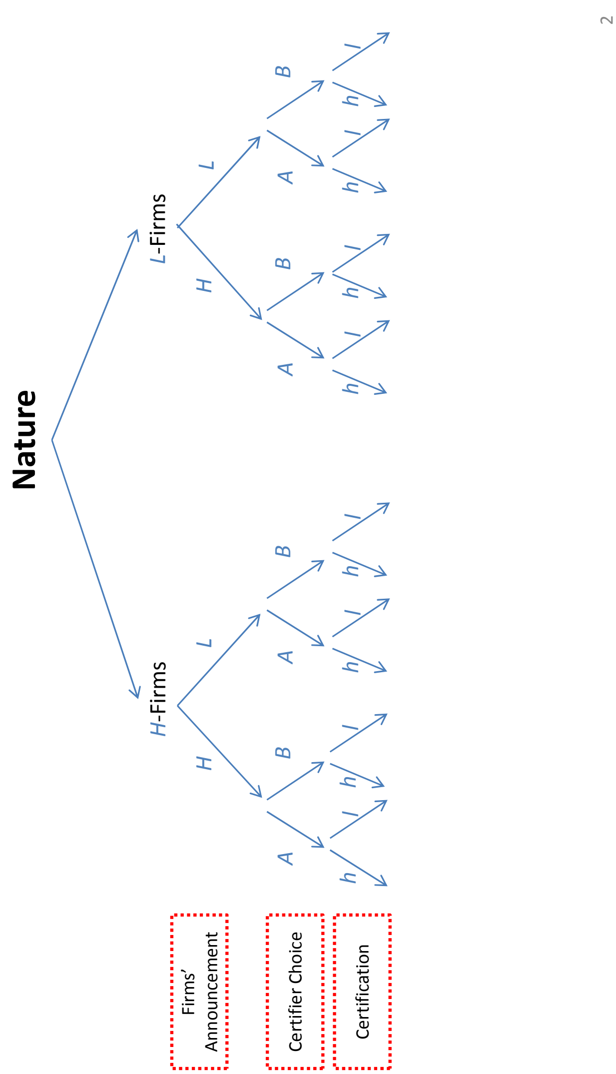

Figure 4.2: Game tree with nonstrategic certifiers

Figure 4.2 gives the sequence of play and players' action spaces. The cost difference between certifier A and B can potentially serve as a screening device to separate H- and L-firms. When c_A is significantly high, L-firms' payoff from investment is not enough to compensate for the cost of hiring such a certifier. To avoid a simple signaling game where the cost of certifier play a critical role in firms' strategies, we assume that L-firms' expected benefit to choosing a type A certifier is positive. By imposing this assumption all through the rest of the paper, we can take a mechanism design approach to investigate litigation and auditor competition.

Assumption 2. $c_A < a(1 - t_A)$.

Model 2: Strategic Certifiers without Litigation We relax the assumption of non-strategic certifiers and consider certifiers as a rational economic agent who determine issuance of certification to maximize their profit. Firms, who seek to maximize their own payoff, offer payment contracts to induce a preferable certification. The strategic aspect of certifiers is reflected in the impact of payment contract on certifiers' certification issuance.

Figure 4.3 describe the new structure of the game.

We follow the same assumptions from the previous section. In addition, we assume that investors can observe the payment contract offered to certifiers. Certifiers are often required to disclose the fee that they receive from their clients.

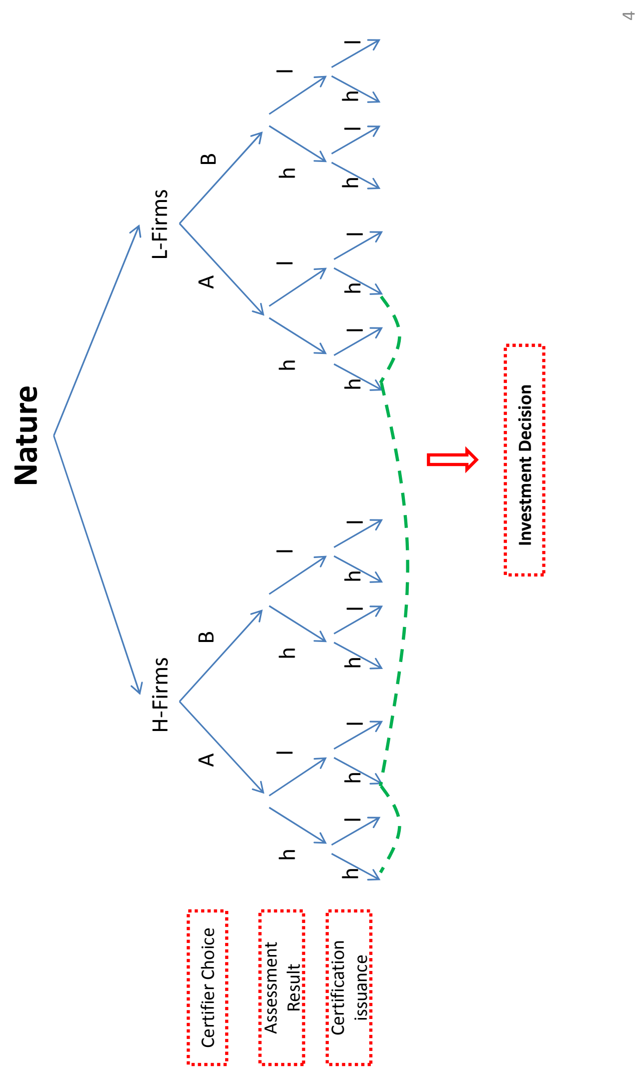

Figure 4.3: Game Tree with Strategic Auditors

Assumption 3. *The audit contract is observable to investors.*

Assumption 4. *If the auditors are indifferent, they will choose actions according to the distribution making the equilibrium as profitable as possible for the operating firms.*

There are two observations to be made about the assumption that the choice is over the equilibrium. First of all, we are treating the choice of contract as a mechanism design problem. Also, we are assuming that the investors know the equilibrium reliability of auditor reports in choosing their best responses. In situations of changing standards/behavior, the equilibrium assumption may be less compelling. However, it forces the operating firms to pay attention to the long run implications of lowering the quality of information available to the investors.

To focus on the strategic property of certifiers, we leave out potential litigation for now, meaning $\xi = 0$. For example, rating agencies including Moody's, Standard & Poor, and Fitch, are protected by the SEC from litigation. They also frequently use First Amendment Right to escape from lawsuits.

Model 3: Strategic Certifiers with Litigation

Auditors' importance has been greatly enhanced since the prosperity of the financial market. The SEC has required publicly traded firms to have their annual statement audited. However, since the Enron scandal, among many others, litigation has been enforced to regulate auditors' behaviors and to align their incentive towards interests of the investors. Empirical studies have provided contrary evidence as to the role of litigation. This model provides a theoretical analysis on the conditions under which litigation is useful in aligning incentives. We define ξ as the expected liability in case of a misreport on L-firms, in other words $\xi = probabilityofbeingsuedXLiability$. Denote $E(Lia)$ as auditors' expected liability, then $E(Lia) = prob(L)(1 - t_A)\xi$.

We provide equilibrium analysis and comparative statics for each model in the following sections.

4.4 Model 1: Nonstrategic Certifiers

In this section, we examine the benchmark model — certifiers truthfully reveal their results without any strategic considerations. Here, certifiers do not have a conflict of interest with either the firms or the investors, and act as an independent and unbiased third party.

We first simplify the game tree by iterative elimination of weakly dominated strategies. We then describe the set of equilibria. After this, we summarize the main results for the game, and end with the details of the arguments.

4.4.1 Equilibrium analysis

4.4.1.1 Firms' announcement

We first make the following statement about firms' announcement strategies.

Lemma 5. *Making L announcement is a weakly dominated strategy for both H and L firms.*

See appendix for proof. Intuitively, announcing H helps H-firms reveal their good performance to the investors. However, because L-firms want to free ride H-firms and obtain investment, they make the same choice to announce H. Consequently, firms' announcement does not provide additional information for investors' decisions. We describe the new game tree with Figure 4.4.

We further assume that H-firms are more motivated to choose type A auditor then L-firms, that is, the expected gain to H-firms from going to A is larger than the expected gain to the L-firms.

Assumption 5. $s_A(1-b) > a(1-t_A)$.

4.4.1.2 Investor Behavior

We begin with ascertaining cutoff beliefs determining investor behavior, and we give the three parameter regions for the different types of equilibria.

Given a choice of certifier, $\gamma \in \{A, B\}$, and the certifier's report, $\rho \in \{h, l\}$, and a prior belief about the type of the operating firm, θ, the investors forms beliefs using Bayes' Law. We now define cutoff beliefs, $\theta_{\gamma,\rho}$, above which investors will invest conditional on observing report ρ from certifier γ. For

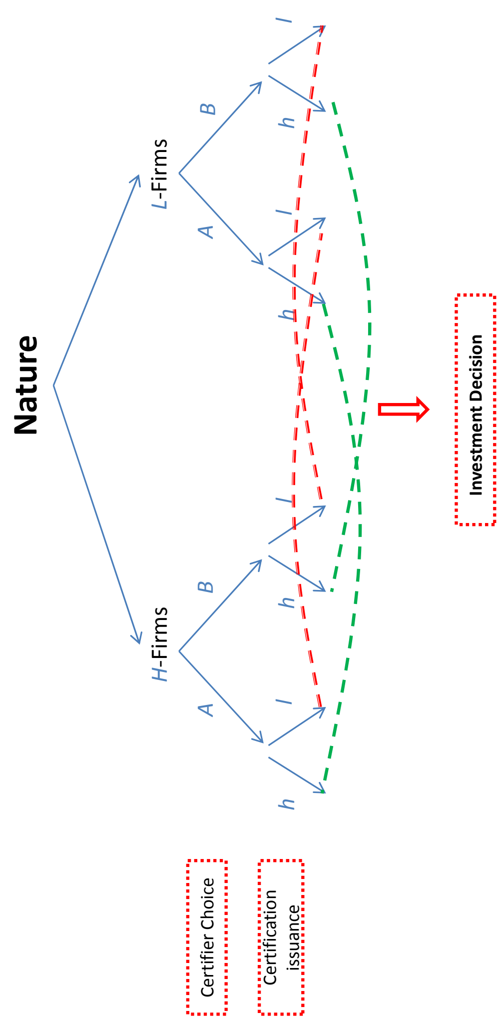

Figure 4.4: Simplified Game Tree with Nonstrategic Auditors

example, $\theta_{B,h}$ solves

$$f_{B,h}(\theta) = \frac{s_B\theta}{s_B\theta+(1-t_B)(1-\theta)} = r, \text{ i.e. } \theta_{B,h} = \frac{1}{1+\frac{1-r}{r}\frac{s_B}{1-t_B}}. \tag{4.1}$$

The other three cutoffs regarding certifier effort and certifier report solve

$$f_{A,h}(\theta) = \frac{s_A\theta}{s_A\theta+(1-t_A)(1-\theta)} = r, \text{ i.e. } \theta_{A,h} = \frac{1}{1+\frac{1-r}{r}\frac{s_A}{1-t_A}}, \tag{4.2}$$

$$f_{B,l}(\theta) = \frac{(1-s_B)\theta}{(1-s_B)\theta+t_B(1-\theta)} = r, \text{ i.e. } \theta_{B,l} = \frac{1}{1+\frac{1-s_B}{t_B}\frac{1-r}{r}}, \tag{4.3}$$

$$f_{A,l}(\theta) = \frac{(1-s_A)\theta}{(1-s_A)\theta+t_A(1-\theta)} = r, \text{ i.e. } \theta_{A,l} = \frac{1}{1+\frac{1-s_A}{t_A}\frac{1-r}{r}}. \tag{4.4}$$

Note that the better/worse the report and the better/worse the auditing accuracy, the lower/higher the cutoffs $\theta_{A,h}$ and $\theta_{B,h}$ are, while the higher/lower are the cutoffs $\theta_{A,l}$ and $\theta_{B,l}$. This can be seen in Figure 4.5.

4.4.1.3 Equilibrium

We restrict our attention to the most relevant equilibrium:

Definition 1. *We define a sensible equilibrium as: (1) investors willing to invest after h reports; (2) both certifiers are active; (3) the rate of return to investment in the operating firm is equal to the market rate of return.*

The first restriction let us focus on nontrivial cases where certifiers' opinion provides guidance to investors. The second restriction allows us to examine the impact of competition on information communication. The third restriction describes a market equilibrium.

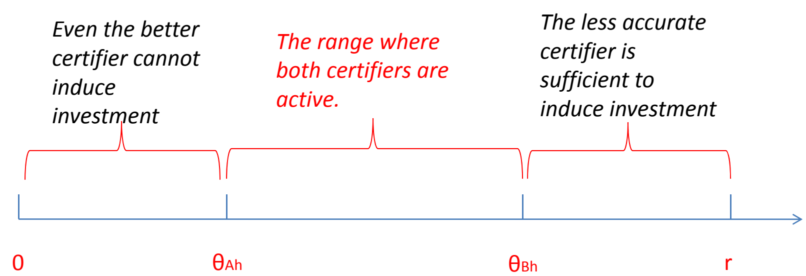

Figure 4.5: Investors' Cutoff Beliefs

These restriction rules out equilibria in region I and III and some equilibrium in region II. There exists a unique equilibrium that satisfies these restrictions and we describe in Proposition 7.

Proposition 7. *There is a unique sensible equilibrium that exists when* $\theta_{A,h} < \mu < \theta_{B,h}$*: the investors make the market rate of return,* r*, the* L*- and the* H*-firms randomize between certifier* A *and* B*, and investors invest iff they see the report* h *from either certifier.*

Proof. See Appendix. □

This equilibrium reflects the current auditing industry, where Big-4 auditing firms and non-big-4 auditing firms both get business in the market. Because of the lower cost of going to a non-big-4 firm, even well performing firms choose non-big-4 auditors. Rational investors take into consideration auditors' random error and treat auditors' unqualified opinion with scrutiny.

4.4.2 Information Quality Analysis

With a focus on the quality of communication between firms and investors, we investigate the impact of competition on the degree of information communicated effectively and the level of efficient investment. Non-strategic certifiers describe the ideal situation for certification. Investors can rely on certifiers' report while taking into account their certification errors. However, the communication between firms and investors are complicated by the competition between two certifiers. We focus on the impact of competition on the quality of information revealed through certifiers. With non-strategic certifiers, the impact on information quality is reflected indirectly through firms' certifier choice. It is easy to observe the following table of information quality.

	H-Firm	L-Firm
$\rho = h$	$(1-\beta)s_A + \beta s_B$	$(1-\alpha)(1-t_A) + \alpha(1-t_B)$
$\rho = l$	$(1-\beta)(1-s_A) + \beta(1-s_B)$	$(1-\alpha)t_A + \alpha t_B$

Denote $Inf_H = (1-\beta)s_A + \beta s_B$, $Inf_L = (1-\alpha)(1-t_A) + \alpha(1-t_B)$ to capture the equilibrium information quality. Inf_H is the probability of a H firm receiving a good report, and Inf_L is the probability of a L firm receiving a good report. Inf_H can be understood as the effectiveness of audit, while Inf_L is the ineffectiveness of audit.

Corollary 5. *As the lower quality competitor, B, increases its certification quality, the proportion of firms choosing certifier B increases and the overall information quality increases.*

Proof. See Appendix. □

We next look at the efficiency of investment in equilibrium. The most efficient allocation of investment would have all of the H-firms receiving investments and none of the L-firms. We define the probability that a H firm receives investment as $Inv_H = (1-\beta)s_A p + \beta s_B q$, and the probability that a L firm receives investment as $Inv_L = (1-\alpha)(1-t_A)p + \alpha(1-t_B)q$. We have the following dependencies.

Corollary 6. *Close competition between certifiers decreases the efficiency of investment, while increases the welfare of both H and L-firms.*

This seemingly counter-intuitive result explains what happened in the industry. Close competition has two impacts on the investment decision. On the one hand, under close competition, both certifiers provide certification with good accuracies. This means investors overall get better information. On the other hand, close competition drives both types of firms to choose B certifier, and especially relatively more L-firms choose B certifier. In other words, in equilibrium, L-firms overall have a higher chance of getting a h certification, which leads to inefficient investment. Meanwhile, both types of firms are better off. H-firms enjoy high quality certification at a lower cost, while L-firms acquire more investment.

4.5 Model 2: Strategic Certifiers without Litigation

We now turn to a more realistic model, where certifiers' report is influenced by firms' payment. Since certifier revenue comes from firms' payment, certifiers may have the tendency to cater to firms' wishes in order to acquire payment and future payments. Certifiers, including the bond rating agencies, Moody's, S&P, Fitch's, are, at least at present, legally protected from liability. The legal theory is that their ratings are "opinions," hence protected by free speech case law.

To examine this issue, we assume that firms can offer a contingent contract to their chosen certifiers. After certifiers have a determination, h or l, they can choose to issue them truthfully or not. [6] Investors observe the contract accepted by the certifiers, and are aware that the certification is strategically chosen by the certifiers.

In the rest of this section, we provide a description of the model, summarize the main results and end with details of arguments.

4.5.1 Equilibria

We next proceed to equilibrium analysis in a fashion analogous to the previous model. Firms contract on certifiers' effort level and certification ex ante. Denote the payment contract as $\{p_{A,h}, p_{A,l}, p_{B,h}, p_{B,l}\}$. The same set of belief cutoffs θ applies here.

The Enron and Anderson case reveals an interesting aspect where certifiers intentionally lie about what they found. In other words, while certifiers still check over the firm's financial documents, their final report does not depends on their work. Both certifiers and firms in this case hope to fool investors through the effort level, but investors, by observing the audit contract and audit report, find out that certifiers are lying and ignores certifiers' report when they make their decisions. Note that, even in this model, firms cannot offer a simple contract that pays more for certain report as it is so obvious that

[6] This is an all-or-nothing choice, and we intend to work on the shadings of meaning, with special attention to the observation that large investors, e.g. hedge funds, hire "forensic auditors" to read auditor reports.

investors do not believe it. A sensible equilibrium would be that firms offer a payment contract that makes certifiers indifferent between lying and not lying. This induces trust from investors in certifiers, however, firms can potentially manipulate certifiers' report by offering small bribes ("toaster oven"), which can be done through letting the certifier perform other services for the same firm. Denote an audit contract as $\{P_{ij}, i \in \{A, B\}, j \in \{h, l\}\}$. We hence focus attention on $P_{A,h} = P_{A,l} = c_A$ or $P_{B,h} = P_{B,l} = c_B$ or both.

Note that, without small bribes that induces the strategic behavior of certifiers, this model reduces to model 2 where firms bear audit costs and choose audit effort and the only equilibrium being firms pool at A and B and investors invest upon seeing h. With bribes, however, firms are able to distort certifiers' reporting probabilities $s_B(s_A)$ and $t_B(t_A)$. This model can therefore address a variety of questions:

– If firms can design the certification technology, in terms of the accuracy (false positive and false negative rate), what would certifiers look like?

– When there are different certifiers, each with different accuracy, which one will firms choose?

– When certifiers can lie in their report, how to describe the equilibrium scenario? To what extent will communication be effective?

Applying the Rao-Blackwell theorem, certifier lying makes their report a worse signal of firms' actual performance. We hereon use notations $\tilde{s}_{A(B)}$ and $\tilde{t}_{A(B)}$ to denote the new audit accuracy. The same structure of analysis and screening of different equilibrium possibilities apply in this case, except that we have to consider auditing accuracies, and consequently $\theta_{A,h}$, $\theta_{B,h}$ as determined endogenously.

Proposition 8. *With strategic certifiers, there is only one stable equilibria with rate of return r and it falls in the middle region of $\theta_{A,h} < \mu < \theta_{B,h}$. certifier A's accuracy is reduced to make $\theta_{A,h} = \mu$; both firms patronize certifier A; investors earn return r.*

4.5.1.1 Equilibrium analysis

With strategic certifiers, firms' incentive to manipulate auditing results can be summarized in the following three scenarios:

(a) when $0 < \mu < \theta_{A,h}$, firms have incentive to increase the accuracies s_A, t_A, which is not possible under Markov scramble. In the pooling equilibrium no investment happens, thus firms have no marginal incentive to influence certifiers' report. In the semi-separating equilibrium, only H-firms receive a non-negative payoff, which positively depends on only the accuracies of certifier A.

(b) when $\theta_{A,h} < \mu < \theta_{B,h}$, there are two possibilities, but they eventually converge to the same scenario under Markov scramble:

   i. both firms patronizing certifier A is a stable equilibrium with rate of return equal to r, the market rate of return. H-firms's payoff positively depends on certifier A's accuracy, while L-firms will have incentive to decrease the accuracy of certifier A, meaning t_A. When t_A lowers down to $\Delta_{t_A} = 1 - \frac{\mu(1-r)}{(1-\mu)r} s_A$, we have $\theta_{A,h} = \mu$.

ii. the other sensible equilibrium with market rate of return is the mixed-pooling equilibrium in case (b). The comparative statics show that both H-firms and L-firms have incentive to decrease the accuracies s_A, t_A, and increase the accuracies s_B, t_B. The latter is not possible under Markov scramble. As s_A and t_A decrease, the cutoff $\theta_{A,h}$ increases while $\theta_{B,h}$ remains the same. Eventually, $\theta_{A,h} = \mu$. Since it is a two-dimension change in accuracies, there seem to be multiple ways to achieve this equilibrium result. Notice that in this case $\alpha = \beta = 0$, which means that both firms will patronize only certifier A.

(c) when $\theta_{B,h} < \mu < r$, there is a pooling equilibrium with both firms choosing certifier A, H firms have incentive to increase s_A, while L firms have incentive to decrease t_A. As a result, $\theta_{A,h}$ goes up until it becomes $\theta_{B,h}$. In this case, $t_A = 1 - \frac{s_A}{s_B}(1 - t_B)$.

It is easy to observe that, when certifiers are driven by expected profit and are flexible in their certification issuance, firms tend to degrade auditing accuracies.

Corollary 7. *Strategic behaviors by certifiers have a negative impact on the accuracies of the more-accurate certifiers, A, and minimize the difference between the two types of certifiers, A and B.*

Notice that, suppose we assume investors do not expect correctly the changes in auditing accuracies, meaning their beliefs $\theta_{A,h}$, $\theta_{B,h}$ remain the same while the actual s, t changes. Analysis will be the same even for the mixed-pooling equilibria. For instance, in the only sensible equilibrium, α and β remain the same as the actual s, t changes, but they do not enter the welfare function of the firms. On the other hand, p, q have to be updated to functions of $\tilde{s}, \tilde{t}$, and they have nothing to do with α, β and thus, the above analysis still hold.

4.5.1.2 Information Quality Analysis

The stable equilibrium requires that $\theta_{A,h} = \mu$, i.e. $\tilde{t}_A = 1 - \frac{\tilde{s}_A}{s_B}(1 - t_B)$. If we substitute $\tilde{t}_A$ in expressions for α β p q, we will have two quick observations: (1) $\alpha = 1$ and $\beta = 0$ do not vary as other parameters change; (2) p and q will depend on μ and r. Notice that $\frac{1-\tilde{t}_A}{\tilde{s}_A} = \frac{\mu(1-r)}{(1-\mu)r}$, and

$$\begin{aligned}
p* &= \frac{C_A(\frac{1}{s_B} - \frac{1}{a(1-t_B)}) + b(\frac{\tilde{s}_A}{s_B} - 1)}{(\frac{1-\tilde{t}_A}{\tilde{s}_A} - \frac{1-t_B}{s_B})\frac{\tilde{s}_A}{1-t_B}}; \\
q* &= \frac{\frac{C_A}{\tilde{s}_A}(1 - \frac{1}{a}\frac{\tilde{s}_A}{1-\tilde{t}_A}) + b(1 - \frac{s_B}{\tilde{s}_A})}{(\frac{\tilde{s}_A}{1-\tilde{t}_A} - \frac{s_B}{1-t_B})\frac{1-t_B}{s_B}}, \\
W_H^* &= \tilde{s}_A p + (1 - \tilde{s}_A)b - c_A = s_B q + (1 - s_B)b, \\
W_L^* &= (1 - \tilde{t}_A)pa - c_A = (1 - t_B)qa.
\end{aligned}$$

	a	b	μ	r	$\tilde{s}_A$	$\tilde{t}_A$	s_B	t_B
$W_H^* = s_B q + (1 - s_B) b$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$+$
$W_L^* = (1 - t_B) q a$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$+$
$W_I = r$	$\cdot$	$\cdot$	$\cdot$	$+$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
$p*$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$+$
$q*$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$+$

Table 4: Comparative Statics for Firms' and Investors' Welfare

Corollary 8. *Close competition increases both firms' welfare.*

This follows the same intuition as we have discussed in the previous section: close competition leads to more ineffective investment and cheaper certifications. In addition, certifiers' strategic behavior enhances the tendency for close competition as it reduces the distance between the two certifiers.

	H-Firm	L-Firm
$\rho = h$	$\tilde{s}_A$	$1 - \tilde{t}_A$
$\rho = l$	$1 - \tilde{s}_A$	$\tilde{t}_A$

The information quality in equilibrium becomes $Inf_H = \tilde{s}_A$ and $Inf_L = \tilde{t}_A$. With simple algebra to compare the above matrix with the previous section, we have the following results.

Corollary 9. *The equilibrium level of information quality is lower when certifiers strategically report.*

4.6 Model 3: Strategic Certifiers with Litigation

We study a different industry structure of certification, where certifiers can be penalized for providing incorrect certification. Auditors are very different from other certifiers, such as rating agencies, in that they are subject to legal liability. By U.S. law, auditing firms are partnerships rather than limited liability corporations. This means that the impulse to maximizing expected profits must take into account the liabilities that arise from the reports. To counter this, auditing firms have a variety of strategies: the law prohibits payments to certifiers explicitly depending on the outcome of the audit; partners not working on an account must certify the work for an account; issuing audit reports with disclaimers.[7] Despite these protections, operating firms have multiple methods of affecting certifiers' profit. Examples include offering future work, e.g. hiring the certifier for additional consulting work, with the idea that an experienced outside certifier may see profitable changes in operational procedures.

Liability and reputation are two major and crucial forces to regulate certifiers' behaviors in the presence of liability, and align their incentive towards the interests of the investors. Both liability and reputation reduce the expected profits of certifiers in different ways. Liability payments occur when the certifiers are sued by investors and are judged liable in a court. Reputation effects through investors' trustworthiness in certifiers. Lawsuits can damage the certifiers' reputation, but reputation loss may happen when the client firms are sued. We model liability as the expected payment when a certain type of certifier issues a certain type of certification. In other words, we include liability as the certifiers' in-confidence level of their certification being

[7]Often in footnotes.

accurate.

We denote the expected liability level for certifier $A(B)$ as $E(LiaA)(E(LiaB))$. The set of contingent payment can now be expressed as

$$P_{A,h} = C_A + E(LiaA);$$
$$P_{A,l} = C_A + E(LiaA);$$
$$P_{B,h} = E(LiaB);$$
$$P_{B,l} = E(LiaB).$$

Notice that, we model contingent contract not as based on auditors' decision, but on the prospect that firms may get sued and have an impact on auditors' reputation or liability. [8] Both H and L firms will offer the same set of payments, otherwise their different payments will be a strong signal of their types. The payments also incorporate the expected liability borne by auditors. Our model reflect one common argument regarding litigation that is, litigation only makes auditing more expensive. While the public may view litigation as a way to regulate the auditors, our analysis shows another aspect to this: litigation in fact serves to reduce operating firms' incentive to put pressure on auditing firms. As we can observe from Lehman's Repo 105 scandal, auditing firms very often choose to accommodate their clients' need through various means, such as requesting comfort letters from attorneys, or

[8] Ernst & Young in Lehman's case suffer reputation loss and possible lawsuit. Investors may hesitate to believe in a report by Ernst & Young.

providing vague statement to protect themselves. The fundamental issue lies in operating firms' interest to misstate.

We rewrite the indifference conditions for investors and two types of firms as follows:

$$\begin{aligned}
pa(1-t_A) - c_A - E(LiaA) &= qa(1-t_B) - E(LiaB); \\
ps_A + (1-s_A)b - c_A - E(LiaA) &= qs_B + (1-s_B)b - E(LiaB); \\
\frac{\mu(1-\beta)}{\mu(1-\beta)+(1-\mu)(1-\alpha)} &= \frac{(1-t_A)(r-\psi\xi)}{s_A(1-r)+(1-t_A)(r-\psi\xi)}; \\
\frac{\mu\beta}{\mu\beta+(1-\mu)\alpha} &= \frac{(1-t_B)(r-\psi\xi)}{s_B(1-r)+(1-t_B)(r-\psi\xi)}.
\end{aligned}$$

Also notice that $E(LiaA) = (1-\mu)(1-\alpha)(1-t_A)\xi$ and $E(LiaB) = \mu(1-\beta)(1-t_B)\xi$. Define $T_A = C_A + \xi((1-\mu)(1-t_A)(1-\alpha) - \mu(1-t_B)(1-\beta))$, and replace C_A with T_A in the first two equations, we can easily solve for everything as

$$\begin{aligned}
\alpha &= \frac{\frac{1-r}{r-\psi\xi}\frac{\mu}{1-\mu} - \frac{1-t_A}{s_A}}{\frac{1-t_B}{s_B} - \frac{1-t_A}{s_A}}; \\
\beta &= \frac{\frac{s_A}{1-t_A} - \frac{r-\psi\xi}{1-r}\frac{1-\mu}{\mu}}{\frac{s_A}{1-t_A} - \frac{s_B}{1-t_B}}; \\
p &= \frac{T_A(\frac{1}{s_B} - \frac{1}{a(1-t_B)}) + b(\frac{s_A}{s_B} - 1)}{\frac{1-t_A}{1-t_B} - \frac{s_A}{s_B}}; \\
q &= \frac{T_A(\frac{1}{s_A} - \frac{1}{a(1-t_A)}) + b(1 - \frac{s_B}{s_A})}{\frac{1-t_B}{1-t_A} - \frac{s_B}{s_A}}.
\end{aligned}$$

For both H and L-type firms, degrading s_A and t_A increases their welfare. Since the above equilibrium falls under the condition of $\theta_{A,h} \leq \mu$, we look for the equilibrium $\{s_A^*, t_A^*\}$ that satisfy $s_A^* = (1 - t_A^*)\frac{1-\mu}{\mu}\frac{r-\psi\xi}{1-r}$. Substitute s_A^* back in to W_L and w_H, it is easy to observe that both types of firms prefer a higher t_A, which is not possible, and a lower s_A. As a result of the Markov scramble, the equilibrium audit quality of auditor A is $t_A^* = t_A$, $s_A^* = (1 - t_A)\frac{1-\mu}{\mu}\frac{r-\psi\xi}{1-r}$.

Proposition 9. *With strategic certifiers, there is only one stable equilibria with rate of return r and it falls in the middle region of $\theta_{A,h} < \mu < \theta_{B,h}$. Certifier A's accuracy is reduced to $t_A^* = t_A$, $s_A^* = (1 - t_A)\frac{1-\mu}{\mu}\frac{r-\psi\xi}{1-r}$; both firms patronize certifier A; investors earn return r.*

4.6.1 Information Quality Analysis

The information quality is as follows.

	H-Firm	L-Firm
$\rho = h$	$\tilde{s}_A$	$1 - \tilde{t}_A$
$\rho = l$	$1 - \tilde{s}_A$	$\tilde{t}_A$

Corollary 10. *Litigation decreases quality of information in equilibrium.*

Besides, we have the following few observations: (1) Strategic Behaviors by auditors increase the false negative rate while decrease the false positive rate; (2) Imposing liability further increase false negative rate; (3) Except for case i with non-strategic auditors, the comparative statics of information quality in the other two cases are the same as $s(t)_A$ or $1 - s(t)_A$.

4.7 Conclusions

We have investigated the communication between firms and investors with the facilitation of two different types of certifiers. We focus on situation where communication turns into cheap talk when there is no intermediaries. The central question we intend to address is how effective is certified communication, under three different scenarios. We first analyze a benchmark case where the certifier has no additional incentive other than truthfully disclose their certified results. This resembles an ideal scenario of auditors. Auditors are usually defined as a independent and unbiased party who provide their third-party opinions on firms' financial performance. We also incorporate two types of certifiers who possess technology with different accuracies. Our analysis sheds light on the competition effect on communication effectiveness. In our second model, we turn to a more realistic case where certifiers are profit-maximizing economic agents. Auditors are paid by their client firms. Bond rating agencies, such as Moody's and Standard and Poor are paid by bond issuers who need a rating. On the other hand, firms often have a preferred certification that they want to obtain from the certifiers, and their preferred certification may differ from their true performance. We analyze firms' influence on the certification result through payment contract, and certifiers' strategic revelation of their certification results. In the third model, we study a more mature certification industry, where certifiers are regulated by litigation to provide truthful certifications. The auditing industry has long been under government regulation, and auditing firms are often sued for issuing misleading audit reports, especially after their client go bankrupt. Litigation has multiple impacts on the resulted communication effectiveness.

We also investigate reputation effects on communication. Certifiers' existence relies on investors' trust in them. In the extreme case of Enron scandal, its certifier, Arthur Anderson, demised after investors discovered the role they played in the bankruptcy of Enron. When investors lose trust in a certifier, their certification can no longer induce any investment and thus has no value to the firms.

This paper builds a framework for further investigation of certifiers' reporting strategies. Certifiers' reporting option is often a continuous set of opinions that are often vague in meanings. A firm's financial statement, which is prepared and certified by auditors, can contain critical information in foot-notes that most investors pay little attention to comprehend. When investors have different interpretations of a vague certification, their investment decisions differ. Vagueness can also be used as a defense of certifiers in case of lawsuits. Certifiers may take into account of different vague opinions and the associated liability cost when issue their certification.

Appendices

Appendix A

Proofs to Results in Chapter 2

A.1 Determining π_L

We divide the denominator of the right-hand side (RHS) of (9) by $\prod_{i=0}^{h-1} w_i$, and rearrange it into:

$$\lambda_H = \frac{1}{1 + (1 - w_h)\left[\frac{1}{\prod_{i=0}^{h-1} w_i} + \frac{1}{\prod_{i=1}^{h-1} w_i} + \ldots + \frac{1}{\prod_{i=h-2}^{h-1} w_i}\right]}.$$

It's easy to see that the RHS is increasing in $w_i, i = 0, 1, \ldots, h$. Also, as π_L increases, w_0 increases according to (4), and thus V_1 increases according to (3); and again w_1 increases due to (4), and so on such that all w_j increase. Thus, RHS is increasing in π_L, or π_L is increasing in λ_H.

On the other hand, when w_h is close to 1, the denominator goes to 1, and thus RHS becomes 1. When any w_j is close to 0, the denominator becomes infinitely large, and thus, the RHS is 0. Therefore, as π_L varies, RHS varies from 0 to 1. Combining with the monotonicity of RHS, given any $\lambda_H \in [0, 1]$, there exists a unique π_L.

A.2 Proof of Existence

Denote $A \equiv k_L(\frac{k_H}{k_L}\frac{2k_L-1}{2k_H-1})^2$ and $B \equiv [k_H(\frac{2k_L-1}{2k_H-1})^2 - \frac{k_L(k_L-k_H)}{k_H}]$. It is easy to verify that $B > A > 0$.

The IC condition $\pi_{h-1}^H \leq \pi_H$ can be rearranged into

$$[k_H(\frac{2k_L-1}{2k_H-1})^2 - \frac{k_L(k_L-k_H)}{k_H}]w_{h-1}^2 - (2k_L-1)w_{h-1} - \pi_L \geq 0,$$

which requires $w_{h-1} \geq \frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B}$ (note $w_{h-1} \geq 0$). Notice that because $w_h \leq 1$ and $w_h = \frac{2k_L-1}{2k_H-1}w_{h-1}$, we must have $w_{h-1} < \frac{2k_H-1}{2k_L-1}$. Therefore, the above IC condition is equivalent to

$$w_{h-1} \in [\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B}, \frac{2k_H-1}{2k_L-1}]. \tag{A.1}$$

Similarly, solving the other IC condition $\pi_h^L < \pi_L$, we get

$$w_{h-1} \in [0, \frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4A\pi_L}}{2A}]. \tag{A.2}$$

A separating equilibrium is supported if w_{h-1} satisfies both (A.1) and (A.2). Notice that $\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B} < \frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4A\pi_L}}{2A}$ because $B > A$ and $\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4x\pi_L}}{2x}$ decreases in x (which can be verified by checking the first-order derivative). Therefore, as long as

$$\frac{(2k_L-1)+\sqrt{(2k_L-1)^2+4B\pi_L}}{2B} < \frac{2k_H-1}{2k_L-1}, \tag{A.3}$$

there exists w_{h-1} satisfing both (A.1) and (A.2). By simple algebra, (A.3) reduces to $\pi_L < \left(\frac{2k_H-1}{2k_L-1}\right)^2 B - (2k_H-1)$, which is equivalent to (by substituting in B)

$$1 - k_H - (k_L - k_H)\frac{k_L}{k_H}(\frac{2k_H-1}{2k_L-1})^2 - \pi_L > 0. \tag{A.4}$$

Given the range for w_{h-1}, we can determine the cutoff value of h on a trial basis. (a) Let $h = i$ ($i \geq 0$). (b) Calculate $\{w_j, V_j\}$ and π_L. (c) Check if (A.4) is satisfied. If yes, we get the equilibrium; otherwise, try $h = i + 1$, and repeat (a)˜(c).

Appendix B

Proofs of Results in Chapter 3

B.1 Proof of Proposition 4

For the case where opportunistic commentators adopt a mixed strategy, notice the expected quality is the weighted average of the success rate s and mk. The ratio of weight for s to mk determines the value. If $s > mk$, the expected quality is increasing in the ratio of weight for s to that for mk. So, to conclude that $r_1 > r_0$ is equivalent to showing:

$$\frac{\mu s}{(1-\mu)\,mk} \geq \frac{\mu\,(1-s)}{(1-\mu)\,(1-mk)}, \tag{B.1}$$

which can be verified as true. If $s < mk$, similarly, we can derive $r_1 > r_0$. $s = mk$ is a special case in which $r_1 = r_0$.

In the equilibrium with effort, it can be similarly shown that $r_1 > r_0$. In the equilibrium with no effort, it is easy to see $r_1 = s > r_0$.

B.2 Proof of Proposition 5

(a) For $w \in [0, \frac{k}{1+k})$, incentive conditions require that $\alpha_1 k + \beta k\,(v_1 - v_0) - c < 0$ and $\alpha_0 k + \beta k\,(v_1 - v_0) - c = 0$. Since $\alpha_1 < \alpha_0$, we need check only the second condition. Notice that we have $v_1 = (1-\alpha_1)\,r_1(w) + \beta v_0$, and

$v_0 = (1-\alpha_0)\, r_0(w) + \beta v_0$. So incentive conditions lead to:

$$(\alpha_0 k - c) + \beta k \left[(1-\alpha_1)\, r_1(w) - (1-\alpha_0)\, r_0(w)\right] = 0. \tag{B.2}$$

Therefore,

$$\alpha_1 = \frac{[1+\beta r_0(w)]\alpha_0 + \beta[r_1\,(w) - r_0(w)] - c/k}{\beta r_1(w)} = H_1(\alpha_0|w) = 1 - \frac{\beta(1-\alpha_0) r_0(w) - \alpha_0 + c/k}{\beta r_1(w)}. \tag{B.3}$$

Since $r_1(w)$ decreases in w and $r_0(w)$ increases in w, the right-hand side is decreasing in w. So, if $H_1(\alpha_0|\frac{k}{1+k}) < \alpha_1 \leq H_1(\alpha_0|0)$, we can get a unique solution to $\alpha_1 = H_1(\alpha_0|w)$.

(b) For $w \in (\frac{k}{1+k}, k]$, incentive conditions require that $\alpha_1 k + \beta k\,(v_1 - v_0) - c = 0$. Noticing that $v_1 - v_0 = (\alpha_1 - \alpha_0)k + (1-\alpha_1)r_1 - (1-\alpha_0)r_0$, we can derive:

$$\alpha_1 = H_2(\alpha_0|w) \equiv \frac{\beta[k - r_0(w)]\alpha_0 + \beta[r_0(w) - r_1\,(w)] + c/k}{1 + \beta k - \beta r_1\,(w)}.$$

Similarly, if $H_2(\alpha_0|\frac{k}{1+k}) < \alpha_1 \leq H_2(\alpha_0|k)$, there is α_1 such that $\alpha_1 = H_2(\alpha_0|w)$.

(c) In the case with $w = \frac{k}{1+k}$, incentive conditions require that $\alpha_1 k + \beta k\,(v_1 - v_0) - c \leq 0$ and $\alpha_0 k + \beta k\,(v_1 - v_0) - c \geq 0$ (with at most one equality at the same time). From the proofs in (a) and (b), we can verify that the incentive conditions hold when $\alpha_1 \leq H_1(\alpha_0|\frac{k}{1+k})$ and $\alpha_1 \leq H_2(\alpha_0|\frac{k}{1+k})$.

B.3 Proof of Corollary 2

For $w \in [0, \frac{k}{1+k}]$, based on Eq.(3.15) and Eq.(3.16), the condition for $r_1(w) < r_0(w)$ is:

$$\mu ss\,(1-\mu)\,(1-w) < \mu s\,(1-\mu)\,w + (1-\mu)\,w[\mu(1-s)s + (1-\mu)\,w]$$

which can be reduced to $(1-\mu)\,w^2 + 2\mu sw - \mu s^2 > 0$ by simple algebra, or, equivalently, $w > \frac{\sqrt{\mu}-\mu}{1-\mu}s = \frac{\sqrt{\mu}}{1+\sqrt{\mu}}s$.

B.4 Proof of Proposition 6

(a) Under (α_0, α_1) with $\alpha_1 \geq \alpha_0$, for $w = 0$, the minimum resource required is trivially 0. For $w \in (0, k)$, under $k \geq s$, we have $k > r_1(w)$ and $\alpha_1 = M(\alpha_0|w)$ in Eq.(3.22) has a negative slope. Substituting $\alpha_1 = M(\alpha_0|w)$ into Eq.(3.24), the coefficient of α_0 is $\frac{-(1-\beta k+\beta r_0(w))}{\beta(k-r_1(w))}n_1 + n_0$. The sign of the coefficient is the same as:

$$-\frac{1}{\beta}(1-\beta k+\beta r_0)n_1 + n_0(k-r_1) = k - \frac{1}{\beta}n_1 - r_0n_1 - r_1n_0.$$

Notice that $r_0n_1 + r_1n_0 = \frac{n_1}{n_0}r_0n_0 + \frac{n_0}{n_1}(r_1n_1 - n_1 + n_1) = n_0 + (\frac{n_1}{n_0} - \frac{n_0}{n_1})r_0n_0 > n_0$, where the second equality is due to $r_0n_0 + r_1n_1 = n_1$ (i.e., the number of commentators with high reputation is constant). Also, notice that $k - \frac{1}{\beta}n_1 < 1 - n_1 = n_0$. Therefore, the coefficient is negative and the optimal solution is the maximum α_0 possible on $\alpha_1 = M(\alpha_0|w)$, which is $\alpha_0 = \alpha_1$. By Figure 3.2, for $w = k$, the optimal solution is $\alpha_1 = \alpha_0 = M(\alpha_0|k)$.

(b) Under (α_0, α_1) with $\alpha_1 \leq \alpha_0$, for $w = 0$, the minimum resource required is trivially 0. For any $w \in (0, \frac{k}{1+k})$, by Proposition 5, $\alpha_1 = H_1(\alpha_0|w)$

defines a line on which any pair (α_0, α_1) yields the same proportion w in equilibrium. Given the positive slope of $\alpha_1 = H_1(\alpha_0|w)$, the coefficient of α_0 in Eq.(3.24) is positive once α_1 is substituted in. Therefore, the optimal solution is the minimum α_0 possible on $\alpha_1 = H_1(\alpha_0|w)$, which is $\alpha_1 = 0$ and α_0 determined by $0 = H_1(\alpha_0|w)$. For $w \in (\frac{k}{1+k}, k)$, under $k \geq s$, $k \geq r_0(w)$ and thus $\alpha_1 = H_2(\alpha_0|w)$ has a positive slope. Substituting $\alpha_1 = H(\alpha_0|w)$ into Eq.(3.24), the coefficient of α_0 is positive, and therefore the optimal solution is the minimum α_0 possible on $\alpha_1 = H_2(\alpha_0|w)$, which is $\alpha_1 = \alpha_0$. By Figure 3.1, for $w = \frac{k}{1+k}$, the optimal solution is clearly$\alpha_1 = 0$ and α_0 determined by $0 = H_1(\alpha_0|\frac{k}{1+k})$. For $w = k$, the optimal solution is $\alpha_1 = \alpha_0 = H_2(\alpha_0|k)$.Proof: For all (α_0, α_1) with $\alpha_1 < \alpha_0$, we consider the case with $w \leq \frac{k}{1+k}$ here. For $w = 0$, the minimum resource required is trivially 0. For any $0 < w \leq \frac{k}{1+k}$, by Proposition 5 $\alpha_1 = H(\alpha_0|w)$ defines a line on which any pair (α_0, α_1) yields the same proportion w in equilibrium. Substituting $\alpha_1 = H(\alpha_0|w)$ into Eq.(3.24), we can see that the coefficient of α_0 (i.e., $\frac{\beta r_0(w)+1}{\beta r_1(w)} n_1 + n_0$) is positive. Therefore, the optimal solution is minimum α_0 possible on $\alpha_1 = H(\alpha_0|w)$. Given the positive slope of $H(\alpha_0|w)$, the optimal solution is $\alpha_1 = 0$ and α_0 determined by $0 = H(\alpha_0|w)$. The other case with $w > \frac{k}{1+k}$ can be similarly analyzed, and the optimal solution is $\alpha_1 = \alpha_0$.

For all (α_0, α_1) with $\alpha_1 \geq \alpha_0$, similarly,for $w = 0$, the minimum resource required is trivially 0. For any $0 < w \leq 1$, $\alpha_1 = M(\alpha_0|w)$ in Eq.(3.22) define a line on which any pair (α_0, α_1) yields the same proportion w in equilibrium. Substituting $\alpha_1 = M(\alpha_0|w)$ into Eq.(3.24), the coefficient of α_0 is

$\frac{-(1-\beta k+\beta r_0(w))}{\beta(k-r_1(w))}n_1+n_0$. Therefore,

(I) When $k \leq s$, from Eq.(3.19), we know $k \leq r_1(w)$ as $r_1(w)$ is a weighted average of s and k. The coefficient of α_0 is positive and thus the optimal solution is at $\alpha_0 = \alpha_1$ (given the positive slope of $M(\alpha_0|w)$).

(II) When $k > s$, we again check the sign of the coefficient. Notice that now $k > r_1(w)$ and the sign of the coefficient is the same as

$$-(1-\beta k+\beta r_0)n_1+n_0\beta(k-r_1) = \beta k - n_1 - \beta r_0 n_1 - \beta r_1 n_0$$

$$= \beta k - n_1 - \beta r_0 - \beta r_1 + \beta r_0 n_0 + \beta r_1 n_1 = \beta k - \beta(r_0+r_1) - n_1(1-\beta) \equiv R(w)$$

where the last step is due to $r_0 n_0 + r_1 n_1 = n_1$ (i.e., the number of commentators with high reputation is constant). Notice that r_0, r_1, and $n_1$are increasing in w. Therefore, $R(w)$ is decreasing in w. Then, given the positive slope of $M(\alpha_0|w)$, if $R(0) < 0$, the coefficient of α_0 is negative and the optimal solution is at $\alpha_0 = \alpha_1$; if $R(k) > 0$, the coefficient of α_0 is positive and the optimal solution is the minimum α_0 possible on $\alpha_1 = M(\alpha_0|w)$; Otherwise, there is a cutoff w^*: When $w > w^*$, the optimal solution is at $\alpha_0 = \alpha_1$, and when $w \leq w^*$, the optimal solution is the minimum α_0 possible on $\alpha_1 = M(\alpha_0|w)$, where $w^*$is determined by $R(w^*) = 0$.

Appendix C

Proofs of Results in Chapter 4

Proof of Lemma 5

C.1 Analysis of Firms' Announcement Strategies

In the following tables, the rows represent the firm statements, $\sigma \in \{L, H\}$, to the auditors, the columns represent the auditor reports, $\rho \in \{l, h\}$, and the entries represent the probabilities of the reports given the statements.

Auditor accuracies for H-firms at auditor $\gamma \in \{A, B\}$

	$\rho = l$	$\rho = h$
$\sigma = L$	$1 - s_{\gamma,L}$	$s_{\gamma,L}$
$\sigma = H$	$1 - s_{\gamma}$	s_{γ}

Thus, $s_{\gamma,L} = Prob(\rho = l|\sigma = L)$ is the probability that the H-firm receives an auditor report of l given that they lie and report that their attractiveness to investors is L rather than H. We assume that for both auditors, $s_{\gamma} > s_{\gamma,L}$. This reflects our assumption that when firms provide accurate information, the auditors are more likely to reach the correct conclusion.

Given that h reports are sometimes strictly better and always at least as good for the firms as l reports, $s_{\gamma} > s_{\gamma,L}$ means that making the statement

$\sigma = L$ is weakly dominated for the H firms. This is important, and will help us drastically reduce the complexity of the game tree.

Auditor accuracies for L-firms at auditor $\gamma \in \{A, B\}$

	$\rho = l$	$\rho = h$
$\sigma = L$	$t_{\gamma,L}$	$1 - t_{\gamma,L}$
$\sigma = H$	t_γ	$1 - t_\gamma$

We assume that for both auditors, $t_\gamma < t_{\gamma,L}$, that is, we assume that the auditors are more likely to reach the true conclusion that the L-firm is in fact an L-firm if the firm provides them with accurate documentation.

We assume throughout that auditor reports are more likely to be correct than incorrect, that A is unambiguously more accurate than auditor B, and that audits are more accurate when the firms provide truthful information, i.e. the following is in force throughout.

Assumption 6. *$s_A > s_B > \frac{1}{2}$, $t_A > t_B > \frac{1}{2}$. $s_{A,L} > s_{B,L} > \frac{1}{2}$, and $t_{A,L} > t_{B,L} > \frac{1}{2}$.*

The "unambiguously more accurate" part of this assumption may seem like a strong assumption, but we argue that it is not. In the first place, if the two accuracies reflect knowledge of the auditing firm, then we would expect the more knowledgeable auditor to be unambiguously more accurate. Second, suppose that the auditor with the higher accuracy for H-firms is also the one with the lower accuracy for L-firms. In this case, disregarding costs, both firms would strictly prefer one auditor to the other.

We have already observed that making the statement $\sigma = L$ is weakly dominated for the H-firm. If one removes the choice of L from the H-firm from the game tree, then when investors see $\sigma = L$, they are certain that it came from an L-firm, hence will not invest in any perfect equilibrium. Removing this investment option from the game tree means that $\sigma = L$ is now weakly dominated for the L-firms. After removing $\sigma = L$ from both firms' choice sets, the firms have two options, A and B.

Proof of Proposition 7. We take these cases in order.

(a) Suppose that $0 \leq \mu < \theta_{A,h}$. We first examine the pure strategy possibilities and then the mixed strategy possibilities.

Pure strategies

It is clear that both firms choosing auditor B and investors not investing is an equilibrium. By considering cases, we show that there can only be one other type of equilibrium.

Other pure strategy combinations are not equilibria. The strategy combination (A, B), A for the L-firm and B for the H-firm, has the low firm paying the higher cost, c_A, but not getting any investment, and this is not a best response for the L-firm. (B, A) has the investors investing with probability 1 to any firm that hires auditor A, so that B is not a best response for the L-firm. (A, A) has both firms paying c_A and receiving no investment, both would rather pay $c_B = 0$ and receive no investment.

Mixed strategies

L playing a pure strategy and H randomizing is not an equilibrium. Whichever auditor L is not picking is only patronized by H-firms, hence investors will invest no matter the report from that firm. This means that L is not playing a best response to investor behavior.

H patronizing B and L randomizing is not an equilibrium. In this case, when investors see any report from auditor A, they know it was the L-firm, hence they will never invest. This means that L patronizing A costs them c_A, and they can avoid this cost by patronizing B.

H patronizing A and L randomizing is ***sometimes*** *an equilibrium.* If $p^* = \frac{c_A}{a(1-t_A)}$ of the time they see an h report from A the investors invest, then the L-firms are indifferent between A and B. For this to be sensible, we must have $p^* < 1$, i.e. $c_A < a(1-t_A)$, which is Assumption 2. For the H-firms to be willing to patronize only A, we must have $s_A p^* + (1-s_A)b - c_A > b$, i.e. $\frac{s_A}{s_A/a(1-t_A)-1} < c_A$. □

(b) Suppose now that $\theta_{A,h} < \mu < \theta_{B,h}$. We first examine the pure strategy possibilities, showing that both firms patronizing auditor A is the only equilibrium possibility, and then turn to the mixed strategy possibilities.

Pure strategies

Both firms patronizing A is a stable equilibrium outcome. If firms L and H patronize A, then, because $\theta_{A,h} < \mu$, the investors are strictly willing to invest when they see report h, hence earn a rate of return higher than r, the market rate of return. For this scenario to be a stable equilibrium, both types of firms must prefer auditor A to auditor B, i.e.,

$$\begin{aligned} s_A + (1 - s_A)b - c_A &> s_B(p_1 + (1 - p_1)b) + (1 - s_B)(p_2 + (1 - p_2)b); \\ (1 - t_A)a - c_A &> (1 - t_B)p_1 a + t_B p_4 a, \end{aligned}$$

in which $p_1 = prob(Investment|B, h)$ and $p_2 = prob(Investment|B, l)$. Since a report h is a weakly better signal than a report l, there are only two possibilities for investors' beliefs: $(p_1 < 1, p_2 = 0)$ or $(p_1 = 1, p_2 > 0)$. When $p_2 = 0$, the above two incentive conditions reduce to

$$\begin{aligned} p_1 &< \frac{s_A}{s_B} - \frac{c_A}{s_B(1 - b)}, \\ p_1 &< \frac{1 - t_A}{1 - t_B} - \frac{c_A}{(1 - t_B)a}. \end{aligned}$$

Since $s_A(1 - b) - c_A > s_B(1 - b)$, $\frac{s_A}{s_B} - \frac{c_A}{s_B(1-b)}$ is greater than 1, while $\frac{1-t_A}{1-t_B} - \frac{c_A}{(1-t_B)a} < 1$. In other words, L-firms have a larger incentive to deviate to B than the H-firms, stability implies that investor beliefs at B must put mass 1 on L-firms, meaning that they will not invest. This is consistent with the belief that $(p_1 < 1, p_2 = 0)$, and therefore, both patronizing A is part of a stable set of equilibria.

Both firms patronizing B is not a stable equilibrium outcome. If firms L and H patronize B, then, because $\mu < \theta_{B,h}$, the investors will never invest. For this set of strategies to be a stable equilibrium, Just below, we show that stability implies that investor beliefs after seeing a firm deviating to

A should put mass 1 on H-firms. This in turn implies that investors invest in any firm patronizing auditor A, meaning that B is not a best response for either firm.

Let $p_3 = prob(Investment|A, h)$ and $p_4 = prob(Investment|A, l)$. Incentive compatibility requires that

$$b > s_A(p_3 + (1 - p_3)b) + (1 - s_A)(p_4 + (1 - p_4)b) - c_A;$$

$$0 > (1 - t_A)p_3 a + t_A p_4 a - c_A.$$

Similarly, since a report h is a weakly better signal than a report l, there are only two possibilities for investors' beliefs: $(p_3 < 1, p_4 = 0)$ or $(p_3 = 1, p_4 > 0)$. When $p_4 = 0$, we have $p_3 < \frac{c_A}{s_A(1-b)} < \frac{c_A}{(1-t_A)a} < 1$. We make two claims. First of all, p_3 must be less than 1, which means that $(p_3 = 1, p_4 > 0)$ is not a set of equilibrium belief. Secondly, H-firms have more incentive to patronize A, which implies that investor beliefs after seeing a firm deviating to A should put mass 1 on H-firms. This in turn implies that investors invest in any firm patronizing auditor A, meaning that B is not a best response for either firm.

Pure strategy separation is not a Nash equilibrium. If firms L and H play (A, B) (respectively (B, A)), then choice of auditor reveals firm type, hence investors ignore reports and invest in any firm patronizing B (respectively A), hence L is not playing a best response.

Mixed strategies

L randomizing and H playing a pure strategy is not a Nash equilibrium. If H patronizes A, then because $\theta_{A,h} < \mu$, investors will always invest if they see a report of h from A. Because $c_A < a(1-t_A)$, this means that L strictly prefers auditor A to B, so that randomizing is not a best response. If H patronizes B, then equilibrium implies that L-firms will always get a negative payoff to patronizing A, but a non-negative payoff to B, hence they are not indifferent.

L playing a pure strategy and H randomizing is not a Nash equilibrium. The investors will invest after any report from whichever auditor L does not patronize, implying that neither L nor H is best responding.

There is always a full support Nash equilibrium in which L and H randomize, and investors invest in any firm receiving an h report.

We use the following notation for the mixed strategies for the firms and the investors,

Firm/Auditor	$\gamma = A$	$\gamma = B$
H	$1-\beta$	β
L	$1-\alpha$	α

Report/source	$\gamma = A$	$\gamma = B$
$\rho = h$	p	q
$\rho = l$	0	0

With H-firms patronizing both types of auditors, investor equilibrium responses must involve never investing if there is an l report, and being indifferent between investing and not investing when there is an h report. The L- and H- type firms have the indifference conditions

$$pa(1-t_A) - c_A = qa(1-t_B) \text{ and } \tag{C.1}$$

$$ps_A + (1-s_A)b - c_A = qs_B + (1-s_B)b. \tag{C.2}$$

Investors indifference conditions are

$$\frac{\mu(1-\beta)}{\mu(1-\beta)+(1-\mu)(1-\alpha)} = \theta_{A,h} = \frac{1}{1+\frac{1-r}{r}\frac{s_A}{1-t_A}} \text{ and} \tag{C.3}$$

$$\frac{\mu\beta}{\mu\beta+(1-\mu)\alpha} = \theta_{B,h} = \frac{1}{1+\frac{1-r}{r}\frac{s_B}{1-t_B}}. \tag{C.4}$$

In other words, we need $\frac{\alpha}{\beta} = \frac{1-r}{r}\frac{\mu}{1-\mu}\frac{s}{1-t}$ and $\frac{1-\alpha}{1-\beta} = \frac{1-r}{r}\frac{\mu}{1-\mu}\frac{s_A}{1-t_A}$. Solving these four equations for α, β, p, and q leads to solution for this mixed-strategy pooling equilibrium as:

$$\alpha = \frac{\frac{1-r}{r}\frac{\mu}{1-\mu} - \frac{1-t_A}{s_A}}{\frac{1-t_B}{s_B} - \frac{1-t_A}{s_A}}; \tag{C.5}$$

$$\beta = \frac{\frac{s_A}{1-t_A} - \frac{r}{1-r}\frac{1-\mu}{\mu}}{\frac{s_A}{1-t_A} - \frac{s_B}{1-t_B}}; \tag{C.6}$$

$$p = \frac{c_A(\frac{1}{s_B} - \frac{1}{a(1-t_B)}) + b(\frac{s_A}{s_B} - 1)}{\frac{1-t_A}{1-t_B} - \frac{s_A}{s_B}}; \tag{C.7}$$

$$q = \frac{c_A(\frac{1}{s_A} - \frac{1}{a(1-t_A)}) + b(1 - \frac{s_B}{s_A})}{\frac{1-t_B}{1-t_A} - \frac{s_B}{s_A}}. \tag{C.8}$$

(c) Suppose finally that $\theta_{B,h} < \mu < r$.

There is a pooling equilibrium in which the firms choose A auditors (stability gives only this possibility by Assumption **??**), and a mixed-pooling equilibrium in which both H and L firms mix between A and B auditors, investors invest with prob 1 and q for A and B auditors' h report, and p' for auditors A's l report. The notation is in the following tables.

Firm/Auditor	$\gamma = A$	$\gamma = B$
H	$1-\beta$	β
L	$1-\alpha$	α

Report/soruce	$\gamma = A$	$\gamma = B$
$\rho = h$	1	q
$\rho = l$	p'	0

Investors' indifference requires

$$\frac{1}{1+\frac{1-\mu}{\mu}\frac{1-\alpha}{1-\beta}} = \frac{1}{1+\frac{1-r}{r}\frac{1-s'}{t'}};$$
$$\frac{1}{1+\frac{1-\mu}{\mu}\frac{\alpha}{\beta}} = \frac{1}{1+\frac{1-r}{r}\frac{s}{1-t}}.$$

Solving for the four leads to

$$\alpha = \frac{\frac{t'}{1-s'} - \frac{\mu}{1-\mu}\frac{1-r}{r}}{\frac{t'}{1-s'} - \frac{1-t}{s}}; \tag{C.9}$$

$$\beta = \frac{\frac{1-\mu}{\mu}\frac{r}{1-r} - \frac{1-s'}{t'}}{\frac{s}{1-t} - \frac{1-s'}{t'}}; \tag{C.10}$$

$$p = \frac{c_H(\frac{1}{s} - \frac{1}{a(1-t)}) + b(\frac{s'}{s} - 1) + \frac{1-t'}{1-t} - \frac{s'}{s}}{\frac{1-s'}{s}(1-b) - \frac{t'}{1-t}}; \tag{C.11}$$

$$q = \frac{c_H(\frac{1}{at'} - \frac{1}{(1-s')(1-b)}) + \frac{s}{1-s'}\frac{b}{1-b} + \frac{1}{1-s'} - \frac{1}{t'}}{\frac{s}{(1-s')(1-b)} - \frac{1-t}{t'}}. \tag{C.12}$$

□

Proof of Corollary 5 We first look at the probability of a H-firm getting report h, $(1-\beta)s_A+\beta s_B$. Simplify expressions of α and β as $\alpha = \frac{R-A}{B-A}$ and $\beta = \frac{\frac{1}{A}-\frac{1}{R}}{\frac{1}{A}-\frac{1}{B}}$. We take derivative of this probability over s_B and examine the sign of the numerator, $(\frac{1}{A}-\frac{1}{B}-\frac{1}{R})(\frac{1}{A}-\frac{1}{B})+(\frac{1}{A}s_B-\frac{1}{B}s_A-\frac{1}{R}(s_A-s_B))(\frac{1}{1-t_B})$. Rearrange, the above expression becomes $(\frac{1}{A}-\frac{1}{R})(\frac{1}{A}-\frac{1}{B})+(\frac{1}{R}-\frac{1}{B})(\frac{s_A}{s_B}-1)\frac{1}{B}$, which is positive.

We show the same for the other three probabilities.

Proof of Corollary 6 We first look at $Inv_H = (1-\beta)s_A p + \beta s_B q$. Rearranging leads to $Inv_H = \frac{c_A(\frac{1}{R}-\frac{1}{aAB})+b\frac{s_A-s_B}{R}}{\frac{1}{A}-\frac{1}{B}}$. Take derivative of Inv_H over

s_B, we have $\frac{\partial Inv_H}{\partial s_B} = \frac{1}{(\frac{1}{A}-\frac{1}{B})^2}(\frac{c_A}{1-t_B}(\frac{1}{R} - \frac{1}{aA^2}) + \frac{b}{R}(\frac{s_A}{1-t_B} - \frac{s_A}{1-t_A}))$. Notice that both terms in the numerator are negative, which leads to $\frac{\partial Inv_H}{\partial s_B} < 0$.

We show the same to Inv_H relative to t_B, and Inv_L relative to s_B, t_B.

Bibliography

Akerlof, George A. 1970. The market for "lemons": Quality uncertainty and the market mehchanism. *The Quarterly Journal of Economics* **84**(3) 488–500.

Ba, Sulin, Paul A. Pavlou. 2002. Evidence of the effect of trust building technology in electronic markets: Price premiums and buyer behavior. *MIS Quarterly* **26**(3) 243–268.

Baer, Justin, Helen Thomas. 2010. Valukas smoothes way to legal action. *Financial Times* .

Bar-Yosef, Sasson, Bharat Sarath. 2005. Auditor size, market segmentation and litigation patterns: A theoretical analysis. *Review of Accounting Studies* **10** 59–92.

Bateman, Patrick J., Peter H. Gray, Brian S. Butler. 2006. Community commitment: How affect, obligation, and necessity drive online behaviors. *Proceedings of Twenty-Seventh International Conference on Information Systems*. Milwaukee, Wisconsin, 983–1000.

Benabou, Roland, Jean Tirole. 2006. Incentives and prosocial behavior. *American Economic Review* **96**(5) 1652–1678.

Carcello, Joseph V., Zoe-Vonna Palmrose. 1994. Auditor litigation and modified reporting on bankrupt clients. *Journal of Accounting Research* **32** 1–30.

Choi, Jong-Hag, Jeong-Bon Kim, Xiaohong Liu, Dan A. Simunic. 2008. Audit pricing, legal liability regimes, and big 4 premiums: Theory and cross-country evidence. *Contemporary Accounting Research* **25**(1) 55–99.

Chromatic, Brian Aker, David Krieger. 2002. *Running Weblogs with Slash*. O'Reilly Media, Inc.

Coskun, Deniz. 2008. Credit rating agencies in a post-enron world: Congress revisits the nrsro concept. *Journal of Banking Regulation* **9**(4) 264–283.

Crawford, Vincent P., Joel Sobel. 1982. Strategic information transmission. *Econometrica* **50**(6) 1431–1451.

Cripps, Martin W., George J. Mailath, Larry Samuelson. 2004. Imperfect monitoring and impermanent reputations. *Econometrica* **72**(2) 407–432.

Dellarocas, Chrysanthos. 2005. Reputation mechanism design in online trading environments with pure moral hazard. *Information Systems Research* **16**(2) 209–230.

DeLone, William H., Ephraim R. McLean. 1992. Information systems success: The quest for the dependent variable. *Information Systems Research* **3**(1) 60–95.

Dye, Ronald A. 1993. Auditing standards, legal liability, and auditor wealth. *The Journal of Political Economy* **101**(5) 887–914.

Fasten, Erik R., Dirk Hofmann. 2010. Two-sided certification: The market for rating agencies. *SFB 649 Discussion Paper* .

Forman, Chris, Anindya Ghose, Batia Wiesenfeld. 2008. Examining the relationship between reviews and sales: The role of reviewer identity disclosure in electronic markets. *Information Systems Research* **19**(3) 291–313.

Frankel, Richard M., Marilyn F. Johnson, Karen K. Nelson. 2002. The relation between auditors' fees for nonaudit services and earnings management. *The Accounting Review* **77** 71–105.

Furchtgott-Roth, Harold, Robert W. Hahn, Anne Layne-Farrar. 2006. The law and economics of regulating ratings firms. *Journal of Competition Law and Economics* **3**(1) 49–96.

Hillegeist, Stephen A. 1999. Financial reporting and auditing under alternative damage apportionment rules. *The Accounting Review* **74**(3) 347–369.

Kreps, David M., Robert Wilson. 1982. Reputation and imperfect information. *Journal of Economic Theory* **27**(2) 253–279.

Krugman, Paul. 2010. Berating the raters. *The New York Times* .

Lampe, Cliff, Paul Resnick. 2004. Slash(dot) and burn: Distributed moderation in a large online conversation space. *Proceedings of ACM CHI 2004*

Conference on Human Factors in Computing Systems. Vienna Austria, 543–550.

Laux, Volker, D. Paul Newman. 2010. Auditor liability and client acceptance decisions. *The Accounting Review* **85**(1) 261–285.

Lizzeri, Alessandro. 1999. Information revelation and certification intermediaries. *The RAND Journal of Economics* **30**(2) 214–231.

Lu, Tong, Haresh Sapra. 2009. Auditor conservatism and investment efficiency. *The Accounting Review* **84**(6) 1933–1958.

Mathis, Jrme, James McAndrews, Jean-Charles Rochet. 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics* **56** 657–674.

Mayzlin, Dina. 2006. Promotional chat on the internet. *Marketing Science* **25**(2) 155–163.

Milgrom, Paul, John Roberts. 1982. Predation, reputation, and entry deterrence. *Journal of Economic Theory* **27**(2) 280–312.

Mookherjee, Dilip, Ivan Png. 1989. Optimal auditing, insurance, and redistribution. *The Quarterly Journal of Economics* **104**(2) 399–415.

Murphy, Megan, Brooke Masters. 2010. Ernst & young faces uk inquiry over audit of lehman brothers. *Financial Times* .

Rapoport, Michael. 2010. Bank of america, citigroup incorrectly hid billions in repo debt. *Wall Street Journal* .

Sangiorgi, Francesco, Jonathan Sokobin, Chester Spatt. 2009. Credit-rating shopping, selection and the equilibrium structure of ratings. *Carnegie Mellon Working Paper* .

Schwartz, Rachel. 1997. Legal regimes, audit quality and investment. *The Accounting Review* **72**(3) 385–406.

Skreta, Vasiliki, Laura Veldkamp. 2009. Ratings shopping and asset complexity: A theory of ratings inflation. *Journal of Monetary Economics* **56**(5) 678–695.

Stahl, Konrad, Roland Strausz. 2009. Certification and exchange in vertically concentrated markets. *Working Paper* .

Stolper, Anno. 2009. Regulation of credit rating agencies. *Journal of Banking & Finance* **33** 1266–1273.

Wasko, Molly McLure, Samer Faraj. 2005. Why should I share? examining social capital and knowledge contribution in electronic networks of practice. *MIS Quarterly* **29**(1) 35–57.

Index

Vita

Hong Xu received the Bachelor of Management degree in Management Information Systems from Tsinghua University in China. She entered the doctoral program in Information Systems at the University of Texas at Austin in August 2005. Her research interests include online reputation, moderation, auditing, and knowledge creation and management.

Permanent address: IROM Department
1 University Station B6500
Austin, TX 78712

This dissertation was typeset with LaTeX[†] by the author.

[†] LaTeX is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's TeX Program.