Copyright

by

Zaher Kassas

2010

The Report Committee for Zaher Kassas
Certifies that this is the approved version of the following report:

**Optimal $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Control of Extremely Large Segmented
Telescopes**

APPROVED BY

SUPERVISING COMMITTEE:

Robert H. Bishop, Supervisor

Ari Arapostathis

Optimal $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Control of Extremely Large Segmented Telescopes

by

Zaher Kassas, B.E.; M.S.

REPORT

Presented to the Faculty of the Graduate School of
The University of Texas at Austin
in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE IN ENGINEERING

THE UNIVERSITY OF TEXAS AT AUSTIN

August 2010

Optimal $\mathcal{H}_2$ and $\mathcal{H}_\infty$ Control of Extremely Large Segmented Telescopes

by

Zaher Kassas, M.S.E.

The University of Texas at Austin, 2010

SUPERVISOR: Robert H. Bishop

Extremely large telescopes (ELTs) are the next generation of ground-based reflecting telescopes of optical wavelengths. ELTs possess an aperture of more than 20 meters and share a number of common features, particularly the use of a segmented primary mirror and the use of adaptive optics systems. In 2005, the European Southern Observatory introduced a new giant telescope concept, named the European Extremely Large Telescope (E-ELT), which is scheduled for operation in 2018. The E-ELT will address key scientific challenges and will aim for a number of notable firsts, including discovering Earth-like planets around other stars in the “habitable zones” where life could exist, attempting to uncover the relationship between black holes and galaxies, measuring the properties of the first stars and galaxies, and probing the nature of dark matter and dark energy. In 2009, a feasibility study, conducted by National Instruments, proved the feasibility of the real-time (RT) control system architecture for the E-ELT’s nearly 1,000 mirror segments with 3,000 actuators and 6,000 sensors. The goal of the RT control system was to maintain a perfectly aligned field of mirrors at all times with a loop-time of 1 ms. The study assumed a prescribed controller algorithm. This research

report prescribes the optimal controller algorithms for large segmented telescopes. In this respect, optimal controller designs for the primary mirror of the E-ELT, where optimality is formulated in the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ frameworks are derived. Moreover, the designed controllers are simulated to show that the desired performance metrics are met.

Table of Contents

Abstract	iv
List of Figures	viii
Chapter 1. Introduction	1
Chapter 2. System Modeling	6
2.1 Optical design	6
2.2 Structure of the primary mirror system	7
2.3 Dynamics of the primary mirror system	9
2.4 Wind disturbance	10
2.5 Edge sensor model	13
2.6 Imaging performance requirements	14
Chapter 3. Optimal $\mathcal{H}_2$ Controller Design for a Seven-Segment System	15
3.1 Optimal $\mathcal{H}_2$ Control Theory	16
3.2 Optimal $\mathcal{H}_2$ problem formulation	19
Chapter 4. Optimal $\mathcal{H}_\infty$ Controller Design for a Seven-Segment System	22
4.1 Optimal $\mathcal{H}_\infty$ Control Theory	22
4.2 Optimal $\mathcal{H}_\infty$ problem formulation	25
Chapter 5. Simulations and Results	27
5.1 Open-Loop Plant Analysis	27
5.2 Optimal $\mathcal{H}_2$ Control Simulations	31
5.3 Optimal $\mathcal{H}_\infty$ Control Simulations	36
Chapter 6. Conclusion and Future Work	40
Appendix	41

Appendix 1. Numerical Values for System Matrices	42
Bibliography	43
Vita	45

List of Figures

1.1	Artist depiction of the European Extremely Large Telescope. Source [2]. . .	2
1.2	Hardware-in-the-loop $M1$ simulator	4
2.1	E-ELT optical system. Source [2].	6
2.2	E-ELT primary mirror ($M1$). Source [2].	7
2.3	Sensor and actuator arrangement on $M1$ mirror segments	8
2.4	Seven-segment system	8
2.5	Bode magnitude plot of wind disturbance reconstruction filter	12
2.6	Effect of relative displacement on the Strehl ratio. Source [9].	14
3.1	Standard control system block diagram configuration	16
3.2	Block diagram of control system with augmented plant	20
4.1	Block diagram of control system with augmented plant	25
5.1	Position of supporting point 1 for segment “0” without control	28
5.2	Position of supporting point 2 for segment “0” without control	28
5.3	Position of supporting point 3 for segment “0” without control	29
5.4	Relative displacement between segment “0” and segment “1” without control	29
5.5	Relative displacement between segment “0” and segment “2” without control	30
5.6	Relative displacement between segment “0” and segment “3” without control	30
5.7	LabVIEW block diagram of closed-loop control system of a 7-segment telescope	32
5.8	Position of supporting point 1 for segment “0” with $\mathcal{H}_2$ control	33
5.9	Position of supporting point 2 for segment “0” with $\mathcal{H}_2$ control	33
5.10	Position of supporting point 3 for segment “0” with $\mathcal{H}_2$ control	34
5.11	Relative displacement between segment “0” and segment “1” with $\mathcal{H}_2$ control	34
5.12	Relative displacement between segment “0” and segment “2” with $\mathcal{H}_2$ control	35
5.13	Relative displacement between segment “0” and segment “3” with $\mathcal{H}_2$ control	35
5.14	Position of supporting point 1 for segment “0” with $\mathcal{H}_\infty$ control	37

5.15	Position of supporting point 2 for segment “0” with $\mathcal{H}_\infty$ control	37
5.16	Position of supporting point 3 for segment “0” with $\mathcal{H}_\infty$ control	38
5.17	Relative displacement between segment “0” and segment “1” with $\mathcal{H}_\infty$ control	38
5.18	Relative displacement between segment “0” and segment “2” with $\mathcal{H}_\infty$ control	39
5.19	Relative displacement between segment “0” and segment “3” with $\mathcal{H}_\infty$ control	39

Chapter 1

Introduction

Understanding the evolution of the universe is a fundamental research direction for astronomers and physicists. Scientists have been studying the universe since the inception of the field of astronomical research 400 years ago. Modern astronomical research requires investigating extremely faint objects that have small angular diameters. Large aperture telescopes provide higher spatial and spectral resolution that enables the detection of the astronomical objects in question. Early hints about how stars and galaxies formed are being discovered with the deployment of large ground- and space-based telescopes [15]. By pushing the capabilities of ground-based telescopes to their limits using extremely large telescopes, astronomers are hoping to answer several lingering questions about our universe.

At the present time, the world's largest telescope is the Keck telescope located at Mauna Kea Observatory in Hawaii. The Keck's primary mirror is 10 meters (m) in diameter and consists of a mosaic of 36 hexagonal segments. The 36 segments are adjusted once per minute to compensate for the changing position of the telescope [4].

In the 1990s, a consortium of universities proposed the first of the next generation of extremely large telescopes, the California Extremely Large Telescope (CELT), which was renamed the Thirty Meter Telescope (TMT) in 2003 [1]. The design concept for the TMT includes a 30 m segmented primary mirror consisting of 1,080 segments with 3,240 primary

mirror actuators with the out-of-plane degrees of freedom (DOF) actively controlled. In addition, it includes a 3 m secondary mirror requiring at least 5 DOF control [11].

Since the end of 2005, the European Southern Observatory (ESO) has been defining a new extremely large telescope named the European Extremely Large Telescope (E-ELT) [2]. The E-ELT will address key scientific challenges and will aim for a number of notable firsts, including discovering Earth-like planets around other stars in the “habitable zones” where life could exist. Other answers that the E-ELT will attempt to uncover include understanding the relationship between black holes and galaxies, measuring the properties of the first stars and galaxies, and probing the nature of dark matter and dark energy. An artist’s depiction of the E-ELT is illustrated in Figure 1.1.

Figure 1.1: Artist depiction of the European Extremely Large Telescope. Source [2].

The E-ELT is a revolutionary 42 m diameter ground-based telescope that will be the largest optical/near-infrared telescope in the world. It is planned to be operational in 2018. The E-ELT will implement adaptive optics technology that incorporates a combination of sensors, actuators, and a control system so that the telescope can maintain the correct mirror shape, or collimation. The correct configuration must be maintained for the telescope to reduce any residual aberrations in the optical design [7].

Controlling the complex system of mirrors requires a vast amount of processing capability. The E-ELT primary segmented mirror, denoted as $M1$, consists of 984 hexagonal mirrors each weighing nearly 150 kg with diameters of 1.4 m for a total 42 m in diameter. In comparison, the diameter of the primary mirror of the Hubble Space Telescope is 2.4 m. In the $M1$ operation, adjacent mirror segments may tilt with respect to other segments. This deviation is monitored using edge sensors and actuator legs that can move the segment 3 degrees in any direction. The 984 mirror segments comprise a system of 3,000 actuators and 6,000 sensors.

Since the control system engineering design can impact the construction characteristics of the telescope, it is critical to assess the feasibility of the proposed control system architecture. For this purpose, a two-step study is conducted. The first study is to build a real-time (RT) simulation of the $M1$ mirror to perform hardware-in-the-loop (HIL) control system testing. The goal was to maintain a perfectly aligned field of mirrors at all times with a loop-time of 1 ms. For the purposes of HIL testing, the controller algorithm was assumed to be prescribed. Consequently, a case study was conducted to verify the feasibility of the control system architecture using LabVIEW deployed to a multi-core PC running the LabVIEW Real-Time Module [14]. The architecture incorporated two Dell Precision®

T7400 Workstations, each with eight cores and a notebook that provided an operator interface. One Workstation simulated the telescope, whereas the other Workstation simulated the controller algorithm. The HIL *M1* simulator is depicted in Figure 1.2. The HIL simulator included a standard network that connects both RT targets to the notebook and a 1 GB time-triggered Ethernet network between the RT targets for exchanging input/output (I/O) data. The required computational power was provided by multi-core implementations, which was supplemented by field-programmable gate arrays (FPGAs) and graphical processing units (GPUs). The study concluded that within the specified 1 ms loop-time, it is feasible to obtain the sensor data, send the data to the controller, compute the control signal, and send the control signal to the actuators [12].

Figure 1.2: Hardware-in-the-loop *M1* simulator

The second study is the topic of this report. This work focuses on prescribing optimal controller algorithms for segmented telescopes. In this respect, optimal controller designs for the primary mirror of the E-ELT, where optimality is formulated in the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ frame-

works are derived. Moreover, the closed-loop system comprised of the designed controllers, the telescope dynamics, and the exogenous disturbances, is simulated. The simulations show that the desired performance metrics, which are defined in terms of imaging performance requirements, are met.

This report is organized as follows. Chapter 2 discusses the optical design of the E-ELT and models the primary mirror system. Chapters 3 and 4 formulate the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control design problems for the primary mirror system, respectively. Chapter 5 presents the designed controller and open-loop and closed-loop simulations of the overall system. Concluding remarks and future work are discussed in Chapter 6.

Chapter 2

System Modeling

2.1 Optical design

The core concept for the E-ELT is a telescope with a 42 m diameter primary mirror covering a field on the sky about a tenth the size of the full Moon. The optical design is based on a five-mirror scheme that results in an exceptional image quality (see Figure 2.1).

Figure 2.1: E-ELT optical system. Source [2].

The primary mirror, denoted as $M1$, consists of 984 segments, each 1.4 m wide, but only 50 mm thick (see Figure 2.2). The control system for the E-ELT includes the control of the primary mirror, control loops for positioning the secondary mirrors, and control loops for adaptive optics. The crucial factor that determines the imaging quality is the performance of the primary mirror control system. This is the topic of this report.

Figure 2.2: E-ELT primary mirror ($M1$). Source [2].

2.2 Structure of the primary mirror system

The $M1$ mirror consists of 984 hexagonal mirror segments. On the back of each segment, 3 displacement actuators are installed to provide 3 out-of-plane degrees of freedom: piston, tip, and tilt. Moreover, each segment is equipped with 6 precision capacitive edge sensors that will measure the relative surface displacements between adjacent segments. Figure 2.3 illustrates the geometry of the actuators and sensors placements on one mirror segment and shows a neighborhood of 3 mirror segments. The optimal choice of the segment size is a function of several factors. Smaller segments are easier to support against gravity,

easier to handle, and allow fewer optical surfaces to be used. However, smaller segments require more actuators and more edge sensors besides other disadvantages [5].

Figure 2.3: Sensor and actuator arrangement on $M1$ mirror segments

In this report, we formulate and solve the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal control design problem of the seven-segment system illustrated in Figure 2.4. The control of the entire telescope becomes an extension of the seven-segment system to as many segments the telescope possesses.

Figure 2.4: Seven-segment system

2.3 Dynamics of the primary mirror system

We adopt a nodal model to describe the dynamics of the mirror segments [8, 9]. In particular, our model consists of localized masses mounted on springs and dampers. In general, a nodal model is a collection of equations of motion governed by second order vector differential equations, given by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{J}\mathbf{q} = \mathbf{B}_w\mathbf{w} + \mathbf{B}_u\mathbf{u},$$

where $\mathbf{q} \in \mathbb{R}^{n_d}$, $\dot{\mathbf{q}} \in \mathbb{R}^{n_d}$, and $\ddot{\mathbf{q}} \in \mathbb{R}^{n_d}$ are the nodal displacements, velocities, and accelerations, respectively; $\mathbf{M} \in \mathbb{R}^{n_d \times n_d}$ is the mass matrix, $\mathbf{D} \in \mathbb{R}^{n_d \times n_d}$ is the damping matrix, $\mathbf{J} \in \mathbb{R}^{n_d \times n_d}$ is the stiffness matrix, $\mathbf{w} \in \mathbb{R}^{n_w}$ is the disturbance to the system, $\mathbf{u} \in \mathbb{R}^{n_u}$ is the control force, and n_d is the number of degrees of freedom the system has (3 in this case), n_w is the number of disturbances affecting the system, and n_u is the number control forces applied to the system. Note that the supporting foundation of the primary mirror is much stiffer than the segments and oscillates at higher frequencies than those that the control system involves. To simplify the modeling procedure, we assume that the supporting foundation is fixed and each segment is supported by three springs with damping. Hence, the state dynamics of each segment are decoupled.

We define the state vector of the i^{th} segment as one representing the displacements and velocities of the three supporting points of the i^{th} segment, i.e.

$$\mathbf{x}_i = \begin{bmatrix} \mathbf{q}_i \\ \dot{\mathbf{q}}_i \end{bmatrix}.$$

Consequently, the nodal representation can be converted into the state-space representation

$$\dot{\mathbf{x}}_i = \mathbf{A}_i\mathbf{x}_i + \mathbf{B}_{1,i}\mathbf{w}_i + \mathbf{B}_{2,i}\mathbf{u}_i,$$

where

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_i^{-1}\mathbf{J}_i & -\mathbf{M}_i^{-1}\mathbf{D}_i \end{bmatrix}, \quad \mathbf{B}_{1,i} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}_i^{-1}\mathbf{B}_{w,i} \end{bmatrix}, \quad \mathbf{B}_{2,i} = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}_i^{-1}\mathbf{B}_{u,i} \end{bmatrix},$$

where $\mathbf{x}_i \in \mathbb{R}^6$ is the segment state vector, $\mathbf{w}_i$ denotes the disturbance inputs, and $\mathbf{u}_i$ denotes the control forces on the i^{th} segment, respectively. The invertibility of the mass matrix $\mathbf{M}$ is guaranteed by construction.

Finally, the dynamics of the complete system with $n + 1$ segments is expressed as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u}, \quad (2.1)$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_n \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} \mathbf{B}_{i,0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{i,1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{B}_{i,n} \end{bmatrix}, \quad i \in \{1, 2\},$$

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_n \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_n \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_n \end{bmatrix}.$$

Numerical values for the system matrices are given in Appendix 1.

2.4 Wind disturbance

The disturbances that affect the primary mirror system can be categorized into optical wavefront disturbances from the atmosphere and mechanical disturbances. The mechanical disturbance sources include gravity deformations, thermal variations, wind, and vibrations

from drives and motors. Among these, the disturbances from the wind are very significant to the operation of a large segmented telescope, and if we can reject these disturbances, the other sources can be readily addressed. For dynamic wind disturbances, we adopt the Von Karman model with the wind pressure power spectral density (PSD) given by

$$\Phi_{pt}^{\nu K}(f) = \frac{C_{pt}^{\nu K}}{\left[1 + \left(\frac{f}{f_0}\right)^2\right]^{\frac{7}{6}}}.$$

We note that the PSD is fully defined by two parameters, the magnitude $C_{pt}^{\nu K}$ and the bandwidth f_0 . For most wind disturbances, the bandwidth is 1 Hz [13]. Based on empirical measured wind PSD, we can identify $C_{pt}^{\nu K} = 10$ [3]. Moreover, based on least-square estimate (LSE) method we can reconstruct the wind disturbance by passing white noise through a filter with transfer function

$$W_{wn}(s) = \frac{0.2017s + 3.162}{0.1196s^2 + 1.32s + 1}. \quad (2.2)$$

The Bode magnitude plot in Equation 2.2 is illustrated in Figure 2.5, which closely matches the measured PSD of wind disturbances [8].

A state-space realization of the filter $W_{wn}(s)$ can be obtained as

$$\begin{aligned} \dot{\boldsymbol{\xi}}_i &= \mathbf{A}_{\xi_i} \boldsymbol{\xi}_i + \mathbf{B}_{\xi_i} w_{0_i} \\ w_i &= \mathbf{C}_{\xi_i} \boldsymbol{\xi}_i, \end{aligned}$$

where

$$\mathbf{A}_{\xi_i} = \begin{bmatrix} -11.04 & -8.36 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{B}_{\xi_i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{C}_{\xi_i} = [1.69 \quad 26.44],$$

where w_{0_i} is the input white noise to the reconstruction filter and w_i is the output of the filter.

For simplicity, we assume that the wind disturbance is applied equally to the three supporting

Figure 2.5: Bode magnitude plot of wind disturbance reconstruction filter

points; hence, the noise disturbance model affecting $n + 1$ segments of the telescope is given by

$$\dot{\xi} = A_{\xi} \xi + B_{\xi} w_0 \quad (2.3)$$

$$w = C_{\xi} \xi, \quad (2.4)$$

where

$$A_{\xi} = \begin{bmatrix} A_{\xi_0} & 0 & \cdots & 0 \\ 0 & A_{\xi_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{\xi_n} \end{bmatrix}, \quad B_{\xi} = \begin{bmatrix} B_{\xi_0} & 0 & \cdots & 0 \\ 0 & B_{\xi_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{\xi_n} \end{bmatrix}, \quad C_{\xi} = \begin{bmatrix} C_{\xi_0} & 0 & \cdots & 0 \\ 0 & C_{\xi_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{\xi_n} \end{bmatrix},$$

$$\xi = \begin{bmatrix} \xi_0 \\ \xi_1 \\ \vdots \\ \xi_n \end{bmatrix}, \quad w_0 = \begin{bmatrix} w_{0_0} \\ w_{0_1} \\ \vdots \\ w_{0_n} \end{bmatrix}, \quad w = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_n \end{bmatrix}.$$

2.5 Edge sensor model

Ideally, the relative displacements between the adjacent segments should be controlled to be less than one tenth of the operating wavelength. At this accuracy level, the best solution is to sense the position of each segment edge with respect to those of its neighbors. Typically two edge sensors are required per edge to provide the relative height and twist of adjacent segments. Edge sensors are best at defining the positions of segments with respect to their neighbors, but not over the scale of the overall mirror because of error propagation. For the control design purposes, we will assume that the outputs that become locally available from sensing at each segment, i , are the direct displacements of the three supporting points, i.e.

$$\mathbf{y}_i = \mathbf{C}_i \mathbf{x}_i + \mathbf{v}_i,$$

where $\mathbf{v}_i$ represents the measurement noise vector, and $\mathbf{C}_i$ has the structure

$$\mathbf{C}_i = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}.$$

For simplicity, we assume that the measurement noise is negligible, which is motivated by the fact that it is of smaller magnitude as compared to the wind disturbance. Hence, if we can address the wind disturbance issue, adding the noise model of the sensors in the design process should be straightforward.

Combining the measurements for the $n+1$ segments, yields the measurement equation

$$\mathbf{y} = \mathbf{C} \mathbf{x}, \tag{2.5}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{C}_n \end{bmatrix}.$$

2.6 Imaging performance requirements

For an optical system, the imaging performance is characterized by its point spread function, i.e. the light intensity distribution resulting from a point source. However, such function is two-dimensional and is very complex. For convenience, we adopt a commonly used practical criterion known as the Strehl ratio. The Strehl ratio is the ratio of the peak intensity in the actual image compared to the peak theoretical diffraction intensity. According to the Maréchal rule, an optical system is considered diffraction limited if the Strehl ratio is 0.8 [4].

Figure 2.6 shows how the Strehl ratio varies according to relative displacement. From Figure 2.6, we note that to maintain a Strehl ratio greater than 0.7, we have to control the relative displacements of the segments at the 1×10^{-7} m order.

Figure 2.6: Effect of relative displacement on the Strehl ratio. Source [9].

Chapter 3

Optimal $\mathcal{H}_2$ Controller Design for a Seven-Segment System

This chapter formulates the $\mathcal{H}_2$ optimal controller design problem for a seven-segment system of the large segmented telescope. The $\mathcal{H}_2$ optimal control problem may be interpreted in the time-domain or the frequency-domain. In the time-domain, it can be viewed as the problem of finding the feasible control law, $\mathbf{u}$, that minimizes a quadratic objective functional weighing the states, $\mathbf{x}$, and the control input, $\mathbf{u}$. In the frequency-domain, it can be viewed as the problem of finding the control law that minimizes the $\mathcal{H}_2$ -norm of the closed-loop transfer function matrix, $\mathbf{F}_{zw}$, between exogenous signals, $\mathbf{w}$, and signals of interest, $\mathbf{z}$. The exogenous signals include all external signals acting on the system dynamics, such as reference, disturbance, and noise signals. The signals of interest follow from desired performance specification and may include certain error signals and the system measured response [10].

In this chapter, we assume the standard control system block diagram configuration, often referred to as the $\mathbf{P}-\mathbf{K}$ configuration, illustrated in Figure 3.1. In this respect, the control system block diagram is first re-arranged into such configuration, with $\mathbf{w}$ representing

the exogenous signals, $\mathbf{z}$ representing the signals of interest, $\mathbf{y}$ representing the measured outputs, and $\mathbf{u}$ representing the control signals. The $\mathbf{P}$ block contains all the uncontrolled dynamics, which include the system dynamics, actuator dynamics, and various pre-designed filters. The $\mathbf{K}$ block contains the controller to be designed.

Figure 3.1: Standard control system block diagram configuration

3.1 Optimal $\mathcal{H}_2$ Control Theory

This section presents the necessary $\mathcal{H}_2$ relevant control theory background. First, Lemma 3.1.1 establishes the connection between the time-domain linear quadratic regulator (LQR) minimization problem and the frequency-domain $\mathcal{H}_2$ -norm transfer function minimization problem. Then, the $\mathcal{H}_2$ optimal control solution is presented.

Lemma 3.1.1. *The problem of finding a state-feedback control law $\mathbf{u} = f(\mathbf{x})$ that minimizes the linear quadratic objective subject to the dynamics of a linear time-invariant system, i.e.*

$$\begin{aligned} \text{minimize} \quad & J = \int_0^\infty \begin{bmatrix} \mathbf{x}^\top(t) & \mathbf{u}^\top(t) \end{bmatrix} \begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^\top & \mathbf{R} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} dt \\ \text{subject to} \quad & \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_2\mathbf{u}, \quad \mathbf{u} \in \mathcal{L}_2, \end{aligned}$$

where $\mathbf{Q} = \mathbf{Q}^\top \geq 0$, $\mathbf{R} = \mathbf{R}^\top > 0$, and $\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^\top & \mathbf{R} \end{bmatrix} \geq 0$ is equivalent to the optimization problem, defined as

$$\begin{aligned} \text{minimize} \quad & J = \|\mathbf{z}\|_2^2 = \|\mathbf{C}_1\mathbf{x} + \mathbf{D}_{12}\mathbf{u}\|_2^2 \\ \text{subject to} \quad & \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_2\mathbf{u}, \quad \mathbf{u} \in \mathcal{L}_2, \end{aligned} \tag{3.1}$$

where $\mathbf{Q} = \mathbf{C}_1^\top \mathbf{C}_1$, $\mathbf{R} = \mathbf{D}_{12}^\top \mathbf{D}_{12}$, $\mathbf{S} = \mathbf{C}_1^\top \mathbf{D}_{12}$, and $\mathbf{z}$ is the signal of interest whose $\mathcal{H}_2$ -norm is to be minimized.

Theorem 3.1.2. *Given a plant in the general feedback formulation shown in Figure 3.1, where the plant dynamics, signals of interest, and the plant outputs are given respectively by*

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u} \\ \mathbf{z} &= \mathbf{C}_1\mathbf{x} + \mathbf{D}_{12}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}_2\mathbf{x} + \mathbf{D}_{21}\mathbf{w}, \end{aligned}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{w} \in \mathbb{R}^{n_w}$, $\mathbf{u} \in \mathbb{R}^{n_u}$, $\mathbf{y} \in \mathbb{R}^{n_y}$ are the plant states, exogenous inputs, control inputs, and plant outputs, respectively. Assume that

- $(\mathbf{A}, \mathbf{B}_2)$ is stabilizable.
- $(\mathbf{A}, \mathbf{C}_1)$ is detectable.

- $(\mathbf{A}, \mathbf{C}_2)$ is detectable.

Define the Hamiltonian matrices

$$\begin{aligned}\mathbf{H}_2 &= \begin{bmatrix} \mathbf{A} - \mathbf{B}_2 \mathbf{D}_{12}^\top \mathbf{C}_1 & -\mathbf{B}_2 \mathbf{B}_2^\top \\ -\mathbf{C}_1^\top \mathbf{C}_1 & -(\mathbf{A} - \mathbf{B}_2 \mathbf{D}_{12}^\top \mathbf{C}_1)^\top \end{bmatrix} \\ \mathbf{J}_2 &= \begin{bmatrix} \mathbf{A}^\top - \mathbf{C}_2^\top \mathbf{D}_{21} \mathbf{B}_1^\top & -\mathbf{C}_2^\top \mathbf{C}_2 \\ -\mathbf{B}_1 \mathbf{B}_1^\top & -(\mathbf{A}^\top - \mathbf{C}_2^\top \mathbf{D}_{12} \mathbf{B}_1)^\top \end{bmatrix}.\end{aligned}$$

Define $\mathbf{X}_R = \text{Ric}(\mathbf{H}_2)$ and $\mathbf{Y}_R = \text{Ric}(\mathbf{J}_2)$ to be the Riccati equation solutions associated with the Hamiltonian matrices $\mathbf{H}_2$ and $\mathbf{J}_2$, respectively. Define the controller gain and observer gains respectively as

$$\begin{aligned}\mathbf{F}_2 &= -[\mathbf{B}_2^\top \mathbf{X}_R + \mathbf{D}_{12}^\top \mathbf{C}_1] \\ \mathbf{L}_2 &= -[\mathbf{Y}_R \mathbf{C}_2^\top + \mathbf{B}_1 \mathbf{D}_{21}^\top].\end{aligned}$$

Then, the $\mathcal{H}_2$ optimal controller $\mathbf{K}(s)$ to problem (3.1) has a state-space realization given by

$$\mathbf{K}(s) \sim \left[\begin{array}{c|c} \mathbf{A} + \mathbf{B}_2 \mathbf{F}_2 + \mathbf{L}_2 \mathbf{C}_2 & -\mathbf{L}_2 \\ \hline \mathbf{F}_2 & \mathbf{0} \end{array} \right].$$

Moreover, the controller $\mathbf{K}(s)$ minimizes the $\mathcal{H}_2$ norm of the transfer function between the exogenous inputs $\mathbf{w}$ and signals of interest $\mathbf{z}$, $\|\mathbf{F}_{zw}\|_2^2$, and the minimum at such minimizer is given by

$$J_{\min} = \|\mathbf{G}_F\|_2^2 + \|\mathbf{G}_L\|_2^2,$$

where $\mathbf{G_F}$ and $\mathbf{G_L}$ represent the closed-loop controller and observer transfer functions that have state-space realizations given respectively as

$$\begin{aligned}\mathbf{G_F}(s) &\sim \left[\begin{array}{c|c} \mathbf{A} + \mathbf{B}_2\mathbf{F}_2 & \mathbf{B}_1 \\ \hline \mathbf{C}_1 + \mathbf{D}_{12}\mathbf{F}_2 & \mathbf{0} \end{array} \right] \\ \mathbf{G_L}(s) &\sim \left[\begin{array}{c|c} \mathbf{A} + \mathbf{L}_2\mathbf{C}_2 & \mathbf{B}_1 + \mathbf{L}_2\mathbf{D}_{21} \\ \hline -\mathbf{F}_2 & \mathbf{0} \end{array} \right].\end{aligned}$$

3.2 Optimal $\mathcal{H}_2$ problem formulation

The state-space model for a seven-segment system was expressed in Equation (2.1), and the outputs of the whole system was formulated in Equation (2.5) as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u} \\ \mathbf{y} &= \mathbf{Cx},\end{aligned}$$

where $\mathbf{x} \in \mathbb{R}^{42}$, $\mathbf{w} \in \mathbb{R}^{21}$, $\mathbf{u} \in \mathbb{R}^{21}$, $\mathbf{y} \in \mathbb{R}^{21}$, $\mathbf{A} \in \mathbb{R}^{42 \times 42}$, $\mathbf{B}_1 \in \mathbb{R}^{42 \times 21}$, $\mathbf{B}_2 \in \mathbb{R}^{42 \times 21}$, and $\mathbf{C} \in \mathbb{R}^{21 \times 42}$.

Since the imaging performance is essentially related to the root mean square (RMS) of the relative displacements of the segments, the performance objective of the controller will be to minimize this RMS value in the presence of the atmospheric disturbances. This will be formulated as an $\mathcal{H}_2$ optimization problem.

Since we are assuming that we are measuring the direct (absolute) displacements, but the primary objective is to regulate the relative displacements between adjacent segments to

zero, we define the variable of primary interest, $\mathbf{z}_1$, as

$$\mathbf{z}_1 = \mathbf{C}_{z_1} \mathbf{x},$$

where $\mathbf{C}_{z_1} \in \mathbb{R}^{6 \times 6}$ is the mapping matrix from the state vector $\mathbf{x}$ that describes the displacements and velocities of the supporting points to the relative displacements between adjacent segments. The mapping matrix $\mathbf{C}_{z_1}$ can be readily determined from the geometry of the segmented mirror system and has the structure

$$\mathbf{C}_{z_1} = \begin{bmatrix} \mathbf{C}_{z_1,1} & \mathbf{0}_{6 \times 3} \end{bmatrix}.$$

Numerical values of $\mathbf{C}_{z_1}$ are given in Appendix 1. The block diagram that describes the control system for our particular system is shown in Figure 3.2, where $\mathbf{x}$ is the plant states, $\mathbf{u}$ is the control input vector, $\mathbf{y}$ is the vector of measurements, $\mathbf{z}_1$ is the vector describing the displacement between the adjacent segments, $\mathbf{w}_0$ is a white noise input, $\mathbf{w}$ is the wind disturbance, $W_{wn}(s)$ is a the white noise filter, $\mathbf{G}$ is the plant dynamics, and $\mathbf{y}$ is the controller to be designed.

Figure 3.2: Block diagram of control system with augmented plant

Note that the actual wind disturbance $\mathbf{w}$ is reconstructed through the white noise filter $W_{wn}(s)$ whose state-space realization was derived in (2.3)-(2.4). The control problem objective is to align seven segments to form a smooth larger segment with minimal efforts.

Next, we will formulate the augmented plant shown in Figure 3.2 into the standard $\mathbf{P}-\mathbf{K}$ block diagram formulation illustrated in Figure 3.1. To this end, we define the exogenous signals and signal of interest respectively as

$$\tilde{\mathbf{w}} \triangleq \mathbf{w}_0, \quad \tilde{\mathbf{z}} \triangleq \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{u} \end{bmatrix}.$$

Consequently, the dynamics of the resulting standard control system block diagram configuration depicted in Figure 3.1 is characterized by

$$\begin{aligned} \dot{\boldsymbol{\chi}} &= \tilde{\mathbf{A}} \boldsymbol{\chi} + \tilde{\mathbf{B}}_1 \tilde{\mathbf{w}} + \tilde{\mathbf{B}}_2 \mathbf{u} \\ \tilde{\mathbf{z}} &= \tilde{\mathbf{C}}_1 \boldsymbol{\chi} + \tilde{\mathbf{D}}_{12} \mathbf{u} \\ \mathbf{y} &= \mathbf{C}_2 \boldsymbol{\chi}, \end{aligned}$$

where $\boldsymbol{\chi} \triangleq [\mathbf{x}^\top \boldsymbol{\xi}^\top]^\top \in \mathbb{R}^{84}$, $\tilde{\mathbf{w}} \in \mathbb{R}^{21}$, $\mathbf{u} \in \mathbb{R}^{21}$, $\tilde{\mathbf{z}} \in \mathbb{R}^{84}$, and $\mathbf{y} \in \mathbb{R}^{21}$, and the system matrices are given by

$$\begin{aligned} \tilde{\mathbf{A}} &= \begin{bmatrix} \mathbf{A} & \mathbf{B}_1 \mathbf{C}_\xi \\ \mathbf{0} & \mathbf{A}_\xi \end{bmatrix}, \quad \tilde{\mathbf{B}}_1 = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_\xi \end{bmatrix}, \quad \tilde{\mathbf{B}}_2 = \begin{bmatrix} \mathbf{B}_2 \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{C}_2 = [\mathbf{C} \quad \mathbf{0}] \\ \tilde{\mathbf{C}}_1 &= [\bar{\mathbf{C}}_1 \bar{\mathbf{C}}_{z_1} \quad \mathbf{0}], \quad \bar{\mathbf{C}}_1 = \begin{bmatrix} \bar{\bar{\mathbf{C}}}_1 \\ \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{C}}_{z_1} = \text{diag}[\mathbf{C}_{z_1}, \dots, \mathbf{C}_{z_1}], \quad \tilde{\mathbf{D}}_{12} = \begin{bmatrix} \bar{\mathbf{D}}_{12} \\ \mathbf{0} \end{bmatrix}, \end{aligned}$$

where in the formulation above the LQR “tuning” matrices are $\bar{\bar{\mathbf{C}}}_1 = \bar{\bar{\mathbf{C}}}_1^\top \geq 0$ and $\bar{\bar{\mathbf{C}}}_1 \in \mathbb{R}^{42 \times 42}$; $\bar{\mathbf{D}}_{12} = \bar{\mathbf{D}}_{12}^\top > 0$ and $\bar{\mathbf{D}}_{12} \in \mathbb{R}^{21 \times 21}$.

Chapter 4

Optimal $\mathcal{H}_\infty$ Controller Design for a Seven-Segment System

This chapter formulates the $\mathcal{H}_\infty$ optimal controller design problem for a seven-segment system of the large segmented telescope. The $\mathcal{H}_\infty$ optimal control problem may be interpreted as the problem of finding the control law that minimizes the $\mathcal{H}_\infty$ -norm of the closed-loop transfer function matrix, $\mathbf{F}_{zw}$, between exogenous signals, $\mathbf{w}$, and signals of interest, $\mathbf{z}$. As in Chapter 3, we will assume the standard $\mathbf{P}$ – $\mathbf{K}$ block diagram configuration (see Figure 3.1).

4.1 Optimal $\mathcal{H}_\infty$ Control Theory

This section presents the necessary $\mathcal{H}_\infty$ relevant control theory background. It is known that the solution to the $\mathcal{H}_\infty$ controller design problem is not unique [6]. There are several ways to derive an $\mathcal{H}_\infty$ controller. One approach is through a Youla parametrization of the closed loop. However, this often leads to a very high-order controller. Another approach is through an optimization-based reformulation of the Riccati equation that uses

linear matrix inequalities. A third approach is through a Riccati-based approach that solves two Riccati equations to find the sub-optimal controller. While the third approach requires several simplifying assumptions, we will adopt this approach as these assumptions are readily satisfied in our formulation [10].

Theorem 4.1.1. *Given a plant in the general feedback formulation shown in Figure 3.1, where the plant dynamics, signals of interest, and plant outputs are given respectively by*

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathbf{u} \\ \mathbf{z} &= \mathbf{C}_1\mathbf{x} + \mathbf{D}_{11}\mathbf{w} + \mathbf{D}_{12}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}_2\mathbf{x} + \mathbf{D}_{21}\mathbf{w} + \mathbf{D}_{22}\mathbf{u},\end{aligned}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{w} \in \mathbb{R}^{n_w}$, $\mathbf{u} \in \mathbb{R}^{n_u}$, $\mathbf{y} \in \mathbb{R}^{n_y}$ are the plant states, exogenous inputs, control inputs, and plant outputs, respectively. Assume that

- $(\mathbf{A}, \mathbf{B}_2)$ is stabilizable.
- $(\mathbf{A}, \mathbf{C}_1)$ is detectable.
- $(\mathbf{A}, \mathbf{C}_2)$ is detectable.
- $\mathbf{D}_{11} = \mathbf{0}$ and $\mathbf{D}_{22} = \mathbf{0}$.
- $\mathbf{D}_{12}^\top \mathbf{C}_1 = \mathbf{0}$ and $\mathbf{D}_{12}^\top \mathbf{D}_{12} = \mathbf{I}$.

- $\mathbf{B}_1 \mathbf{D}_{21}^\top = \mathbf{0}$ and $\mathbf{D}_{21} \mathbf{D}_{21}^\top = \mathbf{I}$.

Given $\gamma \in \mathbb{R}_{++}$. Then, there exists an admissible controller with $\|\mathbf{F}_{zw}\|_\infty < \gamma$ if and only if

- $\mathbf{X}_\infty = \text{Ric}(\mathbf{H}_\infty) \geq 0$ for

$$\mathbf{H}_\infty = \begin{bmatrix} \mathbf{A} & \gamma^{-2} \mathbf{B}_1 \mathbf{B}_1^\top - \mathbf{B}_2 \mathbf{B}_2^\top \\ -\mathbf{C}_1^\top \mathbf{C}_1 & -\mathbf{A}^\top \end{bmatrix}.$$

- $\mathbf{Y}_\infty = \text{Ric}(\mathbf{J}_\infty) \geq 0$ for

$$\mathbf{J}_\infty = \begin{bmatrix} \mathbf{A}^\top & \gamma^{-2} \mathbf{C}_1^\top \mathbf{C}_1 - \mathbf{C}_2^\top \mathbf{C}_2 \\ -\mathbf{B}_1 \mathbf{B}_1^\top & -\mathbf{A} \end{bmatrix}.$$

- $\rho(\mathbf{X}_\infty \mathbf{Y}_\infty) < \gamma^2$.

Further, if the above hold, then the $\mathcal{H}_\infty$ optimal controller $\mathbf{K}(s)$ has a state-space realization given by

$$\mathbf{K}(s) \sim \left[\begin{array}{c|c} \hat{\mathbf{A}}_\infty & -\mathbf{Z}_\infty \mathbf{L}_\infty \\ \hline \mathbf{F}_\infty & \mathbf{0} \end{array} \right],$$

where

$$\hat{\mathbf{A}}_\infty = \mathbf{A} + \gamma^{-2} \mathbf{B}_1 \mathbf{B}_1^\top \mathbf{X}_\infty + \mathbf{B}_2 \mathbf{F}_\infty + \mathbf{Z}_\infty \mathbf{L}_\infty \mathbf{C}_2$$

$$\mathbf{F}_\infty = -\mathbf{B}_2^\top \mathbf{X}_\infty$$

$$\mathbf{L}_\infty = -\mathbf{Y}_\infty \mathbf{C}_2^\top$$

$$\mathbf{Z}_\infty = [\mathbf{I} - \gamma^{-2} \mathbf{Y}_\infty \mathbf{X}_\infty]^{-1}.$$

4.2 Optimal $\mathcal{H}_\infty$ problem formulation

Due to the system setup assumed in Theorem 4.1.1, we will consider the augmented plant shown in Figure 4.1. Consequently, we will formulate the augmented plant shown in Figure 4.1 into the standard **P–K** block diagram configuration in Figure 3.1. For formulation purposes, we will assume that there is an additive sensor noise $\mathbf{v}_0$ and we will design the $\mathcal{H}_\infty$ controller accordingly. However, when simulating the closed-loop system, we will “turn off” such sensor noise for meaningful comparison against the $\mathcal{H}_2$ design.

Figure 4.1: Block diagram of control system with augmented plant

To this end, we define the exogenous signals and signal of interest respectively as

$$\tilde{\mathbf{w}} \triangleq \begin{bmatrix} \mathbf{w}_0 \\ \mathbf{v}_0 \end{bmatrix}, \quad \tilde{\mathbf{z}} \triangleq \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{u} \end{bmatrix}.$$

The dynamics of the resulting standard control system block diagram configuration

depicted in Figure 3.1 is characterized by

$$\begin{aligned}\dot{\boldsymbol{\chi}} &= \tilde{\mathbf{A}} \boldsymbol{\chi} + \tilde{\mathbf{B}}_1 \tilde{\mathbf{w}} + \tilde{\mathbf{B}}_2 \mathbf{u} \\ \tilde{\mathbf{z}} &= \tilde{\mathbf{C}}_1 \boldsymbol{\chi} + \tilde{\mathbf{D}}_{12} \mathbf{u} \\ \mathbf{y} &= \mathbf{C}_2 \boldsymbol{\chi} + \mathbf{D}_{21} \mathbf{w},\end{aligned}$$

where $\boldsymbol{\chi} \triangleq [\mathbf{x}^\top \ \boldsymbol{\xi}^\top]^\top \in \mathbb{R}^{84}$, $\tilde{\mathbf{w}} \in \mathbb{R}^{42}$, $\mathbf{u} \in \mathbb{R}^{21}$, $\tilde{\mathbf{z}} \in \mathbb{R}^{84}$, and $\mathbf{y} \in \mathbb{R}^{21}$, and the system matrices are given by

$$\begin{aligned}\tilde{\mathbf{A}} &= \begin{bmatrix} \mathbf{A} & \mathbf{B}_1 \mathbf{C}_\xi \\ \mathbf{0} & \mathbf{A}_\xi \end{bmatrix}, \quad \tilde{\mathbf{B}}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{B}_\xi & \mathbf{0} \end{bmatrix}, \quad \tilde{\mathbf{B}}_2 = \begin{bmatrix} \mathbf{B}_2 \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{C}_2 = [\mathbf{C} \ \mathbf{0}] \\ \tilde{\mathbf{C}}_1 &= [\bar{\mathbf{C}}_{z_1} \ \mathbf{0}], \quad \bar{\mathbf{C}}_{z_1} = \text{diag}[\mathbf{C}_{z_1}, \dots, \mathbf{C}_{z_1}], \quad \tilde{\mathbf{D}}_{12} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \mathbf{D}_{21} = [\mathbf{0} \ \mathbf{I}].\end{aligned}$$

Chapter 5

Simulations and Results

This chapter presents open-loop plant analysis and closed-loop system simulations with the designed $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers to achieve stabilization of the mirror segments in the presence of wind disturbances. The objective of the control system is to keep the relative displacements between the segments to within $1 \times 10^{-7}\text{m}$ in accordance with the imaging performance requirement specified by the Strehl ratio (see Section 2.6).

5.1 Open-Loop Plant Analysis

First, the seven-segment mirror system was constructed and its poles, zeros, and stability were investigated. The system was found to be stable, strictly proper, with no zeros, and with 42 poles located at $s_{1,2} = -0.078 \pm j21.074$, $s_{3,4} = -0.173 \pm j31.415$, $s_{5,6} = -0.078 \pm j21.074$, where each of the complex-conjugate pole pairs appear with multiplicity 7. Then, the system was simulated in open-loop configuration, i.e. without the controller in the loop. Figures 5.1-5.3 and 5.4-5.6 show the position of segment “0” (x_1 , x_2 , and x_3) and the relative displacements between segment “0” and 3 of its neighbors (z_1 , z_2 , and z_3), respectively, for the open-loop system. It is clear that the desired imaging performance is

not achievable for the open-loop system. Similar observations were made upon plotting the time histories for other mirror segments.

Figure 5.1: Position of supporting point 1 for segment “0” without control

Figure 5.2: Position of supporting point 2 for segment “0” without control

Figure 5.3: Position of supporting point 3 for segment "0" without control

Figure 5.4: Relative displacement between segment "0" and segment "1" without control

Figure 5.5: Relative displacement between segment “0” and segment “2” without control

Figure 5.6: Relative displacement between segment “0” and segment “3” without control

5.2 Optimal $\mathcal{H}_2$ Control Simulations

The underlying assumptions for the $\mathcal{H}_2$ control theory were first checked, namely $(\mathbf{A}, \mathbf{B}_2)$ is stabilizable, $(\mathbf{A}, \mathbf{C}_1)$ is detectable, and $(\mathbf{A}, \mathbf{C}_2)$ is detectable. Then, a LabVIEW code was written to design the $\mathcal{H}_2$ controller. Then, the closed-loop system illustrated in Figure 5.7 was simulated. The initial condition for the positions of the 3 supporting points of each segments were set to be 1×10^{-6} . The LQR tuning matrices were set to $\bar{\mathbf{C}}_1 = \alpha \mathbf{I}_{42 \times 42}$ and $\bar{\mathbf{D}}_{12} = \beta \mathbf{I}_{21 \times 21}$, where $\alpha \in \mathbb{R}_+$ and $\beta \in \mathbb{R}_{++}$ are the tuning parameters. The scalars α and β enter the cost functional to be minimized by weighing the system states and the control input. In the “standard” LQR problem formulation discussed in Lemma 3.1.1, the $\mathbf{Q}$ matrix penalizes the system states, the $\mathbf{R}$ matrix penalizes the control input, and the $\mathbf{S}$ matrix represents the cross-penalty between the system states and the control input.

A straightforward LQR tuning rule starts with setting $\mathbf{R} = \rho \mathbf{I}$. Consequently, $\rho \rightarrow 0$ corresponds to the “cheap control” scenario, where we do not penalize large control action as much, i.e. we expect control inputs to be of larger amplitudes. In such case, if $\mathbf{G}_1(s)$ has a state-space realization

$$\mathbf{G}_1(s) \sim \begin{bmatrix} \mathbf{A} & \mathbf{B}_2 \\ \mathbf{C}_1 & \mathbf{0} \end{bmatrix},$$

and $\mathbf{G}_1(s)$ has l zeros, then as $\rho \rightarrow 0$, l of the n closed-loop poles tend to the left-half plane zeros of $\mathbf{G}_1(s)$ or reflections about the $j\omega$ -axis of the right half-plane zeros of $\mathbf{G}_1(s)$. The remaining $(n - l)$ closed-loop poles tend to $-\infty$. On the other hand, setting the $\mathbf{Q}$ matrix too “large” means that large state deviations from desired regulation is not tolerated [10].

Figures 5.8-5.10 and 5.11-5.13 show the position of segment “0” (x_1 , x_2 , and x_3); and the relative displacement between segment “0” and 3 of its neighbors (z_1 , z_2 , and z_3),

5×10^{-4} , respectively.

Figure 5.7: LabVIEW block diagram of closed-loop control system of a 7-segment telescope

Figure 5.8: Position of supporting point 1 for segment “0” with $\mathcal{H}_2$ control

Figure 5.9: Position of supporting point 2 for segment “0” with $\mathcal{H}_2$ control

Figure 5.10: Position of supporting point 3 for segment “0” with $\mathcal{H}_2$ control

Figure 5.11: Relative displacement between segment “0” and segment “1” with $\mathcal{H}_2$ control

Figure 5.12: Relative displacement between segment “0” and segment “2” with $\mathcal{H}_2$ control

Figure 5.13: Relative displacement between segment “0” and segment “3” with $\mathcal{H}_2$ control

5.3 Optimal $\mathcal{H}_\infty$ Control Simulations

The underlying assumptions for the $\mathcal{H}_\infty$ control theory were first checked, namely $(\mathbf{A}, \mathbf{B}_2)$ is stabilizable; $(\mathbf{A}, \mathbf{C}_1)$ is detectable; $(\mathbf{A}, \mathbf{C}_2)$ is detectable; $\mathbf{D}_{11} = \mathbf{0}$ and $\mathbf{D}_{22} = \mathbf{0}$; $\mathbf{D}_{12}^\top \mathbf{C}_1 = \mathbf{0}$ and $\mathbf{D}_{12}^\top \mathbf{D}_{12} = \mathbf{I}$; and $\mathbf{B}_1 \mathbf{D}_{21}^\top = \mathbf{0}$ and $\mathbf{D}_{21} \mathbf{D}_{21}^\top = \mathbf{I}$. Then, a LabVIEW code was written to design the $\mathcal{H}_\infty$ controller. In designing the $\mathcal{H}_\infty$ controller, we iterated on the parameter $\gamma > 0$ starting from the initial iterate $\gamma^{(0)} = 1$. At each iterate, ν , we used the bisection-type update rule

$$\gamma^{(\nu+1)} = \frac{1}{2} \gamma^{(\nu)}, \quad \nu \in \mathbb{Z}.$$

At each iterate, we checked the necessary and sufficient conditions for existence of an admissible controller with $\|\mathbf{F}_{zw}\|_\infty < \gamma$, namely $\mathbf{X}_\infty = \text{Ric}(\mathbf{H}_\infty) \geq 0$, $\mathbf{Y}_\infty = \text{Ric}(\mathbf{J}_\infty) \geq 0$, and $\rho(\mathbf{X}_\infty \mathbf{Y}_\infty) < \gamma^2$. The terminating condition was whenever any of these conditions was violated. The corresponding γ was found to be $\gamma = 0.0078125$.

Consequently, the closed-loop system illustrated in Figure 5.7 was simulated, but with the $\mathcal{H}_\infty$ replacing the $\mathcal{H}_2$ controller. The initial condition for the positions of the 3 supporting points of each segments were set to be 1×10^{-6} . Figures 5.14-5.16 and 5.17-5.19 show the position of segment “0” (x_1 , x_2 , and x_3); and the relative displacement between segment “0” and 3 of its neighbors (z_1 , z_2 , and z_3), respectively. It is clear that the desired imaging performance was achieved with the designed $\mathcal{H}_\infty$ controller. Similar observations were made upon plotting the time histories for other mirror segments. However, it was noted that $\mathcal{H}_2$ achieved slightly better results. This issue can be addressed by introducing appropriate “weighting” filters and consequently adjusting the signal of interest $\mathbf{z}$ so that both controllers perform comparably.

Figure 5.14: Position of supporting point 1 for segment “0” with $\mathcal{H}_\infty$ control

Figure 5.15: Position of supporting point 2 for segment “0” with $\mathcal{H}_\infty$ control

Figure 5.16: Position of supporting point 3 for segment “0” with $\mathcal{H}_\infty$ control

Figure 5.17: Relative displacement between segment “0” and segment “1” with $\mathcal{H}_\infty$ control

Figure 5.18: Relative displacement between segment “0” and segment “2” with $\mathcal{H}_\infty$ control

Figure 5.19: Relative displacement between segment “0” and segment “3” with $\mathcal{H}_\infty$ control

Chapter 6

Conclusion and Future Work

This research work derived optimal control laws for a segmented telescope system, where optimality was defined in the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ senses. The objective of the control system is to align the segments of the telescope to achieve a desired imaging performance, in the presence of wind disturbances. It was noticed that without the control system, the desired imaging performance was not achievable. However, the designed $\mathcal{H}_2$ and $\mathcal{H}_\infty$ optimal controllers achieved the desired specifications.

The current wind disturbance model assumes no cross-correlation between wind disturbances across segments. Such assumption needs to be relaxed and spatially-correlated model of the wind disturbances needs to be derived and incorporated into the wind disturbance model. Moreover, future research is required to make the control system robust to other external disturbances, such as sensor measurement noise, wavefront disturbances, gravity deformations, etc. Moreover, robustness issues with respect to modeling uncertainties and plant parameters mismatches need to be addressed. Finally, fault detection, isolation, and recovery (FDIR) techniques need to be put in place for robust operation against potential system faults.

Appendix

Appendix 1

Numerical Values for System Matrices

This Appendix lists numerical values for the various system matrices.

$$\begin{aligned}
 \mathbf{A}_i &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -625.0749 & -180.9427 & -180.9427 & -0.2194 & -0.0635 & -0.0635 \\ -180.9427 & -625.0749 & -180.9427 & -0.0635 & -0.2194 & -0.0635 \\ -180.9427 & -180.9427 & -625.0749 & -0.0635 & -0.0635 & -0.2194 \end{bmatrix} \\
 \mathbf{B}_{1,i} &= \mathbf{B}_{2,i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.2194 & 0.0635 & 0.0635 \\ 0.0635 & 0.2194 & 0.0635 \\ 0.0635 & 0.0635 & 0.2194 \end{bmatrix} \\
 \mathbf{C}_i &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \\
 \mathbf{C}_{z1,i} &= \begin{bmatrix} 1.6667 & -0.7778 & 0.1111 & 0 & 0 & 0 \\ 1.6667 & 0.1111 & -0.7778 & 0 & 0 & 0 \\ 0.1111 & 1.6667 & -0.7778 & 0 & 0 & 0 \\ -0.7778 & 1.6667 & 0.1111 & 0 & 0 & 0 \\ -0.7778 & 0.1111 & 1.6667 & 0 & 0 & 0 \\ 0.1111 & -0.7778 & 1.6667 & 0 & 0 & 0 \end{bmatrix}.
 \end{aligned}$$

Bibliography

- [1] <http://www.tmt.org>.
- [2] <http://www.eso.org/public/astronomy/projects/e-elt.html>.
- [3] G.Z. Angeli, M.K. Cho, M. Sheehan, , and L.M. Stepp. Characterization of wind loading of telescopes. In *Proceedings of SPIE*, pages 72–83, February 2002.
- [4] J.N. Aubrun, K.R. Lorell, T.S. Mast, and J.E. Nelson. Dynamic analysis of the actively controlled segmented mirror of the W.M. Keck ten-meter telescope. *IEEE Control Systems Magazine*, 7(6):3–10, 1987.
- [5] P.Y. Bely. *The Design and Construction of Large Optical Telescopes*. Springer-Verlag, New York, 2003.
- [6] J.C. Doyle, K. Glover, P.P Khargonekar, and B.A. Francis. State-space solutions to standard H_2 and H_∞ control problems. *IEEE Transactions on Automatic Control*, 34(8):831–847, 1989.
- [7] A. Glindemann, S. Hippler, T. Berkefeld, and W. Hackenberg. Adaptive optics on large telescopes. *Experimental Astronomy*, 10(1):5–48, 2000.
- [8] S. Jiang. Distributed control of large scale dynamic systems. Master’s thesis, University of Illinois at Urbana-Champaign, 2004.

- [9] S. Jiang, P.G. Voulgaris, L.E. Holloway, and L.A. Thompson. H_2 control of large segmented telescopes. *Journal of Vibration and Control*, 15(6):923–949, 2009.
- [10] U. Mackenroth. *Robust Control Systems: Theory and Case Studies*. Springer-Verlag, New York, 2004.
- [11] D.G. MacMartin. Control challenges for extremely large telescopes. In *Proceedings of SPIE Commercial and Industrial Applications of Smart Structures*, volume 5054, pages 275–286, March 2003.
- [12] S. McCaslin, L. Wenzel, J. Meisel, and M. Cerna. Controlling extremely large telescopes. In *Proceedings of the international conference of high performance computing networking, storage, and analysis*, November 2008.
- [13] M. Quattri, F. Koch, L. Noethe, A.C. Bonnet, and S. Noelting. Owl wind loading characterization: a preliminary study. In *Proceedings of S.P.I.E.*, volume 4840, pages 459–470, January 2003.
- [14] J. Spyromilio. Developing RT control for the world’s largest telescope using LabVIEW with multicore functionality. <http://sine.ni.com/cs/app/doc/p/id/cs-11465>.
- [15] M.P. Ulmer, E.D. Kibblewhite, T. Herter, L.A. Thompson, R. Giovanelli, D.A. Harper, R.G. Kron, J. Mohr, G.J. Stacey, J. Tumlinson, and D.G. York. Large telescope project dedicated to an origins survey. In *Proceedings of S.P.I.E.*, volume 5382, pages 193–204, July 2004.
- [16] K. Zho, , J.C. Doyle, and K. Glover. *Robust and optimal control*. Prentice-Hall Inc., 1996.

Vita

Zaher Kassas received the Bachelor of Engineering degree in Electrical Engineering from The Lebanese American University (LAU) in 2001. While at LAU, he worked at the engineering laboratories as a research assistant, where his research aimed at the application of the Global Positioning System (GPS) in Intelligent Transportation Systems (ITS). He received the Master of Science degree in Electrical and Computer Engineering from The Ohio State University (OSU) in 2003. While at OSU, he worked as a graduate research associate at the Collaborative Center of Control Science, where he was engaged in designing estimation filters for air and ground Uninhibited Autonomous Vehicles (UAVs). In 2004, he joined the Control Design and Dynamical Systems Simulation Research and Development Group at National Instruments (NI). His responsibilities at NI span researching, implementing, and testing algorithms and software for the next generations of the LabVIEW Control Design & Simulation Module, System Identification Toolkit, and PID Toolkit for Real-Time (RT) and Field Programmable Gate Array (FPGA) targets. Since 2006 has been pursuing the Master of Science degree in Aerospace Engineering and the Ph.D. degree in Electrical and Computer Engineering at The University of Texas at Austin.

Permanent address: zkassas@ieee.org

This report was typeset with $\text{\LaTeX}^\dagger$ by the author.

[†] $\text{\LaTeX}$ is a document preparation system developed by Leslie Lamport as a special version of Donald Knuth's $\text{\TeX}$ Program.