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Essays on Technology and Innovation

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The IT age is marked by innovative approaches to the online commerce. Technology as the core of innovation has undergone numerous evolutions through the "creative destruction." Motivated by the phenomena and the challenges in the technology-driven markets, I explore the economic role of innovation from different angles in the following essays.

Chapter 1 focuses on firms' competitive strategies while constructing novel business models in delivering online services. In particular, I am interested in their bundling of marketing services with the core business. In a game theoretic model, I derive competing firms' equilibrium strategies with choices between three business models, no ad-support, ad-support with the optional advertising strategy, and the mandatory advertising strategy, and find that competitive business models can be differentiation-driven or advertising-driven depending on market ad aversion. Interestingly, mandatory advertising weakly dominates optional advertising under certain market conditions. My findings offer new insights to the bundling literature.

Chapter 2 examines the performance-based auction model in the iconic online advertising innovation, keyword auctions. I analyze advertisers' decision of utilizing their existing reputation from a primary auction upon entering a new auction. The short-term and long-term setups are modeled for analyzing seasonal marketing in a new auction and branding a new product, in examining the impact of new market size, performance, and risk on advertisers' decisions. While an optimistic new market encourages reputation stretching, in the long-term setup it further depends on the performance difference between the two markets. A higher risk is found to induce stretching under intensive competition for both cases; in the long-term, stretching decision is determined by the market size.

Chapter 3 examines the connection between business cycles and innovation and offers insights for regulatory innovation policies. Combining endogenous market structure with the dynamic game framework, I study the Markov perfect equilibrium where heterogeneous firms choose their innovation rates. I find that increased per-capita income tends to improve aggregate innovation, while income inequality shocks may reduce innovation conditional on the market structure. I also find subsidies to dampen innovation incentives, and policies such as tax credits that reduce the variable R&D costs to have positive effects on innovation.

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Chapter 1

Competitive Strategies in Business Model Innovation

1.1 Introduction

Successful innovations have generated many insightful discussions focused on the technology aspect. While the fundamental technology is a key component in any innovation, the solution provided to the consumers must be delivered to the market with an effective business model that generates profitable returns [3]. For example, Apple's success in introducing iPod does not only reside in its elegant designs and attractive product attributes; the overwhelming market popularity is largely owing to Apple's business model that combines the mp3 player with iTunes online music store [25]. In this paper, we examine a current area of innovation, online service provision, where business models are flourishing with variant advertising mechanisms. We analyze firms' strategic decisions of adopting advertising support for constructing profitable business models under the forces of market competition and consumers' taste for advertisements.

In the emergence of Cloud computing, the innovative ways of delivering services has had a significant impact on the traditional businesses, such as those in media and software. YouTube has made agreements with studio partners such as CBS and MGM to provide TV network programs in addition to its user-generated contents [42]. A joint venture launched in 2007 by NBC Universal and News Corp, Hulu, is experiencing rapid growth both in content provision and user base, offering numerous high-quality hit TV programs and films online. Netflix, which is known for renting DVD movies by mail, is making more movies available for online streaming. Meanwhile, the sales of DVDs have declined significantly, which has triggered companies such as Disney to turn their investment to the Web-based entertainment [5].

This shift of business paradigm to a service-based model in a networked environment creates challenges for the competing firms. The difference between the contents provided by YouTube, Hulu, and Netflix is transitory as these competitors strive to meet the viewers' demand. The competing business models defined by elaborate advertising strategies through agile technologies and rich interactivity, are providing otherwise substitutable digital products with an opportunity to create differentiation through advertising options, as consumers' heterogeneous ad taste is highly susceptible to ad presentation and contexts.

Advertising has undergone substantial transformation in the past few decades owing to the digitization of advertising channels. Consequently, advertisers' wasted marketing expense and advertising nuisance experienced by users are both declining. YouTube displays the minimizable in-video ads politely on the bottom of the screen. Compared to television, Hulu delivers TV programs with fewer commercial interruptions and with other customization features. A recent survey showed high user satisfaction on Hulu for the amount and quality of commercials shown in the videos (The New York Times, 2008). Leading advertising platforms such as Google and Yahoo! revolutionized advertising by implementing auction-based ranking and specifying a pay-per-click contingent payment contract with advertisers [31]. There is a variety of advertising models, but for the tractability of the current analysis, we divide them into two categories, *optional* and *mandatory advertising*. With optional advertising, the ad-averse users are able to eliminate nuisance (e.g., dismissable ads in YouTube videos described above). As a result, ads do not have any negative impact on users' experience. With mandatory advertising, users cannot disregard or disable ads. The ad-averse users' experience is then affected negatively; the experience of the users who value ads is elevated. For example, Hulu employs mandatory advertising by not allowing users to fast-forward through commercials, users' experience is then affected by their personal tastes for Hulu ads.

We model a duopoly setting, where the competing firms face strategic choices of offering their services without ad-support, with mandatory or optional advertising. Observing consumers' taste for ads, in equilibrium firms may have different strategic aims in constructing their business models - differentiation driven or advertising revenue driven. In a strongly ad-averse market, the equilibrium is differentiation driven: A firm can sustain a profitable market position with mandatory advertising, while the competing firm implements either of the other two strategies. In this way, ad-aversion is leveraged to create differentiation and mitigate price competition. In a moderately or mildly ad-averse market, the equilibrium strategies further depend on the profitability of advertising support, since firms have less leverage from market ad-aversion. At a high level of ad revenue, firms' equilibrium strategies are advertising-driven where both firms adopting advertising support, even with significant ad-aversion. Our findings underscore the point that business model innovations occur in a competitive landscape, where firms need to examine the key market attributes with a strategic lens.

We approach the problem with a bundling setup viewing ad-supported service as a bundle of service and advertisements. Mandatory advertising is equivalent to pure bundling (or fixed bundling) such that only the bundle, the ad-supported service, is available to consumers. The interpretation of optional advertising has the flavor of mixed bundling - consumers are choosing between the service alone and the service with ads. However, while in mixed bundling different prices are assigned to the bundle options, in the context of advertising technology, uniform pricing is more appropriate, because users' choice of ad option and their resultant experience are realized ex post, making it unfit for the ad-supported firm to assign multiple prices. Our framework extends from classic bundling literature by encapsulating unique characteristics of advertisements not applicable to traditional goods. Bundling advertisements creates a potential additional revenue source for the firm. The analysis illustrates that the ad revenue in some cases suppresses equilibrium prices by inducing ad-supported firms to compete more aggressively. The effect of this revenue source combined with other properties in our model yield findings different from those of the past bundling literature.

In Section 1.2, we review the related literature. Section 1.3 presents the model. In Section 1.4 we present the analysis in two duopoly scenarios: 1) One firm is ad-supported and chooses between optional and mandatory advertising, and the other firm is assumed to not adopt advertising; and 2) both firms adopt an ad-support. These cases are consisted of the subgames of the entire problem. These analyses illustrate the subgame equilibria given different competition configurations and lead to the final equilibrium results. In Section 1.5, we obtain these equilibrium strategies and evaluate the welfare implication. We discuss the relaxation of the key assumptions in Section 1.6. And then we conclude.

1.2 Literature Review

The keyword auction literature has shown the efficiency of performance-based auction over the traditional second-price auction in ranking advertisements [10] [28] [30] [46]. Our research takes the angle from the perspective of business model competition when adopting an advertising support. Advertising has traditionally been studied as a firm's strategy to increase consumer demand and price discriminate by promoting its own product [7] [13] [22]. Iyer et al. studied the effect of targeted advertising on firms' marketing and pricing decisions in a competitive environment, and found that by advertising only to certain segments of consumers, firms are able to eliminate costs of ineffective advertising and improve profits [24]. In our work, advertising does not promote the firm's own product, but adds content to the service while financing the firm. Nevertheless, our results connect with Iyer et al.'s findings in that optional advertising in our context can create intense price competition as targeted advertising in their setting has the same effect.

As previously discussed, an ad-supported firm effectively bundles advertisements with its product. Firms bundle to reduce cost or price discriminate, and can use bundling as a competitive tool [19]. In bundling as a competition tool, Chen studied firms using bundling to differentiate their products and reduce price competition [11]. While mixed bundling is the dominant strategy in a monopoly case, Chen showed that it is weakly dominated in a duopoly, which parallels our equilibrium results in the case where both firms are ad-supported [11]. Contrasting with Chen (1997), which found that mixed bundling is weakly dominated by unbundling, we find optional advertising can be dominated by mandatory advertising. And we also show that no ad-support (analogous to unbundling) does not dominate any strategy due to that its equilibrium price is suppressed by the rival's ad revenue.

Fan et al. studied a firm's pricing decisions in the context of online media with an advertising option [15]. Their formulation focuses more on the monopoly pricing strategy of ad-supported and ad-free media products. Our approach is more oriented towards the advertising strategy in a competitive setting; and advertising can improve or lower the product quality depending on consumer preference.

1.3 Model Setup

Consider a duopoly market, where two firms produce perfectly substitutable services, for which consumers have homogeneous valuation with the reservation price r. Assume consumers have unit demand, and r is sufficiently large such that the market is covered. The firms have zero marginal cost.

Firms may acquire advertising support. The consumer valuation for the advertisements, θ, is characterized by the uniform distribution, $\theta \sim U[\alpha, \alpha+1]$, where $\alpha \geq -1$ (i.e., at least some consumers value ads positively), and denoted by the cdf $G(\theta)$. An ad-supported firm derives marginal advertising revenue of $\beta \in (0,1)$ from consumers with $\theta > 0$. Here the implicit assumption is that firms offer contingent payment contracts to the advertisers such that payments are only collected when ads attract users. Thus, consumers who dislike ads are unlikely to click on the ads and are assumed to not contribute to firm's advertising revenues. Consumers' utility function for firm i's service with advertisements at price p_i is

$$u_i(p_i) = r - p_i + \theta. \tag{1.1}$$

In the first stage, firms choose between competing without advertising support (N), with optional advertising (O), and with mandatory advertising (M). And then they engage in price competition. The timeline of the game is as follows:

1. Firms choose from strategies ad-support with optional advertising (O), ad-support with mandatory advertising (M), and no ad-support (N);
2. Firms compete in price;
3. Consumers make purchase decisions;
4. Profits are realized.

There are nine pricing games corresponding to nine possible strategy combinations, among which strategy set (N,N) has a Bertrand outcome with both firms pricing at zero and making zero profits. In the following section, we break down the analysis of the other eight pricing games into two duopolies. In the former case exactly one firm does not adopt advertising support and the competing firm chooses between the advertising strategies, while in the latter case both firms adopt ad-support. Examining these two settings individually allows us to gain additional insights in specific competitive scenarios. Based on the results derived, we analyze the equilibrium strategies in the first stage in Section 1.5.

1.4 Duopoly Analysis

1.4.1 No-Ad and Ad-Supported Business Models

In this section, without loss of generality, only firm 2 is supported by advertising. Firm 2 chooses between the mandatory and optional advertising strategies. We will solve for the equilibrium price in the subgame under each strategy, and then obtain the dominant strategy.

No-Ad vs. Mandatory Advertising - (N,M)

When firm 2 chooses the mandatory advertising strategy, consumers' preference between firm 1's service and firm 2's service with ads depends on the prices as well as their valuation for ads. Based on Equation (1.1), a consumer purchases from firm 1 when $\theta < p_2 - p_1$. The demand function for firm 1 then follows

$$q_1(p_1, p_2) = \begin{cases} G(p_2 - p_1), & \text{if } p_1 \leq r. \\ 0, & \text{if } p_1 > r. \end{cases}$$

The profit functions of the two firms are,

$$\pi_1(p_1, p_2) = G(p_2 - p_1) * p_1 \tag{1.2}$$

$$\pi_2(p_2, p_1) = (1 - G(p_2 - p_1)) * p_2 \tag{1.3}$$

$$+\beta * min\{1 - G(p_2 - p_1), 1 + \alpha\}.$$

Firm 2 will obtain the consumers on the upper end of the valuation continuum, and firm 1 the rest. The relative price levels will only determine to which side of $\theta = 0$ the market split occurs. That is, when firm 2 charges a lower price than firm 1, it will have all consumers with positive θ as well as some with negative θ, while by charging a higher price firm 2 will not get any ad-averse consumers and lose some favor-ad consumers to firm 1. The following lemma summarizes the equilibrium prices and profits at different values of α.

Table 1.1: Equilibrium Prices for Strategy Set (N,M)

	Price
$a < -\frac{1}{2}$	$p_1^* = \frac{1}{3}(1-\alpha)^\dagger$ $p_2^* = \frac{1}{3}(2+\alpha)$
$-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$	$p_1^* = -\alpha$ $p_2^* = -\alpha$
$-\frac{1}{2}(1-\beta) < \alpha < 1-\beta$	$p_1^* = \frac{1}{3}(1-\alpha-\beta)$ $p_2^* = \frac{1}{3}(2+\alpha-2\beta)^\dagger$
$\alpha \geq 1-\beta$	$p_1^* = 0$ $p_2^* = \alpha^\dagger$

Note: The larger value is marked with †.

Lemma 1.4.1. *For the subgame of the strategy set (N,M), the equilibrium prices and profits are shown in Tables 1.1 and 1.2.*

Proposition 1.4.2. *For the subgame of strategy set (N,M), $p_1^* > p_2^*$ if $\alpha < -\frac{1}{2}$, and $p_1^* \leq p_2^*$ otherwise (see Figure 1.1); $\pi_2^* > \pi_1^*$ iff $\alpha > \frac{1}{2+3\beta} - 1$.*

When α is very small (i.e., $\alpha < -\frac{1}{2}$), the ad-averse consumer segment is large. Under mandatory strategy, firm 2 must charge a lower price than firm 1 to attract some of these consumers while getting the consumers who favor

Table 1.2: Equilibrium Profits for Strategy Set (N,M)

	Profit
$a < -\frac{1}{2}$	$\pi_1^* = \frac{1}{9}(1-\alpha)^2$ $\pi_2^* = \frac{1}{9}(2+\alpha)^2$ $+\beta(1+\alpha)$
$-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$	$\pi_1^* = \alpha^2$ $\pi_2^* = (\beta-\alpha)(1+\alpha)^\dagger$
$-\frac{1}{2}(1-\beta) < \alpha < 1-\beta$	$\pi_1^* = \frac{1}{9}(1-\alpha-\beta)^2$ $\pi_2^* = \frac{1}{9}(2+\alpha+\beta)^{2\dagger}$
$\alpha \geq 1-\beta$	$\pi_1^* = 0$ $\pi_2^* = \alpha+\beta^\dagger$

Note: The larger value is marked with †.

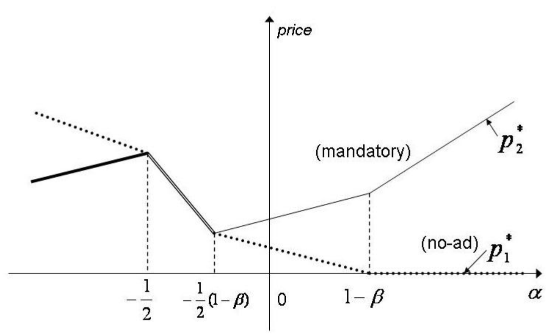

Figure 1.1: Equilibrium Prices for Strategy Set (N,M)

ads. And firm 2 makes a higher profit than firm 1 only when β, the marginal revenue for advertising, is sufficiently large.

For all other values of α, firm 2 obtains a higher equilibrium profit relative to firm 1. For $-\frac{1}{2} \le \alpha \le -\frac{1}{2}(1-\beta)$, two firms set identical prices in equilibrium. Thus, a consumer's choice of firm is determined solely on her valuation for advertisements. As a result, firm 2 gets all consumers with positive θ and firm 1 gets the rest. When the consumer segment that value ads is sufficiently large, firm 2 is willing to give up all ad-averse consumers as well as some favor-ad consumers by charging a higher price than firm 1, and still obtain a higher profit. When α is large enough or advertising is profitable enough, firm 2 can push firm 1's price and profit to zero.

No-Ad vs. Optional Advertising - (N,O)

When firm 2 undertakes the optional strategy, the consumers who have positive valuation for ads get the service with ads; and the consumers who have negative valuation for ads are able to eliminate the disutility from ads, in effect, receive ad-free service from firm 2. The firms compete only in price for ad-averse consumers.

$$\text{Consumer chooses firm 1} \begin{cases} \text{if } p_1 < p_2, & \text{for } \theta < 0. \\ \theta < p_2 - p_1, & \text{for } \theta > 0. \end{cases}$$

Lemma 1.4.3. *The subgame equilibrium for the strategy set (N,O) is as follows,*

1) For $\alpha \le -\frac{1}{2}(1-\beta)$, $p_1^* = p_2^* = 0$, $\pi_1^* = 0$, *and* $\pi_2^* = \beta(1+\alpha)$;

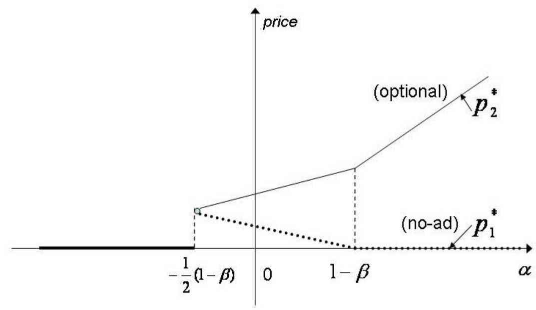

Figure 1.2: Equilibrium Prices for Strategy Set (N,O)

2) For $\alpha > -\frac{1}{2}(1-\beta)$, the equilibrium is identical to the subgame of the strategy set (N,M).

Comparing figures 1.1 and 1.2, the difference between the two cases resides in the smaller values of α, where in the optional case both firms price at 0 as in a Bertrand competition. At $\alpha = -\frac{1}{2}(1-\beta)$, the number of ad-averse consumers is small enough such that firm 2 is only concerned with competing for consumers who value ads, which is consistent with the mandatory case since the option to disable ads is irrelevant for these consumers.

Firm 2's Strategy Choice

Proposition 1.4.4. *In the subgame with a no-ad firm competing with an ad-supported firm, mandatory advertising is a weakly dominant strategy. For $\alpha \leq -\frac{1}{2}(1-\beta)$, mandatory advertising strategy yields a higher profit for firm*

2, both firms obtain positive profits; For other values of α, firm 2 is indifferent between the two strategies.

By making advertisements mandatory, firm 2 differentiates its service from firm 1's service for all consumers, which allows firm 1 to obtain profits from the ad-averse group.

1.4.2 Ad-Supported Competition

In this section, we analyze the symmetric game where both firms have the ad-supported business model and bundle their services with advertisements. Assume consumers are indifferent between the advertisements provided by two firms.

Identical Advertising Strategies - (M,M) or (O,O)

Consider the case where both firms choose mandatory advertising (M,M). A consumer's utility for either firm is Equation (1.1). Clearly, the valuation for advertising has no effect here–she will simply purchase from the firm with the lower price. Thus, the Bertrand competition yields the subgame equilibrium with both firms charging zero price and splitting the advertising revenue, assuming that each firm captures half of the market and makes equal profits that are derived from advertising: $\pi_1^* = \pi_2^* = \frac{1}{2}\beta(1+\alpha)$.

When both firms sell their services with optional advertising (O,O), consumers with negative valuation have the utility function, $u(p_i) = r - p_i$, while consumers with positive valuation still follow Equation (1.1). The subgame exhibits perfect symmetry when the firms adopt identical strategies. Thus, the equilibrium results are the same in both (M,M) and (O,O) cases.

Proposition 1.4.5. *In the subgame of the strategy set (M,M) or (O,O), in equilibrium,* $p_1^* = p_2^* = 0$*,* $\pi_1^* = \pi_2^* = \frac{1}{2}\beta(1+\alpha)$*.*

If prices are not non-negative, the equilibrium profits will be zero with firms charging equal and negative prices. Negative pricing can be reflected in rewards, coupons, credits, or other forms of "payment" offered to the consumers in the transaction. For simplicity, we take the non-negative pricing assumption, while our findings will still hold if this assumption is to be relaxed.

Different Advertising Strategies - (M,O)

Now we consider the case where firms choose different strategies. Without loss of generality, let firm 2 undertake optional advertising. For the consumers with positive valuation for ads, both firms display advertisements, and consumers will choose the firm with a lower price. For the consumers with negative valuation for ads,

$$\begin{aligned} u_1(p_1) = &\quad r - p_1 + \theta, \quad \text{if purchase from firm 1.} \\ u_2(p_2) = &\quad r - p_2, \qquad \text{if purchase from firm 2.} \end{aligned}$$

Consumers' preference follows:

$$\text{Consumer chooses firm 1} \begin{cases} \text{if } p_1 < p_2, & \text{for } \theta \geq 0. \\ \theta > p_1 - p_2, & \text{for } \theta < 0. \end{cases}$$

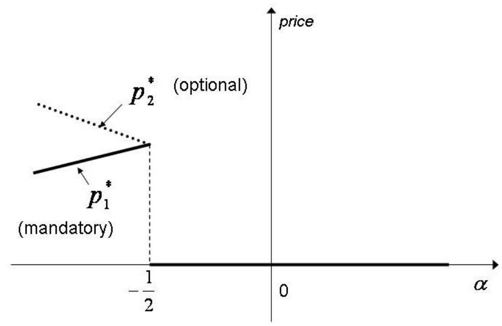

Figure 1.3: Equilibrium Prices for Strategy Set (M,O)

The firms' profit functions are

$$\pi_1(p_1, p_2) = \begin{cases} (1 - G(p_1 - p_2))p_1 + \beta(1 + \alpha), & \text{if } p_1 < p_2; \\ \frac{1}{2}(\beta + p_1)(1 + \alpha), & \text{if } p_1 = p_2; \\ 0, & \text{if } p_1 > p_2; \end{cases}$$

$$\pi_2(p_1, p_2) = \begin{cases} G(p_1 - p_2)p_2, & \text{if } p_1 < p_2. \\ \frac{1}{2}(\beta + p_2)(1 + \alpha) + (-\alpha)p_2, & \text{if } p_1 = p_2. \\ p_2 + \beta(1 + \alpha), & \text{if } p_1 > p_2. \end{cases}$$

Lemma 1.4.6. *The subgame equilibrium for the strategy set (M,O) is as follows:*

1) For $\alpha < -\frac{1}{2}$, $p_1^* = \frac{2+\alpha}{3}$, $p_2^* = \frac{1-\alpha}{3}$, $\pi_1^* = \frac{1}{9}(2+\alpha)^2 + \beta(1+\alpha)$, $\pi_2^* = \frac{1}{9}(1-\alpha)^2$;

2) For $\alpha \geq -\frac{1}{2}$, $p_1^* = p_2^* = 0$, $\pi_1^* = \pi_2^* = \frac{1}{2}\beta(1+\alpha)$.

When the number of ad-averse consumers is large (i.e., $\alpha < -\frac{1}{2}$), firms 2 charges a higher price than firm 1, who in turn obtains some ad-averse consumers. Intuitively, firm 2 gets enough market share from the ad-averse consumers so that it is willing to forgo the split of the rest of the market with firm 1 by raising its price above that of firm 1. For larger α values, both firms price at zero and split the profits.

The competition here reverses that of the no-ad versus ad-supported case with the ad-supported firm choosing optional advertising. In the that setup, Bertrand competition occurs for the ad-averse consumers because they get ad-free services from both firms, while here the firms offer identical bundles (service with ads) for consumers who favor ads. Also, firm 2 gets the higher-value consumers in the previous case, but the lower-value consumers in the current scenario. The cutoff α value that separates the two sets of equilibrium prices, $p_1^* = p_2^* = 0$ and $p_2^* > p_1^*$, is weighted by $(1 - \beta)$ in the previous case. This implies that firm 2 needs a larger consumer segment that it gets for sure in the previous case than in the current case to forgo the part of the market it can split with the other firm. It appears counterintuitive, because the portion of consumers that prefer firm 2's service in the previous case are those who like ads and can generate revenues not just by purchasing the service but also by interacting with the advertisements, while in the mirror-image of the current case, that part of market is made up of consumers who dislike ads and do not contribute to firm 2's advertising revenue. Thus, firm 2 is seemingly "greedier" in the no-ad versus ad-supported case.

The explanation behind the above observation is that advertising revenue can dampen the differentiation effect and lower the equilibrium prices.

When the firm obtaining the higher-value consumers also derives revenues from advertising, its competitive behaviors are more sensitive to the demand change; the stimulated incentives to gain higher market shares end up driving down the equilibrium prices. The implication is that when consumer segment is heavier on the favor-ad side, the firm without an ad support has a smaller leeway to differentiate and charge a positive price than the mandatory ad-support firm does when the ad-averse consumer segment is more dominant.

Comparing the scenario here with the no-ad firm competing with ad-support firm case when the ad-supported firm chooses mandatory advertising, the equilibrium prices for $\alpha < -\frac{1}{2}$ are the same, but here firms charge price 0 for larger α values. Note that the competition for ad-averse consumers is exactly the same for the two cases. As a result, at the same α value in both cases the firm offering its service without ads raises its price above that of the other firm.

Advertising Strategy Choice

Proposition 1.4.7. *In the subgame with both firms being ad-supported, in equilibrium, for $\alpha < -\frac{1}{2}$, one firm chooses optional advertising strategy, and the other one chooses mandatory advertising; For $\alpha \geq -\frac{1}{2}$, both firms choose either optional or mandatory advertising strategy.*

When both firms are ad-supported, choosing the same advertising strategy creates Bertrand competition. Therefore, in this subgame equilibrium, the two firms choose different strategies to differentiate their services. The firm that undertakes optional advertising charges a higher price than the mandatory firm and loses the favor-ad consumer segment to the mandatory firm. The comparison of the profits, however, depends on the marginal advertising revenue. From Proposition 3 and Lemma 3, we see that in the case $\alpha \geq -\frac{1}{2}$, all profits in all cases are identical ($\pi_1^* = \pi_2^* = \frac{1}{2}\beta(1+\alpha)$). Thus, for heavier favor-ad consumer segment, both firms select any of the advertising options.

1.5 Equilibrium Results

We now analyze two firms' strategies among the choices of no ad support, mandatory and optional advertising. From the previous section, the firms' equilibrium strategy choices are dependent on the value of α and also partially on β. The equilibrium in this section suggests that the successful innovations in a competitive environment need different aims given particular market characteristics such as consumers' overall taste.

1.5.1 Strong Ad-Aversion

Lemma 1.5.1. *When $\alpha < -\frac{1}{2}$, the pure-strategy Nash equilibria include (N,M) and (M,O)*[1]*: two firms choose either i) no advertising support and mandatory advertising, or ii) mandatory and optional advertising. The equilibrium strategies in the case of strongly ad-aversion is differentiation-driven.*

[1] The complete set of pure-strategy Nash Equilibria actually also includes (M,N) and (O,M) due to the symmetry of the game. For presentation simplicity, we omit listing symmetric strategies for all results in this section.

Given a large consumer segment that is ad-averse, the competing firms leverage differentiation to attain maximum profits in equilibrium - exactly one firm chooses mandatory advertising, while the other firm competes without an ad-support or with optional advertising. With a strong ad-aversion in the market, this differentiation-driven outcome is not ad-revenue sensitive, because the high-price firm in this scenario does not profit from advertisements. It is interesting to observe that not just competing without ad-support can be profitable, also two ad-supported firms can co-exist in the market at positive prices if implementing different advertising strategies.

In Chen's duopoly bundling setting , mixed bundling is weakly dominated by unbundling [11]. In contrast, here optional advertising (which has similarity with mixed bundling as discussed earlier) is not a dominated strategy. Uniform pricing in the context of advertising reduces the intensity of competition between ad-supported firms; consequently, in competing with mandatory-advertising firm, the firm with optional advertising is able to sustain a higher price and obtain the same level of profits as if to compete without advertising support. Note that this result is not driven by the revenue generated by advertising support; in fact, the optional-advertising firm derive zero revenue from advertising in this case.

1.5.2 Moderate to Mild Ad-Aversion

When the ad-averse consumer segment is of moderate size or smaller, firms' equilibrium strategies further depend on the marginal advertising revenue. Without sufficient ad-aversion, firms have less leverage to create differentiation, thus their competitive strategies may be completely driven by the advertising profitability.

Lemma 1.5.2. Low Advertising Revenue

For $\beta < \frac{1}{3}$,

1) when $-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$, *the pure-strategy equilibria are (N,M) and (O,O);*

For an $\alpha^* \in (-\frac{1}{2}(1-\beta), (1-\beta)]$,

2) when $-\frac{1}{2}(1-\beta) < \alpha < \alpha^*$, *the pure-strategy equilibria are (N,M) and (N,O);*

3) when $\alpha = \alpha^*$, *the pure-strategy equilibria include all strategy sets except (N,N);*

4) when $\alpha > \alpha^*$, *the pure-strategy equilibria are (M,O), (M,M) and (O,O).*

Lemma 1.5.3. High Advertising Revenue

For $\beta > \frac{1}{3}$, *for an* $\alpha^{**} \in (-\frac{1}{2}, -\frac{1}{2}(1-\beta)]$,

1) when $-\frac{1}{2} \leq \alpha < \alpha^{**}$, *the pure-strategy equilibria are (N,M) and (O,O);*

2) when $\alpha = \alpha^{**}$, *the pure-strategy equilibria are (N,M), (M,O), (M,M) and (O,O).*

2) when $\alpha > \alpha^{**}$, *the pure-strategy equilibria are (M,O), (M,M) and (O,O).*

Lemma 5 states that when the marginal ad revenue is low, the firm without ad-support is able to obtain a profitable market position in equilibrium as long as there exist a certain number of ad-averse consumers. With a sufficiently high marginal ad revenue, the tipping point of α at which no ad-support is not an equilibrium strategy is lower. In other words, when advertising generates enough returns, in equilibrium both firms adopt advertising support and, as a result engage in aggressive price competition, even with moderate level of ad-aversion, whereas at low marginal ad revenue equilibrium strategies are only advertising driven with a low ad-aversion in the market.

Proposition 1.5.4. *When $\beta < \frac{1}{3}$ and $\alpha > \alpha^*$, or $\beta > \frac{1}{3}$ and $\alpha > \alpha^{**}$, no advertising support is a dominated strategy; Mandatory advertising is a weakly dominant strategy. The equilibrium strategies are advertising-driven.*

Indicated in Proposition 5, when both firms choose ad-support in equilibrium, no ad-support is a dominated strategy. This contrasts the general results in the conventional bundling studies that unbundling (which is analogous to no ad-support) is an equilibrium strategy [11]. The phenomenon shown in our study is due to the impact of advertising revenue on firms' competitive behavior. As discussed under Lemma 3, advertising revenue can suppress the equilibrium prices due to the ad-supported firm's stimulated incentive to gain a larger market share. As a result, in the presence of an ad-supported opponent, the firm without an ad support may be easily cornered to charging a low or zero price. This combined with the additional revenue associated with an advertising support makes it a dominated strategy to compete without bundling advertisements. Also, mandatory advertising is weakly dominant given the same condition, while with conventional bundling setup there exists no dominant strategy.

Proposition 1.5.5. *When* $-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$*, optional advertising is a weakly dominated strategy.*

Regardless of the level of ad revenue, optional advertising is a weakly dominated strategy, when α is between $-\frac{1}{2}$ and $-\frac{1}{2}(1-\beta)$ implying the favor-ad consumer segment is slightly outweighs the ad-averse segment. At a glance, it may seem counterintuitive because optional advertising provides full customization that suits all consumers' taste. While it clearly would be an optimal strategy in a monopoly market, the forces of competition generate additional tensions here when optional advertising is employed. The key insight is that optional advertising creates little differentiation at certain levels of ad-aversion resulting in intense rivalry and minimal profits for both firms. Thus, mandatory advertising weakly dominates optional advertising. Note that uniform pricing has an opposite effect here compared to the strong ad-aversion case. Its restriction on the firm with optional advertising now intensifies the price competition between the ad-supported firms, because the mandatory-advertising firm gains competitive strength from the increased number of consumers who value ads. This connects with the bundling literature, in which mixed-bundling is a dominated strategy [11].

1.6 Discussion

In this section, we discuss some of the assumptions made throughout our analysis.

Marginal Ad Revenue In the current model, ad-supported firms only derive advertising revenue from consumers who value ads positively. An interesting extension of our setup is to consider an additional positive advertising revenue generated among the ad-averse consumers. This applies in situations such as variation in ad-averse users' interest towards ads (one may become usually ad-loving while engaged in the search for a particular product or service), their accidental clicks on pay-per-click ads, etc. Thus, we will also consider a smaller marginal ad revenue relative to β that is proportional to the demand among ad-averse consumers.

Le $\delta \in [0, 1]$ denote the proportion of ad-averse consumers contributing to the ad revenue. The profit function of the ad-supported firm from Section 1.4.1 will then take the form $\pi_2(p_2, p_1) = (1 - G(p_2 - p_1)) * p_2 + \beta * min\{1 - G(p_2 - p_1), 1 + \alpha\} + \beta * \delta * \max\{0, -G(p_2 - p_1) - \alpha\}$, where $\beta\delta \leq \beta$ can be interpreted as the marginal ad revenue from ad-averse consumers, let us use $b = \beta\delta$. This will change the analysis except for the case in Section 1.4.1 with no-ad firm competing with the optional firm, where no revenue from the ad-averse consumers who disable ads.

In other parts of the analysis, although most of the results now are conditional on new inequalities with b, the main findings still hold with a few additional insights. We here omit the expressions of the new equilibrium prices and profits as they differ slightly from the original results, and discuss the intuitions underlying the results from the modified setup in the following propositions.

Proposition 1.6.1. *Under the strategy set (N,M), when* $b \geq \frac{1}{2}$*, within the range of* $-1 \leq \alpha < -2(1-b)$*,* $p_1^* = -\frac{1}{2}\alpha$*,* $p_2^* = 0$*.*

The above proposition implies that a sufficiently profitable ad-averse segment will lead the mandatory firm to price cut more aggressively while competing with a firm without ad support. In the original model, the mandatory firm does not have the incentive to attract additional ad-averse consumers since $b = 0$.

Proposition 1.6.2. *If* $b \geq \frac{1}{2}$*, the strategy set (M,M) is always one of the pure-strategy equilibria.*

Without b, for small values of α one firm would choose no ad-support or optional ad-support; however, here a large b implies a profitable advertising opportunity thus both firms compete with mandatory advertising. When α is large, both firm choosing mandatory advertising is also an equilibrium under the same intuition as the original model.

The results in this extension reflect the early years of online advertising, when pop-up and banner ads were ubiquitous and concepts of "respective ads" were amorphous. Online services often undertook these advertising mechanisms and offered their products for free, which is consistent with Proposition

7. The introduction of Google and Yahoo!'s pay-per-click advertising model revolutionized online marketing. When ad revenue is only contingent on users' clicks, revenues from ad-averse user became insignificant; given a large-averse segment, in equilibrium the mandatory ad-support firm does not price at zero and the competing firms do not all choose mandatory advertising. The incentive resides in differentiating services; user experience is also improved. This is shown by the original model.

Perfect substitutes To relax the assumption that two firms offer perfectly substitutable services, firms' services can be imperfect substitutes such that consumers may prefer one over the other. In particular, the horizontal differentiation is commonly specified using the Hotelling model. Firm 1 is located at 0 and firm 2 at 1. Consumers are uniformly distributed on the interval $[0,1]$. A consumer at location x incurs transport cost δx to buy from firm 1 and $\delta(1-x)$ to buy from firm 2. The parameter δ measures the degree of service differentiation. The model becomes complicated since consumers now have two dimensions of characteristics (x,θ), in which x and θ could be independent or related.

The distributions of both θ and x are needed to determine the demand for each firm. x and θ may be positive or negative correlated, but in either case the firms' demands are skewed without meaningful implications. If x and θ are independently distributed, the results will merely have additional conditions to the ranges of α to describe the equilibria when consumers' preference for a certain service exceeds some threshold. Overall, the introduction of horizontal differentiation may yield complex solutions that are difficult to interpret, with the main effect of reducing the competition between firms such that Bertrand-type competition under some conditions may no longer occur. We do not believe the added dimension offers new insights for problem considered.

Consumer valuation for the service Another assumption is that consumers have homogeneous valuation for the service offered by the firms with reservation price r. We may relax this assumption by assuming that r follows a distribution function $F(r)$ on $[\underline{r}, \overline{r}]$. Although consumers have heterogeneous reservation price r, for any particular consumer, two firms' services are still perfect substitutes. When comparing the a consumer's utilities between these services offered by the two firms, only the valuation for advertisements θ matters. Therefore, the relaxation of this assumption does not affect the results as long as $\underline{r}$ satisfies some condition relating to α, such that the consumer reservation value for the service does not depart too far below the necessary value relating to α in our current analysis. The varying r values can impact the monopoly case discussed in Section 1.4, where we focus on a relevant r value such that the monopolist enjoys a higher profit than otherwise in a competitive duopoly market. While the quantitative derivations may change with a varying r, the analysis and results remain valid given that the distribution of r satisfies a reasonable condition. Again, modeling r as a variable generates no new idea to the problem on hand.

Without Assuming Positive Ad Valuation In the current model, we have assumed a uniform distribution of ad valuation, which may extend into the positive range depending on the value of the lower bound. One may argue that given the nature of advertisements, consumers may not have a high positive valuation from ads. Here we show that our results are robust even without consumers who have strictly positive valuation for ads.

We consider a variation of θ distribution that does not include any positive values; Instead, the positive values in the original distribution concentrate on zero as a mass point. Thus, $\theta \sim [\alpha, 0]$, characterized by a cdf $F(\theta)$, such that for $\theta < 0$, $F(\theta) = G(\theta)$, and for $\theta = 0$, $F(\theta) = 1$. The results for the analysis where both firms are ad-supported are unaffected by this change of distribution.

For the strategy set (N,M), it can be shown that for $\alpha < -\frac{1}{2}$, $p_1^* = \frac{1-\alpha}{3}$ and $p_2^* = \frac{2+\alpha}{3}$; thus the results are unchanged. For $\alpha > -\frac{1}{2}$, $p_1^* = p_2^* = 0$, $\pi_1^* = 0$, and $\pi_2^* = \beta(1+\alpha)$. The only difference from Lemma 3 is that firm 1 makes zero profit here due to the lack of advertising support. Intuitively, in both scenarios, the users who do not dislike ads are indifferent between the service offered by the competing firms. For the strategy set (N,O), the competing firms have no leverage to create differentiation; thus, the subgame equilibrium result is $p_1^* = p_2^* = 0$, $\pi_1^* = 0$, and $\pi_2^* = \beta(1+\alpha)$, for all α.

Firms' equilibrium strategies are then unchanged for the case of strong ad-aversion. The results become much simpler in the case of moderate to mild ad-aversion, where both firms choose ad-support in equilibrium: (M,M), (M,O), (O,M), and (O,O). When positive ad valuation is removed, firms' equilibrium strategies are not sensitive to advertising profitability: The equilibrium strategies are either differentiation-driven with a strongly ad-averse market, or advertising-driven with moderate to mild ad-aversion.

1.7 Conclusion

Competing firms' strategies in business model innovation is studied in the current paper, which focuses on a recent instance of innovative markets - online service provision with revenue support from advertising. We consider the variations of advertising mechanisms in terms of mandatory and optional advertising strategies, analyze firms' choices of competing without ad support, or ad-supported business models of mandatory or optional advertising in the market rivalry, and identify the driving forces for their equilibrium strategies.

Our findings articulate the relevance of market condition impacted by IT in innovating firms' decisions. Specifically, the customization allowed by interactivity and greater information aggregation over the network push forward an advertising age where ads are more informative and appealing, and consumers' attitudes towards ads are transforming. As firms strive to reap profits through competing business model innovations, consumers' valuation for advertisements and, in some cases, the marginal advertising revenue are both important factors in firms' decision of acquiring advertising support and type of advertising strategy to employ. Given a strongly ad-averse market, the equilibrium strategies are differentiation driven, in that mandatory advertising is always employed by exactly one firm while the other firm either competes without an ad-support or employ optional advertising. However, when the ad taste becomes more positive overall, the equilibrium strategies are sensitive to the profitability of advertisements. Given a sufficient level of advertising returns, the equilibrium business models are advertising driven, which creates a socially optimal outcome.

This work also offers theoretical contributions for the literature of bundling. By considering a special bundled good, advertisements, we derive findings that contrast with those of the conventional bundling framework. The revenue generated by advertising is observed to intensify rivalry and lower equilibrium prices. Moreover, the distinction between mixed-bundling and optional advertising in the pricing option creates an interesting effect that results in mandatory advertising being a dominant strategy and optional advertising being dominated for certain ranges of ad valuation. The implication of this result is that while the business model of service with optional advertising offers full customization that suits the ad taste of all consumers, depending on firms' strategic focus at the level of market ad-aversion, it may be an unprofitable choice.

Thus far, we have been taking consumers' taste for ads as given. In future extension, it will be interesting to endogenize the ad taste such that firms can choose the level of investment in advertising technology to influence consumers' response to ads. Also in the current work, the distribution of consumers' valuation for ads is common knowledge. This assumption is widely used in the literature, and easily interpretable for the scenario where the differentiation of consumer valuation for ads is application dependent. When firms attempt novel advertising models, they may conduct surveys and gather data on consumer feedback that is unavailable to the public, in which case, a model with asymmetric information may be more appropriate.

Chapter 2

Reputation Stretching in Online Auctions

2.1 Introduction

Reputation unarguably plays a major role in environments that involve interactions and exchange. As the Internet opens up a vast market for e-businesses and trading individuals, the myriad of choices as well as the intrinsic anonymity lead to an increased importance of reputation, which essentially serves as an evaluation of one's past performance in most online markets. The studies in reputation have taken a broad range of perspectives as well as in many different settings. We position our question in the performance-based auction, where reputation is combined with the bids to determine the bidders' ranking. In particular, we take the bidders' perspective and investigate the strategy of reputation stretching–the extension of one's existing reputation in one market to a new market–and how it affects bidders' payoffs based on three market factors: the market size, the expected performance, and the risk of the new market.

In the conventional sense, reputation is reflected in a firm's brand. Reputation stretching in the branding context implies producing a new product under an existing brand name. In the online markets, the infrastructure of the reputation systems allows users to have a similar option, that is using the score representing the performance from the previous transactions in new and disparate transactions. Evidentally, eBay permits reputation stretching by assigning only one reputation score to each user. For instance, a seller who sells laptops as well as clothing under the same identity is implicitly using reputation stretching between two types of products. On the other hand, sellers can potentially register separate accounts for selling different products or create new accounts when the current reputation score becomes unsatisfactory; thus the issue of multiple identities arises. Despite eBay's effort to verify user identity in order to prevent the ownership of multiple accounts, it is not difficult for users to cheat the system and start a new reputation from a clean slate. eBay's reputation system is representative of many other online marketplaces (e.g., Amazon.com), where reputation stretching is automatically applied while not stretching (creating a new account) being a feasible alternative.

Another example of online reputation stretching resides in the ranking system used by keyword advertising programs such as Google AdWords, Yahoo! Sponsored Search, Microsoft AdCenter, etc. These keyword advertising programs provide online marketing services, where advertisers specify keywords to which they associate their advertisements and bid to display the ads. While advertisers submit bids on how much they are willing to pay for each click, the ranking of the ads are based on these bids combined with the advertisers' past performances. In particular, the past performance is measured by the historic click-through rate (CTR)–the ratio of the number of clicks an advertisement receives to the number of times it is displayed-which is used as bidders' reputation in terms of their click generating ability. Currently, the advertiser accounts do not include an option that allows them to apply their reputation associated with a keyword market to another; in other words, the advertisers' reputation scores are automatically kept separate for different keyword markets, and the not-stretching strategy is implicit.

The link between advertisers' performances across keyword markets can be discussed on several dimensions. Hence, the reputation stretching issue in the performance-based auction context incorporates a moderate level of complexity that invites research studies with various approaches for different facades of the problem. One may point out that an advertiser's click-through performance is directly related to its brand name and consumers' perception of its product quality. Along the perspective of reputation relating to the brand name, the problem seems quite similar to that of the goods market case. However, we should take into account that commonly-known websites are often ranked well in the organic search results that the sponsored advertising actually provides opportunities for growing businesses to attract new customers, in which case the advertisers' performance may be largely impacted by the effectiveness of the ad targeting strategy. Furthermore, on the topic of advertisers' ability in attracting clicks, the correlation between keyword markets is a natural factor in reputation stretching. We believe examining the connections between keyword markets and their relevance to advertisers' bidding strategy is a well-founded and a separate research issue from the scope of our study.

We analyze the fundamental factors characterizing each market independently, such as the market size, the expected performance, and the risk of the performance in a market, and provide insights to the reputation stretching decision aside from the interdependency elements between markets.

We use the auction framework to model the performance-based auction setting for both the short-term and long-term cases, and analyze our results according to the three factors mentioned above. In both cases, an advertiser has an existing reputation from the base market, and is faced with the reputation stretching decision before the auctions begin. The short-term analysis considers a two-period model, with the new market auction followed by the primary market auction. The short two-period setup provides a clean abstraction of participation in the new market only temporarily. This is representative of firms that are focused on their primary products and only extend to a different market for special events. For example, online florists such as ProFlowers advertise heavily in "flowers"-related keyword markets. They may consider entering the market for the "gift baskets" keyword group for holidays such as Valentine's Day without continuing in that market when the hype is over. A potentially more promotional event is the Olympics, during which certain advertisers may choose to expand from their primary keyword markets for products highly attractive for their association with the occasion. It is clear that further participation in the new markets after the events expire provides no profitable opportunities, given that the firms have not chosen to enter those markets in the past. Therefore, our short-term model emphasizes on the temporary participation with the primary market following the new market auction to capture the effect of reputation stretching on the original market. In the long-term case, participation in the new market persists. The application is straightforward–firms expand their businesses to a new group of keywords in addition to their current advertising campaigns. We take the varied motivations behind such decision as given, and design the model in the infinite horizon where the new market and the primary market take place back to back. Alternatively, one could use a model where the two markets are active simultaneously in each period. Due to the reciprocating effect, the results will be equivalent to the simplified design chosen in our paper with only one market per period and alternating markets in consecutive periods.

We obtained both mirroring and contrasting results in the short- and long-term settings. We find that the advertiser with good reputation stretches if the new market is significantly bigger than the primary market. Also, the advertiser with good reputation is more likely to stretch if the performance of the new market looks sufficiently promising, expecting a positive impact on the primary market. In addition, the performance risk in the new market also plays an important role. In a very competitive primary market, the advertiser behaves like risk-seeker: the higher the risk, the more likely one will stretch. We also investigate the long-term case, where the new market auction and the primary market auction take place alternately in the infinite horizon. Some results are notably different from those in the short-term analysis. When the gap between performances in two markets is big, it is optimal for the advertiser not to stretch. The effect of risk in performance depends on the market sizes. In general, the bidders tend to apply the risk effect to the bigger market.

The rest of the paper is organized as follows. In section 2.2, we briefly review the related literature and compare our model with work that is related to our study. Section 2.3 lays out our model, followed by an analysis of bidding function and bidders' expected payoffs in Section 2.4. We then examine the short-term and long-term expanding cases in section 2.5 and section 2.6, respectively. Finally, we conclude.

2.2 Literature Review

Early literature examined the role of reputation in the interaction between two parties. Having a "good" reputation can mean a seller providing high-quality product at a certain price [26], a firm honoring a high wage after paying the worker lower wage initially [44], a monopoly using predation for new entrants [27], etc. In the moral hazard setting, one party relies on the other party to take an action, while the other party can choose to perform differently to its own short-term advantage; however the former party can then terminate future interactions as a punishment to the other party [26] [44]. Establishing a reputation of performing expected action through repeated interactions is crucial for continuing transactions. In the adverse selection setting, at least one party has imperfect information, and the other party is of a specific type, and shows its type through repeated interactions with the former party [27] [35] [43] [38]. Unlike the moral hazard setting, here the party with a type does not choose among different actions. Its performance is its reputation, which signals the type. In both settings, reputation serves as a powerful tool, using which a mechanism can induce optimal equilibria without governmental and third party intervention.

While the economic role of reputation has been well examined, only a few papers studied reputation stretching. Wernerfelt (1988), Pepall and Richards (2002), and Cabral (2000) considered the reputation stretching (or "umbrella branding" in Wernerfelt's term) problem, where a firm's reputation on an old product can be used to sell a new product by that firm [47] [36] [9]. [47] and [9] considered reputation stretching in an adverse selection setting using the seller-buyer game model, where the seller makes the decision between stretching and not stretching his/her reputation of the base product onto a new product. They show that sellers of higher quality derive higher marginal benefit from stretching (direct reputation effect), but sellers' stretching behavior also depends on the effect of the performance of the new product on the reputation of the old product (reputation feedback effect). [47] and [9] differ in the cost of starting from a new name; the former considers that creating a new name is less costly than stretching from the existing reputation, while the latter assumes that stretching is cost neutral. [36] studied a model in which brand identity is a complementary feature that enhances consumer willingness to pay. They focus on how a firm's established strong brand name can affect the competition in a new market.

While reputation has mostly been examined in the traditional economic setting of goods markets, recently studies explored reputation in electronic commerce as well as online keyword advertising. In online transactions, where users are anonymous and obtaining a new identity is trivial, reputation systems have been shown to reduce fraudulent behaviors for online buyer-seller interactions [21] [33]. Reputation is also used in keyword auctions, which have provided lucrative results for keyword advertising search engines [16] [30]. For example, Liu and Chen (2006) has shown that compared to traditional auctions, where bidders are weighed equally and are only ranked based on their bids, weighted auctions yield higher performance [30]. In keyword auctions, reputation serves as the weighing factor for the bidders; hence, bidders with higher performances are given more weight and are preferred in the auction ranking. The consequent payoff to the auctioneer is optimized with weighted unit price auctions.

Our study connects the idea of reputation stretching with the weighted auction setting, and research the effect of using a bidder's reputation in one auction for a different auction. Instead of using the seller-buyer game as in [9] and many other related studies, we examine reputation stretching in a setting similar to keyword auctions, where multiple bidders compete. While in most other reputation studies, reputation induces a mechanism for establishing trust between players, reputation in our study is a bidder's attribute that determines rankings of auctions. Different factors, such as market size, expected performance and the risk of performance in the new market, are examined and found to produce insightful results. Moreover, we study both a short-term model and a long-term model to gain an in-depth understanding of the effects of reputation on bidders' total payoffs.

2.3 The Model

We consider a two-market multi-period model where one advertiser, specifically indicated by i, faces the decision of whether to stretch its reputation from one market to the other.

Advertisers are heterogeneous in two dimensions (v, q): valuation-per-click or unit-valuation, v, and its reputation characterized by the click-through rate (CTR), q. Valuation per click can be understood as the average revenue the advertiser derives from each click received on its advertisement. The CTR is essentially the probability the advertisement is clicked during each display. It characterizes the degree to which the advertisement attracts users and is modeled as the indicator of the advertiser's performance in a market. Let $s \equiv vq$, which measures the advertiser's total valuation for a given traffic size. Thus, q in the current market impacts the advertiser's payoff and serves as its reputation for the next period. We apply one of the common assumptions in auction theories, Independent Private Value (IPV) assumption, that one's total valuation is her private information, and is independent of others' valuations.

In the first market (or primary market) where advertiser i has an existing reputation, the size of traffic or the market size is k_1, normalized into 1 ($k_1 = 1$). n_1 advertisers (including advertiser i) compete in the first market, and their total valuations satisfy distribution $G_1(s)$, which can be derived from the joint distribution of (v, q). We assume advertiser i has a fixed unit valuation v_i, and reputation q_1 that does not vary from period to period. In other words, q_1 represents the advertiser i's fixed reputation which has stabilized in the primary market.

In the second market (or new market) where advertiser i has no prior reputation, the size of traffic is k_2. In the new market, n_2 advertisers compete, and their total valuations satisfy distribution $G_2(s)$. We assume the advertiser has the same unit valuation v_i as in the primary market, which simply is, and can be shown rigorously as, a normalization, because we allow different market sizes and valuation distributions in two markets. In other words, a model using a different v_i for the new market is equivalent to incorporating the difference into k and $G(s)$, leaving v the same as that in the primary market. The advertiser may have a high or low performances in this market. The expected performance in each period is q_2, having outcomes $q_2 + \epsilon$ and $q_2 - \epsilon$ with equal probability. ϵ indicates the risk of the performance in the new market, and is relatively small such that $q_2 - \epsilon$ is bounded away from 0. New entrants to this market are assigned reputation q_0, since they have no past reputation records. Thus, advertiser i will be assigned q_0 if it chooses not to stretch its reputation from the primary market. In the case of stretching, the advertiser uses the performance in the primary market q_1 as the initial reputation in the new market.

The primary market is constructed to characterize a setting where the advertiser has established its marketing ability and possesses an existing reputation. Most of the uncertainties reside in the new market, where the advertiser has no prior participation, thus decides whether to stretch its existing reputation from the primary market according to several factors that will be analyzed later in this section.

We use the auction setup to model the competitive environment, motivated by Google keyword auctions. We assume that the auctioneer allocates traffic through a unit-price auction. Every advertiser places its bid on the amount it is willing to pay for each click, and the ranking is based on advertisers' scores. The score is the product of the advertiser's bid and its reputation, which can be either performance in the previous period in the case of stretching, or the default initial reputation q_0 otherwise. The bidder with the highest score wins the auction, but only pays the unit price high enough to yield the second highest score when calculated with the winner's reputation. This second-price-like auction model, through its allocation rule, captures the essential mechanisms and impact of keyword auction practice by Google and Yahoo!.

Advertisers are risk neutral, and their objective in each auction is to maximize their expected payoffs. We denote b as the advertiser's unit-price bid in a market, and $\hat{q}$ as its reputation, which can take the value of either its last-period performance or q_0. b and $\hat{q}$ determine its winning probability in this market, $Pr(b, \hat{q})$. Also denote p as the actual unit-price the advertiser pays if it wins. So, conditional on winning, its unit surplus is $v - p$. Then the expected payoff can be written as

$$u(v, b, \hat{q}) = k * E(q) * (v - p) * Pr(b, \hat{q}) \quad (2.1)$$

where $k \in \{k_1, k_2\}$ is the size of traffic or the market size, and $E(q)$ is the expected click-through rate or the expected performance in this market.

The sequence of actions is as follows: Advertiser i first decides whether to stretch in the new market, and then competes in the new market with the reputation based on its stretching decision. Next, it competes in the primary market using that reputation. The realization of q in any period determines the advertiser's payoff in that period; and, if stretching is been chosen, q becomes $\hat{q}$ for the next period, affecting the advertiser's winning probability in that period. For the above sequence of actions, we discuss both the short-term and long-term expanding cases. In the short-term expanding (two-period) model, the advertiser only competes in the new market for one period, which is modeled as a one-shot game. In the long-term expanding (infinitely repeated) model, the advertiser competes in both markets alternately and infinitely.

We focus on the impact of three factors on the stretching decision: the market size, the advertiser's expected performance, and the risk of the performance in the new market.

2.3.1 Bidding Strategies

To begin the analysis, we derive advertisers' equilibrium bidding strategies and their equilibrium payoff in any single auction.

Lemma 2.3.1. *In each auction, bidding the true unit valuation is the (weakly) dominant strategy.*

Upon this truth bidding equilibrium, we obtain the equilibrium winning probability of an advertiser with unit valuation v and reputation $\hat{q}$ in each auction: $Pr(v,\hat{q}) = [G(\hat{q}v)]^{n-1}$. For notational simplicity, we denote $H_l(s) \equiv [G_l(s)]^{n_l-1}$, $l \in \{1,2\}$, which is the cumulative distribution function of the highest total valuation of $n_l - 1$ bidders in market l. $H_l(s)$ roughly embodies the competition in a market. Notice that $H_l(s) \in [0,1]$, and increases in s.

The price an advertiser pays, conditional on its winning, is the expected second-highest total valuation divided by its reputation:

$$\begin{aligned} p(v,\hat{q}) &= \frac{E[s_{1:n-1}|s_{1:n-1} < \hat{q}v]}{\hat{q}} \\ &= \frac{\int_0^{\hat{q}v} tdH(t)}{\hat{q}H(\hat{q}v)} \\ &= v - \frac{\int_0^{\hat{q}v} H(t)dt}{\hat{q}H(\hat{q}v)} \end{aligned}$$

where $s_{1:n-1}$ is the random variable of the highest total valuation among $n-1$ draws from the distribution $G(s)$.

Substituting the above equilibrium winning probability and price in the payoff function (2.1), we can write the equilibrium payoff as

$$U(v,\hat{q}) = k\frac{E(q)}{\hat{q}}\int_0^{\hat{q}v} H(t)dt = kE(q)\int_0^{v} H(\hat{q}t)dt \qquad (2.2)$$

where the second step is a result of integration by substitution.

2.4 Short-Term: Two-Period Setup

In this section, we focus on the case where the advertiser runs short-term business in the new market. Examples of the short-term expanding include advertising for special events (e.g., FIFA World Cup and Olympics), seasonal promotions (e.g., Christmas, Valentine's Day, etc.), where the advertiser does not stay in the new market after the temporary activity. This contrasts with the long-term case discussed in the following section, where the second market is a new business that the advertiser is starting and will continually manage.

We model bidding in this short-term new market as a one-shot game. In the first period, the advertiser competes in the auction for the new market either under a separate name (non-stretching, with the assigned reputation q_0) or under the same name as that in the primary market (stretching, with reputation q_1). In the second period, the advertiser competes in the auction for the primary market again, using reputation q_1 if non-stretching is chosen or the realized performance in the new market as reputation if the strategy of stretching is chosen.

If advertiser i chooses not to stretch, then it is assigned q_0 as the initial reputation for the new market. Therefore, its expected payoff in the new market is $U_2(v_i, q_0) = k_2 q_2 \int_0^{v_i} H_2(q_0 t)dt$ by (2.2), and, in the primary market, $U_1(v_i, q_1) = q_1 \int_0^{v_i} H_1(q_1 t)dt$ (recall that $k_1 = 1$). The total expected payoff in two markets is $U_1(v_i, q_1) + U_2(v_i, q_0)$.

If advertiser i chooses to stretch, it brings the reputation from the primary market q_1 into the new market, and its expected payoff in the new market can be formulated as $U_2(v_i, q_1) = K_2 q_2 \int_0^{v_i} H_2(q_1 t)dt$. The realized performance in the new market is $q_2 + \epsilon$ or $q_2 - \epsilon$ with probability 0.5 each, which not only affects its realized payoff for the current new market but also serves as the reputation for the primary market in the next period. We will refer to the latter as the feedback effect. Clearly, due to the feedback effect, different realized performance/reputations result in different payoffs in the primary market: $U_1(v_i, q_2 + \epsilon)$ or $U_1(v_i, q_2 - \epsilon)$. Notice that at the time the advertiser makes the stretching decision, it only has an expectation about the future payoff that depends on the uncertain performance, but does not know the actual realization. The expected payoff in the primary market is

$$E[U_1(v_i, q)] = \frac{1}{2} q_1 \int_0^{v_i} H_1((q_2 + \epsilon)t)dt + \frac{1}{2} q_1 \int_0^{v_i} H_1((q_2 - \epsilon)t)dt$$

The total expected payoff in both markets for the stretching case is $E[U_1(v_i, q)] + U_2(v_i, q_1)$.

The equilibrium stretching behavior can be determined by comparing the payoffs under stretching and non-stretching cases. The difference in payoffs is

$$
\begin{aligned}
\Delta &\equiv [E[U_1(v_i,q)] + U_2(v_i,q_1)] - [U_1(v_i,q_1) + U_2(v_i,q_0)] \\
&= k_2 q_2 \int_0^{v_i} [H_2(q_1 t) - H_2(q_0 t)]\, dt \\
&\quad + \frac{1}{2} q_1 \int_0^{v_i} [H_1((q_2+\epsilon)t) + H_1((q_2-\epsilon)t) - 2H_1(q_1 t)]\, dt \qquad (2.3)
\end{aligned}
$$

Thus, if $\Delta > 0$, it is optimal for an advertiser to stretch; otherwise, it is optimal not to stretch. Clearly, the relative market size will impact the stretching decision. We conclude the following:

Proposition 2.4.1. *Given q_0, q_2, and ϵ, when the new market is large enough ($k_2 \geq k^*$), if $q_1 > q_0$, it is optimal for the advertiser to stretch; if $q_1 < q_0$, it is optimal not to stretch.*

When the new market is of considerable size relative to the primary market, the payoff in the new market dominates that in the primary market. Thus, for a high-reputation advertiser, the gain from stretching in the new market by getting competitive advantage can out-weigh any possible loss from the negative feedback effect on the primary market; for a low-reputation advertiser, the loss from stretching is too big to be compensated by the possible gain from primary market.

It is worth noting that when the new market is small, it is not clear whether advertisers with high reputation will stretch. In fact, in the case with small enough new market, the advertiser's stretching decision depends on the feedback effect on the primary market: if $q_2 \gg q_1$, the advertiser will stretch expecting that the high reputation in the new market will enhance the competitiveness in the primary market; if $q_2 \ll q_1$, the advertiser will not stretch expecting that the low reputation in the new market will dampen the competitiveness in the primary market.

Proposition 2.4.2. *Given q_0, ϵ, and k_2, for a high reputation advertiser ($q_1 > q_0$), there exists q_2^* such that for any $q_2 \geq q_2^*$ it is optimal for the advertiser to stretch; and the gain from stretching is increasing in q_2. For a low reputation advertiser ($q_1 < q_0$), increase in q_2 has no conclusive effect.*

When the advertiser's reputation is higher than the default reputation, stretching is clearly advantageous for winning in the new market. Also, when the expected performance is high enough, the new market is optimistic. Therefore by stretching, the high-reputation advertiser can potentially further improve the reputation significantly, which in turn benefits its primary market payoff.

However, for an advertiser, whose reputation is lower than the default reputation, its stretching decision depends on the tradeoff between the loss of expected payoff in the new market and possible gain from the reputation feedback effect in the primary market. When the expected performance increases, both the loss and gain increase, so the net effect is inconclusive. It is possible that under some circumstances, a low reputation advertiser also has incentive to stretch (when the feedback effect dominates the direct effect).

Lastly, we find that the risk of the performance in the new market also impacts the stretching decision.

Proposition 2.4.3. *If $H_1(s)$ is convex, the gain from stretching is increasing in the risk in the new market. If $H_1(s)$ is concave, the opposite holds.*

Recall that $H_1(s) = G_1(s)^{n_1-1}$ is the distribution of the highest total valuation of one's competitors, and $G_1(s)$ is the distribution of total valuation in the primary market. Convexity of $G_1(s)^{n_1-1}$ means that the competitors' highest valuation is more likely to appear toward high valuation end, which can be easily satisfied. It can be shown that non-decreasing density function $G_1'(s)$ suffices to guarantee $G_1(s)^{n_1-1}$ to be convex. Also, for most continuous distributions, when n_1 is reasonably big, $G_1(s)^{n_1-1}$ commonly emerges as convex. Both cases can be interpreted as that the market is competitive enough: the former emphasizes the bidders' values are skewed toward the high-end; and the latter simply means enough bidders compete in the market.

As indicated above, when the primary market is competitive, an increase in the risk of the new market performance may benefit the advertiser in the case of stretching. Here increasing the risk has two effects: making the good realization better and making the bad realization worse. Given that the competitor is more skewed toward high valuation, the impact from being better is more significant than that from being worse.

By the results derived in this section, when an advertiser is considering stretching its reputation to a market short term - for example, selling

Olympic coin sets - it can base the decision on its current reputation and the three factors discussed above. First, if the new market is large (e.g., there is an enormous demand for Olympic coins sets since it is a global event), then high-reputation advertiser should stretch to take advantage of the established reputation. Second, if the expected performance in the new market is high, stretching is optimal for high-reputation. Third, in the stretching case, the risk of the performance in the new market may have different implications for the advertiser depending on the competition intensity of the primary market. Facing a fierce competition in the primary market, the advertiser may appreciate the risk of the new market, expecting that possible high performance could bring in a significant revenue increase via the feedback effect.

2.5 Long-Term: Infinite-Horizon Setup

In this section, the new market is not short-lived like in the previous section; rather, it repeats and alternates with the primary market infinitely. This model describes the setting where the advertiser creates new advertising campaigns in the business area dissimilar to its current one, and bids for both campaigns in the alternating manner. Long-term new markets in general application include firms introducing new products, in which case reputation stretching is using the same brand name as before. For example, Sony manufactures various home-entertainment products, such as TVs and DVD players, while producing laptop computers under the same name. As in the long-term expanding case, it continues to participate in both home-entertainment and computing markets; thus, its reputation in the two markets may affect each other. This section studies the advertiser's decision of whether to stretch its reputation on an primary market to the long-term new market, when the effect in the case of stretching will bounce between the two markets throughout the infinite horizon.

We model the long-term expanding case in the way that the new and the primary markets take place one after another from period to period. We assume future revenue discounting is reasonably small so that the impact of the initial reputation in the new market on the overall revenue can be ignored. For instance, in the case of non-stretching the starting reputation for the new market is q_0, but its effect on the overall revenue is gradually replaced by that of the advertiser's actual performance in the new market overtime. Moreover, we continue with the setup that the performance in the new markets can be $q_2 + \epsilon$ or $q_2 - \epsilon$ with equal probability.

We check the payoffs in two consecutive periods, the primary market period and the new market period. In the case of stretching, the payoff in the new market is $k_2 q_2 \int_0^{v_i} H_2(q_1 t)dt$, since here expected performance is q_2, and the reputation is q_1, the performance in the primary market, i.e., $E(q) = q_2$ and $\hat{q} = q_1$ in (2.2). Similarly, we can formulate the payoff in the primary market noting the performance is and the reputation can be $q_2 + \epsilon$ or $q_2 - \epsilon$ with equal probabilities. Therefore, the two-period payoff is

$$\frac{1}{2}q_1 \int_0^{v_i} H_1((q_2+\epsilon)t)dt + \frac{1}{2}q_1 \int_0^{v_i} H_1((q_2-\epsilon)t)dt + k_2 q_2 \int_0^{v_i} H_2(q_1 t)dt$$

In the case of non-stretching, the two-period payoff is

$$q_1 \int_0^{v_i} H_1(q_1 t)dt + \frac{1}{2}k_2 q_2 \int_0^{v_i} H_2((q_2+\epsilon)t)dt + \frac{1}{2}k_2 q_2 \int_0^{v_i} H_2((q_2-\epsilon)t)dt$$

We check the difference in payoffs between stretching and non-stretching:

$$\begin{aligned} \Delta &= \frac{1}{2}q_1 \int_0^{v_i} \left[H_1((q_2+\epsilon)t) + H_1((q_2-\epsilon)t) - 2H_1(q_1 t)\right] dt \\ &+ \frac{1}{2}k_2 q_2 \int_0^{v_i} \left[2H_2(q_1 t) - H_2((q_2+\epsilon)t) - H_2((q_2-\epsilon)t)\right] dt \end{aligned}$$

Hence, if $\Delta > 0$, it is optimal for the advertiser to stretch.

We denote $\int_0^{v_i} \left[2H_2(q_1 t) - H_2((q_2+\epsilon)t) - H_2((q_2-\epsilon)t)\right] dt \equiv D_2(q_1, q_2)$, the difference in the unit surplus between stretching and non-stretching in the new market. Similar to the short-term expanding case, the relative market size impacts the stretching decision, but in a different way.

Proposition 2.5.1. *Given q_2 and ϵ, for an advertiser with q_1 and a big enough market ($k_2 \geq k^*$), if $D_2(q_1, q_2) > 0$, it is optimal for the advertiser to stretch; if $D_2(q_1, q_2) < 0$, it is optimal not to stretch.*

Given the competition structure and the expected performance in the new market, $D_2(q_1, q_2) > 0$ requires that q_1 is sufficiently large, or the advertiser has a good reputation in the primary market.

Here the similar intuition as in Proposition (1) holds: When the new market is large enough, a high-reputation advertiser stretches, and a low-reputation advertiser does not. However, the differences from the short-term case are significant. First, notice that the assigned reputation q_0 does not play a role in the stretching decision, this is due to our assumption that the discounting of future revenue flow is reasonable small. In addition, the stretching decision closely relates to the performance (q_2 and ϵ) as well as the competition ($H_2(s)$) in the new market when the new market is large. In contrast, in the short-term case, the stretching decision critically depends on the difference between q_1 and q_0.

Next we discuss the impact of the performance in the new market (q_2 ,ϵ) on the stretching decision. We focus on the case with the same competition in both markets ($H_1(s) = H_2(s)$).

Proposition 2.5.2. *Given ϵ, k_2 and $H_1(s) = H_2(s)$, for an advertiser with q_1, there exist q_2^L and q_2^H such that under any $q_2 < q_2^L$ or $q_2 > q_2^H$ it is optimal for the advertiser not to stretch. For $q_2 \in [q_2^L, q_2^H]$, the optimal decision on stretching depends on further conditions.*

This proposition states that the gap between the expected performances in two markets determines the advertiser's stretching decision. When the difference between the two markets is large, the advertiser gets a high total payoff keeping the reputations separate, benefiting from the market with the high expected performance. If the advertiser chooses to stretch, the reputations of the two markets merge. The high-reputation will be applied in the low-performing market, which yields a low reputation that will reduce the expected payoff in the high-performing market. As a result, it is optimal not to stretch when the difference between the expected performances from the two markets is large. As a result, when q_2 is sufficiently high, it is optimal not to stretch. When q_1 is sufficiently high, the same reasoning applies in the reverse (stretching from new market to the primary market), due to the symmetry in the long-term expanding case.

Notice that this result is considerably different from that in the short-term case, where stretching is optimal for high-reputation advertisers as long as the new market is optimistic enough. Such difference is mainly driven by the asymmetry in the way reputations in two markets impact payoffs. In particular, in the short-term case, the advertiser does not have a chance to build up its reputation in the new market: q_2 does not impact the reputation in the new market. In contrast, in the long-term case, q_2 can be shifted by the performance in the primary market to affect the reputation in the primary market in the case of stretching, or affect the new market in the case of not stretching.

Proposition 2.5.3. *In the case with the same competition in both markets ($H_1(s) = H_2(s)$) and convex $H_1(s)$, if $q_1 > k_2q_2$, the gain from stretching is increasing in the risk of the performance of the new market; if $q_1 < k_2q_2$, the opposite holds.*

The positive effect from risk is different in the long-term expanding case. Here the risk can impact either the primary market or the new market. If the advertiser stretches, the positive effect hits the primary market; in the non-stretching case, the positive effect hits the new market. Therefore, when the primary market is larger, it is optimal for the advertiser to utilize the effect on the primary market, and stretch; otherwise, not stretching is the optimal choice.

According to the findings in this section, advertisers considering bidding with a second advertising campaign long-term should implement their stretching strategies differently than in the short-term expanding case. The assigned default reputation value here is negligible. Only the size, the performance, and the competitiveness of the new market impact the stretching decision. Moreover, the primary market and the new market will operate in a symmetric fashion; thus the difference between the performances in the two markets matters.

2.6 Conclusion

Motivated by keyword auctions, we analyzed advertisers' reputation stretching decisions in short-term and long-term models. We found that the stretching decision is critically dependent on the market size, the expected performance of the new market, and the risk of performance in the new market. Moreover, some results of the conditions for stretching are significantly different in the short-term and the long-term settings.

In the two-period short-term model, if stretching means using a better reputation, the advertiser is willing to stretch as long as the new market is sufficiently big. And intuitively, a promising new market induces a high-reputation advertiser to stretch. In the long-term case, although a large new market size also leads to stretching, the difference between the performances of the two markets determines the advertiser's stretching behavior, since the effect of stretching is symmetric for the two markets in the infinite horizon. We also found that in both short- and long-term cases, higher risk yields a higher payoff in a competitive market for the following period, thus is desirable for the advertisers. In the short-term case, a higher risk provides the advertiser with more incentive to stretch; but in the long-term case, the advertiser bases its decision on the comparison of the effects of the risk on the two markets, because depending on its stretching decision, the positive risk effect can carry onto either market.

The results derived from our model provide insights for the reputation stretching issue. Our analysis on the risk effect and other factors in both the short-term and long-term cases provided in-depth understanding to reputation stretching decision in the competitive environment. By finding conditions for the optimal stretching decision, our study has taken a preliminary step to designing an optimal reputation system in auction settings. Some work that follows may include auctioneer's strategies in designing auction mechanism given the equilibrium behaviors of the bidders, and further developments of the model to consider different reputation measures.

Chapter 3

Regulatory Policies for Demand-Driven Innovation by Heterogeneous Firms

3.1 Introduction

An economic recession often heightens the awareness for innovation. In responding to the current economic meltdown, the Obama administration has allocated large sums of funding for the development of science and engineering to stimulate innovation efforts [20]. $22.5 billion dollars are distributed among the major research agencies including the National Science Foundation (NSF), the National Institutes of Health (NIH), the Department of Energy (DOE), the National Institute of Standards and Technology (NIST), etc. [34]. In the meantime, the Obama administration is attempting to make the R&D tax credit permanent to increase the incentive for innovation by businesses [45].

In order to evaluate the impact of these research stimuli, it is critical to understand the implications of different innovation policies. In this paper, using a dynamic game framework we analyze innovators' equilibrium decisions and R&D efforts facing economic shocks, and explore the impact of public policies on R&D through reducing innovators' sunk costs and variable costs. Our findings provide theoretical explanations for firms' R&D activities through business cycles, and strong theoretical support for empirical evidence on innovation policies in the forms of government subsidies and tax incentives.

On innovation activities within fluctuating business cycles, one argument states that under unfavorable economic climates firms cut back on R&D in order to focus on their core business and that motivating continued innovation efforts is crucial for reviving the economy [37]; However, others argue that it is exactly the recession that provides the strongest driving force for firms to explore drastically new ideas for a chance to survive and thrive. In the research front, empirical studies has shown strong support for the procyclicality of R&D activities [4], while other recent work demonstrates that recession should foster innovation [2] [8].

Barlevy examined the inefficient procyclical allocation of innovation within business cycles, and analyzed the problem based on the externality of R&D that benefits firms aside from the innovator [4]; Taking a different angle, we look at the heterogeneity in innovators' variable costs. As Schumpeter stated, "[profit] is the premium which capitalism attaches to innovation" [39], entrepreneurs enters the R&D race based on their evaluation of future market profitability with the potential costs, which are conditional on their capital, resources, and capability. We model the impact of business cycles as exogenous income shocks that shift the market demand; In a recession, consumers have lower disposal income and have less desire to purchase the higher quality products. Our results show that more efficient (low variable R&D cost) firms innovate more in a recession due to dampened competition as less efficient (high variable R&D cost) firms perceive lower future profits and exit the innovation race. When less efficient firms innovate in the boom, the efficient firms innovate at a lower intensity in equilibrium, because intensified competition reduces innovation. However, in the latter case the aggregate innovation rate is higher since both types of firms are innovating, which is consistent with the empirical evidence in the literature and reconciles the conflicting view points on innovation activities in a downturn.

Based on the analytical results, we demonstrated several numerical examples and derived insights for innovation policies. By varying the R&D fixed cost, we found that subsidies that directly lower this cost may not stimulate innovation, because it reduces innovating firms' incentive to offset the sunk cost while maximizing profits; in other words, firms become "lazier." However, at a very large fixed cost, the industry only has efficient firms innovating, where the aggregate innovation rate is lower than in an industry where all firms choose to innovate. The empirical literature on subsidy policies also shows this inconsistency [18]. On the other hand, we found that reducing variable R&D cost has a generally favorable effect and encourages both types of firms to innovate at a higher rate. Various R&D tax incentives, such as tax credits, are examples of policies that directly affect the R&D variable cost. Our finding is supported by wide empirical evidence on positive impact of tax incentives on firms' innovation efforts [14] [23] [6].

This work also offers theoretical contributions to the related literature. Foellmi and Zweimuller studied the effect of income inequality on growth using non-homothetic consumer preference [17]. They found that higher income inequality induces innovation - the effect of higher price that results from higher income inequality dominates the effect of larger market size, which occurs under lower income inequality [17]. In their formulation, both the poor and rich consumer segments either all purchase one good or not. Our model incorporates the consumer tastes as well as income levels, under which market segmentation occurs for each income level. This allows for the analysis of richer results, as market segmentation varies with consumer preferences.

We also account for the heterogeneity of competing innovators at different costs; As a result, we are able to contrast different types of firms as innovation rate either changes smoothly or jumps with income, and infer policy implications. Furthermore, the explicit characterization of vertically differentiation shows that the equilibrium can fall under several cases; we found that within the case where high-income consumers only purchase from high-quality firm (low-quality firm only gets low-income consumers), the innovation rate is sensitive to inequality and per-capita income, whereas in the case where both types of consumers purchase from both firms in equilibrium, innovation rate is insensitive to changes in inequality when the per-capita income is held fixed. These imply that the segmentation of consumers at various income levels leads to different results when examining how income parameters affect innovation rate, and are in sharp contrast with the findings in [17].

Studies in the industrial organization literature have examined firm-level R&D issues. However, this line of work has mostly focused on a static model that limits the analysis to a single or finite period model [29] [32], with a few exceptions such as Segal and Whinston's work on anti-trust policy and innovation [40]. We adopt their framework with an extension to include dynamic draws of innovators' types in terms of their variable costs. Moreover, we endogenize entrant's and incumbents' profits using a consumer income distribution and product quality levels to include the demand factor, which is absent from Segal and Whinston's work. Furthermore, deviating from a monopolistic market, we consider a vertically differentiated market with multiple incumbents where successful innovations trigger simultaneous entry and exit. The shifting of business cycles that is reflected in consumer income change then plays a major role in incentizing potential entrants' R&D efforts, as consumers' demands directly determines the future rewards of the innovators.

Based on Shaked and Sutton, the seminal work on market equilibrium with vertical differentiation [41], we relax the assumption of uniform income distribution by generalizing the distribution, and further refine their model with a taste shock for consumers at all income levels. This setup lifts the distribution restriction imposed in most analytical work, in turn permits matching of actual data moments to find results relevant to realistic economic settings.

The rest of the paper is organized as follows. We describe the price competition game and analyze the endogenous market structure in Section 3.2. Then we present the innovation race and analyze the firm's innovation decisions in Section 3.3. Section 3.4 discusses the reaction of equilibrium innovation rate to different income shocks and regulatory policies. Section 3.5 concludes.

3.2 Price Competition and Market Structure

The present paper develops a dynamic model with price competition on differentiated products in each period. This model connects consumers' demand and firms' innovation effort through endogenous market structure. The analysis shows the impact of the aggregate economic conditions from the demand side on aggregate innovation. In this section, we describe the model setup for the static price competition, and analyze firm's pricing strategies and market segmentation based on consumers' preferences. In Section 3.3, we will analyze the firm's innovation behaviors.

Our framework has an infinite horizon, where each discrete period has the discount factor $\beta \in (0, 1)$. In each period, there exists two groups of firms differing in their objectives and actions. The incumbent firms compete in price in the product market, into which the innovations are introduced as the latest generation, or highest quality, good; The potential entrants are the firms making innovation decisions in the R&D race. This section formulates the competition and market structure in the product market among the incumbents. The innovators, prior to successfully innovating and entering the product market, choose whether to enter the R&D race and, if so, the equilibrium level of innovation effort. That is presented in Section 3.3.

Using the dynamic programming approach, we solve for the stationary Markov perfect equilibria of the infinite-horizon game. This section analyzes the existence and uniqueness of Nash equilibrium of the pricing competition game in a static vertical differentiation model. Assuming firms do not collude, the pricing strategies in the analysis here is part of the stationary Markov perfect equilibrium of the dynamic game.

3.2.1 Consumers

The setup here extends Shaked and Sutton [41] by generalizing the consumer income distribution. A continuum of consumers are heterogeneous in their income levels and tastes for the product. Denote a consumer's income by $I \in \{I_H, I_L\}$, such that $I_L < I_H$, and $\triangle = I_H - I_L$,; let $\pi_L \in [0,1]$ and $\pi_H \in [0,1]$ be the proportion of low and high income segments respectively. $\pi_H + \pi_L = 1$. Define income per capita $\overline{I} = I_H\pi_H + I_l\pi_L$, the relative high income ratio $q_h = \frac{I_H}{\overline{I}}$. Thus the triple $(\overline{I}, q_h, \pi_H)$ characterizes the income distribution of the economy. Furthermore, each consumer experiences a taste shock denoted by the random variable z that follows the uniform distribution: $z \sim U[\underline{z}, \overline{z}]$. For simplicity, a consumer's taste is fixed across her life.

In each period, consumers observe firms which produce vertically differentiated, substitute goods as a result of the innovation race, described in section 3.3. Denote $k = 1, ..., n$ as an index of the quality of products, where a higher k represents a higher quality.

The consumers are utility maximizing:

$$\max \;\; U(I, z, k) = u_k * (I + z)$$

where $u_k = e^{ak}$ following [12] and $u_0 < u_1 < ... < u_n$. Each consumer's utility is defined by the utility for consuming a certain quality good weighted by the consumer's disposable income and taste. Let C_k be the relative utility difference between products k and $k-1$, and $C_k > 1$:

$$C_k = \frac{u_k}{u_k - u_{k-1}} = \frac{e^a}{e^a - 1} = C$$

.

Define z_k^j as the indifference taste level in the income segment j, so that the consumer with taste z_k^j is indifferent between product k and $k-1$ at their respective prices. So for $j \in \{L, H\}$,

$$U(I_j - p_k, z_k^j, k) = U(I_j - p_{k-1}, z_k^j, k-1)$$

From here, we derive

$$z_1^j = p_1 C_1 - I_j \quad (3.1)$$

$$z_k^j = p_{k-1}(1 - C_k) + p_k C_k - I_j \quad (3.2)$$

Then consumers within each income segment with taste $z > z_k^j$ has the preference order $(k, p_k) \succ (k-1, p_{k-1})$.

Proposition 3.2.1. *The indifference taste levels z_k^j have the following properties:*

1. $\forall\ k$, $z_k^j > z_{k-1}^j$, *for* $j \in \{L, H\}$;

2. $\forall\ k$, $z_k^H < z_k^L$;

3. $\forall\ k$, $z_k^H + I_H = z_k^L + I_L$, *so* $z_k^H + \triangle = z_k^L$.

3.2.2 Market Structure Analysis

Firms' revenue functions take different forms depending on market segmentation, which is determined by the values of the exogenous parameters (e.g., those for income distribution and taste) and by the equilibrium prices. For example, a firm's revenue function will not include the term describing the low-income segment, if in equilibrium its price does not capture any low-income consumers; and the levels of high and low incomes as well as upper and lower bounds for consumers' taste impact such segmentation in equilibrium. The revenue functions for n firms below are listed by these cases[1].

For $k = 1$, $R_1(p_1, p_2, ..., p_n)$, the revenue of firm 1 given the price of his product p_1, is expressed in terms of the following cases:

---

[1]We consider the ordering $z^H_{k-1} < z^L_{k-1} < z^H_k < z^L_k$, the first and the last inequalities are directly obtained from Proposition 3.2.1. We assume $z^L_{k-1} < z^H_k$, which implies that for some taste levels both low- and high-income consumers will purchase the low-quality good; whereas the reverse would mean that those with high-income and purchase the low-quality good would prefer not purchasing anything if endowed with low-income. We have made this assumption instead of considering the other case $z^H_k < z^L_{k-1}$ due to the reasons that 1) $z^L_{k-1} < z^H_k$ represents a more realistic phenomenon, where low- and high-income consumers' preferences have some overlap; 2) Through simulation we found the current ordering to hold in equilibrium under the range of parameter values that are justified in the related literature (see Section 3.4). The results based on the reverse inequality $z^H_k < z^L_{k-1}$ have also been derived and tested numerically. However, the analysis is omitted here, since this case is not possible in equilibrium under the relevant parameter values.

$$R_1(p_1, p_2, ..., p_n) = \begin{cases} p_1(z_2^L - \underline{z})\pi_L, & z_2^H \leq \underline{z} \text{ and } z_2^L \geq \underline{z}; \\ p_1(z_2^H - \underline{z})\pi_H + p_1(z_2^L - \underline{z})\pi_L, & z_1^L \leq \underline{z} \text{ and } z_2^H \geq \underline{z}; \\ p_1(z_2^H - \underline{z})\pi_H + p_1(z_2^L - z_1^L)\pi_L, & z_1^H \leq \underline{z} \text{ and } z_1^L \geq \underline{z}; \\ p_1(z_2^H - z_1^H)\pi_H + p_1(z_2^L - z_1^L)\pi_L, & z_1^H \geq \underline{z}. \end{cases} \tag{3.3}$$

In the first two cases, the lowest taste consumers among the low-income segment strictly prefer purchasing the low-quality product than not buying – the low-income market is covered; In case 1, all high-income consumers will purchase the high-quality product, whereas in case 2, they are split between two products. In the last two cases, some low-taste consumers in the low-income segment would not purchase even the low-quality product - the low-income market is not covered; In case 3, the high-income segment is covered, whereas in case 4, the high-income market may not be covered.

For $1 < k < n$, $R_k(p_1, p_2, ..., p_n)$, the revenue of firm k given the price of his product p_k, is,

$$R_k(p_1, p_2, ..., p_n) = \begin{cases} p_k(z_{k+1}^H - \underline{z})\pi_H + p_k(z_{k+1}^L - z_k^L)\pi_L, & z_k^H \leq \underline{z} \\ p_k(z_{k+1}^H - z_k^H)\pi_H + p_k(z_{k+1}^L - z_k^L)\pi_L, & z_k^H \geq \underline{z}; \end{cases} \tag{3.4}$$

And for $k = n$,

$$R_n(p_1, p_2, ..., p_n) = p_n(\overline{z} - z_n^H)\pi_H + p_n(\overline{z} - z_n^L)\pi_L. \tag{3.5}$$

The first-order conditions are,

- for $1 < k < n$,

$$\begin{cases} z_{k+1}^L - \pi_L z_k^L - \pi_H \underline{z} - \pi_H \Delta - p_k[(C_{k+1} - 1) + \pi_L C_k] = 0, & z_k^H \leq \underline{z} \\ z_{k+1}^L - z_k^L - p_k[(C_{k+1} - 1) + C_k] = 0, & z_k^H \geq \underline{z}, \end{cases} \tag{3.6}$$

- for $k = n$,

$$(\overline{z} - z_n^H)\pi_H + (\overline{z} - z_n^L)\pi_L - p_n C_n = 0. \tag{3.7}$$

Lemma 3.2.2. *Let* $\overline{z} < min\{2^N \underline{z} + (2^N - 1)I_L - \pi_H \Delta, (2^{N-1}\pi_L + 2)\underline{z} + (2^{N-1}\pi_L + 2^{N-1} - 1)I_L + \pi_H \Delta\}$*, for any Nash equilibrium in this vertically differentiated market, at most N firms (producing products of qualities* n*,* $n-1$*, ..* $n-(N-1)$*) obtain positive market shares.*

We have derived the necessary condition for an N-firm equilibrium in Lemma 3.2.2. To further analyze the existence of such equilibrium, for tractability we apply the lemma to the N=2 case and consider a two-firm market.

Proposition 3.2.3. *Let* $\overline{z} < min\{4\underline{z} + 3I_L - \pi_H \Delta, (2\pi_L + 2)\underline{z} + (2\pi_L + 1)I_L + \pi_H \Delta\}$*, for any Nash equilibrium in this vertically differentiated market, at most two firms (producing products of qualities* n *and* $n - 1$*) obtain positive market shares.*

3.2.3 Two-firm equilibrium

Define $V \equiv \frac{u_2 - u_0}{u_2 - u_1} = \frac{C_2 - 1}{C_1} + 1$. We have

$$p_1 = \frac{z_1^j + I_j}{C_1} \tag{3.8}$$

$$p_2 = \frac{z_2^j + I_j + (z_1^j + I_j)(V-1)}{C_2} \tag{3.9}$$

Referring back to Equations (3.3), we get the following FOCs for firm 1, listed in the order of the corresponding cases:

$$z_2^L = \begin{cases} \underline{z} + (z_1^L + I_L)(V-1) \\ \pi_H \Delta + \underline{z} + (z_1^L + I_L)(V-1) \\ \pi_H(\Delta + \underline{z}) + z_1^L \pi_L + (z_1^L + I_L)(V - 1 + \pi_L) \\ z_1^L + (z_1^L + I_L)V; \end{cases} \tag{3.10}$$

Firm 2's FOC is either of the following ordered as the profit functions:

$$z_2^L = \begin{cases} \frac{1}{2}\left[\overline{z} - I_L - (z_1^L + I_L)(V-1) + (\overline{z} - \underline{z})\frac{\pi_H}{\pi_L}\right] \\ \frac{1}{2}\left[\overline{z} + \pi_H \Delta - I_L - (z_1^j + I_j)(V-1)\right] \end{cases} \tag{3.11}$$

Figure 1 plots firm 1's FOCs for different ranges of z_1^L . Regions 1 through 4 in the figure correspond to the four cases of Equation (3.3); and Regions 5, 6 and 7 are the regions between the adjacent cases. In these regions, in equilibrium one firm varies its price while the other holds its price constant. Note that from Equations (3.10), firm 1's FOCs are expressed as functions $z_2^L(z_1^L)$, which is increasing, whereas from Equations (3.11) firm 2's FOCs are decreasing functions. The point of intersection is the equilibrium taste levels z_1^{L*} and z_2^{L*}, from which equilibrium prices are calculated. In the lemma below, we set conditions under which equilibrium occurs in certain regions.

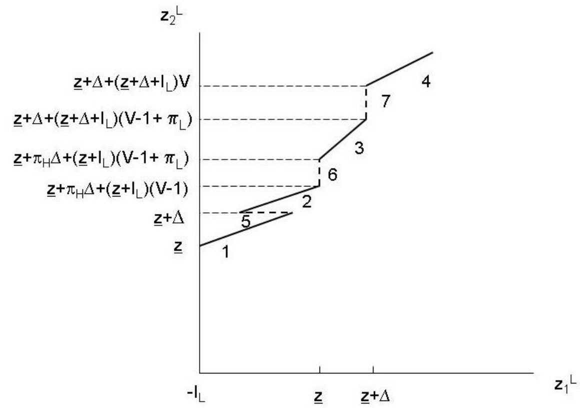

Figure 3.1: Firm 1's First Order Conditions

Lemma 3.2.4. *Assuming* $\underline{z} + I_L \geq \Delta$*, let* $(2\pi_L + \pi_H)\underline{z} + \pi_L I_L < \overline{z} < (2\pi_L + 2)\underline{z} + (2\pi_L + 1)I_L + \pi_H \Delta$*, there exist a unique equilibrium where exactly 2 firms will have positive market shares. The possible regions where the equilibrium lies include Regions 1, 2, 5 and 6. Moreover, both low- and high-income markets are covered (i.e., the equilibrium does not lie in Region 3, 4 or 7).*

The equilibrium region depends on the values of the exogenous parameters. The general results for determining equilibrium region are stated in the following proposition.

Proposition 3.2.5. *When* $\overline{z} \in [(2\pi_L+\pi_H)\underline{z}+\pi_L I_L, (2\pi_L+\pi_H)\underline{z}+\pi_L I_L+3\pi_L\Delta]$*, the equilibrium lies in Region 1. When* $\overline{z} \in [(2\pi_L+\pi_H)\underline{z}+\pi_L I_L+3\pi_L\Delta, 2\underline{z}+I_L+(3\pi_L+\pi_H)\Delta]$*, the equilibrium lies in Region 5. When* $\overline{z} \geq 2\underline{z}+I_L+(3\pi_L+\pi_H)\Delta$*, if* $V \geq \frac{\overline{z}+\underline{z}+2I_L-\pi_H\Delta}{3(\underline{z}+I_L)}$*, then the equilibrium lies in Region 2, otherwise it lies in Region 6.*

Table 3.1: Equilibrium Prices in Regions 1, 2, 5, and 6

	Prices
Region 1	$p_1^* = \frac{\overline{z}-2\underline{z}-I_L+(\overline{z}-\underline{z})\frac{\pi_H}{\pi_L}}{3C(V-1)}$ $p_2^* = \frac{1}{3C}\left[2\overline{z}-\underline{z}+I_L+2(\overline{z}-\underline{z})\frac{\pi_H}{\pi_L}\right]$
Region 5	$p_1^* = \frac{1}{C(V-1)}[(1+\frac{\pi_H}{\pi_L})(\overline{z}-\underline{z})-\underline{z}-I_L-2\Delta]$ $p_2^* = \frac{(\overline{z}-\underline{z})(1+\frac{\pi_H}{\pi_L})-\Delta}{C}$
Region 2	$p_1^* = \frac{\overline{z}-2\underline{z}-\pi_H\Delta-I_L}{3(C-1)}$ $p_2^* = \frac{2\overline{z}-\underline{z}+\pi_H\Delta+I_L}{3C}$
Region 6	$p_1^* = \frac{\underline{z}+I_L}{C}$ $p_2^* = \frac{\overline{z}+\pi_H\Delta+I_L+(\underline{z}+I_L)(V-1)}{2C}$

We derive the equilibrium indifference taste levels, prices, and revenues in these different regions. This characterizes the market structure and profitability given the economic conditions, in particular the consumer income distribution within a period.

Table 3.2: Equilibrium Profits in Regions 1, 2, 5, and 6

	Profits
Region 1	$R_1^* = \frac{\pi_L}{9C(V-1)}\left[\overline{z} - 2\underline{z} - I_L + (\overline{z}-\underline{z})\frac{\pi_H}{\pi_L}\right]^2$ $R_2^* = \frac{\pi_L}{9C}\left[2\overline{z} - \underline{z} + I_L + 2(\overline{z}-\underline{z})\frac{\pi_H}{\pi_L}\right]^2$
Region 5	$R_1^* = \frac{\pi_L\Delta}{C(V-1)}[(1+\frac{\pi_H}{\pi_L})(\overline{z}-\underline{z}) - \underline{z} - I_L - 2\Delta]$ $R_2^* = \frac{1}{C\pi_L}\left[(1+\frac{\pi_H}{\pi_L})(\overline{z}-\underline{z}) - \Delta\right]^2$
Region 2	$R_1^* = \frac{(\overline{z}-2\underline{z}-\pi_H\Delta-I_L)^2}{9(C-1)}$ $R_2^* = \frac{(2\overline{z}-\underline{z}+\pi_H\Delta+I_L)^2}{9C}$
Region 6	$R_1^* = \frac{\underline{z}+I_L}{C}[\overline{z} - \pi_H\Delta - I_L - 2\underline{z} - (\underline{z}+I_L)(V-1)]$ $R_2^* = \frac{[\overline{z}+\pi_H\Delta+I_L+(\underline{z}+I_L)(V-1)]^2}{4C}$

3.3 Innovating Firms

This section describes the innovation race and firms' innovation decisions. Our setup follows the framework developed by Segal and Whinston (2007) with the extension of heterogeneity of innovation costs across firms [40].

There exist M firms who are potential entrants. Every period, they pick up a draw ϵ from a distribution $F(\cdot)$. This draw affects the cost of innovation, which is $\epsilon c(\phi_i(\epsilon))$. $\phi_i(\epsilon) \in (0,1)$ is the innovation rate of firm i with the draw ϵ. $c(\cdot)$ is a concave function.

Potential entrants make decisions in three stages: 1) Entry to innovation race - firms choose whether to innovate; 2) Innovation effort - firms choose the level of R&D, which affects its probability to successfully innovate, hence the chance of market entry; 3) In case of market entry, firms choose their prices, which is described in the equilibrium results in the previous section.

Multiple innovators may succeed in developing new products. However, only one of these innovations is granted a patent. The firm with a patent then enters the product market and becomes an incumbent with the highest quality product. We use the simultaneous entry and exit setup, thus the lowest-quality incumbent is displace upon a new entry. The innovation model connects to the market structure analysis at this point, as the profits of a new entrant is characterized by the equilibrium results derived in Section 3.2.

If a firm chooses to innovate, it incurs a sunk innovation cost f. Let $\pi_M(\phi^I_-)$ denote the probability of a firm successfully creating a new product. $\phi^I_- \in [0,1]^M$ describes the innovation efforts of all the potential entrants. However, each period only one of these firms is granted a patent and enters the market, the probability of actually obtaining the patent is then denoted by $\lambda_{M^l}(\phi, \phi_-)$. $\phi_- \in [0,1]^{M-1}$ denotes the innovation efforts of the rest of potential entrants[2]. The value functions of the firms at different stages are listed below:

$$V^0(\epsilon,\phi_-) = max\{0, -f + V^E(\epsilon,\phi_-)\} \tag{3.12}$$

$$V^E(\epsilon,\phi_-) = max_\phi\{\lambda_{M^l}(\phi,\phi_-)V_J^I + (1-\lambda_{M^l}(\phi,\phi_-))EV^0(\epsilon',\phi'_-) - \epsilon c(\phi)\} \tag{3.13}$$

$$V_i^I(\epsilon,\phi_-) = \pi_{M^l}(\phi_-^I)[R_{i-1} + \beta V_{i-1}^I(\epsilon,\phi_-)] + (1-\pi_{M^l}(\phi_-^I))[R_i + \beta V_i^I(\epsilon,\phi_-)] \tag{3.14}$$

$$i = 2, ..., J$$

$$V_1^I(\epsilon,\phi_-) = \pi_{M^l}(\phi_-)EV^0(\epsilon',\phi'_-) + (1-\pi_{M^l}(\phi_-))[R_1 + \beta V_1^I(\epsilon,\phi_-)] \tag{3.15}$$

$V^0(\epsilon,\phi_-)$ is the value function of firms at the start of the game; $V^E(\epsilon,\phi_-)$ is the value function at Stage 1; $V_i^I(\epsilon,\phi_-)$ and $V_1^I(\epsilon,\phi_-)$ are the value functions for incumbents producing product quality i and the lowest quality product before exiting, respectively. It is easy to show that the dynamic programming problem described by equations (3.12)-(3.15) satisfies the Blackwell sufficient conditions, thus it has a unique fixed point in a bounded space.

Assuming $\epsilon \in \{\epsilon_l, \epsilon_h\}$ follows Bernoulli distribution, the probability of drawing ϵ_h is η . For simplicity, let the number of firms facing low innovation shocks in each period be M^l (by the Law of Large Numbers $M^l \approx (1-\eta)M$).

[2] ϕ_-^h and ϕ_-^l are other firms' innovation efforts for a firm with high or low innovation costs respectively.

Following the formulation for multiple entrants case in Segal and Whinston's work [40], if both types of firms innovate, the probability of at least one firm successfully creating a innovation is,

$$\pi_{M^l}(\phi_-^I) = [1 - (1 - \phi(\epsilon_h))^{M-M^l}(1 - \phi(\epsilon_l)^{M_l}] \tag{3.16}$$

If only low-cost firms innovate,

$$\pi_{M^l}(\phi_-^I) = [1 - (1 - \phi(\epsilon_l))^{M^l}] \tag{3.17}$$

And in the former case, for any one firm, conditional on successful innovation, the probabilities of obtaining a patent for the high- and low-cost firms are

$$\begin{aligned} r(\phi_-^h) &= \sum_{x=0}^{M-M^l-1} \sum_{y=0}^{M^l} [\binom{M-M^l-1}{x}\binom{M^l}{y} \\ &\quad \frac{(\phi_h)^x(1-\phi_h)^{M-M^l-1-x}(\phi_l)^y(1-\phi_l)^{M^l-y}}{x+y+1}] \\ r(\phi_-^l) &= \sum_{x=0}^{M-M^l} \sum_{y=0}^{M^l-1} [\binom{M-M^l}{x}\binom{M^l-1}{y} \\ &\quad \frac{(\phi_h)^x(1-\phi_h)^{M-M^l-1-x}(\phi_l)^y(1-\phi_l)^{M^l-y}}{x+y+1}] \end{aligned}$$

The probability of obtaining a patent for this firm with high or low cost is, respectively, then,

$$\begin{aligned} \lambda_{M^l}(\phi(\epsilon_h), \phi_-^h) &= \phi(\epsilon_h) r(\phi_-^h) \\ \lambda_{M^l}(\phi(\epsilon_l), \phi_-^l) &= \phi(\epsilon_l) r(\phi_-^l) \end{aligned}$$

In the latter case, the conditional probability for a given firm is

$$r(\phi_-^l) = \sum_{k=0}^{k=M^l-1} \left[\frac{1}{k+1} \binom{M^l-1}{k} \phi_l^k (1-\phi_l)^{M^l-1-k} \right]$$

The equilibrium is the fixed point of the following correspondence:

$$\phi(\epsilon) = argmax_{\phi' \in [0,1]}\{0, -f + V^E(\epsilon, \phi')\}$$

In the following section, we discuss the numerical solutions to the firm's dynamic problem and derive insight on the impacts of different innovation policies and aggregate economic conditions on innovation.

3.4 Equilibrium and Comparative Statics

In this section, we will discuss the parameterizations and the comparative statics results based on the numerical analysis. First we show the change in equilibrium innovation rate with respect to different types of income shocks. And then the implications of public policies are discussed according to the results from varying sunk and variable innovation costs.

3.4.1 Parameterization

The aim of our analysis is to provide insight into the qualitative properties of equilibrium innovation rate under the effect of income shocks and different types of innovation policies. Although some parameters are chosen from standard values and previous literature, they are not based on data from some specific industries.

The discount rate $\beta = 0.95$ implies the annual interest rate is approximately 5%. a in the utility function is 1.2. The income per capital $\overline{I}$ is 0.9. The relative high income q_h is 1.11. We assume half of consumers have high income. The upper bound of taste shock $\overline{z}$ is 4.2, while the lower bound of the taste shock $\underline{z}$ is 1.2. The sunk cost of innovation f is set to 5. As for the functional form of innovation cost $c(\cdot)$, we follow Aghion, et al.'s model and use quadratic form, $c(\epsilon) = \epsilon\phi^2$ [1]. Firms with high variable innovation costs have $\epsilon_h = 20$. Firms with low variable innovation costs have $\epsilon_l = 12$. We also assume the number of potential entrants is 10 each period and the number of firms with high innovation costs is 5. We set these numbers relatively small to reduce the computation load.

With the above parameterizations, both types of firms conduct innovation. The innovation rate for the firms with high innovation costs is 0.4641. The innovation effort of the rest of firms is higher, 0.5964, as their innovation costs are lower. The equilibrium prices fall in Region 2 in Section 3.4.2. If we raise the lower bound of taste shock $\underline{z}$ to 2.2 and set $a = 1.4$, then the equilibrium falls into Region 1. The firms with high innovation cost do not innovate. The firms with low innovation cost now invest higher and have innovation rate 0.7729. We discuss firms' innovation behaviors in different regions in the following.

3.4.2 Innovation and Income Shock

We study two types of income shocks in this part of the numerical analysis, income inequality and income per capita. For income inequality, we hold the per-capita income $\overline{I}$ fixed and vary the income gap. For income per capita, we hold the income inequality fixed. These two experiments allow us

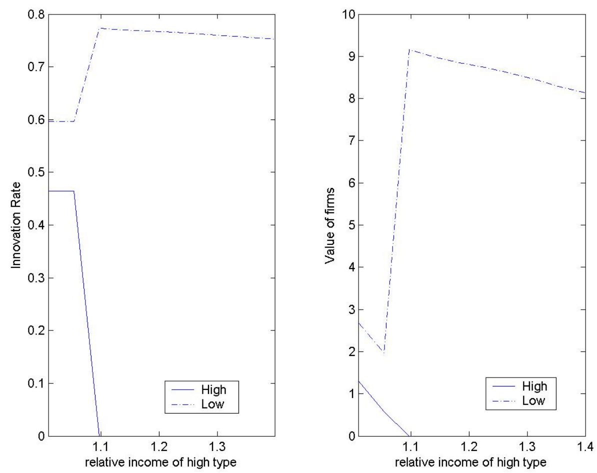

Figure 3.2: Innovation & Income Inequality - Region 1

to isolate the effect of each of the two factors.

With respect to income inequality, holding the income per capita $\overline{I}$ and proportion of low- and high-income segments, π_h, π_l fixed, we vary the relative high income level q_h to reflect the varying inequality. The results contrast sharply between Region 1 and Region 2 (recalling that the region where equilibrium falls is found endogenously by the parameter values that characterize the consumer preference and income, and thus the prices set by firms). In other words, varying income inequality within the range of the region conditions allowed us to examine innovation in industries with certain consumer preferences and yielded notably distinct results.

In Region 1 (see Figure 3.2), where low-income is segmented between two products and high-income is solely captured by the higher quality firm, in-

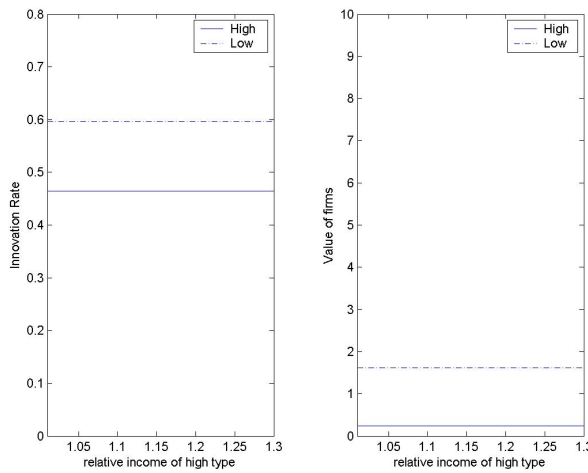

Figure 3.3: Innovation & Income Inequality - Region 2

creasing inequality has an adverse effect on both innovate rate and firm values. The values of both types of firm decline with inequality due to reduced revenue. This may appear counterintuitive, as one would expect steeper inequality to benefit the higher-quality firm, which obtains the entire high-income segment as well as part of the low-income segment. Our finding offers the opposite explanation that with a wider income gap higher-quality firm actually lowers its price in equilibrium in order to reach the low-income level while still gaining the entirety of high-income segment. The aggregate innovation rate shows insignificant change until the high-variable-cost firms drop out of the innovation race, at which point the innovation rate declines significantly.

Region 2's equilibrium results are independent of income inequality shifts when the income per capita is held fixed, due to the condition that both low- and high-income segments are shared between two firms. As a result,

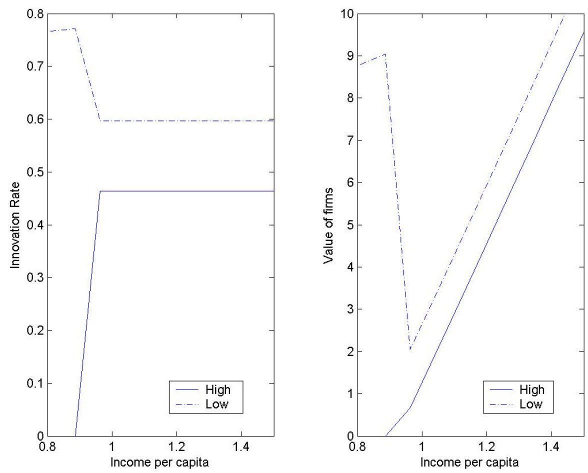

Figure 3.4: Innovation & Income per Capita - Region 1

both firm values and innovation rates are constant in income quality (see Figure 3.3). This result is confirmed by the equilibrium profits in Table 3.2 where these profits are only related to $\overline{I}$ rather than q_h.

We further analyze the impact of per-capita income on innovation while holding fixed the income inequality parameter q_h. The results for the two regions look similar. Increasing per-capita income directly shifts the equilibrium revenues. The value of the firms that have a low variable cost increases first, where the high-variable-cost firms choose to not innovate; as the income level rises further, the value of the low-variable-cost firms drops and then increases in parallel with the high-variable cost firms (see Figures 3.4 & 3.5).

Thus, the improvement in overall income levels has three effects: 1) It encourages high-variable-cost firms to enter the R&D race; and 2) It intensifies

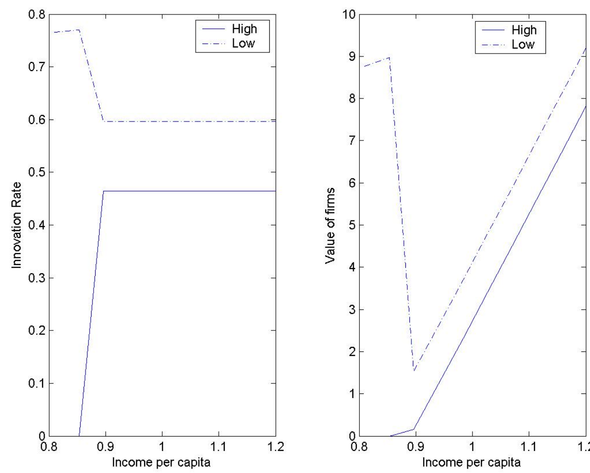

Figure 3.5: Innovation & Income per Capita - Region 2

competition and in turn shifts down low-variable-cost firms' innovation rate; 3) It induces higher innovation rate among the existing innovators. This is linked to the increased demand and equilibrium profits in the product market where the successful innovator enters. Even though increased competition shifts down the equilibrium innovation rate of the low-variable-cost firms upon the entry of the high-variable-cost firms, the aggregate innovation is increased.

Examination of different types of potential entrants reveals the underlying value gaps of the heterogeneous firms within the aggregate innovation rate. The procyclicality of R&D is reaffirmed in the aggregate sense. On the other hand, the argument that recession can also stimulate innovation is reflected in the first effect described above. The high-variable-cost firms drop out of the R&D competition, thus reduced competition gives a boost to those remaining in the R&D race.

3.4.3 Innovation and Policy

We also look at the effects of varying the variable costs and sunk costs of the innovating firms. We vary the variable costs proportionally, since the heterogeneity resides in firms' variable costs.

As Figure 3.6 shows, as the variable cost decreases (examining the x axis from right to left) both the value and innovation rate increase for the low-variable-cost firms, while the high-variable-cost firms do not innovate. Similar to the previous observations with income per capita, further decreasing the variable cost results in a drop of the value and innovation rate of the low-variable-cost firms, as the high-variable-cost firms join the R&D race.

This result has an important R&D tax policy implication. R&D tax incentives can be designed to lower firms' R&D variable costs by providing more tax cuts for more dollars spent on technology innovation. In effect, these policies increase firms' profits for more R&D activities. The similar three effects as those observed with the income per capita are drawn here. And we see that reducing variable R&D costs improves the aggregate innovation.

We also analyzed innovation rate against the innovation sunk cost, which refers to the fixed cost to set up an R&D facility and purchase R&D equipments, which may be applicable to start-up firms or those of small capacity. It seems counterintuitive that increasing fixed cost can encourage innovation rate. This finding is consistent for the low-variable-cost firms, whereas for the high-variable-cost firms the innovation rate drops to zero when the fixed

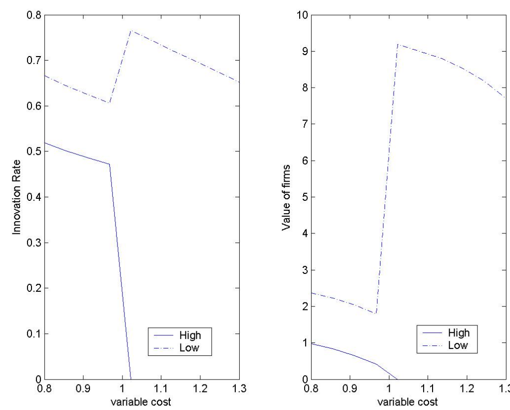

Figure 3.6: Innovation & Variable Costs

cost exceeds a certain threshold (see Figure 3.7).

The partially positive effect of fixed cost is due to that the firms innovate more intensely in order to achieve a higher probability of success to offset the cost. However, above a certain threshold, the level of fixed cost no longer justifies innovation decision for those with high variable costs, in which case the high-variable-cost firms drop out of the R&D race, while the low-variable-cost firms' innovation rate has an upward jump caused by the dampened competition and continues to rise at a low rate.

In the aggregate sense, the optimal innovation is achieved in the first range when both types of firms innovate (see Figure 3.7). The insight here is that subsidies for innovation may not stimulate R&D efforts, because the innovating firms the incentives to compete in an attempt to recover the sunk cost are diminished. In effect, firms become "lazier" as the profitability linked with innovation effort is more easily achieved. Furthermore, when an industry

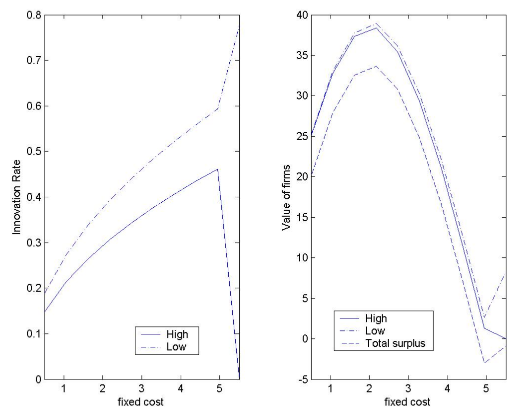

Figure 3.7: Innovation & Fixed Costs

has a very high R&D sunk cost, only specialized or established firms that can efficiently carry out R&D will compete in innovation.

3.5 Concluding Remarks

In this paper we have studied the change in innovation rate given income shocks to understand the innovation decision of firms under the impact of business cycles. We further analyzed the innovation rate while varying heterogeneous innovating firms' sunk and variable R&D costs and derived implications for innovation policies. Our formulation of a rich microfoundation with the dynamic model has several major contributions: 1) Contrasting with Segal and Whinston (2007) we provided an added dimension of business cycle in the analysis of innovation rate; We are able to vary consumer income to find firms' R&D patterns through the fluctuation of business cycles; 2) Through the market structure analysis, we found that equilibrium market segmentation is sensitive to consumer preferences; We in turn derived in-depth insights on innovation and income inequality, which reached beyond the findings in [17]; 3) We modeled multiple types of innovating firms in terms of their variable R&D costs; the results on policies and innovation are consistent with the empirical literature.

We found that effects of income shocks differ by varying either income inequality and per-capita income while holding the other fixed. Under income inequality change, the change in innovation rate has radically different results conditional on the equilibrium region. In Region 1, when high-income segment only consumes high-quality good, the innovation rate is sensitive to the inequality shock due to the asymmetry in market segmentation of the two income levels. It decreases as income levels become more polarized, because equilibrium revenues and prices are lowered to capture the poorer low-income consumers. In Region 2, varying inequality has little effect since both low- and high-income markets are segmented.

Increasing per-capita income has similar results in the two regions, because firms' equilibrium profits increase in both Region 1 and Region 2. Raising the overall income levels has several effects. First, it encourages high-variable-cost firm to enter the R&D race, because the profitability of the market is increased. However, introducing more innovating firms intensifies competition and reduces the other firms' equilibrium innovation rate. Thus there is a downward shift of the low-variable-cost firms' innovation rate as more firms enter the race. Regardless, the aggregate innovation rate increases with such a shock.

Our policy analysis showed consistent findings with the empirical evidence that subsidies tend to have ambiguous effects on innovation, whereas tax incentives have strongly positive impact. We provide the explanation that subsidies often directly compensate firms' sunk R&D costs, thus reduce the premium that firms aim to recover through innovation success. While it moderates innovation efforts, dampened competition may have a positive effect on firms with lower variable costs. R&D tax credits reduce firms' variable costs; therefore, they continuously stimulate innovation efforts while encouraging entry into the R&D race.

Appendices

Appendix A

Proofs of Results in Chapter 1

Proof of Lemma 1.4.1.

Proof. i) Suppose $p_1^* > p_2^*$, firms' profit functions are

$$\pi_1 = (p_2 - p_1 - \alpha) * p_1$$

$$\pi_2 = (1 - p_2 + p_1 + \alpha)p_2 + \beta(1 + \alpha)$$

The first order conditions are

$$p_2 - 2p_1 - \alpha = 0$$

$$1 - 2p_2 + p_1 + \alpha = 0$$

The equilibrium prices are

$$p_1^* = \frac{1 - \alpha}{3}, p_2^* = \frac{2 + \alpha}{3}$$

If $\alpha < -\frac{1}{2}$, $p_1^* > p_2^*$ holds. The equilibrium profits are

$$\pi_1^* = \frac{1}{9}(1 - \alpha)^2, \pi_2^* = \frac{1}{9}(2 + \alpha)^2 + \beta(1 + \alpha)$$

ii) Now suppose $p_1^* < p_2^*$.

$$\pi_1 = (p_2 - p_1 - \alpha) * p_1$$

$$\pi_2 = (1 - p_2 + p_1 + \alpha)(p_2 + \beta)$$

The first order conditions are

$$p_2 - 2p_1 - \alpha = 0$$

$$1 - 2p_2 + p_1 + \alpha - \beta = 0$$

By solving the first order conditions, we get

$$p_1^* = \frac{1 - \alpha - \beta}{3}, p_2^* = \frac{2}{3}(1 - \beta) + \frac{1}{3}\alpha$$

But now $0 < p_1^* < p_2^*$ implies $-\frac{1}{2}(1 - \beta) < \alpha < 1 - \beta$. The equilibrium profits are $\pi_1^* = \frac{1}{9}(1-\beta-\alpha)^2$, $\pi_2^* = \frac{1}{9}(2+\beta+\alpha)^2$, $\pi_2^* - \pi_1^* = \frac{1}{3}(1+2\beta+2\alpha)$, which is positive if $\alpha > -\frac{1}{2}(1 - \beta)$.

iii) For $\alpha \geq 1-\beta$, p_1^* is driven down to 0, maximizing $\pi_2 = (1-p_2+\alpha)(p_2+\beta)$ yields $p_2^* = \frac{1}{2}(1 + \alpha - \beta)$, demand $q_2^* = \frac{1}{2}(1 + \alpha + \beta) > 1$. Since q_2^* is at most 1, $p_2^* \geq \alpha$. From the concavity of π_2 and $\frac{1}{2}(1+\alpha-\beta) \leq \alpha$, we infer that $p_2^* = \alpha$.

iv) For $-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1 - \beta)$, the only possible case is that $p_1^* = p_2^* = p$.

In this case, firms' profit functions are

$$\pi_1 = -\alpha p$$

$$\pi_2 = (1+\alpha)(p+\beta)$$

Let's check if firm 1 deviates. $\forall \epsilon > 0$, if $p_1 = p + \epsilon$, then $\tilde{\pi}_1 = (-\epsilon - \alpha)(p+\epsilon)$,

$$\tilde{\pi}_1 - \pi_1 = -\epsilon(p + \epsilon + \alpha)$$

so if $p + \alpha \geq 0$, $\tilde{\pi}_1 - \pi_1 < 0$, firm 1 will not charge higher price.

If $p_1 = p - \epsilon$, then $\tilde{\pi}_1 = (\epsilon - \alpha)(p - \epsilon)$,

$$\tilde{\pi}_1 - \pi_1 = \epsilon(p - \epsilon + \alpha)$$

so if $p+\alpha \leq 0$, $\tilde{\pi}_1 - \pi_1 < 0$, firm 1 will not charge lower price. Therefore, if $p = -\alpha$, then firm 1 will not deviate.

Now let's check if firm 2 deviates at $p = -\alpha$. If $p_2 = -\alpha + \epsilon$, then $\tilde{\pi}_2 = (1 - \epsilon + \alpha)(-\alpha + \epsilon + \beta)$,

$$\tilde{\pi}_2 - \pi_2 = \epsilon(1 - \epsilon + 2\alpha - \beta)$$

since $\alpha \leq -\frac{1}{2}(1-\beta)$, we have $1 + 2\alpha - \beta \leq 0$, thus $\tilde{\pi}_2 - \pi_2 < 0$, firm 2 will not charge higher price.

If $p_2 = -\alpha - \epsilon$, then $\tilde{\pi}_2 = (1 + \epsilon + \alpha)(-\alpha - \epsilon) + \beta(1+\alpha)$,

$$\tilde{\pi}_2 - \pi_2 = \epsilon(-1 - \epsilon - 2\alpha)$$

since $\alpha \geq -\frac{1}{2}$, we have $-2\alpha - 1 \leq 0$, thus $\tilde{\pi}_2 - \pi_2 < 0$, firm 2 will not charge lower price.

Therefore, $p_1^* = p_2^* = -\alpha$ is a unique equilibrium within this case. The equilibrium profits are $\pi_1^* = \alpha^2$, $\pi_2^* = (1+\alpha)(\beta-\alpha)$. Since $1+\alpha > -\alpha > 0$, $\beta - \alpha > -\alpha > 0$, then $(1+\alpha)(\beta-\alpha) > \alpha^2$.

□

Proof of Proposition 1.4.2.

Proof. i) For $\alpha < -\frac{1}{2}$, clearly $\frac{1}{3}(1-\alpha) > \frac{1}{3}(2+\alpha)$, so $p_1^* > p_2^*$.

For $-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$, $p_1^* = p_2^* = \alpha$.

For $-\frac{1}{2}(1-\beta) < \alpha < 1-\beta$, $p_2^* - p_1^* = \frac{1}{3}(1+2\alpha-\beta) > 0$, since $\alpha > -\frac{1}{2}(1-\beta)$.

For $\alpha \geq 1-\beta$, $p_2^* > p_1^*$.

Therefore, in equilibrium, $p_1^* > p_2^*$ if $\alpha < -\frac{1}{2}$, and $p_1^* \leq p_2^*$ otherwise.

ii) For $\alpha < -\frac{1}{2}$, $\pi_2^* - \pi_1^* = \frac{1+2\alpha+3\beta(1+\alpha)}{3}$, which is positive if $\alpha > \frac{1}{2+3\beta} - 1$.

As shown in the proof for Lemma 1.4.1, for $-\frac{1}{2}(1-\beta) < \alpha < 1-\beta$, $\pi_2^* - \pi_1^* = \frac{1}{3}(1+2\beta+2\alpha) > 0$; and for $-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$, $(1+\alpha)(\beta-\alpha) > \alpha^2$.

For $\alpha > 1-\beta$, $\pi_2^* = \alpha+\beta$ is clearly > 0.

Therefore, $\pi_2^* > \pi_1^*$ iff $\alpha > \frac{1}{2+3\beta} - 1$.

□

Proof of Lemma 1.4.3.

Proof. Suppose $p_1^* < p_2^*$, then firms' profit functions are as follows:

$$\pi_1 = (p_2 - p_1 - \alpha)p_1$$

$$\pi_2 = (1 - p_2 + p_1 + \alpha)(p_2 + \beta)$$

We can see that this case is exactly the same as the mandatory case for $\alpha > -\frac{1}{2}(1-\beta)$ (see the proof for Lemma 1.4.1).

For $\alpha \leq -\frac{1}{2}(1-\beta)$, if $p_1^* > p_2^*$, all consumers will purchase from firm 2, while firm 1 has no market share. So firm 1 would lower its price. Firm 2's profit is $\pi_2 = p_2 + \beta(1+\alpha)$, so it has incentive to raise its price. Therefore, there is no pure strategy equilibrium for $p_1 > p_2$.

Now check if $p_1 = p_2 = p$ is an equilibrium. If $p > 0$, firm 2 has incentive to lower its price to undercut firm 1. But for $p = 0$, neither firm has incentive to deviate. So $p_1 = p_2 = 0$ is the unique equilibrium for $\alpha \leq -\frac{1}{2}(1-\beta)$. □

Proof of Proposition 1.4.4.

Proof. By comparing firm 2's profits under mandatory and optional strategies, it is clear that for $\alpha \leq -\frac{1}{2}(1-\beta)$, mandatory strategy yields higher profits; for other values of α, profits are identical. So mandatory strategy is weakly dominant for firm 2. □

Proof of Proposition 1.4.5.

Proof. It follows straightforward from the perfect substitutability of two firms' services and advertisements that consumers are indifferent between the offerings of the two firms, when they choose the same advertising strategies. The Bertrand competition results in zero price. As firms evenly split the market, they make equal advertising revenue $\frac{1}{2}\beta(1-\alpha)$. $\square$

Proof of Lemma 1.4.6.

Proof. Suppose $p_1^* > p_2^*$. Since $\pi_1 = 0$, $\pi_2 = p_2 + \beta(1+\alpha)$, firm 1 has incentive to lower its price, while firm 2 has incentive to raise price, so this is not an equilibrium.

Suppose $p_1^* < p_2^*$. Now $\pi_1 = (1 - p_1 + p_2 + \alpha)p_1 + \beta(1 + \alpha)$, $\pi_2 = (p_1 - p_2 - \alpha)p_2$, we get the equilibrium prices $p_1^* = \frac{1}{3}(2 + \alpha)$, $p_2^* = \frac{1}{3}(1 - \alpha)$. So $p_2^* > p_1^*$ if $\alpha < -\frac{1}{2}$.

For $\alpha \geq -\frac{1}{2}$, $p_1^* = p_2^* = p$ is the only possible equilibrium. It's easy to verify that if $p > 0$, firm 1 has incentive to undercut. But if $p = 0$, both firms don't deviate and make positive profits $\pi_1^* = \pi_2^* = \frac{1}{2}\beta(1 + \alpha)$. $\square$

Proof of Proposition 1.4.7.

Proof. First we show that $\frac{1}{9}(1-\alpha)^2 > \frac{1}{2}\beta(1+\alpha)$, or equivalently, $\Delta = 2(1-\alpha)^2 - 9\beta(1+\alpha) > 0$. Since $\frac{\partial \Delta}{\partial \alpha} = 4\alpha - (4 + 9\beta) < 0$, it is enough to show that $\Delta > 0$ at $\alpha = -\frac{1}{2}$. Since $0 < \beta < 1$, then $\Delta(-\frac{1}{2}) = \frac{9}{2} - \frac{9}{2}\beta > 0$ holds. So, given that a firm chooses mandatory advertising, the other firm's best response is optional advertising.

We also need to show that $\frac{1}{9}(2+\alpha)^2 + \beta(1+\alpha) > \frac{1}{2}\beta(1+\alpha)$, which always holds since $1+\alpha > 0$ and $\beta > 0$. Thus, given that a firm chooses optional advertising, the other firm's best response is mandatory advertising.

So for $\alpha < -\frac{1}{2}$, the equilibrium would be one firm chooses mandatory strategy and the other firm chooses optional strategy. □

Proof of Lemma 1.5.1.

Proof. We have shown that $\frac{1}{9}(1-\alpha)^2 > \frac{1}{2}\beta(1+\alpha)$ and $\frac{1}{9}(2+\alpha)^2+\beta(1+\alpha) > \frac{1}{2}\beta(1+\alpha)$, for $\alpha < -\frac{1}{2}$. From Table A.1, we can derive the pure-strategy equilibria to be (M,N), (N,M), (M,O), and (O,M). □

		Firm 2		
		N	M	O
	N	0, 0	$\frac{1}{9}(1-\alpha)^2,\ \frac{1}{9}(2+\alpha)^2+\beta(1+\alpha)$	$0,\ \beta(1+\alpha)$
Firm 1	M	$\frac{1}{9}(2+\alpha)^2+\beta(1+\alpha),\ \frac{1}{9}(1-\alpha)^2$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$	$\frac{1}{9}(2+\alpha)^2+\beta(1+\alpha),\ \frac{1}{9}(1-\alpha)^2$
	O	$\beta(1+\alpha),\ 0$	$\frac{1}{9}(1-\alpha)^2,\ \frac{1}{9}(2+\alpha)^2+\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$

Table A.1: Payoff Matrix for $\alpha < -\frac{1}{2}$

Proof of Lemma 1.5.2.

Proof. For this case $\beta < \frac{1}{3}$.

Let $\Delta(\alpha) = \alpha^2 - \frac{1}{2}\beta(1+\alpha)$. When $\beta < \frac{1}{3}$, $\Delta(\alpha) > 0$ for the current α range (see Table A.2). Thus, when $-\frac{1}{2} \leq \alpha \leq -\frac{1}{2}(1-\beta)$, the pure-strategy equilibria are (N,M), (M,N), and (O,O).

Now refer to Table A.3, and let $\Delta(\alpha) = \frac{1}{9}(1-\alpha-\beta)^2 - \frac{1}{2}\beta(1+\alpha)$. $\frac{\partial\Delta}{\partial\alpha} < 0$. Since $\Delta(-\frac{1}{2}(1-\beta)) > 0$ and $\Delta((1-\beta)) < 0$, for an $\alpha^* \in (-\frac{1}{2}(1-\beta), (1-\beta)]$, when $-\frac{1}{2}(1-\beta) < \alpha \leq \alpha^*$, the pure-strategy equilibria are (M,N), (N,M), (N,O), (O,N).

Based on Table A.4, we get when $\alpha > \alpha^*$, the pure-strategy equilibria are (M,M), (M,O), (O,M) and (O,O). □

		Firm 2		
		N	M	O
	N	0, 0	α^2, $(\beta-\alpha)(1+\alpha)$	0, $\beta(1+\alpha)$
Firm 1	M	$(\beta-\alpha)(1+\alpha)$, α^2	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$
	O	$\beta(1+\alpha)$, 0	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$

Table A.2: Payoff Matrix for $-\frac{1}{2} < \alpha \leq -\frac{1}{2}(1-\beta)$

Proof of Lemma 1.5.3.

Proof. For this case $\beta > \frac{1}{3}$.

		Firm 2		
		N	M	O
	N	0, 0	$\frac{1}{9}(1-\alpha-\beta)^2,\ \frac{1}{9}(2+\alpha+\beta)^2$	$\frac{1}{9}(1-\alpha-\beta)^2,\ \frac{1}{9}(2+\alpha+\beta)^2$
Firm 1	M	$\frac{1}{9}(2+\alpha+\beta)^2,\ \frac{1}{9}(1-\alpha-\beta)^2$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$
	O	$\frac{1}{9}(2+\alpha+\beta)^2,\ \frac{1}{9}(1-\alpha-\beta)^2$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$

Table A.3: Payoff Matrix for $-\frac{1}{2}(1-\beta) < \alpha \leq (1-\beta)$

		Firm 2		
		N	M	O
	N	0, 0	$0,\ \alpha+\beta$	$0,\ \alpha+\beta$
Firm 1	M	$\alpha+\beta,\ 0$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$
	O	$\alpha+\beta,\ 0$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$

Table A.4: Payoff Matrix for $\alpha > (1-\beta)$

Let $\Delta(\alpha) = \alpha^2 - \frac{1}{2}\beta(1+\alpha)$. $\Delta(-\frac{1}{2}) > 0$. When $\beta > \frac{1}{3}$, $\Delta(-\frac{1}{2}(1-\beta)) < 0$. Thus, for an $\alpha^{**} \in (-\frac{1}{2}, -\frac{1}{2}(1-\beta)]$, when $-\frac{1}{2} \leq \alpha \leq \alpha^{**}$, the pure-strategy equilibria are (N,M), (M,N), and (O,O) (See Table A.2).

Now refer to Table A.3, and let $\Delta(\alpha) = \frac{1}{9}(1-\alpha-\beta)^2 - \frac{1}{2}\beta(1+\alpha)$. When $\beta > \frac{1}{3}$, $\Delta(\alpha) < 0$ for all α in this range. Thus, based on Table A.4, when $\alpha > \alpha^{**}$, the pure-strategy equilibria are (M,O), (O,M), (M,M), (O,O). □

Proof of Proposition 1.5.4.

Proof. Clearly, when the equilibrium strategies are (M,M), (M,O), (O,M), and (O,O), N is a dominated strategy and M is a weakly dominant strategy. From Lemma 5 and 6, we obtain the conditions that $\beta < \frac{1}{3}$ and $\alpha > \alpha^*$, or $\beta > \frac{1}{3}$ and $\alpha > \alpha^{**}$, such dominance occurs. □

Proof of Proposition 1.5.5.

Proof. From Table A.2, we can see that optional advertising is weakly dominated by mandatory advertising for this range of α. □

Proof of Proposition 1.6.1.

Proof. i) Suppose $p_1^* > p_2^*$, firms' profit functions are

$$\pi_1 = (p_2 - p_1 - \alpha)p_1$$

$$\pi_2 = (1 - p_2 + p_1 + \alpha)p_2 + \beta(1+\alpha) + b(p_1 - p_2)$$

The first order conditions are

$$p_2 - 2p_1 - \alpha = 0$$

$$1 - 2p_2 + p_1 + \alpha - b = 0$$

The equilibrium prices are

$$p_1^* = \frac{1-\alpha-b}{3}, p_2^* = \frac{2+\alpha-2b}{3}$$

If $\max\{-1, -2(1-b)\} < \alpha < -\frac{1}{2}(1-b)$, $p_1^* > p_2^* > 0$ holds. The equilibrium profits are

$$\pi_1^* = \frac{1}{9}(1-\alpha-b)^2, \pi_2^* = \frac{1}{9}\{(2+\alpha)^2 + b^2\} + \beta(1+\alpha) - \frac{1}{9}(5+7\alpha)b$$

ii) When $b \geq \frac{1}{2}$, we get $-1 \leq -2(1-b)$. For $-1 \leq \alpha \leq -2(1-b)$, p_2^* is driven down to 0. Maximizing $\pi_1 = (-p_1 - \alpha)p_1$ yields $p_1^* = -\frac{1}{2}\alpha$, $q_1^* = -\frac{1}{2}\alpha$, and $q_2^* = 1 + \frac{1}{2}\alpha$. $\pi_1^* = \frac{1}{4}\alpha^2$, $\pi_2^* = \beta(1+\alpha) - \frac{1}{2}\alpha b$.

□

Proof of Proposition 1.6.2.

Proof. We focus on the case when $b \geq \frac{1}{2}$.

i) For $-1 \leq \alpha \leq -2(1-b)$, the payoff matrix of firm 1 and firm 2 is as follows:

Since

$$\frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b - \frac{1}{4}\alpha^2 = \frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha(b + \frac{1}{2}\alpha) > 0$$

Table A.5: Payoff Matrix for $-1 \leq \alpha \leq -2(1-b)$ when $b \geq \frac{1}{2}$

		Firm 2		
		N	M	O
	N	0, 0	$\frac{1}{4}\alpha^2,\ \beta(1+\alpha) - \frac{1}{2}\alpha b$	$0,\ \beta(1+\alpha)$
Firm 1	M	$\beta(1+\alpha) - \frac{1}{2}\alpha b,\ \frac{1}{4}\alpha^2$	$\frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b,\ \frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b$	$\beta(1+\alpha) - \frac{1}{2}\alpha b,\ \frac{1}{4}\alpha^2$
	O	$\beta(1+\alpha),\ 0$	$\frac{1}{4}\alpha^2,\ \beta(1+\alpha) - \frac{1}{2}\alpha b$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$

and

$$\beta(1+\alpha) - \frac{1}{2}\alpha b - \frac{1}{2}\beta(1+\alpha) > 0$$

thus "M" is a dominant strategy for both firms. The only equilibrium is (M,M).

ii) For $-2(1-b) < \alpha \leq -\frac{1}{2}(1-b)$, the payoff matrix of firm 1 and firm 2 is as follows:

Table A.6: Payoff Matrix for $-2(1-b) \leq \alpha \leq -\frac{1}{2}(1-b)$

		Firm 2		
		N	M	O
	N	0, 0	$\pi_N^*,\ \pi_M^*$	$0,\ \beta(1+\alpha)$
Firm 1	M	$\pi_M^*,\ \pi_N^*$	$\pi_{MM}^*,\ \pi_{MM}^*$	$\pi_M^*,\ \pi_N^*$
	O	$\beta(1+\alpha),\ 0$	$\pi_N^*,\ \pi_M^*$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$

Note: $\pi_M^* = \frac{1}{9}[(2+\alpha)^2 + b^2] + \beta(1+\alpha) - \frac{1}{9}(5+7\alpha)b$, $\pi_N^* = \frac{1}{9}(1-\alpha-b)^2$
$\pi_{MM}^* = \frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b$.

We show that "M" is a dominant strategy.

First, need to show that $\frac{1}{2}\beta(1+\alpha)-\frac{1}{2}\alpha b > \frac{1}{9}(1-\alpha-b)^2$, or equivalently, $\Delta = \frac{1}{2}\beta(1+\alpha)-\frac{1}{2}\alpha b-\frac{1}{9}(1-\alpha-b)^2 > 0$. Since $\frac{\partial\Delta}{\partial\alpha} = \frac{1}{2}(\beta-b)+\frac{2}{9}(1-\alpha-b) > 0$, it is enough to show that $\Delta \geq 0$ at $\alpha = -2(1-b)$. Since $\frac{1}{2} \leq b \leq 1$, $\Delta|_{\alpha=-2(1-b)} = (2b-1)(1-b+\frac{1}{2}\beta) \geq 0$ holds.

Second, we have shown in the proof of Proposition 2 that $\frac{1}{9}\{(2+\alpha)^2 + b^2\} + \beta(1+\alpha) - \frac{1}{9}(5+7\alpha)b > \beta(1+\alpha)$, so $\frac{1}{9}\{(2+\alpha)^2+b^2\}+\beta(1+\alpha) - \frac{1}{9}(5+7\alpha)b > \frac{1}{2}\beta(1+\alpha)$ holds.

Therefore, "M" is a dominant strategy for both firms. The only equilibrium is (M,M).

iii) For $-\frac{1}{2}(1-b) < \alpha \leq -\frac{1}{2}(1-\beta)$, the payoff matrix of firm 1 and firm 2 is as follows:

Table A.7: Payoff Matrix for $-\frac{1}{2}(1-b) < \alpha \leq -\frac{1}{2}(1-\beta)$

			Firm 2	
		N	M	O
	N	0, 0	α^2, $(\beta-\alpha)(1+\alpha)$	0, $\beta(1+\alpha)$
Firm 1	M	$(\beta-\alpha)(1+\alpha)$, α^2	$\frac{1}{2}\beta(1+\alpha)-\frac{1}{2}\alpha b$, $\frac{1}{2}\beta(1+\alpha)-\frac{1}{2}\alpha b$	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$
	O	$\beta(1+\alpha)$, 0	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha)$, $\frac{1}{2}\beta(1+\alpha)$

Clearly, $(\beta-\alpha)(1+\alpha) > \beta(1+\alpha)$ and $\frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b > \frac{1}{2}\beta(1+\alpha)$.

We also have $\frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b - \alpha^2 = \frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha(b+2\alpha)$. Since $\alpha > -\frac{1}{2}(1-b) \Rightarrow 2\alpha + b > 2b-1 > 0$, then $\frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b > \alpha^2$

Thus "M" is a weakly dominant strategy and there are two equilibria (M,M) and (O,O).

iv) For $-\frac{1}{2}(1-\beta) < \alpha \le 0$, the payoff matrix of firm 1 and firm 2 is as follows:

Table A.8: Payoff Matrix for $-\frac{1}{2}(1-\beta) < \alpha \le 0$

			Firm 2	
		N	M	O
	N	0, 0	$\frac{1}{9}(1-\alpha-\beta)^2,\ \frac{1}{9}(2+\alpha+\beta)^2$	$\frac{1}{9}(1-\alpha-\beta)^2,\ \frac{1}{9}(2+\alpha+\beta)^2$
Firm 1	M	$\frac{1}{9}(2+\alpha+\beta)^2,\ \frac{1}{9}(1-\alpha-\beta)^2$	$\frac{1}{2}\beta(1+\alpha)-\frac{1}{2}\alpha b,\ \frac{1}{2}\beta(1+\alpha)-\frac{1}{2}\alpha b$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$
	O	$\frac{1}{9}(2+\alpha+\beta)^2,\ \frac{1}{9}(1-\alpha-\beta)^2$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$	$\frac{1}{2}\beta(1+\alpha),\ \frac{1}{2}\beta(1+\alpha)$

Note that $\beta \ge b \ge \frac{1}{2}$. Let $\Delta(\alpha) = \frac{1}{2}\beta(1+\alpha) - \frac{1}{9}(1-\alpha-\beta)^2$, then $\frac{\partial\Delta}{\partial\alpha} = \frac{1}{2}\beta + \frac{2}{9}(1-\alpha-\beta) > 0$. Since $\Delta(-\frac{1}{2}(1-\beta)) = \frac{3}{4}\beta - \frac{1}{4} > 0$ and $\Delta(0) = \frac{13}{18}\beta - \frac{1}{9}\beta - \frac{1}{9} > 0$, thus $\frac{1}{2}\beta(1+\alpha) - \frac{1}{2}\alpha b > \frac{1}{2}\beta(1+\alpha) > \frac{1}{9}(1-\alpha-\beta)^2$. Thus "M" is a weakly dominant strategy and there are two equilibria (M,M) and (O,O).

In summary, when $b \ge \frac{1}{2}$, "M" is a weakly dominant strategy for both firms. For $\alpha < -\frac{1}{2}(1-b)$, the unique equilibrium is (M,M) ; for $\alpha \ge -\frac{1}{2}(1-b)$, the equilibria are (M,M) and (O,O). So (M,M) is always one of the equilibria.

□

Appendix B

Proofs of Results in Chapter 2

Proof of Lemma 2.3.1.

Proof. First, it is never optimal for a bidder to bid higher than his/her true unit-valuation. If one can win by bidding the true unit valuation, then there is nothing to gain by bidding higher. Otherwise, winning would necessarily result in a negative payoff, since the bidder would have to pay a price above his valuation. Second, for bidder j with unit-valuation v, bidding less than v is weakly dominated by bidding v. Let s_{-j} denote the highest score among the remaining bidders. If j can win with $b < v$, it does so as well with $b = v$ and without paying more (since by the second-score rule, j pays for his/her yield at the unit price $\frac{s_{-j}}{\hat{q}_j}$). If j does not win with b, it will get zero payoff. It can get at least the same amount by bidding his/her true unit-valuation. Thus bidding one's true valuation is the weakly dominant strategy. □

Proof of Proposition 2.4.1.

Proof. From (2.3), if $q_1 > q_0$, $\Delta(k_2)$ increases in k_2. If $Delta(0) \geq 0$, it is optimal for the advertiser to stretch and $k^* = 0$. If $\Delta(0) < 0$, there exists k^*

such that $\Delta(k^*) = 0$. So, for $k_2 \geq k^*$, $\Delta(k_2) \geq 0$. Thus it is optimal for the advertiser to stretch. Similar analysis holds for the case with $q_1 < q_0$. $\square$

Proof of Proposition 2.4.2.

Proof. From (2.3), if $q_1 > q_0$, it is easy to verify that $\Delta(q_2)$ increases in q_2; and there must exist q_2^* such that, for $q_2 \geq q_2^*$, $\Delta(q_2) \geq 0$. As a result, for any $q_2 \geq q_2^*$, it is optimal for the advertiser to stretch. Due to the monotonicity of $\Delta(q_2)$, the bigger q_2, the more gain from stretching. For the case with $q_1 < q_0$, increase in q_2 has negative impact on the new market but positive impact on the primary market; so there is no conclusive effect. $\square$

Proof of Proposition 2.4.3.

Proof. Notice that $\Delta'(\epsilon) = 0.5q_1 \int_0^{v_i} \left[H_1'(s)|_{(q_2+\epsilon)t} - H_1'(s)|_{(q_2-\epsilon)t}\right] tdt$. In the case with convex $H_1(s)$, $H_1'(s)|_{(q_2+\epsilon)t} > H_1'(s)|_{(q_2-\epsilon)t}$ since $H_1'(s)$ increases in s by the definition of convexity. So $\Delta'(\epsilon) > 0$ and thus Δ increases in ϵ. In the case with concave $H_1(s)$, Δ decreases in ϵ. $\square$

Proof of Proposition 2.5.1.

Proof. The proof is similar to that of Proposition 2.5.1. $\square$

Proof of Proposition 2.5.2.

Proof. Proof: In the case with $H_1(s) = H_2(s)$, $\Delta = 0.5(k_2q_2 - q_1)D_2(q_1, q_2)$. Denote q_2^* as the solution to $k_2q_2 - q_1 = 0$ and q_2^{**} as the solution to $D_2(q_1, q_2) = 0$. Let $q_2^U = max\{q_2^*, q_2^{**}\}$. For any $q_2 > q_2^U(q_1)$, $\Delta < 0$ since $k_2q_2 - q_1$ increases in q_2 and $D_2(q_1, q_2)$ decrease in q_2. Similarly, $q_2^L = min\{q_2^*, q_2^{**}\}$, for any $q_2 < q_2^L(q_1)$, $\Delta < 0$. In both cases, it is optimal for the advertiser not to stretch. □

Proof of Proposition 2.5.3.

Proof. Notice that $\Delta'(\epsilon) = 0.5(q_1 - k_2q_2)\int_0^{v_i}\left[H_1'(s)|_{(q_2+\epsilon)t} - H_1'(s)|_{(q_2-\epsilon)t}\right] tdt$. In the case with convex $H_1(s)$, $H_1'(s)|_{(q_2+\epsilon)t} > H_1'(s)|_{(q_2-\epsilon)t}$ by the definition of convexity. So, if $q_1 > k_2q_2$, $\Delta'(\epsilon) > 0$ and thus Δ increases in ϵ. If $q_1 < k_2q_2$, Δ decreases in ϵ . □

Appendix C

Proofs of Results in Chapter 3

Proof of Proposition 3.2.1.

Proof. Equation (2) - Equation (1) is positive, thus part 1 follows.

Since $I_H > I_L$, from Equations (1) and (2), we get $z_k^H < z_k^L$ for all k.

From Equation (2), part 3 follows immediately. □

Proof of Lemma 3.2.2.

Proof. Equation (4) can be rewritten as $\forall j \in \{L, H\}$,

$$\begin{cases} z_{k+1}^L - 2\pi_L z_k^L - \pi_H \underline{z} - \pi_H \Delta - \pi_L I_L - p_k(C-1) - \pi_L p_{k-1}(C-1) = 0, & z_k^H \leq \underline{z} \\ z_{k+1}^L - 2z_k^L - I_L - p_k(C_{k+1}-1) - p_{k-1}(C_k - 1) = 0, & z_k^H \geq \underline{z}. \end{cases}$$

Since $C_k > 1\ \forall k$, we get $\forall j \in \{L, H\}$,

$$z_{k+1}^j > 2z_k^j + I_j \tag{C.1}$$

$$z_{k+1}^j > 2\pi_L z_k^j + \pi_H \underline{z} + \pi_H \Delta + \pi_L I_j \tag{C.2}$$

From Equation (2), we have

$$\overline{z} - 2z_n^L - I_L + \pi_H \Delta - (C_n - 1)p_{n-1} = 0$$

From here we derive the following condition:

$$\overline{z} > 2z_n^L + I_L - \pi_H \Delta$$

From Conditions (C.1), it follows that

$$\begin{aligned}
\overline{z} &> 4z_{n-1}^L + 3I_L - \pi_H \Delta \\
\overline{z} &> 8z_{n-2}^L + 7I_L - \pi_H \Delta \\
&... \\
\overline{z} &> 2^N z_{n-(N-1)}^L + (2^N - 1)I_L - \pi_H \Delta
\end{aligned}$$

And by Condition (C.2),

$$\begin{aligned}
\overline{z} &> 4\pi_L z_{n-1}^L + (2\pi_L + 1)I_L + 2\pi_H \underline{z} + \pi_H \Delta \\
\overline{z} &> 8\pi_L z_{n-2}^L + (4\pi_L + 3)I_L + 4\pi_H \underline{z} + \pi_H \Delta \\
&... \\
\overline{z} &> 2^N \pi_L z_{n-(N-1)}^L + (2^{N-1}\pi_L + 2^{N-1} - 1)I_L + 2^{N-1}\pi_H \underline{z} + \pi_H \Delta
\end{aligned}$$

By the assumption that $\overline{z} < min\{2^N \underline{z} + (2^N - 1)I_L - \pi_H \Delta, (2^{N-1}\pi_L + 2)\underline{z} + (2^{N-1}\pi_L + 2^{N-1} - 1)I_L + \pi_H \Delta\}$, we get $\underline{z} > z_{n-(N-1)}^L$.

We conclude that for any Nash equilibrium, given $\overline{z} < min\{2^N \underline{z} + (2^N - 1)I_L - \pi_H \Delta, (2^{N-1}\pi_L + 2)\underline{z} + (2^{N-1}\pi_L + 2^{N-1} - 1)I_L + \pi_H \Delta\}$, at most the N firms producing products of qualities n, $n-1$, $n-(N-1)$,..., hold positive market shares. □

Proof of Proposition 3.2.3.

Proof. The proof for Proposition 2 follows immediately from that of Lemma 3.2.2. □

Proof of Lemma 3.2.4.

Proof. Since (10) is increasing and (11) is decreasing, two equations intersect if (11) lies above $\underline{z}$ at $z_1^L = -I_L$. From (11), at $z_1^L = -I_L$

$$z_2^L = \frac{1}{2}\left[\overline{z} - I_L + (\overline{z} - \underline{z})\frac{\pi_H}{\pi_L}\right]$$

If the above is greater than $\underline{z}$, then the condition is satisfied. Thus, we have $\overline{z} \geq (2\pi_L + \pi_H)\underline{z} + \pi_L I_L$. We can also verify that the second order conditions of the profit functions are satisfied, such that both (10) and (11) are concave in p_1 and p_2, respectively, with the other price fixed.

If the FOCs are to intersect in Region 3, Equation (11) at $z_1^L = \underline{z}$ must lie above $\underline{z} + \pi_H\Delta + (\underline{z} + I_L)(V - 1 + \pi_L)$. From here we derive that

$$\begin{aligned} \frac{1}{2}\left[\overline{z} + \pi_H\Delta - I_L - (\underline{z} + I_L)(V-1)\right] &\geq \underline{z} + \pi_H\Delta + (\underline{z} + I_L)(V - 1 + \pi_L) \\ V &\leq \frac{\overline{z} - \pi_H\Delta + \underline{z} + 2I_L - 2\pi_L(\underline{z} + I_L)}{3(\underline{z} + I_L)} \end{aligned}$$

Given the condition from Proposition 1, $\overline{z} < (2\pi_L + 2)\underline{z} + (2\pi_L + 1)I_L) + \pi_H\Delta$, we have $V < 1$. The above condition cannot be satisfied, because $V > 1$; Thus, the equilibrium does not occur in Region 3 or above, which also includes Regions 4 and 7. The two firms' FOCs will intersect in Region 1, 2, 5, or 6, given the condition $2\pi_L + \pi_H)\underline{z} + \pi_L I_L < \overline{z} < (2\pi_L + 2)\underline{z} + (2\pi_L + 1)I_L) + \pi_H \Delta$. And a unique equilibrium exists under this condition. □

Proof of Proposition 3.2.5.

Proof. From the sets of FOCs (10) and (11), we can easily derive z_1^* and z_2^* for Regions 1, 2, 5 and 6, and then directly determine equilibrium prices and profits from Equations (3), (4), (8) and (9). The expressions are listed in Tables 1 & 2. Since in Region 1, $z_2^{L*} - \Delta \leq \underline{z}$, we have $\overline{z} \leq (2\pi_L - \pi_H)\underline{z} + \pi_L I_L + 3\pi_L \Delta$; for Region 2, $\underline{z} \leq z_2^{L*} - \Delta$, solve the inequality will give us $\overline{z} \geq 2\underline{z} + I_L + (3\pi_L + \pi_H)\Delta$. And the range between them indicates Region 5. For Region 6, we can easily find the condition for V from the boundary points of Region 2 and 3. □

Bibliography

[1] Aghion, P., N, Bloom, R. Blundell, R. Griffith, and P. Howitt (2005), "Competition and Innovation: An Inverted-U Relationship", *The Quarterly Journal of Economics*, pp. 701-728.

[2] Aghion, P. and G. Saint-Paul (1998), "Virtues of Bad Times: Interaction between Productivity Growth and Economic Fluctuations," *Macroeconomic Dynamics*, 2(3), pp. 322-344.

[3] Andrew, J., H. Sirkin, and J. Butman (2006), *Payback: Reaping the Rewards of Innovation*, Boston, MA: Harvard Business School Publishing.

[4] Barlevy, G. (2007), "On the Cyclicality of Research and Developmen", *American Economic Review*, 97(4), pp. 1131-1164.

[5] Barnes, B. (2009), "Disney Plans to Cut Costs and Release Fewer DVDs," *The New York Times*, Feb. 4, 2009.

[6] Bloom, N., R. Griffith, and J. Van Reenen (2002), "Do R&D Tax Credit Work? Evidence from a Panel of Countries 1979-1997", *Journal of Public Economics*, 85, pp. 1-31.

[7] Butters, G.R. (1977), "Equilibrium Distributions of Sales and Advertising," *The Review of Economic Studies* 44(3), pp. 465-491.

[8] Caballero, R.J. and M.L. Hammour (1994), "The Cleansing Effect of Recessions," *The American Economic Review*, 84(5), pp. 1350-1368.

[9] Cabral, L.M.B. (2000), "Stretching firm and brand reputation," *RAND Journal of Economics*, 31(4), pp. 658-673.

[10] Chen, J., D. Liu, and A.B. Whinston (2008), "Auctioning Keywords in Online Search," *Journal of Marketing*, forthcoming.

[11] Chen, Y. (1997), "Equilibrium Product Bundling." *The Journal of Business* 70(1), pp. 85-103.

[12] Choi, Y.C., D. Hummels, C. Xiang (2009), "Explaining Import Quality: the Role of the Income Distribution", *Journal of International Economics*, forthcoming.

[13] Esteban, L., A. Gil, J. M. Hernandez (2001), "Informative Advertising and Optimal Targeting in a Monopoly," *Journal of Industrial Economics* 49(2), pp. 161-180.

[14] Falk, M. (2004), "What Drives Business R&D Intensity Across OECD Countries", WIFO Working Paper, No. 236/2004.

[15] Fan, M., S. Kumar, and A.B. Whinston (2008), "Selling or Advertising: Strategies for Providing Digital Media Online," *Journal of Management Information Systems* 24(3), pp. 143-166.

[16] Feng, J, H. Bhargava, and D. Pennock (2007), "Implementing Sponsored Search in Web Search Engines: Computational Evaluation of Alternative Mechanisms," *Informs Journal on Computing*, 19(1), pp. 137-148.

[17] Foellmi, R. and J. Zweimuller (2006), "Income Distribution and Demand-Induced Innovation", *Review of Economic Studies*, 73, pp. 941-960.

[18] Garcia-Quevado, G. (2004),"Do Public Subsidies Complement Business R&D? A Meta-Analysis of Econometric Evidence", Kyklos, 57, pp. 87-102.

[19] Geng, X., M. B. Stinchcombe, and A. B. Whinston. "Product Bundling," *Handbook of Economics and Information Systems*, Terrence Hendershott ed., Elsevier, 2006.

[20] Gorelick, D. (2009), "Obama to Seek Record Levels of Research Investment", *America.gov*, April 27, 2009.

[21] Gregg, D.G. and Scott, J.E. (2006), "The role of reputation systems in reducing online auction fraud," *International Journal of electronic Commerce*, 10(3), pp. 95-120.

[22] Grossman, G.M. and C. Shapiro (1984), "Informative Advertising with Differentiated Products," *The Review of Economic Studies* 51(1), pp. 63-81.

[23] Hall, B.H. and J. Van Reenan (2000), "How Effective are Fiscal Incentives for R&D? A Review of the Evidence", *Research Policy*, 29, pp. 449-469.

[24] Iyer, G., D. Soberman, and J.M. Villas-Boas (2005), "The Targeting of Advertising," *Marketing Science* 24(3), pp. 461-476.

[25] Johnson, M, C. Christensen, and H. Kagermann (2008), "Reinventing Your Business Model," *Harvard Business Review*, Dec. 2008, pp. 51-59.

[26] Klein, B. and Leffler, K. B. (1981), "The role of market forces in assuring contractual performance," *Journal of Political Economy*, 89(4), pp. 615-641.

[27] Kreps, D.M. and Wilson, R. (1982), "Reputation and imperfect information," *Journal of Economic Theory*, 27(2), pp. 253-279.

[28] Lahaie, S. (2006), "An Analysis of Alternative Slot Auction Designs for Sponsored Search," *Proceedings of the 7th ACM Conference on Electronic Commerce*, June, 2006, pp. 218-227.

[29] Lee, T. and L.L. Wilde (1980),"Market Structure and Innovation: A Reformulation", *Quarterly Journal of Economics*, 94(2), pp. 429-436.

[30] Liu, D. and J. Chen (2006), "Designing Online Auctions with Past Performance Information," *Decision Support Systems*, 42(3), pp. 1307-1320.

[31] Liu, D., J. Chen, and A.B. Whinston (2008), "Ex-Ante Information and the Design of Keyword Auctions," *Information Systems Research*, forthcoming.

[32] Loury, G. (1979), "Market Structure and Innovation", *Quarterly Journal of Economics*, 93(3), pp. 395-410.

[33] MacInnes, I., Y. Li and Yurcik, W. (2005), "Reputation and dispute in eBay transactions," *nternational Journal of Electronic Commerce*, 10(1), pp. 27-54.

[34] Mervis, J. (2009), "Amid the Gloom, Researchers Prepare for a Boom in Funding," *Science*, 323, March 6, 2009, pp. 1274-1275.

[35] Milgrom, P. and Roberts, J. (1982), "Predation, reputation, and entry deterrence," *Journal of Economic Theory*, 27(2), pp. 280-312.

[36] Pepall, L.M. and Richards, D.J. (2002), "The simple economics of brand stretching," *Journal of Business*, 75(3), pp. 535-552.

[37] Rigby, D. K., K. Gruver, and J. Allen (2009),"Innovation in Turbulent Times", *Harvard Business Review*, June 2009, pp. 79-86.

[38] Shapiro, C. (1983), "Premiums for high quality products as returns to reputations," *Quarterly Journal of Economics*, 98(4), pp. 659-679.

[39] Schumpeter, J.A. (1918),*The Crisis of the Tax State*, in Economics and Sociology (Schumpeter 1991). p. 113.

[40] Segal, I. and M. Whinston (2007), "Antitrust in Innovative Industries", *American Economic Review*, 97(5), pp. 1703-1730.

[41] Shaked, A. and J. Sutton (1982). "Relaxing Price Competition Through Product Differentiation", *Review of Economic Studies* (49), pp. 3-13.

[42] Stelter, B. (2008), "Web Sites Formula for Success: TV Content With Fewer Ads," *The New York Times*, Oct. 28, 2008.

[43] Tadelis, S. (1999), "What's in a name? Reputation as a tradeable asset," *American Economic Review*, 89(3), pp. 548-563.

[44] Telser, L.G. (1980), "A theory of self-enforcing agreements," *Journal of Business*, 53(1), pp. 27-44.

[45] "The Obama-Biden Plan", *Change.gov*, *http://change.gov/agenda/economy_agenda/*.

[46] Weber, T.A. and Z. Zheng (2007), "A Model of Search Intermediaries and Paid Referrals," *Information Systems Research*, 18(4), pp. 414-436.

[47] Wernerfelt, B. (1988), "Umbrella branding as a signal of new product quality: An example of signalling by posting a bond," *RAND Journal of Economics*, 19(3), pp. 458-466.

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