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THE TEXAS MATHEMATICS TEACHERS' BULLETIN

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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Volume XII, Number 3

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Charles Donnell Rice, M.S., Vanderbilt; D.Sc., University of Geneva, Switzerland, and the founder and editor of the Texas Mathematics Teachers' Bulletin, died May 28, 1928.

THE READJUSTMENT OF COLLEGIATE EDUCATION

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Every one is aware that a great change is now being attempted in the organization of American educational institutions. The fundamental causes of the changes and the advantages or disadvantages are subjects for warm debate among educators. The reorganization has brought about the junior high school, the first of which was established in Berkeley, Calif., in 1910. So rapidly has this institution spread that there are today over thirteen hundred such institutions with many new ones every year. Courses in junior high-school work are given in most of the larger universities and teachers are specializing in this stage of teaching. Similarly has arisen the junior college. The first two to be established were in Peoria and Chicago, Ill., in 1896 and 1897. These were privately endowed schools. In 1902 the first public junior college was opened in connection with the township high school at Joliet, Ill. Since these early beginnings the movement for junior colleges has grown rapidly. There are today about one hundred and ten public junior colleges, 25 state, and 175 to 200 private institutions. In addition, a score or more of the leading universities of the continent have divided their arts colleges into two divisions and are making the first two years a more or less distinct unit in curriculum, administration, and discipline. Moreover, two of our most advanced institutions, Johns Hopkins and Leland Stanford Junior universities, have decided to eliminate the freshman and sophomore years entirely from their organizations.

Changes so profound and so widespread must necessarily have some very powerful and convincing causes. Moreover, we ought to open our eyes to see what readjustments these changes are apt to produce elsewhere. Any one with experience in the reconstructing of an old house, an old garment, an old piece of furniture, knows that when an alteration is made it affects the original in numerous ways which were not noticed at first. So it is with our complicated school mechanism or institutions. It is desirable for us to sketch briefly what has happened and why it has happened. Then we may be in a better position to guess what should be done about it.

From 1820 to 1860 educators were busy constructing our present system. They organized the eight-year elementary school, the four-year high school, and they already had the four-year college. At first these institutions were not articulated but each was wholly unrelated to the other. The new step was to place the high school on top, so to speak, of the eight-year elementary school. The second step was to require a four-year high-school course as preparatory to entrance to college. By 1890 this system of articulated schools was fairly well accomplished. Now the educators looked upon their tower and considered it a beautiful structure. But there was one who saw it was not good.

In 1889 President Charles W. Eliot, President of Harvard, in an epochal address to the Department of Superintendents compared the product of our new American system with the product of the European schools. This table will exhibit the comparison:

	American System	European System
Pre-school	6 years	6 years
Elementary School	8 years	3 years
Secondary School	4 years	9 years
College	4 years	
Professional	3–5 years	3 years
	25–27	21

The American professional man had to spend from four to six years longer in preparing to enter upon his professional career. Moreover, he did not know any more apparently than his European cousin. President Eliot declared that some years were lost in pre-college education, and that the schools must shorten their courses and eliminate the unnecessary dead wood of learning given children in elementary school and high school.

While this process of organization was going on from 1850 to 1890 the American college had changed very greatly in every way. It was, in fact, trying to become a university after the European model. Now in this movement the one outstanding fact of deepest significance was this: the age of college students advanced about two years. In place of entering college at from 15 to 17, they entered from 17 to 19.

The result of President Eliot's criticism was electrical. I do not know of any one statement in American education which caused more discussion or led to so much study and to such far-reaching results. School men began to study the elementary school to see what could be done to shorten the course, especially for the brighter students. A national committee took up the study of secondary education, another committee studied college entrance requirements. The university leaders were equally agitated and began to seek a change in organization. Many advocated a three-year college course. President Harper of Chicago, President Jesse of Missouri, President Jordan of Stanford, and President James of Illinois advocated the junior college.

Another event of far-reaching importance in American education was the founding of Johns Hopkins University in 1876 as a distinct graduate school. Some little graduate work had been done before this time, but now for the first time the American people saw a genuine university of the highest learning. The effect of this new institution was really profound. It made clear to the educators of this country that the four-year arts colleges and universities were not universities in the genuine sense of that term. They were but a poor mixture of secondary work with a slight university flavor.

One may take the first decade of our present century as the great turning point in our educational history. At this time the weaknesses in the old 8-4-4 school system became fully apparent and everywhere an effort was made to correct the situation. But what should be the basis for the building of a new organization? Whether rightly or wrongly they took as the new principle the science of human development, the stages of child growth. The age of puberty was seized upon as the time for elementary school training to end, and for the higher or secondary school training to begin. The result of this was the establishment of the junior high school.

Among other things, its object was to speed up the education of the child.

As a result of this new view many public schools reorganized on the 6-3-3 or the 6-2-4 plan. Or as we had only seven grades in Texas, it was the 6-2-3 or the 6-3-2 plan.

The movement for saving time, set in motion by President Eliot, had also an effect upon the internal organization of the school. The lock-step system was abandoned, bright students were pushed forward as rapidly as they could go, summer schools speeded up their work; non-essentials were eliminated from the courses, and many students began to enter the primary school a year earlier. The consequence of all these developments is apparent. The age of high-school graduation is a year or more earlier than it was twenty-five years ago. The girls are entering college at 16 and 17; the boys at 17 and 18. The significance of this fact cannot be exaggerated. Our young people are now-a-days getting through high school at an age so immature that their parents refuse to send them to college. For a time they sent them back to the high schools to do "post graduate work." But that did not prove wholly satisfactory. The parents were afraid to trust their children to the universities where there were no dormitories, little supervision, personal liberty which the youth did not know how to use wisely, and method of work which they did not know how to profit by.

At this juncture the public junior colleges arrived. They met the needs of a large group who could not go, or whose parents would not let them go, to college.

The addition of the junior college to a public school system complicates the patchwork still more, for the junior college is a two-year institution. As a result of this addition we have in Texas the following types of organization:

8-4-2
6-2-3-2
6-3-2-2

It is apparent to anyone that our school system is falling into an atomistic condition. No institution of two years can hope to make a very deep impression upon the young. For a number of reasons the authorities on the junior college are now advocating that the last two years of the high school and the first two years of the college should be organized together into a new four-year institution. A number of private junior colleges are now practically on this basis, some of State junior colleges are also on this basis, and now we find the public junior colleges of Texas moving in the same direction. This is true at Hillsboro and Edinburg. In these two places the new type of organization is the 5-4-4. This is the most progressive step taken for some time in American education. As I have said, it has the unqualified endorsement of the leading students of school administration everywhere on the continent. The Texas experiment will be watched with the keenest interest.

The basic view of reorganization today may be summed up in the following statements:

1. The dividing point between elementary and secondary education is puberty, normally from 11 to 13 years of age.

2. The freshman and sophomore work is distinctly secondary education.

3. The secondary-school period from the twelfth to the twentieth year should be divided into two stages, early and middle adolescence.

4. University education begins at the present junior year; it signifies specialization and research on the one hand, or strictly high-class professional work on the other.

It would seem that American education is now in process of building its own system, based upon psychological and scientific foundations: The elementary and junior school up to 15 or 16 for all children, the junior college for those who wish to go farther, and the university for professional training.

HIGH-SCHOOL PREPARATION FOR COLLEGE MATHEMATICS

J. E. NELSON

San Antonio Junior College

Colleges continue to complain that too many of their freshmen are unprepared to do college work. It sounds like the same old story of the college teacher blaming the high school, the high school complaining to the grammar school, and so on down to the primary grades. One is led to wonder whether the work done in the different units of our school system could be so organized and correlated that the average student would experience no greater shock when passing from one unit to the next higher, than he would experience on passing from one grade to the next higher in the same unit. It is certain that such correlation does not as yet obtain.

College teachers in all the departments complain that the average freshman comes to college poorly prepared to do college work, and in many instances without having learned the first principles of how to study effectively. More than this, it seems certain that many freshmen have little idea of the thorough preparatory training in high school or the close application after entering college that are necessary if they are to do creditable college work.

In the case of mathematics the vast majority of college freshmen find great difficulty in doing creditable work for the reason that they have a very meager knowledge of algebra. It has been two years since they studied this subject. The plane geometry of the junior year of high school gave little chance for algebra review. No mathematics was studied in the senior year of the high school. The result, therefore, is unsatisfactory even when algebra is the first mathematics undertaken in college, and still more unsatisfactory if trigonometry and analytics are undertaken without the algebra review. The college student's progress in trigonometry and analytics is, other things being equal, generally directly proportional to his knowledge of algebra.

A sufficient knowledge of plane geometry necessary in the study of trigonometry and analytics is found among a large per cent of even those students who present only the three required units in mathematics for college entrance. This is true because only a comparatively few theorems are used, and also perhaps because geometry was the last mathematics subject studied in high school.

The most satisfactory students in college mathematics are those who have done three and one-half or preferably four units of mathematics in the high school. Their trigonometry done during the fourth year of high school served as a review of algebra. Then, too, they are usually students who like the subject. They explain why they took so much mathematics in the high school by saying that it was easy for them. A freshman class composed of such students does so much better work than a class composed of those who take mathematics only because it is required for a degree that it is practically impossible to require the same standard of work for credit from both classes of students. These students presenting more than three entrance units in mathematics constitute not more than 15 per cent of the total enrollment. The mortality rate among the other 85 per cent in any college that pretends to maintain proper standards is so alarmingly high as to make the need of some adjustments plainly evident.

Naturally, the question arises as to what those adjustments should be and how they may best be brought about. I am convinced that the content of the mathematics course of study of the average high school, if satisfactorily completed, will prepare the high-school student to continue his study in any college. If the four units are done in the high school, the order in which these are usually given, namely, algebra in the eighth and ninth, plane geometry in the tenth, and solid geometry and trigonometry in the eleventh grade, seems satisfactory. However, for the approximately 85 per cent who do only three years of mathematics in the high school, the above arrangement should be modified so as to provide for mathematics in the senior year of the high school. Taking the three prescribed units during the last three years of high school but in the same order as above stated would almost surely give better results from the standpoint of preparation for college. Perhaps the arrangement with plane geometry sandwiched between the two years of algebra and giving the second-year algebra in the senior year of the high school would be still more effective.

In addition to effective teaching and the proper arrangement of suitable courses, no doubt much good could be accomplished by an organized effort thoroughly to acquaint the high-school students with the kind of difficulties they are to meet in the first year at college. Most colleges furnish, either voluntarily or on request, a transcript of grades made by its freshmen to the different high schools from which these freshmen have come. Students enrolled in high schools situated near junior or senior colleges have ample opportunity to hear of these difficulties, if not in their own homes, then from their intimate friends who have entered college. The reaction on the high-school teaching due to the presence of a well-standardized college in the same district is always wholesome. In the case of public junior colleges there seems to be no reason why the same coöperation and understanding should not obtain between the high school and college faculties as already obtain between those who teach in the high school and those who teach in the grammar school. With the establishment of more and more public junior colleges the possibilities of narrowing the gap between the high school and college seem real.

BETTER PREPARATION FOR COLLEGE MATHEMATICS

(MISS) PHYLLIS HENRY

Hillsboro Junior College

In the Hillsboro High School, algebra is taught in the eighth and ninth grades, and plane geometry in the tenth, while solid geometry is optional in the eleventh.

It appears that students have forgotten most of their high-school algebra by the time they become first-year college students. I am no longer surprised when I find that students have forgotten decimals; adding, multiplying, and dividing the simplest fractions in arithmetic, and all rules of signs in algebra. I devote the first six weeks of freshman mathematics to a review of Wentworth's *New School Algebra* before beginning trigonometry and analytics. Copies of discarded high-school algebras are placed in the library for their use.

All types of students enter freshman mathematics, but I find that students who are not willing to work usually drop mathematics in about six weeks. Those who stay in the class are ordinarily willing to work. They have no time to play while they are reviewing their algebra. If I find a student who has not quite forgotten everything, I put him to work on the more difficult problems and examine him closely to see if he really understands the subject. I find the review sets that are given at the end of the chapters in the algebra very beneficial to the students; also the following chapters: Theory of Exponents, Radical Expressions, Imaginary Expressions, Quadratic Equations, Simultaneous Quadratics, and Graphs.

As long as eleventh-grade mathematics is optional, there will be many in the freshman class who have not had mathematics for a year and no one will have had algebra for two years. This makes mathematics very difficult for college students. A mistake often made in high schools is giving business arithmetic as a required subject. Students who are intending to go to college could be taking a half a year of advanced arithmetic and be benefited. Business arithmetic belongs in the commercial department. It is too simple for the eleventh grade.

Too often in Texas schools, mathematics is given to the coach, or to someone who cannot teach anything else, or to a principal who has other duties to perform. It seems to be generally thought that mathematics is easy to teach. Mathematics certainly must be thoroughly understood before it can be taught successfully. I am very happy to state that our mathematics department in Hillsboro High School has been strengthened in the last two years. We have a very competent woman teaching algebra and a man teaching geometry who has a master's degree and who majored in mathematics.

If a student has had trigonometry in high school in my mind, it means no more than having had ninth-grade algebra. In both cases it can be followed by college algebra and trigonometry in college. I am not in favor of teaching trigonometry in high school for I have had students who have no conceptions of functions of angles greater than an acute angle. They cannot give a graphic representation of the trigonometric functions for they do not know the value of the functions of 30, 45, 60 in any of the four quadrants. Of course it is very essential that they know trigonometry for college mathematics that follows. It is not the fault of the high-school teacher for he has all types of students in his class and he cannot afford to fail too large a per cent of his students. The high-school teacher therefore does not require as much as a college teacher must.

I would suggest half a year of algebra or advanced arithmetic, and half a year of solid geometry for the eleventh grade, followed by the regular freshman mathematics course of college algebra, trigonometry, and analytical geometry in college.

PREPARATION FOR COLLEGE MATHEMATICS

DEAN CHARLES PURYEAR

Texas Section of the American Mathematical Society, College Station, Texas

The college must deal with the students it admits, not with students having the ideal preparation for college work. In Texas colleges, as in many others, almost all the freshmen are admitted on certificate of graduation from an accredited school, presenting fifteen high-school units including two in algebra and one in plane geometry. In the public schools the algebra is given in the first two years of the high school, and plane geometry in the third year.

Many boys in the high school fail to become thorough in algebra, and even those who do well in the subject naturally forget nearly all about it in the two years preceding admission to college. This fact makes it necessary to give in the freshman year a review of the more important topics in elementary algebra. This decreases the time available for strictly college algebra and makes it difficult to give a satisfactory course in the usual time of one term.

This college, I think, is exceptional in that in all our engineering courses we devote to algebra, not one term but two, the course being given three hours a week throughout the year, accompanied in the first term by trigonometry and in the second by analytics. This has been made possible by the fact that the members of the engineering staff recognize the fact that thoroughness in algebra is essential to success in engineering studies.

I favor the plan of dividing the freshman class into sections on the basis of ability in mathematics. This division may be made on the basis of the results of tests given within the first few days of the session. These tests need be standard tests but may be devised to meet the end in view. If the freshmen be divided into three groups, those in the lowest group should be required, in order to receive credit for the course, to reach a certain minimum attainment; those in the higher group should be encouraged to do considerably more, and the effort should be made to arouse in them enthusiasm for the work.

Then for the first week or two of the session the pace should be rather slow, so as to give the student time to adapt himself to new conditions and new standards. During this period the instructor should devote more time than usual to explaining and demonstrating the work to the class.

It is important that the instructor determine in advance the manner in which he will present the subject, and that he enter the classroom thoroughly prepared at every point. With some assignments it may be wise to send students to the blackboard the first thing; with others it may be wise for him to demonstrate the more difficult parts of the assignment at the beginning of the hour.

The ability to preserve a proper balance between explanations on his part and drill work on the part of the student is a mark of the good instructor. In his explanations the instructor should be clear; he should use mathematical language with exactness; he should seize upon the essential points of the lesson and bring them out clearly.

Assiduous practice at the blackboard is in my opinion one of the best ways of keeping the student up to his work and of gauging his progress. In his blackboard work the student will usually write out the necessary equations and solve them, and will explain them orally. From time to time however he should be required to hand in an exercise in the formal style of a proposition in geometry or of a model solution of a problem in some other subject, with all explanations written in as if he were preparing it for a textbook. As a guide, examples of such models may be pointed out in the text. Slouchy work should not be accepted.

Throughout the course the effect should be made to help the student to acquire the power to think clearly; to analyze a problem, discriminating between the essential and the non-essential elements; to draw logical conclusions, and to express his thoughts clearly. Upon completing his course the student should have some appreciation of the fact that mathematics is one of the great fields of human knowledge, and that it is the foundation for many of the other sciences, pure and applied, and that the progress of the world depends largely upon the application of mathematics.

Under the current practice of admitting students on certificate of graduation from the high school, it always happens that a considerable number are admitted who are not prepared for the freshman course in mathematics.

We have this year followed the plan of the University of Texas in assigning such students to a non-credit course in algebra with the view of preparing them to take up college algebra later on. I think this plan promises good results in giving the poorly prepared student the opportunity to correct his weakness or to show that he is not qualified for further work in mathematics.

I would like now to digress from the text and to say that I think it desirable that discussions of this nature should lead to some tangible result. You are all aware of the general complaint that many students enter college with an unsatisfactory preparation in mathematics, particularly in algebra. The question arises, Can anything be done to better the situation? I suggest that consideration be given to the question of transferring algebra in the high school from the first and second years to the third and fourth. Many pupils drop out of high school in the first two years. Many of these have no aptitude for algebra. Such pupils would be relieved of work in algebra and but little would be lost thereby; and the pupil who goes on to college would be fresher on his algebra upon entering college.

An alternative suggestion is that a review of algebra be given in the last year of the high school for the benefit of those who plan to enter college.

I further suggest that the Texas Section at this meeting take steps to have this matter carefully considered and to enlist the coöperation of the Mathematical Section of the State Teachers' Association and of the Association of Texas Colleges in the effort to bring about such changes as may be thought advisable in this matter. I realize the difficulty in making any change in the curriculum, but I believe it is worth considering.

THE FRENCH SECONDARY SCHOOL

C. D. RICE

The French nation is one compact state under a republican form of government. There are no separate states with powers and privileges apart from the central authority as in the United States or the Swiss Federation. But for political and administrative purposes the country is divided into ninety departments. These divisions have very little local self-government and are controlled principally by the central power at the capitol of the country. For educational purposes the departments are combined, forming seventeen divisions or arrondissements of the country called academies, each of which is presided over by a recteur. In each academy under the recteur there are, besides the primary schools, a university and a number of secondary schools. The universities of the academies are together called the University of France. The largest one, the University of Paris, is more often called the Sorbonne. This is probably the most famous university in the world.

The secondary schools of an academy are determined and controlled by the state. There are two kinds of secondary schools, namely: the lycée and the communal college. Any city that has not given a lycée may, by permission of the state establish a local college. In such case the local community is to provide most of the equipment and salary expense, but the state reserves the right to supervise the work and determine to a large extent its standards. The work of the college is approximately the same as in the lycée, but the teachers of highest grade are reserved by the state for the lycée and for that reason the college is often regarded as a school of lower standing. There are, however, some colleges of high grade in some of the cities. Under certain conditions the state may take over the college and develop it into a lycée. In all these secondary schools there is no free tuition except in cases where scholarships are granted. When the tuition is not sufficient, as in most cases, to pay salaries and expenses, the state pays the difference.

The characteristic feature of French education is the highly centralized power that controls the schools from the highest to the lowest part of the work. The minister of education in his office in Paris knows what is being done each day in the remotest districts of the country.

In the primary school, tuition is free and attendance is compulsory. University education is free to all who may wish to take the lectures. A fee is charged, of course, for cost of material in laboratory work and later there is a fee for the privilege of taking certain examinations. What is said here of French education is true in general of nearly every European country, namely: no tuition for primary and university instruction, but a rather heavy tuition for secondary education. Under such circumstances, parents are careful to make some decision as to the future career of their sons before undertaking their education in an expensive secondary school. For this reason many turn aside at the end of their primary education to study in the schools of commerce or agriculture or in one of the many trade or vocational schools. In doing this they renounce their right to a university career and the many opportunities to which it may lead. The road, so to speak, to the university is by way of the lycée or the college. This decision on the part of a large per cent of the school population reduces very materially the number of secondary schools required for a community as compared with the needs of a like population in our American system. The problem of organizing and supplying highly trained teachers for secondary schools under such a regime is not so great a task as in our system of "mass production."

There are ample opportunities offered for instruction in commercial and industrial lines in every part of the country. The work of vocational and trade schools is largely determined by the industry and manufacturing interests of the local community. Thus for instance the trade schools of Lyon are different from those at Marsailles. France being a rural country, to a large extent, has established a number of fine agricultural schools. In order that these special schools may not be overshadowed in any way by the interest in general education, the schools of commerce are placed under the minister of commerce, those of agriculture under the minister of agriculture, and the military and naval schools and the great polytechnic school are placed under the ministers of war and navy. Our trade and vocational schools of America would likely develop more rapidly under a like management.

In a centralized government like France, many trained men are needed to fill the various bureaus and offices of the state. The majority of those entering the secondary schools are looking forward to some career in the service of the government. A few others expect to serve the public as lawyers and doctors. The state expects its servants to be thorough and competent and the training is made long and rigorous. There is no place for local influence or favoritism as in our system. Students in a lycée are examined by a board made up of university professors and teachers in the academy. More than 40 per cent fail and never reach the university. Many government positions may be filled by graduates of a lycée and thus they may enter life without going further in their education.

Those who pass the lycée may enter the university at once, but their preparation is hardly sufficient to do creditable work. For that reason many return to the lycée and continue their studies from one to two years before beginning the rigorous preparation for the university examinations. The state offers a brilliant career to those who succeed in securing the coveted *state degree,* but the price in years of work must be paid. Here again the percentage of failures is large. In many cases, the number who pass depends upon the number the state will need for its service in the near future and the passing mark is fixed accordingly. The examinations become, in reality, competitive examinations for those places, the number of which is fixed in advance. The state selects in this way the flower of its best intellects for its service. This will explain why so many brilliant authors of the country are serving in some bureau of the state.

The degrees and distinctions given by the French universities to the people of France are *state* degrees and guarantee the holder some position of service in the country. such degrees are of very little interest to a foreigner who may wish to avail himself of the superior advantages of French education and culture, and later return to his own country. For nearly a century Americans in increasing numbers studied in Germany where a degree was given that would be of value to them in their home land. During all these years no one doubted the value of French cûlture, but the distinction obtained was not worth as much at home as that which he could get elsewhere. About the beginning of the present century the faculties of the French universities became convinced that a change would be desirable for themselves as well as for those who would prefer to study with them. A reorganization made it possible for a foreigner to take a doctorate somewhat on the plan as it is given in Germany or in the United States. The state degrees for the French people were left undisturbed. The new degree is called the *university* degree. In no way does it qualify one for service in France as do the *state* degrees. It is becoming more attractive to foreigners who are now going to France in increasing numbers for study and French culture.

The work of the lycée is almost the equivalent of the American high school and the junior college combined. In mathematics there is a continuous development from the primary school to the university. There are no breaks in curriculum and method as found in our schools between grammar grades and high school and later between high school and college. In many cases the sequence of topics studied in passing from the lower to the higher phases of a subject is quite different from that found in many of our American texts. High-school teachers in this country, interested in constructing and revising their courses of study in mathematics will find many helpful suggestions in the Franch Programmes issued each year by the Minister of Education.

In the lycée we find, probably, the best-trained secondary teachers of any country. The preparation required is practically the same as that demanded of the university professor, the difference being that the universities take the more brilliant. The tests for proficiency are made by thorough and rigorous examinations in which no personal or private influence has an opportunity to affect the final outcome. Grades of salaries and positions depend upon the standing made in the final examinations given in the University. The coveted state degree so valuable to a teacher is that of agregé. When the candidate cannot reach that distinction, he must take a position of lower grade and reduced salary.

AN INTRODUCTION TO THE STUDY OF LOGARITHMS

C. W. McClelland

Daniel Baker College

Introduction

The writer of this article has had occasion to teach the use of logarithms to a number of groups of young people and from several different texts and he does not hesitate to say that he has found this subject the hardest to get over of anything included in the usual first-year course in college mathematics. It is easy to find reasons for this state of affairs and he would not attempt to offer a complete solution of the problem of making this job easy. Perhaps the presentation of the theory is made too formal. Or the grief from the struggle of interpolation with the ordinary three-place table makes hopeless any argument about such methods being labor saving.

He would offer, however, the following plan as a satisfactory outline of a method of introducing the subject, the work to be put on the board before the chapter on the use of logarithms has been read by the class, and a class discussion provoked if possible so that all the steps be thoroughly understood. As an assignment for the following day he would suggest that a similar argument be written by the class for the operation of division, to be handed in.

It would seem that some such an introduction could well be included in the text before the more formal treatment is worked out. The writer has before him a brand new text in college algebra in which four full pages are covered with a discussion of such things as the law of exponents extended to fractional forms, the approximate values of such exponents in certain cases and other things before the logarithm of a number is defined. Such a discussion is vital and interesting to a critical reader but it is quite incomprehensible to the average first-year student and possibly serves to make him feel that the whole process is terribly complicated and that he is to be dragged through another painful chapter of pure mathematics.

Outline of an Introduction to the Subject of Logarithms

Problem.—To multiply a number M by a number N.

Discussion of problem.—The method of arithmetic we already know. We will use this method later to check the results of a second method which will now develop.

There is a number x such that $10^x = M$ and a number y such that $10^y = N$. This fact can be proved but in this discussion will be assumed to be true.

Then in place of $M \cdot N$ we might write $10^x \cdot 10^y$.

But there is a law of exponents which we have already studied (reference should here be made to page and serial number of this law in the text used) which says that

$$10^x \cdot 10^y = 10^{x+y}.$$

Hence it follows that

$$M \cdot N = 10^x \cdot 10^y = 10^{x+y}.$$

From the above result we might work out a second method for the problem, somewhat as follows:

(1) Find x such that $10^x = M$.
(2) Find y such that $10^y = N$.
(3) Find the sum $x+y$.
(4) Find a number R such that $10^{x+y} = R$.

Then $R = M \cdot N$.

Discussion of method.—The numbers x, y, $x+y$ as used above are called *logarithms.* The number *10* as used is called *the base of the system* of logarithms. Any number might be used as a base of a system of logarithms but the number *10* is most convenient to use as a base for numerical calculations.

The operations of steps (1), (2), and (4) are carried out by consulting a *table of logarithms* by means of which can be found the value of the power to which the number *10* must be raised to equal any four-place number between zero and one thousand. Such a table may be used to find the same thing for longer numbers by carrying out a process called interpolation.

We have thus found a way in which the process of finding the product of two numbers may be carried out without actually doing any multiplication. The new method is shorter in that fewer figures are handled and hence the chance for a mistake is lessened.

A similar method may be worked out by which the operation of division may be reduced to a subtraction; raising to a power reduced to a multiplication, and finally extracting a root reduced to a division.

Example.—To find $R=32.74\times 9.42$.

(1) $32.74=10^{1.51508}$; therefore $x=1.51508$.

(2) $9.74=10^{0.97405}$; therefore $y=0.97405$.

(3) Then $x+y=1.51508+0.97405=2.48913$.

(4) But $10^{2.48913}=308.41$.

Hence $32.74\times 9.42=308.41$.

Check by the method of arithmetic:

$$\begin{array}{r} 32.74 \\ 9.42 \\ \hline 6548 \\ 13\,096 \\ 294\,66 \\ \hline 308.4108 \end{array}$$

This result shows an agreement with the result of the other method.

Further discussion.—Any class discussion of the method will bring up the question as to the existence of the values of the exponents used in the example worked out and at this time the instructor can show the class enough of the nature of the logarithm to satisfy the normal student that there is such a number for any case. The statement is usually made that logarithms are gotten by the use of formulas derived in higher mathematics, which is not a very satisfactory answer to an interested student but which too frequently is the only one in the repertoire of the instructor.

It is the contention of the writer that at least three class periods could be well spent in the working out of methods for the four operations for which logarithms are most frequently used and in the solution of problems such as used in the example shown above, the instructor furnishing to the students all exponents (logarithms) used before the class drill essential to the rapid use of the tables is begun. The instructor might gradually let the class take up the use of the table, but he should impress on the class by his own example the fact that even though interpolation be necessary the logarithm can be found quickly and without painful mental exertion.

TO OUR HIGH SCHOOLS

EDITOR

It may be well to point out some difficulties we find with students who have had no algebra since the ninth grade have when they attempt college mathematics.

We find that the power to detect and express the factors of an algebraic expression returns to them with little dif ficulty when it has once been given to them.

A working knowledge of exponents is generally poor and has to be taught as a new subject. The radical as a fractional exponent is not clear and there is usually some difficulty in reducing two radicals to the same index. Very few texts give a sufficient number of problems to fix the principles of exponents, and a good collection for supplementary use should be made by the teacher. The importance of logarithms is increasing, and a good knowledge of exponents is necessary to the understanding of the principles involved.

The addition of fractions is often badly understood. In reducing to a common denominator the term *cross multiply* is used but no reason can be given by the student for the use of the term. After cross multiplication, the denominator is often discarded as is done in clearing an equation of fractions, and the two distinct processes are hopelessly confused.

The logic of the manipulation of equations as expressed by a few simple axioms seems to be unknown or forgotten. Clearing equations of fractions is a purely mechanical process. Here again we meet the term *cross multiply* as the only thing to be said to justify the process. Recently in one of my own large freshman classes not a single member could explain why a term could be regarded as transposed under a change of sign.

The word *cancel* is quite generally used with two or three different meanings. The specific process in hand usually showing the meaning intended.

In the solution of simultaneous equations only one method is given, namely, that of addition or subtraction. Apparently students remember no solution by comparison or substitution. The training in those two processes is lost.

In spite of the fact that the State Department specifies that the quadratic should be given in accredited high schools, the number of men entering the freshman class with no knowledge of the quadratic is increasing each year.

In the power of a binomial as $(a+b)^2$ a majority will write a^2+b^2. And there is much confusion as to the difference between a factor and a term.

One last word in conclusion, while I have tried to point out what seem to me several of the obvious faults in our high-school preparation for college mathematics, I would be willing to guarantee that if a thorough review of only two subjects—equations and especially fractions—was given thoroughly to each prospective university man during his last year of high school, that the percentage of failures in mathematics would be cut in half.